CHAPTER 9

Solutions to Chapter-End Problems

A. Key Concepts

Constant and Current Dollars:

9.1 **(a)** current
(b) real
(c) current
(d) real
(e) current
(f) current
(g) real

9.2 **(a)** 400(P/F, 4%, 3) = 400(0.85480) = \$341.92
(b) 400(F/P, 4%, 3) = 400(1.1699) = €467.96
(c) 10(P/F, 4%, 1) = 10(0.94154) = \$9.42
(d) 350 983(P/F, 4%, 10) = 350 983(0.67556) = \$237 110
(e) 1(F/P, 4%, 1000) = $(1.04)^{1000}$ = £1.0798×10^{17}
(f) 1 000 000 000(P/F, 4%, 300) = 1 000 000 000/$(1.04)^{300}$ = \$7762.44

Time Value of Money under Inflation:

9.3 **(a)** 400(P/F, 4%, 3)(P/F, 4%, 3) = 400(0.85480)(0.85480) = \$292.27
(b) 400(F/P, 4%, 3)(F/P, 4%, 3) = 400(1.1699)(1.1699) = €547.47
(c) 10(P/F, 4%, 1)(P/F, 4%, 1) = 10(0.94154)(0.94154) = \$8.86
(d) 350 983(P/F, 4%, 10)(P/F, 4%, 10)
= 350 983(0.67556)(0.67556) = \$160 182
(e) 1(F/P, 4%, 1000)(F/P, 4%, 1000) = 1.04^{2000} = £1.659×10^{34}
(f) 1 000 000 000(P/F, 4%, 300)(P/F, 4%, 300)
= 1 000 000 000/1.04^{600} = \$0.06

9.4 **(a)** Using the present worth factor to convert to current (today) dollars:

Real dollar amount = 10 000(P/F, 10%, 5) = 10 000(0.62092) = 6209

This is worth about \$6209 in real (today) dollars.

(b) We can first convert to real dollars using the present worth factor, and then find the present worth using the present worth factor:

PW = 10 000(P/F, 10%, 5)(P/F, 10%, 5)
= 10 000(0.62092)(0.62092) = \$3855

(c) $MARR_C = (1 + MARR_R)(1 + f) – 1 = (1.1)(1.1) – 1 = 0.21$

The current dollar MARR is 21%.

(d) PW = 10 000(P/F, 21%, 5) = 10 000(0.38554) = 3855

The present worth of the investment is $3855.

9.5 **(a)** The actual dollar MARR, $MARR_A$, is given by

$MARR_A = (1 + MARR_R)(1 + f) – 1 = (1.08)(1.06) – 1 = 0.1448 = 14.48\%$

(b) PW = 1000(P/A, 14.48%, 10) = 1000(5.1199) = 5119.9

The present worth of the annuity is about $5120.

9.6 **(a)** PW = –7500 + 1000(P/A, i, 12)

The actual dollar internal rate of return is between 8% and 8.5% in a spread sheet approximation. Sample trial and error results are shown below:

Interest rate	PW
0.08	36.08
0.081	2.81
0.085	–155.31
0.09	–339.28

(b) If we take the actual dollar internal rate of return to be 8.1%, the real internal rate of return is obtained as 1.081/1.06 – 1 = 0.0198, or about 2.0%.

9.7 **(a)** PW = –90 000 + 10 000(P/A, i, 10)

The actual dollar internal rate of return is between 1.95% and 2.0% in a spread sheet approximation. Sample trial and error results are shown below:

Interest rate	PW
0.019	297.65
0.0195	61.29
0.02	–174.15
0.0205	–408.69

(b) If we take the actual dollar internal rate of return to be 2% the real internal rate of return is obtained as 1.02/1.05 – 1 = –0.02857, or about –2.9%. Notice that when the actual dollar internal rate of return is less than the inflation rate the real internal rate of return is negative.

9.8 **(a)** 1.59(F/P, 4%, 50) = 1.59(50.505) = $80.30
We would expect to pay about $80.30 for the hamburger.

(b) 15 000(F/P, 4%, 50) = 15 000(50.505) = $757 575
We would expect to pay about $757 575 for the automobile.

(c) 180 000(F/P, 4%, 50) = 180 000(50.505) = $9 090 900
We would expect to pay about $9 090 900 for the house.

9.9 1 000 000 = 38 000(F/P, 5%, N)
1.05^N = 1 000 000/38 000 = 26.316
N = ln(26.316)/ln(1.05) = 67.025
The average person will be a millionaire in about 67 years.

B. Applications

9.10 **(a)** 10 000(P/F, 5%, 10)(P/F, 4%, 10)
= 10 000(0.61391)(0.67556) = $4147.33

(b) [10000(P/F, 5%, 5)(P/F, 8%, 5](P/F, 5%, 5)
= [10000(0.78353)(0.68058)](0.78353) = $4178.21

(c) [10000(P/F, 5%, 5)](P/F, 8%, 5)(P/F, 5%, 5)
= [10000(0.78353)](0.68058)(0.78353) = $4178.21

(d) [10000(P/F, 5%, 5)(P/F, 2%, 5][(P/F, 5%, 5)(P/F, 6%, 5)]
= [10000(0.78353)(0.90573)][(0.78353)(0.74726)]= $4155.10

(e) [10000(P/F, 5%, 1)(P/F, 40%, 1](P/F, 5%, 9)
= [10000(0.95238)(0.71429)](0.64461) = $4385.12

Since the present worth of the $10 000 was fairly close for a variety of inflation patterns, and that the average of 4% per year was the most conservative of the calculated values, it probably is reasonable to use an average inflation rate.

