University of Victoria - Stat 260 - Spring 2023
Term Test 2 - Version A
Section A01 - Instructor: Dr. Michelle Edwards

Instructions:

- This paper has 6 pages of questions plus this cover page. Please count your pages and report any discrepancy immediately to the invigilator.
- Answer all questions on the test paper in the space provided. Questions 1 through 8 are short answer and worth 3 marks each. For each question show your work and write your final answer in the space as indicated. Question 9 is a full-answer question. To obtain full marks on this question, clear step-by-step reasoning must be displayed. Marks will be deducted for incoherent or incomplete solutions.
- Unless an exact answer with fewer decimals can be found, give all answers with at least three decimals of accuracy.
- A Sharp EL-510R series calculator may be used. No other calculator may be used.
- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

This page will not be graded.
Feel free to use this page for scrap work.

[3] **1.** In testing of a new computer operating system, the developers measure the number of times the system crashes. Let the random variable X be the number of times the operating system crashes in a day. The cumulative distribution of X is given below:

x	0	1	2	3	4	5	6
$F(x)$	0.17	0.30	0.49	0.64	0.81	0.93	1
$f(x)$	.17	.13	.19	.15	.17	.12	.07

Find the standard deviation of X. That is, find σ_X.

$E(X) = 0(.17) + 1(.13) + 2(.19) + 3(.15) + 4(.17) + 5(.12)$
$+ 6(.07) = 2.66$

$E(X^2) = 0^2(.17) + 1^2(.13) + 2^2(.19) + 3^2(.15) + 4^2(.17)$
$+ 5^2(.12) + 6^2(.07) = 10.48$

$V(X) = E(x^2) - (E(x))^2 = 10.48 - (2.66)^2 = 3.4044$

$\sigma_X = \sqrt{V(X)} = \sqrt{3.4044} = 1.8451$

Answer

1.8451

[3] **2.** A person takes the same city bus to work every day at the same time. Suppose the bus arrives late 40% of the time, and that late arrivals on days are independent of one another. For a random sample of 17 days, what is the probability that the bus arrives late on at least 6 days but no more than 11 days?

binomial $n = 17$, $p = 0.40$

$P(6 \leq X \leq 11) = P(X \leq 11) - P(X \leq 5)$

$= 0.9894 - 0.2639$

$= 0.7255$

Answer

0.7255

[3] **3.** A linguistics professor is studying the effects of autocorrect software on the incidence of typos in large documents. Their research suggests that on average 7 typos exist in a 10 page document. Suppose the professor conducts a study which asks participants to create a 20 page document. For a random participant from the study, if it is known that they make at least 12 typos in their document, what is the probability that they make exactly 15 typos in their document?

Poisson $\lambda = 7(2) = 14$

$$P(X=15 \mid X \geq 12) = \frac{P(X=15 \cap X \geq 12)}{P(X \geq 12)} = \frac{P(X=15)}{1-P(X \leq 11)}$$

$$= \frac{\frac{14^{15} e^{-14}}{15!}}{1-0.2600} = \frac{0.0989}{0.7400} = 0.1337$$

Answer
0.1337

[3] **4.** The weekly demand for propane gas (in 1000s of gallons) from a particular facility is a continuous random variable X with pdf

$$f(x) = \begin{cases} 2\left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Suppose the facility randomly selects their accounting records for one week. What is the probability that the demand for propane that week was between 1500 gallons and 1750 gallons? (Note: An amount of 1000 gallons would correspond to a value of $x = 1$.)

$$P(1.5 \leq X \leq 1.75) = \int_{1.5}^{1.75} f(x)\,dx = \int_{1.5}^{1.75} 2\left(1 - \frac{1}{x^2}\right) dx$$

$$= 2(x + x^{-1}) \Big|_{1.5}^{1.75} = 2\left(1.75 + \frac{1}{1.75}\right) - 2\left(1.5 + \frac{1}{1.5}\right)$$

$$= \frac{65}{14} - \frac{13}{3} = \frac{13}{42} = 0.3095$$

Answer
0.3095

[3] **5.** Let X be the continuous random variable with pdf

$$f(x) = \begin{cases} k(3-x^2) & 3 \le x \le 5 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_3^5 k(3-x^2)\,dx$$

