ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 7: Discounted Cash Flow Analysis I

Present, Future and Annual Worth

September 21, 2021

Version 1

Strongly Recommended Reading

- Solved problems on Present Values, Future Values & Annuities (handout) – It’s fine to skim this one.

(Optional) Case Study

- Hasan, M. et al. (2016). Discounted cash flow analysis of greenhouse solar kilns. *Renewable Energy*, pp. 404-412. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.renene.2016.04.050

Recommended Reading & Viewing

- *Engineering Economics*, 6th edition, 2.6, 3.1-3.5, 3.9
- *Engineering Economics,* 7th edition, 3.1 – 3.4
- Engineering Economics Guy. (2020, October 24). Present Worth and Annual Worth Explained Engineering Economics Live Class Recording [Video File]. https://youtu.be/WYbC1-TsGis
- Tayari, F. (2018). Lesson 1: Investment Decision Making and Compound Interest [Web Page]. Retrieved from https://www.e-education.psu.edu/eme460/node/3
  - **Long and in depth, but makes a good reference. Their notation is slightly different. They write $P/F_{i,N}$ where we use (P/F,i,N), etc.**
- Watts, J. M. Jr. & Chapman, R. E. (2002). Engineering Economics. In Dinenno, P. J., Dysdale, D., Beyler, C. L. & Walton, W. D. (eds.) *SPFE Handbook of Fire Protection Engineering* (3rd edition), 93-104. Retrieved from https://web.archive.org/web/20041107145259/http://www.fire.nist.gov/bfrlpubs/build02/PDF/b02155.pdf
  - **A very short overview of discounted cash flow analysis.** Ignore the short section on Nominal versus effective interest on the top right of page 5-95. **It is misleading and confusing to the point of being borderline incorrect.**
  - (What they call a 'nominal interest rate' isn't an interest rate at all – it's an APR – and what they call the 'effective interest rate' is specifically an interest in per year terms. What's worse, 'nominal interest rate' is an extremely important reserved term in economics, which we'll be using a LOT later in the course: it refers to an interest rate that has not been adjusted for inflation.)

Optional Reading

- Remer, D. S., Tu, J. C., Carson, D. E. & Ganiy, S. A. (1984). The state of the art of present worth analysis of cash flow distributions. *Engineering Costs and Production Economics, 7*(4), 257-278. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1016/0167-188X(84)90044-2
  - **Mostly For ELEC students. Relates Cash flow analysis to Zeta transforms, and derives certain factors.**
- Conn, R. R. (2013). Capitalized Earnings vs. Discounted Cash Flow: Which is the more accurate business valuation tool? Retrieved from http://www.connvaluation.com/caseStudies/Capitalization_vs_DCF.pdf
  - **A business valuator's perspective on the usefulness of the P = A/i approximation.**

Historical Origins of DCFA

- Brackenborough, S., McLean, T. & Oldroyd, D. (2001). The Emergence of Discounted Cash Flow Analysis in the Tyneside Coal Industry c.1700-1820. *The British Accounting Review*, *33*(2), 137-155. https://doi-org.ezproxy.library.uvic.ca/10.1006/bare.2001.0158
  - **An in-depth examination of why British collieries suddenly switched to this kind of analysis around 1801.**
- Edwards, J.R. & Warman, A. (1981). Discounted Cash Flow and Business Valuation in a Nineteenth Century Merger: A Note. *The Accounting Historians Journal*, *8*(2), 37-50. https://www-jstor-org.ezproxy.library.uvic.ca/stable/40697685
  - **A much easier read than Brackenborough et al. (in my opinion), this is an analysis of one early instance of the use of discounted cash flow analysis.**
- Parker, R. H. (1968). Discounted Cash Flow in Historical Perspective. *Journal of Accounting Research*, *6*(1), 58–71. https://doi-org.ezproxy.library.uvic.ca/10.2307/2490123
  - **A good overview of the history of discounted cash flow analysis, which emphasizes the role of engineering economics.**
- Ward, J. (1724). CHAP. XII. Of Compound Interest, and Annuities, &c. In *The Young Mathematician's Guide*. London: A Bettesworth & F. Fayrham, pp.253– 282. https://archive.org/details/youngmathematic02wardgoog/page/n269/mode/2up
  - **An early explanation of present value, future value and annuities, now difficult to read because of its use of pounds, shillings and pence instead of decimal currency.**

Learning Objectives

- Become familiar with basic cash flow elements (Present Value, Future Value, Annuity) and be able to convert between them at will using appropriate conversion factors.
- Become familiar with the notation for conversion factors.

Relevant Solved Problems

- From *Engineering Economics*, 6th edition:
- Elements in general: Case in Point 3.1, 3.1, 3.2, 3.44, 3.45
- Future Value: Example 3.1, 3.3, 3.4, 3.5, 3.18, 3.19, 3.21, 3.23
- Annuities: Example 3.2, Example 3.3, Example 3.4, Close-Up 3.2, Close-Up 3.4, 3.6, 3.7, 3.8, 3.9.a., 3.13, 3.14, 3.15, 3.16, 3.17, 3.24, 3.25, 3.26, 3.27, 3.29, 3.31, 3.33, 3.42, 3.43, 3.47, 3.52

More solved problems

- *Engineering Economics: The Basics*, by Stuart Nielsen, is an inexpensive Kindle text with a large number of solved problems, written by a practicing bridge engineer.
- There are two editions, and I think the extra $2 (for a total price of $4.04) for the expanded second edition is worth it:
- https://www.amazon.ca/Engineering-Economics-Basics-Stuart-Nielsen-ebook/dp/B01N49V8RA/
- The only 'minus' is that in the solutions, Nielsen uses the expanded formulas, instead of the standard (X/Y,i,N) notation, which you ARE expected to use in your own solutions. (It makes following your reasoning much easier, and if you use Excel you should seldom need to make use of the expanded formulas.)
- **The relevant chapters for this lecture are Chapters 4 and 5. All of those problems are useful. You may also want to read Chapter 3, on cash flow diagrams.**
- **Also relevant, from Chapter 11: Examples 11-1, 11-2, 11-6, 11-7, 11-12, 11-13**

Notation Dictionary

(Not provided on quiz/final formula sheet)

- A = Annuity
- F = Future Value
- i = interest rate
- N = the N’th time period
- P = Present Value
- S = Salvage
- Green Text = Excel Formula

- Conversion factors are of the form (X/Y,z)
- Read as: X, given Y and z.
- X is the element we want.
- Y is the element we have.
- z represents additional parameters.
- e.g. (P/F,i,N)
- Present Value, given a Future Value at time N and interest rate i.

