Analysis Worksheet

1. **Include a photo of your raw data with your TA's signature.**

Arfaz Hossain Feb. 18
James Kim

		Time (s)	x (m)	z (m)	v_x (m/s)	v_z (m/s)
1	0	1.100	1.179	0.7330		
2	0.033	1.133	1.129	0.7838		
3	0.067	1.167	1.085	0.8154		
4	0.1	1.200	1.044	0.8429		
5	0.133	1.233	0.9933	0.8704		
6	0.167	1.267	0.9189	0.8682		
7	0.2	1.300	0.9108	0.8716		
8	0.233	1.333	0.8601	0.8619		
9	0.267	1.367	0.8136	0.8386		
10	0.3	1.400	0.7671	0.8112		
11	0.333	1.433	0.7206	0.7617		
12	0.367	1.467	0.6698	0.7160		
13	0.4	1.500	0.6233	0.6626		
14	0.433	1.533	0.5789	0.5808		
15	0.467	1.567	0.5316	0.4988		
16	0.5	1.600	0.5007	0.4159		

2. **Create a table to show all your data as described in the Analysis & Submission section of the lab manual. Clearly indicate which method was used for calculating the speeds and show a sample calculation.**

Time (s)	x (m)	z (m)	Vx (m/s)	Vz (m/s)
0.000	1.1790	0.7330	0	0
0.033	1.1290	0.7858	1.129	0.7858
0.067	1.0950	0.8154	1.095	0.8154
0.100	1.0440	0.8429	1.044	0.8429
0.133	0.9933	0.8704	0.9933	0.8704
0.167	0.9489	0.8682	0.9489	0.8682
0.200	0.9108	0.8746	0.9108	0.8746
0.233	0.8601	0.8619	0.8601	0.8619
0.237	0.8136	0.8386	0.8136	0.8386
0.300	0.7671	0.8112	0.7671	0.8112
0.333	0.7206	0.7647	0.7206	0.7647
0.367	0.6698	0.7160	0.6698	0.7160
0.400	0.6233	0.6526	0.6233	0.6526
0.433	0.5789	0.5808	0.5789	0.5808
0.467	0.5346	0.4983	0.5346	0.4983
0.500	0.5007	0.4159	0	0

The values for velocity in the x-direction and velocity in the z-direction is calculated using the *central difference method.*

3. **Create the four graphs as described in the Analysis & Submission section of the lab manual.**

