

# CHAPTER 4

## Solutions to Chapter-End Problems

### A. Key Concepts

Relations Among Projects:

- 4.1** (a) There are eight mutually exclusive alternatives:  
 {T, TL, TM, TLM, L, LM, M, no new quarry}
- (b) There are now twelve alternatives. Let C stand for the cutter-loader.  
 {T, TL, TLC, TM, TLM, TLMC, L, LC, LM, LMC, M, no new quarry}
- (c) There are now eight feasible combinations. Total first costs are in parentheses.  
 {T (\$0.9 million), TL (\$2.3 million), TM (\$1.9 million), L (\$1.4 million),  
 LC (\$1.8 million), LM (\$2.4 million), M (\$1.0 million), do nothing (\$0)}

- 4.2** If the city does not widen either King or Main, there are two projects, stoplights and no stoplights.

If the city widens King, the following 5 projects are available:

Left Lane on King	no	no	yes	yes	yes
Stoptlights	no	yes	no	yes	yes
Advanced Green on King	no	no	no	no	yes

Similarly, if the city widens Main, there are 5 projects:

Left Lane on Main	no	no	yes	yes	yes
Stoptlights	no	yes	no	yes	yes
Advanced Green on Main	no	no	no	no	yes

The grand total is 12 projects.

### 4.3

Candidates	Financing	Buy Barco	Expand	X division	Feasible?
Do nothing	Ok	ok	ok	ok	yes
A	Ok	ok	ok	ok	yes
B	Ok		ok	ok	no
C		ok	ok	ok	no
D	Ok	ok		ok	no
AB	Ok	ok	ok	ok	yes
AC		ok	ok	ok	no

AD	Ok	ok			no
BC	Ok	ok	ok	ok	yes
BD	Ok		ok	ok	no
CD	Ok	ok		ok	no
ABC	Ok	ok	ok	ok	yes
ABD	Ok	ok	ok		no
ACD	Ok	ok			no
BCD	Ok	ok	ok	ok	yes
ABCD	Ok	ok	ok		no

There are six feasible projects: doing nothing, A, AB, BC, ABC, BCD

Present Worth Comparisons:

$$\begin{aligned}
 4.4 \quad PW &= -28\,000 + 5000(P/A, 15\%, 10) + 3000(P/F, 15\%, 10) \\
 &= -28\,000 + 5000(5.0188) + 3000(0.24718) \\
 &= -2164.46
 \end{aligned}$$

The present worth of the project is about -\$2164

$$\begin{aligned}
 4.5 \quad P &= -200\,000 + 2000(P/A, 0.5\%, 60) + 210\,000(P/F, 0.5\%, 60) \\
 &= -200\,000 + 2000(51.726) + 210\,000(0.74137) \\
 &= 59\,140
 \end{aligned}$$

Since the present worth of this set of cash flows is positive, Nabil should buy the house.

$$\begin{aligned}
 4.6 \quad PW(\text{moderate}) &= -6500 + [3300 - 300(A/G, 8\%, 7)](P/A, 8\%, 7) \\
 &= -6500 + [3300 - 300(2.6937)](5.2064) \\
 &= 6473.78
 \end{aligned}$$

For extensive upgrading:

$$g = -0.2$$

$$i^o = (1 + i)/(1 + g) - 1 = 1.08/0.8 - 1 = 0.35$$

$$\begin{aligned}
 PW(\text{extensive}) &= -10\,550 + 7600(P/A, i^o, 7)/(1 + g) \\
 &= -10\,550 + 7600[(1.35^7 - 1)/(0.35 \times 1.35^7)]/0.8 \\
 &= 13\,271.47
 \end{aligned}$$

Extensive upgrading is better.

$$4.7 \quad \text{At a MARR of 9\%, the present worth of the contract from the first company is:}$$

$$\begin{aligned} P1 &= 10\,000(P/A, 9\%, 5) + 20\,000(P/A, 9\%, 10)(P/F, 9\%, 5) \\ &= 10\,000(3.8896) + 20\,000(6.4176)(0.64993) \\ &= 122\,315.82 \end{aligned}$$

The present worth of the contract from the second company is:

$$\begin{aligned} P2 &= 10\,000(P/A, 9\%, 10) + 3000(A/G, 9\%, 10)(P/A, 9\%, 10) \\ &= (P/A, 9\%, 10)[10\,000 + 3000(A/G, 9\%, 10)] \\ &= 6.4177[10\,000 + 3000(3.7978)] \\ &= 137\,296.42 \end{aligned}$$

The software genius should choose the contract from the second company.

#### 4.8 Using the capitalized cost formula:

$$\begin{aligned} &PW(\text{concrete reservoir} + \text{pipe}) \\ &= 500\,000 + A/i = 500\,000 + 2000/0.08 = 525\,000 \end{aligned}$$

$$\begin{aligned} &PW(\text{earthen dam} + \text{aqueduct}) \\ &= 200\,000 + 12\,000/0.08 + 100\,000(A/F, 8\%, 15)/0.08 \\ &= 350\,000 + 100\,000(0.03683)/0.08 \\ &= 396\,037.5 \end{aligned}$$

The alternative with the least present cost is the earthen dam and aqueduct alternative. It has a present cost of \$396 038 and should be chosen.

#### Annual Worth Comparisons:

$$\begin{aligned} 4.9 \quad AW &= -28\,000(A/P, 15\%, 10) + 5000 + 3000(A/F, 15\%, 10) \\ &= -28\,000(0.19925) + 5000 + 3000(0.04925) \\ &= -431.25 \end{aligned}$$

The annual worth of the project is about -\$431.

$$\begin{aligned} 4.10 \quad A &= -200\,000(A/P, 0.5\%, 60) + 2000 + 210\,000(A/F, 0.5\%, 60) \\ &= -200\,000(0.01933) + 2000 + 210\,000(0.01433) \\ &= 1143 \end{aligned}$$

Since the annual worth of this set of cash flows is positive, Nabil should buy the house.

**4.11** For the hydraulic press:

$$g = 0.15$$

$$i^{\circ} = (1 + i)/(1 + g) - 1 = 1.12/1.15 - 1 = -0.026$$

AW(hydraulic)

$$\begin{aligned} &= -275\,000(A/P, 12\%, 15) + 33\,000 + 19\,250(A/F, 12\%, 15) \\ &\quad - 2000[(P/A, i^{\circ}, 15)/(1 + g)](A/P, 12\%, 15) \\ &= -275\,000(0.14682) + 33\,000 + 19\,250(0.02682) \\ &\quad - 2000[(0.974^{15} - 1)/(-0.026 \times 0.974^{15})](0.14682)/1.15 \\ &= -275\,000(0.14682) + 33\,000 + 19\,250(0.02682) - 2000(18.65228) \\ &= 24\,716.08 \end{aligned}$$

AW(molding)

$$\begin{aligned} &= -185\,000(A/P, 12\%, 10) + 24\,500 - [1000 + 350(A/G, 12\%, 10)] \\ &\quad + 14\,800(A/F, 12\%, 10) \\ &= -185\,000(0.17698) + 24\,500 - [1000 + 350(3.5847)] + 14\,800(0.05698) \\ &= -9652.64 \end{aligned}$$

The hydraulic press is the preferred alternative.

**4.12** The annual cost of operating an auto is:

$$\begin{aligned} A &= 24\,000(A/P, 11\%, 5) + (2000 + 600 + 600 + 1000) + 400(A/G, 11\%, 5) \\ &\quad - 8000(A/F, 11\%, 5) \\ &= 24\,000(0.27057) + 4200 + 400(1.7922) - 8000(0.16057) \\ &= 10\,126.00 \end{aligned}$$

If Tom was to use taxis instead, the annual cost would be  $A = 6600$ .

