

CHAPTER 8

Solutions to Chapter-End Problems

A. Key Concepts

Before- and After-Tax MARR:

8.1 Using: $MARR_{\text{after-tax}} \cong MARR_{\text{before-tax}} \times (1 - t)$:

(a) $MARR_{\text{after-tax}} \cong 0.14 \times (1 - 0.4) = 0.084 = 8.4\%$

(b) $MARR_{\text{after-tax}} \cong 0.14 \times (1 - 0.5) = 0.074 = 7.0\%$

(c) $MARR_{\text{after-tax}} \cong 0.14 \times (1 - 0.6) = 0.056 = 5.6\%$

8.2 Before-tax IRR:

$$5000(P/F, i, 1) + 10\,000(P/F, i, 2) - 12\,000 = 0$$

At $i = 0.14$: LHS = 80.64

At $i = 0.15$: LHS = -90.73

Interpolation: IRR = 14.47%

Approximate after-tax IRR = $0.1447(1 - 0.4) = 0.0868$

The after-tax IRR is approximately 8.68%

IRR Tax Calculations:

8.3 $IRR_{\text{after-tax}} \cong IRR_{\text{before-tax}} \times (1 - t)$

$$= 0.24(1 - 0.40)$$

$$= 0.144$$

The after-tax IRR is approximately 14.4%. For an after-tax MARR of 18%, the project should not be approved. However, for an after-tax MARR of 14%, since the after-tax IRR is an approximation, a more detailed examination would be advisable.

8.4 The CTF is $1 - [(0.40)(0.3)(1 + i^*/2)]/[(i^* + 0.3)(1 + i^*)]$.

Setting disbursements equal to receipts gives:

$$12\,000(\text{CTF}) = [5000(P/F, i^*, 1) + 10\,000(P/F, i^*, 2)](1 - 0.40)$$

Trial and error calculations give: $i^* = 6.226\%$

8.5 **(a)** Class 12, fully expensed in first year

(b) Class 1, 3 or 6, rate 5% to 10%

- (c) Class 10, rate 30%
- (d) Class 8, rate 20%
- (e) Class 9, rate 25%
- (f) Class 8, rate 20%

CTF and CSF:

8.6 (a) $CSF = 1 - (0.5 \times 0.2) / (0.09 + 0.2) = 0.6552$
 $CTF = 1 - [(0.5)(0.2)(1 + 0.09/2)] / [(0.09 + 0.2)(1 + 0.09)] = 0.6694$

(b) $CSF = 1 - (0.35 \times 0.3) / (0.12 + 0.3) = 0.75$
 $CTF = 1 - [(0.35)(0.3)(1 + 0.12/2)] / [(0.12 + 0.3)(1 + 0.12)] = 0.7674$

(c) $CSF = 1 - (0.55 \times 0.05) / (0.06 + 0.05) = 0.75$
 $CTF = 1 - [(0.55)(0.05)(1 + 0.06/2)] / [(0.06 + 0.05)(1 + 0.06)] = 0.7571$

8.7 First year CCA is on \$50 000 (half of capital purchases).

Net income
 $= 110\,000 - 65\,000 - 50\,000(0.2) = 45\,000 - 10\,000 = \$35\,000$

Taxes paid $= 35\,000 \times 0.55 = \$19\,250$

8.8 Second year CCA is on (\$50 000 – \$10 000) from the half that was subject to CCA during first year plus the remaining \$50 000 of the original purchases: (50 000 – 10 000) + 50 000 = 90 000.

Net income
 $= 110\,000 - 65\,000 - 90\,000(0.2) = 45\,000 - 18\,000 = \$27\,000$

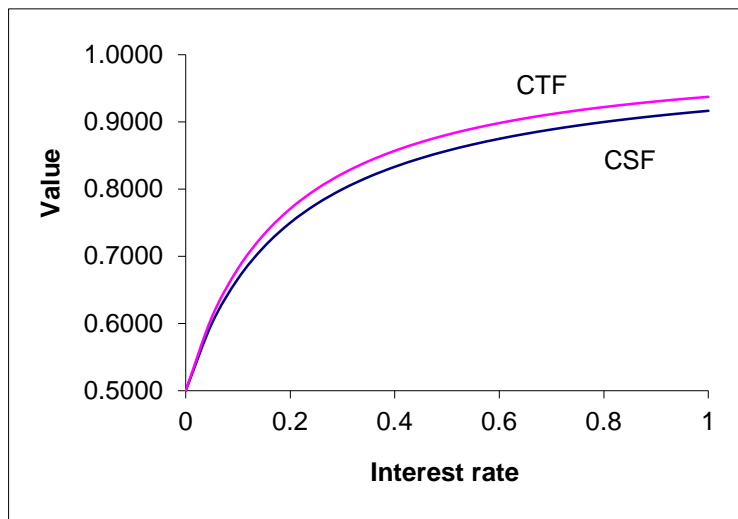
Taxes paid $= 27\,000 \times 0.55 = \$14\,850$

8.9 The UCC at the end of 2016 was \$26 779.

Year	Adjustment	Base UCC	CCA@30 %	Remaining UCC
2012				10000
2013	50000	35000	10500	49500
2014	0	49500	14850	34650
2015	20000	44650	13395	41255
2016	-3000	38255	11477	26779

8.10 The following spreadsheet shows the numbers used for the chart, and the chart:

Interest Rate	CSF	CTF
0	0.5000	0.5000
0.05	0.6000	0.6095
0.1	0.6667	0.6818
0.15	0.7143	0.7329
0.2	0.7500	0.7708
0.25	0.7778	0.8000
0.3	0.8000	0.8231
0.35	0.8182	0.8418
0.4	0.8333	0.8571
0.45	0.8462	0.8700
0.5	0.8571	0.8810
0.55	0.8667	0.8903
0.6	0.8750	0.8984
0.65	0.8824	0.9055
0.7	0.8889	0.9118
0.75	0.8947	0.9173
0.8	0.9000	0.9222
0.85	0.9048	0.9266
0.9	0.9091	0.9306
0.95	0.9130	0.9342
1	0.9167	0.9375



8.11 Using: $MARR_{\text{after-tax}} \cong MARR_{\text{before-tax}} \times (1 - t)$

$$0.052 \cong MARR_{\text{before-tax}} \times (1 - 0.53)$$

$$MARR_{\text{before-tax}} \cong 0.052/0.47 = 11.06\%$$

Similarly, using: $IRR_{\text{after-tax}} \cong IRR_{\text{before-tax}} \times (1 - t)$

$$0.087 \cong IRR_{\text{before-tax}} \times (1 - 0.53)$$

$$IRR_{\text{before-tax}} \cong 0.087/0.47 = 18.51\%$$

Enrique will likely report a before-tax IRR of 18.5 % compared to a before-tax MARR of 11%.

