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Improving Solving Problem Ability with Tower of Hanoi Puzzle

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Abstract:- Several studies have shown that the monochromatic Tower of Hanoi (TOH) Puzzle enhances the solving problemability of students in mathematics. This study aimed is to determine the students' solving problem ability through the TOH puzzle presented in three realias-–the unicolored, bicolored, and multicolored. The study used the randomized block design, with random assignment as strategy in selecting 117 samples participants. Results showed that the female group performed better by time in solving the bicolored realia. This confirms that females have a keener sense of color perception than males. Further, the study also revealed how students themselves developed certain strategies in solving the puzzle. The qualitative data generated through interview showed an improvement in student's problem-solving ability in general. The study recommended that the Tower of Hanoi Puzzle be part of the problem-solving activity in classroom instruction.

*Keywords:*Tower of Hanoi, unicoloredrealia, bicolored realia, multicoloredrealia

Introduction

The world is unquestionably mathematical. A person comes into the physical world where shape, size, dimensions and numbers and their relationship play very important roles. Whatever maybe one's occupation, it will always have a mathematical core which that person cannot escape from. Mathematics is an integral part of in the solution of many problems and for this reason, it is recognized as one of the general education courses in the new curriculum. A lot of occupations require adequate problem-solving skills like architecture and engineering because there is a need to design and construct buildings that are not only pleasing and functional but ones that meet the strict safety requirements (Aufmann, 2018). It is a common knowledge that mathematics is one of the most difficult subjects for students. Among the countries that participated in the 1999 and 2003 Trends in International Mathematics and Science Study (TIMSS), the Philippines ranked third and fourth to the last according to the Growth Revolution magazine.

With the growing need for internationalization, students must be mathematically prepared to meet the demands the times. The relevance of mathematics in this modern world is unstoppable. This is one of the reasons why problem solving activities in school are introduced--it is to prepare students to solve real-life problems. The failure of the teacher to

develop in the students the conceptsin mathematics specifically solving problem is to neglectone of the most important educational responsibilities of a teacher.

But, sad to say that in normal classroom setting, students already have the notion that solving problem is difficult and exhausting, so it has become less appreciated. How can this problem be addressed? With the revised curriculum, GE 3 also known as Mathematics in the Modern World (MMW) is offered to freshmen college students. This course includes topics on solving problem using creative games like Sudoku, puzzle, and riddles which all revolve around mathematical concepts. The Tower of Hanoi (TOH) puzzle is one of them. This puzzle has been used for a number of decades (Simon, 1975). It was invented by a French mathematician Edouard Lucas in 1883 and is a well-known problem-solving task that has been used many times in experimental setting (Anderson, 2001). Games and puzzles provide an excellent environment to explore ideas of computational thinking. The fact that many of these are available both in a non-computerized form and in a computerized form helps to create this excellent learning environment. A modern education prepares students to be productive and responsible adult citizens in a world in which mind/brain and computer working together is a common approach to solving problems and accomplishing tasks.

American philosopher John Dewey expounded a theory of education which is "learning by doing." It is a hand-on approach to learning which means that students must interact with their environment in order to adapt and learn (Anzai, Y. A., 1979).

Solving problem has been an important aspect of mathematics, teaching and learning mathematics. It involves high and low-level thinking (Hoiriyah et al., 2014). With solving problem ability, students can improve their thinking ability, apply procedures, and deepen conceptual understanding (Ranjan&Gunendra, 2013).Learning materials are things that must be prepared by the teacher before implementing learning. The teacher is expected to be able to design learning in order to achieve the stated educational goals (Sapta, Hamid, &Syahputra, 2018). Learning materials are materials that are needed and used in managing the teaching and learning process or a very important tool for teachers to conduct learning efficiently and to improve student learning achievement (Trianto, 2011; Olayinka, 2016).

