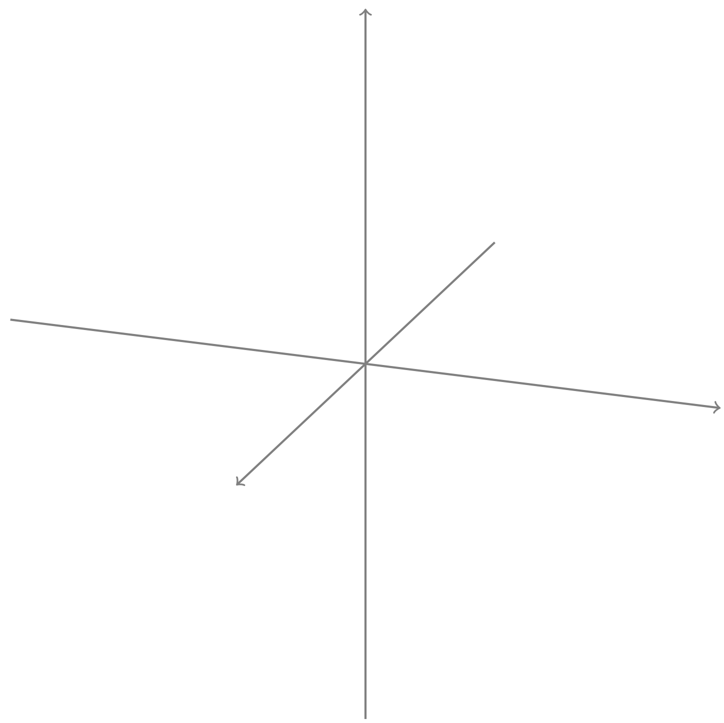
# TAREA 10 AMD. PRODUCTO ESCALAR

**Ejercicio 1.** *Para cada una de las rectas dadas, calcula un vector $v_1$ de la recta y dos vectores $v_2$ y $v_3$ perpendiculares a la recta de forma que la base $B = \{v_1, v_2, v_3\}$ tenga orientación positiva, normalízalos, píntalos en unos ejes coordenados y dibuja un cuadrado de tamaño $4 \times 4$ centrado en el origen con lados paralelos a los vectores $v_2$ y $v_3$. Para hacer los dibujos usa los siguientes ejes coordenados:*



(1) *La recta $r$ dada por* $\begin{cases} 2x + 2y - z = 0 \\ x - y + 3z = 0 \end{cases}$

(2) *La recta $r$ dada por $x = 2\lambda, y = 0, z = -\lambda$ con $\lambda \in \mathbb{R}$.*

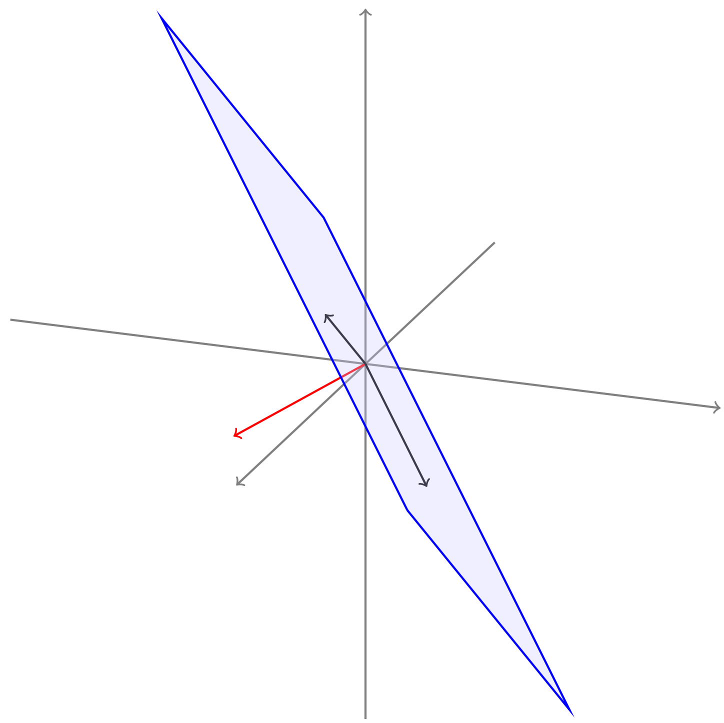
*Solución*

```
n1 = vector(RR, [2, 2, -1])
n2 = vector(RR, [1, -1, 3])
vr = n1.cross_product(n2)

v1 = vr.normalized()
v2 = vector(RR, [vr[1], -vr[0], 0]).normalized()
v3 = v1.cross_product(v2).normalized()
B = block_matrix([[v1.column(), v2.column(), v3.column()]])

square = matrix(RR, [[0, 2, 2], [0, 2, -2], [0, -2, -2], [0, -2, 2]]).T
squareB = [B * square.column(i) for i in range(4)]
```

Recta definida por dos planos cuyas normales son $n_1 = (2, 2, -1)$ $n_2 = (1, -1, 3)$ El vector de la recta $v_r$ es perpendicular a las normales, $v_r = n_1 \times n_2 = (5.00000000000000, -7.00000000000000, -4.00000000000000)$



