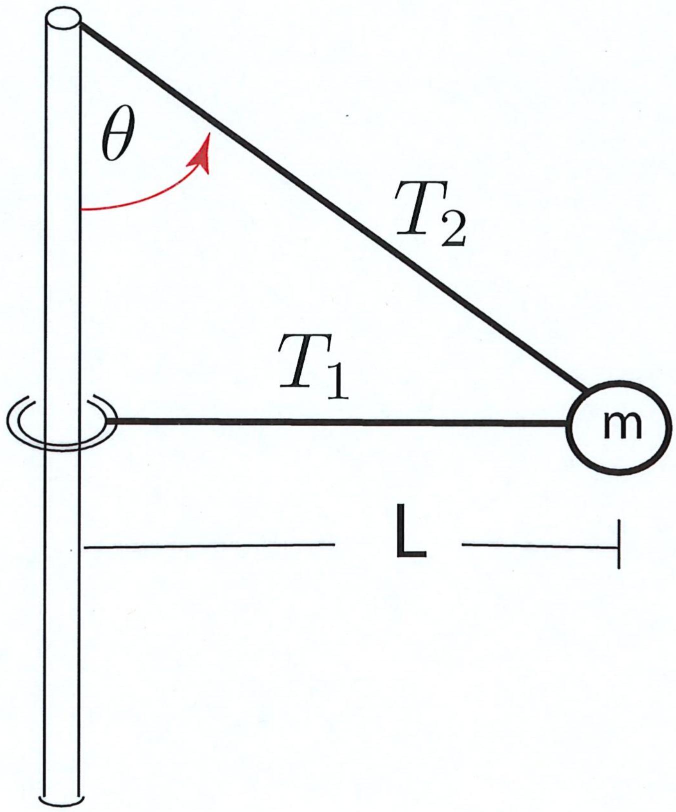
5-6-Example-2nd Law III

Second Law - III

A ball of mass $3kg$ is swinging in a horizontal circle supported by two ropes, as shown in the diagram. Rope 1 is horizontal and under tension T_1, and Rope 2 makes an angle of $\theta = 30°$ with the vertical, as shown.

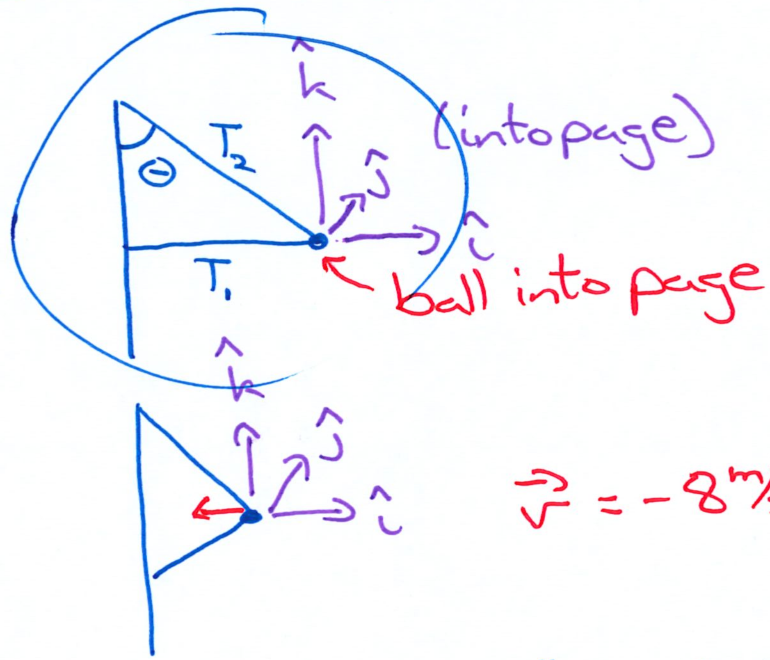


The ball is a distance $L = 2m$ from the pole, and the ball is travelling at a speed of $8\frac{m}{s}$.

- What is T_1?
- What is T_2?

[Can work out $\vec{a}$ → $\vec{a} = \vec{F}_{net}/m$

find forces from two ropes.



