UNIVERSITY OF VICTORIA
EXAMINATIONS APRIL 2002
COMPUTER SCIENCE 225 (S01)

NAME: ____________________ STUDENT NO. ____________

INSTRUCTOR: Frank Ruskey

SECTION: S01

DURATION: 3 Hours

TO BE ANSWERED ON THE PAPER

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS 11 PAGES.

NOTES: (0) CLOSED BOOK EXAM; NO NOTES OR CALCULATORS ALLOWED, (1) ANSWER ALL QUESTIONS, (2) THERE ARE A TOTAL OF 117 MARKS, (3) SCRATCH PAPER IS AVAILABLE FROM THE INVIGILATORS.

Question	Possible marks	Actual marks
1	46	
2	7	
3	16	
4	5	
5	5	
6	5	
7	6	
8	5	
9	6	
10	4	
11	4	
12	4	
13	4	
Total	128	

1. Fill in the blanks: [2 marks each, 46 total]

   - In terms of the structure of the recursive program, mergesort is most similar to which traversal of a binary tree (preorder, inorder, or postorder)?
     ANSWER: ________________________.

   - In terms of the structure of the recursive program, quicksort is most similar to which traversal of a binary tree (preorder, inorder, or postorder)?
     ANSWER: ________________________.

   - What is the running time to insert the items $n, n-1, \ldots, 2, 1$ successively into an initially empty tree of the following three types? Give your solutions as a Θ expressions.
     ANSWER:

     binary search tree: ________________________

     AVL tree: ________________________

     splay tree: ________________________

   - Exactly how many comparisons does it take to verify that a n element array represents a binary heap?

     ANSWER: ________________.

   - A binomial queue with 1027 elments has how many edges?
     ANSWER: ________________

   - Using the standard merging algorithm on two sorted lists, one of size n and the other of size m, what (exactly) is the worst case number of key comparisons used by the algorithm?
     ANSWER: ________________.

   - Name a good algorithm for sorting a small number of elements (e.g. 20 elements).

     ANSWER: ________________________

   - If Algorithm A is $O(n)$ and Algorithm B is $O(n^2)$ does this mean that A is faster than B for all sufficiently large values of n? Explain briefly.

     ANSWER: ____.

   - In a binary (min) heap implemented with arrays, the 3-rd smallest element could appear in which different locations? I.e., if x is the third smallest element and $\texttt{heap}[i] = x$, what are the possible values for i?

     ANSWER: ________________________.

   - What sorting algorithm is $O(n \log n)$ on average, but is $\Theta(n^2)$ in the worst case?

     ANSWER: ________________________.

- What is the most commonly used efficient data structure for implementing dictionaries?
  ANSWER: ______________________
- Given a binary tree with n uniquely labelled nodes, under what condition(s) does the preorder and postorder traversal give the same sequence?

  ANSWER: ______________________
- Given a binary tree with n uniquely labelled nodes, under what condition(s) does the inorder and postorder traversal give the same sequence?

  ANSWER: ______________________
- Give two inequalities that are sufficient to prove that $\Theta(m+n) = max(n,m)$.

  ANSWER: ______________________ and ______________________.
- Name a sorting algorithm that is not comparison based.

  ANSWER: ______________________
- What is the result of running the book's partitioning phase of quicksort with the median-of-three pivot selection rule on the data 8,1,4,9,0,3,5,2,7,6 (pivot is 6)?

  ANSWER: ______________________
- What $O(n+m)$ algorithm would you use to find the shortest path from v to w in an *unweighted* directed graph?
  ANSWER: ______________________
- Let `class SR = { Object o; SR next; }` be the stack record declaration for a linked list implementation of stacks, where `SR top` is a reference to the top of the stack. Give the two java statements (in the correct order) necessary to push a new stack record `SR p` onto the stack.

  ANSWER: ______________________; ______________________;.
- In a standard implementation of postorder traversal of a binary trees, how many times is the test for `null` made on an n node tree?

  ANSWER: ______________________.
- Which operation, UNION or FIND, takes constant time in the worst case when implemented with up-trees (parent arrays)?
  ANSWER: ______________________.
- In the up-tree implementation of the UNION-FIND problem with both balancing and compression, what is the maximum height of any tree in the forest after n operations have been performed?

  ANSWER: ______________________

2. (a) Given the initial data 9,8,7,6,5,4,3,2,1, apply the bottom-up $O(n)$ algorithm to obtain a binary heap. Draw the heap as a tree T and it's array representation.
(b) Extract the minimum element from the heap T and show the resulting tree.
(c) Insert 0 into the heap T and show the resulting tree.
[7 marks]

3. [16 marks] Do a depth-first search of the following directed graph. Assume that the driver loop processes the vertices in alphabetic order. Draw the graph in standard DFS format.

