For the questions #1 – #4, find an **antiderivative** for each of the following functions:

1. (1 point) $t(x) = -4\csc^2(2x)$

(A) $-\frac{2}{3}\csc^3(2x)$ (B) $-16\csc^2(2x)\cot(2x)$ (C) $-2\cot(2x)$ (D) $-2\csc(2x)$

(E) $\frac{2}{3}\csc^3(2x)$ (F) $16\csc^2(2x)\cot(2x)$ (G) $2\cot(2x)$ (H) None of those

$\int -4\csc^2 2x = 2\cot 2x + C$

2. (1 point) $g(x) = -5\cos\left(\frac{3x}{2}\right)$

(A) $-\frac{15}{2}\sin\left(\frac{3x}{2}\right)$ (B) $-\frac{10}{3}\sin\left(\frac{3x}{2}\right)$ (C) $-\frac{15}{2}\cos\left(\frac{3x}{2}\right)$ (D) $-\frac{10}{3}\cos\left(\frac{3x}{2}\right)$

(E) $\frac{15}{2}\sin\left(\frac{3x}{2}\right)$ (F) $\frac{10}{3}\sin\left(\frac{3x}{2}\right)$ (G) $\frac{15}{2}\cos\left(\frac{3x}{2}\right)$ (H) $\frac{10}{3}\cos\left(\frac{3x}{2}\right)$

(I) $\frac{3}{4}\sin^2\left(\frac{3x}{2}\right)$ (J) None of those

$\int -5\cos\frac{3x}{2} = -5\int \cos\left(\frac{3x}{2}\right)dx$ let $\frac{3x}{2} = u$ $\frac{2du}{3} = dx$

$= -5\left(\frac{2}{3}\right)\int \cos u\, du = -5\sin u\left(\frac{2}{3}\right) = \frac{-10}{3}\sin u$

$= \frac{-10}{3}\sin\left(\frac{3x}{2}\right) + C$

For the questions #1 – #4, find an **antiderivative** for each of the following functions:

3. (1 point) $h(x) = \frac{1}{2}e^{7x} - \frac{1}{2}e^{-7x}$

(A) $\frac{1}{14}e^{7x} - \frac{1}{14}e^{-7x}$ (B) $\frac{7}{2}e^{7x} - \frac{7}{2}e^{-7x}$ (C) $14e^{7x} - 14e^{-7x}$ (D) $\frac{\ln(7)}{14}e^{7x} + \frac{\ln(7)}{14}e^{-7x}$

(E) $\frac{1}{14}e^{7x} + \frac{1}{14}e^{-7x}$ (F) $\frac{7}{2}e^{7x} + \frac{7}{2}e^{-7x}$ (G) $14e^{7x} + 14e^{-7x}$ (H) None of those

$$\int \frac{1}{2}e^{7x} - \frac{1}{2}e^{-7x}\,dx$$

$$= \frac{1}{2}\int e^{7x} - \frac{1}{2}\int e^{-7x}$$

$$= \frac{e^{7x}}{14} + \frac{e^{-7x}}{14}$$

4. (1 point) $f(x) = 7x - \frac{5}{x^4}$

(A) $7 + \frac{20}{x^5}$ (B) $7x^2 + \frac{1}{x^5}$ (C) $\frac{7}{2}x^2 + \frac{15}{x^3}$ (D) $\frac{7x^2}{2} + \frac{5}{3x^3}$

(E) $7 - \frac{20}{x^5}$ (F) $7x^2 - \frac{1}{x^5}$ (G) $\frac{7}{2}x^2 - \frac{15}{x^3}$ (H) $\frac{7x^2}{2} - \frac{5}{3x^3}$

(I) $7x^2 - \frac{5}{3x^3}$ (J) None of those

$$= \int 7x - \frac{5}{x^4}\,dx = \int 7x\,dx - 5\int x^{-4}\,dx$$

$$= \frac{7x^2}{2} + \frac{5x^{-3}}{3} + C$$

$$= \frac{7}{2}x^2 + \frac{5}{3x^3} + C$$

$a^x = \frac{\ln x}{\ln a}$

5. (2 points) Calculate $\int_{-1}^{0} ye^{y^2+1}\,dy$.