$$= k\left(3x - \frac{x^3}{3}\right)\Big|_3^5 = k\left(3(5) - \frac{5^3}{3}\right) - k\left(3(3) - \frac{3^3}{3}\right)$$

$$= k\left(15 - \frac{125}{3}\right) - k\left(9 - \frac{27}{3}\right) = k\left(-\frac{80}{3}\right) - 0 = -\frac{80}{3}k$$

$$1 = -\frac{80}{3}k \implies k = -\frac{3}{80} = -0.0375$$

Answer: $-3/80$

[3] **6.** Let the continuous random variable X denote the life in hours of a particular electronic device. The probability density function is

$$f(x) = \begin{cases} \dfrac{20000}{x^3} & x \ge 100 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected life for this particular electronic device. That is, find $E(X)$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{100}^{\infty} x \cdot 20000x^{-3}\,dx$$

$$= \lim_{c\to\infty} \int_{100}^{c} 20000x^{-2}\,dx = \lim_{c\to\infty} -20000x^{-1}\Big|_{100}^{c}$$

$$= \lim_{c\to\infty} -\frac{20000}{c} - \left(-\frac{20000}{100}\right) = 200$$

Answer: 200

[3] **7.** It is estimated that 15% of workers in Canada are self-employed. If a group of 200 workers in Canada are randomly sampled, what is the probability that at most 21 of them are self-employed? Use the normal approximation to the binomial distribution (with the continuity correction factor) to solve this question.

$\mu = np = (200)(.15) = 30$

$\sigma = \sqrt{np(1-p)} = \sqrt{200(.15)(.85)} = \sqrt{25.5}$

Answer: 0.0465

$$\underset{\text{binomial}}{P(X \leq 21)} \approx \underset{\text{normal}}{P(X \leq 21.5)} = P\left(\frac{X-\mu}{\sigma} \leq \frac{21.5-30}{\sqrt{25.5}}\right)$$

$$= P(Z \leq -1.68) = 0.0465$$

[3] **8.** Suppose that students submit requests for help in the Math & Stats Assistance Centre according to a Poisson process. The Assistance Centre reports that they receive an average of 6 questions every 15 minutes. Suppose a student has just asked a question. What is the probability that the time until the next question is more than 4 minutes?

$\lambda = \frac{6}{15} = 2/5$

Answer: 0.2019

exponential

$$P(X > 4) = 1 - P(X \leq 4) = 1 - (1 - e^{-2/5(4)})$$

$$= e^{-8/5}$$

$$= 0.2019$$

(Question 9) The distribution of resistance for resistors of a certain type is known to be normal with a mean of 10 ohms and a standard deviation of 0.27 ohms.

(a) [**2 marks**] If a resistor of this type is selected at random, what is the probability that it has a resistance between 9.7 ohms and 10.4 ohms?

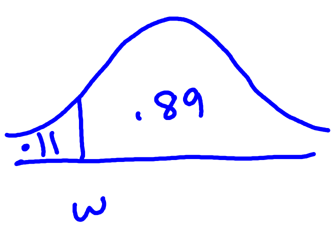
$$P(9.7 \leq X \leq 10.4) = P\left(\frac{9.7-10}{0.27} \leq \frac{X-\mu}{\sigma} \leq \frac{10.4-10}{0.27}\right)$$

$$= P(-1.11 \leq Z \leq 1.48) \quad = P(Z \leq 1.48) - P(Z \leq -1.11)$$

$$= 0.9306 - 0.1335$$

$$= 0.7971$$

(b) [**2 marks**] Find the resistance w for which 89% of resistors of this type have a resistance greater than w.



$$Z = -1.23$$

$$Z = \frac{X-\mu}{\sigma}$$

$$-1.23 = \frac{w-10}{0.27}$$

$$w = (-1.23)(0.27) + 10$$

$$w = 9.6679$$

Recall: The distribution of resistance for resistors of a certain type is known to be normal with a mean of 10 ohms and a standard deviation of 0.27 ohms.

(c) [**2 marks**] For this type of resistors, the component will be called defective if the resistance is less than 9.5 ohms. Suppose 10 independent resistors are selected at random. What is the probability that exactly one of these resistors is defective?

