CHAPTER 5

Solutions to Chapter-End Problems

A. Key Concepts

Choice of Methods:

5.1 Both the leasing costs and demand are on a monthly basis. The annual worth method would be best.

5.2 This is a relatively inexpensive purchase. If a comparison method was used at all, it would probably be payback period.

5.3 If Joan was relatively unsophisticated, a simple payback period would be adequate. If she wanted a more detailed analysis, the annual worth method would probably be best since heating has a natural yearly cycle.

5.4 Since for a large company there is probably detailed information available about the benefits and costs of new scales, IRR would be the likely choice. An alternative would be present worth.

5.5 Payback period is the likely choice since Mona probably won't have the inclination to do a more detailed analysis. If she did want a more detailed analysis, annual worth is a likely choice.

5.6 Payback period is ideal when cash flow is tight and there is limited time to make a decision.

5.7 Lemuel would likely use IRR or present worth, as mandated by the large company he works for.

5.8 An annual worth comparing the monthly leasing costs to expected net savings would probably be the choice.

5.9 Payback period would probably be appropriate since there is a need to have capital recovered quickly.

IRR for Independent Projects:

5.10 **(a)** 1000 = 200(P/A, i, 7)
(P/A, i, 7) = 5

(P/A, 10%, 7) = 4.8683
(P/A, 9%, 7) = 5.0329

Linear interpolation:
i = 9 + (5 – 5.0329)/(4.8683 – 5.0329) = 9.1998

The IRR is about 9.2%.

(b) 1000 = 200(P/A, i, 6)
(P/A, i, 6) = 5

(P/A, 6%, 6) = 4.9172
(P/A, 5%, 6) = 5.0756

Linear interpolation:
i = 5 + (5 – 5.0756)/(4.9172 – 5.0756) = 5.48

The IRR is about 5.48%.

(c) 1000 = 200(P/A, i, 100)
(P/A, i, 100) = 5

This is not available in tables to infer, so trial and error calculations must be done from the basic formula:

$1000 = 200[(1 + i)^{100} – 1]/[i(1 + i)^{100}]$

Such trial and error experiments give an IRR of 20%.

(d) 1000 = 200(P/A, i, 2)
(P/A, i, 2) = 5

Again using trial and error in the formula:

$1000 = 200[(1 + i)^{2} – 1]/[i(1 + i)^{2}]$

The result is an IRR of –44.2%.

5.11 Setting disbursements equal to receipts (present worth) and solving for i*:

8000 = 400(P/A, i*, 30)
(P/A, i*, 30) = 20

(P/A, 2%, 30) = 22.396
(P/A, 3%, 30) = 19.6

The IRR for this project is between 2% and 3%. This is less than NF's MARR, so the windows are not a good investment.

5.12 Setting the PW of disbursements equal to the PW of receipts:

200 000 + 40 000(P/A, i*, 6)
= 40 000/i* + 20 000(P/A, i*, 5)
– 5 000(A/G, i*, 5)(P/A, i*, 5)
40i* + 8i*(P/A, i*, 6) – 4i*(P/A, i*, 5) + i*(A/G, i*, 5)(P/A, i*, 5) = 8

At i* = 12%: LHS = 7.7842
At i* = 13%: LHS = 8.3319

Linearly interpolating:
i* = 12 + (8 – 7.7842)/(8.3319 – 7.7842) ≅ 12.39%

This is a good investment since the IRR exceeds the MARR.

5.13 The IRR for each alternative can be found by solving for i in each of the following equations:

Alternative A:
–100 000 + 25 000(P/F,i,1) + 25 000(P/F, i, 2) + 25 000(P/F, i, 3) + 25 000(P/F, i, 4) + 25 000(P/F, i, 5) = 0

Alternative B:
–100 000 + 5 000(P/F, i, 1) + 10 000(P/F, i, 2) + 20 000(P/F, i, 3) + 40 000(P/F, i, 4) + 80 000(P/F, i, 5) = 0

Alternative C:
–100 000 + 50 000(P/F, i, 1) + 50 000(P/F, i, 2) + 10 000(P/F, i, 3) = 0

Alternative D:
–100 000 +1 000 000(P/F, i, 5) = 0

Using a trial and error approach with a spreadsheet, the IRRs for the individual projects are listed below:

Alternative	**IRR**
A	7.931%
B	11.294%
C	6.044%
D	58.489%

With a MARR of 8%, projects B and D are acceptable, project A is marginally unacceptable, and project C is unacceptable.

5.14 Increment from do-nothing to 1:
100 000 = 160 000(P/F, i*, 1) ⇒ i* = 60% > MARR

Increment is justified; we can accept contract 1.

Increment from 1 to 2:
100 000 = 140 000(P/F, i*, 1) $\Rightarrow$ i* = 40% > MARR

Increment is justified; we can accept contract 2.

Increment from 2 to 3:
50 000 = 55 000(P/F, i*, 1) $\Rightarrow$ i* = 10% < MARR

Increment from 2 to 3 cannot be justified.

Choose Contract #2.

5.15 First, the incremental IRR from do-nothing to A is found by solving for i in:
(P/A, i, 5) = 100 000/50 000 = 2 $\Rightarrow$ IRR = 41.1%

Since the IRR on A exceeds the MARR, A is the current best alternative. B then challenges A. The IRR on the incremental investment between A and B is:

–(400 000 – 100 000) + (150 000 – 50 000)(P/A, i, 5) = 0
(P/A, i, 5) = 300 000/100 000 = 3 $\Rightarrow$ IRR = 19.9%

Because the IRR on the incremental investment is above 10%, project B becomes the current best alternative. Since there are no other projects to challenge B, it should be taken.

5.16 Since alternative A has an IRR that exceeds the MARR, it is acceptable. The incremental IRR to the next cheapest alternative, B, is only 12%, less than the MARR, and so B is rejected. The incremental investment from A to C is 30%, so A is rejected in favour of C. The incremental investment from C to D is less than the MARR, so D is rejected. The incremental investment from C to E is equal to the MARR, and so E is accepted. The incremental investment from E to F is less than the MARR, and so E is the best choice.

5.17 **(a)** Setting disbursements equal to receipts at the end of year 2, taking *cash on hand* forward at the MARR:

8000(F/P, 6%, 1) + 8000 = 10 000(F/P, 6%, 2) + 5500(P/F, *i**, 1)
This could also have been written as:
(10 000(P/F, 6%, 1) – 8000)*(P/F, 6%,1) – 8000 + 5500(P/F, ***I*,*** 1)

Solving for ***I**** gives ***I**** = 4.88% = ERR

(b) Setting disbursements equal to receipts at the end of the three year period, with receipts taken forward at the MARR:

8000(F/P, ***I****, 2) + 8000(F/P, ***I****, 1) = 10 000(F/P, 6%, 3) + 5500

Solving for ***I**** gives ***I**** = 5.76% =approximate ERR. (Recall that the approximate ERR will always be between the accurate ERR and the MARR.)

(c) This is not a good investment

5.18 **(a)** 20 000 + 100 000(P/F, i, 2) = 120 000(P/F, i, 1)
$1 + 5/(1+i)^2 = 6/(1+i)$
$(1+i)^2 + 5 = 6(1+i)$
$i^2 + 2i + 1+ 5 = 6 + 6i$
$i^2 - 4i = 0$
$i(i-4) = 0$
$i = 0$ or $4 \Rightarrow$ IRR of 0% or 400%

Both IRRs are mathematically correct, as can be seen by looking at *project balances* at yearly intervals:

End of year	At i = 0%	At i = 400%
0	20 000	20 000
1	20 000 (1 + 0) – 120 000 = –100 000	20 000(1 + 4) – 120 000 = –20 000
2	–100 000(1 + 0) + 100 000 = 0	–20 000(1 + 4) + 100 000 = 0

These results are not realistic since the money in hand for the first year cannot be invested at the same rate at the rest of the project: it will not be 0 or 400%.

