Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** Practice Questions for Sections 6.3 & 7.2 [Not for

Solve the differential equation.

$$\frac{dy}{dx} = \frac{1}{12}\sqrt{y}\,\mathbf{cos}^2\sqrt{y}$$

Some differential equations can be solved by separating the variables. A differential equation of the form $y' = f(x,y)$ is separable if f can be expressed as a product of a function of x and a function of y.

Rewrite the equation in its differential form.

$$\frac{dy}{dx} = \frac{1}{12}\sqrt{y}\,\mathbf{cos}^2\sqrt{y}$$

$$dy = \frac{1}{12}\sqrt{y}\,\mathbf{cos}^2\sqrt{y}\;dx$$

Separate the variables by collecting all the y-terms with dy and all the x-terms with dx. Divide both sides of the equation by $\frac{1}{12}\sqrt{y}\,\mathbf{cos}^2\sqrt{y}$ to write the equation in the form $h(y)\,dy = g(x)\,dx$. Assume $y > 0$.

$$dy = \frac{1}{12}\sqrt{y}\,\mathbf{cos}^2\sqrt{y}\;dx$$

$$\frac{12\,\mathbf{sec}^2\sqrt{y}\;dy}{\sqrt{y}} = dx$$

Now integrate both sides of the equation. Move the constant to the outside of the integral on the left side.

$$12\int\frac{\mathbf{sec}^2\sqrt{y}\;dy}{\sqrt{y}} = \int dx$$

The left side is not yet in a form that allows integration. Use the substitution method to rewrite the expression inside the integral on the left side in a form which can be integrated. Let $u = \sqrt{y}$.

$$\frac{du}{dy} = \frac{1}{2\sqrt{y}}$$

Solve the equation $\frac{du}{dy} = \frac{1}{2\sqrt{y}}$ for dy to get $dy = 2\sqrt{y}\;du$. Then substitute this for dy inside the integral on the left side.

$$12\int\frac{2\sqrt{y}\,\mathbf{sec}^2\sqrt{y}\;du}{\sqrt{y}} = \int dx$$

Notice that the $\sqrt{y}$ divides out from the numerator and the denominator. Move the constant to the outside of the integral.

$$12\cdot 2\int\frac{\cancel{\sqrt{y}}\,\mathbf{sec}^2\sqrt{y}\;du}{\cancel{\sqrt{y}}} = \int dx$$

$$24\int\mathbf{sec}^2\sqrt{y}\;du = \int dx$$

To integrate on the left side, the denominator must be in terms of u. Remember that $u = \sqrt{y}$.

$24\int \sec^2 u\,du = \int dx$ Replace $\sqrt{y}$ with u.

$24 \cdot \tan u + C_1 = \int dx$ Use the rule $\int \sec^2 u\,du = \tan u + C$ to integrate.

Replace u with $\sqrt{y}$ on the left side. Integrate the right side.

$$24 \tan \sqrt{y} + C_1 = x + C_2$$

After completing the integrations, y is defined implicitly as a function of x. Combine the constants of integration as C.

$24 \tan \sqrt{y} = x + C$

Thus, solving the original differential equation, $\frac{dy}{dx} = \frac{1}{12}\sqrt{y}\cos^2\sqrt{y}$, yields $24 \tan \sqrt{y} = x + C$.