$f(x)$

Useful Equations

- Notation: The orange symbol on a slide indicates a formula sheet formula is introduced there.
- (F/P,i,N) = $(1+i)^N$
- (P/F,i,N) = $(1+i)^{-N}$
- (A/F,i,N) = $\frac{i}{(1+i)^N - 1}$
- (F/A,i,N) = $\frac{[(1+i)^N - 1]}{i}$
- (A/P,i,N) = $\frac{i(1+i)^N}{[(1+i)^N - 1]}$
- (P/A,i,N) = $\frac{(1+i)^N - 1}{[i(1+i)^N]}$
- Capitalized Value = A/i

The main Excel Functions for DCFA

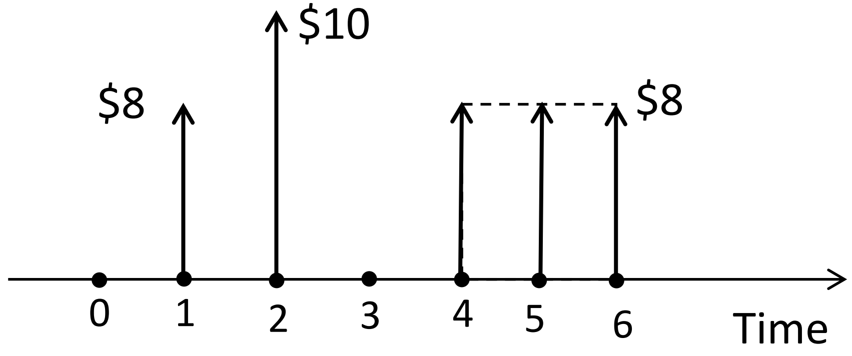
- See the full lecture on how to use Excel for DCFA in Basic Information → Excel Help
- DCFA = Discounted Cash Flow Analysis
- F x (P/F,i,N) =PV(i,N,,-F)
- P x (F/P,i,N) =FV(i,N,,-P)
- F x (A/F,i,N) =PMT(i,N,,-F)
- A x (F/A,i,N) =FV(i,N,-A)
- P x (A/P,i,N) =PMT(i,N,-A)
- A x (P/A,i,N) =PV(i,N,-A)
- The negative signs are there because Excel assumes you're talking about negative numbers (costs and payments).
- Adding your own negative sign cancels that out.

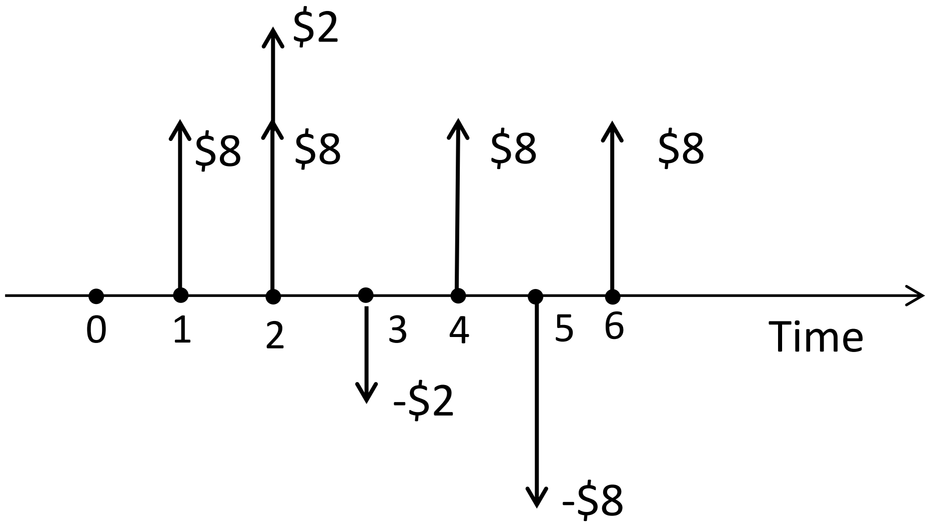
Essentials

Turning oranges and apples into fruit

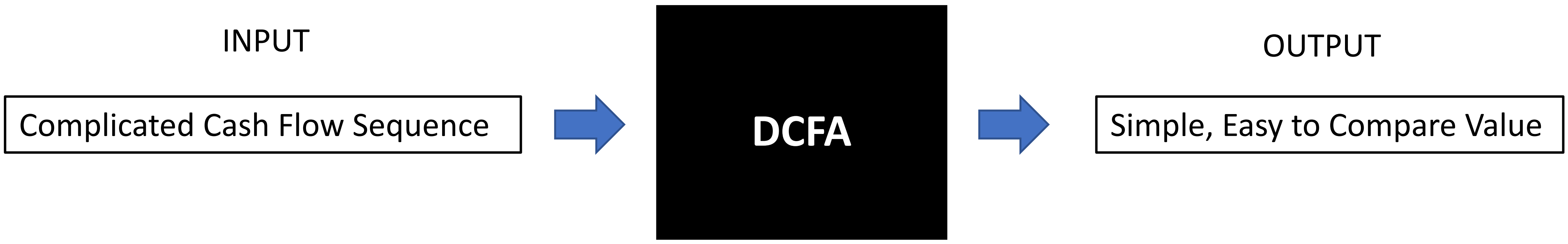
- Engineering projects have complicated cash flow profiles.
- Looking at costs alone… some may have one big cost at the start, the same costs every year, costs that rise by a certain % each year, or a big cost at the end of the project.
- Comparing two projects is like comparing apples and oranges.
- To compare different projects, we need a way of bringing them onto common ground: to turn apples and oranges into fruit.
- Thankfully, we can do that: today we'll look at *conversion factors* (traditionally called ‚discount factors‘) that will let us translate from, say, a yearly payment over 10 years to its equivalent lump-sum payment today.

