ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 35: The Payback Period and Project Crashing

December 2, 2022

Version 1

Recommended Viewing

- The Finance Storyteller. (2018, August 20). Payback period explained [Video File]. https://youtu.be/FJjGi7gsK3A
- AllThingsMathematics. (2017, July 3). FIN 300 – Payback Rule Overview – Ryerson University [Video File]. https://youtu.be/lJKwRvejjV8
- AllThingsMathematics. (2017, July 3). FIN 300 – Discounted Payback rule – Ryerson University [Video File]. https://youtu.be/ndKalS7HGhg
- Engineer4Free. (2015, February 15). Project crashing explained [Video]. https://youtu.be/yLPzSyBQ3-k
- Engineer4Free. (2015, February 16). Project crashing full example (part 1/2) [Video]. https://youtu.be/ZOW2QU_sbJ4
- Engineer4Free. (2015, February 16). Project crashing full example (part 2/2) [Video]. https://youtu.be/0oBUxrpzWhM

Recommended Reading

- *Engineering Economics* 6th edition,
- Chapter 11, Sections 11.3.3.1 and 11.3.3.2
- …especially Example 11.5 starting on p. 403

Optional Reading: Payback Period

- *Engineering Economics*, 6th edition, 4.5
- Abhinav, A., Ahmad, W. & Anand, S. (2015). Online monitoring of power extraction efficiency for minimizing payback period of solar PV system. *2015 IEEE International Conference on Industrial Technology (ICIT)*. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1109/ICIT.2015.7125520
- **Includes a payback period calculation in the context of the use of DC-link electrolytic capacitors in single phase solar PV inverters.**
- Mathews, G. E. & Mathews, E. H. (2016). Household photovoltaics – A Worthwhile Investment? *2016 International Conference on the Domestic Use of Energy (DUE)*. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1109/DUE.2016.7466716
- **Uses the payback period as one metric by which to judge whether household photovoltaics are worthwhile investments.**
- Mazzanti, G., Santini, E. & Romito, D. Z. (2012). Economic analysis of small-size photovoltaic plants in Italy in the framework of the "Conto Energia". *International Symposium on Power Electronics, Electrical Drives, Automation and Motion*. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1109/SPEEDAM.2012.6264602
- **Details on discounted and non-discounted payback periods.**

Optional reading on Implementing Crashing

- Aliyu, A.M. (2012). Project Management using Critical Path Methods: A pragmatic study. *Global Journal of Pure and Applied Sciences, 18*(3-4). https://doi.org/10.4314/gjpas.v18i3-4.11
- Baker, B. M. (1997). Cost/time trade-off analysis for the critical path method: a derivation of the network flow approach. *Journal of the Operational Research Society, 48,* 1241-1244. https://doi.org/10.1057/palgrave.jors.2600489
- Georges, N., Semaan, N. & Rizk, J. (2014). CRASH: An automated tool for schedule crashing. *International Journal of Science, Environment and Technology, 3*(2), 374-394. https://www.ijset.net/journal/264.pdf

Case Studies: Crashing

- Barković, D., Jukić, J. & Lujić, R. (2019). Minimizing the Pessimistic Time of Activity in Overhaul Project. *TEHNIČKI VJESNIK, 26*(2), 391-397. https://doaj.org/article/08cdb10925bf4fcb9241451260b39f39
- Biruk, S. & Jaskowski, P. (2020). Selection of the Optimal Actions for Crashing Processes Duration to Increase the Robustness of Construction Schedules. *Applied Sciences, 10*(22), 8028. https://doi.org/10.3390/app10228028
- Khalaf, W.S., June, L.W., Bakar, M.R.B.A. & Soon, L. L. (2011). A Comparative Study on Time-Cost Trade-Off Approaches within Critical Path Method. *Journal of Applied Sciences, 11*, 920-931. https://scialert.net/abstract/?doi=jas.2011.920.931

Learning Objectives

- Be aware of how to calculate the payback period & discounted payback period.
- To be able to use a CPM diagram to determine which activities to crash, and by how much, in order to shorten a project’s completion time.
- To be able to calculate the cost of crashing a project.

Payback Period

- Companies & managers like things to be done *quickly*, as seen by the popularity of the “payback period” as a measure of the viability of a project.
- Payback Period: How long it takes to earn back your initial costs.
- Many businesses require the payback period to be less than 4 years.
- If annual savings (net benefits from purchase) are constant:
- Payback Period = First Cost / Annual Savings
- e.g. You buy a candy machine for $100. It provides net weekly income of $10. Payback period = 100/10 = 10 weeks.
- If annual savings are not constant, keep adding cash flows until the initial payment is recovered.
- e.g. You buy a candy machine for $100. It provides an initial weekly income of $10, and this goes up by $10 a week (so $20,$30,$40…)
- $10 + $20 + $30 + $40 = $100, so the payback period is 4 weeks.

Pros and Cons

- Pros:
  - Easy to understand and calculate
  - Accounts for the need to recovery capital
  - The future is unknown, so why waste time on precise calculations? GIGO.
- Cons:
  - Discriminates against long-term projects
  - Ignores time value of money, timing of cash flows
  - Ignores expected service life and any benefits after payback

A partial fix: discounted payback period

- Suppose annual savings are A_t for each time period t…
- …and the project costs P up front.
- For each year t starting with t = 1…
- Calculate A_t x (P/F,i,t) and add it to the previous year's total.
- (For Year 1, the previous year's total is 0.)
- Stop once the total is first equal to or greater than P.
- (For constant A, this can be solved analytically: Solve A(P/A,i,N)=P for N)
- This is the *discounted payback period*.
- Pro: Takes the time value of money into account.
- Con: Loses the simplicity which is much of the appeal of the payback period approach.

Discounted Payback Period Example

Week	Net Income	Present Worth	Cumulative PW
0	0	$0.00	$0.00
1	10	$9.09	$9.09
2	20	$16.53	$25.62
3	30	$22.54	$48.16
4	40	$27.32	$75.48
5	50	$31.05	$106.53

< $100 Initial Cost

- The candy machine costs $100 in Week 0 (table only lists income, not costs)
- Net income is $10 in week 1, and goes up by $10 each week (Income = 10t).
- The MARR is 10% a week.
- PW of Week t income is (t x $10)(P/F,10%,t)
- Inspecting table, payback period is between 4 and 5 weeks, but closer to 5.
- (Possible but messy to solve exactly.)

Back to Critical Path Management (CPM)

- Last lecture, we saw that in the early 2000s, at least one large software company was responding to scheduling pressures by having *everyone* work overtime for long periods…
- CPM shows this is probably not required, or a good idea.

- Suppose we DO want to reduce project completion time in a rigorous, efficient manner.
- How do we go about it?

