ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 11: Present & Annual Worth Comparisons

Note: Replacement Chain and Repeated Lifetimes are different names for the *same technique*, and will be used interchangeably in this course. (Both terms are in common use.)

October 1, 2021

Version 1.01 – Fixed two typos on slide 33.

Required Reading

- Sections 17.4.1 – 17.4.3 on pages 370-371 of
- Higbee, C. (1995). *Engineering Cost Analysis*. Oregon: Geo-Heat Center. http://digitallib.oit.edu/digital/collection/geoheat/id/10700/rec/2

Recommended Reading

- *Engineering Economics*, Chapter 4, Sections 4.2-4.4
- Newman, J. (2020, June 29). How to Calculate Amortization [Web Page]. Retrieved from https://www.wikihow.com/Calculate-Amortization
- The risk-return relationship [Web Page]. (n.d.). Retrieved from https://www.getsmarteraboutmoney.ca/invest/investing-basics/understanding-risk/the-risk-return-relationship/

Optional reading on Annual Worth

- 6.1 and 6.2 – Annual Worth Calculations [Web Page]. (n.d.). Retrieved from http://engineering.utep.edu/enge/EE/06/02/1.htm
- Fehr, D. (2017). Equivalent annuity vs. replacement chain approach for mutually exclusive investment projects. *Journal of Finance and Accountancy*, *22*. Retrieved from http://www.aabri.com/manuscripts/162550.pdf
- Kelly, J. J. & Leahy, P. G. (2020). Sizing Battery Energy Storage Systems: Using Multi-Objective Optimization to Overcome the Investment Scale Problem of Annual Worth. *IEEE Transactions on Sustainable Energy*, *11*(4), 2305-2314. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/8907493

Learning Objectives

- Be able to calculate and compare present worth for multiple projects which may have different lifetimes.
- Be able to calculate and compare annual worth for multiple projects which may have different lifetimes.
- Understand when one may wish to use annual worth vs NPV, & vice versa.

Relevant Solved Problems I

- From *Engineering Economics*, 6th edition:
- Present Worth of Independent Projects: Example 4.2, Example 4.3, 4.4, 4.5
- Constructing Mutually Exclusive Alternatives (Optional): Review Problem 4.1, 4.1, 4.2, 4.3, 4.21, 4.23
- Present Worth for Mutually Exclusive Projects: Example 4.4, Example 4.6, Example 4.7, Review Problem 4.1, 4.6, 4.7, 4.8, 4.15, 4.16, 4.24, 4.27, 4.28, 4.29, 4.32, 4.33, 4.36.b., 4.42
- Repeated Lives and Study Period: 4.15, 4.16, 4.28, 4.29, 4.32, 4.36.b., 4.33, 4.42

Relevant Solved Problems II

- From Stuart Nielsen's Engineering Economics: The Basics, 2nd edition:
- Present Worth: Example 5-7, Example 10-3, Example 10-6, Example 11-1, Example 11-2, Example 11-6, Example 11-7
- Annual Worth: Example 10-4, Example 10-5
- Present & Annual Worth comparisons: Example 11-4, Example 11-5, Example 11-8
- (Optional) Future Worth: Example 11-10
- **Note: The chapter 10 questions are benefit-cost ratio questions, but you can solve them as NPV or annual worth questions with just a slight rearrangement (e.g. NPV = PW of Benefits - PW of Costs).**

Notation Dictionary

(Not provided on quiz/final formula sheet)

- AW = Annual Worth
- MARR = Minimum Acceptable Rate of Return
- NPV = Net Present Value
- PW = Present Worth

- Conversion factors are of the form (X/Y,z)
- Read as: X, given Y and z.
- X is the element we want.
- Y is the element we have.
- z represents additional parameters.
- e.g. (P/F,i,N)
- Present Value, given a Future Value at time N and interest rate i.

ESSENTIALS (14 slides)

What should we do with our maple trees?

