Lecture 3

句式分析

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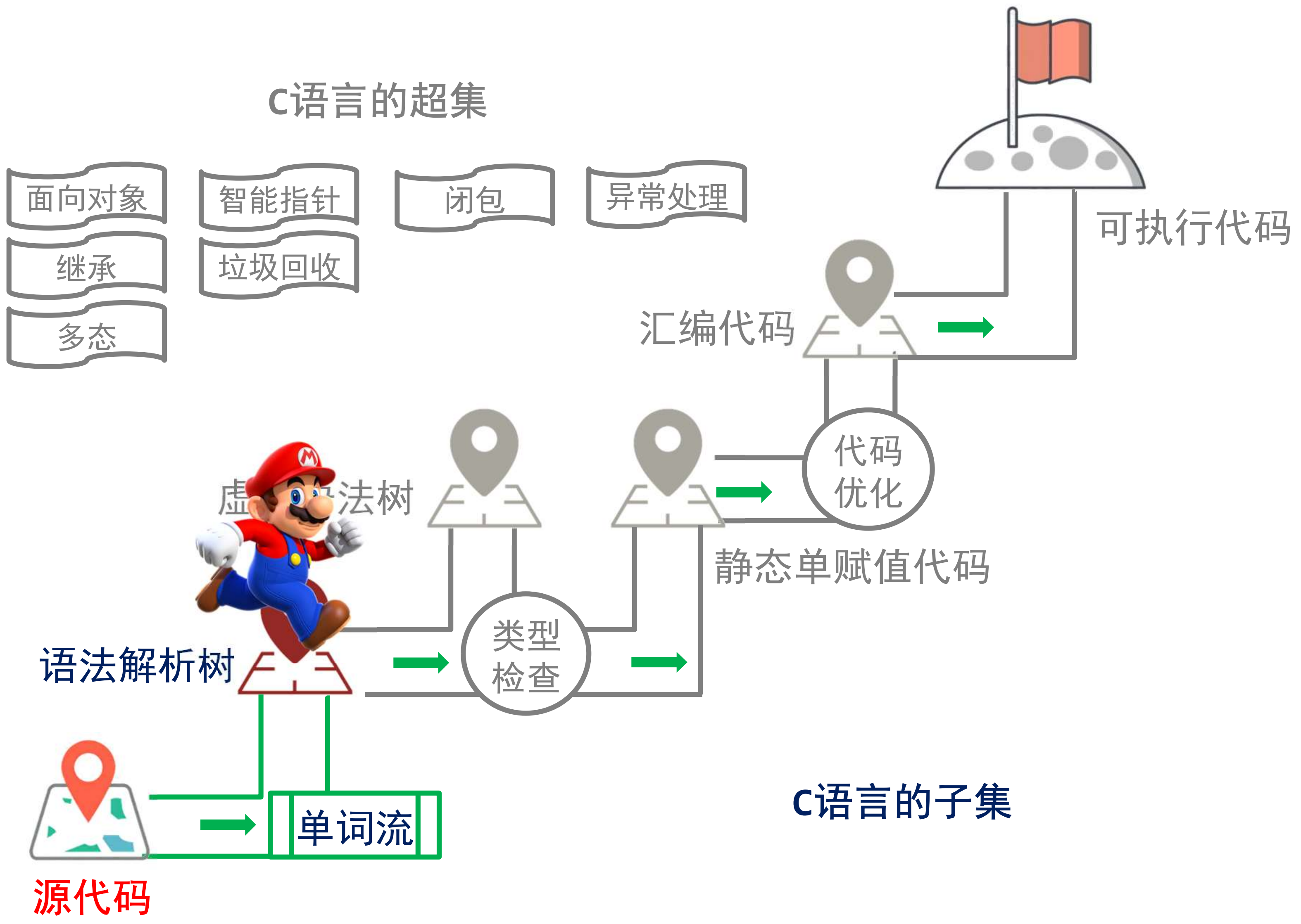
学习地图



C语言的超集
面向对象
智能指针
闭包
异常处理
继承
垃圾回收
多态
可执行代码
汇编代码
代码优化
虚 法树
静态单赋值代码
类型检查
语法解析树
单词流
源代码
C语言的子集

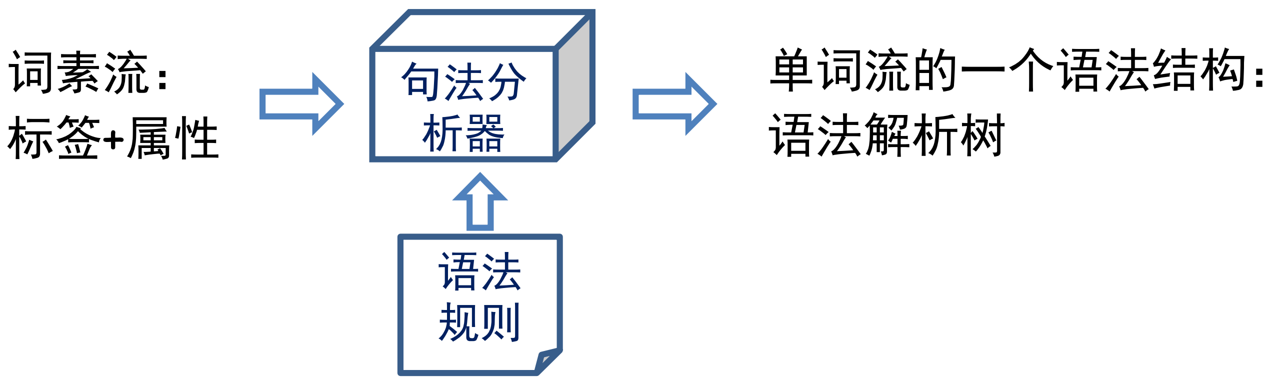
大纲

- 一、句式分析的基本概念
- 二、LLVM案例分析
- 三、自顶向下分析
- 四、自底向上分析
- 五、语法分析工具

一、句式分析的基本概念

问题定义

- 给定一个句子和语法规则，找到可生成该句子的一个语法推导。
- 通过词法分析已经将句子转换为了标签流。
- 语法规则（Grammar）定义了：
  - 什么是语法分析器（parser）可接受的标签组成，
  - 及其语法推导方式。



基本概念

- 一门语言（language）是多个句子（sentences）的集合。
- 句子（sentence）是由终结符（terminal symbols）组成的序列（sequence）。
- 字符串（string）是包含终结符和非终结符的序列。
  - 字符串符号：α, β, γ
  - 非终结符：X, Y, Z
  - 终结符（标签）：a, b, c
- 一条语法（grammar）包括一个开始符号S和多条推导规则（productions）
  - $X \rightarrow \beta$。

语法推导

- 语法G的语言$L(G)$是该语法可推导的所有句子的集合。
- 问题：下列语法是否可推导出句子$aaabbbccc$?

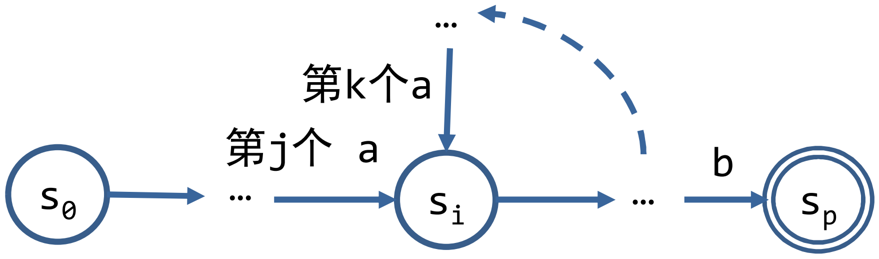
语法规则		推导	
[1]	$S \to aBSc$	[1]	$S \to aBSc$
[2]	$S \to abc$	[1]	$S \to aBaBScc$
[3]	$Ba \to aB$	[3]	$S \to aaBBScc$
[4]	$Bb \to bb$	[2]	$S \to aaBBabccc$
		[3]	$S \to aaBaBbccc$
		[3]	$S \to aaaBBbccc$
		[4]	$S \to aaaBbbccc$
		[4]	$S \to aaabbbccc$

语法表示：使用正则表达式？

- 正则表达式是否可识别四则运算？
  - $y = a \times x + b$
    - $(var|num)\big((+|-|\times|\div)\ (var|num)\big)^*$
  - $y = a \times (x + b)$
    - $('('|var|num)\big((+|-|\times|\div)\ ('('|var|num|')')\big)^*$
    - 可导致单词流被错误接收：
      - $y = (a \times (x + b)$
      - $y = (a \times (x + (b)$
- 正则表达式不能处理括号匹配问题：$(^*)^*$

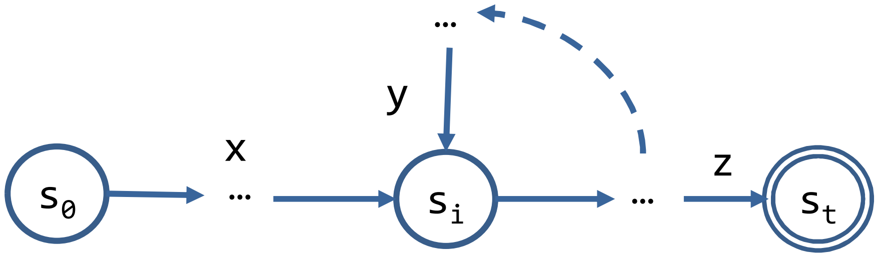
非正则语言

- 不能用正则表达式或有穷自动机表示的语言。
  - 正则语言不能计数，如$L = \{a^n b^n, n > 0\}$
  - 证明：
    - 假设DFA可识别该语言，其包含p个状态；
    - 假设某词素为$a^q b^q, q > p$。
    - 识别该词素需要经过某状态s_i至少两次，分别对应第j和第k个a；
    - 该DFA可同时接受$a^q b^q$和$a^{q+k-j} b^q$，推出矛盾。



正则语言的泵引理（Pumping Lemma）

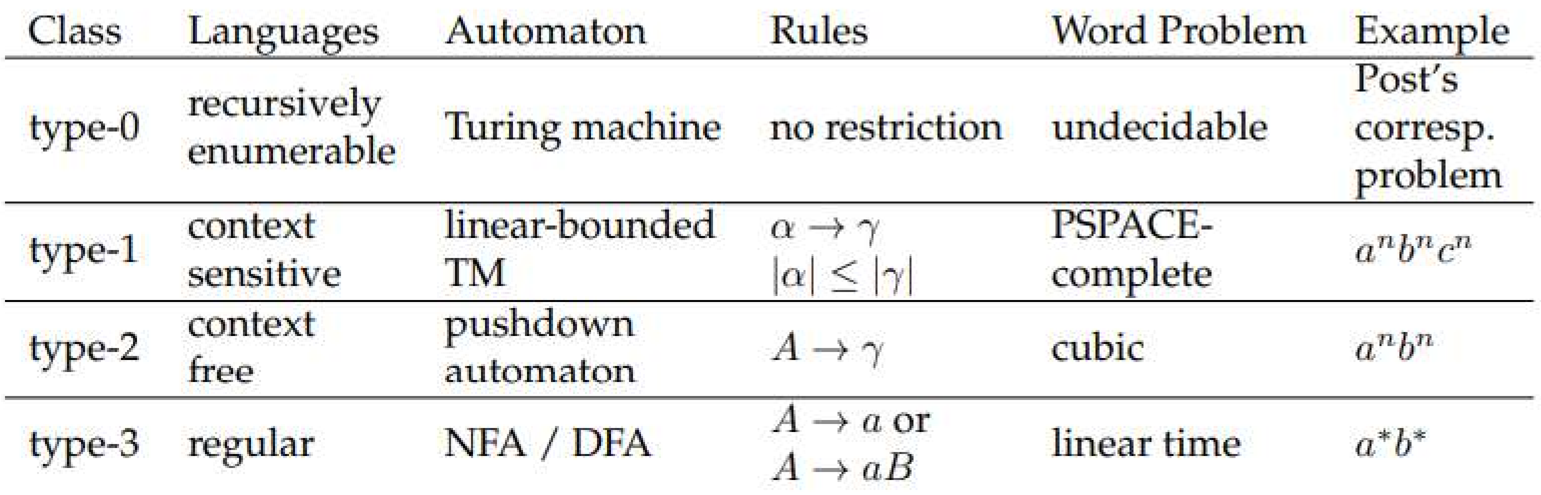
- 词素数量有限的语言一定是正则语言。
- 词素数量无穷多的语言是否为正则语言？
- 某语言$L(r)$是正则语言的必要条件：
  - 任意长度超过p（泵长）的句子都可以被分解为xyz的形式
  - 其中x和z可为空，
  - 子句y被重复任意次（如$xyyz$）后得到的句子仍属于该语言。



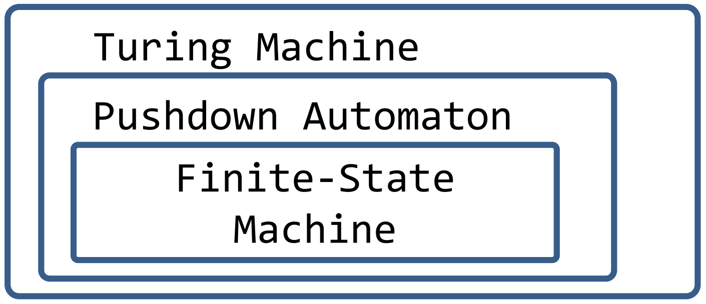
语言分析问题难度

- 通常来说，判断一个句子是否属于某个语言$w \in L(G)$是不能计算的。

Chomsky Hierarchy



Class	Languages	Automaton	Rules	Word Problem	Example
type-0	recursively enumerable	Turing machine	no restriction	undecidable	Post's corresp. problem
type-1	context sensitive	linear-bounded TM	$\alpha \to \gamma$ $\lvert\alpha\rvert \leq \lvert\gamma\rvert$	PSPACE-complete	$a^n b^n c^n$
type-2	context free	pushdown automaton	$A \to \gamma$	cubic	$a^n b^n$
type-3	regular	NFA / DFA	$A \to a$ or $A \to aB$	linear time	$a^* b^*$



上线文无关语法和BNF范式

- 上下文无关语法（CFG/context-free grammar）是一个四元组(T, NT, S, P)
  - T：终结符
  - NT：非终结符
  - S：起始符号
  - P：产生式规则集合$X \rightarrow \gamma$,
    - X 是非终结符
    - γ 是可能包含终结符和非终结符的字符串
- BNF范式（Backus-Naur form）是传统的上下文无关语法表示方法。

$$\boxed{\begin{array}{c} \langle SheepNoise \rangle ::= baa \ \langle SheepNoise \rangle \\ \mid baa \end{array}}$$

上线文无关语法举例

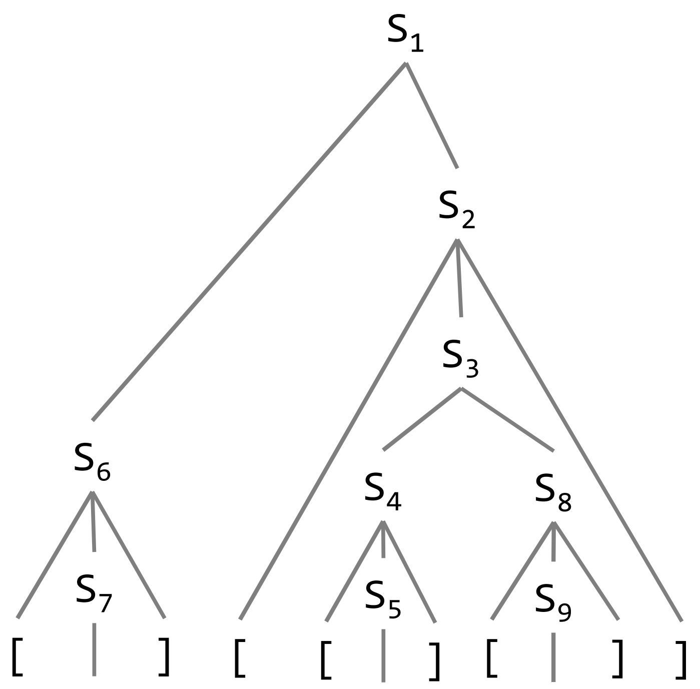
- 给定可生成所有匹配括号对的语法，[ ][[ ][ ]]是该语法的一个推导吗?

语法规则

$$\boxed{\begin{array}{lll}[1] & S \rightarrow \epsilon \\ [2] & \quad\;\; \mid [S] \\ [3] & \quad\;\; \mid \; SS\end{array}}$$

推导

$$\begin{array}{ll}
[3] & S \rightarrow SS \\
[2] & S \rightarrow S[S] \\
[3] & S \rightarrow S[SS] \\
[2] & S \rightarrow S[S[S]] \\
[1] & S \rightarrow S[S[\,]] \\
[2] & S \rightarrow S[[S][\,]] \\
[1] & S \rightarrow S[[\,][\,]] \\
[2] & S \rightarrow [S][[\,][\,]] \\
[1] & S \rightarrow [\,][[\,][\,]]
\end{array}$$



语法解析树

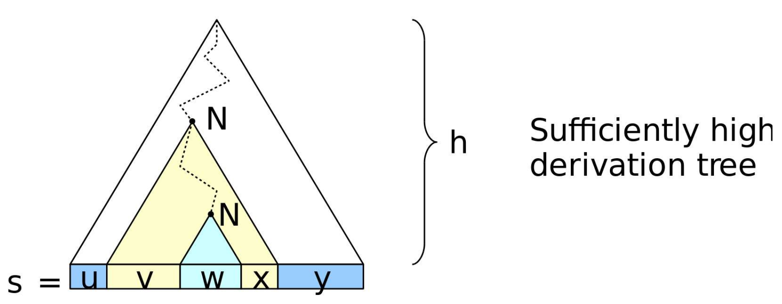
非CFG语言：上下文敏感语法

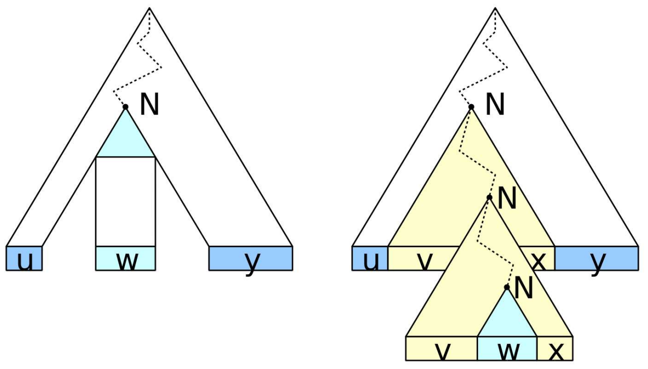
- $L = \{a^n b^n c^n, n > 0\}$不是CFG语言
- 无法用CFG定义
  - $X \to \gamma$
- 可以用上下文敏感语法定义
  - $\alpha A \beta \to \alpha \gamma \beta$
  - α和β不变，A展开

```
[1]  S → aBC
[2]     | aSBC
[3]  CB → CZ
[4]  CZ → WZ
[5]  WZ → WC
[6]  WC → BC
[7]  aB → ab
[8]  bB → bb
[9]  bC → bc
[10] cC → cc
```

非CFG语言的泵引理

- CFG语言的泵引理（必要条件）：
  - 任意长度超过p（泵长）的句子可以被拆分为$uvwxy$，
  - 子句v和x被重复任意次后得到的新句子（如$uvvwxxy$ ）仍属于该语言。
- 正则属于CFG：$uv^n w\epsilon^n\epsilon$





Generating uv^0wx^0y Generating uv^2wx^2y

澄清几个概念

- Regular language：用DFA/NFA可以计算
  - Regular expression
    - 狭义：其表示的都是正则语言，所有正则语言都可以用正则表达式表示（Flex工具）
    - 广义（regex）：字符串匹配工具。
- Context-free language：需要pushdown automaton计算
  - Context-free grammar
  - 正则语言可以用CFG表示：$StartSymbol \to regex$
    - 特性：右侧的非终结符均可替换为终结符
      - $S \to (0?1)^*$
      - $S \to 0S1S|1S0S|\epsilon$
- Context-sensitive language：需要图灵机计算

https://en.wikipedia.org/wiki/Regular_expression

下列语言是否为正则语言？

- 集合表示
  1) $\mathrm{L} = \{\mathrm{a}^{\mathrm{n}}\mathrm{b}^{\mathrm{n}}\}$
  2) $\mathrm{L} = \{\mathrm{a}^{\mathrm{n}}\mathrm{b}^{\mathrm{n}} | \mathrm{n} \le 100\}$
  3) $\mathrm{L} = \{\mathrm{a}^{\mathrm{n}} | \mathrm{n} \ge 1\}$
  4) $\mathrm{L} = \{\mathrm{a}^{2\mathrm{n}} | \mathrm{n} \ge 1\}$
  5) $\mathrm{L} = \{\mathrm{a}^{\mathrm{p}} |\ \mathrm{p\ is\ prime}\}$
- Regex/CFG语法表示
  1) $\mathrm{S} \to (0?1)^*$
  2) $\mathrm{S} \to aT|\epsilon, T \to Sb$
  3) $\mathrm{S} \to 0\mathrm{S}1\mathrm{S}|1\mathrm{S}0\mathrm{S}|\epsilon$
  4) $\mathrm{S} \to \mathrm{A}|\mathrm{B}$
     - $A \to E1A'E$
     - $A' \to A|\epsilon$
     - $B \to E0B'E$
     - $B' \to B|\epsilon$
     - $E \to 0S1S|1S0S|\epsilon$

