SIGMAS

Students in Graduate Mathematics and Statistics

EXAM SALES

Course: MATH 101

Semester: August 2016

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UNIVERSITY OF VICTORIA
EXAMINATIONS AUGUST 2016
MATHEMATICS 101: CALCULUS II

Last Name: Solutions

First Name: SIGMAS

Student ID: V00

Lecture Section:

Duration: 3 hours

Instructors:
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TO BE ANSWERED ON THE PAPER

COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATORS.

Question	Value	Marks
1–17	34	
18	5	
19	5	
20	5	
21	2	
22	5	
23	4	
Total	60	

THIS EXAMINATION PAPER HAS 16 PAGES PLUS FRONT AND BACK COVER PAGES.

INSTRUCTIONS:

1. Your NAME and STUDENT NUMBER must be recorded on your test paper.
2. The only hand calculator permitted is the Sharp EL-510 R/RN/RNB. No other electronic devices are allowed.
3. Questions 1 through 17 are **short answer** questions, worth 2 marks each. Write your final answer in the box provided. For questions requiring numerical answer, **unless otherwise instructed, give your answer as a decimal value rounded to 3 decimal places**. Only answers written in their associated boxes will be marked. For verification purposes, show all calculations on your question paper. Unverified answers may be disallowed.
4. Questions 18 through 23 are **full answer** questions, worth 2 to 5 marks each. For these questions, write out your solutions carefully and completely on the question paper. Marks will be deducted for incomplete or poorly presented solutions.
5. No other aids such as textbooks, notes, etc. are permitted.
6. Cellphones and other communication devices (including smart watches) must be turned off and stored with the rest of your belongings at the front of the room. Headphones may not be worn during the examination.
7. You may use the back of the pages if you require more room.

1. **[2 marks]** Find the derivative of $f(x) = \frac{1}{2}\sinh(2x+1)$ at $x = 1$.

$$f'(x) = \frac{1}{2}\cosh(2x+1)(2)$$
$$= \cosh(2x+1)$$
$$f'(1) = \cosh(2(1)+1) = \cosh(3)$$
$$= 10.068$$

Answer

10.068

2. **[2 marks]** Calculate $\int_0^1 \frac{dx}{\sqrt{4x^2+9}}$. $= I$

$$I = \int_0^1 \frac{dx}{3\sqrt{\left(\frac{2}{3}x\right)^2+1}}$$

$\left[\begin{array}{l} \text{Let } u = \frac{2}{3}x \\ \text{Then } du = \frac{2}{3}dx \end{array}\right]$

$$= \int_{0^*}^{1^*} \frac{3/2\, du}{3\sqrt{u^2+1}} = \frac{1}{2}\int_{0^*}^{1^*} \frac{du}{\sqrt{u^2+1}} = \frac{1}{2}\sinh^{-1}(u)\Big|_{0^*}^{1^*}$$

$$= \frac{1}{2}\left[\sinh^{-1}(2/3) - \sinh^{-1}(0)\right]$$

$$= 0.313$$

Answer

0.313

3. **[2 marks]** Find $\int_1^2 x^5 \ln x \, dx$.

Let $u = \ln x$, $dv = x^5\,dx$
$du = \frac{1}{x}\,dx$, $v = \frac{x^6}{6}$

Integration by Parts:

$$I = \frac{x^6}{6}\cdot \ln x - \int \frac{x^6}{6}\cdot\frac{1}{x}\,dx$$

$$= \left[\frac{x^6}{6}\cdot \ln x - \frac{1}{6}\cdot\frac{1}{6}x^6\right]_1^2$$

$$= \left[\frac{2^6}{6}\cdot \ln 2 - \frac{1}{36}\cdot 2^6\right] - \left[\frac{1}{6}\ln(1) - \frac{1}{36}(1)\right]$$

$$= 5.644$$

Answer

5.644

4. **[2 marks]** Evaluate $\int \sec^6 x \tan^3 x \, dx$.

