

NIALL M. FRASER ELIZABETH M. JEWKES

# ENGINEERING ECONOMICS

FINANCIAL DECISION MAKING  
FOR ENGINEERS

FIFTH EDITION



# List of Symbols

---

|               |  |              |   |
|---------------|--|--------------|---|
| $(A/F,i,N)$   | sinking fund factor  | $BV_{sl}(n)$ | book value at end of period $n$ using straight-line method        |
| $(A/G,i,N)$   | arithmetic gradient to annuity conversion factor                   | <b>C</b>     | present worth of costs  |
| $(A/P,i,N)$   | capital recovery factor  | <b>CCA</b>   | capital cost allowance  |
| $(F/A,i,N)$   | uniform series compound amount factor                              | <b>CSF</b>   | capital salvage factor  |
| $(F/P,i,N)$   | compound amount factor   | <b>CTF</b>   | capital tax factor  |
| $(P/A,g,i,N)$ | geometric gradient to present worth conversion factor              | <i>d</i>     | depreciation rate for declining-balance method                    |
| $(P/A,i,N)$   | series present worth factor  | $D_{db}(n)$  | depreciation amount for period $n$ using declining-balance method |
| $(P/F,i,N)$   | present worth factor   | $D_{sl}(n)$  | depreciation amount for period $n$ using straight-line method     |
| <i>A</i>      | annuity amount, equivalent annual cost                             | <b>EAC</b>   | equivalent annual cost  |
| $A_c$         | current dollars in year $N$  | <b>ERR</b>   | external rate of return   |
| $A_{tot}$     | total annuity for arithmetic gradient to annuity conversion factor | <b>E(X)</b>  | expected value of the random variable, $X$                        |
| $A'$          | base annuity for arithmetic gradient to annuity conversion factor  | <b>F</b>     | future value, future worth  |
| <b>AW</b>     | annual worth   | <i>f</i>     | inflation rate per year   |
| <b>B</b>      | present worth of benefits  | <b>FW</b>    | future worth  |
| <b>BCR</b>    | benefit-cost ratio   | <i>g</i>     | growth rate for geometric gradient                                |
| <b>BCRM</b>   | modified benefit-cost ratio  | <i>i</i>     | current interest rate   |
| $BV_{db}(n)$  | book value at end of period $n$ using declining-balance method     | <i>I</i>     | interest amount   |
|               |  | <i>i'</i>    | real interest rate  |
|               |  | $I_c$        | compound interest amount  |

---

|                         |  |                  |   |
|-------------------------|--|------------------|---|
| $i_e$                   | effective interest rate  | $N$              | number of periods, useful life of an asset                        |
| $i_s$                   | interest rate per subperiod  | $P$              | present value, present worth, purchase price, principal amount    |
| $I_s$                   | simple interest amount   | $PW$             | present worth   |
| $i^o$                   | growth adjusted interest rate                                      | $p(x)$           | probability distribution  |
| <b>IRR</b>              | internal rate of return  | $\Pr\{X = x_i\}$ | alternative expression of probability distribution                |
| <b>IRR<sub>C</sub></b>  | current dollar IRR   | $r$              | nominal interest rate, rating for a decision matrix               |
| <b>IRR<sub>R</sub></b>  | real dollar IRR  | $R_{0,N}$        | real dollar equivalent to $A_N$ relative to year 0, the base year |
| $i^*$                   | internal rate of return  | $S$              | salvage value   |
| $i_e^*$                 | external rate of return  | $t$              | tax rate  |
| $i_{ea}^*$              | approximate external rate of return                                | <b>UCC</b>       | undepreciated capital cost  |
| $I_{0,N}$               | the value of a global price index at year $N$ , relative to year 0 | $X$              | random variable   |
| $m$                     | number of subperiods in a period                                   | $\pi_{01}$       | Laspeyres price index   |
| <b>MARR</b>             | minimum acceptable rate of return                                  |                  |   |
| <b>MARR<sub>C</sub></b> | current dollar MARR  |                  |   |
| <b>MARR<sub>R</sub></b> | real dollar MARR   |                  |   |

# **ENGINEERING ECONOMICS**

**FINANCIAL DECISION MAKING  
FOR ENGINEERS**

**FIFTH EDITION**

*This page intentionally left blank*

**NIALL M. FRASER**

Open Options Corporation

**ELIZABETH M. JEWKES**

University of Waterloo

# **ENGINEERING ECONOMICS**

**FINANCIAL DECISION MAKING  
FOR ENGINEERS**

**FIFTH EDITION**

**PEARSON**

Toronto

Vice-President, Editorial Director: Gary Bennett  
Executive Acquisitions Editor: Cathleen Sullivan  
Senior Marketing Manager: Kim Ukrainec  
Supervising Developmental Editor: Suzanne Schaan  
Developmental Editor: Eleanor MacKay  
Project Manager: Ioana Gagea  
Manufacturing Manager: Susan Johnson  
Production Editor: Ridhi Mathur  
Copy Editor: Sally Glover  
Proofreader: Susan Bindernagel  
Technical Review: Claude Théoret  
Coldread: Karen Alliston  
Index: Belle Wong  
Compositor: Cenveo Publisher Services  
Photo and Permissions Researcher: Joanne Tang  
Art Director: Julia Hall  
Cover and Interior Designer: Anthony Leung  
Cover Image: Rudy Sulgan/Corbis

The image on the cover features the newest edition to Vancouver's transportation network: The Canada Line Rapid Transit. In 2011, Engineers Canada awarded the Canada Line Rapid Transit project the National Award for an Engineering Project.

Credits and acknowledgments for material borrowed from other sources and reproduced, with permission, in this textbook appear on page 515.

If you purchased this book outside the United States or Canada, you should be aware that it has been imported without the approval of the publisher or the author.

---

Copyright © 2013, 2009, 2006, 2000, 1997 Pearson Canada Inc. All rights reserved. Manufactured in the United States of America. This publication is protected by copyright and permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. To obtain permission(s) to use material from this work, please submit a written request to Pearson Canada Inc., Permissions Department, 26 Prince Andrew Place, Don Mills, Ontario, M3C 2T8, or fax your request to 416-447-3126, or submit a request to Permissions Requests at [www.pearsoncanada.ca](http://www.pearsoncanada.ca).

10 9 8 7 6 5 4 3 2 1 [EB]

#### **Library and Archives Canada Cataloguing in Publication**

Engineering economics : financial decision making for engineers / Niall M. Fraser, Elizabeth M. Jewkes.—5th ed.

First-3rd eds. publ. under title: Engineering economics in Canada.

4th ed. publ. under title: Global engineering economics.

Includes bibliographical reference and index.

ISBN 978-0-13-237925-0

1. Engineering economy. I. Fraser, Niall M. (Niall Morris), 1952– II. Jewkes, Elizabeth M. (Elizabeth Marie), 1958–

TA177.4.F725 2012

658.15

C2011-906926-1

**PEARSON**

ISBN 978-0-13-237925-0

# Brief Contents

Preface   xiii

|                   |   |                   |
|-------------------|---|-------------------|
| <b>CHAPTER 1</b>  | Engineering Decision Making   | 1                 |
| <b>CHAPTER 2</b>  | Time Value of Money   | 18                |
| <b>CHAPTER 3</b>  | Cash Flow Analysis  | 45                |
| Appendix 3A       | Derivation of Discrete Compound Interest Factors                        | 81                |
| <b>CHAPTER 4</b>  | Comparison Methods Part 1   | 85                |
| Appendix 4A       | The MARR and the Cost of Capital  | 122               |
| <b>CHAPTER 5</b>  | Comparison Methods Part 2   | 125               |
| <b>CHAPTER 6</b>  | Depreciation and Financial Accounting                                   | 163               |
| Appendix 6A       | Cost Estimation   | 207               |
| <b>CHAPTER 7</b>  | Replacement Decisions   | 215               |
| <b>CHAPTER 8</b>  | Taxes   | 263               |
| Appendix 8A       | Deriving the Capital Tax Factor   | 301               |
| <b>CHAPTER 9</b>  | Inflation   | 304               |
| Appendix 9A       | Computing a Price Index   | 338               |
| <b>CHAPTER 10</b> | Public Sector Decision Making   | 342               |
| <b>CHAPTER 11</b> | Project Management  | 387               |
| <b>CHAPTER 12</b> | Dealing With Uncertainty and Risk                                       | 424               |
| Appendix 12A      | Decision Matrices   | 471               |
| <b>CHAPTER 13</b> | Qualitative Considerations and Multiple Criteria                        | COMPANION WEBSITE |
| <b>APPENDIX A</b> | Compound Interest Factors for Discrete Compounding, Discrete Cash Flows | 477               |
| <b>APPENDIX B</b> | Answers to Selected Problems  | 501               |
| Glossary          | 507   |                   |
| Photo Credits     | 515   |                   |
| Index             | 517   |                   |

*This page intentionally left blank*

# Contents

Preface   xiii

---

## CHAPTER 1   Engineering Decision Making   1

Engineering Economics in Action, Part 1A: Naomi Arrives   2

    1.1   Engineering Decision Making   2

    1.2   What Is Engineering Economics?   3

    1.3   Making Decisions   4

Engineering Economics in Action, Part 1B: Naomi Settles In   5

    1.4   Dealing With Abstractions   6

    1.5   The Moral Question: Three True Stories   9

NET VALUE 1.1: PROFESSIONAL ENGINEERING ASSOCIATIONS   11

    1.6   Uncertainty and Sensitivity Analysis   11

    1.7   How This Book Is Organized   12

Engineering Economics in Action, Part 1C: A Taste of What Is to Come   13

Problems   14

Mini-Case 1.1: R. v. Syncrude Canada Ltd.   16

---

## CHAPTER 2   Time Value of Money   18

Engineering Economics in Action, Part 2A: A Steal For Steel   19

    2.1   Introduction   19

    2.2   Interest and Interest Rates   19

NET VALUE 2.1: PRIME INTEREST RATES   20

    2.3   Compound and Simple Interest   22

    2.4   Effective and Nominal Interest Rates   25

    2.5   Continuous Compounding   28

    2.6   Cash Flow Diagrams   29

    2.7   Equivalence   32

        2.7.1   Mathematical Equivalence   32

        2.7.2   Decisional Equivalence   32

        2.7.3   Market Equivalence   33

Review Problems   34

Summary   38

Engineering Economics in Action, Part 2B: You Just Have to Know When   38

Problems   39

Mini-Case 2.1: Student Credit Cards   44

---

**CHAPTER 3 Cash Flow Analysis 45**

|   |  |           |
|---|--|-----------|
| <b>Engineering Economics in Action, Part 3A: Apples and Oranges</b> | <b>46</b>  |           |
| <b>3.1</b>  | <b>Introduction</b>  | <b>46</b> |
| <b>3.2</b>  | <b>Timing of Cash Flows and Modelling</b>                                | <b>46</b> |
| <b>3.3</b>  | <b>Compound Interest Factors for Discrete Compounding</b>                | <b>47</b> |
| <b>3.4</b>  | <b>Compound Interest Factors for Single Disbursements or Receipts</b>    | <b>48</b> |
| <b>3.5</b>  | <b>Compound Interest Factors for Annuities</b>                           | <b>51</b> |
| <b>3.6</b>  | <b>Conversion Factor for Arithmetic Gradient Series</b>                  | <b>58</b> |
| <b>3.7</b>  | <b>Conversion Factor for Geometric Gradient Series</b>                   | <b>61</b> |
| <b>NET VALUE 3.1: ESTIMATING GROWTH RATES</b>                       | <b>62</b>  |           |
| <b>3.8</b>  | <b>Non-Standard Annuities and Gradients</b>                              | <b>65</b> |
| <b>3.9</b>  | <b>Present Worth Computations When <math>N \rightarrow \infty</math></b> | <b>67</b> |
| Review Problems   | 68   |           |
| Summary   | 71   |           |
| <b>Engineering Economics in Action, Part 3B: No Free Lunch</b>      | <b>72</b>  |           |
| Problems  | 73   |           |
| Mini-Case 3.1: The Canadian Oil Sands                               | 79   |           |
| Appendix 3A Derivation of Discrete Compound Interest Factors        | 81   |           |

---

**CHAPTER 4 Comparison Methods Part 1 85**

|   |  |            |
|---|--|------------|
| <b>Engineering Economics in Action, Part 4A: What's Best?</b>   | <b>86</b>  |            |
| <b>4.1</b>  | <b>Introduction</b>  | <b>86</b>  |
| <b>4.2</b>  | <b>Relations Among Projects</b>                                  | <b>88</b>  |
| <b>4.3</b>  | <b>Minimum Acceptable Rate of Return (MARR)</b>                  | <b>90</b>  |
| <b>4.4</b>  | <b>Present Worth (PW) and Annual Worth (AW) Comparisons</b>      | <b>91</b>  |
| <b>4.4.1</b>  | <b>Present Worth Comparisons for Independent Projects</b>        | <b>91</b>  |
| <b>4.4.2</b>  | <b>Present Worth Comparisons for Mutually Exclusive Projects</b> | <b>94</b>  |
| <b>4.4.3</b>  | <b>Annual Worth Comparisons</b>                                  | <b>94</b>  |
| <b>NET VALUE 4.1: CAR PAYMENT CALCULATORS</b>                   | <b>95</b>  |            |
| <b>4.4.4</b>  | <b>Comparison of Alternatives With Unequal Lives</b>             | <b>97</b>  |
| <b>4.5</b>  | <b>Payback Period</b>  | <b>101</b> |
| Review Problems   | 105  |            |
| Summary   | 109  |            |
| <b>Engineering Economics in Action, Part 4B: Doing It Right</b> | <b>109</b>   |            |
| Problems  | 110  |            |
| Mini-Case 4.1: Rockwell International                           | 121  |            |
| Appendix 4A The MARR and the Cost of Capital                    | 122  |            |

---

**CHAPTER 5 Comparison Methods Part 2 125**

|   |  |            |
|---|--|------------|
| <b>Engineering Economics in Action, Part 5A: What's Best? Revisited</b> | <b>126</b>                                 |            |
| <b>5.1</b>  | <b>Introduction</b>                        | <b>126</b> |
| <b>5.2</b>  | <b>The Internal Rate of Return</b>         | <b>126</b> |
| <b>5.3</b>  | <b>Internal Rate of Return Comparisons</b> | <b>129</b> |

|   |  |     |
|---|--|-----|
| 5.3.1   | IRR for Independent Projects                                   | 129 |
| 5.3.2   | IRR for Mutually Exclusive Projects                            | 132 |
| 5.3.3   | Multiple IRRs  | 136 |
| <b>NET VALUE 5.1: ADDITIONAL PROJECT COMPARISON RESOURCES</b>       |  | 139 |
| 5.3.4   | External Rate of Return Methods                                | 140 |
| 5.3.5   | When to Use the ERR  | 142 |
| <b>5.4</b>  | Rate of Return and Present/Annual Worth Methods Compared       | 143 |
| 5.4.1   | Equivalence of Rate of Return and Present/Annual Worth Methods | 143 |
| 5.4.2   | Why Choose One Method Over the Other?                          | 147 |
| Review Problems   |  | 149 |
| Summary   |  | 152 |
| <b>Engineering Economics in Action, Part 5B: The Invisible Hand</b> |  | 153 |
| Problems  |  | 154 |
| Mini-Case 5.1: The Galore Creek Project                             |  | 161 |

## **CHAPTER 6 Depreciation and Financial Accounting** 163

---

|   |  |     |
|---|--|-----|
| <b>Engineering Economics in Action, Part 6A: The Pit Bull</b> |  | 164 |
| <b>6.1</b>  | Introduction                             | 164 |
| <b>6.2</b>  | Depreciation and Depreciation Accounting | 165 |
| 6.2.1   | Reasons for Depreciation                 | 165 |
| 6.2.2   | Value of an Asset                        | 165 |
| 6.2.3   | Straight-Line Depreciation               | 166 |
| 6.2.4   | Declining-Balance Depreciation           | 168 |
| <b>6.3</b>  | Elements of Financial Accounting         | 172 |
| 6.3.1   | Measuring the Performance of a Firm      | 173 |
| 6.3.2   | The Balance Sheet                        | 173 |
| 6.3.3   | The Income Statement                     | 178 |
| 6.3.4   | Estimated Values in Financial Statements | 180 |
| 6.3.5   | Financial Ratio Analysis                 | 180 |
| <b>NET VALUE 6.1: SECURITIES REGULATORS</b>                   |  | 181 |
| 6.3.6   | Financial Ratios                         | 181 |

|  |  |     |
|--|--|-----|
| Review Problems  |  | 188 |
| Summary  |  | 192 |
| <b>Engineering Economics in Action, Part 6B: Usually the Truth</b> |  | 193 |
| Problems   |  | 193 |
| Mini-Case 6.1: Research In Motion                                  |  | 205 |
| Appendix 6A Cost Estimation  |  | 207 |
| Problems   |  | 212 |

## **CHAPTER 7 Replacement Decisions** 215

---

|   |                                       |     |
|---|---------------------------------------|-----|
| <b>Engineering Economics in Action, Part 7A: You Need the Facts</b> |                                       | 216 |
| <b>7.1</b>  | Introduction                          | 216 |
| <b>7.2</b>  | A Replacement Example                 | 217 |
| <b>7.3</b>  | Reasons for Replacement or Retirement | 220 |
| <b>7.4</b>  | Capital Costs and Other Costs         | 221 |

|  |  |
|--|--|
| <b>NET VALUE 7.1: ESTIMATING SALVAGE VALUES AND SCRAP VALUES</b> | 222  |
| <b>7.5</b>   | Defender and Challenger Are Identical 222                                  |
| <b>7.6</b>   | Challenger Is Different From Defender; Challenger Repeats Indefinitely 227 |
| <b>7.6.1</b>   | Converting From Subcontracted to In-House Production 229                   |
| <b>7.6.2</b>   | The Irrelevance of Sunk Costs 230  |
| <b>7.6.3</b>   | When Capital or Operating Costs Are Non-Monotonic 231                      |
| <b>7.7</b>   | Challenger Is Different From Defender; Challenger Does Not Repeat 234      |
| Review Problems  | 235  |
| Summary  | 240  |
| <b>Engineering Economics in Action, Part 7B: Decision Time</b>   | 241  |
| Problems   | 242  |
| Mini-Case 7.1: Lighting the Way                                  | 261  |

## CHAPTER 8 Taxes 263

---

|  |   |
|--|---|
| <b>Engineering Economics in Action, Part 8A: It's in the Details</b> | 264   |
| <b>8.1</b>   | Introduction 264  |
| <b>8.2</b>   | Personal Income Taxes and Corporate Income Taxes Compared 265 |
| <b>8.3</b>   | Corporate Tax Rates 265                                       |
| <b>8.4</b>   | Before- and After-Tax MARR 268                                |
| <b>8.5</b>   | The Effect of Taxation on Cash Flows 269                      |
| <b>8.5.1</b>   | The Effect of Taxes on First Cost 269                         |
| <b>8.5.2</b>   | The Effect of Taxes on Savings 271                            |
| <b>8.5.3</b>   | The Effect of Taxes on Salvage or Scrap Value 272             |
| <b>8.6</b>   | Present Worth and Annual Worth Tax Calculations 273           |
| <b>8.7</b>   | IRR Tax Calculations 274                                      |
| <b>8.7.1</b>   | Accurate IRR Tax Calculations 274                             |
| <b>8.7.2</b>   | Approximate After-Tax Rate-of-Return Calculations 275         |
| <b>8.8</b>   | Specific Tax Rules in Canada 276                              |
| <b>8.8.1</b>   | The Capital Cost Allowance System 276                         |
| <b>NET VALUE 8.1: CANADA REVENUE AGENCY WEBSITE</b>                  | 276   |
| <b>8.8.2</b>   | Undepreciated Capital Cost and the Half-Year Rule 280         |
| <b>8.8.3</b>   | The Capital Tax Factor and Capital Salvage Factor 283         |
| <b>8.8.4</b>   | Components of a Complete Tax Calculation 285                  |
| Review Problems  | 288   |
| Summary  | 293   |
| <b>Engineering Economics in Action, Part 8B: The Work Report</b>     | 294   |
| Problems   | 294   |
| Mini-Case 8.1: Flat Taxes  | 300   |
| Appendix 8A Deriving the Capital Tax Factor                          | 301   |

## CHAPTER 9 Inflation 304

---

|  |   |
|--|---|
| <b>Engineering Economics in Action, Part 9A: The Inflated Expert</b> | 305   |
| <b>9.1</b>   | Introduction 305                                |
| <b>9.2</b>   | Measuring the Inflation Rate 306                |
| <b>NET VALUE 9.1: STATISTICS CANADA</b>                              | 306   |
| <b>9.3</b>   | Economic Evaluation With Inflation 308          |
| <b>9.3.1</b>   | Converting Between Real and Current Dollars 308 |

|              |  |     |
|--------------|--|-----|
| <b>9.4</b>   | The Effect of Correctly Anticipated Inflation  | 311 |
| <b>9.4.1</b> | The Effect of Inflation on the MARR  | 312 |
| <b>9.4.2</b> | The Effect of Inflation on the IRR   | 314 |
| <b>9.5</b>   | Project Evaluation Methods With Inflation  | 316 |
|              | Review Problems  | 322 |
|              | Summary  | 325 |
|              | Engineering Economics in Action, Part 9B: Exploiting Volatility  | 326 |
|              | Problems   | 326 |
|              | Mini-Case 9.1: Economic Comparison of High Pressure and Conventional Pipelines: Associated Engineering | 337 |
|              | Appendix 9A Computing a Price Index  | 338 |

---

## CHAPTER 10 Public Sector Decision Making 342

|               |  |     |
|---------------|--|-----|
|               | Engineering Economics in Action, Part 10A: New Challenges Down Under | 343 |
| <b>10.1</b>   | Introduction   | 343 |
| <b>10.2</b>   | Market Failure   | 346 |
| <b>10.2.1</b> | Market Failure Defined   | 346 |
| <b>10.2.2</b> | Remedies for Market Failure  | 348 |
| <b>10.3</b>   | Decision Making in the Public Sector                                 | 349 |
| <b>10.3.1</b> | The Point of View Used for Project Evaluation                        | 350 |
|               | NET VALUE 10.1: CANADIAN BENEFIT–COST ANALYSIS GUIDES                | 352 |
| <b>10.3.2</b> | Identifying and Measuring the Costs of Public Projects               | 352 |
| <b>10.3.3</b> | Identifying and Measuring the Benefits of Public Projects            | 353 |
| <b>10.3.4</b> | Benefit–Cost Ratios  | 356 |
| <b>10.3.5</b> | The MARR in the Public Sector  | 362 |
|               | Review Problems  | 363 |
|               | Summary  | 368 |
|               | Engineering Economics in Action, Part 10B: Look at It Broadly        | 369 |
|               | Problems   | 369 |
|               | Mini-Case 10.1: Emissions Trading                                    | 384 |

---

## CHAPTER 11 Project Management 387

|                 |   |     |
|-----------------|---|-----|
|                 | Engineering Economics in Action, Part 11A: Filling a Vacuum | 388 |
| <b>11.1</b>     | Introduction  | 388 |
| <b>11.2</b>     | Project Management Lifecycle                                | 389 |
| <b>11.2.1</b>   | Initiation  | 390 |
| <b>11.2.2</b>   | Planning  | 390 |
| <b>11.2.3</b>   | Execution   | 391 |
| <b>11.2.4</b>   | Monitoring and Controlling                                  | 391 |
| <b>11.2.5</b>   | Closure   | 392 |
| <b>11.3</b>     | Project Management Tools                                    | 392 |
| <b>11.3.1</b>   | Work Breakdown Structure                                    | 392 |
| <b>11.3.2</b>   | Gantt Charts  | 394 |
| <b>11.3.3</b>   | The Critical Path Method                                    | 396 |
| <b>11.3.3.1</b> | Project Scheduling and the Critical Path Method             | 400 |
| <b>11.3.3.2</b> | Project Crashing and Time–Cost Tradeoffs                    | 403 |

|  |     |
|--|-----|
| Review Problems  | 408 |
| Summary  | 412 |
| <b>Engineering Economics in Action, Part 11B: A Crash Course</b> | 412 |
| Problems   | 413 |
| Mini-Case 11.1: No Longer LEEDing                                | 422 |

**CHAPTER 12 Dealing With Uncertainty and Risk** 424

---

|   |     |
|---|-----|
| <b>Engineering Economics in Action, Part 12A: Trees from Another Planet</b> | 425 |
| <b>12.1</b> Introduction  | 425 |
| <b>12.2</b> Sensitivity Graphs  | 426 |
| <b>12.3</b> Break-Even Analysis   | 429 |
| <b>12.3.1</b> Break-Even Analysis for a Single Project                      | 430 |
| <b>12.3.2</b> Break-Even Analysis for Multiple Projects                     | 432 |
| <b>12.4</b> Basic Concepts of Probability                                   | 436 |
| <b>12.5</b> Structuring Decisions With Decision Trees                       | 442 |
| <b>NET VALUE 12.1: DECISION TREE SOFTWARE</b>                               | 446 |
| Review Problems   | 448 |
| Summary   | 454 |
| <b>Engineering Economics in Action, Part 12B: Chances Are Good</b>          | 454 |
| Problems  | 455 |
| Mini-Case 12.1: China Steel Australia Limited                               | 469 |
| Appendix 12A Decision Matrices  | 471 |
| Problems  | 474 |
| <b>Engineering Economics in Action, Part 12C: Moving On</b>                 | 476 |

**CHAPTER 13 Qualitative Considerations and Multiple Criteria** COMPANION WEBSITE

---

|   |  |
|---|--|
| <b>Engineering Economics in Action, Part 13A: Don't Box Them In</b> |  |
| <b>13.1</b> Introduction  |  |
| <b>13.2</b> Efficiency  |  |
| <b>13.3</b> Decision Matrixes                                       |  |
| <b>13.4</b> The Analytical Hierarchy Process                        |  |
| <b>NET VALUE 13.1: AHP SOFTWARE</b>                                 |  |
| <b>13.5</b> The Consistency Ratio for AHP                           |  |
| Review Problems   |  |
| Summary   |  |
| <b>Engineering Economics in Action, Part 13B: Moving On</b>         |  |
| Problems  |  |
| Mini-Case 13.1: Northwind Stoneware                                 |  |

|                   |   |     |
|-------------------|---|-----|
| <b>Appendix A</b> | Compound Interest Factors for Discrete Compounding, Discrete Cash Flows | 477 |
| <b>Appendix B</b> | Answers to Selected Problems  | 501 |
| Glossary          | 507   |     |
| Photo Credits     | 515   |     |
| Index             | 517   |     |

# Preface

Courses on engineering economics are found in engineering curricula in Canada and throughout the world. The courses generally deal with deciding among alternative engineering projects with respect to expected costs and benefits. The topic is so fundamental to engineering knowledge that the Canadian Engineering Accreditation Board requires that all accredited professional engineering programs provide studies in engineering economics. Many engineers have found that a course in engineering economics can be as useful in their practice as any of their more technical courses.

There are several stages to making a good decision. One stage is being able to determine whether a solution to a problem is technically feasible. This is one of the roles of the engineer, who has specialized training to make such technical judgments. Another stage is deciding which of several technically feasible alternatives is best. Deciding among alternatives often does not require the technical competence needed to determine which alternatives are feasible, but it is equally important in making the final choice. Some engineers have found that choosing among alternatives can be more difficult than deciding what alternatives exist.

The role of engineers in Canadian society has changed. In the past, engineers tended to have a fairly narrow focus, concentrating on the technical aspects of a problem and on strictly computational aspects of engineering economics. As a result, many engineering economics texts focused on the mathematics of the subject. Today, engineers are more likely to be involved in many stages of an engineering project, and thus they need to be able to take into account strategic and policy issues.

This book is designed for teaching a course on engineering economics to match engineering practice in Canada today. It recognizes the role of the engineer as a decision maker who has to make and defend sensible decisions. Such decisions must not only take into account a correct assessment of costs and benefits; they must also reflect an understanding of the environment in which the decisions are made.

This book has had four previous editions. The first three editions were focussed on Engineering Economics from a Canadian perspective. Given the increasing globalization of many engineering activities, the title and content were updated in the fourth edition to reflect that engineers increasingly worked in an international setting. Feedback from our users on the fourth edition has brought the fifth edition back to its original Canadian perspective. This book has been the text of choice for many Canadian educators for over 15 years.

Canadian engineers have a unique set of circumstances that warrant a text with a specific Canadian focus. Canadian firms make decisions according to norms and standards that reflect Canadian views on social responsibility, environmental concerns, and cultural diversity. This perspective is reflected in the content and tone of much of the material in this book. Furthermore, Canadian tax regulations are complicated and directly affect engineering economic analysis. These regulations and their effect on decision making are covered in detail in Chapter 8.

This book also relates to students' everyday lives. In addition to examples and problems with an engineering focus, there are a number that involve decisions that many students might face, such as renting an apartment, getting a job, or buying a car. Other examples in the text are adapted from familiar sources such as Canadian newspapers and well-known Canadian companies.

## Content and Organization

Because the mathematics of finance has not changed dramatically over the past number of years, there is a natural order to the course material. Nevertheless, a modern view of the role of the engineer flavours this entire book and provides a balanced exposure to the subject.

Chapter 1 frames the problem of engineering decision making as one involving many issues. Manipulating the cash flows associated with an engineering project is an important process for which useful mathematical tools exist. These tools form the bulk of the remaining chapters. However, throughout the text, students are kept aware of the fact that the eventual decision depends not only on the cash flows, but also on less easily quantifiable considerations of business policy, social responsibility, and ethics.

Chapters 2 and 3 present tools for manipulating monetary values over time. Chapters 4 and 5 show how students can use their knowledge of manipulating cash flows to make comparisons among alternative engineering projects. Chapter 6 provides an understanding of the environment in which the decisions are made by examining depreciation and the role it plays in the financial functioning of a company and in financial accounting. Because students frequently ask "where do the numbers come from?" we have added in this new edition a brief overview of cost estimating as an appendix to Chapter 6.

Chapter 7 deals with the analysis of replacement decisions. Chapters 8 and 9 are concerned with taxes and inflation, which affect decisions based on cash flows. Chapter 10 provides an introduction to public sector decision making.

Chapter 11, new to this edition, provides an introduction to project management. Engineers should have an appreciation of the phases that all engineering projects pass through, as well as the role engineering economics plays in evaluating the economic feasibility of a project.

Most engineering projects involve estimating future cash flows as well as other project characteristics. Since estimates can be made in error and the future is unknown, it is important for engineers to take uncertainty and risk into account as completely as possible. Chapter 12 addresses several approaches for dealing with uncertainty and risk in economic evaluations.

## New to This Edition

In addition to updating material and correcting errors, we have made the following important changes in the fifth edition:

- We have introduced a **Spreadsheet Savvy** feature to each chapter. As a running theme, each feature highlights how spreadsheets and, in particular, Excel functionality, can be used to support the student as he or she learns the basic concepts covered in the chapter. The textbook does not provide a tutorial on the general use of spreadsheets; this task is left to others. However, it does highlight spreadsheet features that are relevant to the engineering economic topics contained in each chapter and instructs students on their use.

While many instructors will want to ensure that students are able to conduct the economic analysis from first principles, in practice, students should also know how to make best use of the tools commonly available in the workplace. In addition to the Spreadsheet Savvy features, for selected examples in each chapter, we have noted where Excel spreadsheets can be used to compute the answer to a problem.

- Chapter 6 now features an appendix on cost estimation. This appendix provides an overview of how cash flows are estimated and how the level of detail contained in the estimation depends on the purpose of the estimation, whether it is for early-stage project assessment or later detailed estimates for use in project monitoring and control.
- Chapter 8 on taxation has undergone a major revision in order to focus on the Canadian tax system. The current chapter benefits from the fourth edition material in that the information was reorganized around generic themes in taxation. The current chapter includes these generic themes, but now also contains an emphasis on Canadian tax rules.
- Chapters 11, 12, and 13 from the fourth edition have been combined and shortened into a new Chapter 12 entitled “Dealing with Uncertainty and Risk.” We removed more advanced topics that will likely be covered in other courses and focused on key decision-making topics that will be useful for any instructor who wants to include an introduction to uncertainty and risk in decision making.
- The text has a completely new Chapter 11 on project management. This chapter responds well to the new Canadian Engineering Accreditation Board (CEAB) accreditation criteria for engineering schools to teach both economics and project management in their curricula. Chapter 11 provides introductory coverage of project management and can be used as the basis for reinforcement and mastery through subsequent course projects, team competitions, and other program components.
- We reorganized and renumbered the problem sets throughout the text. Each chapter-end problem set is divided into three categories: Key Concepts, Applications, and More Challenging Problems. These correspond well with the new CEAB requirement for learning outcome levels of “introduction,” “reinforcement,” and “mastery.”
- Minor changes to all other chapters have been made to update and improve the overall flow and presentation of the material.

## Special Features

We have created special features for this book in order to facilitate the learning of the material and an understanding of its applications:

- **Engineering Economics in Action** boxes near the beginning and end of each chapter recount the fictional experiences of a young engineer at a Canadian company. These vignettes reflect and support the chapter material. The first box in each chapter usually portrays one of the characters trying to deal with a practical problem. The second box demonstrates how the character has solved the problem by applying material discussed in the chapter above. All these vignettes are linked to form a narrative that runs throughout the book. The main character is Naomi, a recent engineering graduate. In the first chapter, she starts her job in the engineering department at Canadian Widgets and is given a decision problem by her supervisor. Over the course of the book, Naomi learns about engineering economics on the job. There are several characters, who relate to one another in various ways, exposing students to practical, ethical, and social issues as well as mathematical problems.

**ENGINEERING ECONOMICS IN ACTION, PART 1A****Naomi Arrives**

Naomi's first day on the job wasn't really her first day on the job. Ever since she had received the acceptance letter three weeks earlier, she had been reading and rereading all her notes about the company. Somehow she had arranged to walk past the plant entrance going on errands that never would have taken her that route in the past. So today wasn't the first time she had walked through that tidy brick entrance to the main offices of Canadian Widgets—she had done it the same way in her imagination a hundred times before.

Clement Sheng, the engineering manager who had interviewed Naomi for the job, was waiting for her at the reception desk. His warm smile and easy manner did a lot to break the ice. He suggested that they could go through the plant on the way to her desk. She agreed enthusiastically. "I hope you remember the engineering economics you learned in school," he said.

Naomi did, but rather than sound like a know-it-all, she replied, "I think so, and I still have my old textbook. I suppose you're telling me I'm going to use it."

"Yes. That's where we'll start you out, anyhow. It's a good way for you to learn how things work around here. We've got some projects lined up for you already, and they involve some pretty big decisions for Canadian Widgets. We'll keep you busy."

- **Spreadsheet Savvy** boxes in the chapters point out elements of Excel that relate to the chapter material and how Excel can be used to support the computations necessary to implement the concepts covered. From the basics of computing interest rates or the present worth of a series of cash flows to a full-blown analysis of major projects, spreadsheets help engineers compute results, evaluate alternatives, document outcomes, and make recommendations to colleagues and other stakeholders.


**S P R E A D S H E E T   S A V V Y**

A spreadsheet program is a useful way of performing calculations that are more complex than can be easily handled using a calculator. In particular, a spreadsheet program allows you to organize data into a grid and perform simultaneous calculations on rows, columns, or other subsets of the data. Spreadsheet programs are used extensively by engineers and are particularly helpful for engineering economics calculations. In this book, we will focus on the popular Microsoft Excel, but other spreadsheet programs have similar functionality.

A sample Excel spreadsheet is shown below. Each cell in the spreadsheet contains either a value (such as a number or text) or a formula. Text was entered into cells A1 and A2 to label rows 1 and 2. Then the integers 1 through 5 were entered into cells B2 through F2. Row 2 computes the cumulative sum of the values in row 1. The summation starts by entering “=B1” in cell B2. Proceeding to the right, cell C2 is the sum of B2 and C1, cell D2 is the sum of C2 and D1, and so forth.

|   | A              | B | C | D | E  | F  |
|---|----------------|---|---|---|----|----|
| 1 | Numbers        | 1 | 2 | 3 | 4  | 5  |
| 2 | Cumulative Sum | 1 | 3 | 6 | 10 | 15 |

In normal use of a spreadsheet, the calculation result (as shown in row 2) appears by default. If you wish to display the formulas in a spreadsheet, click on the Formulas tab and select Show Formulas. The formulas for the above spreadsheet are as follows:

|   | A              | B   | C      | D      | E      | F      |
|---|----------------|-----|--------|--------|--------|--------|
| 1 | Numbers        | 1   | 2      | 3      | 4      | 5      |
| 2 | Cumulative Sum | =B1 | =B2+C1 | =C2+D1 | =D2+E1 | =E2+F1 |

To return to the default display, go back to the Formulas tab and click on Show Formulas again.

- **Close-Up** boxes in the chapters present additional material about concepts that are important but not essential to the chapter.

| CLOSE-UP 2.1  | Interest Periods                              |
|---|---|
| The most commonly used interest period is one year. If we say, for example, “6 percent interest” without specifying an interest period, the assumption is that 6 percent interest is paid for a one-year period. However, interest periods can be of any duration. Here are some other common interest periods: |   |
| Interest Period   | Interest Is Calculated:                       |
| Semiannually  | Twice per year, or once every six months      |
| Quarterly   | Four times a year, or once every three months |
| Monthly   | 12 times per year                             |
| Weekly  | 52 times per year                             |
| Daily   | 365 times per year                            |
| Continuous  | For infinitesimally small periods             |

- In each chapter, a **Net Value** box provides a chapter-specific example of how the internet can be used as a source of information and guidance for decision making.

| NET VALUE 1.1  |  |
|--|--|
| <p><b>Professional Engineering Associations</b></p> <p>Each of the provincial engineering associations has a website that can be a good source of information about engineering practice. At time of publication, the engineering association websites are:</p> <p>Alberta: <a href="http://www.apegga.com">www.apegga.com</a></p> <p>British Columbia: <a href="http://www.apeg.bc.ca">www.apeg.bc.ca</a></p> <p>Engineers Canada: <a href="http://www.engineerscanada.ca">www.engineerscanada.ca</a></p> <p>Manitoba: <a href="http://www.apegm.mb.ca">www.apegm.mb.ca</a></p> <p>New Brunswick: <a href="http://www.apegnb.ca">www.apegnb.ca</a></p> <p>Newfoundland and Labrador: <a href="http://www.pegnl.ca">www.pegnl.ca</a></p> <p>North West Territories &amp; Nunavut: <a href="http://www.napegg.nt.ca">www.napegg.nt.ca</a></p> <p>Nova Scotia: <a href="http://www.engineersnovascotia.ca">www.engineersnovascotia.ca</a></p> <p>Ontario: <a href="http://www.peo.on.ca">www.peo.on.ca</a></p> <p>Prince Edward Island: <a href="http://www.engineerspei.com">www.engineerspei.com</a></p> | <p>Quebec: <a href="http://www.oiq.qc.ca">www.oiq.qc.ca</a></p> <p>Saskatchewan: <a href="http://www.apegs.sk.ca">www.apegs.sk.ca</a></p> <p>These sites contain information such as recent salary surveys, the regional code of ethics along with disciplinary actions, job advertisements, member's directory, news releases and other useful information about the practice of engineering across Canada.</p> <p>Engineers Canada is a National organization of the provincial and territorial associations that regulate the profession of engineering in Canada. Their site contains information about the programs and services that Engineers Canada supports.</p> <p>The Canadian Engineering Accreditation Board (CEAB), established by Engineers Canada, accredits undergraduate engineering programs to ensure they provide the academic requirements needed for becoming a licensed professional engineer in Canada.</p> |

- At the end of each chapter, a **Canadian Mini-Case**, complete with discussion questions, relates interesting stories about how familiar Canadian companies have used engineering economic principles in practice.

**MINI-CASE 4.1****Rockwell International**

The Light Vehicle Division of Rockwell International makes seat-slide assemblies for the automotive industry. It has two major classifications for investment opportunities: developing new products to be manufactured and sold, and developing new machines to improve production. The overall approach to assessing whether an investment should be made depends on the nature of the project.

In evaluating a new product, it considers the following:

1. *Marketing strategy*: Does it fit the business plan for the company?
2. *Work force*: How will it affect human resources?
3. *Margins*: The product should generate appropriate profits.
4. *Cash flow*: Positive cash flow is expected within two years.

In evaluating a new machine, it considers the following:

1. *Cash flow*: Positive cash flow is expected within a limited time period.
2. *Quality issues*: For issues of quality, justification is based on cost avoidance rather than positive cash flow.
3. *Cost avoidance*: Savings should pay back an investment within one year.

## **Additional Pedagogical Features**

- Each chapter begins with a **list of the major sections** to provide an overview of the material that follows.
- **Key terms** are boldfaced where they are defined in the body of the text. For easy reference, all these terms are defined in a glossary near the back of the book.
- Additional material is presented in **chapter appendices** at the ends of Chapters 3, 4, 6, 8, 9, and 12.
- Numerous worked-out **Examples** are given throughout the chapters. Although the decisions have often been simplified for clarity, most of them are based on real situations encountered in the authors' consulting experiences.
- Worked-out **Review Problems** near the end of each chapter provide more complex examples that integrate the chapter material.
- A concise prose **Summary** is given for each chapter.
- Each chapter has 30 to 50 **Problems** of various levels of difficulty covering all of the material presented. Like the worked-out Examples, many of the problems have been adapted from real situations.
- A **spreadsheet icon** indicates where examples involve the use of spreadsheets, which are available on the Instructor's Resource Centre.
- **Tables of Interest Factors** are provided in Appendix A
- **Answers to Selected Problems** are provided in Appendix B.
- For convenience, a **List of Symbols** used in the book is given on the inside of the front cover, and a **List of Formulas** is given on the inside of the back cover.

## **Course Designs**

This book is ideal for a one-term course, but with supplemental material it can also be used for a two-term course. It is intended to meet the needs of students in all engineering programs, including, but not limited to, aeronautical, chemical, computer, electrical, industrial, mechanical, mining, and systems engineering. Certain programs emphasizing public projects may wish to supplement Chapter 10, "Public Sector Decision Making," with additional material.

A course based on this book can be taught in the first, second, third, or fourth year of an engineering program. The book is also suitable for college technology programs. No more than high school mathematics is required for a course based on this text. The probability theory required to understand and apply the tools of uncertainty and risk analysis is provided in Chapter 12. Prior knowledge of calculus or linear algebra is not needed.

This book is also suitable for self-study by a practitioner or anybody interested in the economic aspects of decision making. It is easy to read and self-contained, with many clear examples. It can serve as a permanent resource for practising engineers or anyone involved in decision making.

## Supplements

### For Students

**Companion Website ([www.pearsoncanada.ca/fraser](http://www.pearsoncanada.ca/fraser)):** We have created a robust, password-protected Companion Website to accompany the book. It contains the following items for students and instructors:

- *Practice Quizzes* for each chapter. Students can try these self-test questions, send their answers to an electronic grader, and receive instant feedback. These quizzes were revised for the fifth edition by Claude Théoret at the University of Ottawa.
- *Additional Challenging Problems* with selected solutions. These problems created by John Jones at Simon Fraser University, were provided on the CD-ROM in the previous edition.
- *Excel spreadsheets* for selected Spreadsheet Savvy discussions, examples (designated by a spreadsheet icon in the book), and problems.
- *Chapter 13, “Qualitative Considerations and Multiple Criteria”* from our Fraser et al., *Global Engineering Economics: Financial Decision Making for Engineers* Fourth Edition.
- *Extended Cases*
- *Interest Tables:*
  - Compound Interest Factors for Continuous Compounding, Discrete Cash Flows
  - Compound Interest Factors for Continuous Compounding, Continuous Compounding Periods
- *Glossary Flashcards*
- *Pearson eText.* Pearson eText gives students access to the text whenever and wherever they have access to the Internet. eText pages look exactly like the printed text, offering powerful new functionality for students and instructors. Users can create notes, highlight text in different colours, create bookmarks, zoom, click hyperlinked words and phrases to view definitions, and view in single-page or two-page view. Pearson eText allows for quick navigation to key parts of the eText using a table of contents and provides full-text search.

### For Instructors

The following instructor supplements are available for downloading from a password-protected section of Pearson Canada's online catalogue ([www.pearsoncanada.ca/highered](http://www.pearsoncanada.ca/highered)). Navigate to your book's catalogue page to view a list of those supplements that are available. See your local sales representative for details and access.

**Instructor's Solutions Manual.** The Solutions Manual contains full solutions to all the problems in the book, full solutions to all the additional problems, model solutions to the extended cases on the Companion Website, teaching notes for the Mini-Cases, and Excel spreadsheets for selected examples and problems. This manual was created by the text authors.

**Computerized Testbank (Pearson TestGen).** This bank allows instructors to view and edit all questions, generate tests, print tests in a variety of formats, administer tests on a local area network, and have the tests graded electronically. The questions were revised for the fifth edition by Yuri Yevdokimov at the University of New Brunswick, Fredericton.

**PowerPoint<sup>®</sup> Slides.** PowerPoint slides have been created for each chapter and can be used to help present material in the classroom. They were revised for the fifth edition by Mark Thomas at Centennial College.

**Image Library.** We have compiled all of the figures and tables from the book in electronic format.

**CourseSmart for Instructors.** CourseSmart goes beyond traditional expectations—providing instant, online access to the textbooks and course materials you need at a lower cost for students. And even as students save money, you can save time and hassle with a digital eTextbook that allows you to search for the most relevant content at the very moment you need it. Whether it's evaluating textbooks or creating lecture notes to help students with difficult concepts, CourseSmart can make life a little easier. See how when you visit [www.coursesmart.com/instructors](http://www.coursesmart.com/instructors).

**Technology Specialists.** Pearson's Technology Specialists work with faculty and campus course designers to ensure that Pearson technology products, assessment tools, and online course materials are tailored to meet your specific needs. This highly qualified team is dedicated to helping schools take full advantage of a wide range of educational resources, by assisting in the integration of a variety of instructional materials and media formats. Your local Pearson Education sales representative can provide you with more details on this service program.

**Pearson Custom Library.** For enrollments of at least 25 students, you can create your own textbook by choosing the chapters that best suit your own course needs. To begin building your custom text, visit [www.pearsoncustomlibrary.com](http://www.pearsoncustomlibrary.com). You may also work with a dedicated Pearson Custom editor to create your ideal text—publishing your own original content or mixing and matching Pearson content. Contact your local Pearson Representative to get started.

## Acknowledgments

The authors wish to acknowledge the contributions of a number of individuals who assisted in the development of this text. First and foremost are the hundreds of engineering students at the University of Waterloo who have provided us with feedback on passages they found hard to understand, typographical errors, and examples that they thought could be improved. There are too many individuals to name in person, but we are very thankful to each of them for their patience and diligence.

Other individuals who have contributed strongly to previous editions of the book include Irwin Bernhardt, May Tajima, Peter Chapman, David Fuller, J.B. Moore, Tim Nye, Ron Pelot, Victor Waese, Yuri Yevdokimov, and Peggy Fraser.

During the development process for the new edition, Pearson Canada arranged for the anonymous review of parts of the manuscript by a number of very able reviewers. These reviews were extremely beneficial to us, and many of the best ideas incorporated in the final text originated with these reviewers. We can now thank them by name:

James S. Christie  
Raad Jassim  
John Jones  
Muslim A. Majeed

University of New Brunswick, Saint John  
McGill University  
Simon Fraser University  
Carleton University

|                  |  |
|------------------|--|
| Prashant Mhaskar | McMaster University                      |
| Juan Pernia      | Lakehead University                      |
| Claude Théoret   | University of Ottawa                     |
| Frank Trimmell   | Ryerson University                       |
| Yuri Yevdokimov  | University of New Brunswick, Fredericton |

Finally, we want to express our appreciation to the various editors at Pearson Canada for their professionalism and support during the writing of this book. Our developmental editors for most this edition, Rema Celio and Eleanor MacKay, were able support for the author team. Our very capable and expert production and design team included: Ioana Gagea, Lesley Deugo, Ridhi Mathur, Sally Glover, and Anthony Leung. We remain grateful to Maurice Esses, who played a particularly strong role in bringing the first and second editions to completion; his guidance was instrumental in making it possible for this edition to exist. Finally, we would like to recognize Paul McInnis, the Pearson Education sales representative for this text, for his initial encouragement to write this book and his continued support and feedback over the past 15 years.

To all of the above, thank you again for your help. To those we may have forgotten to thank, our appreciation is just as great, even if our memories fail us. Without doubt, some errors remain in this text in spite of the best efforts of everyone involved. To help us improve for the next edition, if you see an error, please let us know.

Niall M. Fraser  
Elizabeth M. Jewkes

*This page intentionally left blank*



# Engineering Decision Making

**Engineering Economics in Action, Part 1A: Naomi Arrives**

- 1.1 Engineering Decision Making**
- 1.2 What Is Engineering Economics?**
- 1.3 Making Decisions**

**Engineering Economics in Action, Part 1B: Naomi Settles In**

- 1.4 Dealing With Abstractions**
- 1.5 The Moral Question: Three True Stories**
- 1.6 Uncertainty and Sensitivity Analysis**
- 1.7 How This Book Is Organized**

**Engineering Economics in Action, Part 1C: A Taste of What Is to Come**

**Problems**

**Mini-Case 1.1: R. v. Syncrude Canada Ltd.**

## ENGINEERING ECONOMICS IN ACTION, PART 1A

### Naomi Arrives

Naomi's first day on the job wasn't really her first day on the job. Ever since she had received the acceptance letter three weeks earlier, she had been reading and rereading all her notes about the company. Somehow she had arranged to walk past the plant entrance going on errands that never would have taken her that route in the past. So today wasn't the first time she had walked through that tidy brick entrance to the main offices of Canadian Widgets—she had done it the same way in her imagination a hundred times before.

Clement Sheng, the engineering manager who had interviewed Naomi for the job, was waiting for her at the reception desk. His warm smile and easy manner did a lot to break the ice. He suggested that they could go through the plant on the way to her desk. She agreed enthusiastically. "I hope you remember the engineering economics you learned in school," he said.

Naomi did, but rather than sound like a know-it-all, she replied, "I think so, and I still have my old textbook. I suppose you're telling me I'm going to use it."

"Yes. That's where we'll start you out, anyhow. It's a good way for you to learn how things work around here. We've got some projects lined up for you already, and they involve some pretty big decisions for Canadian Widgets. We'll keep you busy."

## 1.1

### Engineering Decision Making

Engineering is a noble profession with a long history. The first engineers supported the military using practical know-how to build bridges, fortifications, and assault equipment. In fact, the term *civil engineer* was coined to make the distinction between engineers who worked on civilian projects and engineers who worked on military problems.

In the beginning, all engineers had to know were the technical aspects of their jobs. Military commanders, for example, would have wanted a strong bridge built quickly. The engineer would be challenged to find a solution to the technical problem, and would not have been particularly concerned about the costs, safety, or environmental impacts of the project. As years went by, however, the engineer's job became far more complicated.

All engineering projects use resources, such as raw materials, money, labour, and time. Any particular project can be undertaken in a variety of ways, with each one calling for a different mix of resources. For example, an incandescent light bulb requires inexpensive raw materials and little labour, but it is inefficient in its use of electricity and does not last very long. On the other hand, a high-efficiency light bulb uses more expensive raw materials and is more expensive to manufacture, but consumes less electricity and lasts longer. Both products provide light, but choosing which is better in a particular situation depends on how the costs and benefits are compared.

Historically, as the kinds of projects engineers worked on evolved and technology provided more than one way of solving technical problems, engineers were more often faced with having to choose among alternative solutions to a problem. If two solutions both dealt with a problem effectively, clearly the less expensive one was preferred. The practical science of engineering economics was originally developed specifically to deal with determining which of several alternatives was, in fact, the most economical.

Choosing the least expensive alternative, though, is not the entire story. Though a project might be technically feasible and the most reasonably priced solution to a problem, if the money isn't available to do it, it can't be done. The engineer has to become aware of the financial constraints on the problem, particularly if resources are very limited. In addition, an engineering project can meet all other criteria, but may cause detrimental environmental effects. Finally, any project can be affected by social and political constraints. For example, a large irrigation project called the Garrison Diversion Unit in North Dakota was effectively cancelled because of political action by Canadians and environmental groups, even though over \$2 000 000 000 had been spent.

Engineers today must make decisions in an extremely complex environment. The heart of an engineer's skill set is still technical competence in a particular field. This permits the determination of possible solutions to a problem. However, necessary to all engineering is the ability to choose among several technically feasible solutions and to defend that choice credibly. The skills permitting the selection of a good choice are common to all engineers and, for the most part, are independent of which engineering field is involved. These skills form the discipline of engineering economics.

## 1.2 | What Is Engineering Economics?

---

Just as the role of the engineer in society has changed over the years, so has the nature of engineering economics. Originally, engineering economics was the body of knowledge that allowed the engineer to determine which of several alternatives was economically best—the least expensive, or perhaps the most profitable. In order to make this determination properly, the engineer needed to understand the mathematics governing the relationship between time and money. Most of this book deals with teaching and using this knowledge. Also, for many kinds of decisions, the costs and benefits are the most important factors affecting the decision, so concentrating on determining the economically “best” alternative is appropriate.

In earlier times, an engineer would be responsible for making a recommendation on the basis of technical and analytic knowledge, including the knowledge of engineering economics, and then a manager would decide what should be done. A manager's decision could be different from the engineer's recommendation because the manager would take into account issues outside the engineer's range of expertise. Recently, however, the trend has been for managers to become more reliant on the technical skills of the engineers, or for the engineers themselves to be the managers. Products are often very complex, manufacturing processes are fine-tuned to optimize productivity, and even understanding the market sometimes requires the analytic skills of an engineer. As a result, it is often only the engineer who has sufficient depth of knowledge to make a competent decision.

Consequently, understanding how to compare costs, although still of vital importance, is not the only skill needed to make suitable engineering decisions. One must also be able to take into account all the other considerations that affect a decision, and to do so in a reasonable and defensible manner.

**Engineering economics**, then, can be defined as the science that deals with techniques of quantitative analysis useful for selecting a preferable alternative from several technically viable ones.

The evaluation of costs and benefits is very important, and it has formed the primary content of engineering economics in the past. The mathematics for doing this evaluation, which is well developed, still makes up the bulk of studies of engineering economics.

However, the modern engineer must be able to recognize the limits and applicability of these economic calculations and must be able to take into account the inherent complexity of the real world.

## 1.3 | Making Decisions

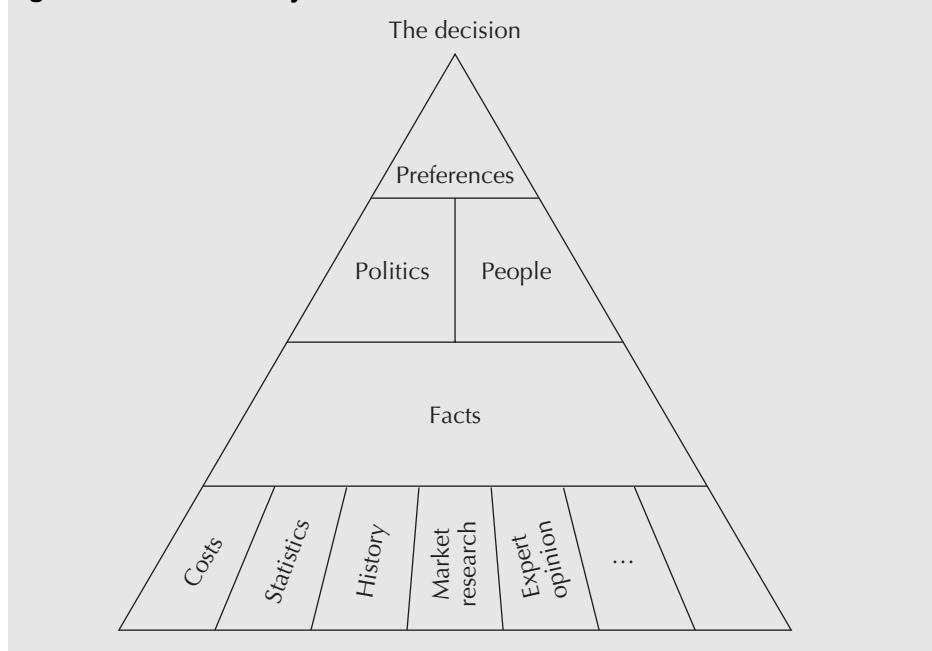
All decisions, except perhaps the most routine and automatic ones or those that are institutionalized in large organizations, are made, in the end, on the basis of belief as opposed to logic. People, even highly trained engineers, do what feels like the right thing to do. This is not to suggest that one should trust only one's intuition and not one's intellect, but rather to point out something true about human nature and the function of engineering economics studies.

Figure 1.1 is a useful illustration of how decisions are made. At the top of the pyramid are preferences, which directly control the choices made. Preferences are the beliefs about what is best, and are often hard to explain coherently. They sometimes have an emotional basis and include criteria and issues that are difficult to verbalize.

The next tier is composed of politics and people. Politics in this context means the use of power (intentional or not) in organizations. For example, if the owner of a factory has a strong opinion that automation is important, this has a great effect on engineering decisions on the plant floor. Similarly, an influential personality can affect decision making. It's difficult to make a decision without the support, either real or imagined, of other people. This support can be manipulated, for example, by a persuasive salesperson or a persistent lobbyist. Support might just be a general understanding communicated through subtle messages.

The next tier is a collection of facts. The facts, which may or may not be valid or verifiable, contribute to the politics and the people, and indirectly to the preferences. At the bottom of the pyramid are the activities that contribute to the facts. These include a

**Figure 1.1 Decision Pyramid**



history of previous similar decisions, statistics of various sorts, and, among other things, a determination of costs.

In this view of decisions, engineering economics is not very important. It deals essentially with facts and, in particular, with determining costs. Many other facts affect the final decision, and even then the decision may be made on the basis of politics, personality, or unstated preferences. However, this is an extreme view.

Although preferences, politics, and people can outweigh facts, usually the relationship is the other way around. The facts tend to control the politics, the people, and the preferences. It is facts that allow an individual to develop a strong opinion, which then may be used to influence others. Facts accumulated over time create intuition and experience that control our “gut feeling” about a decision. Facts, and particularly the activities that develop the facts, form the foundation for the pyramid in Figure 1.1. Without the foundation, the pyramid would collapse.

Engineering economics is important because it facilitates the establishment of verifiable facts about a decision. The facts are important and necessary for the decision to be made. However, the decision eventually made may be contrary to that suggested by analysis. For example, a study of several methods of treating effluent might determine that method A is most efficient and moderately priced, but method B might be chosen because it requires a visible change to the plant which, it is felt, will contribute to the company’s image in environmental issues. Such a final decision is appropriate because it takes into account facts beyond those dealt with in the economic analysis.

## ENGINEERING ECONOMICS IN ACTION, PART 1B

### Naomi Settles In

As Naomi and Clement were walking, they passed the loading docks. A honk from behind told them to move aside so that a forklift could get through. The operator waved in passing and continued on with the task of moving coils of sheet metal into the warehouse. Naomi noticed shelves and shelves of packaging material, dies, spare parts, and other items that she didn’t recognize. She would find out more soon enough. They continued to walk. As they passed a welding area, Clem pointed out the newest recycling project at Canadian Widgets: the water used to degrease the metal was now being cleaned and recycled rather than being used only once.

Naomi became aware of a pervasive, pulsating noise emanating from somewhere in the distance. Suddenly the corridor opened up to the main part of the plant and the noise became a bedlam of clanging metal and thumping machinery. Her senses were assaulted. The ceiling was very high and there were rows of humpbacked metal monsters unlike any presses she had seen before. The tang of mill oil overwhelmed her sense of smell, and she felt the throbbing from the floor knocking her bones together. Clem handed her hearing and eye protectors.

“These are our main press lines.” Clem was yelling right into Naomi’s ear, but she had to strain to hear. “We go right from sheet metal to finished widgets in 12 operations.” A passing forklift blew propane exhaust at her, momentarily replacing the mill-oil odour with hot-engine odour. “Engineering is off to the left there.”

As they went through the double doors into the engineering department, the din subsided and the ceiling came down to normal height. Removing the safety equipment, they stopped for a moment to get some juice at the vending machines. As Naomi looked around, she saw computers on desks more or less sectioned off by acoustic room dividers. As Clem led her farther, they stopped long enough for him to introduce Naomi to Carole Brown, the receptionist and secretary. Just past Carole’s desk and around the corner was Naomi’s desk. It was a nondescript metal desk with a long row of empty shelving above. Clem said her computer would arrive within the week. Naomi noticed that the desk next to hers was empty, too.



"Am I sharing with someone?" she asked.

"Well, you will be. That's for your co-op student."

"My co-op student?"

"Yep. He's a four-month industrial placement from the university. Don't worry, we have enough to do to keep you both busy. Why don't you take a few minutes to settle in while I take care of a couple of things? I'll be back in, say, 15 minutes. I'll take you over to human resources. You'll need a security pass, and I'm sure they have lots of paperwork for you to fill out."

Clem left. Naomi sat and opened the briefcase she had carefully packed that morning. Alongside the brown-bag lunch was her engineering economics textbook. She took it out and placed it on the empty shelf above the desk. "I thought I might need you," she said to herself. "Now, let's get this place organized!"

## 1.4 Dealing With Abstractions

The world is far more complicated than can ever be described in words, or even thought about. Whenever one deals with reality, it is done through models or abstractions. For example, consider the following description:

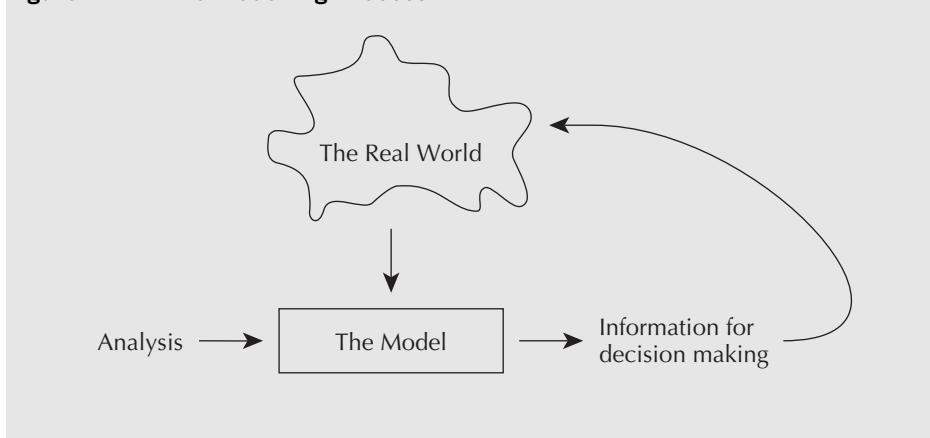
Naomi watched the roll of sheet metal pass through the first press. The die descended and punched six oval shapes from the sheet. These "blanks" dropped through a chute into a large metal bin. The strip of sheet metal jerked forward into the die and the press came down again. Pounding like a massive heart 30 times a minute, the machine kept the operator busy full-time just providing the giant coils of metal, removing the waste skeleton scrap, and stacking blanks in racks for transport to the next operation.

This gives a description of a manufacturing process that is reasonably complete in that it permits one to visualize the process. But it is not absolutely complete. For example, how large and thick were the blanks? How big was the metal bin? How heavy was the press? How long did it take to change a die? These questions might be answered, but no matter how many questions are asked, it is impossible to express all of the complexity of the real world. It is also undesirable to do so.

When one describes something, one does so for a purpose. In the description, one selects those aspects of the real world that are relevant to that purpose. This is appropriate, since it would be very confusing if a great deal of unnecessary information were given every time something was talked or written about. For example, if the purpose of the above description were to explain the exact nature of the blanks, there would be considerably less emphasis on the process and many more details about the blanks themselves.

This process of simplifying the complexities of the real world is necessary for any engineering analysis. For example, in designing a truss for a building, it is usually assumed that the members exhibit uniform characteristics. However, in the real world these members would be pieces of lumber with individual variations: some would be stronger than average and some would be weaker. Since it is impractical to measure the characteristics of each piece of wood, a simplification is made. As another example, the various components of an electric circuit, such as resistors and capacitors, have values that differ from their nominal specifications because of manufacturing tolerances, but such differences are often ignored and the nominal values are the ones used in calculations.

Figure 1.2 illustrates the basic process of modelling that applies in so much of what humans do, and applies especially to engineering. The world is too complicated to express completely, as represented by the amorphous shape at the top of the figure. People extract from the real world a simplification (in other words, a model) that captures information useful and appropriate for a given purpose. Once the model is developed, it is used to analyze a situation and perhaps make some predictions about the real world. The analysis

**Figure 1.2 The Modelling Process**

and the predictions are then related back to the real world to make sure the model is valid. As a result, the model might need some modification so that it more accurately reflects the relevant features of the real world.

The process illustrated in Figure 1.2 is exactly what is done in engineering economics. The model is often a mathematical one that simplifies a more complicated situation but does so in a reasonable way. The analysis of the model provides some information, such as which solution to a problem is cheapest. This information must always be related back to the real problem, however, to take into account the aspects of the real world that may have been ignored in the original modelling effort. For example, the economic model might not have included taxes or inflation, and an examination of the result might suggest that taxes and inflation should not be ignored. Or, as already pointed out, environmental, political, or other considerations might modify any conclusions drawn from the mathematical model.

## EXAMPLE

### 1.1

Naomi's brother Ben has been given a one-year assignment in Whitehorse and he wants to buy a car just for the time he is there. He has three choices, as illustrated in Table 1.1. For each alternative, there is a purchase price, an operating cost (including gas, insurance, and repairs), and an estimated resale value at the end of the year. Which should Ben buy?

**Table 1.1 Buying a Car**

|           | 1980 Corvette | 2007 Toyota Corolla | 2007 BMW 5-Series |
|-----------|---------------|---------------------|-------------------|
| Purchase  | \$12 000      | \$7000              | \$25 000          |
| Operation | \$400/month   | \$300/month         | \$450/month       |
| Resale    | \$13 000      | \$5000              | \$23 000          |

The next few chapters of this book will show how to take the information from Table 1.1 and determine which alternative is economically best. As it turns out, under most circumstances, the Corvette is best. However, in constructing a model of the decision, we must make a number of important assumptions.

For example, how can one be sure of the resale value of something until one actually tries to sell it? Along the same lines, who can tell what the actual maintenance costs will be? There

is a lot of uncertainty about future events that is generally ignored in these kinds of calculations. Despite this uncertainty, estimates can provide insights into the appropriate decision.

Another problem for Ben is getting the money to buy a car. Ben is fairly young and would find it very difficult to raise even \$7000, and perhaps impossible to raise \$25 000. The Corvette might be the best value, but if the money isn't available to take advantage of the opportunity, it doesn't matter. In order to do an economic analysis, we may assume that he has the money available.

If an economic model is judged appropriate, does that mean Ben should buy the Corvette? Maybe not.

A person who has to drive to work every morning would probably not want to drive an antique car. It is too important that the car be reliable (especially in the Yukon in the winter). The operating costs for the Corvette are high, reflecting the need for more maintenance than with the other cars, and there are indirect effects of low reliability that are hard to capture in dollars.

If Ben were very tall, he would be extremely uncomfortable in the compact Toyota Corolla, so even if it were economically best, he would hesitate to resign himself to driving with his knees on either side of the steering wheel.

Ben might have strong feelings about the environmental record of one of the car manufacturers and might want to avoid driving that car as a way of making a statement.

Clearly, there are so many intangibles involved in a decision like this that it is impossible for anyone but Ben himself to make such a personal choice. An outsider can point out to Ben the results of a quantitative analysis given certain assumptions, but cannot authoritatively determine the best choice for him. ■

## S P R E A D S H E E T   S A V V Y



A spreadsheet program is a useful way of performing calculations that are more complex than can be easily handled using a calculator. In particular, a spreadsheet program allows you to organize data into a grid and perform simultaneous calculations on rows, columns, or other subsets of the data. Spreadsheet programs are used extensively by engineers and are particularly helpful for engineering economics calculations. In this book, we will focus on the popular Microsoft Excel, but other spreadsheet programs have similar functionality.

A sample Excel spreadsheet is shown below. Each cell in the spreadsheet contains either a value (such as a number or text) or a formula. Text was entered into cells A1 and A2 to label rows 1 and 2. Then the integers 1 through 5 were entered into cells B2 through F2. Row 2 computes the cumulative sum of the values in row 1. The summation starts by entering “=B1” in cell B2. Proceeding to the right, cell C2 is the sum of B2 and C1, cell D2 is the sum of C2 and D1, and so forth.

|   | A              | B | C | D | E  | F  |
|---|----------------|---|---|---|----|----|
| 1 | Numbers        | 1 | 2 | 3 | 4  | 5  |
| 2 | Cumulative Sum | 1 | 3 | 6 | 10 | 15 |

In normal use of a spreadsheet, the calculation result (as shown in row 2) appears by default. If you wish to display the formulas in a spreadsheet, click on the Formulas tab and select Show Formulas. The formulas for the above spreadsheet are as follows:

|   | A              | B   | C      | D      | E      | F      |
|---|----------------|-----|--------|--------|--------|--------|
| 1 | Numbers        | 1   | 2      | 3      | 4      | 5      |
| 2 | Cumulative Sum | =B1 | =B2+C1 | =C2+D1 | =D2+E1 | =E2+F1 |

To return to the default display, go back to the Formulas tab and click on Show Formulas again.

## 1.5 | The Moral Question: Three True Stories

Complex decisions often have an ethical component. Recognizing this component is important for engineers, since society relies on them for so many things. The following three anecdotes concern real companies—although names and details have been altered for anonymity—and illustrate some extreme examples of the forces acting on engineering decision making.

### EXAMPLE

#### 1.2

The process of making sandpaper is similar to that of making a photocopy. A two-metre-wide roll of paper is coated with glue and given a negative electric charge. It is then passed over sand (of a particular type) that has a positive charge. The sand is attracted to the paper and sticks on the glue. The fact that all the bits of sand have the same type of charge ensures that the grains are evenly spaced. The paper then passes through a long, heated chamber to cure the glue. Although the process sounds fairly simple, the machine that does this, called a maker, is very complicated and expensive. One such machine, costing several million dollars, can support a factory employing hundreds of workers.

Preston Sandpapers, a subsidiary of a large firm, is located in a small town. Its maker was almost 30 years old and desperately needed replacement. However, rather than replace it, the parent company might have chosen to close down the plant and transfer production to one of its sister plants located in a different country.

The chief engineer had a problem. The costs for installing a new maker were extremely high and it was difficult to justify a new maker economically. However, if he could not do so, the plant would close and hundreds of workers would be out of a job, including perhaps himself. What he chose to do was lie. He fabricated figures, ignored important costs, and exaggerated benefits to justify the expenditures. The investment was made and the plant is still operating. ■

### EXAMPLE

#### 1.3

Hespeler Meats is a medium-sized meat processor specializing in deli-style cold cuts and European processed meats. Hoping to expand its product offerings, it decided to add a line of canned pâtés. The company was eligible for a government grant to cover some of the purchase price of the necessary production equipment.

Government support for manufacturing is generally fairly sensible. Support is usually not given for projects that are clearly very profitable, since the company should be able to justify such an expense itself. On the other hand, support is also usually not given for projects that are clearly not very profitable, because taxpayers' money should not be wasted. Support is directed at projects that the company would not otherwise undertake, but that have good potential to create jobs and expand the economy.

Hespeler Meats had to provide a detailed justification for the canned pâté project in order to qualify for the government grant. Its problem was that it had to predict both the expenditures and the receipts for the following five years. This was a product line with which it had no experience and that, in fact, had not been offered in North America by any meat processor. The company had absolutely no idea what its sales would be. Any numbers it picked would be guesses, but to get the grant it had to give numbers.

What it did was select an estimate of sales that, given the equipment expenditures expected, fell exactly within the range of profitability that made the project suitable for government support. Hespeler Meats got the money. As it turned out, the product line was a flop and the canning equipment was sold as scrap five years later. ■

**EXAMPLE****1.4**

When a large metal casting is made, as for the engine block of a car, it has only a rough exterior and often has flash—ragged edges of metal formed where molten metal seeped between the two halves of the mould. The first step in finishing the casting is to grind off the flash and to grind flat surfaces so that the casting can be held properly for subsequent machining.

Galt Casting Grinders (GCG) made the complex specialized equipment for this operation. It had once commanded the world market for this product, but lost market share to competitors. The competitors did not have a better product than GCG but were able to increase market share by adding fancy display panels with coloured lights, dials, and switches that looked very sophisticated.

GCG's problem was that its idea of sensible design was to omit the features the competitors included (and the customers wanted). GCG reasoned that these features added nothing to the capability of the equipment, but did add a lot to the manufacturing cost and to the maintenance costs that would be borne by the purchaser. The company had no doubt that it was unwise, and poor engineering design, to make such unnecessarily complicated displays, so it made no changes.

GCG went bankrupt several years later. ■

In each of these three examples, the technical issues are overwhelmed by the non-technical ones. For Preston Sandpapers, the chief engineer was pressured by his social responsibility and self-interest to lie and to recommend a decision not justified by the facts. In the Hespeler Meats case, the engineer had to choose between stating the truth—that future sales were unknown—which would deny the company a very useful grant, and selecting a convenient number that would encourage government support. For Galt Casting Grinders, the issue was marketing. The company did not recognize that a product must be more than technically good; it must also be saleable.

Beyond these principles, however, there is a moral component to each of these anecdotes. As guardians of knowledge, engineers have a vital responsibility to society to behave ethically and responsibly in all ways. When so many different issues must be taken into account in engineering decision making, it is often difficult to determine what course of action is ethical.

For Preston Sandpapers, most people would probably say that what the chief engineer did was unethical. However, he did not exploit his position simply for personal gain. He was, to his mind, saving a town. Is the principle of honesty more important than several hundred jobs? Perhaps it is, but when the job holders are friends and family, it is understandable that unethical choices might be made.

For Hespeler Meats, the issue is subtler. Is it ethical to choose figures that match the ideal ones to gain a government grant? It is, strictly speaking, a lie, or at least misleading, since there is no estimate of sales. On the other hand, the bureaucracy demands that some numbers be given, so why not pick ones that suit your case?

In the Galt Casting Grinders case, the engineers apparently did no wrong. The ethical question concerns the competitors' actions. Is it ethical to put features on equipment that do no good, add cost, and decrease reliability? In this case and for many other products, this is often done, ethical or not. If it is unethical, the ethical suppliers will sometimes go out of business.

There are no general answers to difficult moral questions. Practising engineers often have to make choices with an ethical component and can sometimes rely on no stronger foundation than their own sense of right and wrong. More information about ethical issues for engineers can be obtained from professional engineering associations.

## N E T   V A L U E   1 . 1

### **Professional Engineering Associations**

Each of the provincial engineering associations has a website that can be a good source of information about engineering practice. At time of publication, the engineering association websites are:

Newfoundland and Labrador: [www.pegnl.ca](http://www.pegnl.ca)

Nova Scotia: [www.engineersnovascotia.ca](http://www.engineersnovascotia.ca)

Prince Edward Island:

[www.engineerspei.com](http://www.engineerspei.com)

New Brunswick: [www.apegnb.ca](http://www.apegnb.ca)

Quebec: [www.oiq.qc.ca](http://www.oiq.qc.ca)

Ontario: [www.peo.on.ca](http://www.peo.on.ca)

Manitoba: [www.apegm.mb.ca](http://www.apegm.mb.ca)

Saskatchewan: [www.apegs.sk.ca](http://www.apegs.sk.ca)

Alberta: [www.apegga.com](http://www.apegga.com)

British Columbia: [www.apeg.bc.ca](http://www.apeg.bc.ca)

Northwest Territories & Nunavut:

[www.napegg.nt.ca](http://www.napegg.nt.ca)

Engineers Canada: [www.engineerscanada.ca](http://www.engineerscanada.ca)

These sites contain information such as recent salary surveys, the regional code of ethics along with disciplinary actions, job advertisements, members' directory, news releases, and other useful information about the practice of engineering across Canada.

Engineers Canada is a national organization of the provincial and territorial associations that regulate the profession of engineering in Canada. Their site contains information about the programs and services that Engineers Canada supports.

The Canadian Engineering Accreditation Board (CEAB), established by Engineers Canada, accredits undergraduate engineering programs to ensure they provide the academic requirements needed for becoming a licensed professional engineer in Canada.

## 1.6 | Uncertainty and Sensitivity Analysis

Whenever people predict the future, errors occur. Sometimes predictions are correct, whether the predictions are about the weather, a ball game, or company cash flow. On the other hand, it would be unrealistic to expect anyone to always be right about things that haven't happened yet.

Although one cannot expect an engineer to predict the future precisely, approximations are very useful. A weather forecaster can dependably say that it will not snow in July in Toronto, for example, even though it may be more difficult to forecast the exact temperature. Similarly, an engineer may not be able to precisely predict the scrap rate of a testing process, but may be able to determine a range of likely rates to help in a decision-making process.

Engineering economics analyses are quantitative in nature, and most of the time the quantities used in economic evaluations are estimates. The fact that we don't have precise values for some quantities may be very important, since decisions may have expensive consequences and significant health and environmental effects. How can the impact of this uncertainty be minimized?

One way to control this uncertainty is to make sure that the information being used is valid and as accurate as possible. The GIGO rule—"garbage in, garbage out"—applies here. Nothing is as useless or potentially dangerous as a precise calculation made from inaccurate data. However, even accurate data from the past is of only limited value when predicting the future. Even with sure knowledge of past events, the future is still uncertain.

Sensitivity analysis involves assessing the effect of uncertainty on a decision. It is very useful in engineering economics. The idea is that, although a particular value for a parameter can be known with only a limited degree of certainty, a range of values can be

assessed with reasonable certainty. In sensitivity analysis, the calculations are done several times, varying each important parameter over its range of possible values. Usually only one parameter at a time is changed so that the effect of each change on the conclusion can be assessed independently of the effect of other changes.

In Example 1.1, Naomi's brother Ben had to choose a car. He made an estimate of the resale value of each of the alternative cars, but the *actual* resale amount is unknown until the car is sold. Similarly, the operating costs are not known with certainty until the car is driven for a while. Before concluding that the Corvette is the right car to buy (on economic grounds, at least), Ben should assess the sensitivity of this decision by varying the resale values and operating costs within a range from the minimum likely amount to the maximum likely amount. Since these calculations are often done on spreadsheets, this assessment is not hard to do, even with many different parameters to vary.

Sensitivity analysis is an integral part of all engineering economics decisions because data regarding future activities are always uncertain. In this text, emphasis is usually given to the structure and formulation of problems rather than to verifying whether the result is robust. In this context, *robust* means that the same decision will be made over a wide range of parameter values. It should be remembered that no decision is properly made unless the sensitivity of that decision to variation in the underlying data is assessed.

A related issue is the number of significant digits in a calculation. Modern calculators and computers can carry out calculations to a large number of decimal places of precision. For most purposes, such precision is meaningless. For example, a cost calculated as \$1.0014613076 is of no more use than \$1.00 in most applications. It is useful, though, to carry as many decimal places as convenient to reduce the magnitude of accumulated rounding-off errors.

In this book, all calculations have been done to as many significant digits as could conveniently be carried, even though the intermediate values are shown with three to six digits. As a rule, only three significant digits are assumed in the final value. For decision-making purposes, this is plenty.

## 1.7 | How This Book Is Organized

There are 11 chapters remaining in this book. The first block, consisting of Chapters 2 to 5, forms the core material of the book. Chapters 2 and 3 of that block provide the mathematics needed to manipulate monetary values over time. Chapters 4 and 5 deal with comparing alternative projects. Chapter 4 illustrates present worth, annual worth, and payback period comparisons, and Chapter 5 covers the internal rate of return (IRR) method of comparison.

The second block, Chapters 6 to 8, broadens the core material. It covers depreciation and analysis of a company's financial statements, when to replace equipment (replacement analysis), and taxation. The appendix to Chapter 6 provides an introduction to how costs and benefits for engineering economic evaluations are estimated.

The third block, Chapters 9 to 11, provides supporting material for the previous chapters. Chapter 9 concerns the effect of inflation on engineering decisions, and Chapter 10 explores how decision making is done for projects owned by or affecting the public, rather than an individual or firm. Chapter 11 provides an overview of project management and how economic analysis fits into the evolution of any engineering project. Finally, Chapter 12 deals with handling uncertainty about important information through sensitivity analysis and with situations where exact parameter values are not known, but probability distributions for them are known.

Each chapter begins with a story about Naomi and her experiences at Canadian Widgets. There are several purposes to these stories. They provide an understanding of engineering practice that is impossible to convey with short examples. In each chapter, the story has been chosen to make clear why the ideas being discussed are important. It is also hoped that the stories make the material a little more interesting.

Throughout the text are boxes that contain information associated with, and complementary to, the text material. One set of boxes contains Close-Ups, which focus on topics of relevance to the chapter material. These appear in each chapter in the appropriate section. There are also Net Value boxes, which tie the material presented to internet resources. These boxes are in the middle sections of each chapter. Another set of boxes presents Mini-Cases, which appear at the end of each chapter, following the problem set. These cases report how engineering economics is used in familiar companies and include questions designed for classroom discussion or individual reflection.

Finally, most chapters contain a Spreadsheet Savvy box. These boxes present a logical sequence of introductory material on how spreadsheet software can be used to facilitate engineering economic evaluations. Because it is widely available, Microsoft Excel is used for illustrative purposes. In addition, solutions to worked examples show the use of Excel's financial functions as well as the solutions obtained with standard compound interest factors.

End-of-chapter appendices contain relevant but more advanced material. Appendices at the back of the book provide tables of important and useful values and answers to selected end-of-chapter problems.

## ENGINEERING ECONOMICS IN ACTION, PART 1C

### A Taste of What Is to Come

Naomi was just putting on her newly laminated security pass when Clem came rushing in. "Sorry to be late," he puffed. "I got caught up in a discussion with someone in marketing. Are you ready for lunch?" She certainly was. She had spent the better part of the morning going through the benefits package offered by Canadian Widgets and was a bit overwhelmed by the paperwork. Dental plan options, pension plan beneficiaries, and tax forms swam in front of her eyes. The thought of food sounded awfully good.

As they walked to the lunchroom, Clem continued to talk. "Maybe you will be able to help out once you get settled in, Naomi."

"What's the problem?" she asked. Obviously Clem was still thinking about his discussion with this person from marketing.

"Well," said Clem, "currently we buy small aluminum parts from a subcontractor. The cost is quite reasonable, but we should consider making the parts ourselves because our volumes are increasing and the fabrication process would not be difficult for us to bring in-house. We might be able to make the parts at a lower cost. Of course, we'd have to buy some new equipment. That's why I was up in the marketing department talking to Prabha."

"What do you mean?" asked Naomi, still a little unsure. "What does this have to do with marketing?"

Clem realized he was making a lot of assumptions about Naomi's knowledge of Canadian Widgets. "Sorry," he said. "I need to explain. I was up in marketing to ask for some demand forecasts so that we would have a better handle on the volumes of these aluminum parts we might need in the next few years. That, combined with some digging on possible equipment costs, would allow us to do an analysis of whether we should make the parts in-house or continue to buy them."

Things made much more sense to Naomi now. Her engineering economics text was certainly going to come in handy.

## PROBLEMS

### A. Key Concepts

- 1.1 In which of the following situations would engineering economics analysis play a strong role, and why?
  - (a) Buying new equipment
  - (b) Changing design specifications for a product
  - (c) Deciding on the paint colour for the factory floor
  - (d) Hiring a new engineer
  - (e) Deciding when to replace old equipment with new equipment of the same type
  - (f) Extending the cafeteria business hours
  - (g) Deciding which invoice forms to use
  - (h) Changing the 8-hour work shift to a 12-hour one
  - (i) Deciding how much to budget for research and development programs
  - (j) Deciding how much to donate for the town's new library
  - (k) Building a new factory
  - (l) Downsizing the company
- 1.2 Starting a new business requires many decisions. List five examples of decisions that might be assisted by engineering economics analysis.
- 1.3 For each of the following items, describe how the design might differ if the costs of manufacturing, use, and maintenance were not important. On the basis of these descriptions, is it important to consider costs in engineering design?
  - (a) A car
  - (b) A television set
  - (c) A light bulb
  - (d) A book

### B. Applications

- 1.4 For each of the following situations, describe what you think you *should* do. In each case, *would* you do this?
  - (a) A fellow student, who is a friend, is copying assignments and submitting them as his own work.
  - (b) A fellow student, who is *not* a friend, is copying assignments and submitting them as her own work.
  - (c) A fellow student, who is your only competitor for an important academic award, is copying assignments and submitting them as his own work.

- (d) A friend wants to hire you to write an essay for school for her. You are dead broke and the pay is excellent.
  - (e) A friend wants to hire you to write an essay for school for him. You have lots of money but the pay is excellent.
  - (f) A friend wants to hire you to write an essay for school for her. You have lots of money and the pay is poor.
  - (g) Your car was in an accident. The insurance adjuster says that the car was totalled and that they will give you only the Blue Book value for it as scrap. They will pick up the car in a week. A friend points out that in the meantime you could sell the almost-new tires and replace them with bald ones from the scrap yard, and perhaps sell some other parts, too.
  - (h) The CD player from your car has been stolen. The insurance adjuster asks you how much it was worth. It was a very cheap one of poor quality.
  - (i) The engineer you work for has told you that the meter measuring effluent discharged from a production process exaggerates and that the measured value must be halved for recordkeeping.
  - (j) The engineer you work for has told you that part of your job is to make up realistic-looking figures reporting effluent discharged from a production process.
  - (k) You observe unmetered and apparently unreported effluent discharged from a production process.
  - (l) An engineer where you work is copying directly from a manufacturer's brochure machine-tool specifications to be included in a purchase request. These specifications limit the possible purchase to the particular one specified.
  - (m) An engineer where you work is copying directly from a manufacturer's brochure machine-tool specifications to be included in a purchase request. These specifications limit the possible purchase to the particular one specified. You know that the engineer's best friend is the salesman for that manufacturer.
- 1.5 Leslie and Sandy, recently married students, are going to rent their first apartment. Leslie has carefully researched the market and decided that, all things considered, there is only one reasonable choice. The two-bedroom apartment in the building at the corner of University and Erb streets is the best value for the money and is also close to school. Sandy, on the other hand, has just fallen in love with the top half of a duplex on Dunbar Road. Which apartment should they move into? Why? Which do you think they will move into? Why?
- 1.6 Describe the process of using the telephone as you might describe it to a six-year-old using it for the first time to call a friend from school. Describe using the telephone to an electrical engineer who just happens never to have seen one before. What is the correct way to describe using a telephone?
- 1.7 (a) Karen has to decide which of several computers to buy for school use. Should she buy the least expensive one? Can she make the best choice on price alone?
- (b) Several computers offer essentially the same features, reliability, service, etc. Among these, can she decide the best choice on price alone?
- 1.8 Ciel is trying to decide whether now is a good time to expand her manufacturing plant. The viability of expansion depends on the economy (an expanding economy means

more sales), the relative value of the currency (a lower-valued currency means more exports), and changes in international trade agreements (lower tariffs also mean more exports). These factors may be highly unpredictable, however. What two things can she do to help ensure she makes a good decision?

- 1.9** Trevor started a high-tech business two years ago and now wants to sell out to one of his larger competitors. Two different buyers have made firm offers. They are similar in all but two respects. They differ in price: the Investco offer would result in Trevor's walking away with \$2 000 000, while the Venture Corporation offer would give him \$3 000 000. The other way they differ is that Investco says it will recapitalize Trevor's company to increase growth, while Trevor thinks that Venture Corporation will close down the business so that it doesn't compete with several of Venture Corporation's other divisions. What would you do if you were Trevor, and why?
- 1.10** Telekom Company is considering the development of a new type of cell phone based on a brand new, emerging technology. If successful, Telekom will be able to offer a cell phone that works even in mountainous areas. Before proceeding with the project, however, what uncertainties associated with the new technology should the company be aware of? Can sensitivity analysis help address these uncertainties?

### C. More Challenging Problem

- 1.11** In Example 1.1 it is stated that in most circumstances, the Corvette is the right economic choice. Under what circumstances would either the Toyota or the BMW be the right choice?

## M I N I - C A S E 1 . 1

### R. v. Syncrude Canada Ltd.

In June 2010, Syncrude Canada Ltd. was found responsible for the death of nearly 1600 ducks in its settling pond in Fort McMurray, Alberta. The company was sentenced to pay an award of approximately \$3 million. The decision received considerable media attention at the time and delivered the message that companies are required to take all reasonable measures to avoid foreseeable harm to wildlife. At the very least, they must comply with their own environmental plans and approvals.

As part of its oil sands operations in Fort McMurray, Syncrude operates the Aurora Settling Basin, an artificial pond about the size of 640 football fields. The settling basin is used to extract water from the tailings that come from processing the tar sand deposits. The tailings contain water, sand, and bitumen, a viscous material that forms lumps and mats that can trap waterfowl, leading to almost certain death.

In the spring, migrating birds are drawn to settling basins because they are indistinguishable from natural water bodies and, because they are warm, they tend to thaw earlier. The Fort McMurray site is on a well-known spring migratory path for many birds, and thus the environmental challenge posed by the presence of the settling basin was foreseeable. Syncrude had developed and submitted to the government a comprehensive waterfowl deterrent and monitoring program that included the use of cannons and effigies to scare birds away from the basin.

Despite the presence of this plan, Syncrude was late in placing the cannons and failed to deploy sufficient resources to prevent approximately 1600 birds from landing in the basin and

subsequently dying. Two competitors of Syncrude had successfully set up their deterrence programs that spring, thus demonstrating the feasibility of predicting and preventing the wildlife contamination. The court held that Syncrude did not deploy the deterrents early or quickly enough. The failure was attributed to the lack of effective documented procedures, inadequate training, reductions in staff, and delays in hiring new staff.

Source: R. v. Syncrude Canada Ltd. (2010). ABPC 229. Canadian Legal Information Institute (CanLII). Retrieved June 13, 2011 from [www.canlii.org/en/ab/abpc/doc/2010/2010abpc229/2010abpc229.pdf](http://www.canlii.org/en/ab/abpc/doc/2010/2010abpc229/2010abpc229.pdf).

### Questions

1. There are probably several companies in your city or province that are known to pollute. Name some of these. For each:
  - (a) What sort of damage do they do?
  - (b) How long have they been doing it?
  - (c) Why is this company still permitted to pollute?
  - (d) What would happen if this company were forced to shut down? Is it ethically correct to allow the company to continue to pollute?
2. Does it make more sense to fine a company for environmental damage or to fine management personally for environmental damage caused by a company? Why?
3. Should the fines for environmental damage be raised enough so that no company is tempted to pollute? Why or why not?
4. Governments can impose fines, give tax breaks, and take other actions that use economics to control the behaviour of companies. Is it necessary to do this whenever a company that pursues profits might do some harm to society as a whole? Why might a company do the socially correct thing even if profits are lost?



# Time Value of Money

**Engineering Economics in Action, Part 2A: A Steal For Steel**

- 2.1** Introduction
- 2.2** Interest and Interest Rates
- 2.3** Compound and Simple Interest
- 2.4** Effective and Nominal Interest Rates
- 2.5** Continuous Compounding
- 2.6** Cash Flow Diagrams
- 2.7** Equivalence
  - 2.7.1** Mathematical Equivalence
  - 2.7.2** Decisional Equivalence
  - 2.7.3** Market Equivalence

**Review Problems**  
**Summary**

**Engineering Economics in Action, Part 2B: You Just Have to Know When**  
**Problems**  
**Mini-Case 2.1: Student Credit Cards**

## ENGINEERING ECONOMICS IN ACTION, PART 2A

### A Steal For Steel

“Naomi, can you check this for me?” Terry’s request broke the relative silence as Naomi and Terry worked together one Tuesday afternoon. “I was just reviewing our J-class line for Clem, and it seems to me that we could save a lot of money there.”

“Okay, tell me about it.” Since Naomi and Terry had met two weeks earlier, just after Naomi started her job, things had been going very well. Terry, an engineering student at the local university, was on a four-month co-op work term industrial placement at Canadian Widgets.

“Well, mostly we use the heavy rolled stock on that line. According to the pricing memo we have for that kind of steel, there is a big price break at a volume that could supply our needs for six months. We’ve been buying this stuff on a week-by-week basis. It just makes sense to me to take advantage of that price break.”

“Interesting idea, Terry. Have you got data about how we have ordered before?”

“Yep, right here.”

“Let’s take a closer look.”

“Well,” Terry said, as he and Naomi looked over his figures, “the way we have been paying doesn’t make too much sense. We order about a week’s supply. The cost of this is added to our account. Every six months we pay off our account. Meanwhile, the supplier is charging us 2 percent of our outstanding amount at the end of each month!”

“Well, at least it looks as if it might make more sense to pay off our bills more often,” Naomi replied.

“Now look at this. In the six months ending last December, we ordered steel for a total cost of \$1 600 000. If we had bought this steel at the beginning of July, it would have only cost \$1 400 000. That’s a saving of \$200 000!”

“Good observation, Terry, but I don’t think buying in advance is the right thing to do. If you think about it . . .”

## 2.1

### Introduction

Engineering decisions frequently involve evaluating tradeoffs among costs and benefits that occur at different times. A typical situation is when we invest in a project today in order to obtain benefits from the project in the future. This chapter discusses the economic methods used to compare benefits and costs that occur at different times. The key to making these comparisons is the use of an interest rate. In Sections 2.2 to 2.5, we illustrate the comparison process with examples and introduce some interest and interest rate terminology. Section 2.6 deals with cash flow diagrams, which are graphical representations of the magnitude and timing of cash flows over time. Section 2.7 explains the equivalence of benefits and costs that occur at different times.

## 2.2

### Interest and Interest Rates

Everyone is familiar with the idea of interest from their everyday activities:

From a furniture store ad: *Pay no interest until next year!*

From a bank: *Now 2.6 percent daily interest on passbook accounts!*

Why are there interest rates? If people are given the choice between having money today and the same amount of money one year from now, most would prefer the money today. If they had the money today, they could do something productive with it in hopes of benefit in the future. For example, they could buy an asset like a machine today, and could

## NET VALUE 2 . 1

**Prime Interest Rates**

When you hear about the interest rate going up or down in the news, it usually refers to the *prime rate*—the interest rate charged by banks to their most creditworthy customers, such as prominent businesspeople. It attracts people's attention because it is the reference point for interest rates charged on many mortgage, personal, and business loans. The prime rate is almost always the same among major banks. The internet makes it easy to collect and compare interest rate information from banks and other financial institutions.

Contrary to what many people may think, the prime rate is *not* set by the Bank of Canada, the country's central bank. Its role, as defined in

the original *Bank of Canada Act* of 1934, is to promote the economic and financial well-being of Canada, but setting the prime rate on behalf of all Canadian banks is not one of its jobs. This doesn't mean that the Bank of Canada is not interested in having control over what the prime rate should be. It does try to influence the rate by setting the target for the short-term interest rates called *overnight rates*, which represent the average interest rates that the Bank of Canada would like to see in the marketplace and are representative of what the prime rate should be. Find more on the Bank of Canada and its role in setting or influencing various interest rates at its website at [www.bankofcanada.ca](http://www.bankofcanada.ca).

