Translational Equilibrium & Newton's first law.

Newton's law

If an object is subject to no net force then (the object won't change how it moves) won't accelerate (direction or speed)

If you observe an object not changing how it moves then you can conclude that the net force on ~~the~~ the object is 0.

this is the sum of all forces on the object

If $\sum \vec{F}$ on thing is 0

If $\sum \vec{F} = 0$ then translational equilibrium

If translational equilibrium then infer $\sum \vec{F} = 0$

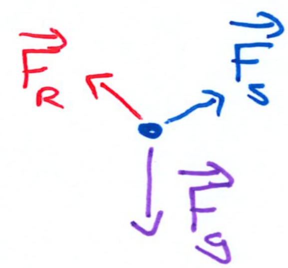
Important caveat:

True in an inertial reference frame. ie: One where the basis vectors don't change in time.

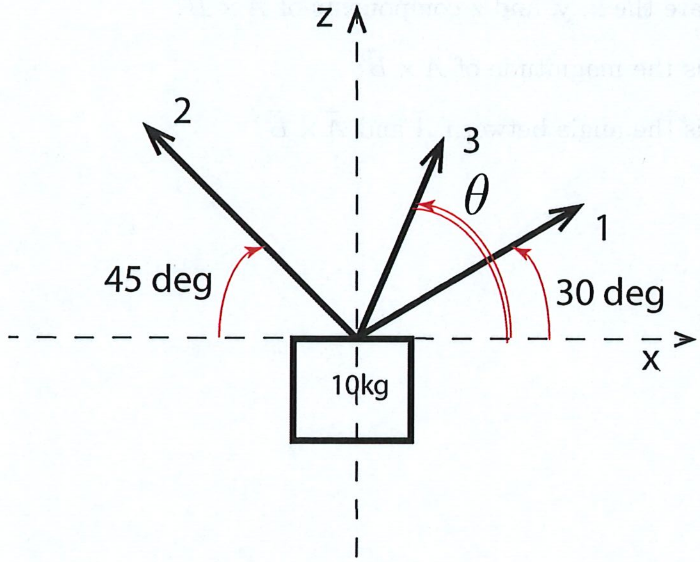
Useful tool for analysis

"Free-body diagram"

- Pretend object is point particle
- List & draw as vectors all forces on object



Translational Equilibrium - I
A $10kg$ box is supported by three ropes as shown in the figure.

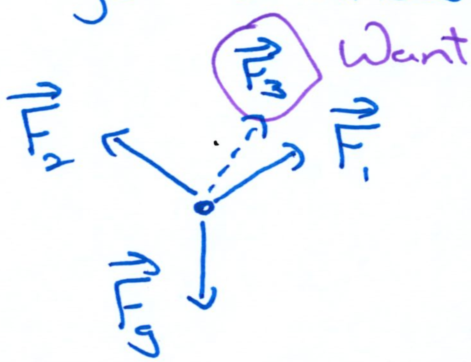


Rope 1 pulls with a force of magnitude $80N$ at an angle of $30°$ with $\hat{\imath}$ as shown. Rope 2 pulls with a force of magnitude $50N$ at an angle of $45°$ with $-\hat{\imath}$ and $45°$ with $\hat{k}$ as shown.

- What is the x-component of the force exerted by rope 3?
- What is the z-component of the force exerted by rope 3?
- What is the magnitude of the force exerted by rope 3?
- Rope 3 is in the xz plane; what angle does it make with $\hat{\imath}$ measured counterclockwise from the x-axis as shown? ie what is θ?

Equilibrium problem
Identify net force is 0

Way 1: Write all forces as vectors with specific components. Vector algebra → get unknown components



b/c equilibrium determine

$\vec{F}_{net} = 0$

$0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_g$

$\vec{F}_g = -mg\hat{k} = -10kg\, 9.8^{N}/_{kg}\, \hat{k} = -98N\hat{k}$

$\vec{F}_1 = 80N\cos 30\,\hat{\imath} + 80N\cos 60\,\hat{k}$

$= \sin 30$

$= 69.3N\hat{\imath} + 40N\hat{k}$

$\vec{F}_2 = 50N\cos 135\,\hat{\imath} + 50N\cos 45\,\hat{k}$

$= -35.4N\hat{\imath} + 35.4N\hat{k}$

$\vec{F}_3 = F_{3x}\hat{\imath} + F_{3z}\hat{k}$

$$0 = (69.3N\hat{i} + 40N\hat{k}) + (-35.4N\hat{i} + 35.4N\hat{k}) + (F_{3x}\hat{i} + F_{3z}\hat{k}) + (-98N\hat{k})$$

