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Find the length of the curve $x = \int_0^y \sqrt{2\,\mathbf{sec}^4 t - 1}\,dt$, on $-\frac{\pi}{6} \le y \le \frac{\pi}{6}$.

The length, L, of a curve on $a \le y \le b$ defined by $x = g(y)$ is $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\,dy$.

To obtain $\frac{dx}{dy}$ for $x = \int_0^y \sqrt{2\,\mathbf{sec}^4 t - 1}\,dt$, apply the Fundamental Theorem of Calculus Part 1.

The Fundamental Theorem states that if $F(x) = \int_a^x f(t)dt$, then $F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$.

Applying the the Fundamental Theorem to $x = \int_0^y \sqrt{2\,\mathbf{sec}^4 t - 1}\,dt$, $F'(y) = \frac{dx}{dy} = \frac{d}{dy}\int_0^y \sqrt{2\,\mathbf{sec}^4 t}\,dt = \sqrt{2\,\mathbf{sec}^4 y - 1}$.

Substituting $\frac{dx}{dy} = \sqrt{2\,\mathbf{sec}^4\,y - 1}$ into $L = \int_a^b \sqrt{\left[1 + \left(\frac{dx}{dy}\right)^2\right]}\,dy$, $L = \sqrt{2}\int_{-\pi/6}^{\pi/6} \left(\mathbf{sec}^2\,y\right) dy$.

Thus, the curve length is $2\sqrt{\frac{2}{3}}$.