MATHEMATICS 101 (Sections A01-A02),
Midterm # 1, September 26, 2017.
September 28, 551 B.C. – Happy birthday, Confucius!
"With education there is no distinction between classes or races of men."
Time: 45 minutes

Last name: _____ StudentID: $V00$_____

First name: _____ Tutorial section number: T_____

Problem #	1 – 2	3	4	5	6	7	8	9	10	TOTAL
Points (max)	2	2	2	4	2	4	3	2	4	25
Score										

- The only calculators allowed on any examination are Sharp EL-510R, Sharp EL-510 RN and Sharp EL-510RNB.
- This test consists of 10 questions and has 10 pages (including this cover, the **Blank page** and a **Formula sheet** on the last page).
  - Questions 1 through 2 are multiple-choice. Write your full answer in this booklet in the provided space. **Clearly mark your final answer among the multiple choises.** You need to show your work for all answers, as we may disallow any answer which is not properly justified.
  - Questions 3 through 10 are long-answer. Write your detailed solutions in space provided in this booklet.
- For the multiple-choice questions, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices.
- Before starting your test enter your Name (Last, First), student ID, and tutorial section number (T01 - T11) on this page.
- If you have finished working on your paper with less than 10 minutes before the end of the examination, please close your paper and **remain seated** until the test time is completed. It is important to minimize the desruptions in the room.
- At the end of 45-minute test, turn-in this booklet.
- This is version B of the Midterm #1.

For the questions #1 – #8, calculate following integrals:

1. (1 point) $\int_0^1 x^2 e^{x^3}\, dx =$

(A) -4.5 (B) -3.5 (C) -2.5 (D) -1.5 (E) -0.5
(F) 0.5 (G) 1.5 (H) 2.5 (I) 3.5 (J) 4.5

2. (1 point) $\int_1^7 \frac{dx}{x-10} =$

(A) -4.1 (B) -3.1 (C) -2.1 (D) -1.1 (E) -0.1
(F) 0.1 (G) 1.1 (H) 2.1 (I) 3.1 (J) 4.1

3. (2 points) $\displaystyle\int_0^{\pi/4} \frac{\tan^2\theta}{\cos^2\theta}\, d\theta =$

4. (2 points) If $\displaystyle\int_0^2 f(t)\, dt = 5$, calculate $\displaystyle\int_1^2 f(4-2t)\, dt =$

5. (4 points) $\int \arctan y \, dy = \int \tan^{-1} y \, dy =$

6. (2 points) $\int \sin(4x)\sin(6x) \, dx =$

7. (4 points) $\int e^{-2\theta} \sin\theta \, d\theta =$

8. (3 points) $\int \frac{dx}{x\sqrt{1-9x^2}} =$

(4) $\int \frac{\log_{10} x}{x}\,dx$ $\log_a x = \frac{1}{x \ln a}$ $\frac{1 \cdot e}{\ln e} = \ln$

$\int \frac{\log_{10} x}{x}\,dx$ $\log_{10} x = \frac{\ln x}{\ln 10}$

$\log_5 7 = \frac{\ln 7}{\ln 5}$

$\int \frac{\ln(x)}{x \ln 10}\,dx = \frac{1}{\ln 10}\int \frac{\ln x}{x}\,dx$ $u = \ln x$, $du = \frac{1}{x \ln e} = \frac{1}{x}$

$= \frac{1}{\ln 10}\int u\,du$

$= \frac{(\ln x)^2}{2 \ln 10} + C$

$\frac{d}{dx} u^2 = 2u$

(5) $\int x^7 \sqrt{x^4 + 1^4}\,dx$ trig sub time 2

oh there's a x^4 but it still can work

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$\int \tan^7(\theta) \sec^3(\theta) \sec^2(\theta)$ $x = \tan(\theta)$

$\int \tan^7(\theta) \sec^6(\theta)$ $dx = \sec^2(\theta)$

$\sqrt{x^4+1}\ \sec^2(\theta)$

$(\tan^2\theta + 1)^2 = \sec^2\theta$

$= \sqrt{\tan^4\theta + 1}$

9. (2 points) Calculate derivative of the function $y = \sin^{-1}(\tanh x)$.
Simplify your answer as far as possible.

