Student: Arfaz Hossain **Date:** 10/30/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 6

Find the value(s) of c that satisfy the equation $\frac{f(b)-f(a)}{b-a}=f'(c)$ in the conclusion of the mean value theorem for the function $f(x)=\sin^{-1}x$ in the interval $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$.

If $f(x)=\sin^{-1}x$ and the interval [a,b] is $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$, what are f(a) and f(b)?

$f(a)=\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)=$ $-\frac{\pi}{4}$, $f(b)=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=$ $\frac{\pi}{4}$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression.)

Evaluate $f'(c)$.

$$f'(c)=\frac{f(b)-f(a)}{b-a}$$

$$=\frac{\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)}{\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)}$$

$$=\frac{\pi}{2\sqrt{2}}$$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression.)

Put $y=f(x)=\sin^{-1}x$, then $\sin y=x$. Differentiate both sides to obtain $\frac{d}{dx}(\sin y)=\frac{d}{dx}(x)$.

Use the chain rule, $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ to evaluate the derivative.

$$\frac{d}{dx}(\sin y)=\frac{d}{dx}(x)$$

$\cos y \frac{dy}{dx} =$ 1

(Simplify your answer.)

Express cos y as a function of x. Use $\sin^2 y+\cos^2 y=1$ to obtain $\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-x^2}$.

Substitute $\cos y=\sqrt{1-x^2}$ in the above.

$$\cos y\frac{dy}{dx}=1$$

$$\sqrt{1-x^2}\frac{dy}{dx}=1$$

$$\frac{dy}{dx}=1\div\sqrt{1-x^2}$$

Put $\frac{dy}{dx}=f'(x)=\frac{1}{\sqrt{1-x^2}}$, then what is $f'(x)$ at $x=c$?

$f'(c)=$ $\frac{1}{\sqrt{1-c^2}}$

(Simplify your answer. Type an exact answer, using radicals as needed.)

From above, $f'(c) = \frac{\pi}{2\sqrt{2}}$. Simplify the expression.

$$\frac{1}{\sqrt{1-c^2}} = \frac{\pi}{2\sqrt{2}}$$

$$1 - c^2 = \frac{8}{\pi^2}$$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression.)

Solve for c.

$$1 - c^2 = \frac{8}{\pi^2}$$

$$c = \pm \sqrt{1 - \frac{8}{\pi^2}}$$

(Simplify your answer. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the expression.)

The values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a} = f'(c)$ for the function $f(x) = \sin^{-1} x$ in the interval $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ are $c = \pm \sqrt{1 - \frac{8}{\pi^2}}$.