CSC 225

Algorithms and Data Structures I

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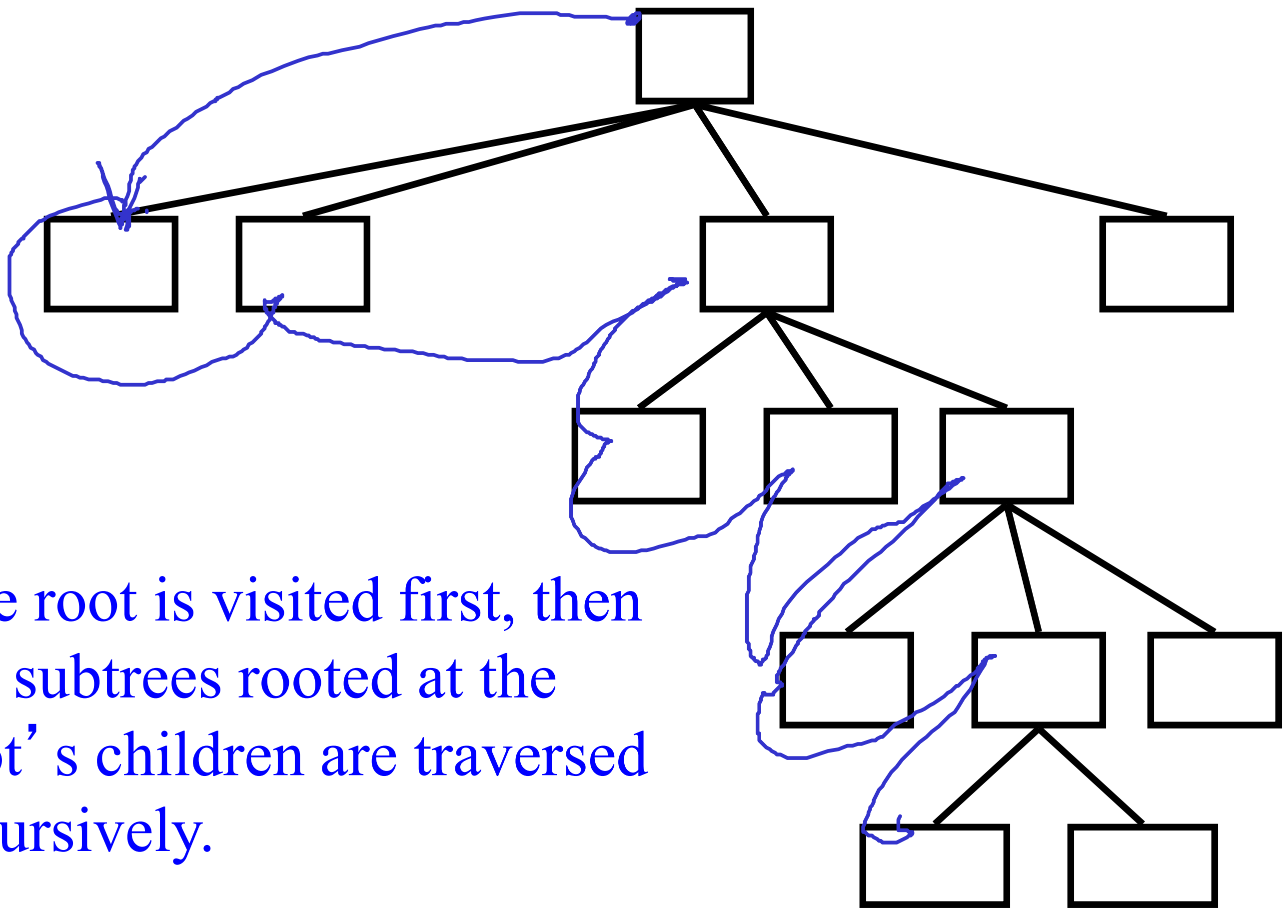
ECS 516

Tree Traversals

- n-ary tree traversals
  - Preorder
  - Postorder
  - Level order

- Binary tree traversals
  - Preorder
  - Postorder
  - Inorder
  - Level order

Preorder Traversal
Depth First Search



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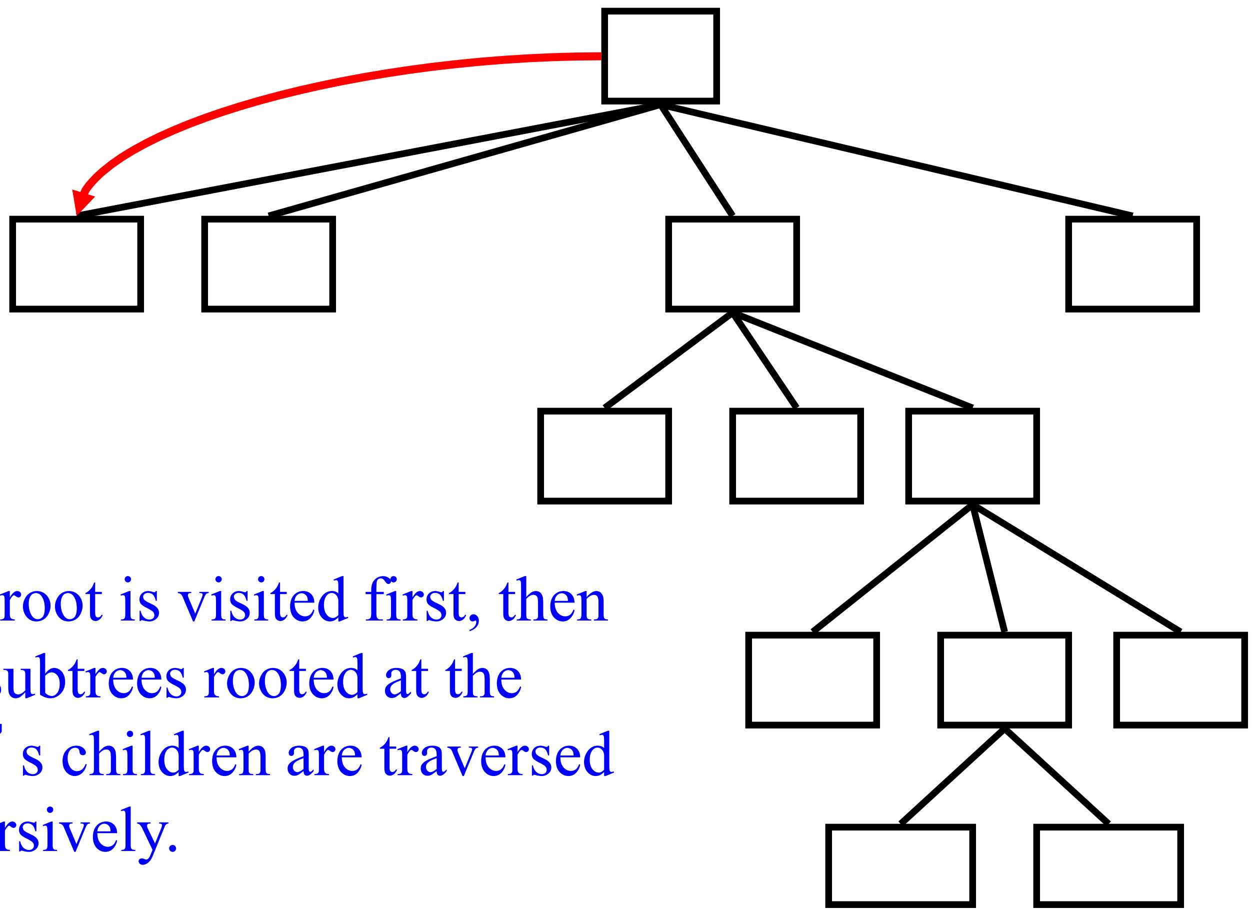

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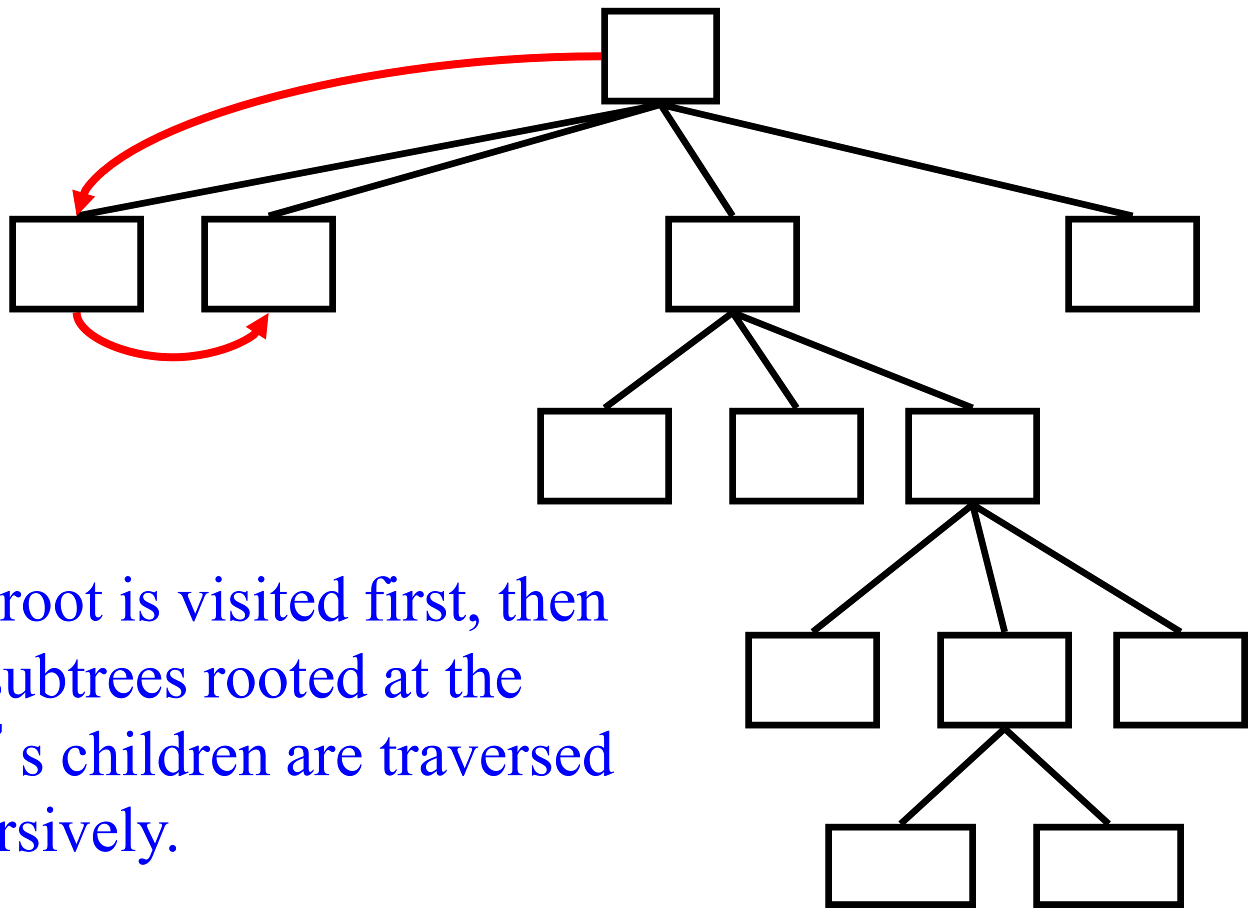

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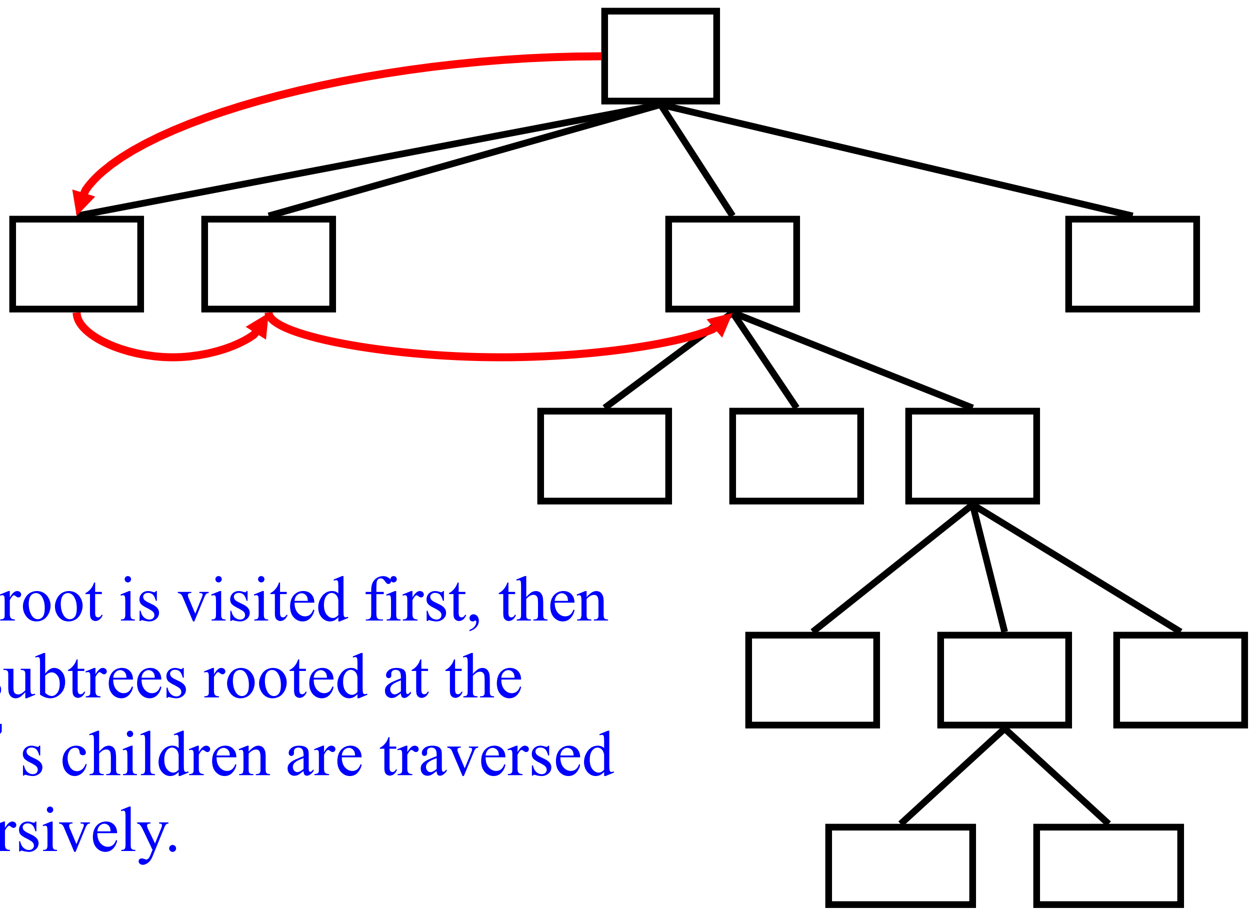

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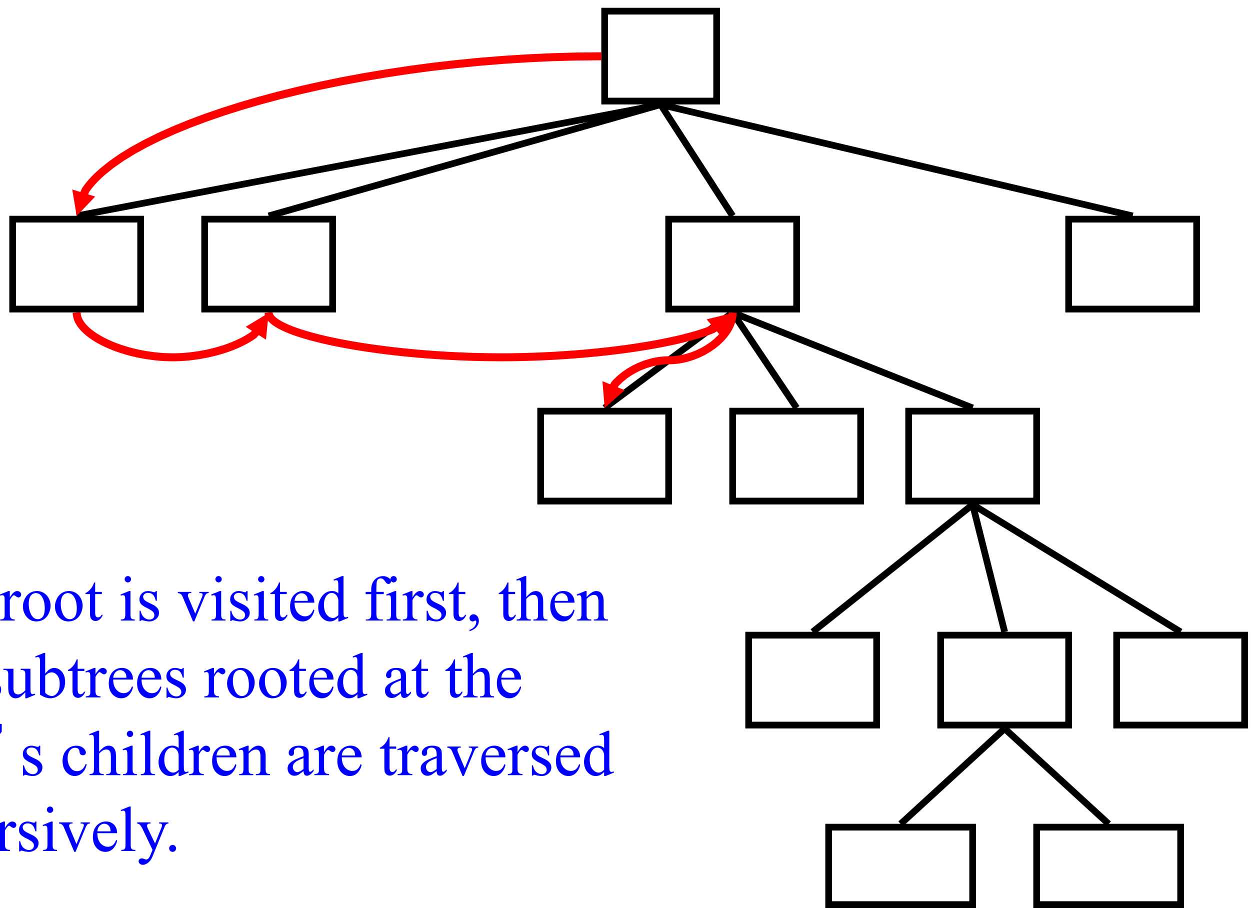

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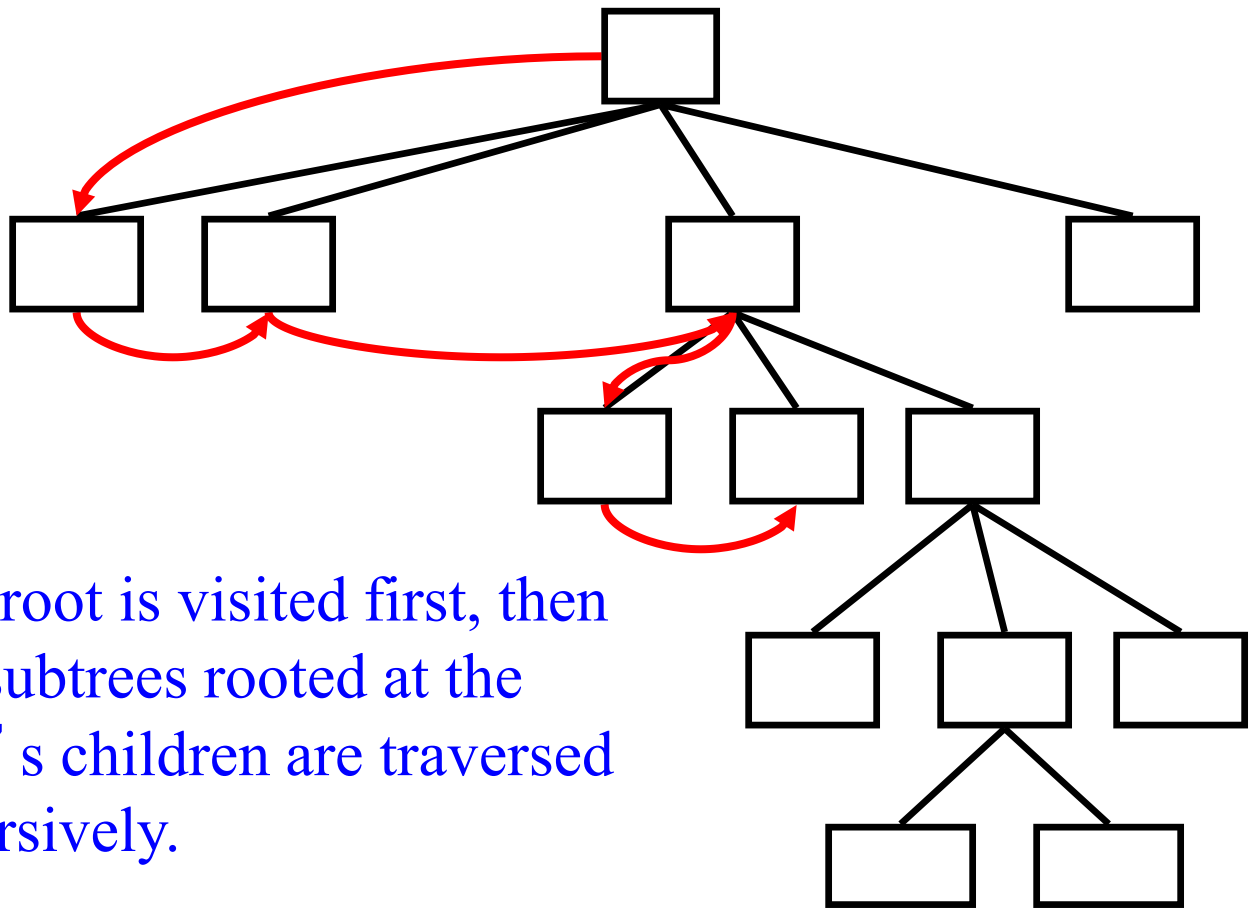

