Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

Consider the series $\sum_{n=0}^{\infty} (-1)^n(10x+3)^n$.

(a) Find the series' radius and interval of convergence.
(b) For what values of x does the series converge absolutely?
(c) For what values of x does the series converge conditionally?

(a) To test a power series for convergence, use the ratio test to find the interval where the series converges absolutely.

Recall that the ratio test states that if Σa_n is any series and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \rho$, then the series converges absolutely if $\rho < 1$, the series diverges if $\rho > 1$ or ρ is infinite, and the test is inconclusive if $\rho = 1$.

Begin by finding the limit below.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(10x+3)^{n+1}}{(-1)^n(10x+3)^n}\right|$$

$$= \lim_{n\to\infty}|10x+3| \qquad \text{Simplify.}$$

Find the limit.

$$\lim_{n\to\infty}|10x+3| = |10x+3|$$

The ratio test states that the series converges absolutely if this limit is less than 1. Thus, find the interval of convergence by solving the inequality $|10x+3| < 1$ for x.

$$-1 < 10x+3 < 1$$

$$-\frac{2}{5} < x < -\frac{1}{5} \qquad \text{Solve for x.}$$

Now test for convergence or divergence at each endpoint of this interval. Begin with $x = -\frac{2}{5}$, by examining the series at this value of x.

$$\sum_{n=0}^{\infty}(-1)^n(10x+3)^n = \sum_{n=0}^{\infty}(-1)^n\left(10\left(-\frac{2}{5}\right)+3\right)^n \qquad \text{Substitute } x = -\frac{2}{5}.$$

$$= \sum_{n=0}^{\infty}(-1)^n(-1)^n \qquad \text{Simplify.}$$

Simplify the series further.

$$\sum_{n=0}^{\infty}(-1)^n(10x+3)^n = \sum_{n=0}^{\infty}(-1)^n(-1)^n$$

$$= \sum_{n=0}^{\infty}(-1)^{2n} \qquad \text{Add exponents.}$$

$$= \sum_{n=0}^{\infty}(1)^n \qquad \text{Simplify.}$$

Thus, at $x = -\frac{2}{5}$ the series is $\sum_{n=0}^{\infty} 1^n = 1 + 1 + 1 + \cdots$. For this series, it is not true that $a_n \to 0$, and so the series diverges by the nth term test.

Now examine the series at the other endpoint, $x = -\frac{1}{5}$.

$$\sum_{n=0}^{\infty} (-1)^n (10x+3)^n = \sum_{n=0}^{\infty} (-1)^n \left(10\left(-\frac{1}{5}\right)+3\right)^n \qquad \text{Substitute } x = -\frac{1}{5}.$$

$$= \sum_{n=0}^{\infty} (-1)^n (1)^n \qquad \text{Simplify.}$$

Thus, at $x = -\frac{1}{5}$ the series is $\sum_{n=0}^{\infty} (-1)^n = 1 - 1 + 1 - \cdots$. For this series, it is not true that $u_n \to 0$, where u_n is the absolute value of the nth term of the series, and so the series diverges by the alternating series test.

Since the series does not converge at each endpoint of the interval, the interval of convergence is as shown below.

$$-\frac{2}{5} < x < -\frac{1}{5}$$

Now find the radius of convergence, R. The convergence of a power series, $\sum_{n=0}^{\infty} c_n (x-a)^n$, is described by one of the following three possibilities. The first possibility is that there is a positive number R such that the series diverges for x with $|x-a| > R$ but converges absolutely for x with $|x-a| < R$. The second possibility is that the series converges absolutely for every x, in which case the radius of convergence is infinity. Finally, the last possibility is that the series converges at $x = a$ and diverges elsewhere, in which case R is equal to zero. Note that the given series falls into the first possibility.

To find the radius of convergence, compute half the width of the interval of convergence.

$$R = \frac{1}{2}\left(-\frac{1}{5} - \left(-\frac{2}{5}\right)\right)$$

$$= \frac{1}{10}$$

(b) A series Σa_n converges absolutely if the corresponding series of absolute values, $\Sigma |a_n|$, converges.

Recall that the interval of absolute convergence was found in part (a) to be $-\frac{2}{5} < x < -\frac{1}{5}$.

(c) A series that converges but does not converge absolutely converges conditionally.

Recall from part (a) that the series converges absolutely for $-\frac{2}{5} < x < -\frac{1}{5}$ and diverges for $x \le -\frac{2}{5}$ and $x \ge -\frac{1}{5}$ because the nth term does not approach zero for those values of x. Thus, there are no values of x for which the given series converges conditionally.