



# Aircraft Structures

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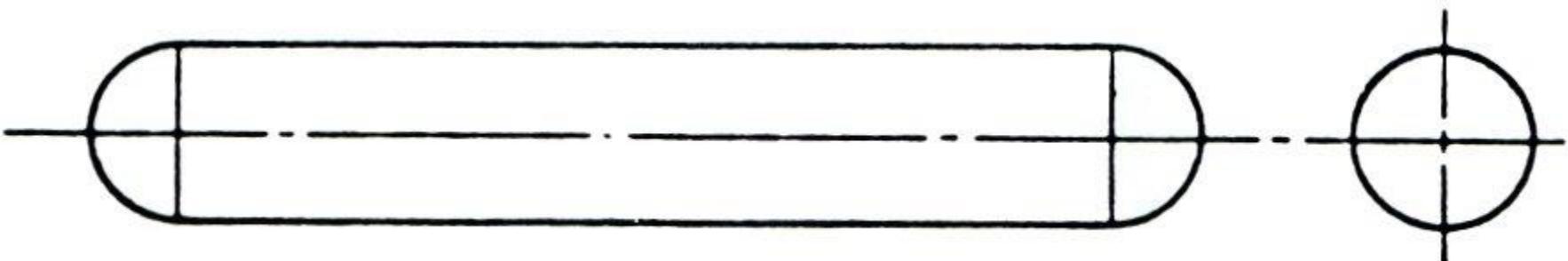


