Math 101 (A01-A04)
Test 3
Version: A
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Time: 120 minutes

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Given name: [redacted] Student ID: V00[redacted]

MC #	right answers	my answers
1	C	C
2	J	J
3	H	J
4	J	J
5	E	A
6	C	C
7	G	G
8	H	I
9	A	H

Test Score		
Question	**Points**	**Score**
Multiple Choice	18	10
Question 10	6	4
Question 11	6	5
Question 12	5	5
Question 13	5	2
Total	40	26

Instructions:

- Before beginning the test, enter your name and ID number on this cover and on the bubble sheet. Be sure to fill in the bubbles for your ID number.
- The only items you should have with you are writing implements, your OneCard, and your calculator. The only calculators permitted are Sharp EL-510R, Sharp EL-510RN, and Sharp EL-510RNB. No notes or any other aids are permitted. You are responsible for ensuring that you do not have any prohibited items with you during the test.
- Write out your solutions carefully and completely on the question paper provided. Marks will not be awarded for final answers that are not supported by appropriate work. This includes multiple choice problems.
- For multiple choice questions, the exact answer may not appear as one of the options. Solve the question, then select the answer *closest* to your answer. If your answer is exactly equidistant from two options, choose the larger answer.
- If you require extra space, use the backs of test pages or the blank page at the end of the test. Be sure to clearly indicate where you have done so. No outside paper is permitted.
- This test has 14 pages, including this cover and the blank page at the end.
- **Fill in "A" in the "Form" field of the bubble sheet now.**

(2) 1. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ whose terms are given by $a_n = \dfrac{4n^3+2n-1}{1-2n^3}$. Determine whether the sequence converges or diverges. If it converges, find its limit.

C

A. -4 B. -3 C. -2 D. -1 E. 0

F. 1 G. 2 H. 3 I. 4 J. The sequence diverges

$$\lim_{n\to\infty} \frac{4n^3+2n-1}{1-2n^3} \quad \frac{\infty}{\infty} \quad \text{l'hop}$$

$$\lim_{n\to\infty} \frac{3\cdot 4\cdot n^2+2}{-2\cdot 3\cdot n^2} \quad \frac{\infty}{\infty} \quad \text{l'hop}$$

$$\lim_{n\to\infty} \frac{24n}{-12n} \quad \text{l'hop}$$

$$\lim_{n\to\infty} \frac{24}{-12} \Rightarrow n\to\infty \quad = -2$$

(2) 2. Consider the sequence $\{a_n\}_{n=1}^{\infty}$ whose terms are given by $a_n = \dfrac{n!}{3^n}$. Determine whether the sequence converges or diverges. If it converges, find its limit.

A. 0 B. 1 C. 2 D. 3 E. 4

F. 5 G. 6 H. 7 I. 8 J. The sequence diverges

$\lim \frac{n!}{3^n} \to \infty$

$\lim_{n\to\infty} \frac{n(n-1)(n-2)(n-3)}{3\cdot 3\cdot 3\quad 3\cdot 3}$, $\frac{n(n-1)(n-2)(n-3)!}{3^n}$ $\to \infty$ v fast

$3^n \to$ goes to ∞ less fast?

$\frac{1}{3}$, $\frac{1}{}$

0.333, 0.222, 0.2222, 0.296, 0.49 → 0.98, 2.50

(: 3. Determine whether the series $\sum_{n=1}^{\infty} 2e^{-2n+2}$ converges or diverges. If it converges, find its value.

A. -1.43 B. -0.58 C. -0.16 D. 0 E. 0.12

F. 0.27 G. 1.76 H. 2.31 I. 3.52 J. The series diverges

$e > 1$

1-

$\sum_{n=1}^{\infty} 2e^{-2(n+1)} = \sum_{n=1}^{\infty} 2e^{-2} \cdot e^{n-1}$

contn

geometric

$\frac{a}{1-r}$

$a = 2e^{-2}$

$r = e$

$\frac{2e^{-2}}{1-e} = \cancel{}$

$2 \quad (e^{-2})\, e^{(n-1)}$

(2) 4. Determine whether the series $\sum_{n=1}^{\infty} \sqrt[n]{3}$ converges or diverges. If it converges, find its value.

