Vectors - I

The vector $\vec{A} = -3m\hat{\imath} + 4m\hat{\jmath}$.

- What is the length of $\vec{A}$? ie what is $\left|\vec{A}\right|$?
- What is the angle between $\vec{A}$ and the positive x-axis ($\hat{\imath}$)?
- What is the angle between $\vec{A}$ and the positive y-axis ($\hat{\jmath}$)?

Vectors - II

The vector $\vec{A}$ makes an angle of 30° with the positive x-axis, and an angle of 120° with the positive y-axis. The length of $\vec{A}$ is $5m$.

- What is the x-component of $\vec{A}$? ie what is A_x?
- What is the y-component of $\vec{A}$? ie what is A_y?

Vectors - III

Point A is at location $5m\hat{\imath} + 4m\hat{\jmath}$. Point B is at location $-3m\hat{\imath} + 2m\hat{\jmath}$.

- What is the vector *from A to B* expressed in components?
- What is the length of the vector from A to B?
- What is the angle between the vector from A to B and the positive x-axis?
- What angle does the vector from A to B make *measured counterclockwise from the positive x-axis*?

Vectors - IV

$\vec{A}$ is $5m$ long and makes an angle of $45°$ measured counterclockwise in the xy plane from $\hat{\imath}$. $\vec{B} = 3m\hat{\imath} - 5m\hat{\jmath}$.

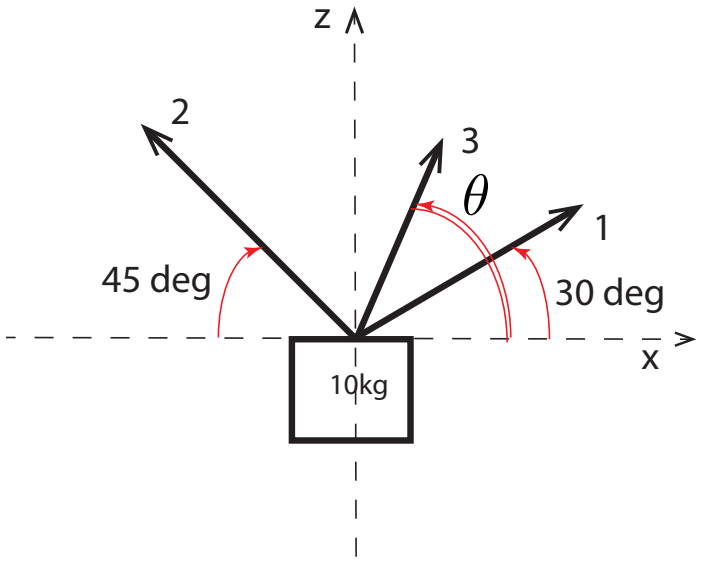
- What is the dot product between $\vec{A}$ and $\vec{B}$?
- What is the angle between $\vec{A}$ and $\vec{B}$?

Vectors - V
$\vec{A} = 3m\hat{\imath} + 2m\hat{\jmath} - 4m\hat{k}$ and $\vec{B} = 2m\hat{\imath} - 3m\hat{\jmath} - 1m\hat{k}$.

- What are the x, y, and z components of $\vec{A} \times \vec{B}$?
- What is the magnitude of $\vec{A} \times \vec{B}$?
- What is the angle between $\vec{A}$ and $\vec{A} \times \vec{B}$?

Translational Equilibrium - I

A $10kg$ box is supported by three ropes as shown in the figure.

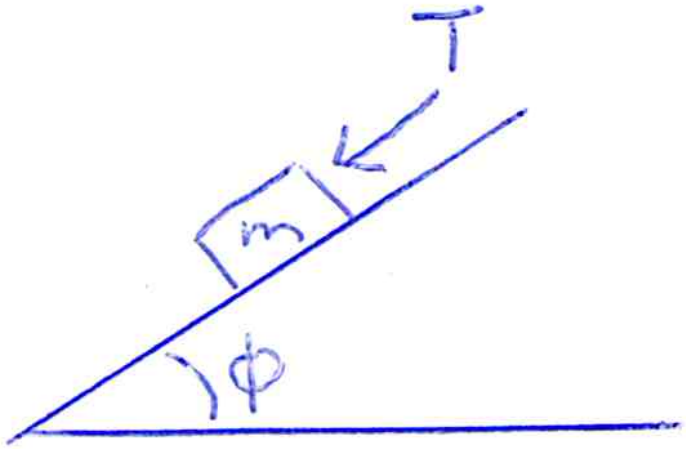


Rope 1 pulls with a force of magnitude $80N$ at an angle of $30°$ with $\hat{\imath}$ as shown. Rope 2 pulls with a force of magnitude $50N$ at an angle of $45°$ with $-\hat{\imath}$ and $45°$ with $\hat{k}$ as shown.

