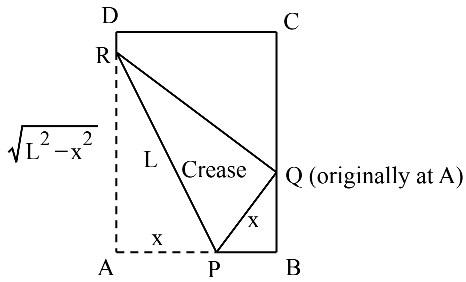
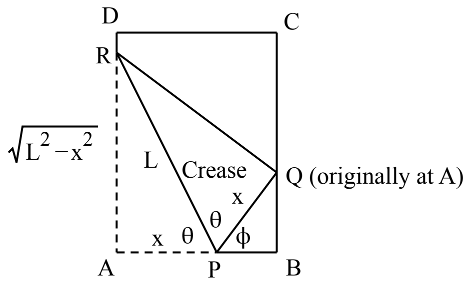
Student: Arfaz Hossain **Date:** 11/14/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 8

A rectangular sheet of 9.9-in.-by-12.3-in. paper is placed on a flat surface. One of the corners is placed on the opposite longer edge as shown in the figure, and held there as the paper is smoothed flat. The problem is to make the length of the crease as small as possible. Call the length L. Complete parts (a) through (c) below.



a. Find L^2 in terms of x. To find the formula for L^2, label the angles θ and ϕ as shown on the figure to the right. Notice that $\phi + 2\theta = 180$.



In terms of x and L, $\cos(\theta) = \frac{x}{L}$.

Therefore, $L = \frac{x}{\cos(\theta)}$.

$$\cos(\phi) = \frac{9.9 - x}{x}$$

Therefore, $\phi = \cos^{-1}\left(\frac{9.9 - x}{x}\right)$.

Solving $\phi + 2\theta = 180$ for θ gives $\theta = 90 - \frac{\phi}{2}$. Express $\cos(\theta)$ in terms of ϕ.

$$\cos(\theta) = \sin\left(\frac{\phi}{2}\right)$$

Now subsitute $\phi = \cos^{-1}\left(\frac{9.9 - x}{x}\right)$ in $\cos(\theta) = \sin\left(\frac{\phi}{2}\right)$.

$$\sin\left(\frac{\phi}{2}\right) = \sin\left(\frac{\cos^{-1}\left(\frac{9.9 - x}{x}\right)}{2}\right)$$

The next step is to apply a trigonometric identity. Using $\sin\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1 - \cos(\alpha)}{2}}$ with $\alpha = \phi = \cos^{-1}\left(\frac{9.9 - x}{x}\right)$, simplify $\sin\left(\frac{\phi}{2}\right)$.

$$\sin\left(\frac{\phi}{2}\right) = \sin\left(\frac{\cos^{-1}\left(\frac{9.9 - x}{x}\right)}{2}\right) = \sqrt{\frac{2x - 9.9}{2x}}$$

Recall that **cos** $(\theta) =$ **sin** $\left(\frac{\phi}{2}\right)$, and therefore, **cos** $(\theta) = \sqrt{\frac{2x - 9.9}{2x}}$. Also recall that $L = \frac{x}{\cos(\theta)}$. Using this information, find a formula for L^2.

$$L^2 = \frac{2x^3}{2x - 9.9}$$

b. What value of x minimizes L^2? To find the value of x that minimizes L^2, take the derivative of $L^2 = \frac{2x^3}{2x - 9.9}$.

$$\frac{dL^2}{dx} = \frac{8x^3 - 59.4x^2}{(2x - 9.9)^2}$$

Now set the numerator of the derivative equal to zero and solve.

$$8x^3 - 59.4x^2 = 0$$

$$x = 0 \qquad \text{or} \qquad x = 7.425$$

The value $x = 0$ is not meaningful since this would be that the paper is not folded. Therefore, the value of x that minimizes L^2 is 7.425.

c. What is the minimum value of L? Substitute $x = 7.425$ in $L^2 = \frac{2x^3}{2x - 9.9}$ and simplify to find the minimum value of L.

The minimum value of L, rounded to two decimal places is 12.86.