MATHEMATICS 101 (Sections A01-A04), T01
Midterm # 1, January 30, 2014.
Time: 2 hours

Cqpo Heather	V00[redacted]	A03 T01
Name (Last, First)	**Student ID**	**Section**

Problems 2 - 6	10	2 marks for each
Problem 7	5	5 marks
Problem 8	5	5 marks
Problem 9	5	5 marks
Total:	25	25 marks

- **As stated in the course outline, the only calculators allowed on any examination are the Sharp EL-510R, RN or RNB.**
- This test consists of 8 questions (numbered 2 through 9) and has 7 pages (including this cover). You need to **show your work** for all questions (2 through 9), as we may disallow any answer which is not properly justified.
- For questions with numerical answers, the exact answer may not be among the options. In that case, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices.
- Before starting your test fill out your name (Last, First), student number (drop V00, fill in last 6 digits), and the tutorial section number (T01 - T28) on the top of this exam paper and on the bubble sheet, using an HB or softer pencil.
- Enter "B" in the bubble sheet as your answer to Question 1 now.

$\cos x \to -\sin x$
$\sin x \to \cos x$

1. Enter "B" in the bubble sheet as your answer to Question 1 now.

2. Evaluate $\int_0^{3\pi/4} \cos(2t)\,dt$.

(A)	-1.00	(B)	-0.75	(C)	-0.50	(D)	-0.25	(E)	0.00
(F)	0.25	(G)	0.50	(H)	0.75	(I)	1.00	(J)	1.50

$2t = u$
$dx\, 2 = du$

$\frac{1}{2}\int_0^{3\pi/4} \cos(u)\, du$

$\frac{1}{2}\left[\sin(u)\right]_0^{3\pi/4}$

$\frac{1}{2}\left[\sin(2t)\right]_0^{3\pi/4}$ $\quad (\sin \frac{6\pi}{4} - \sin 0)\frac{1}{2}$

$-1 \quad - \quad 0$

3. The rate of water flow into an initially empty container is $(100 - 4t)$ liters per minute at time t (in minutes). How much water flows into the container during the interval from $t = 10$ to $t = 20$ minutes?

(A)	200	(B)	400	(C)	600	(D)	800	(E)	1,000
(F)	1,200	(G)	1,400	(H)	1,600	(I)	1,800	(J)	2,000

$100 - 4t$ l/min

$A = \int_{10}^{20} 100 - 4t \; dt$

$100t - \frac{4}{2}t^2 \Big|_{10}^{20}$

$(2000 - 800) - (1000 - \frac{400}{2})$

$1200 - 800 =$

4. Determine the value of the derivative of $y = 5^{x\sqrt{x}}$ at the point $x = 1$.

(A) 0	(B) 2	(C) 4	(D) 6	(E) $\cancel{8}$
(F) 10	(G) 12	(H) 14	(I) 16	(J) Does not exist

$y = 5^{x\sqrt{x}}$

$y = 5^{\sqrt{x^3}}$

$y = 5^{x^{3/2}}$

$(\ln 5)\, 5^{x^{3/2}} \quad \frac{3}{2} x^{1/2}$

$(1.609)(5)(1.5)$

5. Calculate $\displaystyle\int_0^{1/6} \frac{1}{\sqrt{1-9x^2}}\,dx$.

(A) 0.10	(B) 0.13	(C) 0.17	(D) 0.20	(E) 0.25
(F) 0.30	(G) 0.45	(H) 0.60	(I) 0.80	(J) 1.00

$\displaystyle\int_0^{1/6} \frac{1}{\sqrt{1-(3x)^2}}$ $\quad u = 3x$, $du = 3\,dx$

$\displaystyle\frac{1}{3}\int_0^{1/6} \frac{1}{\sqrt{1-u^2}}$

$\frac{1}{3}\left[\sin^{-1} u\right]_0^{1/6}$

$\frac{1}{3}\left[\sin^{-1} 3(\frac{1}{6}) - \sin^{-1} 3(0)\right]$

$\displaystyle\int_0^{1/6} \frac{1}{\sqrt{1-(3x)^2}}$ $\quad 3x = u$, $dx\, 3 = du$

