Stat 260 Welcome!

Instructor: Dr. Michelle Edwards
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Course Website: bright.uvic.ca

Office Hours:
One hour will be held in person and one hour will be held virtually through Zoom. Information on the schedule, location, and Zoom link will be posted on our Brightspace page.

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Required Texts and Materials

The best resource to learn the definitions and material for this course is my collection of lecture notes.

Textbooks:

- *Essentials of Probability & Statistics for Engineers & Scientists* by Walpole, Myers, Myers, & Ye. (Contains suggested problems, can purchase as E-Text from the UVic Bookstore.) → good practice questions
- *Introduction to Probability and Statistics* by Tim Swartz. (Pdf posted on Brightspace, does not have our list of suggested problems.)
  ↳ free textbook on brightspace

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Calculator Requirements

You will want a calculator for your tests and assignments.

The only allowed calculators for in-person tests are the Sharp EL-510R series calculators, which are the approved calculators for the UVic Math & Stats Department.

The calculators can be purchased through the UVic Bookstore.

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Calculation Of Your Final Grade

Your grade in the course will be calculated by:

Item	Date(s)	Weight
Written Assignments	On Crowdmark (best 6 of 8)	12%
R Assignments	On Crowdmark	6%
Online Assignments	On Brightspace	6%
Test 1	Friday February 10th	10%
Test 2	Friday March 10th	10%
Test 3	Tuesday April 4th	10%
Final Exam	TBA	46%

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Format of the Course

Written Assignments

- There will be 8 written assignments on Crowdmark, only your best 6 will count toward your final grade.
- You may handwrite your solutions on paper, or with a tablet, or type your work.

R Assignments

- There will be 3 R Assignments on Crowdmark, they use the (free!) R statistical software.
- Step-by-step instructions will be included with each assignment.
- Your work must by typed (since you will by copying output from the software).

Online Assignment

- There are 3 Online Assignments on Brightspace
- The purpose is to review the course material before each Term Test
- You may attempt each Online Assignment an unlimited number of times, only your highest attempt score will be used
- Questions are randomized and you will receive a new collection of questions on each attempt.

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Tests

- There will be 3 Term Tests (midterms):
  - Friday February 10th
  - Friday March 10th
  - Tuesday April 4th
- Tests will be written during our lecture time.
- More information on each test will be announced closer to the test date.
- We will provide you with a copy of the course formula sheet and stat tables during tests.

Final Exam

- The final exam is cumulative and will be written in person.
- The exam is scheduled by Records to occur during the exam period Tuesday April 11th to Wednesday April 26th.
- Please do not make plans for employment or other activities which may conflict with our exam time during these days until the exam schedule is released in mid-February.
- Deferred exams are only given for excused absences.
- Practice tests (term tests and finals) will be posted on Brightspace.
- We will provide you with a copy of the course formula sheet and stat tables during tests.

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If You Miss A Test

- If you miss a term test due to a legitimate reason (illness, accident, family affliction, etc.) contact me via email ASAP, preferably before/on the test date.
- Documentation for missed tests is not needed. (Documentation for missed final exam may be needed, check the university regulations.)
- There is a time limit for requesting a concession on term work (within one week of grades being returned).
- Reweighting scheme is in the Course Outline. You cannot pass the course if you miss all three term tests.
- Alternate times for tests will not be provided. (i.e. You must write with your section, and there are no make-up tests.)

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If You Miss An Assignment

- If you miss an R Assignment: Hopefully this doesn't happen! Lots of working time (weeks) will be given, they typically take about 1 hour to complete. If something does happen though, contact me ASAP by email.
- If you miss a Written Assignment: Your lowest two scores (including missed assignments) will be dropped. Use these wisely! If you miss 3 or more assignments due to a valid reason, contact me ASAP by email.
- If you miss an Online Assignment: Again, hopefully this doesn't happen. Lots of working time (weeks) will be given and multiple attempts are allowed. If something does happen, contact me ASAP by email.
- Alternate assignments for missed or low-scoring work will not be provided.

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Expectations For This Semester

- We are still navigating some uncertainty brought on by the pandemic. Please be patient with us, instructors usually are told information at the same time as you.
- Please post content and policy questions on the Brightspace Discussion Forum, keeping email to questions of a more sensitive nature (e.g. grades, excused absences, etc.).
- Talking to me in person (in class or in office hours) or posting on the Brightspace Discussion Forum will generally give answers quicker than email.
- I am a sessional instructor (meaning I am not a full time nor a permanent faculty member). To set some boundaries: I will monitor the Discussion Forum every weekday throughout the day, and will reply to email 1-2 times per day. I will not reply to email or Forum posts between Friday evenings and Monday mornings.

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Let's Talk COVID Procedures

Live lectures will not be recorded. Instead, there are a collection of pre-recorded lecture videos which will be made available on Brightspace.

If you miss class you should obtain a copy of the lecture notes from that day. I will not post filled in lecture notes nor will I send copies via email. Instead ask your classmates for a copy of the notes (there is a space to share notes on the Brightspace Discussion Forum), or you can photograph my printed notes during lectures.

Documentation for missing assignments and tests is not needed.

It is VERY IMPORTANT that you do not attend any in-person lecture if you are not feeling well. If you are questioning whether you are well enough to attend class, you are not well enough to attend.

If I am not well enough to attend lecture, there will be an announcement made on Brightspace. Instructions on what to do will be in the Brightspace announcement.

I have to be extremely careful with the COVID risks I take, so I will be wearing a mask while I lecture this semester. I will wear a microphone so I can be heard clearly.

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Finally, Your To Do List:

- If you haven't yet, visit our Brightspace page. (This is how I will take attendance.)
- Read our Course Outline, and the handouts on Undergraduate Policies and Information.
- Get a calculator (required: Sharp EL-510R series).
- Visit Crowdmark and try use the interface. (It will be visible once the first assignment is posted, this should be on Wednesday January 11th.)
- Download the R statistical software. I have videos posted on how to download and use the program if you want more help.

I look forward to a great semester of working with you all!

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Set 1 - Basic Terminology and Concepts

January 7, 2023 6:44 PM

Stat 260 Lecture Notes
Set 1 - Basic Terminology and Concepts

Statistics: Is the development and application of methods to collect, analyze, and interpret data.

Stats teaches us how to make intelligent judgments and informed decisions in the the presence of uncertainty and variation.

Definitions:

- **population:** a collection of objects big group we're interested in
- **sample:** a selection from the population take a few from that group
- **parameter:** a descriptive measure of the population (e.g. mean (μ), standard deviation (σ), variance (σ^2), etc.) comes from population
- **statistic:** a descriptive measure of the sample (e.g. mean ($\bar{x}$), standard deviation (s), variance (s^2), etc.) comes from sample
- **random variable (r.v.):** a characteristic that changes from object to object in the population
  - discrete r.v. - set of all possible values are finite or are infinite and countable (they can be listed in a finite or infinite sequence)
  - continuous r.v. - set of all possible values are infinite and it's impossible to list all the possible values (e.g. it's impossible to list all the decimals between 0 and 0.5.)

parameter/statistic: what we're calculating

→ eye colour - answer changes from person to person
→ thing that we're measuring

How many classes? 0, 1, 2, 3, 4, 5, 6 discrete r.v.
↳ b/c can list all of them
0, 0.1, 0.2, 0.3, 0.4, ... discrete

0 0.1 0.2 0.3 0.4 0.5
0.01
0.001

continuous r.v.

1

discrete r.v. ⇔ finite decimal places
continuous r.v. ⇔ infinite decimal places

⇒ think about it theoretically

first written assignment posted (can do first question)

Stat 260 consists of three main topics:

- **Descriptive Stats (Sets 1-3):** Ways to describe the data set (e.g. charts/graphs, mean (average), median, standard deviation, variance, etc.)
- **Probability (Sets 4-21):** How likely events are to occur. We look at the population to see how likely events will be in the sample.
- **Inferential Stats (Sets 22-31):** Use the sample to make generalizations about the population.

Example 1: Suppose we wish to look at the average height of adults who live in Canada. To do this we create a poll on our Stat 260 Brightspace page and ask the students in the class to report their height in cm.

→ socioeconomic, gender, race, age, sample size

- population: adults who live in Canada
- sample: Stat 260 students who answer
  ↳ is subset of population
- parameter: average height of adults who live in Canada
  Notation: μ
- statistic: average height of Stat 260 students who answer the poll
  Notation: $\bar{x}$
- random variable: height (in cm)
  ↳ the thing that's being measured
  ↳ continuous r.v.
  (b/c theoretically height could have

- random variable: height (in cm)
  ↳ the thing that's being measured
  ↳ continuous r.v.
  (b/c theoretically height could have infinite decimals)

2

Example 2: Suppose we wish to look at the average number of classes current UVic students are registered in this semester. To do this we randomly select 200 students who are registered this semester and from their UVic profile record the number of classes they are currently registered in.

→ good set up

- population: current UVic students
- sample: 200 UVic students measured
- parameter: average # of classes of current UVic students
  Notation: μ
- statistic: average # of classes of 200 UVic Student sample
  Notation: $\bar{x}$
- random variable: number of classes
  ↳ discrete r.v.
  (all whole numbers, finite decimal)

Example 3: Determine if each of the following are discrete or continuous random variables.

(a) A person's height in cm. continuous

(b) The number of courses a student is registered in this semester. discrete

(c) The top running speed of greyhound dogs in km/h. continuous

(d) The price of a cup of coffee in dollars. discrete (only 2 decimal places) → price you have to pay is discrete
↳ but might change based on context

(e) The number of accidents at a particular intersection on a highway in a year. discrete

(f) The lowest temperature in the month. continuous
↳ b/c theoretically infinite

discrete / continuous don't change how you calculate things but changes for probability

3

Stat 260 Lecture Notes
Set 2 - Basic Descriptive Statistics

Knowing what type of data we have affects what can be done to analyze the data. Univariate data is collected from single measurements on subjects.

average height vs eye colour

Suppose we take a sample with n observations $x_1, x_2, x_3, \ldots, x_n$. Each x_i represents the single value that was measured in that observation. This is univariate data (i.e. we measured only one number per each observation). We can display univariate data with a frequency table or a histogram or a boxplot.

Example 1: Measurements of lengths of lizards captured in months of August and October.

```
August measurements:
[1] 7.5 7.2 3.0 12.1 15.1 12.1 11.5 11.8 7.2 13.2 13.6 8.2 9.5 8.4 13.3
[16] 12.5 12.4 2.1 10.7 9.4 6.7 6.8 6.1 8.3 7.9 6.0 7.6 13.2 4.5 9.3
[31] 8.1 3.5 9.0 50.0
```

1st observation

31st observation

n = 34

```
October measurements:
[1] 43.7 37.2 29.0 31.6 47.5 48.3 38.3 19.7 32.5 45.2 36.1 30.5 37.2 50.5 36.9
[16] 44.5 35.9 28.7 37.5 30.2 36.9 43.2 27.0 26.2 41.8 26.4 34.3 28.6 35.9 22.0
[31] 45.4 30.3 29.8 46.1 42.7 31.5 37.4 25.1 27.2 45.0
```

Look at the August measurements. Here we can create a frequency table.

count how many

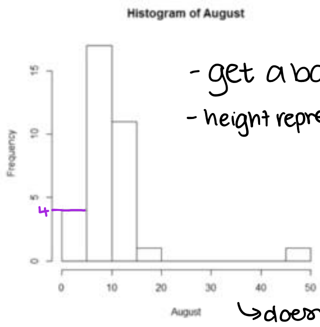
convert into percentage

* 3-4 decimal places

Interval	Frequency	Relative Frequency
equal → $[0, 5)$	||||	4/34 = 0.12 = 12%
$[5, 10)$	卌 卌卌 ||	17/34 = 0.5 = 50%
$[10, 15)$	卌卌 |	11/34 = 0.32 = 32%
$[15, 20)$	|	1/34 = 0.03 = 3%
$[20, 25)$	0	0
$[25, 30)$	0	0
$[30, 35)$	0	0
$[35, 40)$	0	0
$[40, 45)$	0	0
$[45, 50]$	|	1/34 = 0.03 = 3%
	n=34	1 = 100%

1

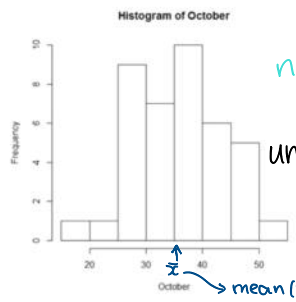
Using this frequency table we can create a histogram. The histogram for the August data looks like this:



- get a bar for each interval
- height represents how many things in interval

↳ doesn't tell you [or)

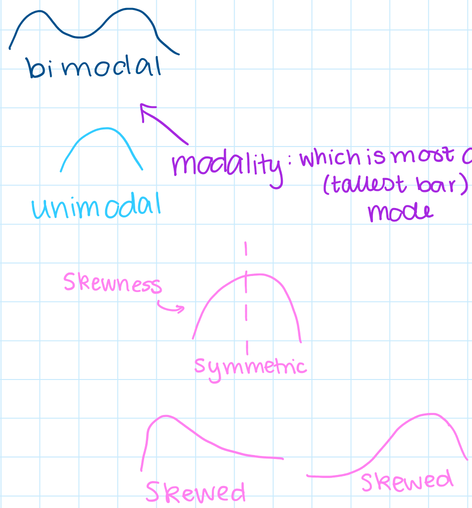
Doing a similar thing for the October data gives a histogram that looks like this:



n = sum of heights of all bars

unimodal

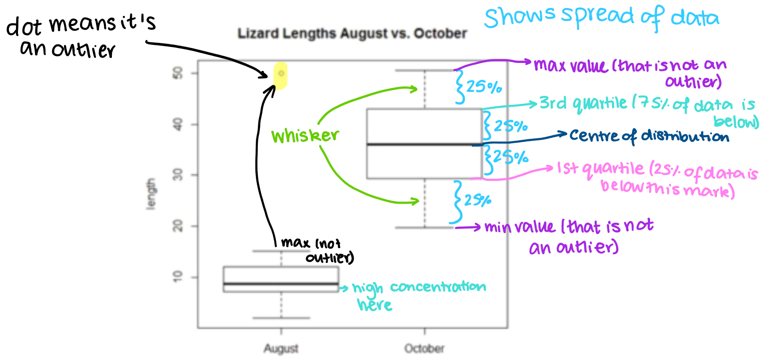
mean (vertical weights)



modality : which is most common (tallest bar) mode

2

It can be difficult to compare two data sets like this, so we could also use a boxplot. The boxplots for the August and October data look like this:



Outlier: number that doesn't fit pattern of what other numbers are doing

→ the 50 from August is an outlier

→ outliers are part of data set, so max for August would still be 50

With this boxplot it is easy to see that October lengths are larger than August lengths.

