ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 25: Decision Trees

November 3, 2021

Version 1

Required Reading

- Magee, J. F. (1964). Decision Trees for Decision Making. *The Harvard Business Review*, pp. 126 – 138. Retrieved from https://hbr.org/1964/07/decision-trees-for-decision-making
  - **A thorough walkthrough of decision tree analysis. Includes a worked example.**
- A Decision Tree Primer – Written by the Instructor. Included with the lecture files.

Recommended Reading

- *Engineering Economics*, Chapter 12, Sections 12.4 - 12.5
- Jha, S. & Powell, A. (2014). A (Gentle) Introduction to Behavioral Economics, *American Journal of Roentgenology*, 203(1), 111-117. http://www.ajronline.org/doi/abs/10.2214/AJR.13.11352

Relevant Solved Problems

- Expected Value: Example 12.4, Example 12.5, 12.9, 12.10, 12.11, 12.12, 12.29, 12.30, 12.31, 12.32
- Decision Trees: 12.13, 12.14, 12.15, 12.33, 12.35, 12.36, 12.37, 12.43, 12.44

Learning Objectives

- To be able to create, use an interpret decision trees in the context of project evaluation.
- (Optional) To become familiar with the concept of expected value, if that is not already the case, and be able to use it when creating and reading decision trees.

ESSENTIALS (15 slides)

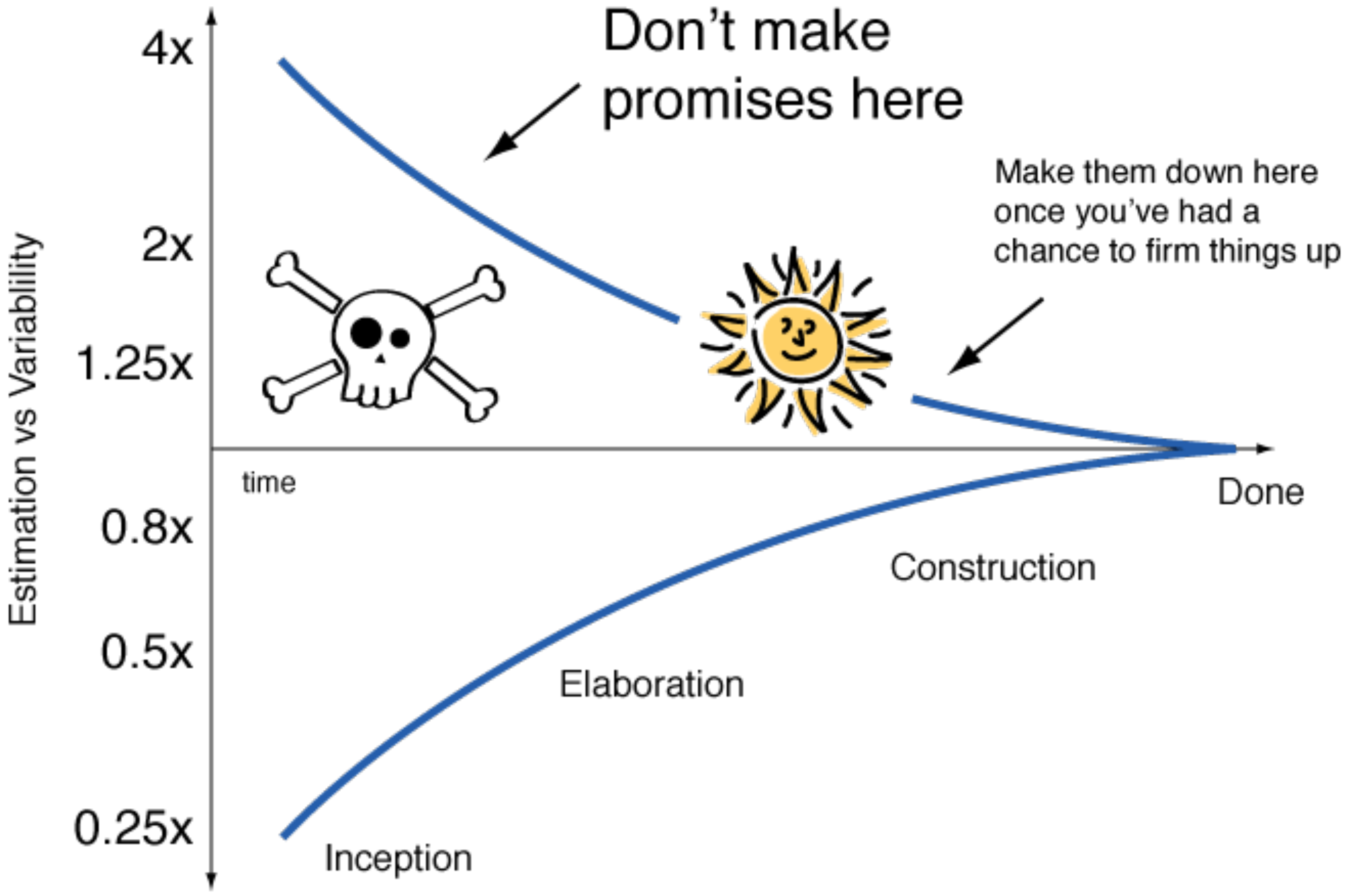
What if we vary more than one parameter?

