Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** Practice Questions for Sections 6.3 & 7.2 [Not for

Solve the differential equation.

$$\frac{dy}{dx} = 2x\sqrt{49-y^2}, \quad -7 < y < 7$$

Some differential equations can be solved by separating the variables. A differential equation of the form $y' = f(x,y)$ is separable if f can be expressed as a product of a function of x and a function of y.

Rewrite the equation in its differential form. Divide both sides of the equation by $\sqrt{49-y^2}$ to write the equation in the form $h(y)dy = g(x)dx$.

$$\frac{dy}{dx} = 2x\sqrt{49-y^2}$$

$$\frac{dy}{\sqrt{49-y^2}} = 2x\,dx$$

Now integrate both sides of the equation. Begin by integrating the left side. Use the rule $\int \frac{du}{\sqrt{a^2-u^2}} = \mathbf{sin}^{-1}\frac{u}{a}$ to integrate.

$$\int \frac{dy}{\sqrt{49-y^2}} = \int 2x\,dx$$

$$\mathbf{sin}^{-1}\frac{y}{7} + C_1 = \int 2x\,dx$$

Integrate the right side. First move the constant to the outside of the integral.

$$\mathbf{sin}^{-1}\frac{y}{7} + C_1 = 2\int x\,dx$$

$$\mathbf{sin}^{-1}\frac{y}{7} + C_1 = 2 \cdot \frac{x^2}{2} + C_2$$

Simplify the right side of the equation.

$$\mathbf{sin}^{-1}\frac{y}{7} + C_1 = x^2 + C_2$$

After completing the integrations, y is defined implicitly as a function of x. Combine the constants of integration as C.

$$\mathbf{sin}^{-1}\frac{y}{7} = x^2 + C$$

This equation can be solved for y. Take the **sin** of both sides of the equation to remove the $\mathbf{sin}^{-1}$ function on y.

$$\mathbf{sin}\left(\mathbf{sin}^{-1}\frac{y}{7}\right) = \mathbf{sin}\left(x^2 + C\right)$$

$$\frac{y}{7} = \mathbf{sin}\left(x^2 + C\right)$$

Multiply both sides by 7 to solve the equation for y.

$$y = 7\,\mathbf{sin}\left(x^2 + C\right)$$

Thus, solving the original differential equation, $\frac{dy}{dx} = 2x\sqrt{49-y^2}$, yields $y = 7\,\mathbf{sin}\left(x^2 + C\right)$.