ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 14: The Internal Rate of Return

October 8, 2021

Version 1.1 – Fixed a typo on slide 28. Added slides 31 and 32.

Required Viewing

- Edspira. (2013, September 18). IRR (Internal Rate of Return) [Video File]. https://youtu.be/OSDDrZZaV8E
- Edspira. (2013, September 18). NPV vs IRR [Video File]. https://youtu.be/6RztxNwerOA

Optional Reading

- *Engineering Economics*, Chapter 5, Sections 5.2, 5.3.1 and 5.3.2
- Farouk, N. (2012). Proposal for Improve the Electrical Power Supply in Port Sudan Town. *International Journal of Advanced Science and Technology*, 42, 91-99. Retrieved from https://web.archive.org/web/20150430042323/http://www.sersc.org/journals/IJAST/vol42/9.pdf
    - **Port Sudan example**. **Now offline but archived on the Web Archive.**
- Riiny, M. (2014). South Sudan Rural Electrification Project. *Global Humanitarian Technology Conference (GHTC) 2014*, 25 – 27. https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6713673
    - **A followup on Port Sudan. As of 2014, South Sudan's power supply was still inadequate. A hybrid approach of mini-grids and portable battery kits was a promising quick fix for bringing electricity to rural areas.**
- *Conflict Between NPV and IRR* [Web Page]. (n.d.). Retrieved from https://financetrain.com/conflict-between-npv-and-irr/
    - **Explains how and why NPV and IRR sometimes yield different results**.
- Palmer, B. (2019, February 16). Should IRR or NPV Be Used for Capital Budgeting? [Web Page]. https://www.investopedia.com/ask/answers/05/irrvsnpvcapitalbudgeting.asp
    - **A comparison of NPV and IRR**.

More optional reading on NPV vs. IRR

- Rich, S. P. & Rose, J. T. (2014). Re-examining an Old Question: Does the IRR Method Implicitly Assume a Reinvestment Rate? *Journal of Financial Education*, 40(1/2), 152 – 166. Retrieved from http://www.jstor.org.ezproxy.library.uvic.ca/stable/24331033
  - **Not required, but it'll clear up, in detail, the difference between the NPV and IRR approaches. It's from an education journal, so it's very readable.**
- **A more recent paper on the topic:**
- Magni, C. A. & Martin, J. D. (2017). The Reinvestment Rate Assumption Fallacy for IRR and NPV [MPRA Paper No. 83889]. Retrieved from https://mpra.ub.uni-muenchen.de/83889/
- **The following web page is also useful**:
- Gallant, C. (2020, April 14). Net Present Value and the Internal Rate of Return [Web Page]. Retrieved from https://www.investopedia.com/exam-guide/cfa-level-1/quantitative-methods/discounted-cash-flow-npv-irr.asp

Case Studies

- Bejbl, J., Bems, J., Kralik, T., Stary, O. & Vastl, J. (2014). New approach to brown coal pricing using internal rate of return methodology. *Applied Energy, 133,* 289-197. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.apenergy.2014.07.082
- Prol, J. L. & Steininger, K. W. (2020). Photovoltaic self-consumption is now profitable in Spain: Effects of the new regulation on prosumers' internal rate of return. *Energy Policy, 146*, 111793. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.enpol.2020.111793
- Talavera, D. L., Nofuentes, G. & Aguilera, J. (2010). The internal rate of return of photovoltaic grid-connected systems: A comprehensive sensitivity analysis. *Renewable Energy, 35*(1), 101-111. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.renene.2009.07.006

Learning Objectives

- Understand the concept behind the IRR.
- Be able to set up the equation for the IRR for a single project. Be able to compare a project's IRR to the MARR and interpret the result.
- Be able to evaluate independent projects using IRR.
- Be aware of approaches for solving this equation, including interpolation.
- Understand how to use the incremental IRR to evaluate mutually exclusive projects.
- Understand why the NPV and IRR are different.

Relevant Solved Problems I

- From Engineering Economics, Chapter 5
- IRR Calculation: Example 5.1, Example 5.2, 5.10, 5.31, 5.32, 5.33, 5.34.a, 5.38.a, 5.39
- IRR and Independent Projects: Example 5.3, 5.11, 5.12, 5.13, 5.20.a, 5.30.a, 5.34.b
- IRR and Mutually Exclusive Projects: Review Problem 5.1, Review Problem 5.2, Example 5.4, Example 5.5, Example 5.6, 5.14, 5.15, 5.16, 5.19, 5.20.b, 5.20.c, 5.21, 5.24, 5.25, 5.26, 5.27, 5.29, 5.30.b, 5.36, 5.40

Relevant Solved Problems II

- From Nielsen’s *Engineering Economics: The Basics* (2nd ed.):
- Chapter 8 (all)
- Example 11-3, Example 11-9

Notation Dictionary

(Not provided on quiz/final formula sheet)

- A = Annuity
- F = Future Value
- IRR = Internal Rate of Return
- ERR = External Rate of Return
- MARR = Minimum Acceptable Rate of Return
- N = the N'th time period
- P = Present Value
- Green Text = Excel Formula

- Conversion factors are of the form (X/Y,z)
- Read as: X, given Y and z.
- X is the element we want.
- Y is the element we have.
- z represents additional parameters.
- e.g. (P/F,i,N)
- Present Value, given a Future Value at time N and interest rate i.

$f(x)$

Formulas Introduced

- Notation: The orange symbol on a slide indicates a formula sheet formula is introduced there.
- $\sum_{t=0}^{N} \frac{\sum C_t^j}{(1+IRR)^t} = 0$

ESSENTIALS (19 slides)

Internal Rate of Return (IRR)

- Why 'Internal'?
- Because it depends only on cash flows internal to the project.
- …not always the best assumption! (more on this later)

- IRR is the rate of return that makes the PW of cash flows zero.
- Intuition: think back to our discussion of MARR…
- PW = 0 means the cash flow is earning the same return as paying \$1 today for $\$(1+i)^N$ in period N.
- (Cost: \$1 Benefit: Future Value of \$1, with a present value of \$1)
- → The IRR is kind of like an effective interest rate for the cash flow.

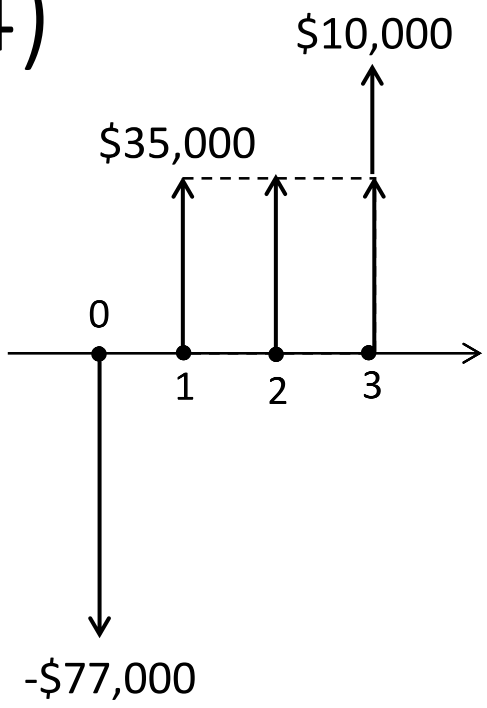
Some simple examples

- Project A: Pay \$100 today, get \$100 tomorrow.
- NPV(i) = -\$100 + \$100 x (P/F,i,1)
- NPV(IRR) = 0 → -\$100 + \$100 x (P/F,IRR,1) = 0
- → 1 = (P/A,IRR,1) → 1 = 1/(1+IRR) → IRR = 0
- Project B: Pay \$100 today, get \$200 tomorrow.
- NPV(IRR) = 0 → -\$100 + \$200 x (P/F,IRR,1) = 0 → IRR = 100%
- **Check**: -\$100 + \$200/(1 + 100%) = -\$100 + \$100 = 0
- Project C: Pay \$100 today, get \$50 tomorrow.
- NPV(IRR) = 0 → -\$100 + \$50 x (P/F,IRR,1) = 0 → IRR = -50%
- **Check**: -\$100 + \$50/(1 – 50%) = -\$100 + \$100 = 0
- (A negative IRR implies you're losing money, period.)

