Math 110

P1Q1

$$A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Let the eigenvalue be λ

$$(A-\lambda I) = \begin{bmatrix} 2-\lambda & 0 & 2 & 0 \\ 0 & 3-\lambda & 0 & 0 \\ 4 & 4 & 4-\lambda & 4 \\ 0 & 0 & 0 & 3-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = -0 + (3-\lambda)\begin{vmatrix} 2-\lambda & 2 & 0 \\ 4 & 4-\lambda & 4 \\ 0 & 0 & 3-\lambda \end{vmatrix} - 0 + 0$$

$$= (3-\lambda)\left\{ +0 - 0 + (3-\lambda)\begin{vmatrix} 2-\lambda & 2 \\ 4 & 4-\lambda \end{vmatrix} \right\}$$

$$= (3-\lambda)(3-\lambda)\{(2-\lambda)(4-\lambda) - 8\}$$

$$= (3-\lambda)(3-\lambda)(2-\lambda)(4-\lambda) - 8(3-\lambda)(3-\lambda)$$

$$= (3^2+\lambda^2-2\cdot 3\cdot\lambda)(2(4-\lambda)-\lambda(4-\lambda)) - 8(3^2+\lambda^2-6\lambda)$$

$$= (9+\lambda^2-6\lambda)(8-2\lambda-4\lambda+\lambda^2) - (72-8\lambda^2+48\lambda)$$

$$= 9(8-2\lambda-4\lambda+\lambda^2)+\lambda^2(8-2\lambda-4\lambda+\lambda^2)-6\lambda(8-2\lambda-4\lambda+\lambda^2)$$

$$-72+8\lambda^2-48\lambda = 72-18\lambda-36\lambda+9\lambda^2+8\lambda^2-2\lambda^3-4\lambda^3+\lambda^4$$

$$-48\lambda+12\lambda^2+24\lambda^2-6\lambda^3 = \lambda^4-12\lambda^3+53\lambda^2-102\lambda+72$$

$$\cancel{(\lambda^2)^2-12\lambda(\lambda^2)+53(\lambda^2)} = \lambda^4-12\lambda^3+45\lambda^2-54\lambda$$

Kilroy

$$= \lambda(\lambda^3-12\lambda^2+45\lambda-54) = \lambda(3-\lambda)^2(\lambda-6)$$

$$= (3-\lambda)^2\left(2(4-\lambda)-\lambda(4-\lambda)-8\right)$$

$$= (3-\lambda)^2(\cancel{8}-2\lambda-4\lambda+\lambda^2-\cancel{8})$$

$$= (3-\lambda)^2(-6\lambda+\lambda^2)$$

$$= (3-\lambda)^2(\lambda^2-6\lambda)$$

$$= \lambda(3-\lambda)^2(\lambda-6)$$

$\lambda = 0, 3, 6$

The eigenvalues are 0, 3, 6 given $\det((A-\lambda I)) = 0$

Given $A = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 4 & 4 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix}$

$\det(A - \lambda I) = \lambda(\lambda-3)^2(\lambda-6) = 0$

where $\lambda = 3, 6, 0$

so, $\boxed{Alg(3) = 2}$

$\boxed{Alg(6) = 1}$

$\boxed{Alg(0) = 1}$

For $\lambda = 0$

$E_0 A = null(A - 0I)$

$$= \left[\begin{array}{cccc|c} 2 & 0 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 3 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{2}R_1,\ \frac{1}{3}R_2,\ \frac{1}{3}R_4} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 4 & 4 & 4 & 4 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 - R_1,\ R_3 - R_2,\ R_3 - R_4} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

We get: $x + z = 0$ or, $x = -z$

$y = 0$

$w = 0$

For $\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -z \\ 0 \\ z \\ 0 \end{bmatrix} = z \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ is a basis vector for the eigenspace of $E_0 A$