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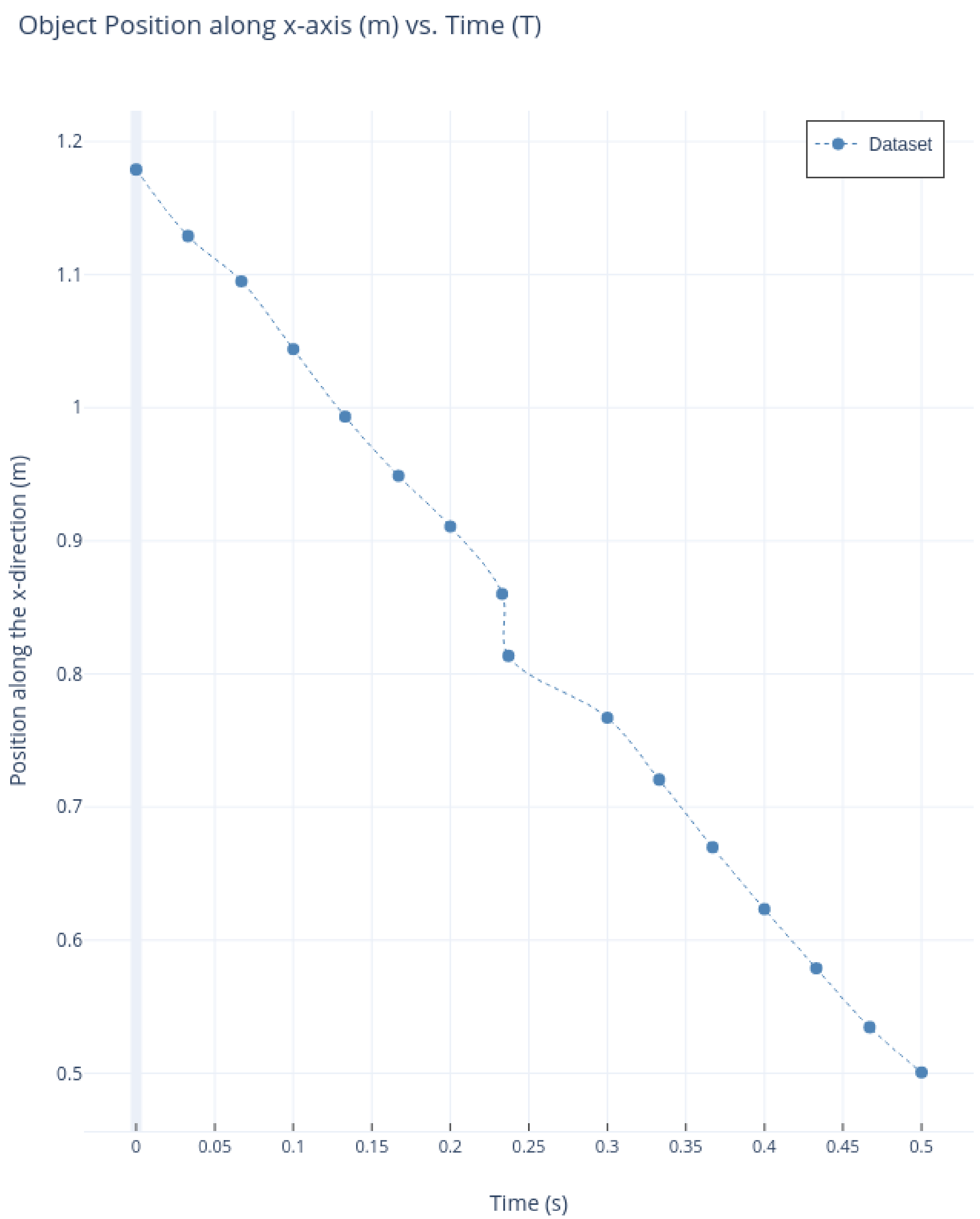
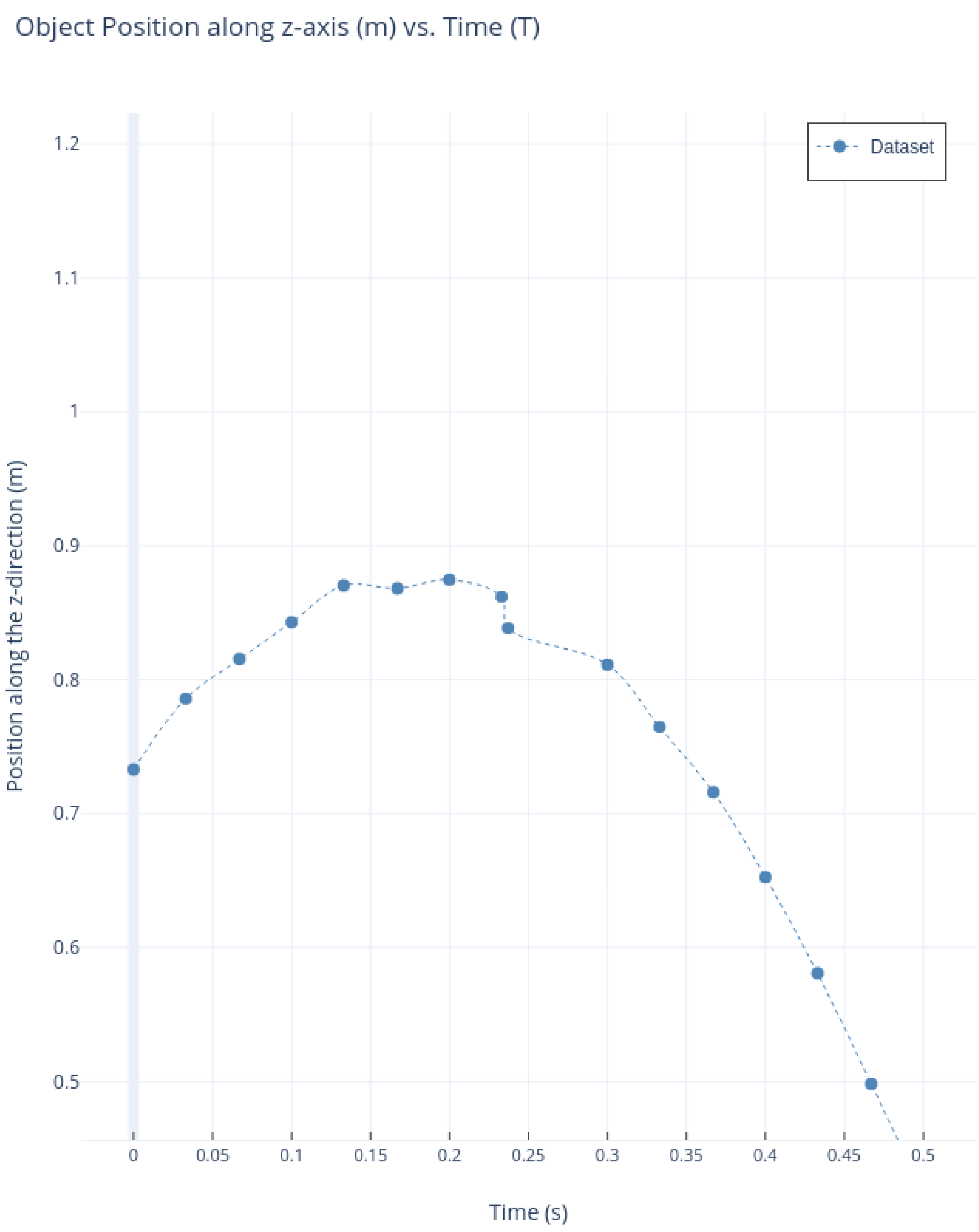
x vs t


