CHAPTER 12

Solutions to Chapter-End Problems

A. Key Concepts

Sensitivity Graphs:

12.1 **(a)** - First cost
- Annual operating and maintenance costs
- Annual savings or revenue
- Salvage value
- Service life

(b) - First cost
- Annual operating and maintenance costs
- Annual revenue
- Shipping cost
- Inflation rate
- Exchange rate
- Tariff

12.2 **(a)** For example, for Canada, as of March 2008: Base figure: 1.4%

Range of variation observed over last 10 years: 0.5% to 4%; this should be reasonable for a relatively short time period

(b) For example, for Canada, as of May 2008: Base figure: $1US per $1CAN

Range of variation observed in last two years: $1.05 to $0.95; this should be reasonable for a relatively short time period

(c) Base figure: use the average annual savings of the equipment you already have

Range of variation: a variation range of 5-10% should be reasonable since the new equipment is similar to the old one; you can probably get good estimate for the range of variation based on the information on the old one

(d) Base figure: use the average annual revenue of a similar internet-based business (if it exists)

Range of variation: since the nature of internet-based business is highly unpredictable at this stage, one should employ a large range of variation such as 5-50%

(e) Base figure: use the book value computed by declining-balance method with depreciation rate of 30%

Range of variation: this depends on the age of the computer, but this could be anywhere between 0 and 50% of the purchase price

12.3 **(a)** Break-even analysis for multiple projects: to see the effect of the future demand on the annual worth of leasing the trucks of different sizes

(b) Break-even analysis for a single project: to see the effect of the uncertain heating expenses on the annual worth of the business

(c) Break-even analysis for a single project: to see the effect of the future demand on the annual worth of the business

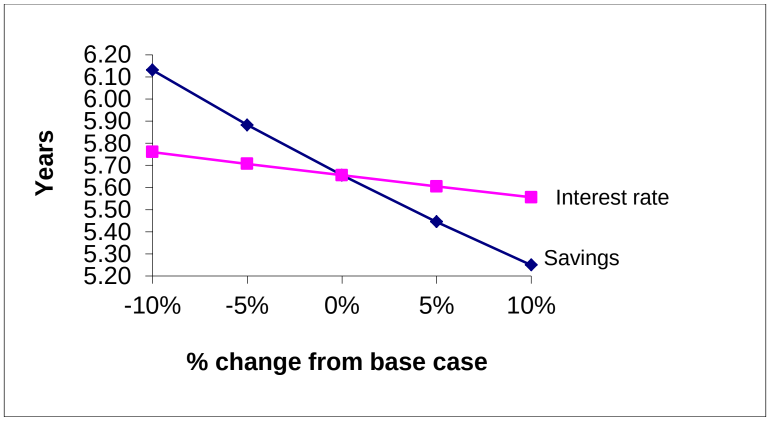
(d) Sensitivity graphs: to see the effect of the uncertain construction cost on the total cost

(e) Break-even analysis for a single project or scenario analysis: break-even analysis for examining the effect of the growth rate and the length of growth separately; scenario analysis is appropriate to examine the growth rate and the length of growth together

12.4 The number of years, N, to save F $ by putting aside A $ per year at an annual interest rate i is N = ln[(iF + A)/A]/ln(1 + i).

With 5% and 10% decreases and increases in the savings per year and the interest rate, Kelowna Go-Karts will take the following number of years to save $50 000:

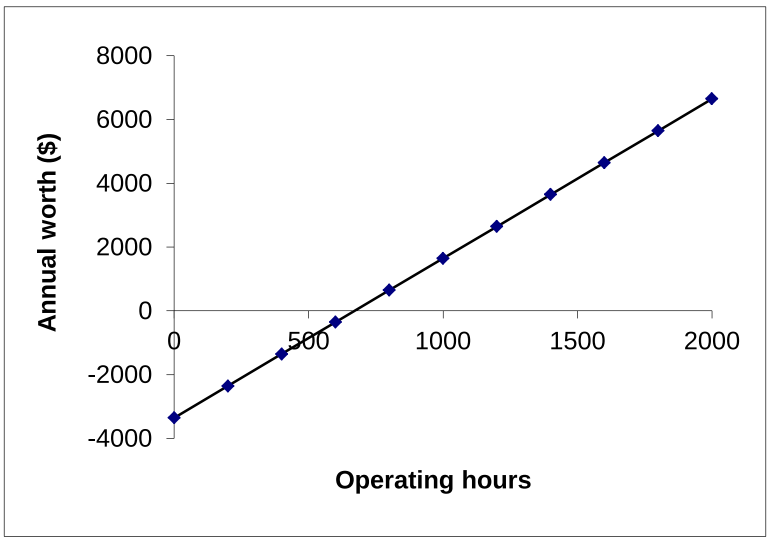
Parameter	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
Savings per year	6300	6650	7000	7350	7700
Interest rate	9.00%	9.50%	10.00%	10.50%	11.00%
Number of years to save $50 000:	**-10%**	**-5%**	**0%**	**5%**	**10%**
With changes to savings per year	6.13	5.88	5.66	5.44	5.25
With changes to interest rate	5.76	5.71	5.66	5.60	5.56



Break Even Analysis:

12.5 **(a)** AW $= -21\,500(A/P, 10\%, 10) + 10(1500) - 5(1500)$
$+ 21\,500(1 - 0.2)^{10}(A/F, 10\%, 10)$
$= -21\,500(0.16275) + 15\,000 - 7500 + 21\,500(0.8)^{10}\times(0.06275)$
$= \$4145.74$

(b) Break-even graph:



The break-even level of operating hours is 670.85 hours.

12.6 **(a)** First, the IRR for the incremental investment from “do nothing” to A is found by solving for i in:

$100\,000(P/A, i, 5) = 50\,000 \Rightarrow (P/A, i, 5) = 2 \Rightarrow IRR = 41.1\%$

A is the current best alternative. The IRR on the incremental investment between A and B is found by solving for i in:

(400 000 – 100 000) + (150 000 – 50 000)(P/A, i, N) = 0
(P/A, i, N) = 300 000/100 000 = 3

This gives an IRR of 19.9%, thus the incremental investment in B is justified.

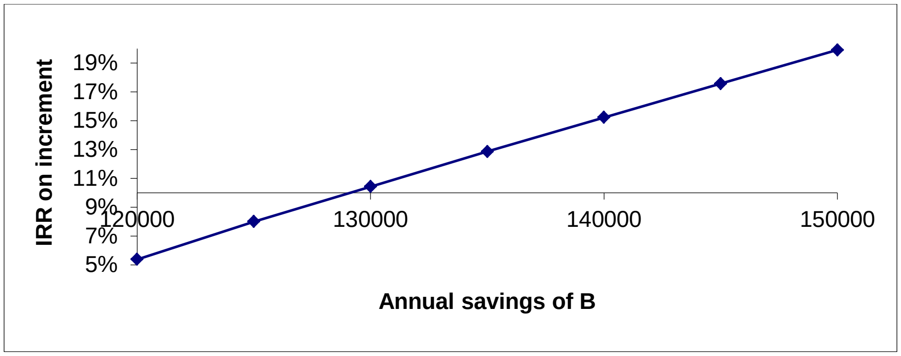
(b) Preference for A over "do nothing" remains unchanged. The incremental investment between A and B is of concern. If the savings per year due to B are X, then the IRR in the incremental investment is found by solving for i in:

(P/A, i%, N) = 300 000/(X – 50 000)

The IRR for various values of X is:

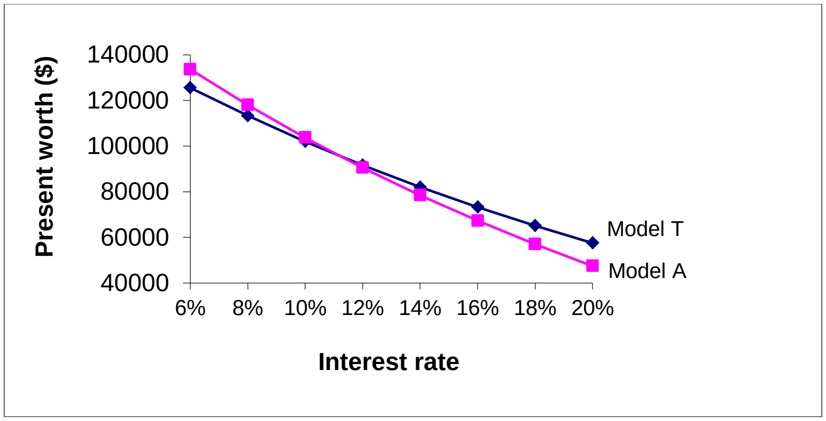
X	(300 000)/(X-50 000)	IRR
120 000	4.286	5.38%
125 000	4.000	8.00%
130 000	3.750	10.42%
135 000	3.529	12.86%
140 000	3.333	15.23%
145 000	3.158	17.57%
150 000	3.000	19.90%

With a MARR of 10%, A is preferred for X (annual savings of B) below about $129 000 and B is preferred for X above $129 000. The amount $129 000 is the break-even annual savings for B. A diagram of savings due to B versus IRR on the incremental investment is:



12.7 From trial and error with a spreadsheet, the break-even interest rate is 11.2% Model T is preferred for a MARR of 16%.

PW(T) = –100 000 + 50 000(P/A, i, 5) + 20 000(P/F, i, 5)
PW(A) = –150 000 + 62 000(P/A, i, 5) + 30 000(P/F, i, 5)



12.8 Since the two models have unequal lives, it is easiest to compare them based on an annual worth computation.

The annual worth of each for a variety of interest rates is:

Depreciation Rate (Model A)	AW Model A	AW Model B
0.3	3365	3809
0.4	3669	3809
0.5	3887	3809
0.6	4033	3809

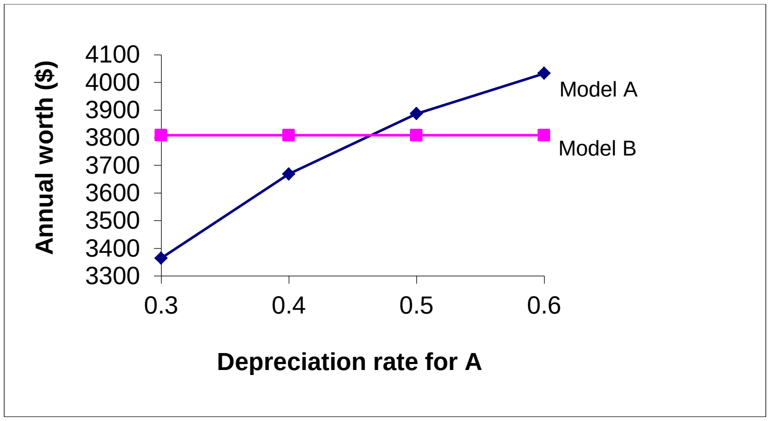
Sample computation for d = 40% depreciation rate for Model A:

AW(model A) $= 8000(A/P, 11\%, 3) + 1000 - 8000(1 - d)^3$
$= 8000(0.3982) + 1000 - 8000(0.6)^3 = 3669$

And for Model B with straight line depreciation at $2500 per year:

AW(model B)
$= 10\,000(A/P, 11\%, 4) + 700 - (10\,000 - 2500\times4)(A/F, 11\%, 4)$
$= 10\,000\ (0.3108) + 700 = 3809$

Since the annual worth of Model A is lower than that of Model B over the range of depreciation rates Julia has estimated for Model A, she should pick Model A. From the above table, we can interpolate the break-even depreciation rate to be 46%. Below this rate, Model A is preferred; above this rate, Model B is preferred. She is indifferent with a depreciation rate of 46%. A break-even chart is as follows:



Probability and Expected Value:

12.9 (a)

E(return on investment)
= Pr{7%}(10 000×0.07) + Pr{10%}(10 000×0.1) + Pr{15%}(10 000×0.15)
= 0.65(700) + 0.25(1000) + 0.1(1500) = 855

The expected return from this investment is $855.

12.10 (a)

E(loss) = Pr{capacity 30}(loss of 20) + Pr{capacity 40}(loss of 10)
+ Pr{capacity 50}(loss of 0) + Pr{capacity 60}(loss of 0)
= 0.2(20) + 0.4(10) + 0.3(0) + 0.1(0)
= 8 calls per hour

The expected loss of customers due to the lack of processing capacity is 8 per hour.

12.11 (a)

E(number of defects, A1)
= 0.3(0) + 0.28(1) + 0.15(2) + 0.15(3) + 0.1(4) + 0.02(5) = 1.53/100 units

E(number of defects, X1000)
= 0.25(0) + 0.33(1) + 0.26(2) + 0.1(3) + 0.05(4) + 0.01(5) = 1.4/100 units

According to the expected number of defects, X1000 seems to be slightly better than A1.

12.12 (a)

E(cost in a summer month)
= 0.4(800) + 0.25(2×800) + 0.2(3×800) + 0.1(3×800 + 1500)
+ 0.05(3×800 + 2×1500)
= $1860 per month

E(cost in a non-summer month)
= 0.45(0) + 0.4(800) + 0.15(2×800) = $560 per month

CB Electronix should consider getting the complete coverage policy because the expected cost is over $500 even in the non-summer months.

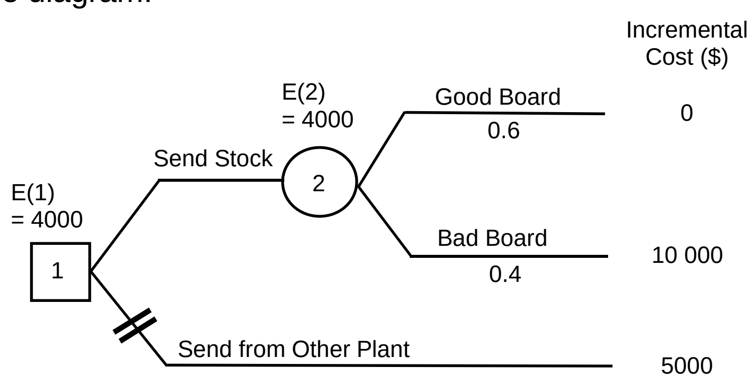
Decision Trees:

12.13 **(a)** p(customer getting a bad board) = 2/5 = 0.4

(b) E(send stock) = p(good board)(0) + p(bad board)(–10 000)
= 0.6(0) + 0.4(–10 000) = –4000

The expected cost of sending the customer one of the five boards is $4000.