# Analysis of fuselages

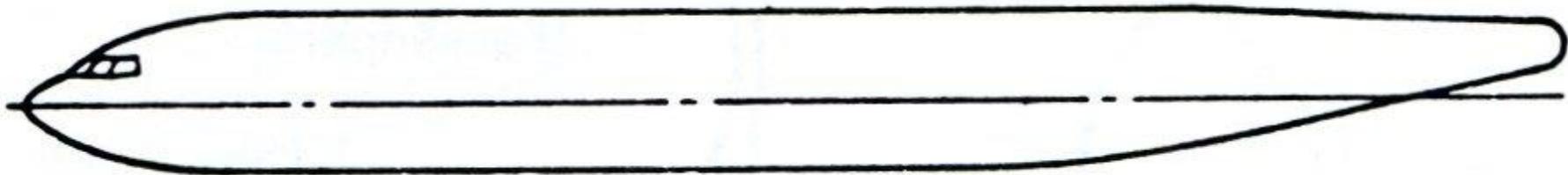


# Fuselage requirements

## ➤ Pressure

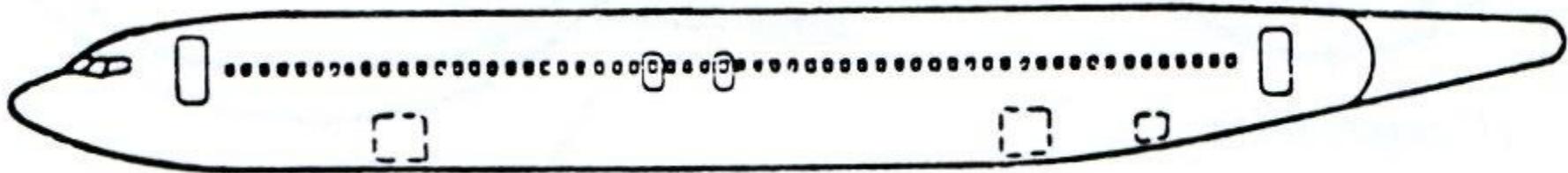


## ➤ Aerodynamics

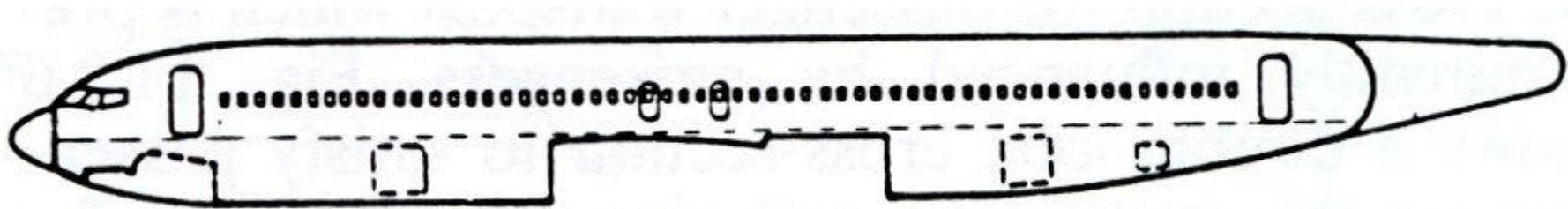


# Fuselage requirements

- Access/comfort



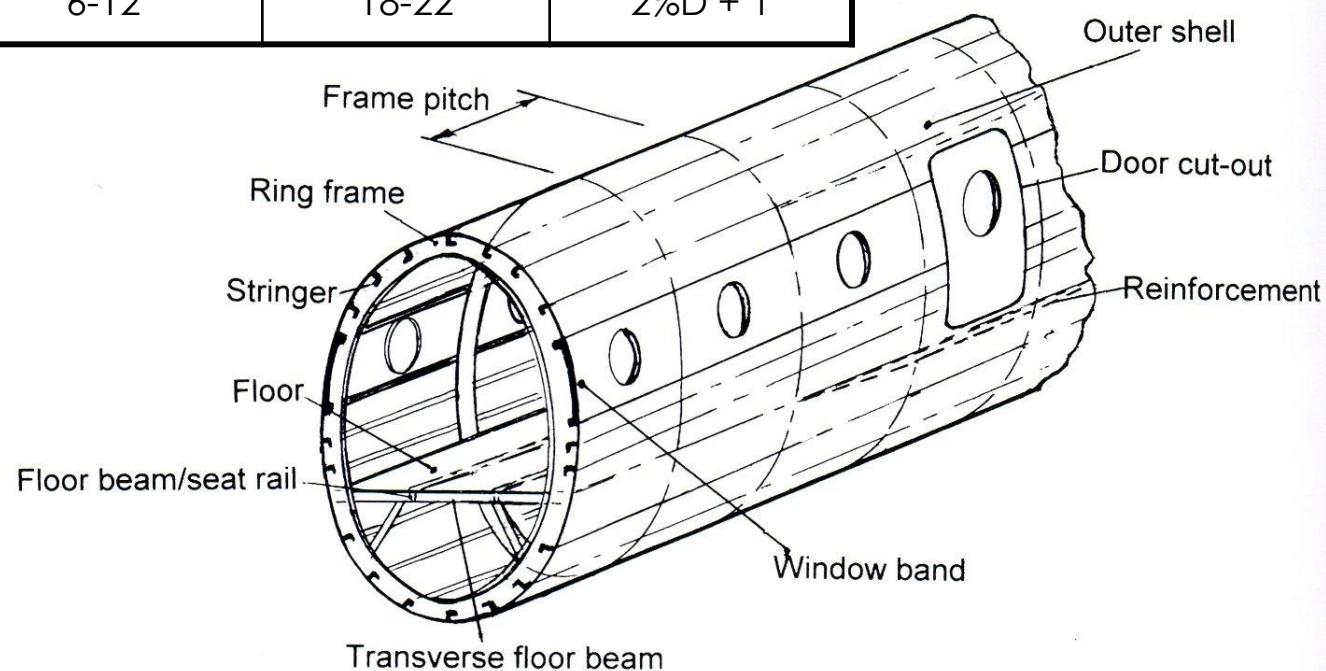
- Structural integration



# Semi-monocoque

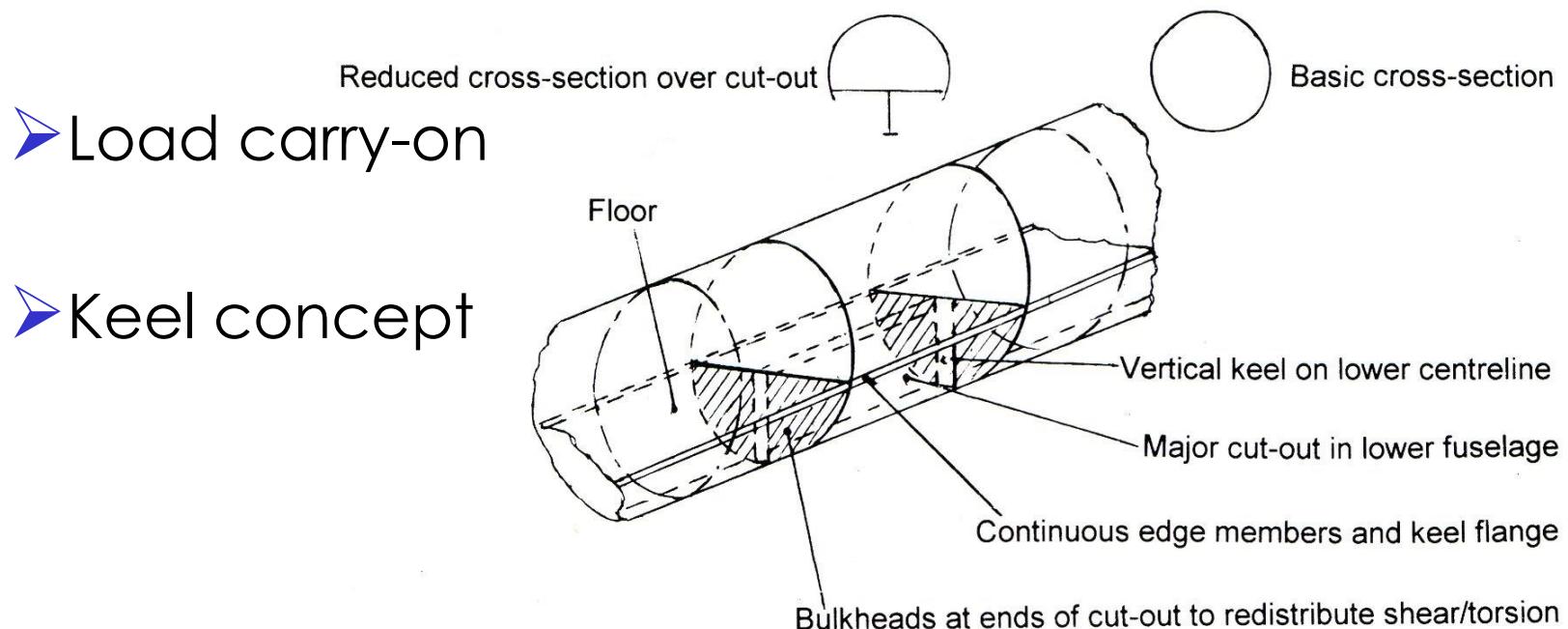
## ➤ Classical ring/stringer layout

(inches)	Stringer pitch	Frame pitch	Frame depth
Small commercial	10-15	24-30	1.5
Fighter/trainer	8-12	15-20	2
Large transport	6-12	18-22	$2\%D + 1$



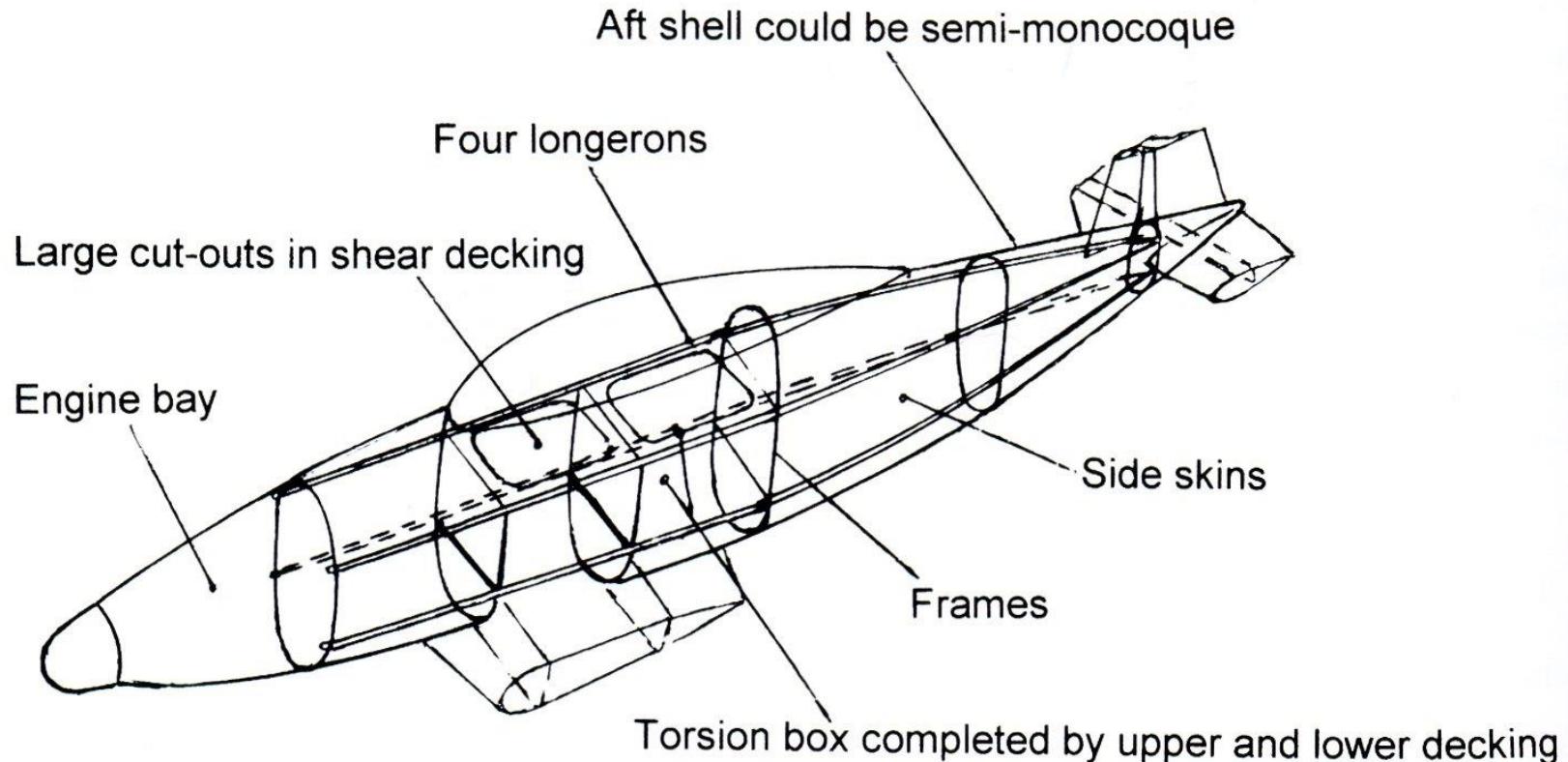
# Cutouts

- Windows (& doors)
  - Window doubles (heavy stringers)
- Large cutouts
  - Pressure sealing

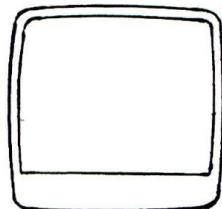


# Discrete boom-longeron

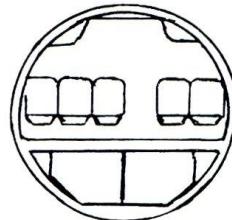
- Use heavy longerons
- For low loading, numerous cutouts



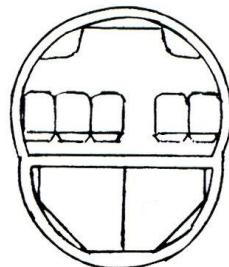
# Cross-section types



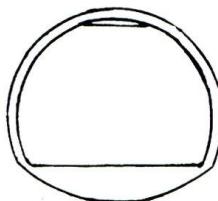
(a) Unpressurized, basically rectangular