- We now have tools to show how sensitive our calculations are to changes in <u>one</u> parameter at a time: Tornado Diagrams, Spider Plots
- We can also vary several parameters at a time, using scenarios.
- These assume there's a single 'true' value out there…
- What if our values are *probabilistic*, so that the 'true' value will be random?
- Over the next few lectures we'll develop tools to deal with that.

Stages to project uncertainty

- Over the life of an engineering project…
- The nature of decisions changes
- Knowledge is accumulated

- →The comfort range of an estimate narrows (asymptotically) toward certainty
- Implication: estimates should be revisited at each stage of a project
- There is a ‘cone of uncertainty’



(Source: AgileNutshell – no longer online)

What % is that?

- Much of the literature on decision-making given uncertainty assumes you have a probability (distribution) for what you’re looking at…
- …so that you can compute expected values.
- How can we get those in a practical project setting, starting from best case / worst case estimates?
- One approach: use standard deviations.
- If we can calculate a standard deviation for our estimate(s)…
- widely available tables (and apps) let us turn those into confidence intervals/likelihoods …
- …given basic assumptions about the distribution.

More commonly…

- When working on a project, you'll have a good idea of the possible range of values for given variables, and their distribution.
- The probability distribution of the COMBINATION of these variables into a present value, IRR, BCR and so on may not be easy to determine.
- The Monte Carlo method of dealing with uncertainty (named after the casino) is to run the calculations many different times, with random variables fitting the desired distributions. (We'll look at this later today.)
- A probability distribution for the complex value can then (hopefully) be inferred from the results.
- For many common cases, this is overkill. For simpler problems, a decision tree is often good enough - or even optimal.

Dealing with certain uncertainties

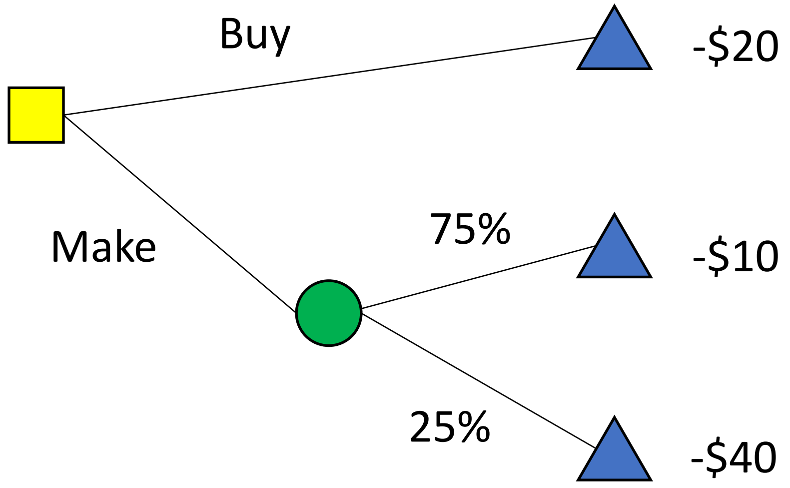
- Popsicle stands do well in hot weather, and poorly in cold weather.
- Suppose you KNOW the chances of a summer season hot or cold (maybe from access to historical weather statistics) – how do you figure out if you should buy, rent or stay out of the popsicle business?
- We use the concept of expected value: the sum of outcomes weighted by the probability of that outcome occurring.
- e.g. If you have a 50/50 chance of winning or losing \$1 on a coin toss, the expected value is 0.50 x \$1 – 0.50 x \$1 = 0.
- We also use decision trees to help clarify the situation.

What’s a decision tree?

- Identifies expected costs/benefits through all possible choices and outcomes of a project.
- **Decision Node**: Square (by convention), and represents a choice. Usually, the first node in the tree is a decision node.
- **Chance Node**: Round (by convention), and represents a known uncertainty, in the sense that we can/must attach probabilities to it.
- **Terminal Node**: Triangular or | (by convention), and represents the expected value of a unique outcome. To be useful, a tree must be comprehensive, and have a terminal node associated with any possible outcome.
- A well-crafted decision tree will usually have a <u>choice</u> on the far left and <u>outcomes</u> on the far right.
- Left → Right ordering usually follows the sequence of events…
- …but available data may dictate deviations.

