

Q₁ (Part 2)

$$\left[\begin{array}{ccc|c} 2 & 0 & k & 1 \\ 0 & 3 & 1 & 2 \\ 0 & k & 0 & 4 \end{array} \right] \xrightarrow{\frac{1}{2} R_1} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} \\ 0 & 3 & 1 & 2 \\ 0 & k & 0 & 4 \end{array} \right]$$

$$\xrightarrow{\frac{1}{3} R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & k & 0 & 4 \end{array} \right] \xrightarrow{R_3 - kR_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & -\frac{k}{3} & 4 - \frac{2k}{3} \end{array} \right]$$

$$\xrightarrow{-\frac{3}{k} R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & (-\frac{3}{k})(4 - \frac{2k}{3}) \end{array} \right] \xrightarrow{R_1 - \frac{k}{2} R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

~~$$\xrightarrow{R_1 - \frac{k}{2} R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{1}{2} - (k-6) \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

$$\xrightarrow{R_1 - \frac{k}{2} R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{k}{2} & \frac{13}{2} - k \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

$$\xrightarrow{R_1 - \frac{k}{2} R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{2} - k \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

$$\xrightarrow{R_2 - \frac{1}{3} R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{2} - k \\ 0 & 1 & 0 & \frac{2}{3} - \frac{2}{k}(k-6) \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

$$\xrightarrow{R_2 - \frac{2}{3} R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{2} - k \\ 0 & 1 & 0 & \frac{2k-12}{k} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$

$$\xrightarrow{R_2 - \frac{2}{3} R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{13}{2} - k \\ 0 & 1 & 0 & \frac{2k-12}{k} \\ 0 & 0 & 1 & \frac{2}{k}(k-6) \end{array} \right]$$~~

$$\frac{2}{k}(k-6) = \frac{2k}{k} - \frac{12}{k} = 2 - \frac{12}{k}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 1 & \frac{13}{2} - k \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 0 & 1 & 2 - \frac{12}{k} \end{bmatrix}$$

$$\frac{2}{3} - \frac{2(k-6)}{3k} = \frac{2}{3} - \frac{2k-12}{3k} = \frac{2k-2k+12}{3k} = \frac{4}{k}$$

$$R_2 - \frac{1}{3}R_3 \rightarrow \begin{bmatrix} 1 & 0 & 1 & \frac{13}{2} - k \\ 0 & 1 & 0 & \frac{4}{k} \\ 0 & 0 & 1 & 2 - \frac{12}{k} \end{bmatrix}$$

~~Here, $z = 2 - \frac{12}{k}$~~

~~or, $z = \frac{2k-12}{k}$~~

~~or, $zk = 2k - 12 \Rightarrow z(k-6)$~~

~~or~~

$$\begin{aligned} z - 2 &= -\frac{12}{k} \\ 2 - z &= \frac{12}{k} \\ k &= \left(\frac{12}{2-z} \right) \end{aligned}$$

Now, $x - y + kz = 1$

$$\Rightarrow x - y + \left(\frac{12}{2-z} \right) z = 1$$

$$\Rightarrow x - y + \frac{12z}{2-z} = 1$$

$$\Rightarrow \frac{x(2-z) - y(2-z) + 12z}{2-z} = 1$$

$$\Rightarrow 2x - xz - 2y + yz + 12z = 2 - z$$

$$\Rightarrow 2x - xz - 2y + yz + 12z - 2 + z = 0$$

\Rightarrow