Tom should not buy the vehicle. He will end up saving about \$3536 per year by hiring taxis.

**4.13** The annual cost of the chemical recovery system is:

$$\begin{aligned} A &= (300\,000 - 75\,000)(A/P, 9\%, 7) + 75\,000(0.09) \\ &= 225\,000(0.19869) + 75\,000(0.09) \\ &= 51\,456 \end{aligned}$$

The net annual benefit is then:  $52\,800 - 51\,456 = 1344$

The net annual benefit of purchasing the chemical recovery system is about \$1344 per year.

$$\begin{aligned}
 4.14 \quad A_A &= 5600 - (14\,000 - 2000)(A/P, 9\%, 7) - 2000(0.09) \\
 &= 5600 - 12\,000(0.19869) - 2000(0.09) \\
 &= 3035.72
 \end{aligned}$$

$$\begin{aligned}
 A_B &= 5600 - (25\,000 - 10\,000)(A/P, 9\%, 10) - 5000(0.09) \\
 &= 5600 - 15\,000(0.15582) - 10\,000(0.09) \\
 &= 2362.7
 \end{aligned}$$

Machine A is best with an annual benefit of about \$3036.

Alternatives with Unequal Lives:

- 4.15 (a)** Twenty years is the least common multiple of the service lives. Find the present worth analysis for a twenty-year period.

Lawn Guy:

$$\begin{aligned}
 P &= 350 + 350(P/F, 5\%, 10) + (60 + 30)(P/A, 5\%, 20) \\
 &= 350 + 350(0.61391) + 90(12.462) \\
 &= 1\,686.47
 \end{aligned}$$

Clip Job:

$$\begin{aligned}
 P &= 120 + 120(P/F, 5\%, 4) + 120(P/F, 5\%, 8) + 120(P/F, 5\%, 12) \\
 &\quad + 120(P/F, 5\%, 16) + (40+60)(P/A, 5\%, 20) \\
 &= 120(1 + 0.82270 + 0.67684 + 0.55684 + 0.45811) + 100(12.462) \\
 &= 1\,667.96
 \end{aligned}$$

There is very little difference between the present worth of the Lawn Guy and the present worth of the Clip Job. However, since the present worth (cost) of the "Clip Job" machine is least, it is economically preferable.

- (b)** Use a 4-year study period, and let  $X$  be the salvage value of the Lawn Guy after 4 years.

Lawn Guy:

$$\begin{aligned}
 P &= 350 + (30 + 60)(P/A, 5\%, 4) - X(P/F, 5\%, 4) \\
 &= 350 + 90(3.5458) - X(0.82270) \\
 &= 669.14 - 0.8227X
 \end{aligned}$$

- 4.16 (a)** With the least common multiple of the service lives method, the present costs are:

Used:

$$P = 475(A/P, 8\%, 3)(P/A, 8\%, 24) = 475(0.38803)(10.529) = 1940.65$$

New:

$$P = 1250 + 1250(P/F, 8\%, 8) + 1250(P/F, 8\%, 16) \\ = 1250(1 + 0.54027 + 0.29189) = 2290.2$$

The used refrigerator is the better buy.

**(b)** With a service period of three years the present worth computations are:

$$\text{Used: } P = 475$$

$$\text{New: } P = 1250 - 1000(P/F, 8\%, 3) = 1250 - 1000(0.79383) = 456.17$$

In this case, the new refrigerator is the better buy.

Payback Period:

$$4.17 \quad \text{Payback period} = 28\,000/5000 = 5.6 \text{ years}$$

$$4.18 \quad \text{Payback period} = 10\,000/(6567 - 2000) = 2.19 \text{ years}$$

$$4.19 \quad \text{Payback Period} = 65\,000\,000/(12 \times 5000 \times 365 \times 0.8) = 3.71 \text{ years}$$

$$4.20 \quad \text{Payback period} \\ = 20\,000/[(3000 + 1000 + 2500) - (700 + 200)] \\ = 20\,000/5600 = 3.6 \text{ years}$$

If Greene Cheese had a maximum payback period of less than 3.6 years, the project would be rejected.

## B. Applications

4.21

Possible combinations	Budget	Lead Time	Resources	Feasible?
Do nothing	<\$200 000	ok	ok	no
A	<\$200 000	ok	ok	no
B	<\$200 000	ok	ok	no
C	<\$200 000	ok	ok	no
D	<\$200 000	ok	ok	no
AB	<\$200 000	ok	>100%	no
AC	ok	same	ok	no
AD	<\$200 000	ok	ok	no
BC	ok	ok	ok	yes
BD	<\$200 000	same	ok	no
CD	ok	ok	ok	yes
ABC	>\$300 000	ok	>100%	no
ABD	ok	ok	>100%	no

ACD	>\$300 000	ok	>100%	no
BCD	ok	ok	>100%	no
ABCD	>\$300 000	ok	>100%	no

There are only two mutually exclusive projects that IQ Computers should consider: (1) making products B and C, or (2) making products C and D.

#### 4.22

Candidates	Testers	Press	Total cost	Feasible?
1	Ok	ok	ok	yes
2	Ok	ok	ok	yes
3	Ok	ok	ok	yes
4	Ok	overhaul	ok	no
12	Ok	ok	ok	yes
13	Ok	ok	ok	yes
14	Ok	ok	ok	yes
23	>1	ok	ok	no
24	Ok	overhaul	ok	yes
34	Ok	overhaul	>\$100 000	no
123	>1	ok	>\$100 000	no
124	Ok	ok	ok	yes
134	Ok	ok	>\$100 000	no
234	>1	overhaul	>\$100 000	no
1234	>1	ok	>\$100 000	no

There are 8 mutually exclusive projects available:  
{1, 2, 3, 12, 13, 14, 24, 124}

#### 4.23

Since at least 2 printing lines must be available at all times, they can either upgrade one line or two lines.

If one line is to be upgraded, there are 7 possible combinations of 3 printing stations:

	Total Cost	Total Time	Feasible?
(1) A	\$7000	10 days	yes
(2) B	\$5000	5 days	yes
(3) C	\$3000	3 days	yes
(4) AB	\$12000	15 days	no
(5) AC	\$10000	13 days	yes
(6) BC	\$8000	8 days	yes
(7) ABC	\$15000	18 days	no

Combinations (4) and (7) are not feasible.

If two lines are to be upgraded, the following combinations must be considered:

	Line 1	Line 2	Total Cost	Total Time	Feasible?
(8)	A	A	\$14000	20 days	no

(9)	A	B	\$12000	15 days	no
(10)	A	C	\$10000	13 days	yes
(11)	A	AC	\$17000	23 days	no
(12)	A	BC	\$15000	18 days	no
(13)	B	B	\$10000	10 days	yes
(14)	B	C	\$8000	8 days	yes
(15)	B	AC	\$15000	18 days	no
(16)	B	BC	\$13000	13 days	yes
(17)	C	C	\$6000	6 days	yes
(18)	C	AC	\$13000	16 days	no
(19)	C	BC	\$11000	11 days	yes
(20)	AC	AC	\$20000	26 days	no
(21)	AC	BC	\$18000	21 days	no
(22)	BC	BC	\$16000	16 days	no

(10), (13), (14), (16), (17), and (19) are feasible.

Nottawasaga Printing can choose to upgrade one or two printing lines. If they decide to upgrade one line, there are 5 feasible mutually exclusive alternatives (see combinations 1, 2, 3, 5, and 6), and if they decide to upgrade two lines, there are 6 feasible mutually exclusive alternatives (see combinations 10, 13, 14, 16, 17, and 19).

