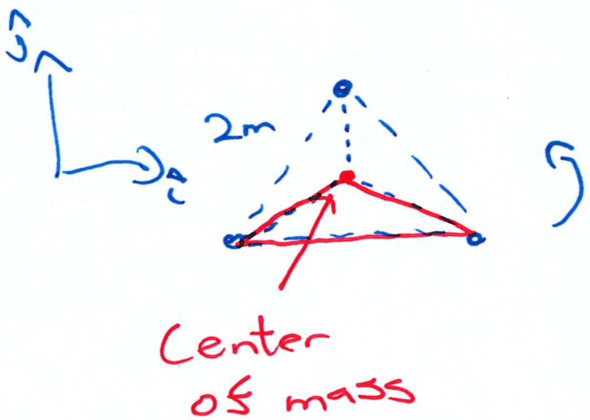
8-5-Example-AM2

Angular Momentum - II

A rigid object is made in the shape of an equilateral triangle with side length $2m$. There is a $1.5kg$ mass at each of the vertices of the triangle, and the object lies in the xy plane. The rest of the rigid object is massless.

The object completes two rotations per second around its center of mass. It is rotating around a line which is parallel to the z-axis.

- What is the moment of inertia of this object around its axis of rotation?
- What is the angular momentum of this rotating object?



$$I = \sum_i m_i |\vec{r}_i|^2$$

$$\vec{L} = \left(\text{rot}^n \text{ rate}\right) I \left(\text{unit vector along rotation}\right)$$

Using trig to get distance

$\cancel{\sin}\cos 30 = \frac{1m}{hyp}$ ← length I want

hyp = 1.155m

For 1st mass $(1.5kg)(1.155m)^2 \sim 2.0kgm^2$

2nd & 3rd same

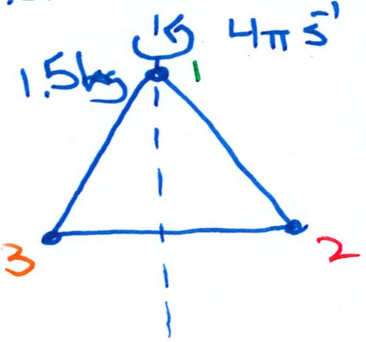
~~8-5 Example Ans~~

$$I = 6.0\,\text{kg}\,\text{m}^2$$

$$\vec{L} = \underbrace{\left(2\,\frac{\text{rot}}{\text{s}}\right)\left(2\pi\,\frac{1}{\text{rot}}\right)}_{\frac{d\theta}{dt} = 2(2\pi)\frac{1}{\text{s}}}\left(6.0\,\text{kg}\,\text{m}^2\right)\hat{k}$$

$$\vec{L} = 75.4\,\text{kg}\,\text{m}^2/\text{s}\;\hat{k}$$

What if the situation was



I different b/c depends on rotation axis

$$I = \sum_{\substack{\text{all}\\\text{masses}}} m_i |\vec{r}_i|^2 = 1.5\,\text{kg}(0\,\text{m})^2 + 1.5\,\text{kg}(1\,\text{m})^2 + 1.5\,\text{kg}(1\,\text{m})^2$$

$$= 3.0\,\text{kg}\,\text{m}^2$$

$$\vec{L} = 37.7\,\text{kg}\,\text{m}^2/\text{s}\;\hat{j}$$

How to see that $\vec{L}$ says something about how the object rotates.

$$\vec{\tau}_{net} = \frac{d}{dt}\vec{L}$$

For rigid, rotating object (along axis of symmetry

$$\vec{L} = \frac{d\theta}{dt}\, I \text{ (unit vec along axis)}$$

rotation rate

moment of inertia

Can determine the direction of $\vec{L}$ or, equiv, axis of rotation by using right-hand rule.

- All parts of rigid object have $\vec{L}$ in same direction
  Use right hand rule to get direction of $\vec{r}\times\vec{v}$ for one part, this points along axis where rotating.

