

Program: BS (CS)
Semester: Spring-2021
Time Allowed: **03 hours**
Course: Probability & Statistics (MT205/MT206)

Examination: **FINAL**
Total Marks: 100, Weightage: **50**
Date: 02 / 07 / 2021
Instructor: Osama Sohrab

NOTE: Attempt all questions.

Question # 01

[15]

The following stem-and-leaf plot records the diastolic blood pressure of a sample of 30 men.

| | | |
|---|--|------------------------------|
| 9 | | 3, 5, 8 |
| 8 | | 6, 7, 8, 9, 9, 9 |
| 7 | | 0, 1, 2, 2, 4, 5, 5, 6, 7, 8 |
| 6 | | 0, 1, 2, 2, 3, 4, 5, 5 |
| 5 | | 4, 6, 8 |

- Compute the sample mean, median and mode.
- Compute the sample standard deviation s .
- What proportion of the data values lies between $\bar{x} + 2s$ and $\bar{x} - 2s$?
- Compare the answer in part (c) to the one prescribed by the empirical rule.

Question # 02

[15]

- What information does the “**odds of an event**” tell us about the event? Briefly explain with example.
- If n people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year?
- A president and a treasurer are to be chosen from a student club consisting of **50** people. How many different choices of officers are possible if one particular student (say **A**) told that he will serve only if he is president.

Question # 03

[20]

- In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for **180** individuals:

| | Nonsmokers | Moderate Smokers | Heavy Smokers |
|------|------------|------------------|---------------|
| H | 21 | 36 | 30 |
| NH | 48 | 26 | 19 |

where **H** and **NH** in the table stand for Hypertension and Nonhypertension, respectively. If one of these individuals is selected at random, find the probability that the person is experiencing hypertension, given that the person is a heavy smoker.

(b) Consider a test that can diagnose kidney cancer. The test correctly detects when a patient has cancer 92% of the time. Also, if a person does not have cancer, the test correctly indicates so 95% of the time. Finally, suppose it is known that 2 in every 10,000 individuals has kidney cancer. Find the probability that a patient has kidney cancer, given that the test indicates he does.

Question # 04

[20]

Let X denote the reaction time, in seconds, to a certain stimulus and Y denote the temperature ($^{\circ}F$) at which a certain reaction starts to take place. Suppose that two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find

(a) $P\left\{0 \leq X \leq \frac{1}{2} \text{ and } \frac{1}{4} \leq Y \leq \frac{1}{2}\right\}$

(b) $P\{X < Y\}$

Question # 05

[20]

(a) Before the computer is assembled, its vital component (motherboard) goes through a special inspection. Only 80% of components pass this inspection. What is the probability that at least 18 of the next 20 components pass inspection?

(b) Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits less than 5 minutes for a bus.

Question # 06

[10]

Suppose that 3 batteries are randomly chosen from a group of 3 new, 4 used but still working, and 5 defective batteries. If X and Y denote, respectively, the number of new and used but still working batteries that are chosen, then find the joint probability mass function of X and Y .

[Hint : Think on possible values of X then Y and $p(x, y) = P\{X = x_i, Y = y_i\}$.]

The END