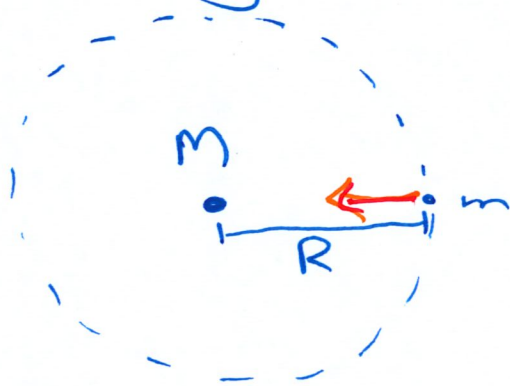


Newtonian gravity & orbits



We found mass of Jupiter by watching moons orbit!



$$\vec{F}_{onm} = \frac{GMm}{R^2} \text{ (towards M)}$$

$$\frac{\vec{F}_{onm}}{m} = \frac{|\vec{v}|^2}{R} \text{ (in to center)}$$

$$\frac{GMm}{R^2} = \frac{m|\vec{v}|^2}{R}$$

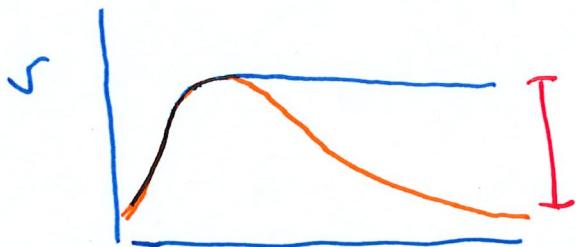
$$\frac{6m}{R} = |\vec{v}|^2$$

measure period of orbit T

$$|\vec{v}| = \frac{2\pi R}{T}$$

$$\frac{6m}{R} = \left(\frac{2\pi R}{T}\right)^2$$

$$T^2 = \frac{(2\pi)^2}{6m} R^3$$



Coulomb Force

Can be attractive or repulsive

Charge can be positive or negative

positive repels positives

" attract negative

negative repels negative.

q_B B
•
 \vec{r}_B

Diagram showing two charges, A and B, with position vectors \vec{r}_A and \vec{r}_B relative to a common origin. The force vector \vec{F}_{onAbyB} is shown pointing from B towards A.

$$\vec{F}_{onAbyB} = \frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{|\vec{r}_A - \vec{r}_B|^2} \left(\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|} \right)$$

Annotations:

- $\frac{1}{4\pi\epsilon_0}$ is circled in red, with a red arrow pointing to it from the value $\frac{1}{4\pi\epsilon_0} \approx 9.0 \times 10^9 \frac{Nm^2}{C^2}$ written below.
- $q_A q_B$ is circled in purple, with a purple arrow pointing to it from the text "magnitudes respective charges".
- $|\vec{r}_A - \vec{r}_B|^2$ is circled in green, with a green arrow pointing to it from the text "(separation)²".
- $\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$ is circled in orange, with an orange arrow pointing to it from the text "unit vector B to A".

Newtonian gravity

$$\vec{F}_{onAbyB} = -G \frac{m_A m_B}{|\vec{r}_A - \vec{r}_B|^2} \left(\frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|} \right)$$

Forces - III

A $3 \times 10^{-3} C$ charge is at $-2m\hat{i} + 3m\hat{j}$. A $5 \times 10^{-3} C$ charge is at $4m\hat{i} + 3m\hat{j}$.

A $-4 \times 10^{-3} C$ charge is at $-4m\hat{i} + 1m\hat{j}$.

- * • What is the Coulomb force on the first charge by the second charge?
- What is the total Coulomb force on the first charge?

Find $\vec{F}_{on1 by 2}$

- Find $\vec{F}_{on1 by 3}$

- Add

A • B

$$\vec{F}_{onA by B} = \frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{|\vec{r}_A - \vec{r}_B|^2} \frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$$

$$q_A = 3 \times 10^{-3} \quad \vec{r}_A = -2m\hat{i} + 3m\hat{j}$$

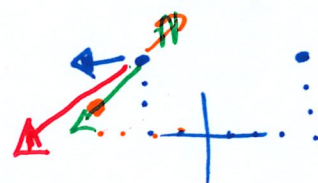
$$q_B = 5 \times 10^{-3} \quad \vec{r}_B = 4m\hat{i} + 3m\hat{j}$$

