Symbolab

Solution

$\sum_{n=1}^{\infty}x^{n}n^{n}$: Radius of convergence is 0, Interval of convergence is $x=0$

Steps

$\sum_{n=1}^{\infty}x^{n}n^{n}$

Use the Root Test to compute the convergence interval

Hide Steps

$\sum_{n=1}^{\infty}x^{n}n^{n}$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|\left(x^{n}n^{n}\right)^{\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\left(x^{n}n^{n}\right)^{\frac{1}{n}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\left(x^{n}n^{n}\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(x^{n}n^{n}\right)^{\frac{1}{n}}$: nx

Hide Steps

$\left(x^{n}n^{n}\right)^{\frac{1}{n}}$

Use the following exponent property: $(a\cdot b)^{n}=a^{n}\cdot b^{n}$

$\left(x^{n}n^{n}\right)^{\frac{1}{n}}=\left(x^{n}\right)^{\frac{1}{n}}\left(n^{n}\right)^{\frac{1}{n}}$

$=\left(x^{n}\right)^{\frac{1}{n}}\left(n^{n}\right)^{\frac{1}{n}}$

Use the following exponent property: $\left(a^{n}\right)^{m}=a^{n\cdot m}$

$\left(x^{n}\right)^{\frac{1}{n}}=x^{n\frac{1}{n}}$, $\left(n^{n}\right)^{\frac{1}{n}}=n^{n\frac{1}{n}}$

$=x^{n\frac{1}{n}}n^{n\frac{1}{n}}$

$x^{n\frac{1}{n}}=x$

Hide Steps

$x^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1

Hide Steps

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=x^{1}$

Apply rule $a^{1}=a$

$=x$

$=n^{n\frac{1}{n}}x$

$n^{n\frac{1}{n}}=n$

Hide Steps

$n^{n\frac{1}{n}}$

Multiply $n\frac{1}{n}$: 1

Hide Steps

$n\frac{1}{n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{1\cdot n}{n}$

Cancel the common factor: n

$=1$

$=n^{1}$

Apply rule $a^{1}=a$

$=n$

$= nx$

$L = \lim_{n\to\infty}\left(|nx|\right)$

$L = |x|\cdot\lim_{n\to\infty}\left(|n|\right)$

$\lim_{n\to\infty}\left(|n|\right) = \infty$ Hide Steps

$\lim_{n\to\infty}\left(|n|\right)$

n is positive when $n\to\infty$. Therefore $|n| = n$

$= \lim_{n\to\infty}\left(n\right)$

Apply the common limit: $\lim_{n\to\infty}\left(n\right) = \infty$

$= \infty$

$L = |x|\cdot\infty$

$|x| = 0 \quad : \quad x = 0$ Hide Steps

$|x| = 0$

Apply absolute rule: If $|u| = 0$ then $u = 0$

$x = 0$

Therefore

$L = \infty, x \neq 0; L = 0, x = 0$

$L = \infty, x \neq 0; L = 0, x = 0$

$L = \infty, x \neq 0; L = 0, x = 0$, therefore the power series converges only for $x = 0$

Radius of convergence is 0, Interval of convergence is $x = 0$

Radius of convergence is 0, Interval of convergence is $x = 0$