

Subject: Chapter 4 (circle)

Date: _____

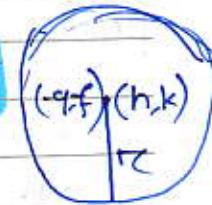
1

$$① x^2 + y^2 + 2gx + 2fy + c = 0 \Rightarrow (x-h)^2 + (y-k)^2 = r^2$$

$$\begin{aligned} h &= -g \\ k &= -f \end{aligned}$$

$$② \text{ centre, } c = (-g, -f) \equiv (h, k)$$

$$\text{constant, } c = h^2 + k^2 - r^2$$



$$③ \text{ radius, } r = \sqrt{g^2 + f^2 - c}$$

constant (as formula shifted to right)

$$r = \sqrt{h^2 + k^2 - c}$$

④ x-axis distance between two points = $2\sqrt{g^2 - c}$

CHORD INTERCEPTS ALONG X-AXIS

x-axis chord

$$⑤ y\text{-axis distance between two points} = 2\sqrt{f^2 - c}$$

y-axis chord

$$⑥ \text{ radius of a circle touching x-axis} = r = |f|$$

..... $f^2 = c$

$$⑦ \text{ radius of a circle touching y-axis} = r = |g|$$

..... $g^2 = c$

$$⑧ \text{ radius of a circle touching two axes} = r = |f| = |g|$$

b

⑨ For a circle to become a circle: coefficient of x^2 & y^2 will be same (1)

• " " " any orz any other

variables not included in ①

formula will be zero/null

• $(g^2 + f^2 - c)$ must be greater orz equal to zero.

$$(g^2 + f^2 - c \geq 0)$$

$$\begin{aligned} \text{x-axis chord} &= 2\sqrt{g^2 - c} \\ \text{y-axis chord} &= 2\sqrt{f^2 - c} \end{aligned}$$

$$⑩ \text{ When a circle touches x-axis, } f = 0$$

$$f^2 = c$$

$$(-g, -f)$$

$$⑪ \text{ When a circle touches y-axis, } g = 0$$

$$g^2 = c$$

$$⑫ \text{ When a circle touches a line } ax + by + c = 0 \text{ it becomes } ah + bk + c = 0$$

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(13)

How to bring centre (h, k) and radius from a circle equation

[18(ii)]

$$\begin{aligned}
 & x^2 + y^2 - 2x + 4y - 31 && \rightarrow \text{a circle eqn} \\
 \Rightarrow & (x^2 - 2x + 1) + (y^2 + 4y + 4) = 1 + 4 + 31 \\
 \Rightarrow & (x-1)^2 + (y+2)^2 = 6^2 && r^2 = (x-h)^2 + (y-k)^2 \\
 & h & k & \downarrow \text{radius}
 \end{aligned}$$

(14) Distance of a point (whose coordinates are given) from a line (whose equation is given)

$$d = r = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$$

(15) m_1 m_2

[Equation $a \times x + b \times y + c = 0$ value, $2 \times x_1 + 0 \times y_1 + c = 0$, we put the values from the point (center) coordinates from the center.
[or we'd have used $(a+bk+c)$ as the line equation in the formula equation]

$$\left\{
 \begin{aligned}
 x &= \frac{(m_1 x_2) + (m_2 x_1)}{m_1 + m_2} \\
 y &= \frac{(m_1 y_2) + (m_2 y_1)}{m_1 + m_2}
 \end{aligned}
 \right.$$

(16) Equation of the line of common chord when the equation of the two circles are given. S_1, S_2

$$\boxed{S_1 - S_2 = 0 = l}$$

(common chord eqn)



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(3)

 S = Circle equation L = Line equation

- (17) Equation of a circle passing through the intersection of a line and another circle both of whose equations are given. $L=0, S_1=0, S=0$

$$\left\{ \begin{array}{l} S_1 + kL = 0 \\ S = 0 \end{array} \right. \quad [k = \text{constant}]$$

- (18) Equation of a tangent of a circle given the coordinates of the point of contact (x_1, y_1) .

$$\left\{ \begin{array}{l} xx_1 + yy_1 + g(x+x_1) + f(y+y_1) + c = 0 \\ (x^2 + y^2 + 2gx + 2fy + c = 0) \rightarrow \text{circle equation} \end{array} \right.$$

- (19) Length of a tangent of a circle given the coordinates of the point of contact (x_1, y_1) .

$$L_1 = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

- (20) Constant of a tangent given the radius of the circle and slope of the tangent.

$$c = \pm r\sqrt{1+m^2}$$

- Equation of a tangent of a circle given the radius and slope of the circle & tangent

$$y = mx \pm r\sqrt{1+m^2}$$

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- (21) Equation of a straight line passing through the origin is

$$\left\{ \begin{array}{l} y = mx \\ \text{or } y - mx = 0 \end{array} \right.$$

If the equation of a tangent is $y = mx$, then putting it in the given equation of a circle, we will find a quadratic equation where,

$$\left\{ b^2 - 4ac = 0 \right.$$

- (22) Equation of a straight line given the coordinates of two points on it $(x_1, y_1), (x_2, y_2)$

$$\left\{ \frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = m \right.$$

- (23) Equation of two perpendicular straight lines given the two slopes.

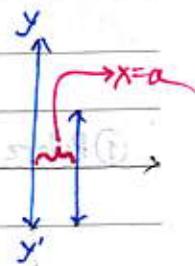
$$\left\{ m_1 \cdot m_2 = -1 \right.$$

- (24) Eqⁿ of a perpendicular and parallel straight lines to a given line,

$$\boxed{\left\{ \begin{array}{l} ax+by+c \\ bx-ay+k \quad (\text{perpendicular}) \\ ax+by+k \quad (\text{parallel}) \end{array} \right.}$$

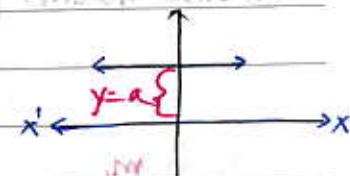
(25) Equation of a straight line parallel to y -axis is

$$\left\{ \begin{array}{l} \text{Direction ratio } \\ \text{constant value} \\ \text{eqn, } x-a=0 \end{array} \right. \quad \text{or, } x=a$$



(26) Equation of a straight line parallel to x -axis is

$$\left\{ \begin{array}{l} \text{y=a } (\text{a is a constant}) \\ \text{eqn, } y-a=0 \end{array} \right.$$



(27) A point on x -axis means $y=0$

A point on y -axis means $x=0$

coordinates of points

(28) Two points of the diameter are given as (x_1, y_1) and (x_2, y_2) . Then circle equation:

$$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$$

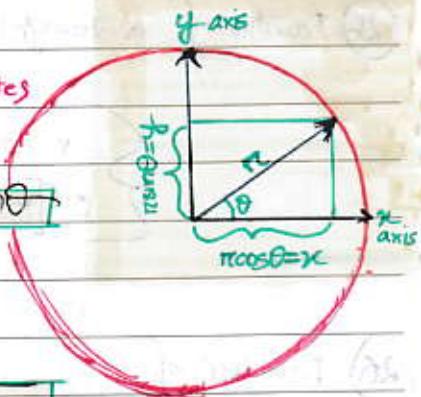
⑥

Subject : Chapter 3 Straight Lines

Date : Chapter 3.1

① Polar Equations :

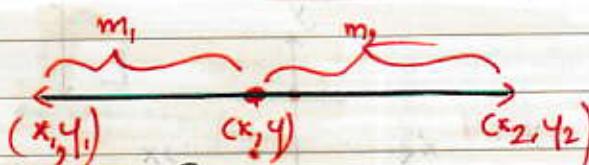
- $P(x,y) = P(r, \theta)$
- $r = \sqrt{x^2 + y^2}$ & $y = r \sin \theta$
- $\tan \theta = \frac{y}{x}$



$$\text{② Line Equation: } d\vec{P} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Chapter 3.2

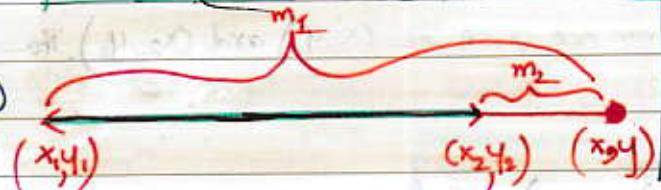
①



Internal intersecting
point of a straight line

$$(x, y) = \left\{ \begin{array}{l} \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2} \right) \end{array} \right\}$$

②



External intersecting
point of a straight line.

$$(x, y) = \left\{ \begin{array}{l} \left(\frac{m_1 x_2 - m_2 x_1}{m_1 - m_2}, \frac{m_1 y_2 - m_2 y_1}{m_1 - m_2} \right) \end{array} \right\}$$

(K:I)

③ Centroid
(G)

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

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Chapter 3.3

① Slope related:
/ Gradient

$$m = \tan \theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{y - y_1}{x - x_1}$$

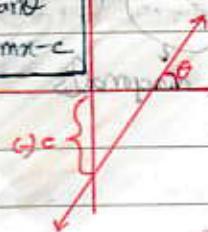
• line equation from points,

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$ax + by + c = 0$$

$$m = \tan \theta$$

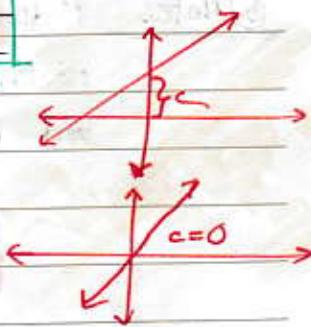
$$y = mx + c$$



• General St. line - slope ; $y = mx + c$

• " " " [$c = y$ intercept]

