Solution

Check convergence of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{9^n}{n^9}$:   diverges

Steps

$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{9^n}{n^9}$

Apply Series Ratio Test: diverges

Hide Steps

$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{9^n}{n^9}$

Series Ratio Test:

If there exists an N so that for all $n \ge N$, $a_n \ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$:

If $L < 1$, then $\sum a_n$ converges

If $L > 1$, then $\sum a_n$ diverges

If $L = 1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{(n+1)+1}\frac{9^{(n+1)}}{(n+1)^9}}{(-1)^{n+1}\frac{9^n}{n^9}}\right|$$

Simplify $\left|\frac{(-1)^{(n+1)+1}\frac{9^{(n+1)}}{(n+1)^9}}{(-1)^{n+1}\frac{9^n}{n^9}}\right|$: $\frac{9\left|n^9\right|}{\left|(n+1)^9\right|}$

Hide Steps

$$= \left|\frac{(-1)^{(n+1)+1}\frac{9^{n+1}}{(n+1)^9}}{(-1)^{n+1}\frac{9^n}{n^9}}\right|$$

Remove parentheses: $(a) = a$

$$= \left|\frac{(-1)^{n+1+1}\frac{9^{n+1}}{(n+1)^9}}{(-1)^{n+1}\frac{9^n}{n^9}}\right|$$

Add the numbers: $1+1=2$

$$=\left|\frac{(-1)^{n+2}\frac{9^{n+1}}{(n+1)^9}}{(-1)^{n+1}\frac{9^n}{n^9}}\right|$$

Multiply $(-1)^{n+1}\frac{9^n}{n^9}:\quad \frac{9^n(-1)^{n+1}}{n^9}$

Hide Steps

$$(-1)^{n+1}\frac{9^n}{n^9}$$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$$=\frac{9^n(-1)^{n+1}}{n^9}$$

$$=\left|\frac{(-1)^{n+2}\frac{9^{n+1}}{(n+1)^9}}{\frac{9^n(-1)^{n+1}}{n^9}}\right|$$

Multiply $(-1)^{n+2}\frac{9^{n+1}}{(n+1)^9}:\quad \frac{9^{n+1}(-1)^{n+2}}{(n+1)^9}$

Hide Steps

$$(-1)^{n+2}\frac{9^{n+1}}{(n+1)^9}$$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$$=\frac{9^{n+1}(-1)^{n+2}}{(n+1)^9}$$

$$=\left|\frac{\frac{9^{n+1}(-1)^{n+2}}{(n+1)^9}}{\frac{9^n(-1)^{n+1}}{n^9}}\right|$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$$=\left|\frac{9^{n+1}(-1)^{n+2}n^9}{(n+1)^9\cdot 9^n(-1)^{n+1}}\right|$$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$$\frac{9^{n+1}}{9^n}=9^{n+1-n}$$

$$=\frac{9^{n-n+1}(-1)^{n+2}n^9}{(-1)^{n+1}(n+1)^9}$$

Add similar elements: $n+1-n=1$

$$= \frac{9(-1)^{n+2}n^9}{(-1)^{n+1}(n+1)^9}$$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$$\frac{(-1)^{n+2}}{(-1)^{n+1}} = (-1)^{n+2-(n+1)}$$

$$= \frac{9(-1)^{n+2-(n+1)}n^9}{(n+1)^9}$$

Add similar elements: $n + 2 - (n+1) = 1$

$$= \left|\frac{9(-1)n^9}{(n+1)^9}\right|$$

Refine

$$= \left|\frac{-9n^9}{(n+1)^9}\right|$$

Apply the fraction rule: $\frac{-a}{b} = -\frac{a}{b}$

$$= \left|-\frac{9n^9}{(n+1)^9}\right|$$

Apply absolute rule: $|-a| = |a|$

$$= \left|\frac{9n^9}{(n+1)^9}\right|$$

Apply absolute rule: $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$$= \frac{|9n^9|}{|(n+1)^9|}$$

Apply absolute rule: $|ax| = a|x|, a \geq 0$

$$|9n^9| = 9|n^9|$$

$$= \frac{9|n^9|}{|(n+1)^9|}$$

$$\lim_{n\to\infty}\left(\frac{9|n^9|}{|(n+1)^9|}\right) = 9$$

Hide Steps

$$\lim_{n\to\infty}\left(\frac{9|n^9|}{|(n+1)^9|}\right)$$

n^9 is positive when $n \to \infty$. Therefore $|n^9| = n^9$

$$= \lim_{n\to\infty}\left(\frac{9n^9}{|(n+1)^9|}\right)$$

$(n+1)^9$ is positive when $n\to\infty$. Therefore $\left|(n+1)^9\right|=(n+1)^9$

$$=\lim_{n\to\infty}\left(\frac{9n^9}{(n+1)^9}\right)$$

$$\lim_{x\to a}[c\cdot f(x)]=c\cdot\lim_{x\to a}f(x)$$

$$=9\cdot\lim_{n\to\infty}\left(\frac{n^9}{(n+1)^9}\right)$$

Simplify $\frac{n^9}{(n+1)^9}$: $\left(\frac{n}{n+1}\right)^9$ *Hide Steps*

$$\frac{n^9}{(n+1)^9}$$

Apply exponent rule: $\frac{a^c}{b^c}=\left(\frac{a}{b}\right)^c$

$$\frac{n^9}{(n+1)^9}=\left(\frac{n}{n+1}\right)^9$$

$$=\left(\frac{n}{n+1}\right)^9$$

$$=9\cdot\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^9\right)$$

$$\lim_{x\to a}[f(x)]^b=\left[\lim_{x\to a}f(x)\right]^b$$

With the exception of indeterminate form

$$=9\left(\lim_{n\to\infty}\left(\frac{n}{n+1}\right)\right)^9$$

Divide by highest denominator power: $\frac{1}{1+\frac{1}{n}}$ *Hide Steps*

$$\frac{n}{n+1}$$

Divide by n

$$=\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}}$$

Refine

$$=\frac{1}{1+\frac{1}{n}}$$

$$=9\left(\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)\right)^9$$

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)},\quad \lim_{x\to a}g(x)\neq 0$$

With the exception of indeterminate form

$= 9\left(\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}\right)^9$

$\lim_{n\to\infty}(1) = 1$ Hide Steps

$\lim_{n\to\infty}(1)$

$\lim_{x\to a} c = c$

$= 1$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$ Hide Steps

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$

$\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
With the exception of indeterminate form

$= \lim_{n\to\infty}(1) + \lim_{n\to\infty}\left(\frac{1}{n}\right)$

$\lim_{n\to\infty}(1) = 1$ Hide Steps

$\lim_{n\to\infty}(1)$

$\lim_{x\to a} c = c$

$= 1$

$\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$ Hide Steps

$\lim_{n\to\infty}\left(\frac{1}{n}\right)$

Apply Infinity Property: $\lim_{x\to\infty}\left(\frac{c}{x^a}\right) = 0$

$= 0$

$= 1 + 0$

Simplify

$= 1$

$= 9\left(\frac{1}{1}\right)^9$

Simplify

$= 9$

$L > 1$, by the ratio test

$=$ diverges

$=$ diverges