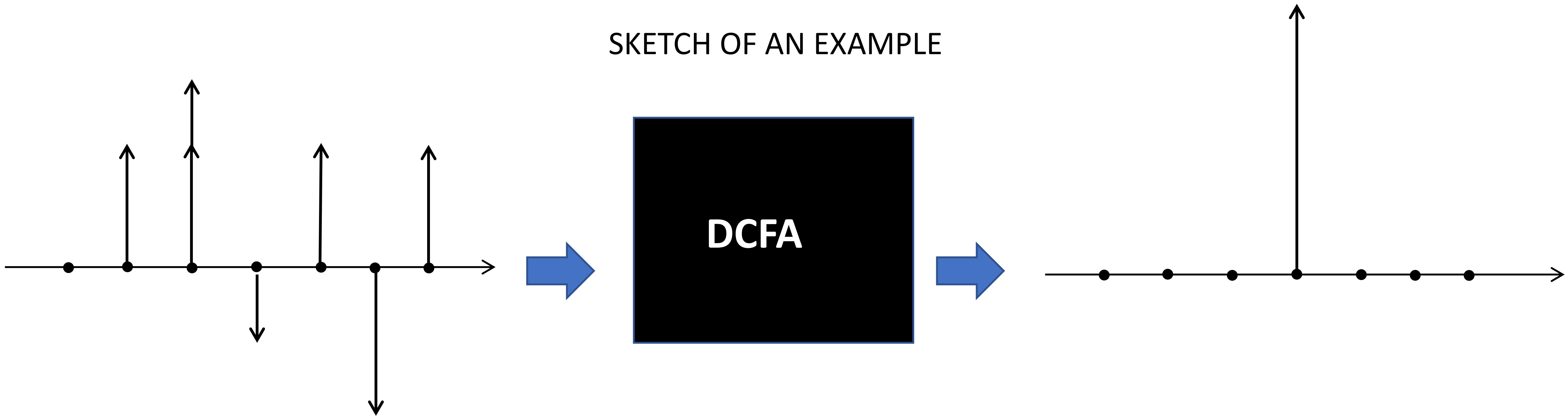
How cash flow diagrams work





- Time's arrow: Time flows left to right.
- Typically, tick-marks between time periods.
- Arrow pointing UP: Cash coming IN to the project. (e.g. getting paid)
- Arrow pointing DOWN: Cash flowing OUT of the project (e.g. buy stuff)
- Often, height of arrow (roughly) proportional to magnitude of cash flow.
- You can STACK arrows to represent multiple cash flows.

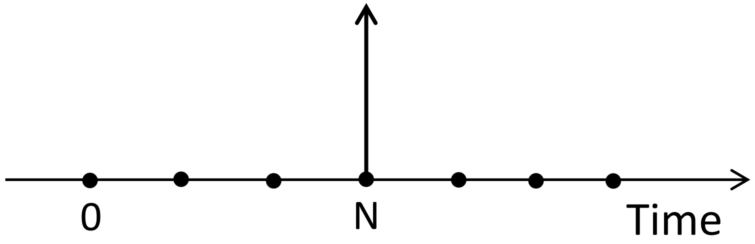




It may help to think of Discounted Cash Flow Analysis (DCFA) as a “black box” that turns complicated cash flows into more convenient equivalents. You’re being trained to understand how the black box works.

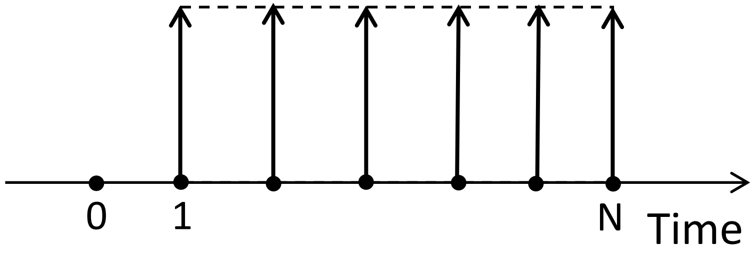
The ‘black box’ uses the discount factors like a library of subroutines. There is seldom a unique way to achieve the required transformation. The fewer discount factors used, the more efficient the black box, in general.

Four basic cash flow elements



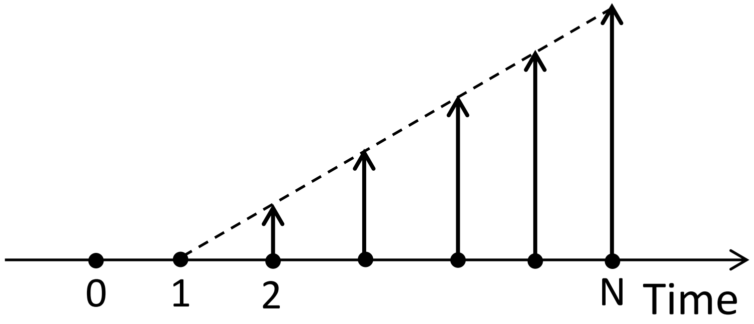
Impulse (Future Value, F)

One Cash Flow in Time N



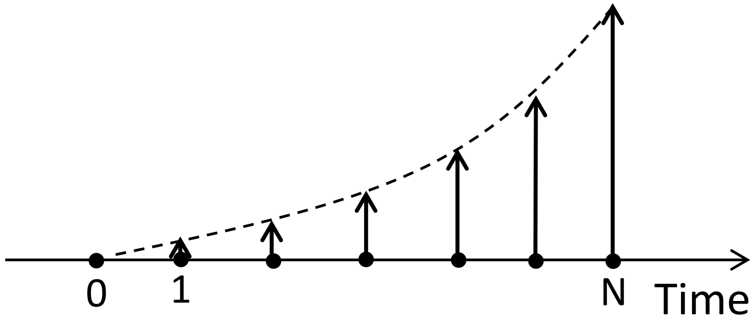
Step (Annuity, A)

A sequence of N equal payments



Ramp (Arithmetic Gradient, G)

A sequence 0,G,2G,3G…



Geometric Gradient Series (Growth/Decay)

A sequence A,(1+g)A,$(1+g)^2$A,…

What we're doing today

- We're learning to convert between those four and the fifth element, present value.
- We'll see a LOT of algebra… but don't sweat the equations.
- The important thing is to understand what those equations *mean*, so you can reduce any cash flow into its basic elements and turn them into whatever you require.

(Not THAT fifth element.)
(Oddly, the tag line still applies.)

