ECON 180 FALL 2022: PROJECT 2

DUE **OCTOBER 11, 2022** by 11:59 PM VICTORIA, B.C. TIME

(Note the slightly extended due date compared to the syllabus)[1]

Honor Code: I guarantee that this submission is **entirely my own work**. I have **cited any outside sources** in APA or IEEE style. **(You must accept this code to receive a mark.)**

Name or Signature for Honor Code: **ARFAZ HOSSAIN**

Last 3 digits of student number: **4826**

Please enter your answers in the spaces and tables provided. Your submission must be in either PDF or Microsoft 365 (Word, etc.) format, so Brightspace can read it properly.

Question		Marks
1	a	75
	b	75
	c	75
	d	75
	e	75
	Q1 (Average)	75
2	a	75
	b	75
	c	75
	d	75
	Q2 (Average)	75
Q1 to Q2	(Q1+Q2)/2	75
3	a	5
	b	5
	c	5
	Q3 (Total)	15
Subtotal	(Q1 to Q2) + Q3	90
Communication		10
Total		100

[1] For those wondering, I extended the due date so there would be one week between the last lecture covered, on September 30, and the due date. I want to make sure you have time to let the material sink in.

Table of Contents

Basic Information and Assumptions

- Note: Like programmers, project planners count from 0.
- It is currently Month 0, Year 0.
- Mandeep will be a student until the end of Year 2 (Month 35)
- To keep things simple, assume Mandeep has no income and no housing costs while studying[2].
- Mandeep will work as an engineer for exactly 40 years (480 months).
- Mandeep will start working as an engineer in Year 3 (Month 36).
- For this project, assume that **Mandeep's MARR is 5.45% per year** (the current prime lending rate in Canada at the time of writing this project).
- While it's important to calculate the final values asked for in each question, I'm not very interested in the numerical results, which will vary by student: I want to see that you understand how to use discount factors such as (P/A,i,N), (P/F,i,N), etc.
- →When asked to show your work, please write your answer using this functional notation – e.g., $300 x (P/A,6%,45). There's no need to show all your arithmetic – I encourage you to use a math program or Excel to do it for you. It's fine to write $300x(P/A,6%,45) on one line, and $300 x 15.4558 on the next. (I used Excel to determine that (P/A,6%,45) was about 15.4558.)
- While you're not given marks for it, it may be useful to start each question by sketching a cash flow diagram (or two) to help you picture the situations you're studying.
- While it's fine to use 'brute force' (finding the present value, etc. of each cash flow, one time period at a time) to check your solution, this will not receive full marks. Part of what I'm testing is your ability to use the discount factors.

[2] This is an unrealistic assumption, but I'm making it to reduce the work you need to do. In a normal term, I usually ask students to consider tuition costs, food, etc. This is an out-of-major course taking place during a time of global crisis and great uncertainty and combined with the unusually late posting time I feel a more streamlined project is appropriate.

Question 1: Housing Costs (Present Value, Annuities, Interest)

a. From APRs to Interest Rates (Lecture 6)

i. For reference only, please fill out your baseline values from Project 1 In the first two columns of the table below. If you did not submit Project 1, you may use the values from the Project 1 sample value spreadsheet.

Once you've written down the APRs, use techniques from this course to calculate the corresponding monthly (per month) interest rate. Remember, you're going from an APR compounded every six months to a per-month interest rate. **To show you know what you're doing, write down the calculation for your baseline monthly mortgage interest rate. This sample calculation (room to answer on the next page) is most of what you'll be marked on in this part of the question.**

Bank Chosen: Royal Bank Of Canada

Value Type	Mortgage Type	Fixed Rate APR (%)	Monthly Interest Rate (%)
Minimum	2 Year Closed	6.020%	0.4954%
Baseline	5 Year Closed	6.140%	0.5053%
Maximum	10 Year Closed	7.000%	0.5750%

Remember that even though it looks like one, the APR is *not* an interest rate. At best, it's an *encrypted* interest rate, and the decryption key is the compounding rate per year, as discussed in class. To go from an APR compounded semi-annually to a monthly interest rate, first you'll need to decrypt the APR into a semi-annual (per 6 months) interest rate, then convert the per six months interest rate into the equivalent per month interest rate[3].

