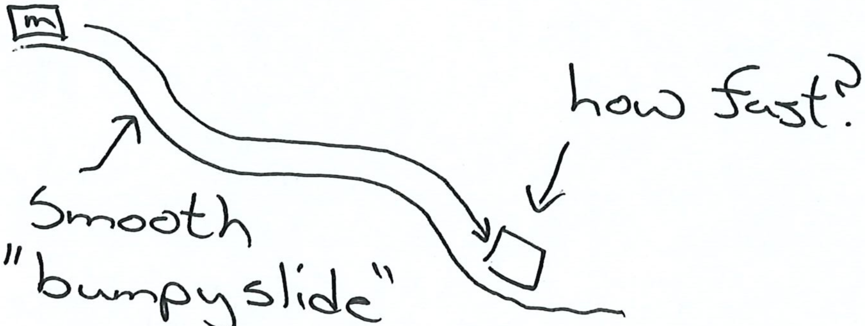
Work & Energy



Clearly $\vec{F}$'s not constant

So → hard to calculate $\vec{v}$

What is we just want speed?

Introduce <u>work</u>

The work done by a constant force $\vec{F}$ on object which is displaced by $\Delta\vec{r}$

$$W = \vec{F} \cdot \Delta\vec{r}$$ Nm ⇝ J

For const forces

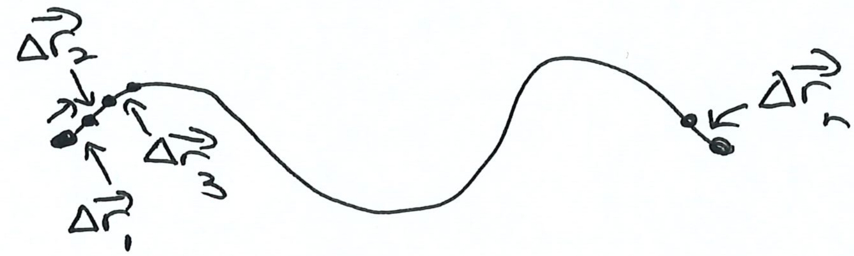
$\vec{F}$ & $\Delta\vec{r}$ same way $W > 0$

opposite $W < 0$

At 90° $W = 0$

Work by a non-constant force?

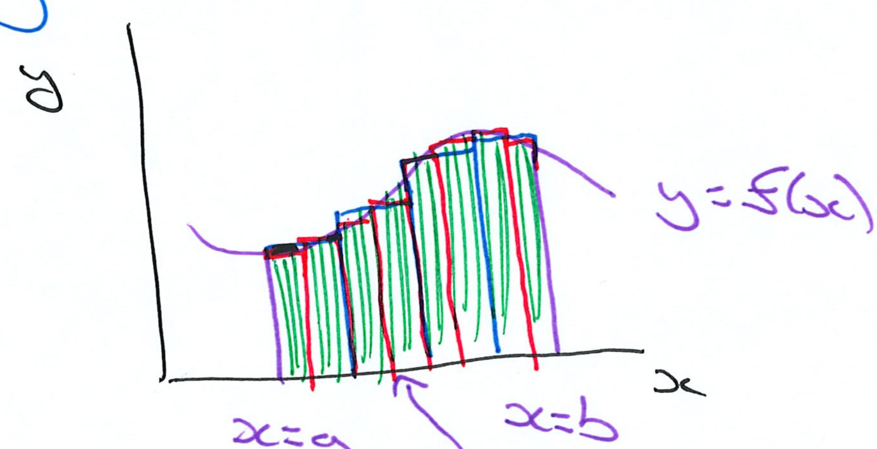
Do an approximation



Break path into n small steps
Where $\vec{F} \approx$ constant during that step

$$W_{total} \approx \vec{F}_1 \cdot \Delta\vec{r}_1 + \vec{F}_2 \cdot \Delta\vec{r}_2 + \ldots + \vec{F}_n \cdot \Delta\vec{r}_n$$

Integration:



What is area?

