# University of Victoria - Stat 260 - Spring 2023
# Term Test 3 - Version A
# Section A01 - Instructor: Dr. Michelle Edwards

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Instructions:

- This paper has 6 pages of questions plus this cover page. Please count your pages and report any discrepancy immediately to the invigilator.
- Answer all questions on the test paper in the space provided. Questions 1 through 8 are short answer and worth 3 marks each. For each question show your work and write your final answer in the space as indicated. Question 9 is a full-answer question. To obtain full marks on this question, clear step-by-step reasoning must be displayed. Marks will be deducted for incoherent or incomplete solutions.
- Unless an exact answer with fewer decimals can be found, give all answers with at least three decimals of accuracy.
- A Sharp EL-510R series calculator may be used. No other calculator may be used.
- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

This page will not be graded.
Feel free to use this page for scrap work.

**(Questions 1 and 2 refer to the following setup)**

Let $X$ denote the number of times a numerical control machine will malfunction on a given day. Let $Y$ denote the number of times a technician is called on an emergency call. The joint probability distribution of $X$ and $Y$ is given by:

| | $p(x,y)$ | $y$ = 0 | 1 | 2 | |
|---|---|---|---|---|---|
| $x$ | 3 | 0.15 | 0.16 | 0.14 | .45 |
| | 5 | 0.11 | 0.19 | 0.25 | .55 |
| | | .26 | .35 | .39 | |

[3] **1.** Find the probability that the technician is called at least once, given that the machine malfunctions 5 times during the day.

$$P(Y \geq 1 \mid X=5) = \frac{P(Y \geq 1 \cap X=5)}{P(X=5)}$$

$$= \frac{0.19 + 0.25}{0.55} = \frac{0.44}{0.55} = 0.80$$

Answer

0.80

[3] **2.** Find the covariance of $X$ and $Y$. That is, find $Cov(X, Y)$.

$E(X) = 3(.45) + 5(.55) = 4.1$

$E(Y) = 0(.26) + 1(.35) + 2(.39) = 1.13$

| $X \cdot Y$ | | $Y$: 0 | 1 | 2 |
|---|---|---|---|---|
| $X$ | 3 | 0 | 3 | 6 |
| | 5 | 0 | 5 | 10 |

$E(X \cdot Y) = 0 + 3(.16) + 6(.14) + 0 + 5(.19) + 10(.25)$
$= 4.77$

$Cov(X,Y) = E(XY) - E(X)E(Y) = 4.77 - (4.1)(1.13)$
$= 0.137$

Answer

0.137

[3] **3.** Let $W$, $X$, and $Y$ be normally distributed random variables with $\sigma_W = 3$, $\sigma_X = 2$, and $\sigma_Y = 1$. Suppose also that $\mu_{4W-3X+2Y} = 9$. Find $P(4W - 3X + 2Y > 25)$.

$$V(4W-3X+2Y) = 16V(W) + 9V(X) + 4V(Y)$$
$$= 16(9) + 9(4) + 4(1) = 184$$

$$\sigma_{4W-3X+2Y} = \sqrt{184}$$

$$P(4W-3X+2Y > 25) = P\left(Z > \frac{25-9}{\sqrt{184}}\right) = P(Z > 1.18)$$

$$= 1 - P(Z \leq 1.18) = 1 - 0.8810 = 0.1190$$

Answer

0.1190

[3] **4.** The diameter of steel rods manufactured from an extruding machine is being investigated. Suppose the manufacturing process creates steel rods with a mean diameter of 8.7 cm and a standard deviation of 0.35 cm.
What is the probability that in a random sample of 60 rods, the mean diameter falls between 8.68 cm and 8.73 cm?

$$P(8.68 \leq \bar{X} \leq 8.73)$$
$$= P\left(\frac{8.68 - 8.7}{0.35/\sqrt{60}} \leq \frac{\bar{X} - \mu}{s/\sqrt{n}} \leq \frac{8.73 - 8.7}{0.35/\sqrt{60}}\right)$$

$$= P(-0.44 \leq Z \leq 0.66) = P(Z \leq 0.66) - P(Z \leq -0.44)$$

$$= 0.7454 - 0.3300$$

$$= 0.4154$$

Answer

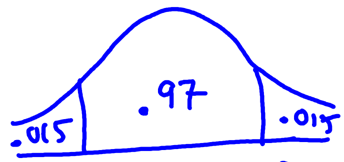
0.4154

[3] **5.** An electric company wishes to report the mean monthly household electricity usage in a particular city. A random sample of 42 households showed an average monthly electricity usage of 877 kWh with a standard deviation of 126 kWh. Find the upper limit of a 97% confidence interval for $\mu$, the true mean monthly household electricity usage.

