

# Introduction to Principles of Microeconomics and Financial Project Evaluation

## Lecture 12: Optimizing Replacement Cycles

October 5, 2021

# Recommended Viewing

- A quick warning: while these videos are pretty good, in my opinion the presenter relies way, way too much on one specific formula for calculating EAC (the one with  $+Si$  in it – you'll know it when you see it).
- The concept is MUCH more general than this, and trying to memorize that one equation can lead you astray in more realistic applications.
- Engineering Economics Guy. (2020, March 30). Introduction to Equivalent Annual Cost - Engineering Economics Lightboard.  
<https://youtu.be/QGJOkgcmdMo>
- Engineering Economics Guy. (2020, March 30). Equivalent Annual Cost Example 1 - Engineering Economics Lightboard. <https://youtu.be/yKIH0kD-iWU>
- Engineering Economics Guy. (2020, March 30). Equivalent Annual Cost Example 2 - Engineering Economics Lightboard.  
[https://youtu.be/4so\\_q61kgzc](https://youtu.be/4so_q61kgzc)

# Recommended Reading

- *Engineering Economics*, 6th edition, 2.7, 7.1 – 7.7
- Schoemaker, M.A., Verlaan, J.G., Vos, R. & Kok, M. (2016). The use of equivalent annual cost for cost-benefit analyses in flood risk reduction strategies. *E3S Web of Conferences*, 7, 20005.  
<https://doi.org/10.1051/e3sconf/20160720005>
- van den Boomen, M., Schoenmaker, R. & Wolfert, A.R.M. (2018). A life cycle costing approach for discounting in age and interval replacement optimisation models for civil infrastructure assets. *Structure and Infrastructure Engineering*, 14(1), 1-13. <https://doi-org.ezproxy.library.uvic.ca/10.1080/15732479.2017.1329843>
- **The only reason the two papers above aren't required reading is their length. They are excellent resources if you want to learn how to use the EAC approach in practice.**

# Electric Bus Charging Station Readings

- Chen, L., Qian, K., Qin, M., Xu, X. & Xia, Y. (2020). A Configuration-Control Integrated Strategy for Electric Bus Charging Station With Echelon Battery System. *IEEE Transactions on Industry Applications*, 56(5), 6019-6028. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/9091801>
- A few short readings about **energy arbitrage**:
- Larson, A. & Konidena, R. (2020, May 5). What Time of Use rate makes sense for residential energy arbitrage? [Web Page]. Retrieved from <https://www.renewableenergyworld.com/2020/05/20/what-time-of-use-rate-makes-sense-for-residential-energy-arbitrage/>
- Stein, J. (2015, September 8). It Doesn't Look Good for Time-of-Use Arbitrage [Web Page]. Retrieved from <https://www.esource.com/es-blog-9-8-15-tou/it-doesnt-look-good-time-use-arbitrage>

# Source for Isuzu diesel truck data

- Taghipour, S. Salari, N. Optimal sustainable vehicle replacement model. *2015 Annual Reliability and Maintainability Symposium (RAMS)*, 1-6. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/7105110>

# Optional Reading (EAC)

- Avinash, N. A., Jaiswal, G. C. & Ballal, M. S. (2014). Economical aspects of remote condition monitoring system for distribution transformer. *2014 International Conference on Power, Automation and Communication (INPAC)*, 45-49. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6981133>
- **Interesting points on estimating operating costs for EAC calculations.**
- Chen, H. & Wang, Z. (2011). The model of concrete structure's durability design based on economic life. *2011 International Conference on Remote Sensing, Environment and Transportation Engineering (RSETE)*, 8687-8689. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/5964201>
- **Concrete structure example; includes an EAC equation with DCFA factors.**
- Horwood, W. S. (1961). Economic life determination. A case study. *Production Engineer*, 40(11), 741 – 746. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/4911395>
- **A case study involving Morris trucks.**
- Lee, S. H., Lee, A. K. & Kim, J. O. (2012). Determining economic life cycle for power transformer based on life cycle cost analysis. *2012 IEEE International Power Modulator and High Voltage Conference (IPMHVC)*, 604-607. Retrieved from <https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6518816>
- **South Korean transformer example.**
- Yatsenko, Y. & Hritonenko, N. (2011). Economic life replacement under improving technology. *International Journal of Production Economics*, 133(2), 596-602. Retrieved from <https://doi-org.ezproxy.library.uvic.ca/10.1016/j.ijpe.2011.04.027>
- **Considers economic lifetime determination in the context of changing technology.**

# Learning Objectives

- Obtain a basic understanding of the basic concepts behind depreciation.
- Gain familiarity with declining balance and straight line depreciation.
- Understand the relationship between Effective Annual Costs and the economic lifetime of an asset.
- Be able to calculate the economic lifetime of a typical asset (one with an initial cost, operating/maintenance costs and a salvage value).
- Be aware of the difficulties in forecasting and estimating maintenance costs.
- Be able to calculate appropriate replacement strategies when i) the challenger and defender are identical, ii) the challenger and defender differ and the challenger repeats forever.
- Be aware of the one-year principle and understand when it may be invoked.

# Notation Dictionary

(Not provided on quiz/final formula sheet)

- AW = Annual Worth
- C(t) = O/M Cost in Year t
- d = depreciation rate of resale value EAC = Effective Annual Cost
- O/M = Operating/Maintenance
- PW = Present Worth
- S(N) = Salvage/Resale value if salvaged in Year N

# Relevant Solved Problems

- From *Engineering Economics*, 6th edition.
- Economic Lifetime: Example 7.1, Example 7.3, Example 7.8, Review Problem 7.1, 7.5, 7.6, 7.16, 7.23.a., 7.28, 7.31, 7.32, 7.35, 7.38, 7.39, 7.40, 7.41, 7.42, 7.44
- Challenger and Defender Identical: Example 7.2, 7.7, 7.13, 7.14, 7.21
- General Replacement: Example 7.7, Example 7.8, Example 7.9, Review Problem 7.2, Review Problem 7.3, 7.1, 7.2, 7.8, 7.17, 7.18, 7.19, 7.20, 7.24, 7.25, 7.26, 7.27, 7.29, 7.30, 7.33, 7.34, 7.36, 7.37
- One-year principle: Example 7.4, Example 7.5, Example 7.6, 7.9, 7.10, 7.11, 7.12, 7.15

ESSENTIALS (16 slides)

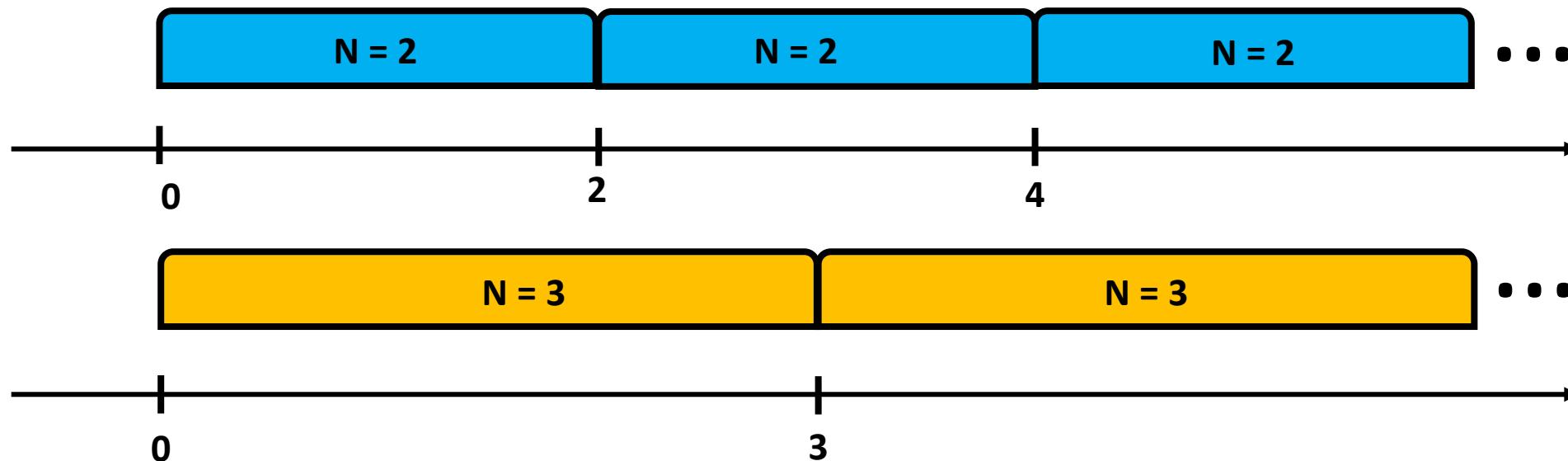
# Your firm needs a forklift

- Suppose your company needs a ~~forklift~~, period...
- ..and it's a mature technology: you can buy the same one over and over.
- Buy forklift, pay operating costs, sell it → Buy forklift...
- You're always buying the *same* forklift.
- Only question: WHEN do you replace your forklifts?
- ~~Replace it sooner~~ → more frequent buying costs
- ~~Replace it later~~ → operating costs pile up & rise (maintenance on old machines vs new ones), resale value falls.

## Two different, mutually exclusive projects:

- Buy a forklift, **keep it two years**, then sell it & replace with same model.
- Buy a forklift, **keep it three years**, then sell it & replace with same model.
- We can represent these two different endless replacement chains with a diagram:

Let  $N$  = Years we keep the forklift



# We know how to deal with this!

- Keep repeating the same project forever → tailor-made for the Annual Worth Comparison approach.
- For each N you're considering...
- Find the net present value of *one* forklift: buying, operating & selling
- Use (A/P,MARR,N) to find the equivalent annual worth (or monthly, or daily, or per millisecond – whatever time scale's most appropriate).
- (Do whatever's easiest to get the equivalent annuity. You don't NEED to find the NPV first if that would mean more work for you, but it's common to do so.)
- Compare your results for the different 'N', and pick the biggest.
- OR... since most of what we're looking at are *costs* (even the resale value can be seen as a delayed rebate on the initial cost)...
- Why not compare equivalent annual costs (EAC) and pick the smallest one?
- EXACTLY THE SAME THING, just relabeled so we don't have to carry as many negative signs.