(A) $\frac{1}{2}(1-e)$ (B) $2(e-e^2)$ (C) $\frac{1}{2}(e-e^2)$ (D) $e-e^2$ (E) $e$

(F) $\frac{1}{2}(e-1)$ (G) $2(e^2-e)$ (H) $\frac{1}{2}(e^2-e)$ (I) $e^2-2e$ (J) None of those

$\int_{-1}^{0} ye^{y^2+1}$

$\cancel{u = y \quad du = 1 \quad v = ye \quad dv = e^{y}}$

let $u = y^2+1$

$\frac{du}{2y} = dy$

$\frac{1}{2}\int_{y=-1}^{y=0} e^u = \frac{1}{2}e^u \Big|_{y=-1}^{y=0} = \frac{e^{y^2}}{2}\Big|_{y=-1}^{y=0}$

$= \frac{1}{2}(e^0 - e^{(-1)^2})$

$= \frac{1}{2}(1-e)$

6. (2 points) Calculate $\int_{\pi/8}^{3\pi/8} \cos\left(2\theta - \frac{\pi}{4}\right)\,d\theta$.

(A) $-0.75$ (B) $-0.5$ (C) $-0.25$ (D) $0.0$

(E) $0.25$ (F) $0.5$ (G) $0.75$ (H) None of those

$\cos 2\theta - \cos\frac{\pi}{4}$

$\int \cos 2\theta - \frac{\pi}{4} = \frac{\sin(2\theta - \frac{\pi}{4})}{2}$

$\frac{\sin\left(\frac{6\pi}{8} - \frac{\pi}{4}\right) - \sin\left(\frac{\pi}{8} - \frac{\pi}{4}\right)}{2}$

$\frac{1 - \text{ans}}{2} = .69$

last minute

7. (2 points) Calculate $\int_{-2}^{-1} 2z|z|\,dz$.

(A) $\frac{14}{3}$ (B) $-\frac{14}{3}$ (C) 7 (D) $-7$ (E) 6

(F) $\frac{4}{3}$ (G) $-\frac{4}{3}$ (H) 3 (I) $-3$ (J) None of those

$$= \int_{-2}^{-1} -2z^2 = \frac{-2}{3}z^3 \Big|_{-2}^{-1} = \frac{-2(-1)^3}{3} + \frac{2(-2)^3}{3}$$

Since bounds are negative, this will always be negative

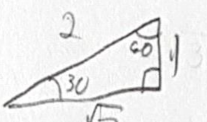
$$= \frac{2}{3} + \left(\frac{-16}{3}\right) = -4.\overline{66} = \boxed{\frac{-14}{3}}$$

8. (2 points) Solve the initial-value problem: $\frac{dz}{dw} = \frac{z^2+1}{w^2+1}$, $z(0) = -\frac{1}{\sqrt{3}}$.

Calculate $z\left(\frac{-1}{\sqrt{3}}\right)$.

(A) $\frac{\pi}{6}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{\pi}{3}$ (D) $\sqrt{3}$ (E) 3

(F) $-\frac{\pi}{6}$ (G) $-\frac{1}{\sqrt{3}}$ (H) $-\frac{\pi}{3}$ (I) $-\sqrt{3}$ (J) None of those




$$\frac{dz}{dw} = \frac{z^2+1}{w^2+1}; \quad \int \frac{dz}{z^2+1} = \int \frac{dw}{w^2+1}; \quad \tan^{-1} z = \tan^{-1} w + C$$

$$z = \tan(\tan^{-1}(w) + C) \quad \therefore \quad \frac{-1}{\sqrt{3}} = \tan(0 + C)$$

$$= \tan\left(0 - \frac{\pi}{6}\right)$$

$$z = \tan\left(\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) - \frac{\pi}{6}\right)$$

$$= \tan\left(\frac{-\pi}{6} - \frac{\pi}{6}\right)$$

dropped negative here $= \tan\frac{\pi}{3} = \sqrt{3}$

9. (2 points) Perform integration by parts on $I = \int xe^{2x}\, dx$ once, so that the new integral is simpler than the old one.

