# Sample/practice exam July 2020, questions

Algorithms and Data Structures I (University of Victoria)

# CSC 225 A01 FALL 2015 (CRN: 10789)
# ALGORITHMS AND DATA STRUCTURES I
# MIDTERM EXAMINATION
# UNIVERSITY OF VICTORIA

1. Student ID: ______________________
2. Name: _________________________
3. DATE: 20 OCTOBER 2015
   DURATION: 45 MINUTES
   INSTRUCTOR: V. SRINIVASAN
4. THIS QUESTION PAPER HAS SIX PAGES INCLUDING THE COVER PAGE.
5. THIS QUESTION PAPER HAS FOUR QUESTIONS.
6. ALL ANSWERS ARE TO BE WRITTEN ON THIS EXAMINATION PAPER.
7. THIS IS A CLOSED BOOK EXAM. NO CALCULATORS OR ANY OTHER AIDS ARE ALLOWED.
8. READ THROUGH ALL THE QUESTIONS AND ANSWER THE EASY QUESTIONS FIRST. KEEP YOUR ANSWERS SHORT AND PRECISE.

| | |
|---|---|
| Q1 (5) | |
| Q2 (3) | |
| Q3 (4) | |
| Q4 (3) | |
| TOTAL (15) = | |

1. **Asymptotic Notation**

(i) Indicate for each pair of expressions (A,B) in the table below, whether A is $O$ or $\Omega$ of $B$. Your answer should be in the form of a "yes" or "no" in each box. [3 Marks]

| A | B | O | $\Omega$ |
|---|---|---|---|
| $n^3$ | $(\log n)^6$ | | |
| $2n^3 + 4n^2$ | $4n^3$ | | |
| $n!$ | $2^n$ | | |
| $n^5$ | $n^4$ | | |
| 10000 | 5 | | |
| $n^5$ | $(1.1)^n$ | | |

(ii) Prove that $2n^3+3n^2$ is $O(n^3)$. Show your calculations and write down the values of the constants $c$ and $n_0$ clearly. [2 Marks]

2. **Solving Recurrence Equations**

(a) Solve the following recurrence equation to get a closed-formula for $T(n)$. [3 Marks]

$$\begin{aligned} T(n) &= 2 \text{ if } n = 1 \\ &= T(n-1) + 2n \text{ if } n \geq 2 \end{aligned}$$

4. **Proof by Induction**

   **The Stacking Game.** You begin with a stack of $n$ boxes. Then you make a sequence of moves. In each move, you divide one stack of boxes into two nonempty stacks. The game ends when you have $n$ stacks, each containing a single box. You earn points for each move; in particular, if you divide one stack of height $a + b$ into two stacks with heights $a$ and $b$, then you score $ab$ points for that move. Your overall score is the sum of the points that you earn for each move. An example of this game for $n = 8$ is shown below. On each line, the underlined stack is divided in the next step. Is there a strategy that you can use to maximize your total score? Prove the claim on the next page to answer this question. [3 Marks]

| Stack Heights | Score in Points |
|---|---|
| $\underline{8}$ | |
| $\underline{5}$ 3 | 15 |
| 3 2 $\underline{3}$ | 6 |
| 3 2 $\underline{2}$ 1 | 2 |
| $\underline{3}$ 2 1 1 1 | 1 |
| 2 1 $\underline{2}$ 1 1 1 | 2 |
| $\underline{2}$ 1 1 1 1 1 1 | 1 |
| 1 1 1 1 1 1 1 1 | 1 |
| Total Score | 28 |

**Claim:** Prove, by induction on $n$, that every way of unstacking $n$ blocks gives a total score of $\frac{n(n-1)}{2}$ points for any $n \geq 1$. Show the base case, the induction hypothesis and calculations inside the induction step clearly.

END OF EXAM