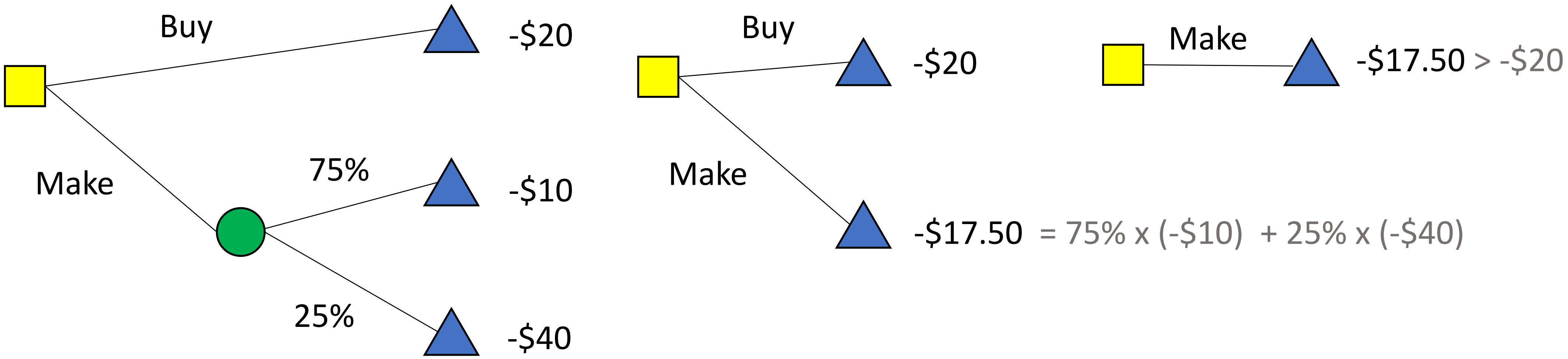
A simple example

- You can buy or make a Halloween costume.
- If you buy the costume, it costs \$20.
- If you make the costume…
  - There's a 75% chance everything will go well and it will cost you only \$10.
  - There's a 25% chance you'll mess it up, and the total cost will be \$40.
- What should you do?



Rolling Back the Decision Tree

- At the very *end* of the decision tree, you have all possible outcomes.
- Starting at the end, you 'roll them back' using expected value at each stage. In our simple example, the expected cost of making your own costume is 75% x 10 + 25% x 40 = \$7.50 + \$10 = \$17.50
- Since this is less than the other option, buy for \$20, we choose to make the costume.
- When rolling back, at CHANCE nodes, calculate the expected value. At CHOICE nodes, choose the preferred outcome.

