SOLVED BENEFIT-COST RATIO PRACTICE PROBLEMS: SOLUTIONS

Version 1.02 (Oct. 14 2018): Fixed a typo in Problem 9's solution.

Note: Many of these involve discounting and inflation, which we'll cover later in the course.

1. **Use incremental benefit cost ratio analysis (and discounting) to choose between the following four mutually exclusive projects**. Show your work. You are mostly being marked on your work.

Project A: Costs $697 in Year 0, gives benefits worth $845 in Year 3.
Project B: Costs $477 in Year 1, gives benefits worth $1,100 in Year 2.
Project C: Costs $396 in Year 2, gives benefits worth $1,028 in Year 1.
Project D: Costs $494 in Year 3, gives benefits worth $1,053 in Year 0.

Background information:

- It is currently Year 0
- All cash flows (costs and benefits) are real.
- Inflation is 3% per year.
- Your **nominal** MARR is 4% per quarter (a quarter is three months long).
- There are 12 months in a year.

a. (2 marks) The **real interest rate** is ___**13.58%**_____ % per year.

Various ways to do this. For now…

i. Find the nominal MARR per year: $(1 + 4\%)^4 - 1 = 16.99\%$
ii. Adjust for inflation:
Real Rate = $(1 + 16.99\%)/(1 + 3\%) - 1 = 13.58\%$

b. (10 marks) The **preferred project** is project ____**D**_____

Work: Use (P/F,13.58%,N) to find the present worth of each project's costs and benefits.

	Present Worth					
Project	Benefit	Cost	BCR	IB	IC	IBCR
C	$905.10	$306.97	2.95	(D - C)	(D - C)	(D - C)
D	$1,053.00	$337.16	3.12	$147.90	$30.19	4.90
B	$852.71	$419.97	2.03			
A	$576.72	$697.00	0.83			

I've arranged them in order of increasing (present worth) cost, for ease of exposition.
Project A can be discarded, because its BCR < 1.
Project B can be discarded, because it is dominated by D (lower costs, higher benefits).
The choice is down to C or D. Incremental benefit is $147.90, incremental cost is $30.19, so the incremental benefit-cost ratio is 4.90, which is > 1. Therefore choose more expensive project, D.

c. (3 marks) **If the projects were independent, and you had a budget of $1,000 in Year 0, which projects would you choose?** Briefly explain your reasoning.

Projects Chosen: __**C and D**_____

Reasoning:

In this case, the rule of thumb that you buy projects in descending order of BCR until you run out of money works: choose projects C and D.

You wouldn't want to choose A, because its BCR < 1.
You would choose D before B, because B is dominated by D.

You can only afford (any) two of B, C and D.

This means your choices are:
C + D: Benefits of $1,958.10
D + B: Benefits of $1,905.71

Pick C + D.

(Students are fine for full marks if they mention and apply the rule of thumb correctly.)

2. (4 marks) Note: The benefits and costs in the following question have already been put into present value terms for you. You are considering 3 mutually exclusive projects:

Project A has costs of $9 and benefits of $15.
Project B has costs of $41 and benefits of $54
Project C has costs of $52 and benefits of $30

Use incremental benefit-cost analysis to choose the preferred project. Show your work.

Preferred Project: _____**B**_____

Work:

We can rule out project C right away, since Benefits < Costs.

IBCR(B – A) = (54 – 15)/(41 – 9) = 39/32 > 1

Incremental benefits exceed incremental costs, so more expensive project (B) is preferred.

3. The table below lists the costs and benefits of five mutually exclusive projects.

Project	Costs	Benefits
A	9	12
B	6	5
C	3	4
D	9	11
E	4	6

a. (5 marks) Calculate the benefit-cost ratio (BCR) of each project:

Project	BCR
A	1.33
B	0.83
C	1.33
D	1.22
E	1.50

b. (5 marks) Use incremental benefit-cost analysis to determine the preferred project. Show your work.

