Symbolab

Solution

The Taylor Series of $\frac{3}{x}$ with center 1: $3-3(x-1)+3(x-1)^2-3(x-1)^3+3(x-1)^4+\ldots$

Steps

Taylor Series

Taylor series of function f(x) at a is defined as:

$$f(x)=f(a)+\frac{f'(a)}{1!}(x-a)+\frac{f''(a)}{2!}(x-a)^2+\frac{f'''(a)}{3!}(x-a)^3+\ldots$$

Apply the Taylor Formula — Hide Steps

Find the derivatives of $f(x)=\frac{3}{x}$, at $a=1$

$f(1)$: 3 — Hide Steps

Take the point $x=1$ and plug it into $\frac{3}{x}$

$=\frac{3}{1}$

Apply rule $\frac{a}{1}=a$

$=3$

$=3+\frac{\frac{d}{dx}\left(\frac{3}{x}\right)(1)}{1!}(x-1)+\frac{\frac{d^2}{dx^2}\left(\frac{3}{x}\right)(1)}{2!}(x-1)^2+\frac{\frac{d^3}{dx^3}\left(\frac{3}{x}\right)(1)}{3!}(x-1)^3+\ldots$

$=3+\frac{\frac{d}{dx}\left(\frac{3}{x}\right)(1)}{1!}(x-1)+\frac{\frac{d^2}{dx^2}\left(\frac{3}{x}\right)(1)}{2!}(x-1)^2+\frac{\frac{d^3}{dx^3}\left(\frac{3}{x}\right)(1)}{3!}(x-1)^3+\ldots$

Evaluate Derivatives — Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)(1)$: -3 — Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)(1)$

Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)=-\frac{3}{x^2}$

$\frac{d}{dx}\left(\frac{3}{x}\right)$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=3\frac{d}{dx}\left(\frac{1}{x}\right)$

Apply exponent rule: $\frac{1}{a}=a^{-1}$

$=3\frac{d}{dx}\left(x^{-1}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right)=a\cdot x^{a-1}$

$=3\left(-1\cdot x^{-1-1}\right)$

Simplify $3\left(-1\cdot x^{-1-1}\right)$: $-\frac{3}{x^2}$ — Hide Steps

$3\left(-1\cdot x^{-1-1}\right)$

Remove parentheses: $(-a)=-a$

$=-3\cdot 1\cdot x^{-1-1}$

Subtract the numbers: $-1-1=-2$

$=-3\cdot 1\cdot x^{-2}$

Apply exponent rule: $a^{-b}=\frac{1}{a^b}$

$x^{-2}=\frac{1}{x^2}$

$=3\cdot 1\cdot\frac{1}{x^2}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=-1\cdot\frac{1\cdot 3}{x^2}$

Refine

$= -\frac{3}{x^2}$

$= -\frac{3}{x^2}$

$= -\frac{3}{x^2}$

Evaluate $-\frac{3}{x^2}$ at point $x = 1$: -3 Hide Steps

Take the point $x = 1$ and plug it into $-\frac{3}{x^2}$

$= -\frac{3}{1^2}$

Apply rule $1^a = 1$

$1^2 = 1$

$= -\frac{3}{1}$

Apply the fraction rule: $\frac{a}{1} = a$

$\frac{3}{1} = 3$

$= -3$

$= -3$

$\frac{d^2}{dx^2}\left(\frac{3}{x}\right)(1)$: 6 Hide Steps

$\frac{d^2}{dx^2}\left(\frac{3}{x}\right)(1)$

$\frac{d^2}{dx^2}\left(\frac{3}{x}\right) = \frac{6}{x^3}$ Hide Steps

$\frac{d^2}{dx^2}\left(\frac{3}{x}\right)$

$\frac{d}{dx}\left(\frac{3}{x}\right) = -\frac{3}{x^2}$ Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 3\frac{d}{dx}\left(\frac{1}{x}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= 3\frac{d}{dx}\left(x^{-1}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 3\left(-1 \cdot x^{-1-1}\right)$