推导（Derivation）的优先级

暂不考虑优先级

$$\begin{array}{ll}
[1] & Expr \rightarrow (Expr) \\
[2] & \quad\quad \mid Expr\ Op\ num \\
[3] & \quad\quad \mid num \\
[4] & Op \rightarrow + \\
[5] & \quad\quad \mid - \\
[6] & \quad\quad \mid \times \\
[7] & \quad\quad \mid \div
\end{array}$$

(a + b) × 2

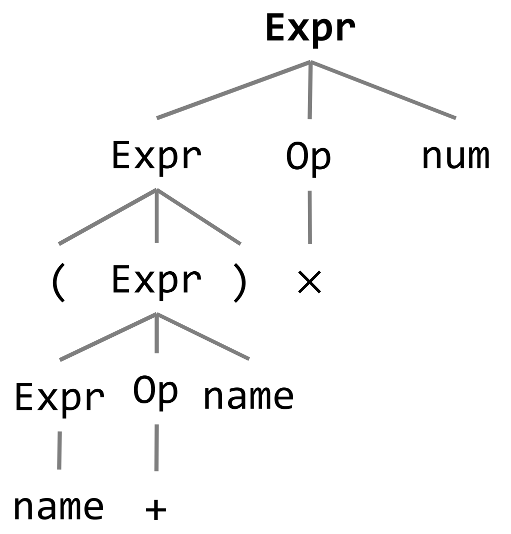
右侧优先推导
（Rightmost Derivation）

$$\begin{array}{ll}
 & Expr \\
[2] & \underset{rm}{\Rightarrow} Expr\ Op\ num \\
[6] & \underset{rm}{\Rightarrow} Expr\ \times\ num \\
[1] & \underset{rm}{\Rightarrow} (Expr) \times\ num \\
[2] & \underset{rm}{\Rightarrow} (Expr\ Op\ num) \times num \\
[4] & \underset{rm}{\Rightarrow} (Expr + num) \times num \\
[3] & \underset{rm}{\Rightarrow} (num + num) \times num
\end{array}$$

(a + b) × 2

$$\begin{array}{ll}
 & Expr \\
[2] & \underset{lm}{\Rightarrow} Expr\ Op\ num \\
[1] & \underset{lm}{\Rightarrow} (Expr)\ Op\ num \\
[2] & \underset{lm}{\Rightarrow} (Expr\ Op\ num) Op\ num \\
[3] & \underset{lm}{\Rightarrow} (num\ Op\ num)\ Op\ num \\
[4] & \underset{lm}{\Rightarrow} (num + num)\ Op\ num \\
[6] & \underset{lm}{\Rightarrow} (num + int) \times num
\end{array}$$

左侧优先推导
（Leftmost Derivation）



语法解析树完全相同

练习：语法推导

给定下列语法和字符串$aa+a*$

1） 写出左推导

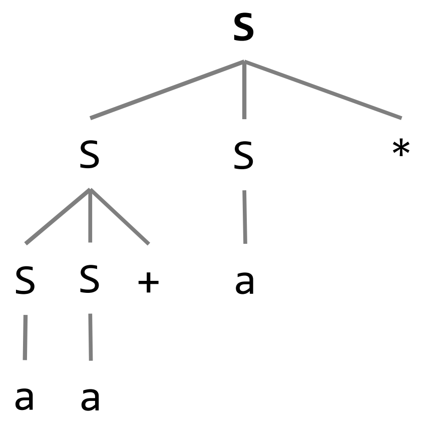
2） 写出右推导

3） 画出语法推导树

[1] $S \rightarrow SS+$
[2] $\quad | SS*$
[3] $\quad |a$

$$S \underset{lm}{\Rightarrow} SS* \underset{lm}{\Rightarrow} SS+S* \underset{lm}{\Rightarrow} aS+S* \underset{lm}{\Rightarrow} aa+S* \underset{lm}{\Rightarrow} aa+a*$$

$$S \underset{rm}{\Rightarrow} SS* \underset{rm}{\Rightarrow} Sa* \underset{rm}{\Rightarrow} SS+a* \underset{rm}{\Rightarrow} Sa+a* \underset{rm}{\Rightarrow} aa+a*$$



练习：语法设计

- 为下列语言设计语法规则。
  1) 所有0和1组成的字符串，每一个0后面紧跟着若干个1
  2) 所有0和1组成的字符串，0和1的个数相同
  3) 所有0和1组成的字符串，0和1的个数不相同

$$S \to (0?\,1)^*$$

$$S \to 0S1S|1S0S|\epsilon$$

$$S \to A|B$$
$$A \to E1A'E$$
$$A' \to A|\epsilon$$
$$B \to E0B'E$$
$$B' \to B|\epsilon$$
$$E \to 0S1S|1S0S|\epsilon$$

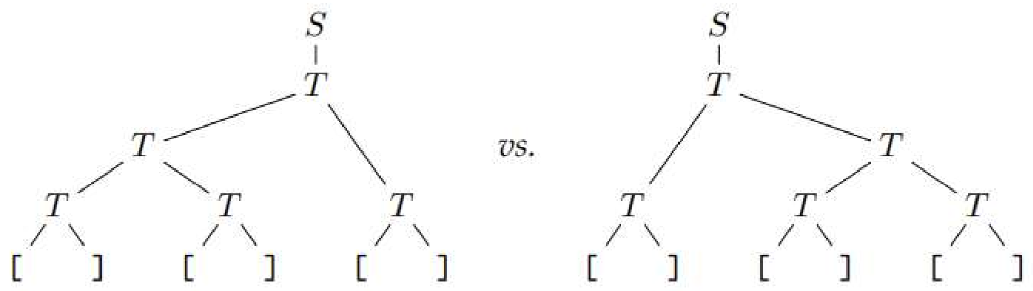
二义性（ambiguity）

- 如果L(G)中的某个句子有一个以上的最左（或最右）推导，那么语法G就有二义性。
  - 语法解析树不同
- 根据下列语法规则如何推导出[ ][ ][ ]?

[1] $S \to \epsilon$
[2] $\quad | \, T$
[3] $T \to []$
[4] $\quad | \, [T]$
[5] $\quad | \, TT$

$$S \to T \to TT \to TTT \to [\,]TT \to T[\,][\,] \to [\,][\,][\,]$$

$$S \to T \to TT \to [\,]T \to [\,]TT \to [\,][\,]T \to [\,][\,][\,]$$



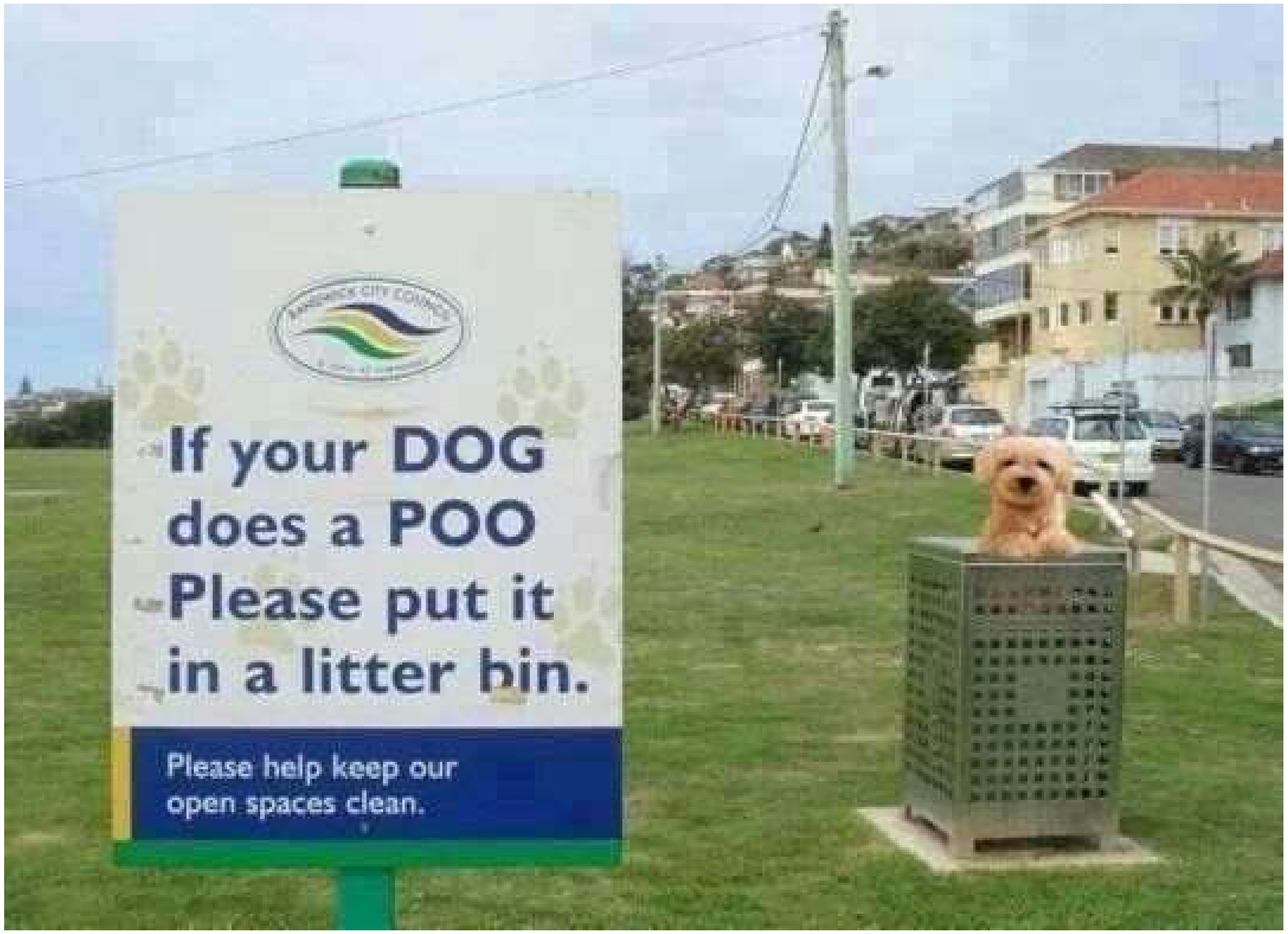
极端情况

- 存在无数棵语法解析树
  - 考虑循环的情况
- 根据下列语法规则如何推导出[ ][[ ][ ]]?

[1]	$S \to \epsilon$
[2]	$\mid [S]$
[3]	$\mid SS$

$$S \to SS \to [S]S \to [\,]S \to [\,][S] \to [\,][S\ S] \to [\,][[S]\ [S]] \to [\,][[\,][\,]]$$

$$S \to SS \to S \to SS \to \cdots$$


RANDWICK CITY COUNCIL
If your DOG does a POO Please put it in a litter bin.
Please help keep our open spaces clean.

消除二义性

[1] $S \to \epsilon$
[2] $\quad | \ [S]$
[3] $\quad | \ SS$

消除循环引起的二义性

$S \to SS \to S$

[1] $S \to \epsilon$
[2] $\quad | \ T$
[3] $T \to []$
[4] $\quad | \ [T]$
[5] $\quad | \ TT$

[1] $S \to \epsilon$
[2] $\quad | \ T$
[3] $T \to UT$
[4] $\quad | \ U$
[5] $U \to [\,]$
[6] $\quad | \ [T]$

推导[ ][ ][ ]的例子

$T \to TT$ 左递归容易引起二义性（回溯语法）

将语义加入语法中：四则运算的例子

```
[1] Expr → Expr Bop Expr
[2]        | num
[3]        | (Expr)
[4]  Bop → +
[5]        | −
[6]        | ×
[7]        | ÷
```

3 + 4 × 5的语义？

$Expr$
$\rightarrow Expr\ Bop\ Expr$
$\rightarrow Expr\ Bop\ Expr\ Bop\ Expr$
$\rightarrow Expr + Expr \times Expr$

$(3+4)\times 5$

$3+(4\times 5)$

将运算符特性加入到语法规则中：

- 优先级：( ) >×/÷>+/−
- 结合性：左结合

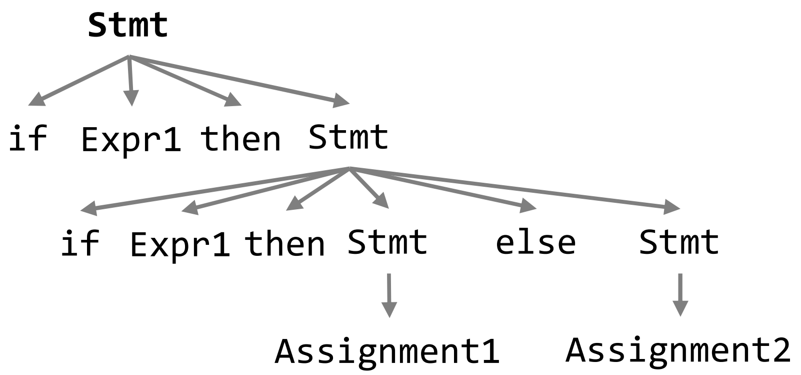
$Expr$
$\rightarrow Expr\ AMop\ Term$
$\rightarrow Term\ AMop\ Term$
$\rightarrow Term\ AMop\ Term\ MDop\ Factor$
$\rightarrow \cdots$

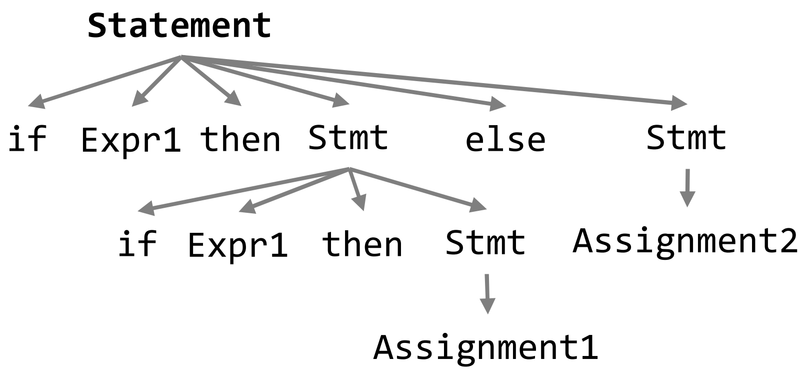
```
[1] Expr → Expr AMop Term
[2]          | Term
[3] Term → Term MDop Factor
[4]          | Factor
[5] Facor → (Expr)
[6]          | num
[7]  AMop → +
[8]          | −
[9]  MDop → ×
[10]         | ÷
```

If-Else嵌套的二义性语法

```
[1]  Stmt → if Expr then Stmt else Stmt
[2]         | if Expr then Stmt
[3]         | Assignment
[4]         | …
```

```
if Expr1 then if Expr2 then Assignment1 else Assignment2
```





```
if Expr1 then
  if Expr2 then
    Assignment1
  else
    Assignment2
```

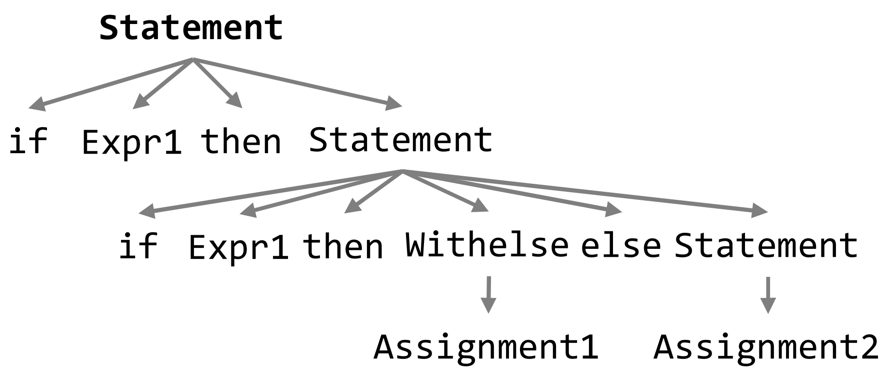
```
if Expr1 then
  if Expr2 then
    Assignment1
else
    Assignment2
```

消除If-Else语法的二义性

- 将语义编码加入到结构中
  - 要求else优先匹配内层if
    - 如果外层出现else，则内部嵌套的if语句一定有匹配的else

[1] $Stmt \rightarrow$ if $Expr$ then $Withelse$ else $Stmt$	[1] 有else配套的if语句，可继续展开[4]
[2] $\quad\quad\quad\quad$ | if $Expr$ then $Stmt$	[2] 无配套else的if语句
[3] $\quad\quad\quad\quad$ | $Assignment$	[3]
[4] $Withelse \rightarrow$ if $Expr$ then $Withelse$ else $Withelse$	[4] 内层有配套else的if语句
[5] $\quad\quad\quad\quad$ | $Assignment$	[5]

```
if Expr1 then if Expr2 then Assignment1 else Assignment 2
```



```
if Expr1 then
  if Expr2 then
    Assignment1
  else
    Assignment2
```

不存在其它推导方式

练习：二义性分析

- 下列If-Else语法是否存在二义性？

```
[1]  Stmt → if Expr then Stmt
[2]            | matchedStmt
[3]  matchedStmt → if Expr then matchedStmt else Stmt
[4]            | Assignment
```

```
if Expr1 then if Expr2 then Assignment1 else Assignment 2
```
✓

```
if Expr1 then if Expr2 then Assignment1 else if Expr3 then Assignment2 else Assignment3
```
✗

```
if Expr1 then
    if Expr2 then
        Assignment1
    else
        if Expr3 then
            Assignment2
        else
            Assignment3
```

```
if Expr1 then
    if Expr2 then
        Assignment1
    else
        if Expr3 then
            Assignment2
else
     Assignment3
```

练习

- 为描述正则语言的正则表达式语法设计一种CFG
  - 支持字符[A-Za-z0-9]
  - 支持连接、或|、闭包*运算
  - 支持()
- 检查语法是否有二义性？

[1] $<regex> ::= <union> \mid <concat> \mid <closure> \mid <term>$
[2] $<union> ::= <regex>$ "|" $<regex>$
[3] $<concat> ::= <regex><regex>$
[4] $<closure> ::= <regex>*$
[5] $<term> ::= <group> \mid <alphanum>$
[6] $<group> ::= (<regex>)$
[7] $<alphanum> ::= A|\ldots|Z|a|\ldots|z|0|\ldots|9$

- 主要问题：
  - 未考虑运算的优先级：比如解析ab|c存在歧义。
  - 一般按照运算符优先级由低到高依次展开

改写

$$
\begin{aligned}
&< regex >::=< union > \mid < concat > \mid < closure > \mid < term >\\
&< union >::=< regex > "|" < regex >\\
&< concat >::=< regex >< regex >\\
&< closure >::=< regex >*\\
&< term >::=< group > \mid < alphanum >\\
&< group >::= (< regex >)\\
&< alphanum >::= A|\dots|Z|a|\dots|z|0|\dots|9
\end{aligned}
$$

$$\Downarrow$$

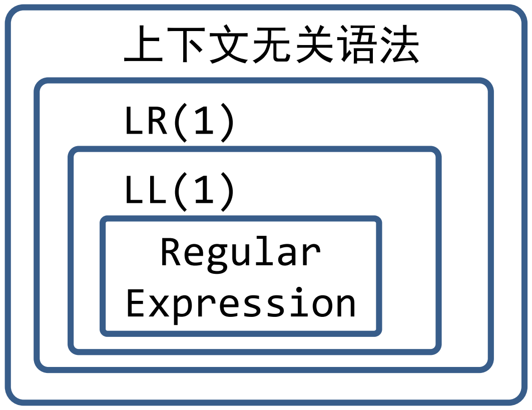
$$
\begin{aligned}
&< regex >::=< union > \mid < concat >\\
&< union >::=< regex > "|" < concat >\\
&< concat >::=< concat >< term > \mid < term >\\
&< term >::=< element >* \mid < element >\\
&< element >::= (< regex >) \mid < alphanum >\\
&< alphanum >::= A|\dots|Z|a|\dots|z|0|\dots|9
\end{aligned}
$$

思考

1) 用正则表达式可以定义所有的正则语言吗？
2) 用有穷自动机（正则表达式模拟器）可以解析任意正则表达式吗？
3) 用CFG可以定义任意正则语言吗？
4) 用CFG可以定义任意CFL语言吗？
5) 用pushdown automaton（CFG模拟器）可以解析任意正则表达式吗？
6) 用pushdown automaton可以解析任意CFG吗？
7) 用通用图灵机可以解析任意CFG吗？
8) 用通用图灵机可以解析任意程序吗？

编译器的任务：找到语法树推导

- 方法：
  - 自顶向下（top-down parser）
  - 自底向上（bottom-up parser）
- 语法难度：CFG>LR(1)>LL(1)>RE
  - 任意CFG需要花费更多时间进行语法分析
    - Earley/CYK算法复杂度O(n^3)
  - LL(1)是LR(1)的一个子集
    - Left-to-Right，Leftmost
    - 前瞻单词1个
    - 适合自顶向下分析
  - LR(1)是无歧义CFG的一个子集
    - Left-to-Right，Rightmost
    - 前瞻单词1个
    - 适合自底向上分析



二、LLVM案例分析

LR(0)

LR(1): SLR/LALR

Clang采用自顶向下分析算法

A single unified parser for C, Objective C, C++, and Objective C++

Clang is the "C Language Family Front-end", which means we intend to support the most popular members of the C family. We are convinced that the right parsing technology for this class of languages is a **hand-built recursive-descent parser**. Because it is plain C++ code, recursive descent makes it very easy for new developers to understand the code, it easily supports ad-hoc rules and other strange hacks required by C/C++, and makes it straight-forward to implement excellent diagnostics and error recovery.