#### 4.24 (a) PW(Smoothie)

$$= -15\,000 + (4200 - 1200)(P/A, i, 12) + 2250(P/F, i, 12)$$

$$= -15\,000 + 3000(P/A, i, 12) + 2250(P/F, i, 12)$$

#### PW(Creamy)

$$= -36\,000 + (10\,800 - 3520)(P/A, i, 12) + 5000(P/F, i, 12)$$

$$= -36\,000 + 7280(P/A, i, 12) + 5000(P/F, i, 12)$$

By letting  $PW(\text{Smoothie}) = PW(\text{Creamy})$ :

$$21\,000 = 4280(P/A, i, 12) + 2750(P/F, i, 12)$$

At  $i = 0.15$ :  $(P/A, i, 12) = 5.4206$ ,  $(P/F, i, 12) = 0.18691$ , and  $LHS = 23\,714$

At  $i = 0.20$ :  $(P/A, i, 12) = 4.4392$ ,  $(P/F, i, 12) = 0.11216$ , and  $LHS = 19\,308$

Using linear interpolation:

$$i = 0.15 + (0.20 - 0.15)[(21\,000 - 23\,714)/(19\,308 - 23\,714)]$$

$$= 0.180798 \cong 18.1\%$$

A MARR of 18.1% makes the two alternatives equivalent in terms of PW.

**(b)** Since the service life differs now for the two alternatives, we cannot use a simple present worth comparison. Either repeated lives or an annual worth comparison would be appropriate. We show the annual worth method here with an assumption that the two alternatives can be repeated.



AW(Smoothie)

$$\begin{aligned}
 &= -15\,000(A/P, 18.1\%, 14) + 4200 - 1200 + 2250(A/F, 18.1\%, 14) \\
 &= -15\,000[0.181(1.181^{14})/(1.181^{14} - 1)] + 3000 - 2250[0.181/(1.181^{14} - 1)] \\
 &= -15\,000(0.2005289) + 3000 - 2250(0.0195289) \\
 &= 36.006525 \cong \$36.01
 \end{aligned}$$

AW(Creamy)

$$\begin{aligned}
 &= -36\,000(A/P, 18.1\%, 12) + 10\,800 - 3520 + 5000(A/F, 18.1\%, 12) \\
 &= -36\,000(0.2094499) + 7280 - 5000(0.0284499) \\
 &= -117.9469 \cong -\$117.95
 \end{aligned}$$

Smoothie is better than Creamy if the service life of Smoothie is 14 years.

**4.25** 
$$\begin{aligned}
 FW &= -28\,000(F/P, 15\%, 10) + 5000(F/A, 15\%, 10) + 3000 \\
 &= -28\,000(4.0456) + 5000(20.304) + 3000 \\
 &= -8756.8
 \end{aligned}$$

The future worth of the project in 10 years is about -\$8757.

Year	Present worth	Cumulative
1	$5000(P/F, 15\%, 1) = 5000(0.86957) = 4348$	4348
2	$5000(P/F, 15\%, 2) = 5000(0.75614) = 3781$	8129
3	$5000(P/F, 15\%, 3) = 5000(0.65752) = 3288$	11417
4	$5000(P/F, 15\%, 4) = 5000(0.57175) = 2859$	14276
5	$5000(P/F, 15\%, 5) = 5000(0.49718) = 2486$	16762
6	$5000(P/F, 15\%, 6) = 5000(0.43233) = 2162$	18924
7	$5000(P/F, 15\%, 7) = 5000(0.37594) = 1880$	20804
8	$5000(P/F, 15\%, 8) = 5000(0.32690) = 1635$	22439
9	$5000(P/F, 15\%, 9) = 5000(0.28426) = 1421$	23860

10	$5000(P/F, 15\%, 10) = 5000(0.24718)$ $= 1236$	25096
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The discounted payback period is greater than 10 years, the life of the project.

- 4.26 (a)** At a production level of 30 000 units per year, the annual worth (cost) of a wax melter from each supplier is computed as follows:

Finedetail:

$$\begin{aligned} A &= 200\,000(A/P, 15\%, 7) + 16\,500 + 2.25(30\,000) - 5000(A/F, 15\%, 7) \\ &= 200\,000(0.24036) + 16\,500 + 67\,500 - 5000(0.09036) \\ &= 131\,620.30 \end{aligned}$$

Simplicity:

$$\begin{aligned} A &= 350\,000(A/P, 15\%, 10) + 35\,500 + 1.06(30\,000) - 20\,000(A/F, 15\%, 10) \\ &= 350\,000(0.19925) + 35\,500 + 1.06(30\,000) - 20\,000(0.04925) \\ &= 136\,053.20 \end{aligned}$$

Based on annual worth, the wax melter from Finedetail is preferred because it has the lowest annual cost.

- (b)** At a production level of 200 000 units per year, the annual worth (cost) for the two wax melters is computed as follows:

Finedetail:

$$\begin{aligned} A &= 200\,000(A/P, 15\%, 7) + 16\,500 + 2.25(200\,000) - 5000(A/F, 15\%, 7) \\ &= 200\,000(0.24036) + 16\,500 + 67\,500 - 5000(0.09036) \\ &= 514\,120.27 \end{aligned}$$

Simplicity:

$$\begin{aligned} A &= 350\,000(A/P, 15\%, 10) + 35\,500 + 1.06(200\,000) - 20\,000(A/F, 15\%, 10) \\ &= 350\,000(0.19925) + 35\,500 + 1.06(200\,000) - 20\,000(0.04925) \\ &= 316\,253.18 \end{aligned}$$

Based on annual worth, the Simplicity wax melter is preferred because it has the lowest annual cost.

- (c)** The annual costs associated with each wax melter for production levels between 20 000 and 200 000 units per year are:

Production level	Annual cost	
	Finedetail	Simplicity
20000	109120	125453
30000	131620	136053
40000	154120	146653
50000	176620	157253
100000	289120	210253
150000	401620	263253
200000	514120	316253

The choice of supplier is quite sensitive to the number produced, with the wax melter from Finedetail being preferred for production of about 35 000 units per year or less, and the wax melter from Simplicity otherwise.

The Finedetail wax melter is *roughly* comparable to the Simplicity wax melter at a production level of 30 000 units per year. Moreover, if production does reach 200 000 units per year, Simplicity's wax melter has much lower annual costs than the Finedetail wax melter. Since the 30 000 is a lower bound on production levels, and it is quite possible that production will be higher, the Simplicity wax melter is the recommended alternative.

**4.27** Use the capitalized cost formula.

Concrete pool:

$$\begin{aligned} P &= 1\,500\,000 + 200\,000(A/F, 5\%, 10)/0.05 \\ &= 1\,500\,000 + 200\,000(0.07951)/0.05 \\ &= 1\,818\,040 \end{aligned}$$

Plastic pool:

$$\begin{aligned} P &= 500\,000 + [100\,000(A/F, 5\%, 5) + 150\,000(A/F, 5\%, 15) + 5000]/0.05 \\ &= 500\,000 + [100\,000(0.18098) + 150\,000(0.04634) + 5000]/0.05 \\ &= 1\,100\,980 \end{aligned}$$

The design with the lowest present cost is the pool with the plastic liner.

**4.28** Using a 6-year study period:

$$\begin{aligned} P_{XJ3} &= 4500[1 + (P/F, 10\%, 3)] - 1000[(P/F, 10\%, 3) + (P/F, 10\%, 6)] \\ &= 4500(1 + 0.75131) - 1000(0.75131 + 0.56447) \\ &= 6565.12 \end{aligned}$$

$$\begin{aligned} P_{Y19} &= 3200[1 + (P/F, 10\%, 2) + (P/F, 10\%, 4)] + 300(P/A, 10\%, 6) \\ &\quad - 1000[(P/F, 10\%, 2) + (P/F, 10\%, 4) + (P/F, 10\%, 6)] \\ &= 3200(1 + 0.82645 + 0.68301) + 300(4.3553) \\ &\quad - 1000(0.82645 + 0.68301 + 0.56447) \\ &= 7262.93 \end{aligned}$$

Based on a 6-year study period, Val should buy the model XJ3 display panel, which has the smallest present cost.

