Aircraft Structures

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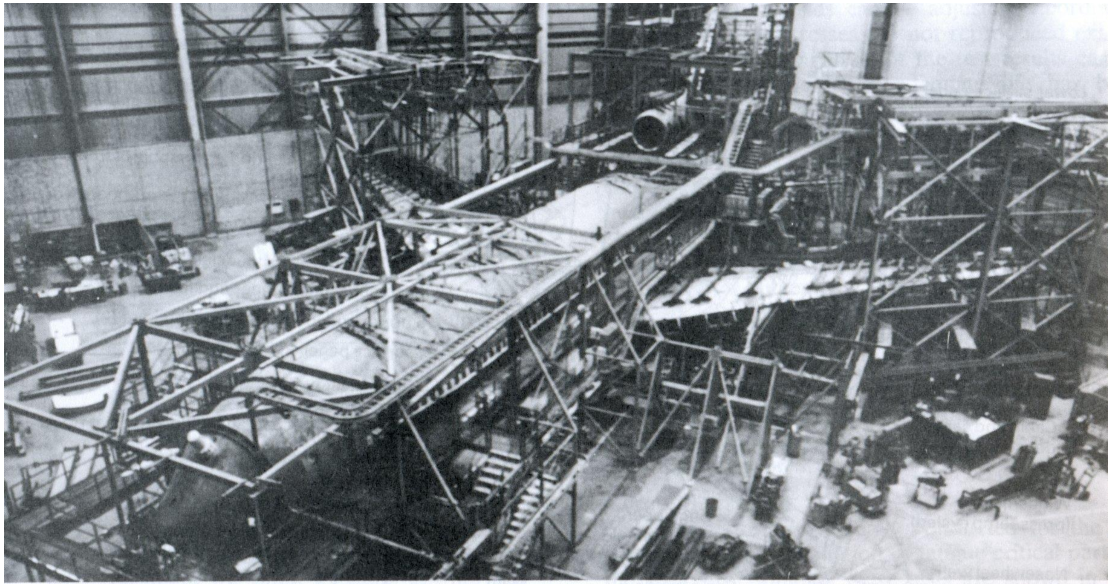
Fatigue design requirements



L1011 fatigue test airplane and test bench

Origins of fatigue

- cyclic fatigue → turbulence
  - fluctuating loads such as repeated pressurization
- corrosion fatigue
  - surface corrosion which penetrates inwards
- fretting fatigue
  - small-scale rubbing and abrasion due to contact
- thermal fatigue → Supersonic aircraft (Concorde)
  - repeated expansion and contraction of the material
- sonic or acoustic fatigue
  - high-frequency stresses due to noise-induced vibrations

Design concepts

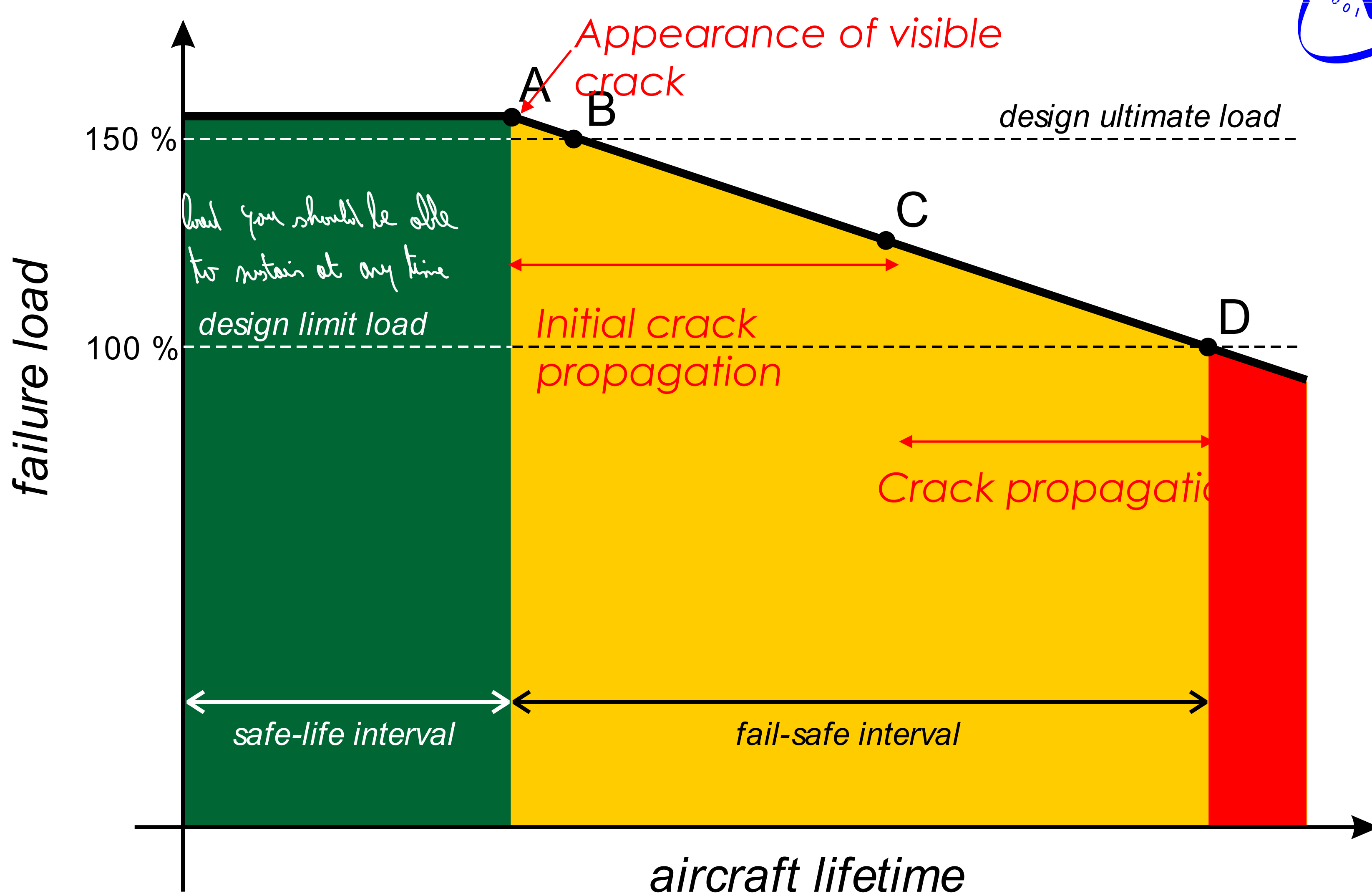
- *safe-life* design → replace a part when it's really damaged
  - no damage during lifetime of the aircraft (or vehicle)
  - rather costly
  - not used anymore for full structures
  - used for critical components
- *fail-safe* design / *damage-tolerant* design
  - damage will appear during lifetime of the aircraft
  - damage will be detected before failure
  - repairs or replacements during servicing operations
  - structural redundancy

CS-25.571 requirements

- *An evaluation of the strength, detail design, and fabrication must show that catastrophic failure due to fatigue, corrosion, or accidental damage, will be avoided throughout the operational life of the aeroplane. This evaluation must be conducted for each part of the structure which could contribute to a catastrophic failure (such as wing, empennage, control surfaces and their systems, the fuselage, engine mounting, landing gear, and their related primary attachments)*

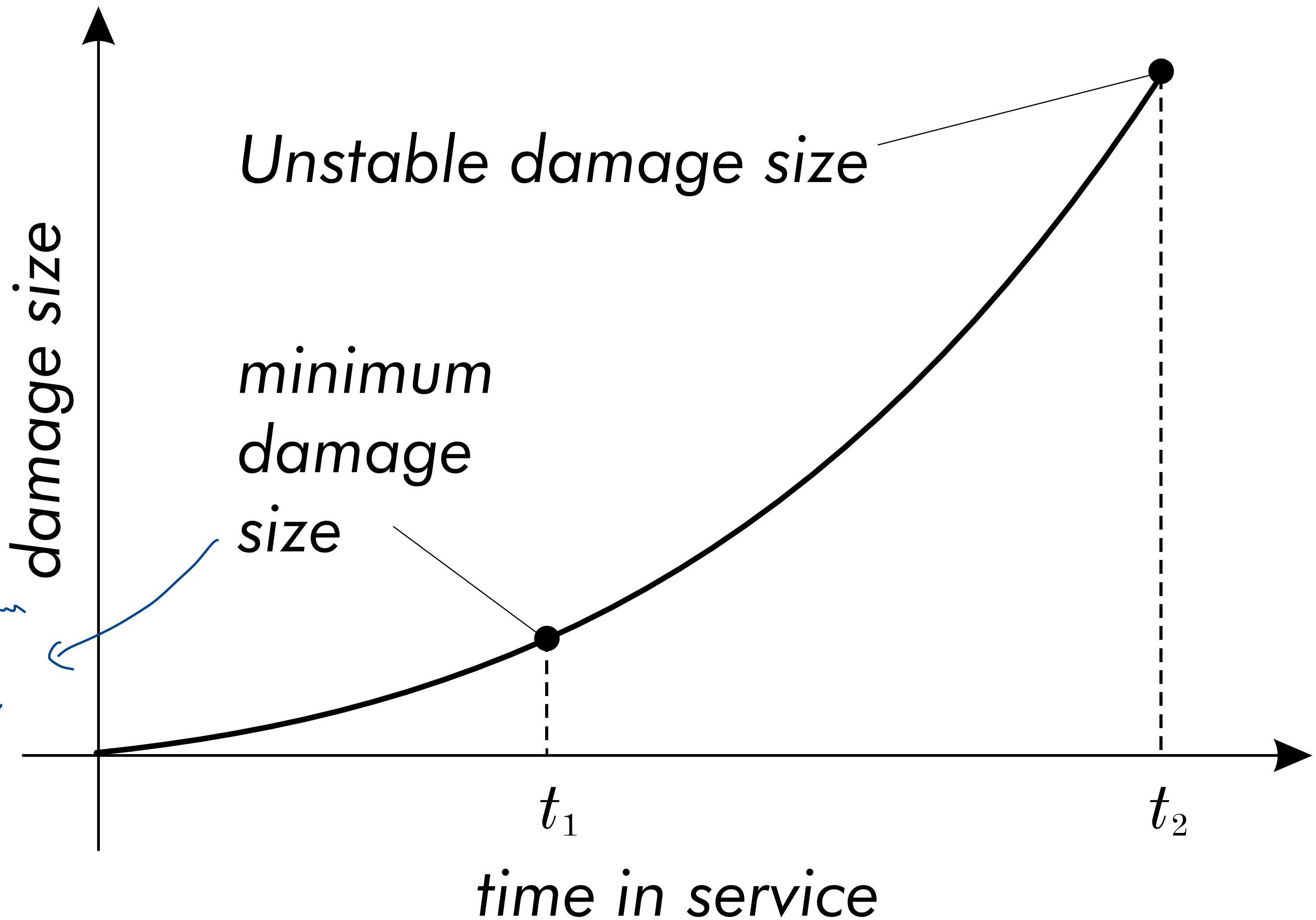
The loads are considered static limit loads

Design for fatigue



Damage-tolerant design

check ppt



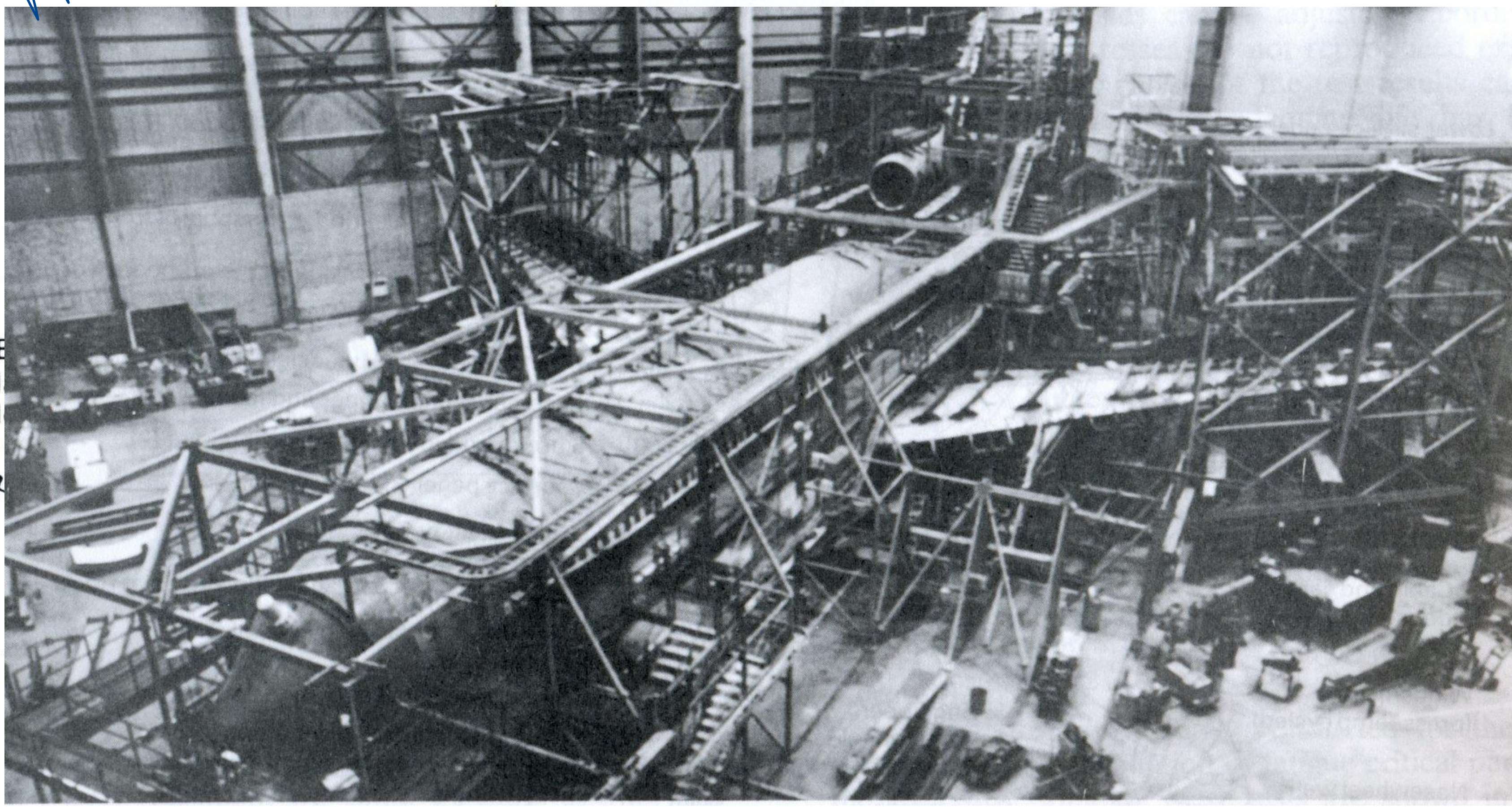
Before this you don't see it

Operational consequences

- Operational requirements
  - fatigue depends on flight cycles, not flight time
  - crack-free life
    - Airbus: 24000 flights or 30000 hours
    - BAe46: 40000 flights
  - economic repair life → lifetime when you can repair the airplane without spending too much
    - usually double crack-free life
- Economical consequences
  - Design influences TBI and TBO
  - TBI and TBO influence operational costs for airlines

