Tax Multipliers in a DSGE Model[*]

Jason DeBacker[†]   Richard W. Evans[‡]   Aerfate Haimiti[§]

First Draft: September 28, 2010
This Version: September 16, 2025

Abstract

This paper analyzes the stabilization properties of tax cuts in a DSGE model.

Consumption taxes have long been favored by tax economists for their relative efficiency vis-a-vis taxes on capital and labor. This paper attempts to argue that consumption taxes should also be favored by macroeconomists for their stabilization properties. Cuts to consumption taxes can quickly and effectively stimulate aggregate demand. The following analysis studies the quantitative impact of tax cuts in a dynamic schocastic general equilibrium (DSGE) model. In addition to consumption taxes, the model nests other tax policies, such as investment tax credits and changes to expensing and depreciation policies. These policies target investment by lowering the after-tax cost of capital. Historically, such instruments have been used by the U.S. to provide counter-cyclical fiscal policy (e.g. the Economic Recovery Act of 1981 increased the ability of businesses to take advantage of the investment tax credit). Despite their use, such policies have not been studied in a DSGE environment.

The goal of this paper is to study specific tax policies in a DSGE environment. In a sense, bring in the tax policies favored by tax economists into a model used by macroeconomists. Recently, a number of researchers have used DSGE models to study the government spending multiplier (e.g., Christiano, Eichenbaum and Rebelo (2010)), but less work has been done on tax multipliers. Those who provide insights on tax multipliers (e.g., Zubairy (2010), Kumhof, Coenen, Muir, Freedman, Mursula, Erceg, Furceri, Lalonde, Linde, Mourougane, Roberts, Laxton, de Resende, Roeger, Snudden, Trabandt and in 't Veld (2010)) focus on multipliers associated with broad taxes on capital and labor income. Such tax cuts are generally not the most effective counter-cyclical policies. We are the first to provide quantitative insights into the the size of the multipliers associated with tax cuts that directly target consumption and investment.

Two recent, but related, events have spurred interest in consumption taxes; the 2007-2009 recession and the large federal budget deficits. In a 2008 *Financial Times* aricle Kotlikoff and Leamer (2008) argue that a national sales tax holiday would be an especially effective response to the dip in aggregate demand experienced during the 2007-2009 financial crisis. While possible, it is difficult to implement such a policy without a national sales tax. Recent debate about the fiscal sustainability of the federal government has brough much

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attention to value added taxes (VAT) as a way to increase revenue with minimal efficiency costs. We hope to show that a tax on consumption has a second positive attribute - a non-zero tax on consumption allows policy makers important flexibility for macro stabilization policies.

Maybe cite some of the micro evidence of effects of sales tax holidays...

Stress:

- lower dwl due to consumption tax than other forms of tax

- faster impact of consumption tax cut

- consumption beats spending because people get to choose what to spend it on (though we can't easily model this)

- current focus on the VAT, which is a consumption tax. Most of talk is about less distortionary way to raise revenue, but it would also allow to a good fiscal stabilizer.

Other novelties:

- we build a model that can handle other tax cuts like bonus depreciation (something simple so don't have to take account of vintage) and investment tax credits- see what the multiplier is for these in short and long run

- Be best if could have heterogenous firms who vary in productivity and have price stickiness. Build in corporate income tax, div tax, cap gains tax in more realistic detail (now all lumped in tax on capital).

1 Environment

1.1 Households

Representative household.

HH maximizes:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, g_t) \tag{1.1}$$

subject to the real budget constraint:

$$(1 + \tau_t^c)c_t + \frac{b_t}{P_t} + i_t = \frac{(1 + r_t(1 - \tau_t^i))b_{t-1}}{P_t} + \frac{(1 - \tau_t^l)w_t l_t}{P_t} + \\ \frac{(1 - \tau_t^k)r_t^k v_t k_{t-1}}{P_t} + \tau_t^k \delta_t^\tau k_{t-1}^\tau + \tau_t^{ic} i_t + \tau_t^k e_t^\tau i_t + \frac{(1 - \tau_t^d)d_t}{P_t} + x_t \tag{1.2}$$

Where all variables pre-determined at time t have subscripts strictly less than time t. c represents consumption, i investment, b government bond holdings, p price level, l labor supply, k capital stock, x government transfers, d dividends, w the nominal wage rate, v intensity of capital utilization. Taxes τ^i, τ^l, τ^k, τ^d are taxes on interest income, labor income, capital income, and dividend income. r is the nominal interest rate on government bonds and r^k is the rental rate on capital. k^τ is the tax basis for the household's capital stock, δ^τ is the rate of depreciation for tax purposes, e^τ is the rate of expensing for tax purposes, and τ^{ic} is the investment tax credit.

The law of motion of the household's capital stock is:

$$k_t = (1 - \delta(v_t))k_{t-1} + i_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) \right] \tag{1.3}$$

I'll assume that the investment adjustment cost function takes the form $S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$. Such a functional form is common in the literature (e.g., Smets and Wouters (2003), Christiano et al. (2010), Zubairy (2010), et al) and has the nice properties that in the steady state $S = 0$, $S' = 0$ and $S'' > 0$, and that in a deterministic steady state, the adjustment costs are zero.

The law of motion of the household's tax basis for it's capital stock is:

$$k_t^\tau = (1 - \delta_t^\tau)k_{t-1}^\tau + i_t(1 - e_t^\tau) \tag{1.4}$$

Assume also that $\delta(v_t)$ takes the form: $\delta(v_t) = \delta_0 + \delta_1(v_t - 1) + \frac{\delta_2}{2}(v_t - 1)^2$. This means that in the deterministic steady state, we can calibrate δ such that $v_t = 1$, and $\delta(v) = \delta_0$ (see Traum and Yang (2010) on this point).

FOC's (to go in appendix):

$$\frac{\partial U}{\partial c_t} : u_c(c_t, l_t, g_t) - \lambda_t(1 + \tau_t^c) = 0 \tag{1.5}$$

$$\frac{\partial U}{\partial l_t} : u_l(c_t, l_t, g_t) + u_c(c_t, l_t, g_t)\frac{w_t(1 - \tau_t^l)}{P_t(1 + \tau_t^c)} = 0 \tag{1.6}$$

$$\frac{\partial U}{\partial i_t} : u_c(c_t, l_t, g_t)\frac{(-1 + \tau_t^{ic} + \tau_t^k e_t^\tau)}{(1 + \tau_t^c)} +$$
$$\beta E_t u_c(c_{t+1}, l_{t+1}, g_{t+1})\frac{1}{(1 + \tau_{t+1}^c)}\left[\frac{(1 - \tau_{t+1}^k)r_{t+1}^k v_{t+1}}{P_{t+1}}\left(1 - S\left(\frac{i_t}{i_{t-1}}\right) - i_t\left(\frac{S'\frac{i_t}{i_{t-1}}}{i_{t-1}}\right)\right) + \tau_{t+1}^k \delta_{t+1}^\tau(1 - e_t^\tau)\right]$$
$$\beta^2 E_t\left[u_c(c_{t+2}, l_{t+2}, g_{t+2})\left(\frac{1 - \tau_{t+2}^k}{1 + \tau_{t+2}^c}\right)\frac{r_{t+2}^k v_{t+2}}{P_{t+2}}\gamma\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right] = 0 \tag{1.7}$$

$$\frac{\partial U}{\partial b_t} : \frac{-u_c(c_t, l_t, g_t)}{P_t(1 + \tau_t^c)} + \beta E_t u_c(c_{t+1}, l_{t+1}, g_{t+1})\left(\frac{(1 + r_{t+1}(1 - \tau_{t+1}^i))}{P_{t+1}(1 + \tau_{t+1}^c)}\right) = 0 \tag{1.8}$$

$$\frac{\partial U}{\partial v_t} : u_c(c_t, l_t, g_t)\left(\frac{(1 - \tau_t^k)r_t^k k_{t-1}}{P_t(1 + \tau_t^c)}\right) - \beta E_t u_c(c_{t+1}, l_{t+1}, g_{t+1})\left(\frac{(1 - \tau_{t+1}^k)r_{t+1}^k v_{t+1}\frac{\partial \delta(v_t)}{\partial v_t}k_{t-1}}{P_{t+1}(1 + \tau_{t+1}^c)}\right) = 0 \tag{1.9}$$