$\bar{x} + z_{\alpha/2} \cdot s/\sqrt{n}$

$= 877 + 2.17 \cdot 126/\sqrt{42}$

$= 919.1896$




$z_{\alpha/2} = z_{.015}$
$= 2.17$

Answer

919.1896

[3] **6.** The accompanying data on cube compressive strength (MPa) of concrete specimens appeared in a study:

92.7 86.0 99.2 95.8 89.0 86.7

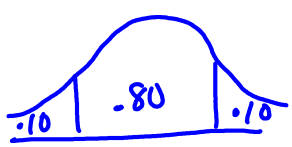
$\bar{x} = 91.5667$
$s = 5.2675$

Calculate the lower limit of an 80% confidence interval for true mean cube compressive strength (MPa) of concrete specimens. If needed, you may assume the data is normally distributed.

$\bar{x} - t_{\nu,\alpha/2} \cdot s/\sqrt{n}$

$= 91.5667 - 1.476 \cdot 5.2675/\sqrt{6}$

$= 88.3926$



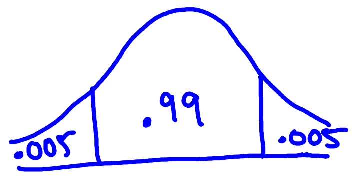

$t_{\nu,\alpha/2} = t_{5,.10}$
$= 1.476$

Answer

88.3926

[3] **7.** A manufacturer of ball bearings is worried that their production line is producing bearings with a surface finish that is rougher than the specifications allow. Suppose a random sample of 350 bearings show that 133 of them are too rough. Use this to determine the upper limit of a 99% confidence interval for the true proportion of bearings the production line creates that are too rough for specifications.

$$\hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$




$z_{\alpha/2} = z_{.005}$
$= 2.576$ (or 2.575 from z table)

$$= 133/350 + 2.576\sqrt{\frac{133/350\,(217/350)}{350}}$$

$$= 0.4468$$

Answer

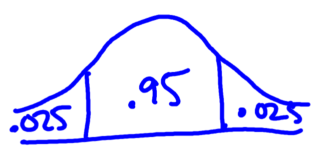
0.4468

[3] **8.** An electrician wants to determine the breakdown voltage of a specific electrical circuit. Suppose 8 random copies of this circuit are studied and the breakdown voltage of each is recorded below:

1470 2200 1900 1510 2500 2280 2130 2000 $s = 361.7986$

Use this data as a pilot study to determine the sample size needed to estimate the true mean breakdown voltage of the circuit to within 35 volts with 95% confidence.

$d = 35$




$z_{\alpha/2} = z_{.025}$
$= 1.96$

$$d = z_{\alpha/2} \cdot s/\sqrt{n}$$

$$n = \left(\frac{z_{\alpha/2} \cdot s}{d}\right)^2 = \left(\frac{1.96\,(361.7986)}{35}\right)^2$$

$$= 410.4968$$

round up, so $n = 411$

Answer

411

**(Question 9)** The manufacturer of a metal stand for large TV sets must make sure that the final product will not fail under the weight of the TV. The company's safety inspectors have set a standard to ensure that the stands can support an average of over 500 pounds. To monitor the production process the inspectors collect a random sample of 59 TV stands from the manufacturer and measure the weight at which the stand fails. The data collected showed a mean of 506 pounds and a standard deviation of 26 pounds. Test the claim that the average failure weight of the TV stand is above 500 pounds.

(a) [**2 marks**] Define the parameter of interest using the correct notation. Then, state the null and alternative hypotheses for this study.

testing $\mu$ - true mean failure weight of TV stand

$H_0: \mu = 500$

$H_1: \mu > 500$

(a) [**1 mark**] Calculate the observed value of the test statistic. State the distribution (and degrees of freedom if needed) it follows.

$n = 59$ is large $\Rightarrow$ $\bar{x}$ is normally distributed

$$z_{obs} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{506 - 500}{26/\sqrt{59}} = 1.77$$

(b) [**1 mark**] Compute the $p$-value or provide a range of appropriate values for the $p$-value.

p-value $= P(Z > 1.77) = 1 - P(Z \leq 1.77)$

$= 1 - 0.9616 = 0.0384$

(c) [**1 mark**] Using your $p$-value, state the strength of evidence against $H_0$.