X = resistance level

$$P(\text{success}) = P(X < 9.5) = P\left(\frac{X-\mu}{\sigma} \le \frac{9.5-10}{0.27}\right) = P(Z \le -1.85)$$
$$= 0.0322$$

Y = # of defective resistors

binomial $n=10$, $p=0.0322$

$$P(Y=1) = \binom{10}{1}(.0322)^1(.9678)^9$$

$$= 0.2398$$

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Term Test 2 - Version B
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[3] **1.** In testing of a new computer operating system, the developers measure the number of times the system crashes. Let the random variable X be the number of times the operating system crashes in a day. The cumulative distribution of X is given below:

x	0	1	2	3	4	5	6
$F(x)$	0.21	0.36	0.53	0.65	0.87	0.94	1
$p(x)$	.21	.15	.17	.12	.22	.07	.06

Find the standard deviation of X. That is, find σ_X.

$E(X) = 0(.21) + 1(.15) + 2(.17) + 3(.12) + 4(.22) + 5(.07) + 6(.06)$
$= 2.44$

Answer

1.8402

$E(X^2) = 0^2(.21) + 1^2(.15) + 2^2(.17) + 3^2(.12) + 4^2(.22) + 5^2(.07) + 6^2(.06) = 9.34$

$V(X) = E(X^2) - (E(X))^2 = 9.34 - (2.44)^2 = 3.3864$

$\sigma_X = \sqrt{V(X)} = \sqrt{3.3864} = 1.8402$

[3] **2.** A person takes the same city bus to work every day at the same time. Suppose the bus arrives late 60% of the time, and that late arrivals on days are independent of one another. For a random sample of 19 days, what is the probability that the bus arrives late on at least 9 days but no more than 14 days?

binomial $n = 19$ $p = 0.60$

Answer

0.8419

$P(9 \leq X \leq 14) = P(X \leq 14) - P(X \leq 8)$

$= 0.9304 - 0.0885$

$= 0.8419$

[3] **3.** A linguistics professor is studying the effects of autocorrect software on the incidence of typos in large documents. Their research suggests that on average 4 typos exist in a 5 page document. Suppose the professor conducts a study which asks participants to create a 20 page document. For a random participant from the study, if it is known that they make at least 13 typos in their document, what is the probability that they make exactly 17 typos in their document?

Poisson $\lambda = 4 \cdot 4 = 16$

$$P(X=17 \mid X \geq 13) = \frac{P(X=17 \cap X \geq 13)}{P(X \geq 13)} = \frac{P(X=17)}{1-P(X \leq 12)}$$

$$= \frac{\frac{16^{17} e^{-16}}{17!}}{1-0.1931} = \frac{0.0934}{0.8069} = 0.1157$$

Answer
0.1157

[3] **4.** The weekly demand for propane gas (in 1000s of gallons) from a particular facility is a continuous random variable X with pdf

$$f(x) = \begin{cases} \frac{3}{4}\left(1 - \frac{1}{x^2}\right) & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Suppose the facility randomly selects their accounting records for one week. What is the probability that the demand for propane that week was between 1500 gallons and 2250 gallons?

$$P(1.5 \leq X \leq 2.25) = \int_{1.5}^{2.25} \frac{3}{4}(1-x^{-2})\,dx$$

$$= \frac{3}{4}(x + x^{-1}) \Big|_{1.5}^{2.25} = \frac{3}{4}\left(2.25 + \frac{1}{2.25}\right) - \frac{3}{4}\left(1.5 + \frac{1}{1.5}\right)$$

$$= 97/48 - 13/8 = 19/48 = 0.3958$$

Answer
0.3958

[3] **5.** Let X be the continuous random variable with pdf

$$f(x) = \begin{cases} k(2 - x^3) & 2 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

$$1 = \int_{-\infty}^{\infty} f(x)dx = \int_2^5 k(2-x^3)dx = k(2x - x^4/4)\Big|_2^5$$

$$= k(10 - 5^4/4) - k(4 - 16/4) = k(-585/4)$$

$$1 = k(-585/4) \Rightarrow k = -4/585 = -0.006838$$

Answer
$-4/585$

[3] **6.** Let the continuous random variable X denote the life in hours of a particular electronic device. The probability density function is