[3] This is the *Same interest rate*, just in terms of a different time scale, just like 60 km/hr and 1 km/minute are the Same speed. Conversion is a bit trickier for interest rates than for speed, because interest (usually) compounds.

Sample Calculation: Baseline APR to Baseline Monthly Mortgage Interest Rate. If you're going for a high communication mark, make sure that your calculation is clearly laid out, in such a way that someone else reading it can understand each step without having to ask additional questions. (Think of the format that YOU would like to see in a long form answer key for this question.)

Usually after APR costs, there are other extra costs to buying a house [1] which I did not include in my calculation towards the interest rate. I also found that RBC's Mortgage Interest Rates are compounded half-yearly (noted in the *Legal Disclaimer Section*). A sample APR to interest rate conversion can look like this:

$$\text{periodical \% interest} = \frac{APR - \frac{\text{APR fees}}{(\text{Loan Total}) \times (\text{length of Loan})}}{\text{Periods in Total}}$$

$$= \frac{6.020}{2}$$

RBC Fixed Mortgage rate has interest rates compounded half yearly

$$\text{\% Interest half yearly} = 3.01\%$$

$$\text{\% interest monthly: } (1 + i_{monthly})^{12} = (1 + i_{half\ yearly})^{2}$$

$$\Rightarrow (1 + i_m)^{12} = (1 + i_{hy})^{2}$$

$$\Rightarrow (1 + i_m)^{6} = 1 + i_{hy} = 1 + 3.01\%$$

$$\Rightarrow i_m = 0.4954\% \text{ monthly.}$$

I'm assuming that the Two-Year Mortgage works such that the compound interest will change for a mortgage type semi-annually, but Mandeep does not have to pay the compounded interest rate as it will remain the same in each two year period.

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1. J. Lee, "What Are APR Fees?" *Bankrate*. https://www.bankrate.com/mortgages/apr-fees.html (accessed Oct. 05, 2022).
2. RBC Royal Bank, "Mortgage Rates," *www.rbcroyalbank.com*. https://www.rbcroyalbank.com/mortgages/mortgage-rates.html (accessed Oct. 05, 2022).

b. Housing costs in Montreal (Lecture 7)

i. For reference only, please fill out your baseline values from Project 1. If you did not submit Project 1, you may use the values from the Project 1 sample value spreadsheet.

City	Baseline Monthly Rent ($)
Montreal	$2,000.00[*]

* I'm using the sample values to accurately represent the values and calculations from my project 1 findings.

ii. Calculate the present value (Month 0 value) of the rent in Montreal.

- Monthly rent remains at its baseline level forever. (We'll relax this in later projects.)
- Mandeep pays rent from Month 36 to Month 515, inclusive. This is a total of 480 months.
- Consider converting Mandeep's MARR (5.45% per year) into an equivalent % per month.
- Show your work.
- When showing your work, you must use correct functional notation such as (P/F,10%,12) for full marks. (Think of how problems are presented in the lecture notes).
- Your solution should make use of (P/A,i,N).

City	Present Value of Rent ($)
Montreal	$340,279.44

Montreal

Mandeep's Yearly MARR = 5.45%

Converting to Monthly MARR = $\mathbf{(1+i_{monthly})^{12} = (1+i_{yearly})^{1}}$ --> $\mathbf{i_{monthly} = 0.4432\%}$

From my understanding, from month 0 to month 35 (exclusive, that's 35 months), Mandeep doesn't have to pay the rent. From month 36 to month 515 (inclusive, 515-36+1=480 total monthly payments) with the same rental rate of $2000.00 (Montreal Value), not accounting for any growth rate.

