# CSC 225: Fall 2022: Lab 9

## 1 Walk, Trail and Path

Let $G = (V, E)$ be an undirected graph with vertex set $V$ and edge set $E$. Let $x, y$ be two (not necessarily distinct) vertices of $G$.

**Walk:** An $x$ - $y$ walk of $G$ is an alternating sequence of vertices and edges starting from $x$ and ending at $y$. The sequence may look like this:

$$x = x_0, e_1, x_1, e_2, \ldots, e_n, x_n = y$$

The *length* of a walk is the number of edges in the walk. In the example above the length of the $x$ - $y$ walk is $n$. There might be repeated vertices and/or repeated edges in a walk.

**Closed and Open Walks:** An $x - y$ walk is *closed* if $x = y$, otherwise it is *open*.

**Trail and Circuit:** A *trail* is an $x - y$ walk where no edge is repeated. A closed trail is called a *circuit*.

**Path and Cycle:** A *path* is an $x - y$ trail where no vertex is repeated. A closed path is called a *cycle*.

### Exercise

Based on the definitions above, answer the following questions.

1. In each of the following pairs, which one is a subset of the other? For example, in the pair "path, circuit", is a path always a circuit? or is a circuit always a path? or neither is true?

- path, circuit
- cycle, trail
- trail, open walk

2. Draw the graph with the following edges and let's call it $T_t$. Try to draw it without crossing edges.

$$\{a, b\}, \{b, c\}, \{c, a\}, \{a, d\}, \{b, e\}, \{c, f\}, \{a, e\}, \{b, f\}, \{c, d\}$$

3. How many $a$ -$c$ paths are there in graph $T_t$ in Exercise 2? How many of those paths have length 4?

4. Let $G$ be an undirected graph and let $x, y$ be two distinct vertices of $G$. If there is an $x - y$ trail in $G$, prove that there is an $x - y$ path in $G$.