$W = 1500$ lbs

$b = 35$ ft

$c = 6$ ft

$v_{st} = 45$ mph $= 39$ knots

$v_A = 95$ mph $= 83$ kts → manoeuvre speed ($= 42{,}2$ m/s)

$v_C = 110$ mph $= 96$ kts → cruising speed

$v_D = 12\cancel{0}$ kts → diving speed

↳ 132 kts! (see later calculations)

$C_{L,max} = 1{,}5$

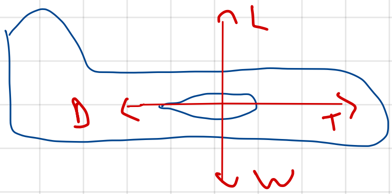
$n_a^+ = 4{,}4$ (max load factor)

$n_c^- = -1{,}76$ $(= 4{,}4 \times 0{,}4)$

$n_D^- = -1$

$S = 19{,}5 \text{ m}^2 = 210 \text{ ft}^2$

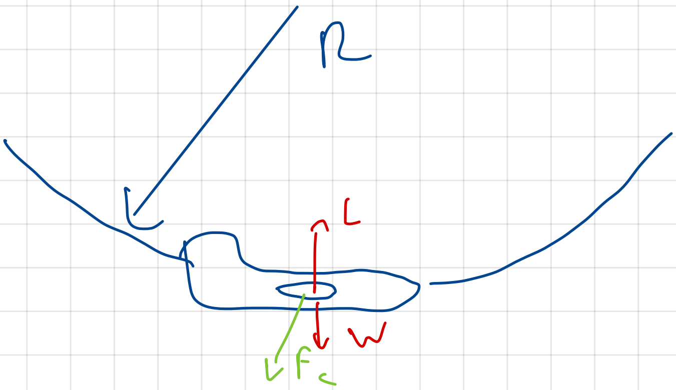
Cruise



$L = W$

$D = T$

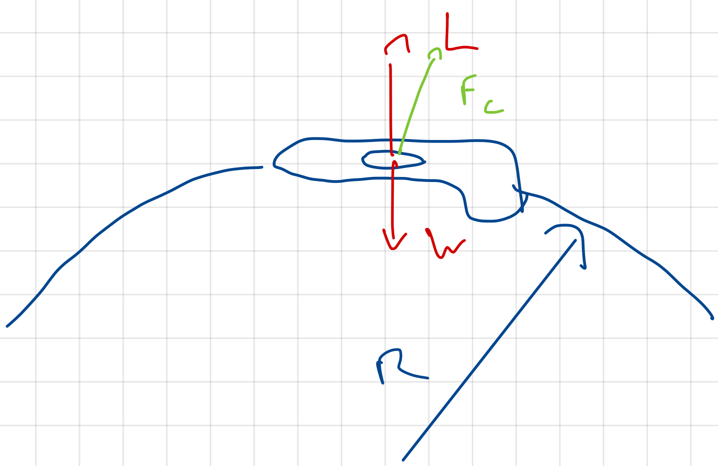
manoeuvre:



$$L = W + F_c = W + \frac{mv^2}{R}$$

$$n = \frac{L}{W} = 1 + \frac{v^2}{Rg} > 1$$

↳ load factor



$$W = L + F_c$$

$$L = W - \frac{mv^2}{R}$$

$$n = 1 - \frac{v^2}{Rg}$$

Stall

$$\left.\begin{aligned} L = W &= \frac{1}{2}\rho V_{st,1}^2 S\, C_{L,max} \\ L = nW &= \frac{1}{2}\rho V_{st,2}^2 S \cdot C_{L,max} \end{aligned}\right\} \quad \frac{n}{\cancel{2}}\cancel{\rho} V_{st,1}^2 \cdot \cancel{S} \cdot \cancel{C_{L,max}} = \frac{1}{\cancel{2}}\cancel{\rho} V_{st,2}^2 \cdot \cancel{S} \cdot \cancel{C_{L,max}}$$

