Student: Arfaz Hossain	**Course:** MATH 100 (A01, A02, A03) Fall 2021
Instructor: UVIC Math	**Book:** Thomas' Calculus Early Transcendentals, 14e
Date: 12/07/21	**Time:** 22:48

Find the limit of the rational function (a) as $x \to \infty$ and (b) as $x \to -\infty$.

$$f(x) = \frac{6x^4 + 6}{x^4 - x^2 + x + 4}$$

---

a. To find the limit of the rational function as x approaches ∞, divide the numerator and denominator by the highest power of x in the denominator, which is x^4.

Divide and simplify.

$$\frac{\dfrac{6x^4}{x^4} + \dfrac{6}{x^4}}{\dfrac{x^4}{x^4} - \dfrac{x^2}{x^4} + \dfrac{x}{x^4} + \dfrac{4}{x^4}} = \frac{6 + \dfrac{6}{x^4}}{1 - \dfrac{1}{x^2} + \dfrac{1}{x^3} + \dfrac{4}{x^4}}$$

The limit, as x approaches ∞, for $\dfrac{1}{x^n}$ when n is any positive number is shown below.

$$\lim_{x \to \infty} \left(\frac{1}{x^n} \right) = 0$$

So, all the terms with x^n in the denominator go to zero as x approaches ∞. Use that information to find the limit of the entire function as x approaches ∞.

$$\lim_{x \to \infty} \left(\frac{6x^4 + 6}{x^4 - x^2 + x + 4} \right) = \lim_{x \to \infty} \left(\frac{6 + \dfrac{6}{x^4}}{1 - \dfrac{1}{x^2} + \dfrac{1}{x^3} + \dfrac{4}{x^4}} \right)$$
$$= 6$$

So, the limit as x approaches ∞ of the function $f(x) = \dfrac{6x^4 + 6}{x^4 - x^2 + x + 4}$ is 6.

b. Using the same process, find the limit of the function as x approaches $-\infty$.

$$\lim_{x \to -\infty} \left(\frac{6x^4 + 6}{x^4 - x^2 + x + 4} \right) = \lim_{x \to -\infty} \left(\frac{6 + \dfrac{6}{x^4}}{1 - \dfrac{1}{x^2} + \dfrac{1}{x^3} + \dfrac{4}{x^4}} \right)$$
$$= 6$$

So, the limit as x approaches $-\infty$ of the function $f(x) = \dfrac{6x^4 + 6}{x^4 - x^2 + x + 4}$ is 6.