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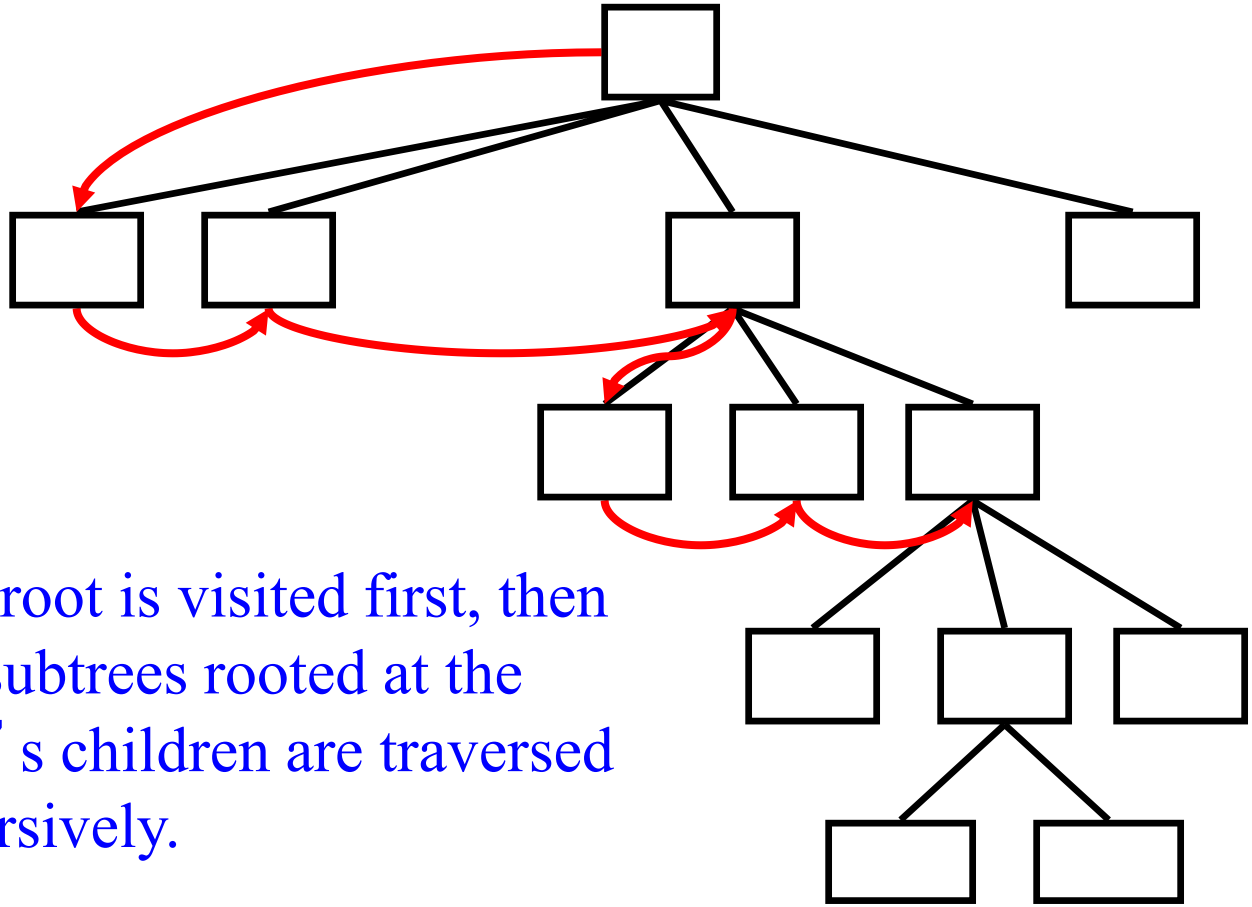

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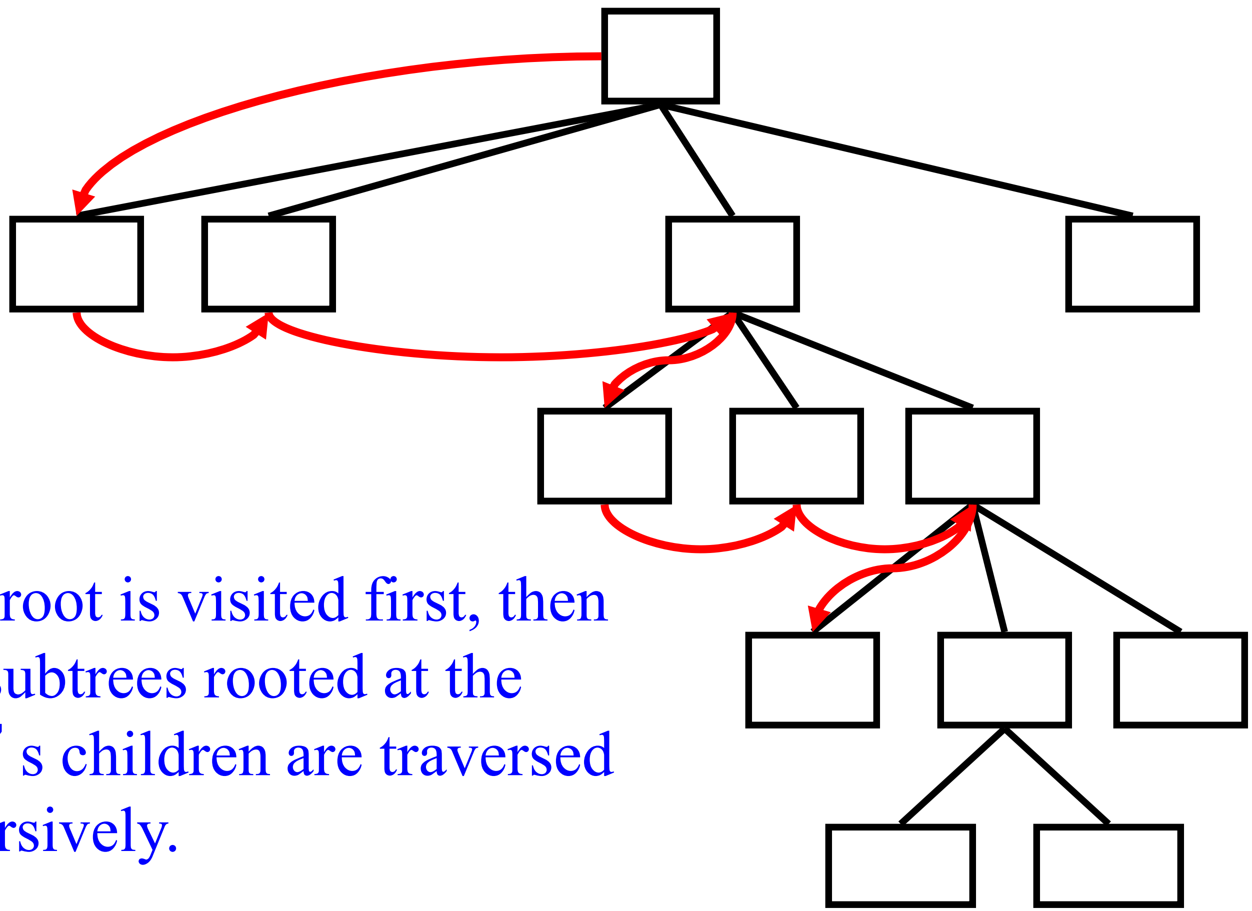

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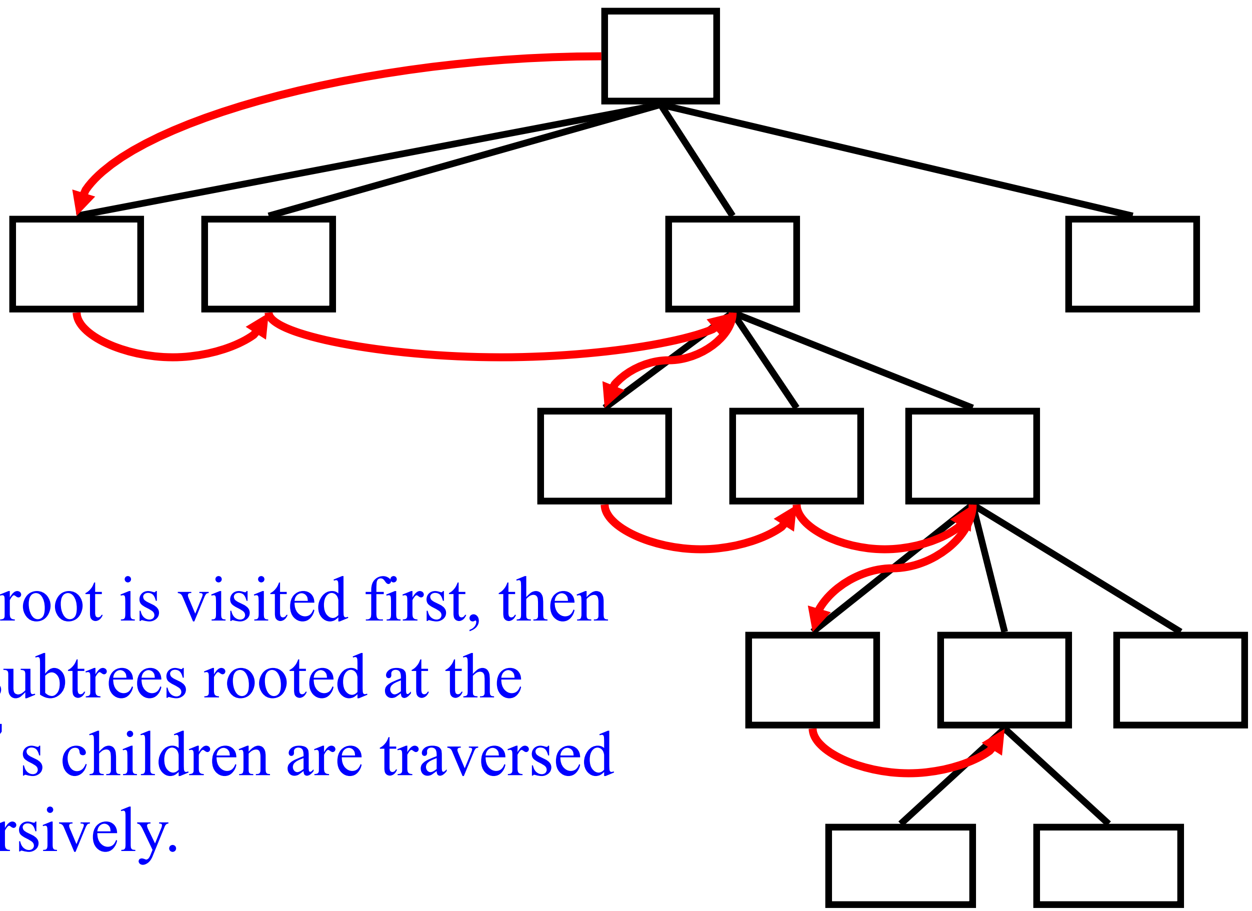

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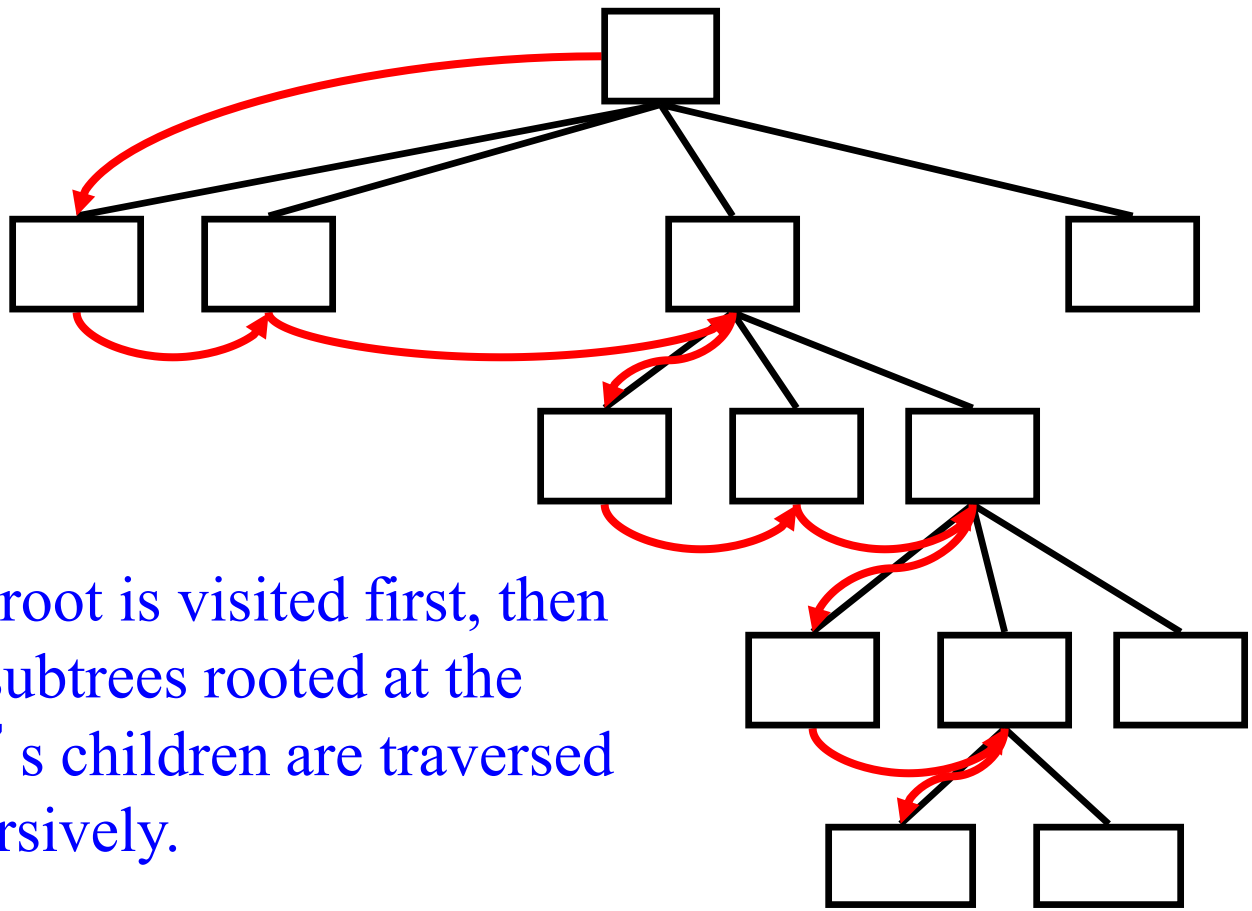

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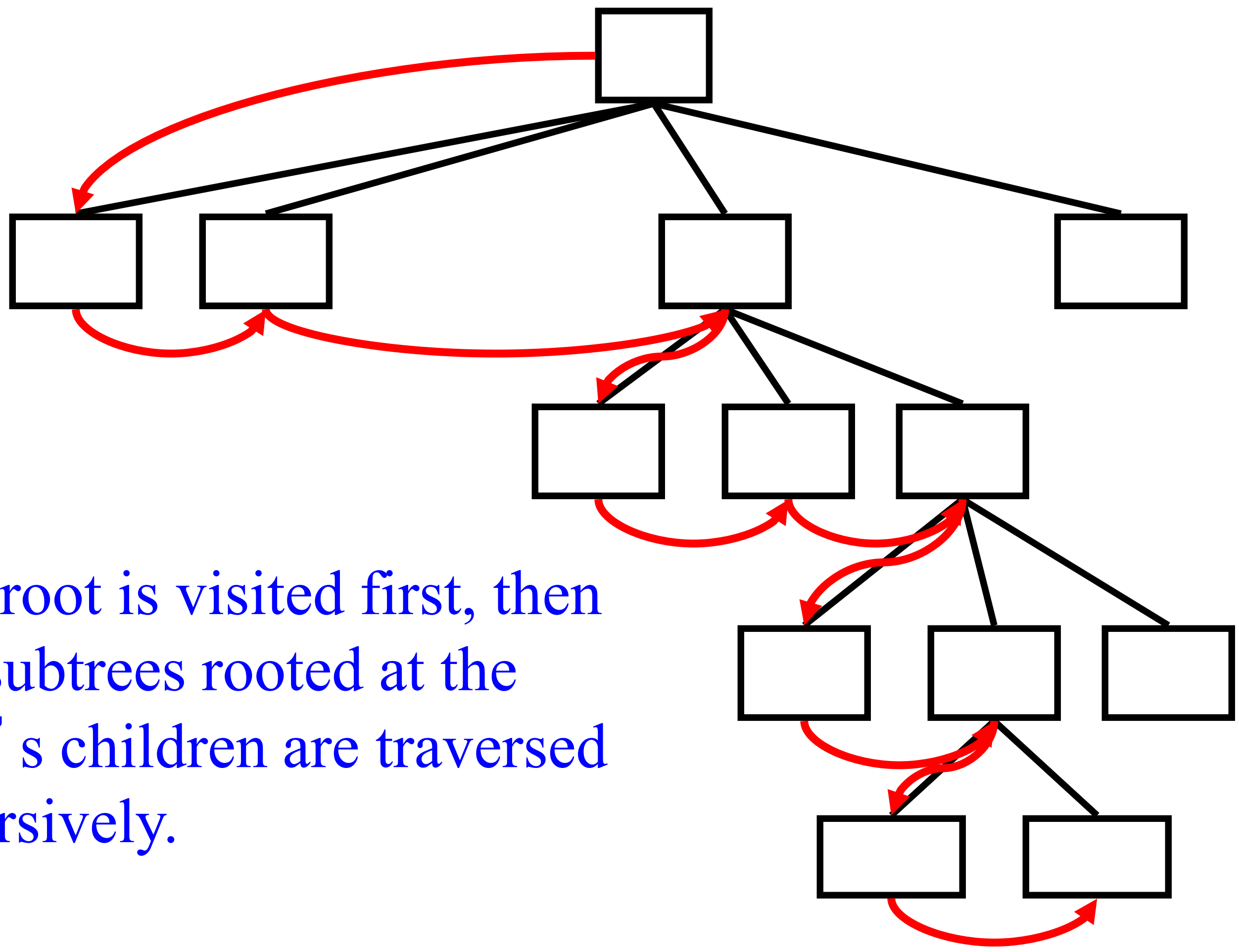

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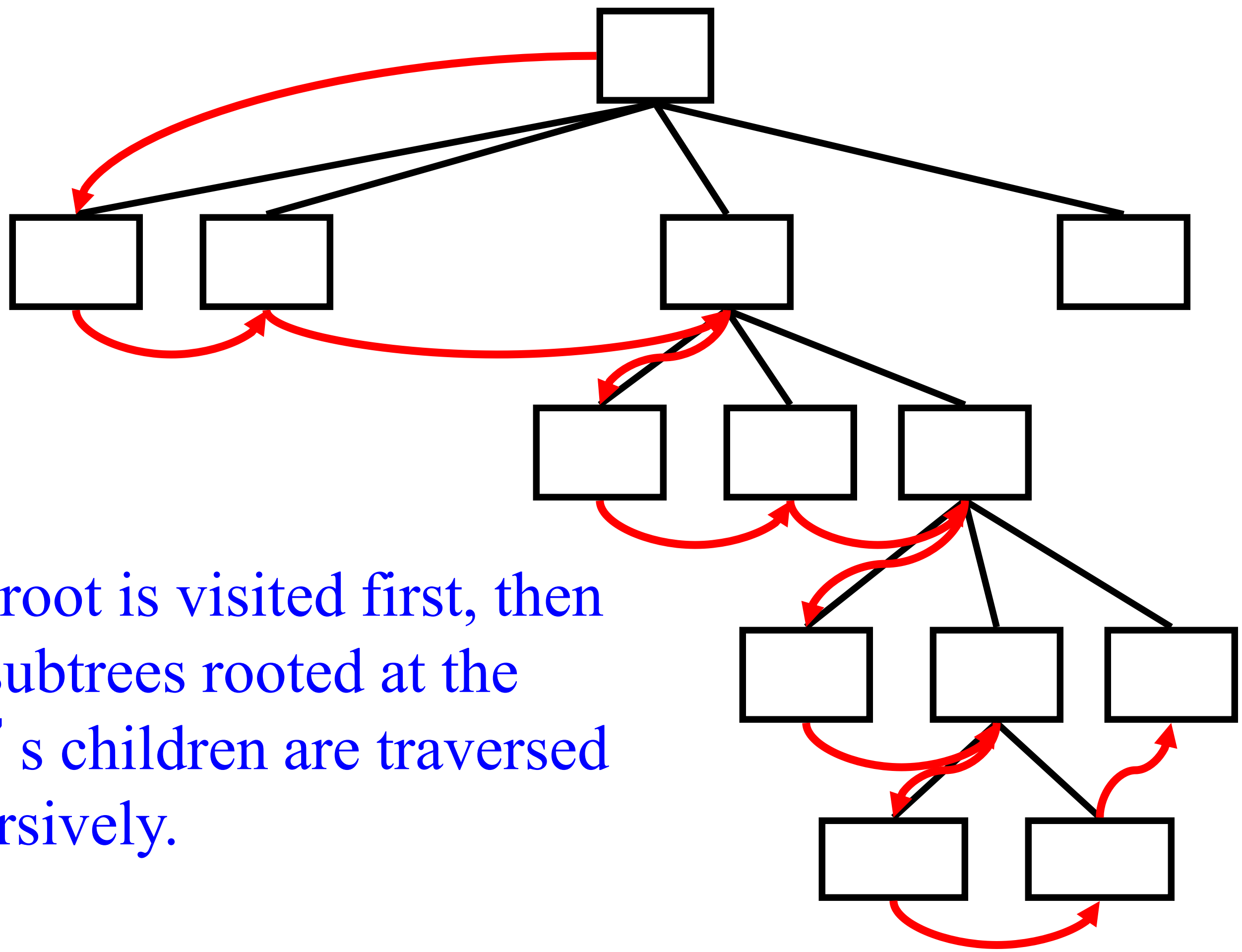

