ECON 180 FALL 2022: PROJECT 3

DUE **NOVEMBER 8, 2022** by 11:59 PM VICTORIA, B.C. TIME
(Note the extended due date compared to the syllabus)[1]

Honor Code: I guarantee that this submission is **entirely my own work**. I have **cited any outside sources** in APA or IEEE style. **(You must accept this code to receive a mark.)**

Name or Signature for Honor Code: Arfaz Hossain

Last 3 digits of student number: 826

Please enter your answers in the spaces and tables provided. Your submission must be in either PDF or Microsoft 365 (Word, etc.) format, so Brightspace can read it properly.

Question		Marks
1	a	75
	b	75
	Q1 (Average)	75
2	a	75
	b	75
	c	75
	d	75
	Q2 (Average)	75
Q1 to Q2	(Q1+Q2)/2	75
3	a	0 (not a typo)
	b	3
	c	3
	d	3
	e	3
	f	3
	Q3 (Total)	15
Subtotal	(Q1 to Q2) + Q3	90
Communication		10
Total		100

[1] I extended the due date to accommodate students with an overwhelming amount of work in October.

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Question 1: EAC (Lectures 12 and 13)

This question is based[2] on a published paper:

Al-Chalabi, H., Lundberg, J., Alireza, A. & Jonsson, A. (2015). Case Study: Model for Economic Lifetime of Drilling Machines in the Swedish Mining Industry. *The Engineering Economist, 60*(2), 138-154. https://doi-org.ezproxy.library.uvic.ca/10.1080/0013791X.2014.952466

During the co-op part of their engineering degree, Mandeep interned for Boliden, a Swedish mining company. As one of their first tasks, the company asked them to determine how often they should replace their drilling machines.[3] Currently, they are replacing them every 120 months, but recently it's been suggested that they could save money by choosing a slightly different replacement period.

The situation is as follows:

- The machines cost $6,000 in month 0.[4]
- Once the company is done with the machines, it sells them.
- Immediately (less than one second) after purchase, the machine's resale value falls to 90% of its purchase price. This is the 'driving the car off the lot' depreciation familiar to car owners.
- After this immediate drop in value, the machines lose d% of their resale value each month. If the resale value is $V in month N, it's (1 – d%)xV in month N+1.
- Mandeep's company contact isn't sure what the value of d is, but they know that the machine's resale value is $50 in month 120.
- The machine is subject to ongoing operating and maintenance costs.
- Operating costs grow by 4.043% per month.
- Maintenance costs grow by 3.895% per month.
- Mandeep's company contact only has operating & maintenance values for month 75 on hand. In month 75, operating costs for the month are $9.89, and maintenance costs for the month are $24.20.
- The mining company's MARR is 10% per **year**[5].

Help Mandeep out by doing the following:

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[2] I have simplified the case study slightly for this project. In particular, I've approximated their Lorentzian cost equations by geometric sequences calibrated to their estimates for months 75 – 120, to make it easier to use the DCFA taught in ECON 180.

[3] A former ECON 180 student once told me that they were assigned a very similar task during an internship at a world-famous mining company.

[4] If the dollar values in this question seem low, it's because the 'currency units' cited in the paper are an encryption of the actual cost, done to avoid revealing their exact costs to the public (and competitors). The drill in question is an Atlas Copco rock drill.

[5] The company in the original paper is Boliden, and they still use a 10% MARR: https://www.boliden.com/investor-relations/financials/financial-targets

a. Write the EAC equation using appropriate notation

Write the EAC(N) equation for the drilling machine using DCFA notation (e.g. (P/F,6%,N)). You will probably want to use (P/A,g,i,N) and (A/P,i,N).

Initial Cost: $6000
MARR = 10% / Year = 0.797414% / Month
Present Value of Operating Cost: Given,

Operating Cost Growth Rate	:	4.043%
Operating Cost (Month 75)	:	$9.89
Operating Cost (Month 1)	:	$Ax(1+4.043\%)^{74}$ = $9.89
		or $A = \$9.89 / (1+4.043\%)^{74}$
		or A = $0.52655

Operating Cost (to Month 0 from Month N) --> **$0.52655x(P/A,4.043%,0.7974%,N)**

Present Value of Maintenance Cost: Given,

Maintenance Cost Growth Rate	:	3.895%
Maintenance Cost (Month 75)	:	$24.20
Maintenance Cost (Month 1)	:	$Ax(1+3.895\%)^{74}$ = $24.20
		or $A = \$24.20 / (1+3.895\%)^{74}$
		or A = $1.4315667

Maintenance Cost (to Month 0 from Month N) --> **$1.4315667x(P/A,3.895%,0.7974%,N)**

Present Value of Salvage Cost: Given,

Salvage Value (Month 0)	:	(90%x$6000) = $5400
Salvage Value (Month 120)	:	$50
Salvage Rate	:	$\$5400x(1-S)^{120}$ = $50
		or $S = 1-(\$50/\$5400)^{1/120}$
		or S = $0.03826
		or S = 3.827% / Month

Since Salvage Value is not adding to the cost of the EAC (N), it's a negative value.

Salvage Value (Month N)	:	$-\$5400x(1-3.827\%)^{120}$

Salvage Value (to Month 0 from Month N)--> **$-\$5400x(1-3.827\%)^{120}x(P/F,0.797\%,N)$**

Total Cost (From Month N to Month 0) = $6000 + [$0.52655x(P/A,4.043%,0.7974%,N)] + [$1.4315667x(P/A,3.895%,0.7974%,N)] – [$\$5400x(1-3.827\%)^{120}x(P/F,0.797\%,N)$]

EAC (N) = [$6000 + {$0.52655x(P/A,4.043%,0.7974%,N)} + {$1.4315667x(P/A,3.895%,0.7974%,N)} – {$\$5400x(1-3.827\%)^{N}x(P/F,0.797\%,N)$}] x (A/P,0.797%,N)

b. Calculate the economic lifetime

Use your answer for part a. to calculate the economic lifetime of the drilling machine to the nearest month. Back up your answer (explain your reasoning) with one or more of a graph[6], a table or a step-by-step analytical/numerical solution. **(I recommend a graph and a table if you're comfortable with Excel.)**

