3-6-Example-RotEqII

Rotational Equilibrium - II

A $3kg$ mass is at $1m\hat{\imath} + 3m\hat{\jmath}$, a $4kg$ mass is at $-2m\hat{\imath} + 1m\hat{\jmath}$, and a $5kg$ mass is at $-1m\hat{\jmath}$.

- What is the location of the center of mass of this assembly?
- Gravity acts in the negative $\hat{k}$ direction; what is the total torque measured around the origin due to the three masses?
- What is the total torque measured around the origin due to a force equal to the total gravitational force acting at the center of mass?

$$\vec{r}_{cm} = \frac{\sum_{\text{all mass}} \vec{r}_i m_i}{\sum_{\text{all}} m_i} = \frac{(1m\hat{\imath}+3m\hat{\jmath})3kg + (-2m\hat{\imath}+1m\hat{\jmath})4kg + (-1m\hat{\jmath})5kg}{3kg+4kg+5kg}$$

$$= \frac{-5kgm\hat{\imath} + 8kgm\hat{\jmath}}{12kg}$$

$$= -0.417m\hat{\imath} + 0.667m\hat{\jmath}$$

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3$$

$$= (1m\hat{\imath} + 3m\hat{\jmath}) \times (-3kg\, 9.8N/kg\, \hat{k})$$

$$+ (-2m\hat{\imath} + 1m\hat{\jmath}) \times (-39.2N\hat{k})$$

$$+ (-1m\hat{\jmath}) \times (-49N\hat{k})$$

$$= 29.4\,\mathrm{Nm}\,\hat{j} - 88.2\,\mathrm{Nm}\,\hat{i}$$

$$+ -78.4\,\mathrm{Nm}\,\hat{j} - 39.2\,\mathrm{Nm}\,\hat{i}$$

$$+ 49\,\mathrm{Nm}\,\hat{i}$$

$$= -49\,\mathrm{Nm}\,\hat{j} - 78.4\,\mathrm{Nm}\,\hat{i}$$

$$= -78.4\,\mathrm{Nm}\,\hat{i} - 49\,\mathrm{Nm}\,\hat{j}$$

is this same as

$$\vec{\tau}_{net} = \vec{r}_{cm} \times \vec{F}_{g,total}$$

$$= (-0.417\,\mathrm{m}\,\hat{i} + 0.667\,\mathrm{m}\,\hat{j}) \times (-117.6\,\mathrm{N}\,\hat{k})$$

$$= -49\,\mathrm{Nm}\,\hat{j} - 78.4\,\mathrm{Nm}\,\hat{i}$$

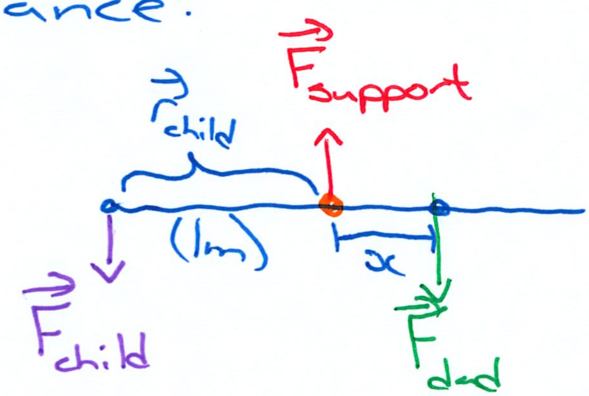
How to solve problem of rotational equilibrium

Apply idea

$\vec{F}_{net} = 0$ & $\underbrace{\vec{\tau}_{net} = 0}_{\text{implies choice of point as origin}}$

If $\vec{F}_{net} = 0$ then $\vec{\tau}_{net} = 0$ for one choice of origin implies $\vec{\tau}_{net} = 0$ for any.

Suppose 2m long bar (m=0), supported in middle. A 20kg child sits one end. Where does 100kg parent sit to balance?



looking at rot equil of bar

For child

$$\left.\begin{array}{l} \uparrow \vec{F}_{bar\,on\,child} \\ \bullet \\ \downarrow \vec{F}_g \end{array}\right\} \text{sum to } 0$$

$$\vec{F}_{bar\,on\,child} = -\vec{F}_{g(on\,child)}$$

3^{rd} law $\vec{F}_{child\,on\,bar} = -\vec{F}_{bar\,on\,child} = \vec{F}_{g\,on\,child}$

On bar 3 forces, where father?

Origin at support

$$\vec{r}_{child} = -1m\,\hat{i} \qquad \vec{F}_{child} = -196N\,\hat{k}$$

$$\vec{r}_{support} = 0\,\hat{i} \qquad \vec{F}_{support} = ?$$

$$\vec{r}_{dad} = x\,\hat{i} \qquad \vec{F}_{dad} = -980N\,\hat{k}$$

$$\vec{\tau}_{net} = \vec{\tau}_{child} + \vec{\tau}_{support} + \vec{\tau}_{dad}$$

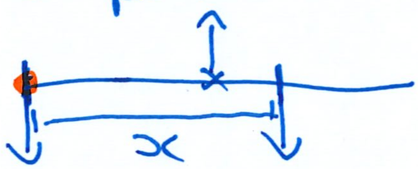
$$= \vec{r}_{child} \times \vec{F}_{child} + 0 + \vec{r}_{dad} \times \vec{F}_{dad}$$

$$= (-1m\,\hat{i}) \times (-196N\,\hat{k}) + (x\,\hat{i}) \times (-980N\,\hat{k})$$

$$0 = -196Nm\,\hat{j} + 980N\,x\,\hat{j}$$

$$x = 0.2m$$

What if pick childs location as origin?



