ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 6: Interest Rate Conversions

September 20, 2022

Version 1

Required Reading

- *Always Red the Fine Print* handout
- Investopedia Staff. (2017). APR and APY: Why your Bank Hopes You Can't Tell the Difference. http://www.investopedia.com/articles/basics/04/102904.asp
  - **The article's 'APY' is close to our use of 'annual interest rate'.**

Recommended Reading & Viewing

- *Practice problems for APR and interest rate conversions* handout
- Thompson, A. (2020, May 4). Why is being poor so expensive? [Video File]. Retrieved from https://youtu.be/cMQMSc89Va4
  - **A YouTuber explains why being poor is expensive, drawing on personal experience.**
- Buckland, J., Robinson, C. & Visano, B. S. (2018). *Payday Lending in Canada in a Global Context*. Retrieved from https://link-springer-com.ezproxy.library.uvic.ca/book/10.1007%2F978-3-319-71213-0
- Collins, D., Morduch, J., Rutherford S. & Ruthven, O. (2009). *Portfolios of the Poor: How the World's Poor Live on $2 a Day*. US: Princeton University Press. https://www-degruyter-com.ezproxy.library.uvic.ca/document/doi/10.1515/9781400829965/html
- APRInterest spreadsheet
  - **This automates APR <-> Interest Rate conversions for you.**

Sources

- Alini, E. (2020, January 24). TD to start charging compound interest on personal credit cards [Web Page]. https://globalnews.ca/news/6451352/td-credit-cards-compound-interest/
- CashAdvantage [Web Site]. (n.d.). https://www.cashadvantage.ca/
- Chequing Account NSF Fees. (2019, May 2). https://www.ratehub.ca/chequing-accounts/chequing-account-nsf-fees
- Dieker, N. & Pratchett, T. (2015, March 12). To Terry Pratchett, Who Gave Us Sam Vimes' 'Boots' Theory of Socioeconomic Unfairness [Web Page]. https://medium.com/the-billfold/to-terry-pratchett-who-gave-us-sam-vimes- boots-theory-of-socioeconomic-unfairness-a33858c1c74c
- Government of Canada. (2020, January 31). Payroll penalties and interest [Web Page]. https://www.canada.ca/en/revenue-agency/services/tax/businesses/topics/payroll/penalties-interest-other-consequences/payroll-penalties-interest.html
- Payday Loans [Web Page]. (n.d.). https://www2.gov.bc.ca/gov/content/family-social-supports/borrowing-money/expensive-loans/payday-loans
- Queen's Printer. (n.d.). Part 6.1 – Business Practices and Consumer Protection Act, Chapter 2. Payday Loans [Web Page]. http://www.bclaws.ca/civix/document/id/lc/statreg/04002_08

Relevant Solved Problems

If a problem talks about '10% interest, compounded monthly', they mean the APR is 10% and m = 12

Note:the *Engineering Economics* text calls 'APR' 'nominal interest rates'. The 'effective interest rate' is just the interest rate.

- From Engineering Economics, 6th ed., Chapter 2
- General Intuition: Example 2.1, 2.45
- Compound Interest: Example 2.2, Review Problem 2.1, 2.3, 2.4, 2.26, 2.27(a), 2.28(b), 2.31, 2.34
- Converting between interest periods: Example 2.5, 2.25, 2.34, 2.47
- (Optional) Simple Interest: Example 2.3, 2.1, 2.2, 2.22, 2.27(b), 2.28(a)
- APR vs Interest Rates: Example 2.4, Example 2.5, Example 2.6, 2.23, 2.24, 2.25(b), 2.29(a)(b), 2.30, 2.32, 2.33, 2.35, 2.36 (challenging), 2.46, 2.48 (Challenging),
- APR and Continuous Compounding: Example 2.7, 2.28(c), 2.34 (challenging), 2.37 (challenging)

