Question 1: [**Cash Flow Diagrams, Discounted Cash Flow Analysis**]

Assumptions:

- Sam's MARR is your baseline interest rate for Project 1, assuming APR was compounded daily (the result from the regular version of Question 2).
- As in the previous project, Sam's degree starts on January 1 of Year 0 (the present) and lasts until December 31 of Year 4. Sam's working life starts on January 1 of Year 5 and lasts for their baseline working life.
- While studying, Sam pays tuition. We will model this[1] by having Sam pay $7,936 every January 1 from Year 0 to Year 4 (5 payments total).
- Sam pays rent on the 1st day of each month.
- While studying, Sam's rent goes up by $100 every January. Sam's rent is at its baseline level from Project 1 for every month of year 0, then is (baseline + $100) for every month of year 1, (baseline + $200) for every month of year 2, etc.
- Practicing engineers typically spend more than university students. Sam continues to pay rent while working, but at a higher level than while studying. Sam's rent in January of Year 5, their first rent paid while working, is double that of their rent in December of Year 4, their last rent while studying. After that, Sam's rent increases by $100 every January for the rest of their working life, so that rent in January of Year 6 is (rent in December of Year 5 + $100) = ((2 x rent in December of Year 4) + $100).
- Sam buys groceries on the 1st day of each week. For simplicity, assume there are exactly 52 weeks in a year. Over the course of a year, food costs go up by 3.4%[2], which we will model by having the price of groceries go up by $(1.034^{1/52} - 1) = 0.06432$ % per week. This means that in the first week of Year 0, Week 0, food costs are at their baseline values, and in Week 1 they are (Baseline x 1.0006432).
- Practicing engineers typically spend more than university students. Sam continues to buy groceries while working, but at a higher level than while studying. Sam's food costs in the first week of Year 5, their first grocery bill paid while working, is double that of their food bill in the last week of Year 4, their last grocery bill while studying. After that, Sam's grocery bill increases by 0.06432% per week for the rest of their working life, so that the food bill in week 2 of Year 5 is equal to (Week 1 of Year 5 bill) x 1.0006432 = (2 x Week 52 of Year 4 bill) x 1.0006432.
- Sam's salary increases by 3% per year[3]. This means they earn the baseline salary from Project 1 in Year 5[4], and (1.03) times this salary in Year 6.

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[1] The yearly cost used is the estimated two-term tuition cost, including fees, for a domestic Engineering student obtained with UVic's Tuition Fee Estimator. To keep things simple, we're ignoring co-op, exact payment dates and that different years may have different numbers of terms.

[2] Approximately the increase in Canada's food costs from April 2019 to April 2020. See https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1810000403

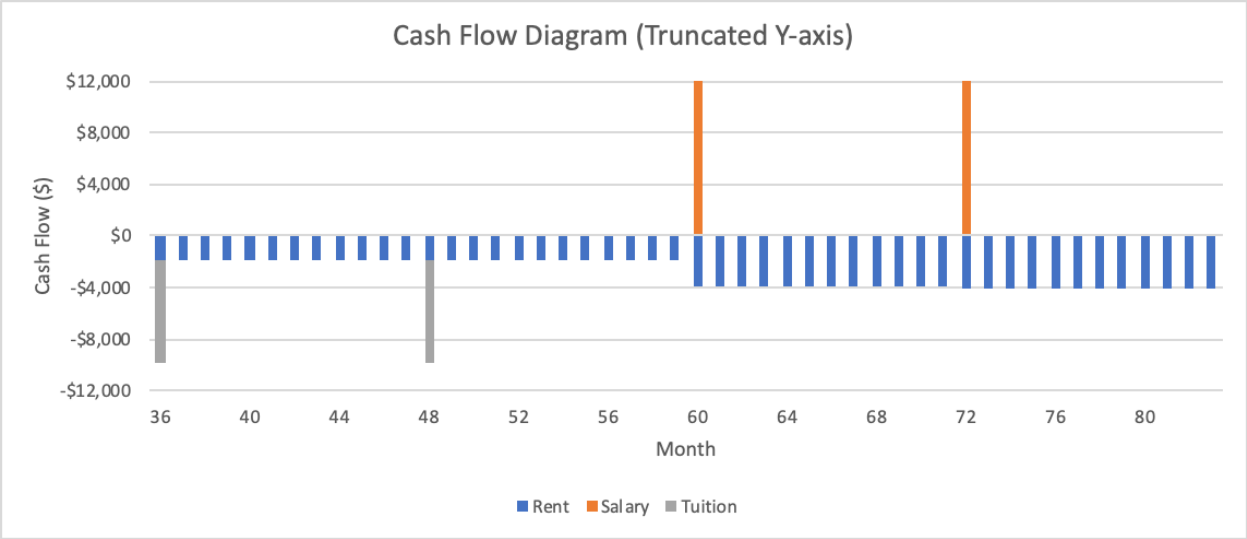
[3] Consistent with estimates for Canadian engineers in 2019, according to estimates by the Conference Board of Canada and others.

[4] Yes, this assumes the baseline starting salary doesn't go up in the next five years.

a. [**Cash Flow Diagram**]

Draw and label a cash flow diagram showing the rent, tuition and salary[5] cash flows from Year 3 to Year 6, inclusive. That is, for the last two years of Sam's degree, and the first two years of their working life. You may use a stacked bar chart, if you wish, like the ones we used in class for the electrical pole example. You may also draw an 'arrow' diagram as seen in the textbook. Each cash flow should be separate – don't add them together to get (for example) a total per month. Remember that we assumed Sam is paid their entire yearly salary each January. Use values for Victoria.

Diagram:



[**Discounted Cash Flow Analysis**]

Let R stand for the baseline monthly rent payment from Project 1, F stand for the baseline weekly food payment from Project 1, T stand for the yearly tuition payment, Y stand for baseline yearly salary while working from Project 1, and WL stand for Sam's baseline working life from Project 1. Using the correct notation (e.g. (P/F,i,N)), write expressions for the following:

b. The present value of Sam's tuition payments. These should be negative, since tuition is a negative cash flow to Sam. Use (P/A,i,N) in your answer.

T + T x (P/A,iyearly,4)

c. The present value of rent costs incurred while Sam is studying. These should be negative, since rent is a negative cash flow to Sam. Use (A/G,i,N) in your answer[6].

[5] I'm omitting weekly food costs to avoid cluttering the diagram.

[6] This requirement for the present value of rent costs is probably the trickiest part of this question, but I've included it because I believe the process of figuring this out will help you develop an intuition for discounted cash flow analysis that goes beyond looking for the appropriate equation to plug numbers into. Don't forget that you can always check your math by having Excel calculate the present value of each rent payment one at a time, then add them up to get the

-(A + G x (A/G,iyearly,5)) x (P/A,iyearly,5) x (F/P,imonthly,13)

Where A = R x (P/A,imonthly,12) and G = $100 x (P/A,imonthly,12)

Each year's rent can be separated into two components: the base rent R, and the addition over the base rent. The base rent always has the same yearly equivalent, since we're repeating the same calculation: find the Month X-1 equivalent of 12 months of rent R, from Month X to Month X+11. Similarly for the rent increases, with each year's increase being like adding an additional $100 to the rent, so that we have an arithmetic gradient with step size G as above. After finding all the yearly equivalents, we have an annuity with 5 payments, and an arithmetic gradient which we can turn into an annuity with the same timing. The total five-payment annuity is (A + G x (A/G,iyearly,5)). We can turn this into a Month -13 value by multiplying it by (P/A,iyearly,5), since the first payment is in Month -1, and (P/A,iyearly,5) will return an equivalent value one year before the first payment. We bring that forward to Year 0 by using (F/P,imonthly,13).

d. The present value of rent costs incurred while Sam is working. These should be negative, since rent is a negative cash flow to Sam. Use (A/G,i,N) in your answer.

