1-5-Theory-Addition
ScalarMult

Vector

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

How to math vectors:

Adding

$$\vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x\hat{i} + B_x\hat{i} + A_y\hat{j} + B_y\hat{j} + A_z\hat{k} + B_z\hat{k}$$

$$= \underline{(A_x + B_y)}\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$$

the x-component of $\vec{A} + \vec{B}$ is $A_x + B_x$

Same rule as

$$(A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y + B_y, A_z + B_z)$$

Multiply a vector by a scalar

(A number without components)

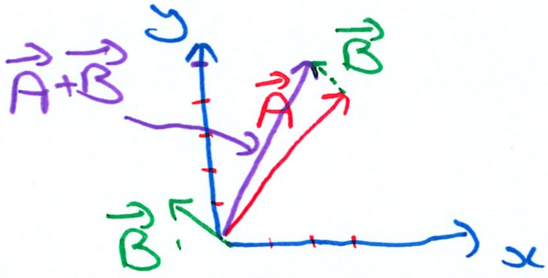
$$c\vec{A} = c(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$
$$= cA_x\hat{i} + cA_y\hat{j} + (cA_z)\hat{k}$$

z-component of $c\vec{A}$

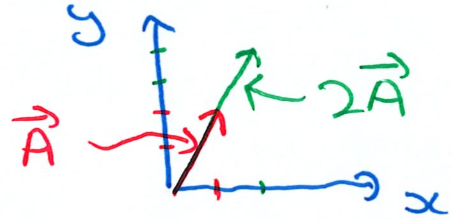
Multiply all components by same scalar.

Geometric picture $(A_z = B_z = 0)$

$$\left.\begin{aligned}\vec{A} &= 3\hat{i} + 4\hat{j}\\ \vec{B} &= -1\hat{i} + 1\hat{j}\end{aligned}\right\} \quad \vec{A}+\vec{B} = 2\hat{i} + 5\hat{j}$$

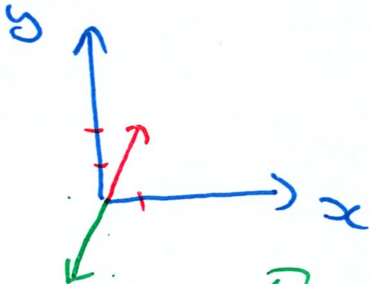


Suppose $\vec{A} = 1\hat{i} + 2\hat{j}$, $c = 2$



$c\vec{A} = 2\hat{i} + 4\hat{j}$

What if $\vec{A} = 1\hat{i} + 2\hat{j}$, $c = -1$



$c\vec{A} = -1\hat{i} - 2\hat{j}$

Opposite direction,
same length

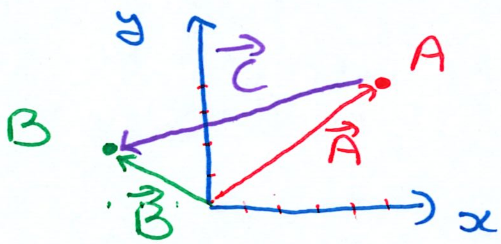
Subtraction

$$\vec{A} - \vec{B} = \vec{A} + (-1\vec{B})$$
$$= (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}$$
$$+ (A_z - B_z)\hat{k}$$

1-6-Example-Vectors III

Vectors - III
Point A is at location $5m\hat{\imath} + 4m\hat{\jmath}$. Point B is at location $-3m\hat{\imath} + 2m\hat{\jmath}$.

- What is the vector *from A to B* expressed in components?
- What is the length of the vector from A to B?
- What is the angle between the vector from A to B and the positive x-axis?
- What angle does the vector from A to B make *measured counterclockwise from the positive x-axis*?



$\vec{A} = 5m\hat{\imath} + 4m\hat{\jmath}$

$\vec{B} = \cancel{5m\hat{\imath}+}$
$-3m\hat{\imath} + 2m\hat{\jmath}$

$\vec{C}$ = "vector from A to B"

$\vec{B} = \vec{A} + \vec{C}$

$\vec{B} - \vec{A} = \underbrace{\vec{C}}_{\text{want}}$

know

$\vec{C} = \vec{B} - \vec{A}$

$= (-3m\hat{i} + 2m\hat{j}) + (-5m\hat{i} - 4m\hat{j})$

$= \underline{-8m\hat{i} - 2m\hat{j}}$

Vector $\vec{A}$ to $\vec{B}$

in general (end up) - (start)

$|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-8m)^2 + (-2m)^2 + (0m)^2}$