$$\begin{aligned} \vec{r}_A - \vec{r}_B &= (-2m\hat{i} + 3m\hat{j}) - (4m\hat{i} + 3m\hat{j}) \\ &= -6m\hat{i} \end{aligned}$$

$$|\vec{r}_A - \vec{r}_B| = 6m$$

$$\vec{F}_{\text{on 1st by 2nd}} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{3 \times 10^{-3} \text{C} \cdot 5 \times 10^{-3} \text{C}}{(6\text{m})^2} \frac{(-6\text{m}\hat{i})}{6\text{m}}$$

$$* = -3.75 \times 10^3 \text{N}\hat{i}$$



$$q_A = 3 \times 10^{-3} \text{C} \quad \vec{r}_A = -2\text{m}\hat{i} + 3\text{m}\hat{j}$$

$$q_B = -4 \times 10^{-3} \text{C} \quad \vec{r}_B = -4\text{m}\hat{i} + 1\text{m}\hat{j}$$

$$\begin{aligned} \vec{r}_A - \vec{r}_B &= (-2\text{m}\hat{i} + 3\text{m}\hat{j}) - (-4\text{m}\hat{i} + 1\text{m}\hat{j}) \\ &= 2\text{m}\hat{i} + 2\text{m}\hat{j} \end{aligned}$$

$$|\vec{r}_A - \vec{r}_B| = \sqrt{(2\text{m})^2 + (2\text{m})^2} = 2.83\text{m}$$

$$\vec{F}_{\text{on 1st by 3rd}} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \frac{3 \times 10^{-3} \text{C} \cdot (-4 \times 10^{-3} \text{C})}{(2.83\text{m})^2} \frac{2\text{m}\hat{i} + 2\text{m}\hat{j}}{2.83\text{m}}$$

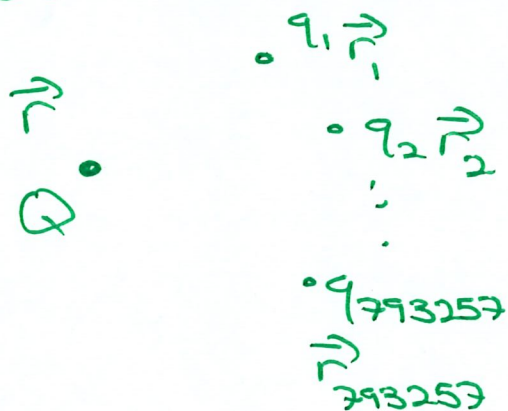
$$* = -9.54 \times 10^3 \text{N}\hat{i} - 9.54 \times 10^3 \text{N}\hat{j}$$

$$\vec{F}_{\text{total on 1st}} = -1.33 \times 10^4 \text{N}\hat{i} - 9.54 \times 10^3 \text{N}\hat{j}$$

Lorentz Force & Magnetism

Describe in terms of a "magnetic field" (\vec{B})

Can express Coulomb Force in terms of "Electric Field"



$$\vec{F}_{\text{net}, Q} = \vec{F}_{1 \text{ on } Q} + \vec{F}_{2 \text{ on } Q} + \dots$$

$$= \frac{1}{4\pi\epsilon_0} \frac{q_1 Q}{|\vec{r} - \vec{r}_1|^2} (\text{vector}) + \frac{1}{4\pi\epsilon_0} \frac{q_2 Q}{|\vec{r} - \vec{r}_2|^2} (\text{vector}) + \dots$$

$$= Q \left[\frac{1}{4\pi\epsilon_0} \left(\text{something depends on } \vec{r} \text{ and all } q_i \text{'s } \vec{r}_i \text{'s} \right) \right]$$

