Aircraft Structures

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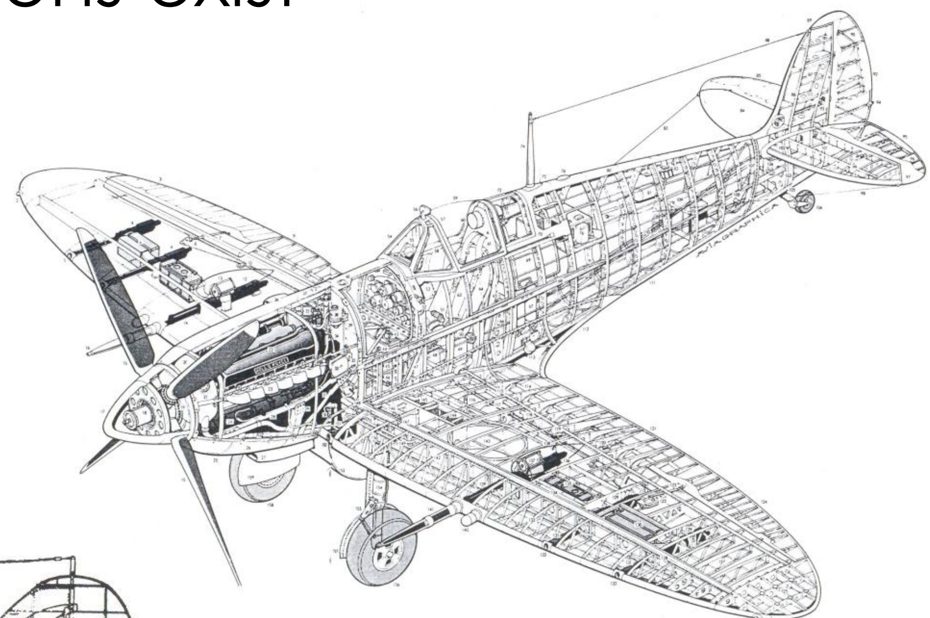
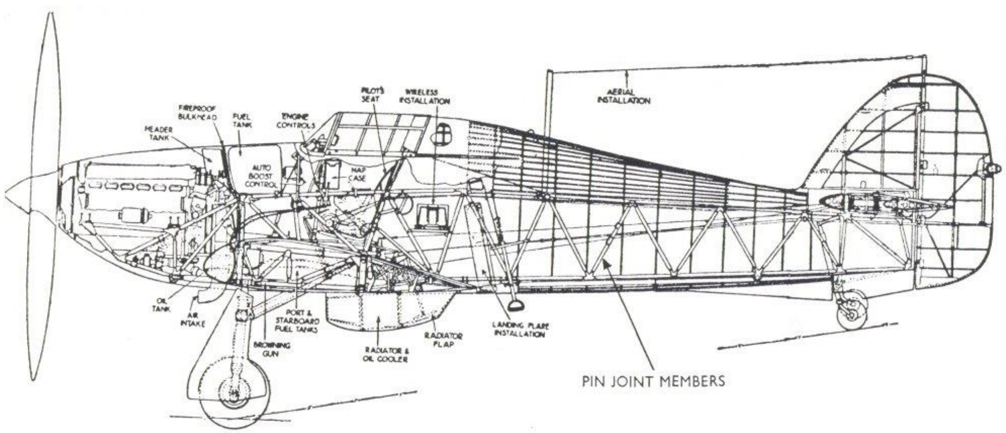
The monocoque aeroplane

Dawn of the aeroplane

- WWII: both conceptions exist
  - Robust trussed aeroplanes
  - More capable monocoques