Object Position along x-axis (m) vs. Time (T)
Dataset
Position along the x-direction (m)
Time (s)
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

z vs t



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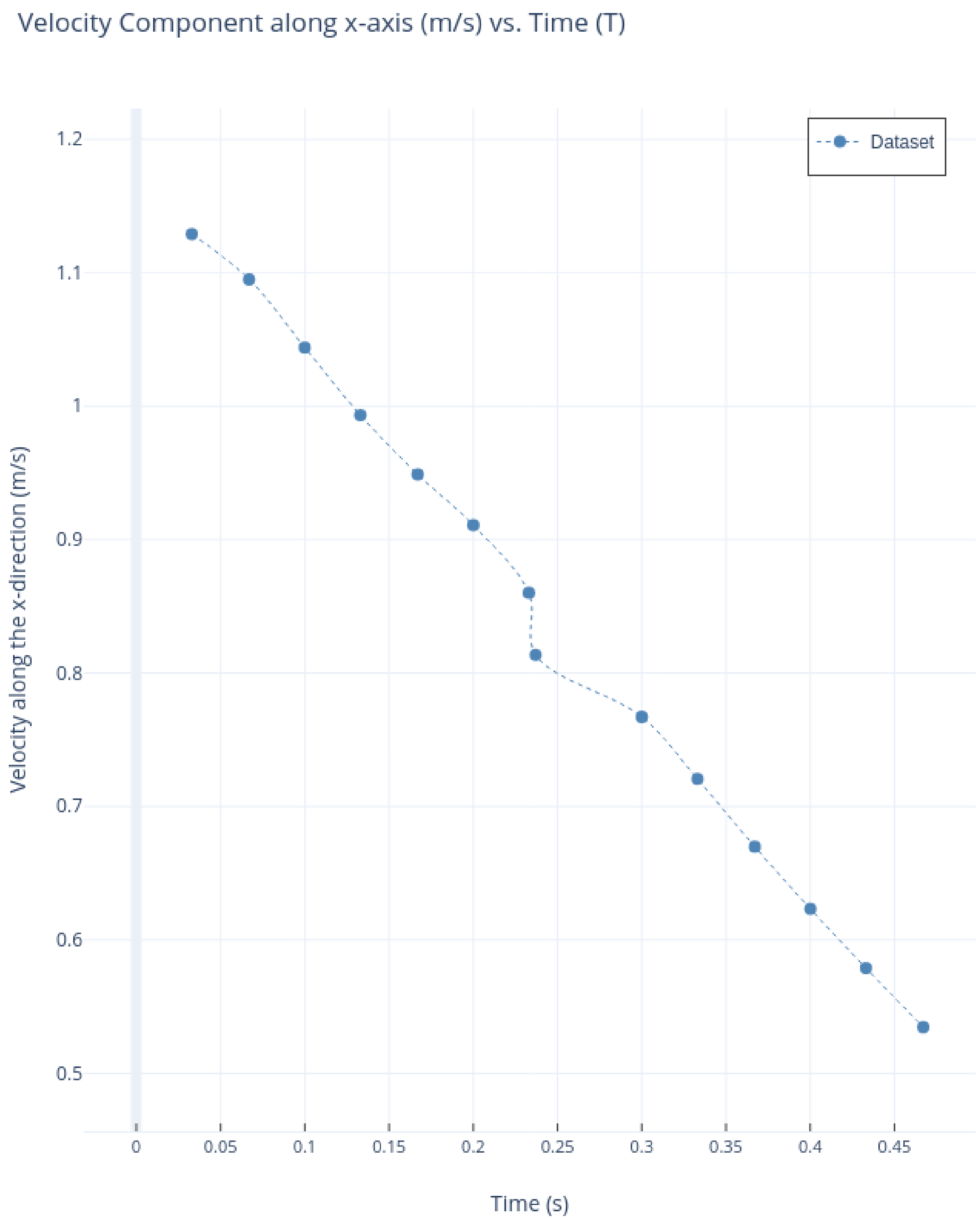
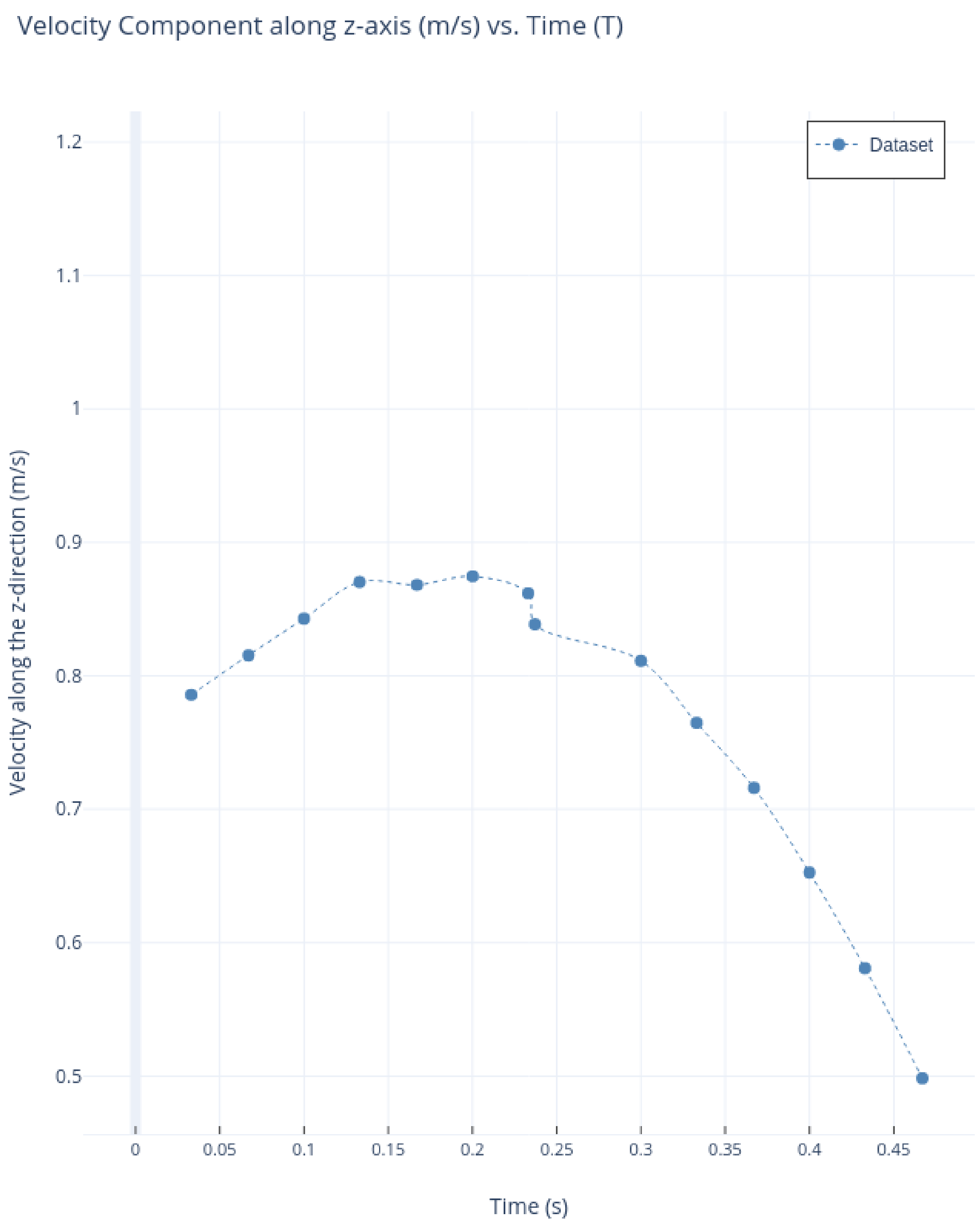
v_x vs t