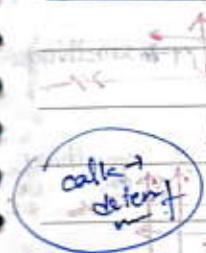
when passing through origin, $y = mx$



• Perpendicular slope / Gradient, $m_1 \cdot m_2 = -1$ ($\tan \alpha \cdot \tan \beta = -1$)

• Parallel, slope / Gradient, $m_1 = m_2$ ($\tan \alpha = \tan \beta$)
Similar

② Area related : • Δ Area of three given points : $(x_1, y_1), (x_2, y_2), (x_3, y_3)$.



clockwise

$$\text{Area} = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Determinant 1

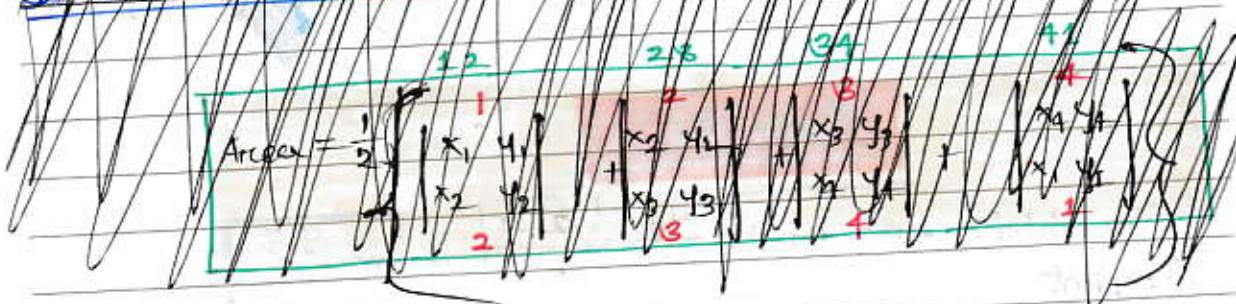
• When passes through same line, $\Delta \text{Area} = 0$.

③ Trigonometry : $\cos \alpha \cos \beta + \sin \alpha \sin \beta = \cos(\alpha - \beta)$

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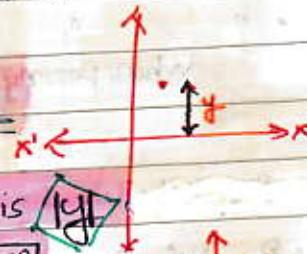
④ Extra Area Related:Area of given points $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$.

⑤ Note: Midpoints of diagonals of a parallelogram, square, rhombus, rectangles are the points where two of the ~~lines~~ intersect and have same co-ordinates.

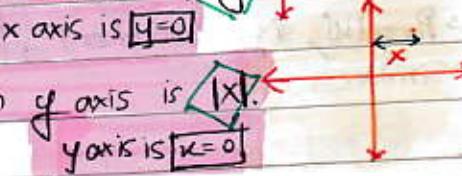
diagonals

Chapter 3.4

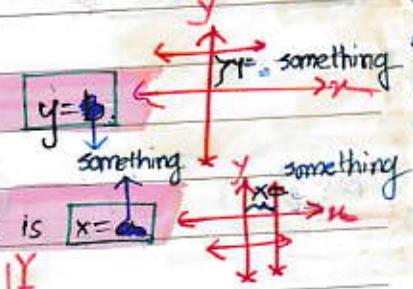
- Distance of a point from x-axis is $|y|$.
x axis is $y=0$



- Distance of a point from y-axis is $|x|$.
y axis is $x=0$



- Equation of a straight line parallel to x-axis is $y=b$.



- Equation of a straight line parallel to y-axis is $x=a$.



- Equation of a point on x-axis is $y=0$.



- Equation of a point on y-axis is $x=0$.



HEART'S

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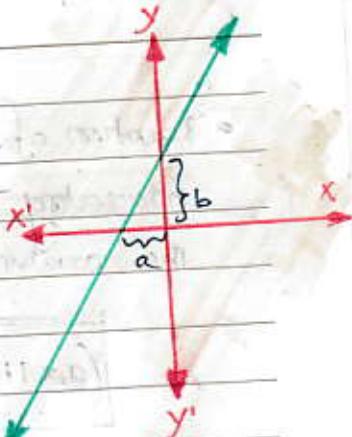
(9)

Chapter 3.5

- Equation of a straight line along with the intercepts.

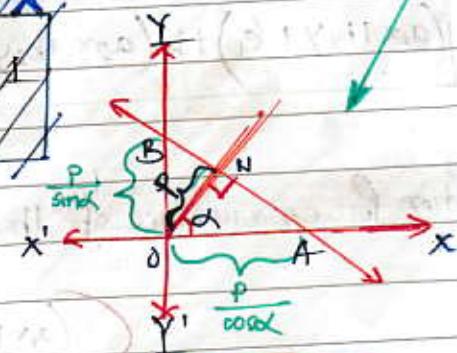
$$\frac{x}{a} + \frac{y}{b} = 1$$

where a is x intercept
 b is y intercept



$$\frac{x}{a} + \frac{y}{b} = 1$$

Proof



$$x \cos \alpha + y \sin \alpha = p$$

This is the equation of the straight line AB.

P is the distance between point N and O. It has to be given.

- For two straight line equation in the form $(ax+by+c)$, if denotes the same straight line

analytic

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Chapter 3.6

$$\tan \theta = \pm \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(middle part will be)

$$\begin{cases} m_1 = \tan A \\ m_2 = \tan B \end{cases}$$

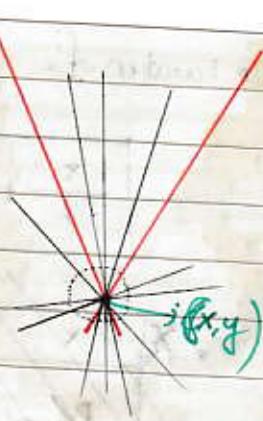
$$\tan(A-B) = \tan C$$

Now we have to find m_2 for this st. line

Angle given (2)
Equation given (m_1 given) (1)

- Equation of lines passing through the intersecting point of two straight lines given the equations of them too.

$$(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$$



- Condition for concurrency of three lines given their equation.

Determinant - 2

Diagram illustrating the condition for the concurrency of three lines:

Three lines are shown converging at a single point. The equations of these lines are:

$$(a_1x + b_1y + c_1 = 0)$$

$$(a_2x + b_2y + c_2 = 0)$$

$$(a_3x + b_3y + c_3 = 0)$$

A red arrow points from the determinant symbol to the following 3x3 matrix:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

• Post Test (Parallel, perpendicular)

• Post Test (Parallel Line Equation)

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Chapters 3-8 / 3, 7

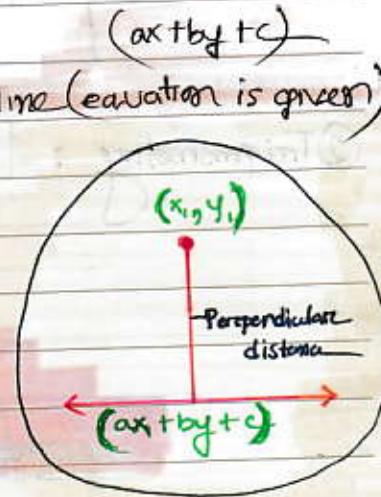
- Perpendicular distance of a point (x_1, y_1) and a line (equation is given)

is

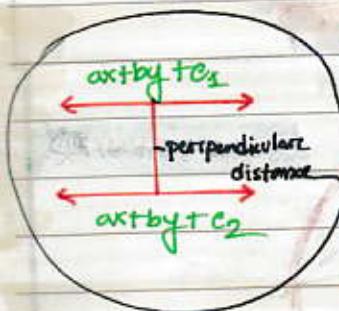
distance
cannot
be
negative

$$d = \pm \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$



- Perpendicular distance between two parallel lines (equations are given)



$$d = \pm \frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

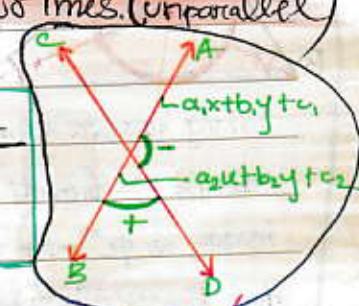
$$(ax + by + c_1)$$

$$(ax + by + c_2)$$

• Post Test: Perpendicular Bisector Line equation

- Equation of the bisector of angles between two lines (unparallel)

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$



- c_1 & c_2 are of same sign

$$\frac{ax + by + c_1}{\sqrt{a^2 + b^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Please
check
Page 13 / PTO
for more

- c_1 & c_2 are of opposite sign

$$\frac{ax + by + c_1}{\sqrt{a^2 + b^2}} = - \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

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$$\text{Centroid, } (x_1, y_1) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

• Incenter :

$$(x_1, y_1) = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

② Trigonometry :

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

(u.v.x)

(v.w.y)

(w.x.z)

(x.y.u)

(y.z.v)

(z.x.w)

(w.y.u)

(y.z.v)

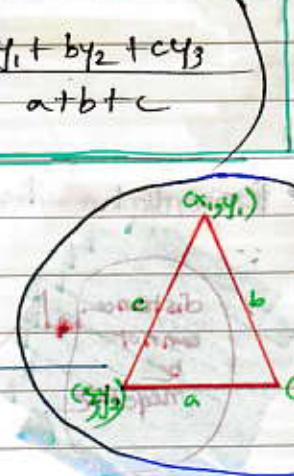
(z.x.w)

(x.y.u)

(y.z.v)

(z.x.w)

$$\sin 2\theta = 2 \sin \theta \cos \theta$$



CIRCLE TYPES

circumcentre, centroid, incentre, orthocentre

CENTROID

Here $AC : CD = 2:1$

Here C is the centroid of the triangle.

Incenter

Here I is the incenter.

Taking bisectors of angles in a triangle and joining a circle inside, we get an in-circle.

Circumcenter

Here O is the circumcenter.