# So, what would this look like?

- Remember: N is how many years we keep the forklift.
- For any given N, we have...
- Initial cost: Given. We're assuming we can repeat this project, so prices are constant (**yes, often an unrealistic assumption**)
- Operating & Maintenance Costs: Start off low, get higher with time for two reasons: more time, & the costs per year get higher as the hardware gets older.
- Resale Value: (a "negative cost") The longer we wait, the less we can sell the forklift for. (**Unless we wait so long it's a collector's item.**)

1

Identify repeating life cycles  
with corrective and preventive  
replacement costs

2

Calculate the present value of  
one life cycle by using  $(P/F, i, t)$

3

Calculate the equivalent annual costs  
over the (expected) cycle length  
by using  $(A/P, i, t)$

Equivalent  
Annual  
Cost

(van den Boomen et al., 2018)

*Under the assumption of identical replacements and repeating life cycle costs,  
the EAC of one life cycle equals the EAC of an infinite number of life cycles*

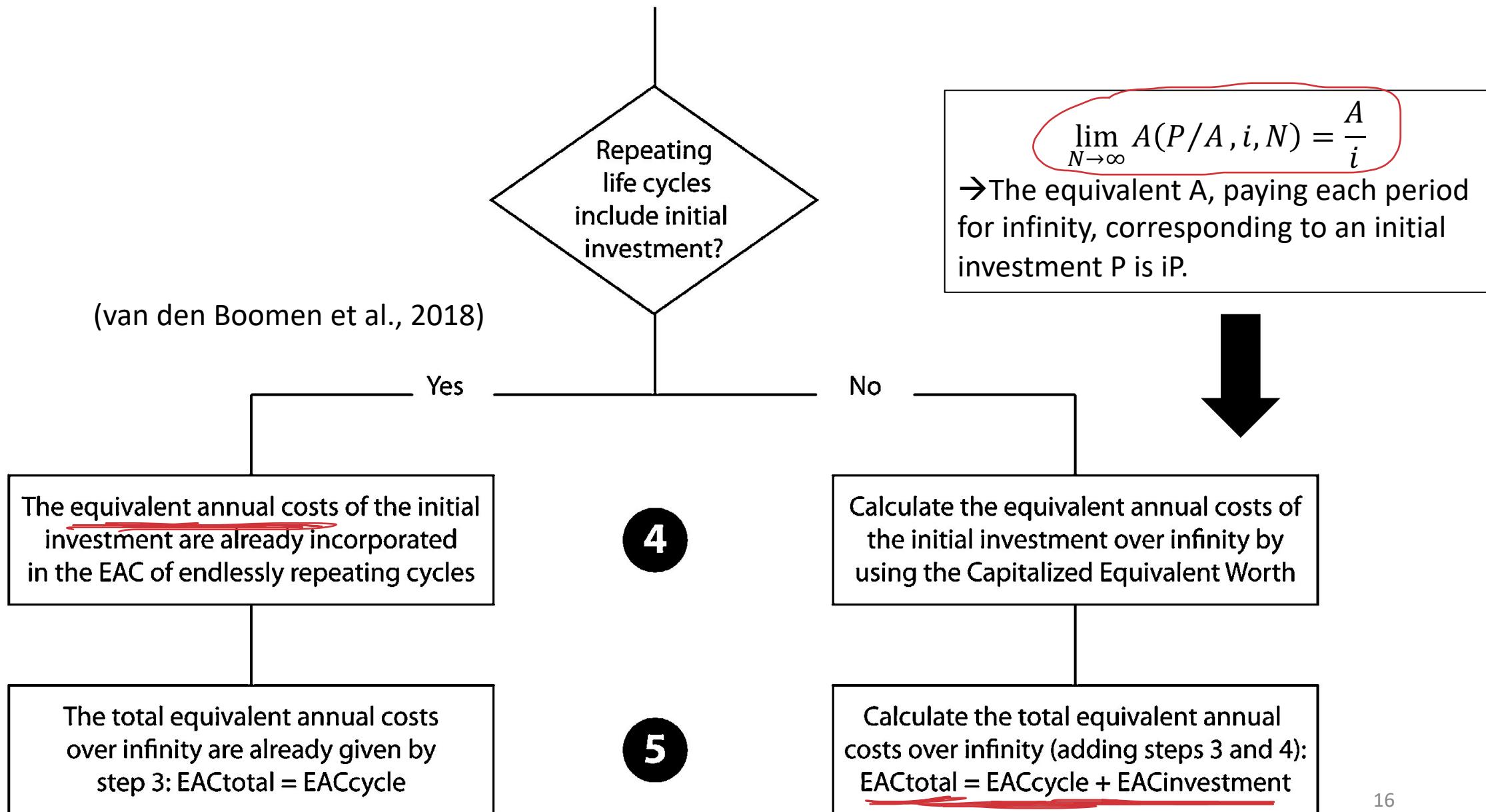
This is **not** the only way  
to perform this kind of  
analysis. I've posted this  
because it's an interesting  
example of the procedure  
as recommended by  
some practitioners.

present to annual

$$NPV \times (A/P, i, N) = EAC(N)$$

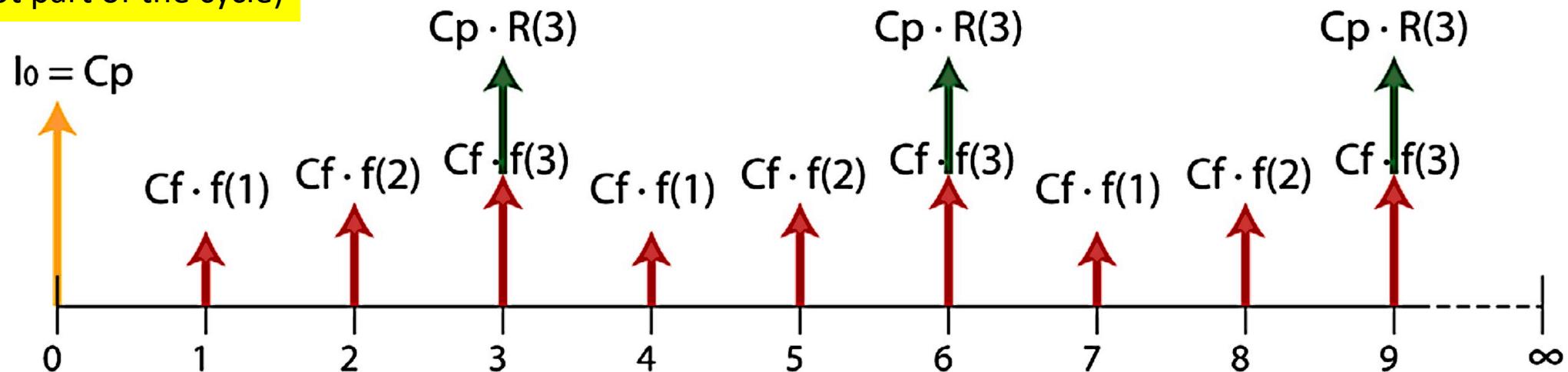
↓  
net present value

*Under the assumption of identical replacements and repeating life cycle costs, the EAC of one life cycle equals the EAC of an infinite number of life cycles*



# What the end result may look like

Note the separate initial cost (not part of the cycle)



**Figure 2.** Cash flow diagram of an age replacement policy for a preventive replacement interval of three years.

Note: Three full cycles and an initial investment are shown.

(van den Boomen, 2018)

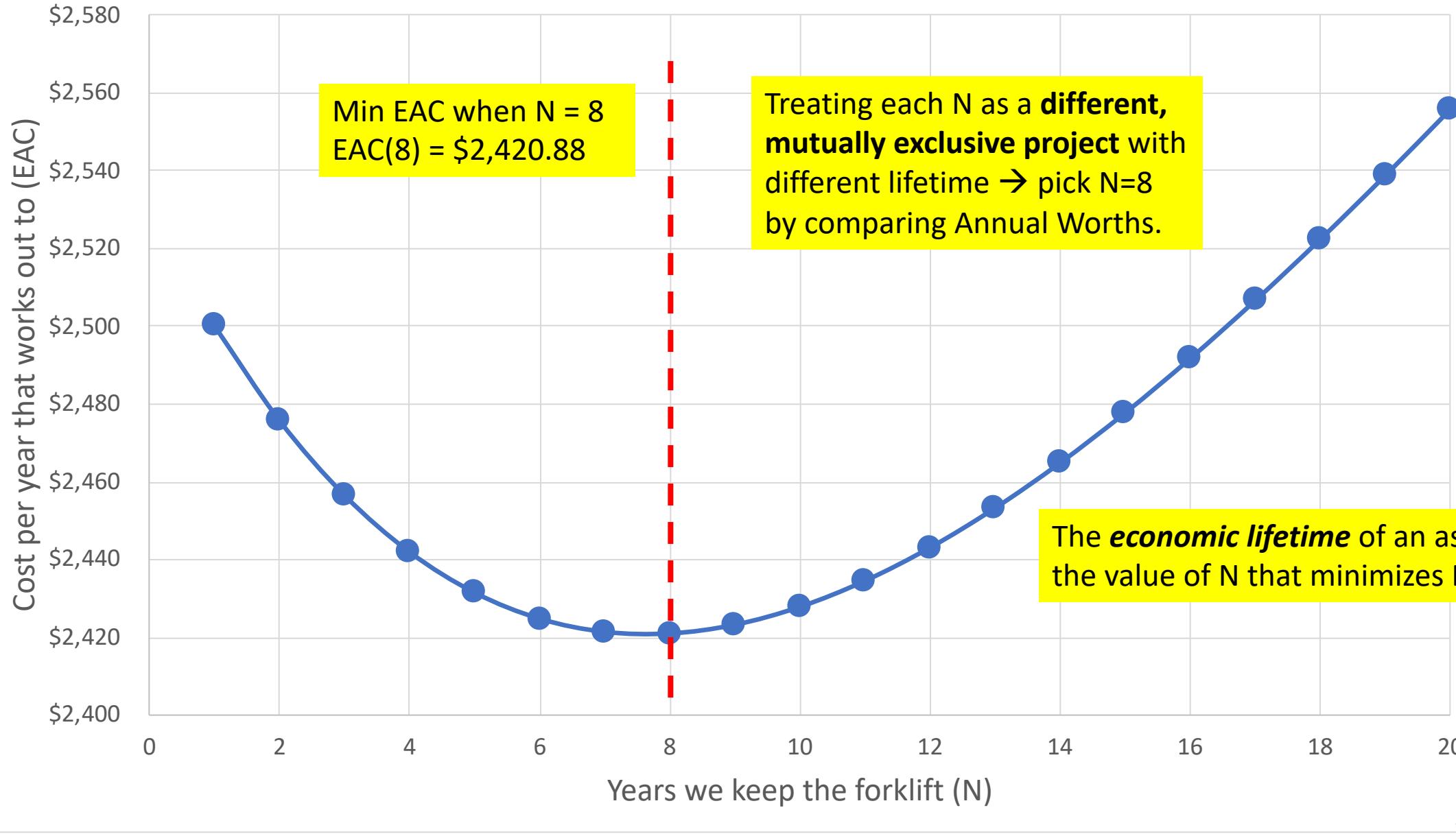
# Let's pick some numbers to work with

- Simple values & functions that keep the flavour of what we need.
- Suppose N is in years, and our MARR is 5% per year...
- Initial Cost:  $\$P = \$10,000$
- Operating & Maintenance Costs: \$1,000 the first year, go up by \$100 thereafter (rigged so it looks like an arithmetic gradient with  $G = \$100$  sitting on top of an annuity with  $A = \$1,000$ ).
- Resale Value: Falls by 10% every year. Call it 'S' for salvage.
  - $\rightarrow S = (1 - 10\%)^N \times P = 0.9^N \times \$10,000$
- Let's find the equivalent annual cost (EAC), given N.

