1. (1 point) Calculate $\frac{(1+i)^3}{1+2i}$.

$$\frac{(1+i)(1+i)(1+i)}{(1+2i)} = \frac{(1+2i-1)\,1+i}{1+2i}$$

(A) $0.4 + 1.2i$ (B) $0.4 - 1.2i$ (C) $-0.4 + 1.2i$ (D) $-0.4 - 1.2i$

(E) $2 + 6i$ (F) $2 - 6i$ (G) $-6 - 2i$ (H) None of those

$$\frac{2i-2}{1+2i}\left(\frac{1-2i}{1-2i}\right) = \frac{2i-4i^2-2+4i}{1-(-4)} = \frac{6i+2}{5}$$

$$.4 + 1.2i$$

2. (1 point) Compute $\sum_{k=1}^{\infty} \frac{2^k + 3^k}{4^k}$, if the series converges.

(A) 3.2 (B) 3.4 (C) 3.6 (D) The series diverges

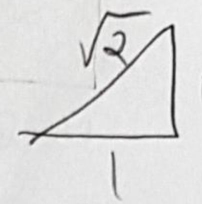
(E) 3.8 (F) 4.0 (G) 4.2 (H) None of those

$$\frac{2^k}{4^k} + \frac{3^k}{4^k} \qquad \frac{\frac{1}{2}}{\frac{1}{2}} + \frac{\frac{3}{4}}{\frac{1}{4}}$$

$$a = \frac{2}{4} \quad a_2 = \frac{3}{4} \qquad 1 + 3 = 4$$

$$r = \frac{2}{4} \quad r_2 = \left(\frac{3}{4}\right)$$

For next two questions, consider the equation $z^5 = \sqrt{2} + \sqrt{2}i$.



3. (1 point) Find the radius r for z and the number n of distinct roots for z.

(A) $r = 1$ and $n = 5$ (B) $r = \sqrt[5]{2}$ and $n = 5$ (C) $r = 1$ and $n = 2$

(D) $r = \sqrt[10]{2}$ and $n = 5$ (E) $r = \sqrt[5]{4}$ and $n = 5$ (F) $r = \sqrt[2]{2}$ and $n = 2$

(G) None of those

$\sqrt{2} = r\cos\frac{\pi}{4}$

$\sqrt{2} = \frac{r}{\sqrt{2}}$

$2 = r$

$z^5 = 2e^{i\pi/4}$

$z = 2^{1/5} e^{i\pi/4\left(\frac{1}{5}\right)}$

$r = \sqrt[5]{2}$ $n = 5$

4. (1 point) Find an angle that corresponds to one of the root for z.

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{5}$ (D) $\frac{\pi}{10}$

(E) $\frac{\pi}{20}$ (F) $\frac{\pi}{2}$ (G) π (H) None of those

$\frac{2\pi}{5}$

$\boxed{\frac{\pi}{20} \quad \frac{9}{20} \quad \frac{17}{20} \quad \frac{5}{4} \quad \frac{33}{20}}$

5. (2 points) Evaluate the limit of the sequence $\{a_n\}_{n=1}^{\infty}$, if it exists, where $a_n = n \sin\left(\frac{1}{bn}\right)$, $b > 2$.

(A) $\frac{1}{b}$ (B) $\sin(b)$ (C) 1 (D) The sequence diverges

(E) b (F) $\sin\left(\frac{1}{b}\right)$ (G) 0 (H) None of those

$-n \le \sin\frac{1}{bn} \le +n$ $\quad \sin\frac{1}{bn} = 0$

$-\infty \le 0 \le \infty$

$\frac{\frac{1}{bn^2}\cos\frac{1}{bn}}{\frac{1}{n^2}} = (n^2 \cos\frac{1}{bn})\frac{1}{b}$ $\quad \frac{-\frac{1}{bn^2}\sin\frac{1}{bn}}{\frac{1}{n}}$

~~$= n^2(1) = \infty$~~

$\boxed{1/b}$

6. (2 points) A sequence is defined recursively by $a_1 = -3$, $a_{n+1} = \dfrac{a_n + 3}{a_n + 1}$, $n \ge 1$. Assume that the sequence converges. Compute the limit of this sequence.

