Symbolab

Solution

$\sum_{n=0}^{\infty}\frac{(x-3)^n}{5^n}$: Radius of convergence is 5, Interval of convergence is $-2<x<8$

Steps

$\sum_{n=0}^{\infty}\frac{(x-3)^n}{5^n}$

Use the Root Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}\frac{(x-3)^n}{5^n}$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|\left(\frac{(x-3)^n}{5^n}\right)^{\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\left(\frac{(x-3)^n}{5^n}\right)^{\frac{1}{n}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\left(\frac{(x-3)^n}{5^n}\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(\frac{(x-3)^n}{5^n}\right)^{\frac{1}{n}}$: $\frac{x-3}{5}$

Show Steps

$L=\lim_{n\to\infty}\left(\left|\frac{x-3}{5}\right|\right)$

$L=\left|\frac{x-3}{5}\right|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$

Show Steps

$L=\left|\frac{x-3}{5}\right|\cdot 1$

Simplify

$L=\frac{|x-3|}{5}$

$L=\frac{|x-3|}{5}$

The power series converges for $L<1$

$\frac{|x-3|}{5}<1$

Find the radius of convergence

Show Steps

Radius of convergence is 5

Find the interval of convergence

Show Steps

Interval of convergence is $-2<x<8$

Radius of convergence is 5, Interval of convergence is $-2<x<8$