Important Intuition

- You can think of a project's cash flow profile of being composed of a *net initial outlay* (often called NIO) at the start of the project (Year 0), followed by other cash flows.
- The IRR is the rate that allows the NIO to duplicate the rest of the cash flows in the project.
- ”I have this pile of money in Year 0. What interest rate that I can lend and borrow this money at would *just* let me build the project's cash flow profile?” That's the IRR.
- Realizing this is what the IRR is actually measuring can help you understand why the NPV is zero at the IRR: because the rest of the cash flow will JUST balance out the initial costs.

<u>A worked example</u> (Rich & Rose, 2014)



- Let's <u>verify</u> the IRR for the following investment:
- An initial cost of $77,000 in Year 0
- Income of $35,000 from Year 1 to Year 3
- Salvage of $10,000 in Year 3
- From Rich & Rose, we know the IRR is 21.9882%
- Let's verify that first, using the previous slide's intuition…
- Then, by calculating the NPV and checking that it's zero.
- If you're wondering why we only have a negative cash flow in Year 0…
- …it's because changing signs in the cash flow profile can really mess up IRR calculations. We'lll deal with that next lecture.

Using the NIO ($77,000) as a 'savings account'

- It helps to think of the initial outlay (NIO) as a savings account that you draw from to replicate the project's cash flows.
- The 'savings account' can be thought of as earning the IRR (21.9882%)
- The following table is lightly adapted from Rich & Rose.

	YEAR 0	YEAR 1	YEAR 2	YEAR 3
NIO (Start of Year)	$77,000	$93,931	$71,889	$45,000
'Withdrawals'	$0	$35,000	$35,000	$45,000
Leaving…	$77,000	$58,931	$36,889	$0
Applying Interest	x 1.219882	x 1.219882	x 1.219882	x 1.219882
NIO (End of Year)	$93,931	$71,889	$45,000	$0

As required, the IRR is just enough to replicate the cash flows by withdrawing from it.

Checking that the NPV = 0

- We have an initial cost, an annuity, and a future salvage value.
- Our N = 3, and as the i we're using the IRR = 0.219882
- NPV = -$77,000 + $35,000 x (P/A,IRR,3) + $10,000 x (P/F,IRR,3)
- NPV = -$77,000 + $35,000 x 2.0426 + $10,000 x 0.5509
- NPV = -$77,000 + $71,491.34 + $5,508.66
- NPV = $0
- As required of an IRR, the NPV = 0.

Remember: IRR and NPV use different concepts!

- The NPV calculated using the IRR as the discount rate is equal to zero, but that does NOT mean they're the same thing!
- To calculate 'the' NPV, you need to use the MARR as the discount rate. In calculating the NPV, you are comparing the project's cash flows to your fallback investment (from which the MARR is derived), and calculating the resulting profit or loss.
- You're answering the questions, "Would I be better off with this project, or with my fallback project, and how big is the difference?"
- In calculating the IRR, you are answering the question, "what interest rate would JUST allow my initial outlay to replicate the project's cash flow profile?"
- When comparing the IRR to the MARR. If IRR > MARR, that means you could *not* replicate those cash flows using the MARR, so the project is beyond what you could 'usually afford'. That's why a project is worthwhile if IRR > MARR.
- If IRR < MARR, you could replicate the project's cash flows with i = MARR, AND have money left over. Since you could already more than afford that cash flow profile at your MARR, the project is not worthwhile.

Why would we want this?

- Suppose as part of a project proposal, you are asked to summarize how successful similar projects have been in the past.
- What published values from different projects will you show?
- Benefit-Cost Ratios? Magnitudes are arbitrary and not directly comparable.
- Present/Annual Worth? Doesn’t allow easy merging of results from projects of different sizes. (e.g. City-wide green energy upgrades vs results for 1 building.)
- The internal rate of return *can* be sensibly reported for different projects at different scales, and compared to the current MARR.
- Yearly % also allows easy comparison to other values commonly reported as yearly rates. (e.g. bond yields, cost of borrowing.)
- IRR is also the highest MARR for which the project is worthwhile:
- If your MARR is uncertain or likely to change, this can be useful.

How to solve for it?

- Analytically (Good luck! Not often tractable.)
- Excel: =IRR(A1:A10), where A1 to A10 are net cash flows.
  - (You can't skip a year: remember to add a 0 if you need to.)
- Numerically: trial and error, numerical solution of analytical equations, interpolation (usually linear)
  - Remember to pick the most convenient of A,F or P. This can speed things up!
- Graphically:
  - derive an expression for the present value of the cash flow using (P/F,i,N) and other conversion factors…
  - …then plot it for reasonable values of i, and zoom in on the spot where the graph crosses the x-axis.

IRR and Independent Projects

- Any project with an IRR>MARR should be accepted.
- Intuition: the 'do nothing' case earns you the MARR, so anything higher than that implies profit.
- In our example, the IRR > MARR, so "the project is considered economically feasible".
- Another take: "The IRR is the (assumed constant) rate of return on the invested capital, period by period. The condition that IRR > [MARR] only means that if, in any given period, an investor invests an amount of capital equal to the capital that remains invested in the project, then the rate of return earned with the former would be greater than the rate earned with the latter." (Magni & Martin, 2017)

IRR and Mutually Exclusive Projects

- Remember incremental BCR analysis? Same thing.
- 1. Rank the projects from lowest to highest first cost.
- 2. Find the *incremental* cash flow profile by subtracting the net flows of the first project (A) from those of the second project (B).
- Let IIRR = Incremental Internal Rate of Return denote the IRR of this incremental cash flow (this is standard terminology).
- 3. Find the IIRR of the (B – A) cash flow. If this IIRR > MARR, keep B and discard A. If IIRR < MARR, keep A and discard B.
- 4. Repeat until there is only one project left.

Simple Example

- Suppose our MARR is 10% per day.
- Consider these mutually exclusive projects:
- Project F: Pay \$100 today, get \$180 tomorrow.
- Project G: Pay \$125 today, get \$225 tomorrow.
- Project H: Pay \$160 today, get \$230 tomorrow.
- By inspection, all projects have an IRR > MARR.
- Which do we choose? Let’s use incremental IRR analysis.
- We start with the two with lowest initial costs, F&G.

Round One: FIGHT! Project F vs Project G

- Project F: -\$100 today, + \$180 tomorrow.
- Project G: -\$125 today, + \$225 tomorrow.
- To choose between them, create an 'incremental flow':
- Project (G – F): (-\$125 – (-\$100)) today, (\$225 - \$180) tomorrow.
- Project (G – F): -\$25 today, +\$45 tomorrow
- NPV(IIRR) = -25 + 45 x (P/A,IIRR,1) = 0
- → 25 = 45/(1+IIRR) → IIRR = 80%
- IRR = 80% > MARR = 10%, so choose the project with the higher initial cost, Project G, and discard the other project.

