Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

Find the Taylor polynomials of orders 0, 1, 2, and 3 generated by f at a.

$f(x) = e^{10x}$, $a = 0$

Let f be a function with derivatives of order for k = 1, 2, ..., N in some interval containing a as an interior point. Then for any integer from 0 through N, the Taylor polynomial of order n generated by f at x = a is the polynomial shown below.

$$\begin{aligned} P_n(x) &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots \\ &\quad + \frac{f^{(k)}(a)}{k!}(x-a)^k + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n \end{aligned}$$

The formula that corresponds to the Taylor polynomial of order 0 generated by f at a = 0 is shown below.

$$P_0(x) = f(0)$$

Find f(0).

$$\begin{aligned} f(0) &= e^{10(0)} \\ &= 1 \end{aligned}$$

Therefore, the Taylor polynomial of order 0 generated by $f(x) = e^{10x}$ at a = 0 is shown below.

$$P_0(x) = 1$$

The formula that corresponds to the Taylor polynomial of order 1 generated by f at a = 0 is shown below.

$$P_1(x) = f(0) + f'(0)x$$

Find $f'(x)$.

$$f'(x) = 10e^{10x}$$

Now find $f'(0)$.

$$f'(0) = 10$$

Recall that f(0) = 1. Use f(0) and $f'(0)$ to find the Taylor polynomial of order 1 generated by $f(x) = e^{10x}$ at a = 0.

$$\begin{aligned} P_1(x) &= f(0) + f'(0)x \\ &= 1 + 10x \end{aligned}$$

The formula for the 2nd order Taylor polynomial of f at a = 0 is shown below.

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$

Recall that $f'(x) = 10e^{10x}$. What is $f''(x)$?

$$f''(x) = 100e^{10x}$$

Now find $f''(0)$.

$$f''(0) = 100$$

Recall that $P_1(x) = 1 + 10x$. Use $P_1(x)$ to find the Taylor polynomial of order 2 generated by $f(x) = e^{10x}$ at a = 0.

$$P_2(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$
$$= 1 + 10x + 50x^2$$

The formula for the 3rd order Taylor polynomial of f at a = 0 is shown below.

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3$$

Recall that $f''(x) = 100\,e^{10x}$. What is $f^{(3)}(x)$?

$$f^{(3)}(x) = 1000\,e^{10x}$$

Now find $f^{(3)}(0)$.

$$f^{(3)}(0) = 1000$$

Recall that $P_2(x) = 1 + 10x + 50x^2$. Use $P_2(x)$ to find the Taylor polynomial of order 3 generated by $f(x) = e^{10x}$ at a = 0.

$$P_3(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3$$
$$= 1 + 10x + 50x^2 + \frac{500}{3}x^3$$

The Taylor polynomials of orders 0, 1, 2, and 3 generated by $f(x) = e^{10x}$ at a = 0 are listed below.

$$P_0(x) = 1$$
$$P_1(x) = 1 + 10x$$
$$P_2(x) = 1 + 10x + 50x^2$$
$$P_3(x) = 1 + 10x + 50x^2 + \frac{500}{3}x^3$$