$\boxed{geo(0) = 1}$

For $\lambda=3$

$$E_3A = null(A-3I)$$

$$= \begin{bmatrix} 2-3 & 0 & 2 & 0 \\ 0 & 3-3 & 0 & 0 \\ 4 & 4 & 4-3 & 4 \\ 0 & 0 & 0 & 3-3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 4 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{-1R_1} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 4 & 4 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 \times \frac{1}{4}} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1/4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_1} \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 9/4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x - 2z = 0$$
$$y + 9/4 z + w = 0$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} +2z \\ -9/4 z + w \\ 1 \\ 1 \end{bmatrix} = z \begin{bmatrix} 2 \\ 9/4 \\ 1 \\ 0 \end{bmatrix} + w \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} 2 \\ 9/4 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ are two basis vectors for E_3A (Eigenvalue 3)

$$\boxed{\therefore geo(3) = 2}$$

For $\lambda = 6$,

$$E_6A = \begin{bmatrix} 2-6 & 0 & 2 & 0 \\ 0 & 3-6 & 0 & 0 \\ 4 & 4 & 4-6 & 4 \\ 0 & 0 & 0 & 3-6 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 & 2 & 0 \\ 0 & -2 & 0 & 0 \\ 4 & 4 & -2 & 4 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\xrightarrow[-\frac{1}{3}R_4]{-\frac{1}{4}R_1,\ -\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 4 & 4 & -2 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow[R_3-4R_4]{R_3-4R_1,\ R_3-4R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

we get $\begin{cases} x - \frac{1}{2}z = 0 \text{ or, } \frac{1}{2}z = x \text{ or } \boxed{z = 2x} \\ \boxed{y = 0} \\ \boxed{w = 0} \end{cases}$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ 2x \\ 0 \end{bmatrix} = x \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$ is the basis vector for the eigenspace of E_6A (Eigenvalue 6)

$$\boxed{geo(6) = 1}$$

Hibroy

$$\boxed{y = 3x + 0}$$

$$\boxed{m = 3}$$

P122

Since Line L goes through [0,0]

When $x=1, y=3$, so, the line also goes through [1, 3]

Directional vector of the line L, $\vec{d} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Now, $\underbrace{\text{proj}_{\begin{bmatrix} 1 \\ 3 \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}}$ $= \dfrac{\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}}{\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

projection of vector $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ onto line $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

$$= \frac{x+3y}{1+9} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \left(\frac{x+3y}{10}\right) \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$= \left(\frac{x}{10} + \frac{3}{10}y\right) \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{x}{10} + \frac{3}{10}y \\ 3 \cdot \frac{x}{10} + 3 \cdot \frac{3}{10}y \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x}{10} & \frac{3y}{10} \\ \frac{3}{10}x & \frac{9}{10}y \end{bmatrix}$$

$[T] = \underbrace{\text{perp}_{\begin{bmatrix} 1 \\ 3 \end{bmatrix}} \vec{v}} = \vec{v} - \text{proj}_{\begin{bmatrix} 1 \\ 3 \end{bmatrix}} \vec{v} = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} \frac{x+3y}{10} \\ \frac{3x+9y}{10} \end{bmatrix}$

Orthogonal projection of vector $\vec{v} = \begin{bmatrix} x \\ y \end{bmatrix}$ onto line $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

$$= \begin{bmatrix} x - \frac{x+3y}{10} \\ y - \frac{3x+9y}{10} \end{bmatrix} = \begin{bmatrix} \frac{10x - x - 3y}{10} \\ \frac{10y - 3x - 9y}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9x - 3y}{10} \\ \frac{y - 3x}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{10}x - \frac{3y}{10} \\ \frac{y}{10} - \frac{3x}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$$

P1Q2

(a) $[T] = \begin{bmatrix} \frac{9}{10} & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10} \end{bmatrix}$

(b) $(A-\lambda I) = \begin{bmatrix} \frac{9}{10}-\lambda & -\frac{3}{10} \\ -\frac{3}{10} & \frac{1}{10}-\lambda \end{bmatrix}$

$\det(A-\lambda I)=0$

$\Rightarrow \left(\frac{9}{10}-\lambda\right)\left(\frac{1}{10}-\lambda\right) - \left(-\frac{3}{10}\right)\left(-\frac{3}{10}\right) = 0$

$\Rightarrow \left(\frac{9-10\lambda}{10}\right)\left(\frac{1-10\lambda}{10}\right) - \frac{9}{100} = 0$

