P&S (MT205) Final Sol.
Spring 2021

Sol 1 (a) $\bar{x} = \dfrac{\sum_{i=1}^{30} x_i}{30} \approx \boxed{74.13}$ Ans

Since $n = 30$ (even) so the sample median is the average of ~~30/2~~ the values in positions $30/2$ and $30/2 + 1$ i.e.

Sample median $= \dfrac{72+74}{2} = \boxed{73}$ Ans

Sample mode is $\boxed{89}$. Ans

(b)

$$s^2 = \left[\sum_{i=1}^{30} x_i^2 - 30\,(74.13)^2\right] / n-1$$

$$\Rightarrow s^2 \approx 160.5618$$

$$\Rightarrow s \approx \sqrt{160.5618}$$

$$\Rightarrow \boxed{s \approx 12.6713}$$ Ans

~~(c)~~

Sol 2 (a)

The odds of an event say A tells how much more likely it is that A occurs that that it does not occur. For example if $P(A) = 3/4$, then $P(A)/(1-P(A)) = 3$, so the odds are 3. This mean that it is 3 times as likely that A occurs as it is that it does not.

Note:
One can describe the above idea in some other wordings. That is also OK.

(b)

If n people are present in a room then the probability that no two of them celebrate their ~~per~~ birthday on the same day of the year is

$$\frac{(365)(364)(363)\ldots(365-n+1)}{(365)^n}$$

(Detail in slides)

(c) Since A will serve only if he is president, we have two situation here: (i) A is selected as the president, which gives 49 possible outcomes for the treasurer's positions, or (ii) officers are selected from the remaining 49 people without A, which has the numbers of choices ${}^{49}P_2 = 2352$. Total choices $= 49 + 2352 = \boxed{2401}$

Sol 3 (a) Let

H denotes Hypertension

NH denotes Nonhypertension

NS denotes nonsmoker

MS denotes moderate smoker and

HS denotes heavy smoker

We want $P(H \mid HS)$

By definition

$$P(H \mid HS) = \frac{P(H \cap HS)}{P(HS)}$$

$P(HS) = 49/180$ $\left[\because P(HS) = \dfrac{\text{Total number of heavy smokers}}{\text{Total number of smokers}}\right]$

$P(H \cap HS) = \dfrac{30}{180}$ $\left[\because P(H \cap HS) = \dfrac{\text{Total Number of heavy smokers that have hypertension}}{\text{Total number of smoker}}\right]$

So

$$P(H \mid HS) = \frac{30/180}{49/180}$$

$$\Rightarrow \boxed{P(H \mid HS) = \frac{30}{49}}$$

P_o = Positive
C = Cancer
N = Negative
H = Healthy

(b) $P(P_o|C) = \dfrac{92}{100}$

$P(N|H) = \dfrac{95}{100} \Rightarrow P(P_o|H) = 1 - \dfrac{95}{100} = \dfrac{5}{100}$

$P(C) = \dfrac{2}{10{,}000} \Rightarrow P(H) = 1 - \dfrac{2}{10{,}000} = \dfrac{9998}{10{,}000}$

Required $P(C|P_o)$

By Bayes Rule We have

$$P(C|P_o) = \frac{P(P_o|C)P(C)}{P(P_o)} \quad \text{--- (i)}$$

Now by Law of Total Probability, we have

$$P(P_o) = P(P_o|C)P(C) + P(P_o|H)P(H)$$

$$\Rightarrow P(P_o) = \left(\frac{92}{100}\right)\left(\frac{2}{10{,}000}\right) + \left(\frac{5}{100}\right)\left(\frac{9998}{10{,}000}\right)$$

$$\simeq 0.0502$$

Substituting values in (i) we get

$$P(C|P_o) = \frac{\left(92/100\right)\left(2/10{,}000\right)}{0.0502}$$

$$\Rightarrow \boxed{P(C|P_o) \simeq 0.0037}$$ which is the Required probability.