- What is the x-component of the force exerted by rope 3?
- What is the z-component of the force exerted by rope 3?
- What is the magnitude of the force exerted by rope 3?
- Rope 3 is in the xz plane; what angle does it make with $\hat{\imath}$ measured counterclockwise from the x-axis as shown? ie what is θ?

Translational Equilibrium - II

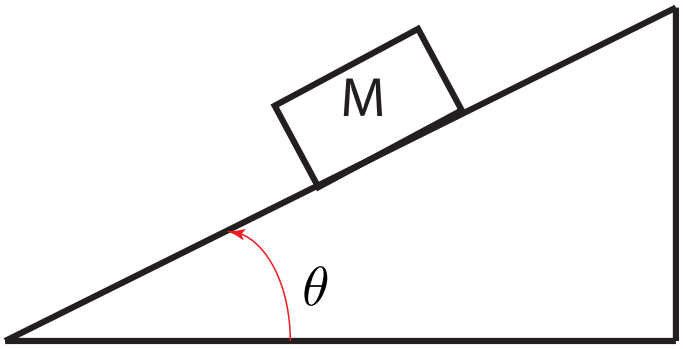
A $5kg$ mass is stationary on a rough slope. The slope makes an angle of $\phi = 25°$ with the horizontal, and the mass is subject to an external force of magnitude $T = 20N$ pushing parallel to the slope and downwards.



- What is the magnitude of the total force the slope exerts on the mass?
- What is the component of the force the slope exerts at $90°$ to its plane? In other words, what is the normal force on the mass?
- What is the component of the force the slope exerts along its plane? In other words, what is the friction force?

Translational Equilibrium - III

A $4kg$ mass is on a rough slope. The slope makes an angle of θ with the horizontal as shown.

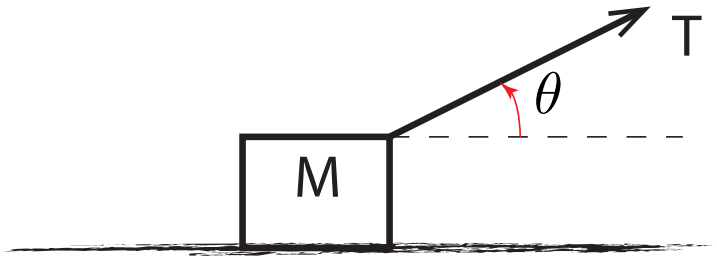


The coefficient of static friction between the mass and the surface is $\mu_s = 0.3$. The coefficient of kinetic friction between the mass and the surface is $\mu_k = 0.2$.

- For what range of angles can the mass be in static equilibrium?
- For what angle θ would the mass slide down the slope at a constant speed?

Translational Equilibrium - IV

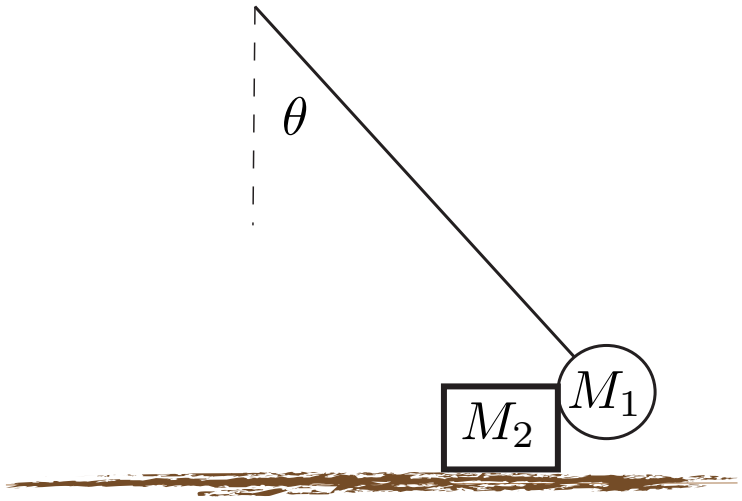
A $20kg$ mass is on a rough horizontal surface. It subject to a force from a rope which pulls up and to the right at an angle $\theta = 20°$. The coefficient of static friction between the mass and the surface is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.



- What is the minimum tension T in the rope (ie the magnitude of the force exerted by the rope) which will cause the mass to start to move?
- What tension T will result in the mass moving at a constant speed along the surface.

Translational Equilibrium - V

A ball of mass $M_1 = 3kg$ is suspended by a rope which makes an angle of $\theta = 20^\circ$ with the vertical. It is touching a block of mass $M_2 = 5kg$ which rests on a rough horizontal surface. The coefficient of static friction between the surface and the block is $\mu_s = 0.3$.



The sides of the block are vertical, so any force between the ball and block is horizontal.

- What is the force that the ball exerts on the block?
- What is the friction force the surface exerts on the block?

Rotational Equilibrium - I
A force of $\vec{F} = 5N\hat{\imath} + 6N\hat{k}$ is being exerted at $\vec{r} = 3m\hat{\imath}$.