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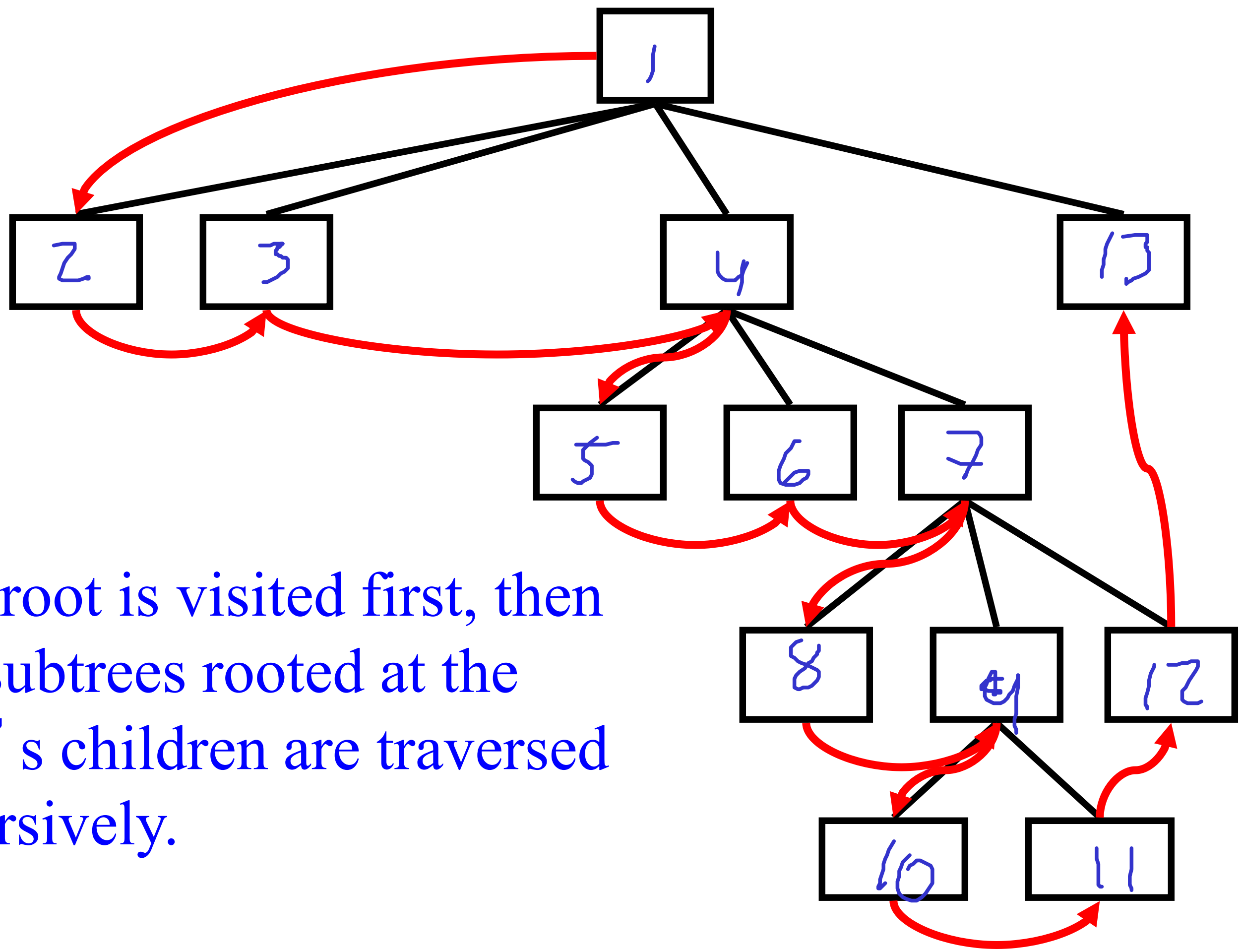

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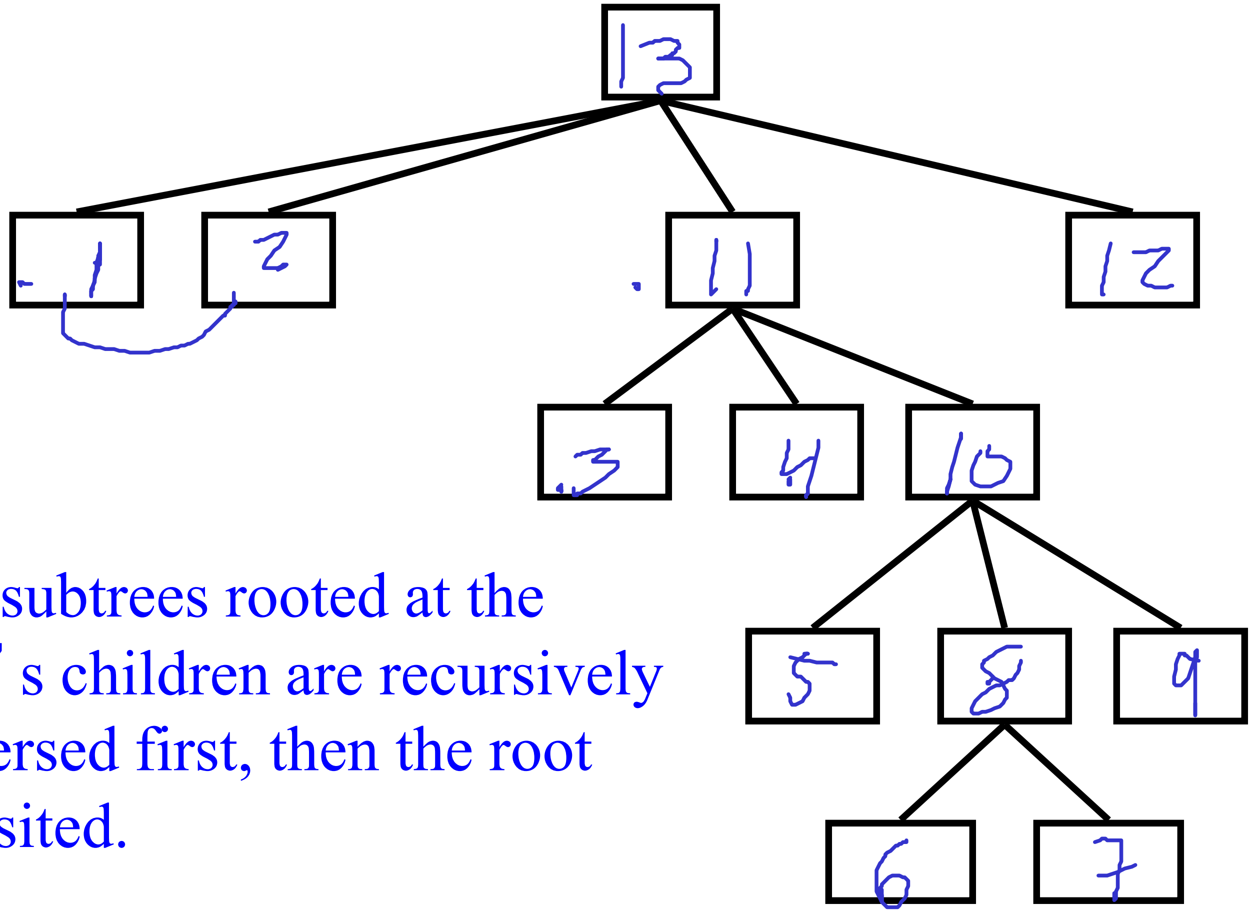

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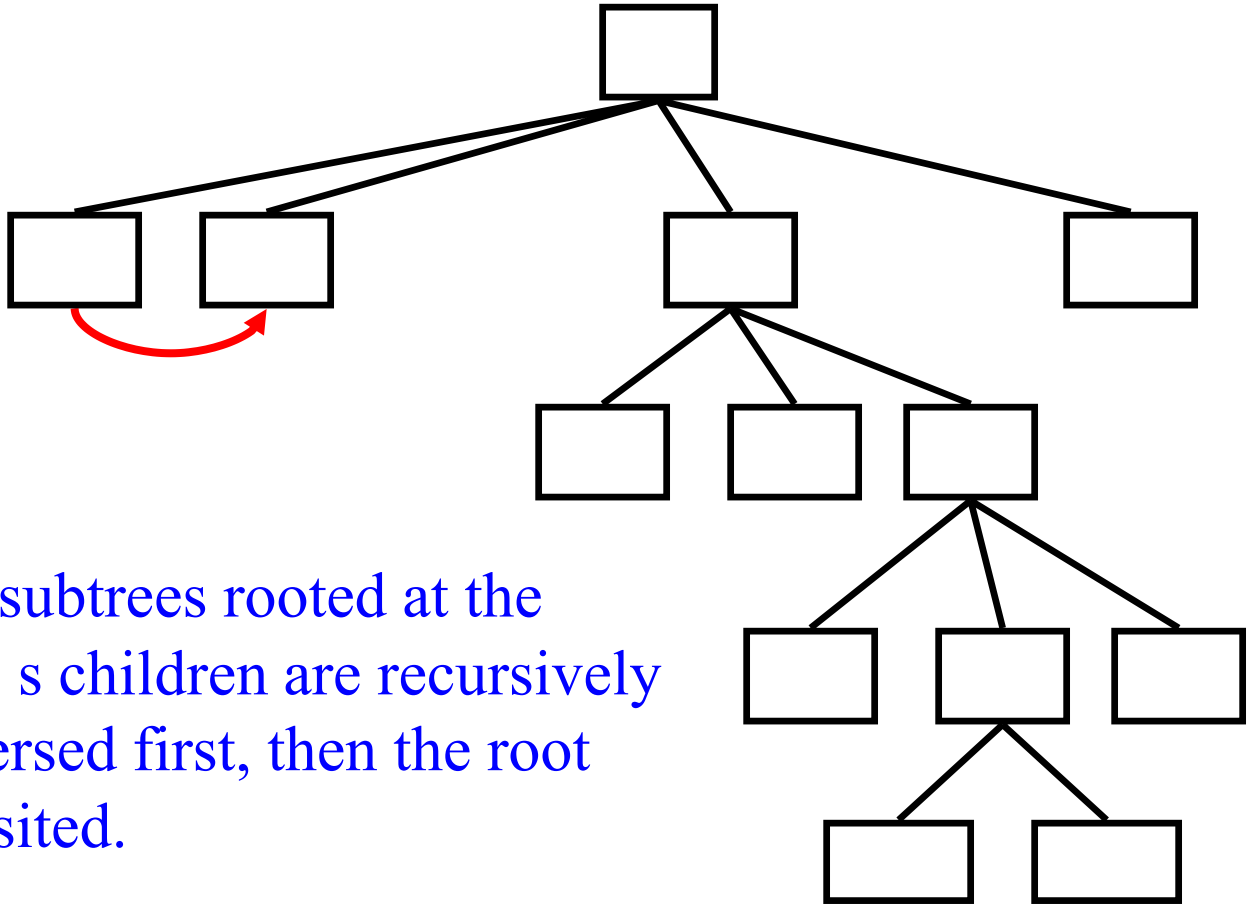

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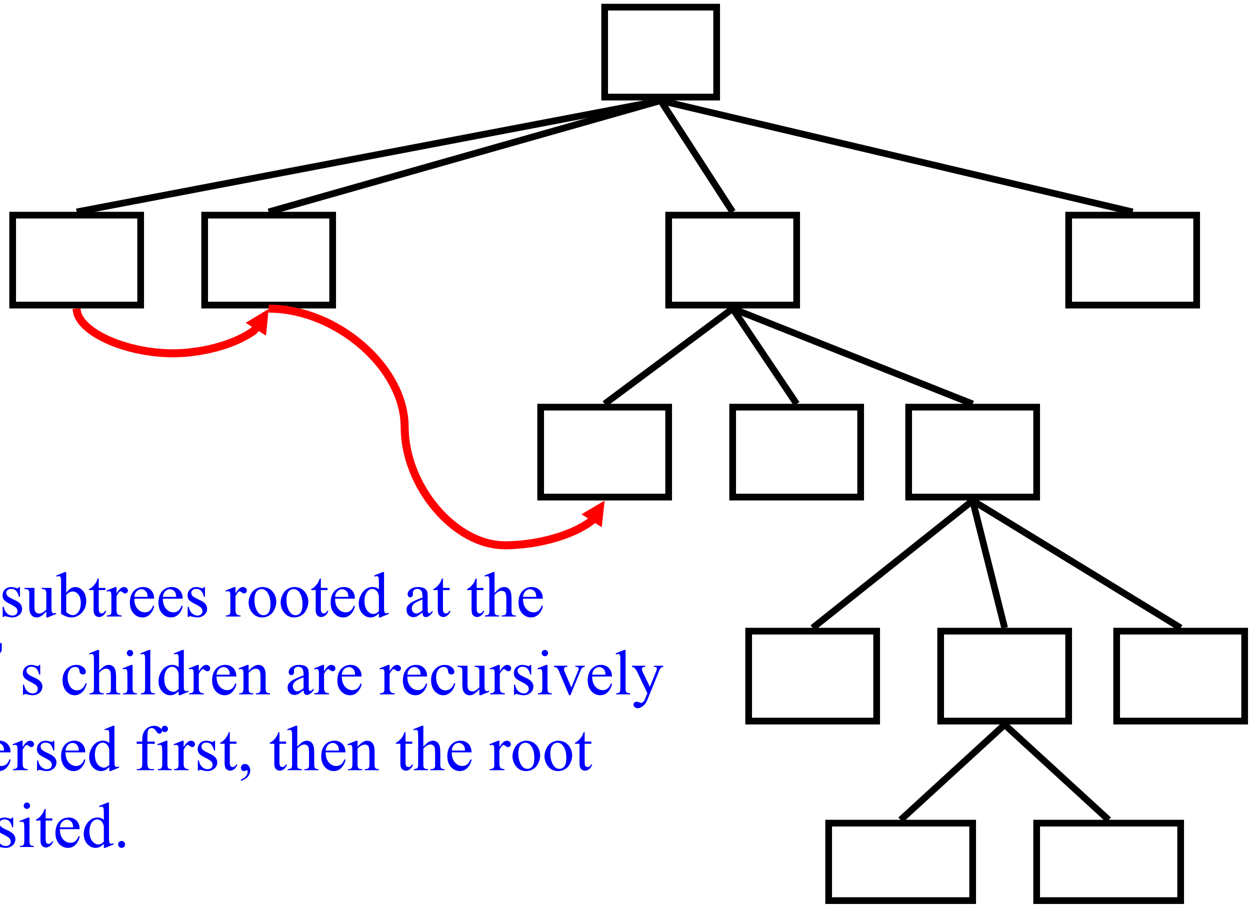

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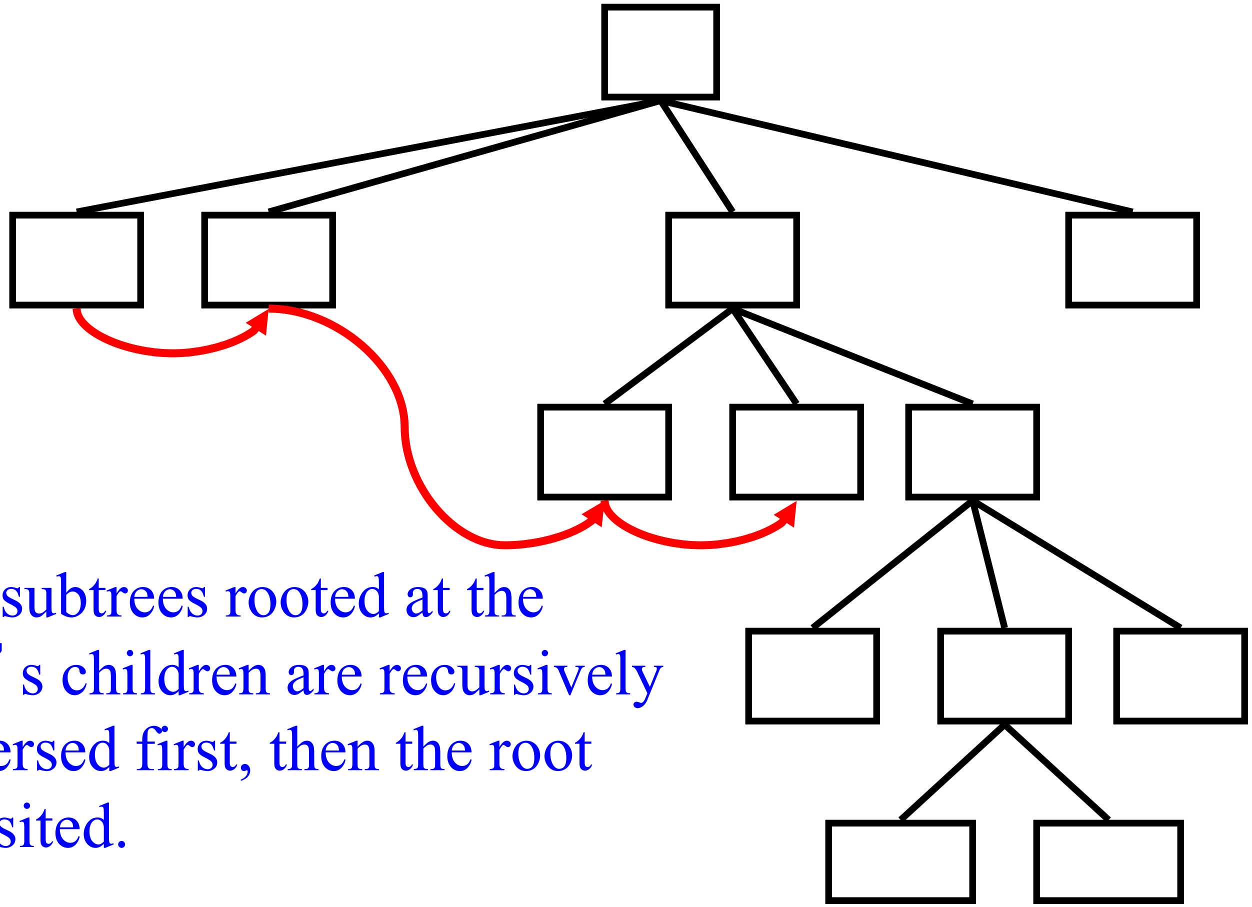

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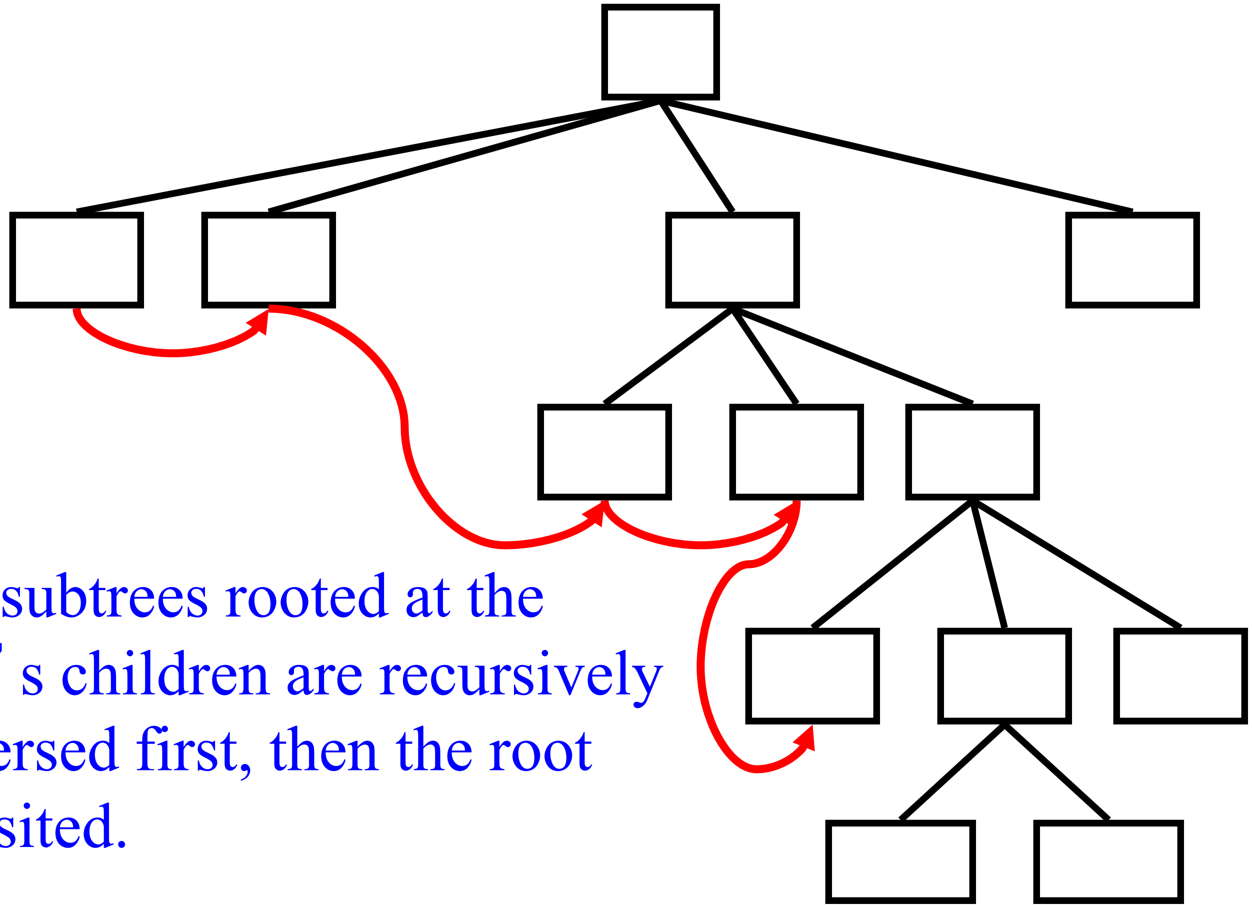

