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Assignment: HW-7 [Sections 10.7 & 10.8]

18. To find the sum of the series $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$, express $\frac{1}{1-x}$ as a geometric series, differentiate both sides of the resulting equation with respect to x , multiply both sides of the result by x , differentiate again, multiply by x again, and set x equal to $\frac{1}{2}$.

Express $\frac{1}{1-x}$ as a geometric series.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (\text{Type an expression using } x \text{ as the variable.})$$

Differentiate both sides of the resulting equation with respect to x . Begin with the left side of the equation.

$$\frac{d}{dx} \left(\frac{1}{1-x} \right) = \frac{1}{(1-x)^2}$$

Differentiate the right side of the equation.

$$\frac{d}{dx} \left(\sum_{n=0}^{\infty} x^n \right) = \sum_{n=1}^{\infty} nx^{n-1} \quad (\text{Type expressions using } x \text{ as the variable.})$$

Multiply both sides of the result by x and differentiate again. Begin with the left side of the equation.

$$\frac{d}{dx} \left(\frac{x}{(1-x)^2} \right) = -\frac{x+1}{(x-1)^3}$$

Differentiate the right side of the equation.

$$\frac{d}{dx} \left(\sum_{n=1}^{\infty} nx^{n-1} \right) = \sum_{n=1}^{\infty} n^2 x^{n-2} \quad (\text{Type expressions using } x \text{ as the variable.})$$

Multiply by x again, and set x equal to $\frac{1}{2}$.

$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} = 6$$