Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Does the series $\sum_{n=1}^{\infty} (-1)^n n^7 \left(\frac{2}{11}\right)^n$ converge absolutely, converge conditionally, or diverge?

A series $\sum a_n$ converges absolutely (is absolutely convergent) if the corresponding series of absolute values, $\sum |a_n|$ converges. If the series converges, but is not absolutely convergent, then the series converges conditionally. Otherwise, the series diverges.

Find the terms of the corresponding series of absolute values.

$$\left|(-1)^n n^7 \left(\frac{2}{11}\right)^n\right| = n^7 \left(\frac{2}{11}\right)^n$$

Determine the behavior of the series of absolute values using the Ratio Test. Let $\sum a_n$ be any series and let $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$. Then the series converges absolutely if $\rho < 1$, the series diverges if $\rho > 1$ or ρ is infinite, or the test is inconclusive if $\rho = 1$.

The term $|a_{n+1}|$ is $(n+1)^7 \left(\frac{2}{11}\right)^{n+1}$.

Find the ratio of terms $\frac{a_{n+1}}{a_n}$.

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^7 \left(\frac{2}{11}\right)^{n+1}}{n^7 \left(\frac{2}{11}\right)^n} = \left(\frac{n+1}{n}\right)^7 \left(\frac{2}{11}\right)$$

Evaluate the limit.

$$\frac{2}{11} \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^7 = \frac{2}{11}$$

The series of absolute values converges because $\rho < 1$.

The given series converges absolutely because the corresponding series of absolute values converges.