Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

Use a geometric series to represent each of the following functions as a power series about x = 0. Find the interval of convergence.

a. $f(x) = \frac{5}{9-x}$ **b.** $g(x) = \frac{2}{x-7}$

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a. Since the problem statement specifies that a geometric series should be used, recall the formula for the sum of a geometric series.

$$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \cdots = \frac{a}{1-r}$$

The sum of the geometric series, $\frac{a}{1-r}$, is very similar in form to the given function with r replaced by x. However, the denominator of f(x) is $9-x$ rather than $1-x$.

Let $r = \frac{x}{9}$. Then multiply the numerator and denominator by 9 to get a denominator of $9-x$.

$$\sum_{n=0}^{\infty} a\left(\frac{x}{9}\right)^n = \frac{a}{1-\frac{x}{9}}$$

$$= \frac{9a}{9-x}$$

Let $a = \frac{5}{9}$ so that the right side becomes f(x).

$\frac{9a}{9-x}$ becomes $\frac{5}{9-x}$

Thus, letting $a = \frac{5}{9}$ and $r = \frac{x}{9}$ in the formula for the geometric series $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ gives the following geometric series.

$$\frac{5}{9-x} = \sum_{n=0}^{\infty} \frac{5}{9}\left(\frac{x}{9}\right)^n$$

A power series about x = 0 is a series in the form $\sum_{n=0}^{\infty} c_n x^n$. Write the series in this form.

$$\sum_{n=0}^{\infty} \frac{5}{9}\left(\frac{x}{9}\right)^n = \sum_{n=0}^{\infty} \frac{5}{9^{n+1}} x^n.$$

Thus, the power series representation for $\frac{5}{9-x}$ about x = 0 is $\sum_{n=0}^{\infty} \frac{5}{9^{n+1}} x^n$.

The radius of convergence for the geometric series $\sum_{n=0}^{\infty} ar^n$ is $|r| < 1$.

Express the inequality $|r| < 1$ as a double inequality without using the absolute value symbol.

$$-1 < r < 1$$

Let $r = \frac{x}{9}$ to get $-1 < \frac{x}{9} < 1$. Express this as inequalities for x by multiplying by 9.

$$-9 < x < 9$$

Thus, the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{5}{9^{n+1}} x^n$ is $(-9,9)$.

b. Using the same procedure as in part (a), the right side of $\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ can be expressed as a fraction equivalent to $\frac{2}{x-7}$ by letting $r = \frac{x}{7}$ and $a = -\frac{2}{7}$.

Specifically, when $r = \frac{x}{7}$ and $a = -\frac{2}{7}$, the formula for the sum of the geometric series becomes the following.

$$\sum_{n=0}^{\infty} \left(-\frac{2}{7}\right)\left(\frac{x}{7}\right)^n = \frac{-\frac{2}{7}}{1-\frac{x}{7}} = \frac{2}{x-7}$$

Write this as a power series.

$$\sum_{n=0}^{\infty} \left(-\frac{2}{7}\right)\left(\frac{x}{7}\right)^n = \sum_{n=0}^{\infty} \frac{-2}{7^{n+1}} x^n$$

The radius of convergence for the geometric series $\sum_{n=0}^{\infty} ar^n$ is $|r| < 1$, or $-1 < r < 1$. When $r = \frac{x}{7}$, the corresponding inequalities for x are shown below.

$$-7 < x < 7$$

Thus, the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{-2}{7^{n+1}} x^n$ is $(-7,7)$.