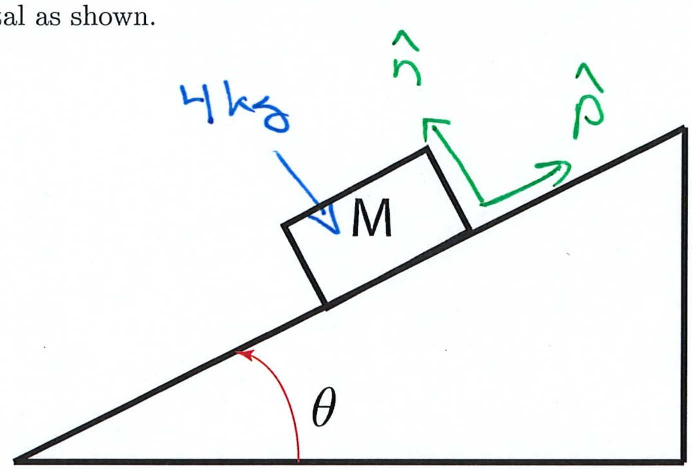
2-11-Example-TransEq III

**Translational Equilibrium - III**

A $4kg$ mass is on a rough slope. The slope makes an angle of $\theta$ with the horizontal as shown.

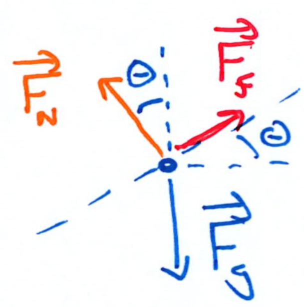



The coefficient of static friction between the mass and the surface is $\mu_s = 0.3$. The coefficient of kinetic friction between the mass and the surface is $\mu_k = 0.2$.

- For what range of angles can the mass be in static equilibrium?
- For what angle $\theta$ would the mass slide down the slope at a constant speed?

Assume equilibrium

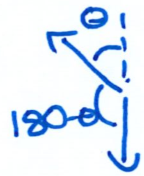
$$\vec{F}_{net} = 0$$




$$0 = \vec{F}_g + \vec{F}_f + \vec{F}_N$$

$\hat{n}$ component of $\vec{F}_{net} = 0$

$$0 = \vec{F}_g \cdot \hat{n} + \vec{F}_f \cdot \hat{n} + \vec{F}_N \cdot \hat{n}$$




$$= |\vec{F}_g| \cos 180-\theta + |\vec{F}_f| \cos 90 + |\vec{F}_N|$$

$\cos 180-\theta = -\cos\theta$


$$0 = -mg\cos\theta + 0 + |\vec{F}_N|$$

$$|\vec{F}_N| = mg\cos\theta \quad ✓ \text{①}$$

$\hat{p}$ component


$$0 = \vec{F}_g \cdot \hat{p} + \vec{F}_f \cdot \hat{p} + \vec{F}_N \cdot \hat{p}$$

$$= (mg)\cos 90+\theta + |\vec{F}_f| + |\vec{F}_N| \cos 90$$

$\cos 90+\theta = -\sin\theta$

$$|\vec{F}_f| = mg\sin\theta \quad ✓ \text{②}$$

① & ② from equilibrium

$$|\vec{F}_f| \le \mu_s |\vec{F}_N|$$

$$mg\sin\theta \le \mu_s mg\cos\theta$$

need $\dfrac{\sin\theta}{\cos\theta} \le \mu_s$ ← 0.3 $\quad \boxed{0 \le \theta \le 16.7}$