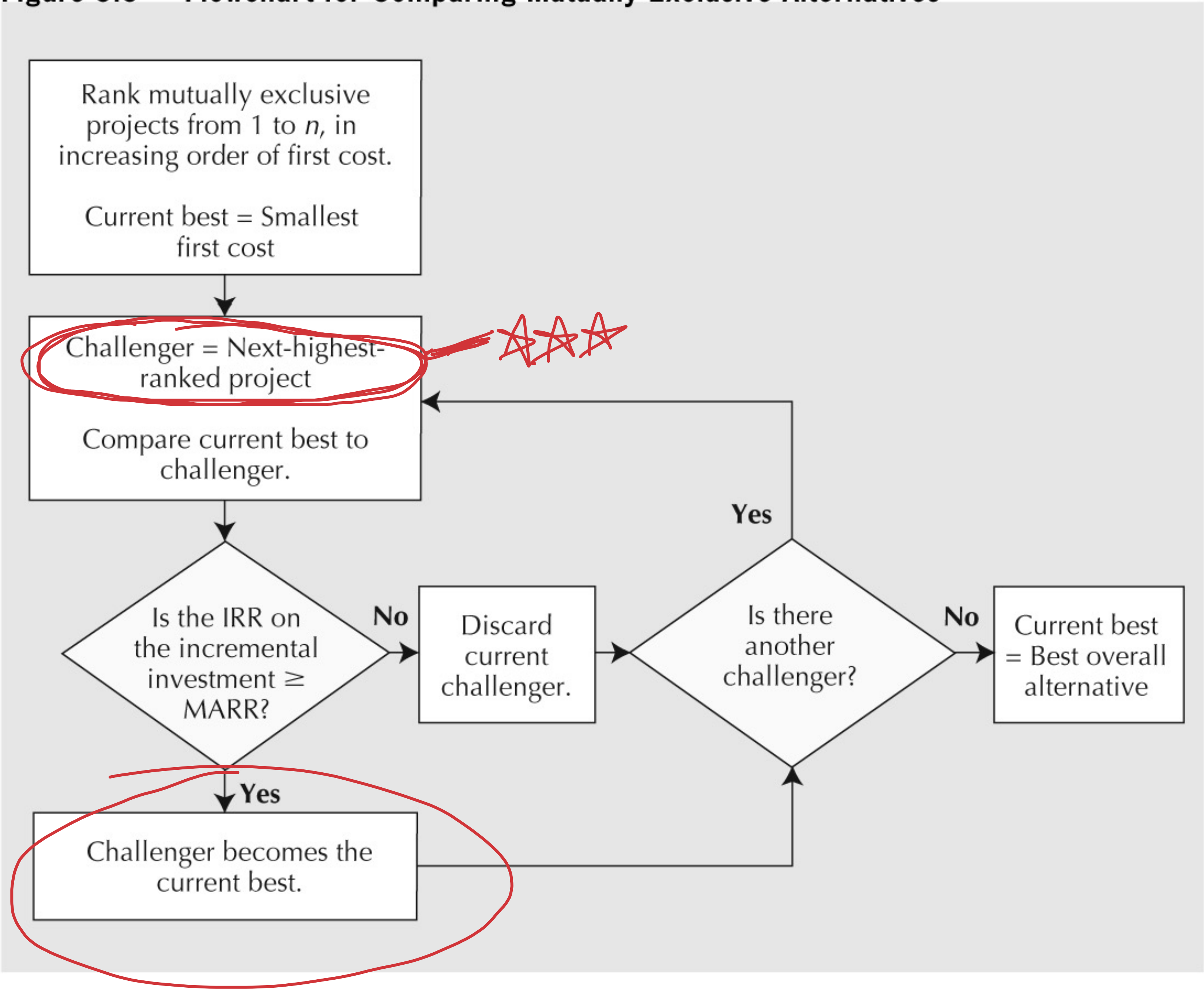
Round Two: FIGHT! Project G vs Project H

- Project G: -$125 today, +$225 tomorrow.
- Project H: -$160 today, +$230 tomorrow.
- To choose between them, create an ‘incremental flow’:
- Project (H – G): (-$160 – (-$125)) today, ($230 - $225) tomorrow.
- Project (H – G): -$35 today, +$5 tomorrow
- NPV(IRR) = -35 + 5 x (P/A,IIRR,1) = 0
- → 35 = 5/(1+IIRR) → IIRR = -86% (approx.)
- IIRR = -86% < MARR = 10%, so choose the project with the lower initial cost, Project G, and discard the other project.
- Only Project G is left, so Project G is the winner.

What if the IIRR doesn't exist?

- As before, that means the NPV of the cash flow you're looking at is always negative.
- →The upgrade to the more expensive project isn't worth it.
- (Assuming costs up front, benefits later.)
- Exception: you'll also get non-existence if there are NO sign changes in your project's cash flows, and it's ALL costs, or ALL benefits – but that should be trivial to spot and interpret.

Figure 5.5 Flowchart for Comparing Mutually Exclusive Alternatives



Why can't we just pick the biggest IRR?

- Consider the following two projects (example 5.4):
- Suppose the MARR is 70%.
- Project A: Pay $1 today for $2 next year. IRR = 100%
- Project B: Pay $1000 today for $1900 next year. IRR = 90%

BUT

- Sure, Project A has the highest rate of return…. Proportional to its initial cost.
- You're missing out on a lot by not upgrading to Project 2.
- Incremental IRR analysis avoids these situations.

Incremental IRR (IIRR) analysis

- Continuing our example…
- Project A: -\$1 in Period 0, \$2 in Period 1.
- Project B: -\$1,000 in Period 0, \$1,900 in Period 1.
- Incremental (B – A) = -\$999 in Period 0, \$1,898 in Period 1.
- From the definition of the IRR, we need the IRR such that
- -999 + 1898(P/F,IRR,1) = 0
- →(P/F,IRR,1) = 999/1898
- →1/(1 + IRR) = 999/1898
- →IRR = 899/999 = 90% (approx.)
- Since this is higher than the MARR of 70%, upgrading to Project B is worth it.

Also: IRR and NPV can disagree on rankings

- Consider this example from Investopedia:
- MARR = 6%, N = 1, two projects, A and B with initial costs and a single future payment at time 1.
- From the table below, note that NPV and IRR *disagree* on the ranking of A & B. B has a higher IRR, but lower NPV.
- Using the *incremental rate of return* reconciles the two approaches.

Year	A	B	A-B
0	-$250,000	-$50,000	-$200,000
1	$280,000	$60,000	$220,000
IRR	12%	20%	10%
NPV (MARR = 6%)	$14,151	$6,604	$7,547

IIRR(A-B) > MARR
→ A is preferred

NPV(A) > NPV(B)
→ A is preferred

A little help interpreting that last slide

- Projects A and B are mutually exclusive projects with the same lifetime. Your MARR is 6% per year.
- Project A NPV = $14,151
- Project B NPV = $6,604

NPV matters here

- →Project A is preferred (same lifetime, higher NPV)
- Individually, Projects A & B have an IRR (12% for A, 20% for B) greater than the MARR, so each of them is worthwhile.
- If you thought you should just pick the project with the higher IRR, you'd make the wrong call: Project B has a higher IRR, but its NPV is less than that of Project A, which has the same lifetime.

