Solution

The Taylor Series of $\tan(x)$ with center $\frac{5\pi}{4}$: $1+2\left(x-\frac{5\pi}{4}\right)+2\left(x-\frac{5\pi}{4}\right)^2+\frac{8}{3}\left(x-\frac{5\pi}{4}\right)^3+\frac{10}{3}\left(x-\frac{5\pi}{4}\right)^4+\dots$

Steps

Taylor Series

Taylor series of function f(x) at a is defined as:

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\dots$$

Apply the Taylor Formula *Hide Steps*

Find the derivatives of $f(x)=\tan(x)$, at $a=\frac{5\pi}{4}$

$f\left(\frac{5\pi}{4}\right)$: 1 *Hide Steps*

Take the point $x=\frac{5\pi}{4}$ and plug it into $\tan(x)$

$=\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)$ *Hide Steps*

$\tan\left(\frac{5\pi}{4}\right)$

Rewrite the angles for $\tan\left(\frac{5\pi}{4}\right)$:

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{4+1}{4}\pi\right)=\tan\left(\left(\frac{4}{4}+\frac{1}{4}\right)\pi\right)=\tan\left(\pi+\frac{1}{4}\pi\right)$

$=\tan\left(\pi+\frac{1}{4}\pi\right)$

Apply the periodicity of tan : $\tan(x+\pi\cdot k)=\tan(x)$

$\tan\left(\pi+\frac{1}{4}\pi\right)=\tan\left(\frac{1}{4}\pi\right)$

$=\tan\left(\frac{1}{4}\pi\right)$

Multiply $\frac{1}{4}\pi$: $\frac{\pi}{4}$ *Hide Steps*

$\frac{1}{4}\pi$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\pi}{4}$

Multiply: $1\pi=\pi$

$=\frac{\pi}{4}$

$=\tan\left(\frac{\pi}{4}\right)$

$=\tan\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\tan\left(\frac{\pi}{4}\right)=1$

$=1$

$$=1+\frac{\frac{d}{dx}(\tan(x))\left(\frac{5\pi}{4}\right)}{1!}\left(x-\frac{5\pi}{4}\right)+\frac{\frac{d^2}{dx^2}(\tan(x))\left(\frac{5\pi}{4}\right)}{2!}\left(x-\frac{5\pi}{4}\right)^2+\frac{\frac{d^3}{dx^3}(\tan(x))\left(\frac{5\pi}{4}\right)}{3!}\left(x-\frac{5\pi}{4}\right)^3+\dots$$

$$=1+\frac{\frac{d}{dx}(\tan(x))\left(\frac{5\pi}{4}\right)}{1!}\left(x-\frac{5\pi}{4}\right)+\frac{\frac{d^2}{dx^2}(\tan(x))\left(\frac{5\pi}{4}\right)}{2!}\left(x-\frac{5\pi}{4}\right)^2+\frac{\frac{d^3}{dx^3}(\tan(x))\left(\frac{5\pi}{4}\right)}{3!}\left(x-\frac{5\pi}{4}\right)^3+\dots$$

Evaluate Derivatives *Hide Steps*

$\frac{d}{dx}(\tan(x))\left(\frac{5\pi}{4}\right)$: 2 *Hide Steps*

$\frac{d}{dx}(\tan(x))\left(\frac{5\pi}{4}\right)$

$\frac{d}{dx}(\tan(x))=\sec^2(x)$ *Hide Steps*

$\frac{d}{dx}(\tan(x))$

$\frac{d}{dx}(\tan(x))=\sec^2(x)$ *Hide Steps*

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$= \sec^2(x)$

$= \sec^2(x)$

Evaluate $\sec^2(x)$ at point $x = \frac{5\pi}{4}$: 2

Hide Steps

Take the point $x = \frac{5\pi}{4}$ and plug it into $\sec^2(x)$

$= \sec^2\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right) = -\sqrt{2}$

Hide Steps

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right) = \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$= \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

Hide Steps

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi + \frac{\pi}{4}\right)$

$= \cos\left(\pi + \frac{\pi}{4}\right)$

Using the summation identity: $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$= \cos(\pi)\cos\left(\frac{\pi}{4}\right) - \sin(\pi)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos(\pi) = (-1)$

Use the following trivial identity: $\sin(\pi) = 0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$= (-1)\frac{\sqrt{2}}{2} - 0 \cdot \frac{\sqrt{2}}{2}$

Simplify

$= -\frac{\sqrt{2}}{2}$

$= \frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

Hide Steps

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b} = -\frac{a}{b}$

$= -\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}} = \frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt{2} = 2^{\frac{1}{2}}$

$= \frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}} = 2^{1-\frac{1}{2}}$

$= 2^{1-\frac{1}{2}}$

Subtract the numbers: $1-\frac{1}{2}=\frac{1}{2}$

$=2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}}=\sqrt[n]{a}$

$2^{\frac{1}{2}}=\sqrt{2}$

$=-\sqrt{2}$

$=-\sqrt{2}$

$=\left(-\sqrt{2}\right)^2$

Simplify Hide Steps

$\left(-\sqrt{2}\right)^2$

Apply exponent rule: $(-a)^n=a^n$, if n is even

$\left(-\sqrt{2}\right)^2=\left(\sqrt{2}\right)^2$

$=\left(\sqrt{2}\right)^2$

Apply radical rule: $\sqrt{a}=a^{\frac{1}{2}}$

$=\left(2^{\frac{1}{2}}\right)^2$

Apply exponent rule: $\left(a^b\right)^c=a^{bc}$

$=2^{\frac{1}{2}\cdot 2}$

$\frac{1}{2}\cdot 2=1$ Hide Steps

$\frac{1}{2}\cdot 2$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 2}{2}$

Cancel the common factor: 2

$=1$

$=2$

$=2$

$=2$

$\frac{d^2}{dx^2}(\tan(x))\left(\frac{5\pi}{4}\right)\;:\;4$ Hide Steps

$\frac{d^2}{dx^2}(\tan(x))\left(\frac{5\pi}{4}\right)$

$\frac{d^2}{dx^2}(\tan(x))=2\sec^2(x)\tan(x)$ Hide Steps

$\frac{d^2}{dx^2}(\tan(x))$

$\frac{d}{dx}(\tan(x))=\sec^2(x)$ Hide Steps

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x))=\sec^2(x)$

$=\sec^2(x)$

$=\frac{d}{dx}\left(\sec^2(x)\right)$

$\frac{d}{dx}\left(\sec^2(x)\right)=2\sec^2(x)\tan(x)$ Hide Steps

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $2\sec(x)\frac{d}{dx}(\sec(x))$ Hide Steps

