Symbolab

Solution

$\sum_{n=0}^{\infty}\left(\frac{x^2+3}{6}\right)^n$: Interval of convergence is $-\sqrt{3}<x<\sqrt{3}$

Steps

$\sum_{n=0}^{\infty}\left(\frac{x^2+3}{6}\right)^n$

Use the Root Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}\left(\frac{x^2+3}{6}\right)^n$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|\left(\frac{x^2+3}{6}\right)^{n\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\left(\frac{x^2+3}{6}\right)^{n\frac{1}{n}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\left(\left(\frac{x^2+3}{6}\right)^n\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(\left(\frac{x^2+3}{6}\right)^n\right)^{\frac{1}{n}}$: $\frac{x^2+3}{6}$

Hide Steps

$\left(\left(\frac{x^2+3}{6}\right)^n\right)^{\frac{1}{n}}$

Use the following exponent property: $(a\cdot b)^n=a^n\cdot b^n$

$\left(\frac{x^2+3}{6}\right)^n=\frac{(x^2+3)^n}{6^n}$, $\left(\frac{(x^2+3)^n}{6^n}\right)^{\frac{1}{n}}=\frac{\sqrt[n]{(x^2+3)^n}}{\sqrt[n]{6^n}}$

$=\frac{\sqrt[n]{(x^2+3)^n}}{\sqrt[n]{6^n}}$

Use the following exponent property: $(a^n)^m=a^{n\cdot m}$

$\sqrt[n]{(x^2+3)^n}=(x^2+3)^{n\frac{1}{n}}$, $\sqrt[n]{6^n}=6^{n\frac{1}{n}}$

$=\frac{(x^2+3)^{n\frac{1}{n}}}{6^{n\frac{1}{n}}}$

$6^{n\frac{1}{n}}=6$

Hide Steps

$6^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1

Hide Steps

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=6^1$

Apply rule $a^1=a$

$=6$

$=\frac{(x^2+3)^{n\frac{1}{n}}}{6}$

$(x^2+3)^{n\frac{1}{n}}=x^2+3$

Hide Steps

$(x^2+3)^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1

Hide Steps

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=(x^2+3)^1$

Apply rule $a^1 = a$

$= x^2 + 3$

$= \frac{x^2+3}{6}$

$L = \lim_{n\to\infty}\left(\left|\frac{x^2+3}{6}\right|\right)$

$L = \left|\frac{x^2+3}{6}\right| \cdot \lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1) = 1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a} c = c$

$= 1$

$L = \left|\frac{x^2+3}{6}\right| \cdot 1$

Simplify

$L = \frac{|x^2+3|}{6}$

$L = \frac{|x^2+3|}{6}$

The power series converges for $L < 1$

$\frac{|x^2+3|}{6} < 1$

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$\frac{|x^2+3|}{6} < 1 \quad : \quad -\sqrt{3} < x < \sqrt{3}$ *Hide Steps*

$\frac{|x^2+3|}{6} < 1$

Multiply both sides by 6

$\frac{6|x^2+3|}{6} < 1 \cdot 6$

Simplify

$|x^2+3| < 6$

Apply absolute rule: If $|u| < a, a > 0$ then $-a < u < a$

$-6 < x^2 + 3 < 6$

$x^2 + 3 > -6 \quad$ and $\quad x^2 + 3 < 6$ *Hide Steps*

$x^2 + 3 > -6 \quad$ and $\quad x^2 + 3 < 6$

$x^2 + 3 > -6 \quad : \quad$ True for all $x \in \mathbb{R}$ *Hide Steps*

$x^2 + 3 > -6$

Subtract 3 from both sides

$x^2 + 3 - 3 > -6 - 3$

Simplify

$x^2 > -9$

If n is even, $u^n \ge 0$ for all u

True for all x

$x^2 + 3 < 6 \quad : \quad -\sqrt{3} < x < \sqrt{3}$ *Hide Steps*

$x^2 + 3 < 6$

Subtract 3 from both sides

$x^2 + 3 - 3 < 6 - 3$

Simplify

$x^2 < 3$

For $u^n < a$, if n is even then $-\sqrt[n]{a} < u < \sqrt[n]{a}$

$-\sqrt{3} < x < \sqrt{3}$

Combine the intervals

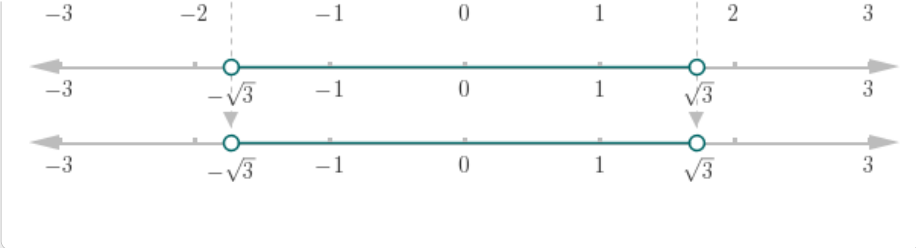
True for all x and $-\sqrt{3} < x < \sqrt{3}$

True for all x and $-\sqrt{3} < x < \sqrt{3}$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals
True for all $x \in \mathbb{R}$ and $-\sqrt{3} < x < \sqrt{3}$

$-\sqrt{3} < x < \sqrt{3}$



$-\sqrt{3} < x < \sqrt{3}$

$-\sqrt{3} < x < \sqrt{3}$

Check the interval end points: $x = -\sqrt{3}$:diverges, $x = \sqrt{3}$:diverges *Hide Steps*

For $x = -\sqrt{3}$, $\sum_{n=0}^{\infty}\left(\frac{(-\sqrt{3})^2+3}{6}\right)^n$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\left(\frac{(-\sqrt{3})^2+3}{6}\right)^n$

Refine

$= \sum_{n=0}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

= diverges

For $x = \sqrt{3}$, $\sum_{n=0}^{\infty}\left(\frac{\sqrt{3}^2+3}{6}\right)^n$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\left(\frac{(\sqrt{3})^2+3}{6}\right)^n$

Refine

$= \sum_{n=0}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

= diverges

$x = -\sqrt{3}$:diverges, $x = \sqrt{3}$:diverges

Therefore

Interval of convergence is $-\sqrt{3} < x < \sqrt{3}$

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