$A \approx$ Add up rectangles

$$A \approx \sum \text{A lot of super thin rectangles}$$

$$= \lim_{\text{width}\to 0} \sum \text{thin rectangles}$$

$$= \int_{x=a}^{b} \underbrace{f(x)}_{\substack{\uparrow \\ \text{height}}} \underbrace{dx}_{\substack{\uparrow \\ \text{width} \\ \text{(super thin)}}}$$

$$W = \int_{\text{path from } \vec{r}_1}^{\vec{r}_2} \vec{F}(\vec{r}, t, \ldots) \cdot d\vec{r}$$

Work by constant force

$$W = \vec{F} \cdot \Delta\vec{r}$$

Work by varying force

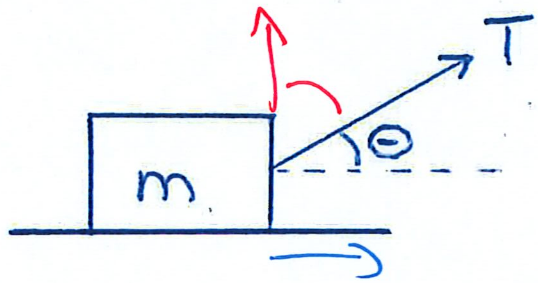
$$W = \int \vec{F} \cdot d\vec{r}$$

9-3-Example-WKE1

Work and Kinetic Energy - I

A $m = 4kg$ mass is on a horizontal rough surface with which it has a coefficient of kinetic friction $\mu_k = 0.2$. The mass is pulled to the right by a rope under tension $T = 15N$ which makes an angle of $\theta = 30°$ with the horizontal.

The box has been pulled a distance $d = 3m$ along the surface.



- What is the work done by the rope?
- What is the work done by friction?
- What is the work done by the normal force?

For constant forces $W = \vec{F} \cdot \Delta\vec{r}$

$\Delta\vec{r} = 3m\,\hat{i}$; $|\Delta\vec{r}| = 3m$

$$W_{rope} = \vec{F}_{rope} \cdot \Delta\vec{r}$$
$$= |\vec{F}_{rope}||\Delta\vec{r}|\cos\phi$$
$$= (T)\,3m\cos 30$$
$$= (15N)(3m)\cos 30$$
$$= 39.0\,J$$

$$W_{fric} = \vec{F}_f \cdot \Delta\vec{r}$$

180

$-\hat{\imath}$

$$= -|\vec{F}_f||\Delta\vec{r}|$$

$\mu_k|\vec{F}_N|$

$\vec{F}_N$ vertical

$$0 = \vec{F}_{net} \cdot \hat{k}$$

$$0 = |\vec{F}_N| + |\vec{F}_{rope}|\cos 60 + (mg)\cos 180$$

$$|\vec{F}_N| = 31.7N$$

$$|\vec{F}_f| = 6.34N$$

$$= -(6.34N)(3m)$$

$$= -19J$$

$$W_{normal} = \vec{F}_N \cdot \Delta\vec{r}$$

$$= (31.7N\hat{k}) \cdot (3m\hat{\imath})$$

$$= 0\ J$$

9-4-Theory-WorkVaryingF

Work by a varying force

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$$

- Parametrize the path
  Express $\vec{r}$ as a function of some parameter s
  $\vec{r}(s_i) = \vec{r}_i$
  $\vec{r}(s_f) = \vec{r}_f$
  $\vec{r}(s)$ with $s_i < s < s_f$ is on path taken.
- Calculate $d\vec{r}$
  $$d\vec{r} = \frac{d\vec{r}(s)}{ds}\, ds$$
- Write $\vec{F}(s)$
  $\vec{F}(\vec{r}) \rightsquigarrow$ know $\vec{r}(s)$
- Calculate $\vec{F} \cdot d\vec{r} \rightsquigarrow g(s)\, ds$
- Do the integral!

9-5-Example-
WKE2

Work and Kinetic Energy - II

A $m = 2kg$ mass starts at the origin and moves along the positive x-axis. While it does so, it is subject to a force which depends on position: $\vec{F} = \left(5\frac{N}{m}x - 1\frac{N}{m^2}x^2\right)\hat{\imath}$.

How much work is done on the mass as it moves from $x = 0m$ to $x = 2m$? From $x = 1m$ to $x = 3m$?

by $\vec{F}$

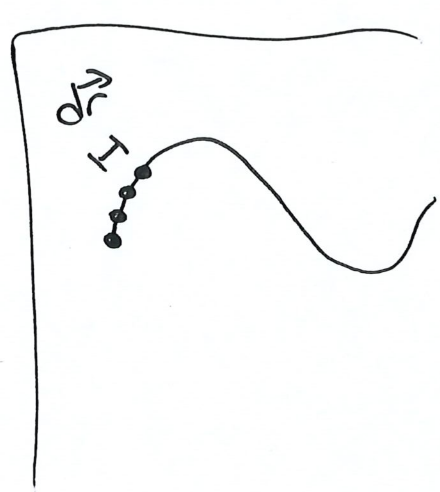
- Parametrize path
- Calculate $d\vec{r}$
- Write $\vec{F}$ in terms of parameter
- Calculate $\vec{F}\cdot d\vec{r}$
- Do integral