Notation

- Conversion factors are written in the form (X/Y,i,N)
- This can be read as ‘X, given Y, i and N’.
- X and Y are two different elements (e.g. F and A), i is the interest rate and N is as shown in the element diagrams.
- If you want X, and have Y, i and N, then the Y that is equivalent to X is given by Y x (X/Y,i,N).
- If you need a conversion factor that’s not given to you, you can treat the notation as if they really were fractions.
- e.g. X/Z = X/Y x Y/Z, and (X/Z,i,N) = (X/Y,i,N) x (Y/Z,i,N)

$f(x)$

Present worth factor: (P/F,i,N)

- Converts F to P: P = F x (P/F,i,N) = PV(i,N,,-F)
- Sample use: present worth of salvage for B/C calculations

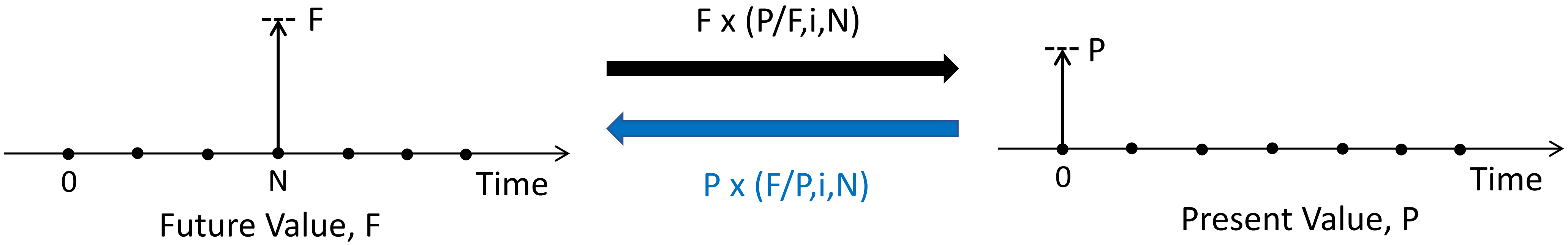
$$(P/F\,,i,N) = \frac{1}{(1+i)^N}$$

- Inverse function (P to F): F = P x (F/P,i,N) = FV(i,N,,-P)

Multiplying by (P/F,i,N) moves the cash flow *backward* by N time periods.

$$(\mathrm{F/P\,,i,N}) = (1+\mathrm{i})^{\mathrm{N}}$$

Multiplying by (F/P,i,N) moves the cash flow *forward* by N time periods.



F x (P/F,i,N) in plain English

Future X (Present to Future)

- Suppose that the relevant discount rate is i.
- You have a cash flow of F and you want it take it *back in time* by N time periods. So you multiply F x (P/F,i,N).
- By ‘back in time’, I mean you’re finding the money you would have had to set aside N time periods before F to have F.
- It’s CALLED the present value, but you don’t need to be bringing the cash flow back to ***the present***.
- Say i = 10% per year, you’re staring at a cash flow of $121 in 1970 and you want to find the equivalent 1968 value.
- You’re taking it back TWO years, so N = 2, and i=10%, and N F = $121.
- So, given i=10%, $121 in 1970 is equal to $121 x (P/F,10%,2) = $100 in 1968.

What about P x (F/P,i,N)?

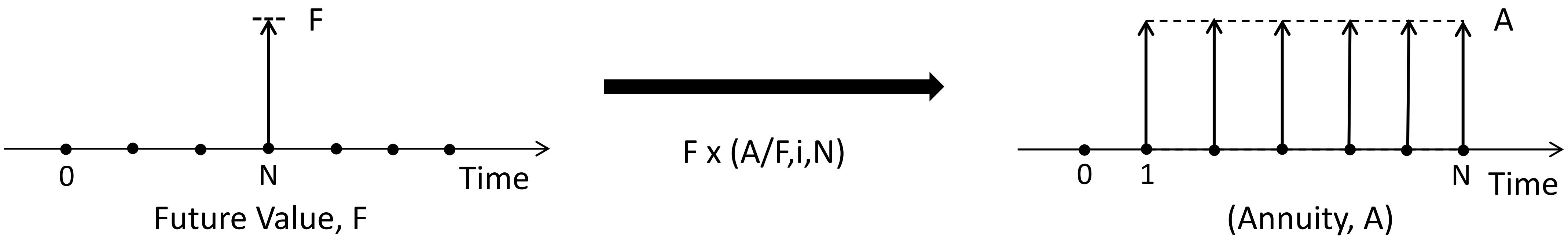
- You can use F x (P/F,i,N) to bring a cash flow F *left* on the timeline by N time periods (‘back in time’ N time periods).
- Similarly, you can use P x (F/i,N) to bring a cash flow P *right* on the timeline by two periods.
- So if i=10% per year, a cash flow of $100 in 1968…
- …is equivalent to $100 x (F/P,10%,2) = $121 in 1970, 2 time periods later. (Save $100 in 1968 at 10% per year, and you’ll have $121 in 1970).

Sinking Fund Factor (A/F,i,N)

$f(x)$

- Converts F to A: A = F x (A/F,i,N) = PMT(i,N,,-F)
- Answers the question: how much do I have to save each period if I want to have F dollars by period N?
- A sinking fund is an interest-bearing account you *sink* money into on a regular basis in order to accumulate a fixed known amount.

$$(A/F,i,N) = \frac{i}{(1+i)^N - 1}$$

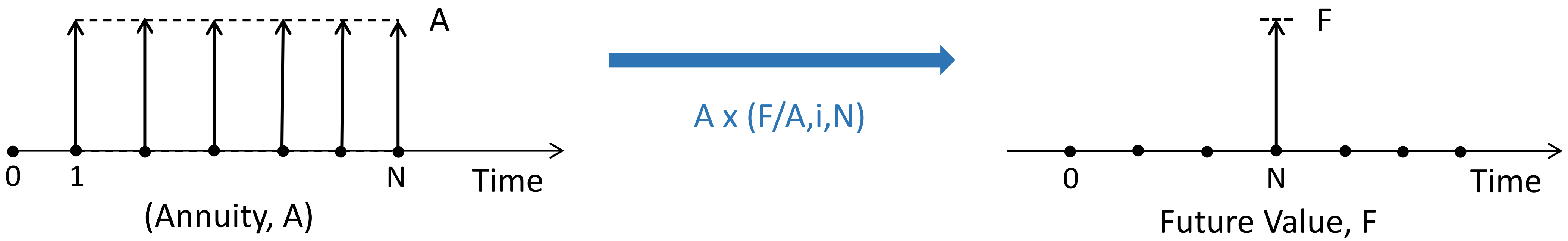


Uniform series compound amount factor (F/A,i,N)

