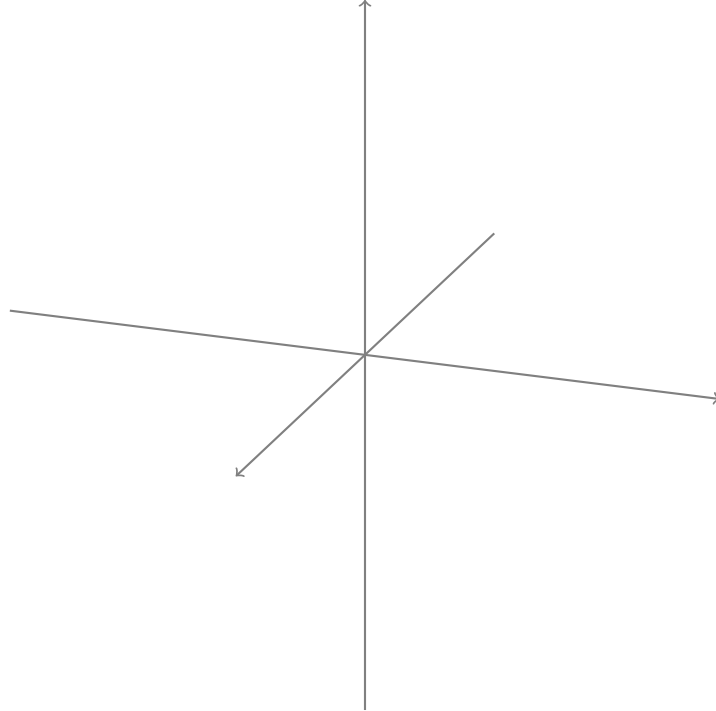


TAREA 10 AMD. PRODUCTO ESCALAR

Ejercicio 1. Para cada una de las rectas dadas, calcula un vector v_1 de la recta y dos vectores v_2 y v_3 perpendiculares a la recta de forma que la base $B = \{v_1, v_2, v_3\}$ tenga orientación positiva, normalízalos, píntalos en unos ejes coordenados y dibuja un cuadrado de tamaño 4×4 centrado en el origen con lados paralelos a los vectores v_2 y v_3 . Para hacer los dibujos usa los siguientes ejes coordenados:



- (1) La recta r dada por $\begin{cases} 2x + 2y - z = 0 \\ x - y + 3z = 0 \end{cases}$
 (2) La recta r dada por $x = 2\lambda, y = 0, z = -\lambda$ con $\lambda \in \mathbb{R}$.

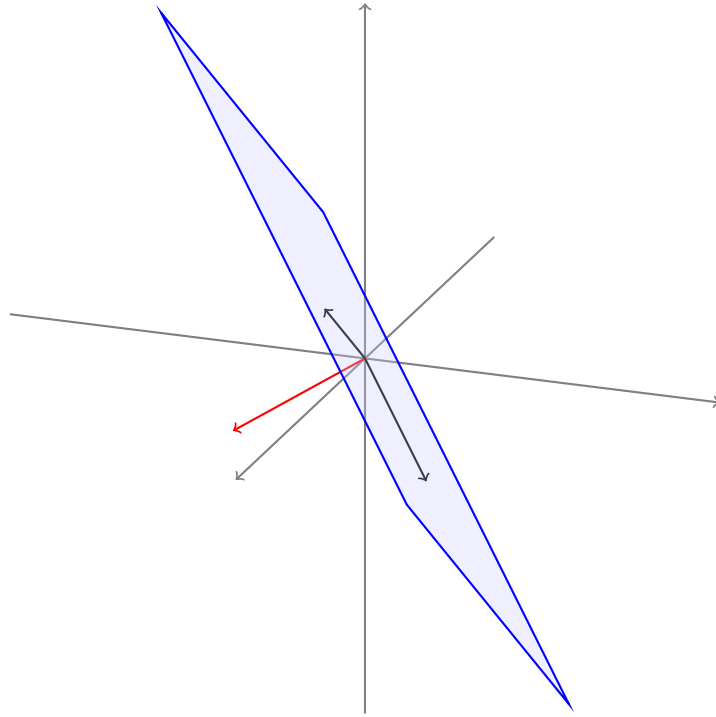
Solución

```
n1 = vector(RR, [2, 2, -1])
n2 = vector(RR, [1, -1, 3])
vr = n1.cross_product(n2)

v1 = vr.normalized()
v2 = vector(RR, [vr[1], -vr[0], 0]).normalized()
v3 = v1.cross_product(v2).normalized()
B = block_matrix([[v1.column(), v2.column(), v3.column()]])

square = matrix(RR, [[0, 2, 2], [0, 2, -2], [0, -2, -2], [0, -2, 2]]).T
squareB = [B * square.column(i) for i in range(4)]
```

Recta definida por dos planos cuyas normales son $n_1 = (2, 2, -1)$ $n_2 = (1, -1, 3)$ El vector de la recta v_r es perpendicular a las normales, $v_r = n_1 \times n_2 = (5.000000000000000, -7.000000000000000, -4.000000000000000)$



Ejercicio 2. Dado el espacio $W = N(H) \leq \mathbb{R}^5$, donde
 $H = \text{matrix}(QQ, [[-1, -1, 1, 0, 2], [2, 0, 1, 1, 0]])$

$$H = \begin{bmatrix} -1 & -1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

Calcula una base de W y obtén una base ortonormal a partir de ella utilizando el método de Gram-Schmidt.