Simplify $3\left(-1 \cdot x^{-1-1}\right)$: $-\frac{3}{x^2}$ Hide Steps

$3\left(-1 \cdot x^{-1-1}\right)$

Remove parentheses: $(-a) = -a$

$= -3 \cdot 1 \cdot x^{-1-1}$

Subtract the numbers: $-1-1 = -2$

$= -3 \cdot 1 \cdot x^{-2}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-2} = \frac{1}{x^2}$

$= 3 \cdot 1 \cdot \frac{1}{x^2}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= -1 \cdot \frac{1 \cdot 3}{x^2}$

Refine

$= -\frac{3}{x^2}$

$= -\frac{3}{x^2}$

$= \frac{d}{dx}\left(-\frac{3}{x^2}\right)$

Hide Steps

$\frac{d}{dx}\left(-\frac{3}{x^2}\right)=\frac{6}{x^3}$

$\frac{d}{dx}\left(-\frac{3}{x^2}\right)$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=-3\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Apply exponent rule: $\frac{1}{a}=a^{-1}$

$=-3\frac{d}{dx}\left(x^{-2}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right)=a\cdot x^{a-1}$

$=-3\left(-2x^{-2-1}\right)$

Simplify $-3\left(-2x^{-2-1}\right)$: $\frac{6}{x^3}$ *Hide Steps*

$-3\left(-2x^{-2-1}\right)$

Apply rule $-(-a)=a$

$=3\cdot\ 2x^{-2-1}$

Subtract the numbers: $-2-1=-3$

$=3\cdot\ 2x^{-3}$

Apply exponent rule: $a^{-b}=\frac{1}{a^b}$

$x^{-3}=\frac{1}{x^3}$

$=3\cdot\ 2\cdot\frac{1}{x^3}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot\ b}{c}$

$=\frac{1\cdot\ 3\cdot\ 2}{x^3}$

Multiply the numbers: $1\cdot\ 3\cdot\ 2=6$

$=\frac{6}{x^3}$

$=\frac{6}{x^3}$

$= \frac{6}{x^3}$

Evaluate $\frac{6}{x^3}$ at point $x = 1$: 6 — Hide Steps

Take the point $x = 1$ and plug it into $\frac{6}{x^3}$

$= \frac{6}{1^3}$

Apply rule $1^a = 1$

$1^3 = 1$

$= \frac{6}{1}$

Apply the fraction rule: $\frac{a}{1} = a$

$= 6$

$= 6$

$\frac{d^3}{dx^3}\left(\frac{3}{x}\right)(1)$: -18 — Hide Steps

$\frac{d^3}{dx^3}\left(\frac{3}{x}\right)(1)$

$\frac{d^3}{dx^3}\left(\frac{3}{x}\right) = -\frac{18}{x^4}$ — Hide Steps

$\frac{d^3}{dx^3}\left(\frac{3}{x}\right)$

$\frac{d}{dx}\left(\frac{3}{x}\right) = -\frac{3}{x^2}$ — Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 3\frac{d}{dx}\left(\frac{1}{x}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= 3\frac{d}{dx}\left(x^{-1}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 3\left(-1 \cdot x^{-1-1}\right)$

Simplify $3\left(-1 \cdot x^{-1-1}\right)$: $-\frac{3}{x^2}$ — Hide Steps

$3\left(-1 \cdot x^{-1-1}\right)$

Remove parentheses: $(-a) = -a$

$= -3 \cdot 1 \cdot x^{-1-1}$

Subtract the numbers: $-1-1 = -2$

$= -3 \cdot 1 \cdot x^{-2}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-2} = \frac{1}{x^2}$

$= 3 \cdot 1 \cdot \frac{1}{x^2}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= -1 \cdot \frac{1 \cdot 3}{x^2}$

Refine

$= -\frac{3}{x^2}$

$= -\frac{3}{x^2}$

$= \frac{d^2}{dx^2}\left(-\frac{3}{x^2}\right)$

$\frac{d}{dx}\left(-\frac{3}{x^2}\right) = \frac{6}{x^3}$ — Hide Steps

$\frac{d}{dx}\left(-\frac{3}{x^2}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= -3\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= -3\frac{d}{dx}\left(x^{-2}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= -3\left(-2x^{-2-1}\right)$