Complete aircraft testing

ppt



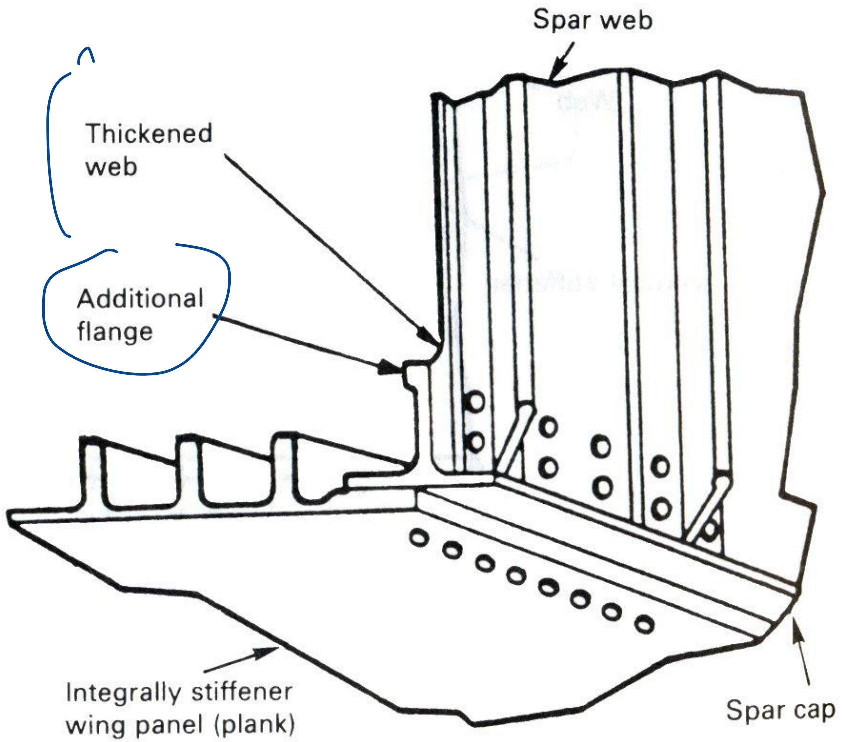
test Concorde on thermal fatigue by heating & cooling

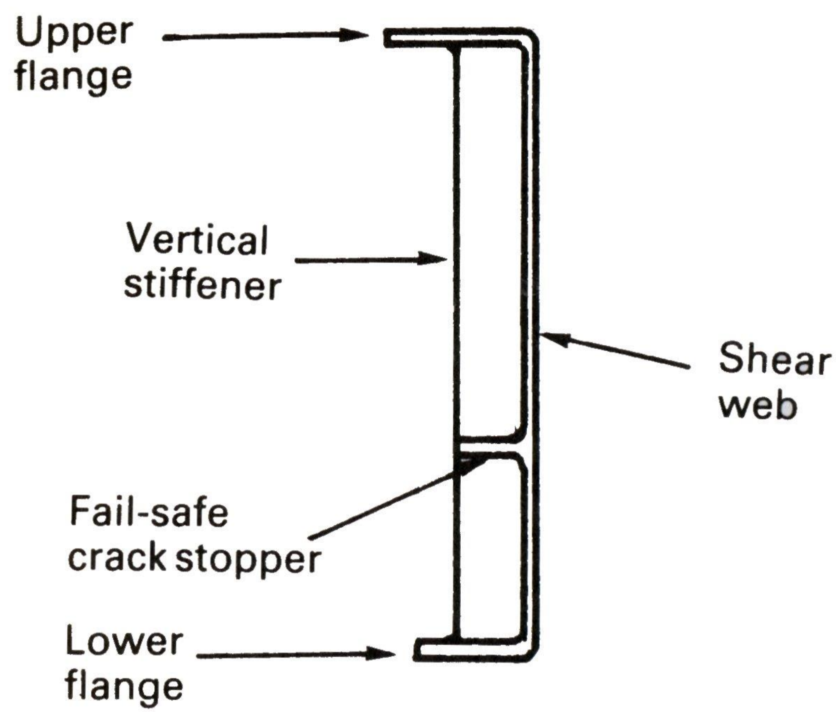
Structural Significant Items

- SSI require stronger design criteria
- Specific criteria apply:
  - 3 to 5 times the fatigue lifetime
  - ½ fatigue lifetime to first inspection
  - 3 times inspection interval with cracks of 5 mm

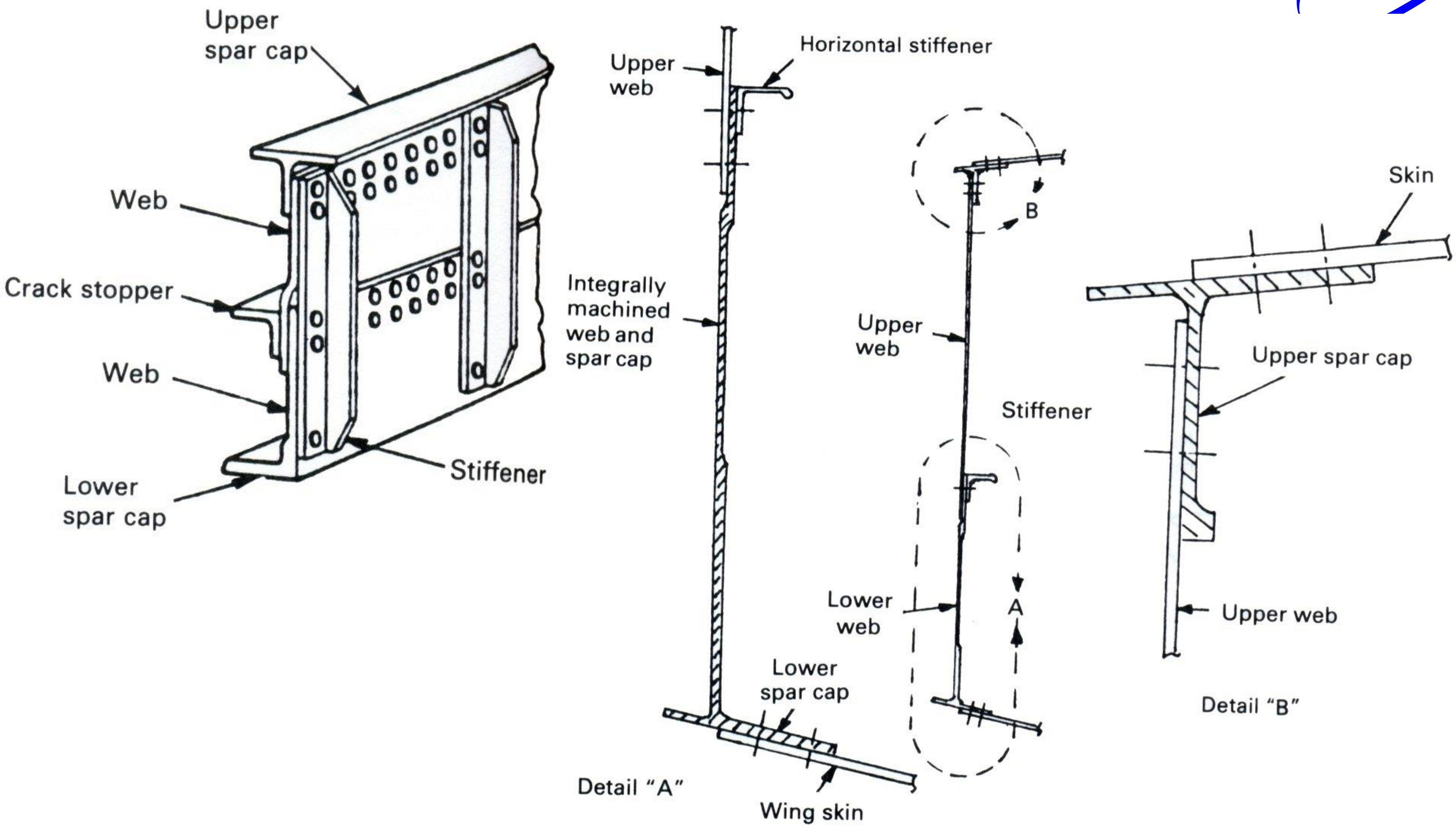
Slow crack growth

add thickness to limit crack growth





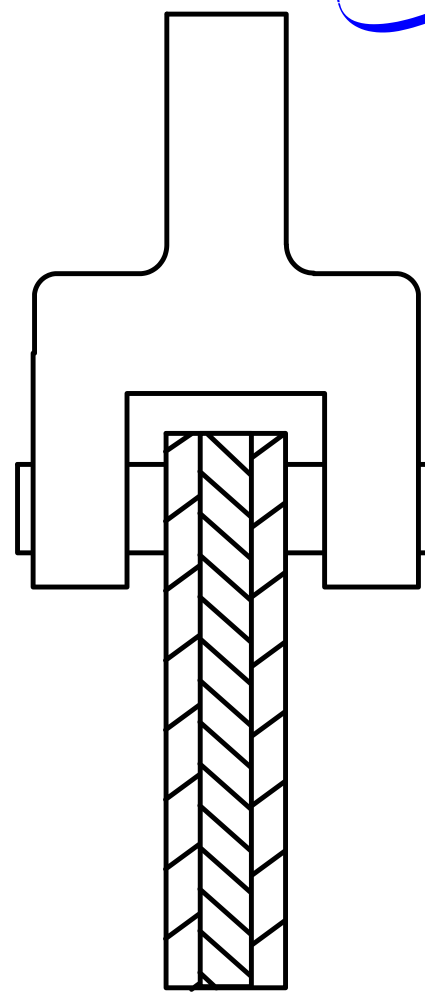
Crack arrest



Multiple load paths

- Redundant structures
- Must be analysed with some component failure
  - stress concentration areas
  - high stress levels areas
  - major load path redistributions areas

in order to see if the structure maintain after failure of component



Analysis of material properties

Ultimate tensile stress measurement
Aluminium bar, 8 mm diameter

Sources of material data

- MIL-HDBK-V-J
  - *metallic materials and elements for aerospace vehicle structures*
    - Tensile & compressive strength, bearing properties
    - moduli of elasticity and shear
    - elongation, total strain at failure and reduction of area
    - stress-strain curves, tangent-modulus curves
    - Creep, fatigue, fatigue-crack propagation and fracture toughness
  - Most widely used official materials database

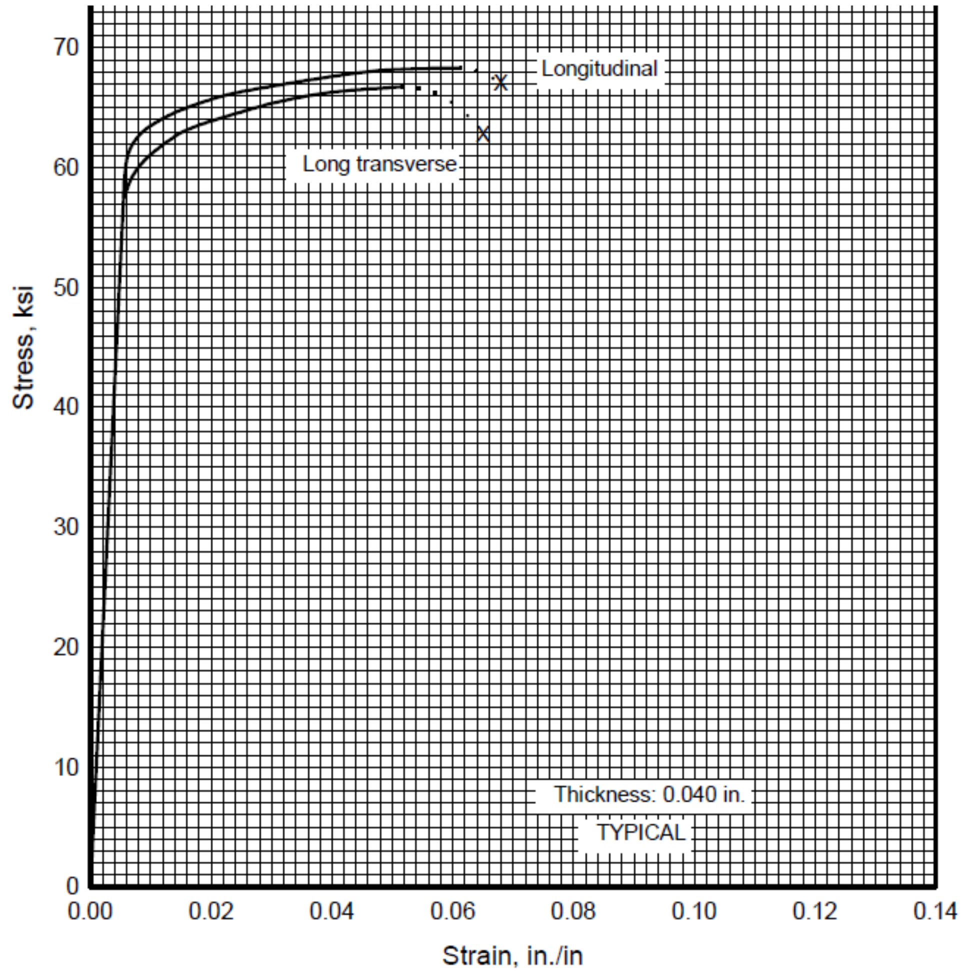
Sources of material data

- MMPDS
  - *Metallic Material Properties, Development and Standardization* (MMPDS 01, January 2003)
  - Published by the FAA
  - Supersedes MIL-HDBK-5-J
  - Intends to incorporate ESDU data
- Other sources
  - ESDU 00932 (*International Metallic Materials Data Handbook*)
  - Industry specifications (e.g. Airbus AIMS)

Material data

- Stress-strain curves
  - When using Imperial units: 1 ksi = 1000 psi (also kip)

« 2024 is a heat-treatable Al-Cu alloy which is available in a wide variety of product forms and tempers. The properties vary markedly with temper ».