- Congratulations! You’ve inherited a stand of maple trees.
- Among your options is starting a maple syrup operation.
- Initial costs would be $80,000
- The trees would provide syrup for 50 years, starting next year.
- Net of operating costs, the syrup would bring in $20,000 a year in revenue.
- After the 50 years were up, you or your heirs could salvage the maple trees for $10,000.
- The next-best use of your money would be to pay off your existing debt, for which you are being charged 15% yearly interest.
- Is the maple syrup operation a good idea?

Setting up the problem

- MARR = 15%, N = 50
- Initial Cost = -P = -$80,000
- Annual revenue = A = $20,000
- Salvage = S = $10,000 in Year 50

- NPV = -$80,000 + $20,000(P/A,15%,50) + $10,000(P/F,15%,50)
- NPV = -$80,000 + $133,210.29 + $9.23 = $53,219.52
- NPV > 0, so the project is worthwhile (Profit > $50,000)

If you’re a lumberjack(/jane), is that okay?

- What about *logging* the trees and selling the lumber?
- Suppose trees are ready for logging in 10 years (N = 10).
- Let initial costs be \$10,000, and operating costs be \$3,000/year.
- P = -\$10,000, A = -\$3,000
- Once the 10 years are up, you can sell the lumber for \$275,000.
- S = \$275,000
- NPV = -\$10,000 - \$3,000(P/A,15%,10) + \$275,000(P/F,15%,10)
- NPV = -\$10,000 - \$15,056.31 + \$67,975.79 = \$42,919.49

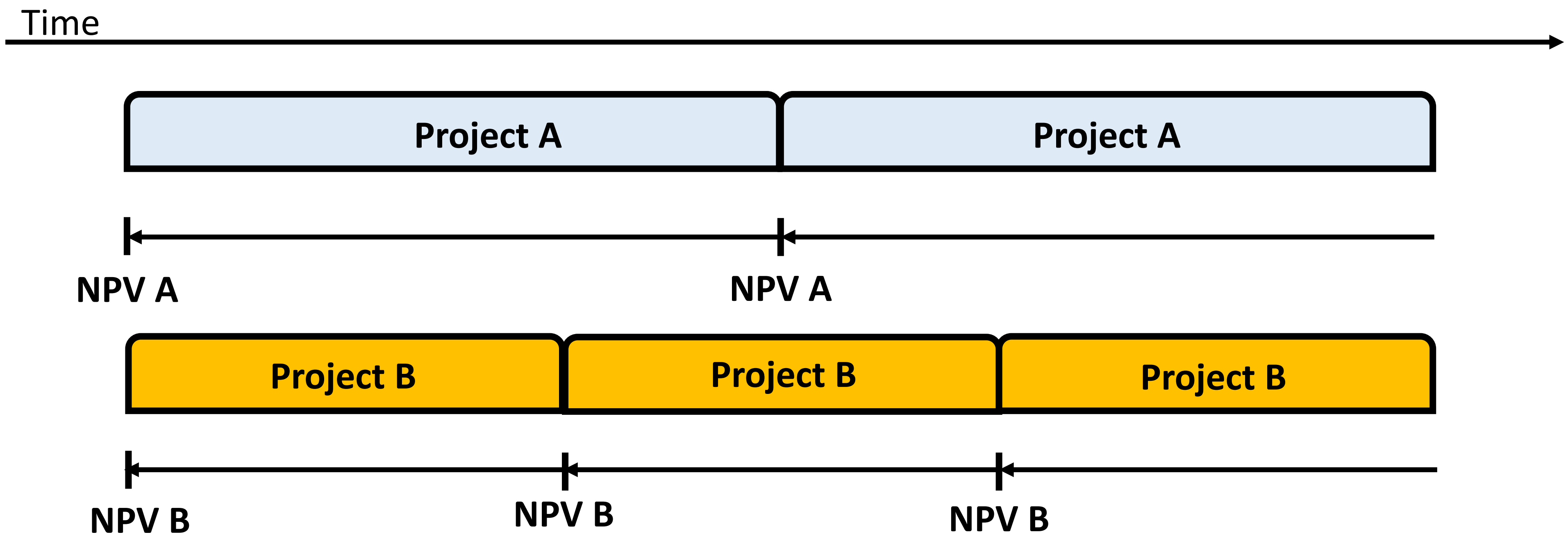
So…. no, then?