Simplify $-3\left(-2x^{-2-1}\right)$: $\frac{6}{x^3}$ *Hide Steps*

$-3\left(-2x^{-2-1}\right)$

Apply rule $-(-a) = a$

$= 3 \cdot 2x^{-2-1}$

Subtract the numbers: $-2-1 = -3$

$= 3 \cdot 2x^{-3}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-3} = \frac{1}{x^3}$

$= 3 \cdot 2 \cdot \frac{1}{x^3}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 3 \cdot 2}{x^3}$

Multiply the numbers: $1 \cdot 3 \cdot 2 = 6$

$= \frac{6}{x^3}$

$= \frac{6}{x^3}$

$= \frac{d}{dx}\left(\frac{6}{x^3}\right)$

$\frac{d}{dx}\left(\frac{6}{x^3}\right) = -\frac{18}{x^4}$ *Hide Steps*

$\frac{d}{dx}\left(\frac{6}{x^3}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 6\frac{d}{dx}\left(\frac{1}{x^3}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= 6\frac{d}{dx}\left(x^{-3}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 6\left(-3x^{-3-1}\right)$

Simplify $6\left(-3x^{-3-1}\right)$: $-\frac{18}{x^4}$ *Hide Steps*

$6\left(-3x^{-3-1}\right)$

Remove parentheses: $(-a) = -a$

$= -6 \cdot 3x^{-3-1}$

Subtract the numbers: $-3-1 = -4$

$= -6 \cdot 3x^{-4}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-4} = \frac{1}{x^4}$

$= 6 \cdot 3 \cdot \frac{1}{x^4}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= -\frac{1 \cdot 6 \cdot 3}{x^4}$

Multiply the numbers: $1 \cdot 6 \cdot 3 = 18$

$= -\frac{18}{x^4}$

$= -\frac{18}{x^4}$

$= -\frac{18}{x^4}$

Hide Steps

Evaluate $-\frac{18}{x^4}$ at point $x=1$: -18

Take the point $x=1$ and plug it into $-\frac{18}{x^4}$

$=-\frac{18}{1^4}$

Apply rule $1^a=1$

$1^4=1$

$=-\frac{18}{1}$

Apply the fraction rule: $\frac{a}{1}=a$

$\frac{18}{1}=18$

$=-18$

$=-18$

$\frac{d^4}{dx^4}\left(\frac{3}{x}\right)(1)$: 72 Hide Steps

$\frac{d^4}{dx^4}\left(\frac{3}{x}\right)(1)$

$\frac{d^4}{dx^4}\left(\frac{3}{x}\right)=\frac{72}{x^5}$ Hide Steps

$\frac{d^4}{dx^4}\left(\frac{3}{x}\right)$

$\frac{d}{dx}\left(\frac{3}{x}\right)=-\frac{3}{x^2}$ Hide Steps

$\frac{d}{dx}\left(\frac{3}{x}\right)$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=3\frac{d}{dx}\left(\frac{1}{x}\right)$

Apply exponent rule: $\frac{1}{a}=a^{-1}$

$=3\frac{d}{dx}\left(x^{-1}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right)=a\cdot x^{a-1}$

$=3\left(-1\cdot x^{-1-1}\right)$

Simplify $3\left(-1\cdot x^{-1-1}\right)$: $-\frac{3}{x^2}$ Hide Steps

$3\left(-1\cdot x^{-1-1}\right)$

Remove parentheses: $(-a)=-a$

$=-3\cdot 1\cdot x^{-1-1}$

Subtract the numbers: $-1-1=-2$

$=-3\cdot 1\cdot x^{-2}$

Apply exponent rule: $a^{-b}=\frac{1}{a^b}$

$x^{-2}=\frac{1}{x^2}$

$=3\cdot 1\cdot\frac{1}{x^2}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=-1\cdot\frac{1\cdot 3}{x^2}$

Refine

$=-\frac{3}{x^2}$

$=-\frac{3}{x^2}$

$=\frac{d^3}{dx^3}\left(-\frac{3}{x^2}\right)$

$\frac{d}{dx}\left(-\frac{3}{x^2}\right)=\frac{6}{x^3}$ Hide Steps

$\frac{d}{dx}\left(-\frac{3}{x^2}\right)$

Take the constant out: $(a\cdot f)'=a\cdot f'$

$=-3\frac{d}{dx}\left(\frac{1}{x^2}\right)$

Apply exponent rule: $\frac{1}{a}=a^{-1}$

$= -3\frac{d}{dx}\left(x^{-2}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= -3\left(-2x^{-2-1}\right)$

