

1 Interpolación

1.1 Polinomio de Taylor

```
# nodos
X = [ k/5 for k in [0..n] ]
# puntos
XY = [ (X[i], f(X[i])) for i in [0..n] ]
# funcion interpolacion
p(x) = RR['x'].lagrange_polynomial(XY)      # magia
```

Error

$$|f^{(n+1)}(x)| \leq M \quad x \in [a, b]$$
$$\varepsilon = \frac{M}{(n+1)!}$$

1.2 Lineal a trozos

$$r(a, b, x) = f(a) + \frac{f(b) - f(a)}{b - a} \cdot (x - a)$$

```
# [nodos] puntos
XY = [ (X[i], f(X[i])) for i in [0..n] ]
# funcion lineal
r(a, b, x) = f(a) + ((f(b) - f(a))/(b - a)) * (x - a)
# trozos de funciones lineales
L = piecewise([[X[i], X[i+1]], r(X[i], X[i+1], x)] for i in [0..n-1]], var = x)      # feardo
```

Error

$$|f''(x)| \leq M \quad x \in [a, b]$$
$$\varepsilon = \frac{M}{8} \cdot \max(\Delta x)$$

2 Integración numérica

2.1 Trapecios

$$\int_a^b f(x) dx \approx \frac{b-a}{n} \cdot \sum_{i=0}^{n-1} \frac{f(x_i) + f(x_{i+1})}{2}$$

```
# nodos
X = [ (a + k*((b-a)/n)) for k in [0..n] ]
# trapecios
trap = ((b-a)/n) * sum( (f(X[i]) + f(X[i+1]))/2 for i in [0..n-1] )
```

Error

$$|f''(x)| \leq M \quad x \in [a, b]$$
$$\varepsilon = \frac{M(b-a)^3}{12n^2}$$

2.2 Simpson

$$\int_a^b f(x) dx \approx \frac{b-a}{6n} \cdot \left(f(a) + 4 \sum_{i=1}^n f(x_{2i-1}) + 2 \sum_{i=1}^{n-1} f(x_{2i}) + f(b) \right)$$

```
# nodos (2n+1)
X = [ (a + k*(b-a)/(2*n)) for k in [0..2*n] ]
# simpson
simpson = ((b-a)/(6*n))*( f(a) + 4*sum(f(X[2*i-1]) for i in [1..n]) +
    2*sum(f(X[2*i]) for i in [1..n-1]) + f(b) )
```

Error

$$|f''''(x)| \leq M \quad x \in [a, b]$$

$$\varepsilon = \frac{M(b-a)^5}{180(2n)^4}$$

3 Integrales útiles

3.1 Área entre funciones

$$\int_a^b g(x) - f(x) dx$$

3.2 Longitud de curva

$$\int_a^b \sqrt{1 + |f'(x)|^2} dx$$

4 Plotting

Para funciones dadas y interpoladas

```
plot(x, a, b)
```

Composición

```
plot(x, a, b) + plot(g, a, b, color='red')
```

Cota

```
plot(x, a, b) + plot(12, a, b)
```

Cota de absoluto de derivada

```
plot(abs(diff(f, x)), a, b) + plot(12, a, b)
```

Error real entre función y su interpolada g por debajo de cota de error e

```
plot(abs(f(x) - g(x)), (x, a, b)) + plot(e, a, b)
```

Función, interpolada y nodos

```
plot(x, a, b) + plot(g, a, b, color='red') + points(XY, color='magenta')
```