UNIVERSITY OF VICTORIA
EXAMINATIONS DECEMBER 2018
MATH 100, SECTIONS [A01], [A02], [A03]

Student ID: V00 _____ Familt (Last) Name: _____

Lecture Section: _____ Given (First) Name: _____

Lecture sections &	**Instructors:**
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Question(s)	Value	Score
Multiple Choice	42	
Question 22	4	
Question 23	5	
Question 24	9	
Question 25	10	
Total	70	

THIS EXAM CONTAINS 18 PAGES (INCLUDING THIS COVER PAGE) AS WELL AS THE BLUE INSTRUCTIONS PAGE AT THE END. COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEGORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATORS.

THIS EXAM CONSISTS OF 21 MULTIPLE-CHOICE AND 4 LONG-ANSWER QUESTIONS.

TO BE ANSWERED: ON THE PAPER AND ON BUBBLE SHEETS

DURATION: 3 HOURS

Do not open this packet until instructed to do so.

1. Identifying information:
   (a) Enter your Student ID, Name, and Lecture Section at the top of **this page** now.
   (b) Fill out and sign the **blue instruction page** on the back of this exam now.
   (c) Enter your Name, Course, Section, Instructor, and Date at the top of your **multiple-choice sheet** now. The multiple choice sheet is on the other side of your name plate.
   (d) Make sure your student number is coded with bubbles on the multiple-choice sheet now.
2. Only the following materials are permitted:
   (a) Your UVic Student Identity One Card (place it on your desk now).
   (b) Pens, pencils, erasers, and a ruler are permitted at your desk. If you have a pencil case it must be stored with your belongings in the front of the room.
   (c) You may only use calculator which model starts with Sharp EL-510. No other calculator is acceptable on this examination.
3. No notes, outside paper, or aid other than the ones listed above is permitted. You are responsible for ensuring that any unauthorized material is stored with your belongings in the front of the room.
4. Show all calculations on this paper for all problems (even the multiple-choice). We may disallow any answer given without appropriate justification.
5. For multiple-choice questions insure that all your answers are correctly filled in on the multiple-choice sheet.
6. For multiple-choice questions requiring numerical answers, the correct answer might not be listed. Choose the value that is **nearest to your answer**; if your answer is exactly between two options, choose the larger of these two choices.
7. Once the exam is over, submit both your examination paper and your multiple-choice sheet.
8. If you need to leave the room during the exam, raise your hand until an invigilator comes to you. Students may not leave during the first 30 minutes or the last 15 minutes of the exam.

Do not open this packet until instructed to do so.
Fill in bubble "A" in the Form field of the multiple-choice sheet now.

1. (2) If $f(x) = \frac{1}{3}x^3 + 6^x$, find $f'(2)$

   A. 22 B. 24 C. 36 D. 40 E. 44
   F. 62 G. 64 H. 66 I. 68 J. 70

2. (2) Find $\lim_{x \to +\infty} \frac{6(1 - e^{-x})}{e^{-x} - 2}$

   A. -6 B. -3 C. -2 D. -1 E. 0
   F. 1 G. 2 H. 3 I. 6 J. Does not exist

3. (2) Compute the difference quotient $\frac{f(2+h) - f(2)}{h}$ for the function $f(x) = \frac{1}{2x+3}$, and simplify your answer.

   A. $\frac{2}{7+2h}$ B. $\frac{2h}{7+2h}$ C. $\frac{2}{7(7+2h)}$ D. $\frac{2h}{7(7+2h)}$ E. 0

   F. $-\frac{2}{7+2h}$ G. $-\frac{2h}{7+2h}$ H. $-\frac{2}{7(7+2h)}$ I. $-\frac{2h}{7(7+2h)}$ J. None of those

4. (2) Find $\lim_{h\to 0} \dfrac{1-\cos(4h)}{3h^2}$

A. -3.6 B. -3.2 C. -2.6 D. -0.6 E. 0.0

F. 0.6 G. 2.6 H. 3.2 I. 3.6 J. Does not exist

5. (2) Find $\displaystyle\int_1^4 \left(e^{-x} + \frac{\ln x}{x}\right)\, dx$

A. 0.61 B. 0.65 C. 0.91 D. 0.94 E. 1.24

F. 1.31 G. 1.34 H. 1.41 I. 1.44 J. 1.74

6. (2) Calculate the area of the region bounded by the curve $y = |2 - 4x|$, the x-axis, $x = -1$, and $x = 1$.