(b) The exact ERR can be calculated in this case by taking the $20 000 forward at the MARR for one year, and then equating the cash flows at that time.

20 000(F/P, 12%, 1) + 100 000(P/F, i, 1) = 120 000
1 + i =100 000/[1 20 000 – 20 000(1.12)] = 1.024 59
i = 2.46% = ERR

The ERR (the correct IRR) is about 2.46%. The project should not be accepted. Calculating an accurate ERR is generally difficult because in complicated cash flows it's difficult to tell when there is "cash in hand."

(c) For the approximate ERR, take all receipts forward at the MARR to the time of the last cash flow, and equate it to the future worth of the disbursements at an unknown interest rate. The unknown interest rate is the approximate ERR:
20 000(F/P, 12%, 2) + 100 000 = 120 000(F/P, i, 1)
1 + i = [20 000(1.2544) +100 000]/1 20 000 = 1.0424
i = 4.24% = ERR (approximate)

Note that the approximate ERR is always between the accurate ERR and the MARR. This is good in that for acceptable projects the approximate ERR always errs on the *conservative* side, i.e., it is always smaller than actual.

B. Applications

5.19 **(a)** Because CB Electonix must buy equipment, the current best is P1, the alternative with the smallest first cost.

	P1	**P2**	**P3**
First cost ($)	0	200 000	850 000
Annual costs ($)	135 000	95 000	0

The first challenger is P2.

P2 – P1:
(200 000 – 0) + (95 000 – 135 000)(P/A, i, 10) = 0
(P/A, i, 10) = 200 000/40 000 = 5

From table lookups:
(P/A, 15%,10) = 5.0187
(P/A, 20%, 10) = 4.1924

IRR is between 15 and 20%; challenge succeeds. P2 becomes the current best. Next, P3 challenges P2.

P3 – P2:
(850 000 – 200 000) + (0 – 95 000)(P/A, i, 10) = 0
(P/A, i, 10) = 650 000/95 000 = 6.842

Table lookups:
(P/A, 7%, 10) = 7.0235
(P/A, 8%, 10) = 6.7100

IRR is between 7% and 8%. Challenge fails.

P2 is the best project.

(b) Consider the last challenge: P3 – P2. Solve for the IRR via linear interpolation:

(P/A, i, N) = 6.842
(P/A, 7%, 10) = 7.0235
(P/A, 8%, 10) = 6.7100

i = 8 + (7 – 8)[(6.842 – 6.7100)/(7.0235 – 6.7100)] = 7.5789

Below 7.58%, P3 becomes preferred.

5.20 **(a)** With a MARR of 16%, do projects 1, 3 and 4 as their IRR's meet or exceed 16%.

(b) For MARR = 15%: The current best is alternative 1, which has the least first cost and IRR > MARR.

Challenge 1 with 2: incremental IRR is 9%; challenge fails.

Challenge 1 with 3: incremental IRR is 17%; challenge succeeds. Hence, 3 is current best.

Challenge 3 with 4: incremental IRR is 13%; challenge fails.

Pick 3.

(c) For MARR = 17%: The current best is alternative 1, which has the least first cost and IRR > MARR.

Challenge 1 with 2: incremental IRR is 9%; challenge fails.

Challenge 1 with 3: incremental IRR is 17%; challenge still succeeds. Hence, 3 is current best.

Challenge 3 with 4: incremental IRR is 13%; challenge fails.

Pick 3.

5.21 Ordered by first cost, the choice are:
1) Free machine, first cost $6000
2) Used machine, first cost $36 000
3) Owned machine, first cost $71 000

The incremental IRR from do-nothing to alternative 1:

6000 = (20 000 –15 000)(P/A, i, 6)
(P/A, i, 6)= 1.2

Noting that (P/A, 40%, 6) = 2.16 and that (P/A, 50%, 6) = 1.8, the free machine is certainly acceptable.

Alternatively, accept free machine because one interprets the “requirement for a filling machine” as meaning that one of the projects must be chosen.

Look at the increment of investment between the free and used machines: an ERR method must be used since the incremental cash flows are not a simple investment.

Year	Free	Used	Increment	FW@MARR	FW@ERR
0	-6000	-36000	-30000		-49697
1	5000	17000	12000	19326	
2	5000	14500	9500	13909	
3	5000	12000	7000	9317	
4	5000	9500	4500	5445	
5	5000	7000	2000	2200	
6	5000	4500	-500		-500
				Net FW	**ERR**
				0.0003776	0.0877646

The ERR is approximately 8.78%. This is less than MARR, so the incremental investment between the free and used machines is not warranted.

Look at the increment of investment between the free and new machines: since the incremental cash flows form a simple investment, the IRR approach can be taken.

Year	Free	Own	Increment	PW@IRR
0	-6000	-71000	-65000	-65000
1	5000	20000	15000	13289
2	5000	20000	15000	11772
3	5000	20000	15000	10429
4	5000	20000	15000	9239
5	5000	20000	15000	8185
6	5000	30000	25000	12085
			IRR	**Net PW**
			0.12879	0.00433138

The IRR on the incremental investment can be determined as approximately 12.88%; hence, the investment is warranted. The new machine should be chosen.

5.22 Since this is not a simple investment, the approximate ERR should be used.

500 000(F/P, 25%, 4) + 1 200 000 = 400 000(F/P, i, 3) + 900 000(F/P, i, 2)

5(2.4414) + 12 = 4(F/P, i, 3) + 9(F/P, i, 2)
4(F/P, i, 3) + 9(F/P, i, 2) = 24.207

For i = 25: LHS = 4(1.9531) + 9(1.5625) = 21.87
For i = 30: LHS = 4(2.197) + 9(1.69) = 23.00

The approximate ERR is well above the MARR and the project should be accepted.

5.23 This problem requires the use of an explicit rate of return on the positive cash flows generated from the agreement. The cash flows associated with the project that must be invested elsewhere are the $450 000 (now) and the $650 000 at the end of the 12th year. The future value of these cash receipts (invested at MARR until the end of the 15th year) is equated to the future value of disbursements:

450 000(F/P, 20%, 15) + 650 000(F/P, 20%, 3)
= 900 000(F/P, i, 10) + 900 000(F/P, i, 5) + 900 000
9(15.407) + 13(1.728) = 18(F/P, i, 10) + 18(F/P, i, 5) + 18
(F/P, i, 10) + (F/P, i, 5) = 7.9515

At i = 20%: LHS = 6.1917 + 2.4883 = 8.68
At i = 15%: LHS = 4.0455 + 2.0113 = 6.057

A linear interpolation gives:
ERR = 15 + (5)(7.95 – 6.057)/(8.68 – 6.057) = 18.61%
(Note that, with a spreadsheet, one can obtain an ERR = 18.77% by trial and error rather than by interpolation.)

Samiran should not accept this deal because the ERR is less than his MARR of 20%. There is only one answer using ERR; it is not the accurate IRR, but nonetheless, it will give the correct recommendation.

5.24 Since one of the two alternatives must be chosen, the least first cost alternative (the custom automated equipment) becomes the current best.

To see if the off-the-shelf standard automated equipment should be chosen, we find the net incremental cash flows to select the off-the-shelf equipment. The least common multiple of service lives is 30 years.