**Ejercicio 2.** *Dado el espacio $W = N(H) \leq \mathbb{R}^5$, donde*

```
H=matrix(QQ,[[-1,-1,1,0,2],[2,0,1,1,0]])
```

$$H = \begin{bmatrix} -1 & -1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix}.$$

*Calcula una base de $W$ y obtén una base ortonormal a partir de ella utilizando el método de Gram-Schmidt.*

*Solución:*

```
HtI = block_matrix([[H.T, 1]])
HtIr = HtI.echelon_form()
HtIr = copy(HtIr)
HtIr.subdivide(2, 2)
A = HtIr.subdivision(1, 1).T
Ar = A.echelon_form()
v1 = A.column(0)
v2 = A.column(1)
v3 = A.column(2)
w1 = v1
w2 = v2 - (((v2.dot_product(w1))/(w1.dot_product(w1))) * w1)
w3 = v3 - (((v3.dot_product(w1))/(w1.dot_product(w1))) * w1) - (((v3.dot_product(w2))/(w2.dot_product(w2))
```

$$[H^T|I] = \left[\begin{array}{rr|rrrrr} -1 & 2 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{rr|rrrrr} 1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & -2 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & -1 & -\frac{1}{2} \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 0 & -1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A es una base de $W$

$v_1 = \left(1,\, 0,\, 0,\, -2,\, \frac{1}{2}\right)$ $v_2 = \left(0,\, 1,\, 0,\, 0,\, \frac{1}{2}\right)$ $v_3 = \left(0,\, 0,\, 1,\, -1,\, -\frac{1}{2}\right)$

$$w_1 = v_1 = \left(1,\, 0,\, 0,\, -2,\, \frac{1}{2}\right)$$

$$w_2 = v_2 - \frac{v_2 \cdot w_1}{w_1 \cdot w_1} w1 = \left(-\frac{1}{21},\, 1,\, 0,\, \frac{2}{21},\, \frac{10}{21}\right)$$

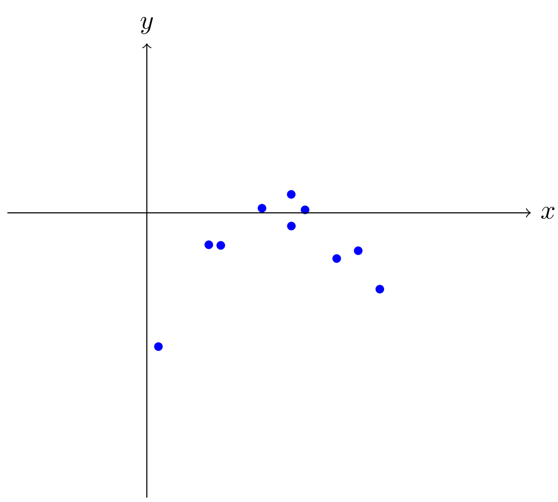
$$w_3 = v_3 - \frac{v_3 \cdot w_1}{w_1 \cdot w_1} w1 - \frac{v_3 \cdot w_2}{w_2 \cdot w_2} w2 = \left(-\frac{9}{26},\, \frac{7}{26},\, 1,\, -\frac{4}{13},\, -\frac{7}{13}\right)$$

**Ejercicio 3.** *Calcula la parábola que mejor se ajusta a los datos que se proporcionan a continuación:*

```
XY = matrix(RR,[[ 0.9791609077659043 ,  -0.430715356970999 ],
                [ 1.5250083079649113 ,  0.061165657369748695 ],
                [ 2.7989027102112827 ,  -0.5015548718775755 ],
                [ 1.912158417289881 ,  0.24440901271557722 ],
                [ 0.8205530309864537 ,  -0.42261180235811696 ],
                [ 1.914008178794904 ,  -0.17482280182386326 ],
                [ 3.0850261801730925 ,  -1.0104522969296543 ],
                [ 0.15356067616723457 ,  -1.7711019287922931 ],
                [ 2.0952887890653016 ,  0.03975209722972092 ],
                [ 2.514022662217597 ,  -0.6049726172196987 ]])

X = XY.column(0)
Y = XY.column(1)
```

*La representación gráfica de estos puntos es:*




*Solución:*

```
B = matrix([[1 for x in X],
[x for x in X],
[x^2 for x in X]]).T

C = (B.T*B)^-1 * B.T * Y
```

Parabola $y = c_0 + c_1 x + c_2 x^2$

$$B = \begin{bmatrix} 1.00000000000000 & 0.979160907765904 & 0.958756083296950 \\ 1.00000000000000 & 1.52500830796491 & 2.32565033936200 \\ 1.00000000000000 & 2.79890271021128 & 7.83385638122806 \\ 1.00000000000000 & 1.91215841728988 & 3.65634981281254 \\ 1.00000000000000 & 0.820553030986454 & 0.673307276661056 \\ 1.00000000000000 & 1.91400817879490 & 3.66342730849378 \\ 1.00000000000000 & 3.08502618017309 & 9.51738653235338 \\ 1.00000000000000 & 0.153560676167235 & 0.0235808812649383 \\ 1.00000000000000 & 2.09528878906530 & 4.39023510958274 \\ 1.00000000000000 & 2.51402266221760 & 6.32030994614365 \end{bmatrix}$$

$$BC = Y$$
$$C = (B^T B)^{-1} B^T Y$$

$$C = (-2.05294994019641,\ 2.35942774111745,\ -0.661378998099879)$$