Project Crash

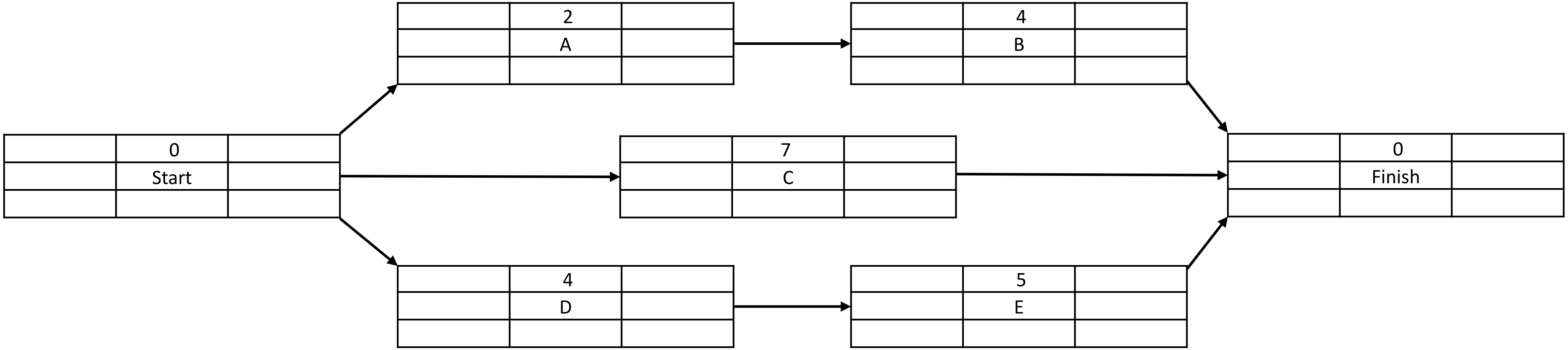
- GIVEN a critical path, a planner can make an informed tradeoff between reduced completion time and extra cost.
- Implicit: best-practice WBS and accurate cost/time estimates.
- Need Crashed and Uncrashed duration and cost data.
- To crash:
  - Identify which critical path activities are the least expensive to hasten.
  - Shorten those activities by the max amount or **until another critical path is created**.
  - Repeat until done or unable to continue.
- We'll be looking at that in detail next lecture.
- For today, we'll just look at a bird's-eye view.

A closer look at multiple critical paths.

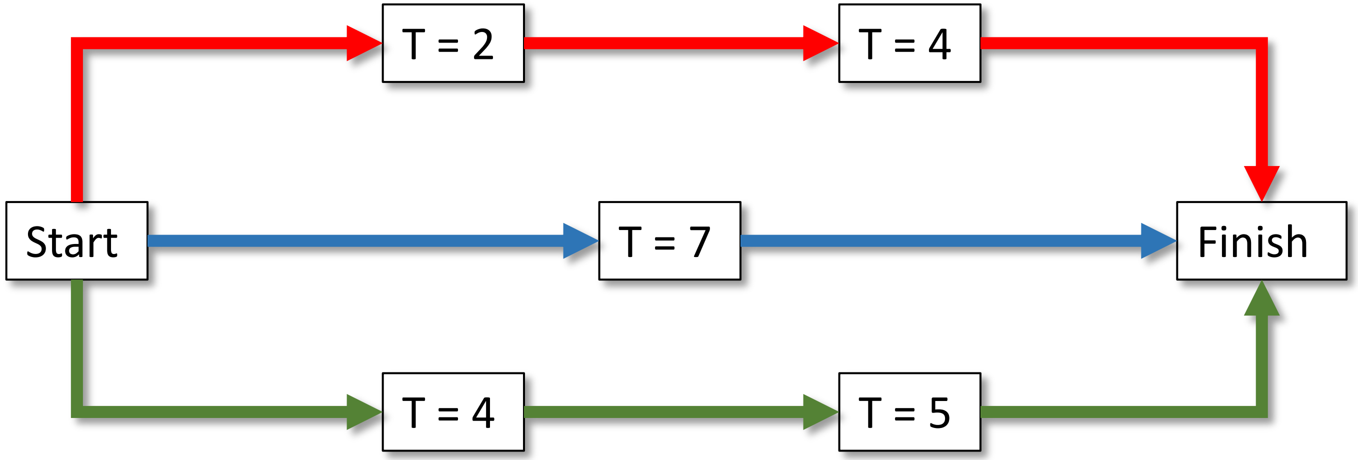
- Students (and project planners!) are often confused about what causes a new critical path to form.
- The following very simple example should clear things up.
- In this example, we start with one critical path, but gradually transform the project into one in with three critical paths.
- Tip: A critical path is a path from the start to the finish of the project. If there's a gap in the path, or if your start and finish nodes aren't critical, something's wrong! Check your work.

Simplifying things a bit

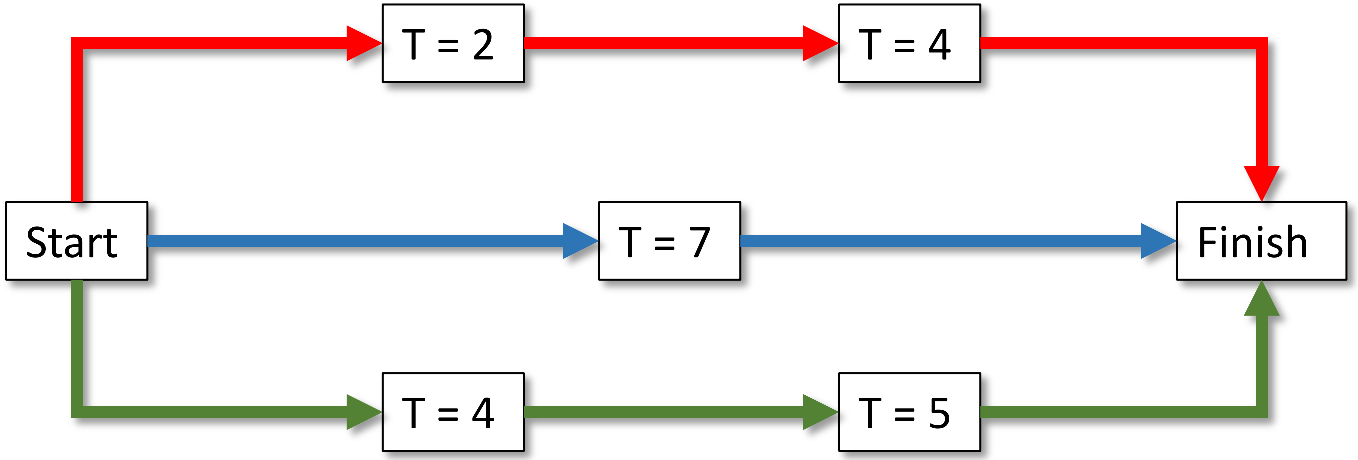
From our original AoN diagram…



To a schematic version

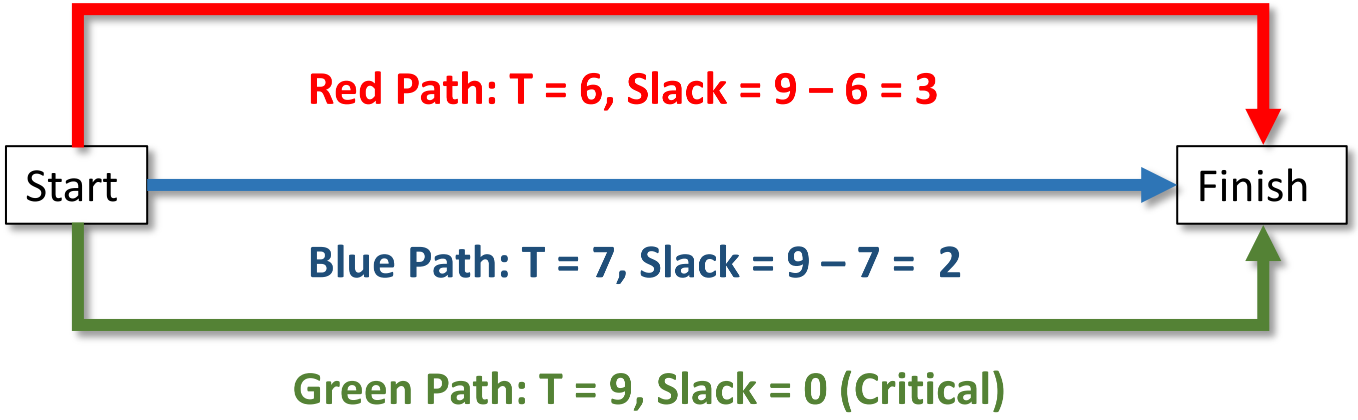


The beginning: three paths, one critical.