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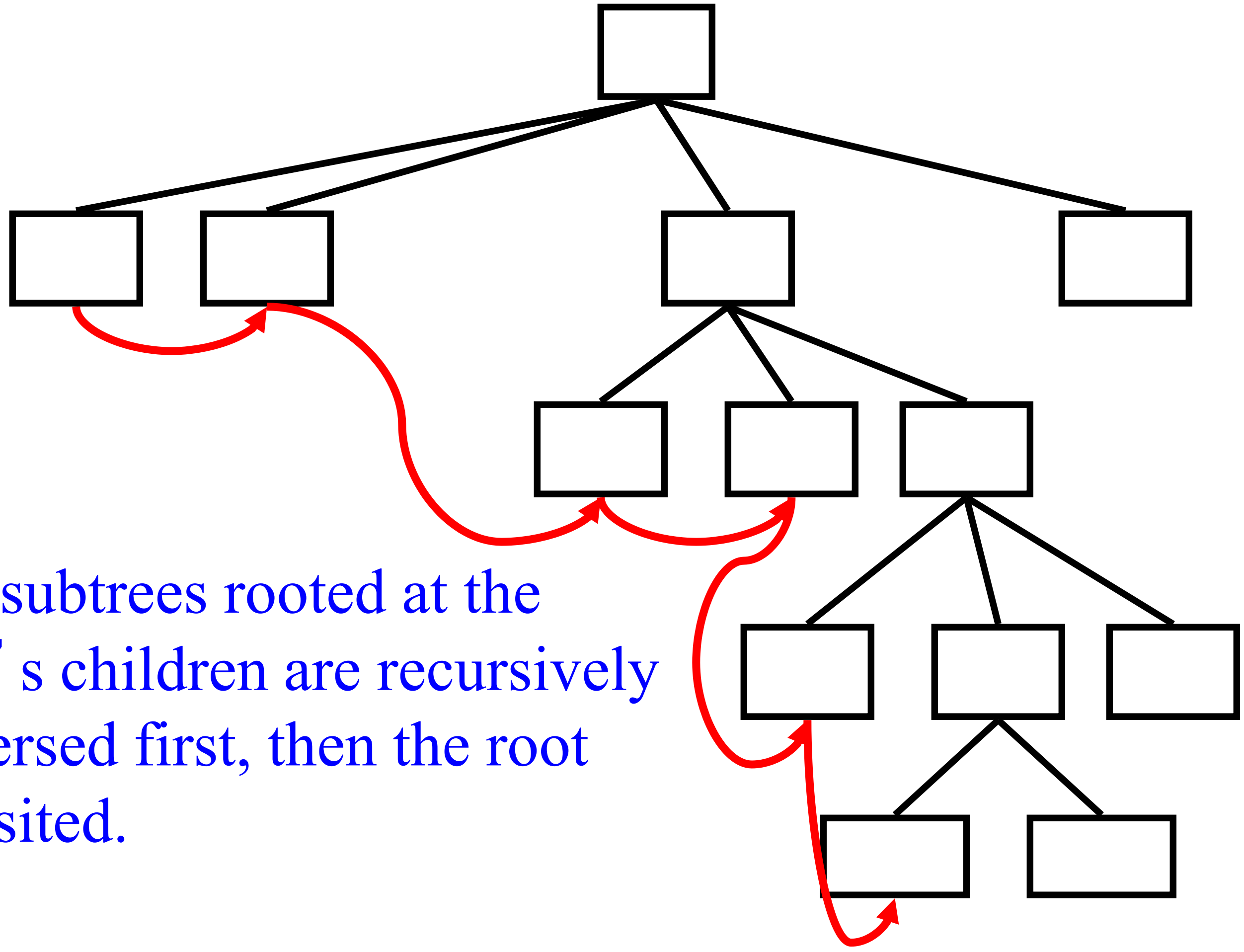

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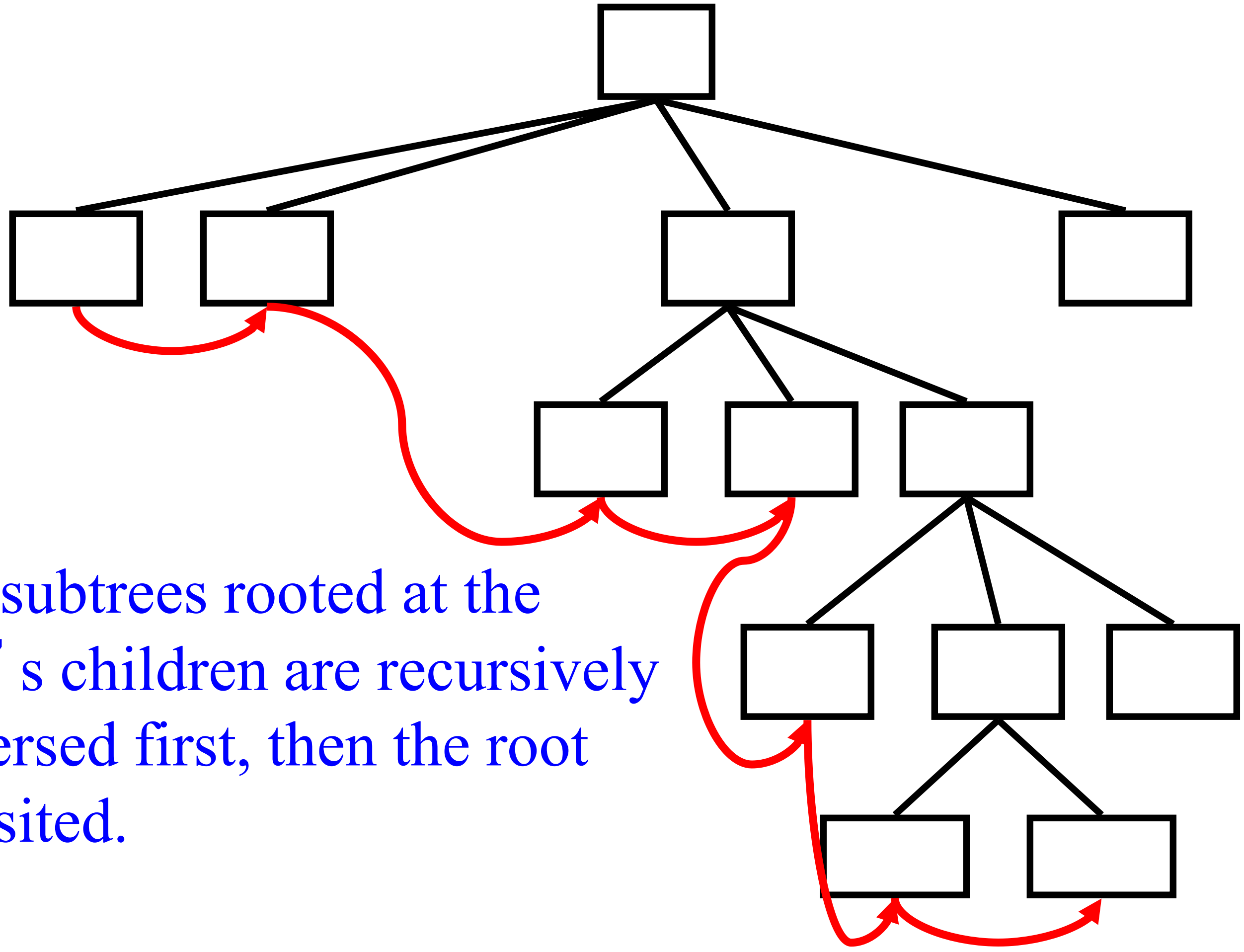

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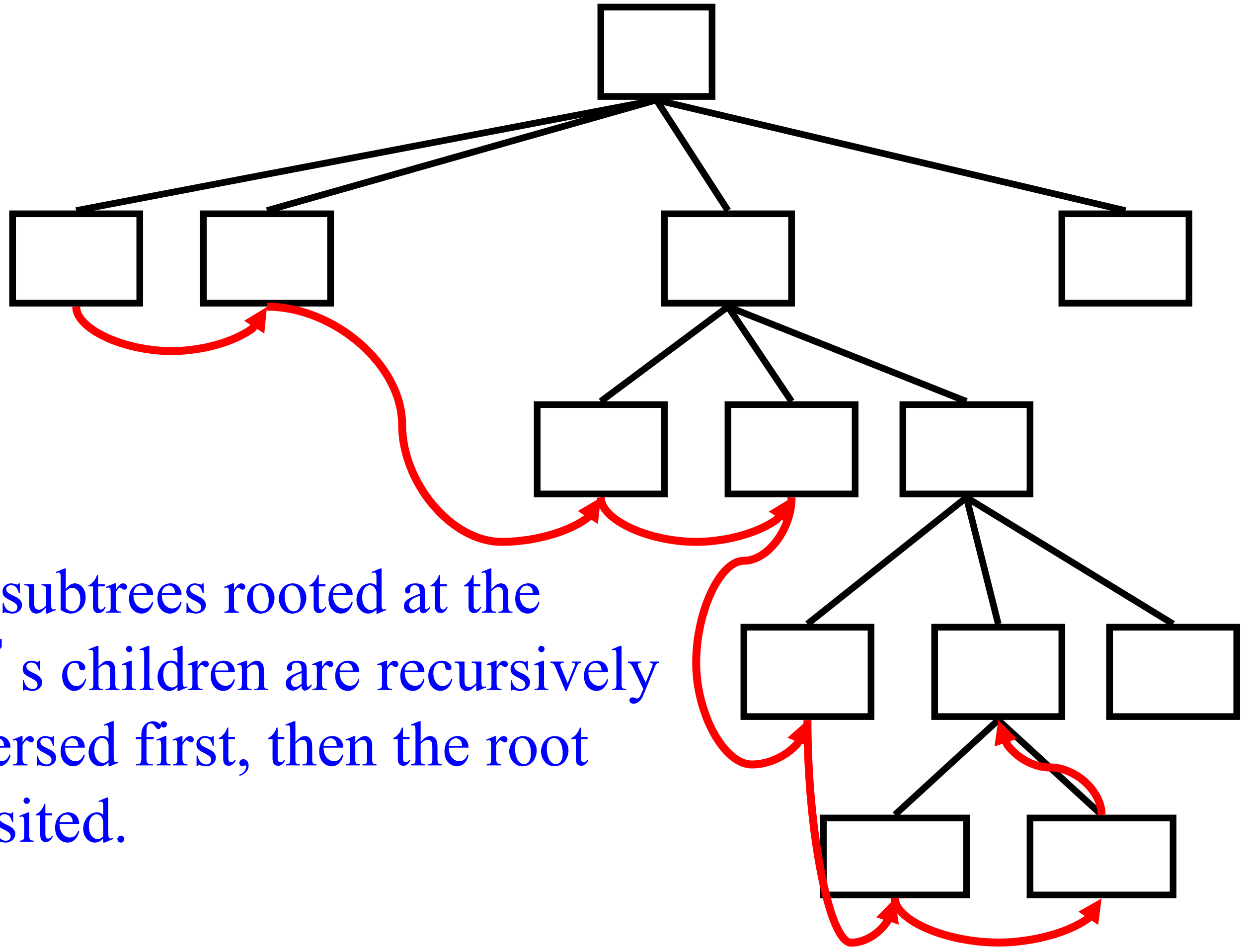

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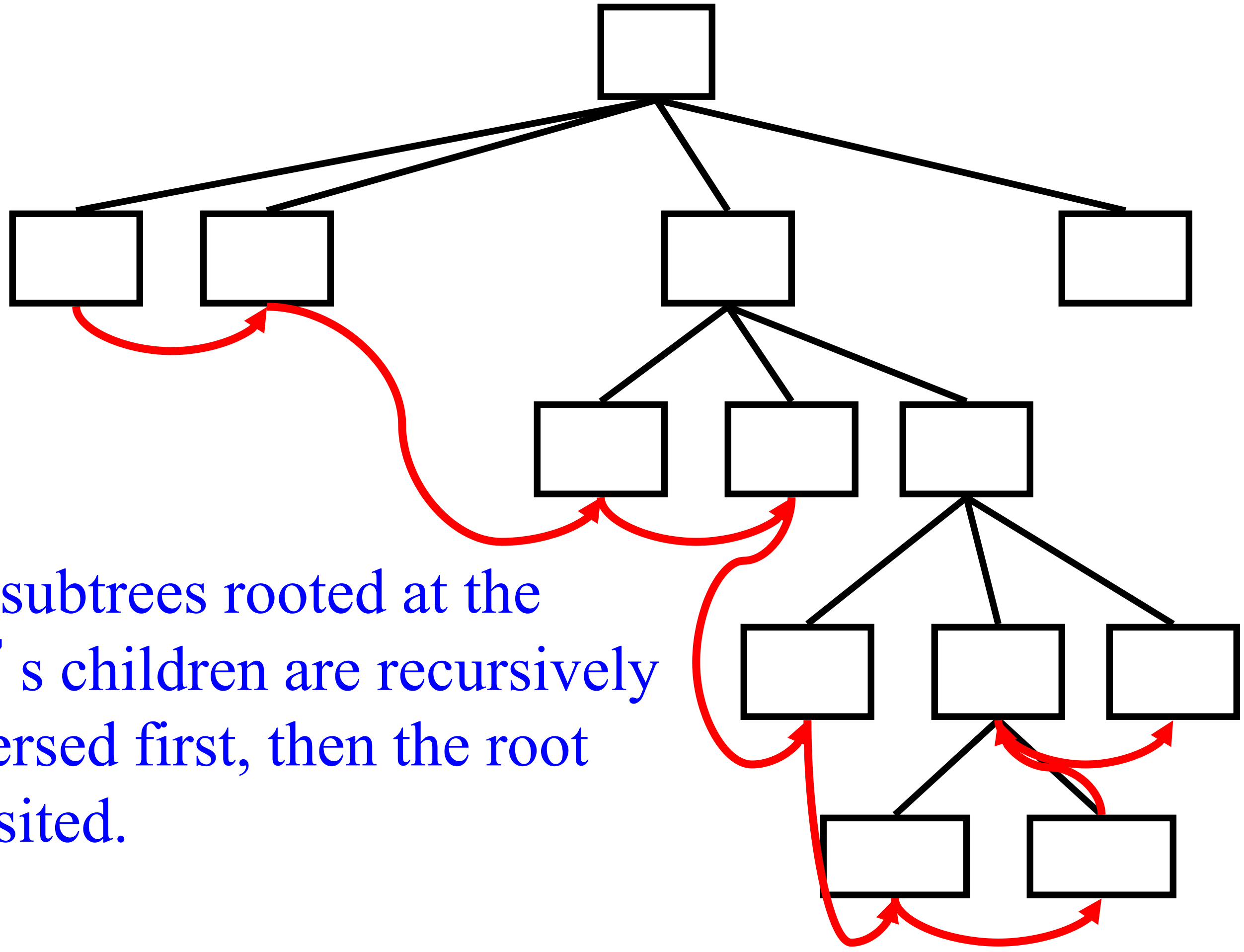

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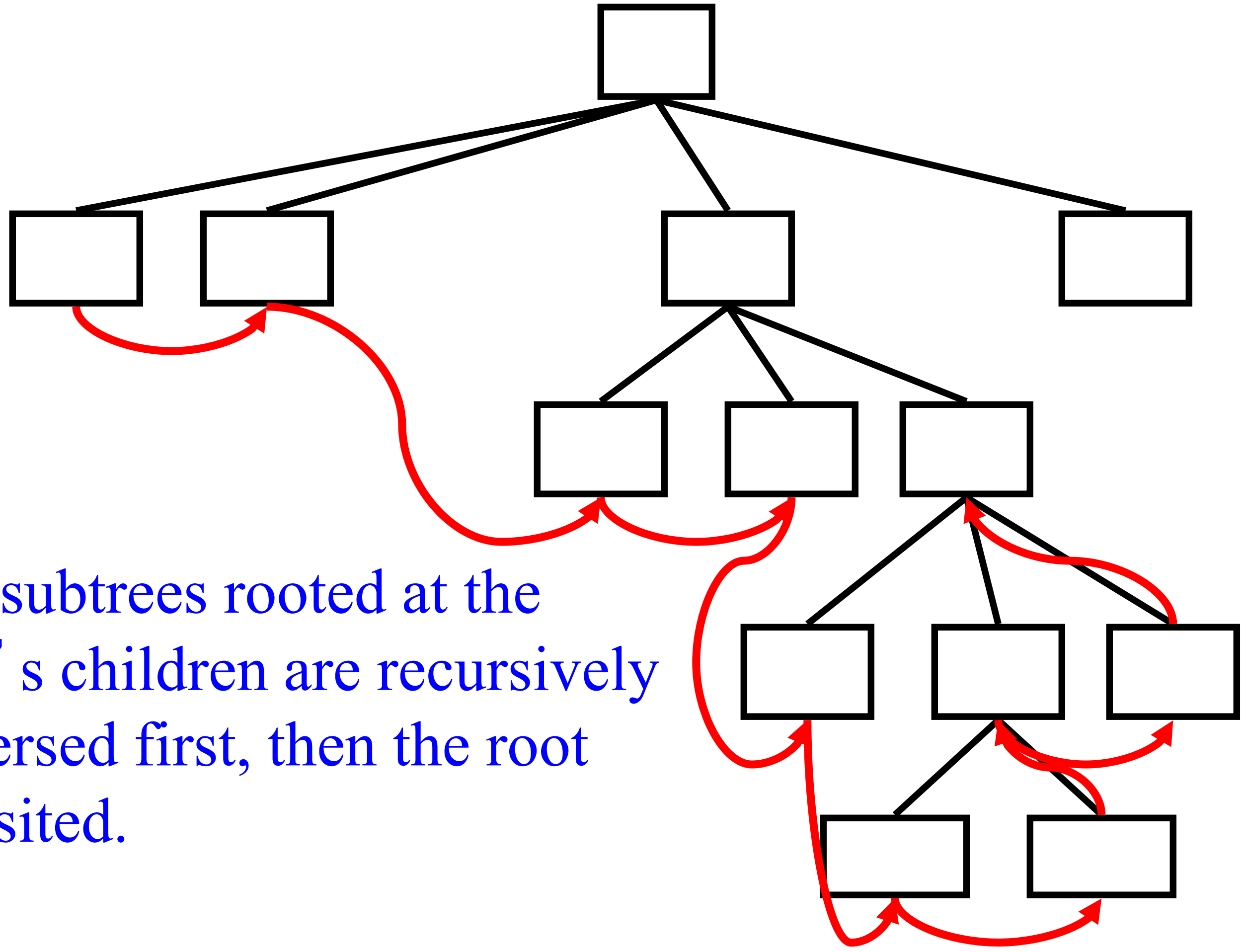

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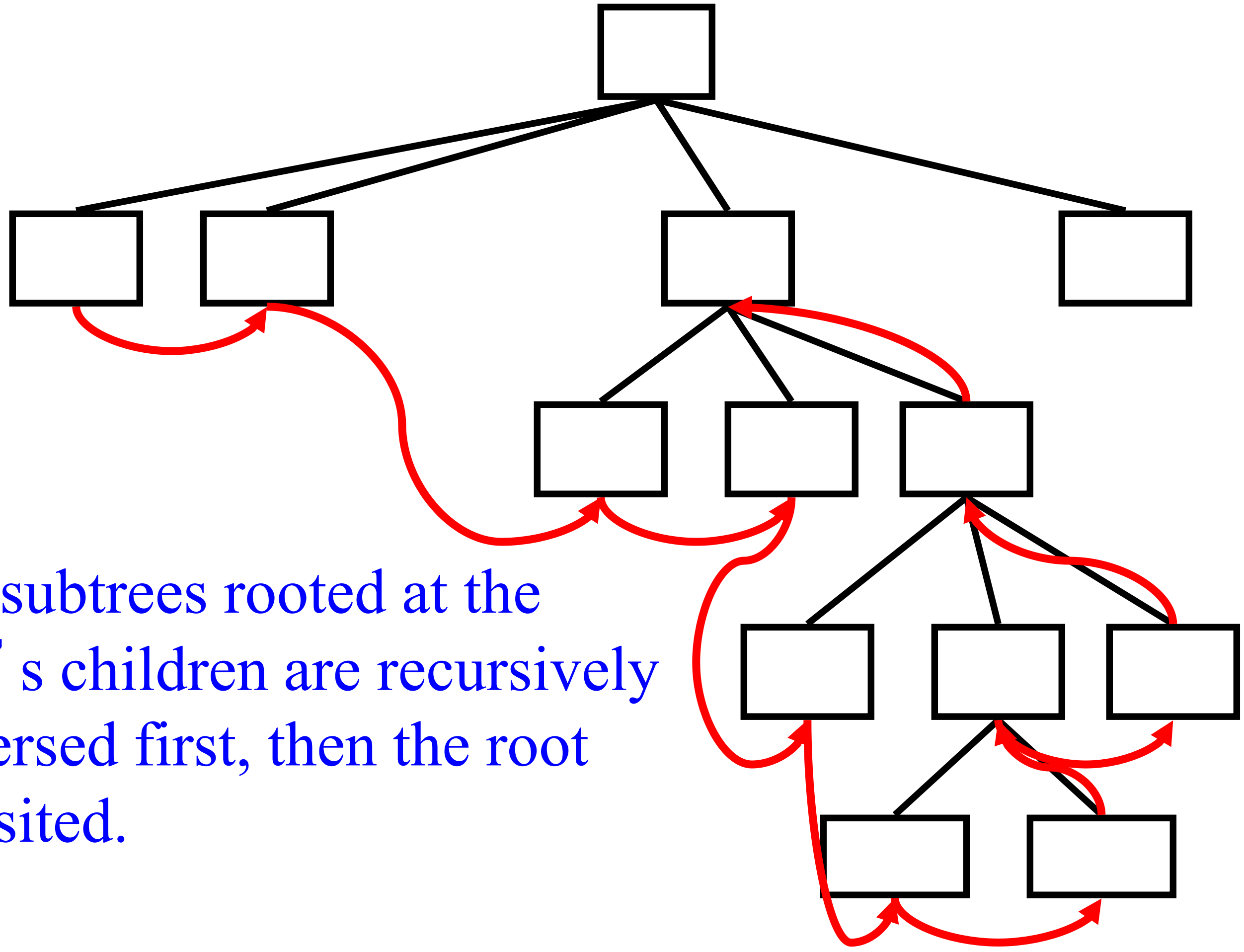