Although mathematics is a very important subject in formal education and is closely related to human life, mathematics is not a topic of interest the student's and many students face difficulties in mathematical problem solving due to inability to acquire many mathematical ability and lack cognitive learning ability (Simamora et al., 2017; Tambychik et al., 2010). Based on preliminary observations at SMP Negeri 13 Medan on 8th – 11st November 2017, the results of interviews with teachers stated that mathematics is a subject that is not in demand by most students. It was also found that many students did not like mathematics because mathematics was too difficult. Other facts also show that student's mathematical problem solving ability are still low. The low mathematical problem solving ability of students can be seen from the results of diagnostic tests in the form of story questions. The low mathematical problem solving ability is an important and urgent problem to be solved.Many researches on student self-efficacy have been carried out. The study results that self-efficacy is closely related to mathematics learning achievement (Ayotola&Adedeji, 2009; Liu &Koirala, 2009; Motlagh et al., 2011). Skaalvik et al. (2015) stated that student motivation was strongly predicted by self-efficacy. So, the student's self-

efficacy should be taken seriously by the teacher. Teachers must find ways to improve students' mathematical learning ability and must emphasize self-efficacy by designing appropriate learning (Ayotola&Adedeji, 2009). The source of students' main beliefs to improve students' self-efficacy is: performance accomplishments/mastery experience; vicarious experience, verbal persuasion and emotional arousal (Bandura, 1994; Schunk&Pajares, 2001; Zimmerman, 2000).

According to Bruner, discovery learning was a learning model that uses inquiry-based constructivist learning theory that occurs in problem solving situations where learners learn through existing knowledge and previous experience to find facts and relationships with new material being studied (Bruner, 1961; Learning Theories, 2017). The view of Neo-Vygotskian, Ylimaki (2010), stated that effective learning lied in activity, context, and culture as collaborative efforts in groups.

In solving problem activities, the teacher serves as a "trainer" for students. Students are asked to "think" more, and create rather than "quote" material (Schoenfeld, 1980).Polya stated that good education was education that provided opportunities for students to find things, which in this case are mathematical concepts by themselves (Schoenfeld, 1987). The concepts in mathematics are not given directly by the teacher to students. Students must be involved in the process of rediscovering the concept. Students are required to create ideas, look for relationships to form concepts. In students actively involved in the discovery of various concepts and principles through solving problem or the results of abstraction of various cultural objects.The mechanism underlying high level mental work is a copy of social interaction (Confrey, 1995; Taylor, 1993). Schoenfeld (2013) said that ideas formed by individuals was often built and refined in collaboration with others. This implies that all cognitive, even high-level work in humans, starts from the culture and that means, students should learn through interaction with adults and more capable peers. Vygotsky stated that, in the implementation of learning required the organization of students in the classroom (Ormrod, 1995). Teachers need to implement learning strategies that allow students to interact with their friends.In discovery learning, students are encouraged to learn on their own independently. Students are actively involved in the discovery of various concepts and principles through problem solving or the results of abstraction of various cultural objects. Concepts and rules in mathematics can be mastered in full by students, when students are actively involved in thinking about, discovering, and reconstructing the mathematical knowledge that is being studied (Ernest, 1991; Wheeler 1970). The teacher encourages and motivates students to gain experience by doing activities that allow students to discover mathematical concepts and principles for themselves. This learning arouses curiosity and fosters motivation in students to work until they find the answer. Students learn to solve problems independently with thinking skills because they have to analyze and manipulate information.Learning materials are essential and significant materials needed in teaching and learning activities in schools to improve teacher efficiency and improve student learning achievement (Nesari&Heidari, 2014;Olayanki, 2016). Learning materials are a number of materials, tools, media, instructions, and guidelines that students and teachers will use to conduct learning activities (Nasution&Sinaga, 2017; Trianto, 2013).

