Tips, Formulae and Shortcuts for Permutations & Combinations

By

CRACKU.IN

Cracku Tip 1 – Permutations & Combinations

- Permutations & Combinations, and Probability are key topics in CAT.
- You don't have to go too deep into these topics, but ensure that you learn the basics well.
- So look through this formula list a few times and understand the formulae.

Cracku Tip 2 – Permutations & Combinations

- The best way to tackle this subject is by solving questions. The more questions you solve, the better you will get at this topic.
- Once you practice good number of sums, you will start to see that all of them are generally variations of the same few themes that are listed in the formula list.
- In this slide, we will look at the important formulae on P&C, and Probability.

Cracku Tip 3 – Permutations & Combinations

- N! = N(N-1)(N-2)(N-3)……1
- 0! = 1! = 1
- $^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$
- $^{n}P_{r} = \frac{n!}{(n-r)!}$

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Cracku Tip 4 – Permutations & Combinations

- **Arrangement** :

n items can be arranged in n! ways

- **Permutation** :

A way of selecting and arranging r objects out of a set of n objects, ${}^{n}P_{r} = \frac{n!}{(n-r)!}$

- **Combination** :

A way of selecting r objects out of n (arrangement does not matter)

$${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$$

Cracku Tip 5 – Permutations & Combinations

- Selecting r objects out of n is same as selecting (n-r) objects out of n, $^nC_r = {}^nC_{n-r}$

- Total selections that can be made from ‘n’ distinct items is given $\sum_{k=0}^{n} {}^nC_k = 2^n$

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Cracku Tip 6 – Permutations & Combinations

Partitioning :

- Number of ways to partition n identical things in r distinct slots is given by $^{n+r-1}C_{r-1}$
- Number of ways to partition n identical things in r distinct slots so that each slot gets at least 1 is given by $^{n-1}C_{r-1}$
- Number of ways to partition n distinct things in r distinct slots is given by r^n
- Number of ways to partition n distinct things in r distinct slots where arrangement matters = $\frac{(n+r-1)!}{(r-1)!}$

Cracku Tip 7 – Permutations & Combinations

Arrangement with repetitions :

If x items out of n items are repeated, then the number of ways of arranging these n items is $\frac{n!}{x!}$ ways. If a items, b items and c items are repeated within n items, they can be arranged in $\frac{n!}{a!\,b!\,c!}$ ways.

Cracku Tip 8 – Permutations & Combinations

Rank of a word :

- To get the rank of a word in the alphabetical list of all permutations of the word, start with alphabetically arranging the n letters. If there are x letters higher than the first letter of the word, then there are at least x*(n-1)! Words above our word.
- After removing the first affixed letter from the set if there are y letters above the second letter then there are y*(n-2)! words more before your word and so on. So rank of word = x*(n-1)! + y*(n-2)! + .. +1

Cracku Tip 9 – Permutations & Combinations

Integral Solutions :

- Number of positive integral solutions to $x_1+x_2+x_3+.....+x_n= s$ where $s \geq 0$ is ${}^{s-1}C_{n-1}$
- Number of non-negative integral solutions to $x_1+x_2+x_3+.....+x_n = s$ where $s \geq 0$ is ${}^{n+s-1}C_{n-1}$

Cracku Tip 10 – Permutations & Combinations

Circular arrangement :

Number of ways of arranging n items around a circle are 1 for n = 1,2 and (n-1)! for n≥3. If its a necklace or bracelet that can be flipped over, the possibilities are $\frac{(n-1)!}{2}$

Derangements :

If n distinct items are arranged, the number of ways they can be arranged so that they do not occupy their intended spot is

$$D = n!\left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \ldots\ldots \frac{(-1)^n}{n!}\right)$$

Cracku Tip 11(i) – Probability

Bayes Theorem (Conditional Probability) for CAT:

Conditional probability is used in case of events which are not independent. In the discussion of probabilities all events can be classified into 2 categories: Dependent and Independent.

Independent events are those where the happening of one event does not affect the happening of the other. For example, if an unbiased coin is thrown 'n' times then the probability of head turning up in any of the attempts will be 1/2. It will not be dependent on the results of the previous outcomes.

Dependent events, on the other hand, are the events in which the outcome of the second event is dependent on the outcome of the first event.

For example, if you have to draw two cards from a deck one after the other, then the probability of second card being of a particular suit will depend on the which card was drawn in the first attempt.

Cracku Tip 11(ii) – Probability

Bayes Theorem (Conditional Probability) for CAT:

Let us first discuss the definition of conditional probability.
Let 'A' and 'B' be two events which are not independent then the probability of occurrence of B given that A has already occurred is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Here, $P(A \cap B)$ is nothing but the probability of occurrence of both A and B.
We often use Bayes theorem to solve problems on conditional probability.
Bayes theorem is defined as follows

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

Here, P(A|B) is the probability of occurrence of A given that B has already occurred.
P(A) is the probability of occurrence of A
P(B) is the probability of occurrence of B

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Important CAT P&C and Probability Questions

Questions

Instructions
For the following questions answer them individually

Question 1

How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?

