Relational Algebra

André Restivo

Loading [MathJax]/jax/output/HTML-CSS/fonts/TeX/fontdata.js

Index

Introduction

Relational Model

- A set of relations defined by their schemas.
- Each relation is composed by attributes and tuples.
- Schema of a relation R with attributes a_1, a_2, a_3, ..., a_n:

? R(a_1, a_2, a_3, ..., a_n)

Relational Model

The cardinality (number of tuples) in relation R:

$$|R|$$

A tuple with attribute values $v_1, v_2, v_3, \ldots, v_n$:

$$t = <v_1, v_2, v_3, \ldots, v_n>$$

Attribute a_i belonging to relation R:

$$R.a_i$$

The domain (possible values) of attribute a_i:

$$Dom(a_i)$$

The *null* value:

$$\perp$$

Example

?

Relation: Employee(id, name, salary, taxes, department)

Tuple: t = <1234, John, 1200, 200, \perp>

Attribute: Employee.name

Domain: Dom(Employee.name) = text

Example

Employee(id, name, salary, taxes, department)

id	name	salary	taxes	departament
1	John Doe	1000	200	1
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp

$|\text{Employee}| = 5$

Unary Operators

Projection

The result of a projection is defined as the set that is obtained when all tuples in R are restricted to the set <a_1, ..., a_n>

? \Pi_{a_1,...,a_n}(R)

Consider L as a list containing attributes from R:

? S = \Pi_{L}(R)

Relation S will only have the attributes from that list.

? If L does not contain a key from R, repeated tuples are eliminated.

Example

? S = \Pi_{name, salary}(Employee)

name	salary
John Doe	1000
Jane Doe	800
John Smith	1200
Jane Roe	1000

? Notice that one line was eliminated.

Projection

Renaming and Arithmetic Operators

The projection operator can also be used to rename attributes:

```
? S = \Pi_{name, wages = salary}(Employee)
```

And calculate simple arithmetic expressions:

```
? S = \Pi_{name, wages = salary - taxes}(Employee)
```

For simplicity the result should be renamed.

Example

? S = \Pi_{name, wages = salary - taxes}(Employee)

id	name	salary	taxes	departament
1	John Doe	1000	200	1
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp

name	wages
John Doe	800
Jane Doe	700
John Smith	850
Jane Roe	800
John Doe	1000

Rename

Renaming the relation R to S:

? \rho_S(R)

Renaming attribute a to atribute x in relation R(a,b,c):

? \rho_{a/x}(R) \Rightarrow R(x,b,c)

Renaming attribute a to atribute x and c to atribute y in relation R(a,b,c):

? \rho_{a/x, c/y}(R) \Rightarrow R(x,b,y)

Selection

Select a set of tuples where a certain condition c holds:

? $S = \sigma_c(R)$

- c is a condition involving attributes from R.
- The condition can contain arithmetic ($+$ $-$ $\times$ $\div$), conditional ($<$ $>$ $\leq$ $\geq$ $\neq$) and logical ($\vee$ $\wedge$ $\neg$) operators.
- S has the same attributes as R.

Example

? $S = \sigma_{salary < 1000 \vee department = 3}(Employee)$

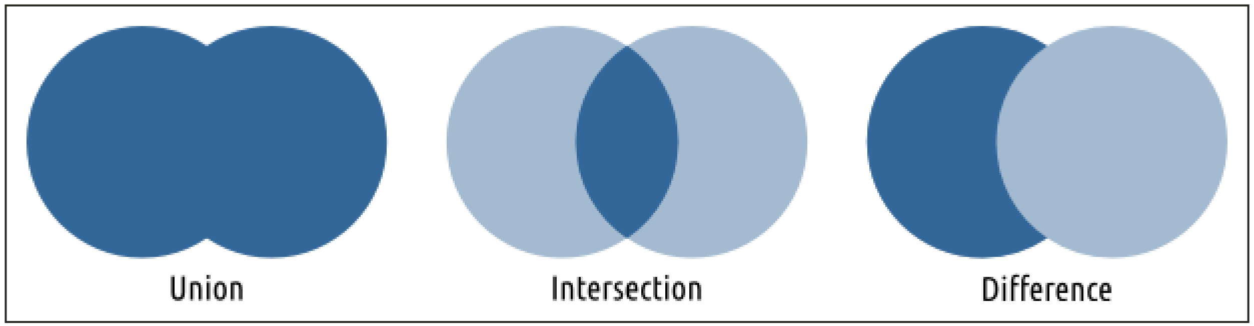
Employees with a salary smaller than 1000 or that work at department 3.

