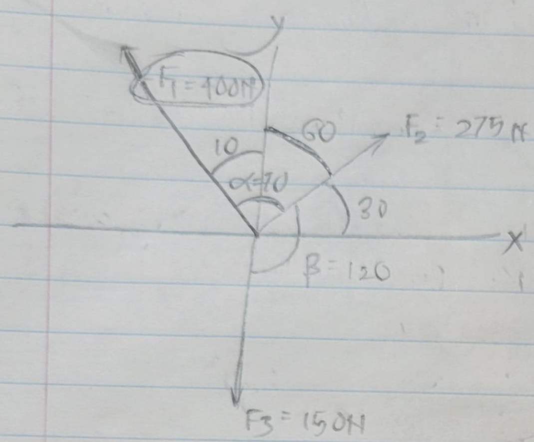
Question 1



$F_1 = 400N$

$F_2 = 275N$

$F_3 = 150N$

$\alpha = 70^\circ$

$\beta = 120^\circ$

$F_{2x} = |F_2| \cos 30\,\hat{i} = 275 \cos 30\,\hat{i}$

$F_{2y} = |F_2| \cos 60\,\hat{j} = 275 \times \cos 60\,\hat{j}$

$F_{3x} = |F_3| \cos 90\,\hat{i} = 0\,\hat{i}$

$F_{3y} = |F_3| \cos 180\,\hat{j} = -|F_3|\,\hat{j}$
$= -150N\,\hat{j}$

$F_{1x} = |F_1| \cos 100 = -69.4592$

$F_{1y} = |F_1| \cos 10 = 393.92$

Finding x and y components of each force

Given $F_1 = 400N$

$F_2 = 275N$

$F_3 = 150N$

$$\vec{R} = \hat{i}\left(0 + |F_2|\cos 30 + |F_1|\cos 100\right)$$

$$+ \hat{j}\left(|F_3|\cos 180 + |F_2|\cos 60 + |F_1|\cos 10\right)$$

$$= 168.697715\,\hat{i} + 381.4231012\,\hat{j}$$

$$= R_x + R_y$$

$$|R| = \sqrt{(R_x)^2 + (R_y)^2}$$

$$= \sqrt{(168.698)^2 + (381.423)^2}$$

$$= 417.0641\ N$$

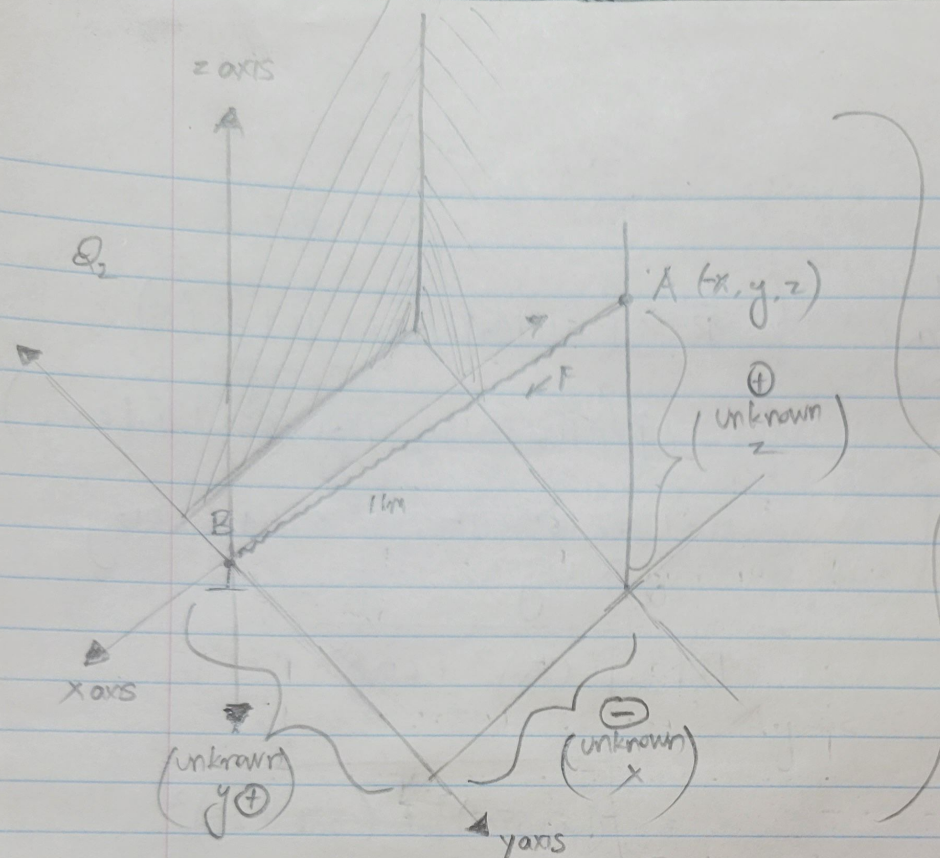
Calculating the angle of F_R with x axis:

$$\tan\theta = \left(\frac{381.423}{168.698}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{381.423}{168.698}\right)$$

$$= 66.14087038^\circ$$

(Ans)