Notation Dictionary

- i = Interest Rate
- F = Future Value
- APR = the simple interest rate that banks are allowed to advertise (or list), even when charging compound interest. Can be thought of as an 'encrypted' interest rate.
- **(The Engineering Economics text calls the 'APR' 'Nominal Interest', but 'Nominal Interest' is a reserved economic term we'll use a lot when talking about inflation.)**
- APY = another name for annual interest rate (% per year)
- N = Time Index (usually, years from present)
- P = Present Value
- m = compounding periods in one year (e.g. m = 12 if APR is compounded monthly, or 365 if compounded daily in a non-leap year)
- Green Text: Equivalent Microsoft Excel expression

$f(x)$

Formulas

- Notation: The orange symbol on a slide indicates a formula sheet formula is introduced there.
- Future Value: $\mathrm{F} = \mathrm{P}(1+\mathrm{i})^{\mathrm{N}}$
- Present Value: $\mathrm{P} = \mathrm{F}(1+\mathrm{i})^{-\mathrm{N}}$
- Interest from APR: $\mathrm{i} = \frac{APR}{m}$
- This gives you interest per compounding period. e.g. An APR of 12%, compounded monthly, corresponds to an interest rate of 1% per month.
- Continuous Compounding: $\mathrm{i}_{\infty} = \mathrm{e}^{\mathrm{APR}} - 1$
- APR = Textbook's 'Nominal Interest Rate'

Learning Objectives

- Be able to convert an interest rate from one period into another period (e.g. daily to yearly and back).
- Understand how simple interest works.
- Understand what a rollover is, in the context of bank loans.
- Understand the difference between APR and the (compound) interest rate.
- Given APR and a compounding period, be able to calculate the corresponding annual interest rate.

Essentials (19 slides)

The basics of present and future value

- Consider a loan of $P for one year, at an interest rate of i .
- Let F be what you must pay back at the end of that year.

$$F = P + iP = P(1 + i)$$

- P dollars today ↔ F dollars next year

i = interest
P = money I invested / borrowed
iP = money I will get / will save after time

- $P = F/(1 + i)$ is the Present Value of F
- Present Value of future money: How much you'd need to set aside today to have that money in the future.
- $F = P(1 + i)$ is the Future Value of P
- Future Value of present money: If you set aside money today, how much will you have in the future?

Example: Lending money for a year

- Suppose you can lend money at 10% interest per year (i = 0.1)
- If you sacrifice $10 today by lending it for a year, you get $11 in a year.
- →The **future value** of $10 today is $11 a year from now.

- Suppose you want to buy something for $11 a year from now.
- You only need to lend $10 today, and you'll be paid back $11 a year from now. (Maybe you lend to a bank, by depositing the money…)
- The **present value** of $11 a year from now is the $10 you'll have to set aside today.
- What happens if you save or borrow money for more than one year?

Compound interest

- In most (not all) real life cases, each period's interest is added to the principal.
- Next period's interest is calculated over the original amount and all accumulated interest.
- If interest is **constant** at a rate i per time period,
- F = (P + i x P) + i x (P + iP) + … = P x (1 + i) x (1 + i) x …
- → $\mathbf{F = P(1+i)^N}$ = FV(i,N,,-P) → Excel
- Example: Suppose you borrow $100 at 10% for 2 years.
- After 1 year, you owe $100 x (1 + 10%) = $110
- After 2 years, you owe $110 x (1 + 10%) = $121
- $F = \$100 \times (1 + 0.1) \times (1 + 0.1) = \$100(1 + 0.1)^2 = \$121$

N = number of Time period

$f(x)$

Application: Saving up

- You want to save up for a $100,000 crane 10 years from now.
- You can lend (or save) money at 10% yearly interest.
- How much money do you need to put in the bank today?
- →F = 100,000, i = 0.1, N = 10, P = to be determined
- $\mathrm{F} = \mathrm{P}(1+\mathrm{i})^{\mathrm{N}} \rightarrow \mathbf{P = F(1+i)^{-N}}$ = PV(i,N,,-F)
- $\mathrm{P} = 100{,}000 \times (1+0.1)^{-10} = \$38{,}554.33$
- Considerably less than $100,000!

