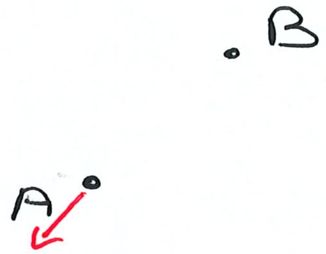
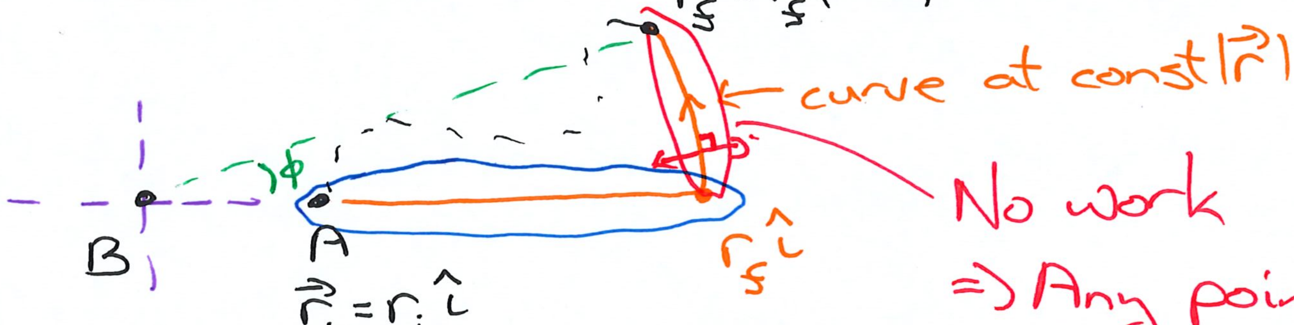
Electric Potential Energy



$$\vec{F}_{\text{on}A\text{by}B} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{|\vec{r}_A-\vec{r}_B|^2}\frac{\vec{r}_A-\vec{r}_B}{|\vec{r}_A-\vec{r}_B|}$$

What is ΔPE for them if B fixed and A moves

$\vec{r}_f = r_f(\cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath})$



$\Rightarrow$ Any point same separation same PE

$\vec{r} = s\hat{\imath} \quad r_i \le s \le r_f$

$d\vec{r} = ds\,\hat{\imath}$

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{s^2}\hat{\imath}$$

$$\vec{F}\cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} q_A q_B \frac{ds}{s^2}$$

$$W_E = \int_{r_i}^{r_f} \frac{1}{4\pi\varepsilon_0} q_A q_B \frac{ds}{s^2}$$

$$= \frac{-1}{4\pi\varepsilon_0} q_A q_B \frac{1}{r_f} - \left(-\frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r_i} \right)$$

$$W_E = -\Delta PE_E$$

$$PE_f = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{r_f} + \cancel{C}$$

by convention 0

$$PE_E = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{|\vec{r}_A - \vec{r}_B|} + C$$

A bunch of charges

$$PE = \sum_{\substack{\text{all charge} \\ \text{pairs}}} \frac{1}{4\pi\varepsilon_0} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

Recall, if look at one charge

$$\vec{F}_{\text{on it}} = q \vec{E}(\vec{r})$$

where q is, where other charges are and what they are

So PE of one charge $\underline{q}$ near lots of others

$$PE = \underbrace{q \sum_i \frac{1}{4\pi\epsilon_0} \frac{q_i}{|\vec{r} - \vec{r}_i|}}_{} + \sum_{\substack{\text{all} \\ \text{other} \\ \text{charges}}} \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{|\vec{r}_i - \vec{r}_j|}$$

$$= q\, V(\vec{r})$$

"encodes" where all other charges are and what they are

$V(\vec{r})$ = "Electric potential"