Categorical data occurs when our recorded data falls into categories. (e.g. eye colour, program major, customer satisfaction). While we cannot do many of the calculations with categorical data that we can do with univariate data, we can display categorical data with a bar chart.

for categories instead of numbers

Suppose we have a sample with observations $x_1, x_2, \ldots, x_n$.

summation

- *sample mean*, $\bar{x} = \dfrac{x_1 + x_2 + \ldots + x_n}{n} = \dfrac{1}{n}\sum_{i=1}^{n} x_i$. The mean is the same thing as the average.
- tilde
  sample median, $\tilde{x}$, is the the middle observation. If there is an even number of observations then there are two middle observations so the median is the midpoint (here, the same as the average) of these two values. Remember that the data must be sorted first!
- *mode*, is the most common value. There may be many modes if many values are tied for occurring the most often.

Example 2: Suppose we have the sample:

$$0, 0, 2, 3, 6, 7, 10, 11, 20.$$

Calculate the mean, median, and mode.

leave as reduced fraction

mean: $\bar{x} = \dfrac{0+0+2+3+6+7+10+11+20}{9} = \dfrac{59}{9}$

median: $\tilde{x} = 6$

mode $= 0$

In general: 3-4 decimals

1 more decimal than the data

Example 3: Suppose we have the sample:

$$1, 1, 2, 8, 15, 15, 25, 100.$$

Calculate the mean, median, and mode.

mean: $\bar{x} = \frac{1+1+2+8+15+15+25+100}{8} = \frac{167}{8} = 20.875$ ← exact

median: $\tilde{x} = 11.5 = \frac{8+15}{2}$

mode = 1 and 15

Without 100 : $\frac{67}{7}$ = mean = 9.57

median = 8

Mean sensitive to outliers

Mode is useful when we're talking about categories

Notes:

- $\bar{x}$ doesn't have to be a value actually observed in the sample. Neither does $\tilde{x}$. (So don't round your final answers for average or median, even if they physically aren't possible for a single observation.)
- $\bar{x}$ is sensitive to outliers (extreme values), $\tilde{x}$ is not. Look at what happens in Example 3 if the value of 100 is removed. The mean of the new data list changes quite a bit, the median of the new data list changes very little.
- The median splits the data in two: 50% of the observations are larger than $\tilde{x}$, and 50% of the observations are smaller than $\tilde{x}$.
- The mean and median tell us where the "center" of our sample data is.
- The mode tells us which observation is most common. The mode can be used for categorical data, whereas $\bar{x}$ and $\tilde{x}$ cannot be used for categorical data.
- In a histogram, the mean (average) occurs at the "balance point" of the picture. ↳ mean is the balance point (almost like the point of equilibrium)

Example 4: Look at the following two samples. How could we describe the difference in these samples?

sample 1:	**sample 2:**
10	10
20	48
49	49
50	50
51	51
80	52
90	90

further out from the centre

min and max are both same

Both samples have $\overline{x} = \widetilde{x} = 50$. Which sample is more spread out?

mean median

We look at *measures of variability* to describe differences between these samples.

variance & standard deviation

- *sample variance*, $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$
- *sample standard deviation*, $s = \sqrt{\text{variance}}$
- *shortcut formula for sample variance*, $s^2 = \underbrace{\dfrac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \dfrac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right]}_{\text{looks longer but easier calculations}}$

Note: The standard deviation s and the mean $\overline{x}$ have the same units as your original data. The variance s^2 uses $(\text{units})^2$.

Standard deviation : same units

variance : $(\text{units})^2$

(kind of like an error measurement)

How far is data from the mean?

$$S^2 = \frac{\sum_{i=1}^{n}\overbrace{(x_i - \bar{x})}^{\text{distance of observation from mean}}{}^2}{n-1}$$

(if it wasn't squared you'd always get 0)

→ course has a formula sheet (for exams)

If only 1 data point then variance = 0

Single number doesn't have a variance but outlier contributes to variance

Example 5: Calculate the sample variance for sample 1 and sample 2 in Example 4. Calculate the sample standard deviation for sample 1 in Example 4.

$$s^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1}$$

Sample 1: 10, 20, 49, 50, 51, 80, 90

$n=7 \quad \bar{x}=50$

x_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
10	-40	1600
20	-30	900
49	-1	1
50	0	0
51	1	1
80	30	900
90	40	1600
	$\Sigma(x_i-\bar{x})=0$	5002

will always get 0 here

$$s^2 = \frac{5002}{7-1} = \frac{5002}{6} = 833.7$$

Sample 2: 10, 48, 49, 50, 51, 52, 90 $\quad n=7$

$$s^2 = \frac{1}{n-1}\left[\Sigma x_i^2 - \frac{1}{n}(\Sigma x_i)^2\right]$$

Shortcut formula

$\Sigma x_i = 350$

$\Sigma(x_i^2) = 10^2+48^2+\dots+90^2 = 20{,}710$

$$s^2 = \frac{1}{7-1}\left[20{,}710 - \frac{1}{7}(350)^2\right] = 535$$

The numbers on their own don't mean much, useful for comparing them

Sample 1: $s^2 = 833.7$

Sample 2: $s^2 = 535$

Variance from sample 1 is larger, so sample 1 is more spread out from the mean.

Standard deviation of sample 1: $s = \sqrt{\text{variance}} = \sqrt{833.7} = 28.874$

Sample values are estimates of the population

Note: The **population variance** σ^2 is calculated using a slightly different formula than the sample variance. Suppose our population has N items and we take a sample of n items from it.

sample mean

- *sample variance*, $s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}$ dividing by n-1 gives the better estimate

population mean

- *population variance*, $\sigma^2 = \dfrac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$ for population you divide by N

In the sample, we don't know the whole picture so the sample variance s^2 is just an estimate of the population variance σ^2. As it turns out (due to technical reasons and a definition we won't cover here) that dividing by $n-1$ gives a better estimate of the population when we perform the sample variance calculation. Dividing by n gives an estimate that underestimates.

Lastly, the quickest way to calculate standard deviation, variance, and the mean is by using the stats functions on your calculator. On tests and assignments it is preferred that you use the calculator stat functions (so you do not need the formulas from Example 5).

Stat mode (1) → Store values enter #, press M+ → Stat functions above ( ) x ÷ $\boxed{\text{RCL}}$ mean $\overline{x}$ St. dev Sx

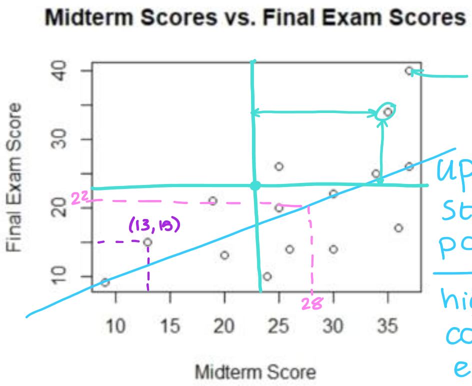
Set 3 - The Correlation Coefficient

January 17, 2023 11:15 AM

Stat 260 Lecture Notes
Set 3 - The Correlation Coefficient

When data arise in pairs, such as (x_1, y_1), (x_2, y_2), (x_3, y_3) ... (x_n, y_n) the structure is called **bivariate**.

Here we may want to see if there is a relationship between the x and y values. To do this we can use a **scatterplot**.



upward trend
straight line
positive relationship,

higher midterm scores correspond to higher final exam scores

can use this model to predict datavalues

Be careful with extrapolating (making predictions). This data only shows what happens on the final exam for students with midterm scores between 9 and 37. The data included here would not be useful for making final exam predictions for midterm scores of, say, 80. (In other words, your data is only useful in making predictions for other data values close to your collection.)

1

Recall: variance $= s^2 = \dfrac{\sum(x_i - \overline{x})^2}{n-1} = \dfrac{\Sigma(x_i - \bar{x})(x_i - \bar{x})}{n-1}$

positive covariance: positive slope

When we work with bivariate data we can calculate the **covariance**, s_{xy}.

$$s_{xy} = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{n-1}$$

can be positive or negative

Example 1: Calculate the covariance for the data (3, 4), (8, 7), (10, 8), (11, 8).

x	y	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$
3	4	-5	-2.75	(-5)(-2.75) = 13.75
8	7	0	0.25	0(0.25) = 0
10	8	2	1.25	(2)(1.25) = 2.5
11	8	3	1.25	(3)(1.25) = 3.75

n describes how many observations

$\bar{x} = 8$ $\bar{y} = 6.75$

$S_{xy} = \dfrac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{n-1} = \dfrac{13.75 + 0 + 2.5 + 3.75}{4-1} = \dfrac{20}{3} = 6.67$

$\bar{x}=8$ $\bar{y}=6.75$

$$S_{xy}=\frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{n-1}=\frac{13.75+0+2.5+3.75}{4-1}=\frac{20}{3}=6.67$$

Kind of meaningless on its own (only tells you positive slope)

The correlation coefficient, r, measures the strength of the linear relationship between x and y values.

$$r=\frac{\sum(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\sum(x_i-\overline{x})^2\sum(y_i-\overline{y})^2}}=\frac{s_{xy}}{s_x s_y}$$

where s_{xy} = the covariance of x and y, s_x = the standard deviation of x values, s_y = the standard deviation of y values

Example 2: Calculate the correlation coefficient using the data from Example 1. (3,4),(8,7),(10,8),(11,8)

$S_{xy}=\frac{20}{3}$

$S_x=3.559$

$S_y=1.893$

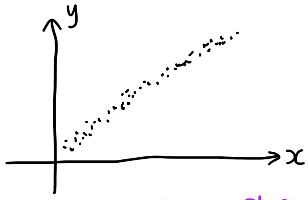
$$\frac{\frac{20}{3}}{(3.559)(1.893)}=\frac{20}{3(3.559)(1.893)}=0.9895$$

Scaling covariance measurement into a percentage

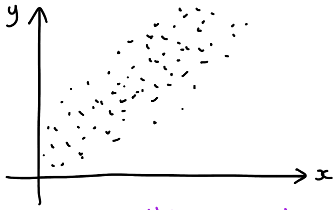
Rule: No matter what we have for x and y values, $-1 \le r \le +1$.

The closer that that correlation coefficient r is to $+1$ or -1, the stronger the linear relationship there is. A positive value of r indicates a positive linear relationship, and a negative value of r indicates a negative linear relationship. A value of 0 indicates no linear relationship.

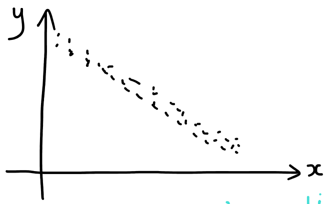
(generally cutoff strong relationship around 0.8)



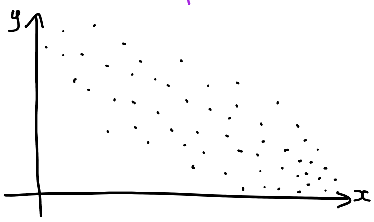
r is positive, strong linear relationship, r is close to +1.



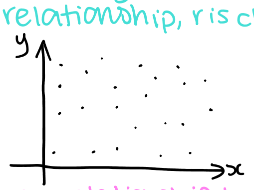
r is positive, weak linear relationship, r is not close to +1.



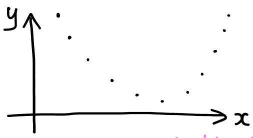
r is negative, strong linear relationship, r is close to −1.



r is negative, weak linear relationship, r is not close to −1.



no relationship between x and y, r is close to 0



there is a relationship between x and y, just not linear. r is close to 0

An exact linear relationship occurs when $r = 1$ or $r = -1$ and in this case we can represent the data as a straight line in the form $y_i = ax_i + b$.

Be careful! Correlation $\neq$ causation. (spurious correlations)

* r tells us nothing about the value of the slope, other than if it is positive or negative

4

Stat 260 Lecture Notes
Set 4 - Basic Set Theory

The sample space $\mathcal{S}$ of a random experiment is a list of all the possible outcomes of the experiment.

Example 1: The students in our Stat 260 class are asked to report what month they were born in.