$$f(x) = \begin{cases} \dfrac{80000}{x^3} & x \geq 200 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected life for this particular electronic device. That is, find $E(X)$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{200}^{\infty} x \cdot \frac{80000}{x^3}\,dx$$

$$= \lim_{c\to\infty} \int_{200}^{c} 80000x^{-2}\,dx = \lim_{c\to\infty} -80000x^{-1}\Big|_{200}^{c}$$

$$= \lim_{c\to\infty} \frac{-80000}{c} + \frac{80000}{200} = 400$$

Answer
400

[3] **7.** It is estimated that 35% of workers in Canada are self-employed. If a group of 400 workers in Canada are randomly sampled, what is the probability that at most 120 of them are self-employed? Use the normal approximation to the binomial distribution (with the continuity correction factor) to solve this question.

$\mu = np = 400(.35) = 140$

$\sigma^2 = np(1-p) = 400(.35)(.65) = 91$

$P(X \leq 120) \approx P(X \leq 120.5) = P\left(Z \leq \frac{120.5 - 140}{\sqrt{91}}\right)$

$= P(Z \leq -2.04) = 0.0207$

Answer

0.0207

[3] **8.** Suppose that students submit requests for help in the Math & Stats Assistance Centre according to a Poisson process. The Assistance Centre reports that they receive an average of 4 questions every 15 minutes. Suppose a student has just asked a question. What is the probability that the time until the next question is more than 5 minutes?

$\lambda = 4/15$ exponential

$P(X > 5) = 1 - P(X \leq 5)$

$= 1 - (1 - e^{-4/15(5)}) = e^{-4/3} = 0.2636$

Answer

0.2636

(Question 9) The distribution of resistance for resistors of a certain type is known to be normal with a mean of 20 ohms and a standard deviation of 0.43 ohms.

(a) **[2 marks]** If a resistor of this type is selected at random, what is the probability that it has a resistance between 19.5 ohms and 20.7 ohms?

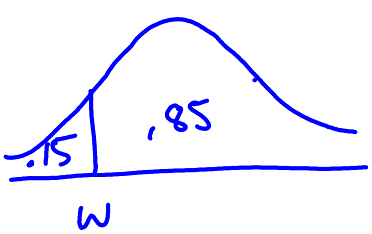
$$P(19.5 \le X \le 20.7) = P\left(\frac{19.5-20}{0.43} \le \frac{X-\mu}{\sigma} \le \frac{20.7-20}{0.43}\right)$$

$$= P(-1.16 \le Z \le 1.63) = P(Z \le 1.63) - P(Z \le -1.16)$$

$$= 0.9484 - 0.1230$$

$$= 0.8254$$

(b) **[2 marks]** Find the resistance w for which 85% of resistors of this type have a resistance greater than w.



$$Z = -1.04$$

$$Z = \frac{X-\mu}{\sigma}$$

$$-1.04 = \frac{w-20}{0.43}$$

$$w = (-1.04)(0.43) + 20 = 19.5528$$

Recall: The distribution of resistance for resistors of a certain type is known to be normal with a mean of 20 ohms and a standard deviation of 0.43 ohms.

(c) **[2 marks]** For this type of resistors, the component will be called defective if the resistance is less than 19.3 ohms. Suppose 12 independent resistors are selected at random. What is the probability that exactly two of these resistors are defective?

X = resistance level

$$P(\text{success}) = P(X < 19.3) = P\left(\frac{X-\mu}{\sigma} \leq \frac{19.3-20}{0.43}\right) = P(Z \leq -1.63)$$
$$= 0.0516$$

Y = # of defective resistors

binomial $n = 12$ $p = 0.0516$

$$P(Y=2) = \binom{12}{2}(.0516)^2(0.9484)^{10}$$

$$= 0.1035$$

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- Unless an exact answer with fewer decimals can be found, give all answers with at least three decimals of accuracy.
- A Sharp EL-510R series calculator may be used. No other calculator may be used.
- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

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[3] **1.** Let the random variable X be the number of classes a current UVic student is registered in this semester. Suppose the cumulative distribution of X is given below:

x	1	2	3	4	5	6
$F(x)$	0.09	0.22	0.39	0.53	0.87	1
$f(x)$	.09	.13	.17	.14	.34	.13

Find the standard deviation of X. That is, find σ_X.