The preferred project is Project ____A_____

Work:

We can rule out projects B (BCR < 1) and D (same cost as A, less benefits). A,C,E are left. In order of increasing cost: C,E,A. Letting IB be incremental benefit and IC be incremental cost,

Project	Benefit	Cost	IB	IC	IBCR
C	4	3			
E	6	4	2	1	2
A	12	9	6	5	1.2

IBCR (B – A) = 2/1 = 2 > 1, so keep the more expensive project, E, and discard C.
IBCR (A – E) = 6/5 = 1.2 > 1, so keep the more expensive project, A, and discard E.

Only project A is left, so A wins (despite having a lower BCR than E, the additional benefits are more than worth the additional costs).

4. The table below lists the costs and benefits of five mutually exclusive projects.

Project	Costs	Benefits
A	6	8
B	9	14
C	7	9
D	7	11
E	10	7

c. (5 marks) Calculate the benefit-cost ratio (BCR) of each project:

Project	BCR
A	1.33
B	1.56
C	1.29
D	1.57
E	0.70

d. (5 marks) Use incremental benefit-cost analysis to determine the preferred project. Show your work.

The preferred project is Project ____B_____

We can rule out projects E (BCR < 1) and C (same cost as D, less benefits). A,B,D are left. In order of increasing cost: A,D,B. Letting IB be incremental benefit and IC be incremental cost,

Project	Benefit	Cost	IB	IC	IBCR
A	8	6			
D	11	7	3	1	3
B	14	9	3	2	1.5

IBCR (D – A) = 3/1 = 3 > 1, so keep the more expensive project, D, and discard A.
IBCR (B – D) = 3/2 = 1.5 > 1, so keep the more expensive project, B, and discard D.

Only project B is left, so B wins (despite having a lower BCR than D, the additional benefits are more than worth the additional costs).

5. Project A has benefits of 12, and costs of 10. Project B has benefits of 24, and costs of 20. Project C has benefits of 32, and costs of 30. All numbers are already in present value terms, so no interest calculations are needed.

Project	PW Benefits	PW Costs	IBCR	Which 'wins''?
A	12	10	-	-
B	24	20	**12/10**	**B vs A, B wins**
C	32	30	**8/10**	**C vs B, B wins**

a. (9 marks) Use incremental benefit-cost ratio analysis to select the best project. Show your work. For full marks, we must see the incremental benefit-cost ratios you calculated, and the decisions you made based on those ratios.

First compare project B to Project A.
IBCR (B – A) = (24 – 12)/(20 – 10) = 12/10 = 1.2 > 1.
The additional benefits exceed additional costs, so keep B and discard A.

Next compare project C to Project B.
IBCR (C – B) = (32 – 24)/(30 – 20) = 8/10 = 0.8 < 1.
The additional benefits are LESS than the additional costs, so keep B and discard C. Only project B is left, so B is the preferred project.

b. (1 mark) Project __**B**_____ is preferred. (This question here for TA convenience.)

6. Project A has benefits of 18, and costs of 12. Project B has benefits of 28, and costs of 24. Project C has benefits of 40, and costs of 35. All numbers are already in present value terms, so no interest calculations are needed.

Project	PW Benefits	PW Costs	IBCR	Which 'wins''?
A	18	12	-	-
B	28	24	**10/12**	**B vs A, A wins**
C	40	35	**22/23**	**C v A, A wins**

c. (9 marks) Use incremental benefit-cost ratio analysis to select the best project. Show your work. For full marks, we must see the incremental benefit-cost ratios you calculated, and the decisions you made based on those ratios.

First compare project B to Project A.
IBCR (B – A) = (28 – 18)/(24 – 12) = 10/12 = 0.83 < 1.
The additional costs exceed additional benefits, so keep A and discard B.