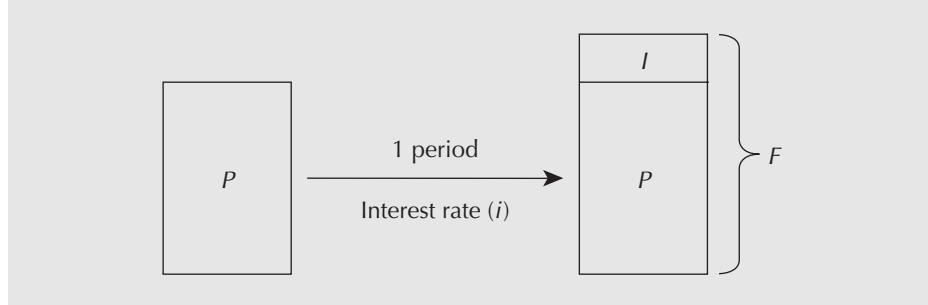
use it to make money from their initial investment. Or they may want to buy a consumer good like a new home theatre system and start enjoying it immediately. What this means is that one dollar today is worth more than one dollar in the future. This is because a dollar today can be invested for productive use, while that opportunity is lost or diminished if the dollar is not available until some time in the future.

The observation that a dollar today is worth more than a dollar in the future means that people must be compensated for lending money. They are giving up the opportunity to invest their money for productive purposes now on the promise of getting more money in the future. The compensation for loaning money is in the form of an interest payment—say,  $I$ . More formally, **interest** is the difference between the amount of money lent and the amount of money later repaid. It is the compensation for giving up the use of the money for the duration of the loan.

An amount of money today,  $P$  (also called the *principal amount*), can be related to a *future amount*  $F$  by the interest amount  $I$  or interest rate  $i$ . This relationship is illustrated graphically in Figure 2.1 and can be expressed as  $F = P + I$ . The interest  $I$  can also be expressed as an interest rate  $i$  with respect to the principal amount so that  $I = Pi$ . Thus

$$\begin{aligned} F &= P + Pi \\ &= P(1 + i) \end{aligned}$$

**Figure 2.1 Present and Future Worth**



**EXAMPLE****2.1**

Samuel bought a one-year guaranteed investment certificate (GIC) for \$5000 from a bank on May 15 last year. The bank was paying 10 percent on one-year guaranteed investment certificates at the time. One year later, Samuel cashed in his certificate for \$5500.

We may think of the interest payment that Samuel got from the bank as compensation for giving up the use of money. When Samuel bought the GIC for \$5000, he gave up the opportunity to use the money in some other way during the following year. On the other hand, the bank got use of the money for the year. In effect, Samuel lent \$5000 to the bank for a year. The \$500 interest was payment by the bank to Samuel for the loan. The bank wanted the loan so that it could use the money for the year. (It may have lent the money to someone else at a higher interest rate.) ■

This leads to a formal definition of interest rates. Divide time into periods like days, months, or years. If the right to  $P$  at the beginning of a time period exchanges for the right to  $F$  at the end of the period, where  $F = P(1 + i)$ ,  $i$  is the **interest rate** per time period. In this definition,  $P$  is called the **present worth** of  $F$ , and  $F$  is called the **future worth** of  $P$ .

**EXAMPLE****2.1 RESTATED**

Samuel invested \$5000 with the bank on May 15 last year. The bank was paying 10 percent on one-year fixed term investments at the time. The agreement gave Samuel the right to claim \$5500 from the bank one year later.

Notice in this example that there was a transaction between Samuel and the bank on May 15 last year. There was an exchange of \$5000 on May 15 a year ago for the right to collect \$5500 on May 15 this year. The bank got the \$5000 last year and Samuel got the right to collect \$5500 one year later. Evidently, having a dollar on May 15 last year was worth more than the right to collect a dollar a year later. Each dollar on May 15 last year was worth the right to collect  $5500/5000 = 1.1$  dollars a year later. This 1.1 may be written as  $1 + 0.1$ , where 0.1 is the interest rate. The interest rate, then, gives the rate of exchange between money at the beginning of a period (one year in this example) and the right to money at the end of the period. ■

The dimension of an interest rate is currency/currency/time period. For example, a 9 percent interest rate means that for every dollar lent, 0.09 dollars (or other unit of money) is paid in interest for each time period. The value of the interest rate depends on the length of the time period. Usually, interest rates are expressed on a yearly basis, although they may be given for periods other than a year, such as a month or a quarter. This base unit of time over which an interest rate is calculated is called the **interest period**. Interest periods are described in more detail in Close-Up 2.1. The longer the interest period, the higher the interest rate must be to provide the same return.

Interest concerns the lending and borrowing of money. It is a parameter that allows an exchange of a larger amount of money in the future for a smaller amount of money in the present, and vice versa. As we will see in Chapter 3, it also allows us to evaluate very complicated exchanges of money over time.

Interest also has a physical basis. Money can be invested in financial instruments that pay interest, such as a bond or a savings account, and money can also be invested directly in industrial processes or services that generate wealth. In fact, the money invested in financial instruments is also indirectly invested in productive activities by the organization

| CLOSE-UP 2.1  | Interest Periods                              |
|---|---|
| The most commonly used interest period is one year. If we say, for example, “6 percent interest” without specifying an interest period, the assumption is that 6 percent interest is paid for a one-year period. However, interest periods can be of any duration. Here are some other common interest periods: |   |
| Interest Period   | Interest Is Calculated:                       |
| Semiannually  | Twice per year, or once every six months      |
| Quarterly   | Four times a year, or once every three months |
| Monthly   | 12 times per year                             |
| Weekly  | 52 times per year                             |
| Daily   | 365 times per year                            |
| Continuous  | For infinitesimally small periods             |

providing the instrument. Consequently, the root source of interest is the productive use of money, as this is what makes the money actually increase in value. The actual return generated by a specific productive investment varies enormously, as will be seen in Chapter 4.

## 2.3 Compound and Simple Interest

We have seen that if an amount,  $P$ , is lent for one interest period at the interest rate,  $i$ , the amount that must be repaid at the end of the period is  $F = P(1 + i)$ . But loans may be for several periods. How is the quantity of money that must be repaid computed when the loan is for  $N$  interest periods? The usual way is one period at a time. Suppose that the amount  $P$  is borrowed for  $N$  periods at the interest rate  $i$ . The amount that must be repaid at the end of the  $N$  periods is  $P(1 + i)^N$ ; that is

$$F = P(1 + i)^N \quad (2.1)$$

This is derived as shown in Table 2.1.

This method of computing interest is called *compounding*. Compounding assumes that there are  $N$  sequential one-period loans. At the end of the first interest period, the borrower owes  $P(1 + i)$ . This is the amount borrowed for the second period. Interest is

**Table 2.1 Compound Interest Computations**

| Beginning<br>of Period | Amount Lent      | Interest Amount       | Amount Owed<br>at Period End              |
|------------------------|------------------|-----------------------|---|
| 1                      | $P$              | $+ Pi$                | $= P + Pi = P(1 + i)$                     |
| 2                      | $P(1 + i)$       | $+ P(1 + i)i$         | $= P(1 + i) + P(1 + i)i = P(1 + i)^2$     |
| 3                      | $P(1 + i)^2$     | $+ P(1 + i)^2i$       | $= P(1 + i)^2 + P(1 + i)^2i = P(1 + i)^3$ |
| :                      | :                |                       |   |
| $N$                    | $P(1 + i)^{N-1}$ | $+ [P(1 + i)^{N-1}]i$ | $= P(1 + i)^N$                            |

required on this larger amount. At the end of the second period  $[P(1 + i)](1 + i)$  is owed. This is the amount borrowed for the third period. This continues so that at the end of the  $(N - 1)$ th period,  $P(1 + i)^{N-1}$  is owed. The interest on this over the  $N^{\text{th}}$  period is  $[P(1 + i)^{N-1}]i$ . The total interest on the loan over the  $N$  periods is

$$I_c = P(1 + i)^N - P \quad (2.2)$$

$I_c$  is called **compound interest**. It is the standard method of computing interest where interest accumulated in one interest period is added to the principal amount used to calculate interest in the next period. The interest period used with the compound interest method is called the **compounding period**.

**EXAMPLE****2.2**

If you were to lend \$100 for three years at 10 percent per year compound interest, how much interest would you get at the end of the three years?

If you lend \$100 for three years at 10 percent compound interest per year, you will earn \$10 in interest in the first year. That \$10 will be lent, along with the original \$100, for the second year. Thus, in the second year, the interest earned will be  $\$11 = \$110(0.10)$ . The \$11 is lent for the third year. This makes the loan for the third year \$121, and  $\$12.10 = \$121(0.10)$  in interest will be earned in the third year. At the end of the three years, the amount you are owed will be \$133.10. The interest received is then \$33.10. This can also be calculated from Equation (2.2):

$$I_c = F - P = 100(1 + 0.1)^3 - 100 = 33.10 \quad \blacksquare = 100*(1 + 0.1)^3 - 100$$

Table 2.2 summarizes the compounding process.

An Excel spreadsheet corresponding to Table 2.2 along with instructions for creating it are provided in the Spreadsheet Savvy box on page 24. ■

**Table 2.2 Compound Interest Computations for Example 2.2**

| Beginning<br>of Year | Amount Lent | Interest Amount      | Amount Owed<br>at Year-End |
|----------------------|-------------|----------------------|----------------------------|
| 1                    | 100         | + $100 \times 0.1$ = | \$110                      |
| 2                    | 110         | + $110 \times 0.1$ = | \$121                      |
| 3                    | 121         | + $121 \times 0.1$ = | \$133.10                   |

If the interest payment for an  $N$ -period loan at the interest rate  $i$  per period is computed without compounding, the interest amount,  $I_s$ , is called *simple interest*. It is computed as

$$I_s = PiN$$

**Simple interest** is a method of computing interest where interest earned during an interest period is not added to the principal amount used to calculate interest in the next period. Simple interest is rarely used in practice, except as a method of calculating approximate interest.



## S P R E A D S H E E T   S A V V Y

One of the most common uses of spreadsheets is to compute the value of formulas. The formulas written into a cell of a worksheet can be as complicated as necessary to write out the calculation desired. It's a good idea to use parentheses often, both to indicate the order of execution of the operations within the formula and to make error checking easier. Use + and – for addition and subtraction, \* for multiplication, / for division, and ^ for exponentiation. Within a formula, you can refer to values that are stored in other cells. For example, if the value for  $P$  is in cell B1, the value for  $i$  is in cell B2, and the value for  $N$  is in cell A8, the expression  $P(1 + i)^{N-1}$  is entered into a worksheet cell as “=B1\*(1 + B2)^(A8 – 1).” Note that the = sign is used to indicate that the expression is to be evaluated by Excel.

Worksheets such as the one shown in Table 2.2 are particularly easy to create in a spreadsheet program. It is a good idea to designate one area of the spreadsheet for parameters and to document each. For example, the values for  $P$  and  $i$  are entered into cells B1 and B2. For documentation, cells A1 and A2 have text indicating what cells B1 and B2 mean. Calculations in other cells of the worksheet can reference the values of  $P$  and  $i$ , and these calculations can be easily updated simply by changing the contents of cells B1 or B2 as needed.

In the example worksheet below, Table 2.2 is reproduced for five periods for a principal amount  $P = \$100$  and an interest rate of  $i = 10$  percent. The values are shown in the left-hand table and the formulas used are shown in the right. Start by entering “=B1” into cell B4. C4 is computed as the product of cells B4 and B2, and D4 as the sum of B4 and C4. Cell B5, the amount owed at the beginning of period 2, is the amount owed at the end of period 1, D4. Once cells C5 and D5 are entered, rows 6 through 8 can be completed by using the Fill Down function: Highlight cells B5:D5 to B8:D8, and under the Home tab in Excel, select Fill Down. This will copy the formulas from cells B5 to D5 down to rows 6 through 8, making the appropriate row number adjustments automatically. Copying and pasting will achieve the same effect.

**Important:** the “\$” entered in cell C4 in reference to B\$2 tells Excel not to adjust the row reference to the interest rate when cell B2 is filled down or pasted.

|   | A                   | B           | C               | D                         |
|---|---------------------|-------------|-----------------|---------------------------|
| 1 | $P =$               | \$100.00    |                 |                           |
| 2 | $i =$               | 10.00%      |                 |                           |
| 3 | Beginning of Period | Amount Lent | Interest Amount | Amount Owed at Period End |
| 4 | 1                   | \$100.00    | \$10.00         | \$110.00                  |
| 5 | 2                   | \$110.00    | \$11.00         | \$121.00                  |
| 6 | 3                   | \$121.00    | \$12.10         | \$133.10                  |
| 7 | 4                   | \$133.10    | \$13.31         | \$146.41                  |
| 8 | 5                   | \$146.41    | \$14.64         | \$161.05                  |

|   | A                   | B           | C               | D                         |
|---|---------------------|-------------|-----------------|---------------------------|
| 1 | $P =$               | 100         |                 |                           |
| 2 | $i =$               | 0.1         |                 |                           |
| 3 | Beginning of Period | Amount Lent | Interest Amount | Amount Owed at Period End |
| 4 | 1                   | =B1         | =B4*B\$2        | =B4+C4                    |
| 5 | 2                   | =D4         | =B5*B\$2        | =B5+C5                    |
| 6 | 3                   | =D5         | =B6*B\$2        | =B6+C6                    |
| 7 | 4                   | =D6         | =B7*B\$2        | =B7+C7                    |
| 8 | 5                   | =D7         | =B8*B\$2        | =B8+C8                    |

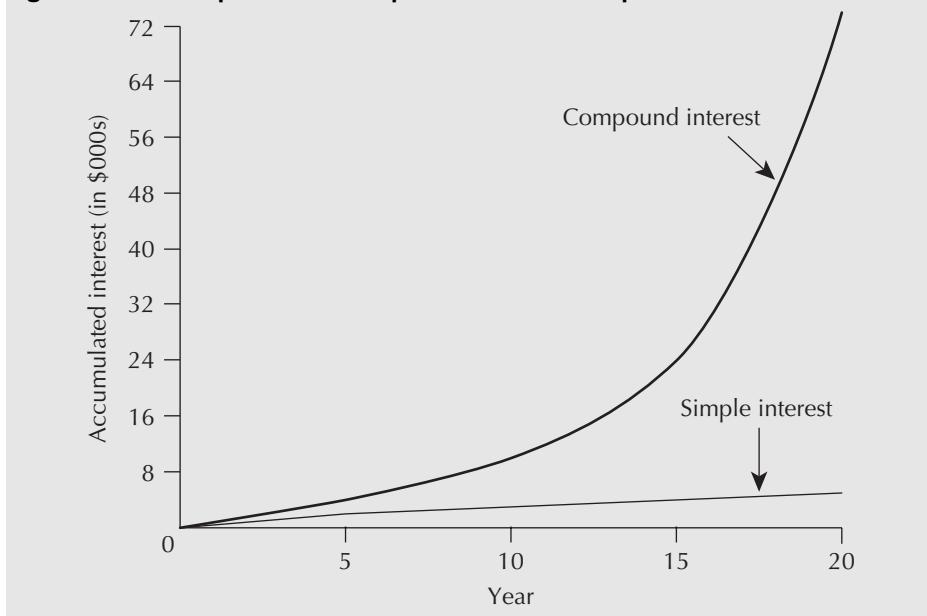
**EXAMPLE**
**2.3**

If you were to lend \$100 for three years at 10 percent per year simple interest, how much interest would you get at the end of the three years?

The total amount of interest earned on the \$100 over the three years would be \$30. This can be calculated by using  $I_s = PiN$ :

$$I_s = PiN = 100(0.10)(3) = 30$$

Interest amounts computed with simple interest and compound interest will yield the same results only when the number of interest periods is one. As the number of periods increases, the difference between the accumulated interest amounts for the two methods increases exponentially.

**Figure 2.2 Compound and Simple Interest at 24% per Year for 20 Years**

When the number of interest periods is significantly greater than one, the difference between simple interest and compound interest can be very great. In April 1993, a couple in Nevada presented the state government with a \$1000 bond issued by the state in 1865. The bond carried an annual interest rate of 24 percent. The couple claimed the bond was now worth several trillion dollars (*Newsweek*, August 9, 1993, p. 8). If one takes the length of time from 1865 to the time the couple presented the bond to the state as 128 years, the value of the bond could have been  $\$908 \text{ trillion} = \$1000(1 + 0.24)^{128}$ . As of 2012, the value would have been \$54 quadrillion!

If, instead of compound interest, a simple interest rate given by  $iN = (24\%)(128) = 3072$  percent were used, the bond would be worth only  $\$31\,720 = \$1000(1 + 30.72)$ . Thus, the difference between compound and simple interest can be dramatic, especially when the interest rate is high and the number of periods is large. The graph in Figure 2.2 shows the difference between compound interest and simple interest for the first 20 years of the bond example. As for the couple in Nevada, the \$1000 bond was worthless after all—a state judge ruled that the bond had to have been cashed by 1872.

The conventional approach for computing interest is the compound interest method rather than simple interest. Simple interest is rarely used, except perhaps as an intuitive (yet incorrect!) way of thinking of compound interest. We mention simple interest primarily to contrast it with compound interest and to indicate that the difference between the two methods can be large.

## 2.4

## Effective and Nominal Interest Rates

Interest rates may be stated for some period, such as a year, while the computation of interest is based on shorter compounding subperiods such as months. In this section we consider the relation between the *nominal* interest rate that is stated for the full period and the *effective* interest rate that results from the compounding based on the subperiods. This relation between nominal and effective interest rates must be understood to

answer questions such as: How would you choose between two investments, one bearing 12 percent per year interest compounded yearly and another bearing 1 percent per month interest compounded monthly? Are they the same?

**Nominal interest rate** is the conventional method of stating the annual interest rate. It is calculated by multiplying the interest rate per compounding period by the number of compounding periods per year. Suppose that a time period is divided into  $m$  equal subperiods. Let there be stated a nominal interest rate,  $r$ , for the full period. By convention, for nominal interest, the interest rate for each subperiod is calculated as  $i_s = r/m$ . For example, a nominal interest rate of 18 percent per year, compounded monthly, is the same as

$$0.18/12 = 0.015 \text{ or } 1.5 \text{ percent per month}$$

**Effective interest rate** is the actual but not usually stated interest rate, found by converting a given interest rate with an arbitrary compounding period (normally less than a year) to an equivalent interest rate with a one-year compounding period. What is the effective interest rate,  $i_e$ , for the full period that will yield the same amount as compounding at the end of each subperiod,  $i_s$ ? If we compound interest every sub-period, we have

$$F = P(1 + i_s)^m$$

We want to find the effective interest rate,  $i_e$ , that yields the same future amount  $F$  at the end of the full period from the present amount  $P$ . Set

$$P(1 + i_s)^m = P(1 + i_e)$$

Then

$$\begin{aligned} (1 + i_s)^m &= 1 + i_e \\ i_e &= (1 + i_s)^m - 1 \end{aligned} \tag{2.3}$$

Note that Equation (2.3) allows the conversion between the interest rate over a compounding subperiod,  $i_s$ , and the effective interest rate over a longer period,  $i_e$ , by using the number of subperiods,  $m$ , in the longer period.

### EXAMPLE

### 2.4

What is the annual effective interest rate equivalent to a nominal rate of 12 percent a year?

The nominal interest rate is given as  $r = 12$  percent, and the number of corresponding periods per year is  $m = 12$ . This gives  $i_s = r/m = 0.12/12 = 0.01$ . Then:

$$\begin{aligned} i_e &= (1 + i_s)^m - 1 \\ &= (1 + 0.01)^{12} - 1 & \blacksquare &= (1 + 0.01)^{12} - 1 \\ &= 0.126825 \\ &\approx 0.127 \text{ or } 12.7\% \end{aligned}$$

An interest rate of 1 percent per month, compounded monthly, is equivalent to an effective rate of approximately 12.7 percent per year, compounded yearly. ■

Interest rates are normally given as nominal rates. We may get the effective (yearly) rate by substituting  $i_s = r/m$  into Equation (2.3). We then obtain a direct means of computing an effective interest rate, given a nominal rate and the number of compounding periods per year:

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1 \quad (2.4)$$

This formula is suitable only for converting from a nominal rate  $r$  to an annual effective rate. If the effective rate desired is for a period longer than a year, then Equation (2.3) must be used.

**EXAMPLE****2.5**

Leona the loan shark lends money to clients at a rate of 5 percent interest per week! What is the nominal interest rate for these loans? What is the effective annual interest rate?

The nominal interest rate is  $5\% \times 52 = 260\%$ . Recall that nominal interest rates are usually expressed on a yearly basis. The effective yearly interest rate can be found by substitution into Equation (2.3):

$$i_e = (1 + 0.05)^{52} - 1 = 12.64$$

Leona charges an effective annual interest rate of about 1264 percent on her loans. ■

**EXAMPLE****2.6**

The Cardex Credit Card Company charges a nominal 24 percent interest on overdue accounts, compounded daily. What is the effective interest rate?

Assuming that there are 365 days per year, we can calculate the interest rate per day using either Equation (2.3) with  $i_s = r/m = 0.24/365 = 0.0006575$  or by the use of Equation (2.4) directly. The effective interest rate (per year) is

$$\begin{aligned} i_e &= (1 + 0.0006575)^{365} - 1 \quad \blacksquare = (1 + 0.24/365)^{365} - 1 \\ &= 0.271 \text{ or } 27.1\% \end{aligned}$$

With a nominal rate of 24 percent compounded daily, the Cardex Credit Card Company is actually charging an effective rate of about 27.1 percent per year. ■

Although there are laws that may require that the effective interest rate be disclosed for loans and investments, it is still very common for nominal interest rates to be quoted. Since the nominal rate will be less than the effective rate whenever the number of compounding periods per year exceeds one, there is an advantage to quoting loans using the nominal rates, since it makes the loan look more attractive. This is particularly true when interest rates are high and compounding occurs frequently.

## 2.5 | Continuous Compounding

As has been seen, compounding can be done yearly, quarterly, monthly, or daily. The periods can be made even smaller—as small as desired; the main disadvantage in having very small periods is having to do more calculations. If the period is made infinitesimally small, we say that interest is compounded *continuously*. There are situations in which very frequent compounding makes sense. For instance, an improvement in materials handling may reduce downtime on machinery. There will be benefits in the form of increased output that may be used immediately. If there are several additional runs a day, there will be benefits several times a day. Another example is trading on the stock market. Personal and corporate investments are often in the form of mutual funds. Mutual funds represent a changing set of stocks and bonds in which transactions occur very frequently, often many times a day.

A formula for **continuous compounding** can be developed from Equation (2.3) by allowing the number of compounding periods per year to become infinitely large:

$$i_e = \lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^m - 1$$

By noting from a definition of the natural exponential function,  $e$ , that

$$\lim_{m \rightarrow \infty} \left(1 + \frac{r}{m}\right)^m = e^r$$

we get

$$i_e = e^r - 1 \quad (2.5)$$

### EXAMPLE

#### 2.7

Cash flow at the Arctic Oil Company is continuously reinvested. An investment in a new data logging system is expected to return a nominal interest of 40 percent, compounded continuously. What is the effective interest rate earned by this investment?

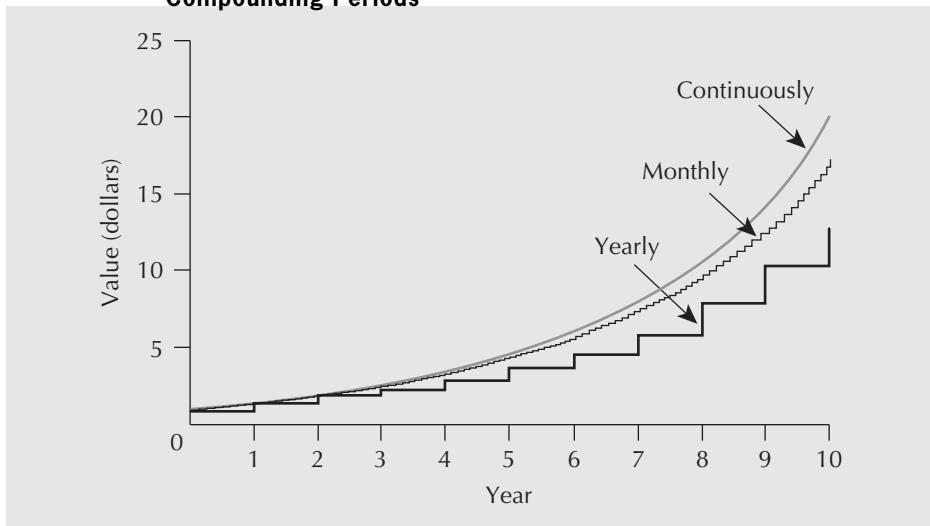
The nominal interest rate is given as  $r = 0.40$ . From Equation (2.5),

$$\begin{aligned} i_e &= e^{0.4} - 1 & \blacksquare &= \exp(0.4) - 1 \\ &= 1.492 - 1 \cong 0.492 \text{ or } 49.2\% \end{aligned}$$

The effective interest rate earned on this investment is about 49.2 percent. ■

Although continuous compounding makes sense in some circumstances, it is rarely used. As with effective interest and nominal interest, in the days before calculators and computers, calculations involving continuous compounding were difficult to do. Consequently, discrete compounding is, by convention, the norm. As illustrated in Figure 2.3, the difference between continuous compounding and discrete compounding is relatively insignificant, even at a fairly high interest rate.

**Figure 2.3** Growth in Value of \$1 at 30% Interest for Various Compounding Periods

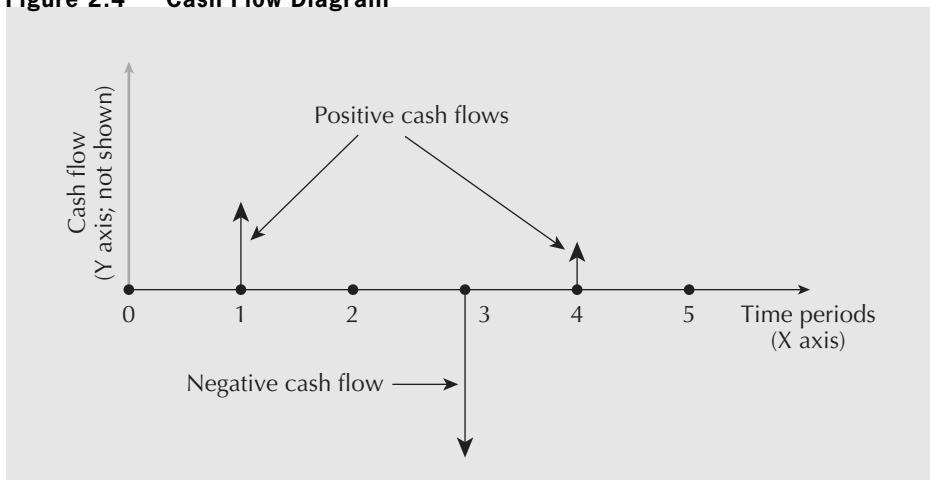


## 2.6 | Cash Flow Diagrams

Sometimes a set of cash flows can be sufficiently complicated that it is useful to have a graphical representation. A **cash flow diagram** is a graph that summarizes the timing and magnitude of cash flows as they occur over time.

On a cash flow diagram, the graph's vertical axis is not shown explicitly. The horizontal (X) axis represents time, measured in periods, and the vertical (Y) axis represents the size and direction of the cash flows. Individual cash flows are indicated by arrows pointing up or down from the horizontal axis, as indicated in Figure 2.4. The arrows that point up represent positive cash flows, or receipts. The downward-pointing arrows represent negative cash flows, or disbursements. See Close-Up 2.2 for some conventions pertaining to the beginning and ending of periods.

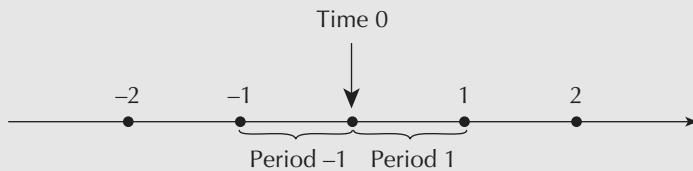
**Figure 2.4** Cash Flow Diagram



**CLOSE-UP 2.2****Beginning and Ending of Periods**

As illustrated in a cash flow diagram (see Figure 2.4), the end of one period is exactly the same point in time as the beginning of the next period. Now is time 0, which is the end of period  $-1$  and also the beginning of period 1. The end of period 1 is the same as the beginning of period 2, and so on.  $N$  years from now is the end of period  $N$  and the beginning of period  $(N + 1)$ .

**Figure 2.5 Beginning and Ending of Periods**

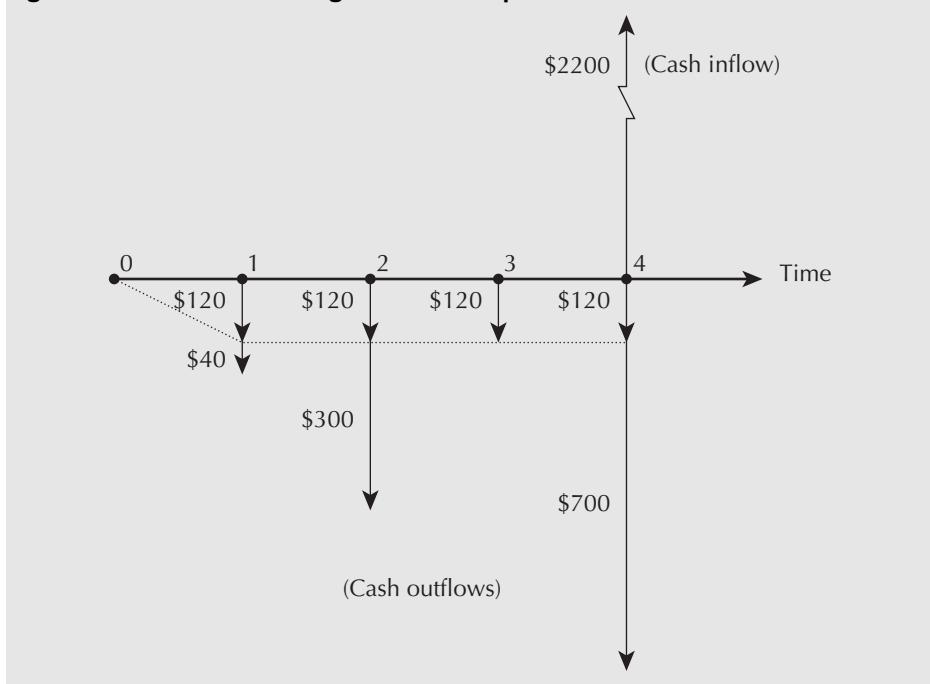
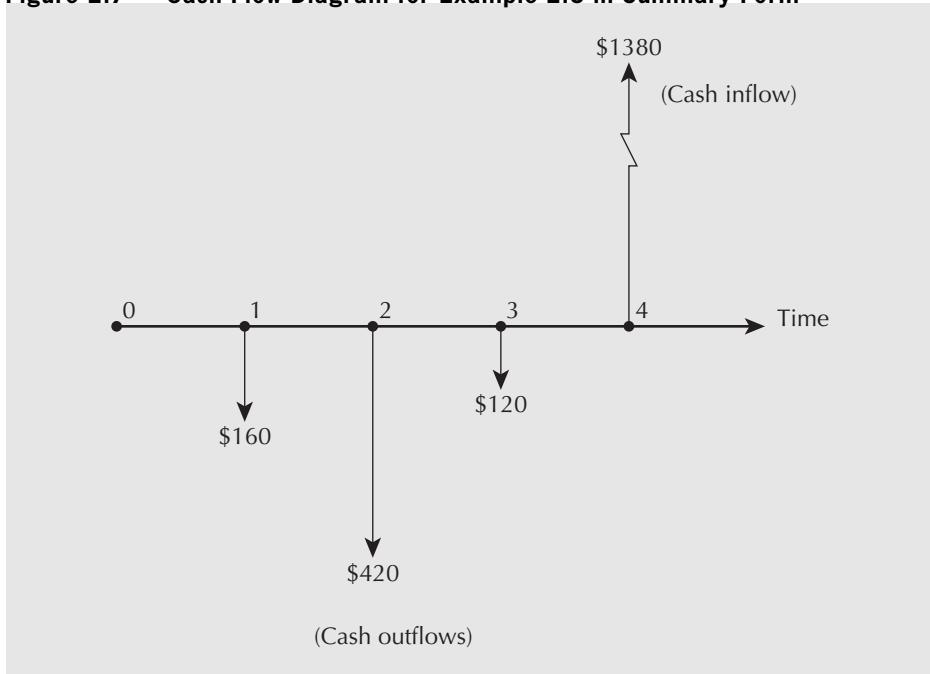
**EXAMPLE****2.8**

Consider Ashok, a recent university graduate who is trying to summarize typical cash flows for each month. His monthly income is \$2200, received at the end of each month. Out of this he pays for rent, food, entertainment, telephone charges, and a credit card bill for all other purchases. Rent is \$700 per month (including utilities), due at the end of each month. Weekly food and entertainment expenses total roughly \$120, a typical telephone bill is \$40 (due at the end of the first week in the month), and his credit card purchases average \$300. Credit card payments are due at the end of the second week of each month.

Figure 2.6 shows the timing and amount of the disbursements and the single receipt over a typical month. It assumes that there are exactly four weeks in a month, and it is now just past the end of the month. Each arrow, which represents a cash flow, is labelled with the amount of the receipt or disbursement.

When two or more cash flows occur in the same time period, the amounts may be shown individually, as in Figure 2.6, or in summary form, as in Figure 2.7. The level of detail used depends on personal choice and the amount of information the diagram is intended to convey.

We suggest that the reader make a practice of using cash flow diagrams when working on a problem with cash flows that occur at different times. Just going through the steps in setting up a cash flow diagram can make the problem structure clearer. Seeing the pattern of cash flows in the completed diagram gives a “feel” for the problem. ■

**Figure 2.6 Cash Flow Diagram for Example 2.8****Figure 2.7 Cash Flow Diagram for Example 2.8 in Summary Form**

## 2.7 Equivalence

We started this chapter by pointing out that many engineering decisions involve costs and benefits that occur at different times. Making these decisions requires that the costs and benefits at different times be compared. To make these comparisons, we must be able to say that certain values at different times are *equivalent*. **Equivalence** is a condition that exists when the value of a cost at one time is equivalent to the value of the related benefit received at a different time. In this section we distinguish three concepts of equivalence that may underlie comparisons of costs and benefits at different times.

With **mathematical equivalence**, equivalence is a consequence of the mathematical relationship between time and money. This is the form of equivalence used in  $F = P(1 + i)^N$ .

With **decisional equivalence**, equivalence is a consequence of indifference on the part of a decision maker among available choices.

With **market equivalence**, equivalence is a consequence of the ability to exchange one cash flow for another at zero cost.

Although the mathematics governing money is the same regardless of which form of equivalence is most appropriate for a given situation, it can be important to be aware of what assumptions must be made for the mathematical operations to be meaningful.

### 2.7.1 Mathematical Equivalence

Mathematical equivalence is simply a mathematical relationship. It says that two cash flows,  $P_t$  at time  $t$  and  $F_{t+N}$  at time  $t + N$ , are equivalent with respect to the interest rate,  $i$ , if  $F_{t+N} = P_t(1 + i)^N$ . Notice that if  $F_{t+N+M}$  (where  $M$  is a second number of periods) is mathematically equivalent to  $P_t$ , then

$$\begin{aligned} F_{t+N+M} &= P_t(1 + i)^{N+M} \\ &= F_{t+N}(1 + i)^M \end{aligned}$$

so that  $F_{t+N}$  and  $F_{t+N+M}$  are also equivalent to each other. The fact that mathematical equivalence has this property permits complex comparisons to be made among many cash flows that occur over time.

### 2.7.2 Decisional Equivalence

For any individual, two cash flows,  $P_t$  at time  $t$  and  $F_{t+N}$  at time  $t + N$ , are equivalent if the individual is indifferent between the two. Here, the implied interest rate relating  $P_t$  and  $F_{t+N}$  can be calculated from the decision that the cash flows are equivalent, as opposed to mathematical equivalence, in which the interest rate determines whether the cash flows are equivalent. This can be illustrated best through an example.

#### EXAMPLE

#### 2.9

Bildmet is an extruder of aluminum shapes used in construction. The company buys aluminum from Alpure, an outfit that recovers aluminum from scrap. When Bildmet's purchasing manager, Greta Kehl, called in an order for 1000 kilograms of metal on August 15, she was told that Alpure was having production difficulties and was running behind schedule. Alpure's manager, Masaaki Sawada, said that he could ship the order immediately if Bildmet required it. However, if Alpure shipped Bildmet's order, it would

not be able to fill an order for another user whom Mr. Sawada was anxious to impress with Alpure's reliability. Mr. Sawada suggested that, if Ms. Kehl would wait a week until August 22, he would show his appreciation by shipping 1100 kilograms then at the same cost to Bildmet as 1000 kilograms now. In either case, payment would be due at the end of the month. Should Ms. Kehl accept Alpure's offer?

The rate of exchange, 1100 to 1000 kilograms, may be written as  $(1 + 0.1)$  to 1, where the  $0.1 = 10$  percent is an interest rate for the one-week period. (This is equivalent to an effective interest rate of more than 14 000 percent per year!) Whether or not Ms. Kehl accepts the offer from Alpure depends on her situation. There is some chance of Bildmet's running out of metal if it doesn't get supplied for a week. This would require Ms. Kehl to do some scrambling to find other sources of metal in order to ship to her own customers on time. Clearly, Ms. Kehl would prefer the 1000 kilograms on the 15th to 1000 kilograms on the 22nd. But there is some minimum amount, larger than 1000 kilograms, that she would accept on the 22nd in exchange for 1000 kilograms on the 15th. This amount would take into account both measurable costs and immeasurable costs such as inconvenience and anxiety.

Let the minimum rate at which Ms. Kehl would be willing to make the exchange be 1 kilogram on the 15th for  $(1 + x)$  kilograms on the 22nd. In this case, if  $x < 10$  percent, Ms. Kehl should accept Alpure's offer of 1100 kilograms on the 22nd. ■

In Example 2.9, the aluminum is a capital good that can be used productively by Bildmet. There is value in that use, and that value can be measured by Greta's willingness to postpone receiving the aluminum. It can be seen that interest is not necessarily a function of exchanges of money at different points in time. However, money is a convenient measure of the worth of a variety of goods, and so interest is usually expressed in terms of money.

### 2.7.3 Market Equivalence

Market equivalence is based on the idea that there is a market for money that permits cash flows in the future to be exchanged for cash flows in the present, and vice versa. Converting a future cash flow,  $F$ , to a present cash flow,  $P$ , is called borrowing money, while converting  $P$  to  $F$  is called lending or investing money. The market equivalence of two cash flows  $P$  and  $F$  means that they can be exchanged, one for the other, at zero cost.

The interest rate associated with an individual's borrowing money is usually a lot higher than the interest rate applied when that individual lends money. For example, the interest rate a bank pays on deposits is lower than what it charges to lend money to clients. The difference between these interest rates provides the bank with income. This means that, for an individual, market equivalence does not exist. An individual can exchange a present worth for a future worth by investing money, but if he or she were to try to borrow against that future worth to obtain money now, the resulting present worth would be less than the original amount invested. Moreover, every time either borrowing or lending occurred, transaction costs (the fees charged or cost incurred) would further diminish the capital.

#### EXAMPLE

#### 2.10

This morning, Averill bought a \$5000 one-year guaranteed investment certificate (GIC) at his local bank. It has an effective interest rate of 7 percent per year. At the end of a year, the GIC will be worth \$5350. On the way home from the bank, Averill unexpectedly

discovered a valuable piece of art he had been seeking for some time. He wanted to buy it, but all his spare capital was tied up in the GIC. He went back to the bank, this time to negotiate a one-year loan for \$5000, the cost of the piece of art. He figured that if the loan came due at the same time as the GIC, he would simply pay off the loan with the proceeds of the GIC.

Unfortunately, Averill found out that the bank charges 10 percent effective interest per year on loans. Considering the proceeds from the GIC of \$5350 one year from now, the amount the bank would give him today is only  $\$5350/1.1 = \$4864$  (roughly), less any fees applicable to the loan. He discovered that for him, market equivalence does not hold. He cannot exchange \$5000 today for \$5350 one year from now, and vice versa, at zero cost. ■

Large companies with good records have opportunities that differ from those of individuals. Large companies borrow and invest money in so many ways, both internally and externally, that the interest rates for borrowing and for lending are very close to being the same, and the transaction costs are negligible. They can shift funds from the future to the present by raising new money or by avoiding investment in a marginal project that would earn only the rate that they pay on new money. They can shift funds from the present to the future by undertaking an additional project or investing externally.

But how large is a “large company”? Established businesses of almost any size, and even individuals with some wealth and good credit, can acquire cash and invest at about the same interest rate, provided that the amounts are small relative to their total assets. For these companies and individuals, market equivalence is a reasonable model, assuming that market equivalence makes calculations easier and still generally results in good decisions.

For most of the remainder of this book, we will be making two broad assumptions with respect to equivalence: first, that market equivalence holds, and second, that decisional equivalence can be expressed entirely in monetary terms. If these two assumptions are reasonably valid, mathematical equivalence can be used as an accurate model of how costs and benefits relate to one another over time. In several sections of the book, when we cover how firms raise capital and how to incorporate non-monetary aspects of a situation into a decision, we will discuss the validity of these two assumptions. In the meantime, mathematical equivalence is used to relate cash flows that occur at different points in time.

## REVIEW PROBLEMS

### **REVIEW PROBLEM 2.1**

Atsushi has had \$800 stashed under his mattress for 30 years. How much money has he lost by not putting it in a bank account at 8 percent annual compound interest all these years?

### **ANSWER**

Since Atsushi has kept the \$800 under his mattress, he has not earned any interest over the 30 years. Had he put the money into an interest-bearing account, he would have far more today. We can think of the \$800 as a present amount and the amount in 30 years as the future amount.

$$\text{Given: } P = \$800$$

$$i = 0.08 \text{ per year}$$

$$N = 30 \text{ years}$$

$$\begin{aligned}\text{Formula: } F &= P(1 + i)^N \\ &= 800(1 + 0.08)^{30} \\ &= 8050.13\end{aligned}$$

Atsushi would have \$8050.13 in the bank account today if he had deposited his \$800 at 8 percent annual compound interest. Instead, he has only \$800. He suffered an opportunity cost of  $\$8050.13 - \$800 = \$7250.13$  by not investing the money. ■

### **REVIEW PROBLEM 2.2**

You want to buy a new computer, but you are \$1000 short of the amount you need. Your aunt has agreed to lend you the \$1000 you need now, provided you pay her \$1200 two years from now. She compounds interest monthly. Another place from which you can borrow \$1000 is the bank. There is, however, a loan processing fee of \$20, which will be included in the loan amount. The bank is expecting to receive \$1220 two years from now based on monthly compounding of interest.

- (a) What monthly rate is your aunt charging you for the loan? What is the bank charging?
- (b) What effective annual rate is your aunt charging? What is the bank charging?
- (c) Would you prefer to borrow from your aunt or from the bank?

### **ANSWER**

- (a) *Your aunt*

Given:  $P = \$1000$

$F = \$1200$

$N = 24$  months (since compounding is done monthly)

Formula:  $F = P(1 + i)^N$

The formula  $F = P(1 + i)^N$  must be solved in terms of  $i$  to answer the question.

$$\begin{aligned}i &= \sqrt[24]{F/P} - 1 \\ &= \sqrt[24]{1200/1000} - 1 \quad \blacksquare = (1200/1000)^{(1/24)} - 1 \\ &= 0.007626\end{aligned}$$

Your aunt is charging interest at a rate of approximately 0.76 percent per month.

*The bank*

Given:  $P = \$1020$  (since the fee is included in the loan amount)

$F = \$1220$

$N = 24$  months (since compounding is done monthly)

$$\begin{aligned}i &= \sqrt[24]{F/P} - 1 \\ &= \sqrt[24]{1220/1020} - 1 \\ &= 0.007488\end{aligned}$$

The bank is charging interest at a rate of approximately 0.75 percent per month.

- (b) The effective annual rate can be found with the formula  $i_e = (1 + r/m)^m - 1$ , where  $r$  is the nominal rate per year and  $m$  is the number of compounding periods per year. Since the number of compounding periods per year is 12, notice that  $r/m$  is simply the interest rate charged per month.

*Your aunt*

$$i = 0.007626 \text{ per month}$$

Then

$$\begin{aligned} i_e &= (1 + r/m)^m - 1 \\ &= (1 + 0.007626)^{12} - 1 \\ &= 0.095445 \end{aligned}$$

The effective annual rate your aunt is charging is approximately 9.54 percent.

*The bank*

$$i = 0.007488 \text{ per month}$$

Then

$$\begin{aligned} i_e &= (1 + r/m)^m - 1 \\ &= (1 + 0.007488)^{12} - 1 \\ &= 0.09365 \end{aligned}$$

The effective annual rate for the bank is approximately 9.37 percent.

- (c) The bank appears to be charging a lower interest rate than that offered by your aunt. This can be concluded by comparing the two monthly rates or the effective annual rates. If you were to base your decision only on who charged the lower interest rate, you would pick the bank, despite the fact that it has a fee. However, although you are borrowing \$1020 from the bank, you are getting only \$1000, since the bank immediately gets its \$20 fee. The cost of money for you from the bank is better calculated as

Given:  $P = \$1000$

$F = \$1220$

$N = 24$  months (since compounding is done monthly)

$$\begin{aligned} i &= \sqrt[24]{F/P} - 1 \\ &= \sqrt[24]{1220/1000} - 1 \\ &= 0.00832 \end{aligned}$$

From this point of view, the bank is charging interest at a rate of approximately 0.83 percent per month and you would be better off borrowing from your aunt. ■

### REVIEW PROBLEM 2.3

At the end of four years, you would like to have \$5000 in a bank account to purchase a used car. What you need to know is how much to deposit in the bank account now. The account pays daily interest. Create a spreadsheet and plot the necessary deposit today as a function of interest rate. Consider nominal interest rates ranging from 5 percent to 15 percent per year, and assume that there are 365 days per year.

### ANSWER

From the formula  $F = P(1 + i)^N$ , we have  $5000 = P(1 + i)^{365 \times 4}$ . This gives

$$P = 5000 \times \frac{1}{(1 + i)^{365 \times 4}}$$

The left spreadsheet in Table 2.3 shows the necessary deposits for a range of interest rates. The right spreadsheet shows the Excel formulas that were used. To construct the spreadsheet, 0.05 was entered into cell A2 and the formula **=A2 + 0.01** was put into cell A3. After then selecting A3 through A12, the Fill Down function was used to complete the remaining cells in column A. Once the formula in cell B2 was entered, the Fill Down function was also used to complete the rest of column B. The references to the interest rates in A3, A4, etc. were automatically incremented by the Fill Down function.

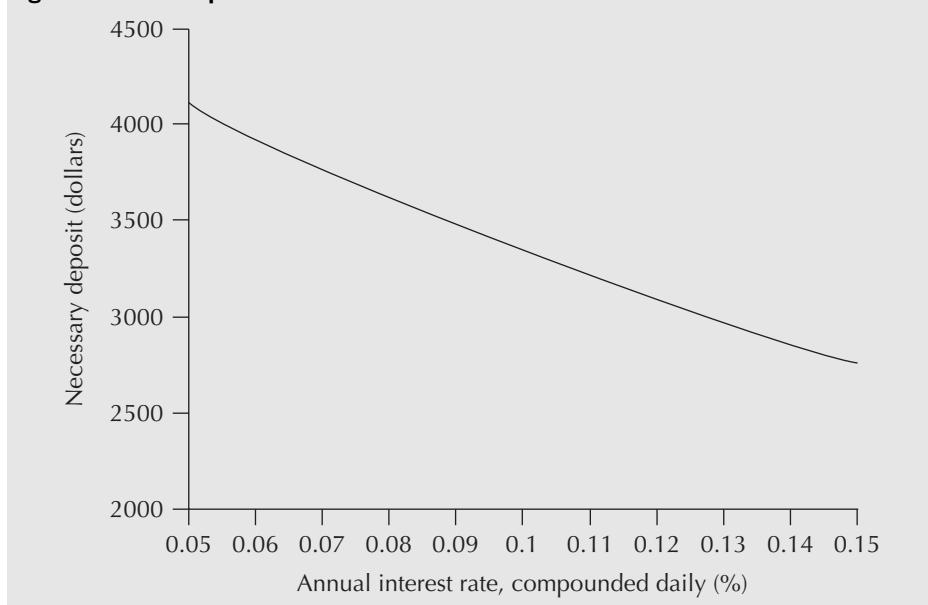
**Table 2.3 Necessary Deposits for a Range of Interest Rates**



|    | A                     | B                         |
|----|-----------------------|---------------------------|
| 1  | Interest Rate $i$ (%) | Necessary Deposit, P (\$) |
| 2  | 0.05                  | 4094                      |
| 3  | 0.06                  | 3933                      |
| 4  | 0.07                  | 3779                      |
| 5  | 0.08                  | 3631                      |
| 6  | 0.09                  | 3489                      |
| 7  | 0.10                  | 3352                      |
| 8  | 0.11                  | 3220                      |
| 9  | 0.12                  | 3094                      |
| 10 | 0.13                  | 2973                      |
| 11 | 0.14                  | 2856                      |
| 12 | 0.15                  | 2744                      |

|    | A                     | B                           |
|----|-----------------------|-----------------------------|
| 1  | Interest Rate $i$ (%) | Necessary Deposit, P (\$)   |
| 2  | =0.05                 | =5000/((1+A2/365)^(365*4))  |
| 3  | =A2+0.01              | =5000/((1+A3/365)^(365*4))  |
| 4  | =A3+0.01              | =5000/((1+A4/365)^(365*4))  |
| 5  | =A4+0.01              | =5000/((1+A5/365)^(365*4))  |
| 6  | =A5+0.01              | =5000/((1+A6/365)^(365*4))  |
| 7  | =A6+0.01              | =5000/((1+A7/365)^(365*4))  |
| 8  | =A7+0.01              | =5000/((1+A8/365)^(365*4))  |
| 9  | =A8+0.01              | =5000/((1+A9/365)^(365*4))  |
| 10 | =A9+0.01              | =5000/((1+A10/365)^(365*4)) |
| 11 | =A10+0.01             | =5000/((1+A11/365)^(365*4)) |
| 12 | =A11+0.01             | =5000/((1+A12/365)^(365*4)) |

**Figure 2.8 Graph for Review Problem 2.3**



The specific implementation of this formula will vary depending on the particular spreadsheet program used. Figure 2.8 is a diagram of the necessary deposits plotted against interest rates.■

## S U M M A R Y

This chapter has provided an introduction to interest, interest rate terminology, and interest rate conventions. Through a series of examples, the mechanics of working with simple and compound interest, nominal and effective interest rates, and continuous compounding were illustrated. Cash flow diagrams were introduced to represent graphically monetary transactions at various points in time. The final part of the chapter contained a discussion of various forms of cash flow equivalence: mathematical, decisional, and market. With the assumption that mathematical equivalence can be used as an accurate model of how costs and benefits relate to one another over time, we now move on to Chapter 3, in which equivalence formulas for a variety of cash flow patterns are presented.

### ENGINEERING ECONOMICS IN ACTION, PART 2B

#### You Just Have to Know When

Naomi and Terry were looking at the steel orders for the J-class line. Terry thought money could be saved by ordering in advance. "Now look at this," Terry said. "In the six months ending last December, we ordered steel for a total cost of \$1 600 000. If we had bought this steel at the beginning of July, it would have cost only \$1 400 000. That's a savings of \$200 000!"

"Good observation, Terry, but I don't think buying in advance is the right thing to do. If you think about it, the rate of return on our \$1 400 000 would be  $200\ 000/1\ 400\ 000$  or about 14.3 percent over six months."

"Yes, but that's over 30 percent effective interest, isn't it? I'll bet we only make 3 percent or 4 percent for money we keep in the bank."

"That's true, but the money we would use to buy the steel in advance we don't have sitting in the bank collecting interest. It would have to come from somewhere else—either money we borrow from the bank, at about 14 percent plus administrative costs, or from our shareholders."

"But it's still a good idea, right?"

"Well, you are right and you are wrong. Mathematically, you could probably show the advantage of buying a six-month supply in advance. But we wouldn't do it for two reasons. The first one has to do with where the money comes from. If we had to pay for six months of steel in advance, we would have a capital requirement beyond what we could cover through normal cash flows. I'm not sure the bank would even lend us that much money, so we would probably have to raise it through equity—that is, selling more shares in the company. This would cost a lot and throw all your calculations way off."

"Just because it's such a large amount of money?"

"That's right. Our regular calculations are based on the assumption that the capital requirements don't take an extraordinary effort."

"You said there were two reasons. What's the other one?"

"The other reason is that we just wouldn't do it."

"Huh?"

"We just wouldn't do it. Right now the steel company's taking the risk—if we can't pay, it is in trouble. If we buy in advance, it's the other way around—if our widget orders dropped, we would be stuck with some pretty expensive raw materials. We would also have the problem of where to store the steel, and other practical difficulties. It makes sense mathematically, but I'm pretty sure we just wouldn't do it."



Terry looked a little dejected. Naomi continued, "But your figures make sense. The first thing to do is find out why we are carrying that account so long before we pay it off. The second thing to do is see if we can't get that price break retroactively. We are good customers and I'll bet we can convince them to give us the price break anyhow, without changing our ordering pattern. Let's talk to Clem about it."

"But, Naomi, why use the mathematical calculations at all, if they don't work?"

"But they do work, Terry. You just have to know when."

## PROBLEMS

### A. Key Concepts

- 2.1 Using 12 percent simple interest per year, how much interest will be owed on a loan of \$500 at the end of two years?
- 2.2 If a sum of \$3000 is borrowed for six months at 9 percent simple interest per year, what is the total amount due (principal and interest) at the end of six months?
- 2.3 What principal amount will yield \$150 in interest at the end of three months when the interest rate is 1 percent simple interest per month?
- 2.4 If 2400 rupees in interest is paid on a two-year simple-interest loan of 12 000 rupees, what is the interest rate per year?
- 2.5 How much will be in a bank account at the end of five years if \$2000 is invested today at 12 percent interest per annum, compounded yearly?
- 2.6 How much is accumulated in each of these savings plans over two years?
  - (a) Deposit €1000 today at 10 percent compounded annually.
  - (b) Deposit €900 today at 12 percent compounded monthly.
- 2.7 Greg wants to have \$50 000 in five years. The bank is offering five-year investment certificates that pay 8 percent nominal interest, compounded quarterly. How much money should he invest in the certificates to reach his goal?
- 2.8 Greg wants to have \$50 000 in five years. He has \$20 000 today to invest. The bank is offering five-year investment certificates that pay interest compounded quarterly. What is the minimum nominal interest rate he would have to receive to reach his goal?
- 2.9 Hans now has \$6000. In three months, he will receive a cheque for \$2000. He must pay \$900 at the end of each month (starting exactly one month from now). Draw a single cash flow diagram illustrating all of these payments for a total of six monthly periods. Include his cash on hand as a payment at time 0.
- 2.10 Margaret is considering an investment that will cost her \$500 today. It will pay her \$100 at the end of each of the next 12 months, and cost her another \$300 one year from today. Illustrate these cash flows in two cash flow diagrams. The first should show each cash flow element separately, and the second should show only the net cash flows in each period.
- 2.11 Heddy is considering working on a project that will cost her \$20 000 today. It will pay her \$10 000 at the end of each of the next 12 months and cost her another \$15 000 at the end of each quarter. An extra \$10 000 will be received at the end of the project, one year from now. Illustrate these cash flows in two cash flow diagrams. The first should show each cash flow element separately, and the second should show only the net cash flow in each period.

- 2.12** Illustrate the following cash flows over 12 months in a cash flow diagram. Show only the net cash flow in each period.

|                      |  |
|----------------------|--|
| <b>Cash Payments</b> | \$20 every three months, starting now  |
| <b>Cash Receipts</b> | Receive \$30 at the end of the first month, and from that point on, receive 10 percent more than the previous month at the end of each month |

## B. Applications

- 2.13** Simple interest of \$190.67 is owed on a loan of \$550 after four years and four months. What is the annual interest rate?
- 2.14** Greg wants to have \$50 000. He will invest \$20 000 today in investment certificates that pay 8 percent nominal interest, compounded quarterly. How long will it take him to reach his goal?
- 2.15** Greg will invest \$20 000 today in five-year investment certificates that pay 8 percent nominal interest, compounded quarterly. How much money will this be in five years?
- 2.16** You bought an antique car three years ago for \$500 000. Today it is worth \$650 000.
- (a) What annual interest rate did you earn if interest is compounded yearly?
  - (b) What monthly interest rate did you earn if interest is compounded monthly?
- 2.17** You have a bank deposit now worth \$5000. How long will it take for your deposit to be worth more than \$8000 if
- (a) The account pays 5 percent actual interest every half-year and is compounded every half-year?
  - (b) The account pays 5 percent nominal interest, compounded semiannually?
- 2.18** Some time ago, you put £500 into a bank account for a “rainy day.” Since then, the bank has been paying you 1 percent per month, compounded monthly. Today, you checked the balance, and found it to be £708.31. How long ago did you deposit the £500?
- 2.19** (a) If you put \$1000 in a bank account today that pays 10 percent interest per year, how much money could be withdrawn 20 years from now?  
 (b) If you put \$1000 in a bank account today that pays 10 percent *simple* interest per year, how much money could be withdrawn 20 years from now?
- 2.20** How long will it take any sum to double itself
- (a) With an 11 percent simple interest rate?
  - (b) With an 11 percent interest rate, compounded annually?
  - (c) With an 11 percent interest rate, compounded continuously?
- 2.21** Compute the effective annual interest rate on each of these investments.
- (a) 25 percent nominal interest, compounded semiannually
  - (b) 25 percent nominal interest, compounded quarterly
  - (c) 25 percent nominal interest, compounded continuously

- 2.22** For a 15 percent effective annual interest rate, what is the nominal interest rate if
- Interest is compounded monthly?
  - Interest is compounded daily (assume 365 days per year)?
  - Interest is compounded continuously?
- 2.23** A Studebaker automobile that cost \$665 in 1934 was sold as an antique car at \$14 800 in 1998. What was the rate of return on this “investment”?
- 2.24** Clifford has  $X$  dollars right now. In five years,  $X$  will be \$3500 if it is invested at 7.5 percent, compounded annually. Determine the present value of  $X$ . If Clifford invested  $X$  dollars at 7.5 percent compounded daily, how much would the value of  $X$  be in 10 years?
- 2.25** The Kovalam Bank advertises savings account interest as 6 percent compounded daily. What is the effective interest rate?
- 2.26** The Bank of Brisbane is offering a new savings account that pays a nominal 7.99 percent interest, compounded continuously. Will your money earn more in this account than in a daily interest account that pays 8 percent?
- 2.27** The Crete Credit Union advertises savings account interest as 5.5 percent compounded weekly and chequing account interest at 7 percent compounded monthly. What are the effective interest rates for the two types of accounts?
- 2.28** Victory Visa, Magnificent Master Card, and Amazing Express are credit card companies that charge different interest on overdue accounts. Victory Visa charges 26 percent compounded daily, Magnificent Master Card charges 28 percent compounded weekly, and Amazing Express charges 30 percent compounded monthly. On the basis of interest rate, which credit card has the best deal?
- 2.29** April has a bank deposit now worth \$796.25. A year ago, it was \$750. What was the nominal monthly interest rate on her account?
- 2.30** May has \$2000 in her bank account right now. She wanted to know how much it would be in one year, so she calculated and came up with \$2140.73. Then she realized she had made a mistake. She had wanted to use the formula for monthly compounding, but instead, she had used the continuous compounding formula. Redo the calculation for May and find out how much will actually be in her account a year from now.
- 2.31** There are two possible investments, A and B. Their cash flows are shown in the table below. Illustrate these cash flows over 12 months in two cash flow diagrams. Show only the net cash flow in each period. Just looking at the diagrams, would you prefer one investment to the other? Comment on this.

|                 | <b>Investment A</b>  | <b>Investment B</b>  |
|-----------------|--|--|
| <b>Payments</b> | \$2400 now and a closing fee of \$200 at the end of month 12 | \$500 every two months, starting two months from now   |
| <b>Receipts</b> | \$250 monthly payment at the end of each month               | Receive \$50 at the end of the first month, and from that point on, receive \$50 more than the previous month at the end of each month |

**2.32** Using a spreadsheet, construct graphs for the loan described in part (a) below.

- (a) Plot the amount owed (principal plus interest) on a simple interest loan of \$100 for  $N$  years for  $N = 1, 2, \dots, 10$ . On the same graph, plot the amount owed on a compound interest loan of \$100 for  $N$  years for  $N = 1, 2, \dots, 10$ . The interest rate is 6 percent per year for each loan.
- (b) Repeat part (a), but use an interest rate of 18 percent. Observe the dramatic effect compounding has on the amount owed at the higher interest rate.

**2.33 (a)** At 12 percent interest per annum, how long will it take for a penny to become a million dollars? How long will it take at 18 percent?

- (b) Show the growth in values on a spreadsheet using 10-year time intervals.

**2.34** Use a spreadsheet to determine how long it will take for a \$100 deposit to double in value for each of the following interest rates and compounding periods. For each, plot the size of the deposit over time, for as many periods as necessary for the original sum to double.

- (a) 8 percent per year, compounded monthly
- (b) 11 percent per year, compounded semiannually
- (c) 12 percent per year, compounded continuously

**2.35** Construct a graph showing how the effective interest rate for the following nominal rates increases as the compounding period becomes shorter and shorter. Consider a range of compounding periods of your choice, from daily compounding to annual compounding.

- (a) 6 percent per year
- (b) 10 percent per year
- (c) 20 percent per year

### C. More Challenging Problems

**2.36** You have just won a lottery prize of \$1 000 000 collectable in 10 yearly installments of \$100 000 starting today. Why is this prize not really \$1 000 000? What is it really worth today if money can be invested at 10 percent annual interest, compounded monthly? Use a spreadsheet to construct a table showing the present worth of each installment and the total present worth of the prize.

**2.37** Suppose in Problem 2.36 that you have a large mortgage you want to pay off now. You propose an alternative but equivalent payment scheme. You would like \$300 000 today and the balance of the prize in five years when you intend to purchase a large piece of waterfront property. How much will the payment be in five years? Assume that annual interest is 10 percent, compounded monthly.

**2.38** You are looking at purchasing a new computer for your four-year undergraduate program. Brand 1 costs \$4000 now, and you expect it will last throughout your program without any upgrades. Brand 2 costs \$2500 now and will need an upgrade at the end of two years, which you expect to be \$1700. With 8 percent annual interest, compounded monthly, which is the less expensive alternative, if they provide the same level of service and will both be worthless at the end of the four years?

- 2.39** You are comparing two investments. The first pays 1 percent interest per month, compounded monthly, and the second pays 6 percent interest per six months, compounded every six months.
- What is the effective semiannual interest rate for each investment?
  - What is the effective annual interest rate for each investment?
  - On the basis of interest rates, which investment do you prefer? Does your decision depend on whether you make the comparison based on an effective six-month rate or an effective one-year rate?
- 2.40** You have \$50 000 to invest in the stock market and have sought the advice of Adam, an experienced colleague who is willing to advise you, for a fee. Adam has told you that he has found a one-year investment for you that provides 15 percent interest, compounded monthly.
- What is the effective annual interest rate based on a 15 percent nominal annual rate and monthly compounding?
  - Adam says he will make the investment for you for a modest fee of 2 percent of the investment's value one year from now. If you invest the \$50 000 today, how much will you have at the end of one year (before Adam's fee)?
  - What is the effective annual interest rate of this investment, including Adam's fee?
- 2.41** You are indifferent about whether you receive \$100 today or \$110 one year from now. The bank pays you 6 percent interest on deposits and charges you 8 percent for loans. Name the three types of equivalence and comment (with one sentence for each) on whether each exists for this situation and why.
- 2.42** June has a small house on a small street in a small town. If she sells the house now, she will likely get \$110 000 for it. If she waits for one year, she will likely get more—say, \$120 000. If she sells the house now, she can invest the money in a one-year guaranteed growth bond that pays 8 percent interest, compounded monthly. If she keeps the house, then the interest on the mortgage payments is 8 percent compounded daily. June is indifferent between the two options: selling the house now and keeping the house for another year. Discuss whether each of the three types of equivalence exists in this case.
- 2.43** Today, an investment you made three years ago has matured and is now worth \$3000. It was a three-year deposit that bore an interest rate of 10 percent per year, compounded monthly. You knew at the time that you were taking a risk in making such an investment because interest rates vary over time and you “locked in” at 10 percent for three years.
- How much was your initial deposit? Plot the value of your investment over the three-year period.
  - Looking back over the past three years, interest rates for similar one-year investments did indeed vary. The interest rates were 8 percent the first year, 10 percent the second, and 14 percent the third. Plot the value of your initial deposit over time as if you had invested at this set of rates, rather than at a constant 10 percent rate. Did you lose out by having locked into the 10 percent investment? If so, by how much?
- 2.44** Marlee has a choice between  $X$  dollars today or  $Y$  dollars one year from now.  $X$  is a fixed value, but  $Y$  varies depending on the interest rate. At interest rate  $i$ ,  $X$  and  $Y$  are mathematically equivalent for Marlee. At interest rate  $j$ ,  $X$  and  $Y$  have decisional equivalence for Marlee. At interest rate  $k$ ,  $X$  and  $Y$  have market equivalence for Marlee. What can be said about the origins, nature, and comparative values of  $i, j$ , and  $k$ ?

**M I N I - C A S E   2 . 1****Student Credit Cards**

Most major banks offer a credit card service for students. In 2011, Canadian credit card interest rates were in the range of 19.5 to 19.9 percent. Credit limits for student cards vary from \$500 and up, depending on the type of card. Many have rewards programs that allow the card-holder, for example, to build up points toward free travel on the reward program airline. Annual fees depend on the features of the card. Basic cards have no annual fee; those with a richer feature set charge fees between \$30 per year and \$120 per year. The approval process for getting a card is relatively simple for university and college students so that they can start building a credit history and enjoy the convenience of having a credit card while still in school.

The printed information does not use the term *nominal* or *effective*, nor does it define the compounding period. However, it is common in the credit card business for the annual interest rate to be divided into daily rates for billing purposes. Hence, the quoted annual rate of 19.7 percent is a nominal rate and the compounding period is daily. The actual effective interest rate is then  $(1 + 0.197/365)^{365} - 1 = 0.2177$ , or 21.77 percent.

**Discussion**

Interest information must be disclosed by law, but lenders and borrowers have some latitude as to how and where they disclose it. Moreover, there is a natural desire to make the interest rate look lower than it really is for borrowers, and higher than it really is for lenders.

In the example of student credit cards, the effective interest rate is 21.77 percent, roughly 2 percent higher than the stated interest rate. The actual effective interest rate could even end up being higher if fees such as late fees, over-the-limit fees, and transaction fees are charged.

**Questions**

1. Go to your local bank branch and find out the interest rate paid for various kinds of savings accounts, chequing accounts, and loans. For each interest rate quoted, determine if it is a nominal or effective rate. If it is nominal, determine the compounding period and calculate the effective interest rate.
2. Have a contest with your classmates to see who can find the organization that will lend money to a student like you at the cheapest effective interest rate, or that will take investments that provide a guaranteed return at the highest effective interest rate. The valid rates must be generally available, not tied to particular behaviour by the client, and not secured to an asset (like a house).
3. If you borrowed \$1000 at the best rate you could find and invested it at the best rate you could find, how much money would you make or lose in a year? Explain why the result of your calculation could not have the opposite sign.



# Cash Flow Analysis

Engineering Economics in Action, Part 3A: Apples and Oranges

- 3.1 Introduction
- 3.2 Timing of Cash Flows and Modelling
- 3.3 Compound Interest Factors for Discrete Compounding
- 3.4 Compound Interest Factors for Single Disbursements or Receipts
- 3.5 Compound Interest Factors for Annuities
- 3.6 Conversion Factor for Arithmetic Gradient Series
- 3.7 Conversion Factor for Geometric Gradient Series
- 3.8 Non-Standard Annuities and Gradients
- 3.9 Present Worth Computations When  $N \rightarrow \infty$

**Review Problems**

**Summary**

Engineering Economics in Action, Part 3B: No Free Lunch

**Problems**

**Mini-Case 3.1: The Canadian Oil Sands**

**Appendix 3A: Derivation of Discrete Compound Interest Factors**

## ENGINEERING ECONOMICS IN ACTION, PART 3A

### Apples and Oranges

The flyer was slick, all right. The information was laid out so anybody could see that leasing palletizing equipment through the Provincial Finance Company (PFC) made much more sense than buying it. It was something Naomi could copy right into her report to Clem.

Naomi had been asked to check out options for automating part of the shipping department. Parts were to be stacked and bound on plastic pallets, then loaded onto trucks and sent to one of Canadian Widgets' sister companies. The saleswoman for the company whose equipment seemed most suitable for Canadian Widgets' needs had included the leasing flyer with her quote.

Naomi looked at the figures again. They seemed to make sense, but there was something that didn't seem right to her. For one thing, if it was cheaper to lease, why didn't everybody lease everything? She knew that some things, like automobiles and airplanes, are often leased instead of bought, but generally companies buy assets. Second, where was the money coming from to give the finance company a profit? If the seller was getting the same amount and the buyer was paying less, how could PFC make money?

"Got a recommendation on that palletizer yet, Naomi?" Clem's voice was cheery as he suddenly appeared at her doorway. Naomi knew that the shipping department was the focus of Clem's attention right now and he wanted to get improvements in place as soon as possible.

"Yes, I do. There's really only one that will do the job, and it does it well at a good price. There is something I'm trying to figure out, though. Christine sent me some information about leasing it instead of buying it, and I'm trying to figure out where the catch is. There has got to be one, but I can't see it right now."

"Okay, let me give you a hint: apples and oranges. You can't add them. Now, let's get the paperwork started for that palletizer. The shipping department is just too much of a bottleneck." Clem disappeared as quickly as he had arrived, leaving Naomi musing to herself.

"Apples and *oranges*? Apples and oranges? Ahh . . . apples and oranges, of course!"

### 3.1

### Introduction

Chapter 2 showed that interest is the basis for determining whether different patterns of cash flows are equivalent. Rather than comparing patterns of cash flows from first principles, it is usually easier to use functions that define *mathematical* equivalence among certain common cash flow patterns. These functions are called *compound interest factors*. We discuss a number of these common cash flow patterns, along with their associated compound interest factors, in this chapter. These compound interest factors are used throughout the remainder of the book. It is, therefore, particularly important to understand their use before proceeding to subsequent chapters.

This chapter opens with an explanation of how cash flow patterns that engineers commonly use are simplified approximations of complex reality. Next, we discuss four simple, discrete cash flow patterns and the compound interest factors that relate them to each other. There is then a brief discussion of the case in which the number of time periods considered is so large that it is treated as though the relevant cash flows continue indefinitely. Appendix 3A presents mathematical derivations of the compound interest factors.

### 3.2

### Timing of Cash Flows and Modelling

The actual timing of cash flows can be very complicated and irregular. Unless some simple approximation is used, comparisons of different cash flow sequences will be very difficult and impractical. Consider, for example, the cash flows generated by a relatively

simple operation like a service station that sells gasoline and supplies and also services cars. Some cash flows, like sales of gasoline and minor supplies, will be almost continuous during the time the station is open. Other flows, like receipts for the servicing of cars, will be on a daily basis. Disbursements for wages may be on a weekly basis. Some disbursements, like those for a manager's salary and for purchases of gasoline and supplies, may be monthly. Disbursements for insurance and taxes may be quarterly or semiannually. Other receipts and disbursements, like receipts for major repairs or disbursements for used parts, may be irregular.

An analyst trying to make a comparison of two projects with different, irregular timings of cash flows might have to record each of the flows of the projects, and then, on a one-by-one basis, find summary equivalent values such as present worth that would be used in the comparison. This activity would be very time-consuming and tedious if it could be done, but it probably could not be done because the necessary data would not exist. If the projects were potential rather than actual, the cash flows would have to be predicted. This could not be done with great precision for either size or timing of the flows. Even if the analysis were of the past performances of ongoing operations, it is unlikely that it would be worthwhile to maintain a databank that contained the exact timing of all cash flows.

Because of the difficulties of making precise calculations of complex and irregular cash flows, engineers usually work with fairly simple models of cash flow patterns. The most common type of model assumes that all cash flows and all compounding of cash flows occur at the ends of conventionally defined periods, such as months or years. Models that make this assumption are called **discrete models**. In some cases, analysts use models that assume cash flows and their compounding occur continuously over time; such models are called **continuous models**. Whether the analyst uses discrete modelling or continuous modelling, the model is usually an approximation. Cash flows do not occur only at the ends of conventionally defined periods, nor are they actually continuous. We emphasize discrete models throughout the book because they are more common and more readily understood by persons of varied backgrounds.

### 3.3 Compound Interest Factors for Discrete Compounding

**Compound interest factors** are formulas that define mathematical equivalence for specific common cash flow patterns. The compound interest factors permit cash flow analysis to be done more conveniently because tables or spreadsheet functions can be used instead of complicated formulas. This section presents compound interest factors for four discrete cash flow patterns that are commonly used to model the timing of receipts and disbursements in engineering economic analysis. The four patterns are:

1. A single disbursement (money spent) or receipt (money received)
2. A set of equal disbursements or receipts over a sequence of periods, referred to as an **annuity**
3. A set of disbursements or receipts that change by a constant *amount* from one period to the next in a sequence of periods, referred to as an **arithmetic gradient series**
4. A set of disbursements or receipts that change by a constant *proportion* from one period to the next in a sequence of periods, referred to as a **geometric gradient series**

The principle of discrete compounding requires several assumptions:

1. Compounding periods are of equal length.
2. Each disbursement and receipt occurs at the end of a period. A payment at time 0 can be considered to occur at the end of period  $-1$ .
3. Annuities and gradients coincide with the ends of sequential periods. (Section 3.8 suggests several methods for dealing with annuities and gradients that do not coincide with the ends of sequential periods.)

Mathematical derivations of six of the compound interest factors are given in Appendix 3A at the end of this chapter.

## 3.4 Compound Interest Factors for Single Disbursements or Receipts

In many situations, a single disbursement or receipt is an appropriate model of cash flows. For example, the salvage value of production equipment with a limited service life will be a single receipt at some future date. An investment today to be redeemed at some future date is another example.

Figure 3.1 illustrates the general form of a single disbursement or receipt. Two commonly used factors relate a single cash flow in one period to another single cash flow in a different period. They are the *compound amount factor* and the *present worth factor*.

The **compound amount factor**, denoted by  $(F/P,i,N)$ , gives the future amount,  $F$ , that is equivalent to a present amount,  $P$ , when the interest rate is  $i$  and the number of periods is  $N$ . The value of the compound amount factor is easily seen as coming from Equation (2.1) on page 22, the compound interest equation, which relates present and future values:

$$F = P(1 + i)^N$$

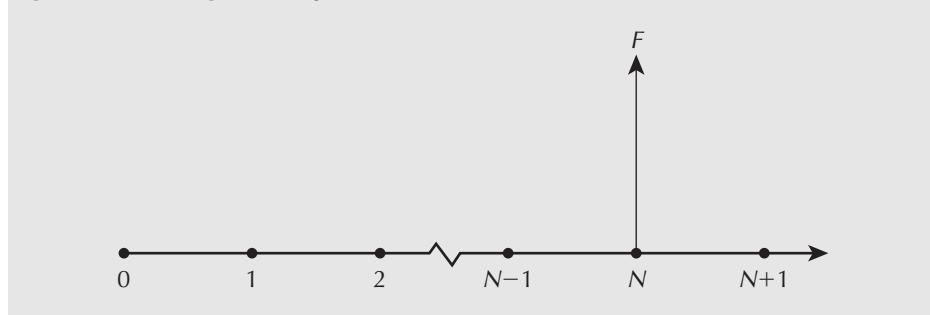
In the symbolic convention used for compound interest factors, this is written

$$F = P(1 + i)^N = P(F/P,i,N) \quad \boxed{F} = FV(i,N,-P)$$

so that the compound amount factor is

$$(F/P,i,N) = (1 + i)^N$$

**Figure 3.1 Single Receipt at End of Period N**



A handy way of thinking of the notation is (reading from left to right): “What is  $F$ , given  $P$ ,  $i$ , and  $N$ ? ”

The compound amount factor is useful in determining the future value of an investment made today if the number of periods and the interest rate are known.

The **present worth factor**, denoted by  $(P/F,i,N)$ , gives the present amount,  $P$ , that is equivalent to a future amount,  $F$ , when the interest rate is  $i$  and the number of periods is  $N$ . The present worth factor is the inverse of the compound amount factor,  $(F/P,i,N)$ . That is, while the compound amount factor gives the future amount,  $F$ , that is equivalent to a present amount,  $P$ , the present worth factor goes in the other direction. It gives the present worth,  $P$ , of a future amount,  $F$ . Since  $(F/P,i,N) = (1 + i)^N$ ,

$$(P/F,i,N) = \frac{1}{(1 + i)^N}$$

The compound amount factor and the present worth factor are fundamental to engineering economic analysis. Their most basic use is to convert a single cash flow that occurs at one point in time to an equivalent cash flow at another point in time. When comparing several individual cash flows that occur at different points in time, an analyst would apply the compound amount factor or the present worth factor, as necessary, to determine the equivalent cash flows at a common reference point in time. In this way, each of the cash flows is stated as an amount at one particular time. Example 3.1 illustrates this process.

Although the compound amount factor and the present worth factor are relatively easy to calculate, some of the other factors discussed in this chapter are more complicated, and it is therefore desirable to have an easier way to determine their values. Many of the compound interest factors are available as built-in functions in spreadsheets (see Table 3.3 at the end of the chapter for a summary). They can also be fairly easily programmed into a calculator.

A traditional and still useful method for determining the value of a compound interest factor is to use tables. Appendix A at the back of the book lists values for all the compound interest factors for a selection of interest rates for discrete compounding periods. The desired compound interest factor can be determined by looking in the appropriate table.

### EXAMPLE

### 3.1

How much money will be in a bank account at the end of 15 years if \$100 is invested today and the nominal interest rate is 8 percent compounded semiannually?

Since a present amount is given and a future amount is to be calculated, the appropriate factor to use is the compound amount factor,  $(F/P,i,N)$ . There are several ways of choosing  $i$  and  $N$  to solve this problem. The first method is to observe that, since interest is compounded semiannually, the number of compounding periods,  $N$ , is 30. The interest rate per six-month period is 4 percent. Then

$$\begin{aligned} F &= 100(F/P, 4\%, 30) \\ &= 100(1 + 0.04)^{30} \\ &= 324.34 \end{aligned}$$

The bank account will hold \$324.34 at the end of 15 years.

Alternatively, we can obtain the same results by using the interest factor tables.

$$\begin{aligned} F &= 100(3.2434) \quad (\text{from Appendix A}) \\ &= 324.34 \end{aligned}$$

A second solution to the problem is to calculate the *effective* yearly interest rate and then compound over 15 years at this rate. Recall from Equation (2.4) on page 27 that the effective interest rate per year is

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

where  $i_e$  = the effective annual interest rate

$r$  = the nominal rate per year

$m$  = the number of periods in a year

$$i_e = (1 + 0.08/2)^2 - 1 = 0.0816$$

where  $r = 0.08$

$m = 2$

When the effective yearly rate for each of 15 years is applied to the future worth computation, the future worth is

$$\begin{aligned} F &= P(F/P,i,N) \\ &= P(1 + i)^N \\ &= 100(1 + 0.0816)^{15} \\ &= 324.34 \end{aligned}$$

Once again, we conclude that the balance will be \$324.34. ■



## S P R E A D S H E E T   S A V V Y

Spreadsheet software applications include a library of built-in functions to make certain types of calculations easier. For example, Excel includes a number of libraries such as statistical, mathematical and trigonometric, and financial. Among the financial functions are several that are useful for economic evaluation of single payment and uniform series cash flow patterns. As an alternative to writing out the full expression to be evaluated using a compound interest factor, the functions can be used to shorten the process. One must take care, however, to fully understand the cash flow pattern captured by each compound interest factor or Excel function so that the structure of the cash flows is not forgotten amid the formulas.

The table below provides a summary of how three Excel functions can be used as alternatives to compound interest factors covered in this chapter. Each function requires a set of arguments that can be either numeric values or references to cells containing numeric values. Warning!: Excel assumes that the cash flows are costs. To get the same signs as the compound interest factors, we have reversed the signs of the amounts  $A$ ,  $F$ , and  $P$  in the Excel functions.

| Compound Interest Factor | Excel Function    |
|--------------------------|-------------------|
| $P = A(P/A,i,N)$         | $P = PV(i,N,-A)$  |
| $P = F(P/F,i,N)$         | $P = PV(i,N,-F)$  |
| $F = A(F/A,i,N)$         | $F = FV(i,N,-A)$  |
| $F = P(F/P,i,N)$         | $F = FV(i,N,-P)$  |
| $A = P(A/P,i,N)$         | $A = PMT(i,N,-P)$ |
| $A = F(A/F,i,N)$         | $A = PMT(i,N,-F)$ |

Below are two examples of how Excel functions and compound interest factors can be used to compute the same quantities. The first table shows the formulas, and the second, the results.

|   | A                 | B                     | C                               | D                                |
|---|-------------------|-----------------------|---------------------------------|----------------------------------|
| 1 | <b>Values:</b>    |                       |                                 |                                  |
| 2 | $i = 0.1$         |                       | $P = 2500$                      |                                  |
| 3 | $N = 15$          |                       | $F = 9400$                      |                                  |
| 4 | <b>Relation</b>   | <b>Excel Function</b> | <b>Compound Interest Factor</b> |                                  |
| 5 | $A = (A/P, i, N)$ | =PMT(B2,B3,-D2)       |                                 | =D2*(B2*(1+B2)^B3)/((1+B2)^B3-1) |
| 6 | $A = (A/F, i, N)$ | =PMT(B2,B3,-D3)       |                                 | =D3*B2/((1+B2)^B3-1)             |

|   | A                 | B                     | C                               | D        |
|---|-------------------|-----------------------|---------------------------------|----------|
| 1 | <b>Values:</b>    |                       |                                 |          |
| 2 | $i = 0.1$         |                       | $P = 2500$                      |          |
| 3 | $N = 15$          |                       | $F = 9400$                      |          |
| 4 | <b>Relation</b>   | <b>Excel Function</b> | <b>Compound Interest Factor</b> |          |
| 5 | $A = (A/P, i, N)$ | \$328.68              |                                 | \$328.68 |
| 6 | $A = (A/F, i, N)$ | \$295.85              |                                 | \$295.85 |

There are several ways to deal with the fact that Excel assumes that the cash flows have the opposite sign to the compound interest factors. The first is to use a minus sign in reference to the cash flows (as on the previous page). This assumes that the reference cell uses a positive sign for a cash receipt and a negative sign for a disbursement. The second way is to refer to the cash flows directly and then reverse the sign of the Excel function (this would be entered as  $= -PMT(B2,B3,D3)$  in cell B6 above, for example). The third method is to change the sign of the cash flow referred to (for example, set D2 =  $-2500$  and D3 =  $-9400$  above). The third method was not used in this example because the compound interest factor computations also make reference to cells D2 and D3.

## 3.5

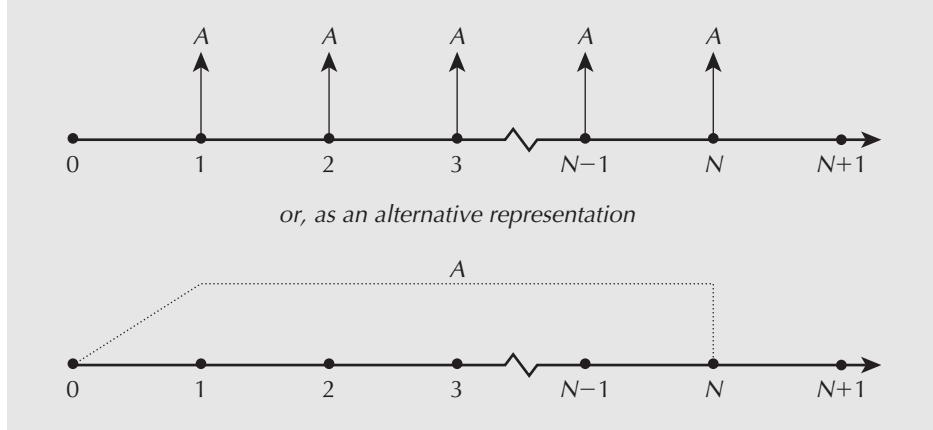
## Compound Interest Factors for Annuities

The next four factors involve a series of uniform receipts or disbursements that start at the end of the first period and continue over  $N$  periods, as illustrated in Figure 3.2. This pattern of cash flows is called an annuity. Mortgage or lease payments and maintenance contract fees are examples of the annuity cash flow pattern. Annuities may also be used to model series of cash flows that fluctuate over time around some average value. Here the average value would be the constant uniform cash flow. This would be done if the fluctuations were unknown or deemed to be unimportant for the problem.

The **sinking fund factor**, denoted by  $(A/F, i, N)$ , gives the size,  $A$ , of a repeated receipt or disbursement that is equivalent to a future amount,  $F$ , if the interest rate is  $i$  and the number of periods is  $N$ . The name of the factor comes from the term **sinking fund**. A sinking fund is an interest-bearing account into which regular deposits are made in order to accumulate some amount.

The equation for the sinking fund factor can be found by decomposing the series of disbursements or receipts made at times 1, 2, . . . ,  $N$ , and summing to produce a total future value. The formula for the sinking fund factor is

$$(A/F, i, N) = \frac{i}{(1 + i)^N - 1}$$

**Figure 3.2 Annuity Over  $N$  Periods**

The sinking fund factor is commonly used to determine how much has to be set aside or saved per period to accumulate an amount  $F$  at the end of  $N$  periods at an interest rate  $i$ . The amount  $F$  might be used, for example, to purchase new or replacement equipment, to pay for renovations, or to cover capacity expansion costs. In more general terms, the sinking fund factor allows us to convert a single future amount into a series of equal-sized payments, made over  $N$  equally spaced intervals, with the use of a given interest rate  $i$ .

The **uniform series compound amount factor**, denoted by  $(F/A,i,N)$ , gives the future value,  $F$ , that is equivalent to a series of equal-sized receipts or disbursements,  $A$ , when the interest rate is  $i$  and the number of periods is  $N$ . Since the uniform series compound amount factor is the inverse of the sinking fund factor,

$$(F/A,i,N) = \frac{(1 + i)^N - 1}{i}$$

The **capital recovery factor**, denoted by  $(A/P,i,N)$ , gives the value,  $A$ , of the equal periodic payments or receipts that are equivalent to a present amount,  $P$ , when the interest rate is  $i$  and the number of periods is  $N$ . The capital recovery factor is easily derived from the sinking fund factor and the compound amount factor:

$$\begin{aligned} (A/P,i,N) &= (A/F,i,N)(F/P,i,N) \\ &= \frac{i}{(1 + i)^N - 1} (1 + i)^N \\ &= \frac{i(1 + i)^N}{(1 + i)^N - 1} \end{aligned}$$

The capital recovery factor can be used to find out, for example, how much money must be saved over  $N$  future periods to “recover” a capital investment of  $P$  today. The capital recovery factor for the purchase cost of something is sometimes combined with the sinking fund factor for its salvage value after  $N$  years to compose the **capital recovery formula**. See Close-Up 3.1.

The **series present worth factor**, denoted by  $(P/A,i,N)$ , gives the present amount,  $P$ , that is equivalent to an annuity with disbursements or receipts in the amount,  $A$ , where the interest rate is  $i$  and the number of periods is  $N$ . It is the reciprocal of the capital recovery factor:

$$(P/A,i,N) = \frac{(1 + i)^N - 1}{i(1 + i)^N}$$

**CLOSE-UP 3.1****Capital Recovery Formula**

Industrial equipment and other assets are often purchased at a cost of  $P$  on the basis that they will incur savings of  $A$  per period for the firm. At the end of their useful life, they will be sold for some salvage value  $S$ . The expression to determine  $A$  for a given  $P$  and  $S$  combines the capital recovery factor (for  $P$ ) with the sinking fund factor (for  $S$ ):

$$A = P(A/P,i,N) - S(A/F,i,N)$$

Since

$$\begin{aligned} (A/F,i,N) &= \frac{i}{(1+i)^N - 1} = \frac{i}{(1+i)^N - 1} + i - i \\ &= \frac{i}{(1+i)^N - 1} + \frac{i[(1+i)^N - 1]}{(1+i)^N - 1} - i \\ &= \frac{i + i(1+i)^N - i}{(1+i)^N - 1} - i = \frac{i(1+i)^N}{(1+i)^N - 1} - i \\ &= (A/P,i,N) - i \end{aligned}$$

then

$$\begin{aligned} A &= P(A/P,i,N) - S[(A/P,i,N) - i] \\ &= (P - S)(A/P,i,N) + Si \quad \blacksquare = PMT(i,N,-(P - S)) + Si \end{aligned}$$

This is the **capital recovery formula**, which can be used to calculate the savings necessary to justify a capital purchase of cost  $P$  and salvage value  $S$  after  $N$  periods at interest rate  $i$ .

The capital recovery formula is also used to determine an annual amount that captures the loss in value of an asset over the time it is owned. Chapter 7 treats this use of the capital recovery formula more fully.

**EXAMPLE****3.2**

The Kelowna Go-Kart Klub has decided to build a clubhouse and track five years from now. It must accumulate \$50 000 by the end of five years by setting aside a uniform amount from its dues at the end of each year. If the interest rate is 10 percent, how much must be set aside each year?

Since the problem requires that we calculate an annuity amount given a future value, the solution can be obtained using the sinking fund factor where  $i = 10$  percent,  $F = \$50\,000$ ,  $N = 5$ , and  $A$  is unknown.

$$\begin{aligned} A &= 50\,000(A/F,10\%,5) \quad \blacksquare = PMT(0.10,5,-50\,000) \\ &= 50\,000(0.1638) \\ &= 8190.00 \end{aligned}$$

The Go-Kart Klub must set aside \$8190 at the end of each year to accumulate \$50 000 in five years. ■

**EXAMPLE****3.3**

A car loan requires 30 monthly payments of \$199.00, starting *today*. At an annual rate of 12 percent compounded monthly, how much money is being lent?

This cash flow pattern is referred to as an **annuity due**. It differs from a standard annuity in that the first of the  $N$  payments occurs at time 0 (now) rather than at the end of the first time period. Annuities due are uncommon—not often will one make the first payment on a loan on the date the loan is received! Unless otherwise stated, it is reasonable to assume that any annuity starts at the end of the first period.

Two simple methods of analyzing an annuity due will be used for this example.

*Method 1.* Count the first payment as a present worth and the next 29 payments as an annuity:

$$P = 199 + A(P/A,i,N) \quad \blacksquare = 199 + PV(i,N,-A)$$

where  $A = 199$ ,  $i = 12\%/12 = 1\%$ , and  $N = 29$ .

$$\begin{aligned} P &= 199 + 199(P/A,1\%,29) \\ &= 199 + 199(25.066) \\ &= 199 + 4988.13 \\ &= 5187.13 \end{aligned}$$

The present worth of the loan is the current payment, \$199, plus the present worth of the subsequent 29 payments, \$4988.13—a total of about \$5187.

*Method 2.* Determine the present worth of a standard annuity at time  $-1$ , and then find its worth at time 0 (now). The worth at time  $-1$  is

$$\begin{aligned} P_{-1} &= A(P/A,i,N) \quad \blacksquare = PV(i,N,-A) \\ &= 199(P/A,1\%,30) \\ &= 199(25.807) \\ &= 5135.79 \end{aligned}$$

Then the present worth now (time 0) is

$$\begin{aligned} P_0 &= P_{-1}(F/P,i,N) \quad \blacksquare = FV(i,N,-P_{-1}) \\ &= 5135.79(F/P,1\%,1) \\ &= 5135.79(1.01) \\ &= 5187.15 \end{aligned}$$

The second method gives the same result as the first, allowing a small margin for the effects of rounding. ■

It is worth noting here that although it is natural to think about the symbol  $P$  as meaning a cash flow at time 0, the present, and  $F$  as meaning a cash flow in the future, in fact these symbols can be more general in meaning. As illustrated in the last example, we can consider any point in time to be the “present” for calculation purposes, and similarly any point in time to be the “future,” provided  $P$  is some point in time earlier than  $F$ . This observation gives us substantial flexibility in analyzing cash flows.

**EXAMPLE****3.4**

Clarence bought a condo for \$94 000 in 2012. He made a \$14 000 down payment and negotiated a mortgage from the previous owner for the balance. Clarence agreed to pay the previous owner \$2000 per month at 12 percent nominal interest, compounded monthly. How long will it take him to pay back the mortgage?

Clarence borrowed only \$80 000, since he made a \$14 000 down payment. The \$2000 payments form an annuity over  $N$  months where  $N$  is unknown. The interest rate per month is 1 percent. We must find the value of  $N$  such that

$$P = A(P/A,i,N) = A\left(\frac{(1+i)^N - 1}{i(1+i)^N}\right) \quad \blacksquare = PV(i,N,-A)$$

or, alternatively, the value of  $N$  such that

$$A = P(A/P,i,N) = P\left(\frac{i(1+i)^N}{(1+i)^N - 1}\right)$$

where  $P = \$80\,000$ ,  $A = \$2000$ , and  $i = 0.01$ .

By substituting the known quantities into either expression, some manipulation is required to find  $N$ . For illustration, the capital recovery factor has been used.

$$A = P\left(\frac{i(1+i)^N}{(1+i)^N - 1}\right) \quad \blacksquare = PMT(i,N,-P)$$

$$2000 = 80\,000\left(\frac{0.01(1.01)^N}{1.01^N - 1}\right)$$

$$2.5 = \frac{(1.01)^N}{(1.01)^N - 1}$$

$$2.5/1.5 = (1.01)^N$$

$$N[\ln(1.01)] = \ln(2.5/1.5)$$

$$N = 51.34 \text{ months}$$

It will take Clarence four years and four months to pay off the mortgage. He will make 51 full payments of \$2000 and will be left with only a fraction of a full payment for his 52nd and last monthly installment. Problem 3.47 asks what his final payment will be. Note that mortgages can be confusing because of the different terms used. See Close-Up 3.2. ■

In Example 3.4, it was possible to use the formula for the compound interest factor to solve for the unknown quantity directly. It is not always possible to do this when the number of periods or the interest rate is unknown. We can proceed in several ways. One possibility is to determine the unknown value by trial and error with a spreadsheet. Another approach is to find the nearest values using tables, and then to interpolate linearly to determine an approximate value. Some calculators will perform the interpolation automatically. See Close-Up 3.3 and Figure 3.3 for a reminder of how linear interpolation works.