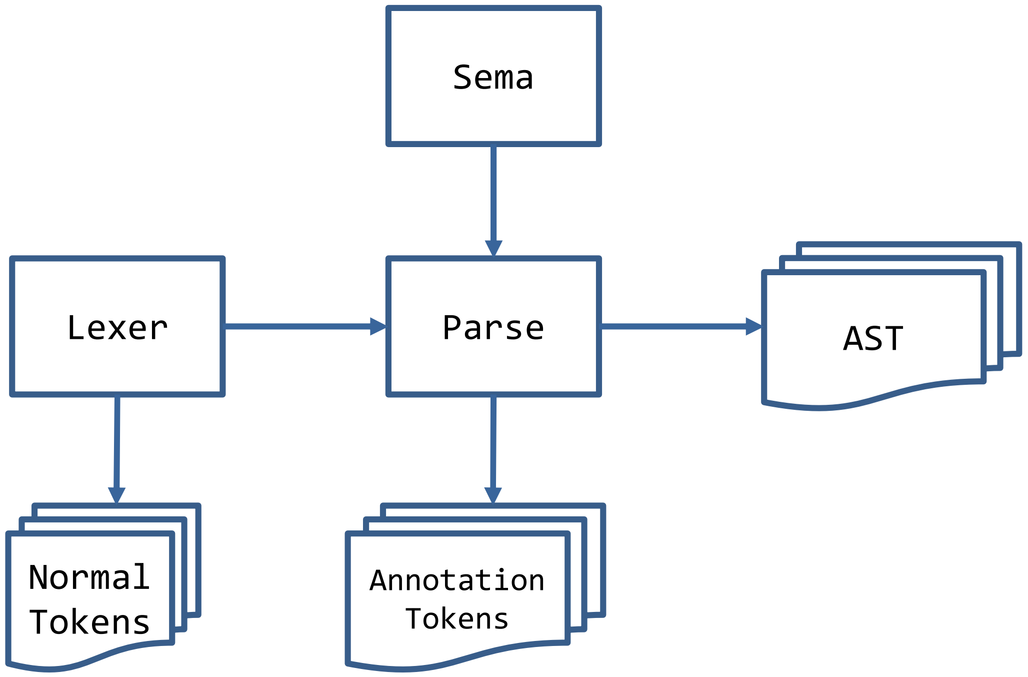
We believe that implementing C/C++/ObjC in a single unified parser makes the end result easier to maintain and evolve than maintaining a separate C and C++ parser which must be bugfixed and maintained independently of each other.

https://clang.llvm.org/features.html#unifiedparser

作业回顾

$$
\begin{array}{lcl}
<program> & ::= & <gdecl>^{*}<function>^{*} \\
<gdecl> & ::= & extern <prototype>; \\
<function> & ::= & <prototype><body> \\
<prototype> & ::= & <type><ident>(<paramlist>) \\
<paramlist> & ::= & \epsilon| <type><ident>[,<type><ident>]^{*} \\
<body> & ::= & \{<stmt>\} \\
<stmt> & ::= & <exp> \\
<exp> & ::= & (<exp>)| <const> | <ident> | \\
 & & <exp><binop><exp> | <callee> \\
<callee> & ::= & <ident> (\epsilon| <exp> [,<exp>]^{*}) \\
<ident> & ::= & [A-Z\_a-z][0-9A-Z\_a-z]^{*} \\
<const> & ::= & <intconst> | <doubleconst> \\
<binop> & ::= & + \mid \text{-} \mid * \mid < \\
<intconst> & ::= & [0-9][0-9]^{*} \\
<doubleconst> & ::= & <intconst> . <intconst> \\
<type> & ::= & int|double
\end{array}
$$

LLVM前端架构

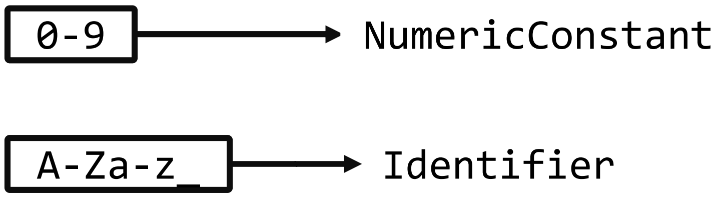


Lexer::LexTokenInternal()

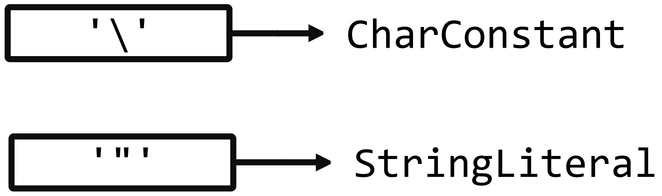
```
bool Lexer::LexTokenInternal(Token &Result, bool TokAtPhysicalStartOfLine) {
LexNextToken:
  // New token, can't need cleaning yet.
  Result.clearFlag(Token::NeedsCleaning);
  Result.setIdentifierInfo(nullptr);
  const char *CurPtr = BufferPtr;
  if (isHorizontalWhitespace(*CurPtr)) {
    do {
      ++CurPtr;
    } while (isHorizontalWhitespace(*CurPtr));
    BufferPtr = CurPtr;
    Result.setFlag(Token::LeadingSpace);
  }
  char Char = getAndAdvanceChar(CurPtr, Result);
  tok::TokenKind Kind;
  switch (Char) {
      case '0': case '1': case '2': case '3': case '4':
      case '5': case '6': case '7': case '8': case '9’:
          MIOpt.ReadToken();
          return LexNumericConstant(Result, CurPtr);
      case …
```

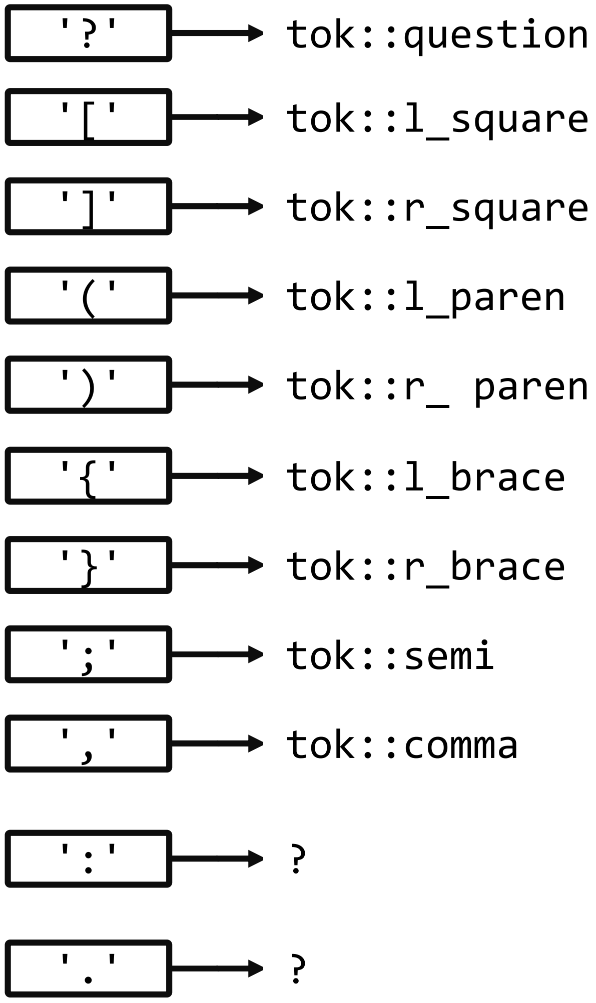
https://github.com/llvm/llvm-project/blob/main/clang/lib/Lex/Lexer.cpp

根据首字符大致分类

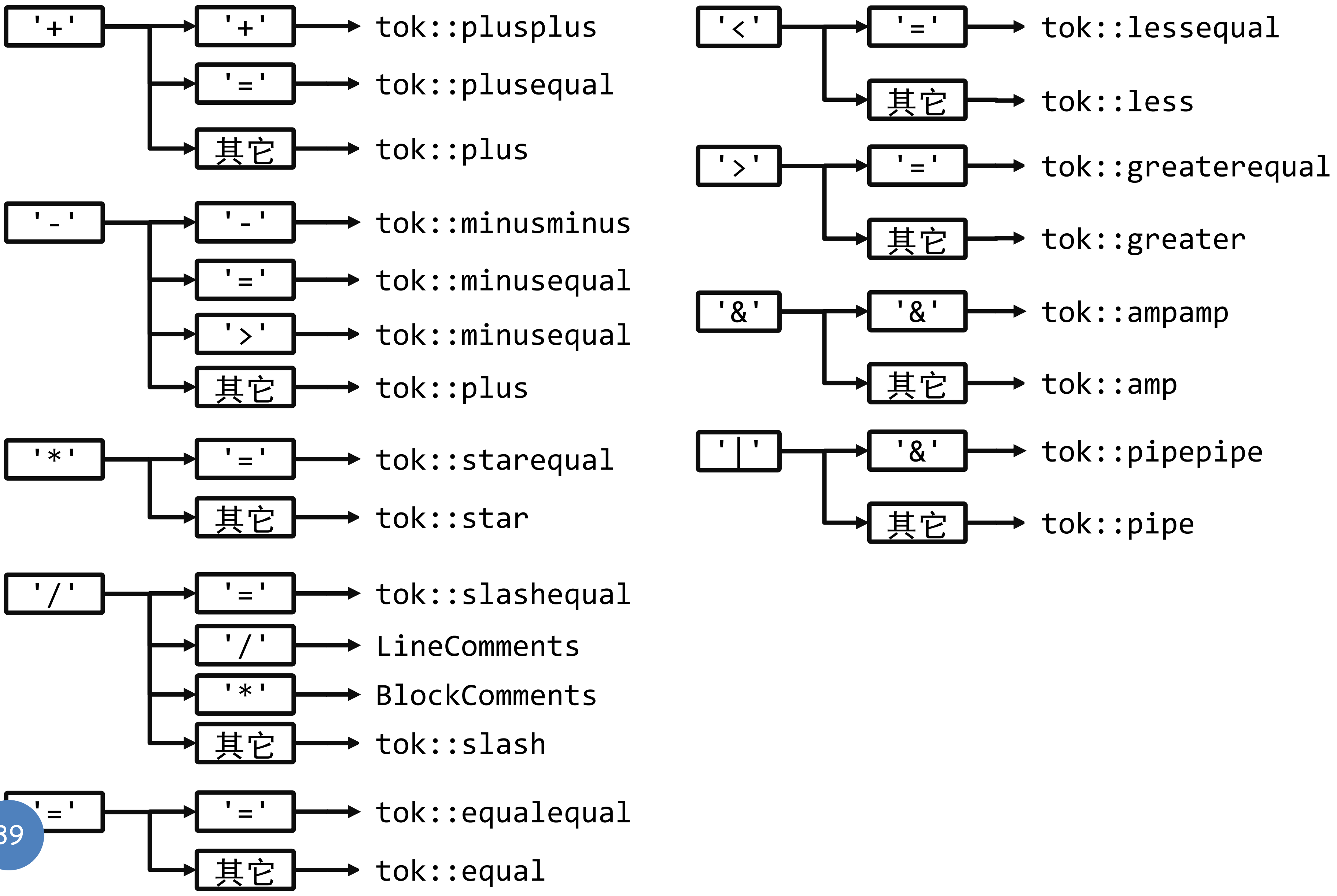


特殊情况：
1） 保留字
2） u/U/L





根据前两个字符大致分类



Token类

- 单个token，供lexer和parser使用，AST中不存在。
- Sizeof(Token)是16字节
- 分为两类：
  - Normal tokens：lexer返回的结果
    - 标识符信息（指向用于检索的哈希值）
    - Token类型（参照TokenKinds.def中的定义）
    - 位置、长度、Flags
    - 缺少了什么？
      - 数值信息
  - Annotation tokens：parser处理后的结果，添加了语义信息

Parser::ParseStatementOrDeclarationAfterAttributes()

```
Parser::ParseStatementOrDeclarationAfterAttributes(
    StmtVector &Stmts, ParsedStmtContext StmtCtx,
    SourceLocation *TrailingElseLoc, ParsedAttributesWithRange &Attrs) {
    tok::TokenKind Kind = Tok.getKind();
    switch (Kind) {
        case tok::at: // May be a @try or @throw statement
        {
            AtLoc = ConsumeToken();  // consume @
            return ParseObjCAtStatement(AtLoc, StmtCtx);
        }
        case tok::identifier: {
           Token Next = NextToken();
           if (Next.is(tok::colon)) { // C99 6.8.1: labeled-statement
               return ParseLabeledStatement(Attrs, StmtCtx);
           }
           if (Next.isNot(tok::coloncolon)) {  … }
           default: {
               …
           }
        }
        …
    }
}
```

https://github.com/llvm/llvm-project/blob/main/clang/lib/Parse/ParseStmt.cpp

根据首字符大致分类

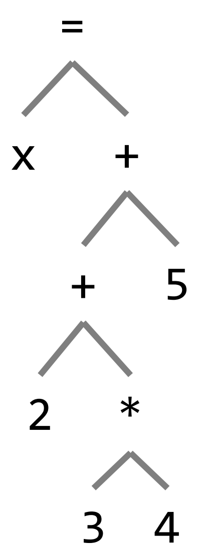
tok::identifier	ParseExprStatement
	ParseDeclaration
tok::l_brace:	ParseCompoundStatement
tok::kw_if:	ParseIfStatement
tok::kw_switch:	ParseSwitchStatement
tok::kw_case:	ParseCaseStatement
tok::kw_while:	ParseWhileStatement
tok::kw_do:	ParseDoStatement
tok::kw_for:	ParseForStatement
tok::kw_break:	ParseBreakStatement
tok::kw_default:	ParseDefaultStatement
tok::kw_continue:	ParseContinueStatement
tok::kw_goto:	ParseGotoStatement
tok::kw_return:	ParseReturnStatement

clang/lib/Parse/ParseStmt.cpp

如何解析表达式?

x = 2 + 3 * 4 + 5

- 根据运算符优先级,
- 假设*>+>=



Pratt Parsing

```
优先级
=: [1,2]
+: [3,4]
*: [5,6]

Parse(token, precedence) {
  left = token.next();
  if left.type != tok::num
    return -1;
  while true:
    op = token.peek();
    if op.tokentype != tok::binop
      return -1;
    lp, rp = Precedence[op];
    if lp < precedence
      break;
    token.next();
    right = Parse(token, rp)
    left = (op, left, right)
  return left
}
```

0 1 2 3 4 5 6 3 4 0

x = 2 + 3 * 4 + 5

Parse(start,0)
op: =,
Parse(=,2)
op: +
Parse(+,4)
op: *
Parse(*,+)
Return left = 4
left = *(3, 4)

0 1 2 3 4 ~~5 6~~ 3 4 0

x = 2 + ~~3 * 4~~ + 5

Return left = +(2, *(3, 4))

0 1 2 ~~3 4~~ ~~5 6~~ 3 4 0

x = ~~2 + 3 * 4~~ + 5

Parse(pre5,+)
Return left = +(+(2, *(3, 4)), 5)

0 1 2 ~~3 4~~ ~~5 6 3 4 0~~

x = ~~2 + 3 * 4 + 5~~

Return left = = (x, +(+(2, *(3, 4)), 5))

三、自顶向下分析

Top-down

Recursive-descent

自顶向下构建语法解析树

如何解析(num+num)×num?

[1] $Expr \rightarrow Expr + Term$
[2] $\quad | \, Expr - Term$
[3] $\quad | \, Term$
[4] $Term \rightarrow Term \times Factor$
[5] $\quad | \, Term \div Factor$
[6] $\quad | \, Factor$
[7] $Facor \rightarrow (Expr)$
[8] $\quad | \, num$
[9] $\quad | \, id$

假设每次都能选对规则

word	cur	Rule	Stack
(	Expr	[3]	Term
(	Term	[4]	Term, ×, Factor
(	Term	[6]	Factor, ×, Factor
(	Factor	[7]	(, Expr,), ×, Factor
(	(	-	Expr,), ×, Factor
num	Expr	[1]	Expr,+, Term,), ×, Factor
num	Expr	[3]	Term,+, Term,), ×, Factor
num	Term	[6]	Factor,+, Term,), ×, Factor
num	Term	[8]	num,+, Term,), ×, Factor
num	num	-	+, Term,), ×, Factor
+	+	-	Term,), ×, Factor
num	Term	[6]	Factor,), ×, Factor
num	Factor	[8]	num,), ×, Factor
num	num	-	), ×, Factor
)	)	-	×, Factor
×	×	-	Factor
num	Factor	[8]	num
num	num	-	null
eof			

自动搜索语法树的算法…

```
输入: 程序单词流 seq;
       CFG语法 rules;
Output: accept: 语法解析树ptree,
        reject;
初始化:
let ptree = start symbol;
let ptr = root;
let st = stack();
st.push(null);

开始:
let word = seq.NextWord();
While (true) do:
  if (!ptr.nodetype().isTerminal())
    For each rule in $\{A \to \beta_1, \dots, \beta_n; A \to \cdots\}$
      ptr.children = $(\beta_1, \dots, \beta_n)$;
      For 1 < i < n+1
        st.push($\beta_{n+2-i}$);
      ptr = $\beta_1$;
  //ptr.nodetype()=terminal
  else if (word == ptr) //单词匹配成功
    word = seq.NextWord(); //下一个单词
    ptr = st.pop()
  else if (word == eof && cur == null)
    accept and return ptree;
  else
    backtrack(); //回溯
```

- 不考虑递归的情况：
  - $A \to \cdots \to A$
- 不考虑左递归的情况：
  - $A \to \cdots \to AX$
- 复杂度高：
  - 单词个数
  - 规则个数
  - 规则生产的符号数
  - 基于栈的回溯

左递归问题

- 对CFG的一个规则来说，其右侧的第一个符号与左侧符号相同或者能够推导出左侧符号。
- 主要问题：可使搜索算法无限递归下去，不终止。

[1] $Expr \rightarrow Expr + Term$
[2] $\quad\quad | \, Expr \; - Term$
[3] $\quad\quad | \, Term$

word	cur	Rule	Stack
(	Expr	[1]	*Expr*, +, *Term*
(	Expr	[1]	*Expr*, +, *Term*, +, *Term*
(	Expr	[1]	*Expr*, +, *Term*, +, *Term*, +, *Term*
			…

消除左递归

- 引入新的非终结符，基本规则：

$$\boxed{\begin{aligned} E &\to E\ \alpha \\ &\mid \beta \end{aligned}} \Rightarrow \boxed{\begin{aligned} E &\to \beta\ E' \\ E' &\to \alpha\ E' \\ &\mid \epsilon \end{aligned}} \qquad \boxed{\begin{aligned} E &\to E\ \alpha \\ &\mid \beta \\ &\mid \gamma \end{aligned}} \Rightarrow \boxed{\begin{aligned} E &\to \beta\ E' \mid \gamma E' \\ E' &\to \alpha\ E' \\ &\mid \epsilon \end{aligned}}$$

