Student: Arfaz Hossain **Date:** 02/28/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3]

Use the Integral Test to determine if the series shown below converges or diverges. Be sure to check that the conditions of the Integral Test are satisfied.

$$\sum_{k=3}^{\infty} \frac{8}{k(\ln k)^2}$$

Let $\{a_n\}$ be a sequence of positive terms. Suppose that $a_n = f(n)$, where f is a continuous, positive, decreasing function of x for all $x \geq N$ (N a positive integer). Then the series $\sum_{n=N}^{\infty} a_n$ and the integral $\int_N^{\infty} f(x)dx$ both converge or both diverge.

Consider the function $f(x) = \frac{8}{x \ln^2 x}$. Notice that this function is continuous, positive, and decreasing for $x \geq 3$.

Find the improper integral.

$$\int_3^{\infty} \frac{8}{x \ln^2 x} dx$$

First, find the indefinite integral.

$$\int \frac{8}{x \ln^2 x} dx$$

Make the substitution $u = \ln x$, $du = \frac{dx}{x}$.

$$\int \frac{8}{x \ln^2 x} dx = \int \frac{8}{u^2} du$$

$$= -\frac{8}{u} + C \qquad \text{Find the integral in terms of u.}$$

$$= -\frac{8}{\ln x} + C \qquad \text{Replace u with } \ln x.$$

Calculate the improper integral.

$$\int_3^{\infty} \frac{8}{x \ln^2 x} dx = \lim_{b \to \infty} \int_3^b \frac{8}{x \ln^2 x} dx$$

$$= \lim_{b \to \infty} -\frac{8}{\ln x}\Bigg]_3^b$$

Simplify.

$$\lim_{b \to \infty} -\frac{8}{\ln x}\Bigg]_3^b = \lim_{b \to \infty}\left(-\frac{8}{\ln b} + \frac{8}{\ln 3}\right) \qquad \text{Use the Fundamental Theorem of Calculus.}$$

$$= 0 + \frac{8}{\ln 3} \qquad \text{Find the limit.}$$

$$= \frac{8}{\ln 3} \qquad \text{Add.}$$

The improper integral is convergent.

Hence, the infinite series is convergent.