Student: Arfaz Hossain	**Instructor:** Muhammad Awais	**Assignment:** Practice Questions for
Date: 02/28/22	**Course:** Math 101 A04 Spring 2022	Sections 6.3 & 7.2 [Not for

Solve the differential equation.

$$2\sqrt{xy}\,\frac{dy}{dx} = 5, \quad x,y > 0$$

Some differential equations can be solved by separating the variables. A differential equation of the form $y' = f(x,y)$ is separable if f can be expressed as a product of a function of x and a function of y.

Separate the variables by collecting all the y-terms with dy and all the x-terms with dx. Since x and y are always greater than zero, solve the equation by separating the variables.

Rewrite the equation in its differential form.

$$2\sqrt{xy}\,\frac{dy}{dx} = 5$$

$$2\sqrt{y}\,dy = \frac{5}{\sqrt{x}}dx$$

Now integrate both sides of the equation. First move the constants to the front of the integrals on each side.

$$\int 2y^{1/2}dy = \int 5x^{-1/2}dx$$

$$2\int y^{1/2}dy = 5\int x^{-1/2}dx$$

Use the rule $\int u^n\,du = \frac{u^{n+1}}{n+1}$ to integrate on both sides.

$$2 \cdot \left(\frac{2}{3}\right)y^{3/2} + C_1 = 5 \cdot 2x^{1/2} + C_2$$

$$\left(\frac{4}{3}\right)y^{3/2} + C_1 = 10x^{1/2} + C_2 \qquad \text{Simplify.}$$

The equation gives y as an implicit function of x. Solve for y as an explicit function of x. Combine the constants of integration as C and solve for y.

$$\left(\frac{4}{3}\right)y^{3/2} = 10x^{1/2} + C$$

$$y^{3/2} = \left(\frac{3}{4}\right)10x^{1/2} + C \qquad \text{Let } C = \frac{3}{4}C \text{ since C is arbitrary.}$$

$$y = \left(\frac{15}{2}x^{1/2} + C\right)^{2/3} \qquad \text{This is the equation in explicit form.}$$

Thus, solving the original differential equation, $2\sqrt{xy}\,\frac{dy}{dx} = 5$, yields $y = \left(\frac{15}{2}x^{1/2} + C\right)^{2/3}$.