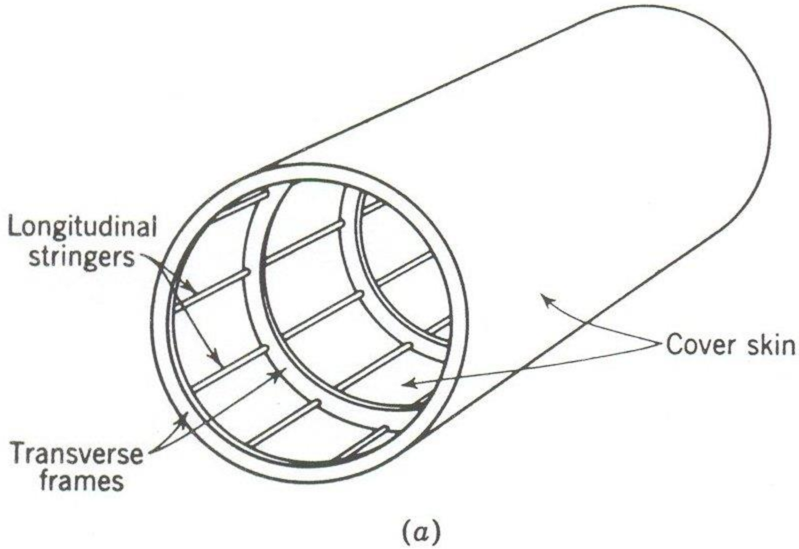


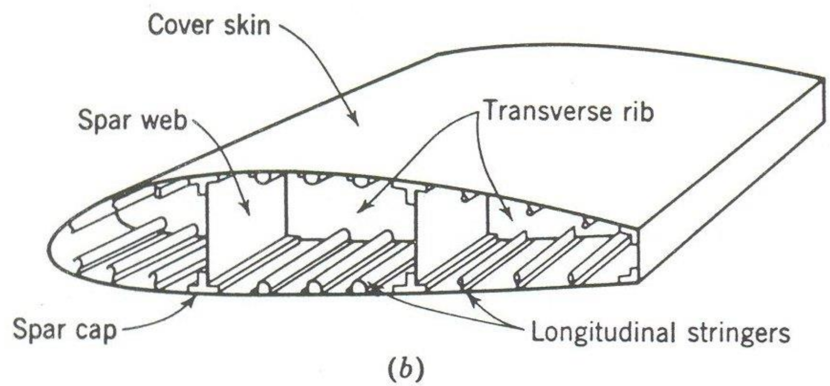
Vickers-Supermarine Spitfire



Hawker Hurricane

Semi-monocoque design

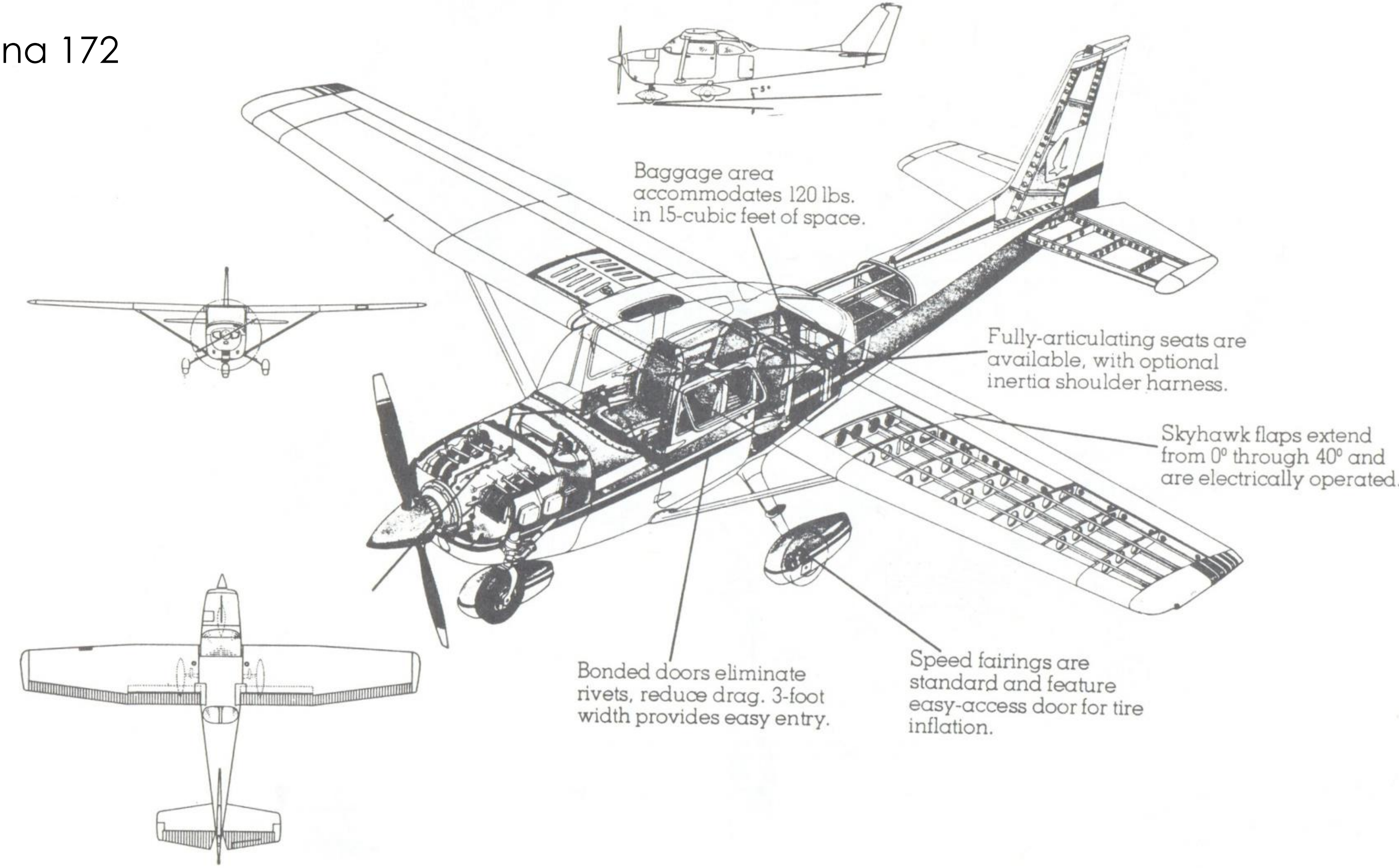




- Semi-monocoque structure:
  - Shear-resistant skin/spar webs
  - Bending-resistant stringer/spars
  - Torsion-resistant “torque box”
  - Pressure-resistant frames/bulkheads
  - Load transfer by ribs/frames

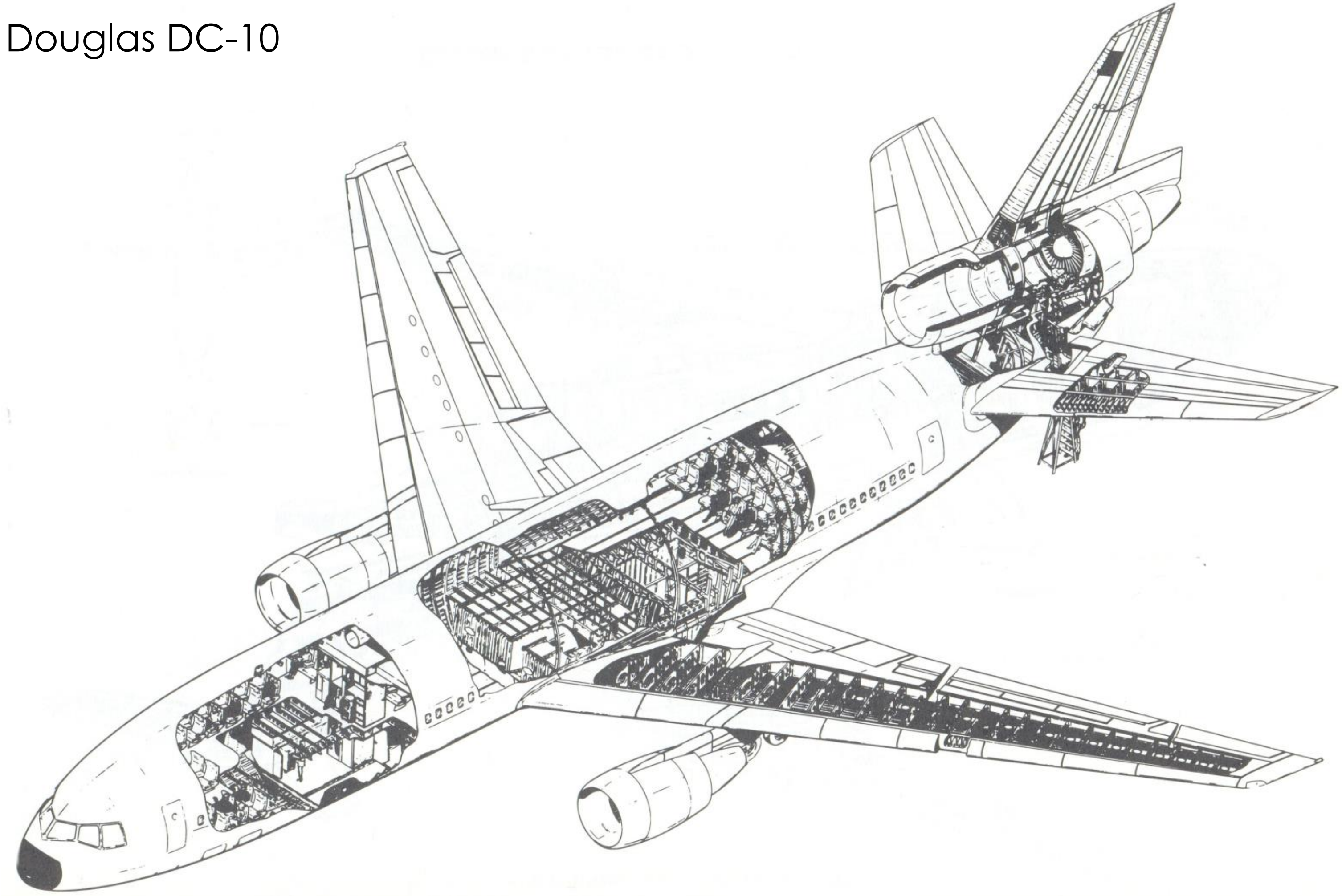
Semi-monocoques (1)