$$\boxed{n\, v_{st,1}^2 = v_{st,2}^2}$$

Our design

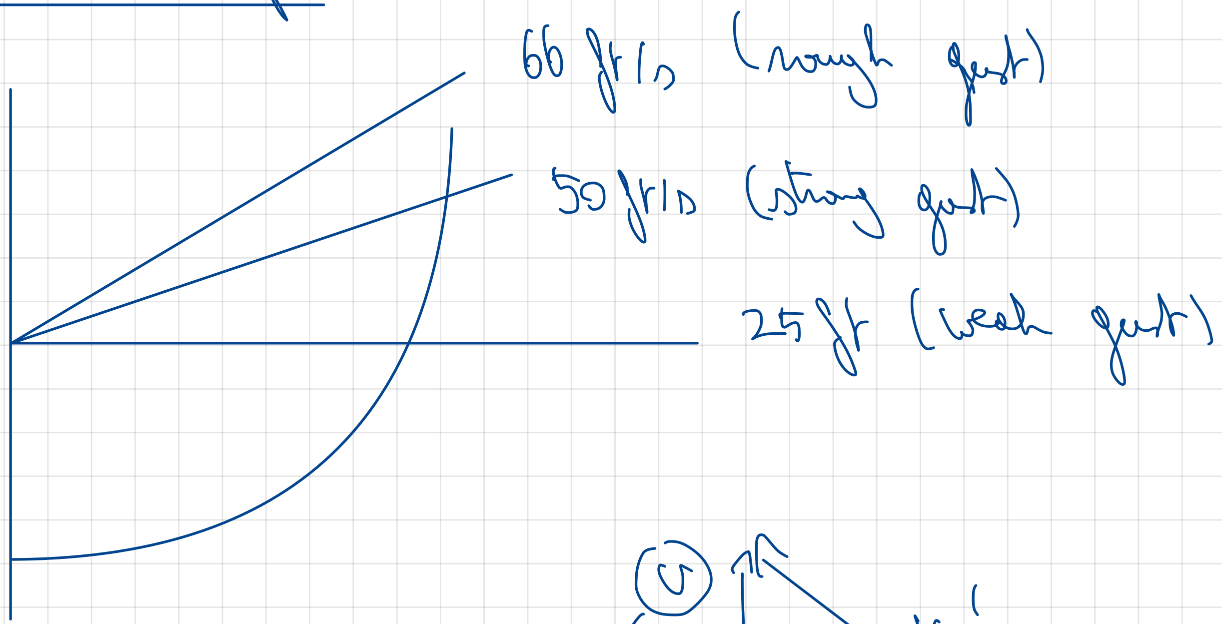
$V_c > 88 \rightarrow V_c = 95$ is good choice (or 96)

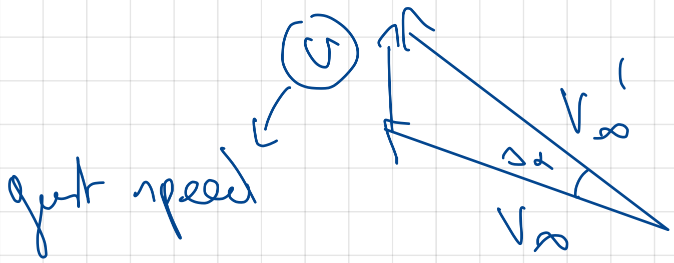
$V_D > 1{,}25\, V_c = 1{,}25 \times 95 = 119$ kts

$> 1{,}50\, V_{c,min} = 1{,}5 \times 88 = 132$ kts

$$V_{st} = \sqrt{\frac{2W}{\rho S C_{L,max}}} = 37{,}5 \text{ kts} \quad (\text{given } V_{st} = 39 \text{ kts} > 37{,}5 \checkmark)$$

Gust envelope





$$L = W = \frac{1}{2}\rho V_\infty S C_L \longrightarrow a(\alpha+\beta)$$

$$L' = \frac{1}{2}\rho V_\infty'^2 S C_L' \longrightarrow \underbrace{\frac{1}{2}\rho V_\infty'^2 S a(\alpha+\beta)}_{W} + \frac{1}{2}\rho V_\infty'^2 S a \Delta\alpha$$

$$C_L' = a(\alpha'' + \Delta\alpha + \beta)$$

$$\Delta\alpha = \min \Delta\alpha = \frac{5}{V_\infty'}$$

$$\Downarrow$$

$$\frac{1}{2}\rho V_\infty^2 S a(\alpha+\beta) + \left(\frac{1}{2}\rho 5 S a\right) V_\infty'$$

<u>Wing lift data</u>

$$C_L = a(\alpha+\beta)$$

$$C_z = a_0(\alpha+\beta)$$

$$a = \frac{a_0}{1 + \frac{a_0}{\pi AR}}$$

(found in slides: $\cancel{C_L}$ C_z $= 0,1\,\alpha + 0.1$)

$$AR = \frac{b^2}{S} = \frac{(35\ ft)^2}{270} = 5,8$$

$$\Rightarrow a = \frac{5,729}{1 + \frac{5,729}{\pi \cdot 5,8}} = 4,36\ /\text{rad}$$

$$C_L = 0,0762\,\alpha + 0,1 \Rightarrow \alpha = \frac{1,4}{0,0762} = 18,47^\circ$$

Lift?

$$n_A^+ = 4.4 = \frac{L}{W} \Rightarrow L = 4.4\,W = 6600 \text{ lb}$$

Moment?

$$M = \frac{1}{2} \rho v^2 S c \cdot C_m$$

$$= \frac{1}{2} \cdot 1{,}225 \cdot 42{,}2^2 \cdot 19{,}5 \cdot 1{,}8 \cdot (-0{,}007)$$

$$= -277 \text{ Nm} = -2400 \text{ lb} \cdot \text{in}$$

Drag?