A. 4.0 B. 4.5 C. 5.0 D. 5.5 E. 6.0
F. 6.5 G. 7.0 H. 7.5 I. 8.0 J. Area is unbounded

7. (2) Define $f(2)$ in order to make $f(x) = \dfrac{x^2 + 2x - 8}{x - 2}$ continuous at $x = 2$.

A. -6 B. -4 C. -2 D. 0 E. 2
F. 4 G. 6 H. 8 I. 10 J. Such value does not exist.

8. (2) Let $f(x) = x^4 - 3x^2 + 1$. Find $f''(-1)$.

A. -10 B. -6 C. -4 D. -3 E. 0
F. 3 G. 4 H. 6 I. 10 J. Does not exist.

9. (2) Calculate the derivative of the function $y = \int_{-1}^{x^2} \left(1 + t^3\right)^4 \, dt$ at $x = 1$.

A. -56 B. -48 C. -36 D. -1 E. 0
F. 1 G. 36 H. 48 I. 56 J. Does not exist.

10. (2) Consider the price function $p(x) = \dfrac{45}{\sqrt{x}}$, where x is the number of items.

Calculate the average price between $x = 100$ and $x = 400$.

A. 1.5 B. 2.0 C. 2.5 D. 3.0 E. 3.5
F. 4.0 G. 4.5 H. 45 I. 900 J. Impossible to determine.

11. (2) Calculate the derivative of the function $y = (x^4 + 1)^x$.

A. $\left(\dfrac{4x^4}{x^4+1} + \ln(x^4+1)\right)$

B. $(x^4+1)^x \left(\dfrac{4x^4}{x^4+1} + \ln(x^4+1)\right)$

C. $(x^4+1)^x \left(\dfrac{1}{x^4+1} + \ln(x^4+1)\right)$

D. $x(x^4+1)^{x-1}$

E. $4x^4(x^4+1)^{x-1}$

F. $(x^4+1)^x \left(\dfrac{4x^4}{x^4+1}\right)$

G. $(x^4+1)^x$

H. $x\ln(x^4+1)$

I. $(x^4+1)^x \left(\dfrac{4x^3}{x^4+1} + \ln(x^4+1)\right)$

J. None of the above

12. (2) Calculate the equation of the line tangent to $y = \dfrac{2x}{x+1}$ at the point with $x-$coordinate 1.

A. $y = 1$ B. $y = \dfrac{1}{2}x - \dfrac{1}{2}$ C. $y = \dfrac{1}{2}x + \dfrac{1}{2}$ D. $y = \dfrac{1}{2}$ E. $y = \dfrac{1}{2}x$

F. $x = 1$ G. $y = -\dfrac{1}{2}x + \dfrac{1}{2}$ H. $y = -\dfrac{1}{2}x - \dfrac{1}{2}$ I. $x = \dfrac{1}{2}$ J. None of the above.

13. (2) Find the area of the region enclosed by the parabolas $y = 4 - x^2$ and $y = 2 - \dfrac{1}{2}x^2$.

A. -6.5 B. -5.5 C. -3.5 D. -2.5 E. 0.0
F. 2.5 G. 3.5 H. 5.5 I. 6.5 J. Area is unbounded

14. (2) Calculate the slope of the graph of $x^2y^2 - 2x = 4 - y$ at the point (1, 2).

A. -1.6 B. -1.4 C. -1.2 D. -1.0 E. 0.0
F. 1.0 G. 1.2 H. 1.4 I. 1.6 J. Does not exist

15. (2) Find the Riemann sum of the function $y = \sqrt{x}$ on the interval $[1, 5]$ using four equal-length subintervals of length 1 and selecting the right-hand endpoint in each subinterval.

A. 0.5 B. 1.5 C. 2.5 D. 3.5 E. 4.5
F. 5.5 G. 6.5 H. 7.5 I. 8.5 J. 9.5

16. (2) Find the maximum value of the function $f(x) = x^3 - 12x + 23$ in the interval $[-1, 3]$.