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx}=\frac{df}{du}\cdot\frac{du}{dx}$

$f=u^2,\;\;u=\sec(x)$

$=\frac{d}{du}\left(u^2\right)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}\left(u^2\right)=2u$ Hide Steps

$\frac{d}{du}\left(u^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right)=a\cdot x^{a-1}$

$=2u^{2-1}$

Simplify

$=2u$

$=2u\frac{d}{dx}\left(\sec\left(x\right)\right)$

Substitute back $u=\sec\left(x\right)$

$=2\sec\left(x\right)\frac{d}{dx}\left(\sec\left(x\right)\right)$

$=2\sec\left(x\right)\frac{d}{dx}\left(\sec\left(x\right)\right)$

$\frac{d}{dx}\left(\sec\left(x\right)\right)=\sec\left(x\right)\tan\left(x\right)$ *Hide Steps*

$\frac{d}{dx}\left(\sec\left(x\right)\right)$

Apply the common derivative: $\frac{d}{dx}\left(\sec\left(x\right)\right)=\sec\left(x\right)\tan\left(x\right)$

$=\sec\left(x\right)\tan\left(x\right)$

$=2\sec\left(x\right)\sec\left(x\right)\tan\left(x\right)$

Simplify $2\sec\left(x\right)\sec\left(x\right)\tan\left(x\right)$: $2\sec^2\left(x\right)\tan\left(x\right)$ *Hide Steps*

$2\sec\left(x\right)\sec\left(x\right)\tan\left(x\right)$

Apply exponent rule: $a^b\cdot a^c=a^{b+c}$

$\sec\left(x\right)\sec\left(x\right)=\sec^{1+1}\left(x\right)$

$=2\sec^{1+1}\left(x\right)\tan\left(x\right)$

Add the numbers: $1+1=2$

$=2\sec^2\left(x\right)\tan\left(x\right)$

$=2\sec^2\left(x\right)\tan\left(x\right)$

$=2\sec^2\left(x\right)\tan\left(x\right)$

Evaluate $2\sec^2\left(x\right)\tan\left(x\right)$ at point $x=\frac{5\pi}{4}$: 4 *Hide Steps*

Take the point $x=\frac{5\pi}{4}$ and plug it into $2\sec^2\left(x\right)\tan\left(x\right)$

$=2\sec^2\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)$

$\sec^2\left(\frac{5\pi}{4}\right)=2$ *Hide Steps*

$\sec^2\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right)=-\sqrt{2}$ *Hide Steps*

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right)=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$ *Hide Steps*

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi+\frac{\pi}{4}\right)$

$=\cos\left(\pi+\frac{\pi}{4}\right)$

Using the summation identity: $\cos\left(x+y\right)=\cos\left(x\right)\cos\left(y\right)-\sin\left(x\right)\sin\left(y\right)$

$=\cos\left(\pi\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\pi\right)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos\left(\pi\right)=\left(-1\right)$

Use the following trivial identity: $\sin\left(\pi\right)=0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$=(-1)\frac{\sqrt{2}}{2}-0\cdot\frac{\sqrt{2}}{2}$

Simplify

$=-\frac{\sqrt{2}}{2}$

$=\frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}}=-\sqrt{2}$ Hide Steps

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b}=-\frac{a}{b}$

$=-\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}}=\frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{2}}$

$=-\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a}=a^{\frac{1}{n}}$

$\sqrt{2}=2^{\frac{1}{2}}$

$=\frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}}=2^{1-\frac{1}{2}}$

$=2^{1-\frac{1}{2}}$

Subtract the numbers: $1-\frac{1}{2}=\frac{1}{2}$

$=2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}}=\sqrt[n]{a}$

$2^{\frac{1}{2}}=\sqrt{2}$

$=-\sqrt{2}$

$=-\sqrt{2}$

$=\left(-\sqrt{2}\right)^2$

Apply exponent rule: $(-a)^n=a^n$, if n is even

$\left(-\sqrt{2}\right)^2=\left(\sqrt{2}\right)^2$

$=\left(\sqrt{2}\right)^2$

Apply radical rule: $\sqrt{a}=a^{\frac{1}{2}}$

$=\left(2^{\frac{1}{2}}\right)^2$

Apply exponent rule: $\left(a^b\right)^c=a^{bc}$

$=2^{\frac{1}{2}\cdot 2}$

$\frac{1}{2}\cdot 2=1$ Hide Steps

$\frac{1}{2}\cdot 2$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 2}{2}$

Cancel the common factor: 2

$=1$

$=2$

$=2\cdot 2\tan\left(\frac{5\pi}{4}\right)$

Hide Steps

$\tan\left(\frac{5\pi}{4}\right)=1$

$\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)$ *Hide Steps*

$\tan\left(\frac{5\pi}{4}\right)$

Rewrite the angles for $\tan\left(\frac{5\pi}{4}\right)$:

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{4+1}{4}\pi\right)=\tan\left(\left(\frac{4}{4}+\frac{1}{4}\right)\pi\right)=\tan\left(\pi+\frac{1}{4}\pi\right)$

$=\tan\left(\pi+\frac{1}{4}\pi\right)$

Apply the periodicity of tan : $\tan(x+\pi\cdot k)=\tan(x)$

$\tan\left(\pi+\frac{1}{4}\pi\right)=\tan\left(\frac{1}{4}\pi\right)$

$=\tan\left(\frac{1}{4}\pi\right)$

Multiply $\frac{1}{4}\pi$: $\frac{\pi}{4}$ *Hide Steps*

$\frac{1}{4}\pi$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\pi}{4}$

Multiply: $1\pi=\pi$

$=\frac{\pi}{4}$

$=\tan\left(\frac{\pi}{4}\right)$

$=\tan\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\tan\left(\frac{\pi}{4}\right)=1$