Economic lifetime: 101 months

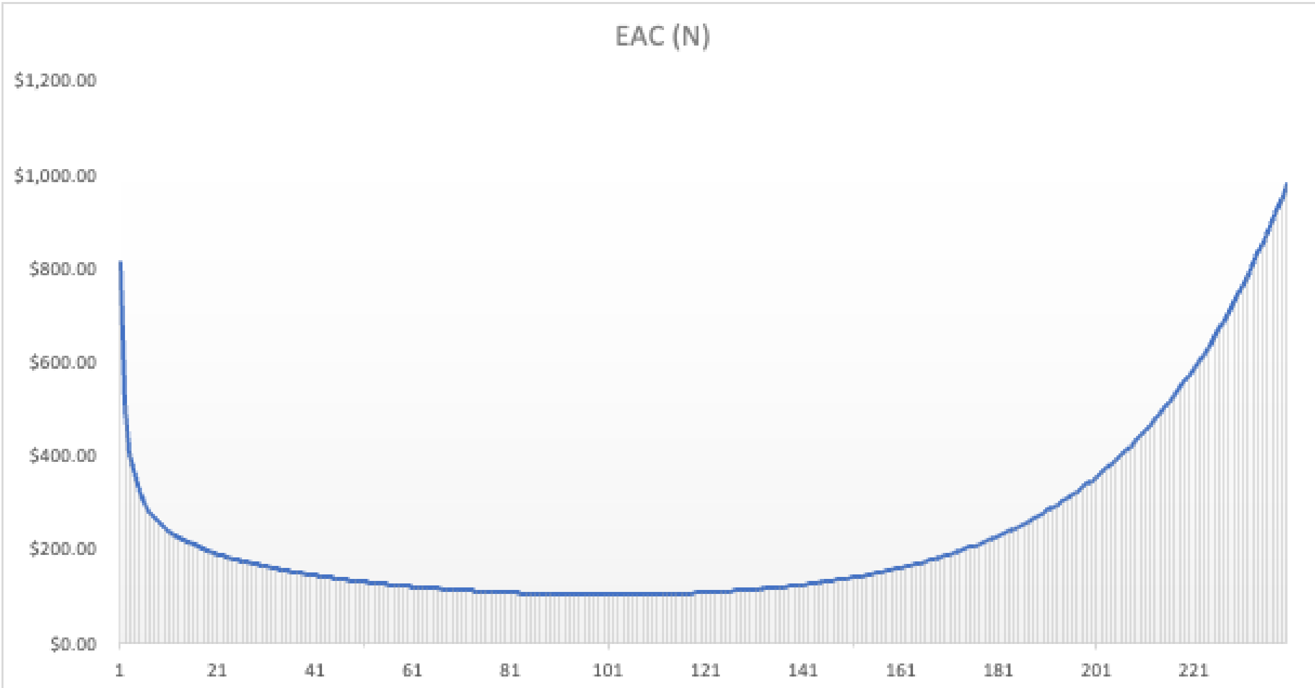
Work backing up your answer:

At exactly 101 months, the price for the cost analysis is the lowest in a time period of 240 months. After that, the capital cost for maintaining and operating keeps increasing. So, it doesn't seem like a viable option to keep the machine, rather to buy a new one at this point. Therefore, the economic lifetime would be 101 months for this machine.

84	$216.19909	$541.2600	$203.63832	$6,553.82080	$107.35
85	$223.68290	$559.3136	$195.84508	$6,587.15143	$107.00
86	$231.40767	$577.9220	$188.35009	$6,620.97960	$106.68
87	$239.38117	$597.1023	$181.14193	$6,655.34151	$106.37
88	$247.61141	$616.8719	$174.20963	$6,690.27373	$106.09
89	$256.10666	$637.2492	$167.54262	$6,725.81320	$105.82
90	$264.87545	$658.2526	$161.13077	$6,761.99727	$105.58
91	$273.92659	$679.9015	$154.96429	$6,798.86375	$105.36
92	$283.26916	$702.2156	$149.03381	$6,836.45097	$105.15
93	$292.91256	$725.2155	$143.33029	$6,874.79779	$104.97
94	$302.86646	$748.9222	$137.84504	$6,913.94365	$104.81
95	$313.14087	$773.3574	$132.56971	$6,953.92861	$104.67
96	$323.74611	$798.5436	$127.49626	$6,994.79344	$104.55
97	$334.69283	$824.5037	$122.61698	$7,036.57957	$104.45
98	$345.99202	$851.2616	$117.92443	$7,079.32922	$104.38
99	$357.65504	$878.8418	$113.41146	$7,123.08540	$104.32
100	$369.69359	$907.2696	$109.07121	$7,167.89197	$104.29
101	$382.11978	$936.5710	$104.89705	$7,213.79368	$104.27
102	$394.94608	$966.7728	$100.88264	$7,260.83622	$104.28
103	$408.18537	$997.9027	$97.02186	$7,309.06624	$104.32
104	$421.85095	$1,029.9893	$93.30884	$7,358.53145	$104.37
105	$435.95656	$1,063.0620	$89.73791	$7,409.28063	$104.45
106	$450.51635	$1,097.1510	$86.30364	$7,461.36369	$104.55
107	$465.54496	$1,132.2875	$83.00080	$7,514.83171	$104.67
108	$481.05747	$1,168.5039	$79.82436	$7,569.73702	$104.81
109	$497.06947	$1,205.8332	$76.76948	$7,626.13321	$104.98
110	$513.59705	$1,244.3097	$73.83151	$7,684.07522	$105.18
111	$530.65679	$1,283.9686	$71.00598	$7,743.61938	$105.39
112	$548.26585	$1,324.8462	$68.28858	$7,804.82347	$105.64
113	$566.44190	$1,366.9800	$65.67518	$7,867.74676	$105.90
114	$585.20321	$1,410.4087	$63.16179	$7,932.45010	$106.20
115	$604.56861	$1,455.1719	$60.74458	$7,998.99594	$106.51
116	$624.55756	$1,501.3108	$58.41989	$8,067.44844	$106.86
117	$645.19013	$1,548.8675	$56.18416	$8,137.87347	$107.23
118	$666.48706	$1,597.8857	$54.03399	$8,210.33876	$107.63
119	$688.46972	$1,648.4103	$51.96611	$8,284.91387	$108.06
120	$711.16021	$1,700.4875	$49.97737	$8,361.67032	$108.51
121	$734.58132	$1,754.1651	$48.06473	$8,440.68166	$109.00
122	$758.75656	$1,809.4922	$46.22530	$8,522.02349	$109.51
123	$783.71022	$1,866.5196	$44.45626	$8,605.77359	$110.06
124	$809.46737	$1,925.2995	$42.75491	$8,692.01198	$110.63

A figure of the graph is on next page.

[6] If you are submitting a graph made in Excel, do NOT just copy-paste it from Excel into Word, as this can cause issues with file dependencies, displaying in Brightspace, etc. After copying the graph in Excel, go to Word and choose Edit → Paste Special. From there, pick a standard image format (jpg, gif, png, pdf, etc.).


EAC (N)
$1,200.00
$1,000.00
$800.00
$600.00
$400.00
$200.00
$0.00
1
21
41
61
81
101
121
141
161
181
201
221

Question 2: Incremental IRR & ERR (Lectures 14, 15, 16)

Data from the 2016 census shows that the salary benefits from graduate degrees in Engineering are surprisingly low. Statistics Canada found[7] that a master's degree in Engineering increased income by 0.7% (for men) and 0.4% (for women), relative to a Bachelor's degree. A doctorate in Engineering increased income by 7.8% (for men) and 1.8% (for women) relative to a bachelor's degree. In this question, you will determine whether it makes sense for Mandeep to spend two years to earn a master's degree, or six years to earn a PhD, given that the average engineering tuition cost in Canada is $7,454 per year as of 2022.