Taking perpendicular bisectors of sides of a triangle and joining a circle outside, we get a circumcircle.

Orthocenter

Here G is the orthocenter.

Diagram showing the altitudes AD, BE, and CF meeting at point G.

Post Test

• Area of a Rhombus = $\frac{1}{2} \times AC \times BD$ (Two diagonals) 

$$\sin 2\theta = (2 \sin \theta \cos \theta)$$

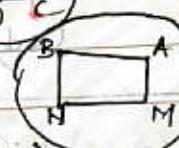
$$\cos 2\theta = (\cos^2 \theta - \sin^2 \theta)$$

$$\cos(A-B) = (\cos A \cos B + \sin A \sin B)$$

• Area of Triangle = $\frac{1}{2} \times BC \times AD$ (base)(height)



• Area of Trapezium = $\frac{1}{2} \times (AM+BN) \times MN$ (Two parallel sides addition) (distance)



• Area of a triangle = $\frac{1}{2} \left| \begin{array}{cccc} a_1 & a_2 & a_3 & a_1 \\ b_1 & b_2 & b_3 & b_1 \end{array} \right|$ [clockwise]

$$= \frac{1}{2} a \left\{ a_1 b_2 + a_2 b_3 + a_3 b_1 - a_2 b_1 - a_3 b_2 - a_1 b_3 \right\}$$

where, $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3) are coordinates of three vertices.

~~ax+by+c~~

$\rightarrow ax+by+c$ (knowing)
(normal)(perpendicular)

$\rightarrow ax+by+c$ (parallel)

Tore Equation

$$\frac{a_1x+b_1y+c_1}{\sqrt{a^2+b^2}}, \frac{a_2x+b_2y+c_2}{\sqrt{a^2+b^2}}$$

The two lines
are

$a_1x+b_1y+c_1=0$ $\rightarrow a_1a_2+b_1b_2 > 0 \rightarrow$ obtuse'ed bisector

$a_2x+b_2y+c_2=0$ $\rightarrow a_1a_2+b_1b_2=0 \rightarrow$ perpendicular-ed

$a_1a_2+b_1b_2 < 0 \rightarrow$ acute'ed

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Subject :

Chap #3 Additional Notes

Date : / /

Test Test

• EQUATION OF PERPENDICULAR BISECTOR of Line joining (a,b) & (c,d)

(a,b)

line joining (a,b) and (c,d)

(c,d)

perpendicular bisector
of the line

$$(a-c)x + (b-d)y = \frac{1}{2}(a^2+b^2 - c^2-d^2)$$

$$(\theta_{200} - \theta_{100}) = \theta_{100}$$

$$(\theta_{100} - \theta_{200}) = \theta_{200}$$

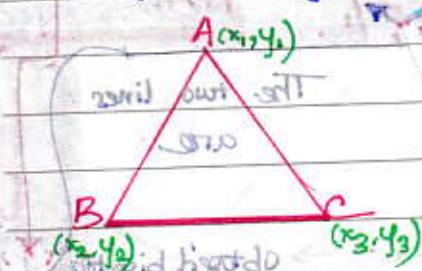
$$(m_{A_1 A_2} + m_{B_1 B_2} - A_{200}) = (B-A)_{200}$$

• EQUATION OF A line parallel to $ax+by+c=0$ at a distance d



$$ax+by+c \pm d\sqrt{a^2+b^2}$$

• Observe / Acute given the 3 vertex coordinates



A is obtuse if

$$(x_1 - x_2)(x_2 - x_3) + (y_1 - y_2)(y_2 - y_3) > 0$$

A is acute if

$$(x_1 - x_2)(x_1 - x_3) + (y_1 - y_2)(y_1 - y_3) < 0$$

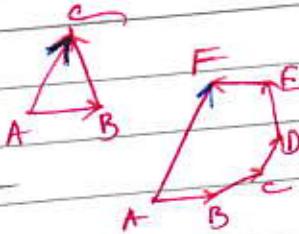
reminds me of
diameter equation
(Chapter 1)

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Subject: Chapter 2: Vectors

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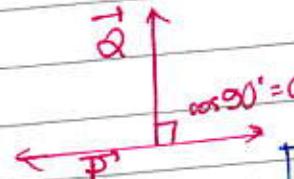
Triangle law: $\vec{AB} + \vec{BC} = \vec{AC}$



Polygon law: $\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE} = \vec{AE}$

Dot Products

① Two vectors \vec{P} & \vec{Q} being perpendicular to each other, their equation is $\vec{P} \cdot \vec{Q} = 0$



② Dot product of two vectors:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Turns it into scalar magnitude

③ $\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = \hat{i} \cdot \hat{i} - \hat{k} \cdot \hat{k} = \hat{i} \cdot \hat{k} = 0$

scalar \rightarrow $\begin{matrix} \hat{i} \\ \hat{j} \\ \hat{k} \end{matrix} \rightarrow 0$
entry = 0

$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$

④ $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
 $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$
 $\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$

$$\begin{aligned} \vec{A} &= A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\ \vec{B} &= B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \\ \vec{A} \cdot \vec{B} &= A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z \end{aligned}$$

$\vec{A} \cdot \vec{B} = AB \cos \theta$

⑤ Projection (scalar):

Projection of \vec{A} along \vec{B} =

$$A \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$$

Projection of \vec{B} along \vec{A} =

$$B \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$$

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$$\cos\theta = \frac{A_x A_y + A_z A_2}{|\vec{A}| |\vec{B}|}$$

⑥ Angle θ between vectors we use dot multiplication.

$$\vec{A} \cdot \vec{B} = AB \cos\theta \Rightarrow \cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} \Rightarrow \theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

Cross Products

$$\begin{array}{l} \leftrightarrow \\ \leftrightarrow \\ \theta = 0^\circ \\ \sin\theta = 0 \end{array}$$

① Two vectors being parallel to each other. $\vec{P} \times \vec{Q} = 0$

② Cross product of two vectors:

$$\vec{A} \times \vec{B} = AB \sin\theta \cdot \hat{n}$$

$$\begin{array}{|c|} \hline \text{sin}\theta = 0 \\ \hline \begin{array}{l} \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \\ \hat{i} \times \hat{j} = \hat{k}; \hat{j} \times \hat{i} = -\hat{k} \\ \hat{j} \times \hat{k} = \hat{i}; \hat{k} \times \hat{j} = -\hat{i} \\ \hat{k} \times \hat{i} = \hat{j}; \hat{i} \times \hat{k} = -\hat{j} \end{array} \\ \hline \end{array}$$

Vector \hat{n} \rightarrow $\hat{i}, \hat{j}, \hat{k}$

$$\begin{array}{l} \text{④ } \vec{A} = x_A \hat{i} + y_A \hat{j} + z_A \hat{k} \\ \vec{B} = x_B \hat{i} + y_B \hat{j} + z_B \hat{k} \end{array}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_A & y_A & z_A \\ x_B & y_B & z_B \end{vmatrix}$$

$\hat{n} = \text{etc}$

⑤ Component / Resolved part (Vector)

$$|\vec{AB}| = AB \cos\theta \rightarrow$$

$$\text{Component of } \vec{A} = A \cos\theta \cdot \hat{b} = \frac{\vec{A} \cdot \vec{B}}{B} \cdot \hat{b}$$

other component in
other direction
(g.)

$$\text{Component of } \vec{B} = B \cos\theta \cdot \hat{a} = \frac{\vec{A} \cdot \vec{B}}{|A|} \cdot \hat{a}$$

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unit vector, \hat{r}

4) Unit vector parallel to a vector =

$$\frac{\vec{A}}{|\vec{A}|} = \frac{\vec{A}}{A}$$

Unit vector parallel to two vectors =

the resultant of

$$\frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|} = \frac{\vec{A} + \vec{B}}{|\vec{A} + \vec{B}|}$$

Some meaning

Unit vector parallel to a vector =

$$\frac{\vec{PQ}}{|\vec{PQ}|} = \frac{\vec{PQ}}{PQ}$$

Unit vector perpendicular to two vectors =

on the plane consisting

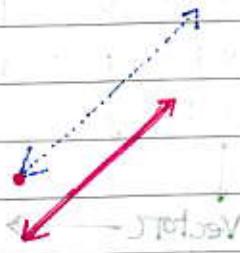
$$\frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$$

Parallel to yz plane = perpendicular to x axis

Parallel to xy plane = perpendicular to z axis

Parallel to xz plane = perpendicular to y axis

5) Vector Equation of a point and a vector parallel to a line passing through

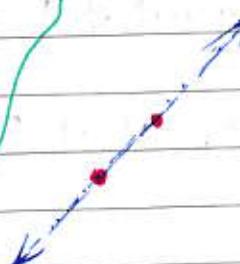


$$\vec{r}_c = \vec{a} + \lambda \vec{b}$$

6) Vector Equation of a line passing through two points \vec{a} & \vec{b}

$$\vec{r} = (1-\lambda)\vec{a} + \lambda\vec{b}$$

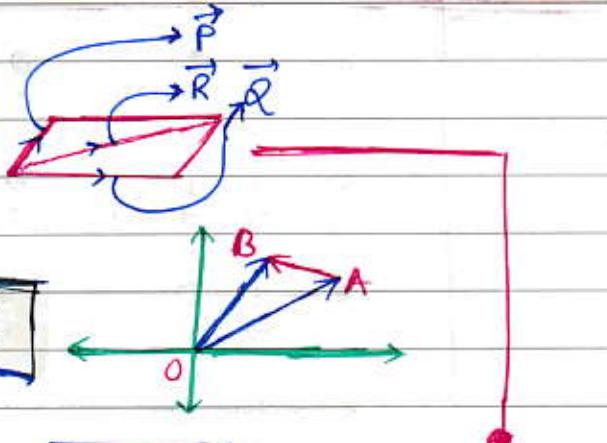
$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$



