Symbolab

## Solution

$\sum_{n=1}^{\infty}\frac{9^nx^{2n}}{n}$: Radius of convergence is $\frac{1}{3}$, Interval of convergence is $-\frac{1}{3}\le x<\frac{1}{3}$

## Steps

$\sum_{n=1}^{\infty}\frac{9^nx^{2n}}{n}$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=1}^{\infty}\frac{9^nx^{2n}}{n}$

Series Ratio Test:

If there exists an $N$ so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{9^{(n+1)}x^{2(n+1)}}{(n+1)}}{\frac{9^nx^{2n}}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{9^{(n+1)}x^{2(n+1)}}{(n+1)}}{\frac{9^nx^{2n}}{n}}\right|\right)$ *Hide Steps*

$L=\lim_{n\to\infty}\left(\left|\frac{\frac{9^{(n+1)}x^{2(n+1)}}{(n+1)}}{\frac{9^nx^{2n}}{n}}\right|\right)$

Simplify $\frac{\frac{9^{(n+1)}x^{2(n+1)}}{(n+1)}}{\frac{9^nx^{2n}}{n}}$: $\frac{9nx^2}{n+1}$ *Hide Steps*

$\frac{\frac{9^{n+1}x^{2(n+1)}}{n+1}}{\frac{9^nx^{2n}}{n}}$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$=\frac{9^{n+1}x^{2(n+1)}n}{(n+1)\cdot 9^nx^{2n}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{9^{n+1}}{9^n}=9^{n+1-n}$

$=\frac{9^{n-n+1}nx^{2(n+1)}}{x^{2n}(n+1)}$

Add similar elements: $n+1-n=1$

$=\frac{9nx^{2(n+1)}}{x^{2n}(n+1)}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{x^{2(n+1)}}{x^{2n}}=x^{2(n+1)-2n}$

$=\frac{9nx^{2(n+1)-2n}}{n+1}$

Add similar elements: $2(n+1)-2n=2$

$=\frac{9nx^2}{n+1}$

$L=\lim_{n\to\infty}\left(\left|\frac{9nx^2}{n+1}\right|\right)$

$L=\left|9x^2\right|\cdot\lim_{n\to\infty}\left(\left|\frac{n}{n+1}\right|\right)$

$\lim_{n\to\infty}\left(\left|\frac{n}{n+1}\right|\right)=1$ *Hide Steps*

$\lim_{n\to\infty}\left(\left|\frac{n}{n+1}\right|\right)$

$\frac{n}{n+1}$ is positive when $n\to\infty$. Therefore $\left|\frac{n}{n+1}\right|=\frac{n}{n+1}$

$=\lim_{n\to\infty}\left(\frac{n}{n+1}\right)$

Divide by highest denominator power: $\frac{1}{1+\frac{1}{n}}$ *Hide Steps*

$\frac{n}{n+1}$

Divide by $n$

$=\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}}$

Refine

$=\frac{1}{1+\frac{1}{n}}$

$=\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)},\quad \lim_{x\to a}g(x)\neq 0$

With the exception of indeterminate form

$=\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1$ *Hide Steps*

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$

$\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$

With the exception of indeterminate form

$=\lim_{n\to\infty}(1)+\lim_{n\to\infty}\left(\frac{1}{n}\right)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$\lim_{n\to\infty}\left(\frac{1}{n}\right)=0$ *Hide Steps*

$\lim_{n\to\infty}\left(\frac{1}{n}\right)$

Apply Infinity Property: $\lim_{x\to\infty}\left(\frac{c}{x^a}\right)=0$

$=0$

$=1+0$

Simplify

$=1$

$=\frac{1}{1}$

Simplify

$=1$

$L=\left|9x^2\right|\cdot 1$

Simplify

$L=9|x|^2$

$L=9|x|^2$

The power series converges for $L<1$

$9|x|^2<1$

Find the radius of convergence

*Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$9|x|^2<1: \quad |x|<\frac{1}{3}$