**CLOSE-UP 3.2****Mortgages**

Mortgages can be a little confusing because of the terminology used. In particular, the word *term* is used in different ways in different countries. It can mean either the duration over which the original loan is calculated to be repaid (called the **amortization period** in Canada). It can also mean the duration over which the loan agreement is valid (otherwise called the *maturity*). The interest rate is a nominal rate, usually compounded monthly.

For example, Salim has just bought a house for \$135 000. He paid \$25 000 down, and the rest of the cost has been obtained from a mortgage. The mortgage has a nominal interest rate of 9.5 percent compounded monthly with a 20-year amortization period. The term (maturity) of the mortgage is three years. What are Salim's monthly payments? How much does he owe after three years?

Salim's monthly payments can be calculated as

$$\begin{aligned} A &= (135\,000 - 25\,000)(A/P, 9.5/12\%, [20 \times 12]) \\ &= 110\,000(A/P, 0.7917\%, 240) \quad \boxed{=} = \mathbf{PMT(0.007917, 240, -110\,000)} \\ &= 110\,000(0.00932) \\ &= 1025.20 \end{aligned}$$

A more direct approach would be to compute the present worth (at the end of three years) of the remaining 204 payments:

$$P = \$1025.2(P/A, 0.079, 204) = 103\,598.$$

Salim's monthly payments would be about \$1025.20. After three years he would have to renegotiate his mortgage at the current interest rate at that time. The amount owed would be

$$\begin{aligned} F &= 110\,000(F/P, 9.5/12\%, 36) - 1025.20(F/A, 9.5/12\%, 36) \\ &= 110\,000(1.3283) - 1025.20(41.47) \quad \boxed{=} = \mathbf{FV(0.007913, 36, -110\,000)} \\ &\quad - \mathbf{FV(0.007913, 36, -1025.20)} \\ &= 103\,598 \end{aligned}$$

After three years, Salim still owes \$103 598.

**CLOSE-UP 3.3****Linear Interpolation**

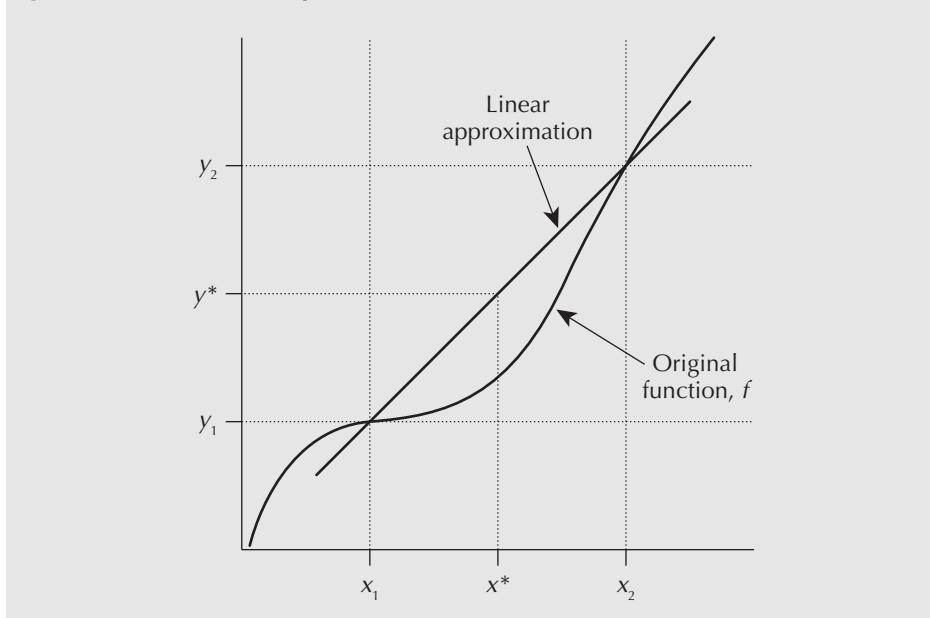
Linear interpolation is the process of approximating a complicated function by a straight line in order to estimate a value for the independent variable based on two sample pairs of independent and dependent variables and an instance of the dependent variable. For example, the function  $f$  in Figure 3.3 relates the dependent variable  $y$  to the independent variable  $x$ . Two sample points,  $(x_1, y_1)$  and  $(x_2, y_2)$ , and an instance of  $y$ ,  $y^*$ , are known, but the actual shape of  $f$  is not. An estimate of the value for  $x^*$  can be made by drawing a straight line between  $(x_1, y_1)$  and  $(x_2, y_2)$ .

Because the line between  $(x_1, y_1)$  and  $(x_2, y_2)$  is assumed to be straight, the following ratios must be equal:

$$\frac{x^* - x_1}{x_2 - x_1} = \frac{y^* - y_1}{y_2 - y_1}$$

Isolating the  $x^*$  gives the linear interpolation formula:

$$x^* = x_1 + (x_2 - x_1) \left[ \frac{y^* - y_1}{y_2 - y_1} \right]$$

**Figure 3.3 Linear Interpolation**
**EXAMPLE 3.5**

Clarence paid off an \$80 000 mortgage completely in 48 months. He paid \$2000 per month, and at the end of the first year made an extra payment of \$7000. What interest rate was he charged on the mortgage?

Using the series present worth factor and the present worth factor, this can be formulated for an unknown interest rate:

$$\begin{aligned} 80\,000 &= 2000(P/A, i, 48) + 7000(P/F, i, 12) \quad \blacksquare = PV(i, 48, -2000) \\ 2(P/A, i, 48) + 7(P/F, i, 12) &= 80 \quad + PV(i, 12, -7000) \\ 2\left[\frac{(1+i)^{48}-1}{i(1+i)^{48}}\right] + 7\left[\frac{1}{(1+i)^{12}}\right] &= 80 \end{aligned} \quad (3.1)$$

Solving such an equation for  $i$  directly is generally not possible. However, using a spreadsheet as illustrated in Table 3.1 can establish some close values for the left-hand side of Equation (3.1), and a similar process can be done using either tables or a calculator. Using a spreadsheet program or calculator, trials can establish a value for the unknown interest rate to the desired number of significant digits.

Once the approximate values for the interest rate are found, linear interpolation can be used to find a more precise answer. For instance, by working from the values of the interest rate that give the LHS (left-hand side) value closest to the RHS (right-hand side) value of 80, which are 1.1 percent and 1.2 percent,

$$\begin{aligned} i &= 1.1 + (1.2 - 1.1)\left[\frac{80 - 80.4141}{78.7209 - 80.4141}\right] \\ &= 1.1 + 0.02 = 1.12 \text{ percent per month} \end{aligned}$$

The nominal interest rate was  $1.12 \times 12 = 13.44$  percent.

The effective interest rate was  $(1.0112)^{12} - 1 = 14.30$  percent. ■

**Table 3.1 Trials to Determine an Unknown Interest Rate**

| Interest Rate <i>i</i> | $2(P/A,i,48) + 7(P/F,i,12)$ |
|------------------------|-----------------------------|
| 0.5%                   | 91.7540                     |
| 0.6%                   | 89.7128                     |
| 0.7%                   | 87.7350                     |
| 0.8%                   | 85.8185                     |
| 0.9%                   | 83.9608                     |
| 1.0%                   | 82.1601                     |
| 1.1%                   | 80.4141                     |
| 1.2%                   | 78.7209                     |
| 1.3%                   | 77.0787                     |
| 1.4%                   | 75.4855                     |
| 1.5%                   | 73.9398                     |

Another interesting application of compound interest factors is calculating the value of a bond. See Close-Up 3.4.

**CLOSE-UP 3.4****Bonds**

**Bonds** are investments that provide an annuity and a future value in return for a cost today. They have a *par* or *face* value, which is the amount for which they can be redeemed after a certain period of time. They also have a *coupon rate*, meaning that they pay the bearer an annuity, usually semiannually, calculated as a percentage of the face value. For example, a coupon rate of 10 percent on a bond with an \$8000 face value would pay an annuity of \$400 each six months. Bonds can sell at more or less than the face value, depending on how buyers perceive them as investments.

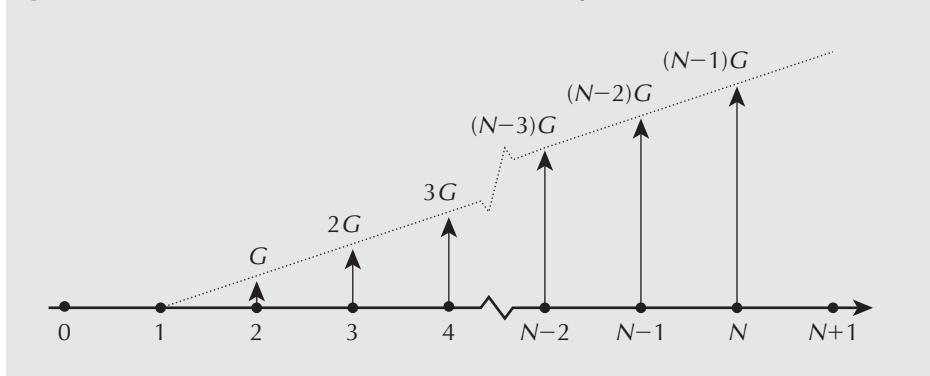
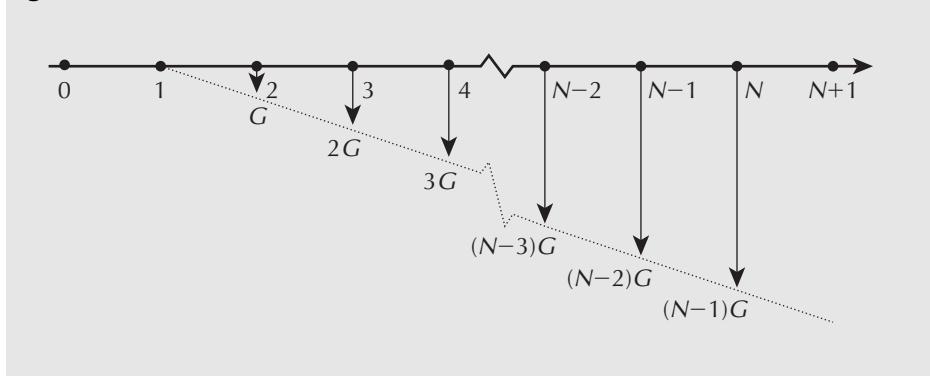
To calculate the worth of a bond today, sum together the present worth of the face value (a future amount) and the coupons (an annuity) at an appropriate interest rate. For example, if money can earn 12 percent compounded semiannually, a bond maturing in 15 years with a face value of \$5000 and a coupon rate of 7 percent is today worth

$$\begin{aligned}
 P &= 5000(P/F, 6\%, 30) + (5000 \times 0.07/2)(P/A, 6\%, 30) \\
 &= 5000(0.17411) + 175(13.765) \\
 &= 3279.43
 \end{aligned}$$

The bond is worth about \$3279 today.

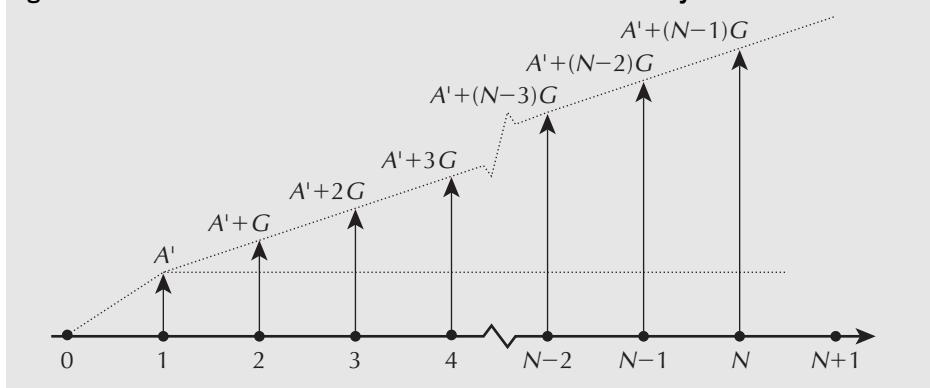
**3.6****Conversion Factor for Arithmetic Gradient Series**

An arithmetic gradient series is a series of receipts or disbursements that starts at zero at the end of the first period and then increases by a constant *amount* from period to period. Figure 3.4 illustrates an arithmetic gradient series of receipts. Figure 3.5 shows

**Figure 3.4 Arithmetic Gradient Series of Receipts****Figure 3.5 Arithmetic Gradient Series of Disbursements**

an arithmetic gradient series of disbursements. As an example, we may model a pattern of increasing operating costs for an aging machine as an arithmetic gradient series if the costs are increasing by (approximately) the same amount each period. Note carefully that the first non-zero cash flow of a gradient occurs at the end of the *second* compounding period, not the first.

The sum of an annuity plus an arithmetic gradient series is a common pattern. The annuity is a base to which the arithmetic gradient series is added. This is shown in Figure 3.6. A constant-amount increase to a base level of receipts may occur where the increase in receipts is due to adding capacity and where the ability to add capacity is

**Figure 3.6 Arithmetic Gradient Series With Base Annuity**

limited. For example, a company that specializes in outfitting warehouses for grocery chains can expand by adding work crews. But the crews must be trained by managers who have time to train only one crew member every six months. Hence, we would have a base amount and a constant amount of growth in cash flows each period.

The **arithmetic gradient to annuity conversion factor**, denoted by  $(A/G,i,N)$ , gives the value of an annuity,  $A$ , that is equivalent to an arithmetic gradient series where the constant increase in receipts or disbursements is  $G$  per period, the interest rate is  $i$ , and the number of periods is  $N$ . That is, the arithmetic gradient series  $0G, 1G, 2G, \dots, (N-1)G$  is given and the uniform cash flow,  $A$ , over  $N$  periods is found. Problem 3.44 asks the reader to show that the equation for the arithmetic gradient to annuity factor is

$$(A/G,i,N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$$

There is often a base annuity  $A'$  associated with a gradient, as illustrated in Figure 3.6. To determine the uniform series equivalent to the *total* cash flow, the base annuity  $A'$  must be included to give the overall annuity:

$$A_{\text{tot}} = A' + G(A/G,i,N)$$

### EXAMPLE

### 3.6

Susan Ng owns an eight-year-old Prius automobile. She wants to find the present worth of repair bills over the four years that she expects to keep the car. Susan has the car in for repairs every six months. Repair costs are expected to increase by \$50 every six months over the next four years, starting with \$500 six months from now, \$550 six months later, and so on. What is the present worth of the repair costs over the next four years if the interest rate is 12 percent compounded monthly?

First, observe that there will be  $N = 8$  repair bills over four years and that the base annuity payment,  $A'$ , is \$500. The arithmetic gradient component of the bills,  $G$ , is \$50, and hence the arithmetic gradient series is \$0, \$50, \$100, and so on. The present worth of the repair bills can be obtained in a two-step process:

*Step 1.* Find the total uniform annuity,  $A_{\text{tot}}$ , equivalent to the sum of the base annuity,  $A' = \$500$ , and the arithmetic gradient series with  $G = \$50$  over  $N = 8$  periods.

*Step 2.* Find the present worth of  $A_{\text{tot}}$  using the series present worth factor.

The 12 percent nominal interest rate, compounded monthly, is 1 percent per month. The effective interest rate per six-month period is

$$i_{\text{6month}} = (1 + 0.12/12)^6 - 1 = 0.06152 \text{ or } 6.152\%$$

*Step 1*

$$\begin{aligned} A_{\text{tot}} &= A' + G(A/G,i,N) \\ &= 500 + 50 \left( \frac{1}{i} - \frac{N}{(1+i)^N - 1} \right) \\ &= 500 + 50 \left( \frac{1}{0.06152} - \frac{8}{(1.06152)^8 - 1} \right) \\ &= 659.39 \end{aligned}$$

*Step 2*

$$\begin{aligned}
 P &= A_{\text{tot}}(P/A, i, N) = A_{\text{tot}} \left( \frac{(1+i)^N - 1}{i(1+i)^N} \right) \\
 &= 659.39 \left( \frac{(1.06152)^8 - 1}{0.06152(1.06152)^8} \right) \\
 &= 4070.09
 \end{aligned}$$

The present worth of the repair costs is about \$4070. ■

## 3.7

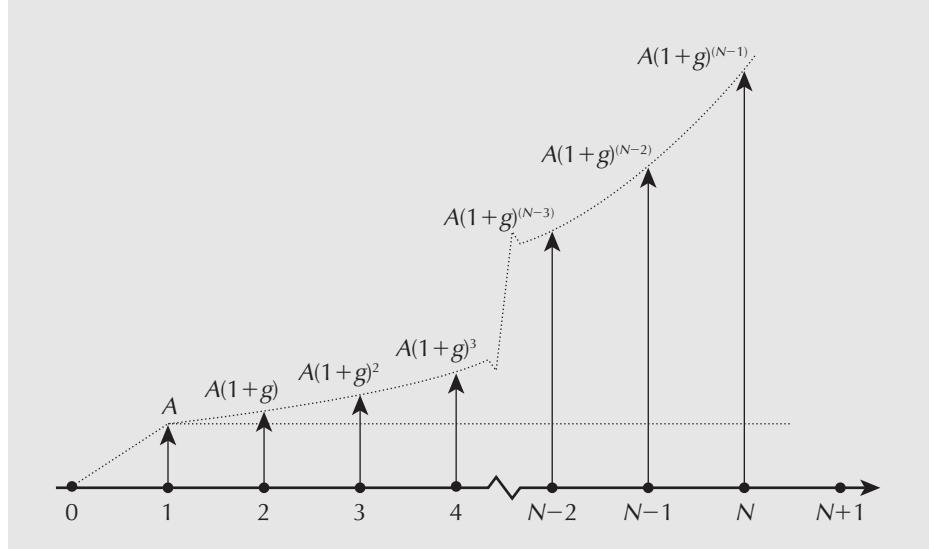
## Conversion Factor for Geometric Gradient Series

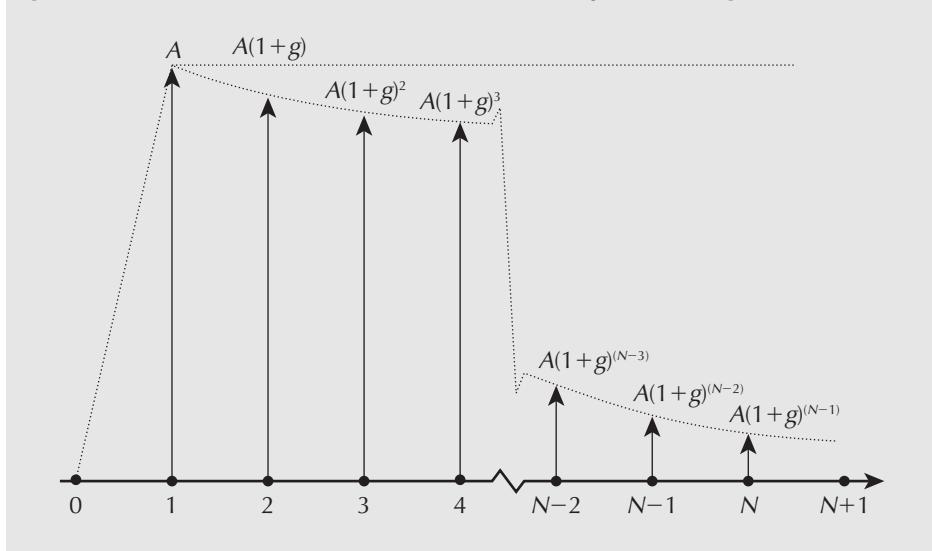
A geometric gradient series is a series of cash flows that increase or decrease by a constant *percentage* each period. The geometric gradient series may be used to model inflation or deflation, productivity improvement or degradation, and growth or shrinkage of market size, as well as many other phenomena.

In a geometric series, the base value of the series is  $A$  and the “growth” rate in the series (the rate of increase or decrease) is referred to as  $g$ . The terms in such a series are given by  $A, A(1+g), A(1+g)^2, \dots, A(1+g)^{N-1}$  at the ends of periods  $1, 2, 3, \dots, N$ , respectively. If the rate of growth,  $g$ , is positive, the terms are increasing in value. If the rate of growth,  $g$ , is negative, the terms are decreasing. Figure 3.7 shows a series of receipts where  $g$  is positive. Figure 3.8 shows a series of receipts where  $g$  is negative.

The **geometric gradient to present worth conversion factor**, denoted by  $(P/A, g, i, N)$ , gives the present worth,  $P$ , that is equivalent to a geometric gradient series where the base receipt or disbursement is  $A$ , and where the rate of growth is  $g$ , the interest rate is  $i$ , and the number of periods is  $N$ .

**Figure 3.7 Geometric Gradient Series for Receipts With Positive Growth**



**Figure 3.8 Geometric Gradient Series for Receipts With Negative Growth**

The present worth of a geometric series is

$$P = \frac{A}{1+i} + \frac{A(1+g)}{(1+i)^2} + \dots + \frac{A(1+g)^{N-1}}{(1+i)^N}$$

where     $A$  = the base amount  
 $g$  = the rate of growth  
 $i$  = the interest rate  
 $N$  = the number of periods  
 $P$  = the present worth

## NET VALUE 3.1

### Estimating Growth Rates

Geometric gradient series can be used to model the effects of inflation, deflation, production rate change, and market size change on a future cash flow. When using a geometric gradient series, the relevant growth rate must be estimated. The internet can be a useful research tool for collecting information such as expert opinions and statistics on trends for national and international activities by product type and industry. For example, a sales growth rate may be estimated by

considering a number of factors: economic condition indicators (e.g., gross domestic product, employment, consumer spending), population growth, raw material cost, and even online shopping and business-to-business trading. Federal governments are keen to assess what has happened in order to predict what will happen. In Canada, such information is collected by Statistics Canada and is available to the public online at [www.statcan.gc.ca](http://www.statcan.gc.ca).

We can define a **growth-adjusted interest rate**,  $i^o$ , as

$$i^o = \frac{1+i}{1+g} - 1$$

so that

$$\frac{1}{1+i^o} = \frac{1+g}{1+i}$$

Then the geometric gradient series to present worth conversion factor is given by

$$(P/A, g, i, N) = \frac{(P/A, i^o, N)}{1+g} \text{ or}$$

$$(P/A, g, i, N) = \left( \frac{(1+i^o)^N - 1}{i^o(1+i^o)^N} \right) \frac{1}{1+g} \quad \boxed{= PV(i, N, -A)/(1+g)}$$

Care must be taken in using the geometric gradient to present worth conversion factor. Four cases may be distinguished:

1.  $i > g > 0$ . *Growth is positive, but less than the rate of interest.* The growth-adjusted interest rate,  $i^o$ , is positive. Tables or functions built into software may be used to find the conversion factor.
2.  $g > i > 0$ . *Growth is positive and greater than the interest rate.* The growth-adjusted interest rate,  $i^o$ , is negative. It is necessary to compute the conversion factor directly from the formula.
3.  $g = i > 0$ . *Growth is positive and exactly equal to the interest rate.* The growth-adjusted interest rate,  $i^o$ , equals zero. As with any case where the interest rate is zero, the present worth of the series with constant terms,  $A/(1+g)$ , is simply the sum of all the  $N$  terms:

$$P = N \left( \frac{A}{1+g} \right)$$

4.  $g < 0$ . *Growth is negative.* In other words, the series is decreasing. The growth-adjusted interest rate,  $i^o$ , is positive. Tables or functions built into software may be used to find the conversion factor.

### EXAMPLE

### 3.7

Tru-Test is in the business of assembling and packaging automotive and marine testing equipment to be sold through retailers to “do-it-yourselfers” and small repair shops. One of its products is tire pressure gauges. This operation has some excess capacity. Tru-Test is considering using this excess capacity to add engine compression gauges to its line. It can sell engine pressure gauges to retailers for \$8 per gauge and expect to be able to produce about 1000 gauges in the first month of production. It also expects that, as the workers learn how to do the work more efficiently, productivity will rise by 0.25 percent per month for the first two years. In other words, each month’s output of gauges will be 0.25 percent more than the previous month’s. The interest rate is 1.5 percent per month. All gauges are sold in the month in which they are produced, and receipts from sales are at the end of each month. What is the present worth of the sales of the engine pressure gauges in the first two years?

We first compute the growth-adjusted interest rate,  $i^o$ :

$$i^o = \frac{1+i}{1+g} - 1 = \frac{1.015}{1.0025} - 1 = 0.01247$$

$$i^o \approx 1.25\%$$

We then make use of the geometric gradient to present worth conversion factor with the uniform cash flow  $A = \$8000$ , the growth rate  $g = 0.0025$ , the growth-adjusted interest rate  $i^o = 0.0125$ , and the number of periods  $N = 24$ .

$$P = A(P/A, g, i, N) = A \left( \frac{(P/A, i^o, N)}{1+g} \right) \quad \blacksquare = PV(i, N, -A)/(1+g)$$

$$P = 8000 \left( \frac{(P/A, 1.25\%, 24)}{1.0025} \right)$$

From the interest factor tables we get

$$P = 8000 \left( \frac{20.624}{1.0025} \right)$$

$$P = 164\,580$$

The present worth of sales of engine compression gauges over the two-year period would be about \$165 000. Recall that we worked with an *approximate* growth-adjusted interest rate of 1.25 percent when the correct rate was a bit less than 1.25 percent. This means that \$164 580 is a slight understatement of the present worth. ■

### EXAMPLE

### 3.8

Emery's company, Dry-All, produces control systems for drying grain. Proprietary technology has allowed Dry-All to maintain steady growth in the U.S. market in spite of numerous competitors. Company dividends, all paid to Emery, are expected to rise at a rate of 10 percent per year over the next 10 years. Dividends at the end of this year are expected to total \$110 000. If all dividends are invested at 10 percent interest, how much will Emery accumulate in 10 years?

If we calculate the growth-adjusted interest rate, we get

$$i^o = \frac{1.1}{1.1} - 1 = 0$$

and it is natural to think that the present worth is simply the first year's dividends multiplied by 10. However, recall that in the case where  $g = i$ , the present worth is given by

$$P = N \left( \frac{A}{1+g} \right) = 10 \left( \frac{110\,000}{1.1} \right) = 1\,000\,000$$

Intuitively, dividing by  $(1+g)$  compensates for the fact that growth is considered to start after the end of the first period, but the interest rate applies to all periods. We want the future worth of this amount after 10 years:

$$F = 1\,000\,000(F/P, 10\%, 10) = 1\,000\,000(2.5937) = 2\,593\,700$$

Emery will accumulate \$2 593 700 in dividends and interest. ■

## 3.8 Non-Standard Annuities and Gradients

As discussed in Section 3.3, the standard assumption for annuities and gradients is that the payment period and compounding period are the same. If they are not, the formulas given in this chapter cannot be applied directly. There are three methods for dealing with this situation:

1. Treat each cash flow in the annuity or gradient individually. This is most useful when the annuity or gradient series is not large.
2. Convert the non-standard annuity or gradient to standard form by changing the compounding period.
3. Convert the non-standard annuity to standard form by finding an equivalent standard annuity for the compounding period. This method cannot be used for gradients.

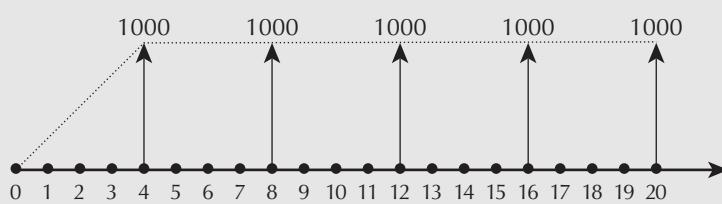
### EXAMPLE

### 3.9

How much is accumulated over 20 years in a fund that pays 4 percent interest, compounded yearly, if \$1000 is deposited at the end of every fourth year?

The cash flow diagram for this set of payments is shown in Figure 3.9.

**Figure 3.9 Non-Standard Annuity for Example 3.9**



*Method 1:* Consider the annuities as separate future payments.

$$\text{Formula: } F = P(F/P, i, N)$$

Known values:  $P = \$1000, i = 0.04, N = 16, 12, 8, 4, \text{ and } 0$

| Year | Future Value         | = |                       |
|------|----------------------|---|-----------------------|
| 4    | $1000(F/P, 4\%, 16)$ | = | $1000(1.8729) = 1873$ |
| 8    | $1000(F/P, 4\%, 12)$ | = | $1000(1.6010) = 1601$ |
| 12   | $1000(F/P, 4\%, 8)$  | = | $1000(1.3685) = 1369$ |
| 16   | $1000(F/P, 4\%, 4)$  | = | $1000(1.1698) = 1170$ |
| 20   | 1000                 | = | <u>1000</u>           |
|      | Total future value   | = | 7013                  |

About \$7013 is accumulated over the 20 years.

*Method 2:* Convert the compounding period from yearly to every four years. This can be done with the effective interest rate formula.

$$\begin{aligned} i_e &= (1 + 0.04)^4 - 1 \\ &= 16.99\% \end{aligned}$$

The future value is then

$$F = 1000(F/A, 16.99\%, 5) = 1000(7.013) \quad \blacksquare = FV(0.1699, 5, -1000) \\ = 7013$$

*Method 3:* Convert the annuity to an equivalent yearly annuity. This can be done by considering the first payment as a future value over the first four-year period and finding the equivalent annuity over that period using the sinking fund factor:

$$A = 1000(A/F, 4\%, 4) \quad \blacksquare = PMT(0.04, 4, -1000) \\ = 1000(0.23549) \\ = 235.49$$

In other words, a \$1000 deposit at the end of the four years is equivalent to four equal deposits of \$235.49 at the end of each of the four years. This yearly annuity is accumulated over the 20 years.

$$F = 235.49(F/A, 4\%, 20) \quad \blacksquare = FV(0.04, 20, -235.49) \\ = 235.49(29.777) \\ = 7012$$

Note that each method produces the same amount, allowing for rounding. When you have a choice in methods as in this example, your choice will depend on what you find convenient or what is the most efficient computationally. ■

### EXAMPLE

### 3.10

This year's electrical engineering class has decided to save up for a class party. Each of the 90 people in the class is to contribute \$0.25 per day, which will be placed in a daily interest (7 days a week, 365 days a year) savings account that pays a nominal 8 percent interest. Contributions will be made *five* days a week, Monday through Friday, beginning on Monday. The money is put into the account at the beginning of each day, and thus earns interest for the day. The class party is in 14 weeks (a full 14 weeks of payments will be made), and the money will be withdrawn on the Monday morning of the 15th week. How much will be saved, assuming everybody makes payments on time?

There are several good ways to solve this problem. One way is to convert each day's contribution to a weekly amount on Sunday evening/Monday morning, and then accumulate the weekly amounts over the 14 weeks:

Total contribution per day is  $0.25 \times 90 = 22.50$

The interest rate per day is  $\frac{0.08}{365} = 0.000219$

The effective interest rate for a one-week period is

$$i = (1 + 0.08/365)^7 - 1 = 0.001535$$

Value of one week's contribution on Friday evening (*annuity due formula*):

$$22.50 \times (F/P, 0.08/365, 1) \times (F/A, 0.08/365, 5)$$

On Sunday evening, this is worth

$$\begin{aligned}[22.50(F/P, 0.08/365, 1)(F/A, 0.08/365, 5)] &\times (F/P, 0.08/365, 2) \\&= 22.50(F/P, 0.08/365, 3)(F/A, 0.08/365, 5)\end{aligned}$$

Then the total amount accumulated by Monday morning of the 15th week is given by:

$$\begin{aligned}[22.50(F/P, 0.08/365, 3)(F/A, 0.08/365, 5)](F/A, 0.1535, 14) \\&= [22.50(1.000658)(5.00219)](14.1406) \\&= 1592.56\end{aligned}$$

The total amount saved would be \$1592.56. ■

### 3.9

### Present Worth Computations When $N \rightarrow \infty$

We have until now assumed that the cash flows of a project occur over some fixed, finite number of periods. For long-lived projects, it may be reasonable to model the cash flows as though they will continue indefinitely. The present worth of an infinitely long uniform series of cash flows is called the **capitalized value** of the series. We can get the capitalized value of a series by allowing the number of periods,  $N$ , in the series present worth factor to go to infinity:

$$\begin{aligned}P &= \lim_{N \rightarrow \infty} A(P/A, i, N) \\&= A \lim_{N \rightarrow \infty} \left[ \frac{(1 + i)^N - 1}{i(1 + i)^N} \right] \\&= A \lim_{N \rightarrow \infty} \left[ \frac{1 - \frac{1}{(1 + i)^N}}{i} \right] \\&= \frac{A}{i}\end{aligned}$$

#### EXAMPLE

#### 3.11

The town of South Battleford is considering building a bypass for truck traffic around the downtown commercial area. The bypass will provide merchants and shoppers with benefits that have an estimated value of \$500 000 per year. Maintenance costs will be \$125 000 per year. If the bypass is properly maintained, it will provide benefits for a very long time. The actual life of the bypass will depend on factors such as future economic conditions that cannot be forecast at the time the bypass is being considered. It is therefore reasonable to model the flow of benefits as though they will continue indefinitely. If the interest rate is 10 percent, what is the present worth of benefits minus maintenance costs?

$$P = \frac{A}{i} = \frac{500\,000 - 125\,000}{0.1} = 3\,750\,000$$

The present worth of benefits net of maintenance costs is \$3 750 000. ■

## REVIEW PROBLEMS

### REVIEW PROBLEM 3.1

The benefits of a revised production schedule for a seasonal manufacturer will not be realized until the peak summer months. Net savings will be \$1100, \$1200, \$1300, \$1400, and \$1500 at the ends of months 5, 6, 7, 8, and 9, respectively. It is now the beginning of month 1. Assume 365 days per year, 30 days per month. What is the present worth (PW) of the savings if nominal interest is

- (a) 12 percent per year, compounded monthly?
- (b) 12 percent per year, compounded daily?

#### ANSWER

$$(a) A = \$1100$$

$$G = \$100$$

$$i = 0.12/12 = 0.01 \text{ per month} = 1\%$$

$$\begin{aligned} \text{PW(end of period 4)} &= (P/A, 1\%, 5)[1100 + 100(A/G, 1\%, 5)] \\ &= 4.8528[1100 + 100(1.9801)] \\ &= 6298.98 \end{aligned}$$

$$\begin{aligned} \text{PW(at time 0)} &= \text{PW(end of period 4)}(P/F, 1\%, 4) \\ &= 6298.98/(1.01)^4 = 6053.20 \end{aligned}$$

The present worth is about \$6053.

$$\begin{aligned} (b) \text{ Effective interest rate } i &= (1 + 0.12/365)^{30} - 1 = 0.0099102 \\ \text{PW(at time 0)} &= \text{PW(end of period 4)}(P/F, i, 4) \\ &= (P/A, i, 5)[1100 + 100(A/G, i, 5)](P/F, i, 4) \\ &= 4.8547[1100 + 100(1.98023)](0.9613) \\ &= 6057.80 \end{aligned}$$

The present worth is about \$6058. ■

### REVIEW PROBLEM 3.2

It is January 1 of this year. You are starting your new job tomorrow, having just finished your engineering degree at the end of last term. Your take-home pay for this year will be \$66 000. It will be paid to you in equal amounts at the end of each month, starting at the end of January. There is a cost-of-living clause in your contract that says that each subsequent January you will get an increase of 3 percent in your yearly salary (i.e., your take-home pay for next year will be  $1.03 \times \$66\ 000$ ). In addition to your salary, a wealthy relative regularly sends you a \$2000 birthday present at the end of each June.

Recognizing that you are not likely to have any government pension, you have decided to start saving 10 percent of your monthly salary and 50 percent of your birthday present for your retirement. Interest is 1 percent per month, compounded monthly. How much will you have saved at the end of five years?

#### ANSWER

Yearly pay is a geometric gradient; convert your monthly salary into a yearly amount by the use of an effective yearly rate. The birthday present can be dealt with separately.

**Salary:**

The future worth (FW) of the salary at the end of the first year is

$$FW(\text{salary, year 1}) = 5500(F/A, 1 \text{ percent}, 12) = 69\,756.00$$

This forms the base of the geometric gradient; all subsequent years increase by 3 percent per year. Savings are 10 percent of salary, which implies that  $A = \$6976.00$ .

$$A = \$6976.00 \quad g = 0.03$$

Effective yearly interest rate  $i_e = (1 + 0.01)^{12} - 1 = 0.1268$  per year

$$i^o = \frac{1 + i_e}{1 + g} - 1 = \frac{1 + 0.1268}{1 + 0.03} - 1 = 0.093981$$

$$\begin{aligned} PW(\text{gradient}) &= A(P/A, i^o, 5)/(1 + g) = 6976(3.8498)/1.03 \\ &= 26\,074 \end{aligned}$$

$$\begin{aligned} FW(\text{gradient, end of five years}) &= PW(\text{gradient})(F/P, i_e, 5) \\ &= 26\,074(1.1268)^5 = 43\,363 \end{aligned}$$

**Birthday Present:**

The present arrives in the middle of each year. To get the total value of the five gifts, we can find the present worth of an annuity of five payments of \$2000(0.5) as of six months prior to employment:

$$PW(-6 \text{ months}) = 2000(0.5)(P/A, i_e, 5) = 3544.6$$

The future worth at  $5 \times 12 + 6 = 66$  months later is

$$FW(\text{end of five years}) = 35544.6(1.01)^{66} = 6836$$

$$\text{Total amount saved} = \$6836 + \$43\,363 = \$54\,199 \blacksquare$$

**REVIEW PROBLEM 3.3**

The Easy Loan Company advertises a “10 percent” loan. You need to borrow \$1000, and the deal you are offered is the following: You pay \$1100 (\$1000 plus \$100 interest) in 11 equal \$100 amounts, starting one month from today. In addition, there is a \$25 administration fee for the loan, payable immediately, and a processing fee of \$10 per payment. Furthermore, there is a \$20 non-optional closing fee to be included in the last payment. Recognizing fees as a form of interest payment, what is the actual effective interest rate?

**ANSWER**

Since the \$25 administration fee is paid immediately, you are only getting \$975. The remaining payments amount to an annuity of \$110 per month, plus a \$20 future payment 11 months from now.

$$\text{Formulas: } P = A(P/A, i, N), P = F(P/F, i, N)$$

$$\text{Known values: } P = \$975, A = \$110, F = \$20, N = 11$$

$$975 = 110(P/A, i, 11) + 20(P/F, i, 11) \quad \blacksquare = PV(i, 11, -110) + PV(i, 11, -20)$$

At  $i = 4$  percent

$$\begin{aligned} 110(P/A, 4\%, 11) + 20(P/F, 4\%, 11) &= PV(0.04, 11, -110) + PV(0.04, 11, -20) \\ &= 110(8.7603) + 20(0.64958) \\ &= 976.62 \end{aligned}$$

At  $i = 5$  percent

$$\begin{aligned} 110(P/A, 5\%, 11) + 20(P/F, 5\%, 11) &= PV(0.05, 11, -110) + PV(0.05, 11, -20) \\ &= 110(8.3062) + 20(0.58469) \\ &= 925.37 \end{aligned}$$

Linearly interpolating gives

$$\begin{aligned} i &= 4 + (5 - 4)(975 - 976.62)/(925.37 - 976.62) \\ &= 4.03 \end{aligned}$$

The effective interest rate is then

$$\begin{aligned} i &= (1 + 0.0403)^{12} - 1 \\ &= 60.69\% \text{ per annum (!)} \end{aligned}$$

Although the loan is advertised as a “10 percent” loan, the actual effective rate is over 60 percent. ■

#### REVIEW PROBLEM 3.4

Ming wants to retire as soon as she has enough money invested in a special bank account (paying 14 percent interest, compounded annually) to provide her with an annual income of \$25 000. She is able to save \$10 000 per year, and the account now holds \$5000. If she just turned 20, and expects to die in 50 years, how old will she be when she retires? There should be no money left when she turns 70.

#### ANSWER

Let Ming’s retirement age be  $20 + x$  so that

$$5000(F/P, 14\%, x) + 10\,000(F/A, 14\%, x) = 25\,000(P/A, 14\%, 50 - x)$$

Dividing both sides by 5000,

$$(F/P, 14\%, x) + 2(F/A, 14\%, x) - 5(P/A, 14\%, 50 - x) = 0$$

At  $x = 5$

$$\begin{aligned} (F/P, 14\%, 5) + 2(F/A, 14\%, 5) - 5(P/A, 14\%, 45) \\ = 1.9254 + 2(6.6101) - 5(7.1232) = -20.4704 \end{aligned}$$

At  $x = 10$

$$\begin{aligned} (F/P, 14\%, 10) + 2(F/A, 14\%, 10) - 5(P/A, 14\%, 40) \\ = 3.7072 + 2(19.337) - 5(7.1050) = 6.8562 \end{aligned}$$

Linearly interpolating,

$$\begin{aligned} x &= 5 + 5 \times (20.4704)/(6.8562 + 20.4704) \\ &= 8.7 \end{aligned}$$

Ming can retire at age  $20 + 8.7 = 28.7$  years old. ■

## S U M M A R Y

In Chapter 3 we considered ways of modelling patterns of cash flows that enable easy comparisons of the worths of projects. The emphasis was on discrete models. Four basic patterns of discrete cash flows were considered:

1. Flows at a single point
2. Flows that are constant over time
3. Flows that grow or decrease at a constant arithmetic rate
4. Flows that grow or decrease at a constant geometric rate

Compound interest factors were presented that defined mathematical equivalence among the basic patterns of cash flows. A list of these factors with their names, symbols, and formulas appears in Table 3.2. Table 3.3 summarizes the Excel equivalents to the compound interest factors. The chapter also addressed the issue of how to analyze non-standard annuities and gradients as well as the ideas of capital recovery and capitalized value.

**Table 3.2 Summary of Useful Formulas for Discrete Models**

| Name  | Symbol and Formula  |
|---|---|
| Compound amount factor                                | $(F/P,i,N) = (1 + i)^N$   |
| Present worth factor                                  | $(P/F,i,N) = \frac{1}{(1 + i)^N}$   |
| Sinking fund factor                                   | $(A/F,i,N) = \frac{i}{(1 + i)^N - 1}$   |
| Uniform series compound amount factor                 | $(F/A,i,N) = \frac{(1 + i)^N - 1}{i}$   |
| Capital recovery factor                               | $(A/P,i,N) = \frac{i(1 + i)^N}{(1 + i)^N - 1}$  |
| Series present worth factor                           | $(P/A,i,N) = \frac{(1 + i)^N - 1}{i(1 + i)^N}$  |
| Arithmetic gradient to annuity conversion factor      | $(A/G,i,N) = \frac{1}{i} - \frac{N}{(1 + i)^N - 1}$   |
| Geometric gradient to present worth conversion factor | $(P/A,g,i,N) = \frac{(P/A,i^o,N)}{1 + g}$<br>$(P/A,g,i,N) = \left( \frac{(1 + i^o)^N - 1}{i^o(1 + i^o)^N} \right) \frac{1}{1 + g}$<br>$i^o = \frac{1 + i}{1 + g} - 1$ |
| Capitalized value formula                             | $P = \frac{A}{i}$   |
| Capital recovery formula                              | $A = (P - S)(A/P,i,N) + Si$   |

**Table 3.3 Compound Interest Factors and Equivalent Excel Computations**

| Name  | Use of Compound Interest Factor | Equivalent Excel Computation     |
|---|---------------------------------|----------------------------------|
| Compound amount factor                                | $F = P(F/P,i,N)$                | $F = FV(i,N, , -P)$              |
| Present worth factor                                  | $P = F(P/F,i,N)$                | $P = PV(i,N, , -F)$              |
| Sinking fund factor                                   | $A = F(A/F,i,N)$                | $A = PMT(i,N, , -F)$             |
| Uniform series compound amount factor                 | $F = A(F/A,i,N)$                | $F = FV(i,N,-A)$                 |
| Capital recovery factor                               | $A = P(A/P,i,N)$                | $A = PMT(i,N,-P)$                |
| Series present worth factor                           | $P = A(P/A,i,N)$                | $P = PV(i,N,-A)$                 |
| Arithmetic gradient to annuity conversion factor      | $A_{tot} = A' + G(A/G,I,N)$     | No equivalent Excel function     |
| Geometric gradient to present worth conversion factor | $P = A(P/A,g,i,N)$              | $P = PV(i^{\circ},N,-A)/(1 + g)$ |
| Capital recovery formula                              | $A = (P - S)(A/P,i,N) + Si$     | $A = PMT(i,N,-(P - S)) + Si$     |

## ENGINEERING ECONOMICS IN ACTION, PART 3B

### No Free Lunch

This time it was Naomi who stuck her head in Clem's doorway. "Here's the recommendation on the shipping palletizer. Oh, and thanks for the hint on the leasing figures. It cleared up my confusion right away."

"No problem. What did you figure out?" Clem had his "mentor" expression on his face, so Naomi knew he was expecting a clear explanation of the trick used by the leasing company.

"Well, as you hinted, they were adding apples and oranges. They listed the various costs for each choice over time, including interest charges, taxes, and so on. But then, for the final comparison, they added up these costs. When they were added, leasing was cheaper."

"So what's wrong with that?" Clem prompted.

"They're adding apples and oranges. We're used to thinking of money as being just money, without remembering that money always has a 'when' associated with it. If you add money at different points in time, you might as well be adding apples and oranges: you have a number but it doesn't mean anything. In order to compare leasing with buying, you first have to change the cash flows into the same money—that is, at the same point in time. That's a little harder to do, especially when there's a complicated set of cash flows."

"So were you able to do it?"

"Yes. I identified various components of the cash flows as annuities, gradients, and present and future worths. Then I converted all of these to a present worth for each alternative and summed them. This is the correct way to compare them. If you do that, buying is cheaper, even when borrowing money to do so. And of course it has to be—that leasing company has to pay for those slick brochures somehow. There's no free lunch."

Clem nodded. "I think you've covered it. Mind you, there are some circumstances where leasing is worthwhile. For example, we lease our company cars to save us the time and trouble of reselling them when we're finished with them. Leasing can be good when it's hard to raise the capital for very large purchases, too. But almost always, buying is better. And you know, it amazes me how easy it is to fall for simplistic cash flow calculations that fail to take into account the time value of money. I've even seen articles in the newspaper quoting accountants who make the same mistake, and you'd think they would know better."

"Engineers can make that mistake, too, Clem. I almost did."

## P R O B L E M S

### A. Key Concepts

- 3.1** St. Agatha Kennels provides dog breeding and boarding services for a nearby city. Most of the income is derived from boarding, with typical boarding stays being one or two weeks. Customers pay at the end of the dog's stay. Boarding is offered only during the months of May to September. Other income is received from breeding golden retrievers, with two litters of about eight dogs each being produced per year in the spring and fall. Expenses include heating, water, and sewage, which are paid monthly, and food, bought in bulk every spring. The business has been neither growing nor shrinking over the past few years.

Joan, the owner of the kennels, wants to model the cash flows for the business over the next 10 years. What cash flow elements (e.g., single payments, annuities, gradients) would she likely consider, and how would she estimate their value? Consider the present to be the first of May. For example, one cash flow element is food. It would be modelled as an *annuity due* over 10 years, and estimated by the amount paid for food over the past few years.

- 3.2** It is September, the beginning of his school year, and Marco has to watch his expenses while he is going to school. Over the next eight months, he wants to estimate his cash flows. He pays rent once a month. He takes the bus to and from school. A couple of times a week he goes to the grocery store for food, and he eats lunch in the cafeteria at school every school day. At the end of every four-month term, he will have printing and copying expenses because of reports that will be due. Over the Christmas holidays after the first term, he will have extra expenses for buying presents, but will also get some extra cash from his parents. What cash flow elements (e.g., single payments, annuities, gradients) should Marco likely consider in his estimates? How should he estimate them?
- 3.3** How much money will be in a bank account at the end of 15 years if \$100 is deposited today and the interest rate is 8 percent compounded annually?
- 3.4** How much should you invest today at 12 percent interest to accumulate \$1 000 000 in 30 years?
- 3.5** You have \$1725 to invest. You know that a particular investment will double your money in five years. How much will you have in 10 years if you invest in this investment, assuming that the annual rate of return is guaranteed for the time period?
- 3.6** Morris paid £500 a month for 20 years to pay off the mortgage on his Glasgow house. If his down payment was £5000 and the interest rate was 6 percent compounded monthly, what was the purchase price of the house?
- 3.7** An industrial juicer costs \$45 000. It will be used for five years and then sold to a remarketer for \$25 000. If interest is 15 percent, what net yearly savings are needed to justify its purchase?
- 3.8** Fred wants to save up for an automobile. What amount must he put in his bank account each month to save \$10 000 in two years if the bank pays 6 percent interest compounded monthly?
- 3.9** How much is accumulated over two years in each of the following savings plans?
- \$40 at the end of each month for 24 months at 12 percent compounded monthly
  - \$30 at the end of the first month, \$31 at the end of the second month, and so forth, increasing by \$1 per month, at 12 percent compounded monthly

- 3.10** A UK lottery prize pays £1000 at the end of the first year, £2000 the second, £3000 the third, and so on for 20 years. If there is only one prize in the lottery, 10 000 tickets are sold, and you could invest your money elsewhere at 15 percent interest, how much is each ticket worth, on average?
- 3.11** Reginald is expecting steady growth of 10 percent per year in profits from his new company. All profits are going to be invested at 20 percent interest. If profits for this year (at the end of the year) total \$10 000, how much will be saved at the end of 10 years?
- 3.12** Reginald is expecting steady growth in profits of 20 percent per year from his new company. All profits are going to be invested at 10 percent interest. If profits for this year (at the end of the year) total \$10 000, how much will be saved at the end of 10 years?
- 3.13** An investment pays \$10 000 every five years, starting in seven years, for a total of four payments. If interest is 9 percent, how much is this investment worth today?
- 3.14** It is May 1. You have just bought \$2000 worth of furniture. You will pay for it in 24 equal monthly payments, starting at the end of May next year. Interest is 6 percent nominal per year, compounded monthly. How much will your payments be?
- 3.15** What is the present worth of the total of 20 payments, occurring at the end of every four months (the first payment is in four months), which are \$400, \$500, \$600, increasing arithmetically? Interest is 12 percent nominal per year, compounded monthly.
- 3.16** City engineers are considering several plans for building municipal aqueduct tunnels. They use an interest rate of 8 percent. One plan calls for a full-capacity tunnel that will meet the needs of the city forever. The cost is \$3 000 000 now and \$100 000 every 10 years thereafter for repairs. What is the total present worth of the costs of building and maintaining the aqueduct?
- 3.17** Goderich Automotive (GA) wants to donate a vacant lot next door to its plant to the city for use as a public park and ball field. The city will accept only if GA will also donate enough cash to maintain the park indefinitely. The estimated maintenance costs are \$18 000 per year and interest is 7 percent. How much cash must GA donate?
- 3.18** A 7 percent, 20-year municipal bond has a \$10 000 face value. I want to receive at least 10 percent compounded semiannually on this investment. How much should I pay for the bond?
- 3.19** If money is worth 8 percent compounded semiannually, how much is a bond maturing in nine years, with a face value of \$10 000 and a coupon rate of 9 percent, worth today?
- 3.20** Trenny has asked her assistant to prepare estimates of cost of two different-sized power plants. The assistant reports that the cost of the 100 MW plant is \$200 000 000, while the cost of the 200 MW plant is \$360 000 000. If Trenny has a budget of only \$300 000 000, estimate how large a power plant she could afford using linear interpolation.
- 3.21** What interest rate will result in \$5000 seven years from now, starting with \$2300 today?
- 3.22** Gwen just bought solar panels to power ventilation at her chicken farm. The panels cost \$2000 and will reduce her electricity bills by \$40 per month. How long will it take her to recoup her investment in the panels if she can earn 12 percent interest, compounded monthly, on her money?

## B. Applications

- 3.23** Martin and Marcy McCormack have just become proud parents of septuplets. They have savings of \$5000. They want to invest their savings so that they can partially support the children's university education. Martin and Marcy hope to provide \$20 000

for each child by the time the children turn 18. What must the annual rate of return on the investment be for Martin and Marcy to meet their goal?

- 3.24** Refer back to the Kelowna Go-Kart Klub problem of Example 3.2 on page 53. The members determined that it is possible to set aside only \$7000 each year, and that they will have to put off building the clubhouse until they have saved the \$50 000 necessary. How long will it take to save a total of \$50 000, assuming that the interest rate is 10 percent? (*Hint:* Use logarithms to simplify the sinking fund factor.)
- 3.25** Yoko has just bought a new computer (\$2000), a printer (\$350), and a scanner (\$210). She wants to take the monthly payment option. There is a monthly interest of 3 percent on her purchase.
- If Yoko pays \$100 per month, how long does it take to complete her payments?
  - If Yoko wants to finish paying in 24 months, how much will her monthly payment be?
- 3.26** Rinku has just finished her first year of university. She wants to tour Europe when she graduates in three years. By having a part-time job through the school year and a summer job during the summer, she plans to make regular weekly deposits into a savings account, which bears 18 percent interest, compounded monthly.
- If Rinku deposits \$15 per week, how much will she save in three years? How about \$20 per week?
  - Find out exactly how much Rinku needs to deposit every week if she wants to save \$5000 in three years.
- 3.27** Seema is looking at an investment in upgrading an inspection line at her plant. The initial cost would be \$140 000 with a salvage value of \$37 000 after five years. Use the capital recovery formula to determine how much money must be saved every year to justify the investment at an interest rate of 14 percent.
- 3.28** Enrique has determined that investing \$500 per month will enable him to accumulate \$11 350 in 12 years, and that investing \$800 per month will enable him to accumulate \$18 950 over the same period. Estimate, using linear interpolation, how much he would have to invest each month to accumulate exactly \$15 000.
- 3.29** Joseph and three other friends bought a \$260 000 house close to the university at the end of August last year. At that time they put down a deposit of \$10 000 and took out a mortgage for the balance. Their mortgage payments are due at the end of each month (September 30, last year, was the date of the first payment) and are based on the assumption that Joseph and friends will take 20 years to pay off the debt. Annual nominal interest is 12 percent, compounded monthly. It is now February. Joseph and friends have made all their fall-term payments and have just made the January 31 payment for this year. How much do they still owe?
- 3.30** A new software package is expected to improve productivity at Grand Insurance. However, because of training and implementation costs, savings are not expected to occur until the third year of operation. At that time, savings of \$10 000 are expected, increasing by \$1000 per year for the following five years. After this time (eight years from implementation), the software will be abandoned with no scrap value. How much is the software worth today, at 15 percent interest?
- 3.31** Clem is saving for a car in a bank account that pays 12 percent interest, compounded monthly. The balance is now \$2400. Clem will be saving \$120 per month from his

salary, and once every four months (starting in four months) he adds \$200 in dividends from an investment. Bank fees, currently \$10 per month, are expected to increase by \$1 per month henceforth. How much will Clem have saved in two years?

- 3.32** Yogajothi is thinking of investing in a rental house. The total cost to purchase the house, including legal fees and taxes, is \$230 000. All but \$30 000 of this amount will be mortgaged. He will pay \$1600 per month in mortgage payments. At the end of two years, he will sell the house and at that time expects to clear \$40 000 after paying off the remaining mortgage principal (in other words, he will pay off all his debts for the house and still have \$40 000 left). Rents will earn him \$2000 per month for the first year and \$2400 per month for the second year. The house is in fairly good condition now, so he doesn't expect to have any maintenance costs for the first six months. For the seventh month, Yogajothi has budgeted \$400. This figure will be increased by \$40 per month thereafter (e.g., the expected month 7 expense will be \$400, month 8, \$440, month 9, \$480, etc.). If interest is 6 percent compounded monthly, what is the present worth of this investment? Given that Yogajothi's estimates of revenue and expenses are correct, should he buy the house?
- 3.33** A new wave-soldering machine is expected to save Burnaby Circuit Boards \$15 000 per year through reduced labour costs and increased quality. The device will have a life of eight years and will have no salvage value after this time. If the company can generally expect to get 12 percent return on its capital, how much could it afford to pay for the wave-soldering machine?
- 3.34** Gail has won a lottery that pays her \$100 000 at the end of this year, \$110 000 at the end of next year, \$120 000 the following year, and so on, for 30 years. Leon has offered Gail \$2 500 000 today in exchange for all the money she will receive. If Gail can get 8 percent interest on her savings, is this a good deal?
- 3.35** Gail has won a lottery that pays her \$100 000 at the end of this year and increases by 10 percent per year thereafter for 30 years. Leon has offered Gail \$2 500 000 today in exchange for all the money she will receive. If Gail can get 8 percent interest on her savings, is this a good deal?
- 3.36** Tina has saved \$20 000 from her summer jobs. Rather than work for a living, she plans to buy an annuity from a trust company and become a beachcomber in Fiji. An annuity will pay her a certain amount each month for the rest of her life and is calculated at 7 percent interest, compounded monthly, over Tina's 55 remaining years. Tina calculates that she needs at least \$5 per day to live in Fiji, and she needs \$1200 for air fare. Can she retire now? How much would she have available to spend each day?
- 3.37** A regional municipality is studying a water supply plan for its tri-city and surrounding area to the end of year 2060. To satisfy the water demand, one suggestion is to construct a pipeline from a major lake some distance away. Construction would start in 2020 and take five years at a cost of \$20 million per year. The cost of maintenance and repairs starts after completion of construction and for the first year is \$2 million, increasing by 1 percent per year thereafter. At an interest rate of 6 percent, what is the present worth of this project?
- Assume that all cash flows take place at year-end. Consider the present to be the end of 2015/beginning of 2016. Assume that there is no salvage value at the end of year 2060.
- 3.38** A French software genius has been offered €10 000 per year for the next five years and then €20 000 per year for the following 10 years for the rights to his new smart phone app. At 9 percent interest, how much is this worth today?
- 3.39** A bank offers a personal loan called "The Eight Percent Plan." The bank adds 8 percent to the amount borrowed; the borrower pays back 1/12 of this total at the end of each month for

a year. On a loan of \$500, the monthly payment is  $540/12 = \$45$ . There is also an administrative fee of \$45, payable now. What is the actual effective interest rate on a \$500 loan?

- 3.40** Shamsir's small business has been growing slowly. He has noticed that his monthly profit increases by 1 percent every two months. Suppose that the profit at the end of this month is \$10 000. What is the present value of all his profit over the next two years? Annual nominal interest is 18 percent, compounded monthly.
- 3.41** A Paradorian bond pays \$500 (Paradorian dollars) twice each year and \$5000 five years from now. I want to earn at least 300 percent *annual* (effective) interest on this investment (to compensate for the very high inflation in Parador). How much should I pay for this bond now?
- 3.42** A bond with a face value of \$5000 pays quarterly interest of 1.5 percent each period. Twenty-six interest payments remain before the bond matures. How much would you be willing to pay for this bond today if the next interest payment is due now and you want to earn 8 percent compounded quarterly on your money?

### C. More Challenging Problems

- 3.43** You have been paying off a mortgage in quarterly payments at a 24 percent nominal annual rate, compounded quarterly. Your bank is now offering an alternative payment plan, so you have a choice of two methods—continuing to pay as before or switching to the new plan. Under the new plan, you would make monthly payments 30 percent of the size of your current payments. The interest rate would be 24 percent nominal, compounded monthly. The time until the end of the mortgage would not change, regardless of the method chosen.
- Which plan would you choose, given that you naturally wish to minimize the level of your payment costs? (*Hint:* Look at the costs over a three-month period.)
  - Under which plan would you be paying a higher effective yearly interest rate?
- 3.44** Derive the arithmetic gradient conversion to a uniform series formula. (*Hint:* Convert each period's gradient amount to its future value, and then look for a substitution from the other compound amount factors.)
- 3.45** Derive the geometric gradient to present worth conversion factor. (*Hint:* Divide and multiply the present worth of a geometric series by  $[1 + g]$  and then substitute in the growth-adjusted interest rate.)
- 3.46** Ruby's business has been growing quickly over the past few years, with sales increasing at about 50 percent per year. She has been approached by a buyer for the business. She has decided she will sell it for half of the value of the estimated sales for the next five years. This year she will sell products worth \$1 456 988. Use the geometric gradient factor to calculate her selling price for an interest rate of 5 percent.
- 3.47** In Example 3.4 on page 55, Clarence bought a \$94 000 condo with a \$14 000 down payment and took out a mortgage for the remaining \$80 000 at 12 percent nominal interest, compounded monthly. We determined that he would make 51 \$2000 payments and then a final payment. What is his final payment?
- 3.48** Clem has a \$50 000 loan. The interest rate offered is 8 percent compounded annually and the repayment period is 15 years. Payments are to be received in equal installments at the end of each year. Construct a spreadsheet (you must use a spreadsheet program) similar to the following table that shows the amount received each year, the portion that is interest, the portion that is unrecovered capital, and the amount that is outstanding

(i.e., unrecovered). Also, compute the total recovered capital, which must equal the original capital amount; this can serve as a check on your solution. Design the spreadsheet so that the capital amount and the interest rate can be changed by updating only one cell for each. Construct:

- (a) The completed spreadsheet for the amount, interest rate, and repayment period indicated
- (b) The same spreadsheet, but for \$75 000 at 10 percent interest (same repayment period)
- (c) A listing showing the formulas used

| Sample Capital Recovery Calculations |                |                   |                   |                     |
|--------------------------------------|----------------|-------------------|-------------------|---------------------|
| Capital amount                       |                |                   | \$50 000.00       |                     |
| Annual interest rate                 |                |                   | 8.00%             |                     |
| Number of years to repay             |                |                   | 15                |                     |
| Payment Periods                      | Annual Payment | Interest Received | Recovered Capital | Unrecovered Capital |
| 0                                    |                |                   |                   | \$50 000.00         |
| 1                                    | \$5841.48      | \$4000.00         | \$1841.48         | 48 158.52           |
| 2                                    |                |                   |                   |                     |
| .                                    |                |                   |                   |                     |
| .                                    |                |                   |                   |                     |
| .                                    |                |                   |                   |                     |
| 15                                   |                |                   |                   | 0.00                |
| Total                                |                |                   | \$50 000.00       |                     |

**3.49** Coastal Shipping is setting aside capital to fund an expansion project. Funds earmarked for the project will accumulate at the rate of \$50 000 per month until the project is completed in two years. Once the project starts, costs will be incurred at the rate of \$150 000 per month over 24 months. Coastal currently has \$250 000 saved. What is the minimum number of months it will have to wait before it can start if money is worth 18 percent nominal, compounded monthly? Assume that

1. Cash flows are all at the ends of months.
2. The first \$50 000 savings occurs one month from today.
3. The first \$150 000 payment occurs one month after the start of the project.
4. The project must start at the beginning of a month.

**3.50** A company is about to invest in a joint venture research and development project with another company. The project is expected to last eight years, but the company will begin its yearly payments immediately (i.e., a payment is made today, and the last payment will be made eight years from today). Salaries will account for \$40 000 of each payment. The remainder will cover equipment costs and facility overhead. The initial (immediate) equipment and facility cost is \$26 000. Each subsequent year, this figure will drop by

\$3000 until a cost of \$14 000 is reached, after which the costs will remain constant until the end of the project.

- (a) Draw a cash flow diagram to illustrate the cash flows for this situation.
  - (b) At an interest rate of 7 percent, what is the total future worth of all project payments at the end of the eight years?
- 3.51** Xiaohang is conducting a biochemical experiment for the next 12 months. In the first month, the expenses are estimated to be \$15 000. As the experiment progresses, the expenses are expected to increase by 5 percent each month. Xiaohang plans to pay for the experiment with a government grant, which is received in six monthly installments, starting a month after the experiment completion date. Determine the amount of the monthly installment so that the total of the six installments pays for all expenses incurred during the experiment. Annual nominal interest is 12 percent, compounded monthly.
- 3.52** The city of Sault Ste. Marie is installing a new swimming pool in the municipal recreation centre. One design being considered is a reinforced concrete pool, which will cost \$1 500 000 to install. Thereafter, the inner surface of the pool will need to be refinished and painted every 10 years at a cost of \$200 000 per refinishing. Assuming that the pool will have essentially an infinite life, what is the present worth of the costs associated with this pool design? The city uses a 5 percent interest rate.
- 3.53** What happens to the present worth of an arithmetic gradient series as the number of periods approaches infinity? Consider all four cases:
- (a)  $i > g > 0$
  - (b)  $g > i > 0$
  - (c)  $g = i = 0$
  - (d)  $g < 0$

### M I N I - C A S E 3 . 1

#### The Canadian Oil Sands

The Canadian Oil Sands are a group of bitumen deposits located in northeastern Alberta. Bitumen is a form of crude oil, and in oil sands (formerly called tar sands) the bitumen is mixed with sand, clay, and water, making it difficult to process into gasoline and other products. The Canadian deposits are huge—comparable in size to all other sources of conventional crude oil in the world combined.

Commercial development of the oil sands began in the 1960s led by familiar Canadian companies such as Suncor and Syncrude. In recent years, international oil companies including Royal Dutch Shell, Chevron, and ExxonMobil have been investing in various projects, as have the national oil companies of China, Japan, and Korea.

Extracting synthetic crude oil from oil sands bitumen for use in refineries is difficult and expensive. If the bitumen can be mined from the surface, the process essentially involves mixing it with hot water so that the oil rises to the top. If the bitumen is too deep underground to extract, steam is pumped down into the bitumen to separate the oil so that it can be more easily pumped out. Both methods require enormous amounts of energy to heat large quantities of water. Oil sands processing currently consumes about 4 percent of Canada's entire natural gas supply.

There are a large number of uncertainties with conventional oil production. Exploration is risky because many wells may need to be drilled before one is found that produces oil. Even when production starts, it is unknown how much oil the well will produce. Oil fields may be in politically or economically unstable countries in which an investment could be expropriated or damaged by war or terrorism. But Canada's oil sands are different. The existence and quantity of the oil is certain; it merely needs to be dug or pumped out. Canada is unlikely to expropriate investments or be involved in a war that might affect them. Even though the cost of production is higher than for conventional oil, the physical and political certainty arguably offsets that extra cost.

Consequently, one might expect that developing oil production projects in the Canadian oil sands would be a relatively reliable and easy process. However, projects in northern Alberta have been anything but straightforward. Some that were announced with great fanfare at their inception were cancelled a few years later—over \$90 billion in projects were cancelled in 2009. In other cases, costs for projects escalated to multiples of the initial estimates. Contrary to expectations, a resource that should provide very predictable profits has turned out to be remarkably unpredictable.

### Discussion

Oil industry projects tend to require anywhere from hundreds of millions to tens of billions of dollars in investments. As one might expect, very capable people think through all aspects of a project—technical, financial, environmental, political—very carefully. Oil companies take planning seriously, and their engineers are very experienced. The \$90 billion of cancelled projects in 2009 does not represent incompetence, but rather something more fundamental—no matter how careful the planning process is, one can never fully predict how the future will turn out.

Three key issues have strongly affected development of the Canadian oil sands. The first is that crude oil is a commodity, meaning that there are so many sources that no one can easily predict or control its price. Even if the cost of production is absolutely certain, the profitability and project viability depends very much on the world price, which itself is subject to many random forces. No one predicted that the price of oil would rise to a peak of US\$145 per barrel in 2008, or drop the same year to below US\$50. Both are extremes outside of the planning window imagined earlier in the same decade. For conventional oil production, costs can be controlled—to a degree. The cost of exploration efforts can be reduced, for example, when prices are low. But oil sands project costs are much more difficult to cut back on once they are started. There are very large capital costs to recover and high direct costs per barrel of oil produced. As such, investment decisions are more vulnerable to low oil prices.

The second confounding issue concerns social, environmental, and political forces. Mining oil sands is very dirty work that damages the physical sites and produces vast quantities of carbon dioxide. Engineers involved in early oil sands projects sometimes completely failed to manage their environmental impact and ended up with unexpected costs for administration, legal bills, remediation, and regulation adherence. Even in recent years, when such costs are taken into account in the planning process, they are somewhat unpredictable because of the inherent nature of social and political intervention. Otherwise benign activities can suddenly become difficult and expensive if they somehow gain media attention. This has happened on several occasions in the oil sands. For example, in 2011, the “Rethink Alberta” movement, formed by several environmental groups, gained international media attention for its efforts to dissuade tourists from visiting Alberta unless new oil sands developments were stopped.

A third problem is a confluence of global trends, which has been compounded by the oil sands' remote location. Between 2003 and 2008, for example, a key global force was the industrialization of China. China's industrialization led to an increase in demand for oil and gas worldwide, causing oil companies to increase supply. This in turn caused a global proliferation

of oil and gas developments, creating a tremendous demand for manufacturing capacity, technical expertise, and specialized equipment that significantly increased the costs of acquiring the resources necessary to develop the oil sands. In particular, due to the isolated location of the sands and a shortage of workers, the cost of labour in Alberta skyrocketed to nearly double the previously estimated amount, far beyond even the most pessimistic expectations.

Even with the best planning and the most reliable process and product, the future remains unpredictable. Economic analyses still need to be done, however. There are sophisticated ways to deal with uncertainty about future cash flows, some of which are discussed in Chapter 12. In most cases, it makes sense to carry out economic analyses with a range of possible values for future cash flows. But, as the oil sands example shows, there are situations when even this approach is unable to account for what the future actually holds.

### Questions

1. For each of the following, comment on how sensible it is to estimate the precise value of future cash flows:
  - (a) Your rent for the next six months
  - (b) Your food bill for the next six months
  - (c) Your medical bills for the next six months
  - (d) A company's payroll for the next six months
  - (e) A company's raw material costs for the next six months
  - (f) A company's legal costs for liability lawsuits for the next six months
  - (g) Canada's costs for funding university research for the next six months
  - (h) Canada's costs for employment insurance for the next six months
  - (i) Canada's costs for emergency management for the next six months
2. Your company is looking at the possibility of buying a new widget grinder for the widget line. The future cash flows associated with the purchase of the grinder are fairly predictable, except for one factor. A significant benefit is achieved with the higher production volume of widgets, which depends on a contract to be signed with a particular important customer. This won't happen for several months, but you must make the decision about the widget grinder now. Discuss some sensible ways of dealing with this issue.

## Appendix 3A Derivation of Discrete Compound Interest Factors

This appendix derives six discrete compound interest factors presented in this chapter. All of them can be derived from the compound interest equation

$$F = P(1 + i)^N$$

### 3A.1 | Compound Amount Factor

In the symbolic convention used for compound interest factors, the compound interest equation can be written

$$F = P(1 + i)^N = P(F/P,i,N)$$

so that the compound amount factor is

$$(F/P,i,N) = (1 + i)^N \quad (3A.1)$$

## 3A.2 | Present Worth Factor

---

The present worth factor,  $(P/F,i,N)$ , converts a future amount  $F$  to a present amount  $P$ :

$$\begin{aligned} P &= F(P/F,i,N) \\ \Rightarrow F &= P\left(\frac{1}{(P/F,i,N)}\right) \end{aligned}$$

Thus the present worth factor is the reciprocal of the compound amount factor. From Equation (3A.1),

$$(P/F,i,N) = \frac{1}{(1+i)^N}$$

## 3A.3 | Sinking Fund Factor

---

If a series of payments  $A$  follows the pattern of a standard annuity of  $N$  payments in length, then the future value of the payment in the  $j^{\text{th}}$  period, from Equation (3A.1), is:

$$F = A(1+i)^{N-j}$$

The future value of all of the annuity payments is then

$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots + A(1+i)^1 + A$$

Factoring out the annuity amount gives

$$F = A[(1+i)^{N-1} + (1+i)^{N-2} + \dots + (1+i)^1 + 1] \quad (3A.2)$$

Multiplying Equation (3A.2) by  $(1+i)$  gives

$$F(1+i) = A[(1+i)^{N-1} + (1+i)^{N-2} + \dots + (1+i)^1 + 1](1+i)$$

$$F(1+i) = A[(1+i)^N + (1+i)^{N-1} + \dots + (1+i)^2 + (1+i)] \quad (3A.3)$$

Subtracting Equation (3A.2) from Equation (3A.3) gives

$$F(1+i) - F = A[(1+i)^N - 1]$$

$$Fi = A[(1+i)^N - 1]$$

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

Thus the sinking fund factor is given by

$$(A/F,i,N) = \frac{i}{(1+i)^N - 1} \quad (3A.4)$$

## 3A.4 | Uniform Series Compound Amount Factor

---

The uniform series compound amount factor,  $(F/A,i,N)$ , converts an annuity  $A$  into a future amount  $F$ :

$$F = A(F/A,i,N)$$

$$\Rightarrow A = F\left(\frac{1}{(F/A,i,N)}\right)$$

Thus the uniform series compound amount factor is the reciprocal of the sinking fund factor. From Equation (3A.4),

$$(F/A,i,N) = \frac{(1 + i)^N - 1}{i}$$

## 3A.5 Capital Recovery Factor

---

If a series of payments  $A$  follows the pattern of a standard annuity of  $N$  payments in length, then the present value of the payment in the  $j$ th period is

$$P_j = A \frac{1}{(1 + i)^j}$$

The present value of the total of all the annuity payments is

$$P = A \left( \frac{1}{(1 + i)} \right) + A \left( \frac{1}{(1 + i)^2} \right) + \dots + A \left( \frac{1}{(1 + i)^{N-1}} \right) + A \left( \frac{1}{(1 + i)^N} \right)$$

Factoring out the annuity amount gives

$$P = A \left[ \left( \frac{1}{(1 + i)} \right) + \left( \frac{1}{(1 + i)^2} \right) + \dots + \left( \frac{1}{(1 + i)^{N-1}} \right) + \left( \frac{1}{(1 + i)^N} \right) \right] \quad (3A.5)$$

Multiplying both sides of Equation (3A.5) by  $(1 + i)$  gives

$$P(1 + i) = A \left[ 1 + \left( \frac{1}{(1 + i)} \right) + \dots + \left( \frac{1}{(1 + i)^{N-2}} \right) + \left( \frac{1}{(1 + i)^{N-1}} \right) \right] \quad (3A.6)$$

Subtracting Equation (3A.5) from Equation (3A.6) gives

$$Pi = A \left[ 1 - \left( \frac{1}{(1 + i)^N} \right) \right]$$

$$P = A \left[ \frac{(1 + i)^N - 1}{i(1 + i)^N} \right]$$

$$A = P \left[ \frac{i(1 + i)^N}{(1 + i)^N - 1} \right]$$

Thus the capital recovery factor is given by

$$(A/P,i,N) = \frac{i(1 + i)^N}{(1 + i)^N - 1} \quad (3A.7)$$

## 3A.6 Series Present Worth Factor

---

The series present worth factor,  $(P/A,i,N)$ , converts an annuity  $A$  into a present amount  $P$ :

$$P = A(P/A,i,N)$$

$$\Rightarrow A = P \left( \frac{1}{(P/A,i,N)} \right)$$

Thus the uniform series compound amount factor is the reciprocal of the sinking fund factor. From Equation (3A.7),

$$(P/A,i,N) = \frac{(1 + i)^N - 1}{i(1 + i)^N}$$

## 3A.7 | Arithmetic and Geometric Gradients

---

The derivation of the arithmetic gradient to annuity conversion factor and the geometric gradient to present worth conversion factor are left as problems for the student. See Problems 3.44 and 3.45.



# Comparison Methods Part 1

Engineering Economics in Action, Part 4A: What's Best?

- 4.1 Introduction
- 4.2 Relations Among Projects
- 4.3 Minimum Acceptable Rate of Return (MARR)
- 4.4 Present Worth (PW) and Annual Worth (AW) Comparisons
  - 4.4.1 Present Worth Comparisons for Independent Projects
  - 4.4.2 Present Worth Comparisons for Mutually Exclusive Projects
  - 4.4.3 Annual Worth Comparisons
  - 4.4.4 Comparison of Alternatives With Unequal Lives
- 4.5 Payback Period

## Review Problems

## Summary

Engineering Economics in Action, Part 4B: Doing It Right

## Problems

### Mini-Case 4.1: Rockwell International

### Appendix 4A: The MARR and the Cost of Capital

## ENGINEERING ECONOMICS IN ACTION, PART 4A

### What's Best?

Naomi waved hello as she breezed by Carole Brown, the receptionist, on her way in from the parking lot one Monday morning. She stopped as Carole caught her eye. "Clem wants to see you right away. Good morning."

After a moment of socializing, Clem got right to the point. "I have a job for you. Put aside the vehicle-life project for a couple of days."

"Okay, but you wanted a report by Friday."

"This is more important. You know that drop forging hammer in the South Shop? The beast is about 50 years old. I don't remember the exact age. We got it used four years ago. We were having quality control problems with the parts we were buying on contract and decided to bring production in-house. Stinson Brothers sold it to us cheap when it upgraded its forging operation. Fundamentally the machine is still sound, but the guides are worn out. The production people are spending too much time fiddling with it instead of turning out parts. Something has to be done. I have to make a recommendation to Ed Burns and Anna Kulkowski, who are going to be making decisions on investments for the next quarter. I'd like you to handle it." Ed Burns was the manager of manufacturing, and Anna Kulkowski was, among other things, the president of Canadian Widgets.

"What's the time frame?" Naomi asked. She was shifting job priorities in her mind and deciding what she would need to postpone.

"I want a report by tomorrow morning. I'd like to have a chance to review what you've done and submit a recommendation to Burns and Kulkowski for their Wednesday meeting." Clem sat back and gave Naomi his best big smile.

Naomi's return smile was a bit weak, as she was preoccupied with trying to sort out where to begin.

Clem laughed and continued with, "It's really not so bad. Dave Sullivan has done most of the work. But he's away and can't finish. His father-in-law had a heart attack on Friday, and he and Helena have gone to Florida to see him."

"What's involved?" asked Naomi.

"Not much, really. Dave has estimated all the cash flows. He's put everything on a spreadsheet. Essentially, there are three major possibilities. We can refurbish and upgrade the existing machine. We can get a manually operated mechanical press that will use less energy and be a lot quieter. Or we can go for an automated mechanical press.

"Since there is going to be down time while we are changing the unit, we might also want to replace the materials-handling equipment at the same time. If we get the automated press, there is the possibility of going the whole hog and integrating materials handling with the press. But even if we automate, we could stay with a separate materials-handling setup.

"Basically, you're looking at a fairly small first cost to upgrade the current beast versus a large first cost for the automated equipment. But if you take the high-first-cost route, you will get big savings down the road. All you have to do is decide what's best."

## 4.1

### Introduction

The essential idea of investing is to give up something valuable now for the expectation of receiving something of greater value later. An investment may be thought of as an exchange of resources now for an expected flow of benefits in the future. Business firms, other organizations, and individuals all have opportunities to make such exchanges. A company may be able to use funds to install equipment that will reduce labour costs in the future. These funds might otherwise have been used on another project or returned to the shareholders or owners. An individual may be able to study to become an engineer. Studying requires that time be given up that could have been used to earn money or to travel. The benefit of study, though, is the expectation of a good income from an interesting job in the future.

Not all investment opportunities *should* be taken. The company considering a labour-saving investment may find that the value of the savings is less than the cost of installing the equipment. Not all investment opportunities *can* be taken. The person spending the next four years studying engineering cannot also spend that time getting degrees in law and science.

Engineers play a major role in making decisions about investment opportunities. In many cases, they are the ones who estimate the expected costs of and returns from an investment. They then must decide whether the expected returns outweigh the costs to see if the opportunity is potentially acceptable. They may also have to examine competing investment opportunities to see which is best. Engineers frequently refer to investment opportunities as **projects**. Throughout most of this text, the term *project* will be used to mean *investment opportunity*.

In this chapter and in Chapter 5, we deal with methods of evaluating and comparing projects, sometimes called **comparison methods**. We start in this chapter with a scheme for classifying groups of projects. This classification system permits the appropriate use of any of the comparison methods. We then turn to a consideration of several widely used methods for evaluating opportunities. The **present worth method** compares projects by looking at the present worth of all cash flows associated with the projects. The **annual worth method** is similar, but converts all cash flows to a uniform series—that is, an annuity. The **payback period method** estimates how long it takes to “pay back” investments. The study of comparison methods is continued in Chapter 5, which deals with the internal rate of return.

We have made six assumptions about all the situations presented in this chapter and in Chapter 5:

1. We have assumed that costs and benefits are always measurable in terms of money. In reality, costs and benefits need not be measurable in terms of money. For example, providing safe working conditions has many benefits, including improvement of worker morale. However, it would be difficult to express the value of improved worker morale objectively in dollars and cents. Such other benefits as the pleasure gained from appreciating beautiful design may not be measurable quantitatively.
2. We have assumed that future cash flows are known with certainty. In reality, future cash flows can only be estimated. Usually the further into the future we try to forecast, the less certain our estimates become. We look at methods of assessing the impact of uncertainty and risks in Chapter 12.
3. We have assumed that cash flows are unaffected by inflation or deflation. In reality, the purchasing power of money typically declines over time. We shall consider how inflation affects decision making in Chapter 9.
4. Unless otherwise stated, we have assumed that sufficient funds are available to implement all projects. In reality, cash constraints on investments may be very important, especially for new enterprises with limited ability to raise capital. We look at methods of raising capital in Appendix 4A.
5. We have assumed that taxes are not applicable. In reality, taxes are pervasive. We shall show how to include taxes in the decision-making process in Chapter 8.
6. Unless otherwise stated, we shall assume that all investments have a cash outflow at the start. These outflows are called *first costs*. We also assume that projects with first costs have cash inflows after the first costs that are at least as great in total as the first costs. In reality, some projects have cash inflows at the start, but involve a commitment of cash outflows at a later period. For example, a consulting engineer may receive an advance payment from a client—a cash inflow—to cover

some of the costs of a project, but to complete the project the engineer will have to make disbursements over the project's life. We shall consider evaluation of such projects in Chapter 5.

## 4.2 Relations Among Projects

---

Companies and individuals are often faced with a large number of investment opportunities at the same time. Relations among these opportunities can range from the simple to the complex. We can distinguish three types of connections among projects that cover all the possibilities. Projects may be

1. Independent,
2. Mutually exclusive, or
3. Related but not mutually exclusive.

The simplest relation between projects occurs when they are **independent**. Two projects are independent if the expected costs and the expected benefits of each project do not depend on whether the other one is chosen. A student considering the purchase of a vacuum cleaner and the purchase of a personal computer would probably find that the expected costs and benefits of the computer did not depend on whether he or she bought the vacuum cleaner. Similarly, the benefits and costs of the vacuum cleaner would be the same whether or not the computer was purchased. If there are more than two projects under consideration, they are said to be independent if all possible pairs of projects in the set are independent. When two or more projects are independent, evaluation is simple. Consider each opportunity one at a time, and accept or reject it on its own merits.

Projects are **mutually exclusive** if, in the process of choosing one, all other alternatives are excluded. In other words, two projects are mutually exclusive if it is impossible to do both or it clearly would not make sense to do both. For example, suppose Bismuth Realty Company wants to develop downtown office space on a specific piece of land. It is considering two potential projects. The first is a low-rise poured-concrete building. The second is a high-rise steel-frame structure with the same capacity as the low-rise building, but it has a small park at the entrance. It is impossible for Bismuth to have both buildings on the same site.

As another example, consider a student about to invest in a computer printer. She can get an inkjet printer or a laser printer, but it would not make sense to get both. She would consider the options to be mutually exclusive.

The third class of projects consists of those that are **related but not mutually exclusive**. For pairs of projects in this category, the expected costs and benefits of one project depend on whether the other one is chosen. For example, Klamath Petroleum may be considering a service station at Fourth Avenue and Main Street as well as one at Twelfth and Main. The costs and benefits from either station will clearly depend on whether the other is built, but it may be possible, and may make sense, to have both stations.

Evaluation of related but not mutually exclusive projects can be simplified by combining them into exhaustive, mutually exclusive sets. For example, the two projects being considered by Klamath can be put into four mutually exclusive sets:

1. Neither station—the “do nothing” option
2. Just the station at Fourth and Main
3. Just the station at Twelfth and Main
4. Both stations

In general,  $n$  related projects can be put into  $2^n$  sets, including the “do nothing” option. Once the related projects are put into mutually exclusive sets, the analyst treats these sets as the alternatives. We can make  $2^n$  mutually exclusive sets with  $n$  related projects by noting that for any single set, there are exactly two possibilities for each project. The project may be *in* or *out* of that set. To get the total number of sets, we multiply the  $n$  twos to get  $2^n$ . In the Klamath example, there were two possibilities for the station at Fourth and Main—accept or reject. These are combined with the two possibilities for the station at Twelfth and Main to give the four sets that we listed.

A special case of related projects is where one project is *contingent* on another. Consider the case where project A could be done alone or A and B could be done together, but B could not be done by itself. Project B is then contingent on project A because it cannot be taken unless A is taken first. For example, the Athens and Manchester Development Company is considering building a shopping mall on the outskirts of town. It is also considering building a parking garage to avoid long outdoor walks by patrons. Clearly, it would not build the parking garage unless it were also building the mall.

Another special case of related projects is due to resource constraints. Usually the constraints are financial. For example, Bismuth may be considering two office buildings at different sites, where the expected costs and benefits of the two are unrelated, but Bismuth may be able to finance only one building. The two office-building projects would then be mutually exclusive because of financial constraints. If there are more than two projects, then all of the sets of projects that meet the budget form a mutually exclusive set of alternatives.

When there are several related projects, the number of logically possible combinations becomes quite large. If there are four related projects, there are  $2^4 = 16$  mutually exclusive sets, including the “do nothing” alternative. If there are five related projects, the number of alternatives doubles to 32. A good way to keep track of these alternatives is to construct a table with all possible combinations of projects. Example 4.1 demonstrates the use of a table.

**EXAMPLE****4.1**

The Small Street Residential Association wants to improve the district. Four ideas for renovation projects have been proposed: (1) converting part of the roadway to gardens, (2) adding old-fashioned light standards, (3) replacing the pavement with cobblestones, and (4) making the street one way. However, there are a number of restrictions. The association can afford to do only two of the first three projects together. Also, gardens are possible only if the street is one way. Finally, old-fashioned light standards would look out of place unless the pavement was replaced with cobblestones. The residential association feels it must do something. It does not want simply to leave things the way they are. What mutually exclusive alternatives are possible?

Since the association does not want to “do nothing,” only  $15 = 2^4 - 1$  alternatives will be considered. These are shown in Table 4.1. The potential projects are listed in rows. The alternatives, which are sets of projects, are in columns. An “x” in a cell indicates that a project is in the alternative represented by that column. Not all logical combinations of projects represent feasible alternatives, as seen in the special cases of contingent alternatives or budget constraints. A last row, below the potential-project rows, indicates whether the sets are feasible alternatives.

The result is that there are seven feasible mutually exclusive alternatives:

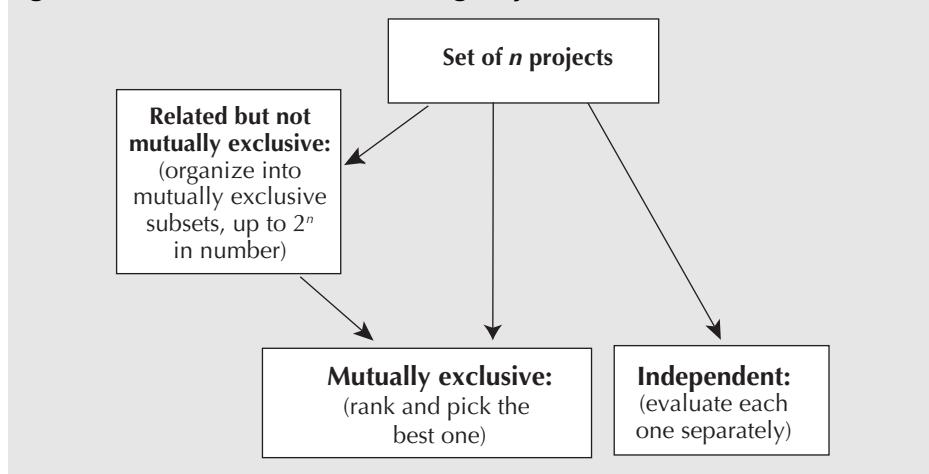
1. Cobblestones (alternative 3)
2. One-way street (alternative 4)
3. One-way street with gardens (alternative 7)

- 4. Cobblestones with lights (alternative 8)
- 5. One-way street with cobblestones (alternative 10)
- 6. One-way street with cobblestones and gardens (alternative 13)
- 7. One-way street with cobblestones and lights (alternative 14) \_\_\_\_\_ ■

**Table 4.1 Potential Alternatives for the Small Street Renovation**

| Potential Alternative | 1  | 2  | 3   | 4   | 5  | 6  | 7   | 8   | 9  | 10  | 11 | 12 | 13  | 14  | 15 |
|-----------------------|----|----|-----|-----|----|----|-----|-----|----|-----|----|----|-----|-----|----|
| Gardens               | x  |    |     |     | x  | x  | x   |     |    |     | x  | x  | x   |     | x  |
| Lights                |    | x  |     |     | x  |    |     | x   | x  |     | x  | x  |     | x   | x  |
| Cobblestones          |    |    | x   |     |    | x  |     | x   |    | x   | x  |    | x   | x   | x  |
| One-way               |    |    |     | x   |    |    | x   |     | x  | x   |    | x  | x   | x   | x  |
| Feasible?             | No | No | Yes | Yes | No | No | Yes | Yes | No | Yes | No | No | Yes | Yes | No |

To summarize our investigation of possible relations among projects, we have a three-fold classification system: (1) independent projects, (2) mutually exclusive projects, and (3) related but not mutually exclusive projects. We can, however, arrange related projects into mutually exclusive sets and treat the sets as mutually exclusive alternatives. This reduces the system to two categories, independent and mutually exclusive. (See Figure 4.1.) Therefore, in the remainder of this chapter we consider only independent and mutually exclusive projects.

**Figure 4.1 Possible Relations Among Projects and How to Treat Them**

## 4.3 | Minimum Acceptable Rate of Return (MARR)

A company evaluating projects will set for itself a lower limit for investment acceptability known as the **minimum acceptable rate of return (MARR)**. The MARR is an interest rate that must be earned for any project to be accepted. Projects that earn at least the MARR are desirable, since this means that the money is earning at least as much as can be

earned elsewhere. Projects that earn less than the MARR are not desirable, since investing money in these projects denies the opportunity to use the money more profitably elsewhere.

The MARR can also be viewed as the rate of return required to get investors to invest in a business. If a company accepts projects that earn less than the MARR, investors will not be willing to put money into the company. This minimum return required to induce investors to invest in the company is the company's **cost of capital**. Methods for determining the cost of capital are presented in Appendix 4A.

The MARR is thus an opportunity cost in two senses. First, investors have investment opportunities outside any given company. Investing in a given company implies forgoing the opportunity of investing elsewhere. Second, once a company sets a MARR, investing in a given project implies giving up the opportunity of using company funds to invest in other projects that pay at least the MARR.

We shall show in this chapter and in Chapter 5 how the MARR is used in calculations involving the present worth, annual worth, or internal rate of return to evaluate projects. Henceforth, it is assumed that a value for the MARR has been supplied.

## 4.4 Present Worth (PW) and Annual Worth (AW) Comparisons

The present worth (PW) comparison method and the annual worth (AW) comparison method are based on finding a comparable basis to evaluate projects in monetary units. With the present worth method, the analyst compares project A and project B by computing the present worths of the two projects at the MARR. The preferred project is the one with the greater present worth. The value of any company can be considered to be the present worth of all of its projects. Therefore, choosing projects with the greatest present worth maximizes the value of the company. With the annual worth method, the analyst compares projects A and B by transforming all disbursements and receipts of the two projects to a uniform series at the MARR. The preferred project is the one with the greater annual worth. One can also speak of *present cost* and *annual cost*. See Close-Up 4.1.

### 4.4.1 Present Worth Comparisons for Independent Projects

The alternative to investing money in an independent project is to "do nothing." Doing nothing doesn't mean that the money is not used productively. In fact, it would be used for some other project, earning interest at a rate at least equal to the MARR.

| CLOSE-UP 4.1  | Present Cost and Annual Cost |
|---|------------------------------|
| <p>Sometimes mutually exclusive projects are compared in terms of present cost or annual cost. That is, the best project is the one with the minimum present worth of cost as opposed to the maximum present worth. Two conditions should hold for this to be valid: (1) All projects have the same major benefit, and (2) the estimated value of the major benefit clearly outweighs the projects' costs, even if that estimate is imprecise. Therefore, the "do nothing" option is rejected. The value of the major benefit is ignored in further calculations since it is the same for all projects. We choose the project with the lowest cost, considering secondary benefits as offsets to costs.</p> |                              |

However, the present worth of any money invested at the MARR is zero, since the present worth of future receipts would exactly offset the current disbursement. Consequently, if an independent project has a present worth greater than zero, it is acceptable. If an independent project has a present worth less than zero, it is unacceptable. If an independent project has a present worth of exactly zero, it is considered *marginally* acceptable.

**EXAMPLE****4.2**

Steve Chen, a third-year electrical engineering student, is really into gaming. He and all of his friends like online multi-player games, but are annoyed by delays caused by slow internet connections. Sometimes they link their own computers into local networks and play, but it's hard to find a place to play and their laptops are not the best machines for gaming. On his work term in Abu Dhabi, Steve noticed that gaming halls are popular. High-level networked game machines are located in malls, and people rent the machines by the hour for playing games either with each other or over a very fast internet connection. Steve expects to be on campus for the next five years and sees an opportunity in setting up a similar business at the mall next to his university. The first cost for equipment, furniture, and software is expected to be \$70 000. Students will be able to rent time on the computers by the hour, and Steve will be able to augment this income with some auxiliary services. Net annual cash flow, after paying for labour, supplies, and other costs, is expected to be \$30 000 a year for five years. When Steve finishes his graduate program in five years, he plans to close the business. The five-year-old equipment and furniture are expected to have zero value. If investors in this type of service enterprise demand a return of 20 percent per year, is this a good investment?

The present worth of the project is

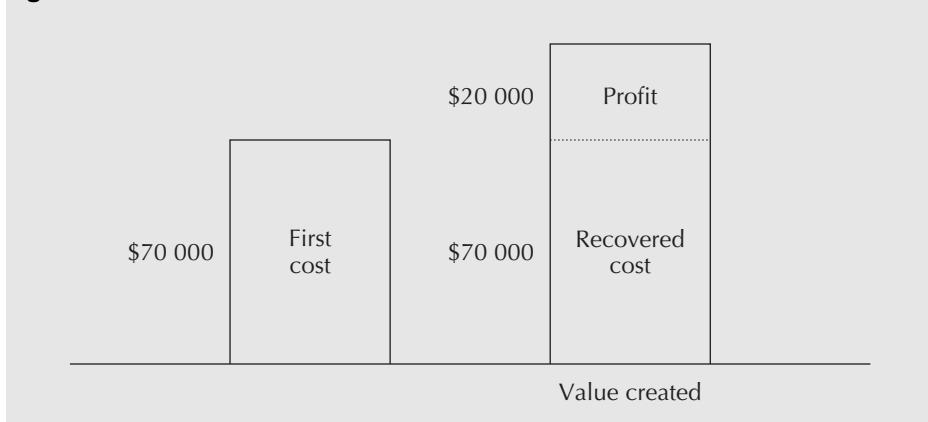
$$\begin{aligned} \text{PW} &= -70\,000 + 30\,000(P/A, 20\%, 5) \quad \square = -70\,000 + PV(0.20, 5, -30\,000) \\ &= -70\,000 + 30\,000(2.9906) \\ &= 19\,718 \\ &\approx 20\,000 \end{aligned}$$

The project is acceptable, since the present worth of about \$20 000 is greater than zero.