举例：

$$\boxed{\begin{array}{lll} [1] & Expr \to & Expr + Term \\ [2] & & \mid Expr\ - Term \\ [3] & & \mid Term \end{array}} \Rightarrow \boxed{\begin{array}{lll} [1] & Expr \to & Term\ Expr' \\ [2] & Expr' \to & + Term\ Expr' \\ [3] & & \mid\ - Term\ Expr' \\ [4] & & \mid \epsilon \end{array}}$$

$$\boxed{\begin{array}{lll} [4] & Term \to & Term \times Factor \\ [5] & & \mid Term \div Factor \\ [6] & & \mid Factor \end{array}} \Rightarrow \boxed{\begin{array}{lll} [5] & Term \to & Factor\ Term' \\ [6] & Term' \to & \times\ Factor\ Term' \\ [7] & & \mid \div\ Factor\ Term' \\ [8] & & \mid \epsilon \end{array}}$$

$$\boxed{\begin{array}{lll} [7] & Facor \to & (Expr) \\ [8] & & \mid num \\ [9] & & \mid name \end{array}} \Rightarrow \boxed{\begin{array}{lll} [9] & Facor \to & (Expr) \\ [10] & & \mid num \\ [11] & & \mid name \end{array}}$$

更多例子

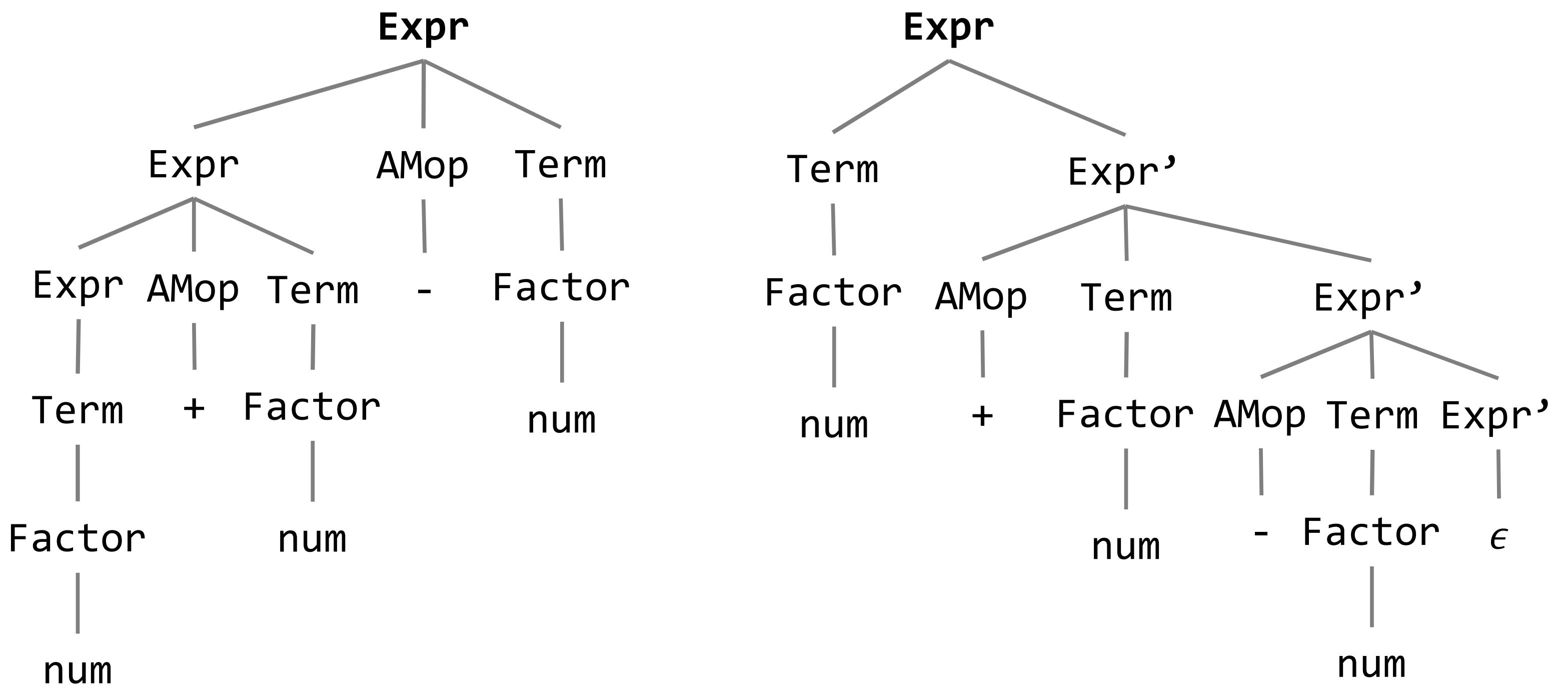
[1] $Expr \rightarrow Expr\ AMop\ Term$
[2] $\quad\quad\quad |\ Term$
[3] $Term \rightarrow Term\ MDop\ Factor$
[4] $\quad\quad\quad |\ Factor$
[5] $Facor \rightarrow (Expr)$
[6] $\quad\quad\quad |\ num$
[7] $AMop \rightarrow +$
[8] $\quad\quad\quad |\ -$
[9] $MDop \rightarrow \times$
[10] $\quad\quad\quad |\ \div$

$\Rightarrow$

[1] $Expr \rightarrow Term\ Expr'$
[2] $Expr' \rightarrow AMop\ Term\ Expr'$
[3] $\quad\quad\quad |\ \epsilon$
[4] $Term \rightarrow Factor\ Term'$
[5] $Term' \rightarrow MDop\ Factor\ Term'$
[6] $\quad\quad\quad |\ \epsilon$
[7] $Facor \rightarrow (Expr)$
[8] $\quad\quad\quad |\ num$
[9] $AMop \rightarrow +$
[10] $\quad\quad\quad |\ -$
[11] $MDop \rightarrow \times$
[12] $\quad\quad\quad |\ \div$

思考

- 是否会改变运算符结合性？如3+4-5。
  - 语法解析树和虚拟语法树的区别



间接左递归问题

$$\boxed{\begin{array}{l} E \rightarrow \alpha \\ \alpha \rightarrow \beta + \\ \beta \rightarrow E \end{array}} \quad \Longrightarrow \quad E \rightarrow E +$$

```
展开所有非终结符NT检测和消除间接左递归

输入: Grammar{T,NT}

开始:
for i=1 to n
  for j=1 to i-1
    if ∃ $NT_i \rightarrow NT_j\,\gamma$
      展开 $NT_i \rightarrow NT_j\,\gamma$中的非终结符$NT_j$
  重写会造成$NT_i$左递归的规则
```

无回溯语法

- 目的：消除语法生成规则选择时的不确定性，避免回溯。
- 思路：如果对于每个非终结符的任意两个生成式，其产生的首个终结符号不同，则在前瞻一个单词的情况下，总能够选择正确的生成式规则。
  - [1] $NT_1 \to NT_i \to \cdots \to \text{term}_1\ NT_p$
  - [2] $NT_1 \to NT_j \to \cdots \to \text{term}_2\ NT_q$
- 预测解析（Predictive Parsing）：LL(1)语法
  - Left-to-Right，Leftmost，前瞻一个字符

消除回溯：提取左因子

- 对一组产生式提取并隔离共同前缀

$$A \to \alpha\beta_1 | \alpha\beta_2 | \dots | \alpha\beta_n | \gamma_1 | \dots | \gamma_j \quad \Rightarrow \quad \begin{array}{l} A \to \alpha B \mid \gamma_1 | \dots | \gamma_j \\ B \to \beta_1 \mid \beta_2 | \dots | \beta_n \end{array}$$

应用举例：

```
[11]  Factor → name
[12]           | name [ArgList]
[13]           | name (ArgList)
[14]  ArgList → Expr MoreArgs
[15]  MoreArgs →, Expr MoreArgs
[16]                  | ϵ
```

⇒

```
[11]  Factor → name Arguments
[12]  Arguments → [ArgList]
[13]                   | (ArgList)
[14]                   | ϵ
[15]  ArgList → Expr MoreArgs
[16]  MoreArgs →, Expr MoreArgs
[17]                   | ϵ
```

无回溯语法的必要性质

$$First^{+}(A \to \beta) = \begin{cases} First(\beta), & if \epsilon \notin First(\beta) \\ First(\beta) \cup Follow(A), & otherwise \end{cases}$$

$$\forall 1 \le i, j \le n, First^{+}(A \to \beta_i) \cap First^{+}\left(A \to \beta_j\right) = \emptyset$$

- 同一非终结符A 的任意两个语法推导$(A \to \beta_i)$和$\left(A \to \beta_j\right)$ 所产生的的首个终结符不能相通。
- $First(\beta)$是从语法符号β推导出的每个子句的第一个终结符的集合，其值域是$T \cup \{\epsilon, \mathrm{eof}\}$。
- 如果$First(\beta)$是$\{\epsilon\}$，则计算紧随A之后出现的终结符的集合$Follow(A)$。

First集合计算

- 对于生成式$A \rightarrow \beta_1\beta_2 \dots \beta_n$ 来说：
  - 如果 $\epsilon \notin First(\beta_1)$， 则$First(A) = First(\beta_1)$
  - 如果 $\epsilon \in First(\beta_1)\& \dots \&\epsilon \in First(\beta_i)$, 则$First(A) = First(\beta_1) \cup \cdots \cup First(\beta_{i+1})$

[1] $Expr \rightarrow Term\ Expr'$
[2] $Expr' \rightarrow +\ Term\ Expr'$
[3] $\quad\quad |\ -\ Term\ Expr'$
[4] $\quad\quad |\ \epsilon$

[5]$Term \rightarrow Factor\ Term'$
[6]$Term' \rightarrow \times\ Factor\ Term'$
[7] $\quad\quad |\ \div\ Factor\ Term'$
[8] $\quad\quad |\ \epsilon$

[9] $Facor \rightarrow (Expr)$
[10] $\quad\quad |\ num$
[11] $\quad\quad |\ id$

	num	id	$+$	$-$	$\times$	$\div$	(	)	ϵ
$Expr$	√	√					√		
$Expr'$			√	√					√
$Term$	√	√					√		
$Term'$					√	√			√
$Facor$	√	√					√		

Follow集合计算

- 紧随非终结符之后出现的所有可能的终结符

[1]	$Expr \rightarrow Term\ Expr'$
[2]	$Expr' \rightarrow +\ Term\ Expr'$
[3]	$\mid -\ Term\ Expr'$
[4]	$\mid \epsilon$

[5]	$Term \rightarrow Factor\ Term'$
[6]	$Term' \rightarrow \times\ Factor\ Term'$
[7]	$\mid \div\ Factor\ Term'$
[8]	$\mid \epsilon$

[9]	$Facor \rightarrow (Expr)$
[10]	$\mid num$
[11]	$\mid id$

	num	id	+	−	×	÷	(	)	ϵ	eof
$Expr$	√	√					√	⊙		⊙
$Expr'$			√	√				⊙	√	⊙
$Term$	√	√	⊙	⊙			√	⊙		⊙
$Term'$			⊙	⊙	√	√		⊙	√	⊙
$Facor$	√	√	⊙	⊙	⊙	⊙	√	⊙		⊙

First+集合计算

$$First^{+}(A \rightarrow \beta) = \begin{cases} First(\beta), & if \epsilon \notin First(\beta) \\ First(\beta) \cup Follow(A), & otherwise \end{cases}$$

	num	id	$+$	$-$	$\times$	$\div$	(	)	ϵ	eof
$Expr$	√	√					√	~~⊙~~		~~⊙~~
$Expr'$			√	√				⊙	√	⊙
$Term$	√	√	~~⊙~~	~~⊙~~			√	~~⊙~~		~~⊙~~
$Term'$			⊙	⊙	√	√		⊙	√	⊙
$Facor$	√	√	~~⊙~~	~~⊙~~	~~⊙~~	~~⊙~~	√	~~⊙~~		~~⊙~~

解析表构造：应用哪条规则可得到目标终结符？

	num	id	+	−	×	÷	(	)	ϵ	eof
$Expr$	√	√					√			
$Expr'$			√	√				⊙	√	⊙
$Term$	√	√					√			
$Term'$			⊙	⊙	√	√		⊙	√	⊙
$Facor$	√	√					√			

[1] $Expr \rightarrow Term\ Expr'$
[2] $Expr' \rightarrow + Term\ Expr'$
[3] $\quad | - Term\ Expr'$
[4] $\quad | \epsilon$

[5] $Term \rightarrow Factor\ Term'$
[6] $Term' \rightarrow \times Factor\ Term'$
[7] $\quad | \div Factor\ Term'$
[8] $\quad | \epsilon$

[9] $Facor \rightarrow (Expr)$
[10] $\quad | num$
[11] $\quad | id$

	num	id	+	−	×	÷	(	)	ϵ	eof
$Expr$	1	1					1			
$Expr'$			2	3				4		4
$Term$	5	5					5			
$Term'$			8	8	6	7		8		8
$Facor$	10	11					9			

应用解析表

	num	id	$+$	$-$	$\times$	$\div$	(	)	*eof*
Expr	1	1					1		
Expr′			2	3				4	4
Term	5	5					5		
Term′			8	8	6	7		8	8
Facor	10	11					9		

(num+num)×num

[1] $Expr \rightarrow Term\ Expr'$
[2] $Expr' \rightarrow +\ Term\ Expr'$
[3] $\qquad\quad |\ -\ Term\ Expr'$
[4] $\qquad\quad |\ \epsilon$

[5] $Term \rightarrow Factor\ Term'$
[6] $Term' \rightarrow \times\ Factor\ Term'$
[7] $\qquad\quad |\ \div\ Factor\ Term'$
[8] $\qquad\quad |\ \epsilon$

[9] $Facor \rightarrow (Expr)$
[10] $\qquad\quad |\ num$
[11] $\qquad\quad |\ id$

word	cur	Rule	Stack
(	Expr	[1]	Term, Expr’
(	Term	[5]	Factor, Term’, Expr’
(	Factor	[9]	(, Expr,), Term’, Expr’
(	(	-	Expr,), Term’, Expr’
num	Expr	[1]	Term, Expr’,), Term’, Expr’
num	Term	[5]	Factor, Term’, Expr’,), Term’, Expr’
num	Factor	[10]	num, Term’, Expr’,), Term’, Expr’
num	num	-	Term’, Expr’,), Term’, Expr’
+	Term’	[8]	Expr’,), Term’, Expr’
+	Expr’	[2]	+, Term, Expr’,), Term’, Expr’
+	+	-	Term, Expr’,), Term’, Expr’
num	Term	[5]	Factor, Term’, Expr’,), Term’, Expr’
num	Factor	[10]	num, Term’, Expr’,), Term’, Expr’
num	num	-	Term’, Expr’,), Term’, Expr’
)	Term’	[8]	Expr’,), Term’, Expr’
)	Expr’	[4]	), Term’, Expr’
)	)	-	Term’, Expr’
+	Term’	[8]	Expr’
…	…	…	…

练习：

- 将正则表达式CFG改写为LL(1)语法并写出应用解析表

$$
\begin{aligned}
&< regex >::=< union > \mid < concat > \\
&< union >::=< regex > \text{"|"} < concat > \\
&< concat >::=< concat >< term > \mid < term > \\
&< term >::=< element >* \mid < element > \\
&< element >::= (< regex >) \mid < alphanum >
\end{aligned}
$$

解答：消除左递归

- 左递归问题一：
  - $<concat>::=<concat><term> \mid <term>$
- 改写结果：
  - $<concat>::=<term><concat'>$
  - $<concat'>::=<term><concat'> \mid \epsilon$
- 和下列形式等价（但转换AST时要注意结合性）：
  - $<concat>::=<term><concat> \mid <term>$

```
[1]  <regex> ::= <concat><regex'>
[2]  <regex'> ::= "|" <concat><regex'>
[3]                 |ε
[4]  <concat> ::= <term><concat'>
[5]  <concat'> ::= <term><concat'>
[6]                 |ε
[7]  <term> ::= <element>*
[8]                 |<element>
[9]  <element> ::= (<regex>)
[10]                |<alphanum>
```

- 左递归问题二（间接左递归）：
  - $<regex>::=<union> \mid <concat>$
  - $<union>::=<regex>\ "|"\ <concat>$
  - $\Rightarrow <regex>::=<regex>\ "|"\ <concat> \mid <concat>$
- 改写结果：
  - $<regex>::=<concat><regex'>$
  - $<regex'>::=\ "|"\ <concat><regex'> \mid \epsilon$

解答：消除回溯

- 回溯问题语法
  - $< term >::=< element >*$
  - $\qquad\qquad | < element >$
- 改写结果：
  - $< term >::=< element >< follow >$
  - $< follow >::=* \ |\epsilon$

```
[1]  <regex> ::= <concat><regex'>
[2]  <regex'> ::= "|" <concat><regex'>
[3]             |ε
[4]  <concat> ::= <term><concat'>
[5]  <concat'> ::= <term><concat'>
[6]             |ε
[7]  <term> ::= <element><follow>
[8]  <follow> ::= *
[9]             |ε
[10]  <element> ::= (<regex>)
[11]             | <alphanum>
```

解答：构建LL(1)解析表

	(	)	$A-Z, a-z, 0-9$	|	$*$	ϵ	eof
regex	√[1]		√[1]				
regex′				√[1]		√	⊙[3]
concat	√[4]		√[4]				
concat′	√[5]		√[5]			√	⊙[6]
term	√[7]		√[7]				
element	√[10]		√[11]				
follow	⊙[9]		⊙[9]		√[8]	√	
alphanum			√[12]				

√:first()
⊙:follow()
[]:语法规则条目

通用自顶向下语法分析算法：Earley算法

- 三种基本操作：
  - 预测（Prediction）：对于每个状态$X \to \alpha \circ Y\beta$，根据语法规则预测$Y \to \circ\gamma$。
  - 扫描（Scanning）：如果下一个待处理的符号是a，并且存在状态 $X \to \alpha \circ a\beta$，则扫描该字符并且将状态变更为$X \to \alpha a \circ \beta$ 。
  - 完成（Completion）：$Y \to \gamma\circ$完成了对Y的分析，进而更新 $X \to \alpha \circ Y\beta$为 $X \to \alpha Y \circ \beta$。

Earley 算法参考

```
DECLARE ARRAY S;

function INIT(words) {
    S ← CREATE-ARRAY(LENGTH(words) + 1)
    for k ← from 0 to LENGTH(words)
        S[k] ← EMPTY-ORDERED-SET
}
function EARLEY-PARSE(words, grammar) {
    INIT(words)
    ADD-TO-SET((γ → •S, 0), S[0])
    for k ← from 0 to LENGTH(words)
        for each state in S[k] { // S[k] can expand during this loop
            if not FINISHED(state) {
                if NEXT-ELEMENT-OF(state) is a nonterminal then
                    PREDICTOR(state, k, grammar)    // non-terminal
                else
                    SCANNER(state, k, words)        // terminal
            }
            else
                COMPLETER(state, k)
        }
    }
    return chart
}
procedure PREDICTOR((A → α•Bβ, j), k, grammar) {
    for each (B → γ) in GRAMMAR-RULES-FOR(B, grammar)
        ADD-TO-SET((B → •γ, k), S[k])
}
procedure SCANNER((A → α•aβ, j), k, words)
    if a ⊂ PARTS-OF-SPEECH(words[k])
        ADD-TO-SET((A → αa•β, j), S[k+1])

procedure COMPLETER((B → γ•, x), k)
    for each (A → α•Bβ, j) in S[x]
        ADD-TO-SET((A → αB•β, j), S[k])
```

https://en.wikipedia.org/wiki/Earley_parser

Earley算法

如何根据下列语法规则解析(3+4)×5?

