Solution

$\sum_{n=0}^{\infty}\frac{22^n x^n}{n!}$: Radius of convergence is ∞, Interval of convergence is $-\infty < x < \infty$

Steps

$\sum_{n=0}^{\infty}\frac{22^n x^n}{n!}$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=0}^{\infty}\frac{22^n x^n}{n!}$

Series Ratio Test:

If there exists an N so that for all $n \ge N$, $a_n \ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$:

If $L < 1$, then $\sum a_n$ converges

If $L > 1$, then $\sum a_n$ diverges

If $L = 1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{22^{(n+1)}x^{(n+1)}}{(n+1)!}}{\frac{22^n x^n}{n!}}\right|$$

Compute $L = \lim_{n\to\infty}\left(\left|\frac{\frac{22^{(n+1)}x^{(n+1)}}{(n+1)!}}{\frac{22^n x^n}{n!}}\right|\right)$ *Hide Steps*

$$L = \lim_{n\to\infty}\left(\left|\frac{\frac{22^{(n+1)}x^{(n+1)}}{(n+1)!}}{\frac{22^n x^n}{n!}}\right|\right)$$

Simplify $\frac{\frac{22^{(n+1)}x^{(n+1)}}{(n+1)!}}{\frac{22^n x^n}{n!}}$: $\frac{22x}{n+1}$ *Hide Steps*

$$\frac{\frac{22^{n+1}x^{n+1}}{(n+1)!}}{\frac{22^n x^n}{n!}}$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$

$$= \frac{22^{n+1}x^{n+1}n!}{(n+1)! \cdot 22^n x^n}$$

Cancel $\frac{22^{n+1}x^{n+1}n!}{(n+1)! \cdot 22^n x^n}$: $\frac{22xn!}{(n+1)!}$ *Hide Steps*

$$\frac{22^{n+1}x^{n+1}n!}{(n+1)! \cdot 22^n x^n}$$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{22^{n+1}}{22^n} = 22^{n+1-n}$$

$$= \frac{22^{n-n+1}x^{n+1}n!}{x^n(n+1)!}$$

Add similar elements: $n+1-n = 1$

$$= \frac{22x^{n+1}n!}{x^n(n+1)!}$$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{x^{n+1}}{x^n} = x^{n+1-n}$$

$$= \frac{22x^{n-n+1}n!}{(n+1)!}$$

Add similar elements: $n+1-n = 1$

$$= \frac{22xn!}{(n+1)!}$$

$$= \frac{22xn!}{(n+1)!}$$

Cancel the factorials: $\frac{n!}{(n+m)!}=\frac{1}{(n+1)\cdot(n+2)\cdots(n+m)}$

$\frac{n!}{(n+1)!}=\frac{1}{(n+1)}$

$=\frac{22x}{n+1}$

$L=\lim_{n\to\infty}\left(\left|\frac{22x}{n+1}\right|\right)$

$L=|22x|\cdot\lim_{n\to\infty}\left(\left|\frac{1}{n+1}\right|\right)$

$\lim_{n\to\infty}\left(\left|\frac{1}{n+1}\right|\right)=0$ *Hide Steps*

$\lim_{n\to\infty}\left(\left|\frac{1}{n+1}\right|\right)$

$\frac{1}{n+1}$ is positive when $n\to\infty$. Therefore $\left|\frac{1}{n+1}\right|=\frac{1}{n+1}$

$=\lim_{n\to\infty}\left(\frac{1}{n+1}\right)$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)},\quad \lim_{x\to a}g(x)\neq 0$

With the exception of indeterminate form

$=\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}(n+1)}$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$\lim_{n\to\infty}(n+1)=\infty$ *Hide Steps*

$\lim_{n\to\infty}(n+1)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty,\ a>0,$ n is odd

$a=1,\ n=1$

$=\infty$

$=\frac{1}{\infty}$

Apply Infinity Property: $\frac{c}{\infty}=0$

$=0$

$L=|22x|\cdot\ 0$

Simplify

$L=0$

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$L<1$ for every x, therefore the power series converges for all x

Radius of convergence is ∞ , Interval of convergence is $-\infty<x<\infty$

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