A 499

B 500

C 375

D 376

Answer: D

Video Solution

Explanation:
We have to essentially look at numbers between 1000 and 4000 (including both).

The first digit can be either 1 or 2 or 3.

The second digit can be any of the five numbers.

The third digit can be any of the five numbers.

The fourth digit can also be any of the five numbers.

So, total is 3*5*5*5 = 375.

However, we have ignored the number 4000 in this calculation and hence the total is 375+1=376

Question 2

What is the number of distinct terms in the expansion of $(a+b+c)^{20}$?

A 231

B 253

C 242

D 210

E 228

Answer: A

Video Solution

Explanation:
The power is 20.

20 has to be divided among a, b and c. This can be done in $^{20+3-1}C_{3-1}$ = $^{22}C_2$ = 231

Option a) is the correct answer.

Question 3

There are 6 boxes numbered 1,2,... 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is

A 5

B 21

C 33

D 60

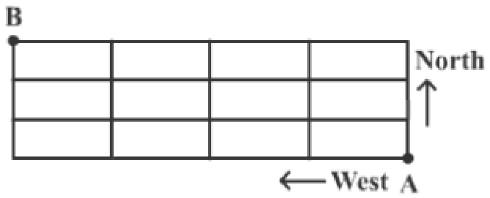
Answer: B

Video Solution

Explanation:
If there is only 1 green ball, it can be done in 6 ways
If there are 2 green balls, it can be done in 5 ways.
.
.
.
If there are 6 green balls, it can be done in 1 way.
So, the total number of possibilities is 6*7/2 = 21

Question 4

In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point B from point A?



A 15

B 35

C 120

D 336

Answer: B

Video Solution

Explanation:
The person has to take 3 steps north and 4 steps west, in whatever way he travels.

Total steps = 7, 3 north and 4 west.

Number of ways = 7!/(4!3!) = 35

Question 5

A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done such that no two adjacent stripes have the same colour is

A 12×81

B 16×192

C 20 × 125

D 24 × 216

Answer: A

Video Solution

Explanation:
The number of ways of selecting a colour for the first stripe is 4. The number of ways of selecting a colour for the second stripe is 3. Similarly, the number of ways of selecting colours for the third, fourth, fifth and sixth stripes are 3, 3, 3 and 3 respectively.
The total number of ways of selecting the colours is, therefore, 4*3*3*3*3*3 = 12*81.

Question 6

Consider the set S = { 1, 2, 3, .., 1000 }. How many arithmetic progressions can be formed from the elements of S that start with l and end with 1000 and have at least 3 elements?

A 3

B 4

C 6

D 7

E 8

Answer: D

Video Solution

Explanation:
The nth term is a + (n-1)d
1000 = 1 + (n-1)d
So, (n-1)d = 999
999 = 3^3 * 37
So, the number of factors is 4*2 = 8
Since there should be at least 3 terms in the series, d cannot be 999.
So, the number of possibilities is 7

Question 7

In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is

A 200

B 216

C 235

D 256

Answer: A

Video Solution

Explanation:

Number of games in which both the players are girls = ${}^{G}C_2$ where G is the number of girls
${}^{G}C_2 = 45$
${}^{10}C_2 = 45$
So, G = 10
Similarly, number of games in which both the players are boys = ${}^{B}C_2$, where B is the number of boys
${}^{B}C_2 = 190$
${}^{20}C_2 = 190$
So, B = 20
So, number of games in which one player is a boy and the other player is a girl is 20*10 = 200

Question 8

If there are 10 positive real numbers $n_1 < n_2 < n_3 ... < n_{10}$, how many triplets of these numbers $(n_1, n_2, n_3), (n_2, n_3, n_4)$ can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number?

A 45

B 90

C 120

D 180

Answer: C

Video Solution

Explanation:
For any selection of three numbers, there is only one way in which they can be arranged in ascending order.

So, the answer is ${}^{10}C_3 = 120$

Question 9

A man has 9 friends: 4 boys and 5 girls. In how many ways can he invite them, if there has to be exactly 3 girls in the invitees?

A 320

B 160

C 80

D 200

Answer: B

Video Solution

Explanation:
Selecting 3 girls from 5 girls can be done in ${}^{5}C_3$ ways => 10 ways

Each of the boys may or may not be selected => 2 * 2 * 2 * 2 = 16 ways

=> 16 * 10 = 160 ways

Question 10

For a scholarship, at most n candidates out of 2n + 1 can be selected. If the number of different ways of selection of at least one candidate is 63, the maximum number of candidates that can be selected for the scholarship is:

A 3

B 4

C 2

D 5

Answer: A

Video Solution

Explanation:
At least one candidate and at most n candidates among 2n+1 candidates =>

$\Rightarrow\ {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+2}C_3 + ... + {}^{2n+1}C_{n-1} + {}^{2n+1}C_n = 63$

We know that ${}^{2n+1}C_0$ and ${}^{2n+1}C_{2n+1}$ are equal to 1.