$0.05 < $ p-value $< 0.10$ so strong evidence

(d) [**1 mark**] Using the significance level $\alpha = 0.05$, state your conclusions about if the TV stand has a failure weight above 500 pounds.

p-value $= 0.0384 \leq \alpha = 0.05 \Rightarrow$ p-value is small $\Rightarrow$ reject $H_0$.

We conclude that the TV stand has a mean failure weight above 500 pounds.

# University of Victoria - Stat 260 - Spring 2023
# Term Test 3 - Version B
# Section A01 - Instructor: Dr. Michelle Edwards

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- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

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Feel free to use this page for scrap work.

**(Questions 1 and 2 refer to the following setup)**

Let $X$ denote the number of times a numerical control machine will malfunction on a given day. Let $Y$ denote the number of times a technician is called on an emergency call. The joint probability distribution of $X$ and $Y$ is given by:

| | $p(x,y)$ | $y$ = 0 | 1 | 2 | |
|---|---|---|---|---|---|
| $x$ | 4 | 0.12 | 0.15 | 0.17 | .44 |
| | 6 | 0.13 | 0.20 | 0.23 | .56 |
| | | .25 | .35 | .40 | |

[3] **1.** Find the probability that the technician is called at least once, given that the machine malfunctions 4 times during the day.

$P(Y \geq 1 | X=4) = P(Y \geq 1 \cap X=4)$

$= \dfrac{0.15 + 0.17}{0.44} = \dfrac{0.32}{0.44} = 0.7273$

Answer

0.7273

[3] **2.** Find the covariance of $X$ and $Y$. That is, find $Cov(X, Y)$.

$E(X) = 4(.44) + 6(.56) = 5.12$

$E(Y) = 0(.25) + 1(.35) + 2(.40) = 1.15$

Answer

0.032

| X·Y | 0 | 1 | 2 |
|---|---|---|---|
| 4 | 0 | 4 | 8 |
| 6 | 0 | 6 | 12 |

$E(XY) = 0 + 4(.15) + 8(.17) + 0 + 6(.20) + 12(.23)$

$= 5.92$

$Cov(X,Y) = E(XY) - E(X)E(Y) = 5.92 - (5.12)(1.15) = 0.032$

[3] **3.** Let $W$, $X$, and $Y$ be normally distributed random variables with $\sigma_W = 1$, $\sigma_X = 3$, and $\sigma_Y = 2$. Suppose also that $\mu_{5W-2X+4Y} = 30.5$. Find $P(5W - 2X + 4Y > 17)$.

$V(5W-2X+4Y) = 25V(X) + 4V(X) + 16V(Y)$

$= 25(1) + 4(9) + 16(4) = 125$

$\sigma_{5W-2X+4Y} = \sqrt{125}$

$P(5W-2X+4Y > 17) = P\left(Z > \frac{17-30.5}{\sqrt{125}}\right) = P(Z > -1.21)$

$= 1 - P(Z \le -1.21) = 1 - 0.1131$

Answer

0.8869

[3] **4.** The diameter of steel rods manufactured from an extruding machine is being investigated. Suppose the manufacturing process creates steel rods with a mean diameter of 7.4 cm and a standard deviation of 0.52 cm.

What is the probability that in a random sample of 55 rods, the mean diameter falls between 7.29 cm and 7.53 cm?

$P(7.29 \le \bar{X} \le 7.53)$

$= P\left(\frac{7.29-7.4}{0.52/\sqrt{55}} \le \frac{\bar{X}-\mu}{s/\sqrt{n}} \le \frac{7.53-7.4}{0.52/\sqrt{55}}\right)$

$= P(-1.57 \le Z \le 1.85) = P(Z \le 1.85) - P(Z \le -1.57)$

$= 0.9678 - 0.0582 = 0.9096$

Answer

0.9096

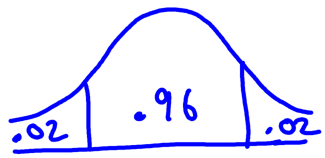
[3] **5.** An electric company wishes to report the mean monthly household electricity usage in a particular city. A random sample of 42 households showed an average monthly electricity usage of 917 kWh with a standard deviation of 132 kWh. Find the lower limit of a 96% confidence interval for $\mu$, the true mean monthly household electricity usage.

