Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3]

Indicate whether the series converges or diverges. If it converges, find its sum.

$$\sum_{k=1}^{\infty} \frac{k!}{112^k}$$

The nth-term test for divergence states that if the series $\sum_{k=1}^{\infty} a_k$ converges, then $\lim_{k\to\infty} a_k = 0$. Equivalently, if $\lim_{k\to\infty} a_k \neq 0$ or if $\lim_{k\to\infty} a_k$ does not exist, then the series diverges.

Let $a_k = \frac{k!}{112^k}$.

Notice the following.

$$\begin{aligned}(k+1)! &= (k+1)(k)(k-1)(k-2)\cdots(3)(2)(1)\\ &= (k+1)k!\end{aligned}$$

Similarly, $112^{k+1} = (112)112^k$.

Using these observations, express a_{k+1} in terms of a_k.

$$\begin{aligned}a_{k+1} &= \frac{(k+1)!}{112^{k+1}}\\ &= \frac{(k+1)k!}{(112)112^k}\\ &= \frac{k+1}{112}\left(\frac{k!}{112^k}\right)\\ &= \frac{k+1}{112}a_k\end{aligned}$$

Next, notice that for $k \geq 112$, we have $\frac{k+1}{112} > 1$.

Thus, for $k \geq 112$, $a_{k+1} = \frac{k+1}{112}a_k > a_k$.

Based on these observations, the sequence $\{a_k\}$ becomes an increasing sequence for $k \geq 112$. Therefore, the limit of this sequence cannot equal zero.

Since $\lim_{k\to\infty} a_k \neq 0$, by the nth-term test for divergence, $\sum_{k=1}^{\infty} \frac{k!}{112^k}$ diverges.