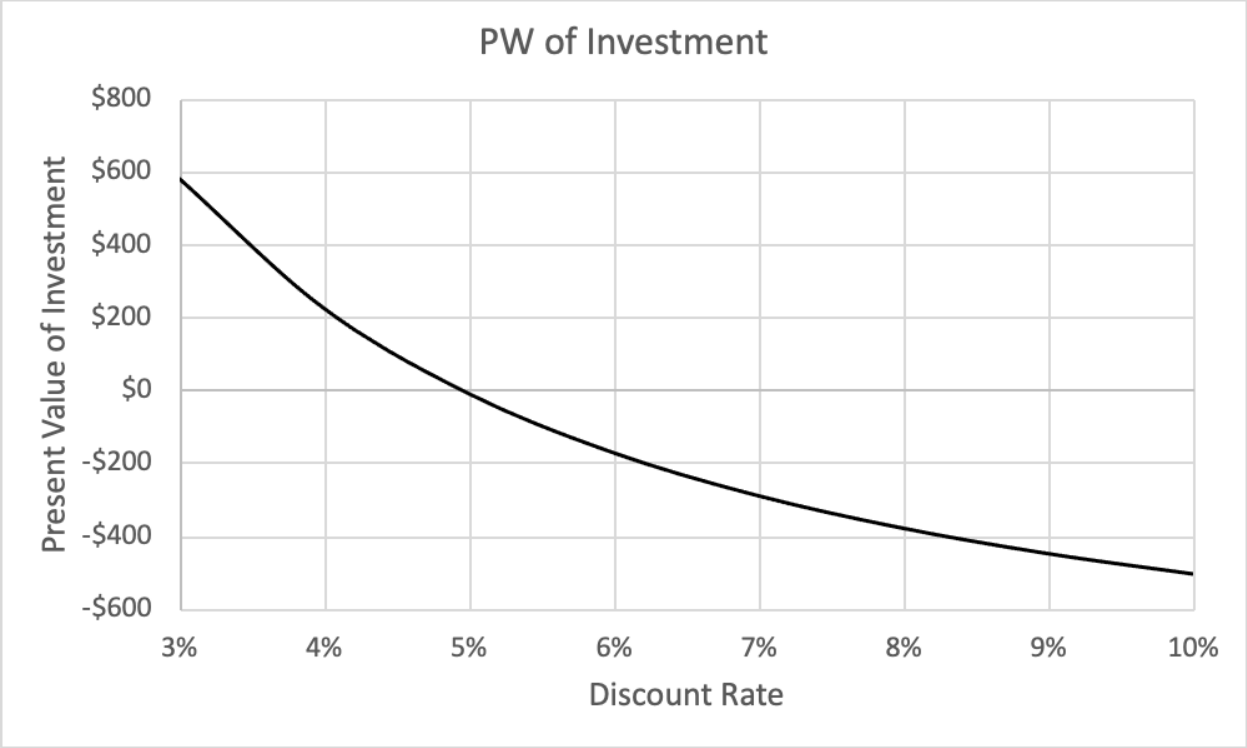
V1.05 (June 25) – Fixed calculations on p. 7 (1995 rent, food/rent for whole year in 2.d)

1**. [IRR/ERR] [Regular version]**

a. [**Lecture 7, Slides 9-11, *Engineering Economics* 5.3.1**]

- An investment costs $1,000 in Year 0 and pays you $50 every year from Year 1 to Year 100.
- **Write down an equation that will let you solve for this investment's IRR.**
- Do not use a summation (Σ) sign. You don't need it.
- Remember, at the IRR, the present value of the investment's cash flows are zero. I've plotted the relevant function for you below. This is similar to Figure 5.3 on p. 137 of the textbook.



Equation (use discount factors, e.g. (P/A,IRR,100)):

-$1,000 + $50 x (P/A,IRR,100) = 0

b. **Solve for the IRR of the investment in part a, to four significant figures.** Use whichever numerical method you prefer (Excel's Goal Seek, Wolfram Alpha, Trial and Error, Linear Interpolation, etc.). As the diagram above shows you, your answer should be close to 5%. I don't need to see your calculations.