B. Applications

8.12 ICC's investments are clearly in CCA class 10, with a depreciation rate (CCA rate) of 30%.

$$CSF = 1 - (0.45 \times 0.3)/(0.10 + 0.3) = 0.6625$$

$$CTF = 1 - [(0.45) \times (0.3) \times (1 + 0.10/2)] / [(0.10 + 0.3) \times (1 + 0.10)] = 0.645625$$

The present worth of the inventory system is then:

$$PW = -2\,300\,000 \times CTF + 880\,000 \times (1 - 0.45) \times (P/A, 10\%, 10)$$

$$+ 200\,000 \times CSF \times (P/F, 10\%, 10)$$

$$= -2\,300\,000 \times 0.6456 + 880\,000 \times 0.55 \times 6.1446 + 200\,000 \times 0.6625 \times 0.38554$$

$$= 1\,540\,133$$

The present worth of the new system is \$1 540 000. ICC should make this investment.

8.13 As of March 2010:

(a) Class 6, 10%

(b) Class 38, 30%

(c) Class 10, 30%, class 45 (45%), class 50 (55%) or class 52, 100%, depending on when it was purchased

(d) Class 46, 30%

(e) Cash register: class 8, rate 20%, scanner, class 12, 100%

$$8.14 \quad P = -55\,000(\text{CTF}) + 17\,000(P/A, 10\%, 6)(1 - 0.54) + 1000(P/F, 10\%, 6)(\text{CSF})$$

$$\text{CTF} = 1 - [(0.54)(0.2)(1 + 0.1/2)]/[(0.1 + 0.2)(1 + 0.1)] = 0.6564$$

$$\text{CSF} = 1 - (0.54 \times 0.2)/(0.1 + 0.2) = 0.64$$

$$P = -55\,000(0.6564) + 17\,000(4.3552)(0.46) + 1000(0.56448)(0.64) = -1\,683$$

The after tax present worth of the chip placer is -\$1683.

8.15 Before-tax IRR:

$$-400\,000 + 85\,000(P/A, i, 10) = 0$$

$$(P/A, i, 10) = 400\,000/85\,000 = 4.71$$

$$(P/A, 15\%, 10) = 5.0187$$

$$(P/A, 20\%, 10) = 4.1924$$

By interpolating: IRR = 16.9%

$$\text{After-tax IRR} = 16.9(1 - 0.45) = 9.3\%$$

Quebec Widgets should not invest.

8.16 The CTF is $1 - [(0.45)(0.2)(1 + i^*/2)]/[(i^* + 0.2)(1 + i^*)]$.

Setting disbursements equal to receipts gives:

$$400\,000(\text{CTF}) = 85\,000(P/A, i^*, 10)(1 - 0.45)$$

Trial and error calculations give: $i^* = 10.1087\%$

The exact IRR is 10.1087%. Canadian Widgets should not invest.

8.17

	1	2	3	4	5	6	7	8	9	10	13
	Class Number	UCC at beginning of year (Col 13 from last year)	Cost of acquisitions during the year	Adjustments	Proceeds of disposition	UCC (2)+(3)-(5)	50% rule	Reduced UCC ((6)-(7))	CCA Rate	CCA (8)*(9)	UCC at year end (6)-(10)+(7)
a	10	10 000	30 000	0	0	40 000	15 000	25 000	0.3	7500	32 500
b	10	10 000	20 000	0	5000	25 000	10 000	15 000	0.3	4500	20 500
c	8	20 000	30 000	0	15 000	35 000	15 000	20 000	0.2	4000	31 000

8.18 The UCC account values are shown in the following spreadsheet.

Year	Adjustment	Base UCC	CCA@20 %	Remaining UCC
2001				0
2002	200 000	100 000	20 000	180 000
2003	0	180 000	36 000	144 000
2004	0	144 000	28 800	115 200
2005	240 000	235 200	47 040	308 160
2006	0	308 160	61 632	246 528
2007	0	246 528	49 306	197 222
2008	0	197 222	39 444	157 778
2009	60 000	187 778	37 556	180 222
2010	0	180 222	36 044	144 178
2011	0	144 178	28 836	115 342
2012	0	115 342	23 068	92 274
2013	0	92 274	18 455	73 819
2014	345 000	246 319	49 264	369 555
2015	-45 000	324 555	64 911	259 644
2016	0	259 644	51 929	207 715

It can be seen that the depreciation for this CCA class for 2016 is \$51 929.

8.19 $t = 0.40$, $i = 0.20$, $d = 0.3$

$$\text{CSF} = 1 - (0.4 \times 0.3) / (0.2 + 0.3) = 0.76$$

$$\text{CTF} = 1 - (0.4 \times 0.3 \times 1.1) / (1 + 0.2)(0.2 + 0.3) = 0.78$$

$$\begin{aligned} \text{AW} &= -45\,000(\text{A/P}, 20\%, 5)\text{CTF} + 4500(\text{A/F}, 20\%, 5)\text{CSF} \\ &\quad - 0.22(750)(250)(1 - 0.4) \\ &= -45\,000(0.33438)(0.78) + 4500(0.13438)(0.76) \\ &\quad - 0.22(750)(250)(1 - 0.4) \\ &= -36\,072 \end{aligned}$$

The machine has a total annual cost of about \$36 072.

8.20 The value of the 30% UCC account at the end of 2016 was about \$12 962.

Year	Adjustment	Base UCC	CCA@30 %	Remaining UCC
2008				0
2009	25000	12500	3750	21250
2010	0	21250	6375	14875
2011	14000	21875	6563	22313
2012	0	22313	6694	15619

Divided

2013	0	15619	4686	10933
2014	28000	24933	7480	31453
2015	-5000	26453	7936	18517
2016	0	18517	5555	12962

8.21 $CTF = 1 - [(0.35)(0.2)(1 + 0.06/2)] / [(0.06 + 0.2)(1 + 0.06)] = 0.7384$

$PW = 380\,000(CTF) = 380\,000(0.7384) = 280\,588$

The present worth of the crane, taking into account all future tax benefits due to CCA, is \$280 588.

8.22 $CTF = 1 - [(0.45)(0.2)(1 + 0.08/2)] / [(0.08 + 0.2)(1 + 0.08)] = 0.6905$
 $CSF = 1 - (0.45 \times 0.2) / (0.08 + 0.2) = 0.6786$

First cost
 $= -230\,000(CTF) = -230\,000(0.6905) = -158\,810$

Savings
 $= 35\,000(P/A, 8\%, 12)(1 - 0.45) = 35\,000(7.5361)(0.55) = 145\,070$

Salvage value
 $= 30\,000(P/F, 8\%, 12)CSF = 30\,000(0.39711)(0.6786) = 8084$

$PW = -158\,810 + 145\,070 + 8084 = -5656$

The project has a present worth of -\$5656; it should not be done.