- NPV > 0, so on its own, the investment is worthwhile…
- …but it's lower than the NPV from the maple syrup project.
- Does that mean we should go for maple syrup production instead?
- Not necessarily…
- The two projects have *different lifetimes* and are not directly comparable.

Unequal project lifetimes

- Two main ways to deal with this:
- Repeated Lives: Use the least common multiple of project lives as the basis of comparison, and replicate cash flows as many times as needed.
- Con: Not the best assumption, if technical change is important.
- Study period: Pick a study period less than or equal to the lifetime of the shortest project. Look only at the years in that period.
- Cons: requires assumptions about salvage value (what the project is worth at the end of the study period – also called ‘**residual value**’.)
- Also: can miss important cash flows that lie outside the study period.
- Caution: these approaches can lead to wildly different results!
- **Course Note: Unless you are told otherwise, use the repeated lives method.**

The Repeated Lifetimes 'Replacement Chain'



- We can't compare NPVs for projects with different lifetimes.
- →Repeat each project to the least common multiple of lifetimes.
- Comparing *those* extended NPVs is legitimate… but lots of math.

Chain lifetime = least common multiple of original project lifetimes.

Approach 1: Repeat Cash Flows

MARR = 10% per year

Year	Original Projects		Making the Chain					Final Replacement Chains	
	Project A	Project B	A Link 1	A Link 2	B Link 1	B Link 2	B Link 3	Chain A	Chain B
0	-$100	-$150	-$100		-$150			-$100	-$150
1	$500	$700	$500		$700			$500	$700
2	$650	$500	$650		$500	-$150		$650	$350
3	$600		$600	-$100		$700		$500	$700
4				$500		$500	-$150	$500	$350
5				$650			$700	$650	$700
6				$600			$500	$600	$500

Brute Force Method		
Year	PV Chain A	PV Chain B
0	-$100.00	-$150.00
1	$454.55	$636.36
2	$537.19	$289.26
3	$375.66	$525.92
4	$341.51	$239.05
5	$403.60	$434.64
6	$338.68	$282.24
Total	**$2,351.18**	**$2,257.48**

Note: The last period of one link is 'Year 0' for the next.

Approach 2: Repeat NPVs

Year	Original Projects		Making the Chain					Final Replacement Chains	
	Project A	Project B	A Link 1	A Link 2	B Link 1	B Link 2	B Link 3	Chain A	Chain B
0	$1,342.52	$899.59	$1,342.52		$899.59			$1,342.52	$899.59
1									
2						$899.59			$899.59
3				$1,342.52				$1,342.52	
4							$899.59		$899.59
5									
6									

Brute Force Method		
Year	PV Chain A	PV Chain B
0	$1,342.52	$899.59
1		
2		$743.46
3	$1,008.66	
4		$614.43
5		
6		
Total	**$2,351.18**	**$2,257.48**

Year	PV Project A	PV Project B
0	-$100.00	-$150.00
1	$454.55	$636.36
2	$537.19	$413.22
3	$450.79	
Total (NPV)	**$1,342.52**	**$899.59**

NPV A	$1,342.52
NPV B	$899.59

Both approaches give the same result:
Project A > Project B, because the Chain NPV is higher.

If it’s a dollar a day, it’s a dollar a day

- Suppose I told you that Project A was like getting \$3 a day for 3 years, and Project B was like getting \$2 a day for 2 years.
- I could have found this by using (A/P,MARR,N) on each project’s NPV, where N is the duration of the project in days, and MARR is in % per day.
- Do I need to go further?
- If I repeat Project A 20 times, it’s like getting \$3 a day for 60 years.
- If I repeat Project B 8 times, it’s like getting \$2 a day for 16 years.
- No matter how often I repeat them, Project A is still like getting \$3 a day, and Project B is like getting \$2 a day.
- Since \$3 a day is better than \$2 a day, Project A is preferred.