Same as above, with three modifications: the initial rent is 2x(R+400), N is equal to the baseline working life (WL), and we need to bring it over to the present using (P/F,imonthly,35) since the first yearly payment will be in the last month of Year 4, which means multiplying by (P/A,iyearly,WL) will give us an equivalent payment in the last month of Year 3, which is Month 47.

-(A + G x (A/G,iyearly,WL)) x (P/A,iyearly,WL) x (P/F,imonthly,47)
Where A = 2x(R+400) x (P/A,imonthly,12) and G = $100 x (P/A,imonthly,12)

e. The present value of food costs incurred while Sam is studying. These should be negative, since food is a negative cash flow to Sam. Use (P/A,g,i,N) (or equivalently, (P/A,i^o,N)/(1+g)) in your answer.

There are 260 weeks in 5 years, but the first week's food bill is already in present value terms. That also means the first term in the annuity is F x (1+g)

F + F x (1+g) x (P/A,g,iweekly,259), where g is as given
We can also write this as F + F x (P/A,i^o,259) since (1+g)/(1+g)=1

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total. If you are not able to figure out how to use (A/G,i,N) in this, but write down a correct present value formula that does not use it, you will get a 68 instead of a 78 for this specific part of the question. **That's a very small penalty, given the number of parts in this question, so don't stress yourself out too much if you're having difficulty with it.** It helps to draw a cash flow diagram, so you can visualize the problem, and it helps to start with a similar, simpler problem first – say, a situation where someone with a MARR of 10% per month pays $100 in Month 1 and Month 2, $200 in Month 3 and Month 4, and $300 in Month 5 and Month 6, and you need to find the present (Month 0) value of those payments using (A/G,i,N).

f. The present value of food costs incurred while Sam is working. These should be negative, since food is a negative cash flow to Sam. Use (P/A,g,i,N) (or equivalently, (P/A,i°,N)/(1+g)) in your answer.

As above, but the first food cost, in Week 260, is $2F \times (1+g)^{259}$ and our N is (52 x WL) - 1. Also, since our calculation will give us a Week 260 value, we need to multiply by (P/F,iweekly,260) to get a Week 0 value.

$$2F \times (1 + g)^{259} \times (1 + (1+g) \times (P/A,g,iweekly,52xWL-1)) \times (P/F,iweekly,260)$$
$$2F \times (1 + g)^{259} \times (1 + (P/A,g,i^{o},52xWL-1)) \times (P/F,iweekly,260)$$

g. The present value of Sam's salary payments across their entire working life. These should be positive, since salary is a positive cash flow to Sam. Use (P/A,g,i,N) (or equivalently, (P/A,i°,N)/(1+g)) in your answer.

There are a total of WL payments, and the first payment is in Year 5. Multiplying Y by (P/A,g,iyearly,WL) will give us a Year 4 payment, so we need to multiply that by (P/F,iyearly,4) to get a Year 0 payment.

$$Y \times (P/A,g,iyearly,WL) \times (P/F,iyearly,4)$$

h. Calculate the Net Present Value (NPV) of studying and working in each of the five cities and fill them in, in the table below. I don't need to see your calculations, since I've already seen the equations you'll use to get them – it's fine to do them in Excel or a math program.

City	NPV ($)
Victoria	-$69,401.64
Vancouver	-$138,032.88
Edmonton	-$74,116.95
Toronto	-$181,696.40
Halifax	-$190,182.42

i. According to your NPV calculations, which city should Sam choose to live and work in? Briefly explain your reasoning.

If Sam MUST choose a city, Victoria. It has the highest NPV. But the negative NPVs suggest Sam shouldn't pay for everything on their credit card..

j. [**Capitalized Value**] The capitalized value of a property is a (very) rough estimate of the maximum price that you should pay for that property. Assuming rent stays at its baseline level forever, given Sam's MARR, what is the *capitalized value* of the rent? That is, what is the present value of an annuity that pays the baseline rent in Victoria each month, starting in Month 1 and continuing literally forever? Show your work. (Fun fact: The average price of a condo in Victoria is currently about $510,000.)

Capitalized value of baseline Victoria rent: $__143,511.95_____

Work:

Capitalized value = A/i. A = $1,561.41, i = 1.088% per month, A/I = $143,511.95

Question 1 88 Challenge: [**MARR**]

This challenge is to be undertaken *in addition to* the regular version of the question. Doing well on it will add an extra 10 points to your Question 1 mark (for example, bringing your mark up to 88 if you'd scored 78 on all parts of the question, or up to a 78 if you'd scored a 68 on all parts of the question.) If you only do well on one of the two parts (perhaps because you skipped one of the two parts), then you will receive an additional 5 points on your Question 1 mark. This is in acknowledgment of the fact that Part 2 requires substantial additional work in the form of reading, and not all students may be willing or able to do this.

Part 1: An alternate MARR for Sam

We're using the interest rate on credit card debt as Sam's MARR, when looking at the decision of which city to study and work in. This is not the only possible choice. Suggest an appropriate alternative MARR for Sam, that is NOT based on some sort of debt (bank loans, student debt, etc.). You do *not* have to calculate this MARR, but you do need to explain how you would obtain it (e.g. "The rate of return on commercial lobster fishing in Newfoundland.") You also need to briefly explain *why* the MARR you have chosen is appropriate for Sam.

Keep in mind the following: Sam is a prospective engineering student, about your age, in your discipline. Sam is a Canadian citizen. Sam does not know the future with certainty. The 'project' Sam is evaluating takes place across two *very* different periods in Sam's life: being a student, and being a working professional.

Suggest an appropriate non-debt-derived MARR for Sam, and briefly explain why it is appropriate.

Lots of possibilities here. The source of the MARR should represent an opportunity cost. Students should be able to explain, at least briefly, why it's appropriate for Sam, and for the evaluation of a long-term project. (Examples could include the rate of return on long-term bonds, average return of an indexed stock fund, rate of return on the family business, real interest rates on savings accounts (if assumed more constant than nominal ones), etc.)

Part 2: Discount rates and pandemic preparedness

In Canada, health care is largely a provincial responsibility. Suppose that the province of British Columbia is considering paying for a biomedical engineering research project that will make it easier for the provincial health care system to deal with future COVID-19-like viruses. This could be in the form of improved ventilators, a new method for mass-producing N-95 masks on short notice, etc. What discount rate (MARR) should the province of British Columbia use when evaluating the costs (and benefits) of such a project?