8.23 $CTF = 1 - [(0.35)(0.3)(1 + 0.12/2)] / [(0.12 + 0.3)(1 + 0.12)] = 0.763$
 $CSF = 1 - (0.35 \times 0.3) / (0.12 + 0.3) = 0.75$

First cost
 $= -65\,000(A/P, 12\%, 5)CTF = -65\,000(0.27741)(0.763) = -13\,758$

Annual savings $= 15\,000(1 - 0.35) = 9750$

Salvage value
 $= 20\,000(A/F, 12\%, 5)CSF = 20\,000(0.15741)(0.75) = 2361$

Annual worth $= -13\,758 + 9750 + 2361 = -1647$

The purchase will result in an annual cost of \$1647 over five years.

8.24 The CTF is $1 - [(0.40)(0.2)(1 + i^*/2)]/[(i^* + 0.2)(1 + i^*)]$

Setting disbursements equal to receipts gives:

$$17\,000(\text{CTF}) = [3000(\text{P/A}, i^*, 7) + 1000(\text{P/F}, i^*, 7)](1 - 0.40)$$

Trial and error calculations give: $i^* = 3.8\%$

Their exact after-tax IRR is 3.737%.

The estimated after-tax MARR is: $\text{MARR}_{\text{after-tax}} \cong 0.1 \times (1 - 0.4) = 6.0\%$

The purchase should not be made since the after-tax IRR is less than the after-tax MARR.

8.25 $\text{CTF} = 1 - [(0.45)(0.2)(1 + 0.1/2)]/[(0.1 + 0.2)(1 + 0.1)] = 0.7136$

$$\text{CSF} = 1 - (0.45 \times 0.2)/(0.1 + 0.2) = 0.7$$

$$\begin{aligned} \text{AW} &= -200\,000(\text{A/P}, 10\%, 20)\text{CTF} \\ &\quad - [7500 + 25\,000(\text{A/P}, 10\%, 5)](1 - 0.45) \\ &\quad + 15\,000(\text{A/F}, 10\%, 20)\text{CSF} \\ &= -200\,000(0.117\,46)(0.7136) \\ &\quad - [7500 + 25\,000(0.26380)](0.55) + 15\,000(0.01746)(0.7) \\ &= -\$24\,333 \end{aligned}$$

The after-tax annual cost of the splitter is $-\$24\,333$.

8.26 $\text{CTF} = 1 - [(0.45)(0.2)(1 + 0.09/2)]/[(0.09 + 0.2)(1 + 0.09)] = 0.7025$

$$\text{CSF} = 1 - (0.45 \times 0.2)/(0.09 + 0.2) = 0.6897$$

The annual cost of the chemical recovery system is then:

$$\begin{aligned} A &= -30\,000(\text{A/P}, 9\%, 7)\text{CTF} + 5280(1 - 0.45) \\ &\quad + 7500(\text{A/F}, 9\%, 7)\text{CSF} \\ &= -30\,000(0.19869)(0.7025) + 5280(0.55) + 7500(0.10869)(0.6897) \\ &= -721 \end{aligned}$$

The chemical recovery system has an after-tax annual cost of about \$721.

8.27 The exact after-tax IRR is calculated by trial and error to be

$$\text{IRR}_{\text{after-tax}} = 8.44\%.$$

$i =$	7%	8%	8.44%	9%	10%
CCTF(old) =	0.5926	0.6071	0.6133	0.6207	0.6333
CCTF(new) =	0.6059	0.6217	0.6283	0.6363	0.6500
(P/A, $i\%$, 6) =	4.7665	4.6229	4.5612	4.4859	4.3553
(P/F, $i\%$, 6) =	0.6663	0.6302	0.6148	0.5963	0.5645

PW(disbursements)	-66651	-68386	-69117	-69998	-71500
=					
PW(receipts) =	72246	70061	69117	67962	65946
PW(total) =	5595	1675	0	-2037	-5554

The before-tax IRR is calculated to be $IRR_{\text{before-tax}} = 18.77\%$

i =	17%	18%	18.77%	19%	20%
(P/A, i%, 6) =	3.5892	3.4976	3.4292	3.4098	3.3255
(P/F, i%, 6) =	0.3898	0.3704	0.3562	0.3521	0.3349
PW(disbursements)	-110000	-110000	-110000	-110000	-110000
) =					
PW(receipts) =	115472	112337	110000	109336	106463
PW(total) =	5472	2337	0	-664	-3537

$$IRR_{\text{after-tax}} \cong IRR_{\text{before-tax}} \times (1 - 0.55) = 0.1877(0.45) = 0.0845 = 8.45\%$$

Since the IRR, through either calculation, is less than the MARR, the backhoe should not be purchased.

8.28 $d = 0.30$, $t = 0.40$

Ordered by first cost, the alternatives are:

- 1) Free machine, first cost \$6 000
- 2) Used machine, first cost = \$36 000
- 3) Owned machine, first cost = \$71 000

Alternative 1:

Note that the first cost is an expense of installation, and not a capital cost, so the after-tax IRR is found by solving for i in:

$$6000(1 - t) = (20\,000 - 15\,000)(P/A, i, 6)(1 - t)$$

$$(P/A, i, 6) = 1.2$$

Noting that $(P/A, 40\%, 6) = 2.16$ and $(P/A, 50\%, 6) = 1.8$, the free machine is acceptable as it has an after-tax IRR of above 50%.

Next, look at the increment of investment between the free and used machines:

Year	Increment: used – free
0	–30000
1	12000

2	9500
3	7000
4	4500
5	2000
6	-500

This is almost a simple investment, so as an approximation, we will use an IRR approach (the negative cash flow in the sixth year is not likely to create multiple IRRs).

Taking present worths gives:

$$-30\,000(\text{CTF}) + \text{PW}(\text{incremental annual flows})(1 - t) = 0$$

With a trial and error approach, we obtain an after-tax IRR of 3.18%, which is below the MARR.

We now find the after-tax IRR on the incremental investment from the free machine to the new machine.

Year	Increment: new – free
0	-65000
1	15000
2	15000
3	15000
4	15000
5	15000
6	25000

This is a simple investment, so the IRR can be found as follows:

$$-65\,000\text{CTF} + \text{PW}(\text{incremental cash flows})(1 - t) + (\text{salvage value})\text{CSF} = 0$$

By trial and error we get an after-tax IRR of 9.88% that is less than the MARR.

Conclusion: the free machine is best.