1.1.1 Utility specification 0

This utility function is similar to that in Christiano et al. (2010), Traum and Yang (2010), and Zubairy (2010).

$$u(c_t, l_t, g_t) = \varepsilon_t^b \left(\frac{[c_t^\zeta (1 - l_t)^{(1-\zeta)}]^{1-\sigma}}{1 - \sigma} + \chi^g \frac{g_t^{1-\sigma_g}}{1 - \sigma_g} \right) \tag{1.10}$$

in which the preference shock is assumed to follow the following AR(1) process:

$$\log(\varepsilon_t^b) = \rho_\varepsilon \log(\varepsilon_{t-1}^b) + \eta_{\varepsilon,t}$$
$$\text{where} \quad \eta_{\varepsilon,t} \sim N(0, \sigma_\varepsilon) \quad \text{and} \quad \rho_\varepsilon \in [0, 1) \tag{1.11}$$

FOC's with this utility function are:

$$\frac{\partial U}{\partial c_t} : \frac{\varepsilon_t^b \zeta c_t^{\zeta(1-\sigma)-1} (1 - l_t)^{(1-\zeta)(1-\sigma)}}{1 + \tau_t^c} = \lambda_t \tag{1.12}$$

$$\frac{\partial U}{\partial l_t} : \left(\frac{1 - \zeta}{\zeta} \right) \left(\frac{c_t}{1 - l_t} \right) = \frac{w_t (1 - \tau_t^l)}{P_t (1 + \tau_t^c)} \tag{1.13}$$

$$\frac{\partial U}{\partial i_t} : \varepsilon_t^b \left(\frac{c_t}{1 - l_t} \right)^{(\zeta-1)(1-\sigma)} c_t^{-\sigma} \left(\frac{\tau_t^{ic} + \tau_t^k e_t^\tau - 1}{1 + \tau_t^c} \right) + \ldots$$

$$\beta E_t \left[\varepsilon_{t+1}^b \left(\frac{c_{t+1}}{1 - l_{t+1}} \right)^{(\zeta-1)(1-\sigma)} c_{t+1}^{-\sigma} \ldots \right.$$

$$\left(\frac{[1 - \tau_{t+1}^k] r_{t+1}^k v_{t+1}}{P_{t+1}(1 + \tau_{t+1}^c)} \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \gamma \left(\frac{i_t}{i_{t-1}} \right) \left(\frac{i_t}{i_{t-1}} - 1 \right) \right] + \frac{\tau_{t+1}^k \delta_{t+1}^\tau (1 - e_{t+1}^\tau)}{1 + \tau_{t+1}^c} \right) \right] + \ldots$$

$$\beta^2 E_t \left[\varepsilon_{t+2}^b \left(\frac{c_{t+2}}{1 - l_{t+2}} \right)^{(\zeta-1)(1-\sigma)} c_{t+2}^{-\sigma} \frac{(1 - \tau_{t+2}^k) r_{t+2}^k v_{t+2}}{P_{t+2}(1 + \tau_{t+2}^c)} \gamma \left(\frac{i_{t+1}}{i_t} - 1 \right) \left(\frac{i_{t+1}}{i_t} \right)^2 \right] = 0 \tag{1.14}$$

$$\frac{\partial U}{\partial b_t} : \varepsilon_t^b \left(\frac{c_t}{1 - l_t} \right)^{(\zeta-1)(1-\sigma)} c_t^{-\sigma} \frac{1}{P_t(1 + \tau_t^c)} = \ldots$$

$$\beta E_t \left[\varepsilon_{t+1}^b \left(\frac{c_{t+1}}{1 - l_{t+1}} \right)^{(\zeta-1)(1-\sigma)} c_{t+1}^{-\sigma} \frac{1 + r_{t+1}(1 - \tau_{t+1}^i)}{P_{t+1}(1 + \tau_{t+1}^c)} \right] \tag{1.15}$$

$$\frac{\partial U}{\partial v_t} : \varepsilon_t^b \left(\frac{c_t}{1 - l_t} \right)^{(\zeta-1)(1-\sigma)} c_t^{-\sigma} \frac{(1 - \tau_t^k) r_t^k k_{t-1}}{P_t(1 + \tau_t^c)} = \ldots$$

$$\beta E_t \left[\varepsilon_{t+1}^b \left(\frac{c_{t+1}}{1 - l_{t+1}} \right)^{(\zeta-1)(1-\sigma)} c_{t+1}^{-\sigma} \frac{(1 - \tau_{t+1}^k) r_{t+1}^k v_{t+1}}{P_{t+1}(1 + \tau_{t+1}^c)} (\delta_1 + \delta_2[v_t - 1]) k_{t-1} \right] \tag{1.16}$$

$$\tag{1.17}$$

1.1.2 Alternative Setup

$$E_0 \sum_{t=0}^\infty \beta^t \left[u(c_t, l_t, g_t) - \lambda_t \left[(1 + \tau_t^c) c_t + \frac{b_t}{P_t} + i_t - \frac{(1 + r_t(1 - \tau_t^i)) b_{t-1}}{P_t} - \frac{(1 - \tau_t^l) w_t l_t}{P_t} - \frac{(1 - \tau_t^k) r_t^k v_t k_{t-1}}{P_t} - \tau_t^k \delta \right. \right.$$

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$$\tag{1.18}$$

FOCS:

$$\frac{\partial U}{\partial c_t} : u_c(c_t, l_t, g_t) - \lambda_t(1 + \tau_t^c) = 0 \tag{1.19}$$

$$\frac{\partial U}{\partial l_t} : u_l(c_t, l_t, g_t) + \frac{\lambda_t w_t(1 - \tau_t^l)}{P_t} = 0 \tag{1.20}$$

$$\frac{\partial U}{\partial b_t} : \frac{\lambda_t}{Pt} = \beta E_t \left[\frac{\lambda_{t+1}(1 + r_{t+1}(1 - \tau_{t+1}^i))}{P_{t+1}} \right] \tag{1.21}$$

$$\frac{\partial U}{\partial v_t} : \frac{\lambda_t(1 - \tau t^k)r_t^k k_{t-1}}{P_t} = Q_t(\delta_1 + \delta_2(v_t - 1))k_{t-1} \tag{1.22}$$

$$\frac{\partial U}{\partial k_t} : Q_t = \beta E_t \left[\frac{\lambda_{t+1}(1 - \tau_{t+1}^k)r_{t+1}^k v_{t+1}}{P_{t+1}} + Q_{t+1}(1 - \delta(v_{t+1})) \right] \tag{1.23}$$

$$\frac{\partial U}{\partial i_t} : \lambda_t(1 - \tau_t^{ic} - \tau_t^k e_t^\tau) = Q_t \left[1 - S\left(\frac{i_t}{i_{t-1}}\right) - S'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}} \right] +$$
$$\beta E_t \left[\lambda_{t+1}\tau_{t+1}^k \delta_{t+1}^\tau(1 - e_t^\tau) + Q_{t+1}S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \right] \tag{1.24}$$

