Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Use limits to find horizontal asymptotes for each function.

a. $y = \frac{x}{13} \tan\left(\frac{13}{x}\right)$ **b.** $y = \frac{10x + e^{9x}}{9x + e^{10x}}$

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a. A line y = b is a horizontal asymptote of the graph of a function y = f(x) if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to -\infty} f(x) = b$.

Note that $\lim_{x\to\infty} \frac{x}{13} \tan\left(\frac{13}{x}\right)$ is in the indeterminate form $\infty \cdot 0$. Convert the limit to a $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, and then use l'Hôpital's rule to evaluate the limit.

Begin by using substitution. Let $h = \frac{13}{x}$. Substitute h into the equation.

$$y = \frac{1}{h} \tan(h)$$

Now set up the limit as shown below.

$$\lim_{x\to\infty} \frac{x}{13} \tan\left(\frac{13}{x}\right) = \lim_{h\to 0^+} \frac{1}{h} \tan(h) = \lim_{h\to 0^+} \frac{\tan(h)}{h}$$

The limit is no longer in the indeterminate form $\infty \cdot 0$. It is now in the form $\frac{0}{0}$, so l'Hôpital's rule can be used to evaluate the limit.

Apply l'Hôpital's rule. Differentiate $f(h) = \tan(h)$.

$$f'(h) = \sec^2(h)$$

Differentiate $g(h) = h$.

$$g'(h) = 1$$

The new limit after applying l'Hôpital's rule is shown below.

$$\lim_{h\to 0^+} \frac{\tan(h)}{h} = \lim_{h\to 0^+} \frac{\sec^2(h)}{1}$$

L'Hôpital's rule does not need to be applied again because the limit is no longer in the form $\frac{\infty}{\infty}$.

To evaluate the limit, substitute 0 for h. Find $\sec^2(0)$.

$$\sec^2(0) = 1$$

Evaluate the limit.

$$\lim_{h\to 0^+} \frac{\sec^2(h)}{1} = \frac{\sec^2(0)}{1} = \frac{1}{1} = 1$$

Therefore, $y = \frac{x}{13}\tan\left(\frac{13}{x}\right)$ has a horizontal asymptote at y = 1.

b. Use l'Hôpital's rule to evaluate the limit. Recall that $x \to a$ may be replaced by the one-sided limits $x \to a^+$ or $x \to a^-$.

First, set up the limit as x approaches positive infinity.

$$\lim_{x\to\infty} \frac{10x + e^{9x}}{9x + e^{10x}}$$

Then apply l'Hôpital's rule.

$$\lim_{x\to\infty} \frac{10x + e^{9x}}{9x + e^{10x}} = \lim_{x\to\infty} \frac{10 + 9e^{9x}}{9 + 10e^{10x}}$$

The limit is still in the indeterminate form $\frac{\infty}{\infty}$ because $\lim_{x\to\infty} e^x = \infty$.

Apply l'Hôpital's rule again.

$$\lim_{x\to\infty} \frac{10 + 9e^{9x}}{9 + 10e^{10x}} = \lim_{x\to\infty} \frac{81e^{9x}}{100e^{10x}}$$

The limit is still in the indeterminate form $\frac{\infty}{\infty}$. To resolve this, rewrite the fraction as shown below.

$$\lim_{x\to\infty} \frac{81e^{(9x - 10x)}}{100} = \lim_{x\to\infty} \frac{81e^{(-x)}}{100} = \lim_{x\to\infty} \frac{81}{100e^x}$$

Evaluate the limit.

$$\lim_{x\to\infty} \frac{81}{100e^x} = 0$$

Now, evaluate the limit as x approaches negative infinity to determine if it is different from the limit as x approaches positive infinity.

$$\lim_{x\to-\infty} \frac{10x + e^{9x}}{9x + e^{10x}}$$

Then apply l'Hôpital's rule as shown below.

$$\lim_{x\to-\infty} \frac{10x + e^{9x}}{9x + e^{10x}} = \lim_{x\to-\infty} \frac{10 + 9e^{9x}}{9 + 10e^{10x}}$$

The limit is no longer in the indeterminate form $\frac{\infty}{\infty}$ because $\lim_{x\to-\infty} e^x = 0$.

Evaluate the limit.

$$\lim_{x\to-\infty} \frac{10 + 9e^{9x}}{9 + 10e^{10x}} = \frac{10 + 9(0)}{9 + 10(0)} = \frac{10}{9}$$

Therefore, $y = \frac{10x + e^{9x}}{9x + e^{10x}}$ has horizontal asymptotes at y = 0 and $y = \frac{10}{9}$.