- What is the torque this force exerts around (about) the origin?
- What is the torque this force exerts around (about) the point $\vec{r} = 1m\hat{\imath} + 1m\hat{k}$?

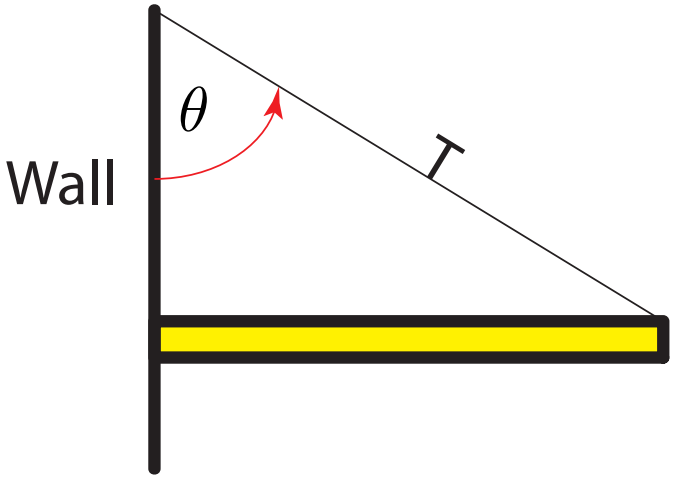
Rotational Equilibrium - II

A $3kg$ mass is at $1m\hat{\imath} + 3m\hat{\jmath}$, a $4kg$ mass is at $-2m\hat{\imath} + 1m\hat{\jmath}$, and a $5kg$ mass is at $-1m\hat{\jmath}$.

- What is the location of the center of mass of this assembly?
- Gravity acts in the negative $\hat{k}$ direction; what is the total torque measured around the origin due to the three masses?
- What is the total torque measured around the origin due to a force equal to the total gravitational force acting at the center of mass?

Rotational Equilibrium - III

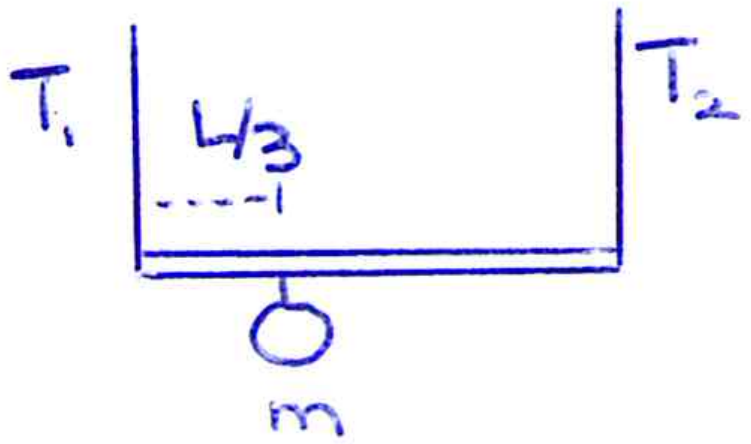
A $10kg$ uniform beam of length $5m$ is held horizontally by a rope. The rope is attached to a vertical wall, and to the far end of the beam; the rope makes an angle of $\theta = 20°$ with the wall.



- What is the tension in the rope? ie what is the magnitude of the force the rope exerts?
- What is the vertical component of the force the wall exerts on the beam?
- What is the horizontal component of the force the wall exerts on the beam?

Rotational Equilibrium - IV

A $12kg$ bar of length $L = 3m$ is held horizontally by two vertical ropes, one at each end. A $6kg$ mass is located $\frac{L}{3} = 1m$ from the left end of the bar.



- What is the tension T_1 in the left-hand rope?
- What is the tension T_2 in the right-hand rope?

Rotational Equilibrium - V

A $6m$-long pole is held upright by three ropes pulling horizontally. The bottom of the pole is at the origin and touching the ground. One rope is attached at $1m$ up from the ground and it pulls with a force of $200N\hat{\imath}$. A second rope is attached $3m$ up from the ground and it pulls with a force of $-150N\hat{\jmath}$. The third rope is attached at the top.

- What is the x-component of the force exerted by the third rope?
- What is the y-component of the force exeerted by the third rope?
- What angle does the third rope make with $\hat{\imath}$ and $\hat{\jmath}$?
- What is the horizontal part of the force that the ground exerts on the pole?

Velocity and Acceleration - I

A particle moves with position as a function of time given by

$$\vec{r}(t) = \left(2m - 3\frac{m}{s}t + 4\frac{m}{s^2}t^2\right)\hat{\imath} + \left(5\frac{m}{s}t - 2\frac{m}{s^3}t^3\right)\hat{\jmath} \tag{1}$$

- What is the velocity of the particle at $t = 2s$? At $t = 4s$?
- What is the acceleration of the particle at $t = 2s$? At $t = 4s$?

