

Lecture 35: A Crashing Example

December 1, 2021

Required Viewing

- Engineer4Free. (2015, February 15). Project crashing explained [Video]. <https://youtu.be/yLPzSyBQ3-k>
- Engineer4Free. (2015, February 16). Project crashing full example (part 1/2) [Video]. https://youtu.be/ZOW2QU_sBJ4
- Engineer4Free. (2015, February 16). Project crashing full example (part 2/2) [Video]. <https://youtu.be/0oBUxrpzWhM>

Recommended Reading

- *Engineering Economics* Chapter 11, Sections 11.3.3.1 and 11.3.3.2
- ...especially Example 11.5 starting on p. 403

Implementing Crashing

- Aliyu, A.M. (2012). Project Management using Critical Path Methods: A pragmatic study. *Global Journal of Pure and Applied Sciences*, 18(3-4). <https://doi.org/10.4314/gjpas.v18i3-4.11>
- Baker, B. M. (1997). Cost/time trade-off analysis for the critical path method: a derivation of the network flow approach. *Journal of the Operational Research Society*, 48, 1241-1244. <https://doi.org/10.1057/palgrave.jors.2600489>
- Georges, N., Semaan, N. & Rizk, J. (2014). CRASH: An automated tool for schedule crashing. *International Journal of Science, Environment and Technology*, 3(2), 374-394. <https://www.ijset.net/journal/264.pdf>

Case Studies: Crashing

- Barković, D., Jukić, J. & Lujić, R. (2019). Minimizing the Pessimistic Time of Activity in Overhaul Project. *TEHNIČKI VJESNIK*, 26(2), 391-397. <https://doaj.org/article/08cdb10925bf4fcb9241451260b39f39>
- Biruk, S. & Jaskowski, P. (2020). Selection of the Optimal Actions for Crashing Processes Duration to Increase the Robustness of Construction Schedules. *Applied Sciences*, 10(22), 8028. <https://doi.org/10.3390/app10228028>
- Khalaf, W.S., June, L.W., Bakar, M.R.B.A. & Soon, L. L. (2011). A Comparative Study on Time-Cost Trade-Off Approaches within Critical Path Method. *Journal of Applied Sciences*, 11, 920-931. <https://scialert.net/abstract/?doi=jas.2011.920.931>

Learning Objectives

- To be able to use a CPM diagram to determine which activities to crash, and by how much, in order to shorten a project's completion time.
- To be able to calculate the cost of crashing a project.

Relevant Solved Problems

- From *Engineering Economics*, 6th ed.
- Critical Path Calculation: Example 11.4, Example 11.5 (first part), Review Problem 11.4, 11.17, 11.18, 11.19, 11.20, 11.24 (b) and (c), 11.31, 11.36 (c), 11.38 (b), 11.39, 11.40 (c)
- Crashing Activities: 11.21, 11.22, 11.23, 11.24 (f), 11.32, 11.33, 11.41

Basic information for our example

Activity	Predecessors	Normal		Crashed	
		Duration (Days)	Cost (\$)	Duration (Days)	Cost (\$)
A	-	2	50	1	400
B	A	4	100	2	900
C	-	7	150	2	300
D	-	4	200	1	600
E	D	5	250	3	1,000

- To start with, we need some basic information: cost and duration for the normal and crashed activities, and predecessors.
- Recall: the 'crashed' values are for when we've pushed the activity as far as it can go. We've reduced the duration by as much as we can.
- The 'crashed cost' is what the activity costs, if we've chosen to reduce the duration by as much as we can. (Additional costs are due to overtime, etc.)

