Newton's 2nd Law

Recall first law says:

If net force 0 then no $\vec{a}$

&

If no $\vec{a}$ then deduce $\vec{F}_{net} = 0$

If object feels net force then will have $\vec{a}$, proportional to net force, inversely proportional to mass

$$\vec{a} = \frac{\vec{F}_{net}}{m}$$

the net force

The acceleration of object

is given by

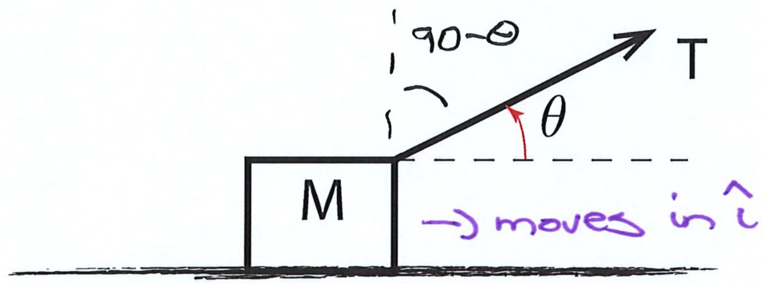
divided by object's mass

Note: $m\vec{a}$ isn't a force

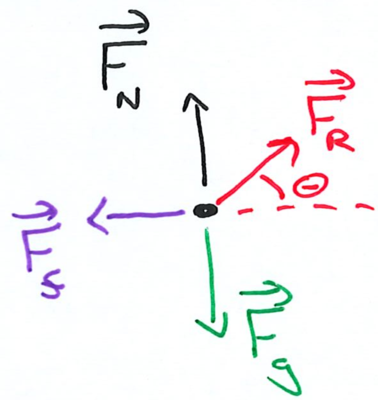
- If know $\vec{a}$ then find $\vec{F}_{net}$ and hence other forces on it.
- If know $\vec{F}_{net}$, m predict acceleration
- If know $\vec{F}_{net}$ & measure $\vec{a}$ based on that find m

Second Law - I

A $20kg$ mass is being pulled over a rough surface with which it has a coefficient of kinetic friction of $\mu_k = 0.2$. It is being pulled in the positive x-direction by a rope which is exerting a force of magnitude $T = 50N$ at an angle of $\theta = 25°$ above the horizontal.



What is the acceleration of this mass?



2nd law $\vec{a} = \dfrac{\vec{F}_{net}}{m}$?

kinetic friction

$$|\vec{F}_f| = \mu_k |\vec{F}_N|$$

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_R + \vec{F}_g + \vec{F}_f$$

$$= |\vec{F}_N|\hat{k} + (T\cos\theta\,\hat{i} + T\sin\theta\,\hat{k}) + (-Mg\hat{k}) + (-|\vec{F}_f|\hat{i})$$

$$\frac{\vec{F}_{Net}}{m} = \frac{1}{m}\left[\left(T\cos\theta - |\vec{F}_f|\right)\hat{\imath} + \left(|\vec{F}_N| + T\sin\theta - Mg\right)\hat{k}\right]$$

2nd law

$$\vec{a} = \left(\qquad \text{''} \qquad \right)$$

$$a_x\hat{\imath} + a_z\hat{k} = \frac{1}{m}\left[\left(T\cos\theta - |\vec{F}_f|\right)\hat{\imath} + \left(|\vec{F}_N| + T\sin\theta - Mg\right)\hat{k}\right]$$

$$a_x = \frac{1}{m}\left(T\cos\theta - |\vec{F}_f|\right)$$

$$0 \quad \cancel{a_z} = \frac{1}{m}\left(|\vec{F}_N| + T\sin\theta - Mg\right)$$

mass stays on surface

if T very big mass pulled off surface

is there a $|\vec{F}_N| > 0$ consistent with $a_z = 0$?

$$|\vec{F}_N| = Mg - T\sin\theta$$

tells us $|\vec{F}_f| = \mu_k|\vec{F}_N| = \mu_k(Mg - T\sin\theta)$

$$a_x = \frac{1}{m}\left(T\cos\theta - \mu_k(Mg - T\sin\theta)\right)$$

$M = 20\text{kg}$ $\quad \theta = 25^\circ$ $\quad T = 50\text{N}$ $\quad \mu_k = 0.2$

$$= 0.517\,\text{m/s}^2$$

What do/can forces depend on?