$\vec{r}_{child} = 0\hat{i}$ $\qquad$ $\vec{F}_{child} = -196N\hat{k}$

$\vec{r}_{support} = 1m\hat{i}$ $\qquad$ $\vec{F}_{support} = ?\ (1176N\hat{k})$

$\vec{r}_{dad} = x\hat{i}$ $\qquad$ $\vec{F}_{dad} = -980N\hat{k}$

$$\vec{\tau}_{net} = 0 = 0 + (1m\hat{i}) \times \underline{\cancel{\vec{F}_{support}}}^{\,1176N\hat{k}} + (\underline{x}\hat{i}) \times (-980N\hat{k})$$

2 unknowns

Bar equilibrium

$$0 = \vec{F}_{child} + \vec{F}_{support} + \vec{F}_{dad}$$

$$0 = (-196N\hat{k}) + \vec{F}_{support} + (-980N\hat{k})$$

$$F_{support} = 1176N\hat{k}$$

$$0 = -1176Nm\hat{j} + 980Nx\hat{j}$$

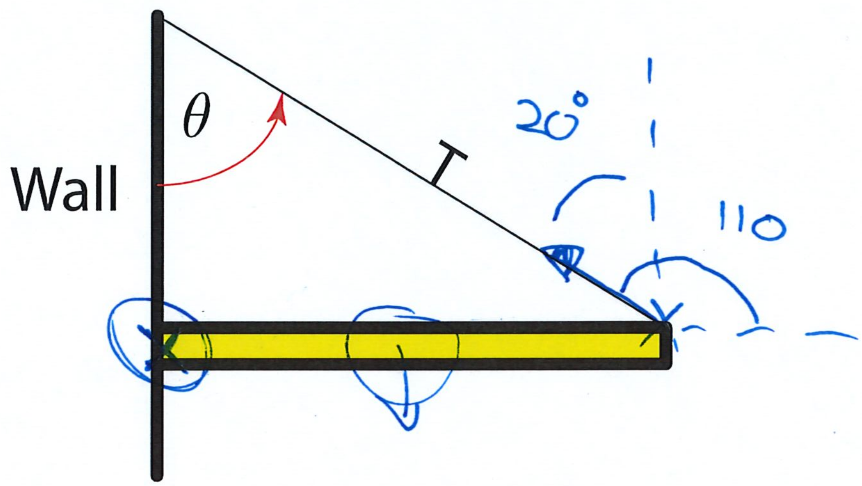
y-comp $\qquad 0 = -1176Nm + 980Nx$

$$x = 1.2m$$

3-9-Example-RotEqIII

Rotational Equilibrium - III

A $10kg$ uniform beam of length $5m$ is held horizontally by a rope. The rope is attached to a vertical wall, and to the far end of the beam; the rope makes an angle of $\theta = 20°$ with the wall.



- What is the tension in the rope? ie what is the magnitude of the force the rope exerts?
- What is the vertical component of the force the wall exerts on the beam?
- What is the horizontal component of the force the wall exerts on the beam?

$\vec{F}_{net} = 0 \qquad \vec{\tau}_{net} = 0$

3 Forces : Wall, Rope, gravity

Wall $\vec{r}_{wall} = 0\hat{\imath}$ $\vec{F}_{wall} = \underline{F_{w,x}}\hat{\imath} + \underline{F_{w,z}}\hat{k}$

Rope $\vec{r}_{rope} = 5m\hat{\imath}$ $\vec{F}_{rope} = T\cos 110\,\hat{\imath} + T\sin 110\,\hat{k}$

or $\cos 20$

$= -0.342T\hat{\imath} + 0.940T\hat{k}$

gravity $\vec{r}_{gravity} = 2.5m\hat{\imath}$ $\vec{F}_g = -98N\hat{k}$

$$\vec{\tau}_{net} = 0 = \vec{\tau}_{wall} + \vec{\tau}_{rope} + \vec{\tau}_g$$

$$= (0\hat{\imath}) \times (F_{w,x}\hat{\imath} + F_{w,z}\hat{k})$$

$$+ (5m\hat{\imath}) \times (-0.342T\hat{\imath} + 0.940T\hat{k})$$

$$+ (2.5m\hat{\imath}) \times (-98N\hat{k})$$

$$= \cancel{0 + 0(-\hat{\jmath}) + 0} - 4.70mT\hat{\jmath}$$

$$+ 245Nm\hat{\jmath}$$

$$T = 521.3N$$

$$\vec{F}_{net} = 0 = \vec{F}_{wall} + \vec{F}_{rope} + \vec{F}_g$$

$$= \left(F_{w,x}\,\hat{i} + F_{w,z}\,\hat{k}\right) + \left(-17.8.3\,\mathrm{N}\,\hat{i} + 49.0\,\mathrm{N}\,\hat{k}\right)$$

$$+ \left(-98\,\mathrm{N}\,\hat{k}\right)$$

$$0 = \underbrace{\left(F_{w,x} - 17.8\,\mathrm{N}\right)}_{0}\hat{i} + \underbrace{\left(F_{w,z} + 49\,\mathrm{N} - 98\,\mathrm{N}\right)}_{0}\hat{k}$$

$$F_{w,x} = 17.8\,\mathrm{N}$$

$$F_{w,z} = 49\,\mathrm{N}$$