# EAC, Element by Element, for our example

- For a given  $N$  in years, and remembering  $\text{MARR} = 5\%$  per year...  
*not actually multiplying*
- Initial Cost:  $P$  is a present value, so  $A = P \times (A/P, \text{MARR}, N)$ 
  - $\$10,000 \times (A/P, 5\%, N)$
- Operating Costs: Annuity on arithmetic gradient, so  $A + G \times (A/G, \text{MARR}, N)$ 
  - $\$1,000 + \$100 \times (A/G, 5\%, N)$
- Resale Value:  $S$  is a future value in time  $N$  so  $A = F \times (A/F, \text{MARR}, N)$ 
  - $0.9^N \times \$10,000 \times (A/F, 5\%, N)$
- $\text{EAC}(N) = \$10,000 \times (A/P, 5\%, N) + \$1,000 + \$100 \times (A/G, 5\%, N) - 0.9^N \times \$10,000 \times (A/F, 5\%, N)$
- Why the negative sign? The 'C' in EAC is Cost. Salvage is a negative cost.

## EAC(N): What's the cost per year if we replace them every N years?



# The short version

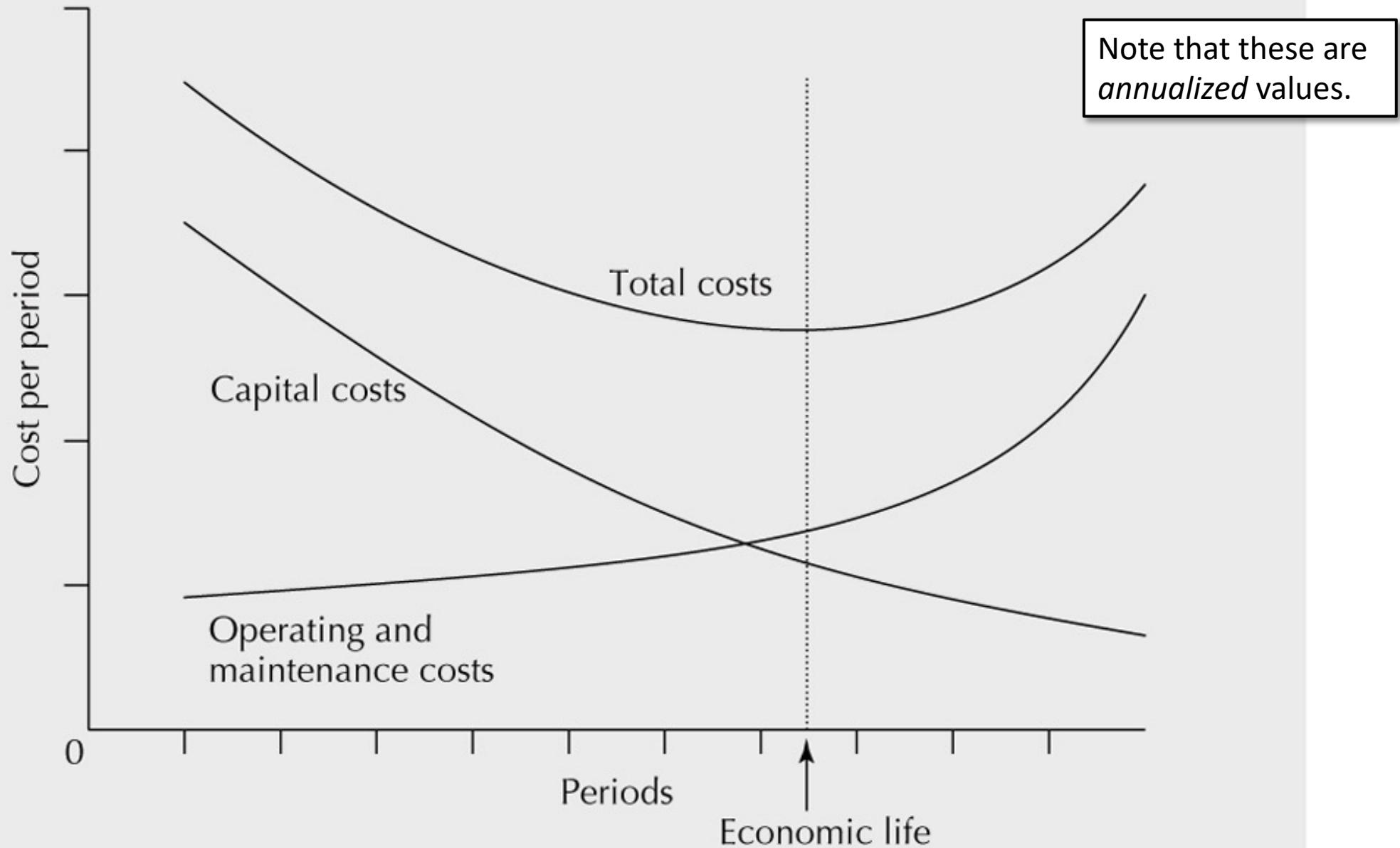
- Total cost of equipment = initial cost + operating costs – resale value
- Varies with N: time from purchase to resale & replacement
- EAC(N) = “Annual” (really, per-period) Worth of Total Cost
- EAC is U-shaped
- The economic lifetime is the time-stamp of the bottom of the U.

*m = 8 for vs*

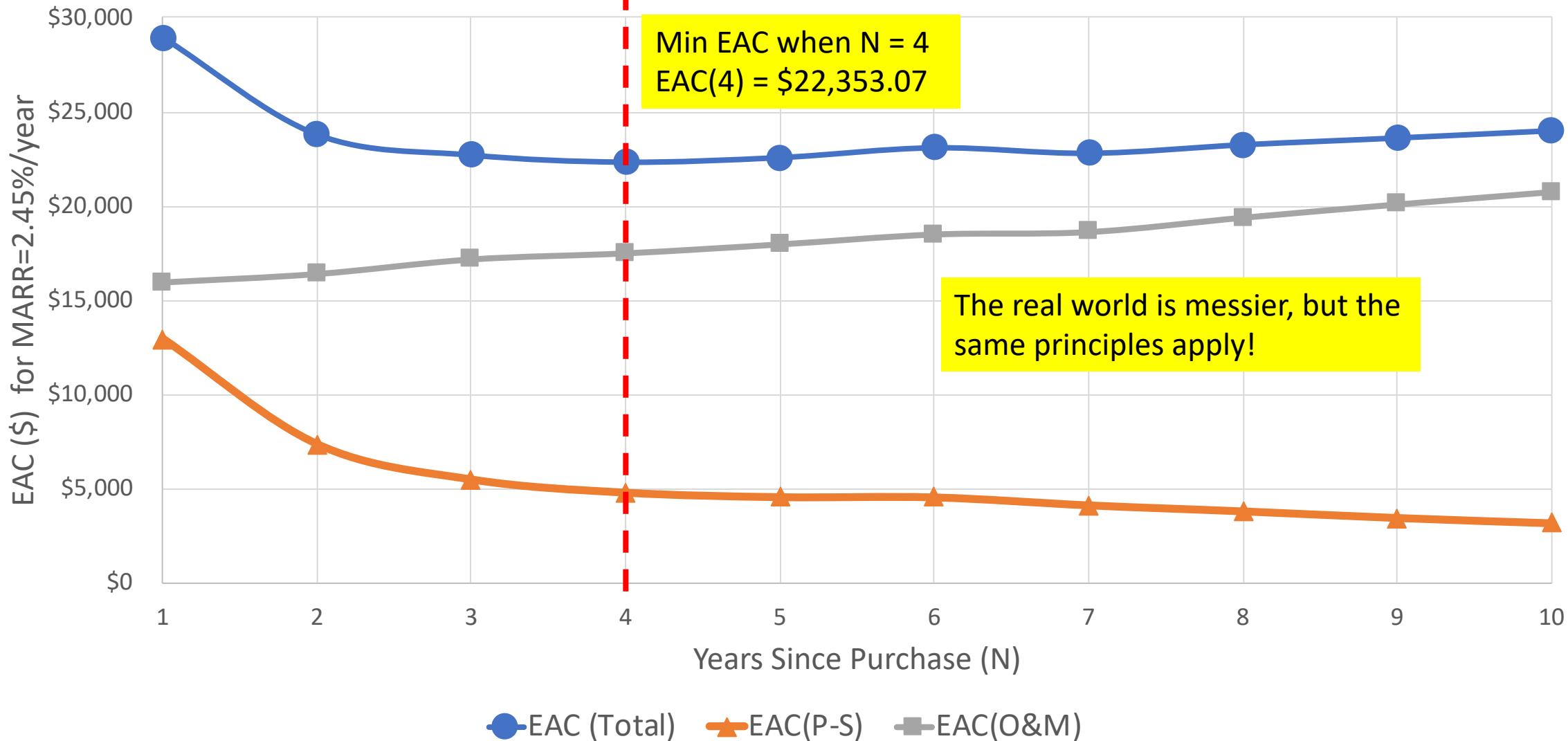
## Why U-shaped?

- Initial cost is a price tag, and spread out over more years as time increases.
- For the first few years, the initial cost dominates total costs, and annual costs will fall with the number of years the asset is kept.
- BUT: Operating and Maintenance costs rise as a machine ages
- Eventually, the rise dominates, and EAC goes up again.

**Figure 7.1 Cost Components for Replacement Studies**



## EAC for an Isuzu N-Series diesel truck (Taghipur & Salari, 2015)



# What are you replacing the item with?

- Itself: (e.g. your firm always replaces a worn-out red stapler with exactly the same model of red stapler)
  - Each 'N' corresponds to a different 'replacement chain' with infinite duration.
  - Each of those replacement chains will have a different annual worth equivalent ( $EAC(N)$ ), because (for example) the bigger the N, the lower the salvage value and the greater the maintenance costs, and the smaller the N, the fewer years of use across which to spread the initial cost.
  - → The optimal replacement period is the bottom of the EAC 'U'.
- But, what if your firm is considering replacing the red stapler with a new electric stapler? What if the *challenger* is different than the *defender*?
- Pretend that if you switch from the defender to the challenger, you'll stick to the challenger for the rest of time, repeating it with itself.
- Run the numbers for  $EAC(N)$  for the challenger (the electric stapler), and find what the equivalent annual cost is at the bottom of its 'U'. If that's better than what you can get from your current asset, the defender (red stapler), switch to the challenger immediately.

