CSC 226 Summer 2023 Lab 2: Probability stuff!

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1 Introduction

In this lab session, we briefly explain

- Sample Space, Event, Probability
- Random Variable, Probability Distribution, Expected Value
- Conditional Probability, Independence

And then you will have time to work on the following 3 questions.
Remember that you should show your work in order to get the mark for the lab.
Please don't hesitate to ask questions!

Question 1. *Suppose you pick a number from the set $S = \{10, 11, 12, \ldots, 99\}$ uniformly at random. What is the probability that digits sum up to 7?*

Since we pick a number unformly at random, probability of all outcome are the same. There we can use the formula

$$P(E) = \frac{n(E)}{n(S)}$$

where $n(E)$ denotes the number of outcomes in which event E happens and $n(S)$ denotes the sample size.

$$n(E) = |E| = |\{16, 25, 34, 43, 52, 61, 70\}| = 7$$

$$n(S) = 99 - 10 + 1 = 90$$

Therefore,

$$P(E) = \frac{n(E)}{n(S)} = \frac{7}{90}$$

Question 2. *Denote X_1 and X_2 to be two fair dice.*

- *Calculate the value of $A = \mathbb{E}\Big[\max\{X_1, X_2\}\Big]$*
- *Calculate the value of $B = \max\Big\{\mathbb{E}[X_1], \mathbb{E}[X_2]\Big\}$*
- *Is A equal to B?*

For A use

$$\begin{aligned}
\mathbb{E}\Big[\max\{X_1, X_2\}\Big] &= \sum_{i=1}^{6} P(\max\{X_1, X_2\} = i) \cdot i \\
&= P(\max\{X_1, X_2\} = 1) \cdot 1 + P(\max\{X_1, X_2\} = 2) \cdot 2 + P(\max\{X_1, X_2\} = 3) \cdot 3 \\
&+ P(\max\{X_1, X_2\} = 4) \cdot 4 + P(\max\{X_1, X_2\} = 5) \cdot 5 + P(\max\{X_1, X_2\} = 6) \cdot 6 \\
&= \frac{1}{36} \cdot 1 + \frac{3}{36} \cdot 2 + \frac{5}{36} \cdot 3 + \frac{7}{36} \cdot 4 + \frac{9}{36} \cdot 5 + \frac{11}{36} \cdot 6 = \frac{161}{36} = 4.472
\end{aligned}$$

For B,

$$\max\left\{\mathbb{E}[X_1], \mathbb{E}[X_2]\right\} = \max\left\{3.5, 3.5\right\} = 3.5$$

A is not equal to B.

Question 3. *A fair dice is rolled, find the probability that the number is five given that it is odd.*

Let A be the event that the dice shows a five, and let B be the event that the dice shows an odd number. Then Using the conditional expectation formula, we get

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

We know $P(A \cap B) = \frac{1}{6}$. In order to see that Observe that if the dice is five, then implies it is odd. And the probability that it's five is 1/6. Moreover, $P(B) = \frac{n(odd)}{n(S)} = 3/6$.

Substituting in the formula, we get

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$$