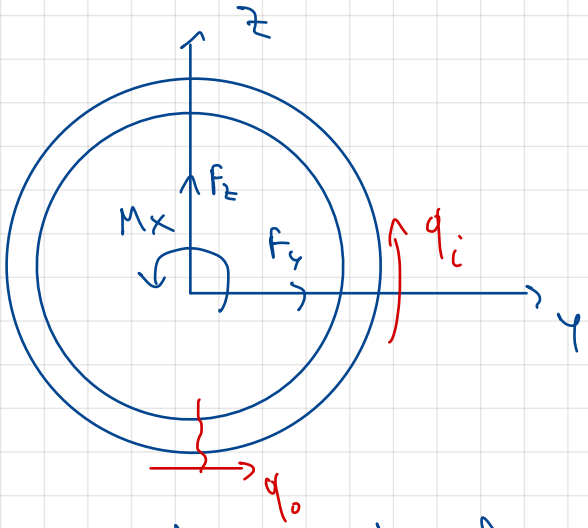
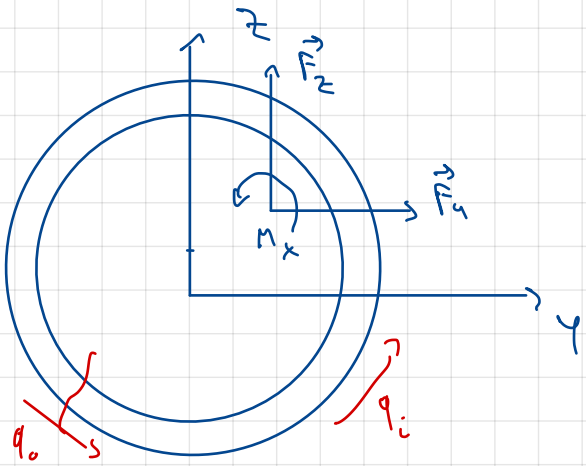
Single cell

Direct method

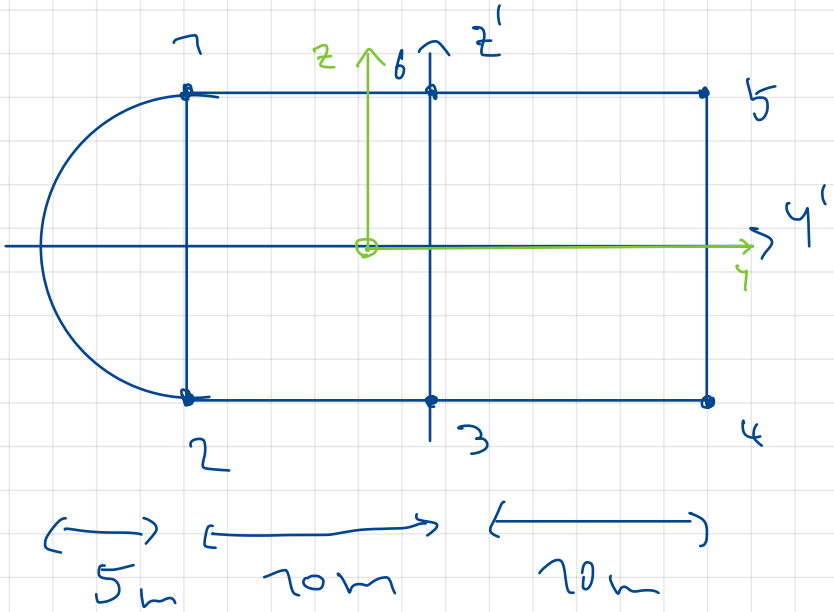


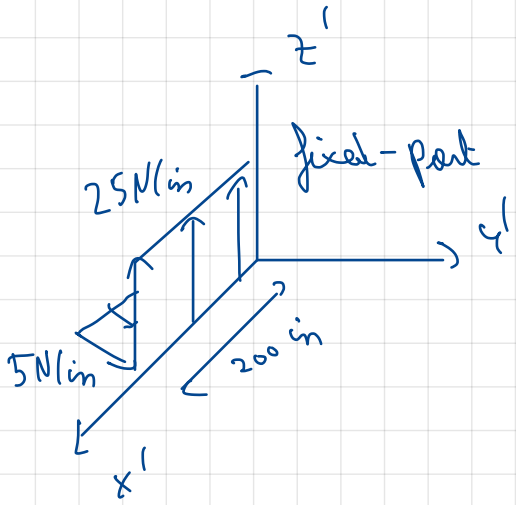
- cut to open the beam
- impose constant q_0
- solve for q_i
- solve for q_0 with moment equilibrium

Shear centre method



- cut to open the beam
- impose cte
- 3 computations
  - ↳ using $\vec{F}_y$ (y-shear flow) $= q^y$
  - ↳ $\vec{F}_z$ (z-shear flow) $= q^z$
  - ↳ M_x (torsion shear flow) $= q^t$
- solve for q^y and q^z (+ shear centre)
- solve for q^t (





Shear stress?

