Aircraft Structures

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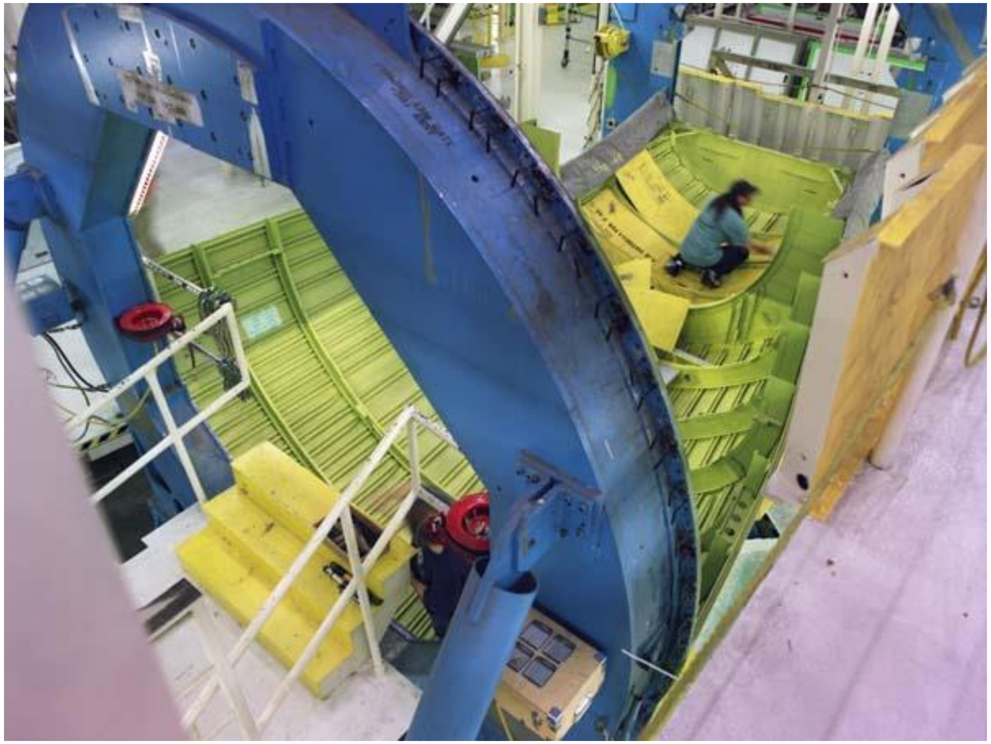
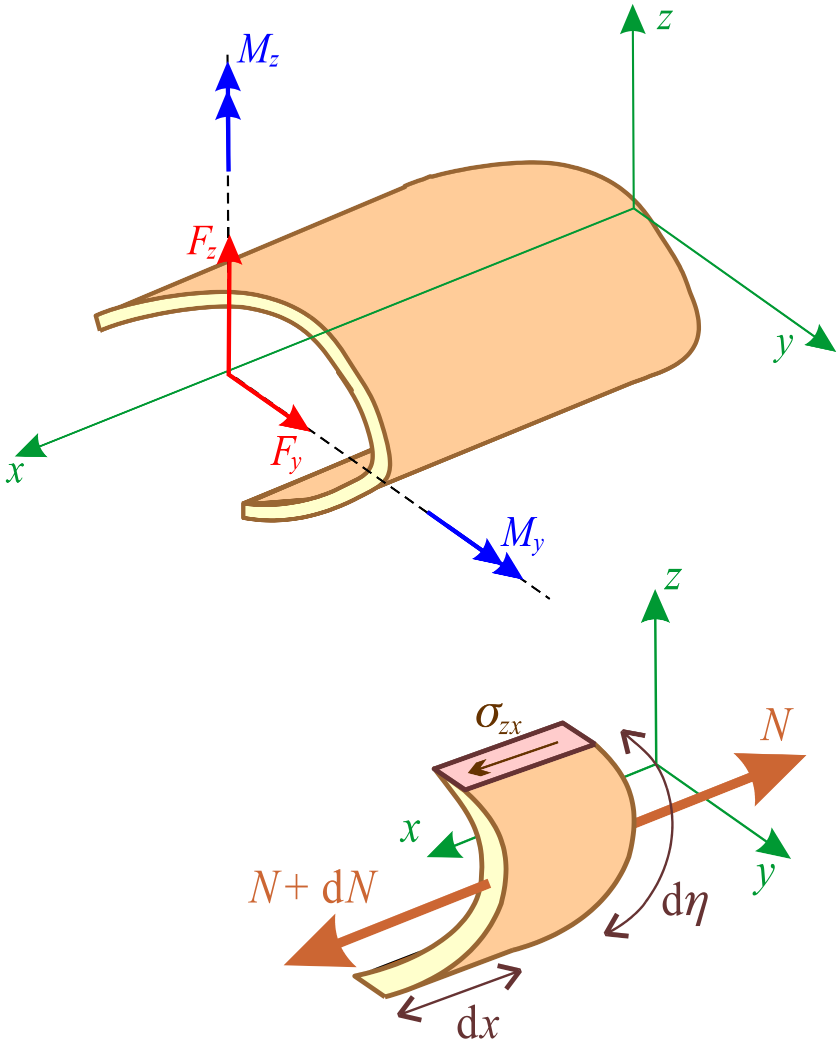
Shear loads on thin-walled open beams



Build-up and major assembly begins on the tail cone Section 48 of the B767-400.

Equilibrium



$$N + dN + \sigma_{zx}\, t\, dx = N$$

$$q = -\frac{\partial N}{\partial x}$$

$$q(\eta) = -\int_{\eta}^{\eta+d\eta} \frac{\partial \sigma_{xx}}{\partial x} t(\eta')d\eta'$$

$$\frac{dq}{d\eta} = -t(\eta)\frac{\partial \sigma_{xx}}{\partial x}$$

Shear flow computation 1

Recall bending theory

$$\sigma_{xx} = \frac{E}{E_0}\frac{F_x^*}{A^*} - \frac{E}{E_0}\frac{M_y^* I_{yz}^* + M_z^* I_{yy}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} y + \frac{E}{E_0}\frac{M_z^* I_{yz}^* + M_y^* I_{zz}^*}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} z - E\alpha\Delta T$$

$$\frac{\partial \sigma_{xx}}{\partial x} = \frac{E}{E_0 A^*}\frac{dF_x^T}{dx} - \frac{E}{E_0}\frac{y}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}}\left(I_{yz}^*\frac{dM_y^*}{dx} + I_{yy}^*\frac{dM_z^*}{dx}\right)$$
$$+ \frac{E}{E_0}\frac{z}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}}\left(I_{yz}^*\frac{dM_z^*}{dx} + I_{zz}^*\frac{dM_y^*}{dx}\right) - E\alpha\frac{d(\Delta T)}{dx}$$

Axial force derivative: combined axial distributed load

$$p_x^T = \frac{dF_x^T}{dx} = \int_A E\alpha\frac{d(\Delta T)}{dx}dA$$

(no distributed axial load)

Shear flow computation 2

Moment derivatives

$$M_y^* = M_y + M_y^T = M_y + \int_A z\,E\,\alpha\,\Delta T\,dA$$

$$\frac{dM_y^*}{dx} = -m_y(x) + F_z + \int_A z\,E\,\alpha\frac{d(\Delta T)}{dx}dA$$

Distributed thermal bending moments

$$m_y^T = -\int_A z\,E\,\alpha\frac{d(\Delta T)}{dx}dA\,, \qquad m_z^T = \int_A y\,E\,\alpha\frac{d(\Delta T)}{dx}dA$$

Use combined distributed bending moments

$$\frac{dM_y^*}{dx} = -m_y(x) + F_z - m_y^T(x) = -m_y^*(x) + F_z$$

$$\frac{dM_z^*}{dx} = -m_z(x) - F_y - m_z^T(x) = -m_z^*(x) - F_y$$

Shear flow computation 3

Modulus-weighted swept cross-sectional properties

$$A_\eta^* = \int_A \frac{E}{E_0} dA_\eta$$

$$Q_{\eta,y}^* = \int_A \frac{E}{E_0} z\, dA_\eta$$

$$Q_{\eta,z}^* = \int_A \frac{E}{E_0} y\, dA_\eta$$

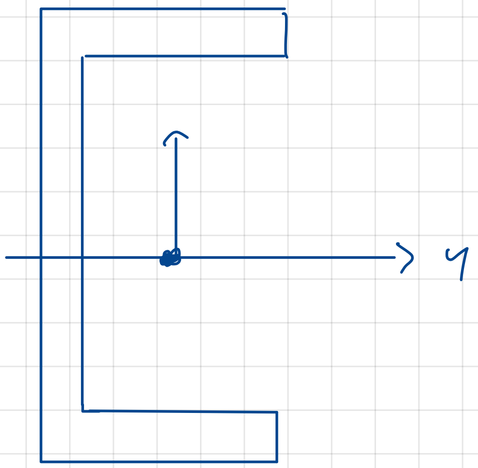
Final expression of the shear flow

$$q(\eta) = q_0 - p_x^T \frac{A_\eta^*}{A^*} + \int_{A_\eta} E\, \alpha \frac{d(\Delta T)}{dx} dA_\eta$$

$$+ \frac{I_{yz}^*\left(F_z^* - m_y^*\right) - I_{yy}^*\left(F_y^* + m_z^*\right)}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} Q_{\eta,z}^* + \frac{I_{yz}^*\left(F_y^* + m_z^*\right) - I_{yy}^*\left(F_z^* - m_y^*\right)}{I_{yy}^* I_{zz}^* - I_{yz}^{*2}} Q_{\eta,y}^*$$

Example

- Shear analysis of a C section
  - h = 10 cm
  - d = 5 cm
  - t = 5 mm
  - L = 1 m
  - E = 50 GPa
  - F_z = 10 000 N
  - Determine the shear flow and shear centre



in te vullen...

mm	A_i	y_i'	z_i'	$A_i y_i'$	$A_i z_i'$	$I_{\bar{y}\bar{y}}$	$I_{\bar{z}\bar{z}}$	$A_i y_i'^2$	$A_i z_i'^2$	$A_i z_i' y_i'$
1	237,5	26,25								
2	525	0								
3	237,5	-50								
Σ										

in te vullen ...

$$y_f' = \frac{12\,468,76}{1000} = 12,47 \text{ mm} \approx 12,5 \text{ mm}$$

$$I_{yy} = \frac{bt^3}{12} \qquad I_{zz} = \frac{b^3 t}{12}$$

$$z_f' = \frac{0}{1000} = 0 \text{ mm}$$

$$I_{y'y'} = 1\,610\,834 \text{ mm}^4$$

$$I_{z'z'} = 4\,17\,708 \text{ mm}^4 = \Sigma I_{\bar{z}\bar{z}} + A_i y_i'^2$$

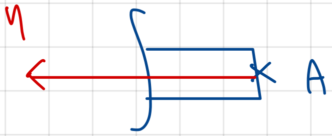
$$I_{yy} = I_{y'y'} - \Sigma A_i z_f' = I_{y'y'}$$

$$I_{zz} = I_{z'z'} - \ldots ? \ldots = 262\,207 \text{ mm}^4$$

$$q(\eta) = q_0 - \frac{(F_y + m_g)}{I_{zz}} Q_{\eta,z} - \frac{(F_z - m_g)}{I_{yy}} Q_{\eta,y}$$

$$\hookrightarrow q(\eta) = q_0 - \frac{F_z}{I_{yy}} Q_{\eta,y}$$

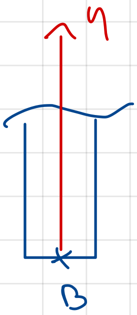
From A to B



$$Q_{\eta,y} = \int_{A_\eta} z\, dA_\eta = \int_0^\eta z\, \overset{t = cte}{t(\eta)}\, d\eta = z t \int_0^\eta d\eta$$

$$Q_{\eta,y} = z t \eta \qquad q_A = 0 \qquad q_B = \underset{0}{q_A} - \frac{\overset{10\,000}{F_z}}{\underset{1670834}{I_{yy}}} \cdot \overset{-50}{z} \cdot t \cdot 50 = 74{,}8\ \frac{N}{mm}$$

