Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Use any method to determine if the series converges or diverges. Give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{(n+8)!}{8!n!8^n}$$

The nth-Term Test states that $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero. Since $\lim_{n\to\infty} \frac{(n+8)!}{8!n!8^n} = 0$, the nth-Term Test is inconclusive. Since the terms contain n as factorials and exponents, use the Ratio Test.

Let $\sum a_n$ be any series and suppose that $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \rho$. The Ratio Test states that the series converges absolutely if $\rho < 1$ and diverges if $\rho > 1$ or ρ is infinite. The test is inconclusive if $\rho = 1$.

In this case, $a_n = \frac{(n+8)!}{8!n!8^n}$. Determine a_{n+1}.

$$a_{n+1} = \frac{(n+9)!}{8!(n+1)!8^{n+1}}$$

The limit for the Ratio Test is as shown below.

$$\lim_{n\to\infty}\left|\frac{\frac{(n+9)!}{8!(n+1)!8^{n+1}}}{\frac{(n+8)!}{8!n!8^n}}\right| = \rho$$

Simplify the limit.

$$\begin{aligned}
\rho &= \lim_{n\to\infty}\left|\frac{\frac{(n+9)!}{8!(n+1)!8^{n+1}}}{\frac{(n+8)!}{8!n!8^n}}\right| \\
&= \lim_{n\to\infty}\left|\frac{(n+9)!}{8!(n+1)!8^{n+1}}\cdot\frac{8!n!8^n}{(n+8)!}\right| \\
&= \lim_{n\to\infty}\left|\frac{8!}{8!}\cdot\frac{8^n}{8^{n+1}}\cdot\frac{n!}{(n+1)!}\cdot\frac{(n+9)!}{(n+8)!}\right| \\
&= \lim_{n\to\infty}\left|\frac{n+9}{8n+8}\right|
\end{aligned}$$

Evaluate this limit.

$$\lim_{n\to\infty}\left|\frac{n+9}{8n+8}\right| = \frac{1}{8}$$

Thus, $\rho = \frac{1}{8}$. Therefore, as $\rho < 1$, the series $\sum_{n=1}^{\infty} \frac{(n+8)!}{8!n!8^n}$ converges.