# A few things to watch out for

- If the defender has already been in place for a while when you consider the challenger – if your firm has owned the red stapler for a few years when purchasing notices an ad for a new electric stapler – you need to take into account that the defender is not a new product:
- You don't need to buy it – it's already there. The 'initial cost' for the old red stapler is not the purchase price, but the resale/salvage income you give up by keeping it, instead of selling it.
- It's possible you've had the defender for so long (maybe because you put off thinking about replacement decisions) that its cost profile doesn't look like a 'U', but like a '/': annualized costs will only go up from now on.
- If that's the case, you don't need to run the full EAC(N) calculations for the defender, since you know the best you can do, in annual cost terms, is keeping it for one more period than selling it.
- →The 'one-year rule of thumb' applies in these cases, and you'll be comparing the (new, not yet purchased) challenger's minimum EAC(N), the annualized cost at its economic lifetime, to the annualized cost corresponding to keeping the defender for one more time period.

# What if I don't plan to repeat the challenger?

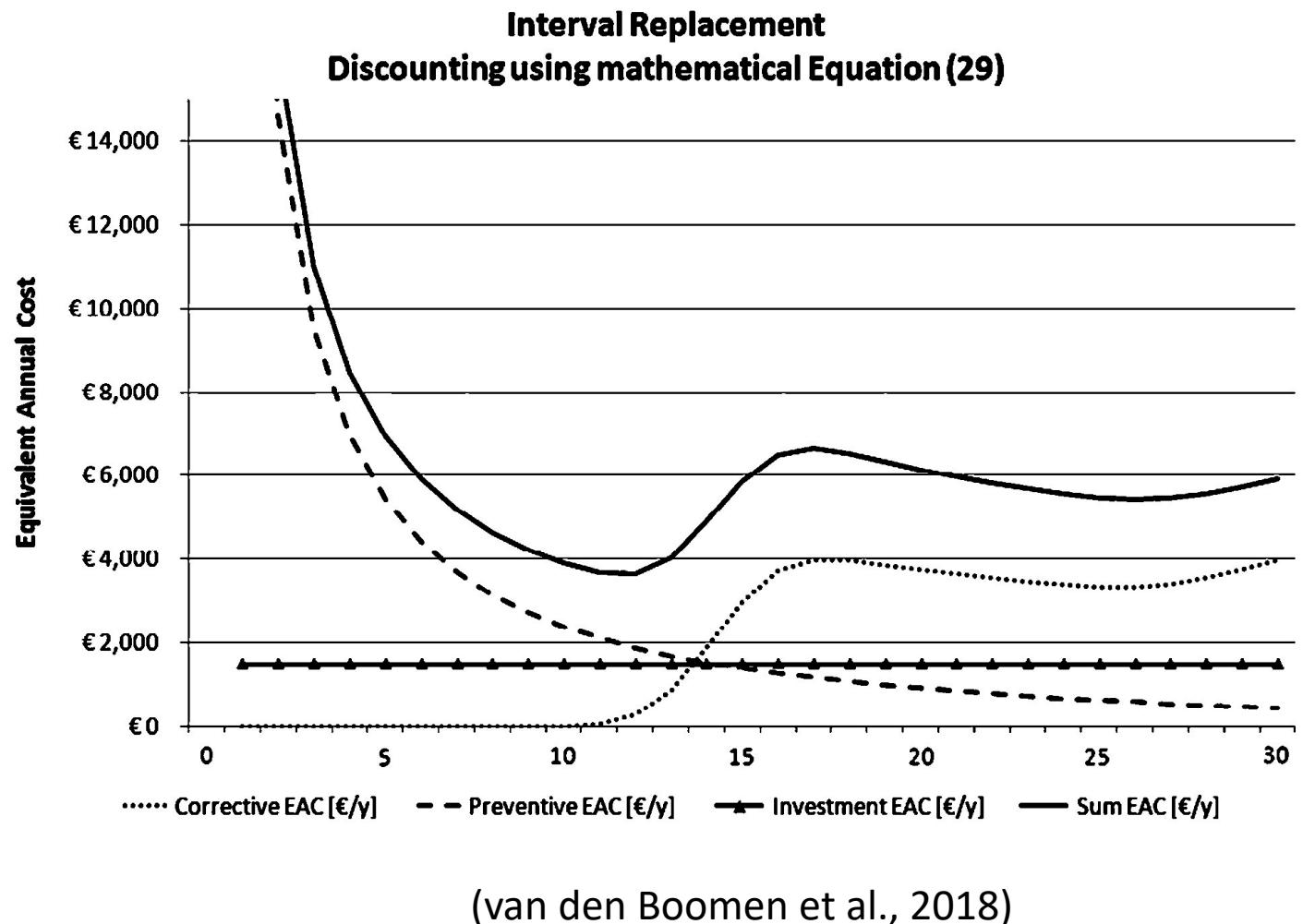
- In some cases, it may not be reasonable to assume that you'll repeat the challenger forever.
- Maybe your firm DOES want to replace its decades-old red stapler with an electric stapler, but isn't willing to commit to replacing the electric stapler with itself, because stapler technology is changing so quickly that by the time replacement comes around, there may be a superior alternative available.
- In that case, you *can't* use the EAC(N) approach.
- Instead, you'll need to consider all possible combinations of purchases & replacements, and pick the combination with the lowest total costs (highest total NPV).
- This can be a *lot* of work. We'll see just how much work, in the after-hours segment.

## AFTER HOURS

- Multiple or no minima in the EAC function (5 slides)
- Challenger  $\neq$  Defender, Challenger repeats (9 slides)
  - Notes on real-world maintenance costs (4 slides)
  - Challenger  $\neq$  Defender, no repeats (7 slides)

# Can there be more than one minimum?

- The effects mentioned above are usually dominant, leading to an ubiquitous U-shape, BUT occasionally you'll see multiple EAC minima.
- If your EAC curve DOES have multiple minima, then focus on the absolute minimum (the value of N with the lowest EAC).