Taking the annuity, A = $2000.00, starting with Month 36 and ending with Month 515, we get a Month 36 value of the rent through **Ax(P/A,i,N)** which basically calculated the summation of all the equally installment payments into a chunk amount for Month 36, accounting all interests. That's $397,241.5117 on month of 36.

Now we have to bring this back to month 0 amount of the rent, as we want to find the present value. We already have i, N (35 months), F ($397,241.5117). With **Fx(P/F,i,N)** we get $340,279.44.

$Ax(P/A,i_{MARR},N_1)x(P/F,i_{MARR},N_2)$ = $340,279.44.

A ($2000.00), i_{MARR} = 5.45%, N_1 = 480, N_2 = 35.

c. Housing costs in Regina (Lecture 7)

i. For reference only, please fill out your baseline house price and mortgage % per month from Project 1. If you did not submit Project 1, you may use the value from the Project 1 sample value spreadsheet.

City	Baseline House Price ($)
Regina	$375,600.00[*]

* I'm using the sample values to accurately represent the values and calculations from my project 1 findings.

Baseline Mortgage interest	0.45887	% per month

ii. Calculate Mandeep's monthly mortgage payments if they choose to buy a house in Regina. **Show your work**, and use correct notation (e.g. (P/A,10%,52)).

- Assume Mandeep takes out a mortgage for the *entire* value of the house. That means that the only housing costs you need to consider in Regina are the mortgage payments. (We'll relax this assumption in later projects.)
- Mandeep buys the house in Month 36. The net cash flow related to housing in month 36 is zero, because the money coming in from the bank and the money going out to pay for the house are exactly the same and cancel out.
- Mandeep's Regina mortgage is for 25 years. Mandeep makes exactly 25 years (300 months) of payments, and the first payment is in Month 37.
- To calculate mortgage payments, you need to split the house price into an equivalent sequence of monthly payments, using (A/P,i,N). The 'i' you use will be the *mortgage interest rate*, **not** Mandeep's MARR, because you want to find the monthly payments the *bank* thinks are equivalent (worth) the initial cost of the house.
- For example: suppose Alex took out a mortgage and bought a house in Month 0 for $10,000. Suppose that the interest rate on the mortgage is 1% per month, and Alex will pay it off over 10 years (120 months), with the first payment due the month after buying the house – month 1. In that case, the amount of each monthly mortgage payment is given by $10,000 x (A/P,1%,120) = $143.47

Mandeep's monthly mortgage payment ($)	$2307.96	per month

From Month of 36 to Month of 336, assuming they pay all the monthly mortgage payments (that's ***N***=25x12=300 months in total), the total cost of the house is $375,600 (we're considering this as the present value, ***P*** of the house). Putting it into equal instalment annuities,

Converting APR compounded semi-annually to monthly mortgage interest:

$(1+i_{monthly})^{12}=(1+i_{yearly})^{2}$ --> $i_{monthly}$ = 0.45887% = ***Mortgage Interest Monthly***

Now,

A = **Px(A/P,$i_{mortgage}$,N)** = $2,307.96 = ***Mortgage Monthly Payments***

iii. Calculate the present (Month 0) value of the mortgage payments. Show your work.

- For this calculation, you *will* use Mandeep's MARR of 5.45% per year (or the monthly equivalent).
- You can treat the mortgage payments as a sequence of 300 equal payments, starting in month 37.
- Your solution should make use of (P/A,i,N).

City	PV of Mortgage ($)
Regina	$326,259.20

Mandeep's Yearly MARR = **5.45%**
Converting to Monthly MARR = $\mathbf{(1+i_{monthly})^{12} = (1+i_{yearly})^{1}}$ --> $\mathbf{i_{monthly} = 0.4432\%}$

We already have the Annuity / ***Mortgage Monthly Installment Payments*** value from 1ii, **$2,307.96** (A). We know that Month 36 to Month 336 or **300** monthly payments ***N*** has to be made for mortgage and the payment started from Month 37 (First month of annuity payment). We want to convert all those 25 years (300 installments) into one big chunk of the present amount as of Month 36 (before any payments).

