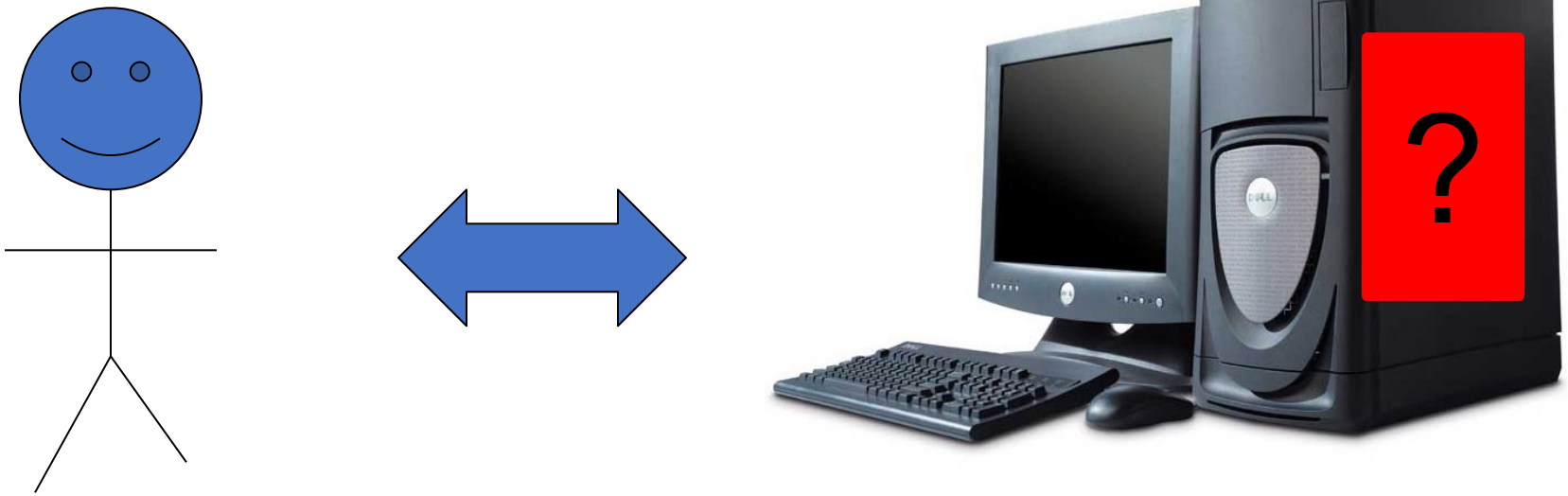
Number Representation

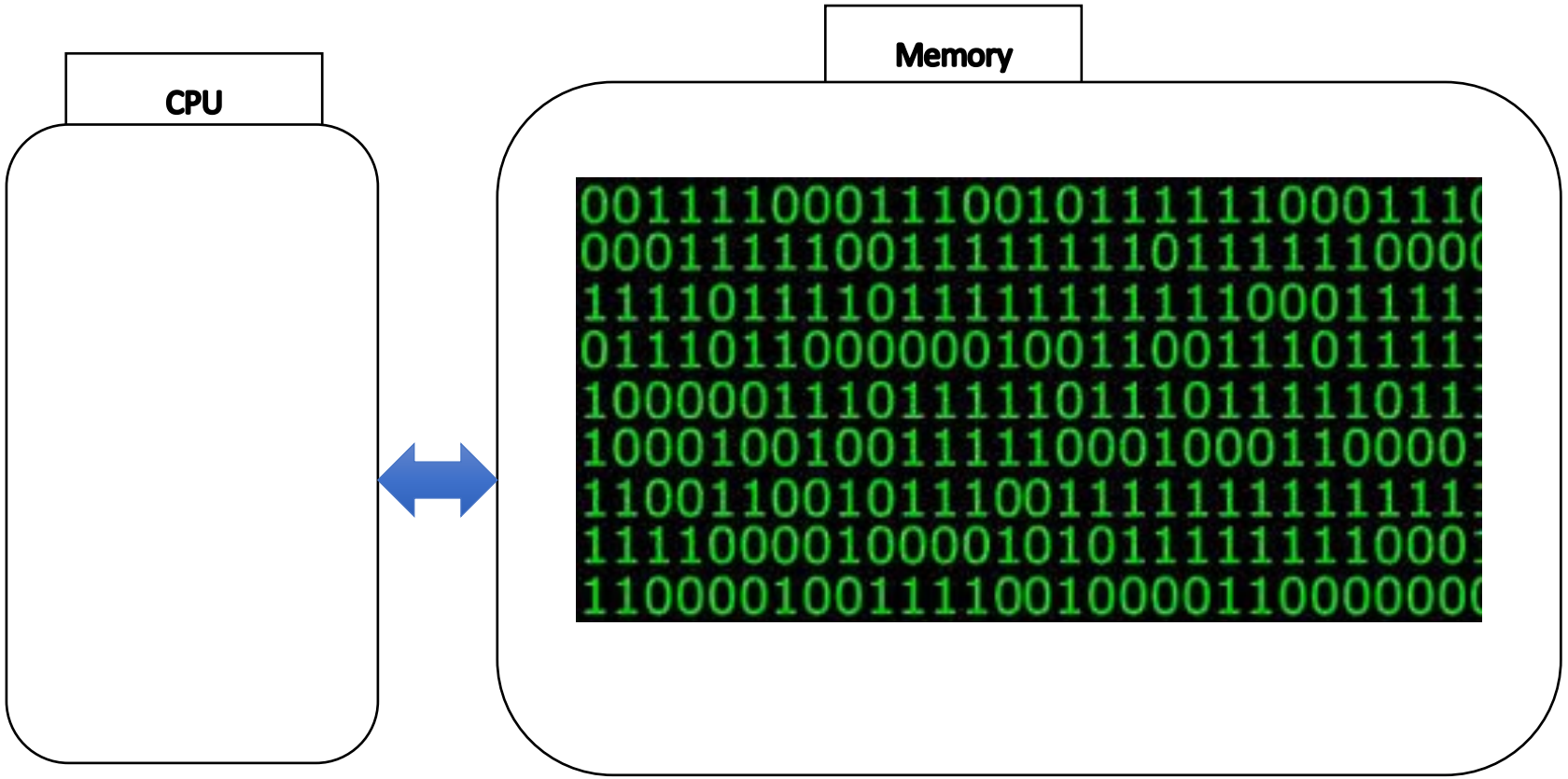
CSC111 – Spring 2022 – Joe Krysl