$$\left[\begin{array}{l}\text{Let } u = \sec x \\ du = \sec x \cdot \tan x \, dx\end{array}\right]$$

$$= \int \sec^5 x \cdot \tan^2 x \cdot \sec x \tan x \, dx$$

$$= \int \sec^5 x \cdot (\sec^2 x - 1)(\sec x \cdot \tan x)\,dx$$

$$= \int u^5 (u^2 - 1)\,du = \int u^7 - u^5 \,du$$

$$= \frac{u^8}{8} - \frac{u^6}{6} + C = \frac{\sec^8 x}{8} - \frac{\sec^6 x}{6} + C$$

Answer

$\frac{1}{8}\sec^8 x - \frac{1}{6}\sec^6 x + C$

5. **[2 marks]** Evaluate $\int_0^1 \frac{4x}{(x+1)(x+2)}\,dx$. $= \int_0^1 \frac{A}{(x+1)} + \frac{B}{(x+2)}\,dx$

$\Rightarrow 4x = A(x+2) + B(x+1)$
$\Rightarrow 4 = A + B$
$\Rightarrow 0 = 2A + B$

$A = -4$
$B = 8$

$I = \int_0^1 \frac{-4}{(x+1)} + \frac{8}{(x+2)}\,dx$

$= \left[-4\ln(x+1) + 8\ln(x+2)\right]_0^1$

$= [-4\ln(1+1) + 8\ln(1+2)] - [-4\ln(0+1) + 8\ln(0+2)]$

$= -4\ln(2) + 8\ln(3) - 8\ln(2)$

$= 0.471$

Answer

0.471

6. **[2 marks]** Evaluate $\int_0^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$.

$\left[\begin{array}{l} \text{Let } u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}}\,dx \end{array}\right]$

$= \lim_{t\to\infty} \int_{0^*}^{t^*} e^{-u} \cdot 2\,du$

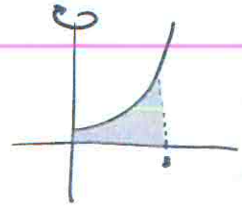
$= \lim_{t\to\infty} -2e^{-u}\Big]_{0^*}^{t^*} = \lim_{t\to\infty} -2e^{-\sqrt{x}}\Big|_{0^*}^{t^*}$

$= \lim_{t\to\infty} -2e^{-\sqrt{t}} + 2e^0 = 2(0) + 2 = 2$

Answer

2

7. **[2 marks]** Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2 + 1$, $x = 3$, $x = 0$ and $y = 0$, about the y-axis.



$$V = \int_a^b 2\pi\, r(x)\, h(x)\, dx$$
$$= 2\pi \int_0^3 x \cdot (x^2+1)\, dx$$
$$= 2\pi \int_0^3 x^3 + x\, dx$$
$$= 2\pi \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^3$$
$$= 2\pi \left[\frac{3^4}{4} + \frac{3^2}{2} - 0 \right]$$
$$= 155.509$$

Answer

155.509

8. **[2 marks]** Find the arclength of the curve $f(x) = \ln(\cos x)$ for $0 \le x \le \pi/4$.

$$f'(x) = \frac{1}{\cos x} \cdot -\sin x = -\tan x$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$
$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx$$
$$= \int_0^{\pi/4} |\sec x|\, dx$$
$$= \int_0^{\pi/4} \sec x\, dx$$
$$= \ln|\sec x + \tan x| \Big|_0^{\pi/4}$$
$$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$$
$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$
$$= 0.881$$

For $0 \le x \le \pi/4$, $|\sec x| = \sec x$

Answer

0.881

9. **[2 marks]** Determine if the sequence $\{a_n\}$ where $a_n = \ln\left(1+\frac{3}{n}\right)^n$ converges or diverges. If it converges, find its limit.

$$L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(1+\frac{3}{n}\right)^n$$

$$= \lim_{n\to\infty} \ln\left(1+\frac{3}{n}\right)^n = \lim_{n\to\infty} n \cdot \ln\left(1+\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \frac{\ln\left(1+\frac{3}{n}\right)^n}{1/n} \overset{L'Hop}{=} \lim_{n\to\infty} \frac{\left(\frac{1}{1+3/n}\right)\left(\frac{-3}{n^2}\right)}{\left(\frac{-1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{3}{1+3/n} = \frac{3}{1+0} = 3$$