*Hide Steps*

$9|x|^2<1$

Divide both sides by $9$

$\frac{9|x|^2}{9}<\frac{1}{9}$

Simplify

$|x|^2<\frac{1}{9}$

Take the square root of both sides of an inequality

$\sqrt{|x|^2}<\sqrt{\frac{1}{9}}$

Simplify

$|x|<\frac{1}{3}$

Therefore

Radius of convergence is $\frac{1}{3}$

Radius of convergence is $\frac{1}{3}$

Find the interval of convergence

*Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $x$

$9|x|^2<1 \quad : \quad -\frac{1}{3}<x<\frac{1}{3}$

*Hide Steps*

$9|x|^2<1$

Find positive and negative intervals

*Hide Steps*

Find intervals for $|x|$

$x\ge 0: x\ge 0, \quad |x|=x$

*Hide Steps*

Rewrite $|x|$ for $x\ge 0$: $\quad |x|=x$

*Hide Steps*

Apply absolute rule: If $u\ge 0$ then $|u|=u$

$|x|=x$

$x<0: x<0, \quad |x|=-x$

*Hide Steps*

Rewrite $|x|$ for $x<0$: $\quad |x|=-x$

*Hide Steps*

Apply absolute rule: If $u<0$ then $|u|=-u$

$|x|=-x$

Identify the intervals:

$x<0, \ x\ge 0$

| | $x<0$ | $x\ge 0$ |
|---|---|---|
| $\lvert x\rvert$ | $-$ | $+$ |

$x<0, \ x\ge 0$

$x<0, \ x\ge 0$

Solve the inequality for each interval

*Hide Steps*

$x<0, \ x\ge 0$

For $x<0$: $\quad -\frac{1}{3}<x<0$

*Hide Steps*

For $x<0$ rewrite $9|x|^2<1$ as $9(-x)^2<1$

$9(-x)^2<1 \quad : \quad -\frac{1}{3}<x<\frac{1}{3}$

*Hide Steps*

$9(-x)^2<1$

Divide both sides by $9$

$\frac{9(-x)^2}{9} < \frac{1}{9}$

Simplify

$(-x)^2 < \frac{1}{9}$

For $u^n < a$, if $n$ is even then $-\sqrt[n]{a} < u < \sqrt[n]{a}$

$-\sqrt{\frac{1}{9}} < -x < \sqrt{\frac{1}{9}}$

If $a < u < b$ then $a < u$ and $u < b$

$-\sqrt{\frac{1}{9}} < -x$ and $-x < \sqrt{\frac{1}{9}}$

$-\sqrt{\frac{1}{9}} < -x \quad : \quad x < \frac{1}{3}$ *Hide Steps*

$-\sqrt{\frac{1}{9}} < -x$

Add $x$ to both sides

$-\sqrt{\frac{1}{9}} + x < -x + x$

Simplify *Hide Steps*

$-\sqrt{\frac{1}{9}} + x < -x + x$

Simplify $-\sqrt{\frac{1}{9}} + x: \quad -\frac{1}{3} + x$ *Hide Steps*

$-\sqrt{\frac{1}{9}} + x$

$\sqrt{\frac{1}{9}} = \frac{1}{3}$ *Hide Steps*

$\sqrt{\frac{1}{9}}$

Apply radical rule: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$, assuming $a \geq 0, b \geq 0$

$= \frac{\sqrt{1}}{\sqrt{9}}$

$\sqrt{9} = 3$ *Hide Steps*

$\sqrt{9}$

Factor the number: $9 = 3^2$

$= \sqrt{3^2}$

Apply radical rule: $\sqrt[n]{a^n} = a$

$\sqrt{3^2} = 3$

$= 3$

$= \frac{\sqrt{1}}{3}$

Apply rule $\sqrt{1} = 1$

$= \frac{1}{3}$

$= -\frac{1}{3} + x$

Simplify $-x + x: \quad 0$ *Hide Steps*

$-x + x$

Add similar elements: $-x + x < 0$

$= 0$

$-\frac{1}{3} + x < 0$

$-\frac{1}{3} + x < 0$

Add $\frac{1}{3}$ to both sides

$-\frac{1}{3} + x + \frac{1}{3} < 0 + \frac{1}{3}$

Simplify

$x < \frac{1}{3}$

$-x<\sqrt{\frac{1}{9}}\quad:\quad x>-\frac{1}{3}$ *Hide Steps*

$-x<\sqrt{\frac{1}{9}}$

Multiply both sides by $-1$ (reverse the inequality)