Constant: eg gravity near surface of earth

$$\vec{F}_g = -mg\hat{k}$$

g: 9.8 N/kg

$$\vec{a} = \frac{\vec{F}_g}{m} = -g\hat{k}$$

g: 9.8 m/s^2

Depend on velocity
- kinetic friction in opposite direction
- Air resistance magnitude depends on speed
- Magnetic force on charged particles

Depend on position
- From rope on ball, direction depends where ball is
- Spring Force depends on how much stretched or compressed

- Electric (Coulomb) force between charced particles
- Gravitational force

Constraint:

- Normal force - can be whatever they need to be.

Second Law - II

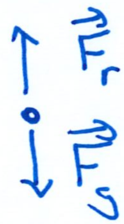
A block of mass $m = 2kg$ falls and is subject to two forces, the downwards force of gravity and an upwards resistive force.

The block's position as a function of time is given by

$$\vec{r}(t) = \left(1000m - 21\frac{m}{s}t - 45m\left(e^{-0.4667s^{-1}t} - 1\right)\right)\hat{k} \tag{3}$$

What is the resistive force on the block at time $t = 3s$? $t = 15s$? $t = 40s$?

$$\vec{a}(t) = \frac{1}{m}\vec{F}_{net}$$



$$\vec{a} = \frac{d^2}{dt^2}\vec{r}(t)$$

$$= \frac{d}{dt}\left(\frac{d}{dt}\vec{r}(t)\right)$$

$$= \frac{d}{dt}\left[\frac{d}{dt}\left(1000m - 21\tfrac{m}{s}t - 45m\left(e^{-0.4667s^{-1}t} - 1\right)\right)\hat{k}\right]$$

$$= \frac{d}{dt}\left[\left(0 - 21\tfrac{m}{s} - 45m\frac{d}{dt}\left(e^{\underline{-0.4667s^{-1}t}} - 1\right)\right)\hat{k}\right]$$

$$\frac{d}{dx}e^{ax} = ae^{ax}$$

$$= \frac{d}{dt}\left[-21\tfrac{m}{s} - 45m\left(-0.4667s^{-1}\right)e^{-0.4667s^{-1}t}\right]\hat{k}$$

$$= \frac{d}{dt}\underbrace{\left(-21\,\mathrm{m/s} + 21\,\mathrm{m/s}\; e^{-0.4667\,s^{-1}\,t}\right)}_{\text{is } \vec{v}(t)}\hat{k}$$

$$\text{at } t = 0\,s \qquad \vec{v} = 0\,\mathrm{m/s}$$

$$\text{at } t = \infty\,s \qquad \vec{v} = -21\,\mathrm{m/s}\,\hat{k}$$

$$= 21\,\mathrm{m/s}\left(-0.4667\,s^{-1}\right) e^{-0.4667\,s^{-1}\,t}\,\hat{k}$$

$$\vec{a}(t) = -9.8\,\mathrm{m/s^2}\; e^{-0.4667\,s^{-1}\,t}\,\hat{k}$$

$$\vec{a}(t) = \frac{1}{m}\left(\vec{F}_g + \vec{F}_r\right)$$

$$m\vec{a}(t) = \vec{F}_g + \vec{F}_r$$

$$-\vec{F}_g + m\vec{a}(t) = \vec{F}_r$$

$$-\left(-m\underbrace{g}_{9.8\,\mathrm{N/kg}}\hat{k}\right) + m\left(-9.8\,\mathrm{m/s^2}\; e^{-0.4667\,s^{-1}\,t}\,\hat{k}\right) = \vec{F}_r$$

$$19.6\,\mathrm{N}\,\hat{k}\left(1 - e^{-0.4667\,s^{-1}\,t}\right) = \vec{F}_R$$

$$\vec{F}_r(3s) = 14.7\,\mathrm{N}\,\hat{k}$$

$$\vec{F}_r(15s) = 19.58\,\mathrm{N}\,\hat{k}$$

$$\vec{F}_r(40s) = 19.6\,\mathrm{N}\,\hat{k}$$