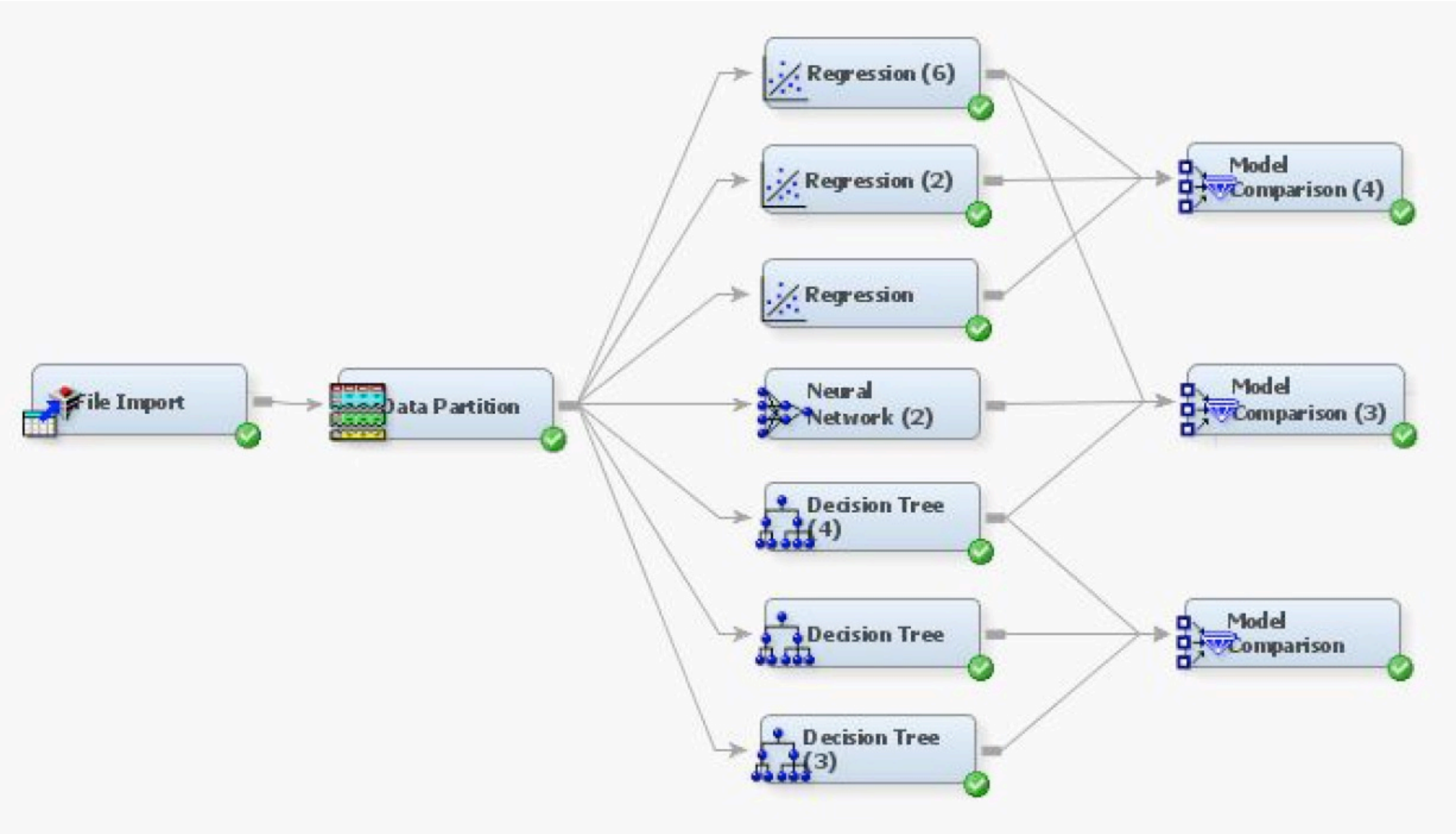


Interpreting the tree

- Number atop line (branch): conditional probability of that branch. (e.g. making the costume, 25% chance of failure.)
- Number at far right: value of a particular outcome. (e.g. Cost of having successfully made the costume, is $10.)
- Square (Decision) Node: Split due to choice (e.g. Make or Buy?)
- Circular (Event) Node: Split due to chance/nature (e.g. Fail or not?)
- Number by Node: expected value of everything connected and to the right of the node.
- Left-most Node: expected value of the entire project. (Obtained by choosing the choice nodes with the highest expected value.)

Decision Trees and Data Mining

- Decision Trees have found new life as a cornerstone of data mining, due to a number of desirable characteristics:
  - Fast to build
  - High precision
  - Simple calculations
  - Output is easy to interpret (and can be written in the form of ‘rules’)
  - Able to deal with missing data values
- One drawback: trees can get ‘explosive’ very quickly.
- Once the tree gets too large, a Monte Carlo simulation should be used. Good thing we’re about to look at that technique!



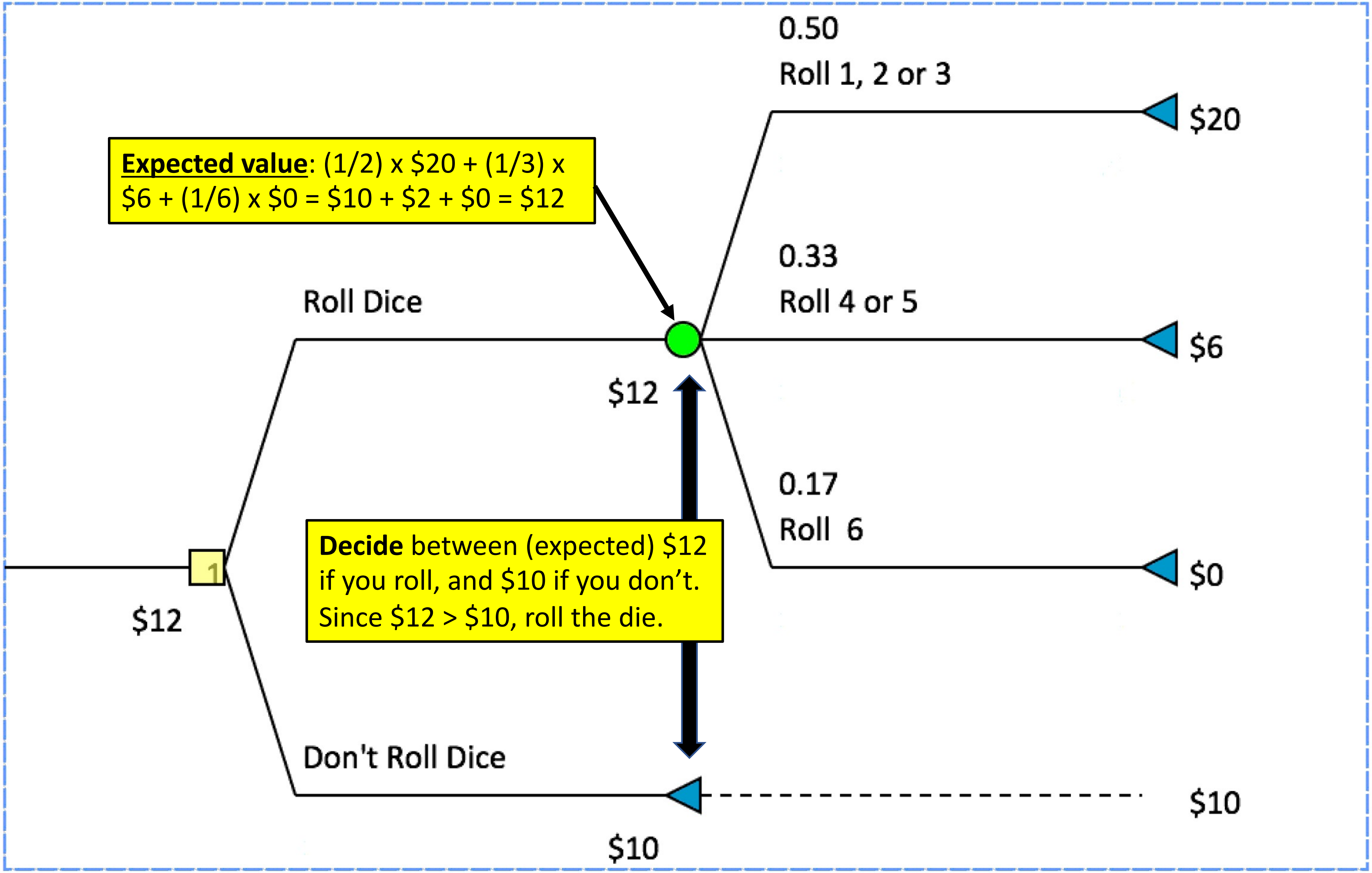
Example of using Decision Trees in data mining using SAS Enterprise (Serban et al.)

