CSc 225, Spring 2003/04

Algorithms and Data Structures Final Examination

April 14, 2004

University of Victoria

Student ID	Name

INSTRUCTIONS

- DURATION: 3 hours
- All answers to be written on examination paper.
- There are 7 questions on 16 pages. Marks for each question are indicated on the question sheet.
- This is a closed books, closed notes, no calculators exam. No aids are allowed.

Q1 (24 marks)		**Q5 (40 marks)**	
Q2 (40 marks)		**Q6 (15 marks)**	
Q3 (10 marks)		**Q7 (20 marks)**	
Q4 (15 marks)			
Total (164)			

Question 1. (24 marks) For **each** of the following statements decide whether it is true (**T**) or false (**F**). Circle the correct answer.

1) $\log(n^{12})$ is $O(n)$.	**T**	**F**
2) $3 \log(n) + \log(\log n)$ is $O(\log n)$.	**T**	**F**
3) $4n^2 + 17n$ is $o(n^3)$.	**T**	**F**
4) To prove correctness of an algorithm, we can show the loop invariant for every branch in the algorithm.	**T**	**F**
5) The running time of the Linear Selection algorithm is $O(\log n)$.	**T**	**F**
6) For an input of size n, every recursive algorithm has a running time of $O(n^2 \log n)$.	**T**	**F**
7) For every input, Merge Sort is faster than Selection Sort.	**T**	**F**
8) Given a reference point, a point set in the Euclidean plane can be sorted in clockwise order around the reference point.	**T**	**F**
9) Radix-sort is a stabil sorting algorithm.	**T**	**F**
10) Every heap is an AVL tree.	**T**	**F**
11) Every binary search tree storing n elements has height at most $O(\log n)$.	**T**	**F**
12) The running time of the BFS traversal is linear in the number of edges in the graph.	**T**	**F**
13) The Floyd-Warshall algorithm to compute the transitive closure for a given directed graph uses the algorithm design technique dynamic programming.	**T**	**F**
14) An undirected graph is a special case of a directed graph.	**T**	**F**
15) P = NP was shown by Muller in 1989.	**T**	**F**

16) Every spanning tree of an (edge-)weighted undirected graph has minimum weight. **T** **F**

17) DFS traversal for a graph G can be used to compute a spanning forest of G. **T** **F**

18) A stack is an abstract data type supporting the methods push(o), pop(), and dequeue(). **T** **F**

19) A tree is an acyclic disconnected graph. **T** **F**

20) The convex hull of a set of points P in the Euclidean plane is a non-intersecting polygon that contains all the $|P|$ points. **T** **F**

21) The longest common subsequence of two given sequences (of length n and m) can be computed in $O(n)$ time. **T** **F**

22) With the link relation we can determine the transitive closure of a directed graph. **T** **F**

23) Baruvka's algorithm for minimum spanning trees is based on adding safe edges to a forest of the given graph. **T** **F**

24) Let $T(n) = T(n/2) + 1$ for $n > 1$ and $T(1) = 12$. Then $T(n)$ is $O(n \log n)$. **T** **F**

Question 2. (40 marks)

a) Define the following terms.

I. Biconnected Component

II. Spanning Tree

III. Convex Hull

IV. Graph traversal

V. Directed graph

b) Answer the following questions regarding the graph given below.

