Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3]

Does the series below converge or diverge? Give a reason for your answer. (When checking your answer, remember there may be more than one way to determine the series' convergence or divergence.)

$$\sum_{n=-3}^{\infty} \frac{-22}{n+4}$$

Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$. If f is a continuous, positive, decreasing function of x for all $x \geq N$ (N is a positive integer), then the Integral Test can be used to determine the convergence of $\sum_{n=N}^{\infty} a_n$. The Integral Test states that the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)dx$ both converge or both diverge.

Notice that the terms in the given series are not positive. However, the series can be rewriten using the constant multiple rule for series. The constant multiple rule says that two series that differ by a constant multiple must both converge or both diverge.

$$\sum_{n=-3}^{\infty} \frac{-22}{n+4} = -22 \sum_{n=-3}^{\infty} \frac{1}{n+4}$$

Let $a_n = \frac{1}{n+4}$, then define a function $f(x) = \frac{1}{x+4}$. This function meets the criteria for the integral test. Find the antiderivative.

$$\int \frac{1}{x+4} dx = \ln(x+4) + C$$

Now evaluate the improper integral.

$$\begin{aligned} \int_{-3}^{\infty} \frac{1}{x+4} dx &= \lim_{b \to \infty} \int_{-3}^{b} \frac{1}{x+4} dx \\ &= \lim_{b \to \infty} \left[\ln(x+4)\right]_{-3}^{b} \\ &= \lim_{b \to \infty} \left[\ln(b+4) - \ln(-3+4)\right] \\ &= \lim_{b \to \infty} \ln(b+4) \\ &= \infty \end{aligned}$$

Since the integral $-22 \int_{-3}^{\infty} \frac{1}{x+4} dx$ diverges, by the integral test, the series $\sum_{n=-3}^{\infty} \frac{-22}{n+4}$ diverges.