One of the gaps of the study is to examine if adding colors to the realias would change or improve the students' performance in solving the puzzle. Although there has been studies about the Tower of Hanoi (TOH), none has been conducted whether color mattered in the the

problem solving ability of freshmen college students. This study determined the performance of the students using three kinds of Tower of Hanoi relias--the unicolored, bicolored and multicolored. Specifically, the study sought answers to the following questions: 1. What are the mean performance of the students in terms of time spent along solving the Tower of Hanoi in three realias? 2. Is there a significant difference between the time spent in solving the Tower of Hanoi using the unicolored, bicolored, and multicoloredrealias among the males? among the females? 3. Is there a significant difference in the performance of the male and the female in solving the Tower of Hanoi using: a. unicoloredrealias? b. using bicolored realias?, and c. using multicoloredrealias? 4. How did the students find the Tower of Hanoi as a teaching strategy in problem solving? 5. What strategies were used by the students in solving the Tower of Hanoi using the unicolored, bicolored and the multicoloredrealias?6. What are the teacher-participants' on the Tower of Hanoi as a teaching strategy?

Materials and Methods

There were three kinds of Tower of Hanoi realias used in the study—the unicolored, bicolored and multicoloredrealias with 5 chips each. The researcher facilitated in the making the therealiasan saw to it that they are of the same specifications. This is to minimize biases during the conduct of the experiment. The puzzle was introduced as one of the creative games and an extended activity on problem solving after the students had been exposed to the usual problem solving activity without the puzzle. In this way, they had the chance to compare which strategy they liked better and why.

Research Design

The study primarily used quantitative approach with qualitative procedure as follow up. The quantitative approach used experimental method of the randomized block design. The researcher used random assignment in selecting the 117 samples. They were grouped into two by gender classification. Thereafter, each group were sub-divided into 3 bearing the 3 colored-based labels; as unicolored, bicolored and multicolored. The same arrangements was done to the female group as well. For the male group, there were 13 participants for each subgroup for a total of 39 males in all. On the other hand, the female group was composed of 26 participant each for a total of 78 females. These samples solved the tower of hanoi using three different kinds of realias. The time spent in solving the puzzle were recorded and kept for the analysis of data. The Analysis of Variance (ANOVA) was used to find the differences in the performance of the males using the three tower of Hanoi realias. ANOVA was also used to find the differences in the performance of the females using the three tower of Hanoi realias. The t-test for independent samples was used to compare the performance of the male and female using each type of realia. Interviews were conducted to find answers to the questions on the strategies used by the students in solving the puzzle using three different realias and to know their feelings towards the TOH as a teaching strategy.

Data Collection

The selected samples solved the tower of hanoi using three different kinds of realias.First, the unicoloredrealia was used in solving, followed by the bicolored realia and lastly, the multicolored. The time spent in solving the puzzle were recorded and kept for the treatment and analysis of data.

The qualitative approach used interview and observation to both the students and teacher-participants. In the interview, the researcher asked how the participants find learning with the TOH puzzle and what strategies they developed to solve the puzzle by themselves.

Moustaka's (1994) structured method of inductive data analysis was used in the study. After each individual transcript was read more than once, all transcripts were read again and again and memos were recorded to immerse the researcher and highlight key concepts. The following steps were followed:

1. Horizontalization was performed by giving equal value and significance to each statement and coding it with a descriptive label.
2. Reduction and elimination of statements that were not a horizon of the experience performed to determine the invariant constituents of the experience. This is to see whether the statement contained a moment that was necessary for understanding the experience and if it could be abstracted or labelled.
3. Clustering was performed to group related constituents together, and each category was given a thematic label. Initial coding resulted in categories of invariant constituents of the experience. This was repeated several times to group and reduce categories until all constituents were clustered and themes of the experience emerged.
4. Final identification of themes were done by rereading the complete transcripts to verify that the theme and accompanying invariants constituents were explicitly expressed and compatible with the students' responses. These themes were used to construct individual and overall textural, structural, and textural-structural descriptions, culminating into an overall essence of the experience. Themes are presented within this text.