By Binomial Expansion

${}^{2n+1}C_0 + {}^{2n+1}C_1$+... + ${}^{2n+1}C_{2n} + {}^{2n+1}C_{2n+1} = (1+1)^{2n+1}$ ---- Eq 1

Also ${}^{2n+1}C_1$ = ${}^{2n+1}C_{2n}$ by symmetry

and ${}^{2n+1}C_2$ = ${}^{2n+1}C_{2n-1}$ and so on

So $\Rightarrow\ {}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + ... + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n} = 63$

Therefore, on substituting these values in Eq 1 we get

1 + 63 + 63 +1 = 2^{2n+1}

2^{2n+1} = 128

2n+1 = 7

Therefore, n=3

As at most n students can be selected, the correct answer is 3.

Question 11

Let AB, CD, EF, GH, and JK be five diameters of a circle with center at 0. In how many ways can three points be chosen out of A, B, C, D, E, F, G, H, J, K, and O so as to form a triangle?

Answer:160

Video Solution

Explanation:
The total number of given points are 11. (10 on circumference and 1 is the center)
So total possible triangles = 11C3 = 165.
However, AOB, COD, EOF, GOH, JOK lie on a straight line. Hence, these 5 triangles are not possible. Thus, the required number of triangles = 165 - 5 = 160

Question 12

In how many ways can 7 identical erasers be distributed among 4 kids in such a way that each kid gets at least one eraser but nobody gets more than 3 erasers?

A 16

B 20

C 14

D 15

Answer: A

Video Solution

Explanation:
We have been given that a + b + c + d = 7
Total ways of distributing 7 things among 4 people so that each one gets at least one = ${}^{n-1}C_{r-1}$ = 6C3 = 20
Now we need to subtract the cases where any one person got more than 3 erasers. Any person cannot get more than 4 erasers since each child has to get at least 1. Any of the 4 childs can get 4 erasers. Thus, there are 4 cases. On subtracting these cases from the total cases we get the required answer. Hence, the required value is 20 - 4 = 16

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Question 13

The numbers 1, 2, ..., 9 are arranged in a 3 X 3 square grid in such a way that each number occurs once and the entries along each column, each row, and each of the two diagonals add up to the same value.

If the top left and the top right entries of the grid are 6 and 2, respectively, then the bottom middle entry is

Answer:3

Video Solution

Explanation:
According to the question each column, each row, and each of the two diagonals of the 3X3 matrix add up to the same value. This value must be 15.

Let us consider the matrix as shown below:

6		2

Now we'll try substituting values from 1 to 9 in the exact middle grid shown as 'x'.

If x = 1 or 3, then the value in the left bottom grid will be more than 9 which is not possible.

x cannot be equal to 2.

If x = 4, value in the left bottom grid will be 9. But then addition of first column will come out to be more than 15. Hence, not possible.

If x=5, we get the grid as shown below:

6	7	2
1	5	9
8	3	4

Hence, for x = 5 all conditions are satisfied. We see that the bottom middle entry is 3.

Hence, 3 is the correct answer.

Question 14

In how many ways can 8 identical pens be distributed among Amal, Bimal, and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens?

Answer:6

Video Solution

Explanation:
After Amal, Bimal and Kamal are given their minimum required pens, the pens left are 8 - (1 + 2 + 3) = 2 pens

Now these two pens have to be divided between three persons so that each person can get zero pens = ${}^{2+3-1}C_{3-1}$ = ${}^{4}C_{2}$ = 6

Question 15

How many four digit numbers, which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6, such that no digit is used more than once and 0 does not occur in the left-most position?

Answer:50

Video Solution

Explanation:
For the number to be divisible by 6, the sum of the digits should be divisible by 3 and the units digit should be even. Hence we have the digits as

Case I: 2, 3, 4, 6
Now the units place can be filled in three ways (2,4,6), and the remaining three places can be filled in 3! = 6 ways.
Hence total number of ways = 3*6 = 18

Case II: 0, 2, 3, 4
case II a: 0 is in the units place => 3! = 6 ways
case II b: 0 is not in the units place => units place can be filled in 2 ways(2,4) , thousands place can be filled in 2 ways (remaining 3 - 0) and remaining can be filled in 2! = 2 ways. Hence total number of ways = 2 * 2 * 2 = 8
Total number of ways in this case = 6 + 8 = 14 ways.

Case III: 0, 2, 4, 6
case III a: 0 is in the units place => 3! = 6 ways

case II b: 0 is not in the units place => units place can be filled in 3 ways(2,4,6) , thousands place can be filled in 2 ways (remaining 3 - 0) and remaining can be filled in 2! = 2 ways. Hence total number of ways = 3 * 2 * 2 = 12
Total number of ways in this case = 6 + 12= 18 ways.

