What you need to know for the PHYS 110 final exam:

This material is grouped by "themes" rather than particular lectures, and should serve as a basis for your review. If you can do all of this, you'll be fine.

1. General Mathematical Skills:
    a. Algebra:
        i. Be able to work through problems symbolically, rather than substituting numbers in at first.
        ii. Be able to take two equations with two unknowns and solve for the unknowns.
    b. Vectors:
        i. Addition/subtraction
        ii. How to multiply by scalars
        iii. How to find lengths
        iv. Find angles between
        v. Express vectors in components given magnitude and direction
        vi. Cross product
        vii. Dot product
    c. Calculus:
        i. Be able to integrate or differentiate polynomials.
        ii. Be able to identify when it's appropriate to do integration or differentiation.
        iii. Be able to differentiate sin(ax), cos(ax), e^{ax}, ln(x).
    d. Linear algebra:
        i. Given a system of two, three, or four linear equations, solve for the unknown quantities. (This comes up in situations like Atwood machines and circuits)
2. Forces. Know how to find the magnitude and direction of:
    a. Normal force (always at 90 degrees to surface)
    b. Kinetic friction $|\vec{F}_k| = \mu_k |\vec{n}|$; direction opposes motion.
    c. Static friction $|\vec{F}_s| \leq \mu_s |\vec{n}|$; direction opposes potential motion, can be less than the maximum.
    d. Springs $|\vec{F}_{spring}| = k|\overrightarrow{\Delta x}|$. It's a restoring force, so the spring wants to return to equilibrium length.
    e. Gravity (near Earth's surface) $\vec{F}_g = -mg\hat{z}$.
    f. Gravity (Newtonian) $|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$, direction is along the vector between the two centers of mass. Always attractive.
    g. Electric force $|\vec{F}_e| = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$, direction along the vector between the two charges. Attractive if the signs are opposite, repulsive if the signs same.
    h. Electric force due to an electric field $\vec{F}_e = q\vec{E}$

      i. Force on a moving charged particle in a magnetic field $\vec{F}_B = q\vec{v}\times\vec{B}$. This is also called the Lorentz force.

3. Statics and Dynamics:
   a. Newton's laws:
      i. Equilibrium implies that the net force is 0
      ii. $\vec{F}_{net} = m\vec{a}$
      iii. $\vec{F}_{12} = -\vec{F}_{21}$; interacting objects exert forces of equal magnitudes in opposite directions on each other.
   b. Rotational equilibrium that's also static requires that the net torque is zero, as well as the net force is zero.
      i. $\vec{\tau} = \vec{r}\times\vec{F}$ where $\vec{r}$ is the vector from the origin to where the force is applied. If the net torque is zero, you can choose the origin wherever you like.
   c. Velocity is the time rate of change of position: $\vec{v} = \frac{d}{dt}\vec{x}$
   d. Acceleration is the time rate of change of velocity: $\vec{a} = \frac{d}{dt}\vec{v} = \frac{d^2}{dt^2}\vec{x}$
   e. For constant acceleration there are useful relationships between position, velocity, displacement, and time:
      i. $\vec{x}(t) = \vec{x}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$
      ii. $\vec{v}(t) = \vec{v}_0 + \vec{a}t$
      iii. $2\vec{a}\cdot\Delta\vec{x} = |\vec{v}(t)|^2 - |\vec{v}_0|^2$
   f. An object moving in a circle at constant speed v must be subject to a net force in toward the center of the circle.
      i. This force is, in magnitude, $|\vec{F}_{net}| = m\frac{v^2}{r}$
      ii. The vector describing the velocity is perpendicular to the vector from the center of the circle to the object.
      iii. The magnitude of the velocity of this object moving in a circle is $|\vec{v}| = \omega r$, ω is the "angular speed"
      iv. The *NET* force supplies the centripetal acceleration. There is no separate "centripetal force".
   g. An object subject to a net force will:
      i. Change its speed if there is a component of the net force along the direction of motion.
      ii. Change its direction if there is a component of the net force perpendicular to the direction of motion.