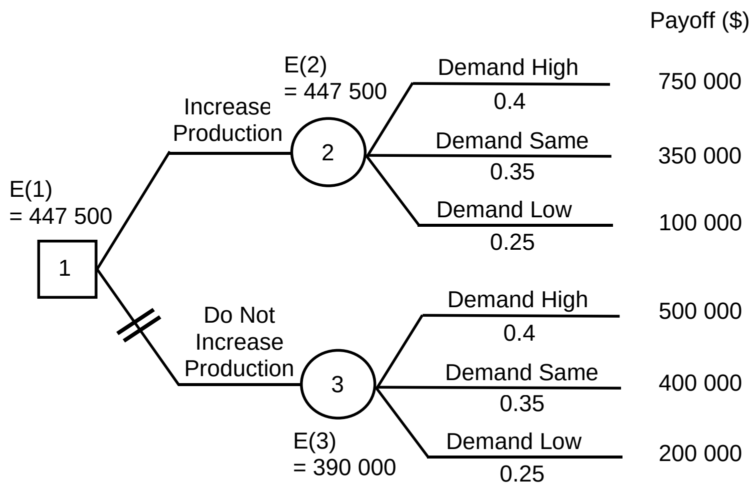
(c) Tree diagram:



E(1) = E(2) = 4000 because E(2) < 5000

Randall should send the customer one from stock.

12.14 Decision Tree diagram:



E(2) = Pr{high demand}(750 000) + Pr{medium demand}(350 000)
+ Pr{low demand}(100 000)
= 0.4(750 000) + 0.35(350 000) + 0.25(100 000)
= 447 500

E(3) = 0.4(500 000) + 0.35(400 000) + 0.25(200 000) = 390 000

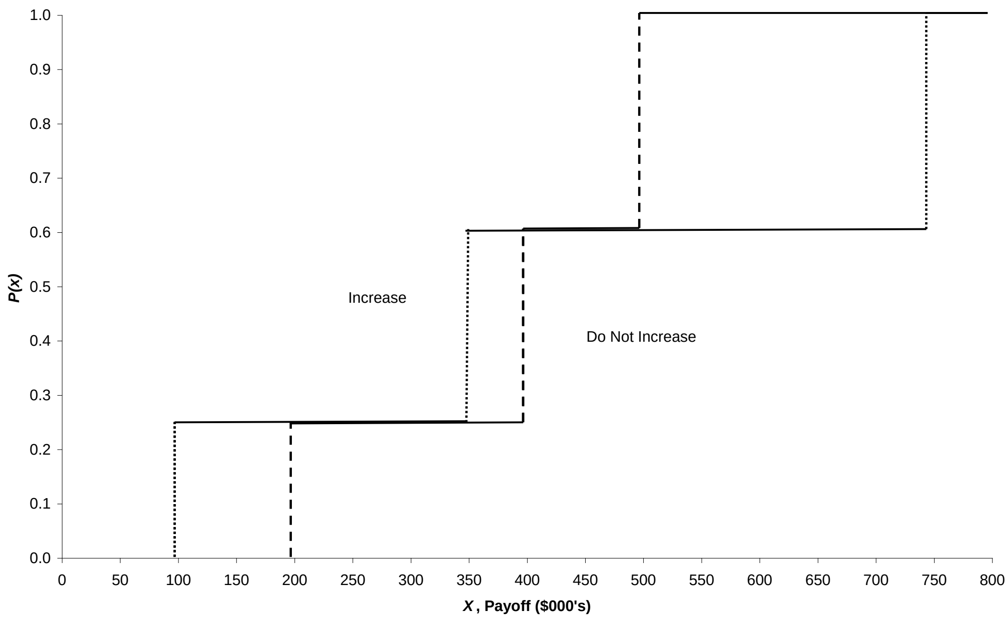
E(1) = E(2) = 447 500 because E(2) > E(3)

SJCF should increase their production.

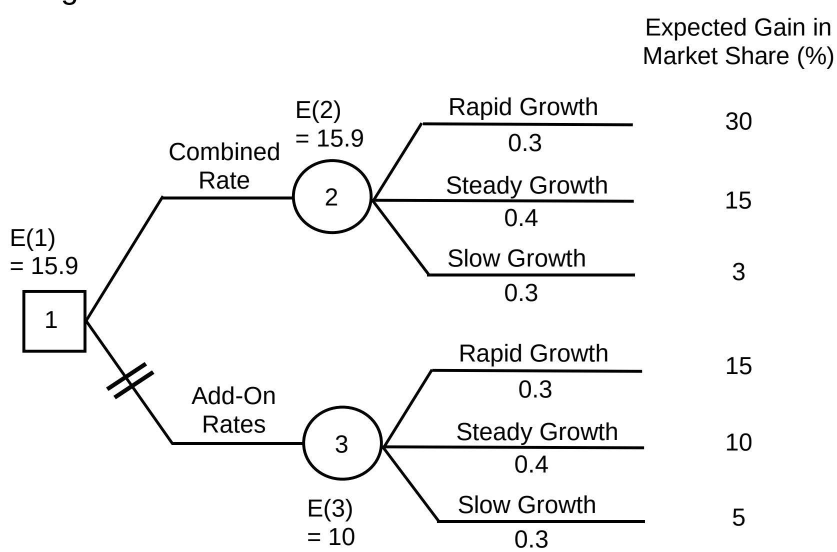
(b) The cumulative risk profile for the two decision alternatives are in the figure below.

Outcome dominance does not exist because one decision is not better than the other for all outcomes (e.g. for the outcome demand low, “do not increase production” is better, but for the outcome demand high, “increase production” is better.)

First degree stochastic dominance does not exist. This can be seen from the cumulative risk profiles below, where Pr(demand ≤ X) is greater for the “increase production” decision for all values of X up to $500, but not for all values of X.



12.15 Tree diagram:



E(2) = Pr{rapid growth}(30) + Pr{steady growth}(15) + Pr{slow growth}(3)
= 0.3(30) + 0.4(15) + 0.3(3) = 15.9

E(3) = 0.3(15) + 0.4(10) + 0.3(5) = 10

E(1) = E(2) = 15.9 because E(2) > E(3)

LOTell should introduce the combined rate.

(b) Several types of dominance reasoning can be applied: mean-variance, outcome or stochastic dominance. Each is covered below:

Mean-variance dominance:

$$\begin{aligned}\text{Var(combined rate)} &= \Pr\{\text{rapid growth}\}(30 - 6.9)^2 \\ &\quad + \Pr\{\text{steady growth}\}(15 - 6.9)^2 + \Pr\{\text{slow growth}\}(3 - 6.9)^2 \\ &= 0.3(533.61) + 0.4(65.61) + 0.3(15.21) = 31\ \$^2\end{aligned}$$

$$\begin{aligned}\text{Var(add-on rates)} &= \Pr\{\text{rapid growth}\}(15 - 5.5)^2 \\ &\quad + \Pr\{\text{steady growth}\}(10 - 5.5)^2 + \Pr\{\text{slow growth}\}(5 - 5.5)^2 \\ &= 8\ \$^2\end{aligned}$$

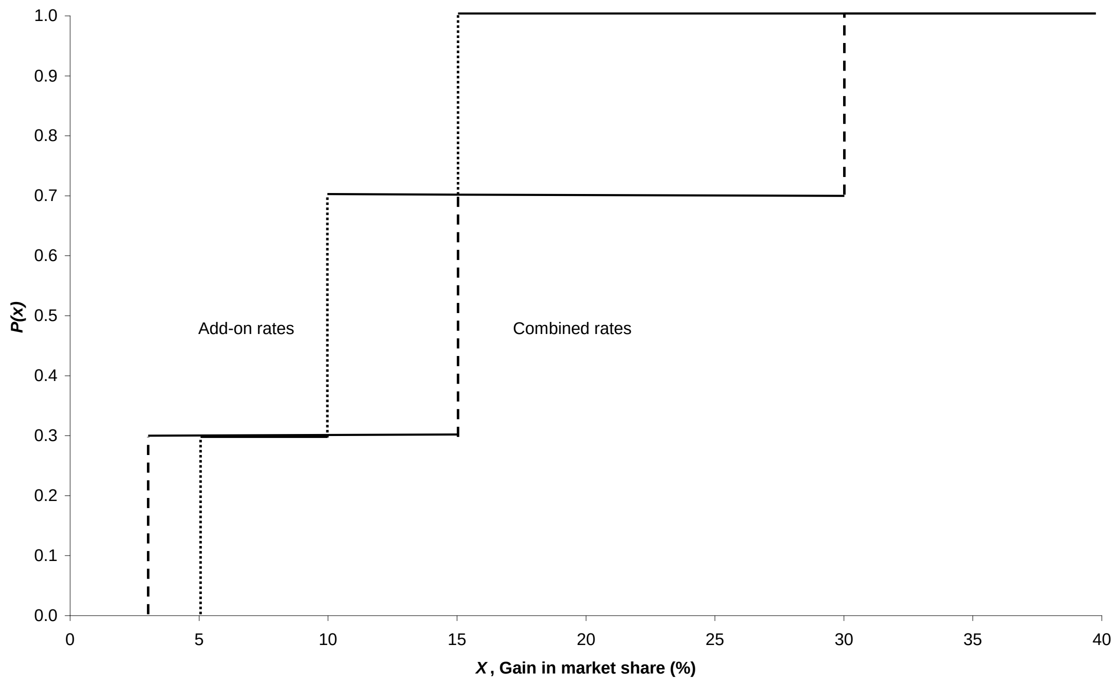
One of the two decisions cannot be eliminated with mean-variance reasoning because the combined rate decision has the higher expected gain in market share, but the add-on rates decision has the lower (better) variance.

Outcome dominance:

Observe that the “add-on rate” decision has a better outcome for slow growth, but the “combined rate” decision has a better outcome for both steady and rapid growth. Since one of the decisions is not better than the other for all possible outcomes, outcome dominance cannot be used to eliminate either decision.

Stochastic Dominance:

The Cumulative risk profiles (CDFs) for the two decisions are shown in the figure below. The “combined rate” decision is dominated for market share outcomes up to 5% (i.e. has a higher probability that demand is less than or equal to X for $X \le 5\%$), but the “add-on rate” decision is dominated for outcomes over 5% market share. Thus, neither decision dominates the other in a first degree stochastic dominance sense.



B. Applications

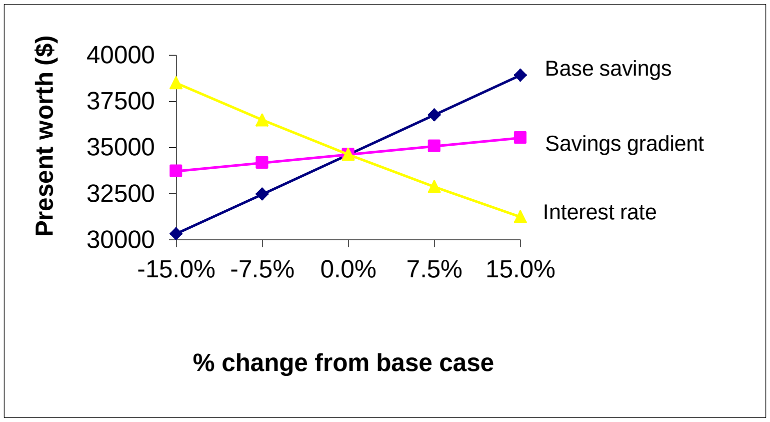
12.16 The present worth of the software when the base annual savings are = 10 000, the gradient is G = 1000, and interest rate is i = 15%:

PW = [10 000 + 1000(A/G, 15%, 6)](P/A, 15%, 6)(P/F, 15%, 2) = 34 617

The other computations are as follows:

Parameter	**-15%**	**-7.5%**	**Base Case**	**7.5%**	**15%**
Base savings	8500	9250	10000	10750	11500
Savings gradient	850	925	1000	1075	1150
Interest rate	12.75%	13.88%	15.00%	16.13%	17.25%
Present Worth:	**-15%**	**-7.5%**	**0%**	**7.5%**	**15%**
Changes to base savings per year	30325	32471	34617	36764	38910
Changes to savings gradient	33717	34167	34617	35068	35518
Changes to interest rate	38484	36485	34617	32872	31238

The sensitivity graph is:



12.17 First, note that all cost figures are given in real dollars, as they do not take into account the effect of inflation.

The municipality uses an actual interest rate of 7% when inflation is expected to be 3%, and hence, their real MARR is:

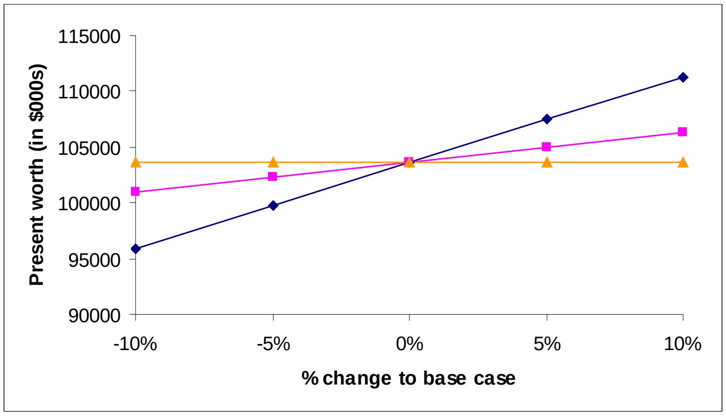
$MARR_R = (1 + MARR_A)/(1 + f) – 1 = 1.07/1.03 – 1 = 0.0388$ or 3.88%

Since all costs are based on current estimates, it is simplest to work with real dollars and the real MARR for the analysis. Note that even if the City's estimates of inflation change by 5% or 10%, the real MARR they use will not change and hence the present worth of the project (in real dollars) will be unaffected by the inflation rate.