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(*) Extras

$$\text{[E.1]} \bullet \boxed{\vec{R} = \vec{P} + \vec{Q}}$$



$$\text{[E.2]} \bullet \boxed{\vec{AB} = \vec{OB} - \vec{OA}}$$

$$\bullet \text{Area Parallelogram} = \boxed{|\vec{P} \times \vec{Q}|}$$

$$\bullet \text{Area Triangle} = \boxed{\frac{1}{2} |\vec{P} \times \vec{Q}|}$$

8 When three vectors, \vec{A} , \vec{B} and \vec{C} are co-planar,

$$\vec{A}(\vec{B} \times \vec{C}) = 0 ; \quad \vec{B}(\vec{C} \times \vec{A}) = 0 ; \quad \vec{C}(\vec{A} \times \vec{B}) = 0$$

$$\vec{A}(\vec{B} \times \vec{C}) \text{ or, } \vec{B}(\vec{C} \times \vec{A}) \text{ or, } \vec{C}(\vec{A} \times \vec{B})$$

$$\textcircled{1} \quad \vec{P} = 5\hat{i} + 5\hat{j} + 5\hat{k}$$

$$|\vec{P}| = \sqrt{5^2 + 5^2 + 5^2}$$

$$= 5\sqrt{3}$$

$$= 5\sqrt{3}$$

$$= 5\sqrt{3}$$

$$\vec{P} = P_x \hat{i} + P_y \hat{j} + P_z \hat{k}$$

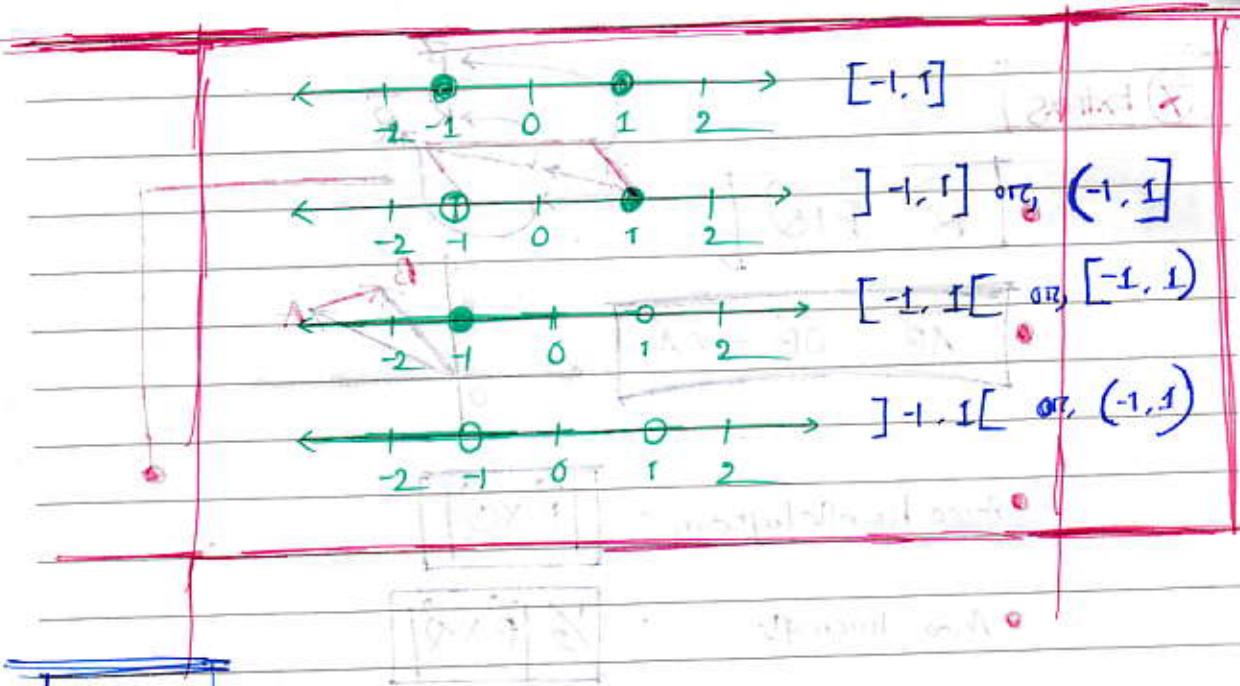
$$|\vec{P}| = \sqrt{P_x^2 \hat{i}^2 + P_y^2 \hat{j}^2 + P_z^2 \hat{k}^2}$$

$$= \sqrt{P_x^2 + P_y^2 + P_z^2}$$

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#Notes:

Domain = \mathbb{R}

① $y = e^x$ For any value of x , positive/negative,
 y is greater than 0,
 but never equal to 0. } Range > 0

Domain = \mathbb{R}

② $y = e^{x^2}$ For any value of x , positive/negative,
 y is equal or greater than 1 } Range ≥ 1
 $y \geq 1$.

③

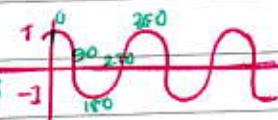
$$-1 \leq \sin \theta \leq 1$$

$$-1 \leq \cos \theta \leq 1$$

$$0 \leq \cos^2 \theta \leq 1$$

→ have positive values
negative

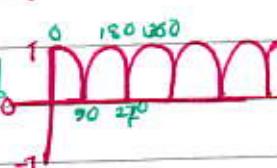
→ no negative values



$$\cos 0 = 1$$

$$\cos 90 = 0$$

$$\cos 180 = -1$$



$$\cos 270 = -1$$

$$\cos 270 = 0$$

$$\cos 360 = 1$$

$$\textcircled{4} \quad \ln u \quad \text{Hence } u > 0$$

$$\textcircled{5} \quad \sqrt{x} \quad \text{Hence } x \geq 0$$

$$\textcircled{6} \quad \ln(x^2) = 2\ln|x|$$

$$\textcircled{7} \quad \ln(xy) = \ln x + \ln y$$

$$\textcircled{8} \quad \ln\left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\textcircled{9} \quad e^{\ln u} = u$$

$$\textcircled{10} \quad \cos(-\theta) = \cos\theta$$

$$\sec(-\theta) = \sec\theta$$

$\cos(-\theta) = \cos\theta$
 $\sec(-\theta) = \sec\theta$

$$\textcircled{11} \quad 2\sin^2\theta = 1 - \cos 2\theta$$

$$2\cos^2\theta = 1 + \cos 2\theta$$

$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$\textcircled{12} \quad 2\cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$\textcircled{13} \quad \cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

\textcircled{14} Denominators $\neq 0$

$$\textcircled{15} \quad f(u) = y$$

$$f^{-1}(y) = u$$

\textcircled{16}

son
set

Father
set

- When a son belongs to two

- father's it's not a function

- When a son of two identities

- belongs to a father of two same

- identities, it's "one one set".

$$f(x_1) = f(u_2) \rightarrow u_1 = u_2$$

- When not enough/all father

- are being able to make son, it's not onto function.

Range \neq co-domain

- When all father makes son it's onto function.

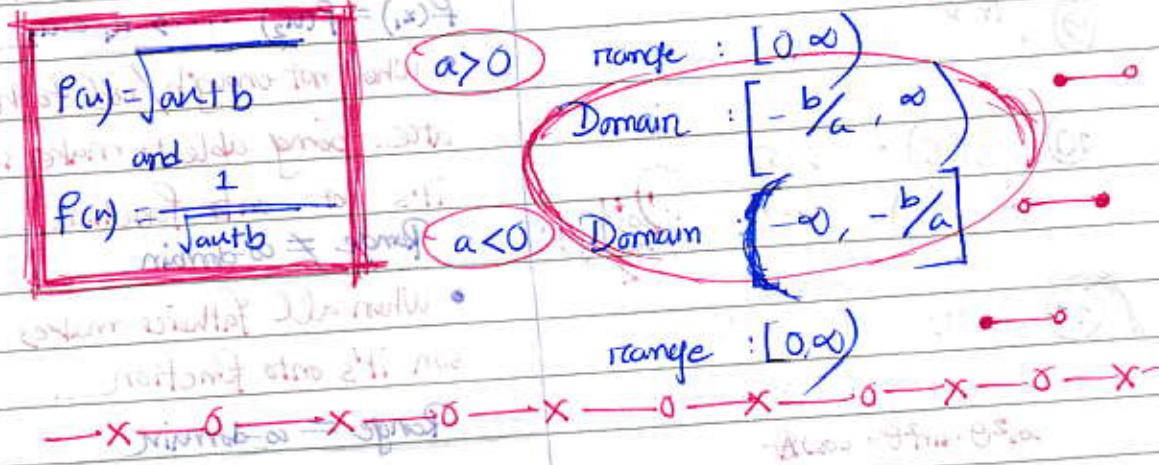
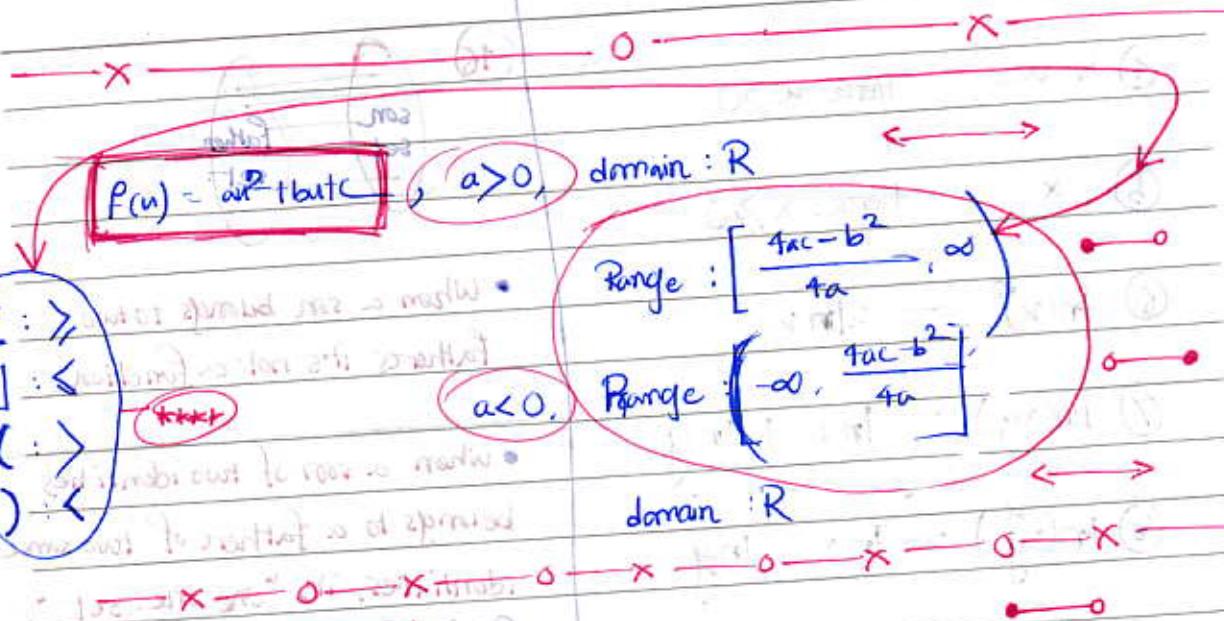
Range = co-domain

