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Assignment: Practice Questions for
Sections 6.3 & 7.2 [Not for

Solve the differential equation.

$$\frac{dy}{dx} = 3e^{x-y}$$

Some differential equations can be solved by separating the variables. A differential equation of the form $y' = f(x,y)$ is separable if it can be expressed as a product of a function of x and a function of y .

Notice that e^{x-y} can also be written as $e^x e^{-y}$. Since e^{-y} is always greater than zero, we can solve the equation by separating the variables. Separate the variables by collecting all the y -terms with dy and all the x -terms with dx .

Rewrite the equation in its differential form.

$$\begin{aligned}\frac{dy}{dx} &= 3e^{x-y} \\ e^y dy &= 3e^x dx\end{aligned}$$

Now integrate both sides of the equation. Begin by integrating the left side. Use the rule $\int e^u du = e^u + C$ to integrate.

$$\begin{aligned}\int e^y dy &= \int 3e^x dx \\ e^y + C_1 &= \int 3e^x dx\end{aligned}$$

Integrate the right side. First move the constant to the outside of the integral. Use the rule $\int e^u du = e^u + C$ to integrate.

$$\begin{aligned}e^y + C_1 &= 3 \int e^x dx \\ e^y + C_1 &= 3e^x + C_2\end{aligned}$$

After completing the integrations, y is defined implicitly as a function of x . Combine the constants of integration as C .

$$e^y = 3e^x + C$$

Also, this equation can be solved for y as an explicit function of x .

$$y = \ln(3e^x + C)$$

Thus, solving the original differential equation, $\frac{dy}{dx} = 3e^{x-y}$, yields $e^y = 3e^x + C$ or $y = \ln(3e^x + C)$.