(a) The present worth of the project is $360 204.

Parameter	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
Annual Construction Costs (in $000s)	18000	19000	20000	21000	22000
Annual Maint. and Rep. costs (in $000s)	1800	1900	2000	2100	2200
Inflation Rate	3.30%	3.15%	3.00%	3.15%	3.30%
Present Worth (in $000s)	**-10%**	**-5%**	**0%**	**5%**	**10%**
Changes to annual construction costs	95949	99784	103619	107454	111289
Changes to annual maint. costs	100927	102273	103619	104965	106310
Changes to inflation rate	103619	103619	103619	103619	103619

(b) The sensitivity graph is as follows:



The present worth of the project is most sensitive to changes in the annual maintenance and repair costs.

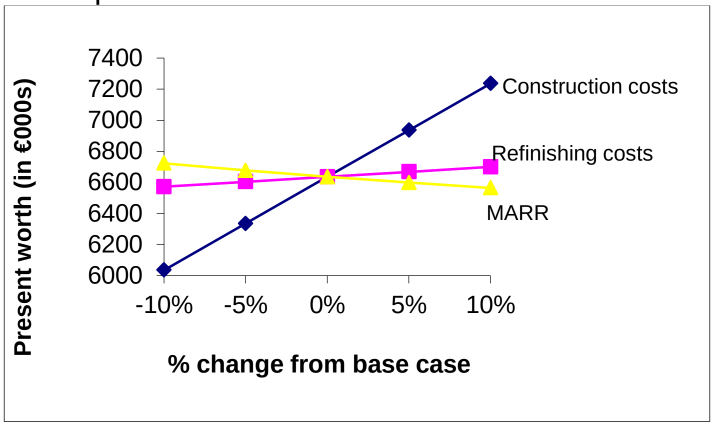
12.18 The present worth of the pool (in $000s) with the "base case" costs and MARR is:

PW = 6000 + [400(A/F, 5%, 10)]/0.05

With one at a time 5% and 10% variations in the first cost, maintenance costs and the MARR, the present worth of costs are:

Parameter	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
Construction costs (in $000s)	5400	5700	6000	6300	6600
Refinishing costs (in $000s)	360	380	400	420	440
MARR	4.50%	4.75%	5.00%	5.25%	5.50%
Present worth of costs (in $000s)	**-10%**	**-5%**	**0%**	**5%**	**10%**
Changes to construction costs	6036	6336	6636	6936	7236
Changes to refinishing costs	6572	6604	6636	6668	6700
Changes to MARR	6723	6677	6636	6599	6565

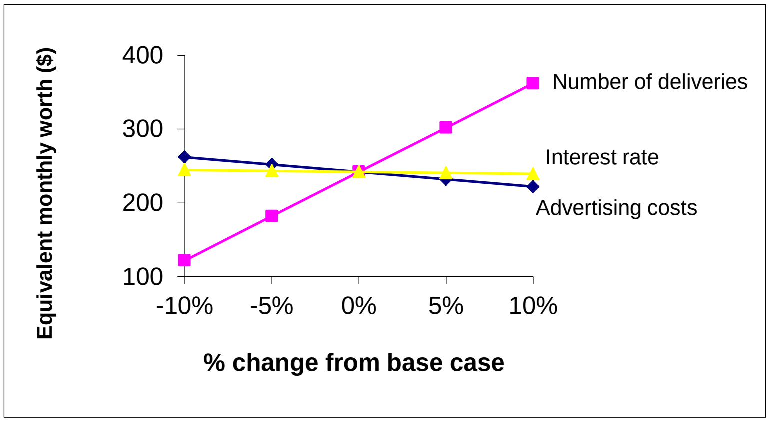
The sensitivity graph shows that the present worth is most sensitive to the first cost of the pool:



12.19 Sample Equivalent Monthly Worth (EMW) Computation (for base case):

EMW = (Monthly Revenues) – (Monthly Costs)
= 300×2 – 100 – [(6000 – 3000)(A/P, 8%/12, 24) + 3000(0.08/12)]
– 600(A/P, 8%/12, 6)
= 242

Parameter	-20%	-10%	Base Case	10%	20%
Monthly Advertising Costs	80	90	100	110	120
Number of Customers per month	240	270	300	330	360
Interest rate (% per month)	0.533%	0.600%	0.667%	0.733%	0.800%
Sensitivity Graph Information	**-10%**	**-5%**	**0.0%**	**5%**	**10%**
Changes to Advertising Costs	262	252	242	232	222
Changes to Number of Deliveries	122	182	242	302	362
Changes in Interest Rate	245	243	242	241	239



The monthly worth is most sensitive to changes in the number of deliveries per month.

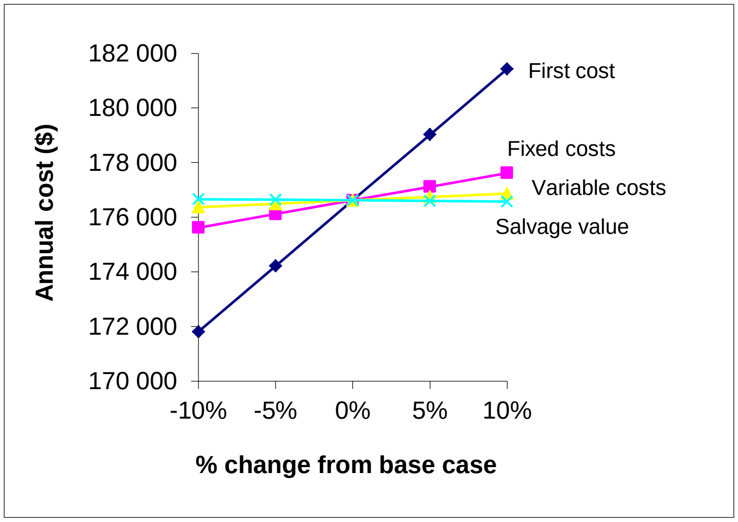
12.20 **(a)** A summary of the costs for Vendor A's device is below at the base value and for 5% and 10% increases and decreases.

Also given is the annual worth of costs for Vendor A's device. A sample computation for the base case is:

AW = 200 000(A/P, 15%, 7) + 10 000 + 6500
+ (0.05 + 0.95 + 1.25)(50 000) – 5000(A/F, 15%, 7)
= 176 620

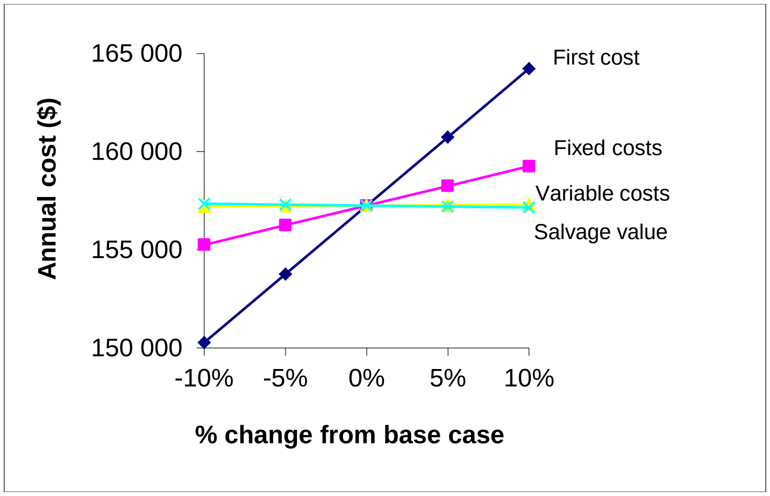
Parameter	-10%	-5%	Base Case	5%	10%
First Cost	180 000	190 000	200 000	210 000	220 000
Annual Maintenance Cost	9 000	9 500	10 000	10 500	11 000
Maintenance cost/unit	0.045	0.048	0.050	0.053	0.055
Labour cost/unit	1.13	1.19	1.25	1.31	1.38
Annual other costs	5 850	6 175	6 500	6 825	7 150
Other Costs/unit	0.86	0.90	0.95	1.00	1.05
Salvage Value	4 500	4 750	5 000	5 250	5 500
Annual costs	**-10%**	**-5%**	**0%**	**5%**	**10%**
Changes in First Cost	171 813	174 217	176 620	179 024	181 427
Changes in Fixed Costs	175 620	176 120	176 620	177 120	177 620
Changes in Variable costs	176 370	176 495	176 620	176 745	176 870
Changes in Salvage Value	176 665	176 643	176 620	176 598	176 575

A sensitivity graph indicating the sensitivity of the annual worth of Vendor A's device to changes in the costs shows that the annual worth is most sensitive to changes in the first cost and then to changes to the fixed costs (maintenance and other).



(b) Vendor B's computations and sensitivity graph are below. As with Vendor A, the annual worth is most sensitive to the first cost and to the fixed annual costs. Also observe that at expected annual production levels, Vendor B's device has a lower annual worth.

Parameter	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
First Cost	315 000	332 500	350 000	367 500	385 000
Annual Maintenance Cost	18 000	19 000	20 000	21 000	22 000
Maintenance cost/unit	0.009	0.010	0.010	0.011	0.011
Labour cost/unit	0.45	0.48	0.50	0.53	0.55
Annual other costs	13 950	14 725	15 500	16 275	17 050
Other Costs/unit	0.50	0.52	0.55	0.58	0.61
Salvage Value	18 000	19 000	20 000	21 000	22 000
Annual cost	**-10%**	**-5%**	**0%**	**5%**	**10%**
Changes in First Cost	150 279	153 766	157 253	160 740	164 227
Changes in Fixed Costs	155 253	156 253	157 253	158 253	159 253
Changes in Variable costs	157 203	157 228	157 253	157 278	157 303
Changes in Salvage Value	157 352	157 302	157 253	157 204	157 155



12.21 **(a)** Let D = the number of deliveries per month. Solve for D in:

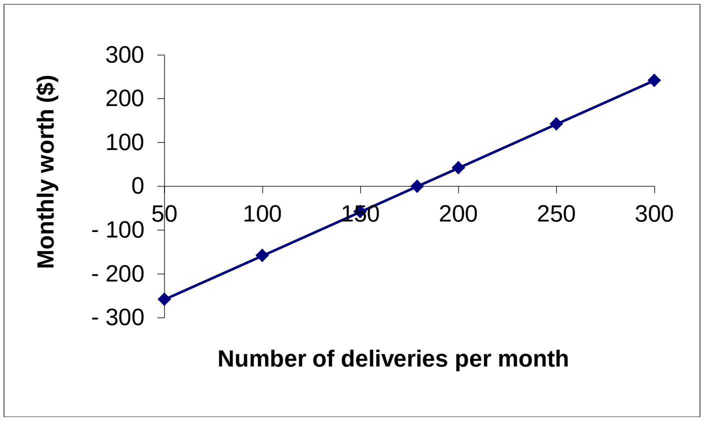
0 = EMW = (Monthly Revenues) – (Monthly Costs)

= 2D – 100 – [(6000 – 3000)(A/P, 8%/12, 24) + 3000(0.08/12)]

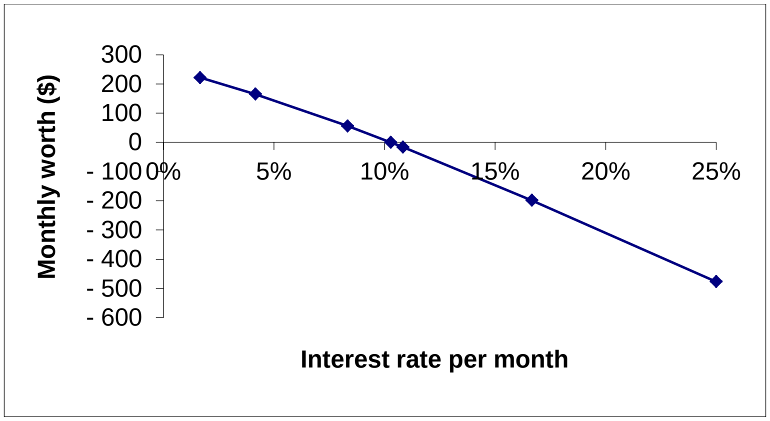
– 600(A/P, 8%/12, 6)

D = 179

The break-even number of deliveries is 179.



(b) Solving for the interest rate is most easily done with a trial and error approach on a spreadsheet. The break-even interest rate is 10.29% *per month*, or 223.5% per year.



