Constant acceleration motion

Differential Equation

$$\frac{d^2}{dt^2}\vec{r}(t) = \vec{a}_o$$

not t; this vector doesn't change.

same

$$\text{if} \quad \frac{d^2}{dx^2} f(x) = c$$

$$\text{then} \quad f(x) = a + bx + \frac{c}{2}x^2$$

can be anything

$$\text{Is} \quad c = \frac{d^2}{dx^2}\left(a + bx + \frac{c}{2}x^2\right)$$

$$= \frac{d}{dx}\left[\frac{d}{dx}\left(a + bx + \frac{c}{2}x^2\right)\right]$$

$$= \frac{d}{dx}\left[0 + b1 + \frac{c}{2}2x\right]$$

$$= \frac{d}{dx}\left[b + cx\right]$$

$$= c \quad \text{yes!}$$

If $\frac{d^2}{dt^2}\vec{r}(t) = \vec{a}_0$

then

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 \left[t-t_0\right] + \frac{1}{2}\vec{a}_0\left[t-t_0\right]^2$$

$\vec{r}_0$ is position at t_0

$\vec{r}(t_0) = \vec{r}_0$

velocity at time $t=t_0$

$\vec{v}(t_0) = \vec{v}_0$

t_0 is time when $\vec{r}_0, \vec{v}_0$ measured.

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = \frac{d}{dt}\left[\vec{r}_0 + \vec{v}_0\left[t-t_0\right] + \frac{1}{2}\vec{a}_0\left[t-t_0\right]^2\right]$$

$$= 0 + \vec{v}_0 + \frac{1}{2}\vec{a}_0 2\left[t-t_0\right]$$

$$\vec{v}(t) = \vec{v}_0 + \vec{a}_0\left[t-t_0\right]$$

$$\vec{r}(t) = \left(x_0\hat{i} + y_0\hat{j} + z_0\hat{k}\right)$$

$$+\left(v_{0,x}\hat{i} + v_{0,y}\hat{j} + v_{0,z}\hat{k}\right)\left[t-t_0\right]$$

$$+\frac{1}{2}\left(a_{0,x}\hat{i} + a_{0,y}\hat{j} + a_{0,z}\hat{k}\right)\left[t-t_0\right]^2$$

$$= \left(x_0 + v_{0,x}[t-t_0] + \frac{1}{2} a_{0,x}[t-t_0]^2 \right)\hat{i}$$

$$+ \left(y_0 + v_{0,y}[t-t_0] + \frac{1}{2} a_{0,y}[t-t_0]^2 \right)\hat{j}$$

$$+ \left(z_0 + v_{0,z}[t-t_0] + \frac{1}{2} a_{0,z}[t-t_0]^2 \right)\hat{k}$$

$$\vec{r}(t)\cdot\hat{i} = x(t) = x_0 + v_{0,x}[t-t_0] + \frac{1}{2} a_{0,x}[t-t_0]^2$$

$$y(t) = y_0 + v_{0,y}[t-t_0] + \frac{1}{2} a_{0,y}[t-t_0]^2$$

$$z(t) = z_0 + v_{0,z}[t-t_0] + \frac{1}{2} a_{0,z}[t-t_0]^2$$

Velocity and Acceleration - II

only

A particle moves in the x-direction with constant acceleration. At time $t = -1s$ it is at $1m\hat{\imath}$ moving with velocity $-3\frac{m}{s}\hat{\imath}$ and subject to an acceleration of $1\frac{m}{s^2}\hat{\imath}$.

- What is the expression for the particle's position as a function of time?
- What is the position and velocity at $t = 4s$?
- What is the minimum value of the x-component of the particle's position and when does it get there?

Know is constant $\vec{a}$ then

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0[t - t_0] + \frac{1}{2}\vec{a}_0[t - t_0]^2$$

$\vec{r}(t_0)$

$\vec{v}(t_0)$

t_0 = when position & velocity measured.

