Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Determine whether the alternating series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n^3}$ converges or diverges.

The Alternating Series Test states that the series $\sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \cdots$ converges if the u_n's are all positive, the positive u_n's are eventually nonincreasing, and $u_n \to 0$.

For n = 1 and increasing to infinity, u_n, or $\frac{3}{n^3}$, will always be positive because n is always positive and n^3 is also always positive.

In order to determine whether $\frac{3}{n^3}$ is eventually nonincreasing, compare u_n, or $\frac{3}{n^3}$, with u_{n+1}. Find u_{n+1} by replacing n with n + 1 in the expression for u_n.

$$u_{n+1} = \frac{3}{(n+1)^3}$$

Notice that $\frac{3}{n^3}$ is greater than or equal to $\frac{3}{(n+1)^3}$ for all n greater than or equal to 1.

Thus, the positive u_n's are eventually nonincreasing.

Evaluate $\lim_{n\to\infty} \frac{3}{n^3}$.

$$\lim_{n\to\infty} \frac{3}{n^3} = 0$$

The series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{n^3}$ meets the three criteria for the Alternating Series Test. Therefore, the series converges.