1.1.3 HH FOC's in the SS

$$\frac{\partial U}{\partial c} : \frac{\zeta \bar{c}^{\zeta(1-\sigma)-1}(1 - \bar{l})^{(1-\zeta)(1-\sigma)}}{(1 + \tau^c)} = \bar{\lambda} \tag{1.25}$$

$$\frac{\partial U}{\partial l} : \frac{(1-\zeta)\bar{c}}{\zeta(1-\bar{l})} = \frac{\bar{w}(1-\tau^l)}{(1+\tau^c)} \tag{1.26}$$

$$\frac{\partial U}{\partial i} : \left(1 - \tau^{ic} - \tau^k e^\tau\right) = \beta \left[(1 - \tau^k)\bar{r}^k + \tau^k \delta^\tau(1 - e^\tau)\right] \tag{1.27}$$

$$\frac{\partial U}{\partial b} : 1 = \beta(1 + \bar{r}(1 - \tau^i)) \tag{1.28}$$

$$\frac{\partial U}{\partial v} : \bar{k} = \beta \delta_1 \tag{1.29}$$

1.1.4 SS of HH constraints/laws of motion

- $\bar{k} = (1 - \delta(\bar{v}))\bar{k} + \bar{i} \implies \bar{k}\delta(\bar{v}) = \bar{i}$, if $\bar{v} = 1$, then $\bar{i} = \bar{k}\delta_0$

- $\delta(\bar{v}) = \delta_0 + \delta_1(\bar{v} - 1) + \frac{\delta_2}{2}(\bar{v} - 1)^2$, if $\bar{v} = 1$, then $\delta(\bar{v}) = \delta_0$

- $S(\bar{i}) = 0$

- $\bar{k}^\tau = (1 - \delta^\tau)\bar{k}^\tau + \bar{i}(1 - e^\tau) \implies \bar{k}^\tau \delta^\tau = \bar{i}(1 - e^\tau)$

- Seems as though can normalize $\bar{P} = 1$ and $\bar{v} = 1$. Problems with this??

1.1.5 SS of non-HH equations

- Resource constraint: $\bar{Y} = \bar{C} + \delta_0 \bar{K} + \bar{G}$

- Government Budget Constraint: $\tau^c \bar{C} + \tau^l \bar{L}\bar{w} - (\bar{r}(1 - \tau^i))\bar{B} + \tau^k \bar{r}^k \bar{K} + \tau^d \bar{D} = \bar{G} + \bar{X} + \tau^{ic}\delta_0 \bar{K} + \tau^k \delta_0 \bar{K}(1 - e^\tau) + \tau^k e^\tau \delta_0 \bar{K}$ It might be that profits of the intermediate goods producers are zero in the SS- in which case, $\bar{D} = 0$.

- Technology shock: $\bar{z} = 1$

- SS conditions from Intermediate Goods Producers:

  - marginal cost: $mc = \frac{\bar{w}(1-\alpha)\bar{r}^{k\alpha}}{\alpha^\alpha(1-\alpha)^{(1-\alpha)}}$

  - labor-capital ratio: $\frac{\bar{K}}{\bar{L}} = \left(\frac{\alpha}{1-\alpha}\right)\frac{\bar{w}}{\bar{r}^k}$

  - output: $\bar{Y} = \bar{K}^\alpha \bar{L}^{(1-\alpha)}$

- Final Goods Producers: $\bar{P} = 1$ (*I think*).

1.1.6 Steady state values of state variables:

From the Calvo pricing equation, we can normalize $\bar{P}$:

$$\bar{P} = 1 \tag{1.30}$$

We can calibrate δ_0 such that we can normalize our SS capital utilization:

$$\bar{V} = 1 \tag{1.31}$$

We can normalize the mean of the TFP process (and know that the disturbance term = 0 in the SS):

$$\bar{Z} = 1 \tag{1.32}$$

We can normalize the mean of the taste shock (and know that the disturbance term = 0 in the SS):

$$\bar{\varepsilon}^b = 1 \tag{1.33}$$

We know that the disturbance term to the price markup is zero in the SS, thus:

$$\bar{\lambda}_p = \lambda_p \tag{1.34}$$

We can solve for the SS interest rate directly from the HH FOC for demand for gov't bonds (or from the Taylor Rule):

$$\bar{r} = \frac{1 - \beta}{\beta(1 - \tau^i)} \tag{1.35}$$

The FOC for the HH investment decision, we can find the SS return on capital:

$$\bar{r}^k = \frac{\left(\frac{1 - \tau^{ic} - \tau^k e^\tau}{\beta}\right) - \tau^k \delta^\tau (1 - e^\tau)}{(1 - \tau^k)} \tag{1.36}$$

The FOC for labor demand from the int goods producer's problem gives the SS wage rate:

$$\bar{w} = (1 - \alpha)\left(\frac{\bar{r}^k}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}} \tag{1.37}$$

Calibration gives the SS labor supply (I think this implies the Frisch elasticity of labor paramter $= 1$ – see Zubairy (2010)):

$$\bar{L} = 0.5 \tag{1.38}$$

The FOC for effective capital demand from the int goods producer's problem gives the SS level of capital:

$$\bar{K} = \left(\frac{\alpha}{1 - \alpha}\right)\frac{\bar{w}\bar{L}}{\bar{r}^k} \tag{1.39}$$

The law of motion for the tax basis of the capital stock gives the SS value for the tax basis of the capital stock:

$$\bar{K}^\tau = \frac{\delta_0 \bar{K}(1 - e^\tau)}{\delta^\tau} \tag{1.40}$$

The law of motion for the capital stock gives SS investment:

$$\bar{I} = \delta_0 \bar{K} \tag{1.41}$$

The int goods producers' production function gives the aggregate level of output in the SS:

$$\bar{Y} = \bar{K}^\alpha \bar{L}^{(1-\alpha)} \tag{1.42}$$

The int goods producers' profit function gives aggregate profits in the SS:

$$\bar{D} = \bar{Y} - \bar{w}\bar{L} - \bar{r}^k \bar{K} \tag{1.43}$$

The HH FOC for labor gives SS consumption:

$$\bar{C} = \frac{\bar{w}(1 - \tau^l)(1 - \bar{L})\zeta}{(1 - \zeta)(1 + \tau^c)} \tag{1.44}$$

The HH FOC for consumption gives SS marginal utility of consumption (the shadow price of consumption):

$$\bar{\lambda} = \frac{\zeta \bar{c}^{\zeta(1-\sigma)-1}(1-\bar{l})^{(1-\zeta)(1-\sigma)}}{(1+\tau^c)} \tag{1.45}$$

The resource constraint gives SS government spending:

$$\bar{G} = \bar{Y} - \bar{C} - \bar{I} \tag{1.46}$$

The gov't budget constraint gives SS bond issuance necessary to finance $\bar{G}$:

$$\bar{B} = \frac{\left[\tau^c \bar{C} + \tau^l \bar{L}\bar{w} + \tau^k \bar{r}^k \bar{K} + \tau^d \bar{D} - \tau^{ic}\bar{I} - \tau^k \delta^\tau \bar{K}^\tau - \tau^k e^\tau \bar{I} - (1-\gamma_x)\bar{G}\right]}{\bar{r}(1-\tau^i)} \tag{1.47}$$

