Math 122 In-Class Assignment 2 - Solutions

1. Consider the following statements:

   - p : m is an even integer
   - q : n is an even integer
   - r : $m + n$ is an even integer
   - s : mn is an even integer

   (a) Write the statement "*if m is an even integer and n is an even integer, then $m + n$ is an even integer*" in symbolic form.
   Solution: $(p \wedge q) \rightarrow r$

   (b) Write the statement $(\neg p \vee \neg q) \rightarrow \neg r$ in plain English.
   Solution: "If m is not an even integer or n is not an even integer, then $m + n$ is not an even integer."
   Or alternatively: "If m is an odd integer or n is an odd integer, then $m + n$ is an odd integer".

   (c) Are the two statements in part (a) and (b) logically equivalent? Explain.
   Solution: No, the statements in (a) and (b) are not logically equivalent. The statement in part (b) is the inverse of the statement in part (a). The inverse is logically equivalent to the converse, which is not logically equivalent to the original implication.
   Or alternatively, you can draw a truth table to show that there is a place where the two statements do not have the same truth value. In particular the instance where p and q are false and r are is true gives that $(p \wedge q) \rightarrow r$ is true, but $(\neg p \vee \neg q) \rightarrow \neg r$ is false. Because there is an instance where the two statements do not have the same truth value, they are not logically equivalent.

   (d) Write the statement "*mn is an even integer when m is an even integer*" in symbolic form. (Hint: if-then.)
   Solution: The statement is equivalent to "*If m is an even integer, then mn is an even integer.*". Symbolically, this is $p \rightarrow s$.