CSC 225 FALL 2022
ALGORITHMS AND DATA STRUCTURES I
ASSIGNMENT 4 - WRITTEN
UNIVERSITY OF VICTORIA

1. [4 marks] Write a sorting algorithm, in pseudocode, that takes an array A with n elements and uses two stacks to sort it. You may only compare array elements to stack elements in your sort, that is, you may not compare array elements to each other. You may assume the following stack ADT methods:

   `push(`o`)`: Insert object o at the top of the stack
   `pop()`: Remove and return element from top of the stack, provided it is not empty
   `top()`: Returns the top element from the stack without removing it, provided it is not empty
   `isEmpty()`: Returns true is the stack is empty and false otherwise

   Analyze the worst-case running time of your algorithm.

2. [4 marks] Show the various steps of Selection Sort, Bubble Sort, Insertion Sort and Mergesort on the example array, $A = [5, 7, 0, 3, 4, 2, 6, 1]$.

3. [4 marks] Consider a version of the quicksort algorithm that uses the element at rank $\lfloor n/2 \rfloor$ as the pivot for a sequence on n elements. What is the running time of this version on a sequence that is already sorted? Describe the kind of sequence that would cause this version of quick-sort to run in $\Theta(n^2)$ time. Justify your answers.

4. [4 marks] Below is the pseudocode implementation of insertion sort that I gave in class. Use a loop invariants proof to show that insertion sort is correct.

```
Algorithm insertionSort(A,n):
    Input: Array A of size n
    Output: Array A sorted
    for k ← 1 to n-1 do
    val ← A[k]
            j ← k–1
            while j ≥ 0 and A[j] > val do
                    A[j+1] ← A[j]
                    j ← j – 1
            end
            A[j+1] = val
    end
end
```

5. [4 marks] Define the ***internal path length***, $I(T)$, of a tree T to be the sum of the depths of all the internal nodes in T. Likewise, define the ***external path length***, $E(T)$, of a tree T to be the sum of the depths of all the external nodes in T. Use induction to show that if T is a proper binary tree with n internal nodes, then $E(T) = I(T) + 2n$.