Cessna 172



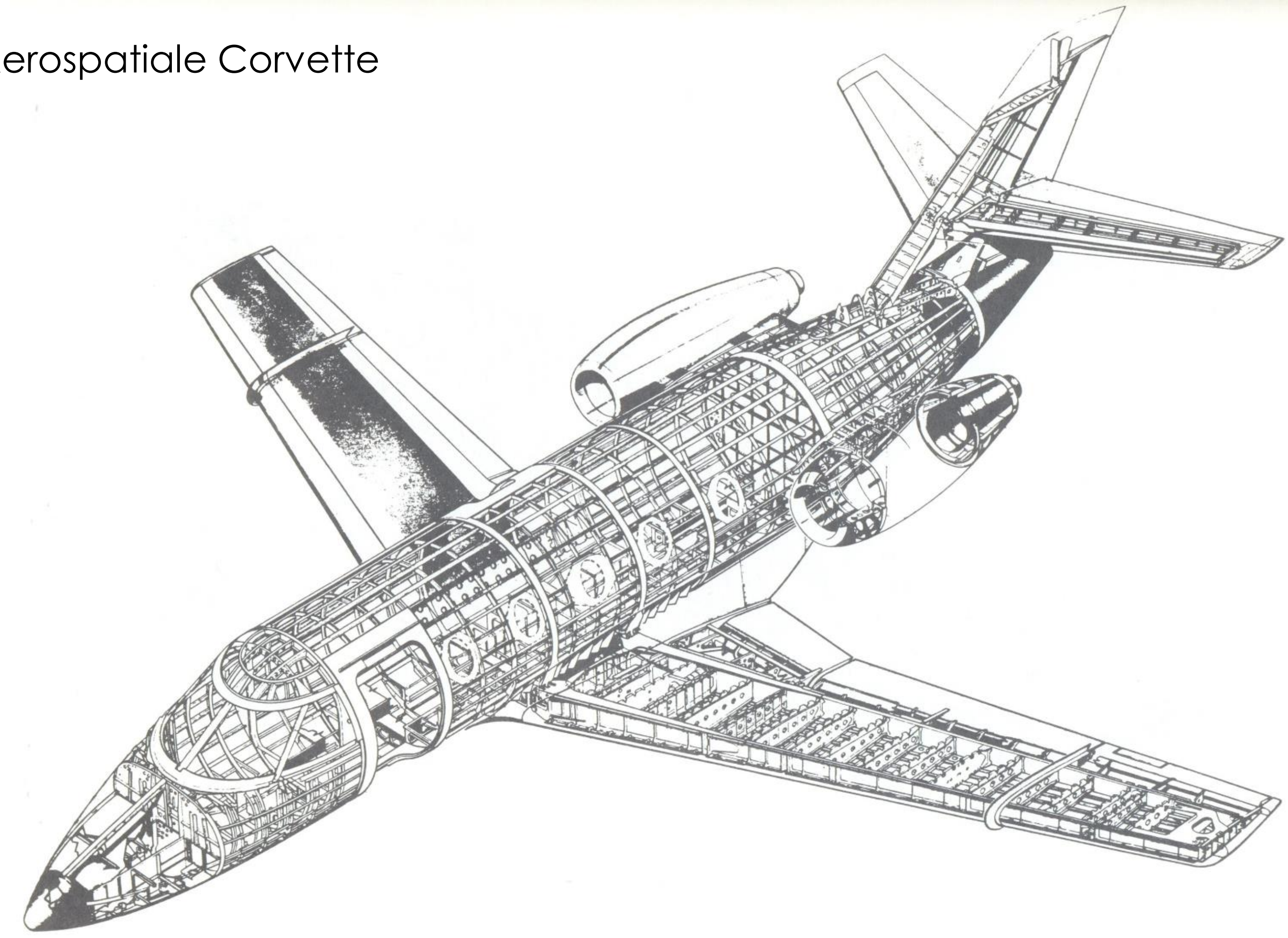
Semi-monocoques (2)