$(-x)(-1)>\sqrt{\frac{1}{9}}(-1)$

Simplify

$x>-\frac{1}{3}$

Combine the intervals

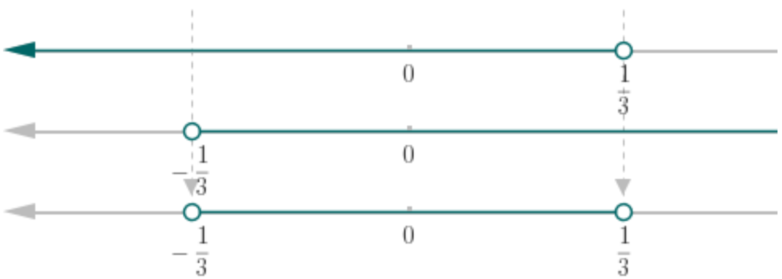
$x<\frac{1}{3}$ and $x>-\frac{1}{3}$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x<\frac{1}{3}$ and $x>-\frac{1}{3}$

$-\frac{1}{3}<x<\frac{1}{3}$




$-\frac{1}{3}<x<\frac{1}{3}$

Combine the intervals

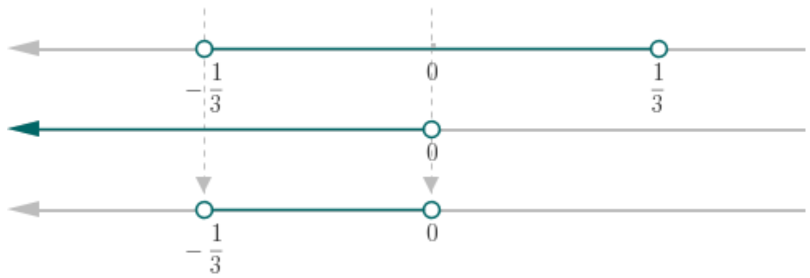
$-\frac{1}{3}<x<\frac{1}{3}$ and $x<0$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$-\frac{1}{3}<x<\frac{1}{3}$ and $x<0$

$-\frac{1}{3}<x<0$




$-\frac{1}{3}<x<0$

For $x\ge 0$: $0\le x<\frac{1}{3}$ *Hide Steps*

For $x\ge 0$ rewrite $9|x|^2<1$ as $9x^2<1$

$9x^2<1\quad:\quad -\frac{1}{3}<x<\frac{1}{3}$ *Hide Steps*

$9x^2<1$

Divide both sides by $9$

$\frac{9x^2}{9}<\frac{1}{9}$

Simplify

$x^2<\frac{1}{9}$

For $u^n<a$, if $n$ is even then $-\sqrt[n]{a}<u<\sqrt[n]{a}$

$-\sqrt{\frac{1}{9}}<x<\sqrt{\frac{1}{9}}$

$\sqrt{\frac{1}{9}}=\frac{1}{3}$ *Hide Steps*

$\sqrt{\frac{1}{9}}$

Apply radical rule: $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}},\quad$ assuming $a\ge 0, b\ge 0$

$=\frac{\sqrt{1}}{\sqrt{9}}$

$\sqrt{9}=3$ *Hide Steps*

$\sqrt{9}$

Factor the number: $9=3^2$

$=\sqrt{3^2}$

Apply radical rule: $\sqrt[n]{a^n}=a$

$\sqrt{3^2}=3$

$=3$

$=\frac{\sqrt{1}}{3}$

Apply rule $\sqrt{1}=1$

$=\frac{1}{3}$

$-\frac{1}{3}<x<\frac{1}{3}$

Combine the intervals

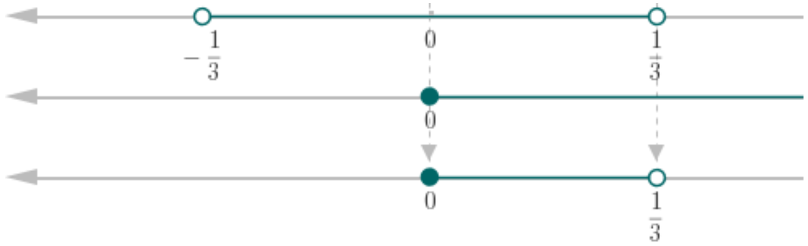
$-\frac{1}{3}<x<\frac{1}{3}$ and $x\ge 0$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$-\frac{1}{3}<x<\frac{1}{3}$ and $x\ge 0$