Comparing annual worths (“equivalent annuities”)

- Comparing annual worths, we *don’t need to worry* about unequal lifetimes!
- I say ‘annual worth’, but as shown, it doesn’t have to be in terms of years – it could be months, days, milliseconds, centuries, etc.
- Also commonly called an ‘equivalent annuity’.
- How to calculate it most efficiently varies by project & context…
- BUT one what that will ALWAYS work…
- For a project with duration of N time periods, calculate the NPV.
- Then AW = NPV x (A/P,MARR,N)

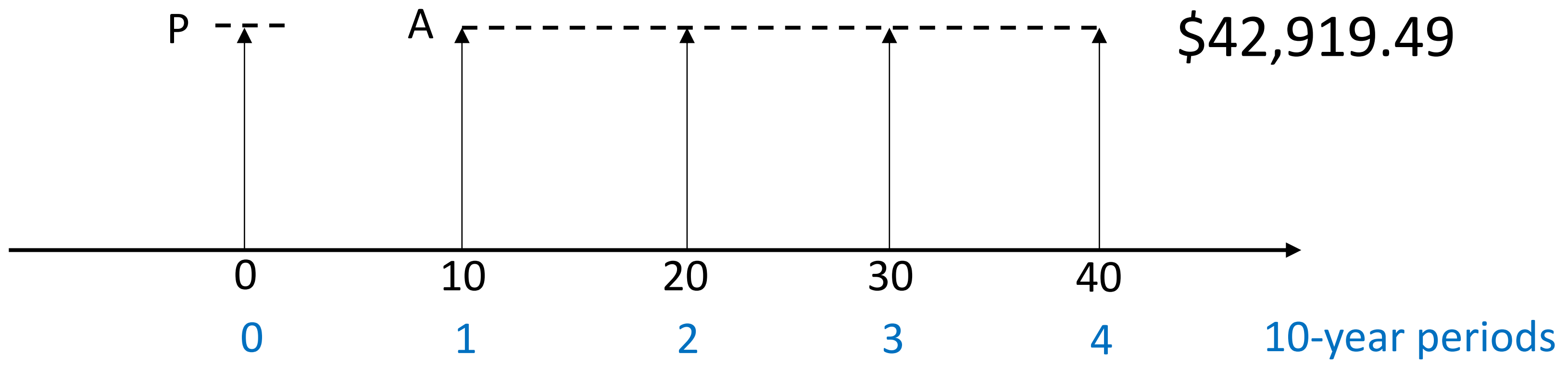
In the *after hours* section we’ll discuss why you’d ever want to use repeated lifetimes when Annual Worth is so easy.

	Annual Worth Approach		
	PV	N	A
Project A	$1,342.52	3	$539.85
Project B	$899.59	2	$518.33
Chain A	$2,351.18	6	$539.85
Chain B	$2,257.48	6	$518.33

The Annual Worth of A > Annual worth of B
Project A is preferred.

The repeated lives of maple trees

- Maple Syrup: N = 50
- Logging: N = 10
- →Repeat Logging 5 times to be able to compare it to maple syrup.
- The final year of the previous project is year 0 of the next one.
- We already know that the NPV for logging (once) is $42,919.49
- →Repeating it 5x is like receiving a payment of $42,919.49 every 10 years. We can make this easy by adjusting the time scale.



Adjusting the Time Scale

- The initial P is easy enough (just $42,919 at Time 0). Let's work on the other 4.
- Payment every 10 years → find the 10-year interest rate.
- Interest = 15%/year, so after 10 years, $1 becomes $1 x $(1 + 15\%)^{10}$.
- → After 10 years, $1 becomes $4.0456, so 10-year interest is 304.56% (rounded).
- A = $42,919.49, N = 4 (10-year-periods)
- →Present Worth = $ 42,919.49 x (P/A,304.56%,4)
- Present Worth of 4 'repeats' = $ 42,919.49 x 0.3271 = $14,039.88
- Total Present Worth = $42,919.49 + $14,039.88 = **$56,959.37**
- This is (slightly) bigger than the NPV from maple syrup production, which was $53,219.52.
- →Logging should be chosen, instead of maple syrup production.

