Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3]

Find a formula for the nth partial sum of the series and use it to determine if the series converges or diverges. If the series converges, find its sum.

$$\sum_{n=1}^{\infty}\left(\frac{3}{n}-\frac{3}{n+1}\right)$$

Let s_n denote the sum of the first n terms of the series. Then the expression for s_1 is as shown below.

$$s_1=\frac{3}{1}-\frac{3}{2}$$

The expression for s_2 is obtained by adding $\left(\frac{3}{n}-\frac{3}{n+1}\right)$ for $n=2$.

$$s_2=\left(\frac{3}{1}-\frac{3}{2}\right)+\left(\frac{3}{2}-\frac{3}{3}\right)$$

Continuing this process, the nth partial sum s_n can be written as shown below.

$$s_n=\left(\frac{3}{1}-\frac{3}{2}\right)+\left(\frac{3}{2}-\frac{3}{3}\right)+\left(\frac{3}{3}-\frac{3}{4}\right)+\cdots+\left(\frac{3}{n-1}-\frac{3}{n}\right)+\left(\frac{3}{n}-\frac{3}{n+1}\right)$$

Notice that the expression has one non-zero term, then a series of pairs of terms with a sum of 0, and finally another non-zero term. Remove the parentheses and simplify by adding all the terms that sum to zero.

$$\begin{aligned}s_n &= \frac{3}{1}-\frac{3}{2}+\frac{3}{2}-\frac{3}{3}+\frac{3}{3}-\frac{3}{4}+\cdots+\frac{3}{n-1}-\frac{3}{n}+\frac{3}{n}-\frac{3}{n+1}\\ s_n &= 3-\frac{3}{n+1}\end{aligned}$$

Evaluate $\lim\limits_{n\to\infty} s_n$.

$$\begin{aligned}\lim_{n\to\infty} s_n &= \lim_{n\to\infty}\left(3-\frac{3}{n+1}\right)\\ &= 3\end{aligned}$$

Therefore, $\sum\limits_{n=1}^{\infty}\left(\frac{3}{n}-\frac{3}{n+1}\right)$ converges and has the sum 3.