From B to C



$$Q_{\eta,y} = \int_{A_\eta} z\, dA_\eta$$

$$Q_{\eta,y} = \int_0^\eta z t\, d\eta' \qquad z = \eta + \overset{-50}{z_B}$$

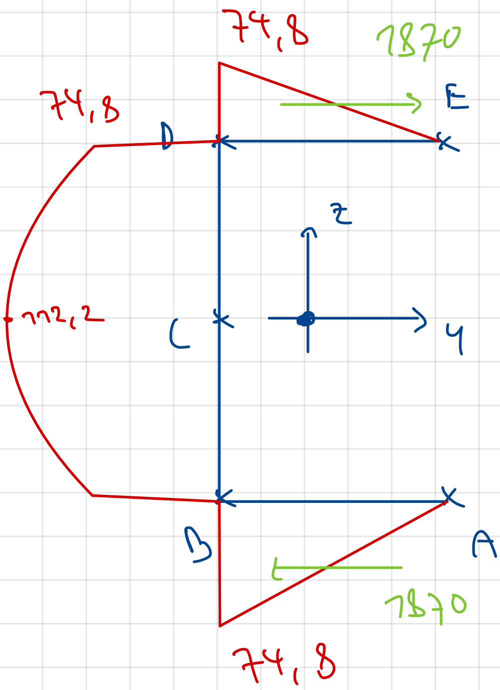
$$Q_{\eta,y} = \int_0^\eta (\eta' + z_B) t\, d\eta' = t\left(\frac{\eta^2}{2} + z_B \cdot \eta\right)$$

$$q(\eta) = q_B - \frac{F_z}{I_{yy}} Q_{\eta,y}$$

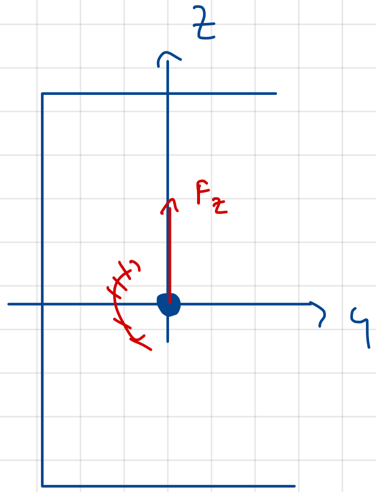
- $$q(C) = q_B - \frac{10\,000}{1\,670\,834} \cdot 5\left(\frac{50^2}{2} + 50 \cdot (-50)\right)$$

$$q(C) = 112{,}2\ N/mm$$

- $$q(D) = q_B - \frac{10\,000}{1\,670\,834} \cdot 5\left(\frac{100^2}{2} - 50 \cdot 100\right) = q_B$$

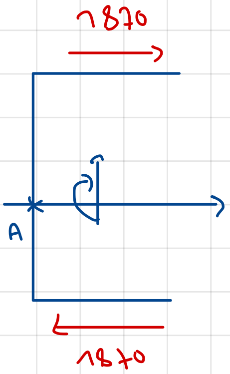


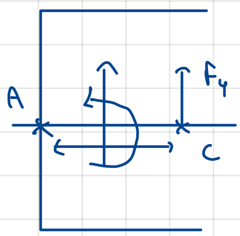
$q_A = 0 \text{ N/mm}$
$q_B = 74{,}8 \text{ N/mm}$
$q_C = 112{,}2 \text{ N/mm}$
$q_D = 74{,}8 \text{ N/mm}$
$q_E = 0 \text{ N/mm}$

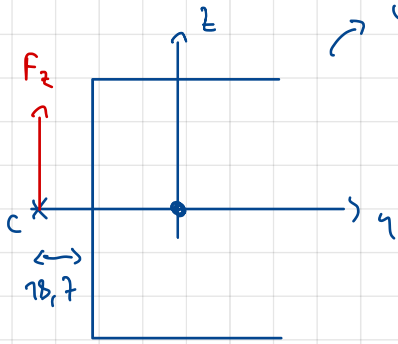


$$H = \int_A^B q(y)\, dy$$

$$V = \int_B^D q(y)\, dy$$





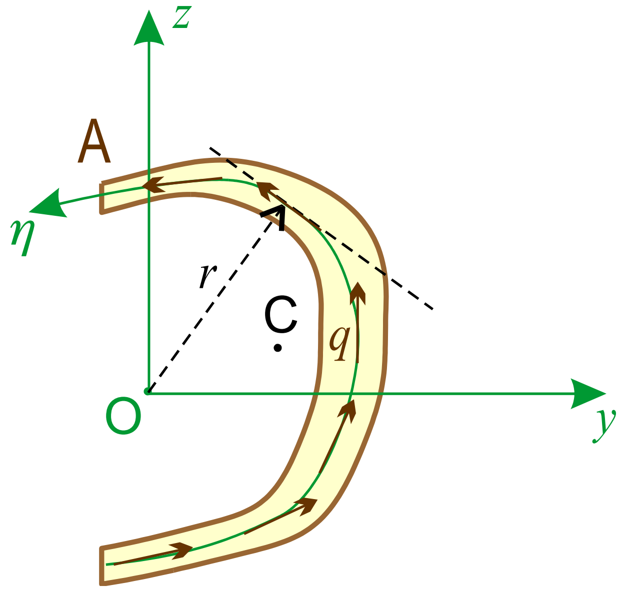


↳ in order to have no torsion!

↳ however, this force can't be physically applied because it's outside the object

$$(1870 \cdot 50) \cdot 2 = -dF_z \Rightarrow d = -\frac{1870 \cdot 50 \cdot 2}{10\,000} = -18{,}7 \text{ mm}$$

Shear centre



Moment equilibrium with the shear flow

$$M_{O,x} = \int_\eta q(\eta)\, r(\eta)\, d\eta$$

$$\overrightarrow{M_O} = \overrightarrow{M_C} + \overrightarrow{OC} \wedge \vec{F}$$

$$M_{O,x} = y_C F_z - z_C F_y$$

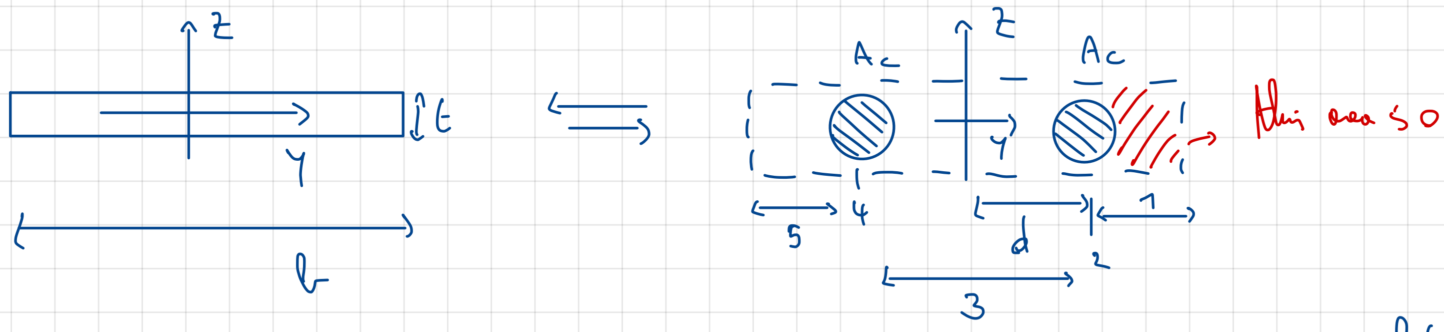
Using linearity (superposition principle)

$$z_C = -\frac{1}{F_y} \int_\eta q^y(\eta)\, r(\eta)\, d\eta$$

$$y_C = \frac{1}{F_z} \int_\eta q^z(\eta)\, r(\eta)\, d\eta$$

Structural idealization

- Purpose
  - Avoid computation of the swept section properties using integrals
- General idea
  - Bundle skin mass into effective lumped masses
  - The skin now has zero mass
  - Swept section properties increase in steps
- Two methods available
  - Rivello method: identical area and inertia
  - Megson method: identical normal stresses



$$A = bt$$

$$I_{zz} = \frac{tb^3}{12}$$

$$I_{yy} = \frac{bt^3}{12} << I_{zz}$$
$$\simeq 0$$

$$A = 2A_c \qquad 2A_c = bt \Rightarrow A_c = \frac{bt}{2}$$

$$I_{zz} = 2A_c \cdot d^2$$

$$\frac{tb^3}{12} = 2 \cdot A_c \cdot d^2$$

$$d^2 = \frac{t b^{\cancel{3}\,2}}{12} \cdot \frac{1}{bt} \Rightarrow d = \frac{b}{\sqrt{12}} = \frac{b}{2\sqrt{3}}$$

A_1, A_2 = Concentrated areas

$$A_1 =$$

$$A_2 =$$

same area

$$\begin{cases} A = bt = 2A_1 + A_2 \\ I_{zz} = \frac{tb^3}{12} = A_1 \cdot \left(\frac{b}{2}\right)^2 \cdot 2 \end{cases}$$

same inertia

$$\begin{cases} bt = 2A_1 + A_2 \rightarrow A_2 = bt - 2A_1 \\ \boxed{A_1 = \frac{bt}{6}} \end{cases} \qquad \boxed{A_2 = \frac{2}{3}bt}$$

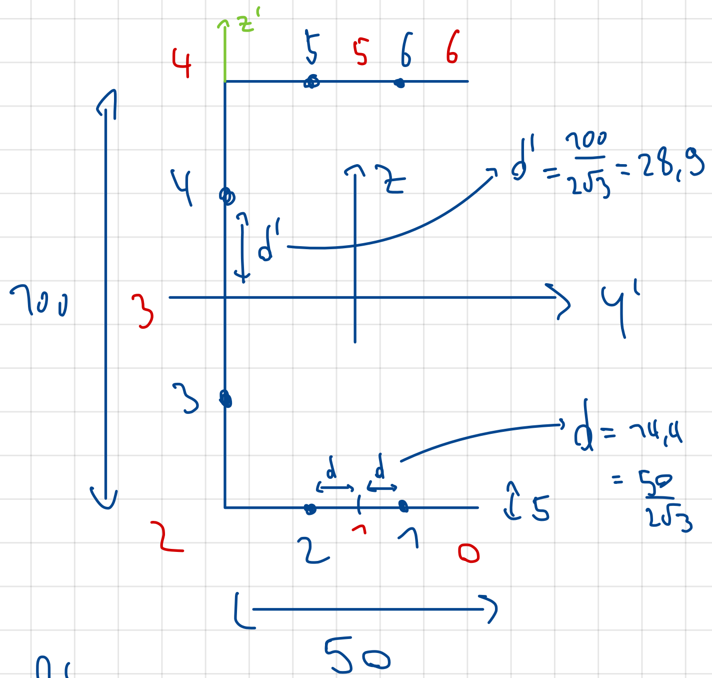
$$Q_{\eta,z} = \int_{A_\eta} y \, dA_\eta \Rightarrow (1) \Rightarrow Q_{\eta,y} = 0$$ Area equal to zero

$$(2) \Rightarrow Q_{\eta,y} = y_2 \cdot A_c$$

$$(5),(3) \Rightarrow Q_{\eta,y} = 0$$

$$Q_{\eta,y} = y_4 \cdot A_c$$

C - section



$= \frac{b_f}{2}$

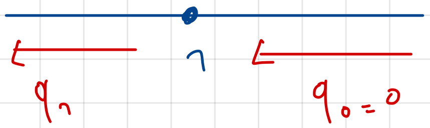
	A_i	y_i'	z_i'
1	125	39,4	-50
2	125	10,6	-50
3	250	0	-28,9
4	250	0	28,9
5	125	10,6	50
6	125	39,4	50

$A_i y_i'$ $A_i z_i'$ $A_i y_i'^2$ $A_i z_i'^2$ $A_i y_i' z_i'$

in te vullen ...