Velocity Component along x-axis (m/s) vs. Time (T)
Dataset
Velocity along the x-direction (m/s)
Time (s)
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

v_z vs t



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4. Create a table for the least squares analysis of the v_X vs t data, as described in the Analysis & Submission section of the lab manual.

Title	Time (s)	Vx (m/s)	T^2	Vx^2	(Vx)(t)
Data	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0330	1.1290	0.0011	1.2746	0.0373
	0.0670	1.0950	0.0045	1.1990	0.0734
	0.1000	1.0440	0.0100	1.0899	0.1044
	0.1330	0.9933	0.0177	0.9866	0.1321
	0.1670	0.9489	0.0279	0.9004	0.1585
	0.2000	0.9108	0.0400	0.8296	0.1822
	0.2330	0.8601	0.0543	0.7398	0.2004
	0.2370	0.8136	0.0562	0.6619	0.1928
	0.3000	0.7671	0.0900	0.5884	0.2301
	0.3330	0.7206	0.1109	0.5193	0.2400
	0.3670	0.6698	0.1347	0.4486	0.2458
	0.4000	0.6233	0.1600	0.3885	0.2493
	0.4330	0.5789	0.1875	0.3351	0.2507
	0.4670	0.5346	0.2181	0.2858	0.2497
	0.5000	0.0000	0.2500	0.0000	0.0000
Average	0.248	0.731	0.09	0.640	0.159

5. Determine the slope and intercept, along with their uncertainties. This should be done by hand using the average column values in the table above.