Answer

3

10. **[2 marks]** Determine if the following series converges or diverges. If it does converge, find its sum.

$$\sum_{n=0}^{\infty} \frac{5^{n+1}}{\pi^{2n}}$$

$$\sum_{n=0}^{\infty} \frac{5\cdot 5^n}{(\pi^2)^n} = \sum_{n=0}^{\infty} 5\left(\frac{5}{\pi^2}\right)^n \quad \left(\text{since } \frac{5}{\pi^2} < 1\right)$$

$$= \frac{5}{1-\frac{5}{\pi^2}}$$

$$= 10.134$$

Answer

10.134

11. **[2 marks]** Determine if the following series converges or diverges. If it does converge, find its sum.

$$\sum_{n=2}^{\infty} \frac{3n}{\ln n}$$

$$\lim_{n\to\infty} \frac{3n}{\ln n} \overset{\text{L'Hop}}{=} \lim_{n\to\infty} \frac{3}{1/n}$$

$$= \lim_{n\to\infty} 3n = \infty \neq 0$$

$\therefore$ Series diverges

Answer

Diverges

12. **[2 marks]** Find the sum of the series $\sum_{n=2}^{\infty} \left(\frac{1}{\ln n} - \frac{1}{\ln(n+1)} \right)$.

$$S_2 = \frac{1}{\ln 2} - \frac{1}{\ln 3}$$

$$S_3 = \left(\frac{1}{\ln 2} - \frac{1}{\ln 3}\right) + \left(\frac{1}{\ln 3} - \frac{1}{\ln 4}\right) = \frac{1}{\ln 2} - \frac{1}{\ln 4}$$

$$S_4 = \left(\frac{1}{\ln 2} - \frac{1}{\ln 4}\right) + \left(\frac{1}{\ln 4} - \frac{1}{\ln 5}\right) = \frac{1}{\ln 2} - \frac{1}{\ln 5}$$

$$S_n = \frac{1}{\ln 2} - \frac{1}{\ln n+1}$$

$$\therefore \sum_{n=2}^{\infty} \frac{1}{n} - \frac{1}{n+1} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{\ln 2} - \frac{1}{\ln(n+1)}$$

$$= \frac{1}{\ln 2} - 0$$

$$= 1.443$$

Answer

1.443

13. **[2 marks]** Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{5n^2}{e^n}(x-2)^n$.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{5(n+1)^2(x-2)^{n+1}}{e^{n+1}}}{\frac{5n^2(x-2)^n}{e^n}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)^2}{n^2}\cdot\frac{(x-2)}{e}\right|$$

$$= \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^2\cdot\frac{|x-2|}{e}$$

$$= \frac{|x-2|}{e} < 1$$

$$-e < x-2 < e$$

$$-e+2 < x < e+2$$

ROC: $e \approx 2.718$

Answer

$e \approx 2.718$

14. **[2 marks]** Use the fact that for $-1 \le x \le 1$, $\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots$ to calculate the following limit.

$$\lim_{x\to 0} \frac{x - \tan^{-1} x}{x^3}$$

$$= \lim_{x\to 0} \frac{x - \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\right]}{x^3}$$

$$= \lim_{x\to 0} \frac{\left[\frac{x^3}{3} - \frac{x^5}{5} + \frac{x^7}{7} - \ldots\right]}{x^3}$$

$$= \lim_{x\to 0} \frac{1}{3} - \frac{x^2}{5} + \frac{x^4}{7} - \ldots$$

$$= \frac{1}{3}$$

Answer

0.333

15. **[2 marks]** Find the constant c_3 in the binomial series $f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \ldots$, for $f(x) = \sqrt{1+x}$ with $-1 < x < 1$.

$$f(x) = \sqrt{1+x} = (1+x)^{1/2}$$

$$c_3 = \binom{\frac{1}{2}}{3} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}$$

$$= \frac{1}{16} = 0.0625$$

Answer

0.063

16. **[2 marks]** Find the coefficient of the $(x-1)^3$ term in the Taylor series for the function $f(x) = 2^x$, centred at $a = 1$.

