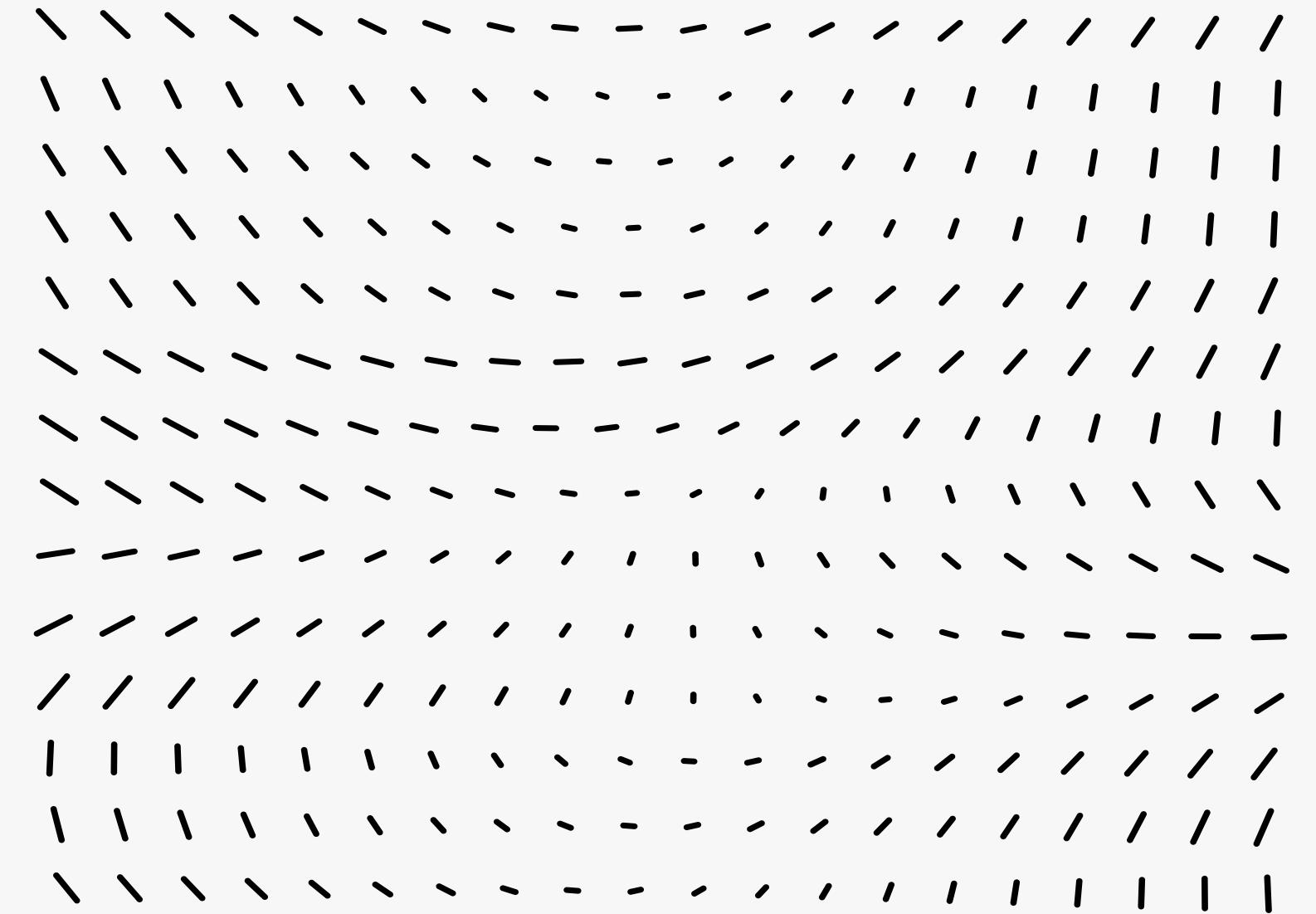


CSC 225 Lab Notes



Counting and discrete mathematics

Reminder:

A standard deck of cards has 52 cards and 4 suits: spades, hearts, clubs, and diamonds. Each suit has 13 cards:

Ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

Note: The above ordering is generally accepted, going from cards worth the least amount of points to most amount of points. Exception: Ace can be both least and highest.

