Student: Arfaz Hossain **Date:** 04/20/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** Practice Questions for Sections 11.4 & 11.5 [Not f

Find the area of the region inside the circle $r = -12\cos\theta$ and outside the circle $r = 6$.

Determine the region's boundaries and find the limits of integration. Since both curves are circles, there are exactly two intersection points. Set the values of the curves equal to each other to determine the limits of integration.

$$-12\cos\theta = 6$$

$$\cos\theta = -\frac{1}{2} \qquad \text{Divide.}$$

Take the inverse of cosine to find θ. Remember to use only positive values of theta, as working clockwise will not include the desired region.

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{2\pi}{3}, \frac{4\pi}{3} \qquad \text{Solve for } \theta.$$

Therefore, the lower limit of integration is $\frac{2\pi}{3}$, and the upper limit of integration is $\frac{4\pi}{3}$.

To find the area of the region, use the following formula, where $r_1 = 6$, and $r_2 = -12\cos\theta$. Substitute the expressions into the formula for area.

$$A = \int_{\alpha}^{\beta} \frac{1}{2}\left(r_2^2 - r_1^2\right) d\theta$$

$$= \int_{2\pi/3}^{4\pi/3} \frac{1}{2}\left((-12\cos\theta)^2 - (6)^2\right) d\theta \qquad \text{Substitute.}$$

Use symmetry to make the integral easier to work with.

$$A = \int_{2\pi/3}^{4\pi/3} \frac{1}{2}\left((-12\cos\theta)^2 - (6)^2\right) d\theta$$

$$= 2\int_{\pi}^{4\pi/3} \frac{1}{2}\left((-12\cos\theta)^2 - (6)^2\right) d\theta \qquad \text{Change the limits of integration using symmetry.}$$

Simplify the integral.

$$A = 2\int_{\pi}^{4\pi/3} \frac{1}{2}\left((-12\cos\theta)^2 - (6)^2\right) d\theta$$

$$= \int_{\pi}^{4\pi/3} \left(144\cos^2\theta - (36)\right) d\theta \qquad \text{Simplify.}$$

Separate into two integrals and integrate.

$$A = 144\int_{\pi}^{4\pi/3} \cos^2\theta\, d\theta - \int_{\pi}^{4\pi/3} 36 d\theta$$

$$= 144\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{\pi}^{4\pi/3} - [36\theta]_{\pi}^{4\pi/3} \qquad \text{Integrate.}$$

Distribute the 144.

$$A = 144\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{\pi}^{4\pi/3} - [36\theta]_{\pi}^{4\pi/3}$$

$$= [72\theta + 36 \sin 2\theta]_{\pi}^{4\pi/3} - [36\theta]_{\pi}^{4\pi/3} \qquad \text{Distribute.}$$

Substitute the upper and lower limits of integration into the expression and simplify. Begin by evaluating the first integral.

$$A_1 = [72(4\pi/3) + 36 \sin 2(4\pi/3)] - [72(\pi) + 36 \sin 2(\pi)] \qquad \text{Substitute.}$$

$$= 96\pi + 18\sqrt{3} - 72\pi - 0$$

$$= 24\pi + 18\sqrt{3} \qquad \text{Simplify.}$$

Now evaluate the second integral.

$$A_2 = -[36(4\pi/3)] - [36(\pi)] \qquad \text{Substitute.}$$

$$= -12\pi \qquad \text{Simplify.}$$

Add the two together to find the area of the region.

$$A = 24\pi + 18\sqrt{3} - 12\pi$$

$$= 12\pi + 18\sqrt{3} \qquad \text{Simplify the expression.}$$

Therefore, the area of the region is $12\pi + 18\sqrt{3}$.