Position of the shear centre!

7, 2 : 2 in²

3, 4, 5, 6 : 7 in²

t = 0,07 in

1| Idealization of the beam (already done here)

2| Geometric properties of the beam
- ↳ Centre of figure
- ↳ Principal moment of inertia

3| Reduce the external forces to the centroid of the beam

4| Shear flow and shear centre

	A_i	y_i'	z_i'	$A_i y_i'$	$A_i z_i'$	$A_i y_i'^2$	$A_i z_i'^2$	$A_i y_i' z_i'$
1	2	-10	5	-20	10	200	50	
2	2	-10	-5	-20	-10	200	50	
3	1	0	-5	0	-5	0	25	
4	1	0	-5	10	-5	100	25	
5	1	10	5	10	5	100	25	
6	1	10	5	0	5	0	25	
Σ	8			-20	0	600	200	0

$y_F = \frac{-20}{8} = -2{,}5\,in$

$z_F = \frac{0}{8} = 0\,in$

$I_{y'y'} = 200\,in^2$

$I_{z'z'} = 600\,in^2$

$I_{yy} = 200 - z_F^2 A = 200\,in^4$

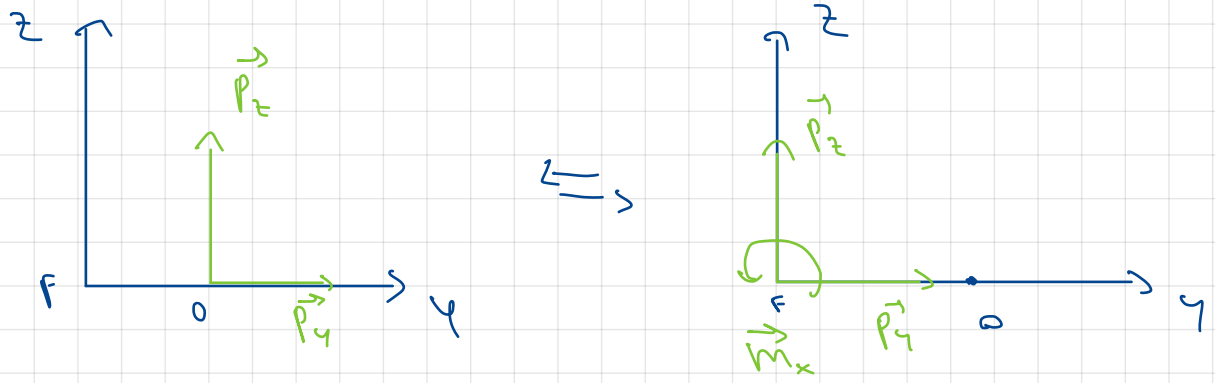
$I_{zz} = 600 - y_F^2 A = 550\,in^4$

Steiner

$p_x = 0\ lbf/in$ $\quad m_x$

$p_y = 5\ lbf/in$ $\quad m_y = 0$

$p_z = 25\ lbf/in$ $\quad m_z = 0$



$m_x = p_z \cdot |y_F|$

$= 25 \cdot 2{,}5 = 62{,}5\ lbf/in$

$$\frac{dF_x}{dx} = -p_x = 0 \Rightarrow F_x = C_1$$

at $x = 200$ in $F_x = 0 \Rightarrow C_1 = 0$

$F_x = 0$

$$\frac{dF_y}{dx} = -p_y = -5 \Rightarrow F_y = -5x + C_2$$

