

Math 101 Tutorial Worksheet 10

There is an associated quiz due on BrightSpace on Tuesday, March 29 at 10:00 PM

1. Evaluate the following expressions and write your answers in the form $a + bi$.

(a) $(3 + 2i)(8 - 2i)$

(b) $\frac{1 + 4i}{3 + 2i}$

(c) $\overline{2i(4 - i)}$

(d) $e^{\frac{\pi}{6}i}$

2. Express the following complex numbers in the exponential form $re^{i\theta}$ with $r \geq 0$ and $-\pi < \theta \leq \pi$, and in the polar form $r(\cos \theta + i \sin \theta)$. Sketch the diagram for each calculation.

(a) $4 - 4i$

(b) $\frac{1}{2} + \frac{1}{2}i$

(c) $2\sqrt{3} - 2i$

(d) $\frac{1 + i}{1 - i}$

3. Find the indicated power using De Moivre's Theorem.

(a) $(1 - \sqrt{3}i)^5$

(b) $(1 - i)^8$

4. Find the indicated roots. Sketch the roots in the complex plane.

(a) The fifth roots of 32

(b) The cube roots of $1 + i$

5. (i) Find the formal Taylor series for $e^{i\theta}$ and $e^{-i\theta}$.

(ii) Using the formal Taylor series for $\sin \theta$ and $\cos \theta$, derive Euler's Formula: $e^{i\theta} = \cos \theta + i \sin \theta$.

(iii) Using (ii), show that $\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$.