$E(X) = 1(.09) + 2(.13) + 3(.17) + 4(.14) + 5(.34) + 6(.13)$

$= 3.9$

$E(X^2) = 1^2(.09) + 2^2(.13) + 3^2(.17) + 4^2(.14) + 5^2(.34) + 6^2(.13)$

$= 17.56$

$V(X) = E(X^2) - (E(X))^2 = 17.56 - (3.9)^2 = 2.35$

$\sigma_X = \sqrt{V(X)} = \sqrt{2.35} = 1.5330$

Answer

1.5330

[3] **2.** In a particular city, it is determined that 30% of homes use electricity as their main heating source (compared to other sources such as oil or natural gas). For a random sample of 20 independent homes in the city, what is the probability that at least 5 but no more than 10 homes use electricity as their main heating source?

binomial $n = 20$ $p = 0.30$

$P(5 \le X \le 10) = P(X \le 10) - P(X \le 4)$

$= 0.9829 - 0.2375$

$= 0.7454$

Answer

0.7454

[3] **3.** A photocopier has an error that will distribute a blank page in the photocopy job every so often. It is determined that this error occurs at an average rate of 2 blank pages for every 75 pages in the photocopy job.
Suppose a person sends a 450 page print job to the photocopier. If it is known that the photocopier distributes at least 9 blank pages, what is the probability that it distributes exactly 14 blank pages?

Poisson $\lambda = 2 \cdot 6 = 12$

$$P(X=14 \mid X \geq 9) = \frac{P(X=14 \cap X \geq 9)}{P(X \geq 9)} = \frac{P(X=14)}{1-P(X \leq 8)}$$

$$= \frac{\frac{12^{14} e^{-12}}{14!}}{1-0.1550} = \frac{0.0905}{0.8450} = 0.1071$$

Answer

0.1071

[3] **4.** A small company offers moving services for small and quick jobs between 1000 and 3000 pounds. The weight of a customer's move (measured in 1000s of pounds) with this service is a continuous random variable X with pdf

$$f(x) = \begin{cases} \frac{5}{32}(x-1)^4 & 1 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Suppose the company randomly selects their accounting records for a random customer. What is the probability that their move with the service was between 1500 and 2500 pounds? (Note: An amount of 1000 pounds would correspond to a value of $x = 1$.)

$$P(1.5 \leq X \leq 2.5) = \int_{1.5}^{2.5} \frac{5}{32}(x-1)^5 \, dx$$

$$= \frac{5}{32} \frac{(x-1)^5}{5} \Big|_{1.5}^{2.5} = \frac{(2.5-1)^5}{32} - \frac{(1.5-1)^5}{32}$$

$$= \frac{243}{1024} - \frac{1}{1024} = \frac{121}{512} = 0.2363$$

Answer

0.2363

[3] **5.** Let X be the continuous random variable with pdf

$$f(x) = \begin{cases} k\sqrt{x} & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_1^4 k\sqrt{x}\,dx = \int_1^4 k\,x^{1/2}\,dx$$

$$= \frac{2k}{3} x^{3/2} \Big|_1^4 = \frac{2k}{3}(8-1) = \frac{14k}{3}$$

$$1 = 14k/3 \implies k = 3/14$$

Answer

$3/14$

[3] **6.** Let the continuous random variable X denote the amount of constant hours a freshly charged battery can be used before it is dead. The probability density function is

$$f(x) = \begin{cases} \dfrac{81000}{x^4} & x \geq 30 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected number of constant hours the freshly charged battery can be used before it is dead. That is, find $E(X)$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{30}^{\infty} x \cdot \frac{81000}{x^4}\,dx$$

$$= \lim_{c \to \infty} \int_{30}^{c} 81000 x^{-3}\,dx = \lim_{c \to \infty} \frac{-81000}{2} x^{-2} \Big|_{30}^{c}$$

$$= \lim_{c \to \infty} -\frac{40500}{c^2} + \frac{40500}{30^2} = 45$$

Answer

45

[3] **7.** It is estimated that 35% of Canadians prefer to use the self-checkout machines when purchasing groceries. If a group of 400 Canadians are randomly sampled, what is the probability that at most 120 of them prefer to use the self-checkout when purchasing groceries?
Use the normal approximation to the binomial distribution (with the continuity correction factor) to solve this question.