(A) ~~$I = 2xe^{2x} - 2\int e^{2x}\, dx$~~ (B) ~~$I = 2xe^{2x} + 2\int e^{2x}\, dx$~~

(C) $I = \frac{x}{2}e^{2x} - \frac{1}{2}\int e^{2x}\, dx$ (D) $I = \frac{x}{2}e^{2x} + \frac{1}{2}\int e^{2x}\, dx$

(E) ~~$I = \frac{1}{2}e^{2x}$~~ $+ \frac{1}{2}\int e^{2x}\, dx$

$I = \int x e^{2x} dx$ $\quad u = x \quad v = \frac{e^{2x}}{2}$

$du = 1 \quad dv = e^{2x}$

$I = \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}$

$I = \frac{xe^{2x}}{2} - \frac{1}{2}\int e^{2x}$

10. (2 points) Calculate $I = \int \sin(5x)\cos(3x)\,dx$.

(A) $I = -\frac{3}{16}\sin(3x)\sin(5x) - \frac{5}{16}\cos(3x)\cos(5x) + C$ (B) $I = -\frac{3}{16}\sin(3x)\sin(5x) + \frac{5}{16}\cos(3x)\cos(5x) + C$

(C) $I = -\frac{5}{16}\sin(3x)\sin(5x) - \frac{3}{16}\cos(3x)\cos(5x) + C$ (D) $I = -\frac{5}{16}\sin(3x)\sin(5x) + \frac{3}{16}\cos(3x)\cos(5x) + C$

(E) $I = -\frac{1}{5}\sin(3x)\sin(5x) - \frac{3}{25}\cos(3x)\cos(5x) + C$ (F) $I = -\frac{1}{5}\sin(3x)\sin(5x) + \frac{3}{25}\cos(3x)\cos(5x) + C$

(G) $I = -\frac{5}{9}\sin(3x)\sin(5x) + \frac{1}{3}\cos(3x)\cos(5x) + C$

$I = \int \sin 5x \cos 3x$

$u = \sin 5x \qquad V = \frac{\sin 3x}{3}$

$du = 5\cos 5x \qquad dv = \cos 3x\, dx$

$I = \frac{\sin 5x \sin 3x}{3} - \frac{5}{3}\int \sin 3x \cos 5x\, dx$

$u = \cos 5x \qquad V = \frac{-\cos 3x}{3}$

$du = -5\sin 5x \qquad dv = \sin 3x\, dx$

$I = \frac{\sin 5x \sin 3x}{3} + \frac{5}{9}\cos 5x \cos 3x - \frac{5}{3}\left(\frac{5}{3}\right)\int \sin 5x \cos 3x\, dx$

$I$

$I = \frac{\sin 5x \sin 3x}{3} + \frac{5}{9}\cos 5x \cos 3x - \frac{25}{9} I$

$\frac{34}{9} I = \frac{\sin 5x \sin 3x}{3} + \frac{5}{9}\cos 5x \cos 3x$

$I = \frac{3}{34} + \frac{1}{34}\cos 5x \cos 3x$

# MATHEMATICS 101 (Sections A01-A05)
## Formula sheet, Spring 2018
## Midterms and Final examinations.

### Table of Integrals

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, (u < a)$

2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, (u > a)$

4. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, (a > 0)$

5. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, (u > a > 0)$

6. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C, (a > u > 0)$

8. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C, (u > 0)$

9. $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + C$

10. $\displaystyle\int \csc u\, du = -\ln|\csc u + \cot u| + C$

### Trigonometric and Hyperbolic Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$
2. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
3. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
4. $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
5. $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
6. $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
7. $\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$
8. $\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$
9. $\cosh^2(x) - \sinh^2(x) = 1$
10. $\sinh(2x) = 2\sinh(x)\cosh(x)$
11. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
12. $\text{sech}^{-1}(x) = \cosh^{-1}\left(\frac{1}{x}\right)$
13. $\text{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$
14. $\coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$