Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Does the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^5}{n^8+8}$ converge absolutely, converge conditionally, or diverge?

A series $\sum a_n$ converges absolutely (is absolutely convergent) if the corresponding series of absolute values, $\sum |a_n|$, converges. If the series converges, but is not absolutely convergent, then the series converges conditionally. Otherwise, the series diverges.

Find the terms of the corresponding series of absolute values.

$$\left|(-1)^{n+1} \frac{n^5}{n^8+8}\right| = \frac{n^5}{n^8+8}$$

Since $\frac{n^5}{n^8+8}$ is a rational function of n, use the Comparison Test to determine if $\sum_{n=1}^{\infty} \frac{n^5}{n^8+8}$ converges.

For the Comparison Test, let $\sum a_n$, $\sum c_n$, and $\sum d_n$ be series with nonnegative terms. For some integer N, let $d_n \le a_n \le c_n$ for all $n > N$.

If $\sum c_n$ converges, then $\sum a_n$ also converges. If $\sum d_n$ diverges, then $\sum a_n$ also diverges.

The exponent in the denominator is 3 greater than the exponent in the numerator. Therefore, to use the Comparison Test, compare $\sum_{n=1}^{\infty} \frac{n^5}{n^8+8}$ to $\sum_{n=1}^{\infty} \frac{1}{n^3}$.

The series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges because a p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$.

Note that $\sum_{n=1}^{\infty} \frac{n^5}{n^8+8} \le \sum_{n=1}^{\infty} \frac{1}{n^3}$ for all $n > 1$. Therefore, the series $\sum_{n=1}^{\infty} \frac{n^5}{n^8+8}$ converges per the Comparison Test.

Therefore, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^5}{n^8+8}$ converges absolutely per the Comparison Test.