The assumption regarding what fraction of the federal budget goes to X gives SS government transfers:

$$\bar{X} = \gamma_x * \left[\tau^c \bar{C} + \tau^l \bar{L}\bar{w} + \bar{B} + \tau^k \bar{r}^k \bar{K} + \tau^d \bar{D} - \tau^{ic}\bar{I} - \tau^k \delta^\tau \bar{K}^\tau - \tau^k e^\tau \bar{I} - \bar{B}\left[1 + \bar{r}(1-\tau^i)\right]\right] \tag{1.48}$$

1.2   Firms

1.2.1   Final Goods Producers

Final goods producers aggregate differentiated inputs to produce a homogenous output. The Dixit-Stiglitz aggregation function is given by;

$$Y_t = \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}, \tag{1.49}$$

where $log(\lambda_{p,t}) = \rho_\lambda log(\lambda_{p,t-1}) + (1-\rho_\lambda)log(\lambda_p) + \eta_t^p$ is the stochastic price markup in the intermediate goods market. We assume that $\eta_t^p \sim N(0, \sigma_\lambda)$

Maximizing profits (or minimizing costs) subject to Equation 1.49 results in demand for intermediate input $y_{i,t}$ of:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t, \tag{1.50}$$

where the price of a unit of the final, homogenous output, P_t, is given by:

$$P_t = \left(\int_0^1 p_{i,t}^{-\frac{1}{\lambda_{p,t}}} di\right)^{-\lambda_{p,t}} \tag{1.51}$$

1.2.2 Working Through the Final Goods Producer's problem

Objective function:

$$\max_{y_{i,t}} P_t Y_t^d - \int_0^1 p_{i,t} y_{i,t} di \tag{1.52}$$

The FOC's are:

$$p_t(1 + \lambda_{p,t}) \left(\int_0^1 y_{i,t}^{\left(\frac{1}{1+\lambda_{p,t}} \right)} di \right)^{\lambda_{p,t}} \left(\frac{1}{1+\lambda_{p,t}} \right) y_{i,t}^{\frac{1}{1+\lambda_{p,t}}-1} - p_{i,t} = 0, \forall i \tag{1.53}$$

Dividing the FOCs for intermediate inputs i and j $\implies$

$$\frac{p_{i,t}}{p_{j,t}} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{1}{1+\lambda_{p,t}}-1} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \tag{1.54}$$

Rearranging:

$$\implies p_{i,t} = \left(\frac{y_{j,t}}{y_{i,t}} \right)^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t}$$

$$\implies p_{i,t} = y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t} y_{i,t}^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \tag{1.55}$$

$$\implies p_{i,t} y_{i,t} = p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}} \right)} y_{i,t}^{\frac{1}{1+\lambda_{p,t}}}$$

Then integrate this condition to yield:

$$\int_0^1 p_{i,t} y_{i,t} di = p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}} \right)} \int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di$$

$$= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}} \right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \tag{1.56}$$

The zero profit condition implies $P_t Y_t = \int_0^1 p_{i,t} y_{i,t} di$. Plugging this into the above and we find:

$$P_t Y_t = p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}} \right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}}$$

$$\implies P_t Y_t = p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \tag{1.57}$$

$$\implies P_t = p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}}$$

Which implies the demand function (just rearranging terms):

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t, \forall i \tag{1.58}$$

To find P_t use the zero profit condition:

$$P_t Y_t = \int_0^1 p_{i,t} y_{i,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di$$

$$\Longrightarrow P_t Y_t = \int_0^1 p_{i,t} \left(\left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t \right)^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di$$

$$\Longrightarrow P_t Y_t = \int_0^1 p_{i,t} \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di$$

$$\Longrightarrow P_t Y_t = \int_0^1 p_{i,t} p_{i,t}^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} P_t^{\frac{(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di \tag{1.59}$$

$$\Longrightarrow P_t Y_t = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, Y_t P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

$$\Longrightarrow P_t = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

$$\Longrightarrow P_t^{\frac{-1}{\lambda_{p,t}}} = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di$$

$$\Longrightarrow P_t = \left(\int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \right)^{-\lambda_{p,t}}$$

1.2.3 Intermediate Goods Producers

Each intermediate-goods producing firm has a monopoly on it's heterogeneous output. Following Calvo (1983), firms can reset prices each period with probability $(1 - \theta)$. Firm's that cannot reset prices must rent labor and capital inputs to meet demand at last period's price, $p_{i,t-1}$. Each intermediate good's producer faces the same production function:

$$y_{i,t} = z_t \tilde{k}_{i,t}^\alpha l_{i,t}^{1-\alpha}, \tag{1.60}$$

where $\tilde{k}_{i,t} = v_t k_{i,t-1}$ are the effective units of capital rented from the households, $l_{i,t}$ is the effective labor rented from households, z_t is a serially correlated shock to firm productivity (which affect all intermediate-goods producers equally).

We assume z_t follows the following AR(1) process:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \eta_{z,t} \quad \text{where} \quad \eta_{z,t} \sim N(0, \sigma_z), \quad \text{and} \quad \rho_z \in [0,1) \tag{1.61}$$

There is no entry or exit.

The problem of intermediate-goods producers who can optimally reset their prices can be solved in two stages. In the first, the demand for factor inputs is found, taking the rental rate for capital, r_t^k, and the wage rate, w_t, as given. This stage yields the relative factor demands and the marginal cost of production. In the second stage, firm's take their marginal cost function as given, and choose the their, $p_{i,t}$ to maximize their expected, discounted profits.

The firms' per-period profit function takes the following form:

$$d_{i,t} = p_{i,t} y_{i,t} - r_t^k \tilde{k}_{i,t} - w_t l_{i,t} \quad \forall i, t \tag{1.62}$$

A firm's objective function when it chooses price is to maximize expected profits in the case that the current price cannot be changed.

$$V = \max_{p_{i,t}, \tilde{k}_{i,t}, l_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^{t+j} \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[p_{i,t} y_{i,t+j} - r_{t+j}^k \tilde{k}_{i,t+j} - w_{t+j} l_{i,t+j} \right] \right) \tag{1.63}$$

FOC's:

$$\frac{\partial V}{\partial \tilde{k}_{i,t}} : \frac{\alpha p_{i,t} y_{i,t}}{\tilde{k}_{i,t}} - r_t^k = 0 \tag{1.64}$$

$$\frac{\partial V}{\partial l_{i,t}} : \frac{(1-\alpha) p_{i,t} y_{i,t}}{l_{i,t}} - w_t = 0 \tag{1.65}$$

Dividing Equation 1.64 by Equation 1.65 and rearranging some terms, one gets:

$$\frac{\tilde{k}_{i,t}}{l_{i,t}} = \left(\frac{\alpha}{1-\alpha} \right) \frac{w_t}{r_t^k} \tag{1.66}$$

Plugging this ratio into Equation 1.60 and setting it equal to 1, one can find the amount of labor needed to produce one unit of output in terms of w_t, rt^k, z_t, and the parameter α. Rearranging 1.66 to put $\tilde{k}_{i,t}$ in terms of $l_{i,t}$, factor input prices, and parameters then writing the cost function $(r_{t+j}^k \tilde{k}_{i,t+j} + w_{t+j} l_{i,t+j})$ in terms of $l_{i,t}$, factor input prices, and parameters, one can substitute in the equation for $l_{i,t}$ This yields the marginal cost of production in real terms (this procedure is described in Fernandez-Villaverde and Rubio-Ramirez (2006)):

$$mc_t = \frac{w_t^{(1-\alpha)} (r_t^k)^\alpha}{p_t z_t \alpha^\alpha (1-\alpha)^{(1-\alpha)}} \tag{1.67}$$

N.B., the marginal cost function, mc_t does not depend upon i. This is because all firms receive the same technology shock and face the same factor input prices.

