MATHEMATICS 100 (Sections A01-A03),
Midterm # 1, September 28, 2017.
September 28, 551 B.C. – Happy birthday, Confucius!
"With education there is no distinction between classes or races of men."
Time: 120 minutes

Last name: _____ StudentID: $V00$_____

First name: _____ Tutorial section number: T_____

Problem #	$1-4$	$5-7$	8	9	10	11	12	13	TOTAL
Points (max)	4	3	2	3	3	2	4	4	25
Score									

- The only calculators allowed on any examination are Sharp EL-510R, Sharp EL-510 RN and Sharp EL-510RNB.
- This test consists of 13 questions and has 9 pages (including this cover, the **Blank page** and a **Formula sheet** on the last page).
  - Questions 1 through 7 are multiple-choice. Write your full answer in this booklet in the provided space. **Clearly mark your final answer among the multiple choises.** You need to show your work for all answers, as we may disallow any answer which is not properly justified.
  - Questions 8 through 13 are long-answer. Write your detailed solutions in space provided in this booklet.
- For the multiple-choice questions, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices.
- Before starting your test enter your Name (Last, First), student ID, and tutorial section number (T01 - T22) on this page.
- If you have finished working on your paper with less than 10 minutes before the end of the examination, please close your paper and **remain seated** until the test time is completed. It is important to minimize the desruptions in the room.
- At the end of 120-minute test, turn-in this booklet.
- This is version A of the Midterm #1.

For the questions #1 − #7, calculate following limits:

1. (1 point) $\lim_{h\to 0} \dfrac{5}{\sqrt{5h+1}+1}$

(A) -2.0 (B) -1.5 (C) -1.0 (D) -0.5 (E) 0.5
(F) 1.0 (G) 1.5 (H) 2.0 (I) 2.5 (J) 3.0

2. (1 point) $\lim_{x\to 0} \dfrac{1+x+\sin x}{3\cos x}$

(A) -3.0 (B) -1.0 (C) -0.6 (D) -0.3 (E) 0.0
(F) 0.3 (G) 0.6 (H) 1.0 (I) 2.0 (J) 3.0

3. (1 point) $\lim_{t\to 4^-} (t - \lfloor t \rfloor)$

(A) -4.0 (B) -3.0 (C) -2.0 (D) -1.0 (E) 0.0
(F) 1.0 (G) 2.0 (H) 3.0 (I) 4.0 (J) 5.0

4. (1 point) $\lim_{t\to 0^-} \dfrac{2t}{\tan t}$

(A) -3.0 (B) -2.0 (C) -1.0 (D) -0.5 (E) 0.0
(F) 0.5 (G) 1.0 (H) 2.0 (I) 3.0 (J) 4.0

5. (1 point) $\lim_{x \to +\infty} \dfrac{\sqrt{x^2+2}}{2x+4}$

(A) -∞ (B) -2.0 (C) -1.0 (D) -0.5 (E) 0.0
(F) 0.5 (G) 1.0 (H) 2.0 (I) 3.0 (J) $+\infty$

6. (1 point) $\lim_{\theta \to -\infty} \dfrac{\cos\theta}{3\theta}$

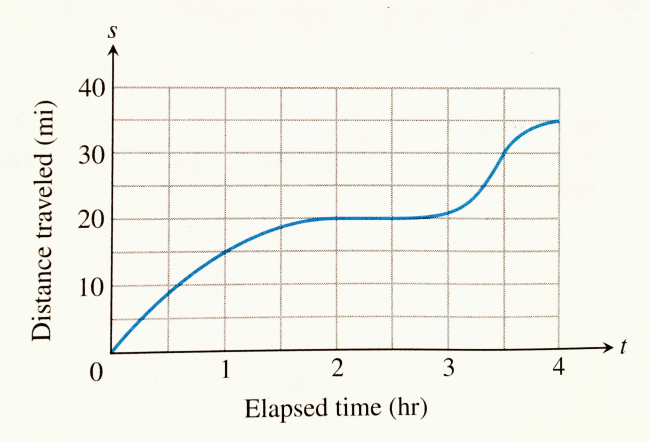
(A) $-\infty$ (B) -3.0 (C) -1.0 (D) -0.3 (E) 0.0
(F) 0.3 (G) 1.0 (H) 2.0 (I) 3.0 (J) $+\infty$

7. (1 point) $\lim_{x \to a} \dfrac{x^2-a^2}{x^4-a^4}$

(A) $-\dfrac{1}{a^2}$ (B) $-\dfrac{a^2}{2}$ (C) $-\dfrac{1}{2a^2}$ (D) $-2a^2$ (E) 0.0

(F) $+2a^2$ (G) $+\dfrac{1}{2a^2}$ (H) $+\dfrac{a^2}{2}$ (I) $+\dfrac{1}{a^2}$ (J) Does not exist

8. (2 points) Graph below shows the total distance s traveled by a bicyclist after t hours.



(a). Estimate the bicyclist's average speed over the time interval [1.0, 2.5].

(b). Estimate the bicyclist's instantaneous speed at the time $t = 0.5$.

9. (3 points) Suppose that $g(x) \leq f(x) \leq h(x)$ for all $x \neq 2$ and suppose that $\lim_{x\to 2} g(x) = \lim_{x\to 2} h(x) = -5$.

(a). Can we conclude anything about the values of f, g, and h at $x = 2$? Justify your answer.

(b). Could $f(2) = 0$? Justify your answer.

(c). Could $\lim_{x\to 2} f(x) = 0$? Justify your answer.

10. (3 points) For what values of a and b, the function $f(x)$ is continuous for all values of x:

$$f(x) = \begin{cases} -3, & x < -1 \\ ax + b, & -1 \leq x \leq 1 \\ 1, & 1 < x \end{cases}$$

11. (2 points)

(a). Give an example of a function $f(x)$ that is defined on the interval $[1, 5]$, and $f(x) \neq 0$ for all x on the interval $[1, 5]$, and $f(1) = -1$, $f(5) = 1$. You can define $y = f(x)$ using a graph or using a formula.

(b). Explain why your function $y = f(x)$ defined in the part (a) does not violate Intermediate Value Theorem.

12. (4 points)

(a). Calculate $\lim_{x \to +\infty} (\sqrt{9x^2 - x} - 3x)$

(b). Using information from part (a) determine the equation of the slant asymptote of the function $y = \sqrt{9x^2 - x}$

Recall: A line $y = mx + b$ is a slant (or oblique) asymptote of the function $y = f(x)$ if $\lim_{x \to -\infty} (f(x) - (mx + b)) = 0$ or $\lim_{x \to +\infty} (f(x) - (mx + b)) = 0$.

13. (4 points) Consider following piecewise-defined function:

$$g(x) = \begin{cases} 2x - 1, & x \geq 0 \\ x^2 - 2x - 1, & x < 0. \end{cases}$$

(a). Determine the left-hand derivative and the right-hand derivative of $g(x)$ at the point $x_0 = 0$ using definition of the one-sided derivative at the point.

(b). Using your results in part (a), determine if the function $g(x)$ is differentiable at the origin. Justify your conclusion.

[BLANK]