Student: Arfaz Hossain **Date:** 04/20/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** Practice Questions for Sections 11.4 & 11.5 [Not f

Find the slope of the curve below at the given points. Sketch the curve along with its tangents at these points.

$$r = \cos 2\theta; \qquad \theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

The slope of a curve $r = f(\theta)$ is given by the formula below.

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

To apply the formula, first determine the derivative of $f(\theta)$.

$$\begin{aligned} f'(\theta) &= (\cos 2\theta)' \\ &= -2\sin 2\theta \end{aligned}$$

Substitute $f(\theta)$ and $f'(\theta)$ into the equation.

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{(r,\theta)} &= \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} \\ &= \frac{(-2\sin 2\theta)\sin\theta + (\cos 2\theta)\cos\theta}{(-2\sin 2\theta)\cos\theta - (\cos 2\theta)\sin\theta} \end{aligned}$$

To find slope when $\theta = 0$, substitute 0 for θ in the equation.

$$\begin{aligned} \frac{(-2\sin 2\theta)\sin\theta + (\cos 2\theta)\cos\theta}{(-2\sin 2\theta)\cos\theta - (\cos 2\theta)\sin\theta} &= \frac{-2\sin 2(0)\sin 0 + \cos 2(0)\cos 0}{-2\sin 2(0)\cos 0 - \cos 2(0)\sin 0} \\ &= \frac{1}{0} \end{aligned}$$

Therefore, the slope is undefined when $\theta = 0$.

To find slope when $\theta = \frac{\pi}{2}$, substitute $\frac{\pi}{2}$ for θ in the equation.

$$\begin{aligned} \frac{(-2\sin 2\theta)\sin\theta + (\cos 2\theta)\cos\theta}{(-2\sin 2\theta)\cos\theta - (\cos 2\theta)\sin\theta} &= \frac{-2\sin 2\left(\frac{\pi}{2}\right)\sin\frac{\pi}{2} + \cos 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2}}{-2\sin 2\left(\frac{\pi}{2}\right)\cos\frac{\pi}{2} - \cos 2\left(\frac{\pi}{2}\right)\sin\frac{\pi}{2}} \\ &= \frac{0}{1} \end{aligned}$$

Therefore, the slope is 0 when $\theta = \frac{\pi}{2}$.

To find slope when $\theta = \pi$, substitute π for θ in the equation.

$$\begin{aligned} \frac{(-2\sin 2\theta)\sin\theta + (\cos 2\theta)\cos\theta}{(-2\sin 2\theta)\cos\theta - (\cos 2\theta)\sin\theta} &= \frac{-2\sin 2(\pi)\sin\pi + \cos 2(\pi)\cos\pi}{-2\sin 2(\pi)\cos\pi - \cos 2(\pi)\sin\pi} \\ &= \frac{-1}{0} \end{aligned}$$

Therefore, the slope is undefined when $\theta = \pi$.

To find slope when $\theta = \frac{3\pi}{2}$, substitute $\frac{3\pi}{2}$ for θ in the equation.

$$\frac{(-2\,\mathbf{sin}\,2\theta)\,\mathbf{sin}\,\theta + (\mathbf{cos}\,2\theta)\,\mathbf{cos}\,\theta}{(-2\,\mathbf{sin}\,2\theta)\,\mathbf{cos}\,\theta - (\mathbf{cos}\,2\theta)\,\mathbf{sin}\,\theta} = \frac{-2\,\mathbf{sin}\,2\left(\frac{3\pi}{2}\right)\mathbf{sin}\,\frac{3\pi}{2} + \mathbf{cos}\,2\left(\frac{3\pi}{2}\right)\mathbf{cos}\,\frac{3\pi}{2}}{-2\,\mathbf{sin}\,2\left(\frac{3\pi}{2}\right)\mathbf{cos}\,\frac{3\pi}{2} - \mathbf{cos}\,2\left(\frac{3\pi}{2}\right)\mathbf{sin}\,\frac{3\pi}{2}}$$

$$= \frac{0}{-1}$$

Therefore, the slope is 0 when $\theta = \frac{3\pi}{2}$.

When a graph has symmetry about the x-axis, if the point (r,θ) lies on the graph, then the point $(r,-\theta)$ or $(-r,\pi-\theta)$ lies on the graph. When a graph has symmetry about the y-axis, if the point (r,θ) lies on the graph, then the point $(r,\pi-\theta)$ or $(-r,-\theta)$ lies on the graph. When a graph has symmetry about the origin, if the point (r,θ) lies on the graph, then the point $(-r,\theta)$ or $(r,\theta+\pi)$ lies on the graph.

Note that this curve has symmetry about the x-axis, y-axis, and origin. To graph the curve, make a short table of values, plot the corresponding points, and use information about symmetry to connect the points with a smooth curve. The calculations are shown rounded to two decimal places as needed.

θ	$r = \cos 2\theta$
0	1
$\frac{\pi}{8}$	0.71
$\frac{\pi}{6}$	0.5

Continue the table. The calculations are shown rounded to two decimal places as needed.

θ	$r = \cos 2\theta$
$\frac{\pi}{4}$	0
$\frac{\pi}{3}$	-0.5
$\frac{3\pi}{8}$	-0.71
$\frac{\pi}{2}$	-1

Recall that the graph is symmetric about the origin. Therefore, the correct graph of the curve is shown below.