# 202201 Math 122 Assignment 1

Due Friday, January 28 at 23:59. Please submit your solutions on your section's Crowdmark page.

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There are five questions of equal value (worth a total of 45 marks), and one bonus question (worth 4 marks). Please feel free to discuss these problems with each other. You may not access any "tutoring" or "help" website in any way. In the end, each person must write up their own solution, in their own words, in a way that reflects their own understanding. Complete solutions are those which are coherently written, and include appropriate justifications.

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1. Consider the statement "*If I had a million dollars, I'd buy you a green dress and a monkey*". Is it logically equivalent to "*If I buy you neither a green dress nor a monkey, then I don't have a million dollars*"? How about to "*I don't have a million dollars, or I'd buy you a green dress and a monkey*"? And what about to "*To buy you a green dress or a monkey, I need to have a million dollars*"? Justify each answer.

2. Your older brother tells you "*To do well on the Math 122 quizzes you need to have gone through the notes at least three times and done at least three of the old quizzes. If you have not gone through the notes at least three times and not done at least three of the old quizzes, then you will not do well on the Math 122 quizzes.*" Write these statements in symbolic form using:

   "$q$: you do well on the Math 122 quizzes",

   "$n$: you have gone through the notes at least three times", and

   "$t$: you have done at least three of the old tests".

   Then, determine whether the two statements in the instructions are logically equivalent. According to your older brother, can a student who has done at least three of the old quizzes but not gone through the notes at all do well on the Math 122 quizzes? Choose one of the following and explain: (i) No; (ii) Yes; (iii) Unclear / can't tell; (iv) Technically not, but why not give it a shot? What does he know? Hard work pays off over time, but laziness pays off now!

3. Let $s_1, s_2, s_3$ be statements. Suppose $s_1 \to s_2$ and $s_2 \to s_3$ and $s_3 \to s_1$ are tautologies.

   (a) If $s_1$ is true, what is the truth value of $s_2$? Explain.

   (b) If $s_1$ is false, what is the truth value of $s_2$? Explain.

   (c) Are $s_1$ and $s_2$ logically equivalent? Why or why not?

   (d) Are the statements $s_1, s_2$ and $s_3$ all logically equivalent? Explain.

4. (a) Show that the statements $p \vee q$ and $p \wedge q$ can be represented using only the logical connective $\rightarrow$ and negation.

   (b) Explain why the result in (a) means every statement has a representation using only the logical connective $\rightarrow$ and negation.

   (c) Represent $p \leftrightarrow q$ using only the logical connective $\rightarrow$ and negation, and justify your answer.

5. (a) Use the Laws of Logic and known logical equivalences to show that $(\neg a \rightarrow b) \wedge [\neg b \vee \neg(a \wedge b)]$ is logically equivalent to $\neg(a \leftrightarrow b)$.

   (b) Show that $p \wedge [(\neg q \leftrightarrow p) \wedge q]$ is a contradiction by using the Laws of Logic and known logical equivalences to show that it is logically equivalent to $\mathbf{0}$.

6. Bonus question [4 marks]. An accused bank robber is on trial in court. The Crown prosecutor says "*If the accused is guilty, then he did not commit this crime alone.*" The defence attorney calmly replies "*No, that's not true.*" The judge then rules that the accused is guilty and sends him to jail. Explain the judge's reasoning.