The second stage, were firms optimally choose price, can be setup as a profit maximization problem, taking mc_t as given. Assuming an interior solution, the problem can be written as:

$$V = \max_{p_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\frac{p_{i,t}}{P_{t+j}} - mc_{t+j} \right) y_{i,t+j} \right] \right), \tag{1.68}$$

subject to:

$$y_{i,t+j} = \left(\frac{p_{i,t}}{P_{t+j}} \right)^{-\left(\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}} \right)} Y_{t+j} \tag{1.69}$$

where Y_{t+j} is the demand for final goods in period $t+j$. N.B. there is no $p_{i,t+j}$ because prices are fixed at their value chose in period t.

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Rewriting the second-stage problem gives:

$$V = \max_{p_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\frac{p_{i,t}}{P_{t+j}} - mc_{t+j} \right) y_{i,t+j} \right] \right)$$

$$\implies V = \max_{p_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\frac{p_{i,t}}{P_{t+j}} - mc_{t+j} \right) \left(\frac{p_{i,t}}{P_{t+j}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} Y_{t+j} \right] \right)$$

$$\implies V = \max_{p_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\frac{p_{i,t}}{P_{t+j}} \right)^{\frac{-1}{\lambda_{p,t+j}}} - \left(\frac{p_{i,t}}{P_{t+j}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} mc_{t+j} \right] Y_{t+j} \right)$$

$$\implies V = \max_{p_{i,t}} E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\lambda_{p,t+j}}} - \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} mc_{t+j} \right] Y_{t+j} \right)$$

(1.70)

In the above, $\pi_t = \frac{P_t}{P_{t-1}}$.

The FOC for this problem (where the intermediate-goods producer is choosing $p_{i,t}$) is given by:

$$\frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left[\frac{-1}{\lambda_{p,t+j}} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right)^{\frac{-1}{\lambda_{p,t+j}}} \left(\frac{1}{p_{i,t}} \right) + \left(\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}} \right) \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} \frac{mc}{p} \right] \right)$$

$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{1}{p_{i,t}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} \left[\frac{-1}{\lambda_{p,t+j}} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right) + \left(\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}} \right) mc_{t+j} \right] Y_{t+j} \right) =$$

(1.71)

Rearranging:

$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{1}{p_{i,t}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} \left[\frac{1}{\lambda_{p,t+j}} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{p_{i,t}}{P_t} \right) \right] Y_{t+j} \right)$$

$$= E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \frac{\lambda_{t+j}}{\lambda_t} \left(\prod_{s=1}^{j} \frac{1}{\pi_{t+s}} \frac{1}{p_{i,t}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} \left[\left(\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}} \right) mc_{t+j} \right] Y_{t+j} \right)$$

(1.72)

Or (using notation without gross inflation):

$$\implies \frac{\partial V}{\partial p_{i,t}} : E_t \left(\sum_{j=0}^{\infty} \beta^j \theta^j \left(\frac{p_{i,t}}{P_{t+j}} \right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}} \frac{Y_{t+j} \lambda_{t+j}}{\lambda_{p,t+j} \lambda_t} \left[\frac{(1+\lambda_{p,t+j}) mc_{t+j} P_{t+j} - p_{i,t}}{p_{i,t} P_{t+j}} \right] \right) = 0 \quad \forall i, t$$

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Which implies:

$$E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}\lambda_t}\left[\frac{(1+\lambda_{p,t+j})mc_{t+j}P_{t+j}}{p_{i,t}P_{t+j}}\right]\right)=$$
$$E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}\lambda_t}\left[\frac{p_{i,t}}{p_{i,t}P_{t+j}}\right]\right) \tag{1.74}$$

Which can be written as:

$$p^*_{i,t}=\frac{E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}}\left[(1+\lambda_{p,t+j})mc_{t+j}\right]\right)}{E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{p_{i,t}}{P_{t+j}}\right)^{-\frac{1+\lambda_{p,t+j}}{\lambda_{p,t+j}}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}P_{t+j}}\right)} \tag{1.75}$$

Assume a symmetric equilibrium ($\implies p^*_{i,t}=p^*_t,\ \forall i$).

Calvo pricing (with zero indexation) implies $P_t^{\frac{-1}{\lambda_{p,t}}}=\theta P_{t-1}^{\frac{-1}{\lambda_{p,t}}}+(1-\theta)p_t^{*\frac{-1}{\lambda_{p,t}}}$ (you can get this from the definition of the Calvo model and the aggregate price index from the final goods producers problem). One can divide this equation through by $P_t^{\frac{-1}{\lambda_{p,t}}}$ and get $1=\theta\left(\frac{P_{t-1}}{P_t}\right)^{\frac{-1}{\lambda_{p,t}}}+(1-\theta)\left(\frac{p_t^*}{P_t}\right)^{\frac{-1}{\lambda_{p,t}}}$. Using the gross inflation variable, π_t defined above, we can write: $\left(\frac{p_t^*}{P_t}\right)=\left(\frac{1-\theta\pi_t^{\frac{1}{\lambda_{p,t}}}}{1-\theta}\right)^{-\lambda_{p,t}}$.

Using this, we can write the equation for p_t^* as:

Which can be written as:

$$p_t^*=\frac{E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{1-\theta\pi_{t+j}^{\frac{1}{\lambda_{p,t+j}}}}{1-\theta}\right)^{1+\lambda_{p,t+j}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}}\left[(1+\lambda_{p,t+j})mc_{t+j}\right]\right)}{E_t\left(\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{1-\theta\pi_{t+j}^{\frac{1}{\lambda_{p,t+j}}}}{1-\theta}\right)^{1+\lambda_{p,t+j}}\frac{Y_{t+j}\lambda_{t+j}}{\lambda_{p,t+j}P_{t+j}}\right)} \tag{1.76}$$

The way most people seem to solve this is to log-linearize the equation. This turns the infinite sum into a difference equation. In addition, the analysis is greatly simplified by assuming that gross inflation (π) equals 1 in the steady state. Assuming that gross inflation equals 1 in the SS, I am getting the following equation for the price chosen by firms who can change price in period t:

$$p_t^*=(1+\lambda_p)\bar{mc}(1-\beta\theta)E_t\left[\sum_{j=0}^{\infty}\beta^j\theta^j\left[\frac{mc_{t+j}-\bar{mc}}{\bar{mc}}+\frac{P_{t+j}-\bar{P}}{\bar{P}}\right]+\frac{\lambda_p}{(1+\lambda_p)}\sum_{j=0}^{\infty}\beta^j\theta^j\left(\frac{\lambda_{p,t+j}-\lambda_p}{\lambda_p}\right)\right] \tag{1.77}$$

Nominal profits from the intermediate-goods producers, $d_{i,t}$ are distributed as dividends to households.

1.3 Government/Fiscal Authority

The government is described by a budget constraint and rules fiscal policy rules. Some, like Zubairy (2010) and Traum and Yang (2010), have equations like Taylor rules for fiscal policy, the parameters of which they estimate using historical data. I don't see the need for this. I'm thinking we just have a projection of tax law into the future (though we can run into problems if that is not sustainable- i.e. can't have increasing debt forever). We perturb policies from this for to see the effects (e.g. lower the consumption tax rate by five percentage points for eight quarters, after which is reverts to it's earlier level). But I can consider other alternatives if needed.