$$\tan(a+b) = \frac{\tan a + \tan b}{1 + \tan a \tan b}$$

$$\sin 2\theta = 2\sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$



$$f(x) = a^2 - x^2 \quad \text{where } a > 0$$

Domain: $[-a, a]$

Range: $[0, a]$



$$f(x) = \frac{ax^2 + bx}{cx + d}$$

Domain: $\mathbb{R} - \left\{ -\frac{d}{c} \right\}$

Range: $\mathbb{R} - \left\{ \frac{a}{c} \right\}$

~ " " द्वारा नियमित जाती हैं

① 32 परे ~ अपेक्षित रूप से बोलते गये हैं।

② " " - alias/Asif indications,

3, 5, 8, 10
10 vs.
4, 6, 7, 9

③ (Row modification) \leftrightarrow (Col modification) \leftrightarrow (column modification)

④ 3, 5, 8, 10 \rightarrow Col first II (C.F.II)

⑤ 1, 3 \rightarrow same beginning \rightarrow Exception subsequence.

⑥ 4, 6, 7, 9 \rightarrow same ending

⑦ 4, 10 \rightarrow same beginning concept

⑧ 10 \rightarrow ending in revolt against 4, 6, 7, 9

⑨ 1, 4 \rightarrow synced in ways

⑩ 5, 7 \rightarrow synced in ways \rightarrow 3 lined \rightarrow some beginning

⑪ 2 \rightarrow 1 lined.

⑫ 8 \rightarrow 2 lined.

1, 3
4, 10
5, 7

$$① 2abc (a+b+c)^3$$

$$② (b^2-ac)(ax^2+2bxy+by^2)$$

$$③ p(p-1)^2(p^2-1)$$

$$④ 4a^2b^2c^2$$

$$⑤ 4x^2y^2z^2$$

$$\times ⑥ 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$$⑦ (a+b+c)^3$$

$$⑧ (1+a^2+b^2)^3$$

$$⑨ (x-y)(y-z)(z-x)(x+y+z)$$

$$\times ⑩ (x-y)(y-z)(z-x)(x+y+z)$$

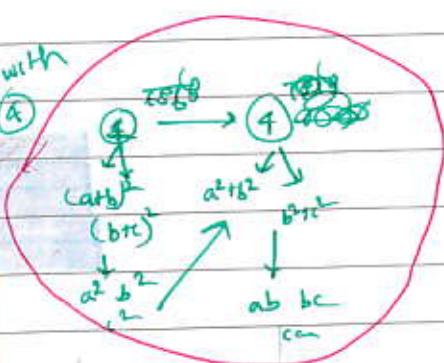
Finding:

$$\left[\begin{matrix} 3 & 5 & 8 & 10 \\ + & 6 & 7 & 9 \end{matrix} \right] \rightarrow c_2' = c_2 - c_3 \rightarrow 4, 5 \\ c_1' = 9 - c_2 \rightarrow 7, 9$$

1

sync with

①



$$(b+c)^2 \quad a^2 \quad a^2$$

$$b^2 \quad (c+a)^2 \quad b^2$$

$$(-b^2 + 1 \times c^2 - a^2)$$

$$c^2 \quad (a+b)^2$$

$$= 2abc(a+b+c)^3$$

$$\begin{aligned} ① \quad c'_2 &= c_2 - c_1 & ② \quad c'_3 &= c_3 - c_1 \\ ③ \quad \text{some} & & & \end{aligned}$$

Exception in
maintaining
subsequence } same in
③

$$\begin{aligned} ③ \quad r'_i &= r_i - r_2 - r_3 & ④ \quad \text{some} & \rightarrow ④ \quad \text{some} \\ ④ \quad \text{some} & & & \end{aligned}$$

4 (असर नहीं होता)

$$\begin{aligned} ⑤ \quad c'_2 &= b \times c_2 & ⑥ \quad c'_3 &= c \times c_3 \\ & \& & \end{aligned}$$

$$\begin{aligned} ⑥ \quad \text{some} & & \\ \text{Follow up} \rightarrow ⑦ \quad c'_2 &= c_2 + c_1 & ⑧ \quad c'_3 &= c_3 + c_1 \\ & \& & \end{aligned}$$

compensation for 1

2



$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = -d(b^2-ac) = -(b^2-ac)(ax^2+2bxy+cy^2)$$

$$① c_3 = -c_1x - c_2y + c_3$$

3

sync with ①

1	1	1	$s_0 + s_1$
$s_0 + s_1 - p$	$p^2 - s_0$	p^4	$p(p-1)^2(p^2-1)$
1	$p^2 - s_0$	p^4	

$$\textcircled{1} \quad c'_2 = c_2 - c_1$$

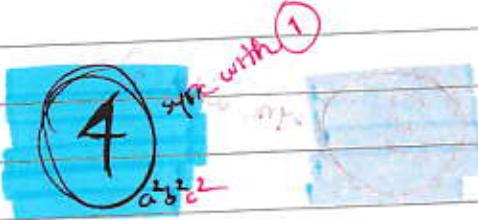
$$c'_3 = c_3 - c_2$$

Exception maintaining
subsequence $\textcircled{1}$
follow zero

② of

* First two steps are first two steps of P1
with exceptions in

$c_3 - c_2 / c_2 - c_1$
third off



Step ① symbolises that
② armed ① by converting
 $(ab)^2$ to (a^2+b^2) . Step 2
follows ①?

$$\begin{array}{c}
 b^2 + c^2 \\
 ab \\
 \hline
 ab(1-q) - c^2 + a^2 \\
 \hline
 ca \\
 bc \\
 \hline
 a^2 + b^2 - q
 \end{array} = 4a^2b^2c^2$$

$$① \pi'_1 = a \times \pi_1 \quad \& \quad \pi'_2 = b \times \pi_2 \quad \& \quad \pi'_3 = c \times \pi_3$$

$$② \pi'_1 = \pi_1 - (\pi_2 + \pi_3) \quad \rightarrow ① \text{ (TJ আছে)}$$

$$③ cf$$

① (পোস্ট)
পুরো সহজে

$$④ c'_2 = c_2 \times b \quad \& \quad c'_3 = c_3 \times c$$

$$⑤ c'_2 = c_2 - c_3 \quad \rightarrow \text{same ending as } ⑥$$

6, 7, 9, 10

similarities with ①

$\langle a^2b^2c^2 \rangle$

π_1 modification first order

$2\pi_1\pi_1$

π_1 observed

c_2 observed after cf

HEARTS

5

symp with 7

$$\begin{array}{c|c|c|c}
 & x^2 & yz & z^2 + zx \\
 \hline
 x^2 + my & & y^2 & z^2 \\
 \hline
 xy & y^2 + yz & zx & = 4x^2y^2z^2
 \end{array}$$

$$\begin{aligned}
 ① \quad \pi'_1 &= \pi_1 + \pi_2 + \pi_3 \\
 ② \quad \pi'_1 &= \pi_1 - \pi_2 \quad \pi'_2 = \pi_2 - \pi_3 \\
 ③ \quad \text{cf} &
 \end{aligned}$$

similarities with 7

→ $\langle 4x^2y^2z^2 \rangle$

TG modification from 2012

$\Sigma \pi_i$

π_i assessed
CAT2 after exp

HEARTS

6

cccc

$$\begin{array}{ccc}
 b+c & c+a & a+b \\
 q+p & p+q & \\
 \hline
 y+z & z+x & x+y
 \end{array}
 = 2 \left| \begin{array}{ccc} a & b & c \\ p & q & r \\ x & y & z \end{array} \right|$$

$$\textcircled{1} \quad c'_1 = c_1 + c_2 + c_3 \quad \xleftarrow{\text{atb + c involved}}$$

$$\textcircled{2} \quad c'_1 = c_1 - c_2$$

$$\textcircled{3} \quad c'_3 = c_3 - c_1$$

$$\textcircled{4} \quad c'_2 = c_2 - c_3 \quad \xrightarrow{\text{some ending as } \textcircled{4}}$$

would go on to be seen in

7, 9, 10

$$\begin{array}{l}
 \begin{array}{c|cc|c}
 a-b-c & 2a & 2a & a \text{ positive} \\
 \hline
 -atb-c & 2b & 2b & b \text{ positive} \\
 \hline
 -a-b+e & 2c & 2c & e \text{ positive}
 \end{array}
 \end{array}$$