Let’s work through a simple example…

- You win a contest, and are presented with the following offer regarding your prize:
- You must choose between accepting $10, or rolling a die.
- The die is a fair, six-sided die. (Each of the numbers between 1 and 6 is equally likely to come up.)
- If you choose to roll the die:
- On a roll of 1, 2 or 3, you earn $20
- On a roll of 4 or 5, you earn $6
- On a roll of 6, you earn $0 (nothing)
- Should you roll a die or take the $10?

What are the *decisions*, outcomes, & payoffs?

- **First**: YOU decide whether to take the \$10, or roll the die.
- This leads to a **single certain payoff of \$10.**
- IF you chose to roll the die, THEN ”Nature” chooses the outcome.
- **Roll 1,2 or 3** (probability = 3/6 = 1/2): Get **\$20**
- **Roll 4 or 5:** (probability = 2/6 = 1/3) Get **\$6**
- **Roll 6:** (probability = 1/6) Get **\$0**
- We now have everything we need to draw our tree.
- One Choice node, one Chance node, 4 possible outcomes.
- (It’s also correct, if less elegant, to consider 7 different outcomes: each of the faces of the die, if it’s rolled, plus the sure \$10 if it is not rolled.)


0.50
Roll 1, 2 or 3
$20
Expected value: (1/2) x $20 + (1/3) x $6 + (1/6) x $0 = $10 + $2 + $0 = $12
0.33
Roll 4 or 5
Roll Dice
$6
$12
0.17
Roll 6
Decide between (expected) $12 if you roll, and $10 if you don't. Since $12 > $10, roll the die.
1
$0
$12
Don't Roll Dice
$10
$10

AFTER HOURS

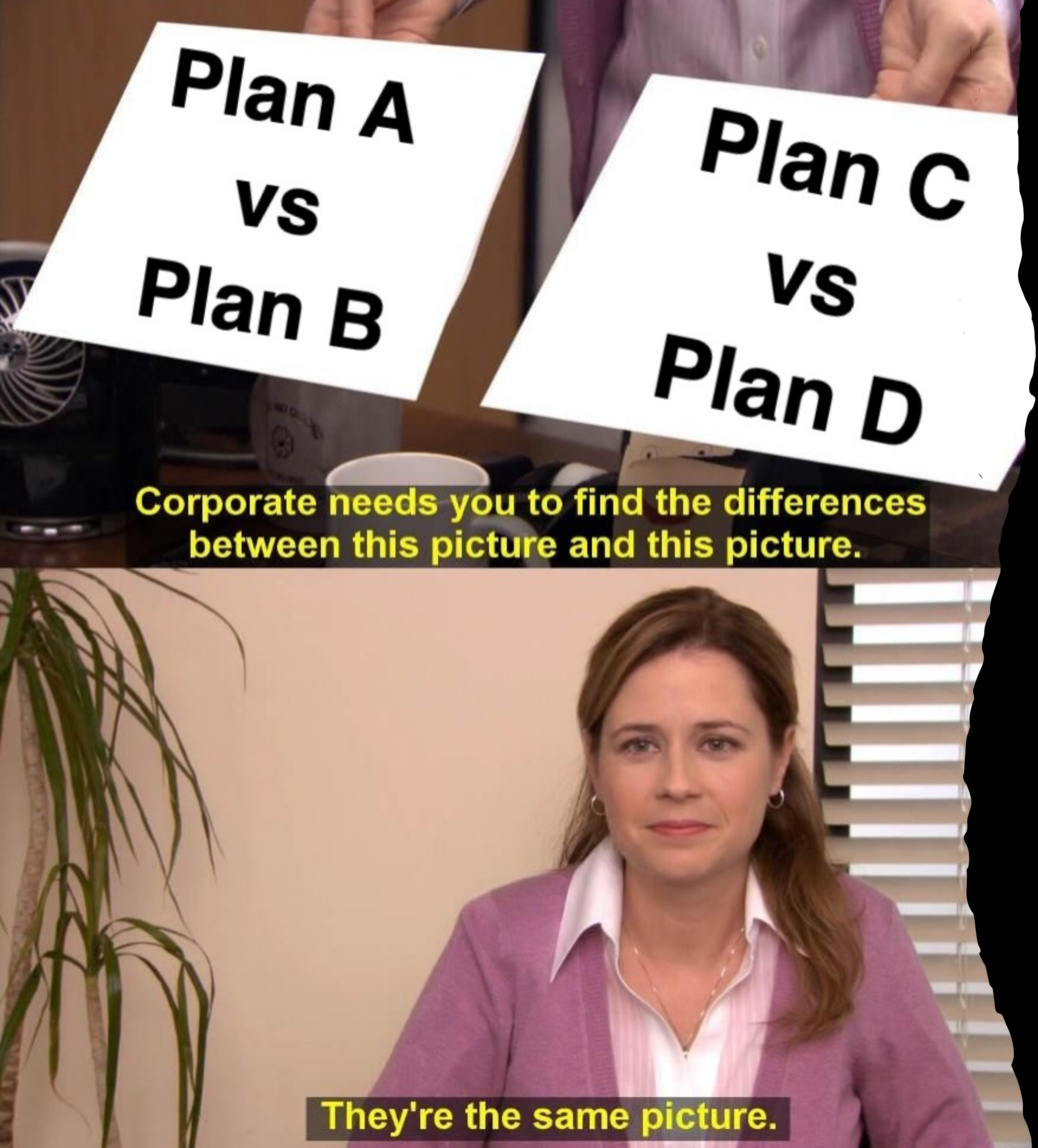
- Prospect theory (14 slides)

