Name:
Student #:

Question [2 marks]
Show that if $p(n) = c_k n^k + c_{k-1} n^{k-1} + \cdots + c_2 n^2 + c_1 n + c_0$ is a polynomial in n, where $c_i \geq 0$ are real constants with $c_k > 0$ and $k \geq 1$ is a natural number, then $\log p(n)$ is $O(\log n)$.

Note: There are a few ways to do this, this is just one of them.

Let $p(n) = c_k n^k + c_{k-1} n^{k-1} + \cdots + c_2 n^2 + c_1 n + c_0$ as defined above. Since each $c_i n^i \leq c_i n^k$, for $i = 0,1, \dots, k$, and $n \geq 1$, then

$$p(n) \leq c_k n^k + c_{k-1} n^k + \cdots + c_2 n^k + c_1 n^k + c_0 n^k$$

$$= (c_k + c_{k-1} + \cdots + c_2 + c_1 + c_0) n^k$$

$$= c n^k$$

where $c = c_k + c_{k-1} + \cdots + c_2 + c_1 + c_0$. So,

$$\log p(n) \leq \log c n^k$$

$$= \log c + \log n^k$$

$$= \log c + k \log n$$

$$\leq \log c \cdot \log n + k \log n$$

Since $\log c \leq \log c \cdot \log n$ for all $n \geq 2$. Thus,

$$\log p(n) \leq (\log c + k) \log n$$

for all $n \geq 2$ and therefore $\log p(n) \in O(\log n)$