

Title: A Representation Approach to the Tower of Hanoi Problem

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Introduction:

- The Tower of Hanoi is a classic puzzle used to demonstrate recursion. However, the recursive solutions can be hard for beginners to understand.
- This paper presents an iterative algorithm to solve the Tower of Hanoi by using a bit-string representation to explicitly encode the directions of disc moves.
- The goals are to develop a straightforward algorithm and analyze the behavior of disc moves based on the representation.

Problem:

- There are 3 pegs (P1, P2, P3) and n discs (D1, D2, ..., Dn) of increasing size.
- The goal is to move all discs from one peg to another, following constraints: only top disc can be moved, never place larger disc over smaller disc, only move one disc at a time.

Representation:

- Directions are encoded as left/right branches of a binary tree, then collapsed into a bit-string using 0=clockwise, 1=anticlockwise.
- Bit-strings are generated recursively. BS(n) is created by prepending and appending inverted BS($n-1$) to BS($n-1$).

Properties:

- Bit-strings have symmetry about the center bit.
- D1 (smallest disc) moves are always first, last, and at odd positions.
- D1 alternates moves with other discs.
- D1 always moves cyclically.
- The bit-string solution is proven optimal, taking $2^n - 1$ steps.
- Discs with odd/even indices move in opposite directions.

Algorithm:

- Iterate $2^n - 1$ times. Move D1 based on bit parity. Move other discs based on D1's peg.
- An iterative C program implements this algorithm.

- The bit-string can also be generated directly using the established properties.

Conclusion:

- Representing directions explicitly leads to an elegant Tower of Hanoi solution.
- The bit-string representation enables analyzing properties of optimal disc moves.