Douglas DC-10



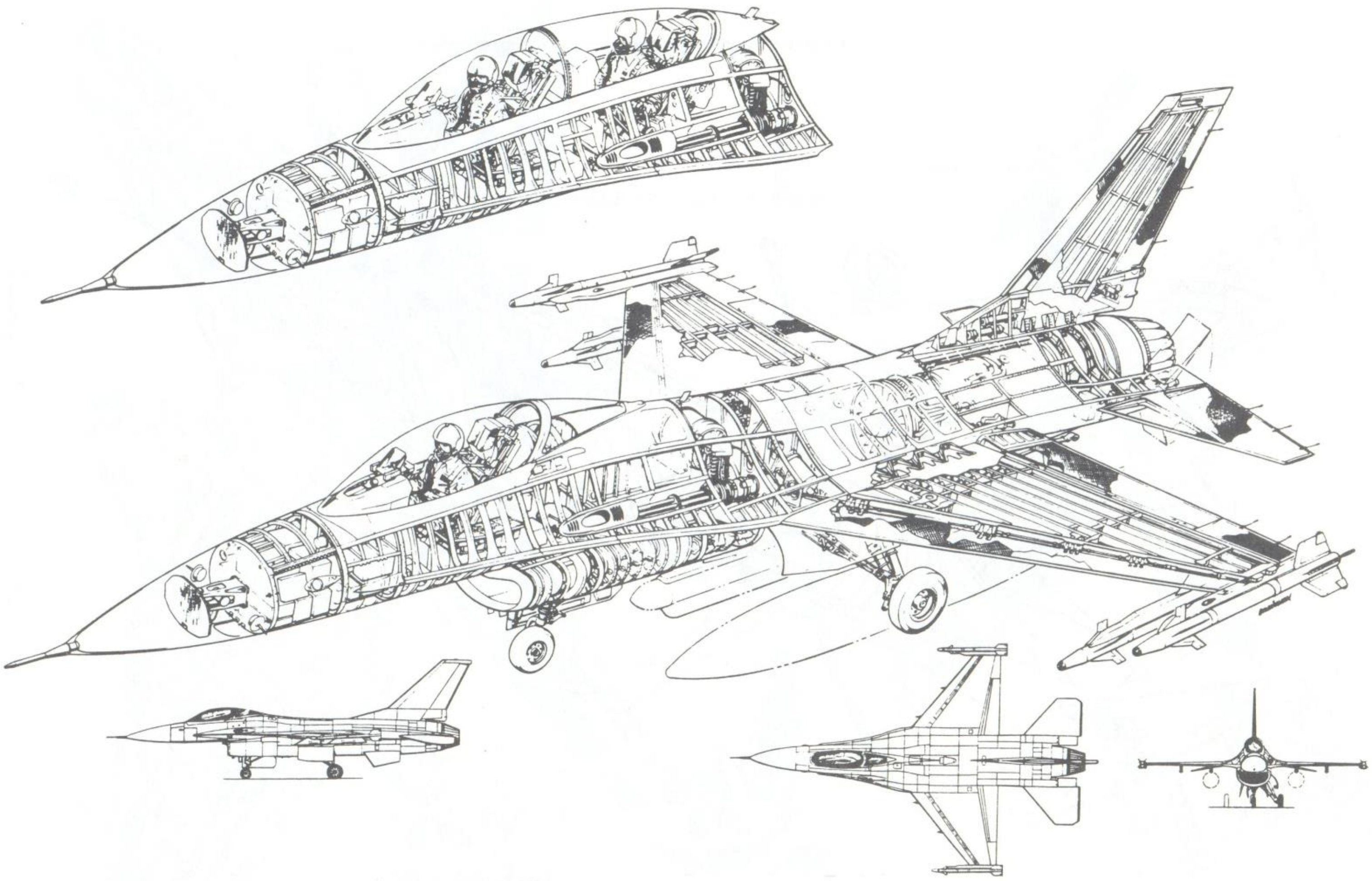
Semi-monocoques (3)

Aerospatiale Corvette

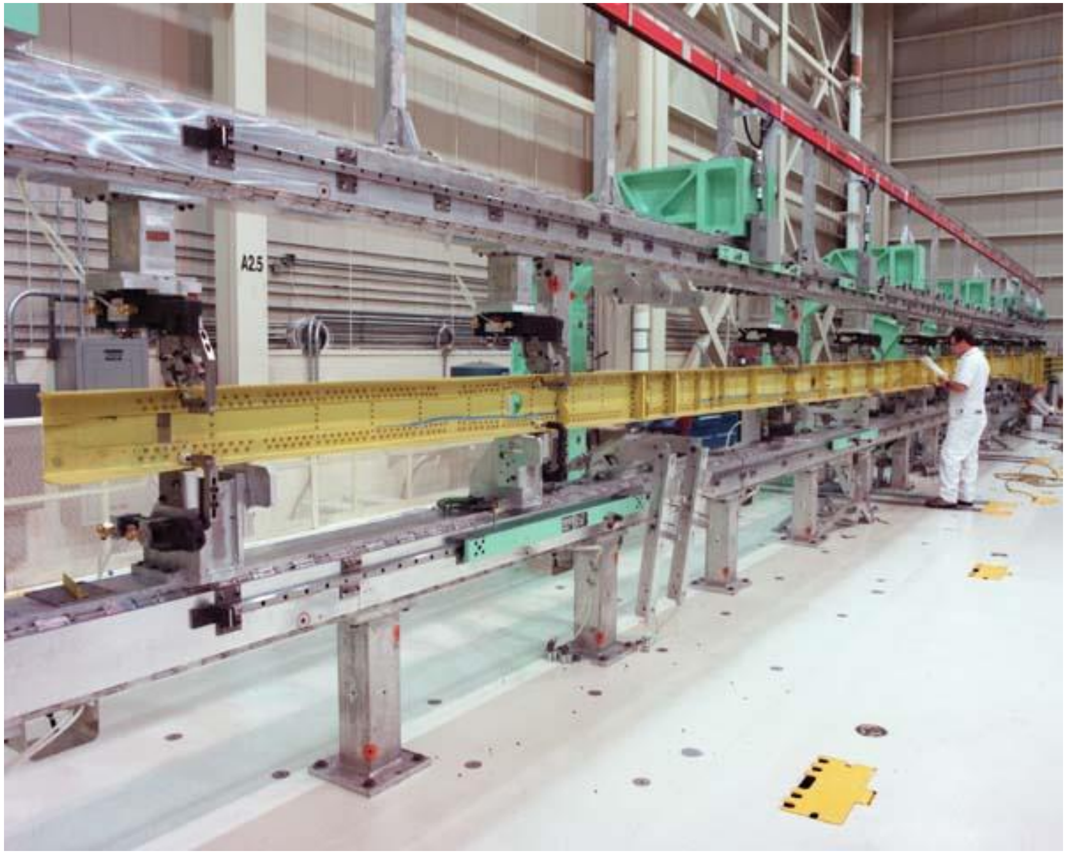


Semi-monocoques (4)

