Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Use an appropriate test to determine whether the series given below converges or diverges.

$$\sum_{n=1}^{\infty} \frac{3}{n\sqrt[n]{n}}$$

To determine whether the series converges or diverges, use the Limit Comparison Test.
Suppose that $a_n > 0$ and $b_n > 0$ for all $n \geq N$ where N is an integer.

1. If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converge or both diverge.

2. If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges.

3. If $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

Let $a_n = \frac{3}{n\sqrt[n]{n}}$. First find b_n by finding the behavior of a_n for large values of n. Find $\lim\limits_{n\to\infty} \sqrt[n]{n}$.

$$\lim_{n\to\infty} \sqrt[n]{n} = 1$$

Therefore, for large values of n, a_n will behave like $\frac{3}{n(1)} = \frac{3}{n}$.

Let $b_n = \frac{3}{n}$. Now find $\lim\limits_{n\to\infty} \frac{a_n}{b_n}$.

$$\frac{a_n}{b_n} = \frac{\left(\frac{3}{n\sqrt[n]{n}}\right)}{\left(\frac{3}{n}\right)}$$

$$= \frac{3n}{3n\sqrt[n]{n}}$$

$$= \frac{1}{\sqrt[n]{n}}$$

Now find $\lim\limits_{n\to\infty} \frac{1}{\sqrt[n]{n}}$. Remember that $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$.

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$$

Therefore $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 1$. Since $1 > 0$, whether $\sum a_n$ converges or diverges depends on whether $\sum b_n$ converges or diverges, according to the Limit Comparison Test. Consider $\sum b_n = \sum\limits_{n=1}^{\infty} \frac{3}{n}$. It diverges because it is a p-series where $p = 1$.

Since $\sum b_n = \sum\limits_{n=1}^{\infty} \frac{3}{n}$ diverges, $\sum a_n = \sum\limits_{n=1}^{\infty} \frac{3}{n\sqrt[n]{n}}$ diverges as well, according to the Limit Comparison Test.