Simplify $-3\left(-2x^{-2-1}\right)$: $\frac{6}{x^3}$ *Hide Steps*

$-3\left(-2x^{-2-1}\right)$

Apply rule $-(-a) = a$

$= 3 \cdot 2x^{-2-1}$

Subtract the numbers: $-2-1 = -3$

$= 3 \cdot 2x^{-3}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-3} = \frac{1}{x^3}$

$= 3 \cdot 2 \cdot \frac{1}{x^3}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 3 \cdot 2}{x^3}$

Multiply the numbers: $1 \cdot 3 \cdot 2 = 6$

$= \frac{6}{x^3}$

$= \frac{6}{x^3}$

$= \frac{d^2}{dx^2}\left(\frac{6}{x^3}\right)$

$\frac{d}{dx}\left(\frac{6}{x^3}\right) = -\frac{18}{x^4}$ *Hide Steps*

$\frac{d}{dx}\left(\frac{6}{x^3}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 6\frac{d}{dx}\left(\frac{1}{x^3}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= 6\frac{d}{dx}\left(x^{-3}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 6\left(-3x^{-3-1}\right)$

Simplify $6\left(-3x^{-3-1}\right)$: $-\frac{18}{x^4}$ *Hide Steps*

$6\left(-3x^{-3-1}\right)$

Remove parentheses: $(-a) = -a$

$= -6 \cdot 3x^{-3-1}$

Subtract the numbers: $-3-1 = -4$

$= -6 \cdot 3x^{-4}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-4} = \frac{1}{x^4}$

$= 6 \cdot 3 \cdot \frac{1}{x^4}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= -\frac{1 \cdot 6 \cdot 3}{x^4}$

Multiply the numbers: $1 \cdot 6 \cdot 3 = 18$

$= -\frac{18}{x^4}$

$= -\frac{18}{x^4}$

$= \frac{d}{dx}\left(-\frac{18}{x^4}\right)$

$\frac{d}{dx}\left(-\frac{18}{x^4}\right) = \frac{72}{x^5}$ *Hide Steps*

$\frac{d}{dx}\left(-\frac{18}{x^4}\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= -18\frac{d}{dx}\left(\frac{1}{x^4}\right)$

Apply exponent rule: $\frac{1}{a} = a^{-1}$

$= -18\frac{d}{dx}\left(x^{-4}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= -18\left(-4x^{-4-1}\right)$

Simplify $-18\left(-4x^{-4-1}\right)$: $\frac{72}{x^5}$ *Hide Steps*

$-18\left(-4x^{-4-1}\right)$

Apply rule $-(-a) = a$

$= 18 \cdot 4x^{-4-1}$

Subtract the numbers: $-4-1 = -5$

$= 18 \cdot 4x^{-5}$

Apply exponent rule: $a^{-b} = \frac{1}{a^b}$

$x^{-5} = \frac{1}{x^5}$

$= 18 \cdot 4 \cdot \frac{1}{x^5}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{1 \cdot 18 \cdot 4}{x^5}$

Multiply the numbers: $1 \cdot 18 \cdot 4 = 72$

$= \frac{72}{x^5}$

$= \frac{72}{x^5}$

$= \frac{72}{x^5}$

Evaluate $\frac{72}{x^5}$ at point $x = 1$: 72 *Hide Steps*

Take the point $x = 1$ and plug it into $\frac{72}{x^5}$

$= \frac{72}{1^5}$

Apply rule $1^a = 1$

$1^5 = 1$

$= \frac{72}{1}$

Apply the fraction rule: $\frac{a}{1} = a$

$= 72$

$= 72$

$= 3 + \frac{-3}{1!}(x-1) + \frac{6}{2!}(x-1)^2 + \frac{-18}{3!}(x-1)^3 + \frac{72}{4!}(x-1)^4 + \ldots$

Refine

$= 3 - 3(x-1) + 3(x-1)^2 - 3(x-1)^3 + 3(x-1)^4 + \ldots$