$\bar{x} - z_{\alpha/2} \cdot s/\sqrt{n}$

$= 917 - 2.054 \cdot 132/\sqrt{42}$

$= 875.1640$

(or 875.2455 using $z_{.02} = 2.05$)




$z_{\alpha/2} = z_{.02}$

$= 2.054$ (or 2.05 from z table)

Answer

875.1640

[3] **6.** The accompanying data on cube compressive strength (MPa) of concrete specimens appeared in a study:

92.1 86.3 98.2 95.0 89.4 85.9

$\bar{x} = 91.15$

$s = 4.8903$

Calculate the upper limit of a 99% confidence interval for true mean cube compressive strength (MPa) of concrete specimens. If needed, you may assume the data is normally distributed.

$\nu = n - 1 = 6 - 1 = 5$

$\bar{x} + t_{\nu,\alpha/2} \cdot s/\sqrt{n}$

$= 91.15 + 4.032 \cdot 4.8903/\sqrt{6}$

$= 99.1997$


$t_{\nu,\alpha/2}$

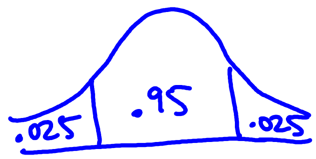
$= t_{5,.005} = 4.032$

Answer

99.1997

[3] **7.** A manufacturer of ball bearings is worried that their production line is producing bearings with a surface finish that is rougher than the specifications allow. Suppose a random sample of 275 bearings show that 77 of them are too rough. Use this to determine the upper limit of a 95% confidence interval for the true proportion of bearings the production line creates that are too rough for specifications.

$$\hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$




Answer: 0.3331

$$= 77/275 + 1.96\sqrt{\frac{77/275\,(198/275)}{275}}$$

$z_{\alpha/2} = z_{.025} = 1.96$

$$= 0.3331$$

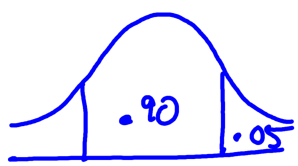
[3] **8.** An electrician wants to determine the breakdown voltage of a specific electrical circuit. Suppose 8 random copies of this circuit are studied and the breakdown voltage of each is recorded below:

1570 2100 1850 1590 2400 2150 2220 1900  $s = 297.6935$

Use this data as a pilot study to determine the sample size needed to estimate the true mean breakdown voltage of the circuit to within 40 volts with 90% confidence.

$d = 40$

$d = z_{\alpha/2} \cdot s/\sqrt{n}$




Answer: 150

$z_{\alpha/2} = z_{.05} = 1.645$

$$n = \left(\frac{z_{\alpha/2} \cdot s}{d}\right)^2 = \left(\frac{1.645\,(297.6935)}{40}\right)^2$$

$$= 149.88$$

round up, so $n = 150$

**(Question 9)** The manufacturer of a metal stand for large TV sets must make sure that the final product will not fail under the weight of the TV. The company's safety inspectors have set a standard to ensure that the stands can support an average of over 400 pounds. To monitor the production process the inspectors collect a random sample of 63 TV stands from the manufacturer and measure the weight at which the stand fails. The data collected showed a mean of 407 pounds and a standard deviation of 32 pounds. Test the claim that the mean failure weight of the TV stand is above 400 pounds.

(a) **[2 marks]** Define the parameter of interest using the correct notation. Then, state the null and alternative hypotheses for this study.

testing $\mu$ - true mean failure weight of TV stand

$H_0: \mu = 400$

$H_1: \mu > 400$

(a) **[1 mark]** Calculate the observed value of the test statistic. State the distribution (and degrees of freedom if needed) it follows.

$n = 63$ is large $\Rightarrow$ $\bar{x}$ is normally distributed

$$z_{obs} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{407 - 400}{32/\sqrt{63}} = 1.74$$

(b) [**1 mark**] Compute the $p$-value or provide a range of appropriate values for the $p$-value.

p-value $= P(Z > 1.74) = 1 - P(Z \leq 1.74)$

$= 1 - 0.9591 = 0.0409$

(c) [**1 mark**] Using your $p$-value, state the strength of evidence against $H_0$.

$0.05 < \text{p-value} < 0.10$ so strong evidence

(d) [**1 mark**] Using the significance level $\alpha = 0.05$, state your conclusions about if the TV stand has a mean failure weight above 400 pounds.

p-value $= 0.0409 \leq \alpha = 0.05 \Rightarrow$ p-value is small $\Rightarrow$ reject $H_0$

We conclude that the TV stand has a mean failure weight above 400 pounds.