Hence the total number of ways = 18 + 14 + 18 = 50 ways

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Question 16

How many numbers with two or more digits can be formed with the digits 1,2,3,4,5,6,7,8,9, so that in every such number, each digit is used at most once and the digits appear in the ascending order?

Answer:502

Video Solution

Explanation:
It has been given that the digits in the number should appear in the ascending order. Therefore, there is only 1 possible arrangement of the digits once they are selected to form a number.
There are 9 numbers (1,2,3,4,5,6,7,8,9) in total.
2 digit numbers can be formed in $9C_2$ ways.
3 digit numbers can be formed in $9C_3$ ways.
....
..9 digit number can be formed in 9C9 ways.

We know that $nC_0 + nC_1 + nC_2 + ... + nC_n = 2^n$
=> $9C_0 + 9C_1 + 9C_2 + ...9C_9 = 2^9$
$9C_0 + 9C_1 + ...9C_9 = 512$

We have to subtract $9C_0$ and $9C_1$ from both the sides of the equations since we cannot form single digit numbers.
=> $9C_2 + 9C_3 + ... + 9C_9 = 512 - 1 - 9$
$9C_2 + 9C_3 + ... + 9C_9 = 502$

Therefore, 502 is the right answer.

Question 17

How many two-digit numbers, with a non-zero digit in the units place, are there which are more than thrice the number formed by interchanging the positions of its digits?

Answer:6

Video Solution

Explanation:
Let 'ab' be the two digit number. Where b $\neq$ 0.

We will get number 'ba' after interchanging its digit.

It is given that 10a+b > 3*(10b + a)

7a > 29b

If b = 1, then a = {5, 6, 7, 8, 9}

If b = 2, then a = {9}

If b = 3, then no value of 'a' is possible. Hence, we can say that there are a total of 6 such numbers.

Question 18

In a tournament, there are 43 junior level and 51 senior level participants. Each pair of juniors play one match. Each pair of seniors play one match. There is no junior versus senior match. The number of girl versus girl matches in junior level is 153, while the number of boy versus boy matches in senior level is 276. The number of matches a boy plays against a girl is

Answer:1098

Video Solution

Explanation:
In a tournament, there are 43 junior level and 51 senior level participants.

Let 'n' be the number of girls on junior level. It is given that the number of girl versus girl matches in junior level is 153.

$\Rightarrow$ nC2 = 153

$\Rightarrow$ n(n-1)/2 = 153

$\Rightarrow$ n(n-1) = 306

=> n^2-n-306 = 0

=> (n+17)(n-18)=0

=> n=18 (rejecting n=-17)

Therefore, number of boys on junior level = 43 - 18 = 25.

Let 'm' be the number of boys on senior level. It is given that the number of boy versus boy matches in senior level is 276.

$\Rightarrow$ mC2 = 276

$\Rightarrow$ m = 24

Therefore, number of girls on senior level = 51 - 24 = 27.

Hence, the number of matches a boy plays against a girl = 18*25+24*27 = 1098

Question 19

With rectangular axes of coordinates, the number of paths from (1, 1) to (8, 10) via (4, 6), where each step from any point (x, y) is either to (x, y+1) or to (x+1, y), is

Answer:3920

Video Solution

Explanation:
The number of paths from (1, 1) to (8, 10) via (4, 6) = The number of paths from (1,1) to (4,6) * The number of paths from (4,6) to (8,10)

To calculate the number of paths from (1,1) to (4,6), 4-1 =3 steps in x-directions and 6-1=5 steps in y direction

Hence the number of paths from (1,1) to (4,6) = $^{(3+5)}C_3$ = 56

To calculate the number of paths from (4,6) to (8,10), 8-4 =4 steps in x-directions and 10-6=4 steps in y direction

Hence the number of paths from (4,6) to (8,10) = $^{(4+4)}C_4$ = 70

The number of paths from (1, 1) to (8, 10) via (4, 6) = 56*70=3920

Question 20

The number of groups of three or more distinct numbers that can be chosen from 1, 2, 3, 4, 5, 6, 7 and 8 so that the groups always include 3 and 5, while 7 and 8 are never included together is

Answer:47

Video Solution

Explanation:
The possible arrangements are of the form

35 _ Can be chosen in 6 ways.

35 _ _ We can choose 2 out of the remaining 6 in $^6C_2 = 15$ways. We remove 1 case where 7 and 8 are together to get 14 ways.

35 _ _ _We can choose 3 out of the remaining 6 in $^6C_3 = 20$ways. We remove 4 cases where 7 and 8 are together to get 16 ways.

35 _ _ _ _We can choose 4 out of the remaining 6 in $^6C_4 = 15$ways. We remove 6 case where 7 and 8 are together to get 9 ways.

35 _ _ _ _ _ We choose 1 out of 7 and 8 and all the remaining others in 2 ways.