For the reliability and the validity of the data collection, commonly identified techniques utilized in this study include the recording of detailed field notes, an audio recorder for accuracy. Final themes, as well as a sample of the invariant constituents of those themes were shown to all participants for review. The students were asked to examine these themes and reflect on the accuracy. The students affirmed that the provided account accurately reflected their perspectives and experiences.

Observation during the conduct of the experiment to solve the Tower of Hanoi was done by mathematics instructors coming from the College of Arts and Sciences to validate the responses given by the students.

The experimental study was conducted at the Mathematics laboratory of the College of Arts and Sciences, Samar State University, Catbalogan City, Samar, Philippines. Interviews were conducted after the experiment to some participants at the Quality Assurance Office where the researcher is holding office. It was scheduled or prearranged according to the availability of time of the interviewees and the interviewer.

Ethical Considerations

To comply with the professional mandate in doing effective research, ethical considerations are employed in this study to ensure minimal lapse in research which can significantly harm human, animal subjects, students and the public.

During the face to face interview, the researcher maintained a relationship of mutual respect and uphold the integrity of every student participant. The interview was conducted during the respondent's free time with snacks. In this way, they felt comfortable and shared their thoughts freely without being judged. They were assured that after the data were analyzed and interpreted, they can check the accuracy of the information gathered by the researcher. They were assured that their anonymity is kept confidential.

Results and Discussions

Findings

Table 1. Comparison of the Performance (Time Spent) of the Male Sample Students Using the Three TOH Realias

Descriptive

TOH Realia	N	Mean	SD
Unicolored	13	34.72	14.37
Bicolored	13	27.97	7.66
Multicolored	13	27.46	15.20
Total	39	30.05	12.96

ANOVA

Sources of Variation	Sum of Squares	df	Mean Square	F	Sig.	Decision
Between Groups	426.11	2	213.06	1.29	0.29	Fail to Reject H_o
Within Groups	5954.86	36	165.41			
Total	6380.98	38				

Table 1 shows the one way ANOVA of the time spent in seconds of the male group using the three types of Tower of Hanoi realiasand it is found to be not significant, (F-value=1.29, p>0.05).

Table 2. Comparison of the Performance (Time Spent) of the Female Sample Students Using the Three TOH Realias

Descriptive

TOH Realia	N	Mean	SD
Unicolored	26	32.73	11.60
Bicolored	26	25.22	8.20
Multicolored	26	33.97	13.07
Total	78	30.64	11.67

ANOVA						
	Sum of Squares	df	Mean Square	F	Sig.	Decision
Between Groups	1165.21	2	582.60	4.69	0.012	Reject H_o
Within Groups	9319.829	75	124.26			
Total	10485.04	77				

Table 2 displays the one way ANOVA of the time spent in seconds of the female group using the three types of Tower of Hanoi realias and this time it is found to be significant, ($F=4.69$, $p<0.05$).It shows the performance of thestudent-participants in each type of TOH realia with the female in the bicolored realia showing the fastest time spent, ($M=25.22$, $SD= 8.20$). They are 7.51 seconds faster than the female unicolored group and 8.75 seconds faster than the female multicolored group.

Table 3. Comparison of the Performance of the Male and Female in the UnicoloredRealia

Descriptive						
	Gender	**N**	**Mean**	**SD**	**Std. Error Mean**	
Unicolored	male	13	34.72	14.37	3.99	
	female	26	32.73	11.6	2.28	
		F	Sig.	t	df	Sig. (2-tailed)
Unicolored	Equal variances assumed	2.74	0.107	0.47	37	0.64

The t-test for independent samples was used to find out the significant difference between the time spent of the male and female in the unicoloredrealia. The difference in time spent of the male and the female in the unicoloredrealia is found to be not significant, (t-value $= 0.47$, $p > 0.05$).