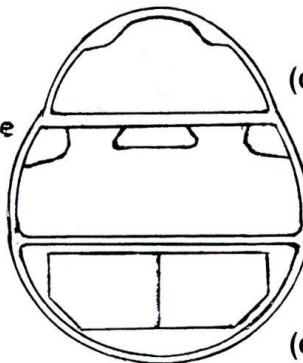
(b) Ideal circular shape



(c) Two-arc cross-section,  
increased below floor space



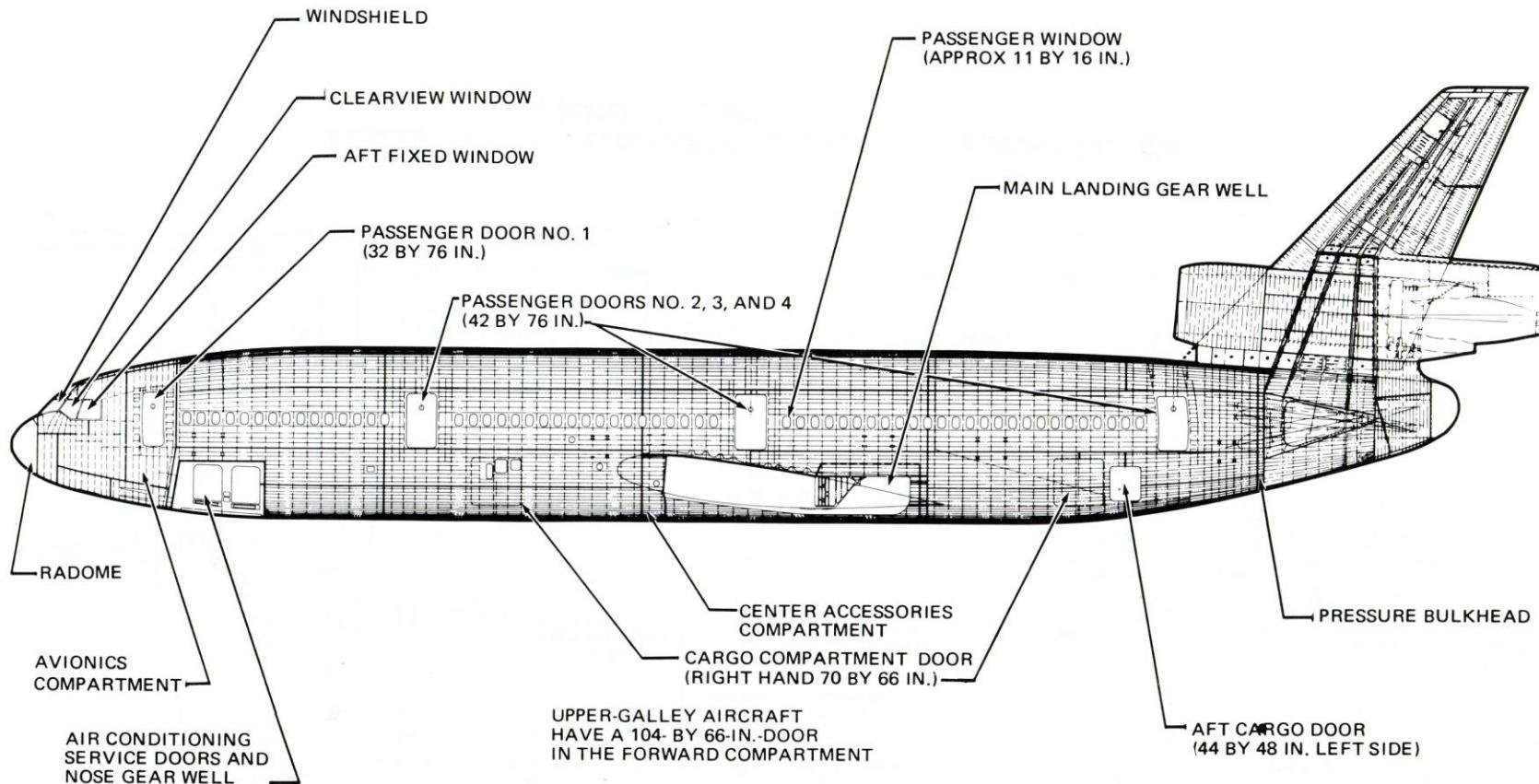
(d) Two-arc cross-section,  
floor line close to ground