12.22 **(a)** A summary of the costs and benefits and the annual worth computations for Alternative A is as follows:

Costs and Benefits:	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
Initial hardware and installation costs	83 250	87 875	92 500	97 125	101 750
Annual Benefits	58 500	61 750	65 000	68 250	71 500
Annual Worths:	**-10%**	**-5%**	**0%**	**5%**	**10%**
Initial hardware and installation costs	11 516	10 595	9 673	8 751	7 830
Annual Benefits	3 173	6 423	9 673	12 923	16 173

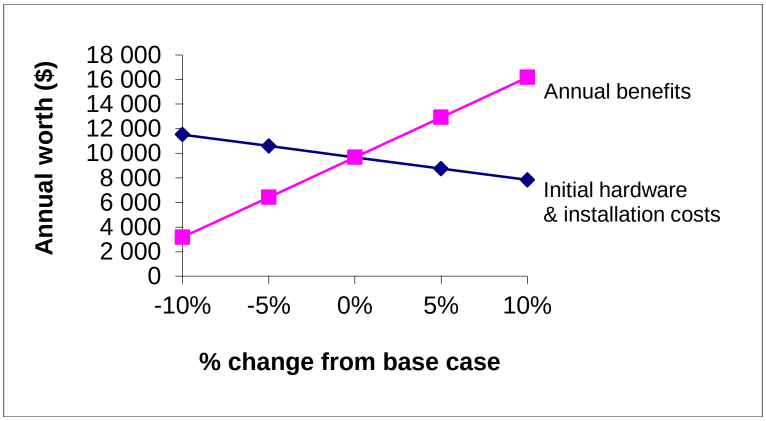
A sample computation for the annual worth of the base case is:

AW(base case)
= 65 000 – [(138 750 + 92 500)(A/P, 15%, 10) + 9250] = 9673

and for the annual worth if the initial hardware and installation costs are 10% higher than the base case:

AW(base case)
= 65 000 – [(138 750 + 101 750)(A/P, 15%, 10) + 9250] = 7830

A sensitivity graph of these results is:

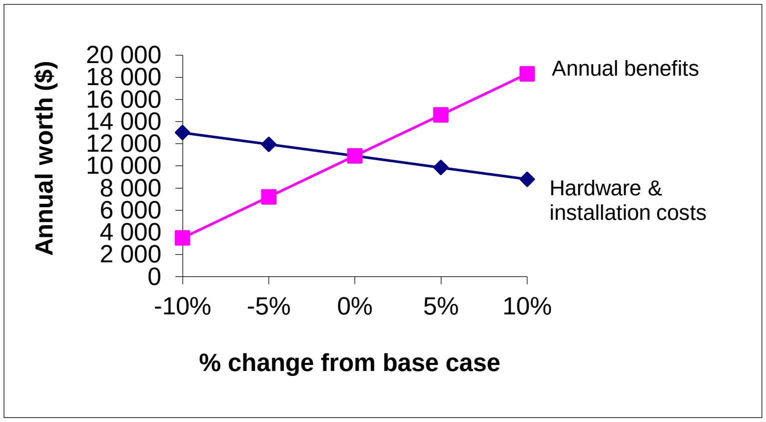


The annual worth is most sensitive to changes in the annual benefits of the network.

(b) A summary of the costs and benefits and the annual worth computations for Alternative B is as follows:

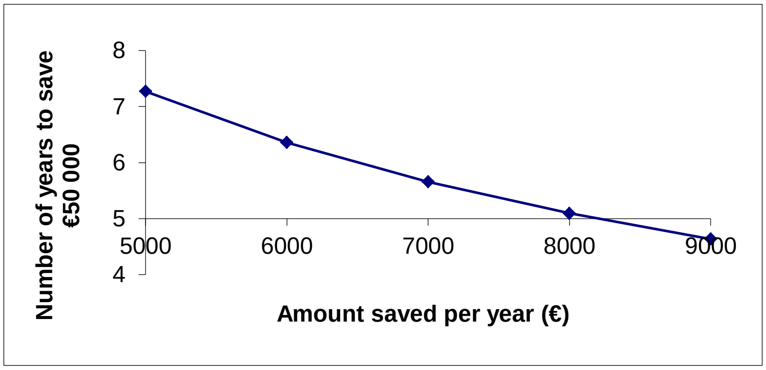
Costs and Benefits ($):	**-10%**	**-5%**	**Base Case**	**5%**	**10%**
Initial hardware and installation costs	94 950	100 225	105 500	110 775	116 050
Annual Benefits	66 600	70 300	74 000	77 700	81 400
Annual Worths ($)	**-10%**	**-5%**	**0%**	**5%**	**10%**
Initial hardware and installation costs	12 999	11 948	10 897	9 846	8 795
Annual Benefits	3 497	7 197	10 897	14 597	18 297

And the corresponding sensitivity graph is:



12.23 A graph showing the number of years required to save $50 000 as a function of the amount saved per year is below.

With annual savings of A and an interest rate i = 0.15, the number of years, N, to save F = $50 000 is N = ln[(iF + A)/A]/ln(1 + i).



The break-even annual savings amount is $8190 (this was obtained with trial and error with the spreadsheet table using the amount saved per year as the input variable). The Go-Kart Klub will need to raise $1190 each year (in addition to the $7000) if they wish to save $50 000 in 5 years.

12.24 First the PW of acquiring the molder and keeping it over its 6-year life is the sum of its capital costs, and operating and maintenance costs:

PW(capital) $= 27\ 000 - BV_{db}(6)(P/F, 15\%, 6)$
$= 27\ 000 - 20\ 000(1 - 0.4)^6(0.4323) = 26\ 596.61$

The present worth of the operating and maintenance costs over the 6-year life can be obtained with the use of a geometric gradient series to present worth conversion factor:

PW(op. and maint.) = 30 000(P/A, i°, 6)/(1 + g)
where i° = (1 + i)/(1 + g) – 1 = 1.15/1.05 – 1 = 0.09524

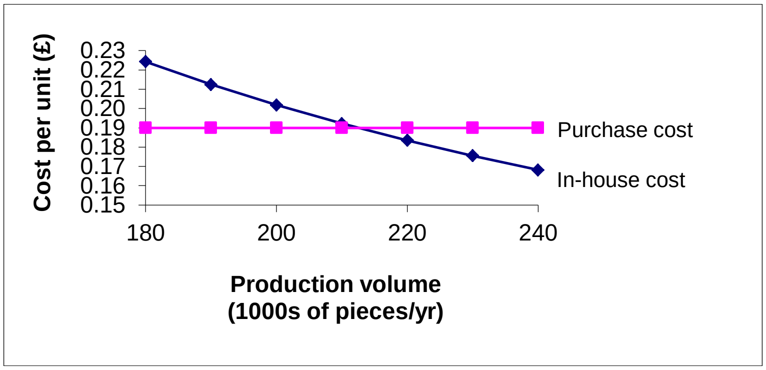
PW(op. and maint.) = 30 000(4.4167)/1.05 = 126 191.43

This gives a present worth of cost for the 6-year planned life of the molding equipment of $150 788 = 126 191.43 + 24 596.61.

The equivalent annual cost can be found from:

EAC(molder) = 152 788(A/P, 15%, 6) = 152 788(0.2642) = 40 366.59

The cost per piece is then EAC(molder)/production volume. Using this, we can construct the break-even analysis. The break-even quantity is 212 500. Below this annual production quantity, Bountiful should continue to purchase from outside. Above this quantity, they should buy the molding equipment.

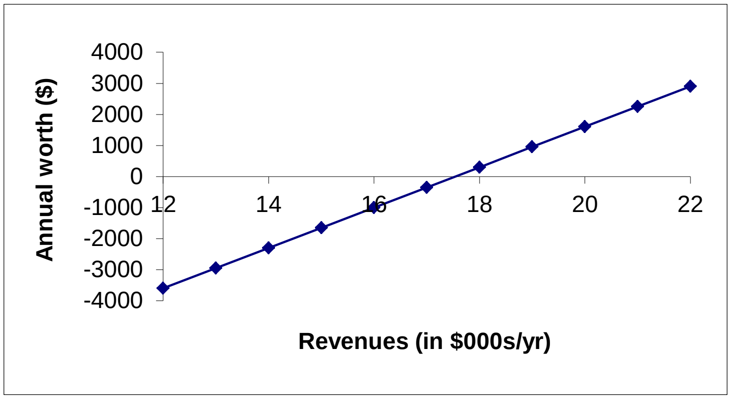


12.25 Under Canadian tax rules, assuming a CCA rate of 30%, first, we need to calculate the Tax Benefit Factor:
TBF = 1 – td/(i + d) = 1 – (0.35)(0.3)/(0.12 + 0.30) = 0.75

AW(first cost) = [–65 000

+ (65 000/2xTBF + 65 000/2(P/F, 12%, 1))(A/P, 12%, 5)
= –13 758
AW(savings) = (Annual Savings)(1 – 0.35)
AW(salvage) = 20 000(1 - TBF)(A/F, 12%, 5)
= 20 000(0.75)(0.15741) = 2361
AW(truck) = –13 758 + 2361 + (Annual Savings)(0.65)

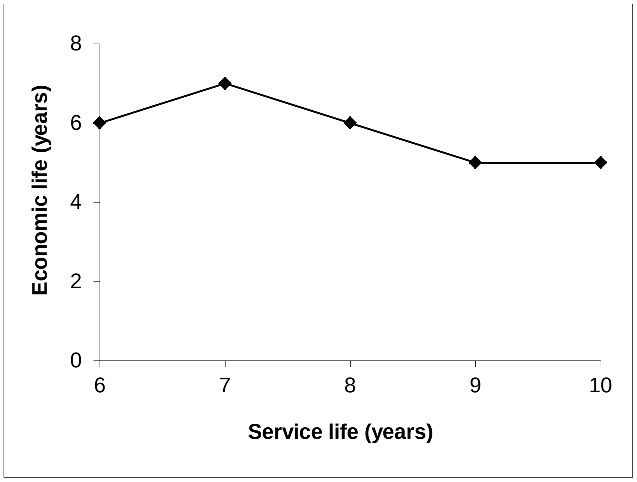
With a spreadsheet, the annual worth of the truck for annual revenues from $12 000 to $22 000 can be calculated. The break-even revenue per year is $17 535.



12.26 **(a)** EAC calculation summary:

Service Life	6	7	8	9	10
Year	**EAC(total)**	**EAC(total)**	**EAC(total)**	**EAC(total)**	**EAC(total)**
1	20 067	19 114	18 400	17 844	17 400
2	17 519	16 621	15 947	15 423	15 004
3	16 733	15 887	15 252	14 758	14 363
4	16 385	15 588	14 990	14 525	14 153
5	16 209	15 459	14 897	**14 460**	**14 110**
6	**16 117**	15 413	**14 885**	14 474	14 146
7		**15 412**	14 916	14 531	14 222
8			14 972	14 610	14 321
9				14 703	14 432
10					14 549

(b) Sensitivity graph:



The sensitivity graph indicates that the economic life is somewhat sensitive to the length of service life. For a service life of 6 or 7 years, it is economical to keep the bottle capping machine until the end of its service life. For a service life longer than 7 years, it is more economical to keep the machine for a shorter period than the service life.

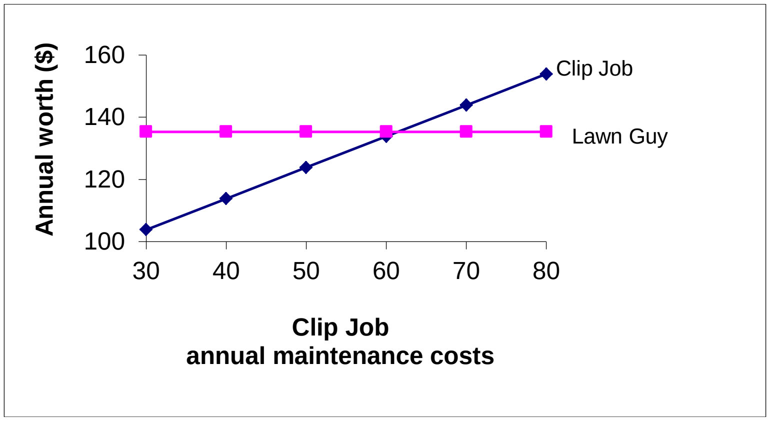
12.27 The annual worth of the Clip Job for annual maintenance costs varying from \$30 to \$80 is as follows:

Annual Maintenance	Annual Worth
30	103.84
40	113.84
50	123.84
60	133.84
70	143.84
80	153.84

For example, with annual maintenance costs of \$50, the annual worth of the Clip Job is:
AW(clip job) = 120(A/P, 5%, 4) + 40 + 50 = 120(0.282) + 90 = 123.84

The annual worth of the Lawn Guy is:
AW(lawn guy) = 350(A/P, 5%, 10) + 60 + 30 = 350(0.1295) + 90 = 135.33

And the maintenance cost for the Clip Job which makes Sam indifferent between the two lawnmowers is $61.50. A break-even graph illustrates the break-even maintenance costs:



Since Sam estimates the maintenance costs to be about $60 per year, and the break-even maintenance cost is close to this amount, we would recommend that Sam purchase the Lawn Guy if he is risk averse and wants to avoid the possibility of $80 maintenance costs for the Clip Job.

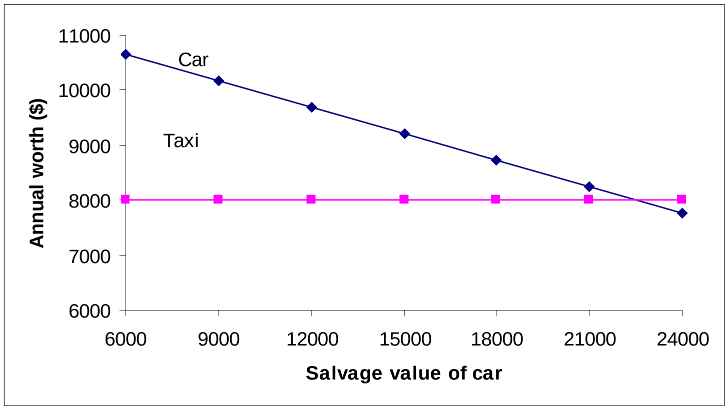
12.28 The annual worth of the car for salvage values varying from $6000 to $24 000 are:

Salvage Value	Annual Worth
6000	10647
9000	10165
12000	9684
15000	9202
18000	8720
21000	8239
24000	7757

For example, the annual worth with a salvage value for $9000 is:

$$\begin{aligned} AW(car) &= 24\,000(A/P, 11\%, 5) + (2000 + 600 + 800 + 1000) \\ &\quad + 400(A/G, 11\%, 5) - 9000(A/F, 11\%, 5) \\ &= 24\,000(0.27057) + 4400 + 400(1.7923) - 9000(0.16057) \\ &= 10\,165 \end{aligned}$$

The annual costs of taking taxis are $8000 (=$7000 + $1000). Based on an annual worth comparison, Ganesh should not buy the car. The salvage value of the car would have to exceed the break-even value of $22 486 before the car will have equal or lower costs than that of taking taxis. Taking into account only the financial aspects of the decision, Ganesh should not buy the car unless he feels there are other benefits (e.g. convenience of having a car) that have not been taken into account.



