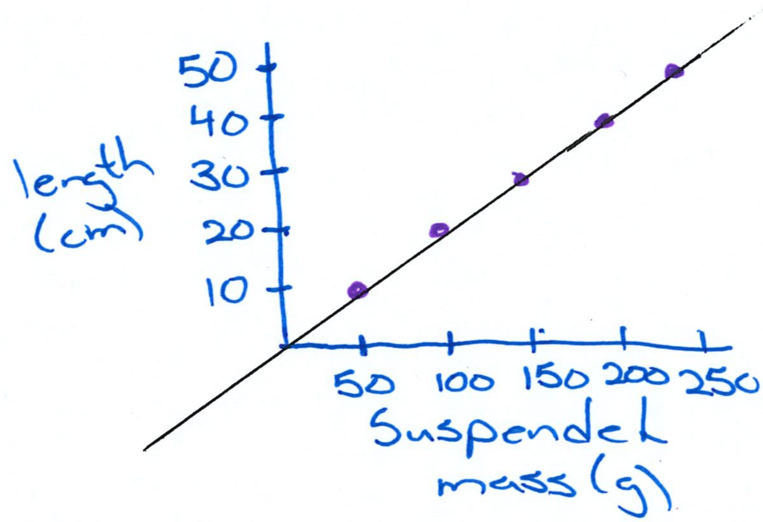
Springs:

In demo

Suspended mass	length	$\|\vec{F}_{spring}\|$
50g	10cm	0.5N
100g	20cm	1 N
150g	29cm	1.5N
200g	39cm	2N
250g	49cm	2.5N

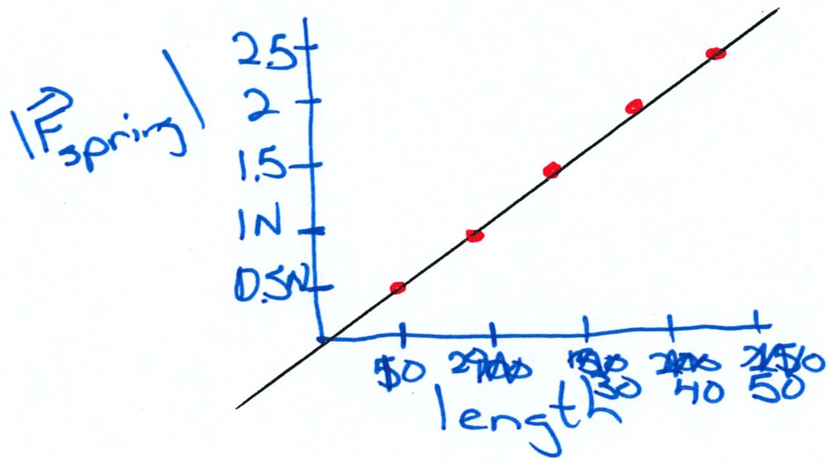


linear!

Equilibrium $\rightarrow \vec{F}_{net} = 0$

mag. of spring force = $|\vec{F}_s|$

$$|\vec{F}_{spring}| = mg$$



$$|\vec{F}_{spring}| = k|\Delta\vec{l}|$$

Spring constant
N/m
depends on
material, shape
etc

change in
spring's
length

A restoring force - push towards spring not stretched or compressed

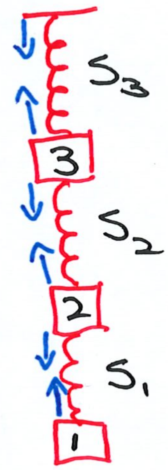
Caveat: Only stretch/compress along spring's axis.

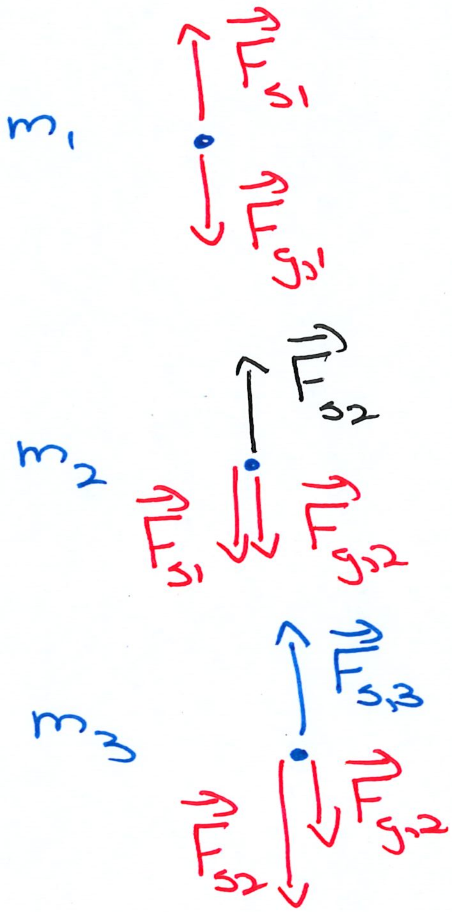
6-4-Example-Forces I

Forces - I

Three identical springs of unstretched length $0.1m$ are used to support three identical $1kg$ masses. The bottom mass is supported by a spring from the middle mass, which is in turn supported by a spring from the upper mass, which is in turn held in place by a spring. $k = 150\,N/m$

