Math & Stats Assistance Centre

MATH 110 Exam Review

December 13, 12pm - 2pm
Location: BWC B150

Here are some MATH 110 problems for practice only. The review session will be based on a selection of these problems, and an answer key will be made available to the MATH 110 instructors to share with you as well.

This is not a complete overview of final exam material, and the Math & Stats Assistance Centre tutors are not involved in creating or evaluating your actual final exam, but we do hope that these materials will be useful to you!

Part 1 (True/False):

1. A linearly independent set of vectors in $\mathbb{R}^n$ has at most n vectors.
2. If A is an $n \times n$ matrix and B is an $n \times p$ matrix such that $AB = \vec{0}$, then B is the 0-matrix.
3. If A is an $n \times n$ matrix and rank$(A) = n$, then Det$(A) = 0$.
4. If A is a square matrix, then AA^t and A^tA are orthogonally diagonalizable.
5. For all vectors $\vec{u}$ and $\vec{v}$ in $\mathbb{R}^n$, $\vec{u} \cdot \vec{v} \geq 0$.
6. Any matrix can be transformed into reduced row echelon form by a finite sequence of elementary row operations.
7. If A is a 4×3 matrix and nullity of A^t is 2, then $\text{rank}(A) = 2$.
8. If B is a 3×5 matrix then the columns of B are linearly dependent as vectors.
9. If A and B are $n \times n$ matrices and B is obtained from A by a sequence of elementary row operations, then Det$(B) =$ Det(A).
10. If $\vec{v}$ is both in the row space and in the column space of a square matrix A, then $\vec{v} = \vec{0}$.
11. Every plane in $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.
12. If V and W are subspaces of $\mathbb{R}^n$, then the set of vectors that are in both V and W is also a subspace of $\mathbb{R}^n$.
13. If V and W are subspaces of $\mathbb{R}^n$, then the set of vectors that are in either V or W or both is also a subspace of $\mathbb{R}^n$.
14. If V and W are subspaces of $\mathbb{R}^n$ and S is the set of vectors of the form $\vec{w} - \vec{v}$, where $\vec{v} \in V$ and $\vec{w} \in W\}$, then S is a subspace of $\mathbb{R}^n$.
15. If matrices M and N have the same size, then $(M + N)^t = M^t + N^t$.

16. If matrices M and N are invertible, and MN exists, then MN is invertible and $(MN)^{-1} = N^{-1}M^{-1}$.
17. If matrix M is invertible, then M^t is invertible.
18. If matrices M, N, and U are square matrices such that $MN = MU$, then $N = U$.
19. If R is the reduced row echelon form of a matrix A, then $Col(A) = Col(R)$.
20. If R is the reduced row echelon form of a matrix A, then $Row(A) = Row(R)$.
21. If $\vec{u}$ and $\vec{v}$ are nonzero, orthogonal vectors in $\mathbb{R}^2$, then they are independent.
22. If $\vec{u}$ and $\vec{v}$ are nonzero, independent vectors in $\mathbb{R}^2$, then they are orthogonal.
23. If A and B are orthogonal $n \times n$ matrices, then so is AB.

Part 2:

1. Let $\vec{u} = \begin{bmatrix} 2 \\ 8 \\ 3 \\ -1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ 7 \\ 2 \\ 9 \end{bmatrix}$. Compute $(\vec{u} - \vec{v}) \cdot (3\vec{v})$.
2. Find the projection of the vector $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ onto the vector $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.
3. Find the distance from the point $Q(4, 2, 4)$ to the plane $2x_1 - x_2 + x_3 = 3$
4. For what values of k is the solution to the following system of linear equations (a) a point, (b) a line, and (c) a plane?
$$\begin{array}{ccccccc} x & + & 2y & + & 3z & = & 1 \\ x & + & ky & + & 3z & = & 1 \\ kx & + & y & + & 3z & = & -2 \end{array}$$
5. If the vector $\vec{w}$ is a linear combination of the vectors $\vec{v_1} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $\vec{v_3} = \begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix}$ with corresponding coefficients $c_1 = 3$, $c_2 = 2$, $c_3 = -1$, find vector $\vec{w}$.
6. Suppose A and B are 4×4 matrices with $\det A = 5$ and $\det B = 3$. Find $\det(AB^2A^t)$.
7. Find the inverse of $\begin{bmatrix} 1 & 0 & -3 \\ 6 & 1 & -13 \\ -1 & 4 & 25 \end{bmatrix}$, or explain why the inverse does not exist.
8. Compute the determinant of the matrix $M = \begin{bmatrix} 2 & -3 & 0 & 1 \\ 5 & 4 & 2 & 0 \\ 1 & -1 & 0 & 3 \\ -2 & 1 & 0 & 0 \end{bmatrix}$ by row-reducing the matrix.

