Symbolab

Solution

$\sum_{n=0}^{\infty}\frac{(x-1)^{2n}}{36^n}$: Interval of convergence is $-5<x<7$

Steps

$\sum_{n=0}^{\infty}\frac{(x-1)^{2n}}{36^n}$

Use the Root Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}\frac{(x-1)^{2n}}{36^n}$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}$: $\frac{(x-1)^2}{36}$

Hide Steps

$\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}$

Use the following exponent property: $(a\cdot b)^n=a^n\cdot b^n$

$\left(\frac{(x-1)^{2n}}{36^n}\right)^{\frac{1}{n}}=\frac{\sqrt[n]{(x-1)^{2n}}}{\sqrt[n]{36^n}}$

$=\frac{\sqrt[n]{(x-1)^{2n}}}{\sqrt[n]{36^n}}$

Use the following exponent property: $(a^n)^m=a^{n\cdot m}$

$\sqrt[n]{(x-1)^{2n}}=(x-1)^{2n\frac{1}{n}}$, $\sqrt[n]{36^n}=36^{n\frac{1}{n}}$

$=\frac{(x-1)^{2n\frac{1}{n}}}{36^{n\frac{1}{n}}}$

$36^{n\frac{1}{n}}=36$

Hide Steps

$36^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1

Hide Steps

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=36^1$

Apply rule $a^1=a$

$=36$

$=\frac{(x-1)^{2n\frac{1}{n}}}{36}$

$(x-1)^{2n\frac{1}{n}}=(x-1)^2$

Hide Steps

$(x-1)^{2n\frac{1}{n}}$

Multiply $2n\frac{1}{n}$: 2

Hide Steps

$2n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 2n}{n}$

Cancel the common factor: n

$=1\cdot 2$

Multiply the numbers: $1\cdot 2=2$

$=2$

$=(x-1)^2$

$= \frac{(x-1)^2}{36}$

$L = \lim_{n\to\infty}\left(\left|\frac{(x-1)^2}{36}\right|\right)$

$L = \left|\frac{(x-1)^2}{36}\right| \cdot \lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1) = 1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a} c = c$

$= 1$

$L = \left|\frac{(x-1)^2}{36}\right| \cdot 1$

Simplify

$L = \frac{|x-1|^2}{36}$

$L = \frac{|x-1|^2}{36}$

The power series converges for $L < 1$

$\frac{|x-1|^2}{36} < 1$

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$\frac{|x-1|^2}{36} < 1 \quad : \quad -5 < x < 7$ *Hide Steps*

$\frac{|x-1|^2}{36} < 1$

Find positive and negative intervals *Hide Steps*

Find intervals for $|x-1|$

$x-1 \geq 0 : x \geq 1, \quad |x-1| = x-1$ *Hide Steps*

$x-1 \geq 0 \quad : \quad x \geq 1$ *Hide Steps*

$x-1 \geq 0$

Add 1 to both sides

$x-1+1 \geq 0+1$

Simplify

$x \geq 1$

Rewrite $|x-1|$ for $x-1 \geq 0$: $|x-1| = x-1$ *Hide Steps*

Apply absolute rule: If $u \geq 0$ then $|u| = u$

$|x-1| = x-1$

$x-1 < 0 : x < 1, \quad |x-1| = -(x-1)$ *Hide Steps*

$x-1 < 0 \quad : \quad x < 1$ *Hide Steps*

$x-1 < 0$

Add 1 to both sides

$x-1+1 < 0+1$

Simplify

$x < 1$

Rewrite $|x-1|$ for $x-1 < 0$: $|x-1| = -(x-1)$ *Hide Steps*

Apply absolute rule: If $u < 0$ then $|u| = -u$

$|x-1| = -(x-1)$

Identify the intervals:

$x < 1, \; x \geq 1$

	$x<1$	$x\geq 1$
$\lvert x-1\rvert$	$-$	$+$

$x < 1, \; x \geq 1$

$x < 1, \; x \geq 1$

Solve the inequality for each interval *Hide Steps*

$x < 1, \; x \geq 1$

For $x < 1$: $\quad -5 < x < 1$ *Hide Steps*

For $x < 1$ rewrite $\frac{|x-1|^2}{36} < 1$ as $\frac{(-(x-1))^2}{36} < 1$

$\frac{(-(x-1))^2}{36} < 1 \quad : \quad -5 < x < 7$ *Hide Steps*

$\frac{(-(x-1))^2}{36} < 1$

Multiply both sides by 36

$\frac{36(-(x-1))^2}{36} < 1 \cdot 36$

Simplify

$(-(x-1))^2 < 36$

For $u^n < a$, if n is even then $-\sqrt[n]{a} < u < \sqrt[n]{a}$

$-\sqrt{36} < -(x-1) < \sqrt{36}$

If $a < u < b$ then $a < u$ and $u < b$

$-\sqrt{36} < -(x-1)$ and $-(x-1) < \sqrt{36}$

$-\sqrt{36} < -(x-1) \quad : \quad x < 7$ *Hide Steps*

$-\sqrt{36} < -(x-1)$

Switch sides

$-(x-1) > -\sqrt{36}$

$\sqrt{36} = 6$ *Hide Steps*

$\sqrt{36}$

Factor the number: $36 = 6^2$

$= \sqrt{6^2}$

Apply radical rule: $\sqrt[n]{a^n} = a$

$\sqrt{6^2} = 6$

$= 6$

$-(x-1) > -6$

Multiply both sides by -1 (reverse the inequality)

$(-(x-1))(-1) < (-6)(-1)$

Simplify

$x - 1 < 6$

Add 1 to both sides

$x - 1 + 1 < 6 + 1$

Simplify

$x < 7$

$-(x-1) < \sqrt{36} \quad : \quad x > -5$ *Hide Steps*

$-(x-1) < \sqrt{36}$

Factor the number: $36 = 6^2$

$-(x-1) < \sqrt{6^2}$

Apply radical rule: $\sqrt[n]{a^n} = a$

$\sqrt{6^2} = 6$

$-(x-1) < 6$

Multiply both sides by -1 (reverse the inequality)

$(-(x-1))(-1) > 6(-1)$

Simplify

$x - 1 > -6$

Add 1 to both sides

$x - 1 + 1 > -6 + 1$

Simplify

$x > -5$

Combine the intervals

$x < 7$ and $x > -5$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x < 7$ and $x > -5$

$-5 < x < 7$

$-5 < x < 7$

Combine the intervals

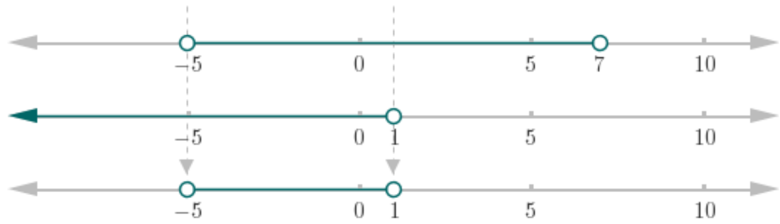
$-5 < x < 7$ and $x < 1$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$-5 < x < 7$ and $x < 1$