for static equilibrium

Constant speed

$\Rightarrow$ kinetic friction

$$|\vec{F}_k| = \mu_k |\vec{F}_N|$$

$$mg\sin\theta = \mu_k mg\cos\theta$$

$$\tan\theta = \mu_k$$

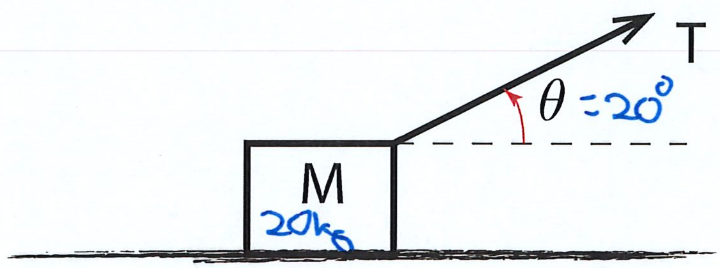
0.2

$$\theta = 11.3^\circ$$

2-12-Example-TransEqN

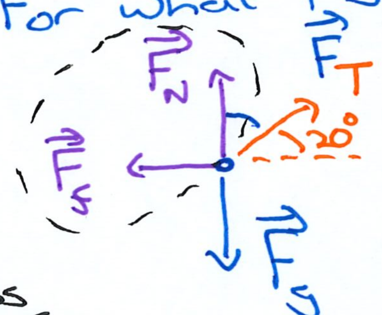
**Translational Equilibrium - IV**

A $20kg$ mass is on a rough horizontal surface. It subject to a force from a rope which pulls up and to the right at an angle $\theta = 20°$. The coefficient of static friction between the mass and the surface is $\mu_s = 0.4$, and the coefficient of kinetic friction is $\mu_k = 0.3$.




- What is the minimum tension $T$ in the rope (ie the magnitude of the force exerted by the rope) which will cause the mass to start to move?
- What tension $T$ will result in the mass moving at a constant speed along the surface.

For what T's can mass be in equilibrium




$\vec{F}$ on mass by surf.

$$0 = \vec{F}_{Net} = \vec{F}_g + \vec{F}_T + \vec{F}_s + \vec{F}_N$$

$$\vec{F}_g = -mg\hat{k}$$

$$\vec{F}_f = -|\vec{F}_f|\hat{i}$$

$$\vec{F}_N = |\vec{F}_N|\hat{k}$$

$$\vec{F}_T = T\cos\theta\,\hat{i} + T\cos(90-\theta)\,\hat{k}$$

$$= T\cos\theta\,\hat{i} + T\sin\theta\,\hat{k}$$

$$0 = \vec{F}_g + \vec{F}_f + \vec{F}_N + \vec{F}_T = -mg\hat{k} - |\vec{F}_f|\hat{i} + |\vec{F}_N|\hat{k}$$

$$+ T\cos\theta\,\hat{i} + T\sin\theta\,\hat{k}$$

x-component

$$0 = -|\vec{F}_f| + T\cos\theta$$

$$|\vec{F}_f| = T\cos\theta$$

z-component

$$0 = -mg + |\vec{F}_N| + T\sin\theta$$

$$|\vec{F}_N| = mg - T\sin\theta$$

Always get $|\vec{F}_N|$ from requirement mass not "fall through" surface

Static

$$|\vec{F}_f| \leq \mu_s |\vec{F}_N|$$

$$T\cos\theta \leq \mu_s (mg - T\sin\theta)$$

$$T\cos\theta + T\mu_s \sin\theta \leq \mu_s mg$$

$$T \leq \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

$\mu_s = 0.4$ $mg = 196N$
$\theta = 20°$

T must do this for static equilibrium

$$T \leq 72.8N$$

if $T > 72.8N$ will slip.

