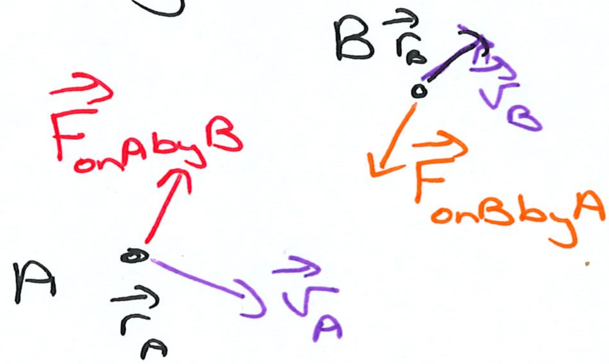
Momentum

Useful to understand complicated situation w/ multiple particles interacting because if isolated, momentum is conserved.

Consider A, B, two particles which only exert force on each other



The only force on A is by B and vice-versa

$$\vec{a}_A = \frac{\vec{F}_{net,A}}{m_A} = \frac{\vec{F}_{onA\,byB}}{m_A}$$

$$\vec{F}_{onAbyB} = m_A \frac{d}{dt}\vec{v}_A$$

Same logic for B

$$\vec{a}_B = \frac{\vec{F}_{net,B}}{m_B} = \frac{\vec{F}_{onBbyA}}{m_B}$$

$$\vec{F}_{onBbyA} = m_B \frac{d}{dt}\vec{v}_B$$

3rd law

$$\vec{F}_{onBbyA} = -\vec{F}_{onAbyB}$$

$$\vec{F}_{onAbyB} = -m_B \frac{d}{dt}\vec{v}_B$$

$$-m_B \frac{d}{dt}\vec{v}_B(t) = m_A \frac{d}{dt}\vec{v}_A(t)$$

m's constant!

$$c\frac{d}{dx}f(x) = \frac{d}{dx}(cf(x))$$

$$-\frac{d}{dt}\left(m_B\vec{v}_B(t)\right) = \frac{d}{dt}\left(m_A\vec{v}_A(t)\right)$$

$$0 = \frac{d}{dt}\left(m_A\vec{v}_A(t)\right) + \frac{d}{dt}\left(m_B\vec{v}_B(t)\right)$$

$$0 = \frac{d}{dt}\left(m_A\vec{v}_A(t) + m_B\vec{v}_B(t)\right)$$

$\Rightarrow m_A\vec{v}_A + m_B\vec{v}_B$ doesn't change

$$m_A\vec{v}_A + m_B\vec{v}_B = \text{const}$$

Define: momentum of object

$$\vec{p}_A = m_A\vec{v}_A \qquad \text{kg m/s}$$

If ~~2~~ any number objects are isolated

(ie only force on one due to other)

$$\vec{p}_{total} = \vec{p}_A + \vec{p}_B + \ldots = \sum m_i\vec{v}_i$$

is a constant, is "conserved"

$\vec{p}$ is conserved $\rightarrow$ laws of physics same everywhere

7-3-Example-
MomentumI

Momentum - I

A $3kg$ mass travels at $10\frac{m}{s}\hat{\imath} - 3\frac{m}{s}\hat{\jmath}$. A $4kg$ mass travels at $-4\frac{m}{s}\hat{\imath} - 2\frac{m}{s}\hat{\jmath}$. A $5kg$ mass travels at $1\frac{m}{s}\hat{\imath} + 2\frac{m}{s}\hat{\jmath}$.

- What is the x-component of the total momentum of these masses?
- What is the y-component of the total momentum of these masses?
- What is the magnitude of the total momentum of these masses?
- What is the velocity of the center of mass of this set of three particles?

- Get $\vec{p}$ for each
- Add them up.