12.29 E(monthly savings)
= 0.25(800 000) + 0.25(1 000 000) + 0.25(1 200 000) + 0.25(1 400 000) = 1 100 000

PW = 1 100 000(P/A, 1%, 24) = 1 100 000(21.243) = 23 367 000

The present worth of the expected monthly savings is about $23 367 000.

12.30 E(revenue)
= 0.1(2.95) + 0.35(3.25) + 0.3(3.50) + 0.15(4.00) + 0.1(5.00)
= $3.58 per parcel

Regional's monthly capacity is 60 000 parcels. So the present worth of the expected revenue over 12 months is:

PW = (3.58×60 000 – 8000)(P/A, 1%, 12)
= (206 800)(11.255) = $2 327 534

12.31 E(life) = p(4 years)(48 months) + p(5 years)(60 months) + p(6 years)(72 months)
= 0.4(48) + 0.4(60) + 0.2(72) = 57.6 months

Using the expected life of 57.6 months, the present worth of the monthly expenses is computed as follows:

PW = (90 + 30 + 20)(P/A, 10/12%, 57.6)
$= 140[(1 + 0.008333)^{57.6} – 1]/[0.008333(1 + 0.008333)^{57.6}]$
= (140)(45.599) = 6383.86

The present worth of the monthly expenses is about €6384 before a major repair.

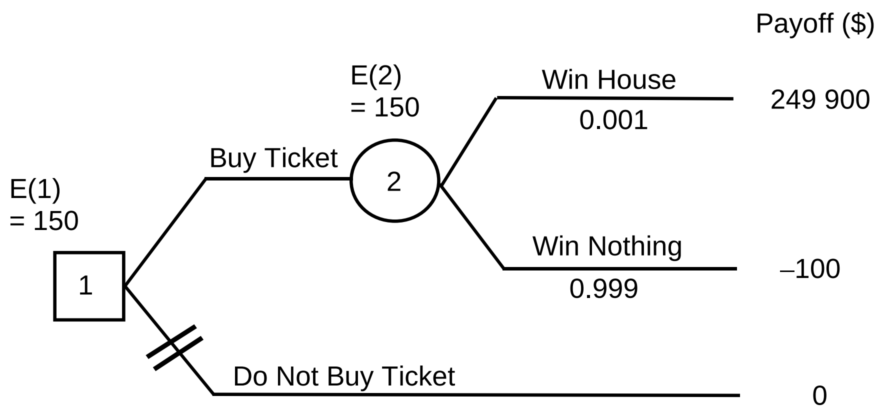
12.32 First, find the annual cost for each scenario:

AC(optimistic) = 75 000
AC(expected) = 240 000(A/F, 15%, 2) = 240 000(0.46512) = 111 628.80
AC(pessimistic) = 500 000(A/F, 15%, 3) = 500 000(0.28798) = 143 990

E(annual cost)
= p(optimistic)(75 000) + p(expected)(111 628.80)
+ p(pessimistic)(143 990)
= 0.15(75 000) + 0.5(111 628.80) + 0.35(143990)
= 117 460.90

The expected annual cost of the vitamin C project is $117 461.

12.33 Tree diagram:



E(2) = p(not win)(–100) + p(win)(250 000 – 100)
= 0.999(–100) + 0.001(249 900)
= 150

E(1) = E(2) = 150 because E(2) > 0

Buying a ticket has a greater expected value than not buying a ticket.

12.34 **(a)** If E(2) = 0.999X + 0.001(250 000 – X) = 0.998X + 250 < 0, then not buying is preferred.

0.998X + 250 < 0
0.998X < –250
X < –250.50

Hence, if the ticket costs more than $250.50, not buying is the preferred option.

(b) If E(2) = (–100)(1 – X) + 249 900X < 0, then not buying is preferred.

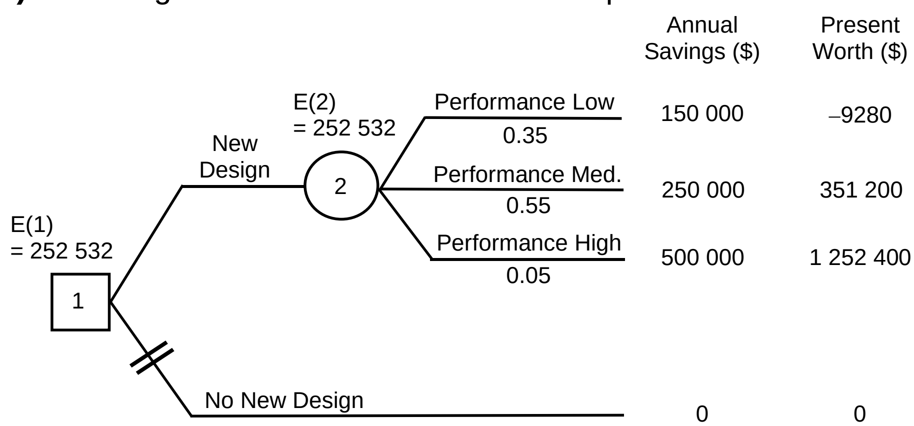
(–100)(1 – X) + 249 900X < 0
–100 + 100X + 249 900X = –100 + 250 000X < 0
250 000X < 100
X < 0.00039984

Hence, if the probability of winning is less than 0.04%, not buying is the preferred option.

12.35 **(a)** PW(high performance) = –550 000 + 500 000(P/A, 12%, 5)
= –550 000 + 500 000(3.6048)
= 1 252 400

PW(medium performance) = –550 000 + 250 000(P/A, 12%, 5) = 351 200
PW(low performance) = –550 000 + 150 000(P/A, 12%, 5) = –9280

(b) Tree diagram: Probabilities do not add up to 100%.

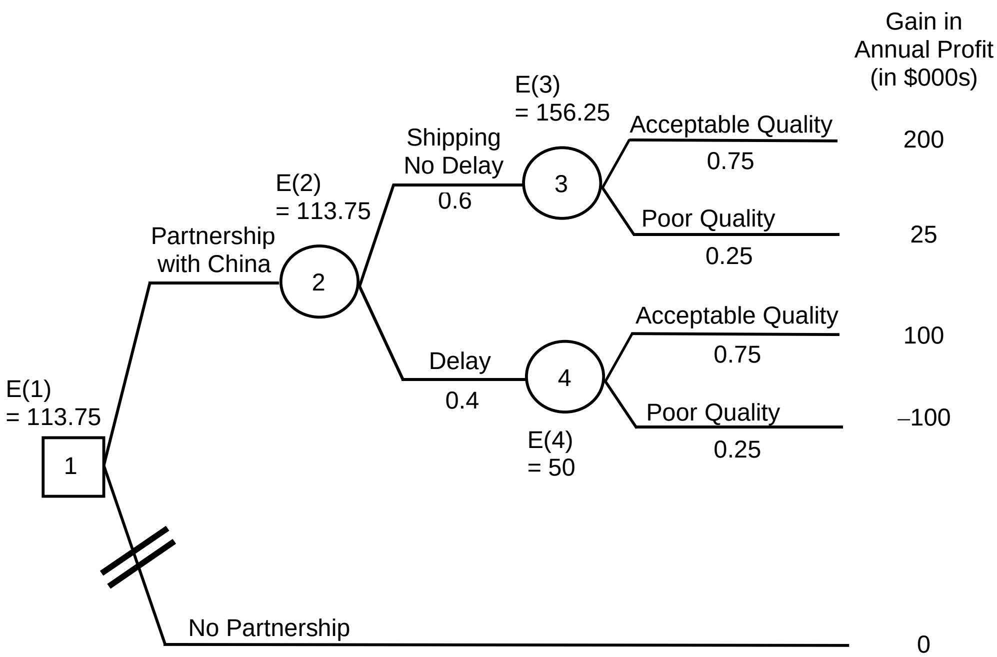


E(2) = p(high performance)(1 252 400)
+ p(medium performance)(351 200) + p(low performance)(–9280)
= 0.05(1 252 400) + 0.55(351 200) + 0.35(–9280)
= 252 532

E(1) = E(2) = 252 532 because E(2) > 0

BB should approve the development of the new robot.

12.36 Tree diagram:



E(3) = p(acceptable quality)(200 000) + p(poor quality)(25 000)
= 0.75(200 000) + 0.25(25 000)
= 156 250

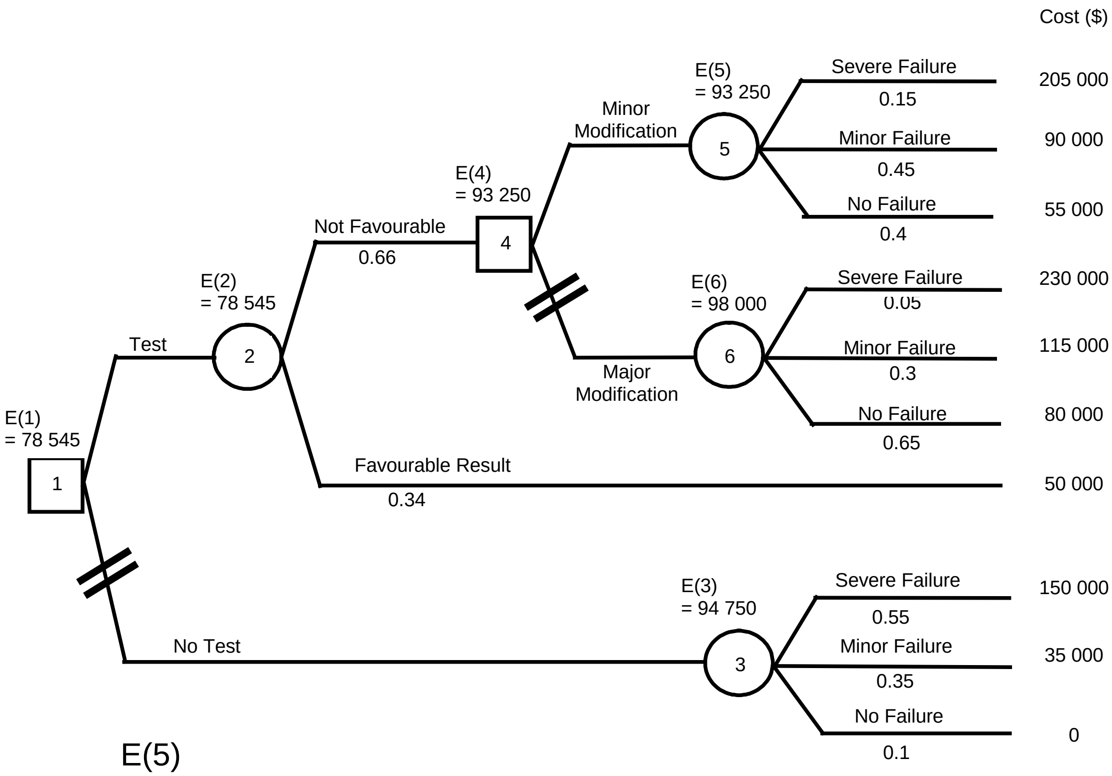
E(4) = p(acceptable quality)(100 000) + p(poor quality)(–100 000)
= 0.75(100 000) + 0.25(–100 000)
= 50 000

E(2) = p(no shipping delay)E(3) + p(shipping delay)E(4)
= 0.6(156 250) + 0.4(50 000)
= 113 750

E(1) = E(2) = 113 750 because E(2) > 0

The partnership with China is still recommended as a result of analyzing shipping delay and quality control possibility.

12.37 Tree diagram:



E(5)
= p(severe failure)(test, modification, and failure costs)
+ p(minor failure)(test, modification, and failure costs)
+ p(no failure)(test and modification costs)
= 0.15(50 000 + 5000 + 150 000) + 0.45(50 000 + 5000 + 35 000)
+0.4(50 000 + 5000)
= 93 250

E(6)
= 0.05(50 000 + 30 000 + 150 000) + 0.3(50 000 + 30 000 + 35 000)
+0.65(50 000 + 30 000)
= 98 000

E(4) = E(5) = 93 250 because E(5) < E(6)

E(2) = p(not favourable)E(4) + p(favourable)(test costs)
= 0.66(93 250) + 0.34(50 000)
= 78 545

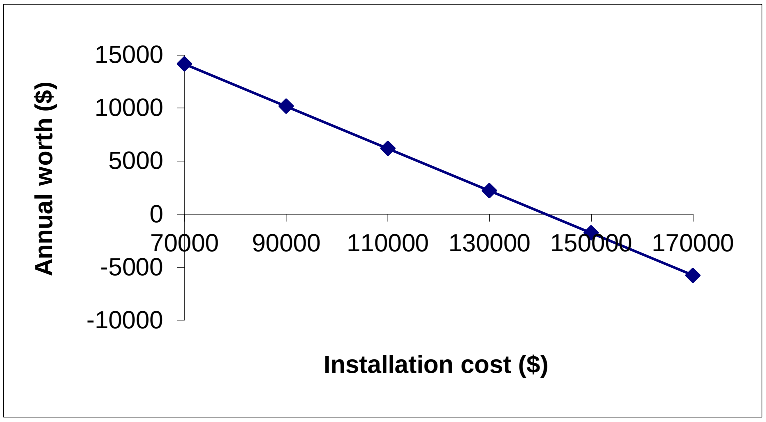
E(3) = p(severe failure)(failure costs) + p(minor failure)(failure costs)
+ p(no failure)(no costs)
= 0.55(150 000) + 0.35(35 000) + 0.1(0)
= 94 750

E(1) = E(2) = 78 545 because E(2) < E(3)

According to decision tree analysis, Rockies should test the upgraded system, and if the result is not favourable, Rockies should apply the minor modification to the system.

