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Assignment: HW-6 [Sections 10.4, 10.5 & 10.6]

Does the series defined below converge or diverge? Give reasons for your answer.

$$a_1 = 6, a_{n+1} = \frac{6}{n} a_n$$

Let $\sum a_n$ be any series and suppose that $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \rho$. The Ratio Test states that the series converges absolutely if $\rho < 1$ and diverges if $\rho > 1$ or ρ is infinite. The test is inconclusive if $\rho = 1$.

Substitute in the appropriate symbolic terms of the series into the limit for the Ratio Test and simplify the expression.

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{6}{n} a_n}{a_n} \right|$$
$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{6}{n} \right|$$

Evaluate $\lim_{n \rightarrow \infty} \left| \frac{6}{n} \right|$.

$$\lim_{n \rightarrow \infty} \left| \frac{6}{n} \right| = 0$$

Thus, $\rho = 0$. This means that the given series converges absolutely by the Ratio Test, and so it converges.