**4.29**  $P_{XJ3} = 4500 - 1900(P/F, 10\%, 2) = 4500 - 1900(0.82645) = 2929.75$

$$\begin{aligned} P_{Y19} &= 3200 + 300(P/A, 10\%, 2) - 1000[(P/F, 10\%, 2)] \\ &= 3200 + 300(1.7355) - 1000(0.82645) = 2894.2 \end{aligned}$$

Based on a 2-year study period, Val should buy the model Y19 display panel, which has the smallest present cost.

**4.30** Let  $P$  = first cost,  $A$  = yearly savings.

Then: Payback period = 3 =  $P/A \Rightarrow P = 3A$

Also: Present worth = 0 =  $-P + A(P/A, i\%, 5)$

By substituting  $P = 3A$  into the present worth expression:

$$\begin{aligned} 0 &= -3A + A(P/A, i\%, 5) \\ (P/A, i\%, 5) &= 3 \end{aligned}$$

By observing the tables, Diana's interest rate was approximately 20%.

**4.31 (a)** To equate costs set:

$$\begin{aligned} (0.08 \times 100 \times 500)/1000 + (2+1)(A/P, 1\%, 1000/500) \\ &= (0.08 \times 90 \times 500)/1000 + (2+3)(A/P, 1\%, N) \\ 4 + 3(0.50757) &= 3.6 + 5(A/P, 1\%, N) \\ (A/P, 1\%, N) &= 1.92271/5 = 0.38454 \end{aligned}$$

$$\begin{aligned} (A/P, 1\%, 2) &= 0.50751 \\ (A/P, 1\%, 3) &= 0.34002 \end{aligned}$$

By linear interpolation:

$$N = 2 + (0.50751 - 0.38454)/(0.50751 - 0.34002) = 2.734$$

The minimum required hours is approximately  $2.734 \times 500 = 1367$  hours.

**(b)** If costs are equated when there are no costs of changing bulbs, the results are:

$$\begin{aligned} (0.08 \times 100 \times 500)/1000 + 1(A/P, 1\%, 2) \\ &= (0.08 \times 90 \times 500)/1000 + 3(A/P, 1\%, N) \\ 4.0 + 0.50757 &= 3.6 + 3(A/P, 1\%, N) \\ (A/P, 1\%, N) &= 0.90757/3 = 0.30252 \end{aligned}$$

$$\begin{aligned} (A/P, 1\%, 3) &= 0.34002 \\ (A/P, 1\%, 4) &= 0.25628 \end{aligned}$$

$$N = 3 + (0.34002 - 0.30252)/(0.34006 - 0.25628) = 3.448$$

The minimum required hours is approximately  $3.448 \times 500 = 1640$  hours.

(c) Since the capital recovery factor is convex in the number of years, the linear interpolation is above the correct curve. Since the capital recovery factor is decreasing in the number of years, the linear interpolation is to the right of the correct curve. This means that the linear interpolation gives an estimate for the required number of years that is too high.

$$\begin{aligned} 4.32 \quad (a) \quad P_T &= -100\,000 + 50\,000(P/A, 11\%, 5) + 20\,000(P/F, 11\%, 5) \\ &= -100\,000 + 50\,000(3.6959) + 20\,000(0.59345) \\ &= 96\,664 \end{aligned}$$

$$\begin{aligned} P_A &= -150\,000 + 60\,000(P/A, 11\%, 5) + 30\,000(P/F, 11\%, 5) \\ &= -150\,000 + 60\,000(3.6959) + 30\,000(0.59345) \\ &= 89\,558 \end{aligned}$$

Model T has the greater PW and therefore should be taken.

(b) By assuming that you can purchase each alternative as many times as necessary, we can construct new projects:

T': buy model T three times, total life 15 years  
 A': buy model A three times, total life 15 years  
 X': buy model X five times, total life 15 years

We only need to compare models T and X since A is already eliminated:

$$\begin{aligned} P_{T'} &= -100\,000[1 + (P/F, 11\%, 5) + (P/F, 11\%, 10)] + 50\,000(P/A, 11\%, 15) \\ &\quad + 20\,000[(P/F, 11\%, 5) + (P/F, 11\%, 10) + (P/F, 11\%, 15)] \\ &= -100\,000(1 + 0.59345 + 0.35218) + 50\,000(7.1909) \\ &\quad + 20\,000(0.59345 + 0.35218 + 0.20900) \\ &= 188\,075 \end{aligned}$$

$$\begin{aligned} P_{X'} &= -200\,000[1 + (P/F, 11\%, 3) + (P/F, 11\%, 6) + (P/F, 11\%, 9) \\ &\quad + (P/F, 11\%, 12)] + 75\,000(P/A, 11\%, 15) + 100\,000[(P/F, 11\%, 3) \\ &\quad + (P/F, 11\%, 6) + (P/F, 11\%, 9) + (P/F, 11\%, 12) + (P/F, 11\%, 15)] \\ &= -200\,000(1 + 0.73119 + 0.53464 + 0.39092 + 0.28584) \\ &\quad + 75\,000(7.1909) + 100\,000(0.73119 + 0.53464 + 0.39092 \\ &\quad + 0.28584 + 0.20900) \\ &= 165\,959 \end{aligned}$$

Model T is still the best because it has the highest present worth.

$$\begin{aligned} 4.33 \quad P_T &= -100\,000 + 50\,000(P/A, 11\%, 3) + 40\,000(P/F, 11\%, 3) \\ &= -100\,000 + 50\,000(2.4437) + 40\,000(0.73119) \end{aligned}$$

$$= 51\,433$$

$$\begin{aligned} P_A &= -150\,000 + 60\,000(P/A, 11\%, 3) + 80\,000(P/F, 11\%, 3) \\ &= -150\,000 + 60\,000(2.4437) + 80\,000(0.73119) \\ &= 55\,117 \end{aligned}$$

$$\begin{aligned} P_X &= -200\,000 + 75\,000(P/A, 11\%, 3) + 100\,000(P/F, 11\%, 3) \\ &= -200\,000 + 75\,000(2.4437) + 100\,000(0.73119) \\ &= 56\,397 \end{aligned}$$

Model X is the best, A is next, and T is worst! This is because using the study period method, the choice of salvage value is critical.

$$\begin{aligned} 4.34 \quad A_T &= -100\,000(A/P, 11\%, 5) + 50\,000 + 20\,000(A/F, 11\%, 5) \\ &= -100\,000(0.27057) + 50\,000 + 20\,000(0.16057) \\ &= 26\,154 \end{aligned}$$

$$\begin{aligned} A_A &= -150\,000(A/P, 11\%, 5) + 60\,000 + 30\,000(A/F, 11\%, 5) \\ &= -150\,000(0.27057) + 60\,000 + 30\,000(0.16057) \\ &= 24\,232 \end{aligned}$$

$$\begin{aligned} A_X &= -200\,000(A/P, 11\%, 3) + 75\,000 + 100\,000(A/F, 11\%, 3) \\ &= -200\,000(0.40921) + 75\,000 + 100\,000(0.29921) \\ &= 23\,079 \end{aligned}$$

Model T is best.

$$\begin{aligned} 4.35 \quad \text{Model T: Payback period} &= 100\,000/50\,000 = 2 \text{ years} \\ \text{Model A: Payback period} &= 150\,000/60\,000 = 2.5 \text{ years} \\ \text{Model X: Payback period} &= 200\,000/75\,000 = 2.667 \text{ years} \end{aligned}$$

Note that all of the following are ignored: life, salvage value, and interest rate. This will in general cause the ranking of alternatives to vary from the correct rankings.

$$\begin{aligned} 4.36 \quad (a) \text{ Machine A:} \\ A &= -1\,500\,000(A/P, 8\%, 5) + (900\,000 - 600\,000) + 100\,000(A/F, 8\%, 5) \\ &= -1\,500\,000(0.25046) + 300\,000 + 100\,000(0.17046) \\ &= -58\,644 \end{aligned}$$

Machine B:

$$\begin{aligned} A &= -2\,000\,000(A/P, 8\%, 10) + (1\,100\,000 - 800\,000) \\ &\quad + 200\,000(A/F, 8\%, 10) \\ &= -2\,000\,000(0.14903) + 300\,000 + 200\,000(0.06903) \\ &= 15\,746 \end{aligned}$$

Only machine B should be purchased.

