Math 101 Tutorial Worksheet 5

There is an associated quiz due on BrightSpace on Tuesday, February 15 at 10:00 PM

1. Find the limits of the following sequences, if they converge.

   (a) $a_n = \ln\left(1 + \dfrac{1}{n}\right)^n$

   (b) $a_n = \sqrt{n}(\sqrt{n+3} - \sqrt{n})$

2. Consider the sequence defined by the recursive formula:

   $$a_{n+1} = \sqrt{1 + a_n} \qquad \text{with} \qquad a_1 = \sqrt{1} \quad \text{and} \quad n \geq 1$$

   Given that the sequence converges, what does it converge to?

3. Determine the length of $y = \dfrac{2}{3}(x-1)^{\frac{3}{2}}$ between $1 \leq x \leq 4$.

4. Find the equation of a curve that passes through the point $(1, 5)$ and has an arc length on the interval $[2, 6]$ given by $\displaystyle\int_2^6 \sqrt{1 + 16x^{-6}}dx$.

5. The Gateway Arch in St. Louis was constructed using the equation

   $$y = -315\left(e^{x/240} + e^{-x/240}\right) + 1260$$

   for the central curve of the arch, where x and y are measured in feet with $|x| \leq 315$. Set up (but do not solve) the arc length equation to estimate the length of the Gateway Arch.

6. The number of bacteria, $N(t)$, after t minutes satisfies the differential equation $\dfrac{dN}{dt} = kN$ for some nonzero constant k. When the colony was checked 60 minutes after beginning of the observations, the number of cells present has increased 4.5 times comparing to the number of cells at the beginning of the observations.

   (a) Determine the constant k that describes the growth of this colony.

   (b) How long will it take for the number of bacteria to reach 8 times the initial population size?