- What is the length of the lowest spring?
- What is the length of the middle spring?
- What is the length of the upper spring?





$$0 = \vec{F}_{s1,1} + \vec{F}_{g,1}$$

$$0 = \vec{F}_{s1,2} + \vec{F}_{s2,2} + \vec{F}_{g,2}$$

$$0 = \vec{F}_{s2,3} + \vec{F}_{s3,3} + \vec{F}_{g,3}$$

$$|\vec{F}_{spring}| = k|\Delta\vec{\ell}|$$

$$\vec{F}_{s1,1} = -\vec{F}_{s1,2}$$

$$\vec{F}_{s2,2} = -\vec{F}_{s2,3}$$

$$\vec{F}_{s1,1} = -\vec{F}_{g,1} = -(-mg\hat{k}) = 9.8N\hat{k}$$

$$\vec{F}_{s2,2} = -\vec{F}_{g,2} - \vec{F}_{s1,2} = -\vec{F}_{g,2} - (-F_{s1,1})$$

$$= -(-mg\hat{k}) - (-9.8N\hat{k})$$

$$= 19.6N\hat{k}$$

$$\vec{F}_{s3,3} = -\vec{F}_{g,3} - \vec{F}_{s2,3} = -\vec{F}_{g,3} - (-F_{s2,2})$$

$$= -(-mg\hat{k}) - (-19.6N\hat{k})$$

$$= 29.4N\hat{k}$$

$$|\vec{F}_{s1}| = k|\Delta\vec{\ell}_1|$$

$$9.8N = 150 N/m \, |\Delta\vec{\ell}_1|$$

$$|\Delta\vec{\ell}_1| = 0.0653m$$ spring 1 $\rightarrow$ 0.1653m

$$|\vec{F}_{s2}| = k|\Delta\vec{\ell}_2|$$

$$19.6N = 150 N/m \, |\Delta\vec{\ell}_2|$$

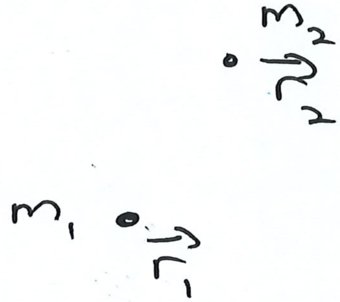
$$|\Delta\vec{\ell}_2| = 0.1307m \qquad spring2 = 0.2307m$$

$$|\Delta\vec{\ell}_3| = 0.1940m \qquad spring3 = 0.294m$$

Newtonian Gravity

$$|\vec{F}_g| \propto \frac{m_1 m_2}{(\text{separation})^2}$$

Attractive.



$6.67\times10^{-11}\ \frac{Nm^2}{kg^2}$

$$\vec{F}_{\text{on 1 by 2}} = -G\frac{m_1 m_2}{|\vec{r}_1-\vec{r}_2|^2}\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}$$

$(\text{separation})^2$

mass 1 towards mass 2

Strictly only true for spherically symmetric or point objects, but humans ≈ spheres

$\vec{r}$ for extended object center of mass

6-6-Example-Forces II

Forces - II

A $200kg$ mass is at $3m\hat{\imath} + 5m\hat{\jmath}$. A $250kg$ mass is at $5m\hat{\imath} - 2m\hat{\jmath}$. A $300kg$ mass is at $2m\hat{\jmath}$.

- What is the gravitational force on the $300kg$ mass by the $200kg$ mass?
- * What is the total gravitational force on the $300kg$ mass?

- Rule for force on one mass by another
- Find $\vec{F}_{on300by200}$
- Find $\vec{F}_{on300by250}$
- Add them!

$$\vec{F}_{onAbyB} = -G\frac{m_A m_B}{|\vec{r}_A - \vec{r}_B|^2}\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$$