Making the right call using IIRR

IIRR Video

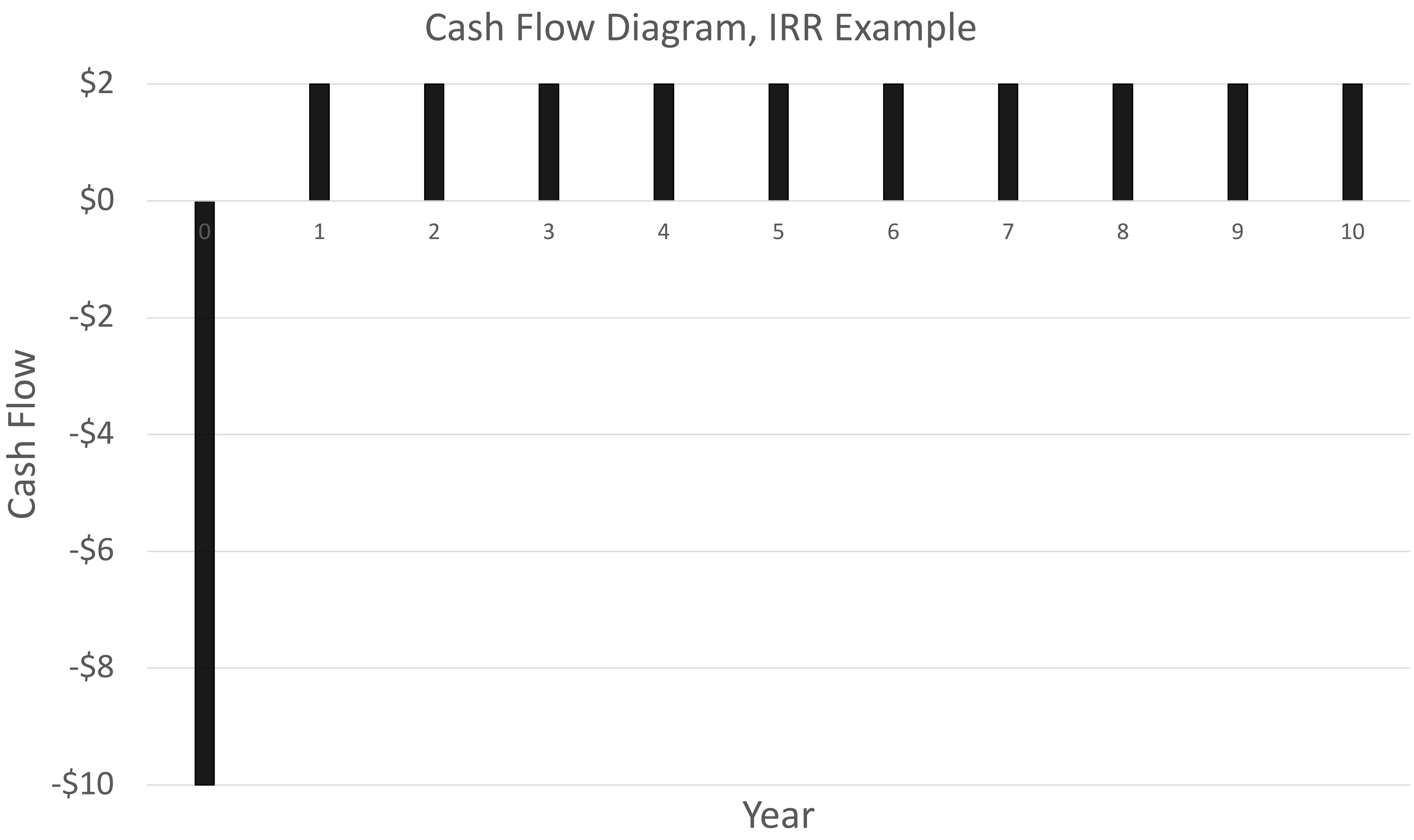
- Project A has a higher net initial outlay than Project B ($250,000, vs $50,000). Let's use the incremental internal rate of return approach:
- Project A: -250,000 today, +280,000 in a year
- Project B: -50,000 today, +60,000 in a year
- Project (A – B) = (-250,000 – (-50,000)) today, (280,000 – 60,000) in a year → -200,000 today, 220,000 in a year.
- NPV Project (A – B) = -200,000 + 220,000/(1+i)
- Setting NPV Project (A – B) = 0 and solving for 'i' will give us the incremental internal rate of return for (A – B): IIRR = 10% > MARR = 6%, so this incremental project is a 'good' project…
- …and since this project represents supersizing from the cheaper project, B, to the more expensive project, A, Project A is preferred, just as the NPV calculation told us. (The NPV of the incremental project is also positive, showing it's worthwile.)

AFTER HOURS

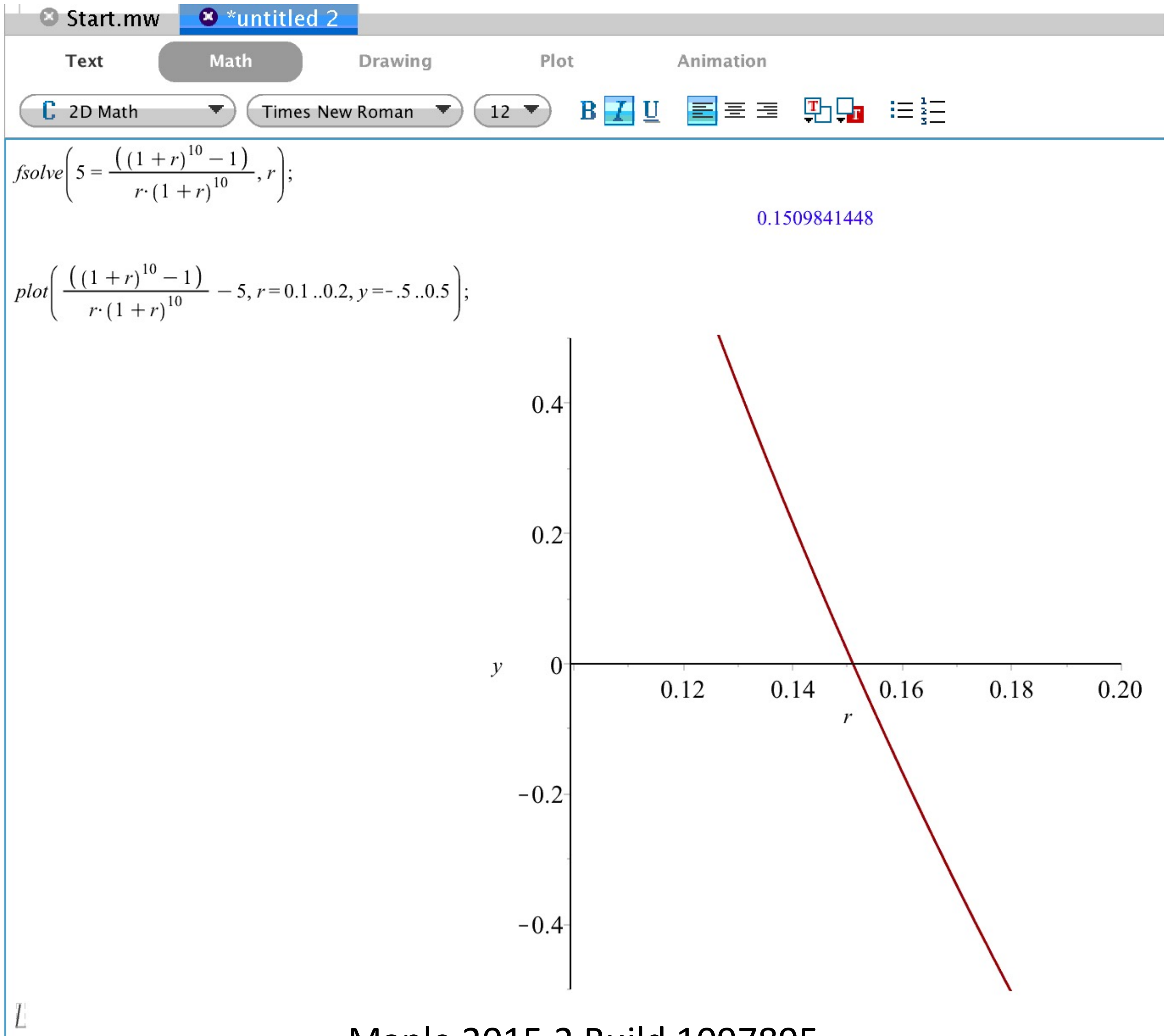
- Numerical Calculation Example (3 slides)
- An IRR may not exist (1 slide)
- What if IRR = MARR? (1 slide)
- Real-world examples: Season Pass & Cable Box (6 slides)
- Aside: The CAP rate (1 slide)
- Very optional Port Sudan case study (17 slides)

Example: Paying for an annuity

- Suppose you pay \$10 in year 0 for an annuity that pays \$2 every year from Year 1 to Year 10. What is its IRR?
- Even this intentionally simple example is painful to solve analytically, and doing so will take many pages of algebra (and factoring a tenth-order polynomial):
- NPV = -P + A(P/A,i,N) for P = \$10, A = \$2, N = 10.
- At the IRR, NPV = 0 and P = A(P/A,IRR,N)
- -10 + 2 x (P/A,IRR,10) = 0 → 5 = $\frac{(1+\mathrm{IRR})^{10}-1}{\mathrm{IRR}(1+\mathrm{IRR})^{10}}$
- In most cases – including this one – it's usually best just to solve numerically.
- Expanding (P/A,IRR,N) and solving numerically for IRR (I used Maple's 'fsolve' and double-checked with Excel's Goalseek), we find that IRR = 15.10% (approx.)
- This is also the value returned by Excel's built-in IRR function.