The Setup

Note: The companion spreadsheet **automates this setup for you**, in the sense that it takes your information and returns a table with the appropriate cash flows. The information below will be useful if you wish to use equations as part of your solution.

For this question, the setup is a bit different to how we thought of income in Project 2. As in previous projects, Mandeep mar is 5.45% per year, and they are going to spend three years finishing their bachelor's degree in engineering, no matter what. However, they also have the option to spend $7,454 a year[8] for two years to earn a Master's degree, or $7,454 a year for six years to earn a Master's degree, followed by a PhD.

Once Mandeep finishes their studies, they will start working as an engineer. If Mandeep completes a bachelor's degree, their starting salary is equal to A, where A is your *highest* baseline salary from Project 1. This time, there is no signing bonus, and Mandeep's salary increases each year at a rate equivalent to 12% every 15 months[9] (about 9.5% per year)[10], which according to Salary Explorer is the average increase in salary for engineers in Canada. Mandeep will work for a total of 40 years if earning a bachelor's degree (from Year 3 to Year 42). For simplicity, we will assume Mandeep is paid their salary once a year.

If Mandeep goes for a master's degree, they will pay $7,454 a year for the two years following the completion of their Bachelor's degree. After that, they will receive income equal to (1+m)

[7] See Table 4 in Wall, K., Zhao, J., Ferguson, S. & Rodriguez, C. (2018). Results from the 2016 Census: Is field of study a factor in the payoff of a graduate degree? [Web Page]. https://www150.statcan.gc.ca/n1/pub/75-006-x/2018001/article/54978-eng.htm

[8] Source: https://www150.statcan.gc.ca/n1/pub/71-607-x/71-607-x2019011-eng.htm

[9] Source: http://www.salaryexplorer.com/charts/canada/engineering/annual-salary-increment-rate-canada-engineering.jpg I can't find Salary Explorer's data source, but scanning posts by Canadian engineers on Reddit suggests that in non-pandemic years, raises of 5% to 7% per year are not unusual.

[10] This conversion is done for you in the companion spreadsheet.

times what they would have earned, the *same year*[11], if they had only had a bachelor's degree, where m is the bonus for having a master's degree (0.7% for men, 0.4% for women). Mandeep will only work for 38 years if they earn a master's degree (from Year 5 to Year 42). This means Mandeep's starting salary in this case is equal to (1+m) times the Year 5 salary they would have earned with a bachelor's degree: A x $(1+8.51\%)^2$ x (1+m).

If Mandeep goes for a doctorate, they will pay $7,454 a year for the six years following the completion of their bachelor's degree (2 years for a Master's, then 4 for a PhD). After that, they will receive income equal to (1+p) times what they would have earned, *the same year*, if they had a bachelor's degree, where p is the bonus for having a doctorate. Mandeep will only work for 34 years if they earn a doctorate (from Year 9 to Year 42).

a. Incremental IRR when perceived as male (Lecture 14)

Mandeep is non-binary. Suppose they receive the same salary benefits from graduate degrees as male Canadian engineers (0.7% higher income for a Master's degree, 7.8% higher income from a doctorate, relative to a bachelor's degree).

Use an ***incremental* IRR approach** to determine Mandeep's preferred project: stop at a bachelor's degree, stop at a master's degree, or earn a PhD. Show your work.

Note: You may use all numerical/spreadsheet methods, all analytical methods (though the problem may not be tractable), or a mix of numerical and analytical methods. In any case, you need to explain your process, as you are not being marked on your final answer, which may vary by student, but on *how* you obtained your final answer.

Hint: The companion spreadsheet will automatically create a table of the relevant cash flows for you. Even if you are taking a mostly analytical approach, you may find this useful for visualization.

Preferred Project: Bachelors

Work from excel is shown on next page.

Even though the Incremental IRR between PhD and bachelor's is almost close to MARR, it isn't greater than MARR. Since the IIRR of both PhD and master's are < (smaller) than bachelors, it doesn't seem like a good investment to go for Masters and PhD after Bachelors.

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[11] This assumption probably over-states the salary benefits from a degree. In this question, I'm erring on the side of making the salary benefits from a degree more attractive.

A	$100,000	INPUT YOUR HIGHEST BASELINE SALARY HERE			
g	9.5%	per year (12% every 15 months)			
M vs B	0.70%	Difference in earnings: Master's vs Bachelor's (Male)			
P vs B	7.80%	Difference in earnings: PhD vs Bachelor's (Male)			
Tuition	$7,454	per year			
MARR	5.45%	per year			

Cash Flows by Project			
Year	Bachelor's	Master's	PhD
0	0	0	0
1	0	0	0
2	0	0	0
3	$100,000	-$7,454	-$7,454
4	$109,489.99	-$7,454	-$7,454
5	$119,880.58	$120,719.74	-$7,454
6	$131,257.24	$132,176.04	-$7,454
7	$143,713.54	$144,719.53	-$7,454
8	$157,351.94	$158,453.40	-$7,454
9	$172,284.62	$173,490.61	$185,722.82
10	$188,634.41	$189,954.85	$203,347.90
11	$206,535.80	$207,981.55	$222,645.59
12	$226,136.03	$227,718.98	$243,774.64
13	$247,596.32	$249,329.49	$266,908.83
14	$271,093.18	$272,990.84	$292,238.45
15	$296,819.90	$298,897.64	$319,971.85
16	$324,988.08	$327,263.00	$350,337.15
17	$355,829.42	$358,320.23	$383,584.12
18	$389,597.60	$392,324.78	$419,986.21
19	$426,570.37	$429,556.37	$459,842.86
20	$467,051.86	$470,321.23	$503,481.91
21	$511,375.04	$514,954.67	$551,262.29
22	$559,904.48	$563,823.81	$603,577.03
23	$613,039.37	$617,330.64	$660,856.44
24	$671,216.74	$675,915.26	$723,571.65
25	$734,915.15	$740,059.55	$792,238.53
26	$804,658.53	$810,291.14	$867,421.89
27	$881,020.54	$887,187.69	$949,740.15
28	$964,629.31	$971,381.71	$1,039,870.40
29	$1,056,172.54	$1,063,565.75	$1,138,554.00
30	$1,156,403.21	$1,164,498.04	$1,246,602.66
31	$1,266,145.77	$1,275,008.79	$1,364,905.14
32	$1,386,302.88	$1,396,007.00	$1,494,434.51
33	$1,517,862.89	$1,528,487.93	$1,636,256.20
34	$1,661,907.94	$1,673,541.29	$1,791,536.76
35	$1,819,622.84	$1,832,360.20	$1,961,553.43
36	$1,992,304.88	$2,006,251.01	$2,147,704.66
37	$2,181,374.42	$2,196,644.04	$2,351,521.63
38	$2,388,386.65	$2,405,105.36	$2,574,680.81
39	$2,615,044.32	$2,633,349.63	$2,819,017.77
40	$2,863,211.77	$2,883,254.25	$3,086,542.29
41	$3,134,930.30	$3,156,874.81	$3,379,454.86
42	$3,432,434.89	$3,456,461.93	$3,700,164.81
IRR		320.587%	75.764%