	Automated		Off-the-shelf			FW	FW
Year	First	Annual	First	Annual	Increment	@MARR	@ERR
0	-15000		-25000		-10000		-133785
1		-6400		-5625	775	9433	
2		-6400		-5625	775	8655	
3		-6400		-5625	775	7940	
4		-6400		-5625	775	7284	
5		-6400		-5625	775	6683	
6		-6400		-5625	775	6131	

7		-6400		-5625	775	5625	
8		-6400		-5625	775	5160	
9		-6400		-5625	775	4734	
10	-15000	-6400		-5625	15775	88410	
11		-6400		-5625	775	3985	
12		-6400		-5625	775	3656	
13		-6400		-5625	775	3354	
14		-6400		-5625	775	3077	
15		-6400	-25000	-5625	-24225		-88607
16		-6400		-5625	775	2590	
17		-6400		-5625	775	2376	
18		-6400		-5625	775	2180	
19		-6400		-5625	775	2000	
20	-15000	-6400		-5625	15775	37345	
21		-6400		-5625	775	1683	
22		-6400		-5625	775	1544	
23		-6400		-5625	775	1417	
24		-6400		-5625	775	1300	
25		-6400		-5625	775	1192	
26		-6400		-5625	775	1094	
27		-6400		-5625	775	1004	
28		-6400		-5625	775	921	
29		-6400		-5625	775	845	
30		-6400		-5625	775	775	
					MARR	**Total FW**	**ERR**
					0.09	0.019482	0.090302

The incremental investment to the off-the-shelf system is not a simple investment. To make a decision, we will use the approximate ERR. Take the positive incremental cash flows forward to the end of 30 years at the MARR (9%) and take the negative cash flows forward to the end of 30 years at the unknown ERR. Setting FW(positive cash flows @ MARR) + FW(negative cash flows @ ERR) = 0, we find the approximate ERR to be 9.03%, which is slightly above the MARR. Since there are no other challengers, the best alternative is (marginally) the off-the-shelf material handling system.

5.25 Since one of the two alternatives must be chosen, the least first cost alternative (the Y19) becomes the current best.

To see if the XJ3 should be chosen, we find the net incremental cash flows to select the XJ3. The length of the study period is 2 years.

	Y19		**XJ3**		
Year	**First/Salv.**	**Annual**	**First/Salv**	**Increment**	**PW @ IRR**
0	-3200		-4500	-1300	-1300
1		-300		300	277
2	1000	-300	1900	1200	1023
				IRR	**Total PW**
				0.083055	0.0050529

The incremental investment to XJ3 is a simple investment. We find the IRR to be 8.3% which is below the MARR. Since there are no other challengers, the best alternative is Y19.

5.26 The lowest first cost project is #3 (first cost of –$175 000). Check the ERR of #3 since it is a non-standard investment:

175 000(F/P, 10%, 3) – 150 000 (F/A, i*, 3) + 300 000 = 0
(F/A, i*, 3) = [175 000(1.331) + 300 000]/150 000 = 3.553
$\Rightarrow$ i* = ERR = 17.4%

Project 3 is acceptable. Now look at the increment of investment to project #2. We still use ERR, but the disbursements and receipts change:

(150 000 – 175 000)(F/P, i*, 3) – (100 000 – 150 000)(F/A, 10%, 3)
+ (230 000 – 300 000) = 0
–25 000(F/P, i*, 3) – (–50 000)(F/A, 10%, 3) + (– 70 000) = 0
–25 000 (F/P, i*, 3) + 50 000(F/A, 10%, 3) – 70 000 = 0
(F/P, i*, 3) = (50 000$\times$3.31 – 70 000)/25 000 = 3.82
$\Rightarrow$ i* = ERR = 56.3%

Project 2 is acceptable. Now look at the increment of investment to project #1. We still use ERR.

(100 000 – 150 000)(F/P, i*, 3) – (75 000 – 100 000)(F/A, 10%, 3)
+ (200 000 – 230 000) = 0
–50 000(F/P, i*, 3) – (–25 000)(F/A, 10%, 3) + (– 30 000) = 0
–50 000(F/P, i*, 3) + 25 000(F/A, 10%, 3) – 30 000 = 0
(F/P, i*, 3) = (25 000$\times$3.31 – 30 000)/50 000 = 1.055
$\Rightarrow$ i* = ERR = 1.80%

The increment from #2 to #1 is not acceptable.

CCC should do project #2.

5.27 The three proposals imply seven mutually exclusive projects:

Project	Proposal	Net first cost	Net annual savings
1	do nothing	0	0
2	A	40 000	20 000
3	B	110 000	30 000
4	C	130 000	45 000
5	AB	150 000	50 000
6	AC	170 000	65 000
7	BC	240 000	75 000

The do-nothing alternative is the current best. First check the incremental investment from do-nothing to Project #2.

40 000 = 20 000(P/A, i*, 4)
(P/A, i*, 4) = 2 $\Rightarrow$ i* = IRR = 34.9%

Project #2 is acceptable. Now check the increment to project #3:

70 000 = 10 000(P/A, i*, 4)
(P/A, i*, 4) = 7$\Rightarrow$ i* = IRR < 0

Project #3 is unacceptable. Now check the increment from #2 to #4:

90 000 = 25 000(P/A, i*, 4)
(P/A, i*, 4) = 3.6 $\Rightarrow$ i* = IRR = 4.4%

Project #4 is unacceptable. Now check the increment from #2 to #5:

110 000 = 30 000(P/A, i*, 4)
(P/A, i*, 4) = 3.66 $\Rightarrow$ i* = IRR = 3.6%

Project #5 is unacceptable. Now check the increment from #2 to #6:

130 000 = 45 000(P/A, i*, 4)
(P/A, i*, 4) = 2.88 $\Rightarrow$ i* = IRR = 14.5%

Project #6 is acceptable. Now check the increment from #6 to #7:

70 000 = 10 000(P/A, i*, 4)
(P/A, i*, 4) = 7 $\Rightarrow$ i* = IRR < 0

Project 7 is unacceptable. Project 6 is the project with the highest first cost for which all increments of investments are justified.

Kool Karavans should invest in proposals A and C.

5.28 The total costs for the checkweigher project are: 30 000 + 5 000$\times$5 = \$55 000. The grant would amount to 55 000$\times$0.3 = \$16 500 in two payments of \$8250.

The total costs for the scheduler project are: 10 000 + 12 000$\times$5 = \$70 000. The grant would amount to 70 000$\times$0.3 = \$21 000 in two payments of \$10 500.

For an incremental IRR comparison, the alternative with the lowest first cost is the scheduler—its first cost is negative. Since there are no net disbursements for this project, it is accepted. The cash flows for the two projects and for the increment of investment from Scheduler to Checkweigher are shown below:

Year	Checkweigher	Scheduler	Increment
0	-21750	500	-22250

1	9000	5000	4000
2	9000	5000	4000
3	9000	5000	4000
4	9000	5000	4000
5	25250	15500	9750

The increment conforms to a simple investment. Calculating the IRR gives:

4000(P/A, i, 5) + 9750(P/F, i, 5) = 22250
16(P/A, i, 5) + 23(P/F, i, 5) = 89
$\Rightarrow$ i = IRR= 4.39%

The increment is below the MARR, so the scheduling project should be done.

5.29 Model A has the least expensive first cost, so since one heating system must be chosen, it can be accepted. Next look at the increment from A to B. The cash flows of models A and B and the increment from A to B are shown below:

Year	Model A	Model B	Increment
0	-500	-3600	-3100
1	-300	500	800
2	-300	500	800
...	...	...	...
9	-300	500	800
10	-300	1500	1800

The IRR of the increment is then calculated from:

3100 = 800(P/A, i, 10) + 1000(P/F, i, 10)
8 (P/A, i, 10) + 10 (P/F, i, 10) = 31
$\Rightarrow$ i = IRR = 23.63%

Since this is greater than the MARR, Model B is preferred over Model A.

Now look at the increment to Model C. The cash flows of models B and C and the increment from B to C are shown below:

Year	Model B	Model C	Increment
0	-3600	-4000	-400
1	500	1000	500
2	500	-3000	-3500
3	500	1000	500
...	...	...	...
9	500	1000	500

10	1500	2000	500

The increment does not followthe pattern of a simple investment, so ERR must be used:

400(F/P, ERR, 10) + 3500(F/P, ERR, 8)
= 500(F/P, 12%, 9) + 500(F/A, 12%,8)
4(F/P, ERR, 10) + 35(F/P, ERR, 8) = 5(2.7731) + 5(12.3) = 75.36
$\Rightarrow$ i = ERR = 8.34%

The ERR is less than the MARR, so Model C is not justified. Jacob should buy model B.