The **Green Path** takes 9 time units to finish.

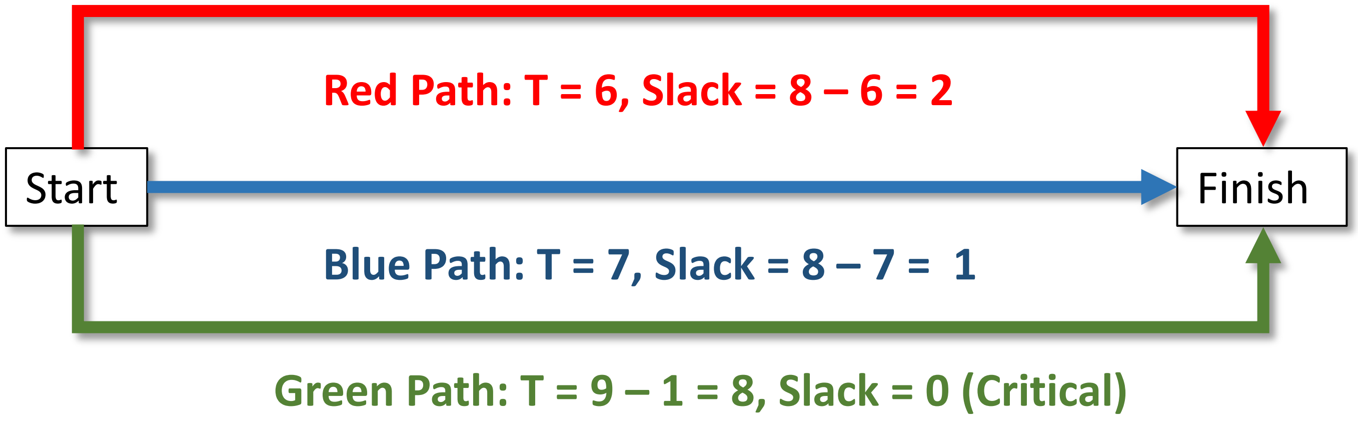
Start → Finish
T = 9



Since the other paths are shorter, it's the **Green Path** that determines completion time (i.e. is critical)

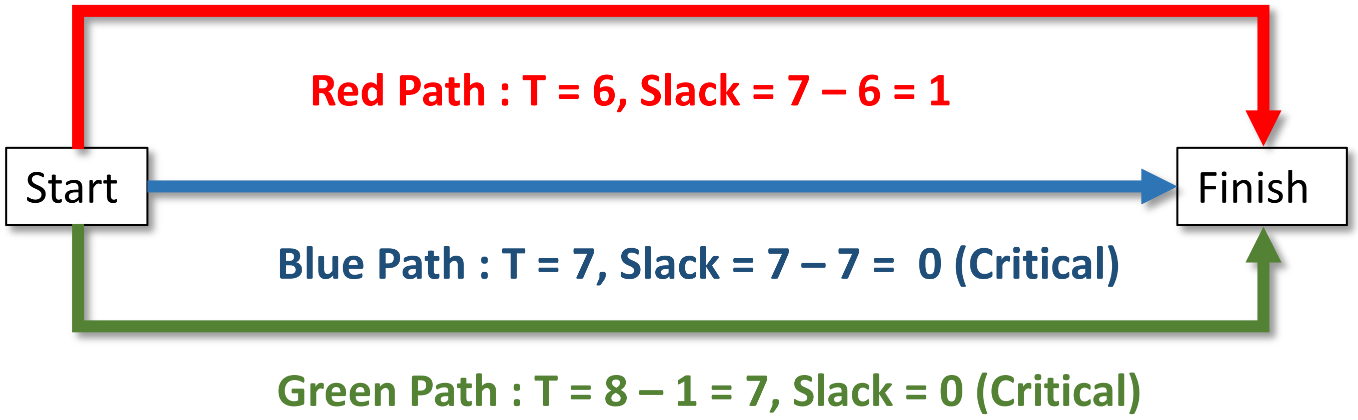
Crashing first by 1, then by 2 time periods.

Start → Finish
T = 8



We need to Crash the **Green Path** to see a reduction in completion time.

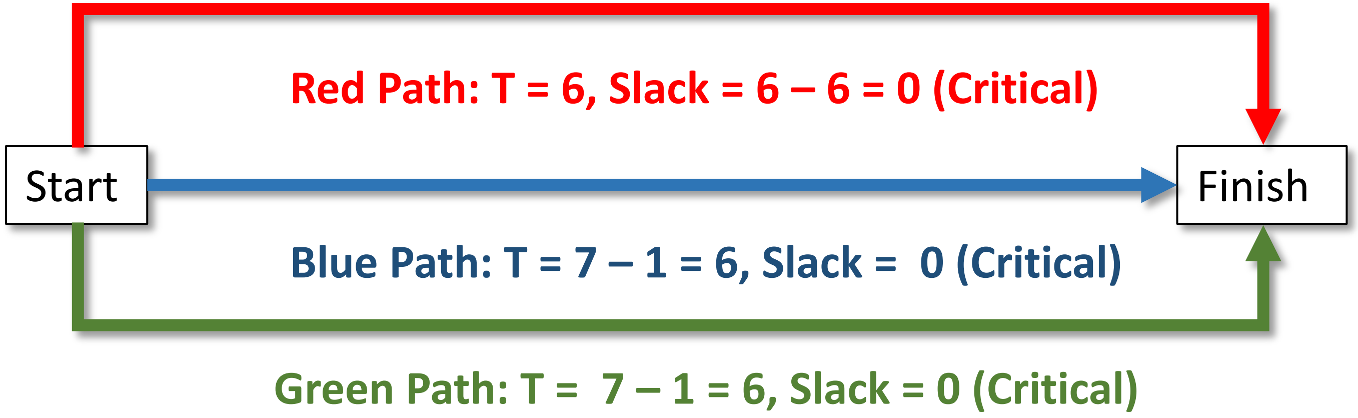
Start → Finish
T = 7



If we crash by *two* time periods, the **Blue Path** becomes critical because its slack is used up.

Crashing first by 3, then by 4 time periods.