Card hands

1. Royal flush consists of 10, J, Q, K, A exactly and must all be of the same suit.

Ans: since there are only 4 suits, there are only 4 ways to make a Royal Flush.

2. A straight flush is 5 cards in consecutive order (with respect to the ^{global} order previously mentioned) (ace both low and high). All cards must be the same suit.

A straight flush can start with:

A, 2, 3, 4, 5, 6, 7, 8, 9, or 10

but notice straight flushes that start with 10 are classified as royal flushes instead. So really, a straight flush can start with

A, 2, 3, 4, 5, 6, 7, 8, or 9

So there are 9 different straight flushes per suit, and 4 suits yielding $9 * 4 = 36$ ways to make a straight flush.

EXAMPLES

1. Flush. A flush is all five cards have to be the same suit (excluding RoyalFlushes and Straight Flushes).

$$\binom{4}{1} \binom{13}{5} = 4 \cdot 1287 = 5148$$

↑
 Choose 1 of
 the 4 suits

Out of 13 cards in 1 suit,
 choose 5

↑
 but this
 number includes
 Royal + straight flushes!

Ans. $\binom{4}{1} \binom{13}{5} - (\# \text{royal} + \text{straight flushes})$

$$= 5148 - (4 + 36)$$

$$= 5148 - 40 = 5108$$

2. Straight: All 5 cards in consecutive order but cannot all be the same suit.

Thought: if all 5 cards were the same suit, then our hand would be either a Royal or straight flush. So we can just subtract 40 from all possible straights!

Ans.

$$\binom{10}{1} \cdot 4^5 - 40 = 10 \cdot 200$$

↑
10 ways to start a straight (ie. 10 different straights)

↑
All 5 cards can be any suit

↑
Subtract Royal and straight flushes from the total

3. Three-of-a-kind : Three cards are the same rank and the other two are different from the three and each other.

Ex.



Ans.

Pick 2 rank from remaining 12 ranks
eg. 2 and 6

$$\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1} \binom{4}{1} = 54912$$

↑
pick the rank of the triplet
eg. 3

↑
pick the 3 suits of the triplet
eg. ♠, ♥, ♦

↑
pick suit for 4th card
eg. 2♥

↑
pick suit for last card
eg. 6♣

4. Two Pair : Two cards are the same rank. Another two cards are the same rank but different rank from the first two cards. The last card is a different rank from the first 4 cards.

Eg.

$2\spadesuit 2\heartsuit \quad 5\clubsuit 5\heartsuit \quad J\clubsuit$

Ans.

Choose first pair's rank and suits
e.g. $2\spadesuit 2\heartsuit$

$\begin{pmatrix} 13 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \begin{pmatrix} 12 \\ 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

choose second pair
e.g. $5\clubsuit 5\heartsuit$

We can select 2 ranks from first 4 cards. So, 2 ranks = 11 remaining ranks in 4 suites.
So, $11 \times 4 = 44$ remaining cards

$\begin{pmatrix} 44 \\ 1 \end{pmatrix} = 123,552$

2
↑
there is only one recognized global ordering.

that pesky last card

Eg. $2\spadesuit 2\heartsuit \quad 5\clubsuit 5\heartsuit \quad J\clubsuit$ is the same as

$5\clubsuit 5\heartsuit \quad 2\spadesuit 2\heartsuit \quad J\clubsuit$

5. One Pair : You have one pair. The remaining 3 cards are all different ranks (from the pair and each other).

Eg-

$2\spadesuit 2\heartsuit \quad 10\clubsuit 8\heartsuit 3\heartsuit$

Ans.

$$\left(\begin{array}{c} 13 \\ 1 \end{array}\right) \left(\begin{array}{c} 4 \\ 2 \end{array}\right) \left(\begin{array}{c} 12 \\ 3 \end{array}\right) \left(\begin{array}{c} 4 \\ 1 \end{array}\right)^3 = 1\ 098\ 240$$