Days available for crashing & crash cost/day

- From last lecture, remember that we assume a linear relationship between crash cost and days crashed.
- It helps to tabulate the information we need, so we can refer back to it.
- Days available for crashing = Normal Duration – Crashed Duration
- Additional cost from maximum crashing = Crashed Cost – Normal Cost
- Therefore, given our assumption...
- Crash Cost / Day = Additional Cost / Days Available
- $$\text{Crash Cost / Day} = \frac{\text{Crashed Cost} - \text{Normal Cost}}{\text{Normal Duration} - \text{Crashed Duration}}$$

Days Available and Crash Cost/Day, Tabulated

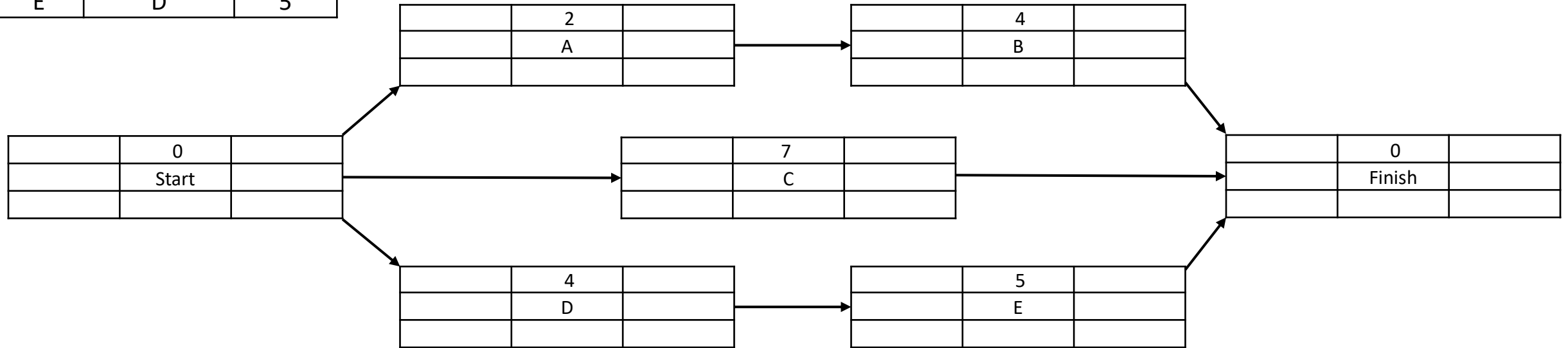
Activity	Predecessors	Normal		Crashed		Crash Days	
		Duration (Days)	Cost (\$)	Duration (Days)	Cost (\$)	Available	\$/Day
A	-	2	50	1	400	1	350
B	A	4	100	2	900	2	400
C	-	7	150	2	300	5	30
D	-	4	200	2	600	2	200
E	D	5	250	3	1,000	2	375

- Very soon, we'll have to calculate the cost of partially crashing an activity (crashing it by less than the maximum).
- This cost will be equal to...
- $\text{Normal Cost} + \text{Days Crashed} \times (\text{Crash Cost} / \text{Day})$

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Drawing the AoN Diagram

ES	T	EF
	ID	
LS	Slack	LF

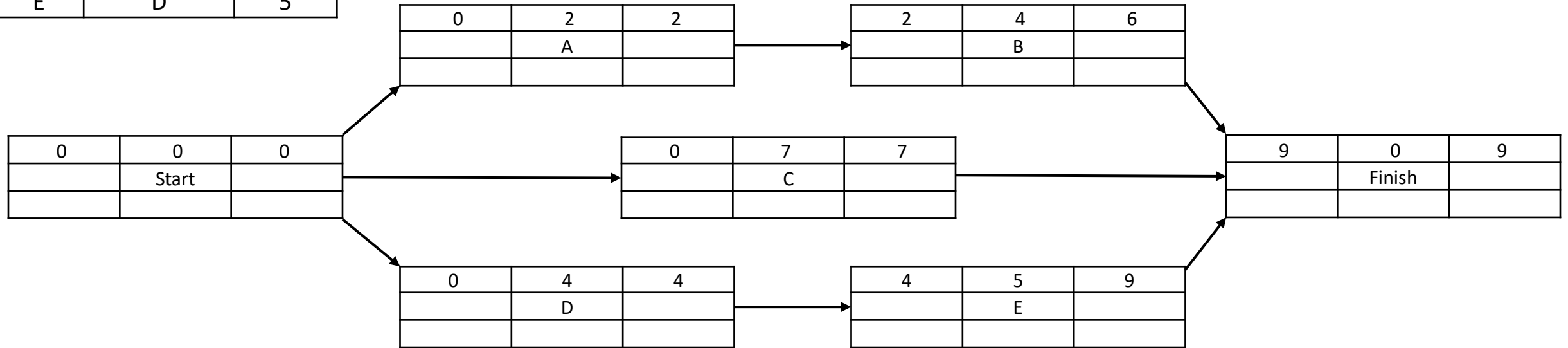


- For the network diagram, I'll be using the grid notation introduced in the previous lecture.
- Since I know I'll be doing that, I'm including the grids, even though I won't be using them yet.
- Since I already have the information, I have included the activity durations.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Forward Pass

