Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Identify the coordinates of any local and absolute extreme points and inflection points. Graph the function.

$f(x) = \mathbf{ln}\left(11 - 2x^2\right)$

Identify the domain and any symmetries the function may have. Since the natural log is only defined for nonnegative numbers. The domain, shown below, is all values that cause $11 - 2x^2$ to be greater than 0.

$$\left(-\sqrt{\frac{11}{2}}, \sqrt{\frac{11}{2}}\right)$$

A function that satisfies $f(x) = f(-x)$ is considered an even function, while a function that satisfies $f(-x) = -f(x)$ is considered odd. Even functions are symmetric about the y-axis and odd functions are radially symmetric about the origin. The function $f(x) = \mathbf{ln}\left(11 - 2x^2\right)$ is an even function.

Knowing now that the function exists over $\left(-\sqrt{\frac{11}{2}}, \sqrt{\frac{11}{2}}\right)$ and that it is symmetric about the y-axis will help in identifying the graph. Next find the critical points of $f(x) = \mathbf{ln}\left(11 - 2x^2\right)$.

The critical points exist where $f'(x) = 0$ or $f'(x)$ is undefined. Find $f'(x)$.

$$\begin{aligned} f(x) &= \mathbf{ln}\left(11 - 2x^2\right) \\ f'(x) &= \frac{4x}{2x^2 - 11} \end{aligned}$$

Now set $f'(x) = \dfrac{4x}{2x^2 - 11}$ equal to 0 and solve for x. Note that $f'(x) = 0$ at $x = 0$.

Now solve for x when $f'(x)$ is undefined. Set the denominator equal to zero and solve for x.

$$\begin{aligned} 2x^2 - 11 &= 0 \\ 2x^2 &= 11 \\ x &= \pm\sqrt{\frac{11}{2}} \end{aligned}$$

Note that the values $x = \pm\sqrt{\frac{11}{2}}$ do not need to be considered because it was determined that $x = \pm\sqrt{\frac{11}{2}}$ are not in the domain of f(x), $\left(-\sqrt{\frac{11}{2}}, \sqrt{\frac{11}{2}}\right)$.

The only critical value to consider is $x = 0$, which does lie in the domain of f(x). To determine the nature of the critical point, apply the Second Derivative Test.

Find $f''(x)$.

$$\begin{aligned} f'(x) &= \frac{4x}{2x^2 - 11} \\ f''(x) &= \frac{-4\left(2x^2 + 11\right)}{\left(2x^2 - 11\right)^2} \end{aligned}$$

Use $f''(x)$ to determine whether the critical point is a maximum or minimum. Maximums occur when $f''(x) < 0$ and minimums when $f''(x) > 0$. Is $f''(0)$ is less than 0 because $f''(0) = -\dfrac{4}{11}$.

There is a maximum at $x = 0$ because $f''(0)$ is less than 0. Since this is the only critical point, the point at $x = 0$ is both a local maximum and an absolute maximum. The coordinates of this point are $(0, \ln 11)$.

Lastly, find the inflection points by finding x-values such that $f''(x)$ is equal to zero or undefined.

Set $f''(x) = \dfrac{-4(2x^2+11)}{(2x^2-11)^2}$ equal to 0 and solve for x. Note that there are no real x-values such that $f''(x) = 0$.

Use a similar process to find out where $f''(x) = \dfrac{-4(2x^2+11)}{(2x^2-11)^2}$ is undefined. Note that $f''(x)$ is undefined at $x = \pm\sqrt{\dfrac{11}{2}}$.

It was determined earlier that the values $x = \pm\sqrt{\dfrac{11}{2}}$ are not in the domain of f(x). Therefore, there are no inflection points.

Using the information gathered in the previous steps to graph the function. Note that the graph to the right exists between $\left(-\sqrt{\dfrac{11}{2}}, \sqrt{\dfrac{11}{2}}\right)$, is symmetric to the y-axis, and has a maximum at $(0, \ln 11)$.