$0\le x<\frac{1}{3}$




$0\le x<\frac{1}{3}$

Combine the intervals

$-\frac{1}{3}<x<0$ or $0\le x<\frac{1}{3}$

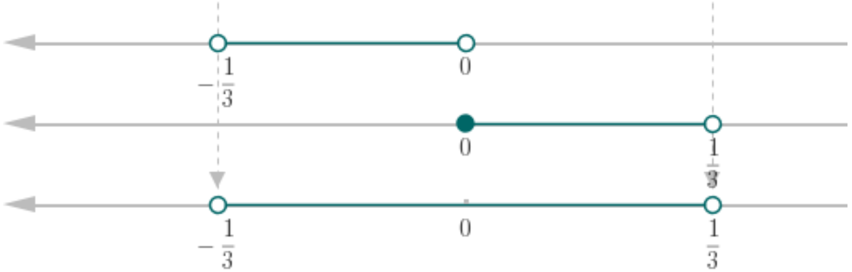
$-\frac{1}{3}<x<0$ or $0\le x<\frac{1}{3}$

Merge Overlapping Intervals *Hide Steps*

The union of two intervals is the set of numbers which are in either interval

$-\frac{1}{3}<x<0$ or $0\le x<\frac{1}{3}$

$-\frac{1}{3}<x<\frac{1}{3}$




The union of two intervals is the set of numbers which are in either interval

$-\frac{1}{3}<x<\frac{1}{3}$ or $x=-\frac{1}{3}$

$-\frac{1}{3}\le x<\frac{1}{3}$

$-\frac{1}{3} \le x < \frac{1}{3}$

$-\frac{1}{3} < x < \frac{1}{3}$

Check the interval end points: $x=-\frac{1}{3}$:converges, $x=\frac{1}{3}$:diverges

*Hide Steps*

For $x=-\frac{1}{3}$, $\sum_{n=1}^{\infty}\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}$: converges

*Hide Steps*

$\sum_{n=1}^{\infty}\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}$

Raabe's Test:

If there exists an $N$ so that for all $n \ge N$, $a_n \ge 0$ and $\lim_{n\to\infty}\left(n\left(\frac{a_n}{a_{n+1}}-1\right)\right)=L$

If $L<1$, then $\sum a_n$ diverges

If $L>1$, then $\sum a_n$ converges

If $L=1$, then the test is inconclusive

$$\lim_{n\to\infty}\left(n\left(\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}-1\right)\right)=1$$

*Hide Steps*

$$\lim_{n\to\infty}\left(n\left(\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}-1\right)\right)$$

*Hide Steps*

$$n\left(\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}-1\right)=1$$

$$n\left(\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}-1\right)$$

$$\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}=\frac{n+1}{n}$$

*Hide Steps*

$$\frac{\frac{9^n\left(-\frac{1}{3}\right)^{2n}}{n}}{\frac{9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}{n+1}}$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$$=\frac{9^n\left(-\frac{1}{3}\right)^{2n}(n+1)}{n\cdot 9^{n+1}\left(-\frac{1}{3}\right)^{2(n+1)}}$$

Apply exponent rule: $\frac{x^a}{x^b}=\frac{1}{x^{b-a}}$

$\frac{9^n}{9^{n+1}}=\frac{1}{9^{n+1-n}}$

$$=\frac{\left(-\frac{1}{3}\right)^{2n}(n+1)}{9^{n-n+1}\left(-\frac{1}{3}\right)^{2(n+1)}n}$$

Add similar elements: $n+1-n=1$

$$=\frac{\left(-\frac{1}{3}\right)^{2n}(n+1)}{9\left(-\frac{1}{3}\right)^{2(n+1)}n}$$

Apply exponent rule: $\frac{x^a}{x^b}=\frac{1}{x^{b-a}}$

$$\frac{\left(-\frac{1}{3}\right)^{2n}}{\left(-\frac{1}{3}\right)^{2(n+1)}}=\frac{1}{\left(-\frac{1}{3}\right)^{2(n+1)-2n}}$$