Velocity and Acceleration - II

A particle moves in the x-direction with constant acceleration. At time $t = -1s$ it is at $1m\hat{\imath}$ moving with velocity $-3\frac{m}{s}\hat{\imath}$ and subject to an acceleration of $1\frac{m}{s^2}\hat{\imath}$.

- What is the expression for the particle's position as a function of time?
- What is the position and velocity at $t = 4s$?
- What is the minimum value of the x-component of the particle's position and when does it get there?

Velocity and Acceleration - III

One particle moves with constant acceleration of $-0.5\frac{m}{s^2}\hat{\imath}$. At time $t = 0s$ it is at the origin moving at $20\frac{m}{s}\hat{\imath}$.

A second particle is at the origin stationary, until at $t = 4s$ it starts to accelerate at a constant $3\frac{m}{s^2}\hat{\imath}$.

- When does the second particle pass the first?
- Where are they when they pass?
- During the period between $t = 0s$ and when they pass, what is the farthest apart they are, and when does that happen?

Velocity and Acceleration - IV

A child kicks a ball at time $t = 0s$. At the instant the ball is kicked it travels at $25\frac{m}{s}$ at an angle of 30° above the horizontal. The ball flies through the air and lands on a school roof. The roof is $4m$ above the level where the ball was kicked from.

- When does the ball land on the roof?
- How far is the ball (horizontally) from the point at which it was kicked?
- What is the ball's velocity just before it hits the roof?

Velocity and Acceleration - V
A mass is moving in uniform circular motion with a position given by

$$\vec{r}(t) = 3m\cos\left(2s^{-1}t\right)\hat{\imath} - 3m\sin\left(2s^{-1}t\right)\hat{\jmath} \tag{2}$$

- What is the velocity at $t = 2s$?
- What is the acceleration at $t = 2s$?
- What is the magnitude of the velocity?
- What is the magnitude of the acceleration?

Velocity and Acceleration - VI

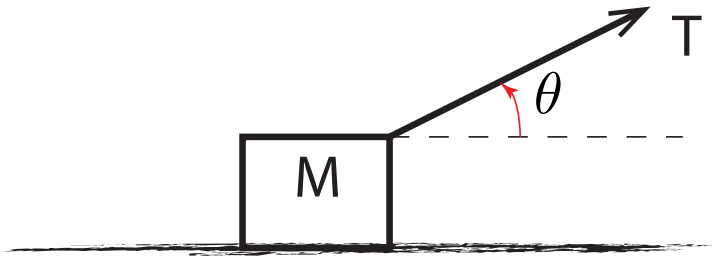
A mass is moving in a circle of radius $4m$ centered at the origin..

At time $t = 0s$ the mass is on the positive y-axis, moving in the negative x-direction. The mass's speed is $8\frac{m}{s}$ and it is slowing down with $\frac{d}{dt}|\vec{v}| = -2\frac{m}{s^2}$.

- What is the x-component of the acceleration of this mass?
- What is the y-component of its acceleration?
- What is the magnitude of the total acceleration of the mass?

Second Law - I

A $20kg$ mass is being pulled over a rough surface with which it has a coefficient of kinetic friction of $\mu_k = 0.2$. It is being pulled in the positive x-direction by a rope which is exerting a force of magnitude $T = 50N$ at an angle of $\theta = 25^\circ$ above the horizontal.



What is the acceleration of this mass?

Second Law - II

A block of mass $m = 2kg$ falls and is subject to two forces, the downwards force of gravity and an upwards resistive force.

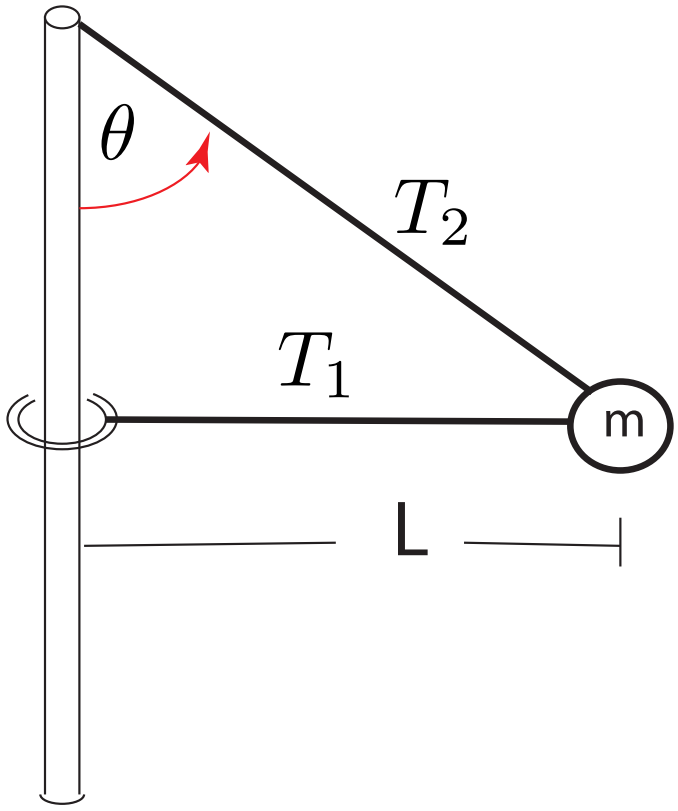
The block's position as a function of time is given by

$$\vec{r}(t) = \left(1000m - 21\frac{m}{s}t - 45m\left(e^{-0.4667s^{-1}t} - 1\right)\right)\hat{k} \tag{3}$$