4. Work and Energy
   a. A force exerted over a displacement does work
      i. For a constant force, $W = \vec{F}\cdot\Delta\vec{x}$
      ii. For a variable force (like springs, electric force, gravitation) $W = \int_{begin}^{end}\vec{F}\cdot d\vec{x}$ along the path taken.
   b. The Work-Energy theorem says that the net work done changes the kinetic energy: $W_{net} = \Delta KE$, with kinetic energy for a point particle given by $KE = \frac{1}{2}m\,|\vec{v}|^2$.

c. Forces can be either conservative or non-conservative
   i. For conservative forces the work done in a given displacement does not depend on the path taken. Key examples include gravity, electric force, and springs.
   ii. For non-conservative forces the work done by the force does depend on the path taken. Key example: friction.

d. Potential energy is defined as $\Delta PE = -W_c$: the work done by a conservative force is negative the change in potential energy. Note that this doesn't tell us anything about the *absolute* value of potential energy. This is usually done by convention.
   i. Gravity near the Earth's surface has $PE = mgz$ where z is the distance above the surface (0 at ground level)
   ii. Newtonian gravity $PE = -G\frac{m_1 m_2}{r}$. This is 0 at $r = \infty$.
   iii. Electric force has $PE = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$. This is 0 at $r = \infty$; note that it is positive at smaller r if the charges are the same sign (the force is repulsive) and negative (like Newtonian gravity) if the charges are opposite signs (the force is attractive).
   iv. Electric potential energy in terms of "voltage" (electric potential) $\Delta PE = q\Delta V$
   v. Springs $PE = \frac{1}{2}k|\Delta\vec{x}|^2$. This is 0 when the spring is neither stretched or compressed from equilibrium.

e. The work-energy theorem can be restated as $W_{NC} = \Delta PE + \Delta KE$. This is very useful in cases where we don't need to know the position as a function of time, and in cases where the force varies in time and space.

f. Power is the time rate of doing work. We can find the instantaneous power used by differentiating the relationship for work. $P = \frac{d}{dt}W = \frac{d}{dt}\vec{F}\cdot\vec{x} = \vec{F}\cdot\vec{v}$. Also, $P_{avg} = \frac{W}{\Delta t}$.

5. Momentum
   a. We define the momentum for a point particle as $\vec{p} = m\vec{v}$.
   b. Net forces change momentum (this is Newton's 2nd law)
      i. $\frac{d}{dt}\vec{p} = \vec{F}_{net}$
      ii. $\Delta\vec{p} = \vec{F}_{average}\Delta t$. We can use this to define an average force on an object.
   c. In a collision (or any interaction) momentum is *always* conserved. This means that the total momentum before and after the collision is always the same. This is true as a *vector*.
   d. For elastic collisions (and *only* elastic collisions) the Kinetic Energy before and after the collisions are the same.

6. Rotation and angular momentum
   a. For any object we can define "angular momentum" as $\vec{L} = \vec{r}\times\vec{p}$.
   b. If there is a net torque, angular momentum changes: $\vec{\tau}_{net} = \frac{d}{dt}\vec{L}$

c. If there is no net torque, angular momentum stays constant.
d. For a rigid rotating object, the magnitude of angular $|\vec{L}| = I\omega$, where I is the moment of inertia (a quantity that measures how "spread out" the mass of the object is; if it's more spread out, I is bigger). For this rotating object, the kinetic energy associated with rotation is $KE = \frac{1}{2}I\omega^2$.

7. Electric current and circuits
   a. A difference in potential will make charged particles move – this motion of charges is called current.
   b. The amount of current going through a resistive element is given by Ohm's law: $\Delta V = IR$. The quantity R is called the "resistance" and depends on the material and its size and shape. The current I is positive in the direction that positive charged particles would move and negative in the other direction.
   c. The rate of energy dissipation in a resistor (the power output) is given by $P = I^2R = \frac{(\Delta V)^2}{R}$.
   d. Circuits with multiple resistors can be simplified using equivalent resistances
      i. $R_{eq} = R_1 + R_2$. (Series)
      ii. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ (Parallel)
   e. Complicated circuits can be analyzed using Kirchoff's laws:
      i. The total voltage change around any loop is 0
      ii. The current into a particular point and the current out of that same point are equal.
   f. Kirchoff's laws give a set of equations which can be solved for the currents through a number of resistors; these are typically linear equations.