Math 122 In-Class Assignment 12 - Solutions

1. Consider the relation $\mathcal{R}$ on the set $A = \{a, b, c, d\}$ given by

$$\mathcal{R} = \{(a,a), (a,b), (b,a), (b,b), (a,d), (d,a), (a,c), (c,a), (d,d)\}$$

Indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

(a) $\mathcal{R}$ is reflexive.
Solution: False. $\mathcal{R}$ is missing (c, c).

(b) $\mathcal{R}$ is symmetric.
Solution: True. Whenever a pair of the form (x, y) is in $\mathcal{R}$, we also have that the pair (y, x) is in $\mathcal{R}$. That is, we have (a, b) and (b, a), (a, d) and (d, a), and (a, c) and (c, a) all in $\mathcal{R}$.

(c) $\mathcal{R}$ is antisymmetric.
Solution: False. $\mathcal{R}$ has (a, b) and (b, a) and $a \neq b$.

(d) $\mathcal{R}$ is transitive.
Solution: False. $\mathcal{R}$ has (c, a) and (a, c) but it is missing (c, c).

(e) $\mathcal{R}$ is an equivalence relation.
Solution: False. $\mathcal{R}$ needs to be reflexive, symmetric, and transitive in order to be an equivalence relation. It is only symmetric.

2. Let $\mathcal{R}$ be the relation on the set $A = \{1, 2, 3, 4, \ldots, 9999\}$ given by

$$\mathcal{R} = \{(a, b) : a \text{ and } b \text{ have the same number of digits}\}$$

For each statement, choose the best response. No reasons are necessary.

(a) $(22, 2222) \in \mathcal{R}$ (Choices: True, False)
Solution: False. 22 has two digits and 2222 has 4 digits so 22 is not related to 2222. Therefore $(22, 2222) \notin \mathcal{R}$.

(b) It can be shown that $\mathcal{R}$ is an equivalence relation. How many equivalence classes does $\mathcal{R}$ have? (Choices: 1, 4, 9, 10, 9999)
Solution: $\mathcal{R}$ has 4 equivalence classes. Elements of $\{1, 2, 3, 4, \ldots, 9999\}$ are related when they have the same number of digits, so each equivalence class will be a list of numbers which have the same number of digits. There are 4 possibilities for the number of digits here, so there are 4 equivalence classes:

- $[1] = \{\text{elements related to } 1\} = \{\text{numbers that have 1 digit}\}$
- $[10] = \{\text{elements related to } 10\} = \{\text{numbers that have 2 digits}\}$
- $[100] = \{\text{elements related to } 100\} = \{\text{numbers that have 3 digits}\}$
- $[1000] = \{\text{elements related to } 1000\} = \{\text{numbers that have 4 digits}\}$