1.4 Fiscal authority equations

The government budget constraint represents 1 equation with two unknowns $\{G_t, X_t\}$. It has 8 tax parameters $\{\tau_t^c, \tau_t^l, \tau_t^k, \tau_t^d, \tau_t^{ic}, \delta_t^\tau, e_t^\tau, \tau_t^i\}$, 1 spending parameter $\{\gamma_{x,t}\}$, and 13 other endogenous aggregate variables $\{C_t, P_t, L_t, B_t, B_{t-1}, w_t, r_t^k, r_t, v_t, K_{t-1}, D_t, I_t, K_{t-1}^\tau\}$.

$$
\begin{aligned}
\tau^c C_t + \frac{\tau_t^l L_t w_t}{P_t} + \frac{B_t}{P_t} + \tau_t^k r_t^k v_t K_{t-1} + \frac{\tau_t^d D_t}{P_t} = ... \\
G_t + X_t + \tau_t^{ic} I_t + \tau_t^k \delta_t^\tau K_{t-1}^\tau + \tau_t^k e_t^\tau I_t + \frac{B_{t-1}\left[1 + r_t(1 - \tau_t^i)\right]}{P_t}
\end{aligned}
\tag{1.78}
$$

Because G_t and X_t are not independently identified, we must make an assumption about how any changes in tax revenue affect spending on public goods versus spending on transfers. We assume here that spending on public goods and on transfers are each a fixed portion of total revenues. If we solve (1.78) for $G_t + X_t$, the other side of the equation represents total revenues. We will assume that $\gamma_{x,t} \in (0,1)$ percent of revenues are spent on transfers and $(1 - \gamma_{x,t})$ percent is spent on public goods.

$$
\begin{aligned}
G_t = (1 - \gamma_{x,t}) & \left[\tau^c C_t + \frac{\tau_t^l L_t w_t}{P_t} + \frac{B_t}{P_t} + \tau_t^k r_t^k v_t K_{t-1} + \frac{\tau_t^d D_t}{P_t} ... \right. \\
& \left. - \tau_t^{ic} I_t - \tau_t^k \delta_t^\tau K_{t-1}^\tau - \tau_t^k e_t^\tau I_t - \frac{B_{t-1}\left[1 + r_t(1 - \tau_t^i)\right]}{P_t} \right]
\end{aligned}
\tag{1.79}
$$

$$
\begin{aligned}
X_t = \gamma_{x,t} & \left[\tau^c C_t + \frac{\tau_t^l L_t w_t}{P_t} + \frac{B_t}{P_t} + \tau_t^k r_t^k v_t K_{t-1} + \frac{\tau_t^d D_t}{P_t} ... \right. \\
& \left. - \tau_t^{ic} I_t - \tau_t^k \delta_t^\tau K_{t-1}^\tau - \tau_t^k e_t^\tau I_t - \frac{B_{t-1}\left[1 + r_t(1 - \tau_t^i)\right]}{P_t} \right]
\end{aligned}
\tag{1.80}
$$

Now we have two equations and two unknowns $\{G_t, X_t\}$.

1.5 Monetary Authority

The monetary authority follows a Taylor rule, targeting the nominal, after-tax interest rate (I'm taking this from Christiano et al. (2010), but feel free to modify to whatever- though I think it's important that we have the zero bound condition):

$$r_{t+1} = max(M_{t+1}, 0) \tag{1.81}$$

Where M_{t+1} (better notation??) is given by:

$$(1 - \tau_t^i)M_{t+1} = \frac{1}{\beta}(1 + \pi_t)^{\phi_1}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_2}\left(\frac{1 + r_t}{1 + \bar{r}}\right)^{\rho_r} - 1 \tag{1.82}$$

$$r_{t+1} = max\left\{\frac{\frac{1}{\beta}(1 + \pi_t)^{\phi_1}\left(\frac{Y_t}{\bar{Y}}\right)^{\phi_2}\left(\frac{1 + r_t}{1 + \bar{r}}\right)^{\rho_r} - 1}{(1 - \tau_t^i}, 0\right\}$$

Where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ and $\phi_1 > 1$, $\phi_2 \in (0, 1)$, and $\rho_r \in (0, 1)$.

1.6 Equilibrium

Standard definition: prices, wages, and interest rates to clear markets (labor, goods, bond)
Market Clearing:

- Household demand for bonds equals gov't supply

- Household labor supply equals intermediate producer labor demand

- Household capital supply equals intermediate producer capital demand

- Household and gov't consumption equals supply from the final goods producer

1.7 Solving the model

Probably best to use some linearization method. We used the AIM algorithm (Anderson and Moore (1985)) in Willis' class and I still have the code for that. Alternatively, we used Christiano's method of undetermined coefficients (Christiano (2002)) in Corbae's class. Chris Sim's has the code for his method (Sims (2002)) available on his website. Uhlig (1998) does a good job summarizing these methods.

1.7.1 Characterizing equations

Altogether, this model has 36 endogenous variables and 28 exogenous parameters. The 36 endogenous variables are characterized by the 37 equations listed in the sections below. By Walras' Law, one of the market clearing conditions is redundant. In the computation, we

discard the resource constraint (1.93).

Endogenous variables: $\left\{c_t, \lambda_t, l_t, i_t, b_t, v_t, k_t, k_t^\tau, \varepsilon_t^b\right\}$

$\left\{Y_t, y_{i,t}^d, P_t, \lambda_{p,t}\right\}$

$\left\{y_{i,t}, \tilde{k}_{i,t}, d_{i,t}, r_t^k, w_t, p_{i,t}, z_t\right\}$

$\{G_t, X_t\}$

$\{r_t\}$

$\{p_t, d_t, x_t, g_t, k_{i,t}, l_{i,t}, C_t, L_t, B_t, K_t, K_t^\tau, D_t, I_t\}$

Exogenous parameters: $\left\{\tau_t^c, \tau_t^i, \tau_t^l, \tau_t^k, \tau_t^{ic}, \tau_t^d, \delta_t^\tau, e_t^\tau\right\}$

$\{\beta, \zeta, \sigma\}$

$\{\delta_0, \delta_1, \delta_2, \gamma\}$

$\{\rho_\varepsilon, \mu_\varepsilon, \sigma_\varepsilon\}$

$\{\rho_\lambda, \mu_\lambda, \sigma_\lambda\}$

$\{\alpha, \theta\}$

$\{\gamma_{x,t}\}$

$\{\beta_r, \phi_1, \phi_2, \rho_r\}$

There are 19 unknown variables (not counting those from the intermediate goods producers and the shocks, $\varepsilon_t^b, \lambda_{p,t}$- for which we will have equations that we haven't specified yet) : $i, k, b, c, k^\tau, \lambda, g, x, d, P, w, z, \varepsilon^b, r, r^k, v, l, Y, M$. These breakdown into the following groups:

- 4 choice variables (Household): i, v, l, c

- 4 prices: P, w, r, r^k

- 2 choice variables (gov't): g, x, reallly $2 + n$ where n is the number tax instruments

- 1 choice variable (monetary authority): M

- 2 exogenous state: z, ε

- 5 endogenous state variable: k, b, d, k^τ, Y

Solve for these unknowns with the following equations:

- 5 FOCs, household $\implies i, v, l, c, b$

- 3 from FOCs, intermediate goods producers $\implies r^k, w, P$ (These FOC's give demand for effective capital and labor. So together with the HH supply of these factors we can use our market clearing conditions to find the eq'm factor prices.)