IRR to 4 significant figures: __**4.961**_____ %

c. **If your MARR is 4% per year, based on the IRR you calculated in part b., is the investment in part a. worth it?** Briefly explain your reasoning. (*Hint: See Slide 26 of Lecture 7 and E.E. 5.3.1.*)

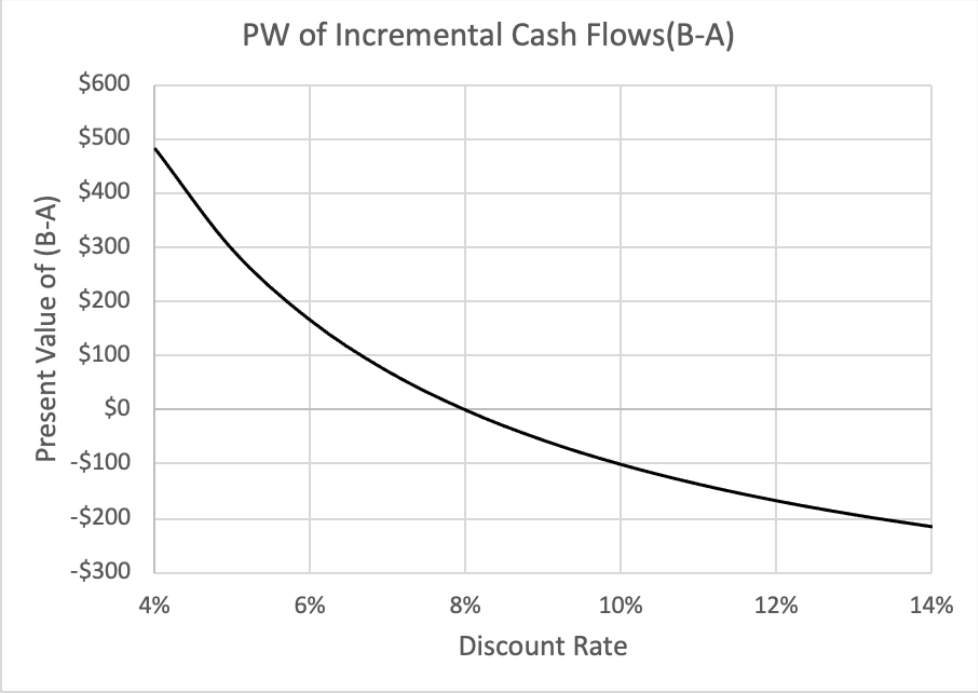
Is the investment worth it? Y/N **YES**

Reasoning:

IRR > MARR, therefore the project is worthwhile.

d. [**Lecture 7, Slides 27-33, *Engineering Economics*, 5.3.2**]

- You are considering two mutually exclusive investments, A and B.
- Investment A costs $1,000 in Year 0 and pays $50 every year from Year 1 to Year 100.
- Investment B costs $1,500 in Year 0 and pays $90 every year from Year 1 to Year 100.
- **Write down an equation that will let you solve for the incremental IRR between investments A and B.**
- Do not use a summation (Σ) sign. You don't need it.
- Remember, at the incremental IRR, the present value of the incremental cash flows is zero; NPV (B-A) = 0. I've plotted the relevant function for you below. This is similar to Figure 5.3 on p. 137 of the textbook.



Equation (use discount factors, e.g. (P/A,IIRR,100)):

-$500 + $40 x (P/A,IIRR,100) = 0

e. **Solve for the incremental IRR of (B-A), to four significant figures.** Use whichever numerical method you prefer (Excel's Goal Seek, Wolfram Alpha, Trial and Error, Linear Interpolation, etc.). As the diagram above shows you, your answer should be close to 8%. I don't need to see your calculations.

Incremental IRR (B – A) to 4 significant figures: __**7.996**____ %

f. **If your MARR is 4% per year, based on the incremental IRR you calculated in part e., which project, A or B, is preferred?** Briefly explain your reasoning. (*Hint: See Slide 28 of Lecture 7 and E.E. 5.3.2.*)

Which Investment is Preferred? A/B/None **B**

Reasoning: Project B. Both projects have IRR > MARR, so individually each is a worthwhile project. The IIRR > MARR, so the additional benefits from the more expensive project are worth it. Go with Project B.

g. [**Lecture 8, Engineering Economics 5.3.4**] Consider the following investment: Your MARR is 5% per year. You receive $1,000 in Year 0, pay $100 per year from Year 1 to Year 50, and receive income of $200 per year from Year 51 to Year 100. Since there is more than one sign change in the cash flow profile, you can't rely on the IRR method.

