



Assignment 01

Course Code: CSE-115

Course Title: Discrete Mathematics

Submitted To: Tasnimatul Jannah

Submitted By:

Name: Abdur Rahim

ID: **231001422E**

Department: ECSE

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Assignment: 1

12. Determine whether each of these function \mathbb{Z} to \mathbb{Z} is one to one.

(a) $f(n) = n-1$

let $f(n_1) = f(n_2)$

$$n_1 - 1 = n_2 - 1$$

$$n_1 = n_2$$

This function is one to one.

(b) $f(n) = n^2 + 1$

let $f(n_1) = f(n_2)$

$$n_1^2 + 1 = n_2^2 + 1$$

$$n_1^2 = n_2^2$$

$$n_1 = \pm n_2$$

This function is not one to one

(c) $f(n) = n^3$

let $f(n_1) = f(n_2)$

$$n_1^3 = n_2^3$$

$$n_1 = n_2$$

This function is one to one

(d) $f(n) = \left\lceil \frac{n}{2} \right\rceil$

let, $f(n_1) = f(n_2)$

$$\frac{n_1}{2} = \frac{n_2}{2}$$

$$n_1 = n_2$$

This function is one to one

13) Which function is Exercise 12 are onto?

(a) $f(n) = n-1$

let $n-1 = m$
 $n = m+1$

This function is onto

(b) $f(n) = n^2 + 1$

let $n^2 + 1 = m$

$$n^2 = m-1$$

$$n = \sqrt{m-1}$$

This function is not onto.

$$\textcircled{c} f(n) = n^3$$

$$\text{Let } n^3 = m \\ n = \sqrt[3]{m}$$

This function is onto

$$\textcircled{d} f(n) = n/2$$

$$\text{Let } n/2 = m \\ n = 2m$$

This function is onto.

$\textcircled{22}$ Determine whether each of these functions is a bijection from \mathbb{R} to \mathbb{R} .

$$\textcircled{a} f(x) = -3x + 4$$

check one to one

$$f(x_1) = f(x_2)$$

$$\Rightarrow -3x_1 + 4 = -3x_2 + 4$$

$$\Rightarrow -3x_1 = -3x_2$$

$$\Rightarrow x_1 = x_2$$

one to one

check onto

$$\text{Let, } -3x + 4 = y$$

$$-3x = y - 4$$

$$x = \frac{y - 4}{-3}$$

onto function.

These function is a bijection.

$$\textcircled{b} f(x) = -3x^2 + 7$$

check one to one

$$f(x_1) = f(x_2)$$

$$-3x_1^2 + 7 = -3x_2^2 + 7$$

$$-3x_1^2 = -3x_2^2$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

not one to one function

that's why These function is not a bijection.

③ $f(x) = (x+1)/(x+2)$

let, $\frac{x_1+1}{x_2+2} = \frac{x_2+1}{x_2+2}$

$\Rightarrow x_1x_2 + x_1 + x_2 + 2 = x_1x_2 + 2x_1 + x_2 + 2$

$\Rightarrow 2x_2 - x_2 = x_1 - x_1$

$\Rightarrow x_1 = x_2$

one to one function.

let, $\frac{x+1}{x+2} = m$

$(x+1) = mx + 2m$

$x - mx = 2m - 1$

$x(1-m) = 2m - 1$

$x = \frac{2m-1}{1-m}$

not onto.

So This function is not a bijection function.

④ $f(x) = x^5 + 1$

let, $f(x_1) = f(x_2)$

$x_1^5 + 1 = x_2^5 + 1$

$\Rightarrow x_1^5 = x_2^5$

$\Rightarrow x_1 = \sqrt[5]{x_2^5}$

$\Rightarrow x_1 = x_2$

one to one

let, $x^5 + 1 = m$

$x^5 = m - 1$

$x = \sqrt[5]{m-1}$

one onto function.

So this function is a bijection.

(29) determine whether each of these is a bijection from \mathbb{R} to \mathbb{R} .

(a) $f(x) = 2x+1$

let $f(x_1) = f(x_2)$

$$2x_1+1 = 2x_2+1$$

$$2x_1 = 2x_2$$

$$x_1 = x_2$$

one to one.

let,

$$2x+1 = m$$

$$2x = m-1$$

$$x = \frac{m-1}{2}$$

onto function.

This function is bijection.

(b) $f(x) = x^2+1$

let, $f(x_1) = f(x_2)$

$$x_1^2+1 = x_2^2+1$$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

not one to one

This function is not bijection

(d) $f(x) = (x^2+1)(x^2+2)$

let, $f(x_1) = f(x_2)$

$$\frac{(x_1^2+1)(x_2^2+1)}{(x_1^2+2)(x_2^2+2)} = \frac{x_2^2+1}{x_2^2+2}$$

$$\Rightarrow x_1^2 x_2^2 + 2x_1^2 + x_2^2 + 2 = x_1^2 x_2^2 + x_1^2 + 2x_2^2 + 2$$

$$\Rightarrow 2x_1^2 - x_1^2 = 2x_2^2 - x_2^2$$

$$\Rightarrow x_1^2 = x_2^2$$

$$\Rightarrow x_1 = \pm x_2$$

This function is not bijection.

(c) $f(x) = x^3$

let, $f(x_1) = f(x_2)$

$$x_1^3 = x_2^3$$

$$x_1 = x_2 \text{ one to one}$$

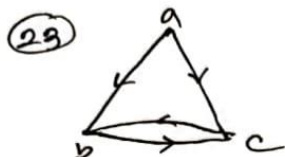
let, $x^3 = m$

$$x = \sqrt[3]{m}$$

onto

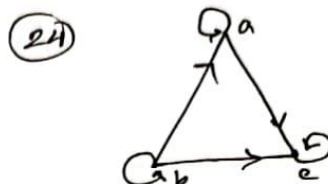
This function is bijection.

31
32 Determine whether the relations represented by the directed graph shown in Exercises (23-25) are reflexive or symmetric.



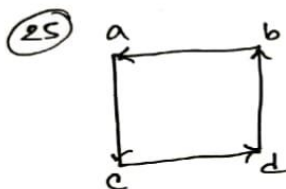
$$R = \{(a, c), (a, b), (b, c), (c, b)\}$$

not reflexive
not symmetric



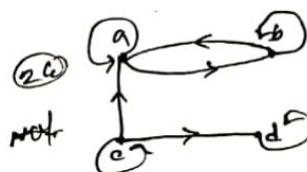
$$R = \{(a, a), (b, b), (c, c), (a, c), (b, c), (c, b)\}$$

reflexive but not symmetric



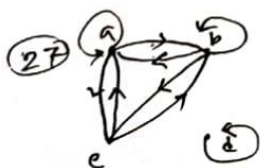
$$R = \{(a, c), (c, a), (c, d), (d, c), (d, b), (b, d)\}$$

not reflexive
not symmetric



$$R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, a), (c, d)\}$$

Reflexive but not symmetric



$$R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (a, c), (c, a), (b, c), (c, b)\}$$

not reflexive
not symmetric



$$R = \{(a, a), (b, b), (c, c), (d, d), (a, b), (b, a), (c, d), (d, c)\}$$

reflexive
symmetric.