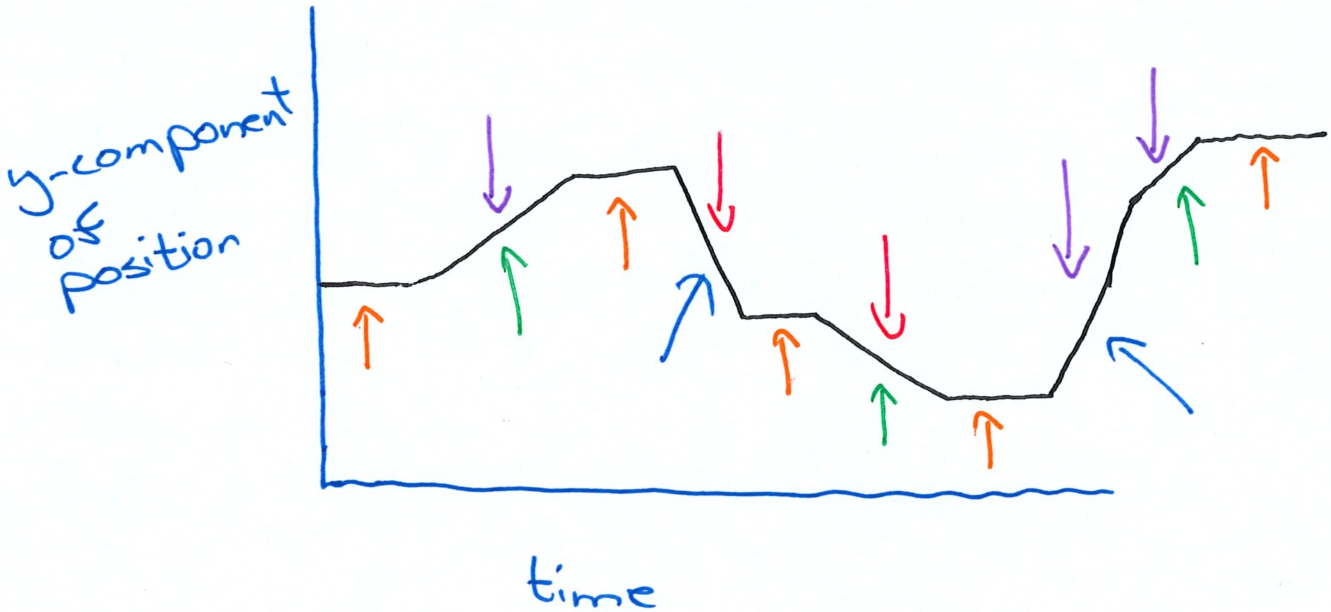
How to describe motion of a point particle.