S = {Jan, Feb, Mar, ..., Dec}

sample space and event both about months

A simple event is a single outcome from the sample space.

e.g. person is born in March

An event is a set of one or more simple events.

e.g. the event $E = \{\text{Jan}, \text{Feb}, \text{Mar}\}$ = person is born in the first three months of the year

There are multiple ways to assign probabilities to events:

capital letters for events

- classical approach: used when possible outcomes are equally likely

theoretical approach

$$P(A) = \frac{\#A}{\#\mathcal{S}} = \frac{\#\text{ of ways event } A \text{ can occur}}{\#\text{ of outcomes in } \mathcal{S}}$$

A = born in Jan

$P(A) = \frac{1 \text{ Jan}}{12 \text{ total months}}$

assuming that being born in any month is equal to being born in any other month.

doesn't take into account real life things

- frequency approach: counts the number of times the event occurs in many, many observations

$$P(A) = \lim_{N\to\infty}\left(\frac{\#\text{ of occurrences of } A}{N}\right) \approx \frac{f}{n}$$

number of things in population

f requency

n umber of observations

$P(A) = \frac{4}{150}$ # w/ Jan birthday / # people asked

In all models, the probability of an event is the sum of its simple events.

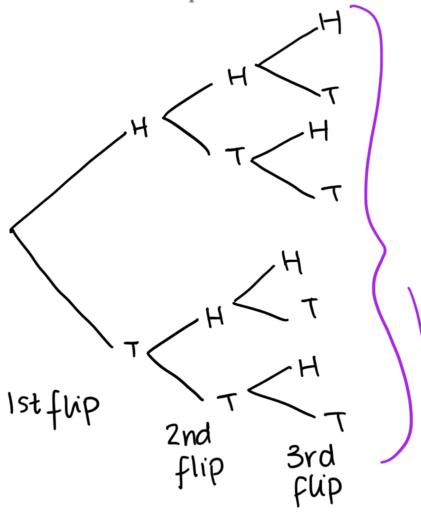
Example 2: Calculate P(born in the first three months) using the classical approach. $E = \{\text{Jan}, \text{Feb}, \text{Mar}\}$

$\boxed{a^{b/c}}$ calculator button for reducing fractions

$$P(E) = P(\text{Jan}) + P(\text{Feb}) + P(\text{Mar}) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$
$$= \frac{3}{12}$$
$$= \frac{1}{4} = 0.25 = 25\%$$

Tree diagrams help us list all possible outcomes in the sample space.

Example 3: Look at the experiment where we flip a fair coin 3 times and we record the sequence of heads and tails.



Outcomes

HHH $H_1 \cap H_2 \cap H_3$
HHT
HTH
HTT
THH
THT
TTH
TTT

# of outcomes = number of endpoints

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$.

$P(\text{all } H) = \frac{1}{8}$

$P(\text{exactly } 1 H) = \frac{3}{8}$

The union of events A and B, denoted $A \cup B$, is read as "A or B". The set $A \cup B$ = outcomes in A or B or in both.

in either or both

$A \cup B$: glue them together

$A \cap B$: the overlap of the sets

The intersection of events A and B, denoted $A \cap B$, is read as "A and B". The set $A \cap B$ = outcomes that are in both A and B.

In other words, the list of events in $A \cup B$ is the lists from A and B combined together as one larger list. The list of events in $A \cap B$ is the overlap in the lists from A and B.

Sometimes we write AB as a shorter version of $A \cap B$.

The event $\emptyset$ is the empty event / impossible event.

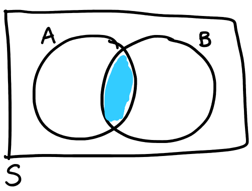
Rule: $P(\emptyset) = 0$.

The events A and B are mutually exclusive (or disjoint) if $A \cap B = \emptyset$. (That is, there is no overlap between events A and B.)

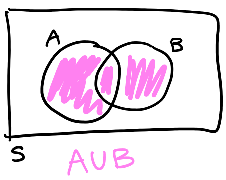
So if A and B are mutually exclusive, then $P(A \cap B) = P(\emptyset) = 0$.

The complement of an event A, denoted $\overline{A}$, is the set of all outcomes in $\mathcal{S}$ that are not in A. "Not A" "$\overline{A}$" "A^c" "A'"

Venn Diagrams help us picture probabilities.

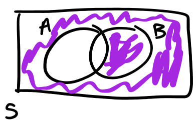


$A \cap B$

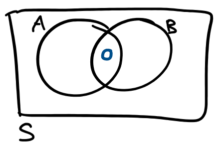


"at least 1 of A or B"

$A \cup B$



$\overline{A}$



A and B mutually exclusive

3

Rule: DeMorgan's Laws

- $\overline{A \cup B} = \overline{A} \cap \overline{B}$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Math 122 : $\neg(A \vee B) \Leftrightarrow \neg A \wedge \neg B$

$\neg(A \wedge B) \Leftrightarrow \neg A \vee \neg B$

Note: the phrase "nor" can be translated as "and not". For example "neither A nor B" is the same as "not A and not B".

$\neg A \wedge \neg B$ $\qquad$ $\neg(A \vee B)$

Set 5 - Introduction to Probability

January 17, 2023 11:16 AM

Stat 260 Lecture Notes
Set 5 - Introduction to Probability

Probability is used to express the likelihood that some event will or will not occur. We measure probability on a scale from 0 to 1, where 0 indicates that it is impossible for the event to occur and 1 indicates that the event is guaranteed to occur. can be interpreted as percentage

For an event A, we denote the probability that A will occur by $P(A)$.

Axioms of Probability:

- For any event A, $P(A) \geq 0$. no negative probabilities
- $P(\mathcal{S}) = 1$, where $\mathcal{S}$ represents the sample space. because S lists all options
- If events $A_1, A_2, A_3, \ldots$, are mutually exclusive (disjoint), then $P(\bigcup_{i=1}^{\infty} A_i) = P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$ → don't overlap in any way

In the last axiom, the fact that the events are disjoint is important. Below we will see how to find the probability of the union when the events are not disjoint.

Notice that an event A and its complement $\overline{A}$ are disjoint and together they comprise all of the sample space $\mathcal{S}$. Therefore we can say that $P(A \cup \overline{A}) = P(A) + P(\overline{A}) = P(\mathcal{S}) = 1$.

Rule: $P(A) + P(\overline{A}) = 1$.

This is often useful in the form $P(A) = 1 - P(\overline{A})$.

← can be helpful if A is super complicated

1

principle of inclusion-exclusion

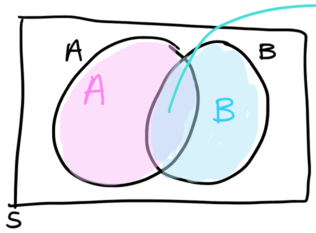
→ these formulas not on formula sheet

Theorem: General Addition Rule

for two sets:

$$P(A \cup B) = \underbrace{P(A) + P(B)}_{\text{middle region is counted twice}} - \underline{P(A \cap B)}$$

so subtract it off



$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

↑ number of items in $A \cup B$ (frequency)

for three sets:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

frequency

2

Example 1: Suppose that when Michelle is on the computer the only activities she does are:

sample space

- event E_1: marking R assignments, occurs with probability p p
- event E_2: prepping lectures, 8 times as likely as R assignments $8p$
- event E_3: answering email, 3 times as likely as R assignments $3p$
- event E_4: updating course website, 2 times as likely as R assignments $2p$
- event E_5: creating new assignments, 6 times as likely as R assignments $6p$

Assume Michelle can only do one activity at a time (i.e. all events are mutually exclusive). A student visits Michelle at a random time and finds that she is working on the computer. What is the probability that she is <u>not</u> marking R assignments?

complement

We know that $E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5 = S$ and all events are disjoint.

$$1 = P(S) = P(E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5)$$
$$= P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5)$$
$$= p + 8p + 3p + 2p + 6p$$
$$= 20p$$

So $1 = 20p \Rightarrow p = \frac{1}{20}$

Want $P(E_1^c) = 1 - P(E_1) = 1 - \frac{1}{20} = \frac{19}{20}$ //

Example 2: In a colony of 160 rabbits

- 104 are grey G
- 105 have straight ears E
- 126 have short fur F
- 90 have short fur & are grey $F \cap G$
- 80 have straight ears & short fur $E \cap F$
- 149 are grey or have straight ears $G \cup E$
- 5 have none of these qualities

What is the probability that a randomly selected rabbit from the colony has all three qualities?

What is the probability that a randomly selected rabbit is grey but has neither of the other two qualities? (i.e. "just grey")

$$P(G \cup E \cup F) = P(G) + P(E) + P(F) - P(G \cap E) - P(G \cap F) - P(E \cap F) + P(G \cap E \cap F)$$

all 3 qualities

$$\frac{155}{160} = \frac{104}{160} + \frac{105}{160} + \frac{126}{160} - \frac{60}{160} - \frac{90}{160} - \frac{80}{160} + P(G \cap E \cap F)$$

$$P(G \cap E \cap F) = \frac{50}{160} = \frac{5}{16} = 0.3125 \;//$$

$$P(G \cup E \cup F) = 1 - P(\overline{G \cup E \cup F}) = 1 - \frac{5}{160} = \frac{155}{160}$$

none of G or E or F

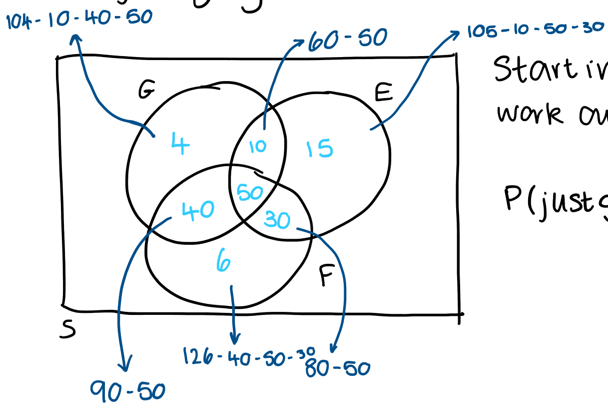
$$P(G \cup E) = P(G) + P(E) - P(G \cap E)$$

$$\frac{149}{160} = \frac{104}{160} + \frac{105}{160} - P(G \cap E)$$

$$P(G \cap E) = \frac{104}{160} + \frac{105}{160} - \frac{149}{160} = \frac{60}{160}$$

4

Want "just grey" $G \cap \bar{E} \cap \bar{F}$



Start in middle and work our way outwards

$$P(\text{just grey}) = \frac{4}{160} = \frac{1}{40} = 0.025$$

$$P(\text{exactly one characteristic}) = \frac{4+15+6}{160} = \frac{25}{160} = \frac{5}{32} = 0.15625$$

5

Set 6 - Conditional Probabilities

January 24, 2023 12:14 PM

Stat 260 Lecture Notes

Set 6 - Conditional Probabilities

↳ given extra bit of information

Example 1: Rolling a 6-sided die.

Suppose we roll a standard 6-sided die and record the number that is facing up. What is the probability of rolling a 3?

$$P(\text{roll a } 3) = \frac{1}{6}$$

→ one # is 3

→ six #s in sample space

$S = \{1,2,3,4,5,6\}$

Now suppose we are told that an odd number was rolled. Now what is the probability of rolling a 3?

$$P\left(\begin{matrix}\text{roll a 3 if we know}\\ \text{an odd \# was rolled}\end{matrix}\right) = \frac{1}{3}$$

→ one # is 3

→ three #s in sample space

new, reduced sample space $S = \{1,3,5\}$

$$P(\text{roll a } 3 \mid \text{odd \#}) = \frac{P(\text{roll a 3 and odd number})}{P(\text{odd number})} = \frac{P(\text{roll a } 3)}{P(\text{odd number})} = \frac{1/6}{3/6} = \frac{1}{3}$$

Instead, suppose we were told that an even number was rolled. What is the probability of rolling a 3?

$$P\left(\begin{matrix}\text{roll a 3 if we know}\\ \text{an even \# was rolled}\end{matrix}\right) = 0 = \frac{0}{3}$$

→ zero #s are 3

→ three #s in S

new, reduced sample space $S = \{2,4,6\}$

Idea: Knowing extra information can change the probability of an outcome. When we know extra info it is called a *conditional probability*. The "given" part of the event is the extra info we are told.

1

Formula: probability of A given B (i.e. the probability that event A will occur given that we know event B has occurred).

"A given B"

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

these use the original sample space

Not on formula sheet

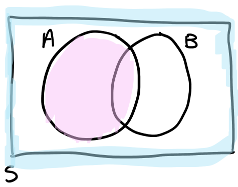
$\left(\frac{\text{probability both}}{\text{probability given part}}\right)$

Notice we could also talk about the probability of B given A:

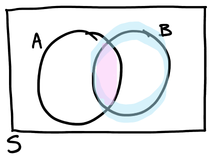
$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}.$$

"B given A"

Where does the formula come from?



