Symbolab

Solution

Check convergence of $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^3\cdot 7^n}$: converges

Steps

$$\sum_{n=1}^{\infty}\frac{(-2)^n}{n^3\cdot 7^n}$$

Apply Series Ratio Test: converges

Hide Steps

$$\sum_{n=1}^{\infty}\frac{(-2)^n}{n^3\cdot 7^n}$$

Series Ratio Test:

If there exists an N so that for all $n\ge N,\quad a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{(-2)^{(n+1)}}{(n+1)^3 7^{(n+1)}}}{\frac{(-2)^n}{n^3 7^n}}\right|$$

Simplify $\left|\frac{\frac{(-2)^{(n+1)}}{(n+1)^3\cdot 7^{(n+1)}}}{\frac{(-2)^n}{n^3\cdot 7^n}}\right|$: $\frac{2|n^3|}{7|(n+1)^3|}$

Hide Steps

$$=\left|\frac{\frac{(-2)^{n+1}}{(n+1)^3\cdot 7^{n+1}}}{\frac{(-2)^n}{n^3\cdot 7^n}}\right|$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$$=\left|\frac{(-2)^{n+1}n^3\cdot 7^n}{(n+1)^3\cdot 7^{n+1}(-2)^n}\right|$$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$$\frac{(-2)^{n+1}}{(-2)^n}=(-2)^{n+1-n}$$

$= \frac{7^{n}(-2)^{n-n+1}n^{3}}{7^{n+1}(n+1)^{3}}$

Add similar elements: $n+1-n=1$

$= \frac{(-2)\cdot 7^{n}n^{3}}{7^{n+1}(n+1)^{3}}$

Apply exponent rule: $\frac{x^{a}}{x^{b}} = \frac{1}{x^{b-a}}$

$\frac{7^{n}}{7^{n+1}} = \frac{1}{7^{n+1-n}}$

$= \frac{(-2)n^{3}}{7^{n-n+1}(n+1)^{3}}$

Add similar elements: $n+1-n=1$

$= \left|\frac{(-2)n^{3}}{7(n+1)^{3}}\right|$

Remove parentheses: $(-a) = -a$

$= \left|\frac{-2n^{3}}{7(n+1)^{3}}\right|$

Apply the fraction rule: $\frac{-a}{b} = -\frac{a}{b}$

$= \left|-\frac{2n^{3}}{7(n+1)^{3}}\right|$

Apply absolute rule: $|-a| = |a|$

$= \left|\frac{2n^{3}}{7(n+1)^{3}}\right|$

Apply absolute rule: $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$= \frac{|2n^{3}|}{|7(n+1)^{3}|}$

Apply absolute rule: $|ax| = a|x|, a \geq 0$

$|7(n+1)^{3}| = 7|(n+1)^{3}|$

$= \frac{|2n^{3}|}{7|(n+1)^{3}|}$

Apply absolute rule: $|ax| = a|x|, a \geq 0$

$|2n^{3}| = 2|n^{3}|$

$= \frac{2|n^{3}|}{7|(n+1)^{3}|}$

Hide Steps

$\lim_{n\to\infty}\left(\frac{2\left|n^3\right|}{7\left|(n+1)^3\right|}\right)=\frac{2}{7}$

$\lim_{n\to\infty}\left(\frac{2\left|n^3\right|}{7\left|(n+1)^3\right|}\right)$

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n^3 is positive when $n\to\infty$. Therefore $\left|n^3\right|=n^3$

$=\lim_{n\to\infty}\left(\frac{2n^3}{7\left|(n+1)^3\right|}\right)$

$(n+1)^3$ is positive when $n\to\infty$. Therefore $\left|(n+1)^3\right|=(n+1)^3$

$=\lim_{n\to\infty}\left(\frac{2n^3}{7(n+1)^3}\right)$

$\lim_{x\to a}\left[c\cdot f(x)\right]=c\cdot\lim_{x\to a}f(x)$

$=\frac{2}{7}\cdot\lim_{n\to\infty}\left(\frac{n^3}{(n+1)^3}\right)$

Simplify $\frac{n^3}{(n+1)^3}$: $\left(\frac{n}{n+1}\right)^3$ *Show Steps*

$=\frac{2}{7}\cdot\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^3\right)$

$\lim_{x\to a}\left[f(x)\right]^b=\left[\lim_{x\to a}f(x)\right]^b$
With the exception of indeterminate form

$=\frac{2}{7}\left(\lim_{n\to\infty}\left(\frac{n}{n+1}\right)\right)^3$

Divide by highest denominator power: $\frac{1}{1+\frac{1}{n}}$ *Show Steps*

$=\frac{2}{7}\left(\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)\right)^3$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)},\quad \lim_{x\to a}g(x)\neq 0$
With the exception of indeterminate form

$=\frac{2}{7}\left(\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}\right)^3$

$\lim_{n\to\infty}(1)=1$ *Show Steps*

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1$ *Show Steps*

$=\frac{2}{7}\left(\frac{1}{1}\right)^3$

Simplify

$= \frac{2}{7}$

$L < 1,$ by the ratio test

$=$ converges

$=$ converges