C. More Challenging Problems:

8.29 (a) $\text{DC}(1) = (360\,000 - 0)/10 = 36\,000$

$$\text{Taxes} = [1\,600\,000 - (1\,300\,000 + 36\,000)] \times 0.5 = 132\,000$$

They would pay about \$132 000.

(b) $\text{DC}(1) = (360\,000 - 0)/5 = 72\,000$

$$\text{Taxes} = [1\,600\,000 - (1\,300\,000 + 72\,000)] \times 0.5 = 114\,000$$

They would pay about \$114 000.

(c) $\text{DC}(1) = 360\,000 \times 0.2 = 72\,000$

$$\text{Taxes} = [1\,600\,000 - (1\,300\,000 + 72\,000)] \times 0.5 = 114\,000$$

They would pay about \$114 000.

(d) $\text{DC}(1) = 360\,000 \times 0.4 = 144\,000$

$$\text{Taxes} = [1\,600\,000 - (1\,300\,000 + 144\,000)] \times 0.5 = 78\,000$$

They would pay about \$78 000.

(e) $\text{DC} = 360\,000$

$$\text{Taxes} = [1\,600\,000 - (1\,300\,000 + 360\,000)] \times 0.5 = -30\,000$$

They would pay no taxes, and would have a tax credit of \$30 000.

- 8.30** The present worth of a year's savings due to CCA for the assets if they were correctly recognised as Class 8 can be calculated using the CTF.

$$CTF = 1 - [(0.5)(0.2)(1 + 0.09/2)]/[(0.09 + 0.2)(1 + 0.09)] = 0.6694$$

$$PW_{\text{savings-8}} = 10\,000 - 10\,000(CTF) = 10\,000 - 10\,000(0.6694) = 3306$$

The present worth of a year's savings due to CCA for the items recognised as Class 12 can be calculated directly from the tax rate:

$$PW_{\text{savings-12}} = (10\,000/2)(P/F, 9\%, 1) = 5000(0.91743) = 4587$$

The present worth of the loss per year is then

$$PW_{\text{loss}} = 4587 - 3306 = 1281$$

The present worth today of such losses over the past three years is then:

$$PW = 1281(F/A, 9\%, 3) = 1281(3.2781) = 4199$$

The present worth of the cost today of this mistake is about \$4199.

- 8.31** $CTF = 1 - [(0.45)(0.05)(1 + 0.15/2)]/[(0.15 + 0.05)(1 + 0.15)] = 0.8948$

Alternative 1:

Using the capitalized value formula:

$$A(\text{first cost}) = P_i(CTF) = 2\,000\,000(0.15)(0.8948) = 268\,440$$

$$A(\text{maintenance}) = 10\,000(1 - 0.45) = 5500$$

$$\begin{aligned} A(\text{paint}) \\ = 15\,000(A/F, 15\%, 15)(1 - 0.45) = 15\,000(0.02102)(0.55) = 173.42 \end{aligned}$$

$$A(\text{total}) = 274\,113$$

Alternative 2:

$$\begin{aligned} A(\text{first costs}) \\ = 1\,250\,000(0.15)(0.8948) + 1\,000\,000(P/F, 15\%, 10)(0.15)(0.8948) \\ = 167\,775 + 150\,000(0.24719)(0.8948) = 200\,953 \end{aligned}$$

The first 10 years incur maintenance costs of \$5000 per year; convert to a PW and spread over infinite life:

$$\begin{aligned} & A(\text{maintenance first 10 years}) \\ & = 5000(P/A, 15\%, 10)(0.15)(1 - 0.45) \\ & = 5000(5.0187)(0.15)(0.55) \\ & = 2070.21 \end{aligned}$$

The maintenance costs change after the renovation to be \$11 000 every year. First convert this to a PW at the end of 10 years, then to PW now, then spread over infinite life:

$$\begin{aligned} & P(\text{maintenance after 10 years}) \\ & = A/i(1 - t) = 11\,000/(0.15)(1 - 0.45) = 40\,333.33 \end{aligned}$$

$$\begin{aligned} & P(\text{maintenance after 10 years, now}) \\ & = 40\,333.33(P/F, 15\%, 10) = 40\,333.33(0.2472) = 9970.40 \end{aligned}$$

$$A = P(\text{maintenance after 10 years})(0.15) = 9970.40(0.15) = 1496$$

Painting costs are every 15 years, starting in 10 years. First calculate the P(paint in 10 years), convert to P(now) and then to A(now):

$$\begin{aligned} & P(\text{paint in 10 years}) \\ & = A/i(1 - t) = 15\,000(A/F, 15\%, 15)/(0.15)(1 - 0.45) \\ & = 15\,000(0.02102)/(0.15)(0.55) \\ & = 1156 \end{aligned}$$

$$P(\text{paint now}) = 1156(P/F, 15\%, 10) = 1156(0.24719) = 285.64$$

$$A = P(\text{paint now})(0.15) = 42.85$$

$$A(\text{total}) = 200\,953 + 2070.21 + 1496 + 43 = 204\,562$$

The annual cost of alternative 2 (\$204 562) is less than that of alternative 1 (\$274 113). Hence, select alternative 2.

8.32 $t = 0.52, d = 0.2, i = 0.11$

$$\begin{aligned} \text{CTF} &= 1 - [(0.52)(0.2)(1 + 0.11/2)]/[(0.11 + 0.2)(1 + 0.11)] = 0.6811 \\ \text{CSF} &= 1 - (0.52 \times 0.2)/(0.11 + 0.2) = 0.6645 \end{aligned}$$

By assuming that you can purchase each alternative as many times as necessary, we can construct new projects:

T': buy model T three times, total life 15 years

A': buy model A three times, total life 15 years

X': buy model X five times, total life 15 years

$$P_{T'} = -100\,000(0.6811)[1 + (P/F, 11\%, 5) + (P/F, 11\%, 10)]$$

$$\begin{aligned}
 &+ 50\,000(1 - 0.52)(P/A, 11\%, 15) \\
 &+ 20\,000(0.6645)[(P/F, 11\%, 5) + (P/F, 11\%, 10) + (P/F, 11\%, 15)] \\
 = &-100\,000(0.6811)(1 + 0.59345 + 0.35218) + 50\,000(0.48)(7.1909) \\
 &+ 20\,000(0.6645)(0.59345 + 0.35218 + 0.20900) \\
 = &55\,410
 \end{aligned}$$

$$\begin{aligned}
 P_{A'} &= -150\,000(0.6811)[1 + (P/F, 11\%, 5) + (P/F, 11\%, 10)] \\
 &+ 60\,000(1 - 0.52)(P/A, 11\%, 15) \\
 &+ 30\,000(0.6645)[(P/F, 11\%, 5) + (P/F, 11\%, 10) + (P/F, 11\%, 15)] \\
 = &-150\,000(0.6811)(1 + 0.59345 + 0.35218) + 60\,000(0.48)(7.1909) \\
 &+ 30\,000(0.6645)(0.59345 + 0.35218 + 0.20900) \\
 = &31\,340
 \end{aligned}$$