Σ 1000

$$y'_F = \frac{72500}{1000} = 72,5\ mm$$

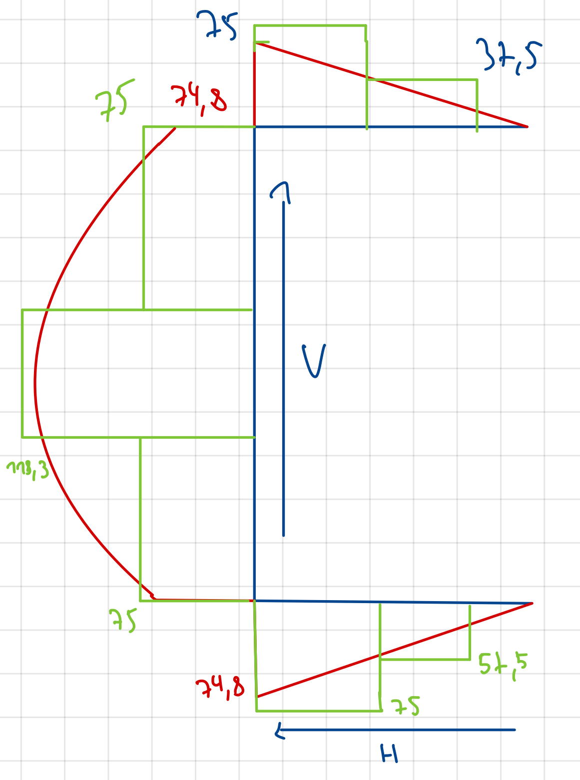


$$q_1 = q_0 = \frac{F_z}{I_{yy}}(A_1 z'_i) = q_1 = -\frac{10000}{1667605}(125 \cdot (-50))$$

$$q_2 = q_1 - \frac{F_y}{I_{yy}}(\underbrace{A_2}_{125} \cdot \underbrace{z'_2}_{-50}) = 74,96\ N/mm$$

$$q_3 = q_2 - \frac{F_z}{I_{yy}}(\underbrace{A_3}_{250}\ \underbrace{z'_3}_{-28,9}) = 118,3\ N/mm$$

$$q_4 = q_3 - \frac{F_z}{I_{yy}}(\underbrace{A_4}_{250} \cdot \underbrace{z'_4}_{28,9}) = 74,96\ N/mm = q_2$$



$$28,9 \cdot 37,5 + 10,6 \cdot 75 = 1878,75$$

$\Downarrow$

1870 with previous method

Example

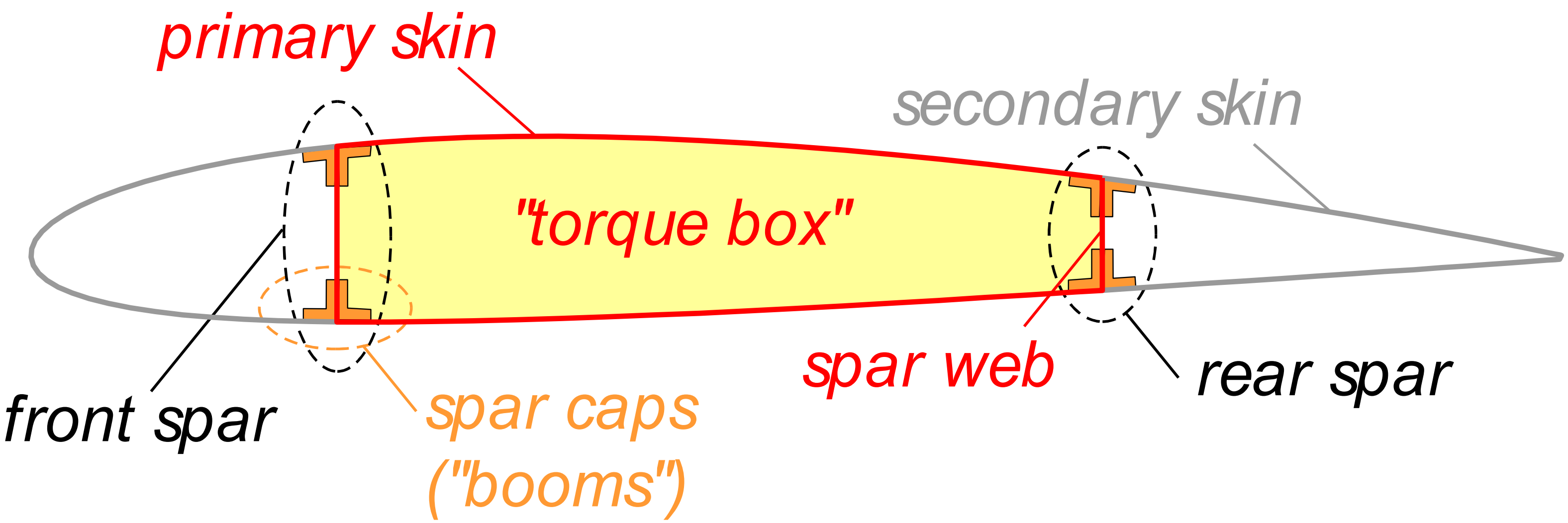
- Perform the C-section analysis using the structural idealization method

Analysis of aircraft wings

The first 787 Dreamliner demonstration wing box is test-ready at Boeing's developmental center in Seattle, Wash. The box represents two thirds of the all-new airplane's wing span and is full-scale in size. It will be used to demonstrate the structural integrity of the design (July 2006).

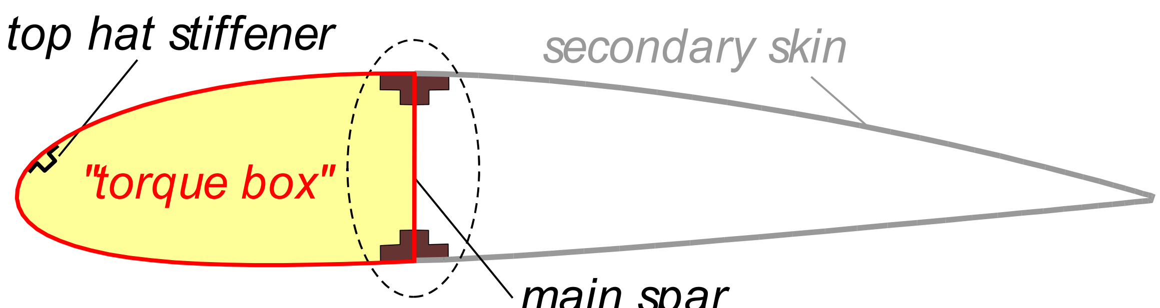
Wing structure

- Semi-monocoque wing basics
  - At least two spars with massive caps: resists bending
  - Torque box with primary skin and webs: resists shear and torsion

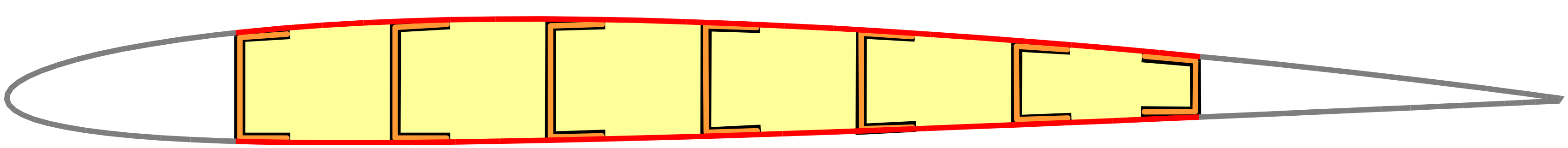


Wing structure

- Concentrated flange wing (monocoque)
  - Single spar mass boom structure (D-nose)

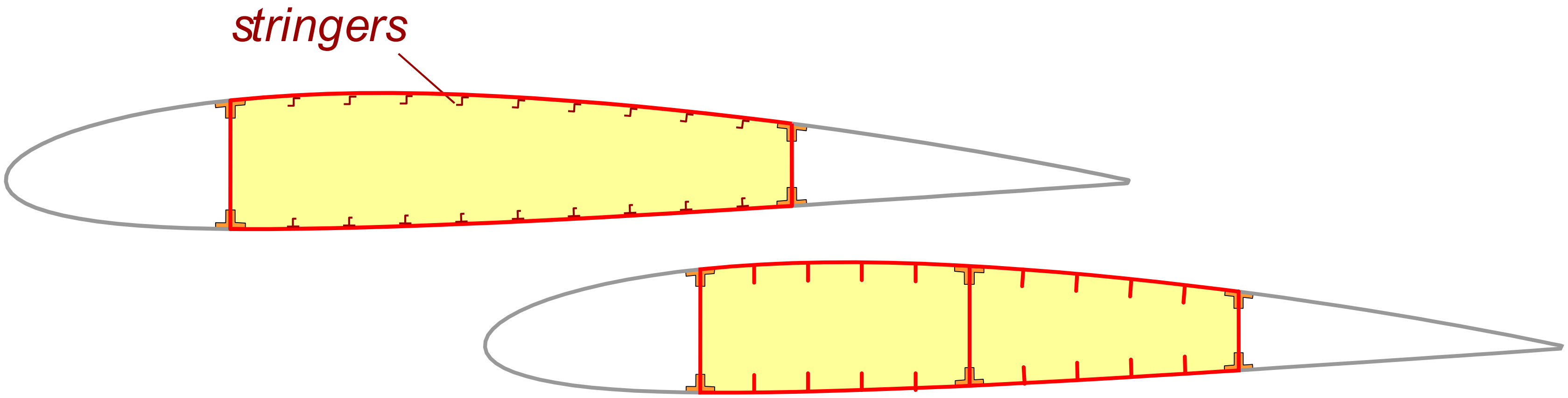


  - Two-spar structure
  - Multispar structure



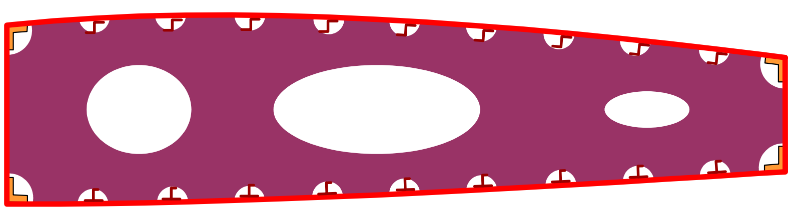
Wing structure

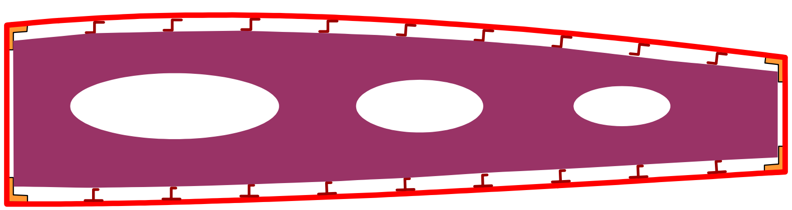
- Distributed flange wing (semi-monocoque)
  - Two-spar or three-spar designs
  - Two skin manufacturing techniques
    - Assembled stringers (welding, riveting)
    - Integrally-machined panels (skin + stringers)



