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Find the length of the curve $x = \frac{y^3}{9} + \frac{3}{4y}$ on $3 \le y \le 5$.

The length L of the curve on $a \le y \le b$ defined by $x = g(y)$ is $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy$.

The derivative with respect to y of $x = \frac{y^3}{9} + \frac{3}{4y}$ is $\frac{dx}{dy} = \frac{y^2}{3} - \frac{3}{4y^2}$.

Expand the square, and simplify.

$$\left[\frac{y^3}{9} - \frac{3}{4y}\right]_3^5 = 1 + \left(\frac{y^2}{3} - \frac{3}{4y^2}\right)^2$$

$$= 1 + \frac{y^4}{9} - \frac{1}{2} + \frac{9}{16y^4}$$

$$= \frac{y^4}{9} + \frac{1}{2} + \frac{9}{16y^4}$$

The expression $\frac{y^4}{9} + \frac{1}{2} + \frac{9}{16y^4}$ is a perfect square and is equal to $\left(\frac{y^2}{3} + \frac{3}{4y^2}\right)^2$.

$$L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\, dy = \int_3^5 \left[\sqrt{\left(\frac{y^2}{3} + \frac{3}{4y^2}\right)^2}\right] dy$$

$$= \int_3^5 \left(\frac{y^2}{3} + \frac{3}{4y^2}\right) dy$$

$$= \left[\frac{y^3}{9} - \frac{3}{4y}\right]_3^5$$

$$= \frac{(5)^3}{9} - \frac{3}{4(5)} - \left[\frac{(3)^3}{9} - \frac{3}{4(3)}\right]$$

$$= \frac{989}{90}$$

Thus, the length of the curve is $\frac{989}{90}$.