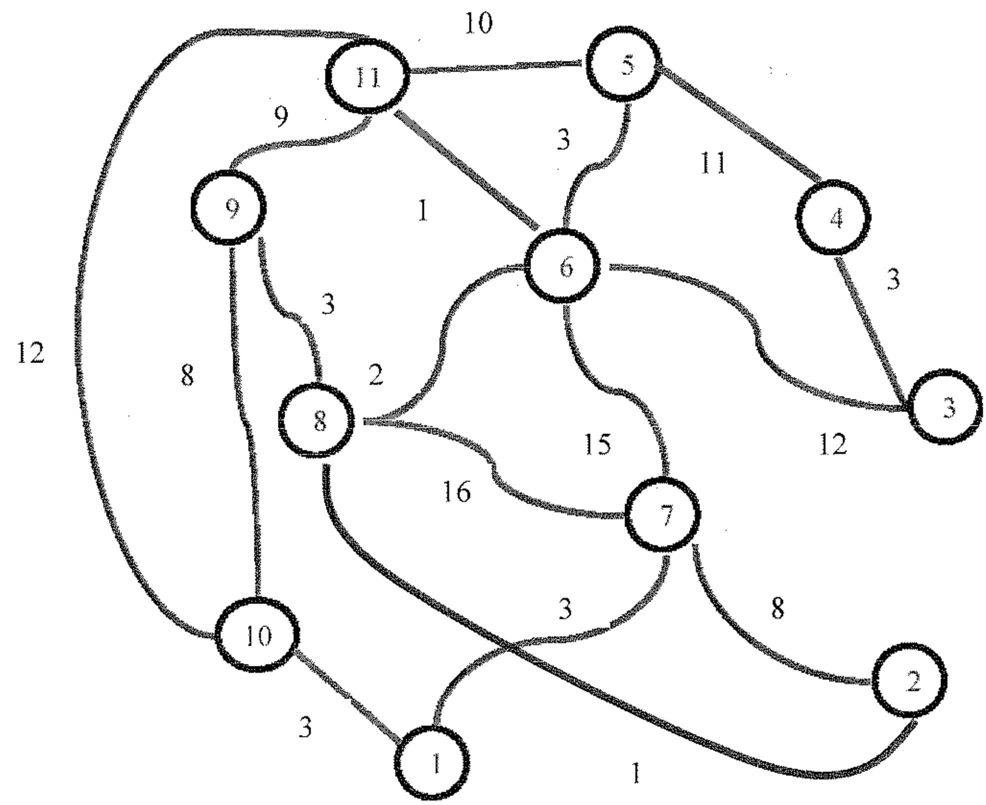
I. Is the graph connected?

II. Is the graph biconnected?

III. Is the graph directed?

IV. Are the edges (3,4) and (2,7) linked?

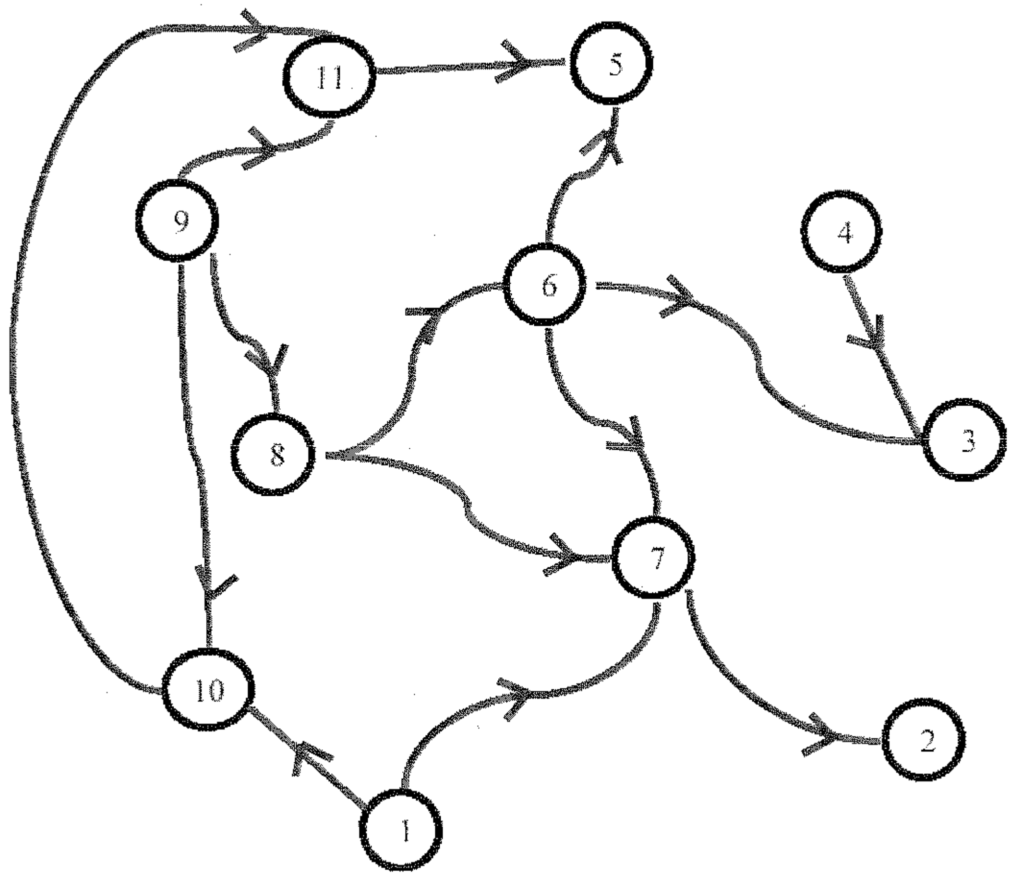
V. Does the graph have cut vertices?