Question

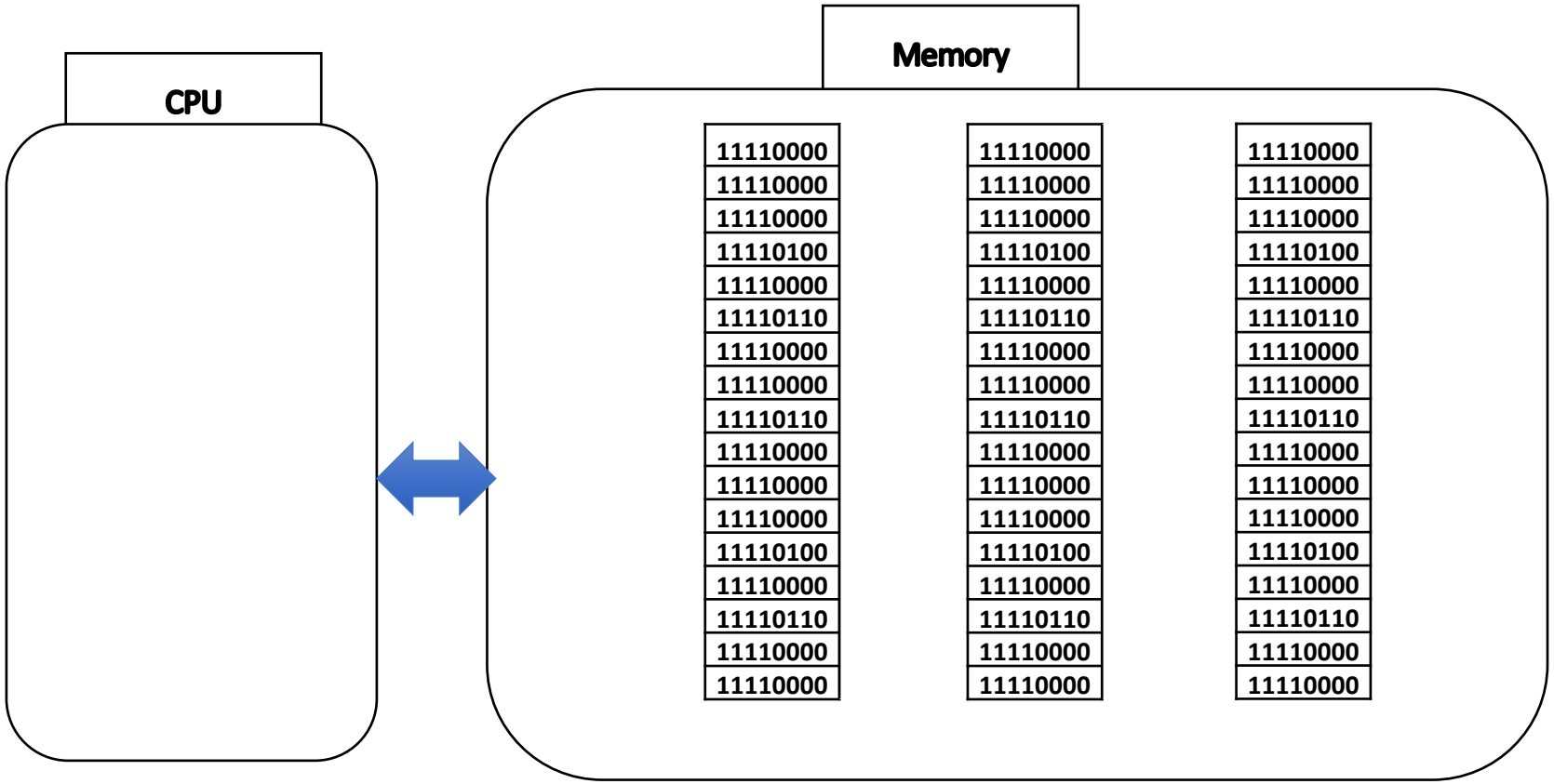
- What computers have you interacted with between the time you woke up this morning and now?



CPU – executes instructions
Memory – holds data
- instructions to be executed
- data used by instructions

