MATHEMATICS 101 (Sections A01-A05),
Midterm # 3, March 21-22, 2018.
Time: 50 minutes

Last name: [illegible] StudentID: V00[illegible]

First name: [illegible] Lecture section number: A04

Problem #	1 − 2	3 − 4	5	6	7	8	9	10	TOTAL
Points (max)	2	2	2	2	2	2	2	2	16
Score									

- Cell phones, books and cheat notes are NOT allowed on this test.
- Only calculators allowed are Sharp EL-510R, RN or RNB
- This test consists of 10 questions and has 10 pages (including this cover, the **Blank page** and a **Formula sheet** on the last page).
  - Questions 1 through 10 are multiple-choice questions. Write your full answer in this booklet in the provided space. **Clearly mark your final answer among the multiple choises.** You need to show your work for all answers, as we may disallow any answer which is not properly justified.
  - All questions 1 through 10 are full marks only, no partial marks.
- Before starting your test enter your Name (Last, First), student ID, and lecture section number (A01 - A05) on this page.
- If you have finished working on your paper with less than 15 minutes before the end of the examination, please close your paper and **remain seated** until the test time is completed. It is important to minimize the disruptions in the room.
- At the end of the 50-minute test, turn-in this booklet and your fully completed bubble sheet.
- This is version C of Midterm #3. Bubble in "Form" **C**, leave "Special" portion empty.

$(1+i)^3 = (1+i)(1+i)(1+i)$

$\cancel{1}+2i+\cancel{i^2}$

$2i(1+i)$

$2i+2i^2$ $\quad \frac{2i-2}{1+2i} = 2+ ?$

1. (1 point) Calculate $\dfrac{(1+i)^3}{1+2i}$.

(A) $0.4+1.2i$ (B) $0.4-1.2i$ (C) $-0.4+1.2i$ (D) $-0.4-1.2i$

(E) $2+6i$ (F) $2-6i$ (G) $-6-2i$ (H) None of those

$(1+i)(1+i)(1+i)$

$1+2i+i^2\,(1+i)$

$1+2i+i^2+i+2i^2+i^3$

$$\frac{1+3i+\overset{-3}{\cancel{3i^2}}+i^3}{1+2i} = \frac{2+3i-i}{}$$

$\dfrac{2+2i}{1+2i}$

$\dfrac{(1+i)^3}{1+2i}$

$(1+i)(1+i)$

$(1+2i+\underbrace{i^2}_{=-1})(1+i)$

$2i(1+i)$

$2i+i^2$

$\dfrac{2i-1}{1+2i} = -1?+i?$

2. (1 point) Compute $\displaystyle\sum_{k=1}^{\infty}\frac{2^k+3^k}{4^k}$, if the series converges.

(A) 3.2 (B) 3.4 (C) 3.6 (D) The series diverges

(E) 3.8 (F) 4.0 (G) 4.2 (H) None of those

$\lim\limits_{k\to\infty}\left(\dfrac{2+3}{4}\right)^k = \left(\dfrac{5}{4}\right)^k$

geometric series with $r=\frac{5}{4}>1$

= diverges

$(2^{\frac{1}{2}})^{\frac{1}{5}} = 2^{\frac{1}{10}}$ $\qquad \sqrt[5]{\sqrt{2}+\sqrt{2}i}$

For next two questions, consider the equation $z^5 = \sqrt{2} + \sqrt{2}i$.

3. (1 point) Find the radius r for z and the number n of distinct roots for z. $\sqrt[9]{2}$

(A) $r = 1$ and $n = 5$ (B) $r = \sqrt[5]{2}$ and $n = 5$ (C) $r = 1$ and $n = 2$

(D) $r = \sqrt[10]{2}$ and $n = 5$ (E) $r = \sqrt[5]{4}$ and $n = 5$ (F) $r = \sqrt[2]{2}$ and $n = 2$

(G) None of those

$z^5 = \sqrt{2} + \sqrt{2i}$

$\sqrt{\sqrt[5]{2} + \sqrt[5]{2}}$

$\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = z^5$

$\sqrt[5]{2\sqrt[5]{2}}$

$\sqrt[10]{2}$

$\sqrt{2+2} = \sqrt{4}$

hm...

