

# Math 110 - Homework 11

## Topic: Diagonalization and orthogonality

Due at 6:00pm (Pacific) on Friday, December 3, submitted through Crowdmark.

### Practice

Before beginning the graded portion of this worksheet, we **strongly** recommend that you practice the basic techniques related to this week's material. Mastering the techniques used in these questions is essential for completing the rest of the worksheet, as well as for success on the tests and exam. The relevant questions this week are from Section 5.1 of the online textbook, as well as the supplemental questions about complex numbers posted on Brightspace.

### MATLAB

Each week we will provide you with a list of new MATLAB commands relevant to the material on the worksheet. You are welcome, and in fact encouraged, to use MATLAB for the calculations in Part II of the worksheet. On Part I you must do the calculations by hand and show your work.

This week's material does not require any new MATLAB commands.

### Graded questions

The questions on the following page are the ones to be submitted for grading. You are permitted to discuss these questions with other students, your tutorial TA, or your instructors - however, the final product that you submit must be written in your own words, and reflect your own understanding. You are **not** permitted to post these questions anywhere on the internet. Your final solutions should be understandable by a student who has been keeping up with this course but does not have any knowledge of the material beyond what we have seen in class - in particular, if you have seen techniques from matrix algebra that have not yet been discussed in the course, do not use them in your solutions.

## Part I: Calculation by hand

For all questions in this section you must show all of the details of your calculations. Credit will be given only if you show the steps by which you obtain your final answer.

1. Let  $A = \begin{bmatrix} 9 & -4 & -8 \\ 12 & -7 & -8 \\ 0 & 0 & -3 \end{bmatrix}$ . Either find an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^{-1}$  or explain why no such matrices exist. If you do find a  $P$  and  $D$  you do not need to calculate  $P^{-1}$ .
2. Let  $L$  be the line in  $\mathbb{R}^3$  that passes through the points  $(0, 0, 0)$  and  $(3, 3, -1)$ .
  - (a) Find a basis for  $L^\perp$ .
  - (b) Describe  $L^\perp$  geometrically.

## Part II: Concepts and connections

In this section you are permitted to use MATLAB to carry out any necessary computations. Almost all of the grades in this section will be awarded for your explanations of *why* you calculated what you did, and what it means. If you use MATLAB to do a calculation, be sure to tell us that you've done so, and also write down both what commands you used and what the output was. If you do use MATLAB for any calculations and it gives you a decimal answer, then give your answers rounded to 2 decimal places.

1. For both parts of this question assume that the matrices are  $3 \times 3$ .
  - (a) If  $A$  is an invertible matrix is it necessarily true that  $A$  is diagonalizable? If so, explain why. If not, give a specific example of a matrix  $A$  that is invertible but not diagonalizable (and explain why your example has these properties).
  - (b) If  $B$  is a diagonalizable matrix is it necessarily true that  $B$  is invertible? If so, explain why. If not, give a specific example of a matrix  $B$  that is diagonalizable but not invertible (and explain why your example has these properties).

2. Find an orthogonal basis for  $\mathbb{R}^4$  that contains the vector  $\begin{bmatrix} 1 \\ 2 \\ -3 \\ 1 \end{bmatrix}$ .