MATH 100, Fall, 2021
Tutorial #6
Linear Approximation and Optimization Part 1

Q1. You know the formula for the volume V and surface area S of a sphere of radius $r > 0$. Develop in your group a formula for surface area in terms of volume: $S = S(V)$.

Assignment: Use linear approximation to estimate the change in surface area ΔS (m^2) when a sphere of radius $r = 8$ (m) has its volume increased by $\Delta V = 0.3$ (m^3).

Q2. For each of the following scenarios, find an example (i.e. a **function**, its **domain** and its **graph**) that satisfies the given conditions. First discuss in your group what kind of graphs might work, then try to come up with simple functions and domains that have those graphs.

a) A continuous function on a domain D that has an absolute maximum, but no absolute minimum.

b) A continuous function on a domain D that has no local maximum or local minimum.

Q3. For each of the following scenarios, find an example (i..e. a **function**, its **domain** and its **graph**) that satisfies the given conditions. First discuss in your group what kind of graphs might work, then try to come up with simple functions and domains that have those graphs.

a) A continuous function on a domain D that has a local minimum, but no absolute minimum.

b) A discontinuous function on a domain $[a, b]$ that has both absolute maximum and absolute minimum.

Q4. During a 2 hour car trip, at some point your speedometer will read exactly the same value as your average speed over the duration of the trip. Discuss this fact and interpret in terms of the mean value theorem in calculus.

Assignment:
Suppose you take a trip, with distance travelled up to time t (hours) given by $d(t) = t^3 + 2t^2$ km. You travel from $t = 0$ to $t = 2$ hours. What is your average speed over the duration of this trip? Find a time $t \in (0, 2)$ when your instantaneous speed equals this average speed.

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Tutorial Worksheet
Tutorial Section (T01, T02 etc)_____
Tutorial Instructor Name:_____
Question Number Attempted (Q1, Q2, etc) Q1

Your Name: KEY Tut 6
Your Student Number: V00_____
Today's Date:_____

Know: $V = \frac{4}{3}\pi r^3$; $S = 4\pi r^2$, $V(8) = \frac{4}{3}\pi \cdot 8^3$

$$\Rightarrow r = \left(\frac{3}{4\pi}V\right)^{1/3} \Rightarrow S = 4\pi\left(\frac{3}{4\pi}V\right)^{2/3}$$

$$= \underline{(4\pi)^{1/3}\, 3^{2/3}\, V^{2/3}} = S(V)$$

$$\Delta S \simeq S'(V)\Delta V = \left[\frac{2\cdot(4\pi)^{1/3}}{3^{1/3}V^{1/3}}\right]_{V=\frac{4}{3}\pi\cdot 8^3} \cdot 0.30.$$

$$= \frac{2}{8}\cdot 0.30 = \frac{3}{40} \simeq 0.075\ m^2$$

Approx increase in surface area given $0.3\ m^3$ increase in volume.

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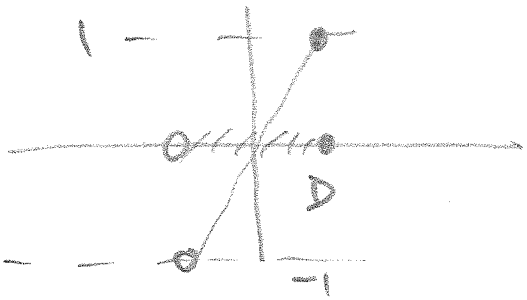
Tutorial Worksheet
Tutorial Section (T01, T02 etc)_____
Tutorial Instructor Name:_____
Question Number Attempted (Q1, Q2, etc) Q2

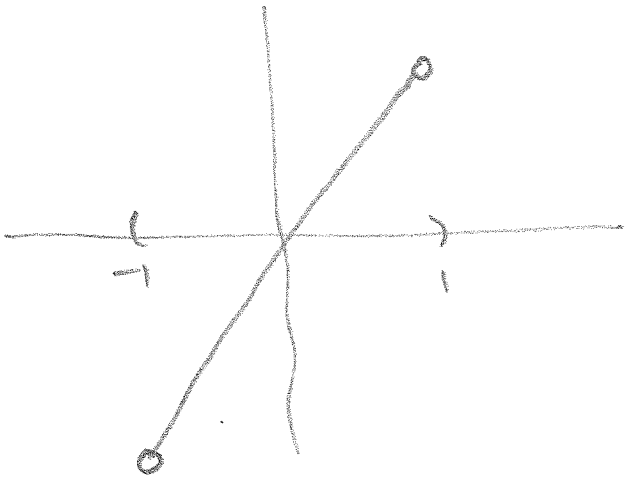
Your Name: KEY: Tut 6
Your Student Number: V00_____
Today's Date:_____

a) $D = (-1, 1]$, $f(x) = x$, cts.



abs. max $f = 1$ (at $x=1$)
" min f DNE.

