Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3]

Does the sequence $\{a_n\}$ converge or diverge? Find the limit if the sequence is convergent.

$$a_n = \frac{\ln n}{5^{1/n}}$$

One way to see if the sequence converges or diverges is to show that it is a nondecreasing sequence and then test to see if it is bounded from above. Another option is to find $\lim_{n\to\infty} a_n$. For this example, the first method is used.

A sequence $\{a_n\}$ with the property that $a_n \le a_{n+1}$ for all n is called a nondecreasing sequence. To test if $\{a_n\}$ is nondecreasing, first find a_{n+1}.

$$a_{n+1} = \frac{\ln(n+1)}{5^{1/(n+1)}}$$

The sequence $\{a_n\}$ is nondecreasing if $\frac{\ln n}{5^{1/n}} \le \frac{\ln(n+1)}{5^{1/(n+1)}}$. This is the case as $\ln(n+1) > \ln n$ and $5^{1/(n+1)} < 5^{1/n}$ for all positive integers n.

Now test if $\{a_n\}$ is bounded from above. A sequence $\{b_n\}$ is bounded from above if there exists a number M such that $b_n \le M$ for all n. The number M is an upper bound for $\{b_n\}$.

Substitute the expression for a_n into the inequality $a_n \le M$. Then multiply both sides by $n^{1/n}$ and find the limit of each side.

$$\frac{\ln n}{5^{1/n}} \le M$$

$$\ln n \le M\left(5^{1/n}\right)$$

$$\lim_{n\to\infty} \ln n \le \lim_{n\to\infty} M\left(5^{1/n}\right)$$

The resulting inequality is $\infty \le M$, as $\lim_{n\to\infty} \ln n = \infty$ and $\lim_{n\to\infty} M\left(5^{1/n}\right) = M$.

There cannot be a number that is greater than or equal to infinity. Therefore, the resulting inequality is false.

The sequence $\{a_n\}$ is not bounded from above as the false inequality shows that there does not exist a number M such that $a_n \le M$ for all n.

The monotonic sequence theorem states that a nondecreasing sequence of real numbers converges if and only if it is bounded from above.

The sequence $\{a_n\}$ diverges as the monotonic sequence theorem implies that a nondecreasing sequence of real numbers that is not bounded from above diverges to infinity.