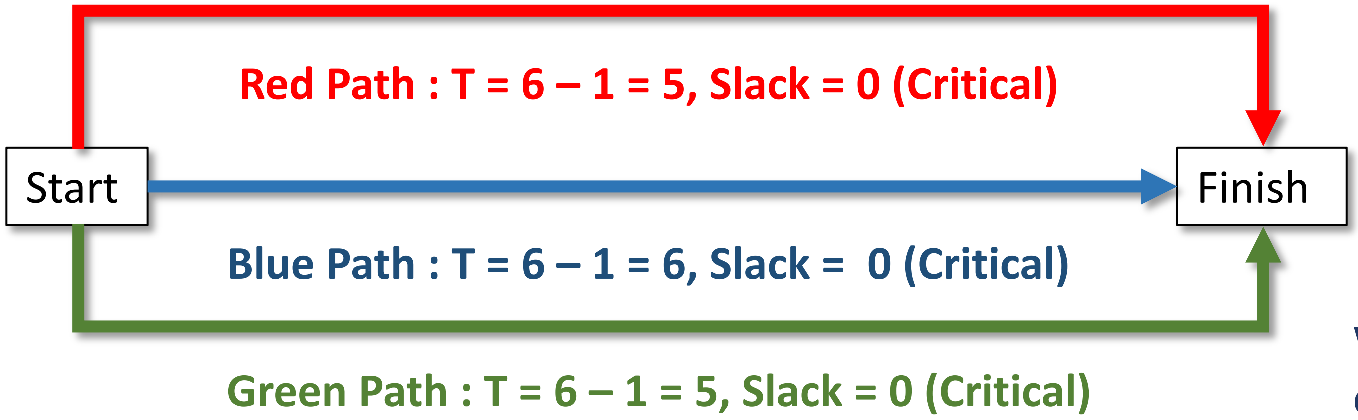
Start → Finish
T = 6



Crashing one path isn't enough – we need to crash *both* the **Green** and **Blue** paths to see a reduction in Finish time.

The end result is that the **Red Path**'s slack is used up, and also becomes critical.

Start → Finish
T = 5



To crash by *four* time periods, we need to crash *all three* paths by 1 time unit.

Without ever explicitly mentioning costs, we've obtained intuition into why schedule-shortening becomes very expensive very quickly!

Basic information for our example

Activity	Predecessors	Normal		Crashed	
		Duration (Days)	Cost ($)	Duration (Days)	Cost ($)
A	-	2	50	1	400
B	A	4	100	2	900
C	-	7	150	2	300
D	-	4	200	1	600
E	D	5	250	3	1,000

- To start with, we need some basic information: cost and duration for the normal and crashed activities, and predecessors.
- Recall: the 'crashed' values are for when we've pushed the activity as far as it can go. We've reduced the duration by as much as we can.
- The 'crashed cost' is what the activity costs, if we've chosen to reduce the duration by as much as we can. (Additional costs are due to overtime, etc.)

Days available for crashing & crash cost/day

- From last lecture, remember that we assume a linear relationship between crash cost and days crashed.
- It helps to tabulate the information we need, so we can refer back to it.
- Days available for crashing = Normal Duration – Crashed Duration
- Additional cost from maximum crashing = Crashed Cost – Normal Cost
- Therefore, given our assumption…
- Crash Cost / Day = Additional Cost / Days Available
- Crash Cost / Day = $\frac{\text{Crashed Cost} - \text{Normal Cost}}{\text{Normal Duration} - \text{Crashed Duration}}$

Days Available and Crash Cost/Day, Tabulated

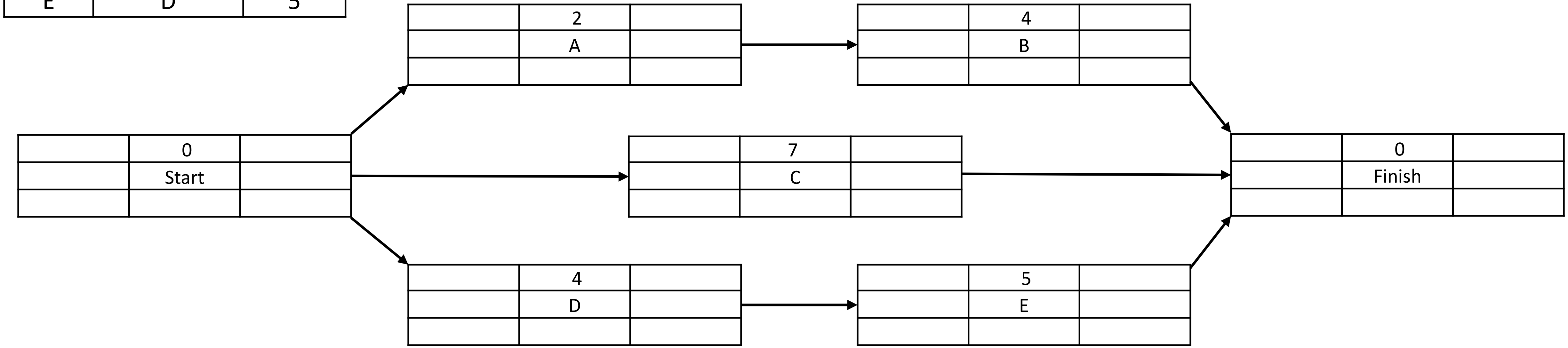
Activity	Predecessors	Normal		Crashed		Crash Days	
		Duration (Days)	Cost ($)	Duration (Days)	Cost ($)	Available	$/Day
A	-	2	50	1	400	1	350
B	A	4	100	2	900	2	400
C	-	7	150	2	300	5	30
D	-	4	200	2	600	2	200
E	D	5	250	3	1,000	2	375

- Very soon, we'll have to calculate the cost of partially crashing an activity (crashing it by less than the maximum).
- This cost will be equal to…
- Normal Cost + Days Crashed x (Crash Cost / Day)

Drawing the AoN Diagram

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

ES	T	EF
	ID	
LS	Slack	LF

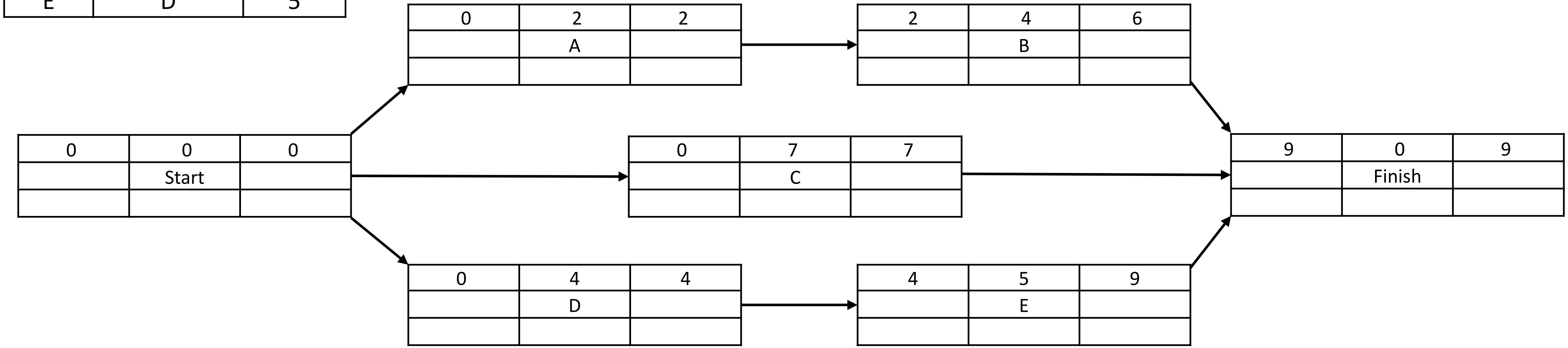


- For the network diagram, I'll be using the grid notation introduced in the previous lecture.
- Since I know I'll be doing that, I'm including the grids, even though I won't be using them yet.
- Since I already have the information, I <u>have</u> included the activity durations.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Forward Pass