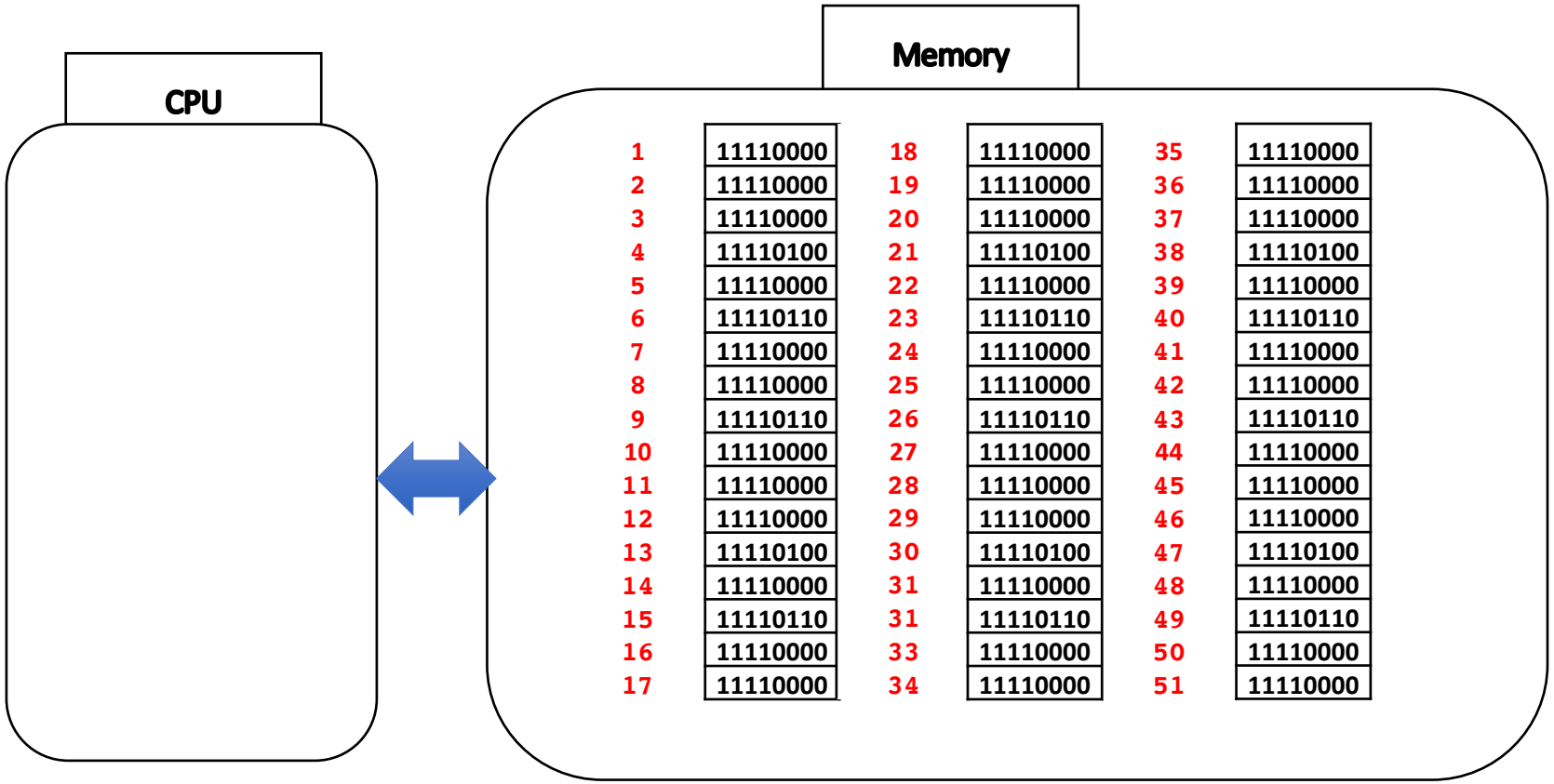


Memory is just a large storage space holding bits
Where a bit can only be a 0 or 1



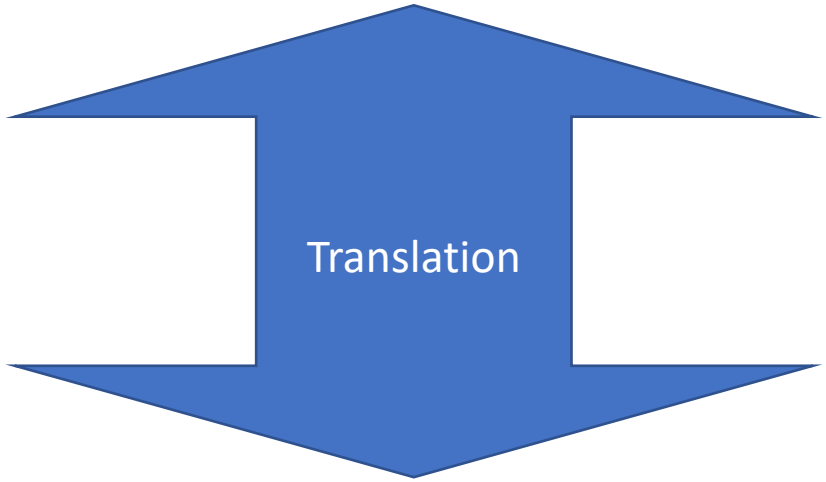
Bits are organized into chunks of 8 bits = 1 byte

These bytes can be:
-instructions in Machine Language that the CPU will load and execute
-data that the instructions will require (input) or have generated (output)