9.11 Real MARR = (1 + 3)/(1 + 2.5) – 1 = 14.3%

9.12 *Years 3-5:*
$MARR_C$ = (1 + 0.12)(1 + 0.1) – 1 = 0.232 = 23.2%
PW(end of year 2) = 25 000(P/A, 23.2%, 3) + 25 000
(in actual dollars)

Year 2:
$MARR_C$ = (1 + 0.12)(1 + 0.05) – 1 = 0.176 = 17.6%

PW(end of year 1) = PW(end of year 2)(P/F, 17.6%, 1) + 25 000
(in actual dollars)

Year 1:
$MARR_C$ = (1 + 0.12)(1 + 0.03) – 1 = 0.1536 = 15.36%
PW(now)
= PW(end of year 1)(P/F, 15.36%, 1)
= {[25 000(P/A, 23.2%, 3) + 25 000](P/F, 17.6%, 1) + 25 000}
$\times$(P/F, 15.36%, 1)
= {[25 000(2.0053) + 25 000](0.85034) + 25 000}(0.86685)
= 77 053 (in today's dollars)

The present worth of the contract is about $77 000.

9.13 First get the current dollar MARR: $MARR_C$ = 1.03/1.1 – 1 = 0.133
The current dollar MARR is about 13.3%.

We then use the series present worth factor:
PW = 2000(P/A, 13.3%, 30) = 14 682.55

The present worth is about $14 683.

9.14 *Method 1*: Use present worth factor to convert to real (today) dollars, and present worth factor to convert to money today.

For example, 4th year amount
= P_4 = 15 000(P/F, 4%, 4)(P/F, 8%, 4)
= 15 000(0.854 80)(0.73503) = 9424.55

Method 2: Combine the inflation rate and interest rate in to an current dollar MARR.

$MARR_C$ = (1 + $MARR_R$)(1 + f) – 1 = 1.08/1.04 – 1 = 0.123 = 12.3%

PW = 15 000(P/A, 12.3%, 10) = 15 000(5.577) = 83 655

The annuity is worth $83 655 today.

9.15 For the real rubles, the present worth is:
PW(real) = 500(P/A, 1.5%, 24) = 500(20.030) = 10 015

For months 13-24:
$MARR_C$ = (1 + 0.015)(1 + 0.2) – 1 = 21.8%

For months 0-12:
$MARR_C$ = (1 + 0.015)(1 + 0.4) – 1 = 42.1%

PW(current)
= 500(P/A, 21.4%, 12)(P/F, 42.1%, 12) + 500(P/A, 42.1%, 12)
= 500(4.2169)(0.01475) + 500(2.3403) = 1201.25

The total present worth of the contract is about 10 015 + 1 201 = 11 216 million rubles.

9.16 The price in 2016 relative to 2013 is 1.1(130/125) = 1.144. The percentage increase is about 14.4%.

9.17 **(a)** The current IRR of the project can be found by solving for i in:

PW = –200 000 + (22 000 – 2 000)(P/A, i, 20) = 0
(P/A, i, 20) = 200 000/20 000 = 10
i = 0.0775 or 7.75%

The current IRR of the project is 7.75%.

(b) The current MARR is

$$\begin{aligned}MARR_C &= MARR_R + f + MARR_R \times f \\ &= 0.04 + 0.03 + 0.04(0.03) = 0.0712 \text{ or } 7.12\%\end{aligned}$$

(c) Yes, they should invest, as the current rate of return exceeds the current MARR.

9.18 **(a) & (b)** The calculation results are summarized in the following table:

Year	Current	Real (f = 0.03)	Real (MARR = 0.04)
0	–200 000	–200 000	–200 000
1	20 000	20 600	19 808
2	20 000	21 218	19 617
19	20 000	35 070	16 646
20	20 000	36 122	16 486
		Total:	161 972

(c) They should invest, as the present worth of the real cash flows at the real MARR is positive ($161 972).

9.19 **(a)**

Year	Real Zerts Cash Flow (2018 Zerts)	Inflation Factor	Actual Zerts Cash Flow	Exchange Rate Factor	Actual Dollar Cash Flow	Real Dollar Cash Flow (2018 dollars)	Present Worth ($)
0	-1 500 000	1	-1 500 000	0.25	-375 000	-375 000	-375 000
1	260 870	1.15	300 000	0.22	67 500	65 534	55 328
2	260 870	1.32	345 000	0.2	69 863	65 852	46 938
3	260 870	1.52	396 750	0.18	72 308	66 172	39 820
4	260 870	1.75	456 263	0.16	74 838	66 493	33 782

5	260 870	2.01	524 702	0.15	77 458	66 816	28 659
6	260 870	2.31	603 407	0.13	80 169	67 140	24 313
7	260 870	2.66	693 918	0.12	82 975	67 466	20 627
						Total:	-125 532

(b) The present worth of the project is –\$125 532 in dollars. It was obtained by using the real MARR and the real dollar cash flow. The real MARR is given by

$MARR_R = (1 + MARR_A)/(1 + f) – 1 = 1.22/1.03 – 1 = 0.184$

We could also have found the present worth in dollars using the current dollar MARR and the actual dollar cash flow.

