Assignment #3

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$$f(x,y) = \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), 0 < x < 1, \; 0 < y < 2$$

(a) Verify that this is a joint density function.

Solution:

$f(x,y) \geq 0$ satisfied!

Now

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dx\,dy = 1$$

let's prove this

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dx\,dy = \int_{0}^{1}\int_{0}^{2} \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dx\,dy$$

$$= \frac{6}{7}\int_{0}^{1}\int_{0}^{2}\left(x^2 + \frac{xy}{2}\right) dx\,dy$$

integerating interms of 'y'

$$= \frac{6}{7}\int_{0}^{1} x^2 y + \frac{xy^2}{2(2)} \Bigg|_{0}^{2} dx$$

$$= \frac{6}{7}\int_{0}^{1} 2x^2 + \frac{x\cancel{(2)^2}}{\cancel{4}}\, dx$$

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$$= \frac{6}{7} \int_0^1 2x^2 + x \, dx$$

Now integerating w.r.t 'x' :

$$= \frac{6}{7} \left(\frac{2x^3}{3} + \frac{x^2}{2} \right) \Big|_0^1$$

$$= \frac{6}{7} \left(\frac{2}{3} + \frac{1}{2} \right)$$

$$= \frac{6}{7} \cdot \frac{7}{6} = 1$$

Hence proved that the given function is a joint density function.

(b) Compute the density function of X.

for $x \in (0,1)$

$$f_X(x) = \int_0^2 f \, dy$$

$$\int_0^2 f \, dy = \int_0^2 \frac{6}{7} \left(x^2 + \frac{xy}{2} \right) dy$$

$$= \frac{6}{7} \int_0^2 x^2 + \frac{xy}{2} \, dy$$

$$= \frac{6}{7} \left(x^2 y + \frac{xy^2}{2(2)} \right) \Big|_0^2$$

$$= \frac{6}{7} \left(2x^2 + \frac{x4}{4} \right)$$

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$$= \frac{6}{7}\left(2x^2 + x\right)$$

(c) find $P\{X > Y\}$

for $x > y$

we have

$$P(X>Y) = \int_0^1 \int_0^x (f) \, dy \, dx$$

$$= \int_0^1 \int_0^x \frac{6}{7}\left(x^2 + \frac{xy}{2}\right) dx \, dy$$

$$= \frac{6}{7} \int_0^1 \int_0^x \left(x^2 + \frac{xy}{2}\right) dx \, dy$$

$$= \frac{6}{7} \int_0^1 \left(x^2 y + \frac{xy^2}{2(2)}\right)\Big|_0^x dx$$

$$= \frac{6}{7} \int_0^1 \left(x^2 . x + \frac{x . x^2}{4}\right) dx$$

$$= \frac{6}{7} \int_0^1 \left(x^3 + \frac{x^3}{4}\right) dx$$

$$= \frac{6}{7} \int_0^1 \frac{5x^3}{4} \, dx$$

$$= \frac{6}{7} \cdot \frac{5}{4} \frac{x^4}{4}\Big|_0^1 = \frac{15}{56} \text{ ans.}$$

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