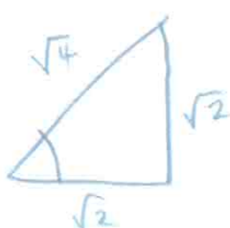
$r = \sqrt{\sqrt{2}^2 + \sqrt{2}^2}$

$r = \sqrt{4}$
of z^5

4. (1 point) Find an angle that corresponds to one of the root for z.

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{5}$ (D) $\frac{\pi}{10}$

(E) $\frac{\pi}{20}$ (F) $\frac{\pi}{2}$ (G) π (H) None of those



yay a $\frac{\pi}{4} - \frac{\pi}{4}$
45-45
triangle!

5. (5 points) Evaluate the limit of the sequence $\{a_n\}_{n=1}^{\infty}$, if it exists, where $a_n = n\sin\left(\frac{1}{bn}\right)$, $b > 2$.

(A) $\frac{1}{b}$ (B) $\sin(b)$ (C) 1 (D) The sequence diverges

(E) b (F) $\sin\left(\frac{1}{b}\right)$ (G) 0 (H) None of those

$\{a_n\}_{n=1}^{\infty}$ $a_n = n\sin\left(\frac{1}{bn}\right)$, $b>2$

$\lim_{n\to\infty} n\sin\left(\frac{1}{bn}\right)$ $n\sin(\cancel{x})$
$\to 1$

6. (2 points) A sequence is defined recursively by $a_1 = -3$, $a_{n+1} = \dfrac{a_n + 3}{a_n + 1}$, $n \geq 1$. Assume that the sequence converges. Compute the limit of this sequence.

(A) -3 (B) -2 (C) $-\frac{1}{2}$ (D) 0

(E) $\frac{1}{2}$ (F) 2 (G) 3 (H) None of those

well uh

ok

$a_1 = -3$, $a_{n+1} = \dfrac{a_n + 3}{a_n + 1}$

-3 $\dfrac{-3+3}{-3+1}$ $\dfrac{0+3}{0+1}$ $\dfrac{3+3}{3+1} = \dfrac{3}{4}$

$\dfrac{-0}{-2}$ $= \dfrac{3}{1}$

$n=2$ $= 3$

$a_3 =$

$\dfrac{\frac{3}{4}+3}{\frac{3}{4}+1} = \dfrac{\frac{15}{4}}{\frac{7}{4}}$

$\frac{60}{28} = \frac{15}{7}$

7 (2 point) Set up the arc length equation for the parametric curve:

$$x = a\cos(t), y = b\sin(t), 0 \le t \le \frac{\pi}{2}.$$

$AL: \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$

(A) $\int_0^{\frac{\pi}{2}} \sqrt{a^2\cos^2(t) + b^2\sin^2(t)}\, dt$

(B) $\int_0^{\frac{\pi}{2}} \sqrt{a^2\sin^2(t) + b^2\cos^2(t)}\, dt$

(C) $\int_0^{\frac{\pi}{2}} \sqrt{a\cos(t) + b\sin(t)}\, dt$

(D) $\int_0^{\frac{\pi}{2}} \sqrt{-a\sin(t) + b\cos(t)}\, dt$

(E) $\int_0^{\frac{\pi}{2}} \sqrt{a^2 + b^2}\, dt$

(F) None of those

$x = a\cos(t)$ $\quad y = b\sin(t)$ $\quad 0 \le t \le \frac{\pi}{2}$

$\frac{dx}{dt} = -a\sin(t)$ $\quad \frac{dy}{dt} = b\cos(t)$

$a^2\sin^2(t)$ $\quad b^2\cos^2(t)$

8. (2 points) Calculate slope of the tangent line to the curve $r = 1 - \cos\theta$ at the point on the curve $(r, \theta) = \left(1, \frac{\pi}{2}\right)$.