Another way to look at the project is to suppose that, once Steve has set up the business off campus, he tries to sell it. If he can convince potential investors, who demand a return of 20 percent a year, that the expectation of a \$30 000 per year cash flow for five years is accurate, how much would they be willing to pay for the network? Investors would calculate the present worth of a 20 percent annuity paying \$30 000 for five years. This is given by

$$\begin{aligned} \text{PW} &= 30\,000(P/A, 20\%, 5) \quad \square = PV(0.20, 5, -30\,000) \\ &= 30\,000(2.9906) \\ &= 89\,718 \\ &\approx 90\,000 \end{aligned}$$

Investors would be willing to pay approximately \$90 000. Steve will have taken \$70 000, the first cost, and used it to create an asset worth almost \$90 000. As illustrated in Figure 4.2, the \$20 000 difference may be viewed as profit. ■

**Figure 4.2 Present Worth as a Measure of Profit**

Let us now consider an example in which the benefit of an investment is a reduction in cost.

**EXAMPLE****4.3**

A mechanical engineer is considering building automated materials-handling equipment for a production line. On the one hand, the equipment would substantially reduce the manual labour currently required to move items from one part of the production process to the next. On the other hand, the equipment would consume energy, require insurance, and need periodic maintenance.

*Alternative 1:* Continue to use the current method. Yearly labour costs are \$9200.

*Alternative 2:* Build automated materials-handling equipment with an expected service life of 10 years.

|                     |                 |
|---------------------|-----------------|
| First cost          | \$15 000        |
| Labour              | \$3300 per year |
| Power               | \$400 per year  |
| Maintenance         | \$2400 per year |
| Taxes and insurance | \$300 per year  |

If the MARR is 9 percent, which alternative is better? Use a present worth comparison.

The investment of \$15 000 can be viewed as yielding a positive cash flow of  $2800 = 9200 - (3300 + 400 + 2400 + 300)$  per year in the form of a reduction in cost.

$$\begin{aligned}
 PW &= -15\,000 + [9200 - (3300 + 400 + 2400 + 300)](P/A, 9\%, 10) \\
 &= -15\,000 + 2800(P/A, 9\%, 10) \quad \blacksquare = -15\,000 + PV(0.09, 10, -2800) \\
 &= -15\,000 + 2800(6.4176) \\
 &= 2969.44
 \end{aligned}$$

The present worth of the cost savings is approximately \$3000 greater than the \$15 000 first cost. Therefore, alternative 2 is worth implementing. ■

#### 4.4.2 Present Worth Comparisons for Mutually Exclusive Projects

It is very easy to use the present worth method to choose the best project among a set of mutually exclusive projects *when the service lives are the same*. One just computes the present worth of each project using the MARR. The project with the greatest present worth is the preferred project because it is the one with the greatest profit.

**EXAMPLE**
**4.4**

Fly-by-Night Aircraft must purchase a new lathe. It is considering four lathes, each of which has a life of 10 years with no scrap value.

| Lathe          | 1         | 2         | 3         | 4         |
|----------------|-----------|-----------|-----------|-----------|
| First cost     | \$100 000 | \$150 000 | \$200 000 | \$255 000 |
| Annual savings | 25 000    | 34 000    | 46 000    | 55 000    |

Given a MARR of 15 percent, which alternative should be taken?

The present worths are:

$$\begin{aligned}
 PW_1 &= -100\,000 + 25\,000(P/A, 15\%, 10) \quad \square = -10\,000 + PV(0.15, 10, -25\,000) \\
 &= -100\,000 + 25\,000(5.0187) \cong 25\,468 \\
 PW_2 &= -150\,000 + 34\,000(P/A, 15\%, 10) \\
 &= -150\,000 + 34\,000(5.0187) \cong 20\,636 \\
 PW_3 &= -200\,000 + 46\,000(P/A, 15\%, 10) \\
 &= -200\,000 + 46\,000(5.0187) \cong 30\,860 \\
 PW_4 &= -255\,000 + 55\,000(P/A, 15\%, 10) \\
 &= -155\,000 + 55\,000(5.0187) \cong 21\,029
 \end{aligned}$$

Lathe 3 has the greatest present worth, and is therefore the preferred alternative. ■

#### 4.4.3 Annual Worth Comparisons

Annual worth comparisons are essentially the same as present worth comparisons, except that all disbursements and receipts are transformed to a uniform series at the MARR, rather than to the present worth. Any present worth  $P$  can be converted to an annuity  $A$  by the capital recovery factor  $(A/P, i, N)$ . Therefore, a comparison of two projects *that have the same life* by the present worth and annual worth methods will always indicate the same preferred alternative. Note that, although the method is called annual worth, the uniform series is not necessarily on a yearly basis.

Present worth comparisons make sense because they compare the worth today of each alternative, but annual worth comparisons can sometimes be more easily grasped mentally. For example, to say that operating an automobile over five years has a present cost of \$20 000 is less meaningful than saying that it will cost about \$5300 per year for each of the following five years.

## NET VALUE 4.1

**Car Payment Calculators**

The internet offers websites that are useful when you are thinking of buying a car—you can learn more about different makes and models, optional features, prices, what's available (used or new) at which dealer, and financing information if a car is to be purchased and not leased. Major car manufacturers, financial services companies, and car information websites make it easy for customers to figure out their financing plans by offering web-based car payment calculators.

A typical monthly payment calculator determines how much a customer pays every month on the basis of the purchase price, down payment,

interest rate, and loan term. This is essentially an annuity calculation. An affordability calculator, on the other hand, gives a present worth (what price of a car you could afford to buy now) or a future worth (total amount of money that would be spent on a car after all payments are made including interest) based on down payment, monthly payment, interest rate, and loan term. The calculators are useful in making instant comparisons among different cars or car companies, studying what-if scenarios with various payment amounts or lengths of the loan, and determining budget limitations. Similar calculators are available for house mortgage payments.

Sometimes there is no clear justification for preferring either the present worth method or the annual worth method. Then it is reasonable to use the one that requires less conversion. For example, if most receipts or costs are given as annuities or gradients, one can more easily perform an annual worth comparison. Sometimes it can be useful to compare projects on the basis of future worths. See Close-Up 4.2.

**CLOSE-UP 4.2**

## Future Worth

Sometimes it may be desirable to compare projects with the **future worth method** on the basis of the future worth of each project. This is most likely to be true for cases where money is being saved for some future expense.

For example, two investment plans are being compared to see which accumulates more money for retirement. Plan A consists of a payment of \$10 000 today and then \$2000 per year over 20 years. Plan B is \$3000 per year over 20 years. Interest for both plans is 10 percent. Rather than convert these cash flows to either present worth or annual worth, it is sensible to compare the future worths of the plans, since the actual dollar value in 20 years has particular meaning.

$$\begin{aligned} FW_A &= 10\,000(F/P, 10\%, 20) + 2000(F/A, 10\%, 20) \\ &= 10\,000(6.7275) + 2000(57.275) \\ &= 181\,825 \end{aligned}$$

$$\begin{aligned} FW_B &= 3000(F/A, 10\%, 20) \\ &= 3000(57.275) \\ &= 171\,825 \end{aligned}$$

Plan A is the better choice. It will accumulate to \$181 825 over the next 20 years.

**EXAMPLE****4.5**

Sweat University is considering two alternative types of bleachers for a new athletic stadium.

*Alternative 1: Concrete bleachers.* The first cost is \$350 000. The expected life of the concrete bleachers is 90 years and the annual upkeep costs are \$2500.

*Alternative 2: Wooden bleachers on earth fill.* The first cost of \$200 000 consists of \$100 000 for earth fill and \$100 000 for the wooden bleachers. The annual painting costs are \$5000. The wooden bleachers must be replaced every 30 years at a cost of \$100 000. The earth fill will last the entire 90 years.

One of the two alternatives will be chosen. It is assumed that the receipts and other benefits of the stadium are the same for both construction methods. Therefore, the greatest net benefit is obtained by choosing the alternative with the lower cost. The university uses a MARR of 7 percent. Which of the two alternatives is better?

For this example, let us base the analysis on annual worth. Since both alternatives have a life of 90 years, we shall get the equivalent annual costs over 90 years for both at an interest rate of 7 percent.

*Alternative 1: Concrete bleachers*

The equivalent annual cost over the 90-year life span of the concrete bleachers is

$$\begin{aligned} AW &= 350\,000(A/P, 7\%, 90) + 2500 \quad \blacksquare = PMT(0.07, 90, -350\,000) + 2500 \\ &= 350\,000(0.07016) + 2500 \\ &= 27\,056 \text{ per year} \end{aligned}$$

*Alternative 2: Wooden bleachers on earth fill*

The total annual costs can be broken into three components:  $AW_1$  (for the earth fill),  $AW_2$  (for the bleachers), and  $AW_3$  (for the painting). The equivalent annual cost of the earth fill is

$$AW_1 = 100\,000(A/P, 7\%, 90) \quad \blacksquare = PMT(0.07, 90, -100\,000)$$

The equivalent annual cost of the bleachers is easy to determine. The first set of bleachers is put in at the start of the project, the second set at the end of 30 years, and the third set at the end of 60 years, but the cost of the bleachers is the same at each installation. Therefore, we need to get only the cost of the first installation.

$$AW_2 = 100\,000(A/P, 7\%, 30) \quad \blacksquare = PMT(0.07, 30, -100\,000)$$

The last expense is for annual painting:

$$AW_3 = 5000$$

The total equivalent annual cost for alternative 2, wooden bleachers on earth fill, is the sum of  $AW_1$ ,  $AW_2$ , and  $AW_3$ :

$$\begin{aligned} AW &= AW_1 + AW_2 + AW_3 \\ &= 100\,000[(A/P, 7\%, 90) + (A/P, 7\%, 30)] + 5000 \\ &= 100\,000(0.07016 + 0.08059) + 5000 \\ &= 20\,075 \end{aligned}$$

The concrete bleachers have an equivalent annual cost of about \$7000 more than the wooden ones. Therefore, the wooden bleachers are the better choice. ■

#### 4.4.4 Comparison of Alternatives With Unequal Lives

When making present worth comparisons, we must always use the same time period in order to take into account the full benefits and costs of each alternative. If the lives of the alternatives are not the same, we can transform them to equal lives with one of the following two methods:

1. Repeat the *service life* of each alternative to arrive at a common time period for all alternatives. Here we assume that each alternative can be repeated with the same costs and benefits in the future—an assumption known as **repeated lives**. Usually we use the *least common multiple* of the lives of the various alternatives. Sometimes it is convenient to assume that the lives of the various alternatives are repeated indefinitely. Note that the assumption of repeated lives may not be valid where it is reasonable to expect technological improvements.
2. Adopt a specified **study period**—a time period that is given for the analysis. To set an appropriate study period, a company will usually take into account the time of required service or the length of time it can be relatively certain of its forecasts. The study period method necessitates an additional assumption about *salvage value* whenever the life of one of the alternatives exceeds that of the given study period. Arriving at a reliable estimate of salvage value may be difficult sometimes.

Because they rest on different assumptions, the repeated lives and the study period methods can lead to different conclusions when applied to a particular project choice.

**EXAMPLE**

#### 4.6 (MODIFICATION OF EXAMPLE 4.3)

A mechanical engineer has decided to introduce automated materials-handling equipment for a production line. She must choose between two alternatives: building the equipment or buying the equipment off the shelf. Each alternative has a different service life and a different set of costs.

*Alternative 1: Build custom automated materials-handling equipment.*

|                     |                 |
|---------------------|-----------------|
| First cost          | \$15 000        |
| Labour              | \$3300 per year |
| Power               | \$400 per year  |
| Maintenance         | \$2400 per year |
| Taxes and insurance | \$300 per year  |
| Service life        | 10 years        |

*Alternative 2: Buy off-the-shelf standard automated materials-handling equipment.*

|                     |                 |
|---------------------|-----------------|
| First cost          | \$25 000        |
| Labour              | \$1450 per year |
| Power               | \$600 per year  |
| Maintenance         | \$3075 per year |
| Taxes and insurance | \$500 per year  |
| Service life        | 15 years        |

If the MARR is 9 percent, which alternative is better?

The present worth of the custom system over its 10-year life is

$$\begin{aligned} \text{PW}(1) &= -15\ 000 - (3300 + 400 + 2400 + 300)(P/A, 9\%, 10) \\ &= -15\ 000 - 6400(6.4176) \quad \blacksquare = -15\ 000 - PV(0.09, 10, -6400) \\ &\approx -56\ 073 \end{aligned}$$

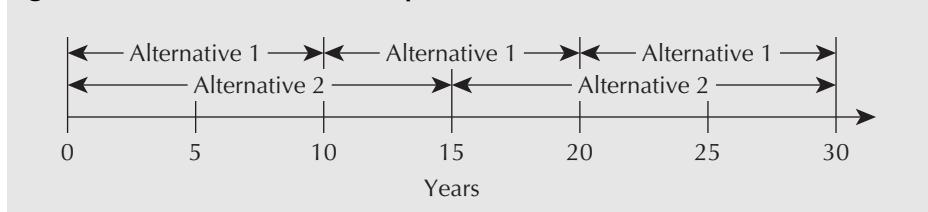
The present worth of the off-the-shelf system over its 15-year life is

$$\begin{aligned} \text{PW}(2) &= -25\ 000 - (1450 + 600 + 3075 + 500)(P/A, 9\%, 15) \\ &= -25\ 000 - 5625(8.0606) \quad \blacksquare = -25\ 000 - PV(0.09, 15, -5625) \\ &\approx -70\ 341 \end{aligned}$$

The custom system has a lower cost for its 10-year life than the off-the-shelf system for its 15-year life, but it would be *wrong* to conclude from these calculations that the custom system should be preferred. The custom system yields benefits for only 10 years, whereas the off-the-shelf system lasts 15 years. It would be surprising if the cost of 15 years of benefits were not higher than the cost of 10 years of benefits. A fair comparison of the costs can be made only if equal lives are compared.

Let us apply the repeated lives method. If each alternative is repeated enough times, there will be a point in time where their service lives are simultaneously completed. This will happen first at the time equal to the least common multiple of the service lives. The least common multiple of 10 years and 15 years is 30 years. Alternative 1 will be repeated twice (after 10 years and after 20 years), while alternative 2 will be repeated once (after 15 years) during the 30-year period. At the end of 30 years, both alternatives will be completed simultaneously. See Figure 4.3.

**Figure 4.3 Least Common Multiple of the Service Lives**



With the same time period of 30 years for both alternatives, we can now compare present worths.

*Alternative 1:* Build custom automated materials-handling equipment and repeat twice.

$$\begin{aligned} \text{PW}(1) &= -15\ 000 - 15\ 000(P/F, 9\%, 10) - 15\ 000(P/F, 9\%, 20) \\ &\quad - (3300 + 400 + 2400 + 300)(P/A, 9\%, 30) \\ \blacksquare &= -15\ 000 - PV(0.09, 10, -15\ 000) - PV(0.09, 20, -15\ 000) - PV(0.09, 30, -6400) \\ &= -15\ 000 - 15\ 000(0.42241) - 15\ 000(0.17843) - 6400(10.273) \\ &\approx -89\ 760 \end{aligned}$$

*Alternative 2:* Buy off-the-shelf standard automated materials-handling equipment and repeat once.

$$\begin{aligned} \text{PW}(2) &= -25\ 000 - 25\ 000(P/F, 9\%, 15) \quad \blacksquare = -25\ 000 \\ &\quad - (1450 + 600 + 3075 + 500)(P/A, 9\%, 30) \quad - PV(0.09, 15, -25\ 000) \\ &= -25\ 000 - 25\ 000(0.27454) - 5625(10.273) \quad - PV(0.09, 30, -5625) \\ &\approx -89\ 649 \end{aligned}$$

Using the repeated lives method, we find little difference between the alternatives. An annual worth comparison can also be done over a period of time equal to the least common multiple of the service lives by multiplying each of these present worths by the capital recovery factor for 30 years.

$$\begin{aligned} \text{AW}(1) &= -89\,760(A/P, 9\%, 30) \quad \blacksquare = -\text{PMT}(0.09, 30, -89\,760) \\ &= -89\,760(0.09734) \\ &\approx -8737 \end{aligned}$$

$$\begin{aligned} \text{AW}(2) &= -89\,649(A/P, 9\%, 30) \\ &= -89\,649(0.09734) \\ &\approx -8726 \end{aligned}$$

As we would expect, there is again little difference in the annual cost between the alternatives. However, there is a more convenient approach for an annual worth comparison if it can be assumed that the alternatives are repeated indefinitely. Since the annual costs of an alternative remain the same no matter how many times it is repeated, it is not necessary to determine the least common multiple of the service lives. The annual worth of each alternative can be assessed for whatever time period is most convenient for each alternative.

*Alternative 1:* Build custom automated materials-handling equipment.

$$\begin{aligned} \text{AW}(1) &= -15\,000(A/P, 9\%, 10) - 6400 \quad \blacksquare = -\text{PMT}(0.09, 10, -15\,000) - 6400 \\ &= -15\,000(0.15582) - 6400 \\ &\approx -8737 \end{aligned}$$

*Alternative 2:* Buy off-the-shelf standard automated materials-handling equipment.

$$\begin{aligned} \text{AW}(2) &= -25\,000(A/P, 9\%, 15) - 5625 \quad \blacksquare = -\text{PMT}(0.09, 15, -25\,000) - 5625 \\ &= -25\,000(0.12406) - 5625 \\ &\approx -8726 \end{aligned}$$

If it cannot be assumed that the alternatives can be repeated to permit a calculation over the least common multiple of their service lives, then it is necessary to use the study period method.

Suppose that the given study period is 10 years because the engineer is uncertain about costs past that time. The service life of the off-the-shelf system (15 years) is greater than the study period (10 years). Therefore, we have to make an assumption about the salvage value of the off-the-shelf system after 10 years. Suppose the engineer judges that its salvage value will be \$5000. We can now proceed with the comparison.

*Alternative 1:* Build custom automated materials-handling equipment (10-year study period).

$$\begin{aligned} \text{PW}(1) &= -15\,000 - (3300 + 400 + 2400 + 300)(P/A, 9\%, 10) \\ &= -15\,000 - 6400(6.4176) \quad \blacksquare = -15\,000 - \text{PV}(0.09, 10, -6400) \\ &\approx -56\,073 \end{aligned}$$

*Alternative 2:* Buy off-the-shelf standard automated materials-handling equipment (10-year study period).

$$\begin{aligned} \text{PW}(2) &= -25\,000 - (1450 + 600 + 3075 + 500)(P/A, 9\%, 10) \\ &\quad + 5000(P/F, 9\%, 10) \end{aligned}$$

$$\begin{aligned}
 &= -25\,000 - 5625(6.4176) + 5000(0.42241) \\
 &\approx -58\,987
 \end{aligned}$$

Using the study period method of comparison, alternative 1 has the smaller present worth of costs at \$56 073 and is, therefore, preferred.

Note that here the study period method gives a different answer than the repeated lives method gives. The study period method is often sensitive to the chosen salvage value. A larger salvage value tends to make an alternative with a life longer than the study period more attractive, and a smaller value tends to make it less attractive.

In some instances, it may be difficult to arrive at a reliable estimate of salvage value. Given the sensitivity of the study period method to the salvage value estimate, the analyst may be uncertain about the validity of the results. One way of circumventing this problem is to avoid estimating the salvage value at the outset. Instead we calculate what salvage value would make the alternatives equal in value. Then we decide whether the actual salvage value will be above or below the break-even value found. Applying this approach to our example, we set  $PW(1) = PW(2)$  so that

$$PW(1) = PW(2)$$

$$56\,073 = -25\,000 - 5625(6.4176) + S(0.42241)$$

where  $S$  is the salvage value.

Solving for  $S$ , we find  $S = 11\,834$ . Is a reasonable estimate of the salvage value above or below \$11 834? If it is above \$11 834, then we conclude that the off-the-shelf system is the preferred choice. If it is below \$11 834, then we conclude that the custom system is preferable. ■

The study period can also be used for the annual worth method if the assumption of being able to indefinitely repeat the choice of alternatives is not justified.

### EXAMPLE

### 4.7

Joan is renting a flat while on a one-year assignment in England. The flat does not have a refrigerator. She can rent one for a £100 deposit (returned in a year) and £15 per month (paid at the end of each month). Alternatively, she can buy a refrigerator for £300, which she would sell in a year when she leaves. For how much would Joan have to be able to sell the refrigerator in one year when she leaves in order to be better off buying the refrigerator than renting one? Interest is at 6 percent nominal, compounded monthly.

Let  $S$  stand for the unknown salvage value (i.e., the amount Joan will be able to sell the refrigerator for in a year). We then equate the present worth of the rental alternative with the present worth of the purchase alternative for the one-year study period:

$$PW(\text{rental}) = PW(\text{purchase})$$

$$-100 - 15(P/A, 0.5\%, 12) + 100(P/F, 0.5\%, 12) = -300 + S(P/F, 0.5\%, 12)$$

$$-100 - 15(11.616) + 100(0.94192) = -300 + S(0.94192)$$

$$S = 127.35$$

If Joan can sell the used refrigerator for more than about £127 after one year's use, she is better off buying it rather than renting one. ■



## S P R E A D S H E E T   S A V V Y

Spreadsheets can be indispensable when it comes to making present worth or annual worth comparisons among projects. It is useful to spend a bit of time thinking about how to set up spreadsheet rows and columns before starting a comparison so you can do the analysis easily and can document your work. A careful setup will also allow you to check the individual components of your work as you go along.

The tables below demonstrate the use of the NPV and PMT Excel functions. The NPV function takes the present worth of a series of cash flows. It assumes that the cash flows start at the end of period 1 and that the interest rate is  $i$ . In contrast to other Excel functions, NPV assumes that the cash flows are receipts, so the sign of cash flows does not need to be reversed. The PMT Excel function computes the annual worth of a single cash flow that occurs at time 0 (assumed to be a disbursement) over  $N$  periods at an interest rate of  $i$ .

The tables show how to compute the present worth of cash flows in cells B5 through to B10 (B5:B10). The two methods shown are used to first compute the present worth of the individual cash flows and then to sum (cell C11), and then to use the NPV function (cell C14). The equivalent annual worth, in cells C12 and C15, is found by using either a compound interest factor (cell C12) or the PMT Excel function (cell C15).

In the top part of the table on the right, cell B2 contains the interest rate. It is good practice to put the interest rate in a cell that is referred to by other formulas rather than to enter it directly into the compound interest formula. The reason for this is twofold: First, the reader can see what the interest rate is for documentation purposes. Second, should you need to do a different computation with another interest rate, it is easier to change one cell than it is to re-enter the formulas.

|    | A                                     | B         | C             |
|----|---------------------------------------|-----------|---------------|
| 1  | Use of Basic Computations:            |           |               |
| 2  | $i =$                                 | 10.00%    |               |
| 3  |                                       |           | Present Worth |
| 4  | Period                                | Cash Flow | $P(P/F,i,N)$  |
| 5  | 0                                     | - 100 000 | - 100 000     |
| 6  | 1                                     | 55 000    | 50 000        |
| 7  | 2                                     | 63 000    | 52 066        |
| 8  | 3                                     | 37 000    | 27 799        |
| 9  | 4                                     | 32 000    | 21 856        |
| 10 | 5                                     | 76 000    | 47 190        |
| 11 | PW                                    |           | 98 911        |
| 12 | AW                                    |           | 26 093        |
| 13 | Use of Excel Functions: (NPV and PMT) |           |               |
| 14 | PW                                    |           | 98 911        |
| 15 | AW                                    |           | 26 093        |

|    | A                                      | B         | C                             |
|----|--|-----------|-------------------------------|
| 1  | Use of Basic Computations:             |           |                               |
| 2  | $i =$                                  | 0.1       |                               |
| 3  |  |           | Present Worth                 |
| 4  | Period                                 | Cash Flow | $P(P/F,i,N)$                  |
| 5  | 0                                      | -100000   | =B5/(1+\$B\$2)^A5             |
| 6  | 1                                      | 55000     | =B6/(1+\$B\$2)^A6             |
| 7  | 2                                      | 63000     | =B7/(1+\$B\$2)^A7             |
| 8  | 3                                      | 37000     | =B8/(1+\$B\$2)^A8             |
| 9  | 4                                      | 32000     | =B9/(1+\$B\$2)^A9             |
| 10 | 5                                      | 76000     | =B10/(1+\$B\$2)^A10           |
| 11 | PW                                     |           | =SUM(C5:C10)                  |
| 12 | AW                                     |           | =C11*B2*(1+B2)^5/((1+B2)^5-1) |
| 13 | Use of Excel Functions: (NPV and PMT): |           |                               |
| 14 | PW                                     |           | =B5+NPV(B2,B6:B10)            |
| 15 | AW                                     |           | =PMT(B2,5,-C14)               |

## 4.5 Payback Period

The simplest method for judging the economic viability of projects is the payback period method. It is a rough measure of the time it takes for an investment to pay for itself. More precisely, the **payback period** is the number of years it takes for an investment to be recouped when the interest rate is assumed to be zero. When annual savings are constant, the payback period is usually calculated as follows:

$$\text{Payback period} = \frac{\text{First cost}}{\text{Annual savings}}$$

For example, if a first cost of \$20 000 yielded a return of \$8000 per year, then the payback period would be  $20\ 000/8000 = 2.5$  years.

If the annual savings are not constant, we can calculate the payback period by deducting each year of savings from the first cost until the first cost is recovered. The number of years required to pay back the initial investment is the payback period. For example, suppose the saving from a \$20 000 first cost is \$5000 the first year, increasing by \$1000 each year thereafter. By adding the annual savings one year at a time, we see that it would take just over three years to pay back the first cost ( $5000 + 6000 + 7000 + 8000 = 26\ 000$ ). The payback period would then be stated as either four years (if we assume that the \$8000 is received at the end of the fourth year) or 3.25 years (if we assume that the \$8000 is received uniformly over the fourth year).

According to the payback period method of comparison, the project with the shorter payback period is the preferred investment. A company may have a policy of rejecting projects for which the payback period exceeds some preset number of years. The length of the maximum payback period depends on the type of project and the company's financial situation. If the company expects a cash constraint in the near future, or if a project's returns are highly uncertain after more than a few periods, the company will set a maximum payback period that is relatively short. As a common rule, a payback period of two years is often considered acceptable, while one of more than four years is unacceptable. Accordingly, government grant programs often target projects with payback periods of between two and four years with the rationale that in this range the grant can justify economically feasible projects that a company with limited cash flow would otherwise be unwilling to undertake.

The payback period need not, and perhaps should not, be used as the sole criterion for evaluating projects. It is a rough method of comparison and possesses some glaring weaknesses (as we shall discuss after Examples 4.8 and 4.9). Nevertheless, the payback period method can be used effectively as a preliminary filter. All projects with paybacks within the minimum would then be evaluated, using either rate of return methods (Chapter 5) or present/annual worth methods.

### EXAMPLE

#### 4.8

Elyse runs a second-hand book business out of her home where she advertises and sells the books over the internet. Her small business is becoming quite successful and she is considering purchasing an upgrade to her computer system that will give her more reliable uptime. The cost is \$5000. She expects that the investment will bring about an annual savings of \$2000, due to the fact that her system will no longer suffer long failures and thus she will be able to sell more books. What is the payback period on her investment, assuming that the savings accrue over the whole year?

$$\text{Payback period} = \frac{\text{First cost}}{\text{Annual savings}} = \frac{5000}{2000} = 2.5 \text{ years}$$

### EXAMPLE

#### 4.9

Pizza-in-a-Hurry operates a pizza delivery service to its customers with two eight-year-old vehicles, both of which are large, consume a great deal of gas, and are starting to cost a lot to repair. The owner, Ray, is thinking of replacing one of the cars with a smaller, three-year-old car that his sister-in-law is selling for \$8000. Ray figures he can save \$3000, \$2000, and \$1500 per year for the next three years and \$1000 per year for the following two years by purchasing the smaller car. What is the payback period for this decision?

The payback period is the number of years of savings required to pay back the initial cost. After three years,  $\$3000 + \$2000 + \$1500 = \$6500$  has been paid back, and this amount is  $\$7500$  after four years and  $\$8500$  after five years. The payback period would be stated as five years if the savings are assumed to occur at the end of each year, or 4.5 years if the savings accrue continuously throughout the year. ■

The payback period method has four main advantages:

1. It is very easy to understand. One of the goals of engineering decision making is to communicate the reasons for a decision to managers or clients with a variety of backgrounds. The reasons behind the payback period and its conclusions are very easy to explain.
2. The payback period is very easy to calculate. It can usually be done without even using a calculator, so projects can be very quickly assessed.
3. It accounts for the need to recover capital quickly. Cash flow is almost always a problem for small to medium-sized companies. Even large companies sometimes can't tie up their money in long-term projects.
4. The future is unknown. The future benefits from an investment may be estimated imprecisely. It may not make much sense to use precise methods like present worth on numbers that are imprecise to start with. A simple method like the payback period may be good enough for most purposes.

But the payback period method has three important disadvantages:

1. It discriminates against long-term projects. No houses or highways would ever be built if they had to pay themselves off in two years.
2. It ignores the effect of the timing of cash flows within the payback period. It disregards interest rates and takes no account of the time value of money. (Occasionally, a discounted payback period is used to overcome this disadvantage. See Close-Up 4.3.)
3. It ignores the expected service life. It disregards the benefits that accrue after the end of the payback period.

#### CLOSE-UP 4.3

#### Discounted Payback Period

In a discounted payback period calculation, the present worth of each year's savings is subtracted from the first cost until the first cost is diminished to zero. The number of years of savings required to do this is the discounted payback period. The main disadvantages of using a discounted payback period include the more complicated calculations and the need for an interest rate.

For instance, in Example 4.8, Elyse had an investment of \$5000 recouped by annual savings of \$2000. If interest were at 10 percent, the present worth of savings would be:

| Year   | Present Worth                               | Cumulative |
|--------|---|------------|
| Year 1 | $2000(P/F, 10\%, 1) = 2000(0.90909) = 1818$ | 1818       |
| Year 2 | $2000(P/F, 10\%, 2) = 2000(0.82645) = 1653$ | 3471       |
| Year 3 | $2000(P/F, 10\%, 3) = 2000(0.75131) = 1503$ | 4974       |
| Year 4 | $2000(P/F, 10\%, 4) = 2000(0.68301) = 1366$ | 6340       |

Thus the discounted payback period is over 3 years, compared with 2.5 years calculated for the standard payback period.

Example 4.10 illustrates how the payback period method can ignore future cash flows.

**EXAMPLE****4.10**

Self Defence Systems of Cape Town is going to upgrade its paper-shredding facility. The company has a choice between two models. Model 007, with a first cost of R500 000 and a service life of seven years, would save R100 000 per year. Model MX, with a first cost of R100 000 and an expected service life of 20 years, would save R15 000 per year. If the company's MARR is 8 percent, which model is the better buy?

*Using payback period as the sole criterion:*

$$\text{Model 007: Payback period} = 500\ 000/100\ 000 = 5 \text{ years}$$

$$\text{Model MX: Payback period} = 100\ 000/15\ 000 = 6.6 \text{ years}$$

It appears that the 007 model is better.

*Using annual worth:*

$$\text{Model 007: AW} = -500\ 000(A/P, 8\%, 7) + 100\ 000 = 3965$$

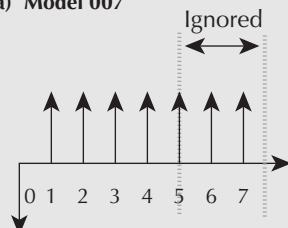
$$\text{Model MX: AW} = -100\ 000(A/P, 8\%, 20) + 15\ 000 = 4815$$

Here, Model MX is substantially better.

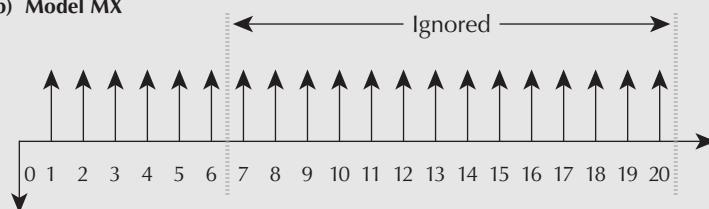
The difference in the results from the two comparison methods is that the payback period method has ignored the benefits of the models that occur after the models have paid themselves off. This is illustrated in Figure 4.4. For Model MX, about 14 years of benefits have been omitted, whereas for model 007, only two years of benefits have been left out. ■

**Figure 4.4 Flows Ignored by the Payback Period**

(a) Model 007



(b) Model MX



## R E V I E W P R O B L E M S

### **REVIEW PROBLEM 4.1**

Tilson Dairies operates several cheese plants. The plants are all old and in need of renovation. Tilson's engineers have developed plans to renovate all of them. Each project would have a positive present worth at the company's MARR. Tilson has \$3.5 million available to invest in these projects. The following facts about the potential renovation projects are available:

| <b>Project</b>      | <b>First Cost</b> | <b>Present Worth</b> |
|---------------------|-------------------|----------------------|
| A: Renovate plant 1 | \$0.8 million     | \$1.1 million        |
| B: Renovate plant 2 | \$1.2 million     | \$1.7 million        |
| C: Renovate plant 3 | \$1.4 million     | \$1.8 million        |
| D: Renovate plant 4 | \$2.0 million     | \$2.7 million        |

Which projects should Tilson accept?

### **ANSWER**

Table 4.2 shows the possible mutually exclusive projects that Tilson can consider.

**Table 4.2 Mutually Exclusive Projects for Tilson Dairies**

| <b>Project</b> | <b>Total First Cost</b> | <b>Total Present Worth</b> | <b>Feasibility</b> |
|----------------|-------------------------|----------------------------|--------------------|
| Do nothing     | \$0.0 million           | \$0.0 million              | Feasible           |
| A              | \$0.8 million           | \$1.1 million              | Feasible           |
| B              | \$1.2 million           | \$1.7 million              | Feasible           |
| C              | \$1.4 million           | \$1.8 million              | Feasible           |
| D              | \$2.0 million           | \$2.7 million              | Feasible           |
| A and B        | \$2.0 million           | \$2.8 million              | Feasible           |
| A and C        | \$2.2 million           | \$2.9 million              | Feasible           |
| A and D        | \$2.8 million           | \$3.8 million              | Feasible           |
| B and C        | \$2.6 million           | \$3.5 million              | Feasible           |
| B and D        | \$3.2 million           | \$4.4 million              | Feasible           |
| C and D        | \$3.4 million           | \$4.5 million              | Feasible           |
| A, B, and C    | \$3.4 million           | \$4.6 million              | Feasible           |
| A, B, and D    | \$4.0 million           | \$5.5 million              | Not feasible       |
| A, C, and D    | \$4.2 million           | \$5.6 million              | Not feasible       |
| B, C, and D    | \$4.6 million           | \$6.2 million              | Not feasible       |
| A, B, C, and D | \$5.4 million           | \$7.3 million              | Not feasible       |

Tilson should accept projects A, B, and C. They have a combined present worth of \$4.6 million. Other feasible combinations that come close to using all available funds are B and D with a total present worth of \$4.4 million, and C and D with a total present worth of \$4.5 million.

Note that it is not necessary to consider explicitly the “leftovers” of the \$3.5 million budget when comparing the present worths. The assumption is that any leftover part of the budget will be invested and provide interest at the MARR, resulting in a zero present worth for that part. Therefore, it is best to choose the combination of projects that has the largest total present worth and stays within the budget constraint. ■

### REVIEW PROBLEM 4.2

City engineers are considering two plans for municipal aqueduct tunnels. They are to decide between the two using an interest rate of 8 percent.

Plan A is a full-capacity tunnel that will meet the needs of the city forever. Its cost is \$3 000 000 now and \$100 000 every 10 years for lining repairs.

Plan B involves building a half-capacity tunnel now and a second half-capacity tunnel in 20 years, when the extra capacity will be needed. Each of the half-capacity tunnels costs \$2 000 000. Maintenance costs for each tunnel are \$80 000 every 10 years. There is also an additional \$15 000 per tunnel per year required to pay for extra pumping costs caused by greater friction in the smaller tunnels.

- (a) Which alternative is preferred? Use a present worth comparison.
- (b) Which alternative is preferred? Use an annual worth comparison.

### ANSWER

#### (a) Plan A: Full-Capacity Tunnel

First, the \$100 000 paid at the end of 10 years can be thought of as a future amount that has an equivalent annuity.

$$\begin{aligned} AW &= 100\,000(A/F, 8\%, 10) \quad \square = -PMT(0.8, 10, -100\,000) \\ &= 100\,000(0.06903) = 6903 \end{aligned}$$

Thus, at 8 percent interest, \$100 000 every 10 years is equivalent to \$6903 every year.

Since the tunnel will have (approximately) an infinite life, the present cost of the lining repairs can be found using the capitalized cost formula, giving a total cost of

$$PW(\text{Plan A}) = 3\,000\,000 + 6903/0.08 = 3\,086\,288$$

#### Plan B: Half-Capacity Tunnels

For the first tunnel, the equivalent annuity for the maintenance and pumping costs is

$$AW = 15\,000 + 80\,000(0.06903) = 20\,522$$

The present cost is then found with the capitalized cost formula, giving a total cost of

$$PW_1 = 2\,000\,000 + 20\,522/0.08 = 2\,256\,525$$

Now, for the second tunnel, basically the same calculation is used, except that the present worth calculated must be discounted by 20 years at 8 percent, since the second tunnel will be built 20 years in the future.

$$\begin{aligned} \text{PW}_2 &= \{2\ 000\ 000 + [15\ 000 + 80\ 000(0.06903)]/0.08\}(P/F, 8\%, 20) \\ &= 2\ 256\ 525(0.21455) \cong 484\ 137 \end{aligned}$$

$$\text{PW}(\text{Plan B}) = \text{PW}_1 + \text{PW}_2 = 2\ 740\ 662$$

Consequently, the two half-capacity aqueducts with a present worth of costs of \$2 740 662 are economically preferable.

**(b) Plan A: Full-Capacity Tunnel**

First, the \$100 000 paid at the end of 10 years can be thought of as a future amount that has an equivalent annuity of

$$\text{AW} = 100\ 000(A/F, 8\%, 10) = 100\ 000(0.06903) = 6903$$

Thus, at 8 percent interest, \$100 000 every 10 years is equivalent to \$6903 every year.

Since the tunnel will have (approximately) an infinite life, an annuity equivalent to the initial cost can be found using the capitalized cost formula, giving a total annual cost of

$$\text{AW}(\text{Plan A}) = 3\ 000\ 000(0.08) + 6903 = 246\ 903$$

*Plan B: Half-Capacity Tunnels*

For the first tunnel, the equivalent annuity for the maintenance and pumping costs is

$$\text{AW} = 15\ 000 + 80\ 000(0.06903) \cong 20\ 522$$

The annual equivalent of the initial cost is then found with the capitalized cost formula, giving a total cost of

$$\text{AW}_1 = 2\ 000\ 000(0.08) + 20\ 522 = 180\ 522$$

Now, for the second tunnel, basically the same calculation is used, except that the annuity must be discounted by 20 years at 8 percent, since the second tunnel will be built 20 years in the future.

$$\begin{aligned} \text{AW}_2 &= \text{AW}_1(P/F, 8\%, 20) \\ &= 180\ 522(0.21455) \cong 38\ 731 \end{aligned}$$

$$\begin{aligned} \text{AW}(\text{Plan B}) &= \text{AW}_1 + \text{AW}_2 \\ &= 180\ 522 + 38\ 731 = 219\ 253 \end{aligned}$$

Consequently, the two half-capacity aqueducts with an annual worth of costs of \$219 253 are economically preferable. ■

### REVIEW PROBLEM 4.3

Fernando Constantia, an engineer at Brandy River Vineyards, has a \$100 000 budget for winery improvements. He has identified four mutually exclusive investments, all of five years' duration, which have the cash flows shown in Table 4.3. For each alternative, he wants to determine the payback period and the present worth. For his recommendation report, he will order the alternatives from most preferred to least preferred in each case. Brandy River uses an 8 percent MARR for such decisions.

**Table 4.3 Cash Flows for Review Problem 4.3**

| Alternative | Cash Flow at the End of Each Year |          |          |          |          |          |           |
|-------------|-----------------------------------|----------|----------|----------|----------|----------|-----------|
|             | 0                                 | 1        | 2        | 3        | 4        | 5        |           |
| A           | -\$100 000                        | \$25 000 | \$25 000 | \$25 000 | \$25 000 | \$25 000 | \$ 25 000 |
| B           | -100 000                          | 5000     | 10 000   | 20 000   | 40 000   |          | 80 000    |
| C           | -100 000                          | 50 000   | 50 000   | 10 000   | 0        |          | 0         |
| D           | -100 000                          |          | 0        | 0        | 0        | 0        | 1 000 000 |

**ANSWER**

The payback period can be found by decrementing yearly. The payback periods for the alternatives are then

- A: 4 years
- B: 4.3125 or 5 years
- C: 2 years
- D: 4.1 or 5 years

The order of the alternatives from most preferred to least preferred using the payback period method with yearly decrementing is: C, A, D, B. The present worth computations for each alternative are:

$$\begin{aligned} \text{A: } \text{PW} &= -100 000 + 25 000(P/A, 8\%, 5) \\ &= -100 000 + 25 000(3.9926) \\ &= -185 \end{aligned}$$

$$\begin{aligned} \text{B: } \text{PW} &= -100 000 + 5000(P/F, 8\%, 1) + 10 000(P/F, 8\%, 2) \\ &\quad + 20 000(P/F, 8\%, 3) + 40 000(P/F, 8\%, 4) + 80 000(P/F, 8\%, 5) \\ &= -100 000 + 5000(0.92593) + 10 000(0.85734) \\ &\quad + 20 000(0.79383) + 40 000(0.73503) + 80 000(0.68059) \\ &= 12 928 \end{aligned}$$

$$\begin{aligned} \text{C: } \text{PW} &= -100 000 + 50 000(P/F, 8\%, 1) + 50 000(P/F, 8\%, 2) \\ &\quad + 10 000(P/F, 8\%, 3) \\ &= -100 000 + 50 000(0.92593) + 50 000(0.85734) \\ &\quad + 10 000(0.79283) \\ &= -2908 \end{aligned}$$

$$\begin{aligned} \text{D: } \text{PW} &= -100 000 + 1 000 000(P/F, 8\%, 5) \\ &= -100 000 + 1 000 000(0.68059) \\ &= 580 590 \end{aligned}$$

The order of the alternatives from most preferred to least preferred using the present worth method is: D, B, A, C. ■

## S U M M A R Y

This chapter discussed relations among projects, and the present worth, annual worth, and payback period methods for evaluating projects. There are three classes of relations among projects: (1) independent, (2) mutually exclusive, and (3) related but not mutually exclusive. We then showed how the third class of projects, those that are related but not mutually exclusive, could be combined into sets of mutually exclusive projects. This enabled us to limit the discussion to the first two classes—*independent* and *mutually exclusive*. Independent projects are considered one at a time and are either accepted or rejected. Only the best of a set of mutually exclusive projects is chosen.

The present worth method compares projects on the basis of converting all cash flows for the project to a present worth. An independent project is acceptable if its present worth is greater than zero. The mutually exclusive project with the highest present worth should be taken. Projects with unequal lives must be compared by assuming that the projects are repeated or by specifying a study period. Annual worth is similar to present worth, except that the cash flows are converted to a uniform series. The annual worth method may be more meaningful and does not require more complicated calculations when the projects have different service lives.

The payback period is a simple method that calculates the length of time it takes to pay back an initial investment. It is inaccurate but very easy to calculate.

## ENGINEERING ECONOMICS IN ACTION, PART 4B

### Doing It Right

Naomi stopped for coffee on her way back from Clem's office. She needed time to think about how to decide which potential forge shop investments were best. She wasn't sure that she knew what "best" meant. She got down her engineering economics text and looked at the table of contents. There were a couple of chapters on comparison methods that seemed to be what she wanted. She sat down with the coffee in her right hand and the text on her lap and hoped for an uninterrupted hour.

One read through the chapters was enough to remind Naomi of the main relevant ideas that she had learned in school. The first thing she had to do was decide whether the investments were independent or not. They clearly were not independent. It would not make sense to refurbish the current forging hammer and replace it with a mechanical press. Where potential investments were not independent, it was easiest to form mutually exclusive combinations as investment options. Naomi came up with seven options. She ranked the options by first cost, starting with the one with the lowest cost:

1. Refurbish the current machine.
2. Refurbish the current machine plus replace the materials-handling equipment.
3. Buy a manually operated mechanical press.
4. Buy a manual mechanical press plus replace the materials-handling equipment.
5. Buy an automated mechanical press.
6. Buy an automated mechanical press plus replace the materials-handling equipment.
7. Buy an automated mechanical press plus integrate it with the materials-handling equipment.

At this point, Naomi wasn't sure what to do next. There were different ways of comparing the options.

Naomi wanted a break from thinking about theory. She decided to take a look at Dave Sullivan's work. She started up her computer and opened up Dave's email. In it Dave apologized for dumping the work on her and invited Naomi to call him in Florida if she needed help. Naomi decided to call him. The phone was answered by



Dave's wife, Helena. After telling Naomi that her father was out of intensive care and was in good spirits, Helena turned the phone over to Dave.

"Hi, Naomi. How's it going?"

"Well, I'm trying to finish off the forge project you started. And I'm taking you up on your offer to consult."

"You have my attention. What's the problem?"

"Well, I've gotten started. I have formed seven mutually exclusive combinations of potential investments."

Naomi went on to explain her selection of alternatives.

"That sounds right, Naomi. I like the way you've organized that. Now, how are you going to make the choice?"

"I've just reread the present worth, annual worth, and payback period stuff, and of those three, present worth makes the most sense to me. I can just compare the present worths of the cash flows for each alternative, and the one whose present worth is highest is the best one. Annual worth is the same, but I don't see any good reason in this case to look at the costs on an annual basis."

"What about internal rate of return?"

"Well, actually, Dave, I haven't reviewed IRR yet. I'll need it, will I?"

"You will. Have a look at it, and also remember that your recommendation is for Burns and Kulkowski. Think about how they will be looking at your information."

"Thanks, Dave. I appreciate your help."

"No problem. This first one is important for you; let's make sure we do it right."

## P R O B L E M S

### A. Key Concepts

**4.1** The Alabaster Marble Company (AM) is considering opening three new quarries. One, designated T, is in Tusksarelooser County; a second, L, is in Lefant County; the third, M, is in Marxbro County. Marble is shipped mainly within a 500-kilometre range of its quarry because of its weight. The market within this range is limited. The returns that AM can expect from any of the quarries depend on how many quarries AM opens. Therefore, these potential projects are related.

- (a) Construct a set of mutually exclusive alternatives from these three potential projects.
- (b) The Lefant County quarry has very rich deposits of marble. This makes the purchase of mechanized cutter-loaders a reasonable investment at this quarry. Such loaders would not be considered at the other quarries. Construct a set of mutually exclusive alternatives from the set of quarry projects augmented by the potential mechanized cutter-loader project.
- (c) AM has decided to invest no more than \$2.5 million in the potential quarries. The first costs are as follows:

| Project       | First Cost    |
|---------------|---------------|
| T quarry      | \$0.9 million |
| L quarry      | 1.4 million   |
| M quarry      | 1.0 million   |
| Cutter-loader | 0.4 million   |

Construct a set of mutually exclusive alternatives that are feasible, given the investment limitation.

- 4.2** The intersection of King and Main streets needs widening and improvement. The possibilities include:
1. Widen King
  2. Widen Main
  3. Add a left-turn lane on King
  4. Add a left-turn lane on Main
  5. Add traffic lights at the intersection
  6. Add traffic lights at the intersection with advanced green for Main
  7. Add traffic lights at the intersection with advanced green for King

A left-turn lane can be installed only if the street in question is widened. A left-turn lane is necessary if the street has traffic lights with an advanced green. The city cannot afford to widen both streets. How many mutually exclusive projects are there?

- 4.3** Yun is deciding among a number of business opportunities. She can
- (a) Sell the X division of her company, Yunco
  - (b) Buy Barzoo's company, Barco
  - (c) Get new financing
  - (d) Expand into Quebec

There is no sense in getting new financing unless she is either buying Barco or expanding into Quebec. She can only buy Barco if she gets financing or sells the X division. She can only expand into Quebec if she has purchased Barco. The X division is necessary to compete in Quebec. What are the feasible projects she should consider?

- 4.4** Margaret has a project with a \$28 000 first cost that returns \$5000 per year over its 10-year life. It has a salvage value of \$3000 at the end of 10 years. If the MARR is 15 percent, what is the present worth of this project?
- 4.5** Nabil is considering buying a house while he is at university. The house costs \$200 000 today. Renting out part of the house and living in the rest over his five years at school will net, after expenses, \$2000 per month. He estimates that he will sell the house after five years for \$210 000. If Nabil's MARR is 6 percent compounded monthly, should he buy the house? Use present worth.
- 4.6** Appledale Dairy is considering upgrading an old ice-cream maker. Upgrading is available at two levels: moderate and extensive. Moderate upgrading costs \$6500 now and yields annual savings of \$3300 in the first year, \$3000 in the second year, \$2700 in the third year, and so on. Extensive upgrading costs \$10 550 and saves \$7600 in the first year. The savings then decrease by 20 percent each year thereafter. If the upgraded ice-cream maker will last for seven years, which upgrading option is better? Use a present worth comparison. Appledale's MARR is 8 percent.
- 4.7** A young software genius is selling the rights to a new video game he has developed. Two companies have offered him contracts. The first contract offers \$10 000 at the end of each year for the next five years, and then \$20 000 per year for the following 10 years. The second offers 10 payments, starting with \$10 000 at the end of the first year, \$13 000 at the end of the second, and so forth, increasing by \$3000 each year (i.e., the tenth payment will be  $\$10\ 000 + 9 \times \$3000$ ). Assume the genius uses a MARR of 9 percent. Which contract should he choose? Use a present worth comparison.

- 4.8** Water supply for an irrigation system can be obtained from a stream in some nearby mountains. Two alternatives are being considered, both of which have essentially infinite lives, provided proper maintenance is performed. The first is a concrete reservoir with a steel pipe system and the second is an earthen dam with a wooden aqueduct. Below are the costs associated with each.

Compare the present worths of the two alternatives using an interest rate of 8 percent. Which alternative should be chosen?

|   | Concrete Reservoir | Earthen Dam |
|---|--------------------|-------------|
| First cost  | \$500 000          | \$200 000   |
| Annual maintenance costs                                    | \$2000             | \$12 000    |
| Replacing the wood portion<br>of the aqueduct each 15 years | N/A                | \$100 000   |

- 4.9** Margaret has a project with a \$28 000 first cost that returns \$5000 per year over its 10-year life. It has a salvage value of \$3000 at the end of 10 years. If the MARR is 15 percent, what is the annual worth of this project?

- 4.10** Nabil is considering buying a house while he is at university. The house costs \$200 000 today. Renting out part of the house and living in the rest over his five years at school will net, after expenses, \$2000 per month. He estimates that he will sell the house after five years for \$210 000. If Nabil's MARR is 6 percent compounded monthly, should he buy the house? Use annual worth.

- 4.11** Midland Metalworking is examining a 750-tonne hydraulic press and a 600-tonne moulding press for purchase. Midland has only enough budget for one of them. If Midland's MARR is 12 percent and the relevant information is as given below, which press should it purchase? Use an annual worth comparison.

|                            | Hydraulic Press                                   | Moulding Press   |
|----------------------------|---|--|
| Initial cost               | \$275 000   | \$185 000  |
| Annual savings             | \$33 000  | \$24 500   |
| Annual maintenance<br>cost | \$2000, increasing by 15%<br>each year thereafter | \$1000, increasing by<br>\$350 each year<br>thereafter |
| Life                       | 15 years  | 10 years   |
| Salvage value              | \$19 250  | \$14 800   |

- 4.12** Tom is considering purchasing a \$24 000 car. After five years, he will be able to sell the vehicle for \$8000. Petrol costs will be \$2000 per year, insurance \$600 per year, and parking \$600 per year. Maintenance costs for the first year will be \$1000, rising by \$400 per year thereafter.

The alternative is for Tom to take taxis everywhere. This will cost an estimated \$6000 per year. Tom will rent a vehicle each year at a total cost (to year-end) of \$600 for the family vacation, if he has no car. If Tom values money at 11 percent annual interest, should he buy the car? Use an annual worth comparison method.

- 4.13** A chemical recovery system costs \$300 000 and saves \$52 800 each year of its seven-year life. The salvage value is estimated at \$75 000. The MARR is 9 percent. What is the net annual benefit or cost of purchasing the chemical recovery system? Use the capital recovery formula.
- 4.14** Savings of \$5600 per year can be achieved through either a \$14 000 machine (A) with a seven-year service life and a \$2000 salvage value, or a \$25 000 machine (B) with a ten-year service life and a \$10 000 salvage value. If the MARR is 9 percent, which machine is a better choice, and for what annual benefit or cost? Use annual worth and the capital recovery formula.
- 4.15** Sam is considering buying a new lawnmower. He has a choice between a Lawn Guy mower and a Bargain Joe's Clip Job mower. Sam has a MARR of 5 percent. The salvage value of each mower at the end of its service life is zero.

|                    | Lawn Guy | Clip Job |
|--------------------|----------|----------|
| First cost         | \$350    | \$120    |
| Life               | 10 years | 4 years  |
| Annual gas         | \$60     | \$40     |
| Annual maintenance | \$30     | \$60     |

- (a) Using the information above, determine which alternative is preferable. Use a present worth comparison and the least common multiple of the service lives.
- (b) For a four-year study period, what salvage value for the Lawn Guy mower would result in its being the preferred choice? What salvage value for the Lawn Guy would result in the Clip Job being the preferred choice?
- 4.16** Sam is buying a refrigerator. He has two choices. A used one, at \$475, should last him about three years. A new one, at \$1250, would likely last eight years. Both have a scrap value of zero. The interest rate is 8 percent.
- (a) Which refrigerator has a lower cost? (Use a present worth analysis with repeated lives. Assume operating costs are the same.)
- (b) If Sam knew that he could resell the new refrigerator after three years for \$1000, would this change the answer in part (a)? (Use a present worth analysis with a three-year study period. Assume operating costs are the same.)
- 4.17** Margaret has a project with a \$28 000 first cost that returns \$5000 per year over its 10-year life. It has a salvage value of \$3000 at the end of 10 years. If the MARR is 5 percent, what is the payback period of this project?
- 4.18** A new gizmo costs \$10 000. Maintenance costs \$2000 per year and labour savings are \$6567 per year. What is the gizmo's payback period?
- 4.19** Building a bridge will cost \$65 million. A round-trip toll of \$12 will be charged to all vehicles. Traffic projections are estimated to be 5000 per day. The operating and maintenance costs will be 20 percent of the toll revenue. Find the payback period (in years) for this project.
- 4.20** A new packaging machine will save Greene Cheese Ltd. \$3000 per year in reduced spoilage, \$2500 per year in labour, and \$1000 per year in packaging material. The new machine will have additional expenses of \$700 per year in maintenance and \$200 per year in energy. If it costs \$20 000 to purchase, what is its payback period? Assume that the savings are earned throughout the year, not just at year-end.

## B. Applications

- 4.21** IQ Computer assembles UNIX workstations at its plant. The current product line is nearing the end of its marketing life, and it is time to start production of one or more new products. The data for several candidates are shown below.

The maximum budget for research and development is \$300 000. A minimum of \$200 000 should be spent on these projects. It is desirable to spread out the introduction of new products, so if two products are to be developed together, they should have different lead times. Resource draw refers to the labour and space that are available to the new products; it cannot exceed 100 percent.

On the basis of the above information, determine the set of feasible mutually exclusive alternative projects that IQ Computers should consider.

|                                | Potential Product |          |           |          |
|--------------------------------|-------------------|----------|-----------|----------|
|                                | A                 | B        | C         | D        |
| Research and development costs | \$120 000         | \$60 000 | \$150 000 | \$75 000 |
| Lead time                      | 1 year            | 2 years  | 1 year    | 2 years  |
| Resource draw                  | 60%               | 50%      | 40%       | 30%      |

- 4.22** Angus Automotive has \$100 000 to invest in internal projects. The choices are:

| Project                 | Cost     |
|-------------------------|----------|
| 1. Line improvements    | \$20 000 |
| 2. New manual tester    | 30 000   |
| 3. New automatic tester | 60 000   |
| 4. Overhauling press    | 50 000   |

Only one tester may be bought and the press will not need overhauling if the line improvements are not made. What mutually exclusive project combinations are available if Angus Auto will invest in at least one project?

- 4.23** Nottawasaga Printing has four printing lines, each of which consists of three printing stations, A, B, and C. The company has allocated \$20 000 for upgrading the printing stations. Station A costs \$7000 and takes 10 days to upgrade. Station B costs \$5000 and takes 5 days, and station C costs \$3000 and takes 3 days. Due to the limited number of technicians, Nottawasaga can only upgrade one printing station at a time. That is, if it decides to upgrade two Bs, the total downtime will be 10 days. During the upgrading period, the downtime should not exceed 14 days in total. Also, at least two printing lines must be available at all times to satisfy the current customer demand. The entire line will not be available if any of the printing stations is turned off for upgrading. Nottawasaga Printing wants to know which line and which printing station to upgrade. Determine the feasible mutually exclusive combinations of lines and stations for Nottawasaga Printing.

- 4.24** Kiwidale Dairy is considering purchasing a new ice-cream maker. Two models, Smoothie and Creamy, are available and their information is given below.

- (a) What is Kiwidale's MARR that makes the two alternatives equivalent? Use a present worth comparison.

|                       | Smoothie | Creamy   |
|-----------------------|----------|----------|
| First cost            | \$15 000 | \$36 000 |
| Service life          | 12 years | 12 years |
| Annual profit         | \$4200   | \$10 800 |
| Annual operating cost | \$1200   | \$3520   |
| Salvage value         | \$2250   | \$5000   |

- (b) It turned out that the service life of Smoothie was 14 years. Which alternative is better on the basis of the MARR computed in part (a)? Assume that each alternative can be repeated indefinitely.
- 4.25 Margaret has a project with a \$28 000 first cost that returns \$5000 per year over its 10-year life. It has a salvage value of \$3000 at the end of 10 years. If the MARR is 15 percent, what is the future worth of this project after 10 years? What is the discounted payback period for this project?
- 4.26 Westmount Waxworks is considering buying a new wax melter for its line of replicas of statues of government leaders. There are two choices of supplier, Finedetail and Simplicity. Their proposals are as follows:

|               | Finedetail                  | Simplicity                  |
|---------------|-----------------------------|-----------------------------|
| Expected life | 7 years                     | 10 years                    |
| First cost    | \$200 000                   | \$350 000                   |
| Maintenance   | \$10 000/year + \$0.05/unit | \$20 000/year + \$0.01/unit |
| Labour        | \$1.25/unit                 | \$0.50/unit                 |
| Other costs   | \$6500/year + \$0.95/unit   | \$15 500/year + \$0.55/unit |
| Salvage value | \$5000                      | \$20 000                    |

Management thinks the company will sell about 30 000 replicas per year if there is stability in world governments. If the world becomes very unsettled so that there are frequent overturns of governments, sales may be as high as 200 000 units a year. Westmount Waxworks uses a MARR of 15 percent for equipment projects.

- (a) Who is the preferred supplier if sales are 30 000 units per year? Use an annual worth comparison.
- (b) Who is the preferred supplier if sales are 200 000 units per year? Use an annual worth comparison.
- (c) How sensitive is the choice of supplier to sales level? Experiment with sales levels between 30 000 and 200 000 units per year. At what sales level will the costs of the two melters be equal?
- 4.27 The City of Brandon is installing a new swimming pool in the municipal recreation centre. Two designs are under consideration, both of which are to be permanent (i.e., lasting forever). The first design is for a reinforced concrete pool that has a first cost of \$1 500 000. Every 10 years, the inner surface of the pool would have to be refinished and painted at a cost of \$200 000.

The second design consists of a metal frame and a plastic liner, which would have an initial cost of \$500 000. For this alternative, the plastic liner must be replaced every 5 years at a cost of \$100 000, and every 15 years the metal frame would need replacement at a cost of \$150 000. Extra insurance of \$5000 per year is required for the plastic liner (to cover repair costs if the liner leaks). The city's cost of long-term funds is 5 percent.

Determine which swimming pool design has the lower present cost.

- 4.28** Val is considering purchasing a new video plasma display panel to use with her notebook computer. One model, the XJ3, costs \$4500 new, while another, the Y19, sells for \$3200. Val figures that the XJ3 will last about three years, at which point it could be sold for \$1000, while the Y19 will last for only two years and will also sell for \$1000. Both panels give similar service, except that the Y19 is not suitable for client presentations. If she buys the Y19, about four times a year she will have to rent one similar to the XJ3, at a total year-end cost of about \$300. Using present worth and the least common multiple of the service lives, determine which display panel Val should buy. Val's MARR is 10 percent.
- 4.29** For Problem 4.28, Val has determined that the salvage value of the XJ3 after two years of service is \$1900. Which display panel is the better choice, on the basis of present worth with a two-year study period?
- 4.30** Diana usually uses a three-year payback period to determine if a project is acceptable. A recent project with uniform yearly savings over a five-year life had a payback period of almost exactly three years, so Diana decided to find the project's present worth to help determine if the project was truly justifiable. However, that calculation didn't help either since the present worth was exactly zero. What interest rate was Diana using to calculate the present worth? The project has no salvage value at the end of its five-year life.
- 4.31** The Biltmore Garage has lights in places that are difficult to reach. Management estimates that it costs about \$2 to change a bulb. Standard 100-watt bulbs with an expected life of 1000 hours are now used. Standard bulbs cost \$1. A long-life bulb that requires 90 watts for the same effective level of light is available. Long-life bulbs cost \$3. The bulbs that are difficult to reach are in use for about 500 hours a month. Electricity costs \$0.08/kilowatt-hour payable at the end of each month. Biltmore uses a 12 percent MARR (1 percent per month) for projects involving supplies.
- What minimum life for the long-life bulb would make its cost lower?
  - If the cost of changing bulbs is ignored, what is the minimum life for the long-life bulb for them to have a lower cost?
  - If the solutions are obtained by linear interpolation of the capital recovery factor, will the approximations underestimate or overstate the required life?
- 4.32** Ridgley Custom Metal Products (RCMP) must purchase a new tube bender. RCMP's MARR is 11 percent. The company is considering two models:

| Model | First Cost | Economic Life | Yearly Net Savings | Salvage Value |
|-------|------------|---------------|--------------------|---------------|
| T     | \$100 000  | 5 years       | \$50 000           | \$20 000      |
| A     | 150 000    | 5 years       | 60 000             | 30 000        |

- (a) Using the *present worth* method, which tube bender should it buy?
- (b) RCMP has discovered a third alternative, which has been added to the table below. Now which tube bender should it buy?

| Model | First Cost | Economic Life | Yearly Net Savings | Salvage Value |
|-------|------------|---------------|--------------------|---------------|
| T     | \$100 000  | 5 years       | \$50 000           | \$ 20 000     |
| A     | 150 000    | 5 years       | 60 000             | 30 000        |
| X     | 200 000    | 3 years       | 75 000             | 100 000       |

- 4.33 RCMP (see Problem 4.32, part (b)) can forecast demand for its products for only three years in advance. The salvage value after three years is \$40 000 for model T and \$80 000 for model A. Using the study period method, which of the three alternatives is best?
- 4.34 Using the annual worth method, which of the three tube benders should RCMP buy? The MARR is 11 percent. Use the data from Problem 4.32, part (b).
- 4.35 What is the payback period for each of the three alternatives from the RCMP problem? Use the data from Problem 4.32, part (b).
- 4.36 Data for two independent investment opportunities are shown below.

|                   | Machine A   | Machine B   |
|-------------------|-------------|-------------|
| Initial cost      | \$1 500 000 | \$2 000 000 |
| Revenues (annual) | \$ 900 000  | \$1 100 000 |
| Costs (annual)    | \$ 600 000  | \$ 800 000  |
| Scrap value       | \$ 100 000  | \$ 200 000  |
| Service life      | 5 years     | 10 years    |

- (a) For a MARR of 8 percent, should either, both, or neither machine be purchased? Use the annual worth method.
- (b) For a MARR of 8 percent, should either, both, or neither machine be purchased? Use the present worth method.
- (c) What are the payback periods for these machines? Should either, both, or neither machine be purchased, based on the payback periods? The required payback period for investments of this type is three years.
- 4.37 Two plans have been proposed for accumulating money for capital projects at Bobbin Bay Lighting. One idea is to put aside \$100 000 per year, independent of growth. The second is to start with a smaller amount, \$80 000 per year, but to increase this in proportion to the expected company growth. The money will accumulate interest at 10 percent, and the company is expected to grow about 5 percent per year. Which plan will accumulate more money in 10 years?

- 4.38** Derek has two choices for a heat-loss prevention system for the shipping doors at Kirkland Manufacturing. He can isolate the shipping department from the rest of the plant, or he can curtain off each shipping door separately. Isolation consists of building a permanent wall around the shipping area. It will cost \$60 000 and will save \$10 000 in heating costs per year. Plastic curtains around each shipping door will have a total cost of about \$5000, but will have to be replaced about once every two years. Savings in heating costs for installing the curtains will be about \$3000 per year. Use the payback period method to determine which alternative is better. Comment on the use of the payback period for making this decision.
- 4.39** Assuming that the wall built to isolate the shipping department in Problem 4.38 will last forever, and that the curtains have zero salvage value, compare the annual worths of the two alternatives. The MARR for Kirkland Manufacturing is 11 percent. Which alternative is better?

### C. More Challenging Problems

- 4.40** CB Electronix needs to expand its capacity. It has two feasible alternatives under consideration. Both alternatives will have essentially infinite lives.

*Alternative 1: Construct a new building of 20 000 square metres now.* The first cost will be \$2 000 000. Annual maintenance costs will be \$10 000. In addition, the building will need to be painted every 15 years (starting in 15 years) at a cost of \$15 000.

*Alternative 2: Construct a new building of 12 500 square metres now and an additional 7500 square metres in 10 years.* The first cost of the 12 500-square-metre building will be \$1 250 000. The annual maintenance costs will be \$5000 for the first 10 years (i.e., until the addition is built). The 7500-square-metre addition will have a first cost of \$1 000 000. Annual maintenance costs of the renovated building (the original building and the addition) will be \$11 000. The renovated building will cost \$15 000 to repaint every 15 years (starting 15 years after the addition is done).

Carry out an annual worth comparison of the two alternatives. Which is preferred if the MARR is 15 percent?

- 4.41** Katie's project has a five-year term, a first cost, no salvage value, and annual savings of \$20 000 per year. After doing present worth and annual worth calculations with a 15 percent interest rate, Katie notices that the calculated annual worth for the project is exactly three times the present worth. What is the project's present worth and annual worth? Should Katie undertake the project?
- 4.42** Nighhigh Newsagent wants to replace its cash register and is currently evaluating two models that seem reasonable. The information on the two alternatives, CR1000 and CRX, is shown in the table.

|                         | CR1000  | CRX     |
|-------------------------|---|---------|
| First cost              | \$680   | \$1100  |
| Annual savings          | \$245   | \$440   |
| Annual maintenance cost | \$35 in year 1, increasing by \$10 each year thereafter | \$60    |
| Service life            | 4 years   | 6 years |
| Scrap value             | \$100   | \$250   |

- (a) If Nighhigh Newsagent's MARR is 10 percent, which type of cash register should it choose? Use the present worth method.
- (b) For the less preferred type of cash register found in part (a), what scrap value would make it the preferred choice?
- 4.43** Xaviera is comparing two mutually exclusive projects, A and B, that have the same initial investment and the same present worth over their service lives. Wolfgang points out that, using the annual worth method, A is clearly better than B. What can be said about the service lives for the two projects?
- 4.44** Xaviera noticed that two mutually exclusive projects, A and B, have the same payback period and the same economic life, but A has a larger present worth than B does. What can be said about the size of the annual savings for the two projects?
- 4.45** Cleanville Environmental Services is evaluating two alternative methods for the disposing of municipal waste. The first involves developing a landfill site near the city. Costs of the site include \$1 000 000 start-up costs, \$100 000 closedown costs 30 years from now, and operating costs of \$20 000 per year. Starting in 10 years, it is expected that there will be revenues from user fees of \$30 000 per year. The alternative is to ship the waste out of the region. An area firm will agree to a long-term contract to dispose of the waste for \$130 000 per year. Using the *annual worth* method, which alternative is economically preferred for a MARR of 11 percent? Would this likely be the actual preferred choice?
- 4.46** Alfredo Auto Parts is considering investing in a new forming line for grille assemblies. For a five-year study period, the cash flows for two separate designs are shown below. Create a spreadsheet that will calculate the present worths for each project for a variable MARR. Through trial and error, establish the MARR at which the present worths of the two projects are exactly the same.

| Cash Flows for Grille Assembly Project |                |          |               |               |          |               |
|--|----------------|----------|---------------|---------------|----------|---------------|
|  | Automated Line |          |               | Manual Line   |          |               |
| Year                                   | Disbursements  | Receipts | Net Cash Flow | Disbursements | Receipts | Net Cash Flow |
| 0                                      | \$1 500 000    | \$ 0     | -\$1 500 000  | \$1 000 000   | \$ 0     | -\$1 000 000  |
| 1                                      | 50 000         | 300 000  | 250 000       | 20 000        | 200 000  | 180 000       |
| 2                                      | 60 000         | 300 000  | 240 000       | 25 000        | 200 000  | 175 000       |
| 3                                      | 70 000         | 300 000  | 230 000       | 30 000        | 200 000  | 170 000       |
| 4                                      | 80 000         | 300 000  | 220 000       | 35 000        | 200 000  | 165 000       |
| 5                                      | 90 000         | 800 000  | 710 000       | 40 000        | 200 000  | 160 000       |

- 4.47** Stayner Catering is considering setting up a temporary division to handle demand created by its city's special tourist promotion during the coming year. The company will invest in tables, serving equipment, and trucks for a one-year period. Labour is employed on a monthly basis. Warehouse space is rented monthly and revenue is generated monthly. The items purchased will be sold at the end of the year, but the salvage values are somewhat uncertain. Given below are the known or expected cash flows for the project.

| Month               | Purchase  | Labour Expenses | Warehouse Expenses | Revenue |
|---------------------|-----------|-----------------|--------------------|---------|
| January (beginning) | \$200 000 |                 |                    |         |
| January (end)       |           | \$ 2000         | \$3000             | \$ 2000 |
| February            |           | 2000            | 3000               | 2000    |
| March               |           | 2000            | 3000               | 2000    |
| April               |           | 2000            | 3000               | 2000    |
| May                 |           | 4000            | 3000               | 10 000  |
| June                |           | 10 000          | 6000               | 40 000  |
| July                |           | 10 000          | 6000               | 110 000 |
| August              |           | 10 000          | 6000               | 60 000  |
| September           |           | 4000            | 3000               | 30 000  |
| October             |           | 2000            | 3000               | 10 000  |
| November            |           | 2000            | 3000               | 5000    |
| December            | Salvage?  | 2000            | 3000               | 2000    |

For an interest rate of 12 percent compounded monthly, create a spreadsheet that calculates and graphs the present worth of the project for a range of salvage values of the purchased items from 0 percent to 100 percent of the purchase price. Should Stayner Catering go ahead with this project?

- 4.48** Alfredo Auto Parts has two options for increasing efficiency. It can expand the current building or keep the same building but remodel the inside layout. For a five-year study period, the cash flows for the two options are shown below. Construct a spreadsheet that will calculate the present worth for each option with a variable MARR. By trial and error, determine the MARR at which the present worths of the two options are equivalent.

| Year | Expansion Option |          |               | Remodelling Option |          |               |
|------|------------------|----------|---------------|--------------------|----------|---------------|
|      | Disbursements    | Receipts | Net Cash Flow | Disbursements      | Receipts | Net Cash Flow |
| 0    | \$850 000        | \$ 0     | -\$850 000    | \$230 000          | \$ 0     | -\$230 000    |
| 1    | 25 000           | 200 000  | 175 000       | 9000               | 80 000   | 71 000        |
| 2    | 30 000           | 225 000  | 195 000       | 11 700             | 80 000   | 68 300        |
| 3    | 35 000           | 250 000  | 215 000       | 15 210             | 80 000   | 64 790        |
| 4    | 40 000           | 275 000  | 235 000       | 19 773             | 80 000   | 60 227        |
| 5    | 45 000           | 300 000  | 255 000       | 25 705             | 80 000   | 54 295        |

- 4.49** Cleanville Environmental Services is considering investing in a new water treatment system. On the basis of the information given below for two alternatives, a fully automated and a partially automated system, construct a spreadsheet for computing the annual worths for each alternative with a variable MARR. Through trial and error, determine the MARR at which the annual worths of the two alternatives are equivalent.

|      | Fully Automated System |          |               | Partially Automated System |          |               |
|------|------------------------|----------|---------------|----------------------------|----------|---------------|
| Year | Disbursements          | Receipts | Net Cash Flow | Disbursements              | Receipts | Net Cash Flow |
| 0    | \$1 000 000            | \$ 0     | -\$1 000 000  | \$650 000                  | \$ 0     | -\$650 000    |
| 1    | 30 000                 | 300 000  | 270 000       | 30 000                     | 220 000  | 190 000       |
| 2    | 30 000                 | 300 000  | 270 000       | 30 000                     | 220 000  | 190 000       |
| 3    | 80 000                 | 300 000  | 220 000       | 35 000                     | 220 000  | 185 000       |
| 4    | 30 000                 | 300 000  | 270 000       | 35 000                     | 220 000  | 185 000       |
| 5    | 30 000                 | 300 000  | 270 000       | 40 000                     | 220 000  | 180 000       |
| 6    | 80 000                 | 300 000  | 220 000       | 40 000                     | 220 000  | 180 000       |
| 7    | 30 000                 | 300 000  | 270 000       | 45 000                     | 220 000  | 175 000       |
| 8    | 30 000                 | 300 000  | 270 000       | 45 000                     | 220 000  | 175 000       |
| 9    | 80 000                 | 300 000  | 220 000       | 50 000                     | 220 000  | 170 000       |
| 10   | 30 000                 | 300 000  | 270 000       | 50 000                     | 220 000  | 170 000       |

- 4.50** Fred has projects to consider for economic feasibility. All of his projects consist of a first cost  $P$  and annual savings  $A$ . His rule of thumb is to accept all projects for which the series present worth factor (for the appropriate MARR and service life) is equal to or greater than the payback period. Is this a sensible rule?

### MINI-CASE 4.1

#### Rockwell International

The Light Vehicle Division of Rockwell International makes seat-slide assemblies for the automotive industry. It has two major classifications for investment opportunities: developing new products to be manufactured and sold, and developing new machines to improve production. The overall approach to assessing whether an investment should be made depends on the nature of the project.

In evaluating a new product, it considers the following:

1. *Marketing strategy:* Does it fit the business plan for the company?
2. *Workforce:* How will it affect human resources?
3. *Margins:* The product should generate appropriate profits.
4. *Cash flow:* Positive cash flow is expected within two years.

In evaluating a new machine, it considers the following:

1. *Cash flow:* Positive cash flow is expected within a limited time period.
2. *Quality issues:* For issues of quality, justification is based on cost avoidance rather than positive cash flow.
3. *Cost avoidance:* Savings should pay back an investment within one year.

### Discussion

All companies consider more than just the economics of a decision. Most take into account the other issues—often called *intangibles*—by using managerial judgment in an informal process. Others, like Rockwell International, explicitly consider a selection of intangible issues.

The trend today is to carefully consider several intangible issues, either implicitly or explicitly. Human resource issues are particularly important since employee enthusiasm and commitment have significant repercussions. Environmental impacts of a decision can affect the image of the company. Health and safety is another intangible with significant effects.

However, the economics of the decision is usually (but not always) the single most important factor in a decision. Also, economics is the factor that is usually the easiest to measure.

### Questions

1. Why do you think Rockwell International has different issues to consider depending on whether an investment is a new product or a new machine?
2. For each of the issues mentioned, describe how it would be measured. How would you determine if it is worth investing in a new product or new machine with respect to that issue?
3. There are two kinds of errors that can be made. The first is that an investment is made when it should not be, and the second is that an investment is not made when it should be. Describe examples of both kinds of errors for both products and machines (four examples in total) if the issues listed for Rockwell International are strictly followed. What are some sensible ways to prevent such errors?

## Appendix 4A The MARR and the Cost of Capital

For a business to survive, it must be able to earn a high enough return to induce investors to put money into the company. The minimum rate of return required to get investors to invest in a business is that business's **cost of capital**. A company's cost of capital is also its minimum acceptable rate of return for projects—its MARR. This appendix reviews how the cost of capital is determined. We first look at the relation between risk and the cost of capital. Then we discuss sources of capital for large businesses and small businesses.

### 4A.1 Risk and the Cost of Capital

There are two main forms of investment in a company, *debt* and *equity*. Investors in a company's debt are lending money to the company. The loans are contracts that give lenders rights to repayment of their loans and to interest at predetermined interest rates. Investors in a company's equity are the owners of the company. They hold rights to the residual after all contractual payments, including those to lenders, are made.

Investing in equity is more risky than investing in debt. Equity owners are paid only if the company first meets its contractual obligations to lenders. This higher risk means that equity owners require an expectation of a greater return on average than the interest rate paid to debt holders. Consider a simple case in which a company has three possible

performance levels—weak results, normal results, and strong results. Investors do not know which level will actually occur. Each level is equally probable. To keep the example simple, we assume that all after-tax income is paid to equity holders as dividends so that there is no growth. The data are shown in Table 4A.1.

We see that, no matter what happens, lenders will get a return of 10 percent:

$$0.1 = \frac{10\,000}{100\,000}$$

**Table 4A.1 Cost of Capital Example**

|   | Possible Performance Levels |                |                |
|---|-----------------------------|----------------|----------------|
|   | Weak Results                | Normal Results | Strong Results |
| Net operating income (\$/year) <sup>1</sup> | 40 000                      | 100 000        | 160 000        |
| Interest payment (\$/year)                  | 10 000                      | 10 000         | 10 000         |
| Net income before tax (\$/year)             | 30 000                      | 90 000         | 150 000        |
| Tax at 40% (\$/year)                        | 12 000                      | 36 000         | 60 000         |
| After-tax income = Dividends (\$/year)      | 18 000                      | 54 000         | 90 000         |
| Debt (\$)                                   | 100 000                     | 100 000        | 100 000        |
| Value of shares (\$)                        | 327 273                     | 327 273        | 327 273        |

<sup>1</sup>Net operating income per year is revenue per year minus cost per year.

Owners get one of three possible returns:

$$5.5\% \left( 0.055 = \frac{18\,000}{327\,273} \right),$$

$$16.5\% \left( 0.165 = \frac{54\,000}{327\,273} \right), \text{ or}$$

$$27.5\% \left( 0.275 = \frac{90\,000}{327\,273} \right)$$

These three possibilities average out to 16.5 percent. If things are good, owners do better than lenders. If things are bad, owners do worse. But their average return is greater than returns to lenders.

The lower rate of return to lenders means that companies would like to get their capital with debt. However, reliance on debt is limited for two reasons.

1. If a company increases the share of capital from debt, it increases the chance that it will not be able to meet the contractual obligations to lenders. This means the company will be bankrupt. Bankruptcy may lead to reorganizing the company or possibly closing the company. In either case, bankruptcy costs may be high.
2. Lenders are aware of the dangers of high reliance on debt and will therefore limit the amount they lend to the company.

## 4A.2 | Company Size and Sources of Capital

---

Large, well-known companies can secure capital both by borrowing and by selling ownership shares with relative ease because there will be ready markets for their shares as well as any debt instruments, like bonds, they may issue. These companies will seek ratios of debt to equity that balance the marginal advantages and disadvantages of debt financing. Hence, the cost of capital for large, well-known companies is a weighted average of the costs of borrowing and of selling shares, which is referred to as the **weighted average cost of capital**. The weights are the fractions of total capital that come from the different sources. If market conditions do not change, a large company that seeks to raise a moderate amount of additional capital can do so at a stable cost of capital. This cost of capital is the company's MARR.

We can compute the after-tax cost of capital for the example shown in Table 4A.1 as follows.

Weighted average cost of capital

$$= 0.1 \left( \frac{100\,000}{427\,273} \right) + 0.165 \left( \frac{327\,273}{427\,273} \right) = 0.150$$

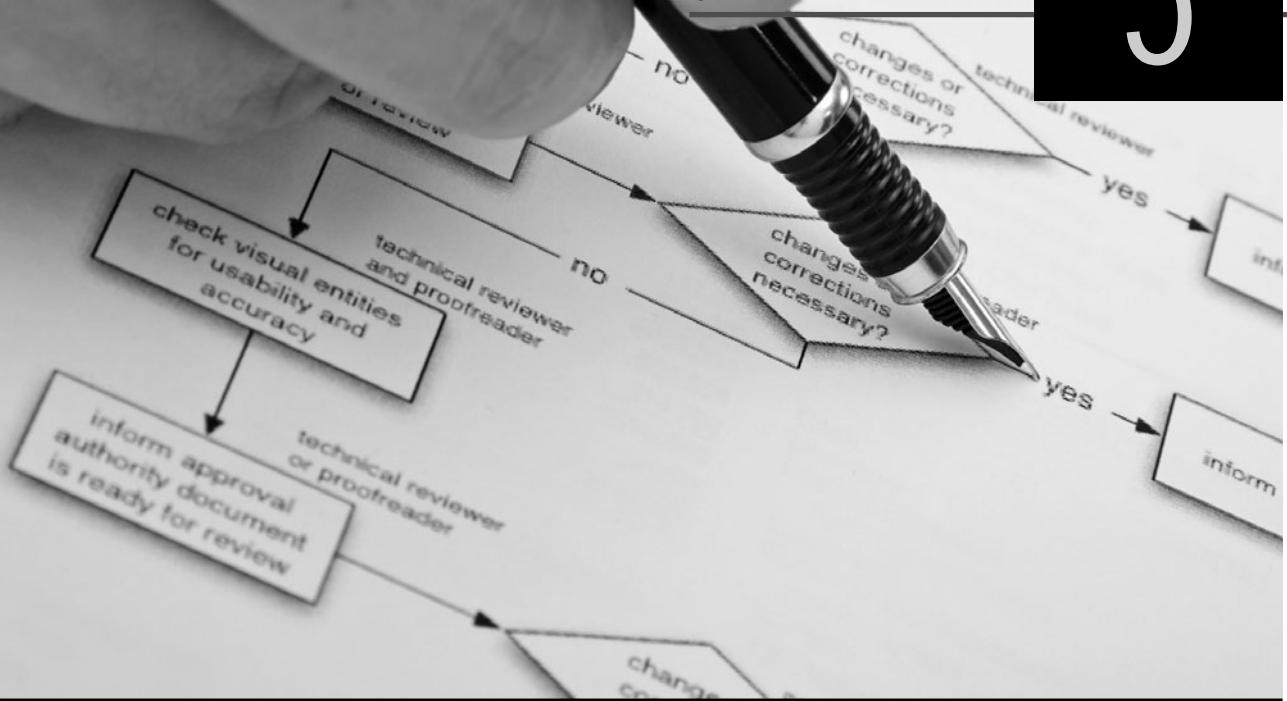
This company has a cost of capital of about 15 percent.

For smaller, less well-known companies, raising capital may be more difficult. Most investors in large companies are not willing to invest in unknown small companies. At start-up, a small company may rely entirely on the capital of the owners and their friends and relatives. Here the cost of capital is the opportunity cost for the investors.

If a new company seeks to grow more rapidly than the owners' investment plus cash flow permits, the next source of capital with the lowest cost is usually a bank loan. Bank loans are limited because banks are usually not willing to lend more than some fraction of the amount an owner(s) puts into a business.

If bank loans do not add up to sufficient funds, then the company usually has two options. One option is the sale of financial securities such as stocks and bonds through stock exchanges that specialize in small, speculative companies. Another option is venture capital. Venture capitalists are investors who specialize in investing in new companies. The cost of evaluating new companies is usually high and so is the risk of investing in them. Together, these factors usually lead venture capitalists to want to put enough money into a small company so that they will have enough control over the company.

In general, new equity investment is very expensive for small companies. Studies have shown that venture capitalists typically require the expectation of at least a 35 percent rate of return after tax. Raising funds on a stock exchange is usually even more expensive than getting funding from a venture capitalist.



# Comparison Methods Part 2

Engineering Economics in Action, Part 5A: What's Best? Revisited

- 5.1 Introduction
- 5.2 The Internal Rate of Return
- 5.3 Internal Rate of Return Comparisons
  - 5.3.1 IRR for Independent Projects
  - 5.3.2 IRR for Mutually Exclusive Projects
  - 5.3.3 Multiple IRRs
  - 5.3.4 External Rate of Return Methods
  - 5.3.5 When to Use the ERR
- 5.4 Rate of Return and Present/Annual Worth Methods Compared
  - 5.4.1 Equivalence of Rate of Return and Present/Annual Worth Methods
  - 5.4.2 Why Choose One Method Over the Other?

## Review Problems

### Summary

Engineering Economics in Action, Part 5B: The Invisible Hand

## Problems

### Mini-Case 5.1: The Galore Creek Project

## ENGINEERING ECONOMICS IN ACTION, PART 5A

### What's Best? Revisited

Clem had said, “I have to make a recommendation to Ed Burns and Anna Kulkowski for their Wednesday meeting on this forging hammer in the South Shop. I’d like you to handle it.” Dave Sullivan, who had started the project, had gone to Florida to see his sick father-in-law. Naomi welcomed the opportunity, but she still had to figure out exactly what to recommend.

Naomi looked carefully at the list of seven mutually exclusive alternatives for replacing or refurbishing the machine. Present worth could tell her which of the seven was “best,” but present worth was just one of several comparison methods. Which method should she use?

Dave would help more, if she asked him. In fact, he could no doubt tell her exactly what to do, if she wanted. But this one she knew she could handle, and it was a matter of pride to do it herself. Opening her engineering economics textbook, she read on.

## 5.1

### Introduction

In Chapter 4, we showed how to structure projects so that they were either independent or mutually exclusive. The present worth, annual worth, and payback period methods for evaluating projects were also introduced. This chapter continues on the theme of comparing projects by presenting a commonly used but somewhat more complicated comparison method called the *internal rate of return*, or IRR.

Although the IRR method is widely used, all the comparison methods have value in particular circumstances. Selecting which method to use is also covered in this chapter. It is also shown that the present worth, annual worth, and IRR methods all result in the same recommendations for the same problem. We close this chapter with a chart summarizing the strengths and weaknesses of the four comparison methods presented in Chapters 4 and 5.

## 5.2

### The Internal Rate of Return

Investments are undertaken with the expectation of a return in the form of future earnings. One way to measure the return from an investment is as a rate of return per dollar invested—in other words, as an interest rate. The rate of return usually calculated for a project is known as the *internal rate of return (IRR)*. The adjective *internal* refers to the fact that the internal rate of return depends only on the cash flows due to the investment. The internal rate of return is that interest rate at which a project just breaks even. The meaning of the IRR is most easily seen with a simple example.

#### EXAMPLE

#### 5.1

Suppose \$100 is invested today in a project that returns \$110 in one year. We can calculate the IRR by finding the interest rate at which \$100 now is equivalent to \$110 at the end of one year:

$$P = F(P/F, i^*, 1)$$

$$100 = 110(P/F, i^*, 1)$$

$$100 = \frac{110}{1 + i^*}$$

where  $i^*$  is the internal rate of return.

Solving this equation gives a rate of return of 10 percent. In a simple example like this, the process of finding an internal rate of return is finding the interest rate that makes the present worth of benefits equal to the first cost. This interest rate is the IRR. ■

Of course, cash flows associated with a project will usually be more complicated than in the example above. A more formal definition of the IRR is stated as follows. The **internal rate of return (IRR)** on an investment is the interest rate,  $i^*$ , such that, when all cash flows associated with the project are discounted at  $i^*$ , the present worth of the cash inflows equals the present worth of the cash outflows. That is, the project just breaks even. An equation that expresses this is

$$\sum_{t=0}^T \frac{(R_t - D_t)}{(1 + i^*)^t} = 0 \quad (5.1)$$

where

$R_t$  = the cash inflow (receipts) in period  $t$

$D_t$  = the cash outflow (disbursements) in period  $t$

$T$  = the number of time periods

$i^*$  = the internal rate of return

Since Equation (5.1) can also be expressed as

$$\sum_{t=0}^T R_t(1 + i^*)^{-t} = \sum_{t=0}^T D_t(1 + i^*)^{-t}$$

it can be seen that, in order to calculate the IRR, one sets the disbursements equal to the receipts and solves for the unknown interest rate. For this to be done, the disbursements and receipts must be comparable as a present worth, a uniform series, or a future worth. That is, use

- PW(disbursements) = PW(receipts) and solve for the unknown  $i^*$ ,
- AW(disbursements) = AW(receipts) and solve for the unknown  $i^*$ , or
- FW(disbursements) = FW(receipts) and solve for the unknown  $i^*$ .

The IRR is usually positive, but it can be negative as well. A negative IRR means that the project is losing money rather than earning it.

We usually solve the equations for the IRR by trial and error, as there is no explicit means of solving Equation (5.1) for projects where the number of periods is large. A spreadsheet provides a quick way to perform trial-and-error calculations; most spreadsheet programs also include a built-in IRR function.

## EXAMPLE

### 5.2

Clem is considering buying a tuxedo. It would cost \$500 but would save him \$160 per year in rental charges over its five-year life. What is the IRR for this investment?

As illustrated in Figure 5.1, Clem's initial cash outflow for the purchase would be \$500. This is an up-front outlay relative to continuing to rent tuxedos. The investment would create a saving of \$160 per year over the five-year life of the tuxedo. These savings can be viewed as a series of receipts relative to rentals. The IRR of Clem's investment can be found by determining what interest rate makes the present worth of the disbursements equal to the present worth of the cash inflows.

$$\text{Present worth of disbursements} = 500$$

$$\text{Present worth of receipts} = 160(P/A, i^*, 5)$$

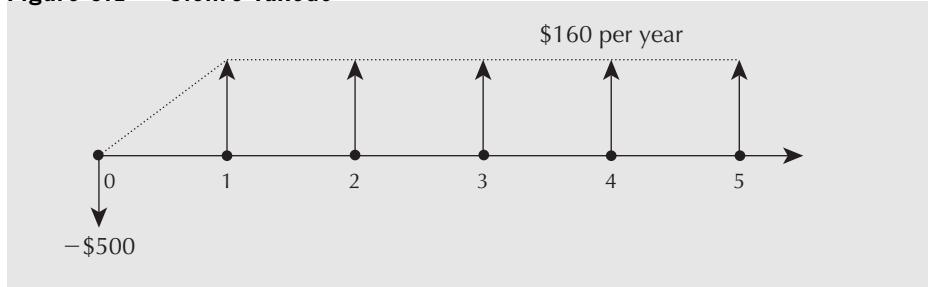
Setting the two equal,

$$500 = 160(P/A, i^*, 5)$$

$$(P/A, i^*, 5) = 500/160$$

$$= 3.125$$

**Figure 5.1 Clem's Tuxedo**



From the interest factor tables, we find that

$$(P/A, 15\%, 5) = 3.3521$$

$$(P/A, 20\%, 5) = 2.9906$$

Interpolating between  $(P/A, 15\%, 5)$  and  $(P/A, 20\%, 5)$  gives

$$\begin{aligned} i^* &= 15\% + (5\%)[(3.125 - 3.3521)/(2.9906 - 3.3521)] \\ &= 18.14\% \end{aligned}$$

An alternative way to get the IRR for this problem is to convert all cash outflows and inflows to equivalent annuities over the five-year period. This will yield the same result as when present worth was used.

$$\text{Annuity equivalent to the disbursements} = 500(A/P, i^*, 5)$$

$$\text{Annuity equivalent to the receipts} = 160$$

Again, setting the two equal,

$$500(A/P, i^*, 5) = 160$$

$$\begin{aligned} (A/P, i^*, 5) &= 160/500 \\ &= 0.32 \end{aligned}$$

From the interest factor tables,

$$(A/P, 15\%, 5) = 0.29832$$

$$(A/P, 20\%, 5) = 0.33438$$

An interpolation gives

$$\begin{aligned} i^* &= 15\% + 5\%[(0.32 - 0.29832)/(0.33438 - 0.29832)] \\ &\cong 18.0\% \end{aligned}$$

Note that there is a slight difference in the answers, depending on whether the disbursements and receipts were compared as present worths or as annuities. This difference is due to the small error induced by the linear interpolation. With Excel, the IRR is found as follows:

$$i^* = \text{IRR } (\text{A1:A6}) = 18.031\%$$

where cells A1:A6 are  $(-500, 160, 160, 160, 160, 160)$ , respectively. ■

## 5.3 Internal Rate of Return Comparisons

In this section, we show how the internal rate of return can be used to decide whether a project should be accepted. We first show how to use the IRR to evaluate independent projects. Then we show how to use the IRR to decide which of a group of mutually exclusive alternatives to accept. We then show that it is possible for a project to have more than one IRR. Finally, we show how to handle this difficulty by using an *external rate of return*.

### 5.3.1 IRR for Independent Projects

Recall from Chapter 4 that projects under consideration are evaluated using the MARR, and that any independent project that has a present or annual worth equal to or exceeding zero should be accepted. The principle for the IRR method is analogous. We will invest in any project that has an IRR equal to or exceeding the MARR. Just as projects with a zero present or annual worth are marginally acceptable, projects with  $\text{IRR} = \text{MARR}$  have a marginally acceptable rate of return (by definition of the MARR).

Also analogous to Chapter 4, when we perform a rate of return comparison on several independent projects, the projects must have equal lives. If this is not the case, then the approaches covered in Section 4.4.4 on page 97 (Comparison of Alternatives With Unequal Lives) must be used.

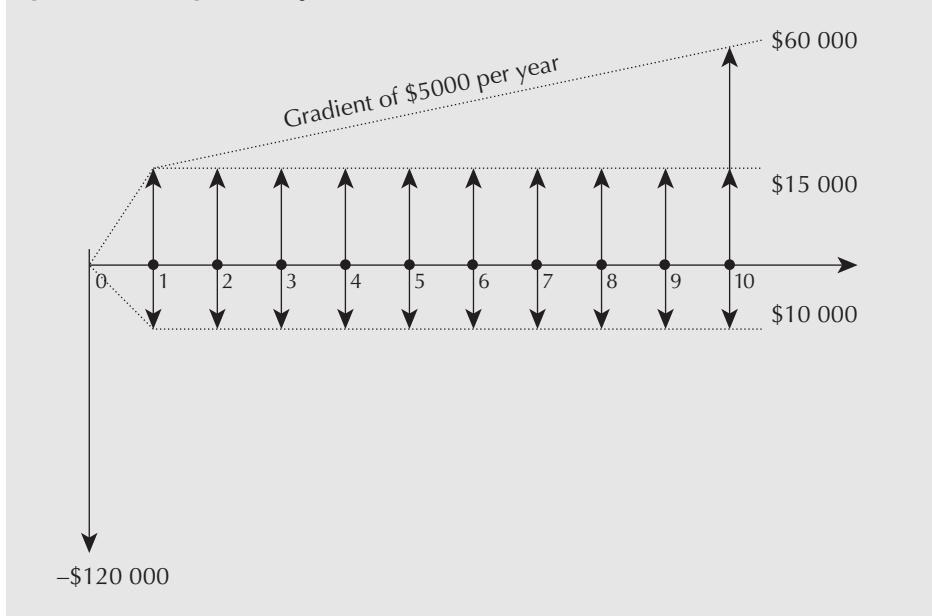
#### EXAMPLE

#### 5.3

The High Society Baked Bean Co. is considering a new canner. The canner costs \$120 000 and will have a scrap value of \$5000 after its 10-year life. Given the expected increases in sales, the total savings due to the new canner, compared with continuing with the current operation, will be \$15 000 the first year, increasing by \$5000 each year thereafter. Total extra costs due to the more complex equipment will be \$10 000 per year. The MARR for High Society is 12 percent. Should it invest in the new canner?

