Symbolab

Solution

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{x}}{6}-1\right)^n$: Interval of convergence is $0<x<144$

Steps

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{x}}{6}-1\right)^n$

Use the Root Test to compute the convergence interval *Hide Steps*

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{x}}{6}-1\right)^n$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|\left(\frac{\sqrt{x}}{6}-1\right)^{n\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\left(\frac{\sqrt{x}}{6}-1\right)^{n\frac{1}{n}}\right|\right)$ *Hide Steps*

$L=\lim_{n\to\infty}\left(\left|\left(\left(\frac{\sqrt{x}}{6}-1\right)^n\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(\left(\frac{\sqrt{x}}{6}-1\right)^n\right)^{\frac{1}{n}}$: $\frac{\sqrt{x}}{6}-1$ *Hide Steps*

$\left(\left(\frac{\sqrt{x}}{6}-1\right)^n\right)^{\frac{1}{n}}$

Use the following exponent property: $(a^n)^m=a^{n\cdot m}$

$\left(\left(\frac{\sqrt{x}}{6}-1\right)^n\right)^{\frac{1}{n}}=\left(\frac{\sqrt{x}}{6}-1\right)^{n\frac{1}{n}}$

$=\left(\frac{\sqrt{x}}{6}-1\right)^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1 *Hide Steps*

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=\left(\frac{\sqrt{x}}{6}-1\right)^1$

Apply rule $a^1=a$

$=\frac{\sqrt{x}}{6}-1$

$L=\lim_{n\to\infty}\left(\left|\frac{\sqrt{x}}{6}-1\right|\right)$

$L=\left|\frac{\sqrt{x}}{6}-1\right|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$L=\left|\frac{\sqrt{x}}{6}-1\right|\cdot 1$

Simplify

$L=\left|\frac{\sqrt{x}}{6}-1\right|$

$L=\left|\frac{\sqrt{x}}{6}-1\right|$

The power series converges for $L<1$

$\left|\frac{\sqrt{x}}{6}-1\right|<1$

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for x

$\left|\frac{\sqrt{x}}{6}-1\right|<1\quad:\quad 0<x<144$ *Hide Steps*

$\left|\frac{\sqrt{x}}{6}-1\right|<1$

Apply absolute rule: If $|u| < a, a>0$ then $-a < u < a$

$-1<\frac{\sqrt{x}}{6}-1<1$

$\frac{\sqrt{x}}{6}-1>-1$ and $\frac{\sqrt{x}}{6}-1<1$ *Hide Steps*

$\frac{\sqrt{x}}{6}-1>-1$ and $\frac{\sqrt{x}}{6}-1<1$

$\frac{\sqrt{x}}{6}-1>-1 \quad : \quad x>0$ *Hide Steps*

$\frac{\sqrt{x}}{6}-1>-1$

Add 1 to both sides

$\frac{\sqrt{x}}{6}-1+1>-1+1$

Simplify

$\frac{\sqrt{x}}{6}>0$

Multiply both sides by 6

$\frac{6\sqrt{x}}{6}>0\cdot 6$

Simplify

$\sqrt{x}>0$

Square both sides

$(\sqrt{x})^2>0^2$

Simplify

$x>0$

Find singularity points

Find non – negative values for radicals: $x\ge 0$ *Hide Steps*

$\sqrt{f(x)} \quad \Rightarrow \quad f(x)\ge 0$

For $\sqrt{x}$: $x\ge 0$

$x\ge 0$

Combine the intervals

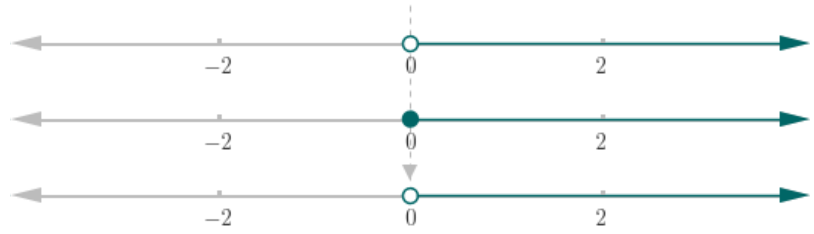
$x>0$ and $x\ge 0$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x>0$ and $x\ge 0$