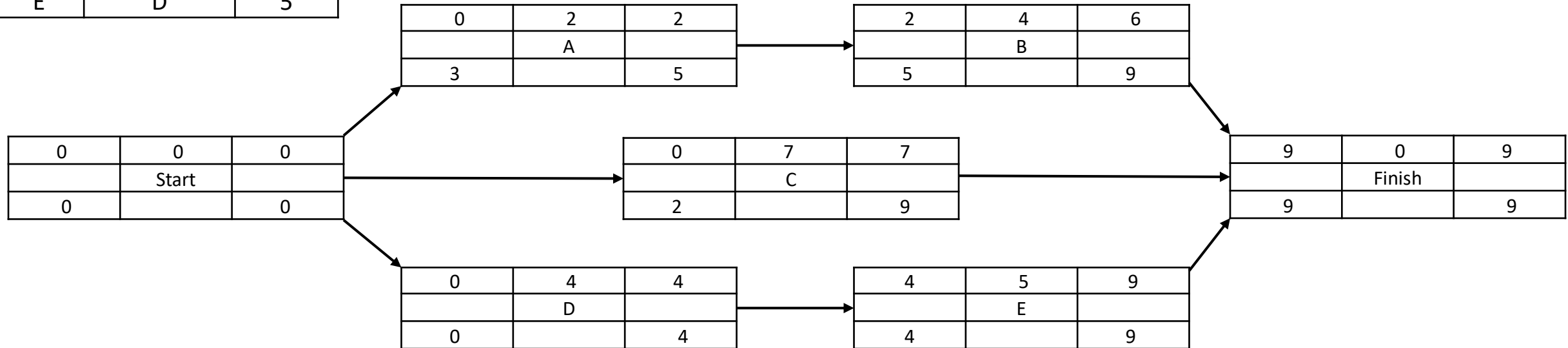
ES	T	EF
	ID	
LS	Slack	LF



Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Backward Pass

ES	T	EF
	ID	
LS	Slack	LF

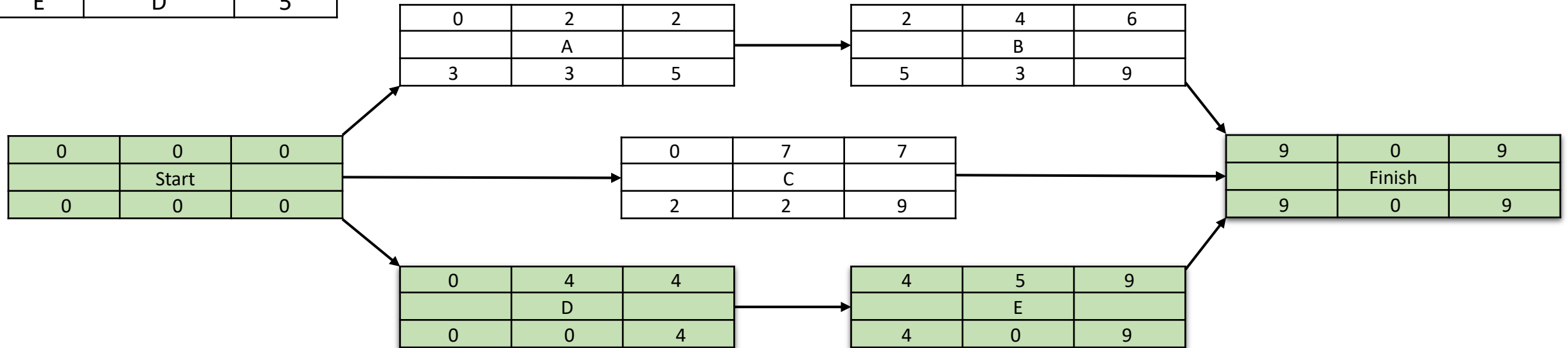


- For the backward pass, we start by assuming the project finishes at its earliest possible finish time (9 in this case).
- We use that as the Finish milestone's latest possible finish time and work backward, seeing how late each activity can start and still have the project finish on time.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Slack & Critical Path