The cash inflows and outflows for this problem are summarized in Figure 5.2. We need to compute the internal rate of return in order to decide if High Society should buy the canner. There are several ways we can do this. In this problem, equating annual outflows and receipts appears to be the easiest approach, because most of the cash flows are already stated on a yearly basis.

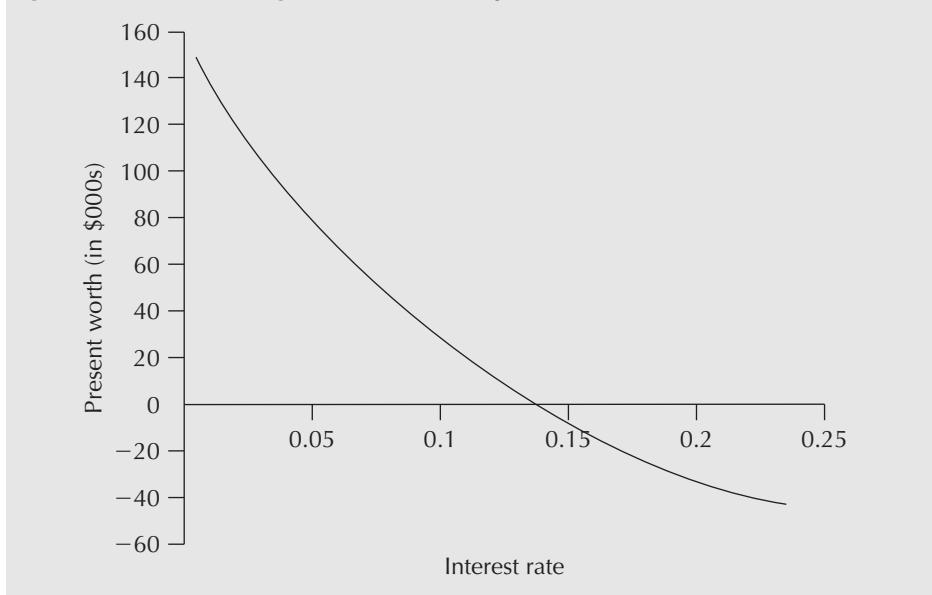
$$\begin{aligned} 5000(A/F, i^*, 10) + 15\,000 + 5000(A/G, i^*, 10) \\ - 120\,000(A/P, i^*, 10) - 10\,000 = 0 \end{aligned}$$

**Figure 5.2 High Society Baked Bean Canner**

Dividing by 5000,

$$(A/F, i^*, 10) + 1 + (A/G, i^*, 10) - 24(A/P, i^*, 10) = 0$$

The IRR can be found by trial and error alone, by trial and error and linear interpolation, or by a spreadsheet IRR function. A trial-and-error process is particularly easy using a spreadsheet, so this is often the best approach. A good starting point for the process is at zero interest. A graph (Figure 5.3) derived from the spreadsheet indicates that the IRR is between 13 percent and 14 percent. This may be good enough for a decision, since it exceeds the MARR of 12 percent.

**Figure 5.3 Estimating the IRR for Example 5.3**

If finer precision is required, there are two ways to proceed. One way is to use a finer grid on the graph—for example, one that covers 13 percent to 14 percent. The other way is to interpolate between 13 percent and 14 percent. We shall first use the interest factor tables to show that the IRR is indeed between 13 percent and 14 percent. Next we will interpolate between 13 percent and 14 percent.

First, at  $i = 13$  percent, we have

$$\begin{aligned}(A/F, 13\%, 10) + 1 + (A/G, 13\%, 10) - 24(A/P, 13\%, 10) \\= 0.05429 + 1 + 3.5161 - 24(0.18429) \\ \approx 0.1474\end{aligned}$$

The result is a bit too high. A higher interest rate will reduce the annual worth of the benefits more than the annual worth of the costs, since the benefits are spread over the life of the project while most of the costs are early in the life of the project.

At  $i = 14$  percent, we have

$$\begin{aligned}(A/F, 14\%, 10) + 1 + (A/G, 14\%, 10) - 24(A/P, 14\%, 10) \\= 0.05171 + 1 + 3.4489 - 24(0.19171) \\ \approx -0.1004\end{aligned}$$

This confirms that the IRR of the investment is between 13 percent and 14 percent. A good approximation to the IRR can be found by linearly interpolating:

$$\begin{aligned}i^* &= 13\% + (0 - 0.1474)/(0.1004 - 0.1474) \\&\approx 13.6\%\end{aligned}$$

To find the IRR using Excel, compute the net cash flows for each period, and then apply the IRR function to the range of cells containing cash flows, as shown in the tables below.



| A        | B             | C          | D       | E                   | F             |
|----------|---------------|------------|---------|---------------------|---------------|
| 1        | IRR =         | 13.60%     |         |                     |               |
| 2        | Net Cash Flow | First Cost | Savings | Additional Expenses | Salvage Value |
| 3 Period |               |            |         |                     |               |
| 4 0      | -120 000      | -120 000   |         |                     |               |
| 5 1      | 5 000         |            | 15 000  | -10 000             |               |
| 6 2      | 10 000        |            | 20 000  | -10 000             |               |
| 7 3      | 15 000        |            | 25 000  | -10 000             |               |
| 8 4      | 20 000        |            | 30 000  | -10 000             |               |
| 9 5      | 25 000        |            | 35 000  | -10 000             |               |
| 10 6     | 30 000        |            | 40 000  | -10 000             |               |
| 11 7     | 35 000        |            | 45 000  | -10 000             |               |
| 12 8     | 40 000        |            | 50 000  | -10 000             |               |
| 13 9     | 45 000        |            | 55 000  | -10 000             |               |
| 14 10    | 55 000        |            | 60 000  | -10 000             | 5 000         |

| A        | B             | C            | D                   | E                   | F             |
|----------|---------------|--------------|---------------------|---------------------|---------------|
| 1        | IRR =         | =IRR(B4:B14) |                     |                     |               |
| 2        | Net Cash Flow | First Cost   |                     |                     |               |
| 3 Period |               |              | Savings             | Additional Expenses | Salvage Value |
| 4 0      | =SUM(C4:F4)   | -120000      | 0                   |                     |               |
| 5 1      | =SUM(C5:F5)   |              | =15000+(A5-1)*5000  | -10000              |               |
| 6 2      | =SUM(C6:F6)   |              | =15000+(A6-1)*5000  | -10000              |               |
| 7 3      | =SUM(C7:F7)   |              | =15000+(A7-1)*5000  | -10000              |               |
| 8 4      | =SUM(C8:F8)   |              | =15000+(A8-1)*5000  | -10000              |               |
| 9 5      | =SUM(C9:F9)   |              | =15000+(A9-1)*5000  | -10000              |               |
| 10 6     | =SUM(C10:F10) |              | =15000+(A10-1)*5000 | -10000              |               |
| 11 7     | =SUM(C11:F11) |              | =15000+(A11-1)*5000 | -10000              |               |
| 12 8     | =SUM(C12:F12) |              | =15000+(A12-1)*5000 | -10000              |               |
| 13 9     | =SUM(C13:F13) |              | =15000+(A13-1)*5000 | -10000              |               |
| 14 10    | =SUM(C14:F14) |              | =15000+(A14-1)*5000 | -10000              | 5000          |

The IRR for the investment is approximately 13.6 percent. Since this is greater than the MARR of 12 percent, the company should buy the new canner. Note again that it was not actually necessary to determine where in the range of 13 percent to 14 percent the IRR fell. It was enough to demonstrate that it was 12 percent or more. ■

In summary, if there are several independent projects, the IRR for each is calculated separately, and, in the absence of budget constraints, those having an IRR equal to or exceeding the MARR should be chosen.

### 5.3.2 IRR for Mutually Exclusive Projects

Choice among mutually exclusive projects using the IRR is a bit more involved. Some insight into the complicating factors can be obtained from an example that involves two mutually exclusive alternatives. It illustrates that the best choice is not necessarily the alternative with the highest IRR.

**EXAMPLE**
**5.4**

Consider two investments. The first costs \$1 today and returns \$2 in one year. The second costs \$1000 and returns \$1900 in one year. Which is the preferred investment? Your MARR is 70 percent.

The first project has an IRR of 100 percent:

$$\begin{aligned} -1 + 2(P/F, i^*, 1) &= 0 \\ (P/F, i^*, 1) &= 1/2 = 0.5 \\ i^* &= 100\% \end{aligned}$$

The second has an IRR of 90 percent:

$$\begin{aligned} -1000 + 1900(P/F, i^*, 1) &= 0 \\ (P/F, i^*, 1) &= 1000/1900 = 0.52631 \\ i^* &= 90\% \end{aligned}$$

If these were independent projects, both would be acceptable since their IRRs exceed the MARR. If one of the two projects must be chosen, it might be tempting to choose the first project, the alternative with the larger rate of return. However, this approach is incorrect because it can overlook projects that have a rate of return equal to or greater than the MARR, but don't have the maximum IRR. In the example, the correct approach is to first observe that the least cost investment provides a rate of return that exceeds the MARR. The next step is to find the rate of return on the more expensive investment to see if the *incremental* investment has a rate of return equal to or exceeding the MARR. The incremental investment is the additional \$999 that would be invested if the second investment was taken instead of the first:

$$\begin{aligned} -(1000 - 1) + (1900 - 2)(P/F, i^*, 1) &= 0 \\ (P/F, i^*, 1) &= 999/1898 = 0.52634 \\ i^* &= 89.98\% \end{aligned}$$

Indeed, the incremental investment has an IRR exceeding 70 percent and thus the second investment should be chosen. ■

The next example illustrates the process again, showing this time how an incremental investment is not justified.

**EXAMPLE**
**5.5**

Monster Meats can buy a new meat slicer system for \$50 000. The company estimates it will save \$11 000 per year in labour and operating costs. The same system with an automatic loader is \$68 000, and will save approximately \$14 000 per year. The life of either system is thought to be eight years. Monster Meats has three feasible alternatives:

| Alternative                       | First Cost | Annual Savings |
|-----------------------------------|------------|----------------|
| “Do nothing”                      | \$ 0       | \$ 0           |
| Meat slicer alone                 | 50 000     | 11 000         |
| Meat slicer with automatic loader | 68 000     | 14 000         |

Monster Meats uses a MARR of 12 percent for this type of project. Which alternative is better?

We first consider the system without the loader. Its IRR is 14.5 percent, which exceeds the MARR of 12 percent. This can be seen by solving for  $i^*$  in

$$-50\,000 + 11\,000(P/A, i^*, 8) = 0$$

$$(P/A, i^*, 8) = 50\,000/11\,000$$

$$(P/A, i^*, 8) = 4.545$$

From the interest factor tables, or by trial and error with a spreadsheet,

$$(P/A, 14\%, 8) = 4.6388$$

$$(P/A, 15\%, 8) = 4.4873$$

By interpolation or further trial and error,

$$i^* \cong 14.6 \text{ percent}$$

The slicer alone is thus economically justified and is better than the “do nothing” alternative.

We now consider the system with the slicer and loader. Its IRR is 12.5 percent, which may be seen by solving for  $i^*$  in

$$-68\,000 + 14\,000(P/A, i^*, 8) = 0$$

$$(P/A, i^*, 8) = 68\,000/14\,000$$

$$(P/A, i^*, 8) = 4.857$$

$$(P/A, 12\%, 8) = 4.9676$$

$$(P/A, 13\%, 8) = 4.7987$$

$$i^* \cong 12.5\%$$

The IRR of the meat slicer and automatic loader is about 12.5 percent, which on the surface appears to meet the 12 percent MARR requirement. But, on the incremental investment, Monster Meats would be earning only 7 percent. This may be seen by looking at the IRR on the *extra*, or *incremental*, \$18 000 spent on the loader.

$$-(68\,000 - 50\,000) + (14\,000 - 11\,000)(P/A, i^*, 8) = 0$$

$$-18\,000 + 3000(P/A, i^*, 8) = 0$$

$$(P/A, i^*, 8) = 18\,000/3000$$

$$(P/A, i^*, 8) = 6$$

$$i^* \cong 7\%$$

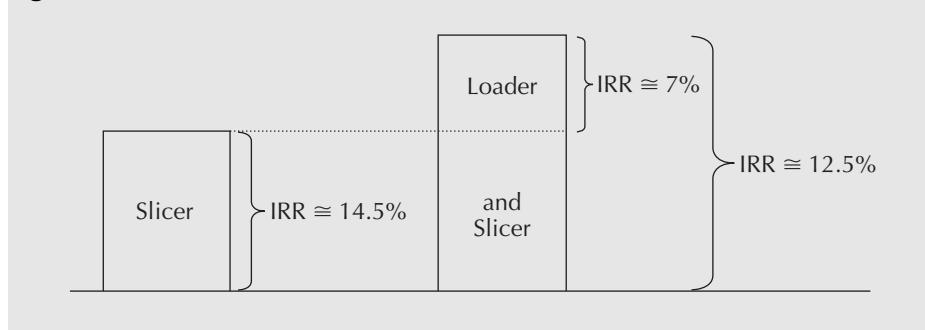
This is less than the MARR; therefore, Monster Meats should not buy the automated loader.

To find the IRR using Excel, compute the net cash flows for each period, and then apply the IRR function to the range of cells containing the cash flows, as shown below:

|    | A      | B        | C           | D           |
|----|--------|----------|-------------|-------------|
| 1  | Time   | Meat     | Slicer with | Incremental |
| 2  | Period | Slicer   | Loader      | Cash Flow   |
| 3  | 0      | - 50 000 | - 68 000    | - 18 000    |
| 4  | 1      | 11 000   | 14 000      | 3 000       |
| 5  | 2      | 11 000   | 14 000      | 3 000       |
| 6  | 3      | 11 000   | 14 000      | 3 000       |
| 7  | 4      | 11 000   | 14 000      | 3 000       |
| 8  | 5      | 11 000   | 14 000      | 3 000       |
| 9  | 6      | 11 000   | 14 000      | 3 000       |
| 10 | 7      | 11 000   | 14 000      | 3 000       |
| 11 | 8      | 11 000   | 14 000      | 3 000       |
| 12 | IRR    | 14.61%   | 12.65%      | 6.88%       |

When the IRR was calculated for the system including the loader, the surplus return on investment earned by the slicer alone essentially subsidized the loader. The slicer investment made enough of a return so that, even when it was coupled with the money-losing loader, the whole machine still seemed to be a good buy. In fact, the extra \$18 000 would be better spent on some other project at the MARR or higher. The relation between the potential projects is shown in Figure 5.4. ■

**Figure 5.4** Monster Meats



The fundamental principle illustrated by the two examples is that, to use the IRR to compare two or more mutually exclusive alternatives properly, we cannot make the decision on the basis of the IRRs of individual alternatives alone; we must take the IRRs of the *incremental* investments into consideration. In order to properly assess the worthiness of the incremental investments, it is necessary to have a systematic way to conduct pair-wise comparisons of projects. Note that before undertaking a systematic analysis of mutually exclusive alternatives with the IRR method, you should ensure that the alternatives have equal lives. If they do not have equal lives, then the methods of Section 4.4.4 (study period or repeated lives methods) must be applied first to set up comparable cash flows.

The first step in the process of comparing several mutually exclusive alternatives using the IRR is to order the alternatives from the smallest first cost to the largest first

cost. Since one alternative must be chosen, accept the alternative with the smallest first cost (which may be the “do nothing” alternative with zero first cost) as the *current best alternative* regardless of its IRR exceeding the MARR. This means that the current best alternative may have an IRR *less* than the MARR. Even if that’s the case, a proper analysis of the IRRs of the incremental investments will lead us to the *correct* best overall alternative. For this reason, we don’t have to check the IRR of any of the individual alternatives.

The second step of the analysis consists of looking at the incremental investments of alternatives that have a higher first cost than the current best alternative. Assume that there are  $n$  projects and they are ranked from 1 (the current best) to  $n$ , in increasing order of first costs. The current best is “challenged” by the project ranked second. One of two things occurs:

1. The incremental investment to implement the challenger does not have an IRR at least equal to the MARR. In this case, the challenger is excluded from further consideration and the current best is challenged by the project ranked third.
2. The incremental investment to implement the challenger has an IRR at least as high as the MARR. In this case, the challenger replaces the current best. It then is challenged by the alternative ranked third.

The process then continues with the next alternative challenging the current best until all alternatives have been compared. The current best alternative remaining at the end of the process is then selected as the best overall alternative. Figure 5.5 on page 137 summarizes the incremental investment analysis for the mutually exclusive projects.

**EXAMPLE****5.6 (REPRISE OF EXAMPLE 4.4)**

Fly-by-Night Aircraft must purchase a new lathe. It is considering one of four new lathes, each of which has a life of 10 years with no scrap value. Given a MARR of 15 percent, which alternative should be chosen?

| Lathe          | 1         | 2         | 3         | 4         |
|----------------|-----------|-----------|-----------|-----------|
| First cost     | \$100 000 | \$150 000 | \$200 000 | \$255 000 |
| Annual savings | 25 000    | 34 000    | 46 000    | 55 000    |

The alternatives have already been ordered from lathe 1, which has the smallest first cost, to lathe 4, which has the greatest first cost. Since one lathe must be purchased, accept lathe 1 as the current best alternative. Calculating the IRR for lathe 1, although not necessary, is shown as follows:

$$100\,000 = 25\,000(P/A,i^*,10)$$

$$(P/A,i^*,10) = 4$$

An approximate IRR is obtained by trial and error with a spreadsheet.

$$i^* \approx 21.4\%$$

The current best alternative is then challenged by the first challenger, lathe 2, which has the next-highest first cost. The IRR of the incremental investment from lathe 1 to lathe 2 is calculated as follows:

$$(150\,000 - 100\,000) - (34\,000 - 25\,000)(P/A,i^*,10) = 0$$

or

$$[150\,000 - 34\,000(P/A,i^*,10)] - [100\,000 - 25\,000(P/A,i^*,10)] = 0$$

$$(P/A,i^*,10) = 50\,000/9000 = 5.556$$

An approximate IRR is obtained by trial and error.

$$i^* \approx 12.4\%$$

Since the IRR of the incremental investment falls below the MARR, lathe 2 fails the challenge to become the current best alternative. The reader can verify that lathe 2 alone has an IRR of approximately 18.7 percent. Even so, lathe 2 is not considered a viable alternative. In other words, the incremental investment of \$50 000 could be put to better use elsewhere. Lathe 1 remains the current best and the next challenger is lathe 3.

As before, the incremental IRR is the interest rate at which the present worth of lathe 3 less the present worth of lathe 1 is zero:

$$[200\,000 - 46\,000(P/A,i^*,10)] - [100\,000 - 25\,000(P/A,i^*,10)] = 0$$

$$(P/A,i^*,10) = 100\,000/21\,000 = 4.762$$

An approximate IRR is obtained by trial and error.

$$i^* \approx 16.4\%$$

The IRR on the incremental investment exceeds the MARR, and therefore lathe 3 is preferred to lathe 1. Lathe 3 now becomes the current best. The new challenger is lathe 4. The IRR on the incremental investment is

$$[255\,000 - 55\,000(P/A,i^*,10)] - [200\,000 - 46\,000(P/A,i^*,10)] = 0$$

$$(P/A,i^*,10) = 55\,000/9000 = 6.11$$

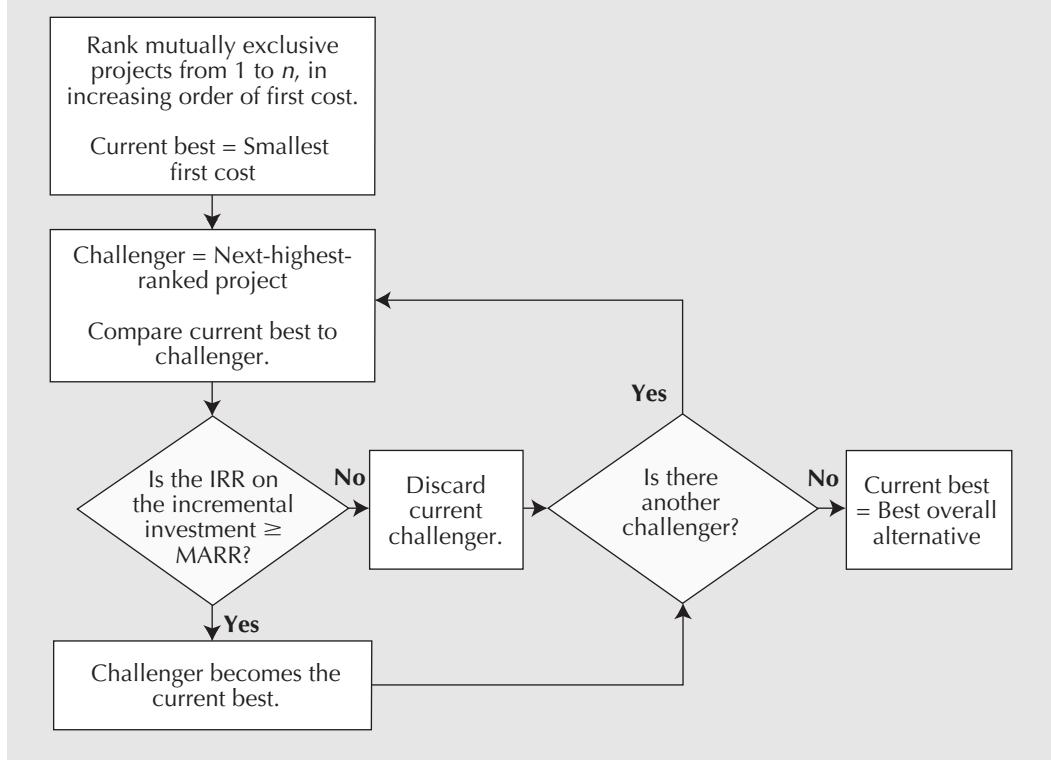
$$i^* \approx 10.1\%$$

The additional investment from lathe 3 to lathe 4 is not justified. The reader can verify that the IRR of lathe 4 alone is about 17 percent. Once again, we have a challenger with an IRR greater than the MARR, but it fails as a challenger because the incremental investment from the current best does not have an IRR at least equal to the MARR. The current best remains lathe 3. There are no more challengers, and so the best overall alternative is lathe 3. ■

In the next section, the issue of multiple IRRs is discussed, and methods for identifying and eliminating them are given. Note that the process described in Figure 5.5 requires that a single IRR (or ERR, as discussed later) be determined for each incremental investment. If there are multiple IRRs, they must be dealt with for *each* increment of investment.

### 5.3.3 Multiple IRRs

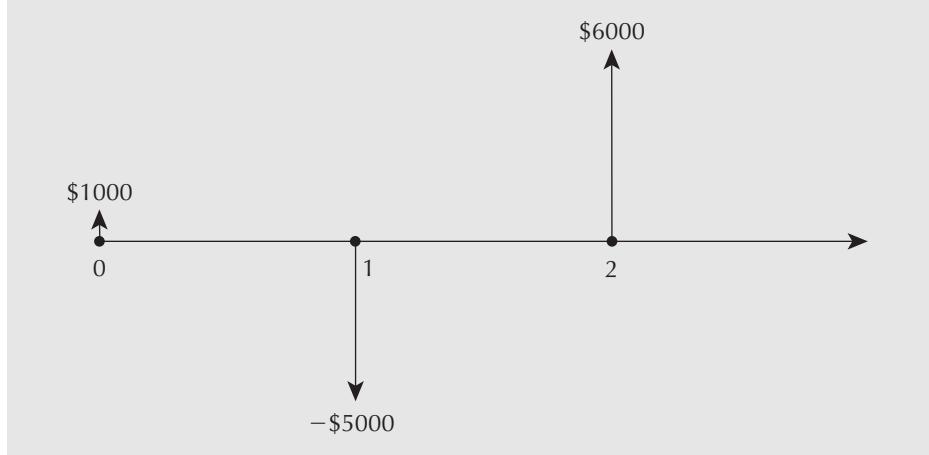
A problem with implementing the internal rate of return method is that there may be more than one internal rate of return. Consider the following example.

**Figure 5.5 Flowchart for Comparing Mutually Exclusive Alternatives****EXAMPLE****5.7**

A project pays \$1000 today, costs \$5000 a year from now, and pays \$6000 in two years. (See Figure 5.6.) What is its IRR?

Equating the present worths of disbursements and receipts and solving for the IRR gives the following:

$$1000 - 5000(P/F, i^*, 1) + 6000(P/F, i^*, 2) = 0$$

**Figure 5.6 Multiple IRR Example**

Recalling that  $(P/F, i^*, N)$  stands for  $1/(1 + i^*)^N$ , we have

$$\begin{aligned} 1 - \frac{5}{1 + i^*} + \frac{6}{(1 + i^*)^2} &= 0 \\ (1 + i^*)^2 - 5(1 + i^*) + 6 &= 0 \\ (1 + 2i^* + i^{*2}) - 5i^* + 1 &= 0 \\ i^{*2} - 3i^* + 2 &= 0 \\ (i^* - 1)(i^* - 2) &= 0 \end{aligned}$$

The roots of this equation are  $i_1^* = 1$  and  $i_2^* = 2$ . In other words, this project has two IRRs: 100 percent and 200 percent! ■

The multiple internal rates of return problem may be stated more generally. Consider a project that has cash flows over  $T$  periods. The **net cash flow**,  $A_t$ , associated with period  $t$  is the difference between cash inflows and outflows for the period (i.e.,  $A_t = R_t - D_t$ , where  $R_t$  is cash inflow in period  $t$  and  $D_t$  is cash outflow in period  $t$ ). We set the present worth of the net cash flows over the entire life of the project equal to zero to find the IRR(s). We have

$$A_0 + A_1(1 + i)^{-1} + A_2(1 + i)^{-2} + \dots + A_T(1 + i)^{-T} = 0 \quad (5.2)$$

Any value of  $i$  that solves Equation (5.2) is an internal rate of return for that project. That there may be multiple solutions to Equation (5.2) can be seen if we rewrite the equation as

$$A_0 + A_1x + A_2x^2 + \dots + A_Tx^T = 0 \quad (5.3)$$

where  $x = (1 + i)^{-1}$ .

Solving the  $T$ th degree polynomial of Equation (5.3) is the same as solving for the internal rates of return in Equation (5.2). In general, when finding the roots of Equation (5.3), there may be as many positive real solutions for  $x$  as there are sign changes in the coefficients—the  $A$ s. Thus, there may be as many IRRs as there are sign changes in the  $A$ s.

We can see the meaning of multiple roots most easily with the concept of **project balance**. If a project has a sequence of net cash flows  $A_0, A_1, A_2, \dots, A_T$ , and the interest rate is  $i'$ , there are  $T + 1$  project balances— $B_0, B_1, B_2, \dots, B_T$ , one at the end of each period  $t$ ,  $t = 0, 1, \dots, T$ . A project balance,  $B_t$ , is the accumulated future value of all cash flows up to the end of period  $t$ , compounded at the rate,  $i'$ . That is,

$$\begin{aligned} B_0 &= A_0 \\ B_1 &= A_0(1 + i') + A_1 \\ B_2 &= A_0(1 + i')^2 + A_1(1 + i') + A_2 \\ B_T &= A_0(1 + i')^T + A_1(1 + i')^{T-1} + \dots + A_T \end{aligned}$$

Table 5.1 shows the project balances at the end of each year for both 100 percent and 200 percent interest rates for the project in Example 5.7. The project starts with a cash inflow of \$1000. At a 100 percent interest rate, the \$1000 increases to \$2000 over the first year. At the end of the first year, there is a \$5000 disbursement, leaving a negative project balance of \$3000. At 100 percent interest, this negative balance increases to \$6000

**Table 5.1 Project Balances for Example 5.7**

| <b>End of Year</b> | <b>At <math>i^* = 100\%</math></b> | <b>At <math>i^* = 200\%</math></b> |
|--------------------|------------------------------------|------------------------------------|
| 0                  | 1000                               | 1000                               |
| 1                  | $1000(1 + 1) - 5000 = -3000$       | $1000(1 + 2) - 5000 = -2000$       |
| 2                  | $-3000(1 + 1) + 6000 = 0$          | $-2000(1 + 2) + 6000 = 0$          |

over the second year. This negative \$6000 is offset exactly by the \$6000 inflow. This makes the project balance zero at the end of the second year. The project balance at the end of the project is the future worth of all the cash flows in the project. When the future worth at the end of the project life is zero, the present worth is also zero. This verifies that the 100 percent is an IRR.

Now consider the 200 percent interest rate. Over the first year, the \$1000 inflow increases to \$3000. At the end of the first year, \$5000 is paid out, leaving a negative project balance of \$2000. This negative balance grows at 200 percent to \$6000 over the second year. This is offset exactly by the \$6000 inflow so that the project balance is zero at the end of the second year. This verifies that the 200 percent is also an IRR!

Looking at Table 5.1, it's actually fairly obvious that an important assumption is being made about the initial \$1000 received. The IRR computation implicitly assumes that the \$1000 is *invested* during the first period at either 100 percent or 200 percent, one of the two IRRs. However, during the first period, the project is not an investment. The project balance is positive. The project is *providing* money, not using it. This money cannot be reinvested immediately in the project. It is simply cash on hand. The \$1000 must be invested elsewhere for one year if it is to earn any return. It is unlikely, however, that the \$1000 provided by the project in this example would be invested in something else at 100 percent or 200 percent. More likely, it would be invested at a rate at or near the company's MARR.

## N E T   V A L U E   5 . 1

### Additional Project Comparison Resources

The internet can be an excellent source of information and examples about the project comparison methods presented in Chapters 4 and 5. However, there is a wide range of perspectives and terminologies that can make it challenging to get clear and consistent information from the web for these methods. At the highest level, a search for information on “capital budgeting” or “corporate finance” will likely be more fruitful than a search for “engineering economics.” In the Wikipedia article on capital budgeting, the annual worth method is referred to as the equivalent annuity method. Broadly, present worth is commonly referred to as net present value, or

NPV. IRR is a widely used term. An alternative for ERR is MIRR, or modified internal rate of return.

With recognition of the different terms and perspectives, one can find examples or explanations for a variety of circumstances. For example, a recent search for “IRR in construction” using Google gave several useful examples along with a lengthy discussion on the details of using Excel for an IRR calculation in a construction problem. A search for “should I use NPV or IRR?” produced hundreds of arguments about when one technique is preferred to another over a wide range of industries.

### 5.3.4 External Rate of Return Methods

To resolve the multiple IRR difficulty, we need to consider what return is earned by money associated with a project that is not invested in the project. The usual assumption is that the funds are invested elsewhere and earn an *explicit rate of return* equal to the MARR. This makes sense, because when there is cash on hand that is not invested in the project under study, it will be used elsewhere. These funds would, by definition, gain interest at a rate at least equal to the MARR. The **external rate of return (ERR)**, denoted by  $i^*_e$ , is the rate of return on a project where any cash flows that are not invested in the project are assumed to earn interest at a predetermined explicit rate (usually the MARR). For a given explicit rate of return, a project can have only one value for its ERR.

It is possible to calculate a precise ERR that is comparable to the IRRs of other projects using an explicit interest rate when necessary. Because the cash flows of Example 5.7 are fairly simple, let us use them to illustrate how to calculate the ERR precisely.

**EXAMPLE**
**5.8 (EXAMPLE 5.7 REVISITED: ERR)**

A project pays \$1000 today, costs \$5000 a year from now, and pays \$6000 in two years. What is its rate of return? Assume that the MARR is 25 percent.

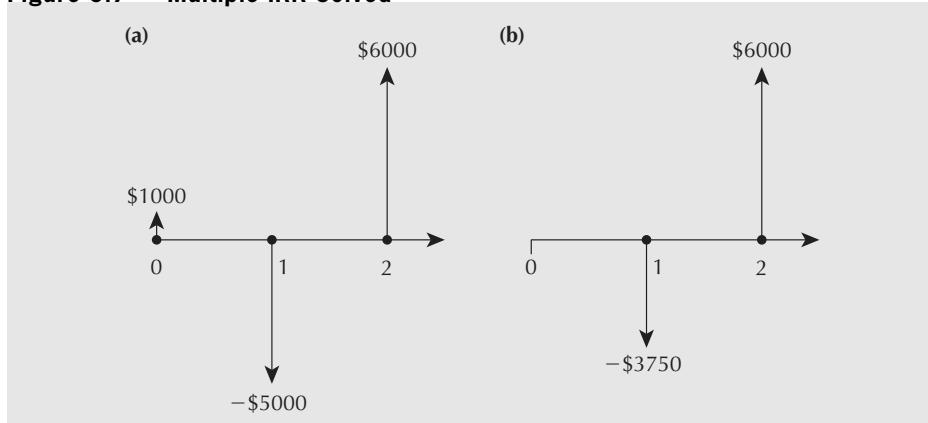
The first \$1000 is not invested immediately in the project. Therefore, we assume that it is invested outside the project for one year at the MARR. Thus, the cumulative cash flow for year 1 is

$$1000(F/P, 25\%, 1) - 5000 = 1250 - 5000 = -\$3750$$

With this calculation, we transform the cash flow diagram representing this problem from that in Figure 5.7(a) to that in Figure 5.7(b). These cash flows provide a single (precise) ERR, as follows:

$$\begin{aligned} -3750 + 6000(P/F, i^*_e, 1) &= 0 \\ (P/F, i^*_e, 1) &= 3750/6000 = 0.625 \\ \frac{1}{1+i^*_e} &= 0.625 \\ 1+i^*_e &= \frac{1}{0.625} = 1.6 \\ i^*_e &= 0.6 \\ \text{ERR} &= 60\% \end{aligned}$$

**Figure 5.7 Multiple IRR Solved**



In general, computing a precise ERR can be a complex procedure because of the difficulty in determining exactly when the explicit interest rate should be applied. In order to do such a calculation, project balances have to be computed for trial ERRs. In periods in which project balances are positive for the trial ERR, the project is a source of funds. These funds would have to be invested outside the project at the MARR. During periods when the project balance is negative for the trial ERR, any receipts would be invested in the project and will typically yield more than the MARR. Whether the project balances are negative or positive will depend on the trial ERRs. This implies that the calculation process requires much experimenting with trial ERRs before an ERR is found that makes the future worth zero. A more convenient, but approximate, method is to use the following procedure:

1. Take all *net* receipts forward at the MARR to the time of the last cash flow.
2. Take all *net* disbursements forward at an unknown interest rate,  $i_{ea}^*$ , also to the time of the last cash flow.
3. Equate the future value of the receipts from Step 1 to the future value of the disbursements from Step 2 and solve for  $i_{ea}^*$ .
4. The value for  $i_{ea}^*$  is the *approximate ERR* for the project.

While Excel does have a modified IRR function called MIRR, it is not the same as the approximate ERR. The Excel MIRR function requires that you specify the rate at which positive cash flows are reinvested, whereas the approximate ERR estimates the return obtained on positive balances.

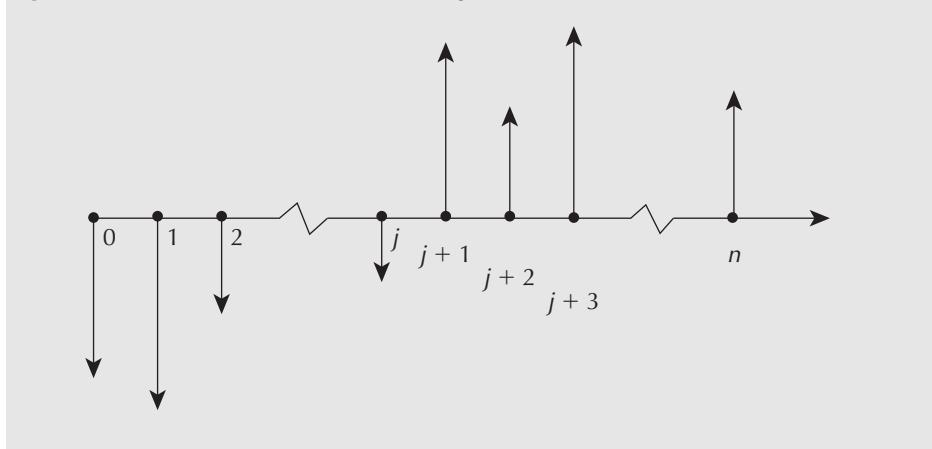
**EXAMPLE****5.9 (EXAMPLE 5.7 REVISITED AGAIN: AN APPROXIMATE ERR)**

To approximate the ERR, we compute the interest rate that gives a zero future worth at the end of the project when all receipts are brought forward at the MARR. In Example 5.7, the \$1000 is thus assumed to be reinvested at the MARR for two years, the life of the project. The disbursements are taken forward to the end of the two years at an unknown interest rate,  $i_{ea}^*$ . With a MARR of 25 percent, the revised calculation is

$$\begin{aligned} 1000(F/P, 25\%, 2) + 6000 &= 5000(F/P, i_{ea}^*, 1) \\ (F/P, i_{ea}^*, 1) &= [1000(1.5625) + 6000]/5000 \\ (F/P, i_{ea}^*, 1) &= 1.5125 \\ 1 + i_{ea}^* &= 1.5125 \\ i_{ea}^* &= 0.5125 \text{ or } 51.25\% \\ \text{ERR} &\equiv 51\% \end{aligned}$$



The ERR calculated using this method is an approximation, since all receipts, not just those that occur when the project balance is positive, are assumed to be invested at the MARR. Note that the precise ERR of 60 percent is different from the approximate ERR of 51 percent. Fortunately, it can be shown that the approximate ERR will always be between the precise ERR and the MARR. This means that whenever the precise ERR is above the MARR, the approximate ERR will also be above the MARR, and whenever the precise ERR is below the MARR, the approximation will be below the MARR as well. This implies that using the approximate ERR will always lead to the correct decision. It should also be noted that an acceptable project will earn *at least* the rate given by the approximate ERR. Therefore, even though an approximate ERR is inaccurate, it is often used in practice because it provides the correct decision as well as a lower bound on the return on an investment, while being easy to calculate.

**Figure 5.8 The General Form of Simple Investments**

### 5.3.5 When to Use the ERR

The ERR (approximate or precise) must be used whenever there are multiple IRRs possible. Unfortunately, it can be difficult to know in advance whether there will be multiple IRRs. On the other hand, it is fortunate that most ordinary projects have a structure that precludes multiple IRRs.

Most projects consist of one or more periods of outflows at the start, followed only by one or more periods of inflows. Such projects are called **simple investments**. The cash flow diagram for a simple investment takes the general form shown in Figure 5.8. In terms of Equations (5.2) and (5.3), there is only one change of sign, from negative to positive in the  $A$ s, the sequence of coefficients. Hence, a simple investment always has a unique IRR.

If a project is not a simple investment, there may or may not be multiple IRRs—there is no way of knowing for sure without further analysis. In practice, it may be reasonable to use an approximate ERR whenever the project is not a simple investment. Recall from Section 5.3.4 that the approximate ERR will always provide a correct decision, whether its use is required or not, since it will underestimate the true rate of return.

However, it is generally desirable to compute an IRR whenever it is possible to do so, and to use an approximate ERR only when there may be multiple IRRs. In this way, the computations will be as accurate as possible.

To reiterate, the approximate ERR can be used to evaluate any project, whether it is a simple investment or not. However, the approximate ERR will tend to be a less accurate rate than the IRR. The inaccuracy will tend to be similar for projects with cash flows of a similar structure, and either method will result in the same decision in the end.



### SPREADSHEET SAVVY

Depending on your approach to doing an economic analysis, the spreadsheet layout may take on a completely different look. The two tables below illustrate how Example 5.6 on page 135 might be answered using (1) the Excel IRR function, and (2) compound interest factors. In the top table, the cash flows for each of the 10 years are listed on each row. The columns represent the alternatives and the incremental cash flows. The second row entries contain the IRR results, as computed using the Excel IRR function. The IRR function has as its arguments the range of cells in which the cash flows are found and an initial guess at the IRR (which is optional).

|        | Cash Flows for the Individual Lathes |           |           |           | Incremental Cash Flows |           |          |
|--------|--------------------------------------|-----------|-----------|-----------|------------------------|-----------|----------|
|        | (1)                                  | (2)       | (3)       | (4)       | (2)-(1)                | (3)-(1)   | (4)-(3)  |
| IRR:   | 21.41%                               | 18.52%    | 18.94%    | 17.13%    | 12.41%                 | 16.40%    | 10.13%   |
| Period | Cash Flows:                          |           |           |           |                        |           |          |
| 0      | - 100 000                            | - 150 000 | - 200 000 | - 255 000 | - 50 000               | - 100 000 | - 55 000 |
| 1      | 25 000                               | 34 000    | 46 000    | 55 000    | 9 000                  | 21 000    | 9 000    |
| :      | 25 000                               | 34 000    | 46 000    | 55 000    | 9 000                  | 21 000    | 9 000    |
| 10     | 25 000                               | 34 000    | 46 000    | 55 000    | 9 000                  | 21 000    | 9 000    |

In the second table, basic computations are used to compute the present worth of each alternative. The rows are used to vary the interest rate. For example, the IRR for lathe 1 is between 21 percent and 22 percent because the present worth becomes zero at some point between these two interest rates. This is sufficient information to move on to compute the IRR on the incremental investment for lathe 2. The IRR on the incremental investment is between 12 percent and 13 percent, which is less than the MARR.

| $i$  | PW of Cash Flows for the Individual Lathes |         |         |         | PW of Incremental Cash Flows |         |         |
|------|--|---------|---------|---------|------------------------------|---------|---------|
|      | (1)  | (2)     | (3)     | (4)     | (2)-(1)                      | (3)-(1) | (4)-(3) |
| 0.10 | 53 614                                     | 58 915  | 82 650  | 82 951  | 5 301                        | 29 036  | 301     |
| 0.11 | 47 231                                     | 50 234  | 70 905  | 68 908  | 3 003                        | 23 674  | -1 997  |
| 0.12 | 41 256                                     | 42 108  | 59 910  | 55 762  | 852                          | 18 655  | -4 148  |
| 0.13 | 35 656                                     | 34 492  | 49 607  | 43 443  | -1 164                       | 13 951  | -6 164  |
| 0.14 | 30 403                                     | 27 348  | 39 941  | 31 886  | -3 055                       | 9 538   | -8 055  |
| 0.15 | 25 469                                     | 20 638  | 30 863  | 21 032  | -4 831                       | 5 394   | -9 831  |
| 0.16 | 20 831                                     | 14 330  | 22 328  | 10 828  | -6 501                       | 1 498   | -11 501 |
| 0.17 | 16 465                                     | 8 393   | 14 296  | 1 223   | -8 073                       | -2 169  | -13 073 |
| 0.18 | 12 352                                     | 2 799   | 6 728   | -7 825  | -9 553                       | -5 624  | -14 553 |
| 0.19 | 8 473                                      | -2 476  | -409    | -16 359 | -10 950                      | -8 882  | -15 950 |
| 0.20 | 4 812                                      | -7 456  | -7 146  | -24 414 | -12 268                      | -11 958 | -17 268 |
| 0.21 | 1 352                                      | -12 161 | -13 512 | -32 026 | -13 513                      | -14 864 | -18 513 |
| 0.22 | -1 920                                     | -16 612 | -19 534 | -39 225 | -14 691                      | -17 613 | -19 691 |

Either approach leads to the conclusion that lathe 3 is the preferred alternative.

## 5.4 Rate of Return and Present/Annual Worth Methods Compared

A comparison of rate of return and present/annual worth methods leads to two important conclusions:

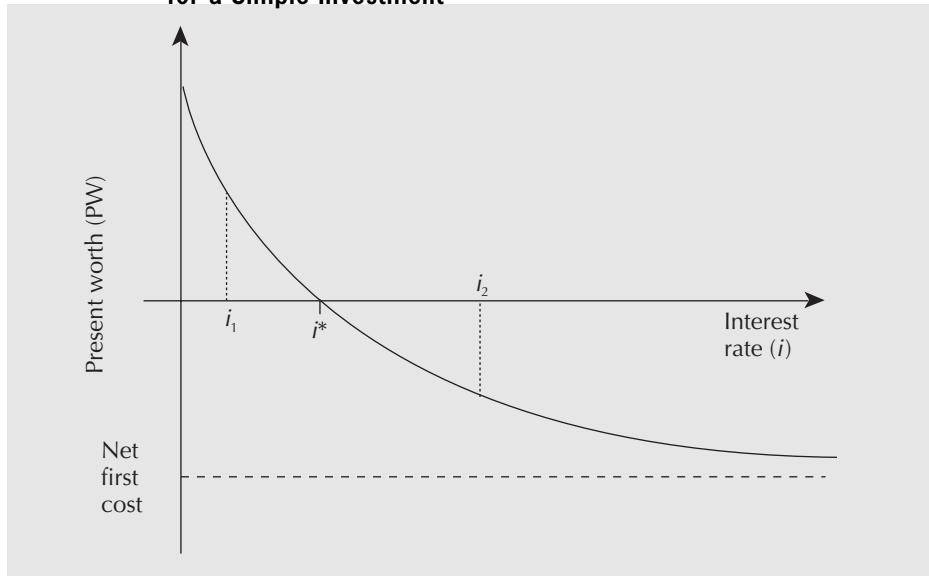
1. The two sets of methods, when properly used, give the same decisions.
2. Each set of methods has its own advantages and disadvantages.

Let us consider each of these conclusions in more detail.

### 5.4.1 Equivalence of Rate of Return and Present/Annual Worth Methods

If an independent project has a unique IRR, the IRR method and the present worth method give the same decision. Consider Figure 5.9. It shows the present worth as a function of the interest rate for a project with a unique IRR. The maximum of the curve lies at the vertical axis (where the interest rate = zero) at the point given by the sum of

**Figure 5.9 Present Worth (PW) as a Function of Interest Rate ( $i$ ) for a Simple Investment**



all undiscounted net cash flows. (We assume that the sum of all the undiscounted net cash flows is positive.) As the interest rate increases, the present worth of all cash flows after the first cost decreases. Therefore, the present worth curve slopes down to the right. To determine what happens as the interest rate increases indefinitely, let us recall the general equation for present worth:

$$PW = \sum_{t=0}^T A_t (1+i)^{-t} \quad (5.4)$$

where

$i$  = the interest rate

$A_t$  = the net cash flow in period  $t$

$T$  = the number of periods

Letting  $i \rightarrow \infty$ , we have

$$\lim_{i \rightarrow \infty} \frac{1}{(1+i)^t} = 0 \text{ for } t = 1, 2, \dots, T$$

Therefore, as the interest rate becomes indefinitely large, all terms in Equation (5.4) approach zero except the first term (where  $t = 0$ ), which remains at  $A_0$ . In Figure 5.9, this is shown by the asymptotic approach of the curve to the first cost, which, being negative, is below the horizontal axis.

The interest rate at which the curve crosses the horizontal axis ( $i^*$  in Figure 5.9), where the present worth is zero, is, by definition, the IRR.

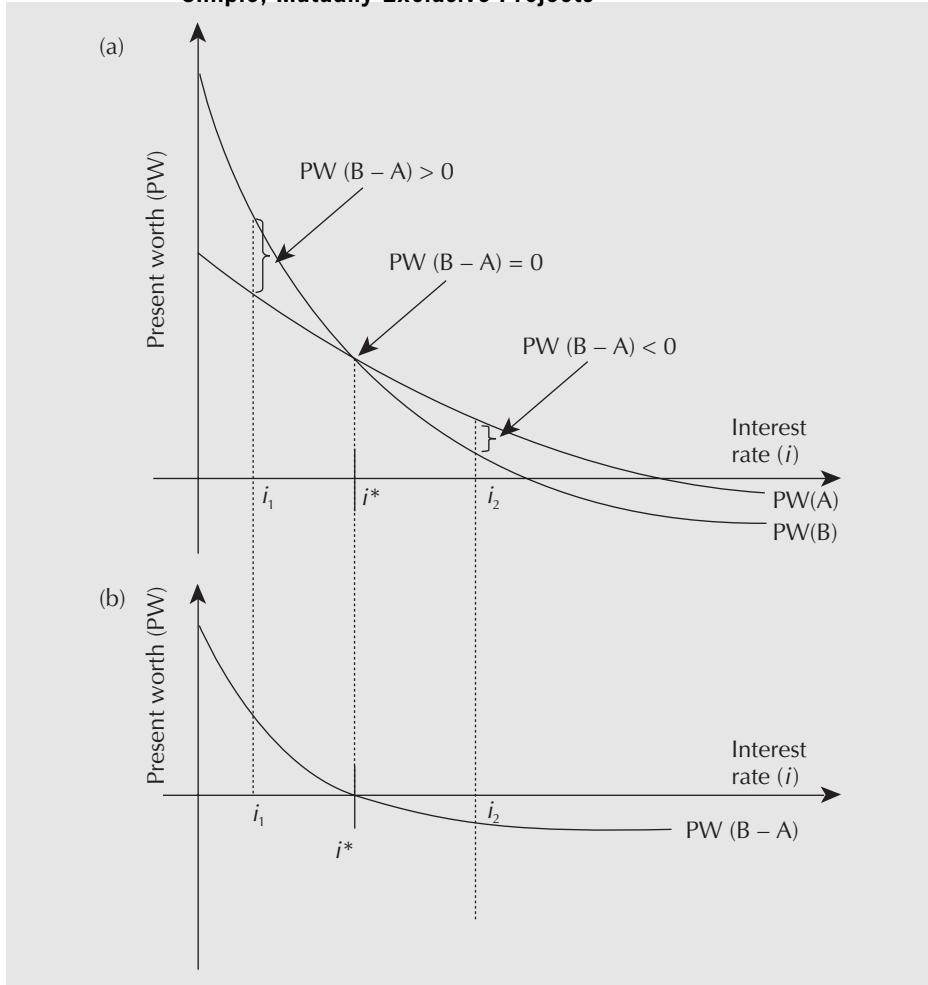
To demonstrate the equivalence of the rate of return and the present/annual worth methods for decision making, let us consider possible values for the MARR. First, suppose the  $MARR = i_1$ , where  $i_1 < i^*$ . In Figure 5.9, this MARR would lie to the left of the IRR. From the graph we see that the present worth is positive at  $i_1$ . In other words, we have

$$IRR > MARR$$

and

$$PW > 0$$

**Figure 5.10 Present Worth as a Function of Interest Rate ( $i$ ) for Two Simple, Mutually Exclusive Projects**



Thus, in this case, both the IRR and PW methods lead to the same conclusion: Accept the project.

Second, suppose the MARR =  $i_2$ , where  $i_2 > i^*$ . In Figure 5.9, this MARR would lie to the right of the IRR. From the graph we see that, at  $i_2$ , the present worth is negative. Thus we have

$$\text{IRR} < \text{MARR}$$

and

$$PW < 0$$

Here, too, the IRR and the PW method lead to the same conclusion: Reject the project.

Now consider two simple, mutually exclusive projects, A and B, where the first cost of B is greater than the first cost of A. If the increment from A to B has a unique IRR, then we can readily demonstrate that the IRR and PW methods lead to the same decision. See Figure 5.10(a), which shows the present worths of projects A and B as a function of the interest rate. Since the first cost of B is greater than that of A, the curve for project B asymptotically approaches a lower present worth than does the curve for project A as the interest rate becomes indefinitely large, and thus the two curves must cross at some point.

To apply the IRR method, we must consider the increment (denoted by  $B - A$ ). The present worth of the increment ( $B - A$ ) will be zero where the two curves cross. This point of intersection is marked by the interest rate,  $i^*$ . We have plotted the curve for the increment ( $B - A$ ) in Figure 5.10(b) to clarify the relationships.

Let us again deal with possible values of the MARR. First, suppose the MARR ( $i_1$ ) is less than  $i^*$ . Then, as we see in Figure 5.10(b), the present worth of ( $B - A$ ) is positive at  $i_1$ . That is, the following conditions hold:

$$\text{IRR}(B - A) > \text{MARR}$$

and

$$\text{PW}(B - A) > 0$$

Thus, according to both the IRR method and the PW method, project B is better than project A.

Second, suppose the MARR =  $i_2$ , where  $i_2 > i^*$ . Then we see from Figure 5.10(b) that the present worth of the increment ( $B - A$ ) is negative at  $i_2$ . In other words, the following conditions hold:

$$\text{IRR}(B - A) < \text{MARR}$$

and

$$\text{PW}(B - A) < 0$$

Thus, according to both methods, project A is better than project B.

In a similar fashion, we could show that the approximate ERR method gives the same decisions as the PW method in those cases where there may be multiple IRRs.

We already noted that the annual worth and present worth methods are equivalent. Therefore, by extension, our demonstration of the equivalence of the rate of return methods and the present worth methods means that the rate of return and the annual worth methods are also equivalent.

## EXAMPLE

### 5.10

A tourist-area resort is considering adding either a parasailing operation or kayak rentals to their other activities. Available space limits them to one of these two choices. The initial costs for parasailing will be \$100 000, with net returns of \$15 000 annually for the 15-year life of the project. Initial costs for kayaking will be \$10 000, with net returns of \$2000 per year for its 15-year life. Assume that both projects have a \$0 salvage value after 15 years, and the MARR is 10 percent.

- (a) Using present worth analysis, which alternative is better?
- (b) Using IRR, which alternative is better?
- (c) The present worths of the two projects are calculated as follows:

$$\begin{aligned}\text{PW}_{\text{para}} &= -100\,000 + 15\,000(P/A, 10\%, 15) \\ &= -100\,000 + 15\,000(7.6061) \\ &= 14\,091.50\end{aligned}$$

$$\begin{aligned}\text{PW}_{\text{kayak}} &= -10\,000 + 2000(P/A, 10\%, 15) \\ &= -10\,000 + 2000(7.6061) \\ &= 5212.20\end{aligned}$$

The parasailing venture has a higher present worth at about \$14 000 and is thus preferred.

- (b) The IRRs of the two projects are calculated as follows:

*Parasailing*

$$100\ 000 = 15\ 000(P/A, i^*, 15)$$

$$(P/A, i^*, 15) = 100\ 000/15\ 000 = 6.67 \rightarrow i^*_{\text{para}} = 12.4\%$$

*Kayaking*

$$10\ 000 = 2000(P/A, i^*, 15)$$

$$(P/A, i^*, 15) = 5 \rightarrow i^*_{\text{kayak}} = 18.4\%$$

One might conclude that, because  $IRR_{\text{kayak}}$  is larger, the resort should invest in the kayaking project, but this is *wrong*. When done correctly, a present worth analysis and an IRR analysis will always agree. The error here is that the parasailing project was assessed without consideration of the increment from the kayaking project. Checking the IRR of the increment (denoted by the subscript “kayak–para”):

$$(100\ 000 - 10\ 000) = (15\ 000 - 2000)(P/A, i^*, 15)$$

$$(P/A, i^*, 15) = 90\ 000/13\ 000 = 6.923 \rightarrow i^*_{\text{kayak-para}} = 11.7\%$$

Since the increment from the kayaking project also exceeds the MARR, the larger parasailing project should be taken. ■

#### 5.4.2 Why Choose One Method Over the Other?

Although rate of return methods and present worth/annual worth methods give the same decisions, each set of methods has its own advantages and disadvantages. The choice of method may depend on the way the results are to be used and the sort of data the decision makers prefer to consider. In fact, many companies, by policy, require that several methods be applied so that a more complete picture of the situation is presented. A summary of the advantages and disadvantages of each method is given in Table 5.2.

**Table 5.2 Advantages and Disadvantages of Comparison Methods**

| Method                | Advantages   | Disadvantages  |
|-----------------------|--|--|
| <b>IRR</b>            | Facilitates comparisons of projects of different sizes<br>Commonly used                                  | Relatively difficult to calculate<br>Multiple IRRs may exist   |
| <b>Present worth</b>  | Gives explicit measure of profit contribution  | Difficult to compare projects of different sizes   |
| <b>Annual worth</b>   | Annual cash flows may have familiar meanings to decision makers  | Difficult to compare projects of different sizes   |
| <b>Payback period</b> | Very easy to calculate<br>Commonly used<br>Takes into account the need to have capital recovered quickly | Discriminates against long-term projects<br>Ignores time value of money<br>Ignores the expected service life |

Rate of return methods state results in terms of *rates*, while present/annual worth methods state results in absolute figures. Many managers prefer rates to absolute figures because rates facilitate direct comparisons of projects whose sizes are quite different. For example, a petroleum company comparing performances of a refining division and a distribution division would not look at the typical values of present or annual worth for projects in the two divisions. A refining project may have first costs in the range of hundreds of *millions*, while distribution projects may have first costs in the range of *thousands*. It would not be meaningful to compare the absolute profits of a refining project and a distribution project. The absolute profits of refining projects will almost certainly be larger than those of distribution projects. Expressing project performance in terms of rates of return permits understandable comparisons. A disadvantage of rate of return methods, however, is the possible complication that there may be more than one rate of return. Under such circumstances, it is necessary to calculate an ERR.

In contrast to a rate of return, a present or annual worth computation gives a direct measure of the profit provided by a project. A company's main goal is likely to earn profits for its owners. The present and annual worth methods state the contribution of a project toward that goal. Another reason that managers prefer these methods is that present worth and annual worth methods are typically easier to apply than rate of return methods.

For completeness of coverage, we note that the payback period method may not give results consistent with rate of return or present/annual worth methods as it ignores the time value of money and the service life of projects. It is, however, a method commonly used in practice due to its ease of use and intuitive appeal.

**EXAMPLE****5.11**

Each of the following scenarios suggests a best choice of comparison method.

- Edward has his own small firm that will lease injection-moulding equipment to make polyethylene containers. He must decide on the specific model to lease. He has estimates of future monthly sales.

The annual worth method makes sense here because Edward's cash flows, including sales receipts and leasing expenses, will probably all be on a monthly basis. As a sole proprietor, Edward need not report his conclusions to others.

- Ramesh works for a large power company and must assess the viability of locating a transformer station at various sites in the city. He is looking at the cost of the building lot, power lines, and power losses for the various locations. He has fairly accurate data about costs and future demand for electricity.

As part of a large firm, Ramesh will likely be obliged to use a specific comparison method. This would probably be IRR. A power company makes many large and small investments, and the IRR method allows them to be compared fairly. Ramesh has the data necessary for the IRR calculations.

- Sehdev must buy a relatively inexpensive log splitter for his agricultural firm. There are several different types that require a higher or lower degree of manual assistance. He has only rough estimates of how this machine will affect future cash flows.

This relatively inexpensive purchase is a good candidate for the payback period method. The fact that it is inexpensive means that extensive data gathering and analysis are probably not warranted. Also, since future cash flows are

relatively uncertain, there is no justification for using a particularly precise comparison method.

4. Ziva will be living in the Arctic for six months, testing her company's equipment under hostile weather conditions. She needs a field office and must determine which of the following choices is economically best: (1) renting space in an industrial building, (2) buying and outfitting a trailer, (3) renting a hotel room for the purpose.

For this decision, a present worth analysis would be appropriate. The cash flows for each of the alternatives are of different types, and bringing them to present worth would be a fair way to compare them. It would also provide an accurate estimate to Ziva's firm of the expected cost of the remote office for planning purposes. ■

## REVIEW PROBLEMS

### REVIEW PROBLEM 5.1

Wei-Ping's consulting firm needs new quarters. A downtown office building is ideal. The company can either buy or lease it. To buy the office building will cost \$6 000 000. If the building is leased, the lease fee is \$400 000 payable at the beginning of each year. In either case, the company must pay city taxes, maintenance, and utilities.

Wei-Ping figures that the company needs the office space for only 15 years. Therefore, it will either sign a 15-year lease or buy the building. If it buys the building, it will then sell the building after 15 years. The value of the building at that time is estimated to be \$15 000 000.

What rate of return will Wei-Ping's firm receive by buying the office building instead of leasing it?

### ANSWER

The rate of return can be calculated as the IRR on the incremental investment necessary to buy the building rather than lease it.

The IRR on the incremental investment is found by solving for  $i^*$  in

$$(6\,000\,000 - 400\,000) - 15\,000\,000(P/F, i^*, 15) = 400\,000(P/A, i^*, 14)$$

$$4(P/A, i^*, 14) + 150(P/F, i^*, 15) = 56$$

For  $i^* = 11$  percent, the result is

$$\begin{aligned} 4(P/A, 11\%, 14) + 150(P/F, 11\%, 15) \\ = 4(6.9819) + 150(0.20900) \\ = 59.2781 \end{aligned}$$

For  $i^* = 12$  percent,

$$\begin{aligned} 4(P/A, 12\%, 14) + 150(P/F, 12\%, 15) \\ = 4(6.6282) + 150(0.1827) \\ = 53.9171 \end{aligned}$$

A linear interpolation between 11 percent and 12 percent gives the IRR

$$i^* = 11\% + (59.2781 - 56)/(59.2781 - 53.9171) = 11.6115\%$$

By investing its money in buying the building rather than leasing, Wei-Ping's firm is earning an IRR of about 11.6 percent. ■

### REVIEW PROBLEM 5.2

The Real S. Tate Company is considering investing in one of four rental properties. Real S. Tate will rent out whatever property it buys for four years and then sell it at the end of that period. The data concerning the properties is shown below:

| Rental Property | Purchase Price | Net Annual Rental Income | Sale Price at the End of Four Years |
|-----------------|----------------|--------------------------|-------------------------------------|
| 1               | \$100 000      | \$ 7200                  | \$100 000                           |
| 2               | 120 000        | 9600                     | 130 000                             |
| 3               | 150 000        | 10 800                   | 160 000                             |
| 4               | 200 000        | 12 000                   | 230 000                             |

On the basis of the purchase prices, rental incomes, and sale prices at the end of the four years, answer the following questions.

- (a) Which property, if any, should Tate invest in? Real S. Tate uses a MARR of 8 percent for projects of this type.
- (b) Construct a graph that depicts the present worth of each alternative as a function of interest rates ranging from 0 percent to 20 percent. (A spreadsheet would be helpful in answering this part of the problem.)
- (c) From your graph, determine the range of interest rates for which your choice in part (a) is the best investment. If the MARR were 9 percent, which rental property would be the best investment? Comment on the sensitivity of your choice to the MARR used by the Real S. Tate Company.

### ANSWER

- (a) Since the “do nothing” alternative is feasible and it has the least first cost, it becomes the current best alternative. The IRR on the incremental investment for property 1 is given by:

$$-100\,000 + 100\,000(P/F, i^*, 4) + 7200(P/A, i^*, 4) = 0$$

The IRR on the incremental investment is 7.2 percent. Because this is less than the MARR of 8 percent, property 1 is discarded from further consideration.

Next, the IRR for the incremental investment for property 2, the alternative with the next-highest first cost, is found by solving for  $i^*$  in

$$-120\,000 + 130\,000(P/F, i^*, 4) + 9600(P/A, i^*, 4) = 0$$

The interest rate that solves the above equation is 9.8 percent. Since an IRR of 9.8 percent exceeds the MARR, property 2 becomes the current best alternative. Now the incremental investments over and above the first cost of property 2 are analyzed.

Next, property 3 challenges the current best. The IRR on the incremental investment for property 3 is

$$\begin{aligned} & (-150\,000 + 120\,000) + (160\,000 - 130\,000)(P/F, i^*, 4) \\ & + (10\,800 - 9\,600)(P/A, i^*, 4) = 0 \\ & -30\,000 + 30\,000(P/F, i^*, 4) + 1200(P/A, i^*, 4) = 0 \end{aligned}$$

This gives an IRR of only 4 percent, which is below the MARR. Property 2 remains the current best alternative and property 3 is discarded.

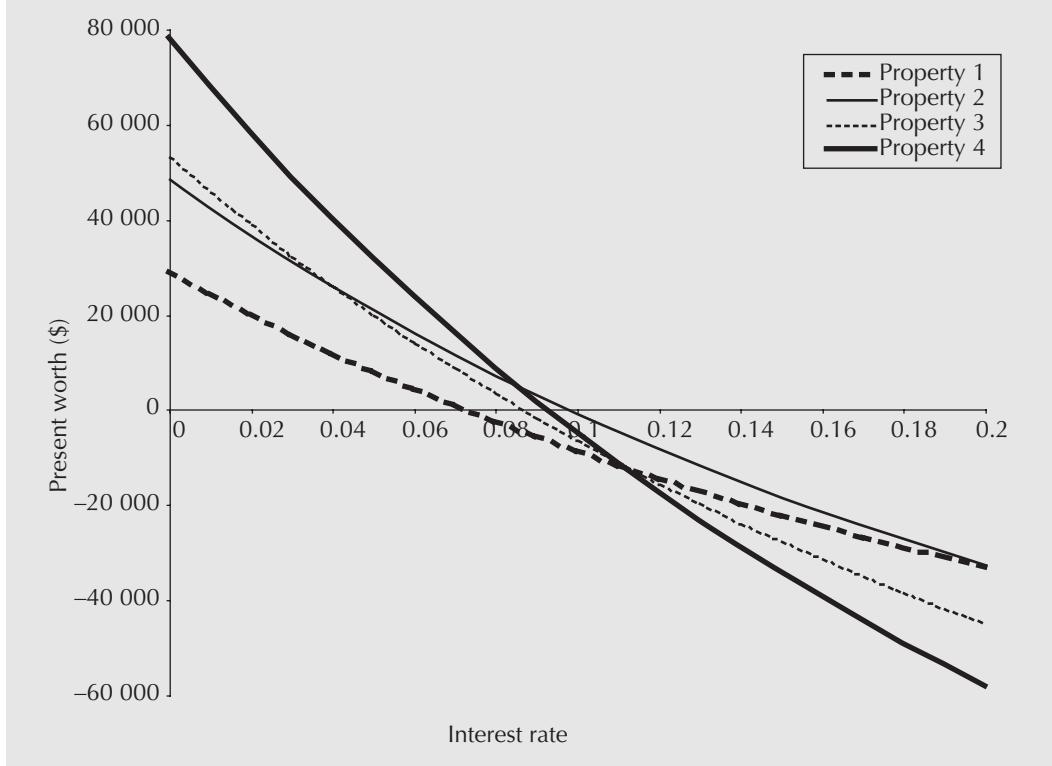
Finally, property 4 challenges the current best. The IRR on the incremental investment from property 2 to property 4 is

$$\begin{aligned} & (-200\,000 + 120\,000) + (230\,000 - 130\,000)(P/F, i^*, 4) \\ & + (12\,000 - 9\,600)(P/A, i^*, 4) = 0 \\ & -80\,000 + 100\,000(P/F, i^*, 4) + 2400(P/A, i^*, 4) = 0 \end{aligned}$$

The IRR on the incremental investment is 8.5 percent, which is above the MARR. Property 4 becomes the current best choice. Since there are no further challengers, the choice based on IRR is the current best, property 4.

- (b) The graph for part (b) is shown in Figure 5.11.
  - (c) From the graph, one can see that property 4 is the best alternative provided that the MARR is between 0 percent and 8.5 percent. This is the range of interest rates over which property 4 has the largest present worth.
- If the MARR is 9 percent, the best alternative is property 2. This can be seen by going back to the original IRR computations and observing that the

**Figure 5.11 Present Worth for Review Problem 5.2**



results of the analysis are essentially the same, except that the incremental investment from property 2 to property 4 no longer has a return exceeding the MARR. This can be confirmed from the diagram (Figure 5.11) as well, since the property with the largest present worth at 9 percent is property 2.

With respect to sensitivity analysis, the graph shows that, for a MARR between 0 percent and 8.5 percent, property 4 is the best choice, and for a MARR between 8.5 percent and 9.8 percent, property 2 is the best choice. If the MARR is above 9.8 percent, no property has an acceptable return on investment, and the “do nothing” alternative would be chosen.■

### REVIEW PROBLEM 5.3

You are in the process of arranging a marketing contract for a new Java applet you are writing. It still needs more development, so your contract will pay you \$5000 today to finish the prototype. You will then get royalties of \$10 000 at the end of each of the second and third years. At the end of each of the first and fourth years, you will be required to spend \$20 000 and \$10 000 in upgrades, respectively. What is the (approximate) ERR on this project, assuming a MARR of 20 percent? Should you accept the contract?

### ANSWER

To calculate the approximate ERR, set

$$\begin{aligned} \text{FW(receipts @ MARR)} &= \text{FW(disbursements @ ERR)} \\ 5000(F/P, 20\%, 4) + 10\,000(F/P, 20\%, 2) + 10\,000(F/P, 20\%, 1) &= 20\,000(F/P, i_{ea}^*, 3) + 10\,000 \\ 5000(2.0736) + 10\,000(1.44) + 10\,000(1.2) &= 20\,000(F/P, i_{ea}^*, 3) + 10\,000 \\ (F/P, i_{ea}^*, 3) &= 1.3384 \end{aligned}$$

$$(1 + i_{ea}^*)^3 = 1.3384$$

$$\begin{aligned} i_{ea}^* &= (1.3384)^{(1/3)} - 1 \\ &= 10.2\% \end{aligned}$$

The (approximate) ERR is 10.2 percent. Since this is below the MARR of 20 percent, the contract should not be accepted.■

### SUMMARY

This chapter presented the IRR method for evaluating projects and also discussed the relationship among the present worth, annual worth, payback period, and IRR methods.

The IRR method consists of determining the rate of return for a sequence of cash flows. For an independent project, the calculated IRR is compared with a MARR, and if

it is equal to or exceeds the MARR it is an acceptable project. To determine the best project of several mutually exclusive ones, it is necessary to determine the IRR of each increment of investment.

The IRR selection procedure is complicated by the possibility of having more than one rate of return because of a cash flow structure that, over the course of a project, requires that capital, eventually invested in the project at some point, be invested externally. Under such circumstances, it is necessary to calculate an ERR.

The present worth and annual worth methods are closely related, and both give results identical to those of the IRR method. Rate of return measures are readily understandable, especially when comparing projects of unequal sizes, whereas present/annual worth measures give an explicit expression of the profit contribution of a project. The main advantage of the payback period method is that it is easy to implement and understand, and takes into account the need to have capital recovered quickly.

## ENGINEERING ECONOMICS IN ACTION, PART 5B

### The Invisible Hand

"Hello." Dave's voice was clear enough over the phone that he could have been in his office down the hall, but Naomi could tell from his relaxed tone that the office was not on his mind.

"Hi, Dave, it's Naomi. Can I bend your ear about that drop forge project again?"

"Oh, hi, Naomi. Sure, what have you got?"

"Well, as I see it, IRR has got to be the way to go. Of course, present worth or annual worth will give the same answer, but I'm sure Ed Burns and Anna Kulkowski would prefer IRR. They have to compare potential investments across different parts of the organization. It's kind of hard to compare net present worths of investments in information systems, where you rarely get above a first cost of \$100 000, with forge investments where you can easily get up to a few hundred thousand first cost. And, as I said before, the drop forge operation isn't one in which the annual cost has any particular significance."

There was a short pause. Naomi suddenly regretted speaking as if she was so sure of herself—but, darn it, she was sure on this one.

"Exactly right," Dave replied. Naomi could feel an invisible hand pat her on the back. "So how exactly would you proceed?"

"Well, I have the options ranked by first cost. The first one is just refurbishing the existing machine. There is no test on that one unless we are willing to stop making our own parts, and Clem told me that was out . . ."

Dave interjected with "You don't mean that you're automatically going to refurbish the existing machine, do you?"

"No, no. The simple refurbishing option is the base. I then go to the next option, which is to refurbish the drop forging hammer and replace the materials-handling system. I compare this with the just-refurbish option by looking at the incremental first cost. I will check to see if the additional first cost has an IRR of at least 15 percent after tax, which, Clem tells me, is the minimum acceptable rate of return. If the incremental first cost has an IRR of at least 15 percent, the combination of refurbishing and replacing the materials-handling system is better than just refurbishing. I then consider the next option, which is to buy the manually operated mechanical press with no change in materials handling. I look at the incremental investment here and see if its IRR is at least 15 percent. To go back a step, if the IRR on replacing materials handling plus refurbishing the old machine did not pay off at 15 percent, I would have rejected that and compared the manually operated mechanical press with the first option, just refurbishing the old machine. I then work my way, option by option, up to the seventh. How does that sound?"



"Well, that sounds great, as far as it goes. Have you checked for problems with multiple IRRs?"

"Well, so far each set of cash flows has been a simple investment, but I will be careful."

"I would also compute payback periods for them in case we are having cash flow problems. If the payback is too long, they may not necessarily take an option even with the incremental IRR being above their 15 percent MARR."

Naomi considered this for a second. "One other question, Dave. What should I do about intangibles?"

"You mean the noise from the forging hammer?"

"Yes. It's important, but you can't evaluate it in dollars and cents."

"Just remind them of it in your report. If they want a more formal analysis, they'll come back to you."

"Thanks, Dave. You've been a big help."

As Naomi hung up the phone, she couldn't help smiling ruefully to herself. She had ignored the payback period altogether—after all, it didn't take either interest or service life into account. "I guess that's what they call practical experience," she said to herself as she got out her laptop.

## P R O B L E M S

### A. Key Concepts

- 5.1 Corral Cartage leases trucks to service its shipping contracts. Larger trucks have cheaper operating costs if there is sufficient business, but are more expensive if they are not full. CC has estimates of monthly shipping demand. What comparison method(s) would be appropriate for choosing which trucks to lease?
- 5.2 The bottom flaps of shipping cartons for Yonge Auto Parts are fastened with industrial staples. Yonge needs to buy a new stapler. What comparison method(s) would be appropriate for choosing which stapler to buy?
- 5.3 Joan runs a dog kennel. She is considering installing a heating system for the interior runs, which will allow her to operate all year. What comparison method(s) would be appropriate for choosing which heating system to buy?
- 5.4 A large food company is considering replacing a scale on its packaging line with a more accurate one. What comparison method(s) would be appropriate for choosing which scale to buy?
- 5.5 Mona runs a one-person company producing custom paints for hobbyists. She is considering buying printing equipment to produce her own labels. What comparison method(s) would be appropriate for choosing which equipment to buy?
- 5.6 Peter is the president of a rapidly growing company. There are dozens of important things to do, and cash flow is tight. What comparison method(s) would be appropriate for Peter to make acquisition decisions?
- 5.7 Lemuel is an engineer working for the electric company. He must compare several routes for transmission lines from a distant nuclear plant to new industrial parks north of the city. What comparison method(s) is he likely to use?
- 5.8 Vicky runs a music store that has been suffering from thefts. She is considering installing a magnetic tag system. What comparison method(s) would be best for her to use to choose among competing leased systems?
- 5.9 Thanh's company is growing very fast and has a hard time meeting its orders. An opportunity to purchase additional production equipment has arisen. What comparison method(s) would Thanh use to justify to her manager that the equipment purchase is prudent?

**5.10** What is the IRR for a \$1000 investment that returns \$200 at the end of each of the next

- (a) 7 years?
- (b) 6 years?
- (c) 100 years?
- (d) 2 years?

**5.11** New windows are expected to save \$400 per year in energy costs over their 30-year life for Fab Fabricating. At an initial cost of \$8000 and zero salvage value, using IRR, are they a good investment? Fab's MARR is 8 percent.

**5.12** An advertising campaign will cost \$200 000 for planning and \$40 000 in each of the next six years. It is expected to increase revenues permanently by \$40 000 per year. Additional revenues will be gained in the pattern of an arithmetic gradient with \$20 000 in the first year, declining by \$5000 per year to zero in the fifth year. What is the IRR of this investment? If the company's MARR is 12 percent, is this a good investment?

**5.13** Refer to Review Problem 4.22 on page 114. Assuming the four investments are independent, use the IRR method to select which, if any, should be chosen. Use a MARR of 8 percent.

**5.14** Aline has three contracts from which to choose. The first contract will require an outlay of \$100 000 but will return \$150 000 one year from now. The second contract requires an outlay of \$200 000 and will return \$300 000 one year from now. The third contract requires an outlay of \$250 000 and will return \$355 000 one year from now. Only one contract can be accepted. If her MARR is 20 percent, which one should she choose?

**5.15** Fantastic Footwear can invest in one of two different automated clicker cutters. The first, A, has a \$100 000 first cost. A similar one with many extra features, B, has a \$400 000 first cost. A will save \$50 000 per year over the cutter now in use. B will save \$150 000 per year. Each clicker cutter will last five years. If the MARR is 10 percent, which alternative is better? Use an IRR comparison.

**5.16** Six mutually exclusive projects, A, B, C, D, E, and F, are being considered. They have been ordered by first costs so that project A has the smallest first cost, F the largest. The data in the table below apply to these projects. The data can be interpreted as follows: the IRR on the incremental investment between project D and project C is 6 percent. Which project should be chosen using a MARR of 15 percent?

| Project | IRR on Overall Investment | IRR on Increments of Investment Compared With Project |     |     |     |
|---------|---------------------------|---|-----|-----|-----|
|         |                           | A   | B   | C   | D   |
| A       | 20%                       |   |     |     |     |
| B       | 15%                       | 12%   |     |     |     |
| C       | 24%                       | 30%   | 35% |     |     |
| D       | 16%                       | 18%   | 22% | 6%  |     |
| E       | 17%                       | 16%   | 19% | 15% | 16% |
| F       | 21%                       | 20%   | 21% | 19% | 18% |
|         |                           |   |     |     | 11% |

**5.17** A cash flow sequence has a receipt of \$10 000 today, followed by a disbursement of \$8000 at the end of this year and again next year, and then a receipt of \$5500 three years from now. The MARR is 6 percent.

- (a) What is the ERR for this set of cash flows?
- (b) What is the approximate ERR for this set of cash flows?
- (c) Would a project with these cash flows be a good investment?

**5.18** Yee Swian has received an advance of \$20 000 on a software program she is writing. She will spend \$120 000 this year writing it (consider the money to have been spent at the end of year 1), and then receive \$100 000 at the end of the second year. The MARR is 12 percent.

- (a) What is the IRR for this project? Does the result make sense?
- (b) What is the precise ERR?
- (c) What is the approximate ERR?

## B. Applications

**5.19** CB Electronix must buy a piece of equipment to place electronic components on the printed circuit boards it assembles. The proposed equipment has a 10-year life with no scrap value.

The supplier has given CB several purchase alternatives. The first is to purchase the equipment for \$850 000. The second is to pay for the equipment in 10 equal installments of \$135 000 each, starting one year from now. The third is to pay \$200 000 now and \$95 000 at the end of each year for the next 10 years.

- (a) Which alternative should CB choose if its MARR is 11 percent per year? Use an IRR comparison approach.
- (b) Below what MARR does it make sense for CB to buy the equipment now for \$850 000?

**5.20** The following table summarizes information for four projects:

| Project | First Cost | IRR on Overall Investment | IRR on Increments of Investment Compared With Project |     |     |
|---------|------------|---------------------------|---|-----|-----|
|         |            |                           | 1   | 2   | 3   |
| 1       | \$100 000  | 19%                       |   |     |     |
| 2       | 175 000    | 15%                       | 9%  |     |     |
| 3       | 200 000    | 18%                       | 17%   | 23% |     |
| 4       | 250 000    | 16%                       | 12%   | 17% | 13% |

The data can be interpreted in the following way: The IRR on the incremental investment between project 4 and project 3 is 13 percent.

- (a) If the projects are independent, which projects should be undertaken if the MARR is 16 percent?
- (b) If the projects are mutually exclusive, which project should be undertaken if the MARR is 15 percent? Indicate what logic you have used.

- (c) If the projects are mutually exclusive, which project should be undertaken if the MARR is 17 percent? Indicate what logic you have used.
- 5.21** There are several mutually exclusive ways Grazemont Dairy can meet a requirement for a filling machine for its creamer line. One choice is to buy a machine. This would cost \$65 000 and last for six years with a salvage value of \$10 000. Alternatively, it could contract with a packaging supplier to get a machine free. In this case, the extra costs for packaging supplies would amount to \$15 000 per year over the six-year life (after which the supplier gets the machine back with no salvage value for Grazemont). The third alternative is to buy a used machine for \$30 000 with zero salvage value after six years. The used machine has extra maintenance costs of \$3000 in the first year, increasing by \$2500 per year. In all cases, there are installation costs of \$6000 and revenues of \$20 000 per year. Using the IRR method, determine which is the best alternative. The MARR is 10 percent.
- 5.22** The following cash flows result from a potential construction contract for Erstwhile Engineering.
1. Receipts of \$500 000 at the start of the contract and \$1 200 000 at the end of the fourth year
  2. Expenditures at the end of the first year of \$400 000 and at the end of the second year of \$900 000
  3. A net cash flow of zero at the end of the third year
- Using an appropriate rate of return method, for a MARR of 25 percent, should Erstwhile Engineering accept this project?
- 5.23** Samiran has entered into an agreement to develop and maintain a computer program for symbolic mathematics. Under the terms of the agreement, he will pay \$900 000 in royalties to the investor at the end of the fifth, tenth, and fifteenth years, with the investor paying Samiran \$450 000 now, and then \$650 000 at the end of the twelfth year.
- Samiran's MARR for this type of investment is 20 percent. Calculate the ERR of this project. Should he accept this agreement on the basis of these disbursements and receipts alone? Are you sure that the ERR you calculated is the only ERR? Why? Are you sure that your recommendation to Samiran is correct? Justify your answer.
- 5.24** Refer to Example 4.6 on page 97, in which a mechanical engineer has decided to introduce automated materials-handling equipment to a production line. Use a present worth approach with an IRR analysis to determine which of the two alternatives is best. The MARR is 9 percent. Use the repeated lives method to deal with the fact that the service lives of the two alternatives are not equal.
- 5.25** Refer to Problem 4.28 on page 116. Val has determined that the salvage value of the XJ3 after two years of service is \$1900. Using the IRR method, which display panel is the better choice? Use a two-year study period. She must choose one of the alternatives.
- 5.26** Three construction jobs are being considered by Clam City Construction (see the following table). Each is characterized by an initial deposit paid by the client to Clam City Construction (CCC), a yearly cost incurred by CCC at the end of each of three years, and a final payment to CCC by the client at the end of three years. CCC has the capacity to do only one of these contracts. Use an appropriate rate of return method to determine which it should do. The company's MARR is 10 percent.

| Job | Deposit (\$) | Cost per Year (\$) | Final Payment (\$) |
|-----|--------------|--------------------|--------------------|
| 1   | 100 000      | 75 000             | 200 000            |
| 2   | 150 000      | 100 000            | 230 000            |
| 3   | 175 000      | 150 000            | 300 000            |

- 5.27** Kool Karavans is considering three investment proposals. Each of them is characterized by an initial cost, annual savings over four years, and no salvage value, as illustrated in the following table. The company can only invest in two of these proposals. If its MARR is 12 percent, which two should it choose?

| Proposal | First Cost (\$) | Annual Savings (\$) |
|----------|-----------------|---------------------|
| A        | 40 000          | 20 000              |
| B        | 110 000         | 30 000              |
| C        | 130 000         | 45 000              |

- 5.28** Development projects done by Standalone Products are subsidized by a government grant program. The program pays 30 percent of the total cost of the project (costs summed without discounting—i.e., the interest rate is zero), half at the beginning of the project and half at the end, up to a maximum of \$100 000. There are two projects being considered. One is a customized checkweigher for cheese products, and the other is an automated production scheduling system. Each project has a service life of five years. Costs and benefits for both projects, not including grant income, are shown below. Only one can be done, and the grant money is certain. PTR has a MARR of 15 percent for projects of this type. Using an appropriate rate of return method, which project should be chosen?

|                 | Checkweigher | Scheduler |
|-----------------|--------------|-----------|
| First cost      | \$30 000     | \$10 000  |
| Annual costs    | 5 000        | 12 000    |
| Annual benefits | 14 000       | 17 000    |
| Salvage value   | 8 000        | 0         |

- 5.29** Jacob is considering the replacement of the heating system for his building. There are three alternatives. All are natural-gas-fired furnaces, but they vary in energy efficiency. Model A is leased at a cost of \$500 per year over a 10-year study period. There are installation charges of \$500 and no salvage value. It is expected to provide energy savings of \$200 per year. Model B is purchased for a total cost of \$3600, including installation. It has a salvage value of \$1000 after 10 years of service, and is expected to provide energy savings of \$500 per year. Model C is also purchased, for a total cost of \$8000, including installation. However, half of this cost is paid now and the other half is paid at the end of two years. It has a salvage value of \$1000 after 10 years and is expected to provide energy savings of \$1000 per year. For a MARR of

12 percent and using a rate of return method, which heating system should be installed? One model must be chosen.

### C. More Challenging Problems

- 5.30** Project X has an IRR of 16 percent and a first cost of \$20 000. Project Y has an IRR of 17 percent and a first cost of \$18 000. The MARR is 15 percent. What can be said about which (if either) of the two projects should be undertaken if (a) the projects are independent and (b) the projects are mutually exclusive?
- 5.31** Charlie has a project for which he had determined a present worth of \$56 740. He now has to calculate the IRR for the project, but unfortunately he has lost complete information about the cash flows. He knows only that the project has a five-year service life and a first cost of \$180 000, that a set of equal cash flows occurred at the end of each year, and that the MARR used was 10 percent. What is the IRR for this project?
- 5.32** Lucy's project has a first cost  $P$ , annual savings  $A$ , and a salvage value of \$1000 at the end of the 10-year service life. She has calculated the present worth as \$20 000, the annual worth as \$4000, and the payback period as three years. What is the IRR for this project?
- 5.33** Patti's project has an IRR of 15 percent, first cost  $P$ , and annual savings  $A$ . She observed that the salvage value  $S$  at the end of the five-year life of the project was exactly half of the purchase price, and that the present worth of the project was exactly double the annual savings. What was Patti's MARR?
- 5.34** Jerry has an opportunity to buy a bond with a face value of \$10 000 and a coupon rate of 14 percent, payable semiannually.
- If the bond matures in five years and Jerry can buy one now for \$3500, what is his IRR for this investment?
  - If his MARR for this type of investment is 20 percent, should he buy the bond?
- 5.35** Refer to Problem 4.15 on page 113. Find which alternative is preferable using the IRR method and a MARR of 5 percent. Assume that one of the alternatives must be chosen. Answer the following questions by using present worth computations to find the IRRs. Use the least common multiple of service lives.
- What are the cash flows for each year of the comparison period (i.e., the least common multiple of service lives)?
  - Are you able to conclude that there is a single IRR on the incremental investment? Why or why not?
  - Which of the two alternatives should be chosen? Use the ERR method if necessary.
- 5.36** Refer to Problem 4.16 on page 113. Use an IRR analysis to determine which of the two alternatives is best. The MARR is 8 percent. Use the repeated lives method to deal with the unequal service lives of the two alternatives.
- 5.37** Zhe develops truss analysis software for civil engineers. He has the opportunity to contract with at most one of two clients who have approached him with development proposals. One contract pays him \$15 000 immediately and then \$22 000 at the end of the project three years from now. The other possibility pays \$20 000 now and \$5000 at the end of each of the three years. In either case, his expenses will be \$10 000 per year. For a MARR of 10 percent, which project should Zhe accept? Use an appropriate rate of return method.

**5.38** The following table summarizes cash flows for a project:

| Year | Cash Flow at End of Year |
|------|--------------------------|
| 0    | -\$5000                  |
| 1    | 3000                     |
| 2    | 4000                     |
| 3    | -1000                    |

- (a) Write out the expression you need to solve to find the IRR(s) for this set of cash flows. Do not solve.
  - (b) What is the maximum number of solutions for the IRR that could be found in part (a)? Explain your answer in one sentence.
  - (c) You have found that an IRR of 14.58 percent solves the expression in part (a). Compute the project balances for each year.
  - (d) Can you tell (without further computations) if there is a unique IRR from this set of cash flows? Explain in one sentence.
- 5.39** Pepper Properties screens various projects using the payback period method. For renovation decisions, the minimum acceptable payback period is five years. Renovation projects are characterized by an immediate investment of  $P$  dollars, which is recouped as an annuity of  $A$  dollars per year over 20 years. The company is considering changing to the IRR method for such decisions. If it changed to the IRR method, what MARR would result in exactly the same decisions as its current policy using payback period?
- 5.40** Three mutually exclusive designs for a bypass are under consideration. The bypass has a 10-year life. The first design incurs a cost of \$1.2 million for a net savings of \$300 000 per annum. The second design would cost \$1.5 million for a net savings of \$400 000 per annum. The third has a cost of \$2.1 million for a net savings of \$500 000 per annum. For each of the alternatives, what range of values for the MARR results in its being chosen? It is not necessary that any be chosen.
- 5.41** Linus's project has cash flows at times 0, 1, and 2. He notices that for a MARR of 12 percent, the ERR falls exactly halfway between the MARR and the IRR, while for a MARR of 18 percent, the ERR falls exactly one-quarter of the way between the MARR and the IRR. If the cash flow is \$2000 at time 2 and negative at time 0, what are the possible values of the cash flow at time 1?
- 5.42** Charro Environmental is considering taking over a contaminated building site on a former military base. In return for \$10 000 000 from the government, it will invest \$8 000 000 per year for the following three years to clean up the site. Once the site is clean (end of year 3), it will receive a further \$15 000 000 from the government. Over the following two years it will invest \$5 000 000 per year constructing a new commercial building on the site. The new building will last forever and net \$5 000 000 per year for Charro from tenant leases (starting at the end of year 6). If Charro's MARR is 10 percent, what is the exact ERR for this project? Should Charro proceed with the project? (Note: All disbursements can be assumed to occur at the end of the year.)

**M I N I - C A S E 5 . 1****The Galore Creek Project**

NovaGold Resources is a former gold exploration company that has recently been transforming itself into a gold producer. Its first independent development is the Galore Creek Project. It is also involved as a partner with Placer Dome in another project, and with Rio Tinto in a third. Galore Creek is expected to produce an average of 7650 kilograms of gold, 51 030 kilograms of silver, and 5 670 000 kilograms of copper over its first five years.

In a news release, NovaGold reported that an independent engineering services company calculated that the project would pay back the US\$500 million mine capital costs in 3.4 years of a 23-year life. They also calculated a pre-tax rate of return of 12.6 percent and an undiscounted after-tax NPV of US\$329 million. All of these calculations were done at long-term average metal prices. At then-current metal prices the pre-tax rate of return almost doubles to 24.3 percent and the NPV (net present value = present worth) increases to US\$1.065 billion.

Source: "Higher Grades and Expanded Tonnage Indicated by Drilling at Galore Creek Gold–Silver–Copper Project," news release, August 18, 2004, NovaGold Resources Inc. site, [www.novagold.net](http://www.novagold.net), accessed May 11, 2008.

### **Discussion**

Companies have a choice of how to calculate the benefits of a project in order to determine if it is worth doing. They also have a choice of how to report the benefits of a project to others.

NovaGold is a publicly traded company. Because of this, when a large and very important project is being planned, not only does NovaGold want to make good business decisions, but it also must maintain strong investor confidence and interest.

In this news release, payback period, IRR, and NPV were used to communicate the value of the Galore Creek project. However, you need to look carefully at the wording to ensure that you can correctly interpret the claims about the economic viability of the project.

### **Questions**

1. “[A]n independent engineering services company calculated that the project would pay back the US\$500 million mine capital costs in 3.4 years of a 23-year life.” There are a variety of costs associated with any project. The payback period here is calculated with respect to “mine capital costs.” This suggests that there might be “non-mine” capital costs—for example, administrative infrastructure, transportation system, etc. It also means that operating costs are not included in this calculation. What do you think is the effect of calculating the payback period on “mine capital costs” alone?
2. “They also calculated a pre-tax rate of return of 12.6 percent. . . .” Taxes reduce the profit from an enterprise and correspondingly reduce the rate of return. As will be seen in Chapter 8, a 50 percent corporate tax rate is fairly common. Thus, if the pre-tax rate is 12.6 percent, the after-tax rate would be about 6.3 percent. Does 6.3 percent seem to you a sufficient return for a capital-intensive, risky project of this nature, given other investment opportunities available?
3. “[A]nd an ‘undiscounted’ after-tax NPV of US\$329 million.” The term *undiscounted* means that the present worth of the project was calculated with an interest rate of 0 percent. Using a spreadsheet, construct a graph showing the present worth of the project for a range of interest rates from 0 percent to 20 percent, assuming the annual returns

for the project are evenly distributed over the 23-year life of the project. Does the reported value of \$US329 million fairly represent a meaningful NPV for the project?

4. The returns for the Galore Creek Project are much more attractive at then-current metal prices, which were significantly higher than long-term average metal prices. Which metal prices are more sensible to use when evaluating the worth of the project?
5. Did NovaGold report its economic evaluation of the Galore Creek Project in an ethical manner?



# Depreciation and Financial Accounting

Engineering Economics in Action, Part 6A: The Pit Bull

## 6.1 Introduction

## 6.2 Depreciation and Depreciation Accounting

### 6.2.1 Reasons for Depreciation

### 6.2.2 Value of an Asset

### 6.2.3 Straight-Line Depreciation

### 6.2.4 Declining-Balance Depreciation

## 6.3 Elements of Financial Accounting

### 6.3.1 Measuring the Performance of a Firm

### 6.3.2 The Balance Sheet

### 6.3.3 The Income Statement

### 6.3.4 Estimated Values in Financial Statements

### 6.3.5 Financial Ratio Analysis

### 6.3.6 Financial Ratios

## Review Problems

## Summary

Engineering Economics in Action, Part 6B: Usually the Truth

## Problems

### Mini-Case 6.1: Research In Motion

### Appendix 6A: Cost Estimation

## Problems

## ENGINEERING ECONOMICS IN ACTION, PART 6A

### The Pit Bull

Naomi liked to think of Terry as a pit bull. Terry had this endearing habit of finding some detail that irked him and not letting go of it until he was satisfied that things were done properly. Naomi had seen this several times in the months they had worked together. Terry would sink his teeth into some quirk of Canadian Widgets's operating procedures and, just like a fighting dog, would not let go until the fight was over.

This time, it was about the disposal of some computing equipment. Papers in hand, he quietly approached Naomi and earnestly started to explain his concern. "Naomi, I don't know what Bill Fisher is doing, but something's definitely not right here. Look at this."

Terry displayed two documents to Naomi. One was an accounting statement showing the book value of various equipment, including some CAD/CAM computers that had been sold for scrap the previous week. The other was a copy of a sales receipt from a local salvage firm for that same equipment.