What is the resistive force on the block at time $t = 3s$? $t = 15s$? $t = 40s$?

Second Law - III

A ball of mass $3kg$ is swinging in a horizontal circle supported by two ropes, as shown in the diagram. Rope 1 is horizontal and under tension T_1, and Rope 2 makes an angle of $\theta = 30^\circ$ with the vertical, as shown.



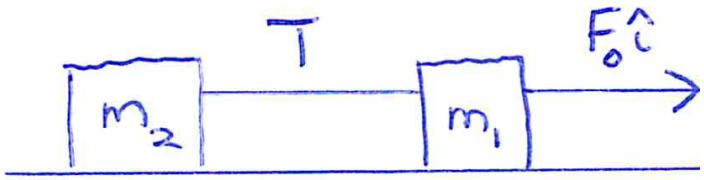
The ball is a distance $L = 2m$ from the pole, and the ball is travelling at a speed of $8\frac{m}{s}$.

- What is T_1?
- What is T_2?

Second Law - IV

A mass $m_1 = 4kg$ is being pulled horizontally by a rope which exerts a force $\vec{F} = F_0\hat{\imath} = 5N\hat{\imath}$. This mass is attached by an inextensible rope to a second mass $m_2 = 8kg$.

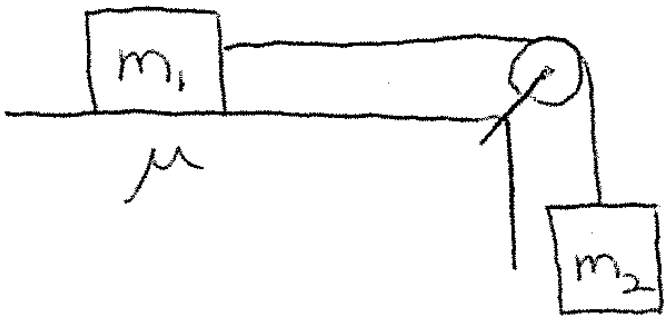
The two masses are on a horizontal frictionless surface.



- What is the tension T in the rope?
- What is the acceleration of m_2?

Second Law - V

A mass $m_1 = 30kg$ is moving to the right over a horizontal surface with which it has a coefficient of kinetic friction of $\mu = 0.1$. The mass is connected via a rope to a second mass $m_2 = 10kg$ which is suspended from a massless frictionless pulley.



- What is the acceleration of m_1?
- What is the acceleration of m_2?
- What is the tension in the connecting rope?

Forces - I

Three identical springs of unstretched length $0.1m$ are used to support three identical $1kg$ masses. The bottom mass is supported by a spring from the middle mass, which is in turn supported by a spring from the upper mass, which is in turn held in place by a spring.

- What is the length of the lowest spring?
- What is the length of the middle spring?
- What is the length of the upper spring?

Forces - II

A $200kg$ mass is at $3m\hat{\imath} + 5m\hat{\jmath}$. A $250kg$ mass is at $5m\hat{\imath} - 2m\hat{\jmath}$. A $300kg$ mass is at $2m\hat{\jmath}$.

- What is the gravitational force on the $300kg$ mass by the $200kg$ mass?
- What is the total gravitational force on the $300kg$ mass?

Forces - III

A $3\times10^{-3}C$ charge is at $-2m\hat{\imath}+3m\hat{\jmath}$. A $5\times10^{-3}C$ charge is at $4m\hat{\imath}+3m\hat{\jmath}$. A $-4\times10^{-3}C$ charge is at $-4m\hat{\imath}+1m\hat{\jmath}$.

- What is the Coulomb force on the first charge by the second charge?
- What is the total Coulomb force on the first charge?

Forces - IV

A $3.0kg$ mass with a charge of $-2 \times 10^{-3}C$ is moving at a velocity of $200\frac{m}{s}\hat{\imath} + 300\frac{m}{s}\hat{\jmath}$ in a region where the magnetic field is $1.2T\hat{\jmath}$.

What is the acceleration of the mass?