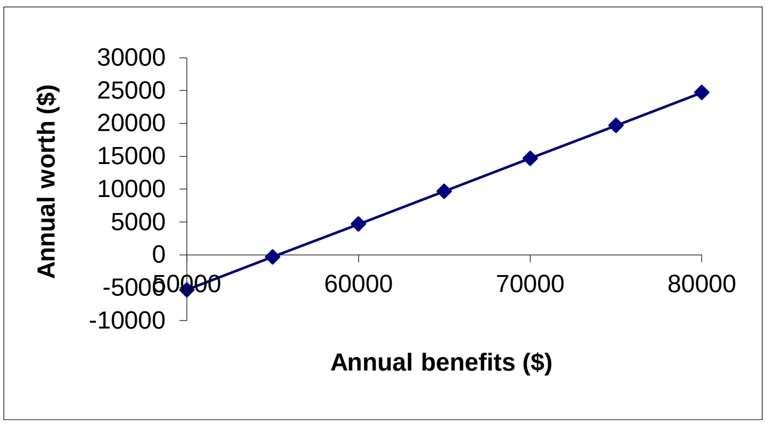
C. More Challenging Problems

12.38 **(a)** The break-even installation cost for Alternative A is $141 045. A break-even chart is shown below:



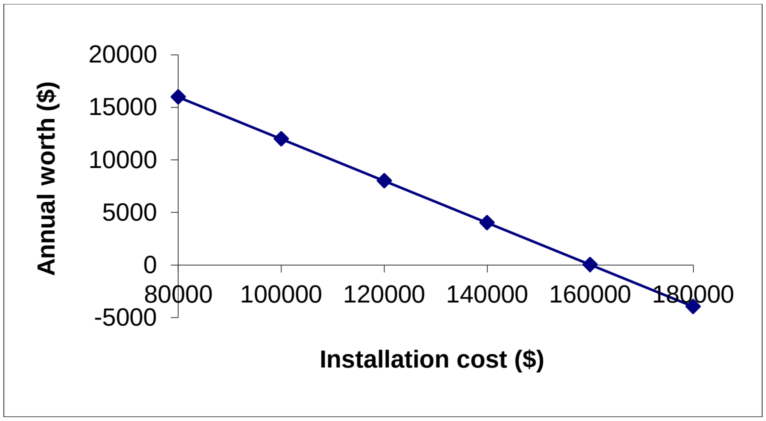
The break-even installation cost is outside the range Merry Metalworks has estimated.

(b) The break-even annual cost for Alternative A is $55 327. A break-even chart is shown below:

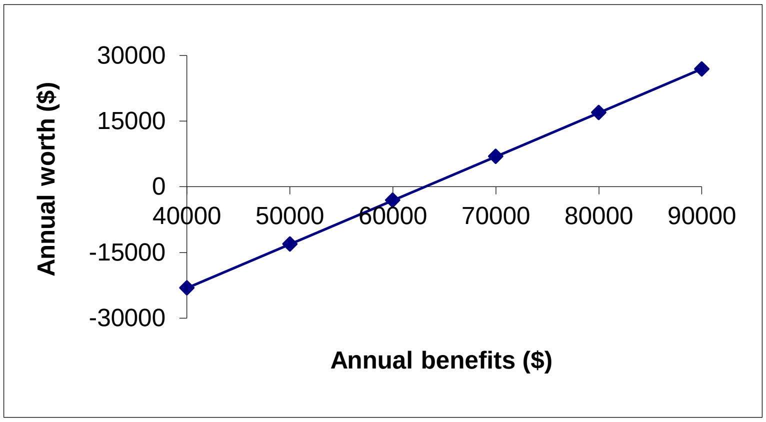


12.39 **(a)** The break-even installation cost for Alternative B is $160 190, which is well above the range of estimated costs.

A break-even chart showing various annual worths is below:

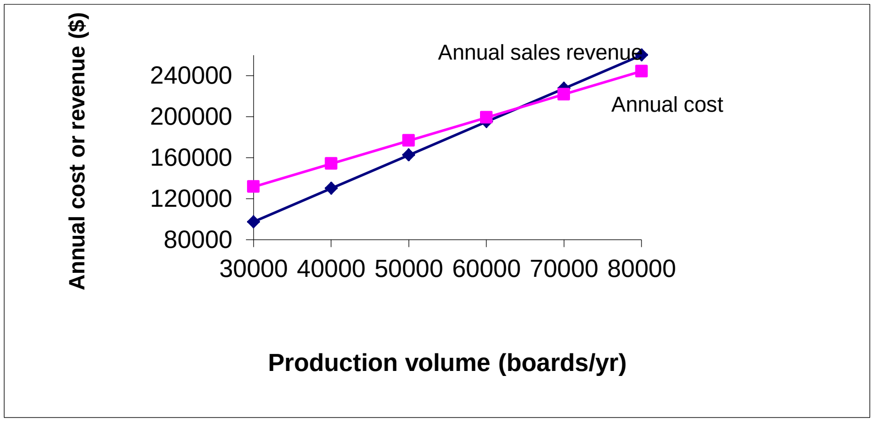


(b) The break-even annual benefits for Alternative B are $63 103. This is within the range of benefits Merry Metalworks has specified, so there is some risk that Alternative B will yield a negative present worth. The break-even chart showing various annual benefits is below:

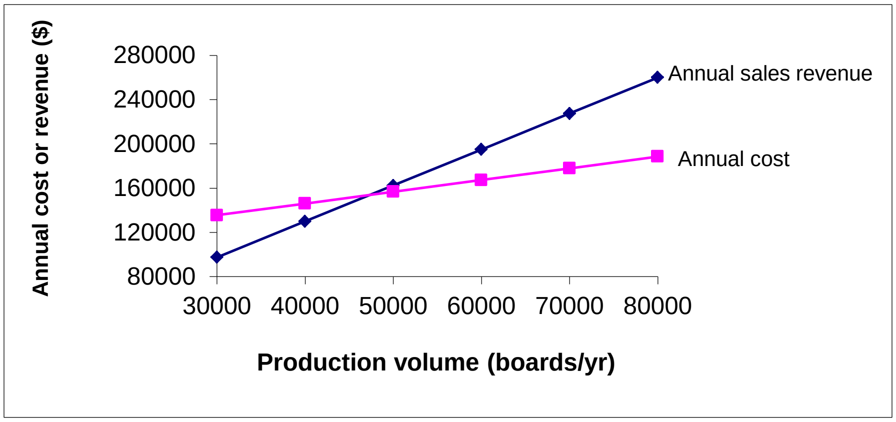


12.40 **(a)** The break-even production volume with a per-unit revenue of $3.25 for Vendor A is 64 120 (obtained by interpolation).

This can be seen in the following break-even graph:



(b) For Vendor B, the break-even volume is 47 375 boards/year.



12.41 Since the lease is an annual amount, it is easiest to compare the two alternatives based on annual costs. This solution assumes Canadian tax rules and a 20% CCA rate.

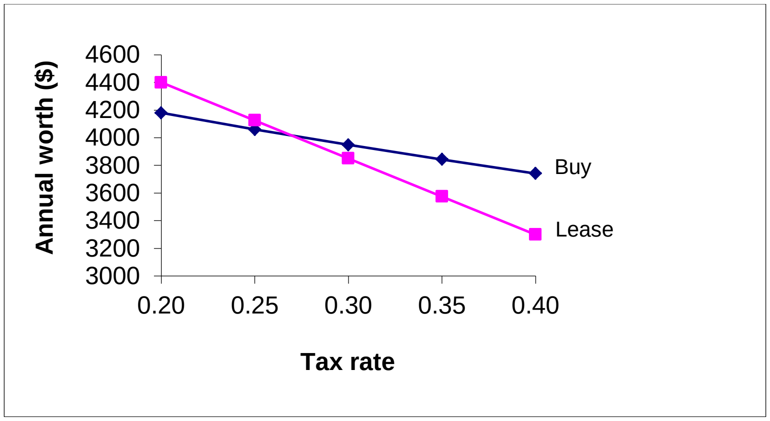
First, the annual worth of the lease decision is $5500 per year, after taxes this is $5500(1 – t) where t is the tax rate. The annual worth of costs for the buy decision depends on Kelly's tax rate.

Tax rate	AW(buy)	TBF	AW(lease)
0.20	4180	0.143	4400
0.25	4060	0.152	4125
0.30	3949	0.158	3850
0.35	3843	0.163	3575
0.40	3742	0.167	3300

For example, at a tax rate of 30% the annual worth of costs for the two alternatives, taking taxes into account is:

AW(buy)
= [15 000 - (15 000/2xTBF + 15 000/2xTBF(P/F, 8%, 1))](A/P, 8%, 5) + [1000 + 400(A/G, 8%, 5)](1 – t) – 2500(1-TBF)
= [15000 - (750x0.158 + 750x0.158x0.54027](0.25046)) + [1000 + 400(1.846 47)](1 – 0.3) – 2500(0.842) = 3949

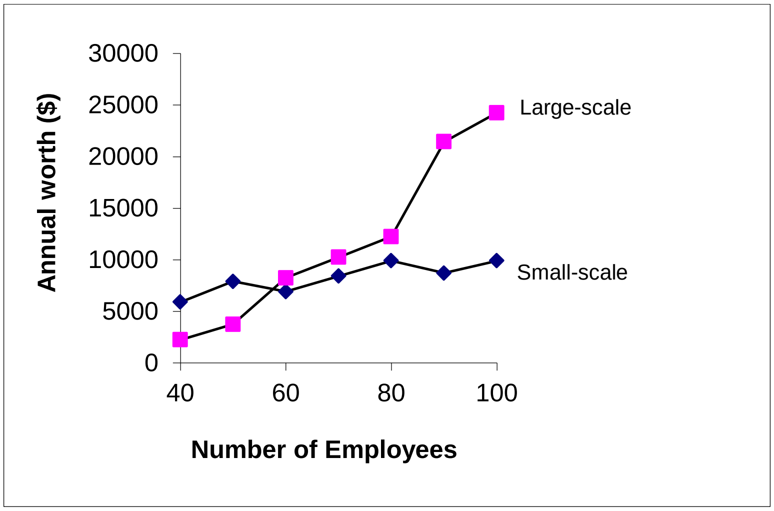
AW(lease) = 5500(1 – t) = 3850
We can see from the table that Kelly should lease if his tax rate is below 27% and buy if he expects his tax rate to be above 27%.



12.42 Break-even graph:

AW(Small) = –(6000 + 1500)(A/P, 12%, 5)
+ (annual savings per employee)×(#employees)

AW(Large) = –(10 000 + 3500)(A/P, 12%, 5)
+ (annual savings per employee)×(#employees)



The small-scale version would be a better choice if Western Insurance wasn't expecting much growth in the next 5 years. However, they are growing. If the growth continues steadily for the next 5 years, the large-scale version seems to be a better choice since it has a greater annual worth than the small-scale version does as long as the average number of employees is greater than 60.

12.43 **(a)** From Problem 12.32:

AC(optimistic) = 75 000
AC(expected) = 111 628.80
AC(pessimistic) = 143 990

AW(optimistic, public accept)
= 1 000 000(A/F, 15%, 1) – AC(optimistic)
= 1 000 000(1) – 75 000
= 925 000

AW(optimistic, not accept)
= 200 000(A/F, 15%, 1) – AC(optimistic)
= 125 000

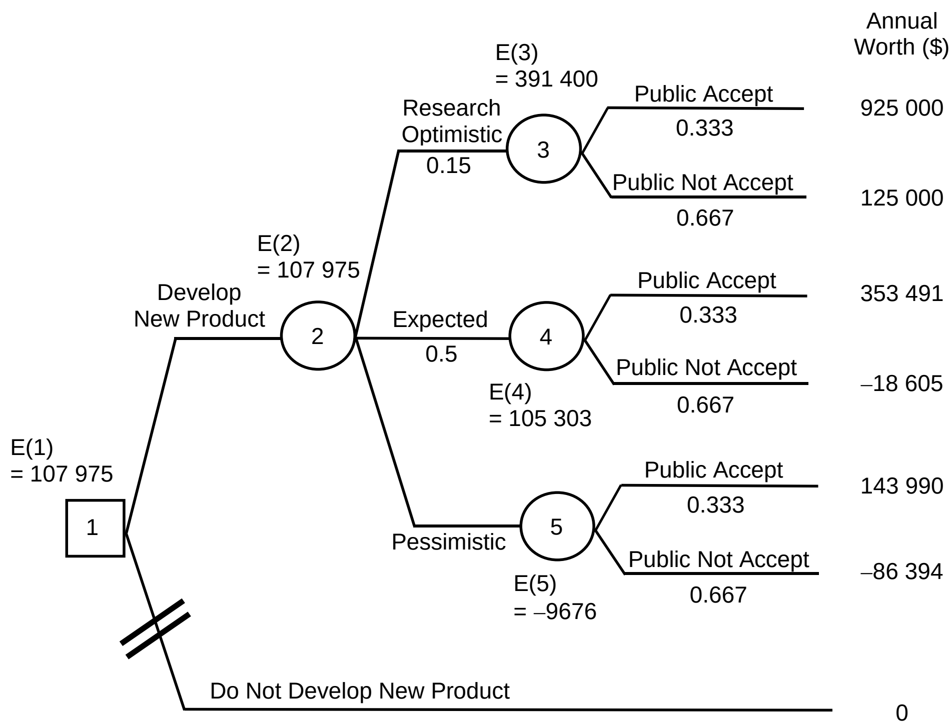
AW(expected, public accept)
= 1 000 000(A/F, 15%, 2) – AC(expected)
= 1 000 000(0.46512) – 111 628.80
= 353 491.20