$t_0 = -1s$

$\vec{r}(-1s) = 1m\hat{\imath}$

$\vec{v}(-1s) = -3 m/s\,\hat{\imath}$

$\vec{a}(-1s)$ = at all times $= 1 m/s^2\,\hat{\imath}$

$$\vec{r}(t) = 1\,\mathrm{m}\,\hat{i} + (-3\,\mathrm{m/s}\,\hat{i})\left[t-(-1\,\mathrm{s})\right] + \frac{1}{2}(1\,\mathrm{m/s^2}\,\hat{i})\left[t-(-1\,\mathrm{s})\right]^2$$

$$= \underline{1\,\mathrm{m}\,\hat{i}} + (\underline{-3\,\mathrm{m/s}\,t})\hat{i} + \underline{(-3\,\mathrm{m})}\hat{i}$$

$$+ \frac{1}{2}(1\,\mathrm{m/s^2}\,t^2)\hat{i} + \frac{1}{2}\underline{(1\,\mathrm{m/s^2}\,\hat{i})(2\,\mathrm{s}\,t)}$$

$$+ \underline{\frac{1}{2}(1\,\mathrm{m/s^2})\hat{i}(1\,\mathrm{s})^2}$$

$$= \underbrace{-1.5\,\mathrm{m}\,\hat{i}}_{\vec{r}(0\,\mathrm{s}) = -1.5\,\mathrm{m}\,\hat{i}} + \underbrace{(-2\,\mathrm{m/s}\,t)\hat{i}}_{\vec{v}(0\,\mathrm{s}) = -2\,\mathrm{m/s}\,\hat{i}} + \underbrace{\frac{1}{2}(1\,\mathrm{m/s^2}\,t^2)\hat{i}}_{\vec{a} = 1\,\mathrm{m/s^2}\,\hat{i}}$$

$$\vec{r}(4\,\mathrm{s}) = 1\,\mathrm{m}\,\hat{i} + (-3\,\mathrm{m/s}\,\hat{i})\left[4\,\mathrm{s}-(-1\,\mathrm{s})\right] + \frac{1}{2}(1\,\mathrm{m/s^2}\,\hat{i})\left[4\,\mathrm{s}-(-1\,\mathrm{s})\right]^2$$

$$= -1.5\,\mathrm{m}\,\hat{i}$$

$$\vec{r}(4\,\mathrm{s}) = -1.5\,\mathrm{m}\,\hat{i} + (-2\,\mathrm{m/s}\;4\,\mathrm{s})\hat{i} + \frac{1}{2}\left(1\,\mathrm{m/s^2}\,(4\,\mathrm{s})^2\right)\hat{i}$$

$$= -1.5\,\mathrm{m}\,\hat{i}$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = \frac{d}{dt}\left(-1.5\,\mathrm{m}\,\hat{i} + (-2\,\mathrm{m/s}\,t)\hat{i} + \frac{1}{2}\,1\,\mathrm{m/s^2}\,t^2\,\hat{i}\right)$$

$$= 0 + -2\,\mathrm{m/s}\,\hat{i} + \frac{1}{2}\,1\,\mathrm{m/s^2}\,\hat{i}\,2t$$

$$= (-2\,\mathrm{m/s} + 1\,\mathrm{m/s^2}\,t)\hat{i}$$

$$\frac{d}{dt}\left[1\,\mathrm{m}\,\hat{\imath} + (-3\,\mathrm{m/s}\,\hat{\imath})[t-(-1\,\mathrm{s})] + \frac{1}{2}(1\,\mathrm{m/s^2}\,\hat{\imath})[t-(-1\,\mathrm{s})]^2\right]$$

$$= 0 + (-3\,\mathrm{m/s}\,\hat{\imath})1 + \frac{1}{2}(1\,\mathrm{m/s^2}\,\hat{\imath})2[t-(-1\,\mathrm{s})]$$

$$= -3\,\mathrm{m/s}\,\hat{\imath} + 1\,\mathrm{m/s^2}[t-(-1\,\mathrm{s})]\hat{\imath}$$

$$= (-2\,\mathrm{m/s} + 1\,\mathrm{m/s^2}\,t)\hat{\imath}$$

$$\vec{v}(4\,\mathrm{s}) = 2\,\mathrm{m/s}\,\hat{\imath}$$

To get min of x-comp of position

[Extrema correspond to derivative vanishing]

min pos^n __when__ $\underbrace{\frac{d}{dt}x(t)}_{v_x(t)} = 0$

$$v_x(t) = 0$$

$$v_x(t) = (-2\,\mathrm{m/s} + 1\,\mathrm{m/s^2}\,\underline{\underline{t}})$$

$$\boxed{0 \text{ at } t = 2\,\mathrm{s}}$$

what is $x(2\,\mathrm{s})$?

$$\vec{r}(2\,\mathrm{s}) = -1.5\,\mathrm{m}\,\hat{\imath} + (-2\,\mathrm{m/s}\;2\,\mathrm{s})\hat{\imath} + \frac{1}{2}(1\,\mathrm{m/s^2})(2\,\mathrm{s})^2$$

$$= -3.5\,\mathrm{m}$$

Velocity and Acceleration - III

One particle moves with constant acceleration of $-0.5\frac{m}{s^2}\hat{\imath}$. At time $t = 0s$ it is at the origin moving at $20\frac{m}{s}\hat{\imath}$.