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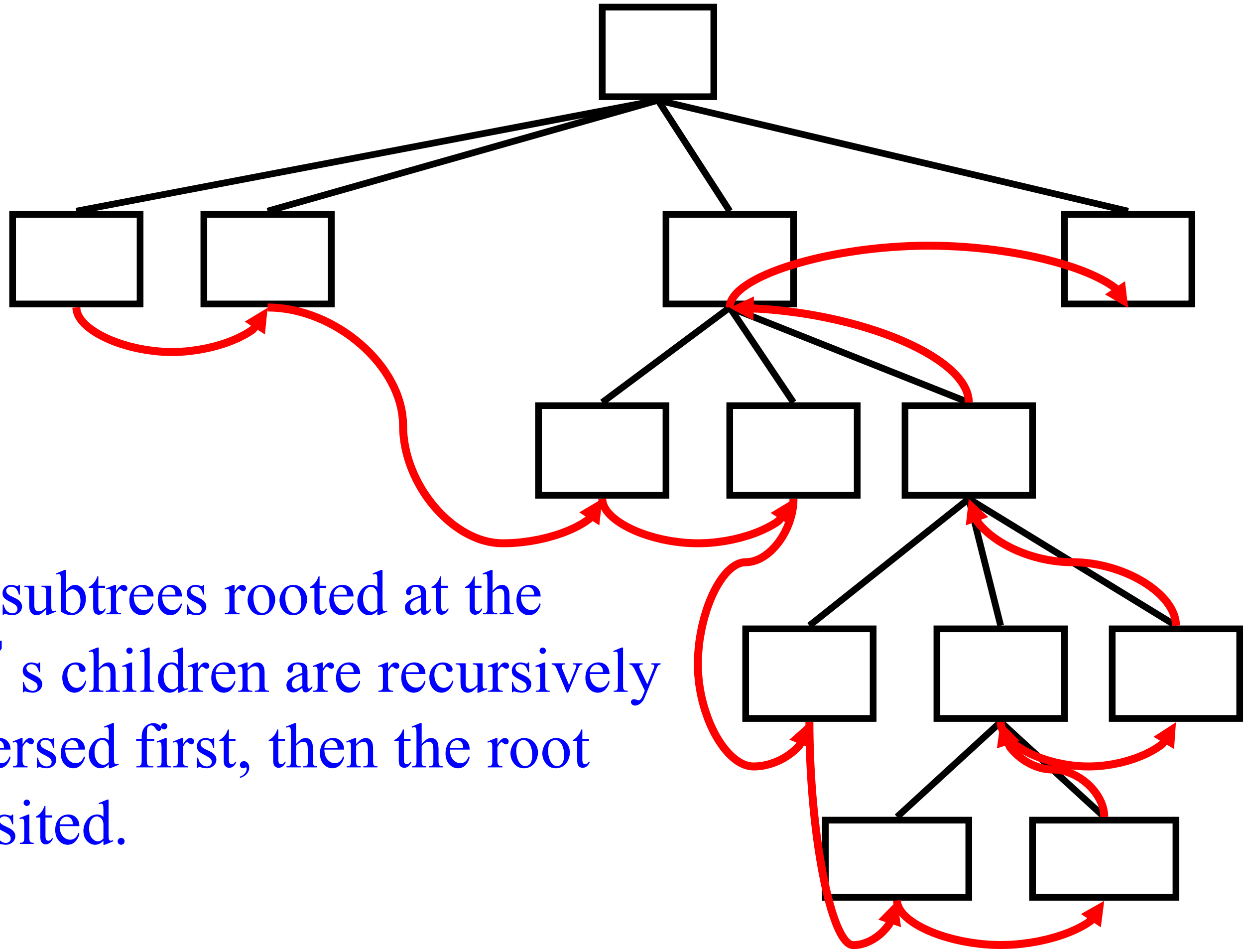

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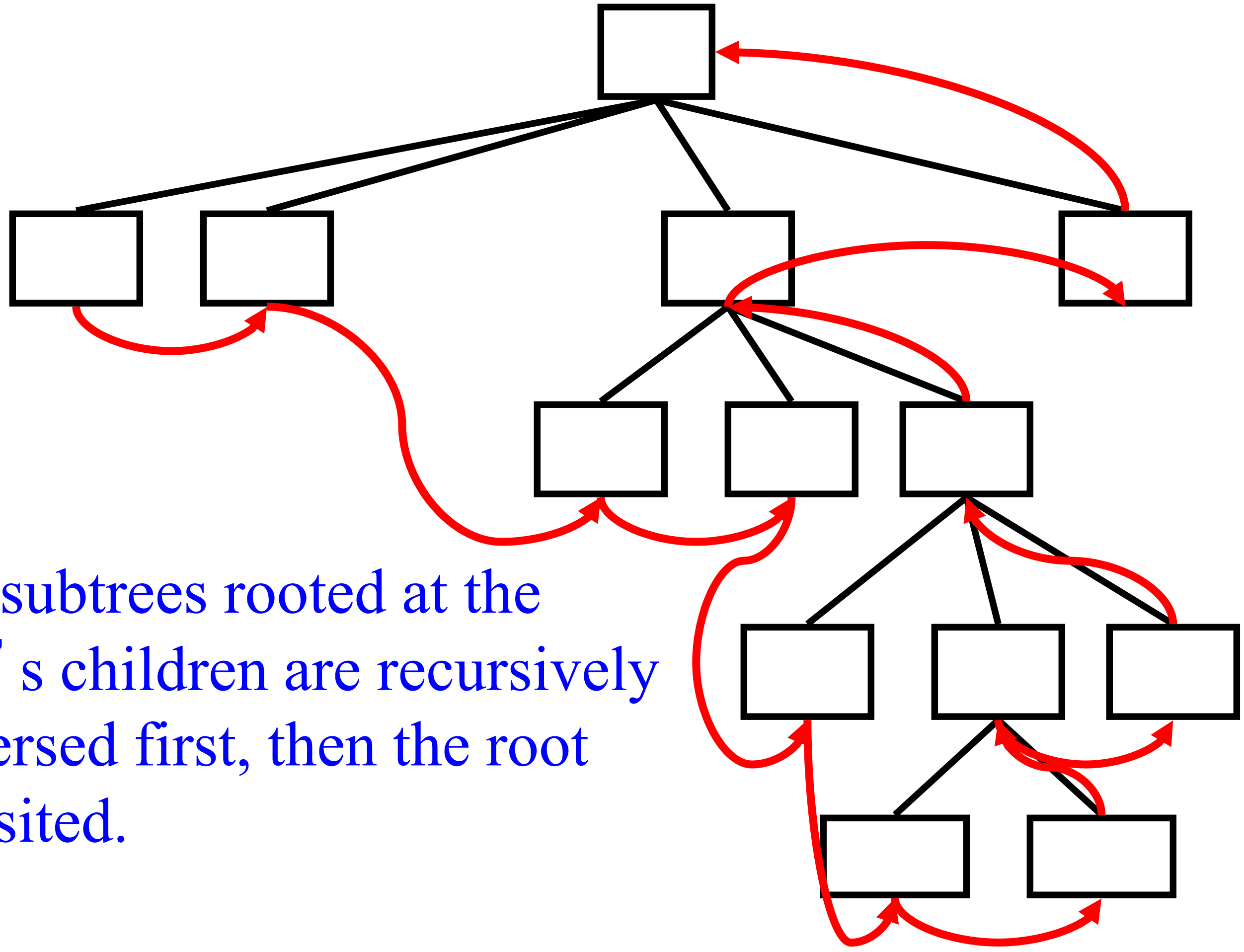

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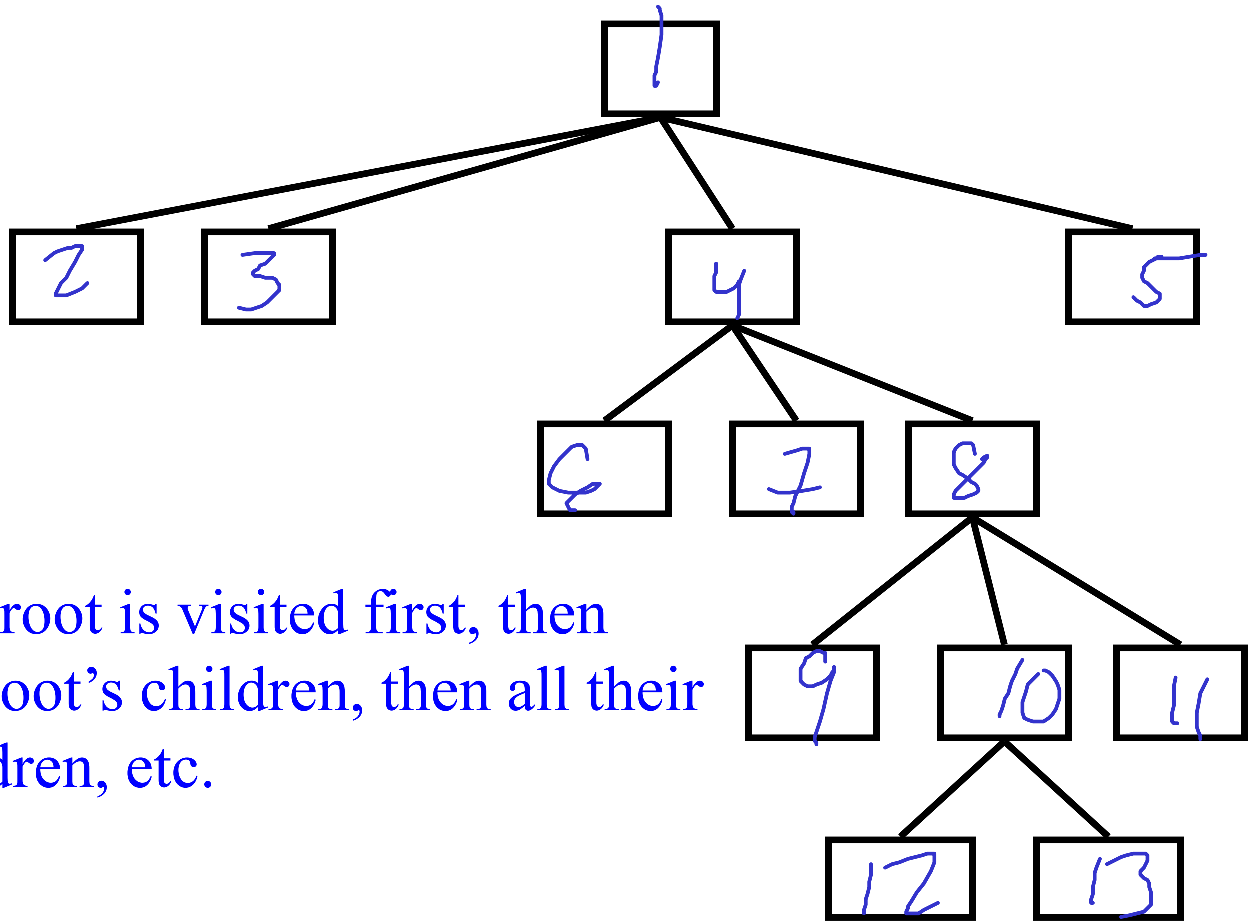
Level-order Traversal
Breadth First Search



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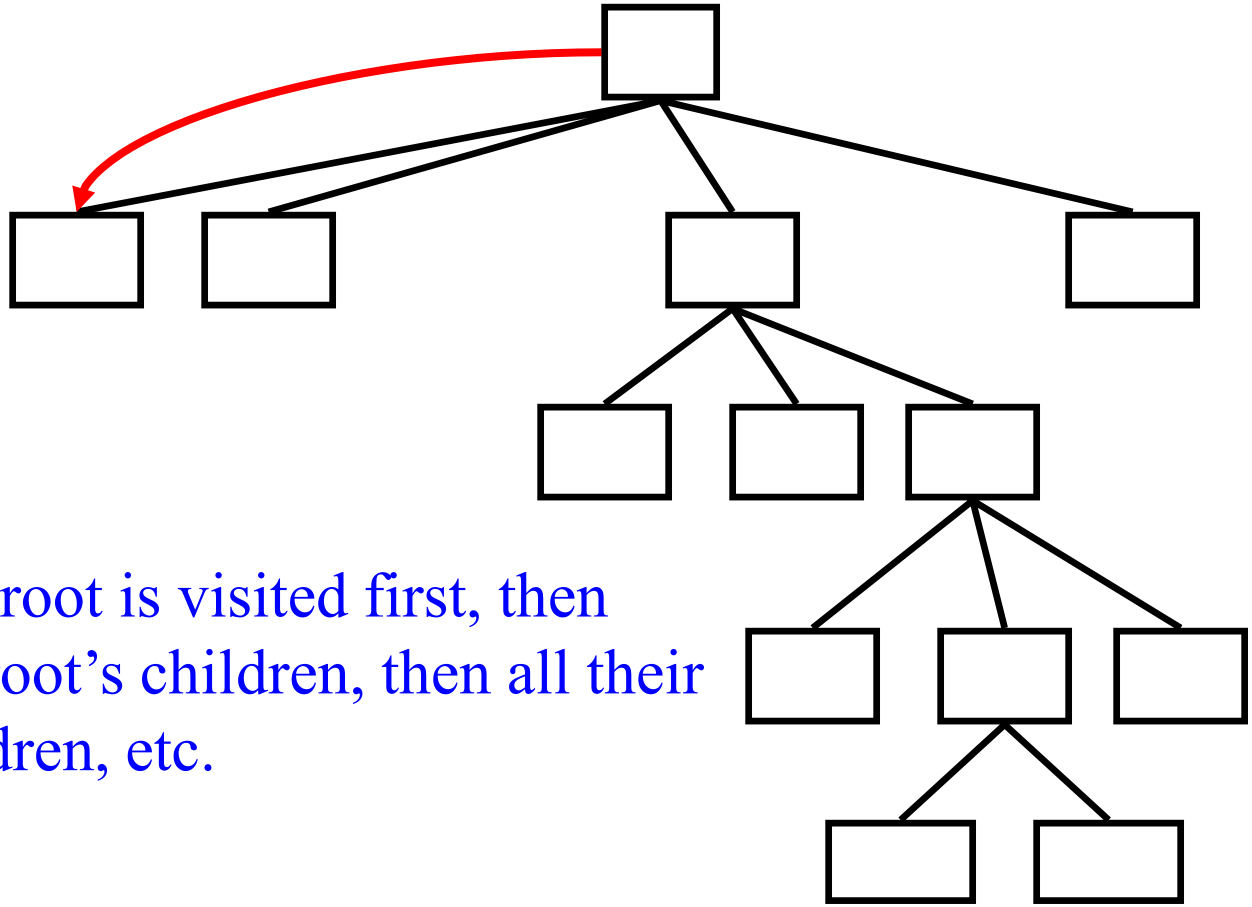

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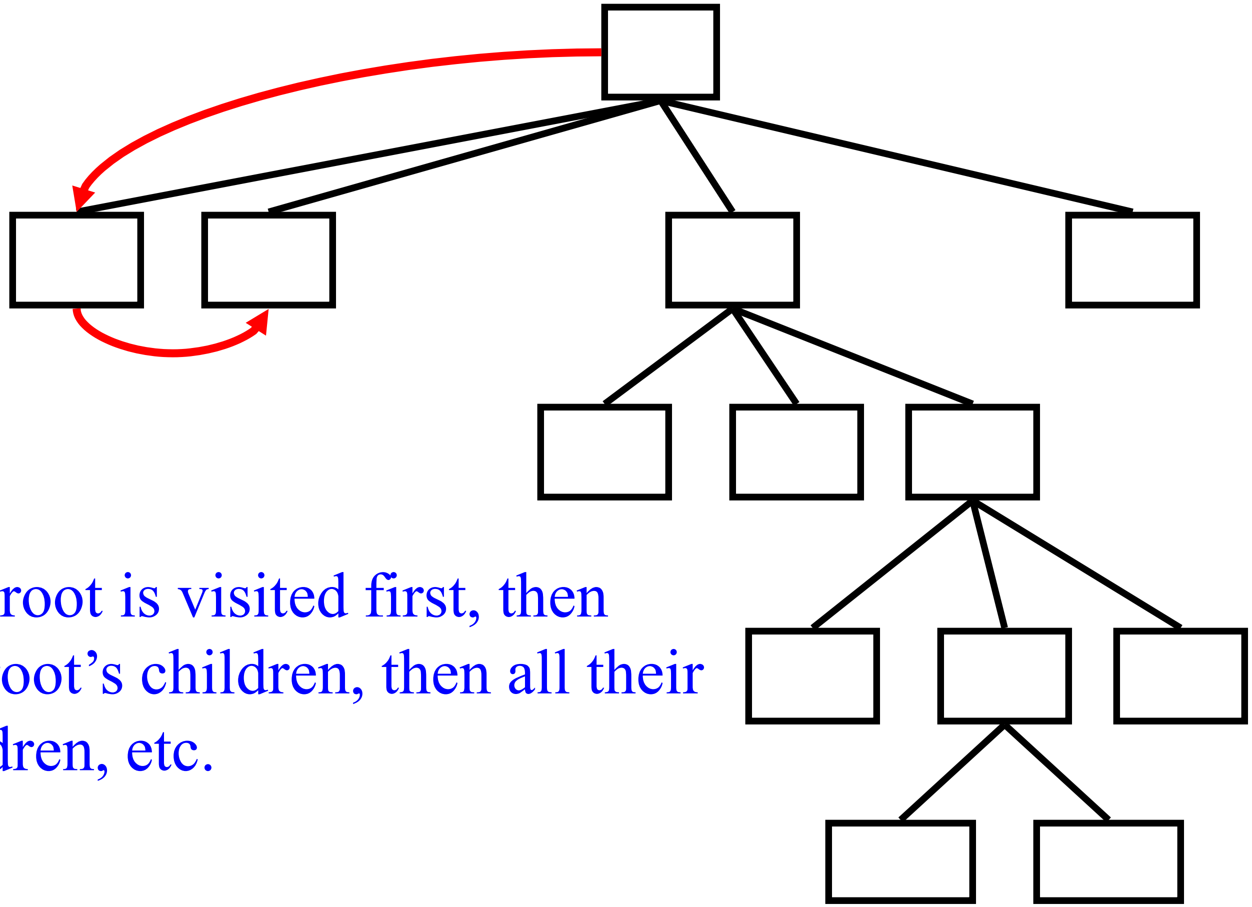

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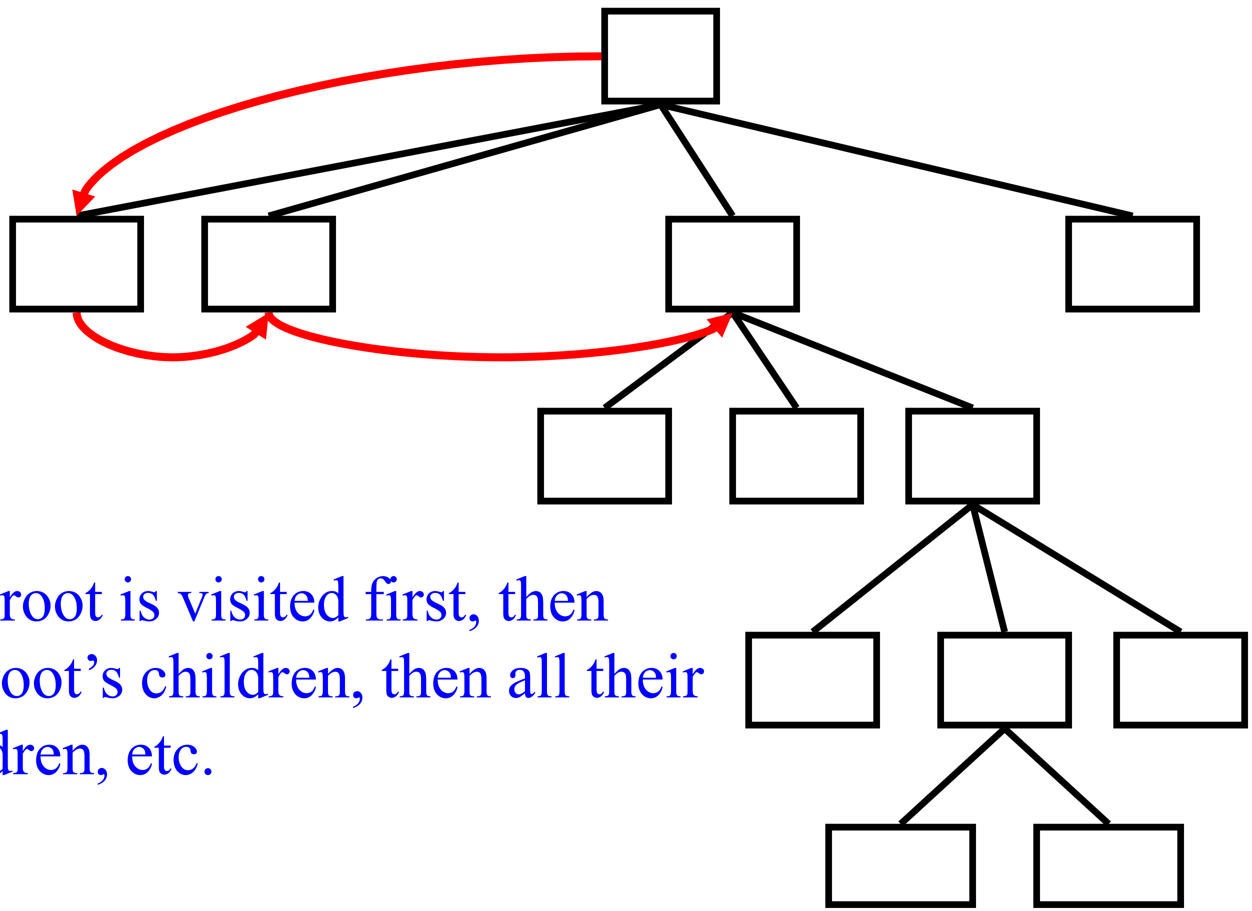

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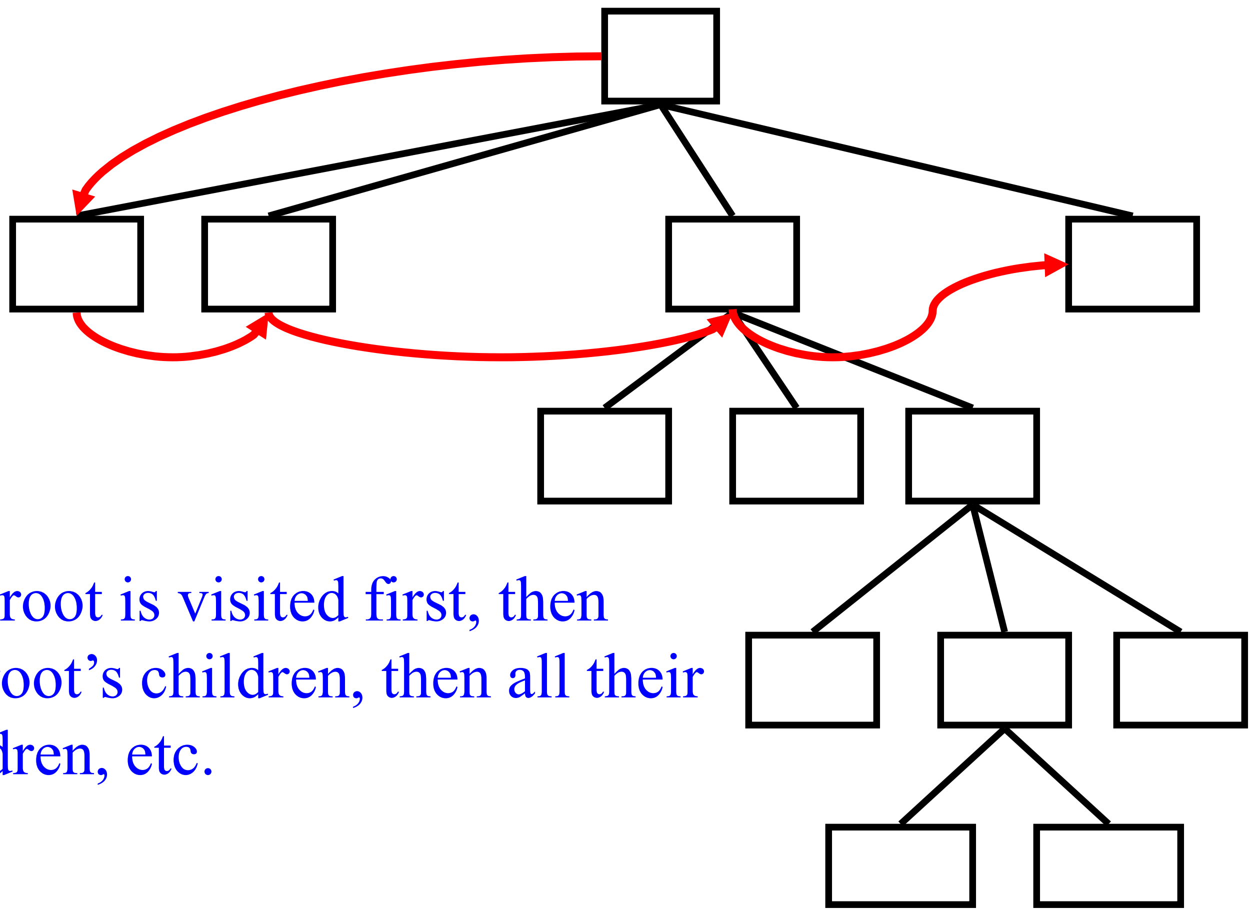