$P(A) = \frac{n(A)}{n(S)}$



$P(A|B) = \frac{n(A \cap B)}{n(S)}$

$= \frac{n(A \cap B)/n(S)}{n(B)/n(S)}$

$= \frac{P(A \cap B)}{P(B)}$

Example 2: A maufacturer wants to see if using a clear coating of paint on their product is connected to if the product rusts.

	rust present (R)	no rust present ($\overline{R}$)	
clear coating used (C)	0.03	0.12	
no clear coating used ($\overline{C}$)	0.17	0.68	

given

If we know that a randomly selected component has a clear coating, what is the probability that it has rust present?

$$P(R|C) = \frac{P(R\cap C)}{P(C)} = \frac{0.03}{0.03+0.12} = \frac{0.03}{0.15} = 0.20$$

If we know that a randomly selected component does not have a clear coating, what is the probability that it has rust present?

$$P(R|\bar{C}) = \frac{P(R\cap \bar{C})}{P(\bar{C})} = \frac{0.17}{0.17+0.68} = \frac{0.17}{0.85} = 0.20$$

Recall: The formula for conditional probability:

$$P(A|B) = \frac{P(A\cap B)}{P(B)} \quad P(B) \neq 0 \qquad\qquad P(B|A) = \frac{P(A\cap B)}{P(A)} \quad P(A) \neq 0$$

From these we get the multiplication rule:

$$P(A\cap B) = P(B)\cdot P(A|B) \qquad\qquad P(A\cap B) = P(A)\cdot P(B|A)$$

look at it by cases

We can partition an event A by looking at where it overlaps with event B: event A can overlap with event B, or it can overlap with event $\overline{B}$. That is, the events $A\cap B$ and $A\cap\overline{B}$ partition the event A. Note too that $A\cap B$ and $A\cap\overline{B}$ are disjoint events, so we can say

$$P(A) = P(A\cap \underline{B}) + P(A\cap \underline{\overline{B}}).$$

know there's no overlap

If we then use the multiplication rule we can say that

$$P(A) = P(A\cap B) + P(A\cap\overline{B}) = P(B)\cdot P(A|B) + P(\overline{B})\cdot P(A|\overline{B}).$$

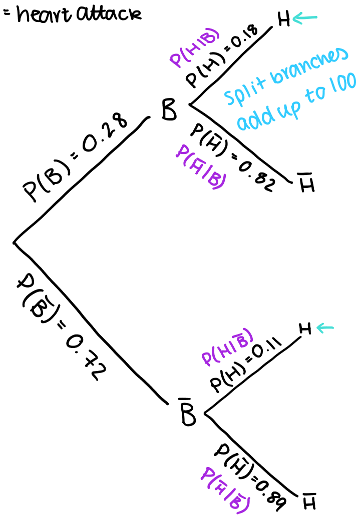
This is best illustrated on a tree diagram.

Example 3: Balding men and heart attacks.

the given part, already in the balding group

A survey was taken of middle aged men and it was found that 28% of them are balding. Of those who are balding, there is an 18% chance that they will have a heart attack in the next 10 years. For those who are not balding, there is an 11% chance that they will have a heart attack within the next 10 years. What is the probability that a middle aged man will have a heart attack in the next 10 years?

B = balding
H = heart attack



$P(H) = P(B \cap H) + P(\bar{B} \cap H)$

$= P(B) \cdot P(H|B) + P(\bar{B}) \cdot P(H|\bar{B})$

$= (0.28)(0.18) + (0.72)(0.11)$

can add together b/c know there's no overlap b/w the cases

numbers have to be comparable before you can add them

Note: multiply going across the branch paths, add cases together moving down the tree

The law of total probability says that to find $P(A)$, we add up the probabilities of a partition of A (i.e. add up all disjoint cases that arrive at A). In symbols:

$$P(A) = \sum_{i=1}^{\infty} P(A \cap B_i) = \sum_{i=1}^{\infty} P(B_i) \cdot P(A|B_i)$$

This is the same thing we did when we added together the different cases from the tree branches in the last example.

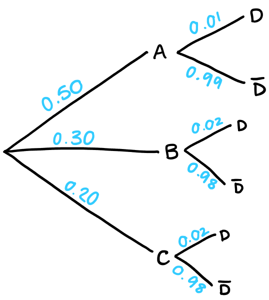
Bayes' Theorem puts together the conditional probability formula along with the multiplication rule. Using tree diagrams for these questions are very useful!

Example 4: TV sets.

TV sets are made at production plants A, B, and C. Suppose 50% are made at plant A, 30% are made at plant B, and 20% are made at plant C. Quality control finds that:

- 1% of plant A TVs are defective.
- 2% of plant B TVs are defective.
- 2% of plant C TVs are defective.

Given that a randomly selected TV is defective, what is the probability that it was produced at plant C?



$$P(C|D) = \frac{P(C \cap D)}{P(D)}$$

$$= \frac{(0.2)(0.02)}{(0.5)(0.01) + (0.3)(0.02) + (0.2)(0.02)}$$

$$= \frac{0.004}{0.015} = 0.267$$

6

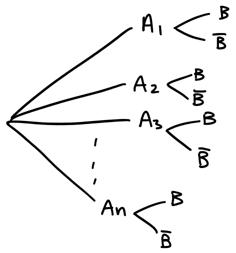
$$\frac{P(C) \cdot P(D|C)}{P(A) \cdot P(D|A) + P(B) \cdot P(D|B) + P(C)\,P(D|C)}$$ Bayes' Theorem

Suppose we have events A_1, A_2, ..., A_n, which are then followed by event B or $\overline{B}$. We can write Bayes' Theorem in symbols as:

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{P(A_1) \cdot P(B|A_1)}{\sum_{i=1}^{n} P(A_i) \cdot P(B|A_i)}$$

adding all the cases together

You don't need to write the symbolic form in your solution, it is much more important to know how to come up with this from using the conditional probability formula and a tree diagram like we did in Example 4.



Sometimes we perform diagnostic tests and the results shown are wrong.

There are 4 options for outcomes in a diagnostic test:

- The condition actually occurs and the test indicates positive for the condition occurring.
  This is a **true positive**, and no error occurs here.
- The condition actually occurs and the test indicates negative for the condition occuring.
  This is a **false negative**, this is an error.
- The condition does not actually occur and the test indicates positive for the condition occurring.
  This is a **false positive**, this is an error.
- The condition does not actually occur and the test indicates negative for the condition occurring.
  This is a **true negative**, and no error occurs here.

false when it's wrong
positive/negative is what the test shows

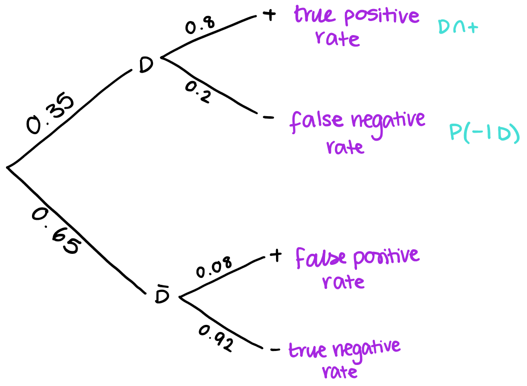
Example 5: Kidney transplants.

A patient receives a kidney transplant. Suppose that 35% of kidney transplants are rejected. During the healing process the patient is tested to see if they are rejecting the kidney. For this particular test the false positive rate is 8% and the false negative rate is 20%. What is the probability that the patient is rejecting the kidney if their test result is positive (i.e. the test indicates they are rejecting the kidney)?

D = actually rejecting kidney

+ = test is + (test indicates rejection)

− = test is − (test indicates no rejection)



$$P(D|+) = \frac{P(D \cap +)}{P(+)} = \frac{(0.35)(0.8)}{(0.35)(0.8) + (0.65)(0.08)}$$

$$= 0.843$$

Remember:

- false positive rate $= P(+|\overline{D})$
- false negative rate $= P(-|D)$
- true positive rate $= P(+|D)$. This is also sometimes called the sensitivity.
- true negative rate $= P(-|\overline{D})$. This is also sometimes called the specificity.

Set 7 - Independent and Mutually Exclusive Events

January 24, 2023 12:15 PM

Stat 260 Lecture Notes
Set 7 - Independent and Mutually Exclusive Events

Idea: Knowing extra information can change the probability of an outcome. This is what we saw with conditional probabilities. Here we look at the case when knowing the extra information does not change the probability of the outcome. This is the idea of having *independent events*.

Definition: Events A and B are *independent* when $P(A|B) = P(A)$ (or when $P(B|A) = P(B)$).

$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A)$

Using this definition along with the conditional probability formula we arrive at the alternate definition that events A and B are independent exactly when

$$P(A \cap B) = P(A) \cdot P(B)$$

Use to check if events are independent

Definition: Events A and B are *mutually exclusive* if they cannot occur at the same time (i.e. there is no overlap between A and B).

In the mathematical sense, we have that A and B are mutually exclusive when $P(A \cap B) = 0$ (i.e. when having both event A and B occur together is impossible).

Example 1: Revisiting the clear coating and rust example.

A maufacturer wants to see if using a clear coating of paint on their product is connected to if the product rusts.

	rust present (R)	no rust present ($\overline{R}$)
clear coating used (C)	0.03	0.12
no clear coating used ($\overline{C}$)	0.17	0.68
	0.20	0.80

Is having rust present independent of using the clear coating?

Check: $P(D \cap G) = P(D) \cdot P(G)$

$P(D \cap G) = 0.03$ same

$P(D) \cdot P(G) = (0.2)(0.15) = 0.03$

So $P(D \cap G) = P(D) \cdot P(G)$ and so D and G are independent events

Rule: If events A and B are independent, then $\overline{A}$ and B are independent too (and A and $\overline{B}$ are independent, and also $\overline{A}$ and $\overline{B}$ are independent).

Is using the clear coating independent of not using the clear coating?

Check: $P(D \cap \overline{D}) = P(D) \cdot P(\overline{D})$

$P(D \cap \overline{D}) = 0$ not the same

$P(D) \cdot P(\overline{D}) = (0.2)(0.8) \neq 0$

So $P(D \cap \overline{D}) \neq P(D) \cdot P(\overline{D})$ so D and $\overline{D}$ are <u>not</u> independent

But D and $\overline{D}$ are mutually exclusive since $P(D \cap \overline{D}) = 0$

Careful! "Mutually exclusive" and "independent" are not the same thing. Here "clear coating" and "no clear coating" are mutually exclusive (since they are disjoint), but they are not independent.

The rule that if A and B are independent then

$$P(A \cap B) = P(A) \cdot P(B)$$

can be extended to more than two events.

Rule: If events $E_1, E_2, E_3, \ldots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot \cdots \cdot P(E_n).$$

Note: When $n \geq 3$ this rule does not work the other way around. That is, just because you have $P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdot \cdots \cdot P(E_n)$ does not guarantee that the events E_1, E_2, $\ldots$, E_n are all independent. (To guarantee independence you would have to do this formula check on all pairs, triples, quadruples, etc.)

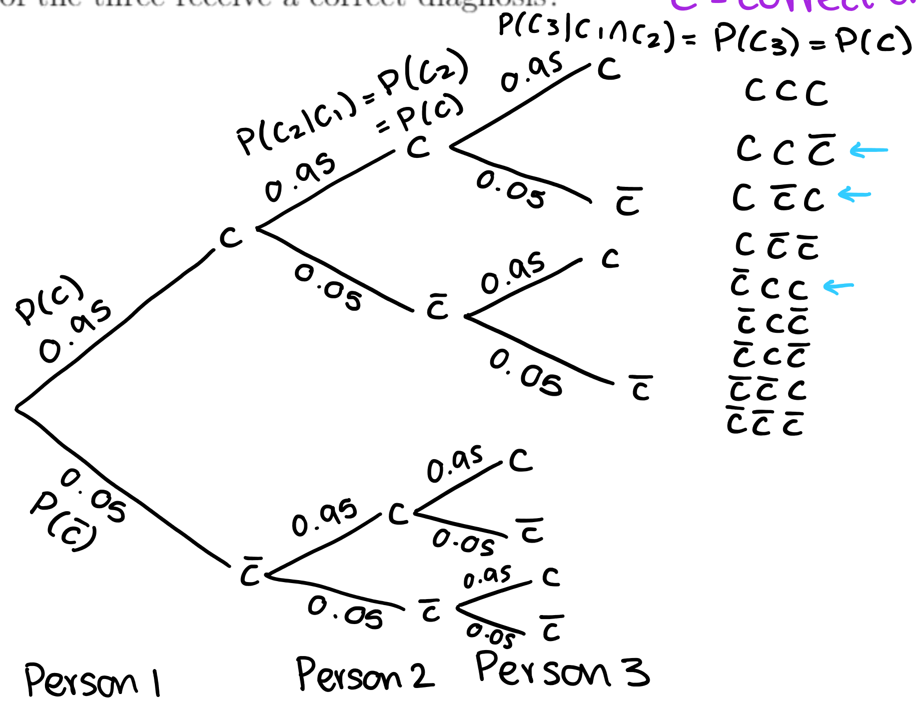
(So the independence formula is an "if and only if" statement for two sets, and just an "if" statement for three or more sets.)

A, B, C check:

$$\begin{cases} P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) \\ P(A \cap B) = P(A) \cdot P(B) \\ P(A \cap C) = P(A) \cdot P(C) \\ P(B \cap C) = P(B) \cdot P(C) \end{cases}$$

Example 2: A diagnostic test is correct 95% of the time. Suppose 3 people are independently tested. What is the probability that exactly two of the three receive a correct diagnosis?