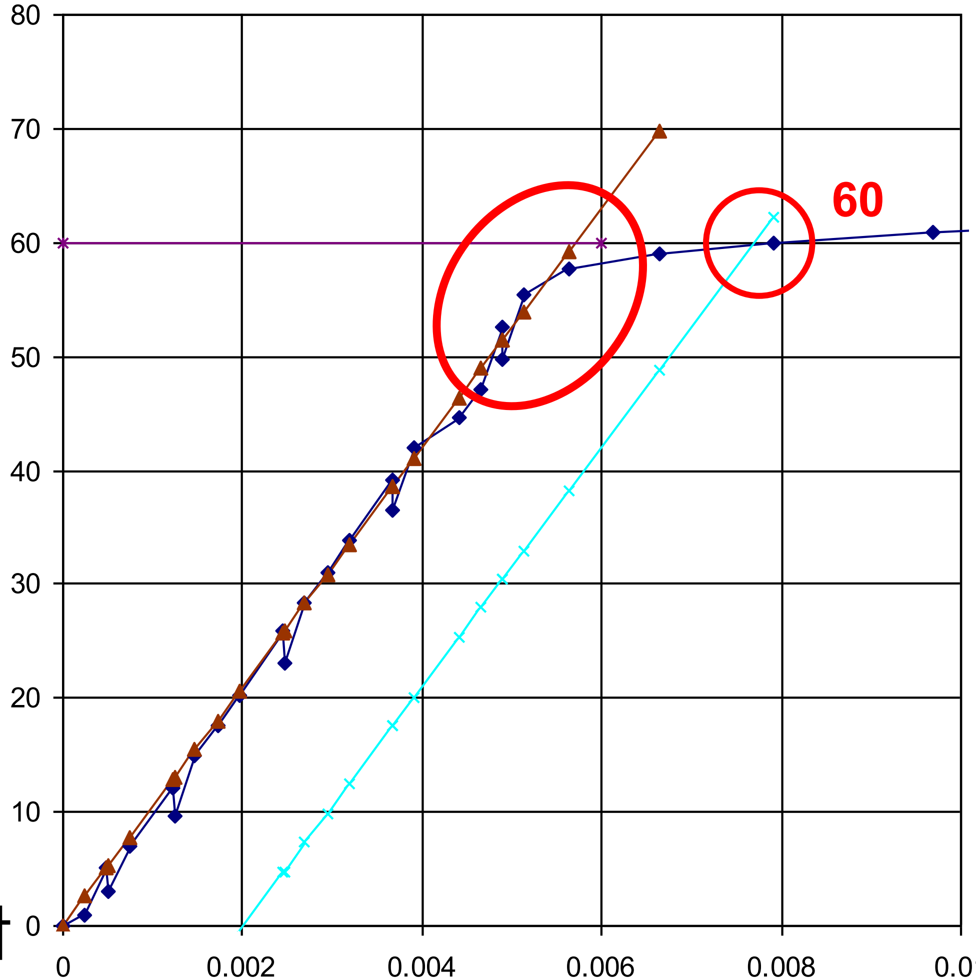


Clad Al 2024-T81 sheet (room temperature)

Material data

- Manipulate data
  - "Engauging"
  - "On-top" plot
- Proportional stress
  - Difficult !
- Yield stress
  - Easier
  - Defined by offset