$300kg = $ "A" $\rightarrow m_A = 300kg \quad \vec{r}_A = 2m\hat{\jmath}$

$200kg = $ "B" $\rightarrow m_B = 200kg \quad \vec{r}_B = 3m\hat{\imath} + 5m\hat{\jmath}$

$$\vec{r}_A - \vec{r}_B = 2m\hat{\jmath} - (3m\hat{\imath} + 5m\hat{\jmath})$$
$$= -3m\hat{\imath} - 3m\hat{\jmath}$$
$$|\vec{r}_A - \vec{r}_B| = \sqrt{(-3m)^2 + (-3m)^2} = 4.243m$$

$$\vec{F}_{\text{on}\,\underset{300}{A}\,\text{by}\,\underset{200}{B}} = -\left(6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\right)\frac{300\,\text{kg}\;200\,\text{kg}}{(4.243\,\text{m})^2}\left(\frac{-3\text{m}\,\hat{i}-3\text{m}\,\hat{j}}{(4.243\,\text{m})}\right)$$

$$= 1.57\times10^{-7}\,\text{N}\,\hat{i} + 1.57\times10^{-7}\,\text{N}\,\hat{j}$$

$$300\,\text{kg} = A \rightarrow m_A = 300\,\text{kg} \quad \vec{r}_A = 2\text{m}\,\hat{j}$$

$$250\,\text{kg} = B \rightarrow m_B = 250\,\text{kg} \quad \vec{r}_B = 5\text{m}\,\hat{i} - 2\text{m}\,\hat{j}$$

$$\vec{r}_A - \vec{r}_B = (2\text{m}\,\hat{j}) - (5\text{m}\,\hat{i} - 2\text{m}\,\hat{j})$$

$$= -5\text{m}\,\hat{i} + 4\text{m}\,\hat{j}$$

$$|\vec{r}_A - \vec{r}_B| = \sqrt{(-5\text{m})^2 + (4\text{m})^2} = \sqrt{41\,\text{m}^2} = 6.403\,\text{m}$$

$$\vec{F}_{\text{on 300 by 250}} = -\left(6.67\times10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\right)\frac{300\,\text{kg}\;250\,\text{kg}}{(6.403\,\text{m})^2}\left(\frac{-5\text{m}\,\hat{i}+4\text{m}\,\hat{j}}{6.403\,\text{m}}\right)$$

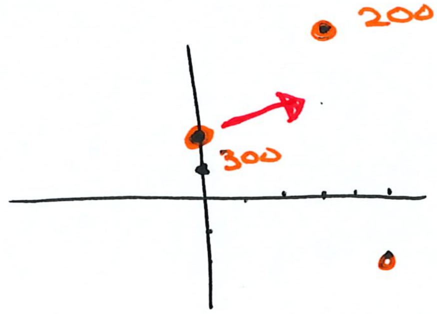
$$= 9.53\times10^{-8}\,\text{N}\,\hat{i} - 7.62\times10^{-8}\,\text{N}\,\hat{j}$$

$$\vec{F}_{\text{net on 300}} = \vec{F}_{\text{on 300 by 200}} + \vec{F}_{\text{on 300 by 250}}$$

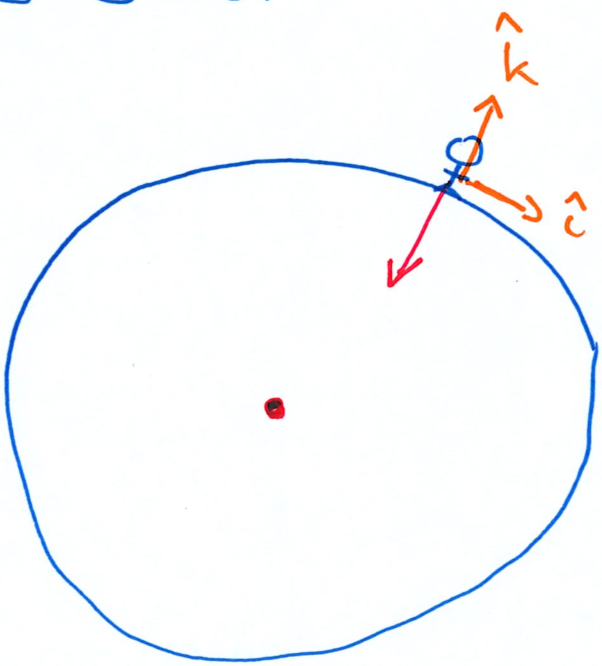
$$= 1.57\times10^{-7}\,\text{N}\,\hat{i} + 1.57\times10^{-7}\,\text{N}\,\hat{j}$$

$$+ \left(0.95\times10^{-7}\,\text{N}\,\hat{i} - 0.76\times10^{-7}\,\text{N}\,\hat{j}\right)$$

$$= 2.52\times10^{-7}\,\text{N}\,\hat{i} + 0.81\times10^{-7}\,\text{N}\,\hat{j}$$



Correspondence between $1/r^2$ Newtonian gravity & $-mg\hat{k}$ near surface.



$$\vec{F} = G\frac{m_E m_p}{R_E^2} \text{(in to center)}$$

$$\vec{F} = -m_p g \hat{k}$$

$$g = \frac{G M_e}{R_E^2} = \frac{6.67\times10^{-11}\frac{Nm^2}{kg^2}\, 5.99\times10^{24}\, kg}{(6.4\times10^{6}\, m)^2}$$

$$\approx 10\, N/kg$$