Wing ribs

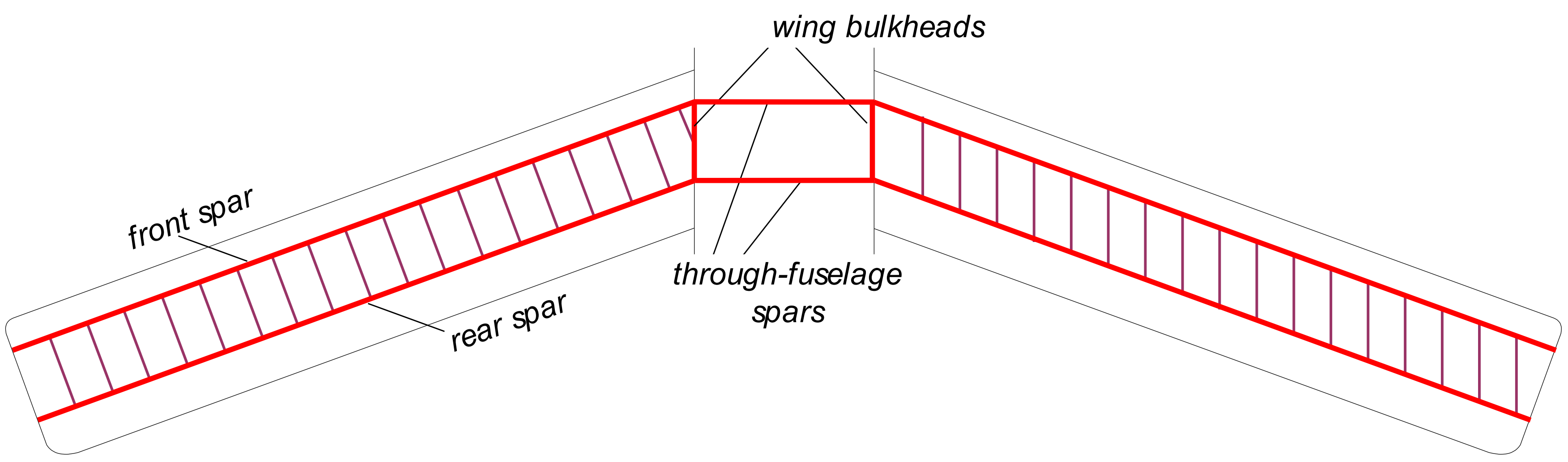
- Roles
  - Provide support for aerodynamic shape
  - Provide entry points for concentrated loads
  - Reduce equivalent column height (buckling)
  - Spaced 24 in (tpt) to 36 in (light) [Roskam]
- Structural integration
  - Holed (piping, access, maintenance)
  - Must not intersect with stringers (holes or clips)





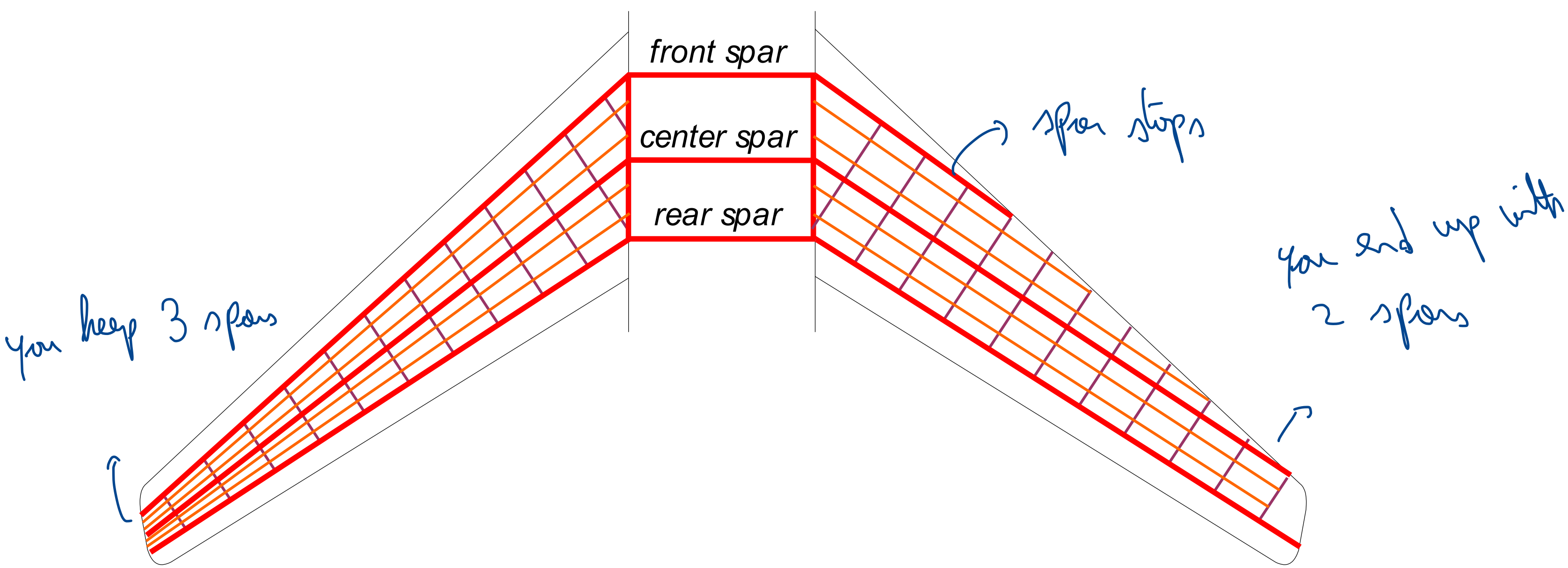
Wing planforms

- Untapered wing (low sweep)
  - Conventional rib layout: easier, lighter
  - Flow-aligned rib layout: more rigid, heavier



Wing planforms

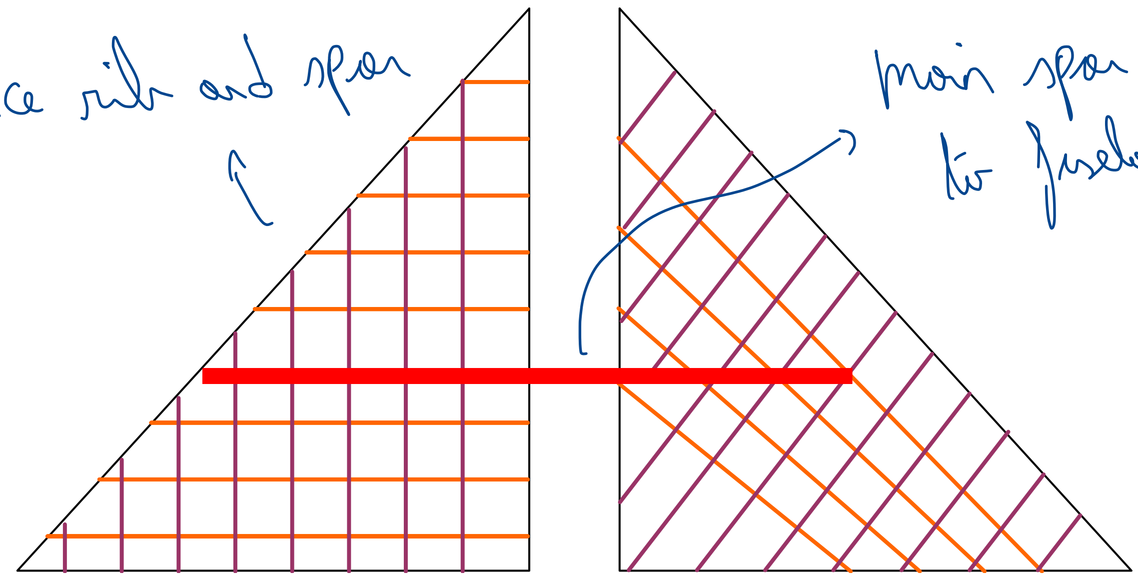
- Tapered wing (high sweep)
  - Converging spars: more difficult to fabricate
  - Parallel spars (*run-out*): easier, but can cause stress concentrations (changes in load paths)



Wing planforms

- Delta wing
  - High flight loads require dense structure
  - Typically multispar: « egg-box » structure
  - Difficult inspection and maintenance

