Practice problem for APR and interest rate conversions

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I. APR[1] to Interest Rates (No Fees)

The following questions all assume there are no fees in the loan the APR applies to, only interest charges. In such cases, the APR is simply equal to the interest % charged per period, times the number of such periods in a year, and therefore the interest rate per period is equal to the APR, divided by the number of periods in a year.

[1] APR stands for Annual Percentage Rate. It is *not* an interest rate, though it is often referred to as such. It is closer to an *encrypted* interest rate, or a hash of the interest rate per period and other information on the relevant loan.

Question I.1

What is the interest rate corresponding to an APR of 12%, compounded monthly?

Answer: There are 12 months in a year, so the interest rate is 12%/12 = 1% per month.

Question I.2

What is the interest rate corresponding to an APR of 7.3%, compounded daily? Assume a non-leap year.

Answer: There are 365 days in a non-leap year, so the interest rate is 7.3%/365 per day = 0.02% per day.

Question I.3

What is the interest rate corresponding to an APR of 7.3%, compounded daily? Assume a leap year.

Answer: There are 366 days in a leap year, so the interest rate is 7.3%/366 per day = 0.0199% per day (approx.).

Question I.4

What is the interest rate corresponding to an APR of 12%, compounded quarterly?

Answer 4: There are 4 quarters in a year, so the interest rate is 12%/4 per quarter = 3% per quarter.

Question I.5

What is the interest rate corresponding to an APR of 10%, compounded yearly?

Answer: There is one year in a year, so the interest rate is 10%/1 per year = 10% per year.

II. Interest rates to APR (No fees)

The following questions all assume there are no fees in the loan the APR applies to, only interest charges.

Question II.1

What is the APR corresponding to an interest rate of 1% per month?

Answer: There are 12 months in a year, so the APR is 1% x 12 = 12%

Question II.2

What is the APR corresponding to an interest rate of 1% per day? Assume a non-leap year.

Answer: There are 365 days in a non-leap year, so the APR is 1% x 365 = 365%

Question II.3

What is the APR corresponding to an interest rate of 1% per day? Assume a leap year.

Answer: There are 366 days in a leap year, so the APR is 1% x 366 = 366%

Question II.4

What is the APR corresponding to an interest rate of 6% per quarter?

Answer: There are 4 quarters in a year, so the APR is 6% x 4 = 24%

Question II.5

What is the APR corresponding to an interest rate of 6% per year?

Answer: There is one year in a year, so the APR is 6% x 1 = 6% per year.

III. APR to Interest Rates (with fees)

This is beyond the scope of what's covered in ECON 180, but important for general knowledge. Sometimes, a loan charges fees in addition to interest. So, you may borrow $100,000 for one year and have to pay an initial fee of $1,000, plus interest equivalent to 0.5% per month. In the absence of the fee, the APR would be 0.5% x 12 = 6%. However, you also have to pay the $1,000 fee, which is 1% of the amount you are borrowing. The APR would be 6% + 1% = 7%. If the loan were for *two* years, now the fees would still be 1% of the borrowed amount, *but* this would be spread over two years, averaging out to 0.5% per year, making the APR 6% + 0.5% = 6.5%.

If there are fees, you have to *add* these to the 'no fee' APR via the following equation:

$$\text{APR} = \text{No Fee APR} + \left(\frac{\text{Total Fees}}{\text{Loan Principal}}\right) \times \left(\frac{1}{\text{Loan length in years}}\right)$$

$$\text{No Fee APR} = (\%\ \text{interest per period}) \times (\text{periods in a year})$$

To go from the APR to the interest rate, first we subtract the 'fees' addition to the APR from the APR, and then divide by the number of interest periods in a year:

$$\%\ \text{interest per period} = \frac{\text{APR} - \left(\frac{\text{Total Fees}}{\text{Loan Principal} \times \text{Loan length in years}}\right)}{\text{Periods in a year}}$$

Question III.1

What is the interest rate corresponding to an APR of 30%, compounded monthly, on a loan of $10,000, for 10 years, which charges a management fee of $100 per year?

Answer: The total fees are $100 per years x 10 years = $1,000. The principal is $10,000. Interest is charged each month, and there are 12 months in a year. The APR is 30%.

% interest per month = (30% - $1,000/($10,000 x 10))/12 = 2.4% (approx.)

Question III.2

What is the interest rate corresponding to an APR of 100%, compounded daily, on a loan of $5,000, for 4 years, one of which is a leap year? Assume a single processing fee of $100 is charged at the start of the loan.