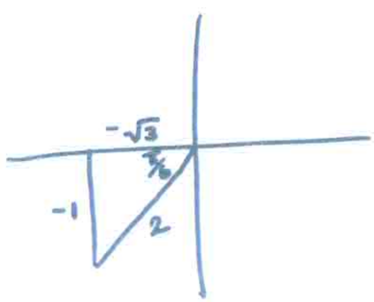
$f(x) = 2^x$	$f(1) = 2^1$
$f'(x) = 2^x \cdot \ln 2$	$f'(1) = 2 \cdot \ln 2$
$f''(x) = 2^x \cdot (\ln 2)^2$	$f''(1) = 2 \cdot (\ln 2)^2$
$f'''(x) = 2^x (\ln 2)^3$	$f'''(1) = 2 \cdot (\ln 2)^3$

Coefficient: $\dfrac{2 \cdot (\ln 2)^3}{3!} = 0.111$

Answer

0.111

17. **[2 marks]** Convert the polar coordinate $(r, \theta) = (2, 7\pi/6)$ to Cartesian coordinates (x, y). **Give your answer in exact values, using radicals if necessary.**



$$x = r\cos\theta = 2 \cdot \cos\left(\frac{7\pi}{6}\right)$$
$$= 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$$

$$y = r\sin\theta = 2\sin\left(\frac{7\pi}{6}\right)$$
$$= 2\left(\frac{-1}{2}\right) = -1$$

Give your answer in exact values, using radicals if necessary

Answer

$(-\sqrt{3}, -1)$

LONG ANSWER PROBLEMS

18. **[5 marks]** Using trigonometric substitution, compute the following indefinite integral,

$$I = \int \frac{dx}{(x^3)\sqrt{9x^2-16}}, \quad \text{for } x \geq 3.$$

$$I = \int \frac{dx}{3x^3\sqrt{x^2 - \frac{16}{9}}} \qquad \left[\begin{array}{l} \text{Let } x = \frac{4}{3}\sec\theta \\ dx = \frac{4}{3}\sec\theta \cdot \tan\theta \, d\theta \\ 0 \leq \theta \leq \frac{\pi}{2} \end{array}\right]$$

$$= \int \frac{\frac{4}{3}\sec\theta\tan\theta \, d\theta}{3\cdot\left(\frac{4}{3}\sec\theta\right)^3\sqrt{\left(\frac{4}{3}\sec\theta\right)^2 - \frac{16}{9}}}$$

$$= \frac{1}{3}\cdot\frac{3^3}{4^3}\int \frac{\tan\theta \cdot d\theta}{\sec^2\theta\sqrt{\sec^2\theta - 1}} = \frac{9}{64}\int \frac{\tan\theta \, d\theta}{\sec^2\theta\sqrt{\tan^2\theta}}$$

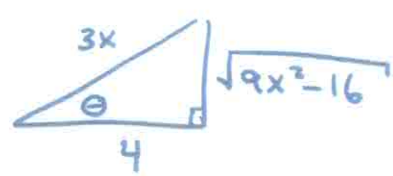
$$= \frac{9}{64}\int \frac{\tan\theta \, d\theta}{\sec^2\theta\,|\tan\theta|} \qquad \left[\begin{array}{l} \text{For } 0 \leq \theta \leq \frac{\pi}{2}, \\ \tan\theta \geq 0 \\ \Rightarrow |\tan\theta| = \tan\theta \end{array}\right]$$

$$= \frac{9}{64}\int \frac{1}{\sec^2\theta} \, d\theta$$

$$= \frac{9}{64}\int \cos^2\theta \, d\theta = \frac{9}{64}\int \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \frac{9}{128}\int 1 + \cos 2\theta \, d\theta = \frac{9}{128}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{9}{128}\left[\theta + \frac{1}{4}\sin\theta\cos\theta\right] + C$$



$$= \frac{9}{128}\left[\arctan\frac{\sqrt{9x^2-16}}{4} + \frac{1}{4}\left(\frac{\sqrt{9x^2-16}}{3x}\right)\left(\frac{4}{3x}\right)\right] + C$$

$$= \frac{9}{128}\left[\arctan\frac{\sqrt{9x^2-16}}{4} + \frac{\sqrt{9x^2-16}}{9x^2}\right] + C$$

19. When a cup of hot coffee is first poured, its temperature is 90°C. The coffee is left to cool in a room with a constant air temperature of 20°C. After 15 minutes, the temperature of the coffee has dropped to 70°C.