Horizontal = stopped

Slope positive = moving up

Slope negative = moving down

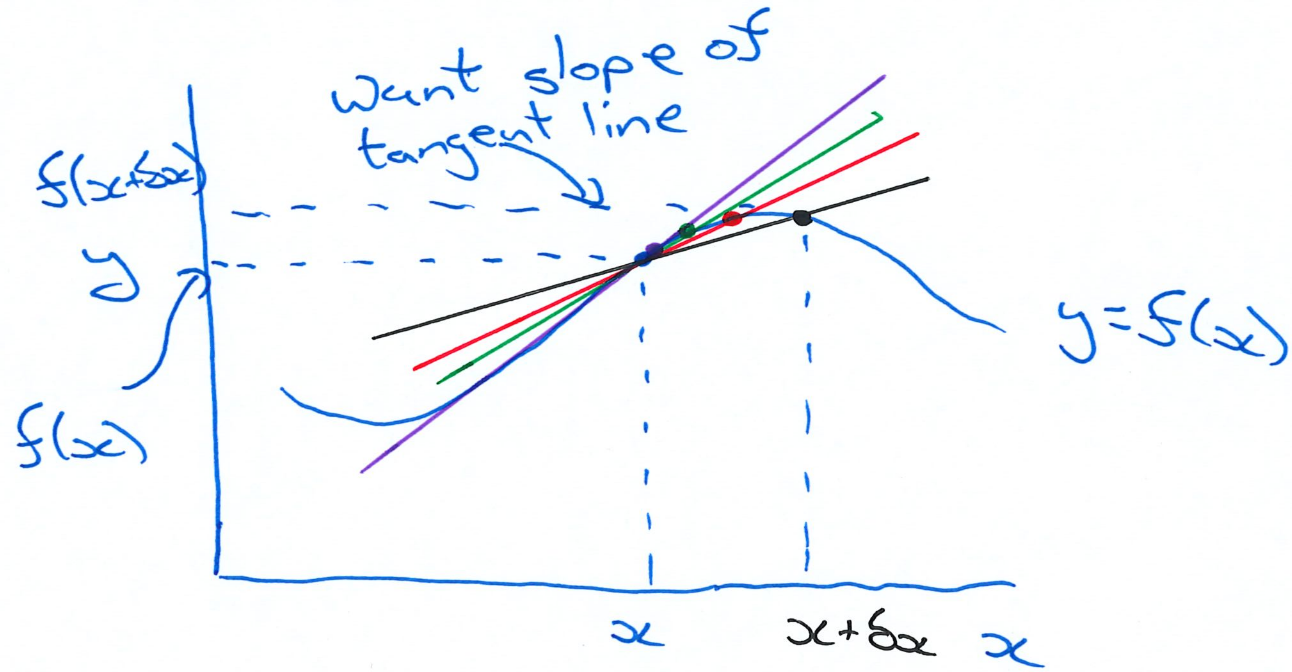
Slope small = moving slow

Slope big = moving fast

Slope of graph of y-component position as a function of time tells motion

How to find slope?

Calculus course



Slope is $\dfrac{f(x+\delta x)-f(x)}{(x+\delta x)-x}$

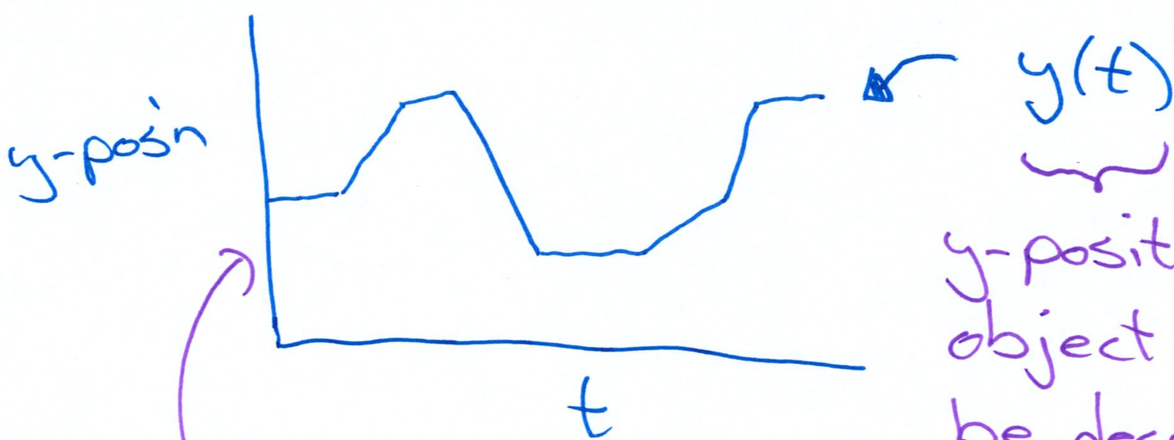
Continue making δx small as it gets smaller, value of slope gets closer to a fixed number

slope of tangent line at x $= \lim\limits_{\delta x \to 0} \dfrac{f(x+\delta x)-f(x)}{\delta x}$

Symbol for "slope of tangent line"

$$\frac{d}{dx} f(x) \equiv \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

derivative tells us slope!

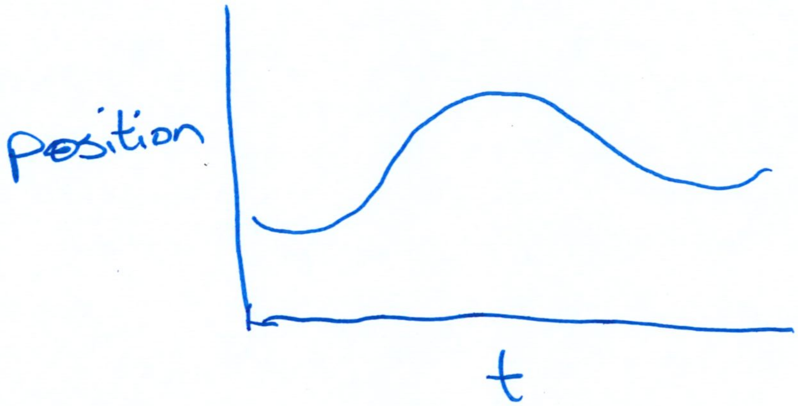


y-position of object can be described as a function of time

Slope $\frac{d}{dt} y(t)$

this is what tells motion!

So far



Slope tells us about motion
Slope big → fast
Slope small → slow
Slope 0 → stopped
± tells us about going up/down or left/right

⇒ derivative of position tells us about motion

Definition of velocity:

$$\vec{v}(t) \equiv \frac{d}{dt}\vec{r}(t)$$

$\vec{r}(t)$

$\underbrace{\qquad}$ vector, describes object's position

the (t) tells us position can change in time

$$\frac{d}{dt}\vec{r}(t) = \frac{d}{dt}\left(x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}\right)$$

x, y, z components can all change

Same!