$=1$

$=2\cdot 2\cdot 1$

Multiply the numbers: $2\cdot 2\cdot 1=4$

$=4$

$=4$

$\frac{d^3}{dx^3}(\tan(x))\left(\frac{5\pi}{4}\right)$: 16 *Hide Steps*

$\frac{d^3}{dx^3}(\tan(x))\left(\frac{5\pi}{4}\right)$

$\frac{d^3}{dx^3}(\tan(x))=-4\sec^2(x)+6\sec^4(x)$ *Hide Steps*

$\frac{d^3}{dx^3}(\tan(x))$

$\frac{d}{dx}(\tan(x))=\sec^2(x)$ *Hide Steps*

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x))=\sec^2(x)$

$=\sec^2(x)$

$=\frac{d^2}{dx^2}\left(\sec^2(x)\right)$

$\frac{d}{dx}\left(\sec^2(x)\right)=2\sec^2(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $2\sec(x)\frac{d}{dx}(\sec(x))$ *Hide Steps*

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx}=\frac{df}{du}\cdot\frac{du}{dx}$

$f=u^2,\ u=\sec(x)$

$=\frac{d}{du}\left(u^2\right)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}\left(u^2\right) = 2u$ Hide Steps

$\frac{d}{du}\left(u^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 2u^{2-1}$

Simplify

$= 2u$

$= 2u\frac{d}{dx}\left(\sec(x)\right)$

Substitute back $u = \sec(x)$

$= 2\sec(x)\frac{d}{dx}\left(\sec(x)\right)$

$= 2\sec(x)\frac{d}{dx}\left(\sec(x)\right)$

$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$ Hide Steps

$\frac{d}{dx}\left(\sec(x)\right)$

Apply the common derivative: $\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$

$= \sec(x)\tan(x)$

$= 2\sec(x)\sec(x)\tan(x)$

Simplify $2\sec(x)\sec(x)\tan(x)$: $2\sec^2(x)\tan(x)$ Hide Steps

$2\sec(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec(x)\sec(x) = \sec^{1+1}(x)$

$= 2\sec^{1+1}(x)\tan(x)$

Add the numbers: $1 + 1 = 2$

$= 2\sec^2(x)\tan(x)$

$= 2\sec^2(x)\tan(x)$

$= \frac{d}{dx}\left(2\sec^2(x)\tan(x)\right)$

$\frac{d}{dx}\left(2\sec^2(x)\tan(x)\right) = -4\sec^2(x) + 6\sec^4(x)$ Hide Steps

$\frac{d}{dx}\left(2\sec^2(x)\tan(x)\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 2\frac{d}{dx}\left(\sec^2(x)\tan(x)\right)$

Apply the Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

$f = \sec^2(x),\ g = \tan(x)$

$= 2\left(\frac{d}{dx}\left(\sec^2(x)\right)\tan(x) + \frac{d}{dx}\left(\tan(x)\right)\sec^2(x)\right)$

$\frac{d}{dx}\left(\sec^2(x)\right) = 2\sec^2(x)\tan(x)$ Hide Steps

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $2\sec(x)\frac{d}{dx}\left(\sec(x)\right)$ Hide Steps

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$f = u^2,\ u = \sec(x)$

$= \frac{d}{du}\left(u^2\right)\frac{d}{dx}\left(\sec(x)\right)$

$\frac{d}{du}\left(u^2\right) = 2u$ Hide Steps

$\frac{d}{du}\left(u^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 2u^{2-1}$

Simplify

$= 2u$

$= 2u\frac{d}{dx}(\sec(x))$

Substitute back $u = \sec(x)$

$= 2\sec(x)\frac{d}{dx}(\sec(x))$

$= 2\sec(x)\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$ Hide Steps

$\frac{d}{dx}(\sec(x))$

Apply the common derivative: $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

$= \sec(x)\tan(x)$

$= 2\sec(x)\sec(x)\tan(x)$

Simplify $2\sec(x)\sec(x)\tan(x)$: $2\sec^2(x)\tan(x)$ Hide Steps

$2\sec(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec(x)\sec(x) = \sec^{1+1}(x)$

$= 2\sec^{1+1}(x)\tan(x)$

Add the numbers: $1 + 1 = 2$

$= 2\sec^2(x)\tan(x)$

$= 2\sec^2(x)\tan(x)$

$\frac{d}{dx}(\tan(x)) = \sec^2(x)$ Hide Steps

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$= \sec^2(x)$

$= 2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$

Simplify $2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$: $-4\sec^2(x) + 6\sec^4(x)$ Hide Steps

$2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$

$2\sec^2(x)\tan(x)\tan(x) = 2\sec^2(x)\tan^2(x)$ Hide Steps

$2\sec^2(x)\tan(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\tan(x)\tan(x) = \tan^{1+1}(x)$

$= 2\sec^2(x)\tan^{1+1}(x)$

Add the numbers: $1 + 1 = 2$

$= 2\sec^2(x)\tan^2(x)$

$\sec^2(x)\sec^2(x) = \sec^4(x)$ Hide Steps

$\sec^2(x)\sec^2(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec^2(x)\sec^2(x) = \sec^{2+2}(x)$

$= \sec^{2+2}(x)$

Add the numbers: $2 + 2 = 4$

$= \sec^4(x)$

$= 2\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right)$

Expand $\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right) \cdot 2$: $2\sec^4(x) + 4\sec^2(x)\tan^2(x)$ Hide Steps

$\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right) \cdot 2$

$= 2\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right)$

Apply the distributive law: $a(b + c) = ab + ac$

$a = 2,\ b = \sec^4(x),\ c = 2\sec^2(x)\tan^2(x)$

$= 2\sec^4(x) + 2 \cdot 2\sec^2(x)\tan^2(x)$

Multiply the numbers: $2 \cdot 2 = 4$

$= 2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

$= 2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

Rewrite using trig identities — Hide Steps

$2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

Use the Pythagorean identity: $\tan^2(x) + 1 = \sec^2(x)$

$\tan^2(x) = \sec^2(x) - 1$

$= 2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$

Simplify $2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$: $6\sec^4(x) - 4\sec^2(x)$ — Hide Steps