$$\begin{aligned}
 P_{X'} &= -200\,000(0.6811)[1 + (P/F, 11\%, 3) + (P/F, 11\%, 6) \\
 &+ (P/F, 11\%, 9) + (P/F, 11\%, 12)] + 75\,000(1 - 0.52)(P/A, 11\%, 15) \\
 &+ 100\,000(0.6645)[(P/F, 11\%, 3) + (P/F, 11\%, 6) \\
 &+ (P/F, 11\%, 9) + (P/F, 11\%, 12) + (P/F, 11\%, 15)] \\
 = &-200\,000(0.6811)(1 + 0.73119 + 0.53464 + 0.39092 + 0.28584) \\
 &+ 75\,000(0.48)(7.1909) + 100\,000(0.6645)(0.73119 + 0.53464 \\
 &+ 0.39092 + 0.28584 + 0.20900) \\
 = &1006
 \end{aligned}$$

Model T is the best choice because it has the highest present worth.

8.33 $t=0.52$, $d = 0.2$, $i = 0.11$

$$CTF = 1 - [(0.52)(0.2)(1 + 0.11/2)] / [(0.11 + 0.2)(1 + 0.11)] = 0.6811$$

$$CSF = 1 - (0.52 \times 0.2) / (0.11 + 0.2) = 0.6645$$

$$\begin{aligned}
 P_T &= -100\,000(0.6811) + 50\,000(1 - 0.52)(P/A, 11\%, 3) \\
 &+ 40\,000(0.6645)(P/F, 11\%, 3) \\
 = &-100\,000(0.6811) + 50\,000(0.48)(2.4437) \\
 &+ 40\,000(0.6645)(0.73119) \\
 = &9974
 \end{aligned}$$

$$\begin{aligned}
 P_{A'} &= -150\,000(0.6811) + 60\,000(1 - 0.52)(P/A, 11\%, 3) \\
 &+ 80\,000(0.6645)(P/F, 11\%, 3) \\
 = &-150\,000(0.6811) + 60\,000(0.48)(2.4437) \\
 &+ 80\,000(0.6645)(0.73119) \\
 = &7084
 \end{aligned}$$

$$\begin{aligned}
 P_{X'} &= -200\,000(0.6811) + 75\,000(1 - 0.52)(P/A, 11\%, 3) \\
 &+ 100\,000(0.6645)[(P/F, 11\%, 3) \\
 = &-200\,000(0.6811) + 75\,000(0.48)(2.4437) + 100\,000(0.6645)(0.73119) \\
 = &340
 \end{aligned}$$

Model T is the best choice because it has the highest present worth.

$$\begin{aligned}
 8.34 \quad AW_T &= -100\,000(0.6811)(A/P, 11\%, 5) + 50\,000(1 - 0.52) \\
 &\quad + 20\,000(0.6645)(A/F, 11\%, 5) \\
 &= -100\,000(0.6811)(0.27057) + 50\,000(0.48) \\
 &\quad + 20\,000(0.6645)(0.16057) \\
 &= 7705
 \end{aligned}$$

$$\begin{aligned}
 AW_A &= -150\,000(0.6811)(A/P, 11\%, 5) + 60\,000(1 - 0.52) \\
 &\quad + 30\,000(0.6645)(A/F, 11\%, 5) \\
 &= -150\,000(0.6811)(0.27057) + 60\,000(0.48) \\
 &\quad + 30\,000(0.6645)(0.16057) \\
 &= 4358
 \end{aligned}$$

$$\begin{aligned}
 AW_X &= -200\,000(0.6811)(A/P, 11\%, 3) + 75\,000(1 - 0.52) \\
 &\quad + 100\,000(0.6645)(A/F, 11\%, 3) \\
 &= -200\,000(0.6811)(0.40921) \\
 &\quad + 75\,000(0.48) + 100\,000(0.6645)(0.29921) \\
 &= 140
 \end{aligned}$$

Model T is the best choice because it has the highest annual worth.

8.35 First find the before-tax IRR for each model:

$$-100\,000 + 50\,000(P/A, i\%, 5) + 20\,000(P/F, i\%, 5) = 0$$

$$\Rightarrow IRR_{T\text{-before-tax}} = 43.11\%$$

i =	42%	43%	43.11%	44%	45%
(P/A, i%, 5) =	1.9686	1.9367	1.9334	1.9057	1.8755
(P/F, i%, 5) =	0.1732	0.1672	0.1666	0.1615	0.1560
PW(disbursements) =	-100000	-100000	-100000	-100000	-100000
PW(receipts) =	101892	100178	100000	98514	96897
PW(total) =	1892	178	0	-1486	-3103

$$-150\,000 + 60\,000(P/A, i\%, 5) + 30\,000(P/F, i\%, 5) = 0$$

$$\Rightarrow IRR_{A\text{-before-tax}} = 31.39\%$$

i =	30%	31%	31.39%	32%	33%
(P/A, i%, 5) =	2.4356	2.3897	2.3723	2.3452	2.3021
(P/F, i%, 5) =	0.2693	0.2592	0.2554	0.2495	0.2403
PW(disbursements) =	-150000	-150000	-150000	-150000	-150000
PW(receipts) =	154214	151156	150000	148198	145337
PW(total) =	4214	1156	0	-1802	-4663

$$-200\,000 + 75\,000(P/A, i\%, 3) + 100\,000(P/F, i\%, 3) = 0$$

$$\Rightarrow IRR_{X\text{-before-tax}} = 24.30\%$$

i =	23%	24%	24.30%	25%	26%
(P/A, i%, 3) =	2.0114	1.9813	1.9724	1.9520	1.9234
(P/F, i%, 3) =	0.5374	0.5245	0.5207	0.5120	0.4999
PW(disbursements) =	-200000	-200000	-200000	-200000	-200000
PW(receipts) =	204591	201046	200000	197600	194248
PW(total) =	4591	1046	0	-2400	-5752

Then calculate the approximate after-tax IRR:

$$IRR_{T\text{-after-tax}} \cong IRR_{T\text{-before-tax}} \times (1 - 0.52) = 0.4311(0.48) = 0.2069 = 20.69\%$$

$$IRR_{A\text{-after-tax}} \cong IRR_{A\text{-before-tax}} \times (1 - 0.52) = 0.3139(0.48) = 0.1507 = 15.07\%$$

$$IRR_{X\text{-after-tax}} \cong IRR_{X\text{-before-tax}} \times (1 - 0.52) = 0.2430(0.48) = 0.1167 = 11.67\%$$