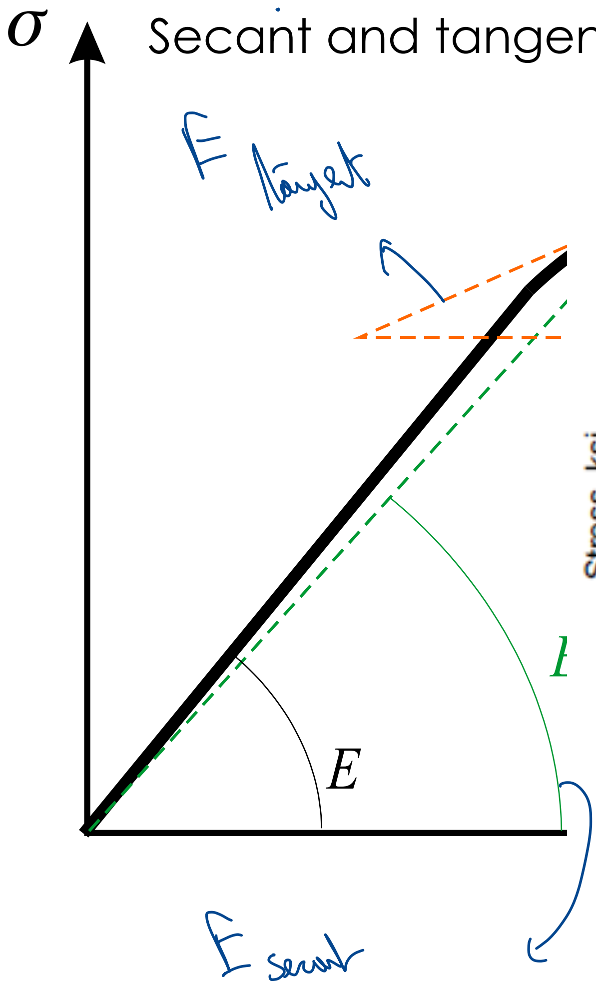
→ 0.002 deformation

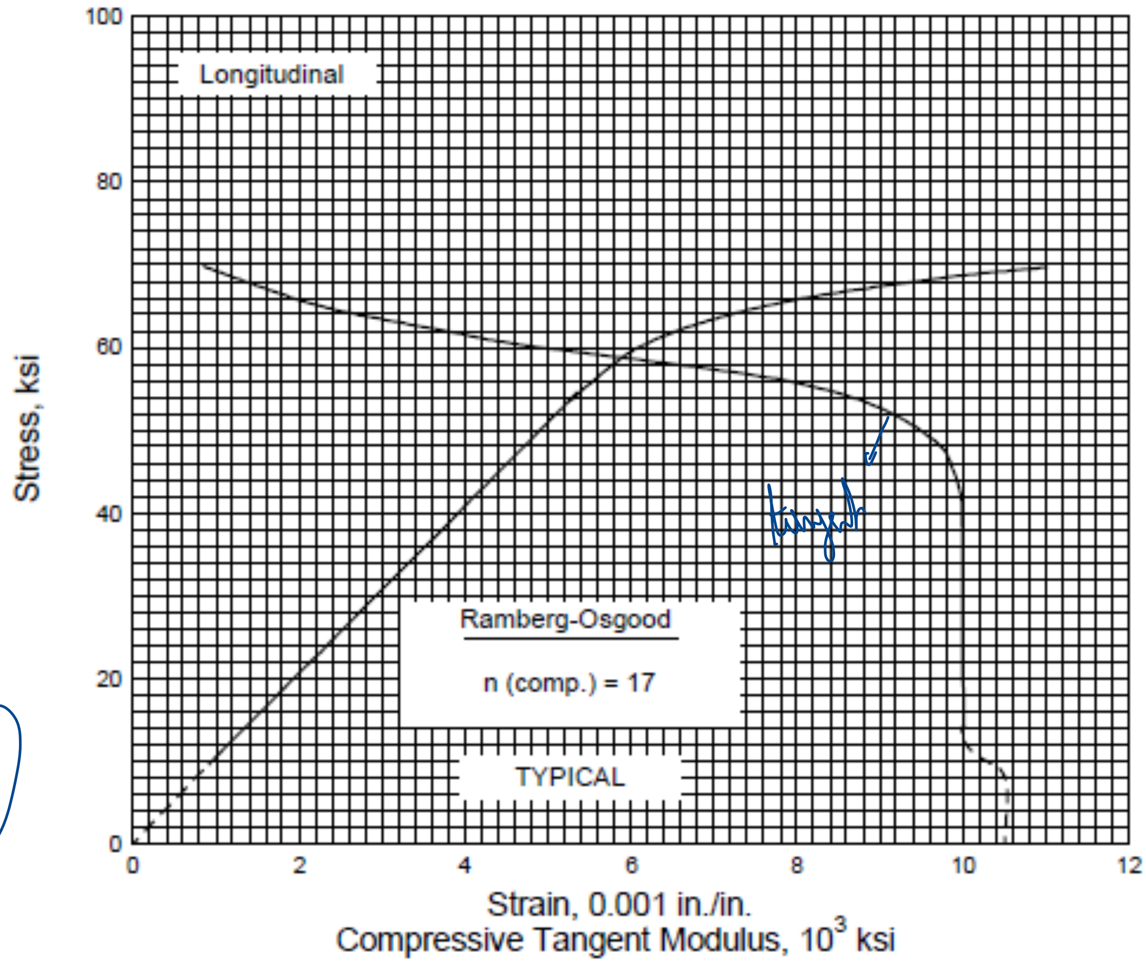


PPT

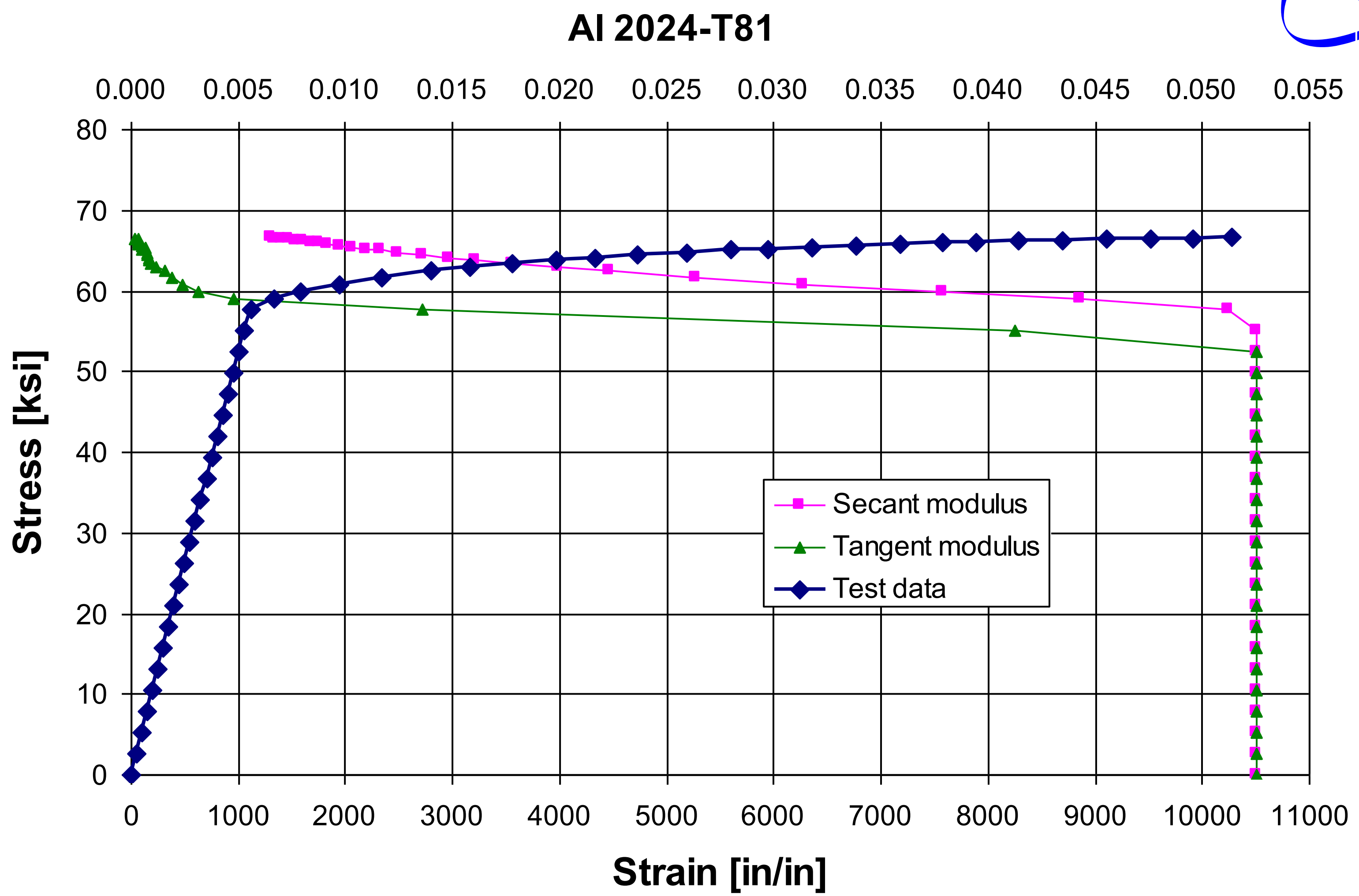
Inelastic stress curve

Secant and tangent moduli

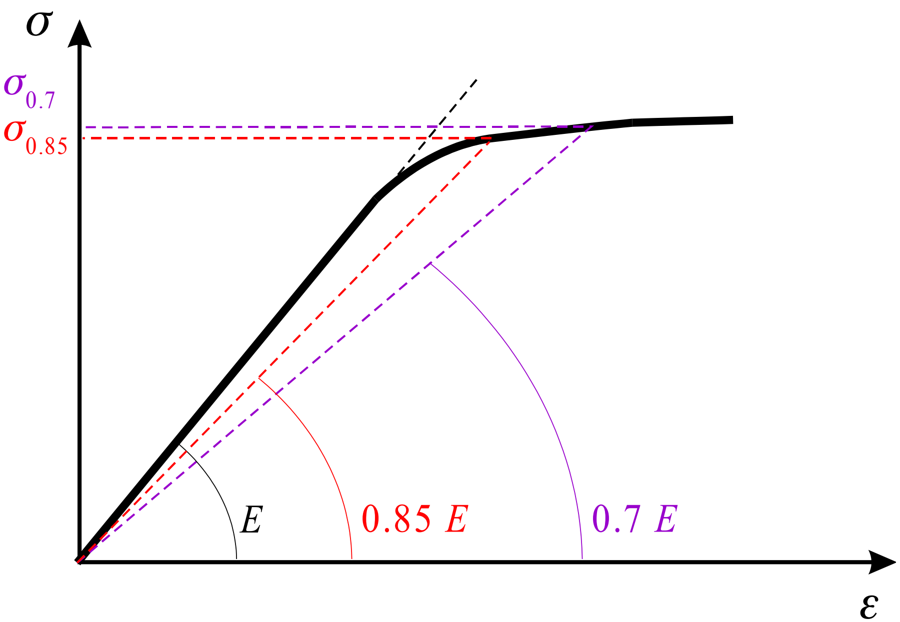




Inelastic stress curve



Ramberg-Osgood model



NACA TN 902, 1943

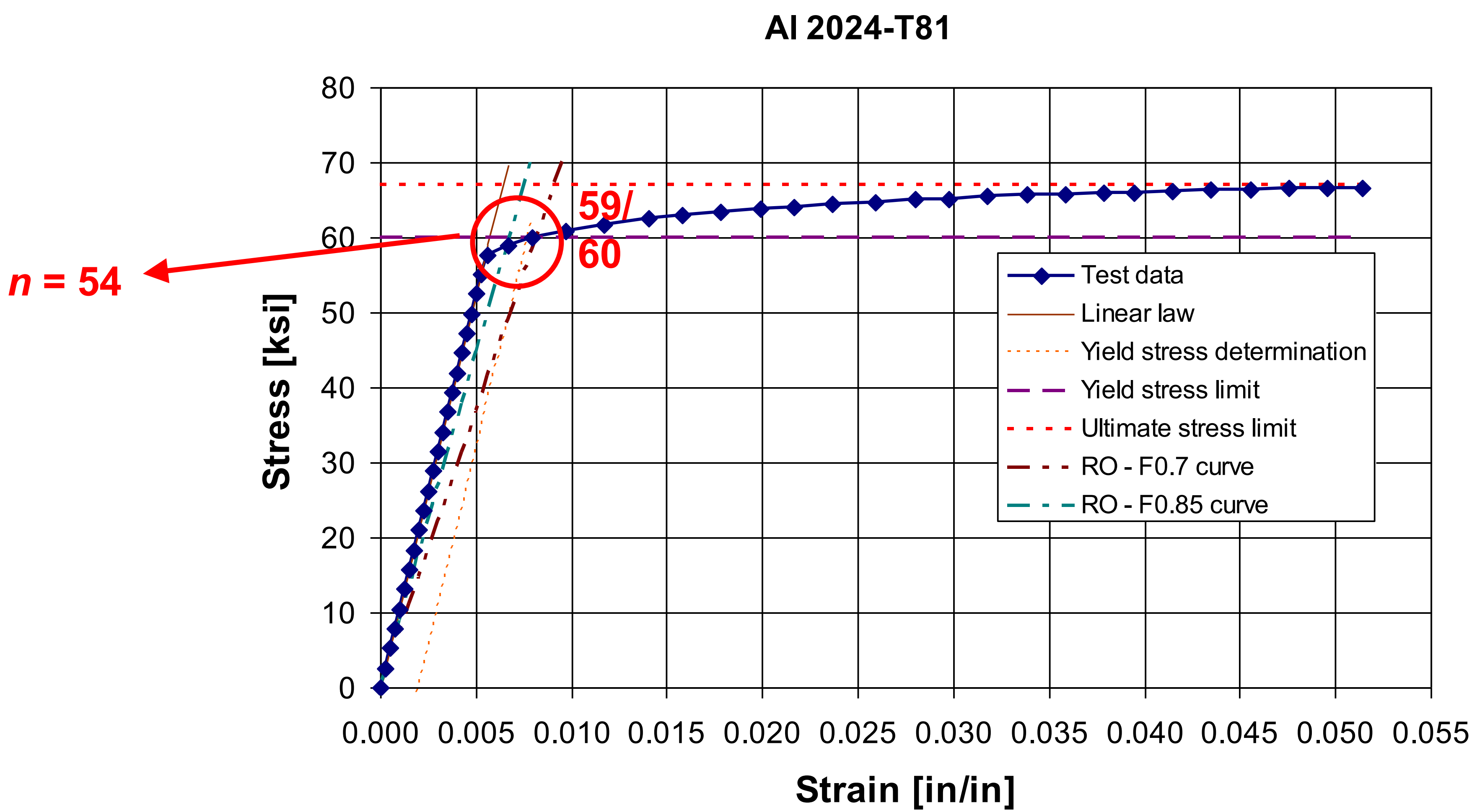
$$n = 1 + \frac{\ln(17/7)}{\ln(\sigma_{0.7}/\sigma_{0.85})}$$

$\sigma_{0,7} \approx \sigma_{cy}$
yield stress

$$\frac{\varepsilon E}{\sigma_{0,7}} = \frac{\sigma}{\sigma_{0,7}}\left[1 + \frac{3}{7}\left(\frac{\sigma}{\sigma_{0,7}}\right)^{n-1}\right]$$

yield stress

Ramberg-Osgood model



Ramberg-Osgood model

$$E_s = \frac{E}{\left[1 + \frac{3}{7}\left(\frac{\sigma}{\sigma_{0.7}}\right)^{n-1}\right]}$$

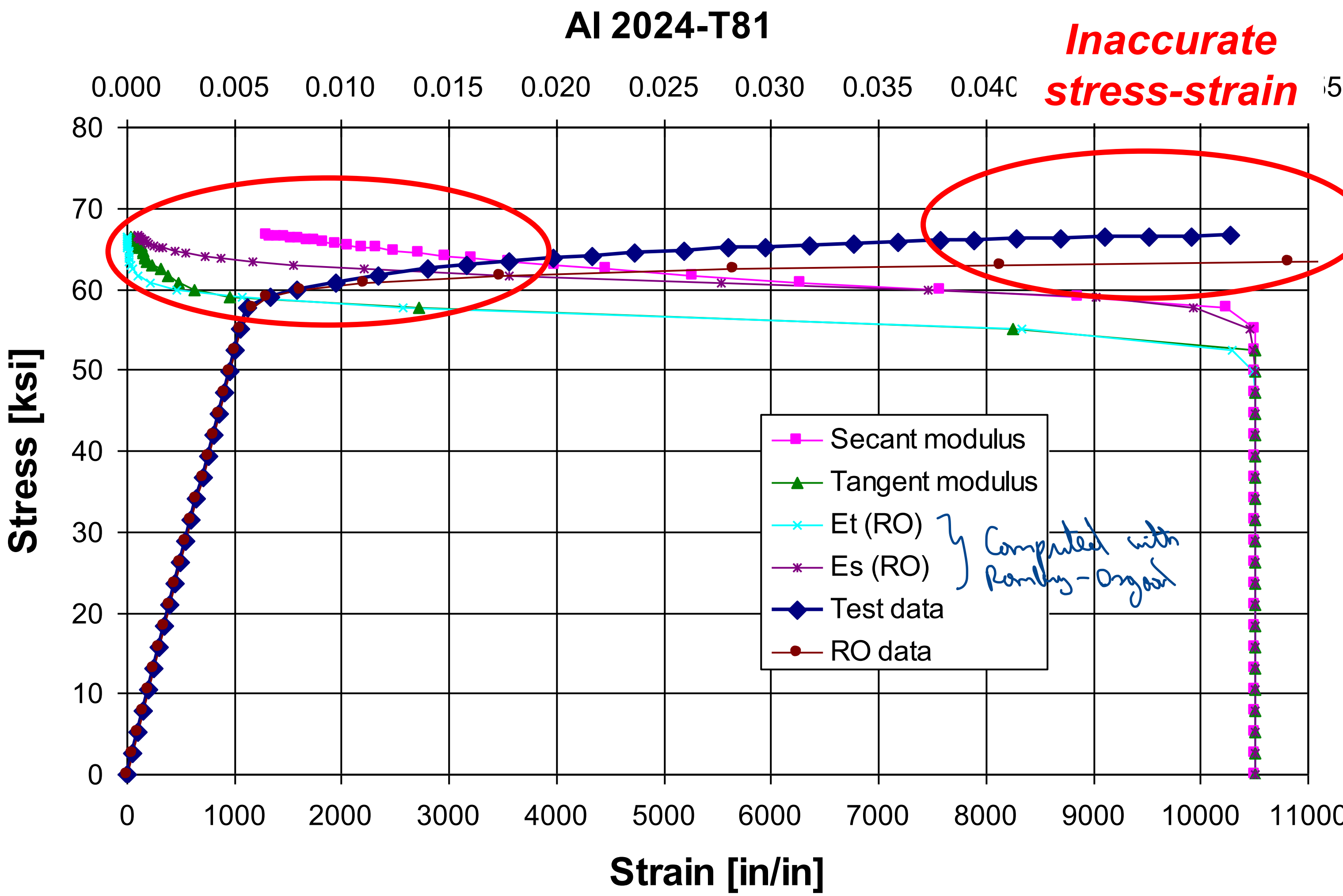
$$E_t = \frac{E}{\left[1 + \frac{3\,n}{7}\left(\frac{\sigma}{\sigma_{0.7}}\right)^{n-1}\right]}$$

Ramberg-Osgood model

n = 54

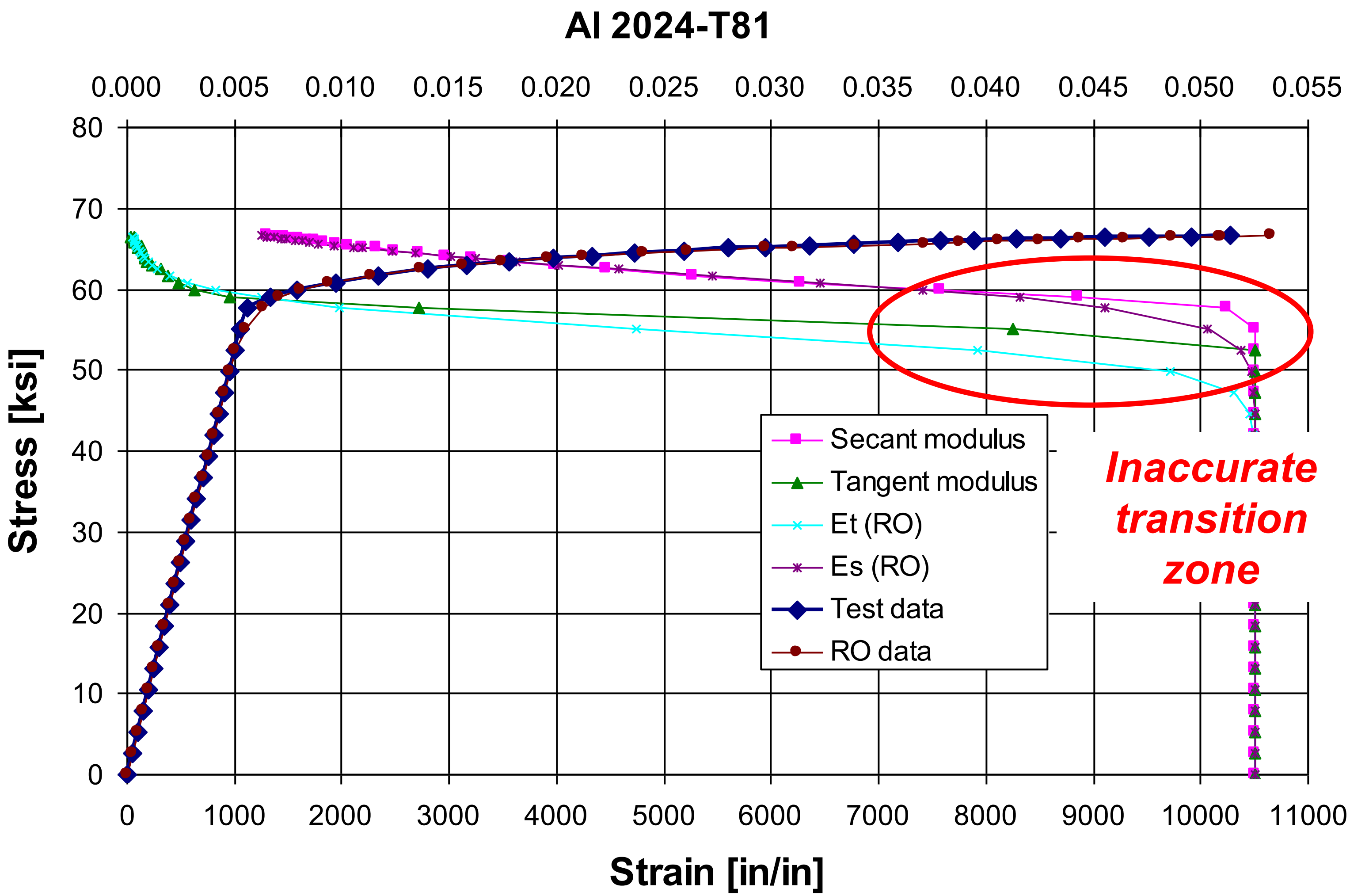
Inaccurate secant modulus

Inaccurate stress-strain



Ramberg-Osgood model

n = 28



Yield & ultimate stresses

- Structural requirements:
  - No failure at ultimate load
  - No permanent deformation (yield) at limit load
- Yield stress criterion
  - Von Mises equivalent stress

$$\sigma_e = \frac{1}{\sqrt{2}}\sqrt{(\sigma_{xx}-\sigma_{yy})^2 + (\sigma_{xx}-\sigma_{zz})^2 + (\sigma_{yy}-\sigma_{zz})^2 + 6\,(\sigma_{xy}+\sigma_{xz}+\sigma_{yz})^2}$$

  - Factor of / Margin of safety concept

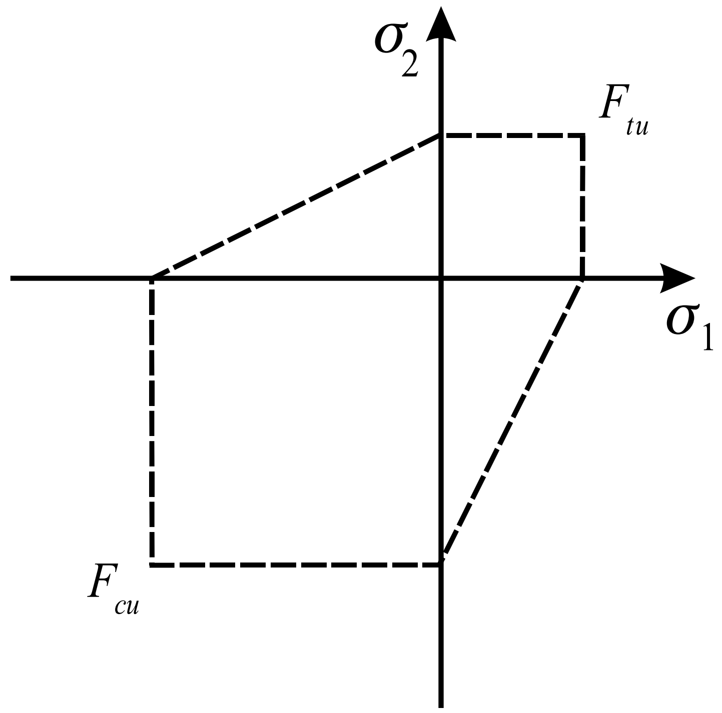
$$F.S._y = \frac{\text{yield load}}{\text{limit load}} \approx 1{,}5$$

$$M.S._y = F.S._y - 1 = 0{,}5$$

↳ the lower the better

Yield & ultimate stresses

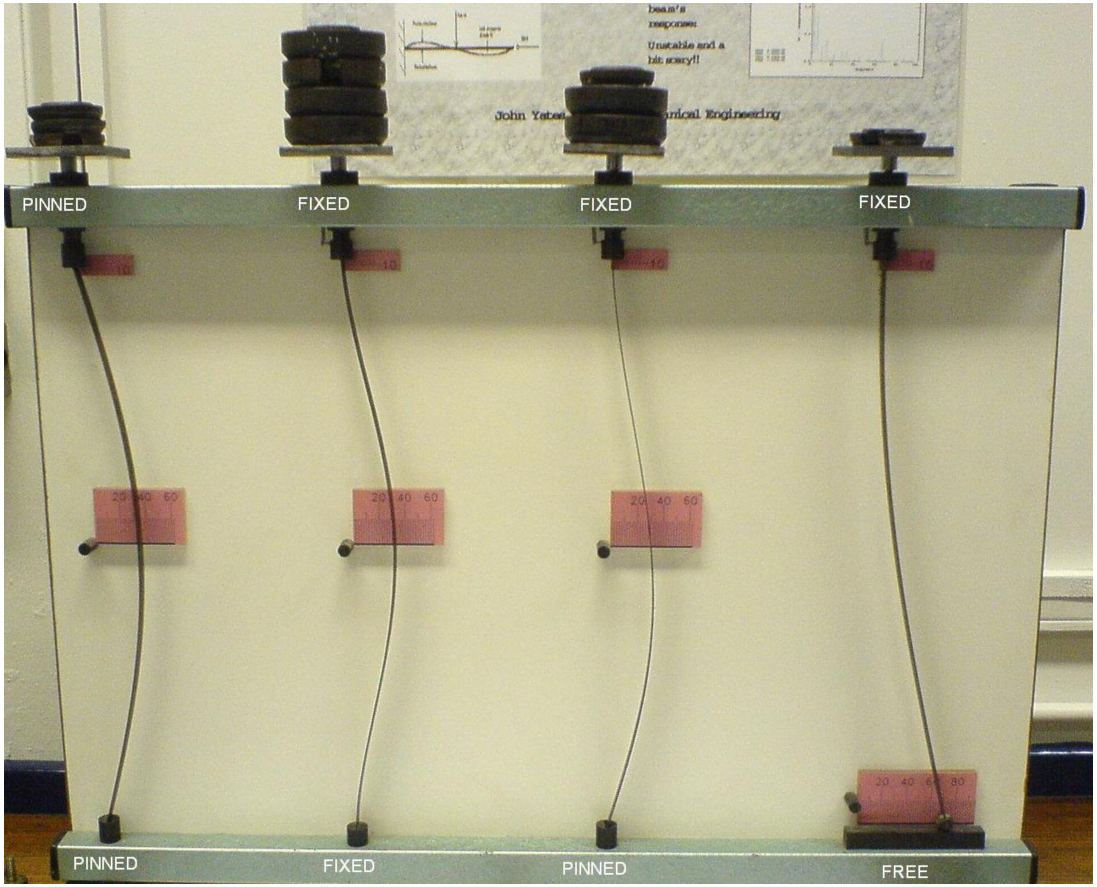
- Ultimate stress criterion
  - Brittle material: Coulomb-Mohr failure envelope
  - Ductile material: use interaction curves
  - Factor of / Margin of safety concept



