# 202201 Math 122 [A01] Quiz #3

February 17th, 2022

Name: ______________________ #V00: ______________

This test has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except where explicitly noted, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F (a) A proof by contradiction shows that an argument is invalid.

   F (b) $1 \in S$ for the set $S = \{\{1\}, 2, \{2\}, \{1, 2\}\}$.

   T (c) $\{x \in \mathbb{Z} : 2x + 7 = 10\} = \emptyset$. $2x+7=10$ $x = 3/2 \notin \mathbb{Z}$

   F (d) $|A| = 4$ for the set $A = \{\emptyset, \{a\}, \{b, c, d\}\}$. $|A| = 3$

2. [3] Prove the following statement for $m$ and $n$ in the universe of integers:

   *"If $mn$ is even, then $m$ is even or $n$ is even."*

   (Hint: The contrapositive.)

Contrapositive: If $m$ is odd and $n$ is odd, then $mn$ is odd.

Proof: Suppose $m$ is odd and $n$ is odd.

Then $\exists k \in \mathbb{Z}$, $m = 2k+1$.

Also $\exists \ell \in \mathbb{Z}$, $n = 2\ell + 1$.

Now $mn = (2k+1)(2\ell+1) = 4k\ell + 2k + 2\ell + 1$

$= 2(2k\ell + k + \ell) + 1$

Since $k, \ell \in \mathbb{Z}$, we have $2k\ell + k + \ell \in \mathbb{Z}$, and so $mn$ is odd. ☒

3. [3] Consider the sets $X = \{a, b, c, d\}$ and $Y = \{b, d, e\}$ in the universe $\mathcal{U} = \{a, b, c, d, e, f, g\}$. Find each of the following:

   (a) $X^c \cap Y^c = \{e, f, g\} \cap \{a, c, f, g\} = \{f, g\}$

   (b) $X \setminus Y^c = \{a, b, c, d\} \setminus \{a, c, f, g\} = \{b, d\}$

   (c) $\mathcal{P}(Y) = \{\emptyset, \{b\}, \{d\}, \{e\}, \{b, d\}, \{b, e\}, \{d, e\}, \{b, d, e\}\}$

4. [2] Use the Laws of Set Theory to show that $(A \backslash B) \cup (A \cap B) = A$.

$$
\begin{aligned}
(A \backslash B) \cup (A \cap B) &= (A \cap B^c) \cup (A \cap B) && \text{definition} \\
&= A \cap (B^c \cup B) && \text{distribution} \\
&= A \cap \mathcal{U} && \text{known set equality} \\
&= A && \text{known set equality}
\end{aligned}
$$

$\blacksquare$

5. [3] Prove that if $A \cup B \subseteq A \cap B$, then $A \subseteq B$. After your proof give an answer to the following: If $A \cup B \subseteq A \cap B$, are the sets $A$ and $B$ equal? No justification is needed here.

<u>Proof</u>: Suppose $A \cup B \subseteq A \cap B$.

Take $x \in A$.

Then $x \in A \cup B$.

Since $A \cup B \subseteq A \cap B$, $x \in A \cap B$.

Then $x \in B$.

Therefore $A \subseteq B$. $\blacksquare$

Yes, when $A \cup B \subseteq A \cap B$ we have $A = B$.
(The proof to show $B \subseteq A$ is similar to the $A \subseteq B$ proof.
Then with $A \subseteq B$ and $B \subseteq A$ we have $A = B$.)

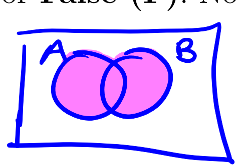
6. [2] Indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

F (a) If $A \cap B = A \cap C$, then $B = C$.

T (b) If $x \in A$, then $\{x\} \in \mathcal{P}(A)$.

F (c) $(A \oplus B) \cup B = B$.

T (d) $A^c \backslash B^c = B \backslash A$.




$\leftarrow (A \oplus B) \cup B$

$A^c \backslash B^c = A^c \cap (B^c)^c = A^c \cap B = B \cap A^c = B \backslash A$