$A \times (P/A, i_{monthly}, N_1) = \$2307.96 \times (P / \$2307.96, 0.4432\%, 300)$
or, $382,562.34
a.k.a Present Value of Month 36 and Future Value of Month 0.

Now, transforming the future value of Month 0 to the current value is:

$F \times (P/F, \mathbf{i_{monthly}}, N_2) = \$382,562.34 \times (P / \$382,562.34, 0.4432\%, 36)$
or, $326,259.20
a.k.a Present Value of Month 0.

In Short,

Present Total Mortgage Value, $PV = A \times (P/A, i_{monthly}, N_1) \times (P/F, \mathbf{i_{monthly}}, N_2) = \$326,259.20$

A = $2,307.96
$\mathbf{i_{monthly} = 0.4432\%}$
$N_1 = 300$
$N_2 = 36$
... (Answer)

d. Housing costs in Victoria (Lectures 6 and 7)

i. For reference only, please fill out your baseline values from Project 1. If you did not submit Project 1, you may use the values from the sample value spreadsheet.

City	Baseline Garden Suite Construction Cost ($)
Victoria	$170,000.00 [*]

* I'm using the sample values to accurately represent the values and calculations from my project 1 findings.

ii. Calculate the present value (Month 0 value) of the cost of building the garden suite, considering the information below.

- Mandeep is required to pay for the garden suite immediately upon graduation.
- Mandeep will pay for the first $10,000 of the garden suite construction cost in cash, from their savings. This payment will be made immediately upon graduation.
- Mandeep will take out a personal loan to pay for the remaining construction cost.
- This loan is offered to Mandeep at a fixed interest rate of **18% per year** [*]. (This is roughly in line with personal loan rates[4] in Canada for someone with a "poor" to "good" credit score, at the time of writing this project. In a later project, you may have the chance to see what happens if the loan rate is different from this value.)
- Mandeep will pay the loan over the course of five years. Payments are monthly, for a total of 5x12 = 60 payments. The bank will deposit the funds directly into Mandeep's account immediately upon graduation. The first payment is due one month after that.
- Calculate monthly loan payments in the same way that you calculated mortgage payments in part b (considering the 5-year repayment schedule, and the 10% interest rate) [*].

* I'm not sure of the interest rate mentioned on the bottom line above. I am using the fixed interest loan rate mentioned on point 4 above as the only interest rate for my calculations.

There is room for your answers and work on the next page.

[4] See Latest personal loan interest rates in Canada. (2022). https://www.finder.com/ca/personal-loan-interest-rates-canada

Mandeep's monthly loan payment ($)	$3,947.74	per month

Remaining payments = $170,000 – $10,000 = $160,000

Given a fixed interest rate of ***18%*** annually, we can derive the monthly interest for Mandeep:

$$(1+i_{yearly})^1=(1+\boldsymbol{i_{monthly}})^{12} \text{ --> } \boldsymbol{i_{monthly}} = 1.38884\% = 0.0138884.$$

Monthly Payments for the loan, given the loan period of 5 years (60 installments, ***N***) and Present Value of $160,000 (***P***) disbursed in equal annuities throughout the loan period:

$$A = \mathbf{P \times (A/P,}\boldsymbol{i_{monthly}}\mathbf{,N)} = \$3,947.74 = \boldsymbol{Monthly\ Loan\ Payments\ (Annuity)}.$$

City	PV of Garden Suite Construction Costs (Loan + $10,000 from savings) ($)
Victoria	$170,000.00

Since he's taking out a loan for next 5 years for 60 installment payments and an annual loan rate of 18% (roughly 1.4% monthly), the present worth of the GSC costs is going to be the exact same as the baseline values, as given.

e. A bonus and a yearly raise (Lectures 7 and 8)

i. For reference only, please fill out your baseline salaries from Project 1. If you did not submit Project 1, you may use the values from the Project 1 sample value spreadsheet.