$x>0$



$x>0$

$\frac{\sqrt{x}}{6}-1<1 \quad : \quad 0\le x<144$ *Hide Steps*

$\frac{\sqrt{x}}{6}-1<1$

Add 1 to both sides

$\frac{\sqrt{x}}{6}-1+1<1+1$

Simplify

$\frac{\sqrt{x}}{6}<2$

Multiply both sides by 6

$\frac{6\sqrt{x}}{6}<2\cdot 6$

Simplify

$\sqrt{x}<12$

Square both sides

$(\sqrt{x})^2<12^2$

Simplify

$x<144$

Find singularity points

Find non – negative values for radicals: $x\ge 0$ *Hide Steps*

$\sqrt{f(x)} \quad \Rightarrow \quad f(x)\ge 0$

For $\sqrt{x}$: $x \geq 0$

$x \geq 0$

Combine the intervals

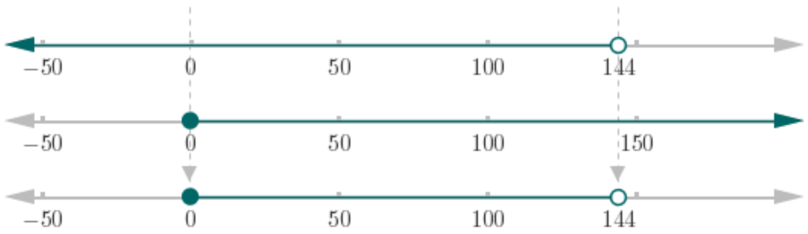
$x < 144$ and $x \geq 0$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x < 144$ and $x \geq 0$

$0 \leq x < 144$



$0 \leq x < 144$

Combine the intervals

$x > 0$ and $0 \leq x < 144$

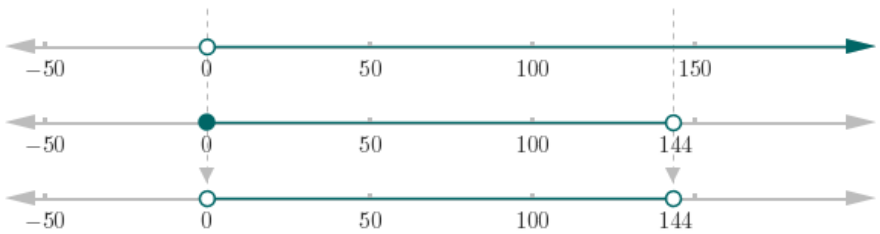
$x > 0$ and $0 \leq x < 144$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x > 0$ and $0 \leq x < 144$

$0 < x < 144$



$0 < x < 144$

$0 < x < 144$

Check the interval end points: $x = 0$:diverges, $x = 144$:diverges *Hide Steps*

For $x = 0$, $\sum_{n=0}^{\infty}\left(\frac{\sqrt{0}}{6} - 1\right)^n$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{0}}{6} - 1\right)^n$

Refine

$= \sum_{n=0}^{\infty}(-1)^n$

Apply Series Geometric Test: diverges *Hide Steps*

$\sum_{n=0}^{\infty}(-1)^n$

Geometric Series:

If the series is of the form $\sum_{n=0}^{\infty} r^n$

If $|r| < 1$, then the geometric series converges to $\frac{1}{1-r}$

If $|r| \geq 1$, then the geometric series diverges

$r = -1$, $|r| = 1 \geq 1$, by the geometric test criteria

$=$ diverges

$=$ diverges

For $x = 144$, $\sum_{n=0}^{\infty}\left(\frac{\sqrt{144}}{6} - 1\right)^n$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\left(\frac{\sqrt{144}}{6} - 1\right)^n$

Refine

$= \sum_{n=0}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

$=$ diverges

$x = 0$:diverges, $x = 144$:diverges

Therefore

Interval of convergence is $0 < x < 144$

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