Table 4. Comparison of the Performance of the Male and Female in the Bicolored Realia

Descriptive					
	Gender	**N**	**Mean**	**Std. Deviation**	**Std. Error Mean**
Bicolored	Male	13	27.97	7.66	2.12
	Female	26	25.22	8.20	1.61
	F	Sig.	t	df	Sig. (2-tailed)

Bicolored	Equal variances assumed	0.021	0.887	1.008	37	0.32

The t-test for independent samples was used to find out the significant difference between the time spent of the male and female in the bicolored realia. It is also found to be not significant, (t-value = 1.008, p > 0.05).

Table 5. Comparison of the Performance of the Male and Female in the MulticoloredRealia

Descriptive						
	Gender	**N**	**Mean**	**SD**	**Std. Error Mean**	
Multicolored	Male	13	27.46	15.20	4.22	
	Female	26	33.97	13.07	2.56	
		F	Sig.	t	df	Sig. (2-tailed)
Multicolored	Equal variances assumed	0.001	0.978	-1.388	37	0.173

The t-test for independent samples was used to find out the significant difference between the time spent of the male and female in the multicoloredrealia. It is also found to be not significant, (t-value = -1.38, p > 0.05).

Table 6

Result: Themes, key concepts from studentsperspectives and supporting quotes

Supporting Quotes	Key Concept	Theme
RQ 4 (How did the students find the Tower of Hanoi (TOH) as a teaching strategy in problem solving?)		
"I remembered the rules and patterns that are incorporated in the TOH and it made my brain function well and at the same time I enjoyed it." (BF7) *"It exercises not only my brain by also my hands. There seems to be a sense of achievement." (MM3)* *"It enhances my reflexes and memory. Very nice strategy to adopt in problem solving." (UM6)*	Improvement of memory, brain exercising and reflex enhancing.	Improved ability in solving problem
"I discovered that patterns are not only found in a series or in nature but also in the TOH puzzle. Great game!" (UF21) *"It is very interactive, fun and at the same time I learned mathematical concepts like finding patterns and the rules." (UF11)*	Integration of mathematical concepts	
"I felt a sense of fulfilment solving the TOH and it is easier to perform than the word problems." (UM3) *"I felt proud of myself solving the puzzle in less than a minute. Grateful to our teacher for sharing this to us." (BF12)*	Sense of fulfilment and achievement	
"I think I was addicted to the puzzle that I kept playing it again and again. It helps improve my memory and reflexes. It lessens my boredom." (BF2) *"It became my stress reliever whenever I'm stressed from my schooling and I found the puzzle an antidote to it." (UM7)*	Stress reliever and lessens boredom.	
"I had fun completing the puzzle at the shortest time and I want to join the TOH contest next school year."(MM4) *"I am enjoying while learning cause I find this interesting and fun. Very excited to play it every time." (MM11)*	Engaging/interactive, fun and exciting.	

RQ5(What strategies were used by the students in solving the tower of Hanoi using the unicolored, bicolored and the multicoloredrealias?)		
"At first, I was confused but when I memorize the pattern, I was able to complete the puzzle. I did one move at a time" (UM9) *"I only remember to put the smaller chips over the bigger ones and to follow the pattern." (BM10)* *"I memorize the rules and do it again and again focusing on the next moves." (MM1)* *"I followed the same rules for all the three realias but bicolored can be solved faster because of the same colors not to be stacked together . It served as a guide to easily solve the TOH" (BF17)* *"I repeat the movement and follow the rules all the times when I make a move. Same colors should not be stacked in bicolored realia" (UF6)* *"No bigger chips over a smaller ones and I did one move at a time." (MF13)*	Becoming focused Enabling to follow patterns/guides	Following Rules
"I changed the position of the realia for easy movement of the hands." (BM5) *"I tried new strategy like facing the realia to me in perpendicular position so that my hands will not cross each other." (UF4)* *"I used both hands In order to solve the puzzle immediately." (MF10)*	Ability to adopt better strategy than the previously known one. Ability to use techniques in hand manipulation	Faster Moves

As shown in the table, the theme that emergedfrom the students' responses under research question No. 4 was: "improved ability in solving problem."The themes that emerged for research question No. 5 about the strategies used by the students in solving the TOH using the three realias were:"following rules; and faster moves." The statements given by the student-participants in the result section used the color-labels in order to protect their identity.