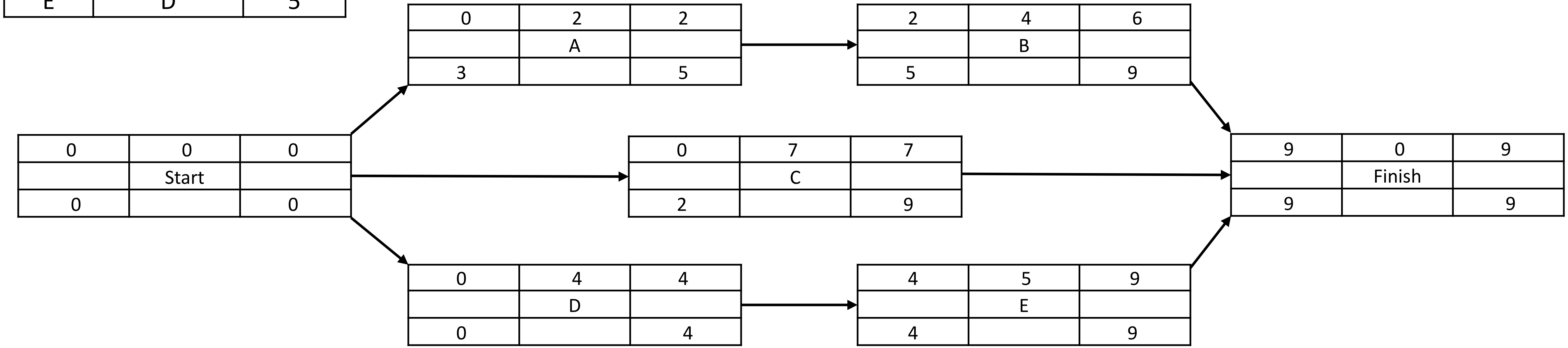
ES	T	EF
	ID	
LS	Slack	LF



Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Backward Pass

ES	T	EF
	ID	
LS	Slack	LF

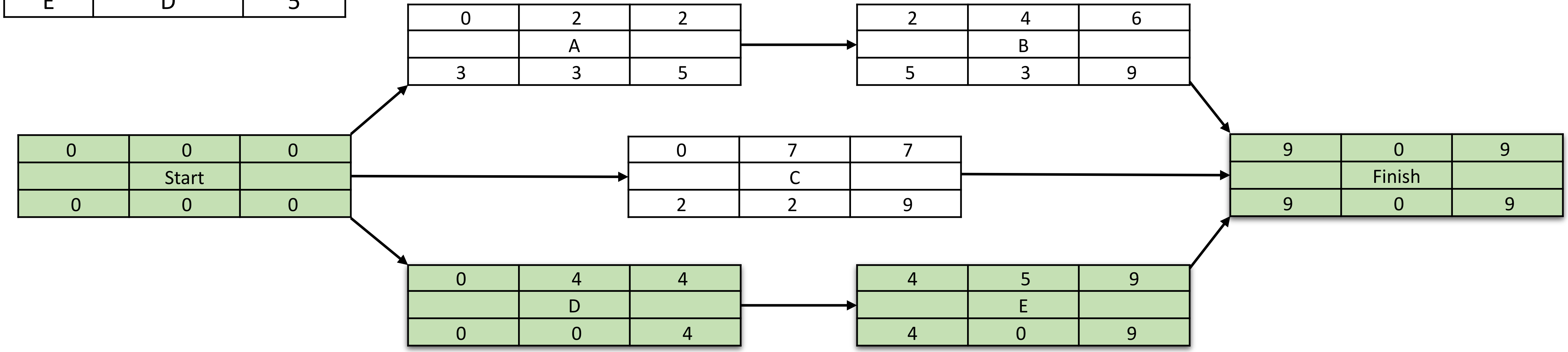


- For the backward pass, we start by assuming the project finishes at its earliest possible finish time (9 in this case).
- We use that as the Finish milestone's latest possible finish time and work backward, seeing how late each activity can start and still have the project finish on time.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Slack & Critical Path

ES	T	EF
	ID	
LS	Slack	LF



- Slack is the difference between early and late start & finish times
- i.e. Slack = (LS-ES) or (LF-EF) [They're off by T, so both give the same result.]
- Activities are on the critical path if and only if their slack is zero. I've shaded these green for convenience.
- The Start and Finish milestones will always be on the critical path.

Normal (Uncrashed) Cost

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0

Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

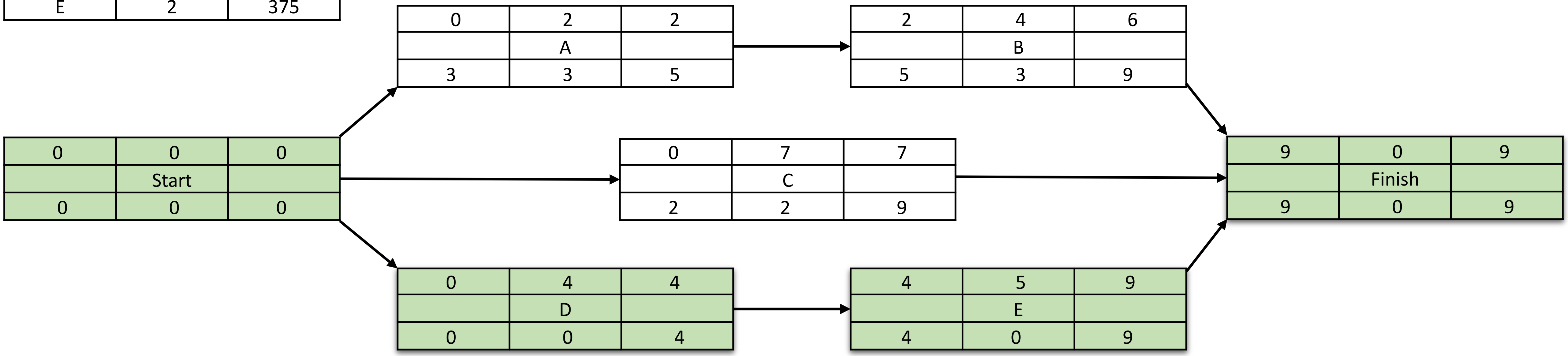
Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750

- We'll need Duration vs Cost information for our tradeoff curve, so it's worthwhile to start writing it down now.
- I find it helpful to keep track of the amount by which I've crashed each activity.
- In combination with the table on the right, listing Normal Cost and Crash Cost/Day…
- …I can write each activity's cost as Normal Cost + Days Crashed x (Crash Cost/Day)
- The total cost of the project is the sum of the costs of all its component activities.