**(b)** Least common multiple of the service lives is 10 years.

Machine A:

$$\begin{aligned} P &= -1\,500\,000 + (900\,000 - 600\,000)(P/A, 8\%, 10) \\ &\quad + (100\,000 - 15\,000)(P/F, 8\%, 5) + 1000(P/F, 8\%, 10) \\ &= -1\,500\,000 + 300\,000(6.7101) - 1\,400\,000(0.68058) \\ &\quad + 100\,000(0.46319) \\ &= -393\,463 \end{aligned}$$

Machine B:

$$\begin{aligned} P &= -2\,000\,000 + (1\,100\,000 - 800\,000)(P/A, 8\%, 10) \\ &\quad + 200\,000(P/F, 8\%, 10) \\ &= -2\,000\,000 + 300\,000(6.7101) + 200\,000(0.46319) \\ &= 105\,668 \end{aligned}$$

Only machine B should be purchased.

**(c)** A:  $1\,500\,000/300\,000 = 5$  years  
 B:  $2\,000\,000/300\,000 = 6.6$  years

Probably neither should be purchased since the payback periods are long, more than four years.

**4.37** Constant amount:

$$F = 100\,000(F/A, 10\%, 10) = 100\,000(15.937) = 1\,594\,000$$

Increasing amount: The growth adjusted interest rate is  $i^0 = 1.1/1.05 - 1 = 4.7619\%$ .

$$\begin{aligned} F &= [80\,000(P/A, 4.7619\%, 10)/1.05](F/P, 10\%, 10) \\ &= 80\,000(7.8118)(2.5937)/1.05 \\ &= 1\,544\,000 \end{aligned}$$

The plan that saves the constant amount will accumulate the most money in 10 years.

**4.38** Payback period for isolation:  $60\,000/10\,000 = 6$  years  
 Payback period for curtains:  $5\,000/3\,000 = 1.67$  years

The payback period for the curtains is much shorter than for the isolation solution.

No threshold for payback was given in the problem, but the payback period for the curtains is less than the standard two-year maximum for acceptable projects.

However, it is not appropriate to use the payback period to compare these alternatives. The service life of isolation is very large, so that saving will be achieved long after the payback period is passed. For the curtains, the service life is almost the same as the payback period.

**4.39** Use the capitalized value formula.

For the isolation alternative:

$$A = 10\,000 - 60\,000(0.11) = 3400$$

For the curtains:

$$A = 3000 - 5000(A/P, 11\%, 2) = 3000 - 5000(0.58393) = 80.35$$

The isolation alternative is best with an annual worth of about \$3400 compared with the curtains of about \$80.

**C. More Challenging Problems**

**4.40** Alternative 1:

Using the capitalized value formula:

$$A(\text{first cost}) = P_i = 2\,000\,000(0.15) = 300\,000$$

$$A(\text{maintenance}) = 10\,000$$

$$A(\text{paint}) = 15\,000(A/F, 15\%, 15) = 15\,000(0.02102) = 315.3$$

$$A(\text{total}) = A(\text{first cost}) + A(\text{maintenance}) + A(\text{paint}) = 310\,315.30$$

Alternative 2:

*First cost:*

$$\begin{aligned} A(\text{first cost}) &= 1\,250\,000(0.15) + 1\,000\,000(P/F, 15\%, 10)(0.15) \\ &= 187\,500 + 150\,000(0.24719) = 224\,578.5 \end{aligned}$$

*Maintenance for the first 10 years:* The first 10 years has maintenance costs of \$5000 per year; convert to a PW and spread over infinite life using the capitalized value formula.

$$A(\text{maintenance first 10 years})$$

$$= 5000(P/A, 15\%, 10)(0.15) = 5000(5.0187)(0.15) = 3764.02$$

*Maintenance after 10 years:* The maintenance costs change after the renovation to be \$11 000 every year. First, convert to a PW at the end of



10 years, then to PW now, then spread over infinite life using the capitalized value formula:

$$P(\text{maintenance after 10 years}) = A/i = 11\,000/(0.15) = 73\,333.33$$

$$\begin{aligned} P(\text{maintenance after 10 years, now}) \\ = 73\,333.33(P/F, 15\%, 10) = 73\,333.33(0.2472) = 18\,128 \end{aligned}$$

$$\begin{aligned} A(\text{maintenance after 10 years}) \\ = P(\text{maintenance after 10 years, now})(0.15) = 18\,128(0.15) = 2719.2 \end{aligned}$$

*Painting costs:* Painting costs are every 15 years, starting in 10 years. Calculate the P(paint in 10 years), convert to P(paint now), and then to A(paint):

$$\begin{aligned} P(\text{paint in 10 years}) \\ = A/i = 15\,000(A/F, 15\%, 15)/(0.15) = 15\,000(0.02102)/(0.15) = 2101 \end{aligned}$$

$$P(\text{paint now}) = 2101(P/F, 15\%, 10) = 2101(0.24719) = 519.35$$

$$A(\text{paint}) = P(\text{paint now})(0.15) = 77.90$$

The total annual cost for alternative 2 is:

$$\begin{aligned} A(\text{total}) \\ = A(\text{first cost}) + A(\text{maintenance first 10 years}) \\ \quad + A(\text{maintenance after 10 years}) + A(\text{painting}) \\ = 224578.5 + 3764.02 + 2719.2 + 77.90 \\ = 231\,139.62 \end{aligned}$$

The annual cost of alternative 2 is less than that of alternative 1. Hence, select alternative 2.

- 4.41** Let     $P$  = first cost  
                $A$  = annual savings  
                $PW$  = present worth  
                $AW$  = annual worth

$$AW = -P(A/P, 15\%, 5) + 20\,000 = -0.29832P + 20\,000$$

$$PW = -P + 20000(P/A, 15\%, 5) = -P + 20000(3.3522) = -P + 67\,044$$

By setting  $PW = 3AW$ :

$$-P + 67\,044 = 3(-0.29832P + 20\,000) = -0.89496P + 60\,000$$

$$P(1 - 0.89496) = 67\,044 - 60\,000$$

$$P = 7044/0.10504 = 67\,060$$

The project's first cost was about \$67 060.

So that:

$$PW = -67\,060 + 67\,044 = -16$$

$$AW = -67\,060(0.29832) + 20\,000 = -5.3$$

Since the Present Worth and Annual Worth are each less than zero, Katie should not undertake the project.

