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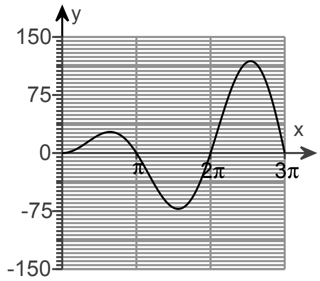
Find the area of the region enclosed by the curve $y = 15x\,\mathbf{sin}\,x$ and the x-axis (see the accompanying figure) for the following intervals.

a. $0 \le x \le \pi$

b. $\pi \le x \le 2\pi$

c. $2\pi \le x \le 3\pi$

d. What pattern do you see here? What is the area between the curve and the x-axis for $n\pi \le x \le (n+1)\pi$, n an arbitrary nonnegative integer?



a. The antiderivative of a function represents the area under the curve. To find the area under the curve from 0 to π for the equation $y = 15x\,\mathbf{sin}\,x$, evaluate $15\int_0^{\pi} x\,\mathbf{sin}\,x\,dx$. Use the integration by parts formula, $\int u\,dv = uv - \int v\,du$, to integrate.

Good choices for u and dv are $u = x$ and $dv = \mathbf{sin}\,x\,dx$.

Use integration by parts to find $\int x\,\mathbf{sin}\,x\,dx$ with $u = x$ and $dv = \mathbf{sin}\,x\,dx$. First determine du.

$du = dx$

Next, find the simplest antiderivative of $\mathbf{sin}\,x$.

$$\begin{aligned} dv &= \mathbf{sin}\,x\,dx \\ \int dv &= \int \mathbf{sin}\,x\,dx \\ v &= -\mathbf{cos}\,x \end{aligned}$$

With $u = x$, $du = dx$, and $v = -\mathbf{cos}\,x$, rewrite the original integral using the formula for integration by parts, $\int u\,dv = uv - \int v\,du$.

$$\begin{aligned} \int x\,\mathbf{sin}\,x\,dx &= -x\,\mathbf{cos}\,x - \int -\,\mathbf{cos}\,x\,dx \\ &= -x\,\mathbf{cos}\,x + \int \mathbf{cos}\,x\,dx \end{aligned}$$

Therefore, $\int x\,\mathbf{sin}\,x\,dx = -x\,\mathbf{cos}\,x + \int \mathbf{cos}\,x\,dx = -x\,\mathbf{cos}\,x + \mathbf{sin}\,x$.

Using this result, evaluate $15\int_0^{\pi} x\,\mathbf{sin}\,x\,dx$.

$$\begin{aligned} 15\int_0^{\pi} x\,\mathbf{sin}\,x\,dx &= 15\left[-x\,\mathbf{cos}\,x + \mathbf{sin}\,x\right]_0^{\pi} \\ &= 15\left[-(\pi)\,\mathbf{cos}\,(\pi) + \mathbf{sin}\,(\pi) - (-(0)\,\mathbf{cos}\,(0) + \mathbf{sin}\,(0))\right] \\ &= 15\pi \end{aligned}$$

Thus, the area of the region enclosed by the curve $y = 15x\,\mathbf{sin}\,x$ from 0 to π is 15π.

b. To find the area between the curve and the x-axis from π to 2π for the equation $y = 15x\,\mathbf{sin}\,x$, evaluate $15\int_{\pi}^{2\pi} x\,\mathbf{sin}\,x\,dx$.

Use the result of the integration from part (a) to evaluate $15\int_{\pi}^{2\pi} x\,\mathbf{sin}\,x\,dx$.

$$\begin{aligned}15\int_{\pi}^{2\pi} x\,\mathbf{sin}\,x\,dx &= 15[-x\,\mathbf{cos}\,x+\mathbf{sin}\,x]_{\pi}^{2\pi}\\ &= 15[-(2\pi)\,\mathbf{cos}\,(2\pi)+\mathbf{sin}\,(2\pi)-(-(\pi)\,\mathbf{cos}\,(\pi)+\mathbf{sin}\,(\pi))]\\ &= -45\pi\end{aligned}$$

Recall that area cannot be a negative value. While the value of the integral from π to 2π is -45π, the area is $|-45\pi|$ which is 45π.

Thus, the area of the region enclosed by the curve $y = 15x\,\mathbf{sin}\,x$ from π to 2π is 45π.

c. To find the area under the curve from 2π to 3π for the equation $y = 15x\,\mathbf{sin}\,x$, evaluate $15\int_{2\pi}^{3\pi} x\,\mathbf{sin}\,x\,dx$.

Use the result of the integration from part (a) to evaluate $15\int_{2\pi}^{3\pi} x\,\mathbf{sin}\,x\,dx$.

$$\begin{aligned}15\int_{2\pi}^{3\pi} x\,\mathbf{sin}\,x\,dx &= 15[-x\,\mathbf{cos}\,x+\mathbf{sin}\,x]_{2\pi}^{3\pi}\\ &= 15[-(3\pi)\,\mathbf{cos}\,(3\pi)+\mathbf{sin}\,(3\pi)-(-(2\pi)\,\mathbf{cos}\,(2\pi)+\mathbf{sin}\,(2\pi))]\\ &= 75\pi\end{aligned}$$

Thus, the area of the region enclosed by the curve $y = 15x\,\mathbf{sin}\,x$ from 2π to 3π is 75π.

d. Use the fact that the intervals in parts (a), (b), and (c) correspond to $n = 0$, $n = 1$, and $n = 2$, respectively, to find an expression for the area in terms of n.

n	Area
0	$15\pi = 15 \bullet 1 \bullet \pi$
1	$45\pi = 15 \bullet 3 \bullet \pi$
2	$75\pi = 15 \bullet 5 \bullet \pi$

Describe the pattern.

As the value of n increases by 1, the factor in the area increases by 2 and is an odd number.

Determine an expression for the factor in the area in terms of n.

n	Area
0	1
1	3
2	5
n	$2n + 1$

Thus, the area between the curve and the x-axis for $n\pi \le x \le (n+1)\pi$ is $15(2n+1)\pi$.