A. 0.1 B. 0.2 C. 0.3 D. 0.4 E. 0.5

F. 1.0 G. 1.2 H. 1.4 I. 2.0 J. The series diverges

$\sum_{n=1}^{\infty} 3^{1/n}$ ← a rule for this

←

$\sum_{n=1}^{\infty} x^{1/n} = 1 \leftarrow x > 1$

$3 + 3^{1/2} + 3^{1/3} + 3^{1/4} + \ldots$

nth term test for divergence

$\lim_{n \to \infty} 3^{1/n} = 1$

$\lim 3^{1/n} = 1 \neq 0$

div

$\lim_{n \to \infty}$

$p > 1$

(!) 5. Suppose that p is a positive constant. For which values of p does the series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ converge?

A. $p > 1$ B. $p \geq 1$ C. $p < 1$ D. $p \leq 1$ E. $p > 2$

F. $p \geq 2$ G. $p < 2$ H. $p \leq 2$ I. All values of p J. No values of p

ratio test

$\lim_{n\to\infty} \left| \frac{\frac{n+2}{(n+1)^p}}{\frac{n+1}{n^p}} \right| = \lim_{n\to\infty} \left| \frac{n+2}{(n+1)^p} \cdot \frac{n^p}{n+1} \right|$ $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$

nth test

$p < 1 \Rightarrow$ converg

$p = 1 \Rightarrow$ inconclu

$p > 1 \Rightarrow$ diverg

P series

$\lim_{n\to\infty} \frac{n+1}{n} = 1 \leftarrow$ diverg

(2) 6. Consider the following three series:

limgoes to 1 not 0

un canbe negative

$\Rightarrow$ (♦) : $\sum_{n=1}^{\infty} \frac{(-1)^n}{1-\frac{1}{n}}$ (♠) : $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sin(n)}$ (♣) : $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ ✓

For which of the above series can we use the alternating series test to show that the series converges?

A. (♦) only B. (♠) only C. (♣) only

D. (♦) and (♠), but not (♣) E. (♦) and (♣), but not (♠) F. (♠) and (♣), but not (♦)

G. (♦), (♠), and (♣) H. None of these series

alternating series test
un must be positive
un must be non-increasin
$u_n \to 0$

♦ $(-1)^n \left(\frac{1}{1-1/n}\right)$ doesn't exist at $n=1$ $\frac{1}{1-1/1} = \frac{1}{1-1} =$ no good

♠ $(-1)^n \left(\frac{1}{\sin(n)}\right) \leftarrow$ can be negative — no good

♣ $(-1)^n \left(\frac{1}{n^2}\right)$ always true ✓

$\frac{(-1)^1}{1-\frac{1}{1}} + \frac{(-1)^2}{1-\frac{1}{2}}$

$\Downarrow$

indeterminate

$\lim_{n\to\infty} \frac{1}{1-\frac{1}{n}} = 1$

$\lim_{n\to\infty} \frac{1}{n^2} \Rightarrow \frac{1}{\infty} \to 0$ ✓

$\frac{d}{dn} \frac{1}{n^2} = \frac{d}{dn} n^{-2} = -2n^{-3} = \frac{-2}{n^3} \leq 0$ non-increasing ✓

13.

(1 (a) Let $f(x) = \sum_{n=0}^{\infty} x^n$. Write a formula for $f(x)$ that is *not* a power series, valid for x in the interval $(-1, 1)$. No justification is required.

$f(x) = \sum_{n=0}^{\infty} x^n = \cancel{\pm\sqrt{x}}$ $\quad x^{1/s}$ $\quad |x| < 1$ $\quad 0$

(2 (b) Let $g(x) = \sum_{n=1}^{\infty} nx^n$. Show that $g(x) = \frac{x}{(1-x)^2}$ for all x in $(-1, 1)$. Justify your answer.