Thus, total number of cases = 6+14+16+9+2 = 47.

Alternatively,

The arrangement requires a selection of 3 or more numbers while including 3 and 5 and 7, 8 are never included together. We have cases including a selection of only 7, only 8 and neither 7 nor 8.

Considering the cases, only 7 is selected.

We can select a maximum of 7 digit numbers. We must select 3, 5, and 7.

Hence we must have (3, 5, 7) for the remaining 4 numbers we have

Each of the numbers can either be selected or not selected and we have 4 numbers :

Hence we have _ _ _ _ and 2 possibilities for each and hence a total of 2*2*2*2 = 16 possibilities.

SImilarly, including only 8, we have 16 more possibilities.

Cases including neither 7 nor 8.

We must have 3 and 5 in the group but there must be no 7 and 8 in the group.

Hence we have 3 5 _ _ _ _.

For the 4 blanks, we can have 2 possibilities for either placing a number or not among 1, 2, 4, 6.

= 16 possibilities

But we must remove the case where neither of the 4 numbers are placed because the number becomes a two-digit number.

Hence 16 - 1 = 15 cases.

Total = 16+15+16 = 47 possibilities

Question 21

How many three-digit numbers are greater than 100 and increase by 198 when the three digits are arranged in the reverse order?

Answer:70

Video Solution

Explanation:
Let the numbers be of the form 100a+10b+c, where a, b, and c represent single digits.

Then (100c+10b+a)-(100a+10b+c)=198

99c-99a=198

c-a = 2.

Now, a can take the values 1-7. a cannot be zero as the initial number has 3 digits and cannot be 8 or 9 as then c would not be a single-digit number.

Thus, there can be 7 cases.

B can take the value of any digit from 0-9, as it does not affect the answer. Hence, the total cases will be $7 \times 10 = 70$.

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Question 22

The number of ways of distributing 15 identical balloons, 6 identical pencils and 3 identical erasers among 3 children, such that each child gets at least four balloons and one pencil, is

Answer:1000

Video Solution

Explanation:
This question is an application of the product rule in probability and combinatorics.

In the product rule, if two events A and B can occur in x and y ways, and for an event E, both events A and B need to take place, the number of ways that E can occur is xy. This can be expanded to 3 or more events as well.

Event 1: Distribution of balloons

Since each child gets at least 4 balloons, we will initially allocate these 4 balloons to each of them.

So we are left with 15 - 4 x 3 = 15 - 12 = 3 balloons and 3 children.

Now we need to distribute 3 identical balloons to 3 children.

This can be done in ${}^{n+r-1}C_{r-1}$ ways, where n = 3 and r = 3.

So, number of ways = ${}^{3+3-1}C_{3-1} = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$

Event 2: Distribution of pencils

Since each child gets at least one pencil, we will allocate 1 pencil to each child. We are now left with 6 - 3 = 3 pencils.

We now need to distribute 3 identical pencils to 3 children.

This can be done in ${}^{n+r-1}C_{r-1}$ ways, where n = 3 and r = 3.

So, number of ways = ${}^{3+3-1}C_{3-1} = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$

Event 3: Distribution of erasers

We need to distribute 3 identical erasers to 3 children.

This can be done in ${}^{n+r-1}C_{r-1}$ ways, where n = 3 and r = 3.

So, number of ways = ${}^{3+3-1}C_{3-1} = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10$

Applying the product rule, we get the total number of ways = 10 x 10 x 10 = 1000.

Question 23

A four-digit number is formed by using only the digits 1, 2 and 3 such that both 2 and 3 appear at least once. The number of all such four-digit numbers is

Answer:50

Video Solution

Explanation:
The question asks for the number of 4 digit numbers using only the digits 1, 2, and 3 such that the digits 2 and 3 appear at least once.

The different possibilities include :

Case 1:The four digits are (2, 2, 2, 3). Since the number 2 is repeated 3 times. The total number of arrangements are :

$\frac{4!}{3!}$ = 4.

Case 2: The four digits are 2, 2, 3, 3. The total number of four-digit numbers formed using this are :

$\frac{4!}{2!\cdot 2!} = 6$

Case 3: The four digits are 2, 3, 3, 3. The number of possible 4 digit numbers are :

$\frac{4!}{3!}$ = 4

Case4: The four digits are 2, 3, 3, 1. The number of possible 4 digit numbers are :

$\frac{4!}{2!} = 12$

Case5: Using the digits 2, 2, 3, 1. The number of possible 4 digit numbers are :

$\frac{4!}{2!} = 12$

Case 6: Using the digits 2, 3, 1, 1. The number of possible 4 digit numbers are :

$\frac{4!}{2!} = 12$

A total of 12 + 12 + 12 + 4 + 6 + 4 = 50 possibilities.