$$D = \frac{1}{2} \rho V^2 S C_D$$

$$\hookrightarrow C_D = C_{D_0} + C_{D_i} = 0{,}148$$

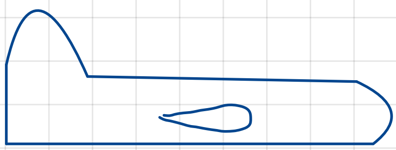
$$C_{D_0} = 0{,}078$$

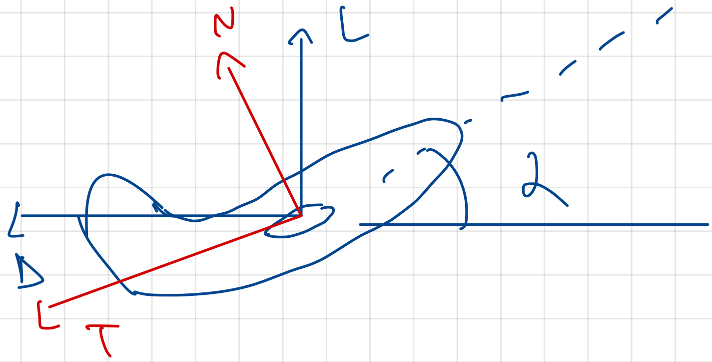
$$C_{D_i} = kC_L^2 = 0{,}064 \cdot 1{,}5^2 = 0{,}13$$

$$k = \frac{1}{\pi AR e} = \frac{1}{\pi\, 5{,}8 \cdot 0{,}845} = 0{,}064$$

$$= \frac{1}{2} 1{,}225 \cdot 42{,}2^2 \cdot 19{,}5 \cdot 0{,}148 \approx 3000 \text{ N}$$

$\Rightarrow$





$N = L\cos\alpha + D\sin\alpha$

$T = D\cos\alpha - L\sin\alpha$

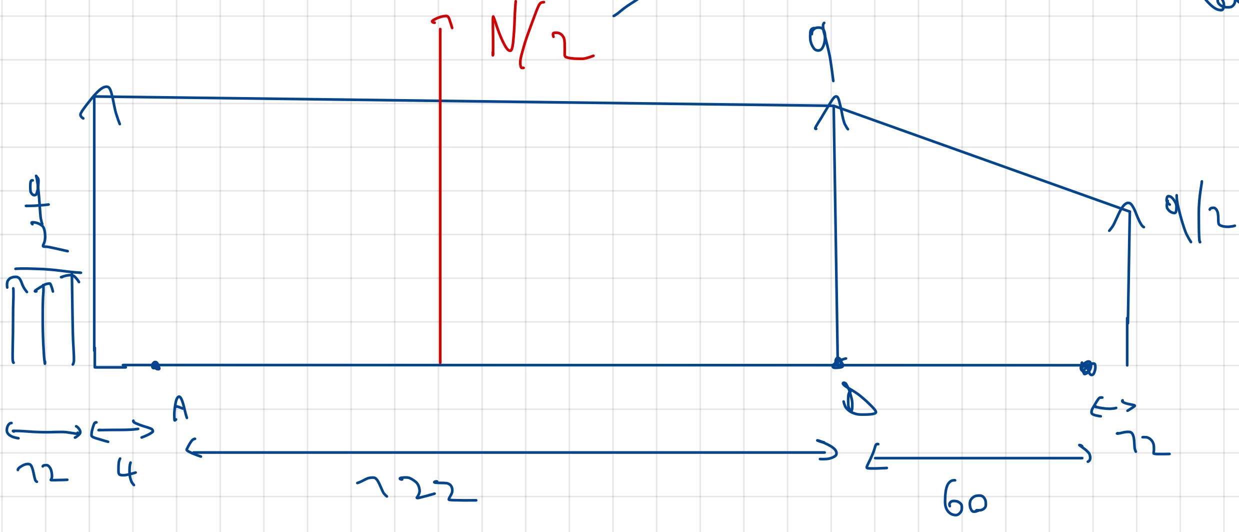
$\alpha = 77°$

$N = 6500$ lb (normal)

$T = -7328$ lb (tangential)

