Potential Energy

- Recall: A force is conservative if the work done by the force does not depend on path chosen
- Recall: Work-Energy theorem

$$\Delta KE = \underbrace{W_{net}}_{\text{add up all forces}}$$

$$\Delta KE = W_{cons} + W_{NC}$$

$$\Delta KE + (-W_{cons}) = W_{NC}$$

If no non-cons. forces then

$$\Delta KE + \underbrace{(-W_{cons})}_{\text{call it } \Delta PE} = 0$$

$$\Delta KE + \Delta PE = 0$$

$$KE_f - KE_i + PE_f - PE_i = 0$$

$$KE_f + PE_f = KE_i + PE_i$$

Note: Since ΔPE doesn't depend on path it only depends on position $PE(\vec{r})$

PE is a function of object's location

10-3-Example-PE1

Potential Energy - I

1 A mass $5kg$ moves from $1m$ above the Earth's surface to $7m$ above the Earth's surface?

- How much work did gravity do on the mass?
- What is the change in gravitational potential energy of the mass?

$$-\Delta PE = W_c$$

$$-\Delta PE_g = W_g$$

$$\vec{F}_g = -mg\hat{k}$$

$$\Delta\vec{r} = (x_f\hat{i} + z_f\hat{k}) - (x_i\hat{i} + z_i\hat{k})$$

$$= (x_f - x_i)\hat{i} + (z_f - z_i)\hat{k}$$

$$W_g = \vec{F}_g \cdot \Delta\vec{r}$$

$$= (-mg\hat{k}) \cdot \left[(x_f - x_i)\hat{i} + (z_f - z_i)\hat{k}\right]$$

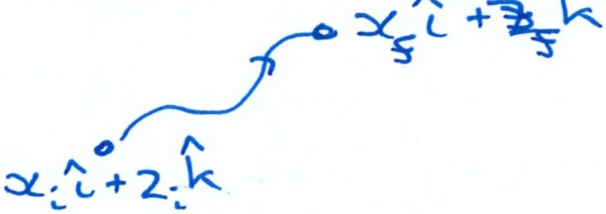
$$= -mg(z_f - z_i) \longrightarrow W_g = -29.4J$$

$$\Delta PE_g = -W_g = -(-mg(z_f - z_i))$$

$$= mg(z_f - z_i) \qquad \Delta PE_g = 29.4J$$

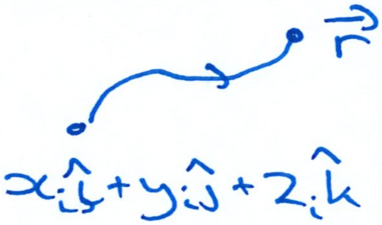
Last Example:

Found



$\Delta PE = mg(z_f - z_i)$

Potential Energy is a function of position



$\Delta PE = mg(z - z_i)$

going from $\vec{r}_i$ to $\vec{r}$

fixed by "where started" but essentially arbitrary

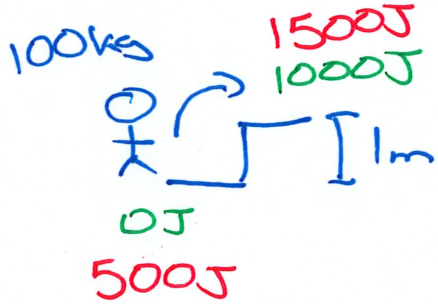
$PE(\vec{r}) = mgz + C$

$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$

analogous to a definite integral

analogous to indefinite integral

PE is defined up to an arbitrary constant



$\Delta PE \approx 1000J$

Useful fact:

$$\vec{F}_c = -\nabla PE(\vec{r})$$

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

"Partial Derivative" take derivative by z but pretend all other variables (x, y) constant