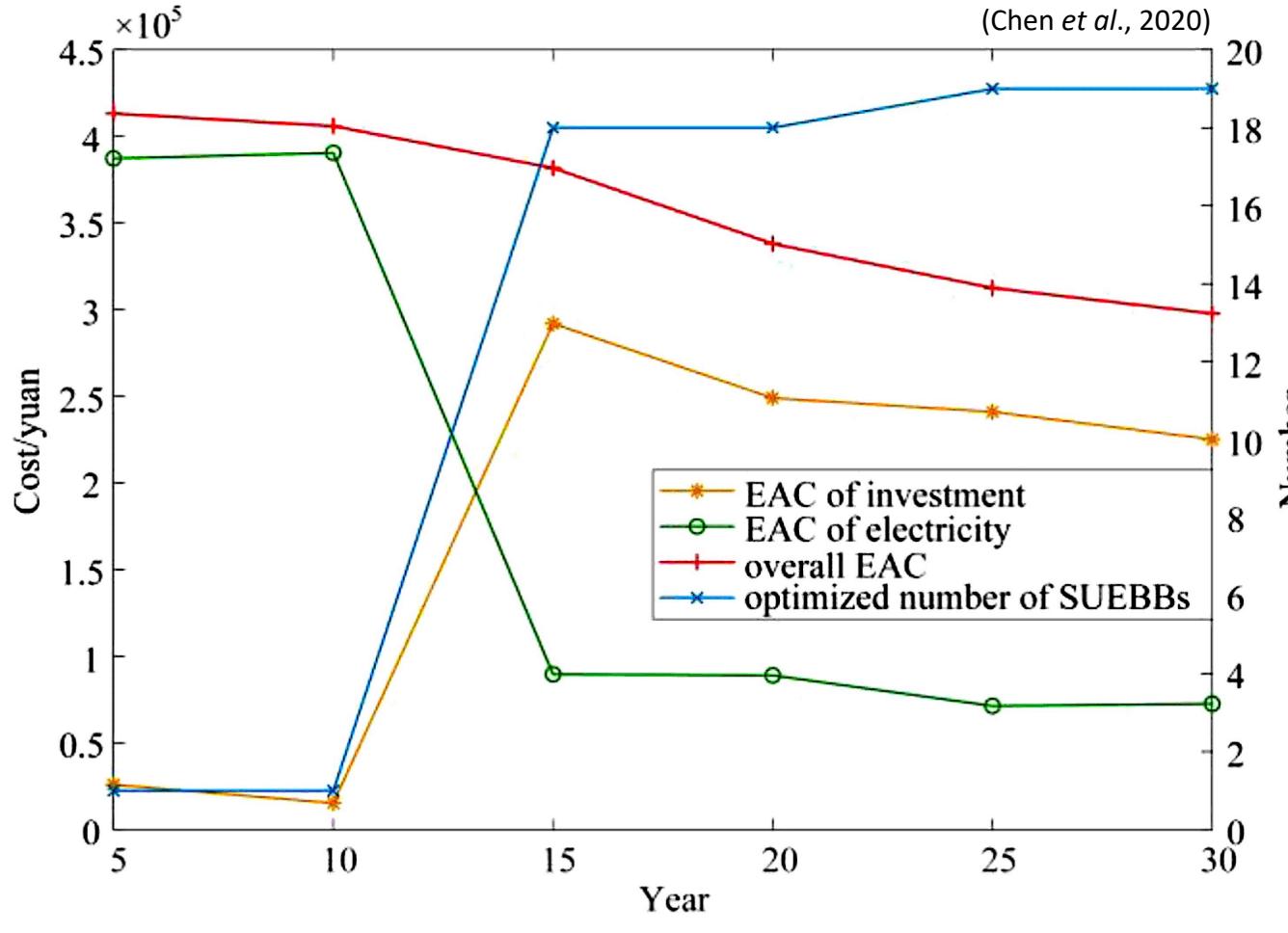


## There could also be no minima

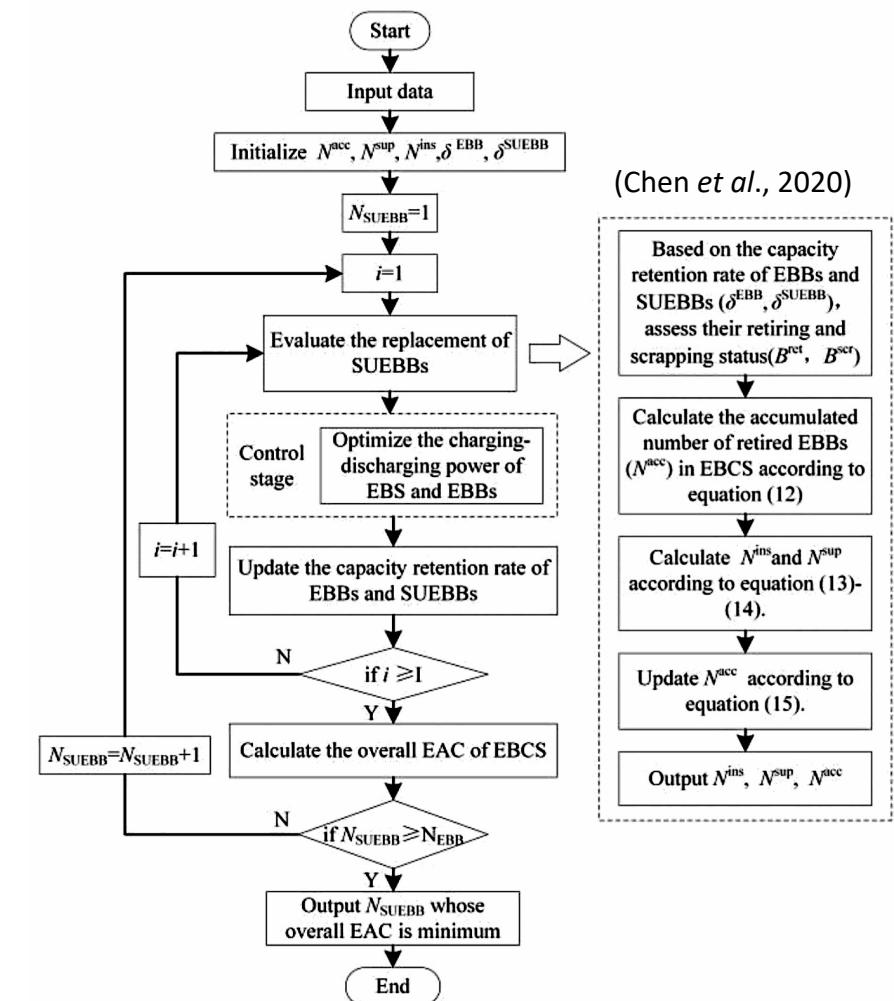
- It could also be you find **no minimum** for a reasonable time period.
- Case in point: 2020 paper on Electric Bus Charging Stations.
- The function didn't reach its minimum within a 30-year planning period...
- Let's take a look.

# EAC of a “Second Use Technology” for Electric Bus Charging (Chen et al., 2020)

SUEBB = Second-Used Electric Bus Battery



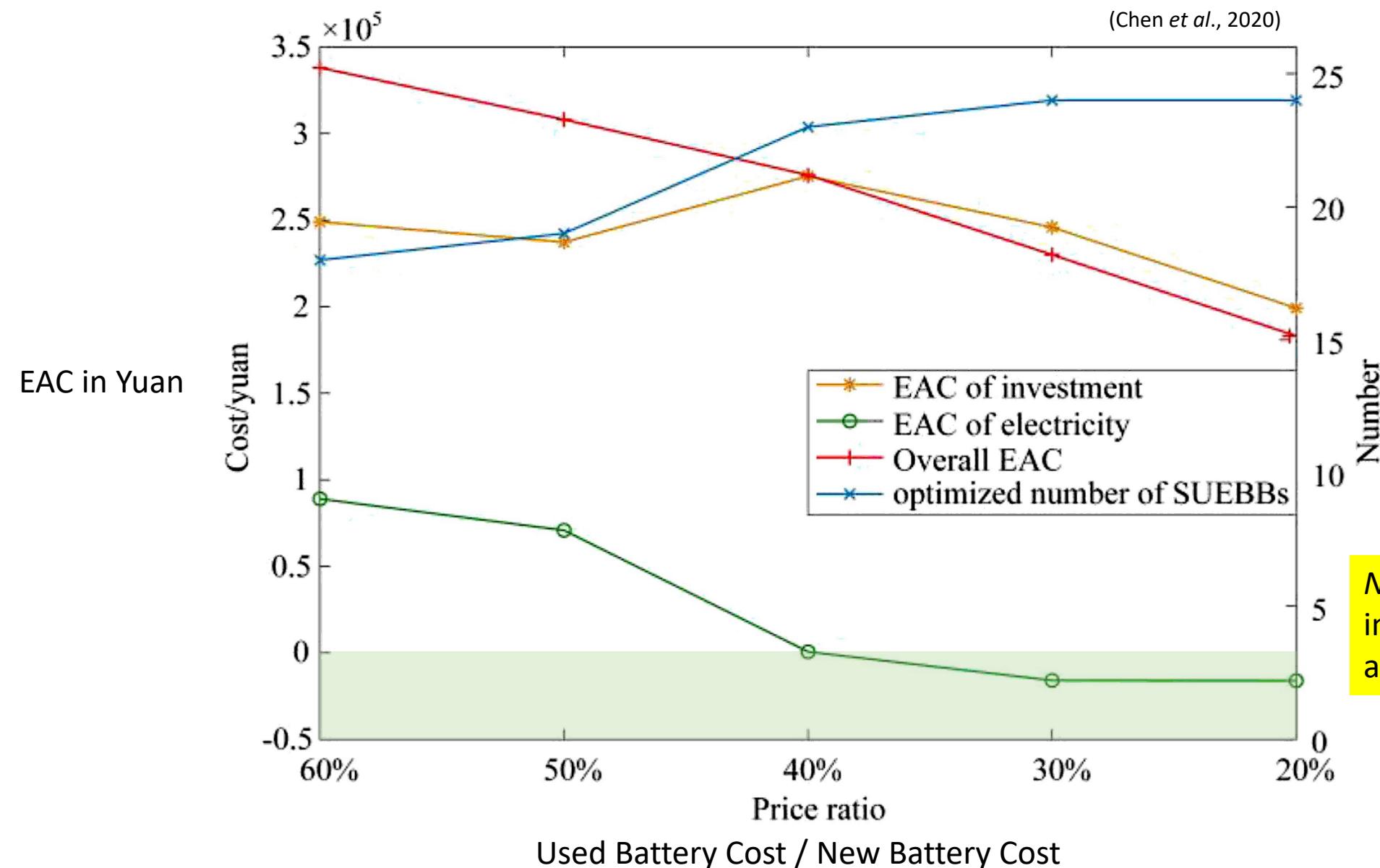
Note: The study's main goal was optimizing the # of batteries, not charger lifespan.



Solution Algorithm

# How did practitioners interpret this?

- EAC never stops falling during this range of N.
- Optimal # batteries goes up, esp. sharply between 10 & 15 years.
- The reason: **arbitrage** – buy low, sell high.
- Batteries can store power when it's cheap (off-peak) and sell it back to the grid when it's expensive (off-peak).
- Under certain conditions, buying more batteries brings in more income this way than they cost (in per year terms) → rise in batteries, because they lower EAC via arbitrage.
- BUT this flattens out quickly after N=15, due to supply limits.
- In this specific case, the batteries are **second use batteries**: cheap while they're available, but when you're out, you're out, or have to pay high prices for the last few remaining. BUT prices could change in the future...
- The authors investigated this, leading to a second graph:



This ***scenario analysis*** is a kind of ***sensitivity analysis*** we'll learn about later in the course.

# of Batteries

Negative EAC of electricity in some scenarios, due to arbitrage.

JOIN IN

HERE COMES A  
NEW CHALLENGER!

What if there's a new forklift available?

# You know what to do... don't you?

- You're 2 years into your current forklift's life, and a new model appears.
- It's the same kind of investment.
- You believe you'll be able to repeat it forever if you switch.
- You run the same numbers, and find it has a minimum EAC of \$2,400.00
- (For what it's worth, this is when N=4, so you'd replace it every 4 years.)
- Since this is less than the minimum EAC of your current forklift...
- You should switch right away, right?
- You'd switch from a project costing \$2,420 a year to one costing \$2,400 a year, on average (and accounting for the time value of money).

# Not so fast: be mindful

- You're not comparing apples with apples.
- Your forklift is *already in place*.
- You've *already paid its initial cost*.
- That cost is *sunk*, and shouldn't affect decision-making.
- Turns out mindfulness (reminding yourself that you live in the *now*) CAN be helpful in engineering economics!
- Place yourself in the *now*:
- Do you replace a forklift you've already paid for, NOW, with a newer model that's cheaper in the long run?
- The existing forklift is the defender, and the new model is the challenger.

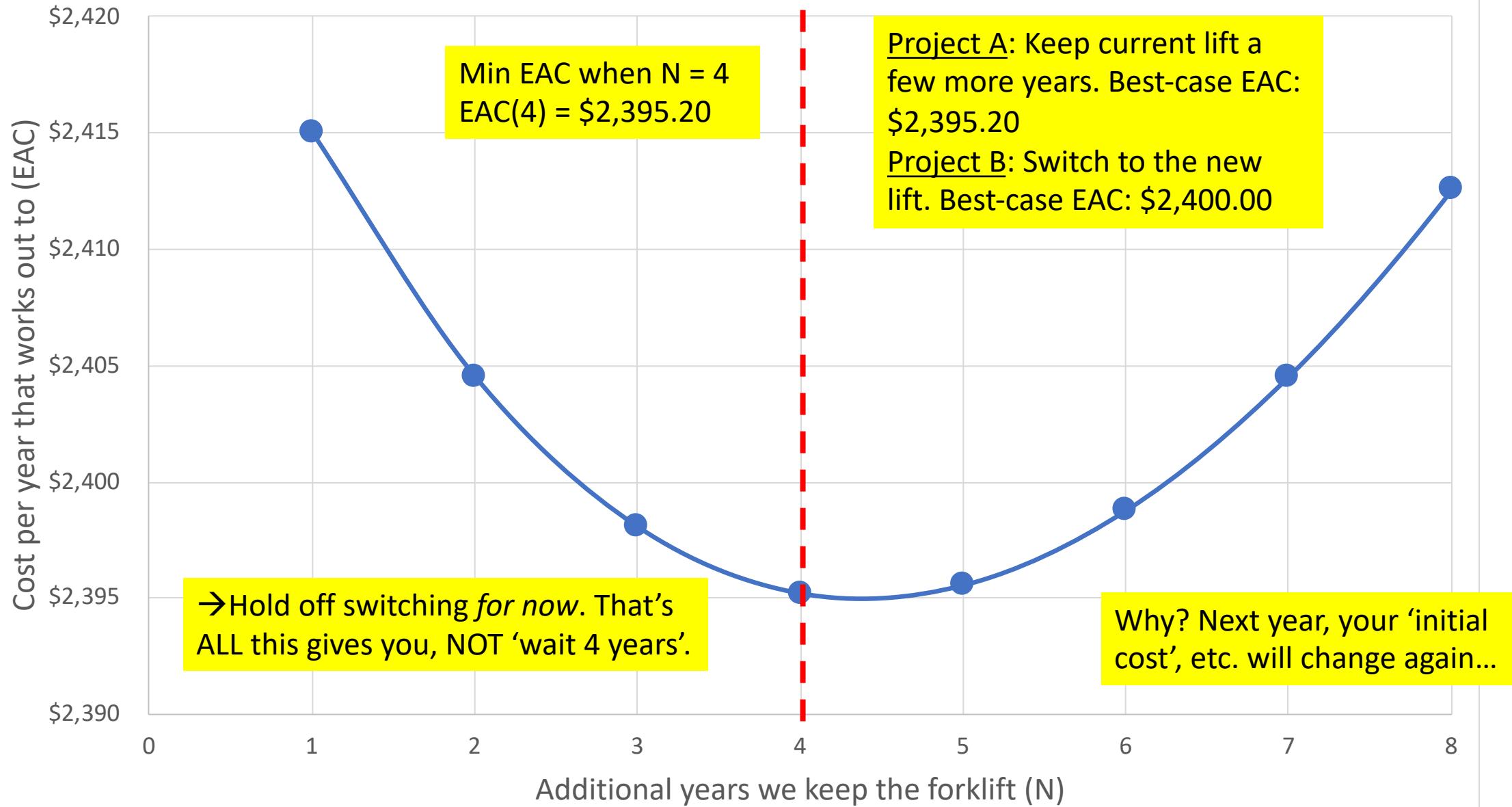
# What's the cost profile of the current 'lift?