# University of Victoria - Stat 260 - Spring 2023
# Term Test 3 - Version A
# Section A02 - Instructor: Dr. Michelle Edwards

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**(Questions 1 and 2 refer to the following setup)**

A car visits a mechanic. Let $X$ denote the number of headlight bulbs that need to be replaced. Let $Y$ denote the number of doors the car has (i.e. if it is a 2-door car or a 4-door car). The joint probability distribution of $X$ and $Y$ is given by:

| | $p(x,y)$ | $x$ = 0 | 1 | 2 | |
|---|---|---|---|---|---|
| $Y$ | 2 | 0.24 | 0.17 | 0.11 | .52 |
| | 4 | 0.21 | 0.14 | 0.13 | .48 |
| | | .45 | .31 | .24 | |

[3] **1.** Find the probability that the car needs at least one headlight bulb replaced, given that it is a 2-door car.

$$P(X \geq 1 \mid Y=2) = \frac{P(X \geq 1 \cap Y=2)}{P(Y=2)}$$

$$= \frac{0.17+0.11}{0.52} = \frac{0.28}{0.52} = 0.5385$$

Answer

0.5385

[3] **2.** Find the covariance of $X$ and $Y$. That is, find $Cov(X, Y)$.

$E(X) = 0(.45) + 1(.31) + 2(.24) = 0.79$

$E(Y) = 2(.52) + 4(.48) = 2.96$

| | $X \cdot Y$ | $X$ = 0 | 1 | 2 |
|---|---|---|---|---|
| $Y$ | 2 | 0 | 2 | 4 |
| | 4 | 0 | 4 | 8 |

$E(XY) = 0 + 2(.17) + 4(.11) + 0 + 4(.14) + 8(.13)$
$= 2.38$

$Cov(X,Y) = E(XY) - E(X)E(Y) = 2.38 - (0.79)(2.96) = 0.0416$

Answer

0.0416

[3] **3.** Let $W$, $X$, and $Y$ be independent normally distributed random variables with $\sigma_W = 1$, $\sigma_X = 3$, and $\sigma_Y = 2$. Suppose also that $\mu_{3W-2X+4Y} = 25$. Find $P(3W - 2X + 4Y > 10)$.

$$V(3W-2X+4Y) = 9V(W) + 4V(X) + 16V(Y)$$

$$= 9(1) + 4(9) + 16(4) = 109$$

Answer

0.9251

$$\sigma_{3W-2X+4Y} = \sqrt{109}$$

$$P(3W-2X+4Y > 10) = P\left(Z > \frac{10-25}{\sqrt{109}}\right) = P(Z > -1.44)$$

$$= 1 - P(Z \leq -1.44) = 1 - 0.0749 = 0.9251$$

[3] **4.** Health Canada reports that adults drink an average of 1.5 cups of caffeinated beverages every day, with a standard deviation of 0.3 cups. Suppose that 140 Canadian adults were randomly sampled. What is the probability that the sample mean amount of caffeinated beverages they consumed daily was between 1.47 cups and 1.54 cups?

$$P(1.47 \leq \bar{X} \leq 1.54)$$

$$= P\left(\frac{1.47-1.5}{0.3/\sqrt{140}} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{1.54-1.5}{0.3/\sqrt{140}}\right)$$

Answer

0.8239

$$= P(-1.18 \leq Z \leq 1.58) = P(Z \leq 1.58) - P(Z \leq -1.18)$$

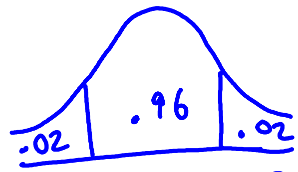
$$= 0.9429 - 0.1190 = 0.8239$$

[3] **5.** A tire manufacturer wishes to determine the stopping distrance of their new model of winter tires in icy road conditions. A random sample of 58 measurements in icy road conditions showed a mean stopping distance of 65.2 m with a standard deviation of 7.1 m. Find the lower limit of a 96% confidence interval for $\mu$, the true mean stopping distrance of the tires in icy road conditions.

$\bar{x} - z_{\alpha/2} \cdot s/\sqrt{n}$

$= 65.2 - 2.054 \cdot 7.1/\sqrt{58}$

$= 63.2851$




$z_{\alpha/2} = z_{.02}$
$= 2.054$
(or 2.05 from z table)

Answer

63.2851

[3] **6.** The accompanying data on kilowatt hours used per year for a particular model of small appliance was included in a technical report:

48.9 53.0 41.3 61.4 57.6 42.5

$\bar{x} = 50.7833$
$s = 8.07698$

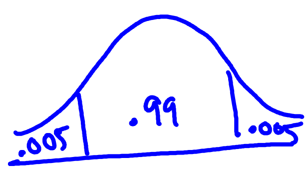
Calculate the upper limit of a 99% confidence interval for true mean amount of kilowatt hours used per year for this small appliance.