$$= Q \vec{E}(\vec{r})$$

\nwarrow
 $\frac{N}{C}$

describes
magnitude &
direction of
 \vec{F} on Q at \vec{r}

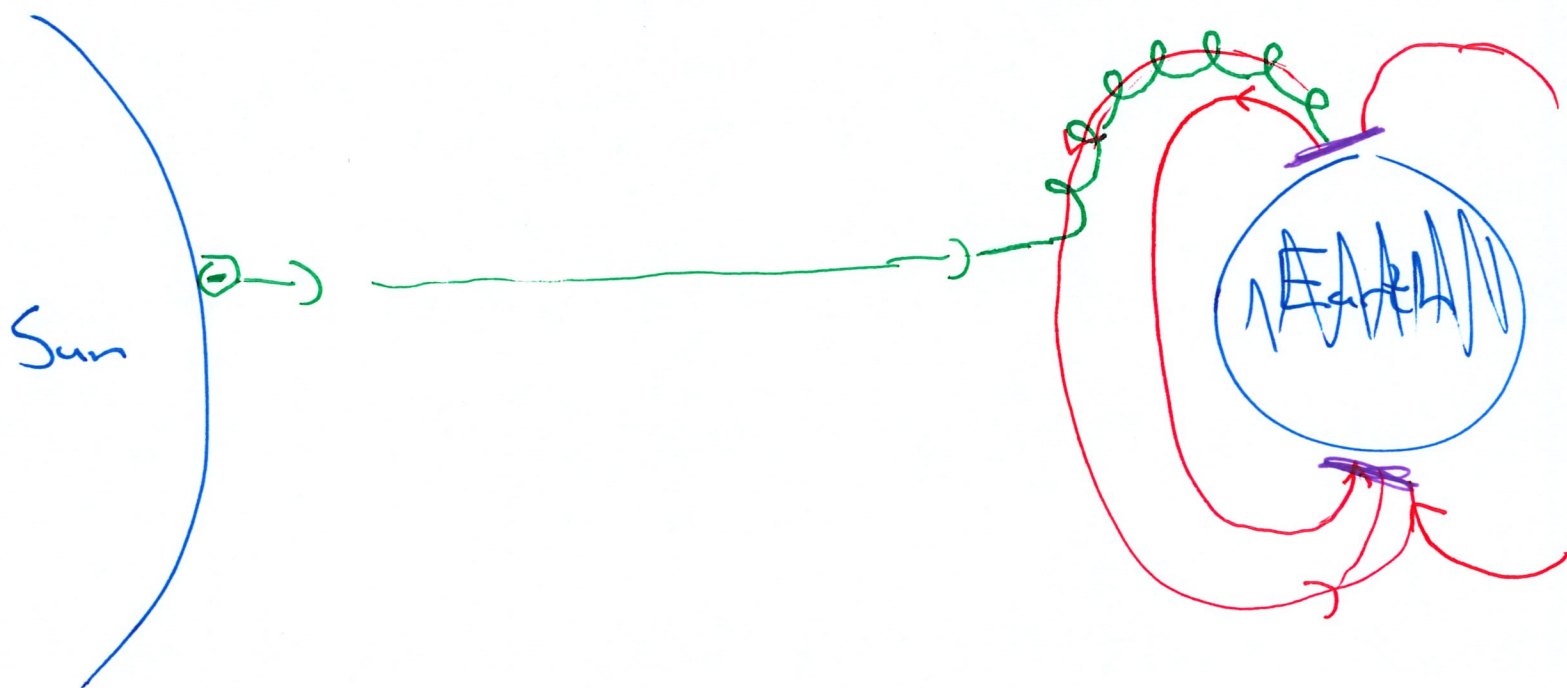
Similarly

\vec{B}

← based on positions, charges and velocities of all other charges

Charged particle moving ~~near~~ in a \vec{B}

$$\vec{F} = q \vec{v} \times \vec{B}$$



6-13 Example-Forces IV

Forces - IV

A 3.0kg mass with a charge of $-2 \times 10^{-3}\text{C}$ is moving at a velocity of $200\frac{\text{m}}{\text{s}}\hat{i} + 300\frac{\text{m}}{\text{s}}\hat{j}$ in a region where the magnetic field is $1.2\text{T}\hat{j}$.

What is the acceleration of the mass?

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\text{N} \quad \text{C} \frac{\text{m}}{\text{s}} \quad \left(\frac{\text{N}}{\text{C} \frac{\text{m}}{\text{s}}} \right) \sim \text{T}$$

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m} = \frac{q\vec{v} \times \vec{B}}{m}$$

$$= \frac{(-2 \times 10^{-3}\text{C})}{3\text{kg}} (200\frac{\text{m}}{\text{s}}\hat{i} + 300\frac{\text{m}}{\text{s}}\hat{j}) \times (1.2\text{T}\hat{j})$$

$$= \frac{-2 \times 10^{-3}\text{C}}{3\text{kg}} \left(240 \underbrace{\text{T} \frac{\text{m}}{\text{s}}}_{\text{N/C}} \hat{k} + 0 \right)$$

$$= -0.16\frac{\text{m}}{\text{s}^2} \hat{k}$$