$\displaystyle\frac{1}{3}\int_0^{1/6} \frac{1}{\sqrt{1-u^2}}\,du$

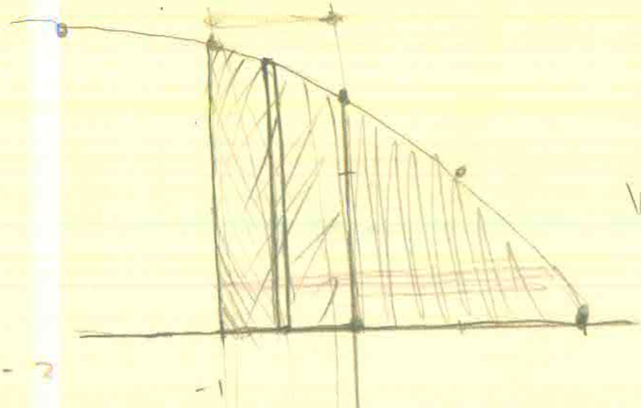
$\frac{1}{3}\left[\sin^{-1}(3x)\right]_0^{1/6}$

. Calculate the volume of the solid formed by rotating the region bounded by the curves $y = \sqrt{2-x}$, $x = -1$ and x-axis, about the x-axis.

(A)	1	(B)	5	(C)	10	(D)	15	(E)	20
(F)	23	(G)	26	(H)	29	(I)	32	(J)	35

$x = 0 \Rightarrow \sqrt{2}$
$x = 1$
$x = -1$
$x = -2 \Rightarrow 2$

washers



$$V = \int_{-1}^{2} \pi \left(\sqrt{2-x}\right)^2 \, dx$$

$$V = \pi \int_{-1}^{2} 2-x \quad dx$$

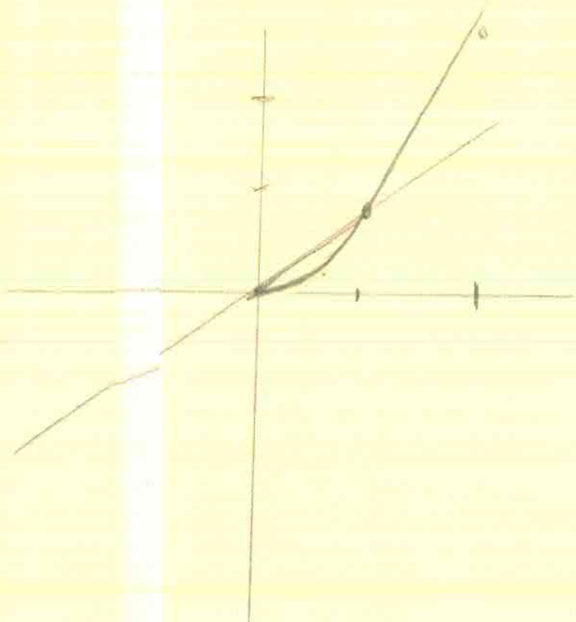
$$V = \pi \left[2x - \tfrac{1}{2}x^2\right]_{-1}^{2}$$

$$(4 - 2) - (-2 - \tfrac{1}{2})$$

$$2 \quad -(-2.5)$$

$$4.5\pi \approx 14.1$$

. (5 points) Find the minimum length of fence needed to enclose a field bounded by the curves $y = x$ and $y = x^{3/2}$.



① Length of $y = x$ from $0 \to 1$

② Length of $y = x^{3/2}$ from $0 \to 1$

① $a^2 + b^2 = c^2$

$\sqrt{1^2 + 1^2} = c$

$\boxed{\sqrt{2} = c}$

② $\int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$y = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

$\left(\frac{dy}{dx}\right)^2 = \frac{3}{2}^2 \left(x^{\frac{1}{2}}\right)^2$

$\frac{9}{4} x$

$\sqrt{1 + \frac{9}{4} x}$

$1 + \frac{9}{4} x = u$

$\frac{9}{4}\, dx = du$

$\frac{4}{9} \int_0^1 (u)^{1/2}\, du$

$\frac{2}{3} u^{3/2} \to \frac{2}{3}\left(1 + \frac{9}{4}x\right)^{3/2}$

$\frac{4}{9} \cdot \frac{2}{3} \left[\left(1 + \frac{9}{4}x\right)^{3/2}\right]_0^1$

