Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Identify the coordinates of any local and absolute extreme points and inflection points. Graph the function.

$$y = 12x^3 - 3x^4 = x^3(12 - 3x)$$

To graph the function, first determine the domain of the function and any symmetries the curve may have.

The domain of the function $y = 12x^3 - 3x^4$ is $(-\infty, \infty)$.

The graph of the function has no symmetry.

Find the derivatives y' and y''. First find y'.

$$\begin{aligned} y &= 12x^3 - 3x^4 \\ y' &= 36x^2 - 12x^3 \end{aligned}$$

Find y''.

$$\begin{aligned} y' &= 36x^2 - 12x^3 \\ y'' &= 72x - 36x^2 \end{aligned}$$

Next, find the critical point(s) of $y = f(x)$ by solving $y' = 0$.

Solve $y' = 0$ for x.

$$\begin{aligned} y' &= 0 \\ 36x^2 - 12x^3 &= 0 \\ 12x^2(3 - x) &= 0 \\ x &= 0, 3 \end{aligned}$$

Since y' exists over the domain of y, the critical points are only at $x = 0$ and $x = 3$.

To determine the behavior at the critical points, use the Second Derivative Test for Local Extrema to determine whether any local extrema occur at the critical points.

If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.
If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at $x = c$.
If $f'(c) = 0$ and $f''(c) = 0$ or $f''(c)$ fails to exist, then the function f may have a local maximum, a local minimum, or neither at $x = c$.

Determine the behavior of the function at the critical points.

Note that y has a local maximum at $x = 3$, and may have a local maximum, a local minimum, or neither at $x = 0$.

Therefore, (3,81) is a local maximum, but (0,0) is not yet known.

Find where the curve is increasing and where it is decreasing. The critical points subdivide the domain of $y = 12x^3 - 3x^4$ to create nonoverlapping open intervals on which y' is either positive or negative. Determine the sign of y' over these intervals.

Interval	$x < 0$	$0 < x < 3$	$3 < x$
Sign of y'	+	+	−

If $y' > 0$ at any point in an open interval, then the curve is increasing on that interval. If $y' < 0$ at any point in an open interval, then the curve is decreasing on that interval. Determine the behavior of the curve.

Interval	$x < 0$	$0 < x < 3$	$3 < x$
Sign of y'	+	+	−
Behavior of the curve	increasing	increasing	decreasing

At a point of inflection, either y'' is 0 or y'' fails to exist. Since the domain of y'' is $(-\infty,\infty)$, there are no values of x where y'' does not exist. Find any potential inflection points by setting the second derivative equal to 0, and solve for x.

$$\begin{aligned} y'' &= 0 \\ 72x - 36x^2 &= 0 \\ 36x(2-x) &= 0 \\ x &= 0, 2 \qquad \text{Solve for x.} \end{aligned}$$

The inflection points are at $x = 0$ and $x = 2$. Use these points to define the intervals where the curve is concave up or concave down. Determine the sign of y'' over these intervals.

Interval	$x<0$	$0<x<2$	$2<x$
Sign of y''	−	+	−

If $y'' > 0$ at any point in an open interval, then the curve is concave up on that interval. If $y'' < 0$ at any point in an open interval, then the curve is concave down on that interval. Determine the concavity of the curve.

Interval	$x<0$	$0<x<2$	$2<x$
Sign of y''	−	+	−
Concavity of the curve	down	up	down

Therefore, the inflection points are (2,48) and (0,0).

Determine the asymptotes of the given function.

There are no asymptotes of the given function as it is defined for all real numbers.

Therefore, the graph of $y = 12x^3 - 3x^4$ is as shown to the right.