C = correct diagnosis



$$P(\text{exactly 2 correct}) = \underbrace{P(CC\bar{C})}^{C\cap C\cap \bar{C}} + P(C\bar{C}C) + P(\bar{C}CC)$$

$$= (0.95)(0.95)(0.05) + (0.95)(0.05)(0.95) + (0.5)(0.95)(0.95)$$

$$= 0.135375$$

since independent

$$P(CC\bar{C}) = P(C\cap C\cap \bar{C}) = P(C)\cdot P(C)\cdot P(\bar{C})$$

$$= (0.95)(0.95)(0.05)$$

What is the probability that none of the three receive a correct diagnosis?

$$P(\bar{c}\bar{c}\bar{c}) = P(\bar{c}) \cdot P(\bar{c}) \cdot P(\bar{c})$$

$$= (0.05)(0.05)(0.05)$$

$$= 0.000125$$

What is the probability that at least one of the three receive a correct diagnosis?

$$P(\text{at least } 1) = 1 - P(\bar{c}\bar{c}\bar{c}) = 1 - 0.000125$$

$$= 0.999875$$

or add up probabilities from the first 7 branch path cases (the places where 1 or more are correct)

Set 8 - Random Variables

January 24, 2023 12:15 PM

Stat 260 Lecture Notes
Set 8 - Random Variables

HHT

$X = \#$ of H $\quad X = 2$

has to be a number

A **random variable (r.v.)** (usually we denote it by X) is a function or a rule that assigns a number to each outcome of the experiment. Back in Set 1 we saw that random variables can be discrete or continuous.

can list all possible x values

The **probability mass function** (**pmf**), or **probability distribution**, is a table, formula, or graph that describes the possible values of the r.v. and the probability that each value will occur.

Think of the pmf as a function f.

$f(2) = P(x = 2)$

r.v. specific values

$$f(x) = P(X = x)$$

function represents a probability

A pmf for a a discrete r.v. X must meet the requirements:

1. $f(x) = P(X = x)$ is defined for all values of x.
2. $f(x) = P(X = x) \geq 0$ for all values of x.
3. $\sum_{\text{all}x} f(x) = \sum_{\text{all}x} P(X = x) = 1$ (the sum of all probabilities is 1).

Continuous probability distributions are studied in a later Set.

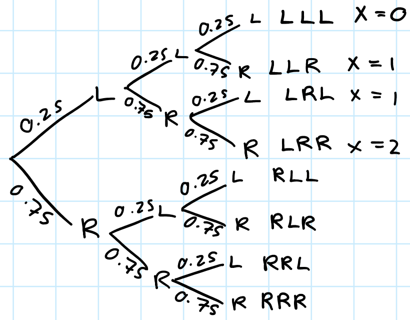
1

Example 1: Dominant writing hands.

Suppose 25% of people are left-handed. Suppose we independently sample 3 people and count how many are right handed.

→ so will be an actual number

Let the r.v. X be the number of right-handed people in the 3 sampled. The possible values of X are 0, 1, 2, 3.



After some work we can find the pmf:

how many right handed

x	0	1	2	3
$f(x) = P(X = x)$	0.015625	0.140625	0.421875	0.421875
	$(0.25)^3$	$3(0.25)^2(0.75)$	$3(0.25)(0.75)^2$	$(0.75)^3$

Notice that all probabilities are ≥ 0 and that

$$\sum_x f(x) = 0.015625 + 0.140625 + 0.421875 + 0.421875 = 1.$$

→ won't always be the same, just because of the specific numbers in this example

(a) Find $P(X = 2)$.

(b) Find $P(X \geq 1)$.

(c) Find $P(X \leq 2 \mid X \geq 1)$.

a) $P(X=2) = 0.421875$

b) $P(X \geq 1) = P(X=1) + P(X=2) + P(X=3)$
$= 0.140625 + 0.421875 + 0.421875$

or $P(X \geq 1) = 1 - P(X=0) = 1 - 0.015625 = 0.984375$

c) $P(X \leq 2 \mid X \geq 1) = \dfrac{P(X \leq 2 \cap X \geq 1)}{P(X \geq 1)} = \dfrac{P(1 \leq X \leq 2)}{P(X \geq 1)}$

overlap

$= \dfrac{P(X=1) + P(X=2)}{P(X \geq 1)}$

$= \dfrac{0.140625 + 0.421875}{0.984375} = 0.5714$

2

The cumulative distribution function (cdf) of a r.v. X is defined as $F(x) = P(X \leq x)$.

So for a value c, $F(c) = P(X \leq c) = \sum_{x \leq c} P(X = x) = \sum_{x \leq c} f(x)$.

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

pmf $f(x) = P(X = x)$

cdf $F(x) = P(X \leq x)$

Example 2: Find the cdf for the dominant writing hand example.

pmf

x	$f(x) = P(X=x)$
0	0.015625
1	0.140625
2	0.421875
3	0.421875

⇒ cdf

x	$F(x) = P(X \leq x)$
0	0.015625
1	0.15625
2	0.578125
3	1

→ last entry of cdf table is 1

$F(0) = P(X \leq 0) = P(X=0) = f(0) = 0.015625$

$F(1) = P(X \leq 1) = P(X=0) + P(X=1) = f(0) + f(1) = 0.015625 + 0.140625$
$= 0.15625$

$F(2) = P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.578125$

$cdf \Rightarrow pmf$

Example 3: The cdf for an experiment is given below. Find the pmf.

x	$F(x) = P(X \le x)$
0	0.15
1	0.38
2	0.74
3	0.92
4	0.98
5	1

pmf

x	$f(x) = P(X = x)$
0	0.15
1	0.23
2	0.36
3	0.18
4	0.06
5	0.02

probabilities sum to 1

$f(3) = P(X=3) = P(X \le 3) - P(X \le 2)$
$= 0.92 - 0.74 = 0.18$

Example 4: Using the distribution from Example 3, find:

(a) $P(X = 2)$

(b) $P(X \ge 3)$

(c) $P(1 < X \le 4)$

a) $P(X=2) = 0.36$

pmf

b) $P(X \ge 3) = P(X \le 5) - P(X \le 2)$
$= 1 - 0.74$
$= 0.26$

c) $P(1 < X \le 4) = P(2 \le X \le 4) = P(X \le 4) - P(X \le 1)$
$= 0.98 - 0.38$
$= 0.6$

can also do using the other method

Rules for discrete r.v.s:

- $P(X \ge x) = 1 - P(X < x)$
- $P(X > x) = 1 - P(X \le x)$
- $P(a \le X \le b) = P(X \le b) - P(X < a)$

Stat 260 Lecture Notes
Set 9 - Expected Value

Let X be a random variable. The *expected value* of X, denoted $E(X)$, is the long-run theoretical average value of X.

Example 1: Suppose we had a population of values:

$$X: \quad 2, \quad 4, \quad 6, \quad 6, \quad 4, \quad 4, \quad 2, \quad 3, \quad 5, \quad 5$$

Find the population mean and find the pmf for this distribution.

mean: $\mu = \frac{2+4+6+6+4+4+2+3+5+5}{10} = 4.1$

$= \frac{(2+2)+3+(4+4+4)+(5+5)+(6+6)}{10}$

$= \frac{2\cdot2+3\cdot1+4\cdot3+5\cdot2+6\cdot2}{10}$

$= 2\left(\frac{2}{10}\right)+3\left(\frac{1}{10}\right)+4\left(\frac{3}{10}\right)+5\left(\frac{2}{10}\right)+6\left(\frac{2}{10}\right)$

connection b/w mean and pmf

pmf:

x	2	3	4	5	6
$f(x)$	2/10	1/10	3/10	2/10	2/10

Definition: Let X be a discrete random variable with pmf $f(x)$. The *expected value* or *mean value* of X is given by

$$\mu = E(X) = \sum_{\text{all } x} x \cdot f(x) = \sum_{\text{all } x} x \cdot P(X = x)$$

Note: Sometimes we write μ_X to clarify that we are talking about the r.v. X.

Example 2: A one-year life insurance policy of \$250,000 is sold to an 18 year old person for \$350. According to Vital Statistics, the probability that an 18 year old will live another year is 0.998936. What is the expected value of the policy to the life insurance company?

$E(x)$

r.v. X = how much money company makes

	person lives	person dies
x	350	350 - 250,000
$f(x)$	0.998936	0.001064

$$E(X) = \sum x \cdot f(x)$$
$$= 350(0.998936) + (350 - 250{,}000)(0.001064)$$
$$= 84$$

So in the long run, on average the company makes \$84 for every policy sold to an 18 year old.

* if $E(x)$ was negative means company is losing money on average.

We can find the expected value for a function of X.

Example 3: Suppose we have the discrete random variable X with pmf:

x^2+5	25^2+5	45^2+5	65^2+5
x^2	25^2	45^2	65^2
x	25	45	65
$f(x)$	1/2	1/3	1/6

← # of units sold

price = $3x+7$

expected cost = $E(3x+7)$

↳ when we might use smt other than only x

(a) Find $E(X)$.

(b) Find $E(X^2)$.

(c) Find $E(X^2+5)$.

a) $E(X) = \sum x \cdot f(x)$

$= 25(\frac{1}{2}) + 45(\frac{1}{3}) + 65(\frac{1}{6})$

$= \frac{115}{3} = 38.33$

b) $E(x^2) = \sum x^2 \cdot f(x)$ → only square the x, not $f(x)$

$= 25^2(\frac{1}{2}) + 45^2(\frac{1}{3}) + 65^2(\frac{1}{6})$

$= 1691.67$

c) $E(x^2+5) = \sum (x^2+5) \cdot f(x)$

$= (25^2+5)(\frac{1}{2}) + (45^2+5)(\frac{1}{3}) + (65^2+5)(\frac{1}{6})$

$= 1696.67$ +5

$$E(x^2+5) = E(x^2) + 5$$

Notice then that for $g(x)$ (a function of the r.v. X) we have that $E(g(x)) = \sum_{\text{all } x} g(x) \cdot f(x)$.

$$E(g(x)) = \sum g(x) \cdot f(x)$$

↑ probabilities

$$E(\sqrt{x}) = \sum \sqrt{x} \cdot f(x)$$

Cutoff for test 1

↳ only first 2 formulas on left of formula sheets

only first 2 formulas
on left of formula sheets

$s^2 =$

$r =$

Stat 260 Lecture Notes

Set 10 - Variance, and Expected Value and Variance Rules

The *variance* of a r.v. X with pmf $f(x)$ is

$$\begin{aligned} V(X) = \sigma_X^2 = \sigma^2 &= E(\overbrace{(X-\mu)^2}^{g(x)}) \\ &= \sum_{\text{all } x} (x-\mu)^2 \cdot f(x) \\ &= \sum_{\text{all } x} (x-\mu)^2 \cdot P(X=x) \end{aligned}$$

Example 1: The discrete random variable X has pmf as follows:

x	25	45	65
$f(x)$	1/2	1/3	1/6

Find the variance of X. That is, find $V(X)$.

$V(X) = E((x-\mu)^2)$

$\mu = E(X) = \sum x \cdot f(x) = 25(1/2) + 45(1/3) + 65(1/6) = \frac{115}{3} = 38.33$

$V(X) = E\left((x-\mu)^2\right) = \sum (x-\mu)^2 \cdot f(x)$

$= (25-38.33)^2(1/2) + (45-38.33)^2(1/3) + (65-38.33)^2(1/6$

$= 222.22$

1

There is a shortcut formula to calculate $V(X)$.

In general $E(x^2) \neq (E(x))^2$

$$
\begin{aligned}
V(X) &= E(X^2) - (E(X))^2 \\
&= E(X^2) - \mu^2 \\
&= \left(\sum_{\text{all } x} x^2 \cdot f(x) \right) - \mu^2
\end{aligned}
$$

on formula sheet

Example 2: Calculate $V(X)$ from Example 1 again, but use the shortcut formula.

pmf

x	25	45	65
$f(x)$	1/2	1/3	1/6

$V(X) = E(x^2) - (E(x))^2$

$E(X) = \sum x \cdot f(x) = 38.33$

$E(x^2) = \sum x^2 \cdot f(x) = 1691.67$

$V(X) = 1691.67 - (38.33)^2 = 222.4811$

rounding errors

without rounding: 222.22

variance ≥ 0

Recall: standard deviation $= \sqrt{\text{variance}}$.

The *standard deviation* of r.v. X is $\sigma_X = \sigma = \sqrt{\sigma_X^2} = \sqrt{V(X)}$.

In the last example, $\sigma = \sqrt{V(X)} = \sqrt{222.22}$
$= 14.9071$

Standard deviation ≥ 0

2

μ and $E(x)$ are the same

Rules for Expected Value:

can add constant on outside

- for a constant c, $E(X + c) = E(X) + c$
- $E(c) = c$ average of all same number just is that number
- $E(cX) = c \cdot E(X)$ can pull out constant

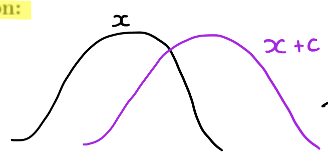
Putting these together we get the rule

$$E(aX + b) = a \cdot E(X) + b$$

→ all the rules in one!

where a and b are constants.

Rules for Variance and Standard Deviation:



variance: how spread out is data from the mean

↳ moving over doesn't change spread

variance

- for a constant c, $V(X + c) = V(X)$
- $V(c) = 0$ no spread
- $V(cX) = c^2 \cdot V(X)$ have to square if taking constant out

standard deviation $\sqrt{\text{variance}}$

- $\sigma_{X+c} = \sigma_X$
- $\sigma_c = 0$ (so SD rules are just $\sqrt{\ }$ of variance rules)
- $\sigma_{cX} = |c| \cdot \sigma_X$ SD ≥ 0

Putting these together we get the rules

$$V(aX + b) = V(aX) = a^2 V(X)$$

one line versions

$$\sigma_{aX+b} = \sigma_{aX} = |a| \cdot \sigma_X$$

where a and b are constants.

adding rules

Rule: For random variables X and Y, we have that $E(X + Y) = E(X) + E(Y)$. → can add separately

Rule: For random variables X and Y that are independent, we have that $V(X + Y) = V(X) + V(Y)$.

