Design and Analysis of Algorithm
Assignment # 02

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(a) $T(n) = 4T(n/2) + n$

Solution:

Let $T(n) = 4T(n/2) + n$

where $a = 4$ $b = 2$ and $f(n) = n$

Now finding $n^{\log_b a}$

$= n^{\log_2 4}$ $= n^2$

So we need to subtract 1 from 2 in order to make it equal to $f(n)$.

so let $\varepsilon = 1$ and

$f(n) = O(n^{\log_3 a-\varepsilon})$ case 1 applies

Hence

$T(n) = \Theta(n^{\log_b a})$ when $f(n) = O(n^{\log_b a-\varepsilon})$

Thus

$T(n) = \Theta(n^2)$ ans,

(b)

$T(n) = 4T(n/2) + n^2$

where

$a = 4$, $b = 2$ and $f(n) = n^2$

Now finding $n^{\log_b a}$

$= n^{\log_2 4} = n^2 = f(n)$ Hence 2nd case applies.

i.e

$T(n) = \Theta(n^{\log_b a} \log n)$ wher $f(n) = \Theta(n^{\log_b a})$

so $T(n) = \Theta(n^2 \log_2 n)$

(c)

$T(n) = 4T(n/2) + n^3$

where $a = 4$, $b = 2$ and $f(n) = n^3$

finding $n^{\log_b a}$

$= n^{\log_2 4} = n^2$

let $\epsilon = 1$

so

$n^{2+\epsilon} = n^{2+1} = n^3 = f(n)$

Now fullfilling the second part of this case

$a f(n/b) \le c \cdot f(n)$ for $c < 1$

$4 f(n/2) \le c \cdot n^3$

$4 \cdot (n/2)^3 \le c \cdot n^3$

$\cancel{4} \cdot n^3 / \cancel{8}_2 \le c \cdot n^3$

$n^3/2 \le c \cdot n^3$ Hence case 3 applies

i.e $T(n) = \Theta(n^3)$ ans.

(4) $T(n) = 2T(n/4) + n^{1/2}$

where $a = 2$, $b = 4$ and $f(n) = n^{1/2}$

finding $n^{\log_b a}$

$= n^{\log_4 2} = n^{1/2} = f(n)$

Hence case 2 applies i.e:

$T(n) = \Theta(n^{\log_b a} \log n)$ where $f(n) = \Theta(n^{\log_b a})$

Thus

$T(n) = \Theta(n^{1/2} \log_2 n)$ Ans