(e) Three-arc cross-section

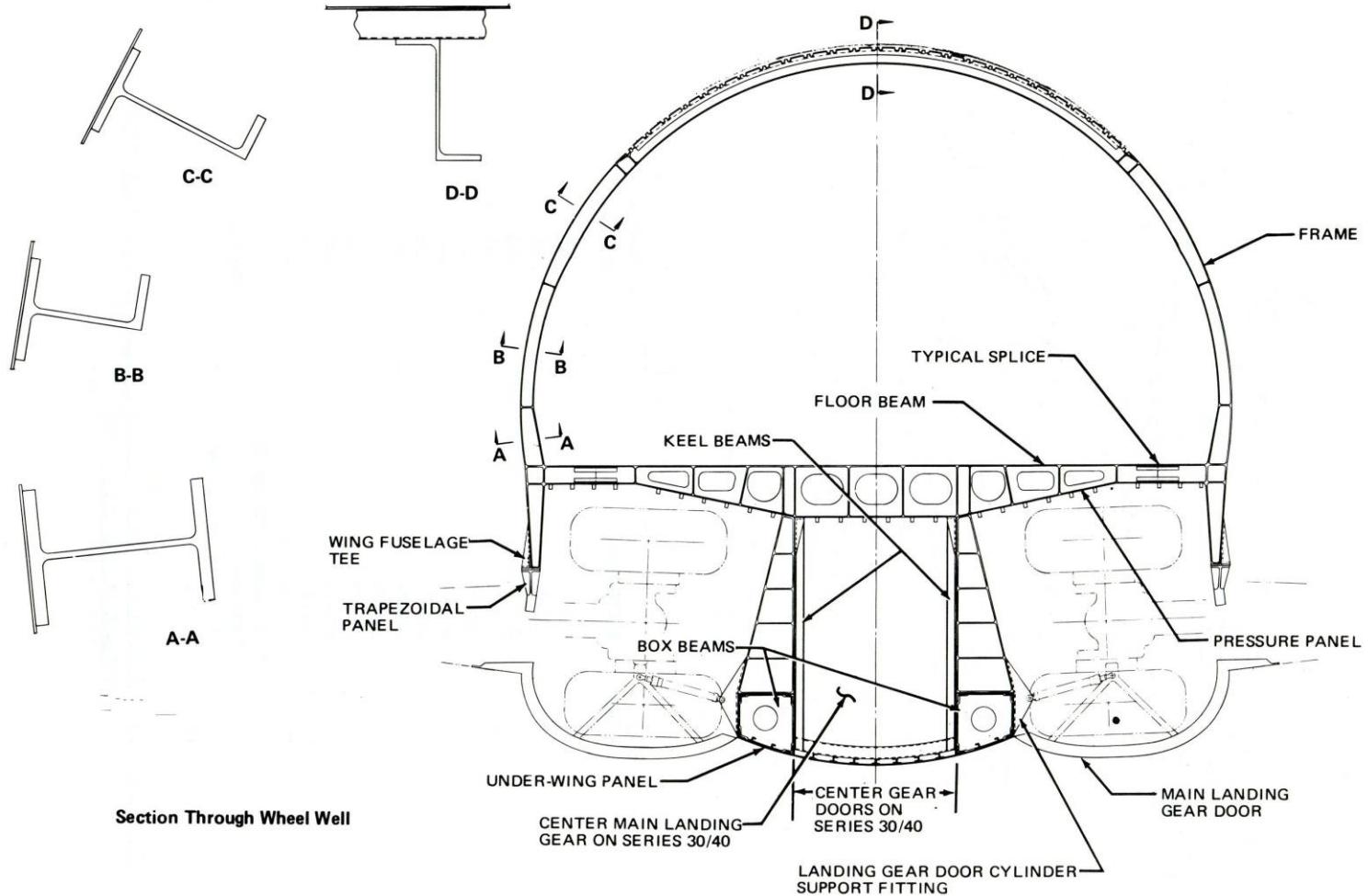
# Fuselage examples

## ➤ DC-10



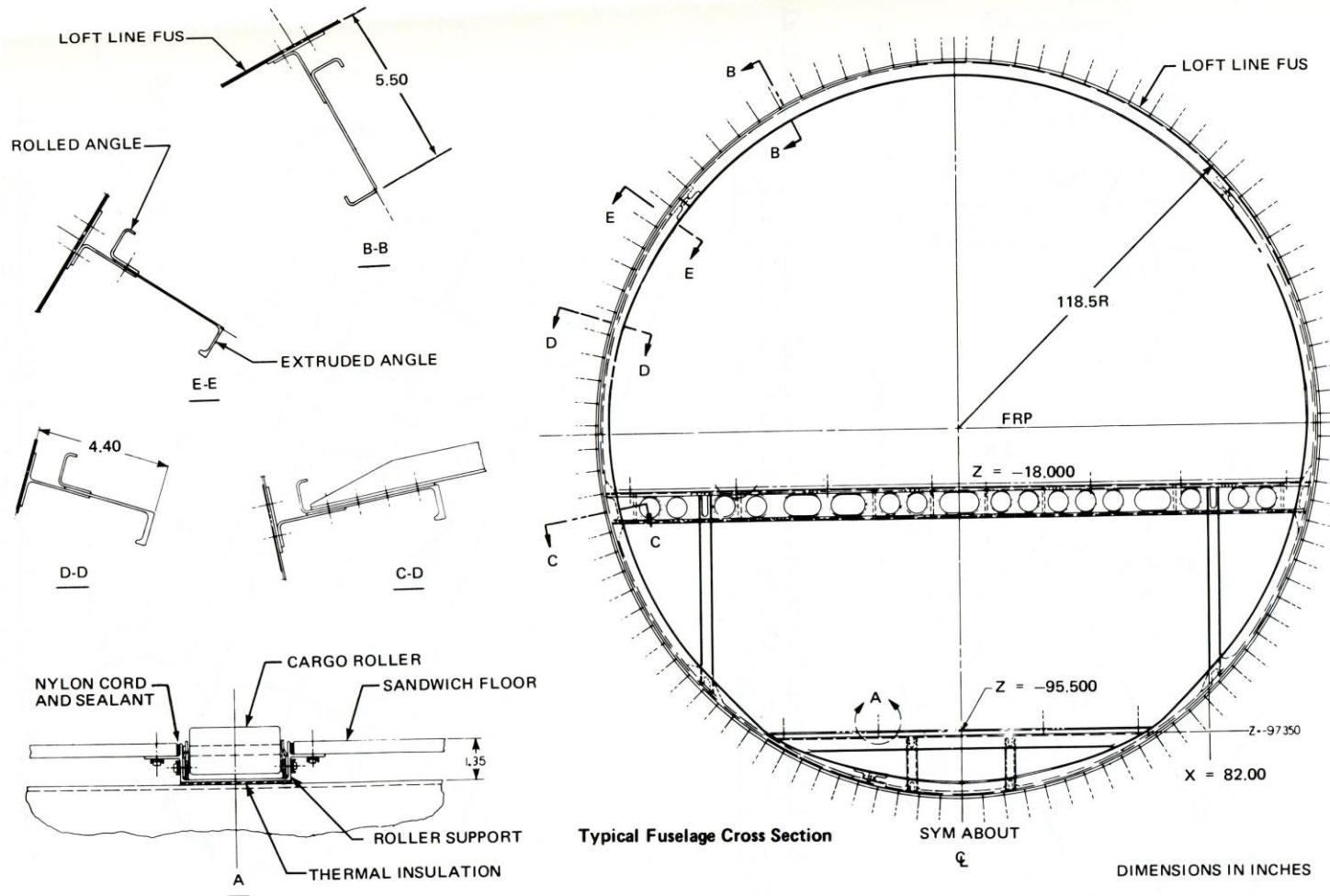
# Fuselage examples

## ► DC-10



# Fuselage examples

## ► DC-10



# Analysis of shells



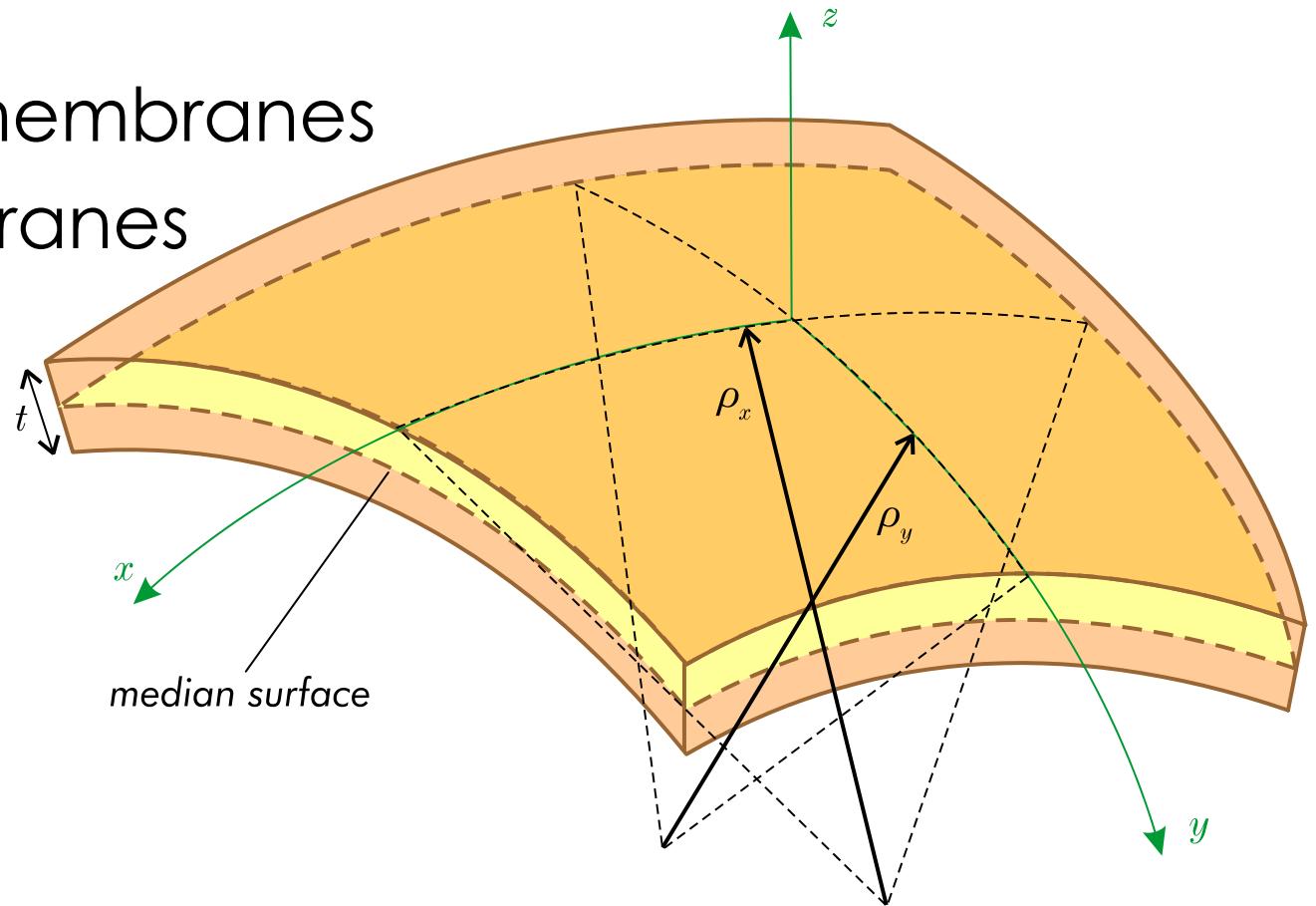
Aug. 4, 2009

An 18m (59ft)-long fuselage demonstrator was assembled in Hamburg and is close to the final design of the A350 fuselage, as it will be used for certification of the design principle.

# Shells

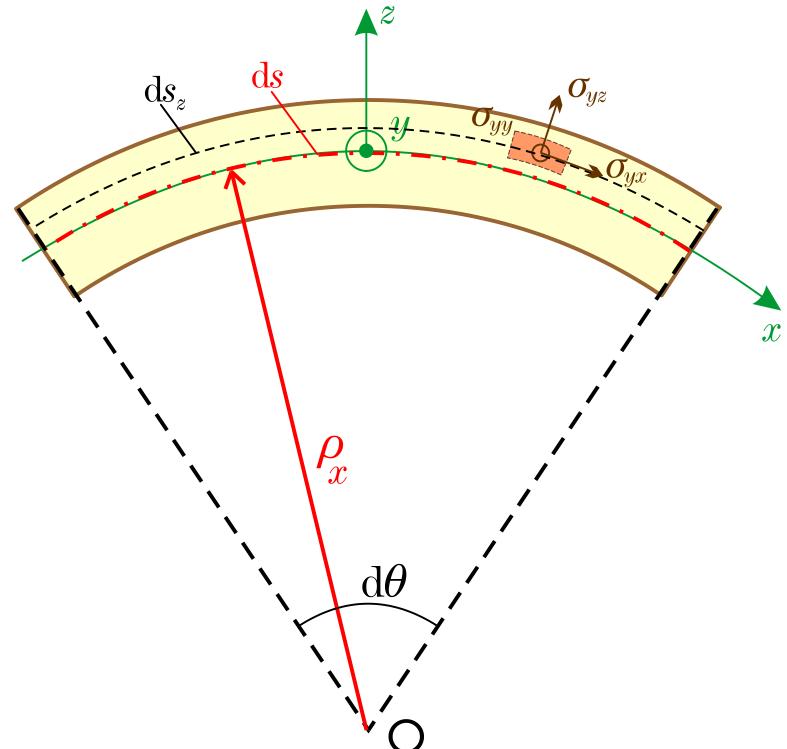
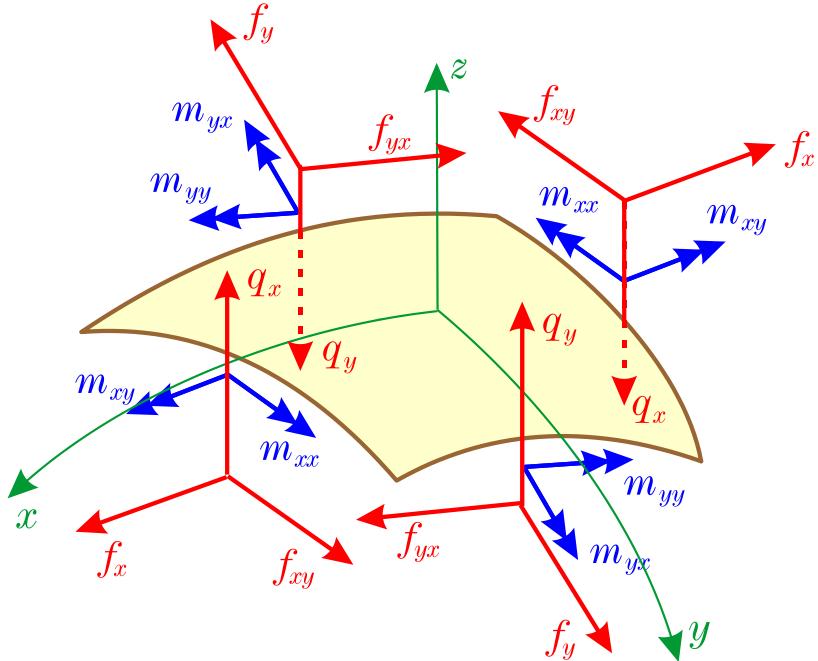
➤ Curved surfaces even without loads