Cash Flow Diagram, IRR Example
$2
$0
-$2
-$4
-$6
-$8
-$10
Cash Flow
0
1
2
3
4
5
6
7
8
9
10
Year



Maple 2015.2 Build 1097895

	A	B	C
1	Year	Cash Flow	
2	0	($10)	
3	1	$2	
4	2	$2	
5	3	$2	
6	4	$2	
7	5	$2	
8	6	$2	
9	7	$2	
10	8	$2	
11	9	$2	
12	10	$2	
13	IRR	15.10%	=IRR(B2:B12)

Microsoft Excel for Mac, v. 16.16.3

Two numerical solutions

Warning: An IRR *may not exist*

- This posed a difficulty for students in Summer 2020, when using real-world data to evaluate living/studying in different cities.
- If a project is such that there is NO real value of the discount rate for which the NPV = 0 or more, the IRR will not exist.
- If this happens (or you suspect it's happening, because your numerical solution won't converge), try evaluating the NPV using a 0% interest rate (just adding up all the cash flows).
- If this is negative, and your project has the standard 'costs now, benefits' later form, the IRR does not exist.
- (We'll see later that if you're using the IRR, your cash flow profile only has one sign change.)
- **If the IRR does not exist, your project is never worthwhile. (NPV is always negative, no matter what the MARR is.)**

What if IRR = MARR? (OPINION)

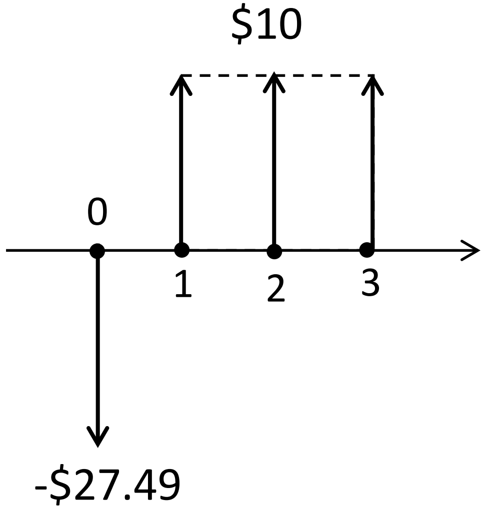
- Your textbook says the project is then ‘marginally acceptable’ and you should go through with it.
- From a practical real-world perspective, I disagree:
- The MARR is derived from your fallback project, and represents a rate of return you’re confident you can get from these resources.
- If you’re running an IRR calculation in the first place, it’s probably because you’re dealing with a novel or unfamiliar situation.
- Issues of uncertainty and randomness, in **my opinion**, will almost always tilt the scales in favor of “the devil you know”.
- →If IRR = MARR, do NOT proceed with the project, ***if*** the risk from the project you’re considering is greater than the risk of the project you derived your MARR from.
- **I’m NOT an engineer, though, so always check with your firm, co-workers and supervisors regarding what the policy is at your workplace!**

Real-World Example: Buying a Season Pass

- Pillars of Eternity II was released for PC/Mac in May 2018.
- Suppose it is currently May 2018. You know:
- It will have 3 expansions, each of which will cost $10 at release.
- The expansions will be available in July, September and November 2018.
- You can buy a season pass including all 3 expansions for $27.49.
- Should you buy the season pass?

What is our project?

- If May is Month 0, the expansions are released in Months 2, 4 and 6.
- We pay \$27.49 today, and receive something worth \$10 in each of months 2, 4 and 6.
- If we use ‘two months’ as our period, our project is an initial cost of \$27.49, and an annuity with A = \$10 and N = 3.
- NPV = $-\$27.49 + \$10 \times (P/A, i_{2\text{-month}}, 3)$
- At the IRR, NPV = 0, so we need to solve for
- $0 = -\$27.49 + \$10 \times (P/A, IRR_{2\text{-month}}, 3)$



Is it worth it?

- I used Excel's GoalSeek to solve for the IRR. It's 4.5%.
- Check: -27.49 + 10 x (P/A,4.5%,3) = -27.49 + 10 x 2.749 = 0, q.e.d.
- This is the 2-month IRR, though. To more easily compare it to interest rates as usually reported, let's find the 1-year IRR.
- Over the course of 1 year, there are 6 2-month periods, so
- $\$1x(1 + IRR_{1\text{-year}}) = \$1x(1 + IRR_{2\text{-month}})^6$
- $IRR_{1\text{-year}} = (1 + IRR_{2\text{-month}})^6 – 1 = (1 + 4.5\%)^6 - 1 = 30.2\%$ a year (approx.)
- As long as you'd definitely be buying all the expansions, anyway, the Season Pass is a good deal if your MARR < 30.2%
- Since BC student loans currently have a rate of about 3.5% for the provincial portion and 6% for the federal portion, the season pass is probably worth it for student gamers (ignoring the cost of lost sleep and study time).

Real-world example: Buying a cable box

Gateway HDPVR

The Gateway HDPVR is the hub of the operation, allowing you to connect up to six TVs. Additional Portals are available for an added cost.

Easy Own	Rental	Purchase
$10* /MO	$15* /MO	$348*
for 36 months	monthly	one time payment

Retrieved May 22, 2017 from shaw.ca

- You absolutely MUST have this box.
- Should you buy, rent, or rent to own?
- We can calculate the IRR to help us decide.

Alternatives

1. Sort by First Cost: Buy > Rent to Own
2. Incremental Flows: Buy – Rent to Own
3. Inc. NPV = -\$348 – (-\$15 x (P/A,i,36)).
4. Solve for Inc. IRR by setting Inc. NPV = 0

- Ignore the \$15/month rental option for now.
- You can either buy, or rent to own.
- If you buy, you pay \$348 now…
- ...BUT you don't have to pay the \$10/month for 36 months.
- Incremental NPV = -348 + 10 x (P/A,i,36) and at the IRR, NPV = 0, so
- 0 = -348 + 10 x (P/A,IRR,36)
- 34.8 = (P/A,IRR,36)
- Solving numerically (Excel GoalSeek, Maple, Interpolation...) we find
- Inc. IRR = 0.1844% a month, or $(1 + 0.1844\%)^{12} - 1 = 2.24\%$ a year.
- →If your MARR is higher than 2.24%, you should rent to own.
- (Not surprising: ignoring interest, only a \$12 difference over 12 months.)
- You can do this for cars or magazines, too!
- Useful if you only have a rough idea of your MARR.

What about that \$15 rental?

1. Sort by First Cost: Buy > Rent
2. Incremental Flows: Buy – Rent
3. Inc. NPV = -\$348 – (-\$15/i).
4. Solve for Inc. IRR by setting Inc. NPV = 0

- Obviously a bad deal if you plan to keep the box for any length of time, but HOW bad, exactly?
- Suppose you pay \$348 today. By doing so, you avoid paying \$15/month <u>forever</u> (ignore rent-to-own).
- Capitalized Value = A/i, A = \$15
- Incremental NPV = -\$348 + \$15/i and NPV = 0 when i = IRR, so…
- 0 = -348 + 15/IRR → IRR = 15/348 = 4.31%/month
- Inc. IRR = $(1 + 4.31\%)^{12} - 1$ = 65.93% a year.
- If your MARR is less than 65%, you're better off buying than renting (literally) forever.
- For this special case (initial cost followed by constant, infinite payments), the IRR is called the 'Cap Rate'.

