Q:1. (a).

$$\int_1^3 \frac{y-3}{\sqrt{y^2-6y+10}}\,dy$$

$$y^2-6y+10 = y^2-6y+9+1$$
$$= (y-3)^2+1$$

$$= \int_1^3 \frac{y-3}{\sqrt{(y-3)^2+1}}\,dy$$

Let $y-3 = u \Rightarrow dy = du$

when; $y=1,\ u=-2$,

$y=3,\ u=0$

$$\Rightarrow \int_1^3 \frac{(y-3)}{\sqrt{(y-3)^2+1}}\,dy = \int_{-2}^0 \frac{u}{\sqrt{u^2+1}}\,du$$

Let $u^2+1 = w \Rightarrow (2u)du = dw$

when, $u=-2,\ w=5$

$u=0,\ w=1$

$$\Rightarrow \int_{-2}^0 \frac{u}{\sqrt{u^2+1}}\,du = \frac{1}{2}\int_5^1 \frac{1}{\sqrt{w}}\,dw = \frac{1}{2}\left[\frac{\sqrt{w}}{1/2}\right]\Bigg|_{w=5}^{w=1} = 1-\sqrt{5}$$

(b) $$\int \frac{3^{\sqrt{2x}}}{\sqrt{2x}}\,dx$$

Let $u = \sqrt{2x}$

$$\Rightarrow du = \frac{1}{\sqrt{2x}}\,dx$$

$$\Rightarrow \int \frac{3^{\sqrt{2x}}}{\sqrt{2x}}\,dx = \int 3^u\,du$$

$$= \frac{3^u}{\ln(3)} + C$$

$$= \frac{3^{\sqrt{2x}}}{\ln(3)} + C$$

(c) $$\int_{-1}^{2} \sqrt{x^2 - 6x + 9}\, dx$$

$$= \int_{-1}^{2} \sqrt{(x-3)^2}\, dx$$

let $u = (x-3) \Rightarrow du = dx$

when; $x = -1,\ u = -4$

$x = 2,\ u = -1$

$$\Rightarrow \int_{-1}^{2} \sqrt{(x-3)^2}\, dx = \int_{-4}^{-1} \sqrt{u^2}\, du = -\int_{-4}^{-1} u\, du = -\left[\frac{u^2}{2}\right]\Bigg|_{-4}^{-1}$$

$$= -\left[\frac{1}{2} - \frac{16}{2}\right]$$

$$= 15/2$$

(d) $\int \frac{\log_2(t)}{t}\, dt = \frac{1}{\ln(2)} \int \frac{\ln(t)}{t}\, dt$

Let $u = \ln(t)$

$\Rightarrow du = \frac{1}{t}\, dt$

$$\Rightarrow \int \frac{\log_2(t)}{t}\, dt = \frac{1}{\ln(2)} \int u\, du = \frac{1}{\ln(2)} \cdot \frac{u^2}{2} + C$$

$$= \frac{1}{2\ln(2)} \left[\ln(t)\right]^2 + C$$

(e).

$$\int_{-2}^{1} \frac{2r^3 + 7r^2 + 8r + 28}{r^2+4} \, dr$$

$$\begin{array}{r|l}
 & 2r + 7 \\ \hline
r^2+4 & 2r^3 + 7r^2 + 8r + 28 \\
 & -\underline{(2r^3 \qquad + 8r)} \\
 & 7r^2 \qquad + 28 \\
 & -\underline{(7r^2 \qquad + 28)} \\
 & 0
\end{array}$$

$$\Rightarrow \int \frac{2r^3 + 7r^2 + 8r + 28}{r^2+4} \, dr = \int_{-2}^{1} (2r+7) \, dr = \left(r^2 + 7r\right)\Big|_{r=-2}^{r=1} = 18$$

Q: 2 (a).

$$F(x) = \int_0^x e^{t^2}\, dt$$

$$\Rightarrow \quad \frac{dF}{dx} = \frac{d}{dx}\int_0^x e^{t^2}\, dt = e^{x^2}$$

(b).

$$G(x) = \int_{2x}^{3x+1} \sin(t^4)\, dt$$

$$= \int_{2x}^{0} \sin(t^4)\, dt + \int_0^{3x+1} \sin(t^4)\, dt$$

$$= -\int_0^{2x} \sin(t^4)\, dt + \int_0^{3x+1} \sin(t^4)\, dt$$

$$\Rightarrow \quad \frac{dG}{dx} = -\frac{d}{dx}\int_0^{2x} \sin(t^4)\,dt + \frac{d}{dx}\int_0^{3x+1} \sin(t^4)\,dt$$

$$= -\sin\left[(2x)^4\right]\frac{d}{dx}(2x) + \sin\left[(3x+1)^4\right]\frac{d}{dx}(3x+1)$$

$$= -2\sin\left[16x^4\right] + 3\sin\left[(3x+1)^4\right]$$