General Dynamics F-16

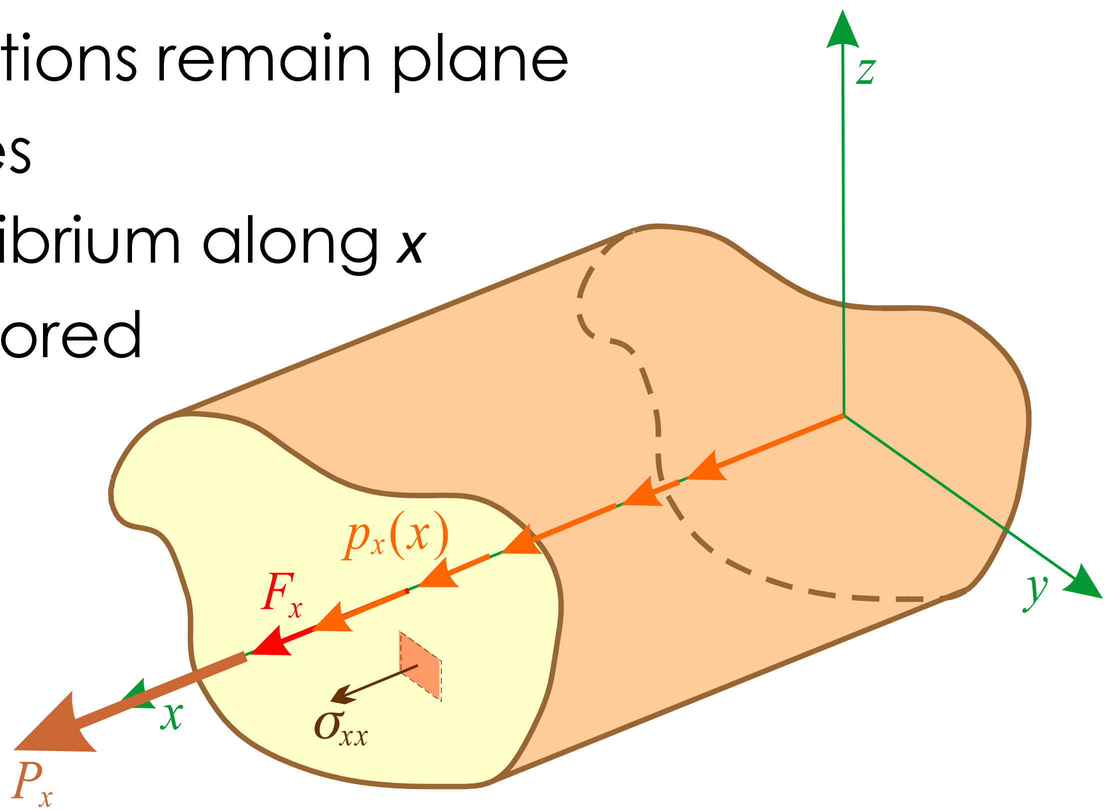


Extension of beams



Rod or bar hypothesis

- Pure extension
  - Transverse stresses negligible (axial stress field)
  - Transverse sections remain plane
  - No body forces
  - Average equilibrium along *x*
  - Equilibrium ignored along *y* and *z*



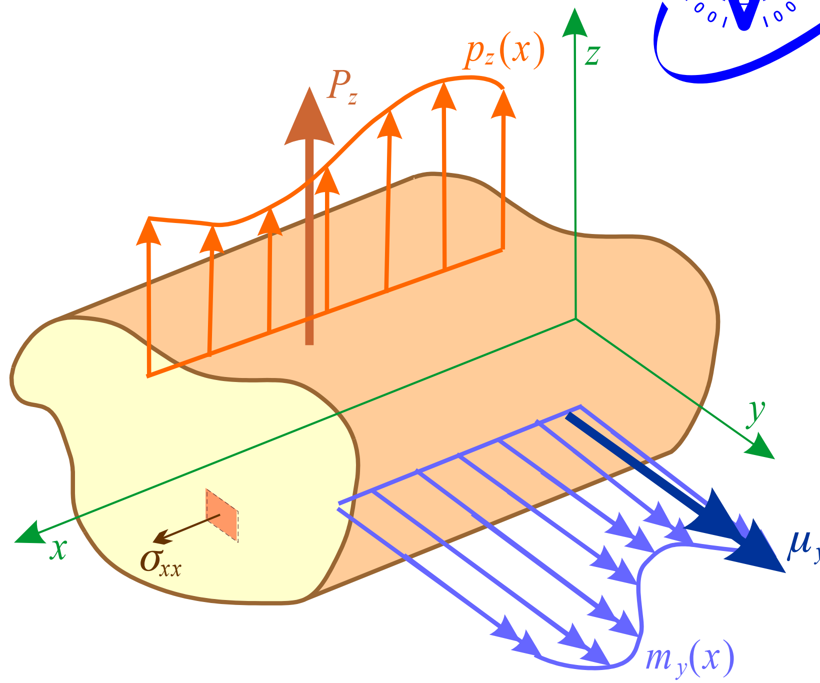
Pure extension

- Well-known simple problem
  - Useful to introduce new concepts
- Heterogeneous structures
  - Several materials with different properties
  - Concept of modulus-weighted properties
- Inclusion of thermal effects
  - Concept of thermal stresses and strains
  - Duhamel-Neumann stress-strain relations
  - Thermal loads (forces, and later moments)

Bending of advanced beams

Bernoulli-Euler hypothesis

- Simple bending
  - Cross-sections remain planar
  - Constant bending moment
  - Axial stress field
  - No body forces
  - Average equilibrium along *x* and *z*