**4.42 (a)** We must use repeated lives.

$$PW(\text{one CR1000})$$

$$= -680 + 245(P/A, 10\%, 4) - [35 + 10(A/G, 10\%, 4)](P/A, 10\%, 4)$$

$$+ 100(P/F, 10\%, 4)$$

$$= -680 + 245(3.1699) - [35 + 10(1.3812)](3.1699) + 100(0.68301)$$

$$= 10.1973$$

$$PW(\text{CR1000 for 12 years})$$

$$= PW(\text{one CR1000})[1 + (P/F, 10\%, 4) + (P/F, 10\%, 8)]$$

$$= 10.1973(1 + 0.68301 + 0.46651)$$

$$= 21.92$$

$$PW(\text{one CRX})$$

$$= -1100 + (440 - 60)(P/A, 10\%, 6) + 250(P/F, 10\%, 6)$$

$$= -1100 + 380(4.3553) + 250(0.56447) = 696.1315$$

$$PW(\text{CRX for 12 years})$$

$$= PW(\text{one CRX})[1 + (P/F, 10\%, 6)]$$

$$= 696.1315(1 + 0.56447) = 1089.08$$

CRX is the preferred choice since it has the highest PW.

**(b)** Let  $S$  be the scrap value to be solved for. By letting  $PW(\text{CR1000 for 12 years}) = PW(\text{CRX for 12 years})$ :

$$1089.08 = (-58.10366 + 0.68301S)[1 + (P/F, 10\%, 4) + (P/F, 10\%, 8)]$$

$$1089.08 = (-58.10366 + 0.68301S)(2.14952)$$

$$S = 826.88$$

However, the first cost of CR1000 is only \$680. It is unlikely that any increase in the scrap value of CR1000 would make it the preferred choice over CRX.

**4.43** The service life of A is longer than that of B.

**4.44** Let  $F_i$  be the first cost,  $A_i$  be the annual savings, and  $N_i$  be the economic life,  $i = A, B$ .

Two projects have the same payback period, so  $F_A/A_A = F_B/A_B$  (\*1).

Two projects have the same economic life, so  $N_A = N_B$  (\*2).

$$PW_A = -F_A + A_A(P/A, i, N_A) \Leftrightarrow PW_A/A_A = -F_A/A_A + (P/A, i, N_A)$$

$$PW_B = -F_B + A_B(P/A, i, N_B) \Leftrightarrow PW_B/A_B = -F_B/A_B + (P/A, i, N_B)$$

From (\*1) and (\*2),  $PW_A/A_A = PW_B/A_B$ . Since  $PW_A > PW_B$  (given information), it must be that  $A_A > A_B$  in order to satisfy the relationship,  $PW_A/A_A = PW_B/A_B$ .

**4.45** Contracting out is already expressed as an annual cost, so it is sufficient to calculate the annual cost of the landfill alternative:

$$\begin{aligned} A &= 1\,000\,000(A/P, 11\%, 30) + 100\,000(A/F, 11\%, 30) + 20\,000 \\ &\quad - 30\,000(F/A, 11\%, 20)(A/F, 11\%, 30) \\ &= 1\,000\,000(0.11502) + 100\,000(0.00502) + 20\,000 \\ &\quad - 30\,000(64.203)(0.00502) \\ &= 125\,351 \end{aligned}$$

Since the annual cost of the landfill site is about \$4650 less per year, on economic grounds, it is the best choice. Other issues would likely affect the final decision, especially because the two alternatives are similar in price. For example, local citizens would probably not like to have a new landfill site near their properties.

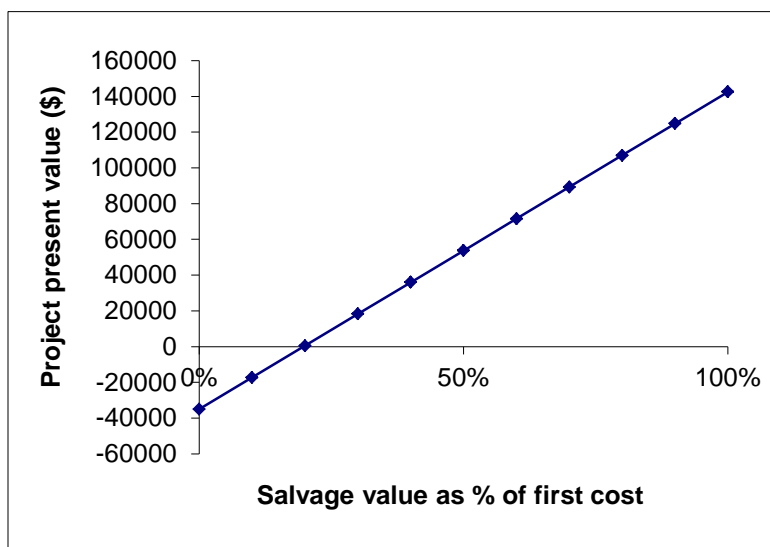
**4.46** The MARR at which the present worths of the two projects are the same is about 12.23%.

Year	Automated Line			Manual Line		
	Disbursement	Receipts	Net Cash Flow	Disbursement	Receipts	Net Cash Flow
0	1500000	0	-1500000	1000000	0	-1000000
1	50000	300000	250000	20000	200000	180000
2	60000	300000	240000	25000	200000	175000
3	70000	300000	230000	30000	200000	170000
4	80000	300000	220000	35000	200000	165000
5	90000	800000	710000	40000	200000	160000
	Present worth		- 386544			- 386544
	MARR		12.2295%			

**4.47** As shown below, only if the salvage value for the purchased items is less than about 20% of the purchase price does Stayner Catering lose money.

This seems unlikely over only a year of usage, so Stayner Catering should proceed with the project.

Month	Purchase	Labour	Warehouse	Revenue	PW	Salvage%	Net PW
January (beg.)	-200000				-200000	0%	-35040
January (end)		-2000	-3000	2000	-2970	10%	-17291
February		-2000	-3000	2000	-2941	20%	458
March		-2000	-3000	2000	-2912	30%	18207
April		-2000	-3000	2000	-2883	40%	35956
May		-4000	-3000	10000	2854	50%	53705
June		-10000	-6000	40000	22609	60%	71454
July		-10000	-6000	110000	87675	70%	89203
August		-10000	-6000	60000	40633	80%	106952
September		-4000	-3000	30000	21030	90%	124701
October		-2000	-3000	10000	4526	100%	142450
November		-2000	-3000	5000	0		
December	?	-2000	-3000	2000	-2662		



**4.48** The present worths of the two options are equivalent at MARR = 6.2878%. The present worth with this MARR is \$38 433.

Year	Expand:				Remodel:				
	Disbursement	Receipt	Net	Indiv.PW	Disbursement	Receipt	Net	Indiv.PW	1/(1+MARR)^N
0	850000	0	-850000		230000	0	-230000		
1	25000	200000	175000	164647	9000	80000	71000	66800	0.940841752
2	30000	225000	195000	172611	11700	80000	68300	60458	0.885183203
3	35000	250000	215000	179056	15210	80000	64790	53958	0.832817316
4	40000	275000	235000	184134	19773	80000	60227	47191	0.783549303
5	45000	300000	255000	187985	25704.9	80000	54295.1	40026	0.737195899
			<b>PW</b>	<b>38433</b>			<b>PW</b>	<b>38433</b>	<b>MARR</b>
									0.062878

**4.49** The annual worths of the two options are equivalent at MARR = 16.40925%. The annual worth with this MARR is \$220 806.