$2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$

Expand $4\sec^2(x)\left(\sec^2(x) - 1\right)$: $4\sec^4(x) - 4\sec^2(x)$ — Hide Steps

$4\sec^2(x)\left(\sec^2(x) - 1\right)$

Apply the distributive law: $a(b-c) = ab - ac$

$a = 4\sec^2(x),\ b = \sec^2(x),\ c = 1$

$= 4\sec^2(x)\sec^2(x) - 4\sec^2(x)\cdot 1$

$= 4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$

Hide Steps
Simplify $4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$: $4\sec^4(x) - 4\sec^2(x)$

$4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$

$4\sec^2(x)\sec^2(x) = 4\sec^4(x)$ — Hide Steps

$4\sec^2(x)\sec^2(x)$

Apply exponent rule: $a^b\cdot a^c = a^{b+c}$

$\sec^2(x)\sec^2(x) = \sec^{2+2}(x)$

$= 4\sec^{2+2}(x)$

Add the numbers: $2 + 2 = 4$

$= 4\sec^4(x)$

$4\cdot 1\cdot\sec^2(x) = 4\sec^2(x)$ — Hide Steps

$4\cdot 1\cdot\sec^2(x)$

Multiply the numbers: $4\cdot 1 = 4$

$= 4\sec^2(x)$

$= 4\sec^4(x) - 4\sec^2(x)$

$= 4\sec^4(x) - 4\sec^2(x)$

$= 2\sec^4(x) + 4\sec^4(x) - 4\sec^2(x)$

Add similar elements: $2\sec^4(x) + 4\sec^4(x) = 6\sec^4(x)$

$= 6\sec^4(x) - 4\sec^2(x)$

$= 6\sec^4(x) - 4\sec^2(x)$

$= -4\sec^2(x) + 6\sec^4(x)$

$= -4\sec^2(x) + 6\sec^4(x)$

$= -4\sec^2(x) + 6\sec^4(x)$

Evaluate $-4\sec^2(x) + 6\sec^4(x)$ at point $x = \frac{5\pi}{4}$: 16 — Hide Steps

Take the point $x = \frac{5\pi}{4}$ and plug it into $-4\sec^2(x) + 6\sec^4(x)$

$= -4\sec^2\left(\frac{5\pi}{4}\right) + 6\sec^4\left(\frac{5\pi}{4}\right)$

Hide Steps

$4\sec^2\left(\frac{5\pi}{4}\right)=8$

$4\sec^2\left(\frac{5\pi}{4}\right)$

$\sec^2\left(\frac{5\pi}{4}\right)=2$ Hide Steps

$\sec^2\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right)=-\sqrt{2}$ Hide Steps

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right)=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$ Hide Steps

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi+\frac{\pi}{4}\right)$

$=\cos\left(\pi+\frac{\pi}{4}\right)$

Using the summation identity: $\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

$=\cos(\pi)\cos\left(\frac{\pi}{4}\right)-\sin(\pi)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos(\pi)=(-1)$

Use the following trivial identity: $\sin(\pi)=0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$=(-1)\frac{\sqrt{2}}{2}-0\cdot\frac{\sqrt{2}}{2}$

Simplify

$=-\frac{\sqrt{2}}{2}$

$=\frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}}=-\sqrt{2}$ Hide Steps

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b}=-\frac{a}{b}$

$=-\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}}=\frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{2}}$

$=-\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a}=a^{\frac{1}{n}}$

$\sqrt{2}=2^{\frac{1}{2}}$

$=\frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}}=2^{1-\frac{1}{2}}$

$=2^{1-\frac{1}{2}}$

Subtract the numbers: $1-\frac{1}{2}=\frac{1}{2}$

$=2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$2^{\frac{1}{2}} = \sqrt{2}$

$= -\sqrt{2}$

$= -\sqrt{2}$

$= \left(-\sqrt{2}\right)^2$

Apply exponent rule: $(-a)^n = a^n$, if n is even

$\left(-\sqrt{2}\right)^2 = \left(\sqrt{2}\right)^2$

$= \left(\sqrt{2}\right)^2$

Apply radical rule: $\sqrt{a} = a^{\frac{1}{2}}$

$= \left(2^{\frac{1}{2}}\right)^2$

Apply exponent rule: $\left(a^b\right)^c = a^{bc}$

$= 2^{\frac{1}{2} \cdot 2}$

$\frac{1}{2} \cdot 2 = 1$ *Hide Steps*

$\frac{1}{2} \cdot 2$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 2}{2}$

Cancel the common factor: 2

$= 1$

$= 2$

$= 4 \cdot 2$

Multiply the numbers: $4 \cdot 2 = 8$

$= 8$

$6\sec^4\left(\frac{5\pi}{4}\right) = 2^2 \cdot 6$ *Hide Steps*

$6\sec^4\left(\frac{5\pi}{4}\right)$

$\sec^4\left(\frac{5\pi}{4}\right) = 2^2$ *Hide Steps*

$\sec^4\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ *Hide Steps*

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right) = \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$= \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ *Hide Steps*

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi + \frac{\pi}{4}\right)$

$= \cos\left(\pi + \frac{\pi}{4}\right)$

Using the summation identity: $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$= \cos(\pi)\cos\left(\frac{\pi}{4}\right) - \sin(\pi)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos(\pi) = (-1)$

Use the following trivial identity: $\sin(\pi) = 0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$= (-1)\frac{\sqrt{2}}{2} - 0 \cdot \frac{\sqrt{2}}{2}$

Simplify

$= -\frac{\sqrt{2}}{2}$

$= \frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$ *Hide Steps*

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b} = -\frac{a}{b}$

$= -\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}} = \frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt{2} = 2^{\frac{1}{2}}$

$= \frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}} = 2^{1-\frac{1}{2}}$

$= 2^{1-\frac{1}{2}}$

Subtract the numbers: $1 - \frac{1}{2} = \frac{1}{2}$

$= 2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$2^{\frac{1}{2}} = \sqrt{2}$

