Negative Binomial Distribution

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Introduction

- A negative binomial distribution (also called the Pascal Distribution) is a discrete probability distribution for random variables in a negative binomial experiment.
- Negative Binomial Distribution is the distribution of the number of trials needed to get the ‘rth’ success.

Difference

- Binomial Distribution:

  Binomial Distribution is the distribution of number of successes in a fixed number of independent Bernoulli Trials.

- Negative Binomial Distribution:

  It is the distribution of the number of trials needed to get a fixed number of success.

Difference

- The random variable is the number of repeated trials, X, that produce a certain number of successes, r. In other words, it's the number of failures before a success. This is the main difference from the binomial distribution: with a regular binomial distribution, you're looking at the number of successes. With a negative binomial distribution, it's the number of failures that counts.

Probabibilities

- P(Success)=p, this stays constant from trial to trial.
- p(Failure)= 1-p
- X represents the trial number of the rth success

Probability Mass Function (PMF)

For the rth success to occur on the xth trial:

- The first x-1 trials must result in r-1 success

$$\binom{x-1}{r-1} p^{r-1} (1-p)^{(x-1)-(r-1)}$$

- The xth trial must be a success which has the probability of p.

Probability Mass Function (PMF)

- The probability the rth success occurs on the xth trial is:

$$P(X=x) = p \times \binom{x-1}{r-1} p^{r-1} (1-p)^{(x-1)-(r-1)}$$

$$= \binom{x-1}{r-1} p^{r} (1-p)^{x-r}$$

r is the number of successes and

p = the probability of success.

Mean and Variance

- Mean is given by:

  rq / p, where q = 1 – p.

- The variance is:

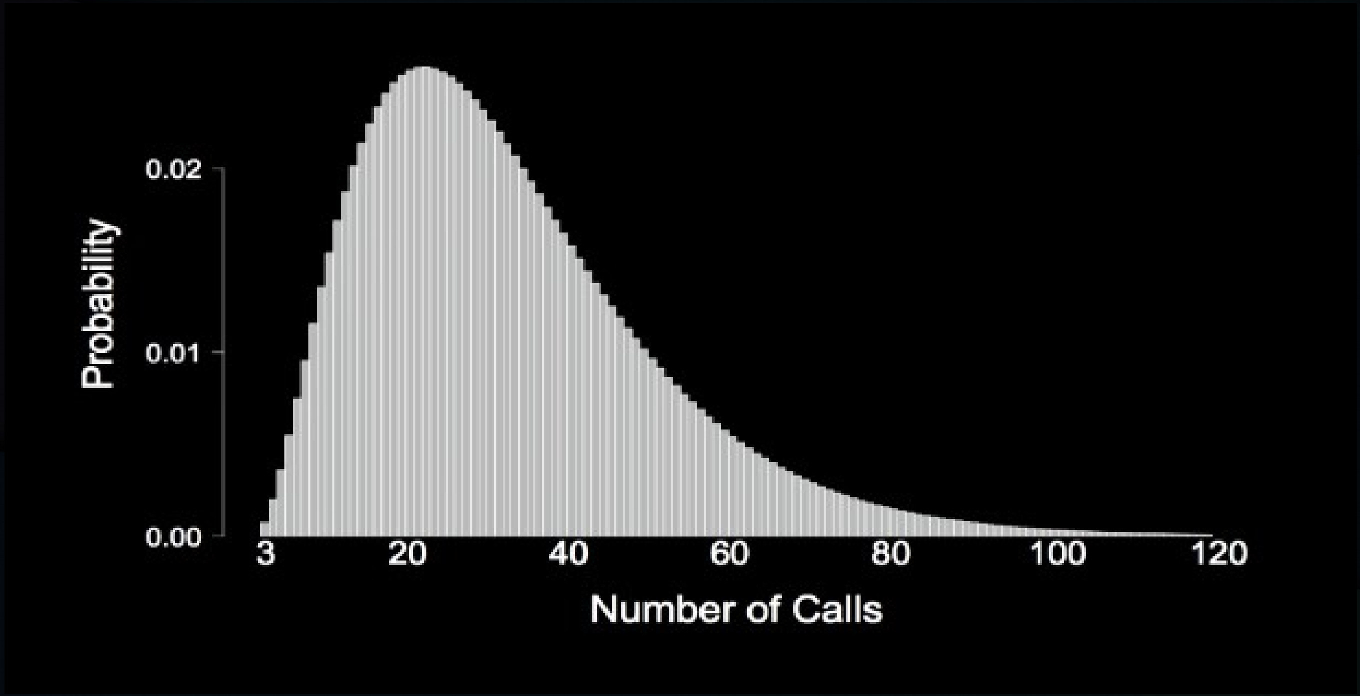
  rq/p2

Use of Negative Binomial Distribution

- Suppose that we flip a fair coin and we ask the question, "What is the probability that we get three heads in the first X coin flips?" This is a situation that calls for a negative binomial distribution.

  The coin flips have two possible outcomes, the probability of success is a constant 1/2, and the trials they are independent of one another. We ask for the probability of getting the first three heads after X coin flips.
- The geometric distribution is negative binomial distribution where the number of successes (r) is equal to 1.

Graph



Problem

A person conducting telephone surveys must get three more complicated surveys before their job is finished.

On each randomly dialed number there is a 9% chance of reaching an adult who will complete the survey.

What is the probability the 3rd completed survey occurs on the 10^{th} call?

Solution

$$P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$r = 3,\ p = 0.09$

$$P(X=10) = \binom{10-1}{3-1} 0.09^3 (1-0.09)^{10-3}$$

$$= \underline{0.01356}$$