Alternatively

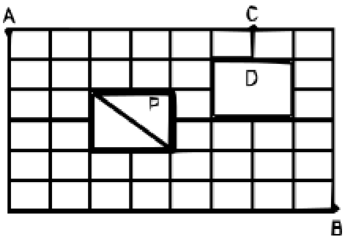
We have to form 4 digit numbers using 1,2,3 such that 2,3 appears at least once
So the possible cases :

1	2	3
2	1	1
1	2/1	1/2
0	3/1	1/3
0	2	2

Now we get $\frac{4!}{2!} \times 3$= 36 (When one digit is used twice and the remaining two once)
$\frac{4!}{3!} \times 2$ = 8 (When 1 is used 0 times and 2 and 3 is used 3 times or 1 time)
$\frac{4!}{2! \times 2!} = 6$(When 2 and 3 is used 2 times each)
So total numbers = 36+8+6 =50

Instructions

Directions for the next two questions: The figure below shows the plan of a town. The streets are at right angles to each other. A rectangular park (P) is situated inside the town with a diagonal road running through it. There is also a prohibited region (D) in the town.



Question 24

Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is

[CAT 2008]

A 60

B 75

C 45

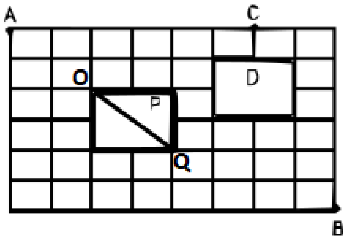
D 90

E 72

Answer: D

Video Solution

Explanation:



The shortest route from A to B is via the diagonal OQ in the square P. One can travel from A to O in 4!/2!*2! ways. The shortest way from O to Q is through the diagonal only.From Q to B can be travelled in 6!/4!*2! ways.

The total number of ways is, therefore, (4!/2!*2!) * (6!/4!*2!) = 6*15 = 90

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Question 25

Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path. Then the number of possible shortest paths that she can choose is

[CAT 2008]

A 1170

B 630

C 792

D 1200

E 936

Answer: A

Video Solution

Explanation:
The shortest route from A to B is via the diagonal in the square P. A to the north-west corner of square P can be travelled in 4!/2!*2! ways.
Number of ways to travel from the south-east corner of square P to B is 6!/4!*2!.
B to the north-east corner of D can be travelled in 6!/5! ways. From there to C can be travelled in 2 ways. There is 1 other way of travelling from B to C (along the perimeter of the field).

The total number of ways is, therefore, (4!/2!*2!) * (6!/4!*2!) * (6!/5! * 2 + 1)
= 6*15*13 = 1170

Instructions

For the following questions answer them individually

Question 26

A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition

A $11 \leq e \leq 66$

B $10 \leq e \leq 66$

C $11 \leq e \leq 65$

D $0 \leq e \leq 11$

Answer: A

Video Solution

Explanation:

Take any 12 points.

The maximum number of edges which can be drawn through these 12 points are $^{12}C_2$ = 66

The minimum number of edges which can be drawn through these 12 points are 12-1 = 11 as the resulting figure need not be closed. It might be open.

Question 27

N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 minutes, what is N?

[CAT 2004]

A 5

B 7

C 9

D None of the above

Answer: B

Video Solution

Explanation:

Total number of pairs is $^NC_2$. Number of pairs standing next to each other = N. Therefore, number of pairs in question = $^NC_2$ - N

= 28/2 = 14.

If N = 7,

$^7C_2$ - 7 = 21 - 7 = 14.

N = 7.

CAT Syllabus (Download PDF)

Question 28

There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?

[CAT 2006]

A 144

B 180

C 192

D 360

E 716

Answer: A

Video Solution

Explanation:
If the first task is assigned to either person 3 or person 4, the second task can be assigned in only 1 way. If the first task is assigned to either person 5 or person 6, the second task can be assigned in 2 ways. Therefore, the number of ways in which the first two tasks can be assigned is 2*1 + 2*2 = 6.
The other 4 tasks can be assigned to 4 people in 4! ways.
The total number of ways of assigning the 6 tasks is, therefore, 6*4! = 144.

Question 29

Let S be the set of five-digit numbers formed by the digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd positions are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?

A 228

B 216

C 294

D 192

Answer: B

Video Solution

Explanation:
When the odd numbers occupy places 1 and 3, only 2 or 4 can be in the 5th place. Odd numbers can occupy places 1 and 3 in 3C2*2! = 6 ways. When 2 is at the 5th place, the other odd number and 4 can be arranged in the remaining places in 2 ways. So, 2 occurs at the end 6*2 = 12 times. Similarly, 4 occurs 12 times.

If odd numbers occupy places 1 and 5, then 2 or 4 should come in the 3rd place. The other two numbers can then be arranged in 2 ways in the remaining blanks. So, if 1 is in the first place and 5 is in the 5th place, the other numbers can be arranged in 2*2 = 4 ways. Similar for 1 and 3; 5 and 1; 3 and 1; 5 and 3; 3 and 5. So, 5 occurs 8 times, 1 8 times and 3 8 times. Similar is the case when odd numbers are placed in 3rd and 5th places.