- Shells
- Shell-membranes
- Membranes



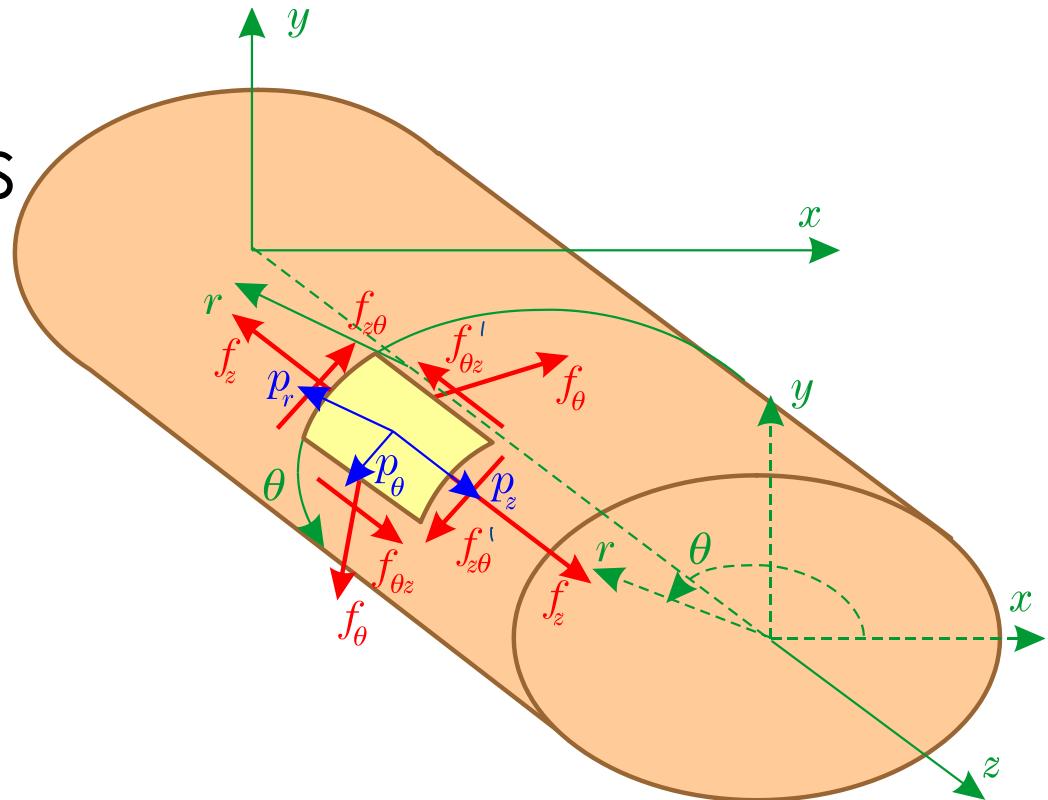
# Internal resultants

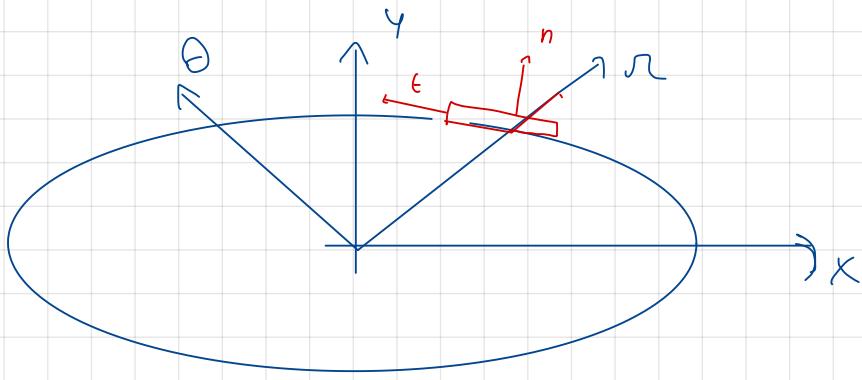
- Stress-based convention (as plates)
- Internal resultants are not « reciprocal »
  - True only under thin shell assumption



# Cylindrical shells

- Equilibrium equations
- Stresses
- Strains
- Displacements





$$1) \oint z \rho d\theta - \oint z \rho d\theta + \oint \theta_z dz - \oint \theta_z dz + p_z d_z \rho d\theta = 0$$

↓ Taylor

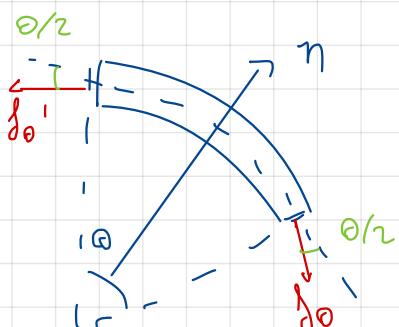
$$\left( \oint z + \frac{\partial \oint z}{\partial z} dz \right) \rho d\theta - \oint z \rho d\theta + \left( \oint \theta_z + \frac{\partial \oint \theta_z}{\partial \theta} d\theta \right) dz - \oint \theta_z dz + p_z \rho d_z \rho d\theta = 0$$

$$\boxed{\frac{\partial \oint z}{\partial z} + \frac{\partial \oint \theta_z}{\partial \theta} \cdot \frac{1}{\epsilon} = -p_z}$$

$$2) \oint \theta_z' dz - p_\theta dz + (\oint z_{\theta_0}' - \oint z_\theta) \rho d\theta + p_\theta \rho dz d\theta = 0$$

$$\Leftrightarrow \frac{\partial \oint \theta_z}{\partial z} d\theta dz + \frac{\partial \oint z_\theta}{\partial z} dz \rho d\theta + p_\theta dz d\theta = 0$$

$$\Leftrightarrow \boxed{\frac{1}{\rho} \frac{\partial \oint \theta_z}{\partial \theta} + \frac{\partial \oint z_\theta}{\partial z} = -p_\theta} \quad (2)$$



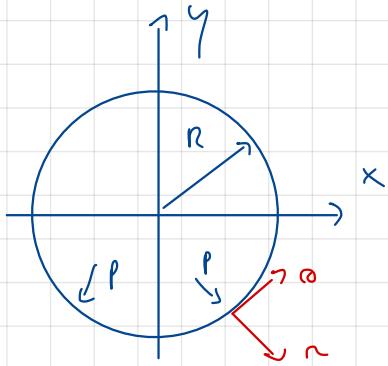
$$(\oint \theta_z' + \oint \theta_z) d\theta \sin\left(\frac{\theta_0}{2}\right) - p_r \rho dz d\theta = 0$$

$$\cancel{\left( \frac{\partial \oint \theta_z}{\partial z} d\theta + 2 \oint \theta_z \right) \frac{dz d\theta}{2}} - p_r \rho dz d\theta = 0$$