(num+num)×num

[1] $Expr \rightarrow Expr + Term$
[2] $\quad | \, Expr - Term$
[3] $\quad | \, Term$
[4] $Term \rightarrow Term \times Factor$
[5] $\quad | \, Term \div Factor$
[6] $\quad | \, Factor$
[7] $Facor \rightarrow (Expr)$
[8] $\quad | \, num$
[9] $\quad | \, id$

no	production	origin	comment
s(0) = ∘(num+num)×num			
1	$Expr \rightarrow \circ\, Expr + Term$	s(0)	start rule
2	$Expr \rightarrow \circ\, Expr - Term$	s(0)	start rule
3	$Expr \rightarrow \circ\, Term$	s(0)	start rule
4	$Term \rightarrow \circ\, Term \times Factor$	s(0)	Predict from [0][3]
5	$Term \rightarrow \circ\, Term \div Factor$	s(0)	Predict from [0][3]
6	$Term \rightarrow \circ\, Factor$	s(0)	Predict from [0][3]
7	$Facor \rightarrow \circ\, (Expr)$	s(0)	Predict from [0][6]
8	$Facor \rightarrow \circ\, num$	s(0)	Predict from [0][6]
9	$Facor \rightarrow \circ\, id$	s(0)	Predict from [0][6]
s(1) = (∘num+num)×num			
1	$Facor \rightarrow (\circ\, Expr)$	s(0)	Scan from [0][7]
2	$Expr \rightarrow \circ\, Expr + Term$	s(1)	Predict from [1][1]
3	$Expr \rightarrow \circ\, Expr - Term$	s(1)	Predict from [1][1]
4	$Expr \rightarrow \circ\, Term$	s(1)	Predict from [1][1]
5	$Term \rightarrow \circ\, Term \times Factor$	s(1)	Predict from [1][4]
6	$Term \rightarrow \circ\, Term \div Factor$	s(1)	Predict from [1][4]
7	$Term \rightarrow \circ\, Factor$	s(1)	Predict from [1][4]
8	$Facor \rightarrow \circ\, (Expr)$	s(1)	Predict from [1][7]
9	$Facor \rightarrow \circ\, num$	s(1)	Predict from [1][7]
10	$Facor \rightarrow \circ\, id$	s(1)	Predict from [1][7]

Earley算法

[1] $Expr \rightarrow Expr + Term$
[2] $\quad | \; Expr \; - Term$
[3] $\quad | \; Term$
[4] $Term \rightarrow Term \;\times\; Factor$
[5] $\quad | \; Term \div Factor$
[6] $\quad | \; Factor$
[7] $Facor \rightarrow (Expr)$
[8] $\quad | \; num$
[9] $\quad | \; id$

no	production	origin	comment
s(2) = (num∘+num)×num			
1	$Facor \rightarrow num \circ$	s(1)	Scan from [1][9]
2	$Term \rightarrow Factor \circ$	s(1)	Complete [1][7]
3	$Term \rightarrow Term \circ \times Factor$	s(1)	Complete [1][5]
4	$Term \rightarrow Term \circ \div Factor$	s(1)	Complete [1][6]
5	$Expr \rightarrow Term \circ$	s(1)	Complete [1][4]
6	$Expr \rightarrow Expr \circ +Term$	s(1)	Complete [1][2]
7	$Expr \rightarrow Expr \circ -Term$	s(1)	Complete [1][3]
8	$Facor \rightarrow (Expr \circ)$	s(0)	Complete [1][1]
s(3) = (num+∘num)×num			
1	$Expr \rightarrow Expr + \circ Term$	s(1)	Scan from [2][6]
2	$Term \rightarrow \circ Term \times Factor$	s(3)	Predict from [3][1]
3	$Term \rightarrow \circ Term \div Factor$	s(3)	Predict from [3][1]
4	$Term \rightarrow \circ Factor$	s(3)	Predict from [3][1]
5	$Facor \rightarrow \circ (Expr)$	s(3)	Predict from [3][4]
6	$Facor \rightarrow \circ num$	s(3)	Predict from [3][4]
7	$Facor \rightarrow \circ id$	s(3)	Predict from [3][4]

Earley算法

[1] $Expr \rightarrow Expr + Term$
[2] $\quad | \, Expr \, - Term$
[3] $\quad | \, Term$
[4] $Term \rightarrow Term \, \times \, Factor$
[5] $\quad | \, Term \div Factor$
[6] $\quad | \, Factor$
[7] $Facor \rightarrow (Expr)$
[8] $\quad | \, num$
[9] $\quad | \, id$

no	production	origin	comment
s(4) = (num+num∘)×num			
1	$Facor \rightarrow num \circ$	s(3)	Scan from [3][6]
2	$Term \rightarrow Factor \circ$	s(3)	Complete [3][7]
3	$Term \rightarrow Term \circ \times \, Factor$	s(3)	Complete [3][5]
4	$Term \rightarrow Term \circ \div Factor$	s(3)	Complete [3][6]
5	$Expr \rightarrow Term \circ$	s(3)	Complete [3][4]
6	$Expr \rightarrow Expr \circ +Term$	s(3)	Complete [3][2]
7	$Expr \rightarrow Expr \circ -Term$	s(3)	Complete [3][3]
8	$Expr \rightarrow Expr + Term \circ$	s(1)	Complete [3][1]
9	$Facor \rightarrow (Expr \circ)$	s(0)	Complete [1][1]
s(5) = (num+num)∘×num			
1	$Facor \rightarrow (Expr) \circ$	s(0)	Scan from [4][9]
2	$Term \rightarrow Factor \circ$	s(0)	Complete [0][6]
3	$Term \rightarrow Term \circ \times \, Factor$	s(0)	Complete [0][4]
4	$Term \rightarrow Term \circ \div Factor$	s(0)	Complete [0][5]
5			
6			
7			

Earley算法

[1] $Expr \rightarrow Expr + Term$
[2] $\quad | \, Expr - Term$
[3] $\quad | \, Term$
[4] $Term \rightarrow Term \times Factor$
[5] $\quad | \, Term \div Factor$
[6] $\quad | \, Factor$
[7] $Facor \rightarrow (Expr)$
[8] $\quad | \, num$
[9] $\quad | \, id$

no	production	origin	comment
s(6) = (num+num)×∘num			
1	$Term \rightarrow Term \times \circ\ Factor$	s(0)	Scan from [6][3]
2	$Facor \rightarrow \circ\ (Expr)$	s(0)	Predict from [6][1]
3	$Facor \rightarrow \circ\ num$	s(6)	Predict from [6][1]
4	$Facor \rightarrow \circ\ id$	s(6)	Predict from [6][1]
s(7) = (num+num)×num∘			
1	$Facor \rightarrow num\ \circ$	s(6)	Scan from [6][4]
2	$Term \rightarrow Term \times\ Factor\ \circ$	s(0)	Complete [6][1]
3	$Expr \rightarrow Term\ \circ$	s(0)	Complete [0][3]
4			
5			
6			
7			

Earley算法能否解析非CFG语法?

练习：

- 应用Earley算法解析正则num+num-num
- 应用Earley算法解析正则 (a|b)*

[1] $Expr \to Expr + Term$
[2] $\quad\quad \mid Expr - Term$
[3] $\quad\quad \mid Term$
[4] $Term \to Term \times Factor$
[5] $\quad\quad \mid Term \div Factor$
[6] $\quad\quad \mid Factor$
[7] $Facor \to (Expr)$
[8] $\quad\quad \mid num$
[9] $\quad\quad \mid id$

[1] $<\text{regex}>::=<\text{union}>$
[2] $\quad\quad \mid <\text{concat}>$
[3] $<\text{union}>::=<\text{regex}>\text{"|"}<\text{concat}>$
[4] $<\text{concat}>::=<\text{concat}><\text{term}>$
[5] $\quad\quad \mid <\text{term}>$
[6] $<\text{term}>::=<\text{element}>*$
[7] $\quad\quad \mid <\text{element}>$
[8] $<\text{element}>::=(<\text{regex}>)$
[9] $\quad\quad \mid <alphanum>$

自顶向下解析算法小结

- Earley算法
  - 支持所有CFG
  - 复杂度$O(n^3)$
    - 无歧义语法：$O(n^2)$
- 能否有更快的算法？
  - 要求CFG为LL(1)
    - 不存在左递归
    - 不存在回溯

A programmer's wife asks him to go to the grocery. She says "Get a gallon of milk. If they have eggs, get 12." The programmer returns with 12 gallons of milk.

A programmer is going to the grocery store and his wife tells him, "Buy a gallon of milk, and if there are eggs, buy a dozen." So the programmer goes, buys everything, and drives back to his house. Upon arrival, his wife angrily asks him, "Why did you get 13 gallons of milk?" The programmer says, "There were eggs!"

四、自底向上分析

SLR/LALR/LR(1)

基本思路：基于规约的方法

- 如果在句法分析栈的上边缘找到β且$A \rightarrow \beta$，则将其规约为A；
- 否则移进

[1] $S \rightarrow \epsilon$
[2] $\quad\quad | \ [S]S$

	当前栈	待输入
		[ ]
第1步:	[	]
第2步:	[S	]
第3步:	[S]	
第4步:	[S]S	
第5步:	S	

移进和规约（Shift-Reduce）

Shift
移进表示：

$$\frac{w: \beta}{wa: \beta a}\text{shift}$$

Reduce
规约表示：

$$[r]\ X \rightarrow \alpha$$

$$\frac{w: \beta\alpha}{w: \beta X}\text{reduce(r)}$$

示例：

$$\frac{\epsilon: \beta_0}{[: \beta_0[}\text{shift}$$

$$\frac{[: \beta_1}{[]: \beta_1]}\text{shift}$$

$$\cdots$$

$$\frac{[][[][]: \beta_7}{[][[][]]: \beta_7]}\text{shift}$$

$$[1]\ S \rightarrow \epsilon$$

$$\frac{w: \beta}{w: \beta S}\text{reduce([1])}$$

$$[2]\ S \rightarrow [S]S$$

$$\frac{w: \beta[S]S}{w: \beta S}\text{reduce([2])}$$

移进-规约应用

[1] $S \rightarrow \epsilon$
[2] $\quad\mid [S]S$

最右推导

	ǀǀ	[][[][]]	$\epsilon : \epsilon$	shift [
[	ǀǀ	[][[][]]	$[: [$	reduce([1])
[S	ǀǀ	][[][]]	$[: [S$	shift
[S]	ǀǀ	[[][]]	$[] : [S]$	shift
[S][	ǀǀ	[][]]	$[][: [S][$	shift
[S][[	ǀǀ	][]]	$[][[: [S][[$	reduce([1])
[S][[S	ǀǀ	][]]	$[][[: [S][[S$	shift
[S][[S]	ǀǀ	[]]	$[][[] : [S][[S]$	shift
[S][[S][	ǀǀ	]]	$[][[][: [S][[S][$	reduce([1])
[S][[S][S	ǀǀ	]]	$[][[][: [S][[S][S$	shift
[S][[S][S]	ǀǀ	]	$[][[][] : [S][[S][S]$	reduce([1])
[S][[S][S]S	ǀǀ	]	$[][[][] : [S][[S][S]S$	reduce([2])
[S][[S]S	ǀǀ	]	$[][[][] : [S][[S]S$	reduce([2])
[S][S	ǀǀ	]	$[][[][] : [S][S$	shift
[S][S]	ǀǀ		$[][[][]] : [S][[S]S]$	reduce([1])
[S][S]S	ǀǀ		$[][[][]] : [S][S]S$	reduce([2])
[S]S	ǀǀ		$[][[][]] : [S]S$	reduce([2])
S	ǀǀ		$[][[][]] : S$	

LR(0)句柄分析

$$\boxed{\begin{array}{ll} [1] & E \rightarrow E + T \\ [2] & \quad\ \mid\ T \\ [3] & T \rightarrow T \times F \\ [4] & \quad\ \mid\ F \\ [5] & F \rightarrow (E) \\ [6] & \quad\ \mid id \end{array}} \xRightarrow{\text{增强语法}} \boxed{\begin{array}{l} Goal \rightarrow E \\ E \rightarrow E + T \\ \quad\ \mid\ T \\ T \rightarrow T \times F \\ \quad\ \mid\ F \\ F \rightarrow (E) \\ \quad\ \mid\ id \end{array}}$$

增强语法有唯一的目标符号Goal，不会出现在产生式的右侧。

$$Goal \rightarrow E \xRightarrow{\text{句柄状态}} \begin{array}{l} Goal \rightarrow\circ E \\ Goal \rightarrow E \circ \end{array}$$

可应用$E \rightarrow E + T$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} E \rightarrow\circ E + T \\ E \rightarrow E \circ +T \\ E \rightarrow E +\circ T \\ E \rightarrow E + T \circ \end{array}$$

可应用$E \rightarrow T$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} E \rightarrow\circ T \\ E \rightarrow T \circ \end{array}$$

可应用$T \rightarrow T \times F$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} T \rightarrow\circ T \times F \\ T \rightarrow T \circ\times F \\ T \rightarrow T \times\circ F \\ T \rightarrow T \times F \circ \end{array}$$

可应用$T \rightarrow F$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} T \rightarrow\circ F \\ T \rightarrow F \circ \end{array}$$

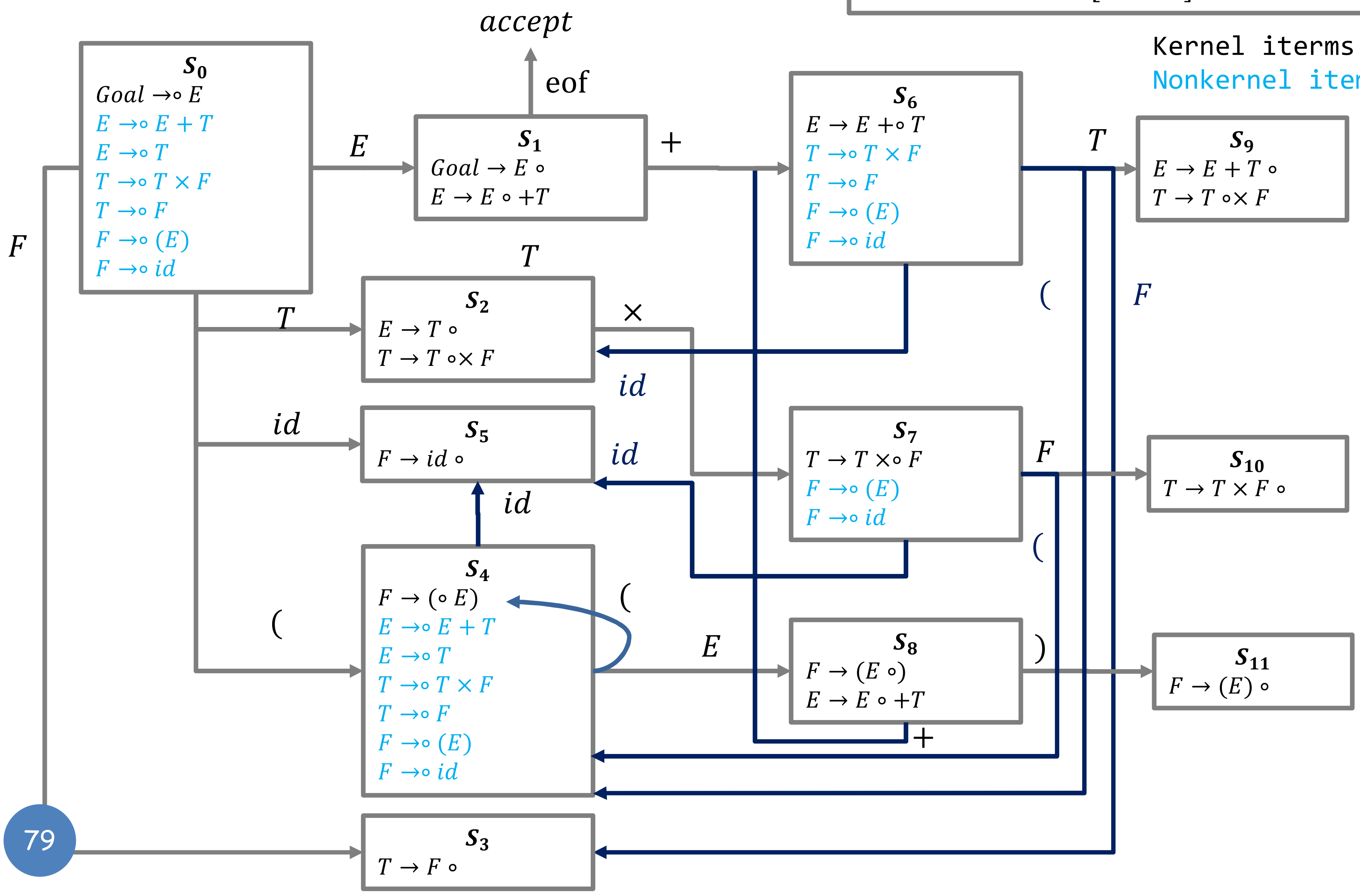
可应用$F \rightarrow (E)$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} F \rightarrow\circ (E) \\ F \rightarrow (\circ E) \\ F \rightarrow (E \circ) \\ F \rightarrow (E) \circ \end{array}$$

可应用$F \rightarrow id$规约的句柄状态 $\Longrightarrow$
$$\begin{array}{l} F \rightarrow\circ id \\ F \rightarrow id \circ \end{array}$$

LR(0)自动机构建

```
While (S has changed)
  for each item $[A \to \beta \circ C\delta, a] \in S$
    for each production $[C \to \lambda] \in G$
      if $[C \to\circ \lambda] \notin S$
        $S \leftarrow S \cup [C \to\circ \lambda]$
```

Kernel iterms
Nonkernel items



LR(0)自动机的状态转移关系表

迭代	规范项	id	+	×	(	)	eof	E	T	F
0	S_0	S_5	$\emptyset$	$\emptyset$	S_4	$\emptyset$	$\emptyset$	S_1	S_2	S_3
1	S_1	$\emptyset$	S_6	$\emptyset$	$\emptyset$	$\emptyset$	$accept$	$\emptyset$	$\emptyset$	$\emptyset$
	S_2	$\emptyset$	$\emptyset$	S_7	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	S_3	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	S_4	S_5	$\emptyset$	$\emptyset$	S_4	$\emptyset$	$\emptyset$	S_8	S_2	S_3
	S_5	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
2	S_6	S_5	$\emptyset$	$\emptyset$	S_4	$\emptyset$	$\emptyset$	$\emptyset$	S_9	S_3
	S_7	S_5	$\emptyset$	$\emptyset$	S_4	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	S_{10}
	S_8	$\emptyset$	S_6	$\emptyset$	$\emptyset$	S_{11}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
3	S_9	$\emptyset$	$\emptyset$	S_7	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	S_{10}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
	S_{11}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

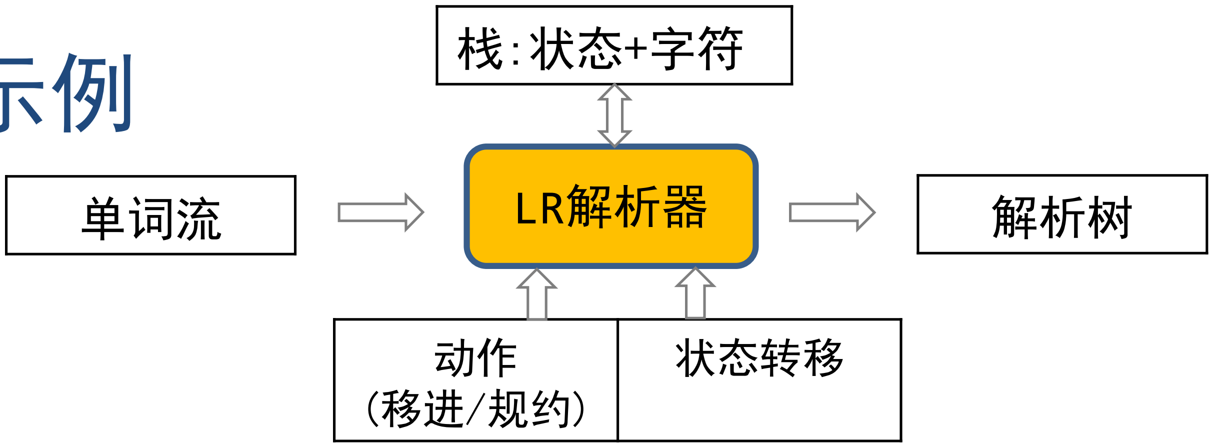
构建SLR解析表

移进条件：如果$A \to \alpha \circ a\beta \in S_i$，并且$Goto(S_i, a) = S_j$，设置$Action(S_i, a) = “shift\ j”$

规约条件：如果$A \to \alpha \circ\in S_i$，$\forall a \in Follow(A)$，设置$Action(S_i, a) = “reduce\ A \to \alpha”$

规范项	Action						Goto		
	id	+	×	(	)	eof	E	T	F
S_0	shift S_5			shift S_4			S_1	S_2	S_3
S_1		shift S_6				acept			
S_2		reduce [2]	shift S_7		reduce [2]	reduce [2]			
S_3									
S_4	shift S_5			shift S_4			S_8	S_2	S_3
S_5		reduce [6]	reduce [6]		reduce [6]	reduce [6]			
S_6	shift S_5			shift S_4				S_9	S_3
S_7	shift S_5			shift S_4					S_{10}
S_8		shift S_6			shift S_{11}				
S_9		reduce [1]	shift S_7		reduce [1]	reduce [1]			
S_{10}		reduce [3]	reduce [3]		reduce [3]	reduce [3]			
S_{11}		reduce [5]	reduce [5]		reduce [5]	reduce [5]			