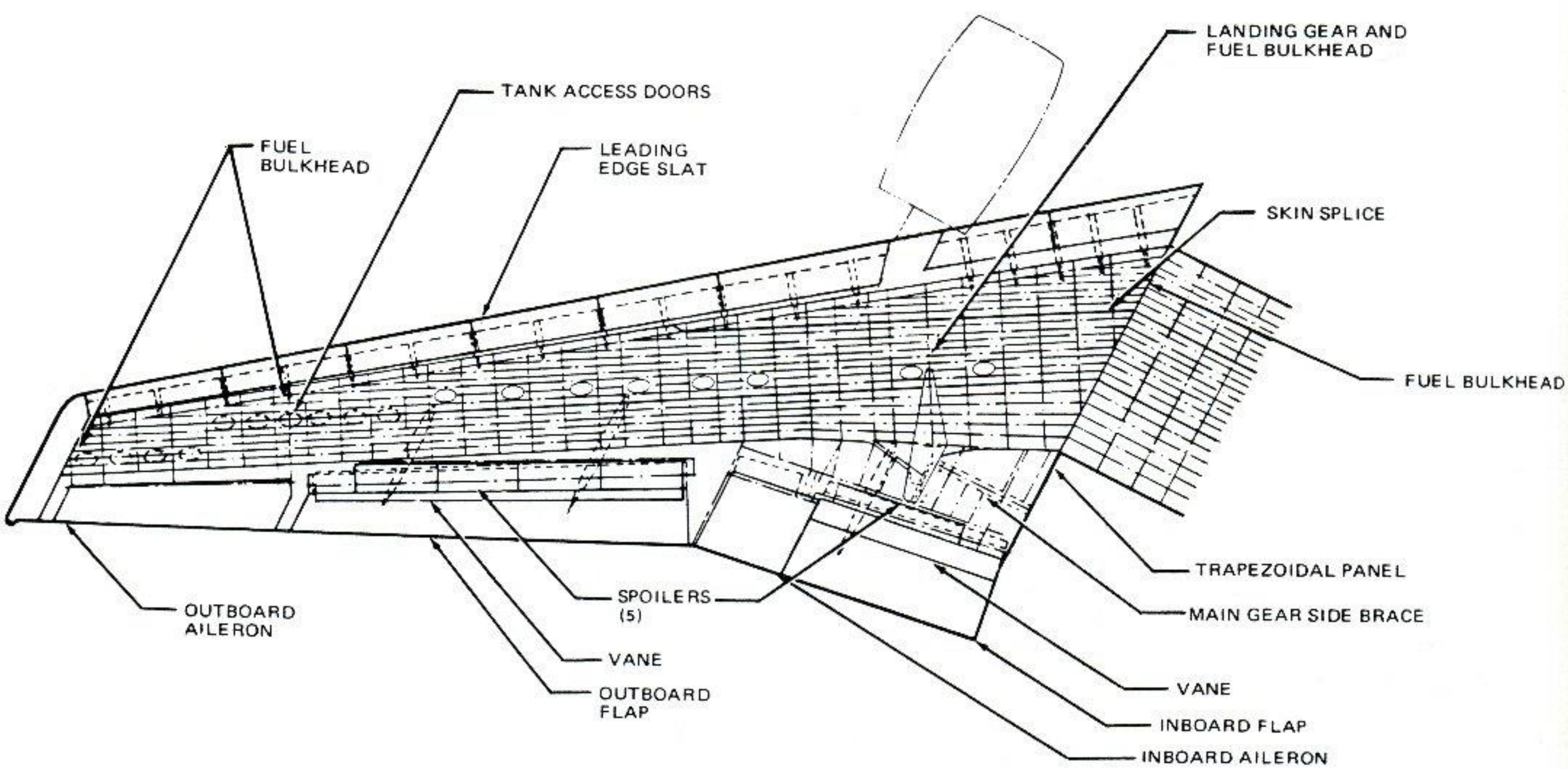
no difference rib and spar



main spar fixing wing to fuselage

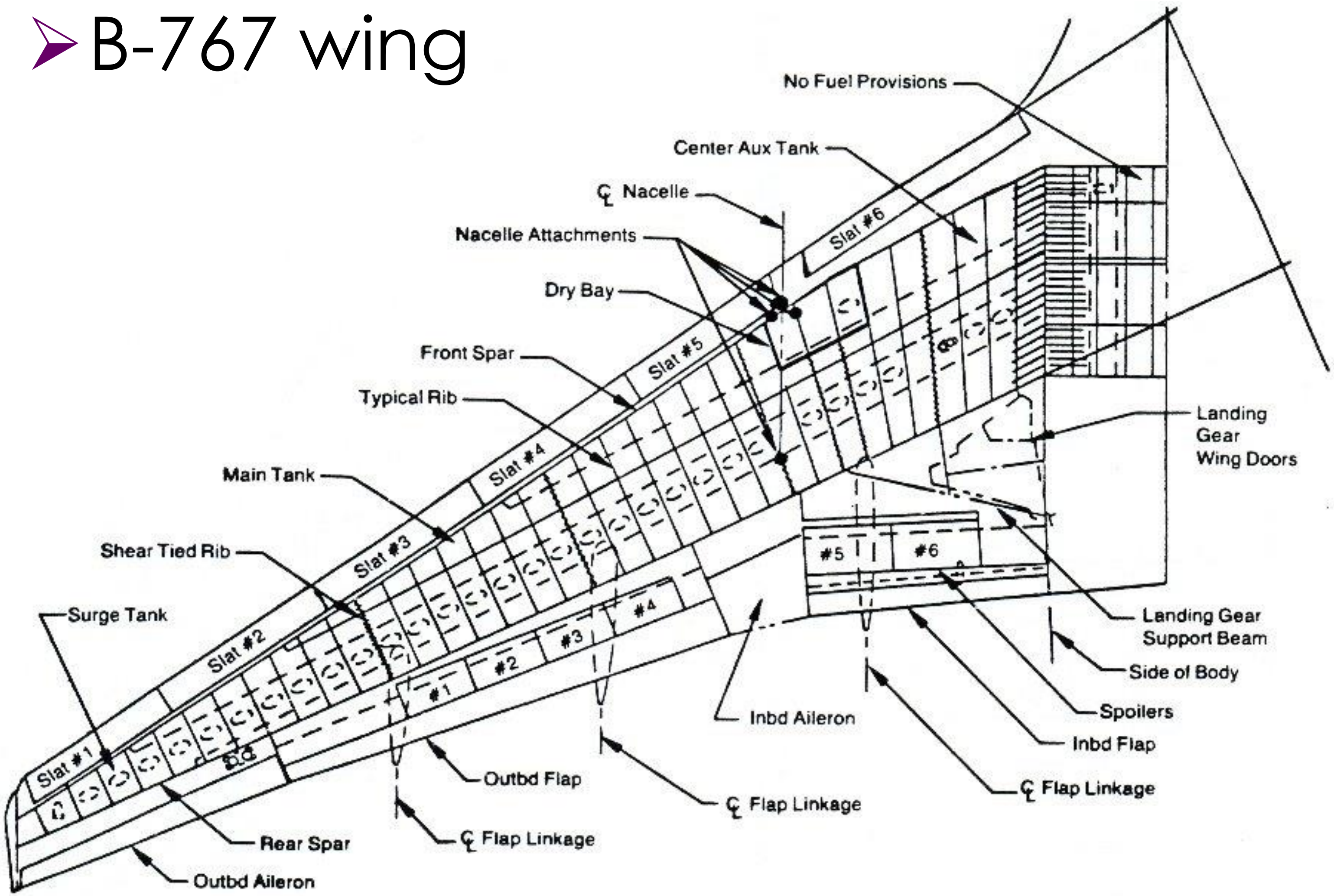
Wing examples

- DC-10 wing



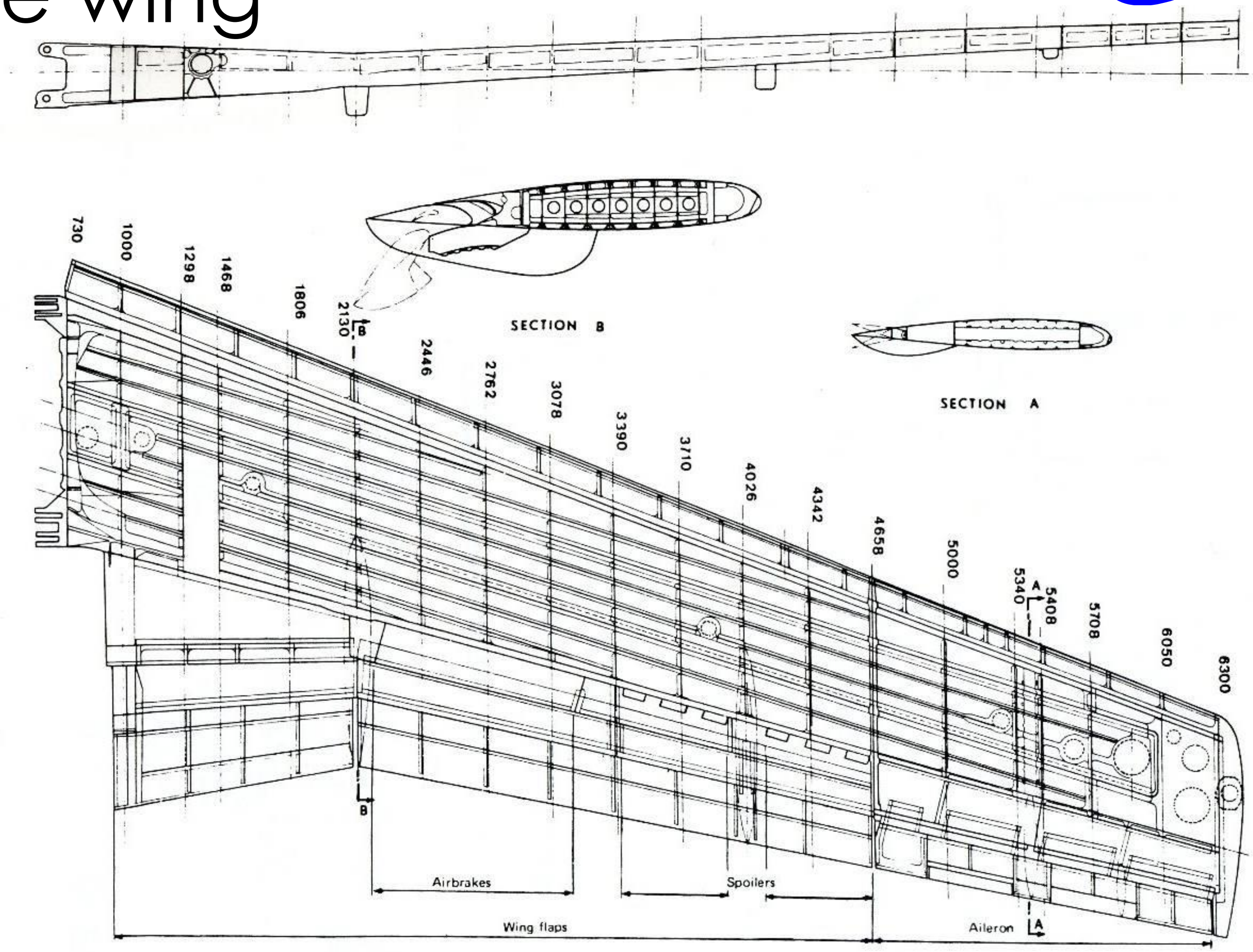
Wing examples

- B-767 wing



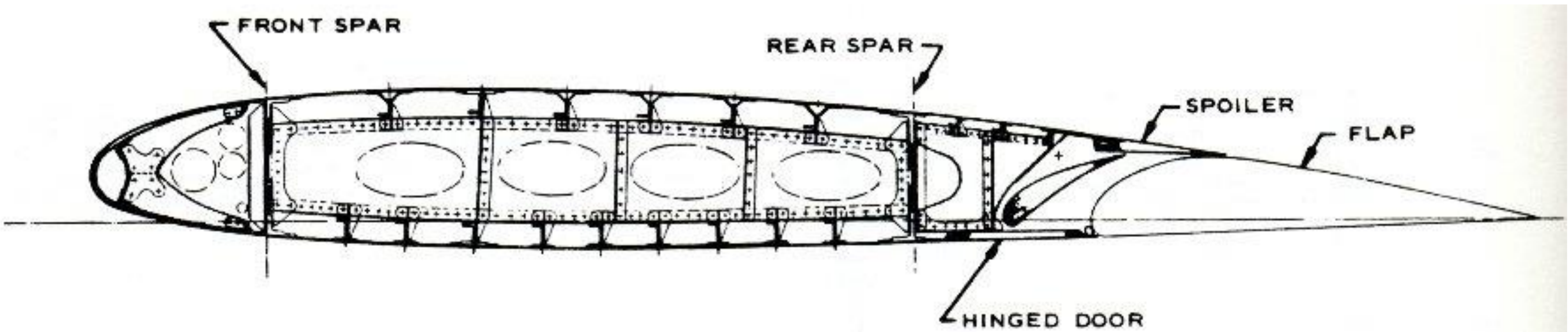
Wing examples

- Corvette wing



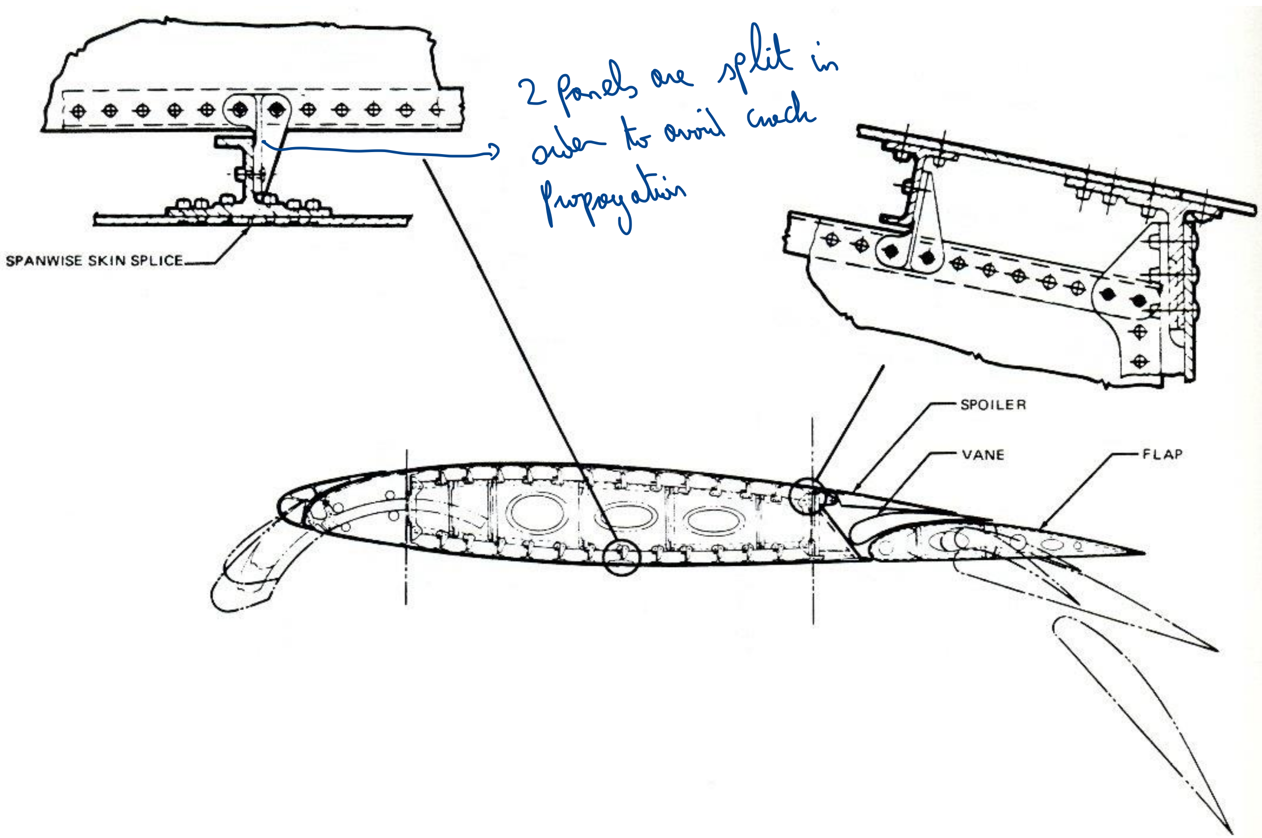
Wing examples

- DC9 wing section
  - Assembled ribs



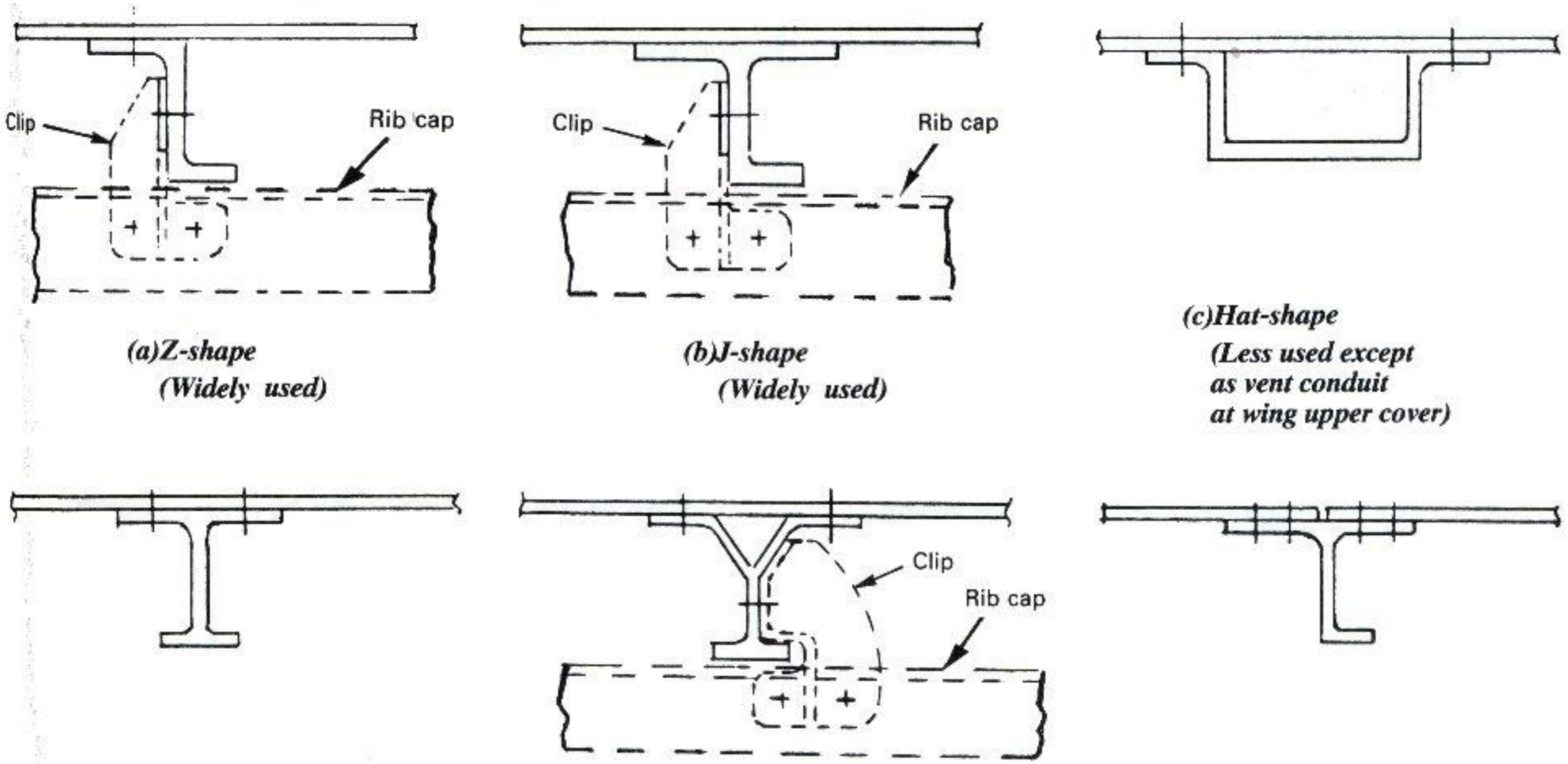
Wing examples

- DC10 wing section
  - Assembled ribs