$= -\sqrt{2}$

$= -\sqrt{2}$

$= (-\sqrt{2})^4$

Apply exponent rule: $(-a)^n = a^n$, if n is even

$(-\sqrt{2})^4 = (\sqrt{2})^4$

$= (\sqrt{2})^4$

Apply radical rule: $\sqrt{a} = a^{\frac{1}{2}}$

$= \left(2^{\frac{1}{2}}\right)^4$

Apply exponent rule: $(a^b)^c = a^{bc}$

$= 2^{\frac{1}{2} \cdot 4}$

$\frac{1}{2} \cdot 4 = 2$ *Hide Steps*

$\frac{1}{2} \cdot 4$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 4}{2}$

Multiply the numbers: $1 \cdot 4 = 4$

$= \frac{4}{2}$

Divide the numbers: $\frac{4}{2} = 2$

$= 2$

$= 2^2$

$= 2^2 \cdot 6$

$= -8 + 2^2 \cdot 6$

Hide Steps

Simplify

$-8+6\cdot\ 2^2$

$6\cdot\ 2^2=24$ Hide Steps

$6\cdot\ 2^2$

$2^2=4$

$=6\cdot\ 4$

Multiply the numbers: $6\cdot\ 4=24$

$=24$

$=\ -8+24$

Add/Subtract the numbers: $-8+24=16$

$=16$

$=16$

$=16$

$\frac{d^4}{dx^4}(\tan(x))\left(\frac{5\pi}{4}\right)\ :\ 80$ Hide Steps

$\frac{d^4}{dx^4}(\tan(x))\left(\frac{5\pi}{4}\right)$

$\frac{d^4}{dx^4}(\tan(x))=\ -8\sec^2(x)\tan(x)+24\sec^4(x)\tan(x)$ Hide Steps

$\frac{d^4}{dx^4}(\tan(x))$

$\frac{d}{dx}(\tan(x))=\sec^2(x)$ Hide Steps

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x))=\sec^2(x)$

$=\sec^2(x)$

$=\frac{d^3}{dx^3}(\sec^2(x))$

$\frac{d}{dx}(\sec^2(x))=2\sec^2(x)\tan(x)$ Hide Steps

$\frac{d}{dx}(\sec^2(x))$

Apply the chain rule: $2\sec(x)\frac{d}{dx}(\sec(x))$ Hide Steps

$\frac{d}{dx}(\sec^2(x))$

Apply the chain rule: $\frac{df(u)}{dx}=\frac{df}{du}\cdot\frac{du}{dx}$

$f=u^2,\ u=\sec(x)$

$=\frac{d}{du}(u^2)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}(u^2)=2u$ Hide Steps

$\frac{d}{du}(u^2)$

Apply the Power Rule: $\frac{d}{dx}(x^a)=a\cdot x^{a-1}$

$=2u^{2-1}$

Simplify

$=2u$

$=2u\frac{d}{dx}(\sec(x))$

Substitute back $u=\sec(x)$

$=2\sec(x)\frac{d}{dx}(\sec(x))$

$=2\sec(x)\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(\sec(x))=\sec(x)\tan(x)$ Hide Steps

$\frac{d}{dx}(\sec(x))$

Apply the common derivative: $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

$= \sec(x)\tan(x)$

$= 2\sec(x)\sec(x)\tan(x)$

Simplify $2\sec(x)\sec(x)\tan(x)$: $2\sec^2(x)\tan(x)$ *Hide Steps*

$2\sec(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec(x)\sec(x) = \sec^{1+1}(x)$

$= 2\sec^{1+1}(x)\tan(x)$

Add the numbers: $1+1=2$

$= 2\sec^2(x)\tan(x)$

$= 2\sec^2(x)\tan(x)$

$= \frac{d^2}{dx^2}\left(2\sec^2(x)\tan(x)\right)$

$\frac{d}{dx}\left(2\sec^2(x)\tan(x)\right) = -4\sec^2(x) + 6\sec^4(x)$ *Hide Steps*

$\frac{d}{dx}\left(2\sec^2(x)\tan(x)\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 2\frac{d}{dx}\left(\sec^2(x)\tan(x)\right)$

Apply the Product Rule: $(f \cdot g)' = f' \cdot g + f \cdot g'$

$f = \sec^2(x),\ g = \tan(x)$

$= 2\left(\frac{d}{dx}\left(\sec^2(x)\right)\tan(x) + \frac{d}{dx}(\tan(x))\sec^2(x)\right)$

$\frac{d}{dx}\left(\sec^2(x)\right) = 2\sec^2(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $2\sec(x)\frac{d}{dx}(\sec(x))$ *Hide Steps*

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$f = u^2,\ u = \sec(x)$

$= \frac{d}{du}\left(u^2\right)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}\left(u^2\right) = 2u$ *Hide Steps*

$\frac{d}{du}\left(u^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 2u^{2-1}$

Simplify

$= 2u$

$= 2u\frac{d}{dx}(\sec(x))$

Substitute back $u = \sec(x)$

$= 2\sec(x)\frac{d}{dx}(\sec(x))$

$= 2\sec(x)\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}(\sec(x))$

Apply the common derivative: $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

$= \sec(x)\tan(x)$

$= 2\sec(x)\sec(x)\tan(x)$

Simplify $2\sec(x)\sec(x)\tan(x)$: $2\sec^2(x)\tan(x)$ *Hide Steps*

$2\sec(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec(x)\sec(x) = \sec^{1+1}(x)$

$= 2\sec^{1+1}(x)\tan(x)$

Add the numbers: $1+1=2$

$= 2\sec^2(x)\tan(x)$

$= 2\sec^2(x)\tan(x)$

$\frac{d}{dx}(\tan(x)) = \sec^2(x)$ Hide Steps

$\frac{d}{dx}(\tan(x))$

Apply the common derivative: $\frac{d}{dx}(\tan(x)) = \sec^2(x)$

$= \sec^2(x)$

$= 2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$

Simplify $2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$: $\quad -4\sec^2(x) + 6\sec^4(x)$ Hide Steps

$2\left(2\sec^2(x)\tan(x)\tan(x) + \sec^2(x)\sec^2(x)\right)$

$2\sec^2(x)\tan(x)\tan(x) = 2\sec^2(x)\tan^2(x)$ Hide Steps

$2\sec^2(x)\tan(x)\tan(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\tan(x)\tan(x) = \tan^{1+1}(x)$