ES	T	EF
	ID	
LS	Slack	LF



- Slack is the difference between early and late start & finish times
- i.e. $\text{Slack} = (\text{LS} - \text{ES})$ or $(\text{LF} - \text{EF})$ [They're off by T, so both give the same result.]
- Activities are on the critical path if and only if their slack is zero. I've shaded these green for convenience.
- The Start and Finish milestones will always be on the critical path.

Normal (Uncrashed) Cost

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750

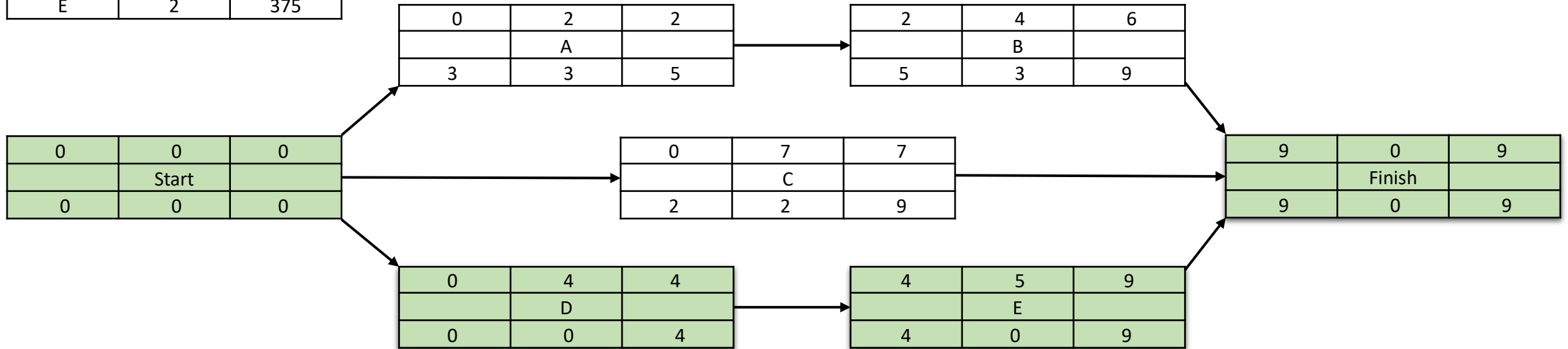
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

- We'll need Duration vs Cost information for our tradeoff curve, so it's worthwhile to start writing it down now.
- I find it helpful to keep track of the amount by which I've crashed each activity.
- In combination with the table on the right, listing Normal Cost and Crash Cost/Day...
- ...I can write each activity's cost as $\text{Normal Cost} + \text{Days Crashed} \times (\text{Crash Cost/Day})$
- The total cost of the project is the sum of the costs of all its component activities.

Considering crashing...