Wing examples

- Stringers design
  - Assembled ribs



(a)Z-shape (Widely used)

(b)J-shape (Widely used)

(c)Hat-shape (Less used except as vent conduit at wing upper cover)

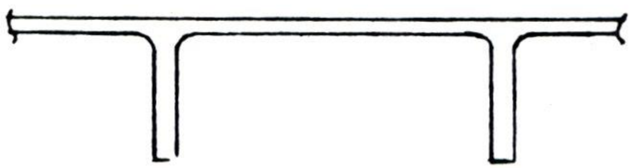
(d)I-shape (Less used)

(e)Y-shape (Less used)

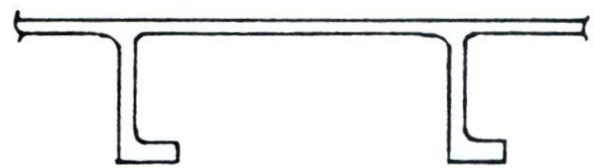
(f)J-shape for panel splice

Wing examples

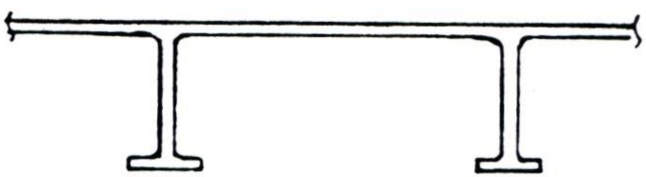
- Stringers design
  - Integrally-machined panels



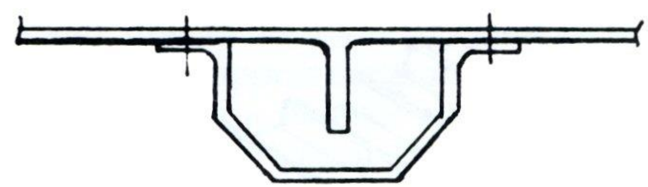
(a)Integral blade section
(Widely used)