$= 2\sec^2(x)\tan^{1+1}(x)$

Add the numbers: $1+1=2$

$= 2\sec^2(x)\tan^2(x)$

$\sec^2(x)\sec^2(x) = \sec^4(x)$ Hide Steps

$\sec^2(x)\sec^2(x)$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$\sec^2(x)\sec^2(x) = \sec^{2+2}(x)$

$= \sec^{2+2}(x)$

Add the numbers: $2+2=4$

$= \sec^4(x)$

$= 2\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right)$

Expand $\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right) \cdot 2$: $\quad 2\sec^4(x) + 4\sec^2(x)\tan^2(x)$ Hide Steps

$\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right) \cdot 2$

$= 2\left(\sec^4(x) + 2\sec^2(x)\tan^2(x)\right)$

Apply the distributive law: $a(b+c) = ab + ac$

$a = 2,\ b = \sec^4(x),\ c = 2\sec^2(x)\tan^2(x)$

$= 2\sec^4(x) + 2 \cdot 2\sec^2(x)\tan^2(x)$

Multiply the numbers: $2 \cdot 2 = 4$

$= 2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

$= 2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

Rewrite using trig identities Hide Steps

$2\sec^4(x) + 4\sec^2(x)\tan^2(x)$

Use the Pythagorean identity: $\tan^2(x) + 1 = \sec^2(x)$

$\tan^2(x) = \sec^2(x) - 1$

$= 2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$

Simplify $2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$: $\quad 6\sec^4(x) - 4\sec^2(x)$ Hide Steps

$2\sec^4(x) + 4\sec^2(x)\left(\sec^2(x) - 1\right)$

Expand $4\sec^2(x)\left(\sec^2(x) - 1\right)$: $\quad 4\sec^4(x) - 4\sec^2(x)$ Hide Steps

$4\sec^2(x)\left(\sec^2(x) - 1\right)$

Apply the distributive law: $a(b-c) = ab - ac$

$a = 4\sec^2(x),\ b = \sec^2(x),\ c = 1$

$= 4\sec^2(x)\sec^2(x) - 4\sec^2(x)\cdot 1$

$= 4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$

Hide Steps

Simplify $4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$: $4\sec^4(x) - 4\sec^2(x)$

$4\sec^2(x)\sec^2(x) - 4\cdot 1\cdot\sec^2(x)$

$4\sec^2(x)\sec^2(x) = 4\sec^4(x)$ Hide Steps

$4\sec^2(x)\sec^2(x)$

Apply exponent rule: $a^b\cdot a^c = a^{b+c}$

$\sec^2(x)\sec^2(x) = \sec^{2+2}(x)$

$= 4\sec^{2+2}(x)$

Add the numbers: $2+2=4$

$= 4\sec^4(x)$

$4\cdot 1\cdot\sec^2(x) = 4\sec^2(x)$ Hide Steps

$4\cdot 1\cdot\sec^2(x)$

Multiply the numbers: $4\cdot 1 = 4$

$= 4\sec^2(x)$

$= 4\sec^4(x) - 4\sec^2(x)$

$= 4\sec^4(x) - 4\sec^2(x)$

$= 2\sec^4(x) + 4\sec^4(x) - 4\sec^2(x)$

Add similar elements: $2\sec^4(x) + 4\sec^4(x) = 6\sec^4(x)$

$= 6\sec^4(x) - 4\sec^2(x)$

$= 6\sec^4(x) - 4\sec^2(x)$

$= -4\sec^2(x) + 6\sec^4(x)$

$= -4\sec^2(x) + 6\sec^4(x)$

$= \frac{d}{dx}\left(-4\sec^2(x) + 6\sec^4(x)\right)$

$\frac{d}{dx}\left(-4\sec^2(x) + 6\sec^4(x)\right) = -8\sec^2(x)\tan(x) + 24\sec^4(x)\tan(x)$ Hide Steps

$\frac{d}{dx}\left(-4\sec^2(x) + 6\sec^4(x)\right)$

Apply the Sum/Difference Rule: $(f\pm g)' = f'\pm g'$

$= -\frac{d}{dx}\left(4\sec^2(x)\right) + \frac{d}{dx}\left(6\sec^4(x)\right)$

$\frac{d}{dx}\left(4\sec^2(x)\right) = 8\sec^2(x)\tan(x)$ Hide Steps

$\frac{d}{dx}\left(4\sec^2(x)\right)$

Take the constant out: $(a\cdot f)' = a\cdot f'$

$= 4\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $2\sec(x)\frac{d}{dx}(\sec(x))$ Hide Steps

$\frac{d}{dx}\left(\sec^2(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$

$f = u^2,\ u = \sec(x)$

$= \frac{d}{du}\left(u^2\right)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}\left(u^2\right) = 2u$ Hide Steps

$\frac{d}{du}\left(u^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a\cdot x^{a-1}$

$= 2u^{2-1}$

Simplify

$= 2u$

$= 2u\frac{d}{dx}(\sec(x))$

Substitute back $u=\sec(x)$

$=2\sec(x)\frac{d}{dx}(\sec(x))$

$=2\sec(x)\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(\sec(x))=\sec(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}(\sec(x))$

Apply the common derivative: $\frac{d}{dx}(\sec(x))=\sec(x)\tan(x)$

$=\sec(x)\tan(x)$

$=4\cdot\ 2\sec(x)\sec(x)\tan(x)$

Simplify $4\cdot\ 2\sec(x)\sec(x)\tan(x)$: $8\sec^2(x)\tan(x)$ *Hide Steps*

$4\cdot\ 2\sec(x)\sec(x)\tan(x)$

Multiply the numbers: $4\cdot\ 2=8$

$=8\sec(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b\cdot a^c=a^{b+c}$

$\sec(x)\sec(x)=\sec^{1+1}(x)$

$=8\sec^{1+1}(x)\tan(x)$

Add the numbers: $1+1=2$

$=8\sec^2(x)\tan(x)$

$=8\sec^2(x)\tan(x)$

$\frac{d}{dx}\left(6\sec^4(x)\right)=24\sec^4(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}\left(6\sec^4(x)\right)$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=6\frac{d}{dx}\left(\sec^4(x)\right)$