9.20 The present worths were computed with current dollar cash flows and the current dollar MARR.

2% Inflation Cash Flows:					
	In-House			**Purchase**	
	Current Dollar	**Act Dollar Other**	**PW**	**Current Dollar**	**PW**
Year	**Labour Cost**	**Operat. Cost**	**(all Op. cost)**	**Purchase**	**(Purchase)**
1	262 500	225 000	406 250	750 000	625 000
2	278 460	234 090	355 938	765 000	531 250
3	295 390	243 547	311 885	780 300	451 563
4	313 350	253 387	273 310	795 906	383 828
5	332 402	263 623	239 529	811 824	326 254
			1 586 912	**Total PW:**	2 317 895
Total PW in-house, including first cost:			1 786 912		

(a) \$278 460
(b) \$243 547
(c) Purchase PW = \$2 317 895, In-house PW = \$1 786 912

9.21 **(a)** The PW of the project is obtained by subtracting the PW of operating cost and the first cost from the PW of revenues.

Notice that the present worths are obtained using the current dollar MARR since the current dollar MARR and the real MARR are the same under the assumption of zero inflation. The present worths of operating and cost and revenues are shown below.

Metcan No Inflation Cash Flows:				
	Operating		**PW**	**PW**
Year	**Cost**	**Revenue**	**Oper. Cost**	**Revenue**
1	332 500	398 125	277 083	331 771
2	332 500	402 106	230 903	279 240
3	332 500	406 127	192 419	235 027
4	332 500	410 189	160 349	197 815
5	332 500	414 290	133 624	166 494
6	332 500	418 433	111 354	140 132
7	332 500	422 618	92 795	117 945
8	332 500	426 844	77 329	99 270
9	332 500	431 112	64 441	83 552
10	332 500	435 423	53 701	70 323
f =	0	**Total:**	1 393 997	1 721 571
i =	0.2		**PW =**	152 574

This gives a present worth for the project under the assumption of zero inflation of $152 574.

(b) The present worths are shown below. They were obtained with current dollar cash flows and the current dollar MARR.

Inflation rate	PW
0	152574
0.01	108775
0.02	63019
0.03	15213
0.04	–34742

Cash flows under the assumption of 1% inflation are shown below.

Metcan 1% Inflation Cash Flows:				
	Operating		PW	PW
Year	Cost	Revenue	Oper. Cost	Revenue
1	332 500	398 125	277 083	331 771
2	335 825	402 106	233 212	279 240
3	339 183	406 127	196 287	235 027
4	342 575	410 189	165 208	197 815
5	346 001	414 290	139 050	166 494
6	349 461	418 433	117 034	140 132
7	352 955	422 618	98 503	117 945
8	356 485	426 844	82 907	99 270
9	360 050	431 112	69 780	83 552
10	363 650	435 423	58 732	70 323
f =	0.01	**Total:**	1 437 795	1 721 571
i =	0.2		**PW =**	108 775

(c) The project is only marginally profitable with inflation at 3%. We checked at 3.5%, and found a project present worth of –$490. That is, inflation must be less than 3.5% for this project to break even. Inflation rates above 3.5% are well within the normal range. A conservative decision, therefore, would be to reject the project.

9.22 See the table:

	In-house	Contract	PW	PW
Year	**current $**	**current $**	**in-house**	**contract**
0	25 000		25 000	
1	108 000	55 000	88 525	45 082
2	113 400	55 000	76 189	36 952
3	119 070	55 000	65 573	30 289
4	125 024	63 700	56 435	28 754
5	131 275	63 700	48 572	23 569
6	137 838	63 700	41 803	19 319
		Total:	402 097	183 965

The present worths were computed with current dollar cash flows and the current dollar MARR. The contract has the lowest costs for the company.

C. More Challenging Problems

9.23 Depends on specific circumstances

9.24 Depends on specific circumstances

9.25 **(a)** Comment on real yen receipts: These are shown in yen of year 1. They might have been shown in yen of year 0. Either would have been correct.

Year	Receipts (year 1 yen)	Inflation Factor	Current Yen Receipts	Rate Factor	Current Dollar Receipts	Receipts (Yr 0 Dollars)	Worth of Receipts ($)
0		1		0.015			
1	30 000 000	1	30 000 000	0.0153	459 000	445 631	376 230
2	33 000 000	1.01	33 330 000	0.0156	520 148	490 289	349 468
3	36 300 000	1.02	37 029 630	0.0159	589 442	539 423	324 610
4	39 930 000	1.03	41 139 919	0.0162	667 968	593 481	301 520
5	43 923 000	1.041	45 706 450	0.0166	756 954	652 955	280 073
6	48 315 300	1.051	50 779 866	0.0169	857 796	718 390	260 151
7	53 146 830	1.062	56 416 431	0.0172	972 071	790 383	241 646
8	58 461 513	1.072	62 678 655	0.0176	1 101 571	869 590	224 457
						Total:	2 358 153

The present worth of receipts is $2 358 153. This was obtained using the current dollar receipts and the current dollar MARR, 22%.

(b) The present worth of the cash outflow is $1 997 069, which was computed using current dollar cash outflow and the current dollar MARR.