	Masters - Bachelors	PhD - Bachelors
	-$107,454	-$107,454
	-$116,944	-$116,944
	$839	-$127,335
	$919	-$138,711
	$1,006	-$151,168
	$1,101	-$164,806
	$1,206	$13,438
	$1,320	$14,713
	$1,446	$16,110
	$1,583	$17,639
	$1,733	$19,313
	$1,898	$21,145
	$2,078	$23,152
	$2,275	$25,349
	$2,491	$27,755
	$2,727	$30,389
	$2,986	$33,272
	$3,269	$36,430
	$3,580	$39,887
	$3,919	$43,673
	$4,291	$47,817
	$4,699	$52,355
	$5,144	$57,323
	$5,633	$62,763
	$6,167	$68,720
	$6,752	$75,241
	$7,393	$82,381
	$8,095	$90,199
	$8,863	$98,759
	$9,704	$108,132
	$10,625	$118,393
	$11,633	$129,629
	$12,737	$141,931
	$13,946	$155,400
	$15,270	$170,147
	$16,719	$186,294
	$18,305	$203,973
	$20,042	$223,331
	$21,945	$244,525
	$24,027	$267,730
IIRR	0.62%	5.19%

b. Incremental IRR when perceived as female (Lecture 14)

Mandeep is non-binary. Suppose they receive the same salary benefits from graduate degrees as female Canadian engineers (0.4% higher income for a Master's degree, 1.8% higher income from a doctorate, relative to a bachelor's degree).

Use an ***incremental* IRR approach** to determine Mandeep's preferred project: stop at a bachelor's degree, stop at a Master's degree, or earn a PhD. Show your work.

Note: You may use all numerical/spreadsheet methods, all analytical methods (though the problem may not be tractable), or a mix of numerical and analytical methods. In any case, you need to explain your process, as you are not being marked on your final answer, which may vary by student, but on *how* you obtained your final answer.

Hint: The companion spreadsheet will automatically create a table of the relevant cash flows for you. Even if you are taking a mostly analytical approach, you may find this useful for visualization.

Preferred Project: Bachelors

Work from excel is shown on next page.

Even though the Incremental IRR between PhD and bachelor's is almost close to MARR, it isn't greater than MARR. Since the IIRR of both PhD and masters are < (smaller) than bachelors, it doesn't seem like a good investment to go for master's and PhD after Bachelors.

A	$100,000	Already linked to the salary you input in part a.
g	9.49%	per year (12% every 15 months)
M vs B	0.40%	Difference in earnings: Master's vs Bachelor's (Female)
P vs B	1.80%	Difference in earnings: PhD vs Bachelor's (Female)
Tuition	$7,454	per year
MARR	5.45%	per year

Cash Flows by Project			
Year	Bachelor's	Master's	PhD
0	0	0	0
1	0	0	0
2	0	0	0
3	$100,000	-$7,454	-$7,454
4	$109,489.99	-$7,454	-$7,454
5	$119,880.58	$120,360.10	-$7,454
6	$131,257.24	$131,782.26	-$7,454
7	$143,713.54	$144,288.39	-$7,454
8	$157,351.94	$157,981.34	-$7,454
9	$172,284.62	$172,973.76	$175,385.74
10	$188,634.41	$189,388.95	$192,029.83
11	$206,535.80	$207,361.95	$210,253.45
12	$226,136.03	$227,040.57	$230,206.48
13	$247,596.32	$248,586.70	$252,053.05
14	$271,093.18	$272,177.56	$275,972.86
15	$296,819.90	$298,007.18	$302,162.66
16	$324,988.08	$326,288.04	$330,837.87
17	$355,829.42	$357,252.74	$362,234.35
18	$389,597.60	$391,155.99	$396,610.36
19	$426,570.37	$428,276.66	$434,248.64
20	$467,051.86	$468,920.07	$475,458.80
21	$511,375.04	$513,420.54	$520,579.79
22	$559,904.48	$562,144.10	$569,982.76
23	$613,039.37	$615,491.52	$624,074.07
24	$671,216.74	$673,901.61	$683,298.64
25	$734,915.15	$737,854.81	$748,143.62
26	$804,658.53	$807,877.16	$819,142.38
27	$881,020.54	$884,544.63	$896,878.91
28	$964,629.31	$968,487.83	$981,992.64
29	$1,056,172.54	$1,060,397.23	$1,075,183.64
30	$1,156,403.21	$1,161,028.83	$1,177,218.47
31	$1,266,145.77	$1,271,210.35	$1,288,936.39
32	$1,386,302.88	$1,391,848.09	$1,411,256.33
33	$1,517,862.89	$1,523,934.35	$1,545,184.43
34	$1,661,907.94	$1,668,555.57	$1,691,822.28
35	$1,819,622.84	$1,826,901.34	$1,852,376.05
36	$1,992,304.88	$2,000,274.10	$2,028,166.37
37	$2,181,374.42	$2,190,099.92	$2,220,639.16
38	$2,388,386.65	$2,397,940.20	$2,431,377.61
39	$2,615,044.32	$2,625,504.49	$2,662,115.11
40	$2,863,211.77	$2,874,664.62	$2,914,749.58
41	$3,134,930.30	$3,147,470.02	$3,191,359.04
42	$3,432,434.89	$3,446,164.63	$3,494,218.71

	Masters - Bachelors	PhD - Bachelors
	-$107,454	-$107,454
	-$116,944	-$116,944
	$480	-$127,335
	$525	-$138,711
	$575	-$151,168
	$629	-$164,806
	$689	$3,101
	$755	$3,395
	$826	$3,718
	$905	$4,070
	$990	$4,457
	$1,084	$4,880
	$1,187	$5,343
	$1,300	$5,850
	$1,423	$6,405
	$1,558	$7,013
	$1,706	$7,678
	$1,868	$8,407
	$2,046	$9,205
	$2,240	$10,078
	$2,452	$11,035
	$2,685	$12,082
	$2,940	$13,228
	$3,219	$14,484
	$3,524	$15,858
	$3,859	$17,363
	$4,225	$19,011
	$4,626	$20,815
	$5,065	$22,791
	$5,545	$24,953
	$6,071	$27,322
	$6,648	$29,914
	$7,278	$32,753
	$7,969	$35,861
	$8,725	$39,265
	$9,554	$42,991
	$10,460	$47,071
	$11,453	$51,538
	$12,540	$56,429
	$13,730	$61,784
IIRR	-$0.0128	-$0.0062

c. Incremental Approximate ERR (Lectures 14, 15)

As in part b., assume that Mandeep receives the same salary benefits from a master's degree as Canadian female engineers. This time, if Mandeep goes for a master's degree, they will also receive a $2,500 LOUD Scholarship in Year 2. LOUD Scholarships are given to Canadian LGBTQ+ post-secondary students[12].