at $x = 200$ in $\Leftrightarrow F_y = 0 = -1000 + C_2$

$F_y = 1000 - 5x$ $\Rightarrow C_2 = 1000$

$$\frac{dF_z}{dx} = -p_z = -25 \Rightarrow F_z = 5000 - 25x \text{ lb}$$

$$\frac{dM_x}{dx} = -m_x = -62,5$$

$$M_x = -62,5x + C_3$$

at $x = 200$ in $\Rightarrow M_x = 0 = -12500 + C_3$

$M_x = -62,5x + 12500$

$$\frac{dM_y}{dx} = -m_y + F_z = 5000 - 25x$$

$M_y = -12,5x^2 + 5000x + C_4$

at $x = 200$ in $\Rightarrow M_y = 0 \Rightarrow C_4 = 500\,000 - 1\,000\,100$

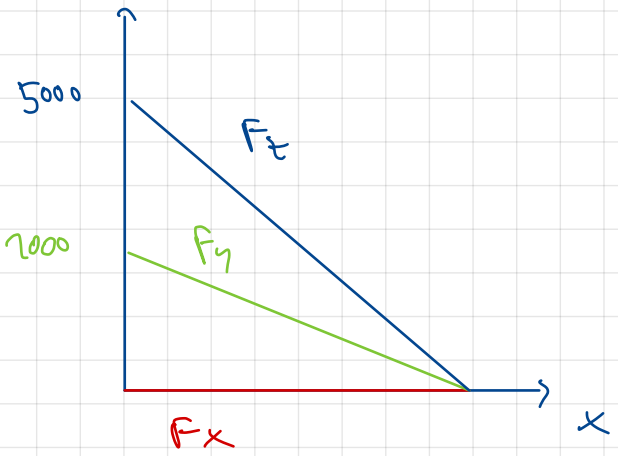
$M_y = -12,5x^2 + 5000x - 500\,000$

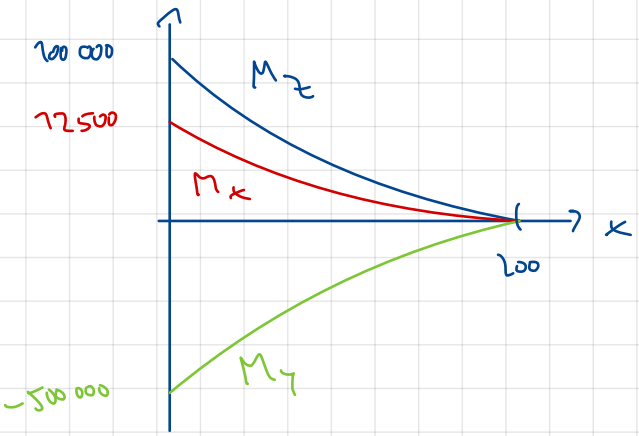
$$\frac{dM_z}{dx} = -m_z - F_y = 5x - 1000$$

$M_z = 2,5x^2 - 1000x + C_5$

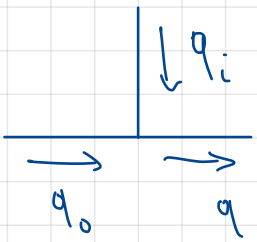
at $x = 200$ in $M_z = 0 \Rightarrow C_5 = 200\,000 - 100\,000$

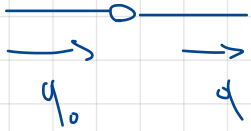
$M_z = 2,5x^2 - 1000x + 100\,000$





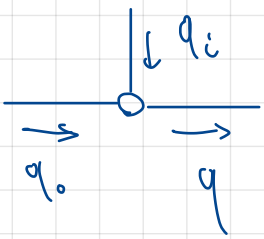
$$q = q_0 - \frac{F_z}{I_{44}} Q_{\eta,\eta} - \frac{F_y}{I_{zz}} Q_{\eta,\eta}$$





$$\sum q_i = -\frac{F_z}{I_{44}}$$

$$q - q_0 - q_i = 0$$



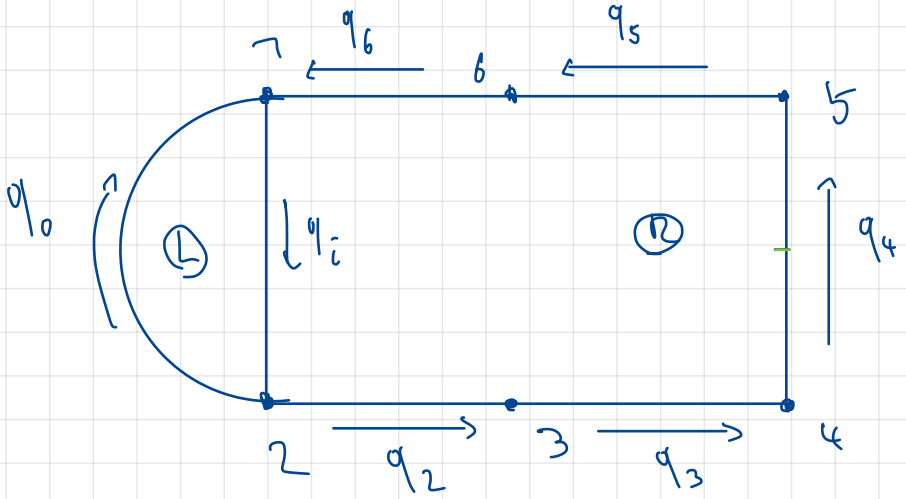
$$q - q_0 - q_i = \frac{F_z}{I_{44}} Q_{\eta,\eta}$$

