CSC 225 Lab Notes

CSC 225 Lab 1

Counting and discrete mathematics

Reminder:

A standard deck of cards has 52 cards and 4 suits: spades, hearts, clubs, and diamonds.
Each suit has 13 cards:

Ace, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

Note: The above ordering is generally accepted, going from cards worth the least amount of points to most amount of points. Exception: Ace can be both least and highest.

$\boxed{\text{Card hands}}$

1. Royal flush consists of 10, J, Q, K, A exactly and must all be of the same suit.

   Ans: since there are only 4 suits, there are only 4 ways to make a Royal Flush.

2. A straight flush is 5 cards in consecutive order (with respect to the global order previously mentioned) (ace both low and high).
   All cards must be the same suit.
   A straight flush can start with:

   A, 2, 3, 4, 5, 6, 7, 8, 9, $\boxed{\text{or } 10}$

   but notice straight flushes that start with 10 are 10, J, Q, K, A and thus are classified as royal flushes instead.

   So really, a straight flush can start with

   A, 2, 3, 4, 5, 6, 7, 8, or 9

So there are 9 different straight flushes per suit, and 4 suits yielding $9*4 = 36$ ways to make a straight flush.

EXAMPLES

1. Flush. A flush is all five cards have to be the same suit (excluding Royal Flushes and Straight Flushes).

$$\binom{4}{1}\binom{13}{5} = 4 \cdot 1287 = 5148$$

Choose 1 of the 4 suits

Out of 13 cards in 1 suit, Choose 5

but this number includes Royal + Straight flushes!

Ans. $\binom{4}{1}\binom{13}{5}$ − (# royal + straight flushes)

$$= 5148 - (4 + 36)$$

$$= 5148 - 40 = 5108$$

2. Straight: All 5 cards in consecutive order but cannot all be the same suit.

Thought: if all 5 cards were the same suit, then our hand would be either a Royal or Straight flush.
So we can just subtract 40 from all possible straights!

Ans.

$$\binom{10}{1} 4^5 - 40 = 10\,200$$

10 ways to start a straight (ie. 10 different straights)

All 5 cards can be any suit

Subtract Royal and straight flushes from the total

3. Three-of-a-kind : Three cards are the same rank and the other two are different from the three and each other.

Ex.

3♠ 3♥ 3♦ 2♥ 6♣

Ans.

pick 2 rank from remaining 12 ranks eg. 2 and 6

$$\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1} = 54\,912$$

pick the rank of the triplet eg. 3

Pick the 3 suits of the triplet eg. ♠♥♦

pick suit for 4th card eg. 2♥

pick suit for last card eg. 6♣

4. Two Pair : Two cards are the same rank. Another two cards are the same rank but different rank from the first two cards. The last card is a different rank from the first 4 cards.

Eg. 2♦ 2♥ 5♠ 5♦ J♣

Ans.

$$\frac{\boxed{\binom{13}{1}\binom{4}{2}}\boxed{\binom{12}{1}\binom{4}{2}}}{2}\binom{44}{1} = 123,552$$

Choose first pair's rank and suits eg. 2♦ 2♥

choose second pair eg. 5♠ 5♦

we cant select 2 ranks from first 4 cards. so, 2 ranks = 11 remaining ranks in 4 suites. So, 11 x 4 = 44 remaining cards.

that pesky last card

there is only one recognized global ordering.

eg. 2♦ 2♥ 5♠ 5♦ J♣ is the same as 5♠ 5♦ 2♦ 2♥ J♣

5. One Pair : You have one pair. The remaining 3 cards are all different ranks (from the pair and each other).

eg. 2♦ 2♥ 10♠ 8♥ 3♥

Ans.

$$\underbrace{\binom{13}{1}\binom{4}{2}}_{\text{Choose the pair}}\underbrace{\binom{12}{3}\binom{4}{1}^3}_{\text{choose the ranks and suits of remaining 3 cards}} = 1\ 098\ 240$$

eg. 2♦ 2♥

eg. 10♠ 8♥ 3♥

Extra Problems

1. There are 5 flavours of ice cream: A, B, C, D, E (the person who named them was very practical. You can choose 3 scoops. How many different bowls can you make if

   a) No flavour can be repeated?

   Ans. $\binom{5}{3} = 10$

   b) Flavours can be repeated?

   Ans. $\binom{5}{1}^3 = 125$

   c) Flavours can be repeated, but order doesn't matter?

   Ans. relies on you knowing the formula for counting combinations with repetition.

   order does not matter.

This formula is $\binom{n+r-1}{r} = \dfrac{(n+r-1)!}{r!\,(n-1)!}$

where n is the number of options (ie. ice cream flavours) and r is how many you are allowed to choose (ie. 3 scoops).

So short answer is plug-and-play:

$$\binom{5+3-1}{3} = \binom{7}{3} = 35.$$

But if you hate random formulas like me, continue reading for the long answer.

LONG ANSWER

Since order does not matter, we can simplify things by creating a global order and only counting bowls that follow that order. Let's say our global order is

A, B, C, D, E

(who would've guessed lol)

Now let's put these ice creams in boxes in a row.

A	B	C	D	E

Imagine there is a robot that moves from left to right. At each point, you can tell it to either take a scoop of the ice cream at its current location or move right.

For example, to get a bowl with flavours A A C, you tell it O O → → O → →

where O means scoop and → means go to the next flavour box (move right). You also always have to leave the robot in the right most position.

Note: you can make any flavour combination this way. Furthermore, for any 3 scoop-bowl, you will always need 3 scoop instructions O and 4 arrow instructions →

So the problem now becomes "how many different ways can I arrange 3 O's and 4 →'s?"

We have 7 total positions in an instruction and we can choose 3 positions for where the O's go.

eg.

pos.	1	2	3	4	5	6	7
		O			O	O	

The rest will automatically get filled in with the remaining 4 →'s.

eg.

pos.	1	2	3	4	5	6	7
	→	O	→	→	O	O	→

So the answer is $\binom{7}{3} = 35$

2. I have 7 pairs of socks in my drawer, one of each colour of the rainbow. How many socks do I have to draw out in order to guarantee that I have grabbed at least one pair?

Ans. After grabbing 7 socks, in the worst case scenario, I have grabbed a sock of each colour. Thus, by the Pigeon hole principle, the 8th sock must match with one of the previous socks.

So I must draw 8 socks to guarantee a pair.