Brute Force Checking

- It's also correct to calculate V(P/F,15%,N) for N=0,10,20,30,40, then add them all up to get the total NPV for the repeated lifetimes:

N	V	V(P/F,i,N)
0	$42,919.49	$42,919.49
10	$42,919.49	$10,609.04
20	$42,919.49	$2,622.39
30	$42,919.49	$648.22
40	$42,919.49	$160.23
Total		**$56,959.37**

A side note on study periods:

The logging project provides a clear example of why **using study periods instead of repeated lives must be done with care.**

Since logging only pays off in Year 10, a study period shorter than 10 years would conclude there were only losses to be had from logging. The repeated lives method correctly includes all cash flows.

Using the Study Period Method

- Pick a length of the study period: this is often equal to the lowest project lifetime being considered – in this case, 10 years.
- Cash flows beyond the study period are *ignored*, but an 'residual value' (often treated as a salvage or potential resale value) for longer-lived alternatives must be included.
- (Despite this, many published studies using study period comparisons incorrectly neglect to calculate a residual value…)
- You can also leave the residual value as a variable, and later see if the value needed to make the option with the residual the most preferred option 'makes sense'. That's what we'll do for this example.

Study period of 10 years

- Logging: NPV = $42,919.49, as we calculated earlier.
- Maple syrup:
  - Initial cost of $80,000: PV = -$80,000
  - Annual revenue of $20,000 from Years 1 to 10: PV = $20,000 x (P/A,15%,10)
    - PV = $20,000 x (P/A,15%,10) = $100,375.37
  - Residual value 'R' in Year 10: PV = R x (P/F,15%,10) = R/4 (approx.)
  - NPV = -$80,000 + $100,375.37 + R/4 = $20,375.37 + R/4
- How big does R have to be for Syrup to be preferred? Solve at equality:
- $42,919.49 = $20,375.37 + R/4
- R = 4 x ($42,919.49 - $20,375.37) = $90,176.47
- From our original information, we know the salvage value of the maple syrup operation is $10,000, so this threshold value seems high (but more assumptions and work would be needed to know for sure).

AFTER HOURS

- How a loan is paid off (3 slides)
- Why would we ever use repeated lifetimes? (8 slides)
- Investment scale & annual worth (5 slides)

Aside on Amortization

- The (A/P,i,N) factor has another important, common use:
- If you borrow \$P at interest i, and pay it over N time periods…
- …if the loan has been ***amortized*** into equal payments…
- …each payment is equal to \$P x (A/P,i,N)
- As always, i here is in terms of the time scale matching the N.
- Monthly payments → i is in % per month, etc.
- E.g. Fixed rate mortages are required by law to be compounded half-yearly, but banks take monthly payments: they find the monthly interest rate equivalent to the relevant 6-month rate, & amortize.

How a loan is paid off

- So you borrowed \$P, and are now paying A = P x (A/P,i,N) , N times.
- First payment: You start out owing P, and pay A.
- That A is broken down into payment of the *interest*, and the principal.
- A = (Paying all interest due) + (Paying off part of the principal)
- A = (i x P) + (A – i x P)
- Principal owing goes from P to (P – (A – i xP))
- Second payment: You start out owing (P – (A – i xP)), and pay A.
- Pay interest of i x (P – (A – i xP)), & the rest is taken off your principal..
- …and so it goes.