Hide Steps

Apply the chain rule: $4(\sec(x))^3\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}\left(\sec^4(x)\right)$

Apply the chain rule: $\frac{df(u)}{dx}=\frac{df}{du}\cdot\frac{du}{dx}$

$f=u^4,\ u=\sec(x)$

$=\frac{d}{du}\left(u^4\right)\frac{d}{dx}(\sec(x))$

$\frac{d}{du}\left(u^4\right)=4u^3$ *Hide Steps*

$\frac{d}{du}\left(u^4\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right)=a\cdot x^{a-1}$

$=4u^{4-1}$

Simplify

$=4u^3$

$=4u^3\frac{d}{dx}(\sec(x))$

Substitute back $u=\sec(x)$

$=4(\sec(x))^3\frac{d}{dx}(\sec(x))$

$=4(\sec(x))^3\frac{d}{dx}(\sec(x))$

$\frac{d}{dx}(\sec(x))=\sec(x)\tan(x)$ *Hide Steps*

$\frac{d}{dx}(\sec(x))$

Apply the common derivative: $\frac{d}{dx}(\sec(x))=\sec(x)\tan(x)$

$=\sec(x)\tan(x)$

$=6\cdot\ 4(\sec(x))^3\sec(x)\tan(x)$

Simplify $6\cdot\ 4\sec^3(x)\sec(x)\tan(x)$: $24\sec^4(x)\tan(x)$ *Hide Steps*

$6\cdot\ 4\sec^3(x)\sec(x)\tan(x)$

Multiply the numbers: $6\cdot\ 4=24$

$=24\sec^3(x)\sec(x)\tan(x)$

Apply exponent rule: $a^b\cdot\ a^c=a^{b+c}$

$\sec^3(x)\sec(x)=\ \sec^{3+1}(x)$

$=24\sec^{3+1}(x)\tan(x)$

Add the numbers: $3+1=4$

$=24\sec^4(x)\tan(x)$

$=24\sec^4(x)\tan(x)$

$=-8\sec^2(x)\tan(x)+24\sec^4(x)\tan(x)$

$=-8\sec^2(x)\tan(x)+24\sec^4(x)\tan(x)$

Evaluate $-8\sec^2(x)\tan(x)+24\sec^4(x)\tan(x)$ at point $x=\frac{5\pi}{4}$: 80 *Hide Steps*

Take the point $x=\frac{5\pi}{4}$ and plug it into $-8\sec^2(x)\tan(x)+24\sec^4(x)\tan(x)$

$=-8\sec^2\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)+24\sec^4\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)$

$8\sec^2\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)=16$ *Hide Steps*

$8\sec^2\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)$

$\sec^2\left(\frac{5\pi}{4}\right)=2$ *Hide Steps*

$\sec^2\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right)=-\sqrt{2}$ *Hide Steps*

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right)=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$=\frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$ *Hide Steps*

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi+\frac{\pi}{4}\right)$

$=\cos\left(\pi+\frac{\pi}{4}\right)$

Using the summation identity: $\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$

$=\cos(\pi)\cos\left(\frac{\pi}{4}\right)-\sin(\pi)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos(\pi)=(-1)$

Use the following trivial identity: $\sin(\pi)=0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$

$=(-1)\frac{\sqrt{2}}{2}-0\cdot\frac{\sqrt{2}}{2}$

Simplify

$=-\frac{\sqrt{2}}{2}$

$=\frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}}=-\sqrt{2}$ *Hide Steps*

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b} = -\frac{a}{b}$

$= -\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}} = \frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

$= -\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a} = a^{\frac{1}{n}}$

$\sqrt{2} = 2^{\frac{1}{2}}$

$= \frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}} = 2^{1-\frac{1}{2}}$

$= 2^{1-\frac{1}{2}}$

Subtract the numbers: $1 - \frac{1}{2} = \frac{1}{2}$

$= 2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}} = \sqrt[n]{a}$

$2^{\frac{1}{2}} = \sqrt{2}$

$= -\sqrt{2}$

$= -\sqrt{2}$

$= (-\sqrt{2})^2$

Apply exponent rule: $(-a)^n = a^n$, if n is even

$(-\sqrt{2})^2 = (\sqrt{2})^2$

$= (\sqrt{2})^2$

Apply radical rule: $\sqrt{a} = a^{\frac{1}{2}}$

$= \left(2^{\frac{1}{2}}\right)^2$

Apply exponent rule: $\left(a^b\right)^c = a^{bc}$

$= 2^{\frac{1}{2} \cdot 2}$

$\frac{1}{2} \cdot 2 = 1$ *Hide Steps*

$\frac{1}{2} \cdot 2$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 2}{2}$

Cancel the common factor: 2

$= 1$

$= 2$

$= 8 \cdot 2\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right) = 1$ *Hide Steps*

$\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right)$ *Hide Steps*

$\tan\left(\frac{5\pi}{4}\right)$

Rewrite the angles for $\tan\left(\frac{5\pi}{4}\right)$:

$\tan\left(\frac{5\pi}{4}\right) = \tan\left(\frac{4+1}{4}\pi\right) = \tan\left(\left(\frac{4}{4} + \frac{1}{4}\right)\pi\right) = \tan\left(\pi + \frac{1}{4}\pi\right)$

$= \tan\left(\pi + \frac{1}{4}\pi\right)$

Apply the periodicity of tan : $\tan(x + \pi \cdot k) = \tan(x)$

$\tan\left(\pi + \frac{1}{4}\pi\right) = \tan\left(\frac{1}{4}\pi\right)$

$= \tan\left(\frac{1}{4}\pi\right)$

Multiply $\frac{1}{4}\pi$: $\frac{\pi}{4}$ *Hide Steps*

$\frac{1}{4}\pi$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1\pi}{4}$

Multiply: $1\pi = \pi$

$= \frac{\pi}{4}$

$= \tan\left(\frac{\pi}{4}\right)$

$= \tan\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\tan\left(\frac{\pi}{4}\right) = 1$