- Converts A to F: A = A x (F/A,i,N) = FV(i,N,-A)
- Reciprocal of the sinking fund factor:

$$(F/A,i,N) = 1/(A/F,i,N)$$

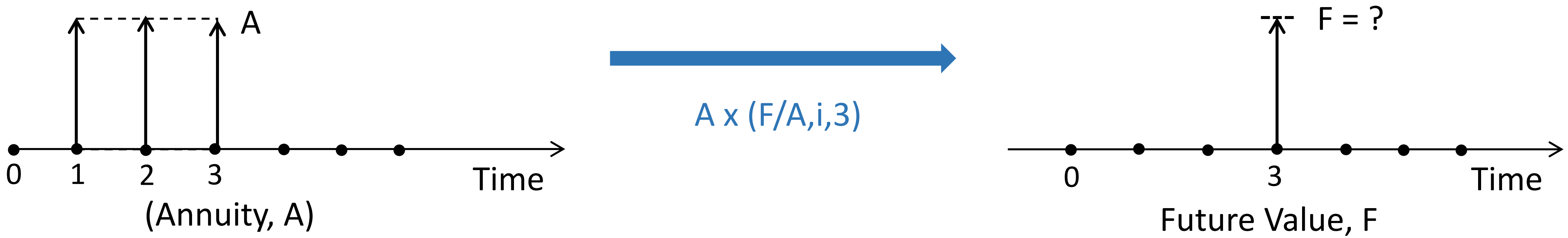
- Answers the question: if I save A per year, starting in year 1, how much money will have at the end of year N?



Deriving (F/A,i,N)

- All of the other discount factors we will be looking at can be derived from (P/F,i,N) and its inverse, (F/P,i,N). They're all shorthand for repeated application of present & future value calculations.
- I'll derive (F/A,i,N) as a representative example.
- A x (F/A,i,N) is the future (Year N) value of a series of N payments of A each. The first payment is in Year 1, the last payment is in Year N.
- Suppose the present is Year 0.
- If N = 1, we have a payment of $A in Year 1.
- By inspection, the Year 1 value of a payment of $A in Year 1 is A.
- Let's make things slightly more complicated by letting N = 3.

Year 3 value of an annuity with N = 3



- We have a payment of A in each of Years 1, 2 and 3.
- The relevant interest rate is i per year.
- We want the Year 3 value of this series of payments.
- From the point of view of Year 1, Year 3 is 2 years in the future, so…
- Year 3 value of $A in Year 1 = A x (F/P,i,2)
- From the point of view of Year 2, Year 3 is 1 year in the future, so…
- Year 3 value of $A in Year 2 = A x (F/P,i,1)
- From the point of view of Year 3, Year 3 is the present, so…
- Year 3 value of $A in Year 3 = A
- Total future value = A + A x (F/P,i,1) + A x (F/P,i,2) = A + A x (1 + i) + A x $(1+i)^2$

Extending this to an arbitrary N…

- F = A + A x (F/P,i,1) + A x (F/P,i,2) + … A x (F/P,i,N-1)
- F = A x (1 + (1+i) + $(1+i)^2$ + … + $(1+i)^{N-1}$)
- Now a little massaging…
- F x (1 + i) = A x ((1+i) + $(1+i)^2$ + … $(1+i)^{N-1}$ + $(1+i)^N$)
- Subtracting our previous equation for F…
- F x (1+i) - F = A x $(1+i)^N$ – A x 1
- iF = A x ($(1+i)^N$ – 1) → F = A x ($(1+i)^N$ – 1)/i
- Since by definition F = A x (F/A,i,N) → (F/A,i,N) = $\frac{[(1+i)^N - 1]}{i}$

$F \times (A/F,i,N)$ in Plain English

- Suppose the relevant discount rate is i (say, 10% per year).
- You have a cash flow of F in time N. Maybe $100 in 1995.
- You choose an N, say N=5, and multiply it by (A/F,i,N).
- What do you get?
- You get the magnitude of a sequence of N payments in a row, with the last payment being in the time period of your original F payment.
- $100 x (A/F,10%,5) = $16.38 (to the nearest cent)
- That means that if i=10% per year, getting $100 in 1995 is equivalent to getting $16.38 in each of 1995, 1994, 1993, 1992 and 1991.

What about A x (F/A,i,N)?

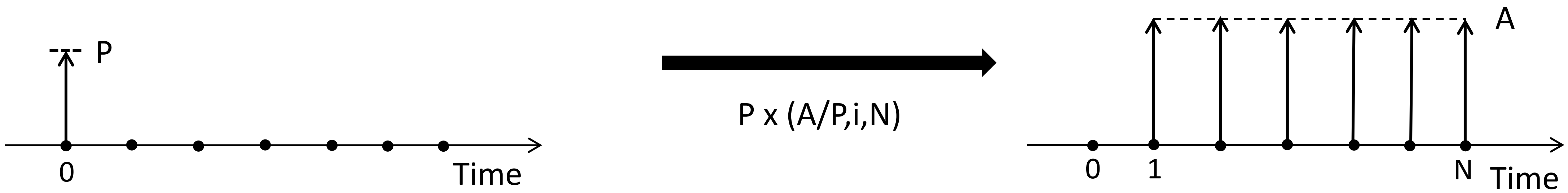
- Suppose the relevant discount rate is i per time period.
- You have a sequence of N payments in a row of magnitude A.
- Maybe you get paid $200 on each of Monday, Tuesday, Wednesday, Thursday and Friday. A = $200, N=5.
- If you calculate A x (F/A,i,N), it turns the sequence of payments into a single equivalent payment on the *last* time period of the original sequence.
- Suppose i = 10% per day. $200 x (F/A,10%,5) = $1,221.02
- That says that, given i=10% per day, getting paid $1,221.02 on Friday is equivalent to getting paid $200 a day from Monday to Friday (inclusive).