Momentum - I

A $3kg$ mass travels at $10\frac{m}{s}\hat{\imath} - 3\frac{m}{s}\hat{\jmath}$. A $4kg$ mass travels at $-4\frac{m}{s}\hat{\imath} - 2\frac{m}{s}\hat{\jmath}$. A $5kg$ mass travels at $1\frac{m}{s}\hat{\imath} + 2\frac{m}{s}\hat{\jmath}$.

- What is the x-component of the total momentum of these masses?
- What is the y-component of the total momentum of these masses?
- What is the magnitude of the total momentum of these masses?
- What is the velocity of the center of mass of this set of three particles?

Momentum - II

A lump of clay of mass $4kg$ travels with velocity $8\frac{m}{s}\hat{\imath}$ on a horizontal frictionless surface. It strikes and sticks to a ball of mass $1kg$ which travels on the same surface with velocity $18\frac{m}{s}\hat{\jmath}$.

- What speed does the clay and ball combo move at after the collision?
- What angle does the clay and ball combo's motion make with the x-axis after the collision?
- What is the change in momentum of the clay in the collision?
- What is the change in momentum of the ball in the collision?

Momentum - III

A ball of mass $3kg$ travels with velocity $10\frac{m}{s}\hat{\imath}$ on a horizontal frictionless surface. A second ball of mass $4kg$ travels with velocity $-9\frac{m}{s}\hat{\imath}$. The balls collide; they interact for a period of $0.2s$.

After the collision the $3kg$ ball travels at $-8\frac{m}{s}\hat{\jmath}$.

- What is the speed of $4kg$ ball after the collision?
- What angle does the $4kg$ ball's velocity make with the positive x-axis (with $\hat{\imath}$)?
- What is the change in the momentum of the $4kg$ ball during the interaction?
- What was the average force on the $4kg$ ball?

Momentum - IV

A rocket consists of a $2kg$ mass and $10kg$ of fuel. The fuel is burned in such a way that $0.1kg$ of the fuel is exhausted at a time, at a velocity of $-200\frac{m}{s}\hat{\imath}$ measured relative to the rocket.

The rocket starts at rest.

- What is the velocity of the rocket after there have been three fuel emissions?
- How would you calculate the velocity of the rocket after all the fuel has been emitted?
- If the fuel were emitted continuously, what would the final velocity of the rocket be?

Angular Momentum - I

A ball of mass $3kg$ travels with constant velocity $3\frac{m}{s}\hat{\imath} + 2\frac{m}{s}\hat{k}$. At time $t = 0s$ it is at $2m\hat{\imath}$.

- What is the angular momentum measured around the origin?
- What is the angular momentum measured around $-1m\hat{\imath} - 2m\hat{k}$?
- What is the angular momentum measured around $1m\hat{\imath} + 2m\hat{\jmath}$?

Angular Momentum - II

A rigid object is made in the shape of an equilateral triangle with side length $2m$. There is a $1.5kg$ mass at each of the vertices of the triangle, and the object lies in the xy plane. The rest of the rigid object is massless.

The object is rotated twice per second around its center of mass along a line which is parallel to the z-axis.

- What is the moment of inertia of this object around its axis of rotation?
- What is the angular momentum of this rotating object?

Angular Momentum - III

A wheel is in the xy plane and is free to rotate around its axle which is oriented along the z-axis. The axle is centered at the origin. The wheel has radius $0.6m$, and moment of inertia $0.72kgm^2$.

The wheel is subject to a force $\vec{F} = 2N\hat{\imath}$ which is exerted at $\vec{r} = 0.6m\hat{\jmath}$. The wheel is at rest at $t = 0s$.

- What is the torque that the force exerts?
- What is the angular momentum of the wheel at $t = 3s$?
- What is the rate of rotation at $t = 3s$?
- What if the force were exerted at $\vec{r} = 0.2m\hat{\jmath}$?

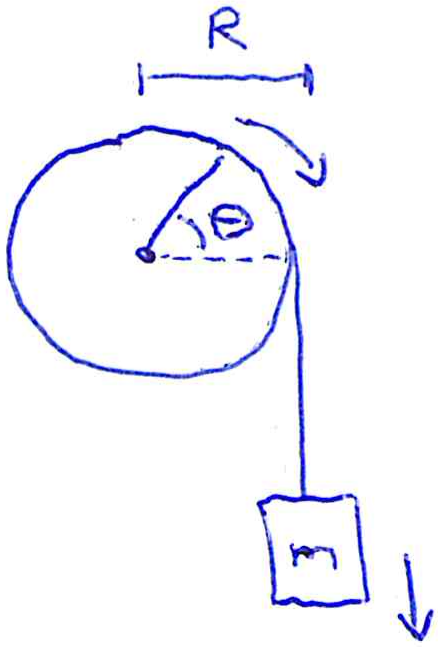
Angular Momentum - IV

A skater is rotating with their arms out at a rate of two times per second. They then draw in their arms. What is their final rotation rate?