id	name	salary	departament
1	John Doe	1000	1
2	Jane Doe	800	2
3	John Smith	1200	2
4	Jane Roe	1000	3
5	John Doe	1000	$\perp$

id	name	salary	departament
2	Jane Doe	800	2
4	Jane Roe	1000	3

Set Operators

Union, Intersection and Difference



Union, Intersection and Difference

The two relations involved must be union-compatible:

- they must have the same number of attributes
- the domain of each attribute must be the same in both R and S

?

$$R \cup S = \{x: x \in R \vee x \in S\}$$

$$R \cap S = \{x: x \in R \wedge x \in S\}$$

$$R - S = \{x: x \in R \wedge x \notin S\}$$

Union Example

Employee

id	name	salary	taxes	departament
1	John Doe	1000	200	1
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp

Manager

id	name	salary	taxes	departament
1	John Doe	1000	200	1
6	Big Boss	5000	0	\perp

Employee \cup Manager

id	name	salary	taxes	departament
1	John Doe	1000	200	1
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp
6	Big Boss	5000	0	\perp

Intersection Example

Employee

|id|name|salary|taxes|departament |-|-|javascript/#46
|1 | John Doe | 1000 | 200 | 1 |2 | Jane Doe | 800 | 100 | 2
|3 | John Smith | 1200 | 350 | 2 |4 | Jane Roe | 1000 | 200
| 3 |5 | John Doe | 1000 | 0 | \perp

Manager

id	name	salary	taxes	departament
1	John Doe	1000	200	1
6	Big Boss	5000	0	\perp

Employee \cap Manager

id	name	salary	taxes	departament
1	John Doe	1000	200	1

Difference Example

Employee

id	name	salary	taxes	departament
1	John Doe	1000	200	1
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp

Manager

id	name	salary	taxes	departament
1	John Doe	1000	200	1
6	Big Boss	5000	0	\perp

Employee - Manager

id	name	salary	taxes	departament
2	Jane Doe	800	100	2
3	John Smith	1200	350	2
4	Jane Roe	1000	200	3
5	John Doe	1000	0	\perp

Joins

Cartesian Product

The cartesian product $R \times S$ is the set of all ordered pairs (r, s) where $r \in R$ and $s \in S$.

$$R \times S = \{ < r,s > : r \in R, s \in S \}$$

The cartesian product between relations $R(a_1,...,a_n)$ and $S(b_1,...,b_m)$ is a relation with $n+m$ attributes $(a_1, ..., a_n, b_1, ..., b_m)$ where there is a tuple for each possible combination of tuples from R and S.

The cardinality of the resulting relation is equal to the product between the cardinalities of the original relations:

? $|R \times S| = |R| \times |S|$

Example

Employee

id	name	departament
1	John Doe	1
2	Jane Doe	2
3	John Smith	2
4	John Doe	\perp

Department

number	designation
1	Marketing
2	Accounting

Employee \times Department

id	name	department	number	designation
1	John Doe	1	1	Marketing
2	Jane Doe	2	1	Marketing
3	John Smith	2	1	Marketing
4	John Doe	\perp	1	Marketing
1	John Doe	1	2	Accounting
2	Jane Doe	2	2	Accounting
3	John Smith	2	2	Accounting
4	John Doe	\perp	2	Accounting

Conditional Join

A cartesian product between relations R and S followed by a selection on condition c:

$$R \bowtie_c S$$

The same as a cartesian product followed by a selection:

$$R \bowtie_c S = \sigma_c(R \times S)$$

Allows the combination of relations that are associated by a foreign key.

Example

Employee

id	name	departament
1	John Doe	1
2	Jane Doe	2
3	John Smith	2
4	John Doe	\perp

Department

number	designation
1	Marketing
2	Accounting

Employee \bowtie_{department=number} Department

id	name	department	number	designation
1	John Doe	1	1	Marketing
2	Jane Doe	2	2	Accounting
3	John Smith	2	2	Accounting

Natural Join

A particular case of a join where the condition is the equality of attributes on both relations having the same name.

? $R \bowtie S$

Attributes used in the condition are merged together.