Example: P = \$1,000, N = 12, i = 1%

Month	Principal	Payment	Interest Payment	Principal Payment	Ending Principal
1	\$1,000.00	\$88.85	\$10.00	\$78.85	\$921.15
2	\$921.15	\$88.85	\$9.21	\$79.64	\$841.51
3	\$841.51	\$88.85	\$8.42	\$80.43	\$761.08
4	\$761.08	\$88.85	\$7.61	\$81.24	\$679.84
5	\$679.84	\$88.85	\$6.80	\$82.05	\$597.79
6	\$597.79	\$88.85	\$5.98	\$82.87	\$514.92
7	\$514.92	\$88.85	\$5.15	\$83.70	\$431.22
8	\$431.22	\$88.85	\$4.31	\$84.54	\$346.68
9	\$346.68	\$88.85	\$3.47	\$85.38	\$261.30
10	\$261.30	\$88.85	\$2.61	\$86.24	\$175.07
11	\$175.07	\$88.85	\$1.75	\$87.10	\$87.97
12	\$87.97	\$88.85	\$0.88	\$87.97	\$0.00

- A = \$1,000 x (A/P,1%,12) = \$88.85

Why would we ever use **repeated lifetimes** if comparing **annual worths** works the same, and is so much easier?

The answer has to do with *risk* and return.

The MARR in a world without risk

- Suppose your fallback investment is risk-free, and gives you \$110 one year later for every \$100 you put in today.
- →Risk-free MARR = 10%
- Suppose a project has flows of -\$100 in Year 0 and +\$121 in Year 1.
- By inspection, better than your fallback: 21% vs 10% per year.
- NPV = -\$100 + \$121 x (P/F,10%,1) = \$10
- Intuition: you'd have had to put the same resources (\$100 today) plus another \$10 in your fallback project to get the same return.
- What if the project is risky, and has a 50% chance of total failure?

Risk vs Return

- This risk → 1 year from now, 50% chance of \$121, 50% chance of \$0
- NPV(success) = -\$100 + \$121 x (P/F,10%,1) = \$10
- NPV(failure) = -\$100 + \$0 x (P/F,10%,1) = -\$100
- NPV(expected flows) = -\$100 + (50% x \$121 + 50% x 0) x (P/F,10%,1)
- NPV(expected flows) = -\$100 + \$60.5 x (P/F,10%,1) = -\$45
- Here, we adjusted cash flows for risk & evaluated at the risk-free MARR.
- Another approach: require a *risk premium* on top of the MARR.

A simple risk premium calculation

- This gets very complicated, very quickly, but here it's simple:
- If I'm spending \$100 today to get \$F one year from now…
- …if there's only a 50% of getting F (and 50% of getting nothing)…
- How big does F have to be for me to invest, if my MARR is 10%/year?
- NPV = -100 + (50% x F + 50% x 0) x (P/F,10%,1)
- NPV = -100 + (50% x F)/(1+10%) = -100 + (.5/1.1) x F
- To be at least as good as my fallback project, I need NPV≧0
- NPV = 0 → F = 100/(.5/1.1) = 220 → 120%/year interest
- →My MARR for a project this risky is 120%, not 10%.
- **In general, firms demand a higher return from riskier projects.**

Why are we talking about this?

- Why are we bringing modified MARRs and NPV into it?
- Because when considering a set of mutually exclusive projects…
- …some of these projects may have *different levels of risk*.
- →We could legitimately be using *different MARRs* to evaluate them.
- Not talked about much in textbooks, BUT important, because:
- **When comparing projects using different MARRs, Repeated Lifetimes NPV and Annual Worth comparisons may disagree on project rankings.**

Our test projects (Fehr, 2017)

Year	Project A	Project B
0	-$35.00	-$50.00
1	$50.00	$50.10
2	$50.00	$50.10
3	$50.00	$50.10
4		$50.10
5		$50.10
NPV (3%)	$106.43	$179.44
AW (3%)	$37.63	$39.18
RL NPV	$449.18	$467.76

MARR (1 year)	3%
MARR (3 years)	9.27%
MARR (5 years)	15.93%

- Risk-free MARR = 3%/year
- Both projects are risk-free
- LCM of lifetimes = 15
- Repeat A x 5, Repeat B x 3
- Equivalent Annual Worth for:
- A: NPVA x (A/P,MARR,3)
- B: NPVB x (A/P,MARR,5)
- Repeated Lifetimes NPV for:
- A: NPVA + NPVA x (P/A,$MARR_{3\text{-year}}$,4)
- B: NPVB + NPVB x (P/A,$MARR_{5\text{-year}}$,2)

What if Project B is risky?