Answer 2: This is the closest thing you'll see to a trick question in this problem set. Interest is compounded once a day. Since the leap year has a different number of days per year, what the financial institution would do to calculate the APR would be to multiply the number of days in the loan by the % interest charged each day, then divide by 4 to get an average 'per year' value. The 'No fee' component of the APR would be:

$$\frac{1}{4} \times (\%\ interest\ per\ day) \times (366 + 3 \times 365)$$

The 'fee' addition would be calculated as usual:

$$\frac{\$100\ fee}{\$5{,}000\ principal \times 4\ years}$$

Putting it all together, and noting that the APR is 100%:

$$100\% = \frac{1}{4} \times (\%\ interest\ per\ day) \times (1{,}461) + \frac{100}{5{,}000 \times 4}$$

$$1 = 365.25 \times (\%\ interest\ per\ day) + 0.005$$

$$(\%\ interest\ per\ day) = \frac{.995}{365.25} = 0.00272 = .272\%\ (approx.)$$

Within rounding error, this is exactly what you would obtain by assuming every year has 365 days, so it's not too bad of a simplifying assumption, in most cases.

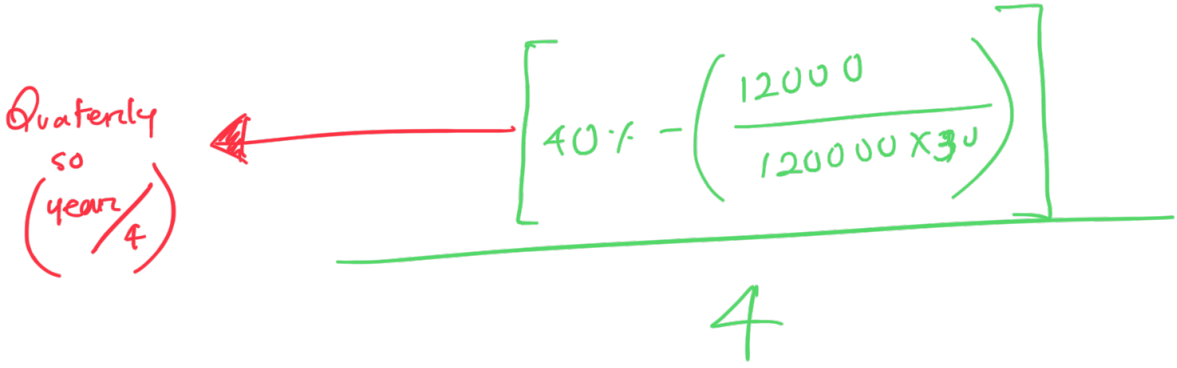
Question III.3

What is the interest rate corresponding to an APR of 40%, compounded quarterly, if the loan is for 30 years, the principal is $120,000, and there is a $100 fee per quarter?

The APR is 40%. There are 4 quarters in a year, so 30 x 4 = 120 quarters in the loan period, and a fee of $100 is charged each quarter, so the total fees are $12,000.

Interest % per quarter = (40% - $12,000/($120,000 x 30 years))/(4 quarters per year)

Interest % per quarter = 9.92%



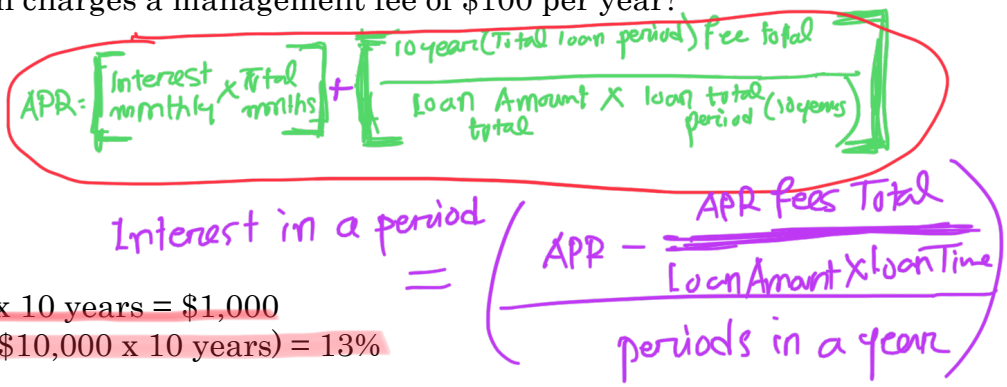
IV. Interest Rates to APR (with fees)

Question IV.1

What is the APR corresponding to an interest rate of 1% per month, on a loan of $10,000, for 10 years, which charges a management fee of $100 per year?