Year	Real Dollar Cash Outflow (Yr 0 dollars)	Current Dollar Cash Outflow	Present Worth of Cash Outflow ($)
0	200 000	200 000	200 000
1	339 806	350 000	286 885
2	373 786	396 550	266 427
3	411 165	449 291	247 428
4	452 282	509 047	229 783
5	497 510	576 750	213 397
6	547 261	653 458	198 179
7	601 987	740 368	184 047
8	662 185	838 837	170 922
		Total:	1 997 069

9.26 **(a)** PW = –1 800 000 + 550 000(P/A, i, 5)

The real Ibernian IRR, based on a spreadsheet approximation, is just over 16.0%. Sample trial and error results are shown below:

Interest rate	PW
0.1575	11414
0.16	862
0.161	–3331

0.1625	–9591

(b) The current pound (CP) internal rate of return is given by

$IRR_{AP} = (1 + IRR_{RP})(1 + f_I) – 1$

where IRR_{RP} is the real IRR in pounds and f_I is the inflation rate in Ibernia.

If we take the real IRR in Ibernian pounds to be 16%, we get the current pound IRR to be 0.276, or 27.6%

(c) The current dollar IRR is given by

$IRR_A = (1 + IRR_{RP})(1 + f_I)(1 + f_e) – 1$

where IRR_{RP} is the real IRR in pounds, f_I is the inflation rate in Ibernia, and f_e is the rate of exchange between the dollar and the Ibernian pound.

If we take the real IRR in Ibernian pounds to be 16%, we get 21.2% as the current dollar IRR.

Alternative method: Get the current dollar IRR by converting current Ibernian cash flows into actual dollars and computing the current dollar IRR directly. Sample trial and error results are shown below:

Interest rate	PW
0.2075	24899
0.21	12176
0.2125	–430
0.215	–12922

We must compare the 21.2% with Leftway's current dollar MARR. This is obtained with the relationship,

$MARR_C = (1 + MARR_R)(1 + f) – 1 = 1.15(1.025) – 1 = 0.179$

Leftway's current dollar MARR is about 18%. This is well below the actual dollar IRR. The project is, therefore, acceptable.

9.27 **(a)** The real MARR is given by

$MARR_R = (1 + MARR_C)/(1 + f) – 1 = 0.113$

This gives 11.3%.

(b) PW = –220 000 + (50 000 + 30 000)(P/A, i, 4) + 20 000(P/F, i, 4)

The real IRR is between 19% and 19.5%. This is based on a spreadsheet approximation. Sample trial and error results are shown below:

Interest rate	PW
0.185	3272.06
0.19	1060.22
0.1925	–30
0.195	–1115.6

(c) The current IRR is given by $IRR_C = (1 + IRR_R)(1 + f) – 1$. If we take the real IRR turn as 19.25%, we get the current IRR as 26.4%.

(d) The current IRR may be computed with a spreadsheet by first computing the current cash flows using the inflation rate. We get an current IRR to be between 26% and 26.5%. Sample trial and error results are shown below:

Interest rate	PW
0.255	3735.71
0.26	1640.94
0.265	–421.57
0.27	–2452.5

(e) The present worth may be obtained with the current cash flows and the current MARR or with real cash flows and the real MARR.

PW(savings, real) = 80 000(P/A, 11.3%, 4) = 246 614.21
PW(scrap, real) = 20 000(P/F, 11.3%, 4) = 13 033.15
PW(first cost) = 220 000
PW(total) = 39 647.37

We get the present worth of about $39 650.

9.28 **(a)** PW = –15 500 000 + 325 000×(35 – 12 – 7.75)(P/A, i, 6)

The real internal rate of return is close to 22.5%. Sample trial and error results are shown below:

Interest rate	PW
0.22	195 948
0.225	9098
0.23	–173 788
0.235	–353 700

(b) We want the inflation rate that will make the real MARR equal to 22.5%.

$1 + MARR_R = (1 + MARR_C)/(1 + f)$
$f = (1 + MARR_C)/(1 + MARR_R) – 1 = 1.25/1.225 – 1 = 0.0204$

We see that inflation must exceed 2.0% per year for this project to break even.

(c) We can get the present worths by discounting the real cash flows with the real MARR's that are implied by the three inflation rates, 1%, 2%, and 3%.

Inflation rate	$MARR_R$	PW
0.01	0.2376	–446503
0.02	0.2255	–8940
0.03	0.2136	441011

(d) This is a close call. We see that we need to have inflation over 2.0% per year for the project to break even. This is on the low side historically for inflation rates. But there have been countries with periods when inflation has been this low.

9.29 **(a)** We first get the effect of reductions in operating cost and revenue on cash flows.

We get the present worth of these flows by discounting at the observed MARR, 20%.

No Inflation Cash Flow:				
	Operating		**PW**	**PW**
Year	**Cost**	**Revenue**	**Oper. Cost**	**Revenue**
1	4 262 500	6 050 000	3 552 083	5 041 667
2	4 219 875	5 959 250	2 930 469	4 138 368
3	4 177 676	5 869 861	2 417 637	3 396 910
4	4 135 899	5 781 813	1 994 550	2 788 297
5	4 094 540	5 695 086	1 645 504	2 288 727
6	4 053 595	5 609 660	1 357 541	1 878 664
7	4 013 059	5 525 515	1 119 971	1 542 070
8	3 972 929	5 442 632	923 976	1 265 782
9	3 933 199	5 360 993	762 280	1 038 996
10	3 893 867	5 280 578	628 881	852 843
		Total:	17 332 893	24 232 325
	i =	0.2	**PW =**	-600 568

We subtract the present worth of variable costs and the first cost from the present worth of revenue. This gives a present worth of about –€601 000.