Write down an equation you can solve for the approximate ERR. Use discount factors, such as (P/A,ERR,50).

The last time period is Year 100, so bring everything forward to there.
$1,000 in Year 0 @ MARR: $1,000 x (F/P,5%,100)
-$100 from Year 0 to Year 50 @ ERR: -$100 x (P/A,ERR,50) x (F/P,ERR,100) (could also use -$100 x (F/A,ERR,50) x (F/P,ERR,50)).
$200 from Year 51 to Year 100 @ MARR: $200 x (F/A,5%,50) (could also use $200 x (P/A,5%,50) x (F/P,5%,50)).

Total:

$1,000 x (F/P,5%,100) + $200 x (F/A,5%,50) - $100 x (P/A,ERR,50) x (F/P,ERR,100) = 0

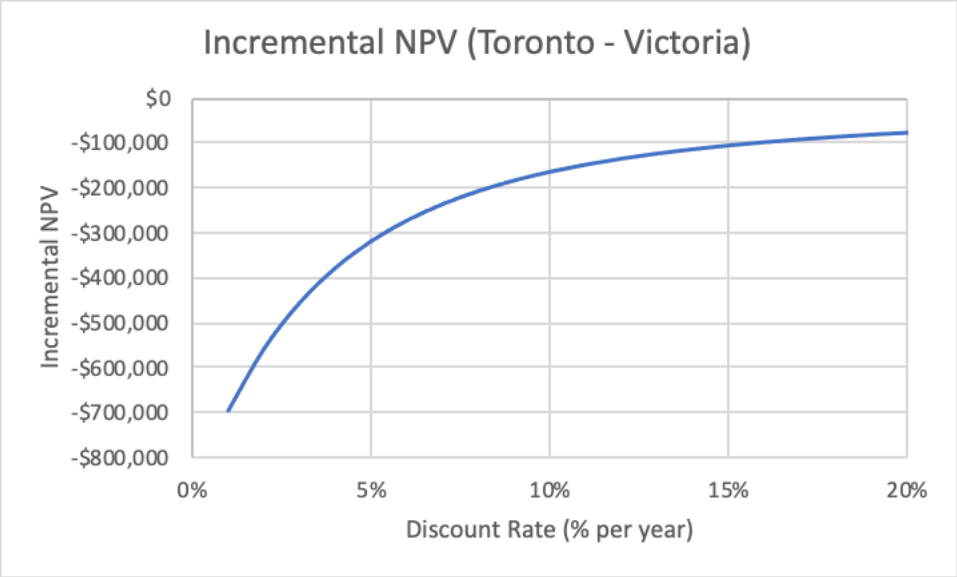
h**. Use numerical methods to calculate the approximate ERR to four significant figures**. You may use whichever numerical method you wish, and I don't need to see your calculations.

Approximate ERR to 4 significant figures: ___**4.591**_____ %

1. [**88 Challenge Version**][**IRR/ERR**][**Lectures 7 & 8, *Engineering Economics* 5.3.1 – 5.3.4**]

a. Suppose Sam must[1] choose between living in Victoria, or living in Toronto, and all of the assumptions are the same as in Question 1 of Project 2. **Plot the present value of the incremental cash flows between Victoria and Toronto as a function of the discount rate**, allowing the discount rate to vary between 1% per year and 20% per year (remember, you can calculate weekly and monthly interest rates as a function of the yearly interest rate).

[Hint: This is just like the plot in part 1.d of the regular version. The only difference is that you are using a more complicated present value equation, which you already should have from Project 2. If you are not confident in your present value equation for project 2, use a placeholder function for now. When the answer key for Project 2 is posted on the 11th, you can use the present value function from that.]