7 *similar with 5*

$$\begin{array}{c|ccc|c}
 & a-b-c & 2a & 2a & a^3 + b^3 + c^3 \\
 \hline
 25 & b-c-a & 2b & & = (a+b+c)^3 \\
 & 2c & 2c & c-a-b &
 \end{array}$$

$$\text{① } \pi'_1 = \pi_1 + \pi_2 + \pi_3$$

$$\text{② cf}$$

$$\text{③ } c'_1 = c_1 - c_2 \quad \& \quad c'_2 = c_2 - c_3$$

similar with 5

*same ending as 9,
with 20*

(8)

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & 2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

There's no c

All Elements must present

$$① c_1 = c_1 - bc_3$$

$$c_2 = c_2 + ac_3$$

c only

②

cf

2 and 8

↓
1 lined
↓
2 lined

c_3' =
modification

c_3 in the
modification
 c_1 and c_2

SA = Summation
Addition

⑨

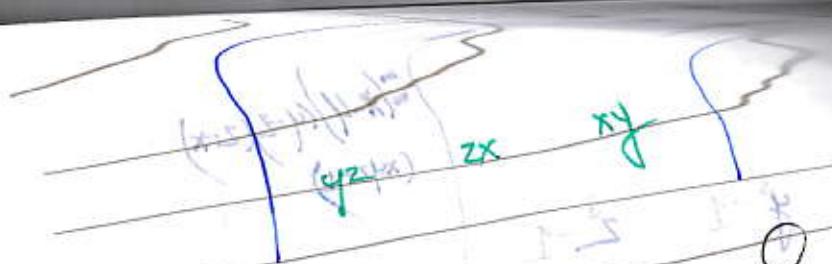
(OP)

	x	y	z	
	x^2	y^2	z^2	s_2
$(x^2 - 1)$	x^2	y^2	z^2	$= (x-1)(y-1)(z-1)$
$x^3 - 1$	$y^3 - 1$	$z^3 - 1$		

① SA

② cf in sa1 & $\overbrace{sa2 \equiv sa1}$ Just 1, 1, 1 [Top]

③ $c'_1 = c_1 - c_2$ & $c'_2 = c_2 - c_3$



$$① c'_1 = c_1 \times n \quad c'_2 = c_2 \times y \quad c'_3 = c_3 \times z$$

② cf [from bottom]

③ 1, 1, 1 [in top]

$$④ c'_1 = c_1 - c_2 \quad c'_2 = c_2 - c_3$$

⑤ cf

$$⑥ c'_1 = c_1 - c_2$$

everywhere
used in every part

4, 6.
with respect.

⑦ cf

beginning
everywhere

HEAR

(10)



	x	y	z	
x ²	y ²	z ²		= (x-y)(y-z)(z-x)
(x+y+z)(x-y+z) =				(xy+yz+zx)
(yz+zx)	zx	xy		

① $c'_1 = c_1 \times n$ & $c'_2 = c_2 \times n$ & $c'_3 = c_3 \times n$

② cf [from bottom]

③ 1, 1, 1 [in top]

④ $c'_1 = c_1 - c_2$ & $c'_2 = c_2 - c_3$

⑤ cf

⑥ $c'_1 = c_1 - c_2$

⑦ cf

$c'_2 = c_2 - c_3$ popular
used in writing of
4, 6,
GTE, GETE,

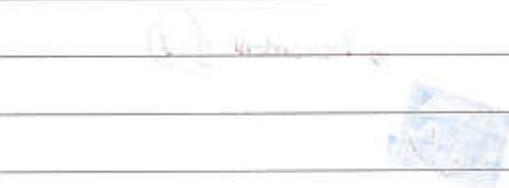
4 beginning
4 ending

Subject : _____

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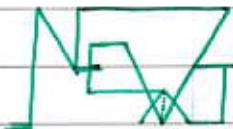
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Date : _____



1 2 3 4 5 6 7 8 9 10

1 2 3 4 5 6 7 8 9 10



(Next)

These 10's are main.



Note : I couldn't find 10 in ~~KAMR~~ KAMR

11

Descendent of 7

$$\begin{array}{c|ccc|c}
 & a+b+2c & a & b & \\
 \hline
 c & & b+c+2a & & \\
 \hline
 & 0a, b & & a, c & = 2(a+b+c) \\
 c & a & & c+a+2b & \\
 \hline
 \end{array}$$

① $c'_1 = c_1 + c_2 + c_3$

② cf

~~Opposite to first last fabric is 5th~~

③ $\pi'_1 = \pi_1 - \pi_2 \quad \& \quad \pi'_2 = \pi_2 - \pi_3$

Opposite to 7
beginning + ending

11, 17, 18, 8, 5, 7

Also 19 → Exception

(12)

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$$\cancel{a+b+c} \quad a \quad 1 \quad b+c$$

$$1 \quad bc \quad bc(b+c) = b \quad 1 - c+a \quad abc = 0$$

$$1 \quad ca \quad ca(c+a) \quad c \quad 1 - a+b$$

$$1 \quad ab \quad ab(a+b)$$

$$\left. \begin{array}{l} \text{① } r'_1 = r_1 \times a \quad \& \quad r'_2 = r_2 \times b \quad \& \quad r'_3 = r_3 \times c \end{array} \right\} \text{beginning of P}$$

② cf

$$\left. \begin{array}{l} \text{③ } c'_1 = c_1 + c_3 \\ \qquad \qquad \qquad \left. \begin{array}{l} c_1 \Rightarrow c_2 \\ + \Rightarrow - \end{array} \right\} \text{ending} \end{array} \right\} c'_2 = c_2 - c_3$$

10/15, 16/21, 22

13.

11 Ending
5 middle

0	1 a	a^2	($a+b+c$)d	d
1 b	b	b^2	($a+b+c$)d	d
1 c	c	c^2	($a+b+c$)d	d

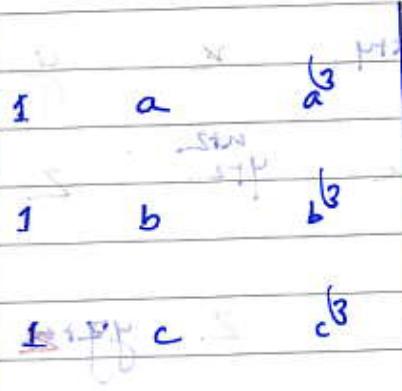
$$\textcircled{1} \quad TC_1' = \pi_1 - TC_2, \quad TC_2' = \pi_2 - \pi_3$$

} 11 Ending
5 middle

13, 14, 20

14..

11



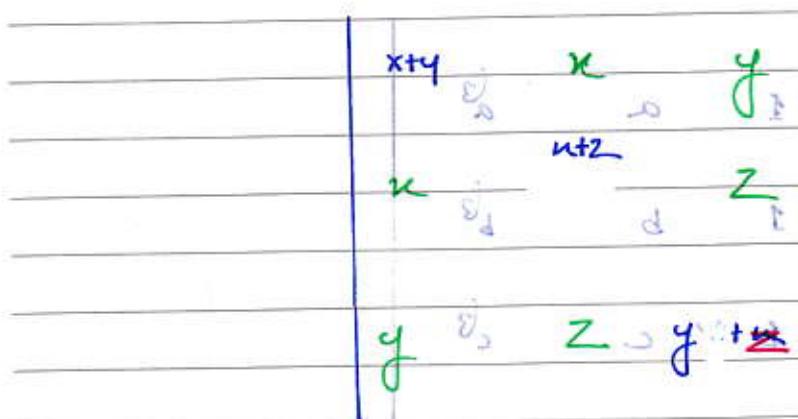
$$\textcircled{1} \quad \pi_1' = \pi_1 - \pi_2, \quad \pi_2' = \pi_2 - \pi_3$$

} 11 Ending
5 middle

13, 14, 20

15..!

11, 12



11 beginning

$$\text{① } c_1 = c_1 - c_2 - c_3 \quad \left. \begin{array}{l} \\ \end{array} \right\} 11 \text{ beginning}$$

$$\text{② } r_2 = r_2 - r_3 \quad \left. \begin{array}{l} \\ \end{array} \right\} 12 \text{ ending w} \\ 4 \text{ ending w}$$

$$c_3 = c_2 - c_3$$

VIOLATION

r₂ / 15, 16 / 21, 22

Subject : _____

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Date :

16. vi

$a+n$	$b+n$	$c+n$
$a+y$	$b+y$	$c+y$
a^2	b^2	c^2

$$\textcircled{1} \quad c'_1 = c_1 - c_2, \quad c'_2 = c_2 - c_3$$

\textcircled{2} cf

$$\textcircled{3} \quad c'_1 = c_1 - c_2$$

\textcircled{4} cf

12/15, 16/21, 22

\textcircled{5}

Subject : _____

IP

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Date : _____

17. DE

$1+u_1$	u_2	u_3
$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2}$	$\frac{1}{2} + \frac{1}{2}$
u_1	$1+\frac{1}{2}$	$1+\frac{1}{2} + \frac{1}{2}$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
u_1	u_2	$1+u_3$

$$\textcircled{1} \quad e'_1 = e_1 + c_2 + c_3$$

$$\textcircled{2} \quad cf$$

$$\textcircled{3} \quad \tau'_1 = \tau_1 - \tau_2, \quad \tau'_2 = \tau_2 - \tau_3$$

11, 17, 18, 19, 2, 5, 7

Subject : _____

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Date :

18.11

	1	-a	a^2	(1)
	a^2	1	-a	
	-a	1	a^2	
				(1)

$$\textcircled{1} \quad e'_1 = e_1 + e_2 + e_3$$

$$\textcircled{2} \quad e'_f$$

$$\textcircled{3} \quad r'_1 = r_1 - r_2, \quad r'_2 = r_2 - r_3$$

\textcircled{4} Extension / विस्तृति

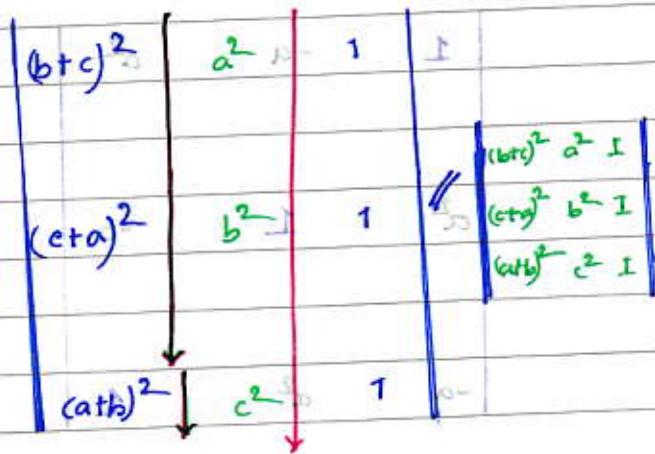
11, 17, 18, 19, 8, 5, 7

Subject : _____

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Date : _____

19. 25



$$\textcircled{1} \quad c_1' = c_1 - c_2$$

$$\textcircled{2} \quad cf$$

$$\textcircled{3} \quad \tau\tau'_1 = \tau\tau_1 - \tau\tau_2, \quad \tau\tau'_2 = \tau\tau_2 - \tau\tau_3$$

11, 17, 18, 19, 5, 7

Subject : _____

Date :

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20.

