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Find the series' interval of convergence and, within this interval, the sum of the series as a function of x.

$$\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n}$$

First, use the root test, $\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$, to find the series' interval of convergence. Begin by taking the nth root of the nth term in the series.

$$\begin{aligned}\rho &= \lim_{n\to\infty} \sqrt[n]{\frac{(x-2)^{2n}}{9^n}} \\ &= \frac{(x-2)^2}{9}\end{aligned}$$

A series will only converge when $\rho < 1$. Solve the inequality $\rho = \frac{(x-2)^2}{9} < 1$ for x to find the interval of convergence.

$$\begin{aligned}\frac{(x-2)^2}{9} &< 1 \\ (x-2)^2 &< 9 \\ |x-2| &< 3 \qquad \text{Using } \sqrt{x^2} = |x|. \\ -1 &< x < 5\end{aligned}$$

Next, test for convergence at each endpoint, $x = -1$ and $x = 5$. Substitute each back into the expression for the series, $\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n}$, and use the integral test to determine whether the series converges or diverges at each point.

Begin with $x = -1$. Substitute this back into the original series.

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n} &= \sum_{n=0}^{\infty} \frac{(-1-2)^{2n}}{9^n} \\ &= \sum_{n=0}^{\infty} \frac{(-3)^{2n}}{9^n} \\ &= \sum_{n=0}^{\infty} (1)^n\end{aligned}$$

Now test for convergence using the nth-term test for divergence. Determine the value of $\lim_{n\to\infty} 1^n$.

$$\lim_{n\to\infty} 1^n = 1$$

Since $\lim_{n\to\infty} 1^n$ is different from zero, the series $\sum_{n=0}^{\infty} (1)^n$ diverges.

Repeat the process for $x = 5$.

$$\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n} = \sum_{n=0}^{\infty} \frac{(5-2)^{2n}}{9^n}$$
$$= \sum_{n=0}^{\infty} \frac{(3)^{2n}}{9^n}$$
$$= \sum_{n=0}^{\infty} 1$$

The series becomes $\sum_{n=0}^{\infty} 1$ at $x = 5$. As was the case at $x = -1$, the series diverges. Therefore, the interval of convergence is $-1 < x < 5$.

Now find the sum of the series $\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n}$. Since the series is written in the form $\sum_{n=1}^{\infty} ar^n$, it is a geometric series.

Find the ratio of the series, r.

$$r = \frac{(x-2)^2}{9}$$

The sum of a geometric series is given by the expression $\sum ar^{n-1} = \frac{1}{1-r}$. Substitute and solve for the sum.

$$\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n} = \frac{1}{1-\left((x-2)^2/9\right)}$$

Substitute $r = \frac{(x-2)^2}{9}$.

$$= \frac{9}{9-(x-2)^2} = \frac{9}{5+4x-x^2}$$

The sum of the series is $\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{9^n} = \frac{9}{5+4x-x^2}$.