A second particle is at the origin stationary, until at $t = 4s$ it starts to accelerate at a constant $3\frac{m}{s^2}\hat{\imath}$.

- When does the second particle pass the first?
- Where are they when they pass?
- During the period between $t = 0s$ and when they pass, what is the farthest apart they are, and when does that happen?

For first particle

$$\vec{r}_1(t) = \vec{r}_0 + \vec{v}_0[t - t_0] + \frac{1}{2}\vec{a}_0[t - t_0]^2$$

$$= 0m\hat{\imath} + 20\,m/s\,\hat{\imath}[t - 0s] + \frac{1}{2}(-0.5\,m/s^2\,\hat{\imath})[t - 0s]^2$$

$$= (20\,m/s\,t - 0.25\,m/s^2\,t^2)\hat{\imath}$$

For 2nd particle before $t = 4s$

$$\vec{r}_2(t) = 0m\hat{\imath} \qquad (t < 4s)$$

$$\vec{r}_2(t) = 0m\hat{\imath} + 0\,m/s\,\hat{\imath}[t - 4s] + \frac{1}{2}(3\,m/s^2\,\hat{\imath})[t - 4s]^2$$

$$= 1.5\,m/s^2[t - 4s]^2\,\hat{\imath} \qquad (t \geq 4s)$$

pass → same location

There is t_p when

$$\vec{r}_1(t_p) = \vec{r}_2(t_p)$$

assuming after 4s

$$\left(20\,\text{m/s}\,t_p - 0.25\,\text{m/s}^2\,t_p^2\right) = \left(1.5\,\text{m/s}^2\right)\left(t_p - 4\,\text{s}\right)^2$$

~~20 m/s t_p~~

$$20\,\text{m/s}\,t_p - 0.25\,\text{m/s}^2\,t_p^2 = 1.5\,\text{m/s}^2\,t_p^2 - 12\,\text{m/s}\,t_p + 24\,\text{m}$$

$$0 = 1.75\,\text{m/s}^2\,t_p^2 - 32\,\text{m/s}\,t_p + 24\,\text{m}$$

quadratic for t_p

$$t_p = \frac{-(-32\,\text{m/s}) \pm \sqrt{(-32\,\text{m/s})^2 - 4(1.75\,\text{m/s}^2)(24\,\text{m})}}{2(1.75\,\text{m/s}^2)}$$

$$= \frac{32\,\text{m/s} \pm 29.26\,\text{m/s}}{3.5\,\text{m/s}^2}$$

$$= 17.5\,\text{s} \text{ or } \cancel{0.783\,\text{s}}$$

$$\vec{r}_2(17.5\,\text{s}) = 1.5\,\text{m/s}^2\left[17.5\,\text{s} - 4\,\text{s}\right]^2\hat{\imath} = 273\,\text{m}\,\hat{\imath}$$

Vector from $\vec{r}_2$ to $\vec{r}_1$ is

$\vec{r}_1(t) - \vec{r}_2(t)$

max x-component at max separation

$\left(20\,\tfrac{m}{s}t - 0.25\,\tfrac{m}{s^2}t^2\right)\hat{i} - 1.5\,\tfrac{m}{s^2}\left[t - 4s\right]^2\hat{i}$

$\left(20\,\tfrac{m}{s}t - 0.25\,\tfrac{m}{s^2}t^2 - 1.5\,\tfrac{m}{s^2}t^2 + 12\,\tfrac{m}{s}t - 24m\right)$

x-comp of separation

$\frac{d}{dt}$ x-comp of sep.

$= \frac{d}{dt}\left(-1.75\,\tfrac{m}{s^2}t^2 + 30\,\tfrac{m}{s}t - 24m\right)$

$= -3.5\,\tfrac{m}{s^2}t + 30\,\tfrac{m}{s}$

0 at $t = 8.57s$

thats when sep. is maximum

$-1.75\,\tfrac{m}{s^2}(8.57s)^2 + 30\,\tfrac{m}{s}(8.57s) - 24m$

$= 104.6m$

Could also have gotten this by saying "max sep." when $\vec{v}$'s same