MATH 100, Fall, 2021
Tutorial #5
More derivatives

Q1 Did you ever wonder what the 'G' is in the 'DRG' button on your calculator? It is angle measured in 'gradians'. See the following Wikipedia page: `https://en.wikipedia.org/wiki/Gradian`. Let $\sin_G$ denote the sine function with respect to angles measured in grads. So now we have three sine functions $\sin_D$ (sine in degrees), $\sin = \sin_R$ (sine in radians) and $\sin_G$ (sine in grads).

Assignment:

> Suppose your calculator is broken and you only have the radian $\sin_R$ function available. Find a formula that computes $\sin_G(x)$ using only the regular $\sin_R$ function and then use this formula to write down the derivative law for the function $\sin_G$ in terms of $\cos_G$.

Q2 Let C be the curve in the xy plane defined by $x^4 + 2x^2 - y^2 = -1$. The point $(x, y) = (-1, -2)$ lies on this curve. You do not need to check this, but you might discuss briefly in your group how you would ...

Assignment:

> Use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y and then use this to find an equation of the tangent line to C at the point $(x, y) = (-1, -2)$.

Q3 Consider the same curve C as in question 1. If we restrict our attention to a subset of C it is sometimes possible to approach the tangent line problem with 'explicit' differentiation:

Assignment:

> Assuming that $y < 0$, find a formula for y in terms of x and use this to find an equation of the tangent line to C when $x = -1$ and $y = -2$.

Q4 A point on the unit circle at an angle θ above the positive x axis from the origin has x coordinate given by $x(\theta) = \cos(\theta)$. The angle θ varies with time according to $\theta(t) = 2t + \frac{1}{2}t^2$.

Assignment:

> Find $\frac{dx}{dt}$ as a function of t.
>
> Find $\frac{dx}{dt}$ when $t = -3$. In which direction is the x-coordinate moving at this time?

Q5 Suppose that $y > 0$. Recall that applying the chain rule to $\ln(y(x))$ gives the logarithmic differentiation rule $\frac{d}{dx}(\ln(y)) = \frac{1}{y}\frac{dy}{dx}$.

Assignment:

Let
$$y = \frac{x^2(x+3)^6 e^{-3x}}{(x^2+1)^{\frac{1}{4}}}.$$
Use logarithmic differentiation to compute an expression for y' in terms of x (no need to simplify fully).