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Evaluate the integral below using any appropriate algebraic method or trigonometric identity.

$$\int_{-30}^{0} \sqrt{\frac{30+y}{30-y}}\,dy$$

Note that none of the basic integration formulas have a square root in both the numerator and the denominator. There are several basic integration formulas that have a square root only in the denominator. The first step is to write an equivalent expression with square roots only in the denominator.

Multiply the numerator and denominator by $30 + y$.

$$\int_{-30}^{0} \sqrt{\frac{30+y}{30-y}}\,dy = \int_{-30}^{0} \sqrt{\frac{(30+y)}{(30-y)} \cdot \frac{(30+y)}{(30+y)}}\,dy$$

$$= \int_{-30}^{0} \sqrt{\frac{(30+y)^2}{900-y^2}}\,dy \qquad \text{Simplify the radicand.}$$

Write the numerator without any radical symbols.

$$\sqrt{(30+y)^2} = |30+y|$$

Note that since y lies in the interval $[-30,0]$, $30 + y$ is always nonnegative. The given integral can be transformed as follows.

$$\int_{-30}^{0} \sqrt{\frac{30+y}{30-y}}\,dy = \int_{-30}^{0} \sqrt{\frac{(30+y)^2}{900-y^2}}\,dy \qquad \text{Multiply radicand by } \frac{30+y}{30+y}.$$

$$= \int_{-30}^{0} \frac{\sqrt{(30+y)^2}}{\sqrt{900-y^2}}\,dy \qquad \text{Split into two square roots.}$$

$$= \int_{-30}^{0} \frac{|30+y|}{\sqrt{900-y^2}}\,dy \qquad \text{Use } \sqrt{a^2} = |a|.$$

$$= \int_{-30}^{0} \frac{30+y}{\sqrt{900-y^2}}\,dy \qquad \text{Remove the absolute value symbols from the numerator.}$$

The resulting integral is similar to several of the basic integration formulas. However, the basic integration formulas contain a single term in the numerator and this integral is a sum of two terms. To transform this integral into a form where any integrand containing a square root in the denominator has just a single term in the numerator, use the Sum Rule.

$$\int_a^b (f(x)+g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

The integral is now the sum of two simpler integrals.

$$\int_{-30}^{0} \sqrt{\frac{30+y}{30-y}}\,dy = I_1 + I_2 \text{ where } I_1 = \int_{-30}^{0} \frac{30}{\sqrt{900-y^2}}\,dy \text{ and } I_2 = \int_{-30}^{0} \frac{y}{\sqrt{900-y^2}}\,dy.$$