CSC 226 SPRING 2022
ALGORITHMS AND DATA STRUCTURES II
PRACTICE MIDTERM
UNIVERSITY OF VICTORIA

1. For the algorithm LinearSelect, derive the recurrence equation for the running times when it uses groups of size 3, 5 and 7. Among these three sizes, which size will you choose in your implementation? Why?

2. Solve the following recurrence equations using the Master Theorem.

   $T(n) = 216T(n/6) + n^{2.5}$

3. Consider the experiment of tossing a standard six-sided dice two times.
   a) What is the size of sample space associated with this experiment?
   b) Let A be the event that the two dice show the same number. What is $Pr(A)$?
   c) Let B be the event that the number on the first die is larger than the number on the second. What is $Pr(B)$?
   d) Let C be the even that the sum of the dice is even. What is $Pr(C)$?

4. Consider the following randomized algorithm that searches for the index of a value x that is present in an unsorted array A consisting of n elements.

   Pick a random index i in A. If $A[i] = x$, then output i. Otherwise, continue the search by picking a new random index in A. We continue picking random indices in A until we find an index j such that $A[j] = x$ or until we have checked every element of A.

   What is the expected number of indices in A that will be picked by the algorithm before the index of x is found and it terminates?

5. Draw the hash table resulting from hashing the keys 10, 22, 31, 4, 15, 28, 17, 88, 59 using hash function $h(k) = k \bmod 11$, assuming collisions are handled by each of the following:
   a) Separate chaining.
   b) Linear probing.
   c) Quadratic probing up to the point where the method fails because no empty slot is found.
   d) Double hashing using the secondary hash function $h'(k) = 1 + (k \bmod 10)$.

6. Prove that in any forest G with n vertices and k connected components, the number of edges $m = n - k$.

7. Prove that every directed acyclic graph (DAG) has a vertex with in-degree 0 called the source vertex.