Fully Automated System:					Partially Automated System:				
Year	Disburse.	Receipt	Net	Indiv.PW	Disburse.	Receipt	Net	Indiv.PW	1/(1+MARR)^N
0	1000000	0	-1000000		650000	0	-650000		
1	30000	300000	270000	231940	30000	220000	190000	163217	0.859038264
2	30000	300000	270000	199246	30000	220000	190000	140210	0.737946739
3	80000	300000	220000	139463	35000	220000	185000	117276	0.633924485
4	30000	300000	270000	147033	35000	220000	185000	100745	0.544565389
5	30000	300000	270000	126307	40000	220000	180000	84204	0.467802506
6	80000	300000	220000	88409	40000	220000	180000	72335	0.401860253
7	30000	300000	270000	93208	45000	220000	175000	60412	0.345213334
8	30000	300000	270000	80069	45000	220000	175000	51897	0.296551463
9	80000	300000	220000	56045	50000	220000	170000	43307	0.254749054
10	30000	300000	270000	59087	50000	220000	170000	37203	0.218839185
			<b>PW</b>	220806			<b>PW</b>	220806	<b>MARR</b>
									0.1640925

#### 4.50

Let  $Q$  = payback period. Then

$$P/A = Q, P = AQ$$

$$PW = -P + A(P/A, i, n)$$

$$= A(P/A, i, n) - AQ$$

$$= A[(P/A, i, n) - Q]$$

Consequently, when  $(P/A, i, n) \geq Q$ ,  $PW \geq 0$ , otherwise  $PW < 0$

Fred's rule is sensible.

## Notes for Case-in-Point 4.1

- 1) All of these are important
- 2) No, he should not
- 3) If he sets his MARR too low, he may not be able to distinguish truly valuable investment and overspend. If he sets his MARR too high, he may miss valuable investments and underspend.

## Notes for Mini-Case 4.1

- 1) It is a reasonable way of distinguishing among investments. For example, a new machine doesn't generally have associated marketing issues, and may have to be purchased even if there is a net cost. New products are very dependent on marketing issues, and clearly have to make a profit.
- 2) Examples:
  - *Marketing strategy*: see if it is consistent with other products. If net effects are positive, it may be worth investing in.
  - *Work force*: measured in number and type of people affected. If the people required can be appropriately obtained and trained in an economically feasible way, it may be worth investing in.
  - *Margins*: measured in dollars. If the rate of return is greater than the MARR, it may be worth investing in.
  - *Cash flow*: measured as dollar earnings per period. If it is positive within two years for a new product, or within a "reasonable" period for new equipment, it may be worth investing in.
  - *Quality issues*: measured as defect or error rate. If reduced with the new equipment, it may be worth investing in.
  - *Cost avoidance*: payback period. If the payback period is one year or less, it may be worth investing in.
- 3) Examples:

*An investment is made when it should not be:*

- **Product**: A product is made that pollutes heavily and incurs unexpected legal costs.
- **Equipment**: Equipment is purchased without taking into account marketing information, and the task for which it was purchased disappears due to lack of demand for the product produced.

*An investment is not made when it should be:*

- Product: A product that requires more than two years of market development is not made in spite of high possible profits later.
- Equipment: Necessary production equipment that only has negative cash flow would never be purchased.

The sensible way to deal with such errors is to recognize that each investment has its own special characteristics, and the listed considerations should only be guidelines, not strict rules.

## Solutions to All the Additional Problems

**Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.**

### 4S.1

The new company has a total of \$1 500 000: 500 000 at 10% interest, 500 000 from investors who expect 15% growth, and \$500 000 from you, where you also expect 15% growth. So your weighted cost of capital (WCC) is

$$\text{WCC} = (1 \times 0.15 + 0.5 \times 0.1) / 1.5.$$

Your MARR should be at least as great as the WCC.

### 4S.2

Each of the windmills will produce  $2800 \times 365 \times 24 \times 0.25$  kWh/year = 6 132 000 kWh/year.

This represents an income of  $6\,132\,000 \times 0.049 = \text{£}300\,000/\text{year}$ .

So the payback period for a single windmill is about five years. The payback period for the project as a whole will be longer, as we see from this table:

End of Year	Cash Out (000)	Cash In (000)	Balance(000)	Total (000)
1	1500	0	-1500	-1500
2	1500	300	-1200	-2700
3	1500	600	-900	-3600
4	1500	900	-600	-4200
5	0	1200	1200	-3000
6	0	1200	1200	-1800

End of Year	Cash Out (000)	Cash In (000)	Balance(000)	Total (000)
7	0	1200	1200	-600
8	0	1200	1200	600

This shows that the payback period is seven and a half years.

For the nuclear power station, once the station comes online it will generate

Energy =  $10\,000 \times 365 \times 24 \times 0.8$  kWh/year = 70 000 000 kWh/year.

This will produce an income of £3 430 000 per year, from which Victoria deducts the £300 000 spent on uranium fuel to obtain a net income of £3 130 000 per year. So after the plant comes online at the end of the third year, it will be a further  $7000 / 3130 = 2.24$  years before the project becomes profitable. This shows that the payback period is 5.2 years.

Based on payback period, the nuclear power station looks like the better investment. However, note that if Victoria were to define the first project as building a single windmill, it would then appear that this was the better investment, since the payback time would be slightly shorter than that for the power station. Then, having chosen to build a single windmill, she could next year consider building one additional windmill. This would again have a payback period of five years, so again would be preferred to the nuclear power station. This illustrates the weakness of the “payback period” method of comparing projects.

Victoria now goes on to consider the “present worth” method of comparison, giving all figures in thousands of pounds:

$$\begin{aligned}
 \text{PW(windmill)} &= -1500(P/A, 0.2, 4)(F/P, 0.2, 1) \\
 &\quad + 300(P/A, 0.2, 20) + 300(P/A, 0.2, 20)(P/F, 0.2, 1) \\
 &\quad + 300(P/A, 0.2, 20)(P/F, 0.2, 2) + 300(P/A, 0.2, 20) \\
 &\quad (P/F, 0.2, 3) + 100((P/F, 0.2, 21) + (P/F, 0.2, 22) + \\
 &\quad (P/F, 0.2, 23) + (P/F, 0.2, 24))
 \end{aligned}$$

Note the factor of  $(F/P, 0.2, 1)$  in the construction costs is needed to represent the fact that this series starts at the *beginning* of the first year. There are several ways we could have represented the energy income. One alternative would have been to show it as an arithmetic increasing gradient over the first four years, followed by a uniform series, followed by an arithmetic decreasing gradient. The last line represents salvage income.

$$\begin{aligned}
 \text{PW (windmill)} &= -1500(P/A, 0.2, 4)(F/P, 0.2, 1) \\
 &\quad + 300(P/A, 0.2, 20)(1 + (P/F, 0.2, 1) + (P/F, 0.2, 2) + \\
 &\quad (P/F, 0.2, 3)) + 100((P/F, 0.2, 21) + (P/F, 0.2, 22) + \\
 &\quad (P/F, 0.2, 23) + (P/F, 0.2, 24))
 \end{aligned}$$



$$\begin{aligned}
 &= -1500(2.5887)(1.2000) \\
 &\quad + 300(4.8700)(1 + (0.8333) + (0.6944) + (0.5787)) \\
 &\quad + 100((0.02174) + (0.01811) + (0.01509) + (0.01258)) \\
 &= -4660 + 4538 + 6.75 \\
 &= -115.25
 \end{aligned}$$

The present worth of the windmill project turns out to be negative, showing that Victoria should not commit the company to doing it at all.

Victoria now calculates the present worth of the nuclear project, again in thousands of pounds:

$$\begin{aligned}
 \text{PW (nuclear)} &= -2333(P/A, 0.2, 3)(F/P, 0.2, 1) + 3130(P/A, 0.2, 20)(P/F, 0.2, 3) - \\
 &\quad 1000(P/F, 0.2, 23) \\
 &= -2333(2.1065)(1.2) + 3130(4.8700)(0.5787) - 1000(0.01509) \\
 &= -5897 + 8821 - 15.1 = 2909
 \end{aligned}$$

So the present worth of the nuclear project is about £2 909 000.