On Improved Abilityin Solving Problem

The theme of "improved ability in solving problem" was felt the improvement of memory, brain exercising, and reflex enhancing on the students as stated in the following:

"I remembered the rules and patterns that are incorporated in the TOH and it made my brain function well and at the same time I enjoyed it."
"It exercises not only my brain by also my hands. There seems to be a sense of achievement."
"It enhances my reflexes and memory. Very nice strategy to adopt in problem solving."
"I discovered that patterns are not only found in a series or in nature but also in the TOH puzzle. Great game!"
"It is very interactive, fun and at the same time I learned mathematical concepts like finding patterns and the rules."

This feeling of intellectual improvement also impacted on the participant's inner pleasure and satisfaction as revealed in the following statements:

"At first, I was confused but when I memorize the pattern, I was able to complete the puzzle. I did one move at a time"
"I only remember to put the smaller chip over the bigger ones and to follow the pattern."
"I memorize the rules and do it again and again focusing on the next moves."

On Following Rules

The theme of "following rules" is elucidated in the participants becoming focused and enabling to follow patterns/guide, as stated in the following comments:

"At first, I was confused but when I memorize the pattern, I was able to complete the puzzle. I did one move at a time"
"I only remember to put the smaller chips over the bigger ones and to follow the pattern."
"I memorize the rules and do it again and again focusing on the next moves."
"I followed the same rules for all the three realias but bicolored can be solved faster because of the same colors not to be stacked together . It served as a guide to easily solve the TOH."
"I repeat the movement and follow the rules all the times when I make a move. Same colors should not be stacked in bicolored realia"
"No bigger chips over a smaller ones and I did one move at a time."

On Faster Moves

The theme of "faster moves" is defined by the participants ability to adopt better strategy than the previously known one, and ability to use techniques in hand manipulation as shown in the following statements:

"I changed the position of the realia for easy movement of the hands."
"I tried new strategy like facing the realia to me in perpendicular position so that my hands will not cross each other."
"I used both hands In order to solve the puzzle immediately."

Teacher-Participants Observations

Observation during the conduct of the experiment to solve the Tower of Hanoi was done by mathematics instructors coming from the College of Arts and Sciences. The comments given were used to enhance the validity of the responses given by the student-participants. They are as follows:

"I can see patterns especially in the movement of the smallest chip—first move is to the last dowel, next to the center and it goes back to its original position then it is repeated again and again." (Teacher 1)

"A disk can be put on an empty stack but no bigger disk is placed over a smaller one." (Teacher 2)
"The sequence is repeatedly done until the puzzle is completely solved. If the sequence was not followed and number of minimum moves is not obtained, the puzzle became difficult to solve." (Teacher 3)

Thus, in general, the teacher-participants observed the elements of iteration/recursion, procedural and following the rule.

Discussion

Based on the results of the study, it was found out that the female group accomplished significantly faster in the bicolored realia than in the other realias. Thus, color can be a significant factor that enhances the problem solving ability among the female group if it not

extremely excessive. It also helps in comprehension (Johnson, 1992), and in readership (Business Papers in Color, 1989). Women can see more range of colors than men (Jain, Verma, Mittal, Singh&Munjal, 2010). Apparently, the beautiful world is more colourful to the female.

As a follow-up procedure, interview results revealed from the experiences of the participants that the students were able to attain improvement in solving problem by following rules and developing faster moves. This is supported in the study of Ranjan&Gunendra (2013) particularly in applying procedure. Following rules through finding patterns enhances students' self efficiency in learning mathematics (Ayotola&Adedeji, 2009; Liu &Koirala, 2009; Motlagh et al., 2011). Further, finding patterns as a form of self-sufficiently is associated with students motivation (Skaalvik et al., 2015), and emotional arousal (Zimmerman, 2000). Furthermore, the bicolored TOH confirms the recursive solution found by (Chaugule, 2015) with the use of algorithm.