"I don't like criticizing my fellow workers, but I really am afraid that Bill might be doing something wrong." Bill Fisher was the buyer responsible for capital equipment at Canadian Widgets, and he also disposed of surplus assets. "You know the CAD/CAM workstations they had in engineering design? Well, they were replaced recently and sold. Here is the problem. They were only three years old, and our own accounting department estimated their value as about \$5000 each." Terry's finger pointed to the evidence on the accounting statement. "But here"—his finger moved to the guilty figure on the sales receipt—"they were actually sold for \$300 each!" Terry sat back in his chair. "How about that!"

Naomi smiled. Unfortunately, she would have to pry his teeth out of this one. "Interesting observation, Terry. But you know, I think it's probably okay. Let me explain."

## 6.1

### Introduction

Engineering projects often involve an investment in equipment, buildings, or other assets that are put to productive use. As time passes, these assets lose value, or *depreciate*. The first part of this chapter is concerned with the concept of depreciation and several methods that are commonly used to model depreciation. Depreciation is taken into account when a firm states the value of its assets in its financial statements, as seen in the second half of this chapter. It also forms an important part of the decision of when to replace an aging asset and when to make cyclic replacements, as will be seen in Chapter 7, and has an important impact on taxation, as we will see in Chapter 8.

With the growth in importance of small technology-based enterprises, many engineers have taken on broad managerial responsibilities that include financial accounting. Financial accounting is concerned with recording and organizing the financial data of businesses. The data cover both *flows over time*, like revenues and expenses, and *levels*, like an enterprise's resources and the claims on those resources at a given date. Even engineers who do not have broad managerial responsibilities need to know the elements of financial accounting to understand the enterprises with which they work.

In the second part of this chapter, we explain two basic financial statements used to summarize the financial dimensions of a business. We then explain how these statements can be used to make inferences about the financial health of the firm.

## 6.2 | Depreciation and Depreciation Accounting

### 6.2.1 Reasons for Depreciation

An asset starts to lose value as soon as it is purchased. For example, a car bought for \$20 000 today may be worth \$18 000 next week, \$15 000 next year, and \$1000 in 10 years. This loss in value, called **depreciation**, occurs for several reasons.

*Use-related physical loss:* As something is used, parts wear out. An automobile engine has a limited life span because the metal parts within it wear out. This is one reason why a car diminishes in value over time. Often, use-related physical loss is measured with respect to *units of production*, such as thousands of kilometres for a car, hours of use for a light bulb, or thousands of cycles for a punch press.

*Time-related physical loss:* Even if something is not used, there can be a physical loss over time. This can be due to environmental factors affecting the asset or to endogenous physical factors. For example, an unused car can rust and thus lose value over time. Time-related physical loss is expressed in units of time, such as a 10-year-old car or a 40-year-old sewage treatment plant.

*Functional loss:* Losses can occur without any physical changes. For example, a car can lose value over time because styles change so that it is no longer fashionable. Other examples of causes of loss of value include legislative changes, such as for pollution control or safety devices, and technical changes. Functional loss is usually expressed simply in terms of the particular unsatisfied function.

### 6.2.2 Value of an Asset

Models of depreciation can be used to estimate the loss in value of an asset over time, and also to determine the remaining value of the asset at any point in time. This remaining value has several names, depending on the circumstances.

**Market value** is usually taken as the actual value an asset can be sold for in an open market. Of course, the only way to determine the actual market value for an asset is to sell it. Consequently, the term *market value* usually means an *estimate* of the market value. One way to make such an estimation is by using a depreciation model that reasonably captures the true loss in value of an asset.

**Book value** is the depreciated value of an asset for accounting purposes, as calculated with a depreciation model. The book value may be more or less than market value. The depreciation model used to arrive at a book value might be controlled by regulation for some purposes, such as taxation, or simply by the desirability of an easy calculation scheme. There might be several different book values for the same asset, depending on the purpose and depreciation model applied. We shall see how book values are reported in financial statements later in this chapter.

**Scrap value** can be either the actual value of an asset at the end of its physical life (when it is broken up for the material value of its parts) or an estimate of the scrap value calculated using a depreciation model.

**Salvage value** can be either the actual value of an asset at the end of its useful life (when it is sold) or an estimate of the salvage value calculated using a depreciation model.

It is desirable to be able to construct a good model of depreciation in order to state a book value of an asset for a variety of reasons:

1. In order to make many managerial decisions, it is necessary to know the value of owned assets. For example, money may be borrowed using the firm's assets

as collateral. In order to demonstrate to the lender that there is security for the loan, a credible estimate of the assets' value must be made. A depreciation model permits this to be done. The use of depreciation for this purpose is explored more thoroughly in the second part of this chapter.

2. One needs an estimate of the value of owned assets for planning purposes. In order to decide whether to keep an asset or replace it, you have to be able to judge how much it is worth. More than that, you have to be able to assess how much it will be worth at some time in the future. The impact of depreciation in replacement studies is covered in Chapter 7.
3. Government tax legislation requires that taxes be paid on company profits. Because there can be many ways of calculating profits, strict rules are made concerning how to calculate income and expenses. These rules include a particular scheme for determining depreciation expenses. This use of depreciation is discussed more thoroughly in Chapter 8.

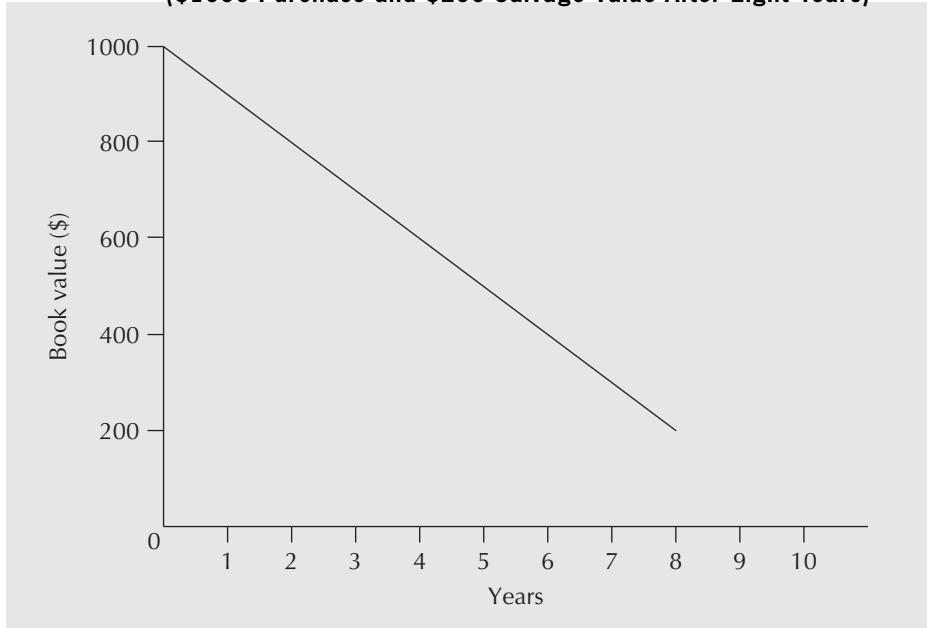
To match the way in which certain assets depreciate and to meet regulatory or accuracy requirements, many different depreciation models have been developed over time. Of the large number of depreciation schemes available (see Close-Up 6.1), straight-line and declining-balance are certainly the most commonly used. Straight-line depreciation is popular primarily because it is particularly easy to calculate. The declining-balance method is required by tax law in many jurisdictions for determining corporate taxes, as is discussed in Chapter 8. In particular, straight-line and declining-balance depreciation are the only ones necessary for corporate tax calculations in Canada and most other countries. Consequently, these are the only depreciation methods presented in detail in this book.

### 6.2.3 Straight-Line Depreciation

The **straight-line method of depreciation** assumes that the rate of loss in value of an asset is constant over its useful life. This is illustrated in Figure 6.1 for an asset worth

| CLOSE-UP 6.1             |  | Depreciation Methods |
|--------------------------|--|----------------------|
| Method                   | Description  |                      |
| Straight-line            | The book value of an asset diminishes by an equal <i>amount</i> each year.   |                      |
| Declining-balance        | The book value of an asset diminishes by an equal <i>proportion</i> each year.   |                      |
| Sum-of-the-years'-digits | An accelerated method, like declining-balance, in which the depreciation rate is calculated as the ratio of the remaining years of life to the sum of the digits corresponding to the years of life. |                      |
| Double-declining-balance | A declining-balance method in which the depreciation rate is calculated as $2/N$ for an asset with a service life of $N$ years.  |                      |
| 150%-declining-balance   | A declining-balance method in which the depreciation rate is calculated as $1.5/N$ for an asset with a service life of $N$ years.  |                      |
| Units-of-production      | The depreciation rate is calculated per unit of production as the ratio of the units produced in a particular year to the total estimated units produced over the asset's lifetime.                  |                      |

**Figure 6.1 Book Value Under Straight-Line Depreciation  
(\$1000 Purchase and \$200 Salvage Value After Eight Years)**



\$1000 at the time of purchase and \$200 eight years later. Graphically, the book value of the asset is determined by drawing a straight line between its first cost and its salvage or scrap value.

Algebraically, the assumption is that the rate of loss in asset value is constant and is based on its original cost and salvage value. This gives rise to a simple expression for the depreciation charge per period. We determine the depreciation per period from the asset's current value and its estimated salvage value at the end of its useful life,  $N$  periods from now, by

$$D_{sl}(n) = \frac{P - S}{N} \quad (6.1)$$

where

$D_{sl}(n)$  = the depreciation charge for period  $n$  using the straight-line method

$P$  = the purchase price or current market value

$S$  = the salvage value after  $N$  periods

$N$  = the useful life of the asset, in periods

Similarly, the book value at the end of any particular period is easy to calculate:

$$BV_{sl}(n) = P - n \left[ \frac{P - S}{N} \right] \quad (6.2)$$

where

$BV_{sl}(n)$  = the book value at the end of period  $n$  using straight-line depreciation

**EXAMPLE****6.1**

A laser cutting machine was purchased four years ago for \$380 000. It will have a salvage value of \$30 000 two years from now. If we believe a constant rate of depreciation is a reasonable means of determining book value, what is its current book value?

From Equation (6.2), with  $P = \$380\,000$ ,  $S = \$30\,000$ ,  $N = 6$ , and  $n = 4$ ,

$$BV_{sl}(4) = 380\,000 - 4 \left[ \frac{380\,000 - 30\,000}{6} \right]$$

$$BV_{sl}(4) = 146\,667$$

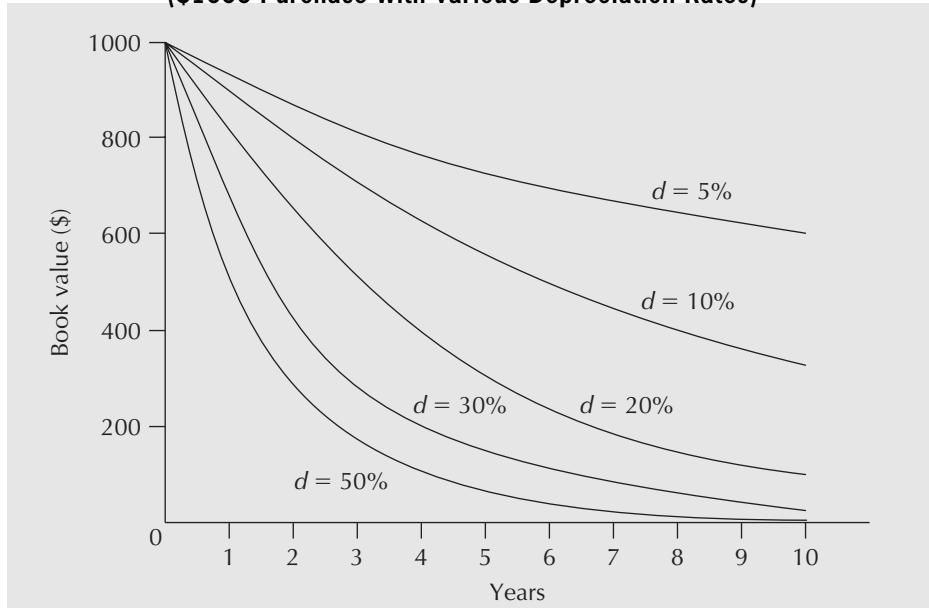
The current book value for the cutting machine is \$146 667. ■

The straight-line depreciation method has the great advantage of being easy to calculate. It is also easy to understand and is in common use. The main problem with the method is that its assumption of a constant rate of loss in asset value is often not valid. Thus, book values calculated using straight-line depreciation will frequently be different from market values. For example, the loss in value of a car over its first year (say, from \$20 000 to \$15 000) is clearly more than its loss in value over its fifth year (say, from \$6000 to \$5000). The declining-balance method of depreciation covered in the next section allows for “faster” depreciation in earlier years of an asset’s life.

#### 6.2.4 Declining-Balance Depreciation

**Declining-balance depreciation** (also known as *reducing-balance depreciation*) models the loss in value of an asset over a period as a constant fraction of the asset’s current book value. In other words, the depreciation charge in a particular period is a constant proportion (called the depreciation rate) of its closing book value from the previous period. The effect of various depreciation rates on book values is illustrated in Figure 6.2.

**Figure 6.2 Book Value Under Declining-Balance Depreciation (\$1000 Purchase With Various Depreciation Rates)**



Algebraically, the depreciation charge for period  $n$  is simply the depreciation rate multiplied by the book value from the end of period  $(n - 1)$ . Noting that  $BV_{db}(0) = P$ ,

$$D_{db}(n) = BV_{db}(n - 1) \times d \quad (6.3)$$

where

$D_{db}(n)$  = the depreciation charge in period  $n$  using the declining-balance method

$BV_{db}(n)$  = the book value at the end of period  $n$  using the declining-balance method

$d$  = the depreciation rate

Similarly, the book value at the end of any particular period is easy to calculate by noting that the remaining value after each period is  $(1 - d)$  times the value at the end of the previous period.

$$BV_{db}(n) = P(1 - d)^n \quad (6.4)$$

where

$P$  = the purchase price or current market value

In order to use the declining-balance method of depreciation, we must determine a reasonable depreciation rate. By using an asset's current value,  $P$ , and a salvage value,  $S$ ,  $n$  periods from now, we can use Equation (6.4) to find the declining balance rate that relates  $P$  and  $S$ .

$$\begin{aligned} BV_{db}(n) &= S = P(1 - d)^n \\ (1 - d) &= \sqrt[n]{\frac{S}{P}} \\ d &= 1 - \sqrt[n]{\frac{S}{P}} \end{aligned} \quad (6.5)$$

### EXAMPLE

### 6.2

Paquita wants to estimate the scrap value of a smokehouse 20 years after purchase. She feels that the depreciation is best represented using the declining-balance method, but she doesn't know what depreciation rate to use. She observes that the purchase price of the smokehouse was \$245 000 three years ago, and an estimate of its current salvage value is \$180 000. What is a good estimate of the value of the smokehouse after 20 years?

From Equation (6.5),

$$\begin{aligned} d &= 1 - \sqrt[n]{\frac{S}{P}} \\ &= 1 - \sqrt[3]{\frac{180\,000}{245\,000}} \\ &= 0.097663 \end{aligned}$$

Then, by using Equation (6.4), we have

$$BV_{db}(20) = 245\,000(1 - 0.097663)^{20} = 31\,372$$

An estimate of the salvage value of the smokehouse after 20 years using the declining-balance method of depreciation is \$31 372. ■



## S P R E A D S H E E T   S A V V Y

Excel provides functions to facilitate depreciation calculations. In particular, the SLN and DB functions compute straight-line and declining-balance depreciation.  $SLN(P,S,N)$  computes the depreciation charge for any period, where  $P$  is the first cost of the asset and  $S$  is its salvage value at the end of  $N$  time periods assumed to be the useful life of the asset. For the declining-balance method, the depreciation in period  $t$  is found with  $DB(P,S,N,t)$ , where  $P$ ,  $S$ , and  $N$  are as previously defined. To illustrate the use of DB, the computations from Table 6.1 on page 171 have been duplicated here. The first table shows the results of using the DB function to compute declining-balance depreciation:

| Assumptions:  |                     |                                |                      |
|---------------|---------------------|--------------------------------|----------------------|
| P =           | \$250 000           |                                |                      |
| S =           | 10 000              |                                |                      |
| $N$ (years) = | 6                   |                                |                      |
|               |                     | Declining Balance Depreciation |                      |
| Year          | Depreciation Charge | Accumulated Depreciation       | Remaining Book Value |
| 0             |                     |                                | \$250 000            |
| 1             | \$103 750           | \$103 750                      | 146 250              |
| 2             | 60 694              | 164 444                        | 85 556               |
| 3             | 35 506              | 199 950                        | 50 050               |
| 4             | 20 771              | 220 721                        | 29 279               |
| 5             | 12 151              | 232 871                        | 17 129               |
| 6             | 7 108               | 239 980                        | 10 020               |

The second table shows the underlying formulas:

|    | A                   | B                              | C                        | D                    |
|----|---------------------|--------------------------------|--------------------------|----------------------|
| 1  | <b>Assumptions:</b> |                                |                          |                      |
| 2  | P =                 | 250000                         |                          |                      |
| 3  | S =                 | 10000                          |                          |                      |
| 4  | $N$ (years) =       | 6                              |                          |                      |
| 5  |                     | Declining Balance Depreciation |                          |                      |
| 6  | Year                | Depreciation Charge            | Accumulated Depreciation | Remaining Book Value |
| 7  | 0                   |                                |                          | 250000               |
| 8  | 1                   | =DB(\$B\$2,\$B\$3,\$B\$4,A8)   | =C7+B8                   | =D7-B8               |
| 9  | 2                   | =DB(\$B\$2,\$B\$3,\$B\$4,A9)   | =C8+B9                   | =D8-B9               |
| 10 | 3                   | =DB(\$B\$2,\$B\$3,\$B\$4,A10)  | =C9+B10                  | =D9-B10              |
| 11 | 4                   | =DB(\$B\$2,\$B\$3,\$B\$4,A11)  | =C10+B11                 | =D10-B11             |
| 12 | 5                   | =DB(\$B\$2,\$B\$3,\$B\$4,A12)  | =C11+B12                 | =D11-B12             |
| 13 | 6                   | =DB(\$B\$2,\$B\$3,\$B\$4,A13)  | =C12+B13                 | =D12-B13             |

The table was completed by entering the parameter values, cells B2:B4, followed by cell D7 and B8:D8. The remainder of the table was completed by filling down from row 8. There are some slight differences between the results obtained with the DB function and those obtained using explicit formulas (see Table 6.1) because the DB function rounds the depreciation rate used to three significant digits.

This example is a good illustration of the potential difficulties associated with using built-in functions. For one thing, errors can be introduced, either through rounding or because of an incorrect sign on a parameter. Also, as seen in this example, the built-in functions may not offer any time or complexity advantage—it would have been just as easy or easier to construct Table 6.1 from basic formulas. Built-in functions are a tool that can help under the right circumstances, but should be used with discretion.

The declining-balance method has a number of useful features. For one thing, it matches the observable loss in value that many assets have over time. The rate of loss is expressed in one parameter, the depreciation rate. It is relatively easy to calculate, although perhaps not quite as easy as the straight-line method. In particular, it is often required to be used for taxation purposes, as discussed in detail in Chapter 8.

**EXAMPLE****6.3**

Sherbrooke Data Services has purchased a new mass storage system for \$250 000. It is expected to last six years, with a \$10 000 salvage value. Using both the straight-line and declining-balance methods, determine the following:

- The depreciation charge in year 1
- The depreciation charge in year 6
- The book value at the end of year 4
- The accumulated depreciation at the end of year 4

This is an ideal application for a spreadsheet solution. Table 6.1 illustrates a spreadsheet that calculates the book value, depreciation charge, and accumulated depreciation for both depreciation methods over the six-year life of the system.

**Table 6.1 Spreadsheet for Example 6.3**



| Year | Straight-Line Depreciation |                          |            |
|------|----------------------------|--------------------------|------------|
|      | Depreciation Charge        | Accumulated Depreciation | Book Value |
| 0    |                            |                          | \$250 000  |
| 1    | \$40 000                   | \$ 40 000                | 210 000    |
| 2    | 40 000                     | 80 000                   | 170 000    |
| 3    | 40 000                     | 120 000                  | 130 000    |
| 4    | 40 000                     | 160 000                  | 90 000     |
| 5    | 40 000                     | 200 000                  | 50 000     |
| 6    | 40 000                     | 240 000                  | 10 000     |

| Year | Declining-Balance Depreciation |                          |            |
|------|--------------------------------|--------------------------|------------|
|      | Depreciation Charge            | Accumulated Depreciation | Book Value |
| 0    |                                |                          | \$250 000  |
| 1    | \$103 799                      | \$103 799                | 146 201    |
| 2    | 60 702                         | 164 501                  | 85 499     |
| 3    | 35 499                         | 200 000                  | 50 000     |
| 4    | 20 760                         | 220 760                  | 29 240     |
| 5    | 12 140                         | 232 900                  | 17 100     |
| 6    | 7 100                          | 240 000                  | 10 000     |

The depreciation charge for each year with the *straight-line* method is \$40 000:

$$D_{sl}(n) = (250\ 000 - 10\ 000)/6 = 40\ 000$$

The depreciation rate for the *declining-balance* method is

$$d = 1 - \sqrt[n]{\frac{S}{P}} = 1 - \sqrt[6]{\frac{10\ 000}{250\ 000}} = 0.4152$$

The detailed calculation for each of the questions is as follows:

- (a) Depreciation charge in year 1

$$D_{sl}(1) = (250\ 000 - 10\ 000)/6 = 40\ 000$$

$$D_{db}(1) = BV_{db}(0)d = 250\ 000(0.4152) = 103\ 799.11$$

- (b) Depreciation charge in year 6

$$D_{sl}(6) = D_{sl}(1) = 40\ 000$$

$$D_{db}(6) = BV_{db}(5)d = 250\ 000(0.5848)^5(0.4152) = 7099.82$$

- (c) Book value at the end of year 4

$$BV_{sl}(4) = 250\ 000 - 4(250\ 000 - 10\ 000)/6 = 90\ 000$$

$$BV_{db}(4) = 250\ 000(1 - 0.4152)^4 = 29\ 240.17$$

- (d) Accumulated depreciation at the end of year 4

$$\text{Using the straight-line method: } P - BV_{sl}(4) = 160\ 000$$

$$\text{Using the declining-balance method: } P - BV_{db}(4) = 220\ 759.83$$

In summary, depreciation affects economic analyses in several ways. First, it allows us to estimate the value of an owned asset, as illustrated in the above examples. We shall see in the next part of this chapter how these values are reported in a firm's financial statements. Next, the capability of estimating the value of an asset is particularly useful in replacement studies, which is the topic of Chapter 7. Finally, in Chapter 8, we cover aspects of tax systems that affect decision making; in particular, we look at the effect of depreciation expenses.

## 6.3 Elements of Financial Accounting

How well is a business doing? Can it survive an unforeseen temporary drop in cash flows? How does a business compare with others of its size in the industry? Answering these questions and others like them is part of the accounting function. The accounting function has two parts, financial accounting and management accounting. **Financial accounting** is concerned with recording and organizing the financial data of a business, which include revenues and expenses, and an enterprise's resources and the claims on those resources. **Management accounting** is concerned with the costs and benefits of the various activities of an enterprise. The goal of management accounting is to provide managers with information to help in decision making.

Engineers have always played a major role in management accounting, especially in a part of management accounting called *cost* accounting. Cost accounting involves determining the costs of products, processes, and services and is an important foundation for estimating the cost of a proposed engineering project. We provide an introduction to cost

estimation in Appendix 6A. The topic of management accounting is not covered further, as it is difficult to provide useful information without taking more than a chapter. Instead, we focus on financial accounting.

With the growth in importance of small technology-based enterprises, many engineers have taken on broad managerial responsibilities that include financial accounting. The object of financial accounting is to provide information to internal management and interested external parties. Internally, management uses financial accounting information for processes such as budgeting, cash management, and management of long-term debt. External users include actual and potential investors and creditors who wish to make rational decisions about an enterprise. External users also include government agencies concerned with taxes and regulation.

Areas of interest to all these groups include an enterprise's revenues and expenses, and assets (resources held by the enterprise) and liabilities (claims on those resources).

In the next few sections, we discuss two basic summary financial statements that give information about these matters: the *balance sheet* and the *income statement*. These statements form the basis of a financial report, which is usually produced on a monthly, quarterly, semiannual, or yearly basis. Following the discussion of financial statements, we shall consider the use of information in these statements when making inferences about an enterprise's performance compared with industry standards and with its own performance over time.

### 6.3.1 Measuring the Performance of a Firm

The flow of money in a company is much like the flow of water in a network of pipes or the flow of electricity in an electrical circuit, as illustrated in Figure 6.3. In order to measure the performance of a water system, we need to determine the flow through the system and the pressure in the system. For an electrical circuit, the analogous parameters are current and voltage. Flow and current are referred to as *through variables*, and are measured with respect to time (flow is litres per second and current is amperes, which are coulombs per second). Pressure and voltage are referred to as *across variables*, and are measured at a point in time.

The flow of money in an organization is measured in a similar way with the income statement and balance sheet. The income statement represents a *through variable* because it summarizes revenues and expenses over a period of time. It is prepared by listing the revenues earned during a period and the expenses incurred during the same period, and by subtracting total expenses from total revenues, arriving at a net income. An income statement is always associated with a particular period of time, be it a month, quarter, or year.

The balance sheet, in contrast to the income statement, is a snapshot of the financial position of a firm at a particular point in time, and so represents an *across variable*. The financial position is summarized by listing the assets of the firm, its liabilities (debts), and the equity of the owner or owners.

### 6.3.2 The Balance Sheet

A **balance sheet** (also called a *position statement*) is a snapshot of an enterprise's financial position at a particular point in time, normally the last day of an accounting period. A firm's financial position is summarized in a balance sheet by listing its assets, liabilities, and owners' (or shareholders') equity. The heading of the balance sheet gives the name of the enterprise and the date.

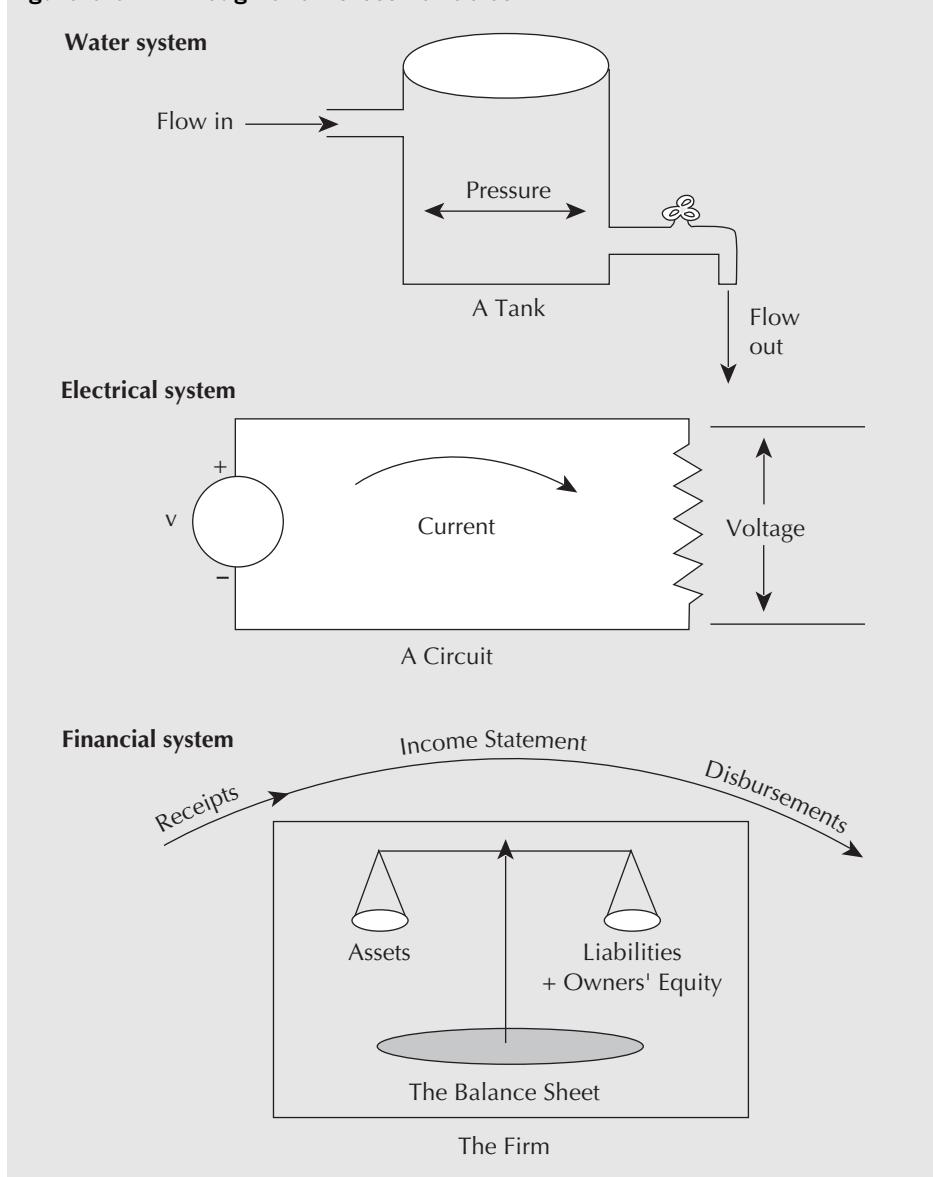
**Figure 6.3 Through and Across Variables****EXAMPLE****6.4**

Table 6.2 shows a balance sheet for the Major Electric Company, a manufacturer of small electrical appliances.

The first category of financial information in a balance sheet reports the **assets** of the enterprise. These are the economic resources owned by the enterprise or, more simply, everything that the enterprise owns. Assets are classified on a balance sheet as current assets and long-term assets. **Current assets** are cash and other assets that could be converted to cash within a relatively short period of time, usually a year or less. Inventory and accounts receivable are examples of non-cash current assets. **Long-term assets** (also called fixed assets or non-current assets) are assets that are not expected to be converted to cash in the short term, usually taken to be one year. Indeed, it may be

**Table 6.2 Balance Sheet for the Major Electric Company**

| <b>Major Electric Company</b><br><b>Balance Sheet as of November 30, 2013</b> |                           |           |
|---|---------------------------|-----------|
| <b>Assets</b>   |                           |           |
| <i>Current Assets</i>   |                           |           |
| Cash  | \$ 39 000                 |           |
| Accounts receivable   | 27 000                    |           |
| Raw materials inventory   | 52 000                    |           |
| Finished goods inventory  | <u>683 000</u>            |           |
| Total Current Assets  | <u>\$ 801 000</u>         |           |
| <i>Long-Term Assets</i>   |                           |           |
| Equipment   | \$6 500 000               |           |
| Less accumulated depreciation   | 4 000 000                 | 2 500 000 |
| Buildings   | 1 750 000                 |           |
| Less accumulated depreciation   | 150 000                   | 1 600 000 |
| Land  | <u>500 000</u>            |           |
| Total Long-Term Assets  | <u>\$ 4 600 000</u>       |           |
| <b>Total Assets</b>   | <b><u>\$5 401 000</u></b> |           |
| <b>Liabilities and Owners' Equity</b>   |                           |           |
| <i>Current Liabilities</i>  |                           |           |
| Accounts payable  | \$ 15 000                 |           |
| Loan due December 31, 2013  | <u>75 000</u>             |           |
| Total Current Liabilities   | <u>90 000</u>             |           |
| <i>Long-Term Liabilities</i>  |                           |           |
| Loan due December 31, 2018  | <u>1 000 000</u>          |           |
| <b>Total Liabilities</b>  | <b><u>\$1 090 000</u></b> |           |
| <i>Owners' Equity</i>   |                           |           |
| Common stock: 1 000 000 shares<br>at \$3 par value per share                  | \$ 3 000 000              |           |
| Retained earnings   | <u>1 311 000</u>          |           |
| <b>Total Owners' Equity</b>   | <b><u>\$4 311 000</u></b> |           |
| <b>Total Liabilities and Owners' Equity</b>                                   | <b><u>\$5 401 000</u></b> |           |

difficult to convert long-term assets into cash without selling the business as a going concern. Equipment, land, and buildings are examples of long-term assets.

An enterprise's **liabilities** are claims, other than those of the owners, on a business's assets or, simply put, everything that the enterprise owes. Debts are usually the most important liabilities on a balance sheet. There may be other forms of liabilities as well. A commitment to the employees' pension plan is an example of a non-debt liability. As with assets, liabilities may be classified as current or long-term. **Current liabilities** are liabilities that are due within some short period of time, usually a year or less. Examples of current liabilities are debts that are close to maturity, accounts payable to suppliers, and taxes due. **Long-term liabilities** are liabilities that are not expected to draw on the business's current assets. Long-term loans and bonds issued by the business are examples of long-term liabilities.

The difference between a business's assets and its liabilities is the amount due to the owners—their equity in the business. That is, owners' equity is what is left over from assets after claims of others are deducted. We have, therefore,

$$\text{Owners' equity} = \text{Assets} - \text{Liabilities}$$

or

$$\text{Assets} = \text{Liabilities} + \text{Owners' equity}$$

**Owners' equity** is the interest of the owner or owners of a firm in its assets. For the basic types of ownership structure in business organizations, see Close-Up 6.2. Owners' equity usually appears as two components on a balance sheet of a corporation. The first is the par value of the owners' shares. When an enterprise is first organized, it is authorized to issue a certain number of shares. **Par value** is the price per share set by the corporation at the time the shares are originally issued. At any time after the first sale, the shares may be traded at prices that are greater than or less than the par value, depending on investors' expectations of the return that will be earned by the business in the future. There is no reason to expect the market price to equal the par value for very long after the shares are first sold. Nonetheless, the amount recorded in the balance sheet is the original par value. New shares sold any time after the first issue may have a par value of their own, distinct from those of the original issue.

The second part of owners' equity usually shown on the balance sheet is retained earnings. **Retained earnings** includes the cumulative sum of earnings from normal

## CLOSE-UP 6.2

### Types of Business Ownerships

There are three basic ways to structure a business organization.

A **sole proprietorship** (sole trader, single proprietor) is a business owned by one person. It is the simplest and least regulated form of business to start (in essence, all you need is a business name), and accounts for the largest number of businesses. Under a sole proprietorship, the owner keeps all the profits, but at the same time, has *unlimited liability*; that is, the owner is personally responsible for all business debts and the creditors can come after even his or her personal assets in order to recoup debts.

A **partnership** is a business owned by two or more people (partners). In a *general partnership*, the partners run the business together and share all profits and losses (*unlimited liability*) according to a partnership agreement. In a *limited partnership*, some partners are involved only as investors (limited partners) and they let one or more general partners take charge of day-to-day operation. The limited partners have *limited liability*; that is, they are only liable for up to the amount of their investment and their personal assets are protected from the creditors of business debts.

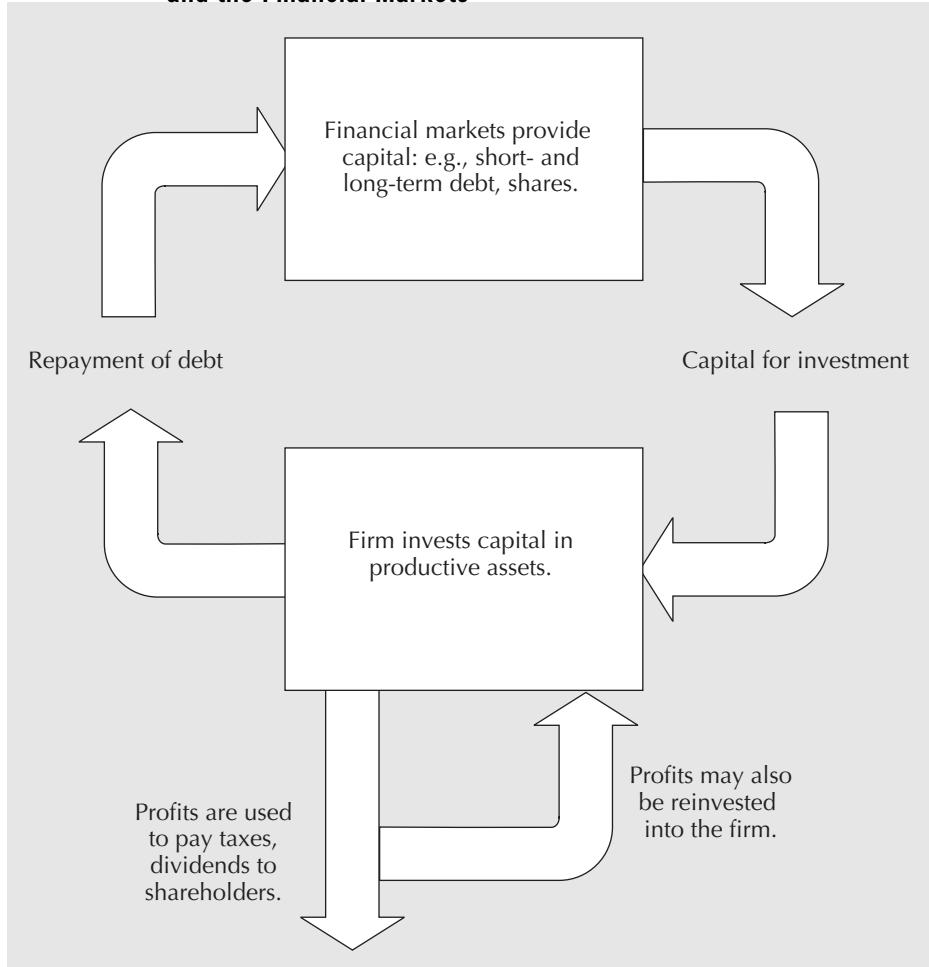
A **corporation** is owned by shareholders. The shareholders elect the board of directors, and the board is responsible for selecting the managers to run the business in the interest of shareholders. A corporation is set up as a business entity with its own rights and responsibilities, separate from the owners. This means that the corporation is responsible for its own debts, and the owners have limited liability (up to the amount of their investment). In the United Kingdom, a corporation is called either a *limited company* (Limited or Ltd.) or *public limited company* (PLC). Other terms include *Aktiengesellschaft* (AG; Germany, Austria, Switzerland), *société anonyme* (SA; France), and *naamloze vennootschap* (NV; The Netherlands).

Generally speaking, the main advantage of incorporating a business is its relative ease in raising capital for a growth opportunity. Sole proprietorships and partnerships, although easier to start up than corporations, are both disadvantaged by personal liability issues, limited availability of equity, and difficulty of ownership transfer, all of which contribute to potential difficulties in raising sufficient funds for growth.

operations in addition to gains (or losses) from transactions like the sale of plant assets or investments—the proceeds of which have been reinvested in the business (i.e., not paid out as dividends). Firms retain earnings mainly to expand operations through the purchase of additional assets. Contrary to what one may think, retained earnings do not represent cash. They may be invested in assets such as equipment and inventory.

The balance sheet gets its name from the idea that the total assets are equal in value to or *balanced by* the sum of the total liabilities and the owners' equity. A simple way of thinking about it is that the capital used to buy each asset has to come from debt (liabilities) and/or equity (owners' equity). At a company's start-up, the original shareholders may provide capital in the form of equity, and there will also likely be debt capital. The general term used to describe the markets in which short- or long-term debt and equity are exchanged is the **financial market**. The capital provided through a financial market finances the assets and capital used for production. As the company undertakes its business activities, it will make or lose money. As it does so, assets will rise or fall and equity and/or debt will rise or fall correspondingly. For example, if the company makes a profit, the profits will either pay off debts, be invested in new assets, or be paid as dividends to shareholders. Figure 6.4 provides an overview of the sources and uses of capital in an organization. ■

**Figure 6.4 Cash Flow Relationship Between the Company's Assets and the Financial Markets**



**EXAMPLE****6.5**

Ian Claymore is the accountant at Major Electric. He has just realized that he forgot to include in the balance sheet for November 30, 2013, a government loan of \$10 000 to help in the purchase of a \$25 000 test stand (which he also forgot to include). The loan is due to be repaid in two years. When he revises the statement, what changes should he make?

The government loan is a long-term liability because it is due in more than one year. Consequently, an extra \$10 000 would appear in long-term liabilities. This extra \$10 000 must also appear as an asset for the balance to be maintained. The \$25 000 value of the test stand would appear as equipment, increasing the equipment amount to \$6 525 000. The \$15 000 extra must come from a decrease in cash from \$39 000 to \$24 000.

Depending on the timing of the purchase, depreciation for the test stand might also be recognized in the balance sheet. Depreciation would reduce the net value of the equipment by the depreciation charge. The same amount would be balanced in the liabilities section by a reduction in retained earnings. ■

### 6.3.3 The Income Statement

An **income statement** summarizes an enterprise's revenues and expenses over a specified accounting period. Months, quarters, and years are commonly used as reporting periods. As with the balance sheet, the heading of an income statement gives the name of the enterprise and the reporting period. The income statement first lists revenues, by type, followed by expenses. Expenses are then subtracted from revenues to give income (or profit) before taxes. Income taxes are then deducted to obtain net income. See Close-Up 6.3 for a measure used for operating profit.

The income statement summarizes the revenues and expenses of a business over a period of time. However, it does not directly give information about the generation of cash. For this reason, it may be useful to augment the income statement with a **statement of changes in financial position** (also called a cash flow statement, a statement of sources and uses of funds, or a funds statement), which shows the amounts of cash generated by a company's operation and by other sources, and the amounts of cash used for investments and other non-operating disbursements.

**CLOSE-UP 6.3****Earnings Before Interest and Income Tax (EBIT)**

Income before taxes *and* before interest payments—that is, total revenue minus operating expenses (all expenses except for income tax and interest)—is commonly referred to as the **earnings before interest and taxes** (EBIT). EBIT measures the company's operating profit, which results from making sales and controlling operating expenses. Due to its focus on operating profit, EBIT is often used to judge whether there is enough profit to recoup the cost of capital (see Appendix 4A).

**EXAMPLE****6.6**

The income statement for the Major Electric Company for the year ended November 30, 2013, is shown in Table 6.3.

We see that Major Electric's largest source of revenue was the sale of goods. The firm also earned revenue from the sale of management services to other companies. The largest

**Table 6.3 Income Statement for the Major Electric Company**

| <b>Major Electric Company<br/>Income Statement</b><br>for the Year Ended November 30, 2013 |             |                    |
|--|-------------|--------------------|
| <b>Revenues</b>  |             |                    |
| Sales  | \$7 536 000 |                    |
| Management fees earned   | 106 000     |                    |
| <b>Total Revenues</b>  |             | <b>\$7 642 000</b> |
| <b>Expenses</b>  |             |                    |
| Cost of goods sold   | \$6 007 000 |                    |
| Selling costs  | 285 000     |                    |
| Administrative expenses  | 757 000     |                    |
| Interest paid  | 86 000      |                    |
| <b>Total Expenses</b>  |             | <b>\$7 135 000</b> |
| Income before taxes  | \$507 000   |                    |
| Taxes (at 40%)   | 202 800     |                    |
| <b>Net Income</b>  |             | <b>\$304 200</b>   |

expense was the cost of the goods sold. This includes the cost of raw materials, production costs, and other costs incurred to produce the items sold. Sometimes firms choose to list cost of goods sold as a *negative revenue*. The cost of goods sold will be subtracted from the sales to give a net sales figure. The net sales amount is the listed revenue, and the cost of goods sold does not appear as an expense.

The particular entries listed as either revenues or expenses will vary, depending on the nature of the business and the activities carried out. All revenues and expenses appear in one of the listed categories. For Major Electric, for example, the next item on the list of expenses, selling costs, includes delivery cost and other expenses such as salespersons' salaries. Administrative expenses include those costs incurred in running the company that are not directly associated with manufacturing. Payroll administration is an example of an administrative expense.

Subtracting the expenses from the revenues gives a measure of profit for the company over the accounting period—one year in the example. However, this profit is taxed at a rate that depends on the company's particular characteristics. For Major Electric, the tax rate is 40 percent, so the company's profit is reduced by that amount. ■

**EXAMPLE****6.7**

Refer back to Example 6.5. Ian Claymore also forgot to include the effects of the loan and test stand purchase in the income statement shown in Example 6.6. When he revises the statement, what changes should he make?

Neither the loan nor the purchase of the asset appears directly in the income statement. The loan itself is neither income nor expense; if interest is paid on it, this is an expense. The test stand is a depreciable asset, which means that only the depreciation for the test stand appears as an expense. ■

### 6.3.4 Estimated Values in Financial Statements

The values in financial statements appear to be authoritative. However, many of these values are estimates based on the **cost principle of accounting**. The cost principle of accounting states that assets are to be valued on the basis of their cost as opposed to market or other values. For example, the \$500 000 given as the value of the land held by Major Electric is what Major Electric paid for the land. The market value of the land may now be greater or less than \$500 000.

The value of plant and equipment is also based on cost. The value reported in the balance sheet is given by the initial cost minus accumulated depreciation. If Major Electric tried to sell the equipment, it might get more or less than this because depreciation models only approximate market value. For example, if there were a significant improvement in new equipment offered now by equipment suppliers compared with when Major Electric bought its equipment, Major Electric might get less than the \$2 500 000 shown on the balance sheet.

Consider the finished goods inventory as another example of the cost principle of accounting. The value reported is Major Electric's manufacturing cost for producing the items. The implicit assumption being made is that Major Electric will be able to sell these goods for at least the cost of producing them. Their value may be reduced in later balance sheets if it appears that Major Electric cannot sell the goods easily.

The value shown for accounts receivable is clearly an estimate. Some fraction of accounts receivable may never be collected by Major. The value in the balance sheet reflects what the accountant believes to be a conservative estimate based on experience.

In summary, when examining financial statement data, it is important to remember that many reported values are estimates. Most firms include their accounting methods and assumptions within their periodic reports to assist in the interpretation of the statements.

### 6.3.5 Financial Ratio Analysis

**Performance measures** are calculated values that allow conclusions to be drawn from data. Performance measures drawn from financial statements can be used to answer such questions as:

1. Is the firm able to meet its short-term financial obligations?
2. Are sufficient profits being generated from the firm's assets?
3. How dependent is the firm on its creditors?

**Financial ratios** are one kind of performance measure that can answer these questions. They give an analyst a framework for asking questions about the firm's liquidity, asset management, leverage, and profitability. Financial ratios are ratios between key amounts taken from the financial statements of the firm. While financial ratios are simple to compute, they do require some skill to interpret, and they may be used for different purposes. For example, internal management may be concerned with the firm's ability to pay its current liabilities or the effect of long-term borrowing for a plant expansion. An external investor may be interested in past and current earnings to judge the wisdom of investing in the firm's stock. A bank will assess the riskiness of lending money to a firm before extending credit.

To properly interpret financial ratios, analysts commonly make comparisons with ratios computed for the same company from previous financial statements (a **trend analysis**) and with industry standard ratios. This is referred to as **financial ratio analysis**.

Industry standards can be obtained from various commercial and government websites and publications. In Canada, Statistics Canada ([www.statcan.gc.ca](http://www.statcan.gc.ca)) publishes *Financial and*

## NET VALUE 6.1

### Securities Regulators

Countries that trade in corporate stocks and bonds (collectively called *securities*) generally have a regulatory body to protect investors, ensure that trading is fair and orderly, and facilitate the acquisition of capital by businesses.

In Canada, the Canadian Securities Administrators (CSA) coordinates regulators from each of Canada's provinces and territories, and also educates Canadians about the securities

industry, the stock markets, and how to protect investors from scams by providing a variety of educational materials on securities and investing. In the United States, the Securities Exchange Commission (SEC) regulates securities for the country. The SEC has been particularly active in enforcement in recent years.

Canada: [www.securities-administrators.ca](http://www.securities-administrators.ca)

United States: [www.sec.gov](http://www.sec.gov)

*Taxation Statistics for Enterprises*, which lists financial data from the balance sheets and income statements as well as selected financial ratios for numerous industries. In the United States, Standard & Poor's *Industry Surveys* and Dun & Bradstreet's *Industry Handbook* are classic commercial sources of information, and less recent information for some industries can be found at the U.S. Census Bureau site ([www.census.gov](http://www.census.gov)). Examples of a past industry-total balance sheet and income statement (in millions of dollars) for the electronic products manufacturing industry are shown in Tables 6.4 and 6.5. These statistics allow an analyst to compare an individual firm's financial statements with those of the appropriate industry.

We shall see in the next section how the financial ratios derived from industry-total financial data can be used to assess the health of a firm.

### 6.3.6 Financial Ratios

Numerous financial ratios are used in the financial analysis of a firm. Here we introduce six commonly used ratios to illustrate their use in a comparison with ratios from the industry-total data and in trend analysis. To facilitate the discussion, we shall use Tables 6.6 and 6.7, which show the balance sheets and income statements for Electco Electronics, a small electronics equipment manufacturer.

The first two financial ratios we address are referred to as **liquidity ratios**. Liquidity ratios evaluate the ability of a business to meet its current liability obligations. In other words, they help us evaluate its ability to weather unforeseen fluctuations in cash flows. A company that does not have a reserve of cash or other assets that can be converted into cash easily may not be able to fulfill its short-term obligations.

A company's net reserve of cash and assets easily converted to cash is called its working capital. **Working capital** is simply the difference between total current assets and total current liabilities:

$$\text{Working capital} = \text{Current assets} - \text{Current liabilities}$$

The adequacy of working capital is commonly measured with two ratios. The first, the **current ratio**, is the ratio of all current assets relative to all current liabilities. The current ratio may also be referred to as the **working capital ratio**.

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}}$$

**Table 6.4 Example of Industry-Total Balance Sheet (in Millions of \$)**

| Electronic Product Manufacturing Balance Sheet |                 |
|--|-----------------|
| Assets   |                 |
| <i>Assets</i>                                  |                 |
| Cash and deposits                              | \$ 2 547        |
| Accounts receivable and accrued revenue        | 9 169           |
| Inventories                                    | 4 344           |
| Investments and accounts with affiliates       | 10 066          |
| Portfolio investments                          | 1 299           |
| Loans  | 839             |
| Capital assets, net                            | 4 966           |
| Other assets                                   | 2 898           |
| <b>Total assets</b>                            | <b>\$36 128</b> |
| <b>Liabilities and Owners' Equity</b>          |                 |
| <i>Liabilities</i>                             |                 |
| Accounts payable and accrued liabilities       | \$ 8 487        |
| Loans and accounts with affiliates             | 3 645           |
| Borrowings:                                    |                 |
| Loans and overdrafts from banks                | 1 408           |
| Loans and overdrafts from others               | 1 345           |
| Bankers' acceptances and paper                 | 232             |
| Bonds and debentures                           | 2 487           |
| Mortgages                                      | 137             |
| Deferred income tax                            | (247)           |
| Other liabilities                              | 1 931           |
| <b>Total liabilities</b>                       | <b>\$19 426</b> |
| <i>Owners' Equity</i>                          |                 |
| Share capital                                  | 9 177           |
| Contributed surplus and other                  | 415             |
| Retained earnings                              | 7 110           |
| <b>Total owners' equity</b>                    | <b>\$16 702</b> |
| <b>Current assets</b>                          | <b>\$17 663</b> |
| <b>Current liabilities</b>                     | <b>11 692</b>   |

Electco Electronics had a current ratio of  $4314/2489 = 1.73$  in 2011 (Table 6.8). Ordinarily, a current ratio of 2 is considered adequate, although this determination may depend a great deal on the composition of current assets. It also may depend on industry standards computed from industry-total financial statements such as those shown in Tables 6.4 and 6.5. In the case of Electco, the industry standard is 1.51, which is listed in Table 6.8. It would appear that Electco had a reasonable amount of liquidity in 2011 from the industry's perspective.

A second ratio, the acid-test ratio, is more conservative than the current ratio. The **acid-test ratio** (also known as the **quick ratio**) is the ratio of quick assets to current liabilities:

$$\text{Acid-test ratio} = \frac{\text{Quick assets}}{\text{Current liabilities}}$$

**Table 6.5 Example of Industry-Total Income Statement (in Millions of \$)**

| Electronic Product Manufacturing Income Statement |               |
|---|---------------|
| <b>Operating revenues</b>                         | \$50 681      |
| Sales of goods and services                       | 49 357        |
| Other operating revenues                          | 1324          |
| <b>Operating expenses</b>                         | <b>48 091</b> |
| Depreciation, depletion,<br>and amortization      | 1501          |
| Other operating expenses                          | 46 590        |
| <b>Operating profit</b>                           | <b>2590</b>   |
| <b>Other revenue</b>                              | <b>309</b>    |
| Interest and dividends                            | 309           |
| <b>Other expenses</b>                             | <b>646</b>    |
| Interest on short-term debt                       | 168           |
| Interest on long-term debt                        | 478           |
| <b>Gains (Losses)</b>                             | <b>(122)</b>  |
| On sale of assets                                 | 25            |
| Others  | (148)         |
| <b>Profit before income tax</b>                   | <b>2130</b>   |
| Income tax  | 715           |
| Equity in affiliates' earnings                    | 123           |
| <b>Profit before extraordinary gains</b>          | <b>1538</b>   |
| Extraordinary gains/losses                        | —             |
| <b>Net profit</b>                                 | <b>1538</b>   |

The acid-test ratio recognizes that some current assets—for example, inventory and prepaid expenses—may be more difficult to turn into cash than others. *Quick assets* are cash, accounts receivable, notes receivable, and temporary investments in marketable securities—those current assets considered to be highly *liquid*.

The acid-test ratio for Electco for 2011 was  $(431 + 2489)/2489 = 1.17$ . Normally, an acid-test ratio of 1 is considered adequate, as this indicates that a firm could meet all its current liabilities with the use of its *quick* current assets if it were necessary. Electco appears to meet this requirement.

The current ratio and the acid-test ratio provide important information about how liquid a firm is, or how well it is able to meet its current financial obligations. The extent to which a firm relies on debt for its operations can be captured by what are called **leverage** or **debt-management ratios**. An example of such a ratio is the **equity ratio**. It is the ratio of total owners' equity to total assets. The smaller this ratio, the more dependent the firm is on debt for its operations and the higher are the risks the company faces.

$$\begin{aligned} \text{Equity ratio} &= \frac{\text{Total owners' equity}}{\text{Total liabilities} + \text{Total owners' equity}} \\ &= \frac{\text{Total owners' equity}}{\text{Total assets}} \end{aligned}$$

**Table 6.6 Balance Sheets for Electco Electronics for Years Ended 2011, 2012, and 2013**

| <b>Electco Electronics<br/>Year-End Balance Sheets<br/>(in thousands of dollars)</b> |                    |                    |                    |
|--|--------------------|--------------------|--------------------|
|  | <b>2011</b>        | <b>2012</b>        | <b>2013</b>        |
| <b>Assets</b>  |                    |                    |                    |
| <i>Current assets</i>  |                    |                    |                    |
| Cash   | 431                | 340                | 320                |
| Accounts receivable  | 2489               | 2723               | 2756               |
| Inventories  | 1244               | 2034               | 2965               |
| Prepaid services   | 150                | 145                | 149                |
| Total current assets   | <u>4314</u>        | <u>5242</u>        | <u>6190</u>        |
| <i>Long-term assets</i>  |                    |                    |                    |
| Buildings and equipment<br>(net of depreciation)                                     | 3461               | 2907               | 2464               |
| Land   | 521                | 521                | 521                |
| Total long-term assets   | <u>3982</u>        | <u>3428</u>        | <u>2985</u>        |
| <b>Total Assets</b>  | <b>8296</b>        | <b>8670</b>        | <b>9175</b>        |
| <b>Liabilities</b>   |                    |                    |                    |
| <i>Current liabilities</i>   |                    |                    |                    |
| Accounts payable   | 1493               | 1780               | 2245               |
| Bank overdraft   | 971                | 984                | 992                |
| Accrued taxes  | 25                 | 27                 | 27                 |
| Total current liabilities  | <u>2489</u>        | <u>2791</u>        | <u>3264</u>        |
| <i>Long-term liabilities</i>   |                    |                    |                    |
| Mortgage   | 2489               | 2455               | 2417               |
| <b>Total Liabilities</b>   | <b>4978</b>        | <b>5246</b>        | <b>5681</b>        |
| <b>Owners' Equity</b>  |                    |                    |                    |
| Share capital  | 1825               | 1825               | 1825               |
| Retained earnings  | 1493               | 1599               | 1669               |
| <b>Total Owners' Equity</b>  | <b><u>3318</u></b> | <b><u>3424</u></b> | <b><u>3494</u></b> |
| <b>Total Liabilities and Owners' Equity</b>  | <b>8296</b>        | <b>8670</b>        | <b>9175</b>        |

The equity ratio for Electco in 2011 was  $3318/8296 = 0.40$  and for the industry was 0.46, as shown in Table 6.8. Electco has paid for roughly 60 percent of its assets with debt; the remaining 40 percent represents equity. This is close to the industry practice as a whole and would appear acceptable.

Another group of ratios is called the **asset-management ratios** or **efficiency ratios**. They assess how efficiently a firm is using its assets. The inventory-turnover ratio is an example. The **inventory-turnover ratio** specifically looks at how efficiently a firm is using its resources to manage its inventories. This is reflected in the number of times that

**Table 6.7 Income Statements for Electco Electronics for Years Ended 2011, 2012, 2013**

| Electco Electronics<br>Income Statements<br>(in thousands of dollars) |        |        |        |
|---|--------|--------|--------|
|   | 2011   | 2012   | 2013   |
| <b>Revenues</b>   |        |        |        |
| Sales   | 12 440 | 11 934 | 12 100 |
| <b>Total Revenues</b>   | 12 440 | 11 934 | 12 100 |
| <b>Expenses</b>   |        |        |        |
| Cost of goods sold<br>(excluding depreciation)                        | 10 100 | 10 879 | 11 200 |
| Depreciation  | 692    | 554    | 443    |
| Interest paid   | 346    | 344    | 341    |
| <b>Total Expenses</b>   | 11 138 | 11 777 | 11 984 |
| Profit before taxes   | 1302   | 157    | 116    |
| Taxes (at 40%)  | 521    | 63     | 46     |
| <b>Profit before extraordinary items</b>                              | 781    | 94     | 70     |
| Extraordinary gains/losses  | 70     | —      | —      |
| <b>Profit after taxes</b>   | 851    | 94     | 70     |

**Table 6.8 Industry-Standard Ratios and Financial Ratios for Electco Electronics**

| Financial Ratio              | Industry Standard | Electco Electronics |       |       |
|------------------------------|-------------------|---------------------|-------|-------|
|                              |                   | 2011                | 2012  | 2013  |
| Current ratio                | 1.51              | 1.73                | 1.88  | 1.90  |
| Quick ratio                  | —                 | 1.17                | 1.10  | 0.94  |
| Equity ratio                 | 0.46              | 0.40                | 0.39  | 0.38  |
| Inventory-turnover ratio     | 11.36             | 10.00               | 5.87  | 4.08  |
| Return-on-total-assets ratio | 4.26%             | 9.41%               | 1.09% | 0.76% |
| Return-on-equity ratio       | 9.21%             | 23.53%              | 2.75% | 2.00% |

its inventories are replaced (or turned over) per year. The inventory-turnover ratio provides a measure of whether the firm has more or less inventory than normal.

$$\text{Inventory-turnover ratio} = \frac{\text{Sales}}{\text{Inventories}}$$

Electco's turnover ratio for 2011 was  $12\,440/1244 = 10$  turns per year (Table 6.9). This is reasonably close to the industry standard of 11.36 turns per year as shown in Table 6.8. In 2011, Electco invested roughly the same amount in inventory per dollar of sales as the industry did, on average.

Two points should be observed about the inventory-turnover ratio. First, the sales amount in the numerator has been generated over a period of time, while the inventory amount in the denominator is for one point in time. A more accurate measure of inventory turns would be to approximate the average inventory over the period in which sales were generated.

A second point is that sales refer to market prices, while inventories are listed at cost. The result is that the inventory-turnover ratio as computed above will be an overstatement of the true turnover. It may be more reasonable to compute inventory turnover based on the ratio of cost of goods sold to inventories. Despite this observation, traditional financial analysis uses the sales-to-inventories ratio.

The next two ratios give evidence of how productively assets have been employed in producing a profit. The **return-on-assets (ROA) ratio** or **net-profit ratio** is the first example of a **profitability ratio**:

$$\text{Return-on-assets ratio} = \frac{\text{Net income (before extraordinary items)}}{\text{Total assets}}$$

Electco had a return on assets of  $781/8296 = 0.0941$  or 9.41 percent in 2011. Table 6.8 shows that the industry-total ROA for 2011 was 4.26 percent. Note the comments on extraordinary items in Close-Up 6.4.

The second example of a profitability ratio is the **return-on-equity (ROE) ratio**:

$$\text{Return-on-equity ratio} = \frac{\text{Net income (before extraordinary items)}}{\text{Total equity}}$$

The return-on-equity ratio looks at how much profit a company has earned in comparison to the amount of capital that the owners have tied up in the company. It is often compared to how much the owners could have earned from an alternative investment and used as a measure of investment performance. Electco had a ROE of  $781/3318 = 0.2354$  or 23.54 percent in 2011, whereas the industry-standard ROE was 9.21 percent, as shown in Table 6.8. The year 2011 was an excellent one for the owners at Electco from their investment point of view.

Overall, Electco's performance in 2011 was similar to that of the electronic product manufacturing industry as a whole. One exception is that Electco generated higher profits than the norm; it may have been extremely efficient in its operations that year.

The rosy profit picture painted for Electco in 2011 does not appear to extend into 2012 and 2013, as a trend analysis shows. Table 6.8 shows the financial ratios computed for 2012 and 2013 with those of 2011 and the industry standard.

For more convenient reference, we have summarized the six financial ratios we have dealt with and their definitions in Table 6.9.

#### CLOSE-UP 6.4

#### Extraordinary Items

**Extraordinary items** are gains and losses that do not typically result from a company's normal business activities, are not expected to occur regularly, and are not recurring factors in any evaluations of the ordinary operations of the business. For example, cost or loss of income caused by natural disasters (floods, tornadoes, ice storms, etc.) would be extraordinary loss. Revenue created by the sale of a division of a firm is an example of extraordinary gain. Extraordinary items are reported separately from regular items and are listed net of applicable taxes.

**Table 6.9 A Summary of Financial Ratios and Definitions**

| Ratio                                     | Definition   | Comments  |
|---|--|---|
| Current ratio<br>(Working capital ratio)  | $\frac{\text{Current assets}}{\text{Current liabilities}}$ | A liquidity ratio   |
| Acid-test ratio<br>(Quick ratio)          | $\frac{\text{Quick assets}}{\text{Current liabilities}}$   | A liquidity ratio<br>(Quick assets = Current assets – Inventories – Prepaid items)      |
| Equity ratio                              | $\frac{\text{Total equity}}{\text{Total assets}}$          | A leverage or debt-management ratio   |
| Inventory-turnover ratio                  | $\frac{\text{Sales}}{\text{Inventories}}$                  | An asset management or efficiency ratio   |
| Return-on-assets ratio (Net-profit ratio) | $\frac{\text{Net income}}{\text{Total assets}}$            | A profitability ratio (excludes extraordinary items)                                    |
| Return-on-equity ratio                    | $\frac{\text{Net income}}{\text{Total equity}}$            | A profitability ratio (measure of investment performance; excludes extraordinary items) |

Electco's return on assets has dropped significantly over the three-year period. Though the current and quick ratios indicate that Electco should be able to meet its short-term liabilities, there has been a significant buildup of inventories over the period. Electco is not selling what it is manufacturing. This would explain the drop in Electco's inventory turns.

Coupled with rising inventory levels is a slight increase in the cost of goods sold over the three years. From the building and equipment entries in the balance sheet, we know that no major capital expenditures on new equipment have occurred during the period. Electco's equipment may be aging and in need of replacement, though further analysis of what is happening is necessary before any conclusions on this issue can be drawn.

A final observation is that Electco's accounts receivable seems to be growing over the three-year period. Since there may be risks associated with the possibility of bad debt, Electco should probably investigate the matter.

In summary, Electco's main problem appears to be a mismatch between production levels and sales levels. Other areas deserving attention are the increasing cost of goods sold and possible problems with accounts receivable collection. These matters need investigation if Electco is to recover from its current slump in profitability.

We close the section on financial ratios with some cautionary notes on their use. First, since financial statement values are often approximations, we need to interpret the financial ratios accordingly. In addition, accounting practices vary from firm to firm and may lead to differences in ratio values. Wherever possible, look for explanations of how values are derived in financial reports.

A second problem encountered in comparing a firm's financial ratios with the industry-standard ratios is that it may be difficult to determine what industry the firm best fits into. Furthermore, within every industry, large variations exist. In some cases, an analyst may construct a relevant "average" by searching out a small number of similar firms (in size and business type) that may be used to form a customized industry average.

Finally, it is important to recognize the effect of seasonality on the financial ratios calculated. Many firms have highly seasonal operations with natural high and low periods of activity. An analyst needs to judge these fluctuations in context. One solution to this problem is to adjust the data seasonally through the use of averages. Another is to collect financial data from several seasons so that any deviations from the normal pattern of activity can be picked up.

Despite our cautionary words on the use of financial ratios, they do provide a useful framework for analyzing the financial health of a firm and for answering many questions about its operations.

## REVIEW PROBLEMS

### REVIEW PROBLEM 6.1

Joan is the sole proprietor of a small lawn-care service. Last year, she purchased an eight-horsepower chipper-shredder to make mulch out of small tree branches and leaves. At the time it cost \$760. She expects that the value of the chipper-shredder will decline by a constant amount each year over the next six years. At the end of six years, she thinks that it will have a salvage value of \$100.

Construct a table that gives the book value of the chipper-shredder at the end of each year, for six years. Also indicate the accumulated depreciation at the end of each year. A spreadsheet may be helpful.

### ANSWER

The depreciation charge for each year is

$$D_{sl}(n) = \frac{(P-S)}{N} = \frac{760 - 100}{6} = 110 \quad n = 1, \dots, 6$$

This is the requested table:

| Year | Depreciation Charge | Book Value | Accumulated Depreciation |
|------|---------------------|------------|--------------------------|
| 0    |                     | \$760      |                          |
| 1    | \$110               | 650        | \$110                    |
| 2    | 110                 | 540        | 220                      |
| 3    | 110                 | 430        | 330                      |
| 4    | 110                 | 320        | 440                      |
| 5    | 110                 | 210        | 550                      |
| 6    | 110                 | 100        | 660                      |

**REVIEW PROBLEM 6.2**

A three-year-old extruder used in making plastic yogurt cups has a current book value of \$168 750. The declining-balance method of depreciation with a rate  $d = 0.25$  is used to determine depreciation charges. What was its original price? What will its book value be two years from now?

**ANSWER**

Let the original price of the extruder be  $P$ . The book value three years after purchase is \$168 750. This means that the original price was

$$\begin{aligned} BV_{db}(3) &= P(1 - d)^3 \\ 168\,750 &= P(1 - 0.25)^3 \\ P &= 400\,000 \end{aligned}$$

The original price was \$400 000.

The book value two years from now can be determined either from the original purchase price (depreciated for five years) or the current value (depreciated for two years):

$$BV_{db}(5) = 400\,000(1 - 0.25)^5 = 94\,921.88$$

or

$$BV_{db}(2) = 168\,750(1 - 0.25)^2 = 94\,921.88$$

The book value two years from now will be \$94 921.88. ■

**REVIEW PROBLEM 6.3**

You have been given the following data from the Fine Fishing Factory for the year ending December 31, 2013. Construct an income statement and a balance sheet from the data.

|                                 |           |
|---------------------------------|-----------|
| Accounts payable                | \$ 27 500 |
| Accounts receivable             | 32 000    |
| Advertising expense             | 2500      |
| Bad debts expense               | 1100      |
| Buildings, net                  | 14 000    |
| Cash                            | 45 250    |
| Common stock                    | 125 000   |
| Cost of goods sold              | 311 250   |
| Depreciation expense, buildings | 900       |
| Government bonds                | 25 000    |
| Income taxes                    | 9350      |
| Insurance expense               | 600       |
| Interest expense                | 500       |
| Inventory, December 31, 2013    | 42 000    |
| Land                            | 25 000    |
| Machinery, net                  | 3400      |
| Mortgage due May 30, 2015       | 5000      |
| Office equipment, net           | 5250      |
| Office supplies expense         | 2025      |
| Other expenses                  | 7000      |
| Total expenses                  | 84 500    |

|                   |         |
|-------------------|---------|
| Prepaid expenses  | 3000    |
| Retained earnings | ?       |
| Salaries expense  | 69 025  |
| Sales             | 421 400 |
| Taxes payable     | 2500    |
| Wages payable     | 600     |

**ANSWER**

Solving this problem consists of sorting through the listed items and identifying which are balance sheet entries and which are income statement entries. Then, assets can be separated from liabilities and owners' equity, and revenues from expenses.

| <b>Fine Fishing Factory<br/>Balance Sheet<br/>as of December 31, 2013</b> |                   |
|---|-------------------|
| <b>Assets</b>   |                   |
| <b>Current assets</b>   |                   |
| Cash  | \$ 45 250         |
| Accounts receivable   | 32 000            |
| Inventory, December 31, 2013  | 42 000            |
| Prepaid expenses  | 3000              |
| Total current assets  | \$ 122 250        |
| <b>Long-term assets</b>   |                   |
| Land  | 25 000            |
| Government bonds  | 25 000            |
| Machinery, net  | 3400              |
| Office equipment, net   | 5250              |
| Buildings, net  | 14 000            |
| Total long-term assets  | \$ 72 650         |
| <b>Total assets</b>   | <b>\$194 900</b>  |
| <b>Liabilities</b>  |                   |
| <b>Current liabilities</b>  |                   |
| Accounts payable  | \$ 27 500         |
| Taxes payable   | 2500              |
| Wages payable   | 600               |
| Total current liabilities   | \$ 30 600         |
| <b>Long-term liabilities</b>  |                   |
| Mortgage due May 30, 2015   | 5000              |
| Total long-term liabilities   | \$ 5000           |
| <b>Total liabilities</b>  | <b>\$ 35 600</b>  |
| <b>Owners' Equity</b>   |                   |
| Common stock  | \$ 125 000        |
| Retained earnings   | 34 300            |
| <b>Total owners' equity</b>   | <b>\$ 159 300</b> |
| <b>Total liabilities and owners' equity</b>                               | <b>\$194 900</b>  |

| <b>Fine Fishing Factory<br/>Income Statement<br/>for the Year Ending December 31, 2013</b> |                  |
|--|------------------|
| <b>Revenues</b>  |                  |
| Sales  | \$421 400        |
| Cost of goods sold   | 311 250          |
| Net revenue from sales   | <u>\$110 150</u> |
| <b>Expenses</b>  |                  |
| Salaries expense   | 69 025           |
| Bad debts expense  | 1100             |
| Advertising expense  | 2500             |
| Interest expense   | 500              |
| Insurance expense  | 600              |
| Office supplies expense  | 2025             |
| Other expenses   | 7000             |
| Depreciation expense, buildings  | 900              |
| Depreciation expense, office equipment   | 850              |
| Total Expenses   | <u>\$ 84 500</u> |
| <b>Income before taxes</b>   |                  |
| Income taxes   | <u>\$ 25 650</u> |
| <b>Income after taxes</b>  |                  |
|  | <b>\$ 16 300</b> |

#### REVIEW PROBLEM 6.4

Perform a financial ratio analysis for the Major Electric Company using the balance sheet and income statement from Sections 6.3.2 (p. 173) and 6.3.3 (p. 178). Industry standards for the ratios are as follows:

| Ratio                    | Industry Standard |
|--------------------------|-------------------|
| Current ratio            | 1.80              |
| Acid-test ratio          | 0.92              |
| Equity ratio             | 0.71              |
| Inventory-turnover ratio | 14.21             |
| Return-on-assets ratio   | 7.91%             |
| Return-on-equity ratio   | 11.14%            |

#### ANSWER

The ratio computations for Major Electric are:

$$\text{Current ratio} = \frac{\text{Current assets}}{\text{Current liabilities}} = \frac{801\,000}{90\,000} = 8.9$$

$$\text{Acid-test ratio} = \frac{\text{Quick assets}}{\text{Current liabilities}} = \frac{66\,000}{90\,000} = 0.73$$

$$\text{Equity ratio} = \frac{\text{Total equity}}{\text{Total assets}} = \frac{4\,311\,000}{5\,401\,000} = 0.7982 \cong 0.80$$

$$\begin{aligned}\text{Inventory-turnover ratio} &= \frac{\text{Sales}}{\text{Inventories}} = \frac{7\,536\,000}{683\,000} \\ &= 11.03 \text{ turns per year}\end{aligned}$$

$$\begin{aligned}\text{Return-on-total-assets ratio} &= \frac{\text{Profits after taxes}}{\text{Total assets}} = \frac{304\,200}{5\,401\,000} \\ &= 0.0563 \text{ or } 5.63\% \text{ per year}\end{aligned}$$

$$\begin{aligned}\text{Return-on-equity ratio} &= \frac{\text{Net income}}{\text{Total equity}} = \frac{304\,200}{4\,311\,000} \\ &= 0.0706 \text{ or } 7.06\% \text{ per year}\end{aligned}$$

A summary of the ratio analysis results follows:

| Ratio                    | Industry Standard | Major Electric |
|--------------------------|-------------------|----------------|
| Current ratio            | 1.80              | 8.90           |
| Acid-test ratio          | 0.92              | 0.73           |
| Equity ratio             | 0.71              | 0.80           |
| Inventory-turnover ratio | 14.21             | 11.03          |
| Return-on-assets ratio   | 7.91%             | 5.63%          |
| Return-on-equity ratio   | 11.14%            | 7.06%          |

Major Electric's current ratio is well above the industry standard and well above the general guideline of 2. The firm appears to be quite liquid. However, the acid-test ratio, with a value of 0.73, gives a slightly different view of Major Electric's liquidity. Most of Major Electric's current assets are inventory; thus, the current ratio is somewhat misleading. If we look at the acid-test ratio, Major Electric's quick assets are only 73 percent of its current liabilities. The firm may have a difficult time meeting its current debt obligations if it has unforeseen difficulties with its cash flow.

Major Electric's equity ratio of 0.80 indicates that it is not heavily reliant on debt and therefore is not at high risk of going bankrupt. Major Electric's inventory turns are lower than the industry norm, as are its ROA and ROE.

Taken together, Major Electric appears to be in reasonable financial shape. One matter the company should probably investigate is why its inventories are so high. With lower inventories, it could improve its inventory turns and liquidity, as well as its return on assets. ■

## S U M M A R Y

This chapter opened with a discussion of the concept of depreciation and various reasons why assets lose value. Two popular depreciation models, straight-line and declining-balance, were then presented as methods commonly used to determine book value of capital assets and depreciation charges.

The second part of the chapter dealt with the elements of financial accounting. We first presented the two main financial statements: the balance sheet and the income statement.

Next, we showed how these statements can be used to assess the financial health of a firm through the use of ratios. Comparisons with industry norms and trend analysis are normally part of financial analysis. We closed with cautionary notes on the interpretation of the ratios.

The significance of the material in this chapter is twofold. First, it sets the groundwork for material in Chapters 7 and 8, replacement analysis and taxation. Second, and perhaps more importantly, it is increasingly necessary for all engineers to have an understanding of depreciation and financial accounting as they become more and more involved in decisions that affect the financial positions of the organizations in which they work.

## ENGINEERING ECONOMICS IN ACTION, PART 6B

### Usually the Truth

Terry showed Naomi what he thought was evidence of wrongdoing by a fellow employee. Naomi said, "Interesting observation, Terry. But you know, I think it's probably okay. Let me explain. The main problem is that you are looking at two kinds of evaluation here, book value and market value. The book value is what an asset is worth from an accounting point of view, while the market value is what it sells for."

Terry nodded. "Yes, I know that. That's true about anything you sell. But this is different. We've got a \$5000 estimate against a \$300 sale. You can't tell me that our guess about the sales price can be that far out!"

"Yes, it can, and I'll tell you why. That book value is an estimate of the market value that has been calculated according to very particular rules. Corporate tax rules require us to use a particular depreciation scheme for all of our assets. Now, the reality is that things decline in value at different rates, and computers lose value really quickly. We could, for our own purposes, determine a book value for any asset that is a better estimate of its market value, but sometimes it's too much trouble to keep one set of figures for tax reasons and another for other purposes. So often everything is given a book value according to the tax rules, and consequently sometimes the difference between the book value and the market value can be a lot."

"But surely the government can see that computers in particular don't match the depreciation scheme they mandate. Or are they just ripping us off?"

"Well, they can see that. They are always tweaking the rules, and sometimes they make changes to help make book values more realistic. But it's hard to keep up with technology when it changes so fast."

Terry smiled ruefully. "So our accounting statements don't really show the truth?"

Naomi smiled back. "I guess not, if by 'truth' you mean market value. But usually they're close. Usually."

## PROBLEMS

### A. Key Concepts

- 6.1** For each of the following, state whether the loss in value is due to use-related physical loss, time-related physical loss, or functional loss:
- Albert sold his two-year-old computer for \$500, but he paid \$4000 for it new. It wasn't fast enough for the new software he wanted.
  - Beatrice threw out her old tennis shoes because the soles had worn thin.
  - Claudia threw out her old tennis shoes because she is jogging now instead.
  - Day-old bread is sold at half-price at the neighbourhood bakery.
  - Egbert sold his old lawnmower to his neighbour for \$20.

- (f) Fred picked up a used overcoat at the thrift store for less than 10 percent of the new price.
  - (g) Gunther notices that newspapers cost \$0.50 on the day of purchase, but are worth less than \$0.01 each as recyclable newsprint.
  - (h) Harold couldn't get the price he wanted for his house because the exterior paint was faded and flaking.
- 6.2 For each of the following, state whether the value is a market value, book value, scrap value, or salvage value:
- (a) Inta can buy a new stove for \$800 at Joe's Appliances.
  - (b) Jacques can sell his used stove to Inta for \$200.
  - (c) Kitty can sell her used stove to the recyclers for \$20.
  - (d) Liam can buy Jacques's used stove for \$200.
  - (e) Noriko is adding up the value of the things she owns. She figures her stove is worth at least \$200.
- 6.3 A company that sells a particular type of web-indexing software has had two larger firms approach it for a possible buyout. The current value of the company, based on recent financial statements, is \$4.5 million. The two bids were for \$4 million and \$7 million, respectively. Both bids were bona fide, meaning they were real offers. What is the market value of the company? What is its book value?
- 6.4 A new industrial sewing machine costs in the range of \$5000 to \$10 000. Technological change in sewing machines does not occur very quickly, nor is there much change in the functional requirements of a sewing machine. A machine can operate well for many years with proper care and maintenance. Discuss the different reasons for depreciation and which you think would be most appropriate for a sewing machine.
- 6.5 Communications network switches are changing dramatically in price and functionality as changes in technology occur in the communications industry. Prices drop frequently as more functionality and capacity are achieved. A switch only six months old will have depreciated since it was installed. Discuss the different reasons for depreciation and which you think would be most appropriate for this switch.
- 6.6 Ryan owns a five-hectare plot of land in the countryside. He has been planning to build a cottage on the site for some time, but has not been able to afford it yet. However, five years ago, he dug a pond to collect rainwater as a water supply for the cottage. It has never been used and is beginning to fill in with plant life and garbage that has been dumped there. Ryan realizes that his investment in the pond has depreciated in value since he dug it. Discuss the different reasons for depreciation and which you think would be most appropriate for the pond.
- 6.7 An asset costs \$14 000 and has a scrap value of \$3000 after seven years. Calculate its book value using straight-line depreciation
- (a) After one year
  - (b) After four years
  - (c) After seven years
- 6.8 An asset costs \$14 000. At a depreciation rate of 20 percent, calculate its book value using the declining-balance method

- (a) After one year  
 (b) After four years  
 (c) After seven years
- 6.9** (a) An asset costs \$14 000. What declining-balance depreciation rate would result in the scrap value of \$3000 after seven years?  
 (b) Using the depreciation rate from part (a), what is the book value of the asset after four years?
- 6.10** Using a spreadsheet program, chart the book value of a \$14 000 asset over a seven-year life using declining-balance depreciation ( $d = 0.2$ ). On the same chart, show the book value of the \$14 000 asset using straight-line depreciation with a scrap value of \$3000 after seven years.
- 6.11** Using a spreadsheet program, chart the book value of a \$150 000 asset for the first 10 years of its life at declining-balance depreciation rates of 5 percent, 20 percent, and 30 percent.
- 6.12** Construct an income statement and a balance sheet from the scrambled entries for Paradise Pond Company from the years 2012 and 2013 shown in the table below.

| Paradise Pond Company                | 2012   | 2013   |
|--------------------------------------|--------|--------|
| Accounts receivable                  | \$ 675 | \$ 638 |
| Less: Accumulated depreciation       | 1500   | 1800   |
| Accounts payable                     | 300    | 225    |
| Bonds                                | 900    | 900    |
| Cash                                 | 300    | 225    |
| Common shares                        | 450    | 450    |
| Contributed capital                  | 900    | 900    |
| Cost of goods sold                   | 1750   | 2125   |
| Depreciation expense                 | 550    | 500    |
| Income taxes                         | 200    | 300    |
| Interest expense                     | 125    | 150    |
| Inventories                          | 825    | 938    |
| Net plant and equipment              | 1800   | 2100   |
| Net revenue from sales               | 1250   | 1500   |
| Operating expenses                   | 075    | 100    |
| Plant and equipment                  | 3300   | 3900   |
| Profit after taxes                   | 300    | 450    |
| Profit before taxes                  | 500    | 750    |
| Retained earnings                    | 1050   | 1200   |
| Sales                                | 3000   | 3625   |
| Total assets                         | 3600   | 3900   |
| Total current assets                 | 1800   | 1800   |
| Total current liabilities            | 300    | 450    |
| Total expenses                       | 750    | 750    |
| Total liabilities and owners' equity | 3600   | 3900   |
| Total long-term assets               | 1800   | 2100   |
| Total long-term liabilities          | 900    | 900    |
| Total owners' equity                 | 2400   | 2550   |
| Working capital loan                 | 000    | 225    |

- 6.13** At the end of last month, Paarl Manufacturing had \$45 954 in the bank. It owed the bank \$224 000 because of the mortgage. It also had a working capital loan of \$30 000. Its customers owed \$22 943 and it owed its suppliers \$12 992. The company owned property worth \$250 000. It had \$123 000 in finished goods, \$102 000 in raw materials, and \$40 000 in work in progress. Its production equipment was worth \$450 000 when new (partially paid for by a large government loan due to be paid back in three years) but had accumulated a total of \$240 000 in depreciation—\$34 000 worth last month.

The company has investors who put up \$100 000 for their ownership. It has been reasonably profitable; this month the gross income from sales was \$220 000, and the cost of the sales was only \$40 000. Expenses were also relatively low; salaries were \$45 000 last month, while the other expenses were depreciation, maintenance at \$1500, advertising at \$3400, and insurance at \$300. In spite of \$32 909 in accrued taxes (Paarl pays taxes at 55 percent), the company had retained earnings of \$135 000.

Construct a balance sheet (as of the end of this month) and income statement (for this month) for Paarl Manufacturing. Should the company release some of its retained earnings through dividends at this time?

- 6.14** Salvador Industries bought land and built its plant 20 years ago. The depreciation on the building is calculated using the straight-line method, with a life of 30 years and a salvage value of \$50 000. Land is not depreciated. The depreciation for the equipment, all of which was purchased at the same time the plant was constructed, is calculated using declining-balance at 20 percent. Salvador currently has two outstanding loans: one for \$50 000 due December 31, 2013, and another one for which the next payment is due in four years.

| <b>Salvador Industries</b>               |           |            |
|--|-----------|------------|
| <b>Balance Sheet as of June 30, 2013</b> |           |            |
| <b>Assets</b>                            |           |            |
| Cash                                     |           | \$ 350 000 |
| Accounts receivable                      |           | 2 820 000  |
| Inventories                              |           | 2 003 000  |
| Prepaid services                         |           | 160 000    |
| Total Current Assets                     |           |            |
| Long-Term Assets                         |           |            |
| Building                                 | \$200 000 |            |
| Less accumulated depreciation            |           |            |
| Equipment                                | \$480 000 |            |
| Less accumulated depreciation            |           |            |
| Land                                     |           | 540 000    |
| Total Long-Term Assets                   |           |            |
| <b>Total Assets</b>                      |           |            |
| <b>Liabilities and Owners' Equity</b>    |           |            |
| Current Liabilities                      |           |            |
| Accounts payable                         |           | \$ 921 534 |
| Accrued taxes                            |           | 29 000     |
| Total                                    |           |            |

|   |             |  |
|---|-------------|--|
| Long-Term Liabilities                       |             |  |
| Mortgage                                    | \$1 200 000 |  |
|   | 318 000     |  |
| Total Long-Term Liabilities                 |             |  |
| <b>Total</b>                                |             |  |
| <b>Owners' Equity</b>                       |             |  |
| Capital stock                               | \$1 920 000 |  |
|   |             |  |
| <b>Total Owners' Equity</b>                 |             |  |
| <b>Total Liabilities and Owners' Equity</b> |             |  |

During April 2013, there was a flood in the building because a nearby river overflowed its banks after unusually heavy rain. Pumping out the water and cleaning up the basement and the first floor of the building took a week. Manufacturing was suspended during this period and some inventory was damaged. Because of lack of adequate insurance, this unusual and unexpected event cost the company \$100 000 net.

- (a) Fill in the blanks and complete a copy of the balance sheet and income statement here, using any of the above information you feel is necessary.

| Salvador Industries<br>Income Statement for the Year Ended June 30, 2013 |             |  |
|--|-------------|--|
| <b>Income</b>  |             |  |
| Gross income from sales  | \$8 635 000 |  |
| Less   | 7 490 000   |  |
| Total income   |             |  |
|  |             |  |
| Depreciation   | 70 000      |  |
| Interest paid  | 240 000     |  |
| Other expenses   | 100 000     |  |
| Total expenses   |             |  |
| Income before taxes  |             |  |
| Taxes at 40%   |             |  |
|  |             |  |
| Net income   |             |  |

- (b) Show how information from financial ratios can indicate whether Salvador Industries can manage an unusual and unexpected event such as the flood without threatening its existence as a viable business.

- 6.15** Fraser Phraser operates a small publishing company. He is interested in getting a loan for expanding his computer systems. The bank has asked Phraser to supply it with his financial statements from the past two years. His statements appear below. Comment on Phraser's financial position with regard to the loan based on the results of financial ratio analysis.

| <b>Fraser Phraser Company<br/>Comparative Balance Sheets<br/>for Years Ending 2012 and 2013<br/>(in thousands of dollars)</b> |                |                |
|---|----------------|----------------|
|   | <b>2012</b>    | <b>2013</b>    |
| <b>Assets</b>   |                |                |
| <i>Current Assets</i>   |                |                |
| Cash  | 22 500         | 1250           |
| Accounts receivable   | 31 250         | 40 000         |
| Inventories   | 72 500         | <u>113 750</u> |
| Total Current Assets  | <u>126 250</u> | <u>155 000</u> |
| <i>Long-Term Assets</i>   |                |                |
| Land  | 50 000         | 65 000         |
| Plant and equipment   | 175 000        | 250 000        |
| Less: Accumulated depreciation  | 70 000         | 95 000         |
| Net plant and equipment   | 105 000        | <u>155 000</u> |
| Total Long-Term Assets  | <u>155 000</u> | <u>220 000</u> |
| <b>Total Assets</b>   | <u>281 250</u> | <u>375 000</u> |
| <b>Liabilities and Owners' Equity</b>   |                |                |
| <i>Current Liabilities</i>  |                |                |
| Accounts payable  | 26 250         | 55 000         |
| Working capital loan  | 42 500         | <u>117 500</u> |
| Total Current Liabilities   | <u>68 750</u>  | <u>172 500</u> |
| <i>Long-Term Liabilities</i>  |                |                |
| Mortgage  | 71 875         | <u>57 375</u>  |
| Total Long-Term Liabilities   | <u>71 875</u>  | <u>57 375</u>  |
| <i>Owners' Equity</i>   |                |                |
| Common shares   | 78 750         | 78 750         |
| Retained earnings   | 61 875         | 66 375         |
| Total Owners' Equity  | <u>140 625</u> | <u>145 125</u> |
| <b>Total Liabilities and Owners' Equity</b>   | <u>281 250</u> | <u>375 000</u> |

| <b>Fraser Phraser Company<br/>Income Statements<br/>for Years Ending 2012 and 2013<br/>(in thousands of dollars)</b> |               |                |
|--|---------------|----------------|
|  | <b>2012</b>   | <b>2013</b>    |
| <b>Revenues</b>  |               |                |
| Sales  | 156 250       | 200 000        |
| Cost of goods sold   | <u>93 750</u> | <u>120 000</u> |
| Net revenue from sales   | <u>62 500</u> | <u>80 000</u>  |

| <b>Expenses</b>            |               |               |
|----------------------------|---------------|---------------|
| Operating expenses         | 41 875        | 46 250        |
| Depreciation expense       | 5625          | 12 500        |
| Interest expense           | 3750          | 7625          |
| Total expenses             | <u>51 250</u> | <u>66 375</u> |
| <b>Income before taxes</b> | 11 250        | 13 625        |
| Income taxes               | <u>5625</u>   | <u>6813</u>   |
| <b>Net income</b>          | 5625          | 6813          |

## B. Applications

- 6.16** A machine has a life of 30 years, costs \$245 000, and has a salvage value of \$10 000 using straight-line depreciation. What depreciation rate will result in the same book value for both the declining-balance and straight-line methods at the end of year 20?
- 6.17** A new press brake costs York Steel \$780 000. It is expected to last 20 years, with a \$60 000 salvage value. What rate of depreciation for the declining-balance method will produce a book value after 20 years that equals the salvage value of the press?
- 6.18** (a) Using straight-line depreciation, what is the book value after four years for an asset costing \$150 000 that has a salvage value of \$25 000 after 10 years? What is the depreciation charge in the fifth year?  
 (b) Using declining-balance depreciation with  $d = 20$  percent, what is the book value after four years for an asset costing \$150 000? What is the depreciation charge in the fifth year?  
 (c) What is the depreciation rate using declining-balance for an asset costing \$150 000 that has a salvage value of \$25 000 after 10 years?
- 6.19** Julia must choose between two different designs for a safety enclosure, which will be in use indefinitely. Model A has a life of three years, a first cost of \$8000, and maintenance of \$1000 per year. Model B will last four years, has a first cost of \$10 000, and has maintenance of \$800 per year. A salvage value can be estimated for model A using a depreciation rate of 40 percent and declining-balance depreciation, while a salvage value for model B can be estimated using straight-line depreciation and the knowledge that after one year its salvage value will be \$7500. Interest is at 14 percent. Using a present worth analysis, which design is better?
- 6.20** Adventure Airline's new baggage handling conveyor costs \$250 000 and has a service life of 10 years. For the first six years, depreciation of the conveyor is calculated using the declining-balance method at the rate of 30 percent. During the last four years, the straight-line method is used for accounting purposes in order to have a book value of zero at the end of the service life. What is the book value of the conveyor after seven, eight, nine, and ten years?
- 6.21** Molly inherited \$5000 and decided to start a lawn-mowing service. With her inheritance and a bank loan of \$5000, she bought a used ride-on lawnmower and a used truck. For five years, Molly had a gross income of \$30 000, which covered the annual operating costs and the loan payment, both of which totalled \$4500. She spent the rest of her income personally. At the end of five years, Molly found that her loan was paid off but the equipment was wearing out.  
 (a) If the equipment (lawnmower and truck) was depreciating at the rate of 50 percent according to the declining-balance method, what is its book value after five years?

- (b) If Molly wanted to avoid being left with a worthless lawnmower and truck and with no money for renewing them at the end of five years, how much should she have saved annually toward the second set of used lawnmower and used truck of the same initial value as the first set? Assume that Molly's first lawnmower and truck could be sold at their book value. Use an interest rate of 7 percent.
- 6.22** Enrique is planning a trip around the world in three years. He will sell all of his possessions at that time to fund the trip. Two years ago, he bought a used car for \$12 500. He observes that the market value for the car now is about \$8300. He needs to know how much money his car will add to his stash for his trip when he sells it three years from now. Use the declining-balance depreciation method to tell him.
- 6.23** Ben is choosing between two different industrial fryers using an annual worth calculation. Fryer 1 has a five-year service life and a first cost of \$400 000. It will generate net year-end savings of \$128 000 per year. Fryer 2 has an eight-year service life and a first cost of \$600 000. It will generate net year-end savings of \$135 000 per year. If the salvage value is estimated using declining-balance depreciation with a 20 percent depreciation rate and the MARR is 12 percent, which fryer should Ben buy?
- 6.24** Movit Manufacturing has the following alphabetized income statement and balance sheet entries from the year 2013. Construct an income statement and a balance sheet from the information given.

|                                |         |
|--------------------------------|---------|
| Accounts payable               | \$ 7500 |
| Accounts receivable            | 15 000  |
| Accrued wages                  | 2850    |
| Cash                           | 2100    |
| Common shares                  | 150     |
| Contributed capital            | 3000    |
| Cost of goods sold             | 57 000  |
| Current assets                 |         |
| Current liabilities            |         |
| Deferred income taxes          | 2250    |
| Depreciation expense           | 750     |
| General expense                | 8100    |
| GICs                           | 450     |
| Income taxes                   | 1800    |
| Interest expense               | 1500    |
| Inventories                    | 18 000  |
| Land                           | 3000    |
| Less: Accumulated depreciation | 10 950  |
| Long-term assets               |         |
| Long-term bonds                | 4350    |
| Long-term liabilities          |         |
| Mortgage                       | 9450    |
| Net income after taxes         | 2700    |
| Net income before taxes        | 4500    |
| Net plant and equipment        | 7500    |

|                                      |        |
|--------------------------------------|--------|
| Net sales                            | 76 500 |
| Operating expenses                   |        |
| Owners' equity                       |        |
| Prepaid expenses                     | 450    |
| Selling expenses                     | 4650   |
| Total assets                         | 46 500 |
| Total current assets                 | 36 000 |
| Total current liabilities            | 15 000 |
| Total expenses                       | 15 000 |
| Total liabilities and owners' equity | 46 500 |
| Total long-term assets               | 10 500 |
| Total long-term liabilities          | 16 050 |
| Total owners' equity                 | 15 450 |
| Working capital loan                 | 4650   |

- 6.25** Calculate for Movit Manufacturing in Problem 6.24 the financial ratios listed in the table below. Using these ratios and those provided for 2011 and 2012, conduct a short analysis of Movit's financial health.

| Movit Manufacturing<br>Financial Ratios |      |      |       |
|---|------|------|-------|
| Ratio                                   | 2013 | 2012 | 2011  |
| Current ratio                           |      | 1.90 | 1.60  |
| Acid-test ratio                         |      | 0.90 | 0.75  |
| Equity ratio                            |      | 0.40 | 0.55  |
| Inventory-turnover ratio                |      | 7.00 | 12.00 |
| Return-on-assets ratio                  |      | 0.08 | 0.10  |
| Return-on-equity ratio                  |      | 0.20 | 0.18  |

### C. More Challenging Problems

- 6.26** Dick noticed that the book value of an asset he owned was exactly \$500 higher if the value was calculated by straight-line depreciation over declining-balance depreciation exactly halfway through the asset's service life, and the scrap value at the end of the service life was the same by either method. If the scrap value was \$100, what was the purchase price of the asset?
- 6.27** A company had net sales of \$20 000 last month. From the balance sheet at the end of last month and an income statement for the month, you have determined that the current ratio was 2.0, the acid-test ratio was 1.2, and the inventory turnover was two per month. What was the value of the company's current assets?
- 6.28** In the last quarter, the financial-analysis report for XYZ Company revealed that the current, quick, and equity ratios were 1.9, 0.8, and 0.37, respectively. In order to

improve the firm's financial health based on these financial ratios, the following strategies are considered by XYZ for the current quarter:

- (i) Reduce inventory
  - (ii) Pay back short-term loans
  - (iii) Increase retained earnings
- (a) Which strategy (or strategies) is effective for improving each of the three financial ratios?
- (b) If only one strategy is considered by XYZ, which one seems to be most effective? Assume no other information is available for analysis.