To answer this question, you will need some additional background. Please read the following two[7] articles:

CADTH. (2017). 7. Discounting. In *Guidelines for the Economic Evaluation of Health Technologies: Canada* (4th ed.). Ottawa: CADTH. Retrieved from https://www.cadth.ca/dv/guidelines-economic-evaluation-health-technologies-canada-4th-edition

You only need to read Section 7, on Discounting. This section explains that the Canadian Agency for Drugs and Technologies in Health recommends the use of "the real [inflation-adjusted] rate of interest on provincial government bonds" for discounting both the costs and benefits of health-related projects.

Paulden, M. (2014). Time Preference and Discounting. In *Encyclopedia of Health Economics*. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1016/B978-0-12-375678-7.00506-X

This article is a deep dive into the issues around discounting the benefits and costs of programs involving public health. A few terms you might want to know: "real" = adjusted for inflation (more on this later in the course), "utility"[8] = happiness, satisfaction, well-being, "marginal" = of the next, so "marginal rate of return" = rate of return on the next unit of resources expended, which is different from the average rate of return, since the first few workers (for example) added to a project tend to contribute more than the 5000th or 6000th worker. Similarly, "marginal utility of consumption" = happiness/satisfaction added by the next unit consumed. This tends to fall with how much you consume, hence "diminishing marginal utility of consumption" – that first slice of cake is great, but the tenth slice in a row isn't as satisfying, and my in fact reduce your utility by making you feel sick.

Based on what you have learned in ECON 180, and from the readings in this project, what discount rate(s) should the province of British Columbia use when evaluating biomedical engineering projects that may help in a future pandemic? I'm not asking for numbers, but rather for a

[7] If after reading the two articles you feel you still need more background, I recommend Severens, J.L. & Milne, R. J. (2004). Discounting Health Outcomes in Economic Evaluation: The Ongoing Debate. *Value In Health*, 7(4), 397-401. Retrieved from http://onlinelibrary.wiley.com/doi/10.1111/j.1524-4733.2004.74002.x/abstract

[8] For more on utility, and a very readable introduction to how economists think about it, see Jha, S. & Powell, A. (2014). A (Gentle) Introduction to Behavioral Economics, *American Journal of Roentgenology*, 203(1), 111-117. Retrieved from http://www.ajronline.org/doi/abs/10.2214/AJR.13.11352

description of the rate(s) to be used (e.g. "The real interest rate on provincial government bonds," if you agree with CADTH.) Briefly explain your reasoning, drawing on material from ECON 180 and the readings.

Lots of possibilities here. What the student writes down should make sense for long(-ish) term biomedical engineering projects. The MARR should represent an appropriate opportunity cost from the point of society and/or the provincial government (pure rate of time preference can also be part, but not all, of the MARR chosen).

Examples could include a Ramsey-style social discount rate combining items representing diminishing marginal utility of consumption, pure time preference and catastrophic risk, discounting costs and benefits differently (benefits less than costs) – though this has issues such as infinite delay, real interest rate faced by the provincial government, rate of 'crowding out' of private investment, etc.

Question 2: [**Repeated Lifetimes, Annual Worth**]

Suppose that Sam has decided to live and work in Victoria. Just before accepting UVic's offer of admission, they receive a counter-offer from a Victoria-based firm.

The counter-offer is as follows:

- Instead of getting their degree and working as an engineer after graduating, Sam will work for the firm on a non-renewable ten-year contract. That is, Sam will work from the start of Year 0 to the end of Year 9.
- The firm will pay Sam the yearly same salary they would have received in their first year working as an engineer. (The baseline salary for Victoria from Project 1). They will be paid this salary every January 1, from Year 0 to Year 9.
- The firm will not give Sam a raise, but *will* give them a signing bonus, to be paid on January 1 of Year 0, of half a year's salary. They will also receive a termination bonus of half a year's salary on January 1 of Year 9. These bonuses are in addition to the salaries paid on those dates.
- While working for the firm, Sam will, of course, *not* have to pay tuition.
- While working for the firm, Sam WILL pay for rent and food. Since they are employed, these expenses will be higher than for a university student.
- Sam's first rent and grocery bills (in Month 0 and Week 0) will be double the baseline values for Victoria from Project 1. After that, as before, rent increases by $100 every January 1 starting on January 1, Year 1, and the food bill goes up by 0.06432% per week, starting in Week 1.

Using the correct notation (e.g. (P/F,i,N)), write an expressions for the present value of the following cash flows if Sam accepts the firm's offer. Note that this will *only* include Years 0 to Year 9. The 'project' of working for the firm is only ten years long.

a. Present Value of Food Costs (negative):

Same reasoning as before, but now we only have 10x5 = 520 weeks to look at, and only 519 of those are part of the annuity. No need for further PV conversions, since we're already in the present.

$$-2F \times (1 + (1+g) \times (P/A,g,iweekly,519)$$
$$-2F \times (1 + (P/A,g,i^{o},519)$$

b. Present Value of Rent Costs (negative):

Same reasoning as before, but 10 years instead of 5, and base rent is 2R:
$$-(A + G \times (A/G,iyearly,10)) \times (P/A,iyearly,10) \times (F/P,imonthly,13)$$
Where $A = 2xR \times (P/A,imonthly,12)$ and $G = \$100 \times (P/A,imonthly,12)$

c. Present Value of Salary and Bonuses (positive):

One and a half times Y in Year 0, then Y from Year 1 to 9, and 0.5Y more in Year 9.

$$1.5Y + Y \times (P/A,iyearly,9) + 0.5Y \times (P/F,iyearly,9)$$

Use the expressions above to calculate the Net Present Value of accepting the firm's offer. I don't need to see your calculations:

d. NPV of accepting the firm's offer: $___181,261.69_____

[**Repeated Lifetimes**]

Now, use a *repeated lifetimes* analysis to determine whether Sam should accept the firm's offer. The length of the 'reject the offer' project is equal to 5 years (of study) + Sam's baseline working life. The length of the 'accept' project is equal to ten years.

e. Least common multiple of project lifetimes: __90___ years

Let NPV_{accept} be the NPV of accepting the firm's offer. Write down an expression, using the correct notation, for the repeated lifetimes present value, including all necessary repetitions. Below the expression, calculate the corresponding numerical value.

Nine repetitions, once every ten years. One NPVaccept in Year 0, and then an annuity of them with N=9 and the time period being '10 years'.

$$\text{Repeated Lifetimes NPV} = NPVaccept + NPVaccept \times (P/A,i10years,9)$$
$$i10years = (1+iyearly)10-1$$

f. NPV_{accept} (Common Lifetime): $____249,280.83_____

g. Let NPV_{reject} be the NPV of rejecting the firm's offer. (You can look this value up in the NPV table you filled out for Question 1.) Write down an expression, using the correct notation, for the repeated lifetimes present value, including all necessary repetitions. Below the expression, calculate the corresponding numerical value.

Two repetitions, once every 45 years.

Repeated Lifetimes NPV = NPVreject + NPVreject x (P/F,iyearly,45)

NPV_{reject} (Common Lifetime): $____-69,602.61___

h. Based on your calculations, should Sam accept or reject the firm's offer? Briefly explain your reasoning.

Accept. The repeated lifetimes NPV for the firm's offer is greater than that from rejecting it (not to mention positive!).