$\vec{v} = 8\,m/s\,\hat{j}$

$\vec{v} = -8\,m/s\,\hat{i}$

"Snapshot in time" at that time make correspondence between x, y, z directions & "into center" and "along velocity"

$\hat{v} = \hat{j}$

$\hat{c} = -\hat{i}$

3 forces $\vec{F}_1, \vec{F}_2, \vec{F}_g$

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_g$$

$$= -T_1\hat{i} + (T_2)(\underbrace{\cos 90+\theta}_{-\sin\theta}\,\hat{i} + \cos\theta\,\hat{k})$$

$$-mg\hat{k}$$

$$= (-T_1 - T_2\sin\theta)\hat{i} + (T_2\cos\theta - mg)\hat{k}$$

No $\hat{j}$ component! $\Rightarrow a_y = 0$

$\Rightarrow$ speed not changing!

$$a_x\hat{i} + a_z\hat{k} = \frac{1}{m}\left((-T_1 - T_2\sin\theta)\hat{i} + (T_2\cos\theta - mg)\hat{k}\right)$$

a_z: 0

b/c motion horizontal

z-comp $\quad 0 = \frac{1}{m}(T_2\cos\theta - mg)$

$$T_2 = mg/\cos\theta$$

$$a_x = \frac{1}{m}\left(-T_1 - \left(\frac{mg}{\cos\theta}\right)\sin\theta\right)$$

?

ball going in to circle

component of accel towards center of circle $|\vec{v}|^2/R$

$$+\frac{|\vec{v}|^2}{R} = \frac{1}{m}\left(+T_1 + (mg)\tan\theta\right)$$

$$T_1 = \frac{m|\vec{v}|^2}{R} - mg\tan\theta$$

$$T_2 = mg/\cos\theta$$

$m = 3kg$ $\theta = 30^\circ$

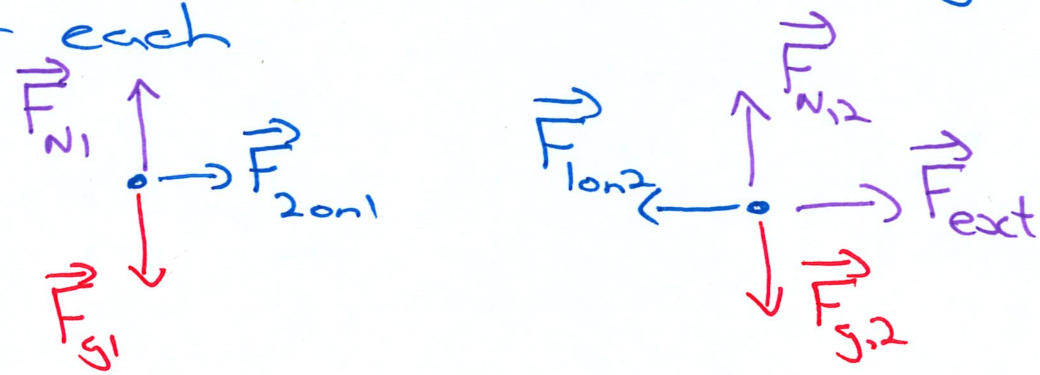
$R = 2m$ $|\vec{v}| = 8 m/s$

$T_1 = 79N$

$T_2 = 33.9N$

Applying Newton's 2nd law to multi-object systems.

Make independent free-body diagrams for each



$$\frac{\vec{F}_{net,1}}{m_1} = \vec{a}_1 \quad ; \quad \frac{\vec{F}_{net,2}}{m_2} = \vec{a}_2$$