$\mu = np = 400(.35) = 140$

$\sigma^2 = np(1-p) = (400)(.35)(.65) = 91$

$P(X \leq 120) \approx P(X \leq 120.5) = P\left(\frac{X-\mu}{\sigma} \leq \frac{120.5-140}{\sqrt{91}}\right)$

$= P(Z \leq -2.04) = 0.0207$

Answer: 0.0207

[3] **8.** Suppose that the number of times a particular type of LED bulb needs to be replaced follows a Poisson process. It is determined that the bulbs need to be replaced at an average rate of 2 bulbs every 15 years. Suppose one of these LED bulbs has just been replaced. What is the probability that it will be at least 9 years before the bulb needs to be replaced?

exponential $\lambda = 2/15$

$P(X \geq 9) = 1 - (1 - e^{-2/15(9)})$

$= e^{-6/5}$

$= 0.3012$

Answer: 0.3012

(Question 9) The inside diameter of a machined piston ring is normally distributed with a mean of 7.5 cm and a standard deviation of 0.39 cm.

(a) [**2 marks**] If a piston ring from this machining process is randomly selected, what is the probability that it has an inside diameter between 6.7 cm and 8.1 cm?

$$P(6.7 \le X \le 8.1) = P\left(\frac{6.7-7.5}{0.39} \le \frac{X-\mu}{\sigma} \le \frac{8.1-7.5}{0.39}\right)$$

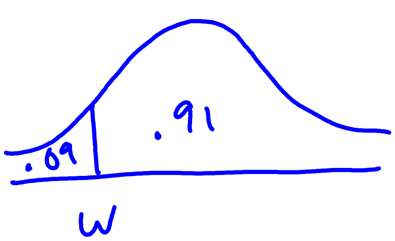
$$= P(-2.05 \le Z \le 1.54)$$

$$= P(Z \le 1.54) - P(Z \le -2.05)$$

$$= 0.9382 - 0.0202$$

$$= 0.9180$$

(b) [**2 marks**] Find the inside diameter w for which 91% of piston rings from this machining process have an inside diameter greater than w.



$$Z = -1.34$$

$$Z = \frac{X-\mu}{\sigma}$$

$$-1.34 = \frac{w-7.5}{0.39}$$

$$w = (-1.34)(.39) + 7.5 = 6.9774$$

Recall: The inside diameter of a machined piston ring is normally distributed with a mean of 7.5 cm and a standard deviation of 0.39 cm.

(c) **[2 marks]** For this machining process, a piston ring will be called defective if it has an inside diameter less than 6.8 cm. Suppose 11 independent piston rings from this machining process are selected at random. What is the probability that exactly two of them are defective?

X = inside diameter of piston ring

$$p = P(\text{success}) = P(X \leq 6.8) = P\left(\frac{X-\mu}{\sigma} \leq \frac{6.8-7.5}{0.39}\right)$$

$$= P(Z \leq -1.79) = 0.0367$$

Y = # of defective piston rings

binomial $n = 11$ $p = 0.0367$

$$P(Y=2) = \binom{11}{2}(.0367)^2(0.9633)^9$$

$$= 0.0529$$

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Section A02 - Instructor: Dr. Michelle Edwards

Instructions:

- This paper has 6 pages of questions plus this cover page. Please count your pages and report any discrepancy immediately to the invigilator.
- Answer all questions on the test paper in the space provided. Questions 1 through 8 are short answer and worth 3 marks each. For each question show your work and write your final answer in the space as indicated. Question 9 is a full-answer question. To obtain full marks on this question, clear step-by-step reasoning must be displayed. Marks will be deducted for incoherent or incomplete solutions.
- Unless an exact answer with fewer decimals can be found, give all answers with at least three decimals of accuracy.
- A Sharp EL-510R series calculator may be used. No other calculator may be used.
- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

This page will not be graded.
Feel free to use this page for scrap work.