$\cancel{= 0}$   
(small)  
projection in  
the  $n$ -direction  
is the same twice

$$\Leftrightarrow \boxed{\oint \theta_z = p_n \cdot \rho} \quad (3)$$

# Cylindrical shell



$$f_\theta = pR \Rightarrow \sigma_{\theta\theta} = \frac{f_\theta}{E} = \frac{pR}{E}$$

(2):  $\frac{1}{R} \frac{\partial f_z}{\partial \theta} + \frac{\partial f_{z\theta}}{\partial z} = 0 \Rightarrow f_{z\theta} = \text{cte} = 0$  (at the edge then resultant = 0)  
 " 0  $\hookrightarrow f_{z\theta} = 0$

$$\frac{\partial f_z}{\partial z} + \frac{\partial f_{z\theta}}{\partial \theta} \cdot \frac{1}{R} = 0$$

" 0  $\hookrightarrow f_z = \text{cte} = \frac{pR}{2} \rightarrow \sigma_{zz} = \frac{p \cdot R}{2E}$   
 (thin wall)

$$f_z \cdot 2\pi R = p \cdot \pi \cdot R^2$$

$$f_z = \frac{p \cdot R}{2}$$

$$\begin{pmatrix} \epsilon_{\theta\theta} \\ \epsilon_{zz} \end{pmatrix} = \frac{1}{E} \begin{pmatrix} \gamma & -\nu \\ -\nu & \gamma \end{pmatrix} \begin{pmatrix} \sigma_{\theta\theta} \\ \sigma_{zz} \end{pmatrix}$$

$$\epsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \nu \cdot \sigma_{zz} = \frac{p \cdot R}{E \cdot E} - \nu \cdot \frac{pR}{2E} = (1-\nu) \cdot \frac{pR}{2E}$$

$$\epsilon_{zz} = -\nu \cdot \sigma_{\theta\theta} + \frac{\sigma_{zz}}{E} = -\nu \cdot \frac{pR}{2E} + \frac{p \cdot R}{2E} = (1-\nu) \frac{p \cdot R}{2E}$$

## Cylindrical shell (circular cross section)

$$\frac{\partial u_z}{\partial z} = \varepsilon_{zz} \Rightarrow u_z = (1 - 2\nu) \frac{p \cdot R}{2E} \cdot z$$

$$\frac{1}{R} \cdot \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{R} = \varepsilon_{\theta\theta} \quad u_r = R \varepsilon_{\theta\theta} = (1 - \nu) \frac{p \cdot R^2}{2E}$$

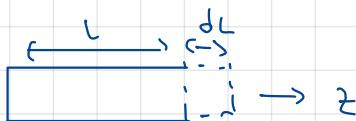
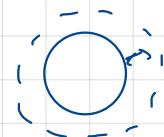
$$\frac{\partial u_\theta}{\partial z} + \frac{u_r}{R} \frac{\partial u_z}{\partial \theta} = \varepsilon_{\theta z} = 0 \quad \text{beware the pressurized cabin will only inflate}$$

$$= 0$$

It can't truth

$$\downarrow \quad \downarrow \quad \downarrow$$

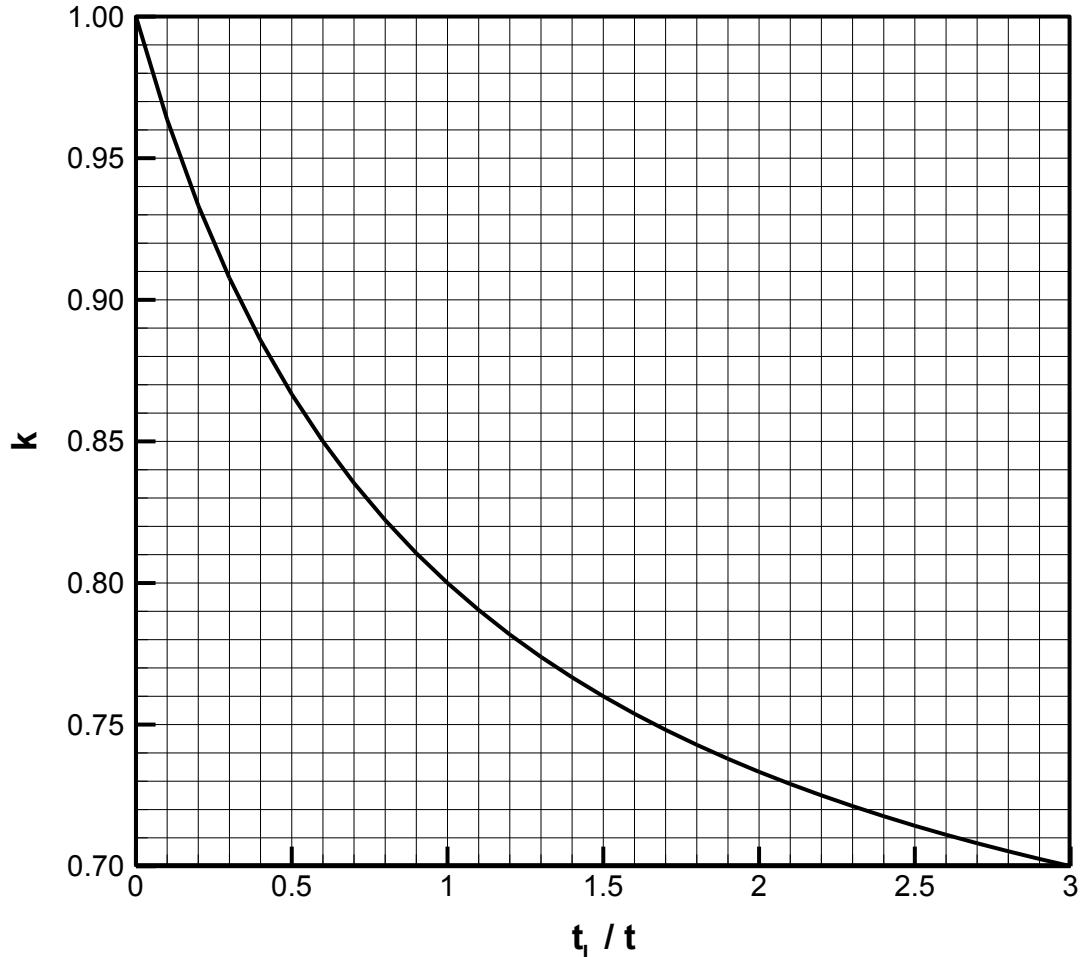
$$\sigma_{\theta z} \neq 0$$



# Application to fuselages

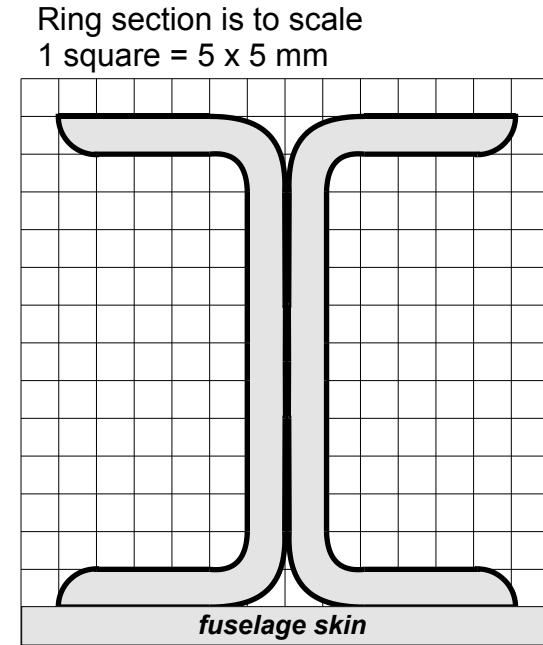
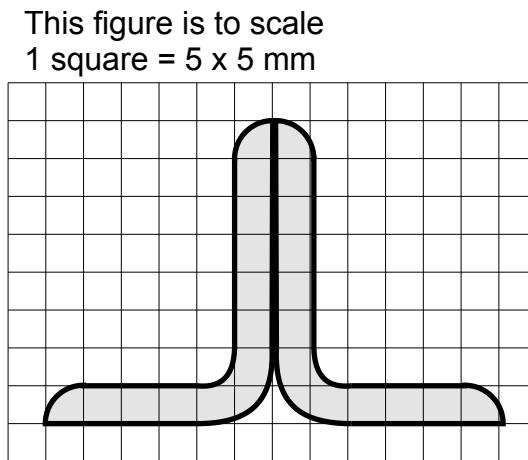
## ➤ Flügge approach