$F_z = 200$ lb (5000)

$\downarrow \quad \cdot 25$

$F_z = 5000$

	A_i	η_i	z_i	$A_i\eta_i$	$A_i z_i$
1	2	-7,5	5	-75	70
2	2	-7,5	-5	-75	-70
3	1	2,5	-5	2,5	-5
4	1	12,5	-5	12,5	-5
5	1	12,5	5	12,5	5
6	1	2,5	5	2,5	5
				$Q_{\eta,z}$	$Q_{\eta,\eta}$



$q_0 = q_L$

$q_2 = q_R$

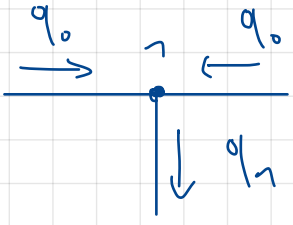
q_L and q_R are the two unknowns

$q_3 = q_2 - (-5) \Rightarrow q_3 = q_R + 5$

$q_4 = q_3 - (-5) \Rightarrow q_4 = q_R + 10$

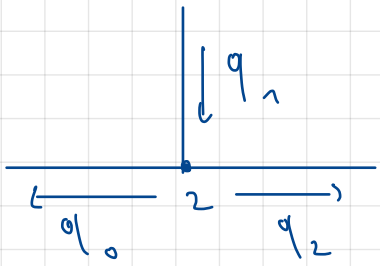
q_i : independent for $i = -Q_{4,47}$

q_i	q_L	q_R	q
0	1	0	0
1	1	1	-10
2	0	1	0
3	0	1	5
4	0	1	10
5	0	1	5
6	0	1	0



$q_1 = q_6 + q_0 - 10$

$q_i = q_R - q_L - 10$



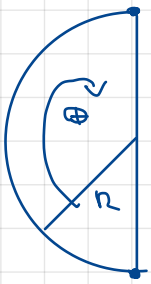
$q_2 + q_0 - q_1 = -(-10)$

$q_R + q_L - q_i - q_R + 10 = 10$

$$\theta_i' = \frac{1}{2S_i}\oint \frac{q_i}{Gt}\,dy = 0 \quad \text{because } F_z \text{ is applied at S (shear center)}$$

$$2Gt\,\theta_i' = \frac{1}{S_i}\oint q_i\,dy = 0$$

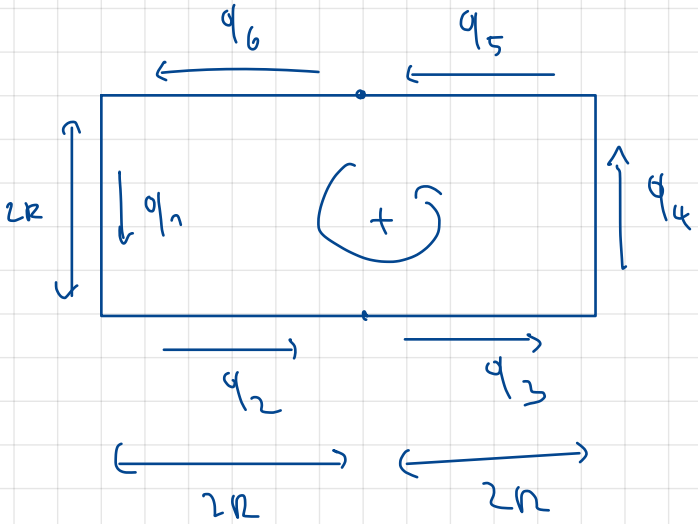
Left cell



$$\frac{1}{S_i}\oint q_i\,dy = 0 \qquad \frac{1}{\frac{\pi R^2}{2}}\left(2Rq_1 + \pi R q_0\right) = 0$$

$$2q_L + 2q_R - 2\alpha + \pi q_L = 0 \Rightarrow \boxed{(2+\pi)q_L + 2q_R = 2\alpha}$$