As a result of this scholarship, the *incremental* cash flow between the 'Bachelor's Degree' and 'Master's Degree' project now has two sign changes. Use an **incremental approximate ERR method** to determine whether Mandeep should stick to a bachelor's degree or go for a Master's degree.

This is identical to the incremental IRR method (Lecture 14), except that you're solving for the approximate ERR (Lecture 15) of the incremental flows, instead of the IRR.

Hint: Again, the companion spreadsheet will set up the cash flows for each project for you (but not the incremental flows). The incremental approximate ERR approach is the same as the incremental IRR approach, except you use an approximate ERR instead of an IRR.

Preferred Project: Masters

Work from excel is shown on next page.

The ERR shows that with the new addition of scholarships, the ERR is much higher than the MARR. Therefore, taking the masters route seems to be a worth decision.

[12] "LOUD stands for Leadership, Opportunity, Unity & Diversity." See https://loudbusiness.com/apply-for-a-scholarship

Year	Until	Masters	Net OutFlow	Net InFlow	PV of OutFlow	FV of Inflow	
0	42	$0.00	$0.00	$0.00	$0.00	$0.00	
1	41	$0.00	$0.00	$0.00	$0.00	$0.00	
2	40	$2,500.00	$0.00	$2,500.00	$0.00	$20,883.50	
3	39	-$7,454.00	$7,454.00	$0.00	$6,356.96	$0.00	
4	38	-$7,454.00	$7,454.00	$0.00	$6,028.41	$0.00	
5	37	$120,360.10	$0.00	$120,360.10	$0.00	$857,444.78	
6	36	$131,782.26	$0.00	$131,782.26	$0.00	$890,295.12	
7	35	$144,288.39	$0.00	$144,288.39	$0.00	$924,404.03	
8	34	$157,981.34	$0.00	$157,981.34	$0.00	$959,819.71	
9	33	$172,973.76	$0.00	$172,973.76	$0.00	$996,592.23	
10	32	$189,388.95	$0.00	$189,388.95	$0.00	$1,034,773.58	
11	31	$207,361.95	$0.00	$207,361.95	$0.00	$1,074,417.72	
12	30	$227,040.57	$0.00	$227,040.57	$0.00	$1,115,580.72	
13	29	$248,586.70	$0.00	$248,586.70	$0.00	$1,158,320.74	
14	28	$272,177.56	$0.00	$272,177.56	$0.00	$1,202,698.22	
15	27	$298,007.18	$0.00	$298,007.18	$0.00	$1,248,775.88	
16	26	$326,288.04	$0.00	$326,288.04	$0.00	$1,296,618.86	
17	25	$357,252.74	$0.00	$357,252.74	$0.00	$1,346,294.80	
18	24	$391,155.99	$0.00	$391,155.99	$0.00	$1,397,873.92	
19	23	$428,276.66	$0.00	$428,276.66	$0.00	$1,451,429.14	
20	22	$468,920.07	$0.00	$468,920.07	$0.00	$1,507,036.15	
21	21	$513,420.54	$0.00	$513,420.54	$0.00	$1,564,773.58	
22	20	$562,144.10	$0.00	$562,144.10	$0.00	$1,624,723.04	
23	19	$615,491.52	$0.00	$615,491.52	$0.00	$1,686,969.28	
24	18	$673,901.61	$0.00	$673,901.61	$0.00	$1,751,600.28	
25	17	$737,854.81	$0.00	$737,854.81	$0.00	$1,818,707.43	
26	16	$807,877.16	$0.00	$807,877.16	$0.00	$1,888,385.58	
27	15	$884,544.63	$0.00	$884,544.63	$0.00	$1,960,733.23	
28	14	$968,487.83	$0.00	$968,487.83	$0.00	$2,035,852.66	
29	13	$1,060,397.23	$0.00	$1,060,397.23	$0.00	$2,113,850.05	
30	12	$1,161,028.83	$0.00	$1,161,028.83	$0.00	$2,194,835.68	
31	11	$1,271,210.35	$0.00	$1,271,210.35	$0.00	$2,278,924.02	
32	10	$1,391,848.09	$0.00	$1,391,848.09	$0.00	$2,366,233.94	
33	9	$1,523,934.35	$0.00	$1,523,934.35	$0.00	$2,456,888.87	
34	8	$1,668,555.57	$0.00	$1,668,555.57	$0.00	$2,551,016.97	
35	7	$1,826,901.34	$0.00	$1,826,901.34	$0.00	$2,648,751.29	
36	6	$2,000,274.10	$0.00	$2,000,274.10	$0.00	$2,750,230.00	
37	5	$2,190,099.92	$0.00	$2,190,099.92	$0.00	$2,855,596.56	
38	4	$2,397,940.20	$0.00	$2,397,940.20	$0.00	$2,964,999.91	
39	3	$2,625,504.49	$0.00	$2,625,504.49	$0.00	$3,078,594.70	
40	2	$2,874,664.62	$0.00	$2,874,664.62	$0.00	$3,196,541.54	
41	1	$3,147,470.02	$0.00	$3,147,470.02	$0.00	$3,319,007.14	
42	0	$3,446,164.63	$0.00	$3,446,164.63	$0.00	$3,446,164.63	
					$12,385.38	$71,036,639.45	$7,647,612.54
					PVoutFlowTotal	FVinFlowTotal	PVinFlowTotal
		PVinFlowTotal is	$12,385.38	for PVoutFlowTotal at the rate of		22.88244246%	ERR

d. A calibration exercise (Lecture 16)

Maintain the assumptions and approach (incremental approximate ERR) from 1.c. What is the minimum size of a scholarship in Year 2 that would convince Mandeep to go for a master's degree instead of stopping with a bachelor's degree? Show your work (or explain what you did, clearly, if you used numerical methods such as Excel's GoalSeek).

Note: In some cases, this value could be negative, if Mandeep would choose to go for a master's degree even without a scholarship.

Minimum scholarship in Year 2: -$8,487,645.46, or 0

Work from excel is shown on next page. With GoalSeek, I looked for the amount adjusted in the present value for which the present value of the income is the same as the present value of all the money earned throughout all the years. The minimum amount found is large, which, adjusted for the present price, just means that if Mandeep incurs a loss of this amount, then choosing masters wouldn't be such a good idea. For this, I also gave 0 as an alternate answer as if there is no scholarship, then with the data gathered from 2B, 2C and 2D, we can look at Incremental IRR, Incremental Cash Flows throughout the years to decide that going for Masters wouldn't be a good idea.