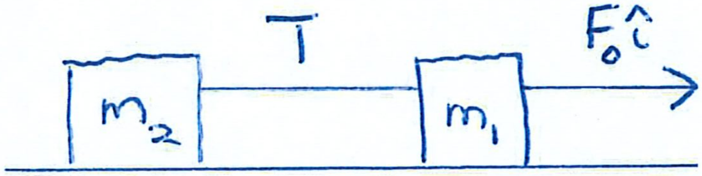
often related

$$\vec{a}_1 = \vec{a}_2$$

Second Law - IV

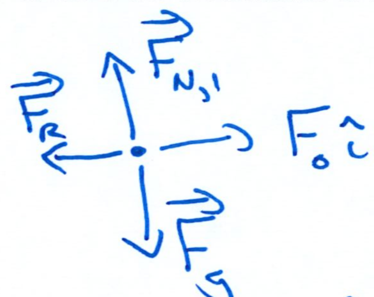
A mass $m_1 = 4kg$ is being pulled horizontally by a rope which exerts a force $\vec{F} = F_0\hat{\imath} = 5N\hat{\imath}$. This mass is attached by an inextensible rope to a second mass $m_2 = 8kg$.

The two masses are on a horizontal frictionless surface.



- What is the tension T in the rope?
- What is the acceleration of m_2?

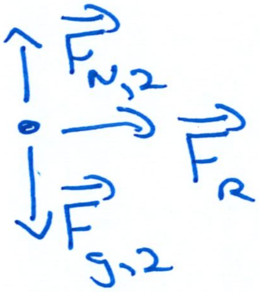
m_1:



$$\vec{F}_{net,1} = -m_1 g\hat{k} + |\vec{F}_{N,1}|\hat{k} + F_0\hat{\imath} - T\hat{\imath}$$

$$\vec{a}_1 = \frac{1}{m_1}\left((F_0 - T)\hat{\imath} + (\cancel{|\vec{F}_{N,1}| - m_1 g})\hat{k}\right)$$

m_2



$$\vec{F}_{net,2} = -m_2 g\hat{k} + |\vec{F}_{N,2}|\hat{k} + T\hat{\imath}$$

$$\vec{a}_2 = \frac{1}{m_2}\left(T\hat{\imath} + \cancel{(|\vec{F}_{N,2}| - m_2 g)\hat{k}}\right)$$

Expect $a_{1z} = 0$ & $a_{2z} = 0$

$$a_{2x} = \frac{1}{m_2}T$$

$$a_{1x} = \frac{1}{m_1}(F_0 - T)$$

$$\boxed{\begin{aligned} a_{1x} &= \frac{1}{m_2}T \\ a_{1x} &= \frac{1}{m_1}(F_0 - T) \end{aligned}}$$

