

Student: Arfaz Hossain
Date: 04/20/22

Instructor: Muhammad Awais
Course: Math 101 A04 Spring 2022

Assignment: Practice Questions for
 Sections 11.4 & 11.5 [Not f

Graph the lemniscate below. What symmetries does the curve have?

$$r^2 = -\sin(4\theta)$$

When a graph has symmetry about the x-axis, if the point (r, θ) lies on the graph, then the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph. When a graph has symmetry about the y-axis, if the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph. When a graph has symmetry about the origin, if the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph.

To identify the symmetries of the curve, determine which of these conditions, if any, the given curve satisfies. First, determine if the curve has symmetry about the x-axis. This graph is not symmetric about the x-axis because (r, θ) lies on the graph, however neither the point $(r, -\theta)$ nor the point $(-r, \pi - \theta)$ lies on the graph.

Next, determine if the curve has symmetry about the y-axis. This graph is not symmetric about the y-axis because (r, θ) lies on the graph, however neither the point $(r, \pi - \theta)$ nor the point $(-r, -\theta)$ lies on the graph.

Next, determine if the curve has symmetry about the origin.

$$\begin{aligned} (r, \theta) \text{ on the graph} &\rightarrow r^2 = -\sin(4\theta) \\ &\rightarrow (-r)^2 = -\sin(4\theta) \\ &\rightarrow (-r, \theta) \text{ on the graph} \end{aligned}$$

Therefore, the curve has symmetry about the origin. To graph the curve, make a short table of values, plot the corresponding points, and use information about symmetry to connect the points with a smooth curve. Notice that r is only defined when $-\sin(4\theta)$ is positive, or $\sin(4\theta)$ is negative, that is, for values of θ between $\frac{\pi}{4}$ and $\frac{\pi}{2}$, or between $\frac{3\pi}{4}$ and π . The calculations are shown rounded to two decimal places as needed.

θ	$r = \sqrt{-\sin(4\theta)}$
$\frac{\pi}{4}$	± 0
$\frac{5\pi}{16}$	± 0.84
$\frac{3\pi}{8}$	± 1

Continue the table. The calculations are shown rounded to two decimal places as needed.

θ	$r = \sqrt{-\sin(4\theta)}$
$\frac{7\pi}{16}$	± 0.84
$\frac{\pi}{2}$	± 0

Note that these values repeat between $\frac{3\pi}{4}$ and π . Recall that the graph is symmetric about the origin. Therefore, the correct graph of the curve is shown below.

