Symbolab

Solution

$\sum_{n=0}^{\infty}\frac{-5}{6}\left(\frac{x}{6}\right)^n$: Radius of convergence is 6, Interval of convergence is $-6<x<6$

Steps

$\sum_{n=0}^{\infty}\frac{-5}{6}\left(\frac{x}{6}\right)^n$

Use the Ratio Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}\frac{-5}{6}\left(\frac{x}{6}\right)^n$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{-5}{6}\left(\frac{x}{6}\right)^{(n+1)}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{-5}{6}\left(\frac{x}{6}\right)^{(n+1)}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\frac{\frac{-5}{6}\left(\frac{x}{6}\right)^{(n+1)}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}\right|\right)$

Simplify $\frac{\frac{-5}{6}\left(\frac{x}{6}\right)^{(n+1)}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}$: $\frac{x}{6}$

Hide Steps

$\frac{\frac{-5}{6}\left(\frac{x}{6}\right)^{n+1}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}$

Apply the fraction rule: $\frac{-a}{b}=-\frac{a}{b}$

$=\frac{-\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}}{\frac{-5}{6}\left(\frac{x}{6}\right)^n}$

Apply the fraction rule: $\frac{-a}{b}=-\frac{a}{b}$

$=\frac{-\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}}{-\frac{5}{6}\left(\frac{x}{6}\right)^n}$

Apply the fraction rule: $\frac{-a}{-b}=\frac{a}{b}$

$=\frac{\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}}{\frac{5}{6}\left(\frac{x}{6}\right)^n}$

$\left(\frac{x}{6}\right)^n=\frac{x^n}{6^n}$

Hide Steps

$\left(\frac{x}{6}\right)^n$

Apply exponent rule: $\left(\frac{a}{b}\right)^c=\frac{a^c}{b^c}$

$=\frac{x^n}{6^n}$

$=\frac{\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}}{\frac{5}{6}\cdot\frac{x^n}{6^n}}$

Multiply $\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}$: $\frac{5x^{n+1}}{6^{n+2}}$

Hide Steps

$\frac{5}{6}\left(\frac{x}{6}\right)^{n+1}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{5\left(\frac{x}{6}\right)^{n+1}}{6}$

$\left(\frac{x}{6}\right)^{n+1}=\frac{x^{n+1}}{6^{n+1}}$

Hide Steps

$\left(\frac{x}{6}\right)^{n+1}$

Apply exponent rule: $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

$= \frac{x^{n+1}}{6^{n+1}}$

$= \frac{5 \cdot \frac{x^{n+1}}{6^{n+1}}}{6}$

Multiply $5 \cdot \frac{x^{n+1}}{6^{n+1}}$: $\frac{5x^{n+1}}{6^{n+1}}$ Hide Steps

$5 \cdot \frac{x^{n+1}}{6^{n+1}}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{x^{n+1} \cdot 5}{6^{n+1}}$

$= \frac{\frac{5x^{n+1}}{6^{n+1}}}{6}$

Apply the fraction rule: $\frac{\frac{b}{c}}{a} = \frac{b}{c \cdot a}$

$= \frac{x^{n+1} \cdot 5}{6^{n+1} \cdot 6}$

$6^{n+1} \cdot 6 = 6^{n+2}$ Hide Steps

$6^{n+1} \cdot 6$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$6^{n+1} \cdot 6 = 6^{n+1+1}$

$= 6^{n+1+1}$

Add the numbers: $1 + 1 = 2$

$= 6^{n+2}$

$= \frac{5x^{n+1}}{6^{n+2}}$

$= \frac{\frac{5x^{n+1}}{6^{n+2}}}{\frac{5}{6} \cdot \frac{x^n}{6^n}}$

Apply the fraction rule: $\frac{\frac{b}{c}}{a} = \frac{b}{c \cdot a}$

$= \frac{x^{n+1} \cdot 5}{6^{n+2} \cdot \frac{5}{6} \cdot \frac{x^n}{6^n}}$

Multiply $6^{n+2} \cdot \frac{5}{6} \cdot \frac{x^n}{6^n}$: $30x^n$ Hide Steps

$6^{n+2} \cdot \frac{5}{6} \cdot \frac{x^n}{6^n}$

Multiply fractions: $a \cdot \frac{b}{c} \cdot \frac{d}{e} = \frac{a \cdot b \cdot d}{c \cdot e}$