[3] **1.** Let the random variable X be the number of classes a current UVic student is registered in this semester. Suppose the cumulative distribution of X is given below:

x	1	2	3	4	5	6
$F(x)$	0.08	0.21	0.34	0.56	0.89	1
$f(x)$	.08	.13	.13	.22	.33	.11

Find the standard deviation of X. That is, find σ_X.

$E(X) = 1(.08) + 2(.13) + 3(.13) + 4(.22) + 5(.33) + 6(.11)$
$= 3.92$

$E(X^2) = 1^2(.08) + 2^2(.13) + 3^2(.13) + 4^2(.22) + 5^2(.33) + 6^2(.11)$
$= 17.5$

$V(X) = E(X^2) - (E(X))^2 = 17.5 - (3.92)^2 = 2.1336$

$\sigma_X = \sqrt{V(X)} = \sqrt{2.1336} = 1.4607$

Answer

1.4607

[3] **2.** In a particular city, it is determined that 40% of homes use electricity as their main heating source (compared to other sources such as oil or natural gas). For a random sample of 19 independent homes in the city, what is the probability that at least 6 but no more than 11 homes use electricity as their main heating source?

binomial $n = 19$ $p = 0.40$

$P(6 \leq X \leq 11) = P(X \leq 11) - P(X \leq 5)$
$= 0.9648 - 0.1629$
$= 0.8217$

Answer

0.8217

[3] **3.** A photocopier has an error that will distribute a blank page in the photocopy job every so often. It is determined that this error occurs at an average rate of 3 blank pages for every 70 pages in the photocopy job.
Suppose a person sends a 350 page print job to the photocopier. If it is known that the photocopier distributes at least 11 blank pages, what is the probability that it distributes exactly 14 blank pages?

Poisson $\lambda = 3(5) = 15$

$$P(X=14 \mid X \geq 11) = \frac{P(X=14 \cap X \geq 11)}{P(X \geq 11)} = \frac{P(X=14)}{1-P(X \leq 10)}$$

$$\frac{\frac{15^{14} e^{-15}}{14!}}{1-0.1185} = \frac{0.1024}{0.8815} = 0.1162$$

Answer
0.1162

[3] **4.** A small company offers moving services for small and quick jobs between 1000 and 4000 pounds. The weight of a customer's move (measured in 1000s of pounds) with this service is a continuous random variable X with pdf

$$f(x) = \begin{cases} \frac{4}{81}(x-1)^3 & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Suppose the company randomly selects their accounting records for a random customer. What is the probability that their move with the service was between 2500 and 3500 pounds? (Note: An amount of 1000 pounds would correspond to a value of $x = 1$.)

$$P(2.5 \leq X \leq 3.5) = \int_{2.5}^{3.5} f(x)\,dx = \int_{2.5}^{3.5} \frac{4}{81}(x-1)^3\,dx$$

$$= \frac{4}{81}\frac{(x-1)^4}{4}\Big|_{2.5}^{3.5} = \frac{(3.5-1)^4}{81} - \frac{(2.5-1)^4}{81}$$

$$= \frac{(2.5)^4}{81} - \frac{(1.5)^4}{81} = \frac{34}{81} = 0.4198$$

Answer
0.4198

[3] **5.** Let X be the continuous random variable with pdf

$$f(x) = \begin{cases} k\sqrt{x} & 4 \leq x \leq 9 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

$$1 = \int_{-\infty}^{\infty} f(x)\,dx = \int_4^9 k\sqrt{x}\,dx = \int_4^9 k x^{1/2}\,dx$$

$$= k\frac{2}{3}x^{3/2}\Big|_4^9 = \frac{2}{3}k\left(9^{3/2} - 4^{3/2}\right)$$

$$= 2/3\, k\,(27 - 8) = 38/3\, k$$

$$1 = 38/3\, k \Rightarrow k = 3/38 = 0.0789$$

Answer

$3/38$

[3] **6.** Let the continuous random variable X denote the amount of constant hours a freshly charged battery can be used before it is dead. The probability density function is

$$f(x) = \begin{cases} \dfrac{3200}{x^3} & x \geq 40 \\ 0 & \text{elsewhere} \end{cases}$$