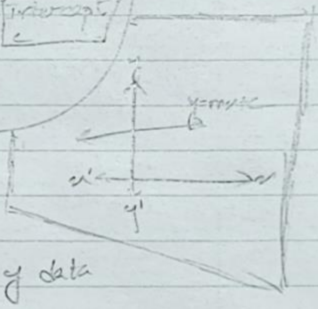
Lab3 Question 5,7

$y = mx + c$

$\Leftrightarrow y = Ax + B$

slope m

intercept c



Equation for slope is

$$\boxed{\text{Slope, } A = \frac{\overline{xy} - (\bar{x})(\bar{y})}{(\overline{x^2}) - (\bar{x})^2}}$$

For x data points in the x axis and y data points in the y axis.

Equation for intercept is

$$\boxed{B = \frac{(\overline{x^2})(\bar{y}) - (\bar{x})(\overline{xy})}{(\overline{x^2}) - (\bar{x})^2}}$$

For x data points in the x axis and y data points in the y axis.

what each data points in our graph would represent	V_x Vs t	V_z Vs t
	$x = t$; $y = V_x$	$x = t$; $y = V_z$
$\bar{x}$	$\bar{t}$	$\bar{t}$
$\bar{y}$	$\overline{V_x}$	$\overline{V_z}$
same: $\bar{x} \cdot \bar{y}$	$\bar{t} \cdot \overline{V_x}$	$\bar{t} \cdot \overline{V_z}$
same: $\overline{xy}$	$\overline{V_x \cdot t}$	$\overline{V_z \cdot t}$
$(\overline{x^2})$	$\overline{t^2}$	$\overline{t^2}$
$(\overline{y^2})$	$\overline{V_x^2}$	$\overline{V_z^2}$

$N = 16$

$\bar{t} = 0.248 (s)$

$\overline{V_x} = 0.731 (m/s)$

$\overline{V_z} = 0.6738 (m/s)$

$\overline{t\, V_x} = 0.159$

$\overline{t \cdot V_z} = 0.1566$

$\overline{t^2} = 0.085$

$\overline{V_x^2} = 0.6405$

$\overline{V_z^2} = 0.0852$

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Physics 110 Lab 3: Using Graphical Analysis

$N = 16$

Lab3 Question 5,7

Uncertainties

$$\Delta_{total} = \sum_{i=1}^{N} \left(y_i^2 + A^2 x_i^2 + B^2 - 2Ay_i x_i - 2By_i + 2ABx_i \right)$$

$$= N\left(\overline{y^2} + A^2(\overline{x^2}) + B^2 - 2A\overline{xy} - 2B\overline{y} + 2AB(\overline{x}) \right)$$

for $\boxed{V_x \text{ vs } t}$

$$\Delta_{total} = N\left(\overline{V_x^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_x t}) - 2B(\overline{V_x}) + 2AB(\overline{t}) \right)$$

for $\boxed{V_z \text{ vs } t}$

$$\Delta_{total} = N\left(\overline{V_z^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_z \cdot t}) - 2B(\overline{V_z}) + 2AB(\overline{t}) \right)$$

$$\delta_\Delta = \sqrt{\frac{\Delta_{total}}{N-2}}$$

$$\delta_A = \delta_\Delta \sqrt{\frac{1}{N(\overline{x^2} - (\overline{x})^2)}}$$

$$\delta_B = \delta_\Delta \sqrt{\frac{\overline{x^2}}{N(\overline{x^2} - (\overline{x})^2)}}$$

for $\boxed{V_x \text{ vs } t}$, and $\boxed{V_z \text{ vs } t}$

$$\delta_A = \delta_\Delta \sqrt{\frac{1}{N(\overline{t^2} - (\overline{t})^2)}}$$

$$\delta_B = \delta_\Delta \sqrt{\frac{1}{N(\overline{t^2} - (\overline{t})^2)}}$$

Q5

- $$\boxed{A = \frac{(\overline{t \cdot V_x}) - (\overline{t})(\overline{V_x})}{(\overline{t^2}) - (\overline{t})^2}} = \frac{0.159 - (0.248)\cdot(0.731)}{0.09 - (0.248)^2} = -\frac{2786}{2937}$$

$$\approx -0.948586$$
$$\approx -0.9486$$

- $$\boxed{B = \frac{(\overline{t^2})(\overline{V_x}) - (\overline{t})(\overline{t \cdot V_x})}{(\overline{t^2}) - (\overline{t})^2}} = \frac{(0.085)(0.731) - (0.248)(0.159)}{(0.085) - (0.248)^2}$$

$$\approx 0.9662495749$$
$$\approx 0.9663$$

- $$\boxed{\Delta_{total} = N\left(\overline{V_x^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_x t}) - 2B(\overline{V_x}) + 2AB(\overline{t})\right)}$$

$$= 16\Big(\underline{0.6405} + \underline{(-0.9486)^2(0.085)} + \underline{(0.9663)^2}$$
$$\underline{-2(-0.9486)(0.159)} \underline{-2(0.9663)(0.731)}$$
$$\underline{+2(-0.9486)(0.9663)(0.248)}\Big)$$

$$= 1.376803$$

- $$\boxed{\delta_A = \sqrt{\frac{\Delta_{total}}{N-2}}} = \sqrt{\frac{1.88098}{16-2}} = \sqrt{\frac{1.88098}{14}} = 0.313597$$