Considering crashing…

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	5	30
D	2	200
E	2	375

ES	T	EF
	ID	
LS	Slack	LF

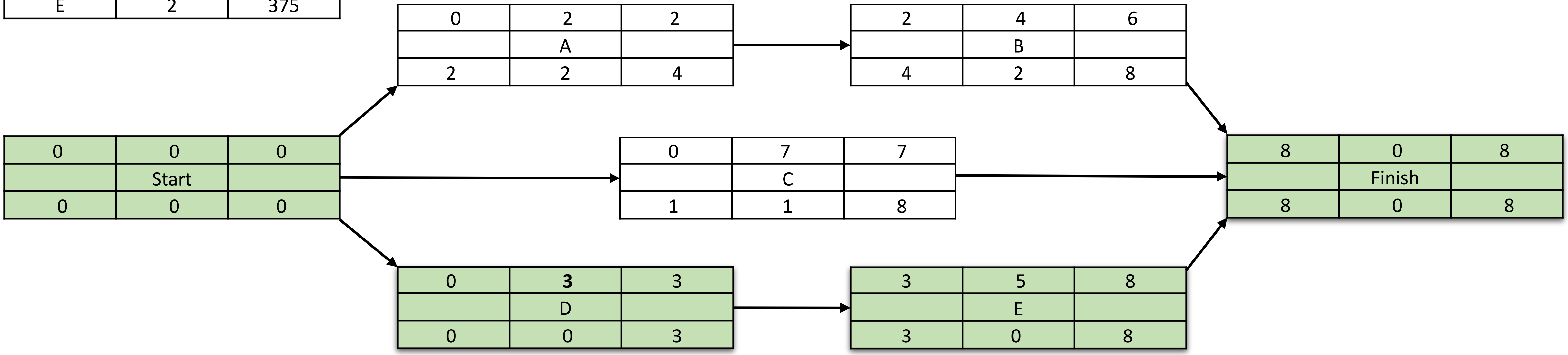


- Suppose we need to reduce the duration of the project by 1, to 8 days.
- We need to crash one of the activities on the critical path: D or E.
- D is cheapest, since it only costs $200 per day. Let’s do that.

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	5	30
D	**1**	200
E	2	375

Reduce Duration to 8

ES	T	EF
	ID	
LS	Slack	LF



- The duration of D has gone from 4 to 3, and we’ve used up one of the 2 days of crashing D has available.
- Note the ripple effect on the slack available to non-critical activities.

Continuing our cost calculations…

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0

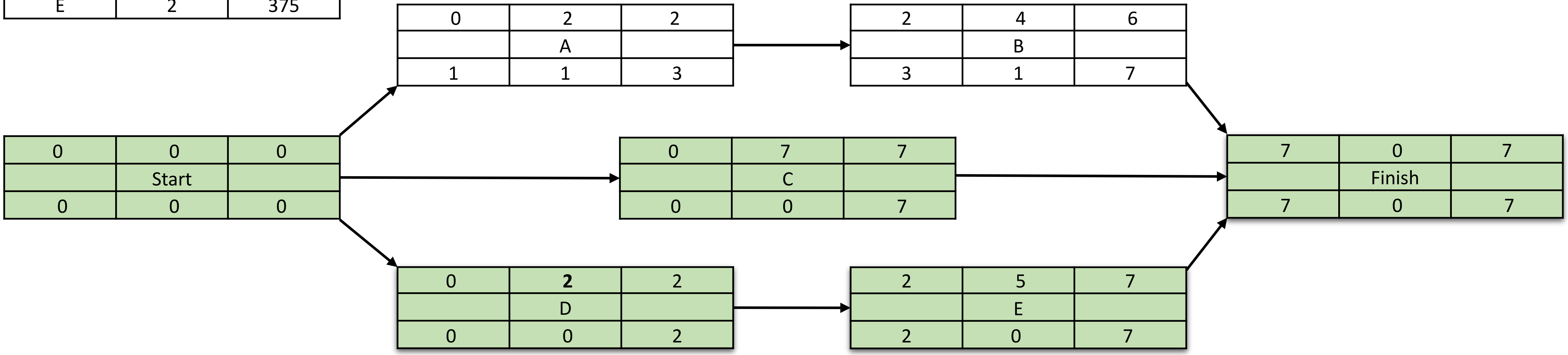
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950

Reduce Duration to 7

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	5	30
D	**0**	200
E	2	375

ES	T	EF
	ID	
LS	Slack	LF



- As before, we need to crash one of the activities on the critical path.
- Again, D is cheapest, so we use up our last available D day.
- Note that C's slack is now entirely gone, so C is critical.

Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0

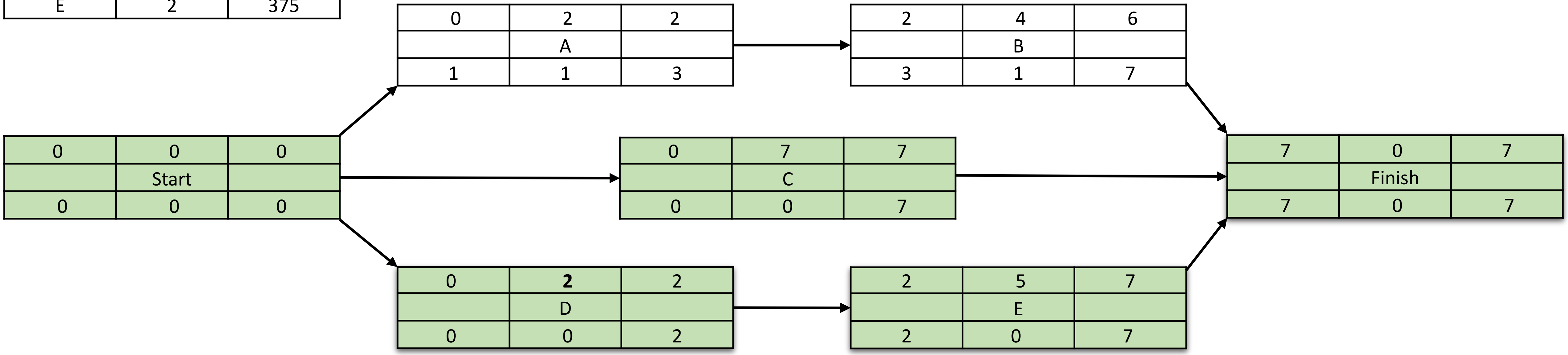
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	5	30
D	**0**	200
E	2	375

Reduce Duration to 6? How?

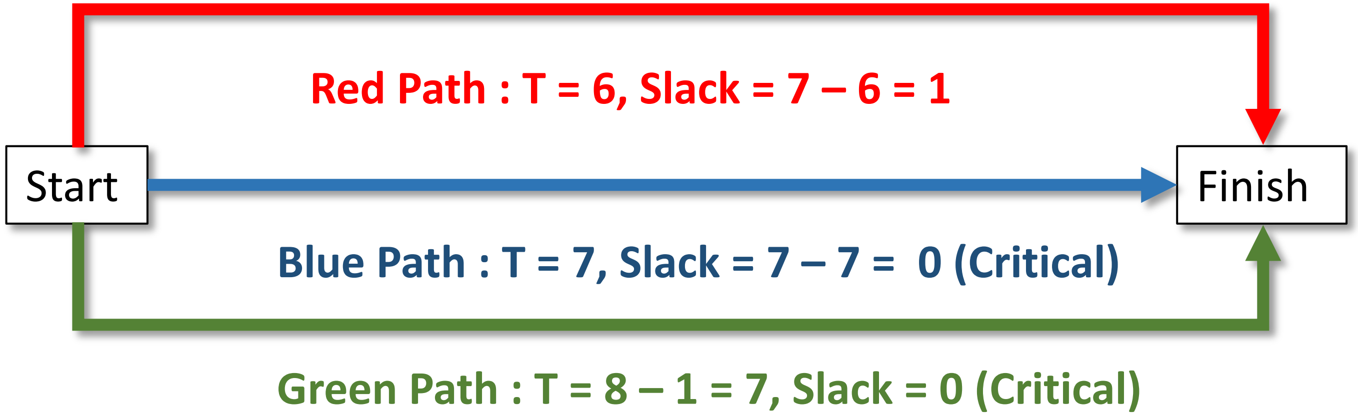
ES	T	EF
	ID	
LS	Slack	LF



- Three activities (other than the milestones) are now critical: C, D & E.
- It’s tempting to say ‘crash C by 1, since it’s the cheapest’…
- Problem: there are two critical PATHS. We need to crash BOTH.
- Recall last lecture…