$$0 = (69.3N - 35.4N + F_{3x})\hat{i} + (40N + 35.4N + F_{3z} - 98N)\hat{k}$$

x-comp $\quad 0 = 69.3N - 35.4N + F_{3x}$

$-33.9N = F_{3x}$

z-comp $\quad 0 = 40N + 35.4N - 98N + F_{3z}$

$22.6N = F_{3z}$

$$|\vec{F}_3| = \sqrt{(F_{3x})^2 + (F_{3y})^2 + (F_{3z})^2} = \sqrt{(-33.9N)^2 + 0^2 + (22.6N)^2}$$

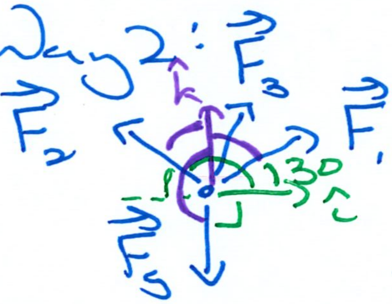
$$= 40.7N$$

$F_{3x} = |\vec{F}_3|\cos\theta$ ← btw $\vec{F}_3$ & $\hat{i}$

$-33.9N = 40.7N\cos\theta$

$\theta \approx 146°$

Way 2:



$$0 = \vec{F}_g + \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$\vec{F}_3 = -\vec{F}_g - \vec{F}_1 - \vec{F}_2$$

x-component

$$\vec{F}_3 \cdot \hat{i} = (-\vec{F}_g - \vec{F}_1 - \vec{F}_2) \cdot \hat{i}$$

$$F_{3x} = -\vec{F}_g \cdot \hat{i} - \vec{F}_1 \cdot \hat{i} - \vec{F}_2 \cdot \hat{i}$$

$$= -|\vec{F}_g| \cos 90 - 80N \cos 30 - 50N \cos 135$$

$$= -33.9N$$

z-component

$$\vec{F}_3 \cdot \hat{k} = (-\vec{F}_g - \vec{F}_1 - \vec{F}_2) \cdot \hat{k}$$

$$F_{3z} = -\vec{F}_g \cdot \hat{k} - \vec{F}_1 \cdot \hat{k} - \vec{F}_2 \cdot \hat{k}$$

$$= -98N \cos 180 - 80N \cos 60$$

$$- 50N \cos 45$$

$$= 22.6N$$

Contact Forces

The force two objects exert on each other when they touch.

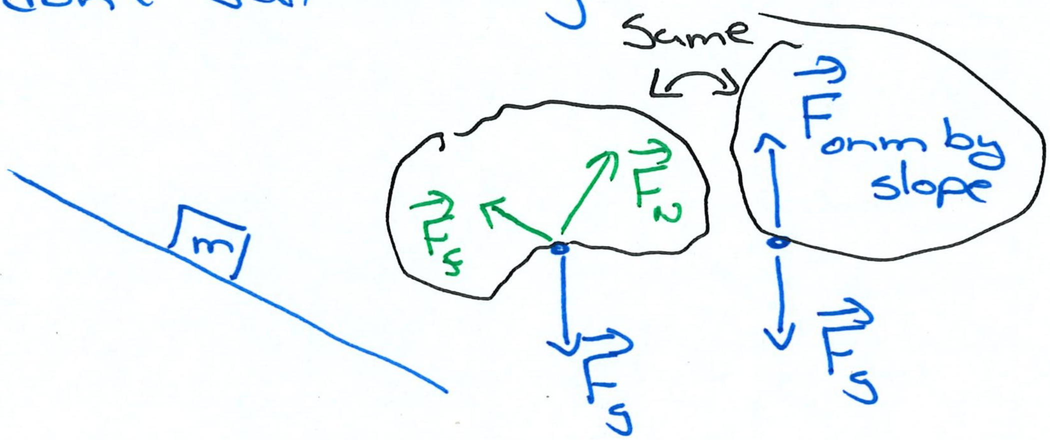
Obey Newton's 3rd law:

When two objects (A and B) interact the force A exerts on B is same magnitude & opposite direction to force B exerts on A.

$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$$

Contact forces are a "force of constraint" they are whatever they need to be to prevent things sliding through each other.

In this course when we talk about a "surface" we mean an ideal surface things don't fall through



Convention:

Call the component of force by a surface at 90° to surface
"normal force"

Call any part of $\vec{F}_{\text{by surface}}$ along surface
"friction force"