$\Rightarrow \frac{9(1-10\lambda)-10\lambda(1-10\lambda)}{100} - \frac{9}{100} = 0$

$\Rightarrow \frac{9-90\lambda-10\lambda+100\lambda^2-9}{100} = 0$

$\Rightarrow -100\lambda+100\lambda^2 = 0$

$\Rightarrow \lambda(-100+100\lambda)=0$

$\boxed{\therefore \lambda=0}$

$-100+100\lambda=0$

$\boxed{\therefore \lambda = \frac{100}{100} = 1}$

The eigenvalues are 1 and 0

(b) Answer: 0, 1

(c) For $\lambda=0$

We want to find null$(T-0I)$

$(T-0I) = \begin{bmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{bmatrix}$

$\xrightarrow[10R_2]{10R_1} \left[\begin{array}{cc|c} 9 & -3 & 0 \\ -3 & 1 & 0 \end{array}\right]$

$\xrightarrow{\frac{1}{9}R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{3} & 0 \\ -3 & 1 & 0 \end{array}\right]$

$\xrightarrow{R_2+3R_1} \left[\begin{array}{cc|c} 1 & -1/3 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x - \frac{1}{3}y = 0$

$\Rightarrow 3x - y = 0$

$\Rightarrow 3x = y$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 3x \end{bmatrix} = x\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is the basis vector for eigenspace of the eigenvalue of 0.

Hilroy

For $\lambda = 1$

$$\text{null}(A - 1\lambda) = \left[\begin{array}{cc|c} \frac{9}{10} - 1 & -\frac{3}{10} & 0 \\ -\frac{3}{10} & \frac{1}{10} - 1 & 0 \end{array}\right]$$

$$= \left[\begin{array}{cc|c} -\frac{1}{10} & -\frac{3}{10} & 0 \\ -\frac{3}{10} & -\frac{9}{10} & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \times (-10)]{R_1 \times (-10)} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 3 & 9 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

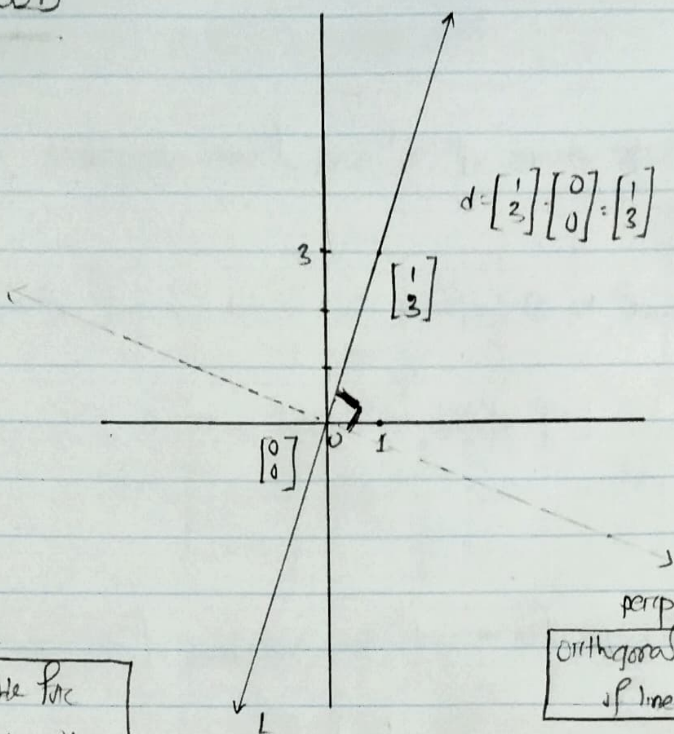
$$x + 3y = 0$$

$$\Rightarrow x = -3y$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3y \\ y \end{bmatrix} = y \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ is the basis vector for eigenspace of eigenvalue 1

P1Q2D



$d = \begin{bmatrix}1\\2\end{bmatrix} - \begin{bmatrix}0\\0\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix}$