Next compare project C to Project A.
IBCR (C – A) = (40 – 18)/(35 – 12) = 22/23 = 0.96 < 1.
The additional benefits are LESS than the additional costs, so keep A and discard C. Only project A is left, so A is the preferred project.

d. (1 mark) Project __**A**__ is preferred. (This question here for TA convenience.)

7. **It is currently Year 0**. There is **no inflation**. Your **MARR is 10% per year**. Consider the following two mutually exclusive projects:

	Cash Flows	
Year	Project A	Project B
0	-100	-200
1	200	
2		1000

(4 marks) Use incremental Benefit-Cost Ratio analysis and discounting to determine the preferred project. Show your work and fill in the blanks below.

Incremental Benefit-Cost Ratio (IBCR): _____6.45_____

Preferred Project: _____B_____

Work:

PV of Project A costs: 100
PV of Project A benefits: 200/1.1 = 181.82

PV of Project B costs: 200
PV of Project B benefits: $1000/1.1^2 = 826.45$

Incremental Benefits: 826.45 – 181.82 = 644.63
Incremental Costs: 200 – 100 = 100

IBCR = 644.63/100 = 6.45 > 1, so Project B is preferred.

8. **It is currently Year 0**. There is **no inflation**. Your **MARR is 10% per year**. Consider the following two mutually exclusive projects:

	Cash Flows	
Year	Project A	Project B
0	-200	-100
1		500
2	2000	

(4 marks) Use incremental Benefit-Cost Ratio analysis and discounting to determine the preferred project. Show your work and fill in the blanks below.

Incremental Benefit-Cost Ratio (IBCR): _____11.98_____

Preferred Project: _____A_____

Work:

PV of Project A costs: 200
PV of Project A benefits: $2000/1.1^2 = 1652.89$

PV of Project B costs: 100
PV of Project B benefits: $500/1.1 = 454.55$

Incremental Benefits: $1652.89 – 454.55 = 1,198.35$
Incremental Costs: $200 – 100 = 100$

IBCR = $1,198.35/100 = 11.98 > 1$, so Project A is preferred.

9. Your firm's MARR is 5% a year. Your firm is trying to decide between two machines, Machine X and Machine Y. You MUST purchase either Machine X or Machine Y. Machine X costs $10,000 in Year 0, has costs of $1,500 a year from Years 1 to 20, and will be salvaged in Year 20 for $2,500. Machine X also provides income of $300 in Year 1. This goes up by $200 a year, so that income is $500 in Year 2, $700 in Year 3, etc. The final payment is in Year 20. Machine Y has a lifetime of 5 years. Machine Y's benefits have a present worth of $25,000. Machine Y's only cost is an initial cost of $5,000.

a. (5 marks) Calculate the present worth of Machine X. (One lifetime, no repeats.)

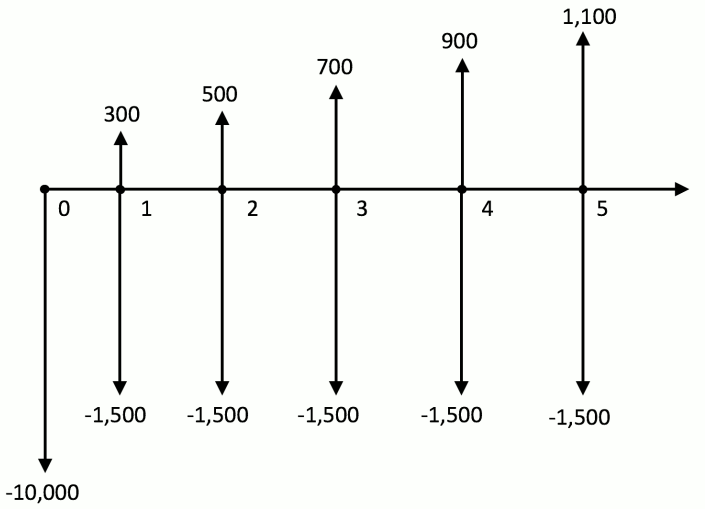
Present Worth: ____-$4,314.75 (negative)_____

$-P - C \times (P/A, i, N) + A \times (P/A, i, N) + G \times (A/G, i, N) \times (P/A, i, N) + S \times (P/F, i, N)$

(This can of course be simplified.)