$\frac{4}{9} \cdot \frac{2}{3} \ (5.859 - 1)$

$\boxed{1.4397}$

① + ②

$\boxed{\sqrt{2} + 1.4397 = 2.854 \text{ units}}$

$\left(\frac{3}{2} x^{\frac{1}{2}}\right)^2$

$\frac{9}{4} x$

$2u^{3/2}$

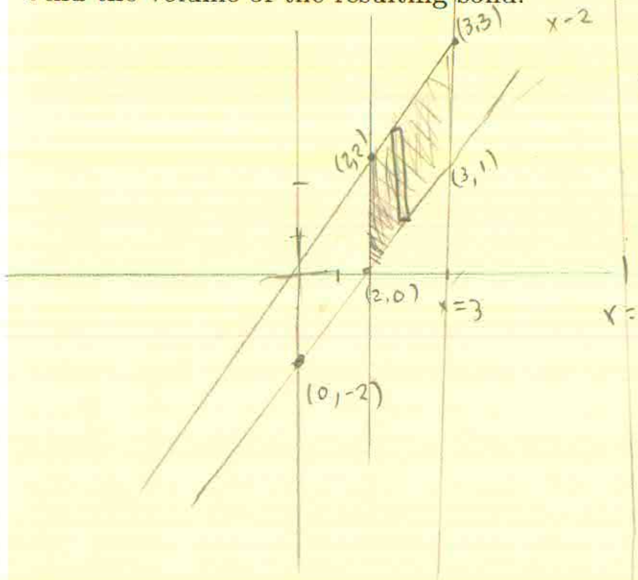
$\frac{3}{2} 2u^{1/2}$

5/5

(5 points) The region bounded by the curves $y = x - 2$, $y = x$, $x = 2$ and $x = 3$ is revolved around the line $x = 4$.

Find the volume of the resulting solid.

Shells method



$V = \int$ (height)(circumfrence) (width)

$[(x)-(x-2)]$ $2\pi(4-x)$ dx

$V = \int_2^3 ((x) - x + 2)\ 2\pi(4-x)\ dx$

$V = 2\pi \int_2^3 2(4-x)\ dx$

$V = 4\pi \int_2^3 (4-x)\ dx$

$= 4\pi \cdot \left[4x - \frac{1}{2}x^2\right]_2^3$

$(12 - \frac{9}{2}) - (8 - 2)$

$7.5 - 6$

$4\pi\ [1.5] = \boxed{6\pi}$

5/5

$\frac{2}{3}x^3$

$3 \frac{2}{3}x^2$

h

πr^2

8 sl(t)

$8\pi - 2\pi = 6\pi$

). (5 points) Compute:

[1 point] (a) value of $\sec^{-1}(4)$

$\sec^{-1} 4 = \theta$

$\sec \theta = 4$

$\frac{1}{\cos\theta} = 4$

$\cos\theta = \frac{1}{4}$ ✓

$\cos^{-1} \frac{1}{4} = 1.318$ ✓

1

[2 points] (b) derivative of $g(x) = \cosh(2x)$ using definition of the hyperbolic cosine, and express the results in terms of a hyperbolic function;

$g(x) = \cosh(2x)$

$= \frac{e^{2x} + e^{-2x}}{2}$ ✓

$\frac{d}{dx}\left(\frac{e^{2x} + e^{-2x}}{2}\right)$

$\frac{d}{dx}\left(\frac{1}{2}e^{2x} + \frac{1}{2}e^{-2x}\right)$

$\frac{1}{2}e^{2x}(2) + \frac{1}{2}e^{2x}(-2)$ ✓

$\rightarrow e^{2x} - e^{2x}$ or $2\left(\frac{e^{2x} - e^{2x}}{2}\right) = 2\sinh(2x)$ ✓

2

[2 points] (c) derivative of $f(x) = \sinh^{-1}(3x)$ using properties of the inverse functions.

$\sinh^{-1}(3x) = y$

$\frac{d}{dx}(\sinh y = 3x)$

$\cosh y \cdot \frac{dy}{dx} = 3$

$\frac{dy}{dx} = \frac{3}{\cosh y}$ ✓

Unit hyperbola

↓

$\cosh^2 x - \sinh^2 x = 1$

$\cosh x = \sqrt{1 + \sinh^2 x}$ ✓

$\frac{dy}{dx} = \frac{3}{\sqrt{\sinh^2 y + 1}}$ ✓

because $\sin y = 3x$

$\frac{dy}{dx} = \frac{3}{\sqrt{(3x)^2 + 1}}$ ✓

2

$\frac{d}{dx}(\sinh^{-1}(3x)) \Rightarrow \frac{3}{\sqrt{(3x)^2 + 1}}$

$x^2 + y^2 = 1$