- Suppose you've already paid the first two years of operating costs, which were \$1,000 and \$1,100.
- You're two years into ownership, so operating costs are \$1,200 this coming year and will be \$1,300 the year after that.
- Annuity of \$1,200 with an arithmetic gradient with  $G=\$100$  atop it.
- What's the 'initial cost'? Not what you paid for it two years ago...
- What IS an "initial cost," conceptually?
- What you have to give up *today* to get something.
- What do you have to give up today to get something you already have?
- The opportunity of NOT having it – the opportunity of selling it.
- → The 'initial cost' is the current resale value of the forklift.

## After that, it all falls into place

- If we sold the forklift today, two years into its life, we could only get  $\$10,000 \times 0.9^2 = \$8,100$  for it. *That's our 'initial cost'.*
- Resale value, as before, keeps falling by 10% per year.
- Now we have all we need to figure out whether we should replace our forklift *today* with the new one.
- We re-do the numbers, with N as the additional life in years of our defender. Find the minimum EAC for our defender.
- Compare this to the minimum EAC we calculated for the challenger.
- If the challenger's minimum EAC is less, go with the challenger.
- Otherwise, wait a bit (maybe until you'd normally replace the 'lift, since comparing new forklift to new forklift, with no costs sunk in either case, the new one wins.)

## Defender EAC(N): What's the cost per year if we keep it an *additional* N years?



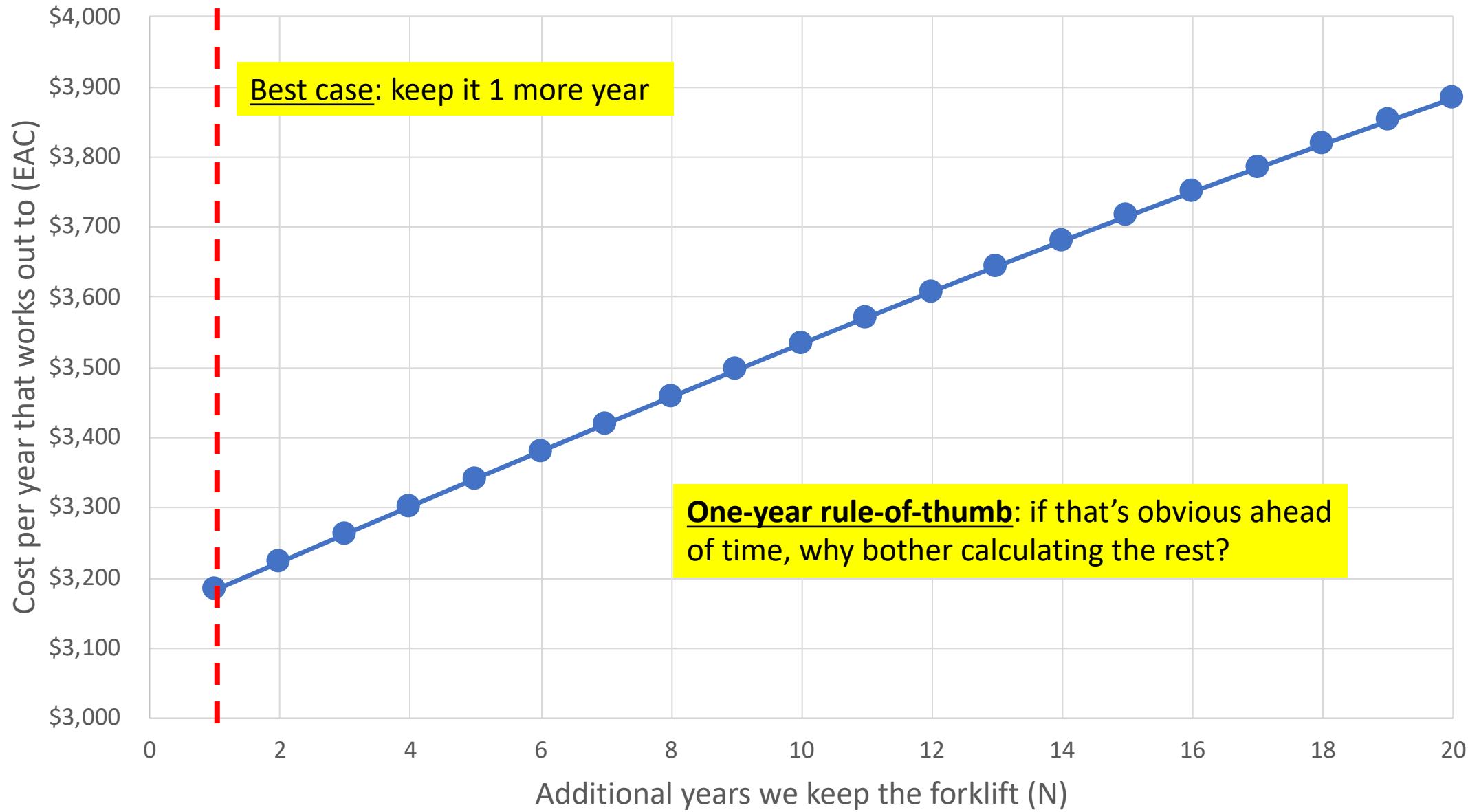
# Do I switch to the shiny new thing *now*?

- If you're using this method, it's because you're looking at a project you intend to keep repeating for a long time (otherwise use NPV).
- Calculate the best-case EAC for the new shiny thing (**the challenger**): switch to it & keep replacing it at the N that gives it its minimum EAC.
- Calculate the best-case EAC for your *existing* thing (**the defender**), remembering to be mindful & stay in the present: costs in the past are over and done with. What's the EAC of keeping it one *more* year? Two *more* years? Etc.
- Compare best-case EAC of the challenger to the best-case EAC of the defender. **RIGHT NOW**, go for whichever has the lowest EAC.
- Defender? Stay with the defender **for now**. Challenger? Switch at once.
- When do you revisit the choice? Your institution will tend to have 'spending seasons'...

# The one-year rule-of-thumb

- NOT something to rely on all the time, but if you REALLY want to avoid calculations (maybe out in the field w/no computer):
- If your defender is *so old* that not only is the initial cost sunk, but you're not giving up much by keeping it in place (low resale/scrap value)...
- ...and by comparison your maintenance costs are very high & rising...
- THEN why bother running the numbers for more than the 'Should I keep the asset one more year/quarter/month' case?
- You're WAY past the point where spreading out big initial costs over more years helps the EAC; all EAC for higher N will just be bigger.
- → Compare the challenger's best-case EAC to the EAC from keeping the defender for just *one more year*.
- BUT you'd better be VERY SURE about this: if you can, best to run the numbers.

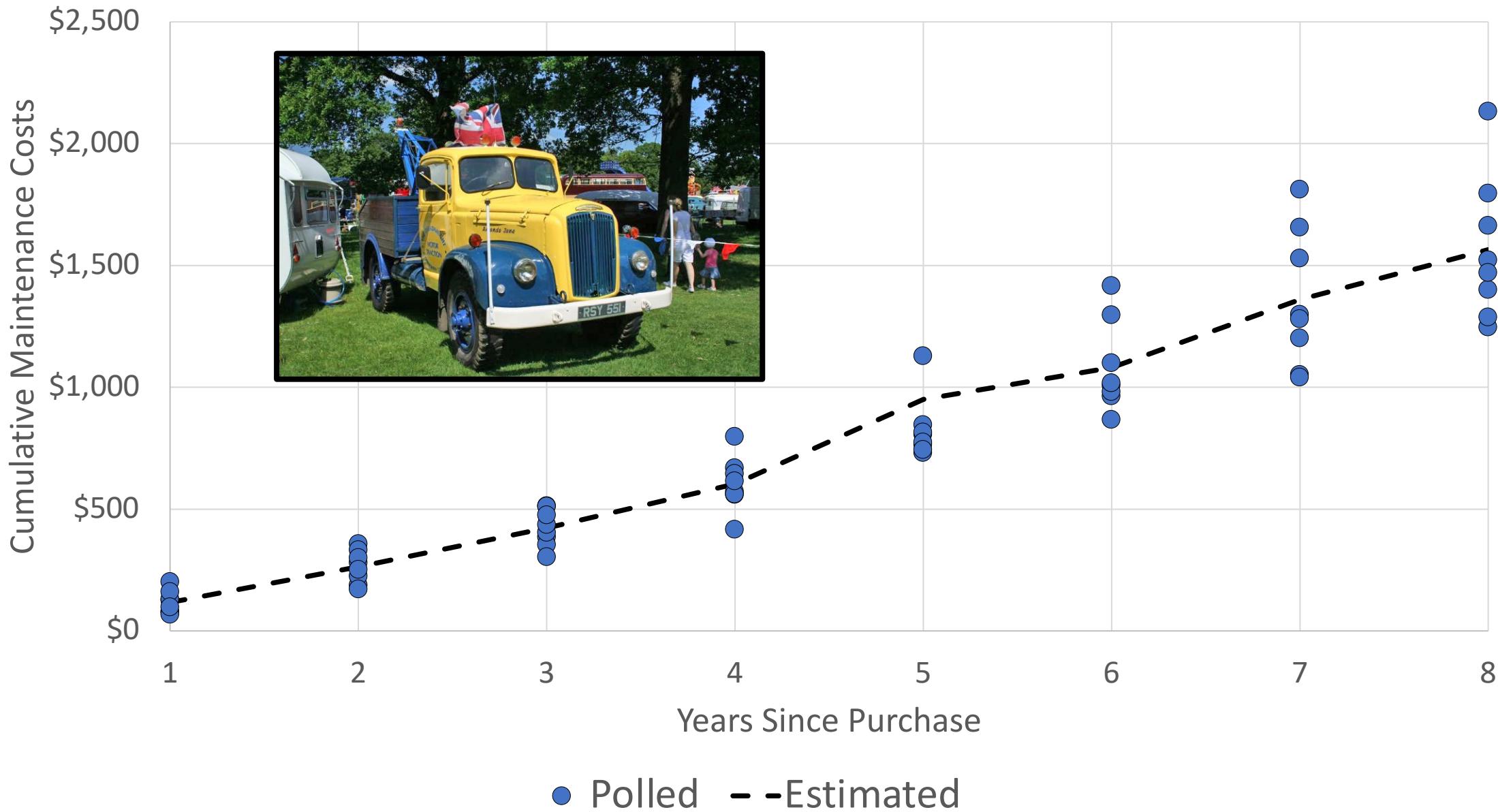
## Defender EAC(N) for a 20-year-old forklift