$m = 3$

$\tan\theta = 3$

$\theta = \tan^{-1} 3 = 71.5°$

$x = 0 \quad y = 0 \quad \therefore c = 0$

$\boxed{y = mx + c = 3x + 0}$

when $x = 1$, $y = 3(1) = 3$

$\therefore x = 1, y = 3$

perp L

$\boxed{\text{orthogonal projection of line L}}$

$D\vec{v} = \lambda\vec{v}$

$(\underbrace{D}_{\text{Matrics}}\vec{v} - \lambda I)\underbrace{\vec{v}}_{\text{Vectore}} = 0$

The only way possible for a product of a matrix with a non-zero vectore to become 0, is possible if the lineare transformation associated w/ that matrics squishes matrices into a lower dimention. We know this to be $\boxed{\det(A - \lambda I) = 0}$ where λ is the scalare value for the squishification, which in effect does what the lineare transformation of the vectore did w/ the matrics.

The lineare transformation happened to the vectore is

$$[T] = \begin{bmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{bmatrix}$$

$T\vec{v} - (\lambda I)\vec{v} = 0$

$\Rightarrow (T - \lambda I)\vec{v} = 0$

$\det(T - \lambda I) = 0$

so, $\underbrace{T\vec{v}}_{\text{change of vectore V due to the lineare transformation}} = \underbrace{\lambda\vec{v}}_{\text{Change due to the scalar multiplication of vectore}}$

$\boxed{\text{Matrics multication of Vectore}}$

When oure eigenvalue is 0, it means that the transformation happened to the Line L, through matrics [T] is the same as the transformation happened due to the scalar multiplication of the vectore. It means that

$$\det\begin{bmatrix} 9/10 - 0 & -3/10 \\ -3/10 & 1/10 - 0 \end{bmatrix} = 0$$

which means $(T - 0\lambda)$ squishes the line to a $\vec{0}$ matrics. The eigenbasis for the eigenspace is

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, which means if $\vec{v}$ is a lineare dependent vectore of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$,

it is vanished to 0 as it's the same amont of transformation

by the matrix $[T]$ of the orthogonal projection of the vectore.

when the eigenvalue is 1, the eigenbasis of the eigenspace is $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

When $\vec{v}$ is a lineare combination of $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$, the vectore remain perpendiculare to the line L, as it remains unchanged by the lineare transformation.

P2Q1

(a) Let D be ourn $n\times n$ diagonal matrix

$$D = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \text{ and } e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$De_1 = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} d_{11} \\ 0 \\ 0 \end{bmatrix}$$

similarly $De_2 = \begin{bmatrix} 0 \\ d_{22} \\ 0 \end{bmatrix}, De_3 = \begin{bmatrix} 0 \\ 0 \\ d_{33} \end{bmatrix}$

Now,

$De_1 = \lambda e_1$

$$\begin{bmatrix} d_{11} \\ 0 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} d_{11} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \lambda \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\therefore d_{11} = \lambda}$

$De_2 = \lambda e_2$

$$\begin{bmatrix} 0 \\ d_{22} \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ d_{22} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \lambda \\ 0 \end{bmatrix}$$

$\therefore \boxed{d_{22} = \lambda}$

$De_3 = \lambda e_3$

$$\begin{bmatrix} 0 \\ 0 \\ d_{33} \end{bmatrix} = \lambda \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ d_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \lambda \end{bmatrix}$$

$\Rightarrow d_{33} - \lambda = 0$

$\Rightarrow \boxed{d_{33} = \lambda}$

Given, D being ourn diagonal matrix, will have the eigenvalues to be the corresponding diagonal entries (d_{11}, d_{22}, d_{33}) if and only if the eigenvector is a standard basis vectors (That is e_1, e_2, e_3)

That is for all k, λ_k is an eigenvalue for all diagonal matrix with respect to some basis vectors $e_1, e_2, \ldots, e_k$. $\boxed{De_k = \lambda_k e_k}$

Hilroy

$\therefore e_1, e_2, e_3$ is the eigenvector for ourn 3×3 diagonal matrix D.