C. More Challenging Problems

5.30 If the projects are independent, they should both be undertaken (provided there are funds) because they each have an IRR which exceeds the MARR.

If the projects are mutually exclusive, then an incremental analysis must be performed. Initially, project Y would be chosen as the current best alternative because it has the smallest first cost and it has an IRR greater than the MARR. Next, the IRR on the incremental investment between Y and X would need to be found to determine if the incremental investment meets the MARR requirements. To find the IRR on the incremental investment, the cash flows associated with each project must be available so that the difference between the two projects can be determined. Since this information is not available, a choice cannot be made.

5.31 We know that

56 740 = –180 000 + X(P/A, 10%, 5)

where X is the unknown cash flow at the end of each of the five years. Solving for X:

X = 236 740/3.7908 = 62 452

We calculate IRR from: 180 000 = X(P/A, i*, 1)

Substituting in the known value for X and solving for i*:
(P/A, i*, 5) = 180 000/62 452 = 2.882 $\Rightarrow$ i* = IRR = 21.7%

The IRR is 21.7%.

5.32 We have:
20 000 = 4000(P/A, MARR, 10)
(P/A, MARR, 10) = 5 $\Rightarrow$ MARR = 15.1%

20 000 = –P + A(P/A, 15.1%, 10) + 1000(P/F, 15.1%, 10)

By substituting P = 3A:
20 000 = –3A + 5A + 1000(0.245046)
2A = 20 000 – 245.046
A = 19 754.95
P = 19 754.95$\times$3 = 59 264.86

Calculate the IRR from:
59 264.86 = 19 754.95(P/A, i*, 10) + 1000(P/F, i* 10)

$\Rightarrow$ i* = IRR = 0.478 = 47.8%

The IRR for Lucy's project is 47.8%.

5.33 From the IRR calculation:
P = A(P/A, 15%, 5) + S (P/F, 15% 5)

Since S = 1/2P:
2S – S(0.49718) = A(3.3522)
S = A(3.3522)/1.50282 = 2.2306A

From the present worth calculation:
PW = –P + A(P/A, MARR, 5) + S(P/F, MARR, 5)

Since PW = 2A, P = 2S, and S = 2.2306A:
2A = –2(2.2306A) + A(P/A, MARR, 5) + 2.2306A(P/F, MARR, 5)

Dividing by A:
2 = –4.4612 + (P/A, MARR, 5) + 2.2306(P/F, MARR, 5)
(P/A, MARR, 5) + 2.2306(P/F, MARR, 5) = 6.4612
$\Rightarrow$ MARR = 3.19%

Patti's MARR is about 3.19%

5.34 **(a)** A bond with a face value of $10 000 and a coupon rate of 14% pays interest of $1400 per year ($700 every six months). A bond which matures 5 years from now has 10 remaining payments. The IRR on Jerry's investment is the interest rate that solves:

10 000(P/F, i, 10) + 700(P/A, i, 10) = 3500
14.29(P/F, i, 10) + (P/A, i, 10) = 5

By using a spreadsheet, an IRR of 25.46% (per six months) can be found.

Now, effective interest per year can be calculated:
IRR = $(1 + 0.2546)^2$ – 1 = 0.574 = 57.4%

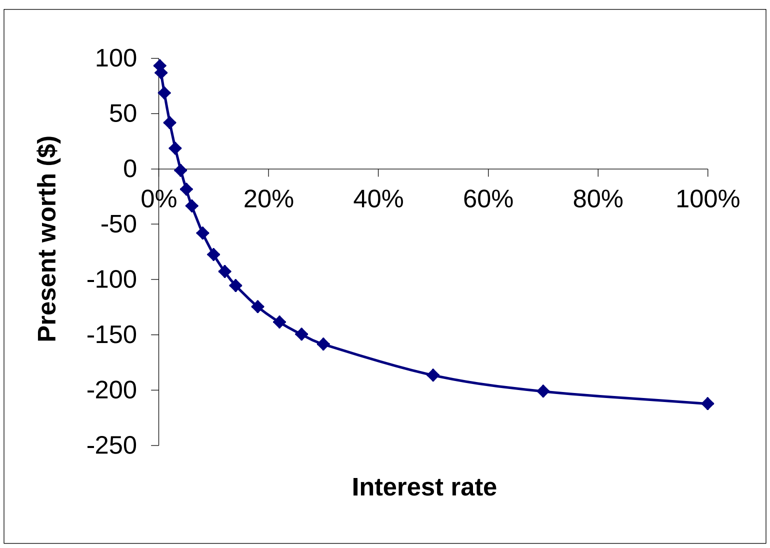
(b) Jerry's IRR for this investment is 57.4%. If his MARR is 20%, he should buy the bond.

5.35 **(a)** With a present worth analysis and a least common multiple of service life of 20 years, the IRR can be found with the following analysis.

Since one of the two must be chosen, the least expensive option, the "Clip Job" becomes the current best and the "Lawn Guy" is the challenger. We need to look to see if there is potential for multiple IRR's on the incremental cash flows. Over a 20 year period (the least common multiple of service life), the cash flows for the two projects, and the incremental cash flows are as follows:

	Lawn Guy		Clip Job		
Year	**First**	**Annual**	**First**	**Annual**	**Increment**
0	-350		-120		-230
1		-90		-100	10
2		-90		-100	10
3		-90		-100	10
4		-90	-120	-100	130
5		-90		-100	10
6		-90		-100	10
7		-90		-100	10
8		-90	-120	-100	130
9		-90		-100	10
10	-350	-90		-100	-340
11		-90		-100	10
12		-90	-120	-100	130
13		-90		-100	10
14		-90		-100	10
15		-90		-100	10
16		-90	-120	-100	130
17		-90		-100	10
18		-90		-100	10
19		-90		-100	10
20		-90		-100	10

(b) The incremental cash flows show that the incremental investment is not a simple investment. Hence, we need to determine if there are multiple IRRs. A plot of the PW of the incremental investment is below.



One IRR of 3.93% is revealed, but no others. There still may be multiple IRRs, so we calculate project balances at 3.93% (rightmost column in table below). This shows that the project has positive balances and hence we still don't know for sure if there are multiple IRRs. Therefore, we should use the approximate ERR method.

Year	Project balance
0	-230
1	-229
2	-228
3	-227
4	-106
5	-100
6	-94
7	-88
8	39
9	50
10	-288
11	-289
12	-170
13	-167
14	-164
15	-160
16	-36
17	-28
18	-19
19	-10
20	0

(c) Applying the approximate ERR method, we take the positive cash flows on the incremental investment forward to the end of the 20 year period at the MARR of 5%. The negative cash flows are brought forward at the unknown approximate ERR. The approximate ERR is 4.7%, which is less than the MARR of 5%. Thus, the incremental investment is not justified and the Clip Job is the preferred choice.

Year	Increment	FW @ MARR	FW @ ERR
0	-230		-577
1	10	25	
2	10	24	
3	10	23	
4	130	284	
5	10	21	
6	10	20	
7	10	19	
8	130	233	
9	10	17	
10	-340		-538
11	10	16	
12	130	192	
13	10	14	
14	10	13	
15	10	13	
16	130	158	
17	10	12	
18	10	11	
19	10	11	
20	10	10	
		Total FW	**ERR**
		0.00185	0.047028

5.36 Since one of the two alternatives must be chosen, the least first cost alternative (the used refrigerator) becomes the current best.

To see if the new refrigerator should be chosen, we find the net incremental cash flows to select the new refrigerator. The least common multiple of service lives is 24 years.

