| | | |
|---|---|---|
| **Student:** Arfaz Hossain | **Instructor:** Muhammad Awais | **Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3] |
| **Date:** 02/28/22 | **Course:** Math 101 A04 Spring 2022 | |

Find a formula for the nth partial sum of the series and use it to find the series' sum if the series converges.

$$\frac{5}{1 \cdot 2} + \frac{5}{2 \cdot 3} + \frac{5}{3 \cdot 4} + \cdots + \frac{5}{n(n+1)} + \cdots$$

Given a sequence of numbers $\{a_n\}$, an expression of the form $a_1 + a_2 + a_3 + \cdots + a_n + \cdots$ is an infinite series. The number $a_n$ is the nth term in the series. The sum of the first n terms, called the partial sum, can be written as $s_n = a_1 + a_2 + a_3 + \cdots + a_n$. Add the terms one at a time from the beginning to see if there is a pattern in how the partial sum grows, then determine if there seems to be a number to which the sum converges.

Find the partial sum of the first three terms.

| **Partial sum** | **Value** |
|---|---|
| $S_1 = \frac{5}{2}$ | $\frac{5}{2}$ |
| $S_2 = \frac{5}{2} + \frac{5}{6}$ | $\frac{10}{3}$ |
| $S_3 = \frac{5}{2} + \frac{5}{6} + \frac{5}{12}$ | $\frac{15}{4}$ |

Compare the three sums. Notice each can be written as a number subtracted from 5.

| **Partial sum** | **Value** | **Suggestive expression** |
|---|---|---|
| $S_1 = \frac{5}{2}$ | $\frac{5}{2}$ | $5 - \frac{5}{2}$ |
| $S_2 = \frac{5}{2} + \frac{5}{6}$ | $\frac{10}{3}$ | $5 - \frac{5}{3}$ |
| $S_3 = \frac{5}{2} + \frac{5}{6} + \frac{5}{12}$ | $\frac{15}{4}$ | $5 - \frac{5}{4}$ |

Now compare the value of each denominator with n in each partial sum. Notice how the denominator of each consecutive term increases by 1 and is 1 more than the given value of n.

Write an expression for the partial sum of the nth term.

$$S_n = 5 - \frac{5}{(n+1)}$$

To determine if the series converges, find the limit as n approaches infinity.

$$\lim_{n \to \infty} \frac{5}{(n+1)} = 0$$

Because the limit of $\frac{5}{(n+1)}$ is zero, the series converges. Thus, the sum of the series is 5.