Year	Until	Masters	Net OutFlow	Net InFlow	PV of OutFlow	FV of Inflow	
0	42	$0.00	$0.00	$0.00	$0.00	$0.00	
1	41	$0.00	$0.00	$0.00	$0.00	$0.00	
2	40	-$8,487,645.46	$8,487,645.46	$0.00	$7,632,978.90	$0.00	
3	39	-$7,454.00	$7,454.00	$0.00	$6,356.96	$0.00	
4	38	-$7,454.00	$7,454.00	$0.00	$6,028.41	$0.00	
5	37	$120,360.10	$0.00	$120,360.10	$0.00	$857,444.78	
6	36	$131,782.26	$0.00	$131,782.26	$0.00	$890,295.12	
7	35	$144,288.39	$0.00	$144,288.39	$0.00	$924,404.03	
8	34	$157,981.34	$0.00	$157,981.34	$0.00	$959,819.71	
9	33	$172,973.76	$0.00	$172,973.76	$0.00	$996,592.23	
10	32	$189,388.95	$0.00	$189,388.95	$0.00	$1,034,773.58	
11	31	$207,361.95	$0.00	$207,361.95	$0.00	$1,074,417.72	
12	30	$227,040.57	$0.00	$227,040.57	$0.00	$1,115,580.72	
13	29	$248,586.70	$0.00	$248,586.70	$0.00	$1,158,320.74	
14	28	$272,177.56	$0.00	$272,177.56	$0.00	$1,202,698.22	
15	27	$298,007.18	$0.00	$298,007.18	$0.00	$1,248,775.88	
16	26	$326,288.04	$0.00	$326,288.04	$0.00	$1,296,618.86	
17	25	$357,252.74	$0.00	$357,252.74	$0.00	$1,346,294.80	
18	24	$391,155.99	$0.00	$391,155.99	$0.00	$1,397,873.92	
19	23	$428,276.66	$0.00	$428,276.66	$0.00	$1,451,429.14	
20	22	$468,920.07	$0.00	$468,920.07	$0.00	$1,507,036.15	
21	21	$513,420.54	$0.00	$513,420.54	$0.00	$1,564,773.58	
22	20	$562,144.10	$0.00	$562,144.10	$0.00	$1,624,723.04	
23	19	$615,491.52	$0.00	$615,491.52	$0.00	$1,686,969.28	
24	18	$673,901.61	$0.00	$673,901.61	$0.00	$1,751,600.28	
25	17	$737,854.81	$0.00	$737,854.81	$0.00	$1,818,707.43	
26	16	$807,877.16	$0.00	$807,877.16	$0.00	$1,888,385.58	
27	15	$884,544.63	$0.00	$884,544.63	$0.00	$1,960,733.23	
28	14	$968,487.83	$0.00	$968,487.83	$0.00	$2,035,852.66	
29	13	$1,060,397.23	$0.00	$1,060,397.23	$0.00	$2,113,850.05	
30	12	$1,161,028.83	$0.00	$1,161,028.83	$0.00	$2,194,835.68	
31	11	$1,271,210.35	$0.00	$1,271,210.35	$0.00	$2,278,924.02	
32	10	$1,391,848.09	$0.00	$1,391,848.09	$0.00	$2,366,233.94	
33	9	$1,523,934.35	$0.00	$1,523,934.35	$0.00	$2,456,888.87	
34	8	$1,668,555.57	$0.00	$1,668,555.57	$0.00	$2,551,016.97	
35	7	$1,826,901.34	$0.00	$1,826,901.34	$0.00	$2,648,751.29	
36	6	$2,000,274.10	$0.00	$2,000,274.10	$0.00	$2,750,230.00	
37	5	$2,190,099.92	$0.00	$2,190,099.92	$0.00	$2,855,596.56	
38	4	$2,397,940.20	$0.00	$2,397,940.20	$0.00	$2,964,999.91	
39	3	$2,625,504.49	$0.00	$2,625,504.49	$0.00	$3,078,594.70	
40	2	$2,874,664.62	$0.00	$2,874,664.62	$0.00	$3,196,541.54	
41	1	$3,147,470.02	$0.00	$3,147,470.02	$0.00	$3,319,007.14	
42	0	$3,446,164.63	$0.00	$3,446,164.63	$0.00	$3,446,164.63	
					$7,645,364.28	$71,015,755.95	$7,645,364.28
					PVoutFlowTotal	FVinFlowTotal	PVinFlowTotal

Question 3: (Challenge) Supply & Demand (Lectures 17, 18, 19, 20)

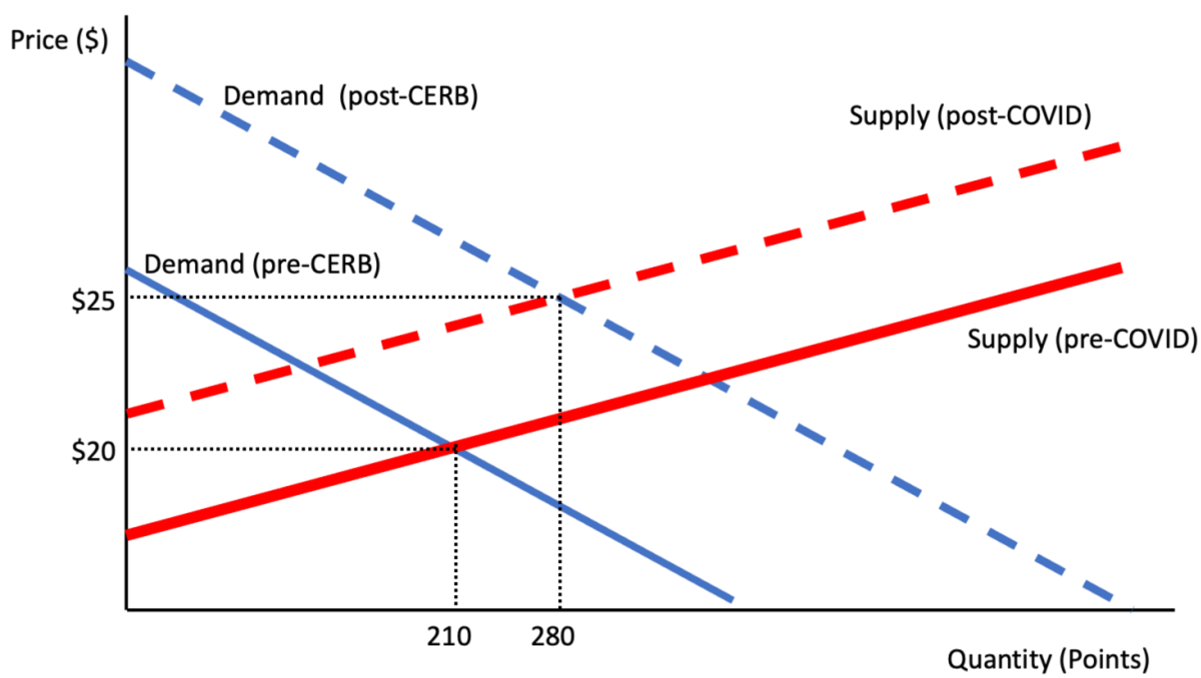
Each of parts b,c,d,e,f is worth 3 marks, for a total of 15 marks.

