The Laws of Logic

Two statements s_1 and s_2 are *logically equivalent* if $s_1 \leftrightarrow s_2$ is a tautology, that is, s_1 and s_2 have the same truth table (up to the order of the rows)

If s_1 and s_2 are logically equivalent, we write $s_1 \Leftrightarrow s_2$. Note that $s_1 \leftrightarrow s_2$ is a statement and can in general be true or false, and $s_1 \Leftrightarrow s_2$ indicates the (higher level) fact that it is a tautology.

Logically equivalent statements are "the same" in the sense that logically equivalent statements can be freely substituted for each other without changing the meaning of a compound statement.

Here are some basic logical equivalences. Each of the following can be verified (proved) with a truth table. It is a good idea to memorize them, so that they are at your fingertips when needed. In what follows, **1** denotes a statement that is always true (i.e. a tautology), and **0** denotes a statement that is always false (i.e. a contradiction).

iCAD

- Idempotence: $p \vee p \Leftrightarrow p, \quad p \wedge p \Leftrightarrow p$
- Commutative: $p \wedge q \Leftrightarrow q \wedge p, \quad p \vee q \Leftrightarrow q \vee p$
- Associative: $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r), \quad (p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$
- Distributative: $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r), \quad p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$
- Double Negation: $\neg(\neg p) \Leftrightarrow p$
- DeMorgan's Laws: $\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q, \quad \neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$
- Identity: $p \wedge \mathbf{1} \Leftrightarrow p, \quad p \vee \mathbf{0} \Leftrightarrow p$
- Dominance: $p \wedge \mathbf{0} \Leftrightarrow \mathbf{0}, \quad p \vee \mathbf{1} \Leftrightarrow \mathbf{1}$

Idempotence, one variable
Commutative, two variable
Associative, three variable

in all bi-statements,
same signs used in individual statements

Distributative: two chaos to three chaos,
three variables to four variables

DeMorgan's Law, in a nutshell:

"not" destroys the "brackets",
which in turn changes the signs (read "chaos")

and // 0 > p > 1 // precedence
or // 1 > p > 0 // precedence
(and prefers p) // (or prefers 1)
among p and 1
(and loves 0) // (or hates 0)

No need to remember
the name of identity and dominance,
but memorize the statements

The following are some other useful logical equivalences.

chaotic

(p thakle q) EQUIVALANT ((breakup) p thakbena q thakle)

- $p \rightarrow q \Leftrightarrow \neg p \vee q$
- $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p) \Leftrightarrow (\neg p \vee q) \wedge (p \vee \neg q)$

(p sufficient q) and (p necessary p)

(p thakbena q thakle) and (p thakle q thakbena)

It is apparent that the Laws of Logic come in pairs. The *dual* of a statement is obtained by replacing $\vee$ by $\wedge$; $\wedge$ by $\vee$; **0** by **1**; and **1** by **0**, wherever they occur. It is a theorem of logic that if s_1 is logically equivalent to s_2, then the dual of s_1 is logically equivalent to the dual of s_2.