9. Let W be the subspace of $\mathbb{R}^3$ spanned by vectors $w_1 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $w_2 = \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$. Find a basis for $W^{\perp}$

10. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 9 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 & 3 \\ 3 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 21 \\ 9 \\ -1 \end{bmatrix}$. Determine whether the systems $A\vec{x} = \vec{b}$ and $D\vec{x} = \vec{b}$ are consistent or inconsistent. If consistent, solve the system.

11. If $\vec{u_1} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $\vec{u_2} = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$, for which values of k is the vector $\vec{w} = \begin{bmatrix} k \\ 2 \\ 1 \end{bmatrix}$ in the $span$ of $\{\vec{u_1}, \vec{u_2}\}$? If W is the subspace of $\mathbb{R}^3$ spanned by vectors $\vec{u_1}, \vec{u_2}$, find a basis for $W^{\perp}$.

12. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y + z \\ 5x + 3z \\ 2|z| \end{bmatrix}$. Is this transformation a linear transformation? If so, what is the standard matrix of T?

    (b) Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by $S\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - 2y + \sqrt{7}z \\ y - 5x + 3z \\ 2z + 0.1x \end{bmatrix}$. Is this transformation a linear transformation? If so, what is the standard matrix of S?

13. $B = \left\{ v_1 = \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, v_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right\}$ is an orthonormal basis of $\mathbb{R}^3$. Find constants c_1, c_2, c_3 such that $w = c_1v_1 + c_2v_2 + c_3v_3$.

14. Find the standard matrix that performs the **clockwise** rotation of vectors in $\mathbb{R}^2$ about the origin by $\pi/4$ radians, then reflects resulting vectors over line $y = -x$, and then projects resulting vectors on $y-$axis. Is this linear transformation invertible?

15. The vectors $\vec{x_1} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{x_2} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ form a basis for a subspace W of $\mathbb{R}^3$.

    Apply the Gram-Schmidt Process to obtain an orthonormal basis for W.

16. Consider the following matrix

$$A = \begin{bmatrix} 1 & 1 & -4 & 3 & -6 \\ 1 & 0 & -1 & 3 & -4 \\ 2 & 1 & -5 & 6 & -10 \\ -1 & -2 & 7 & -3 & 8 \end{bmatrix}$$

    (a) Determine the rank and nullity of matrix A

    (b) Give a basis for the row space of A

    (c) Give a basis for the column space of A

(d) Give a basis for the nullspace of A.

17. For the matrix $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

(a) find an invertible matrix P and a diagonal matrix D such that $A = P^{-1}DP$.

(b) Find formulae for calculating $B = A^k$ for all values of $k \geq 1$.

18. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$.

(a) Find the characteristic polynomial of A, the eigenvalues of A, and the eigenvector(s) corresponding to each eigenvalue.

(b) Is A diagonalizable?

19. Let S be the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that takes a vector $\vec{v}$ and rotates it $\pi/2$ radians. If $A\vec{x} = S(\vec{x})$, what are the real eigenvalues of A?

20. Let T be the linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ that takes a vector $\vec{v}$ and projects it onto the xy-plane. If $B\vec{x} = T(\vec{x})$, what are the real eigenvalues of B?

21. Find the polar form of the complex number $z = 1 + i$. Find z^7.

22. What are the solutions to the equation $z^4 - 4i = 0$?

23. Find all real and complex roots of $-x^3 - 6x + 4x^2 + 24$.