3kg: $\vec{p} = m\vec{v}$

$\vec{p}_3 = (3kg)(10 m/s\,\hat{\imath} - 3 m/s\,\hat{\jmath})$

$= 30\,kg\,m/s\,\hat{\imath} - 9\,kg\,m/s\,\hat{\jmath}$

4kg: $\vec{p}_4 = (4kg)(-4 m/s\,\hat{\imath} - 2 m/s\,\hat{\jmath})$

$= -16\,kg\,m/s\,\hat{\imath} - 8\,kg\,m/s\,\hat{\jmath}$

5kg $\vec{p}_5 = (5kg)(1 m/s\,\hat{\imath} + 2 m/s\,\hat{\jmath})$

$= 5\,kg\,m/s\,\hat{\imath} + 10\,kg\,m/s\,\hat{\jmath}$

$$\vec{P}_{total} = \vec{P}_3 + \vec{P}_4 + \vec{P}_5$$

$$= \left(30\,kg\,m/s\,\hat{\imath} - 9\,kg\,m/s\,\hat{\jmath}\right)$$

$$+ \left(-16\,kg\,m/s\,\hat{\imath} - 8\,kg\,m/s\,\hat{\jmath}\right)$$

$$+ \left(5\,kg\,m/s\,\hat{\imath} + 10\,kg\,m/s\,\hat{\jmath}\right)$$

$$= 19\,kg\,m/s\,\hat{\imath} - 7\,kg\,m/s\,\hat{\jmath}$$

$$\vec{P}_{total} \cdot \hat{\imath} = P_{total,x} = 19\,kg\,m/s$$

$$P_{total,y} = -7\,kg\,m/s$$

$$|\vec{P}_{total}| = \sqrt{(19\,kg\,m/s)^2 + (-7\,kg\,m/s)^2}$$

$$= 20.25\,kg\,m/s$$

$$\vec{v}_{cm} = \frac{d}{dt}\vec{r}_{cm} = \frac{d}{dt}\left(\frac{m_3\vec{r}_3 + m_4\vec{r}_4 + m_5\vec{r}_5}{m_3 + m_4 + m_5}\right)$$

$$\vec{v}_{cm} = \frac{1}{m_3 + m_4 + m_5}\frac{d}{dt}\left(m_3\vec{r}_3 + m_4\vec{r}_4 + m_5\vec{r}_5\right)$$

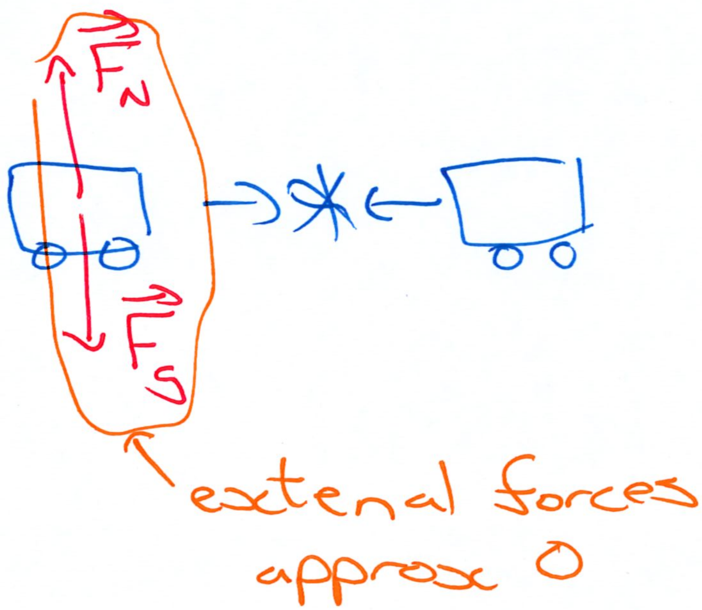
$$= \frac{1}{m_3 + m_4 + m_5}\underbrace{\left(m_3\vec{v}_3 + m_4\vec{v}_4 + m_5\vec{v}_5\right)}_{\vec{P}_{total}}$$

$$= 1.58\,m/s\,\hat{\imath} - 0.58\,m/s\,\hat{\jmath}$$

Collisions

Since objects on ground etc looks like not isolated → don't expect $\vec{p}$ conserved.