Right cell



$$\frac{1}{8R^2}\left(2R\left(q_2+q_3+q_5+q_6+q_1+q_4\right)\right) = 0$$

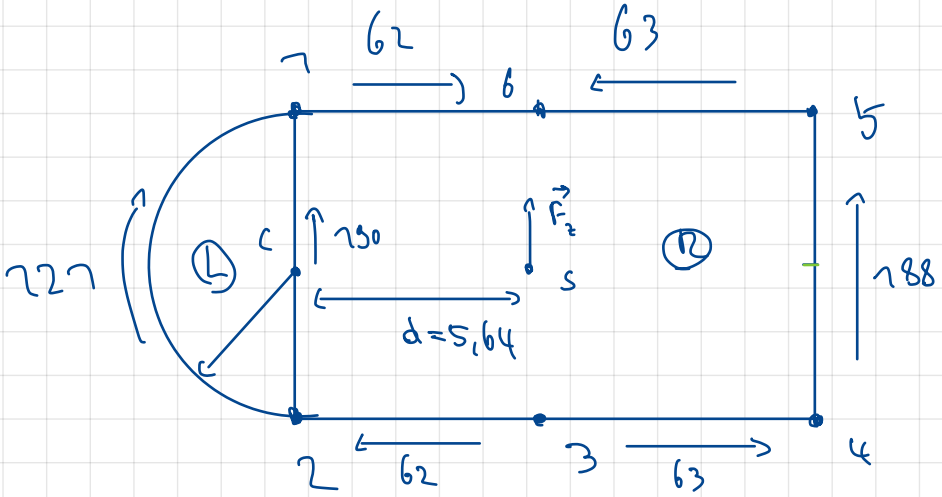
$$q_L + 6q_R + 7\alpha = 0$$

$$q_L + 6q_R = -7\alpha$$

$$\begin{cases} q_L + 6q_R = 0 \\ (2+\pi)q_L + 2q_R = 2\alpha \end{cases} \Leftrightarrow \begin{cases} q_L = 4{,}853\ \alpha/\text{in} \\ q_R = -2{,}475\ \alpha/\text{in} \end{cases}$$

q_i	q_L	q_R	q^i	q_i	q_i
0	1	0	0	4,853	727
1	1	1	-10	-7,662	-790
2	0	1	0	-2,475	-62
3	0	1	5	2,525	63
4	0	1	10	7,525	788
5	0	1	5	2,525	63
6	0	1	0	-2,475	-62

$F_z = 200\,lb$ $F_z = 5000\,lb$



$$M_c(F_z) = d F_z$$

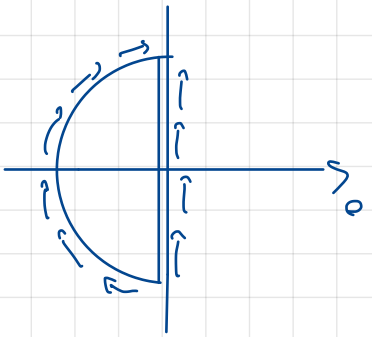
$$M_c(q) = -(727.\pi R).R - (62.2R)R + (63.2R).R + (788.2R).4R + (63.2R).R - (62.2R).R$$

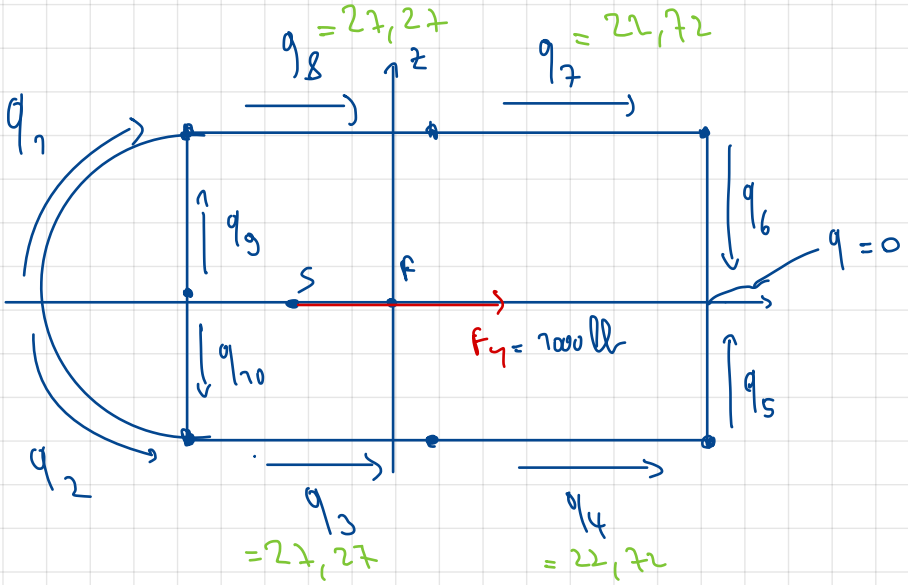
$$M_c(q) = -727.\pi R^2 - 248R^2 + 252R^2 + 1504R^2$$