(b) The internal rate of return is between 17% and 17.5%. This is both a real and an current dollar internal rate of return since we assume inflation is zero. Sample trial and error results are shown below:

Interest rate	PW
1.165	262576
1.17	128455
1.175	–1836
1.18	–128438

(c) The required inflation rate is given by

$1 + IRR_C = (1 + IRR_R)(1 + f)$
$f = (1 + MARR_C)/(1 + MARR_R) – 1 = 1.2/1.175 – 1 = 0.0213 = 2.13\%$

(d) The project probably should be rejected. The break-even requires an inflation rate of over 2.5%. While this is not high by historical standards, there have been many countries with periods with lower inflation rates.

9.30 **(a)** The cash flows as well as present worths of operating costs and revenues under the assumption of zero inflation are shown below.

No Inflation Cash Flows:				
	Operating		**PW**	**PW**
Year	**Cost**	**Revenue**	**Oper. Cost**	**Revenue**
1	1 200 000	1 500 000	960 000	1 200 000
2	1 248 000	1 530 000	798 720	979 200
3	1 297 920	1 560 600	664 535	799 027
4	1 349 837	1 591 812	552 893	652 006
5	1 403 830	1 623 648	460 007	532 037
6	1 459 983	1 656 121	382 726	434 142
7	1 518 383	1 689 244	318 428	354 260
8	1 579 118	1 723 029	264 932	289 076
9	1 642 283	1 757 489	220 423	235 886
10	1 707 974	1 792 639	183 392	192 483
f =	0	**Total:**	4 806 057	5 668 118
i =	0.25		**PW =**	**-37 939**

We see that the project present worth is negative under the assumption of zero inflation.

(b) The present worths are shown below.

Inflation rate	**PW**
0	–37939
0.01	–18801
0.02	998

Cash flows under the assumption of 2% inflation are shown below.

2% Inflation Cash Flows:				
	Operating		**PW**	**PW**
Year	**Cost**	**Revenue**	**Oper. Cost**	**Revenue**
1	1 200 000	1 500 000	960 000	1 200 000
2	1 272 960	1 560 600	814 694	998 784
3	1 350 356	1 623 648	691 382	831 308
4	1 432 458	1 689 244	586 735	691 914
5	1 519 551	1 757 489	497 926	575 894
6	1 611 940	1 828 492	422 560	479 328
7	1 709 946	1 902 363	358 602	398 954
8	1 813 910	1 979 218	304 324	332 058

9	1 924 196	2 059 179	258 261	276 378
10	2 041 187	2 142 369	219 171	230 035
f =	0.02	**Total:**	5 113 655	6 014 654
i =	0.25		**PW =**	998

(c) We see that for inflation rates lower than 2%, the project is not profitable. A conservative decision would be to reject the project. While 2% is on the low side historically, there have been countries with periods with inflation this low.

9.31 **(a)** Under the assumptions of fixed real prices and no inflation we have simply

PW = (Revenue – Operating cost)(P/A, i, N) – (First cost)

For wood veneer, we have:
PW = (960 000 – 375 000)(P/A, 25%, 10) – 2 050 000 = $39 744

For plastic top, we have:
PW = (1 170 000 – 405 000)(P/A, 25%, 10) – 2 700 000 = $31 435

(b) For 2% inflation we can work with real cash flows and the real MARR = 1.25/1.02 – 1 = 0.22549. For the plastic tops, the equation from part (a) is used with the new real MARR of 22.549% to find a present worth of $248 557. For wood veneer top desks, we can get the changing real values of cash flows. We can then work with these and the real MARR. These cash flows and present worths are shown below.

2% Inflation: Wood Cash Flows (real dollars)				
	Operating		**PW**	**PW**
Year	**Cost**	**Revenue**	**Oper. Cost**	**Revenue**
1	375 000	960 000	306 000	783 360
2	393 750	969 600	262 181	645 614
3	413 438	979 296	224 637	532 089
4	434 109	989 089	192 469	438 527
5	455 815	998 980	164 907	361 416
6	478 606	1 008 970	141 292	297 865
7	502 536	1 019 059	121 059	245 488
8	527 663	1 029 250	103 724	202 322
9	554 046	1 039 542	88 870	166 745
10	581 748	1 049 938	76 144	137 425
		Total:	1 681 283	3 810 851

We then subtract the present worth of operating costs and first cost from the present worth of revenue to get the project present worth as $79 568.

9.32 **(a)** We must get the annuities that are equivalent to the first costs for both systems. We then add this to the operating costs per unit.

These are shown below.

Annual worth comparison for fixed prices:					
	Inflation	Real	Fixed	Operating	Total
	rate (/year)	MARR	cost (/year)	cost (/unit)	cost (/unit)
Manual	0%	20%	11926	10.50	11.30
Automated	0%	20%	57245	9.00	12.82

(b) The automated system has a higher first cost than the manual system. However, its operating costs are lower and stable. The effect of the higher first cost is reduced by a lower real MARR. Therefore, a higher expected inflation rate, which implies a lower real MARR, favours the automated system. As well, it is clear that a higher rate of increase in the real operating cost for the manual system favours the automated system. The manual system has a lower present value of costs for zero inflation and a 4% per year increase in manual operating costs. But the automated system is better for a 2% inflation rate and 4% rate of increase in manual operating costs and for no inflation combined with a 5% rate of increase in manual operating costs. A sample table for 2% inflation and 4% rate of increase in manual operating cost is shown below. The table contains real cash flows. The present worths are computed with the real MARR of 17.6%.