(a) **[3 marks]** Recall that Newton's Law of Heating/Cooling states that the rate of change in temperature H of the coffee can be modelled as

$$\frac{dH}{dt} = -k(H - H_0) = -K(H-20)$$

where H is the temperature of the coffee t minutes after being poured, and H_0 is the room temperature.

Solve the above differential equation to find an equation for the coffee's temperature t minutes after being poured. Use the given initial conditions to find an exact value for k.

$$\int \frac{dH}{H-20} = \int -K\,dt$$

$$\ln|H-20| = -Kt + C$$

$$H - 20 = e^{-Kt+C}$$

$$H(t) = Ae^{-Kt} + 20$$

At $t = 0$

$$A(0) = 90 = Ae^{-K(0)} + 20$$

$$\Rightarrow 70 = A$$

At $t = 15$:

$$A(15) = 70 = 70e^{-15K} + 20$$

$$\Rightarrow 50 = 70e^{-15K}$$

$$5/7 = e^{-15K}$$

$$-15K = \ln(5/7)$$

$$K = \frac{-1}{15}\ln(5/7)$$

$$\boxed{H(t) = 70e^{\frac{1}{15}\ln(5/7)} + 20}$$

(b) **[2 marks]** At what time t (in minutes) will the coffee be 50°C?

$$50 = 70e^{\frac{1}{15}\ln(5/7)t} + 20$$

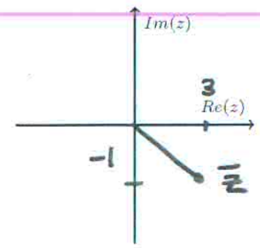
$$3/7 = e^{\frac{1}{15}\ln(5/7)t}$$

$$\ln(3/7) = \frac{1}{15}\ln(5/7)t$$

$$t = \frac{15\ln(3/7)}{\ln(5/7)} = 37.773 \text{ mins}$$

20. Consider the complex number $z = \sqrt{3} + i$.

(a) **[1 mark]** Find $\bar{z}$ and then plot $\bar{z}$ on the provided Argand diagram.



$\bar{z} = \sqrt{3} - i$

(b) **[2 marks]** Write z in polar form (r, θ), expressing the angle θ in terms of π.

$r^2 = a^2 + b^2 = (\sqrt{3})^2 + 1^2 = 4$

$r = 2$

$\theta = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

(c) **[2 marks]** Find z^5. Present your final answer in Cartesian $a + bi$ form, with a and b as exact (unrounded) values, using radicals if necessary.

$z^5 = [re^{i\theta}]^5 = r^5 e^{i(5\theta)}$

$= r^5[\cos(5\theta) + i\sin(5\theta)]$

$= 2^5\left[\cos\left(5\cdot\frac{\pi}{6}\right) + i\sin\left(5\cdot\frac{\pi}{6}\right)\right]$

$= 32\left[\left(\frac{-\sqrt{3}}{2}\right) + i\left(\frac{1}{2}\right)\right]$

$= 16(-\sqrt{3} + i)$

21. **[2 marks]** Determine whether the following series converges conditionally, converges absolutely or diverges.

$$\sum_{n=1}^{\infty}(-1)^n\left(\frac{\sin n}{n^3}\right)$$

Consider $\displaystyle\sum_{n=1}^{\infty}\left|(-1)^n\frac{\sin n}{n^3}\right| = \sum_{n=1}^{\infty}\frac{|\sin n|}{n^3}$

Since $0 \leq |\sin n| \leq 1$,

$$0 \leq \frac{|\sin n|}{n^3} \leq \frac{1}{n^3}.$$

$\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^3}$ is a convergent p-series.

By the Comparison Test, since $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^3}$ converges, so does $\displaystyle\sum_{n=1}^{\infty}\frac{|\sin n|}{n^3}$.