Now $\vec{\tau}$ changes $\vec{L}$

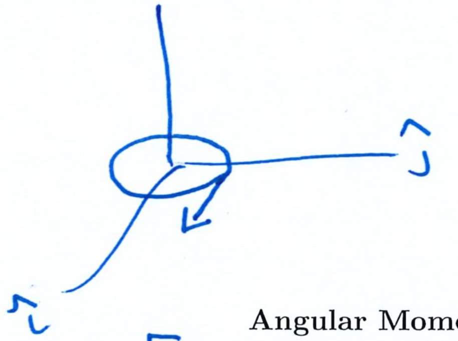
$$\frac{d}{dt}\vec{L} = \frac{d^2\theta}{dt^2} I \text{ (unit vec along rot}^{n}\text{ axis)}$$
$$+ \frac{d\theta}{dt} I \frac{d}{dt}\text{(unit vector)}$$

$\vec{\tau}_{net}$ can change rotation rate

$\vec{\tau}_{net}$ can change axis of rotation
ie change direction around which object rotating.

Example: a top

8-7-Example-AM3



Angular Momentum - III

A wheel is in the xy plane and is free to rotate around its axle which is oriented along the z-axis. The axle is centered at the origin. The wheel has radius $0.6m$, and moment of inertia $0.72kgm^2$.

The wheel is subject to a force $\vec{F} = 2N\hat{\imath}$ which is exerted at $\vec{r} = 0.6m\hat{\jmath}$. The wheel is at rest at $t = 0s$.

- What is the torque that the force exerts?
- What is the angular momentum of the wheel at $t = 3s$?
- What is the rate of rotation at $t = 3s$?
- What if the force were exerted at $\vec{r} = 0.2m\hat{\jmath}$?

$\vec{\tau} = \vec{r} \times \vec{F}$ (around center of wheel)

$= (0.6m\hat{\jmath}) \times (2N\hat{\imath})$

$= -1.2Nm\hat{k}$

At $t = 0s$, $\vec{L} = 0$ b/c at rest

$\frac{d}{dt}\vec{L} = -1.2Nm\hat{k}$

$\vec{L} = (-1.2Nm\hat{k})t \qquad t \geq 0s$

$\vec{L}(3s) = -3.6\underbrace{Nms}_{kgm^2/s}\hat{k}$

know

$\vec{L} = \left(\frac{d\theta}{dt}\right) I \left(\text{unit vec along rot}^n \text{ axis}\right)$

$= -3.6\,Nm\,\hat{k}$

$= \underbrace{3.6\,Nm}_{\left(\frac{d\theta}{dt}\right) I} \left(-\hat{k}\right)$

$3.6\,Nm$

$\left(\frac{d\theta}{dt}\right) 0.72\,kgm^2$

$\frac{d\theta}{dt} = 5\,s^{-1}$ ~ 0.8 rotations in 1s

If $\vec{F}$ at $0.2m\hat{j}$

$\vec{\tau} = -0.4\,Nm\,\hat{k}$

$\vec{L}(3s) = -1.2\,kgm^2/s$

$\frac{d\theta}{dt} = 1.67\,s^{-1}$ ~ 0.27 rotatins in 1s

Angular Momentum

$$\vec{\tau}_{net} = \frac{d}{dt}\vec{L}$$

$$\vec{L} = \left(\frac{d\theta}{dt}\right) I \text{ (unit vec along axis of rot)}$$