Wait, that doesn't feel right…

- Think it's odd someone would treat 12 *expected* dollars as if they were *certain* dollars for decision making?
- You're not alone.
- This is a simple example for an introductory lecture, but there are entire sub-disciplines in economics that study (among other things) ways in which how humans value payoffs deviate from mathematical expectations.
- One of the most important: **behavioral economics** (a field in which psychologists have won Nobel prizes in economics).
- An early result of behavioral economics was the introduction of *prospect theory*.

Two cities with two plans

- Two cities in the same province need to choose a vaccination plan for COVID-19. Each city has two plans to choose from.
- Without vaccination, 600 people in each city are expected to die. The provincial planner is aware of this.
- City I has to choose between Plan A and Plan B.
- Plan A: 200 people will be saved.
- Plan B: 1/3 chance that 600 people are saved, and a 2/3 chance that no one is saved.
- City II has to choose between Plan C and Plan D.
- Plan C: 400 people will die.
- Plan D: 1/3 chance no one dies, 2/3 chance 600 people die.
- What plans should the provincial planner choose?



Two cities?

- You probably noticed A=C, B=D.
- It's the same choice in each city, just phrased differently.
- The planner should therefore choose A&C, or B&D.
- Anything else is inconsistent.

Survey Says…

- Behavioral economists Kahneman & Tversky ran this experiment with real people.
- In the study, 72% chose Treatment A for City I…
- …and 78% chose Treatment D for City II.
- But remember: A = C and B = D.
- This result violates the invariance of traditional preferences (the idea that *what* you say matters, not *how* you say it)…. But why?
- Phrasing of Plan A: SAVE 200 people, with certainty, vs a chance of saving people.
- Phrasing of Plan D: A chance no one DIES, vs certainty that 400 people die.
- Look closely at the options: one was phrased (framed) in terms of gain, and one was framed in terms of loss.
- Plan A vs Plan B: Certain gains vs uncertain gains, certain gains win.
- Plan C vs Plan D: Uncertain losses vs certain losses, uncertain losses win.
- Humans react differently to risk related to loss, and risk related to gain.

Gains and Losses

- When it comes to gains, we are <u>risk averse</u>: we prefer certainty.
- When it comes to losses, we are <u>risk seeking</u>: we prefer uncertainty.
- Another set of Kahneman/Tversky experiments illustrates this.
- Would you rather have A) $450, or B) 50% chance of $1,000 and 50% chance of $0?
- Most chose A, preferring a certain gain to an uncertain one.
- Would you rather have C) $3,000 taken from you, or D) an 80% chance of losing $4,000, and a 20% chance of losing nothing?
- Most chose D, preferring an uncertain loss to a certain loss.