$$=\frac{n+1}{9\left(-\frac{1}{3}\right)^{-2n+2(n+1)}n}$$

Add similar elements: $2(n+1)-2n=2$

$$=\frac{n+1}{9\left(-\frac{1}{3}\right)^{2}n}$$

$9\left(-\frac{1}{3}\right)^2n=9\left(\frac{1}{3}\right)^2n$ *Hide Steps*

$9\left(-\frac{1}{3}\right)^2n$

$\left(-\frac{1}{3}\right)^2=\left(\frac{1}{3}\right)^2$ *Hide Steps*

$\left(-\frac{1}{3}\right)^2$

Apply exponent rule: $(-a)^n=a^n$, if $n$ is even

$\left(-\frac{1}{3}\right)^2=\left(\frac{1}{3}\right)^2$

$=\left(\frac{1}{3}\right)^2$

$=9\left(\frac{1}{3}\right)^2n$

$$=\frac{n+1}{9\left(\frac{1}{3}\right)^{2}n}$$

$\left(\frac{1}{3}\right)^2=\frac{1}{3^2}$ *Hide Steps*

$\left(\frac{1}{3}\right)^2$

Apply exponent rule: $\left(\frac{a}{b}\right)^c=\frac{a^c}{b^c}$

$=\frac{1^2}{3^2}$

Apply rule $1^a=1$

$1^2=1$

$=\frac{1}{3^2}$

$$=\frac{n+1}{9\cdot\frac{1}{3^2}n}$$

Multiply $9\cdot\frac{1}{3^2}n:\quad n$ *Hide Steps*

$9\cdot\frac{1}{3^2}n$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot 9n}{3^2}$

Multiply the numbers: $1\cdot 9=9$

$=\frac{9n}{3^2}$

Factor $9$: $3^2$ *Hide Steps*

Factor $9=3^2$

$=\frac{3^2n}{3^2}$

Cancel the common factor: $3^2$

$=n$

$$=\frac{n+1}{n}$$

$$=n\left(\frac{n+1}{n}-1\right)$$

Apply the distributive law: $a(b-c)=ab-ac$

$a=n,\ b=\frac{n+1}{n},\ c=1$

$= n\frac{n+1}{n} - n \cdot 1$

$= \frac{n+1}{n}n - 1 \cdot n$

Simplify $\frac{n+1}{n}n - 1 \cdot n$: $1$

Hide Steps

$\frac{n+1}{n}n - 1 \cdot n$

$\frac{n+1}{n}n = n+1$

Hide Steps

$\frac{n+1}{n}n$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{(n+1)n}{n}$

Cancel the common factor: $n$

$= n+1$

$1 \cdot n = n$

Hide Steps

$1 \cdot n$

Multiply: $1 \cdot n = n$

$= n$

$= n+1-n$

Group like terms

$= n-n+1$

Add similar elements: $n-n=0$

$= 1$

$= 1$

$= \lim_{n\to\infty}(1)$

$\lim_{x\to a}c = c$

$= 1$

$L > 1$, by the Raabe's test

$=$ converges

For $x = \frac{1}{3}$, $\sum_{n=1}^{\infty}\frac{9^n\left(\frac{1}{3}\right)^{2n}}{n}$: diverges

Hide Steps

$\sum_{n=1}^{\infty}\frac{9^n\left(\frac{1}{3}\right)^{2n}}{n}$

Refine

$= \sum_{n=1}^{\infty}\frac{1}{n}$

Apply Cauchy's Convergence Condition: diverges

Hide Steps

$\sum_{n=1}^{\infty}\frac{1}{n}$

Cauchy's Convergence Condition:

$\sum a_n$ converges, if, and only if
For every $\epsilon > 0$ there is a natural number $N$ such that $|a_{n+1} + a_{n+2} + \ldots + a_{n+p}| < \epsilon$, $\forall\, n > N$ and $p \ge 1$

Taking $S_{2n} - S_n = \sum_{n=1}^{2n}\frac{1}{n} - \sum_{n=1}^{n}\frac{1}{n} = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n} \ge \frac{1}{2n} + \frac{1}{2n} + \ldots$ -
Therefore there cannot be found a number $N$ that satisfies Cauchy's condition

$=$ diverges

$=$ diverges

$x = -\frac{1}{3}$:converges, $x = \frac{1}{3}$:diverges

Therefore

Interval of convergence is $-\frac{1}{3} \le x < \frac{1}{3}$

Interval of convergence is $-\frac{1}{3} \le x < \frac{1}{3}$

Radius of convergence is $\frac{1}{3}$, Interval of convergence is $-\frac{1}{3} \le x < \frac{1}{3}$