Assignment 02

Part 1 (Question 1)

$$\vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

(a) $\vec{v} - \vec{w}$

$$= \begin{bmatrix} -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 \\ 3-2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

(b) $2\vec{v} + (3\vec{w} - \vec{v})$

$$= 2\begin{bmatrix} -1 \\ 3 \end{bmatrix} + \left(3\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)$$

$$= \begin{bmatrix} -2 \\ 6 \end{bmatrix} + \begin{bmatrix} 6 \\ 6 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 6 \end{bmatrix} + \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

(c) $\vec{w} - 2(\vec{v} + 3(\vec{w} - \vec{v}))$

$$3(\vec{w} - \vec{v}) = 3\left(\begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} -1 \\ 3 \end{bmatrix}\right)$$

$$= 3\begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 9 \\ -3 \end{bmatrix}$$

$$2(\vec{v} + 3(\vec{w} - \vec{v}))$$

$$= 2\vec{v} + 2 \cdot 3(\vec{w} - \vec{v})$$

$$= 2\begin{bmatrix} -1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 9 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -2 \\ 6 \end{bmatrix} + \begin{bmatrix} 18 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 18-2 \\ 6-6 \end{bmatrix} = \begin{bmatrix} 16 \\ 0 \end{bmatrix}$$

$$\vec{w} - 2(\vec{v} + 3(\vec{w} - \vec{v}))$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 16 \\ 0 \end{bmatrix} = \begin{bmatrix} 2-16 \\ 2-0 \end{bmatrix}$$

$$= \begin{bmatrix} -14 \\ 2 \end{bmatrix}$$

Part 1 (Question 2)

We have to find a vector in R_3 i.e $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ that cannot be written as a linear combination of $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix}$.

What we're essentially looking for is:

$$a\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} + b\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + c\begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \neq \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

In order to find the scalar values of a, b and c, we have to turn it into a matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & x \\ -3 & 1 & -7 & y \\ 2 & 1 & 3 & z \end{array}\right]$$

$$\xrightarrow{R_2 + 3R_1} \left[\begin{array}{ccc|c} 1 & -2 & 4 & x \\ 0 & -5 & 5 & y+3x \\ 2 & 1 & 3 & z \end{array}\right]$$

$$\xrightarrow{R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -2 & 4 & x \\ 0 & -5 & 5 & y+3x \\ 0 & 5 & -5 & z-2x \end{array}\right]$$

$$\xrightarrow{R_3 + R_2} \left[\begin{array}{cccc} 1 & -2 & 4 & x \\ 0 & -5 & 5 & y+3x \\ 0 & 0 & 0 & z+y+x \end{array}\right]$$

$$\xrightarrow{\frac{1}{5}R_2}\left[\begin{array}{ccc|c} 1 & -2 & 4 & x \\ 0 & -1 & 1 & \frac{y+3x}{5} \\ 0 & 0 & 0 & x+y+z \end{array}\right]$$

$$\xrightarrow{(-1)R_2}\left[\begin{array}{ccc|c} 1 & -2 & 4 & x \\ 0 & 1 & -1 & -\frac{y+3x}{5} \\ 0 & 0 & 0 & x+y+z \end{array}\right]$$

$$\xrightarrow{R_1+2R_2}\left[\begin{array}{ccc|c} 1 & 0 & 2 & x+2\left(-\frac{y+3x}{5}\right) \\ 0 & 1 & -1 & -\frac{y+3x}{5} \\ 0 & 0 & 0 & x+y+z \end{array}\right]$$

$$\longrightarrow\left[\begin{array}{ccc|c} 1 & 0 & 2 & -\left(\frac{x+2y}{5}\right) \\ 0 & 1 & -1 & -\left(\frac{y+3x}{5}\right) \\ 0 & 0 & 0 & (x+y+z) \end{array}\right]$$

$$x+2\left(-\frac{y+3x}{5}\right)$$

$$= x+\left(\frac{-2y-6x}{5}\right)$$

$$= \frac{5x-2y-6x}{5}$$

$$= \frac{-x-2y}{5}$$

$$= -\left(\frac{x+2y}{5}\right)$$

If $(x+y+z) \neq 0$, then $\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix}$ will not have a vector of linear combination in $\mathbb{R}^3$.

Part 2 (Question 1)

$\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ -2 \\ 6 \\ -2 \end{bmatrix}$ can be written as the

linear combination of $\begin{bmatrix} -6 \\ 4.5 \\ 15 \\ -24 \\ 15 \end{bmatrix}$ like

$$a\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \\ -2 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \\ 5 \end{bmatrix} + c\begin{bmatrix} 2 \\ -1 \\ -2 \\ 6 \\ -2 \end{bmatrix} = \begin{bmatrix} -6 \\ 4.5 \\ 15 \\ -24 \\ 15 \end{bmatrix}$$

which can be expressed in augmented matrix as:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -6 \\ -1 & 1 & -1 & 4.5 \\ 2 & 3 & -2 & 15 \\ -1 & -1 & 6 & -24 \\ -2 & 5 & -2 & 15 \end{array}\right]$$

putting the matrix (as "a") on MATLAB for rref(a), we get,

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

where, $a = 1$

$b = 2$

$c = -3.5$

Here we can conclude that the vector $\begin{bmatrix} -6 \\ 4.5 \\ 15 \\ -24 \\ 15 \end{bmatrix}$ can be written as

a linear combination of $\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \\ -2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \\ -2 \\ 6 \\ -2 \end{bmatrix}$ as,

$$\begin{bmatrix} 1 \\ -1 \\ 2 \\ -1 \\ -2 \end{bmatrix} + 2 \begin{bmatrix} 0 \\ 1 \\ 3 \\ -1 \\ 5 \end{bmatrix} - 3.5 \begin{bmatrix} 2 \\ -1 \\ -2 \\ 6 \\ -2 \end{bmatrix} = \begin{bmatrix} -6 \\ 4.5 \\ 15 \\ -24 \\ 15 \end{bmatrix}$$

(Ans)

Part 2 (Question 2)

let $\vec{\omega_1} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $\vec{\omega_2} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$, $\vec{\omega_3} = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

According to the question,

$$a \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + b \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \quad \text{———} (1)$$

where a and b are any scalar values.

We need to show that $3\vec{\omega_1} + 5\vec{\omega_2} - 4\vec{\omega_3}$ can be written as a linear equation.

Now, $3\vec{\omega_1} + 5\vec{\omega_2} - 4\vec{\omega_3}$

(plotting (1))

$$= 3\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + 5\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} - 4\left(a\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + b\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right)$$

$$= (3-4a)\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + (5-4b)\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$= \vec{\omega_1}(3-4a) + \vec{\omega_2}(5-4b)$$

(a and b can be any numbers/scalar values)

Therefore, we can conclude that $3\vec{\omega_1} + 5\vec{\omega_2} - 4\vec{\omega_3}$ can also be written as a linear combination of $\vec{\omega_1}$ and $\vec{\omega_2}$.