Capital recovery factor (A/P,i,N)

$f(x)$

- Converts P to A: A = P x (A/P,i,N) = PMT(i,N,-P)
- Used in the capital recovery formula, which we'll look at shortly.

$$(A/P\,,i,N)=\frac{i(1+i)^N}{(1+i)^N-1}$$



Present Value, P

(Annuity, A)

In his 'Wealth of Nations', Adam Smith used A/PiN factory as his example for the effect of the division of labor.

Deriving the capital recovery factor

- We can derive it in two steps.
- Note we have P, and want A.
- First, convert P to F: F = P(F/P,i,N)
- Next, convert F to A: A = F(A/F,i,N) = P(A/F,i,N)(F/P,i,N)

$$\rightarrow (A/P\,,i,N) = (A/F\,,i,N)(F/P\,,i,N) = \frac{i(1+i)^N}{(1+i)^N - 1}$$

- It's as if we could *chain* the 'fractions' together!
- A/P = A/F x F/P
- This technique is VERY useful.

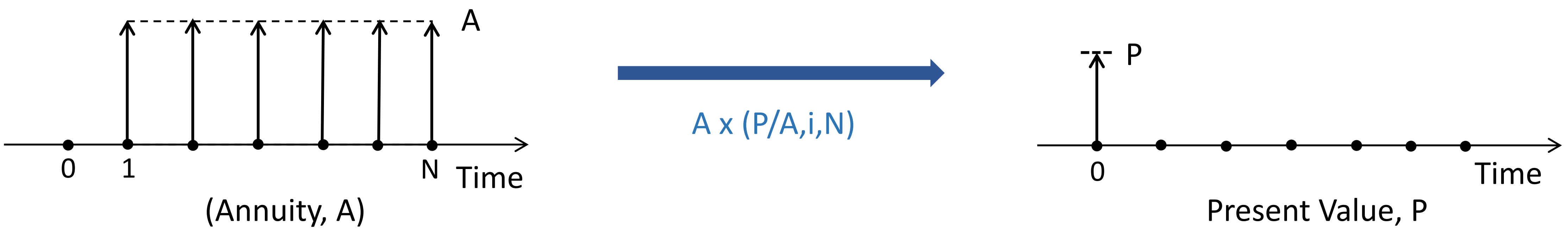
$f(x)$

Series present worth factor (P/A,i,N)

- Misses an awesome chance to be called the ‘PAiN factor’.
- Converts A to P: P = A x (P/A,i,N) = PV(i,N,-A)
- Reciprocal of the capital recovery factor:

$$(P/A,i,N) = 1/(A/P,i,N)$$

- Answers the question: how much should I spend today on something that gives me A dollars a year for N years?



A x (P/A,i,N) in plain English

- You have a sequence of N payments in a row of magnitude A.
- Suppose you get paid \$200 on each of **Monday, Tuesday, Wednesday, Thursday and Friday**. A = \$200, N=5.
- If you calculate A x (P/A,i,N), it turns the sequence of payments into a single equivalent payment, **one period before** the first payment in the original sequence.
- Suppose i = 10% per day. \$200 x (P/A,10%,5) = \$758.16
- The first period in the sequence is Monday. One period before Monday is Sunday.
- That says that, given i=10% per day, getting paid \$200 a day from Monday to Friday (inclusive) is equivalent to getting paid \$758.16 on **Sunday**.

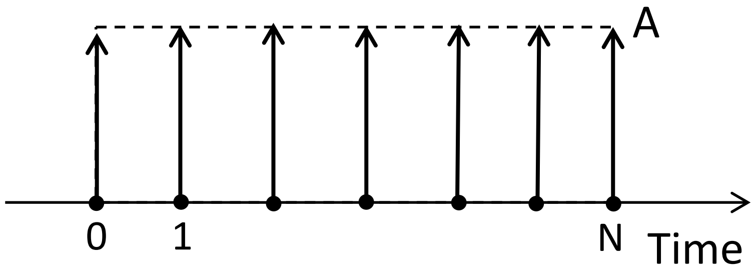
P x (A/P,i,N) in plain English

- Suppose the relevant discount rate is i (say, 10% per year)
- You have a cash flow of F in time N. Maybe $100 in 1995.
- You choose an N, say N=5, and multiply it by (A/P,i,N).
- What do you get?
- You get the magnitude of a sequence of N payments in a row, with the <u>**first**</u> payment being **one period after** your original P payment.
- $100 x (A/P,10%,5) = $26.38 (to the nearest cent)
- The original payment is in 1995, and 1995 + one year = 1996.
- That means that if i=10% per year, getting **$100 in 1995** is equivalent to getting **$26.38** in each of **1996**, **1997**, **1998**, **1999** and **2000**.

(Optional) After Hours

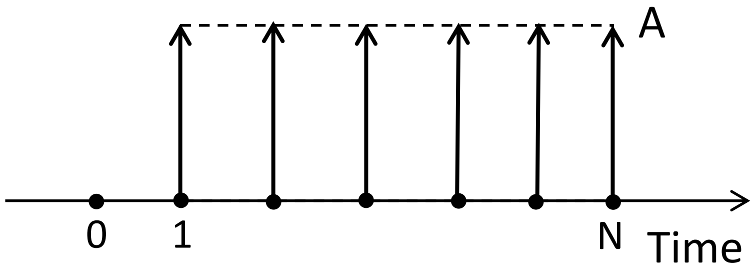
Worked examples: A few cases to watch out for…

Case 1: A payment of A every year, starting today and going to year N.

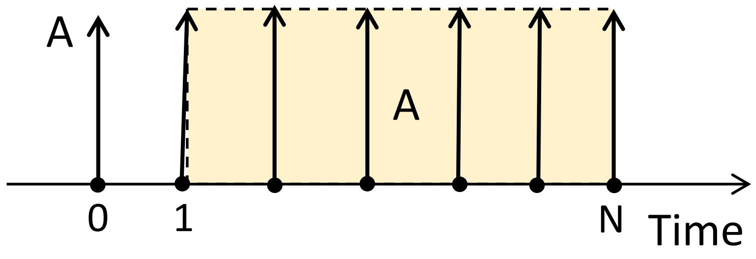


Case 1

- This looks like an annuity of length N, but annuities start being positive in year 1, not year 0.
- Solution: Split the cash flow into a present payment of A, and an annuity of A with length N.