Bernoulli-Euler hypothesis

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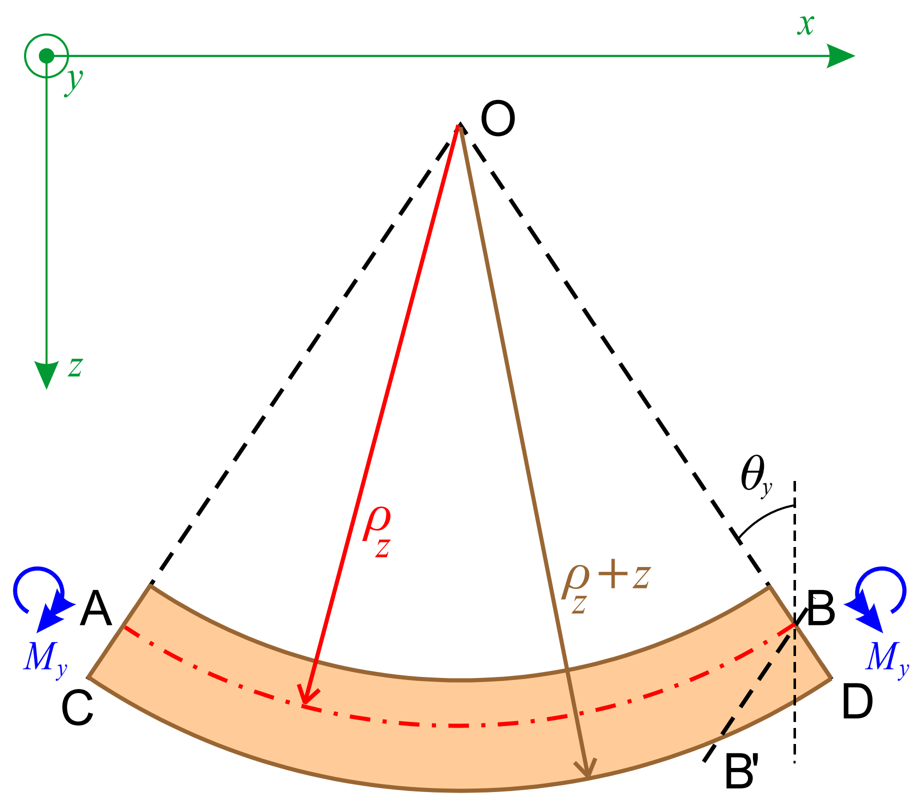


Simple bending solution

$$\varepsilon_{xx} = \frac{z}{\rho} \tag{B4.1}$$

$$\sigma_{xx} = E\varepsilon_{xx} \tag{B4.2}$$

$$\sigma_{xx} = \frac{M_y z}{I_{yy}} \tag{B4.3}$$

$$\frac{\partial u}{\partial x} = \varepsilon_{xx} = \frac{\sigma_{xx}}{E} = \frac{M_y z}{EI_{yy}} \tag{B4.4}$$

$$\frac{1}{\rho} = \pm \frac{\frac{\partial^2 w}{\partial x^2}}{\left[1+\left(\frac{\partial w}{\partial x}\right)^2\right]^{3/2}} \tag{B4.5}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{M_y(x)}{EI_{xx}} \tag{B4.6}$$

Combined bending

$$\varepsilon_{xx} = \varepsilon_0 + \frac{z}{\rho_z} - \frac{y}{\rho_y} \quad \text{(B5.1)}$$

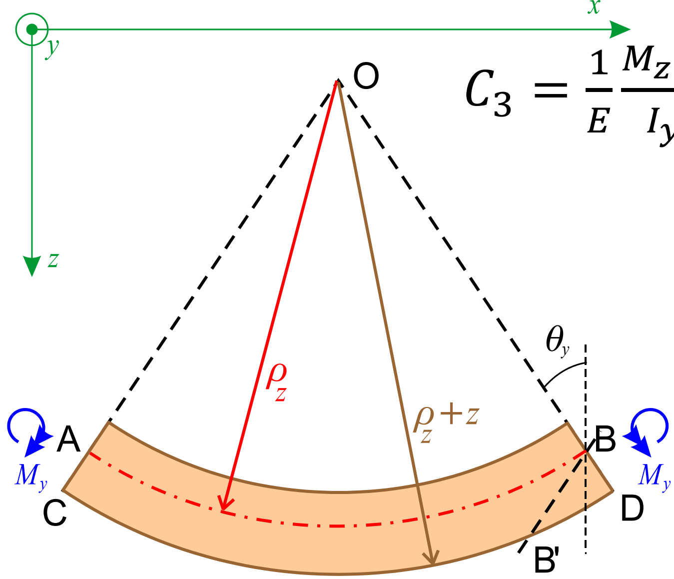
$$\varepsilon_{xx} = C_1 + C_2 y + C_3 z \quad \text{(B5.2)}$$

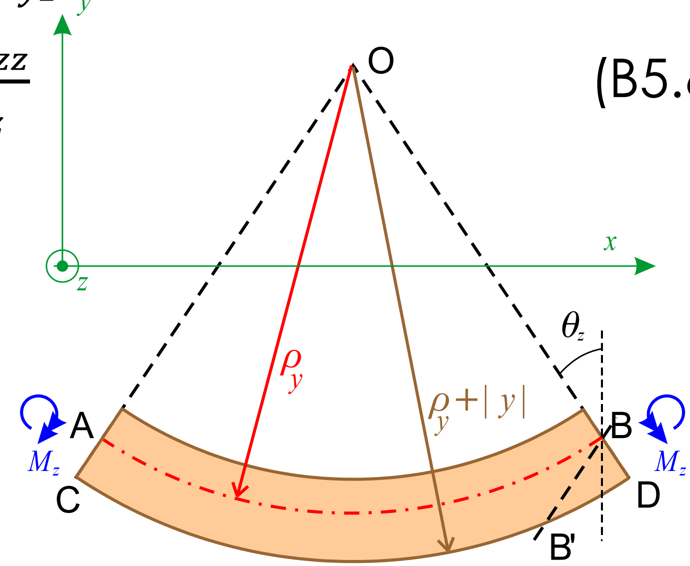
$$\sigma_{xx} = E\varepsilon_{xx} = EC_1 + EC_2 y + EC_3 z \quad \text{(B5.3)}$$

$$C_1 = \frac{F_x}{EA} \quad \text{(B5.4)}$$

$$C_2 = -\frac{1}{E}\frac{M_y I_{yz} + M_z I_{yy}}{I_{yy} I_{zz} - I_{yz}^2} \quad \text{(B5.5)}$$

$$C_3 = \frac{1}{E}\frac{M_z I_{yz} + M_y I_{zz}}{I_{yy} I_{zz} - I_{yz}^2} \quad \text{(B5.6)}$$





Solution

$$\sigma_{xx} = \frac{F_x}{A} - \frac{M_y I_{yz} + M_z I_{yy}}{I_{yy} I_{zz} - I_{yz}^2} y + \frac{M_z I_{yz} + M_y I_{zz}}{I_{yy} I_{zz} - I_{yz}^2} z \quad \text{(B5.7)}$$

$$\varepsilon_{xx} = \frac{F_x}{EA} - \frac{1}{E} \frac{M_y I_{yz} + M_z I_{yy}}{I_{yy} I_{zz} - I_{yz}^2} y + \frac{1}{E} \frac{M_z I_{yz} + M_y I_{zz}}{I_{yy} I_{zz} - I_{yz}^2} z \quad \text{(B5.8)}$$

$$\frac{z}{y} = \tan\lambda = \frac{M_y I_{yz} + M_z I_{yy}}{M_z I_{yz} + M_y I_{zz}} \quad \text{(B5.9)}$$