Aside: ‘Cap Rate’ vs IRR

- The rate that sets Initial Cost = A/i is almost as widely used as the IRR.
- Very common to use both in realty:
- ‘Cap Rate’ is used as a quick-and-dirty comparison.
- Good when “investing in single tenant properties with a long term lease” due to stability/regularity of income and costs.
- For offices/malls, IRR is more common: since it uses the NPV, it can take more information into account, including changing conditions and a finite investment lifespan.
- For more on when to use one over the other, see
- https://www.creconsult.net/market-trends/cap-rate-vs-irr-whats-the-difference/

Everything after this point is OPTIONAL
(even more than other After Hours material)

It’s just a fun example to see that these methods really ARE used by real-world engineers.

…

Sometimes.

Proposed Power Plant in Port Sudan Town

- Until 2005, the main sources of electric power in Port Sudan were four power stations with 5.47 MW of (planned) electrical power.
- In practice, dust and humidity meant this potential was seldom reached. (Decreased engine power, accelerated corrosion and insulation breakdown, short circuits during rainy season.)
- Proposal: new combined cycle power plant with two 100 MW gas turbine generators, each with a heat recovery steam generator. A common steam turbine generator would provide an additional 100 MW.

- Since 2011, capital of the new state of Red Sea.
- Population: About 500,000
- Temperatures range from 11 to 46 degrees Celsius
- 63% relative humidity
- 72% possibility of sunshine

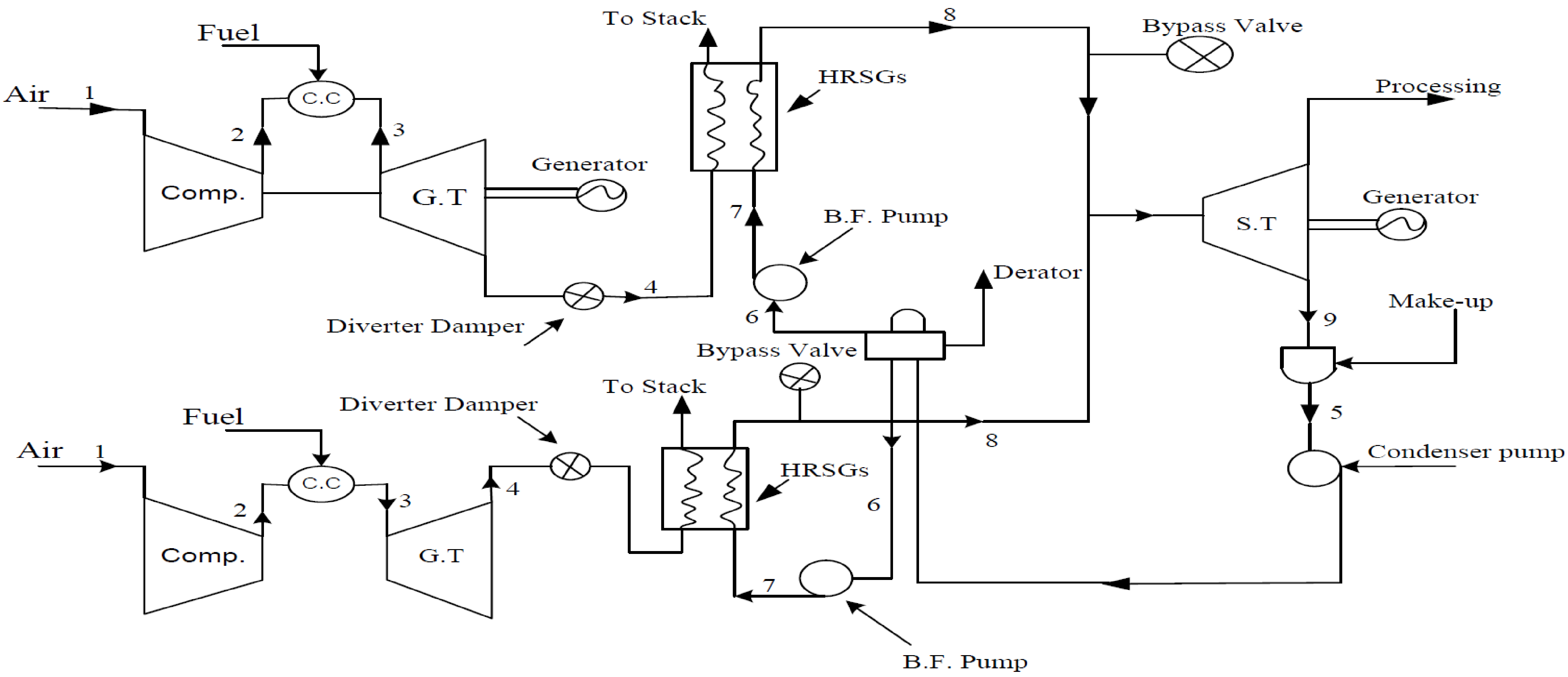
National Electricity Company, Port Sudan

Photo source: Google Earth, Panoramio

Port Sudan

Map courtesy of Lonely Planet

Waste oil outside a Port Sudan power station, 2007 (UN Environment Programme)


Fuel
Air
1
2
C.C
3
Comp.
G.T
Generator
To Stack
HRSGs
8
Bypass Valve
Processing
7
B.F. Pump
S.T
Generator
4
6
Derator
Diverter Damper
Bypass Valve
Make-up
9
5
To Stack
Diverter Damper
Fuel
Air
1
C.C
2
3
4
HRSGs
6
8
Condenser pump
Comp.
G.T
7
B.F. Pump

<3

Calculating the IRR: setting up the problem

- Basic assumptions:
  - Useful life of the proposed plant is 30 years
  - Salvage value is intentionally ignored
  - MARR = 30%, based on other available investment opportunities
- Plant costs can be divided into capital (first) costs C, and annual operation costs A_C.
- Plant revenue can be expressed in annual terms, A_R.
- Annual Profits = Annual Revenue – Annual Costs = (A_R - A_C)
- Profits are taxed at a rate of t%.

$$\rightarrow PW = -C + (1 - t)(A_R - A_C)(P/A, i, 30)$$

Capital Cost Estimation

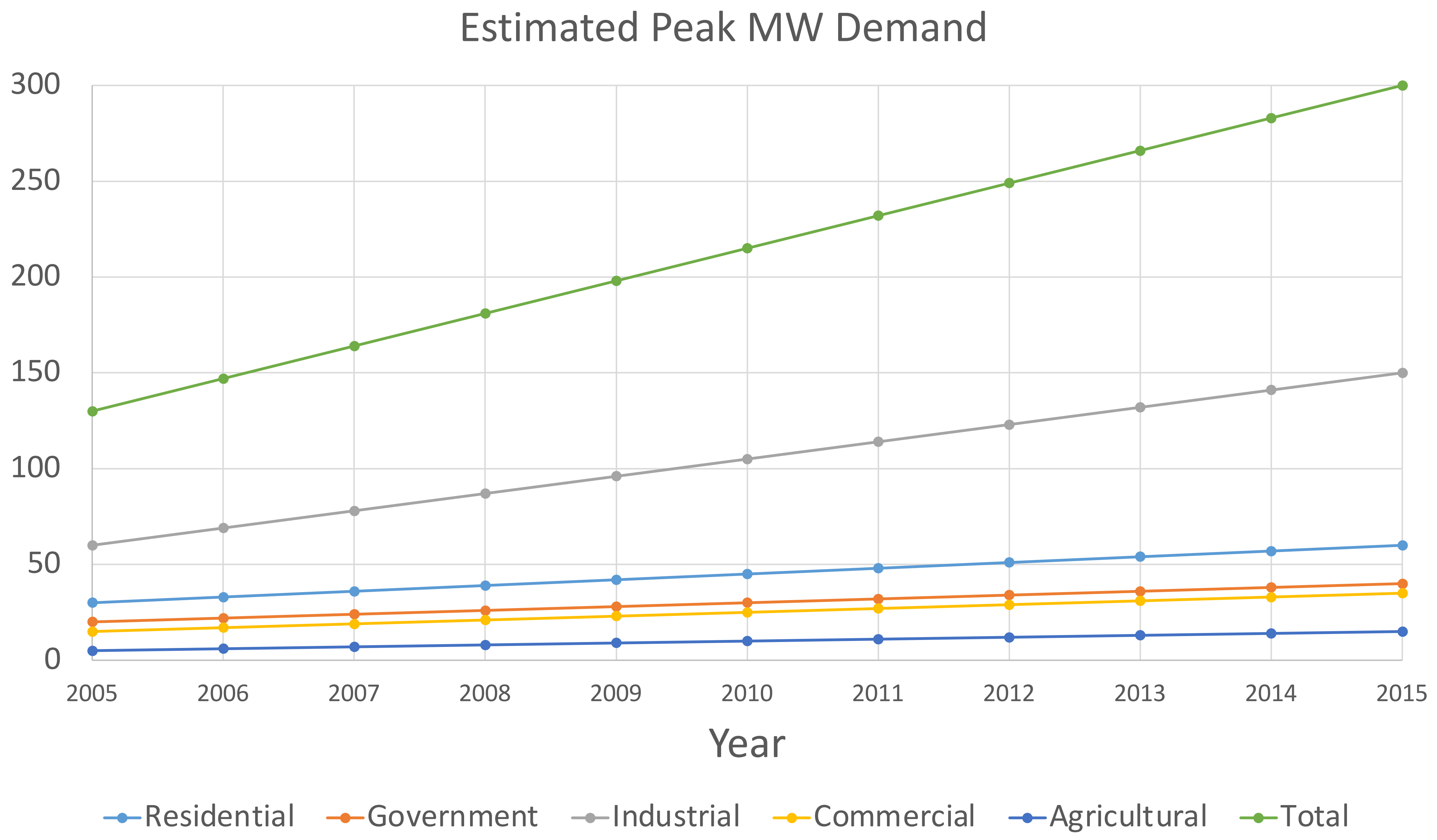
- Capital Cost = investment in the installation of a complete plant.
- About $1 million / MW
- 300 MW plant → Average cost is about $300 million (ballpark)
- Assuming $1 million/used capacity, not potential…
- Average operation will be 214 MW over lifetime → $214 million
- (We'll derive the 214 MW value later.)
- Engineering consultancy: ~15% of the fixed cost
- Civil works: ~5% of the fixed cost
- → C = $263 million