$$PE(\vec{r}) = mgz + C$$

$$\vec{F}_g = -\nabla PE(\vec{r})$$

$$= -\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(mgz + C)$$

$$= -(\hat{i}\,0 + \hat{j}\,0 + \hat{k}\,mg)$$

$$= -mg\hat{k}$$

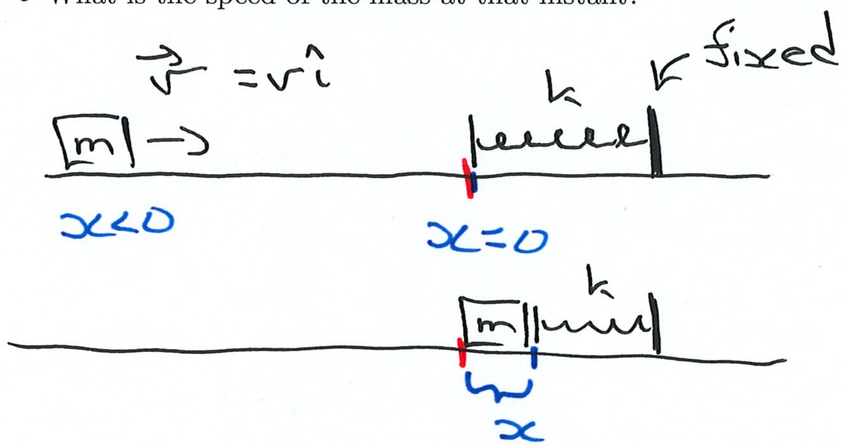
10-5-Example-PE2

Potential energy - II

A $2kg$ mass slides at $5\frac{m}{s}$ on a horizontal frictionless surface. It collides with an ideal spring with spring constant $500\frac{N}{m}$. It contacts the end of the spring and begins to compress it.

When the spring has been compressed by $0.2m$,

- How much work has the spring done?
- What is the change in the potential energy stored in the spring?
- What is the change in the mass's kinetic energy?
- What is the speed of the mass at that instant?



$\Delta PE = -W_c$

$\Delta PE_s = -W_s$

force by spring change

$W_s = \int \vec{F}_s \cdot d\vec{r}$

Parametrize the path

$\vec{r}(s) = \cancel{}\ s\hat{\imath}$

$\underset{\downarrow}{0\,m} \qquad \underset{\downarrow}{0.2\,m}$

$x_i \leq s \leq x_f$

Find $d\vec{r} = \left(\frac{d\vec{r}(s)}{ds}\right) ds$

$= (1\hat{\imath})\,ds$

$= ds\,\hat{\imath}$

Find $\vec{F}_s(s)$

$|\vec{F}_s| = k|\Delta\vec{\ell}|$

direction opposite
a restoring force

500N/m

spring compressed by s

$\vec{F}_s = -ks\hat{\imath}$

Find $\vec{F}\cdot d\vec{r} = (-ks\hat{\imath})\cdot(ds\hat{\imath})$

$= (-ks)(ds)$

$= -ks\,ds$

$W_s = \int_{x_i}^{x_f}(-ks\,ds) = -k\int_{x_i}^{x_f} s\,ds$

$= -k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right)$

$\int_a^b x\,dx = \left.\frac{x^2}{2}\right|_a^b = \frac{b^2}{2} - \frac{a^2}{2}$

Put in #'s $= -10\,J$

What is ΔPE_s?

$$\Delta PE_s = -W_s \rightarrow 10J$$

$$\Delta PE_s = PE_f - PE_i = -\left(-k\left(\frac{x_f^2}{2} - \frac{x_i^2}{2}\right)\right)$$

$$PE_f - PE_i = \frac{kx_f^2}{2} - \frac{kx_i^2}{2} + C - C$$

$$PE_f = \frac{kx_f^2}{2} + C$$

$$PE_i = \frac{kx_i^2}{2} + C$$

$$\boxed{PE_s = \frac{k}{2}|\Delta\vec{l}|^2 + C}$$

Often pick $C=0$ so $PE=0$ at unstretched length

$$\Delta KE = W_{net} = W_c + \cancel{W_{nc}}$$

0 b/c frictionless

$$= -\Delta PE$$

$$= -10J$$

$$KE_f - KE_i = -10J$$

$$KE_f - \frac{1}{2}2kg(5 m/s)^2 = -10J$$

$$KE_f = 15J$$

$$\frac{1}{2}(2kg)|\vec{v}_f|^2 = 15J \Rightarrow |\vec{v}_f| = 3.9 m/s$$