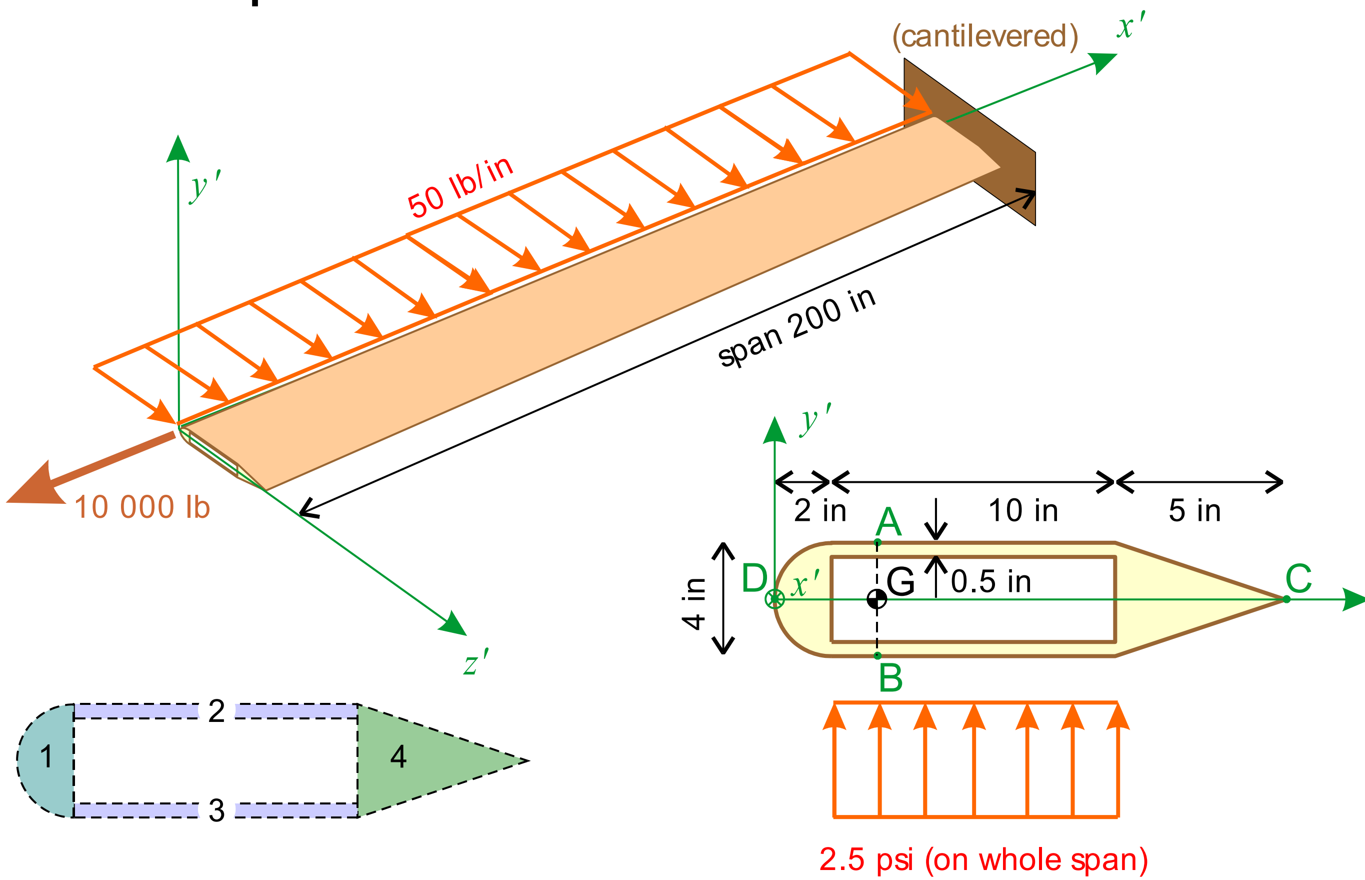
$$\frac{\partial u}{\partial x} = \varepsilon_{xx} \quad \text{(B5.10)}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{E} \frac{M_y I_{yz} + M_z I_{yy}}{I_{yy} I_{zz} - I_{yz}^2} \quad \text{(B5.11)}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{1}{E} \frac{M_z I_{yz} + M_y I_{zz}}{I_{yy} I_{zz} - I_{yz}^2} \quad \text{(B5.12)}$$

Application

➢ the blade is homogeneous with $E = 10^7$ psi



Modulus-weighed properties

$$\sigma_{xx} = \varepsilon_{xx} E - E\alpha\Delta T \tag{B6.1}$$

$$\varepsilon_{xx} = \varepsilon_0 + \frac{z}{\rho_z} - \frac{y}{\rho_y} \tag{B6.2}$$

$$\varepsilon_{xx} = C_1 + C_2 y + C_3 z \tag{B6.3}$$

$$F_x = \int_A \sigma_{xx}\, dA = \int_A [EC_1 - E\alpha\Delta T + EC_2 y + EC_3 z]\, dA \tag{B6.4}$$

$$A^* = \int_A \frac{E}{E_0} dA \tag{B6.5}$$

$$y_F^* = \frac{1}{A^*}\int_A y \frac{E}{E_0} dA \tag{B6.6}$$

$$z_F^* = \frac{1}{A^*}\int_A z \frac{E}{E_0} dA \tag{B6.7}$$

Modulus-weighed properties

$$M_y = \int_A z\sigma_{xx}\, dA = E_0 \int_A \left[\frac{E}{E_0} C_1 z + \frac{E}{E_0} C_2 yz + \frac{E}{E_0} C_3 z^2\right] dA - \int_A zE\alpha\Delta T\ dA \qquad \text{(B6.8)}$$

$$M_z = \int_A -y\sigma_{xx}\, dA = -E_0 \int_A \left[\frac{E}{E_0} C_1 y + \frac{E}{E_0} C_2 y^2 + \frac{E}{E_0} C_3 yz\right] dA + \int_A yE\alpha\Delta T\ dA \qquad \text{(B6.9)}$$