AW(expected, not accept)
= 200 000(A/F, 15%, 2) – AC(expected)
= –18 604.80

AW(pessimistic, public accept)
= 1 000 000(A/F, 15%, 3) – AC(pessimistic)
= 1 000 000(0.28798) – 143 990
= 143 990

AW(pessimistic, not accept)
= 200 000(A/F, 15%, 3) – AC(pessimistic)
= –86 394

(b) Tree diagram:



$$\begin{aligned} E(3) &= p(\text{public accept})(925\,000) + p(\text{not accept})(125\,000) \\ &= 0.333(925\,000) + 0.667(125\,000) \\ &= 391\,400 \end{aligned}$$

$$E(4) = 0.333(353\,491.20) + 0.667(–18\,604.80) = 105\,303.17$$

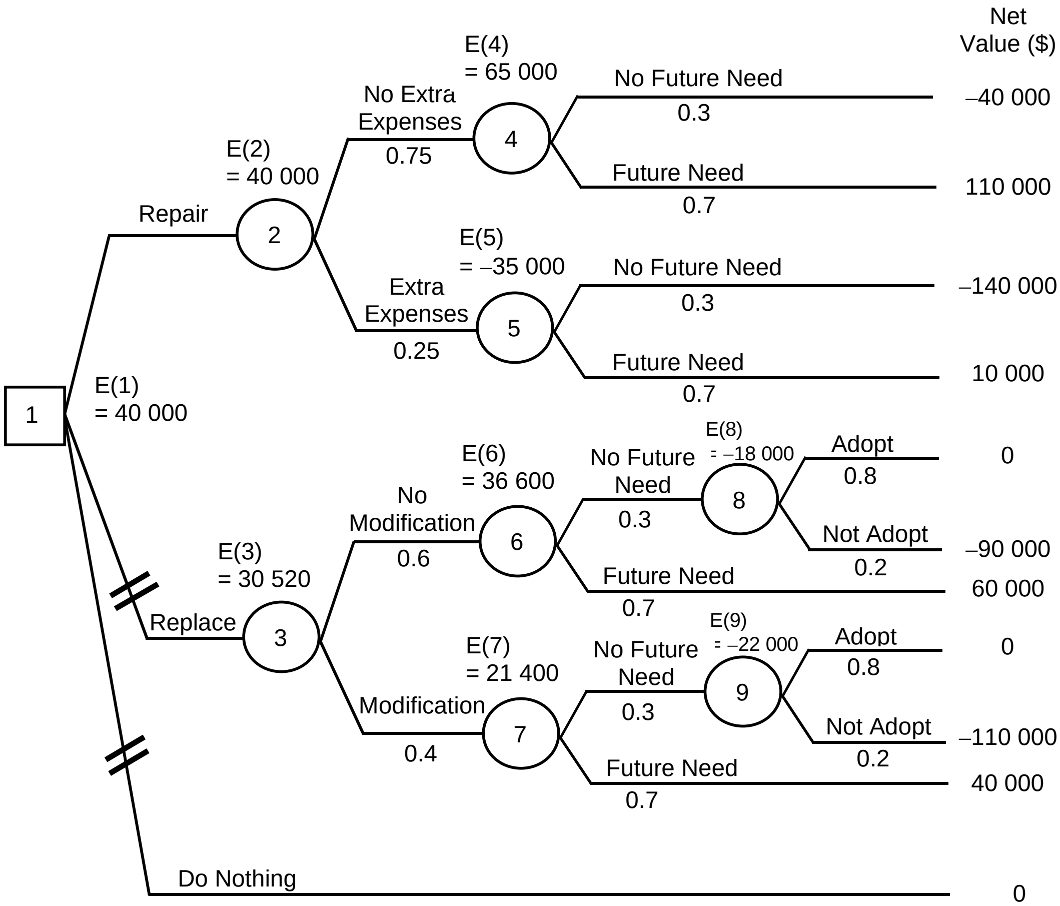
$$E(5) = 0.333(143\,990) + 0.667(–86\,394) = –9676.13$$

$$\begin{aligned} E(2) &= p(\text{optimistic})E(3) + p(\text{expected})E(4) + p(\text{pessimistic})E(5) \\ &= 0.15(391\,400) + 0.5(105\,303.17 + 0.35(–9676.13) \\ &= 107\,974.94 \end{aligned}$$

$E(1) = E(2) = 107\,974.94$ because $E(2) > 0$

Pharma-Excel should proceed with the development of the new product.

12.44 Tree diagram:



Based on the tree diagram, the net value for each terminal position (from top to bottom of the tree) is calculated as follows:

Terminal position 1 = cost of repair = –40 000
Terminal position 2 = PW(benefit) – (cost of repair) = 110 000
Terminal position 3 = (cost of repair) + (extra expenses) = –140 000
Terminal position 4 = PW(benefit) – (costs of repair & extra expenses)
= 10 000
Terminal position 5 = complete recovery of investment = 0
Terminal position 6 = cost of replacement = –90 000
Terminal position 7 = PW(benefit) – (cost of replacement) = 60 000
Terminal position 8 = complete recovery of investment = 0
Terminal position 9 = (cost of replacement) + (cost of modification)
= –110 000
Terminal position 10 = PW(benefit) – (replacement&modification costs)
= 40 000
Terminal position 11 = do nothing = 0

E(8) = p(adopt)(0) + p(not adopt)(–90 000) = –18 000
E(9) = p(adopt}(0) + p(not adopt)(–110 000) = –22 000

E(4) = p(not needed)(–40 000) + p(needed)(110 000) = 65 000
E(5) = p(not needed)(–140 000) + p(needed)(10 000) = –35 000
E(6) = p(not needed)E(8) + p(needed)(60 000) = 36 600
E(7) = p(not needed)E(9) + p(needed)(40 000) = 21 400

E(2) = p(no extra expenses)E(4) + p(extra expenses)E(5) = 40 000
E(3) = p(no modification)E(6) + p(modification)E(7) = 30 520

E(1) = E(2) = 40 000 because E(2) > E(3) and E(2) > 0

BBB should repair the production line.

12.45 The Result of Sensitivity Analysis

Probability Asian demand ↑	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
Probability Asian demand ↓	**0.9**	**0.8**	**0.7**	**0.6**	**0.5**
E(4)	0.8755	1	1.125	1.25	1.375
E(5)	–0.85	–0.7	–0.55	–0.4	–0.25
E(6)	0.525	0.55	0.575	0.6	0.725
E(7)	0.28	0.23	0.24	0.22	0.2
E(2) = 0.6E(4) + 0.4E(5)	0.185	0.32	0.455	0.59	0.725
E(3) = 0.6E(6) + 0.4E(7)	0.427	0.422	0.441	0.448	0.515
E(1) = max{E(2), E(3)}	0.427	0.422	0.455	0.59	0.725
Partnership or no partnership?	no	no	partner	partner	partner

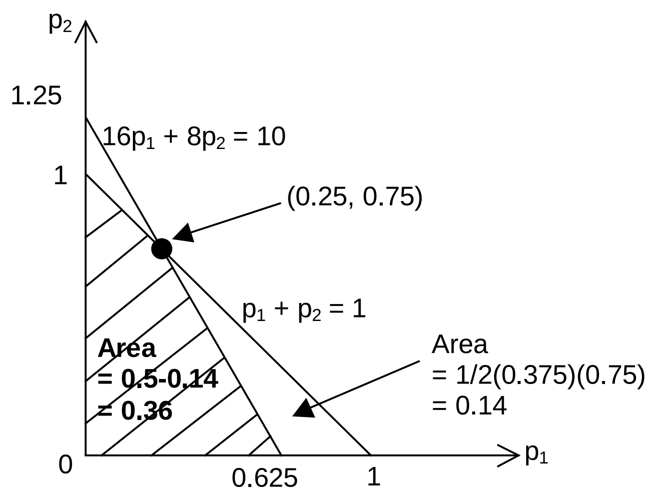
As long as the probability that Asian demand increases is greater than or equal to 0.3, then the partnership with China seems to be the preferred option. Also, the higher the probability, the more confidence in the partnership option.

12.46 **(a)** E(2) = p(rapid growth)E(3) + p(steady growth)E(4) + p(slow growth)E(5)

$= 16p_1 + 8p_2 + 0p_3$

$= 16p_1 + 8p_2$

(b) Graphically, all possible values of p_1 and p_2 are represented by the area that satisfy $p_1 + p_2 \leq 1$, $p_1 \geq 0$, and $p_2 \geq 0$. Furthermore, the values of p_1 and p_2 that *also* satisfy $16p_1 + 8p_2 < 10$ are represented by the shaded region in the graph.



From the graph, we can identify the estimate of probabilities that imply introducing the package now is the preferred option (see the shaded region). For example, $(p_1, p_2) = (0.25, 0.5)$ would lead to the decision to introduce now. Note that whenever the values of p_1 and p_2 do not add up to 1, then it implies that $p_3 = 1 - p_1 - p_2$. Also from the graph, we have the information that roughly 0.36/0.5 = 72% of all possible (p_1, p_2) values indicate that introducing the package now is the preferred option. Another observation is that, for (p_1, p_2) values with high p_1 value (e.g., $p_1 > 0.625$), the option to wait for the survey result is preferred. The possible interpretation for this decision is, if the market growth is likely to be rapid, then waiting for 3 months would not jeopardize LOTell's opportunity to gain more market share even in the presence of the competitors.

12,47

US Salary	75000
US Salary Inflation	5.00%
Number of staff:	20
Turnover Rate:	25.00%per year
Recruiting and Training Costs	0.7 per person
Salary	600000 per year
Salary Inflation	18.21%
Post-training productivity	70.00%
Productivity while training	50.00%
Project Management Costs	15.00%
Conversion to $	40
MARR	0.25%
Outsourcing setup:	400,000
PW(Benefits of Outsourcing)	2,935

Parameter Values			PW(Benefits of Outsourcing)		
-10%	Base Case	10%	-10%	Base Case	10%
4.50%	5.00%	5.50%	937,992	1,064,322	1,193,606
11.25%	12.50%	13.75%	1,260,899	1,064,322	855,758
22.50%	25.00%	27.50%	1,203,150	1,064,322	918,041
63.00%	70.00%	77.00%	1,116,097	1,064,322	1,012,547
45.00%	50.00%	55.00%	969,083	1,064,322	1,142,244
63.00%	70.00%	77.00%	596,900	1,064,322	1,446,758
13.50%	15.00%	16.50%	1,095,200	1,064,322	1,033,443
22.50%	25.00%	27.50%	1,247,302	1,064,322	901,575
36.00	40.00	44.00	505,063	1,064,322	1,521,897
360,000	400,000	440,000	1,104,322	1,064,322	1,024,322

It appears that the productivity rate and the exchange rate have the greatest impact on the PW of outsourcing. Next is the salary growth rate in India.

The exchange rate needs to go to about 33.01INR per USD for the PW(benefits) to drop to zero.
The productivity needs to drop to about 55.86% for the PW(benefits) to drop to zero.
The growth rate needs to be about 18.2% for the PW(benefits) to drop to zero.

Appendix 12A Solutions

12A.1 (a) The decision matrix:

Criteria	Weight	A	B	C	D	E	F	G
C1		81900	36500	31800	31100	28200	16100	11500
Normalized	1.5	10.0	3.6	2.9	2.8	2.4	0.7	0.0
C2		25500	10300	11600	10500	7400	3500	7100
Normalized	2.0	10.0	3.1	3.7	3.2	1.8	0.0	1.6
C3		65	12.2	45	35	30	10	28
Normalized	2.5	0.0	9.6	3.6	5.5	6.4	10.0	6.7
C4		8.6	3	6.3	14.1	6	11.8	4.5
Normalized	3.0	5.0	0.0	3.0	10.0	2.7	7.9	1.4

C5	1.0	3	6	7	6	10	4	4
Score		53.1	41.5	36.7	60.2	41.1	53.8	28.1

The best subway route from this data is D.

(b) The revised decision matrix:

Criteria	Weight	A	B	C	D	E	F	G
C1		81900	36500	31800	31100	28200	16100	11500
Normalized	2.0	10.0	3.6	2.9	2.8	2.4	0.7	0.0
C2		25500	10300	11600	10500	7400	3500	7100
Normalized	2.0	10.0	3.1	3.7	3.2	1.8	0.0	1.6
C3		65	12.2	45	35	30	10	28
Normalized	2.0	0.0	9.6	3.6	5.5	6.4	10.0	6.7
C4		8.6	3	6.3	14.1	6	11.8	4.5
Normalized	2.0	5.0	0.0	3.0	10.0	2.7	7.9	1.4
C5	2.0	3	6	7	6	10	4	4
Score		56.1	44.5	40.3	54.8	46.4	45.2	27.4

Yes. In this case, A is slightly better.

12A.2 **(a)** Only Jobs 1, 3, 7, and 8 can not be dominated.

		#1	#3	#7	#8
Criterion	Weight	Spinoff	Soutel	Ring	Jones
Pay	4	1700	2200	2200	2700
Home	2.5	2	80	250	500
Studies	2	3	4	5	3
Size	1.5	5	150	300	20

(b) The normalized decision matrix:

		#1	#3	#7	#8
Criterion	Weight	Spinoff	Soutel	Ring	Jones
Pay	4	0.0	5.0	5.0	10.0
Home	2.5	10.0	8.4	5.0	0.0
Studies	2	6.0	8.0	10.0	6.0
Size	1.5	10.0	5.1	0.0	9.5
	Scores:	52.0	64.7	52.6	66.2

With the given information, Job 8 scores the highest and is therefore the best.