**6.29** The end-of-quarter balance sheet for XYZ Company indicated that the current ratio was 1.8 and the equity ratio was 0.45. It also indicated that the long-term assets were twice as much as the current assets, and half of the current assets were highly liquid. The total equity was \$68 000. Since the current ratio was close to 2, XYZ feels that the company had a reasonable amount of liquidity in the last quarter. However, if XYZ wants more assurance, which financial ratio would provide further information? Using the information provided, compute the appropriate ratio and comment on XYZ's concern.

**6.30** A potentially very large customer for Milano Metals wants to fully assess the financial health of the company in order to decide whether to commit to a long-term, high-volume relationship. You have been asked by the company president, Roch, to review the company's financial performance over the past three years and make a complete report to him. He will then select from your report information to present to the customer. Consequently, your report should be as thorough and honest as possible.

Research has revealed that in your industry (sheet metal products), the average value of the current ratio is 2.79, the equity ratio is 0.54, the inventory turnover is 4.9, and the net-profit ratio is 3.87. Beyond that information, you have access to only the balance sheet (on the next page) and the income statement shown here, and should make no further assumptions. Your report should be limited to the equivalent of about 300 words.

| <b>Milano Metals<br/>Income Statement<br/>for the Years Ending December 31, 2011, 2012, and 2013<br/>(in thousands of dollars)</b> |             |             |             |
|--|-------------|-------------|-------------|
|  | <b>2011</b> | <b>2012</b> | <b>2013</b> |
| Total revenue  | 9355        | 9961        | 8470        |
| Less: Costs  | 8281        | 9632        | 7654        |
| Net revenue  | 1074        | 329         | 816         |
| Less:  |             |             |             |
| Depreciation   | 447         | 431         | 398         |
| Interest   | 412         | 334         | 426         |
| Income taxes   | 117         | 21          | 156         |
| Net income from operations   | 98          | (457)       | (164)       |
| Add: Extraordinary item  | 770         | —           | (1832)      |
| Net income   | 868         | (457)       | (1996)      |

| <b>Milano Metals<br/>Consolidated Balance Sheets<br/>December 31, 2011, 2012, and 2013<br/>(in thousands of dollars)</b> |               |             |             |
|--|---------------|-------------|-------------|
|  | <b>Assets</b> |             |             |
|  | <b>2011</b>   | <b>2012</b> | <b>2013</b> |
| <i>Current Assets</i>  |               |             |             |
| Cash   | 19            | 19          | 24          |
| Accounts receivable  | 779           | 884         | 1176        |
| Inventories  | 3563          | 3155        | 2722        |
|  | <u>4361</u>   | <u>4058</u> | <u>3922</u> |
| <i>Fixed Assets</i>  |               |             |             |
| Land   | 1136          | 1064        | 243         |
| Buildings and equipment  | 2386          | 4682        | 2801        |
|  | <u>3552</u>   | <u>5746</u> | <u>3044</u> |
| <i>Other Assets</i>  | <u>3413</u>   | <u>....</u> | <u>3</u>    |
| <b>Total Assets</b>  | <b>8296</b>   | <b>9804</b> | <b>6969</b> |
| <b>Liabilities and Owners' Equity</b>  |               |             |             |
|  | <b>2011</b>   | <b>2012</b> | <b>2013</b> |
| <i>Current Liabilities</i>   |               |             |             |
| Due to bank  | 1431          | 1 929       | 2040        |
| Accounts payable   | 1644          | 1 349       | 455         |
| Wages payable  | 341           | 312         | 333         |
| Income tax payable   | 562           | 362         | 147         |
| <i>Long-Term Debt</i>  | <u>2338</u>   | <u>4743</u> | <u>2528</u> |
| <b>Total Liabilities</b>   | <b>6316</b>   | <b>8695</b> | <b>5503</b> |
| <i>Owners' Equity</i>  |               |             |             |
| Capital stock  | 1194          | 1191        | 1091        |
| Retained earnings  | 786           | (82)        | 375         |
| <b>Total Owners' Equity</b>  | <b>1980</b>   | <b>1109</b> | <b>1466</b> |
| <b>Total Liability and Owners' Equity</b>  | <b>8296</b>   | <b>9804</b> | <b>6969</b> |

- 6.31** The Milano Metals income statement and balance sheets shown for Problem 6.30 were in error. A piece of production equipment was sold for \$100 000 cash in 2013 and was not accounted for. Which items on these statements must be changed, and (if known) by how much?
- 6.32** The Milano Metals income statement and balance sheets shown for Problem 6.30 were in error. An extra \$100 000 in sales was made in 2013 and not accounted for. Only half of the payments for these sales have been received. Which items on these statements must be changed, and (if known) by how much?
- 6.33** A friend of yours is thinking of purchasing shares in Petit Ourson SA in the near future and decided that it would be prudent to examine its financial statements for the past two years before making a phone call to his stockbroker. The statements are shown below.

Your friend has asked you to help him conduct a financial ratio analysis. Fill out the financial ratio information on a copy of the third table on the next page. After comparison with industry standards, what advice would you give your friend?

| <b>Petit Ourson SA</b><br><b>Comparative Balance Sheets</b><br><b>for the Years Ending 2012 and 2013</b><br><i>(in thousands of dollars)</i> |                    |                    |
|--|--------------------|--------------------|
|  | <b>2012</b>        | <b>2013</b>        |
| <b>Assets</b>  |                    |                    |
| <i>Current Assets</i>  |                    |                    |
| Cash   | 500                | 375                |
| Accounts receivable  | 1125               | 1063               |
| Inventories  | <u>1375</u>        | <u>1563</u>        |
| Total Current Assets   | <u>3000</u>        | <u>3000</u>        |
| <i>Long-Term Assets</i>  |                    |                    |
| Plant and equipment  | 5500               | 6500               |
| Less: Accumulated depreciation   | 2500               | 3000               |
| Net plant and equipment  | <u>3000</u>        | <u>3500</u>        |
| Total Long-Term Assets   | <u>3000</u>        | <u>3500</u>        |
| <b>Total Assets</b>  | <u><u>6000</u></u> | <u><u>6500</u></u> |
| <b>Liabilities and Owners' Equity</b>  |                    |                    |
| <i>Current Liabilities</i>   |                    |                    |
| Accounts payable   | 500                | 375                |
| Working capital loan   | <u>000</u>         | <u>375</u>         |
| Total Current Liabilities  | <u>500</u>         | <u>750</u>         |
| <i>Long-Term Liabilities</i>   |                    |                    |
| Bonds  | <u>1500</u>        | <u>1500</u>        |
| Total Long-Term Liabilities  | <u>1500</u>        | <u>1500</u>        |
| <i>Owners' Equity</i>  |                    |                    |
| Common shares  | 750                | 750                |
| Contributed capital  | 1500               | 1500               |
| Retained earnings  | 1750               | 2000               |
| Total Owners' Equity   | <u>4000</u>        | <u>4250</u>        |
| <b>Total Liabilities and Owners' Equity</b>  | <u><u>6000</u></u> | <u><u>6500</u></u> |

| <b>Petit Ourson SA</b><br><b>Income Statements</b><br><b>for the Years Ending 2012 and 2013</b><br><i>(in thousands of dollars)</i> |             |             |
|---|-------------|-------------|
|   | <b>2012</b> | <b>2013</b> |
| <b>Revenues</b>   |             |             |
| Sales   | 3000        | 3625        |
| Cost of goods sold  | <u>1750</u> | <u>2125</u> |
| Net revenue from sales  | <u>1250</u> | <u>1500</u> |

| <b>Expenses</b>            |            |            |
|----------------------------|------------|------------|
| Operating expenses         | 75         | 100        |
| Depreciation expense       | 550        | 500        |
| Interest expense           | 125        | 150        |
| Total expenses             | <u>750</u> | <u>750</u> |
| <b>Income Before Taxes</b> | <u>500</u> | <u>750</u> |
| Income taxes               | 200        | 300        |
| <b>Net Income</b>          | 300        | 450        |

| <b>Petit Ourson SA</b>   |                      |             |             |
|--------------------------|----------------------|-------------|-------------|
| <b>Financial Ratios</b>  |                      |             |             |
|                          | <b>Industry Norm</b> | <b>2012</b> | <b>2013</b> |
| Current ratio            | 4.50                 |             |             |
| Acid-test ratio          | 2.75                 |             |             |
| Equity ratio             | 0.60                 |             |             |
| Inventory-turnover ratio | 2.20                 |             |             |
| Return-on-assets ratio   | 0.09                 |             |             |
| Return-on-equity ratio   | 0.15                 |             |             |

**6.34** Like all companies, Exco has to pay taxes on its profits. However, for each additional dollar in depreciation Exco claims, Exco's taxes are reduced by  $t$ , since the depreciation expense offsets income, thus reducing profits subject to tax. Exco is required by tax law to use declining-balance depreciation to calculate the depreciation of assets on a year-by-year basis, with a depreciation rate of  $d$ . This depreciation calculation continues forever, even if the asset is sold or scrapped.

The actual depreciation for a particular asset is best represented as straight-line over  $y$  years, with zero salvage value. If the asset costs  $P$  dollars and interest is  $i$ , what is the present worth of all tax savings Exco will incur by purchasing the asset? (*Hint:* The depreciation savings in the first year is  $Ptd$ .)

## MINI-CASE 6.1

### Research In Motion

The BlackBerry was a device far ahead of its time when it was first introduced in 1999 by Waterloo, Ontario-based Research In Motion (RIM) as a pager with email capability. The device was immediately successful and the consumer response, particularly from business professionals, was so great that RIM was designated the world's fastest growing company by *Fortune* magazine in 2009. RIM now controls about 10 percent of the smartphone market and its products are favourites of businesspeople around the world.

As a publicly traded company, RIM is required to make its financial statements public to provide information to its shareholders and potential investors. Table 6MC.1 shows the balance sheets as reported in RIM's annual reports for the years 2008 through 2011.

**Table 6MC.1**

| Research In Motion Annual Report Balance Sheets, 2008–2011<br>(in US\$ millions) |      |      |        |        |
|--|------|------|--------|--------|
|  | 2008 | 2009 | 2010   | 2011   |
| <b>Assets</b>  |      |      |        |        |
| Cash and equivalents   | 1184 | 836  | 1551   | 1791   |
| Short-term investments   | 421  | 683  | 361    | 330    |
| Total cash and short-term investments  | 1605 | 1518 | 1912   | 2121   |
| Accounts receivable  | 1175 | 2112 | 2594   | 3955   |
| Other receivables  | 75   | 158  | 206    | 324    |
| Total receivables  | 1249 | 2270 | 2800   | 4279   |
| Inventory  | 396  | 682  | 660    | 618    |
| Deferred tax assets, current   | 91   | 184  | 194    | 229    |
| Other current assets   | 136  | 187  | 247    | 241    |
| Total current assets   | 3477 | 4842 | 5813   | 7488   |
| Gross property, plant, and equipment   | 1137 | 1923 | 2878   | 3899   |
| Accumulated depreciation   | −431 | −588 | −921   | −1395  |
| Net property, plant, and equipment   | 706  | 1335 | 1957   | 2504   |
| Goodwill   | 115  | 138  | 151    | 508    |
| Long-term investments  | 739  | 721  | 958    | 577    |
| Deferred tax assets, long term   | 5    | 0    | 0      | 0      |
| Other intangibles  | 470  | 1067 | 1326   | 1798   |
| Total assets   | 5511 | 8101 | 10 205 | 12 875 |
| <b>Liabilities &amp; Equity</b>  |      |      |        |        |
| Accounts payable   | 271  | 448  | 615    | 832    |
| Accrued expenses   | 690  | 1239 | 1260   | 1936   |
| Current portion of long-term debt/capital lease                                  | 0    | 0    | 0      | 0      |
| Current income taxes payable   | 475  | 362  | 96     | 179    |
| Other current liabilities, total   | 0    | —    | 378    | 575    |
| Unearned revenue, current  | 37   | 54   | 68     | 108    |
| Total current liabilities  | 1474 | 2115 | 2432   | 3630   |
| Long-term debt   | 7    | 0    | 0      | 0      |
| Deferred tax liability non-current   | 65   | 88   | 141    | 276    |
| Other non-current liabilities  | 31   | 24   | 29     | 31     |
| Total liabilities  | 1578 | 2227 | 2602   | 3937   |
| Common stock   | 2170 | 2208 | 2372   | 2359   |
| Additional paid-in capital   | 80   | 120  | 0      | 0      |
| Retained earnings  | 1653 | 3546 | 5274   | 6749   |
| Treasury stock   | 0    | 0    | −94    | −160   |

|                                |      |      |        |        |
|--------------------------------|------|------|--------|--------|
| Comprehensive income and other | 30   | 1    | 51     | -10    |
| Total equity                   | 3934 | 5874 | 7603   | 8938   |
| Total Liabilities and Equity   | 5511 | 8101 | 10 205 | 12 875 |

### Discussion

Companies that raise capital by selling their shares to the public gain the resources to make acquisitions, enter new markets, invest in technological developments, and defend themselves from legal challenges. However, this capability comes at a cost, since by reporting their financial statements and conforming to all of the other reporting requirements necessary by law, they also give useful information to their competitors. Since most large companies in Canada are public companies, extensive financial information is available for analysis by investors or any interested party.

### Questions

1. Use the financial ratios introduced in this chapter to examine the financial status and trends evident in the RIM balance sheets from Table 6MC.1. What do you conclude? In particular, are there any areas of concern?
2. Based on the trends you see in the balance sheets, calculate an estimate for the balance sheet entries that might appear in the annual report for 2012. Find the actual 2012 balance sheet for RIM through a web search and compare it to your estimate. Can you explain the differences?
3. Acquire the most recent equivalent balance sheet for RIM and compare it with the balance sheets in Table 6MC.1. How has RIM been performing since 2011? Have the trends evident in Table 6MC.1 continued?

## Appendix 6A Cost Estimation

### 6A.1 | Introduction

Throughout this text, the costs and benefits of projects are usually given without explanation of where the numbers came from. In real life, engineers are often involved in decisions early when there may be little data available. Consequently, in order to make or recommend a good economic decision, the various costs of a project have to be estimated. As a project progresses, cost estimates of increasing accuracy will be required. In general, the basic cost estimating methods used at each stage of a project will depend on the information available when the estimates are prepared, what the estimate is used for, and the accuracy needed. In this appendix, we explain various types of cost estimates and when and how they are used for engineering projects.

### 6A.2 | Cost Concepts

In order to develop cash flow estimates associated with an engineering project, a number of cost concepts should be defined. One set distinguishes costs based on how they relate to the level of project output or activity. **Fixed costs** are independent of the level of output or activity. In contrast, **variable costs** depend on the level of output. For example, a bleach plant will have fixed costs such as equipment depreciation, plant insurance, and

the salaries of security and administrative personnel. Bulldozers would be considered a fixed cost for a road construction project, as would development computers for software projects. Variable costs for a bleach plant would include raw materials, waste treatment, and the cost of utilities that relate to production volume.

**Total costs** are the sum of fixed and variable costs. **Average costs**, which are total costs divided by the output level, are different from **marginal costs**, which are the total variable costs per unit. Average costs are normally used to express the overall cost of producing an item, while marginal costs, the change in total variable costs divided by the change in units produced, are used in decisions relating to production volumes. For example, consider a product that sells for \$15 per unit, has marginal costs of \$10, and annual fixed costs of \$2 million. The break-even sales volume is thus  $2\,000\,000/(15 - 10) = 400\,000$  units. With sales less than the break-even volume, the product is running at a loss; with sales higher than the break-even volume, there are profits. If the production volume has been set at 750 000 units per year, the average cost per unit is \$12.67:

$$\text{Average cost per unit} = \frac{\text{Total Costs}}{\text{Total Volume}} = \frac{2\,000\,000 + 750\,000(10)}{750\,000} = 12.67$$

In estimating the cost of projects, we can also distinguish between **direct costs** and **indirect costs**. Direct costs can be directly traced back to a “cost object.” A **cost object** could be a project, department, activity, or product—anything for which costs data are tracked. Architectural fees would be considered a direct cost of a building project, as would the cost of subcontracting development tasks for a software project. In contrast, indirect costs cannot be easily traced back to a particular cost object. For example, the salary of a plant manager is considered an indirect cost for a specific product or product line made in the plant. The plant manager is responsible for all products made in the plant, not just one specific product or product line, so the manager’s salary is considered a common cost for all products. In estimating the cost of a project, some portion of indirect costs is allocated. We will return to the question of how this is done later in this appendix.

## 6A.3

### Cost Estimation Methods and Classification of Cost Estimates

Most organizations use some type of classification scheme to describe different types of cost estimates used during the lifecycle of a project. The method used for developing a cost estimate depends on how accurate it needs to be, which in turn depends on the purpose of the estimate. In the early stage of a project, management needs rough cost estimates to judge whether the project is viable. Estimates at this stage are typically formed by expert opinion, experience, or high-level models. They are based on a statement of requirements and a high-level description of possible alternatives. The time required to prepare such *order of magnitude* or *indicative* estimates may be several minutes to several hours, and the estimates could be as high as 20 to 30 percent in error of the actual costs.

If a project passes initial screening, more detailed cost estimates will be necessary to conduct an economic or more thorough financial analysis. At this point, the engineering plans will be partially designed and the process technology or functionality will be selected so that the necessary equipment, materials, and labour inputs can be more clearly defined. For cost estimates at this stage, a cost estimator uses data on actual costs of **analogous projects** and various **cost estimation relationships**. Estimating using analogous projects involves determining the historical costs of a similar project and then adjusting these costs for inflation and location-specific differences. Cost estimating relationships (CERs) are parametric models that relate various project design characteristics

to aggregate costs. Estimates using CERs involve selecting and applying the appropriate models and then adjusting (as necessary) for inflation or location-specific differences. Estimates at this stage are typically within 10 to 20 percent of actual costs.

At the most detailed level, when the estimates are to be used for cost control or negotiation with vendors, they will rely on completed drawings and specifications and quotations for actual equipment, materials, and labour. These estimates should be within 5 percent of the actual costs. The effort and cost of putting together such detailed or substantive estimates will be as much as 100 times the effort of early cost estimates.

More information about cost estimation classifications and the attributes of each successive level of detail can be found from the Treasury Board of Canada (<http://www.tbs-sct.gc.ca>) or organizations such as ASTM International (<http://www.astm.org>), a group that specializes in the development of standards. In what follows, we focus on only the most basic concepts related to cost estimation.

## 6A.4 Adjusting for Inflation, Location, and Other Factors

When an estimate is based on the costs of projects carried out in the past or under different circumstances, these costs need to be adjusted, or normalized, in order to more accurately estimate the costs of the current project. These adjustments are typically made through the use of indexes that scale the costs based on the comparison of the two projects. The general form for the adjustment is as follows:

$$\frac{\text{Estimated Cost for } P}{\text{Cost for } P_0} = \frac{\text{Index for } P}{\text{Index for } P_0}$$

where  $P_0$  refers to the analogous project and  $P$  is the current project. The most common adjustments are for changes in price over time (inflation) and differences in cost due to location.

Other adjustments may need to be made for an analogous project. In particular, it is important to understand what items are included in the historical costs. For example, if the historical costs of a plant included removal of existing infrastructure and the current plant is on a clear site, then these costs need to be taken out of the historical costs. If the historical costs did not include provision for certain government regulations, then these will need to be added to the current project costs after other adjustments have been made.

### Example 6A.1

Natasha is preparing an order of magnitude cost estimate for a new bleach plant. The bleach will be sold to neighbouring cities for use in their water treatment systems. It has been eight years since her company built a similar plant and she knows it cost \$100 million then. However, that was in a region with building costs that tend to run about 25 percent higher than where the new plant will be built. She feels that the consumer price index (CPI) is a reasonable estimate of how costs have increased in the past eight years. The CPI now is 127.7, and back then was 104.2. Also, new pollution abatement regulations will add about 3 percent to the overall cost of the plant as compared to eight years ago. What order of magnitude cost estimate should Natasha provide?

### Answer:

Natasha has an analogous project to base her cost estimates on. However, she needs to make three adjustments to the costs of the old project: one for inflation, one for the

difference in location, and one for the new pollution equipment. Because the adjustments are in the form of indexes, she can apply each one in succession:

$$\frac{\text{Estimated Cost for } P}{\text{Cost for } P_0} = \left( \frac{\text{CPI now}}{\text{CPI 8 years ago}} \right) \left( \frac{\text{Costs in new location}}{\text{Costs in old location}} \right) \left( \frac{\text{Costs with abatement}}{\text{Costs without abatement}} \right)$$

$$\frac{\text{Estimated Cost for } P}{100\,000\,000} = \left( \frac{127.7}{104.2} \right) \left( \frac{1.00}{1.25} \right) \left( \frac{1.03}{1.00} \right)$$

From these computations, the estimated cost for the new plant,  $P$ , is \$100 009 835. This is only slightly more than the old plant.

## 6A.5 Cost Estimating Relationships

A cost estimating relationship is an equation that relates cost to one or more independent variables, sometimes called **cost drivers**. A cost estimating relationship can be as simple as a linear function of one variable or it can be a more complex relationship involving multiple input variables that relate to costs in a nonlinear manner. As an example of a simple linear relationship, construction costs are often approximated by the cost per square foot multiplied by the number of square feet to be built.

Cost estimating relationships are developed using statistical analysis of data from many projects. Numerous commercial databases are available to assist with this process and can be an excellent source of cost estimating relationships, along with parameter values that are typical within an industry.

Two commonly used cost estimating relationships are the linear and power relationships. The linear cost estimating relationship, or *unit model*, estimates cost as a linear function of an input factor or factors. Examples are the cost per kilometre to build a two-lane highway, network cable costs per drop, and construction cost per square foot. This is a very simple model of costs, but it can be useful for the purposes of constructing rough estimates. Cost estimates can be built up by summing over a set of component unit cost estimates.

It is important to note that unit cost estimating relationships assume that no economies of scale exist—that is, the cost per unit is the same regardless of the scale or size of the project. This may be a reasonable assumption within a certain range of output, but if there are economies (or diseconomies) of scale, then a more appropriate model may be the *power-sizing*, or *capacity-factored*, model. This model estimates the cost of a new facility based on the cost of a similar facility by the following relationship:

$$\frac{\text{Cost of facility 2}}{\text{Cost of facility 1}} = \left( \frac{\text{Capacity of facility 2}}{\text{Capacity of facility 1}} \right)^x$$

where the exponent  $x$  is referred to as the *power*, or *capacity factor*. This type of cost estimating relationship is commonly used for manufacturing and petrochemical industries to capture scale economies that are known to exist. The value of the exponent depends on the industry, but a value of 0.6 is often used in the absence of information.

In the software industry, development efforts are also approximated with estimating relationships. Several order of magnitude models are popular, one of which predicts the effort required, in person-months, as a function of the number of lines of code:

$$\text{Effort} = a(\text{KLOC})^b$$

where  $a$  and  $b$  are parameters that depend on the level of requirements and KLOC is in thousands of lines of coding. As the level of requirements increases, typical values for  $a$  range from 2.4 to 3.5, and for  $b$  from 1.05 to 1.20. The effort level is then converted to

cost by using measures of programmer productivity and labour cost. Other cost components of a software development project typically include hardware and software tools as well as travel and training, but effort costs usually dominate.

## 6A.6 Dealing with Indirect Costs

Cost estimating relationships primarily deal with the direct costs of a project—costs that arise directly as a result of the size, volume, or other design parameters. It is common practice to also allocate a portion of indirect costs, as they are shared across many activities. There are various cost accounting methods employed for this purpose, but they are beyond the scope of this appendix. They result in the use of an overhead rate, whereby indirect costs are estimated as a fraction of certain direct costs. Industry publications, scholarly journals, and cost estimating software packages are good sources for appropriate industry and project specific rates.

### Example 6A.2

To bring together many of the cost estimating components into an example, consider a situation where you are asked to develop an order of magnitude cost estimate for the first cost and annual costs of a chemical plant. It is to have a capacity of 4.5 million litres per year of output. A similar plant, built very recently, is producing 4 million litres per year and had a first cost of \$41 880 000. Initial engineering estimates of the costs of raw materials needed and waste treatment are in Table 6A.1.

**Table 6A.1**

| Order of Magnitude Cost Estimation |                     |                   |
|------------------------------------|---------------------|-------------------|
| <b>Assumptions:</b>                |                     |                   |
| Production capacity (litres/year)  |                     | 4 500 000         |
| Estimated capital cost (\$)        |                     | 45 000 000        |
| <b>Variable Costs:</b>             |                     |                   |
| Direct material costs (\$/litre)   | <b>Unit Cost</b>    | <b>Total Cost</b> |
| Waste treatment costs (\$/litre)   | 6.50                | 29 250 000        |
| Total variable costs (\$/litre):   | 0.09                | 405 000           |
|                                    | 6.59                | 29 655 000        |
| <b>Fixed Costs:</b>                |                     |                   |
| Capital costs (\$/yr)              | = 45 000 000/10     | 4 500 000         |
| Operating staff salaries (\$/yr)   | = 12 * 80 000       | 960 000           |
| Administration/management (\$/yr)  | = 1.8 * 960 000     | 1 728 000         |
| Maintenance/utilities/misc (\$/yr) | = 0.04 * 45 000 000 | 1 800 000         |
| Total Fixed Costs (\$/yr)          |                     | 8 988 000         |
| <b>Total Costs (\$/yr)</b>         |                     | <b>38 643 000</b> |
| Average cost (\$/l)                |                     | 8.59              |
| Marginal cost (\$/l)               |                     | 6.59              |

As a first approximation, the cost of the plant can be estimated using the power or capacity-sizing model. You are told that an appropriate sizing exponent is 0.61, which leads to an estimated capital cost of about \$45 million.

$$\frac{\text{Estimated cost of new plant}}{41.88} = \left( \frac{4.5}{4} \right)^{0.61}$$

The variable production and waste treatment costs can be estimated directly from unit cost estimates. Fixed depreciation costs have been estimated based on a simple assumption of straight line depreciation over 10 years. The operating staff salaries, considered a fixed cost because they supervise the process regardless of output volumes, are estimated on the basis of three shifts daily, each with four people who are paid \$80 000 per year (this includes benefits). Indirect costs of administrative salaries, supplies, and miscellaneous expenses are estimated to be 1.8 times the salary costs based on historical experience. The cost of maintenance and utilities of such plants has historically run at about 4 percent of the capital cost of the plant.

From these rough estimates, the plant can be expected to cost \$45 million to build and to have total annual costs of \$38.64 million.

## PROBLEMS

- 6A.1** Consider the costs associated with operating a restaurant in a mall. Which of the costs listed below would be considered fixed and which would be considered variable?
- (a) Rental of restaurant space
  - (b) Purchase cost of tables, chairs, and decorative items
  - (c) Food items
  - (d) Insurance
  - (e) Lights and heating
  - (f) Taxes
  - (g) Part-time staff salaries
  - (h) Cleaning supplies
- 6A.2** A friend has asked for some help in estimating the cost of a possible renovation to a house he has just purchased. The basic footprint of the house is 30 by 40 feet. He'd like to replace the roof and install new siding. What process would you use to make this estimate? What would your estimate be?
- 6A.3** CANCO has received an order for 85 OCNAC Systems to be built and delivered one year from today. One hundred and twenty-five of the same systems were built two years ago, so there is recent cost history. The data from the previous order are as follows:

**Direct Labour Hours:**

|                       |                                |
|-----------------------|--------------------------------|
| Plant A: 18 000 hours | Purchased parts: \$10 000 000  |
| Plant B: 12 000 hours | Tooling Costs: \$15 000        |
| Plant C: 12 000 hours | Test Equipment Costs: \$25 000 |

- Manufacturing support labour costs: 18 000 hours from one mechanical engineer, one management engineer, and one electrical engineer at each of the three plant locations.

- Engineering support labour: 8000 hours from two engineers at Plant A and one engineer each at Plants B and C.

Several changes have occurred to production processes that will affect costs:

- A new production process for part D-56 has reduced the assembly time at Plant A by five hours per system.
- Plant B is operating at capacity, so five parts from each system will need to be produced in Plant C. Each of the five parts takes three hours to build.
- Part number D-25, previously assembled at Plant A, is being purchased from an outside vendor at \$200 per unit. Assembly time at Plant A for this part was four hours per unit.

You have been asked to prepare a rough cost estimate. Assume that inflation has been 3 percent per year over the two years since the last project was completed.

- How many direct labour hours per unit do you estimate for Plant A? For Plant B? For Plant C?
- Manufacturing labour costs are \$70 per hour. What do you estimate to be the total manufacturing labour costs at each of the plants?
- The costs of tooling and test equipment are expected to be proportional to the order size. What estimates would you recommend in each of these categories, allowing for inflation?
- To allow for quality and shrinkage, a 5 percent volume allowance is used for purchased parts. Allowing for inflation and new parts, what do you expect the cost of purchased parts to be?
- Indirect support labour costs are semi-variable: Halving the production quantity reduces costs by 25 percent; doubling production increases costs by 50 percent. Manufacturing support labour costs are \$75 per hour and engineering support labour costs are \$85 per hour. What do you estimate the total support labour to cost?
- What is your estimate of total costs for the new order?

- 6A.4** This question deals with cost estimation for a software development project and is based on the COCOMO II (<http://sunset.usc.edu/csse/research/COCOMOII>). The basic cost estimating relationship (CER) is:

$$\text{Effort (in person-months)} = A \times \text{KLOC}^B \times M$$

$A = 2.94$  is based on a large dataset of previous development projects.

$M$  is an effort multiplier that captures characteristics of the project and process. For early design cost estimates, the COCOMO II model assumes  $M$  to be a composite of seven measures. For this problem, a nominal value of 1 will be used.

The scale factor,  $B$ , determines how the costs of the project scale with the number of thousands of lines of code (KLOC). The value of  $B$  depends on five project characteristics:

| Scale Factor                   | Interpretation   |
|--------------------------------|--|
| Precededness (PREC)            | Previous experience with this type of project.<br>0 = Substantial experience; 5 = no previous experience |
| Development Flexibility (FLEX) | Degree to which the client has set the development process   |

|                         |   |
|-------------------------|---|
|                         | 0 = client has set only general goals; 5 = prescribed process   |
| Risk Resolution (RESL)  | Extent to which risk analysis has been carried out<br>0 = comprehensive analysis; 5 = little analysis           |
| Team Cohesion (TEAM)    | Reflects how well the development team works together<br>0 = seamless communication; 5 = difficult interactions |
| Process Maturity (PMAT) | Reflects maturity of development process in the organization<br>0 = highly mature; 5 = no controls              |

To find  $B$ , each factor is rated on a scale of very low (5) to very high (0). The factor ratings are added and the sum is divided by 100 and then added to a base level of 1.01. The results will thus vary between 1.01 and 1.26.

The software project you have been asked to estimate is still in its early design stage. You have already determined that it will require approximately 130 000 lines of code. Other scale factors are:

| Scale Factor                   | Evaluation   |
|--------------------------------|--|
| Precedentedness (PREC)         | This is a new project; there is little experience with this type of application. Rating: low (4) |
| Development Flexibility (FLEX) | The client has set no constraints on the process to be used. Rating: very high (1)               |
| Risk Resolution (RESL)         | No risk analysis has been carried out. Rating: very low (1)                                      |
| Team Cohesion (TEAM)           | No information. Rating: average (3)  |
| Process Maturity (PMAT)        | Some process controls are in place. Rating: average (3)  |

- (a) Using the scale factors, find  $B$ .
- (b) If the cost per month of development is \$10 000, what do you estimate to be the development cost for this project? Assume  $M = 1$ .
- (c) From the scale factors provided above, what characteristics would you ascribe to a project that will have close to a linear relationship between effort and size?
- (d) How would you characterize a software development project in which there are strong diseconomies in the project size?



# Replacement Decisions

**Engineering Economics in Action, Part 7A: You Need the Facts**

- 7.1** Introduction
- 7.2** A Replacement Example
- 7.3** Reasons for Replacement or Retirement
- 7.4** Capital Costs and Other Costs
- 7.5** Defender and Challenger Are Identical
- 7.6** Challenger Is Different From Defender; Challenger Repeats Indefinitely
  - 7.6.1** Converting From Subcontracted to In-House Production
  - 7.6.2** The Irrelevance of Sunk Costs
  - 7.6.3** When Capital or Operating Costs Are Non-Monotonic
- 7.7** Challenger Is Different From Defender; Challenger Does Not Repeat

## **Review Problems**

### **Summary**

**Engineering Economics in Action, Part 7B: Decision Time**

## **Problems**

### **Mini-Case 7.1: Lighting the Way**

## ENGINEERING ECONOMICS IN ACTION, PART 7A

### You Need the Facts

"You know the 5-stage progressive die that we use for the Admiral Motors rocker arm contract?" Naomi was speaking to Terry, her co-op student, one Tuesday afternoon. "Clem asked me to look into replacing it with a 10-stage progressive die that would reduce the hand finishing substantially. It's mostly a matter of labour cost saving, but there is likely to be some quality improvement with the 10-stage die as well. I would like you to come up with a ballpark estimate of the cost of switching to the 10-stage progressive die."

Terry asked, "Don't you have the cost from the supplier?"

"Yes, but not really," said Naomi. "The supplier is Hamilton Tools. They've given us a price for the machine, but there are a lot of other costs involved in replacing one production process with another."

"You mean things like putting the machine in place?" Terry asked.

"Well, there's that," responded Naomi. "But there is also a lot more. For example, we will lose production during the changeover. That's going to cost us something."

"Is that part of the cost of the 10-stage die?"

"It's part of the first cost of switching to the 10-stage die," Naomi said. "If we decide to go ahead with the 10-stage die and incur these costs, we'll never recover them—they are sunk. We have already incurred those costs for the 5-stage die and it's only two years old. It still has lots of life in it. If the first costs of the 10-stage die are large, it's going to be hard to make a cost justification for switching to the 10-stage die at this time."

"Okay, how do I go about this?" Terry asked.

Naomi sat back and chewed on her yellow pencil for about 15 seconds. She leaned forward and began. "Let's start with order-of-magnitude estimates of what it's going to cost to get the 10-stage die in place. If it looks as if there is no way that the 10-stage die is going to be cost-effective now, we can just stop there."

"It sounds like a lot of fuzzy work," said Terry.

"Terry, I know you like to be working with mathematical models. I'm also sure that you can read the appropriate sections on replacement models in an engineering economics book. But none of those models is worth anything unless you have data to put in it. You need the models to know what information to look for. And once you have the information, you will make better decisions using the models. But you do need the facts."

## 7.1

### Introduction

Survival of businesses in a competitive environment requires regular evaluation of the plant and equipment used in production. As these assets age, they may not provide adequate quality, or their costs may become excessive. When a plant or piece of equipment is evaluated, one of four mutually exclusive choices will be made.

1. An existing asset may be kept in its current use without major change.
2. An existing asset may be overhauled so as to improve its performance.
3. An existing asset may be removed from use without replacement by another asset.
4. An existing asset may be replaced with another asset.

This chapter is concerned with methods of making choices about possible replacement of long-lived assets. While the comparison methods developed in Chapters 4 and 5 for choosing between alternatives are often used for making these choices, the issues of replacement deserve a separate chapter for several reasons. First, the relevant costs for making replacement decisions are not always obvious, since there are costs associated with taking the replaced assets out of service that should be considered. This was ignored in the

studies in Chapters 4 and 5. Second, the service lives of assets were provided to the reader in Chapters 4 and 5. As seen in this chapter, the principles of replacement allow the calculation of these service lives. Third, assumptions about how an asset might be replaced in the future can affect a decision now. Some of these assumptions are implicit in the methods analysts use to make the choices. It is therefore important to be aware of these assumptions when making replacement decisions.

The chapter starts with an example to introduce some of the basic concepts involved in replacement decisions. Following this is a discussion of the reasons why a long-lived asset may be replaced. We then develop the idea of the *economic life* of an asset—the service life that minimizes the average cost of using the asset. This is followed by a discussion of replacement with an asset that differs from the current asset, in which the built-in cost advantage of existing assets is revealed. Finally, we look at the case of replacement where there may be a series of replacements for a current asset, each of which might be different.

We will not consider the implications of taxes for replacement decisions in this chapter. This is postponed until Chapter 8. We assume in this chapter that no future price changes due to inflation are expected. The effect of expected price changes on replacement decisions will be considered in Chapter 9.

## 7.2 A Replacement Example

We introduce some of the basic concepts involved in replacement decisions through an example.

### EXAMPLE

#### 7.1

Sergio likes hiring engineering students to work in his landscaping business during the summer because they are such hard workers and have a lot of common sense. The students are always complaining about maintenance problems with the lawnmowers, which are subject to a lot of use and wear out fairly quickly. His routine has been to replace the machines every five years. Clarissa, one of the engineering student workers, has suggested that replacing them more often might make sense, since so much time is lost when there is a breakdown, in addition to the actual repair cost.

"I've checked the records and have made some estimates that might help us figure out the best time to replace the machines," Clarissa reports. "Every time there is a breakdown, the average cost to fix the machine is \$60. In addition to that, there is an average loss of two hours of time at \$20 per hour. As far as the number of repairs required goes, the pattern seems to be zero repairs the first season we use a new lawnmower. However, in the second season, you can expect about one repair, two repairs in the third season, four repairs in the fourth, and eight in the fifth season. I can see why you only keep each mower five years!"

"Given that the cost of a new lawnmower has averaged about \$600 and that they decline in value at 40 percent per year, and assuming an interest rate of 5 percent, I think we have enough information to determine how often the machines should be replaced," Clarissa concludes authoritatively.

How often should Sergio replace his lawnmowers? How much money will he save?

To keep things simple for now, let's assume that Sergio has to have lawns mowed every year for an indefinite number of years into the future. If he keeps each lawnmower for, say, three years rather than five years, he will end up buying lawnmowers more

frequently, and his overall capital costs for owning the lawnmowers will increase. However, his repair costs should decrease at the same time since newer lawnmowers require fewer repairs. We want to find out which replacement period results in the lowest overall costs—this is the replacement period Sergio should adopt and is referred to as the *economic life* of the lawnmowers.

We could take any period of time as a study period to compare the various possible replacement patterns using a *present worth* approach. The least common multiple of the service lives method (see Chapter 3) suggests that we would need to use a  $3 \times 4 \times 5 = 60$ -year period if we were considering service lives between one and five years. This is an awkward approach in this case, and even worse in situations where there are more possibilities to consider. It makes much more sense to use an *annual worth* approach. Furthermore, since we typically analyze the costs associated with owning and operating an asset, annual worth calculated in the context of a replacement study is commonly referred to as **equivalent annual cost (EAC)**. In the balance of the chapter, we will therefore use EAC rather than annual worth. However, we should not lose sight of the fact that EAC computations are nothing more than the annual worth approach with a different name adopted for use in replacement studies.

Returning to Sergio's replacement problem, if we calculate the EAC for the possibility of a one-year, two-year, three-year (and so on) replacement period, the pattern that has the lowest EAC would indicate which is best for Sergio. It would be the best because he would be spending the least, keeping both the cost of purchase and the cost of repairs in mind.

This can be done in a spreadsheet, as illustrated in Table 7.1. The first column, "Replacement Period," lists the possible replacement periods being considered. In this case, Sergio is looking at periods ranging from one to five years. The second column lists the salvage value of a lawnmower at the end of the replacement period, in this case estimated using the declining-balance method with a rate of 40 percent. This is used to compute the entries of the third column. "EAC Capital Costs" is the annualized cost of purchasing and salvaging each lawnmower, assuming replacement periods ranging from one to five years. Using the capital recovery formula (refer back to Close-Up 3.1 on page 53), this annualized cost is calculated as:

$$\text{EAC}(\text{capital costs}) = (P - S)(A/P, i, N) + Si$$

where

$\text{EAC}(\text{capital costs})$  = annualized capital costs for an  $N$ -year replacement period

$P$  = purchase price

$S$  = salvage value of the asset at the end of  $N$  years

**Table 7.1 Total Equivalent Annual Cost Calculations for Lawnmower Replacement Example**

| Replacement Period (Years) | Salvage Value | EAC Capital Costs | Annual Repair Costs | EAC Repair Costs | EAC Total |
|----------------------------|---------------|-------------------|---------------------|------------------|-----------|
| 1                          | \$360.00      | \$270.00          | \$ 0.00             | \$ 0.00          | \$270.00  |
| 2                          | 216.00        | 217.32            | 100.00              | 48.78            | 266.10    |
| 3                          | 129.60        | 179.21            | 200.00              | 96.75            | 275.96    |
| 4                          | 77.76         | 151.17            | 400.00              | 167.11           | 318.27    |
| 5                          | 46.66         | 130.14            | 800.00              | 281.64           | 411.79    |

$S$  can be calculated in turn as

$$BV_{db}(n) = P(1 - d)^n$$

For example, in the case of a three-year replacement period, the calculation is

$$\begin{aligned} \text{EAC(capital costs)} &= [600 - 600(1 - 0.40)^3](A/P, 5\%, 3) + [600(1 - 0.40)^3](0.05) \\ &= (600 - 129.60)(0.36721) + (129.60)(0.05) \\ &\approx 179.21 \end{aligned}$$

The “average” annual cost of repairs (under the heading “EAC Repair Costs”), assuming for simplicity that the cash flows occur at the end of the year in each case, can be calculated as:

$$\begin{aligned} \text{EAC(repairs)} &= [(60 + 40)(P/F, 5\%, 2) + (60 + 40)(2)(P/F, 5\%, 3)](A/P, 5\%, 3) \\ &= [100(0.90703) + 200(0.86584)](0.36721) \\ &\approx 96.75 \end{aligned}$$

The total EAC is then:

$$\text{EAC(total)} = \text{EAC(capital costs)} + \text{EAC(repairs)} = 179.22 + 96.75 \approx 275.96$$

Examining the EAC calculations in Table 7.1, it can be seen that the EAC is minimized when the replacement period is two years (and this is only slightly cheaper than replacing the lawnmowers every year). So this is saying that if Sergio keeps his lawnmowers for two years and then replaces them, his average annual costs over time will be minimized. We can also see how much money Clarissa’s observation can save him by subtracting the expected yearly costs from the estimate of what he is currently paying. This shows that on average Sergio saves  $\$411.79 - \$266.10 = \$145.69$  per lawnmower per year by replacing his lawnmowers on a two-year cycle over replacing them every five years. ■

The situation illustrated in Example 7.1 is the simplest case possible in replacement studies. We have assumed that the physical asset to be replaced is identical to the one replacing it, and such a sequence continues indefinitely into the future. By **asset**, we mean any machine or resource of value to an enterprise. An existing physical asset is called the **defender**, because it is currently performing the value-generating activity. The potential replacement is called the **challenger**. However, it is not always the case that the challenger and the defender are the same. It is generally more common that the defender is outmoded or less adequate than the challenger, and also that new challengers can be expected in the future.

This gives rise to several cases to consider. Situations like Sergio’s lawnmower problem are relatively uncommon—we live in a technological age where it is unlikely that a replacement for an asset will be identical to the asset it is replacing. Even lawnmowers improve in price, capability, or quality in the space of a few years. A second case, then, is that of a defender that is different from the challenger, with the assumption that the replacement will continue in a sequence of identical replacements indefinitely into the future. Finally, there is the case of a defender different from the challenger, which is itself different from another replacing it further in the future. All three of these cases will be addressed in this chapter.

Before we look at these three cases in detail, let’s look at why assets have to be replaced at all and how to incorporate various costs into the replacement decision.

## 7.3 | Reasons for Replacement or Retirement

If there is an ongoing need for the service an asset provides, at some point it will need replacement. **Replacement** becomes necessary if there is a cheaper way to get the service the asset provides, or if the service provided by the existing asset is no longer adequate.

An existing asset is **retired** if it is removed from use without being replaced. This can happen if the service that the asset provides is no longer needed. Changes in customer demand, changes in production methods, or changes in technology may result in an asset no longer being necessary. For example, the growth in the use of MP3 players for audio recordings has led manufacturers of compact discs to retire some production equipment since the service it provided is no longer needed.

There may be a cheaper way to get the service provided by the existing asset for several reasons. First, productive assets often deteriorate over time because of wearing out in use or simply because of the effect of time. As a familiar example, an automobile becomes less valuable with age (older cars, unless they are collectors' cars, are worth less than newer cars with the same mileage) or if it has high mileage (the kilometres driven reflect the wear on the vehicle). Similarly, production equipment may become less productive or more costly to operate over time. It is usually more expensive to maintain older assets. They need fixing more often, and parts may be harder to find and cost more.

Technological or organizational change can also bring about cheaper methods of providing service than the method used by an existing asset. For example, the technological changes associated with the use of computers have improved productivity. Organizational changes, both within a company and in markets outside the company, can lead to lower-cost methods of production. A form of organizational change that has become very popular is the specialist company. These companies take on parts of the production activities of other companies. Their specialization may enable them to have lower costs than the companies can attain themselves. See Close-Up 7.1.

The second major reason why a current asset may be replaced is inadequacy. An asset used in production can become inadequate because it has insufficient capacity to meet growing demand or because it no longer produces items of high enough quality. A company may have a choice between adding new capacity parallel to the existing capacity or replacing the existing asset with a higher capacity asset, perhaps one with more advanced technology. If higher quality is required, there may be a choice between upgrading an existing piece of

### CLOSE-UP 7.1

### Specialist Companies

**Specialist companies** concentrate on a limited range of very specialized products. They develop the expertise to manufacture these products at minimal cost. Larger firms often find it more economical to contract out production of low-volume components instead of manufacturing the components themselves.

In some industries, the use of specialist companies is so pervasive that the companies apparently manufacturing a product are simply assembling it; the actual manufacturing takes place at dozens or sometimes hundreds of supplier firms.

A good example of this is the automotive industry. In North America, auto makers support an extremely large network of specialist companies, linked by computer. A single specialist company might supply brake pads, for example, to all three major auto manufacturers. Producing brake pads in huge quantities, the specialist firm can refine its production process to extremes of efficiency and profitability.

equipment or replacing it with equipment that will yield the higher quality. In either case, contracting out the work to a specialist is one possibility.

In summary, there are two main reasons for replacing an existing asset. First, an existing asset will be replaced if there is a cheaper way to get the service that the asset provides. This can occur because the asset ages or because of technological or organizational changes. Second, an existing asset will be replaced if the service it provides is inadequate in either quantity or quality.

## 7.4 Capital Costs and Other Costs

When a decision is made to acquire a new asset, it is essentially a decision to purchase the capacity to perform tasks or produce output. **Capacity** is the ability to produce, often measured in units of production per time period. Although production requires capacity, it is also important to understand that just acquiring the capacity entails costs that are incurred whether or not there is actual production. Furthermore, a large portion of the capacity cost is incurred early in the life of the capacity. There are two main reasons for this:

1. Part of the cost of acquiring capacity is the expense incurred over time because the assets required for that capacity gradually lose their value. This expense is often called the **capital cost** of the asset. It is incurred by the difference between what is paid for the assets required for a particular capacity and what the assets could be resold for some time after purchase. The largest portion of the capital costs typically occurs early in the life of the asset.
2. Installing a new piece of equipment or new plant sometimes involves substantial up-front costs, called **installation costs**. These are the costs of acquiring capacity, excluding the purchase cost, which may include disruption of production, training of workers, and perhaps a reorganization of other processes. Installation costs are not reversible once the capacity has been put in place.

For example, if Sergio bought a new lawnmower to accommodate new landscaping clients, rather than for replacement, he would be increasing the capacity of his lawn-mowing service. The capital cost of the lawnmower in the first year would be its associated loss in market value over that year. The installation cost would probably be negligible.

It is worth noting that, in general, the total cost of a new asset includes both the installation costs and the cost of purchasing the asset. When we compute the capital costs of an asset over a period of time, the first cost (usually denoted by  $P$ ) includes the installation costs. However, when we compute a salvage value for the asset as it ages, we do *not* include the installation costs as part of the depreciable value of the asset, since these costs are expended upon installation and cannot be recovered at a later time.

The large influence of capital costs associated with acquiring new capacity means that, once the capacity has been installed, the *incremental* cost of continuing to use that capacity during its planned life is relatively low. This gives a defender a cost advantage during its planned life over a challenger.

In addition to the capital and installation costs, the purchase of an asset carries with it future **operating and maintenance costs**. Operating costs might include electricity, gasoline, or other consumables, and maintenance might include periodic servicing and repairs. Also, it is worth noting that a challenger may also give rise to changes in revenues as well as changes in costs that should not be neglected.

The different kinds of costs discussed in this section can be related to the more general ideas of fixed and variable costs. **Fixed costs** are those that remain the same,

regardless of actual units of production. For example, capital costs are usually fixed—once an asset is purchased, the cost is incurred even if there is zero production. **Variable costs** are costs that change depending on the number of units produced. The costs of the raw materials used in production are certainly variable costs, and to a certain degree operating and maintenance costs are as well.

With this background, we now look at the three different replacement cases:

1. Defender and challenger are identical and repeat indefinitely.
2. Challenger repeats indefinitely, but is different from defender.
3. Challenger is different from defender, but does not repeat.

## N E T   V A L U E   7 . 1

### **Estimating Salvage Values and Scrap Values**

An operating asset can have considerable value as long as it continues to perform the function for which it is intended. However, if it is replaced, its salvage value greatly depends on what is done with it. It is likely that the salvage value is different from whatever depreciation value is calculated for accounting or taxation purposes. To make a good replacement decision, it is desirable to have an accurate estimate of the actual salvage value of the asset.

The internet can be very helpful in estimating salvage values. A search for the asset by type, year, and model may reveal a similar asset in a used market. Or a more general search may reveal a broker for used assets of this nature who might be contacted to provide an estimate of salvage value based on the broker's experience with similar assets. In both of these cases, not only can a salvage value be determined, but also a channel for disposing of the asset is found. Even if the asset is scrapped, its value can be estimated from, for example, posted values for metal scrap available on the web.

## 7.5 | Defender and Challenger Are Identical

All long-lived assets eventually require replacement. Consequently, the issue in replacement studies is not *whether* to replace an asset, but *when* to replace it. In this section we consider the case where there is an ongoing need for a service provided by an asset and where the asset technology is not changing rapidly. (This is the case for Sergio's landscaping example.)

Several assumptions are made.

1. The defender and challenger are assumed to be technologically identical. It is also assumed that this remains true for the company's entire planning horizon.
2. The lives of these identical assets are assumed to be short relative to the time horizon over which the assets are required.
3. Relative prices and interest rates are assumed to be constant over the company's time horizon.

These assumptions are quite restrictive. In fact, there are only a few cases where the assumptions strictly hold (cable used for electric power delivery is an example). Nonetheless, the idea of economic life of an asset is still useful to our understanding of replacement analysis.

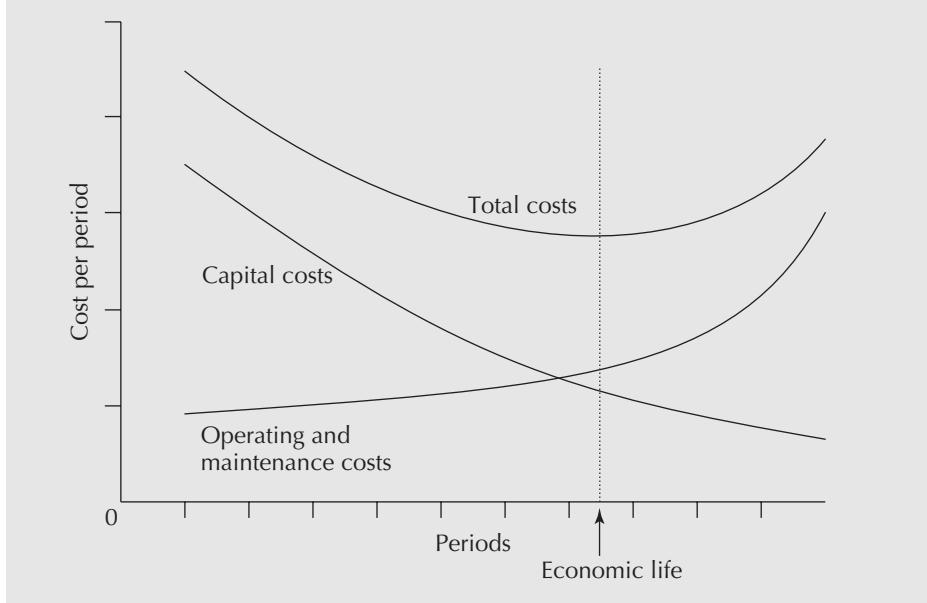
Assumptions 1 and 2 imply that we may model the replacement decision as being repeated an indefinitely large number of times. The objective is then to determine a

minimum-cost lifetime for the assets, a lifetime that will be the same for all the assets in the sequence of replacements over the company's time horizon.

We have seen that the relevant costs associated with acquiring a new asset are the capital costs, installation costs (which are often pooled with the capital costs), and operating and maintenance costs. It is usually true that operating and maintenance costs of assets—plant or equipment—rise with the age of the asset. Offsetting increases in operating and maintenance costs is the fall in capital costs per year that usually occurs as the asset life is extended and the capital costs are spread over a greater number of years. The rise in operating and maintenance costs per year, and the fall in capital costs per year as the life of an asset is extended, work in opposite directions. In the early years of an asset's life, the capital costs per year (although decreasing) usually, but not always, dominate total yearly costs. As the asset ages, increasing operating and maintenance costs usually overtake the declining annual capital costs. This means that there is a lifetime that will minimize the *average cost* (adjusting for the time value of money) per year of owning and using long-lived assets. This is referred to as the **economic life** of the asset.

These ideas are illustrated in Figure 7.1. Here we see the capital costs per period decrease as the number of periods the asset is kept increases because assets tend to depreciate in value by a smaller amount each period of use. On the other hand, the operating and maintenance costs per period increase because older assets tend to need more repairs and have other increasing costs with age. The economic life of an asset is found at the point where the rate of increase in operating and maintenance costs per period equals the rate of decrease in capital costs per period, or equivalently where the total cost per period is minimized.

**Figure 7.1 Cost Components for Replacement Studies**



### EXAMPLE

### 7.2

The Jiffy Printer Company produces printers for home use. Jiffy is considering installing an automated plastic moulding system to produce parts for the printers. The moulder itself costs \$20 000 and the installation costs are estimated to be \$5000. Operating and maintenance costs are expected to be \$30 000 in the first year and to rise

at the rate of 5 percent per year. Jiffy estimates depreciation with a declining-balance model using a rate of 40 percent, and uses a MARR of 15 percent for capital investments. Assuming that there will be an ongoing need for the moulder, and assuming that the technology does not change (i.e., no cheaper or better method will arise), how long should Jiffy keep a moulder before replacing it with a new model? In other words, what is the economic life of the automated moulding system?

Determining the economic life of an asset is most easily done with a spreadsheet. Table 7.2 shows the development of the equivalent annual costs for the automated plastic moulding system of Example 7.2.

In general, the EAC for an asset has two components:

$$\text{EAC} = \text{EAC(capital costs)} + \text{EAC(operating and maintenance)}$$

If

$P$  = the current value of an asset = (for a new asset) purchase price + installation costs

$S$  = the salvage value of an asset  $N$  years in the future

then

$$\text{EAC(capital costs)} = [P - (P/F, i, N)S] (A/P, i, N)$$

which in Close-Up 3.1 was shown to be equivalent to

$$\text{EAC(capital costs)} = (P - S)(A/P, i, N) + Si$$

In the first column of Table 7.2 is the life of the asset, in years. The second column shows the salvage value of the moulding system as it ages. The equipment costs \$20 000 originally, and as the system ages the value declines by 40 percent of current value each

**Table 7.2 Computation of Total Equivalent Annual Costs of the Moulding System With a MARR = 15%**



| Life in Years | Salvage Value | EAC Capital Costs | EAC Operating and Maintenance Costs | EAC Total   |
|---------------|---------------|-------------------|-------------------------------------|-------------|
| 0             | \$20 000.00   |                   |                                     |             |
| 1             | 12 000.00     | \$16 750.00       | \$30 000.00                         | \$46 750.00 |
| 2             | 7200.00       | 12 029.07         | 30 697.67                           | 42 726.74   |
| 3             | 4320.00       | 9705.36           | 31 382.29                           | 41 087.65   |
| 4             | 2592.00       | 8237.55           | 32 052.47                           | 40 290.02   |
| 5             | 1555.20       | 7227.23           | 32 706.94                           | 39 934.17   |
| 6             | 933.12        | 6499.33           | 33 344.56                           | 39 843.88   |
| 7             | 559.87        | 5958.42           | 33 964.28                           | 39 922.70   |
| 8             | 335.92        | 5546.78           | 34 565.20                           | 40 111.98   |
| 9             | 201.55        | 5227.34           | 35 146.55                           | 40 373.89   |
| 10            | 120.93        | 4975.35           | 35 707.69                           | 40 683.04   |

year, giving the estimated salvage values listed in Table 7.2. For example, the salvage value at the end of the fourth year is

$$\begin{aligned} BV_{db}(4) &= 20\,000(1 - 0.4)^4 \\ &\approx 2592 \end{aligned}$$

The next column gives the equivalent annual capital costs if the asset is kept for  $N$  years,  $N = 1, \dots, 10$ . This captures the loss in value of the asset over the time it is kept in service. As an example of the computations, the equivalent annual capital cost of keeping the moulding system for four years is

$$\begin{aligned} EAC(\text{capital costs}) &= (P - S)(A/P, 15\%, 4) + Si \\ &= (20\,000 + 5000 - 2592)(0.35027) + 2592(0.15) \\ &\approx 8238 \end{aligned}$$

Note that the installation costs have been included in the capital costs, as these are expenses incurred at the time the asset is originally put into service. Table 7.2 illustrates that the equivalent annual capital costs decline as the asset's life is extended. Next, the equivalent annual operating and maintenance costs are found by converting the stream of operating and maintenance costs (which are increasing by 5 percent per year) into a stream of equal-sized annual amounts. Continuing with our sample calculations, the EAC of operating and maintenance costs when the moulding system is kept for four years is

$$\begin{aligned} EAC(\text{operating and maintenance costs}) &= 30\,000 [(P/F, 15\%, 1) + (1.05)(P/F, 15\%, 2) + (1.05)^2(P/F, 15\%, 3) \\ &\quad + (1.05)^3(P/F, 15\%, 4)] (A/P, 15\%, 4) \\ &\approx 32\,052 \end{aligned}$$

Notice that the equivalent annual operating and maintenance costs increase as the life of the moulding system increases.

Finally, we obtain the total equivalent annual cost of the moulding system by adding the equivalent annual capital costs and the equivalent annual operating and maintenance costs. This is shown in the last column of Table 7.2. We see that at a six-year life the declining equivalent annual installation and capital costs offset the increasing operating and maintenance costs. In other words, the economic life of the moulder is six years, with a total EAC of \$39 844. ■

While it is *usually* true that capital cost per year falls with increasing life, it is not always true. Capital costs per year can rise at some point in the life of an asset if the decline in value of the asset is not smooth or if the asset requires a major overhaul.

If there is a large drop in the value of the asset in some year during the asset's life, the cost of holding the asset over that year will be high. Consider the following example.

### EXAMPLE

### 7.3

An asset costs \$50 000 to buy and install and has a resale value of \$40 000 after installation. It then declines in value by 20 percent per year until the fourth year, when its value drops from over \$20 000 to \$5000 because of a predictable wearing-out of a major component. Determine the equivalent annual capital cost of this asset for lives ranging from one to four years. The MARR is 15 percent.

The computations are summarized in Table 7.3. The first column gives the life of the asset in years. The second gives the salvage value of the asset as it ages. The asset loses 20 percent of its previous year's value each year except in the fourth, when its value drops to \$5000. The last column summarizes the equivalent annual capital cost of the asset. Sample computations for the third and fourth years are:

EAC(capital costs, three-year life)

$$\begin{aligned} &= (P - S)(A/P, 15\%, 3) + Si \\ &= (40\,000 + 10\,000 - 20\,480)(0.43798) + 20\,480(0.15) \\ &\approx 16\,001 \end{aligned}$$

EAC(capital costs, four-year life)

$$\begin{aligned} &= (P - S)(A/P, 15\%, 4) + Si \\ &= (40\,000 + 10\,000 - 5000)(0.35027) + 5000(0.15) \\ &\approx 16\,512 \end{aligned}$$

**Table 7.3 EAC of Capital Costs for Example 7.3**

| Life in Years | Salvage Value | EAC Capital Costs |
|---------------|---------------|-------------------|
| 0             | \$40 000      |                   |
| 1             | 32 000        | \$25 500          |
| 2             | 25 600        | 18 849            |
| 3             | 20 480        | 16 001            |
| 4             | 5000          | 16 512            |

The large drop in value in the fourth year means that there is a high cost of holding the asset in the fourth year. This is enough to raise the average capital cost per year. ■

In summary, when we replace one asset with another with an identical technology, it makes sense to speak of its economic life. This is the lifetime of an individual asset that will minimize the average cost per year of owning and using it. In the next section, we deal with the case where the challenger is different from the defender.

## S P R E A D S H E E T S A V V Y

As already demonstrated in earlier Spreadsheet Savvy boxes, there are many useful features in spreadsheet software that make economic evaluations much easier to carry out. Spreadsheet software is also used to prepare and present results to others. Reports, whether formal or informal, communicate more clearly and persuasively when they are easy to read and formatted in a professional manner. Some of the basic formatting features of Excel are covered here.

The most basic formatting choices are font, font size, and effects such as **bold**, *italics*, or underline. In Excel 2010, these formatting choices can be found under the Home tab under Font. Simply select the cells to be formatted, choose the appropriate format, and Excel will apply it to the selected cells. Alternatively, by right-clicking on the selected cells, a pop-up box with a number of alternatives will appear, one of which is Format Cells. Selecting this will lead you to a range of formatting

options for the selected cells, including font, alignment, border, fill, and number. The table below illustrates some common formatting choices from these alternatives.

|   | A             | B           | C             | D                               |
|---|---------------|-------------|---------------|---------------------------------|
| 1 |               |             |               |                                 |
| 2 |               | Defender    |               |                                 |
| 3 | Life in Years |             | Salvage Value | Operating and Maintenance Costs |
| 4 | 0             | \$49 000.00 |               |                                 |
| 5 | 1             | 36 500.23   | \$17 000.97   |                                 |
| 6 | 2             | 19 875.19   | 21 320.23     |                                 |
| 7 | 3             | 15 656.78   | 26 806.56     |                                 |
| 8 | 4             | 6 742.10    | 33 774.75     |                                 |

Cells B2:D3 are formatted as “text” (Number tab). These cells also have their background colour set to grey (Fill tab). Cells B3:D3 are centred horizontally and have “wrap text” control selected (Number tab) so that if the text in the column is too long to appear on one line, it “wraps” and appears on two lines. “Defender” was entered into cell B2. Cells B2:D2 were selected and the option chosen was Centre Across Selection, thus centring “Defender” across the three selected columns.

Cells B4:B8 are formatted as numbers with zero decimal places (Number tab) and have been centred horizontally (Alignment tab). The formatting for cells C4:D8 is custom: They are right (indent) aligned with four spaces indented (Alignment tab). Custom formatting has also been applied (Number tab) so that two decimal places appear and a space is used as a thousands separator rather than a comma.

Custom formats can be applied by selecting the Custom formatting option (Number tab) and entering a format style. In this case, \$### ###.00 was used for cells C4 and D5 and ### ###.00 for the remaining numeric cells. The \$ places a leading dollar sign, and number signs are used to display as many or as few numeric characters as needed (up to 999 999). Two decimal places will always appear, regardless of the number of significant digits stored in the cell.

## 7.6

### Challenger Is Different From Defender; Challenger Repeats Indefinitely

The decision rule that minimizes cost in the case where a defender is faced by a challenger that is expected to be followed by a sequence of identical challengers is as follows.

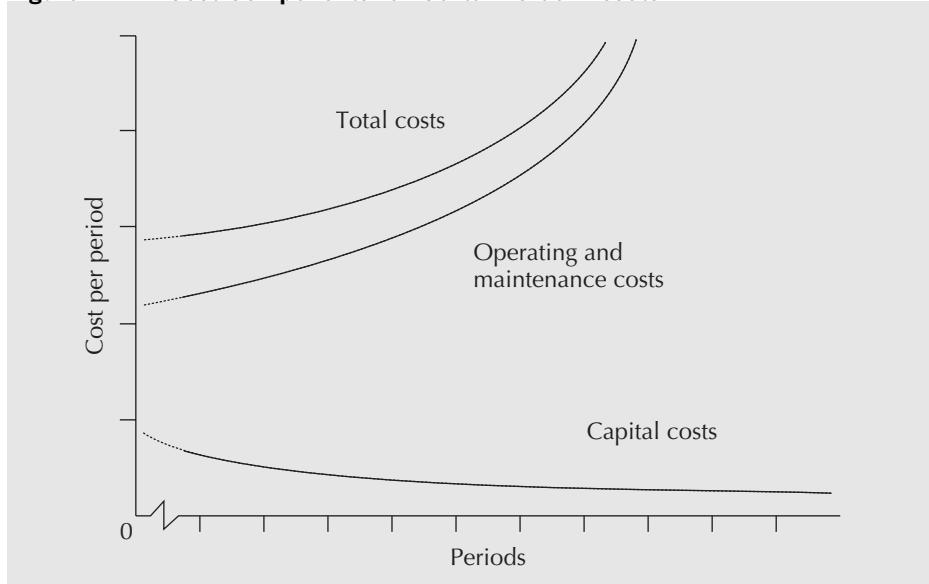
1. Determine the economic life of the challenger and its associated EAC.
2. Determine the remaining economic life of the defender and its associated EAC.
3. If the EAC of the defender is greater than the EAC of the challenger, replace the defender now. Otherwise, do not replace now.

In many cases, the computations in step 2 can be reduced somewhat. For assets that have been in use for several years, the yearly capital costs will typically be low compared to the yearly operating costs—the asset’s salvage value won’t change much from year to year but the operating costs will continue to increase. If this is true, as it often is for assets being replaced, the **one year principle** can be used. This principle states that if the capital costs

for the defender are small compared to the operating costs, and the yearly operating costs are monotonically increasing, the economic life of the defender is one year and its total EAC is the cost of using the defender for one more year.

The principle thus says that if the EAC of keeping the defender one more year exceeds the EAC of the challenger at its economic life, the defender should be replaced immediately. The advantage of the one year principle is that there is no need to find the service life for the defender that minimizes the EAC—it is known in advance that the EAC is minimized in a one-year period. The principle is particularly useful because for most assets the operating costs increase smoothly and monotonically as the asset is kept for longer and longer periods of time, while the capital costs decrease smoothly and monotonically. For a defender that has been in use for some time, the EAC (operating costs) will typically dominate the EAC (capital costs), and thus the total EAC will increase over any additional years that the asset is kept. This is illustrated in Figure 7.2, which can be compared to Figure 7.1 on page 223.

**Figure 7.2 Cost Components for Certain Older Assets**



For older assets that conform to this pattern of costs, it is only necessary to check whether the defender needs replacing due to costs over the next year, because in subsequent years the case for the defender will only get worse, not better. If there is an uneven yearly pattern of operating costs, the one year principle cannot be used, because the current year might give an unrealistic value due to the particular expenses in that year.

### EXAMPLE

### 7.4

An asset is three years old. Yearly operating and maintenance costs are currently \$5000 per year, increasing by 10 percent per year. The salvage value for the asset is currently \$60 000 and is expected to be \$50 000 one year from now. Can the one year principle be used?

The capital costs are not low (and thus the EAC(capital costs) for any given service life is not low) compared to the operating and maintenance costs. Even though costs have a regular pattern, the one year principle cannot be used. ■

**EXAMPLE****7.5**

An asset is 10 years old. Yearly operating and maintenance costs are currently \$5000 per year, increasing by 10 percent per year. The salvage value of the asset is currently \$8000 and is expected to be \$7000 one year from now. Can the one year principle be used?

The capital costs are low compared to the operating and maintenance costs, and costs have a regular pattern. The one year principle can be used. ■

**EXAMPLE****7.6**

An asset is 10 years old. Operating and maintenance costs average \$5000 per year, increasing by 10 percent per year. However, most of the operating and maintenance costs consist of a periodic overhaul costing \$15 000 that occurs every three years. The salvage value of the asset is currently \$8000 and is expected to be \$7000 one year from now. Can the one year principle be used?

The capital costs are low compared to the operating and maintenance costs but the operating and maintenance costs do not have an even pattern from year to year. The one year principle cannot be used. ■

The one year principle can be used when it is clear that the key conditions—low capital costs and a regular year-to-year pattern of monotonically increasing operating and maintenance costs—are satisfied. Where a situation is ambiguous, it is always prudent to fully assess the EAC of the defender.

To fully explore the case of a defender being replaced by a challenger that repeats, as well as to explore some other ideas useful in replacement analysis, the next three subsections cover examples to illustrate the analysis steps. In the first, we examine the situation of replacing subcontracted capacity with in-house production. This is an example of replacing a service with an asset. In the second example, the issue of sunk costs is examined by considering the replacement of an existing productive asset with a different challenger. In the final example, we look at the situation of making replacement decisions when there are irregular cash flows.

### 7.6.1 Converting From Subcontracted to In-House Production

**EXAMPLE****7.7**

Currently, the Jiffy Printer Company of Example 7.2 pays a custom moulder \$0.25 per piece (excluding material costs) to produce parts for its printers. Demand is forecast to be 200 000 parts per year. Jiffy is considering installing the automated plastic moulding system described in Example 7.2 to produce the parts. Should it do so now?

In Jiffy's situation, the *defender* is the current technology: a subcontractor. The *challenger* is the automated plastic moulding system. In order to decide whether Jiffy is better off with the defender or the challenger, we need to compute the unit cost (cost per piece) of production with the automated moulder and compare it to the unit cost for the subcontracted parts. If the automated system is better, the challenger wins and should replace the defender.

From Example 7.2:

$$\text{EAC}(\text{moulder}) = 39\,844$$

Dividing the EAC by the expected number of parts needed per year allows us to calculate the unit cost as  $39\,838/200\,000 = 0.1992$ .

When the unit cost of in-house production is compared with the \$0.25 unit cost of subcontracting the work, in-house production is cheaper, so Jiffy should replace the subcontracting with an in-house automated plastic moulding system. ■

This example has illustrated the basic idea behind a replacement analysis when we are considering the purchase of a new asset as a replacement for current technology. The cost of the replacement must take into account the capital costs (including installation) and the operating and maintenance costs over the life of the new asset.

In the next subsection, we see how some costs are no longer relevant in the decision to replace an *existing* asset.

### 7.6.2 The Irrelevance of Sunk Costs

Once an asset has been installed and has been operating for some time, the costs of installation and all other costs incurred up to that time are no longer relevant to any decision to replace the current asset. These costs are called **sunk costs**. Only those costs that will be incurred in keeping and operating the asset from this time on are relevant. This is best illustrated with an example.

#### EXAMPLE

#### 7.8

Two years have passed since the Jiffy Printer Company from Example 7.7 installed an automated moulding system to produce parts for its printers. At the time of installation, it expected to be producing about 200 000 pieces per year, which justified the investment. However, its actual production has turned out to be only about 150 000 pieces per year. Its unit cost is  $\$39\ 844/150\ 000 = \$0.2656$  rather than the \$0.1992 it had expected. It estimates the current market value of the moulder at \$7200. In this case, maintenance costs do not depend on the actual production rate. Should Jiffy sell the moulding system and go back to buying from the custom moulder at \$0.25 per piece?

In the context of a replacement problem, Jiffy is looking at replacing the existing system (the defender) with a different technology (the challenger). Since Jiffy already has the moulder and has already expended considerable capital on putting it into place, it may be better for Jiffy to keep the current moulder for some time longer. Let us calculate the cost to Jiffy of keeping the moulding system for one more year. This may not be the optimal length of time to continue to use the system, but if the cost is less than \$0.25 per piece it is cheaper than retiring or replacing it now.

The reason why the cost of keeping the moulder an additional year may be low is that the capital costs for the two-year-old system are now low compared with the costs of putting the capacity in place. The capital cost for the third year is simply the loss in value of the moulder over the third year. This is the difference between what Jiffy can get for the system now, at the end of the second year, and what it can get a year from now when the system will be three years old. Jiffy can get \$7200 for the system now. Using the declining-balance depreciation rate of 40 percent to calculate a salvage value, we can determine the capital cost associated with keeping the moulder over the third year.

Applying the capital recovery formula from Chapter 3, the EAC for capital costs is

$$\begin{aligned} \text{EAC}(\text{capital costs, third year}) &= (P - S)(A/P, 15\%, 1) + Si \\ &= [7200 - 0.6(7200)](1.15) + 0.6(7200)(0.15) \\ &\approx 3960 \end{aligned}$$

Recall that the operating and maintenance costs started at \$30 000 and rose at 5 percent each year. The operating and maintenance costs for the third year are

$$\begin{aligned}\text{EAC}(\text{operating and maintenance, third year}) &= 30\,000(1.05)^2 \\ &\cong 33\,075\end{aligned}$$

The total cost of keeping the moulder for the third year is the sum of the capital costs and the operating and maintenance costs:

$$\begin{aligned}\text{EAC}(\text{third year}) &= \text{EAC}(\text{capital costs, third year}) + \\ &\quad \text{EAC}(\text{operating and maintenance, third year}) \\ &= 3960 + 33\,075 \\ &\cong 37\,035\end{aligned}$$

Dividing the annual costs for the third year by 150 000 units gives us a unit cost of \$0.247 for moulding in-house during the third year. Not only is this lower than the average cost over a six-year life of the system, it is also lower than what the subcontracted custom moulder charges. Similar computations would show that Jiffy could go two more years after the third year with in-house moulding. Only then would the increase in operating and maintenance costs cause the total unit cost to rise above \$0.25.

Given the lower demand, we see that installing the automated moulding system was a mistake for Jiffy. The average lifetime costs for in-house moulding were greater than the cost of subcontracting, but once the system was installed, it was not optimal to go back to subcontracting immediately. This is because the capital cost associated with the purchase and installation of an asset (which becomes sunk after its installation) is disproportionately large as compared with the cost of using the asset once it is in place. ■

That a defender has a cost advantage over a challenger, or over contracting out during its planned life, is important. It means that if a defender is to be removed from service during its life for cost reasons, the average lifetime costs for the challenger or the costs of contracting out must be considerably lower than the average lifetime costs of the defender.

Just because well-functioning defenders are not often retired for cost reasons does not mean that they will not be retired at all. Changes in markets or technology may make the quantity or quality of output produced by a defender inadequate. This can lead to its retirement or replacement even when it is functioning well.

### 7.6.3 When Capital or Operating Costs Are Non-Monotonic

Sometimes operating costs do not increase smoothly and monotonically over time. The same can happen to capital costs. When the operating or capital costs are not smooth and monotonic, the one year principle does not apply. The reason why the principle does not apply is that there may be periodic or one-time costs that occur over the course of the next year (as in the case where periodic overhauls are required). These costs may make the cost of keeping the defender for one more year greater than the cost of installing and using the challenger over its economic life. However, there may be a life longer than one year over which the cost of using the defender is less than the cost of installing and using a challenger. Consider this example concerning the potential replacement of a generator.

#### EXAMPLE

#### 7.9

The Colossal Construction Company uses a generator to produce power at remote sites. The existing generator is now three years old. It cost \$11 000 when purchased. Its current salvage value of \$2400 is expected to fall to \$1400 next year and to \$980 the year after, and to continue declining at 30 percent of current value per year. Its ordinary operating and maintenance costs are now \$1000 per year and are expected to rise by

\$500 per year. There is also a requirement to do an overhaul that will cost \$1000 this year and every third year thereafter.

New fuel-efficient generators have been developed, and Colossal is thinking of replacing its existing generator. It is expected that the new generator technology will be the best available for the foreseeable future. The new generator sells for \$9500. Installation costs are negligible. Other data for the new generator are summarized in Table 7.4.

**Table 7.4 Salvage Values and Operating Costs for New Generator**

| End of Year | Salvage Value | Operating Cost |
|-------------|---------------|----------------|
| 0           | \$9500        |                |
| 1           | 8000          | \$1000         |
| 2           | 7000          | 1000           |
| 3           | 6000          | 1200           |
| 4           | 5000          | 1500           |
| 5           | 4000          | 2000           |
| 6           | 3000          | 2000           |
| 7           | 2000          | 2000           |
| 8           | 1000          | 3000           |

Should Colossal replace the existing generator with the new type? The MARR is 12 percent.

We first determine the economic life for the challenger. The calculations are shown in Table 7.5.

Sample calculations for the EAC of keeping the challenger for one, two, and three years are as follows:

$$\begin{aligned}
 \text{EAC}(1 \text{ year}) &= (P - S)(A/P, 12\%, 1) + Si + 1000 \\
 &= (9500 - 8000)(1.12) + 8000(0.12) + 1000 \\
 &\approx 3640
 \end{aligned}$$

**Table 7.5 Economic Life of the New Generator**

| End of Year | Salvage Value | Operating Costs | EAC             |
|-------------|---------------|-----------------|-----------------|
| 1           | \$8000        | \$1000          | \$3640.00       |
| 2           | 7000          | 1000            | 3319.25         |
| 3           | 6000          | 1200            | 3236.50         |
| 4           | 5000          | 1500            | <b>3233.07*</b> |
| 5           | 4000          | 2000            | 3290.81         |
| 6           | 3000          | 2000            | 3314.16         |
| 7           | 2000          | 2000            | 3318.68         |
| 8           | 1000          | 3000            | 3393.52         |

\*Lowest equivalent annual cost.

$$\begin{aligned}
 \text{EAC}(2 \text{ years}) &= (P - S)(A/P, 12\%, 2) + Si + 1000 \\
 &= (9500 - 7000)(0.5917) + 7000(0.12) + 1000 \\
 &\approx 3319
 \end{aligned}$$

$$\begin{aligned}
 \text{EAC}(3 \text{ years}) &= (P - S)(A/P, 12\%, 3) + Si + 1000 + 200(A/F, 12\%, 3) \\
 &= (9500 - 6000)(0.41635) + 6000(0.12) + 1000 \\
 &\quad + 200(0.29635) \\
 &\approx 3237
 \end{aligned}$$

As the number of years increases, this approach for calculating the EAC becomes more difficult, especially since in this case the operating costs are neither a standard annuity nor an arithmetic gradient. An alternative is to calculate the present worths of the operating costs for each year. The EAC of the operating costs can be found by applying the capital recovery factor to the sum of the present worths for the particular service period considered. This approach is particularly handy when using spreadsheets.

By either calculation, we see in Table 7.5 that the economic life of the generator is four years.

Next, to see if and when the defender should be replaced, we calculate the costs of keeping the defender for one more year. Using the capital recovery formula:

$$\begin{aligned}
 \text{EAC} (\text{keep defender 1 more year}) &= \text{EAC}(\text{capital costs}) + \text{EAC}(\text{operating costs}) \\
 &= (2400 - 1400)(A/P, 12\%, 1) + 1400(0.12) + 2000 \\
 &\approx 3288
 \end{aligned}$$

The equivalent annual cost of using the defender one more year is \$3288. This is more than the yearly cost of installing and using the challenger over its economic life. Since the operating costs are not smoothly increasing, we need to see if there is a longer life for the defender for which its costs are lower than for the challenger. This can be done with a spreadsheet, as shown in Table 7.6.

We see that, for an additional life of three years, the defender has a lower cost per year than the challenger, when the challenger is kept over its economic life. Therefore, the defender should not be replaced at this time. Next year a new evaluation should be performed. ■

**Table 7.6 Equivalent Annual Cost of Additional Life for the Defender**



| Additional Life in Years | Salvage Value | Operating Costs | EAC         |
|--------------------------|---------------|-----------------|-------------|
| 0                        | \$2400        |                 |             |
| 1                        | 1400          | \$2000          | \$3288      |
| 2                        | 980           | 1500            | 2722        |
| 3                        | 686           | 2000            | <b>2630</b> |
| 4                        | 480           | 3500            | 2872        |
| 5                        | 336           | 3000            | 2924        |
| 6                        | 235           | 3500            | 3013        |

## 7.7 Challenger Is Different From Defender; Challenger Does Not Repeat

In this section, we no longer assume that challengers are alike. We recognize that future challengers will be available and we expect them to be better than the current challenger. We must then decide if the defender should be replaced by the current challenger. Furthermore, if it is to be replaced by the current challenger, *when* should the replacement occur? This problem is quite complex. The reason for the complexity is that, if we believe that challengers will be improving, we may be better off skipping the current challenger and waiting until the next improved challenger arrives. The difficulties are outlined in Example 7.10.

**EXAMPLE**
**7.10**

Rita is examining the possibility of replacing the kiln controllers at the Burnaby Insulators plant. She has information about the existing controllers and the best replacement on the market. She also has information about a new controller design that will be available in three years. Rita has a five-year time horizon for the problem. What replacement alternatives should Rita consider?

One way to determine the minimum cost over the five-year horizon is to determine the costs of *all* possible combinations of the defender and the two challengers. This is impossible, since the defender and challengers could replace one another at any time. However, it is reasonable to consider only the combinations of the period length of one year. Any period length could be used, but a year is a natural choice because investment decisions tend, in practice, to follow a yearly cycle. These combinations form a mutually exclusive set of investment opportunities (see Section 4.2 on page 88). If no time horizon were given in the problem, we would have had to assume one, to limit the number of possible alternatives.

The possible decisions that need to be evaluated in this case are shown in Table 7.7.

For example, the first row in Table 7.7 (Alternative 1) means to keep the defender for the whole five-year period. Alternative 2 is to keep the defender for four years, and then purchase the challenger four years from now and keep it for one year. Alternative 15 is to replace the defender now with the first challenger, keep it for three years, then replace it with the second challenger and keep the second challenger for the remaining two years.

To choose among these possible alternatives, we need information about the following for the defender and both challengers:

1. Costs of installing the challengers
2. Salvage values for different possible lives for all three kiln controllers
3. Operating and maintenance costs for all possible ages for all three

With this information, the minimum-cost solution is obtained by computing the costs for all possible decision alternatives. Since these are mutually exclusive projects, any of the comparison methods of Chapters 4 and 5 are appropriate, including present worth, annual worth, and IRR. The effects of sunk costs are already included in the enumeration of the various replacement possibilities, so looking at the benefits of keeping the defender is already automatically taken into account. ■

**Table 7.7 Possible Decisions for Burnaby Insulators**

| Decision Alternative | Defender Life in Years | First Challenger Life in Years | Second Challenger Life in Years |
|----------------------|------------------------|--------------------------------|---------------------------------|
| 1                    | 5                      | 0                              | 0                               |
| 2                    | 4                      | 1                              | 0                               |
| 3                    | 4                      | 0                              | 1                               |
| 4                    | 3                      | 2                              | 0                               |
| 5                    | 3                      | 1                              | 1                               |
| 6                    | 3                      | 0                              | 2                               |
| 7                    | 2                      | 3                              | 0                               |
| 8                    | 2                      | 2                              | 1                               |
| 9                    | 2                      | 1                              | 2                               |
| 10                   | 1                      | 4                              | 0                               |
| 11                   | 1                      | 3                              | 1                               |
| 12                   | 1                      | 2                              | 2                               |
| 13                   | 0                      | 5                              | 0                               |
| 14                   | 0                      | 4                              | 1                               |
| 15                   | 0                      | 3                              | 2                               |

The difficulty with this approach is that the computational burden becomes great if the number of years in the time horizon is large. On the other hand, it is unlikely that information about a future challenger will be available under normal circumstances. In Example 7.10, Rita knew about a controller that wouldn't be available for three years. In real life, even if somehow Rita had inside information on the supplier research and marketing plans, it is unlikely that she would be confident enough of events three years away to use the information with complete assurance. Normally, if the information were available, the challenger itself would be available, too. Consequently, in many cases it is reasonable to assume that challengers in the planning future will be identical to the current challenger, and the decision procedure to use is the simpler one presented in the previous section.

## REVIEW PROBLEMS

### REVIEW PROBLEM 7.1

Kenwood Limousines runs a fleet of vans that ferry people from several outlying cities to a major international airport. New vans cost \$45 000 each and depreciate at a declining-balance rate of 30 percent per year. Maintenance for each van is quite expensive, because they are in use 24 hours a day, seven days a week. Maintenance costs, which

are about \$3000 the first year, double each year the vehicle is in use. Given a MARR of 8 percent, what is the economic life of a van?

### ANSWER

Table 7.8 shows the various components of this problem for replacement periods from one to five years. It can be seen that the replacement period with the minimum equivalent annual cost is two years. Therefore, the economic life is two years.

**Table 7.8 Summary Computations for Review Problem 7.1**



| Year | Salvage Value | Maintenance Costs | Equivalent Annual Costs |             |               |
|------|---------------|-------------------|-------------------------|-------------|---------------|
|      |               |                   | Capital                 | Maintenance | Total         |
| 0    | \$45 000      |                   |                         |             |               |
| 1    | 31 500        | \$3000            | \$17 100                | \$ 3000     | \$20 100      |
| 2    | 22 050        | 6000              | 14 634                  | 4442        | <b>19 076</b> |
| 3    | 15 435        | 12 000            | 12 707                  | 6770        | 19 477        |
| 4    | 10 805        | 24 000            | 11 189                  | 10 594      | 21 783        |
| 5    | 7563          | 48 000            | 9981                    | 16 970      | 26 951        |

As an example, the calculation for a three-year period is:

EAC(capital costs)

$$\begin{aligned}
 &= (45\,000 - 15\,435)(A/P, 8\%, 3) + 15\,435(0.08) \\
 &= 29\,565(0.38803) + 15\,435(0.08) \\
 &\approx 12\,707
 \end{aligned}$$

EAC(maintenance costs)

$$\begin{aligned}
 &= [3000(F/P, 8\%, 2) + 6000(F/P, 8\%, 1) + 12\,000] (A/F, 8\%, 3) \\
 &= [3000(1.1664) + 6000(1.08) + 12\,000](0.30804) \\
 &\approx 6770
 \end{aligned}$$

$$\begin{aligned}
 \text{EAC(total)} &= \text{EAC(capital costs)} + \text{EAC(maintenance costs)} \\
 &= 12\,707 + 6770 = 19\,477. \blacksquare
 \end{aligned}$$

### REVIEW PROBLEM 7.2

Canadian Widgets makes rocker arms for car engines. The manufacturing process consists of punching blanks from raw stock, forming the rocker arm in a 5-stage progressive die, and finishing in a sequence of operations using hand tools. A recently developed 10-stage die can eliminate many of the finishing operations for high-volume production.

The existing 5-stage die could be used for a different product, and in this case would have a salvage value of \$20 000. Maintenance costs of the 5-stage die will total \$3500 this year and are expected to increase by \$3500 per year. The 10-stage die will cost \$89 000 and will incur maintenance costs of \$4000 this year, increasing by \$2700 per year thereafter. Both dies depreciate at a declining-balance rate of 20 percent per year. The net yearly benefit of the automation of the finishing operations is expected to be \$16 000 per year. The MARR is 10 percent. Should the 5-stage die be replaced?

**ANSWER**

Since there is no information about subsequent challengers, it is reasonable to assume that the 10-stage die would be repeated. The EAC of using the 10-stage die for various periods is shown in Table 7.9.

A sample EAC computation for keeping the 10-stage die for two years is as follows:

$$\text{EAC}(\text{capital costs, two-year life})$$

$$\begin{aligned} &= (P - S)(A/P, 10\%, 2) + Si \\ &= (89\,000 - 56\,960)(0.57619) + 56\,960(0.10) \\ &\approx 24\,157 \end{aligned}$$

$$\text{EAC}(\text{maintenance costs, two-year life})$$

$$\begin{aligned} &= [4000(F/P, 10\%, 1) + 6700](A/F, 10\%, 2) \\ &= [4000(1.1) + 6700](0.47619) \\ &\approx 5286 \end{aligned}$$

**Table 7.9 EAC Computations for the Challenger in Review Problem 7.2**



| Life<br>in Years | Salvage<br>Value | Maintenance<br>Costs | Equivalent Annual Costs |             |               |
|------------------|------------------|----------------------|-------------------------|-------------|---------------|
|                  |                  |                      | Capital                 | Maintenance | Total         |
| 0                | \$89 000         |                      |                         |             |               |
| 1                | 71 200           | \$ 4000              | \$26 700                | \$ 4000     | \$30 700      |
| 2                | 56 960           | 6700                 | 24 157                  | 5286        | 29 443        |
| 3                | 45 568           | 9400                 | 22 021                  | 6529        | 28 550        |
| 4                | 36 454           | 12 100               | 20 222                  | 7729        | 27 951        |
| 5                | 29 164           | 14 800               | 18 701                  | 8887        | 27 589        |
| 6                | 23 331           | 17 500               | 17 411                  | 10 004      | 27 415        |
| 7                | 18 665           | 20 200               | 16 314                  | 11 079      | <b>27 393</b> |
| 8                | 14 932           | 22 900               | 15 377                  | 12 113      | 27 490        |

$$\text{EAC}(\text{total, two-year life})$$

$$\begin{aligned} &= 24\,157 + 5286 \\ &= 29\,443 \end{aligned}$$

Completing similar computations for other lifetimes shows that the economic life of the 10-stage die is seven years and the associated equivalent annual costs are \$27 393.

The next step in the replacement analysis is to consider the annual cost of the 5-stage die (the defender) over the next year. This cost is to be compared with the economic life EAC of the 10-stage die—that is, \$27 393. Note that the cost analysis of the defender should include the benefits generated by the 10-stage die as an operating cost for the 5-stage die, as this \$16 000 is a cost of *not* changing to the 10-stage die.

Since the capital costs are low and operating costs are monotonically increasing, the one year principle applies. The EAC of the capital and operating costs of keeping the defender one additional year are found as follows:

$$\text{Salvage value of 5-stage die after one year} = 20\,000(1 - 0.2) = \$16\,000$$

$$\begin{aligned}\text{EAC(capital costs, one additional year)} \\ &= (P - S)(A/P, 10\%, 1) + Si \\ &= (20\,000 - 16\,000)(1.10) + 16\,000(0.10) \\ &\approx 6000\end{aligned}$$

$$\begin{aligned}\text{EAC(maintenance and operating costs, one additional year)} \\ &= 3500 + 16\,000 \\ &\approx 19\,500\end{aligned}$$

$$\begin{aligned}\text{EAC(total, one additional year)} \\ &= 19\,500 + 6000 \\ &\approx 25\,500\end{aligned}$$

The 5-stage die should not be replaced this year because the EAC of keeping it one additional year (\$25 500) is less than the optimal EAC of the 10-stage die (\$27 393). The knowledge that the 5-stage die should not be replaced this year is usually sufficient for the immediate replacement decision. However, if a different challenger appears in the future, we would want to reassess the replacement decision.

It may also be desirable to estimate when in the future the defender might be replaced, even if it is not being replaced now. This can be done by calculating the equivalent annual cost of keeping the defender additional years until the time we can determine when it should be replaced. Table 7.10 summarizes those calculations for additional years of operating the 5-stage die.

**Table 7.10 EAC Computations for Keeping the Defender Additional Years**



| Additional Life in Years | Salvage Value | Maintenance and Operating Costs | Equivalent Annual Costs |           |          |
|--------------------------|---------------|---------------------------------|-------------------------|-----------|----------|
|                          |               |                                 | Capital                 | Operating | Total    |
| 0                        | \$20 000      |                                 |                         |           |          |
| 1                        | 16 000        | \$19 500                        | \$6000                  | \$19 500  | \$25 500 |
| 2                        | 12 800        | 23 000                          | 5429                    | 21 167    | 26 595   |
| 3                        | 10 240        | 26 500                          | 4949                    | 22 778    | 27 727   |
| 4                        | 8192          | 30 000                          | 4544                    | 24 334    | 28 878   |
| 5                        | 6554          | 33 500                          | 4202                    | 25 836    | 30 038   |
| 6                        | 5243          | 37 000                          | 3913                    | 27 283    | 31 196   |
| 7                        | 4194          | 40 500                          | 3666                    | 28 677    | 32 343   |
| 8                        | 3355          | 44 000                          | 3455                    | 30 018    | 33 473   |

As an example of the computations, the EAC of keeping the defender for two additional years is calculated as

$$\text{Salvage value of 5-stage die after two years} = 16\ 000(1 - 0.2) = 12\ 800$$

EAC(capital costs, two additional years)

$$\begin{aligned} &= (P - S)(A/P, 10\%, 2) + Si \\ &= (20\ 000 - 12\ 800)(0.57619) + 12\ 800(0.10) \\ &\approx 5429 \end{aligned}$$

EAC(maintenance and operating costs, two additional years)

$$\begin{aligned} &= [19\ 500(F/P, 10\%, 1) + (16\ 000 + 7000)](A/F, 10\%, 2) \\ &= [19\ 500(1.1) + 23\ 000](0.47619) \\ &\approx 21\ 167 \end{aligned}$$

EAC(total, two additional years)

$$= 5429 + 21\ 167 = 26\ 595$$

Further calculations in this manner will predict that the defender should be replaced at the end of the second year, given that the challenger remains the same during this time. This is because the EAC of keeping the defender for two years is less than the optimal EAC of the 10-stage die, but keeping the defender three years or more is more costly. ■

### **REVIEW PROBLEM 7.3**

Avril bought a computer three years ago for \$3000, which she can now sell on the open market for \$300. The local Mr. Computer store will sell her a new HAL computer for \$4000, including the new accounting package she wants. Her own computer will probably last another two years, and then would be worthless. The new computer would have a salvage value of \$300 at the end of its economic life of five years. The net benefits to Avril of the new accounting package and other features of the new computer amount to \$800 per year. An additional feature is that Mr. Computer will give Avril a \$500 trade-in on her current computer. Interest is 15 percent. What should Avril do?

### **ANSWER**

There are a couple of things to note about this problem. First, the cost of the new computer should be taken as \$3800 rather than \$4000. This is because, although the price was quoted as \$4000, the dealer was willing to give Avril a \$500 trade-in on a used computer that had a market value of only \$300. This amounts to discounting the price of the new computer to \$3800. Similarly, the used computer should be taken to be worth \$300, and not \$500. The \$500 figure does not represent the market value of the used computer, but rather the value of the used computer combined with the discount on the new computer. One must sometimes be careful to extract from the available information the best estimates of the values and costs for the various components of a replacement study.