Reference Points and the Status Quo

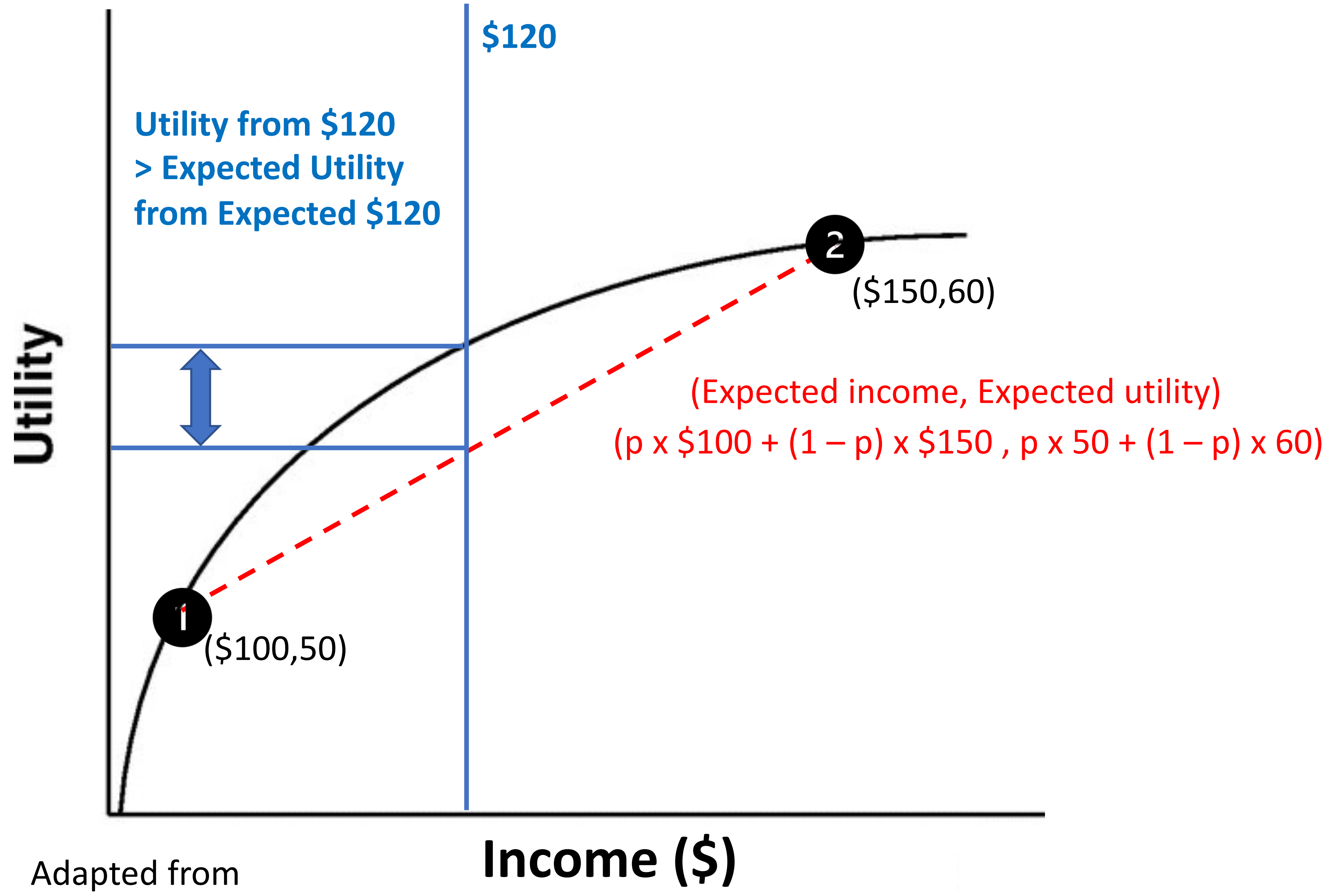
- As the two-city example shows, whether our minds work in the (risk-averse) gain domain or in the (risk-seeking) loss domain depends on how the situation is framed.
- Gains and losses are relative to a reference point. This reference point tends to be the *status quo*, the way things are now.
- Treatment A: ‘200 people will be saved’… relative to an implicit reference point of no one being saved. We’re in the gain domain.
- Treatment C: ‘400 people will die’… relative to an implicit reference point of no one dying. We’re in the loss domain.

Prospect Theory and Status Quo Bias

- These characteristics are the core of Prospect Theory:
- Risk aversion in the gain domain
- Risk seeking in the loss domain
- Gain or loss relative to a reference point matters more than absolute gain or loss.
- Loss aversion → We dislike losing more than we like winning.
- Where uncertainty exists, potential losses from a change will often overwhelm potential gains. The status quo has inertia.
- Incumbents tend to win elections, and negotiations often break down as each party over-values their concessions.
- Let’s take a look at how this works.

Some notes on expected utility (satisfaction)