Choose the pair
eg. $2\spadesuit\ 2\heartsuit$

choose the ranks and suits of remaining 3 cards
eg. $10\clubsuit\ 8\heartsuit\ 3\heartsuit$

Extra Problems

1. There are 5 flavours of ice cream: A, B, C, D, E (the person who named them was very practical). You can choose 3 scoops. How many different bowls can you make if

a) No flavour can be repeated?

$$\text{Ans. } \left(\begin{array}{c} 5 \\ 3 \end{array}\right) = 10$$

b) Flavours can be repeated?

$$\text{Ans. } \left(\begin{array}{c} 5 \\ 1 \end{array}\right)^3 = 125$$

c) Flavours can be repeated, but order doesn't matter?

Ans. relies on you knowing the formula for counting combinations with repetition.

↑
Order does not matter.

This formula is
$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

Where n is the number of options (ie. ice cream flavours) and r is how many you are allowed to choose (ie. 3 scoops).

So short answer is plug-and-play:

$$\binom{5+3-1}{3} = \binom{7}{3} = 35.$$

But if you hate random formulas like me, continue reading for the long answer.

LONG ANSWER

Since order does not matter, we can simplify things by creating a global order and only counting bowls that follow that order. Let's say our global order is

A, B, C, D, E

(Who would've guessed (6!))

Now let's put these ice creams in boxes in a row.

A	B	C	D	E
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Imagine there is a robot that moves from left to right. At each point, you can tell it to either take a scoop of the ice cream at its current location or move right.

For example, to get a bowl with flavours AAC, you tell it $\text{OO} \rightarrow \rightarrow \text{O} \rightarrow \rightarrow$

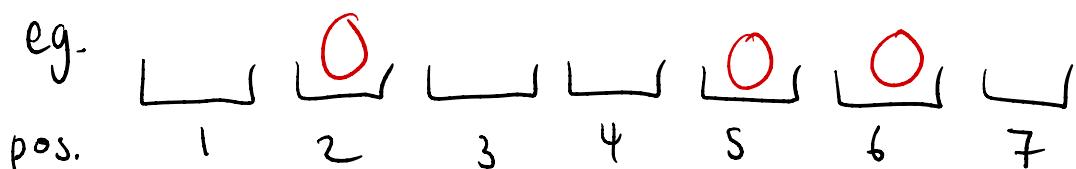
Where \textcircled{O} means scoop and \rightarrow means go to the next flavour box (move right). You also always have to leave the robot in the right most position.

Note: you can make any flavour combination this way.

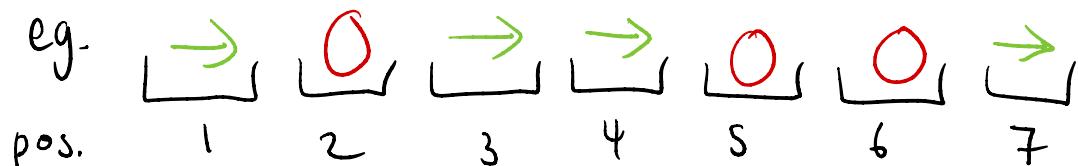
Furthermore, for any 3 scoop-bowl, you will always need 3 scoop instructions \textcircled{O} and 4 arrow instructions \rightarrow .

So the problem now becomes "how many different ways can I arrange 3 \textcircled{O} 's and 4 \rightarrow 's?"

We have 7 total positions in an instruction and we can choose 3 positions for where the \textcircled{O} 's go.



The rest will automatically get filled in with the remaining 4 \rightarrow 's.



So the answer is $\binom{7}{3} = 35$

2. I have 7 pairs of socks in my drawer, one of each colour of the rainbow. How many socks do I have to draw out in order to guarantee that I have grabbed at least one pair?

Ans. After grabbing 7 socks, in the worst case scenario, I have grabbed a sock of each colour. Thus, by the Pigeon hole principle, the 8th sock must match with one of the previous socks.

So I must draw 8 socks to guarantee a pair.