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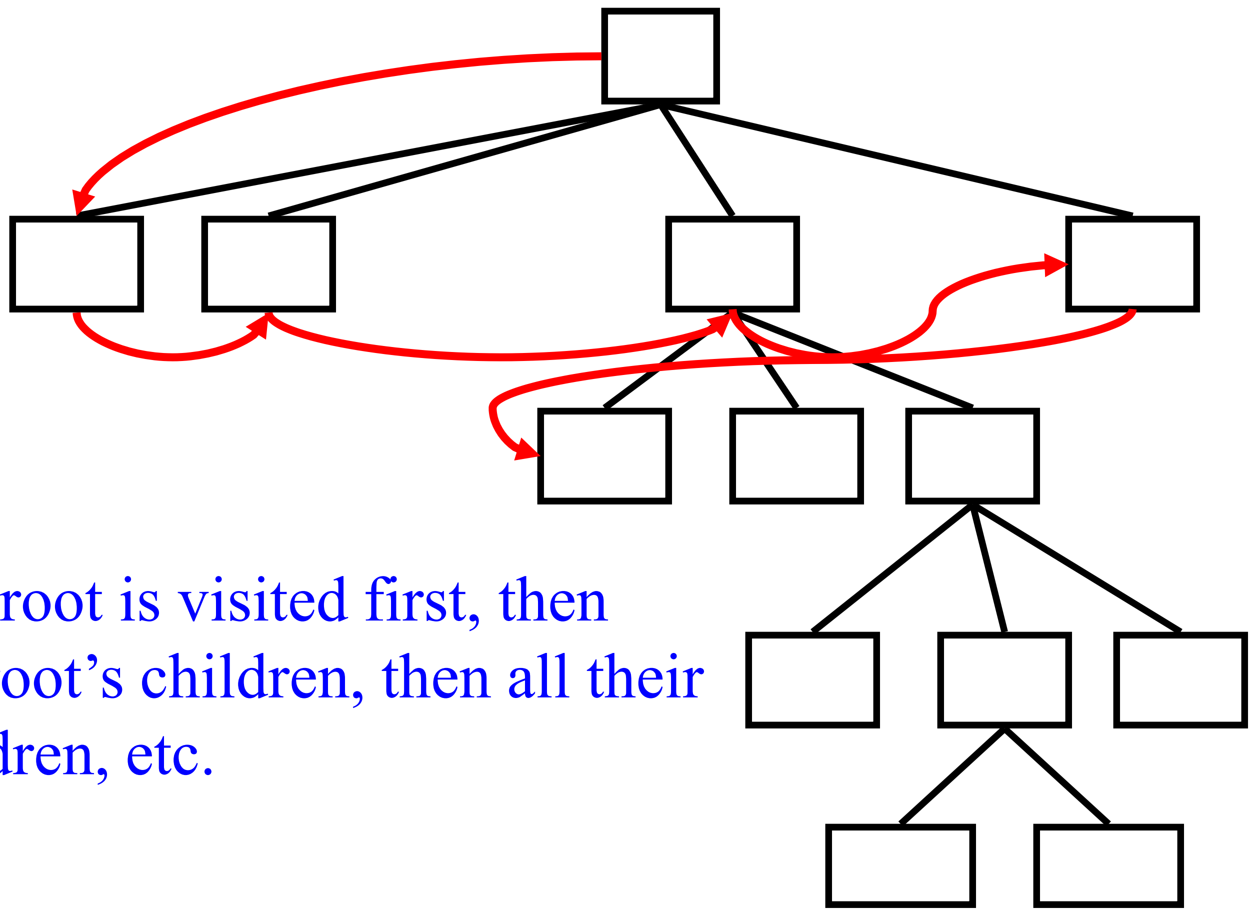

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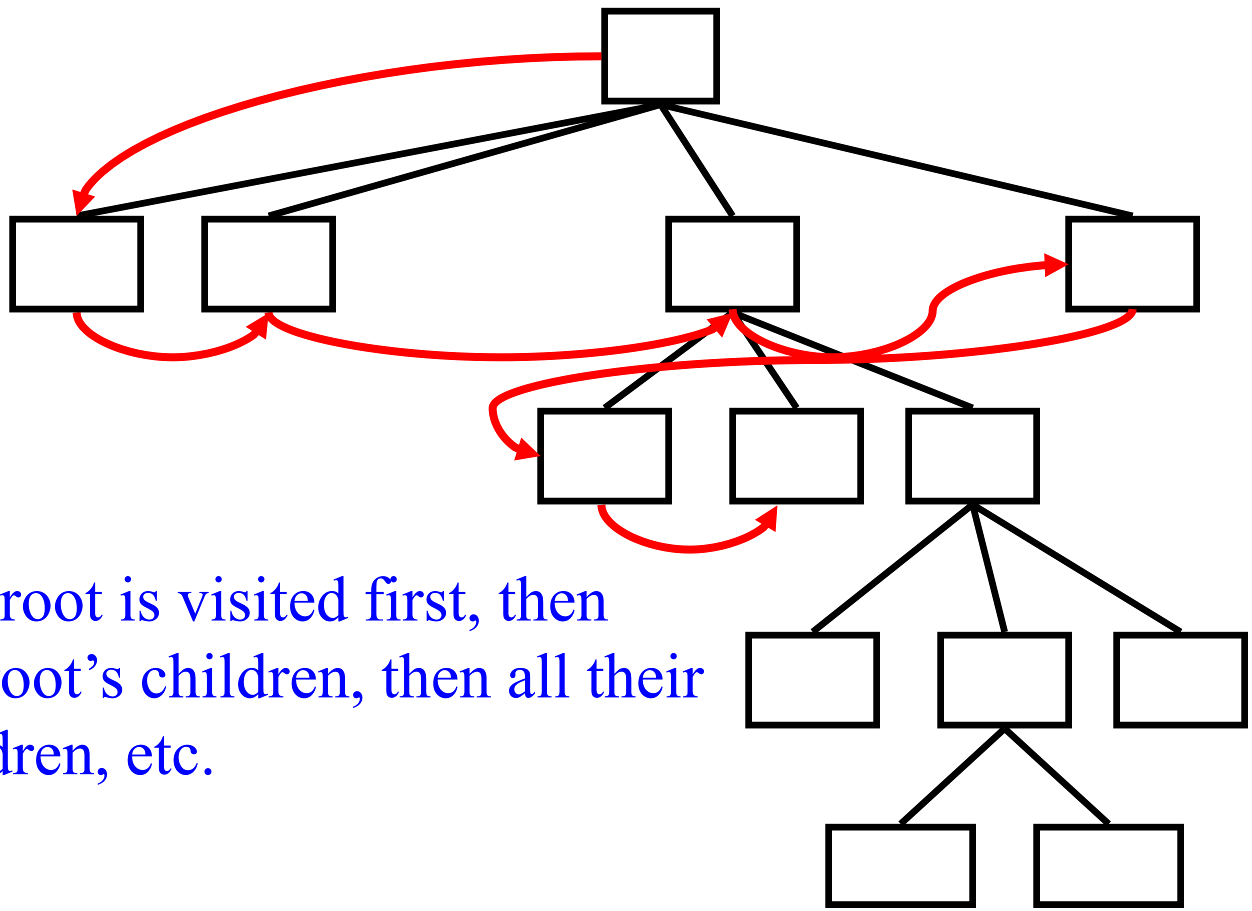

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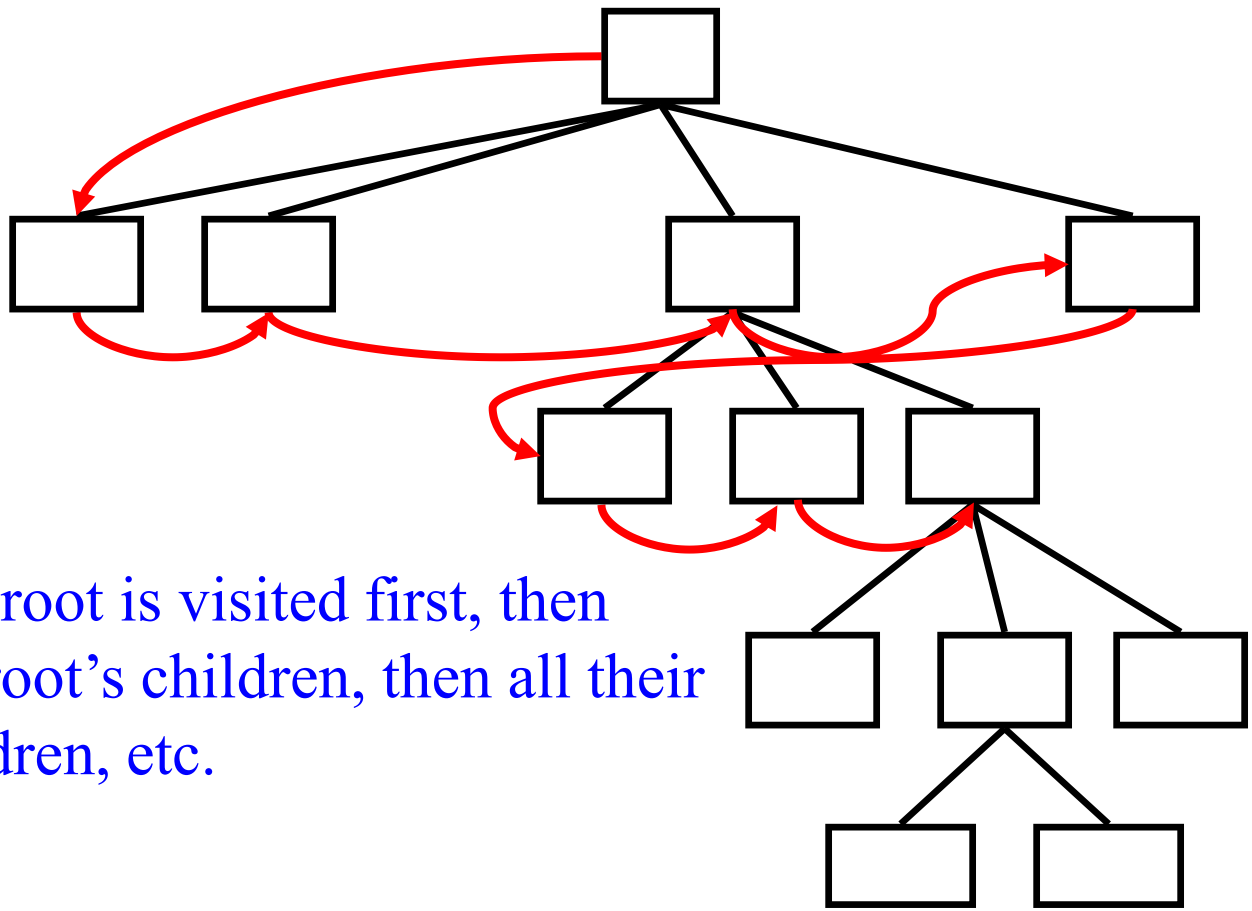

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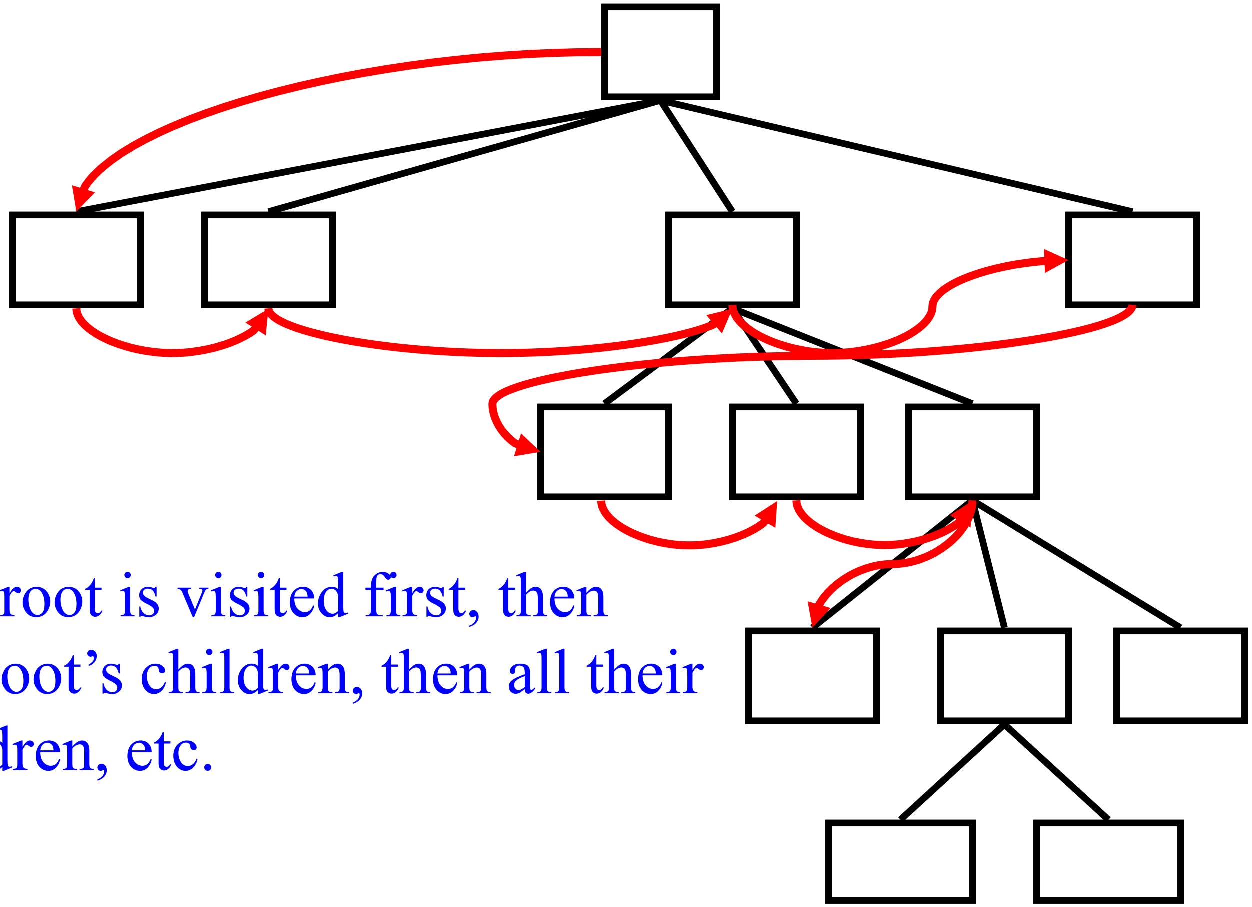

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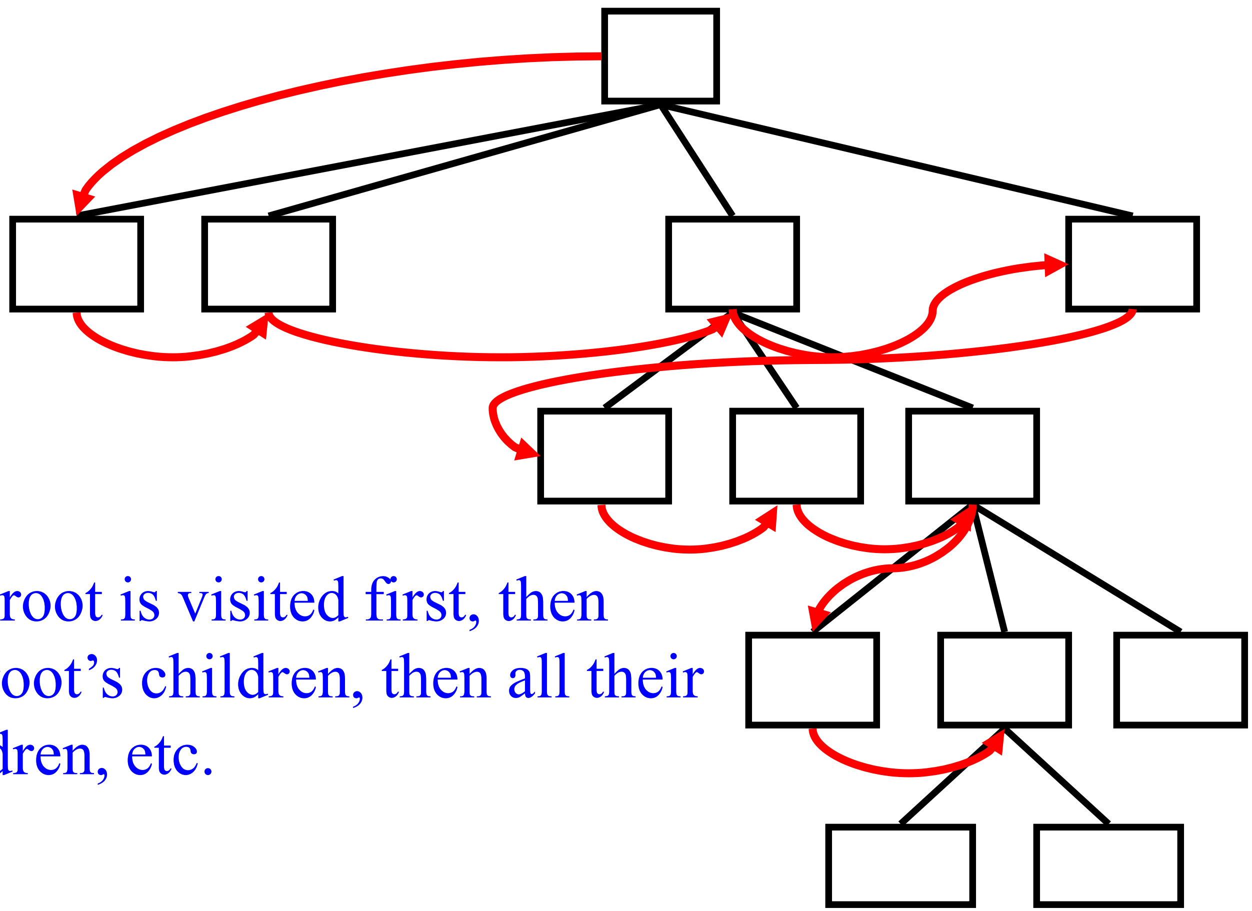

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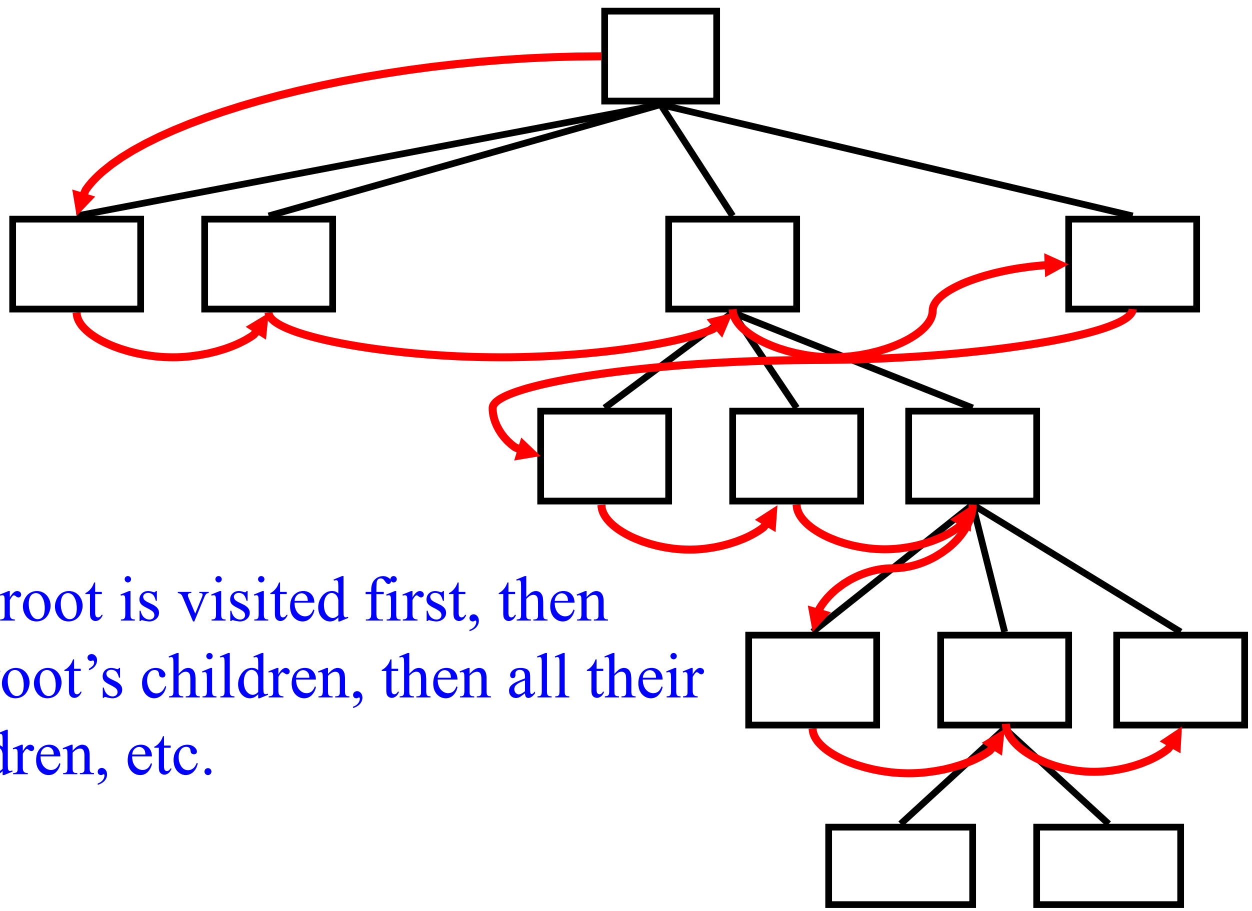