$df = \nu = n - 1 = 6 - 1 = 5$

$\bar{x} + t_{\nu,\alpha/2} \cdot s/\sqrt{n}$

$= 50.7833 + 4.032 \cdot 8.07698/\sqrt{6}$

$= 64.0785$




$t_{\nu,\alpha/2} = t_{5,.005} = 4.032$

Answer

64.0785

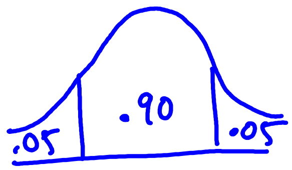
[3] **7.** A fabric manufacturer has had issues with their orders of raw materials arriving late. Suppose a random sample of 140 raw material orders show that 77 of them arrived late. Use this to determine the lower limit of a 90% confidence interval for the true proportion of raw material orders that arrive late.

$\hat{p} = 77/140$

$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$= 77/140 - 1.645\sqrt{\dfrac{77/140\,(63/140)}{140}}$

$= 0.4808$




$z_{\alpha/2} = z_{.05}$
$= 1.645$

Answer
0.4808

[3] **8.** A machine process creates metal pieces that are cylindrical in shape. Suppose 8 random cylinders from the process are sampled and the diameter of each (measured in cm) is recorded below:

1.93 1.72 2.14 2.06 1.99 1.67 2.23 2.01 $s = 0.19298$

Use this data as a pilot study to determine the sample size needed to estimate the true mean diameter of the cylinder pieces to within 0.05 cm with 95% confidence.

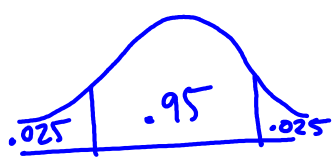
$d = 0.05$

$d = z_{\alpha/2} \cdot s/\sqrt{n}$

$n = \left(\dfrac{z_{\alpha/2} \cdot s}{d}\right)^2 = \left(\dfrac{1.96\,(0.19298)}{0.05}\right)^2$

$= 57.22612$

round up, so $n = 58$




$z_{\alpha/2} = z_{.025} = 1.96$

Answer
58

**(Question 9)** The manufacturer of a particular brand of fishing wire advertises a breaking strength of 15 kg. The company's quality control department measures a random sample of 47 pieces of wire and determines that the mean breaking strength is 15.3 kg with a standard deviation of 0.94 kg. Test the claim that the mean breaking strength of the fishing wire is at least 15 kg.

(a) **[2 marks]** Define the parameter of interest using the correct notation. Then, state the null and alternative hypotheses for this study.

testing $\mu$ - the true mean breaking strength of the wire

$H_0: \mu = 15$

$H_1: \mu > 15$

(a) **[1 mark]** Calculate the observed value of the test statistic. State the distribution (and degrees of freedom if needed) it follows.

$n = 47$ is large $\Rightarrow$ $\bar{x}$ is normally distributed

$$z_{obs} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{15.3 - 15}{0.94/\sqrt{47}} = 2.19$$

(b) **[1 mark]** Compute the $p$-value or provide a range of appropriate values for the $p$-value.

$p$-value $= P(Z > 2.19) = 1 - P(Z \leq 2.19) = 1 - 0.9857$
$= 0.0143$

(c) **[1 mark]** Using your $p$-value, state the strength of evidence against $H_0$.

$0.01 \leq p$-value $< 0.05$ so strong evidence against $H_0$.

(d) **[1 mark]** Using the significance level $\alpha = 0.025$, state your conclusions about if the fishing wire has a mean breaking strength over 15 kg.

$p$-value $= 0.0143 \leq \alpha = 0.025 \Rightarrow p$-value is small $\Rightarrow$ reject $H_0$.

We conclude that there is enough evidence to say the mean breaking strength of the wire is over 15 kg.

