Introduction to Logarithms

In its simplest form, a logarithm answers the question:

How many of *one number* do we multiply to get *another number?*

Example: How many 2s do we multiply to get 8?

Answer: $\mathbf{2 \times 2 \times 2 = 8}$, so we had to multiply 3 of the **2**s to get **8**

So the logarithm is 3

How to Write it

We write "the number of 2s we need to multiply to get 8 is 3" as:

$$\log_2(8) = 3$$

So these two things are the same:

$$\underbrace{2 \times 2 \times 2}_{3} = 8 \quad \leftrightarrow \quad \log_{\underbrace{2}_{\text{base}}}(8) = 3$$

The number we multiply is called the "base", so we can say:

- "the logarithm of 8 with base 2 is 3"
- or "log base 2 of 8 is 3"
- or "the base-2 log of 8 is 3"

Notice we are dealing with three numbers:

- the **base**: the number we are multiplying (a "2" in the example above)
- how often to use it in a multiplication (3 times, which is the **logarithm**)
- The number we want to get (an "8")

More Examples

Example: What is $\log_5(625)$ ... ?

We are asking "how many 5s need to be multiplied together to get 625?"

$\mathbf{5 \times 5 \times 5 \times 5 = 625}$, so we need 4 of the 5s

Answer: $\log_5(625) = 4$

Example: What is $\log_2(64)$ ... ?

We are asking "how many 2s need to be multiplied together to get 64?"

$\mathbf{2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64}$, so we need 6 of the 2s

Answer: $\log_2(64) = 6$

Exponents

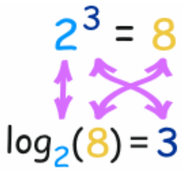
Exponents and Logarithms are related, let's find out how ...

exponent 2^3 base	The **exponent** says **how many times** to use the number in a multiplication. In this example: $\mathbf{2^3 = 2 \times 2 \times 2 = 8}$ *(2 is used 3 times in a multiplication to get 8)*

So a logarithm answers a question like this:

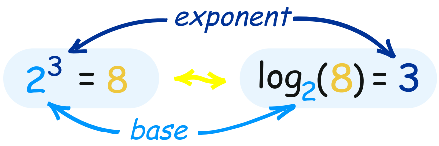
$$2^{?} = 8$$

In this way:



The logarithm tells us what the exponent is!

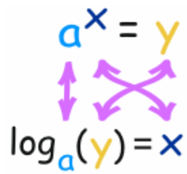
In that example the "base" is 2 and the "exponent" is 3:



So the logarithm answers the question:

What exponent do we need
(for one number to become another number) **?**

The **general** case is:



Example: What is $\log_{10}(100)$ ... ?

$$10^2 = 100$$

So an exponent of 2 is needed to make 10 into 100, and:

$$\log_{10}(100) = 2$$

Example: What is $\log_3(81)$ ... ?

$$3^4 = 81$$

So an exponent of 4 is needed to make 3 into 81, and:

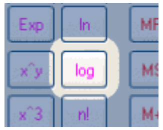
$$\log_3(81) = 4$$

Common Logarithms: Base 10

Sometimes a logarithm is written **without** a base, like this:

$$\log(100)$$

This ***usually*** means that the base is really 10.



It is called a "common logarithm". Engineers love to use it.

On a calculator it is the "log" button.

It is how many times we need to use 10 in a multiplication, to get our desired number.

Example: $\log(1000) = \log_{10}(1000) = 3$

Natural Logarithms: Base "e"

Another base that is often used is [e (Euler's Number)] which is about 2.71828.

This is called a "natural logarithm". Mathematicians use this one a lot.

On a calculator it is the "ln" button.

It is how many times we need to use "e" in a multiplication, to get our desired number.

Example: $\ln(7.389) = \log_e(7.389) \approx 2$

Because $2.71828^2 \approx 7.389$

But Sometimes There Is Confusion ... !

Mathematicians use "log" (instead of "ln") to mean the natural logarithm. This can lead to confusion:

Example	Engineer Thinks	Mathematician Thinks	
log(50)	$\log_{10}(50)$	$\log_e(50)$	confusion
ln(50)	$\log_e(50)$	$\log_e(50)$	no confusion
$\log_{10}(50)$	$\log_{10}(50)$	$\log_{10}(50)$	no confusion

So, be careful when you read "log" that you know what base they mean!

Logarithms Can Have Decimals

All of our examples have used whole number logarithms (like 2 or 3), but logarithms can have decimal values like 2.5, or 6.081, etc.

Example: what is $\mathbf{\log_{10}(26)}$ **... ?**

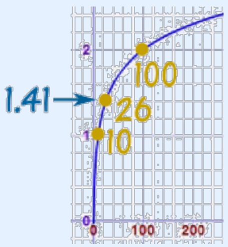
Get your calculator, type in **26** and press **log**

Answer is: **1.41497...**

The logarithm is saying that $10^{1.41497...} = 26$
(10 with an exponent of **1.41497...** equals 26)

This is what it looks like on a graph:

See how nice and smooth the line is.



Read [Logarithms Can Have Decimals] to find out more.

Negative Logarithms

Negative? But logarithms deal with multiplying.
What is the opposite of multiplying? **Dividing!**

A negative logarithm means how many times **to divide** by the number.

We can have just one divide:

Example: What is $\log_8(0.125)$ ... ?

Well, $1 \div 8 = 0.125$,

So $\log_8(0.125) = -1$

Or many divides:

Example: What is $\log_5(0.008)$ ... ?

$\mathbf{1 \div 5 \div 5 \div 5 = 5^{-3}}$,

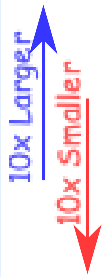
So $\log_5(0.008) = -3$

It All Makes Sense

Multiplying and Dividing are all part of the same simple pattern.

Let us look at some Base-10 logarithms as an example:

Number	How Many 10s	Base-10 Logarithm



.. etc..			
1000	$1 \times 10 \times 10 \times 10$	$\log_{10}(1000)$	$= 3$
100	$1 \times 10 \times 10$	$\log_{10}(100)$	$= 2$
10	1×10	$\log_{10}(10)$	$= 1$
1	1	$\log_{10}(1)$	$= 0$
0.1	$1 \div 10$	$\log_{10}(0.1)$	$= -1$
0.01	$1 \div 10 \div 10$	$\log_{10}(0.01)$	$= -2$
0.001	$1 \div 10 \div 10 \div 10$	$\log_{10}(0.001)$	$= -3$
.. etc..			

Looking at that table, see how positive, zero or negative logarithms are really part of the same (fairly simple) pattern.

The Word

"Logarithm" is a word made up by Scottish mathematician John Napier (1550-1617), from the Greek word *logos* meaning "proportion, ratio or word" and *arithmos* meaning "number", ... which together makes "ratio-number" !

Question 1 Question 2 Question 3 Question 4 Question 5
Question 6 Question 7 Question 8 Question 9 Question 10