- 1 from the profit function of intermediate goods producers $\implies d$

- 1 from FOCs, final goods producer $\implies Y$

- 1 from the zero profit condition for the final goods producer $\implies P$ (This together with the FOC's from the intermediate goods producers gives you the aggregate price index.)

- 1 Gov't budget constraint ($+n$ fiscal policy rules) $\implies X$, ($+n$ tax rates)

- 1 Resource constraint $\implies G$

- 2 laws of motion $\implies k^\tau, k$

- 3 exog shock process $\implies z, \varepsilon^b, \lambda_p$

- 1 Taylor rule for monetary authority $\implies r, M$ (There is the Taylor Rule plus the fact that $r = max(M, 0)$ that determines both these.)

So we have 19 unknowns and 19 equations. We should be able to plug this into Dynare and solve the model.

1.7.2   Market clearing and resource constraint

The following 14 market clearing conditions close the model. The last equation (1.93) is a resource constraint, and can be thought of as goods market clearing. By Walras' Law, one of the following conditions is redundant.

$$c_t = C_t \tag{1.83}$$

$$\int_0^1 l_{i,t} di = l_t = L_t \tag{1.84}$$

$$i_t = I_t \tag{1.85}$$

$$\int_0^1 \tilde{k}_{i,t} di = v_t k_t = V_t K_t \tag{1.86}$$

$$k_t^\tau = K_t^\tau \tag{1.87}$$

$$\left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di \right) = Y_t^d = Y_t \tag{1.88}$$

$$\left(\int_0^1 d_{i,t} \right) = d_t = D_t \tag{1.89}$$

$$\left(\int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \right)^{-\lambda_{p,t}} = P_t \tag{1.90}$$

$$g_t = G_t \tag{1.91}$$

$$x_t = X_t \tag{1.92}$$

$$Y_t = C_t + I_t + G_t \tag{1.93}$$

1.8   Calibration/Parameterization

Many of these DSGE papers estimate the model (e.g., Traum and Yang (2010), Zubairy (2010), Smets and Wouters (2003)), but I think all we need to do is calibrate (or even just parameterize at Christiano et al. (2010) do. Not really sure why need to estimate yet another time if other have done it. Idea is to pick parameters close to other models finding large spending multipliers and show that similar assumptions/parameters also show that certain tax policies also have really large multipliers. Likely want to calibrate to European economy because want a place with a consumption tax. UK is probably a good bet since they have their own central bank and a VAT of about 20%.

- δ_0= annual depreciation rate ($\sim$10%), equals the aggregate SS investment rate ($\frac{\bar{I}}{K}$).

- $\delta_1 =$

- $\delta_2 =$

- $\delta^\tau =$ Not sure how set- varies by asset type and front loaded, but probably just set to 12-14%- something higher than econ depreciation

- $\tau^l =$ See Jones (2002) or just do statutory rate for median household or use tax calculator to get marginal rate for median household

- $\tau^i =$ assume = labor income tax

- $\tau^d =$ See Jones (2002) or just do statutory rate for median household or use tax calculator to get marginal rate for median household

- $\tau^k =$ See Jones (2002)

- $\tau^c =$ See Jones (2002)

- $\tau^{ic} = 0$, most years for most investments this is zero

- $e^\tau = 0$, in reality expensing for tax > 0 for many small businesses at most times, but we can set to zero

- $\gamma =$

- $\zeta =$ I think this is the Frisch elasticity of labor. Christiano et al. (2010) set to 0.29.

- $\sigma =$ coeff of relative risk aversion. Often set to 2.

- $\chi^g =$

- $\sigma_g =$

- $\sigma_\varepsilon =$

- $\rho_\varepsilon =$

- $\sigma_\lambda =$

- $\lambda_p =$ mean markup $= \frac{1+\lambda_p}{\lambda_p}$, Basu and Fernald (1995) give U.S. evidence of avg markup of 10-14% which implies $\lambda_p \sim 8$.

- β = set to match (real?) after-tax interest rate (given tax on interest income)

- α = capital share of output (30-35%)

- σ_z = Estimate log of production function using aggregate data (typical business cycle accounting) to get residual. Fit the residual to AR(1) to get σ_z and ρ_z.

- ρ_z = Estimate log of production function using aggregate data (typical business cycle accounting) to get residual. Fit the residual to AR(1) to get σ_z and ρ_z.

- θ = fraction of firms changing price

- ϕ_1 = estimate a log-log specification of the Taylor rule

- ϕ_2 = estimate a log-log specification of the Taylor rule

- ρ_r = estimate a log-log specification of the Taylor rule

- $\bar{L}$ = fraction of hours worked ($0.3 \sim \frac{8}{24}$, 0.5 corresponds to Frisch elasticity = 1??)

- $\frac{\bar{G}}{Y}$ = historical average

- $\frac{\bar{B}}{Y}$ = historial average

- $\frac{\bar{X}}{Y}$ = historical average

TABLE: Parameritization

2 Short Run Multipliers

1. FIGURES: impulse response functions for tax cuts (capital, income, consumption, investment) and gov't spending increase (% changes in output, emp, cons, inv, int rates, inflation) to policy change

2. TABLE: table with size of multipliers

3 Long Run Multipliers

FIGURE: graph of multiplier from each of fiscal policy measures over time

4 Sensitivity to Government Financing

FIGURE: graph of multiplier over time given how gov't financing temporary tax cuts (with higher levels of debt going forward or with paying back by increasing taxes). Maybe 3D graph with multiplier, years from cut, and year to pay back debt as axes.

5 Sensitivity to Monetary Policy

Talk about multipliers at and away from the zero-bound. What happens if hold interest rates constant?

6 Dead-weight Loss and Fiscal Policy

1. TABLE: table of DWL for each of policy responses

2. FIGURE: graph of DWL over time for policy response

Do some revenue neutral changes in tax policy and calculate welfare....

7 Sensitivity to Parameters

How do results change with change to key parameters...

8 Discussion

Note that hard to know preferences for gov't spending, but imagine that people know better what want, cutting taxes may be politically easier than increasing spending. Note other weaknesses of the modeling assumptions.

9 Conclusion

10 Questions:

1. Do those with models of the multiplier usually have transfers to households or gov't actually buying stuff?

2. Do I want to say anything about optimal consumption tax being non-zero (like want non-zero inflation) because can more easily do tax cut to respond to recession than could do negative tax rate (spending)?

3. Did Europe (or even US states) have any changes to consumption taxes in response to the recession? *A: I don't think so. In fact, I believe some states cancelled their sales tax holidays due to budget shortfalls.*

4. Do we want to have rules for fiscal policy as in Zubairy (2010)? Again, not sure why we need it if not trying to match data. Just need to estimate SS levels and propose levels that will revert to after aggregate shock.

11 Does anyone do this?

1. Kumhof et al. (2010) shows impulse response functions to a consumption tax cut. This paper seems to find that consumption taxes do well compared to other taxes and transfers, but don't do as well as gov't consumption and gov't investment

2. Leeper, Plante and Traum (2010) have consumption taxes, but don't allow them to vary with gov't debt

3. Zubairy (2010) doesn't not have consumption taxes, only capital and labor income taxes. Says few even have distortionary taxes in models of multipliers.

4. Traum and Yang (2010) allow for consumption taxes and allow them to vary. Not answering the same question as I want, but good to look at what they say about consumption taxes. In particular, how with cons taxes the consumer and producer price index will vary and what crowding out happens with consumption tax changes.

5. Coenen and Straub (2005) have consumption taxes, but don't use them as a policy instrument. They don't say anything interesting about consumption taxes.