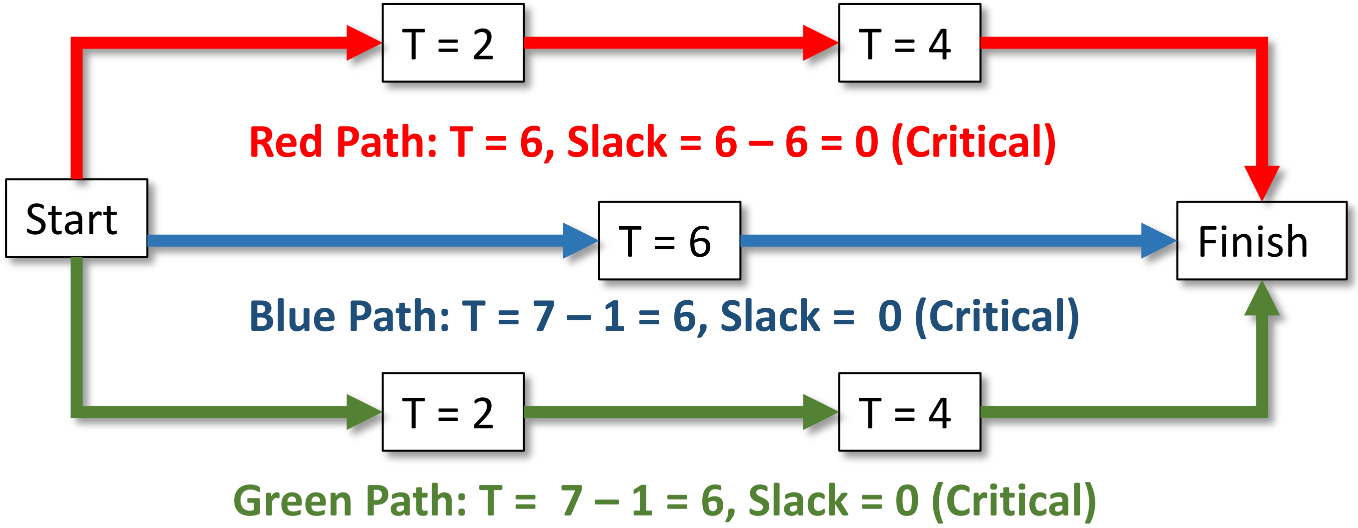
Reducing duration from T = 7 to T = 6

Start → Finish
T = 7



Crashing one path isn't enough – we need to crash *both* the **Green** and **Blue** paths to see a reduction in Finish time.

Start → Finish
T = 6



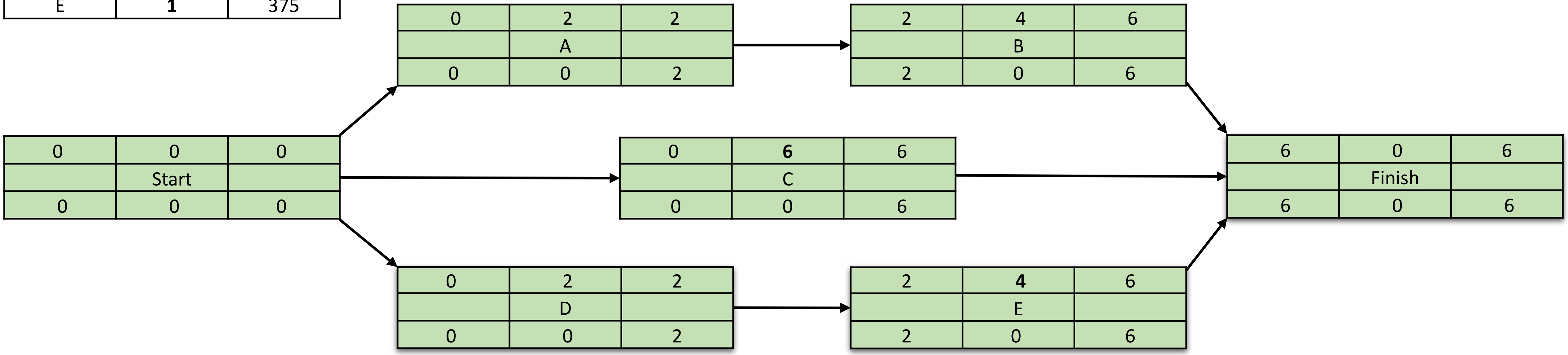
If we try to JUST crash the **Blue** path (Activity C), the **Green** path will still be 7 days long, so the whole project will still be 7 days long.

We've used up D's available days, so E is the only Green path activity left → Crash C and E

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	**4**	30
D	0	200
E	**1**	375

Reduce Duration to 6

ES	T	EF
	ID	
LS	Slack	LF



- We reduced the duration of C from 7 to 6, and E from 5 to 4.
- We also lowered the slack in A and B to zero…
- →ALL activities are now critical, and there are three critical paths.

Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0
6	0	0	1	2	1

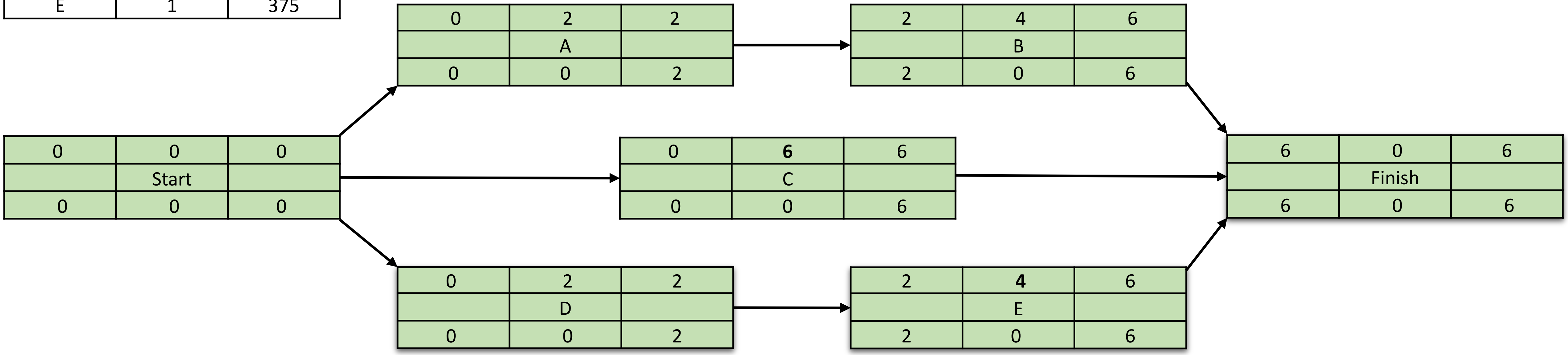
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150
6	50	100	180	600	625	1,555

Activity	Crash Days	
	Available	$/Day
A	1	350
B	2	400
C	4	30
D	0	200
E	1	375

Reduce Duration to 5?