SLR应用示例



Stack	Symbols	Input	Action
S_0		id×id \$	shift id, goto S_5
S_0S_5	id	×id \$	reduce by $F \rightarrow id$, back to S_0, goto S_3
S_0S_3	F	×id \$	reduce by $T \rightarrow F$, back to S_0, goto S_2
S_0S_2	T	×id \$	shift ×, goto S_7
$S_0S_2S_7$	T ×	id \$	shift id, goto S_5
$S_0S_2S_7S_5$	T × id	\$	reduce by $F \rightarrow id$, back to S_7, goto S_{10}
$S_0S_2S_7S_{10}$	T × F	\$	reduce by $T \rightarrow T \times F$, back to $S_7S_2S_0$, goto S_2
S_0S_2	T	\$	reduce by $E \rightarrow T$, back to S_0, goto S_1
S_0S_1	E	\$	

练习

- 下面的语法是否是LL(1)？是否是SLR(1)

[1] $S \rightarrow AaAb$
[2] $\quad |BbBa$
[3] $A \rightarrow \epsilon$
[4] $B \rightarrow \epsilon$

是LL(1)
$First^+(S \rightarrow AaAb) = \{a\}$
$First^+(S \rightarrow BbBa) = \{b\}$

$\boldsymbol{S_0}$
$S' \rightarrow \circ\, S$
$S \rightarrow \circ\, AaAb$
$S \rightarrow \circ\, BbAa$
$A \rightarrow \circ$
$B \rightarrow \circ$

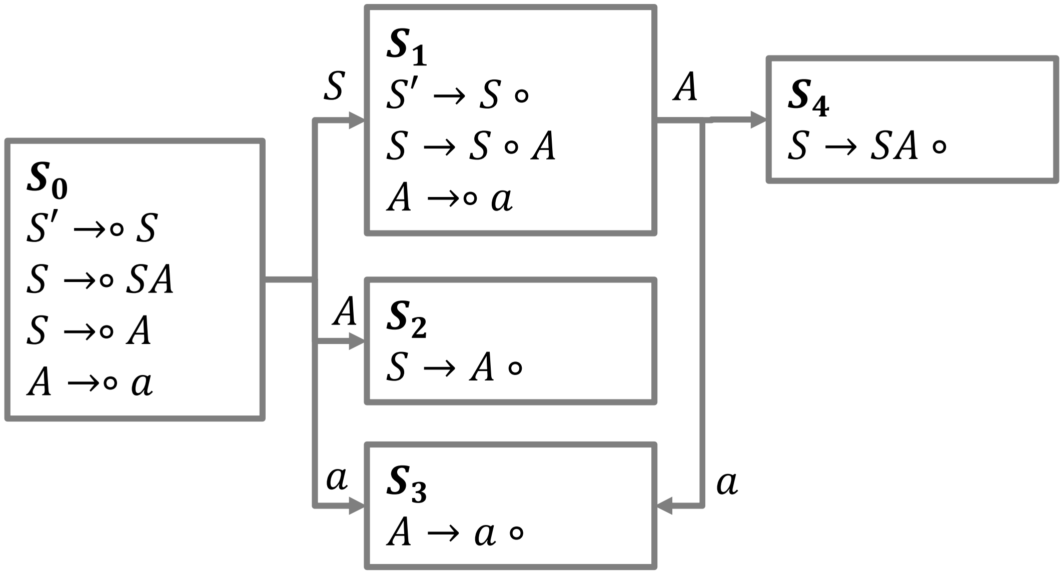
不是SLR(1)
$Follow(A) = Follow(B) = \{a, b\}$
$Action(S_0, a) = reduce[3]$ 或 $reduce[4]$

练习

- 下面的语法是否是LL(1)？ 是否是SLR(1)

[1] $S \rightarrow SA$
[2] $\quad\quad |A$
[3] $A \rightarrow a$

不是LL(1)
$First^{+}(S \rightarrow SA) = \{a\}$
$First^{+}(S \rightarrow A) = \{a\}$

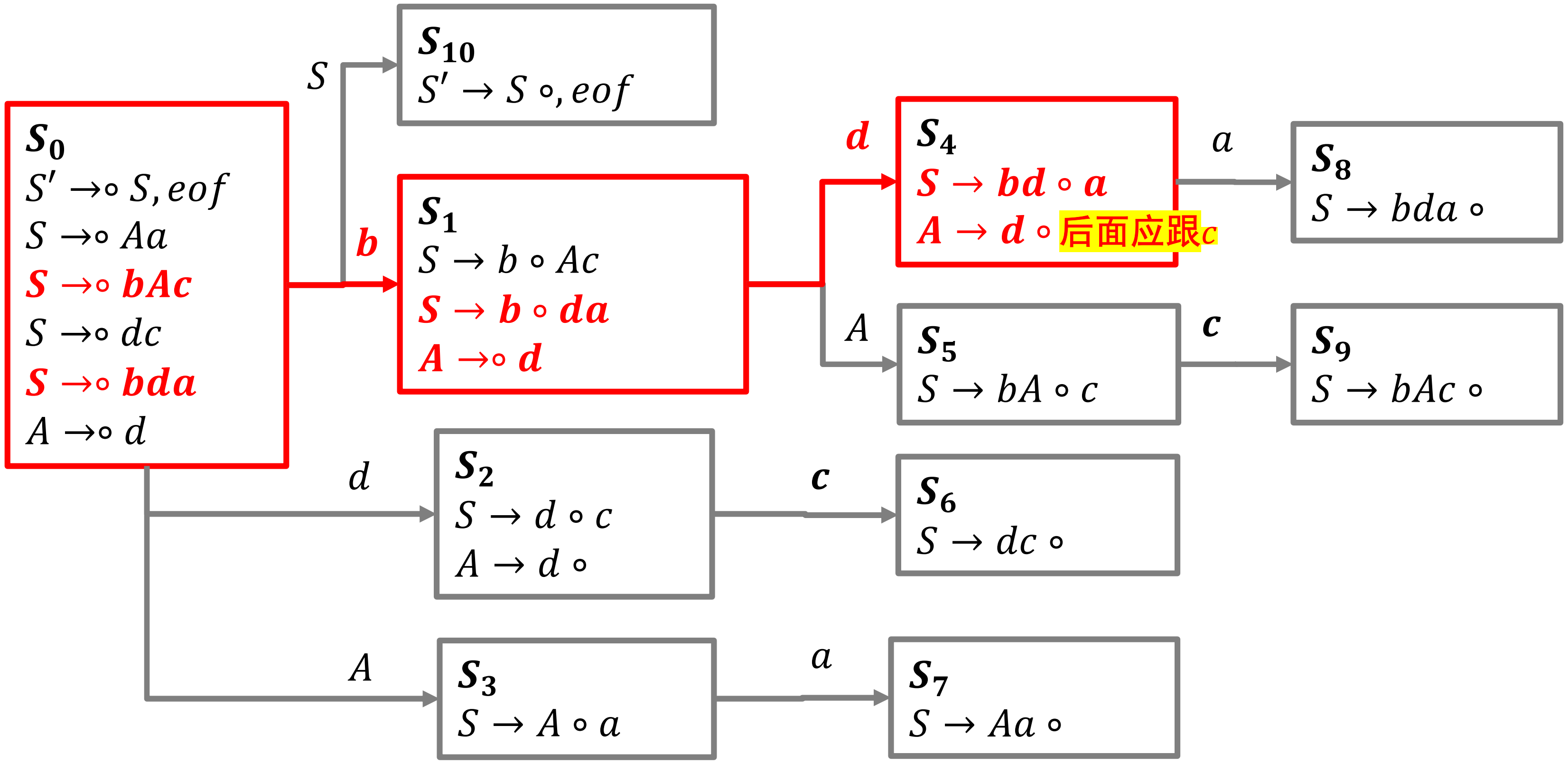


是SLR(1)

二义性语法：移进–规约冲突

- 利用SLR解析表解析bda时存在移进-规约冲突
  - S_4下一个字符为a，可移进
  - $a \in Follow(A)$，可规约

```
[1] S → Aa
[2]     |bAc
[3]     |dc
[4]     |bda
[5] A → d
```

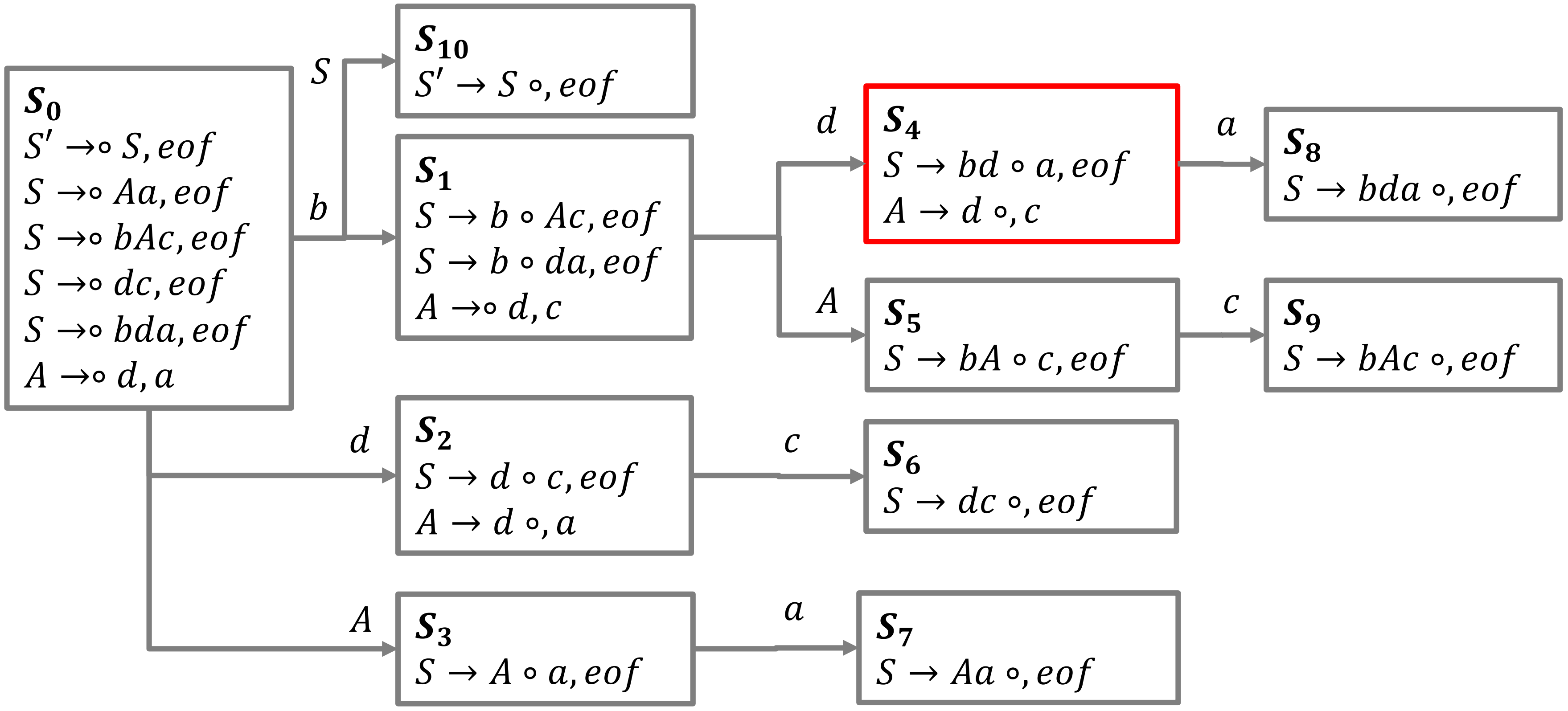


S_{10}: $S' \to S \circ, eof$
S_0: $S' \to \circ S, eof$; $S \to \circ Aa$; $S \to \circ bAc$; $S \to \circ dc$; $S \to \circ bda$; $A \to \circ d$
S_1: $S \to b \circ Ac$; $S \to b \circ da$; $A \to \circ d$
S_4: $S \to bd \circ a$; $A \to d \circ$ 后面应跟c
S_8: $S \to bda \circ$
S_5: $S \to bA \circ c$
S_9: $S \to bAc \circ$
S_2: $S \to d \circ c$; $A \to d \circ$
S_6: $S \to dc \circ$
S_3: $S \to A \circ a$
S_7: $S \to Aa \circ$
S
b
d
a
A
c
d
c
A
a

LR(1)自动机构造

```
[0] S′ → S
[1] S → Aa
[2]      |bAc
[3]      |dc
[4]      |bda
[5] A → d
```

1) 构造其LR(1)项的全集
2) 迭代过程
   - 通过闭包找到规范族
   - 分析规范族之间的状态转移关系



S_0: $S' \to \circ S, eof$; $S \to \circ Aa, eof$; $S \to \circ bAc, eof$; $S \to \circ dc, eof$; $S \to \circ bda, eof$; $A \to \circ d, a$
S_{10}: $S' \to S \circ, eof$
S_1: $S \to b \circ Ac, eof$; $S \to b \circ da, eof$; $A \to \circ d, c$
S_2: $S \to d \circ c, eof$; $A \to d \circ, a$
S_3: $S \to A \circ a, eof$
S_4: $S \to bd \circ a, eof$; $A \to d \circ, c$
S_5: $S \to bA \circ c, eof$
S_6: $S \to dc \circ, eof$
S_7: $S \to Aa \circ, eof$
S_8: $S \to bda \circ, eof$
S_9: $S \to bAc \circ, eof$
S
b
d
A
c
a

$$\begin{tikzcd}
S_0 \arrow[r, "S"] \arrow[dr, "b"] \arrow[ddr, "d"] \arrow[dddr, "A"] & S_{10} & & \\
& S_1 \arrow[r, "d"] \arrow[dr, "A"] & S_4 \arrow[r, "a"] & S_8 \\
& S_2 \arrow[dr, "c"] & S_5 \arrow[r, "c"] & S_9 \\
& S_3 \arrow[dr, "a"] & S_6 & \\
& & S_7 &
\end{tikzcd}$$

得到LR(1)解析表

规范项	Action					Goto	
	a	b	c	d	eof	S	A
S_0		shift S_1		shift S_2		S_{10}	S_3
S_1				shift S_4			S_5
S_2	reduce [5]		shift S_6				
S_3	shift S_7						
S_4	shift S_8		reduce [5]				
S_5			shift S_9				
S_6					reduce [3]		
S_7					reduce [1]		
S_8					reduce [4]		
S_9					reduce [2]		
S_{10}					*accept*		

应用LR(1)解析表

Stack	**Symbols**	**Input**	**Action**
S_0		bda \$	shift b, goto S_1
S_0S_1	b	da \$	shift d, goto S_4
$S_0S_1S_4$	bd	a \$	shift d, goto S_8
$S_0S_1S_4S_8$	bda	\$	reduce by $S \rightarrow bda$, back to S_0, goto S_{10}
S_0S_{10}	S	\$	accept

什么情况下容易出现移进-规约冲突？

```
[1]  S → bAc
[2]      |bda
[3]      |Aa
[4]  A → d
```

```
[1]  S → β₁Xβ₂
[2]      |β₁β₃β₄
[3]  X → β₃
```

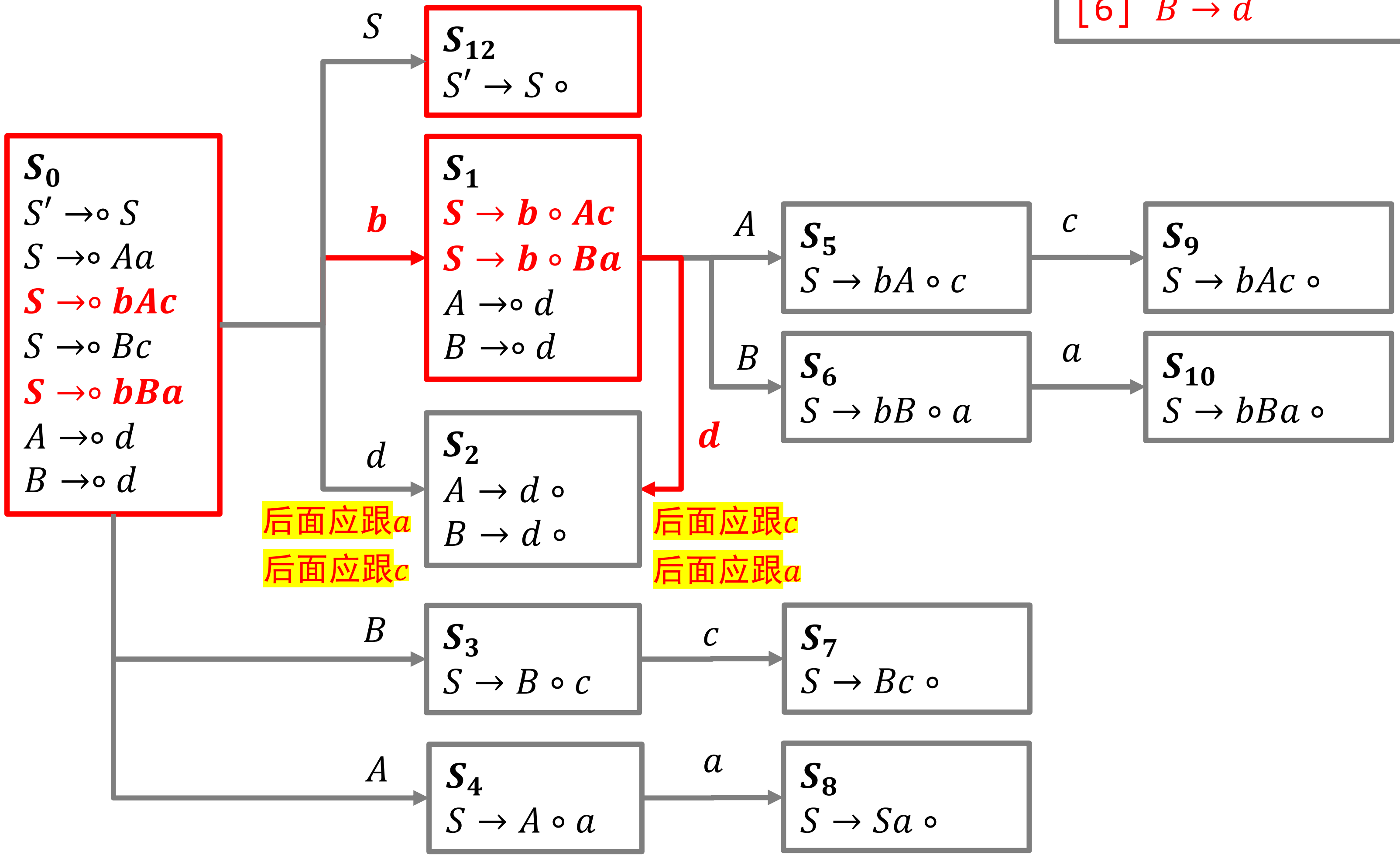
- 同一非终结符的两条规则：
  - 拥有共同的起始字符串β_1；
  - β_1后面分别为非终结符$X\beta_2$和字符串$\beta_3\beta_4$；
  - 存在规则$X \rightarrow \beta_3$。
  - $Follow(X) \cap First(\beta_4) \neq \emptyset$
- 或存在两种推导满足上述条件，如：

```
[1]  S → β₀β₁Xβ₂
[2]  S → β₀Y
[3]  Y → β₁β₃β₄
[3]  X → β₃
```

二义性语法：规约-规约冲突

- 解析bda时存在规约($A \to d$)-规约($B \to d$)冲突
  - $a \in Follow(A)$ 且 $a \in Follow(B)$
- 解析da、dc等其它句子时存在同样的问题。