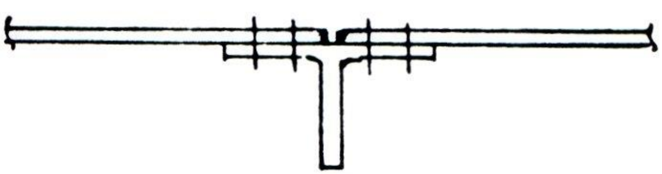
(b)Integral Z-section



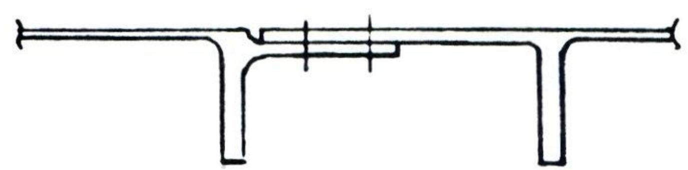
(c)Integral T-section



(d)Blade section with reinforcement



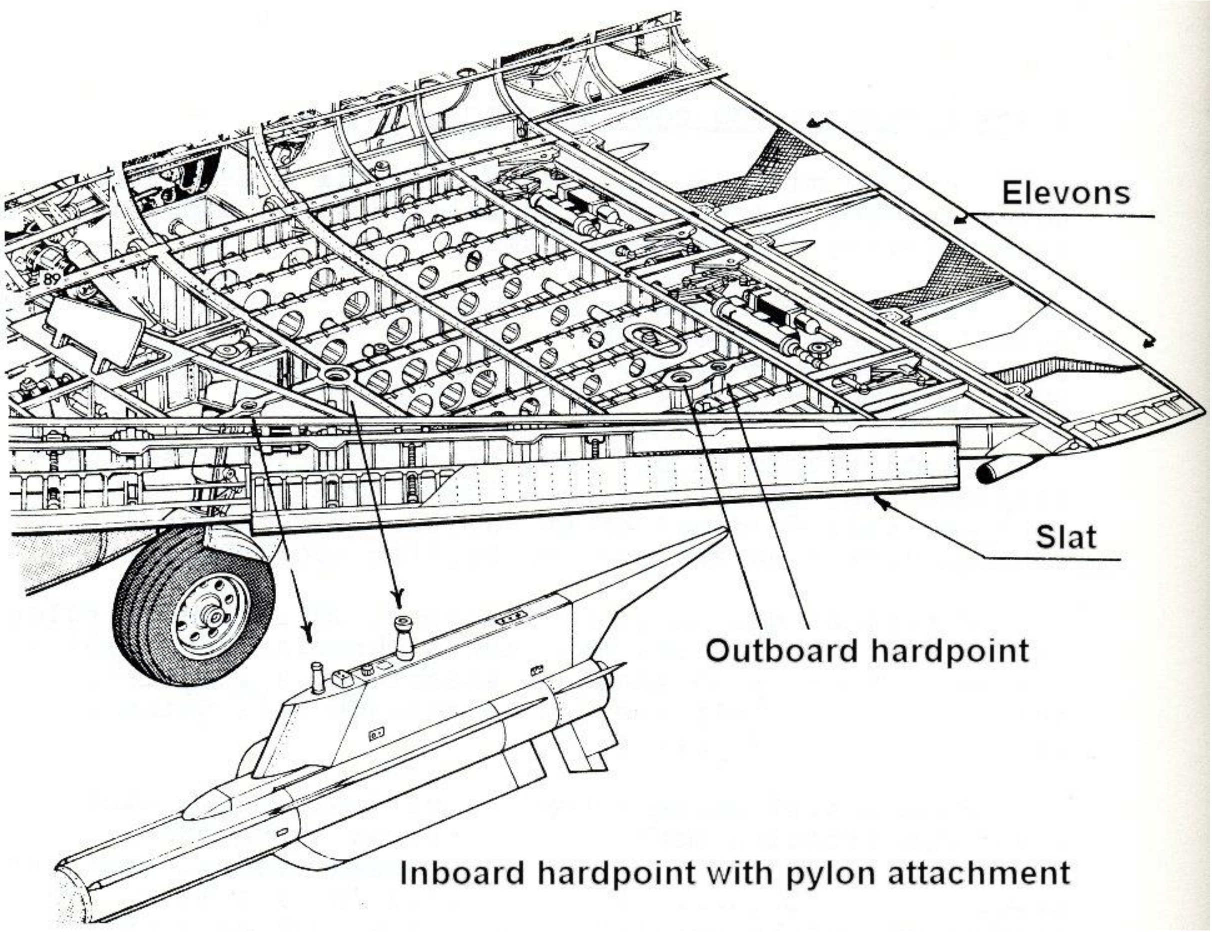
(e)Splice configuration



(f)Splice configuration (avoid)

Wing examples

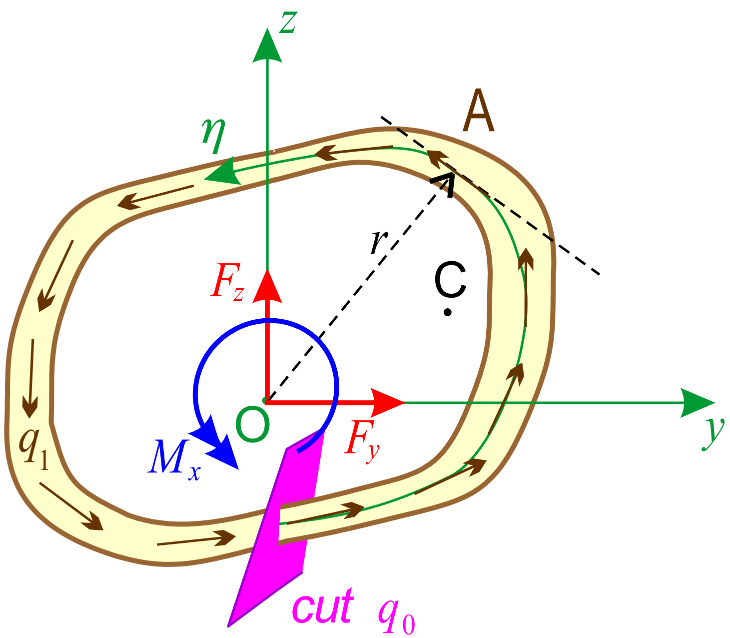
- Structural hardpoint



Shear loads on thin-walled closed beams

767-400ER Wings fly overhead, on their way to be joined with the airplane.

The single-cell beam



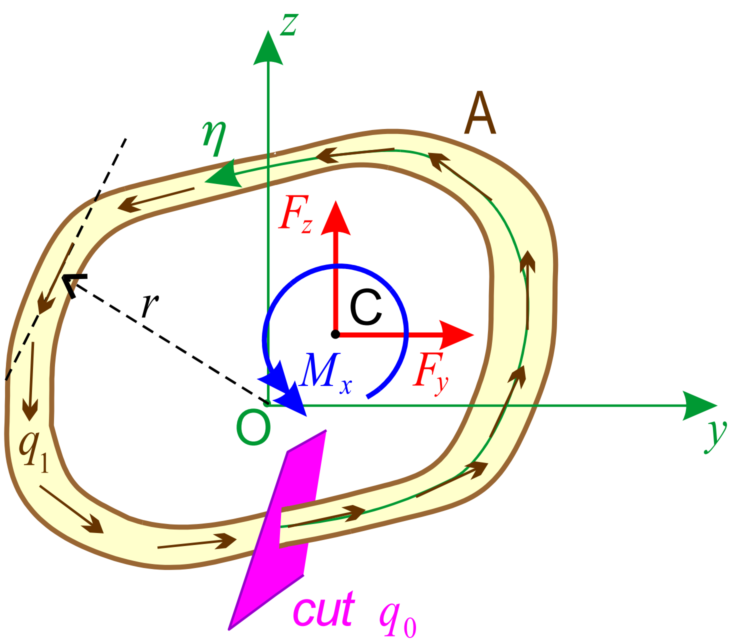
- Direct method
  - cut to open beam
  - impose constant q_0
  - solve for q_1
  - solve for constant q_0
  - no determination of SC

Satisfies force equilibrium $\quad q = q_0 + q_1$

Moment equilibrium $\quad M_x = \oint_C [q_0 + q_1(\eta)]\, r(\eta)\, d\eta$

(Bredt's first formula) $\quad q_0 = \frac{1}{2S}\left(M_x - \oint_C \; q_1(\eta)\, r(\eta)\, d\eta\right)$

The single-cell beam



- Shear centre method
  - cut to open beam
  - split in three parts:
    - q^t due to torsion (M_x) only
    - q^y due to y-shear(F_y) only
    - q^z due to z-shear (F_z) only
  - solve for q^y and q^z (no twist)
  - solve for shear centre
  - solve for torsion flux

The single-cell beam

Split in torsion and shear flows $q = q^t + q^y + q^z$

Open section shear q_1 as before

Unknown "cut" shear q_0 from no twist $\frac{1}{2\,S}\oint_C \frac{q_0^y + q_1^y(\eta)}{G\,t} d\eta = 0$

Centre of shear location $M_x + y_C F_z - z_C F_y = \oint_C [q^t + q^y(\eta) + q^z(\eta)]\, r(\eta)\, d\eta$

$$z_C = -\frac{1}{F_y}\oint_C q^y(\eta) r(\eta)\, d\eta$$

$$y_C = -\frac{1}{F_z}\oint_C q^z(\eta) r(\eta)\, d\eta$$

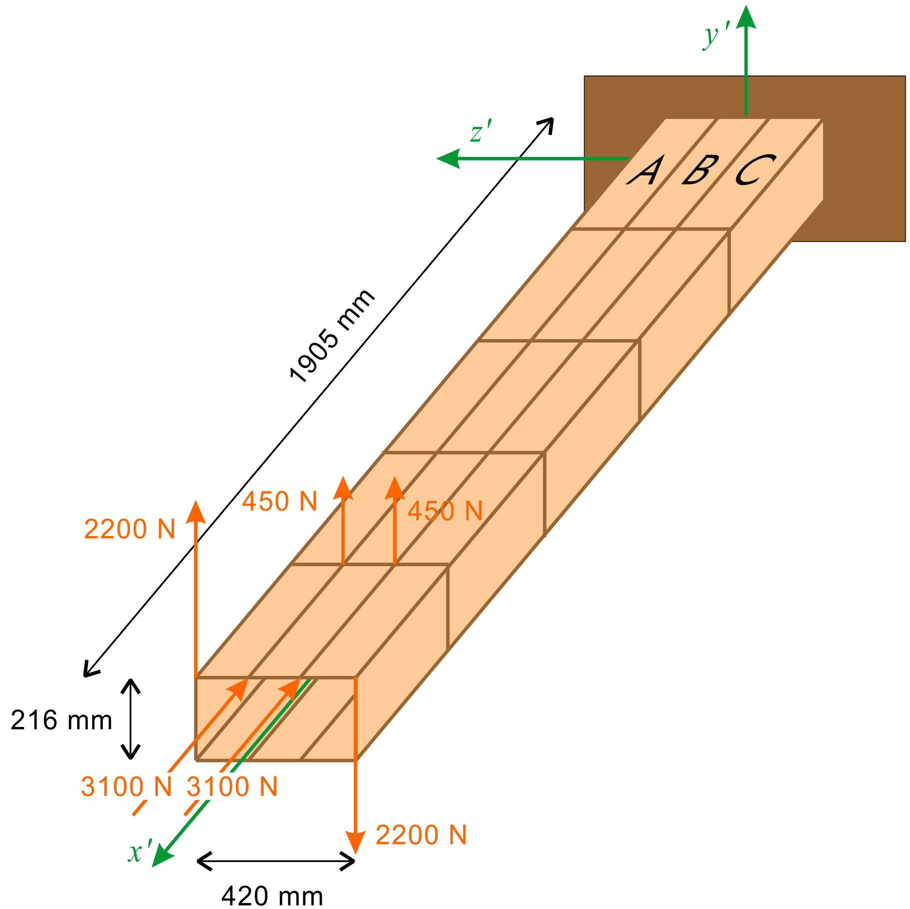
Torsion computation about the centre of shear (F_i cause no torsion)

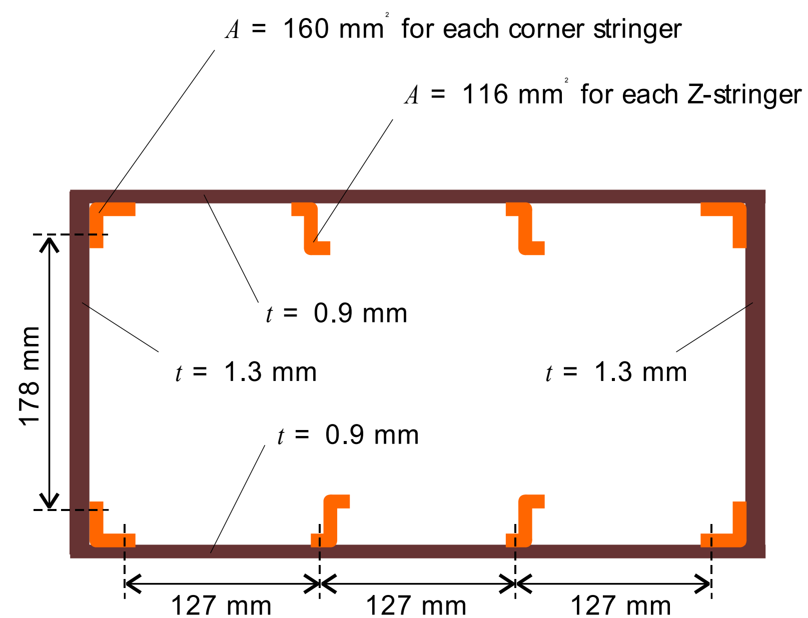
$$q^t = \frac{M_x}{2\,S}$$

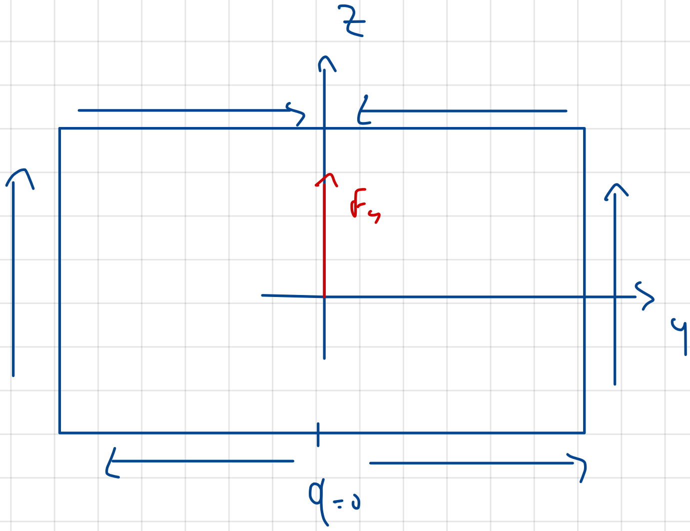
Exercise (home)

