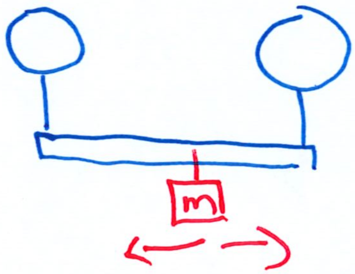
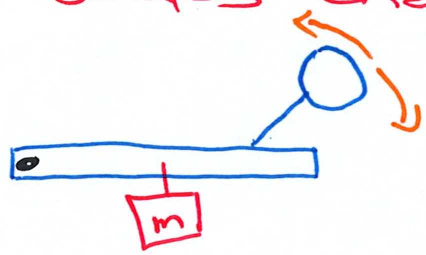
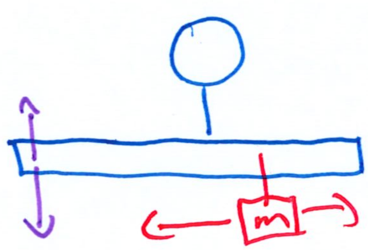
Torque & rotational equilibrium



As m moved left & right readings on scales changed



As direction of scale changed the reading changed (required force changed)



Required direction of force changed as m moved.

This shows that (for extended objects) the equilibrium conditions care about <u>both</u>

- what force is
- where excerted

New quantity: Torque

$$\vec{\tau} = \vec{r} \times \vec{F}$$

vector from "origin" to where force happens

force

Condition for rotational equilibrium

$$\vec{\tau}_{net} = 0$$

3-4-Example-RotEq I

Rotational Equilibrium - I
A force of $\vec{F} = 5N\hat{\imath} + 6N\hat{k}$ is being exerted at $\vec{r} = 3m\hat{\imath}$.

- What is the torque this force exerts around (about) the origin?
- What is the torque this force exerts around (about) the point $\vec{r} = 1m\hat{\imath} + 1m\hat{k}$?

x2 x2

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$F_x = 5N$

$F_y = 0$

$F_z = 6N$

$$\vec{r}\times\vec{F} = (yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$$

$x = 3m$

$y = 0m$

$z = 0m$

$$= (0m\,6N - 0m\,0N)\hat{i} + (0m\,5N - 3m\,6N)\hat{j} + (3m\,0N - 0m\,5N)\hat{k}$$

$$= 0Nm\,\hat{i} - 18Nm\,\hat{j} + 0Nm\,\hat{k}$$

y-direction

$|\vec{\tau}| = 18\,Nm$

Recall

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\Theta$$

$$|\vec{r}\times\vec{F}| = (3m)\sqrt{(5N)^2+(6N)^2}\,\sin\underbrace{\Theta}_{50.2^\circ} = 18\,Nm$$

What is Θ

$$\vec{r}\cdot\vec{F} = |\vec{r}||\vec{F}|\cos\Theta$$

$$15\,Nm = (3m)\sqrt{(5N)^2+(6N)^2}\,\cos\Theta$$

$$\Theta = 50.2^\circ$$

Around $1m\hat{i}+1m\hat{k}$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

vector from place
you measure $\vec{\tau}$ around
to location of force

$$(3m\hat{i}) - (1m\hat{i} - 1m\hat{k})$$

$$\vec{\tau} = (2m\hat{i}+1m\hat{k})\times(5N\hat{i}+6N\hat{k})$$

$$= \cancel{2m\,5N\,\hat{i}\times\hat{i}} + 2m\,6N\,\underbrace{\hat{i}\times\hat{k}}_{-\hat{j}}$$

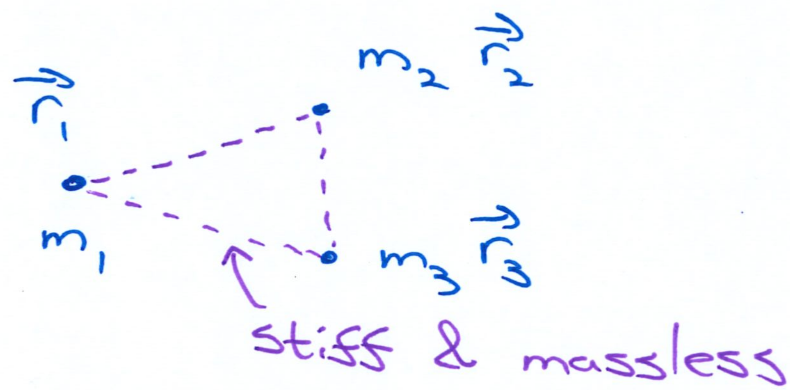
$$+ 1m\,5N\,\underbrace{\hat{k}\times\hat{i}}_{\hat{j}} + \cancel{1m\,6N\,\hat{k}\times\hat{k}}$$

$$= -7\,Nm\,\hat{j}$$

Center of Mass

Key question: Where does gravity act on a rigid object?

all component bits maintain same orientation to each other.



What is $\vec{\tau}$ measured around origin

$(\vec{\tau} = \vec{r} \times \vec{F})$

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3$$

$$= \vec{r}_1 \times (-m_1 g \hat{k}) + \vec{r}_2 \times (-m_2 g \hat{k}) + \vec{r}_3 \times (-m_3 g \hat{k})$$

$$= (m_1 \vec{r}_1) \times (-g \hat{k}) + m_2 \vec{r}_2 \times (-g \hat{k}) + m_3 \vec{r}_3 \times (-g \hat{k})$$

$$= (m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3) \times (-g\hat{k}) \frac{m_1+m_2+m_3}{m_1+m_2+m_3}$$

$$= \underbrace{\left[\frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1+m_2+m_3}\right]}_{\text{Where it's like gravity happens}} \times \underbrace{\left[-(m_1+m_2+m_3)g\hat{k}\right]}_{\text{total } \vec{F}_g}$$

Center of mass

$$\vec{r}_{cm} = \left(\frac{m_1\vec{r}_1 + \ldots}{m_1 + \ldots}\right)$$

$$= \frac{\sum m_i\vec{r}_i}{\sum m_i}$$

Continuous

$$\vec{r}_{cm} = \frac{\int \vec{r}\, dm}{\int dm}$$