6. Christiano et al. (2010) don't have consumption taxes and only focus on spending multipliers. I don't even think they have taxes. Fiscal and monetary policy rules look simple though- might want to take these.

12   Links

- http://www.econbrowser.com/archives/2010/03/policy_in_dsges.html

- Kumhof et al. (2010) paper: http://www.imf.org/external/pubs/ft/wp/2010/wp1073.pdf

- http://delong.typepad.com/sdj/2009/07/cracking-chistiano-eichenbaum-and-rebelos-big-mu.html

- http://www.econbrowser.com/archives/multipliers/index.html

- Traum and Yang (2010) paper: http://www.nber.org/papers/w15160

13   MORE QUESTIONS:

1. Why do Christiano et al. (2010) have a subsidy to correct for monopoly of int goods producers? I don't see others with this.

2. What is capital tax equivalent to? See what Zubairy (2010) estimates from, but it's like a corp income tax, cap gains tax, div tax rolled into one.

3. Do Multiplier of: gov't spending (baseline to compare), cut in: cap tax, labor tax, cons tax, invest tax credit/bonus deprec/accelerated expensing

4. Can I do bonus deprec with my model- or do I need to keep track of vintage/basis since not partial expensing?

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TECHNICAL APPENDIX

T-1 Derivation of monopolistic competition intermediate goods demand and aggregate price Equations

The profit maximization approach to solving for each intermediate good demand equation $y_{i,t}$ and the aggregate price equation P_t is the following.

$$\max_{y_{i,t}} P_t Y_t^d - \int_0^1 p_{i,t} y_{i,t} di \tag{T.1.1}$$

The FOC's are:

$$p_t(1+\lambda_{p,t}) \left(\int_0^1 y_{i,t}^{\left(\frac{1}{1+\lambda_{p,t}}\right)} di \right)^{\lambda_{p,t}} \left(\frac{1}{1+\lambda_{p,t}} \right) y_{i,t}^{\frac{1}{1+\lambda_{p,t}}-1} - p_{i,t} = 0, \forall i \tag{T.1.2}$$

Dividing the FOCs for intermediate inputs i and $j \implies$

$$\frac{p_{i,t}}{p_{j,t}} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{1}{1+\lambda_{p,t}}-1} = \left(\frac{y_{i,t}}{y_{j,t}} \right)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \tag{T.1.3}$$

Rearranging:

$$\begin{aligned}
\implies p_{i,t} &= \left(\frac{y_{j,t}}{y_{i,t}} \right)^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t} \\
\implies p_{i,t} &= y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} p_{j,t} y_{i,t}^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}} \\
\implies p_{i,t} y_{i,t} &= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} y_{i,t}^{\frac{1}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.4}$$

Then integrate this condition to yield:

$$\begin{aligned}
\int_0^1 p_{i,t} y_{i,t} di &= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} \int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di \\
&= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.5}$$

The zero profit condition implies $P_t Y_t = \int_0^1 p_{i,t} y_{i,t} di$. Plugging this into the above and we find:

$$\begin{aligned}
P_t Y_t &= p_{j,t} y_{j,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \\
\implies P_t Y_t &= p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} \\
\implies P_t &= p_{j,t} y_{j,t}^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} (Y_t)^{\frac{-\lambda_{p,t}}{1+\lambda_{p,t}}}
\end{aligned} \tag{T.1.6}$$

Which implies the demand function (just rearranging terms):

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t, \forall i \tag{T.1.7}$$

To find P_t use the zero profit condition:

$$P_t Y_t = \int_0^1 p_{i,t} y_{i,t}^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di$$

$$\implies P_t Y_t = \int_0^1 p_{i,t} \left(\left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t\right)^{\left(\frac{\lambda_{p,t}}{1+\lambda_{p,t}}\right)} (Y_t)^{\frac{1}{1+\lambda_{p,t}}} di$$

$$\implies P_t Y_t = \int_0^1 p_{i,t} \left(\frac{p_{i,t}}{P_t}\right)^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di$$

$$\implies P_t Y_t = \int_0^1 p_{i,t} p_{i,t}^{\frac{-(1+\lambda_{p,t})}{\lambda_{p,t}}} P_t^{\frac{(1+\lambda_{p,t})}{\lambda_{p,t}}} Y_t di \tag{T.1.8}$$

$$\implies P_t Y_t = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, Y_t P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

$$\implies P_t = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di \, P_t^{\frac{1+\lambda_{p,t}}{\lambda_{p,t}}}$$

$$\implies P_t^{\frac{-1}{\lambda_{p,t}}} = \int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di$$

$$\implies P_t = \left(\int_0^1 p_{i,t}^{\frac{-1}{\lambda_{p,t}}} di\right)^{-\lambda_{p,t}}$$

The cost minimization approach to solving for each intermediate good demand equation $y_{i,t}$ and the aggregate price equation P_t is the following.

$$\min_{y_{i,t}} \int_0^1 p_{i,t} y_{i,t} di \quad \text{s.t.} \quad Y_t \leq \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}} \tag{T.1.9}$$

The Lagrangian for this minimization problem is the following,

$$\mathcal{L} = \int_0^1 p_{i,t} y_{i,t} di + \lambda_y \left[Y_t - \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}\right] \tag{T.1.10}$$

in which the multiplier λ_y has the interpretation of being the marginal cost of an extra unit of aggregated output. That is, λ_y is the price of aggregate output P_t.

$$\mathcal{L} = \int_0^1 p_{i,t} y_{i,t} di + P_t \left[Y_t - \left(\int_0^1 y_{i,t}^{\frac{1}{1+\lambda_{p,t}}} di\right)^{1+\lambda_{p,t}}\right] \tag{T.1.11}$$

We need to finish this....

T-2 Derivation of real marginal costs

Dividing the period profits equation (**??**) by aggregate prices P_t gives the following real total costs function.

$$tc_t = \frac{r_t^k}{P_t}\tilde{k}_{i,t} + \frac{w_t}{P_t}l_{i,t} \tag{T.2.1}$$

Substituting in the expression for $\tilde{k}_{i,t}$ from the capital-labor ratio equation (**??**) gives real total costs in terms of α, w_t, P_t, and $l_{i,t}$.

$$tc_t = \frac{1}{1-\alpha}\left(\frac{w_t}{P_t}\right)l_{i,t} \tag{T.2.2}$$

Because the intermediate goods production exhibits constant returns to scale, we can find the real marginal cost mc_t by finding the amount of labor necessary to produce 1 unit of output, and substituting that value into the real total costs equation (T.2.2).

$$z_t\tilde{k}_{i,t}^{\alpha}l_{i,t}^{1-\alpha} = 1 \tag{T.2.3}$$

Next, substitute in the equation for capital $\tilde{k}_{i,t}$ in terms of labor from the capital-labor ratio equation (**??**).

$$z_t\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]l_{i,t}\right)^{\alpha}l_{i,t}^{1-\alpha} = 1 \quad \Rightarrow \quad l_{i,t} = \frac{1}{z_t}\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]\right)^{-\alpha} \tag{T.2.4}$$

Lastly, substitute the amount of labor necessary to produce one unit of output from (T.2.4) into the expression for real total costs (T.2.2) to get real marginal costs.

$$mc_t = \frac{1}{1-\alpha}\left(\frac{w_t}{P_t}\right)\frac{1}{z_t}\left(\frac{\alpha}{1-\alpha}\left[\frac{w_t}{r_t^k}\right]\right)^{-\alpha} = \frac{w_t^{1-\alpha}(r_t^k)^{\alpha}}{z_tP_t\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{T.2.5}$$