Graphing Quadratic Equations

$$ax^2 + bx + c = 0$$

A Quadratic Equation in Standard Form
(**a**, **b**, and **c** can have any value, except that **a** can't be 0.)

Here is an example:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Graphing

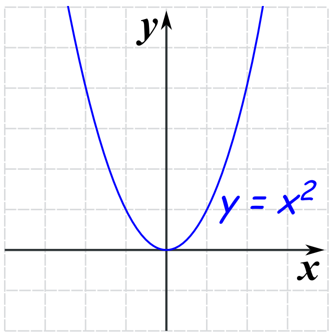
You can graph a Quadratic Equation using the Function Grapher, but to **really understand** what is going on, you can make the graph yourself. Read On!

The Simplest Quadratic

The simplest Quadratic Equation is:

$$f(x) = x^2$$

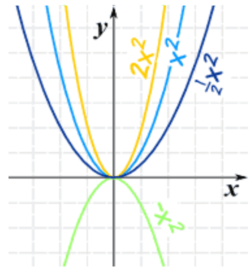
And its graph is simple too:



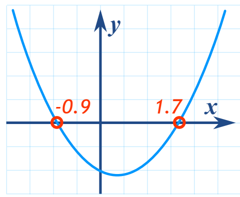
This is the curve $f(x) = x^2$
It is a parabola.

Now let us see what happens when we introduce the "a" value:

$$f(x) = ax^2$$



- Larger values of **a** squash the curve inwards
- Smaller values of **a** expand it outwards
- And negative values of **a** flip it upside down



Play With It

Now is a good time to play with the "Quadratic Equation Explorer" so you can see what different values of **a**, **b** and **c** do.

The "General" Quadratic

Before graphing we **rearrange** the equation, from this:

$$f(x) = ax^2 + bx + c$$

To this:

$$f(x) = a(x-h)^2 + k$$

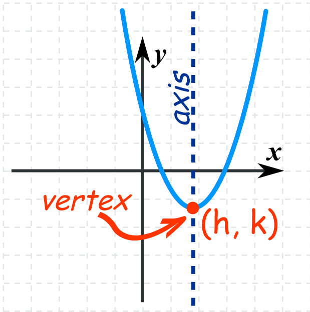
Where:

- $h = -b/2a$
- $k = f(h)$

In other words, calculate **h** (= −b/2a), then find **k** by calculating the whole equation for **x=h**

But Why?

The wonderful thing about this new form is that h and k show us the very lowest (or very highest) point, called the **vertex**:



And also the curve is symmetrical (mirror image) about the **axis** that passes through **x=h**, making it easy to graph

So ...

- **h** shows us how far left (or right) the curve has been shifted from x=0
- **k** shows us how far up (or down) the curve has been shifted from y=0

Lets see an example of how to do this:

Example: Plot f(x) = $2x^2 - 12x + 16$

First, let's note down:

- **a = 2,**
- **b = −12,** and
- **c = 16**

Now, what do we know?

- a is positive, so it is an "upwards" graph ("U" shaped)
- a is 2, so it is a little "squashed" compared to the $\mathbf{x^2}$ graph

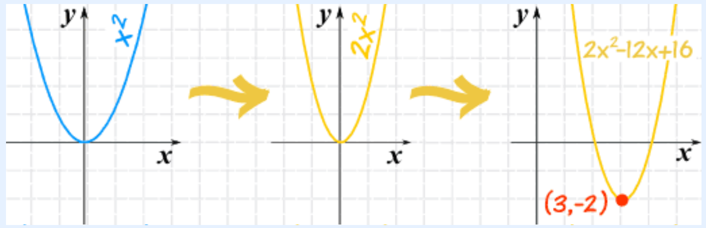
Next, let's calculate h:

h = −b/2a = −(−12)/(2x2) = **3**

And next we can calculate k (using h=3):

k = ***f*(3)** = $2(3)^2$ − 12·3 + 16 = 18−36+16 = **−2**

So now we can plot the graph (with real understanding!):

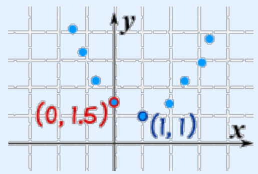


We also know: the **vertex** is (3,−2), and the **axis** is x=3

From A Graph to The Equation

What if we have a graph, and want to find an equation?

Example: you have just plotted some interesting data, and it looks Quadratic:



Just knowing those two points we can come up with an equation.

Firstly, we know **h** and **k** (at the vertex):

(h, k) = (1, 1)

So let's put that into this form of the equation:

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-1)^2 + 1$$

Then we calculate "a":

We know the point **(0, 1.5)** so:	f(0) = 1.5
And $\mathbf{a(x-1)^2 + 1}$ at x=0 is:	$f(0) = a(0-1)^2 + 1$
They are both **f(0)** so make them equal:	$a(0-1)^2 + 1 = 1.5$
Simplify:	a + 1 = 1.5

$$a = 0.5$$

And so here is the resulting Quadratic Equation:

$$f(x) = 0.5(x-1)^2 + 1$$

Note: This may not be the **correct** equation for the data, but it’s a good model and the best we can come up with.

Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10