Model the skater as a vertical cylinder of radius $0.1m$ with a moment of inertia of $0.8kgm^2$, and that their hands are $0.5kg$ at the end of $1m$ long massless arms.

Angular Momentum - V

A disk of radius $R = 0.5m$ and moment of inertia $I = 10kgm^2$ has a light rope wrappd around it. That rope is attached to a mass $m = 2kg$.



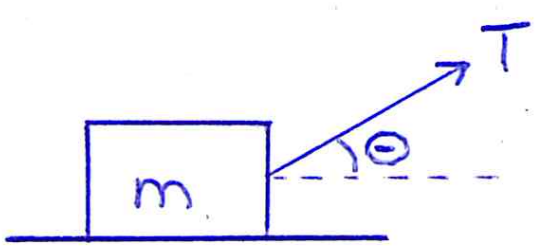
The rope does not slip, and the disk is free to rotate without friction.

- What is the acceleration of the mass m?
- What is the *angular acceleration* of the disk?
- What is the tension in the rope?

Work and Kinetic Energy - I

A $m = 4kg$ mass is on a horizontal rough surface with which it has a coefficient of kinetic friction $\mu_k = 0.2$. The mass is pulled to the right by a rope under tension $T = 15N$ which makes an angle of $\theta = 30^\circ$ with the horizontal.

The box has been pulled a distance $d = 3m$ along the surface.



- What is the work done by the rope?
- What is the work done by friction?
- What is the work done by the normal force?

Work and Kinetic Energy - II

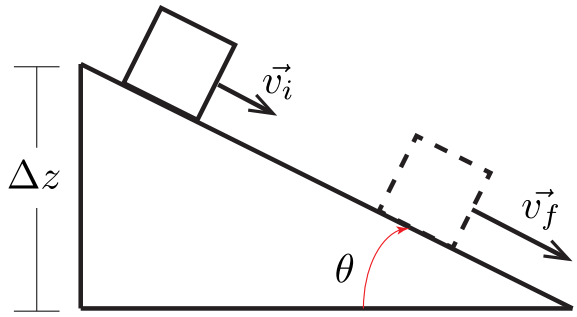
A $m = 2kg$ mass starts at the origin and moves along the positive x-axis. While it does so, it is subject to a force which depends on position: $\vec{F} = \left(5\frac{N}{m}x - 1\frac{N}{m^2}x^2\right)\hat{\imath}$.

How much work is done on the mass as it moves from $x = 0m$ to $x = 2m$? From $x = 1m$ to $x = 3m$?

Work and Kinetic Energy - III

A $2kg$ block is on a rough slope. It is sliding down the slope, and has a coeffiecient of kinetic friction of 0.1 with the slope. The slope makes an angle of $\theta = 20°$ with the horizontal as shown.

The mass initially has a speed of $2\frac{m}{s}$. The vertical component of its displacement was $\Delta z = -1m$ (ie it went down by $1m$).



- How much work is done by gravity?
- How much work is done by friction?
- How much work is done by the normal force?
- What is the block's final kinetic energy?
- What is the block's final speed?

Work and Kinetic Energy - IV

A block of mass $3kg$ is on a flat rough surface with which it has a coefficient of kinetic friction of 0.3.

The block starts at $-1m\hat{\imath}$ and moves to $1m\hat{\imath}$. How much work is done by friction is the path:

- is the straight line from $-1m\hat{\imath}$ to $1m\hat{\imath}$?
- is a triangle from $-1m\hat{\imath}$ to $1m\hat{\jmath}$ to $1m\hat{\imath}$?
- is the half-circle of radius $1m$ centered at the origin going through $1m\hat{\jmath}$?

Potential Energy - I

A mass $5kg$ moves from $1m$ above the Earth's surface to $7m$ above the Earth's surface?

- How much work did gravity do on the mass?
- What is the change in gravitational potential energy of the mass?

Potential energy - II

A $2kg$ mass slides at $5\frac{m}{s}$ on a horizontal frictionless surface. It collides with an ideal spring with spring constant $500\frac{N}{m}$. It contacts the end of the spring and begins to compress it.

When the spring has been compressed by $0.2m$,

- How much work has the spring done?
- What is the change in the potential energy stored in the spring?
- What is the change in the mass's kinetic energy?
- What is the speed of the mass at that instant?

Potential energy - III

A rocket is launched straight up from the surface of a planet which has a mass of $1.0 \times 10^{24} kg$ and a radius of $1.6 \times 10^{6} m$. The planet has not atmosphere. The rocket has an initial velocity (straight up) of $8000 \frac{m}{s}$.

- What is the change in the rocket's potential energy as it moves to $2.4 \times 10^{6} m$ from the center of the planet?
- What is its speed at this distance from the planet?
- What is the rocket's maximum distance from the planet?

Potential energy - IV

A charge $1.0 \times 10^{-6} C$ is at $2m\hat{\imath}$. A charge $-1.5 \times 10^{-6} C$ is at $3m\hat{\jmath}$.