The teacher-participants observation on the aspect of iteration/recursion, procedural and following the rules confirms and validates the quantitative and qualitative data in the study. The students use of finding patterns and the teacher observation of iteration/recursion find validity of these separate experiences.

Conclusion

Solving with the Tower of Hanoi puzzle is more appealing and engaging on the part of the students. It increased their self-confidence and gave them a sense of fulfilment. The tiring, exhausting and less appreciated activity, turned out to be fun and exciting in order to exercise their reflexes in a creatively-mathematical way.Thus, improving their solving problem ability.

Implication

Solving the Tower of Hanoi gives the students a strong foundation for higher mathematics such as Algorithm which is one of the major courses of Information Technology students. Lessons on recursion and iterative processes which are applied in solving the TOH would be a greathelp to them. By continuously solving the TOH even at home may serve as a way of doing brain exercise. This would help the teacher in adding variation to the usual boring classroom solving problem activity while at the same time integrating mathematical concepts into the teaching-learning process. The community may utilize the puzzle to entertain and exercise the brain during free-times and in other recreational activities and contests where games are used. TOH may also be considered as children's toys so that they may help in the young mind's problem-solving abilities at the same time offer activities for fun and enjoyment. Relatively, it may offer alternative recreational activities to children who at this digital age, have been lured and addicted to playing digital gadgets which have been found to have unhealthy and deleterious effects. (www.worldvision.org.ph)

Recommendation

Although, TOH is suggested by the Commission of Higher Education (CHED) to be integrated in the solving problem activity in Mathematics in the Modern World course, as per observation, not all teachers are doing it. With the result of this study, it is strongly recommended that they should be encouraged to include the Tower of Hanoi puzzle as one of the solving problem activities in the teaching-learning process. Even high school teachers may also consider this recommendation. Furthermore, the study may also serve as a baseline for future researches like exploring the learning experiences of the students in solving the TOH and generating theory with TOH puzzle as a teaching strategy in problem solving.

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Appendix A

1. Time Spent in Solving the Tower of Hanoi Puzzle (Male)

	Label/Code	Unicolor	Label/Code	Bicolor	Label/Code	Multicolor
1	**UM1**	24.9	**BM1**	17.75	**MM1**	16.49
2	**UM2**	31.44	**BM2**	24.65	**MM2**	33.01
3	**UM3**	48.88	**BM3**	43.92	**MM3**	25.65
4	**UM4**	47.96	**BM4**	29.52	**MM4**	20.51
5	**UM5**	19.22	**BM5**	18.89	**MM5**	21.45
6	**UM6**	22.29	**BM6**	22.79	**MM6**	26.16
7	**UM7**	56.23	**BM7**	21.57	**MM7**	15.33
8	**UM8**	24.28	**BM8**	37.18	**MM8**	59.27
9	**UM9**	44.24	**BM9**	33.21	**MM9**	18.96
10	**UM10**	23.39	**BM10**	29.67	**MM10**	26.9
11	**UM11**	13.37	**BM11**	34.41	**MM11**	14.53
12	**UM12**	51.09	**BM12**	23.17	**MM12**	59.99
13	**UM13**	44.03	**BM13**	26.92	**MM13**	18.79