$$F.S._u = \frac{\text{ultimate load}}{\text{limit load}}$$

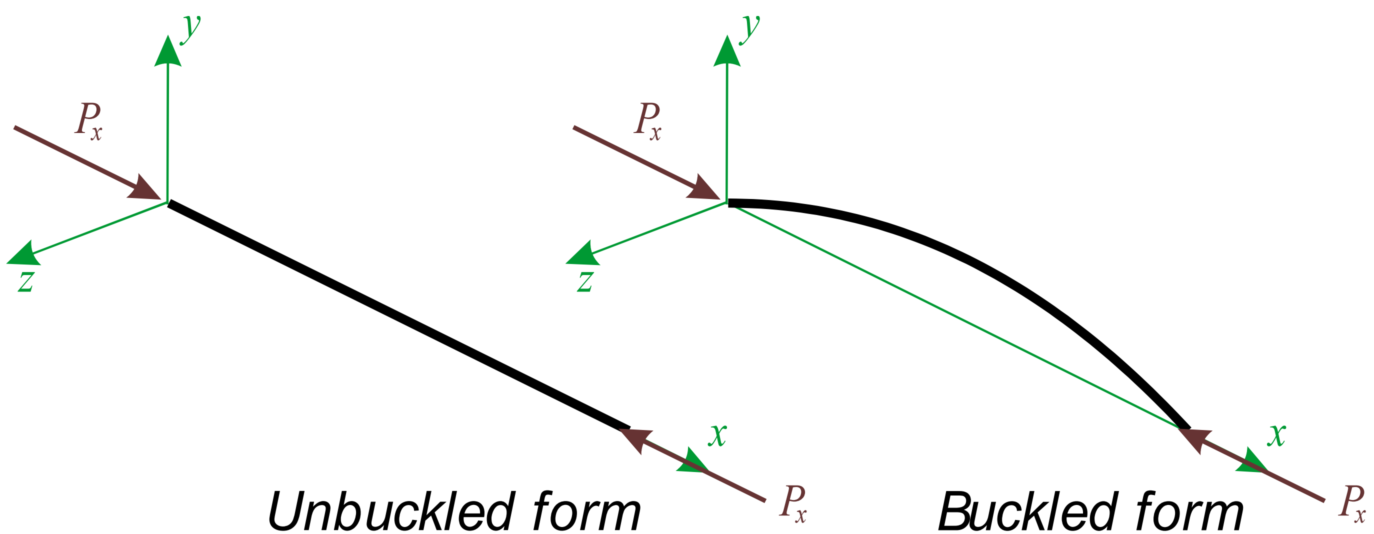
$$M.S._u = F.S._u - 1$$

Elastic column buckling



Canonical problem

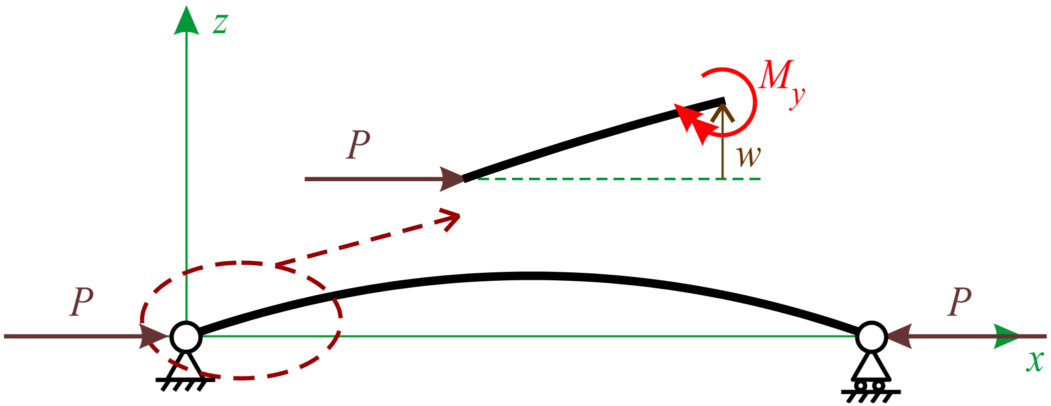
- Euler buckling theory (1744):
  - Instability in compression
  - Valid for long, slender columns
- Primary instability of columns



Euler buckling formula

- Differential equation for buckling:

$$\frac{\partial^2 w}{\partial x^2} = -\frac{M_y}{EI_{yy}}$$



- Eigenvalue problem
  - Deformed shapes (eigenfunctions): $w_n = A_n \sin \dfrac{n\,\pi\,x}{L}$
  - Critical loads (eigenvalues): $\lambda\, P_0 = EI_{yy} \left(\dfrac{n\,\pi}{L} \right)^2$

Buckling equations

- Consider minimal critical load

$$P_{cr} = \frac{\pi^2 EI_{min}}{L^2}$$

- Critical stress formula

$$\sigma_{cr} = \frac{\pi^2 EI_{yy}}{S\,L^2}$$

$$\rho = \sqrt{\frac{I_{yy}}{S}}$$

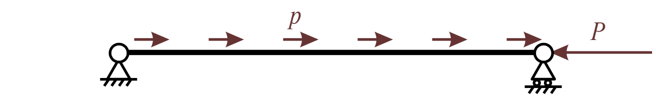
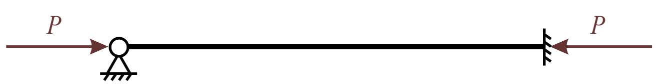
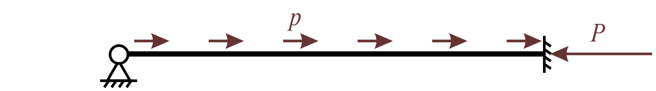
Radius of gyration

$$\sigma_{cr} = \frac{\pi^2 E_c}{(L/\rho)^2}$$

$$\lambda = L/\rho$$

Slenderness ratio

Fixity coefficients

Beam loading configuration		c	$c^{-1/2}$
	*Pinned-pinned Loaded at ends*	1	1
	*Pinned-pinned Uniformly loaded*	1.87	0.731
	*Fixed-fixed Loaded at ends*	4	0.5
	*Fixed-fixed Uniformly loaded*	7.5	0.365
	*Pinned-fixed Loaded at ends*	2.05	0.7
	*Pinned-fixed Uniformly loaded*	6.08	0.406
	*Free-fixed Loaded at ends*	0.25	2.000
	*Free-fixed Uniformly loaded*	0.794	1.12

$$\sigma_{cr} = c\,\frac{\pi^2 E_c}{\underbrace{(L/\rho)}_{\text{slenderness}}{}^2}$$

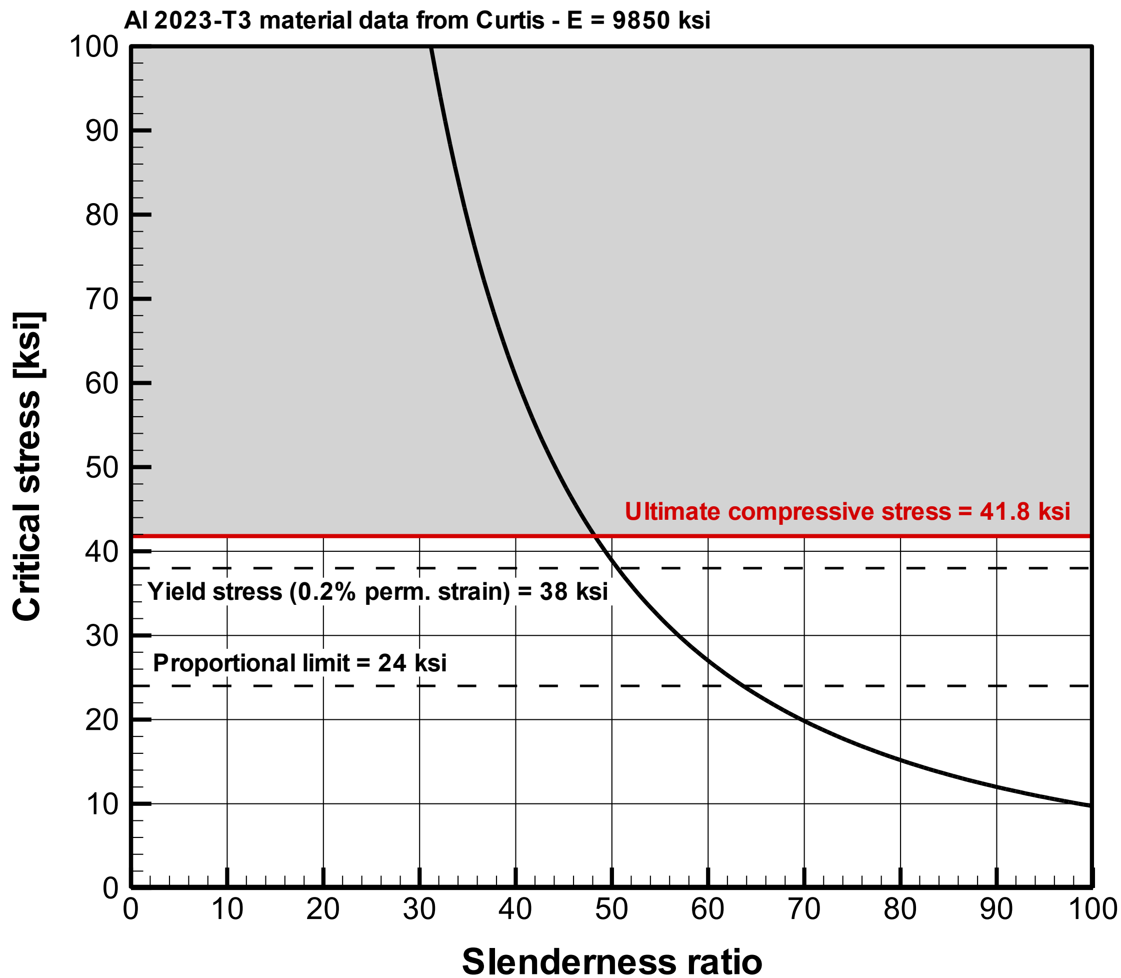
$$\sigma_{cr} = \frac{\pi^2 E_c}{(L'/\rho)^2}$$

$$L' = \frac{L}{\sqrt{c}}$$

Limit of Euler theory

- Column curv
  - Critical stress vs slenderne
  - Type $1/\lambda^2$
  - Limited by ultimate stre

PPT



Column yield stress: usually

Non-uniform columns

- Requires numerical or approximate method
  - Pure numerical analysis (eigenvalue problems)
  - Moment distribution method (Bruhn)
  - Newmark's parabolic approximation (Niu)
  - Martin's iterative finite difference method

Martin method

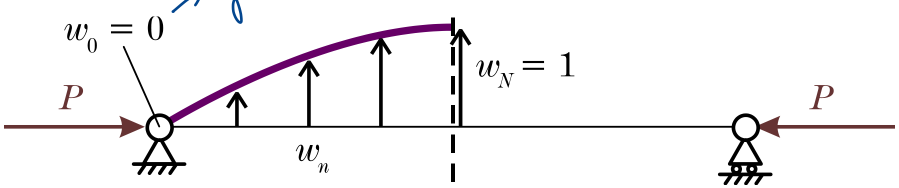
- Finite difference approximation

$$\frac{(I_n/I_0)}{w_n}\left(2\,w_n - w_{n-1} - w_{n+1}\right) = \frac{P\,\Delta^2}{E\,I_0} = K$$