Another way to look into this is,

for a diagonal matrix D, $\boxed{D\vec{V} = \lambda\vec{V}} \rightarrow \boxed{D\vec{V} = (\lambda I)\vec{V}}$

$$(D - \lambda I) = \begin{bmatrix} d_{11} - \lambda & 0 & 0 \\ 0 & d_{22} - \lambda & 0 \\ 0 & 0 & d_{33} - \lambda \end{bmatrix}$$

$\therefore \det(D - \lambda I) = 0$

or, $(d_{11} - \lambda)(d_{22} - \lambda)(d_{33} - \lambda) = 0$

$$\boxed{\text{or } \lambda = d_{11}, d_{22}, d_{33}}$$

Here, this can be only possible if the vector in question $\vec{V}$ is a standard basis vector $(e_1, e_2, e_3 \cdots e_k)$ representing each columns of Matrices D, as $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ where e_1, e_2 and e_3 represents the columns of the identity matrics

$\cancel{Ae_1 = \lambda e_1}$

P2Q1B Yes, A must be diagonal

A is a 3x3 matrix where e_1, e_2 and e_3 are the eigenvectors of A.

Here, $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Ae_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$ $Ae_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$ $Ae_3 = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$ where e_1, e_2 and e_3 are the eigen vectors of A

~~So, $A\vec{e_k} = \lambda \vec{e_k}$ for some λ which is the eigenvalue~~

~~or, $(A - \lambda I)\vec{e_k} = \vec{0}$~~

From $A\vec{V} = \lambda \vec{V}$ we get,

~~or, $\det(A - \lambda I) = 0$~~

(1) $A \cdot e_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \lambda_1 e_1$ or, $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} = \begin{bmatrix} \lambda_1 \\ 0 \\ 0 \end{bmatrix}$

$\therefore \boxed{\lambda_1 = a_{11}}, \boxed{a_{11} = 0}, \boxed{a_{21} = 0}$

(2) $Ae_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \lambda_2 e_2$ or, $\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} = \lambda e_2 = \begin{bmatrix} 0 \\ \lambda_2 \\ 0 \end{bmatrix}$

or, $\boxed{\lambda_2 = a_{22}}, \boxed{a_{12} = 0}, \boxed{a_{32} = 0}$

(3) $Ae_3 = \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \lambda_3 e_3$ or, $\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix} = \lambda_3 e_3 = \begin{bmatrix} 0 \\ 0 \\ \lambda \end{bmatrix}$

or, $\boxed{\lambda_3 = a_{33}}, \boxed{a_{13} = 0}, \boxed{a_{23} = 0}$

Hilary

Therefore our Matrix A has to be

$\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$

where,

$\lambda_1 = a_{11}$

$\lambda_2 = a_{22}$

$\lambda_3 = a_{33}$

$\therefore$ Matrix A must be diagonal.

P222

Given B is a 5×5 matrix. For finding the eigenvector of B,

We compute $\vec{BV} = \lambda\vec{v}$; λ being the eigenvalues and $\vec{v}$ being the eigenvector.

For each eigenvectors of the Matrics, [B],

we put the matrics on MATLAB and got λ to be

$$\lambda_1 = \frac{1}{64337137533864} + 10i$$

$$\lambda_2 = \frac{1}{6433713753384} - 10i$$

$$\lambda_3 = 10 + 2i$$

$$\lambda_4 = 10 - 2i$$

$$\lambda_5 = 10 + 10i$$

$$\lambda_6 = 10 - 10i$$

This means that for the eigenvectors of B the eigenvalues are not real numbers.

It mean that more standard co-ordinate system, the scalar multiple of the vectors $\lambda\vec{v}$ w/r/t all the linear transformations in our system will give us a scalar multiple which will be an imaginary number in our coordinate system.

$B\vec{v} = \lambda\vec{v}$

since λ is a complex numbers, scalar multiple of λ with $\vec{v}$ will give us complex numbers most of the time, unless the scalar multiplication w/ the entry gives us a real number.

So, Every eigenvector of [B] must have atleast one entry which is not a real number.

$\therefore$ no real eigenvalue

Since there eigenvalue is complex numbers, the vectors represented by the matrices in the equation $\boxed{B\vec{V} = \lambda\vec{V}}$ must have at least one entries that are complex number