Each byte is stored in a memory space each with a unique **address**

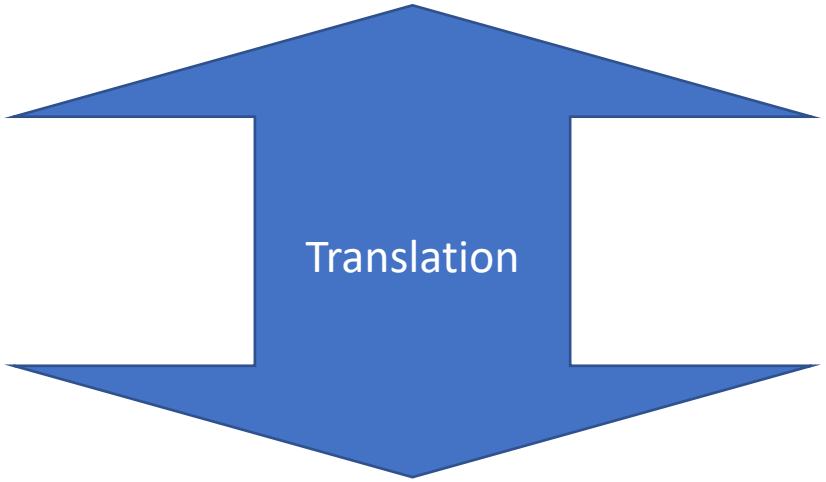
Hello



Bonjour

Both words have the same meaning,
they are just represented in 2 different ways

00001101



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Both numbers have the same meaning,
they are just represented in 2 different ways

Recall, Base 10 number system

Each digit in a base 10 number can be one of the following 10 numbers: 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9

Example. Base 10 number 9789 has a 9, 7, 8 and 9

Based on the positions of these digits,
9789 can be broken down as:

$= 9*1000 + 7*100 + 8*10 + 9*1$

$= 9*10^3 + 7*10^2 + 8*10^1 + 9*10^0$

$= 9000 + 700 + 80 + 9$

$= 9789$

We can leverage this unit breakdown to convert base-2 numbers to base-10 numbers

Base 2 (binary) –> Base 10 (decimal)

Each digit in a base-2 number can be one of the following 2 numbers: 0 or 1

Example. Base 2 number 1101 has an 1, 0, 1 and 1

Based on the positions of these digits,
1011 can be broken down as:

$= 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

$= 1*8 + 1*4 + 0*2 + 1*1$

$= 8 + 4 + 0 + 1$

$= 13$

1011_2 –translated to base 10—> 13_{10}

Base 10 (decimal) –> Base 2 (binary)

Example. 14_{10}

To translate base 10 to base 2:

Repeatedly divide by 2,
recording the remainders.

Write the remainders
bottom to top, left to right.

Base 10 (decimal) –> Base 2 (binary)

To translate base 10 to base 2:

Repeatedly divide by 2, recording the remainders.