Q_2



Taking B as our point of origin

$\langle 0, 0, 0 \rangle$

Given $\bar{F} = [600\hat{i} - 200\hat{j} - 500\hat{k}]$

$\overline{AB} = 11m$

$|\bar{F}| = \sqrt{(600)^2 + (200)^2 + (500)^2}$

$= 806.2257748\ N$

Since B is our origin,

$\overline{AB} = (\vec{B} - \vec{A}) = (0\hat{i} + 0\hat{j} + 0\hat{k}) - (-x\hat{i} + y\hat{j} + z\hat{k})$

$= (+x\hat{i} - y\hat{j} - z\hat{k})$

(i)

$\mu_F = \left[\left(\frac{600}{806.23}\right)\hat{i} + \left(\frac{200}{806.23}\right)\hat{j} + \left(\frac{500}{806.23}\right)\hat{k}\right]$

Since Force and $\overline{AB}$ are in the same direction, we can use the same unit vector for $\overline{AB}$. $|\overline{AB}| = 11m$

(ii)

comparing (i) & (ii)

$\overline{AB} = |\overline{AB}|\mu_F$

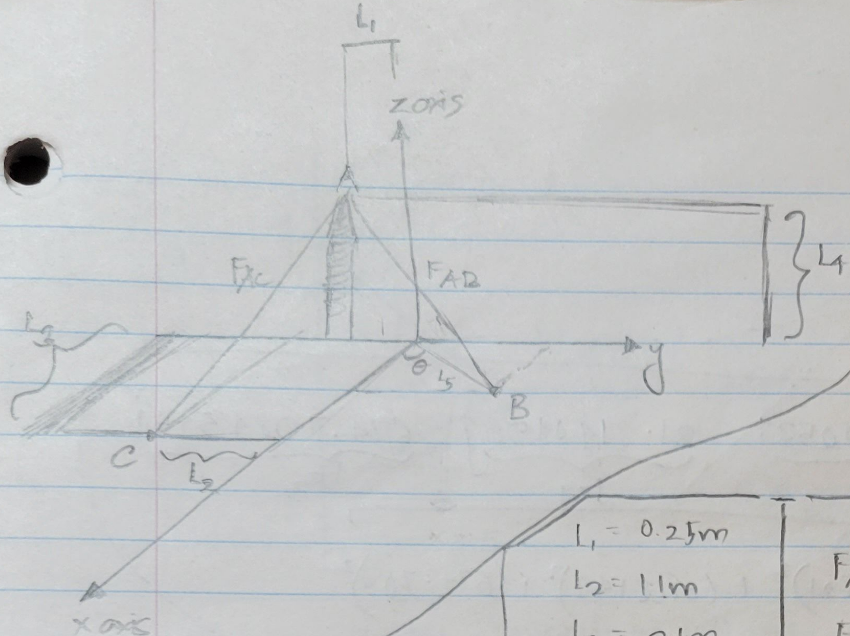
$= 11 \times \left[\frac{600}{806.23}\hat{i} + \frac{200}{806.23}\hat{j} + \frac{500}{806.23}\hat{k}\right]$

$= 8.1862\,m\,\hat{i} + 2.7274\,m\,\hat{j} + 6.8218\,m\,\hat{k}$

$x = -8.1862\,m$

$y = 2.7274\,m$ Ans

$z = 6.8218\,m$



$L_1 = 0.25m$	$F_{AB} = 300N$
$L_2 = 1.1m$	$F_{AC} = 500N$
$L_3 = 2.1m$	
$L_4 = 3.25m$	
$L_5 = 1.6m$	$\theta = 30°$

①

$A = -L_1\hat{j} + L_4\hat{k} = -0.25m\,\hat{j} + 3.25m\,\hat{k}$

$B = L_5\cos\theta\,\hat{i} + L_5\sin\theta\,\hat{j} = \frac{4\sqrt{3}}{5}\hat{i} + \frac{4}{5}\hat{j}$

