202201 Math 122 [A01] Quiz #1

January 20th, 2022

There are 2 pages and 5 questions. There are 12 marks available. The working time limit is 25 minutes, plus you have 5 minutes of time to upload solutions to Crowdmark. This gives the total time limit for the test as 30 minutes. A late penalty of 2% per minute will be applied to work submitted after the allowed time limit. Math and Stats standard calculators are allowed, but unnecessary! Except when indicated, it is necessary to show clearly organized work in order to receive full or partial credit. For the true/false questions, enter your choice directly in Crowdmark. (You may write your true/false answer in the blank space provided if you wish, but the answer you enter on Crowdmark will be the one that is graded as your final answer.)

1. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F (a) Knowing that an if-then statement such as $p \to q$ is true means that you can say the hypothesis p is true. May have p & q both false.

   F (b) If $p \wedge q$ is false, then $p \leftrightarrow q$ is false. p & q both false gives $p \wedge q$ false but $p \leftrightarrow q$ true

   T (c) There are truth values of p and q so that the statements $\neg p \vee q$ and $p \leftrightarrow \neg q$ are both true.
   p false and q true gives $\neg p \vee q$ true and $p \leftrightarrow \neg q$ true

   F (d) The statement $p \wedge (q \vee r)$ is logically equivalent to the statement $(p \wedge q) \vee r$.
   Look at $p:0, q:0, r:1$ $p \wedge (q \vee r)$ is false, $(p \wedge q) \vee r$ is true

2. [3] For the statement "*if mn is irrational, then m is irrational or n is irrational*", write each of the following in plain English:
   $p \to (q \vee r)$
   p: mn is irrational, q: m is irrational, r: n is irrational

   (a) Contrapositive: $\neg(q \vee r) \to \neg p \iff (\neg q \wedge \neg r) \to \neg p$

   If m is rational and n is rational, then mn is rational.

   (b) Converse: $(q \vee r) \to p$

   If m is irrational or n is irrational, then mn is irrational.

   (c) Negation: $\neg(p \to (q \vee r)) \iff \neg(\neg p \vee (q \vee r))$
   $\iff p \wedge \neg(q \vee r)$
   $\iff p \wedge \neg q \wedge \neg r$

   mn is irrational and m is rational and n is rational.

3. Let m, e, and f be the following statements:

- m : Michelle orders oat milk in her coffee;
- e : there is an extra fee;
- f : Michelle orders flavour in her coffee

Write each of the following in symbolic form using only m, e, f, $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, and brackets.

(a) [1] There is an extra fee when Michelle orders a flavoured coffee with oat milk.

$(m \wedge f) \rightarrow e$

(b) [1] Michelle orders oat milk in her coffee or a flavoured coffee but not both.

$(m \vee f) \wedge \neg(m \wedge f) \Leftrightarrow (m \wedge \neg f) \vee (\neg m \wedge f) \Leftrightarrow \neg(m \leftrightarrow f)$

4. Consider the statements $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$.

(a) [2] Use a truth table to determine if the statements $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are logically equivalent. Explain how your truth table shows your conclusion.

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$q \rightarrow r$	$p \rightarrow (q \rightarrow r)$
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	0	1	0	1
0	1	1	0	1	1	1
1	0	0	0	1	1	1
1	0	1	0	1	1	1
1	1	0	1	0	0	0
1	1	1	1	1	1	1

Columns for $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are the same so the two statements are logically equivalent.

(b) [1] Based on your answer to part (a), is the statement $[(p \wedge q) \rightarrow r] \leftrightarrow [p \rightarrow (q \rightarrow r)]$ a tautology? Explain.

Yes. Since truth values of $(p \wedge q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ always agree, $[(p \wedge q) \rightarrow r] \leftrightarrow [p \rightarrow (q \rightarrow r)]$ is a tautology. (Also because the two statements are logically equivalent.)

5. [2] Use the blank to indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

T (a) The statement $p \leftrightarrow \neg p$ is a contradiction. p and $\neg p$ never have same truth value.

F (b) The the negation of the statement $a \rightarrow b$ is $\neg a \rightarrow \neg b$. $\neg a \rightarrow \neg b$ is the Inverse negation is $a \wedge \neg b$.

T (c) If $s_1 \Leftrightarrow \neg s_2$, then $s_2 \Leftrightarrow \neg s_1$. $s_1 \Leftrightarrow \neg s_2$ means s_1 always has opposite truth value of s_2.

F (d) If two statements are not logically equivalent, then one statement is the negation of the other. not logically equivalent means there is a place where truth values disagree, negation means truth values always disagree.