Structural layout

half of the load because half of the wing

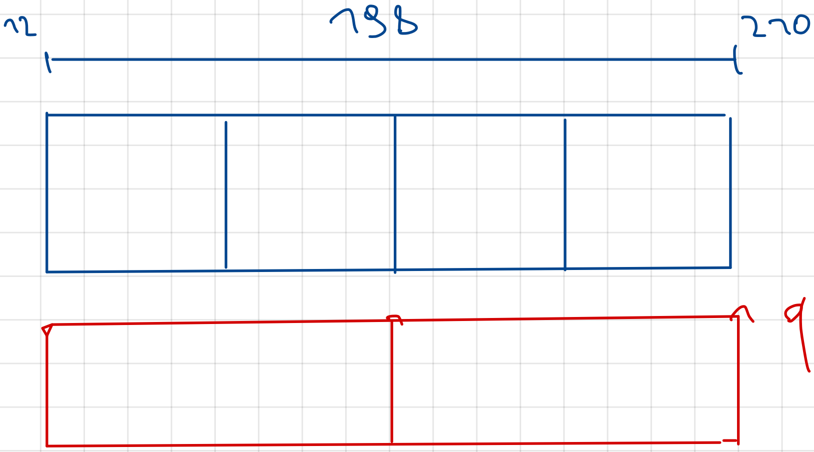


$$q \cdot 726 + \frac{q}{2} \cdot 72 + \frac{q}{2} \cdot \frac{72}{2} + 72 \cdot \frac{q}{2} = N = 6500\text{ lb}$$

$$\Rightarrow q \cdot 786 = \frac{N}{2} \Rightarrow q_N = \frac{3250}{786}$$

$$= 77{,}5 \approx 78\text{ lb/in}$$

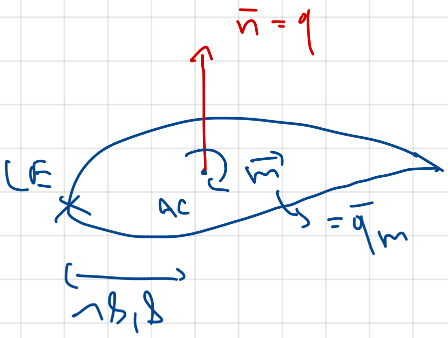
Drag distribution



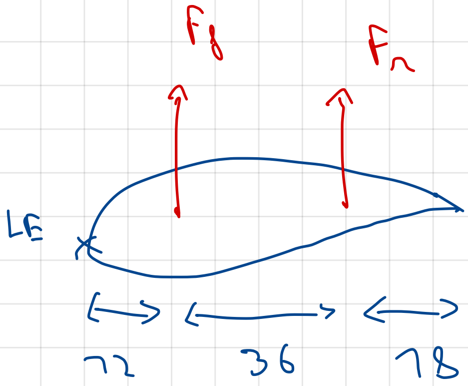
$$q_T = -\frac{7328}{2 \cdot 798} = -3{,}35 \text{ lb/in}$$

Moment

$$q_m = -\frac{2400}{2 \cdot 798} = 6 \text{ . lb.in/in}$$



$\longleftrightarrow$



$F_f = ?$

$F_r = ?$

$q_n = 18$ lb/in

$q_t = -3{,}35$ lb/in

$q_m = -6$ lb/in

$$\begin{cases} q_n = F_f + F_r \\ 18{,}8\, q_n - q_m = 12\, F_f + 48\, F_r \end{cases}$$

$$F_r = q_n - F_f$$

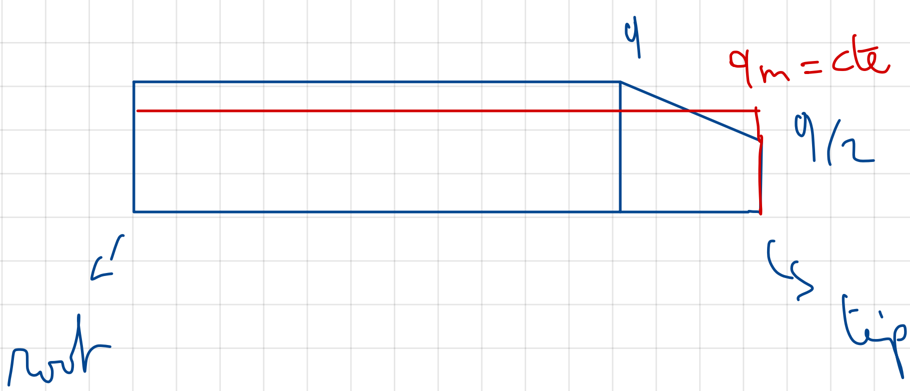
$$12 F_f - 48 F_f + 48 q_n = 18{,}8\, q_n - q_m$$