Annual costs

- Annual fuel costs = operating days/yr x kg/KWH x hr/day x $/kg
- = 360 x 1,000 x 24 x 0.3 = $118,770,000
- Fixed maintenance: ~ $2400/MW = $513,600 /year
- Overhead maintenance: ~$4/MWh = $7,395,840 /year
- Annual depreciation: Capital costs net of engineering consultancy, divided by expected lifetime = $7,696,660
- (Haven't seen it yet, but this is a type of straight-line depreciation.)
- Expected unexpected expenses: 10% of all other annual costs
- → A_C = $147,813,710

Annual Revenue

- As an explicit simplification, assume $/MW is the same for all consumers, and equal to $140.
- Power generation rises with population and economic expansion.
- Demand forecasts (also in the paper) suggest a 10% yearly increase from 130 MW in 2005 to 300 MW (full capacity) in 2015 and thereafter.
- →Average output is 214 MW/year for the life of the project.
- The plant will run 24 hr/day, 360 days/year
- A_R = 214 x 140 x 24 x 360 = $258,854,400


Estimated Peak MW Demand
300
250
200
150
100
50
0
2005
2006
2007
2008
2009
2010
2011
2012
2013
2014
2015
Year
Residential
Government
Industrial
Commercial
Agricultural
Total

Profit and Loss Account

Annual Values	
Yearly Revenue, A_R	$258,854,400
Yearly Operating Costs, A_C	$147,813,710
Yearly Profits (Revenue – Costs)	**$111,040,690**
Taxes (t = 25%)	$27,760,172.50
After-tax Profits	**$83,280,517.50**

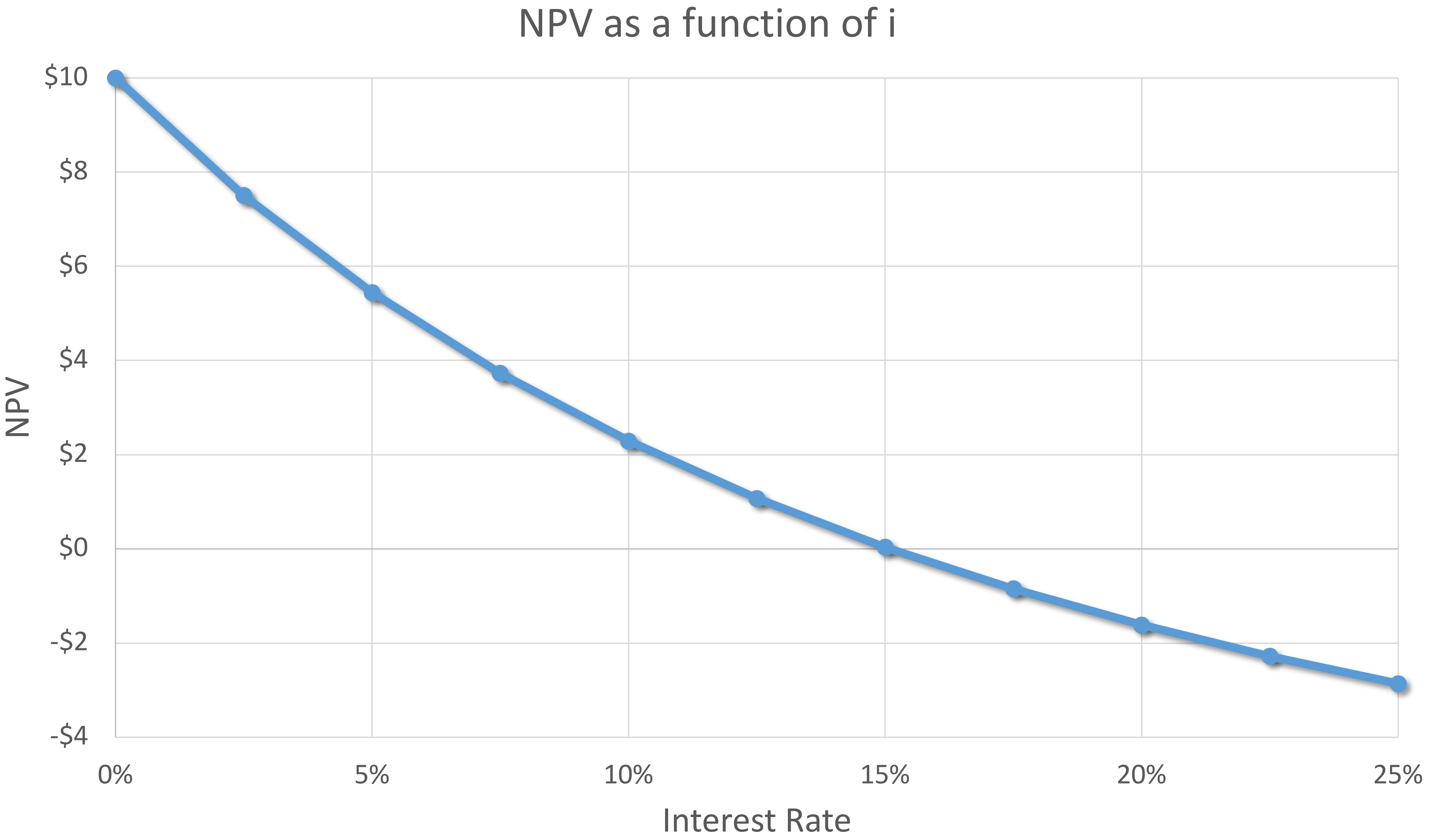
$\rightarrow (1 - t)(A_R - A_C) = \$83,280,517.50$