c) Determine a minimum spanning tree of the graph above. What is its weight?

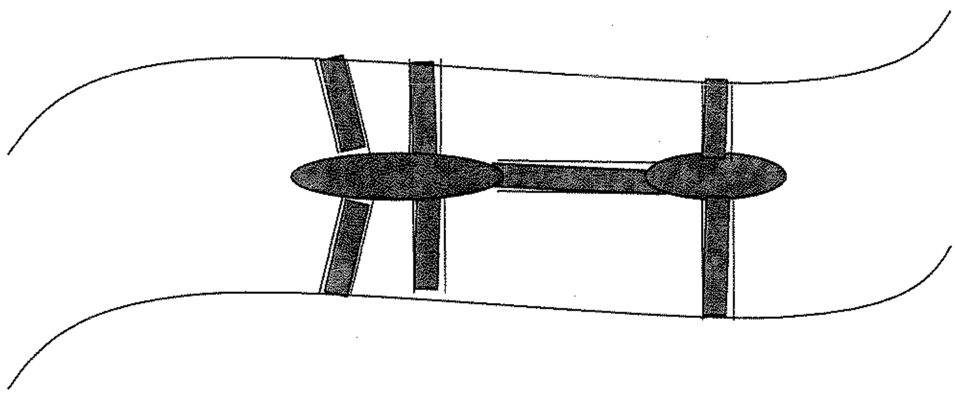
d) Perform a BFS traversal on the graph above starting at vertex 1. Assume that, in the traversal, the adjacent vertices of a given vertex are returned in increasing order of the vertex names. List the *edges* of the graph in visited order.

Question 3. (10 marks)



What is the transitive closure of the graph above? List all the edges that do not belong to the graph but belong to the transitive closure.

Question 4. (15 marks)
The Kőnigsberg-bridge puzzle is universally accepted as the problem that gave birth to graph theory. It was solved by the great Swiss-born mathematician Leonhard Euler. The problem asked, whether one could, in a single stroll, cross all seven bridges of the city exactly once and return to a starting point. Following is a sketch of the river with its two islands and seven bridges.



a) State the problem as a graph problem.

b) Does this problem have a solution?

Question 5. (40 marks)

Consider the following algorithm.

```
Algorithm ComparisonCountingSort(A[0..n-1])
for i ← 0 to n-1 do
        Count[i] ← 0
        for j ← i+1 to n-1 do
                if A[i] < A[j] then
                        Count[j] ← Count[j]+1
                else
                        Count[i] ←Count[i]+1
        for i←0 to n-1 do
                S[Count[i]] ←A[i]
return S
```

a) Given is the input sequence A: 27 3 4 8 9 17 1

Illustrate on this example how ComparisonCountingSort works.

b) ComparisonCountingSort returns the S, the sorted array of A. Prove correctness of the algorithm above!

c) Is this algorithm stable? Argue convincingly.

d) Describe in pseudocode the algorithm for Merge Sort.

Question 6. (15 marks)

Describe the algorithm design technique Divide and Conquer. Give two examples of algorithms where this technique is used.

Question 7. (20 marks)

(a) What is the largest number of elements that can be stored in an AVL tree T that satisfies the following property? The height of T is at least as large as the number of elements in the tree.

(b) Describe in pseudo-code the method of inserting a new element into a heap. (You can assume a vector representation.)