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A metal beam was brought from the outside cold into a machine shop where the temperature was held at 75°F. After 10 min, the beam warmed to 35°F and after another 10 min it was 55°F. Use Newton's Law of Cooling to estimate the beam's initial temperature.

Newton's Law of Cooling states that if H is the temperature of an object at time t, H_0 is the temperature at $t = 0$, and H_S is the constant surrounding temperature, then $H - H_S = \left(H_0 - H_S\right) e^{-kt}$, where k is a constant.

The surrounding temperature is $H_S = 75^\circ$F.

After 10 minutes, the temperature of the beam is 35°F. That is, $H = 35$ when $t = 10$. Newton's Law of Cooling therefore yields the following.

$$\begin{aligned} H - 75 &= \left(H_0 - 75\right) e^{-10k} \\ 35 - 75 &= \left(H_0 - 75\right) e^{-10k} \\ -40 &= \left(H_0 - 75\right) e^{-10k} \end{aligned}$$

Solve this equation for H_0.

$$\begin{aligned} -40 &= \left(H_0 - 75\right) e^{-10k} \\ \frac{-40}{e^{-10k}} &= H_0 - 75 \\ H_0 &= 75 - 40\, e^{10k} \end{aligned}$$

Thus, $H_0 = 75 - 40\, e^{10k}$.

After another 10 minutes, the temperature of the beam is 55°F. That is, $H = 55$ when $t = 20$. Newton's Law of Cooling therefore yields the following.

$$\begin{aligned} H - 75 &= \left(H_0 - 75\right) e^{-20k} \\ 55 - 75 &= \left(H_0 - 75\right) e^{-20k} \\ -20 &= \left(H_0 - 75\right) e^{-20k} \end{aligned}$$

Solve this equation for H_0.

$$\begin{aligned} -20 &= \left(H_0 - 75\right) e^{-20k} \\ \frac{-20}{e^{-20k}} &= H_0 - 75 \\ H_0 &= 75 - 20\, e^{20k} \end{aligned}$$

Thus, $H_0 = 75 - 20\, e^{20k}$.

Because $H_0 = 75 - 40\, e^{10k}$ and $H_0 = 75 - 20\, e^{20k}$, these two expressions must be equal. That is, $75 - 40\, e^{10k} = 75 - 20\, e^{20k}$.

Solve this equation for k.

$$75 - 40e^{10k} = 75 - 20e^{20k}$$

$$-40e^{10k} = -20e^{20k}$$ Subtract 75 from both sides.

$$\frac{-40e^{10k}}{-20e^{10k}} = \frac{-20e^{20k}}{-20e^{10k}}$$ Divide both sides by $-20e^{10k}$.

$$2 = e^{10k}$$ Simplify.

$$\ln 2 = \ln e^{10k}$$ Take the natural logarithm of both sides.

$$\ln 2 = 10k$$ Apply the identity $\ln e^{x} = x$.

$$k = \frac{\ln 2}{10}$$ Divide both sides by 10 and simplify.

Thus, $k = \frac{\ln 2}{10}$.

Recall that $H_0 = 75 - 40e^{10k}$. Use this to compute H_0.

$$\begin{aligned} H_0 &= 75 - 40e^{10k} \\ &= 75 - 40e^{(10 \ln 2)/10} \\ &= 75 - 40e^{\ln 2} \\ &= -5 \end{aligned}$$

Thus, the beam's initial temperature was -5°F.