Answer:

Interest per period = 1%
Periods per year = 12
Principal = $10,000
Loan length in years = 10
Total fees = $100 per year x 10 years = $1,000
APR = (1% x 12) + $1,000/($10,000 x 10 years) = 13%



Question IV.2

What is the APR corresponding to an interest rate of 1% per day, on a loan of $5,000, for 1 (non-leap) year? Assume a single processing fee of $100 is charged at the start of the loan.

Answer:

Interest per period = 1%
Periods per year = 365
Principal = $5,000
Loan length in years = 1
Total fees = $100
APR = (1% x 365) + $100/($5,000 x 1 year) = 367%

Question IV.3

What is the APR corresponding to an interest rate of 6% per quarter, if the loan is for 30 years, the principal is $120,000, and there is a $100 fee per quarter?

Interest per period = 6%
Periods per year = 4
Principal = $120,000
Loan length in years = 30
Total fees = $100 x 30 x 4 = $12,000

APR = (6% x 4) + $12,000/($120,000 x 30 years) = 24.33% (approx.)

V. Converting interest rates between time periods

This is as simple as setting equal things equal to each other. Each time interest is applied, the principal is multiplied by (1 + i), where i is the interest rate – so pick a time period, and set up an equation where interest is applied for that total time period on each side.

Yearly to monthly and back? 1 year = 12 months, so

$$(1 + i_{yearly}) = (1 + i_{monthly})^{12}$$

Quarterly to monthly? 1 year = 4 quarters = 12 months, so

$$(1 + i_{quarterly})^{4} = (1 + i_{monthly})^{12}$$

Or you could just use 1 quarter = 3 months:

$$(1 + i_{quarterly}) = (1 + i_{monthly})^{3}$$

Monthly to daily and back in a non-leap year? There are 365 days and 12 months in a year.

$$(1 + i_{daily})^{365} = (1 + i_{monthly})^{12}$$

And in a leap year?

$$(1 + i_{daily})^{366} = (1 + i_{monthly})^{12}$$

Some banks give monthly statements, but seem to charge different interest rates each month – that can happen if they're actually compounding interest daily, and taking into account that each month has a different number of days. That's easy enough to deal with. In the words of the old rhyme:

"30 days hath September, April, June and November":

$$(1 + i^{daily})^{30} = (1 + i_{30days})$$

February has 28 or 29 days:

$$(1 + i_{daily})^{28} = (1 + i_{28days}),\ (1 + i_{daily})^{29} = (1 + i_{29days})$$

"And when short February's done, all the rest have 31!"

$$(1 + i_{daily})31 = (1 + i_{31days})$$

What about converting between centuries and millennia? There are ten centuries in a millennium, so (throwing in yearly interest rates as a bonus):

$$(1 + i_{millennium}) = (1 + i_{century})^{10} = (1 + i_{yearly})^{1000}$$

That's all there is to it!

Question V.1

Convert 1% interest per month to a yearly equivalent, and 20% interest per year to a monthly equivalent.

Answer:

$(1 + i_{yearly}) = (1 + i_{monthly})^{12}$

For an interest rate of 1% per month:

$i_{yearly} = (1 + i_{monthly})^{12} - 1 = 1.01^{12} - 1 = 0.1268 = 12.68\%$ (approx.)

For an interest rate of 20% per year:

$i_{monthly} = (1 + i_{yearly})^{1/12} - 1 = 1.2^{1/12} - 1 = 0.0153 = 1.53\%$ per month (approx.)

Question V.2

Convert 3% interest per quarter into an equivalent interest rate per century, and an equivalent interest rate per week. Assume every year has exactly 365 days.

There are 4 quarters in a year, so 400 in a century.

$(1 + i_{century}) = (1 + i_{quarterly})^{400}$

For a quarterly interest rate of 3%,

$i_{century} = (1 + 3\%)^{400} - 1 = 1.03^{400} - 1 = 136{,}422.7182 = 13{,}642{,}271.82\%$ per century (approx.)

As for the weekly interest rate… There are 7 days in a week. What can we do with that? We're asked to assume there are 365 days in a year – but we also know there are four quarters in a year. So, in one year, there are (365/7) weeks and 4 quarters.

$(1 + i_{quarterly})^4 = (1 + i_{weekly})^{365/7}$

For a quarterly interest rate of 3%,

$i_{weekly} = ((1 + 3\%)^4)^{7/365} - 1$
$i_{weekly} = 1.03^{28/365} - 1 = 0.00227 = 0.227\%$ (approx.)