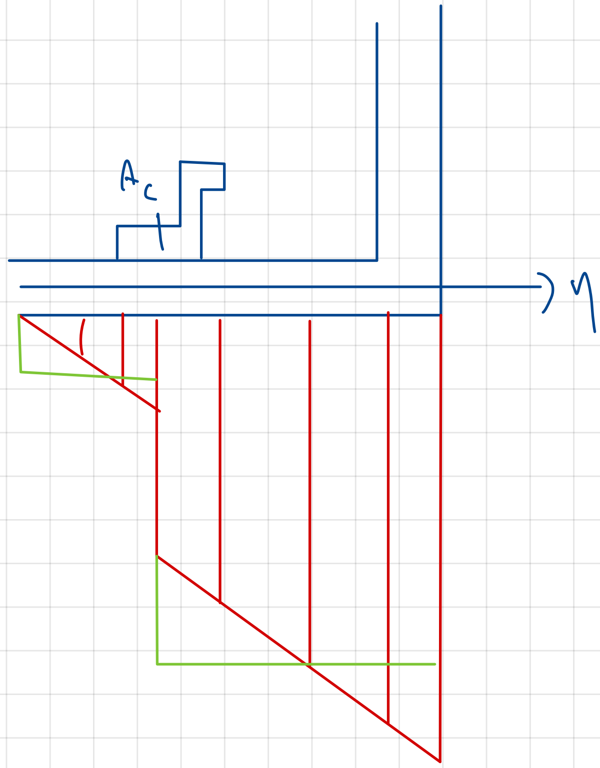
- Compute the normal and shear stresses in the following beam, in particular close to panels A, B and C (which will later be analyzed vs buckling)
- Material properties of Al 2024-T81 are:
  - E = 72395 MPa, ν = 0.3, n = 10,
  - σ_{cu} = 427 MPa
  - $\sigma_{0.7}$ = 386 MPa
  - σ_{cy} = 379 MPa.

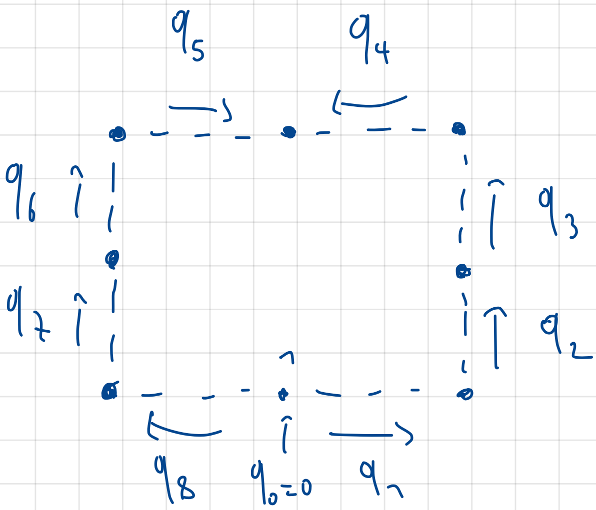
Exercise (home)







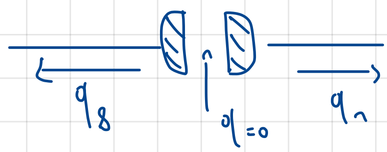


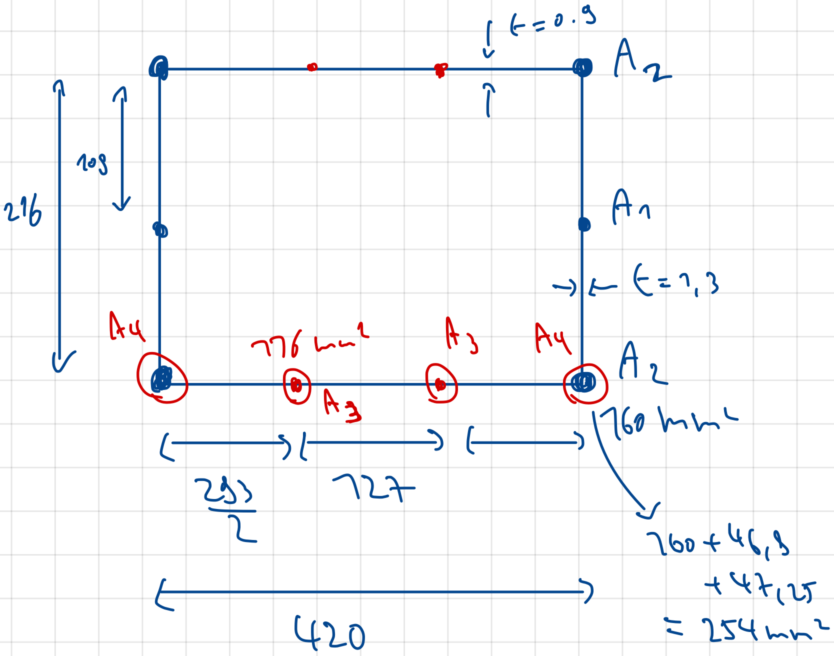


$$q_1 = -q_8$$

$$q_1 = q_8 - \frac{F_z}{I_{yy}}(A_1 z_1)$$

$$2q_1 = -\frac{F_y}{I_{yy}}(A_1 z_1)$$





$$A_1 = \frac{2}{3} bt = \frac{2}{3} \cdot 216 \cdot 1{,}3 = 187{,}2$$

$$A_2 = \frac{bt}{6} = 46{,}8$$

$$\begin{cases} 2(A_3 + A_4) = b \cdot t \\ \dfrac{t b^3}{12} = \left[\left(\dfrac{b}{6}\right)^2 \cdot A_3 + \left(\dfrac{b}{2}\right)^2 \cdot A_4 \right] \cdot 2 \end{cases}$$

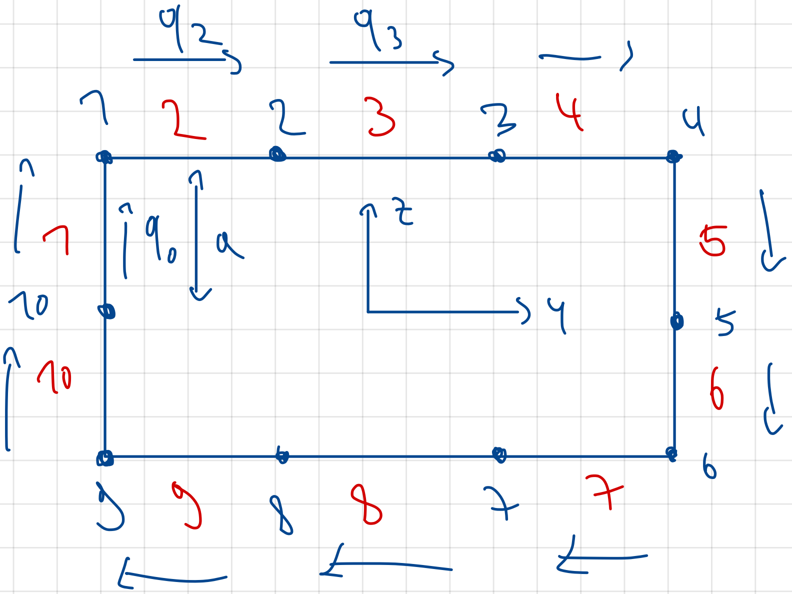
$$\begin{cases} A_3 + A_4 = \dfrac{bt}{2} \\ \dfrac{t b^3}{24} = \dfrac{b^2}{36} A_3 + \dfrac{b^2}{4} A_4 \end{cases}$$

$$\begin{cases} A_3 + A_4 = \dfrac{bt}{2} \\ \dfrac{tb}{24} = \dfrac{A_3}{36} + \dfrac{A_4}{4} \end{cases} \qquad \begin{cases} A_3 = \dfrac{bt}{2} - A_4 \\ \dfrac{tb}{24} = \dfrac{bt}{72} - \dfrac{A_4}{36} + \dfrac{A_4}{4} \end{cases}$$

$$\Rightarrow 3bt = bt - 2 \cdot A_4 + 18 \cdot A_4$$

$$\Rightarrow A_4 = \frac{bt}{8} = \frac{420 \cdot 0{,}9}{8} = 47{,}25$$

$$\Rightarrow A_3 = \frac{3bt}{8} = \frac{3 \cdot 420 \cdot 0{,}9}{8} = 141{,}75$$



	A_i	y_i	z_i	$A_i z_i^2$	$A_i \cdot z_i$
1	254	-210	108		27432
2	258	-70	108		27864
3	258	70	108		27864
4	254	210	108		27432
5	187	210	0		0
6	254	210	-108		-27432
7	258	70	-108		27864
8	258	-70	-108		27864
9	254	-210	-108		-27432
10	187	-210	0		0
Σ				23887872	

$$q(y) = q_0 - \frac{F_z}{I_{yy}} Q_{y,y}$$

$$q_2 = q_0 - \underbrace{\frac{F_z}{I_{yy}}}_{\alpha} Q_{y,y}$$

$F_z \rightarrow 10\,000\ N$

$Q_{y,y} \rightarrow 27432$

$$q_2 = q_0 - 11{,}48\ N/mm = q_9$$

$$q_3 = q_2 - \alpha Q_{y,y}$$

$\hookrightarrow 27864\ N/mm\ (A_2 z_2)$

$$q_3 = q_0 - 23{,}14 = q_8$$

$$q_4 = q_0 - 34{,}87 = q_7$$

$$q_5 = q_0 - 46{,}29$$

$$q_6 = q_0 - 46{,}29 \qquad (q_6 = q_5 - \alpha \overset{0}{Q_{y,y}})$$

$$q_{10} = q_0$$

$$M_x = 0 = (a q_0) \cdot \frac{b}{2} + \left(\frac{b}{3} \cdot q_2\right) + \left(\frac{b}{3} \cdot q_3\right) \cdot a + \left(\frac{b}{3} \cdot q_4\right) \cdot a + \left(\frac{b}{3} \cdot q_4\right) \cdot a$$

$$+ (q_5 \cdot a) \cdot b + (q_6 \cdot a) \cdot b + (q_7 \cdot q_8 \cdot q_9) \cdot \frac{b}{3}\, a + q_{10} \cdot a \cdot \frac{b}{2}$$