For the questions #1 − #4, calculate the following limits.
Do not round your answer. Leave your answer exact.

1. (1 point) $\lim_{x\to 1} \dfrac{x^3-1}{\ln x}$

(A) -3 (B) -2 (C) -1 (D) $-\dfrac{1}{2}$ (E) Does not exist

(F) $\dfrac{1}{2}$ (G) 1 (H) 2 (I) 3 (J) None of those

2. (1 point) $\lim_{t\to \pi} \dfrac{\sin t}{t^2-\pi^2}$

(A) $-\dfrac{1}{2\pi}$ (B) -2 (C) -1 (D) $-\dfrac{1}{2}$ (E) $\dfrac{4}{25}$

(F) $\dfrac{1}{2\pi}$ (G) 2 (H) 1 (I) $\dfrac{1}{2}$ (J) None of those

3. (1 point) $\lim_{x\to 1} \dfrac{x^{1/3}-1}{x^{2/3}-1}$

(A) -3 (B) -2 (C) -1 (D) $-\dfrac{1}{2}$ (E) Does not exist

(F) $\dfrac{1}{2}$ (G) 1 (H) 2 (I) 3 (J) None of those

4. (1 point) $\lim_{x\to\infty} \left(1+\sin\dfrac{1}{3x}\right)^{x}$

(A) $\dfrac{1}{3}$ (B) 1 (C) 2 (D) 0 (E) Does not exist

(F) $e^{1/3}$ (G) e (H) e^{2} (I) e^{3} (J) None of those

5. (1 point) Find differential dy at $x = 1$ when $y = x^5 + 37x$, and $dx = 0.2$

(A) 0.2 (B) 1 (C) 3.8 (D) 4.2 (E) 8.4

(F) 12.0 (G) 24.0 (H) 38.0 (I) 42.0 (J) None of those

6. (2 points) For the function $f(x) = \dfrac{1}{x}$ on the interval $[a, b] = [1, 2]$ find the value c that satisfies the Mean Value Theorem: $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

(A) $-\sqrt{2}$ (B) -1 (C) $-\dfrac{1}{2}$ (D) $-\dfrac{1}{4}$ (E) 0

(F) $\dfrac{1}{4}$ (G) $\dfrac{1}{2}$ (H) 1 (I) $\sqrt{2}$ (J) None of those

7. (2 points) A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 360 m of wire at your disposal, what is the largest area you can enclose?

(A) 90 (B) 100 (C) 180 (D) 270 (E) 400

(F) $16,000$ (G) $16,200$ (H) $16,400$ (I) $16,600$ (J) None of those

8. (2 points) Find a point of inflection $(c, f(c))$ for the function $f(x) = (x^2 - 1)^{2/3}$ with $c > 0$.

The derivatives of this function are $f'(x) = \dfrac{4x}{3(x^2-1)^{1/3}}$, $f''(x) = \dfrac{4(x^2-3)}{9(x^2-1)^{4/3}}$.

(A) $(1, 0)$ (B) $\left(\sqrt{3}, 2^{2/3}\right)$ (C) $\left(\sqrt{3}, 0\right)$ (D) None of those

(E) $(0, 1)$ (F) $\left(\sqrt{3}, -2^{2/3}\right)$ (G) $(3, 4)$ (H) No inflection point with $c > 0$

9. (2 points) Find an interval on which function $f(x) = xe^{-x^2}$ is increasing.
   The derivative of this function is $f'(x) = e^{-x^2}(1 - 2x^2)$.

(A) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{1}{\sqrt{2}}, +\infty\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, 0\right)$ (D) $\left(-\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right)$

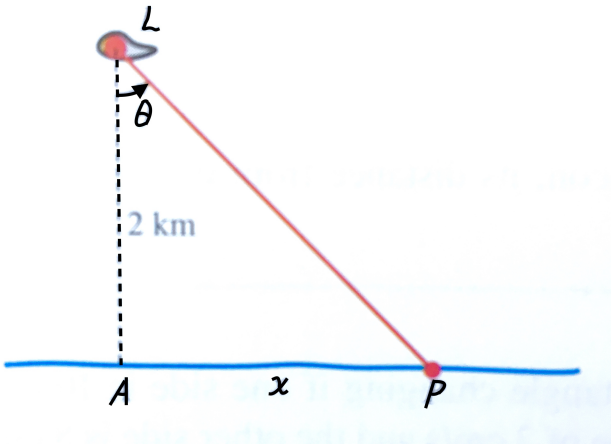
(E) $\left(-\infty, +\frac{1}{\sqrt{2}}\right)$ (F) $\left(+\frac{1}{\sqrt{2}}, +\infty\right)$ (G) $\left(0, +\frac{1}{\sqrt{2}}\right)$ (H) None of those

10. (2 points) Perform one iteration of the Newton's method to find the x coordinate of the point of intersection of the two curves $y = x^3$ and $y = x+1$ starting from the best guess $x_0 = 1.5$. Insure that you round your answer only once, when you are selecting the multiple choice answer.

(A) 1.25 (B) 1.30 (C) 1.35 (D) 1.40 (E) 1.45

(F) 1.50 (G) 1.55 (H) 1.60 (I) 1.65 (J) 1.70

11. (2 points) A lighthouse L is located on a small island 2 km from the nearest points A on a long, straight shoreline. If the lighthouse lamp rotates at 3 revolutions per minute, how fast is the illuminated spot P on the shoreline moving along the shoreline when it is 4 km from A?



You might find the following information useful:

1) one revolution corresponds to 2π radians.

2) $1 + \tan^2 x = \sec^2 x$

(A) 10 (B) 15 (C) 48 (D) 60 (E) 6π

(F) 10π (G) 15π (H) 48π (I) 60π (J) None of those