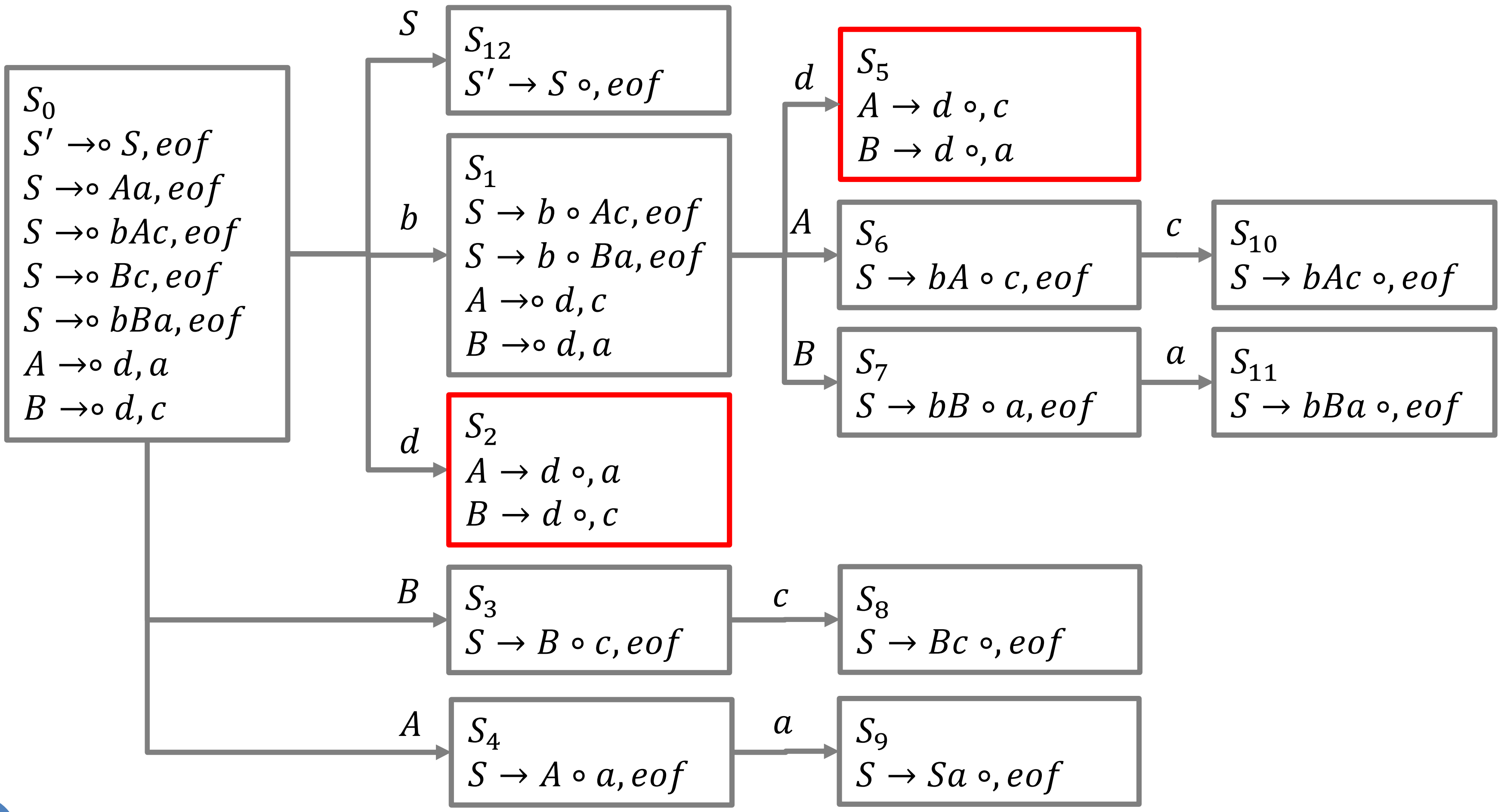
[1] $S \to Aa$
[2] $\quad |bAc$
[3] $\quad |Bc$
[4] $\quad |bBa$
[5] $A \to d$
[6] $B \to d$



S_{12}: $S' \to S\circ$
S_0: $S' \to\circ S$, $S \to\circ Aa$, $S \to\circ bAc$, $S \to\circ Bc$, $S \to\circ bBa$, $A \to\circ d$, $B \to\circ d$
S_1: $S \to b\circ Ac$, $S \to b\circ Ba$, $A \to\circ d$, $B \to\circ d$
S_2: $A \to d\circ$, $B \to d\circ$
S_3: $S \to B\circ c$
S_4: $S \to A\circ a$
S_5: $S \to bA\circ c$
S_6: $S \to bB\circ a$
S_7: $S \to Bc\circ$
S_8: $S \to Sa\circ$
S_9: $S \to bAc\circ$
S_{10}: $S \to bBa\circ$
S, b, d, B, A, c, a
后面应跟a
后面应跟c
后面应跟c
后面应跟a

LR(1)自动机构造

[1] $S \to Aa$
[2] $\quad |bAc$
[3] $\quad |Bc$
[4] $\quad |bBa$
[5] $A \to d$
[6] $B \to d$



什么情况下容易出现规约-规约冲突？

```
[1] $S \to Aa$
[2]     $|bAc$
[3]     $|Bc$
[4]     $|bBa$
[5] $A \to d$
[6] $B \to d$
```

```
[1] $S \to \beta_1 X \beta_2$
[2]     $|\beta_1 Y \beta_3$
[3] $X \to \beta_4$
[4] $Y \to \beta_4$
```

- 同一非终结符的两条规则：
  - 拥有共同的起始字符串β_1；
  - β_1后面分别为非终结符$X\beta_2$和字符串$Y\beta_3$；
  - 存在规则$X \to \beta_4$和$Y \to \beta_4$ 。
  - $Follow(X) \cap Follow(Y) \neq \emptyset$
- 或存在两种推导满足上述条件，如

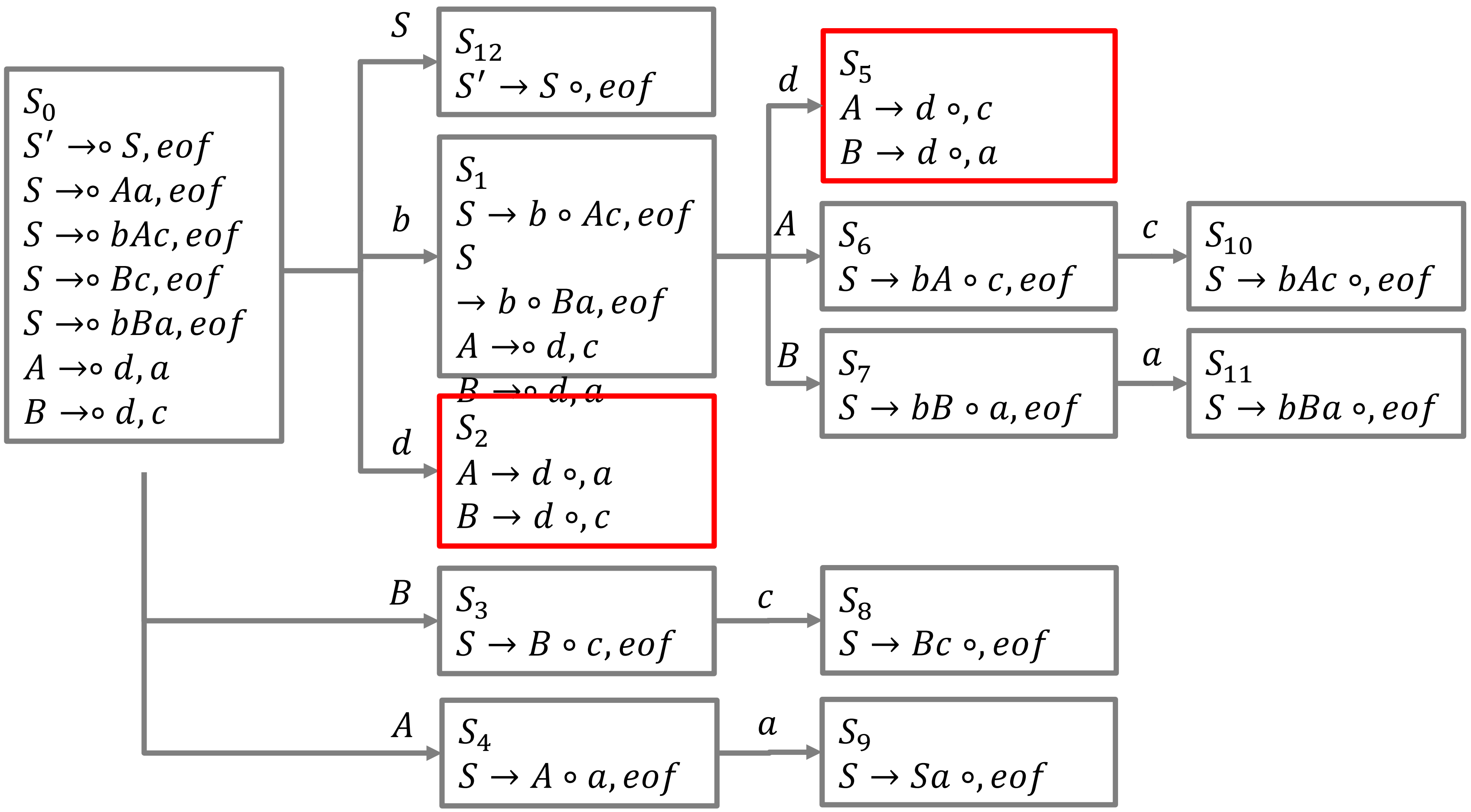
```
[1] $S \to \beta_0 X$
[2]     $|\beta_0 \beta_1 Y \beta_3$
[3] $X \to \beta_1 Z \beta_2$
[4] $Y \to \beta_4$
[5] $Z \to \beta_4$
```

思考：SLR和LR如何选取移进、规约操作？

- SLR维护当前的栈顶句柄信息
  - 通过构造LR(0)自动机和下个字符判断是否可以移进
  - 需要规约时根据Follow判断是否可行
- 经典LR(1)思路类似：
  - 自动机构造时考虑Follow信息；
  - 但LR(1)的规范项和规范集数量很多
- 折中思路：LALR（Lookahead LR）
  - 自动机构造时考虑Follow信息
  - 同时精简规范集

LALR构造思路

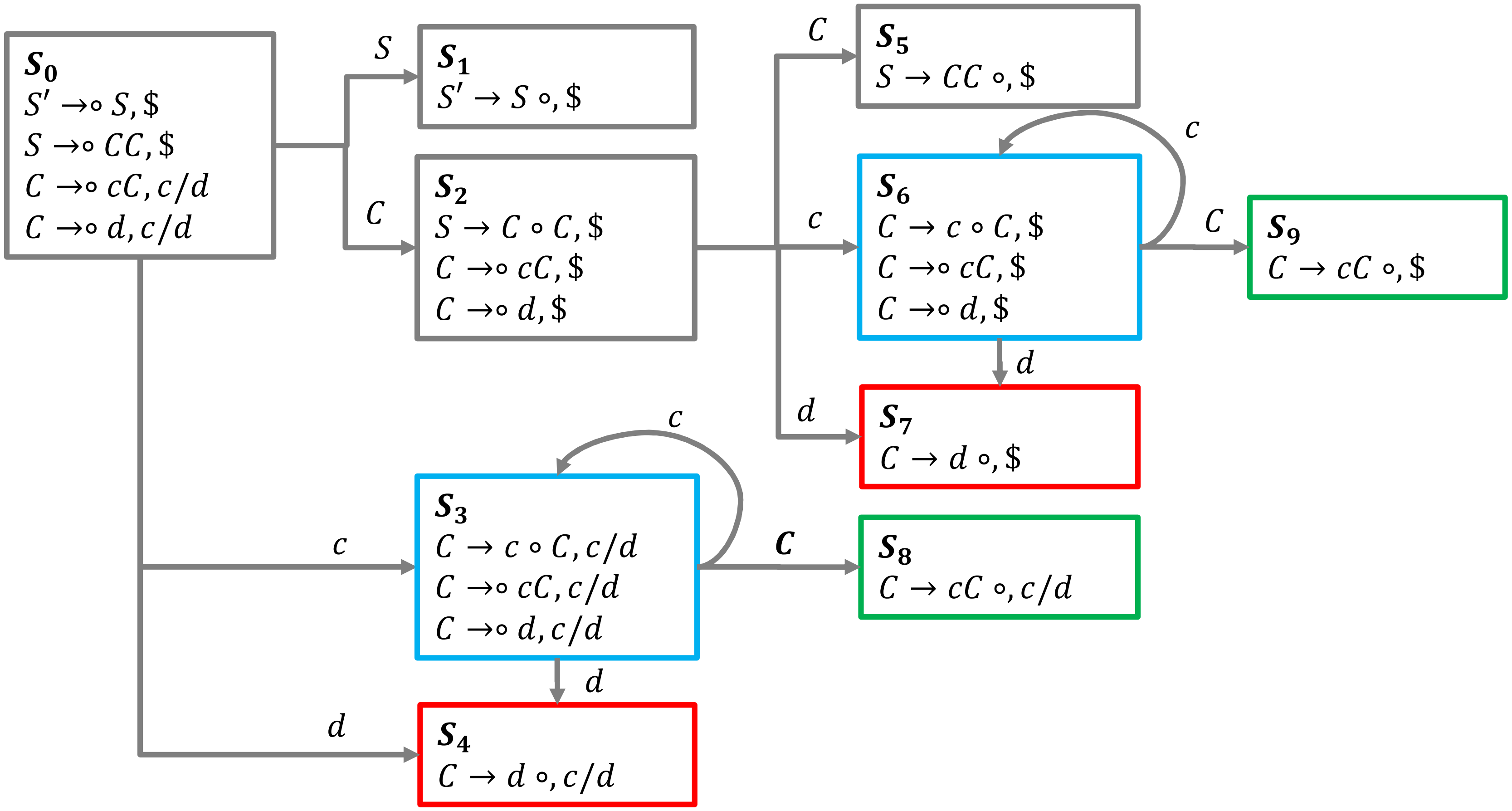
- 合并句柄状态完全相同的状态集
- 下面LR(1)自动机S_2和S_5可以合并，但合并后存在规约-规约冲突
  - 该语法不是LALR



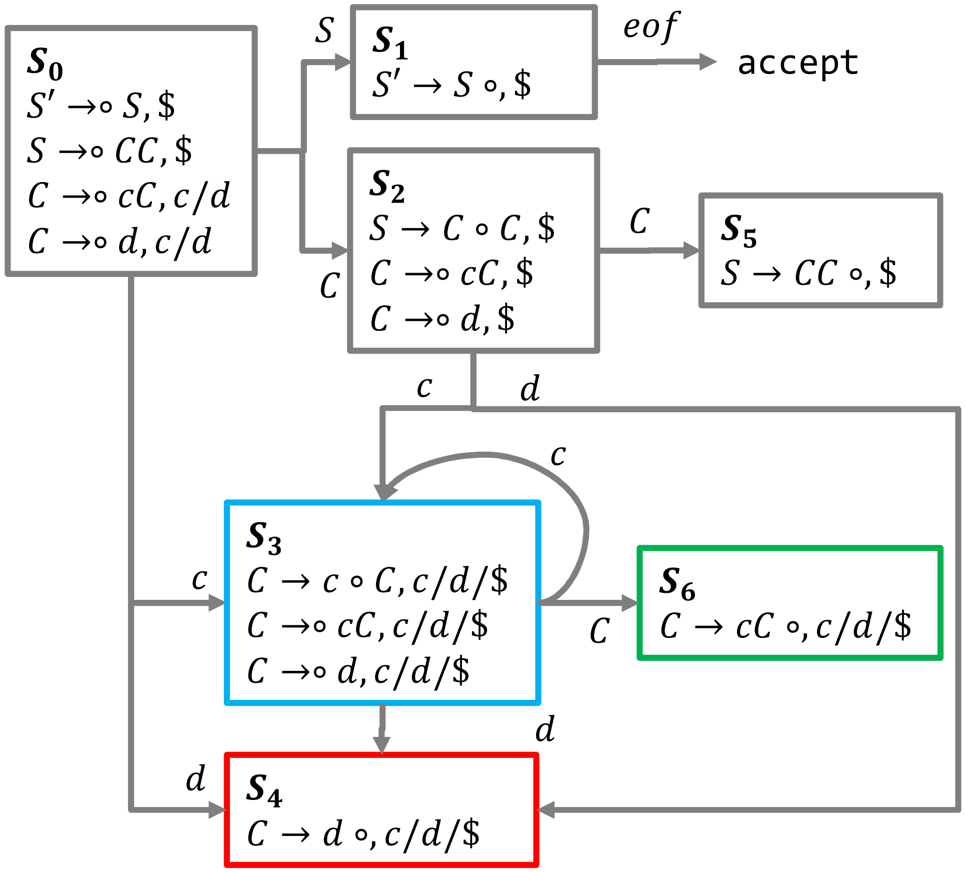
LALR语法举例

[1] $S' \to S$
[2] $S \to CC$
[3] $C \to cC$
[4] $\quad\quad | \, d$

- 可以合并的规范集
  - S3和S6、S4和S7、S8和S9;
  - Follow项取并集。



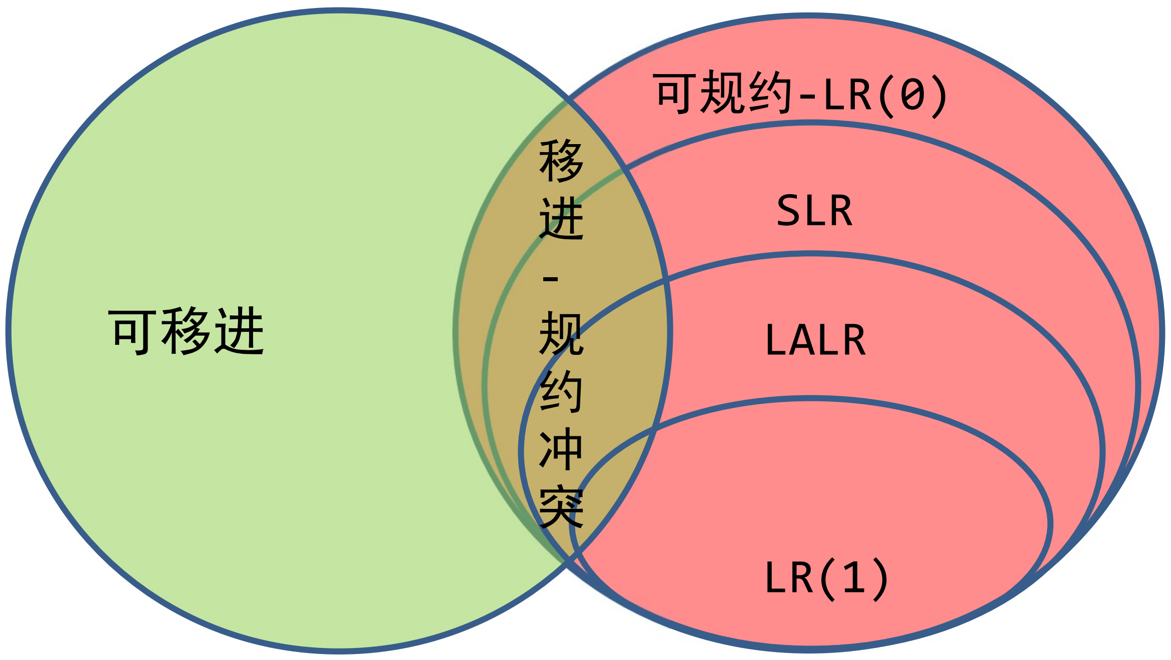
LALR解析表



规范项				Goto	
	c	d	eof	S	C
S_0	shift S_3	shift S_4		S_1	S_2
S_1			accept		
S_2	shift S_3				S_5
S_3	shift S_3	shift S_4			S_6
S_4	reduce [4]	reduce [4]	reduce [4]		
S_5			reduce [2]		
S_6	reduce [3]	reduce [3]	reduce [3]		

几种语法的关系

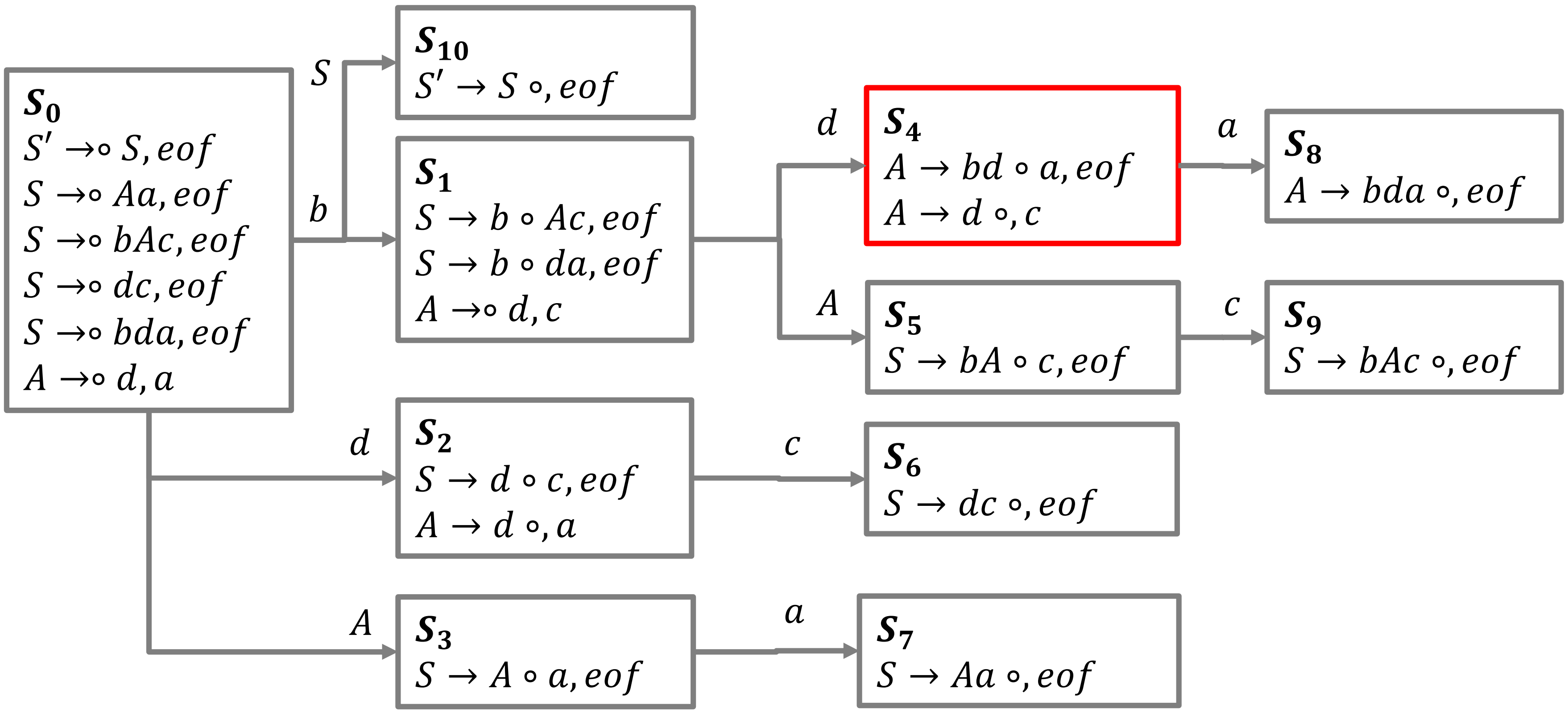
- 语法表达能力：LR(1)>LALR(1)>SLR
  - 规约条件严苛：LR(1)>LALR(1)>SLR
  - 移进条件同LR(0)?