$C = L_3\hat{i} - L_2\hat{j} = 2.1m\,\hat{i} - 1.1m\,\hat{j}$

②

$\overline{AB} = \overline{B} - \overline{A}$

$= \frac{4\sqrt{3}}{5}\hat{i} + \frac{21}{20}\hat{j} - 3.25m\hat{k}$ (m, m)

$\overline{AC} = \overline{C} - \overline{A}$

$= 2.1m\hat{i} - 0.85\hat{j} - 3.25\hat{k}$ (m, m)

$|\overline{AB}| = 3.685783m$

$|\overline{AC}| = 3.96169\,m$

③

$\mu_{AC} = \frac{\overline{AC}}{|\overline{AC}|}$ $\quad$ $\mu_{AB} = \frac{\overline{AB}}{|\overline{AB}|}$

$F_{AB} = |F_{AB}|\mu_{AC}$

$= 300(0.375942\hat{i}$
$+ 0.284878\hat{j}$
$- 0.8817665\hat{k})$

$= 112.7826\hat{i}$
$+ 85.4634\hat{j}$
$- 264.52995\hat{k}$

$F_{AC} = |F_{AC}|\mu_{AB}$

$= 500\left(\frac{2.1}{3.96}\hat{i} - \frac{0.85}{3.96}\hat{j} - \frac{3.25}{3.96}\hat{k}\right)$

$= 265.03840 53\hat{i}$
$- 107.2774498\hat{j}$
$- 410.1788844\hat{k}$

4

$$\bar{F}_R = \underbrace{377.8210053}_{F_{Rx}}\hat{i} - \underbrace{21.8140498}_{F_{Ry}}\hat{j} - \underbrace{674.7084344}_{F_{Rz}}\hat{k}$$

$$|\bar{F}_R| = \sqrt{(377.821)^2 + (21.8144)^2 + (674.708)^2}$$

$$= 773.5990201\ N$$

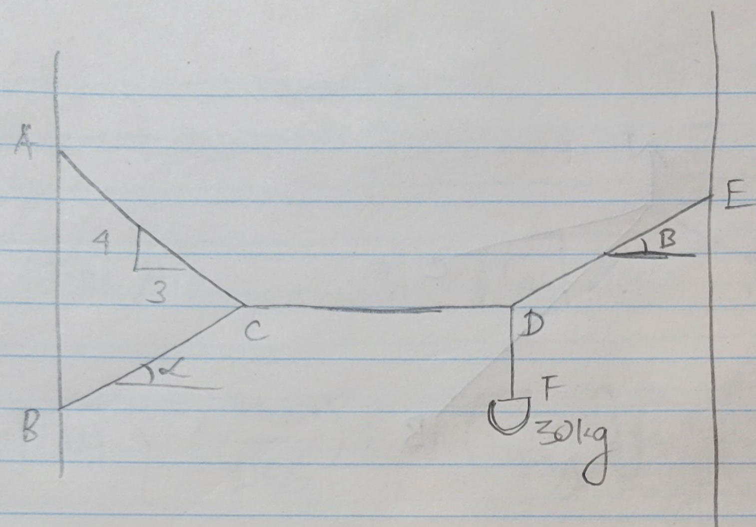
5

$$\alpha = \cos^{-1}\left(\frac{F_{Rx}}{|F_R|}\right) = 60.79^\circ$$

$$\beta = \cos^{-1}\left(\frac{F_{Ry}}{|\bar{F}_R|}\right) = 91.6158^\circ$$

$$\gamma = \cos^{-1}\left(\frac{F_{Rz}}{|F_R|}\right) = 150.7115139^\circ$$

④



Taking upwards as y⊕ axis

Taking right as x⊕ axis

At point D

①

Given $\alpha = 45°$

$\beta = 35°$

$\Sigma F_y = 0$

$\Rightarrow F_{DF} - F_{DE}\sin\beta = 0$

$\Rightarrow (30)(9.80665) - F_{DE}\sin 35 = 0$

$\Rightarrow F_{DE} = 512.9211755\ N$

②

$\Sigma F_x = 0$

$\Rightarrow -F_{CD} + F_{DE}\cos\beta = 0$

$\Rightarrow F_{CD} = 420.1604295$ N

③ At point C

$\Sigma F_x = 0$

$\Rightarrow -F_{CD} + F_{CA}\left(\frac{3}{5}\right) + F_{CB}\cos 45 = 0$

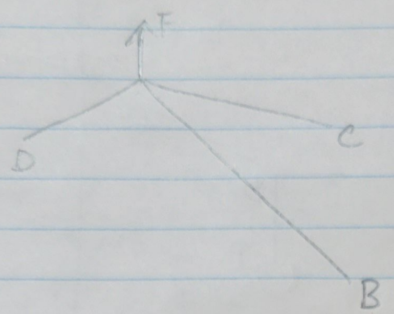
$\Rightarrow F_{CA}\left(\frac{3}{5}\right) + F_{CB}\cos 45 = F_{CD} = 420.160\,N$ — ⓘ