- Rings
- Stringers
- Skin



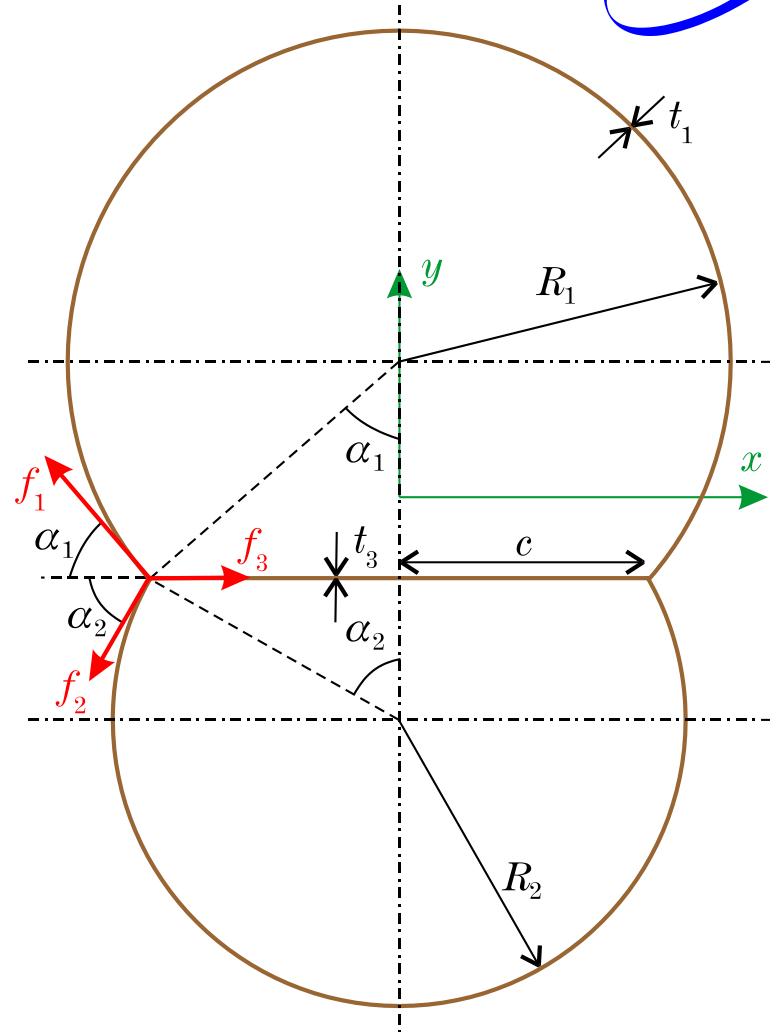
# Exercise

- Consider an ERJ fuselage geometry
  - 37 000 ft ISA with 8000 ft cabin altitude
    - FL 370:  $\delta = 0.2138$ ; FL 80:  $\delta = 0.7428$
  - Cylindrical, 238 cm diameter, fuselage
    - Stringers as shown, every 20°
    - rings as shown, 50 cm pitch
    - Skin 5 mm

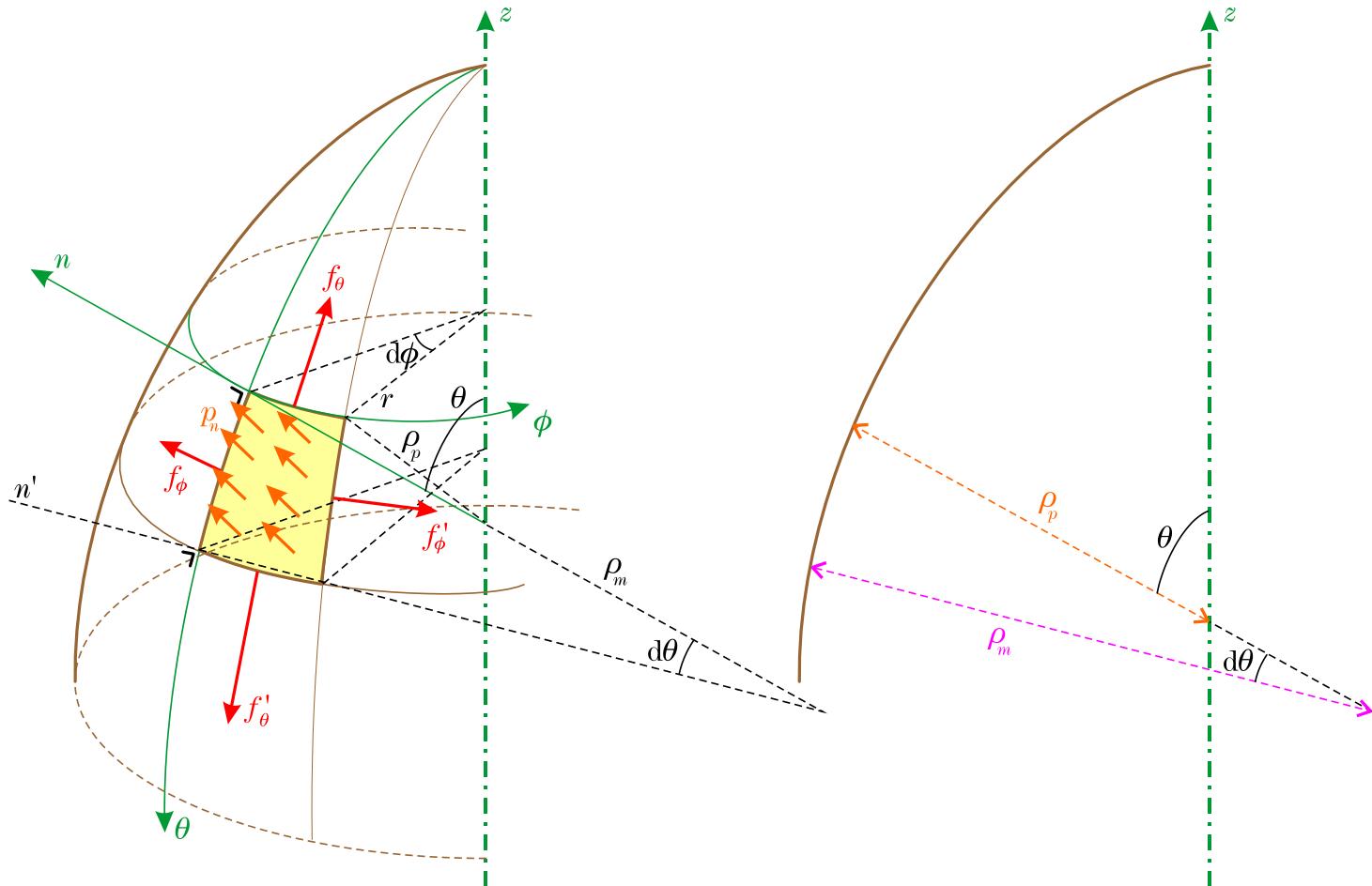


# Double fuselage case

- The running weight is independent of fuselage geometry for given total enclosed area