$$-36 F_f = (18{,}8 - 48)\, q_n - q_m$$

$$F_f = \frac{(18{,}8-48)\,18 + 6}{-36}$$

$$= 14{,}43 \text{ lb/in}$$

$$\Rightarrow F_r = q_n - F_f = 3{,}6 \text{ lb/in}$$



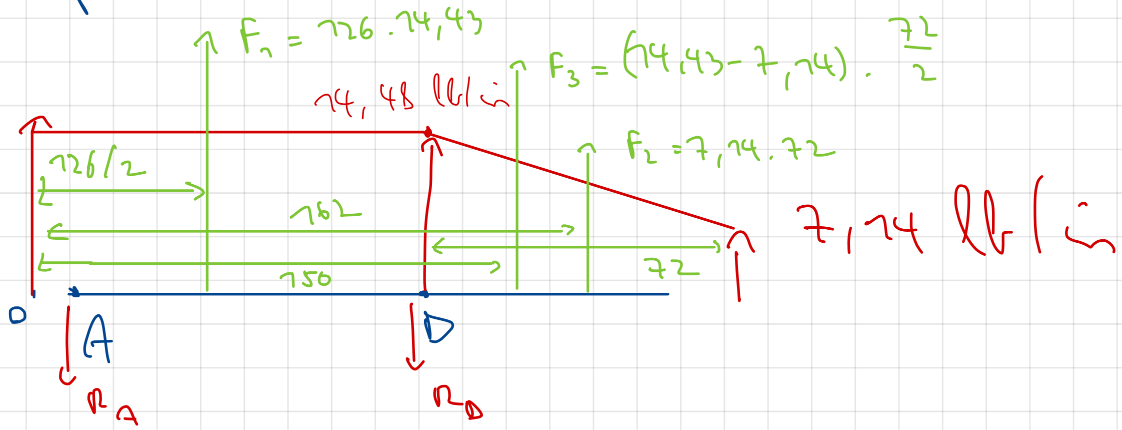
at root: $F_y = 14{,}43$ lb/in

$F_x = 3{,}6$ lb/in

at tip: $F_y = \dfrac{(18{,}8-48)\cdot 9+6}{-36} = 7{,}14$ lb/in

$F_x = 1{,}85$ lb/in

<u>tip</u>



force & moment eqlb to find R_A & R_B

$$\begin{cases} F_1 + F_2 + F_3 = R_A + R_D & (1) \\ -4R_A - 126\,R_D + F_1\cdot 63 + F_2\cdot 162 + F_3\cdot 150 = 0 & (2) \end{cases}$$

(2) - 4 . (1)

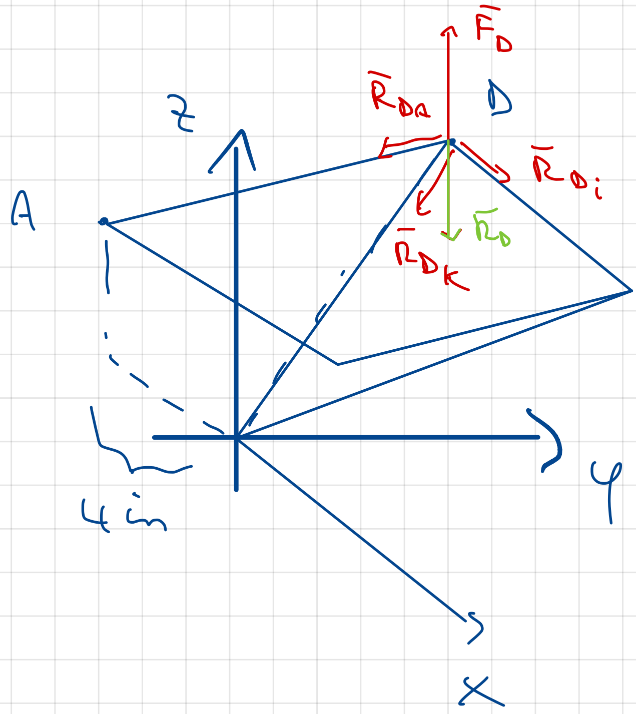
$$122\,R_D = 59F_1 + 158F_2 + 146F_3$$

$$R_D = \frac{59F_1 + 158F_2 + 146F_3}{122} = 1841 \text{ lb}$$

(1): $R_A = F_1 + F_2 + F_3 - R_D$

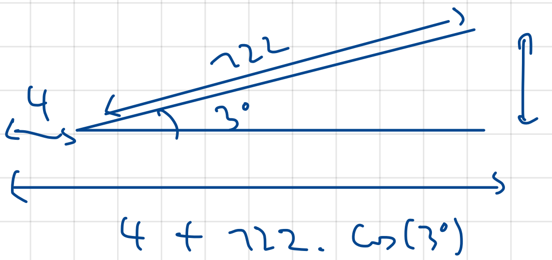
$= (126\cdot 14{,}13) + (7{,}14\cdot 72) + (14{,}43-7{,}14)\cdot 36$

$- 1841 \quad = 716$ lb



$$\bar{F}_D + \bar{R}_{DA} + \bar{R}_{DK} + \bar{R}_{DI} = \bar{0}$$

	x	y	z
A	-10	4	52
D	-10	125,8	58,4
I	26	125,8	57,9
F	26	4	57,5
K	0	0	0



$$D_z = 52 + \sin(3°) . 122 = 58,4$$

$$\vec{D_A} = (-10 + 10 \,;\, 4 - 125,8 \,;\, 52 - 58,4)$$

$$= (0 \,;\, -121,8 \,;\, -6,4)$$

$$\|\vec{D_A}\| = \sqrt{(-121,8)^2 + (-6,4)^2}$$

$$= 121,97$$

$$\vec{e}_{DA} = (0 \,;\, -0,9986 \,;\, -0,0523)$$

	Cos to x	Cos to y	Cos to z
$\vec{e}_{DA}$	0	−0,9986	−0,0523
$\vec{e}_{DI}$	0,9999	0	−0,0739
$\vec{e}_{DK}$	0,0779	−0,9048	−0,4798
$F_D \to \vec{e}_{\perp}$	0,0739	−0,0523	0,9885

