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Use the Ratio Test to determine if the following series converges absolutely or diverges.

$$\sum_{n=1}^{\infty} \frac{13^n}{n!}$$

Let $a_n = \frac{13^n}{n!}$. Since a_n has both powers of 13 and n!, use the Ratio Test to determine convergence or divergence.

Let $\sum a_n$ be any series and suppose that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$. The Ratio Test states that the series converges absolutely if $\rho < 1$ and diverges if $\rho > 1$ or ρ is infinite. The test is inconclusive if $\rho = 1$.

First, determine $\left|\frac{a_{n+1}}{a_n}\right|$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{\left(13^{n+1}\right)}{(n+1)!}}{\frac{\left(13^n\right)}{n!}}\right|$$

$$= \left|\frac{13^{n+1}}{(n+1)!} \cdot \frac{n!}{13^n}\right|$$

Simplify the factorials.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{13^{n+1}}{13^n} \cdot \frac{n!}{(n+1)!}\right|$$

$$= \left|\frac{13^{n+1}}{13^n} \cdot \frac{1}{n+1}\right|$$

Determine $\frac{13^{n+1}}{13^n}$.

$$\frac{13^{n+1}}{13^n} = 13$$

Therefore, $\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{13}{n+1}\right|$.

Evaluate $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{13}{n+1}\right|$$

$$= 0$$

Therefore, $\rho = 0$. By the Ratio Test, the series $\sum_{n=1}^{\infty} \frac{13^n}{n!}$ converges absolutely.