P = 10,000 : Initial, present value cost
C = 1,500 : Negative annuity (cost)
i = 5% : MARR
N = 20 : Project lifetime
S = 2,500 : Salvage
A = 300 : Positive annuity (income)
G = 200 : Arithmetic gradient (income)

b. (5 marks) Draw a cash flow diagram for the first 5 years (only) of Machine X's lifetime.



c. (6 marks) Use incremental cost benefit ratio analysis to determine which machine is preferred: The incremental benefit cost ratio is _-0.026 (about – 0.03) _____ , therefore Machine __Y____ is preferred, because (20 words or less) ___the IBCR is less than zero_(can also point out it's less than one, instead)_____.

PV Benefits X = PV of Income + Salvage = A x $(P/A, i, N)$ + G x (A/G,i,N) x $(P/A, i, N)$ + S x $(P/F, i, N)$
PV Benefits X = 3,738.66 + 19,697.68 + 942.22 = 24,378.57
PV Costs X = Initial Cost + PV Costs = P + C x $(P/A, i, N)$
PV Costs X = 10,000 + 18,693.32 = 28,693.32

PV Benefits Y = 25,000
PV Costs Y = 5,000

IBCR (X – Y) = Incremental Benefits / Incremental Costs

Incremental Benefits = 24,378.57 – 25,000 = -621.43
Incremental Cost = 28,693.32 – 5,000 = 23,693.32

IBCR(X – Y) = -621.43/23,693.32 = -0.026 (about -0.03).

10. Your firm's MARR is 10% a year. Your firm is trying to decide between two machines, Machine X and Machine Y. You MUST purchase either Machine X or Machine Y. Machine X costs $1,000 in Year 0, has costs of $2,500 a year from Years 1 to 25, and will be salvaged in Year 25 for $500. Machine X also provides income of $2,000 in Year 1. This goes up by $100 a year, so that income is $2,100 in Year 2, $2,200 in Year 3, etc. The final payment is in Year 25. Machine Y has a lifetime of 50 years. Machine Y's benefits have a present worth of $25,000. Machine Y's only cost is an initial cost of $5,000.

a. (5 marks) Calculate the present worth of Machine X. (One lifetime, no repeats.)

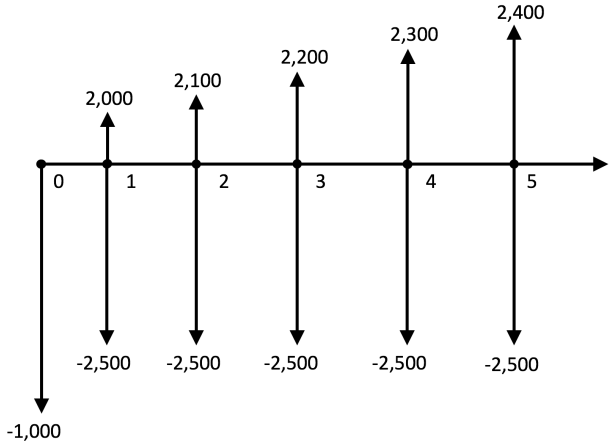
Present Worth: _____$1,277.27_____

$-P - C \times (P/A, i, N) + A \times (P/A, i, N) + G \times (P/A, i, N) \times (P/A, i, N) + S \times (P/F, i, N)$

(This can of course be simplified.)