Example

Employee

id	name	number
1	John Doe	1
2	Jane Doe	2
3	John Smith	2
4	John Doe	\perp

Department

number	designation
1	Marketing
2	Accounting

Employee \bowtie Department

id	name	number	designation
1	John Doe	1	Marketing
2	Jane Doe	2	Accounting
3	John Smith	2	Accounting

Semijoin

A join where only attributes from one of the relations is kept.

? $R \ltimes S = \Pi_R (R \bowtie S)$

? $R \rtimes S = \Pi_S (R \bowtie S)$

Examplejavascript/#46

Employee

id	name	number
1	John Doe	1
2	Jane Doe	2
3	John Smith	2
4	John Doe	\perp

Department

number	designation
1	Marketing
2	Accounting
3	Transports

Employee \ltimes Department

id	name	number
1	John Doe	1
2	Jane Doe	2
3	John Smith	2

Employee \rtimes Department

number	designation
1	Marketing
2	Accounting

Outer Join

A join operation where unmatched tuples are part of the result set. This tuples can come from the R relation (left), the S relation (right) or from both (full).

Left outer join:

? R ? _c S

Right outer join:

? R ?_c S

Full outer join:

? R ?_c S

Example

Employee

id	name	number
1	John Doe	1
2	Jane Doe	2
3	John Smith	2
4	John Doe	\perp

Department

number	designation
1	Marketing
2	Accounting
3	Transports

Employee ? Department

id	name	number	designation
1	John Doe	1	Marketing
2	Jane Doe	2	Accounting
3	John Smith	2	Accounting
4	John Doe	\perp	\perp

Employee ? Department

id	name	number	designation
1	John Doe	1	Marketing
2	Jane Doe	2	Accounting
3	John Smith	2	Accounting
\perp	\perp	3	Transports

Division

Division

The restrictions of tuples in R to the attribute names unique to R for which it holds that all their combinations with tuples in S are present in R.

? $R(a,b,c) \div S(b,c)$

In this example, the result of the division will have one attribute a (the one that does not exist in S), containing the values of a that are combined with all values of S.

Example

Works

id	name	number	designation
1	John Doe	1	Big Rocket
1	John Doe	2	Thingamabob
1	John Doe	3	Take a Nap
2	Jane Doe	2	Thingamabob
2	Jane Doe	3	Take a Nap
3	Jack Doe	1	Big Rocket
3	Jack Doe	2	Thingamabob

Project

number	designation
1	Big Rocket
2	Thingamabob
3	Take a Nap

Works \div Project

id	name
1	John Doe

Division without Division

? $R(a_1,...,a_n,b_1,...,b_m) \div S(b_1, ..., b_m)$

? $R \div S = \Pi_{a_1,...,a_n}(R) - \Pi_{a_1,...,a_n} (\Pi_{a_1,...,a_n}(R) \times S - R)$

Explanation

All tuples from the first n attributes of R:

? \Pi_{a_1,...,a_n}(R)

? \Pi_{id,name}(Works)

id	name
1	John Doe
2	Jane Doe
3	Jack Doe

Explanation

Cartesian product with S gives all possible combinations of those attributes with the tuples of S:

? \Pi_{a_1,...,a_n}(R) \times S

? \Pi_{id,name}(Works) \times Project

id	name	number	designation
1	John Doe	1	Big Rocket
2	Jane Doe	1	Big Rocket
3	Jack Doe	1	Big Rocket
1	John Doe	2	Thingamabob
2	Jane Doe	2	Thingamabob
3	Jack Doe	2	Thingamabob
1	John Doe	3	Take a Nap
2	Jane Doe	3	Take a Nap
3	Jack Doe	3	Take a Nap

Explanation

Removing the original R tuples we get the tuples that are not present in the original R relation:

? \Pi_{a_1,...,a_n}(R) \times S - R

? \Pi_{id,name}(Works) \times Project - Works

id	name	number	designation
2	Jane Doe	1	Big Rocket
3	Jack Doe	3	Take a Nap

Explanation

Projecting again we get the first n attributes of those tuples:

? \Pi_{a_1,...,a_n} (\Pi_{a_1,...,a_n}(R) \times S - R)

? \Pi_{id,name} (\Pi_{id,name}(Works) \times Project - Works)

id	name
2	Jane Doe
3	Jack Doe

Explanation

? $\Pi_{a_1,...,a_n}(R) - \Pi_{a_1,...,a_n} (\Pi_{a_1,...,a_n}(R) \times S - R)$

? $\Pi_{id,name}(Works) - \Pi_{id,name} (\Pi_{id,name}(Works) \times Project - Works)$

id	name
1	John Doe