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Evaluate the integral using any appropriate algebraic method or trigonometric identity.

$$\int \frac{dx}{9e^{-2x} + e^{2x}}$$

To transform $\int \frac{dx}{9e^{-2x} + e^{2x}}$ to a basic form, multiply the integrand by $\frac{e^{2x}}{e^{2x}}$.

$$\frac{e^{2x}}{e^{2x}} \cdot \frac{dx}{9e^{-2x} + e^{2x}} = \frac{e^{2x}dx}{9 + \left(e^{2x}\right)^2}$$

Substitute this last expression into the original integral. Multiplying the integrand by 2 and the integral by $\frac{1}{2}$ and replacing 9 with 3^2 gives $\frac{1}{2}\int \frac{2e^{2x}dx}{3^2 + \left(e^{2x}\right)^2}$.

With $u = e^{2x}$ and $du = 2e^{2x}dx$, this transforms into $\frac{1}{2}\int \frac{du}{3^2 + u^2}$.

Notice that this has an integral of the form $\int \frac{du}{a^2 + u^2}$.

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$$

With $a = 3$ and $u = e^{2x}$, $\int \frac{dx}{9e^{-2x} + e^{2x}} = \frac{1}{6}\tan^{-1}\left(\frac{e^{2x}}{3}\right) + C$.