Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Graph the rational function below, considering the domain, symmetry, critical points, intervals where the function is increasing or decreasing, inflection points, concavity, asymptotes, and intercepts where applicable.

$$y = -\frac{x^2 - 81}{x + 3}$$

First, find the domain of the function $f(x) = -\frac{x^2 - 81}{x + 3}$.

The domain is $(-\infty, -3) \cup (-3, \infty)$.

To determine whether the graph of the function has symmetry, evaluate $f(-x)$.

Since $f(-x) = -\frac{x^2 - 81}{-x + 3}$ does not equal $f(x)$ or $-f(x)$, the graph of $y = f(x)$ has no symmetry about the y-axis or the origin.

Now, find any intercepts of the function. To find any x-intercepts, let $y = 0$ and solve for x. Recall that a rational function equals zero at the values for which its numerator equals zero, provided such values are in the function's domain.

$$-\frac{x^2 - 81}{x + 3} = 0$$
$$x^2 - 81 = 0$$
$$x = -9 \text{ or } x = 9$$

Since -9 and 9 are both in the domain, the graph has x-intercepts at $x = -9$ and $x = 9$. That is, the graph passes through $(-9, 0)$ and $(9, 0)$.

To find the y-intercept, let $x = 0$ and simplify.

$$-\frac{0^2 - 81}{0 + 3} = 27$$

So, the graph has a y-intercept at $y = 27$. That is, the graph passes through $(0, 27)$.

Next, calculate the derivatives of y. Find y'.

$$y = -\frac{x^2 - 81}{x + 3}$$
$$y' = -\frac{x^2 + 6x + 81}{(x + 3)^2}$$

Now, find y''.

$$y' = -\frac{x^2 + 6x + 81}{(x + 3)^2}$$
$$y'' = \frac{144}{(x + 3)^3}$$

Find the critical points. The critical points of a function are interior points of the domain where the first derivative is zero or undefined.

Recall that $y' = -\frac{x^2 + 6x + 81}{(x + 3)^2}$. Identify the values of x, if any, for which $y' = 0$.

Because the equation $x^2 + 6x + 81 = 0$ has no real solutions, there are no such values of x.

Identify the values of x in the domain, if any, for which y' is undefined.

Because the denominator is $(x+3)^2$, y' is undefined at $x = -3$.

Since $x = -3$ is not in the domain, there are no critical points.

Notice that because there are no critical points, the function does not have any local extrema.

Now, determine where f is increasing or decreasing.

Suppose that f is continuous on [a,b] and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on (a,b). If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on (a,b).

Recall that the domain of f(x) is $(-\infty,-3) \cup (-3,\infty)$. Use the fact that $y' = -\dfrac{x^2+6x+81}{(x+3)^2}$ to construct a sign chart for y'.

Interval	$x < -3$	$x > -3$
Sign of y'	$-$	$-$
Behavior of y	decreasing	decreasing

Next, find any inflection points. An inflection point occurs at each point (c,f(c)) in the domain where the concavity changes and $f''(c) = 0$ or $f''(c)$ fails to exist.

Recall that $y'' = \dfrac{144}{(x+3)^3}$. Identify the values of x, if any, for which $y'' = 0$.

Because the equation $144 = 0$ has no solution, there are no such values of x.

Identify the values of x, if any, for which y'' is undefined.

Because the denominator is $(x+3)^3$, y' is undefined at $x = -3$.

It follows that $x = -3$ is the only place where $y = -\dfrac{x^2-81}{x+3}$ can change concavity.

Now, determine the curve's concavity using the second derivative. Let $y = f(x)$ be twice-differentiable on an interval I. If $f''(x) > 0$ on I, then the graph of f over I is concave up. If $f''(x) < 0$ on I, then the graph of f over I is concave down.

Recall that the domain of f(x) is $(-\infty,-3) \cup (-3,\infty)$. Use the fact that $y'' = \dfrac{144}{(x+3)^3}$ to construct a sign chart for y'.

Interval	$x < -3$	$x > -3$
Sign of y''	$-$	$+$
Behavior of y	concave down	concave up

Next, determine any asymptotes. To find the vertical asymptotes, evaluate the one-sided limits at any point not in the domain of the function.

Because the domain of $f(x) = -\dfrac{x^2-81}{x+3}$ is $(-\infty,-3) \cup (-3,\infty)$, the limits $\lim\limits_{x \to -3^-} f(x)$ and $\lim\limits_{x \to -3^+} f(x)$ must be found. These limits will be easier to find if the function is rewritten as shown below.

$$f(x) = -\frac{x^2-81}{x+3} = -x+3+\frac{72}{x+3}$$

Evaluate the one-sided limits.

$$\lim_{x \to -3^-} \left(-x+3+\frac{72}{x+3}\right) = -\infty$$

$$\lim_{x \to -3^+} \left(-x+3+\frac{72}{x+3}\right) = \infty$$

So, the graph of y has a vertical asymptote at $x = -3$.

To find any horizontal or oblique asymptotes, evaluate the limits of the function as x approaches $-\infty$ and ∞.

Again, these limits will be easier to find using the rewritten form of the function. Evaluate the limits.

$$\lim_{x \to -\infty} \left(-x + 3 + \frac{72}{x+3} \right) = \infty$$

$$\lim_{x \to \infty} \left(-x + 3 + \frac{72}{x+3} \right) = -\infty$$

Since these limits are infinite, the graph of y does not have a horizontal asymptote.

As x approaches $-\infty$ or ∞, the graph of $y = -x + 3 + \frac{72}{x+3}$ approaches the line $y = -x + 3$. This is because

$$\lim_{x \to \pm\infty} \frac{72}{x+3} = 0.$$

Thus, $y = -x + 3$ is an oblique asymptote.

Finally, use the information about domain, symmetry, intercepts, critical points, intervals where the function increases or decreases, inflection points, concavity, and asymptotes to sketch the graph of y.

The correct graph is shown here. Notice that it shows all of the characteristics identified previously.