[**Annual Worth**]

i. Write down an expression, using the correct notation, for the annual worth of accepting the offer. Below the expression, calculate the corresponding numerical value.

NPVaccept x (A/P,iyearly,10)

(Why not use (A/P,iyearly,90)? Because that would assume you have the 10 years of cash flows, then 25 years of nothing whatsoever. It's giving you the equivalent, spread over 90 years, of that one set of 10 years of cash flows that gave you NPVaccept.)

A_{accept} (Common Lifetime): $___34,572.84_____

j. Write down an expression, using the correct notation, for the annual worth of rejecting the offer. Below the expression, calculate the corresponding numerical value.

NPVreject x (A/P,iyearly,45)

A_{reject} (Common Lifetime): $____-9,653.27_____

k. Based on your calculations, should Sam accept or reject the firm's offer? Briefly explain your reasoning.

Accept. Annual worth is higher for accepting (and positive!).

Question 2 88 Challenge: [**Study Period**]

For the 88 Challenge, do this question *in addition to* the regular version of the question.

Use a 10-year study period (Year 0 to Year 9) to determine whether Sam should accept the firm's offer. Recall how this method works: For each project, you calculate the net present value of the cash flows during the study period. If a project's lifetime exceeds the study period, you need to add a *residual value* to the final time period of the study period. (In this case, the residual would be an extra 'cash flow' in Year 9 for the 'reject the firm's offer' project.)

You've already calculated the NPV of accepting the firm's offer and working for them from Year 0 to Year 9. For reference, write it down below.

NPV of accepting the firm's offer: $_____181,261.69_____

Next, calculate the present value of the first ten years of cash flows (Year 0 to Year 9) in the 'Reject the firm's offer' project. This time period includes all of Sam's degree, plus the first four years of their working life. I don't need to see your calculations.

PV of Cash Flows in Year 0 to 9: $_____-$107,355.69_____

Now we need a residual. There is no one-size-fits-all way to calculate a residual, so I will ask you to use simplified version of two methods that are in common use.

Residual Method 1: The U.S. Department of Energy[9] has the following to say about residuals: "The residual value is the proportion of the initial cost equal to the remaining years of service divided by the initial cost. […] Where the useful life of equipment or materials extends beyond the study period, there is a residual value. For code measures analyzed, the longest useful life defined is 40 years[,] […] [which is] longer than the 30-year study period. […] A residual value of the unused life of a cost item is calculated at the last year of the study period for components with longer lives than the study period. So, for example, a measure with a 40-year life in a 30-year study period would have a residual value of 25% of its first cost."

We are dealing with an engineering student, not a lifeless piece of equipment, so I'll adapt this procedure slightly. The perspective we'll take is that if society wants an engineer, it has to pay for the degree and the cost of living while earning the degree. Consider the equivalent of the 'first cost' to be the present value of costs incurred while getting a degree: rent, food and tuition. Sam's 'useful life' as an engineer is equal to their baseline working life. Our study period catches the first five years of Sam's working life. Therefore, letting WL stand for the baseline working life,

Residual 1 = (PV of costs while studying) x (WL – 5 years) / (WL)

[9] Hart, R. & Liu, B. (2015). *Methodology for Evaluating Cost-Effectiveness of Commercial Energy Code Changes*. Richland, Washington: Pacific Northwest National Laboratory.

Treating this residual as an additional positive cash flow in Year 9[10] of the 'Reject the offer' project, calculate the NPV of the 'Reject the offer' project. Show your work:

a. NPV of 'Reject the offer' using Residual 1: $___-66,735.63_____

PV Costs while studying = PV Tuition + PV Rent + PV Food (all previously calculated)
PV Costs while studying = $149,408.70, and WL = 40 years
Residual = PV Costs while studying x (40 – 5)/40 = $130,732.61
NPV of 'Reject' = (PV Cash Flows Years 0 to 9) + Residual x (P/F,iyearly,9)
NPV of 'Reject' = -$66,735.63

b. Based on this calculation, should Sam accept or reject the firm's offer? Briefly explain your reasoning.

Yes. PV of accepting the firm's offer is greater (and positive).

Residual Method 2: Another common method is to use "fair market value" at the end of the study period as a measure of the residual. That is, to answer the question, "If I were to sell this asset/project at the end of the study period, what would I get for it?"[11] We will use a slightly simplified version of this, and ask: "If the cash flows beyond the study period represented an investment opportunity, how much would an investor be willing to pay for this opportunity?" For our investor, I'll choose an Ontario energy company, for the simple reason that they recently calculated their costs of capital[12], which we can use as a MARR. In particular, we'll use the 8.52% per year that they calculated as their Return on Equity (RoE)[13]. In general, the most an investor with a MARR of 8.52% per year would be willing to pay for a project is the present value of that project evaluated at their MARR. In our case, we'll take Year 9 as the present, and calculate the present (Year 9) value of the cash flows in the 'Reject' project, from Year 10 to the end of Sam's working life, using a MARR of 8.52% per year. This residual is again treated like a positive Year 9 cash flow.

[10] Yes, it would probably make more sense to just keep these as a present value – that is, a year 0 payment – but in the examples I've seen in the literature, the pro-rated first cost is added as a cash flow in the final year of the study period. Take this example from another U.S. Department of Energy text (from 1977): "'B' costs $1,300 and has an economic life of 12 years.. […] [H]alt the study period at 8 years and determine the residual value of 'B' at the end of year 9. […] 4 years remaining life ÷ 12 years life = 33.3% of life; therefore, 33.3% x $1,300 = $433 residual value for 'B' at the end of year 8." Division of Construction, Planning & Support. (1977). *Life Cycle Costing Emphasizing Energy Conservation: Guidelines For Investment Analysis* [May 1977 revision]. Energy Research & Development Administration.

[11] For some examples, see Tuovila, A. (2020, January 31). Residual Value [Web Page]. Retrieved from https://www.investopedia.com/terms/r/residual-value.asp

[12] Long, C. E. (2019, October 31). Re: 2020 Cost of Capital Parameters [Online Document]. Retrieved from https://www.oeb.ca/sites/default/files/Ltr-2020-Cost-of-Capital-Update-20191031.pdf

[13] RoE is equal to a company's net income over its shareholder equity. "If all the capital needed by a company were derived from stockholders, then the required rate of return on equity would dominate the MARR decision." Drummond, C. K. (2018). *Financial Decision-Making for Engineers*. U.S.: Yale University Press. We'll learn about RoE later. For now, see Fuhrmani, R. (2019, June 24). How to Calculate Return on Equity – RoE [Web Page]. Retrieved from https://www.investopedia.com/ask/answers/070914/how-do-you-calculate-return-equity-roe.asp

Residual 2 = Year 9 value of 'Reject' project cash flows from Year 10 onward, evaluated using a MARR of 8.52% per year.