This question is based on the following article:

Mathew, N., Wong, J. S. H. & Krausz, M. (2021). An Inside Look at B.C.'s Illicit Drug Market During the COVID-19 Pandemic. *BC Medical Journal*, *63*(1), 9-13. https://bcmj.org/articles/inside-look-bcs-illicit-drug-market-during-covid-19-pandemic

Among other interesting bits of information, the article reveals that because of COVID-19, the wholesale price of opioids went up by about 13% (from $1,500 per ounce to $1,700 per ounce), while the retail price went up by 25% (from $20 per point to $25 per point[13]). At first blush, this suggests that far from the burden of the extra $200 cost per ounce being shared by buyers and sellers, more than 100% of the cost was passed through to buyers. Another item of information in the paper is a possible key to the puzzle: in addition to seller costs going up, the income of opioid buyers was temporarily increased by CERB payments.

In this question, you will use supply and demand analysis to investigate the situation described in the article. The relevant diagram, as described in Tool 8 of A Tiny Toolkit, is as follows:



[13] A point is a tenth of a gram.

a. Read the Article & a few helpful tools (No Marks)

I strongly suggest you read or skim the source article cited above. Doing so should make the questions easier to understand.

You will also want to read Tool 8 and Tool 9 on pages 24 – 33 of *A Tiny Toolkit* :

https://onlineacademiccommunity.uvic.ca/willmore/wp-content/uploads/sites/5845/2022/08/atinytoolkit.pdf

b. Derive the Pre-COVID demand curve for opioids (Basic algebra – see hint)

Assume, as in the diagram above, that supply and demand can be represented by straight lines. Suppose that before COVID, **the equilibrium quantity of opioids bought and sold per week is 210 points**[14], the **equilibrium price is $20 per point**, and **the price elasticity of demand[15] is -0.8** (so for every 1% increase in the price of opioids, quantity demanded falls by 0.8%).

Use this information[16] to derive the pre-COVID demand curve for opioids. Show your work.

Pre-COVID Demand Function: Q = 378 - (8.4 * P)

Pre-COVID Inverse Demand Function: P = (378 - Q) / 8.4

(The inverse function is in 'graphing form': recall that supply & demand diagrams have price on the vertical access. Calculating this here will be useful later.)

Hint 1: As the Besanko & Brauetigam text linked in the footnote correctly points out, you can derive the demand function in the form Q = a – bP, where a = (1- ε)Q* and b = -εQ*/P*. Here, P* = equilibrium price, Q* = equilibrium quantity, and ε = price elasticity of demand.

P*	= $20	a	= (1 + Ɛ)Q* = (1+0.8)(210) = 378
Q*	= 210	b	= - ƐQ*/P* = 0.8(210/20) = $8.4
Ɛ	= -0.8	**Q**	**= (a - bP) = 378 – 8.4P**
		P	**= (378 - Q) / 8.4**

[14] John Doe mentions that pre-COVID, he was working 7 days a week and making up to $600 dollars a day. If we assume (to keep things simple) that he means revenue and not profit, and that all the money was from opioids, then ($600/day)/($20/point) = 30 points/day = 210 points/week.

[15] This is taken from an estimate for the price elasticity of demand for heroin, in Olmstead, T.A. et al. (2015). The price elasticity of demand for heroin: Matched longitudinal experimental evidence. *Journal of Health Economics, 41*, 59-71. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.jhealeco.2015.01.008

[16] Need a refresher? Section 2.5 of Besanko & Braeutigam walks you through this, in the sub-section titled 'Fitting Linear Demand Curves Using Quantity, Price, and Elasticity Information', on pages 55-56. Chapter 2 of B&B can be found at https://higheredbcs.wiley.com/legacy/college/besanko/0471457698/chaps/ch02.pdf

c. Solve for the (inverse) supply and demand equations (Basic algebra – see hint)

You've already solved for pre-COVID demand (and inverse demand, i.e. graphing form demand). Now, you also need to solve for after-COVID demand, pre-COVID supply, and after COVID supply. This is easier than it sounds. (See the long hint in blue.)

Before-CERB Inverse Demand (from part b.): P = (378-Q) * 8.4

After-CERB Inverse Demand: P = (504-Q) / 8.96

Before-COVID Inverse Supply: P = ((4.3/70)*(Q-210)) +20

After-COVID Inverse Supply: P = ((4.3/70)*(Q-280)) +25

After COVID Inverse Demand using the same elasticity:

P*	: $25		
Q*	: 280		
Ɛ	: -0.8		
a	: (1 - Ɛ)Q*	= (1+0.8)(280)	= 504
b	: - ƐQ*/P	= 0.8(280/25)	= $8.96
Q	: a – bP	= 504 – 8.96P	
P	**: (a – Q) / b**	**= (504 – Q) / 8.96**	

Before COVID Inverse Supply:

Point 1	: (210, 20)		
Point 2	: (280, 25-.7)	= (280, 24.3)	
Slope, m	: (y2 – y1)/(x2 – x1)	= (24.2 – 20)/(280 – 210)	= 4.3/70
Slope-Intercept Equation	: P = S(Q – Q*) + P*		
P	**: (4.3/70)*(Q – 210) + 20**		

After COVID Inverse Supply:

Point 1	: (210, 20 + 0.7)	= (210, 20.7)	
Point 2	: (280, 25)		
Slope, m	: (y2 – y1)/(x2 – x1)	= (25 – 20.7)/(280 – 210)	= 4.3/70
Slope-Intercept Equation	: P	= S(Q – Q*) + P*	
P	**: (4.3/70)*(Q – 280) + 25**		

Hint: Doing the math from what the graph shows us

What are you trying to find when you're trying to find the equation of 'the supply curve'? (There are two: before COVID, and after COVID).

You're trying to find the equation of a line.

What do you need to find the equation of a line? Slope and intercept, or two points the line goes through.

Do you have the slope & intercept? Not without more work...

Do you have two points?

Well... Yes. You have two points, total, that you know the supply curves cross.

You have the before-COVID equilibrium (Q=210,P=20) and the post-COVID, post-CERB equilibrium (Q=280,P=25).

By the definition of what the equilibrium IS (supply & demand cross there), you know that the first point has to be on the pre-COVID supply curve, and the second point has to be on the post-COVID supply curve.

But that's ONE point per curve. You need two.

Good news: remember that the increase in drug wholesale costs of $200 per ounce works just like a 'tax' of $0.70 per point paid by producers. From lectures 16-19, you may remember what this does to the supply curve: it shifts it up by the amount of the tax.

