Computer Science 225
Final Exam
August 9, 1991
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Question 1
part a
Solve the following recurrence relation where T(n, m) = 1 for all n, m < 0:
T(n,m) = 1 + T(n‹m, 2m)

Question 1
part b
Prove that the following recurrence relation is O(n log n) for all a > 1.

```
T(n) = n + aT(floor(n/a))
T(l) = l,T(O) = O
```

Question 2
Write a Pascal procedure that takes a parameter T, a binary tree and returns the value true if T is a complete binary tree and returns the value false otherwise. Note: you may use any of the data structures defined in class and you may also assume the fol lowing type declarations:

```
type
        ptr = ^cell;
        cell = record
                data:integer;
                left,right:ptr;
        end;
```

Question 3
part a
Construct an AVL binary search tree by successively adding the following values: 13 8 19 4 11 9 21 25 17 18

part b
Draw the the threads that would be on this tree if the tree were threaded.

Question 4
Construct the huffman codes for each of the characters that occur in the example sentence below. Note that you do not need to include a carriage return or the ". " in your code.

this sentence is the example sentence.

CODES:

```
a=_____ c=_____ e=_____ h=_____ i=_____
l=_____ m=_____ n=_____ p=_____ s=_____
t=_____ x=_____ blank=_____
```

Question 5
Imagine that you need to solve a *graph* problem, and you know that although there are over two hundred thousand vertices, there are not more than five hundred edges. Design and code an implenentation of the following abstract data type so that you c ould solve the problem in question. Note: your mark will be affected by the time complexity of your solution.

procedure InitGraph(var G:graph);
function adjacent(var G:graph; u.v:vertex):boolean;
procedure addedge(var G:graph: u,v:vertex);

Question 6
Write a Pascal function called *IsDoubleLeftPossible* that takes a single parameter T of type *ptr* and returns the value true if the tree with root T can have a double left rotation performed on it at the node T and returns the value false othe rwise.

Question 7
Using the heapsort algorithm, sort the following list of numbers: 12 7 3 18 9 l4 6 2 11 8. Show sufficient work so that it can be determined that you understand the algorithm.

Question 8
Part a
Find the minimum cost spanning tree of the weighted graph below and indicate the work done to determine the tree:

Part b Find the shortest path from vertex A to vertex I.

Question 9
Binary guess question (true/false)
a) On average, all operations on a hash table are O(l)
b) On average, all operations on a nearly full hash table are 0(1)
c) It is possible to find the value of the k the largest element from amongst n elements in worst case O(n) time.
d) The best program that reads in a sequence of n test scores and writes them out in sorted order would take order O(n log n) time.
e) Your answers to questions a to d are all correct.
f) Implementing an n x n array always takes n2 space.
g) The best possible implementation of quick sort has worst case order n2 time complexity.
h) Every implementation of mergesort requires a minimum of order n space in addition to the space required to store the data being sorted.
i) The problem of determining if a graph has a cycle of length n‹1 where n is the number of

vertices is NP‹complete.
j) Your answers to questions a to d are all correct.