Symbolab

Solution

$\sum_{n=1}^{\infty}3(x+7)^{n-1}$: Radius of convergence is 1, Interval of convergence is $-8<x<-6$

Steps

$\sum_{n=1}^{\infty}3(x+7)^{n-1}$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=1}^{\infty}3(x+7)^{n-1}$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{3(x+7)^{(n+1)-1}}{3(x+7)^{n-1}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{3(x+7)^{(n+1)-1}}{3(x+7)^{n-1}}\right|\right)$ *Hide Steps*

$L=\lim_{n\to\infty}\left(\left|\frac{3(x+7)^{(n+1)-1}}{3(x+7)^{n-1}}\right|\right)$

Simplify $\frac{3(x+7)^{(n+1)-1}}{3(x+7)^{n-1}}$: $x+7$ *Hide Steps*

$\frac{3(x+7)^{(n+1)-1}}{3(x+7)^{n-1}}$

Remove parentheses: $(a)=a$

$=\frac{3(x+7)^{n+1-1}}{3(x+7)^{n-1}}$

$1-1=0$

$=\frac{3(x+7)^{n}}{3(x+7)^{n-1}}$

Divide the numbers: $\frac{3}{3}=1$

$=\frac{(x+7)^{n}}{(x+7)^{n-1}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{(x+7)^{n}}{(x+7)^{n-1}}=(x+7)^{n-(n-1)}$

$=(x+7)^{n-(n-1)}$

Add similar elements: $n-(n-1)=1$

$=x+7$

$L=\lim_{n\to\infty}\left(|x+7|\right)$

$L=|x+7|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$L=|x+7|\cdot 1$

Simplify

$L=|x+7|$

$L=|x+7|$

The power series converges for $L<1$

$|x+7|<1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$|x+7|<1$

Therefore

Radius of convergence is 1

Radius of convergence is 1

Find the interval of convergence Hide Steps

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$ solve for x

$|x+7|<1 \quad : \quad -8<x<-6$ Hide Steps

$|x+7|<1$

Apply absolute rule: If $|u|<a,\ a>0$ then $-a<u<a$

$-1<x+7<1$

$x+7>-1$ and $x+7<1$ Hide Steps

$x+7>-1$ and $x+7<1$

$x+7>-1 \quad : \quad x>-8$ Hide Steps

$x+7>-1$

Subtract 7 from both sides

$x+7-7>-1-7$

Simplify

$x>-8$

$x+7<1 \quad : \quad x<-6$ Hide Steps

$x+7<1$

Subtract 7 from both sides

$x+7-7<1-7$

Simplify

$x<-6$

Combine the intervals

$x>-8$ and $x<-6$

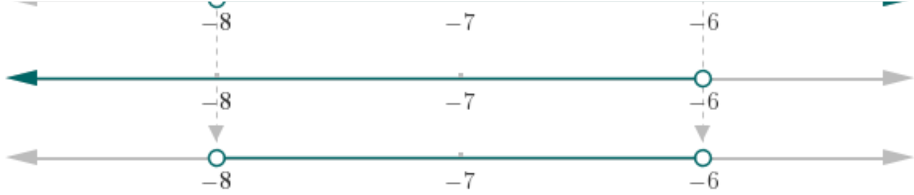
$x>-8$ and $x<-6$

Merge Overlapping Intervals Hide Steps

The intersection of two intervals is the set of numbers which are in both intervals

$x>-8$ and $x<-6$

$-8<x<-6$



$-8<x<-6$

$-8<x<-6$

Check the interval end points: $x=-8$:diverges, $x=-6$:diverges Hide Steps

For $x=-8,\ \sum_{n=1}^{\infty} 3((-8)+7)^{n-1}$: diverges Hide Steps

$\sum_{n=1}^{\infty} 3((-8)+7)^{n-1}$

Refine

$=\sum_{n=1}^{\infty} 3(-1)^{n-1}$

Apply the constant multiplication rule: $\sum c\cdot a_n = c\cdot\sum a_n$

$=3\cdot\sum_{n=1}^{\infty}(-1)^{n-1}$

Apply Series Divergence Test: diverges Hide Steps

$\sum_{n=1}^{\infty}(-1)^{n-1}$

Series Divergence Test:

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ diverges

$\lim_{n\to\infty}\left((-1)^{n-1}\right)=$ diverges Hide Steps

$\lim_{n\to\infty}\left((-1)^{n-1}\right)$

Apply Limit Divergence Criterion: diverges Hide Steps

$\lim_{n\to\infty}\left((-1)^{n-1}\right)$

Limit Divergence Criterion Test:

If two sequences exist, $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ with

$x_n \neq c$ and $y_n \neq c$

$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c$

$\lim_{n\to\infty} f(x_n) \neq \lim_{n\to\infty} f(y_n)$

Then $\lim_{x\to c} f(x)$ does not exist

$c=\infty$, $x_n=2k, y_n=2k+1$

$\lim_{k\to\infty}(2k)=\infty$ *Hide Steps*

$\lim_{k\to\infty}(2k)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty, a>0,$ n is odd

$a=2, n=1$

$=\infty$

$\lim_{k\to\infty}(2k+1)=\infty$ *Hide Steps*

$\lim_{k\to\infty}(2k+1)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty, a>0,$ n is odd

$a=2, n=1$

$=\infty$

$\lim_{n\to\infty}x_n=\lim_{n\to\infty}y_n=c=\infty$

$\lim_{k\to\infty}\left((-1)^{2k-1}\right)=-1$ *Hide Steps*

$\lim_{k\to\infty}\left((-1)^{2k-1}\right)$

$(-1)^{2k-1}=(-1), \forall k\in\mathbb{Z}$

$=\lim_{k\to\infty}(-1)$

$\lim_{k\to\infty}(-1)=-1$ *Hide Steps*

$\lim_{k\to\infty}(-1)$

$\lim_{x\to a}c=c$

$=-1$

$=-1$

$\lim_{k\to\infty}\left((-1)^{(2k+1)-1}\right)=1$ *Hide Steps*

$\lim_{k\to\infty}\left((-1)^{(2k+1)-1}\right)$

$(-1)^{(2k+1)-1}=1, \forall k\in\mathbb{Z}$

$=\lim_{k\to\infty}(1)$

$\lim_{k\to\infty}(1)=1$ *Hide Steps*

$\lim_{k\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$=1$

$\lim_{n\to\infty}f(x_n)\neq\lim_{n\to\infty}f(y_n)$

Therefore $\lim_{n\to\infty}\left((-1)^{n-1}\right)$ is divergent at $n\to\infty$

$=$ diverges

$=$ diverges

By the divergence test criteria

$=$ diverges

$=3$diverges

$=$ diverges

For $x=-6, \sum_{n=1}^{\infty}3((-6)+7)^{n-1}$: diverges *Hide Steps*

$\sum_{n=1}^{\infty}3((-6)+7)^{n-1}$

Refine

$=\sum_{n=1}^{\infty}3$

Every infinite sum of a non – zero constant diverges

$=$ diverges

$x=-8$:diverges, $x=-6$:diverges

Therefore

Interval of convergence is $-8<x<-6$

Interval of convergence is $-8<x<-6$

Radius of convergence is 1, Interval of convergence is $-8<x<-6$