CSC 226 SUMMER 2023
ALGORITHMS AND DATA STRUCTURES II
ASSIGNMENT 2
UNIVERSITY OF VICTORIA

1. A coin has 1/4 chance of having heads, 3/4 chance of having tails.
   a) What is the expected number of coin tosses needed to get a heads? Why?
   b) Suppose that such a coin is tossed n times. What is the probability that the number of heads equals the number of tails? Write the expression using $\binom{n}{x}$ notation.

2. Draw the hash table resulting from hashing the keys 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, and 5, using hash function $h(k) = (2k + 5) \bmod 11$, assuming collisions are handled by each of the following:
   a) Separate chaining.
   b) Linear probing.
   c) Quadratic probing up to the point where the method fails because no empty slot is found.
   d) Double hashing using the secondary hash function $h'(k) = 7 - (k \bmod 7)$.

3. Prove that any connected, undirected graph has a vertex whose removal, along with its incident edges, will not disconnect the graph by designing a DFS based algorithm to find such a vertex.

4. Let (x, y, w) denote the edge $\{x, y\}$ with weight w. The graph for the parts (a)and (b) is given by nodes $V = \{A, B, C, D, E\}$ and weighted edges
   $(A, B, 7), (A, C, 5), (A, D, 1), (B, C, 4), (B, D, 7), (B, E, 1), (C, E, 3), (D, E, 2)$.

   Show how to construct a minimum spanning tree using Prim's algorithm. List the edges and nodes in order of when the edge is added to the tree. The first node in T is A. Give the initial values of $D(v)$ for each node v. Each time a node is added to T, give the D values which have changed as a result.

5. (a) Show that you can rescale the edge weights of a graph G by adding a positive constant to all of them without affecting the MST.

   (b) Show that Prim's algorithm still work correctly if the graph G contains edges with negative edge weights.