Moreover, since $\displaystyle\sum_{n=1}^{\infty}\left|\frac{(-1)^n\sin n}{n^3}\right|$ converges,

$\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\sin n}{n^3}$ converges absolutely.

22. Consider the parametric curve $x = t + e^t$, $y = 1 - e^t$ for $-\infty < t < \infty$.

(a) **[2 marks]** Find the slope $\left(\frac{dy}{dx}\right)$ of the curve at $t = 0$.

$$\frac{dx}{dt} = 1 + e^t, \qquad \frac{dy}{dt} = -e^t$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-e^t}{1+e^t} = y'$$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{-e^0}{1+e^0} = \frac{-1}{1+1} = \frac{-1}{2}$$

(b) **[3 marks]** Find $\frac{d^2y}{dx^2}$ in terms of t.

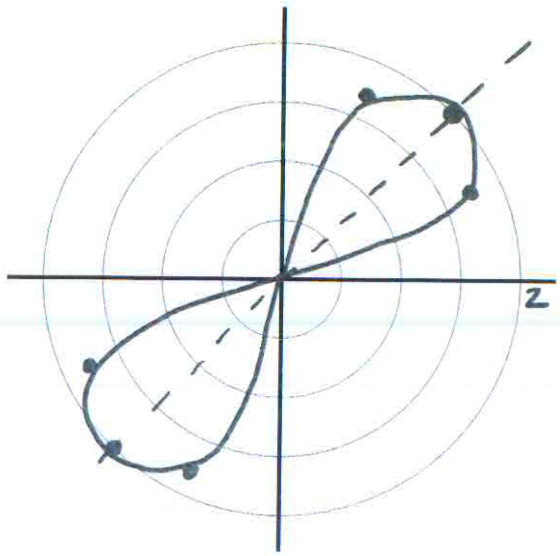
$$\frac{dy'}{dt} = \frac{-e^t(1+e^t) - (e^t)(-e^t)}{(1+e^t)^2}$$

$$= \frac{-e^t - e^{2t} + e^{2t}}{(1+e^t)^2} = \frac{-e^t}{(1+e^t)^2}$$

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{\frac{-e^t}{(1+e^t)^2}}{(1+e^t)} = \frac{-e^t}{(1+e^t)^3}$$

23. Consider the lemniscate $r^2 = 4\sin(2\theta)$.

(a) **[2 marks]** Sketch the curve by **making a table** with values of θ and r, and then **plot** the corresponding points.



θ	r^2	r
0	0	0
$\pi/6$	3.464	± 1.861
$\pi/4$	4	± 2
$\pi/3$	3.464	± 1.861
$\pi/2$	0	0

(b) **[2 marks]** Find the area contained in one loop of the lemnsicate.

$$A = 2\int_0^{\pi/4} \frac{1}{2} r^2 \, d\theta$$

$$= \int_0^{\pi/4} (4\sin(2\theta))\, d\theta$$

$$= 4\int_0^{\pi/4} \sin(2\theta)\, d\theta$$

$$= 4\left[\left(\frac{-1}{2}\right)\cos(2\theta)\right]_0^{\pi/4}$$

$$= -2\cos\left(\frac{\pi}{2}\right) + 2\cos(0)$$

$$= 0 + 2 = 2$$

Math 101 — Table of Integrals

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C$$

$$\int \frac{du}{1+u^2} = \tan^{-1} u + C$$

$$\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1} |u| + C$$

$$\int \frac{du}{\sqrt{u^2+1}} = \sinh^{-1} u + C$$

$$\int \frac{du}{\sqrt{u^2-1}} = \cosh^{-1} u + C$$

$$\int \frac{du}{1-u^2} = \begin{cases} \tanh^{-1} u + C & \text{if } |u| < 1 \\ \coth^{-1} u + C & \text{if } |u| > 1 \end{cases}$$

$$\int \frac{du}{u\sqrt{1-u^2}} = -\operatorname{sech}^{-1}|u| + C$$

$$\int \frac{du}{u\sqrt{1+u^2}} = -\operatorname{csch}^{-1}|u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$