(3) $\int t^2 \cos t \, dt$

$u = t^2 \quad dv = \cos(t)\,dt$
$du = 2t\,dt \quad V = \sin(t)$

$= t^2\sin(t) - \int 2t\sin(t)\,dt \quad u = 2t \quad dv = \sin(t)\,dt$
$du = 2\,dt \quad v = -\cos(t)$

$= t^2\sin(t) + 2t\cos(t) + \int -\cos(t)2\,dt$

$-\cos(t)$

$= t^2\sin(t) + 2t\cos(t) + 2\sin(t) + C$

① $\int_1^{\sqrt{2}} x\, 2^{(x^2)}\, dx$

$\frac{d}{dx} a^x = a^x \ln|a|$

$u = 2^{(x^2)}$ $du = 2^{x^2} \ln 2$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$\int \frac{d}{dx} a^x dx = \int a^x \ln a\, dx$

$a^x = \ln a \int a^x dx$

redo!

$\frac{a^x}{\ln a} = \int a^x dx$

$\int_1^{\sqrt{2}} \frac{du}{2} 2^u$

$\int_1^{\sqrt{2}} \frac{du}{2} 2^u$

$\frac{2^u}{2\ln|2|}\Big|_1^{\sqrt{2}}$

$\frac{2^{x^2}}{2\ln|2|}\Big|_0^{\sqrt{2}} = \frac{4}{2\ln|2|} - \frac{2}{2\ln|2|}$

$= \frac{2}{2\ln|2|} = \frac{1}{\ln|2|}$ ✓

② Solve in terms of x and y redo

$\frac{dy}{dx} = \sqrt{y}\cos^2\sqrt{y}$

$dy = \sqrt{y}\cos^2(\sqrt{y})\,dx$

$\frac{dy}{\sqrt{y}\cos^2(\sqrt{y})} = dx$

$\int \frac{dy}{\sqrt{y}\cos^2(\sqrt{y})} = x + C$ $u = \sqrt{y}$

$du = \frac{1\,dy}{\sqrt{y}}$

$\int \frac{du}{\cos^2(u)} = x + C$

$\int \sec^2(u) = x + C$

$\tan(u) = x + C$

$\tan(\sqrt{y}) + C = x + C$

10. (4 points) A bank account earns interest continuously at a rate of 10% per year of the current balance, B(t). Assume that the initial deposit is \$500, and that no other deposits or withdrawals are made.

(a). Derive general formula for calculating balance in the bank account at the time t, from given differential equation $\frac{dB}{dt} = 0.1B$ taking into account the initial deposit sum. Show all your work.

(b). Determine how long will it take for the balance in the bank account to reach $\$3,000$?

MATHEMATICS 101 (Sections A01-A02)
Formula sheet, Fall 2017
Midterms and Final examinations.

Table of Integrals

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, (u < a)$
2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$
3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, (u > a)$
4. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, (a > 0)$
5. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, (u > a > 0)$
6. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$
7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C, (a > u > 0)$
8. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C, (u > 0)$
9. $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + C$
10. $\displaystyle\int \csc u\, du = -\ln|\csc u + \cot u| + C$

Trigonmetric and Hyperbolic Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$
2. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
3. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
4. $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
5. $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
6. $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
7. $\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$
8. $\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$
9. $\cosh^2(x) - \sinh^2(x) = 1$
10. $\sinh(2x) = 2\sinh(x)\cosh(x)$
11. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
12. $\text{sech}^{-1}(x) = \cosh^{-1}\left(\frac{1}{x}\right)$
13. $\text{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$
14. $\coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$