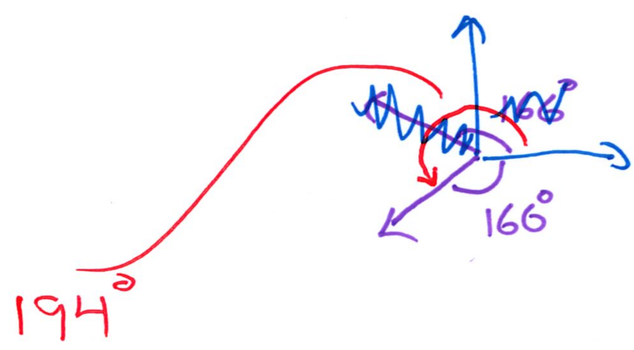
$= 8.25m$

$C_n = |\vec{C}|\cos\Theta$ angle between $\vec{C}$ & $\hat{i}$

$C_x = |\vec{C}|\cos\Theta$ we want

$-8m = 8.25m\cos\Theta$

$\Theta = 166^\circ$



Ways to multiply vectors.

Dot product (AKA "Inner Product" or "Scalar Product")

Takes two vectors gives a number (scalar)

$$\underbrace{\vec{A}\cdot\vec{B}}_{\text{"}\vec{A}\text{ dot }\vec{B}\text{"}} = |\vec{A}||\vec{B}|\cos\Theta$$

magnitudes of $\vec{A}$ & $\vec{B}$

angle between $\vec{A}$ & $\vec{B}$

How it works for unit vectors:

$$\hat{\imath}\cdot\hat{\imath} = (1\hat{\imath}+0\hat{\jmath}+0\hat{k})\cdot(1\hat{\imath}+0\hat{\jmath}+0\hat{k})$$

$$= |\hat{\imath}||\hat{\imath}|\cos 0 \qquad \Rightarrow \hat{\imath}\,\hat{\imath}$$

$|\hat{\imath}| = \sqrt{1^2+0^2+0^2}$

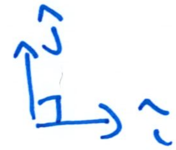
$$\hat{\imath}\cdot\hat{\imath} = 1$$

$$\hat{\jmath}\cdot\hat{\jmath} = |\hat{\jmath}||\hat{\jmath}|\cos 0 = 1$$

$|\hat{\jmath}| = \sqrt{0^2+1^2+0^2}$

$$\hat{k}\cdot\hat{k} = \ldots = 1$$

$$\hat{i}\cdot\hat{j} = |\hat{i}||\hat{j}|\cos 90$$
$$= 0$$
$$\hat{i}\cdot\hat{k} = 0$$
$$\hat{j}\cdot\hat{k} = 0$$



$$\vec{A}\cdot\vec{A} = |\vec{A}||\vec{A}|\cos 0$$
$$= |\vec{A}|^2$$

$$\vec{A}\cdot\vec{A} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(A_x\hat{i} + A_y\hat{j} + A_z\hat{k})$$
$$= \cancel{(A_x\hat{i})\cdot(A_x\hat{i}) + (A_y\hat{j})\cdot(A_x\hat{i}) + (A_z\hat{k})\cdot(A_x\hat{i})}$$
$$\cancel{+ (A_y\hat{j})\cdot(A_x\hat{i}) + (A_y\hat{j})\cdot(A_y\hat{j})} + A$$
$$= (A_x\hat{i})\cdot(A_x\hat{i}) + \cancel{(A_x\hat{i})\cdot(A_y\hat{j})} + \cancel{(A_x\hat{i})\cdot(A_z\hat{k})}$$
$$+ \cancel{(A_y\hat{j})\cdot(A_x\hat{i})} + (A_y\hat{j})\cdot(A_y\hat{j}) + \cancel{(A_y\hat{j})\cdot(A_z\hat{k})}$$
$$+ \cancel{(A_z\hat{k})\cdot(A_x\hat{i})} + \cancel{(A_z\hat{k})\cdot(A_y\hat{j})} + (A_z\hat{k})\cdot(A_z\hat{k})$$
$$= A_x^2 + A_y^2 + A_z^2$$

Same analysis

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$
$$= A_x B_x + A_y B_y + A_z B_z$$

Components:

$$\vec{A}\cdot\hat{\imath} = |\vec{A}||\hat{\imath}|\cos\theta$$ ↙ between $\vec{A}$ & $\hat{\imath}$

$$= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot\hat{\imath}$$
$$= A_x 1 + A_y 0 + A_z 0$$
$$= A_x$$

$\vec{A}\cdot\hat{n}$ = component of $\vec{A}$ in direction $\hat{n}$

$\hat{n}$ Use ^ to indicate unit vector $|\hat{n}| = 1$