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| **Student:** Arfaz Hossain<br>**Date:** 04/20/22 | **Instructor:** Muhammad Awais<br>**Course:** Math 101 A04 Spring 2022 | **Assignment:** Practice Questions for Sections 11.4 & 11.5 [Not f |

Graph the curves $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ and $r = \frac{11}{2} - 5\,\mathbf{sin}\,\theta$.

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First, establish what symmetries $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ has. The curve is symmetric about the x-axis if, when the point $(r,\theta)$ lies on the graph, the point $(r,-\theta)$ or $(-r,\pi-\theta)$ lies on the graph.

Test to see if the curve is symmetric about the x-axis. In the equation $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ substitute $(r,-\theta)$ or $(-r,\pi-\theta)$ for $(r,\theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identities $\mathbf{cos}\,(-\theta) = \mathbf{cos}\,(\theta)$ and $\mathbf{cos}\,(\pi-\theta) = -\,\mathbf{cos}\,(\theta)$. The equation does not change, so the curve is symmetric about the x-axis.

The curve is symmetric about the y-axis if, when the point $(r,\theta)$ lies on the graph, the point $(r,\pi-\theta)$ or $(-r,-\theta)$ lies on the graph.

Test to see if the curve is symmetric about the y-axis. In the equation $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ substitute $(r,\pi-\theta)$ or $(-r,-\theta)$ for $(r,\theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identities $\mathbf{cos}\,(\pi-\theta) = -\,\mathbf{cos}\,(\theta)$ and $\mathbf{cos}\,(-\theta) = \mathbf{cos}\,(\theta)$. The equation changes, so the curve is not symmetric about the y-axis.

The curve is symmetric about the origin if, when the point $(r,\theta)$ lies on the graph, the point $(-r,\theta)$ or $(r,\theta+\pi)$ lies on the graph.

Test to see if the curve is symmetric about the origin. In the equation $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ substitute $(-r,\theta)$ or $(r,\theta+\pi)$ for $(r,\theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identity $\mathbf{cos}\,(\theta+\pi) = -\,\mathbf{cos}\,(\theta)$. The equation changes, so the curve is not symmetric about the origin.

Now that the symmetry about the x-axis has been established, it is possible to graph the curve by finding r for $\theta$ values ranging from 0 to $\pi$, and then reflect the plot about the x-axis to get the whole graph.

For each value of $\theta$, the corresponding value of $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$ has been calculated, rounded to two decimal places.

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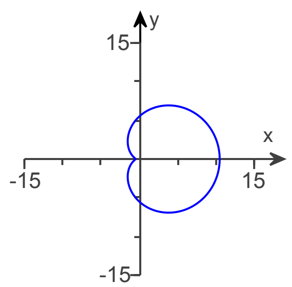
| $\theta = 0$ | $r = 10.5$ | $\theta = \frac{\pi}{3}$ | $r = 8$ | $\theta = \frac{3\pi}{4}$ | $r = 1.96$ |
| $\theta = \frac{\pi}{6}$ | $r = 9.83$ | $\theta = \frac{\pi}{2}$ | $r = 5.5$ | $\theta = \frac{5\pi}{6}$ | $r = 1.17$ |
| $\theta = \frac{\pi}{4}$ | $r = 9.04$ | $\theta = \frac{2\pi}{3}$ | $r = 3$ | $\theta = \pi$ | $r = 0.5$ |

Use these values to sketch the curve $r = \frac{11}{2} + 5\,\mathbf{cos}\,\theta$.




Next, establish what symmetries $r = \frac{11}{2} - 5\,\mathbf{sin}\,\theta$ has. Test to see if the curve is symmetric about the x-axis. The curve is symmetric about the x-axis if, when the point $(r,\theta)$ lies on the graph, the point $(r,-\theta)$ or $(-r,\pi-\theta)$ lies on the graph.

Test to see if the curve is symmetric about the x-axis. In the equation $r = \frac{11}{2} - 5\sin\theta$ substitute $(r, -\theta)$ or $(-r, \pi - \theta)$ for $(r, \theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identities $\sin(-\theta) = -\sin(\theta)$ and $\sin(\pi - \theta) = \sin(\theta)$. The equation changes, so the curve is not symmetric about the x-axis.

The curve is symmetric about the y-axis if, when the point $(r, \theta)$ lies on the graph, the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph.

Test to see if the curve is symmetric about the y-axis. In the equation $r = \frac{11}{2} - 5\sin\theta$ substitute $(r, \pi - \theta)$ or $(-r, -\theta)$ for $(r, \theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identities $\sin(\pi - \theta) = \sin(\theta)$ and $\sin(-\theta) = -\sin(\theta)$. The equation does not change, so the curve is symmetric about the y-axis.

The curve is symmetric about the origin if, when the point $(r, \theta)$ lies on the graph, the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph.

Test to see if the curve is symmetric about the origin. In the equation $r = \frac{11}{2} - 5\sin\theta$ substitute $(-r, \theta)$ or $(r, \theta + \pi)$ for $(r, \theta)$ and see if the resulting equation is equivalent to the original one. It is helpful to make use of the identity $\sin(\theta + \pi) = -\sin(\theta)$. The equation changes, so the curve is not symmetric about the origin.

Now that the symmetry about the y-axis has been established it is possible graph the curve by finding r for $\theta$ values ranging from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$, and then reflect the plot about the y-axis to get the whole graph.

For each value of $\theta$, the corresponding value of $r = \frac{11}{2} - 5\sin\theta$ has been calculated, rounded to two decimal places.

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| $\theta = -\frac{\pi}{2}$ | $r = 10.5$ | $\theta = -\frac{\pi}{6}$ | $r = 8$ | $\theta = \frac{\pi}{4}$ | $r = 1.96$ |
| $\theta = -\frac{\pi}{3}$ | $r = 9.83$ | $\theta = 0$ | $r = 5.5$ | $\theta = \frac{\pi}{3}$ | $r = 1.17$ |
| $\theta = -\frac{\pi}{4}$ | $r = 9.04$ | $\theta = \frac{\pi}{6}$ | $r = 3$ | $\theta = \frac{\pi}{2}$ | $r = 0.5$ |

Use these values to sketch the curve $r = \frac{11}{2} - 5\sin\theta$.