2% Inflation Cash Flows (real dollars):				
	Manual		Automatic	
		PW		PW
Year	Oper. Cost	Oper. Cost	Oper. Cost	Oper. Cost
1	157 500	133 875	135 000	114 750
2	163 800	118 346	135 000	97 538
3	170 352	104 617	135 000	82 907
4	177 166	92 482	135 000	70 471
5	184 253	81 754	135 000	59 900
6	191 623	72 270	135 000	50 915
7	199 288	63 887	135 000	43 278
8	207 259	56 476	135 000	36 786
9	215 550	49 925	135 000	31 268
10	224 172	44 134	135 000	26 578
	Total:	817 766		614 391
	PW(Total)=	867 766	**PW(Total)=**	854 391

Since an inflation rate of 2% per year is at the low end of the typical range, the automated system appears to be the preferred alternative.

9.33 The cost of living clause clearly lowers risk for workers who are protected against inflation that is unknown at the time of the contract signing. It is not clear what the effect is for employers. There are two possibilities. (1) If revenues do not go up and down with general inflation the contract increases their risk. This is because inflation that leads to an increase in wages without an offsetting increase in revenue would lead to lower real net revenue. (2) If revenues do go up and down with inflation, the same inflation that would lead to rise in current dollar wage rates would also lead to an increase in revenue. Therefore, there would be no additional risk for employers in the second case.

9.34 First, note that the real MARR for Mid-Atlantic is the same, whether the money is invested in Columbo or Avalon. However, because of the different inflation rates, the actual MARR will be larger in Columbo rather than Avalon:

Actual MARR in Columbo is: 0.1 + 0.04 + (0.1 x 0.04) = 0.144
Actual MARR in Avalon is: 0.1 + 0.02 + (0.1 x 0.02) = 0.122

To accumulate \$200 000 = £100 000 in 10 years, the amount set aside in Columbo would be:

200 000(P/F, 0.144, 10) = 200 000 x 0.260459 = \$52 092 = £26 046

The amount set aside in Avalon would be:

100 000(P/F, 0.122, 10) = 100 000 x 0.31628 = £31 628= \$63 256

Consequently, the amount of \$52 092 should be set aside in Columbo.

This may seem counter-intuitive. Since pounds lose value more slowly, it would be natural to assume that Mid-Atlantic should keep its money in pounds. But since the exchange rate is fixed, and the actual MARR is larger in Columbo, Mid-Atlantic is actually better of investing its money in Columbo.

Consequently, when looking at the two investment opportunities, regardless of which one Mid-Atlantic pursues, it should exchange pounds for dollars wherever possible. Thus although the investment and receipts in Avalon are in pounds, the equivalent amounts in dollars should then be used for calculation purposes under the assumption that pounds and dollars will be exchanged immediately as needed or received.

The present worth ten years from now of \$200 000 invested in Columbo:

P_C = 0.3(P/A, 0.144, 5) x 200 000 = 0.3 x 3.586987 x 200 000 = \$215 219 = £107 610

The present worth ten years from now of \$200 000 = £100 000 invested in Avalon (using the Columbo actual MARR since the pound receipts are immediately converted to dollars):

P_A = 0.24(P/A, 0.144, 7) x 100 000= 0.23 x 4.534959 x 100 000 = 1.043 x 100 000 = £104 300 = \$208 600

Consequently, Mid-Atlantic should make its investment in Avalon, converting all receipts from pounds to dollars as they are received.

Notes for Case-in-Point 9.1

1) There is no difference.

2) To avoid embarrassing the government.

3) No correct answer – this depends on the judgement of the student and current general perceptions of the economy.

Notes for Mini-Case 9.1

1) No inflation rate can change the decision of the high-pressure pipeline over the conventional pipeline, if only the capital cost is considered, since both projects would be affected in the same way. However, the difference in the savings would have been significantly smaller with a 4% inflation rate for the same actual dollar capital costs.

2) The student should be able to demonstrate that there is a significant opportunity to control the best choice using a selected inflation rate when the cash flow structures of projects under consideration are very different.

3) Perhaps to demonstrate that inflation was being considered. If he was well informed, he would have used actual dollars, but he may have mistakenly used real values.

Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

9S.1

It is worth an actual dollar.

9S.2

It is bad news for the retired CEO, unless her pension is inflation-linked.

If the insurance company is a life insurance company, inflation is good news, since life insurance companies usually promise a fixed sum on death and that sum can now be paid out in inflated dollars. But the question leaves open the possibility that the company insures against fire or theft; in which case, if the amount to be paid out is set at replacement cost, the news is neither good nor bad. (Since the replacement cost in real dollars will be constant.)

For the mortgage company, it is bad news: they have lent a sum of money in the past, and they're now going to get paid back in deflated dollars. (If the mortgages are variable-rate, they may be able to put the rates up to cushion the effect.)

9S.3

We have three options to consider: living in Canada and saving money in a Canadian bank; living in Placidia and saving money in a Placidian bank; and living in Placidia and saving money in a Canadian bank. (Nothing in the question forbids keeping our money under the mattress rather than in the bank, but this is obviously not a good idea.)