# University of Victoria - Stat 260 - Spring 2023
# Term Test 3 - Version B
# Section A02 - Instructor: Dr. Michelle Edwards

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Instructions:

- This paper has 6 pages of questions plus this cover page. Please count your pages and report any discrepancy immediately to the invigilator.
- Answer all questions on the test paper in the space provided. Questions 1 through 8 are short answer and worth 3 marks each. For each question show your work and write your final answer in the space as indicated. Question 9 is a full-answer question. To obtain full marks on this question, clear step-by-step reasoning must be displayed. Marks will be deducted for incoherent or incomplete solutions.
- Unless an exact answer with fewer decimals can be found, give all answers with at least three decimals of accuracy.
- A Sharp EL-510R series calculator may be used. No other calculator may be used.
- A formula sheet and statistical tables are provided. No other aids such as books, notes, scrap paper, electronic devices, etc. are permitted.
- You have 50 minutes to write the test.

This page will not be graded.
Feel free to use this page for scrap work.

**(Questions 1 and 2 refer to the following setup)**

A car visits a mechanic. Let $X$ denote the number of headlight bulbs that need to be replaced. Let $Y$ denote the number of doors the car has (i.e. if it is a 2-door car or a 4-door car). The joint probability distribution of $X$ and $Y$ is given by:

| | $p(x,y)$ | $x$ = 0 | 1 | 2 | |
|---|---|---|---|---|---|
| $y$ | 2 | 0.21 | 0.16 | 0.12 | .49 |
| | 4 | 0.25 | 0.15 | 0.11 | .51 |
| | | .46 | .31 | .23 | |

[3] **1.** Find the probability that the car needs at least one headlight bulb replaced, given that it is a 4-door car.

$$P(X \geq 1 \mid Y=4) = \frac{P(X\geq 1 \cap Y=4)}{P(Y=4)}$$

$$= \frac{0.15+0.11}{0.51} = \frac{0.26}{0.51} = 0.5098$$

Answer

0.5098

[3] **2.** Find the covariance of $X$ and $Y$. That is, find $Cov(X, Y)$.

$E(X) = 0(.46) + 1(.31) + 2(.23) = 0.77$

$E(Y) = 2(.49) + 4(.51) = 3.02$

| $X \cdot Y$ | $x$ = 0 | 1 | 2 |
|---|---|---|---|
| $y$ = 2 | 0 | 2 | 4 |
| 4 | 0 | 4 | 8 |

$E(XY) = 0 + 2(.16) + 4(.12) + 0 + 4(.15) + 8(.11)$
$= 2.28$

$Cov(X,Y) = E(XY) - E(X)E(Y) = 2.28 - (0.77)(3.02) = -0.0454$

Answer

$-0.0454$

[3] **3.** Let $W$, $X$, and $Y$ be independent normally distributed random variables with $\sigma_W = 2$, $\sigma_X = 1$, and $\sigma_Y = 3$. Suppose also that $\mu_{4W-2X+3Y} = 29$. Find $P(4W - 2X + 3Y > 14)$.

$V(4W-2X+3Y) = 16V(W) + 4V(X) + 9V(Y)$

$= 16(4) + 4(1) + 9(9) = 149$

$\sigma_{4W-2X+3Y} = \sqrt{149}$

$P(4W-2X+3Y > 14) = P\left(Z > \frac{14-29}{\sqrt{149}}\right) = P(Z > -1.23)$

$= 1 - P(Z \leq -1.23) = 1 - 0.1093 = 0.8907$

Answer

0.8907

[3] **4.** Health Canada reports that adults drink an average of 2 cups of caffeinated beverages every day, with a standard deviation of 0.4 cups. Suppose that 160 Canadian adults were randomly sampled. What is the probability that the sample mean amount of caffeinated beverages they consumed daily was between 1.97 cups and 2.04 cups?

$P(1.97 \leq \bar{X} \leq 2.04)$

$= P\left(\frac{1.97-2}{0.4/\sqrt{160}} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{2.04-2}{0.4/\sqrt{160}}\right)$

$= P(-0.95 \leq Z \leq 1.26) = P(Z \leq 1.26) - P(Z \leq -0.95)$

$= 0.8962 - 0.1711$

$= 0.7251$

Answer

0.7251

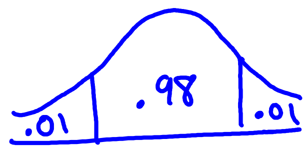
[3] **5.** A tire manufacturer wishes to determine the stopping distrance of their new model of winter tires in icy road conditions. A random sample of 54 measurements in icy road conditions showed a mean stopping distance of 73.1 m with a standard deviation of 6.4 m. Find the upper limit of a 98% confidence interval for $\mu$, the true mean stopping distrance of the tires in icy road conditions.