$-5<x<1$



$-5<x<1$

For $x\ge 1$: $\quad 1\le x<7$ *Hide Steps*

For $x\ge 1$ rewrite $\frac{|x-1|^2}{36}<1$ as $\frac{(x-1)^2}{36}<1$

$\frac{(x-1)^2}{36}<1 \quad : \quad -5<x<7$ *Hide Steps*

$\frac{(x-1)^2}{36}<1$

Multiply both sides by 36

$\frac{36(x-1)^2}{36}<1\cdot 36$

Simplify

$(x-1)^2<36$

For $u^n<a$, if n is even then $-\sqrt[n]{a}<u<\sqrt[n]{a}$

$-\sqrt{36}<x-1<\sqrt{36}$

If $a<u<b$ then $a<u$ and $u<b$

$-\sqrt{36}<x-1$ and $x-1<\sqrt{36}$

$-\sqrt{36}<x-1 \quad : \quad x>-5$ *Hide Steps*

$-\sqrt{36}<x-1$

Switch sides

$x-1>-\sqrt{36}$

$\sqrt{36}=6$ *Hide Steps*

$\sqrt{36}$

Factor the number: $36=6^2$

$=\sqrt{6^2}$

Apply radical rule: $\sqrt[n]{a^n}=a$

$\sqrt{6^2}=6$

$=6$

$x-1>-6$

Add 1 to both sides

$x-1+1>-6+1$

Simplify

$x>-5$

$x-1<\sqrt{36} \quad : \quad x<7$ *Hide Steps*

$x-1<\sqrt{36}$

Factor the number: $36=6^2$

$x-1<\sqrt{6^2}$

Apply radical rule: $\sqrt[n]{a^n}=a$

$\sqrt{6^2}=6$

$x-1<6$

Add 1 to both sides

$x-1+1<6+1$

Simplify

$x<7$

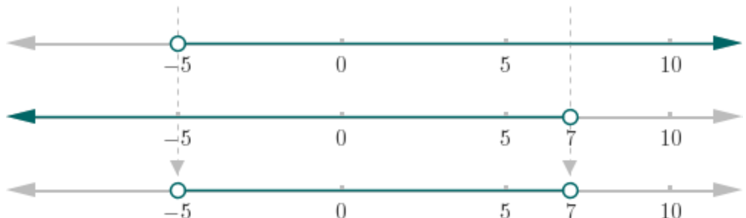
Combine the intervals

$x>-5$ and $x<7$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals
$x>-5$ and $x<7$

$-5<x<7$



$-5 < x < 7$

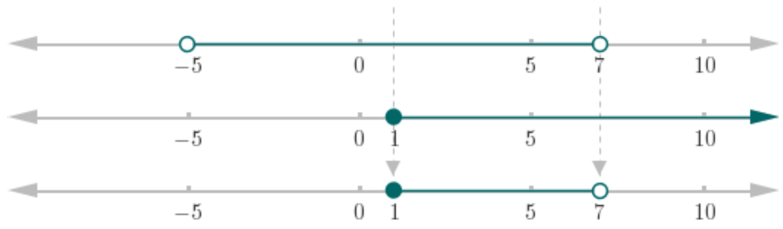
Combine the intervals

$-5 < x < 7$ and $x \ge 1$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals
$-5 < x < 7$ and $x \ge 1$

$1 \le x < 7$



$1 \le x < 7$

Combine the intervals

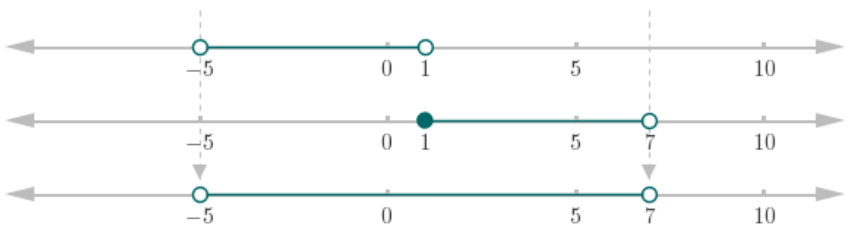
$-5 < x < 1$ or $1 \le x < 7$

$-5 < x < 1$ or $1 \le x < 7$

Merge Overlapping Intervals *Hide Steps*

The union of two intervals is the set of numbers which are in either interval
$-5 < x < 1$ or $1 \le x < 7$

$-5 < x < 7$



$-5 < x < 7$

$-5 < x < 7$

Check the interval end points: $x = -5$:diverges, $x = 7$:diverges *Hide Steps*

For $x = -5$, $\sum_{n=0}^{\infty} \frac{((-5)-1)^{2n}}{36^n}$: diverges *Hide Steps*

$\sum_{n=0}^{\infty} \frac{((-5)-1)^{2n}}{36^n}$

Refine

$= \sum_{n=0}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

$=$ diverges

For $x = 7$, $\sum_{n=0}^{\infty} \frac{(7-1)^{2n}}{36^n}$: diverges *Hide Steps*

$\sum_{n=0}^{\infty} \frac{(7-1)^{2n}}{36^n}$

Refine

$= \sum_{n=0}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

$=$ diverges

$x = -5$:diverges, $x = 7$:diverges

Therefore

Interval of convergence is $-5 < x < 7$

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Interval of convergence is $-5 < x < 7$