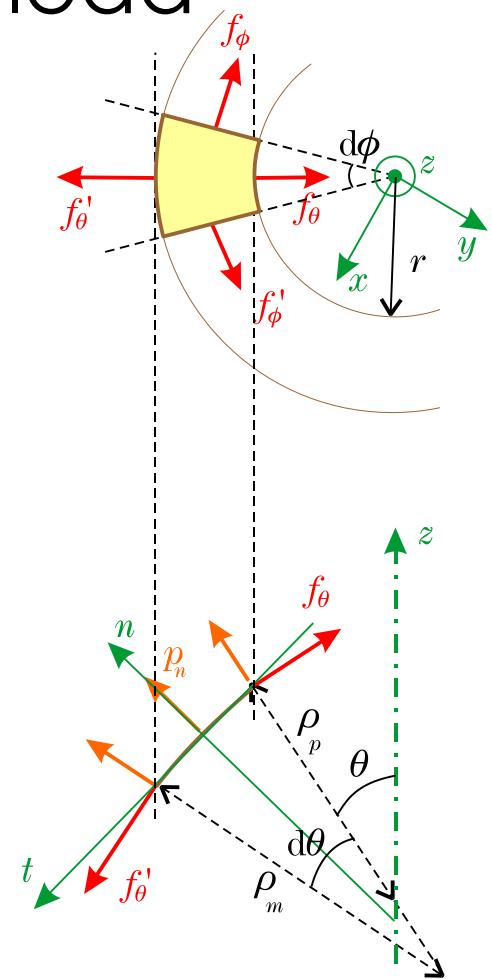
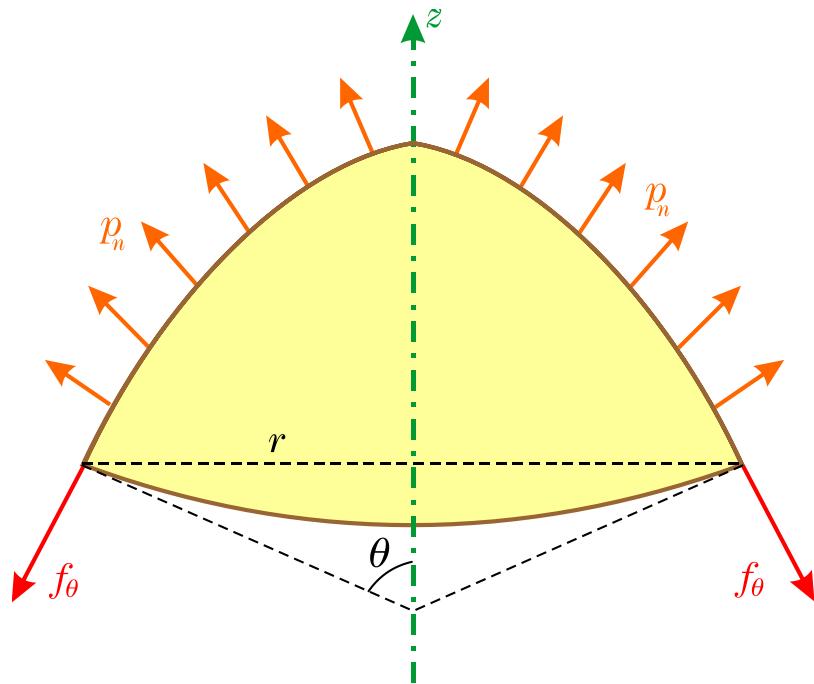


# Axisymmetric shells



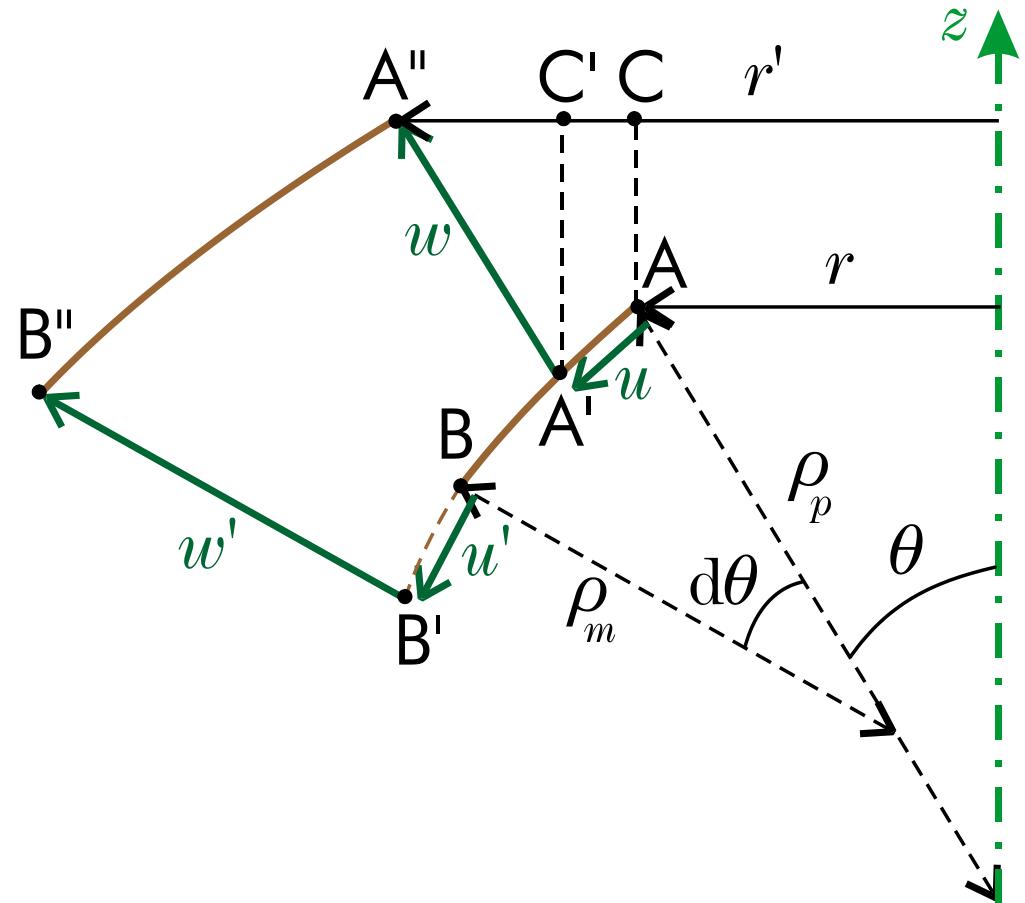
# Axisymmetric shells

- Equilibrium under pressure load



# Axisymmetric shells

## ➤ Strains and displacements



# Exercise

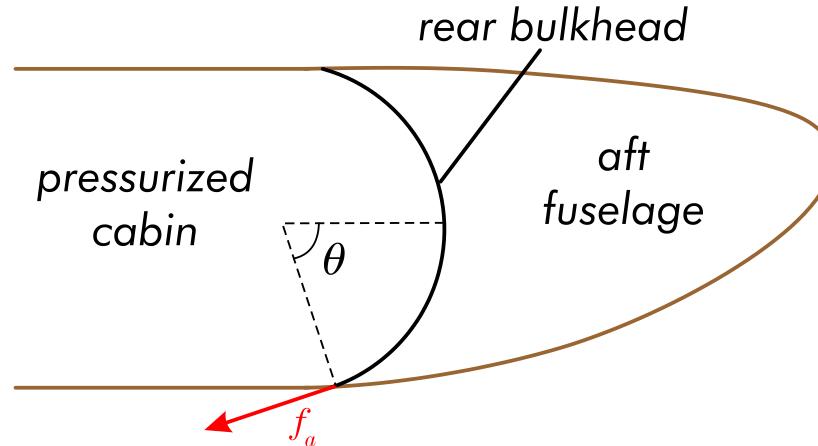
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- Consider an ERJ fuselage geometry
  - 37 000 ft ISA with 8000 ft cabin altitude
    - FL 370:  $\delta = 0.2138$ ; FL 80:  $\delta = 0.7428$
  - Cylindrical, 238 cm diameter, fuselage
- Determine the shell thickness for the pressure bulkhead considering the use of a spherical bulkhead

# Bulkhead troubles

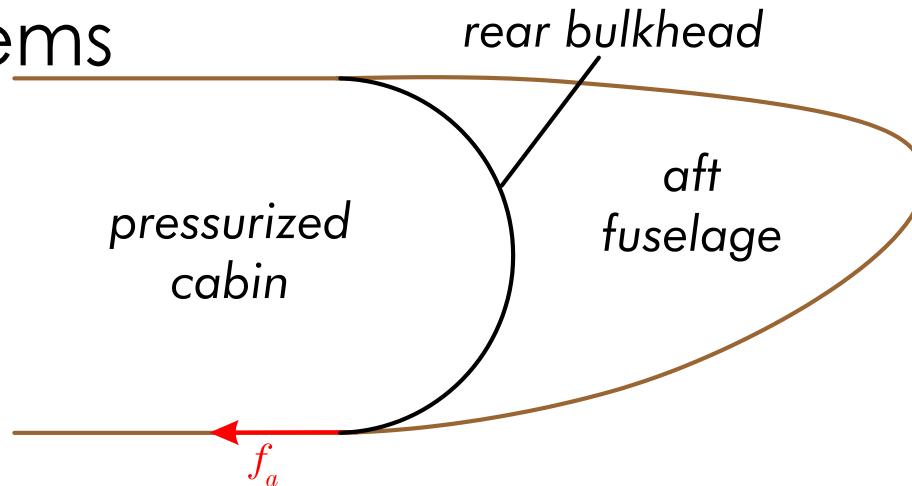
## ➤ Spherical caps

- non-axial meridional forces (shear forces and bending in the cylinder)
- Discontinuity in hoop stresses, strains and displacements
- Compressive stresses if radius/depth is high



# Bulkhead troubles

- Hemispherical caps
- Discontinuity in hoop stresses (factor 2!)
  - Large discontinuity in displacements (shear and moments in the joint)
  - Requires reinforcing ring
- May pose manufacturing/assembly problems



# Exercise

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- Investigate the previous phenomena using the ERJ results previously obtained for the cylinder and bulkhead cases