- $$\boxed{\delta_A = \delta_\Delta\sqrt{\frac{1}{N(\overline{t^2} - (\overline{t})^2)}}} = (0.3665)\sqrt{\frac{1}{16(0.085 - (0.248)^2)}}$$

$$= 0.5105259$$

- $$\boxed{\delta_B = \delta_\Delta\sqrt{\frac{\overline{t^2}}{N(\overline{t^2} - (\overline{t})^2)}}} \quad (0.3665)\sqrt{\frac{0.085}{16(0.085 - (0.248)^2)}} = 0.148916$$

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6. Create a table for the least squares analysis of the v_Z vs t data, as described in the Analysis & Submission section of the lab manual.

Title	Time (s)	Vz (m/s)	T^2	Vz^2	(Vz)(t)
Data	0.0000	0.0000	0.0000	0.0000	0.0000
	0.0330	0.7858	0.0011	0.0011	0.0259
	0.0670	0.8154	0.0045	0.0045	0.0546
	0.1000	0.8429	0.0100	0.0100	0.0843
	0.1330	0.8704	0.0177	0.0177	0.1158
	0.1670	0.8682	0.0279	0.0279	0.1450
	0.2000	0.8746	0.0400	0.0400	0.1749
	0.2330	0.8619	0.0543	0.0543	0.2008
	0.2370	0.8386	0.0562	0.0562	0.1987
	0.3000	0.8112	0.0900	0.0900	0.2434
	0.3330	0.7647	0.1109	0.1109	0.2546
	0.3670	0.7160	0.1347	0.1347	0.2628
	0.4000	0.6526	0.1600	0.1600	0.2610
	0.4330	0.5808	0.1875	0.1875	0.2515
	0.4670	0.4983	0.2181	0.2181	0.2327
	0.5000	0.0000	0.2500	0.2500	0.0000
Average	0.2481	0.6738	0.0852	0.0852	0.1566

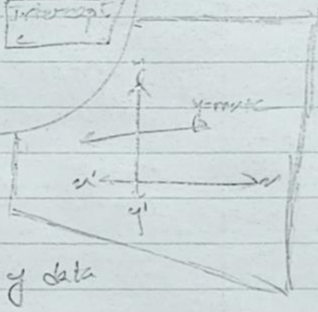
7. **Determine the slope and intercept, along with their uncertainties. This should be done by hand using the average column values in the table above.**

Lab3 Question 5,7

$y = mx + c$

$\Rightarrow y = Ax + B$

slope m ; intercept c



Equation for slope is

$$\boxed{\text{Slope, } A = \frac{\overline{xy} - (\bar{x})(\bar{y})}{(\overline{x^2}) - (\bar{x})^2}}$$

For x data points in the x axis and y data points in the y axis.

Equation for intercept is

$$\boxed{B = \frac{(\overline{x^2})(\bar{y}) - (\bar{x})(\overline{xy})}{(\overline{x^2}) - (\bar{x})^2}}$$

For x data points in the x axis and y data points in the y axis.

what each data points in our graph would represent	V_x vs t	V_z vs t
	$x = t$; $y = V_x$	$x = t$; $y = V_z$
$\bar{x}$	$\bar{t}$	$\bar{t}$
$\bar{y}$	$\overline{V_x}$	$\overline{V_z}$
same $\bar{x}\cdot\bar{y}$	$\overline{t\cdot V_x}$	$\overline{t\cdot V_z}$
$\overline{xy}$	$\overline{V_x\cdot t}$	$\overline{V_z\cdot t}$
$(\overline{x^2})$	$\overline{t^2}$	$\overline{t^2}$
$(\overline{y^2})$	$\overline{V_x^2}$	$\overline{V_z^2}$

$N = 16$

$\bar{t} = 0.248 (s)$

$\overline{V_x} = 0.731 (m/s)$

$\overline{V_z} = 0.6738 (m/s)$

$\overline{t\, V_x} = 0.159$

$\overline{t\cdot V_z} = 0.1566$

$\overline{t^2} = 0.085$

$\overline{V_x^2} = 0.6405$

$\overline{V_z^2} = 0.0852$

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$N=16$

Lab3 Question 5,7

Uncertainties

$$\Delta_{total} = \sum_{i=1}^{N}\left(y_i^2 + A^2 x_i^2 + B^2 - 2Ay_i x_i - 2By_i + 2ABx_i\right)$$

$$= N\left(\overline{y^2} + A^2(\overline{x^2}) + B^2 - 2A\overline{xy} - 2B\overline{y} + 2AB(\overline{x})\right)$$

for $\boxed{V_x \text{ vs } t}$

$$\Delta_{total} = N\left(\overline{V_x^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_x t}) - 2B(\overline{V_x}) + 2AB(\overline{t})\right)$$