Math fact

$$\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$= \left(\frac{d}{dt}x(t)\right)\hat{i} + \left(\frac{d}{dt}y(t)\right)\hat{j} + \left(\frac{d}{dt}z(t)\right)\hat{k}$$

$$\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k}$$

"velocity"
a vector
m/s

$$v_x(t) = \frac{d}{dt}x(t)$$

$$v_y(t) = \frac{d}{dt}y(t)$$

$$v_z(t) = \frac{d}{dt}z(t)$$

Define "speed"

$$|\vec{v}(t)| = \sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)}$$

Know that

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$

Key thought: Equilibrium same as "motion doesn't change"

motion described by velocity

Define acceleration

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

acceleration is time derivative of velocity

$$\vec{a}(t) = \frac{d}{dt}\left(\frac{d}{dt}\vec{r}(t)\right) = \frac{d^2}{dt^2}\vec{r}(t)$$

Velocity and Acceleration - I

A particle moves with position as a function of time given by

$$\vec{r}(t) = \left(2m - 3\frac{m}{s}t + 4\frac{m}{s^2}t^2\right)\hat{\imath} + \left(5\frac{m}{s}t - 2\frac{m}{s^3}t^3\right)\hat{\jmath} \tag{1}$$

- What is the velocity of the particle at $t = 2s$? At $t = 4s$?
- What is the acceleration of the particle at $t = 2s$? At $t = 4s$?

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$

$$v_x(t) = \frac{d}{dt}x(t)$$

$$v_y(t) = \frac{d}{dt}y(t)$$

$$v_x(t) = \frac{d}{dt}\left(2m - 3\frac{m}{s}t + 4\frac{m}{s^2}t^2\right)$$

$$= \frac{d}{dt}2mt^0 - \frac{d}{dt}\left(3\frac{m}{s}t^1\right) + \frac{d}{dt}\left(4\frac{m}{s^2}t^2\right)$$

$$= (2m)0 - 3\frac{m}{s}1t^0 + 4\frac{m}{s^2}2t^1$$

$$= -3\frac{m}{s} + 8\frac{m}{s^2}t$$

$$\boxed{\frac{d}{dx}\left(f(x)+g(x)\right) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)}$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

$$\frac{d}{dt}t^n = nt^{n-1}$$

$$v_y(t) = \frac{d}{dt}\left(5\tfrac{m}{s}t - 2\tfrac{m}{s^3}t^3\right)$$

$$= 5\tfrac{m}{s}1t^0 - 2\tfrac{m}{s^3}3t^2$$

$$= 5\,m/s - 6\tfrac{m}{s^3}t^2$$

$$\vec{v}(t) = \left(-3\,m/s + 8\,m/s^2\,t\right)\hat{i} + \left(5\,m/s - 6\tfrac{m}{s^3}t^2\right)\hat{j}$$

$$\vec{v}(2s) = \left(-3\,m/s + 8\,m/s^2\,2s\right)\hat{i} + \left(5\,m/s - 6\,m/s^3(2s)^2\right)\hat{j}$$

$$= (13\,m/s)\hat{i} + (-19\,m/s)\hat{j}$$

$$\vec{v}(4s) = \left(-3\,m/s + 8\,m/s^2\,4s\right)\hat{i} + \left(5\,m/s - 6\,m/s^2(4s)^2\right)\hat{j}$$

$$= (29\,m/s)\hat{i} + (-91\,m/s)\hat{j}$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt}\left[\left(-3\,m/s + 8\,m/s^2\,t\right)\hat{i} + \left(5\,m/s - 6\,m/s^3\,t^2\right)\hat{j}\right]$$

$$= 0 + \left(8\,m/s^2\,\hat{i}\right)1t^0 + 0 + \left(-6\,m/s^3\,\hat{j}\right)2t^1$$

$$= 8\,m/s^2\,\hat{i} - 12\,m/s^3\,t\,\hat{j}$$

$$\vec{a}(2s) = 8\,m/s^2\,\hat{i} - 24\,m/s^2\,\hat{j}$$

$$\vec{a}(4s) = 8\,m/s^2\,\hat{i} - 48\,m/s^2\,\hat{j}$$

To get $\vec{a}(2s)$ dont find $\vec{v}(2s)$ & then do $\frac{d}{dt}(\vec{v}(2s))$ use $\vec{v}(t)$ to do deriv.

Know $a_x(t) = \frac{d}{dt}v_x(t)$

etc