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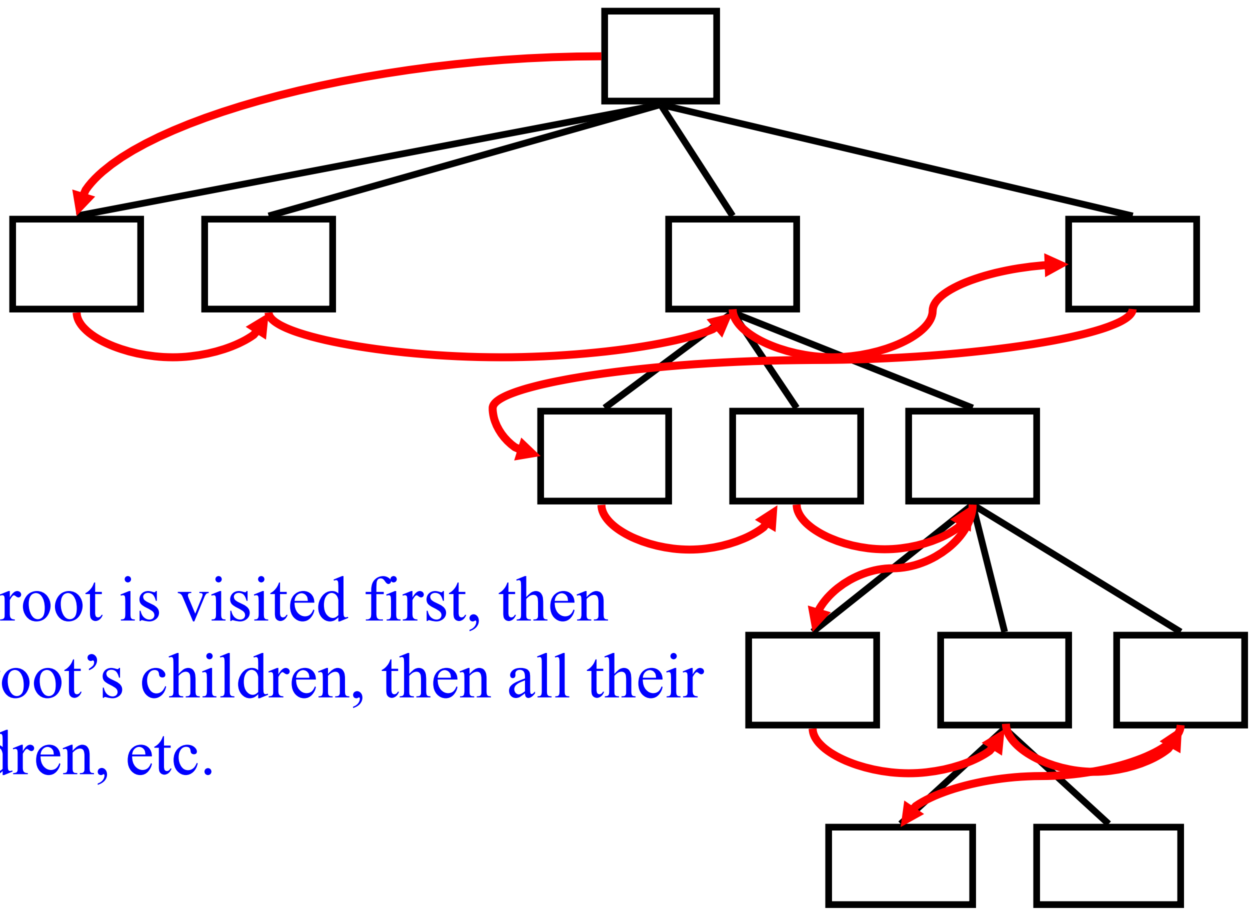

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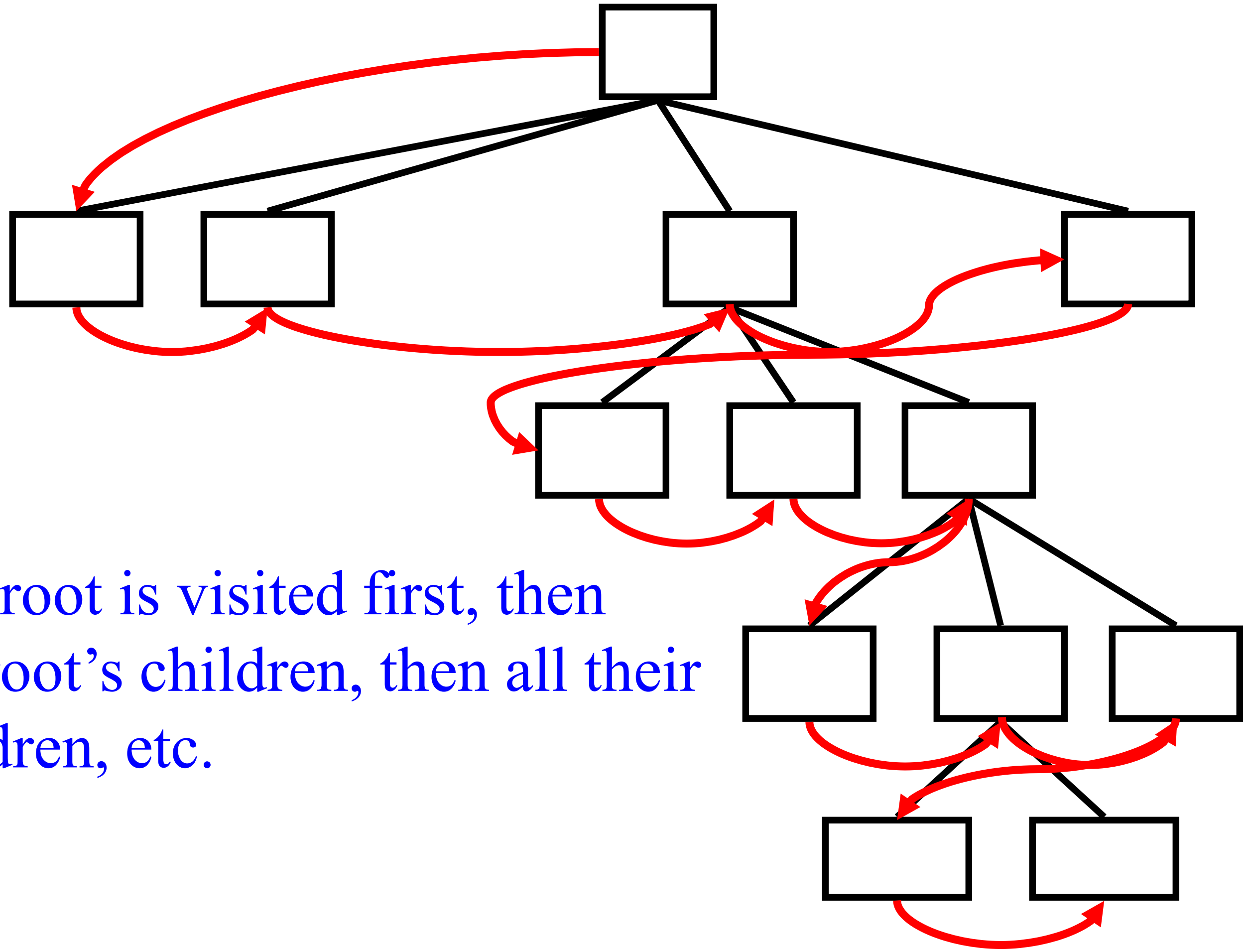

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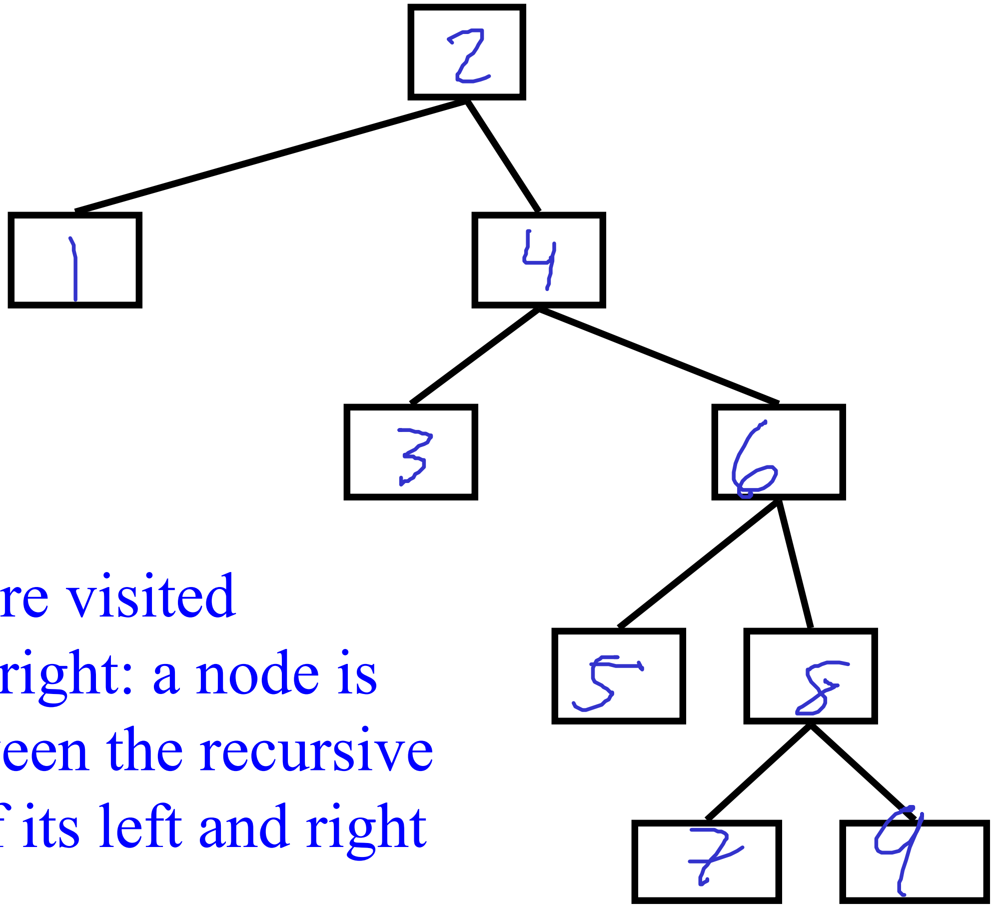
Inorder Traversal for Binary Trees



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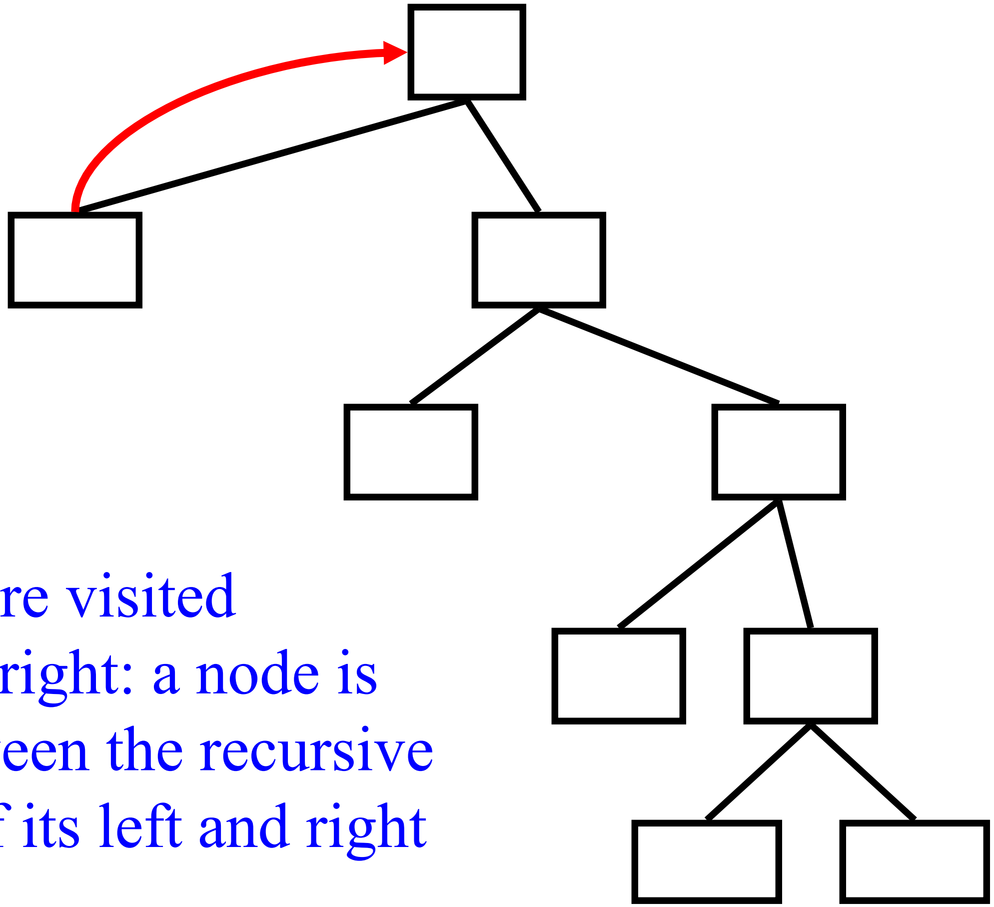

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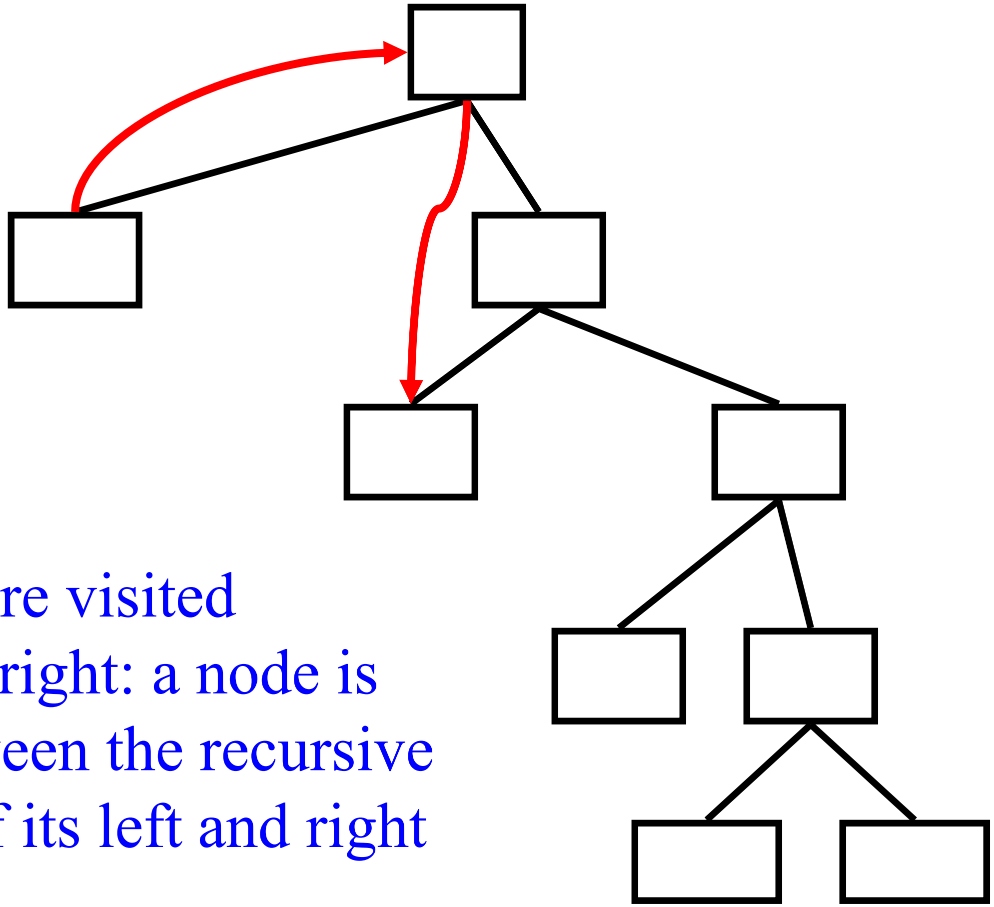

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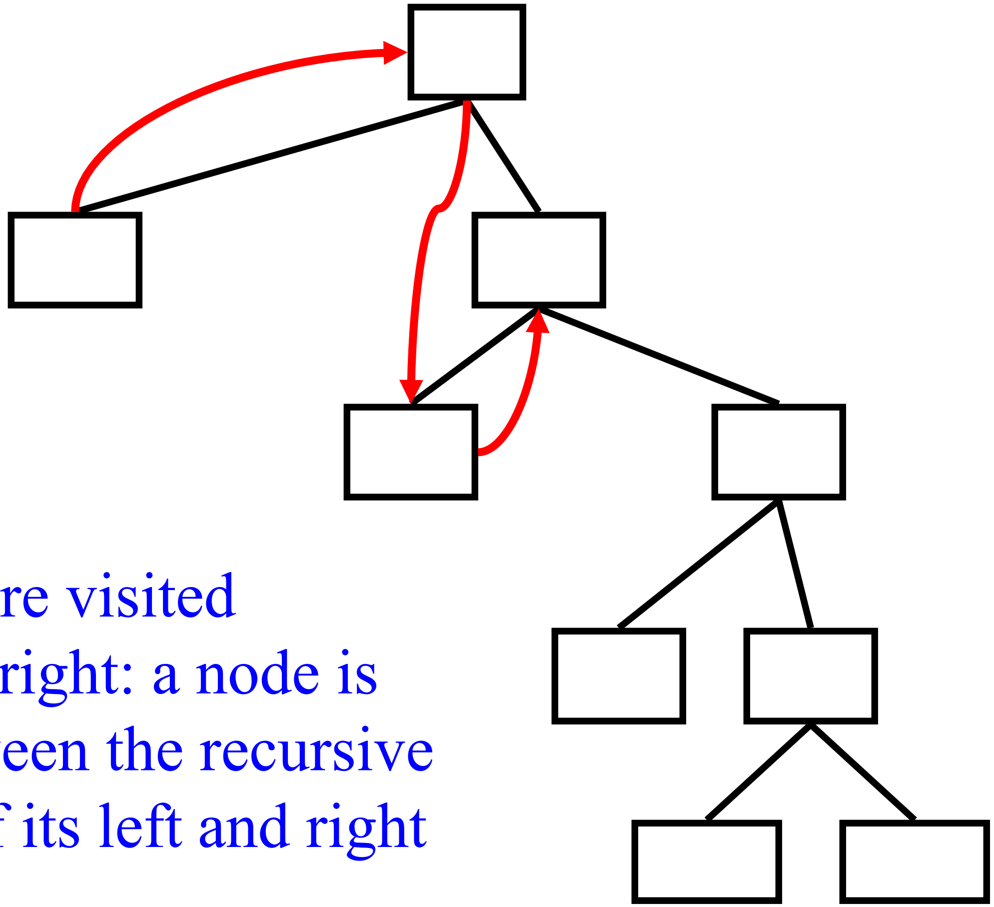

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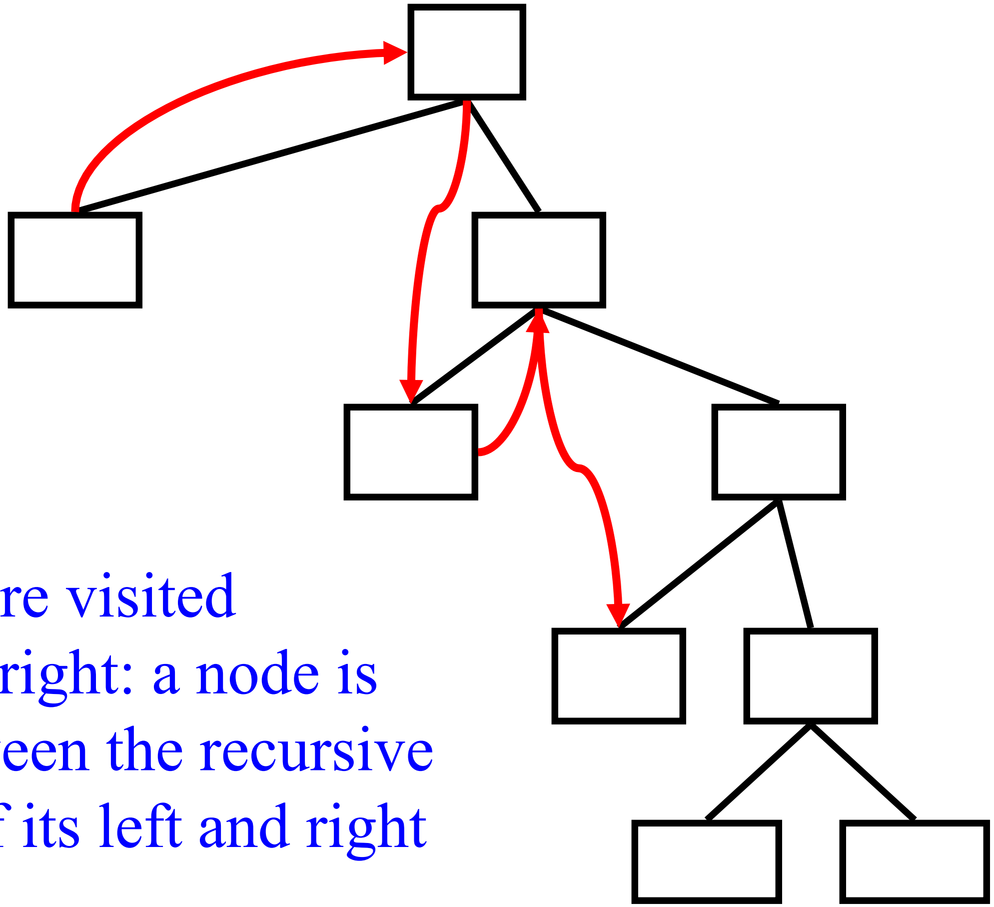