P = 1,000 : Initial, present value cost
C = 2,500 : Negative annuity (cost)
i = 10% : MARR
N = 25 : Project lifetime
S = 500 : Salvage
A = 2,000 : Positive annuity (income)
G = 100 : Arithmetic gradient (income)

b. (5 marks) Draw a cash flow diagram for the first 5 years (only) of Machine X's lifetime.



c. (6 marks) Use incremental cost benefit ratio analysis to determine which machine is preferred: The incremental benefit cost ratio is _-0.002_ , therefore Machine __Y____ is preferred, because (20 words or less) the IBCR is less than zero_(can also point out it's less than one, instead)_

PV Benefits X = PV of Income + Salvage = A x $(P/A, i, N)$ + G x $(P/A, i, N)$ + S x $(P/F, i, N)$
PV Benefits X = $18,154 + $6,769.64 + $46.15 = $24,969.87
PV Costs X = Initial Cost + PV Costs = P + C x $(P/A, i, N)$
PV Costs X = 1,000 + 22,692.60 = 23,962.60

PV Benefits Y = 25,000
PV Costs Y = 5,000

IBCR (X – Y) = Incremental Benefits / Incremental Costs

Incremental Benefits = 24,969.87 – 25,000 = -30.13
Incremental Cost = 23,962.60 – 5,000 = 18,692.60

IBCR(X – Y) = -621.43/23,693.32 = -0.002.

11. This question deals with benefit-cost analysis. **Parts a) and b) are not connected**.

a. The following table lists costs and benefits for four **mutually exclusive** projects. **Select the preferred project and show your work.** Unsupported answers will receive no marks. (10 marks)

Project	Costs	Benefits
A	30	56
B	34	63
C	37	69
D	32	62

We need to calculate the incremental benefit-cost ratio (IBCR) for each pair of alternatives, starting with the two cheapest (A and D). If the IBCR < 1, scrap the more expensive option and keep the cheaper. Otherwise, keep the cheaper and scrap the more expensive. Repeat until there is only one project left.

As shown in the table below, the preferred project is C.

Project	Costs	Benefits	IC	IB	IBCR	
A	30	56				
D	32	62	2	6	3	D - A
B	34	63	2	1	0.5	B - D
C	37	69	5	7	1.4	C - D

(D –A) : IBCR > 1, D wins.
(B – D): IBCR < 1, D wins.
(C – D): IBCR > 1, C wins.

b. "When faced with independent projects and a budget, you should start with the cheapest, then buy projects in order of increasing cost until your run out of money."

Is this statement true or false? Briefly explain your reasoning using concepts from ECON 180. (5 marks)

False.

Looking just at costs doesn't take benefits into accounts: need to check if costs > benefits
Even if all projects have benefits > costs, you'll want to maximize the benefits your budget buys you.

12. This question deals with benefit-cost analysis. **Parts a) and b) are not connected**.

a. The following table lists costs and benefits for four **mutually exclusive** projects. **Select the preferred project and show your work.** Unsupported answers will receive no marks. (10 marks)

Project	Costs	Benefits
A	44	97
B	33	70
C	51	102
D	39	82

We need to calculate the incremental benefit-cost ratio (IBCR) for each pair of alternatives, starting with the two cheapest (B and D). If the IBCR < 1, scrap the more expensive option and keep the cheaper. Otherwise, keep the cheaper and scrap the more expensive. Repeat until there is only one project left.

As shown in the table below, the preferred project is A.

Project	Costs	Benefits	IC	IB	IBCR	
B	33	70				
D	39	82	6	12	2	D-B
A	44	97	5	15	3	A-D
C	51	102	7	5	0.71	C-A

(D – B): IBCR > 1, D wins.
(A – D): IBCR > 1, A wins.
(C – A): IBCR < 1, A wins.

b. "When faced with independent projects and a budget, you should start with the project with the highest benefits, then buy projects in order of decreasing benefits until your run out of money."

Is this statement true or false? Briefly explain your reasoning using concepts from ECON 180. (5 marks)

False.

You also need to look at costs, and make sure that benefits > costs.

Even if benefits > costs for all projects, sorting by highest benefits (or BCR) won't always give the best results.

You should check all affordable combinations fo projects for the one that provides highest benefits, given the budget.