				FW	FW
Year	Automated	Off-the-shelf	Increment	@MARR	@ERR
0	-475	-1250	-775		-3670
1			0		
2			0		
3	-475		475	2391	
4			0		
5			0		
6	-475		475	1898	
7			0		
8		-1250	-1250		-3525
9	-475		475	1507	
10			0		
11			0		
12	-475		475	1196	
13			0		
14			0		
15	-475		475	950	
16		-1250	-1250		-2099
17			0		
18	-475		475	754	
19			0		
20			0		
21	-475		475	598	
22			0		
23			0		
24			0		
			MARR	Total FW	ERR
			0.08	0.000685	0.066939

The incremental investment to the new refrigerator is not a simple investment. To make a decision, we will use the approximate ERR. Take the positive incremental cash flows forward to the end of 24 years at the MARR (8%) and take the negative cash flows forward to the end of 24 years at the unknown ERR. Setting FW(positive cash flows @ MARR) + FW(negative cash flows @ ERR) = 0, we find the approximate ERR to be 6.69% which is slightly below the MARR. Since there are no other challengers, the best alternative is the used refrigerator.

5.37 Since the "do nothing" option is possible, we need to find the IRR on the least first cost alternative to begin. Both options have a zero first cost, so we can begin with either, say contract 1. The net cash flows of the two contracts are summarized below:

Year	0	1	2	3
Contract 1	15 000	–10 000	–10 000	12 000
Contract 2	20 000	–5000	–5000	–5000

The incremental investment from "do nothing" to contract 1 is not a simple project, so use the approximate ERR method:

$(15\ 000 - 0)(F/P, 10\%, 3) + 12\ 000 = (10\ 000 - 0)[(F/P, i^*, 2) + (F/P, i^*, 1)]$
$(F/P, i^*, 2) + (F/P, i^*, 1) = [15\ 000(1.3310) + 12\ 000]/10\ 000$
$(1 + i^*)^2 + (1 + i^*) = 3.1965$

At i* = 30%: LHS = 2.99
At i* = 40%: LHS = 3.36

The approximate ERR is between 30% and 40%, and is clearly above the MARR. Contract 1 becomes the current best alternative.

To see if the second contract should be chosen, we find the net incremental cash flows between contract 1 to contract 2: 5000, 5000, 5000, and 5000. All cash flows are positive, so the incremental investment has an infinite IRR. Hence, contract 2 is superior.

5.38 **(a)** Solve for i in:
–5000 + 3000(P/F, i,1) + 4000(P/F, i, 2) – 1000(P/F, i, 3) = 0

(b) The number of changes in sign of the cash flows is 2. Therefore, the maximum number of IRRs is two.

(c)

Year	Project balance B(i)
0	B(0) = –5000
1	B(1) = B(0)(1.1458) + 3000 = –2729
2	B(2) = B(1)(1.1458) + 4000 = 873.1118
3	B(3) = B(2)(1.1458) – 1000 = 0.4115 ≅ 0

(d) Due to the fact that this is not a simple investment and the fact that one project balance is positive, we cannot be sure that the IRR of 14.58% is unique.

5.39 The current policy requires P/A ≤ 5 ⇒ P ≤ 5A

We need –P + A(P/A, MARR, 20) ≥ 0 ⇒ P ≤ A(P/A, MARR, 20)

So that: 5A = A(P/A, MARR, 20) or (P/A, MARR, 20) = 5

For MARR = 20%: (P/A, MARR, 20) = 4.6755
For MARR = 15%: (P/A, MARR, 20) = 6.2593

By linear interpolation, MARR ≅ 19%

The equivalent MARR is about 19%.

5.40 First calculate the incremental IRR from do-nothing to alternative 1.

1 200 000 = 300 000(P/A, i%, 10)
(P/A, i, 10) = 4
⇒ i = IRR_1 = 21.4%

From #1 to #2:

1 500 000 –1 200 000 = (400 000 – 300 000)(P/A, i%, 10)
300 000 = 100 000(P/A, i%, 10)
(P/A, i%, 10) = 3
$\Rightarrow$ i = IRR_{2-1} = 31.1%

From #1 to #3:

2 100 000 – 1 200 000 = (500 000 – 300 000)(P/A, i%, 10)
900 000 = 200 000(P/A, i%, 10)
(P/A, i%, 10) = 4.5
$\Rightarrow$ i = IRR_{3-1} = 18.0%

From #2 to #3:

2 100 000 –1 500 000 = (500 000 – 400 000)(P/A, i%, 10)
600 000 = 100 000(P/A, i%, 10)
(P/A, i%, 10) = 6
$\Rightarrow$ i = IRR_{3-2} = 10.6%

Starting at the alternative with the lowest first cost, #1, the possibilities can be enumerated:

If MARR > 21.4%, 1 is rejected; else it is accepted.
If 1 is rejected,
  If MARR > 21.4%, 2 is rejected; else it is accepted.
  If 2 is rejected, none are chosen; else 2 is the final choice.
If 1 is accepted, 2 is also accepted.
  If MARR > 10.6, 2 is the final choice; else 3 is the final choice

Summarizing the above:

0 ≤ MARR <10.6%: Choose 3
10.6% ≤ MARR < 21.4%: Choose 2
21.4% ≤ MARR: Choose none

5.41 We have: ERR = (12 + IRR)/2 and ERR = 18 + [(IRR – 18)/4]

so that

(12 + IRR)/2 = 18 + [(IRR – 18)/4]
24 + 2IRR = 72 + IRR – 18

IRR = 72 – 18 –24 = 30
ERR = (12 + 30)/2 = 21%

Let X_j be the cash flow at time j. From the IRR calculation:
$X_0 = X_1(P/F, 30\%, 1) + 2000(P/F, 30\%, 2)$
$X_0 = X_1(0.76923) + 2000(0.59172)$
$X_0 = X_1(0.76923) + 1183.44$

Case 1: assume X_1 is a receipt (positive):

From the ERR calculation:
$2000 = X_0(F/P, 21\%, 2) + X_1(F/P, 12\%, 1)$
$2000 = X_0(1.4641) + X_1(1.12)$

so that

$2000 = [X_1(0.76923) + 1183.44](1.4641) + X_1(1.12)$
$2000 = X_1(1.12622) + 1732.67 + X_1(1.12)$
$267.3255 = X_1(2.24622)$
$X_1 = \$119.01$ and $X_0 = 1274.99$

Case 2: assume X_1 is a disbursement (negative):

From the ERR calculation:
$2000 = X_0(F/P, 21\%, 2) + X_1(F/P, 21\%, 1)$
$2000 = X_0(1.4641) + X_1(1.21)$

so that

$2000 = [X_1(0.76923) + 1183.44](1.4641) + X_1(1.21)$
$2000 = X_1(1.12622) + 1732.67 + X_1(1.21)$
$267.3255 = X_1(2.33622)$
$X_1 = \$114.27$

This contradicts the assumption the X_1 is a disbursement. There is only one possible value for X_1, \$119.01.

5.42 In order to calculate the exact ERR, we must calculate when there is capital available that is not bound up in the project. This capital is invested outside the project at the MARR.

For year 1, \$10 000 000 is invested at the MARR
For year 2, the future value after 1 year of the \$10 000 000 less the \$8 000 000 costs at the end of year one is invested at the MARR
Amount invested at MARR at end of year 1 is:

10 000 000(F/P, 10%, 1) - 8 000 000 = 10 000 000 (1.1) -8 000 000 = 3 000 000

Clearly it is unlikely that there will be a remaining positive cash flow balance at the end of the second year.