units Volts $1V = 1 J/C$

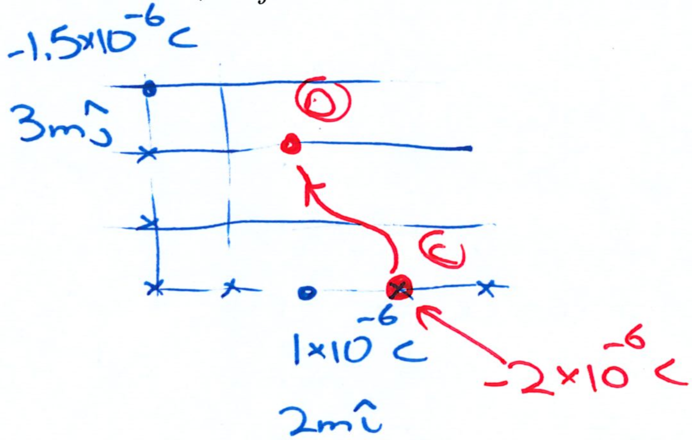
$$\vec{E}(\vec{r}) = -\nabla V(\vec{r})$$

if V changes (in space) a q will feel a force pushing it in direction V changes

Potential energy - IV

A charge $1.0 \times 10^{-6} C$ is at $2m\hat{\imath}$. A charge $-1.5 \times 10^{-6} C$ is at $3m\hat{\jmath}$.

How much work must be done to move a charge of $-2 \times 10^{-6} C$ from $3m\hat{\imath}$ to $2m\hat{\imath} + 2m\hat{\jmath}$?



Find $\Delta PE \rightarrow W_c = -\Delta PE$

$$PE = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{|\vec{r}_A - \vec{r}_B|}$$

$$PE_i = \left(\frac{1}{4\pi\varepsilon_0} q\right)\left[\frac{1\times10^{-6}C}{|3m\hat{i} - 2m\hat{i}|} + \frac{-1.5\times10^{-6}C}{|3m\hat{i} - 3m\hat{j}|}\right]$$

(of red charge)

$q = -2\times10^{-6}C$

$$= \left(-1.8\times10^{4}\frac{Nm^2}{C}\right)\left[1\times10^{-6}\frac{C}{m} - 3.53\times10^{-7}\frac{C}{m}\right]$$

$$= -1.16\times10^{-2} J$$

$$PE_{f} = 9\times10^{9}\frac{Nm^2}{C^2}\left[\frac{(-2\times10^{-6}C)(1\times10^{-6}C)}{|2m\hat{i}+2m\hat{j}-2m\hat{i}|} + \frac{(-2\times10^{-6}C)(-1.5\times10^{-6}C)}{|2m\hat{i}+2m\hat{j}-3m\hat{j}|}\right]$$

$$= 9\times10^{9}\frac{Nm^2}{C^2}\left[-1\times10^{-12}\frac{C^2}{m} + 1.34\times10^{-12}\frac{C^2}{m}\right]$$

$$= 3.07\times10^{-3}J$$

$$\Delta PE = 3.07\times10^{-3}J - (-1.16\times10^{-2}J)$$

$$= 1.47\times10^{-2}J$$

$$W = -1.47\times10^{-2}J$$

Going $C \rightarrow D$

$$\Delta PE = 1.47\times10^{-2}J = \underset{\downarrow}{\overset{-2\times10^{-6}C}{q}}\Delta V$$

$$\Delta V = -7.35\times10^{3}\,J/C$$

$$V_D - V_C = \quad ''$$

Conditions for <u>closest</u> approach
(In case where objects don't touch)

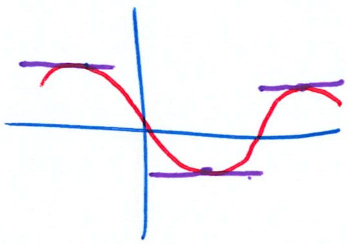
Distance between two objects

$$|\vec{r}_1 - \vec{r}_2|$$

Closest → extremum

$f(x)$ has extremum at x_0

$$\Rightarrow \left.\frac{d}{dx} f(x)\right|_{x_0} = 0$$



$$\frac{d}{dt} |\vec{r}_1 - \vec{r}_2| = 0$$

$$\frac{d}{dt} \sqrt{(\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}_1 - \vec{r}_2)}$$

$$\frac{d}{dx} f(g(x))$$

$$= f'(g(x))\, g'(x)$$

$$= \underbrace{\frac{1}{2} \frac{1}{\sqrt{(\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}_1 - \vec{r}_2)}}}_{f'(g(\,))} \underbrace{\frac{d}{dt}\left[(\vec{r}_1 - \vec{r}_2) \cdot (\vec{r}_1 - \vec{r}_2)\right]}_{g'(\,)}$$