(4) (a)

$$P\{0 \le X \le \tfrac{1}{2} \text{ and } \tfrac{1}{4} \le Y \le \tfrac{1}{2}\}$$

$$= \int_{1/4}^{1/2}\int_{0}^{1/2} 4xy\,dx\,dy = \int_{0}^{1/2} \frac{4x^2y}{2}\Big|_{x=0}^{x=1/2} dy$$

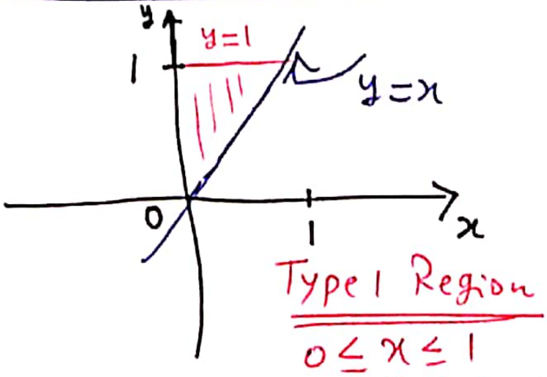
$$= \int_{1/4}^{1/2} 2x^2y\Big|_{x=0}^{x=1/2} dy = \int_{1/4}^{1/2} 2\left(\tfrac{1}{4}\right)y\,dy = \int_{1/4}^{1/2} \tfrac{1}{2}y\,dy$$

$$= \frac{1}{2}\frac{y^2}{2}\Big|_{1/4}^{1/2} = \frac{1}{4}y^2\Big|_{1/4}^{1/2} = \frac{1}{4}\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{1}{4}\left[\frac{4-1}{16}\right]$$

$$= \frac{1}{4}\left[\frac{3}{16}\right] = \frac{3}{64}$$

So $\boxed{P\{0 \le X \le \tfrac{1}{2} \text{ and } \tfrac{1}{4} \le Y \le \tfrac{1}{2}\} = \tfrac{3}{64} \approx 0.0469}$

(b)



Type 1 Region

$0 \le x \le 1$

$x \le y \le 1$

$$P\{X<Y\} = P\{Y>X\}$$

$$= \int_{0}^{1}\int_{x}^{1} 4xy\,dy\,dx$$

$$= \int_0^1 (2xy^2)\Big|_x^1 dx = \int_0^1 (2x - 2x^3)\,dx = \left[x^2 - \frac{2x^4}{4}\right.$$

$$= 1 - \frac{1}{2} = 1/2$$

$\Rightarrow \boxed{P\{X<Y\} = 1/2}$

Sol5 (a) If we take any component either it will pass the inspection (success) or not (Failure). We can think of inspection of 20 components as 20 trials (single trial is testing 1 component with possible outcome Success or Failure).

Let X is the random variable which denote number of components which pass the inspection ~~the~~ Clearly X is a binomial random variable with $p = 80/100$ (Probability of success).

We want $P\{X \geq 18\}$ when $n = 20$.

So $P\{X \geq 18\} = P\{X=18\} + P\{X=19\} + P\{X=20\}$

$$= \binom{20}{18}\left(80/100\right)^{18}\left(\frac{20}{100}\right)^{2} + \binom{20}{19}\left(\frac{80}{100}\right)^{19}\left(\frac{20}{100}\right)^{1} + \binom{20}{20}\left(\frac{80}{100}\right)^{20}\left(\frac{20}{100}\right)^{0}$$

$$\Rightarrow \boxed{P\{X \geq 18\} \approx 0.2061}$$

(b)

$$P\{10 < X < 15\} + P\{25 < X < 30\} = \frac{5}{30} + \frac{5}{30} = \frac{10}{30} = \boxed{\frac{1}{3}}$$

Sol 6

$P\{X=i, Y=j\}$

i \ j	0	1	2	3
0	$\frac{10}{220}$	$\frac{40}{220}$	$\frac{30}{220}$	$\frac{4}{220}$
1	$\frac{30}{220}$	$\frac{60}{220}$	$\frac{18}{220}$	
2	$\frac{15}{220}$	$\frac{12}{220}$		
3	$\frac{1}{220}$			

Detail

p(0,0) is the probability of the event that we take no battery from new as well as from used but working (that is we take 3 batteries from 5 defective batteries.)

So $p(0,0) = \dfrac{\binom{5}{3}}{\binom{12}{3}}$ and so on.

The End