8.36 Using trial and error in a spreadsheet program results in:

$$IRR_{T\text{-after-tax}} = 20.64\%$$

i =	19%	20%	20.64%	21%	22%
CCTF(old) =	0.7333	0.7400	0.7441	0.7463	0.7524
CCTF(new) =	0.7546	0.7617	0.7660	0.7684	0.7747
(P/A, i%, 5) =	3.0576	2.9906	2.9490	2.9260	2.8636
(P/F, i%, 5) =	0.4190	0.4019	0.3913	0.3855	0.3700
PW(disbursement s) =	-75462	-76167	-76599	-76835	-77471
PW(receipts) =	79529	77722	76599	75979	74295
PW(total) =	4067	1556	0	-857	-3176

$$IRR_{A\text{-after-tax}} = 14.67\%$$

i =	13%	14%	14.67%	15%	16%
CCTF(old) =	0.6848	0.6941	0.7001	0.7029	0.7111
CCTF(new) =	0.7030	0.7129	0.7192	0.7222	0.7310
(P/A, i%, 5) =	3.5172	3.4331	3.3783	3.3522	3.2743
(P/F, i%, 5) =	0.5428	0.5194	0.5043	0.4972	0.4761
PW(disbursements) =	-105447	-106935	-107886	-108335	-109655
PW(receipts) =	112448	109688	107886	107025	104457
PW(total) =	7001	2753	0	-1310	-5198

$$IRR_{X\text{-after-tax}} = 11.08\%$$

i =	10%	11%	11.08%	12%	13%
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CCTF(old) =	0.6533	0.6645	0.6653	0.6750	0.6848
CCTF(new) =	0.6691	0.6811	0.6820	0.6924	0.7030
(P/A, i%, 3) =	2.4869	2.4437	2.4405	2.4018	2.3612
(P/F, i%, 3) =	0.7513	0.7312	0.7297	0.7118	0.6931
PW(disbursements) =	-133818	-136228	-136406	-138482	-140595
PW(receipts) =	138613	136563	136406	134511	132465
PW(total) =	4794	335	0	-3971	-8130

8.37 First calculate the before-tax IRR of the alternative with the cheapest first cost, T.

$$-100\,000 + 50\,000(P/A, i\%, 5) + 20\,000(P/F, i\%, 5) = 0$$

$$\Rightarrow \text{IRR}_{T\text{-before-tax}} = 43.11\%$$

i =	42%	43%	43.11%	44%	45%
(P/A, i%, 5) =	1.9686	1.9367	1.9334	1.9057	1.8755
(P/F, i%, 5) =	0.1732	0.1672	0.1666	0.1615	0.1560
PW(disbursements) =	-100000	-100000	-100000	-100000	-100000
PW(receipts) =	101892	100178	100000	98514	96897
PW(total) =	1892	178	0	-1486	-3103

Then calculate the after-tax IRR:

$$\text{IRR}_{T\text{-after-tax}} \cong \text{IRR}_{T\text{-before-tax}} \times (1 - 0.52) = 0.4311(0.48) = 0.2069 = 20.69\%$$

This is higher than the after-tax MARR, so that alternative T is acceptable. Next check the before-tax IRR of the increment of investment from T to A:

$$[-150\,000 - (-100\,000)] + (60\,000 - 50\,000)(P/A, i\%, 5) + (30\,000 - 20\,000)(P/F, i\%, 5) = 0$$

$$-50\,000 + 10\,000(P/A, i\%, 5) + 10\,000(P/F, i\%, 5) = 0$$

$$\Rightarrow \text{IRR}_{A-T \text{ before-tax}} = 5.73\%$$

i =	4%	5%	5.73%	6%	6%
(P/A, i%, 5) =	4.4518	4.3295	4.2432	4.2124	4.2124
(P/F, i%, 5) =	0.8219	0.7835	0.7568	0.7473	0.7473
PW(disbursements) =	-50000	-50000	-50000	-50000	-50000
PW(receipts) =	52737	51130	50000	49596	49596
PW(total) =	2737	1130	0	-404	-404

Then calculate the after-tax IRR:

$$IRR_{A-T \text{ after-tax}} \cong IRR_{A-T \text{ before-tax}} \times (1 - 0.52) = 0.0573(0.48) = 0.0258 = 2.75\%$$

Alternative A is rejected because the after-tax IRR is less than the after-tax MARR. We next look at the increment of investment from T to X. We have to be careful since X has a duration of 3 years, while T has 5 years. We could use a 15-year study period, or a 3-year study period using the information from Problem 8.33, or look at the increment on an annual basis. Using a 3-year study period:

$$\begin{aligned} & [-200\,000 - (-100\,000)] + (75\,000 - 50\,000)(P/A, i\%, 3) \\ & + (100\,000 - 40\,000)(P/F, i\%, 3) = 0 \\ & -100\,000 + 25\,000(P/A, i\%, 3) + 60\,000(P/F, i\%, 3) = 0 \\ \Rightarrow IRR_{X-T \text{ before-tax}} &= 13.29\% \end{aligned}$$

i =	12%	13%	13.29%	14%	15%
(P/A, i%, 3) =	2.4018	2.3612	2.3495	2.3216	2.2832
(P/F, i%, 3) =	0.7118	0.6931	0.6877	0.6750	0.6575
PW(disbursements) =	-100000	-100000	-100000	-100000	-100000
PW(receipts) =	102753	100612	100000	98539	96532
PW(total) =	2753	612	0	-1461	-3468

Then calculate the after-tax IRR:

$$IRR_{X-T \text{ after-tax}} \cong IRR_{X-T \text{ before-tax}} \times (1 - 0.52) = 0.1329(0.48) = 0.0598 = 6.38\%$$

Since the after-tax IRR of the incremental investment to X from T is less than the MARR, Model T should be chosen.

8.38 d = 0.30, t = 0.40

Ordered by first cost, the alternatives are:

- 1) Free machine, first cost \$6 000
- 2) Used machine, first cost = \$36 000
- 3) Owned machine, first cost = \$71 000

The incremental IRR from do-nothing to alternative 1:

$$\begin{aligned} 6000 &= (20\,000 - 15\,000)(P/A, i, 6) \\ (P/A, i, 6) &= 1.2 \end{aligned}$$

Noting that $(P/A, 40\%, 6) = 2.16$ and $(P/A, 50\%, 6) = 1.8$, the free machine has a before-tax IRR of greater than 50%. That is, the free machine has an

approximate after-tax IRR of greater than $50\% \times (1 - 0.4) = 30\%$. The free machine is acceptable.