Some perspective

(Source: http://xkcd.com/947/)

P : Principal I : Interest

- $F = P \times (1 + i)^N$
- P = $1,000
- i = 0.02 per year
- N = 10
- → F = $1,000 x $(1.02)^{10}$
- F = $1,218.99

What if you *don't* want % per year?

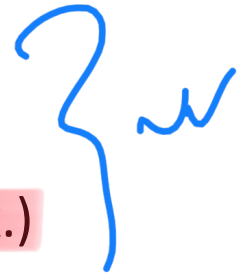
- What if you want your interest rate in % per month, or % per day, instead?
- Before answering that, let's talk about speed.
- Suppose a car travels at a constant speed of 60 km/hour.
- That's the same as traveling at 1 km/minute:
- 60 minutes = 1 hour
- 1 km/minute x 60 minutes = 60 km/hour x 1 hour
- To formally solve for km/minute, we could set equal things equal to each other. Let S be the speed to be solved for:
- (S km/minute) x 60 minutes/hour = (60 km/hour) x 1 hour
- S = 60/60 km/minute = 1 km/minute
- This is the *same speed*, just written in terms of km/minute instead of km/hour.

The same applies to interest rates.

- Math's a bit more complicated due to compound interest (cumulative growth rather than constant speed)…
- …BUT it's the same idea. (Students often get confused about this.)
- Suppose a loan charges 12% interest every 6 months, so that a debt of
- $100 becomes $112.00, 6 months later.
- →Interest rate = 12% per 6 months
- What's the *per month* interest rate?
- It's *not* 2%. You can't just divide 12% by 6.
- Why? Because that ignores the *growth* due to compound interest.
- Interest of 2%/month would mean that after 6 months…
- A $100 debt would become $100 x $(1+2\%)^6$ = $112.62 > $112.00

Let's take it from the top.

- If interest of 12% is charged once every six months…
- At the end of six months, a debt of $100 will become $100 x (1+12%).
- If interest of i% is charged once every month…
- At the end of six months, a debt of $100 becomes $100 x $(1+i)^6$.
- To find the *per month interest rate* that is equivalent to charging 12% interest every 6 months…
- We need to set those two debts equal to each other:
- $\$100 \times (1+12\%) = \$100 \times (1+i)^6$
- $(1 + i)^6 = (1+12\%)$
- $i = (1 + 12\%)^{1/6} - 1 = 1.91\%$ per month (approx.)



Spelling it out: how to convert between time periods

- Let A and B be the two time periods you're dealing with, and the associated interest rates are i_A and i_B.
- Over the course of one 'A' time period, the accumulated interest is:
- **$(1 + i_A)$ in terms of 'per A' interest**
- **$(1 + iB)^{A/B}$ in terms of 'per B' interest**
- **Set them equal to each other and solve.**
- **e.g. $(1+i_{yearly}) = (1+i_{monthly})^{12}$, or $(1+i_{daily}) = (1+i_{weekly})^{1/7}$**
- **That's it! DON'T TRY TO MEMORIZE EQUATIONS FOR THIS!!!**
- **Understand the basic intuition** behind what you're doing, and the result pops out.

Example: Quarterly to 30-Year rates

- You're a planner for a power company making long-term investments.
- Most of your interest rates are quarterly, but you want to know what the 30-year equivalent is (since that's how long you can borrow for).
- Suppose the quarterly rate is 1%.
- There are 4 quarters in a year, and 4 x 30 = 120 quarters in 30 years.
- We want to know: if we borrow at a quarterly rate of 1%, and don't pay the loan back for 30 years, it's as if we were borrowing at what 30-year rate?
- $(1 + i_{quarterly})^{120} = (1 + i_{30years})$
- → $i_{30years} = (1+i_{quarterly})^{120} - 1 = (1+1\%)^{120} - 1 = 2.30 = 230\%$ per 30 years
- Just for fun, let's calculate the per year rate. There are 4 quarters in a year.
- $(1 + i_{quarterly})^{4} = (1 + i_{yearly})$ → $i_{yearly} = (1+1\%)^{4} - 1 = .0406 = 4.06\%$ per year

1 x 30 = 120

What if you DO pay the loan back?