[Legacy Version – 100% Challenge] Students also have to solve for the incremental IRR, which should match the horizontal axis crossing shown in their graph, similar to the regular version of question 1. Also the usual interpretation: if IIRR > MARR, go for the more expensive project, otherwise the cheaper.

b. Suppose Sam chooses to live in Victoria. All of the assumptions are the same as in Question 1 of Project 2, but in addition, Sam receives a gift of $40,000 from a relative in Month 0. There is now more than one sign change in Sam's cash flow profile. **Use numerical methods to calculate the approximate ERR for Sam's 'project' of studying and working in Victoria, to four significant figures, and answer a few simple questions so we know what you're doing**. I don't need to see your calculations.

[1] Assume 'Don't live in either' is not an option. Even if the NPV is negative for both, Sam must choose the 'least bad' alternative.

Which cash flows are you bringing forward at the MARR? (e.g. Rent, Tuition – these are not the right answers, btw.)

Relative's gift net of first rent, grocery, tuition payments. Salary net of each year's first rent, food charges.

Which cash flows are you bringing forward at an approximate ERR to be solved for?

Tuition, food and rent charges, except for the first of each year during the working life, and the first ones in Year 0 (those are part of the net positive flows in those years).

What time period are you bringing them forward to? (e.g. 'The last year of Sam's studies' – not the right answer, btw.)

The time period of the last cash flow – Week 51 of the last year of Sam's working life (final grocery bill).

Approximate ERR to 4 significant figures: ___13.38_____ %

2. [**Inflation and Price Indices**][**Regular Version**]

a. [**Lecture 10, Slide 59**] Statistics Canada calculates a price index for rented accommodation in Victoria, which works like the CPI. In 1995, this index was 89.30. In 2020, this index is 126.2. Using this data, **calculate the average yearly inflation for rented accommodation in Victoria from 1995 to 2020, to four significant figures.** (You are answering the question, what annual rate of inflation f would make something that cost $89.30 in 1995 cost $126.20 in 2020?). Show your work.

Solve $89.30 x $(1 + f)^{25}$ = $126.20
$f = (126.2/89.3)^{(1/25)} - 1 = 1.393\%$

Average yearly inflation for rentals in Victoria, 1995-2020: ___**1.393**_____ %

b. [**Lecture 11**] Consider the following situation:

- Sam's nominal MARR is equal to their baseline interest rate from Project 1.
- Suppose that rent in Victoria is constant in real terms, at its baseline level from Project 1. (So if your baseline rent was $1,000 a month, rent will always be $1,000 in real terms.).
- Suppose yearly inflation for rental property is equal to the value you calculated in part a., and Month 0 is the base month for inflation calculations.
- **Calculate the present value of the first two years (24 months) of Sam's rent payments, to the nearest cent**. Show your work. (Hint: Remember you can use a real MARR with real cash flows, or nominal MARR with nominal flows. Both will give the same result.)

For my values… Rent: $1,561.41, Nominal MARR = 1.0882% per month
f = 1.383% per year = (1+1.383%)1/12-1 = .1154% per month
Real MARR = (1 + Nominal MARR)/(1+f) – 1 = 0.9717% per month
PV = $1,561.41 + $1,5641.1 x (P/A,0.9717%,23) = 33,605.02

(Nominal: Turn the cash flows into nominal cash flows by treating them like a geometric gradient with A = Rent, g = monthly inflation. Use nominal interest rate as MARR. Works out to the same calculation.)

Present value of the first two years of Sam's rent payments: $__**33,605.02**___

c. [**Lecture 10, Slides 54-56**] Below are the rent and food price indices for Victoria in 1995 and 2020. **Assuming your baseline values for rent and weekly food costs are 2020 values, use the price index information to calculate the 1995 cost of rent and food to the nearest cent.** (Recall that P_x/CPI_x is constant for all years x. You're looking for P_{1995}, and using the rented accommodation and Food price indices instead of the CPI.)