Example 3: Say $X_1, X_2, \ldots, X_n$ are all random variables with expected value μ. $E(X_1) = E(X_2) = \ldots = E(X_n) = \mu$

Look at $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$.

Find $E(\overline{X})$.

$$E(\bar{x}) = E\left(\frac{x_1+x_2+\ldots+x_n}{n}\right) = E\left(\frac{x_1}{n} + \frac{x_2}{n} + \ldots + \frac{x_n}{n}\right)$$

$$= E\left(\frac{x_1}{n}\right) + E\left(\frac{x_2}{n}\right) + \ldots + E\left(\frac{x_n}{n}\right)$$

$$= E\left(\frac{1}{n}\cdot x_1\right) + E\left(\frac{1}{n}\cdot x_2\right) + \ldots + E\left(\frac{1}{n}\cdot x_n\right)$$

$$= \frac{1}{n}E(x_1) + \frac{1}{n}E(x_2) + \ldots + \frac{1}{n}E(x_n)$$

$$= \frac{1}{n}\cdot\mu + \frac{1}{n}\cdot\mu + \ldots + \frac{1}{n}\cdot\mu$$

$$= n\cdot\frac{1}{n}\cdot\mu$$

$$= \mu$$

So $E(\bar{x}) = \mu = E(x)$. ■

expected value of average is same as expected value of single x.

Therefore we have the rule that $E(\overline{X}) = \mu_{\overline{X}} = \mu$. Following similar arguments (and using a couple extra assumptions) we can show the rules that $V(\overline{X}) = \dfrac{\sigma_X^2}{n}$ and $\sigma_{\overline{X}} = \dfrac{\sigma_X}{\sqrt{n}}$. (Remember for notation: $V(X) = \sigma_X^2$ and $\sigma_X = \sigma$.)

We will use these rules lots in our later Sets!

Stat 260 Lecture Notes

Set 11 - The Binomial Distribution

Let's revisit two examples we've seen in previous sets.

Example (Set 7): A diagnostic test is correct 95% of the time. Suppose 3 people are independently tested. What is the probability that exactly two of the three receive a correct diagnosis?

Example (Set 8): Suppose 25% of people are left-handed. Suppose we independently sample 3 people and count how many are right-handed. What is the probability that exactly two people are right handed?

Both these questions can be solved by using a tree diagram. We want to generalize the solution for these common question setups.

In these questions the random variable X = the number of "successes" in the 3 trials.
In the Set 7 question success = correct diagnosis.
In the Set 8 question success = right-handed.

These are **binomial experiments**. X is a **binomial random variable**.

We have a binomial experiment if:

1. There is a fixed number of trials, n. $n=3$
2. Each trial results in one of two possible outcomes: a success (S) or a failure (F).
3. The trials are independent and the probability of success in each trial is the same.
   In right-handed example:
   $p = P(\text{success in a single trial})$ $P(\text{right-handed}) = 0.75$
   $1 - p = P(\text{failure in a single trial})$ $1-p = P(\text{left-handed}) = 0.25$
4. The random variable X counts the number of successes in n trials.

"X is distributed as"

If X is a binomial random variable we write $X \sim \text{binomial}(n, p)$.

Each trial of a binomial experiment is a Bernoulli trial - there are only two outcomes.

The pmf for a binomial random variable X is given by the formula

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

prob. x successes → on formula sheet

prob all other $n-x$

" n choose x "

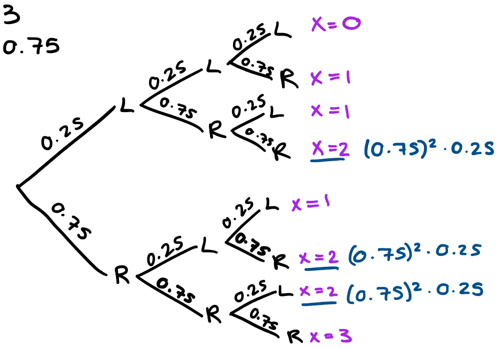
branches w/ x successes

where $x = 0, 1, 2, \ldots, n$.

X = # of right-handed

n = 3

p = 0.75



Look at P(X = 2):

$P(X=2) = 3 \cdot (0.75)^2 \cdot (0.25)$

probability of 2 right-handed

# of branches where x = 2

probability of other 1 is left-handed

From counting: $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ ← counts # of ways to select x objects from a collection of n objects. → n trials, which x of them are successes

nCr on calculator (above 5)

We looked at the pmf of the right handed example in Set 8. Check that this pmf for the binomial distribution also gives the values in the pmf table we found there.

Example 1: A carrier of TB has a 10% chance of passing on the disease to strangers. Suppose a carrier is in close contact with 20 strangers in a day.

→ independence

(a) What is the probability that at least one of the strangers contracts the disease?

(b) If at least one stranger gets the disease, what is the probability that at most three get the disease?

What is the random variable?

X = # of people who get the disease.

X is binomial

$n = 20$ ↳ fixed # of trials

"success" = gets the disease ↳ only 2 options (gets it or doesn't)

$p = 0.10$ ↳ equal chance for each trial

using the complement

a) $P(X \geq 1) = 1 - P(X=0)$

$= 1 - \binom{20}{0}(0.10)^0(0.90)^{20}$ $f(x) = \binom{n}{x}p^x(1-p)^{n-x}$

$= 1 - 0.1216$

$= 0.8784$

b) $P(X \leq 3 \mid X \geq 1) = \dfrac{P(X \leq 3 \cap X \geq 1)}{P(X \geq 1)} = \dfrac{P(1 \leq X \leq 3)}{P(X \geq 1)}$

$= \dfrac{P(X=1) + P(X=2) + P(X=3)}{P(X \geq 1)}$

$= \dfrac{\binom{20}{1}(0.1)^1(0.9)^{19} + \binom{20}{2}(0.1)^2(0.9)^{18} + \binom{20}{3}(0.1)^3(0.9)^{17}}{0.8784}$

$= \dfrac{0.2702 + 0.2852 + 0.1901}{0.8784}$

$= 0.8487$

Rules for the binomial distribution:

x | $f(x)$

$\cancel{E(x) = \sum x \cdot f(x)}$

don't use if you know it's binomial

- $E(X) = n \cdot p$
- $V(X) = n \cdot p(1-p)$
- $\sigma_X = \sqrt{n \cdot p(1-p)}$

$\sigma = \sqrt{V(x)}$

In Example 1, the expected number of the 20 strangers who contract the disease is:

$$E(x) = \underset{n}{20}(\underset{p}{0.10}) = 2$$

For the cdf of a binomial random variable X we have tables with results already calculated (see page 1 of the stats table package).

pmf: $P(x=x) = \binom{n}{x} p^x (1-p)^{n-x}$

Cdf: $P(x \le x)$ stat table

Example 2: Suppose we have a binomial experiment with $n = 15$, $p = 0.3$, and the random variable X counts the number of successes. Find:

(a) $P(X \leq 4)$

(b) $P(X < 2)$

(c) $P(X = 5)$

(d) $P(X \leq 7 \mid X \geq 5)$

a) $P(X \leq 4) = 0.5155$ from stat tables
cdf

Note: if $p = 0.37$ (or any p not on the table) then do the calculation by hand

b) $P(X<2) = P(X \leq 1) = 0.0353$
cdf

c) $P(X=5) = \binom{15}{5}(0.3)^5(0.7)^{10} = 0.2061$
pmf

OR

$P(X=5) = P(X \leq 5) - P(X \leq 4)$ → using stat tables

$= 0.7216 - 0.5155$

$= 0.2061$

d) $P(X \leq 7 \mid X \geq 5) = \dfrac{P(X \leq 7 \cap X \geq 5)}{P(X \geq 5)} = \dfrac{P(5 \leq X \leq 7)}{P(X \geq 5)}$

convert to cdf using complement

$= \dfrac{P(X \leq 7) - P(X \leq 4)}{1 - P(X \leq 4)}$

$= \dfrac{0.9500 - 0.5155}{1 - 0.5155}$ from stat table

$= 0.8968$

Distribution Tables

February 13, 2023 1:47 PM

Tables for STAT 260

What if P = 0.37?
↳ do calculation by hand

Table A.1 Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
1	0	0.9000	0.8000	0.7500	0.7000	0.6000	0.5000	0.4000	0.3000	0.2000	0.1000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0	0.8100	0.6400	0.5625	0.4900	0.3600	0.2500	0.1600	0.0900	0.0400	0.0100
	1	0.9900	0.9600	0.9375	0.9100	0.8400	0.7500	0.6400	0.5100	0.3600	0.1900
	2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0	0.7290	0.5120	0.4219	0.3430	0.2160	0.1250	0.0640	0.0270	0.0080	0.0010
	1	0.9720	0.8960	0.8438	0.7840	0.6480	0.5000	0.3520	0.2160	0.1040	0.0280
	2	0.9990	0.9920	0.9844	0.9730	0.9360	0.8750	0.7840	0.6570	0.4880	0.2710
	3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0	0.6561	0.4096	0.3164	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
	1	0.9477	0.8192	0.7383	0.6517	0.4752	0.3125	0.1792	0.0837	0.0272	0.0037
	2	0.9963	0.9728	0.9492	0.9163	0.8208	0.6875	0.5248	0.3483	0.1808	0.0523
	3	0.9999	0.9984	0.9961	0.9919	0.9744	0.9375	0.8704	0.7599	0.5904	0.3439
	4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0	0.5905	0.3277	0.2373	0.1681	0.0778	0.0313	0.0102	0.0024	0.0003	0.0000
	1	0.9185	0.7373	0.6328	0.5282	0.3370	0.1875	0.0870	0.0308	0.0067	0.0005
	2	0.9914	0.9421	0.8965	0.8369	0.6826	0.5000	0.3174	0.1631	0.0579	0.0086
	3	0.9995	0.9933	0.9844	0.9692	0.9130	0.8125	0.6630	0.4718	0.2627	0.0815
	4	1.0000	0.9997	0.9990	0.9976	0.9898	0.9688	0.9222	0.8319	0.6723	0.4095
	5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0	0.5314	0.2621	0.1780	0.1176	0.0467	0.0156	0.0041	0.0007	0.0001	0.0000
	1	0.8857	0.6554	0.5339	0.4202	0.2333	0.1094	0.0410	0.0109	0.0016	0.0001
	2	0.9842	0.9011	0.8306	0.7443	0.5443	0.3438	0.1792	0.0705	0.0170	0.0013
	3	0.9987	0.9830	0.9624	0.9295	0.8208	0.6563	0.4557	0.2557	0.0989	0.0159
	4	0.9999	0.9984	0.9954	0.9891	0.9590	0.8906	0.7667	0.5798	0.3446	0.1143
	5	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844	0.9533	0.8824	0.7379	0.4686
	6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0	0.4783	0.2097	0.1335	0.0824	0.0280	0.0078	0.0016	0.0002	0.0000	
	1	0.8503	0.5767	0.4449	0.3294	0.1586	0.0625	0.0188	0.0038	0.0004	0.0000
	2	0.9743	0.8520	0.7564	0.6471	0.4199	0.2266	0.0963	0.0288	0.0047	0.0002
	3	0.9973	0.9667	0.9294	0.8740	0.7102	0.5000	0.2898	0.1260	0.0333	0.0027
	4	0.9998	0.9953	0.9871	0.9712	0.9037	0.7734	0.5801	0.3529	0.1480	0.0257
	5	1.0000	0.9996	0.9987	0.9962	0.9812	0.9375	0.8414	0.6706	0.4233	0.1497
	6		1.0000	0.9999	0.9998	0.9984	0.9922	0.9720	0.9176	0.7903	0.5217
	7			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

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Tables for STAT 260

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8	0	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8322	0.5695
	8				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.8658	0.6126
	9					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000		
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.3222	0.0702
	8		1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.6242	0.2639
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.8926	0.6513
	10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000			
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.1611	0.0185
	8		1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.3826	0.0896
	9			1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.6779	0.3026
	10					1.0000	0.9995	0.9964	0.9802	0.9141	0.6862
	11						1.0000	1.0000	1.0000	1.0000	1.0000

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Tables for STAT 260

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000		
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000
13	0	0.2542	0.0550	0.0238	0.0097	0.0013	0.0001	0.0000			
	1	0.6213	0.2336	0.1267	0.0637	0.0126	0.0017	0.0001	0.0000		
	2	0.8661	0.5017	0.3326	0.2025	0.0579	0.0112	0.0013	0.0001		
	3	0.9658	0.7473	0.5843	0.4206	0.1686	0.0461	0.0078	0.0007	0.0000	
	4	0.9935	0.9009	0.7940	0.6543	0.3530	0.1334	0.0321	0.0040	0.0002	
	5	0.9991	0.9700	0.9198	0.8346	0.5744	0.2905	0.0977	0.0182	0.0012	0.0000
	6	0.9999	0.9930	0.9757	0.9376	0.7712	0.5000	0.2288	0.0624	0.0070	0.0001
	7	1.0000	0.9988	0.9944	0.9818	0.9023	0.7095	0.4256	0.1654	0.0300	0.0009
	8		0.9998	0.9990	0.9960	0.9679	0.8666	0.6470	0.3457	0.0991	0.0065
	9		1.0000	0.9999	0.9993	0.9922	0.9539	0.8314	0.5794	0.2527	0.0342
	10			1.0000	0.9999	0.9987	0.9888	0.9421	0.7975	0.4983	0.1339
	11				1.0000	0.9999	0.9983	0.9874	0.9363	0.7664	0.3787
	12					1.0000	0.9999	0.9987	0.9903	0.9450	0.7458
	13						1.0000	1.0000	1.0000	1.0000	1.0000
14	0	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000			
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001			
	2	0.8416	0.4481	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000		
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000