- Suppose you don't let the loan sit for 30 years.
- Instead, each quarter you take out a $100 loan (say) and pay it back the following quarter.
- This way, you avoid paying interest on the interest.
- How much interest, as a % of that $100 loan, would you end up paying?
- Interest is 1% per quarter, and there are 120 quarters.
- All together, you'd pay 120 x 1% = 120% interest on the repeated $100 loan over 30 years.
- High, but much less than 230%.
- Per year, it's 4 x 1% = 4%, instead of 4.6%.
- This kind of interest, where you don't pay interest on interest, is called simple interest. It's very rare…
- ...AND it's the rate banks often quote you on loans.
- Yes, even though you pay compound interest.
- They're allowed to say they're charging you 4% **APR**, when you'll have to pay 4.06% interest per year.

Spelling it out: Simple Interest

- It would be very nice if interest payments worked as follows…
- You want to borrow $P today and the interest rate is i % a year.
- → You pay i% of $P, or iP, per year in interest payments.
- If you take N years to pay off the loan, the total amount you pay is

- **F = P + N x iP = P(1 + iN)**

- This is simple, intuitive… and very rarely happens
- (Notable Exception: **SOME** Canadian credit cards, like RBC's)
- Instead, we'll almost always see *compound interest*…
- ...but this is the rate banks list as an APR ('Annual Percentage Rate')
- (Real-world APRs also include the impact of loan-related fees. For this course, we'll assume most fees are negligible relative to the size of the loan, so can be safely ignored – e.g. a 50-dollar fee for a $1 million loan.)

Word of the day: “rollover”

- BC Business Practices and Consumer Protection Act, Chapter 2, Part 6.1:

> **“rollover”** means any of the following:
>
> (a) the extension or renewal of a payday loan that imposes additional fees or charges on the borrower, other than interest;
>
> (b) The advancement of a new payday loan to pay out an existing payday loan;

- Banks are allowed to pretend that they only offer short-term loans (periods less than a year), and that rollovers are a courtesy. This is the compounding period
- Without rollovers, you never pay interest on interest, which is why the bank is able to advertise the simple rate as **APR**, when it charges the compound rate.
- (If you’re *saving* with them, they’ll advertise the compound rate, since it’s higher!)

What does the ~~Fox~~ Bank say?

- Banks charge compound interest on most loans.
- For historical reasons, they're allowed to advertise simple interest as the APR.
- If a bank charges you 1% interest a month, they're allowed to say they're charging you 12% a year.
- What do you ACTUALLY pay?
- Suppose you borrow $100 for a year…
- $\$100\times(1+1\%)^{12}=\$100\times1.01^{12}=\$112.68$
- Since you pay $12.68 interest on $100…
- The interest rate is 12.68% per year (or 1% per month).
- For an APR of 12%, the interest rate could be even higher if a bank charges interest weekly, daily or even continuously.

The APR is an *encrypted* interest rate

- You can think of the APR as an encrypted interest rate.
- The decryption key is the number of compounding periods per year.
- Call this ‘m’.
- **Interest rate per compounding period = APR/m**
- Although the APR is allegedly in yearly terms (the ‘A’ stands for ‘Annual’), it doesn’t always decrypt into a ‘per year’ interest rate.
- Instead, it gives you the interest rate *per compounding period*.
- You can then turn that into yearly, per century, daily, etc. in the usual way, if you need to.