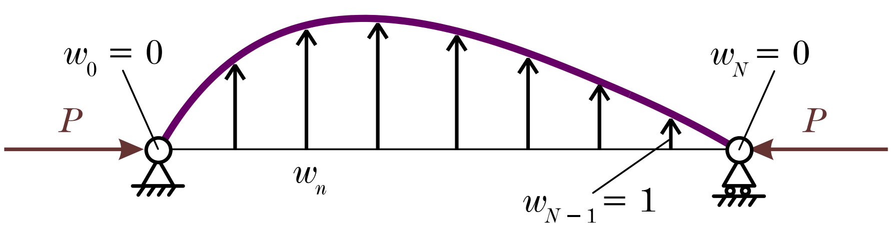
you search for this

- Iteration until zero deflection at end

find P such that $w_0 = 0$



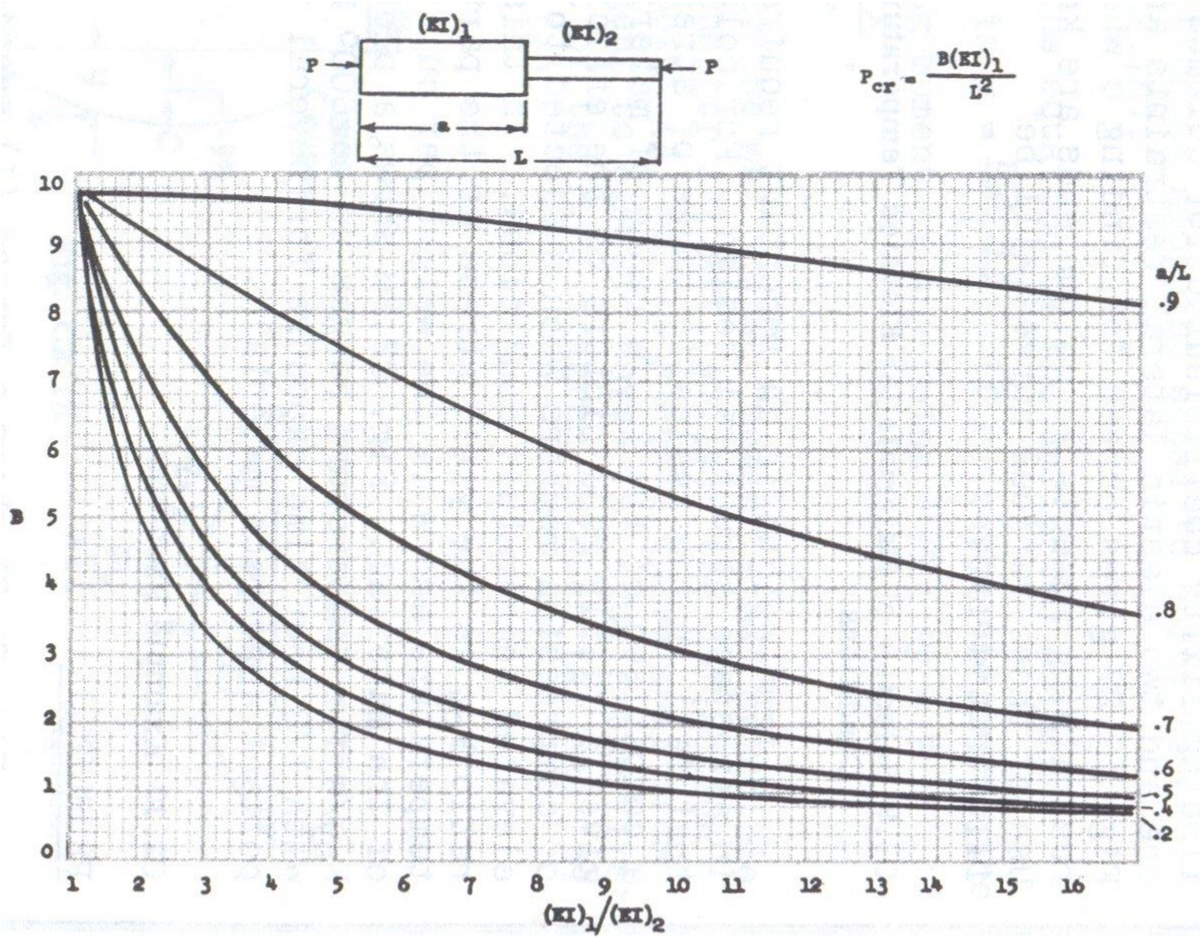
$$P_{cr} = \frac{K\,EI_0}{\Delta^2}$$



$$C = \frac{K\,N^2}{\pi^2}$$

Stepped column charts

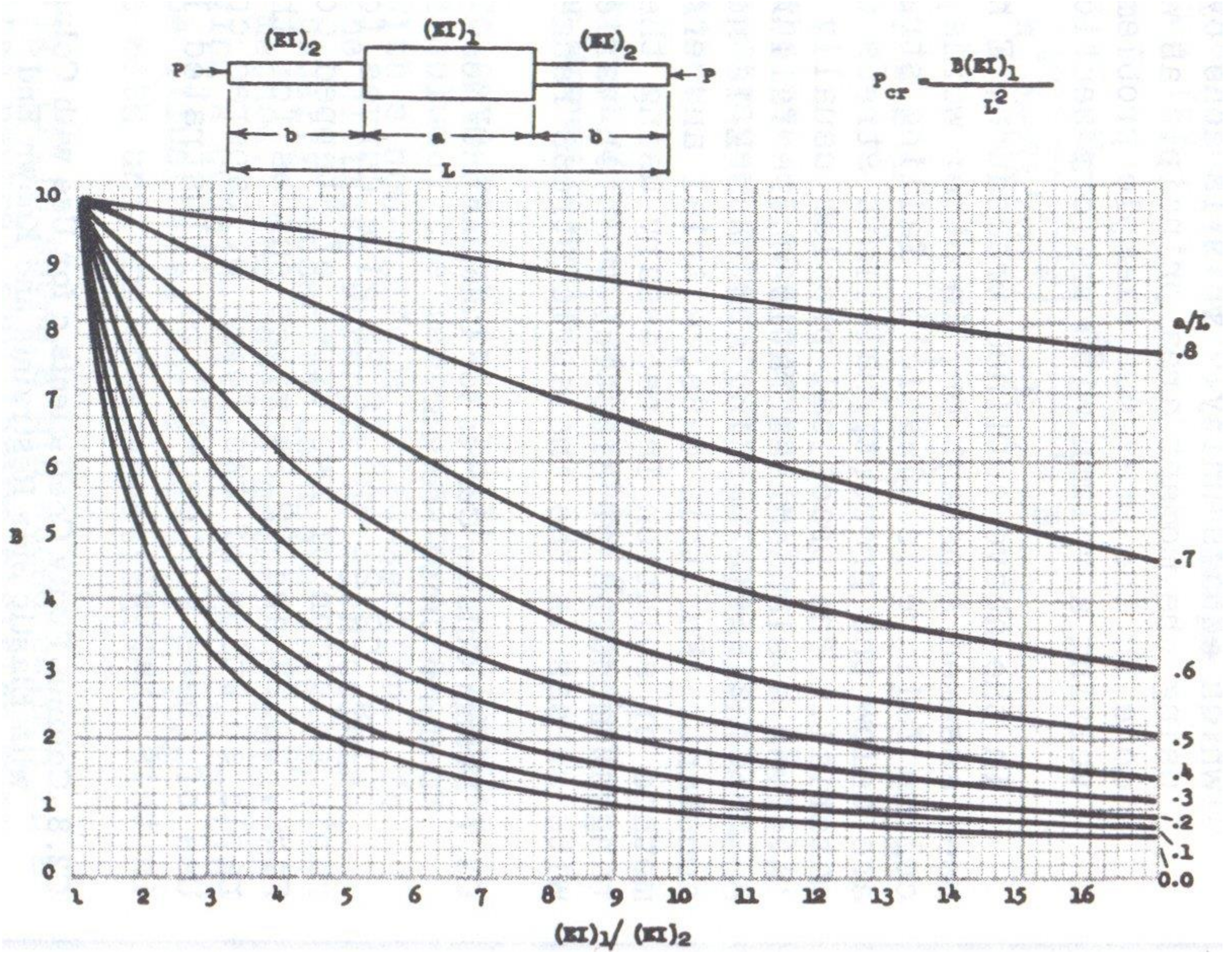
➢ Stepped asymmetric column



[Bruhn]

Stepped column charts

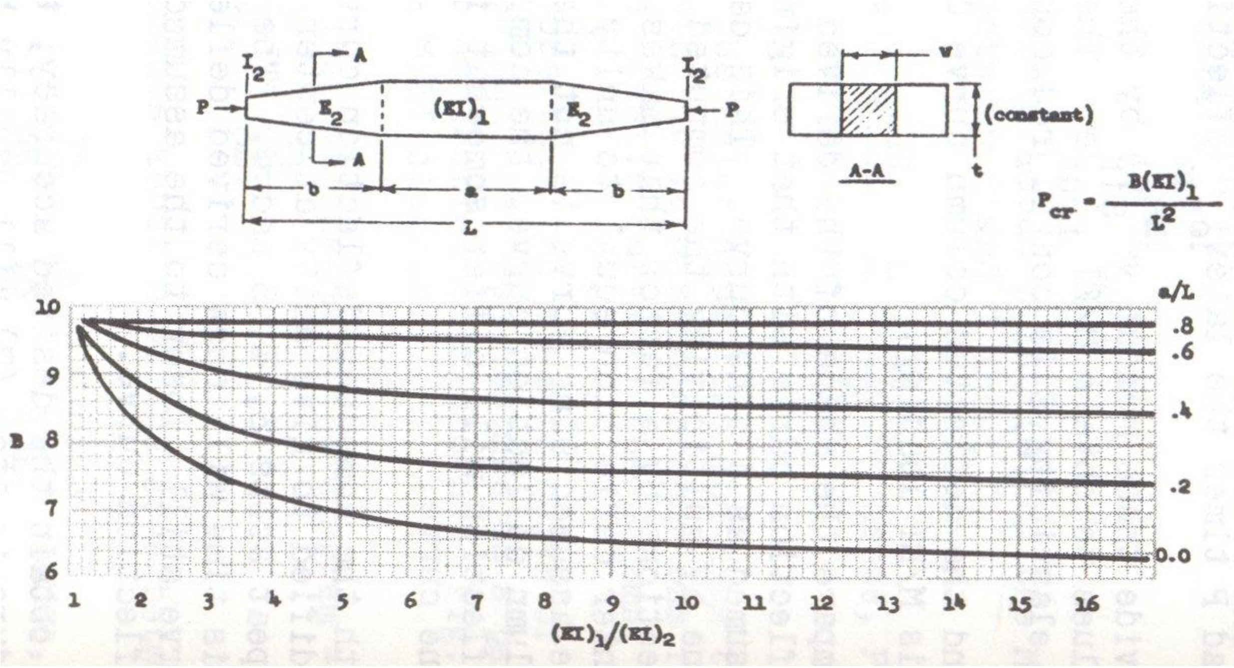
➢ Stepped symmetric column



[*Bruhn*]

Tapered column charts

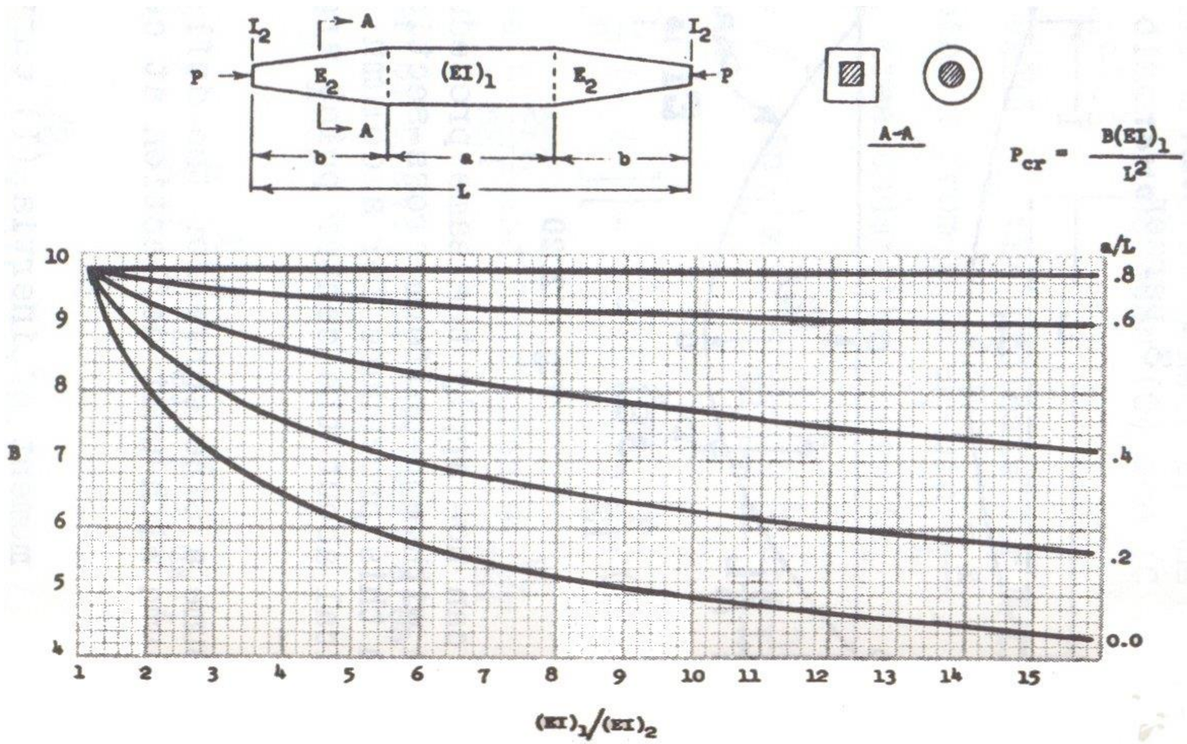
➢ Tapered flat column



[*Bruhn*]

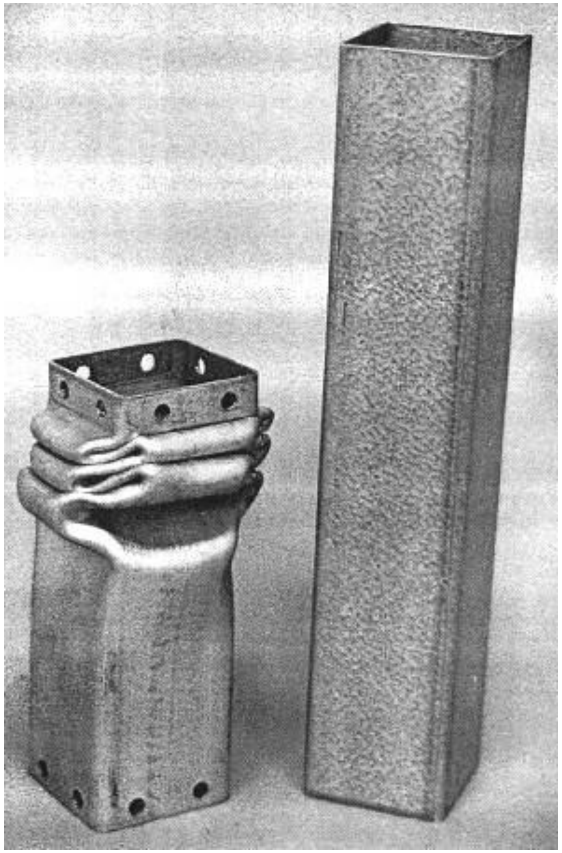
Tapered column charts

- Tapered conical/prismatic column



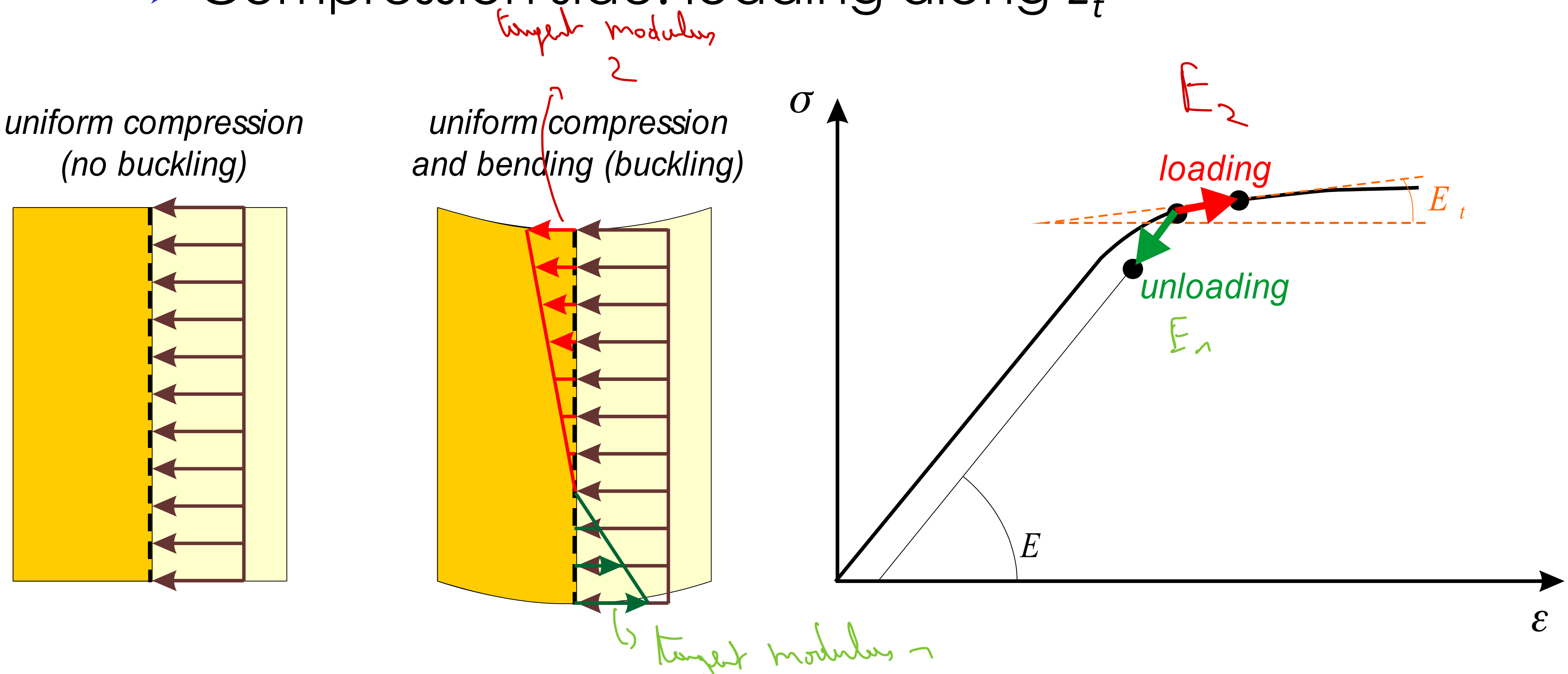
[*Bruhn*]