City	Baseline Yearly Salary ($)
Victoria	$73,277
Montreal	$84,186
Regina	$76,255

ii. Calculate the present (Year 0) value of Mandeep's income, given the following assumptions:

- To keep things simple, assume that Mandeep gets paid once a year, at the start of the year.
- This means you can use 'years' as your time scale.
- Mandeep's first salary payment is in year 3
- There are a total of 40 salary payments.
- In year 3, in addition to the year's salary, Mandeep receives a one-time bonus equal to one fourth of the starting salary.
- Mandeep's salary increases by 3.5% each year, so that their year 4 salary is 1.035 x their Year 3 salary, etc.
- Show your work for the Victoria calculation (all other calculations will be the same except for the numbers). Your answer should make use of (P/A,g,i,N).

City	PV of Mandeep's Income ($)
Victoria	$1,855,509.35
Montreal	$2,131,745.43
Regina	$1,930,917.82

[Show your work for the Victoria calculation]

	MANDEEP MARR	5.45%	
City	**Baseline Yearly Salary ($)**	N	40
Victoria	$73,277	g, Growth Rate (yearly)	3.50%
Montreal	$84,186	io (yearly)	1.8841%
Regina	$76,255		
	VICTORIA	MONTREAL	REGINA
	Salary No Bonus Year 2 =	Salary No Bonus Year 2 =	Salary No Bonus Year 2 =
	$ 1,976,713.76	$ 2,270,993.96	$ 2,057,048.02
	Salary No Bonus Year 0 =	Salary No Bonus Year 0 =	Salary No Bonus Year 0 =
	$ 1,777,667.85	$ 2,042,315.40	$ 1,849,912.82
	Y3 Bonus Equivalent Y0 =	Y3 Bonus Equivalent Y0 =	Y3 Bonus Equivalent Y0 =
	$15,623.13	$17,948.99	$16,258.05
	Total =	Total =	Total =
	$ 1,793,290.97	$ 2,060,264.39	$ 1,866,170.87

Mandeep's MARR is currently 5.45%. This is our interest rate, ***I***.

We have a geometric gradient figure in the salary of Mandeep for ***B***, 40 years with a ***g***, growth rate of 3.50%. We can use these values for turning the 40 years' worth of salary into the present value, ***P***. Since the salary starts from 3rd year, we'll get the present value of the gradient structure on the 2nd year.

Here, io = {(1+i)/(1+g)} -1 = ((1+5.45%) / (1+3.50%)) -1 = 1.8841%

We get **A x (P/A,g,i,40)** -> A x ((P/A,io,40)/(1+g)) = $1,976,713.76 (Victoria), $2,270,993.96 (Montreal), $2,057,048.02 (Regina). This is the year 2 equivalent of the total salary.

We want to bring it left 2 more years, that's **F x (P/F,i,2)** = $1,777,667.85 (Victoria), $2,042,315.40 (Montreal), $1,849,912.82 (Regina). This is the year 0 equivalent of the total salary.

We also want to add the year 3 salary bonus and bring it back to year 1. Although I'm not sure if this bonus is recurring, I'm just going to add the year 3 quarter equivalent here, that's **(A/4) x (P/F,i,3) =** $15,623.13 (Victoria), $17,948.99 (Montreal), 16,258.05 (Regina); A being the Annuity of the Gradient Scale.

Adding all equivalent Year 0 values, we get $1,793,290.97 (Victoria), $2,060,264.39 (Montreal), $1,866,170.87 (Regina).