$$d = \frac{1508R^2 - 727\pi R^2}{F_z} = \frac{1508.25 - 727.\pi.25}{5000} = 5,64\ \text{in (figura c)}$$

Horizontal: $62.2R - 63.2R - 62.2R + 63.2R = 0$

Vertical: $780.2R + 785.2R.727 = 7500 + 7850 + 7270 = 4950 \approx 5000$ lb



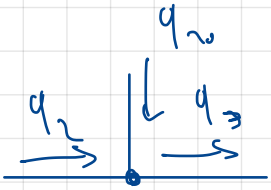


F_z $\quad q_5 = q_6 = 0$

F_y $\quad q_1 = q_2 = 0$

M_x $\quad q_9 = q_{10} = 0$

$50.2R.2 = 7000$ lb



$$q_3 - q_{10} - q_2 = -\frac{F_y}{I_{zz}} Q_{yz}$$

($q_{10} = 0$, $q_2 = 0$)

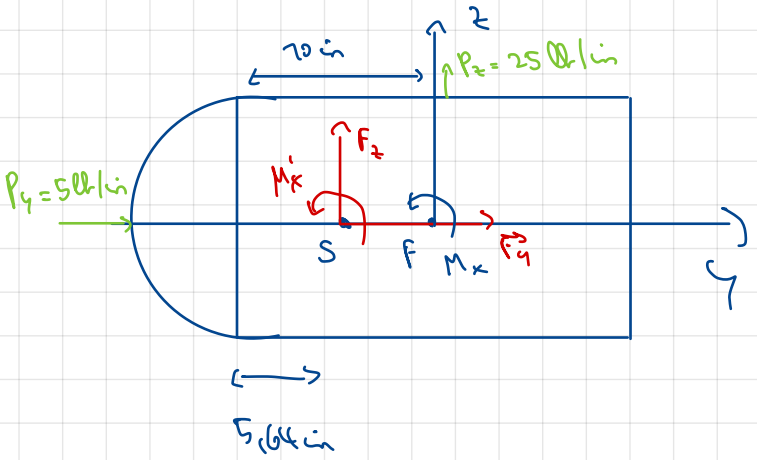
$$q_8 = q_3 = -\frac{7000}{550} \cdot (-15) = 27,27 \text{ lb/in}$$

$$q_4 = q_5 - \frac{F_y}{I_{zz}} Q_{yz}$$

$$q_7 = q_4 = 27,27 - \frac{7000}{550} \cdot 2,5 = 22,72 \text{ lb/in}$$

	$A_i y_i$
1	-15
2	-15
3	2,5
4	-12,5
5	-12,5
6	2,5

$M_x = 7500$ lb in



$M_x = p_y(70 - 5,64)$

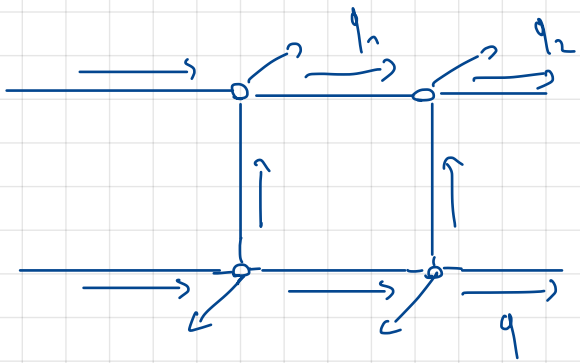
$\frac{dM_x}{dx} = -m_x = -4,36.25 = -108$