Price Index, Victoria		
Year	Rent	Food
1995	89.3	85.2
2020	126.2	148.7

P1995 = P2020 x Index1995/Index2020

Victoria Prices		
	Weekly Food Bill	Monthly Rent
1995 dollars	$148.60	$1,108.86
2020 dollars[2]	$210	$1,561.41

d. [**Lecture 10, Slides 45-49, *Engineering Economics* 9.A**] **Calculate a Laspeyres index for 1995, to two decimal places, using 2020 as the base year**, the prices from part c. and Sam's first month of food and rent costs as the basket. The two items in the basket are 1 x Monthly Rent, and 4 x Weekly Food Bill. Show your work. (Hint: You don't need to consider the value of money.)

Total spending in 2020: $210x4 + $1,561.41x1 = $840 + $1,561.41 = $2,401.41
Share of spending on food: 840/2,401.41 = 34.98%
Share on rent = 1,561.41/2,401.41 = 65.02%
L1995 = (34.98% x (148.60/210) + 65.02% x (1,108.86/1561.41)) x 100 = 70.93

Laspeyres Index for 1995: _____**70.93**_____

[2] These are just your baseline values for Victoria from Project 1.

2. [**88 Challenge Version**][**Inflation**]

a. [**Lecture 10, Slide 59**] The following table contains price index information for Victoria from Statistics Canada. The price indices shown work like the CPI, but they are for food and rent, not for all consumer items.

	1995		2020	
	Rent	Food	Rent	Food
Victoria	89.3	85.2	126.2	148.7

Using the price index data above, calculate the average yearly inflation in Victoria from 1995 to 2020, to four significant figures. (Hint: Letting CX be the price index in Year X, you are answering the question, what annual rate of inflation f would make something that cost C_{1995} in 1995 cost C_{2020} in 2020?) Show your work.

Rent: Solve $\$89.30 \times (1+f)^{25} = \$126.20$
$f = (126.2/89.3)^{(1/25)} - 1 = 1.393\%$

Food: Solve $\$85.20 \times (1+f)^{25} = \$148.70$
$f = (148.70/85.20)^{(1/25)} - 1 = 1.393\%$

Average Yearly Inflation (%)		
City	Rent	Food
Victoria	**1.393%**	**2.253%**

b. . [**Lecture 11**] Consider the following situation:

- Sam's nominal MARR is equal to their baseline interest rate from Project 1.
- Suppose that rent and weekly food costs in Victoria are constant in real terms, at their baseline levels from Project 1. (So if your baseline rent was $1,000 a month, rent will always be $1,000 in real terms.). This is true even after Sam starts working (no doubling).
- Suppose yearly inflation for rent and food are equal to the values you calculated in part a., and Month 0/Week 0/Year 0 is the base month/week/year for inflation calculations.
- As in project 2, Sam has (nominal) tuition payments of $7,936 every January 1 from Year 0 to Year 4, earns the (nominal) baseline salary from Project 1 in Year 5, and this salary increases (in nominal terms) by 3% a year for the rest of Sam's working life.

- **Calculate the present value of Sam studying and working in Victoria until the end of their working life**. Show your work. (Hint: Remember you can use a real MARR with real cash flows, or nominal MARR with nominal flows. Both will give the same result.)

Tuition, Salary are exactly as before in Project 2. Let R = baseline rent, F = baseline food cost. Cash flows for rent, food are constant in real terms, so easiest to work with real rates.

Real MARR for rent = (1+i)/(1+frent) – 1 = 12.30% per year. Letting this be rR, the monthly equivalent is rRm = $(1+rR)^{1/12} - 1$ = 0.9717% per month. There are 540 months in our 45-year project. Present value is R + R x (P/A,rRm,539) = -$161,372.69 for my values.

Real MARR for food = (1+i)/(1+ffood) – 1 = 11.36% per year. Letting this be rF, the weekly (7-day) equivalent is rFw = $(1+rF)^{7/365} - 1$ = 0.2066% per week. There are 2340 weeks (52 weeks a year) in our 45-year project. Present value is R + R x (P/A,rFw,2339) = -$101,056.47 for my values.

Tuition	-$29,003.90
Salary	$440,642.65
Rent	-$161,372.69
Food	-$101,056.47
Total	$149,209.59

Could also use the nominal MARR, and treat the rent and food as a geometric gradient where the growth rate is their respective inflations. Works out to the same calculation (recall (P/A,g,i,N) = (P/A,io,N)/(1+g), and in this case io would be the real interest rate, and our formula would be, for example, R + R x (1+fR) x (P/A,g,i,N) = R + R x (P/A,rRm,N), for the appropriate N.)