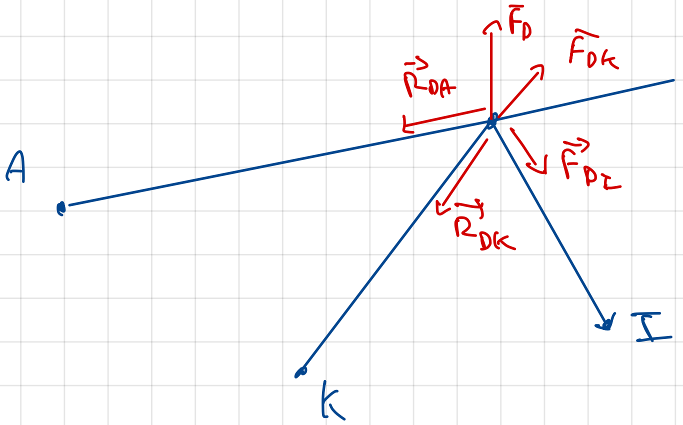
$$\vec{e}_{\perp} = \vec{e}_{DI} \times \vec{R}_{DE}$$

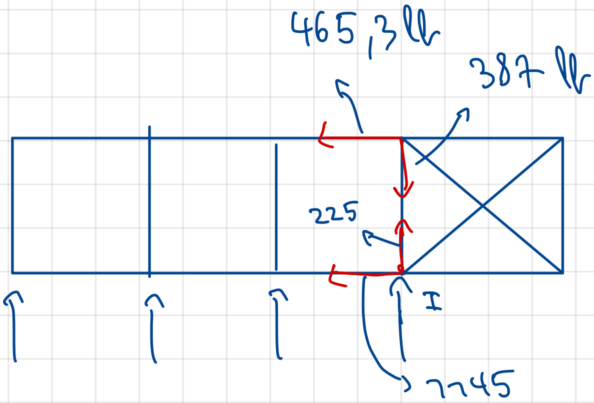
$$/\vec{e}_x \quad 0 \cdot R_{DA} + 0,9999\, R_{DI} + 0,0779\, R_{DK} + 0,0739\, F_D = 0$$

$$\Rightarrow \quad R_{DA} = -4653 \text{ lb}$$

$$R_{DK} = 5028 \text{ lb}$$

$$R_{DI} = -387 \text{ lb}$$





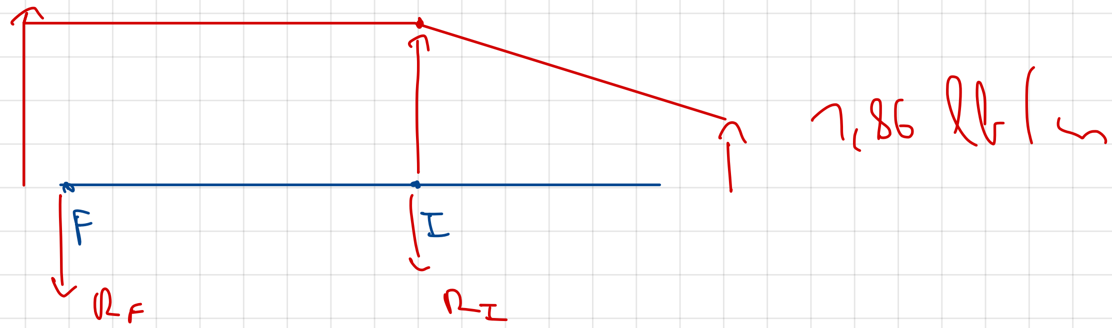
By knowing the reaction forces you can dimension the trusses