On the whole, 4 occurs 12 times, 2 occurs 12 times, 5, 3 and 1 each occur 16 times. The total is, therefore, 48+24+80+48+16 = 216

Question 30

Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English, and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose?

A 5

B 10

C 9

D 15

Answer: C

Video Solution

Explanation:
Consider there are 6 people numbered 1-3 englishmen and 3-6 frenchmen, let 3 know both english and french.
First call would be between 1-3 then 2-3 such that 3 know secret of all 3 englishmen.
Let 3 call 4 .
Similarly there would be call between 4-5 then 4-6 such that 4 know secret of all 3 frenchmen.
Now 3 would call 4 . Such that 3 and 4 would know secret of all 6 members.
Now to let this know to 1,2,5,6 more 4 calls would be required.

Hence, minimum calls required would be 9.

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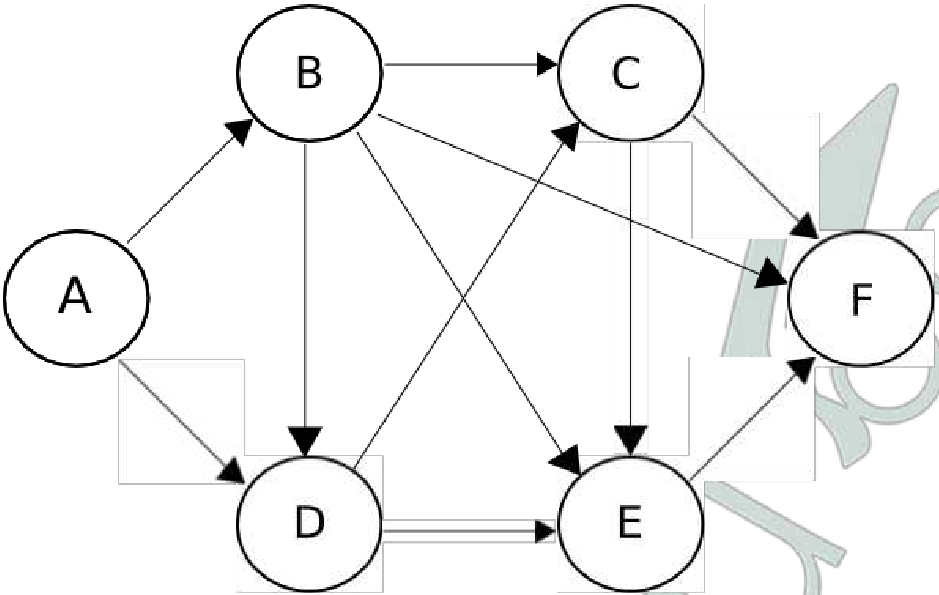
Question 31

The figure below shows the network connecting cities A, B, C, D, E and F. The arrows indicate permissible direction of travel. What is the number of distinct paths from A to F?



A 9

B 10

C 11

D None of these

Answer: B

Video Solution

Explanation:
The distinct paths are:

A -> D -> C -> F
A -> D -> E -> F
A -> D -> C -> E -> F
A -> B -> D -> C -> F
A -> B -> D -> E -> F
A -> B -> D -> C -> E -> F
A -> B -> C -> F
A -> B -> E -> F
A -> B -> F
A -> B -> C -> E -> F

So, the total number of distinct paths is 10

Question 32

One red flag, three white flags and two blue flags are arranged in a line such that,

A. no two adjacent flags are of the same colour

B. the flags at the two ends of the line are of different colours.

In how many different ways can the flags be arranged?

A 6

B 4

C 10

D 2

Answer: A

Video Solution

Explanation:
The three white flags can be arranged in the following two ways:

__ W __ W __ W or W __ W __ W __

In the blanks, the 2 blue and one red flag can be arranged in 3 ways.
So, the total number of arrangements is 2*3 = 6

Question 33

Sam has forgotten his friend's seven-digit telephone number. He remembers the following: the first three digits are either 635 or 674, the number is odd, and the number nine appears once. If Sam were to use a trial and error process to reach his friend, what is the minimum number of trials he has to make before he can be certain to succeed?

A 10000

B 2430

C 3402

D 3006

Answer: C

Video Solution

Explanation:
Consider cases : 1) Last digit is 9: No. of ways in which the first 3 digits can be guessed is 2. No. of ways in which next 3 digits can be guessed is 9*9*9. So in total the number of ways of guessing = 2*9*9*9 = 1458.

2) Last digit is not 9: the number 9 can occupy any of the given position 4, 5, or 6, and there shall be an odd number at position 7.

So in total, the number of guesses = 2*3*(9*9*4) = 1944+1458 = 3402

CAT Percentile Predictor

Question 34

There are three cities A, B and C. Each of these cities is connected with the other two cities by at least one direct road. If a traveller wants to go from one city (origin) to another city (destination), she can do so either by traversing a road connecting the two cities directly, or by traversing two roads, the, first connecting the origin to the third city and the second connecting the third city to the destination. In all there are 33 routes from A to B (including those via C). Similarly there are 23 routes from B to C (including those via A). How many roads are there from A to C directly?