Present value of studying & working in Victoria: $__149,209.59_____

c. Among the project files is a spreadsheet with the price of numerous food items by year, from 1995 to 2019. **Pick one of the following: a recipe from the Canadian Cookbook, a recipe that has meaning to you, or the diet you put together for Project 1, if you answered the challenge versions of Question 1**. Adjust the list of ingredients, if necessary, to match the items you are given prices for. You only need three to eight ingredients, and it's fine to omit minor ingredients (such as salt) you don't have prices for. **Find the cost of buying this list of ingredients in 2019, and in 1995.** (For example, if one of your ingredients is 500g of Round Steak, the spreadsheet says that in 1995 1kg of Round Steak cost $10.07, so the cost of 500g of Round Steak in 1995 would be $5.035.)

Recipe or Diet Chosen: _Standard breakfast (eggs, bacon, orange juice)___

(3-8 items)	Item	Quantity	1995 Cost	2019 Cost
1	**Bacon (2 slices)**	**24 g**	**$0.1378**	**$0.3418**
2	**Eggs**	**2**	**$0.2667**	**$0.5383**
3	**Orange Juice**	**250 mL**	**$0.4150**	**$1.040**
4				
5				
6				
7				
8				
Total			**$0.8194**	**$1.9201**

d. [**Lecture 10, Slides 45-49, *Engineering Economics* 9.A**] **Calculate a Laspeyres index for 1995, to two decimal places, using 2019 as the base year**, the prices from part c. and your list of ingredients as the basket. Show your work.

Shares of spending in 1995:
Bacon: 0.3418/1.9201 = 17.80%
Eggs: 0.5383/1.9201 = 28.04%
Orange Juice: 1.040/1.9201 = 54.16%
L is weighted average of 2019/1995 Cost ratios for each item, with 2019 shares as weights.
L = (17.80% x 0.1378/0.3418 + 28.04% x 0.2667/0.5383 + 54.16% x 0.4150/1.040) x 100
L = 42.68

Laspeyres Index for 1995: _____**42.68**_____

3. [**Supply/Demand**] (**Challenge Question**)

Each part of this question adds 6 marks to your assignment mark if done correctly. Assignment mark. Your main assignment mark is only calculated from questions 1 and 2. If you score 88 on question 1, and 88 on question 2, your assignment mark is 88. Completing this question correctly can bring that up to 100.

a. **Read the three following news stories**, which talk about the impact of a vacant home tax and COVID-19 on the British Columbia (and Canadian) housing market.

Mackie, J. (2019, August 3). Vacancy taxes prompt sale of luxury condos in Downtown Vancouver [Web Page]. Retrieved from https://theprovince.com/business/real-estate/vacancy-taxes-prompt-sale-of-luxury-condos-in-downtown-vancouver/wcm/e5bc57d7-bf6a-4619-80a8-ac3e2f62baab

- A pre-pandemic article that looks at the vacancy tax from the point of view of people who currently own homes they leave vacant much of the year. (Supply)

Zochodne, G. (2019, September 3). Foreign buyer vacancy tax unlikely to derail housing rebound, real estate market watchers say [Web Page]. Retrieved from https://business.financialpost.com/news/economy/foreign-buyer-vacancy-tax-unlikely-to-derail-housing-rebound-real-estate-market-watchers-say

- A pre-pandemic article that looks at the vacancy tax from the point of view of people who may wish to buy homes in which they will only live for part of the year. (Demand)

Rockel, N. (2020, April 23). For B. C. real estate, will COVID-19 bring down the house? [Web page]. Retrieved from https://www.bcbusiness.ca/For-BC-real-estate-will-COVID-19-bring-down-the-house

- An article written during the pandemic that looks at the impact of COVID-19 on B.C.'s housing market.

