202201 Math 122 Assignment 3

Due: Sunday, March 6, 2022 at 23:59. Please submit on your section's Crowdmark page.

There are five questions of equal value (worth a total of 45 marks), plus a left over question from Assignment 2 (worth 9 marks). There are 4 bonus marks available if the solutions are typeset with LaTeX. Information on obtaining and using LaTeXis available on the cross-listed Brightspace page.

Please feel free to discuss these problems with each other. You may not access any "tutoring" or "help" website in any way. In the end, each person must write up their own solution, in their own words, in a way that reflects their own understanding. Complete solutions are those which are coherently written, and include appropriate justifications.

$*$ (Assignment 2, Question 5) Answer each question True or False, and write a sentence or two to briefly explain your reasoning. Let $A = \{1, \{1\}, 2, \{\emptyset\}, \{\{1\}, \{2\}\}, \{\{1\}, 2\}\}$.

(a) $\{2\} \in A$

(b) $\{1, 2\} \subsetneq A$

(c) $\{\{1, \{2\}\}\} \subseteq A$

(d) $\emptyset \in A$

(e) $A \cap \mathcal{P}(A) = \emptyset$

(f) $\{2\} \in \mathcal{P}(A)$

(g) $\emptyset$ is the only set with no non-empty proper subset.

1. Prove that for all sets A, B and C, $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$ by:

   (a) using set-builder notation and showing the LHS and RHS are defined by logically equivalent expressions (give reasons for each step);

   (b) using the Laws of Set Theory (give reasons for each step);

2. Let A and B be sets.

   (a) Prove that if $A \subseteq B$, then $A \setminus B = \emptyset$.

   (b) Prove that if $A \setminus B = \emptyset$, then $A \cup B = B$.

   (c) Prove that if $A \cup B = B$, then $A \subseteq B$.

   (d) Are the three statements (i) $A \subseteq B$, (ii) $A \setminus B = \emptyset$, and (iii) $A \cup B = B$ all logically equivalent? Explain your reasoning.

3. Let A and B be sets. Use the same method as in 2. to show that the three statements (a) $A = B$, (b) $A \cup B = A \cap B$, and (c) $A \oplus B = \emptyset$ are all logically equivalent.

4. Let A, B, C be sets. In each part, if the given statement is true then prove it, and it it is false then give a counterexample. In the case that the statement is false and a Venn diagram suggests that one of the sets is a subset of the other, state the relationship that's suggested.

   (a) For all sets A, B, C, $(A \setminus B) \setminus C = A \setminus (B \setminus C)$.

   (b) For all sets A, B, C, $(A \oplus B^c) \oplus C = A \oplus (B^c \oplus C)$.

5. Let A_1, A_2, A_3, A_4 be sets.

   (a) Give a definition of $A_1 \cup A_2 \cup A_3 \cup A_4$ and $A_1 \cap A_2 \cap A_3 \cap A_4$.

   (b) Prove that $(A_1 \cup A_2 \cup A_3 \cup A_4)^c = A_1^c \cap A_2^c \cap A_3^c \cap A_4^c$.

   (c) Write down the dual of the statement in (b). No proof is required.