Notes for Case-in-Point 12.1

1-4) These are all difficult questions and the answers depend on the views of the individual student.

Notes for Mini-Case 12.1

1) 10% does make sense as an abstract deviation for sensitivity analysis purposes because it provides a way to compare the effect of changes to different parameters. It can also be easily scaled to approximate other particular variation values. Although a change of 10% of the assumed value is a sensible amount for many parameters, for cost of borrowing, which is already a percentage, a 1% change is simply more meaningful. Again, it can be easily scaled to approximate other particular variations.

2) No. Although a 10% change in revenue had the largest effect on profit, it doesn't take in to account how likely a 10% change in revenue is compared to the chance of other parameters varying by 10%. For example, even though the effect of a 10% change on cost of sales is less, it may be much more likely to occur, making it the larger risk.

3) You would have to research the likelihood of each of the relevant parameters varying by a significant amount. You might focus particularly on the risks of changes to revenue, cost of sales and cost of borrowing, but all of the parameters would merit some deeper evaluation.

4) Examples include: researching land values, establishing building costs early, determining expected usage through surveys, etc., researching similar facility fee methods and revenues, get an early commitment on subsidies, etc.

Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

12S.1

Suppose you deposit $100 with the bank. The six equally likely possible outcomes are that you get back nothing, $110, $120,…, $150. So your expected return is:
(110 + 120 + 130 + 140 + 150)/6 = 108.3

This is equivalent to an interest rate of 8.3%.

12S.2

Consider the fate of a single ship. To build and provision it required 100 000 gold pieces. The three possible outcomes are that it is lost, that it returns empty—which is an outcome worth 50 000 gold pieces, since that is what it cost to build—or that it returns laden with merchandise—which outcome is worth 300 000 gold pieces, the value of the merchandise plus the ship itself.

So the expected value at the end of the year is
$0 \times 0.25 + 50\,000 \times 0.25 + 300\,000 \times 0.5 = 162\,500$

This represents a 62.5% return on your original investment.

As you increase the number of ships in your fleet, the rate of return remains the same, but the expected variance is reduced. There is less chance of finding yourself at the extreme ends of the distribution and either losing everything or getting a return much greater than 62.5%.

12S.3

You have a 50% chance of getting the first question right, so your expected income from the first round is
¥100 000 × 0.5 = ¥50 000

To get the ¥200 000 prize for the second round, you must have both passed the first round and got the second question right. The combined probability of this is $0.5 \times 0.5 = 0.25$, so your expected income in the second round is
¥200 000 ×0.25 = ¥50 000

By similar reasoning, your expected income in each subsequent round is ¥50 000, so your total expected income after ten rounds is ¥500 000, and this is what it would be rational to pay for a chance to play the game.
In the variant where there is no upper limit to the series of questions, it would at first appear that you have an infinite series of payouts, each of expected value ¥50 000, and that it would therefore be worth paying an infinite amount of money for the privilege of playing. However, this reasoning is not correct. Even Japanese game shows have an upper limit to their budget. For example, the maximum feasible payout is not likely to exceed ¥450 000 000 000, since this was Japan's Gross Domestic Product in 2006. But you would reach this payout after just 22 rounds of the game, so your maximum expected winnings would still only be 22 × ¥50 000 = ¥1 100 000, and it would not be rational to pay more than this to play.

12S.4

The first thing to do is to calculate the value of a launch now, three months from now, and so on. The value of a launch now is an infinite series of payments of $100 000/month, where the interest rate is 2%. The capitalized cost of this series is $100 000 / 0.02 = $5 000 000. The capitalized cost of the same series, starting 3 months later, is

$5 000 000(*P/F,0.02,3*) = 5 000 000 × 0.9423.

A six-month delay gives a capitalized cost of 5 000 000 × 0.888; nine months,
5 000 000 × 0.8368; a year, 5 000 000 × 0.7885.

(Alternatively, we could observe that all options lead to the same outcome at the end of one year. At that time, the satellite is up and earning; a one-year study period would be equally reasonable.)

So the expected value of a launch with the U.S. rocket is
−1 200 000 + 5 000 000(0.5 + 0.2 × 0.9423 + 0.1 × 0.888 + 0.05 × 0.8368 + 0.15 × 0.7885)
= −1 200 000 + 5 000 000 × 0.937 = 3.487 million.
Variance = $867 000

Using the Russian rocket gives us
−900 000 + 5 000 000(0.5 + 0.1 × 0.9423 + 0.4 × 0.7885)
= −900 000 + 5 000 000 × 0.9096= 3.648 million.
Variance = $1 009 000

Using the Chinese rocket gives us
−750 000 + 5 000 000(0.4 + 0.15 × 0.9423 + 0.1 × 0.888 + 0.05 × 0.8368 + 0.3 × 0.7885)

$= -750\ 000 + 5\ 000\ 000 \times 0.9085 = 3.792$ million.
Variance = \$1 084 000

Using the European rocket gives us
$-1\ 000\ 000 + 5\ 000\ 000(0.7 + 0.3 \times 0.7885)$
$= -1\ 000\ 000 + 5\ 000\ 000 \times 0.9365 = 3.682$ million.
Variance = \$960 000

Using the Japanese rocket gives us
$-1\ 500\ 000 + 5\ 000\ 000(0.9 + 0.05 \times 0.9423 + 0.05 \times 0.7885)$
$= -1\ 500\ 000 + 5\ 000\ 000 \times 0.9865 = 3.492$ million.
Variance = \$735 000

So the Chinese launcher looks like the best bet from the viewpoint of expected value.

Should we also take variance into account?

One way to think about this question is to consider that, if we change our choice in order to get a lower variance, we are paying a premium of several hundred thousand dollars to reduce our uncertainty. Why might we want to do this? Well, if the company is short of cash, and a twelve-month delay might cause the company to fail, we might reasonably attach a higher importance to staying in business than to maximizing expected profit. Secondly, in making plans for the year ahead, it would be convenient to be fairly sure what our cash flows are going to be. Are either of these considerations worth losing \$300 000 in expected profit? There is no algorithm that will tell us.

If our company's resources are large compared with the launch costs, and if we launch satellites on a regular basis, we will probably consider that variance in individual cases will average out in the long term, and hence make a decision based solely on expected value. However, if the company's entire resources are just sufficient for one launch, it may be worth paying a premium in exchange for a lower probability of going out of business.

This is one example of a case where it is worthwhile to distinguish the *utility* of a cash flow from its *value*. To explain this distinction, a comparison may be helpful. You are offered a chance to play one of two games. It costs ten cents to play either of them. Game 1 involves tossing a coin once; you get \$1 if it comes up heads. Game 2 involves tossing a coin twice; you get \$4 if there are 2 heads, otherwise you get nothing. Which game would you rather play?

Now consider two different games. It costs all you possess to play either of them. Game 3 involves tossing a coin once; you get ten times what you possess if it comes up heads. Game 4 involves tossing a coin twice; you get forty times what you possess if there are 2 heads, otherwise you get nothing. Which game would you rather play?

The idea here is that, while the value of a dollar is always a dollar, the *utility* of having ten dollars rather than nothing is much greater than the utility of having a million and ten dollars rather than a million; the first ten dollars may save you from starving, whereas ten dollars on top of a million just allows you to leave a larger tip. There is a branch of economics, *utility theory*, which provides a basis for including these considerations in decision-making; however, it is beyond the scope of this text.

12S.5

To answer this question, we need to choose a study period. How long do we expect the second company to be making this model of communications device? Mobile communications is a fast-changing field, so it may not be that long—certainly not as long as ten years. On the other hand, there is also the question of our reputation with the other company. If we maintain our reputation, they may use our batteries in the successor to the current device. Knowledge of the market would be useful here. Are there many other customers for our batteries, or does this customer dominate the market?

It might be reasonable to get a high and a low estimate. For the low estimate, assume that the other company is only one of a number of possible customers, and that they are only going to be making this model for another three years. For the high estimate, assume that we really need to maintain a good reputation with this company, and that fulfilling this order successfully may lead to repeat business for many years to come—ten years, for example.

<u>Three-Year Study</u>

Suppose the cost of inspecting a battery is c. Then our expected profit on inspected batteries is

$2000 \times (5-c) \times (P/A,0.25,3) = 2000 \times (5-c) \times 1.952 = 3904 \times (5-c)$

If on the other hand we choose not to do the inspection, we definitely get the first year's profit:
Profit in First Year, no inspection: $2000 \times 5 \times (P/F,0.25,1)$.

The chance that the first lot of batteries contains no defectives is $0.9999^{2000} = 0.819$. So we have a 0.819 chance of getting an additional $2000 \times 5 \times (P/F,0.25,2)$. And we have a 0.819×0.819 chance of getting an additional $2000 \times 5 \times (P/F,0.25,3)$. So our expected profit is

$2000 \times 5 \times ((P/F,0.25,1) + 0.819(P/F,0.25,2) + 0.671(P/F,0.25,3))$
$= 2000 \times 5 \times (0.8 + 0.819 \times 0.64 + 0.671 \times 0.512)$
$= 10\,000 \times 1.67 = 16\,700$
This gives the equation $3904 \times (5-c) = 16\,700$

which can be solved to give c = 0.73. So it is worth spending about $0.73 per battery on inspection.

Ten-Year Study

If we do a ten-year study, the expected profit on inspected batteries is

$$2000 \times (5-c) \times (P/A, 0.25, 10) = 2000 \times (5-c) \times 3.571 = 7142(5-c)$$

With no inspection, our expected profit is
$2000 \times 5 \times 0.75 \times (1 + \sum^{9}_{i=1} (0.819 \times 0.75)^{i})$
$= 7500 \times (1 + (1.6149-1) / (0.614 \times 1.6149))$
$=7500 \times 2.607 = 19\,550$
Solving for *c*, we get $2.26.

So our overall conclusion is that it is worth spending at least $0.75 per battery. If we are concerned about establishing a long-term relationship as a supplier to the other company, it may be worth spending up to $2.25 per battery.

12S.6

The following C source code is a working example of the Monte Carlo method, applied to the problem of calculating π.

```
main()
{
        int   i, j;
        float pi, piEst, realI, realJ, xSquared, ySquared;
        double  x,y;
        FILE *outdata;

   pi = 4.0 * atan(1.0);
   srand48(1.0);

   if ((outdata=fopen("monty3","w")) == NULL)
            {
            printf("failed to open the write file\n");
               exit(1);
            }

        j = 0;

        for (i=1; i<= 1000000; i++)
              {
                 x = 2 * drand48() - 1;
                 y = 2 * drand48() - 1;

                 xSquared = x * x;
                 ySquared = y * y;

                 if  ((xSquared + ySquared) < 1.0)j++;
```

```
                    realI = i;
                    realJ = j;

                    piEst = 4.0 * realJ/realI;

                    if (i%1000==0)fprintf(outdata,"%d %f  %f
\n",i,pi,piEst);
               }

}
```

You can copy this code to your own machine. To run it, you may need to replace the calls to srand48 and drand48 with a call to your local random-number generator.

To apply this method to an economics problem, you would replace x and y with inflation rate, selling price, or other variables. The current version of the code will give you a random distribution of values between −1 and 1, which will probably not match the possible values of the variables. If the actual range for selling price of a product is $4.00 to $6.00, the expression
*price = 4.00 + 2 * drand48();*
will give you a uniform distribution of prices over this range.

But what if you know that the distribution is not uniform, which it may not be? This can be handled by approximating the probability function with a histogram, and mapping the interval (0,1) to the cumulative value of the histogram. Thus, for example, if there is a 10% probability that the price will be in the range ($4.00, 4.50), you can use the expression
if (0 < drand48() < 0.1) price = $4.25
and so on for the rest of the range.

Other Applications

The Monte Carlo method has applications to many other areas of engineering. For example, it has been used to simulate the turbulent combustion of a diesel engine spray injection; the spray and the compressed air are represented as made up of a large number of “packets,” each of which has a weighted chance of mixing with another packet of fuel, air, or burned gases. Many runs of this model give a reasonable approximation to actual performance.

12S.7

It is possible to use the “Goldilocks Strategy” of examining the best case, worst case, and median case for each strategy. However, this is not as good as using the Monte Carlo method—for some of the input parameters, the PW of each option is a nonlinear function of the parameter value, so looking at three points does not give you an adequate idea of the overall distribution of PW.

It is absolutely essential to use the same time period, starting and ending at the same times, for both strategies. It seems tempting to start the time period for the lightweight-display strategy later, perhaps when it goes into production, in order to make the comparison “fair.” But you cannot escape the fact that money has a time value so easily.

Anything common to both the strategies can be left out. For example, the nine months between the present and the heavyweight-display strategy going in to production can be omitted, since it is the same for both strategies. You could also omit the fixed costs of $30 000/month, since they are the same in both cases.

Once you have set up the MC method, it’s as easy to run 1 000 000 trials as it is to run 100. You should run at least 10 000 to get an adequate sample.

After you have 10 000 or more present-worths for each strategy, how should you best display them?

One possibility is to just calculate the mean value of each strategy. (Plotting a graph of the mean value versus number of trials will show the evolution of this mean value with number of trials; this is one way of deciding whether you’ve done enough trials.) This choice involves throwing away almost all the information you have generated. You may still have enough—if one PW is clearly higher than the other, that would be the sensible choice—but you can make better use of the additional values you calculated.

The first improvement would be to calculate the variance as well as the mean for each strategy. If the strategy with the higher mean also has a very high variance, it might not be the best choice. Suppose, for example, that it gives a 30% possibility of going out of business, while the strategy with the lower mean value gives no possibility of going out of business.

A more informative picture of the distribution of present value can be obtained by plotting a histogram of the results for each strategy, grouping present value into about ten intervals and counting the number of cases that fall into each interval.