For year 4, $15 000 000 less the future worth (at the beginning of year 4) of prior cash flows is invested at the MARR, if any

The future worth of cash flows at the beginning of year four (call it α) is:

α = [10 000 000 (F/P, 10%, 1) - 3 000 000] (F/P, i*, 2) + 3 000 000 (F/P, 10%, 1) (F/P, i*, 1) + 8 000 000 (F/A, i*, 3)

Observing through trial and error that α < 15 000 000, there is a positive cash flow balance of 15 000 000 - α, which is then is invested at the MARR over year 4. Trial and error calculations reveals there is negative cash flow balance at the end of year 4, and no positive cash flow balances thereafter. Consequently the end of year 4 is an appropriate point in time to calculate the exact ERR:

[15 000 000 - α](F/P, i*,1) - 5 000 000 - 5 000 000(P/F,i*,1) + 5 000 000/i*

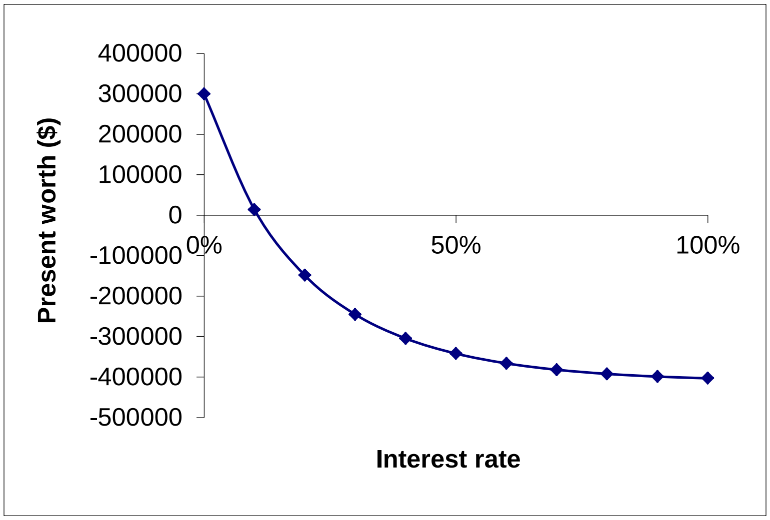
Calculating through spreadsheet trial and error gives i* = ERR = 21.9%

Note: Appendix 5A is on the companion website for this text.

5A.1 **(a)** This project is not simple since there are net receipts at years 2 and 5.

(b) The X axis is crossed once. There is at least one IRR at 0.10674; there may be more.

Interest rate	PW
0%	300000
10%	13969
20%	-148573
30%	-245097
40%	-304462
50%	-341975
60%	-366147
70%	-381906
80%	-392212
90%	-398895
100%	-403125



(c) Using project balances over the 5 years gives the following:

Year	Project balance B(i)
0	B(0) = –300 000
1	B(1) = B(0)(1.10674) – 500 000 = –832 022
2	B(2) = B(1)(1.10674) + 700 000 = –220 832
3	B(3) = B(2)(1.10674) – 400 000 = –644 404
4	B(4) = B(3)(1.10674) – 100 000 = –813 187
5	B(5) = B(4)(1.10674) + 900 000 = 13.41 ≅ 0

Since the project balance is never positive, we can conclude that the IRR of 10.674% is the only IRR. Since it is less than the MARR, the project should not be accepted.

(d) The approximate ERR is calculated as:

–300 000(F/P, ERR, 5) – 500 000(F/P, ERR, 4) + 700 000(F/P, 15%, 3) – 400 000(F/P, ERR, 2) – 100 000(F/P, ERR, 1) + 900 000 = 0

–3(F/P, ERR, 5) – 5(F/P, ERR, 4) + 7(F/P, 15%, 3) – 4(F/P, ERR, 2) – (F/P, ERR, 1) + 9 = 0

3(F/P, ERR, 5) + 5(F/P, ERR, 4) + 4(F/P, ERR, 2) + (F/P, ERR, 1) = 7(1.5209) + 9 = 19.6463

$\Rightarrow$ ERR = 0.1257 = 12.57%

The project should not be accepted; the rate of return is less than the MARR.

5A.2 Project 1 is a simple investment characterized by a disbursement of $3000 in the first period and receipts of $900 for each of four periods. Simple investments will always have a single IRR. The project balances method can be used to demonstrate this for the problem's cash flows.

An IRR of 7.715% can be found by solving for i in the expression: –3000 = 900(P/A, i, 4). To determine if this is the only IRR, the project balances for each period (at an interest rate of 7.715%) are:

Year	Project balance B(i)
0	B(0) = –3000
1	B(1) = B(0)(1.07715) + 900 = –2331
2	B(2) = B(1)(1.07715) + 900 = –1611
3	B(3) = B(2)(1.07715) + 900 = –835
4	B(4) = B(3)(1.07715) + 900 = 0.58 ≅ 0

Since all are non-positive, the project's (unique) IRR is 7.715%.

Project 2: An IRR of 19.52% can be found by some trial and error (a spreadsheet chart is also helpful in showing what interest rate gives rise to a zero present worth).

Year	Project balance B(i)
0	B(0) = –1500
1	B(1) = B(0)(1.1952) + 7000 = 5207
2	B(2) = B(1)(1.1952) – 9000 = –2777
3	B(3) = B(2)(1.1952) + 2900 = –419
4	B(4) = B(3)(1.1952) + 500 = –0.788 ≅ 0

The project balances method indicates that there may be multiple IRRs. A second IRR of 177% can also be found with some trial and error, or with a spreadsheet.

Project 3: One IRR is 23.8%. The project balances at the end of each period are:

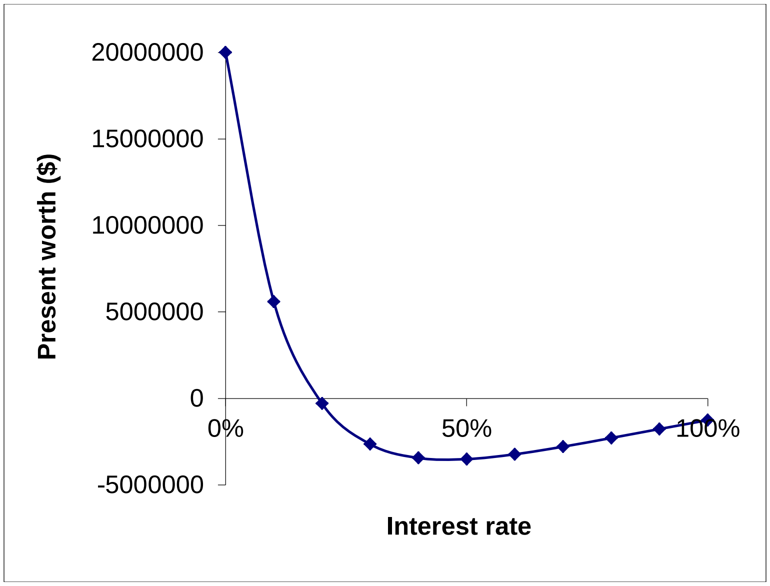
Year	Project balance B(i)
0	B(0) = 600
1	B(1) = B(0)(1.238) – 2000 = –1257
2	B(2) = B(1)(1.238) + 500 = –1056
3	B(3) = B(2)(1.238) + 500 = –807
4	B(4) = B(3)(1.238) + 1000 = 0.934 ≅ 0

Since not all project balances are non-negative, this method indicates that 23.8% may not be a unique IRR (another is 187.2%).

5A.3 **(a)** This project is not simple since there is a receipt at time 0.

(b) The X axis is crossed once. There is at least one IRR at 0.19238; there may be more.

Interest rate	PW
0%	20000000
10%	5598869
20%	-287640
30%	-2648811
40%	-3442349
50%	-3506749
60%	-3226833
70%	-2788768
80%	-2286023
90%	-1766525
100%	-1254883



(c) Using project balances over the 10 years gives the following:
Since there are positive project balances, we cannot conclude that the IRR at 19.238% is the only IRR. We cannot tell if the project should be accepted.