Solución:

```
HtI = block_matrix([[H.T, 1]])
HtIr = HtI.echelon_form()
HtIr = copy(HtIr)
HtIr.subdivide(2, 2)
A = HtIr.subdivision(1, 1).T
Ar = A.echelon_form()
v1 = A.column(0)
v2 = A.column(1)
v3 = A.column(2)
w1 = v1
w2 = v2 - (((v2.dot_product(w1))/(w1.dot_product(w1))) * w1)
w3 = v3 - (((v3.dot_product(w1))/(w1.dot_product(w1))) * w1) - (((v3.dot_product(w2))/(w2.dot_product(w2)))
```

$$[H^T | I] = \left[\begin{array}{cc|ccccc} -1 & 2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[\begin{array}{cc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & -2 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & -1 & -\frac{1}{2} \end{array} \right]$$

$$A = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & -1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array} \right] \rightarrow \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

A es una base de W

$$v_1 = (1, 0, 0, -2, \frac{1}{2}) \quad v_2 = (0, 1, 0, 0, \frac{1}{2}) \quad v_3 = (0, 0, 1, -1, -\frac{1}{2})$$

$$w_1 = v_1 = \left(1, 0, 0, -2, \frac{1}{2}\right)$$

$$w_2 = v_2 - \frac{v_2 \cdot w_1}{w_1 \cdot w_1} w_1 = \left(-\frac{1}{21}, 1, 0, \frac{2}{21}, \frac{10}{21}\right)$$

$$w_3 = v_3 - \frac{v_3 \cdot w_1}{w_1 \cdot w_1} w_1 - \frac{v_3 \cdot w_2}{w_2 \cdot w_2} w_2 = \left(-\frac{9}{26}, \frac{7}{26}, 1, -\frac{4}{13}, -\frac{7}{13}\right)$$

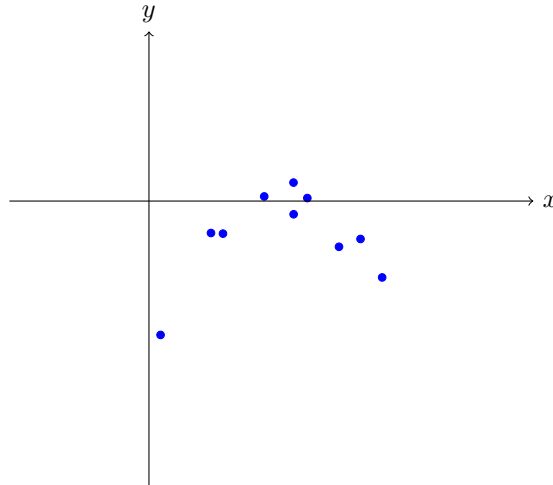
Ejercicio 3. *Calcula la parábola que mejor se ajusta a los datos que se proporcionan a continuación:*

```
XY = matrix(RR, [[ 0.9791609077659043 , -0.430715356970999 ],
                  [ 1.5250083079649113 , 0.061165657369748695 ],
                  [ 2.7989027102112827 , -0.5015548718775755 ],
                  [ 1.912158417289881 , 0.24440901271557722 ],
                  [ 0.8205530309864537 , -0.42261180235811696 ],
                  [ 1.914008178794904 , -0.17482280182386326 ],
                  [ 3.0850261801730925 , -1.0104522969296543 ],
                  [ 0.15356067616723457 , -1.7711019287922931 ],
                  [ 2.0952887890653016 , 0.03975209722972092 ],
                  [ 2.514022662217597 , -0.6049726172196987 ]])
```

```
X = XY.column(0)
```

```
Y = XY.column(1)
```

La representación gráfica de estos puntos es:



Solución:

```
B = matrix([[1 for x in X],
            [x for x in X],
            [x^2 for x in X]]).T
```

```
C = (B.T*B)^-1 * B.T * Y
```

Parabola $y = c_0 + c_1x + c_2x^2$

$$B = \begin{bmatrix} 1.00000000000000 & 0.979160907765904 & 0.958756083296950 \\ 1.00000000000000 & 1.52500830796491 & 2.32565033936200 \\ 1.00000000000000 & 2.79890271021128 & 7.83385638122806 \\ 1.00000000000000 & 1.91215841728988 & 3.65634981281254 \\ 1.00000000000000 & 0.820553030986454 & 0.673307276661056 \\ 1.00000000000000 & 1.91400817879490 & 3.66342730849378 \\ 1.00000000000000 & 3.08502618017309 & 9.51738653235338 \\ 1.00000000000000 & 0.153560676167235 & 0.0235808812649383 \\ 1.00000000000000 & 2.09528878906530 & 4.39023510958274 \\ 1.00000000000000 & 2.51402266221760 & 6.32030994614365 \end{bmatrix}$$

$$BC = Y$$

$$C = (B^T B)^{-1} B^T Y$$

$$C = (-2.05294994019641, 2.35942774111745, -0.661378998099879)$$

