MATH 100, Fall, 2021
Tutorial #4
Derivatives and Instantaneous Rates

Q1. a) Complete the following statement (so as to be true): A function $f = f(x)$ is differentiable at $x = c$ if and only if the **the right-hand derivative**[1] of f exists at $x = c$ and the **the left-hand derivative** of f exists at $x = c$ and ....

b) Find an example of an f and c as in part a) where both left- and right-hand derivatives exist, but f is NOT differentiable at $x = c$. Show that your example works by computing both one-sided derivatives and explaining why f is not differentiable.

Q2 Find the derivative of $y = e^x \left[\dfrac{1}{x^2} - x^{e-1} \right]$ as a function of $x > 0$, then $y'(1)$ as an exact expression. Finish up by computing an approximation to $y'(1)$ rounded to three decimal places.

Q3 Transport Canada developed a model for a car's stopping distance on dry, paved roads as follows

$$s(v) = 0.245v + 0.008v^2$$

where s = stopping distance in metres and v = speed in kilometers per hour.

a) Compute $s'(50)$. What are the units? Interpret the number $s'(50)$ in terms of increased stopping distance on a city street. Do the same for $s'(100)$ on a highway.

b) Use the computation in part a) and the tangent line to the curve s to **estimate** how much extra distance you will need to stop if you are speeding in a 50km per hour zone at 55km/hour (extra, compared to not speeding).

Q4 Let $y = \frac{1}{\cos x} + \frac{1}{\cot x}$ for $-\pi/4 < x < \pi/4$. Find (exact answer) $y'(\pi/6)$. Simplify as much as possible.

Q5 a) Find all points x on the interval $(-\pi, \pi)$ where the slope of the tangent line to the curve $y = \tan x$ is parallel to the line $y = 4x$.

b) Make a sketch of the tangent function and the line from part a) then add in all the tangent lines you found in part a). Use colours!

[1] See textbook page 128 for definition of RH derivative.