Based on what you learn in the articles, draw the following diagrams:

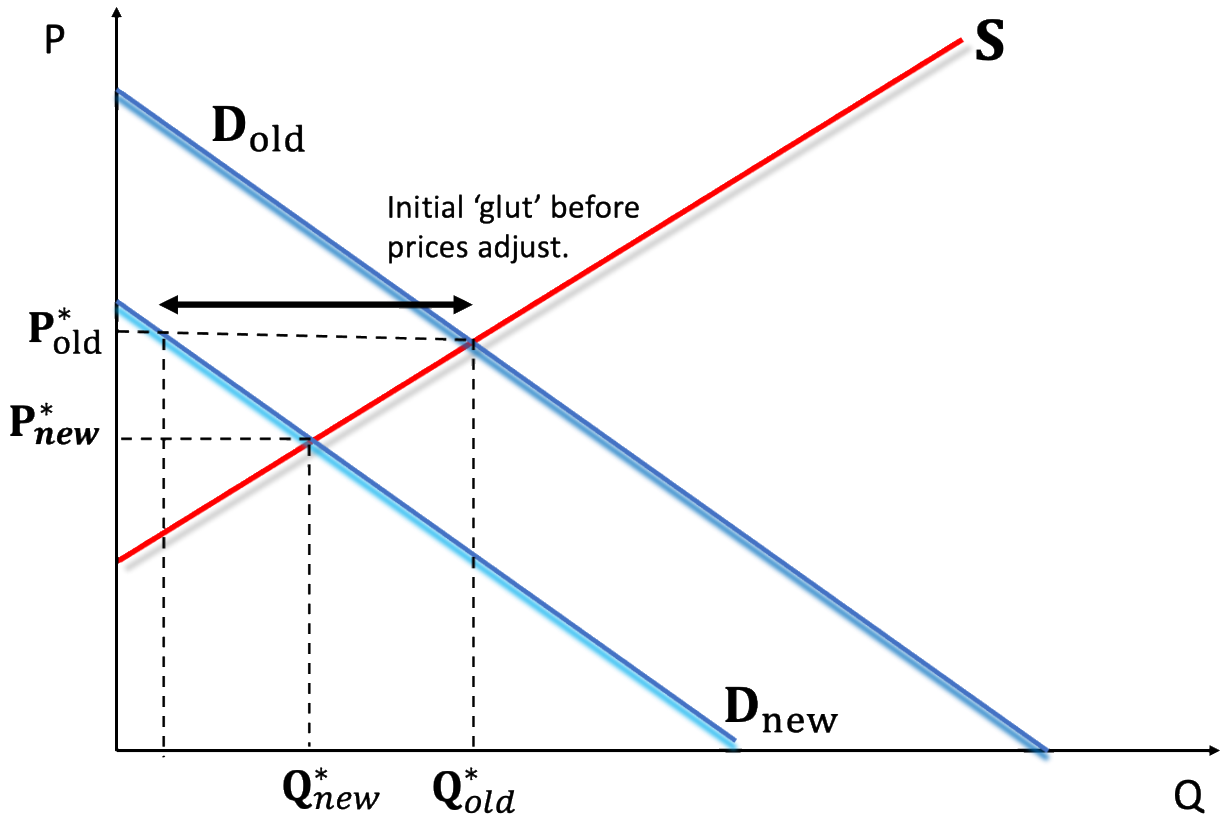
i. **Draw and label a supply and demand diagram that explains how the introduction of a vacant homes tax affected the market for high-end homes in Vancouver *before* the pandemic. Below the diagram, briefly explain what is going on in your diagram, and what the impact of the vacancy tax was on equilibrium price and quantity.**

We have two pre-pandemic articles:

Mackie: Increase in supply due to former part-time residents selling condos leads to a 'glut' as the market moves to the new equilibrium. Can be seen as supply curve shifting to the right.

Zochodne: Implies that there hasn't been a big dip in demand. If there HAD been a chill, it's warming up again, says the article. Also, supply/demand conditions are more balanced (less glut) suggesting market is near its new equilibrium.

How to model this? Most obvious way is probably Demand shifting down due to the tax, equilibrium moving from old to new (lower price, lower quantity). The second article suggests we're seeing rising prices, which may mean demand is shifting up as some new sector of the population enters the housing market, replacing the absentee homeowners, and bringing quantity closer to its original equilibrium level.