$1 \cdot 0^1 + 2 \cdot 0^2 + \cdots = 0$ ✓

when $x = 0$ $\quad \frac{0}{1-0^2} = 0$ ✓

when $x = \frac{1}{2}$ $\left\{ \frac{1}{2} + 2 \cdot \frac{1}{2}^2 + 3 \cdot \frac{1}{2}^3 \cdots = \frac{\frac{1}{2}}{(1-\frac{1}{2})^2} = 2 \right.$ ✓

$a + ar + ar^2$

(2) (c) Find the exact value of $\sum_{n=1}^{\infty} \frac{n}{2^n}$. Justify your answer.

$n \cdot 2^{(-n)}$

$\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5}$

$\frac{1}{2} + \frac{1}{2} + \frac{1}{8} + \frac{1}{4} + \frac{5}{32} + \frac{6}{64} + \frac{7}{128}$

Math 101

Formula Sheet

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1 Trigonometric Identities

$\sin(-\theta) = -\sin(\theta)$ $\quad$ $\cos(-\theta) = \cos(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$ $\quad$ $\sec^2(\theta) = 1 + \tan^2(\theta)$ $\quad$ $\csc^2(\theta) = 1 + \cot^2(\theta)$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\quad$ $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$\cos^2(\theta) = \dfrac{1+\cos(2\theta)}{2}$ $\quad$ $\sin^2(\theta) = \dfrac{1-\cos(2\theta)}{2}$

$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\quad$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$\tan(A+B) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$

$\sin\left(A - \frac{\pi}{2}\right) = -\cos(A)$ $\quad$ $\cos\left(A - \frac{\pi}{2}\right) = \sin(A)$

$\sin\left(A + \frac{\pi}{2}\right) = \cos(A)$ $\quad$ $\cos\left(A + \frac{\pi}{2}\right) = -\sin(A)$

$\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$

$\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$

$\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$

2 Hyperbolic identities

$\cosh^2(x) - \sinh^2(x) = 1$ $\quad$ $\tanh^2(x) = 1 - \operatorname{sech}^2(x)$ $\quad$ $\coth^2(x) = 1 + \operatorname{csch}^2(x)$

$\sinh(2x) = 2\sinh(x)\cosh(x)$ $\quad$ $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$

$\cosh^2(x) = \dfrac{\cosh(2x)+1}{2}$ $\quad$ $\sinh^2(x) = \dfrac{\cosh(2x)-1}{2}$

3 Integrals

$$\int \tan(x)\,dx = \ln|\mathrm{s}$$

$$\int \sec(x)\,dx = \ln|\mathrm{se}$$

$$\int \frac{1}{x\sqrt{x^2-a^2}}\,dx =$$

$$\int \frac{1}{x\sqrt{a^2-x^2}}\,dx =$$

$$\int \frac{1}{x\sqrt{a^2+x^2}}\,dx =$$

12.

(1) (a) Suppose that $f(x)$ is an infinitely differentiable function. St for the Taylor series of $f(x)$ centred at $x = a$.

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}($$

$$= \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k \leftarrow$$

(1) (b) Suppose that $f(x)$ is an infinitely differentiable function, an State the general formula for the order N Taylor polyno $x = a$.

$$P_N(a) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3}}{3}$$

$$\cdots + \frac{f^{(N-2)}(a)}{(N-2)!}(x-a)^{(N-2)} + \frac{f^{(N-1)}(a)}{(N-1)!}(x-a)^{(N-1)} +$$

(3) (c) Find the Taylor series of $f(x) = e^{2x+1}$ centred at $x = \pi$. Ju

$$f(\pi) = e^{2\pi+1}$$

$$f'(x) = \frac{d}{dx} e^{(2x+1)} = 2e^{2x+1} \Rightarrow f$$

$$f''(x) = \frac{d}{dx} 2e^{2x+1} = 4e^{2x+1} \Rightarrow f''$$

$$f'''(x) = \frac{d}{dx} 4e^{2x+1} = 8e^{2x+1} \Rightarrow f^3$$

$$e^{2\pi+1} + 2e^{2\pi+1}(x-\pi) + \frac{4e^{2\pi+1}}{2!}(x-\pi)^2$$

$$a + ar + ar^2 + ar^2 +$$

$$= \sum_{k=0}^{\infty} \frac{f^k(\pi)}{k!}(x-\pi)^k = \sum_{k=0}^{\infty} \frac{2^n \cdot e^{2\pi+1}}{n!}(x-$$

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