A. -28 B. -23 C. -14 D. -3 E. 2
F. 4 G. 7 H. 14 I. 34 J. 39

17. (2) Suppose that $f''(x) = -\sin x + \cos x$ and that $f'(0) = -1$ and $f(0) = 1$. Find $f\left(\frac{\pi}{4}\right)$.

A. -2.8 B. -1.0 C. -0.4 D. 0.0 E. 0.4

F. 0.6 G. 0.8 H. 1.0 I. 2.8 J. 3.6

18. (2) Let $f(x) = \tan^{-1}(\sqrt{x})$. Find $f'(9)$.

A. -0.020 B. -0.016 C. -0.012 D. 0.000 E. 0.012

F. 0.016 G. 0.020 H. 0.033 I. 0.090 J. $f'(9)$ does not exist

19. (2) Calculate $\sum_{k=1}^{23} (\ln(k) - \ln(k+1))$.

A. -3.17 B. -3.13 C. -3.09 D. -2.30 E. 0.00
F. 0.69 G. 2.30 H. 3.09 I. 3.13 J. 3.17

20. (2) Use Newton's method to find an approximate solution of $xe^x = 2$. Start with $x_0 = 0$ and use two iterations of Newton's method to find x_2.

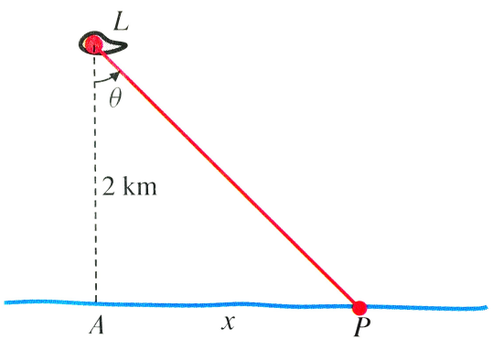
A. -1.73 B. -1.42 C. -1.35 D. 0.03 E. 0.16
F. 0.37 G. 0.94 H. 1.35 I. 1.42 J. 1.73

21. (2) Calculate $\int_0^1 x^2(4-x^3)^3\,dx$.

A. 5; B. 10; C. 15; D. 20; E. 25;
F. 150; G. 175; H. 200; I. 225; J. 2,100.

LONG - ANSWER SECTION

22. (4) A lighthouse sits 2 km offshore and its beam of light rotates counterclockwise at a constant rate of 3 full circles per minute. At what rate is the image of the beam moving down the shore line when the image is 1 km from the spot on the shore nearest the lighthouse?