So the post-COVID supply curve is the pre-COVID curve shifted up by $0.70. Which means that for any point (Q,P) on the old supply curve, there's a point (Q,P+0.7) on the new supply curve, and for any point (Q,P) on the new supply curve, there's a point (Q,P-0.7) on the old supply curve.

So (280,25-0.7) must be on the old supply curve, and (210,20+0.7) must be on the new supply curve.

So, boom: two points for each of the two lines you need. From there, just use the usual high-school level techniques to find the equation of a line from two points.

For how to find the equation of a line from two points, take a peek at this YouTube video:

McLogan, B. (2011, January 27). The equation of a line given two points [Video File]. https://youtu.be/4vXqMsvPSv4

There are even sites that can automate this for you:

Timur. (2019). Line equation from two points [Web Page]. https://planetcalc.com/8110/

(Don't forget that 'graphing form' supply & demand curves have P on the left, so P=f(Q), and we have P on the horizontal axis, even though the theoretical relationship is closer to Q being the dependent variable. That's why we say that we graph 'inverse demand' and 'inverse supply'.)

From there, you can use similar techniques to find the equation of post-CERB Demand. It's your pre-CERB demand curve, which you have an equation for in the form P=f(Q) (inverse demand), shifted up by some amount Y. And you know it must cross the new equilibrium, (Q,P) = (280,25). So, it must be the case that when 25 = f(280) + Y, where f(Q) is the original inverse demand curve.

d. What would the price have been without CERB? (Lectures 17 – 20)

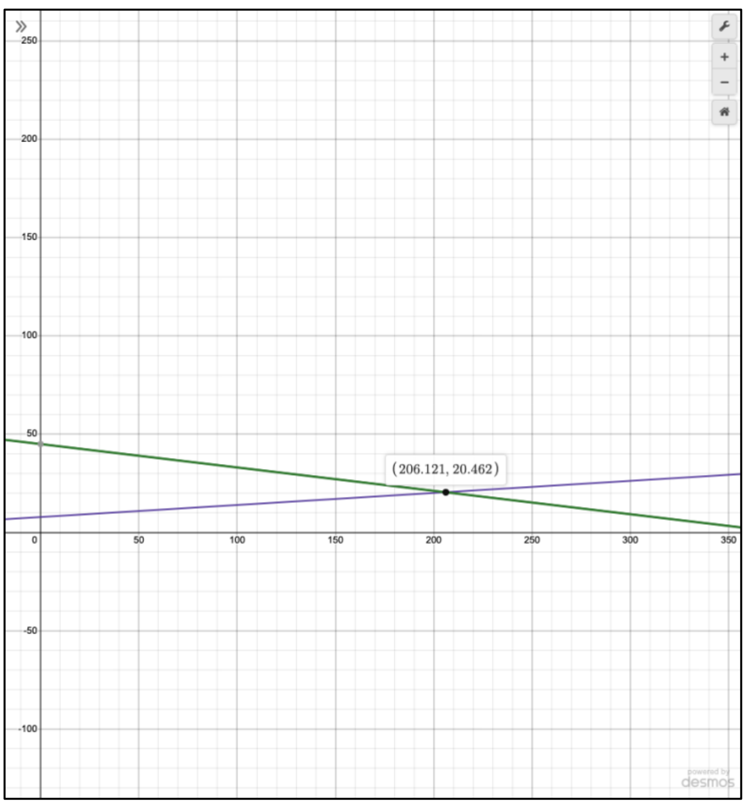
Note: An increase in the wholesale price of opioids works similarly to a tax. Parts ii & iii rely on your knowledge of supply & demand and tax incidence from lectures 17-20. If you need a refresher, I recommend this YouTube video and (short) journal article:

- You Will Love Economics. (2018, October 5) Micro: Unit 1.5 – Excise Taxes and Tax Incidence. https://youtu.be/L7rHOwkUD9A
- Zupan, Mark A. (1988). Teaching Tools: The Relative Size of Supply/Demand Elasticity and Tax Incidence. *Economic Inquiry*, *26*(2), 361-363. http://search.proquest.com.ezproxy.library.uvic.ca/scholarly-journals/features/docview/1297364525/se-2?accountid=14846

John Doe mentioned that demand was "stable" during the pandemic, apart from the extra spending induced by CERB.

(3 marks) What would the retail price of a point of opioids have been during the pandemic, in the absence of CERB? (Hint: You want to solve for the equilibrium price represented by the crossing of the pre-CERB demand curve with the post-COVID supply curve.)

No-CERB price of opioids during COVID: $20.46 per point



Plotting **(378 - Q) / 8.4** & (**(4.3/70)*(Q-280)) +25**, after-CERB Inverse Supply and Before-CERB Inverse Supply, we find that the intersect point for P is $20.46/point for a value of Q = 206.121.

e. Relative elasticities (Lectures 19 and 20)

(3 marks) Based on your answer to part d., in the absence of CERB, which would have been more elastic – supply of opioids or demand for opioids? Briefly explain your reasoning. (Hint: the party with the lowest elasticity ends up paying most of the tax. If we look at the hypothetical 'no CERB' case, what percentage of the added $0.70/point cost of opioids is paid by opioid consumers via higher prices?)

Which is more elastic, supply or demand? Supply

On the supply & demand diagram, the supply curve shifts up by $0.70 per point. The equilibrium price was $20 and during Covid, the equilibrium became $20.46 per point. Change in Equilibrium Price becomes $0.46 per point, so 0.46/0.7 = 0.6571 or 65.71%, the surpass of the additional cost ends up on the consumers. Because of this, the demand curve is less elastic than that of the supply curve.

f. Elasticity and the COVID/CERB equilibrium (Lectures 19 and 20)

(3 marks) Consider the post-COVID, post-CERB equilibrium. If there were a further small increase in the wholesale cost of opioids, who would end up paying for most of it – the buyers or the seller? Briefly explain your reasoning and back up your answer with calculations.

(Hint: This is a tax incidence question. You have everything you need to calculate the price elasticity of supply and price elasticity of demand at the post-COVID, post-CERB equilibrium, using the calculus definitions of elasticity. Once you have those, you can easily calculate the burdens on the buyers and on the sellers from a small increase in seller cost.)

Who would pay for most of the added cost? Buyers

P^* = \$25
Q^* = 280
P_S = ((4.3/70)*(Q - 280)) + 25
or Q = ((P_S - 25)*(70/4.3)) + 280
P_B = (504 – Q)/8.96
or Q = 504 - PB x 8.96

Using Formula [$\varepsilon = d(Q_S^*/d(P_S^*)*(P^*/Q^*)$], we get the elasticity of Sellers and Buyers:
ε_S = (1 - (- 8.96))*(25/280) = 0.89
ε_B = (1 – (70/4.3)) * (25/280) = -0.45

The elasticity of Sellers is more than the buyers as they (buyers) pay more of the added costs than the sellers.