Year	Project balance B(i)
0	B(0) = 10 000 000
1	B(1) = B(0)(1.19238) – 20 000 000 = –8 076 200
2	B(2) = B(1)(1.19238) – 10 000 000 = –19 629 899
3	B(3) = B(2)(1.19238) + 5 000 000 = –18 406 299
4	B(4) = B(3)(1.19238) + 5 000 000 = –16 947 303
5	B(5) = B(4)(1.19238) + 5 000 000 = –15 207 625
6	B(6) = B(5)(1.19238) + 5 000 000 = –13 133 267
7	B(7) = B(6)(1.19238) + 5 000 000 = –10 659 845
8	B(8) = B(7)(1.19238) + 5 000 000 = –7 710 586
9	B(9) = B(8)(1.19238) + 5 000 000 = 0
10	B(10) = B(9)(1.19238) + 5 000 000 = 0

(d) The approximate ERR is calculated as:

10 000 000(F/P, 30%, 10) – 20 000 000(F/P, ERR, 9)
– 10 000 000(F/P, ERR, 8) + 5 000 000(F/A, 30%, 8) = 0
2(13.786) – 4(F/P, ERR, 9) – 2(F/P, ERR, 8) + 23.858 = 0
4(F/P, ERR, 9) + 2(F/P, ERR, 8) = 51.43
2(F/P, ERR, 9) + (F/P, ERR, 8) =25.715
$\Rightarrow$ ERR = 0.280 36 = 28.036%

The project should not be accepted; the rate of return is less than the MARR.

(e) The exact ERR can be determined by taking the \$10 000 000 advance forward one year at the MARR, and then solving for the IRR since the resulting cash flow has the form of a simple investment.

$$10\,000\,000(F/P, 30\%, 1) - 20\,000\,000 - 10\,000\,000(P/F, i, 1)$$
$$+ 5\,000\,000(P/A, i, 8)(P/F, i, 1) = 0$$
$$2(1.3) - 4 - 2(P/F, i, 1) + (P/A, i, 8)(P/F, i, 1) = 0$$
$$[(P/A, i, 8) - 2](P/F, i, 1) = 1.4$$
$$\Rightarrow i = IRR = 0.2126 = 21.26\%$$

The project should not be accepted; the rate of return is less than the MARR.

Notes for Case-in-Point 5.1

1) Yes, because they are real contributors to value. However, they may be temporary and should be included with caution

2) This is a personal judgement; no right answer

3) This is a personal judgement; no right answer

Notes for Mini-Case 5.1

1) The payback period would appear to be more attractive

2) 6.3 percent would be a less attractive return on investment

3) Such a graph shows that not discounting the revenue is highly misrepresentative of the value of the project

4) Long-term metal prices

5) No

Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

5S.1

The present worth of payments is (in thousands of dollars)

50 + 5((*P/F*, *i*, 2) + (*P/F*, *i*, 4) + (*P/F*, *i* ,6))

The present worth of receipts is

25(*P/A*, *i*, 8) + 10(*P/F*, *i*, 8)

To find the IRR, we could set these two quantities equal to each other and solve for *i*. However, an easier solution method is to note that the present worth of the investment is positive for an interest rate of zero, and becomes negative as the interest rate increases without limit. Since the MARR is 20%, the question is whether the graph of present worth versus interest rate crosses the axis at an interest rate greater than 20%. Evaluating the present worth at *i*=20%, we find it is positive. So the IRR must be greater than 20%, and the project is therefore acceptable.

5S.2

The approach here is to construct a table of incremental rates of return:

Alternative	*Initial Cost*	*Revenue*	*IRR*
Waterfront ("W")	10 000	11 200	12%
Signal Hill ("X")	12 000	13 800	15%
Cape Grace ("Y")	15 000	16 950	13%
Camps Bay ("Z")	20 000	22 500	12.5%
W to X	2000	2600	30%
X to Y	3000	3150	5%
X to Z	8000	8700	8.7%

All the projects individually have rates of return greater than 10%, so if they are not mutually exclusive, Cape Town Pizza should do all of them.
If they are mutually exclusive, we find that the incremental rate of return upgrading from Waterfront to Signal Hill is 30%, while the rates of return for upgrading from Signal Hill to anything else are less than 10%. So the company should open its franchise in Signal Hill.

5S.3

If Ntombela does accept the offer, he is getting money for nothing, which is effectively an infinite rate of return. But he still needs to consider the incremental benefit of turning it down.

Consider the incremental benefit to Ntombela of not accepting Van Den Akker's offer. Refusing his offer means that opening his hotel in Pretoria has an effective cost of R 11 000 000: R 10 000 000 of his own money, and R 1 000 000 that he could otherwise have received. So his incremental rate of return would be i, where

$$11\,000\,000 = 8\,000\,000(P/F,i,1) + 9\,000\,000(P/F,i,2)$$

which gives us i=33.4%. So if Ntombela thinks he can get a higher rate of return than this from a Cape Town hotel—that is, if his MARR is above 33.4%—he should accept the offer.

5S.4

The IRR for the project is the solution to

$$-200+300 / (1+i) + 100 / (1+i)^2 - 100 / (1+i)^3 = 0$$

which gives IRR = 61%. This number is suspiciously high and rests on the assumption that we could invest the receipts at the MARR, which may not be possible. So it is desirable to calculate the ERR as a check.

The future worth of the project incomes at the end of the project is

$$300 \times 1.1^2 + 100 \times 1.1$$

and the future worth of the expenditures is

$$100 + 200(1 + j)^3$$

where j is the ERR. Solving this gives us j=23%. This is greater than 10%, so the project should go ahead.

5S.5

Time	*Taxi*	*Canal*	*Taxi - Canal*
Now	−58 500	−48 500	−10 000
Over next 10 years	6648	--	6648
Ten years hence	30 000	138 000	−108 000

From this table, we know that the incremental IRR must satisfy

$-10\ 000 + -6648(P/A,i,10) - 108\ 000(P/F,i,10) = 0$

This has two solutions, i=19% and i=60%. So we can conclude that the taxi is preferable to the canal if the MARR is between 19% and 60%.

However, we should recognize that the "i=60%" solution is obtained on the unrealistic assumption that the firm's capital can be invested externally at 60% interest. So it would be quite reasonable to discount this solution and say that the taxi is preferable as long as the MARR is above 19%. Better still, we should calculate the ERR and base our decision on that.

We should also look at the separate rates of return for the canal and the taxi; they are 11% and 8%, respectively. So if our MARR is above 19%, we would not want to do either project.

The correct conclusion, then, is that the investor will never want to buy the water taxi. In the range where the taxi is attractive, the canal is more attractive, while in the range where the taxi is more attractive than the canal, both options lose money.

5S.6

The net cash flows involved are Rs 40 000 income, an expenditure of Rs 150 000 at the end of the first year, and Rs 135 000 income at the end of the second year. So to find his IRR, Jamal will find i to be:

$40\ 000 - 150\ 000(P/F,i,1) + 135\ 000(P/F,i,2) = 0$

from which we deduce

$4(1+i)^2 - 15(1+i) + 13.5 = 0.$

This equation has two solutions: IRR = 50%, or IRR = 125%. The latter solution seems unreasonably high: to have realised this rate of return, Jamal would have had to invest his initial payment at 125% interest, and there is no reason to think such an opportunity was available.

To obtain the approximate ERR, we find the rate at which the future worth of the project would be zero, given that all receipts are brought to the end of the period at the explicit or `auxiliary' rate of return, 12.5%. This is the solution to:

$40\ 000(F/P,0.125,2) - 150\ 000(F/P, i^*_{ae}, 1) + 135\ 000 = 0$

from which we deduce:

$4(1.2656) – 15(1+ i^*_{ae}) + 13.5 = 0$, and hence $i^*_{ae} = 23.7\%$.