Find the expected number of constant hours the freshly charged battery can be used before it is dead. That is, find $E(X)$.

$$E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx = \int_{40}^{\infty} x \cdot \frac{3200}{x^3}\,dx$$

$$= \lim_{c\to\infty} \int_{40}^{c} 3200 x^{-2}\,dx = \lim_{c\to\infty} -3200x^{-1}\Big|_{40}^{c}$$

$$= \lim_{c\to\infty} \frac{-3200}{c} + \frac{3200}{40} = 80$$

Answer

80

[3] **7.** It is estimated that 40% of Canadians prefer to use the self-checkout machines when purchasing groceries. If a group of 350 Canadians are randomly sampled, what is the probability that at most 130 of them prefer to use the self-checkout when purchasing groceries?
Use the normal approximation to the binomial distribution (with the continuity correction factor) to solve this question.

$\mu = np = 350(.40) = 140$

$\sigma = \sqrt{np(1-p)} = \sqrt{350(.40)(.60)} = \sqrt{84}$

$\underset{\text{binomial}}{P(X \leq 130)} \approx \underset{\text{normal}}{P(X \leq 130.5)} = P\left(\frac{X-\mu}{\sigma} \leq \frac{130.5-140}{\sqrt{84}}\right)$

$= P(Z \leq -1.04) = 0.1492$

Answer
0.1492

[3] **8.** Suppose that the number of times a particular type of LED bulb needs to be replaced follows a Poisson process. It is determined that the bulbs need to be replaced at an average rate of 3 bulbs every 16 years. Suppose one of these LED bulbs has just been replaced. What is the probability that it will be at least 10 years before the bulb needs to be replaced?

$\lambda = 3/16$

exponential

$P(X \geq 10) = 1 - P(X \leq 10) = 1 - (1 - e^{-3/16(10)})$

$= e^{-30/16} = e^{-15/8} = 0.1534$

Answer
0.1534

(Question 9) The inside diameter of a machined piston ring is normally distributed with a mean of 8.5 cm and a standard deviation of 0.46 cm.

(a) **[2 marks]** If a piston ring from this machining process is randomly selected, what is the probability that it has an inside diameter between 7.7 cm and 9.4 cm?

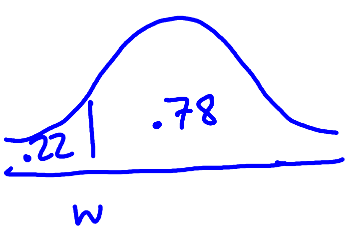
$$P(7.7 \leq X \leq 9.4) = P\left(\frac{7.7-8.5}{0.46} \leq \frac{X-\mu}{\sigma} \leq \frac{9.4-8.5}{0.46}\right)$$

$$= P(-1.74 \leq Z \leq 1.96) = P(Z \leq 1.96) - P(Z \leq -1.74)$$

$$= 0.9750 - 0.0409$$

$$= 0.9341$$

(b) **[2 marks]** Find the inside diameter w for which 78% of piston rings from this machining process have an inside diameter greater than w.



$$Z = -0.77$$

$$Z = \frac{X-\mu}{\sigma}$$

$$-0.77 = \frac{w-8.5}{0.46}$$

$$w = (-0.77)(0.46) + 8.5$$

$$w = 8.1458$$

Recall: The inside diameter of a machined piston ring is normally distributed with a mean of 8.5 cm and a standard deviation of 0.46 cm.

(c) **[2 marks]** For this machining process, a piston ring will be called defective if it has an inside diameter less than 7.6 cm. Suppose 13 independent piston rings from this machining process are selected at random. What is the probability that exactly one of them are defective?

X = inside diameter of piston ring

$$p = P(\text{success}) = P(X \leq 7.6) = P\left(\frac{X-\mu}{\sigma} \leq \frac{7.6-8.5}{0.46}\right)$$

$$= P(Z \leq -1.96) = 0.0250$$

Y = # of defective piston rings

binomial $n=13$ $p=0.0250$

$$P(Y=1) = \binom{13}{1}(.0250)^1(.9750)^{12}$$

$$= 0.2398$$