A: → D → B
B: → A
C: → B → A
D: → A
E: → H → C → G
F: → H → B
G: → F
H: → G → F
I: → H

List below the DFS and postorder numberings of the vertices:

Vertices:	A	B	C	D	E	F	G	H	I
DFS numbering:									
postorder numbering									

List the tree edges: ______________________

List the back edges: ______________________

List the forward edges: ______________________

List the cross edges: ______________________

List the strongly connected components: ______________________

List the topological sort (ignore back edges) derived from the postorder numbering

____

4. In Huffman's algorithm we observed that the frequency at the root of the Huffman tree was the sum of the frequencies at the leaves. Prove this statement by mathematical induction on the number of leaves. [5 marks]

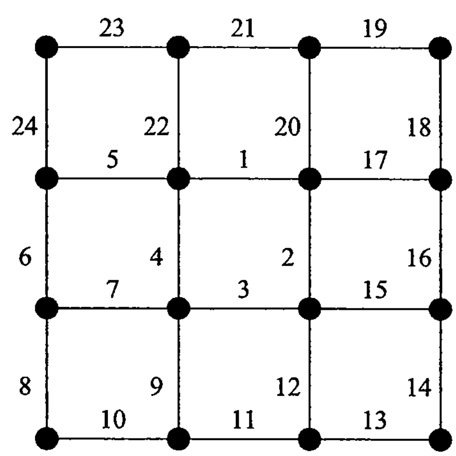
5. How would you efficiently determine if an undirected graph has a cycle *and* print the vertices along a cycle if it has a cycle? [5 marks]

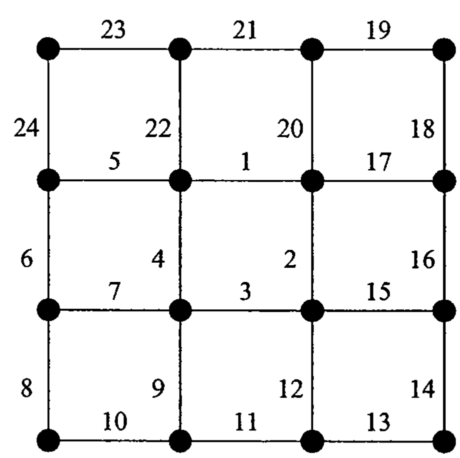
6. Describe an algorithm that takes as input a set S of n integers, and an integer M, and determines whether there are two integers $x, y \in X$ such that $x + y = M$. Your algorithm should run in expected time $O(n)$. Explain why it has that running time. HINT: use hashing. [5 marks]

7. (a) What is the exact number of comparisons, call it $T(n)$, used in the worst case when inserting a new element into a sorted array of n elements? Assume that you are using binary search to find where to insert the new element and that all elements are unique.
   (b) Write a recurrence relation that describes $T(n)$. NOTE: There is no need to relate your answers to (a) and (b).
   (c) Prove that no comparison based algorithm could use fewer comparisons. [6 marks]

8. Prove that the *minimum* depth (= length of path to root) of a leaf in an AVL tree is $\Theta(\log n)$. Note that we are asking for the minimum depth, not the maximum depth. This shows that the *best* case running time of a look-up in an AVL tree is also $\Theta(\log n)$. [5 marks]

9. Show, by marking the edges, the spanning tree that results after 11 edges have been added to the tree by both Kruskal's (on the left) and Prim's (on the right) algorithms. For Prim's algorithm start at the upper left corner. [6 marks]

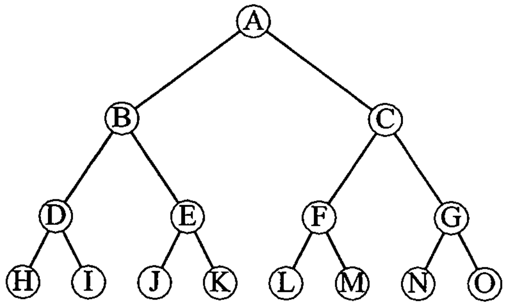




10. [12 marks] Below is the DFS code for *directed* graphs that was presented in class and posted on the course web site. The method is used to classify the edges according to whether they are tree, back, forward, or cross. Fill in the correct type of edge to print at the four underlined places in the code below. [4 marks]

```
void dfs ( int v ) {
   num[v] = ++cn;
   visited[v] = true;
   System.out.println( "Visiting "+v );
   for (Edge p=adj[v]; p != null; p = p.link ) {
      int w = p.vert;
      if ( !visited[w] ) {

         System.out.println( "____________ edge: ("+v+","+w+")" );
         dfs( w );
      } else {
         if ( num[v] > num[w] && pst[w] == -1 )

            System.out.println( "____________ edge: ("+v+","+w+")" );
         if ( num[v] > num[w] && pst[w] != -1 )

            System.out.println( "____________ edge: ("+v+","+w+")" );
         if ( num[v] < num[w] )

            System.out.println( "____________ edge: ("+v+","+w+")" );
      }
   }
   pst[v] = ++cp;
}
```

11. Draw the ordered forest that corresponds to the following binary tree. The forest should look familiar; why? [4 marks]



12. Draw the binary search tree whose preorder traversal is F,E,B,A,D,C,I,G,H. What is the result of a postorder traversal of that tree? [4 marks]

13. Consider a circular doubly linked list with a header node, made from the `DoubleNodes` defined below. Fill in the code for deleting an element from the list. Fill in the code for inserting that deleted node back into the list at the same spot it was at before (assuming that the list hasn't changed in the interim). FYI these have been referred to as "dancing links." [4 marks]

```
class DoubleNode {

  DoubleNode next, prev;

  void delete() {

     ___________________________________________________________;

     ___________________________________________________________;
  }

  void undelete() {

     ___________________________________________________________;

     ___________________________________________________________;
  }
}
```

END