MATHEMATICS 101 [S02]
Term Exam 3
Spring 1998

Name: _____ Student Number: _____

Instructor: Bob Harrison Duration: 50 minutes

THIS QUESTION PAPER HAS **6 PAGES** plus COVER SHEET.

INSTRUCTIONS:

1. **Make sure you have a complete paper, then put your name in 2 places:**
   (a) on the top of this page,
   (b) on the back of the exam, at the top of the page.
2. You may use a basic scientific calculator; you may not use a graphing calculator or a programmable calculator or any calculator or device that can store text or formulas. No other aids such as books, notes, or formula sheets are allowed.
3. Do all your work on the test pages, using the backs of pages if necessary.
4. Problems 1 through 6 are multiple-choice questions.
   For questions requiring numerical answers, the choices are listed in numerically increasing order. Choose the answer *nearest* to your result. *If* the two nearest answers are exactly equidistant from your result, choose the larger of these two answers.
   There is no penalty for wrong answers. In fact, for Term Exams, if your answer is wrong but you have *shown your method*, I'll consider part marks.
   Note. On the Final Exam, there are no part marks for multiple-choice questions.
5. Problems 7 through 9 are full-answer questions.
   Your solutions must show your method and all of your steps. Unjustified or incompletely justified final answers, though they may be correct, may be assigned 0 marks.

PAGE	VALUE	MARK
1	10	
2	10	
3	10	
4	6	
5	7	
6	7	
TOTAL	50	

[5] 1. Find the sum of the series $\sum_{n=0}^{\infty} \frac{2^n + 3^n}{5^n}$. Circle the answer nearest to your result.

(A) Diverges (B) 0 (C) 1 (D) 2 (E) 3
(F) 4 (G) 5 (H) 6 (I) 7 (J) 8

[5] 2. Use the definition of the sum of a series to find the sum of the series $\sum_{n=4}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right)$. Circle the answer nearest to your result.

(A) Diverges (B) 0.0 (C) 0.1 (D) 0.2 (E) 0.3
(F) 0.4 (G) 0.5 (H) 0.6 (I) 0.7 (J) 0.8

[5] 3. Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{n}{10^n} x^{2n}$.

Circle the answer nearest to your result.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
(F) 5 (G) 6 (H) 7 (I) 8 (J) ∞

[5] 4. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{2^n n!}{(2n)!} (x-3)^n$.

Circle the answer nearest to your result.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
(F) 5 (G) 6 (H) 7 (I) 8 (J) ∞

[5] 5. Let $f(x) = \sum_{n=2}^{\infty} \frac{n^2 - 1}{(n+2)!} x^n$. Find the value of $f^{(3)}(0)$. Circle the answer nearest to your result.

(A) 0.0 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4
(F) 0.5 (G) 0.6 (H) 0.7 (I) 0.8 (J) 0.9

[5] 6. Find the coefficient of x^{10} in the power series representation of $f(x) = x\,e^{2x^3}$. Circle the answer nearest to your result.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4
(F) 5 (G) 6 (H) 7 (I) 8 (J) 9

[2] 7. (a) *State* the definition of $P_n(x)$, the nth-degree Taylor polynomial of $f(x)$ at $x = a$.

[4] (b) Find $P_4(x)$, the 4th-degree Taylor polynomial of $f(x) = \sin x$ at $x = \dfrac{\pi}{2}$.

[2] 8. (a) *State* the power series representation of $\frac{1}{1-x}$ and *state* its interval of convergence.

[5] (b) Find the power series representation of $f(x) = \frac{1}{1-3x} + \frac{1}{1+2x}$ and determine its open interval of convergence.

[2] 9. (a) *State* the power series representation of $\cos(x)$ and *state* its interval of convergence.

[5] (b) Find the power series representation of $f(x) = \int_0^x \cos(t^2)\,dt$ and determine its open interval of convergence.