2. Time Spent in Solving the Tower of Hanoi Puzzle (Female)

	Label/Code	Unicolor	Label/Code	Bicolor	Label/Code	Multicolor
1	**UF1**	27.81	**BF1**	15.66	**MF1**	25.70
2	**UF2**	38.00	**BF2**	23.41	**MF2**	22.31
3	**UF3**	18.22	**BF3**	20.49	**MF3**	20.39
4	**UF4**	41.10	**BF4**	44.48	**MF4**	29.32
5	**UF5**	26.83	**BF5**	25.63	**MF5**	28.21
6	**UF6**	24.90	**BF6**	23.76	**MF6**	40.23
7	**UF7**	19.10	**BF7**	13.57	**MF7**	19.51
8	**UF8**	30.51	**BF8**	16.23	**MF8**	32.09
9	**UF9**	36.12	**BF9**	13.17	**MF9**	14.46
10	**UF10**	15.07	**BF10**	34.77	**MF10**	46.28
11	**UF11**	49.39	**BF11**	28.40	**MF11**	43.00
12	**UF12**	39.76	**BF12**	30.52	**MF12**	28.09
13	**UF13**	37.42	**BF13**	22.10	**MF13**	50.50
14	**UF14**	25.35	**BF14**	44.47	**MF14**	23.10
15	**UF15**	42.56	**BF15**	23.50	**MF15**	30.17
16	**UF16**	36.54	**BF16**	28.76	**MF16**	29.98
17	**UF17**	59.25	**BF17**	25.25	**MF17**	21.58
18	**UF18**	50.13	**BF18**	25.61	**MF18**	59.37
19	**UF19**	47.91	**BF19**	24.06	**MF19**	26.05
20	**UF20**	24.79	**BF20**	29.07	**MF20**	59.15
21	**UF21**	31.40	**BF21**	21.80	**MF21**	34.29

22	**UF22**	18.62	**BF22**	35.32	**MF22**	34.95
23	**UF23**	24.96	**BF23**	12.59	**MF23**	59.85
24	**UF24**	41.32	**BF24**	22.66	**MF24**	42.69
25	**UF25**	17.34	**BF25**	27.28	**MF25**	42.10
26	**UF26**	26.63	**BF26**	23.24	**MF26**	19.88

Appendix B

Sample Transcript 1

Interviewer: How did you find the Tower of Hanoi (TOH) as a teaching strategy in problem solving?

Participants:

"I felt a sense of fulfilment solving the TOH and it is easier to perform than the word problems." (UM3)
"I think I was addicted to the puzzle that I kept playing it again and again. It helps improve my memory and reflexes. It lessens my boredom." (BF2)
"Remembering the rules and patterns are incorporated in the TOH and it made my brain function well and at the same time I enjoyed it.' (BF7)
"It exercises not only my brain by also my hands. There seems to be a sense of achievement." (MM3)
"It enhances my reflexes and memory. Very nice strategy to adopt in problem solving." (UM6)
"I felt proud of myself solving the puzzle in less than a minute. Grateful to our teacher for sharing this to us." (BF12)
"It became my stress reliever whenever I'm stressed from my schooling and I found the puzzle an antidote to it." (UM7)
"I had fun completing the puzzle at the shortest time and I want to join the TOH contest next school year." (MM4)
"I am enjoying while learning cause I find this interesting and fun. Very excited to play it every time." (MM11)
"I discovered that patterns are not only found in a series or in nature but also in the TOH puzzle. Great game!" (UF21)
"It is very interactive, fun and at the same time I learned mathematical concepts like finding patterns and the rules." (UF11)

Interviewer: What strategies did you use in solving the Tower of Hanoi?

Participants:

"At first, I was confused but when I memorize the pattern, I was able to complete the puzzle. I did one move at a time" (UM9)
"I only remember to put the smaller chips over the bigger ones and to follow the pattern." (BM10)

"I memorize the rules and do it again and again focusing on the next moves." (MM1)
"I followed the same rules for all the three realias but bicolored can be solved faster because of the same colors not to be stacked together . It served as a guide to easily solve the TOH". (BF17)
"I repeat the movement and follow the rules all the times when I make a move. Same colors should not be stacked in bicolored realia" (UF6)
"No bigger chips over a smaller ones and I did one move at a time." (MF13)
"I changed the position of the realia for easy movement of the hands." (BM5)
"I tried new strategy like facing the realia to me in perpendicular position so that my hands will not cross each other." (UF4)
"In order to solve the puzzle immediately, both hands should be used." (MF10)