	Year	Project A	Project B
	0	-$35.00	-$50.00
	1	$50.00	$50.10
	2	$50.00	$50.10
	3	$50.00	$50.10
	4		$50.10
	5		$50.10
NPV	NPV (3%)	$106.43	
	NPV (6%)		$161.04
AW	AW (3%)	$37.63	
	AW (6%)		$38.23
RL NPV		$449.18	$371.30

- Let Project B be risky, with a higher MARR of 6%.
- Repeating the exercise, but using this rate for Project B:
- Repeated Lifetimes NPV & Annual Worth ***don't agree***.

MARR (1 year)	3%	6%
MARR (3 years)	9.27%	
MARR (5 years)		33.82%

What do you do in this case? (Fehr, 2017)

- (Fehr, 2017) suggests you use the repeated lifetimes NPV approach.
- Why? Because of the NPV measures the *profit* of a project, in terms of much better a use of resources it is than the project your MARR is derived from.
- "[T]he replacement chain [(repeated lifetimes)]approach is best suited because it directly measures value added by the projects."
- This applies to "the very special case in which projects are mutually exclusive, have unequal lives [...] and have different risk characteristics."
- "In such a case, students must be aware that the standard textbook solution methodology may lead to non-optimal decision making."

"The Investment Scale Problem of Annual Worth"

- From Kelly & Leahy (2020).
- Suppose you have two projects, S and T.
- You've calculated the equivalent annual worth using (A/P,i,N).
- So that you can easily do an IBCR analysis on the per-year values, you've also calculated the AW of Benefits and Costs separately.
- AW = AW (Benefits) – AW (Costs)
- Looks good, right? Both projects have AW > 0, and Project S is better.

Project	AW Benefits	AW Costs	AW
T	2,000	1,000	1,000
S	2,002,000	2,000,000	2,000

A quick IBCR, &hints of trouble (Kelly & Leahy, 2020)

Project	Benefits	Costs	IB	IC	IBCR
T	2,000	1,000			
S	2,002,000	2,000,000	2,000,000	1,999,000	1.0005

Project	AW Benefits	AW Costs	AW	AW as % of Cost
T	2,000	1,000	1,000	100%
S	2,002,000	2,000,000	2,000	**0.1%**

- “Project S is given as the best option with an AW twice that of project T.”
- “However, the capital expenditure of Project S is 2000 times that of Project T. As access to capital is limited in real-world cases, clearly Project T is the preferred option.”

What does this imply?

- According to Kelly & Leahy, *if y*ou’re going to use Annual Worth to choose between mutually exclusive projects,
- *then* you need to make sure the projects have similar, fixed budgets; the scale of the investment must be the same.
- Problem: this is often not the case in engineering. Kelly & Leahy ran into problems when working with Battery Energy Storage Systems (BESS), where different options can have vastly different scales.
- Why? Partly of how the options were *found*: they set an optimizing algorithm loose to look for options, and it returned only *optimal* combinations of parameters, <u>not</u> results for incremental changes.
- (If you ask a math program to return the parameter value that maximizes a function, it won’t return other values, even ones that give results very close to the maximum.)

Solutions?