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	5	30
D	2	200
E	2	375

ES	T	EF
	ID	
LS	Slack	LF

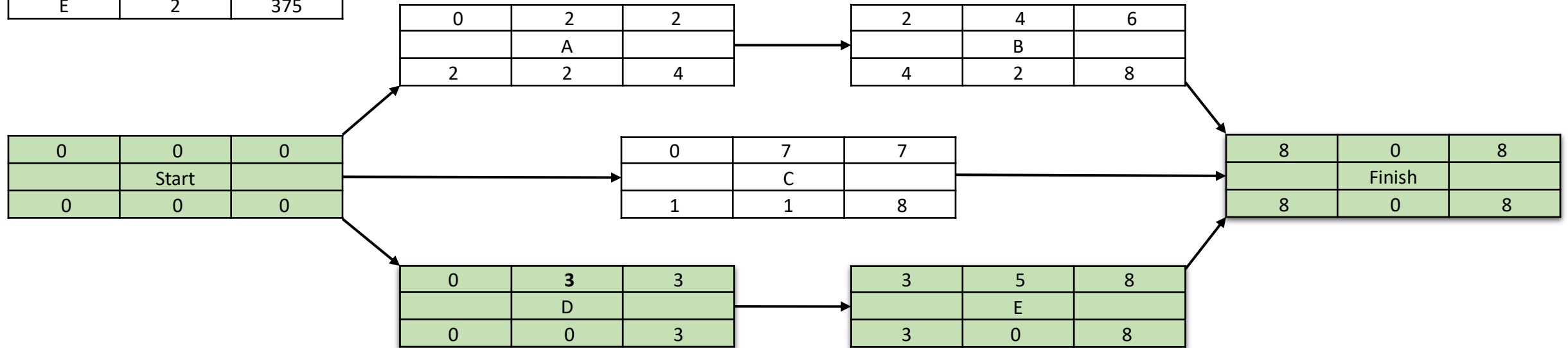


- Suppose we need to reduce the duration of the project by 1, to 8 days.
- We need to crash one of the activities on the critical path: D or E.
- D is cheapest, since it only costs \$200 per day. Let's do that.

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	5	30
D	1	200
E	2	375

Reduce Duration to 8

ES	T	EF
	ID	
LS	Slack	LF



- The duration of D has gone from 4 to 3, and we've used up one of the 2 days of crashing D has available.
- Note the ripple effect on the slack available to non-critical activities.

Continuing our cost calculations...

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0

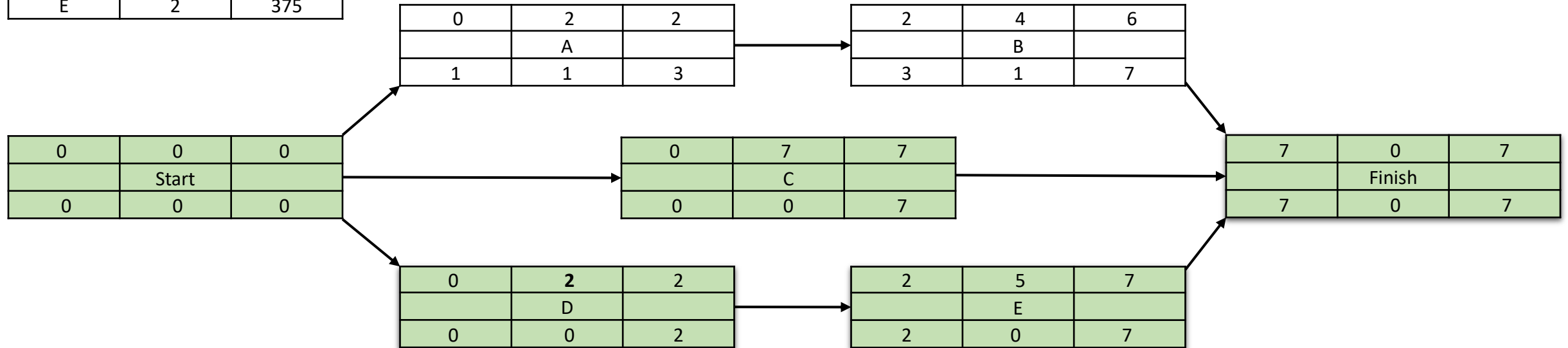
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	5	30
D	0	200
E	2	375

Reduce Duration to 7

ES	T	EF
	ID	
LS	Slack	LF



- As before, we need to crash one of the activities on the critical path.
- Again, D is cheapest, so we use up our last available D day.
- Note that C's slack is now entirely gone, so C is critical.

Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0

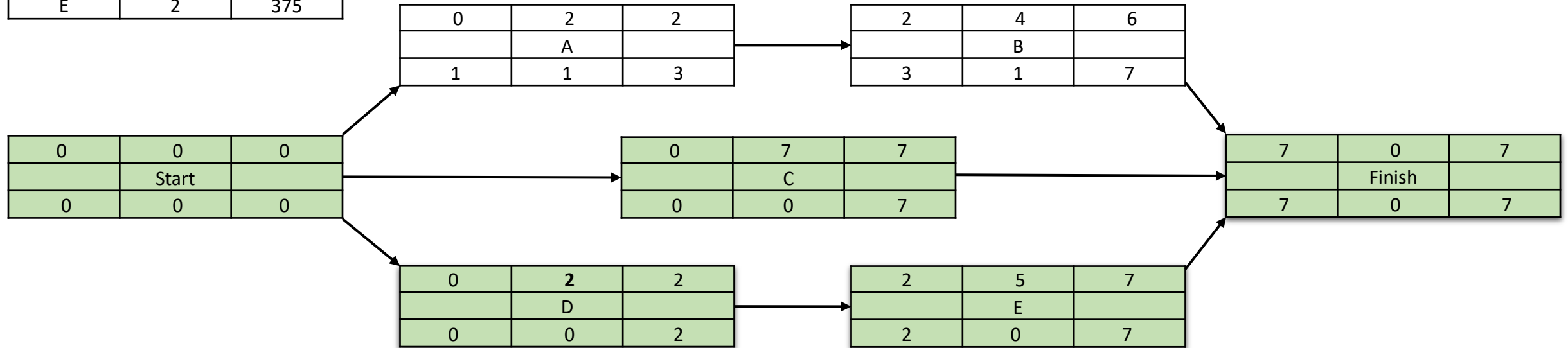
Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	5	30
D	0	200
E	2	375

Reduce Duration to 6? How?

ES	T	EF
	ID	
LS	Slack	LF



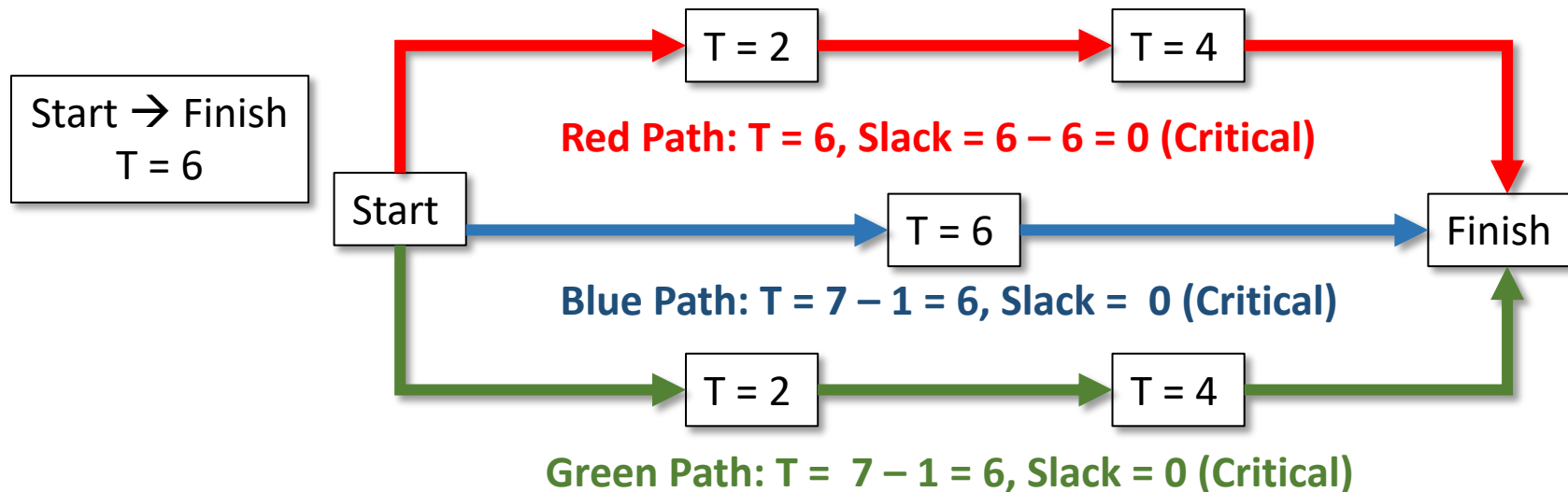
- Three activities (other than the milestones) are now critical: C, D & E.
- It's tempting to say 'crash C by 1, since it's the cheapest'...
- Problem: there are two critical PATHS. We need to crash BOTH.
- Recall last lecture...