Treating this residual as an additional positive cash flow in Year 9 of the 'Reject the offer' project, calculate the NPV of the 'Reject the offer' project. Show your work:

c. NPV of 'Reject the offer' using Residual 2: $__-67,802,.98_____

NPV of Year 10 onward cash flows = NPV during Sam's WL – NPV first 5 years of WL
We have both of these previously calculated for Sam's MARR. Re-do same calculations for a MARR of 8.52% (see Sheet 2 88 help, where I've just changed the MARR appropriately for Q1.)
Then just need to multiply by (F/P,8.52%,9) to see how much our investor would pay in Year 9.
(NPV of Year 10 onward cash flows) x (F/P,8.52%,9) = $127,297.41 = our residual.

NPV (Years 0 to 9) + Residual x (P/F,iyearly,9) = NPV of 'Reject the offer'
NPV of 'Reject the offer' = -67,802,.98

d. Based on this calculation, should Sam accept or reject the firm's offer? Briefly explain your reasoning

Yes. PV of accepting the firm's offer is greater (and positive).

e. One last question: Based on what you have learned in class and while doing this assignment, which approach do you think is more appropriate for helping Sam decide whether to accept or reject the firm's offer – repeated lifetimes, or study period? Briefly explain your reasoning.

Up to the student. Any good discussion should get the mark. Repeated lifetimes: logically consistent, but assumes repeatability which can be very unrealistic. Study period: no repeatability requirement, but residuals are necessary and can vary, with sometimes no consensus on 'right' one.

Question 3: [**Replacement Decisions**]

I do NOT suggest doing this question by hand. A math or spreadsheet program (such as Excel) is highly recommended.

Suppose Sam decides to buy a car in Year 0. Sam pays the full price of the car, P, in Year 0. Maintenance costs are in the form of an arithmetic gradient. There are no maintenance costs in Year 0 or Year 1. Maintenance costs are G in year 2, and go up by G every year thereafter. (2G in Year 3, 3G in Year 4, etc.) There are other costs of A each year (fuel, fees and insurance). These costs are first incurred in Year 0. The car's resale (salvage) value falls by 15% each year, so that the salvage value in Year 1 is 85% of P, and in year 2 is 72.25% (=85% x 85%) of P. Sam's MARR is equal to the baseline interest rate you calculated in Project 1, assuming that APR was compounded daily.

Preliminary Data Gathering:

Obtain information on a car of your choice by going to https://carcosts.caa.ca/ and looking up the car you think Sam would buy. Please use the following settings: 'British Columbia' as the province, New car, 20,000 km annual mileage. Year, Make, Model and Trim are up to you.

For the obvious reasons, you are not allowed to pick a 2020 Honda Accord EX-L 4D Sedan, since that was used for the partial sample answer.

Note that a partial sample answer is available to you at the end of this assignment. Look through it to see what is expected.

Write down your choices below:

Year	2020
Make	Honda
Model	Accord
Trim	EX-L 4D Sedan

Fill out the following table, which takes information provided by the web site and turns it into forms more compatible with this course's explanation of EAC. **You don't need to calculate P,G and A yourself. I've attached a small Excel sheet that will do it for you, and explains the reasoning behind the calculations**.

Item	Value ($)	Calculation
Yearly interest rate, i	13.80%	From Project 1
Monthly car payment (a)	$524.48	From site
P	**$32,785.45**	(a x (P/A,f,60))/0.85[14]
Maintenance cost (b)	$1,065.93	From site
G	**$1,068.02**	(b x (A/P,i,4) + b)/(A/G,i,4)
Insurance costs (c)	$2,503.46	From site
Licence & Registration Fees (d)	$72.00	From site
Fuel Cost per Year (e)	$1,486.71	From site
A	**$4,062.17**	c + d + e

f: Monthly interest rate corresponding to 5% interest a year ($f = 1.05^{1/12} - 1$)
i: Sam's MARR , in % per year terms

According to the CAA, what are the estimated total annual driving costs for your chosen car?

Estimated Total Annual Driving Costs: ___$9,828.80_____ /year

Regular Version of the Question:

Using the correct notation (e.g. (P/F,i,N)) and the symbols given (P, G and A), write an expression for the Equivalent Annual Cost of the car, letting N stand for the Life in Years[15] as used in the tables in Chapter 7 of *Engineering Economics*. To make this simple, we'll break it up into pieces:

a. EAC of the Initial Cost:

P x (A/P,iyearly,N)

b. EAC of Salvage (a negative cost):

$-P \times 0.85^N \times (P/F,iyearly,N) \times (A/P,iyearly,N)$
Or, even easier: $-P \times 0.85^N \times (A/F,iyearly,N)$

[14] Sharp-eyed students may notice this is off, given the timing. I'd changed the payments to be from Month 1 to Month 60 while creating the helper spreadsheet, but neglected to change the project text. Since it's calculated for you, there should be no difficulties, though.
[15] For those without access to the textbook, N = 3 it means that you are considering the situation where you sell the car three years from now. Note that in our example, the car is paid for in full in the present, Year 0.

c. EAC of Cumulative Maintenance Costs[16] to time N:

G x (A/G,iyearly,N)

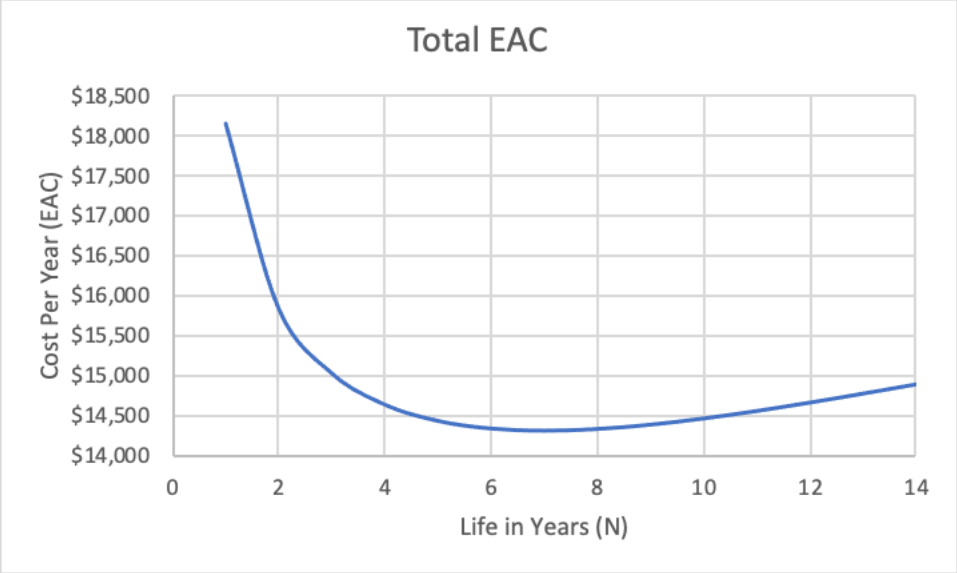
d. EAC of Additional Costs (Fuel,Insurance,Fees)[17]:

A + A x (A/P,iyearly,N) (An annuity A plus a Year 0 A, annualized)

e. Use your equations to fill out the following table. I don't need to see your calculations:

N	EAC Initial Cost	EAC Salvage	EAC Cum. Maint.	EAC Add. Costs	Total EAC
1	$37,332.41	-$27,867.64	$0.00	$8,687.72	$18,152.49
2	$19,876.66	-$11,075.71	$499.38	$6,524.92	$15,825.25
3	$14,090.65	-$5,861.02	$975.81	$5,808.02	$15,013.46
4	$11,221.88	-$3,484.35	$1,429.47	$5,452.58	$14,619.57
5	$9,519.77	-$2,206.47	$1,860.63	$5,241.68	$14,415.62

f. Draw an EAC diagram, preferably with Excel. The vertical axis should be Cost Per Year (EAC), and the horizontal axis should be N (Life in Years). The range of N should be large enough to show the EAC curve's distinctive 'U' shape –we should be able to clearly spot the bottom of the U. This may mean your final N value is considerably greater than 5.