So, our final answer stands from **[A x (P/A,g,i,40) x (P/F,i,2)] + [(A/4) x (P/F,i,3)]**

Question 2: Project Comparisons

a. Net Present Value Comparisons (Lecture 10)

Note: This question is intentionally very simple.

Using the same assumptions as in Question 1, calculate the Net Present Value of living & working in Victoria, Montreal & Regina. (This is the present value of income minus the present value of housing costs.) This calculation can be done very quickly by referring back to your answers for Question 1.

There's no need to show your work, unless you want to[5], because I've given you the formula:

NPV = PV of Income – PV of Housing

City	PV Income	PV Housing	NPV
Victoria	$1,855,509.35	$170,000.00	$1,685,509.35
Montreal	$2,131,745.43	$340,279.44	$1,791,465.99
Regina	$1,930,917.82	$326,259.20	$1,604,658.62

Based on your calculations, which is the preferred city for Mandeep to work in? Briefly explain your reasoning.

Preferred City: Montreal

Reasoning:

	City	PV Income	PV Housing (Sorted)	IBCR	Choices
A	Victoria	+ $1,855,509.35	- $170,000.00	-	-
B	Regina	+ $1,930,917.82	- $326,259.20	0.48	Victoria, Regina = Victoria
C	Montreal	+ $2,131,745.43	- $340,279.44	1.62	Victoria, Montreal = Montreal

Incrementally speaking, it's safe to choose Montreal over Victoria, but considering the NPV, I think I'd still choose Montreal as it has higher NPV value over other cities.

[5] In fact, if you use the companion spreadsheet, the relevant calculations are automated for you.

b. Working as a deckhand (Lectures 7 and 10)

Suppose that Mandeep is considering an alternative to graduating and working as an engineer. In particular, they are considering dropping out of school immediately, and working for six (6) months as **a deckhand on a fishing trawler**.

If Mandeep works as a deckhand for six (6) months:

- They will quit school immediately (Time 0)
- Mandeep will earn $25 an hour[6]. Assume there are 176 working hours in a month.
- Mandeep will be paid twice a month by direct deposit. The first payment will be half a month from now, the second payment will be one month from now, etc.
- The amount of each payment is $25 x (176/2) = $2,200 per half-month.
- Room and board are included as perks of this particular job (Mandeep will live on the ship while working), and for the purposes of this project, Mandeep will not have any other costs[7].

Calculate the NPV of this six-month project. Show your work. Things to keep in mind: it will be useful to calculate the 'per half-month' version of the MARR.

NPV of working as a deckhand for six (6) months (12 half-months): $21,681.66

Show your work (using appropriate notation, e.g., $50x(P/A,12%,60)):

The MARR of Mandeep is 5.45%, which is calculated annually. There are 24 half-months a year, 12 half-months in half a year or, 6 months.
$(1+i_{semi\text{-}monthly})^{24}=(1+i_{MARR})^{1}$ -> $i_{semi\text{-}monthly}$ = 0.2213559686 = 0.0022135%

In 6 months, there are 12 half months. Excluding one, 11.

NPV = $2200 + $2200 x $(P/A, i_{semi\text{-}monthly}, 11)$ = $21,681.66

[6] This is roughly in line with current Canadian deckhand salaries: https://ca.indeed.com/career/deckhand/salaries
[7] A simplifying assumption to make things easier for you.

c. Repeated Lifetimes (Lecture 11)

Assume that, if Mandeep stays in school, they will live and work in Victoria after graduation. Use a repeated lifetime, net present value approach to determine which project is preferred: the 43-year project of going to school then working in Victoria, or the six-month project of working on the fishing trawler.

Note that you already have the (43-year) NPV of living and working in Victoria from part a., and the (6 month) NPV of working as a deckhand from part b. There's no need to recalculate them. You can refer to them as NPV_{Van} and NPV_{fish}, and substitute the appropriate numerical values when you have to.