b) $D = (-1, 1)$; $f(x) = x$, cts



No local max ✓
No local min ✓

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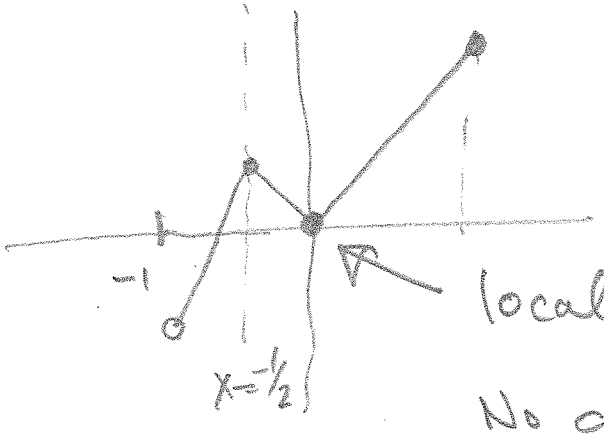
Tutorial Worksheet
Tutorial Section (T01, T02 etc)_____
Tutorial Instructor Name:_____
Question Number Attempted (Q1, Q2, etc) Q3

Your Name: KEY Tut 6
Your Student Number: V00_____
Today's Date:_____

a) $D = (-1, 1]$ $\quad f(x) = \begin{cases} |x| & -\frac{1}{2} \le x \le 1 \\ 2x + \frac{3}{2} & -1 < x < -\frac{1}{2} \end{cases}$

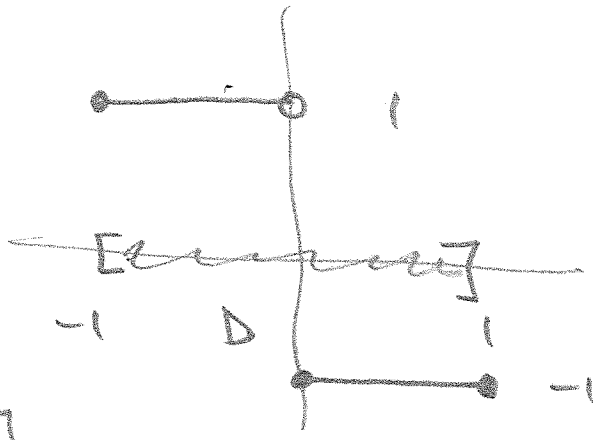


continuous f

local min $f = 0$ at $x = 0$

No abs min

b)



Discontinuous f

$D = [-1, 1]$

$$f(x) = \begin{cases} 1 & -1 \le x < 0 \\ -1 & 0 \le x \le 1 \end{cases}$$

$\text{Abs } f_{max} = 1$

$\text{Abs } f_{min} = -1$

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Tutorial Worksheet
Tutorial Section (T01, T02 etc)_____
Tutorial Instructor Name:_____
Question Number Attempted (Q1, Q2, etc) Q4

Your Name: KEY Tut 6
Your Student Number: V00_____
Today's Date:_____

$d(0) = 0$, $d(2) = 8 + 8 = 16$ (km)

Average speed: $\frac{d(2) - d(0)}{2} = 8$ km/hr.

$d'(t) = 3t^2 + 4t$

Want $d'(t) = 8 \iff 3t^2 + 4t - 8 = 0$

$\iff t = \frac{-4 \pm \sqrt{16 + 4 \cdot 3 \cdot 8}}{6}$

$\iff t = \frac{2}{3}(\pm\sqrt{7} - 1)$ take (+) why?

$t_* = \frac{2}{3}(\sqrt{7} - 1) \approx 1.097$.

Check: $3t_*^2 + 4t_* = 7.998 \approx 8$.

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Tutorial Worksheet
Tutorial Section (T01, T02 etc)_____
Tutorial Instructor Name:_____
Question Number Attempted (Q1, Q2, etc) Q5

Your Name: KEY. TUT 6
Your Student Number: V00_____
Today's Date:_____

Soln Related Rates $V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

Divide by V

$$\frac{\frac{dV}{dt}}{V} = \frac{3x^2 \frac{dx}{dt}}{V} = \frac{3x^2 \frac{dx}{dt}}{x^3} = 3\frac{\frac{dx}{dt}}{x}$$

Want $100 \frac{\frac{dV}{dt}}{V} \leq 10 \quad \left(\frac{\frac{dV}{dt}}{V} \leq \frac{1}{10}\right)$

$\Leftrightarrow 3\frac{\frac{dx}{dt}}{x} \leq \frac{1}{10}$ or $\frac{\frac{dx}{dt}}{x} \leq \frac{1}{30}$

Percentage: $100 \frac{dx}{dt}/x \leq \frac{100}{30} \approx 3.33\%$. rate of % decrease in x.

Soln. log. diff.

$V = x^3 \Rightarrow$ ln. $\ln V = 3 \ln x$.

differentials: $\frac{1}{V} dV = 3 \cdot \frac{1}{x} dx$.

or $\frac{dV}{V} = 3\frac{dx}{x}$. now solve as above

$\Rightarrow \frac{dx}{x} \leq 3.33\%$.