(A) $-\frac{1}{3}$ (B) $-\frac{1}{2}$ (C) -1 (D) 0 (E) Tangent line is vertical

(F) $\frac{1}{3}$ (G) $\frac{1}{2}$ (H) 1 (I) 2 (J) None of those

$r = 1$

$\theta = \frac{\pi}{2}$

$r = 1 - \cos\theta$

$dr = \sin\theta$

$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{dy}{dx}$

$= \frac{r(\theta)\sin\theta}{r(\theta)\cos\theta}$

$= \frac{r'(\theta)\sin\theta + r(\theta)\cos\theta}{r'(\theta)\cos\theta - r(\theta)\sin\theta}$

$\frac{\sin^2\theta + (1-\cos\theta)\cos\theta}{\sin\theta\cos\theta + (1-\cos\theta)\sin\theta}$

$= \frac{\sin^2\theta + 1 - \cos^2\theta}{\cancel{\sin\theta\cos\theta} - \sin\theta + \cancel{\sin\theta\cos\theta}}$

$= \frac{2\sin^2\theta}{\sin(\theta)} = \frac{4}{1} = 4$

$(\sin(\theta))\sin(\theta) + (1-\cos\theta)\cos\theta$

$= \frac{\sin^2\theta + \cos\theta - \cos^2\theta}{\cos(\theta)\sin(\theta) - (1-\cos(\theta))\sin\theta}$

$\frac{\sin^2\theta + \cos\theta - \cos^2\theta}{\sin(\theta)}$

$\frac{1 + 0 - 0}{1} = 1$

10. (2 points) Determine whether or not $\sum_{n=1}^{\infty}\left(\frac{e^n - n^e}{e^n}\right)$ converges, giving appropriate (and correct) justification.

(A) Converges, by the n-th Terms test.

(B) Diverges, by the n-th Terms test.

(C) Converges, by a telescoping sum argument.

(D) Diverges, by a telescoping sum argument.

(E) Converges, by the Integral Test.

(F) Diverges, by the Integral Test.

(G) Diverges, since it is a harmonic series.

(H) None of those

$\sum_{n=1}^{\infty}\left(\frac{e^n - n^e}{e^n}\right)$

$\lim\left(\left(\frac{e^n}{e^n}\right) - \frac{n^e}{e^n}\right)$

$\lim_{n\to\infty}\left(1 - \frac{n^e}{e^n}\right)$

$= 1 - \frac{n^e}{e^n} = \frac{\infty}{\infty}$

$1 - \cancel{\frac{en^{e-1}}{e^n}} = 0?$

$\int_1^{\infty} \frac{e^n - n^e}{e^n}$

$\int (1)^n - \int \frac{n^e}{e^n}$

$= n - \frac{n^{e+1}}{}$

$\left(\frac{e^n - n^e}{e^n}\right)$ $\left(\frac{e-1}{e}\right) + \left(\frac{e^2 - 2^e}{e^2}\right)$

$= 1 - \frac{1}{e} + 1 - 2^e +$

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That felt too easy
Did I fail?
or am I actually decent?

MATHEMATICS 101 (Sections A01-A05)
Formula sheet, Spring 2018
Midterms and Final examinations.

Table of Integrals

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, (u < a)$

2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, (u > a)$

4. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, (a > 0)$

5. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, (u > a > 0)$

6. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C, (a > u > 0)$

8. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C, (u > 0)$

9. $\displaystyle\int \sec u \, du = \ln|\sec u + \tan u| + C$

10. $\displaystyle\int \csc u \, du = -\ln|\csc u + \cot u| + C$

Trigonmetric and Hyperbolic Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$
2. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
3. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
4. $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
5. $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
6. $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
7. $\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$
8. $\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$
9. $\cosh^2(x) - \sinh^2(x) = 1$
10. $\sinh(2x) = 2\sinh(x)\cosh(x)$
11. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
12. $\text{sech}^{-1}(x) = \cosh^{-1}\left(\frac{1}{x}\right)$
13. $\text{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$
14. $\coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$