$\bar{x} + z_{\alpha/2} \cdot s/\sqrt{n}$

$= 73.1 + 2.326 \cdot 6.4/\sqrt{54}$

$= 75.1258$

(or 75.1293 using $z_{.01} = 2.33$)




$z_{\alpha/2} = z_{.01}$

$= 2.326$

(or 2.33 from $z$ table)

Answer

75.1258

(or 75.1293)

[3] **6.** The accompanying data on kilowatt hours used per year for a particular model of small appliance was included in a technical report:

47.6 52.0 43.5 58.7 60.8 45.2

$\bar{x} = 51.3$

$s = 7.1727$

Calculate the lower limit of a 90% confidence interval for true mean amount of kilowatt hours used per year for this small appliance. If needed, you may assume the data is normally distributed.

$df = \nu = n-1 = 6-1 = 5$

$\bar{x} - t_{\nu,\alpha/2} \cdot s/\sqrt{n}$

$= 51.3 - 2.015 \cdot \frac{7.1727}{\sqrt{6}}$

$= 45.3996$


$t_{\nu,\alpha/2} = t_{5,.05} = 2.015$

Answer

45.3996

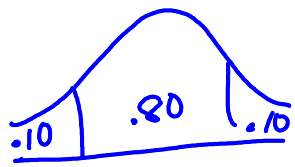
[3] **7.** A fabric manufacturer has had issues with their orders of raw materials arriving late. Suppose a random sample of 180 raw material orders show that 117 of them arrived late. Use this to determine the upper limit of an 80% confidence interval for the true proportion of raw material orders that arrive late.

$\hat{p} = 117/180$

$\hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$= 117/180 + 1.282\sqrt{\dfrac{117/180\,(63/180)}{180}}$

$= 0.6956$




$z_{\alpha/2} = z_{.10}$
$= 1.282$
(or 1.28 from z table)

Answer
0.6956

[3] **8.** A machine process creates metal pieces that are cylindrical in shape. Suppose 8 random cylinders from the process are sampled and the diameter of each (measured in cm) is recorded below:

1.91 1.83 2.07 2.13 1.89 1.97 2.02 2.25 $s = 0.1384$

Use this data as a pilot study to determine the sample size needed to estimate the true mean diameter of the cylinder pieces to within 0.03 cm with 99% confidence.

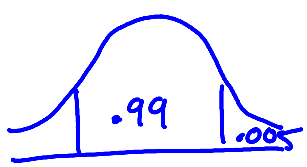
$d = 0.03$

$d = z_{\alpha/2} \cdot s/\sqrt{n}$

$n = \left(\dfrac{z_{\alpha/2} \cdot s}{d}\right)^2 = \left(\dfrac{2.576\,(0.1384)}{.03}\right)^2$

$= 141.23$

round up, so $n = 142$




$z_{\alpha/2} = z_{.005} = 2.576$
(or 2.575 from z table)

Answer
142

**(Question 9)** The manufacturer of a particular brand of fishing wire advertises a breaking strength of 17 kg. The company's quality control department measures a random sample of 53 pieces of wire and determines that the mean breaking strength is 17.3 kg with a standard deviation of 0.92 kg. Test the claim that the mean breaking strength of the fishing wire is at least 17 kg.

(a) **[2 marks]** Define the parameter of interest using the correct notation. Then, state the null and alternative hypotheses for this study.

testing $\mu$ - true mean breaking strength of the wire

$H_0: \mu = 17$

$H_1: \mu > 17$

(a) **[1 mark]** Calculate the observed value of the test statistic. State the distribution (and degrees of freedom if needed) it follows.

$n = 53$ is large $\Rightarrow$ $\bar{x}$ is normally distributed

$$z_{obs} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{17.3 - 17}{0.92/\sqrt{53}} = 2.37$$

(b) **[1 mark]** Compute the $p$-value or provide a range of appropriate values for the $p$-value.

p-value $= P(Z > 2.37) = 1 - P(Z \leq 2.37) = 1 - 0.9911$
$= 0.0089$

(c) **[1 mark]** Using your $p$-value, state the strength of evidence against $H_0$.

p-value $< 0.01$ so very strong evidence against $H_0$

(d) **[1 mark]** Using the significance level $\alpha = 0.025$, state your conclusions about if the fishing wire has a mean breaking strength over 17 kg.

p-value $= 0.0089 \leq \alpha = 0.025 \Rightarrow$ p-value is small $\Rightarrow$ reject $H_0$

We conclude there is enough evidence to say the mean breaking strength of the wire is over 17 kg.