$= 1$

$= 8 \cdot 2 \cdot 1$

Multiply the numbers: $8 \cdot 2 \cdot 1 = 16$

$= 16$

$24\sec^4\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right) = 2^2 \cdot 24$ *Hide Steps*

$24\sec^4\left(\frac{5\pi}{4}\right)\tan\left(\frac{5\pi}{4}\right)$

$\sec^4\left(\frac{5\pi}{4}\right) = 2^2$ *Hide Steps*

$\sec^4\left(\frac{5\pi}{4}\right)$

$\sec\left(\frac{5\pi}{4}\right) = -\sqrt{2}$ *Hide Steps*

$\sec\left(\frac{5\pi}{4}\right)$

Express with sin, cos

$\sec\left(\frac{5\pi}{4}\right) = \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$= \frac{1}{\cos\left(\frac{5\pi}{4}\right)}$

$\cos\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$ *Hide Steps*

$\cos\left(\frac{5\pi}{4}\right)$

Write $\cos\left(\frac{5\pi}{4}\right)$ as $\cos\left(\pi + \frac{\pi}{4}\right)$

$= \cos\left(\pi + \frac{\pi}{4}\right)$

Using the summation identity: $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$= \cos(\pi)\cos\left(\frac{\pi}{4}\right) - \sin(\pi)\sin\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\cos(\pi) = (-1)$

Use the following trivial identity: $\sin(\pi) = 0$

Use the following trivial identity: $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

Use the following trivial identity: $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$= (-1)\frac{\sqrt{2}}{2} - 0 \cdot \frac{\sqrt{2}}{2}$

Simplify

$= -\frac{\sqrt{2}}{2}$

$= \frac{1}{-\frac{\sqrt{2}}{2}}$

$\frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$ *Hide Steps*

$\frac{1}{-\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{a}{-b}=-\frac{a}{b}$

$=-\frac{1}{\frac{\sqrt{2}}{2}}$

Apply the fraction rule: $\frac{1}{\frac{b}{c}}=\frac{c}{b}$

$\frac{1}{\frac{\sqrt{2}}{2}}=\frac{2}{\sqrt{2}}$

$=-\frac{2}{\sqrt{2}}$

Apply radical rule: $\sqrt[n]{a}=a^{\frac{1}{n}}$

$\sqrt{2}=2^{\frac{1}{2}}$

$=\frac{2}{2^{\frac{1}{2}}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{2^1}{2^{\frac{1}{2}}}=2^{1-\frac{1}{2}}$

$=2^{1-\frac{1}{2}}$

Subtract the numbers: $1-\frac{1}{2}=\frac{1}{2}$

$=2^{\frac{1}{2}}$

Apply radical rule: $a^{\frac{1}{n}}=\sqrt[n]{a}$

$2^{\frac{1}{2}}=\sqrt{2}$

$=-\sqrt{2}$

$=-\sqrt{2}$

$=(-\sqrt{2})^4$

Apply exponent rule: $(-a)^n=a^n$, if n is even

$(-\sqrt{2})^4=(\sqrt{2})^4$

$=(\sqrt{2})^4$

Apply radical rule: $\sqrt{a}=a^{\frac{1}{2}}$

$=\left(2^{\frac{1}{2}}\right)^4$

Apply exponent rule: $\left(a^b\right)^c=a^{bc}$

$=2^{\frac{1}{2}\cdot 4}$

$\frac{1}{2}\cdot 4=2$ Hide Steps

$\frac{1}{2}\cdot 4$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 4}{2}$

Multiply the numbers: $1\cdot 4=4$

$=\frac{4}{2}$

Divide the numbers: $\frac{4}{2}=2$

$=2$

$=2^2$

$=2^2\cdot 24\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right)=1$ Hide Steps

$\tan\left(\frac{5\pi}{4}\right)$

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{\pi}{4}\right)$ Hide Steps

$\tan\left(\frac{5\pi}{4}\right)$

Rewrite the angles for $\tan\left(\frac{5\pi}{4}\right)$:

$\tan\left(\frac{5\pi}{4}\right)=\tan\left(\frac{4+1}{4}\pi\right)=\tan\left(\left(\frac{4}{4}+\frac{1}{4}\right)\pi\right)=\tan\left(\pi+\frac{1}{4}\pi\right)$

$=\tan\left(\pi+\frac{1}{4}\pi\right)$

Apply the periodicity of tan : $\tan(x+\pi\cdot k)=\tan(x)$

$\tan\left(\pi+\frac{1}{4}\pi\right)=\tan\left(\frac{1}{4}\pi\right)$

$=\tan\left(\frac{1}{4}\pi\right)$

Multiply $\frac{1}{4}\pi$: $\frac{\pi}{4}$ *Hide Steps*

$\frac{1}{4}\pi$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\pi}{4}$

Multiply: $1\pi=\pi$

$=\frac{\pi}{4}$

$=\tan\left(\frac{\pi}{4}\right)$

$=\tan\left(\frac{\pi}{4}\right)$

Use the following trivial identity: $\tan\left(\frac{\pi}{4}\right)=1$

$=1$

$=2^2\cdot 24\cdot 1$

Multiply the numbers: $24\cdot 1=24$

$=2^2\cdot 24$

$=-16+2^2\cdot 24$

Simplify *Hide Steps*

$-16+24\cdot 2^2$

$24\cdot 2^2=96$ *Hide Steps*

$24\cdot 2^2$

$2^2=4$

$=24\cdot 4$

Multiply the numbers: $24\cdot 4=96$

$=96$

$=-16+96$

Add/Subtract the numbers: $-16+96=80$

$=80$

$=80$

$=80$

$=1+\frac{2}{1!}\left(x-\frac{5\pi}{4}\right)+\frac{4}{2!}\left(x-\frac{5\pi}{4}\right)^2+\frac{16}{3!}\left(x-\frac{5\pi}{4}\right)^3+\frac{80}{4!}\left(x-\frac{5\pi}{4}\right)^4+\dots$

Refine

$=1+2\left(x-\frac{5\pi}{4}\right)+2\left(x-\frac{5\pi}{4}\right)^2+\frac{8}{3}\left(x-\frac{5\pi}{4}\right)^3+\frac{10}{3}\left(x-\frac{5\pi}{4}\right)^4+\dots$