**Title:** An Evolutionary Approach to Tower of Hanoi Problem

**Authors:** Jie Li and Rui Shen from Shandong Aerospace Electro-Technology Institute, China

**Objective**:
- To apply an evolutionary algorithm (EA) approach to find valid solutions for the Tower of Hanoi problem
- To test the approach on extensions of the classic puzzle involving multiple pegs

**Background**:
- The classic Tower of Hanoi puzzle involves 3 pegs and n disks of increasing size that must be moved from one peg to another by following certain rules
- It has been studied extensively with applications in computer programming, psychology, robotics, planning, etc.
- A natural extension is the multi-peg Tower of Hanoi problem with more than 3 pegs which exponentially increases the complexity.

**Approach**:
- Uses a direct integer encoding where each disk move is a gene in the EA chromosome
- Allows variable length chromosomes but fixes length for each experiment
- Applies an elitist selection scheme
- Employs a hierarchical mutation mechanism to mutate top individual to generate new populations
- Fitness function considers:
  - Final state score
  - Number of invalid moves
  - Location of first invalid move
- Building block integers increase as number of pegs increases

**Experiments**:
- Tested for 3, 4 and 5 peg Tower of Hanoi
- For 3 pegs:
  - Tested up to 5 disks in 50 runs each
  - Tried individual lengths of optimal (2^n - 1) and longer
  - Solved up to 5 disks within 0.14 to 22 seconds on average
- For 4 pegs:
  - Tested up to 8 disks based on presumed optimal lengths
  - Solved up to 8 disks in under 6 seconds on average
- For 5 pegs:
  - Tested up to 9 disks based on presumed optimal lengths
  - Solved up to 9 disks but took over 80 minutes on average

**Results**:
- The EA is capable of finding valid solutions for smaller multi-peg problems

- Performance and execution time degrade rapidly as complexity increases
- Setting length based on presumed optimal solutions reduces time

**Conclusions**:
- The proposed evolutionary approach works for simpler Tower of Hanoi cases
- It struggles and takes longer to find solutions as complexity increases
- Future work should apply heuristics and modify fitness and mutations to improve search