① Find $\vec{r}(s)$ which has right properties

$\vec{r}(s) = s\hat{\imath}$ for s between 0m & 2m
or 1m & 3m

② Find $d\vec{r} = \left(\frac{d\vec{r}(s)}{ds}\right)ds$

$= (\hat{\imath})ds$



Write $\vec{F}(s)$

When m is at $x\hat{i}$ force is

$$\vec{F} = \left(5\frac{N}{m}\underbrace{\cancel{x}}_{s} - 1\frac{N}{m^2}\underbrace{\cancel{x^2}}_{s^2}\right)\hat{i}$$

$$= \left(5\frac{N}{m}s - 1\frac{N}{m^2}s^2\right)\hat{i}$$

Find $\vec{F}\cdot d\vec{r}$

$$= \left[\left(5\frac{N}{m}s - 1\frac{N}{m^2}s^2\right)\hat{i}\right]\cdot\left[ds\,\hat{i}\right]$$

$$= \left(5\frac{N}{m}s - 1\frac{N}{m^2}s^2\right)ds$$

$$W = \int_{s_i}^{s_f}\vec{F}\cdot d\vec{r}$$

$$= \int_{0m}^{2m}\left(5\frac{N}{m}s\,ds - 1\frac{N}{m^2}s^2\,ds\right)$$

$$= \int_{0m}^{2m}5\frac{N}{m}s^1\,ds - \int_{0m}^{2m}1\frac{N}{m^2}s^2\,ds$$

$$= 5\frac{N}{m}\frac{s^2}{2}\Big|_{0m}^{2m} - 1\frac{N}{m^2}\frac{1}{3}s^3\Big|_{0m}^{2m}$$

$$= 10\,Nm - 2.67\,Nm$$

$$= 7.33\,J$$

$$\boxed{\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C}$$

Between 1m & 3m

$$W = \int_{1m}^{3m} \vec{F} \cdot d\vec{r}$$

...

$$= 5\frac{N}{m}\frac{s^2}{2}\Big|_{1m}^{3m} - 1\frac{N}{m^2}\frac{1}{3}s^3\Big|_{1m}^{3m}$$

$$= 20J - 8.67J$$

$$= 11.33J$$

Chose simple param what is picked

$$\vec{r} = (e^s - 1)m\hat{i} \qquad s=0 \qquad r = 0\hat{i}$$

$$2\cancel{m} = (e^s - 1)\cancel{m}$$

$$3 = e^s \Rightarrow \ln 3 = s$$

other param

$$\vec{r} = (e^s - 1)m\hat{i} \qquad 0 \leq s \leq \ln 3$$

$$d\vec{r} = \frac{d\vec{r}}{ds}ds$$

$$= e^s(1m)\hat{i}\,ds$$

$$= e^s ds\, 1m\,\hat{i}$$

What is $\vec{F}(s) = \left(5\frac{N}{m}x - 1\frac{N}{m^2}x^2\right)\hat{i}$

$$= \left[5N(e^s-1) - 1N(e^s-1)^2\right]\hat{i}$$

$\vec{F}(s)\cdot d\vec{r}$

$$= \left[5N(e^s-1) - 1N(e^s-1)^2\right]\hat{i}\cdot e^s ds\, 1m\,\hat{i}$$

$$= 5J(e^s-1)e^s ds - 1J(e^s-1)^2 e^s ds$$

$$W = \int_0^{\ln 3} 5J(e^{2s}-e^s)ds - \int_0^{\ln 3} 1J(e^s-1)^2 e^s ds$$

$$= \int_0^{\ln 3} 5J(e^{2s}-e^s)ds$$

$$- \int_0^{\ln 3} 1J\left(e^{3s} - 2e^{2s} + e^s\right)ds$$

$$\boxed{\int e^{ax} = \frac{1}{a}e^{ax} + C}$$

$$= 5J\left(\frac{1}{2}e^{2s} - e^s\right)\Big|_0^{\ln 3}$$

$$- 1J\left(\frac{1}{3}e^{3s} - e^{2s} + e^s\right)\Big|_0^{\ln 3}$$

$$= 5J\left(\frac{1}{2}e^{2\ln 3} - \frac{1}{2}e^{2\cdot 0} - e^{\ln 3} + e^{0}\right)$$

$$-1J\left(\frac{1}{3}e^{3\ln 3} - \frac{1}{3}e^{3\cdot 0} - e^{2\ln 3} + e^{2\cdot 0} + e^{\ln 3} - e^{0}\right)$$

$$= 5J(2.0)$$

$$-1J(2.67)$$

$$= 7.33J$$