Tables for STAT 260

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
15	0	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000				
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000			
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000		
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000
16	0	0.1853	0.0281	0.0100	0.0033	0.0003	0.0000				
	1	0.5147	0.1407	0.0635	0.0261	0.0033	0.0003	0.0000			
	2	0.7892	0.3518	0.1971	0.0994	0.0183	0.0021	0.0001			
	3	0.9316	0.5981	0.4050	0.2459	0.0651	0.0106	0.0009	0.0000		
	4	0.9830	0.7982	0.6302	0.4499	0.1666	0.0384	0.0049	0.0003		
	5	0.9967	0.9183	0.8103	0.6598	0.3288	0.1051	0.0191	0.0016	0.0000	
	6	0.9995	0.9733	0.9204	0.8247	0.5272	0.2272	0.0583	0.0071	0.0002	
	7	0.9999	0.9930	0.9729	0.9256	0.7161	0.4018	0.1423	0.0257	0.0015	0.0000
	8	1.0000	0.9985	0.9925	0.9743	0.8577	0.5982	0.2839	0.0744	0.0070	0.0001
	9		0.9998	0.9984	0.9929	0.9417	0.7728	0.4728	0.1753	0.0267	0.0005
	10		1.0000	0.9997	0.9984	0.9809	0.8949	0.6712	0.3402	0.0817	0.0033
	11			1.0000	0.9997	0.9951	0.9616	0.8334	0.5501	0.2018	0.0170
	12				1.0000	0.9991	0.9894	0.9349	0.7541	0.4019	0.0684
	13					0.9999	0.9979	0.9817	0.9006	0.6482	0.2108
	14					1.0000	0.9997	0.9967	0.9739	0.8593	0.4853
	15						1.0000	0.9997	0.9967	0.9719	0.8147
	16							1.0000	1.0000	1.0000	1.0000

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Tables for STAT 260

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
17	0	0.1668	0.0225	0.0075	0.0023	0.0002	0.0000				
	1	0.4818	0.1182	0.0501	0.0193	0.0021	0.0001	0.0000			
	2	0.7618	0.3096	0.1637	0.0774	0.0123	0.0012	0.0001			
	3	0.9174	0.5489	0.3530	0.2019	0.0464	0.0064	0.0005	0.0000		
	4	0.9779	0.7582	0.5739	0.3887	0.1260	0.0245	0.0025	0.0001		
	5	0.9953	0.8943	0.7653	0.5968	0.2639	0.0717	0.0106	0.0007	0.0000	
	6	0.9992	0.9623	0.8929	0.7752	0.4478	0.1662	0.0348	0.0032	0.0001	
	7	0.9999	0.9891	0.9598	0.8954	0.6405	0.3145	0.0919	0.0127	0.0005	
	8	1.0000	0.9974	0.9876	0.9597	0.8011	0.5000	0.1989	0.0403	0.0026	0.0000
	9		0.9995	0.9969	0.9873	0.9081	0.6855	0.3595	0.1046	0.0109	0.0001
	10		0.9999	0.9994	0.9968	0.9652	0.8338	0.5522	0.2248	0.0377	0.0008
	11		1.0000	0.9999	0.9993	0.9894	0.9283	0.7361	0.4032	0.1057	0.0047
	12			1.0000	0.9999	0.9975	0.9755	0.8740	0.6113	0.2418	0.0221
	13				1.0000	0.9995	0.9936	0.9536	0.7981	0.4511	0.0826
	14					0.9999	0.9988	0.9877	0.9226	0.6904	0.2382
	15					1.0000	0.9999	0.9979	0.9807	0.8818	0.5182
	16						1.0000	0.9998	0.9977	0.9775	0.8332
	17							1.0000	1.0000	1.0000	1.0000
18	0	0.1501	0.0180	0.0056	0.0016	0.0001	0.0000				
	1	0.4503	0.0991	0.0395	0.0142	0.0013	0.0001				
	2	0.7338	0.2713	0.1353	0.0600	0.0082	0.0007	0.0000			
	3	0.9018	0.5010	0.3057	0.1646	0.0328	0.0038	0.0002			
	4	0.9718	0.7164	0.5187	0.3327	0.0942	0.0154	0.0013	0.0000		
	5	0.9936	0.8671	0.7175	0.5344	0.2088	0.0481	0.0058	0.0003		
	6	0.9988	0.9487	0.8610	0.7217	0.3743	0.1189	0.0203	0.0014	0.0000	
	7	0.9998	0.9837	0.9431	0.8593	0.5634	0.2403	0.0576	0.0061	0.0002	
	8	1.0000	0.9957	0.9807	0.9404	0.7368	0.4073	0.1347	0.0210	0.0009	
	9		0.9991	0.9946	0.9790	0.8653	0.5927	0.2632	0.0596	0.0043	0.0000
	10		0.9998	0.9988	0.9939	0.9424	0.7597	0.4366	0.1407	0.0163	0.0002
	11		1.0000	0.9998	0.9986	0.9797	0.8811	0.6257	0.2783	0.0513	0.0012
	12			1.0000	0.9997	0.9942	0.9519	0.7912	0.4656	0.1329	0.0064
	13				1.0000	0.9987	0.9846	0.9058	0.6673	0.2836	0.0282
	14					0.9998	0.9962	0.9672	0.8354	0.4990	0.0982
	15					1.0000	0.9993	0.9918	0.9400	0.7287	0.2662
	16						0.9999	0.9987	0.9858	0.9009	0.5497
	17						1.0000	0.9999	0.9984	0.9820	0.8499
	18							1.0000	1.0000	1.0000	1.0000

Tables for STAT 260

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^{r} b(x; n, p)$

		p									
n	r	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
19	0	0.1351	0.0144	0.0042	0.0011	0.0001					
	1	0.4203	0.0829	0.0310	0.0104	0.0008	0.0000				
	2	0.7054	0.2369	0.1113	0.0462	0.0055	0.0004	0.0000			
	3	0.8850	0.4551	0.2631	0.1332	0.0230	0.0022	0.0001			
	4	0.9648	0.6733	0.4654	0.2822	0.0696	0.0096	0.0006	0.0000		
	5	0.9914	0.8369	0.6678	0.4739	0.1629	0.0318	0.0031	0.0001		
	6	0.9983	0.9324	0.8251	0.6655	0.3081	0.0835	0.0116	0.0006		
	7	0.9997	0.9767	0.9225	0.8180	0.4878	0.1796	0.0352	0.0028	0.0000	
	8	1.0000	0.9933	0.9713	0.9161	0.6675	0.3238	0.0885	0.0105	0.0003	
	9		0.9984	0.9911	0.9674	0.8139	0.5000	0.1861	0.0326	0.0016	
	10		0.9997	0.9977	0.9895	0.9115	0.6762	0.3325	0.0839	0.0067	0.0000
	11		1.0000	0.9995	0.9972	0.9648	0.8204	0.5122	0.1820	0.0233	0.0003
	12			0.9999	0.9994	0.9884	0.9165	0.6919	0.3345	0.0676	0.0017
	13			1.0000	0.9999	0.9969	0.9682	0.8371	0.5261	0.1631	0.0086
	14				1.0000	0.9994	0.9904	0.9304	0.7178	0.3267	0.0352
	15					0.9999	0.9978	0.9770	0.8668	0.5449	0.1150
	16					1.0000	0.9996	0.9945	0.9538	0.7631	0.2946
	17						1.0000	0.9992	0.9896	0.9171	0.5797
	18							0.9999	0.9989	0.9856	0.8649
	19							1.0000	1.0000	1.0000	1.0000
20	0	0.1216	0.0115	0.0032	0.0008	0.0000					
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000				
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002				
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000			
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003			
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000		
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0006	
	10		0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0026	0.0000
	11		0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0100	0.0001
	12		1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.0321	0.0004
	13			1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.0867	0.0024
	14				1.0000	0.9984	0.9793	0.8744	0.5836	0.1958	0.0113
	15					0.9997	0.9941	0.9490	0.7625	0.3704	0.0432
	16					1.0000	0.9987	0.9840	0.8929	0.5886	0.1330
	17						0.9998	0.9964	0.9645	0.7939	0.3231
	18						1.0000	0.9995	0.9924	0.9308	0.6083
	19							1.0000	0.9992	0.9885	0.8784
	20								1.0000	1.0000	1.0000

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Tables for STAT 260

Table A.2 Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6							1.0000	1.0000	1.0000

$P(X \le 2)$

	μ								
r	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
0	0.3679	0.2231	0.1353	0.0821	0.0498	0.0302	0.0183	0.0111	0.0067
1	0.7358	0.5578	0.4060	0.2873	0.1991	0.1359	0.0916	0.0611	0.0404
2	0.9197	0.8088	0.6767	0.5438	0.4232	0.3208	0.2381	0.1736	0.1247
3	0.9810	0.9344	0.8571	0.7576	0.6472	0.5366	0.4335	0.3423	0.2650
4	0.9963	0.9814	0.9473	0.8912	0.8153	0.7254	0.6288	0.5321	0.4405
5	0.9994	0.9955	0.9834	0.9580	0.9161	0.8576	0.7851	0.7029	0.6160
6	0.9999	0.9991	0.9955	0.9858	0.9665	0.9347	0.8893	0.8311	0.7622
7	1.0000	0.9998	0.9989	0.9958	0.9881	0.9733	0.9489	0.9134	0.8666
8		1.0000	0.9998	0.9989	0.9962	0.9901	0.9786	0.9597	0.9319
9			1.0000	0.9997	0.9989	0.9967	0.9919	0.9829	0.9682
10				0.9999	0.9997	0.9990	0.9972	0.9933	0.9863
11				1.0000	0.9999	0.9997	0.9991	0.9976	0.9945
12					1.0000	0.9999	0.9997	0.9992	0.9980
13						1.0000	0.9999	0.9997	0.9993
14							1.0000	0.9999	0.9998
15								1.0000	0.9999
16									1.0000

Tables for STAT 260

Table A.2 (continued) Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$ $\rightarrow \mu = \lambda$

	μ								
r	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
0	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18		1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19			1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20						0.9999	0.9998	0.9996	0.9991
21						1.0000	0.9999	0.9998	0.9996
22							1.0000	0.9999	0.9999
23								1.0000	0.9999
24									1.0000

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Tables for STAT 260

Table A.2 (continued) Poisson Probability Sums $\sum_{x=0}^{r} p(x;\mu)$

	μ								
r	**10.0**	**11.0**	**12.0**	**13.0**	**14.0**	**15.0**	**16.0**	**17.0**	**18.0**
0	0.0000	0.0000	0.0000						
1	0.0005	0.0002	0.0001	0.0000	0.0000				
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000		
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317
25		0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554
26		1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718
27			0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827
28			1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897
29				1.0000	0.9999	0.9996	0.9989	0.9973	0.9941
30					0.9999	0.9998	0.9994	0.9986	0.9967
31					1.0000	0.9999	0.9997	0.9993	0.9982
32						1.0000	0.9999	0.9996	0.9990
33							0.9999	0.9998	0.9995
34							1.0000	0.9999	0.9998
35								1.0000	0.9999
36									0.9999
37									1.0000

Tables for STAT 260

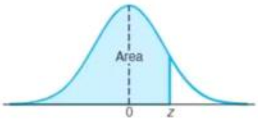

Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

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Table A.3 (continued) Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

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Tables for STAT 260

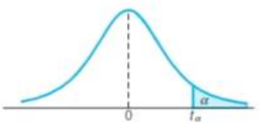

Table A.4 Critical Values of the t-Distribution

v	α = 0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

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Tables for STAT 260

Table A.4 (continued) Critical Values of the t-Distribution

	α						
v	0.02	0.015	0.01	0.0075	0.005	0.0025	0.0005
1	15.894	21.205	31.821	42.433	63.656	127.321	636.578
2	4.849	5.643	6.965	8.073	9.925	14.089	31.600
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.850
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.689
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.660
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.290

Stat 260 Lecture Notes
Set 12 - The Poisson Distribution

↳ name of math guy, not French fish

A **Poisson random variable** X counts the number of events (successes/arrivals/etc.) in an interval of time/length/space/etc.

A Poisson random variable had pmf

what it's equal to

$$f(x) = P(X = x) = \frac{\lambda^x}{x!} e^{-\lambda}$$

→ λ

→ on formula sheet!

where $x = 0, 1, 2, 3, \ldots$.

let a thing run for time and see how many successes happen

In the setup of these problems we will be given some base unit of measurement (day / week / year / km / m / mL / km^2 / acre / etc.).
λ = the average number of event occurrences in our interval of interest

Careful! The average given to us in the question setup may be for an interval other than the interval we want to calculate a probability for. We may need to scale the average in the setup to match our desired probability interval.

Notation: We write $X \sim \text{Poisson}(\lambda)$ if the r.v. X follows a Poisson distribution with parameter λ.