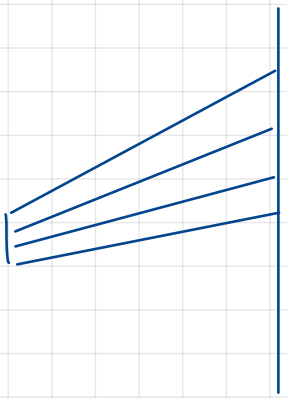
$M_x = -108x + C$

at $x = 200$ in, $M_x = 0 \Rightarrow C = 27600$

$M_x = 27600$ lb.in at rest ($x = 0$)

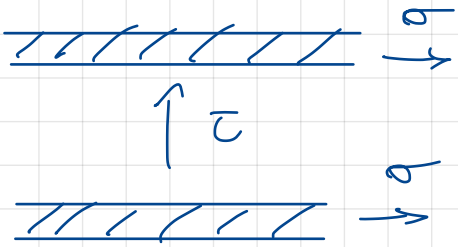
tapered beam

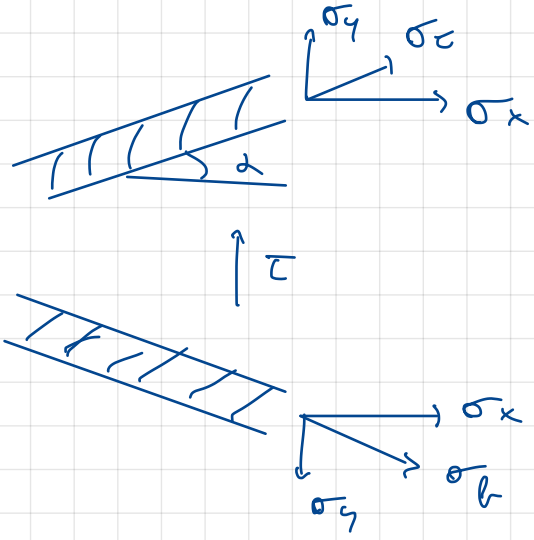




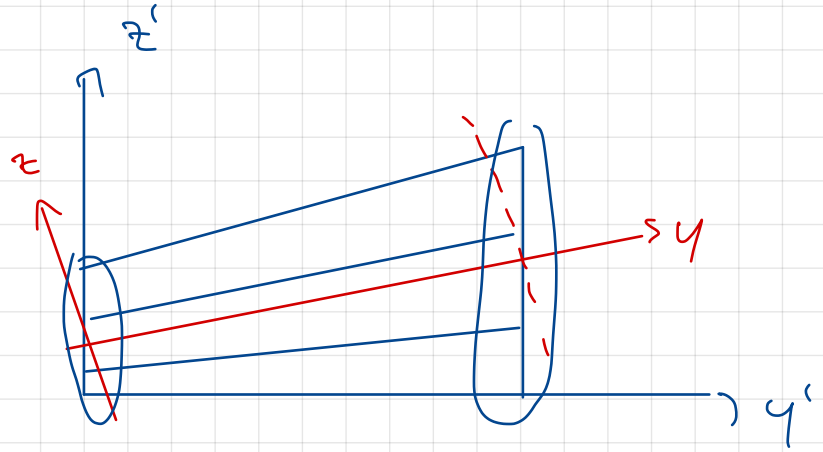
non tapered beam

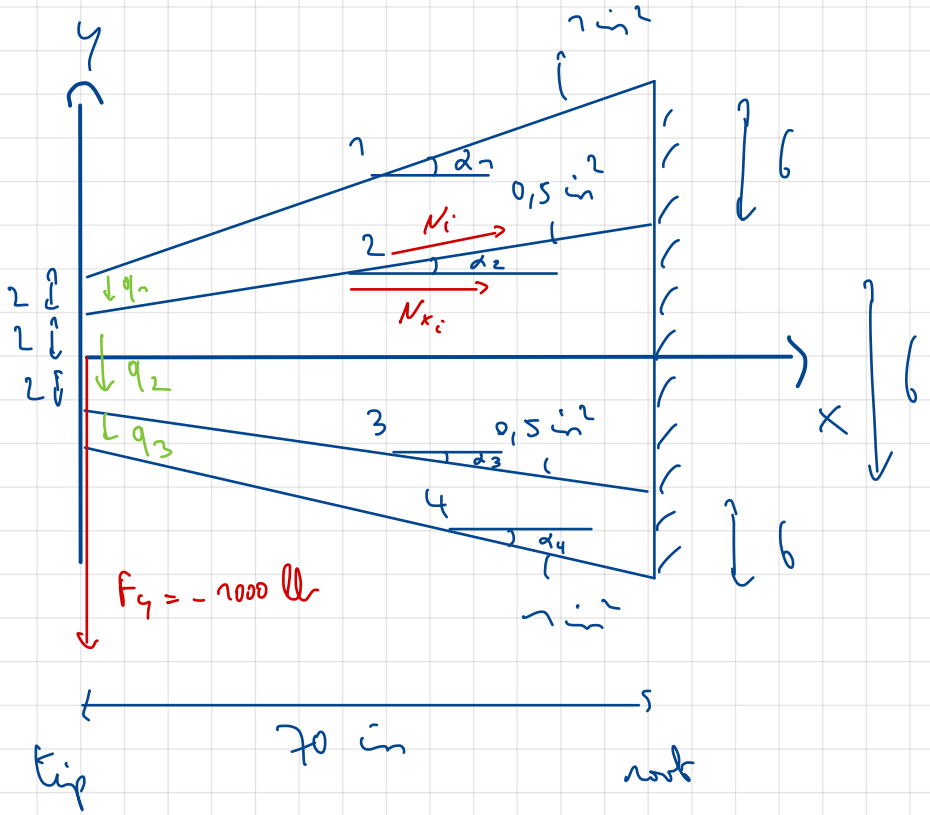
tapered beam



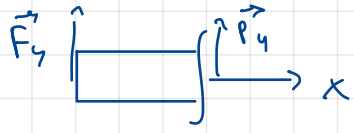


Tapered beam exercise (slide)





$P_y = -1000$ at $x = 0$



$F_y + P_y = 0 \Rightarrow F_y = -P_y$

$F_y = C = +1000$ lb

$$\frac{dM_y}{dx} = -M_y - F_y = 1000 \qquad x=0,\ M_y=0$$

$$M_y = -1000x + C_1 \Rightarrow C_1 = 0$$

$$M_y = -1000x$$

Tip

	A_i	y_i	$A_i y_i$	$A_i y_i^2$
1	1	3	3	9
2	0,5	1	0,5	0,5
3	0,5	1	0,5	0,5
4	1	3	3	9
	3			19

Root

	A_i	y_i	$A_i y_i$	$A_i y_i^2$
1	1	9	9	81
2	0,5	3	1,5	4,5
3	0,5	3	1,5	4,5
4	1	9	9	81
	3			171

tip

$M_z = 0$ lb.in

$F_y = 1000$ lb

$q = q_0 - \frac{F_y}{I_{zz}} Q_{yz}$

q_0 ↓

q_1 ↑