1

 $\log u \quad \log v \quad \log w$
 $\log u - \log v + \log w = 0$
 $\log_{3u} \quad \log_{3v} \quad \log_{3w}$

$$\textcircled{1} \quad r'_{\bar{2}} = r_2 - r_1 \quad , \quad r'_{\bar{3}} = r_3 - r_1$$

① First step
back to beginning

Exceptional
deal breaking

13,14,20

Subject : _____

Date : _____

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21.

$$\begin{array}{ccccc} -a^2 & \text{pol} & ab & \text{pol} & ac \\ ab & \text{pol} & -b^2 & \text{pol} & bc \\ ac & \text{pol} & bc & \text{pol} & -c^2 \end{array}$$

$= 4a^2b^2c^2$

① cf

② cf

③ cf

12/15, 16/21, 22

Subject : _____

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Date :

22.

$$\begin{vmatrix} 1 & x-a & y-b \\ 1 & x_1-a & y_1-b \\ 1 & x_2-a & y_2-b \end{vmatrix} = \begin{vmatrix} 1 & x & y \\ 1 & x_1 & y_1 \\ 1 & x_2 & y_2 \end{vmatrix}$$

① SA

② SA

12/15, 16/21, 22

$c_1 \wedge c_2$ fuchs 2006

$(b+c)^2$		①	$c_1 c_2 / c/c$	$1+a^2+b^2$		③	
$(c+a)^2$				$1-a^2+b^2$			

a b		②	c	x y z		④	
b c				$x^2 y^2 z^2$			

1 1 1		⑤		x y z		⑥	
1 p p ²			c	$x^2 y^2 z^2$			
1 p ² p ⁴				$y^2 z x x y$			

b^2+c^2		⑦	$\pi/\pi c/c$	$a+b+c$		⑧	
a^2+b^2				$b+c+a$			

a^2		⑨	π/π	$b+c+a$		⑩	
b^2				$c+a+b$			

a^2		⑪	π/π	$b+c+a$		⑫	
b^2				$c+a+b$			

$a-b-c$		⑬	π/π	a^2		⑭	
$b-c-a$				b^2			
$c-a-b$				c^2			

2x8

$c_1 = c_2 - c_3$ 8 first
 $c_2 = c_3 - c_4$ 8 second

7's ending \equiv
 9's ending \equiv 16's beginning
 $[c_1 = c_1 - c_2; c_2 = c_2 - c_3]$

X

 π/π $c_2 = c_3 - c_4 \Leftrightarrow c_4 = c_1 - c_2$ π/π π/π

same

HEARTS

c rights ended
All 8 rights of sequence

Subject:

$$\begin{array}{c} b+a \\ a+c \\ b+c \end{array}$$

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Date: 20/1/2018

$$\begin{array}{c} x+y \\ x+z \\ y+z \end{array}$$

(15)

6 first step
crc

$$\begin{array}{c} 1 x-a \\ 1 x-a \\ 1 x_2-a \end{array}$$

(22)

X

$$\begin{array}{c} a+b+c \\ a+b+c \\ a^2+b^2+c^2 \\ 1 x_n \end{array}$$

(16)

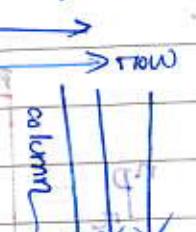
c/c

9's ending = 16's beginning

$$\begin{array}{c} 1 x_1 \\ 1 x_2 \\ 1 x_3 \end{array}$$

(17)

c/c



$$\begin{array}{c} 1-a \\ 1-a \\ -a \end{array}$$

(18)

c/c

transferring 1st digit to minitabutab

transferring 2nd digit to minitabutab

$$\begin{array}{c} (b+a)^2 \\ (a+c)^2 \\ (a+b)^2 \end{array}$$

(19)

c/c

2nd digit to minitabutab

$$\begin{array}{c} log x \log y \log z \\ log x \log y \log z \\ log x \log y \log z \end{array}$$

(20)

c/c

3rd digit to minitabutab

$$\begin{array}{c} -a^2 \\ -b^2 \\ -c^2 \end{array}$$

(21)

III

HEARTS

Permutation

\downarrow

Arrangement

Permutations of

$${}^n P_r = \frac{n!}{(n-r)!} = {}^n C_r \times r!$$

n different things
taken r at a time

$${}^n P_r = \frac{n!}{(n-r)!}$$

- Permutations of things all different:
taken r at a time

- Permutations of things not all different:
taken all at a time

$$\cancel{{}^n P_r} = \frac{n!}{x! y! z!}$$

x kind things
 y kind things
 z kind things

- Permutations involving repetitions
taken all at a time

$${}^n r^r = \cancel{{}^n P_r}$$

$$\checkmark n! = (n)(n-1)(n-2) \dots 3, 2, 1$$

$$\checkmark (n-2)! = (n-2)(n-3)(n-4) \dots 3, 2, 1$$

think n: want to send a packet through a network of 3 nodes.

Combination

selection

AB or, BA

Combinations of

$${}^n C_r = \frac{n!}{r!(n-r)!} = \frac{n!}{r!} = \frac{n!}{(n-r)!}$$

- ${}^n C_r = {}^n C_{n-r}$ proof ~~not available~~

- ${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$ book proof**

Restricted Combinations

- Combinations of n different things taken r at a time in which p always occurs

$${}^{n-p} C_{r-p}$$

- Combinations of n different things taken r at a time in which p never occurs.

not included

not selected

$$\boxed{n-p} \quad C_r$$

- Combinations of n different things taken any number at a time

$$\boxed{2^n - 1}$$

1 set of case being left out and 1 set of case being selected

cases where all are left out

- Combinations of P kind things

\rightarrow $\boxed{P \text{ kind things}}$

\rightarrow $\boxed{Q \text{ kind things}}$

\rightarrow $\boxed{R \text{ kind things}}$

\rightarrow $\boxed{k \text{ different things taken any number at a time}}$

examples & practice in units to go next month there will be no examination

2020-20

$$\boxed{2^k (P+1)(Q+1)(R+1) - 1}$$

case where all are left out

HEARTS

Division into Groups

- The number of ways in which $(m+n)$ different things can be divided into two groups: m group & n group

$$nC_m = \frac{m+n}{m} C_m \times 1 = \frac{(m+n)!}{m! n!}$$

case of
n things
being left out

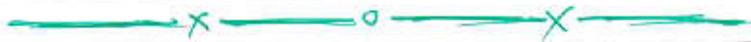
m group and
 n group can
arrange themselves
between them.

When $m=n$,

$$\frac{(m+m)!}{2! (m!) (m!)} = \frac{(2m)!}{2! (m!)^2}$$

when $m=n$ &
equally distributed
among two persons,

$$\frac{(m+m)!}{m! m!} = \frac{(2m)!}{(m!)^2}$$



Subject : _____

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Date :

① 5.1 Exercise math : 13(iv), 14(i), 14(ii) misklief!

5.2 Exercise math : 14(iv), 14(i), 14(ii)

Opmerking: de eerste 3 vragen zijn te makkelijk.

Opmerking: vragen nr 1 & 2 zijn eveneens te makkelijk.

nr. 3 en 4 zijn:

1. Deze vragen
behandelen alleen
eenig wat problemen

2. Deze vragen
behandelen alleen
eenig wat problemen

"Trigonometric Ratios"

6.1

- D. Sexagesimal system, $\frac{1}{90^\circ}$ [unit is $\frac{1}{90^\circ}$ th of a right angle] 0°
- B. Circular System, $\frac{1}{\pi}$ [unit is the angle subtended where the portion of circumference is equal to the radius] 0°
- G₁. Centesimal System, $\frac{1}{100\text{grad}}$ [unit is $\frac{1}{100}$ th of a right angle] 0^g