A 6

B 3

C 5

D 10

Answer: A

Video Solution

Explanation:
The possible roads are:

A -> B Let this be x
A -> C Let this be y
B -> C Let this be z

From the information given, x + yz = 33 -> (1)
z + xy = 23 -> (2)

From the options, if y = 10, x =2 and z = 3 from (2), but it doesn't satisfy (1)
If y = 5, x = 4 and z = 3 from (2) but they don't satisfy (1)
A possible set of numbers for (x,y,z) are (3,6,5)
Number of roads from A -> C = 6

Instructions

Directions for the next two questions: Answer the questions based on the following information.

Each of the 11 letters A, H, I, M, O, T, U, V, W, X and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

Question 35

How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

A 7,920

B 330

C 14,640

D 4,19,430

Answer: A

Video Solution

Explanation:
The number of ways in which this can be done is 11*10*9*8 = 7920

Question 36

How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

A 990

B 2,730

C 12,870

D 15,600

Answer: C

Video Solution

Explanation:
If there are 3 symmetric letters, it can be formed in 11*10*9 ways

If there are 2 symmetric letters, it can be formed in 11C2 * 15C1 * 3! ways

If there is only 1 symmetric letter, the password can be formed in 15C2*11C1*3! ways

Total = 990+330*15+630*11 = 12870 ways

Important Verbal Ability Questions for CAT (Download PDF)

Instructions
For the following questions answer them individually

Question 37

How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8?

A 486

B 1,084

C 728

D None of these

Answer: C

Video Solution

Explanation:
According to given condiitons, number of 1 digit nos. = 2, number of 2 digit nos. = 2*3 , number of 3 digit nos. = $2*3^2$, number of 4 digit nos. = $2*3^3$, number of 5 digit numbers. = $2*3^4$, Number of 6 digit nos. = $2*3^5$.

Total summation 2*(1+3+9+27+81+243) = 728 .

Question 38

Ten straight lines, no two of which are parallel and no three of which pass through any common point, are drawn on a plane. The total number of regions (including finite and infinite regions) into which the plane would be divided by the lines is

A 56

B 255

C 1024

D not unique

Answer: A

Video Solution

Explanation:
If there are 'm' non-parallel lines, then the maximum number of regions into which the plane is divided is given by

[m(m+1)/2]+1

In this case, 'm' = 10

So, the number of regions into which the plane is divided is (10*11/2) + 1 = 56

Question 39

In how many ways is it possible to choose a white square and a black square on a chessboard so that the squares must not lie in the same row or column?

A 56

B 896

C 60

D 768

Answer: D

Video Solution

Explanation:
First a black square can be selected in 32 ways. Out of remaining rows and columns, 24 white squares remain. 1 white square can them be chosen in 24 ways. So total no. of ways of selection is 32*24 = 768.

Question 40

There are six boxes numbered 1, 2, 3, 4, 5, 6. Each box is to be filled up either with a white ball or a black ball in such a manner that at least one box contains a black ball and all the boxes containing black balls are consecutively numbered. The total number of ways in which this can be done equals.

A 15

B 21

C 63

D 64

Answer: B

Video Solution

Explanation:
Total ways when all 6 boxes have only black balls = 1
Total ways when 5 boxes have black balls = 2
Total ways when 4 boxes have black balls = 3
Total ways when 3 boxes have black balls = 4
Total ways when 2 boxes have black balls = 5
Total ways when only 1 box has black ball = 6
So total ways of putting a black ball such that all of them come consecutively = (1+2+3+4+5+6) = 21

Question 41

A player rolls a die and receives the same number of rupees as the number of dots on the face that turns up. What should the player pay for each roll if he wants to make a profit of one rupee per throw of the die in the long run?

A Rs. 2.50

B Rs. 2

C Rs.3.50

D Rs. 4

Answer: A

Video Solution

Explanation:
The expected money got by the player = 1*1/6 + 2*1/6 + 3*1/6 + 4*1/6 + 5*1/6 + 6*1/6 = 21/6 = Rs 3.5

So, the player has to pay 3.5 - 1 = Rs 2.5 to get a profit of Re 1 in the long run.

Question 42

In how many ways can eight directors, the vice chairman and chairman of a firm be seated at a round table, if the chairman has to sit between the the vice chairman and a specific director?

A $9! \times 2$

B $2 \times 8!$

C $2 \times 7!$

D None of these

Answer: C

Video Solution

Explanation:
Chariman, Vice-Chairman and the director can be made as a group such that Chairman sits between the Vice-Chairman and the director. This group can be formed in 2 ways.

Each of the remaining 7 directors and the group can be arranged in 7! ways.

=> Total number of ways = 2 * 7!.