Inelastic column buckling



Beyond elasticity

- Buckling implies an asymmetry
  - Tension side: unloading along *E*
  - Compression side: loading along E_t



Analysis of the problem

- The beam must be treated as an inhomogeneous beam
  - Modulus-weighed section properties
  - Shift of neutral axis

$$\mathcal{A}^* = \mathcal{A}_u + \frac{E_t}{E}\mathcal{A}_l$$

$$\mathcal{I}^*_{yy} = \mathcal{I}_u + \frac{E_t}{E}\mathcal{I}_l$$

- The differential equation for buckling is unchanged

$$\frac{\partial^2 w}{\partial x^2} = -\frac{1}{E}\frac{M^*_y}{\mathcal{I}^*_{yy}}$$

Modified Euler equation

- Engesser (1869)
  - Use the tangent modulus in Euler equation

$$\sigma_{cr} = \frac{\pi^2 E_t}{(L/\rho)^2}$$

  - Requires an iterative procedure
  - Valid if loading on both sides
- Considère (1889) – Engesser (1889)
  - Use the reduced modulus in Euler equation

$$\sigma_{cr} = \frac{\pi^2 E_r}{(L/\rho)^2}$$

  - Valid for loading and unloading
  - Reduced modulus is section-dependent

Reduced modulus

- Defined from the beam equation to use the true area moment of inertia

$$E_r \, \mathcal{I} = E \, \mathcal{I}^*_{yy} = E_t \, \mathcal{I}_l + E \, \mathcal{I}_u$$

inertia

- Requires to locate the neutral axis

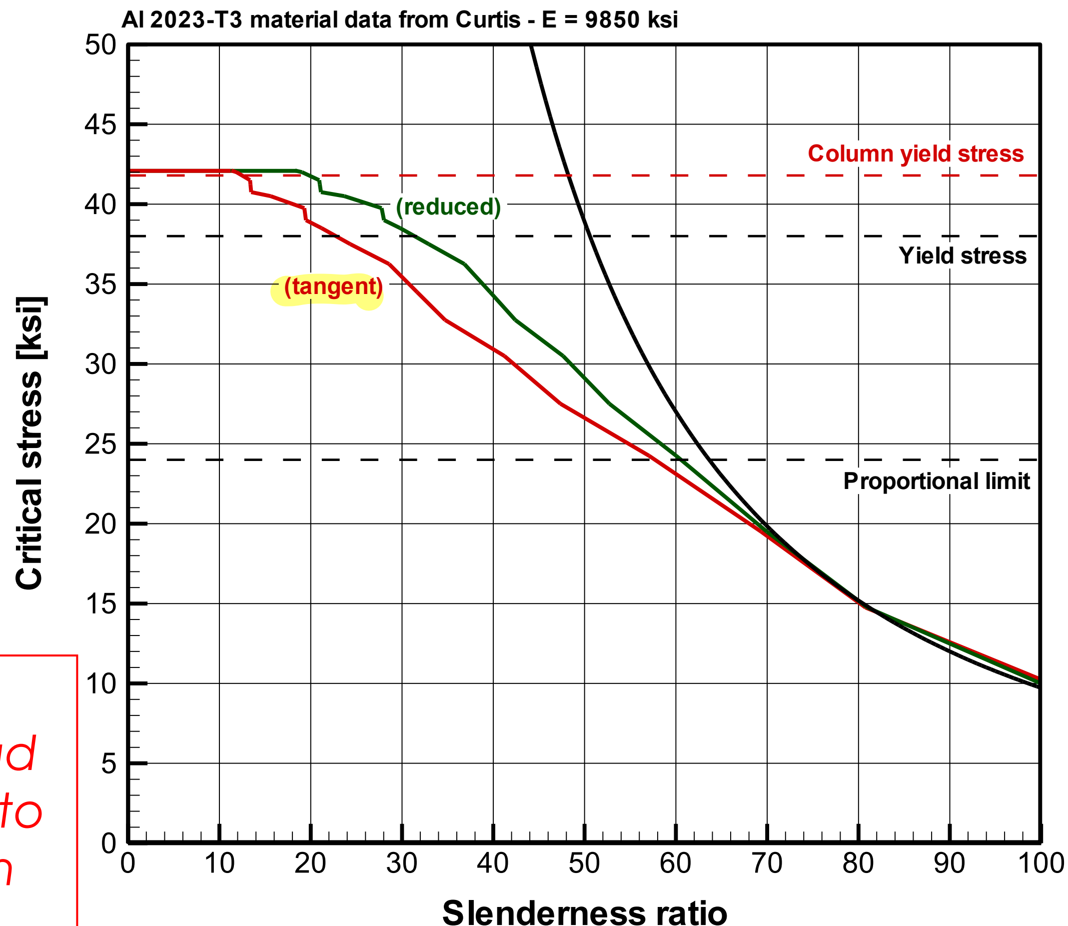
$$z^*_F = \frac{1}{\mathcal{A}^*} \left(\int_{\mathcal{A}_u} z \, \mathrm{d}\mathcal{A} + \frac{E_t}{E} \int_{\mathcal{A}_l} z \, \mathrm{d}\mathcal{A} \right)$$

- Results are section-dependent
  - Rectangular section $\quad \dfrac{E_r}{E} = \dfrac{4\,e}{\left(1+\sqrt{e}\,\right)^2}$
  - Zero-web I beam $\quad \dfrac{E_r}{E} = \dfrac{2\,e}{1+e}$

↳ beam with concentrated mass at both ends

Column curves

- Clear deviation
- Experience ?
  - Reduced (von Kármán)
  - Tangent (Wolford, …)
  - Shanley (1947



a perfect column will buckle under the tangent modulus load when the axial load is allowed to increase during buckling, which is the practical case.

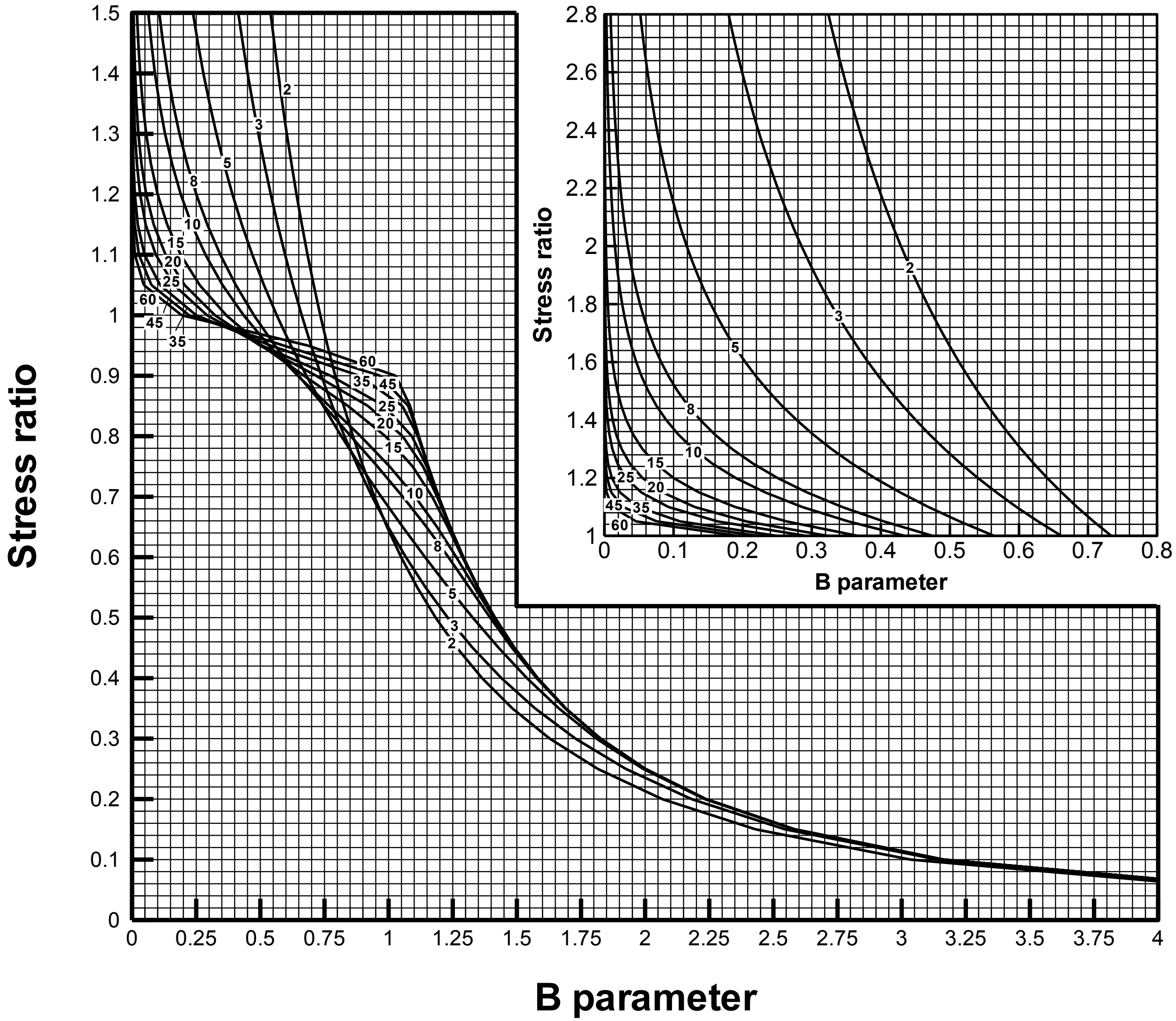
Cozzone-Melcon chart

- Limit of Engesser equation
  - Requires iterations, or
  - Requires full E_t material data
- Cozzone & Melcon (1946)
  - Take advantage of the Ramberg-Osgood material model
  - Provide a chart for direct computation of the buckling stress ratio $\sigma/\sigma_{0.7}$

    σ_c
  - Require to compute B

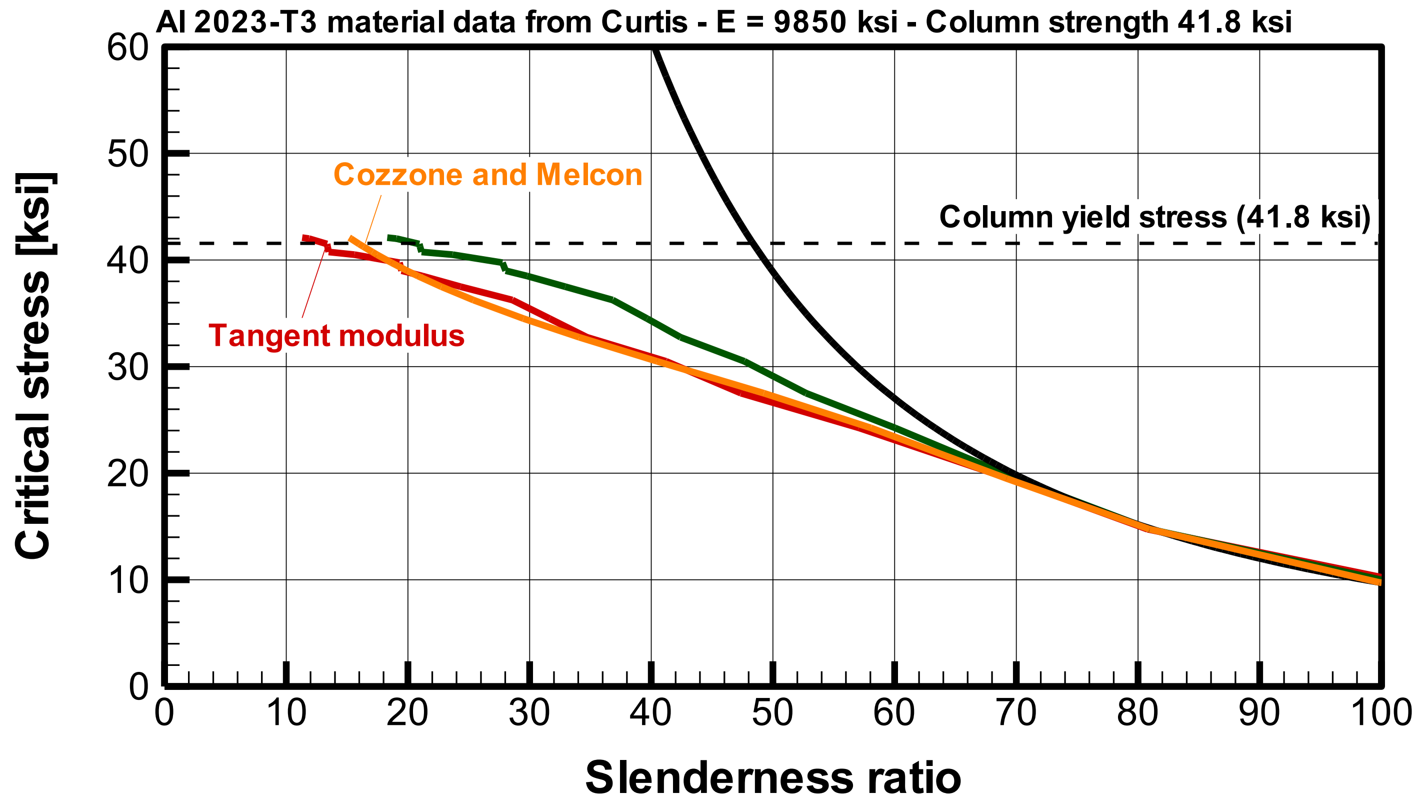
$$B = \frac{(L/\rho)}{\pi}\sqrt{\frac{\sigma_{0.7}}{E_c}}$$