If we want a more exact figure for his ERR, we must keep track of his accounts over the two-year period. Suppose that he had invested the initial payment at the rate we know he had available, 12.5%, then after a year this would have yielded Rs 45 000. Paying for the renewal of his work permit would have consumed all of this, plus a further Rs 105 000 from the re-mortgaging of his house. So his exact ERR on his funds would be given by:

$105\,000 + 135\,000(P/F, i^*_e, 1) = 0$

which can be solved to get $i^*_e = 28.3\%$

5S.7

We have the difficulty that Yan's continuous cash flows change from year to year, while all the formulae we have for calculating the present value of a continuous, continuously compounded cash flow assume that the flow rate is constant over the study period. To deal with this, note that we can describe Yan's cash flows has an inflow of 元50 000 000 a year, throughout the three years; superimposed on this, an outflow of 元65 000 000 a year for the first two years; and superimposed upon this, an inflow of 元35 000 000 in the first year. So the present value of Yan's cash flows is, in millions of Yuan:

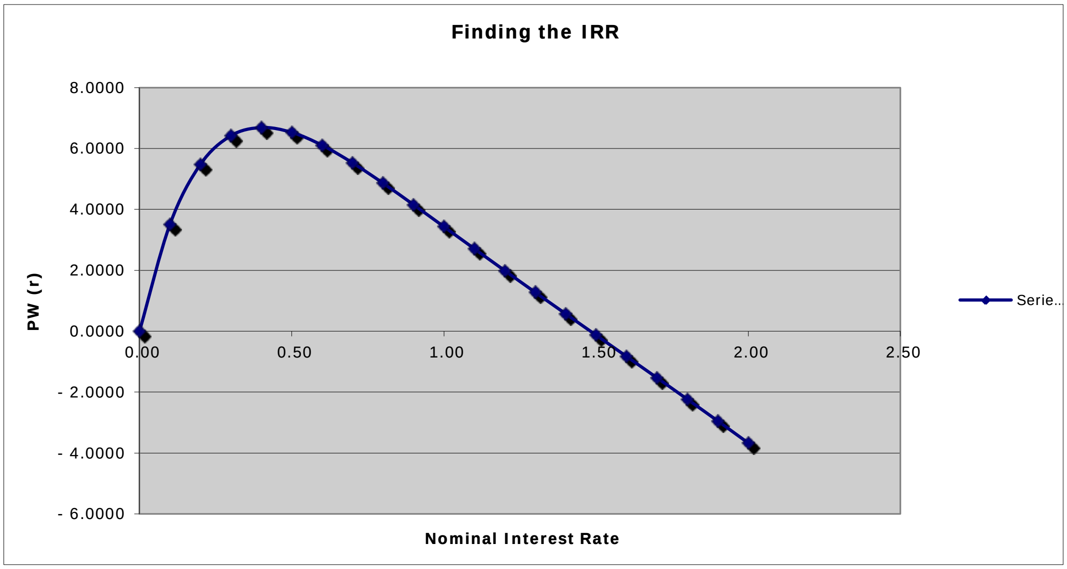
$PW = −10 + 50(e^{3r}−1) / re^{3r} − 65(e^{2r}−1) / re^{2r} + 35(e^r−1) / re^r$

Where r is the nominal per-year internal rate of return.

We multiply through by r and plot a graph, using the attached spreadsheet, **5S_7(IRR).xls**:

$PW(r) = −10r + 50(e^{3r}−1) / e^{3r} − 65(e^{2r}−1) / e^{2r} + 35(e^r−1) / e^r = 0$

(Multiplying through by r avoids a possible divide-by-zero error when r=0. For r>0, the graph of PW(r) will cross the x-axis at the same point as the graph of PW, so both equations have the same solution.)



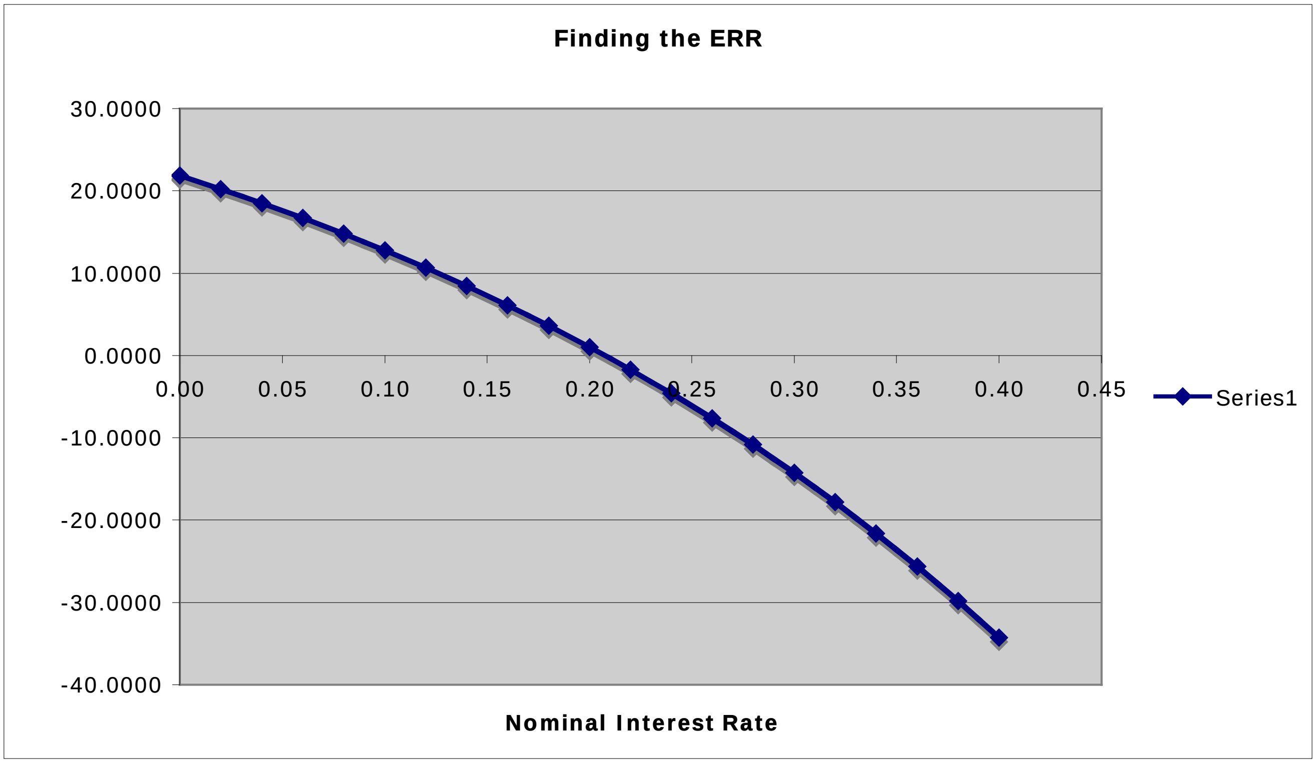
From the graph, we see that PW(*r*) passes through zero at a nominal interest rate of about 150% per year. This seems unreasonably high, so we would like to calculate the external rate of return. Finding the exact external rate of return would be rather difficult—we would have to find the exact point in the second year in which Yan's cash balance became negative—so we settle for the approximate rate of return. To do this, we will again describe Yan's cash flows as a superimposed series: an inflow of 元20 000 000 over the three years, plus an outflow of 元35 000 000 000 for the last two years, plus an inflow of 元65 000 000 in the final year. We then write down an expression for the final value of the casino investment, on the assumption that all receipts are invested at Yan's external rate of return, 15%, and that this rate is available as a continuous cash flow, continuously compounded:

$$FW = -10e^{3r} + 20(e^{3(0.15)} - 1) / r - 35(e^{2r} - 1) / r + 65(e^{0.15} - 1)/r$$

Multiplying through by r and equating to zero, we have

$$FW(r) = -10e^{3r}\, r + 20(e^{0.45} - 1) - 35(e^{2r} - 1) + 65(e^{0.15} - 1) = 0$$

This is graphed in the accompanying worksheet, **5S_7(ERR).xls**, and we see that the approximate ERR for Yan's casino project is about 21%.


Finding the ERR
30.0000
20.0000
10.0000
0.0000
-10.0000
-20.0000
-30.0000
-40.0000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
Series1
Nominal Interest Rate