

## AMD Curso 2023-2024

### Tarea Semana 3

**Ejercicio 1.** *Calcula, usando la reducción de matrices, el inverso de 76860393 en  $\mathbb{Z}_{433362740}$ .*

*Solución.-*

```
A = matrix(ZZ, [[x1], [n1]])
Ap = block_matrix([[A, 1]])
Ar = Ap.echelon_form()
Ar = copy(Ar)
Ar.subdivide([], 1)
S = Ar.subdivision(0, 1)
R = S * A
```

$$A' = [A|I] = \left( \begin{array}{cc|cc} 76860393 & & 1 & 0 \\ 433362740 & & 0 & 1 \end{array} \right)$$
$$\left( \begin{array}{cc|cc} 1 & 332767437 & -59019001 & \\ 0 & 433362740 & -76860393 & \end{array} \right)$$

El  $\text{mcd}(76860393, 433362740) = 1$ , por tanto es invertible.

$$\left( \begin{array}{cc|cc} 332767437 & -59019001 & & \\ 433362740 & -76860393 & & \end{array} \right) \left( \begin{array}{cc} 76860393 \\ 433362740 \end{array} \right) = \left( \begin{array}{cc} 1 \\ 0 \end{array} \right)$$

La expresión del mcd de 76860393 y 433362740

$$332767437 \cdot 76860393 + (-59019001) \cdot 433362740 = 1$$

$$332767437 \cdot 76860393 = 1$$

$$76860393^{-1} = 332767437 \text{ en } (\mathbb{Z}_{433362740})$$

**Ejercicio 2.** *Dada la ecuación  $19165x + 18666y = 145$ , indica si tiene solución y (caso de tener) calcula todas las posibles soluciones.*

*Solución.-*

```
A = matrix(ZZ, [[a2], [b2]])
Ap = block_matrix([[A, 1]])
Ar = Ap.echelon_form()
R = Ar[:, 1:] * A
```

$$A' = [A|I] = \left( \begin{array}{cc|cc} 19165 & & 1 & 0 \\ 18666 & & 0 & 1 \end{array} \right)$$

reducida

$$\left( \begin{array}{cc|cc} 1 & 16459 & -16899 & \\ 0 & 18666 & -19165 & \end{array} \right)$$

$\text{mcd}(19165, 18666) = 1$  es divisor de 145, por tanto hay solución.

$$\begin{pmatrix} 16459 & -16899 \\ 18666 & -19165 \end{pmatrix} \begin{pmatrix} 19165 \\ 18666 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Desarrollar

$$16459 \cdot 19165 + (-16899) \cdot 18666 = 1$$

$$18666 \cdot 19165 + (-19165) \cdot 18666 = 0$$

Multiplicar primera relación por  $m$  para tener  $m$ , y segunda relación por  $t$

$$16459 \cdot 19165 \cdot 145 + (-16899) \cdot 18666 \cdot 145 = 145$$

$$18666 \cdot 19165 \cdot t + (-19165) \cdot 18666 \cdot t = 0$$

Sumar las dos

$$19165(2386555 + 18666t) + 18666(-2450355 - 19165t) = 145$$

Extraer  $x$  e  $y$

$$x = 2386555 + 18666t$$

$$y = -2450355 - 19165t$$

**Ejercicio 3.** *Obtener todas las soluciones del siguiente sistema de congruencias:*

$$n \equiv 1792944400 \pmod{4959384998}$$

$$n \equiv 16054429802 \pmod{18622220374}$$

*Solución.-*

```
A = matrix(ZZ, [[m31], [-m32]])
Ap = block_matrix([[A, 1]])
Ar = Ap.echelon_form()
Ar = copy(Ar)
Ar.subdivide([], 1)
As = Ar[:, 1:]
R = Ar[:, :1]

mcd = R[0, 0]

a3n = 1792944400
b3n = 16054429802
d = b3n - a3n
```

$$F = d / \text{mcd}$$

$$c3 = a3n + (m31 * As[0, 0] * F)$$

$$cm3 = m31 * As[1, 0]$$

$$\text{mcd}(4959384998, 18622220374)$$

$$[A|I] = \left( \begin{array}{cc|cc} 4959384998 & & 1 & 0 \\ -18622220374 & & 0 & 1 \end{array} \right)$$

reducida

$$\left( \begin{array}{cc|cc} 2002174 & & 5884 & 1567 \\ & 0 & 9301 & 2477 \end{array} \right)$$

$$\text{mcd}(4959384998, 18622220374) = 2002174 \neq 1 \text{ es un caso 2}$$

$$n = 1792944400 + 4959384998x$$

$$n = 16054429802 + 18622220374y$$

$$1792944400 + 4959384998x = 16054429802 + 18622220374y$$

$$4959384998x - 18622220374y = 14261485402 = d$$

$$F = \frac{14261485402}{2002174} = 7123$$

El mcd es divisor de  $d$ , por tanto hay solución

$$\begin{pmatrix} 5884 & 1567 \\ 9301 & 2477 \end{pmatrix} \cdot \begin{pmatrix} 4959384998 \\ -18622220374 \end{pmatrix} = \begin{pmatrix} 2002174 \\ 0 \end{pmatrix}$$

$$5884 \cdot 4959384998 + 1567 \cdot (-18622220374) = 2002174$$

$$9301 \cdot 4959384998 + 2477 \cdot (-18622220374) = 0$$

Multiplicar la de arriba para dar  $d$ , y abajo por  $t$

$$5884 \cdot 4959384998 \cdot 7123 + 1567 \cdot (-18622220374) \cdot 7123 = 14261485402$$

$$9301 \cdot 4959384998 \cdot t + 2477 \cdot (-18622220374) \cdot t = 0$$

Al sumarlas

$$4959384998(41911732 + 9301t) - 18622220374(11161741 + 2477t) = 14261485402$$

$$x = 41911732 + 9301t$$

$$y = 11161741 + 2477t$$

$$\begin{aligned}n &= 1792944400 + 4959384998x = 1792944400 + 4959384998(41911732 + 9301t) \\&= 207856416713940936 + 46127239866398t \\&= 207856416713940936 \pmod{46127239866398}\end{aligned}$$