Set Theory Definitions

- A *set* is a well-defined collection of objects called *elements* or *members* of the set.
  - If x is a member of the set S, we write $x \in S$; if x isn't a member of S we write $x \notin S$.
  - Two sets A and B are *equal*, written $A = B$, if they have exactly the same elements; that is $(x \in A) \Leftrightarrow (x \in B)$.

- A set can be specified in several equivalent ways:
  - By listing its elements between brackets; for example, $A = \{2, 3, 4, 5, 6, 7, 8\}$.
  - By listing enough elements to establish a pattern, and using an ellipsis where appropriate; for example, $B = \{2, 3, \ldots, 8\}$.
  - Using *set-builder notation* to specify the collection of all objects of a given type that make a given statement true; for example, $C = \{n \in \mathbb{Z} : 2 \leq n \leq 8\}$. Here $\mathbb{Z}$, *the set of integers* is the universe of n.

  Notice that $A = B = C$.

- Some sets are denoted by special symbols. The set of:
  - *natural numbers* is $\mathbb{N} = \{1, 2, 3, \ldots\}$.
  - *integers* is $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$.
  - *rational numbers* is $\mathbb{Q} = \{a/b : a, b \in \mathbb{Z}, \text{ and } b \neq 0\}$.
  - *real numbers* is $\mathbb{R}$, the collection of all numbers that have a decimal expansion.
  - *complex numbers* is $\mathbb{C} = \{a + bi : a, b \in \mathbb{R}, \text{ and } i^2 = -1\}$.

- The *empty set* is the set that has no elements, that is $\{\}$. It is commonly denoted by $\emptyset$.

- We say A is a *subset* of B, and write $A \subseteq B$ when every element of A is also an element of B. That is, $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$.
  - Note: For sets A and B, we have $A = B \Leftrightarrow (A \subseteq B) \wedge (B \subseteq A)$.

- When $A \subseteq B$ and B has at least one element which is not in A, we say A is a *proper subset* of B and write $A \subsetneq B$. Thus, $A \subsetneq B \Leftrightarrow (A \subseteq B) \wedge (A \neq B)$.