Kinetic friction

$$|\vec{F}_f| = \mu_k |\vec{F}_N|$$

⋮

$\mu_k = 0.3$

$$T = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

$$T = 56.4N$$

Newton's 3rd law

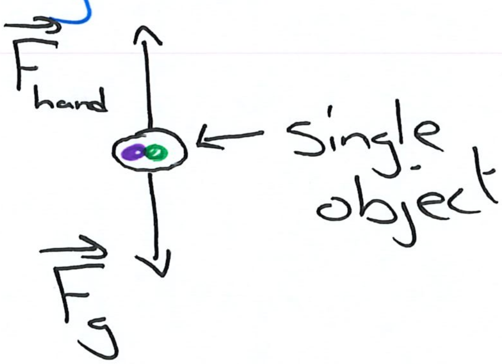
When two objects interact the force that A exerts on B is same in magnitude & opposite in direction as force B exerts on A

$$\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$$

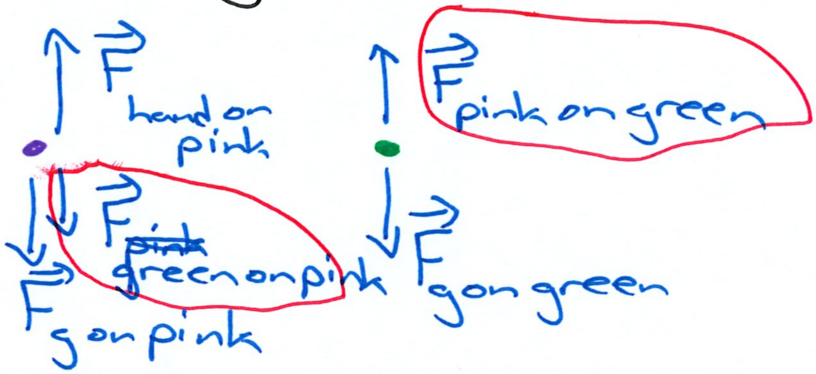
Can often analyse compound objects in static equilibrium

"Demo"

Holding thing made up of two magnets



separately

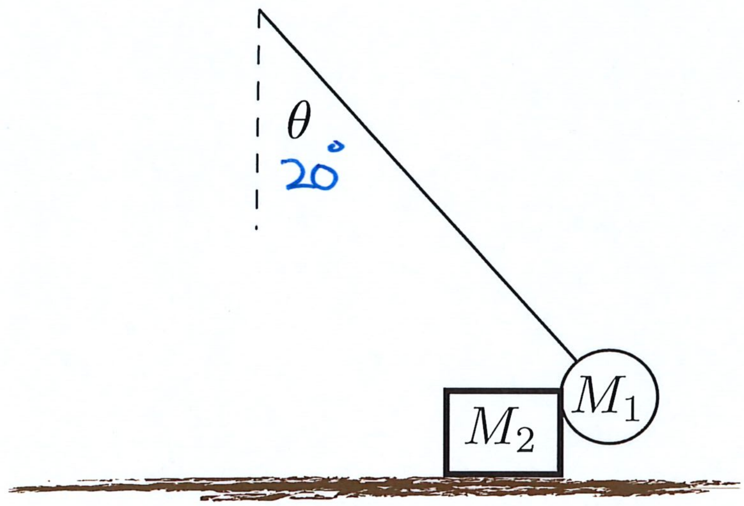



Equilibrium of total, total force on pink & green, internal forces cancel b/c of 3rd law

2-14-ExampleTransEqV

**Translational Equilibrium - V**

A ball of mass $M_1 = 3kg$ is suspended by a rope which makes an angle of $\theta = 20°$ with the vertical. It is touching a block of mass $M_2 = 5kg$ which rests on a rough horizontal surface. The coefficient of static friction between the surface and the block is $\mu_s = 0.3$.




The sides of the block are vertical, so any force between the ball and block is horizontal.

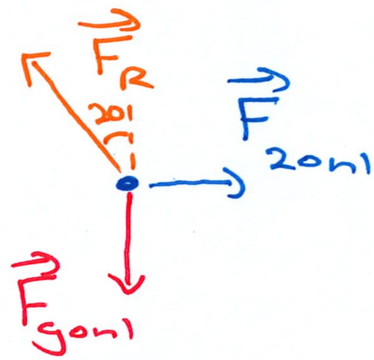
- What is the force that the ball exerts on the block?
- What is the friction force the surface exerts on the block?