ES	T	EF
	ID	
LS	Slack	LF

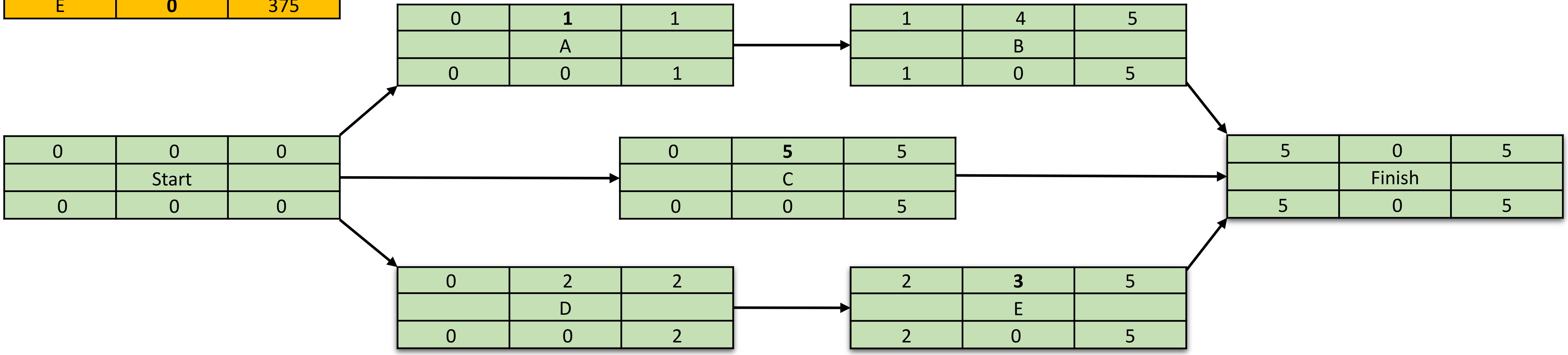


- Three critical paths: A&B, C, D&E
- To reduce project duration by 1, we have to crash all three paths by 1 each.
- D has no crash days left, so C and E are mandatory.
- For the A&B path, A is cheaper than B, so choose A.

Reduce Duration to 5

Activity	Crash Days	
	Available	$/Day
A	**0**	350
B	2	400
C	**3**	30
D	0	200
E	**0**	375

ES	T	EF
	ID	
LS	Slack	LF



- This is as far as we can go: We've run out of crash days for the D&E critical path.
- Even though we have crash days left for B & C, that doesn't help us. Reducing the duration of those activities won't affect project duration, because the D&E critical path remains binding with a length of 5.

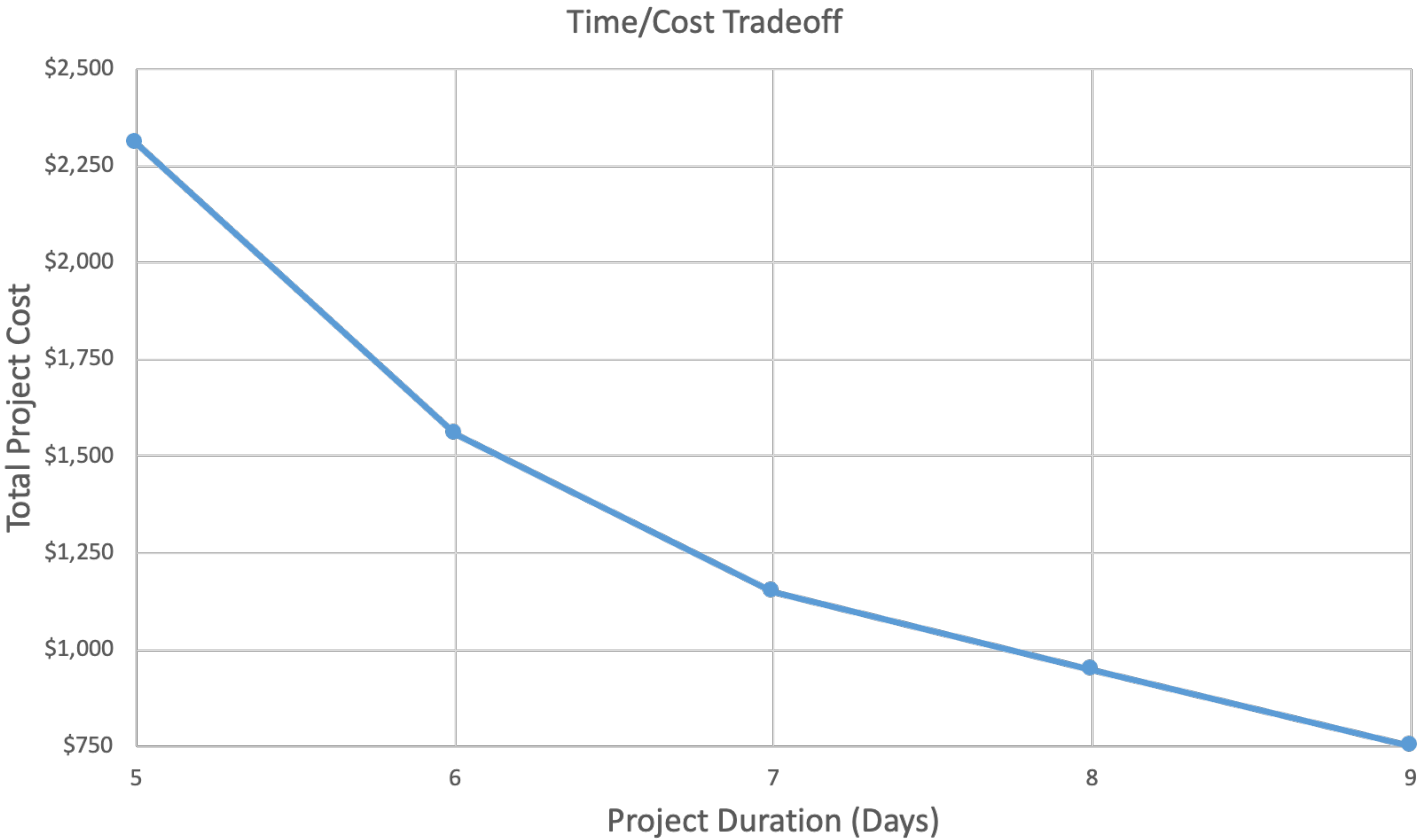
Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0
6	0	0	1	2	1
5	1	0	2	2	2

Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150
6	50	100	180	600	625	1,555
5	400	100	210	600	1,000	2,310

Duration	Project Cost
9	750
8	950
7	1,150
6	1,555
5	2,310


Time/Cost Tradeoff
$2,500
$2,250
$2,000
$1,750
$1,500
$1,250
$1,000
$750
Total Project Cost
5
6
7
8
9
Project Duration (Days)