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Date: 03/07/22

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Course: Math 101 A04 Spring 2022

Assignment: HW-6 [Sections 10.4, 10.5 & 10.6]

Determine if the series converges or diverges. Give a reason for your answer.

$$\sum_{n=3}^{\infty} \frac{10}{\ln(\ln n)}$$

To determine the convergence or divergence of the series, use the comparison test.

Let $\sum a_n$ be a series with no negative terms. The series $\sum a_n$ converges if there is a convergent series $\sum c_n$ with $a_n \leq c_n$ for all $n > N$, for some integer N . The series $\sum a_n$ diverges if there is a divergent series of nonnegative terms $\sum d_n$ with $a_n \geq d_n$ for all $n > N$, for some integer N .

Notice that the expression $\frac{10}{\ln(\ln n)}$ is positive for $n \geq 3$ and $\ln n < n$ for all n .

Since $\ln n < n$ and both values are positive, the relationship shown below is true.

$$\frac{1}{\ln(\ln n)} > \frac{1}{n}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is the divergent harmonic series.

Since $\frac{10}{\ln(\ln n)} \geq \frac{1}{n}$ for all $n \geq 3$ and the series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, the series $\sum_{n=3}^{\infty} \frac{10}{\ln(\ln n)}$ diverges by the comparison test.