The simplest approach is to use a spreadsheet, **9S_3.xls**:

Year	Can. Income ($)	Pla. Income (P)	Pla. Income ($)	COL ($)	COL (P)	Banked ($)	Banked (P)	FW ($)	FW (P)
1	45 000	55 000	47 826	10 000	11 500	35 000	43 500	42 543	52 875
2	47 250	61 600	46 578	10 000	13 225	37 250	48 375	43 122	56 000
3	49 613	68 992	45 363	10 000	15 209	39 613	53 783	43 673	59 296
4	52 093	77 271	44 180	10 000	17 490	42 093	59 781	44 198	62 770
5	54 698	86 544	43 027	10 000	20 114	44 698	66 430	44 698	66 430
Total (P)									297 371
Total ($)								218 233	147 846

From the spreadsheet results, we see that the Canadian job will give us $218 233 in the bank at the end of five years. The Placidian job will give us an impressive P 297 371, but this converts back to a rather less impressive $147 846. So, all other things being equal, we should take the Canadian job.
Moving the Placidian paycheques back to Canada does help, but not enough to change our decision.

As we see from the table below, the future worth of this strategy is $196 226.

Year	Pla. Income (P)	Pla. Income ($)	COL (P)	Banked (P)	Banked (P to $)	FW(P to $)
1	55 000	47 826	11 500	43 500	37 826	45 978
2	61 600	46 578	13 225	48 375	36 578	42 344
3	68 992	45 363	15 209	53 783	35 363	38 988
4	77 271	44 180	17 490	59 781	34 180	35 889
5	86 544	43 027	20 114	66 430	33 027	33 027
Total (P)						
Total ($)						196 226

9S.4

To find the after-tax cost of the asset, we adjust the initial cost using the tax benefit factor, or "*TBF*" (see page 283 of the text). The *TBF* is:

$TBF = td / (i+d) = 0.4 \times 0.2 / (0.1+0.2) = 0.267$

If depreciation began in the first year of purchase, the after-tax initial cost would be reduced by the factor (1−*TBF*). But under current Canadian tax law, we have to take into account the "half-year rule"; from Example 8.9 in the text, we note that application of this rule implies that the effective reduction in purchase cost is given by the factor:

(1−*TBF* × 0.5 × (1 + (*P/F*,*i*,1))), which we shall call "1−*TBF*.*" In this case,
$1-TBF^* = 1-TBF \times 0.5 \times (1 + (P/F,0.1,1))$
$= 1 – 0.267 \times 0.5 \times (1 + 0.9091)$
$= 1–0.254$
$= 0.745$
If there is no inflation, the present worth of switching from buying to leasing is

$$\begin{aligned}PW &= 100\,000(1-TBF^*) - (1-t)(20\,000)(P/A,0.1,8) – 5000(1 - TBF^*)(P/F,0.1,8)\\ &= 74\,500 – 0.6 \times 20\,000 \times 5.3349 – 5000 \times 0.745 \times 0.4665\\ &= 8743\end{aligned}$$

The present worth of making the switch is $8743, so you should do it.

Introducing inflation makes the question more complicated; we work out the after-tax, deflated present cost of each option in the following tables (which were generated using the spreadsheet **9S_4.xls**).

Buying

Year	Annual CCA	Tax Saving	Residual Tax Saving	Discounted	Deflated
			0		−100 000
1	10 000	4000	0	3636	3 306
2	18 000	7200	0	5950	4 918
3	14 400	5760	0	4328	3 251
4	11 520	4608	0	3147	2 150
5	9 216	3686	0	2289	1 421
6	7 373	2949	0	1665	940
7	5 898	2359	0	1211	621
8	4 719	1887	0	881	411
9	3 775	1510	0	640	272
10	3 020	1208	2944	1601	617
Total					−82 094

Where does the "Residual Tax Saving" column come from? For the tenth year, the depreciation is $3020, which saves 3020 × 0.4 = $1324 in taxes. But there is still some money left in the asset class, the undepreciated capital cost of the asset, or UCC, which will continue to shield us from taxes indefinitely.

$UCC = 90\,000 \times 0.8^8 = 15\,099$

The inflation-adjusted interest rate is (1.1 × 1.1 −1.0) = 0.21, and the TBF based on this interest rate is 0.195, so we save 15 099 × 0.195 = $2944 for the year-11-to-eternity value of the tax shield.

Leasing

There is some possibility of getting different answers, depending on whether you consider the lease cost to inflate from $20 000 at the end of year 1 or from $20 000 at the beginning of year 1. There is also a choice as to whether you assume that the $5000 purchase price in year 8 is in real or actual dollars. The solution below assumes that they are actual.

We again use the spreadsheet, doing all calculations in actual dollars and deflating them in the right-hand column. The “After-Tax Price” column represents the cost of purchasing the asset at the end of the eighth year, reduced by the factor (1−*TBF*), which we have just calculated to be 1−0.195 = 0.805.

Year	Lease Cost	Tax Saving	After-Tax Price	Discounted	Deflated
0					
1	−20 000	8 000	0	−10 909	−9 917
2	−22 000	8 800	0	−10 909	−9 016
3	−24 200	9 680	0	−10 909	−8 196
4	−26 620	10 648	0	−10 909	−7 451
5	−29 282	11 713	0	−10 909	−6 774
6	−32 210	12 884	0	−10 909	−6 158
7	−35 431	14 172	0	−10 909	−5 598
8	−38 974	15 590	−4025	−12 787	−5 965
Total					−53 110

We conclude from this comparison that inflation increases the attractiveness of leasing compared to buying.