$= \frac{5x^n \cdot 6^{n+2}}{6 \cdot 6^n}$

$6 \cdot 6^n = 6^{1+n}$ Hide Steps

$6 \cdot 6^n$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$6 \cdot 6^n = 6^{1+n}$

$= 6^{1+n}$

$= \frac{5 \cdot 6^{n+2} x^n}{6^{1+n}}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{6^{n+2}}{6^{n+1}} = 6^{n+2-(n+1)}$

$= 5 \cdot 6^{n+2-(n+1)} x^n$

Add similar elements: $n + 2 - (n + 1) = 1$

$= 5 \cdot 6x^n$

Multiply the numbers: $5 \cdot 6 = 30$

$= 30x^n$

$= \frac{5x^{n+1}}{30x^n}$

Cancel the common factor: 5

$= \frac{x^{n+1}}{6x^n}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{x^{n+1}}{x^n} = x^{n+1-n}$

$= \frac{x^{n+1-n}}{6}$

Add similar elements: $n + 1 - n = 1$

$= \frac{x}{6}$

$L = \lim_{n\to\infty}\left(\left|\frac{x}{6}\right|\right)$

$L = \left|\frac{x}{6}\right| \cdot \lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1) = 1$ Hide Steps

$\lim_{n\to\infty}(1)$

$\lim_{x\to a} c = c$

$= 1$

$L = \left|\frac{x}{6}\right| \cdot 1$

Simplify

$L = \frac{|x|}{6}$

$L = \frac{|x|}{6}$

The power series converges for $L < 1$

$\frac{|x|}{6} < 1$

Find the radius of convergence Hide Steps

To find radius of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for $|x-a|$

$\frac{|x|}{6} < 1: \quad |x| < 6$ Hide Steps

$\frac{|x|}{6} < 1$

Multiply both sides by 6

$\frac{6|x|}{6} < 1 \cdot 6$

Simplify

$|x| < 6$

Therefore

Radius of convergence is 6

Radius of convergence is 6

Find the interval of convergence Hide Steps

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$\frac{|x|}{6} < 1 \; : \; -6 < x < 6$ Hide Steps

$\frac{|x|}{6} < 1$

Multiply both sides by 6

$\frac{6|x|}{6} < 1 \cdot 6$

Simplify

$|x| < 6$

Apply absolute rule: If $|u| < a, a > 0$ then $-a < u < a$

$-6 < x < 6$

$-6 < x < 6$

Check the interval end points: $x = -6$:diverges, $x = 6$:diverges Hide Steps

For $x = -6, \sum_{n=0}^{\infty} \frac{-5}{6}\left(\frac{(-6)}{6}\right)^n$: diverges Hide Steps

$\sum_{n=0}^{\infty} \frac{-5}{6}\left(\frac{(-6)}{6}\right)^n$

Refine

$=\sum_{n=0}^{\infty} -\frac{5}{6}(-1)^n$

Apply the constant multiplication rule: $\sum c\cdot a_n = c\cdot \sum a_n$

$=-\frac{5}{6}\cdot \sum_{n=0}^{\infty}(-1)^n$

Apply Series Geometric Test: diverges

Hide Steps

$\sum_{n=0}^{\infty}(-1)^n$

Geometric Series:

If the series is of the form $\sum_{n=0}^{\infty} r^n$

If $|r|<1$, then the geometric series converges to $\frac{1}{1-r}$

If $|r|\ge 1$, then the geometric series diverges

$r=-1, |r|=1\ge 1,$ by the geometric test criteria

$=$ diverges

$=-\frac{5}{6}$diverges

$=$ diverges

For $x=6, \sum_{n=0}^{\infty}\frac{-5}{6}\left(\frac{6}{6}\right)^n$: diverges

Hide Steps

$\sum_{n=0}^{\infty}\frac{-5}{6}\left(\frac{6}{6}\right)^n$

Refine

$=\sum_{n=0}^{\infty} -\frac{5}{6}$

Every infinite sum of a non – zero constant diverges

$=$ diverges

$x=-6$:diverges, $x=6$:diverges

Therefore

Interval of convergence is $-6<x<6$

Interval of convergence is $-6<x<6$

Radius of convergence is 6, Interval of convergence is $-6<x<6$