and $a_{1x} = a_{2x}$

$$\frac{1}{m_2}T = \frac{1}{m_1}(F_0 - T)$$

$$\frac{1}{m_2}T + \frac{1}{m_1}T = \frac{F_0}{m_1}$$

$$\frac{m_1 + m_2}{m_1 m_2}T = \frac{F_0}{m_1}$$

$$T = \frac{m_2}{m_1 + m_2}F_0$$

$T = 3.33\text{N}$

$$a_{1x} = \frac{1}{m_2}\left(\frac{m_2}{m_1 + m_2}F_0\right)$$

$$= \frac{1}{(m_1 + m_2)}F_0$$

$$= 0.417\ \text{m/s}^2$$

$$T = m_2 a_{1x}$$

$$a_{1x} = \frac{1}{m_1}\left(F_0 - m_2 a_{1x}\right)$$

$$m_1 a_{1x} + m_2 a_{1x} = F_0$$

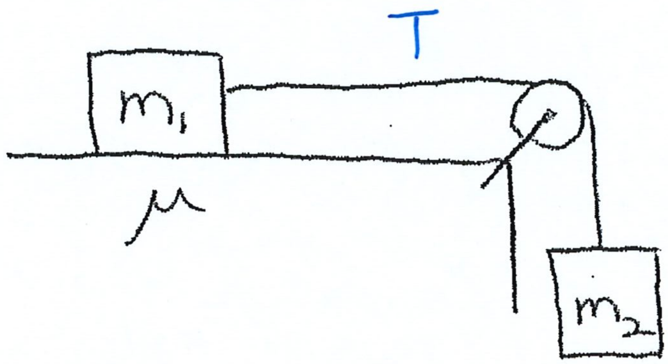
$$a_{1x} = \frac{F_0}{(m_1 + m_2)}$$

$$T = m_2 \frac{F_0}{(m_1 + m_2)}$$

5-10-Example-2ndLawV

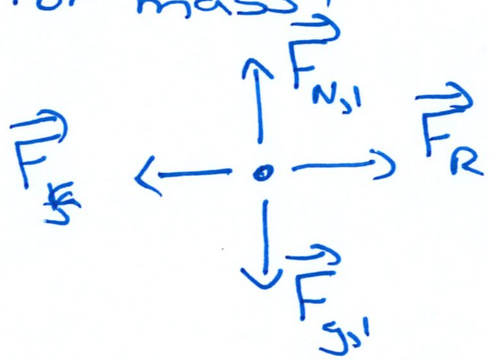
Second Law - V

A mass $m_1 = 30kg$ is moving to the right over a horizontal surface with which it has a coefficient of kinetic friction of $\mu = 0.1$. The mass is connected via a rope to a second mass $m_2 = 10kg$ which is suspended from a massless frictionless pulley.

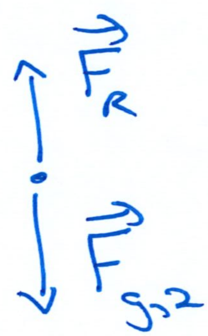


- What is the acceleration of m_1?
- What is the acceleration of m_2?
- What is the tension in the connecting rope?

For mass 1



For mass 2



$$\vec{a}_1 = \frac{1}{m_1}\left(\vec{F}_s + \vec{F}_{N,1} + \vec{F}_{g,1} + \vec{F}_R\right)$$

$$= \frac{1}{m_1}\left(-|\vec{F}_s|\hat{\imath} + |\vec{F}_{N,1}|\hat{k} + (-m_1 g\hat{k}) + T\hat{\imath}\right)$$

$$= \frac{1}{m_1}\left((T - |\vec{F}_s|)\hat{\imath} + (|\vec{F}_{N,1}| - m_1 g)\hat{k}\right)$$

$$a_{1z} = 0 \Rightarrow 0 = \frac{1}{m_1}\left(|\vec{F}_{N,1}| - mg\right)$$

$$|\vec{F}_{N,1}| = m_1 g$$

$$|\vec{F}_s| = \mu|\vec{F}_{N,1}|$$

$$\vec{a}_1 = \frac{1}{m_1}(T - \mu m_1 g)\hat{\imath}$$ only $\vec{a}$ in x

$$\vec{a}_2 = \frac{1}{m_2}\left(\vec{F}_R + \vec{F}_{g,2}\right)$$

$$= \frac{1}{m_2}\left(T\hat{k} - m_2 g\hat{k}\right)$$ only $\vec{a}$ in z

$$|\vec{a}_1| = |\vec{a}_2|$$

$$"a" = \vec{a}_1 \cdot \hat{\imath} = \vec{a}_2 \cdot (-\hat{k})$$

$$a = \frac{1}{m_1}(T - \mu_1 m_1 g)$$

$$a = g - \frac{T}{m_2}$$

Solve for $T = m_2 g - m_2 a$

$$a = \frac{1}{m_1}\left((m_2 g - m_2 a) - \mu_1 m_1 g\right)$$

$$m_1 a + m_2 a = m_2 g - \mu_1 m_1 g$$

$$a = \frac{m_2 g - \mu m_1 g}{(m_1 + m_2)} = 1.715 \, m/s^2$$

$$g - \frac{T}{m_2} = \frac{m_2 g - \mu_1 m_1 g}{m_1 + m_2}$$

$$m_2 g - T = \frac{m_2^2 g - \mu m_1 m_2 g}{m_1 + m_2}$$

$$T = m_2 g - \left(\frac{m_2^2 g - \mu m_1 m_2 g}{m_1 + m_2}\right)$$

$$= \frac{m_1 m_2 g + \mu m_1 m_2 g}{m_1 + m_2} = 80.85 N$$

$\vec{a}_1 = 1.715 \, m/s^2 \, \hat{i}$

$\vec{a}_2 = -1.715 \, m/s^2 \, \hat{k}$

$T = 80.85 \, N$