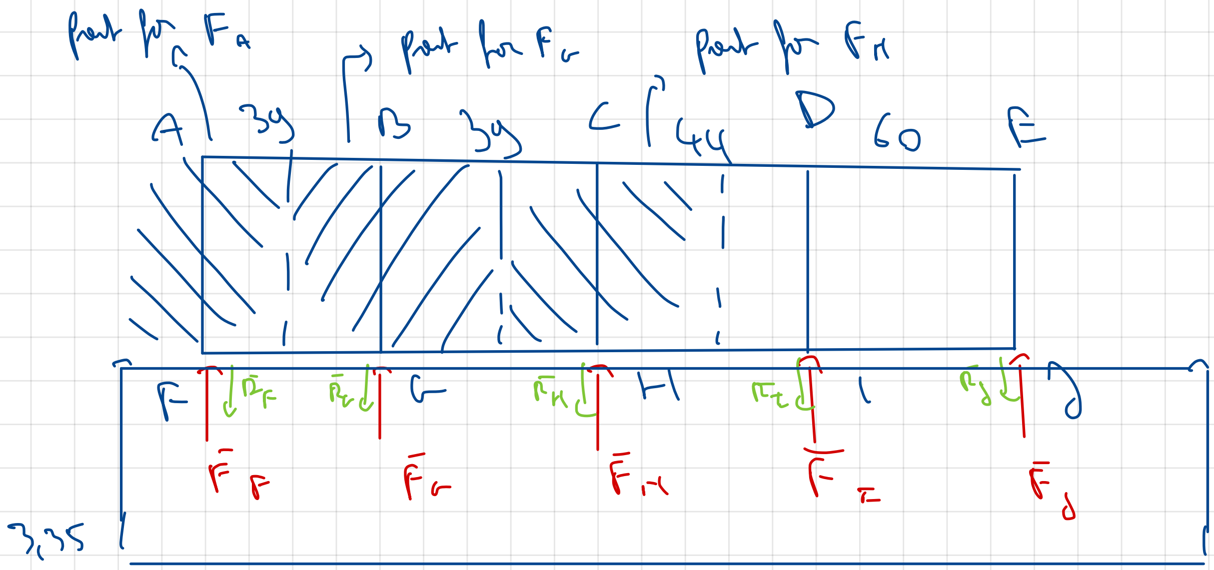
$R_{IF} = -1745$ lb

$R_{ID} = -225$ lb

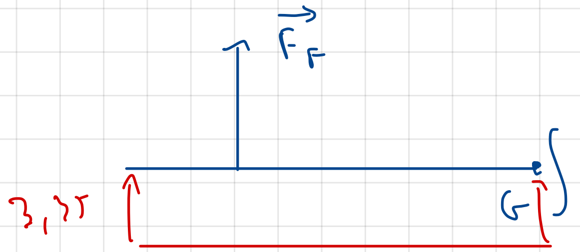
$R_{IK} = 1254$ lb

Roof





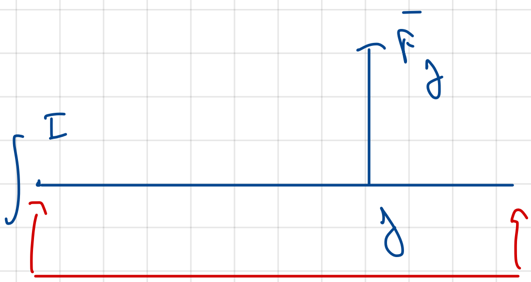
$$F_G = 39 \cdot 3{,}35 = 130 \text{ lb}$$