$$\frac{d}{dt}\left[(\vec{r}_1-\vec{r}_2)\cdot(\vec{r}_1-\vec{r}_2)\right]$$

$$(\vec{v}_1-\vec{v}_2)\cdot(\vec{r}_1-\vec{r}_2) + (\vec{r}_1-\vec{r}_2)\cdot(\vec{v}_1-\vec{v}_2)$$

$$= 2(\vec{r}_1-\vec{r}_2)\cdot(\vec{v}_1-\vec{v}_2)$$

$$\frac{d}{dt}|\vec{r}_1-\vec{r}_2| = \frac{(\vec{r}_1-\vec{r}_2)\cdot(\vec{v}_1-\vec{v}_2)}{|\vec{r}_1-\vec{r}_2|}$$

3 ways for zero

$\vec{r}_1 = \vec{r}_2$ (at same spot)

$\vec{v}_1 = \vec{v}_2$ (at same velocity)

dot product makes it 0

relative velocity & separation vector at 90° to each other.

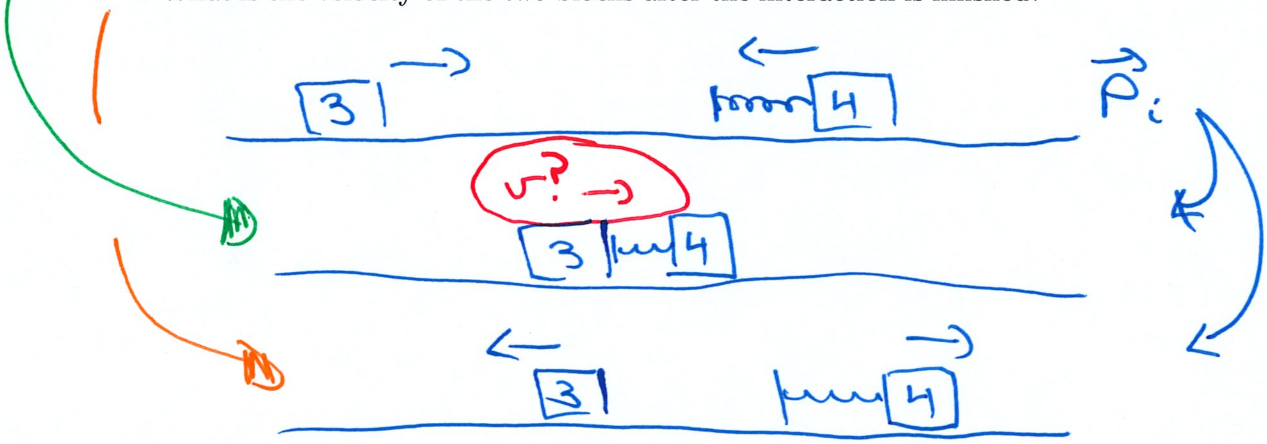
11-2-Example-Synthesis1

Synthesis - I

A block of mass $3kg$ moves at $8\frac{m}{s}\hat{\imath}$. A block of mass $4kg$ moves at $-4\frac{m}{s}\hat{\imath}$.

The $4kg$ block has a spring with spring constant $800\frac{N}{m}$ on its front, and it is initially uncompressed. The blocks hit each other, compressing the spring, and the spring then pushes them apart. The blocks only move in the x-direction.

- * What is the compression of the spring when the two blocks are closest together?
- What is the velocity of the two blocks after the interaction is finished?