9S.5

We will again use a spreadsheet—**9S_5.xls**— to solve the problem. All figures in the spreadsheet are in actual dollars until we deflate them in the right-hand column.

We begin by ensuring that we have identified each cash flow correctly. The company’s annual revenue of $110 000 is in real dollars, so we inflate it to actual dollars in the left-hand column. The machine depreciates by $15 000 actual dollars every year, which yields a tax saving of $15,000*t* =$6900. Operating costs are $52 000/year, and we should expect these to rise with inflation. The lease cost of the facilities is $6000 actual dollars for the first five years—if it had been in real dollars, there would be no need to re-negotiate it—and $8029 actual dollars for the next ten years.

Year	Annual Revenue	Depreciation Tax Saving	Operating Costs	Lease Costs	Discounted	Deflated
0	−150 000	0	0	0	−150 000	−150 000
1	110 000	6900	−52 000	−6000	56 635	53 429
2	116 600	6900	−55 120	−6000	57 674	51 329
3	123 596	6900	−58 427	−6000	58 735	49 315
4	131 012	6900	−61 933	−6000	59 818	47 382
5	138 872	6900	−65 649	−6000	60 924	45 526
6	147 205	6900	−69 588	−8029	60 450	42 615
7	156 037	6900	−73 763	−8029	61 664	41 010
8	165 399	6900	−78 189	−8029	62 899	39 464
9	175 323	6900	−82 880	−8029	64 156	37 974
10	185 843	6900	−87 853	−8029	65 436	36 539
Total						294 582

We conclude that the present worth of the proposal is $294 582.

9S.6

The first part of this question can be solved most simply by doing all our calculations in real pounds. The present worth of the machine and its associated cash flows are then

PW = −10 000 + 2000(P/A,0.2,0.1,7) – 1000(P/A,0.1,7)
We calculate the growth-adjusted interest rate, i^0:

i^0 = 1.1 / 1.2 − 1 = −0.0833

Using the appropriate formula, we find that the conversion factor is

$$(P/A,0.2,0.1,7) = \left(\frac{(1-(1-0.0833)^7)}{0.0833(1-0.0833)^7}\right)\frac{1}{1.2}$$

= 10.06 / 1.2
= 8.39

Thus the present worth, in real pounds, is

PW = −10 000 + 2000 × 8.39 −1000 × 4.8684
= −10 000 + 16 780 − 4868
= 1912
So the pre-tax present worth of the lathe is £1912.

When we come to calculate the after-tax present worth, it will be more convenient to work in actual pounds and create a spreadsheet, **9S_6.xls**:

Year	After-tax Annual	Depreciation	After-tax	Total Cash	Total Cash	Present

	Revenue	Tax Saving	Operating Costs	Flow (Actual)	Flow(Real)	Value
0	−10 000	0	0	−10 000	−10 000	−10 000
1	1 000	1250	−1000	1 250	1 179	1 123
2	1 296	938	−1060	1 174	1 044	947
3	1 680	703	−1124	1 259	1 057	913
4	2 177	527	−1191	1 513	1 199	986
5	2 821	396	−1262	1 954	1 460	1 144
6	3 656	297	−1338	2 615	1 843	1 375
7	4 738	661	−1419	3 981	2 648	1 882
Total						−1 629

We now calculate the annual revenue in actual dollars, increasing it by a factor of (1.2)(1.08) every year. Revenues are taxed at 50%, so we multiply each year's revenue by a factor of 0.5. The tax saving due to depreciation is $(1-t) \times 0.25$ of the WDV every year, but we must take into account the fact that there is still a balance left in the WDV at the end of the seventh year, which will continue to act as a tax shield into the indefinite future. Specifically, at the end of year 7 we have a remaining WDV of:

$$WDV = 10\,000(0.75^7) = 1334$$

This will give us an additional saving of £1334(*TBF*), where the *TBF* is found from:

$$TBF = td\,/\,(i+d)$$

The interest rate used in this calculation must be the actual interest rate, which is related to the real interest rate, i', by

$$i = (1 + i')(1 + f) - 1 = 1.05 \times 1.08 - 1 = 0.13$$

So $TBF = td\,/\,(i+d) = 0.5 \times 0.25\,/\,(0.13 + 0.25) = 0.33$

So we add the amount $1334 \times 0.33 =$ £439 to our tax savings for the final year.

Operating costs can be deducted from our pre-tax income, so their after-tax value is reduced by a factor of 0.5. We then sum the annual cash flows, deflate them to real pounds, and bring them back to the present using the real MARR.

We see that the after-tax analysis gives a different picture: the purchase of the lathe no longer looks like a good investment. The underlying reason is that the tax benefits of depreciating the lathe over time have been reduced by inflation, while the running costs and revenue have not.

9S.7

We take a five-year study period and consider all cash flows to occur at year's end. For the second position, we note that the inflation rate exactly matches the raises, so the present worth is just

PW(aerospace) = 520 000(P/A,0.15,5)
= 520 000(3.3522)
= 1 743 140

For the faculty position, the initial payment is 450 000, and the present worth of payments in subsequent years changes by a factor of 1.08 / ((1.05)(1.15)) every year. This can be simplified to a factor of 1 / (1 + 0.118). So the present worth of the position is

PW(faculty) = 450 000(P/A,0.118,5)
= 450 000(3.6227)
= 1 630 220

So, all other things being equal, you should take the aerospace job.