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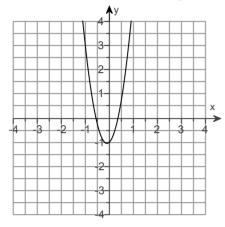
Date: 11/14/21 Course: MATH 100 (A01, A02, A03) Fall Assignment: Assignment 8

2021

Use Newton's method to estimate the solutions of the equation $5x^2 + x - 1 = 0$. Start with $x_0 = -1$ for the left solution and $x_0 = 1$ for the right solution. Find x_2 in each case.

Newton's method uses a 'guess' at a solution, perhaps from a graph, to get the first approximation termed x_0 . Successive approximations, $x_1, x_2, ..., x_n, x_{n+1}$ are calculated by $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$.

The function $f(x) = 5x^2 + x - 1$ is graphed below.



Because Newton's method involves a lot of computation, the guess values, x_0 , are chosen to simplify the computation x_1 . Appropriate choices for x_0 for this exercise are -1 for the left zero of f(x) and 1 for the right zero of f(x).

$$f(x)$$
 is $5x^2 + x - 1$

$$f'(x) = 10x + 1$$

$$x_1 \text{ for the left 0 of f(x) is } x_1 = x_0 - \frac{f\left(x_0\right)}{f'\left(x_0\right)} = -1 - \frac{5(-1)^2 + 1(-1) - 1}{10(-1) + 1} = -\frac{2}{3}.$$

Substituting
$$x_1 = -\frac{2}{3}$$
 into $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \approx -0.5686$.

$$x_1$$
 for the right 0 of f(x) is $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \frac{6}{11}$.

The third approximation, rounded to four decimal places, x_2 , such that $f(x_2) = 0$, is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{6}{11} - \frac{(5)\left(\frac{6}{11}\right)^2 + (1)\left(\frac{6}{11}\right) - 1}{(10)\left(\frac{6}{11}\right) + (1)} \approx 0.3854.$$