Next, look at the increment of investment between the free and used machines:

Year	Increment: used – free
0	–30000
1	12000
2	9500
3	7000
4	4500
5	2000
6	–500

An ERR method was used since the incremental cash flows were not a simple investment. From the result of Problem 5.21, the ERR was found to be 8.78%. Then the approximate after-tax ERR is computed as $8.78\% \times (1 - 0.4) = 5.268\% < \text{MARR}$. The incremental investment between the free and used machines is not warranted.

We now look at the increment of investment between the free and new machines:

Year	Increment: new – free
0	–65000
1	15000
2	15000
3	15000
4	15000
5	15000
6	25000

This is a simple investment. From the result of Problem 5.21, the IRR for this incremental investment was found to be 12.88%. The approximate after-tax IRR is then computed as $12.88\% \times (1 - 0.4) = 7.728\% < \text{MARR}$. Hence, the new machine is not a good investment also.

Conclusion: the free machine is best.

- 8.39 a)** Setting disbursements, adjusted by the tax rate, equal to receipts at the end of year 2, taking cash on hand forward at the MARR:

$$8000(0.5)(F/P, 6\%, 1) + 8000(0.5) = 10\,000(0.5)(F/P, 6\%, 2) + 5500(0.5)(P/F, i^*, 1)$$

Solving for i^* gives $i^* = 4.88\% = \text{ERR}$

Note that the tax rate is immaterial to the calculation, as long as there are no depreciable assets involved in the cash flows.

- b)** Setting disbursements equal to receipts at the end of the three year period, with receipts taken forward at the MARR:

$$8000(0.5)(F/P, i^*, 2) + 8000(0.5)(F/P, i^*, 1) = 10\,000(0.50)(F/P, 6\%, 3) + 5500(0.5)$$

Solving for i^* gives $i^* = 5.76\% = \text{approximate ERR}$. (Recall that the approximate ERR will always be between the accurate ERR and the MARR.)

- c)** This is not a good investment

Notes for Case-in-Point 8.1

- 1)** Pros include the government having money to pursue social programs. Cons include businesses and high-income individuals leaving the country
- 2)** Pros include having a positive environment for business and individuals to generate wealth, which in turn creates more tax revenue. Cons include the government having less money in the short term for social programs
- 3-4)** Depends on the views of the individual student.

Notes for Mini-Case 8.1

- 1) People may have different views, but Accountants, and government tax department workers would likely be against flat taxes, while most other groups mentioned would either be neutral about flat taxes, or favour them.
- 2) The stock market would likely rise, both because corporate profits would likely increase and because the national economy will be more efficient.
- 3) Companies might make marginally more investment, but there would not be a dramatic change in behaviour. There is no reason society should be particularly better off or worse off.

Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

8S.1

The best approach is to calculate the present worth of the purchase price and salvage value, convert this to an equivalent annual cost, then add on the operating expenses, which are already expressed as an equivalent annual cost.

Under Australian tax law, the annual depreciation over the twelve-year life will be $150\% / 12 = 12.5\%$. So the present worth of the purchase is

$$PW = -9600 + 9600(P/A, -12.5\%, 8\%, 12) \times 0.5 \times 0.125$$

We recall that $(P/A, g, i, N) = (P/A, i^0, N) / (1 + g)$

$$\text{where } i^0 = (1+i)/(1+g) - 1 = (1+0.08) / (1-0.125) - 1 = 23.4\%$$

$$\text{So the PW of the purchase is } -9600 + 9600 \times 3.9307 \times 0.5 \times 0.125 = -7241$$

After 12 years, the book value of the machine will be $9600 (1-0.125)^{12} = \$954$. The salvage value is greater than this, so the difference of \$656 is taxable income, on which the company pays $\$656 \times 0.5 = \328 in taxes. So the after-tax present worth of the salvage value is $328(P/F, 0.08, 12) = \$130$

We convert the after-tax present worths of purchase and salvage to an annuity over twelve years, and subtract the after-tax annual cost of running the machine:

$$AW = (-7241 + 130)(A/P, 0.08, 12) - 2100(0.5) = -1993$$

So it costs $-\$1993$ to run the machine. (We cannot conclude from this whether or not the company should buy it—that depends on the value to the company of the service it provides.)

8S.2

The best way to solve this is to create a spreadsheet, such as **8S_2.xls**. The first three columns of this spreadsheet are taken from Table 8.8 in the textbook. The difference in allowable depreciation for each year is simply the difference between the second and third column, multiplied by the total cost of the equipment, \$800 000, and divided by 100

to convert from percentages to fractions. Since the company is taxed at 50%, it saves half the difference in allowable depreciation every year. Finally, we find the total present value of each year's savings, which is \$12 940.

8S.3

The simplest way of approaching this question is to consider the difference in cash flows that results from selling the old machine and buying the new one.

To deal with the initial and salvage costs, we calculate the tax benefit factor, or "TBF" (see page 283 of the text). The TBF is:

$$\text{TBF} = td / (i+d) = 0.52 \times 0.25 / (0.2+0.25) = 0.289$$

So if we sell the old machine after five years, the after-tax value of the price we receive is

$$1000 \times (1-0.289) = \text{£}711.$$

Similarly, if we buy the new machine and sell it for salvage after five years, its after-tax value to the company at that time will be £4267.

So the present cost of selling the old machine and buying the new one, together with the resultant savings in operating costs, is:

$$\begin{aligned} \text{PW} &= (24\,000-8000)(1-\text{TBF}) - (6000-1000)(1-\text{TBF})(P/F\,0.2,5) - (19\,000-12\,000)(1-t)(P/A,0.2,5) \\ &= 16\,000 \times 0.711 - 5000 \times 0.711 \times 0.4019 - 7000 \times 0.48 \times 2.9906 \\ &= 11\,376 - 1429 - 10\,048 \\ &= -101 \end{aligned}$$

Buying the new machine leaves the company just £101 better off. In principle, this implies the company should buy it, but it might be more reasonable to say that financial analysis does not identify a clear winner—the margins of error in our estimates of salvage price and operating costs are almost certainly more than £101.

8S.4

The pre-tax version of this problem can be solved by application of the formulae we already know, but, since the after-tax version is most conveniently solved by use of a spreadsheet, we shall use the same method for both versions. We assume that all operating and maintenance costs are paid at the end of the year in which they occur.

From spreadsheet **8S_4a(PreTax).xls**, we see that the Mandela option has the lowest present cost, and should therefore be chosen. We then repeat the study for the after-tax case (spreadsheet **8S_4b(AfterTax).xls**). The waterproofing option now looks like the best; the fact that we can deduct the cost of the waterproofing treatment from the tax bill in the first year gives it an advantage.