It is worth noting that the salvage and decommissioning costs have very little effect on the present worth, since they occur so far in the future. One reason that neither project looks very attractive is that the company has a high MARR for this type of enterprise—large, long-horizon projects of this kind are often undertaken by governments, which have lower values for MARR.

### 4S.3

The difference in annual costs between the old system and the new system is R 2 000 000 a year. So Dube should express all the one-time costs incurred by the replacement as their annual equivalents and see if they come to less than R 2 000 000.

We are not told what study period to use. It should certainly be longer than five years, but perhaps it would not be reasonable to think that a computer-based system will still be useable in fifty years. We decide to do a first analysis assuming a thirty-year life; it might then be prudent to consider the results for a twenty-year and a forty-year life and see if the answer changes.

We express all figures in thousands of rands:

$$\text{EUAC} = (6600 - 900)(A/P, 0.1, 30) + 2000(A/P, 0.1, 30)(P/F, 0.1, 0.5)$$

$$+ 2000((P/F,0.1,6) + (P/F,0.1,11) + (P/F,0.1,16) + (P/F,0.1,21) + (P/F,0.1,26))(A/P,0.1,30))$$

Note that we deal with the present costs of the R 2 000 000 additional wage bill for the current year by considering them to occur halfway through the year.

$$\begin{aligned} &= (5700)(0.1061) + 2000(0.1061)(0.9530) \\ &\quad + 2000(0.5645 + 0.3505 + 0.2176 + 0.1351 + 0.8391)(0.1061) \\ &= 605 + 202 + 287 = 1094 \end{aligned}$$

We see that John Dube will certainly save money by replacing the system. The margin is sufficiently large that we would not expect the conclusion to change with a ten-year change to the study period.

#### 4S.4

The feasible alternatives are:

1. Hire swim coach
2. Build sheep pens
3. Build sheep dip
4. Build sheep dip and sheep pens
5. Lease fields

The combinations (1,2), (2,5), (3,5), and (4,5) are also feasible. Their present worths can be calculated by summing the present worths of their component projects.

We proceed to calculate the present worth of each alternative, taking a study period of ten years. All figures are given in thousands of dollars:

##### 1. Hire Swim Coach:

$$PW = -120(P/A,0.1,10) + 50(A/G,0.1,10)(P/A,0.1,10)(F/P,0.1,1)$$

Note that the formula for an arithmetic gradient assumes that the first non-zero cash flow of the gradient occurs at the beginning of the *second* compounding period, not the first, so the factor  $(F/P,0.1,1)$  must be included to bring the cash flows one year closer to the present.

$$\begin{aligned} &= -120(6.1446) + 50(3.7255)(6.1446)(1.1) \\ &= 521.7 \end{aligned}$$

So the present worth of hiring the swim coach is \$521 700.

##### 2. Build Sheep Pens:

$$PW = -400 + 150(P/A,0.1,10) = -400 + 150(6.1446) = 521.7$$

So the present worth of building sheep pens is also \$521 700.

**3. Build Sheep Dip:**

$$PW = -450 + 60(P/A, 0.1, 10) = -450 + 60(6.1446) = -81.3$$

So the present worth of converting the pool to a sheep dip is -\$81 300.

**4. Build Sheep Pens and Sheep Dip:**

$$PW = -850 + (100+60+90)(P/A, 0.1, 10) = -850 + 250(6.1446) = 686.1$$

So the present worth of building sheep pen *and* a sheep dip is \$686 100.

**5. Lease Fields:**

$$PW = 350 - 50(P/A, 0.1, 10) - 200(P/F, 0.1, 10)$$

$$= 350 - 50(6.1446) - 200(0.3855) = -34.3$$

So leasing the fields to the Flinders Mining Company has a net present worth of -\$34 300.

Since project 5 has a negative present worth, the only combination worth considering is (1, 2). Projects 1 and 2 have no effect on each other's profitability, so the present worth of the combination is the sum of the individual present worths, which is \$1 043 400. This is better than the second-best alternative, building sheep pens and a sheep dip, so the best choice—from the economic point of view—is to hire the swim coach and convert the University's Philosophy Department to sheep pens.

## 4S.5

The weighted cost of capital to the company is

$$WCC = (0.08 (6\,000\,000 - x) + 0.2 x) / 6\,000\,000$$

where  $x$  is the total equity in the company.

The lowest value of MARR is obtained when the company raises as much money as possible through a bank loan. The bank will loan at most half of the Rs 6 000 000, in which case  $x = 3\,000\,000$  and the WCC is

$$WCC = (0.08 (6\,000\,000 - 3\,000\,000) + 0.2 (3\,000\,000)) / 6\,000\,000 \\ = 0.14$$

The highest value of MARR is obtained when the company sells the greatest number of shares, which is Rs 2 500 000. The total equity is then Rs 5 000 000, and the cost of capital is

$$WCC = (0.08 (6\,000\,000 - 5\,000\,000) + 0.2 (5\,000\,000)) / 6\,000\,000 \\ = 0.18$$

We now calculate the present worth of each of the three projects, giving all figures in millions of Rupees:

*Project A:*

$$PW = -2(P/A, i, 3) + 11(P/F, i, 3)$$

*Project B:*

$$PW = -4 - 2(P/F, i, 1) + (1 + 0.5(A/G, i, 6))(P/A, i, 6) + 1.8(P/F, i, 6)$$

*Project C:*

$$PW = -1.3 - (4 \times 0.8)(P/A, i, 6) + 2.5(P/A, 0.25, i, 6)$$

These three functions are calculated on the accompanying spreadsheet, **4S\_5.xls**, for values of  $i$  in the range 0.14 to 0.18.

<i>Interest Rate</i>	<i>PW(1)</i>	<i>PW(2)</i>	<i>PW(3)</i>
<b>0.1400</b>	2.7814	3.0799	3.0273
<b>0.1500</b>	2.6662	2.7919	2.8196
<b>0.1600</b>	2.5555	2.5184	2.6231
<b>0.1700</b>	2.4489	2.2584	2.4370
<b>0.1800</b>	2.3464	2.0112	2.2607

We see from the results that the best project depends on the MARR; at 14%, Project 2 is the best. For an MARR of 15–16%, Project 3 leads, while at an interest rate of 17–18%, the company should choose Project 1.

#### 4S.6

Under the first, lower leverage scheme, your debt to the bank by the end of the year is  $\$400\,000(F/P, 0.12, 1) = \$448\,000$ . If the company has grown as you expected, then after paying this off, the company's assets are  $\$2\,000\,000 - 448\,000 = \$1\,552\,000$ . You own one fifth of the company, so your personal wealth is now  $\$310\,400$ .

If you had followed the high-leverage scheme instead, you would have paid off a debt of  $\$896\,000$ , leaving the company with  $\$1\,104\,000$ . You own one half of this, so you have  $\$553\,000$ .

If, on the other hand, the company falls in value over the year, the low-leverage scheme will leave the company with  $\$900\,000 - 448\,000 = \$452\,000$ . Your one-fifth share of this is worth  $\$90\,400$ .

If you've followed the high-leverage scheme, your company is worth  $\$4000$  at the end of the year, and your half share of this is worth  $\$2000$ . Also, your fellow investors are not happy with you.

#### 4S.7

First, the company must find the weighted cost of capital. The effective interest rate on the common stock is 9%, so the WCC =  $0.8 \times 0.09 + 0.2 \times 0.12 = 9.6\%$ .

So if the investment is to earn twice the cost of capital, it has to earn 19.2% interest. So the most it can pay for the machine is  $15\,000(P/A, 0.192, 8) + 2000(P/F, 0.192, 8) = 55\,276 + 490 = \$55\,766$ .