Reducing duration from $T = 7$ to $T = 6$



Crashing one path isn't enough – we need to crash *both* the **Green** and **Blue** paths to see a reduction in Finish time.

If we try to JUST crash the **Blue** path (Activity C), the **Green** path will still be 7 days long, so the whole project will still be 7 days long.

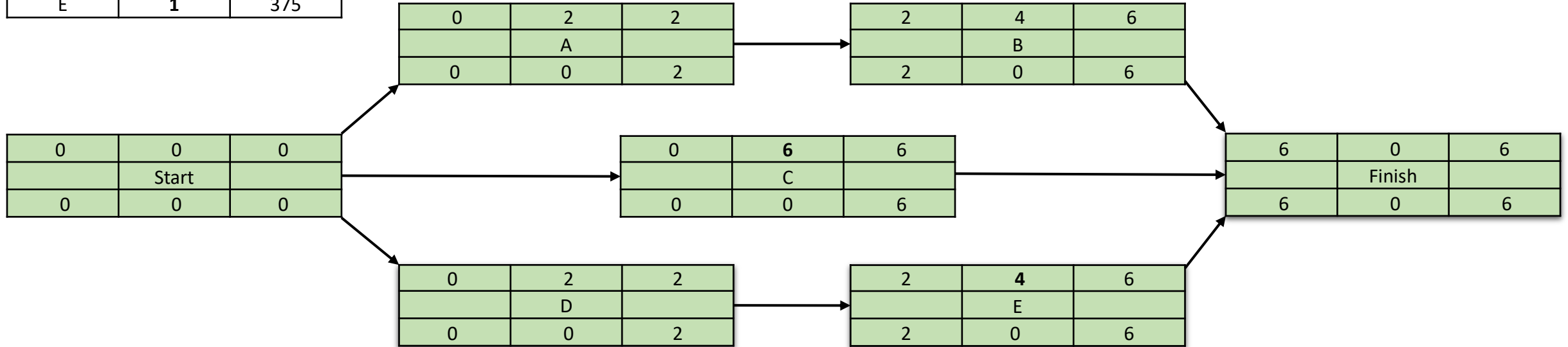


We've used up D's available days, so E is the only Green path activity left → Crash C and E

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	4	30
D	0	200
E	1	375

Reduce Duration to 6

ES	T	EF
	ID	
LS	Slack	LF



- We reduced the duration of C from 7 to 6, and E from 5 to 4.
- We also lowered the slack in A and B to zero...
- → ALL activities are now critical, and there are three critical paths.

Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0
6	0	0	1	2	1

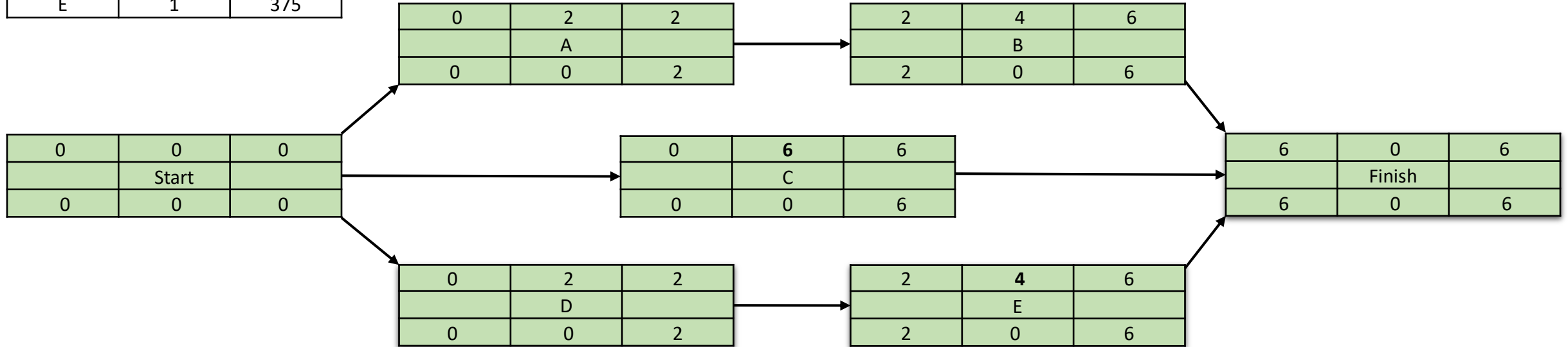
Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150
6	50	100	180	600	625	1,555

Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Activity	Crash Days	
	Available	\$/Day
A	1	350
B	2	400
C	4	30
D	0	200
E	1	375

Reduce Duration to 5?

ES	T	EF
	ID	
LS	Slack	LF

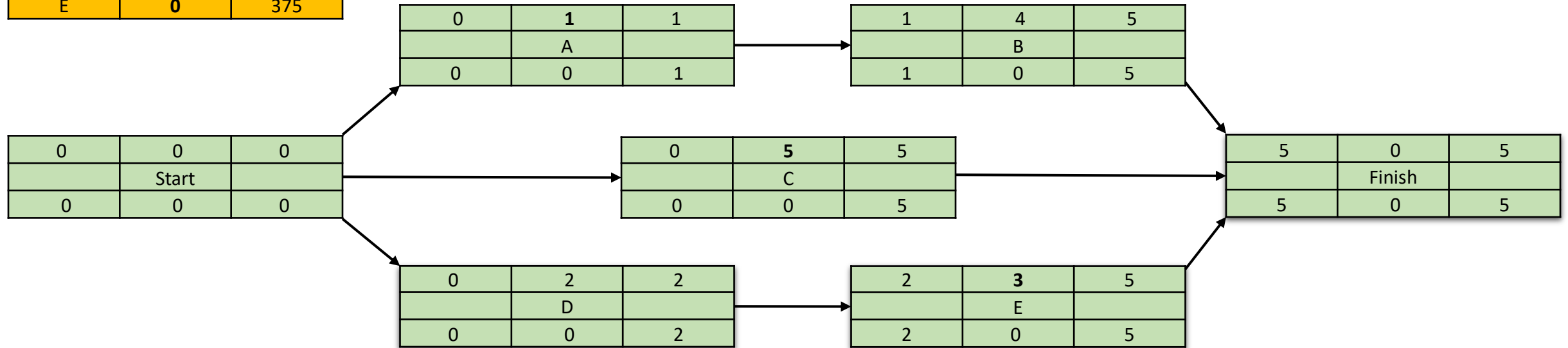


- Three critical paths: A&B, C, D&E
- To reduce project duration by 1, we have to crash all three paths by 1 each.
- D has no crash days left, so C and E are mandatory.
- For the A&B path, A is cheaper than B, so choose A.

Activity	Crash Days	
	Available	\$/Day
A	0	350
B	2	400
C	3	30
D	0	200
E	0	375

Reduce Duration to 5

ES	T	EF
	ID	
LS	Slack	LF



- This is as far as we can go: We've run out of crash days for the D&E critical path.
- Even though we have crash days left for B & C, that doesn't help us. Reducing the duration of those activities won't affect project duration, because the D&E critical path remains binding with a length of 5.

Cost Calculations

Duration	Days Crashed Per Activity				
	A	B	C	D	E
9	0	0	0	0	0
8	0	0	0	1	0
7	0	0	0	2	0
6	0	0	1	2	1
5	1	0	2	2	2

Activity	Normal Cost	Crash Cost/Day
A	50	350
B	100	400
C	150	30
D	200	200
E	250	375

Duration	Cost of Activity					
	A	B	C	D	E	Total
9	50	100	150	200	250	750
8	50	100	150	400	250	950
7	50	100	150	600	250	1,150
6	50	100	180	600	625	1,555
5	400	100	210	600	1,000	2,310

Duration	Project Cost
9	750
8	950
7	1,150
6	1,555
5	2,310

Time/Cost Tradeoff