Calculating the IRR by hand: initial guesses

- PW = -C + (1 – t)(A_R - A_C)(P/A,i,30)
- → PW = -263,000,000 + 83,280,517.50(P/A,i,30)
- When i = IRR, PW = 0
- At a guess, the IRR is somewhere between the MARR (30%) and 40%.
- PW (30%) = -263,000,000 + 83,280,517.5 (P/A, 0.3, 30)
- = -263,000,000+83,280,517.5 x 3.3321 = 14,499,012.40
- PW(40%) = -263,000,000 + 83,280,517.5(P/A, 0.4, 30)
- =- 263,000,000+83,280,517.5 x 2.4999 = - 54,807,034.30

Aside: Linear Interpolation

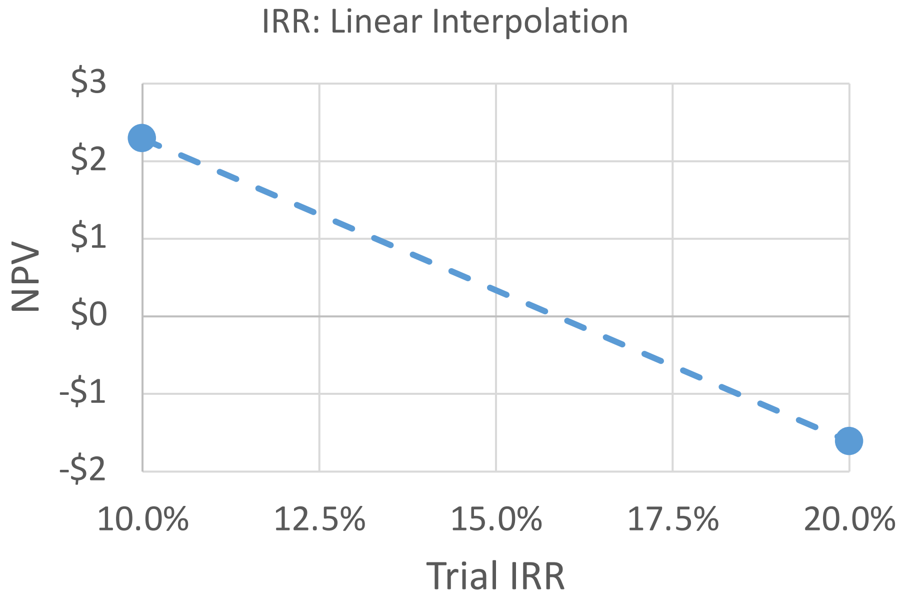
- For two trial values of i, calculate the NPV. (Ideally, the NPV will be near zero in both cases, but positive for one and negative for the other.)
- Assume a straight line connecting the (i,NPV) points.
- Use the slope of that line, and the coordinates of one point, to find the interest rate i corresponding to NPV = 0 on that line.
- That will be your linearly interpolated IRR.
- Accuracy improves the closer your trial values are to the true IRR, so plotting NPV as a function of i beforehand may be wise. (It'll let you find sensible trial values.)


NPV as a function of i
$10
$8
$6
$4
$2
$0
-$2
-$4
NPV
0%
5%
10%
15%
20%
25%
Interest Rate

Interpolation Example

Trial i	NPV
10%	$2.29
20%	-$1.62

Rise	-$3.90
Run	10%
m	-$39.04



Slope = (y - y1)/(x - x1)
We want the x such that y = 0.
Solving, x = x1 - y1/m

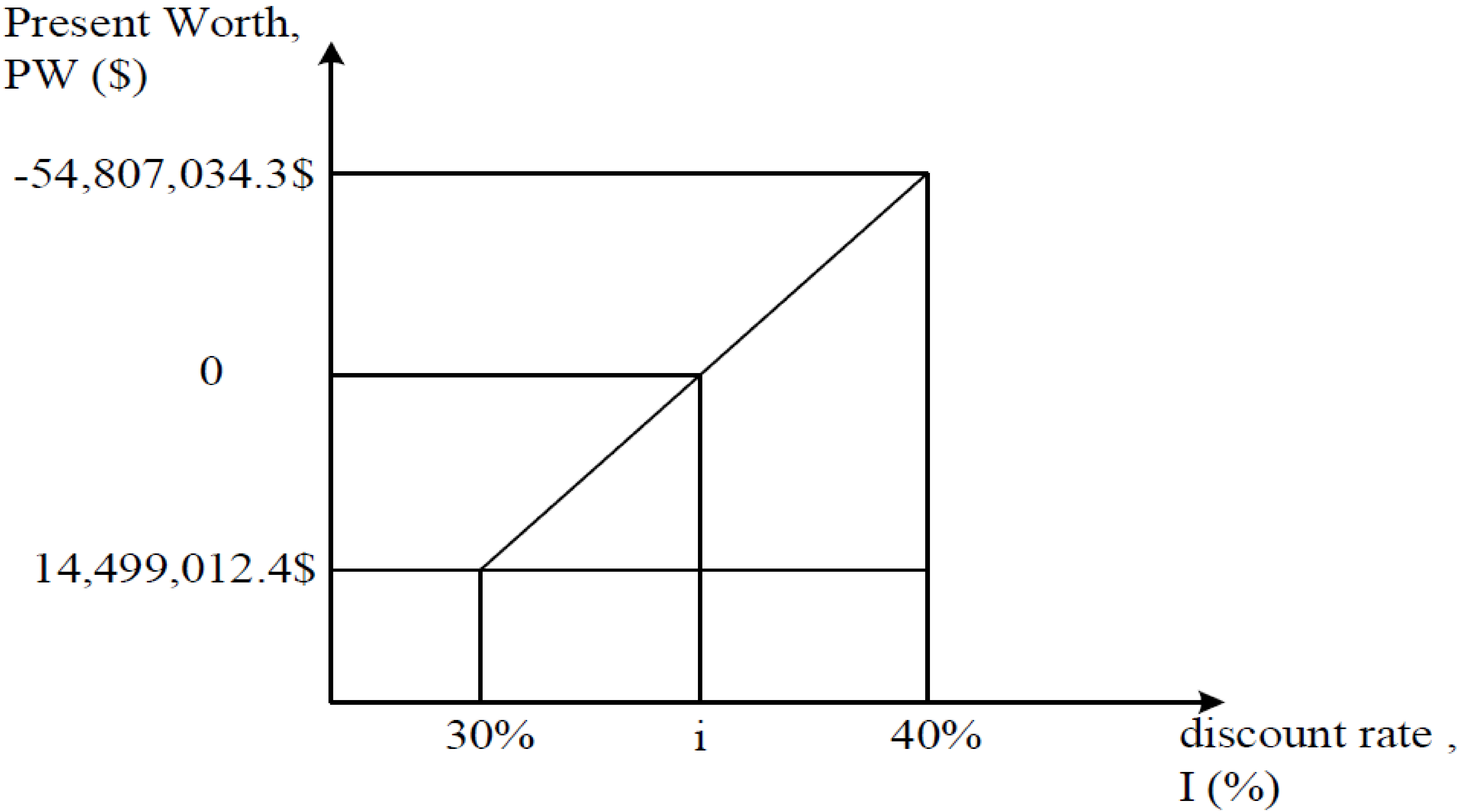
x	15.86%	Using x1=10%, y1=$2.29
x	15.86%	Using x1=20%, y1=-$1.62

IRR	15.86%	Interpolation

- Intentionally using less-than-optimal initial values for teaching purposes.
- While this method can be quick, its accuracy relies on good trial values.
- Values distant from the IRR give less accurate results…
- …and if you already know where the IRR is, the interpolation may not provide much value added.

Interpolation for the Port Sudan project

- We have two (i,PW) points: (0.3,14.5m) and (0.4,-54.8m).
- We want a third point: (i,PW) = (IRR,0)
- If we imagine a line running through all three points, we have one (linear) equation and one unknown: the IRR.
- Solving, (full values used in the actual calculation):
- IRR = 0.3+ ((0 – 14.5m) / (-54.8m – 14.5m))x (0.4 – 0.3) = 0.321
- → The IRR is about 32.1 %
- This is higher than the MARR (of 30%), so the project is worthwhile.


Present Worth,
PW ($)
-54,807,034.3$
0
14,499,012.4$
30%
i
40%
discount rate ,
I (%)