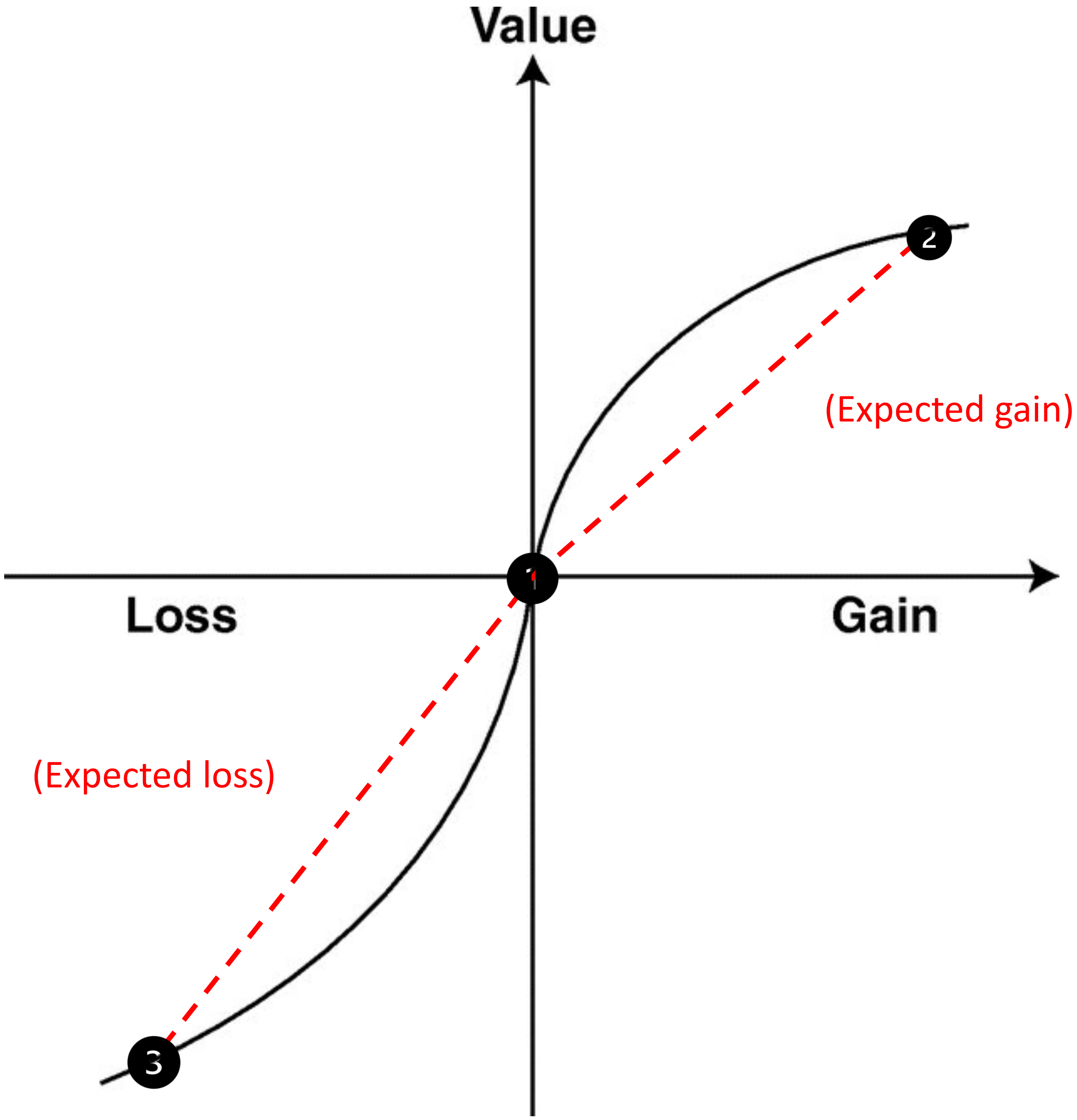
- Let’s suppose that you currently have \$100, but there’s a chance that you could earn an extra \$50, bringing you up to \$150.
- Suppose your utility U(\$100) = 50, and U(\$150) = 60.
- If p is your chance of winning the \$50, then…
- Expected income = p x \$150 + (1 – p) x \$100
- Expected utility = p x 60 + (1 – p) x 50
- If you were to plot this for all possible values of p on the (x,y)=(\$,U) plane, you would get a straight line going from (\$100,50) when p = 0 to (\$150,60) when p = 1.
- For a traditional utility function, this line is *everywhere below* the utility function between \$100 and \$150.
- (Traditional utility functions sees each succeeding dollar add less and less to utility – this is called the diminishing marginal utility of income.)
- →The utility from an *expected* gain is less than the utility from an *actual* gain.
- This is one way of looking at where ‘we like our gains certain’ comes from.



Adapted from
(Jha & Powell, 2014)

What does prospect theory do differently?

- Losses & gains are treated differently, so:
- *Origin* of the utility curve is at the current situation (status quo).
- →*Origin changes* as situation changes.
- Utility function is S-shaped, with the twist about the origin.
- →For gains, utility works like we just saw: **expected** utility from **expected** income of $120 (say) is less than utility from $120.
- BUT for *losses*, the expected utility line is *above* the utility function.
- This accounts for humans preferring *uncertain* losses to certain ones.



Adapted from
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Framing Matters

- *Framing* refers to **how** you present information.
- If you want to see something happen – if you want to convince someone to take action – it is **not enough** to present raw information and results.
- You need to be very careful about **how** you present that information.
- There are many aspects to framing, but we will focus on one of the most basic concerns:
- Framing things in terms of gains, vs framing things in terms of losses.
- This leads people receiving these messages to operate in the *gain domain*, or in the *loss domain*.

Prospect Theory and Incentive Design (Jha/Powell)

- A US association made modest payments to physicians who adhered to a number of quality and safety standards.
- Due to the small size of the payments, they didn't work well as incentives. The association decided to use prospect theory:
- At the start of the year, it would give physicians one half of their total possible annual reward.
- The association then said it would take away the second payment unless physicians adhered to the standards.
- It worked. This phrasing put physicians into the loss domain, by creating an expectation of reward, and then threatening its removal.

Aside: The Endowment Effect

- The association's policy makes use of what is known as the endowment effect: "An item is valued higher once possessed than before the purchase." (Jha/Powell)

- When car dealerships let you take a test drive, or infomercials promise a free trial of an exercise machine, they're taking advantage of this well-known effect.