for $\boxed{V_z \text{ vs } t}$

$$\Delta_{total} = N\left(\overline{V_z^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_z \cdot t}) - 2B(\overline{V_z}) + 2AB(\overline{t})\right)$$

$$\delta_\Delta = \sqrt{\frac{\Delta_{total}}{N-2}}$$

$$\delta_A = \delta_\Delta\sqrt{\frac{1}{N(\overline{x^2} - (\overline{x})^2)}}$$

$$\delta_B = \delta_\Delta\sqrt{\frac{\overline{x^2}}{N(\overline{x^2} - (\overline{x})^2)}}$$

for $\boxed{V_x \text{ vs } t}$, and $\boxed{V_z \text{ vs } t}$

$$\delta_A = \delta_\Delta\sqrt{\frac{1}{N(\overline{t^2} - (\overline{t})^2)}}$$

$$\delta_B = \delta_\Delta\sqrt{\frac{1}{N(\overline{t^2} - (\overline{t})^2)}}$$

Question 7

$$A = \frac{\overline{t \cdot V_z} - (\bar{t})(\overline{V_z})}{(\overline{t^2}) - (\bar{t})^2} = \frac{0.1566 - (0.248)(0.6738)}{0.085 - (0.248)^2} = -\frac{2188}{1695} = -0.1468$$

$$B = \frac{(\overline{t^2})(\overline{V_z}) - (\bar{t})(\overline{t \cdot V_z})}{(\overline{t^2}) - (\bar{t})^2} = \frac{(0.085)(0.6738) - (0.248)(0.1566)}{0.085 - (0.248)^2}$$

$$= 0.7846527066$$

$$\Delta_{total} = N\left(\overline{V_z^2} + A^2(\overline{t^2}) + B^2 - 2A(\overline{V_z t}) - 2B(\overline{V_z}) + 2AB(\bar{t})\right)$$

$$= 16\left(0.0652 + (-0.12446)^2(0.085) + (0.21648)^2 - 2(-0.12446)(0.1566) - 2(0.2184773211)(0.6738) + 2(-0.12446)(0.218478)(0.248)\right)$$

$$= 1.141687$$

$$\delta_\Delta = \sqrt{\frac{\Delta_{total}}{N-2}} = \sqrt{\frac{1.141687}{14}} = 0.285568$$

$$\delta_A = \delta_\Delta \sqrt{\frac{1}{N(\overline{t^2} - (\bar{t})^2)}} = 0.285568 \sqrt{\frac{1}{16(0.085 - (0.248)^2)}}$$

$$= 0.464652$$

$$\delta_B = \delta_\Delta \sqrt{\frac{\overline{t^2}}{N(\overline{t^2} - (\bar{t})^2)}} = 0.285568 \sqrt{\frac{0.085}{16(0.085 - (0.248)^2)}}$$

$$= 0.135606$$

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8. State the initial horizontal and vertical components of velocity and acceleration of the projectile. (Simply state the quantities using proper significant figures.)

Title	Time (s)	x (m)	z (m)	Vx (m/s)	Vz (m/s)	T^2	Vx^2	Vz^2	(Vx)(t)	(Vz)(t)
	0.000	1.1790	0.7330	0	0	0.000	0	0	0	0
	0.033	1.1290	0.7858	1.129	0.7858	0.001	1.274641	0.61748164	0.037257	0.025931
	0.067	1.0950	0.8154	1.095	0.8154	0.004	1.199025	0.66487716	0.073365	0.054632
	0.100	1.0440	0.8429	1.044	0.8429	0.010	1.089936	0.71048041	0.1044	0.08429
	0.133	0.9933	0.8704	0.9933	0.8704	0.018	0.98664489	0.75759616	0.1321089	0.115763
	0.167	0.9489	0.8682	0.9489	0.8682	0.028	0.90041121	0.75377124	0.1584663	0.144989
	0.200	0.9108	0.8746	0.9108	0.8746	0.040	0.82955664	0.76492516	0.18216	0.17492
Data	0.233	0.8601	0.8619	0.8601	0.8619	0.054	0.73977201	0.74287161	0.2004033	0.200823
	0.237	0.8136	0.8386	0.8136	0.8386	0.056	0.66194496	0.70324996	0.1928232	0.198748
	0.300	0.7671	0.8112	0.7671	0.8112	0.090	0.58844241	0.65804544	0.23013	0.24336
	0.333	0.7206	0.7647	0.7206	0.7647	0.111	0.51926436	0.58476609	0.2399598	0.254645
	0.367	0.6698	0.7160	0.6698	0.7160	0.135	0.44863204	0.512656	0.2458166	0.262772
	0.400	0.6233	0.6526	0.6233	0.6526	0.160	0.38850289	0.42588676	0.24932	0.26104
	0.433	0.5789	0.5808	0.5789	0.5808	0.187	0.33512521	0.33732864	0.2506637	0.251486
	0.467	0.5346	0.4983	0.5346	0.4983	0.218	0.28579716	0.24830289	0.2496582	0.232706
	0.500	0.5007	0.4159	0	0	0.250	0	0	0	0
Average	0.248	0.836	0.746	0.731	0.674	0.085	0.640	0.530	0.159	0.157