Each mass in equilibrium

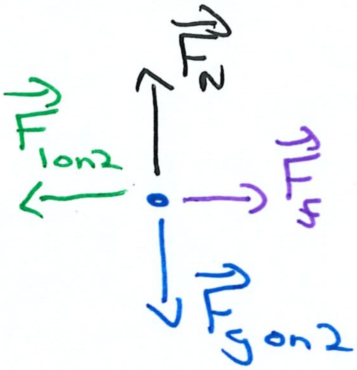
Net force on each is 0

Each mass has ~~its~~ own free-body diagram

$m_1$




$m_2$




Know $\vec{F}_{gon1}$, $\vec{F}_{gon2}$

Want $\vec{F}_{1on2}$, $\vec{F}_f$

Also don't know

$\vec{F}_R$, $\vec{F}_{2on1}$, $\vec{F}_N$

① Use $\vec{F}_{gon1}$ & angle to get $\vec{F}_R$, $\vec{F}_{2on1}$

② Once have $\vec{F}_{2on1}$ by 3rd law get $\vec{F}_{1on2}$

③ Have $\vec{F}_{1on2}$ use equilibrium of $m_2$ to get $\vec{F}_f$.

For $m_1$ know $0 = \vec{F}_{Net} = \vec{F}_R + \vec{F}_{2on1} + \vec{F}_{gon1}$

x-component

$$0 \cdot \hat{\imath} = \vec{F}_R \cdot \hat{\imath} + \vec{F}_{2on1} \cdot \hat{\imath} + \vec{F}_{gon1} \cdot \hat{\imath}$$

$$0 = |\vec{F}_R| \cos 110 + |\vec{F}_{2on1}| \cos 0 + |\vec{F}_{gon1}| \cos 90$$

$$0 = |\vec{F}_R| \cos 110 + |\vec{F}_{2on1}|$$

z - component

$$0 \cdot \hat{k} = \cancel{|\vec{F}_R|} \, \vec{F}_R \cdot \hat{k} + \vec{F}_{2on1} \cdot \hat{k} + \vec{F}_g \cdot \hat{k}$$

$$0 = |\vec{F}_R| \cos 20 + \underbrace{0}_{\text{b/c } \cos 90} + (mg)\cos 180$$

$$0 = |\vec{F}_R| \cos 20 - m_1 g \qquad m_1 = 3\,kg$$

$$|\vec{F}_R| = 31.3N$$

x-comp

$$0 = (31.3N)\cos 110 + |\vec{F}_{2on1}|$$

$$|\vec{F}_{2on1}| = 10.7N$$

Use 3rd law

$$\vec{F}_{2on1} = -\vec{F}_{1on2} \Rightarrow |\vec{F}_{2on1}| = |\vec{F}_{1on2}|$$

$$|\vec{F}_{1on2}| = 10.7N$$

direction $-x$ $\qquad \vec{F}_{1on2} = -10.7N\,\hat{i}$

Find $\vec{F}_f$ only horizontal

Know $0 = \underbrace{\vec{F}_{1on2} + \vec{F}_f}_{x\text{-components}} + \underbrace{\vec{F}_N + \vec{F}_{gon2}}_{\text{vertical}}$

$$\vec{F}_f = 10.7N\,\hat{i}$$

Note that $|\vec{F}_N|$:

z-component of net force on $m_2$

$$0 = \vec{F}_{1on2} \cdot \hat{k} + \vec{F}_s \cdot \hat{k} + \vec{F}_N \cdot \hat{k} + \vec{F}_{gon2} \cdot \hat{k}$$

$$= 0 + 0 + |\vec{F}_N| \cos 0 + m_2 g \cos 180$$

$$|\vec{F}_N| = 49N$$

$$\underset{(must)}{|\vec{F}_s| \le} \mu_s |\vec{F}_N| = 14.7N \quad \checkmark$$