[16] For N=1, these costs are $0. From N=2 onward, you can use the fact that these costs are a well-behaved geometric gradient and apply (A/G,i,N) to get the equivalent annuity.

[17] Though labeled 'A', this is a cost that is *also* incurred in Year 0. That means you can't just say 'it's A, because it's already in per year terms'. Annuities are only positive from Year 1 to Year N – you need to account for the extra payment in Year 0.

g. What is the economic lifetime of the car? Briefly explain your reasoning.

Economic Lifetime: __7___ years

EAC is at a minimum when N=7 (bottom of the 'U').

h. What is the EAC at the car's economic lifetime?

EAC at the car's economic lifetime: $ ___ 14,293.76____

Suppose that Sam keeps his car for the duration of his studies, and right after graduation – on January 1 of Year 5 – Sam is offered the opportunity by his firm to rent a company car, which is a brand new version of the model Sam is driving. The rental fee is paid once a year, on January 1, and is equal to the Estimated Total Annual Driving Costs that the CAA listed for your model. The firm will cover all other costs, so that the ONLY car-related cost Sam will have each year is the rental fee. Should Sam accept, the first rental fee is due immediately. Use EAC analysis to decide what Sam should do.

i. Does the one-year rule apply? Briefly explain your reasoning.

No. Resale value is still fairly high compared to Maintenance costs, implying the downward-slope inducing capital costs may still have influence.

j. Should Sam keep his old car for a while longer, or switch to the company car immediately on the first day of Year 5? Briefly explain your reasoning. (Hint: Remember that for this question, the 'present' is Year 5, not Year 0. You'll probably want to re-do at least some of your EAC calculations.)

Keep. I ran the numbers for the defender's situation (EAC at economic lifetime is about 16k), but in this case, even when new the EAC at the economic lifetime of the car was higher than the estimated average driving costs (~14k vs. 10k), and the EAC of the rental approaches the latter asymptotically.

Partial Worked Out Sample Answer to Question 3 (Regular Version)

When I filled out the form at https://carcosts.caa.ca/ , this is what it looked like:

From there, I was able to fill out the required fields in the answer as follows:

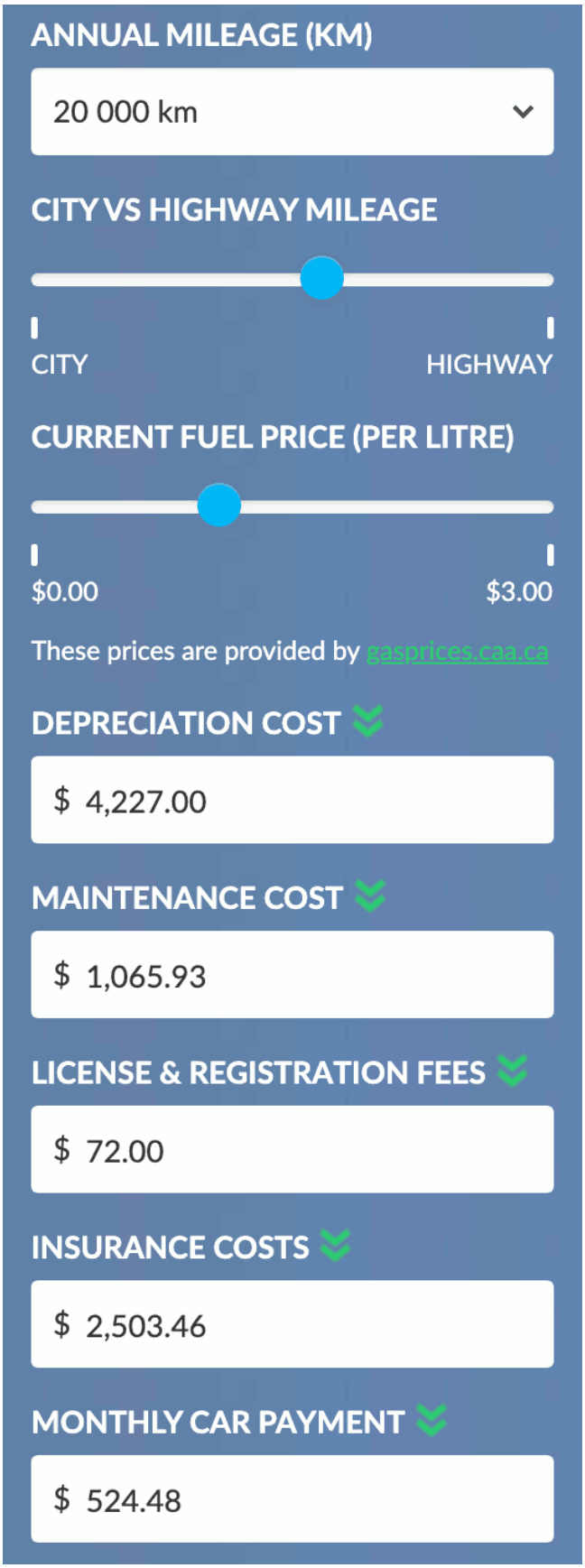
Year	2020
Make	Honda
Model	Accord
Trim	EX-L 4D Sedan

After clicking the 'KNOW YOUR COSTS' button, I was sent to a results screen. The results were split across two sections of the page. On the left was this image:

That gave me two things I needed: the Estimated Total Annual Driving Costs, and the Fuel Cost per Year. I could now fill out the Annual driving costs field:

Estimated Total Annual Driving Costs: ____$9,828.80___ /year

I could also fill in one row of the table. The rest of the information for the table was on the right, and looked as follows:


ANNUAL MILEAGE (KM)
20 000 km
CITY VS HIGHWAY MILEAGE
CITY
HIGHWAY
CURRENT FUEL PRICE (PER LITRE)
$0.00
$3.00
These prices are provided by gasprices.caa.ca
DEPRECIATION COST
$ 4,227.00
MAINTENANCE COST
$ 1,065.93
LICENSE & REGISTRATION FEES
$ 72.00
INSURANCE COSTS
$ 2,503.46
MONTHLY CAR PAYMENT
$ 524.48