Vx vs T		Vz vs T	
A	-0.94859	A	-0.446987
B	0.96625	B	0.784653

Uncertainty	Δ total	$\delta\Delta$	δA	δB
Vx vs T	1.376803	0.313597	0.510259	0.148916
Vz vs T	1.141687	0.285568	0.464652	0.135606

Since we are using a *Velocity Vs. Time* graph chart, the slope of our graphs should be the acceleration of the object and the intercept should be the velocity at time, t=0 or, initial velocity. Since we used to measure the components in the respective x and z direction, we should be able to derive the slope, A in each graph as the Horizontal and Vertical Acceleration, intercept, B in each graph as the Initial Horizontal and Vertical Velocity respectively.

In short,

A (Vx vs T) = Horizontal Acceleration = -0.94859 m/s^2 = A_x
B (Vx vs T) = Initial Horizontal Velocity = 0.96624 m/s = B_{Vx} or, Vx_o
A (Vz vs T) = Vertical Acceleration = -0.44698 m/s^2 = A_z
B (Vz vs T) = Initial Vertical Velocity = 0.78465 m/s = B_{Vz}

9. State the expected values of the components of the acceleration. Perform a statistical test for whether your measured acceleration component values agree with your expectations.

Expected Acceleration Formula in the x-axis,

$$A_{x(exp)} = \frac{Vx - Vo}{t} = \frac{Vx - B_{Vx}}{t} = \frac{0.7305625 - 0.96625}{0.5} = -0.471375$$

$$A_{z(exp)} = \frac{Vz - Vo}{t} = \frac{Vz - B_{Vz}}{t} = \frac{0.6738375 - 0.7846527}{0.5} = -0.2216304$$

The formula used for measuring the numerical accuracy of an experiment that follows a Gaussian probability distribution is:

$$t = \frac{x_1 - x_2}{\sqrt{\delta_{x_1} - \delta_{x_2}}}$$

Let the two quantities x_1 and x_2 be A_x and $A_{x(exp)}$,

$$t_{Ax} = \frac{A_x - A_{x(exp)}}{\delta_{Ax}} = \frac{(-0.94859) - (-0.471375)}{0.510259} = -0.935235100532665 \approx -0.9$$

Now, let the two quantities x_1 and x_2 be A_z and $A_{z(exp)}$,

$$t_{Az} = \frac{A_z - A_{z(exp)}}{\delta_{Az}} = \frac{(-0.44699) - (-0.2216304)}{0.464652} = -0.484999889684925 \approx -0.5$$

If the value of the numerical accuracy test, or t-test stays between -2 and 2, such that $\mathbf{-2 < |t| < 2}$ we can conclude that the findings from that given experiment is consistent. Here, both values we found from A and B stays between -2 and 2. Therefore, the result from our experiment is consistent with our findings.

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10. Respond to the following questions/instructions:

(a) Were any assumptions or approximations involved in performing these calculations? List them and state how you think they might affect the results if they were not valid.

Yes, some approximations were taken while carrying out this experiment, such as

(1) Vx at time t=0 and t=0.5 are taken to be 0 (m/s) due to the *central difference method*.
(2) The gravitational acceleration and other forces involved in interfering with the object's upward and downward projectile were taken to be 0, resulting in the acceleration components in each time to be 0 (m/s^2).
(3) Expected acceleration values in each axis were taken from the measured average velocity components in each component.

(b) What do your statistical tests indicate? What are the implications of the results?

Statistical Analysis tests in both acceleration components show that the measured amount is consistent with the expected values of acceleration. t_{Ax}, statistical analysis of the expected and measured values in the acceleration component in the x-axis is found to be -0.9, indicating that the result from our findings in this experiment is consistent in both cases (also for t_{Az}).

(c) What is the initial speed of your projectile? What is the initial angle from horizontal of the projectile?

The initial speed of the projectile is V = $\sqrt{V_x{}^2 + V_z{}^2}$

$= \sqrt{0.96625^2 + 0.784653^2}$

= 1.244716591 (m/s)

≈ 1.24 (m/s)

where,

$V_{x(o)} = B_{Vx}$ = Initial Velocity at x-axis

$V_{z(o)} = B_{Vz}$ = Initial Velocity at z-axis

Again,

$V_x = |V| \cos\theta$

$\Leftrightarrow \cos\theta = \frac{V_x}{|V|}$

$\Leftrightarrow \theta = \cos^{-1}(\frac{V_x}{|V|})$

$\Leftrightarrow \theta = \cos^{-1}(\frac{0.96625}{1.244716591}) = 39.078671° \approx 39.1°$

Answer: Initial Speed, V = 1.24 (m/s); $\theta = 39.1°$