Question 1

Determine, and prove, a good upper bound for the following recurrence relation.

$$T(n) = T(n-1) + T(n-2) + 1$$
$$T(1) = 1$$

Question 2 (Connect the dots) Using only the following functions:

(a) $g(n) = 1$
(b) $g(n) = \log n$
(c) $g(n) = n$
(d) $g(n) = n \log n$
(e) $g(n) = n^{3/2}$
(f) $g(n) = n^2$
(g) $g(n) = 2^n$
(h) $g(n) = n^n$

give the best lower bound on the left and the best upper bound on the right. Use the letters to indicate the correct function.

$\Omega(g(n))$ $O(g(n))$

_____(1)_____ $T(n) = \sum_{i=1}^{n} (\log i)^2$

_____(2)_____ $T(n) = \sum_{i=1}^{n} \sqrt{i}$

_____(3)_____ $T(n) = \sum_{i=1}^{n} \frac{n}{i}$

_____(4)_____ $T(n) = \frac{a^2 n}{\log n}$

_____(5)_____ $T(n) = n! - (n-1)!$

_____(6)_____ $T(n) = \frac{n!}{2^n}$

Question 3: (a) Add the following elements in turn to an AVL tree: 13 8 11 5 2 9 10 7 6 15

(b) Show the AVL tree that would result if the 2 was deleted.

Question 4

part a

Draw the graph with the following adjacency matrix:

	A	B	C	D	E	F	G	H
A	-1	9	2	-1	4	7	-1	5
B	9	-1	7	11	-1	10	-1	-1
C	2	7	-1	4	-1	3	8	-1
D	-1	11	4	-1	3	1	-1	6
E	4	-1	-1	3	-1	-1	9	2
F	7	10	3	1	-1	-1	16	-1
G	-1	-1	8	-1	9	16	-1	-1
H	5	-1	-1	6	2	-1	-1	-1

Question 4

part b Showing sufficient work, using Prims algorithm, determine the minimum cost spanning tree for the above graph starting at node A

Question 5

Short Answers. Give the best, shortest answer.

(a) In an AVL tree where the distance from the root to the most distant leaf is 6, what is the minimum distance that a leaf could be from the root?

(b) In a complete binary tree where the distance from the root to the most distant leaf is 6, what is the minimum distance that a leaf could be from the root?

(c) Give a one sentence description of how you would implement quicksort to ensure that its time complexity was O(n log n) worst case.

(d) In two sentences or less, discuss the advantages and disadvantages of a hash table of size 97 versus a hash table of size 120.

(e) Discuss the similarities, in one sentence, between Dijkstra's shortest path algorithm and a BFS.

Question 5 (Continued)

(f) If the following is a sequence of Huffman codes then translate the sequence 111001001010, and if it is not then explain why it is not.

a: 100	h: 1110	t: 1010	o: 110
e: 0	i: 11111	w: 1011	period: 10110

(g) Draw the tree with weight sequence 9 4 2 1 1 1 3 1 1

(h) Draw the binary tree that is equivalent to the tree you drew for part (g)

Question 6 Note that this question relates to undirected graphs, and that if your answer to either part b, part c, or part d requires any additional functions or procedures, you must write those additional routines.

Part a Give the declarations necessary to implement a graph with weighted edges where the node names are three character codes such as YVR. Note that you may assume that the number of vertices is less than 40. Although the type for vertex is given in Pascal, you do not need to write your answer in Pascal.

```
type
   VertexName = Packed Array[1..3] of char;
```

Part b Write a function that takes the Graph and a vertex name and returns the value true if the vertex is in the graph.

Part c Write a procedure to add an edge to the graph.

Part d Write a function that takes the graph and two vertices and returns the weight of the edge between the two vertices or -1 if no edge exists.

Question 7

Write a procedure that takes two parameters, a complete binary tree represented using the array implementation and the position of a node in the tree and returns the position of the node that would occur next in an inorder traversal.