225 Final Exam - August 8, 1995

ROELANTS

This final exam is closed book. You have three hours to complete it. Your mark on programming questions will be affected by the time and space complexity of your solution. Questions 1,2, and 3 are worth 12 marks, question 4 is worth 10, question 5 is worth 25, question 6 is worth 15 and question 7 is worth 14.

Question 1 (Part a) As a function of *a* determine the number of comparisons that would be done by the following code segment:

```
x = 3;
do
  x = x + 7;
while (x <= a);
```

(Part b) Let $g(n) = n^3$ and let $g(n)$ be $\mathbf{O}(f(n))$. Determine the **order** of the recurrence

$$T(n) = f(n) + 7T(n/2) \qquad n>0$$
$$T(0) = 0$$

Note that you may express the order of T(n) as a function of n, f(n) and g(n).

Question 2: (Part a) Write a procedure that takes two parameters, HEAP and ELEMENT, removes the element from the top of the heap and sets ELEMENT to the value that was at the top of the heap. Your procedure must preserve the heap property.

(Part b) Indicate exactly, as a function of the size of the heap, the maximum number of comparisons your code would require to remove the element to the heap.

Question 3: For this question imagine that you are implementing a dictionary where the words in the dictionary are at most 10 characters in length and the meanings could be up to 100 characters in length.

(Part a) If you know that there will be approximately 500 entries in the dictionary, what data structure would you use to implement the dictionary and why?

(Part b) Design a good hash function for a dictionary that would contain about 100 entries

Question 4 (Part a) Indicate all the values of the prefix function of the Knuth Morrison Pratt pattern matching algorithm for the following pattern: *abababcabac*

(Part b) What type of programming technique is KMP pattern matching?

(Part c) As a function of *n*, the string length, and *m*, the pattern length, what is the time complexity of the KMP pattern matching algorithm?

Question 5 Short Answers. Give the best, shortest answer.

CSC 225 Final 1995 section K01

(a) What are the minimum and maximum number of characters that could be represented by a Huffman code where one character has a code of length 2 and the maximum length code requires 5 bits.

(b) The "level order representation" of an ordered tree is constructed by labelling each node with the number of siblings it has and then listing the siblings using a level order traversal (or BFS search). Is this a valid representation of an ordered tree? Justify your answer.

(c) How many ordered trees are there with 4 nodes?

(d) What is the time complexity of sorting a list of test scores? Justify your answer.

(e) If space was the primary concern, what data structure would be best suited to the implementation of a connected planar graph?

(f) Would the same data structure still be appropriate if there were 20,000 vertices and 30 edges? Justify your answer.

(g) What is the maximum number of nodes, as a function of k, in an ordered tree where all nodes have at most 4 children, and where all nodes with no children are on the bottom level and the tree has k levels.

(h) What is the minimum number of nodes in a tree with the properties described in part g.

(i) What is the time complexity of a parallel algorithm that uses n processors and determines the value of smallest element?

(j) The theoretical optimal time complexity for a parallel algorithm that uses n processors and solves a problem that requires order n^3 sequential time is order n^2. Why?

Question 6 The following is the adjacency matrix for a weighted graph:

	A	B	C	D	E	F	G	H	I
A	0	12	11	0	0	0	0	0	0
B	12	0	0	2	6	8	16	0	0
C	11	0	0	1	2	0	0	7	9
D	0	2	1	0	0	0	0	0	0
E	0	6	2	0	0	3	4	0	0
F	0	8	0	0	3	0	5	0	0
G	0	16	0	0	4	5	0	13	10
H	0	0	7	0	0	0	13	0	7
I	0	0	9	0	0	0	10	7	0

(Part a) Draw the graph described by the above adjacency matrix

(Part b) What is the shortest path between vertex 2 and vertex 7?

(Part c) Showing sufficient work, construct the minimum cost spanning tree using Prims algorithm starting at vertex 2. Indicate the order that edges are processed and which edges are not used.

Question 7 (Part a) Give the declarations for a data structure to implement a tree for the purposes of decoding a Huffman coded string.

(Part b) Write a procedure that takes a decoding tree T and a linked list of bits L and writes out the characters that are represented by the bits stored in L.