

CSC 225: Fall 2022: Lab 2

1 Solving Recurrence Equations

Determine the closed form of the following recurrence equations.

$$\text{a) } T(n) = \begin{cases} 1, & \text{if } n = 1 \\ T(n-1) + n, & \text{if } n \geq 2 \end{cases}$$

$$\text{b) } T(n) = \begin{cases} 1, & \text{if } n = 0 \\ 2T(n-1), & \text{if } n \geq 1 \end{cases}$$

2 Proof Techniques

Prove each of the following identities using induction.

$$\text{a) } \sum_{i=1}^n (2i-1) = n^2 \text{ for all } n \geq 1.$$

$$\text{b) } \sum_{i=0}^n i^2 = \frac{n(n+1)(2n+1)}{6} \text{ for all } n \geq 0.$$

3 Loop Invariants

Consider the Algorithm `arrayFind`, given below, which searches an array A for an element x . Prove that `arrayFind` is correct using induction (loop invariants).

Algorithm `arrayFind`(x, A, n):

Input: An element x and an n -element array, A .

Output: The index i such that $x = A[i]$ or -1 if no element of A is equal to x .

$i \leftarrow 0$

while $i < n$ **do**

if $x = A[i]$ **then**

return i

else

$i \leftarrow i + 1$

return -1