(A) -3 (B) -2 (C) $-\frac{1}{2}$ (D) 0

(E) $\frac{1}{2}$ (F) 2 (G) 3 (H) None of those

$A = \frac{A+3}{A+1}$

1 2 3 4 5 6

$\frac{0}{-2}$ $\frac{3}{1}$ $\frac{6}{4}$

1.73205

$A^2 + A = A + 3$

$A^2 = 3$

$(A+\sqrt{3})(A-3) = 0$

$A = \pm\sqrt{3}$

7. (2 point) Set up the arc length equation for the parametric curve:

$$x = a\cos(t), y = b\sin(t), 0 \le t \le \frac{\pi}{2}.$$

(A) $\int_0^{\frac{\pi}{2}} \sqrt{a^2\cos^2(t) + b^2\sin^2(t)}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{a^2\sin^2(t) + b^2\cos^2(t)}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{a\cos(t) + b\sin(t)}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{-a\sin(t) + b\cos(t)}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{a^2 + b^2}\, dt$

(F) None of those

$x' = (-a\sin t) \qquad y' = (b\cos t)$

$\sqrt{a^2\sin^2 t + b^2\cos^2 t}$

$\sqrt{}$

8. (2 points) Calculate slope of the tangent line to the curve $r = 1 - \cos\theta$ at the point on the curve $(r, \theta) = \left(1, \frac{\pi}{2}\right)$.

(A) $-\frac{1}{3}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) Tangent line is vertical

(F) $\frac{1}{3}$ (G) $\frac{1}{2}$ (H) 1 (I) 2 (J) None of those

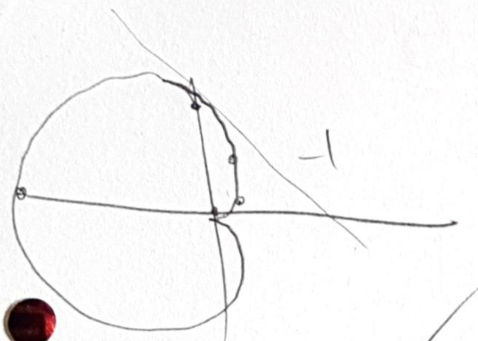
$$\frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$r = 1 - \cos\theta = 1$

$r' = \sin\theta = 1$

$\sin\theta = 1$

$\cos\theta = 0$



$$\frac{1 + 0}{0 - 1} = -1$$

θ	$1 - \cos\theta$
$\frac{\pi}{6}$	.16
$\frac{\pi}{3}$	.5
$\frac{\pi}{4}$	.29
$\frac{\pi}{2}$	1
π	

$\frac{3\pi}{4}$

$\frac{5\pi}{6}$

0

9. (2 points) Compute $\sum_{n=1}^{\infty} (e^{1/n} - e^{1/(n+1)})$, if the series converges.

(A) ~~e~~ (B) $e-1$ 1,78 (C) ~~1~~ (D) ~~The series diverges~~

(E) $e^{1/2}$ 1,646 (F) ~~$e+1$~~ (G) ~~0~~ (H) ~~None of those~~

$e^{\frac{1}{n}} - e^{\frac{1}{n+1}} \approx 1 - 1 = 0$ could converge

$e^{\frac{1}{n}} - e^{\frac{1}{n+1}}$

1.0695 ~~5.99~~ .2531 .1158 .0626

1.501 close to $e^{\frac{1}{2}}$

10. (2 points) Determine whether or not $\sum_{n=1}^{\infty}\left(\frac{e^n - n^e}{e^n}\right)$ converges, giving appropriate (and correct) justification.

(A) Converges, by the n-th Terms test.

(B) Diverges, by the n-th Terms test.

(C) Converges, by a telescoping sum argument.

(D) Diverges, by a telescoping sum argument.

(E) Converges, by the Integral Test.

(F) Diverges, by the Integral Test.

(G) Diverges, since it is a harmonic series.

(H) None of those

$$\frac{e^n}{e^n} - \frac{n^e}{e^n}$$

H

$$1 - \frac{en^{e-1}}{e^n}$$

$$\int \frac{e^x}{e^x} - \int \frac{x^e}{e^x}$$

$$= \int_1^\infty 1$$

$$= x\Big]_1^\infty = \infty - 1$$

diverges by integral test

MATHEMATICS 101 (Sections A01-A05)
Formula sheet, Spring 2018
Midterms and Final examinations.

Table of Integrals

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, (u < a)$

2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, (u > a)$

4. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, (a > 0)$

5. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, (u > a > 0)$

6. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C, (a > u > 0)$

8. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C, (u > 0)$

9. $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + C$

10. $\displaystyle\int \csc u\, du = -\ln|\csc u + \cot u| + C$

Trigonmetric and Hyperbolic Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$
2. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
3. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
4. $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
5. $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
6. $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
7. $\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$
8. $\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$
9. $\cosh^2(x) - \sinh^2(x) = 1$
10. $\sinh(2x) = 2\sinh(x)\cosh(x)$
11. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
12. $\text{sech}^{-1}(x) = \cosh^{-1}\left(\frac{1}{x}\right)$
13. $\text{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$
14. $\coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$