How much work must be done to move a charge of $-2 \times 10^{-6} C$ from $3m\hat{\imath}$ to $2m\hat{\imath} + 2m\hat{\jmath}$?

Synthesis - I

A block of mass $3kg$ moves at $8\frac{m}{s}\hat{\imath}$. A block of mass $4kg$ moves at $-4\frac{m}{s}\hat{\imath}$. The $4kg$ block has a spring with spring constant $800\frac{N}{m}$ on its front, and it is initially uncompressed. The blocks hit each other, compressing the spring, and the spring then pushes them apart. The blocks only move in the x-direction.

- What is the compression of the spring when the two blocks are closest together?
- What is the velocity of the two blocks after the interaction is finished?

Synthesis - II

A planet orbits a star. At a particular instant the planet is $1.5 \times 10^{11} m$ away from the star and the planet travels at $2.5 \times 10^4 \frac{m}{s}$. The velocity vector is at 90° to the vector from the star to the planet.

What is the closest that the planet gets to the star?

Synthesis - III

A clay lump with a mass of $0.50kg$ is suspended by a rope which is $0.8m$ long. A ball of mass $0.10kg$ is thrown at the lump of clay, and sticks to it. Just prior to the ball hitting the ball travelled horizontally with a speed of $12\frac{m}{s}$.

What is the tension in the rope just after the ball hits and sticks to the clay?

DC cirucuits - I

An electron moves from a location where it is at potential $1V$ to a place where it is at potential $3V$.

- What is the change in the electron's potential energy?
- Assuming the electron's kinetic energy has not changed, how much work did the electron do?

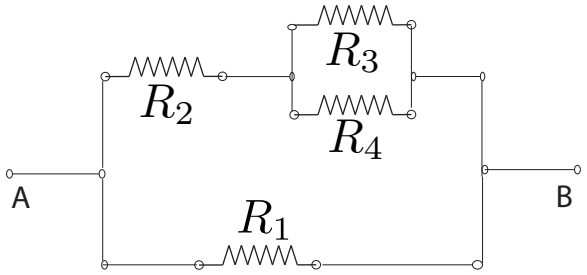
DC circuits - II

A $1.5m$ long piece of metal is oriented in the x-direction. It is moved with a velocity $20\frac{m}{s}\hat{j}$ in a region where the magnetic field is $0.4T\hat{k}$.

What is the difference in energy for an electron moving from one end to the other?

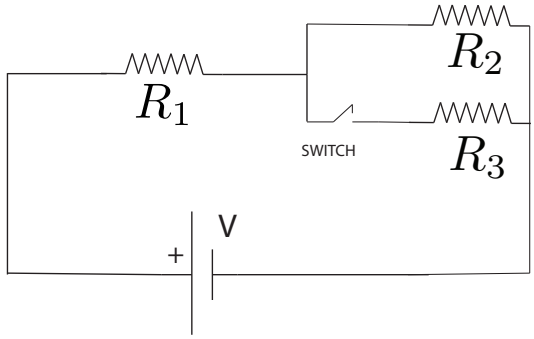
DC circuits - III

In the diagram $R_1 = 100\Omega$, $R_2 = 50\Omega$, $R_3 = 150\Omega$ and $R_4 = 120\Omega$. What is the equivalent resistance between A and B?



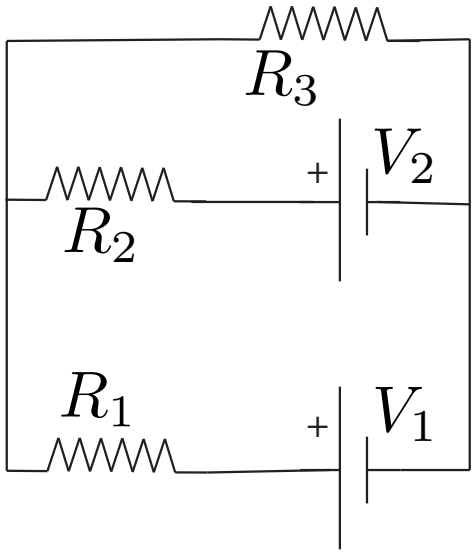
DC circuits - IV

A battery with terminal voltage of $20V$ is connected to three resistors and a switch. $R_1 = 60\Omega$, $R_2 = 120\Omega$, and $R_3 = 90\Omega$.



- When the switch is open so no current flows through R_3 what is the current in each resistor.
- When the switch is open so no current flows through R_3 what is the energy dissipation rate in each resistor.
- Repeat these with the switch closed.

DC circuits - V
Consider the circuit shown below:



If $R_1 = 10\Omega$, $R_2 = 80\Omega$, $R_3 = 20\Omega$, $V_1 = 15V$, and $V_2 = 3V$, find the current through and energy dissipated in each resistor.