Converting from APR to interest rates

- When you see an APR listed, you should also look for the *compounding period*. (e.g. 'Compounded yearly', or 'compounded monthly'.)
- **APR = Interest rate per period x periods per year**
- → Interest rate = APR/(periods per year) per period.
- APR of 36%, compounded monthly? 12 months/year
- →Interest = (36%/12) per month = **3% per month**
- APR of 36%, compounded half-yearly? 2 half-years in a year
- →Interest = (36%/2) per half-year = **18% per half-year**
- The APR + compounding period gives you ONE interest rate almost automatically: interest per compounding period.
- If you want interest in terms of other time periods (e.g. per year), you know how to do that.

APRs can include more than just interest considerations (fees, etc.) but for this course we'll keep things simple.

Greater than 25 Year Amortization

Fixed Mortgage Rates[1]

Term	Special Offers	APR[2]
Closed		
1 Year	3.090%	3.190%
5 Year	3.390%	3.410%
7 Year	3.690%	3.710%

1. Interest rate compounded half-yearly, not in advance. Interest rates are subject to change without notice at any time.

2. The annual percentage rate (APR) is based on a $ 250,000 mortgage for the applicable term assuming a processing fee of $250 (which includes fees associated with determining the value of the property). If there are no cost of borrowing charges, the APR and the interest rate will be the same.

3. Interest rate is compounded monthly, not in advance. This rate may change at any time without notice. Royal Bank of Canada prime rate is an annual variable rate of interest announced by Royal Bank of Canada from time to time as its prime rate.

Variable Mortgage Rates[3]

Retrieved from http://www.rbcroyalbank.com/mortgages/mortgage-rates.html
September 10, 2017

Term	Special Offers	APR[2]
Closed		
5 Year	RBC Prime Rate + 0.000%	3.220%

Slightly Simplified Canadian Examples

- Toronto Dominion Credit Cards: Compounded monthly
- RBC & CIBC Credit Cards: NOT COMPOUNDED (!)
- These banks don't charge interest on unpaid credit card interest.
- (This could change in the future.)
- Fixed Rate Mortgages: Compounded half-yearly (by law)
- RBC Variable Rate Mortgages: Compounded monthly
- Money owed to the CRA: Compounded daily
- Slightly more complicated than this: e.g. TD cards calculate interest on a daily basis but only compound monthly. See required reading for full details.

Suppose each of these advertises a 24% APR

- RBC Credit Cards: Not compounded, so interest is **24% per year**
- TD Credit Cards & RBC Variable Rate Mortgages: compounded monthly
- → Interest = (24/12)% per month = *2% per month*.
- $(1 + 2\%)^{12} = (1 + i_{yearly})$ → $i_{yearly} = 1.02^{12} - 1 =$ **26.82% per year**
- Fixed Rate Mortgages: compounded half-yearly
- → Interest = (24/2) % per 6 months = *12% per 6 months*
- $(1 + 12\%)^{2} = (1 + i_{yearly})$ → $i_{yearly} = 1.12^{2} - 1 =$ **25.44% per year**
- Debt to CRA: compounded daily; assume non-leap year
- →Interest = (24/365) % per day = *0.0658% per day (approx.)*
- $(1 + (24/365)\%)^{365} = (1 + i_{yearly})$ → $i_{yearly} = (1 + (24/365)\%)^{365} - 1 =$ **27.11% per year**

After Hours (Optional)

- Payday Loans (8 slides)

How much does a payday loan actually cost?

(CashAdvantage)

- Payday loans are short-term loans due ‘on your next payday’.
- Due to competition and BC laws, almost all providers offer first-time borrowers $300 for a fixed fee of $45, payable on payday.
- This is advertised as ’15% interest’… but is it?
- By BC law, employees must be paid at least twice a month.
- → Maximum loan period is 2 weeks.

Wage regulations: https://www.labour.gov.bc.ca/esb/facshts/paying_wages.htm

What's the equivalent annual rate?