Cozzone-Melcon chart



Cozzone-Melcon chart

- Gives good agreement with exact tangent modulus analysis

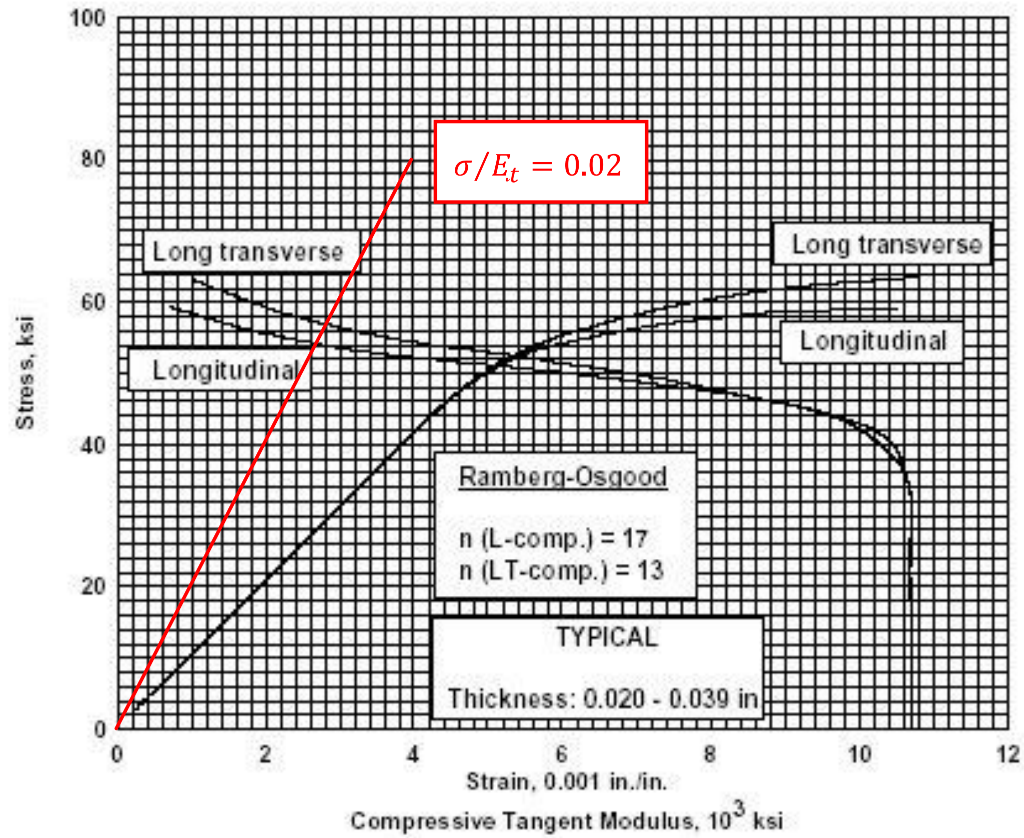


Use of Et charts

- Alternative method (MIL-HDBK-V)
- Superpose equation:

$$\frac{\sigma}{E_t} = \frac{\pi^2}{(L/\rho)^2} = K$$

on the tangent modulus chart



Johnson-Euler curves

- Analytical fit from ultimate stress to the Euler equation
- Material-dependent through ***n***
  - 1 for steel
  - 2 for aluminium

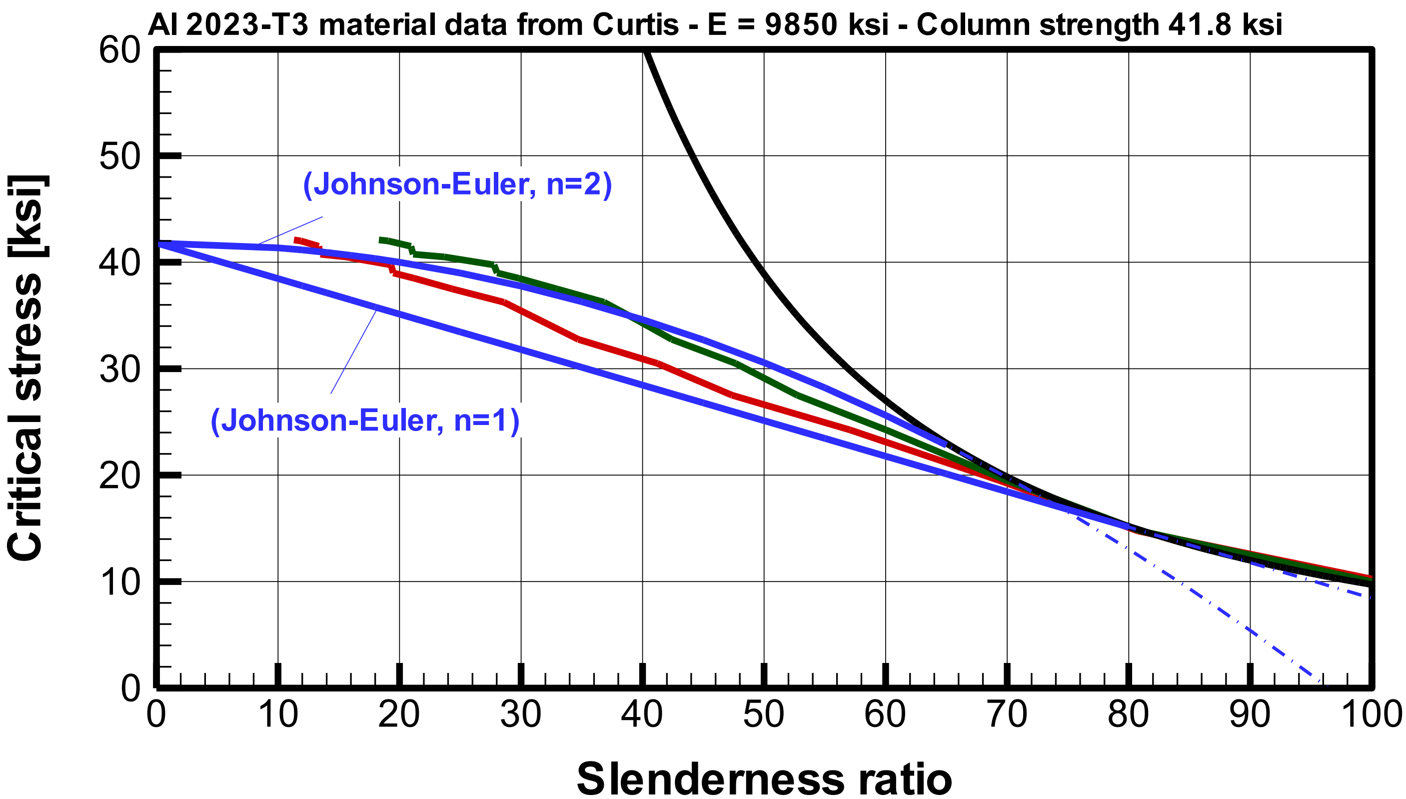
$$\sigma = \sigma_{cu} - K\lambda^n$$

- Slopes match at a critical slenderness

$$\begin{cases} \sigma = \sigma_{co} - \dfrac{\sigma_{cu}{}^2}{4\pi^2 E}(L/\rho)^2 \\ (L/\rho)_{cr} = \pi\sqrt{\dfrac{2E}{\sigma_{cu}}} \end{cases}$$

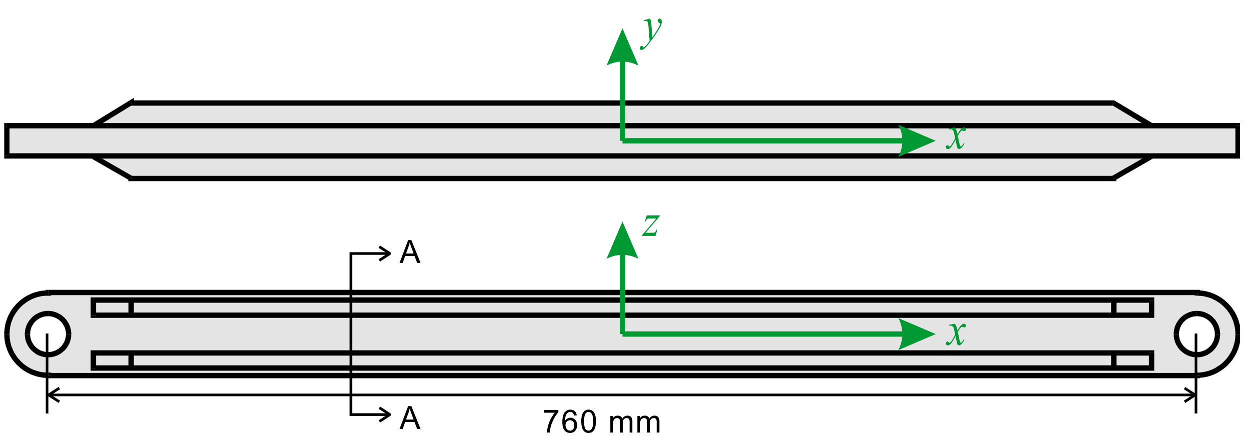
Johnson-Euler curves

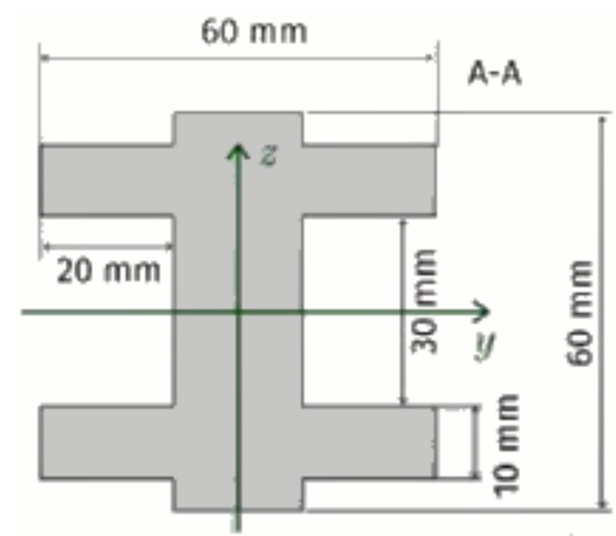
- Fair agreement in this case
  - Using the parabola may not be



Example exercise

- The following part can be subjected to compression loads. Determine the buckling stress. If inelastic buckling occurs, compare the various methods. The material is Aluminium alloy 7079-T6.

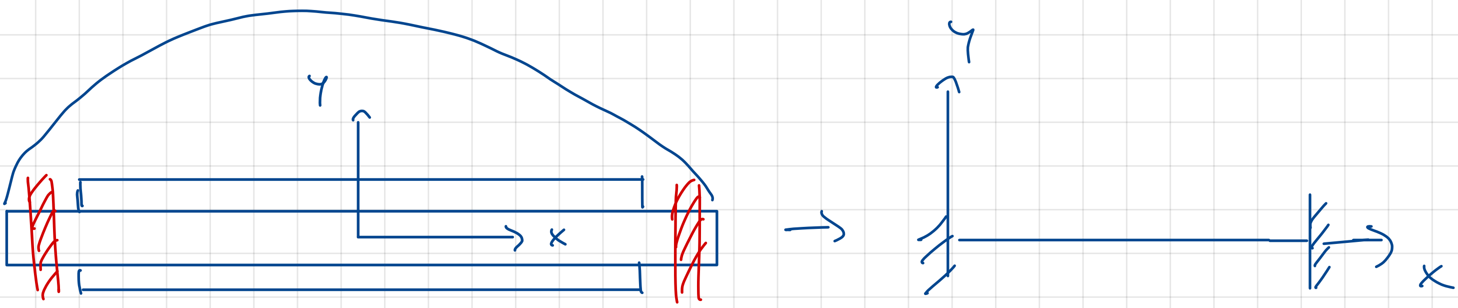


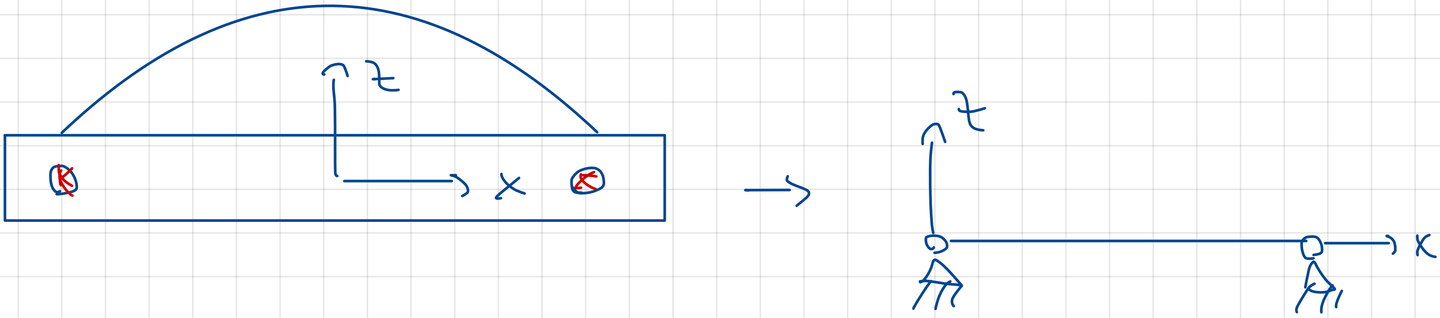


$\sigma_{cy} = 408\ \text{MPa}$ $\quad E_c = 72395\ \text{MPa}$

$\sigma_{cu} = 462\ \text{MPa}$

$n = 26$



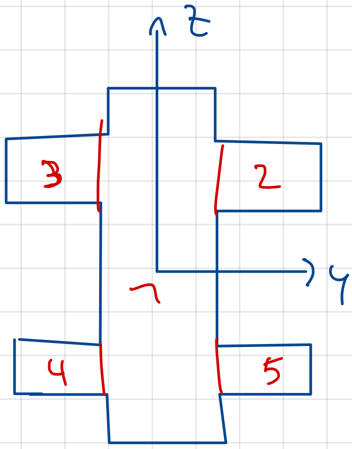


$A = ?$

$I_{yy} =$

$I_{zz} =$

	A	I_{yy}	I_{zz}	y_F	z_F	$A y_F^2$	$A z_F^2$	$A y_F z_F$
1								
2								
3								
4								
5								



$\dfrac{b t^3}{12} = I_{yy} \qquad \dfrac{t b^3}{12} = I_{zz}$

Another exercise

- Compute the critical load and determine the stresses in each part.
  - Material properties of Al 2024-T81 are:
    - $E = 72395$ MPa,
    - $\sigma_{\text{cu}} = 427$ MPa,
    - $\sigma_{\text{cy}} = 379$ MPa,
    - $\sigma_{0.7} = 386$ MPa,
    - $n = 10$.