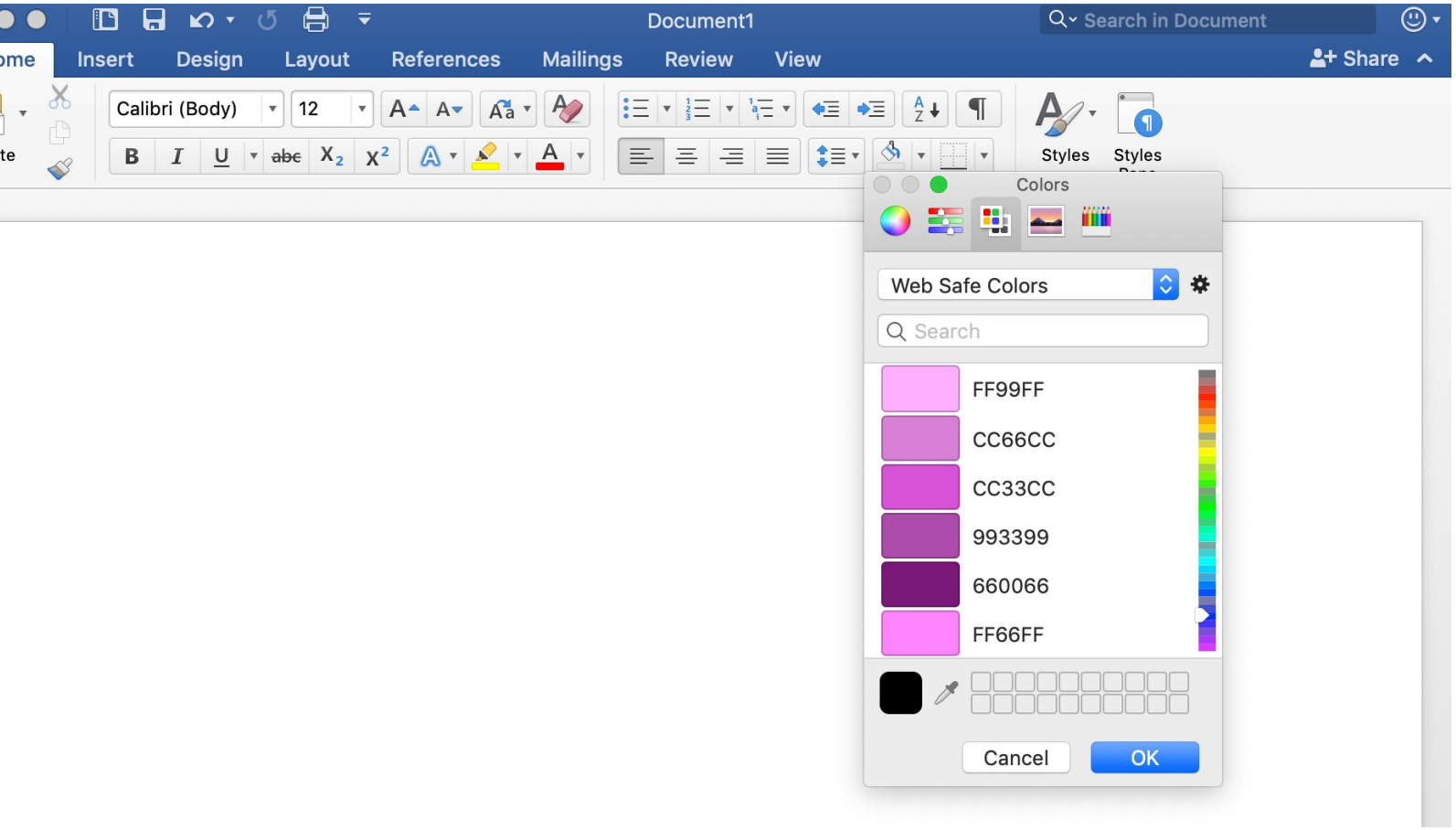
Write the remainders bottom to top, left to right.

Example. 14_{10}

		Remainder
2	14	
2	7	0
2	3	1
2	1	1
	0	1

14_{10} –translated to base 2—> 1110_2

Colours often represented with hexadecimal number system



Digits can be:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E or F

What base is this number system?

Base 16 (hexadecimal) –> Base 10 (decimal)

Example. Base 16 number A9C7 has an A, 9, C and 7

Based on the positions of these digits,
A9C7 can be broken down as:

$= A*16^3 + 9*16^2 + C*16^1 + 7*16^0$

$= A*4096 + 9*256 + C*16 + 7*1$

$= 10*4096 + 9*256 + 12*16 + 7*1$

$= 40960 + 2304 + 192 + 7$

$= 43463$

$A9C7_{16}$ –translated to base 10—> 43463_{10}

Base 10 (decimal) –> Base 16 (hexadecimal)

Example. 108_{10}

To translate base 10 to base 16:

Repeatedly divide by 16,
recording the remainders.

Write the remainders
bottom to top, left to right.

Base 16 (hexadecimal) –> Base 2 (binary)

To translate base 10 to base 16:

Repeatedly divide by 16, recording the remainders.

Write the remainders bottom to top, left to right.

Example. 108_{10}

		Remainder
16	108	
16	6	12 = C
	0	6

108_{10} –translated to base 16—> $6C_{16}$

Base 16 (hexadecimal) –> Base 2 (binary)

Example. $6C_{16}$

To translate base 16 to base 2:

Each Hex digit represents four bits and is equal to a power of 2.

Determine which powers of two sum to the hex digit.

Write below digit

Base 16 (hexadecimal) –> Base 2 (binary)

To translate base 16 to base 2:

Each Hex digit represents four bits and is equal to a power of 2.

Determine which powers of two sum to the hex digit.

Write below digit

Example. $6C_{16}$

6	C
0110	1100

01101000

$6C_{16}$ –translated to base 2—> 01101000_2

Base 2 (binary) –> Base 16 (hexadecimal)

To translate base 2 to base 16:

Each Hex digit is represented by four bits.

Sum the powers of two each bit represents.

Write hex below digit

Example. 11000011_2

1100 0011

C 3

11000011_2 –translated to base 16—> $C3_{16}$