# A note on maintenance costs

- It's often difficult to obtain or predict yearly maintenance costs with certainty – they must be estimated.
- Even when pasts costs are known and measured, uncertainty increases with time.
- Example 1: Maintenance costs known for a fleet of 7 identical model Morris trucks from 1952 to 1960. Author used the mean as an estimate. Note divergence as time goes by.
- Example 2: S. Korean power plant in operation since 1972. Operating costs only known for 2002 to 2009. Rest must be estimated via least-squares regression.
- “Garbage In, Garbage Out” – Keep this in mind & revise your estimates as appropriate.

# Maintenance Costs for a Morris Flatbed Truck



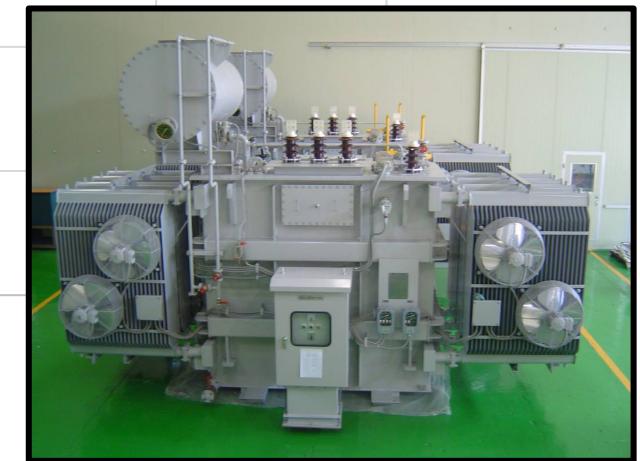
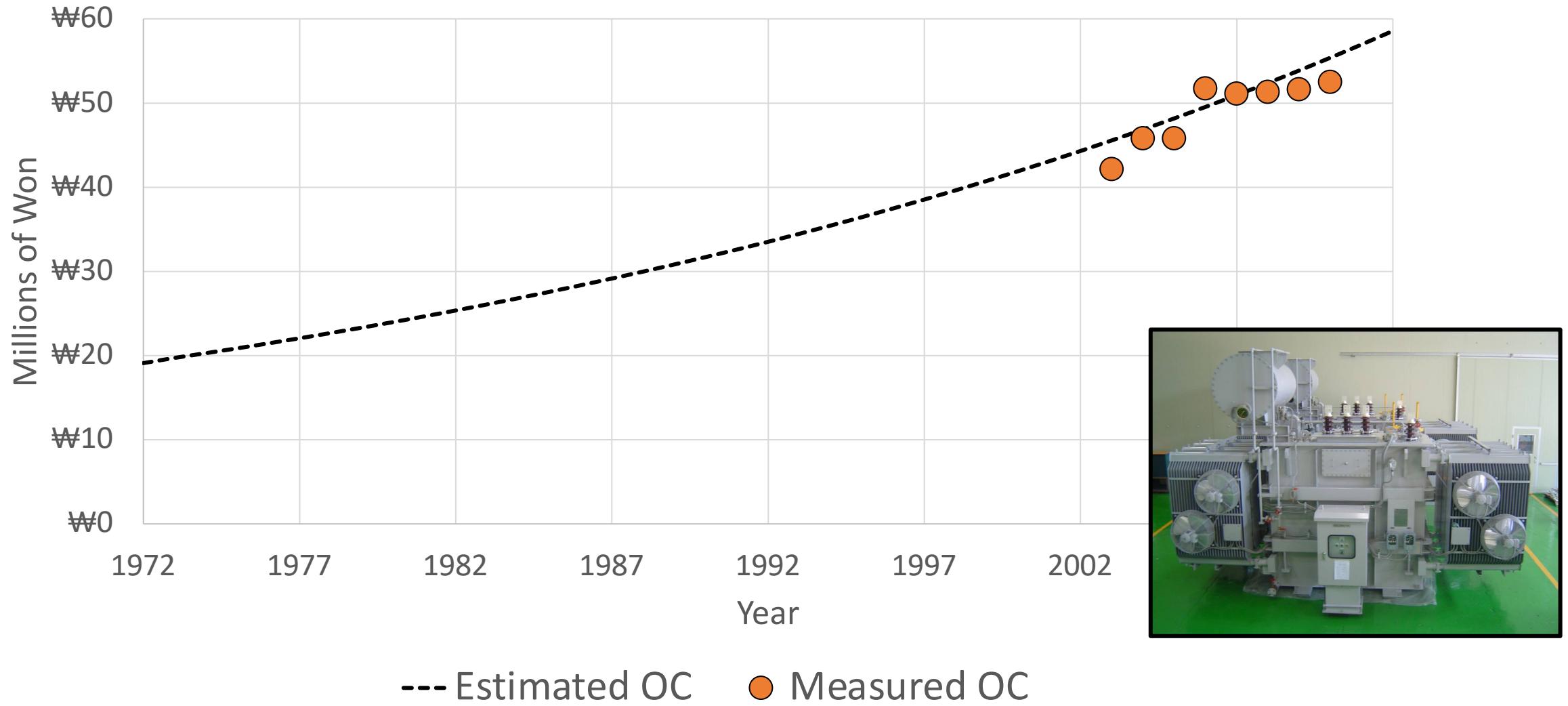
# A South Korean Power Transformer

- Bought in 1972
- Original Cost: 235 million Won
- Estimated Operating Cost in Year t:  
 $19.18 \times 1.0283^t$
- (Found by least-squares regression.)

Pictured: A different South Korean Power Transformer.



# Estimated vs Measured Operating Costs for a South Korean Power Transformer



# Challenger =/= Defender, no repeats

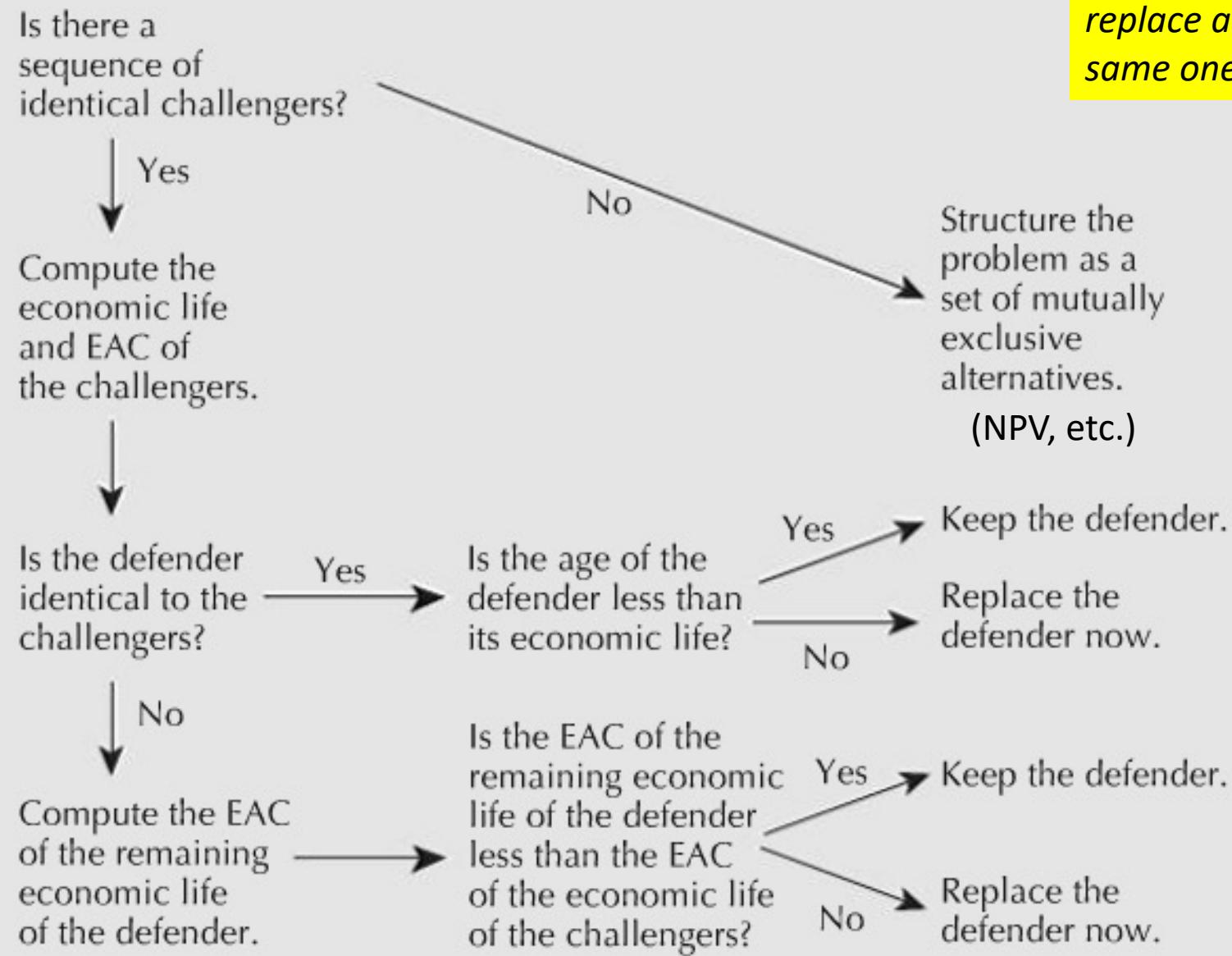
- Surprisingly, this can get very tricky.
- It most often comes up in the context of technological change, when certain challengers aren't available until a certain time period.
- (e.g. You own an iPad 32, the iPad 37 is available now, but you're pretty sure next year the iPad 38 will be out, despite no official announcement, and the iPad 39 two years after that.)
- Suppose you're looking at a five-year planning horizon (project life)
- Currently using Defender (D)
- Challenger A is available now: Max life = 5 years
- Challenger B is available 3 years from now: Max life = 2 years