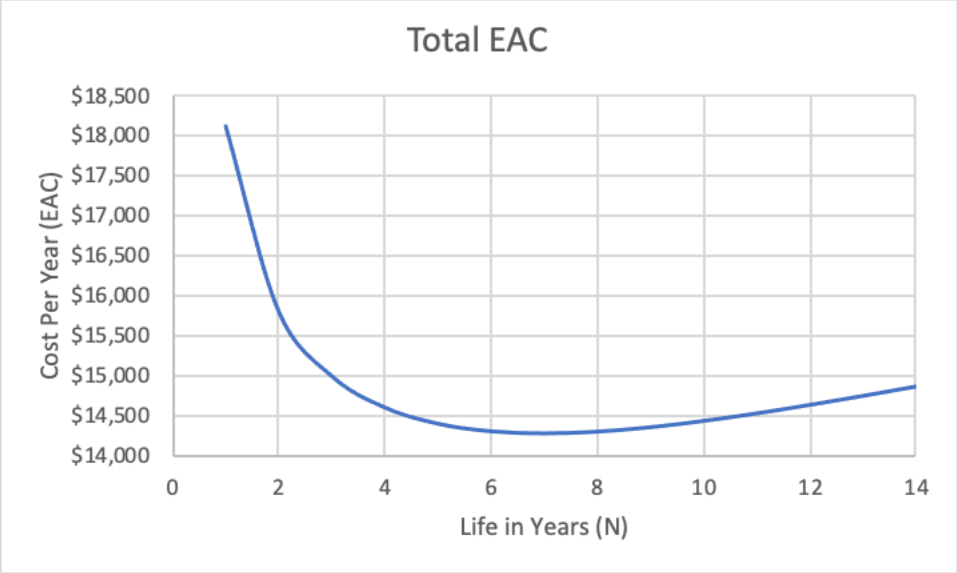
Using the information provided, I filled in the table and let the Excel sheet calculate the required values for me.

Item	Value	Calculation
Yearly interest rate, i	13.80%	From Project 1
Monthly car payment (a)	$524.48	From site
P	$32,785.45	(a x (P/A,f,60))/0.85
Maintenance cost (b)	$1,065.93	From site
G	$1,068.02	(b x (P/A,i,4) + b)/(A/G,i,4)
Insurance costs (c)	$2,503.46	From site
Licence & Registration Fees (d)	$72.00	From site
Fuel Cost per Year (e)	$1,486.71	From site
A	$4,062.17	c + d + e

For the obvious reasons, I won't show you my EAC equations, but when I plugged in my values, these were the results for N=1 to N=5:

N	EAC Initial Cost	EAC Salvage	EAC Cum. Maint.	EAC Add. Costs	Total EAC
1	$37,310.91	-$27,867.64	$0.00	$8,685.05	$18,128.33
2	$19,859.86	-$11,079.11	$499.54	$6,522.84	$15,803.13
3	$14,075.13	-$5,864.69	$976.21	$5,806.10	$14,992.75
4	$11,206.77	-$3,487.69	$1,430.23	$5,450.71	$14,600.02
5	$9,504.74	-$2,209.34	$1,861.83	$5,239.82	$14,397.06

My sample car didn't reach the bottom of its EAC 'U' until N=7, when it was about $14,277, so I plotted it from N=1 to N=14 and set the minimum value of the horizontal axis to $14,500.



For completeness, here's what the spreadsheet I used to plot that graph looked like. Once I'd set up the equations for the N=2 row (N=1 was special, since it had 0 Maintenance costs), I just had Excel fill down and the calculations were done in a second. Even the N counter was automated, as I told Excel to add 1 to the previous row's counter number.

N	EAC Initial Cost	EAC Salvage	EAC Cum. Maint.	EAC Add. Costs	Total EAC	Delta
1	$37,310.91	-$27,867.64	$0.00	$8,685.05	$18,128.33	
2	$19,859.86	-$11,079.11	$499.54	$6,522.84	$15,803.13	-$2,325.20
3	$14,075.13	-$5,864.69	$976.21	$5,806.10	$14,992.75	-$810.38
4	$11,206.77	-$3,487.69	$1,430.23	$5,450.71	$14,600.02	-$392.73
5	$9,504.74	-$2,209.34	$1,861.83	$5,239.82	$14,397.06	-$202.95
6	$8,385.65	-$1,455.87	$2,271.37	$5,101.17	$14,302.31	-$94.75
7	$7,599.41	-$985.44	$2,659.23	$5,003.75	$14,276.95	-$25.36
8	$7,020.97	-$680.00	$3,025.88	$4,932.08	$14,298.93	$21.97
9	$6,580.82	-$476.06	$3,371.83	$4,877.54	$14,354.13	$55.21
10	$6,237.23	-$337.00	$3,697.65	$4,834.97	$14,432.84	$78.71
11	$5,963.62	-$240.67	$4,003.94	$4,801.07	$14,527.97	$95.13
12	$5,742.28	-$173.08	$4,291.37	$4,773.65	$14,634.22	$106.25
13	$5,560.93	-$125.19	$4,560.61	$4,751.18	$14,747.52	$113.30
14	$5,410.76	-$90.98	$4,812.38	$4,732.57	$14,864.73	$117.21

The 'Delta' column is a helper column. I had Excel subtract the previous row's Total EAC from the current row's Total EAC. When the negative sign disappeared from the Delta entries, I knew I had found the bottom of the 'U', since that meant EAC was now going up. (I've added the red by hand for ease of reading. Excel formats will generally indicate a negative number by (brackets), red brackets, red or a negative sign, but not red AND a negative sign.)

88 Challenge Version: The regular version of the question simplified things by assuming Sam paid for the car up front, in cash. They also paid for gas and insurance once a year. For the challenge version of the question, we loosen these training wheels.

- As in the regular version of Question 3, Sam's MARR is as you determined in the regular version of Project 1's Question 2. The car dealer's MARR is 5% per year.
- Sam pays 15% of the car's value (P) as a down payment on January 1 in Year 0.
- Sam then pays a monthly car payment equal to that listed on the CAA page for 60 months (5 years). The first payment is in Month 0, and the last payment is in Month 59. Sam pays this on the first day of each month.
- After Month 59, there are no more monthly car payments.
- If Sam wants to sell the car before the end of the 60 months, Sam must first pay the remaining value of the loan.
    - Note: The remaining value of the loan is not the sum of the remaining payments, but a smaller amount – the present value of the remaining payments on the loan, where 'present' is the time at which Sam pays off the loan, evaluated at the car dealer's interest rate (assumed to be 5% per year).
- Sam is only able to sell the car on the first day of the year. (This is so that you only need to consider N in terms of years, not months or days.) If Sam sells their car before the 60 months are up, they also pay the remaining value of their loan on the same day.
- The price Sam receives for selling the car is equal to the resale/salvage value as detailed in the regular version of Question 3.
- Sam pays for gas once a week, on the first day of the week. Assume there are exactly 52 weeks in a year. Each week, starting with Week 0 (the present), Sam pays for 1/52 of the total year's fuel costs as listed on the CAA site.
- Sam pays for insurance once a month, on the first day of the month. Each month they pay 1/12 of the yearly insurance costs listed on the CAA site.
- Maintenance costs are as in the regular version of Question 3.
- License & Registration Fees are paid on January 1 of each year, as in the regular version of Question 3.