$\vec{p}$ will be conserved

Work-energy theorem

$\Delta PE + \Delta KE = 0$

Know KE_i spring compressed

$\rightarrow \Delta PE \leftarrow$ depend on compression

2^{nd} case: Spring back to normal

$\Delta PE = 0$

$\Rightarrow \Delta KE = 0 \quad \Rightarrow KE_f = KE_i$

$\vec{P}_i = \vec{P}_{closest}$

$= m_3 \vec{v}_{3,c} + m_4 \vec{v}_{4,c}$

Know closest $\rightarrow \vec{v}_3 = \vec{v}_4$

$\vec{P}_i = m_3 \vec{v} + m_4 \vec{v}$ (get $\vec{v}$)

$3kg \, 8 \, m/s \, \hat{i} + 4kg(-4 \, m/s \, \hat{i}) = 3kg \, \vec{v} + 4kg \, \vec{v}$

$8 \, kg \, m/s \, \hat{i} = 7kg \, \vec{v}$

$1.143 \, m/s \, \hat{i} = \vec{v}$

$\Delta PE + \Delta KE = 0$

$\frac{1}{2} k (comp)^2 - \frac{1}{2} k \, 0^2 + \left(\frac{1}{2} 3kg (1.143 \, m/s)^2 + \frac{1}{2} 4kg (1.143 \, m/s)^2 \right)$

$- \left(\frac{1}{2} 3kg (8 \, m/s)^2 + \frac{1}{2} 4kg (4 \, m/s)^2 \right) = 0$

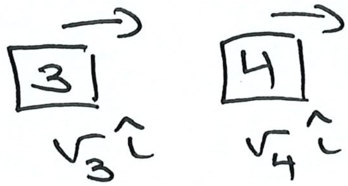
$\left(\frac{1}{2} 800 \, N/m \right) (comp)^2 + 4.57 J - 128 J = 0$

$\left(\frac{1}{2} 800 \, N/m \right) (comp)^2 = 123.4 J$

$(comp)^2 = 0.309 \, m^2$

$comp = 0.55 m$

v's after interaction



$$\vec{P}_{before} = \vec{P}_{after}$$

$$8\,kg\,\tfrac{m}{s}\,\hat{i} = 3\,kg\,v_3\hat{i} + 4\,kg\,v_4\hat{i}$$

$$2\,\tfrac{m}{s} = \frac{3}{4}v_3 + v_4$$

$$\boxed{v_4 = 2\,\tfrac{m}{s} - \frac{3}{4}v_3}$$

$$KE_{before} = KE_{after}$$

$$\underbrace{\frac{1}{2}3\,kg\left(8\,\tfrac{m}{s}\right)^2 + \frac{1}{2}4\,kg\left(4\,\tfrac{m}{s}\right)^2}$$

$$128\,J = \frac{1}{2}3\,kg\,v_3^2 + \frac{1}{2}4\,kg\,v_4^2$$

$$256\,J = 3\,kg\,v_3^2 + 4\,kg\,v_4^2$$

$$\boxed{64\,\tfrac{m^2}{s^2} = \frac{3}{4}v_3^2 + v_4^2}$$

$$64\,m^2/s^2 = \frac{3}{4}v_3^2 + \left(2\,m/s - \frac{3}{4}v_3\right)^2$$

$$= \frac{3}{4}v_3^2 + 4\,m^2/s^2 - 2(2\,m/s)\left(\frac{3}{4}v_3\right) + \frac{9}{16}v_3^2$$

$$0 = \frac{21}{16}v_3^2 - 60\,m^2/s^2 - 3\,m/s\,v_3$$

$$= \frac{21}{16}v_3^2 \underbrace{- 3\,m/s}_{b}\,v_3 - 60\,m^2/s^2$$

$$v_3 = \frac{-(-3\,m/s) \pm \sqrt{(-3\,m/s)^2 - 4\left(\frac{21}{16}\right)(60\,m^2/s^2)}}{2\left(\frac{21}{16}\right)}$$

$$= \frac{3\,m/s \pm \sqrt{324\,m^2/s^2}}{\cancel{2}\left(\frac{21}{\cancel{16}\,8}\right)}$$

$$= \frac{3\,m/s \pm 18\,m/s}{(21/8)}$$

$+ \rightarrow \cancel{8\,m/s}$

$- \rightarrow v_3 = -5.71\,m/s$

$$v_4 = 2\,m/s - \frac{3}{4}(-5.71\,m/s)$$

$$= 6.29\,m/s$$

$\leftarrow$ $\boxed{3}$ $-5.71\,m/s\,\hat{i}$

$\rightarrow$ $\boxed{4}$ $6.29\,m/s\,\hat{i}$