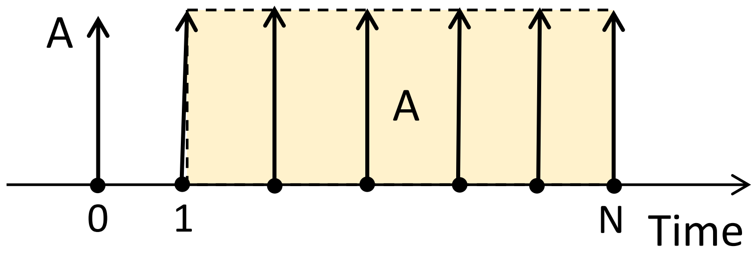


(Annuity, A)



Case 1

Finding the present value for Case 1

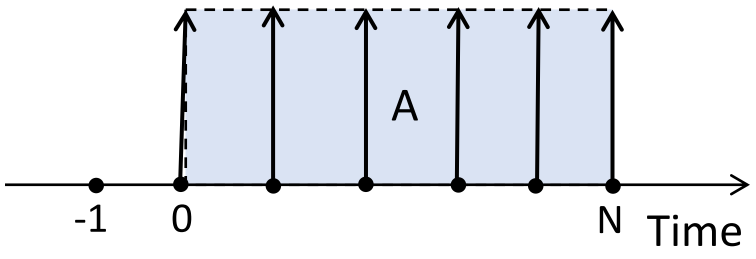


Case 1

- Two items: a lump sum payment of A at time 0, and an annuity A of length N starting at time zero (first period in an annuity is 0).
- The lump sum A is already in present value terms. As for the annuity, we have A and want P, so we'll (P/A,i,N).

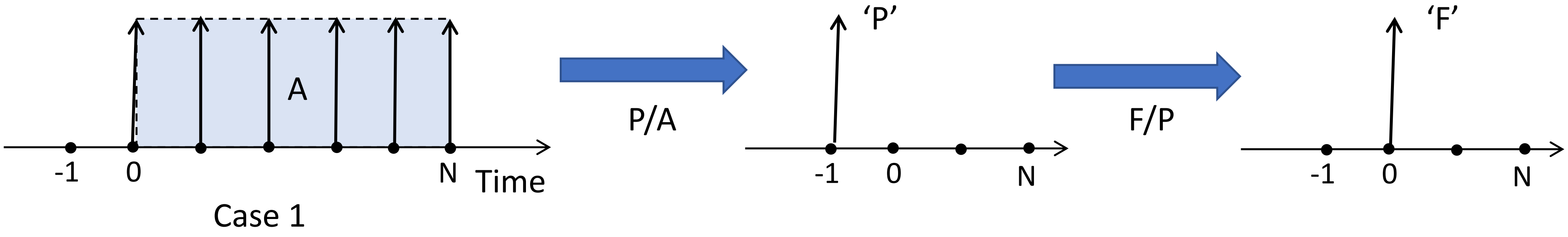
→ P = A + A x (P/A,i,N)

- A second possible approach:
- Consider the annuity as starting in period -1, and having a length (N+1).



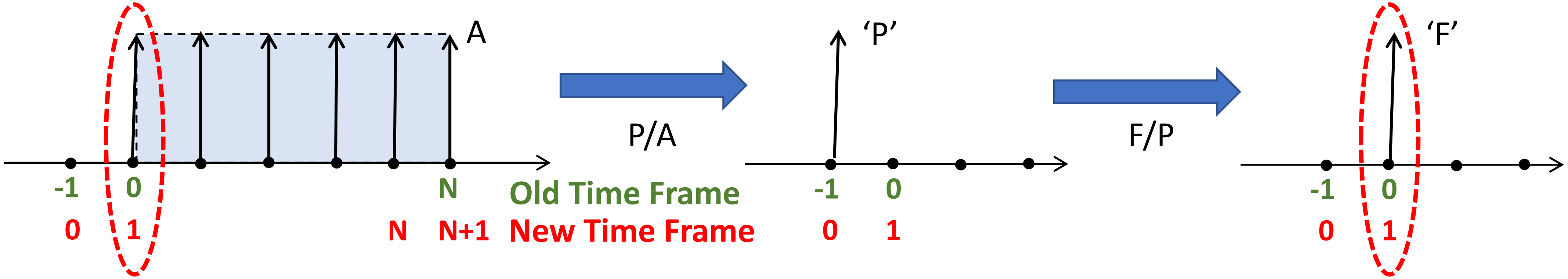
Case 1

Using the second approach



- First, we turn the annuity into a 'present value' from the point of view of Year -1. Then we turn this P into a 'future value' at Year 0.
- A to P: P = A x (P/A,i,N + 1)
- P to F: F = P x (F/P,i,1) (N = 1 since we're jumping forward one year)
- → P = A X (F/P,i,1) x (P/A,i,N + 1)

Put another way…



- Since TIMING is the only reason we can't treat these cash flows as a single annuity, let's just change the timing by **re-labeling the time periods** as being one period earlier. (In other words, treating some other year, in this case the original time frame's Year -1, as being Year 0.)
- We need to re-phrase our objective using the **new time frame**, as well: in terms of the **original time frame**, we're looking for a present (Year 0) value. In terms of our new time frame, we're looking for a Year 1 value, since Year 1 in the new time frame is equivalent to Year 0 in the **old time frame**.
- In the **new time frame**, we start with a well-behaved annuity that is positive from Year 1 to Year N and has a height A. We want to transform it into an equivalent single payment in Year 1. First, we turn it into a present (Year 0) value using A x (P/A,i,N), then we bring the Year 0 value 1 year into the future using (F/P,i,1), so the final value we're looking for is A x (P/A,i,N) x (F/P,i,1).