$$39\ \bar{F}_F = 43 \cdot 3{,}35 \cdot \frac{43}{2}$$

$$\bar{F}_F = \frac{43}{39} \cdot 3{,}35 \cdot 21{,}5$$

$$= 79{,}4 \text{ lb}$$



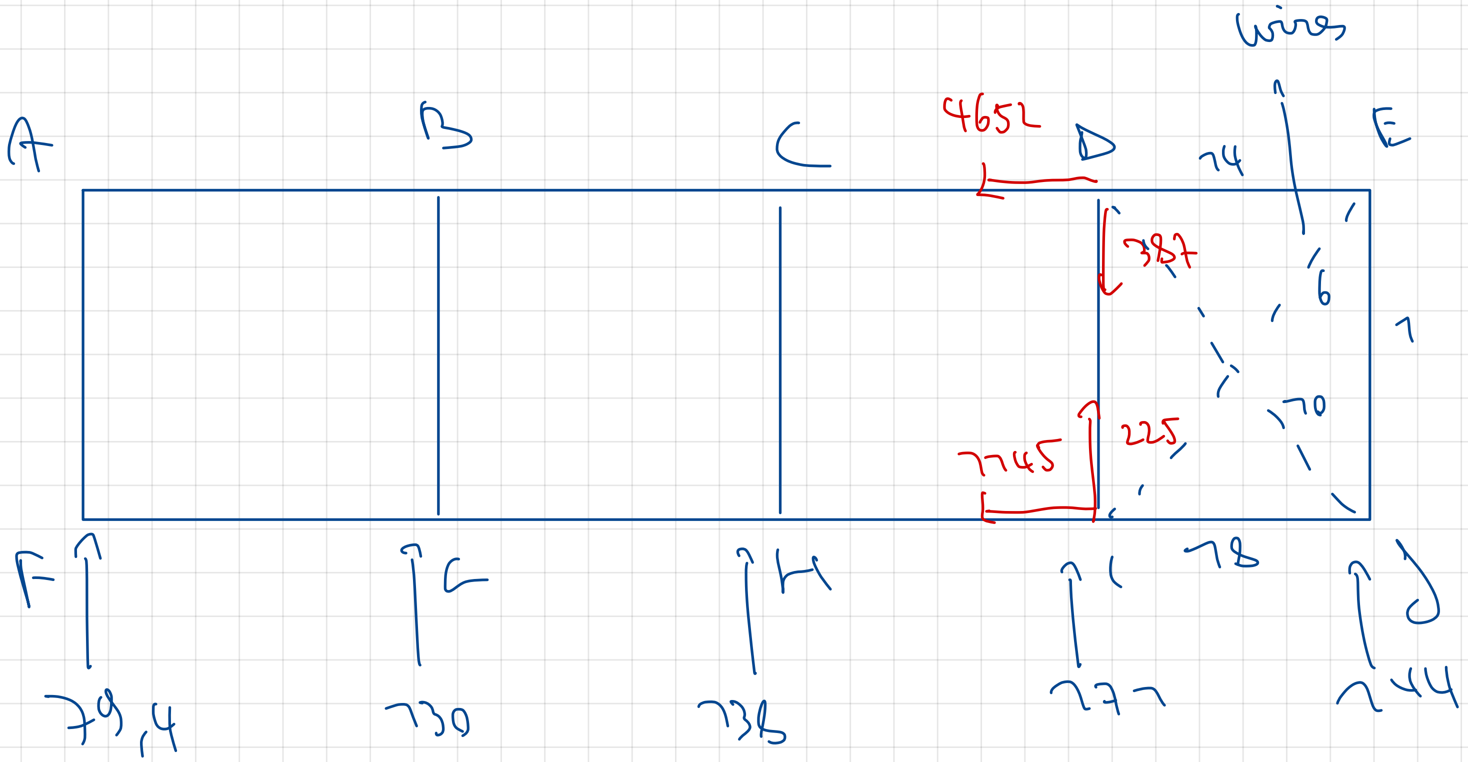
$$60\ F_J = 72 \cdot 3{,}35 \cdot \frac{72}{2}$$

$$F_J = 144 \text{ lb}$$

$$F_H = 138 \text{ lb}$$

$$F_I = 177 \text{ lb}$$

$$F_F = \left(4 + \frac{39}{2}\right) \cdot 9 = 78{,}7$$ (alternative way to calculate) faster but less accurate

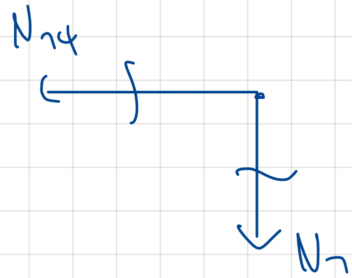


red = force due to the normal force

Only one of 6 or 70 is in tension, so only one works at a time because compression doesn't exist in wires

Suppose 10 is in tension:

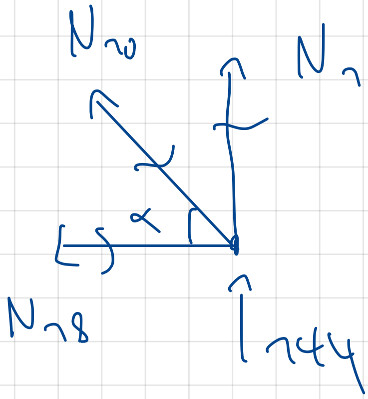
E



$N_1 = 0$

$N_{14} = 0$

J



$N_1 = 0$

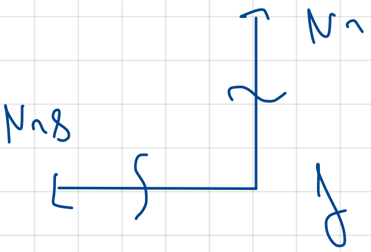
$/e_x = -N_{18} - N_{10}\cos\alpha = 0 \Rightarrow N_{18} = -N_{10}\cos\alpha$

$/e_y = N_{10}\sin\alpha + 144 = 0 \qquad N_{10} = -\dfrac{144}{\sin\alpha} < 0$

$\downarrow$

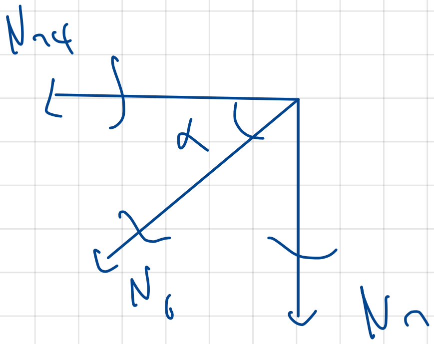
Compression so doesn't exist, we throw 10 and keep 6 !

$\Rightarrow$ 6 is in tension



$N_1 = -144$

$N_{18} = 0$



$$\vec{R}_y = -N_1 - N_6 \sin\alpha = 0$$

$$N_6 = \frac{-N_1}{\sin\alpha} > 0$$

$$\vec{R}_x = -N_{14} - N_6 \cos\alpha = 0$$