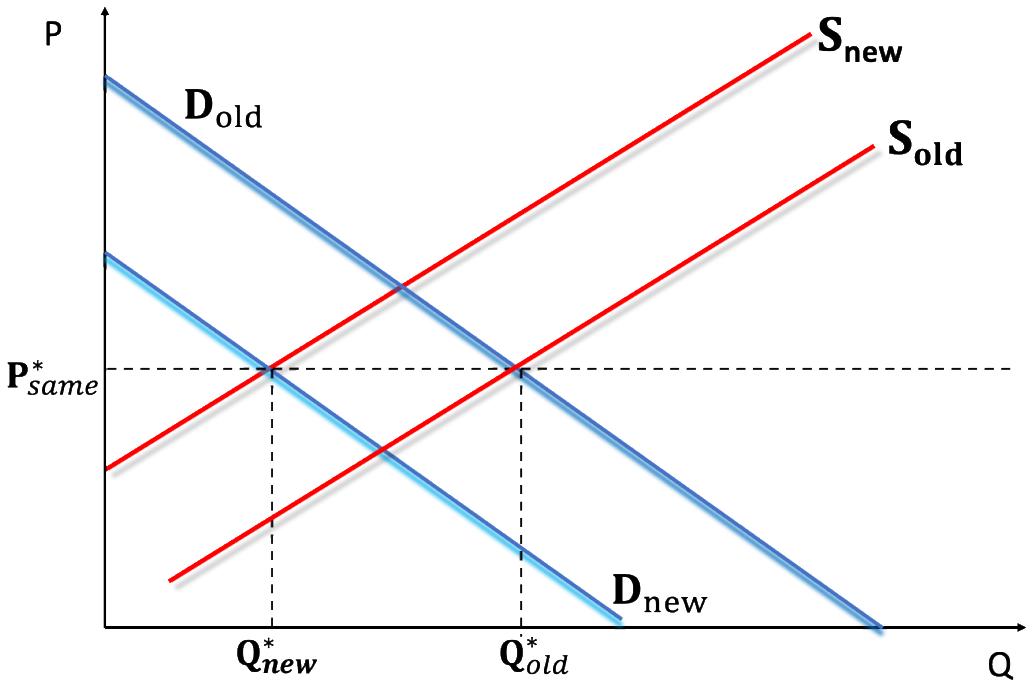


The above is just one possibility among many.

Any reasonable attempt that is consistent with the articles should get the marks. TAs should be kind here, as the engineering students didn't get too much time to practice supply & demand.

ii. **Draw and label a supply and demand diagram that explains how the global pandemic may affect the market for high-end homes in Vancouver in the near future (say, the next year or two). Below the diagram, briefly explain what is going on in your diagram, and what the impact of the pandemic is likely to be on equilibrium price and quantity.** Note that a vacant homes tax was already in place[3] in Vancouver when the pandemic hit.

The during-pandemic article mentions that "the residential market [is] losing both buyer and sellers," and that "property values are holding steady". That's a big neon sign saying that both supply and demand are falling (demand shifting down, supply shifting left) in such a way that price is unchanged. In terms of the curves, they're both shifting left. For any given price, there's less quantity demanded, and less quantity supplied. Drawing the diagram, we see that this should result in a fall in the equilibrium quantity. From the article: "As well as fewer buyers, Hasman has seen a substantial decline in listings."



[3] Vacancy Tax (Empty Homes Tax) Bylaw 11674, applicable from January 1, 2017. See https://vancouver.ca/your-government/vacancy-tax-bylaw.aspx

b. As a result of the global pandemic, firms and employees have learned that many jobs can be performed remotely. **Based on what you know about tax incidence and elasticity, what do you think will be the impact of the global pandemic on who feels the burden of a property tax on rental property? Briefly explain your reasoning.**

There are now more substitutes for housing: for example, someone working for a company headquartered in Vancouver no longer needs to live in Vancouver. This means that for any given % increase in, say, Vancouver housing prices, we'll see a greater % fall in quantity demanded than before, as some people will choose to live elsewhere. This means the price elasticity of demand has increased, and now renters should be seeing a *reduced* burden of the property tax relative to landlords.