<b>Choice</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>Total</b>
<b>1</b>	5	0	0	<b>5</b>
<b>2</b>	4	1	0	<b>5</b>
<b>3</b>	4	0	1	<b>5</b>
<b>4</b>	3	2	0	<b>5</b>
<b>5</b>	3	1	1	<b>5</b>
<b>6</b>	3	0	2	<b>5</b>
<b>7</b>	2	2	1	<b>5</b>
<b>8</b>	2	1	2	<b>5</b>
<b>9</b>	1	4	0	<b>5</b>
<b>10</b>	1	2	2	<b>5</b>
<b>11</b>	0	5	0	<b>5</b>
<b>12</b>	0	4	1	<b>5</b>
<b>13</b>	0	3	2	<b>5</b>

- Once you have all the combinations, compute the costs for each...
- ...then run your favourite kind of analysis on them (AW,PW,IRR, etc.)

Thankfully (?), we don't usually know enough about future challengers to run this kind of analysis. In most situations, it's fine to assume a challenger will replace itself indefinitely.

**Figure 7.3 The Replacement Decision Making Process**



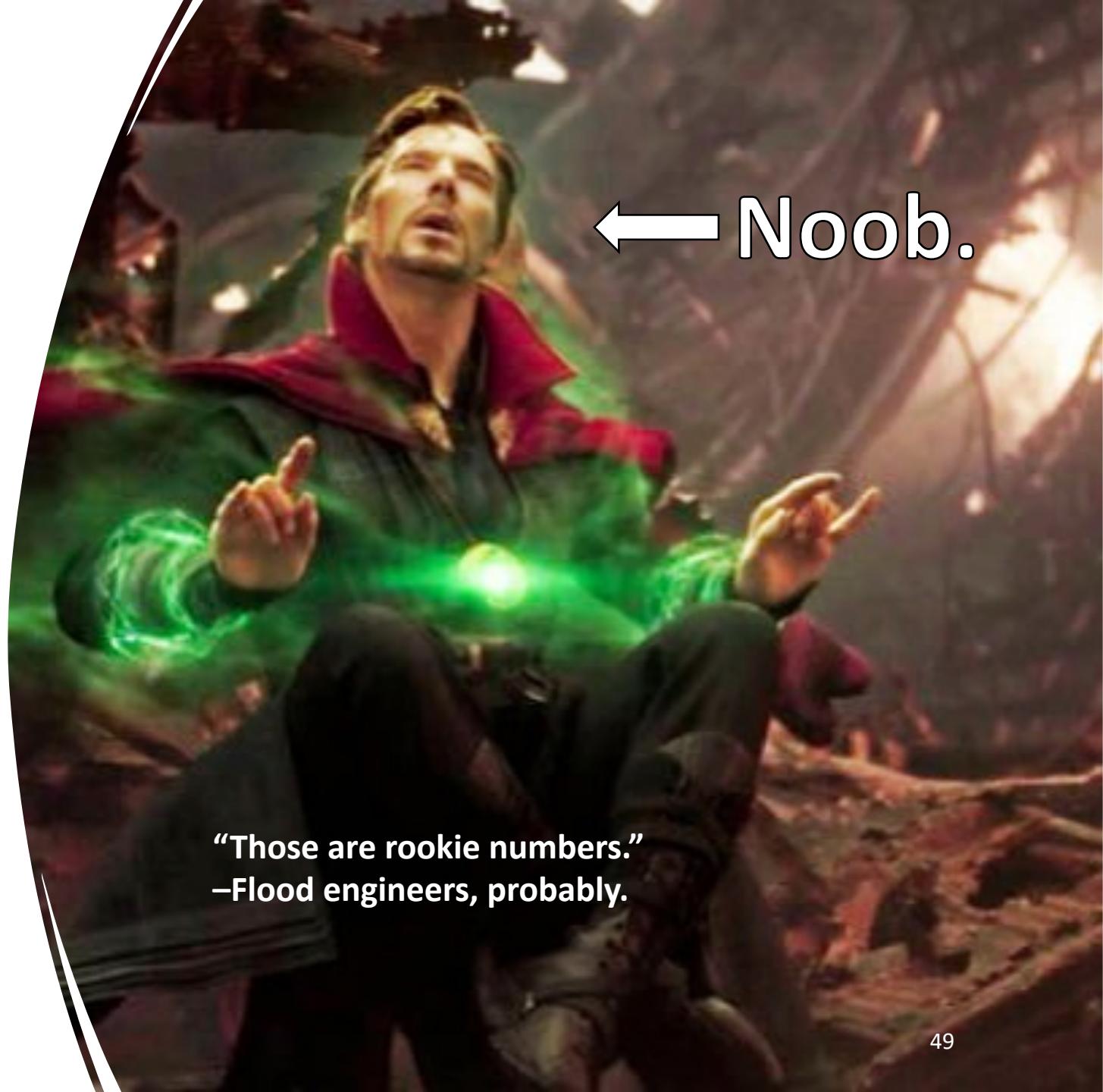
What we've learned today is appropriate *when you're going to replace an asset with exactly the same one over and over.*

Not the case? Use a different technique.

# Case Study: Flood Risk Reduction

From (Schoemaker et al., 2005)

- Flood engineering problem: too many options.
- Suppose engineers have to reinforce (or choose not to reinforce) a dike with 8 sections, once a year from 2016 to 2050.
- It can be shown (see paper) there are  $2.82 \times 10^{12}$  different strategies to be considered, and  $9.87 \times 10^{13}$  computations that must be done to find the *one, optimal strategy*.
- This is a *simple* flood risk project, and orders of magnitude more challenging than the 14 million possibilities examined by Dr. Strange!



# EAC to the Rescue (Schoemaker et al., 2005)

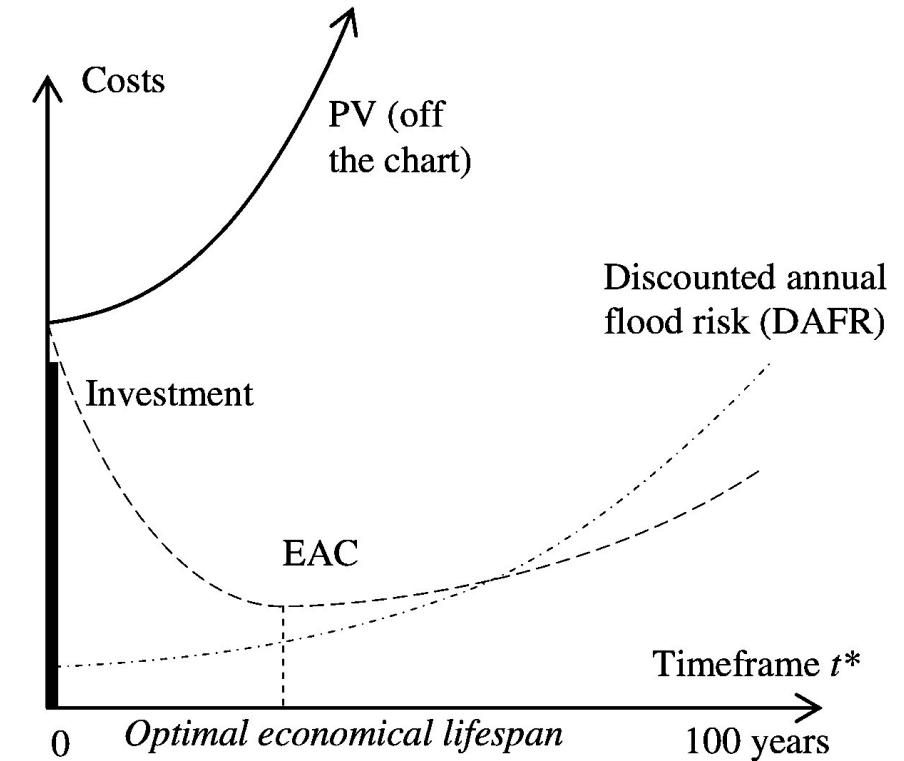
- We can use the EAC approach to dramatically reduce calculations.
- Calculate the minimum EAC for each possible "combination of measures", and find the combination with the lowest cost at its economic lifetime. That's your challenger.
- What's the defender? Flood risk (do nothing for one more year).
- Flood risk only gets worse each year you do nothing, so we can appeal to the one-year rule, and only need to calculate the EAC of doing nothing for one more year.
- → "[A] combination of measures is implemented as soon as the EAC of the optimal combination of measures is lower than the annual flood risk." (Schoemaker et al., 2005)
- The slight twist from the procedure in the rest of this lecture is that we *don't* find the economic lifetime first. Instead...

## In a bit more depth

- Using similar language to the paper:
- for  $t = 1$  to  $N$ , find  $\min(EAC[\text{comb}, t])$
- $\text{comb}$  = combination with the lowest  $EAC(t)$ , including the ‘do nothing’ option (let risk rise for one more year)
- For low  $t$ , ‘do nothing’ is probably the winner, but eventually some combination of preventive measures will have a lower  $EAC(t)$  than ‘do nothing’.
- As soon as that happens, implement the combination in question.
- **Reading the paper? Keep in mind they use *continuous* discounting because of their analytical approach – their time intervals are infinitesimal, so  $e^t$  pops up a lot in their discounting formulas, which are the limits of the discrete-time ones used in ECON 180.**

## What does this buy us?

- Considering the NPV of  $x$  combination of measures and  $m$  possible implementation timings  $\rightarrow m(1+m)^x$  computations needed.
- (Combinatorics, see paper's equations (2) – (5)).
- Using this EAC approach, that goes down to  $2^x m t^*$ , where  $t^*$  = the number of years for which the EACs are evaluated before finding the switching point between 'do nothing' & 'do something'.
- For  $t^*=10$ , the original  $9.78 \times 10^{13}$  calculations become 89,600.



**Figure 1.** Determination of optimal economical lifespan using the EAC

(Schoemaker et al., 2005)