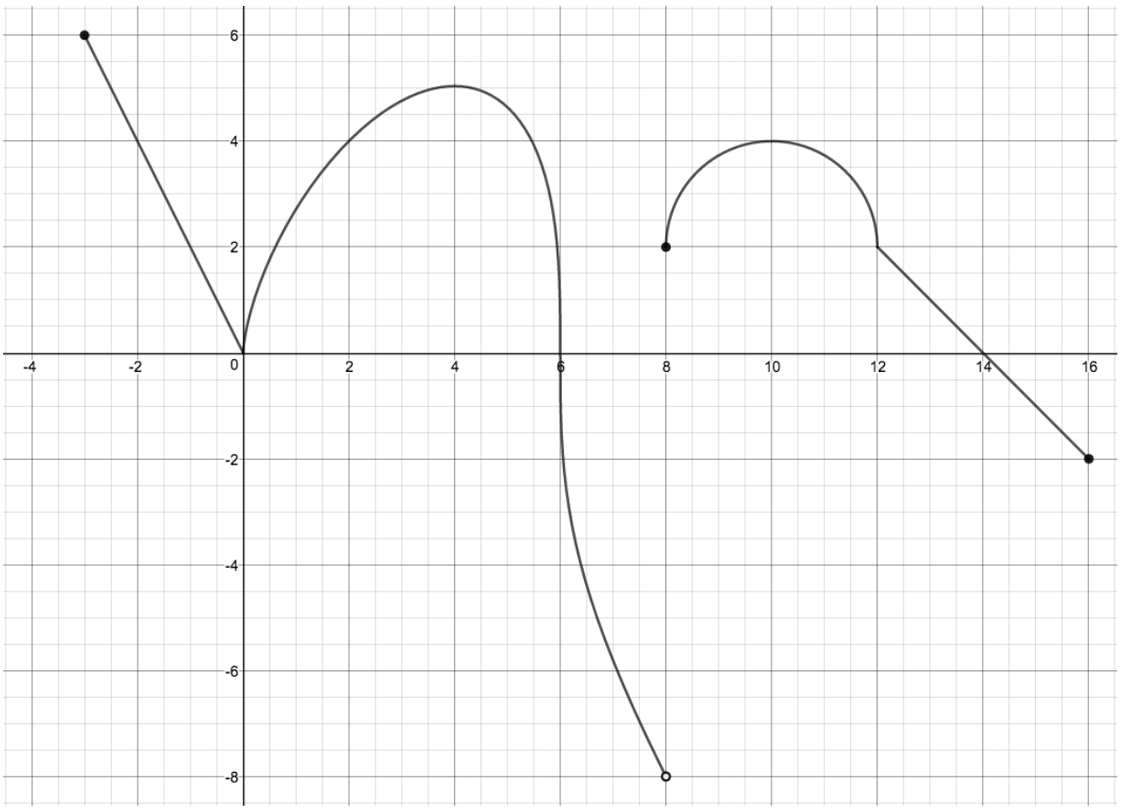


Your answer:

23. (5) A rectangular box of 864 cm^3 is to be constructed with an open top and a square base. Find the dimensions of the box that minimizes its surface area.
Show that the determined dimensions of the box guaranty the minimum surface area.

Your answer:

24. (9) Below is the graph of a function g:



Answer the questions in this page and in the next two pages about this function g. Some of the answers will necessarily only be approximate. Whenever possible, calculate answers exactly. This question does not require justification of each answer.

A) What is $g(8)$?

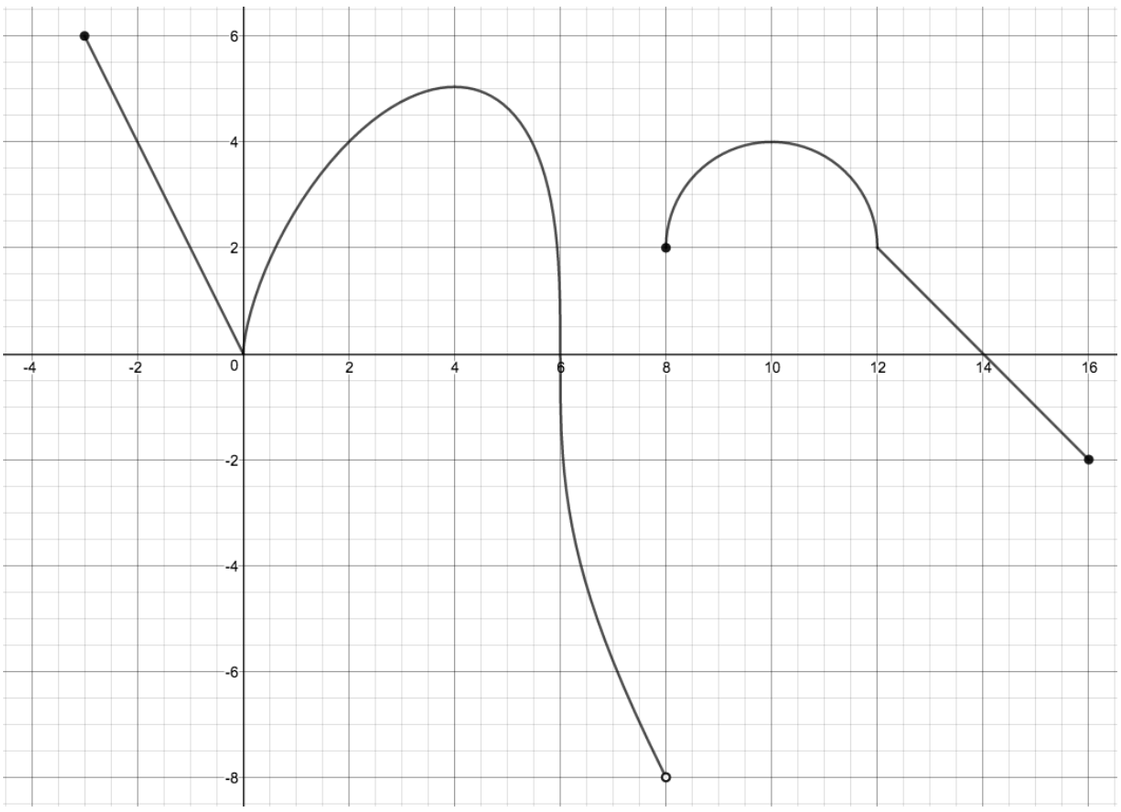
Your answer:

B) What is $g'(-1)$?

Your answer:

C) Give us one value of x such that $1 < x < 7$ and $g'(x)$ does not exist.

Your answer:



D) Give us one value of x such that clearly $g'(x) \neq 0$ and $g''(x) < 0$.

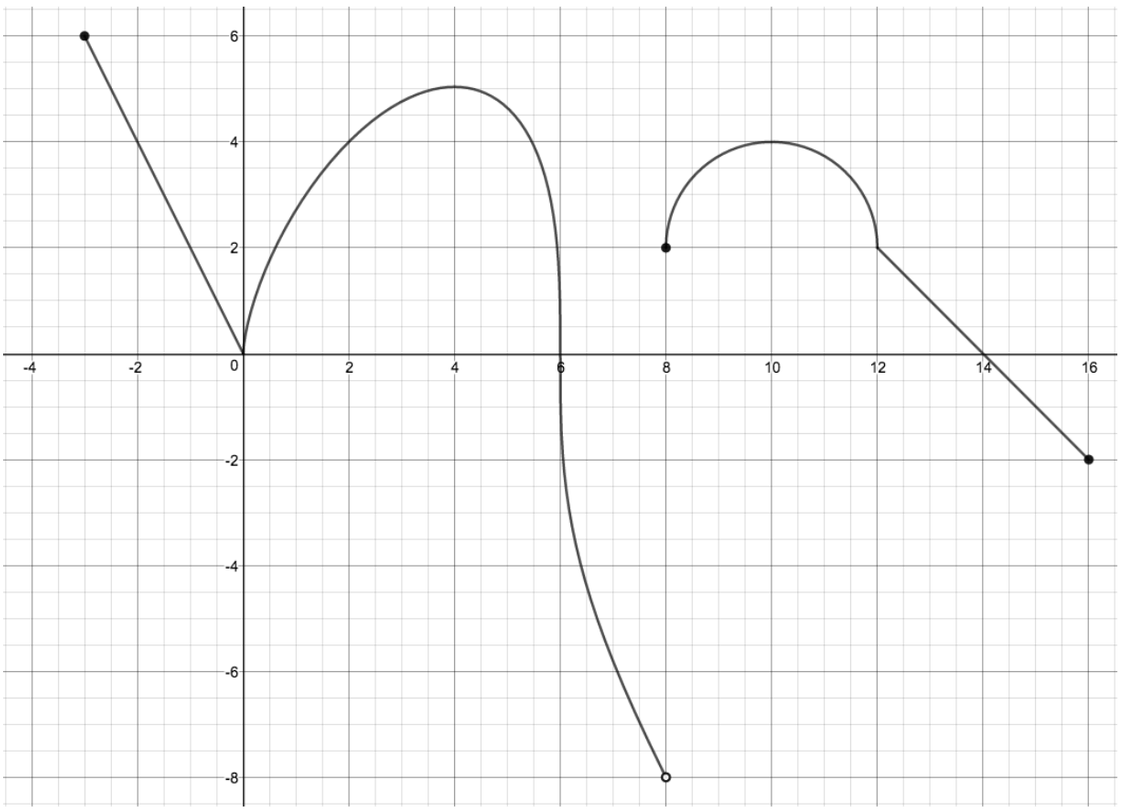
Your answer:

E) Calculate $\lim_{x \to -2} \dfrac{g(x) - 4}{3x + 6}$

Your answer:

F) Calculate $\lim_{x \to 0} g(8 - x^2)$

Your answer:



G) Calculate $\int_{12}^{16} g(x)\ dx$

Your answer:

H) Calculate $\int_{8}^{12} g(x)\ dx$

Your answer:

I) Calculate $\dfrac{d}{dx}\ g(x^3 - 1)$ at $x = -1$

Your answer:

25. (10) Let

$$f(x) = \frac{x}{4 - x^2}$$

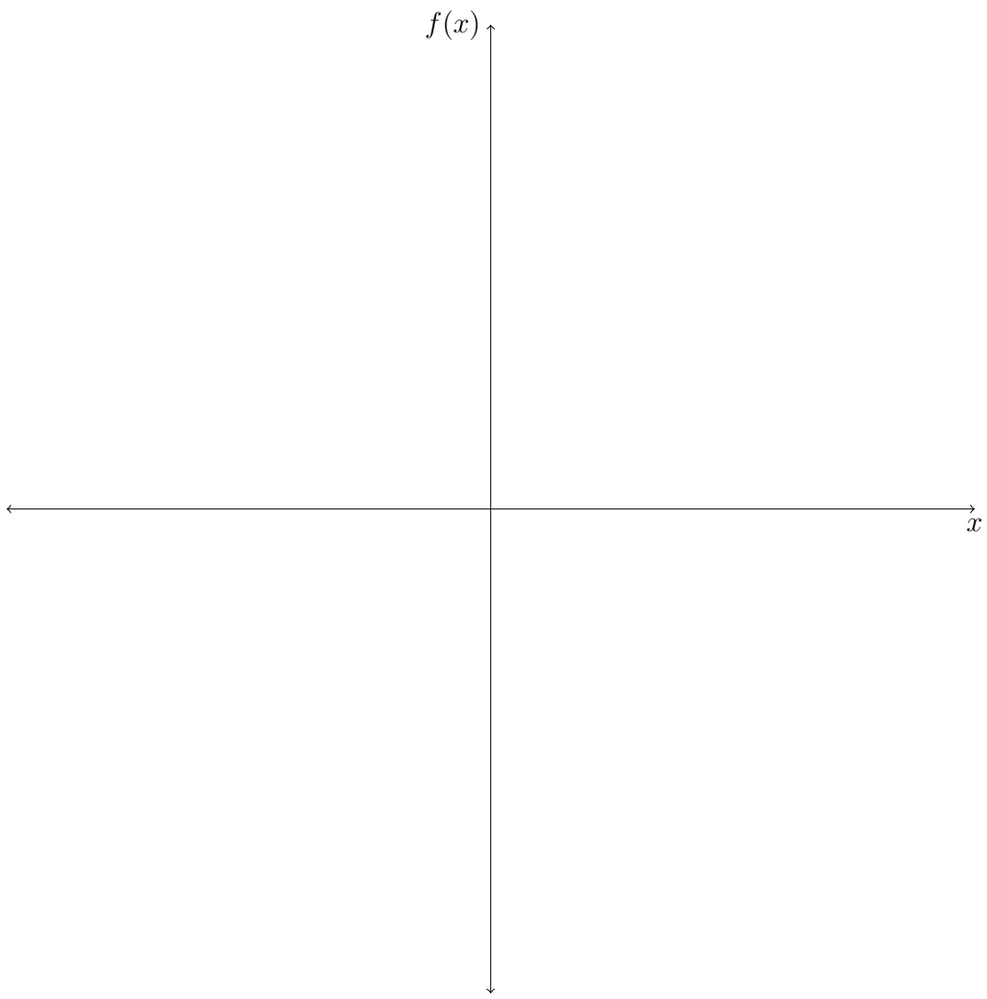
and notice that

$$f'(x) = \frac{x^2 + 4}{(4 - x^2)^2} \text{ and } f''(x) = \frac{2x(x^2 + 12)}{(4 - x^2)^3}.$$

Sketch the graph of f.

Important instructions:

- Your graph should contain all the important characteristics. All the important points should be shown in the graph.
- Your analysis should be clearly explained.
- You will only get points for the parts that are **both** correct in the analysis and in the graph. Use next page, if you need more space for analysis.
- Use an appropriate scale.



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END OF EXAMINATION