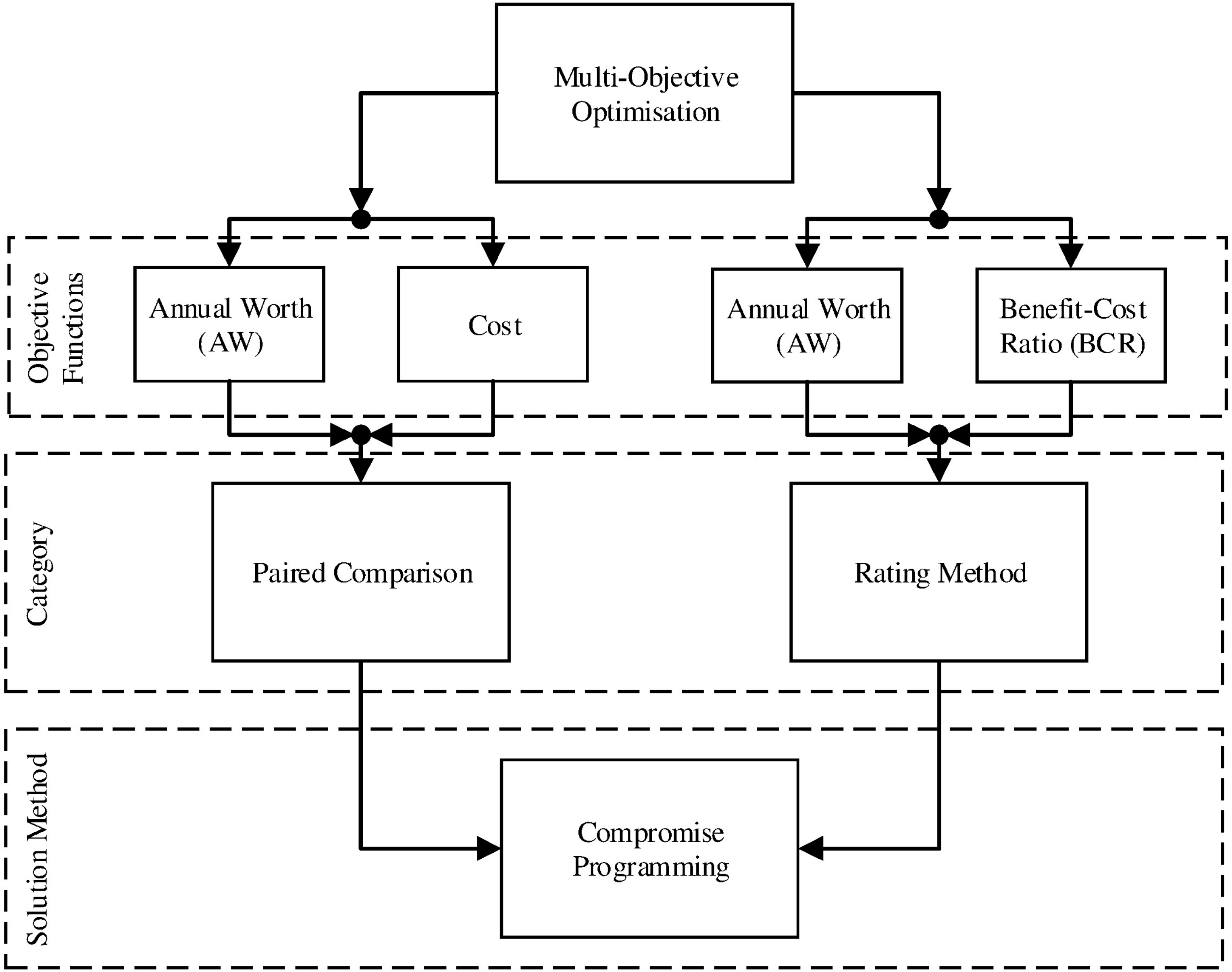
$$B_{BESS} = \left(B_{Grid}^{BESS^+} - C_{Grid}^{BESS^+}\right) - OC$$

$$C_{BESS} = E_{Cost} E_{BESS}$$

(Kelly & Leahy, 2020)

Note use of operating costs as negative benefits

- You could bake an investment size constraint directly into your search algorithm: maximize annual worth, while minimizing costs (or giving lower points for higher costs).
- You could include *other* project evaluation methods:
- Considered: Benefit-Cost Ratios (BCR), time-discounted Payback Periods, internal rate of return (IRR) (we'll see this in a later lecture)
- Went with Cost, AW & BCR, despite some qualms.
- Didn't seem aware of *incremental* **Benefit-Cost analysis**…



"Compromise Programming" (Kelly & Leahy, 2020)