Preferred Project: Victoria

Show your work (using appropriate notation, e.g., \$50x(P/A,12%,60)):

The MARR of Mandeep is 5.45%, which is calculated annually. There are 24 half-months a year, 12 half-months in half a year or, 6 months.
$(1+i_{semi\text{-}monthly})^{24}=(1+i_{MARR})^{1}$ -> $i_{semi\text{-}monthly}$ = 0.2213559686 = 0.0022135%

In 43 years, there are 1032 half months. Excluding one, 1031.

NPV_{Fish} = \$2200 + \$2200 x (P/A, $i_{semi\text{-}monthly}$, 1031) = \$889,999.65
$NPV_{Victoria}$ = \$1,685,509.35

Victoria	Fish-Deck
\$1,685,509.35	\$889,999.65

Repeating the lifetimes of each project, we can see living, working, and studying as an engineer will be a better option for Mandeep.

d. Annual Worth Comparisons (Lecture 11)

We're again comparing living & working in Victoria for 43 years to working as a deckhand for six months, but this time we're using an Annual Worth comparison.

Note that you already have the (43-year) NPV of living and working in Victoria from part a., and the (6 month) NPV of working as a deckhand on a fishing trawler from part b. There's no need to recalculate them. You can refer to them as $NPV_{Victoria}$ and $NPV_{Deckhand}$, and substitute the appropriate numerical values when you have to.

i. Calculate the annual worth of living and working in Victoria for 43 years. Show your work, using appropriate notation.

Annual worth: $63,842.23

NPV_{Fish} = $2200 + $2200 x (P/A, $i_{semi\text{-}monthly}$, 1031) = $889,999.65
$NPV_{Victoria}$ = $1,685,509.35

Annual Worth (AW) = $NPV_{Victoria}$ x (P/A,2.45%,43) = $63,842.23
EXCEL = PMT(2.45%,43, $NPV_{Victoria}$)

ii. Calculate the annual worth of working as a deckhand for 6 months. Show your work, using appropriate notation.

Annual worth: $33,710.62

Annual Worth (AW) = NPV_{Fish} x (P/A,2.45%,43) = $33,710.62

iii. Based on your calculations in part i. and part ii., which is the preferred project? Briefly explain your reasoning.

Preferred Project: Studying and working in Victoria

Reasoning: Annual Worth, Net Present Value is higher.

Question 3: (Challenge) Salary (Gradients, What's missing?)

a. (5 marks) From a geometric gradient to an arithmetic gradient. (Lecture 8)

Let's go back to the situation from question 1.e (where you calculated the present value of Mandeep's salary).

Suppose that Mandeep is in Victoria, but instead of Mandeep's salary going up by 3.5% each year, it goes up by a constant amount G per year, so Mandeep's salary in Year 4 is equal to their Year 3 starting salary plus G, etc. Calculate the value of G needed to make the present value of this stream of income equal to the present value of Mandeep's income that you calculated (for Victoria) in question 1.e.

- Apart from replacing a yearly raise of 3.5% with a yearly raise of G dollars, everything else is exactly as in question 1.e. This includes the signing bonus of ¼ of the starting salary.
- Show your work. Your answer should make use of (A/G,i,N).

Required value of G: $158486.6666
PV of Mandeep's income (Victoria) = $1,855,509.35

Without the geometric grad increment of the salary from Year 3, everything is the same. We just want the value of G.

$[\{ Gx(A/G,i_{MARR},40)*(P/F,i_{MARR}, 2) \} + (A/4)*(P/F,i_{MARR}, 3)] = \$1,855,509.35$

Solving for G,

$[G \times \{ (1/i_{MARR}) – (40/(1 + i_{MARR})^{40}-1) \}] / (1 + i_{MARR})^{40} = (\$1,855,509.35 - \$15623.13)$

... (Some math later) ...