rotation rate

$$I = \sum m_i |\vec{r}_i|^2$$

distance to rotation axis

For momentum

$\vec{p}_{total}$ = const $\neq$ no change in motion

Figure skating

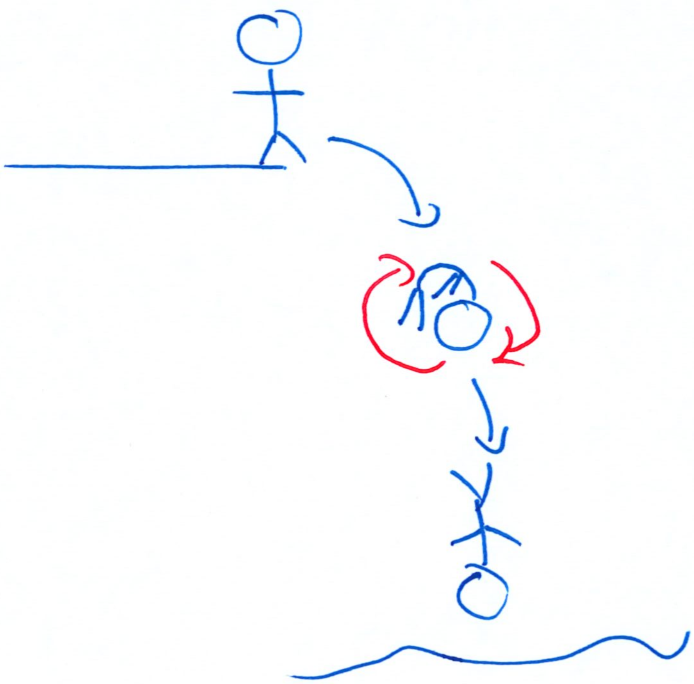
As arms & legs brought in I decreases

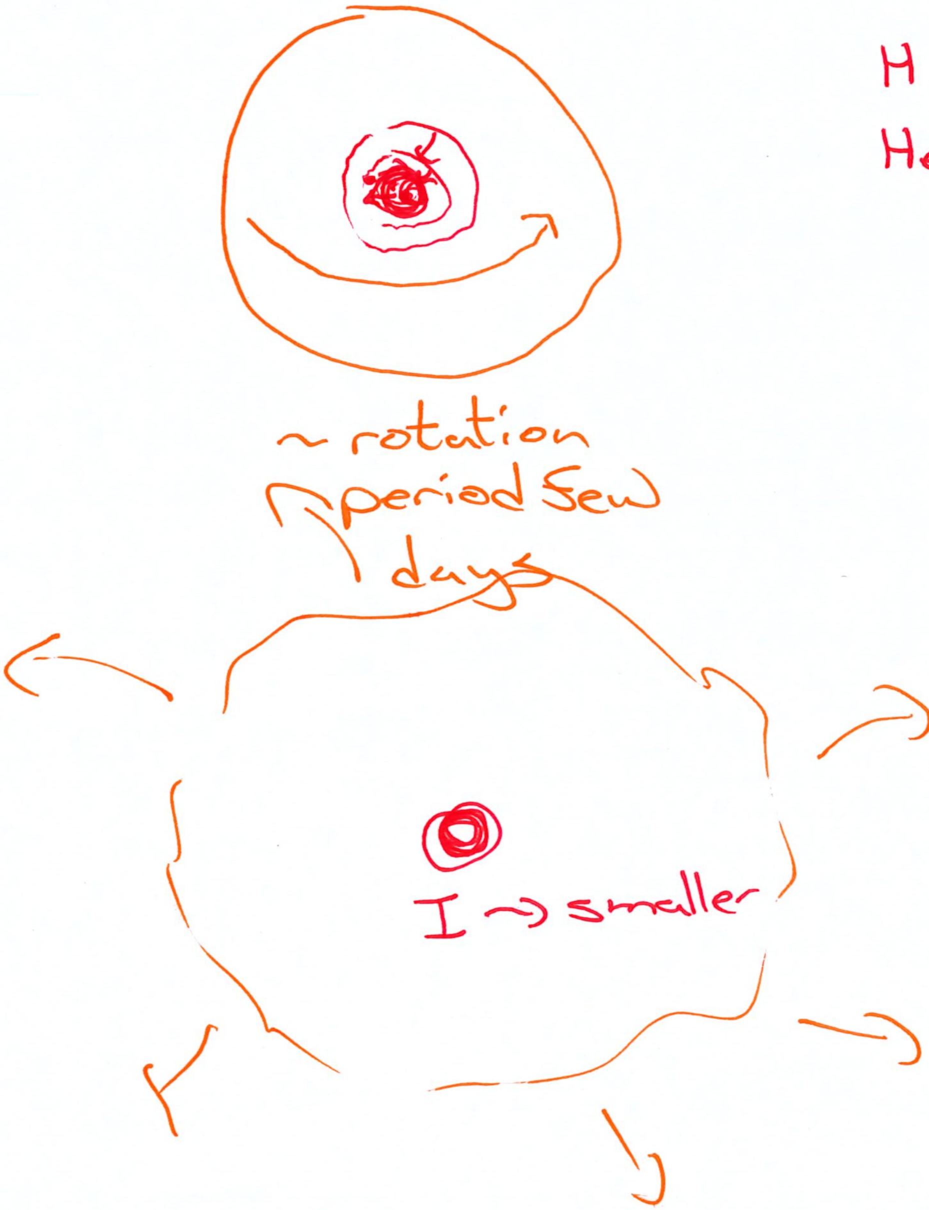
if no $\vec{\tau}$ $\rightarrow$ $\vec{L} =$ const

$$\left(\frac{d\theta}{dt}\right)_{init} I_{init} (\hat{z}) = \left(\frac{d\theta}{dt}\right)_{final} I_{final} (\hat{z})$$

must get bigger to compensate

smaller





$H \to He + \text{energy}$

$He \to C + \text{energy}$

$\vdots$

$\to Fe + \text{energy}$