Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Identify the coordinates of any local and absolute extreme points and inflection points. Graph the function.

$$y = 24x^3 - 3x^4 = x^3(24 - 3x)$$

To graph the function, first determine the domain of the function and any symmetries the curve may have.

The domain of the function $y = 24x^3 - 3x^4$ is $(-\infty,\infty)$.
(Type your answer in interval notation.)

The graph of the function has no symmetry.

Find the derivatives y' and y''. First find y'.

$$y = 24x^3 - 3x^4$$
$$y' = 72x^2 - 12x^3$$

Find y''.

$$y' = 72x^2 - 12x^3$$
$$y'' = 144x - 36x^2$$

Next, find the critical point(s) of $y = f(x)$ by solving $y' = 0$.

Solve $y' = 0$ for x.

$$\begin{aligned} y' &= 0 \\ 72x^2 - 12x^3 &= 0 \\ 12x^2(6 - x) &= 0 \\ x &= 0,6 \end{aligned}$$

(Use a comma to separate answers as needed.)

Since y' exists over the domain of y, the critical points are only at $x = 0$ and $x = 6$.

To determine the behavior at the critical points, use the Second Derivative Test for Local Extrema to determine whether any local extrema occur at the critical points.

If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
If $f'(c) = 0$ and $f''(c) = 0$ or $f''(c)$ fails to exist, then the function f may have a local maximum, a local minimum, or neither at $x = c$.

Determine the behavior of the function at the critical points. Choose the correct answer below.

- **A.** y has a local maximum at $x = 6$, and a local minimum at $x = 0$.
- **B.** y has a local maximum at $x = 6$, and may have a local maximum, a local minimum, or neither at $x = 0$.
- **C.** y has a local minimum at $x = 6$, and a local maximum at $x = 0$.
- **D.** y has a local minimum at $x = 6$, and may have a local maximum, a local minimum, or neither at $x = 0$.

Find where the curve is increasing and where it is decreasing. The critical points subdivide the domain of $y = 24x^3 - 3x^4$ to create nonoverlapping open intervals on which y' is either positive or negative. Determine the sign of y' over these intervals.

Interval	$x<0$	$0<x<6$	$6<x$
Sign of y'	+	+	−

If $y'>0$ at any point in an open interval, then the curve is increasing on that interval. If $y'<0$ at any point in an open interval, then the curve is decreasing on that interval. Determine the behavior of the curve.

Interval	$x<0$	$0<x<6$	$6<x$
Sign of y'	+	+	−
Behavior of the curve	increasing	increasing	decreasing

At a point of inflection, either y'' is 0 or y'' fails to exist. Since the domain of y'' is $(-\infty,\infty)$, there are no values of x where y'' does not exist. Find any potential inflection points by setting the second derivative equal to 0, and solve for x.

$$y''=0$$
$$36x(4-x)=0$$
$$x = 0,4$$ Solve for x.

(Use a comma to separate answers as needed.)

The inflection points are at $x=0$ and $x=4$. Use these points to define the intervals where the curve is concave up or concave down. Determine the sign of y'' over these intervals.

Interval	$x<0$	$0<x<4$	$4<x$
Sign of y''	−	+	−

If $y''>0$ at any point in an open interval, then the curve is concave up on that interval. If $y''<0$ at any point in an open interval, then the curve is concave down on that interval. Determine the concavity of the curve.

Interval	$x<0$	$0<x<4$	$4<x$
Sign of y''	−	+	−
Concavity of the curve	down	up	down

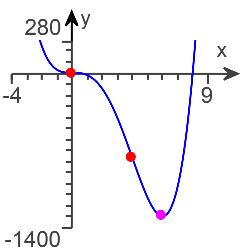
Therefore, the inflection point(s) is/are (4,768),(0,0) .
(Use a comma to separate answers as needed. Type an ordered pair. Do not use commas in the individual coordinates.)

Determine the asymptotes of the given function.

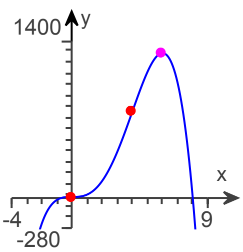
- **A.** There is an asymptote at $y=0$.
- **B.** There are asymptotes at $y=0$ and $x=6$.
- **C.** There is an asymptote at $x=6$.
- **D.** There are no asymptotes.

Choose the correct graph of $y=24x^3-3x^4$.

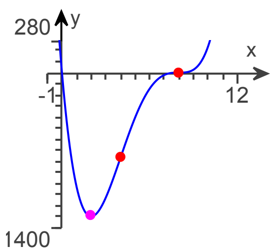
A.



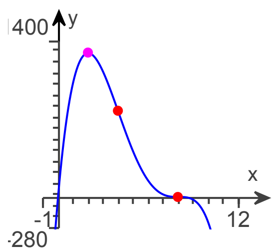
B.



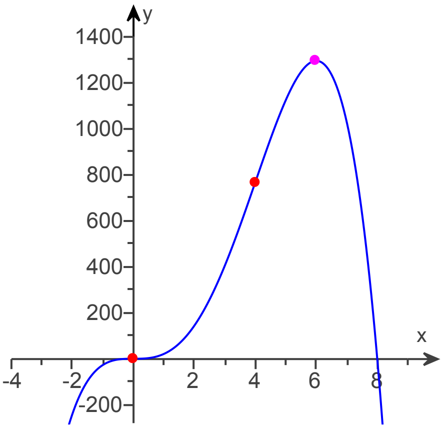
C.



D.



Therefore, the graph of $y = 24x^3 - 3x^4$ is as shown to the right.



YOU ANSWERED: extrema

(0,0),(4,0)