Non-tangential maneuver exercise (ASTD2e2)

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1 Problem Statement

Given the radii R_1 and R_2 of the initial and the final circular orbit and the time of flight TOF, write a Mathematica code to find the maneuvers that should be performed. Namely compute Δv_1, Δv_2 and ϕ. Give your own example by choosing some proper R_1, R_2 and TOF.

2 Solution

Since we are given the initial and the final altitude of the circular orbit and the time of flight, we have to calculate the transfer orbit so that it meets the desired time of flight. Let us assume that the starting point A at a radiusR_1 is the pericenter of the transfer orbit and point B is the point of the transfer orbit with distance equal to R_2. The time of flight is given by the equation:

$$TOF = \sqrt{\frac{\mu}{a^3}}(E_B - e\sin E_B) \tag{1}$$

and at point B the following equation has to be met:

$$R_2 = a\,(1 - e\cos E_B) \tag{2}$$

Note that we have three unknowns in the above two equations and the initial condition which states that the transfer orbit's semi major axis has to be $a \geq \frac{R_1+R_2}{2}$. We can compute also add the following two relationships from the two body problem which state a relationship between the eccentric anomaly and the eccentricity, namely:

$$\cos\nu = \frac{\cos E - e}{1 - e\cos E} \tag{3}$$

and

$$\sin\nu = \frac{\sqrt{1-e^2}\sin E}{1 - e\cos E} \tag{4}$$

Considering these four equations, we can solve them numerically using Mathematica and use relevant searching conditions to obtain the semi-major axis and eccentricity of the transfer orbit. We can then compute:

$$\Delta v_1 = v_a - v_{c1} \tag{5}$$

where

$$v_A = \sqrt{\frac{\mu}{a}\frac{1+e}{1-e}} \tag{6}$$

$$v_{c1} = \sqrt{\frac{\mu}{R_1}} \tag{7}$$

and

$$\Delta v_2 = |\vec{v}_{C2} - \vec{v}_B| = \sqrt{v_{C2}^2 + v_B^2 - 2v_{C2}v_B\cos\varphi} \tag{8}$$

where

$$\mathbf{v}_B = \sqrt{\mu\left(\frac{2}{R_2} - \frac{1}{a}\right)} \tag{9}$$

$$v_{c2} = \sqrt{\frac{\mu}{R_2}} \tag{10}$$

$$\cos\varphi = \frac{R_1 v_A}{R_2 v_B} \tag{11}$$

The attached notebook **ASTD2e2_Retselis.nb** implements the above mentioned procedure. It initializes the system properties and defines the initial conditions. For the initial conditions, the problem from ASTD_CompuLab 1 is used, which can be found in Table 1.

Table 1: Chosen parameters for ASTD2e2

Input parameter	**Value**
R_1	322 km
R_2	35786 km
Time of Flight	12000 sec

After that, the properties of the Hohman transfer eclipse are used as initial searching conditions for Mathematica's FindRoot command. We also make sure to add the relevant conditions that have to be met, and search for solutions in the second quadrant given the selected time of flight. The results are then computed and printed and finally the transfer is visualized. The results are:

$$\Delta v_1 = 2310.59\, m/s \tag{12}$$
$$\Delta v_2 = 2345.14\, m/s \tag{13}$$
$$\phi = 48.7739° \tag{14}$$
$$Total\ impulse\ \Delta v = 4655.74\, m/s \tag{15}$$

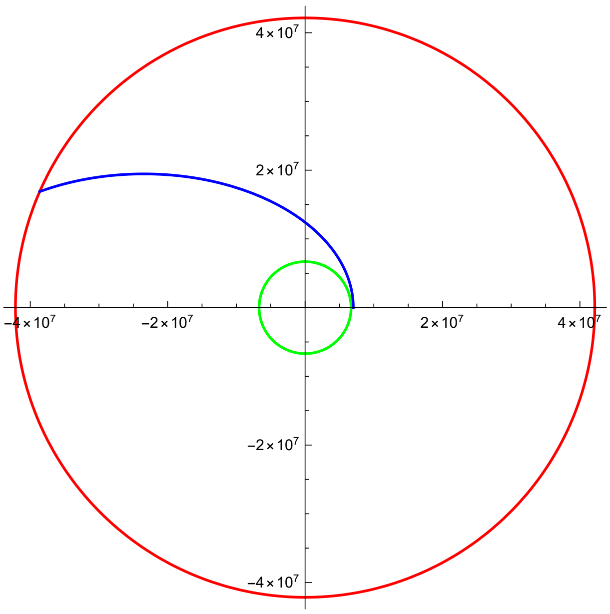

Figure 1: Visualization of the computed non tangential orbit transfer