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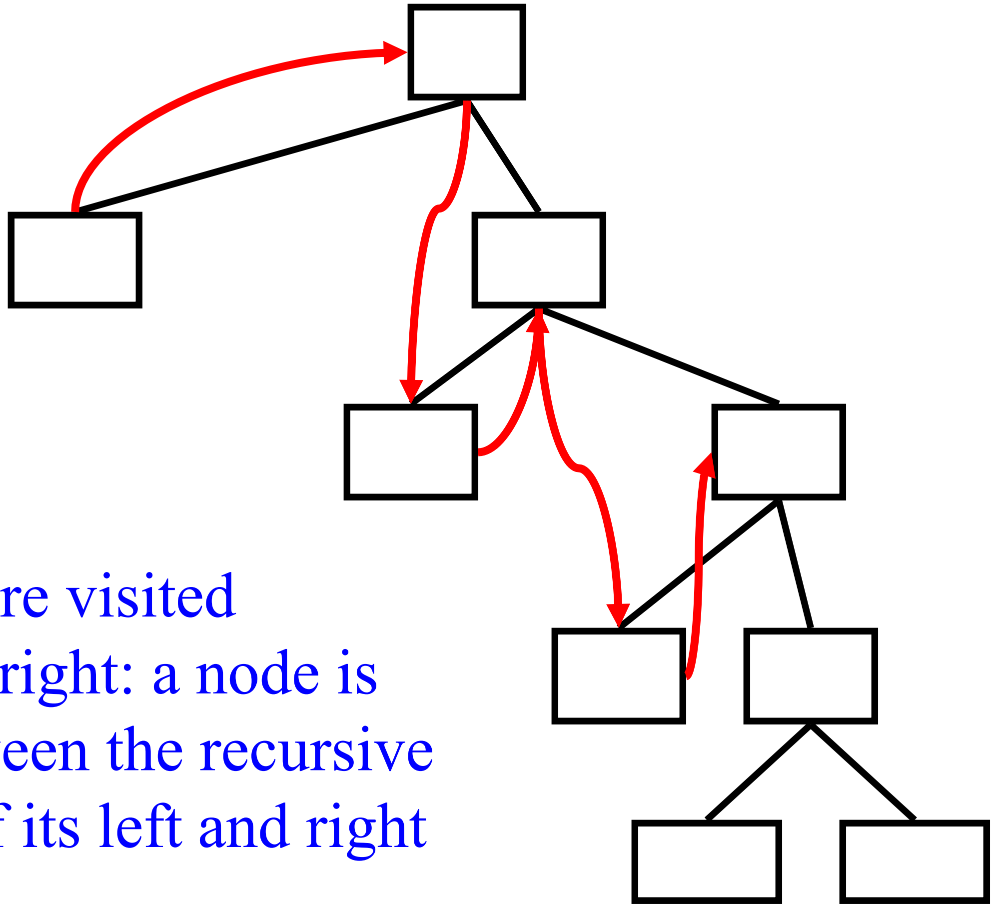

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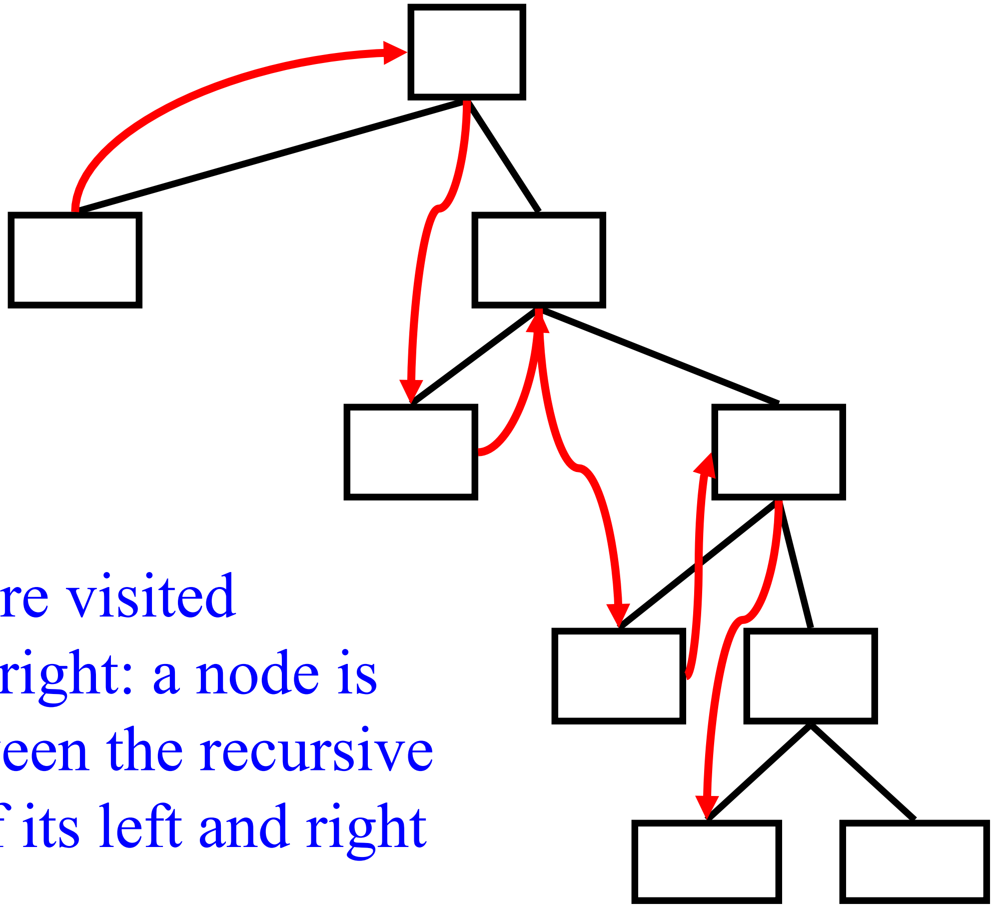

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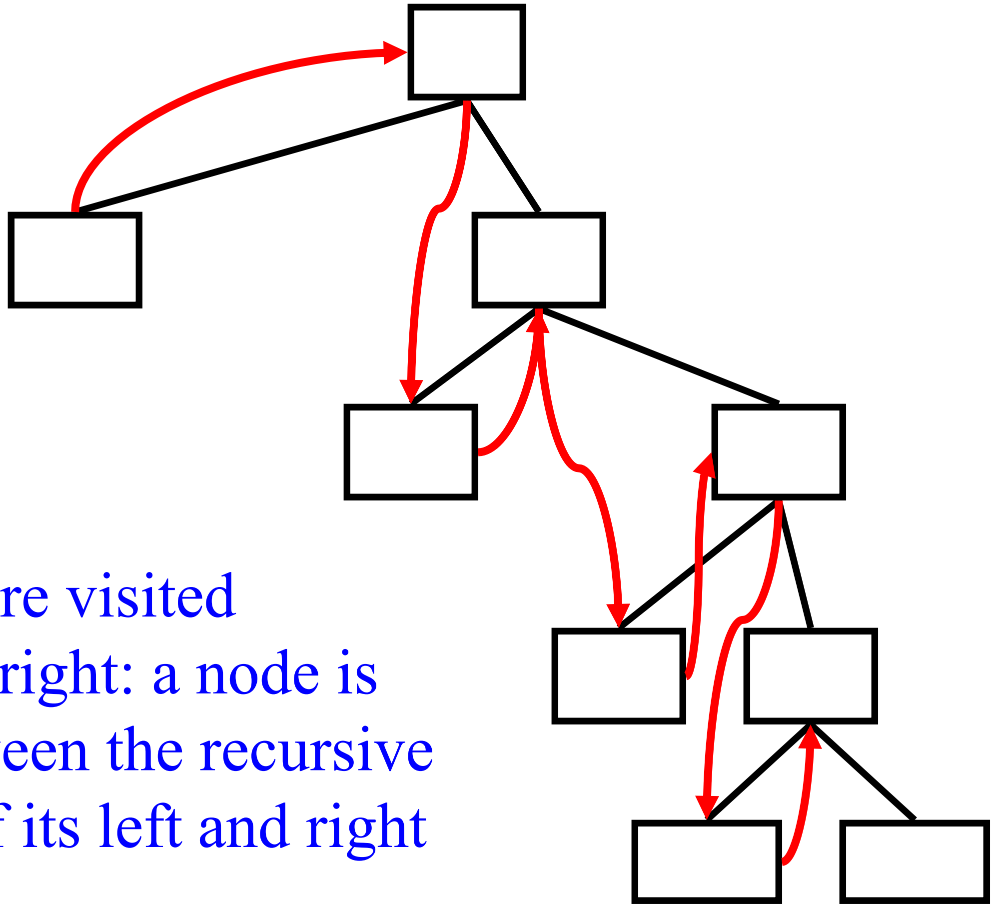

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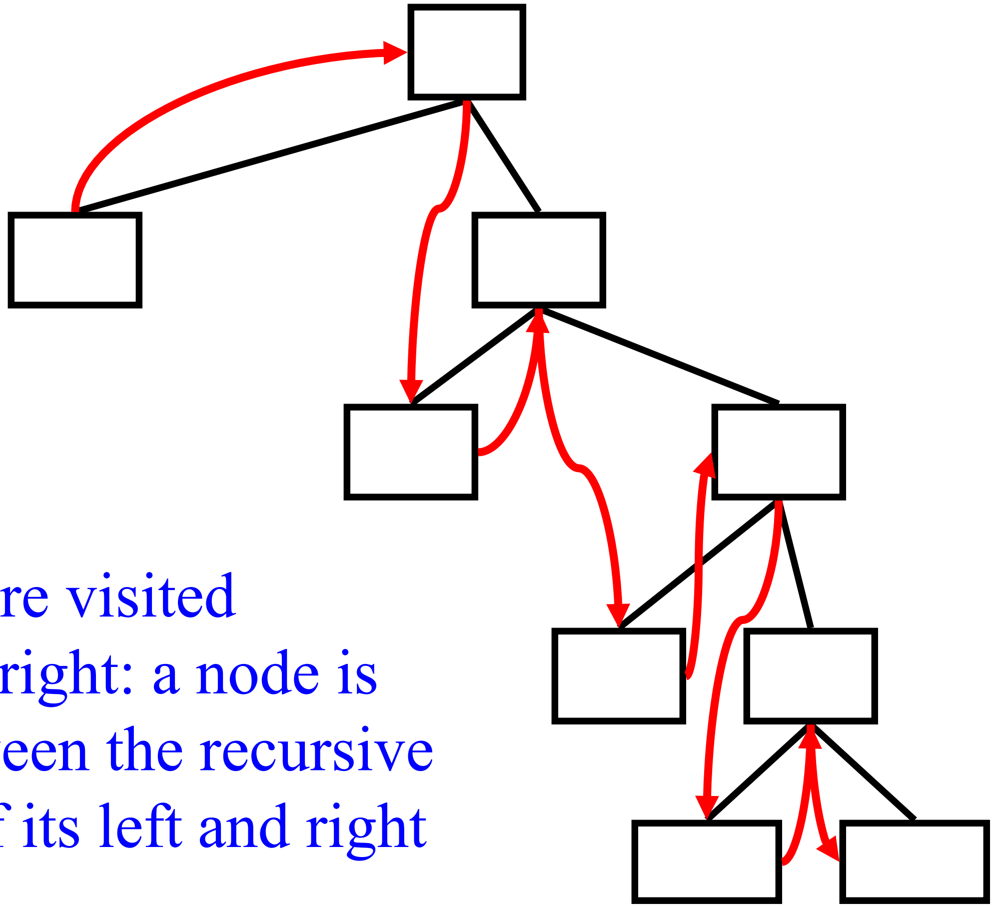

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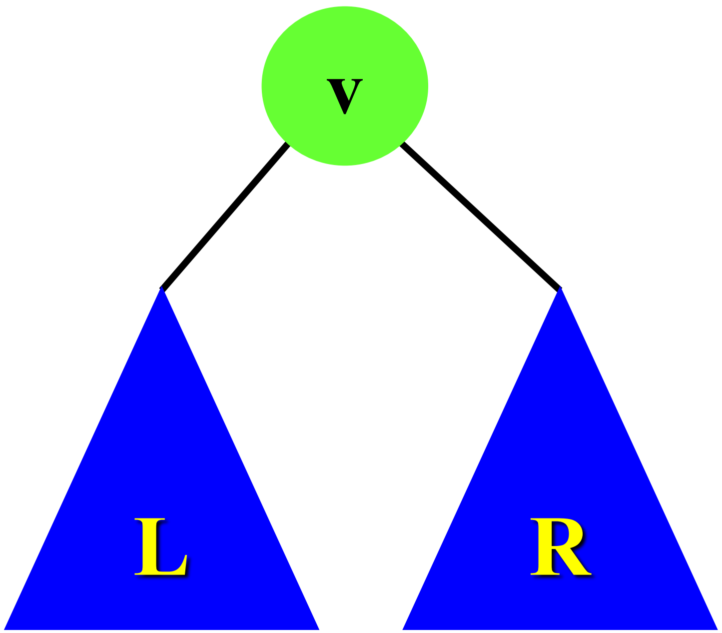
Binary Tree Traversals



- Preorder
  - v, L, R
- Postorder
  - L, R, v
- Inorder
  - L, v, R
- Levelorder
  - v, vL, vR, L, R

Preorder Traversal
Depth First Search (DFS)

Algorithm preorder(Tree T, Node v):
processNode(v)
if T.isInternal(v) **then**
preorder(T, T.leftChild(v))
preorder(T, T.rightChild(v))
end
end

Inorder Traversal

Algorithm inorder(Tree T, Node v):
    if T.isInternal(v) **then**
        inorder(T, T.leftChild(v))
    end
    processNode(v)
    if T.isInternal(v) **then**
        inorder(T, T.rightChild(v))
    end
end

Postorder Traversal

Algorithm postorder(Tree T, Node v):
  if T.isInternal(v) **then**
    postorder(T, T.leftChild(v))
    postorder(T, T.rightChild(v))
  end
  processNode(v)
end

Tree Traversal Numberings

pre = 1;   in = 1;   post = 1

algorithm eulerNumberings(Tree *T*, Node *v*)

  if *v* != null **then**

    v.pre = pre;   pre = pre + 1

    eulerNumberings(*T*,*T*.leftChild(*v*))

    v.in = in;   in = in + 1

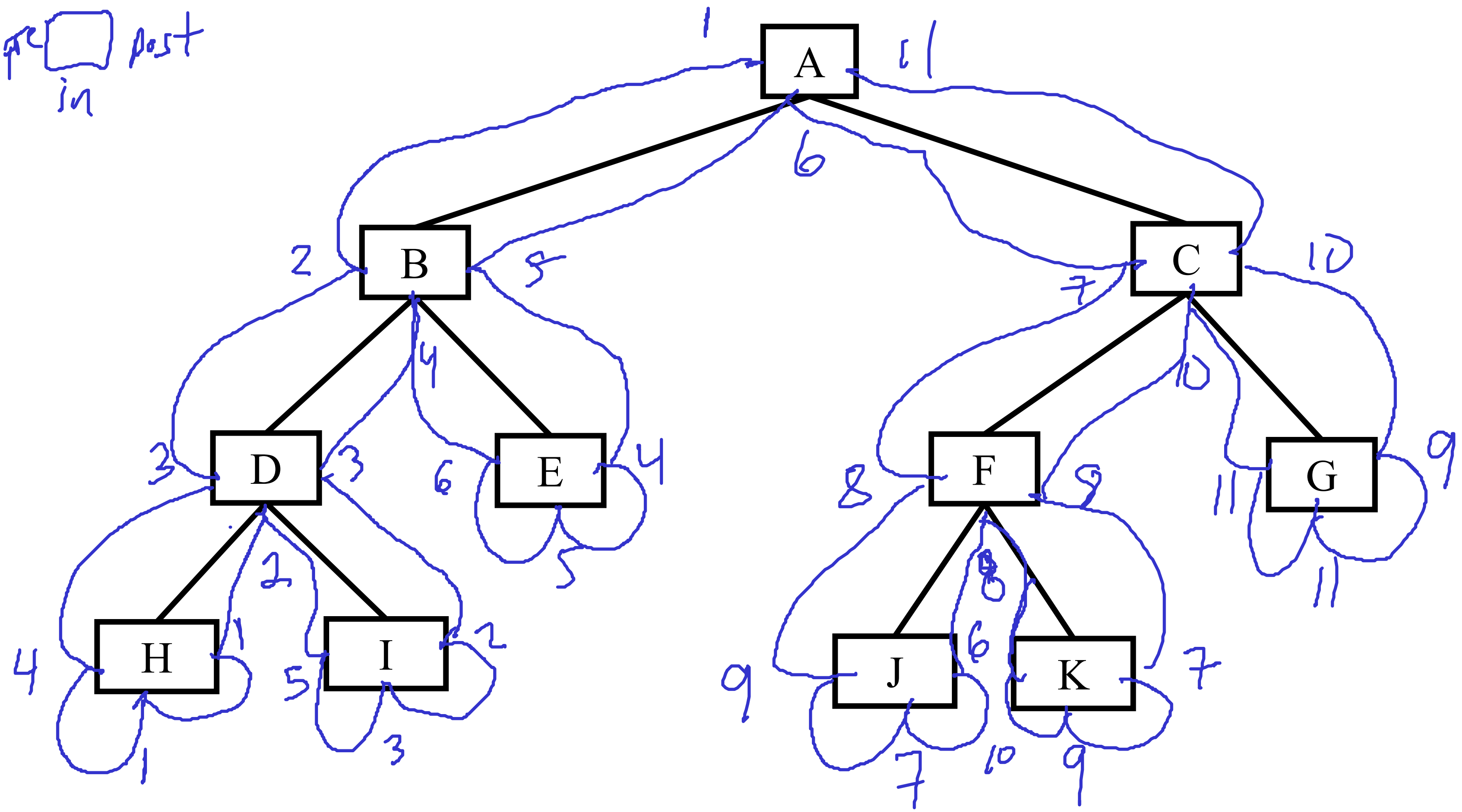
    eulerNumberings(*T*,*T*.rightChild(*v*))

    v.post = post;   post = post + 1

  end

end

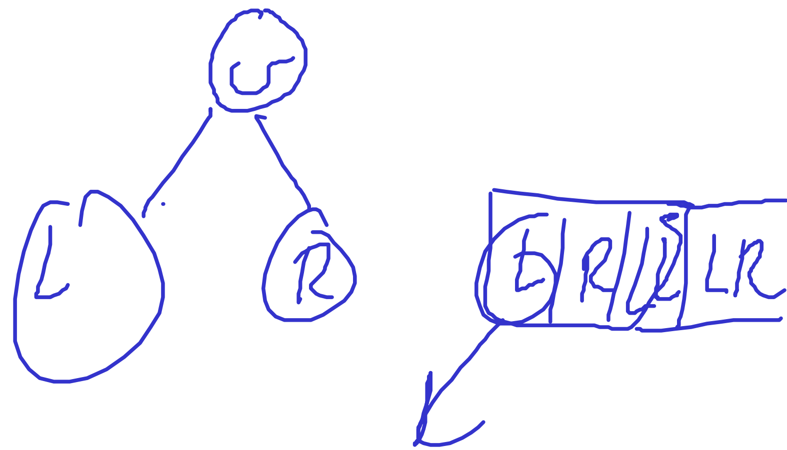
Tree Traversal Numbering Example



Levelorder Traversal
Breadth First Search (BFS)

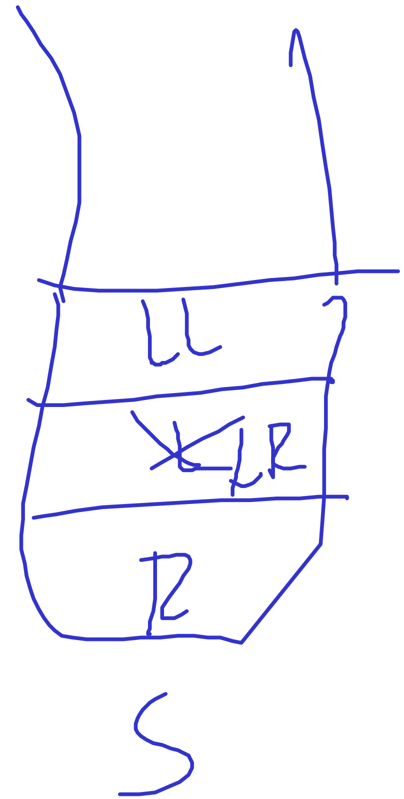
Algorithm levelorder(Tree T, Node r):

```
Algorithm levelorder(Tree T, Node r):
    Queue Q
    Q.enqueue(T.r)
    while !Q.empty() do
        v ← Q.dequeue()
        processNode(v)
        if T.isInternal(v) then
                Q.enqueue(T.leftChild(v))
                Q.enqueue(T.rightChild(v))
        end
    end
end
```



Non-recursive Preorder Traversal
Depth First Search (DFS)

Algorithm dfs(Tree T, Node r)
    Stack S
    S.push(r)
    while !S.empty() **do**
        $v \leftarrow S$.pop()
        processNode(v)
        if T.isInternal(v) **then**
                S.push(T.rightChild(v))
                S.push(T.leftChild(v))
        end
    end
end



Running Time of Tree Traversals

- Each node is visited a fixed number of times (i.e., 3 times)
- **Theorem**
  - The time complexity of Preorder, Inorder, Postorder is $O(n)$

$T(n) = T(n-1) + O(1)$

$\in \underline{O(n)}$

Tree Traversal Summary

Preorder	Depth-first	Stack
Inorder	Symmetric	Stack
Postorder	Bottom-up	Stack
Levelorder	Breadth-first	Queue

Data Structures for *binary* Trees

Operation	Time with linked structure
positions, elements, traversals (iterators): pre-, in-, post-, level-order	$O(n)$
size, isEmpty	O(1)
swapElements, replaceElement	O(1)
leftChild, rightChild, sibling	O(1)
isInternal, isExternal, isRoot	O(1)
root, parent, child	O(1)