$$q_1 = 0 - \frac{1000}{19} \cdot 3 = -157,9 \text{ lb/in}$$

$$q_2 = q_1 - \frac{1000}{19} \cdot 0,5 = -184,8 \text{ lb/in}$$

$$q_3 = q_2 - \frac{1000}{19} \cdot (-0,5) = -157,9 \text{ lb/in}$$

Root

$M_y = -70\,000$ lb in

$F_y = 7000$ lb

$$\sigma = -\frac{M_z}{I_{zz}} y$$

$$\sigma_1 = +\frac{70\,000}{777} \cdot 9$$

	σ_i	$N_{x_i} = \sigma_i A_i$	α_i [°]	N_i	N_{y_i}
1	3684	3684	4,9	3697	315,8
2	7224	674	7,6	674,2	77,7
3	-7228	-674	-7,6	-674,2	77,7
4	-3684	-3684	-4,9	-3697	315,8
					665,8

- $N_{x_i} = \sigma_i A_i = \int_A \sigma_{xx}\, dA$

$N_i = \frac{N_{x_i}}{\cos \alpha_i}$

$N_{y_i} = \sin(\alpha_i) N_i$

- $\alpha_1 = \text{Bytan}\left(\frac{9-3}{70}\right)$

$\alpha_2 = \text{Bytan}\left(\frac{3-1}{70}\right)$

$F_y = T_w + \sum N_{y_i}$

$T_w = F_y - \sum N_{y_i}$

$T_w = 7000 - 665,8$

$= 334,2$ lb

$$q_3 = q_1 = -\frac{334,2}{777} \cdot 9 = -17,58 \text{ lb/in}$$

$$q_2 = q_1 - \frac{334,2}{777} \cdot 7,5 = -26,57 \text{ lb/in} \quad \text{Root}$$