- It's true that 15% x 300 = $45, but this is a 2-week (14 day) loan.
- Assume a non-leap year.
- There are 365/14 two-week periods in one year.
- $(1 + 15\%)^{365/14} = (1 + i_{yearly}) \rightarrow i_{yearly} = 1.15^{365/14} - 1 =$ **3,724% per year**
- The B.C. government disagrees…
- "A payday loan with a cost of $15 per $100 borrowed […] is the same as an annual interest rate of 391% if you get a paycheque every two weeks." (Payday Loans)
- Ignoring compound interest, (15 % x 365/14) = 391.07% per year

Why can we ignore compound interest?

- B.C. law: “A payday lender in British Columbia, or one that provides loans in British Columbia even if located elsewhere, is not allowed to: Give you more than one loan at a time [or] Roll over an existing loan into a new loan with new charges.” (Payday Loans)
- So you CAN’T automatically take out a new loan after 2 weeks for the amount owed.
- There’s also a maximum payday loan amount of $1,500 or 50% of your paycheque, whichever is smaller.
- And they can charge at most 30%/year on unpaid principal.

A modified one-year loan

- Suppose you borrow $300 from a payday lender, and don't pay for a year: what's the charge?
- $45 fee + 30% x $300 = $45 + $90 = $135 interest paid on $300
- $135/$300 = **45% per year**
- Doesn't look much better if you assume the 30% is only paid on one year minus the initial two week loan period (=365 – 14 days).
- $(1+30\%) = (1 + i_{daily})^{365} \rightarrow i_{daily} = 1.3^{1/365} - 1 = 0.0719\%$/day (approx.)
- $\$300 \times (1 + 0.0719\%)^{(365-14)} = \$386.09$
- →$45 + $86.09 = $131.09 interest paid on $300

But wait, there's more!

- "Your bank or credit union will most likely also charge you a Non-Sufficient Funds (NSF) fee or an overdraft fee […]if there isn't enough money in your account when the payday lender tries to withdraw what you owe." (Payday Loans)
- According to ratehub.ca, NSF fees are between $25 and $48.
- Best case: $25 + $131.09 = $156.09 interest on $300: 52.03 %/year
- Worst case: $48 + $135 = $183 interest on $300: 61% per year
- **It gets worse**: The above ignores interest on late payment of the NSF fee.
- If you're living paycheque to paycheque and have bad credit, you'll get charged **MUCH** higher rates than wealthy people.
- And if you fail to pay this loan? NSF & sent to collections → lower credit score → lower ability to get reasonable loans.

Being Poor is *expensive*!

- The ‘Boots’ theory of Socioeconomic Unfairness (Terry Pratchett):
- Suppose someone earns $38 a month.
- Cheap boots cost $10, & last 1 year
- Good boots cost $50, & last 10 years
- The person earning $38/month will buy the cheap boots:
- Over 10 years, spent $10 x 10 = $100 on boots
- A rich person will buy the good boots:
- Over 10 years, spent $50 x 1 = $50 on boots
- **The intuition is similar to the payday loan situation.**

Why aren't payday loans illegal?

- In B.C., they're legal, but heavily regulated.
- What happens if you BAN them?
- Lots of people with bad credit scores STILL need loans…
- …but (almost?) no one is willing to officially lend to them.
- Demand for unofficial loans will either create its own supply (shadow economy of moneylenders, poor people forming saving rings)…
- …OR it won't.
- In one case, lack of official status may lead to unpleasant ways of enforcing repayment; in the other, lack of loans may lead to hardship.
- **Think we could do better? You're not alone.**

Interested in learning more?

- Buckland, J., Robinson, C. & Visano, B. S. (2018). *Payday Lending in Canada in a Global Context*. https://link-springer-com.ezproxy.library.uvic.ca/book/10.1007%2F978-3-319-71213-0
- The whole book is ***free to download*** to UVic students (PDF, ePUB).
- A good introductory source on **how the world's poor save** is
- Collins, D., Morduch, J., Rutherford S. & Ruthven, O. (2009). *Portfolios of the Poor: How the World's Poor Live on $2 a Day*. US: Princeton University Press. https://www-degruyter-com.ezproxy.library.uvic.ca/document/doi/10.1515/9781400829965/html