Q_2 (Part 2)

We got this from the properties mentioned in the question

$$\begin{cases} x+y+z = 15 \\ x+z = 2y \\ y+x = 8 \\ z-y = \text{Chris's Erdős Number} \\ \left(x+\frac{1}{x}\right)+\left(y+\frac{1}{y}\right)+\left(z+\frac{1}{z}\right) = 1110 \end{cases}$$

$$\begin{array}{r} xyz \\ (+)\ zyx \\ \hline 1110 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 1 & 15 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & 0 & 8 \end{bmatrix}$$

$$\xrightarrow{r_1 \rightleftharpoons r_3} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & 1 & 15 \end{bmatrix} \xrightarrow{r_3 - r_1} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 0 & -3 & 1 & -8 \\ 0 & 0 & 1 & 7 \end{bmatrix} \xrightarrow{4r_1 + r_2} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 4 & 1 & 1 & 24 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\xrightarrow{r_2 - r_3} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 4 & 1 & 0 & 17 \\ 0 & 0 & 1 & 7 \end{bmatrix} \xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 1 & 0 & 8 \\ 3 & 0 & 0 & 9 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\xrightarrow[(1/3)r_2]{r_2 \rightleftharpoons r_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 1 & 1 & 0 & 8 \\ 0 & 0 & 1 & 7 \end{bmatrix} \xrightarrow{r_2 - r_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

Here, $x=3$, $y=5$, $z=7$. Chris's number is $(z-y) = (7-5) = 2$ (Ans)