$\Sigma F_y = 0$

$\Rightarrow \left(\frac{4}{5}\right)F_{CA} - F_{CB}\sin 45 = 0$ — ⓘⓘ → ⓘ + ⓘⓘ $\Rightarrow \left(\frac{7}{5}\right)F_{CA} + 0 = 420.160\,N$

$\Rightarrow F_{CA} = \left(\frac{420.160 \times 5}{7}\right)$

$= 300.11459\,N$



$A = 5m\,\hat{k}$

$B = 4.5m\,\hat{i} + 4m\,\hat{j}$

$C = -4.5m\,\hat{i} + 6.5\,\hat{j}$

$D = -6m\,\hat{i} - 2.5m\,\hat{j}$

$$\bar{AB} = \begin{bmatrix} 4.5 \\ 4 \\ 0 \end{bmatrix} m - \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} m = \begin{bmatrix} 4.5 \\ 4 \\ -5 \end{bmatrix} m$$

$$\bar{AC} = \begin{bmatrix} -4.5 \\ 6.5 \\ 0 \end{bmatrix} m - \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} m = \begin{bmatrix} -4.5 \\ 6.5 \\ -5 \end{bmatrix} m$$

$$\bar{AD} = \begin{bmatrix} -2.5 \\ -6 \\ 0 \end{bmatrix} m - \begin{bmatrix} 0 \\ 0 \\ -5 \end{bmatrix} m = \begin{bmatrix} -2.5 \\ -6 \\ -5 \end{bmatrix} m$$

$$|\bar{AB}| = \frac{7\sqrt{5}}{2}\ m \qquad \mu_{AB} = \frac{\bar{AB}}{|AB|}$$

$$|\bar{AC}| = \frac{5\sqrt{14}}{2}\ m \qquad \mu_{AC} = \frac{\bar{AC}}{|AC|}$$

$$|\bar{AD}| = \frac{\sqrt{269}}{2}\ m \qquad \mu_{AD} = \frac{\bar{AD}}{|AD|}$$

$\overline{T_{AB}} = |\overline{T_{AB}}|\, \mu_{AB}$ $\qquad$ $\overline{T_{AC}} = |\overline{T_{AC}}|\, \mu_{AC}$ $\qquad$ $|\overline{T_{AD}}|\, \mu_{AD} = \overline{T_{AD}}$

$$\overline{T_{AB}} = |\overline{T_{AB}}| \left\{ \left(\frac{4.5}{7\sqrt{5}/2}\right)\hat{i} + \left(\frac{4}{7\sqrt{5}/2}\right)\hat{j} + \left(\frac{-5}{7\sqrt{5}/2}\right)\hat{k} \right\}$$

$$\overline{T_{AC}} = |\overline{T_{AC}}| \left\{ \frac{-4.5}{\frac{5\sqrt{15}}{2}}\hat{i} + \left(\frac{6.5}{5\sqrt{15}/2}\right)\hat{j} + \left(\frac{-5}{5\sqrt{15}/2}\right)\hat{k} \right\}$$

$$\overline{T_{AD}} = |\overline{T_{AD}}| \left\{ \frac{-2.5}{\frac{\sqrt{269}}{2}}\hat{i} + \frac{-6}{\frac{\sqrt{269}}{2}}\hat{j} + \frac{-5}{\frac{\sqrt{269}}{2}}\hat{k} \right\}$$

$$\begin{bmatrix} 4.5/7\sqrt{5}/2 & 4.5/(5\sqrt{15}/2) & -2.5/\sqrt{269}/2 \\ 4/7\sqrt{5}/2 & 6.5/(5\sqrt{15}/2) & -6/\sqrt{269}/2 \\ -5/7\sqrt{5}/2 & -5/(5\sqrt{15}/2) & -5/\sqrt{269}/2 \end{bmatrix} \begin{bmatrix} T_{AB} \\ T_{AC} \\ T_{AD} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ -950 \end{bmatrix}$$