$$I_{yy}^* = \int_A z^2 \frac{E}{E_0} dA \qquad \text{(B6.10)}$$

$$I_{zz}^* = \int_A y^2 \frac{E}{E_0} dA \qquad \text{(B6.11)}$$

$$I_{yz}^* = \int_A yz \frac{E}{E_0} dA \qquad \text{(B6.12)}$$

Thermal loads

$$F_x^T = \int_A E\alpha\Delta T \, dA \quad \text{(B6.13)}$$

$$M_y^T = \int_A zE\alpha\Delta T \, dA \quad \text{(B6.14)}$$

$$M_z^T = -\int_A yE\alpha\Delta T \, dA \quad \text{(B6.15)}$$

$$F_x + F_x^T = F_x^* \quad \text{(B6.16)}$$

$$M_y + M_y^T = M_y^* \quad \text{(B6.17)}$$

$$M_z + M_z^T = M_z^* \quad \text{(B6.18)}$$

Final relations

$$\varepsilon_{xx} = \frac{F_x^*}{E_0 A^*} - \frac{1}{E_0}\frac{M_y^* I_{yz}^* + M_z^* I_{yy}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} y + \frac{1}{E_0}\frac{M_z^* I_{yz}^* + M_y^* I_{zz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} z \qquad \text{(B6.19)}$$

$$\sigma_{xx} = \frac{E F_x^*}{E_0 A^*} - \frac{E}{E_0}\frac{M_y^* I_{yz}^* + M_z^* I_{yy}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} y + \frac{E}{E_0}\frac{M_z^* I_{yz}^* + M_y^* I_{zz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} z - E\alpha\Delta T \qquad \text{(B6.20)}$$

$$\sigma_{xx} = E(\varepsilon_{xx} - \alpha\Delta T) \qquad \text{(B6.21)}$$

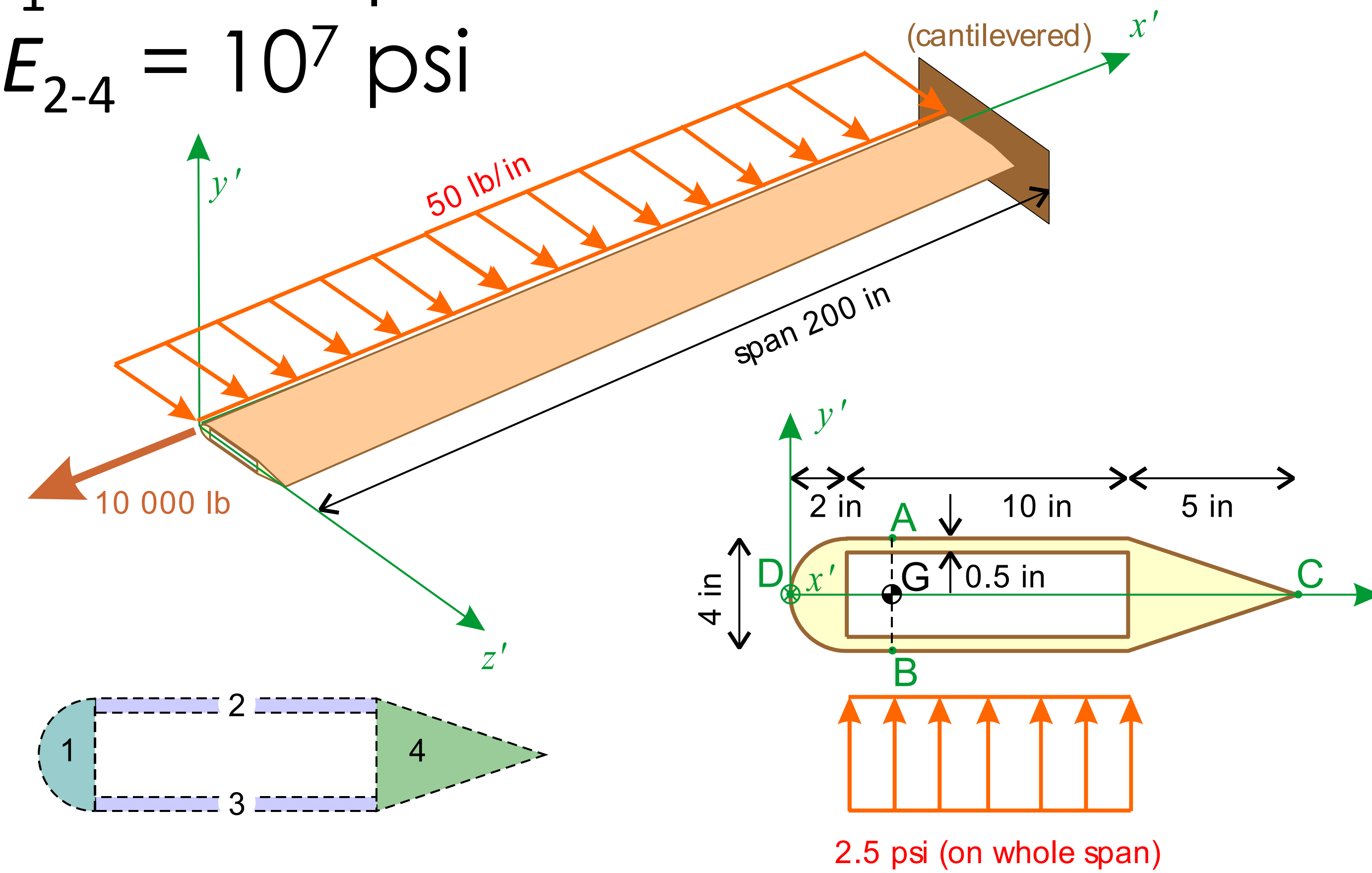
$$\frac{\partial u}{\partial x} = \varepsilon_{xx} \qquad \text{(B6.22)}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{E_0}\frac{M_y^* I_{yz}^* + M_z^* I_{yy}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} \qquad \text{(B6.23)}$$

$$\frac{\partial^2 w}{\partial x^2} = -\frac{1}{E_0}\frac{M_z^* I_{yz}^* + M_y^* I_{zz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} \qquad \text{(B6.24)}$$

Application

- the blade is heterogeneous with $E_1 = 3.10^7$ psi and $E_{2\text{-}4} = 10^7$ psi



Application

- the blade is heterogeneous with $\alpha_1 = 5\ 10^{-6}$ /°F and $\alpha_{2\text{-}4} = 6.5\ 10^{-6}$ /°F
- $\Delta T = 0.10\ x$ °F