G = $158486.6666

Checking with the original formula for the question: *Gx(A/G,i,40)*(P/F,i,2) + (A/4)*(P/F,i,3)*

- **Gx(A/G,i,40)** for finding the Annuity in Year 2 in question: $2,045,898.75.
- **Fx(P/F,i,2),** *F being Year 2 Annuity*, we bring it back to Year 0: $1,839,886.23.
- **(A/4)*(P/F,i,3)** as the Year 3 Bonus brought back to Year 0: $15,623.13.
- **Adding Year 0 Annuity and Year 0 Bonus, we get $1,855,509.36**

b. (5 marks) A more realistic payment scheme (Lectures 7 and 8)

Suppose that Mandeep is in Victoria. The situation is exactly as in question 1.e, except that Mandeep is paid once a month, instead of once a year. Mandeep's salary still goes up by 3.5% a year, starting with the first salary payment each year. In the first year of work, Mandeep's monthly salary is equal to 1/12 of the baseline yearly salary. In the second year of work, Mandeep's monthly salary is equal to 1.035 times their Year 1 monthly salary, and so on. The sign-on bonus of ¼ of Mandeep's starting yearly salary is still paid all at once.

Timing notes: Assume that the bonus is paid at the start of month 36, and the first salary payment is at the start of month 37. (The bonus gets Mandeep through the first month, and at the start of the next month, Mandeep gets paid for the work done the previous month. There is a total of 40x12 = 480 salary payments.)

Calculate the present value of Mandeep's income[8] and show your work. You must use (P/A,g,i,N) in your answer. **(Hint: You'll also want to use (P/A,i,N), and you may find a use for both the 'per year' and 'per month' versions of the MARR.)**

Present value of Mandeep's income: $1,866,031.37 (Victoria)

Mandeep's MARR is currently 5.45% annually, 0.4432% monthly. This is our interest rate, ***I***.

We have a geometric gradient figure in the salary of Mandeep for ***B***, 480 months with a ***g***, growth rate of 0.28709%. We can use these values for turning the 40 years' worth of salary into the present value, ***P***. Since the salary starts from 3rd year, we'll get the present value of the gradient structure on the 2nd year.

Here, io = {(1+i)/(1+g)} -1 = ((1+ 0.4432019219%) / (1+ 0.28709%)) -1 = 0.001557%
We get **A x (P/A,g,i,480)** -> A x ((P/A,io,40)/(1+g)) = $2,057,598.92 (Victoria). This is the month 24 equivalent of the total salary.

We want to bring it left 24 more months, that's **F x (P/F,i,24)** = $1,850,408.25 (Victoria). This is the year 0 equivalent of the total salary. Bringing the month 36 salary to month 0: $15,623.13.

Adding all equivalent Month 0 values, we get **$1,866,031.37** (Victoria).

So, our final answer stands from **[A x (P/A,g,i,480) x (P/F,i,24)] + [(A/4) x (P/F,i,36)]**

[8] This will NOT be the same as in 1.e, because the time distribution of payments is different. Getting $12,000 now is not the same as getting $1,000 a month, starting today.

b. (5 marks) What's missing? (No specific lecture)

Questions 1 and 2 deal with Mandeep's situation. For teaching purposes, the analysis has been simplified from what would be done in a professional, realistic analysis. Given that you are one of UVic's finest problem solvers, by now you probably have some thoughts about what's missing from this analysis. This is your place to share your ideas (and get marks for it). What's something that you think is missing from our analysis of Mandeep's situation, but should definitely be added if we were doing it 'for real'? For full marks, you should attempt an implementation (i.e. try it out, at least in a limited fashion).

This question is intended to be fun, not stressful. It's a sandbox for you to show your stuff to an audience of TAs and be rewarded for it. Please don't spend hours on it unless you really want to. Even a simple, 15-minute answer, is enough for at the very least 1 mark out of 5.

1. Recurring Salary Bonuses every frequent year.
2. Change of workplace, unexpected rise/fall of salaries, career change.
3. Inflation, unexpected change in plan.