Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n (n+5)^n}{(7n)^n}$ converge absolutely, converge conditionally, or diverge?

A series $\sum a_n$ converges absolutely (is absolutely convergent) if the corresponding series of absolute values, $\sum |a_n|$, converges. If the series converges, but is not absolutely convergent, then the series converges conditionally. Otherwise, the series diverges.

Find the terms of the corresponding series of absolute values.

$$\left|(-1)^n \frac{(n+5)^n}{(7n)^n}\right| = \frac{(n+5)^n}{(7n)^n}$$

Since the numerator and denominator of $\frac{(n+5)^n}{(7n)^n}$ both involve a power of n, use the Root Test to determine if $\sum_{n=1}^{\infty} \frac{(n+5)^n}{(7n)^n}$ converges.

For the Root Test, let $\sum a_n$ be any series and suppose that $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \rho$. If $\rho < 1$, then the series converges absolutely. If $\rho > 1$ or ρ is infinite, then the series diverges. If $\rho = 1$, then the test is inconclusive.

First, find $\sqrt[n]{|a_n|}$ for the series $\sum_{n=1}^{\infty} \frac{(n+5)^n}{(7n)^n}$.

$$\sqrt[n]{\left|\frac{(n+5)^n}{(7n)^n}\right|} = \left|\frac{n+5}{7n}\right|$$

Now find $\lim_{n\to\infty} \sqrt[n]{|a_n|}$.

$$\lim_{n\to\infty} \left|\frac{n+5}{7n}\right| = \frac{1}{7}$$

Since $\frac{1}{7} < 1$, the series $\sum_{n=1}^{\infty} \frac{(n+5)^n}{(7n)^n}$ converges per the Root Test.

Therefore, the given series, $\sum_{n=1}^{\infty} \frac{(-1)^n (n+5)^n}{(7n)^n}$, converges absolutely.