8S.5

Consider a study period of five years. Suppose that the salvage value of the workstations drops at the same rate as their cost. If we were doing a pre-tax analysis, the present cost of keeping your current equipment another N years would be $\$100\,000 (P/A, 0.2, 0.2, N)$.

However, you can now deduct the cost of your repairs from your pre-tax cash flow, which effectively halves their cost. Adjusting the MARR to its after-tax value of 0.1, the after-tax present cost is now:

$$\$50\,000 (P/A, 0.2, 0.1, N)$$

The conversion factor can be evaluated as

$$(P/A, 0.2, 0.1, N) = ((1 + i^0)^N - 1) / (i^0 (1 + i^0)^N (1 + g))$$

$$\text{where } i^0 = (1 + i) / (1 + g) - 1 = 1.1 / 1.2 - 1 = -0.083$$

The pre-tax present cost of bringing in low-end workstations after N years would be

$$\$500\,000((0.6N)(P/F, 0.2, N) - (0.6\,5)(P/F, 0.2, 5))$$

To find the after-tax cost, we adjust both the initial cost and the salvage revenue using the tax benefit factor, or “TBF” (see page 283 of the text). The TBF is:

$$\text{TBF} = td / (i + d) = 0.5 \times 0.3 / (0.1 + 0.3) = 0.375$$

And we reduce the income from our salvage revenue by a factor $(1 - \text{TBF})$ to take into account the loss of future depreciation tax benefits. When we correct the purchase cost, we have to take into account the “half-year rule”; from Example 8.9 in the text, we note

that application of this rule implies that the effective reduction in purchase cost is given by the factor:

$(1 - \text{TBF} \times 0.5 \times (1 + (P/F, i, 1)))$, which we shall call " $1 - \text{TBF}^*$ ". In this case,

$$\begin{aligned} 1 - \text{TBF}^* &= 1 - \text{TBF} \times 0.5 \times (1 + (P/F, 0.1, 1)) \\ &= 1 - 0.375 \times 0.5 \times (1 + 0.9091) \\ &= 1 - 0.358 \\ &= 0.642 \end{aligned}$$

So the after-tax cost of bringing in low-end workstations after N years is

$$\$500\,000((0.6N)(P/F, 0.1, N)(1 - \text{TBF}^*) - (0.65)(P/F, 0.1, 5)(1 - \text{TBF}))$$

Similarly, the pre-tax present cost of bringing in high-end workstations after N years would be:

$$\begin{aligned} &\$750\,000((0.7N)(P/F, 0.2, N) - (0.75)(P/F, 0.2, 5)) - \\ &100\,000(P/A, 0.2, (5 - N))(P/F, 0.2, N) \end{aligned}$$

We adjust the initial and salvage prices by the factors $(1 - \text{TBF}^*)$ and $(1 - \text{TBF})$ respectively, and note that the benefit of the salaries saved is reduced by the factor $(1 - t)$, or 50%.

So the after-tax cost of bringing in high-end workstations after N years is

$$\begin{aligned} &\$750\,000((0.7N)(P/F, 0.2, N)(1 - \text{TBF}^*) - (0.75)(P/F, 0.2, 5)(1 - \text{TBF})) - 50\,000 \\ &(P/A, 0.1, (5 - N))(P/F, 0.1, N) \end{aligned}$$

Lastly, the pre-tax present cost of starting to rent workstations after N years would be

$$\$100\,000(P/A, 0.2, (5 - N))(P/F, 0.2, N)$$

Since the rental cost can be deducted from our pre-tax cash flow, the after-tax cost of rental is reduced by the factor $(1 - t)$. So the present value of the after-tax rental cost is:

$$\$50\,000(P/A, 0.1, (5 - N))(P/F, 0.1, N)$$

The present value of keeping your current equipment N years, then switching to one of the three alternatives, is calculated in the accompanying spreadsheet, **8S_5.xls**. All figures on the spreadsheet are in thousands of dollars. Comparing the last three columns, we see that the best strategy is to wait two years, then replace your old machines with high-end workstations.

8S.6

The leverage factor is defined as debt/assets. So at the beginning of the three years, it is $250 / 350 = 0.714$; at the end of the three years it is $(250 \times 1.13) / (350 \times 1.153) = 0.625$.

The increase in your wealth is $350 \times 1.153 - 250 \times 1.13 - 100\,000 = \$99\,556$.

An alternative acceptable solution is to assume that you pay the interest on the \$250 000 every year, so that your indebtedness does not increase. Under this assumption, your leverage factor after three years would be 0.47, and your personal wealth has increased by \$182 300.

After three years, we can assume that the shareholders, including yourself, expect a continuing return of 15% on their investment. Then the weighted cost of capital is

$$WCC = (332\,750 \times 0.1 + 400\,000 \times 0.15) / 732\,750 = 0.127$$

The lowest reasonable figure for the after-tax MARR would then be 0.127. It would be prudent to increase this somewhat, to account for the uncertainty in any investment. So you could take 15% as an after-tax MARR. Your pre-tax MARR should then be $0.15 / 0.8 = 0.187$ (or 0.19, rounding off.)

To determine whether to buy or rent the chromatograph you should carry out an after-tax analysis, since the tax consequences of the two choices are different. We use the “TBF*” factor, as defined in Solution 8S.5:

Buying

The TBF* is $1 - (1 + 0.5 \times 0.15) / (1 + 0.15) \times (0.2 \times 0.2) / (0.15 + 0.2) = 0.89$.

So the present value of buying is

$$PV = -150\,000 \times 0.89 + 50\,000 \times 0.89 \times (P/F, 0.15, 5) = -133\,500 + 22\,120 = -111\,400$$

Leasing

The present value of leasing is

$$\begin{aligned} PV &= -50\,000 \times (P/A, 0.15, 5) \times 0.8 \\ &= -50\,000 \times 3.352 \times 0.8 = -134\,000 \end{aligned}$$

So it is better to buy.

After five years, the annual rate of return is $(1\,200\,000 / 400\,000)^{0.2} = 25\%$. To see if it is still better to buy, repeat the after-tax analysis of the two options with a tax rate of 50%:

Buying

The TBF* is now $1 - (1 + 0.5 \times 0.15) / (1 + 0.15) \times (0.5 \times 0.2) / (0.15 + 0.2) = 0.73$.

So the present value of buying is

$$\begin{aligned} PV &= -150\,000 \times 0.73 + 50\,000 \times 0.73 \times (P/F, 0.15, 5) \\ &= -109\,500 + 18\,150 = -91\,350 \end{aligned}$$

Leasing

The present value of leasing is

$$\begin{aligned} PV &= -50\,000 \times (P/A, 0.15, 5) \times 0.5 \\ &= -50\,000 \times 3.352 \times 0.5 = -83\,800 \end{aligned}$$

So leasing is now the more attractive option.