Using the correct notation (e.g. (P/F,i,N)) and the symbols given (P, G and A), write an expression for the Equivalent Annual Cost of the car, letting N stand for the Life in Years[18] as used in the tables in Chapter 7 of *Engineering Economics*. To make this simple, we'll break it up into pieces:

Assume selling the car takes priority: First thing done in the year. (So no fuel, etc. costs.)

a. EAC of the Down Payment:

$$0.15 \times P \times (A/P,i_{yearly},N)$$

[18] For those without access to the textbook, N = 3 it means that you are considering the situation where you sell the car three years from now. Note that in our example, the car is paid for in full in the present, Year 0.

b. EAC of the Car Payments (N<5):

x1: PV Paid by Year N: a x (1 + (P/A,imonthly,12N-1))
x2: Year N Owed: a x (1 + (P/A,fmonthly,12x(5-N)-1))
x3: PV of Year N Owed: x3 x (P/F,iyearly,N)
EAC of Car Payments: (x1 + x3) x (A/P,iyearly,N)

c. EAC of the Car Payments (N ≥5):

All 60 car payments have been made, so…
(a + a x (P/A,imonthly,59))x(A/P,iyearly,N)

d. EAC of Salvage (a negative cost):

$-P \times 0.85^{N} \times (A/F,iyearly,N)$

e. EAC of Cumulative Maintenance Costs[19] to time N:

Car sale takes priority, so $0 in Year 1, Year 2, otherwise
G x (A/G,iyearly,N) – (N-1)xGx(A/F,iyearly,N)
We're subtracting the annual equivalent of the missing 'step' of height (N-1).

f. EAC of Fuel Cost:

(e/52) x (F/A,iweekly,52) x (F/P,iweekly,1)
That is, first we take Weeks 0-51 of fuel costs and make it a Week 51 value…
Then we move that to the first week of Year 1. Now it behaves like an annuity. The calculation repeats each year, so we have the same payment each year, from Year 1 to Year N.

g. EAC of Insurance:

(c/12) x (F/A,imonthly,12) x (F/P,imonthly,1)
Same reasoning as above.

h. EAC of License & Registration Fees:

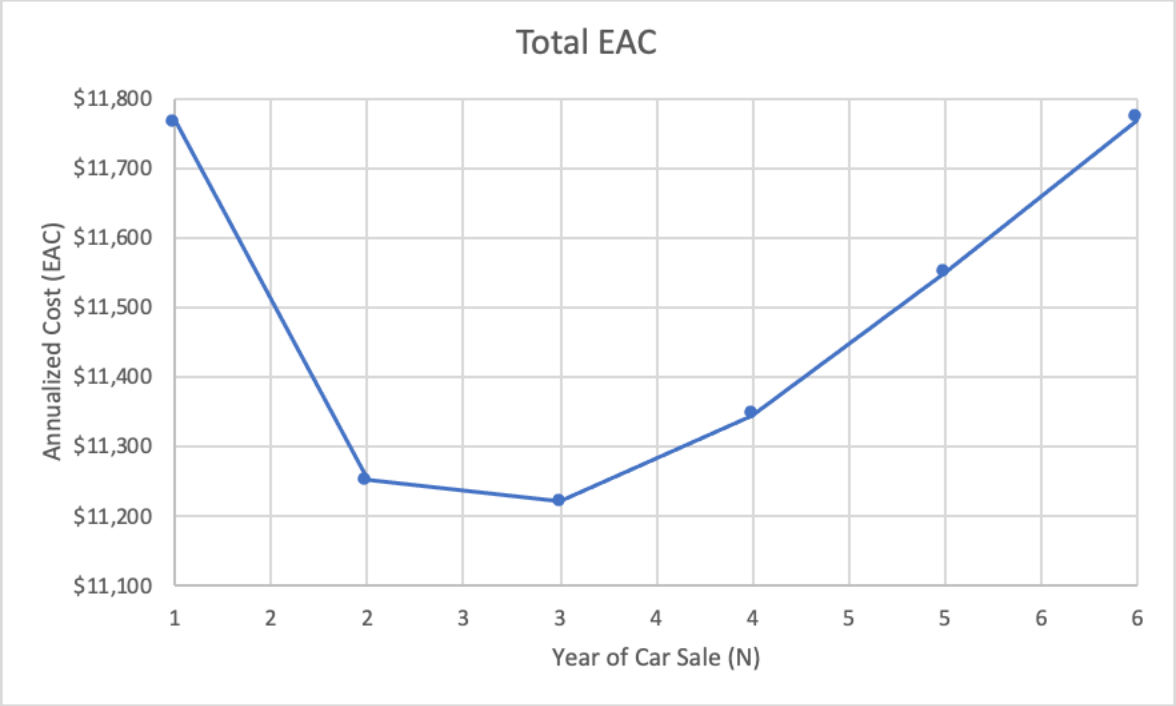
d x (F/P,iyearly,1)
Same reasoning as above. Car sale takes priority, so N=1 only pays insurance for 0, etc.

[19] For N=1, these costs are $0. From N=2 onward, you can use the fact that these costs are a well-behaved geometric gradient and apply (A/G,i,N) to get the equivalent annuity.

i. Use your equations to fill out the following table: (I don't need to see your calculations.)

N	EAC Initial Cost	EAC Salvage	EAC Cum. Maint.	EAC Other Costs[20]	Total EAC
1	$5,599.86	-$27,867.64	$0.00	$34,033.51	$11,765.74
2	$2,981.50	-$11,075.71	$0.00	$19,345.67	$11,251.46
3	$2,113.60	-$5,861.02	$354.01	$14,614.40	$11,220.99
4	$1,683.28	-$3,484.35	$777.14	$12,369.39	$11,345.46
5	$1,427.97	-$2,206.47	$1,212.65	$11,116.23	$11,550.38

j. Draw an EAC diagram, preferably with Excel. The vertical axis should be Cost Per Year (EAC), and the horizontal axis should be N (Life in Years). The range of N should be large enough to show the EAC curve's distinctive 'U' shape –we should be able to clearly spot the bottom of the U. This may mean your final N value is considerably greater than 5.



k. What is the economic lifetime of the car? Briefly explain your reasoning.

Economic Lifetime: _____3____ years

Lowest EAC

l. What is the EAC at the car's economic lifetime?

EAC at the car's economic lifetime: $ 11,220.99

[20] EAC Car Payments + EAC Additional Payments

Suppose that Sam keeps his car for the duration of his studies, and right after graduation – on January 1 of Year 5 – Sam is offered the opportunity by his firm to rent a company car, which is a brand new version of the model Sam is driving. The rental fee is paid once a year, on January 1, and is equal to the Estimated Total Annual Driving Costs that the CAA listed for your model. The firm will cover all other costs, so that the ONLY car-related cost Sam will have each year is the rental fee. Should Sam accept, the first rental fee is due immediately. Use EAC analysis to decide what Sam should do.

m. Does the one-year rule apply? Briefly explain your reasoning.

Yes. By this time salvage values are very small compared to maintenance & other costs. Importantly, all car payments are *sunk*. Also, maintenance costs *start* at 4G (for Year 6 sale, where you had to pay Year 5's maintenance costs).

n. Should Sam keep his old car for a while longer, or switch to the company car immediately on the first day of Year 5? Briefly explain your reasoning. (Hint: Remember that for this question, the 'present' is Year 5, not Year 0. You'll probably want to re-do at least some of your EAC calculations.)

Yes. One-year rule holds and defender's EAC from one more year is greater than the rental offer's EAC, which tends asymptotically towards the average yearly cost from the CAA.