Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Determine whether the following series converges.

$$\sum_{k=2}^{\infty} (-1)^k \frac{k^2-6}{k^2+7}$$

Since the given series is an alternating series, use the alternating series test to test for convergence. The alternating series $\sum(-1)^k a_k$ converges provided the terms of the series are nonincreasing in magnitude ($0 < a_{k+1} \le a_k$ for k greater than some index N) and $\lim_{k\to\infty} a_k = 0$.

Identify the expression a_k for the given alternating series.

$$a_k = \frac{k^2-6}{k^2+7}$$

Determine whether the terms of the series are nonincreasing in magnitude for k greater than some index N. To do so, find the first derivative of $\frac{k^2-6}{k^2+7}$ with respect to k.

$$\frac{d\left(\frac{k^2-6}{k^2+7}\right)}{dk} = \frac{26k}{\left(k^2+7\right)^2}$$

The terms of the series are increasing in magnitude for k greater than some index N because the first derivative is always positive for $k \ge 2$.

Therefore, nothing can be concluded because alternating series $\sum_{k=2}^{\infty} (-1)^k \frac{k^2-6}{k^2+7}$ does not satisfy the conditions for the Alternating Series Test.

Since the Alternating Series Test cannot be used to determine whether the series converges, try another test to determine whether the series converges or diverges. A simple divergence test is the nth-Term Test for Divergence.

The nth-Term Test for Divergence states that $\sum_{n=1}^{\infty} a_n$ if $\lim_{n\to\infty} a_n$ fails to exist or is different from 0.

Evaluate the limit.

Since $\lim_{k\to\infty} \frac{k^2-6}{k^2+7} = 1$, $\lim_{k\to\infty} (-1)^k \frac{k^2-6}{k^2+7}$ does not exist because the values apporach both 1 and -1 as $n\to\infty$.

Since the limit does not exist, $\sum_{k=2}^{\infty} (-1)^k \frac{k^2-6}{k^2+7}$ diverges.

Therefore, although the series fails the Alternating Series Test, it diverges by the nth-Term Test for Divergence.