In case of something like vehicles colliding, the force they exert on each other is larger than the other relevent forces in problem



7-5-Example-Momentum II

Momentum - II

A lump of clay of mass $4kg$ travels with velocity $8\frac{m}{s}\hat{\imath}$ on a horizontal frictionless surface. It strikes and sticks to a ball of mass $1kg$ which travels on the same surface with velocity $18\frac{m}{s}\hat{\jmath}$.

- What speed does the clay and ball combo move at after the collision?
- What angle does the clay and ball combo's motion make with the x-axis after the collision?
- What is the change in momentum of the clay in the collision?
- What is the change in momentum of the ball in the collision?

No net external forces

$\Rightarrow$ use $\vec{p}$ = conserved.

$$\vec{P}_{total,\,before} = \vec{P}_{total,\,after}$$

$$\vec{P}_{clay,b} + \vec{P}_{ball,b} = \vec{P}_{clay,a} + \vec{P}_{ball,a}$$

$$= m_{clay}\vec{v}_{clay,a} + m_{ball}\vec{v}_{ball,a}$$

know

$$\vec{P}_{clay,b} + \vec{P}_{ball,b} = (m_{clay} + m_{ball})\vec{v}_{after}$$

$$\vec{P}_{clay,b} = 4kg(8\,m/s\,\hat{i}) = 32\,kg\,m/s\,\hat{i}$$

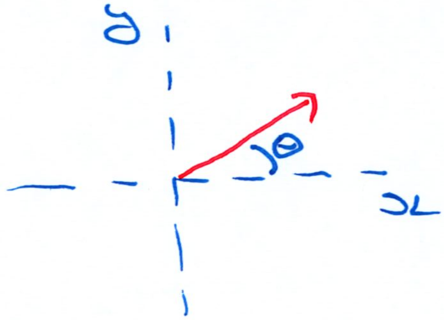
$$\vec{P}_{ball,b} = 1kg(18\,m/s\,\hat{j}) = 18\,kg\,m/s\,\hat{j}$$

$$\vec{P}_{total,before} = 32\,kg\,m/s\,\hat{i} + 18\,kg\,m/s\,\hat{j}$$

$$(32\,kg\,m/s\,\hat{i} + 18\,kg\,m/s\,\hat{j}) = (4kg + 1kg)\vec{v}_{after}$$

$$\vec{v}_{after} = 6.4\,m/s\,\hat{i} + 3.6\,m/s\,\hat{j}$$

$$|\vec{v}_{after}| = \sqrt{(6.4\,m/s)^2 + (3.6\,m/s)^2} \approx 7.34\,m/s$$



$$\cos\theta = \frac{6.4\,m/s}{7.34\,m/s}$$

$$\theta = 29.4^\circ$$

$$\Delta\vec{P}_{clay} = \vec{P}_{clay,a} - \vec{P}_{clay,b}$$

$$= (4kg)(6.4\,m/s\,\hat{i} + 3.6\,m/s\,\hat{j}) - 32\,kg\,m/s\,\hat{i}$$

$$= -6.4\,kg\,m/s\,\hat{i} + 14.4\,kg\,m/s\,\hat{j}$$

$$\Delta\vec{p}_{ball} = \vec{p}_{ball,a} - \vec{p}_{ball,b}$$

$$= (1\,kg)(6.4\,m/s\,\hat{i} + 3.6\,m/s\,\hat{j}) - 18\,kg\,m/s\,\hat{j}$$

$$= 6.4\,kg\,m/s\,\hat{i} - 14.4\,kg\,m/s\,\hat{j}$$

Note

$$\Delta\vec{p}_{ball} = -\Delta\vec{p}_{clay}$$