Brute force testing: A = 10, N = 5, i = 0.1

Year	Flow	PV
0	10	$10.00
1	10	$9.09
2	10	$8.26
3	10	$7.51
4	10	$6.83
5	10	$6.21
Total		**$47.91**
Approach 1		**$47.91**
Approach 2		
	P/A	$43.55
	F/P	**$47.91**

Excel Formulas Used

PV = PV(i,Year,,-Flow)

Approach 1 = A + PV(i,N,-A)

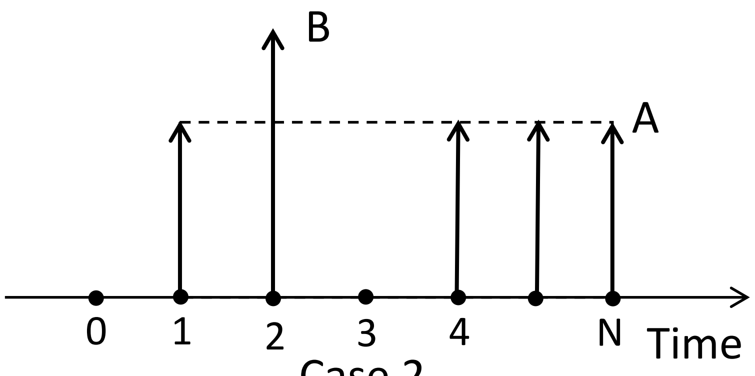
Approach 2:

P/A = PV(i,N+1,-A)

F/P = FV(i,1,-P/A)

By 'Brute Force', I mean inelegantly using repeated applications of the present value formula. This is inefficient, but gets the job done.

Case 2: Fluctuating Payments

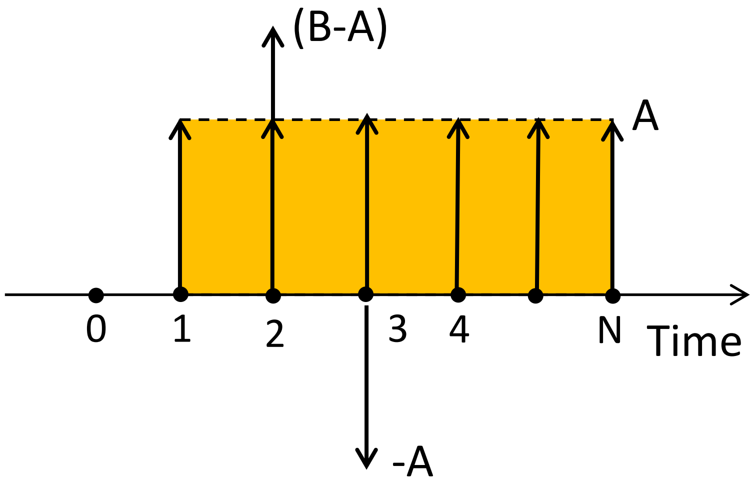


Case 2

- This is almost an annuity, but has an extra-large payment in Year 2, and nothing in Year 3.

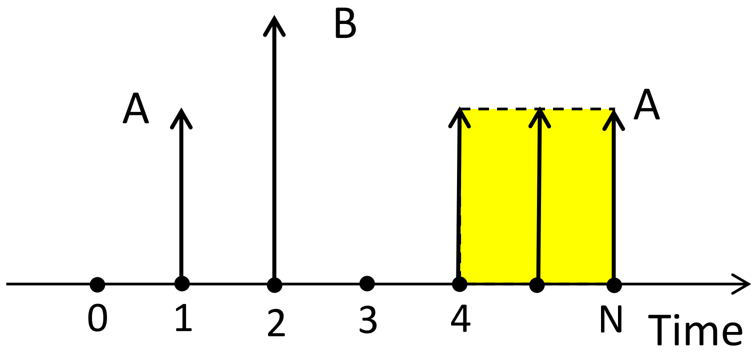
Approach 1: Split the cash flow into an annuity of length N, treat the extra payment in Year 2 as a lump sum, and add a negative lump sum payment in year 3 to compensate for the one added by the assumption of an annuity.

We use (P/A) to turn our annuity A into P, and (P/F) to convert our two future payments into P.



$$P = A(P/A,i,N) + (B - A)(P/F,i,2) - A(P/F,i,3)$$

A second approach



- We could also split the cash flow into an annuity starting in Year 3 (annuities are only positive starting in Year 1), a future payment of A in Year 1 and a future payment of B in Year 2.

- Starting with the annuity:

  In year 3, its present value will be P = A(P/A,i,N – 3)

- From the point of view of Year 0, that's a future value, so we use P/F to bring it to the present: P = F(P/F,i,3) = A(P/F,i,3)(P/A,i,N – 3)

- Adding in the single payments,

  P = A(P/F,i,1) + B(P/F,i,2) + A(P/F,i,3)(P/A,i,N – 3)

Brute Force Testing: A=10, B=15, i=0.1, N=5

Year	Flow	PV
0	0	$0.00
1	10	$9.09
2	15	$12.40
3	0	$0.00
4	10	$6.83
5	10	$6.21
Total		**$34.53**
Approach 1		**$34.53**
Approach 2		
	P/A	$17.36
	P/(P/A)	$13.04
	P/F (A)	$9.09
	P/F (B)	$12.40
	Total	**$34.53**

Excel Formulas Used

PV = PV(i,Year,,-Flow)

Approach 1 = PV(i,N,-A) + PV(i,2,,A-B) + PV(i,3,,A)

Approach 2:

P/A = PV(i,N-3,-A)
P/(P/A) = PV(i,3,,-P/A)
P/F(A) = PV(i,1,,-10)
P/F(A) = PV(i,2,,-15)
Total = P/(P/A) + P/F(A) + P/F(B)

Case 3: Capitalized Value of a Perpetual Annuity

- What's the present value of an annuity that lasts forever?
- Consider an annuity that pays A per year at rate i.

$$P = A(P/A, i, N) = A\frac{(1+i)^N - 1}{i(1+i)^N} = A\left(\frac{1}{i} - \frac{1}{i(1+i)^N}\right)$$

$$\lim_{n\to\infty} P = \lim_{n\to\infty} A\left(\frac{1}{i} - \frac{1}{i(1+i)^N}\right) = A\left(\frac{1}{i} - \frac{1}{\infty}\right) = \frac{A}{i}$$

- The Capitalized Value of A at rate i = $\frac{A}{i}$

And we care, why?

- These days, mostly because it's a handy back-of-the-envelope calculation.

- e.g. You shouldn't pay more than A/i for an investment that pays A per year.
- e.g. If a building is expected to last decades, and has maintenance costs of $M/year, the present value of maintenance costs is about $M/i.

- GIGO (Garbage in, Garbage Out): Capitalized Value is used often by property evaluators. If your best guess for a property's income is 'what's it been every year for the last 10 years', and your best guess of how long the property will last is 'a long time', the A/i formula may yield better results than more complicated analyses.
- (See (Conn, 2013) for a business valuator's perspective.)