→ "is distributed as"

Example 1: The average number of students that email Michelle each day is 3.

base unit is 1 day

x = # of emails
X is poisson

(a) What is the probability that exactly 2 students email Michelle today?

→ let it run for time unit, so Poisson

→ 1 day
$\lambda = 3 \cdot 1 = 3$

$P(X=2) = \frac{3^2}{2!} e^{-3} = 0.2240$

(b) What is the probability of exactly 18 students email Michelle next week?

→ different time interval

$\lambda = 3 \cdot 7 = 21$
avg per day
1 week = 7 days

$P(X=18) = \frac{21^{18}}{18!} e^{-21} = 0.0747$

1

(c) What is the probability at least 3 students email Michelle today?

$\lambda = 3 \cdot 1$

$P(X \geq 3) = 1 - P(X \leq 2)$ use the complement

$= 1 - P(X=0) - P(X=1) - P(X=2)$

$= 1 - \frac{3^0}{0!}e^{-3} - \frac{3^1}{1!}e^{-3} - \frac{3^2 e^{-3}}{2!}$

$= 0.5768$

OR

$P(X \geq 3) = 1 - P(X \leq 2) = 1 - 0.4232 = 0.5768$

cdf, use stat tables!

Rules for the Poisson distribution:

expected value and variance the same

- $E(X) = \lambda$ → if you know it's Poisson then do it this way
- $V(X) = \lambda$
- $\sigma_X = \sqrt{\lambda}$

↳ $\sigma = \sqrt{\text{variance}}$

~~x | f(x)~~

~~$E(x) = \Sigma x \cdot f(x)$~~

In the last example, the average number of student emails in 1 day is 3.
In 4 days, the expected number of emails is

$E(X) = \underbrace{3 \cdot 4}_{\lambda} = 12$

Example 1 continued: The average number of students that email Michelle each day is 3.

(d) What is the probability of at least 17 student emails in the next 5 days?

$\lambda = 3 \cdot 5 = 15$

$$P(X \geq 17) = 1 - P(\overset{\text{cdf!}}{X \leq 16})$$
$$= 1 - 0.6641$$
$$= 0.3359$$

(e) What is the probability of exactly 14 student emails in the next 5 days?

$P(X = 14)$

do on own :)

Answer: 0.1024

(f) What is the probability that over the next 5 days, exactly 3 of those days will have at least two student emails?

n

→ $P(X=3)$

→ success

↳ something is different ???

$p = P(\text{success in 1 trial})$ 1 trial = 1 day

$p = P(\text{at least 2 emails in one day})$

$\lambda = 3 \cdot 1 = 3$

Poisson $P(y \geq 2) = 1 - P(y \leq 1)$

$= 1 - 0.1991$

$= 0.8009 \leftarrow p$

binomial $n=5$
$p = 0.8009$

success = day has at least 2 emails

$P(x=3) = \binom{5}{3}(0.8009)^3(0.1991)^2$

pmf

$\binom{n}{x} p^x (1-p)^{n-x}$

$= 0.2036$

Overall binomial question

need to use Poisson to find $p = P(\text{success})$

4

5 days

3 of 5 days do something specific

Binomial: do 5 days, 3 of the 5 have success

The Poisson pmf is approximately equal to the binomial pmf when n is large and p is small. Therefore we can estimate the binomial using a Poisson approximation. no stat table

Since for the binomial distribution we have $\mu = E(X) = np$, we use the transformation that $\lambda = E(X) = np$ in the Poisson approximation.

In other words binomial$(n, p) \approx$ Poisson$(\overset{\lambda}{np})$ for large n and small p. (Note: There is no definitive cutoff of when we have a good enough approximation, but generally we say the Poisson distribution is a good approximation of the binomial distribution if $n \geq 100$, $p \leq 0.01$, and $np \leq 20$.)

Example 2: Suppose 1% of a large population has a disease. In a group of 200 people, what is the probability that at most 2 have the disease?

success = have disease

binomial exact: $n=200$ $p=0.01$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{200}{0}(0.1)^0(0.99)^{200} + \binom{200}{1}(0.01)^1(0.99)^{199} + \binom{200}{2}(0.01)^2(0.99)^{198}$$

$$= 0.6767$$

approximate: use Poisson with $\lambda = np = 200(0.01) = 2$

$\lambda = 2$

$P(X \leq 2) = 0.6767$ (from stat table)

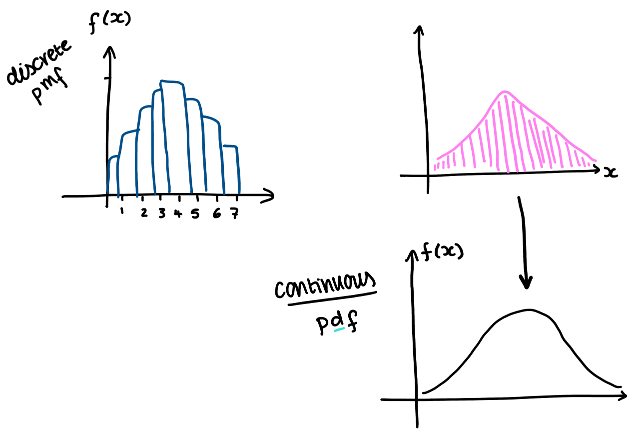
Set 13 and 14 - Continuous Random Variables

February 13, 2023 1:44 PM

Stat 260 Lecture Notes
Sets 13 and 14 - Continuous Random Variables

Recall: A continuous random variable X has an infinite number of possible values and it's impossible to list them all.

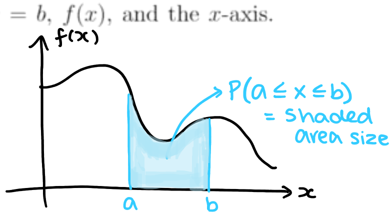
For a discrete random variable we could draw a picture of the pmf $f(x)$ - it looks like a histogram. Imagine making the bars of this histogram thinner and thinner. The top edges of the bars smooth out to a curve - a function. For a continuous random variable X the probability density function (pdf) $f(x)$ is this function.



pmf = discrete
pdf = continuous

Rules for the pdf of a continuous random variable X:

- $f(x) \geq 0$ for all x values. → always above the x-axis (discrete version: probabilities ≥ 0)
- The area bounded by the graph of $f(x)$ and the x-axis is 1. That is $\int_{-\infty}^{\infty} f(x)\,dx = 1$. (discrete: pmf probabilities sum to 1)
- $P(a \leq X \leq b)$ = area between $x = a$, $x = b$, $f(x)$, and the x-axis. That is, $P(a \leq X \leq b) = \int_a^b f(x)\,dx$.



1

Rule: If X is a continuous random variable then for a constant c, $P(X = c) = 0$.

so many choices that getting a specific one is unlikely (infinitely many)

This can be derived from $P(X = c) = P(c \leq X \leq c) = \int_c^c f(x)\,dx = 0$.

Since for a constant c we have that $P(X = c) = 0$, we therefore have that $P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$.

can drop equality b/c chance of being equal to a or b is 0

Careful! This only applies to continuous random variables. We cannot use this rule if we are working with the binomial distribution or the Poisson distribution (as they are both discrete distributions).

Since $P(X = c) = 0$ for a continuous random variable, when we have a continuous random variable we usually deal with problems like $P(a \leq X \leq b)$ or $P(X \leq a)$ or $P(X \geq a)$.

The cumulative distribution function (cdf) $F(x)$ for a continuous random variable is defined by

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(y)\,dy$$

dummy variable

where $f(x)$ is the pdf of the random variable X.

random variable is defined by

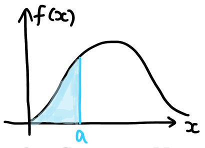
$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(y)\, dy$$ dummy variable

where $f(x)$ is the pdf of the random variable X.

Say $x = a$. Then $F(a)$ is the area under the pdf curve $f(x)$ to the left of the value $x = a$.

$F(a) = P(X \le a) = \text{shaded area} = \int_{-\infty}^{a} f(x)dx$

cdf



Rule: Suppose X is a continuous random variable with pdf $f(x)$ and cdf $F(x)$. Then at every x where the derivative $F'(x)$ exists, we have that $f(x) = F'(x)$.

pdf is derivative of cdf
cdf is integral of pmf

Even with using calculus, finding areas under the pdf $f(x)$ curve to solve things like $P(X \le a)$ can be difficult (some integrals may require advanced

techniques such as numerical approximation). In situations like this we often use cdf tables (our stat tables) to look up values for the cdf $F(x) = P(X \le x)$. If we have knowledge of the exact function $F(x)$ for our cdf, we could also evaluate this function at specific x values to calculate probabilities. (For example, if we wanted to find $P(X \le 2)$ we could evaluate the function $F(x)$ at $x = 2$, so we could find $F(2)$.)

Note: For a discrete random variable X, where w is next smallest value below a. → often this is just a-1

$P(a \le X \le b) = P(X \le b) - P(X < a) = P(X \le b) - P(X \le w)$

not including a

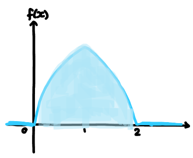
but for a continuous random variable X we have

$P(a \le X \le b) = P(X \le b) - P(X < a)$

$= P(X \le b) - P(X \le \underline{a})$ → include a

Example 1: Say X is a continuous random variable with pdf

$$f(x) = \begin{cases} c(4x - 2x^2) & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}.$$



What is the value of c?

We know $1 = \int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} \cancel{0dx} + \int_{0}^{2} c(4x-2x^2)dx + \int_{2}^{\infty} \cancel{0dx}$

$= \int_{0}^{2} c(4x-2x^2)dx = \left[c\left(\frac{4x^2}{2} - \frac{2x^3}{3}\right)\right]\Big|_0^2$

$= c\left(\frac{4(2^2)}{2} - \frac{2(2^3)}{3}\right) - 0$

$= \frac{8c}{3}$

So $1 = \frac{8c}{3}$

$\Rightarrow c = \frac{3}{8}$.

Find $P(X > 1)$.

$$P(X>1) = P(X \geq 1) = \int_1^\infty f(x)\,dx = \int_1^2 \frac{3}{8}(4x - 2x^2)\,dx$$

$$= \left[\frac{3}{8}\left(\frac{4x^2}{2} - \frac{2x^3}{3}\right)\right]_1^2$$

$$= \frac{3}{8}\left(\frac{4(2)^2}{2} - \frac{2(2)^3}{3}\right) - \frac{3}{8}\left(\frac{4(1)^2}{2} - \frac{2(1)^3}{3}\right) = \frac{1}{2}$$

don't have to know how to graph functions

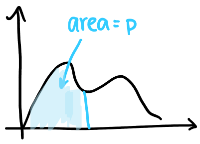
Percentiles: Let p be a value between 0 and 1. The $(100p)^{th}$ **percentile** of the distribution of a continuous random variable X, denoted by $\eta(p)$ is defined by

scales decimal

$$p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\ dy$$

x_0

In other words, $\eta(p)$ is the x value where $F(x) = p$, or rather where $P(X \leq x) = p$.



100pth percentile
say it is x_0

Solve for x_0

Example 2: Suppose the pdf of a continuous random variable X is

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}.$$

We can find $F(x)$:

This means that the $(100p)^{th}$ percentile $x = \eta(p)$ satisfies:

For the 50^{th} percentile (that is, when $p = 0.50$) we need to solve:

The **median** $\widetilde{\mu}$ is the 50^{th} percentile. (So using the notation, that is that $\eta(0.50) = \widetilde{\mu}$.) So half the area under $f(x)$ is to the left of $x = \widetilde{\mu}$ and half of the area is to the right.

Expected Value and Variance:

For a discrete random variable X: $\mu_X = E(X) = \sum x \cdot f(x) = \sum x \cdot P(X = x)$

For a continuous random variable X: $\mu_X = E(X) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$.

(Recall: Geometrically, $E(X)$ is the x value that would "balance" the graph of $f(x)$.)

Example 3: Say the pdf of a continuous random variable X is

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find $E(X)$.

For a discrete random variable X: $E(g(X)) = \sum g(x) \cdot f(x) = \sum g(x) \cdot P(X = x)$

For a continuous random variable X: $E(g(X)) = \int_{-\infty}^{\infty} g(x) \cdot f(x)\, dx$.

Just like before, $E(aX + b) = aE(X) + b$.

For a discrete random variable X: $\sigma_X^2 = V(X) = E((X-\mu)^2)$

For a continuous random variable X: $\sigma_X^2 = V(X) = E((X-\mu)^2) = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\, dx$.

The shortcut formula still holds: $V(X) = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx - \left(\int_{-\infty}^{\infty} x \cdot f(x)\, dx\right)^2$.

The evaluations of $\int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$ and $\int_{-\infty}^{\infty} x \cdot f(x)\, dx$ are why Math 101 is a corequisite for this course - note that integration by parts may be a useful technique here.

The standard deviation is still $\sigma_X = \sqrt{V(X)}$.

The Uniform Distribution:

X is a **uniform random variable** if it has pdf $f(x) = c$ where c is a constant.

More specifically, this means that $f(x) = \dfrac{1}{B-A}$ where the possible X values are in the interval $[A, B]$.

For a uniform random variable X, we have that $P(a \leq X \leq b) = \dfrac{b-a}{B-A}$.

Example 5: Suppose a person is just as likely to arrive at the bus stop any time between 7am and 7:30am. What is the probability that they arrive between 7:05am and 7:15am?

Example 6: Let X have a uniform distribution on the interval $[A, B]$. What is the cdf of X? That is, find $F(x)$.