举例：LALR，非SLR(1)语法

- 构造SLR解析表则解析bda时存在移进-规约冲突
- LALR解析方法可以避免冲突

[1] $S \rightarrow Aa$
[2] $\quad |bAc$
[3] $\quad |dc$
[4] $\quad |bda$
[5] $A \rightarrow d$



S_0
$S' \rightarrow \circ S, eof$
$S \rightarrow \circ Aa, eof$
$S \rightarrow \circ bAc, eof$
$S \rightarrow \circ dc, eof$
$S \rightarrow \circ bda, eof$
$A \rightarrow \circ d, a$
S
b
d
A
S_{10}
$S' \rightarrow S \circ, eof$
S_1
$S \rightarrow b \circ Ac, eof$
$S \rightarrow b \circ da, eof$
$A \rightarrow \circ d, c$
S_2
$S \rightarrow d \circ c, eof$
$A \rightarrow d \circ, a$
S_3
$S \rightarrow A \circ a, eof$
d
A
c
a
S_4
$A \rightarrow bd \circ a, eof$
$A \rightarrow d \circ, c$
S_5
$S \rightarrow bA \circ c, eof$
S_6
$S \rightarrow dc \circ, eof$
S_7
$S \rightarrow Aa \circ, eof$
a
c
S_8
$A \rightarrow bda \circ, eof$
S_9
$S \rightarrow bAc \circ, eof$

思考

- LL(1)语法一定是LR(1)吗？为什么？
    - 不一定是SLR(1)
    - 不一定是LALR(1)

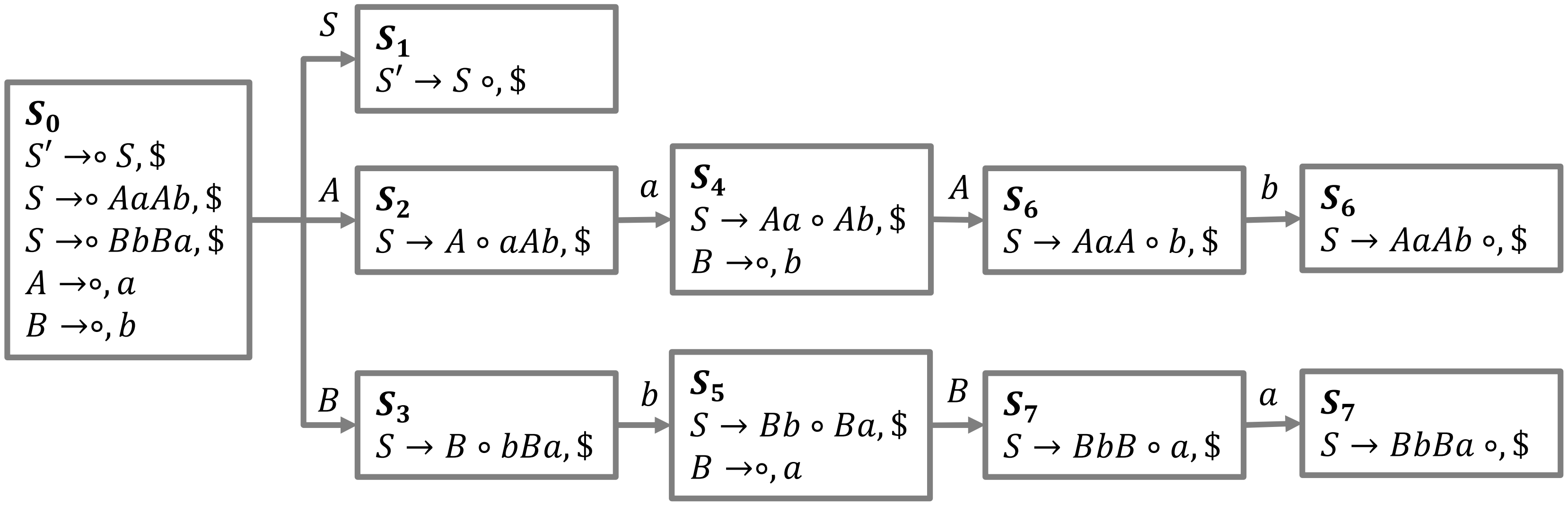
举例说明：LL(1)非SLR(1)

[1] $S \to AaAb$
[2] $\quad |BbBa$
[3] $A \to \epsilon$
[4] $B \to \epsilon$

是LL(1)
- $First^{+}(S \to AaAb) = \{a\}$
- $First^{+}(S \to BbBa) = \{b\}$

不是SLR(1)
- $Follow(A) = Follow(B) = \{a, b\}$
- $Action(S_0, a) = reduce[3]$ 或 $reduce[4]$

是LR(1)



举例说明：LL(1)非LALR(1)

[1] $S \rightarrow aAaAb$
[2] $\quad\quad |bBbBa$
[3] $A \rightarrow b$
[4] $B \rightarrow b$

练习

- 下列语法是否是LR(1) ？

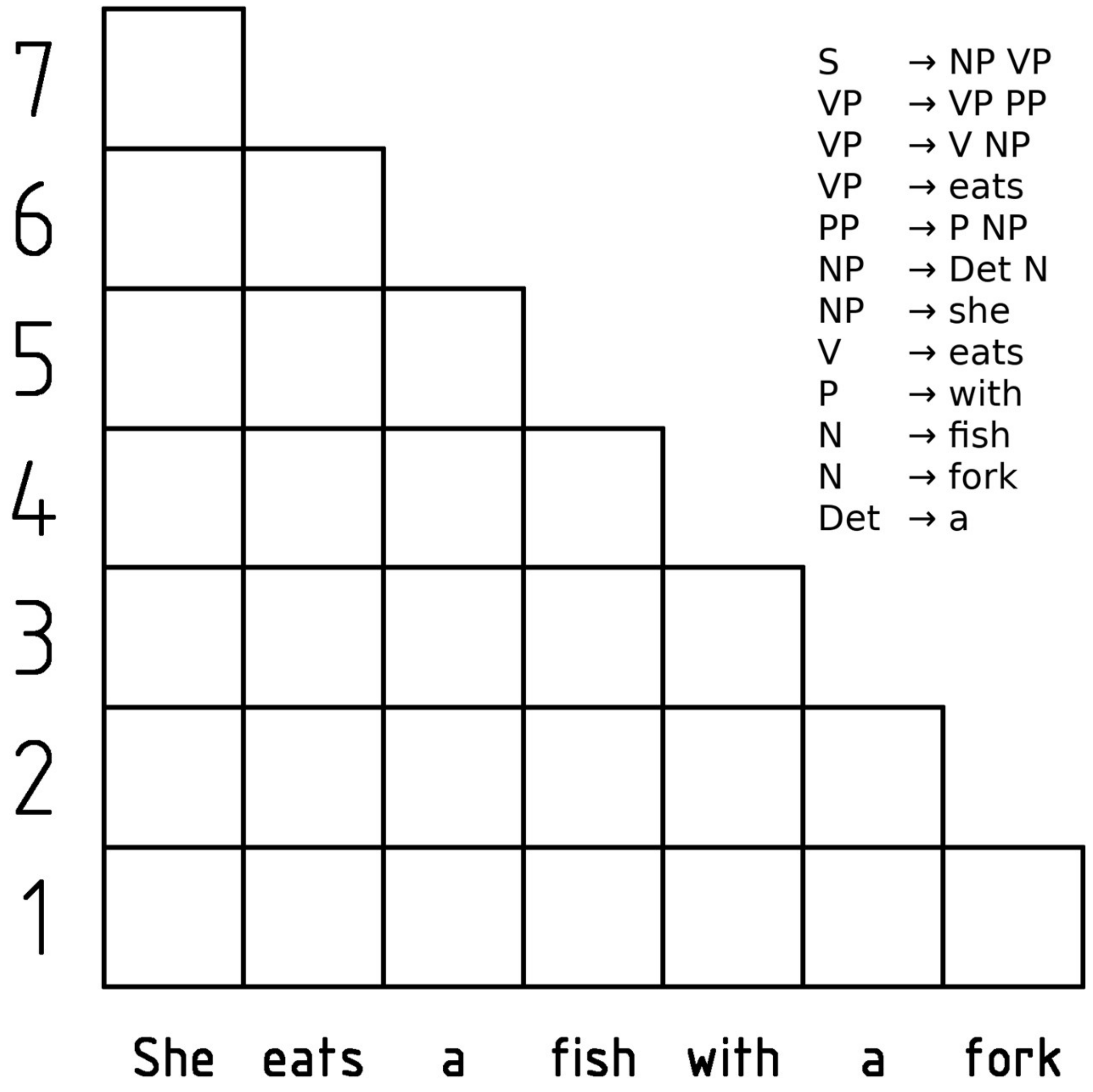
$$
\begin{aligned}
&<regex>::=<union>\ |\ <concat>\\
&<union>::=<regex>\ "|"\ <concat>\\
&<concat>::=<concat><term>\ |\ <term>\\
&<term>::=<element>*\ |\ <element>\\
&<element>::=(<regex>)|\ <alphanum>
\end{aligned}
$$

$$
\begin{array}{ll}
[1] & <regex>::=<concat><regex'>\\
[2] & <regex'>::=\ "|"\ <concat><regex'>\\
[3] & \qquad\qquad |\epsilon\\
[4] & <concat>::=<term><concat'>\\
[5] & <concat'>::=<term><concat'>\\
[6] & \qquad\qquad |\epsilon\\
[7] & <term>::=<element><follow>\\
[8] & <follow>::=*\\
[9] & \qquad\qquad |\epsilon\\
[10] & <element>::=(<regex>)\\
[11] & \qquad\qquad |<alphanum>
\end{array}
$$

如果LR（1）不够用怎么办？

- LR(1)解析表存在冲突
- GLR（Generalized LR）
  - 遇到冲突时分别尝试两种解析指令
  - 复制栈状态，维护解析搜索树

通用自底向上CFG分析： CYK算法



S	→	NP VP
VP	→	VP PP
VP	→	V NP
VP	→	eats
PP	→	P NP
NP	→	Det N
NP	→	she
V	→	eats
P	→	with
N	→	fish
N	→	fork
Det	→	a

CYK解析算法伪代码参考

```
INIT:
    Gramma: $R_1, R_2, \dots R_r$
    String to parse: w = $w_1, w_2, \dots, w_1$
    P[n,n,r] inited with false

Foreach i = 1 to n:
    Foreach $R_r$->$a_i$
        P[1,i,r] = True
Foreach l = 2 to n:
    Foreach i = 1 to n–l+1:
        Foreach j = 1 to l-1:
            Foreach $R_r$->$R_a$ $R_b$
                If P[j,i,a] and P[l-j,i+j,b]:
                    P[l,i,r]= True
If P[n,1,1]:
    w is a string of the language
Else:
    w is not a string of the language
```

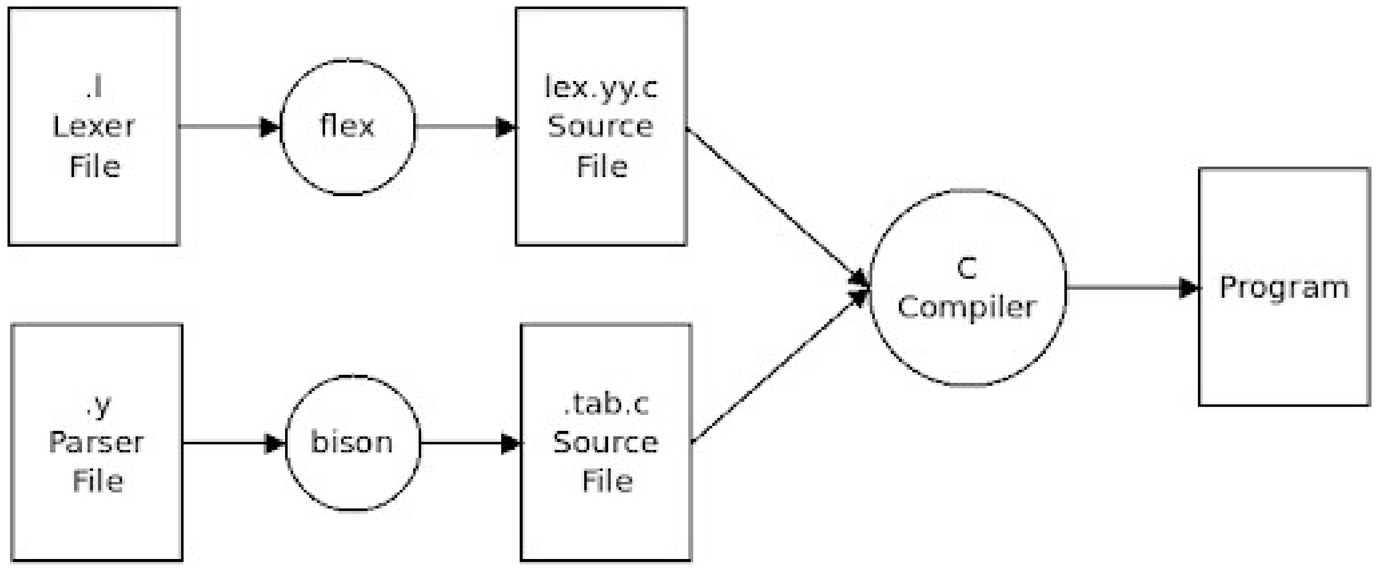
参考资料

- 《编译原理（第2版）》第四章：Syntax Analysis;
- 《编译器设计（第2版）》
  - 第三章：语法分析器。

五、语法分析工具

Bison

- 语法分析工具YACC(POSIX)/Bison (GNU)
  - 默认采用LALR(1)解析
  - 支持LR(1)等方法



https://www.gnu.org/software/bison/manual/html_node/

计算器程序示例

```
Main.c 计算器文件
Lexer.l 词法定义
Parser.y 语法定义
Expression.h 文件
Expression.c 功能函数
```

Lex文件

```
%{ /* Lexer.l file */
#include "Expression.h"
#include "Parser.h"
#include <stdio.h> %}

%option outfile="Lexer.c"
header-file="Lexer.h"
%option warn nodefault
%option reentrant noyywrap never-interactive nounistd
%option bison-bridge

%%
[ \r\n\t]* { continue; /* Skip blanks. */ }
[0-9]+ { sscanf(yytext, "%d", &yylval->value); return TOKEN_NUMBER; }
"*" { return TOKEN_STAR; }
"+" { return TOKEN_PLUS; }
"(" { return TOKEN_LPAREN; }
")" { return TOKEN_RPAREN; }
. { continue; /* Ignore unexpected characters. */}

%%
int yyerror(const char *msg) {
    fprintf(stderr, "Error: %s\n", msg);
    return 0;
}
```

```
%{ /* * Parser.y file * */
#include "Expression.h"
#include "Parser.h"
#include "Lexer.h"
int yyerror(SExpression **expression, yyscan_t scanner, const char *msg) { /* Add error
handling routine as needed */ }
%}

%code requires { typedef void* yyscan_t; }
%output "Parser.c"
%defines "Parser.h"
%define api.pure
%lex-param { yyscan_t scanner }
%parse-param { SExpression **expression }
%parse-param { yyscan_t scanner }
%union { int value; SExpression *expression; }
%token TOKEN_LPAREN "("
%token TOKEN_RPAREN ")"
%token TOKEN_PLUS "+"
%token TOKEN_STAR "*"
%token <value> TOKEN_NUMBER "number"
%type <expression> expr


/* Precedence (increasing) and associativity */
%left "+"
%left "*" %

%%
input : expr { *expression = $1; } ;
expr : expr[L] "+" expr[R] { $$ = createOperation( eADD, $L, $R ); }
    | expr[L] "*" expr[R] { $$ = createOperation( eMULTIPLY, $L, $R ); }
    | "(" expr[E] ")" { $$ = $E; }
    | "number" { $$ = createNumber($1);
} ; %%
```

main.c

```c
int yyparse(SExpression **expression, yyscan_t scanner);
SExpression *getAST(const char *expr) {
    SExpression *expression;
    yyscan_t scanner;
    YY_BUFFER_STATE state;
    if (yylex_init(&scanner)) return NULL;
    state = yy_scan_string(expr, scanner);
    if (yyparse(&expression, scanner)) return NULL;
    yy_delete_buffer(state, scanner);
    yylex_destroy(scanner);
    return expression;
}

int evaluate(SExpression *e) {
    switch (e->type) {
        case eVALUE: return e->value;
        case eMULTIPLY: return evaluate(e->left) * evaluate(e->right);
        case eADD: return evaluate(e->left) + evaluate(e->right);
        default: /* should not be here */ return 0;
    }
}

int main(void) {
    char expr[256];
    scanf("%s",expr);
    SExpression *e = getAST(test);
    int result = evaluate(e);
    printf("Result of '%s' is %d\n", test, result);
    deleteExpression(e);
    return 0;
```

Expression.h

```
#ifndef __EXPRESSION_H__
#define __EXPRESSION_H__

typedef enum tagEOperationType {
    eVALUE,
    eMULTIPLY,
    eADD
} EOperationType;

typedef struct tagSExpression {
    EOperationType type; /* /< type of operation */
    int value; /* /< valid only when type is eVALUE */
    struct tagSExpression *left; /* /< left side of the tree */
    struct tagSExpression *right; /* /< right side of the tree */
} SExpression;

SExpression *createNumber(int value);
SExpression *createOperation(EOperationType type, SExpression *left, SExpression *right);
void deleteExpression(SExpression *b);

#endif
```

Expression.c

```
static SExpression *allocateExpression() {
    SExpression *b = (SExpression *)malloc(sizeof(SExpression));
    if (b == NULL) return NULL;
    b->type = eVALUE;
    b->value = 0;
    b->left = NULL;
    b->right = NULL;
    return b;
}
SExpression *createNumber(int value) {
    SExpression *b = allocateExpression();
    if (b == NULL) return NULL;
    b->type = eVALUE;
    b->value = value;
    return b;
}
SExpression *createOperation(EOperationType type, SExpression *left, SExpression *right) {
    SExpression *b = allocateExpression();
    if (b == NULL) return NULL;
    b->type = type;
    b->left = left;
    b->right = right;
    return b;
}
void deleteExpression(SExpression *b) {
    if (b == NULL) return;
    deleteExpression(b->left);
    deleteExpression(b->right);
    free(b);
}
```

总结

一、句式分析的基本概念

二、LLVM案例分析

三、自顶向下分析

- LL(1)语言
- 通用算法：Earley算法

四、自底向上分析

- SLR、LALR、LR(1)语言
- 通用算法：GLR算法、CYK算法

五、语法分析工具