Sexagesimal to Circular : $\theta^\circ = \left(\frac{\pi}{180} \times \theta^\circ \right)^\circ$

degree to radian

$180^\circ = 2 \times \text{right angle}$

Circular to Sexagesimal : $\theta^\circ = \left(\frac{180}{\pi} \times \theta^\circ \right)^\circ$

radian to degree

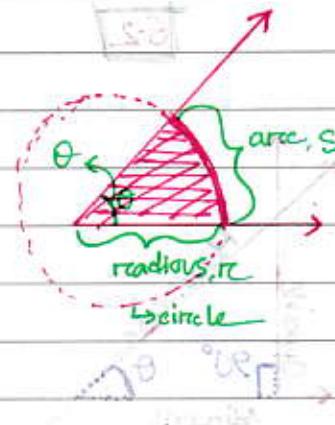
Sexagesimal to Centesimal : $\theta^g = \left(\theta^\circ \times \frac{10}{9} \right)^g$

degree to grad

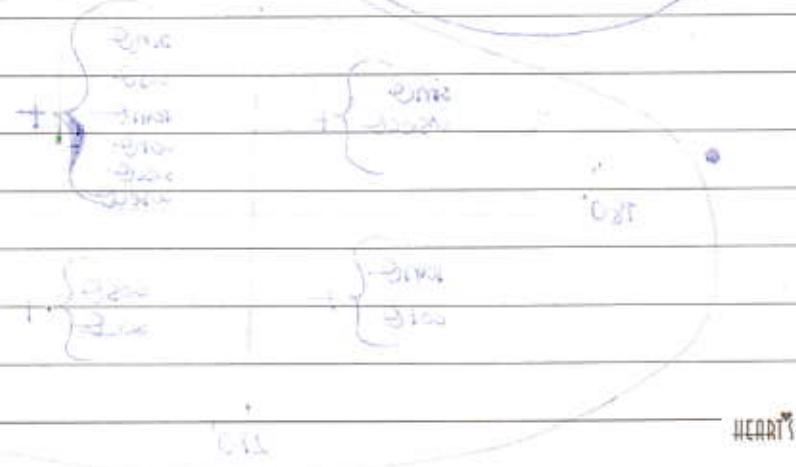
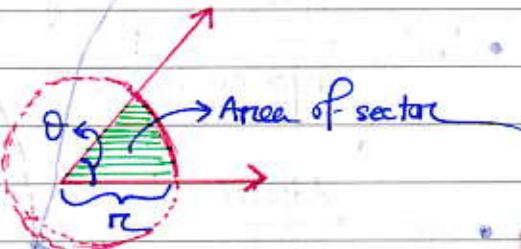
$\frac{D}{180} = \frac{G_1}{200} = \frac{\theta}{\pi}$

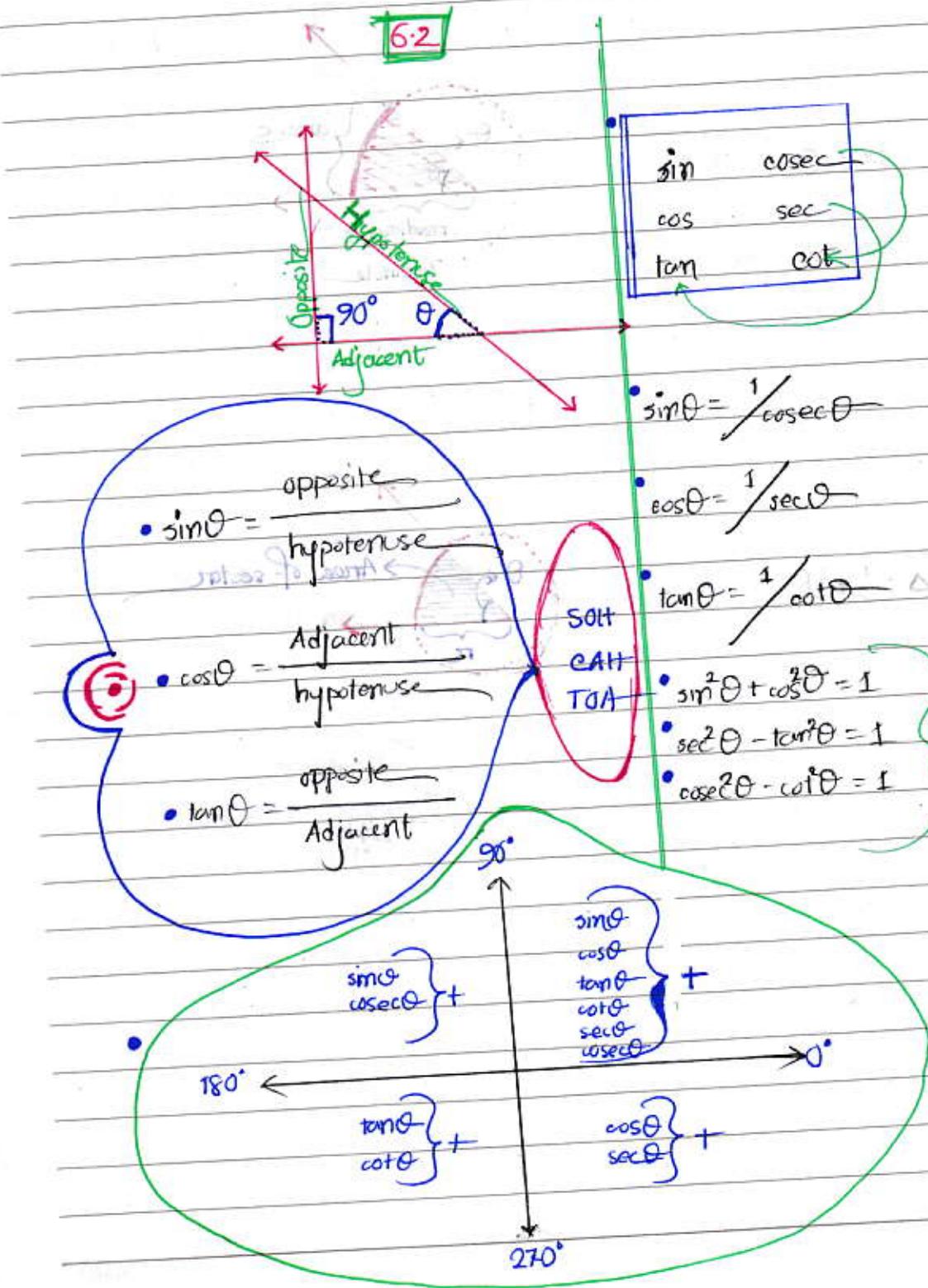
θ = radian angle

- $s = r\theta$



- $\Delta = \frac{1}{2}r^2\theta$

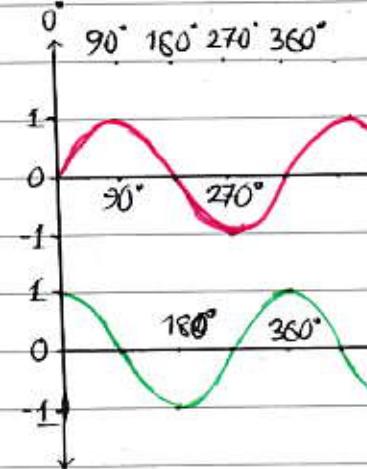




6.3

- $\sin\theta$ starts from 0
- $\cos\theta$ starts from 1

$$[\sin 0 = 0] \quad [\cos 0 = 1]$$



Defined Values

$$\sqrt{\frac{n}{4}}$$

$$\sin\theta, n=0, 1, 2, 3, 4$$

$$\cos\theta, n=4, 3, 2, 1, 0$$

	0	30	45	60	90
$\sin\theta$	$\sqrt{\frac{0}{4}} \parallel 0$	$\sqrt{\frac{1}{4}} \parallel \frac{1}{2}$	$\sqrt{\frac{2}{4}} \parallel \frac{1}{\sqrt{2}}$	$\sqrt{\frac{3}{4}} \parallel \frac{\sqrt{3}}{2}$	$\sqrt{\frac{4}{4}} \parallel 1$
$\cos\theta$	$\sqrt{\frac{4}{4}} \parallel 1$	$\sqrt{\frac{3}{4}} \parallel \frac{\sqrt{3}}{2}$	$\sqrt{\frac{2}{4}} \parallel \frac{1}{\sqrt{2}}$	$\sqrt{\frac{1}{4}} \parallel \frac{1}{2}$	$\sqrt{\frac{0}{4}} \parallel 0$

m number of ways

① A work done with ~~m~~ and n number of ways. $m \times n$ ② A work done with first, m number of ways, then n number of ways. $m \times n$.③ Permutation general formula (1) : ${}^n P_{rc} = \frac{n!}{(n-rc)!}$ arrangement of rc elements from n elements④ Permutation formula (2) : $\frac{n!}{p! q! r!}$ arrangement of P separated p elements, q elements, r elements from n elements.⑤ Permutation formula (3) : n^r arrangement of different elements ~~that~~ that can be taken rc times at a time from n elements⑥ Permutation formula (4) : $(n-1)!$

arrangement of elements in a circle with different orientation in both sides

⑦ Permutation formula (5) : $\frac{1}{2} (n-1)!$

arrangement of elements in a circle with same orientation in both sides

⑧ Combination formula (1) : ${}^n C_{rc} = \frac{n!}{rc!(n-rc)!}$

selection of n things taken rc at a time

⑨ Combination formula (2) : ${}^n C_{rc} = \frac{{}^n P_{rc}}{rc!}$ In ⑩ Combination formula (3) : ${}^n C_{rc} = {}^n C_{n-rc}$ In ⑪ Combination formula (4) : ${}^n C_{rc} + {}^n C_{rc-1} = {}^{n+1} C_{rc}$ $\text{In Prove Important}$ ⑫ Combination formula (5) : $2^n - 1$ Combination of n things taken 1 at a time. HEARTS

(13) Greatest nC_r value is① When rC is even, $rC = \frac{n}{2}$ ② When rC is odd, $rC = \frac{n+1}{2}$ (14) Number of diagonals of a n -sided geometric shape : $(nC_2 - n)$

$$\begin{array}{|c|} \hline \text{X} \\ \hline \end{array} \quad n=4$$

$$nC_2 - n = 4C_2 - 4 = 2$$

(15) Combination formula (8) : $\frac{nC_r}{nC_{r+1}} = \frac{r+1}{n-r}$ (16) Combination formula (9) : if $nC_n = nC_y$
then $n=y$ and $n-y=n$ (17) Selection/Combination of n types of work which
can be done in p types of ways : $p^n - 1$