First, we need to determine the EAC of the challenger over its economic life. We are told that the economic life is five years, and hence the EAC computations are as follows:

$$\begin{aligned}\text{EAC(capital costs)} &= (3800 - 300)(A/P, 15\%, 5) + 300(0.15) \\ &= 3500(0.29832) + 45 \\ &\approx 1089\end{aligned}$$

$$\text{EAC(operating costs)} = 0$$

$$\text{EAC(challenger, total)} = 1089$$

Now we need to check the equivalent annual cost of keeping the existing computer one additional year. A salvage value for the computer for one year was not given. However, we can check to see if the EAC for the defender over two years is less than for the challenger. If it is, this is sufficient to retain the old computer.

$$\begin{aligned}\text{EAC(capital costs)} &= (300 - 0)(A/P, 15\%, 2) + 0(0.15) \\ &= 300(0.61512) + 0 \\ &\approx 185\end{aligned}$$

$$\text{EAC(operating costs)} = 800$$

$$\text{EAC(defender, total over 2 years)} = 985$$

Avril should hang on to her current computer because its EAC over two years is less than the EAC of the challenger over its five-year economic life. ■

## SUMMARY

This chapter is concerned with replacement and retirement decisions. Replacement can be required because there may be a cheaper way to provide the same service, or the nature of the service may have changed. Retirement can be required if there is no longer a need for the asset.

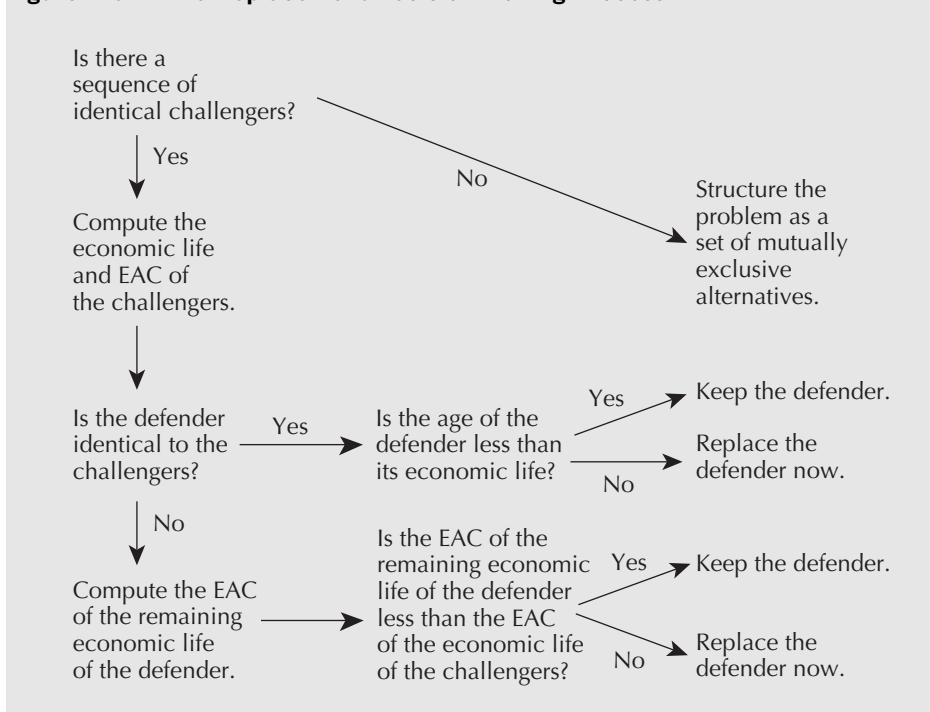
If an asset is replaced by a stream of identical assets, it is useful to determine the economic life of the asset, which is the replacement interval that provides the minimum annual cost. The asset should then be replaced at the end of its economic life.

If there is a challenger that is different from the defender, and future changes in technology are not known, one can determine the minimum EAC of the challenger and compare this with the cost of keeping the defender. If keeping the defender for any period of time is cheaper than the minimum EAC of the challenger, the defender should be kept. Often it is sufficient to assess the cost of keeping the defender for one more year.

Defenders that are still functioning well have a significant cost advantage over challengers or over obtaining the service performed by the defender from another source. This is because there are installation costs and because the capital cost per year of an asset diminishes over time.

Where future changes in technology are expected, decisions about when and whether to replace defenders are more complex. In this case, possible replacement decisions must be enumerated as a set of mutually exclusive alternatives and subjected to any of the standard comparison methods.

Figure 7.3 provides a summary of the overall procedure for assessing a replacement decision.

**Figure 7.3 The Replacement Decision Making Process**

## ENGINEERING ECONOMICS IN ACTION, PART 7B

### Decision Time

Naomi, Dave, and Clem were meeting in Clem's office. They had just finished a discussion of their steel-ordering policy. Clem turned to Naomi and said, "Okay. Let's look at the 10-stage progressive die. Where does that stand?"

Naomi said, "It looks possible. Did you get a chance to read Terry's report?"

"Yes, I did," Clem answered. "Was it his idea to use the 5-stage die for small runs so that we don't have to take a big hit from scrapping it?"

"Actually, it was," Naomi said.

"The kid may be a little intense," Clem said, "but he does good work. So where does that leave us?"

"Well, as I said, it looks possible that the 10-stage die will pay off," Naomi responded. "We have to decide what the correct time horizon is for making the analysis. Then we need more precise estimates of the costs and salvage value for the 10-stage die."

Clem turned in his chair and asked, "What do you think, Dave?"

Dave straightened himself in his chair and said, "I really don't know. How much experience has Hamilton Tools had at making dies this complicated?"

Naomi answered, "Not much. If we took them up on their proposal, we'd be their second or third customer."

"What do you have in mind, Dave?" Clem asked.

"Well, if it's only the second or third time they've done something like this, I think we can expect some improvements over the next couple of years. So maybe we ought to wait," he replied.

"That makes sense," Clem responded. "I'd like you two to work on this. Give Tan Wang at Hamilton Tools a call. He'll know if anything is in the works. Get him to give you an estimate of what to expect. Then I want you to consider some possibilities. You know: 'Replace now.' 'Wait one year.' 'Wait two years.' And so on. Don't make it too complicated. Then evaluate the different possibilities. I want a recommendation for next week's meeting. It's getting to be decision time."

## P R O B L E M S

### A. Key Concepts

- 7.1 Last year, Clairbrook Canning Co. bought a fancy colour printer, which cost \$20 000, for special printing jobs. Fast changes in colour printing technology have resulted in almost identical printers being available today for about one-quarter of the cost. Should CCC consider selling its printer and buying one of the new ones?
- 7.2 Maryhill Mines has a pelletizer that it is considering for replacement. Every three years it is overhauled at considerable cost. It is due for an overhaul this year. Evelyn, the company's mining engineer, has calculated that the sum of the operating and capital costs for this year for the pelletizer are significantly more than the EAC for a new pelletizer over its service life. Should the existing pelletizer be replaced?
- 7.3 Which of the following is a capital cost?
- A new printer
  - Paper for a new printer
  - Cabling to link data sensor nodes for a production management system
  - Transportation costs to move a new kiln from the manufacturer to a cement plant
  - Power to run cooling fans for a new snack food fryer
  - The cost of a building extension that was constructed to allow the placement of a new forge for a steel mill
  - Replacement cutting blades for a chipper
  - Software necessary to run a new chip-placing machine
  - Software updates to other equipment to allow it to interface with a new chip-placing machine
- 7.4 Freeport Brothers have recently purchased a process management system. Cabling installed in the plant connects data collection nodes to a central server. The costs of this system included the following:

|                                     |                   |
|-------------------------------------|-------------------|
| Five data collection nodes          | \$4500 each       |
| New server                          | \$6000            |
| 60-metre cable                      | \$11.40 per metre |
| Cabling hardware                    | \$188             |
| Node software                       | \$1190 per node   |
| Server software                     | \$1950            |
| 10 hours of hardware installer time | \$20 per hour     |
| 25 hours of software installer time | \$60 per hour     |

If Freeport Brothers wanted to calculate a replacement interval for such a process management system, what would be the capital cost for the first year at a depreciation rate of 30 percent?

- 7.5 Determine the economic life for each of the items listed on the next page. Salvage values can be estimated by the declining-balance method using an annual rate of 20 percent. The MARR is 8 percent.

|        | Purchase | Installation | Operating  |
|--------|----------|--------------|--|
| Item 1 | \$10 000 | \$2000       | \$300 first year, increasing by \$300 per year   |
| Item 2 | \$20 000 | \$2000       | \$200 first year, increasing by \$200 per year   |
| Item 3 | \$30 000 | \$3000       | \$2000 first year, increasing by \$2000 per year |

- 7.6 A new bottle-capping machine costs \$45 000, including \$5000 for installation. The machine is expected to have a useful life of eight years with no salvage value at that time (assume straight-line depreciation). Operating and maintenance costs are expected to be \$3000 for the first year, increasing by \$1000 each year thereafter. Interest is 12 percent.
- (a) Construct a spreadsheet that has the following headings: Year, Salvage Value, Maintenance Costs, EAC(Capital Costs), EAC(Operating Costs), and EAC(Total Costs). Compute the EAC(Total Costs) if the bottle capper is kept for  $n$  years,  $n = 1, \dots, 8$ .
  - (b) Construct a chart showing the EAC(Capital Costs), EAC(Operating Costs), and EAC(Total Costs) if the bottle capper were to be kept for  $n$  years,  $n = 1, \dots, 8$ .
  - (c) What is the economic life of the bottle capper?
- 7.7 A water pump to be used by the city's maintenance department costs \$10 000 new. A running-in period, costing \$1000 immediately, is required for a new pump. Operating and maintenance costs average \$500 the first year, increasing by \$300 per year thereafter. The salvage value of the pump at any time can be estimated by the declining-balance rate of 20 percent. Interest is at 10 percent. Using a spreadsheet, calculate the EAC for replacing the pump after one year, two years, etc. How often should the pump be replaced?
- 7.8 The water pump from Problem 7.7 is being considered to replace an existing one. The current one has a salvage value of \$1000 and will retain this salvage value indefinitely.
- (a) Operating costs are currently \$2500 per year and rise by \$400 per year. Should the current pump be replaced? When?
  - (b) Operating costs are currently \$3500 per year and rise by \$200 per year. Should the current pump be replaced? When?
- 7.9 An asset has operating and maintenance costs of \$3000 per year, increasing by 5 percent per year. The asset is currently worth \$350 000. Can the one year principle be used?
- 7.10 An asset has operating and maintenance costs of \$30 000 per year, decreasing by 5 percent per year. The asset is currently worth \$20 000. Can the one year principle be used?
- 7.11 An asset has operating and maintenance costs that average \$22 000 per year. If a main bearing fails—which can happen at any time—that particular year's cost will exceed \$40 000 (this possibility has been included in the average). Operating and maintenance

costs are increasing regularly at about 3 percent per year, and the asset is currently worth \$66 000. Can the one year principle be used?

- 7.12** An asset has maintenance costs that average \$4000 per year and negligible operating costs. The asset is in an inaccessible location and consequently only actually receives maintenance once every four years. The asset has zero salvage or scrap value and the maintenance costs are rising at about the same rate as inflation. Can the one year principle be used?

## B. Applications

- 7.13** Gerry likes driving small cars and buys nearly identical ones whenever the old one needs replacing. Typically, he trades in his old car for a new one costing about \$15 000. A new car warranty covers all repair costs above standard maintenance (standard maintenance costs are constant over the life of the car) for the first two years. After that, his records show an average repair expense (over standard maintenance) of \$2500 in the third year (at the end of the year), increasing by 50 percent per year thereafter. If a 30 percent declining-balance depreciation rate is used to estimate salvage values, and interest is 8 percent, how often should Gerry get a new car?
- 7.14** Gerry (see Problem 7.13) has observed that the cars he buys are somewhat more reliable now than in the past. A better estimate of the repair costs is \$1500 in the third year, increasing by 50 percent per year thereafter, with all other information in Problem 7.13 being the same. Now how often should Gerry get a new car?

- 7.15** For each of the following cases, determine whether the one year principle would apply.
- A defender has been in use for seven years and has negligible salvage value. Operating costs are \$400 per year for electricity. Once every five years it is overhauled at a cost of \$1000.
  - A defender has been in use for seven years and has negligible salvage value. Operating costs are \$400 per year for electricity. Once a year it is overhauled at a cost of \$1000.
  - A defender has been in use for two years and has negligible salvage value. Operating costs are \$400 per year for electricity. Once a year it is overhauled at a cost of \$1000.
  - A defender has been in use for seven years and has current salvage value of \$4000. Its value one year from now is estimated to be \$4000. Operating costs are \$400 per year for electricity. Once a year it is overhauled at a cost of \$1000.
  - A defender has been in use for seven years and has current salvage value of \$4000. Its value one year from now is estimated to be \$2000. Operating costs are \$400 per year for electricity. Once a year it is overhauled at a cost of \$1000.

- 7.16** A roller conveyor system used to transport cardboard boxes along an order-filling line costs \$100 000 plus \$20 000 to install. It is estimated to depreciate at a declining-balance rate of 25 percent per year over its 15-year useful life. Annual maintenance costs are estimated to be \$6000 for the first year, increasing by 20 percent every year thereafter. In addition, every third year, the rollers must be replaced at a cost of \$7000. Interest is at 10 percent.
- Construct a spreadsheet that has the following headings: Year, Salvage Value, Maintenance Costs, EAC(Capital Costs), EAC(Maintenance Costs), and EAC(Total Costs). Compute the EAC(Total Costs) if the conveyor were to be kept for  $n$  years,  $n = 1, \dots, 15$ .

- (b) Construct a chart showing the EAC(Capital Costs), EAC(Maintenance Costs), and EAC(Total Costs) if the conveyor were to be kept for  $n$  years,  $n = 1, \dots, 15$ .
- (c) What is the economic life of the roller conveyor system?
- 7.17** Brockville Brackets (BB) has a three-year-old robot that welds small brackets onto car-frame assemblies. At the time the robot was purchased, it cost \$300 000 and an additional \$50 000 was spent on installation. BB acquired the robot as part of an eight-year contract to produce the car-frame assemblies. The useful life of the robot is 12 years, and its value is estimated to decline by 20 percent of current value per year, as shown in the first table below. Operating and maintenance costs estimated when the robot was purchased are also shown in the table.

BB has found that the operating and maintenance costs for the robot have been higher than anticipated. At the end of the third year, new estimates of the operating and maintenance costs are shown in the second table on the next page.

BB has determined that the reason the operating and maintenance costs were in error was that the robot was positioned too close to existing equipment for the mechanics to repair it easily and quickly. BB is considering moving the robot farther away from some adjacent equipment so that mechanics can get easier access for repairs. To move the robot will cause BB to lose valuable production time, which is estimated to have a cost of \$25 000. However, once complete, the move will lower maintenance costs to what had originally been expected for the remainder of the contract (e.g., \$40 000 for the fourth year, increasing by 10 percent per year thereafter). Moving the robot will not affect its salvage value.

If BB uses a MARR of 15 percent, should it move the robot? If so, when? Remember that the contract exists only for a further five years.

| Defender, When New |               |                                 |
|--------------------|---------------|---------------------------------|
| Life (Years)       | Salvage Value | Operating and Maintenance Costs |
| 0                  | \$300 000     |                                 |
| 1                  | 240 000       | \$40 000                        |
| 2                  | 192 000       | 40 000                          |
| 3                  | 153 600       | 40 000                          |
| 4                  | 122 880       | 40 000                          |
| 5                  | 98 304        | 44 000                          |
| 6                  | 78 643        | 48 400                          |
| 7                  | 62 915        | 53 240                          |
| 8                  | 50 332        | 58 564                          |
| 9                  | 40 265        | 64 420                          |
| 10                 | 32 212        | 70 862                          |
| 11                 | 25 770        | 77 949                          |
| 12                 | 20 616        | 85 744                          |

| Costs for Three-Year-Old Defender |               |                                 |
|-----------------------------------|---------------|---------------------------------|
| Additional Life (Years)           | Salvage Value | Operating and Maintenance Costs |
| 0                                 | \$153 600     |                                 |
| 1                                 | 122 880       | \$50 000                        |
| 2                                 | 98 304        | 55 000                          |
| 3                                 | 78 643        | 60 500                          |
| 4                                 | 62 915        | 66 550                          |
| 5                                 | 50 332        | 73 205                          |

**7.18** Consider Brockville Brackets from Problem 7.17 but assume that it has a contract to produce the car assemblies for an indefinite period. If it does not move the robot, the operating and maintenance costs will be higher than expected. If it moves the robot (at a cost of \$25 000), the operating and maintenance costs are expected to be what were originally expected for the robot. Furthermore, BB expects to be able to obtain new versions of the existing robot for an indefinite period in the future; each is expected to have an installation cost of \$50 000.

- (a) Construct a spreadsheet table showing the EAC(total costs) if BB keeps the current robot in its current position for  $n$  more years,  $n = 1, \dots, 9$ .
- (b) Construct a spreadsheet table showing the EAC(total costs) if BB moves the current robot and then keeps it for  $n$  more years,  $n = 1, \dots, 9$ .
- (c) Construct a spreadsheet table showing the EAC(total costs) if BB is to buy a new robot and keep it for  $n$  years,  $n = 1, \dots, 9$ .
- (d) On the basis of your answers for parts (a) through (c), what do you advise BB to do?

**7.19** Nico has a 20-year-old oil-fired hot air furnace in his house. He is considering replacing it with a new high-efficiency natural gas furnace. The oil-fired furnace has a scrap value of \$500, which it will retain indefinitely. A maintenance contract costs \$300 per year, plus parts. Nico estimates that parts will cost \$200 this year, increasing by \$100 per year in subsequent years. The new gas furnace will cost \$4500 to buy and \$500 to install. It will save \$500 per year in energy costs. The maintenance costs for the gas furnace are covered under guarantee for the first five years. The market value of the gas furnace can be estimated from straight-line depreciation with a salvage value of \$500 after 10 years. Using a MARR of 10 percent, should the oil furnace be replaced?

**7.20** Jack and Jill live in the suburbs. Jack is a self-employed house painter who works out of their house. Jill works in the city, to which she regularly commutes by car. The car is a four-year-old import. Jill could commute by bus. They are considering selling the car and getting by with the van Jack uses for work.

The car cost \$12 000 new. It dropped about 20 percent in value in the first year, and after that it fell by about 15 percent per year. The car is now worth about \$5900. They expect it to continue to decline in value by about 15 percent of current value every year. Operating and other costs are about \$2670 per year. They expect this to rise by about 7.5 percent per year. A commuter pass costs \$112 per month and is not expected to increase in cost.

Jack and Jill have a MARR of 10 percent, which is what Jack earns on his business investments. Their time horizon is two years because Jill expects to quit work at that time.

- (a) Will commuting by bus save money?
- (b) Can you advise Jack and Jill about retiring the car?

**7.21** Ener-G purchases new turbines at a cost of \$100 000. Each has a 15-year useful life and must be overhauled periodically at a cost of \$10 000. The salvage value of a turbine declines by 15 percent of current value each year, and operating and maintenance costs (including the cost of the overhauls) of a typical turbine are as shown in the table below (the costs for the fifth and tenth years include a \$10 000 overhaul, but an overhaul is not done in the fifteenth year since this is the end of the turbine's useful life).

| Defender, When New, Overhaul Every Five Years |               |                                 |
|---|---------------|---------------------------------|
| Life (Years)                                  | Salvage Value | Operating and Maintenance Costs |
| 0   | \$100 000     |                                 |
| 1   | 85 000        | \$15 000                        |
| 2   | 72 250        | 20 000                          |
| 3   | 61 413        | 25 000                          |
| 4   | 52 201        | 30 000                          |
| 5   | 44 371        | 45 000                          |
| 6   | 37 715        | 20 000                          |
| 7   | 32 058        | 25 000                          |
| 8   | 27 249        | 30 000                          |
| 9   | 23 162        | 35 000                          |
| 10  | 19 687        | 50 000                          |
| 11  | 16 734        | 25 000                          |
| 12  | 14 224        | 30 000                          |
| 13  | 12 091        | 35 000                          |
| 14  | 10 277        | 40 000                          |
| 15  | 8735          | 45 000                          |

- (a) Construct a spreadsheet that gives, for each year, the EAC(operating and maintenance costs), EAC(capital costs), and EAC(total costs) for the turbines. Interest is 15 percent. How long should Ener-G keep each turbine before replacing it, given a five-year overhaul schedule? What are the associated equivalent annual costs?
- (b) If Ener-G were to overhaul its turbines every six years (at the same cost), the salvage value and operating and maintenance costs would be as shown in the table on the next page. Should Ener-G switch to a six-year overhaul cycle?

| Defender, When New, Overhaul Every Six Years |               |                                 |
|--|---------------|---------------------------------|
| Life (Years)                                 | Salvage Value | Operating and Maintenance Costs |
| 0  | \$100 000     |                                 |
| 1  | 85 000        | \$15 000                        |
| 2  | 72 250        | 20 000                          |
| 3  | 61 413        | 25 000                          |
| 4  | 52 201        | 30 000                          |
| 5  | 44 371        | 35 000                          |
| 6  | 37 715        | 50 000                          |
| 7  | 32 058        | 20 000                          |
| 8  | 27 249        | 25 000                          |
| 9  | 23 162        | 30 000                          |
| 10   | 19 687        | 35 000                          |
| 11   | 16 734        | 40 000                          |
| 12   | 14 224        | 55 000                          |
| 13   | 12 091        | 25 000                          |
| 14   | 10 277        | 30 000                          |
| 15   | 8 735         | 35 000                          |

- 7.22** The BBBB Machine Company makes a group of metal parts on a turret lathe for a local manufacturer. The current lathe is now six years old. It has a planned further life of three years. The contract with the manufacturer has three more years to run as well. A new, improved lathe has become available. The challenger will have lower operating costs than the defender.

The defender can now be sold for \$1200 in the used-equipment market. The challenger will cost \$25 000 including installation. Its salvage value after installation, but before use, will be \$20 000. Further data for the defender and the challenger are shown in the tables that follow.

| Defender                 |               |                |
|--------------------------|---------------|----------------|
| Additional Life in Years | Salvage Value | Operating Cost |
| 0                        | \$1200        |                |
| 1                        | 600           | \$20 000       |
| 2                        | 300           | 20 500         |
| 3                        | 150           | 21 012.50      |

| Challenger       |               |                |
|------------------|---------------|----------------|
| Life<br>in Years | Salvage Value | Operating Cost |
| 0                | \$20 000      |                |
| 1                | 14 000        | \$13 875       |
| 2                | 9800          | 14 360.63      |
| 3                | 6860          | 14 863.25      |

BBBB is not sure if the contract it has with the customer will be renewed. Therefore, BBBB wants to make the decision about replacing the defender with the challenger using a three-year study period. BBBB uses a 12 percent MARR for this type of investment.

- (a) What is the present worth of costs over the next three years for the defender?
  - (b) What is the present worth of costs over the next three years for the challenger?
  - (c) Now suppose that BBBB did not have a good estimate of the salvage value of the challenger at the end of three years. What minimum salvage value for the challenger at the end of three years would make the present worth of costs for the challenger equal to that of the defender?
- 7.23** Suppose, in the situation described in Problem 7.22, BBBB believed that the contract with the manufacturer would be renewed. BBBB also believed that all challengers after the current challenger would be identical to the current challenger. Further data concerning these challengers are given on the next page. Recall that a new challenger costs \$25 000 installed.

BBBB was also advised that machines identical to the defender would be available indefinitely. New units of the defender would cost \$17 500, including installation. Further data concerning new defenders are shown in the table on the next page. The MARR is 12 percent.

- (a) Find the economic life of the challenger. What is the equivalent annual cost over that life?
- (b) Should the defender be replaced with the challenger or with a new defender?
- (c) When should this be done?

| Challenger    |               |                |
|---------------|---------------|----------------|
| Life in Years | Salvage Value | Operating Cost |
| 0             | \$20 000.00   |                |
| 1             | 14 000.00     | \$13 875.00    |
| 2             | 9800.00       | 14 369.63      |
| 3             | 6860.00       | 14 863.25      |
| 4             | 4802.00       | 15 383.46      |
| 5             | 3361.40       | 15 921.88      |
| 6             | 2352.98       | 16 479.15      |
| 7             | 1647.09       | 17 055.92      |
| 8             | 1152.96       | 17 652.87      |
| 9             | 807.07        | 18 270.73      |
| 10            | 564.95        | 18 910.20      |
| 11            | 395.47        | 19 572.06      |
| 12            | 276.83        | 20 257.08      |

| Defender When New |               |                |
|-------------------|---------------|----------------|
| Life in Years     | Salvage Value | Operating Cost |
| 0                 | \$15 000.00   |                |
| 1                 | 9846.45       | \$17 250.00    |
| 2                 | 6463.51       | 17 681.25      |
| 3                 | 4242.84       | 18 123.28      |
| 4                 | 2785.13       | 18 576.36      |
| 5                 | 1828.24       | 19 040.77      |
| 6                 | 1200.11       | 19 516.79      |
| 7                 | 600.00        | 20 004.71      |
| 8                 | 300.00        | 20 504.83      |
| 9                 | 150.00        | 21 017.45      |
| 10                | 150.00        | 21 542.89      |
| 11                | 150.00        | 22 081.46      |
| 12                | 150.00        | 22 633.49      |
| 13                | 150.00        | 23 199.33      |

- 7.24 An existing piece of equipment has the following pattern of salvage values and operating and maintenance costs:

| <b>Defender</b>                |                      |                          |                          |  |                  |
|--------------------------------|----------------------|--------------------------|--------------------------|--|------------------|
| <b>Additional Life (Years)</b> | <b>Salvage Value</b> | <b>Maintenance Costs</b> | <b>EAC Capital Costs</b> | <b>EAC Operating and Maintenance Costs</b> | <b>EAC Total</b> |
| 0                              | \$10 000             |                          |                          |  |                  |
| 1                              | 8000                 | \$2000                   | \$3500                   | \$2000                                     | \$5500           |
| 2                              | 6400                 | 2500                     | 3174                     | 2233                                       | 5407             |
| 3                              | 5120                 | 3000                     | 2905                     | 2454                                       | 5359             |
| 4                              | 4096                 | 3500                     | 2682                     | 2663                                       | 5345             |
| 5                              | 3277                 | 4000                     | 2497                     | 2861                                       | 5359             |
| 6                              | 2621                 | 4500                     | 2343                     | 3049                                       | 5391             |
| 7                              | 2097                 | 5000                     | 2214                     | 3225                                       | 5439             |
| 8                              | 1678                 | 5500                     | 2106                     | 3391                                       | 5497             |
| 9                              | 1342                 | 6000                     | 2016                     | 3546                                       | 5562             |

A replacement asset is being considered. Its relevant costs over the next nine years are shown below.

There is a need for the asset (either the defender or the challenger) for the next nine years.

- (a) What replacement alternatives are there?
- (b) What replacement timing do you recommend?

| <b>Challenger</b>              |                      |                          |                          |  |                  |
|--------------------------------|----------------------|--------------------------|--------------------------|--|------------------|
| <b>Additional Life (Years)</b> | <b>Salvage Value</b> | <b>Maintenance Costs</b> | <b>EAC Capital Costs</b> | <b>EAC Operating and Maintenance Costs</b> | <b>EAC Total</b> |
| 0                              | \$12 000             |                          |                          |  |                  |
| 1                              | 9600                 | \$1500                   | \$4200                   | \$1500                                     | \$5700           |
| 2                              | 7680                 | 1900                     | 3809                     | 1686                                       | 5495             |
| 3                              | 6144                 | 2300                     | 3486                     | 1863                                       | 5349             |
| 4                              | 4915                 | 2700                     | 3219                     | 2031                                       | 5249             |
| 5                              | 3932                 | 3100                     | 2997                     | 2189                                       | 5186             |
| 6                              | 3146                 | 3500                     | 2811                     | 2339                                       | 5150             |
| 7                              | 2517                 | 3900                     | 2657                     | 2480                                       | 5137             |
| 8                              | 2013                 | 4300                     | 2528                     | 2613                                       | 5140             |
| 9                              | 1611                 | 4700                     | 2419                     | 2737                                       | 5156             |

**7.25** Chatham Automotive purchased new electric forklifts to move steel automobile parts two years ago. These cost \$75 000 each, including the charging stand. In practice, it was found that they did not hold a charge as long as claimed by the manufacturer, so operating costs are very high. This also results in their currently having a salvage value of about \$10 000.

Chatham is considering replacing them with propane models. The new ones cost \$58 000. After one year, they have a salvage value of \$40 000, and thereafter decline in value at a declining-balance depreciation rate of 20 percent, as does the electric model from this time on. The MARR is 8 percent. Operating costs for the electric model are \$20 000 over the first year, rising by 12 percent per year. Operating costs for the propane model will initially be \$10 000 over the first year, rising by 12 percent per year. Should Chatham Automotive replace the forklifts now?

**7.26** Suppose that Chatham Automotive (Problem 7.25) can get a \$14 000 trade-in value for its current electric model when it purchases a new propane model. Should it replace the electric forklifts now?

**7.27** A joint former cost \$60 000 to purchase and \$100 000 to install seven years ago. The market value now is \$33 000, and this will decline by 12 percent of current value each year for the next three years. Operating and maintenance costs are estimated to be \$34 000 this year, and are expected to increase by \$500 per year.

- How much should the EAC of a new joint former be over its economic life to justify replacing the old one sometime in the next three years? The MARR is 10 percent.
- The EAC for a new joint former turns out to be \$10 300 for a 10-year life. Should the old joint former be replaced within the next three years? If so, when?
- Is it necessary to consider replacing the old joint former more than three years from now, given that a new one has an EAC of \$10 300?

**7.28** Northwest Aerocomposite manufactures fibreglass and carbon fibre fairings. Its largest water-jet cutter will have to be replaced some time before the end of four years. The old cutter is currently worth \$49 000. Other cost data for the current and replacement cutters can be found in the tables that follow. The MARR is 15 percent. What is the economic life of the new cutter, and what is the equivalent annual cost for that life? When should the new cutter replace the old?

| Challenger    |               |                                 |
|---------------|---------------|---------------------------------|
| Life in Years | Salvage Value | Operating and Maintenance Costs |
| 0             | \$90 000      |                                 |
| 1             | 72 000        | \$12 000                        |
| 2             | 57 600        | 14 400                          |
| 3             | 46 080        | 17 280                          |
| 4             | 36 864        | 20 736                          |
| 5             | 29 491        | 24 883                          |
| 6             | 23 593        | 29 860                          |
| 7             | 18 874        | 35 832                          |
| 8             | 15 099        | 42 998                          |
| 9             | 12 080        | 51 598                          |

| Defender      |               |                                 |
|---------------|---------------|---------------------------------|
| Life in Years | Salvage Value | Operating and Maintenance Costs |
| 0             | \$49 000      |                                 |
| 1             | 36 500        | \$17 000                        |
| 2             | 19 875        | 21 320                          |
| 3             | 15 656        | 26 806                          |
| 4             | 6742          | 33 774                          |

- 7.29** Northwest Aerocomposite in Problem 7.28 found out that its old water-jet cutter may be overhauled at a cost of \$14 000 now. The cost information for the old cutter after an overhaul is shown in the table below.

| Defender With an Overhaul |               |                                 |
|---------------------------|---------------|---------------------------------|
| Life (Years)              | Salvage Value | Operating and Maintenance Costs |
| 0                         | \$55 000      |                                 |
| 1                         | 40 970        | \$16 500                        |
| 2                         | 22 310        | 20 690                          |
| 3                         | 17 574        | 26 013                          |
| 4                         | 7568          | 32 775                          |

Should Northwest overhaul the old cutter? If an overhaul takes place, when should the new cutter replace the old? Assume that the cost information for the replacement cutter is as given in Problem 7.28.

- 7.30** The water pump from Problem 7.7 has an option to be overhauled once. It costs \$1000 to overhaul a three-year-old pump and \$2000 to overhaul a five-year-old pump. The major advantage of an overhaul is that it reduces the operating and maintenance costs to \$500, which will increase again by \$300 per year thereafter. Should the pump be overhauled? If so, should it be overhauled in three years or five years?

### C. More Challenging Problems

- 7.31** If the operating costs for an asset are  $\$500 \times 2^n$  and the capital costs are  $\$10\,000 \times (0.8)^n$ , where  $n$  is the life in years, what is the economic life of the asset?
- 7.32** A certain machine costs \$25 000 to purchase and install. It has salvage values and operating costs as shown in the table on the next page. The salvage value of \$20 000 listed at time 0 reflects the loss of the installation costs at the time of installation. The MARR is 12 percent.
- What is the economic life of the machine?
  - What is the equivalent annual cost over that life?

| Costs and Salvage Values for Various Lives |               |                |
|--|---------------|----------------|
| Life in Years                              | Salvage Value | Operating Cost |
| 0  | \$20 000.00   |                |
| 1  | 16 000.00     | \$ 3000.00     |
| 2  | 12 800.00     | 3225.00        |
| 3  | 10 240.00     | 3466.88        |
| 4  | 8192.00       | 3726.89        |
| 5  | 6553.60       | 4006.41        |
| 6  | 5242.88       | 4306.89        |
| 7  | 4194.30       | 4629.90        |
| 8  | 3355.44       | 4977.15        |
| 9  | 2684.35       | 5350.43        |
| 10   | 2147.48       | 5751.72        |
| 11   | 1717.99       | 6183.09        |
| 12   | 1374.39       | 6646.83        |
| 13   | 1099.51       | 7145.34        |
| 14   | 879.61        | 7681.24        |
| 15   | 703.69        | 8257.33        |
| 16   | 562.95        | 8876.63        |
| 17   | 450.36        | 9542.38        |
| 18   | 360.29        | 10 258.06      |

Now assume that the MARR is 5 percent.

- (c) What is the economic life of the machine?
- (d) What is the equivalent annual cost over that life?
- (e) Explain the effect of decreasing the MARR.

**7.33** You own several copiers that are currently valued at \$10 000 all together. Annual operating and maintenance costs for all copiers are estimated at \$9000 next year, increasing by 10 percent each year thereafter. Salvage values decrease at a rate of 20 percent per year.

You are considering replacing your existing copiers with new ones that have a suggested retail price of \$25 000. Operating and maintenance costs for the new equipment will be \$6000 over the first year, increasing by 10 percent each year thereafter. The salvage value of the new equipment is well approximated by a 20 percent drop from the suggested retail price per year. Furthermore, you can get a trade-in allowance of \$12 000 for your equipment if you purchase the new equipment at its suggested retail price. Your MARR is 8 percent. Should you replace your existing equipment now?

- 7.34** The Brunswick Table Top Company makes tops for tables and desks. The company now owns a seven-year-old planer that is experiencing increasing operating costs. The defender has a maximum additional life of five years. The company is considering replacing the defender with a new planer.

The new planer would cost \$30 000 installed. Its value after installation, but before use, would be about \$25 000. The company has been told that there will be a new-model planer available in two years. The new model is expected to have the same first costs as the current challenger, but is expected to have lower operating costs. Data concerning the defender and the two challengers are shown in the tables that follow. Brunswick Table has a 10-year planning period and uses a MARR of 10 percent.

| <b>Defender</b>                 |                      |                       |
|---------------------------------|----------------------|-----------------------|
| <b>Additional Life in Years</b> | <b>Salvage Value</b> | <b>Operating Cost</b> |
| 0                               | \$4000               |                       |
| 1                               | 3000                 | \$20 000              |
| 2                               | 2000                 | 25 000                |
| 3                               | 1000                 | 30 000                |
| 4                               | 500                  | 35 000                |
| 5                               | 500                  | 40 000                |

| <b>First Challenger</b> |                      |                       |
|-------------------------|----------------------|-----------------------|
| <b>Life in Years</b>    | <b>Salvage Value</b> | <b>Operating Cost</b> |
| 0                       | \$25 000             |                       |
| 1                       | 20 000               | \$16 800              |
| 2                       | 16 000               | 17 640                |
| 3                       | 12 800               | 18 522                |
| 4                       | 10 240               | 19 448                |
| 5                       | 8192                 | 20 421                |
| 6                       | 6554                 | 21 442                |
| 7                       | 5243                 | 22 514                |
| 8                       | 4194                 | 23 639                |
| 9                       | 3355                 | 24 821                |
| 10                      | 2684                 | 26 062                |

| Second Challenger |               |                |
|-------------------|---------------|----------------|
| Life in Years     | Salvage Value | Operating Cost |
| 0                 | \$25 000      |                |
| 1                 | 20 000        | \$12 000       |
| 2                 | 16 000        | 12 600         |
| 3                 | 12 800        | 13 230         |
| 4                 | 10 240        | 13 892         |
| 5                 | 8192          | 14 586         |
| 6                 | 6554          | 15 315         |
| 7                 | 5243          | 16 081         |
| 8                 | 4194          | 16 885         |
| 9                 | 3355          | 17 729         |
| 10                | 2684          | 18 616         |

- (a) What are the combinations of planers that Brunswick can use to cover requirements for the next 10 years? For example, Brunswick may keep the defender one more year, then install the first challenger and keep it for nine years. Notice that the first challenger will not be installed after the second year when the second challenger becomes available. You may ignore combinations that involve installing the first challenger after the second becomes available. Recall also that the maximum additional life for the defender is five years.

(b) What is the best combination?

- 7.35 You estimate that your two-year-old car is now worth \$12 000 and that it will decline in value by 25 percent of its current value each year of its eight-year remaining useful life. You also estimate that its operating and maintenance costs will be \$2100, increasing by 20 percent per year thereafter. Your MARR is 12 percent.

- (a) Construct a spreadsheet showing (1) additional life in years, (2) salvage value, (3) operating and maintenance costs, (4) EAC(operating and maintenance costs), (5) EAC(capital costs), and (6) EAC(total costs). What additional life minimizes the EAC(total costs)?

- (b) Now you are considering the possibility of painting the car in three years' time for \$2000. Painting the car will increase its salvage value. By how much will the salvage value have to increase before painting the car is economically justified? Modify the spreadsheet you developed for part (a) to show this salvage value and the EAC(total costs) for each additional year of life. Will painting the car extend its economic life?

- 7.36 A long-standing principle of computer innovations is that computers double in power for the same price, or, equivalently, halve in cost for the same power, every 18 months. Auckland Data Services (ADS) owns a single computer that is at the end of its third year of service. ADS will continue to buy computers of the same power as its current one. Its

current computer would cost \$80 000 to buy today, excluding installation. Given that a new model is released every 18 months, what replacement policy should ADS adopt for computers over the next three years? Other facts to be considered are:

1. Installation cost is 15 percent of purchase price.
2. Salvage values are computed at a declining-balance depreciation rate of 50 percent.
3. Annual maintenance cost is estimated as 10 percent of accumulated depreciation or as 15 percent of accumulated depreciation per 18-month period.
4. ADS uses a MARR of 12 percent.

**7.37** Tiny Bay Freight Company (TBFC) wants to begin business with one delivery truck. After two years of operation, the company plans to increase the number of trucks to two, and after four years, plans to increase the number to three. TBFC currently has no trucks. The company is considering purchasing one type of truck that costs \$30 000. The operating and maintenance costs are estimated to be \$7200 per year. The resale value of the truck will decline each year by 40 percent of the current value. The company will consider replacing a truck every two years. That is, the company may keep a truck for two years, four years, six years, and so on. TBFC's MARR is 12 percent.

- (a) What are the possible combinations for purchasing and replacing trucks over the next five years so that TBFC will meet its expansion goals and will have three trucks in hand at the end of five years?
- (b) Which purchase/replacement combination is the best?

*Problems 7.38 through 7.41 are concerned with the economic life of assets where there is a sequence of identical assets. The problems explore the sensitivity of the economic life to four parameters: the MARR, the level of operating cost, the rate of increase in operating cost, and the level of first cost. In each problem there is a pair of assets. The assets differ in only a single parameter. The problem asks you to determine the effect of this difference on the economic life and to explain the result. All assets decline in value by 20 percent of current value each year. Installation costs are zero for all assets. Further data concerning the four pairs of assets are given in the table that follows.*

| Asset Number | First Cost | Initial Operating Cost | Rate of Operating Cost Increase | MARR |
|--------------|------------|------------------------|---------------------------------|------|
| A1           | \$125 000  | \$30 000               | 12.5%/year                      | 5%   |
| B1           | 125 000    | 30 000                 | 12.5%/year                      | 25%  |
| A2           | 100 000    | 30 000                 | \$2000/year                     | 15%  |
| B2           | 100 000    | 40 000                 | \$2000/year                     | 15%  |
| A3           | 100 000    | 30 000                 | 5%/year                         | 15%  |
| B3           | 100 000    | 30 000                 | 12.5%/year                      | 15%  |
| A4           | 75 000     | 30 000                 | 5%/year                         | 15%  |
| B4           | 150 000    | 30 000                 | 5%/year                         | 15%  |

**7.38** Consider Assets A1 and B1. They differ only in the MARR.

- (a) Determine the economic lives for Assets A1 and B1.
- (b) Create a diagram showing the EAC(capital), the EAC(operating), and the EAC(total) for Assets A1 and B1.
- (c) Explain the difference in economic life between A1 and B1.

**7.39** Consider Assets A2 and B2. They differ only in the level of initial operating cost.

- (a) Determine the economic lives for Assets A2 and B2.
- (b) Create a diagram showing the EAC(capital), the EAC(operating), and the EAC(total) for Assets A2 and B2.
- (c) Explain the difference in economic life between A2 and B2.

**7.40** Consider Assets A3 and B3. They differ only in the rate of increase of operating cost.

- (a) Determine the economic lives for Assets A3 and B3.
- (b) Create a diagram showing the EAC(capital), the EAC(operating), and the EAC(total) for Assets A3 and B3.
- (c) Explain the difference in economic life between A3 and B3.

**7.41** Consider Assets A4 and B4. They differ only in the level of first cost.

- (a) Determine the economic lives for Assets A4 and B4.
- (b) Create a diagram showing the EAC(capital), the EAC(operating), and the EAC(total) for Assets A4 and B4.
- (c) Explain the difference in economic life between A4 and B4.

**7.42** This problem concerns the economic life of assets where there is a sequence of identical assets. In this case there is an opportunity to overhaul equipment. Two issues are explored. The first concerns the optimal life of equipment. The second concerns the decision of whether to replace equipment that is past its economic life. Consider a piece of equipment that costs \$40 000 to buy and install. The equipment has a maximum life of 15 years. Overhaul is required in the fourth, eighth, and twelfth years. The company uses a MARR of 20 percent. Further information is given in the table below.

- (a) Show that the economic life for this equipment is seven years.
- (b) Suppose that the equipment is overhauled in the eighth year rather than replaced. Show that keeping the equipment for three more years (after the eighth year), until it next comes up for overhaul, has lower cost than replacing the equipment immediately.

*Hint for part (b):* The comparison must be done fairly and carefully. Assume that under either plan the replacement is kept for its optimal life of seven years. It is easier to compare the plans if they cover the same number of years. One way to do this is to consider an 11-year period as shown on the next page.

| Year | Salvage Value | Operating Cost | Overhaul Cost |
|------|---------------|----------------|---------------|
| 0    | \$15 000      |                |               |
| 1    | 12 000        | \$ 2000        |               |
| 2    | 9600          | 2200           |               |
| 3    | 7680          | 2420           |               |
| 4    | 7500          | 2662           | \$ 2500       |
| 5    | 6000          | 2000           |               |
| 6    | 4800          | 2200           |               |
| 7    | 3840          | 2420           |               |
| 8    | 4500          | 2662           | 32 500        |
| 9    | 3600          | 2000           |               |
| 10   | 2880          | 2800           |               |
| 11   | 2304          | 3920           |               |
| 12   | 2000          | 5488           | 17 500        |
| 13   | 1200          | 4000           |               |
| 14   | 720           | 8000           |               |
| 15   | 432           | 16 000         |               |

| Year | Plan A         | Plan B         |
|------|----------------|----------------|
| 0    |                |                |
| 1    | Defender       | Replacement #1 |
| 2    | Defender       | Replacement #1 |
| 3    | Defender       | Replacement #1 |
| 4    | Replacement #1 | Replacement #1 |
| 5    | Replacement #1 | Replacement #1 |
| 6    | Replacement #1 | Replacement #1 |
| 7    | Replacement #1 | Replacement #1 |
| 8    | Replacement #1 | Replacement #2 |
| 9    | Replacement #1 | Replacement #2 |
| 10   | Replacement #1 | Replacement #2 |
| 11   | Replacement #2 | Replacement #2 |

First, show that the present worth of costs over the 11 years is lower under plan A than under plan B. Second, point out that the equipment that is in place at the end of the eleventh year is newer under plan A than under plan B.

- (c) Why is it necessary to take into account the age of the equipment at the end of the 11-year period?

- 7.43** Northfield Metal Works is a household appliance parts manufacturer that has just won a contract with a major appliance company to supply replacement parts to service shops. The contract is for five years. Northfield is considering using three existing manual punch presses or a new automatic press for part of the work. The new press would cost \$225 000 installed. Northfield is using a five-year time horizon for the project. The MARR is 25 percent for projects of this type. Further data concerning the two options are shown in the tables that follow.

| Automatic Punch Press |               |                |
|-----------------------|---------------|----------------|
| Life in Years         | Salvage Value | Operating Cost |
| 0                     | \$125 000     |                |
| 1                     | 100 000       | \$25 000       |
| 2                     | 80 000        | 23 750         |
| 3                     | 64 000        | 22 563         |
| 4                     | 51 200        | 21 434         |
| 5                     | 40 960        | 20 363         |

| Hand-Fed Press           |               |                |
|--------------------------|---------------|----------------|
| Additional Life in Years | Salvage Value | Operating Cost |
| 0                        | \$10 000      |                |
| 1                        | 9000          | \$25 000       |
| 2                        | 8000          | 25 000         |
| 3                        | 7000          | 25 000         |
| 4                        | 6000          | 25 000         |
| 5                        | 5000          | 25 000         |

Note that the hand-fed press values are for each of the three presses. Costs must be multiplied by three to get the costs for three presses. Northfield is not sure of the salvage values for the new press. What salvage value at the end of five years would make the two options equal?

- 7.44 (a)** Referring to Figure 7.1 on page 223, why is the economic life different from the intersection of the reducing capital costs line and the increasing operating and maintenance costs line? Under what circumstances would the economic life correspond with the intersection of the lines?
- (b)** Referring to Figure 7.2 on page 228, the point at which total costs are lowest is not shown on the graph, but rather would have appeared on a section of the graph to the left of the part illustrated. As seen in Figure 7.1, the asset should have been replaced when total costs were lowest. Even if this wasn't done, every year of additional life the total costs of an asset in Figure 7.2 are increasing. However, as pointed out in the text, the situation illustrated in Figure 7.2 is very common, justifying the one year rule for such assets. How can it be that asset costs commonly have the structure seen in Figure 7.2?

**M I N I - C A S E 7 . 1****Lighting the Way**

In March 2011, Sears Canada announced that it was banning the sale of inefficient incandescent and halogen lighting in all of its full-line and Sears Home stores in Canada ([www.marketnews.ca/content/index/page?pid=9187](http://www.marketnews.ca/content/index/page?pid=9187), accessed September 28, 2011). It also announced that by August of the same year, it would replace more than 130 000 incandescent spotlights in its stores with LED (light-emitting diode) bulbs. It was estimated that this would save more than 16 million kWh (kilowatt hours) of electricity per year and reduce Sears's carbon footprint by 3611 tonnes annually.

"Sears Canada constantly strives to be a leader in sustainability by minimizing its environmental impact across all areas of its business, while helping Canadians choose products that save them money, reduce their eco footprint and create healthier homes," said Dr. James Gray-Donald, sustainability leader and associate vice-president of Sears Canada. "Replacing inefficient lighting products with LED and CFL options is a major move to save customers energy and money, while empowering them to take sustainable steps at home."

**Discussion**

We are all familiar with the routine of replacing a burned-out light bulb. For over a century, it has been a simple matter of finding a bulb of the same size and wattage and screwing it into place. However, the assumption that one could replace an old light bulb with an identical one has been amended in recent years. Now there are many different choices of technologies, each of which has different properties regarding life span, environmental impact, energy usage, appearance, and starting time.

For example, incandescent light bulbs are the traditional mainstay. They are cheap and give a strong light but are energy-wasteful and don't last very long. CFL (compact fluorescent) bulbs are longer lasting and more energy efficient, but contain toxic mercury and are slow to reach maximum brightness. LED bulbs are very long lasting (in terms of years of use) and can be designed to shine any desired colour, but require DC power and do not work well with dimmers.

Along with the new technologies has come a change in the very principle of replacing a light bulb. In the past, a light bulb would either function or not. One would only consider replacing it if it burned out. But it is now considered reasonable (and often admirable) to replace a perfectly functioning light bulb with another one that uses a different technology.

**Questions**

1. A typical incandescent light bulb will last about 1000 hours. Would it likely have been necessary for Sears to replace working incandescent bulbs with LEDs, or could they have simply replaced them as they burned out and still have met their August deadline? Under what circumstances would it make economic sense to replace the incandescent bulbs as soon as possible?
2. Clearly Sears Canada gains brand value by visibly being environmentally conscious. Do you think the real driver here is gaining favourable publicity? To guide you in your answer, a standard incandescent bulb produces about 15 lumens per watt, while a comparable LED produces 80 to 140 lumens per watt, and the current price of electricity is about \$0.65 per kWh.

3. Sears will be able to buttress its environmental credentials and perhaps save money, but it seems peculiar that it would constrain its customers' choices, too. If customers really wanted to buy an incandescent bulb, they could possibly switch to a different retailer to get it and Sears could lose business. Which of the following possible reasons for this make sense to you and why?
- (a) Incandescent bulbs are in short supply because they are no longer being manufactured.
  - (b) Profit margins are higher on the more expensive CFL and LED bulbs.
  - (c) Customers don't buy incandescent bulbs anyhow.
  - (d) Sears is afraid of being sued for contributing to climate change.
  - (e) Sears expects incandescent bulbs to be made illegal anyhow, so it might as well be at the front of the trend.
  - (f) Shelf space is expensive, and with so many competing technologies something has to be left out.
  - (g) Incandescent bulbs are simply not fashionable, and Sears is all about fashion.



# Taxes

**Engineering Economics in Action, Part 8A:** It's in the Details

- 8.1** Introduction
- 8.2** Personal Income Taxes and Corporate Income Taxes Compared
- 8.3** Corporate Tax Rates
- 8.4** Before- and After-Tax MARR
- 8.5** The Effect of Taxation on Cash Flows
  - 8.5.1** The Effect of Taxes on First Cost
  - 8.5.2** The Effect of Taxes on Savings
  - 8.5.3** The Effect of Taxes on Salvage or Scrap Value
- 8.6** Present Worth and Annual Worth Tax Calculations
- 8.7** IRR Tax Calculations
  - 8.7.1** Accurate IRR Tax Calculations
  - 8.7.2** Approximate After-Tax Rate-of-Return Calculations
- 8.8** Specific Tax Rules in Canada
  - 8.8.1** The Capital Cost Allowance System
  - 8.8.2** Undepreciated Capital Cost and the Half-Year Rule
  - 8.8.3** The Capital Tax Factor and Capital Salvage Factor
  - 8.8.4** Components of a Complete Tax Calculation

**Review Problems**

**Summary**

**Engineering Economics in Action, Part 8B:** The Work Report

**Problems**

**Mini-Case 8.1: Flat Taxes**

**Appendix 8A: Deriving the Capital Tax Factor**

## ENGINEERING ECONOMICS IN ACTION, PART 8A

### It's in the Details

"Details, Terry. Sometimes it's all in the details." Naomi pursed her lips and nodded sagely. Terry and Naomi were sitting in the coffee room together. The main break periods for the line workers had passed, so they were alone except for a maintenance person on the other side of the room who was enjoying either a late breakfast or an early lunch.

"Uh, okay, Naomi. What is?"

"Well," Naomi replied, "you know that rocker arm die deal? The one where we're upgrading to a 10-stage die? The rough replacement study you did seems to have worked out okay. We're going to do something, sometime, but now we have to be a little more precise."

"What do we do?" Terry was interested now. He liked things precise and detailed.

"The main thing is to make sure we are working with the best numbers we can get. I'm getting good cost figures from Tan Wang at Hamilton Tools for this die and future possibilities, and we'll also work out our own costs for the changeover. Your cost calculations are going to have to be more accurate, too."

"You mean to more significant digits?" Naomi couldn't tell whether Terry was making a joke or if the idea of more significant digits really did thrill him. She decided it was the former.

"Ha, ha. No, I mean that we had better explicitly look at the tax implications of the purchase. Our rough calculations ignored taxes, and taxes can have a significant effect on the choice of best alternative. And when lots of money is at stake, the details matter."

## 8.1

### Introduction

Taxes are fees paid by individuals or businesses to support their government. Taxes are generally the primary source of government revenue, and are used to provide public goods and services such as highways and dams, police services, the military, water treatment, education, health care, and other social programs.

Taxes can have a significant impact on the economic viability of a project because they change the actual cash flows experienced by a company. A vital component of a thorough economic analysis will therefore include the tax implications of an investment decision. This chapter provides an introduction to corporate taxes and shows how they can affect engineering economics decisions.

When a firm makes an investment, the income from the project will affect the company's cash flows. If the investment yields a profit, the profits will be taxed. Since the taxes result as a direct consequence of the investment, they reduce the net profits associated with that investment. In this sense, taxes associated with a project are a disbursement. If the investment yields a loss, the company may be able to offset the loss from this project against the profits from another and end up paying less tax overall. As a result, when evaluating a loss-generating project, the net savings in tax can be viewed as a negative disbursement. Income taxes thus reduce the benefits of a successful project, while at the same time reducing the costs of an unsuccessful project.

Calculating the effects of income taxes on the viability of a project can be complicated. Prior to exploring the details of such calculations, it is helpful to look at the context of corporate income taxes. In the next two sections we will compare corporate income taxes to the more familiar personal income taxes, and also describe how corporate tax rates are determined.

## 8.2

# Personal Income Taxes and Corporate Income Taxes Compared

There are substantial differences between personal income taxes and corporate income taxes. Most adults are familiar with the routine of filing income tax returns. For most people, the procedure is rather simple: An employer provides a statement reporting the person's income, income tax already paid, and other amounts for the previous year. These amounts are assembled on the tax form and are used to calculate the total tax owed or the amount of a refund. The tax return is then submitted to the appropriate government tax agency for processing.

Corporate taxes are similar in some ways, but there are several substantial differences. One main difference has to do with the tax rate. Personal income taxes usually exhibit a **progressive tax rate**, meaning that people who earn more are charged a larger percentage of their income. For example, if a person's taxable income is very low, he or she might not pay any income taxes. At a moderate income, an individual might be expected to pay, say, 25 percent of his or her income as income tax. A very-high-income individual in some countries might pay 50 percent or more of his or her income in income tax. Although the exact rate of taxation changes from year to year, and varies from country to country, individual income taxes in developed countries are almost always progressive.

In contrast, corporate taxes are typically levied according to a proportional or "flat" tax. This means that corporations pay the same tax rate, regardless of their income level. Again, rates can change over time and between countries, but the rate will be the same whether a company makes a small profit or a very large profit. The main exception is that most countries give smaller companies favourable tax treatment to encourage new businesses. However, once the business is of a certain size, the tax rate is constant.

Another difference between personal income tax and corporate income tax has to do with how the tax is calculated. The income used to determine the taxes an individual pays is reduced by deductions or **tax credits**, which are real or nominal costs that are not taxed or are taxed at a reduced rate. For example, excess medical expenses, tuition fees, and pension plan contributions are generally all considered eligible deductions, along with a substantial "basic personal amount." Thus an individual's "taxable income" can be quite a bit less than his or her actual income.

A corporation's taxes are calculated in quite a different manner. Net income for tax purposes is calculated by subtracting **expenses**, which are either real costs associated with performing the corporation's business or a portion of the capital expense for an asset, from gross income. Consequently, a company that makes no profit (income less expenses) may pay no income taxes.

Finally, personal and corporate taxes differ in complexity. In particular, a company's taxes will usually be complicated by issues concerning **capital expenses**, which are purchases of assets of significant value. Such assets have a strong effect on the income taxes paid by a company, but how they are treated for tax purposes is complex. This is usually not an issue of concern to individuals.

## 8.3

# Corporate Tax Rates

In this chapter, we are concerned only with corporate taxes, and in particular the impact of corporate income taxes on the viability of an engineering project. Table 8.1 compares corporate taxes among several countries, but the actual tax rate applied in any circumstance can be fairly complicated and can depend on the size of the firm, whether it is a

**Table 8.1 Corporate Tax Rates Around the World**

|           |        |                |        |
|-----------|--------|----------------|--------|
| Argentina | 35%    | Germany        | 25%    |
| Australia | 30%    | India          | 30–40% |
| Austria   | 25%    | Ireland        | 12.5%  |
| Brazil    | 34%    | Israel         | 31%    |
| Canada    | 36.1%  | South Africa   | 29%    |
| China     | 25%    | United Kingdom | 28%    |
| France    | 33.33% | United States  | 15–35% |

manufacturer, its location, and a variety of other factors. Comparing the tax rates in different countries is difficult. For example, some countries provide a full range of health-care services from their tax revenues, while others do not. Certain health-care costs may be, instead, a significant expense for a company. In some countries the total corporate tax is a combination of federal tax and state or provincial tax. Tax rates change over time, and there are sometimes short-term tax reductions or incentives (see Close-Up 8.1) for companies in specific industries. Rather than focusing on the implications of such differences, our concern here is with the basic approach used in determining the impact of taxes on a project. For special tax rules, it is best to check with the appropriate tax agency or a tax specialist.

It is also worth noting that tax rules can change suddenly. For example, in the past 30 years there have been fundamental changes to the corporate tax rules in Canada and other countries. These changes can and have had a significant impact on investments made by companies. Less-significant new rules are introduced every year due to changes in technology, current economic conditions, or political factors. In particular, there are usually opportunities for small technological companies to take advantage of beneficial tax rules, as detailed in Close-Up 8.2.

### CLOSE-UP 8.1

#### Incentives

Governments sometimes try to influence corporate behaviour through the use of *incentives*. These incentives include grants to certain types of projects; for example, projects undertaken in particular geographic areas, or projects providing employment to certain categories of people.

Other incentives take the form of tax relief. For example, several countries allow pollution equipment to be fully expensed in the year of purchase. The ability to depreciate pollution equipment quickly makes it a more desirable investment for a company and a beneficial investment for society as a whole.

The exact form of incentives changes from year to year as governments change and as the political interests of society change. In most companies there is an individual or department that keeps track of possible programs affecting company projects.

Incentives must be considered when assessing the viability of a project. Grant incentives provide additional cash flow to the project that can be taken into account like any other cash flow element. Tax incentives may be more difficult to assess since sometimes, for example, they use other forms of depreciation or may result in different tax rates for different parts of the project.

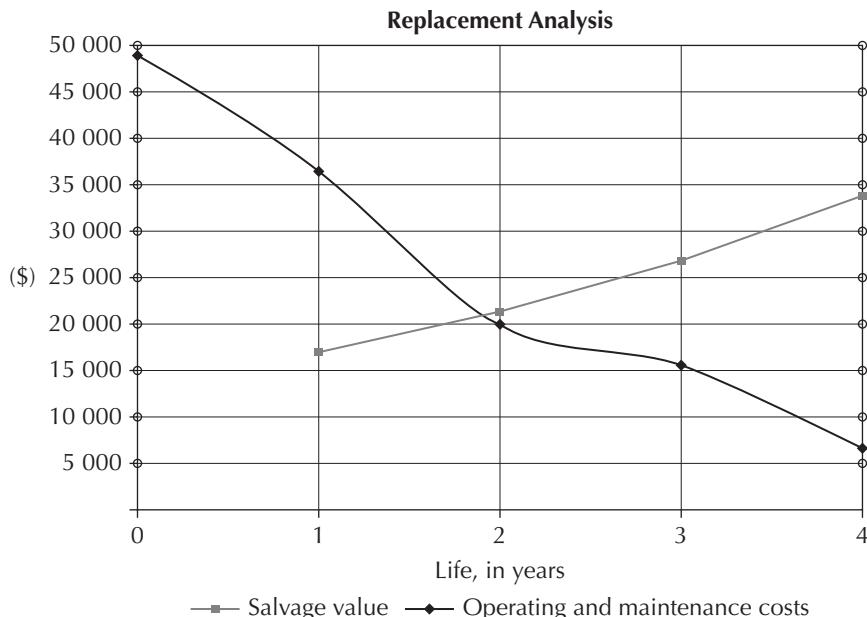
**CLOSE-UP 8.2****Small Company Tax Rules**

Special tax advantages are given to smaller companies, such as a technically oriented start-up. This may be of particular interest to engineers interested in starting their own firm rather than working for a larger company. The purpose of these incentives is to give a strong motivation to incorporate a new company and provide more after-tax income for reinvestment and expansion.

In Canada, the **small business deduction** applies to small Canadian-controlled private corporations and reduces the effective tax rate for a small Canadian company to less than 20 percent.

**S P R E A D S H E E T   S A V V Y**

In Chapters 8 through 11, the Spreadsheet Savvy feature will focus on the use of Excel charts and formatting features useful in the preparation of engineering reports. Spreadsheet software is an excellent tool for preparing charts to summarize the results of an economic analysis. Creating charts with Excel is straightforward. Probably the most commonly used is the X-Y (scatter) chart. It plots a series of Y values as a function of corresponding X values. The scatter chart below was created from the Chapter 7 Spreadsheet Savvy table.



To begin, highlight cells B3:D8 containing the titles and data for Life in Years, Salvage Value, and Operating and Maintenance Costs. Select Insert from the ribbon, and then X-Y (scatter) as the desired chart type. Of the options, the chart above shows “straight lines and markers.” By default, the chart will be inserted as an object into the current worksheet. By clicking anywhere on the chart, the ribbon will indicate three chart editing options: Design, Layout, and Format. Under Design, the layout and style of an existing chart can be changed, as well as its location. Under the Layout tab on the ribbon, the location of the legend was changed to below the chart, and the font was changed to enlarge it from its default value. Also under Layout, the gridlines were adjusted to

show both horizontal and vertical gridlines for the major axis markers. Finally, a chart title was added, as well as both horizontal and vertical axis labels. These options all appear under the Layout tab on the ribbon.

The formatting of the X and Y axes was also changed. When the chart was first made, the maximum Y value displayed was 60 000, and for the X axis it was 5. By hovering over either of the axes or the axes labels and right-clicking, several editing options become available. With the font option for the Y axis, the decimal points were removed and the font size enlarged (to 12pt Arial) and italicized. Under format axis, the maximum value displayed was set to 50 000. Similar changes were made to the X axis, with the maximum value set to 4 and the font size changed to 12pt Arial, italicized. With some practice, most analysts find charts to be a fast and effective tool for summarizing and communicating results from an economic analysis.

## 8.4 Before- and After-Tax MARR

Taxes have a significant effect on engineering decision making, so much so that they cannot be ignored. In this text, so far, it seems as though no specific tax calculations have been done. In fact, they have been implicitly incorporated into the computations through the use of a before-tax MARR, though we have not called it such.

The basic logic is as follows. Since taxes have the effect of reducing profits associated with a project, we need to make sure that we set an appropriate MARR for project acceptability. If we do not explicitly account for the impact of taxes in the project cash flows, then we need to set a MARR high enough to recognize that taxes will need to be paid. This is the *before-tax* MARR. If, on the other hand, the impact of taxes is explicitly accounted for in the cash flows of a project (i.e., reduce the cash flows by the tax rate), then the MARR used for the project should be lower, since the cash flows already take into account the payment of taxes. This is the *after-tax* MARR.

In fact, we can express an approximate relationship:

$$\text{MARR}_{\text{after-tax}} \approx \text{MARR}_{\text{before-tax}} \times (1 - t) \quad (8.1)$$

where  $t$  is the corporate tax rate. The *before-tax* MARR means that the MARR has been chosen high enough to provide an acceptable rate of return without explicitly considering taxes. In other words, since all profits are taxed at the rate  $t$ , the *before-tax* MARR has to include enough returns to meet the *after-tax* MARR and, in addition, provide the amount to be paid in taxes. As we can see from the above equation, the after-tax MARR will generally be lower than the before-tax MARR. We will see later in this chapter how the relationship given in this equation is a simplification but a reasonable approximation of the effect of taxes. In practice, the before- and after-tax MARRs are often chosen independently and are not directly related by this equation. Generally speaking, if a MARR is given without specifying whether it is on a before- or after-tax basis, it can be assumed to be a before-tax MARR.

### EXAMPLE

#### 8.1

Saskatchewan Gold Mines (SGM) has been selecting projects for investment on the basis of a before-tax MARR of 12 percent. Sherri feels that some good projects have been missed because the effects of taxation on the projects have not been examined in enough detail, so she proposed reviewing the projects on an after-tax basis. What would be a good choice of after-tax MARR for her review? SGM pays 45 percent corporate taxes.

Although the issue of selecting an after-tax MARR is likely to be more complicated, a reasonable choice for Sherri would be to use Equation (8.1) as a way of calculating an after-tax MARR for her review. This gives

$$\text{MARR}_{\text{after-tax}} = 0.12 \times (1 - 0.45) = 0.066 = 6.6\%$$

A reasonable choice for after-tax MARR would be 6.6 percent. ■

It is important to know when to use the after-tax MARR and when to use a before-tax MARR. In general, if you are doing an approximate calculation without taking taxes into account explicitly, then you should use a before-tax MARR. Thus, all the examples in this book up to this point were appropriately done using a before-tax MARR, boosted to account for the fact that profits would be taxed. However, when considering taxes explicitly in calculations, the after-tax MARR should be used. There is no need to increase the decision threshold when the profits that are lost to taxes are already taken into account.

## 8.5 | The Effect of Taxation on Cash Flows

There is a set of cash flows that arises whenever an investment is made. Consider the following example.

### EXAMPLE

#### 8.2(a)

Ebcon Corp. is considering purchasing a small device used to test printed circuit boards that has a first cost of \$45 000. The tester is expected to reduce labour costs and improve the defect detection rate so as to bring about savings of \$23 000 per year. Additional operating costs are expected to be \$7300 per year. The salvage value of the tester will be \$5000 in five years. The corporate tax rate is  $t = 42$  percent and the after-tax MARR is 12 percent. ■

There are three key cash flow elements in this example.

- First cost: the \$45 000 investment. This is a negative cash flow made at time zero.
- Net annual savings: the \$23 000 per year savings less the \$7300 additional operating costs for a net \$15 700 positive cash flow at the end of each of years one to five.
- Salvage value: the \$5000 residual value of the tester at the end of its service life in five years.

Each of these cash flow elements is affected by taxes, and in different ways.

In this section we will explore the influence of corporate taxes on the cash flows for this investment under simplified tax rules that are not very common in Canada. The actual rules of the Canadian tax system are quite complicated—probably the most complicated in the world—and are better dealt with after the basic principles have been established.

### 8.5.1 The Effect of Taxes on First Cost

Companies are taxed on net profits, which are revenues less expenses. Consequently, when an expense is incurred, less tax is paid. From this perspective, although the tester in

Example 8.2(A) had a first cost of \$45 000, if this first cost could be recognized as an immediate expense, there would be a tax savings of  $\$45\,000 \times 0.42 = \$18\,900$ . However, the tax benefits are usually claimed at the end of a tax year. Assuming that the tester was purchased at the beginning of the year, and the tax savings accrue at the end of that same year, then the present worth of the first cost of the tester would only be  $-\$45\,000 + \$18\,900(P/F, 12\%, 1) = -\$45\,000 + \$18\,900(0.89286) \approx -\$28\,000$ .

However, by tax rules, capital purchases such as production equipment cannot usually be fully claimed as an expense in the year in which the purchase occurred. The logic is based on the idea of depreciation. As we saw in Chapter 6, assets retain value over time. Their loss in value, as opposed to their cost, is what tax rules generally recognize as an eligible expense.

Given a choice, a firm would want to write off (i.e., depreciate) an investment as quickly as possible. Since depreciation is considered an expense and offsets revenue, the earlier the expense is recognized, the earlier the tax savings will accrue. Since money earlier in time is worth more than the same amount of money later in time, there is good reason to depreciate capital assets as quickly as allowable. This effect of reducing taxes can be considerable.

Although companies would, of course, like to reduce taxes by writing off their assets as quickly as possible, there are also undesirable effects of doing this. For one thing, it may lead to accounting documents that are not representative of a firm's actual financial situation. For example, if valuable capital assets are assigned—for tax reasons—to have an excessively low book value, this may underrepresent the true value of the company as a whole. It may also create an opportunity for inefficient business decisions. From the perspective of the government imposing the taxes, it can lead to companies improperly exploiting the tax rules.

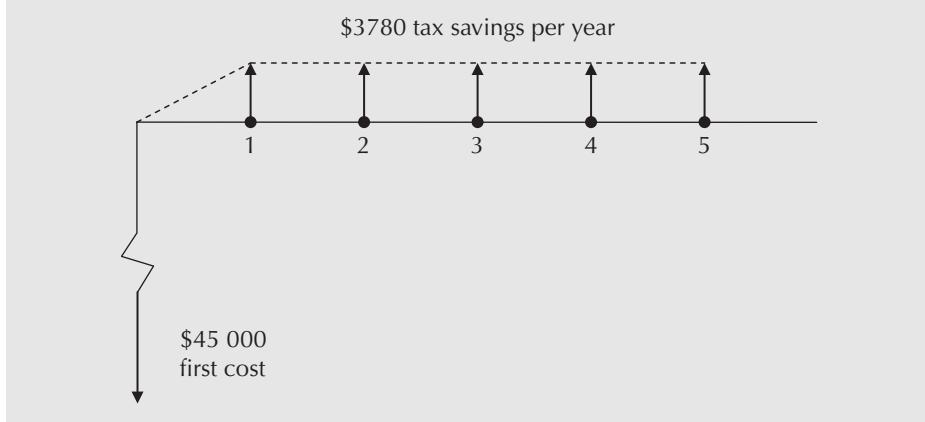
To counter these undesirable effects, tax authorities have carefully defined the depreciation methods they permit for use in computing taxes. The permitted methods for claiming depreciation expenses may not reflect the true depreciation of an asset, as they are rule-oriented rather than truly attempting to recognize the diminishment in value of an asset. Consequently, the depreciation charge is referred to by a different name—the *capital cost allowance*—to recognize that the book value implied by the taxation-related depreciation charges does not necessarily represent a good estimate of the asset's market value. However, there is usually a fair correspondence between capital cost allowance and depreciation, and many firms do not distinguish between them in calculations for their financial statements. It is therefore reasonable to think of them as being the same thing for most purposes.

**EXAMPLE****8.2(B)**

Tax rules for Ebcon Corp. (see Example 8.2(A)) specify that the capital allowance for the tester be calculated as straight-line depreciation over the life of the tester. What is the present worth of the tester's first cost?

This seems like an odd question, but it results from the fact that in a taxed company, the first cost immediately gives rise to future tax savings. This example is consistent with one of the depreciation methodologies occasionally permitted as an investment incentive in Canada.

As illustrated in Figure 8.1, the first cost of \$45 000 gives rise to benefits in the form of an annuity of  $45\,000 \times 0.42/5 = \$3780$ . In a sense, the presence of taxes transforms the \$45 000 purchase into a more complex set of cash flows consisting of not only the first cost but also a substantial annuity.

**Figure 8.1 The Beneficial Effects of a First Cost (for Example 8.2)**

The present worth of the first cost is then:

$$PW_{\text{first cost}} = -45\,000 + 3780(P/A, 12\%, 5) = -45\,000 + 3780(3.6048) \approx -31\,400$$

The present worth of the tester's first cost is about -\$31 400. ■

Note that this result is more costly for Ebcon by about \$31 400 – \$28 000 = \$3400 than if Ebcon could expense the entire tester in the year of purchase. This difference is the present worth of the increased taxes that the government receives by not allowing Ebcon to count the tester as an expense in the year that it was purchased.

### 8.5.2 The Effect of Taxes on Savings

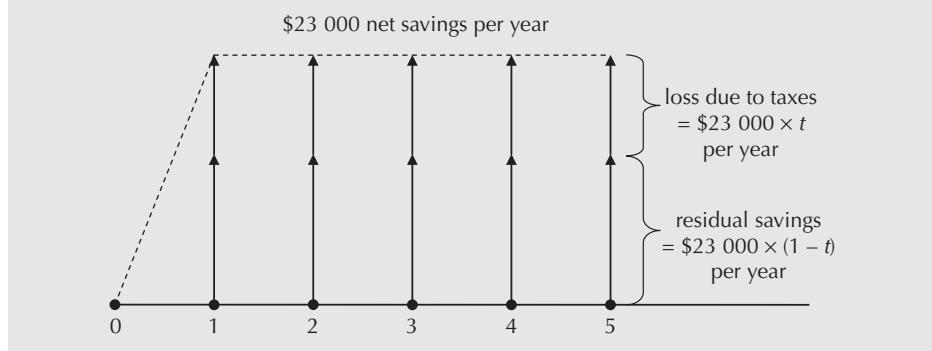
Although in a taxed business environment the first cost is reduced, this is balanced by the fact that the savings from the investment are also reduced. An underlying assumption is that any investment decision is made in a profitable company for which this decision is a relatively small part of the overall business. Consequently, if any money is saved, those savings increase the profits of the firm. And since profits are taxed at the tax rate, the net savings are reduced proportionally.

**EXAMPLE**
**8.2(C)**

What is the present worth of the annual net savings created by the tester?

The approach used is to reduce the net annual savings of the tester by the tax rate, as illustrated in Figure 8.2, and then bring the amount to a present worth:

$$\begin{aligned} PW_{\text{savings}} &= (23\,000 - 7300) \times (1 - 0.42) \times (P/A, 12\%, 5) \\ &= 15\,700 \times 0.58 \times 3.6048 \\ &\approx 32\,800 \end{aligned}$$

**Figure 8.2 Loss of Net Savings Due to Taxes (for Example 8.2)**

The present worth of net savings created by the tester is about \$32 800. ■

### 8.5.3 The Effect of Taxes on Salvage or Scrap Value

When an asset is salvaged or scrapped, unless the value received is zero, money comes into the firm as income (we do not deal with the complications of negative salvage values in this text). Any money received that is in excess of the asset's remaining book value is new revenue and is taxable. Under the depreciation scheme described for Example 8.2, at the end of the tester's service life, its book value for tax purposes is zero. In this case, all money obtained by salvaging or selling it counts as revenue and is taxed at the tax rate. This is illustrated in Figure 8.3.

**Figure 8.3 Loss of Salvage Revenue Due to Taxes (for Example 8.2)**

In other circumstances, at the time an asset is sold, it may have remaining book value for taxation purposes. In this case, different things happen depending on the jurisdiction. The specific treatment in Canada will be discussed later.

However, for Example 8.2, the situation is straightforward.

**EXAMPLE**
**8.2(D)**

What is the present worth of the salvage value of the tester?

This amount is fully taxable since the book value of the tester will be zero. The approach used is to reduce the amount by the tax rate and then bring the result to a present worth:

$$\begin{aligned} PW_{\text{salvage}} &= 5000 \times (1 - 0.42) \times (P/F, 12\%, 5) \\ &= 5000 \times 0.58 \times 0.56743 \\ &= 1646 \end{aligned}$$

The present worth of the salvage value of the tester is \$1646. ■

## 8.6 Present Worth and Annual Worth Tax Calculations

As seen in the previous three sections, a complete tax calculation consists of recognizing how the existence of taxes affects each of the components of an investment. For a present worth comparison, each of the component cash flows is converted to a present worth and summed. In particular:

First cost: Add the present worth of tax savings due to depreciation expenses to the first cost.

Savings or expenses: Multiply by  $(1 - t)$  and convert to present worth.

Salvage or Scrap value: For this simple example, multiply by  $(1 - t)$  and convert to present worth. In general, things are more complicated, as we will see once we look at the details of the Canadian tax system.

### EXAMPLE

#### 8.2(E)

What is the present worth of the tester investment decision?

$$\begin{aligned} \text{PW}_{\text{first cost}} &= -45\,000 + 3780(P/A, 12\%, 5) \\ &= -45\,000 + 3780(3.6048) \\ &= -31\,374 \end{aligned}$$

$$\begin{aligned} \text{PW}_{\text{savings}} &= (23\,000 - 7\,300) \times (1 - 0.42) \times (P/A, 12\%, 5) \\ &= 15\,700 \times 0.58 \times 3.6048 \\ &= 32\,825 \end{aligned}$$

$$\begin{aligned} \text{PW}_{\text{salvage}} &= 5\,000 \times (1 - 0.42) \times (P/F, 12\%, 5) \\ &= 5\,000 \times 0.58 \times 0.56743 \\ &= 1\,646 \end{aligned}$$

$$\begin{aligned} \text{PW}_{\text{total}} &= -31\,374 + 32\,825 + 1\,646 \\ &= 3097 \end{aligned}$$

The present worth of the tester is \$3097. This is an acceptable investment at the given after-tax MARR. ■

Very similar calculations can be done if an annual worth comparison is desired, except that each of the component cash flows is converted to an annual worth and summed. In particular:

First cost: Convert the first cost to an annuity and add it to the annual tax savings due to depreciation expenses.

Savings or expenses: Multiply by  $(1 - t)$  and convert to annual worth.

Salvage or Scrap value: For this simple example, multiply by  $(1 - t)$  and convert to annual worth. In general, things are more complicated, as we will see once we look at the details of the Canadian tax system.

**EXAMPLE****8.2(F)**

What is the annual worth of the tester?

$$\begin{aligned} AW_{\text{first cost}} &= -45\,000(A/P, 12\%, 5) + 3780 \\ &= -45\,000 \times (0.27741) + 3780 \\ &= -8703 \end{aligned}$$

$$\begin{aligned} AW_{\text{savings}} &= (23\,000 - 7300) \times (1 - 0.42) \\ &= 15\,700 \times 0.58 \\ &= 9106 \end{aligned}$$

$$\begin{aligned} AW_{\text{salvage}} &= 5000 \times (1 - 0.42) \times (A/F, 12\%, 5) \\ &= 5000 \times 0.58 \times 0.15741 \\ &= 456 \end{aligned}$$

$$\begin{aligned} AW_{\text{total}} &= -8703 + 9106 + 456 \\ &= 859 \end{aligned}$$

The annual worth of the tester is \$859 per year. This is an acceptable investment at the given after-tax MARR. ■

**8.7****IRR Tax Calculations****8.7.1 Accurate IRR Tax Calculations**

As has been demonstrated, the effect of taxation is to modify the pre-tax cash flows. The after-tax IRR is simply the IRR calculated on the after-tax cash flows.

**EXAMPLE****8.2(G)**

What is the accurate after-tax IRR of the tester investment decision?

We answer this question by setting the sum of present worth of receipts and the present worth of disbursements to zero and solving for the unknown interest rate,  $i^*$ :

$$\begin{aligned} 45\,000 &= (45\,000/5) \times 0.42 \times (P/A, i^*, 5) + 15\,700 \\ &\quad \times (1 - 0.42) \times (P/A, i^*, 5) + 5000(1 - 0.42)(P/F, i^*, 5) \\ 45\,000 &= (3780 + 15\,700 \times 0.58) \\ &\quad \times (P/A, i^*, 5) + 5000 \times 0.58 \times (P/F, i^*, 5) \end{aligned}$$

Solving through trial and error results in:  $i^* = \text{IRR}_{\text{after-tax}} = 14.7\%$

This exceeds the acceptance threshold of after-tax MARR of 12 percent and therefore is an acceptable investment.

Alternatively, set the sum of the annual worth of receipts and the annual worth of the disbursements to zero and solve for  $i^*$ :

$$\begin{aligned} 45\,000(A/P, i^*, 5) &= 45\,000/5 \times 0.42 + 15\,700 \\ &\quad \times (1 - 0.42) + 5000(1 - 0.42) \times (A/F, i^*, 5) \end{aligned}$$

$$45\ 000 = [3780 + 15\ 700 \times 0.58 + 5000 \times 0.58 \times (A/F, i^*, 5)] / (A/P, i^*, 5)$$

$$= [12\ 886 + 5000 \times 0.58 \times (A/F, i^*, 5)] / (A/P, i^*, 5)$$

Solving through trial and error results in:  $i^* = \text{IRR}_{\text{after-tax}} = 14.7\%$

This exceeds the acceptance threshold of after-tax MARR of 12 percent and therefore is an acceptable investment. ■

### 8.7.2 Approximate After-Tax Rate-of-Return Calculations

The IRR is probably one of the most popular means of assessing the desirability of an investment. Unfortunately, a detailed analysis can be somewhat involved. However, an approximate IRR analysis when taxes are explicitly considered can be very easy. The formula to use is:

$$\text{IRR}_{\text{after-tax}} \cong \text{IRR}_{\text{before-tax}} \times (1 - t) \quad (8.2)$$

The reasons for this are exactly the same as described in Section 8.4 for the before-and after-tax MARR. It is an approximation that works because the IRR represents the percentage of the total investment that is net income. Since the tax rate is applied to net income, it correspondingly reduces the IRR by the same proportion. It is not exactly correct because it assumes that expenses offset receipts in the year that they occur. Consequently, if the after-tax IRR is close to the after-tax MARR, a more precise calculation is advisable so that the correct decision is made.

#### EXAMPLE

#### 8.2(H)

What is the approximate after-tax IRR of the tester investment?

We first find the before-tax IRR by setting the present worth of the disbursements equal to the receipts and solving for the interest rate:

$$45\ 000 = 15\ 700(P/A, i, 5) + 5000(P/F, i, 5)$$

Through trial and error, we find that the before-tax IRR is 23.8 percent. The after-tax IRR is then calculated as:

$$\begin{aligned} \text{IRR}_{\text{after-tax}} &= \text{IRR}_{\text{before-tax}} \times (1 - t) \\ &= 0.238(1 - 0.42) \\ &= 0.13804 \end{aligned}$$

The approximate IRR of 13.8 percent is somewhat lower than the accurate IRR of 14.7 percent. If the after-tax MARR were 14 percent rather than 12 percent, it would have given rise to an incorrect investment decision. ■

When doing an after-tax IRR computation in practice, the approximate after-tax IRR can be used as a first pass on the IRR computation. If the approximate after-tax IRR turns out to be close to the after-tax MARR, a precise after-tax IRR computation may be required to make a fully informed decision about the project.

#### EXAMPLE

#### 8.3

Essen Industries pays 40 percent corporate income taxes. Its after-tax MARR is 18 percent. A project has a before-tax IRR of 24 percent. Should the project be approved? What would your decision be if the after-tax MARR were 14 percent?

$$\begin{aligned}\text{IRR}_{\text{after-tax}} &= \text{IRR}_{\text{before-tax}} \times (1 - t) \\ &= 0.24(1 - 0.40) \\ &= 0.144\end{aligned}$$

The after-tax IRR is approximately 14.4 percent. For an after-tax MARR of 18 percent, the project should not be approved. However, for an after-tax MARR of 14 percent, since the after-tax IRR is an approximation, a more detailed examination would be advisable. ■

In summary, we can simplify after-tax IRR computations by using an easy approximation. The approximate after-tax IRR may be adequate for decision making in many cases, but in others a detailed after-tax analysis may be necessary.

## 8.8 Specific Tax Rules in Canada

The tax calculations illustrated in Examples 8.2(A) through 8.2(G) were based on a tester that was subject to straight-line depreciation over its life. It would be convenient if the Canadian tax system permitted straight-line depreciation for all investments, as is the case in some countries, but our system is more complex. In Canada, most assets must be depreciated using the declining-balance method where the depreciation rate depends on the type of asset. The regulations are set out in the capital cost allowance (CCA) system.

### 8.8.1 The Capital Cost Allowance System

When a firm buys a depreciable asset for use in its business, a capital expense is incurred. This is the expenditure associated with the purchase of a long-term depreciable asset. (Almost all tangible assets are depreciable; the primary exception to this is land.) Since the capital asset loses value over time, the firm may deduct the capital expense over a period of years by claiming a depreciation expense each year of the asset's useful life. The depreciation is recorded in two ways. First, it is recorded in the firm's balance sheet as a reduction in the book value of the asset. Second, it is recorded as a depreciation expense on the income statement. In this way, depreciation reduces the before-tax income even though there has been no out-of-pocket cash expense.

### NET VALUE 8.1

#### **Canada Revenue Agency Website**

The Canada Revenue Agency website ([www.cra-arc.gc.ca](http://www.cra-arc.gc.ca)) is a very rich and complete source of information about tax rules. However, because there are so many different types of users and special cases, it is not always easy to find the information you are looking for.

There is a built-in search engine that can be very helpful but may provide more hits than are useful, and the order of the responses may not be convenient. For example, a recent key phrase of

"CCA rates" gave CCA rates of interest to farmers and fishers, along with several other more specific rates, before presenting a link to a description of CCA rates generally applicable in business.

Another approach may be to seek out publications directed at your area of interest. For example, the publication TA4002(E) Business and Professional Income provides very good information about taxes for businesses that would be of interest to anyone who would like to start their own business.

In general, a firm will want to “write off” (i.e., depreciate) a capital investment as quickly as possible. This is because depreciation is considered an expense that offsets revenue and thus reduces net income. Since net income is taxed, taxes can be deferred or reduced by depreciating assets quickly. The effect of deferring taxes can be considerable. To counter this effect, the Canadian tax system defines a specific amount of depreciation that companies may claim in any year for any one depreciable asset. This amount is called the **capital cost allowance, CCA**. In this section, we demonstrate how to apply CCA rules to investment decisions and compare the CCA to depreciation claimed for accounting records.

**EXAMPLE****8.4**

In the imaginary country of Monovia, companies can depreciate their capital asset purchases as fast as they wish. Clive Cutler, owner of Monovia Manufacturing, has just bought equipment worth \$200 000. Two spreadsheets (Tables 8.2 and 8.3) illustrate the effect of different depreciation strategies over a five-year period assuming the following:

1. Income is \$300 000 per year.
2. Expenses excluding depreciation are \$100 000 per year.
3. The tax rate is 50 percent.
4. Available cash is invested at 10 percent interest.
5. The salvage value of the equipment after five years is zero.

Table 8.2 illustrates the case in which the equipment is fully depreciated in the first year, although it generates revenue over its five-year life. In Table 8.3, straight-line depreciation is used over the five-year life.

When we look at the effects of depreciation on economic analyses, it is important to distinguish between expenses that represent a cash outflow and expenses that do not. Purchasing an asset such as a piece of equipment will produce a cash outflow at the time the purchase is made. In particular, the balance sheet will reflect a transfer out of current assets (cash) and a transfer into fixed assets (equipment) and perhaps to current liabilities (bank loan).

Depreciation, on the other hand, does not actually represent a cash outflow, although it is recorded as an expense in the income statement. For example, in Table 8.2, writing off the entire cost of the equipment in its first year produced a depreciation expense of \$200 000 in that year. There was no actual cash outflow due to the depreciation (although there was for the actual purchase of the asset), but depreciation caused the net income to be reduced to zero for that year, even though \$200 000 in cash was actually available.

**Table 8.2 Full Depreciation in One Year**

| Year                            | 1         | 2         | 3         | 4         | 5         |
|---------------------------------|-----------|-----------|-----------|-----------|-----------|
| Income                          | \$300 000 | \$300 000 | \$300 000 | \$300 000 | \$300 000 |
| Expenses excluding depreciation | 100 000   | 100 000   | 100 000   | 100 000   | 100 000   |
| Depreciation expense            | 200 000   | 0         | 0         | 0         | 0         |
| Net income                      | 0         | 200 000   | 200 000   | 200 000   | 200 000   |
| Taxes                           | 0         | 100 000   | 100 000   | 100 000   | 100 000   |
| Profit                          | 0         | 100 000   | 100 000   | 100 000   | 100 000   |
| Cash                            | 200 000   | 100 000   | 100 000   | 100 000   | 100 000   |
| Accumulated cash                | 200 000   | 320 000   | 452 000   | 597 200   | 756 920   |

**Table 8.3 Straight-Line Depreciation Over Five Years**

| Year                            | 1         | 2         | 3         | 4         | 5         |
|---------------------------------|-----------|-----------|-----------|-----------|-----------|
| Income                          | \$300 000 | \$300 000 | \$300 000 | \$300 000 | \$300 000 |
| Expenses excluding depreciation | 100 000   | 100 000   | 100 000   | 100 000   | 100 000   |
| Depreciation expense            | 40 000    | 40 000    | 40 000    | 40 000    | 40 000    |
| Net income                      | 160 000   | 160 000   | 160 000   | 160 000   | 160 000   |
| Taxes                           | 80 000    | 80 000    | 80 000    | 80 000    | 80 000    |
| Profit                          | 80 000    | 80 000    | 80 000    | 80 000    | 80 000    |
| Cash                            | 120 000   | 120 000   | 120 000   | 120 000   | 120 000   |
| Accumulated cash                | 120 000   | 252 000   | 397 200   | 556 920   | 732 612   |

Investing the \$200 000 for the second year at 10 percent interest produces an accumulated cash amount of \$220 000. Adding this to the profit of \$100 000 for the second year gives accumulated cash of \$320 000 at the end of the second year. Continuing in this fashion produces accumulated cash of \$756 920 at the end of the five-year period.

In contrast, if the equipment is depreciated on a straight-line basis over five years, only \$732 612 in cash is accumulated. This can be seen by working through the expenses, net income, taxes, and profit for each year. For example, in year 1, a (straight-line) depreciation expense of  $\$200\,000/5 = \$40\,000$  is claimed. This reduces net income by \$40 000 to \$160 000 and leaves after-tax profits of \$80 000. Now, as before, the depreciation expense of \$40 000 is not a cash outflow, so the cash actually available to invest at the end of the first year is the \$80 000 profit plus \$40 000. Since the depreciation expense is constant with the straight-line method, a cash amount of \$120 000 will be available for investment at the end of each of the five years. As such, at the end of five years, the accumulated cash will be \$732 612. This is \$24 308 less than when the equipment was fully depreciated in the first year because taxes were delayed by depreciating more of the asset's value earlier. The extra income that was available earlier for investment allowed more interest to accumulate over the five-year period. The \$24 308 is significant and illustrates why faster depreciation is preferred to slower depreciation.

As we discussed in Chapter 6, there are several generally accepted depreciation methods. The most prevalent methods in Canada are straight-line and declining-balance. For the purposes of preparing financial statements for investors, a firm may use any or all of the generally accepted methods for calculating depreciation expenses, provided that the method used is the same from period to period. However, if companies had the freedom to depreciate as they wanted to for tax purposes, they would depreciate their assets immediately, since they would get the largest benefit because of tax savings.

Governments have a different perspective. They would prefer to receive taxes as quickly as possible and would want companies to depreciate assets as slowly as possible to keep taxable income as high as possible and produce the most taxes. In order to limit the depreciation amount that companies use for tax purposes, the Canadian government has established a maximum level of capital cost expense (i.e., depreciation) that a company can claim each year, referred to as the firm's CCA. The CCA system specifies the amount and timing of depreciation expenses on capital assets. According to this system, the declining-balance method of depreciation must be used for claiming capital costs associated with most tangible assets. Straight-line depreciation is only used for certain intangible assets. We are mainly concerned with tangible assets, so our discussion will focus on the declining-balance method of depreciation.

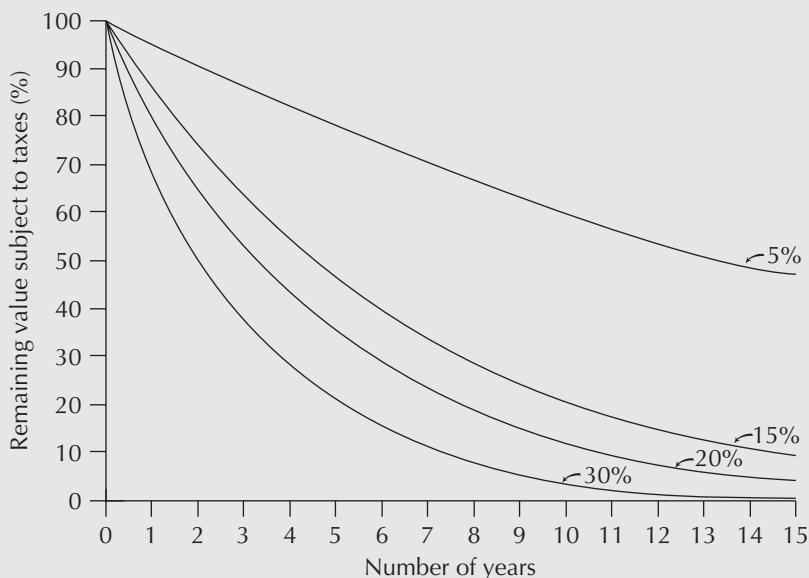
**Table 8.4 Sample CCA Rates and Classes**

| CCA Rate (%) | Class   | Description   |
|--------------|---------|---|
| 4 to 10      | 1, 3, 6 | Buildings and additions   |
| 20           | 8       | Machinery, office furniture, and equipment  |
| 25           | 9       | Aircraft, aircraft furniture, and equipment   |
| 30           | 10      | Passenger vehicles, vans, trucks, computers, and systems software                                     |
| 40           | 16      | Taxis, rental cars, and freight trucks; coin-operated video games                                     |
| 100          | 12      | Dies, tools, and instruments that cost less than \$500; computer software other than systems software |

The **CCA system** specifies the maximum rate a firm can use to depreciate its assets for tax purposes—the **CCA rate**. To implement the CCA system, a firm’s assets are grouped by **CCA asset class**, for which a specified CCA rate is used to compute the CCA. For example, all assets classified as office equipment (desks, chairs, filing cabinets, copiers, and the like) are grouped together, and depreciation expenses are based on the total remaining undepreciated cost of all assets in that class. Some examples of CCA rates and CCA asset classes are given in Table 8.4.

In addition to these standard rates, special rates may be set by the government to encourage certain kinds of investments as an incentive. Close-up 8.2 on page 267 discussed the general idea of government incentives.

Figure 8.4 illustrates how the remaining value of an asset subject to taxes diminishes within the standard range of CCA rates.

**Figure 8.4 Effect of Different CCA Rates**

While the capital cost allowance and depreciation expenses are conceptually similar, it is important to distinguish between the two. Recall from Chapter 6 that, in determining net income, depreciation expenses are deducted from revenues. This is the net income for accounting purposes. For tax purposes, however, we need to determine taxable net income. We start with the accounting net income, add back the depreciation expense for accounting purposes, and then deduct the CCA. This will lead to a figure for net income that is different from that used for accounting purposes. Given the complexities of the tax system, it is possible that this difference can be a large amount. For our purposes, we need only to distinguish between the depreciation for accounting purposes and the depreciation the firm can claim via the capital cost allowance, which is depreciation for tax purposes. ■

### 8.8.2 Undepreciated Capital Cost and the Half-Year Rule

The basis for calculating the capital cost allowance for assets in a particular asset class is the total **undepreciated capital cost (UCC)** of the assets included in that class. The capital cost of an asset when it is purchased is the total cost of acquiring the asset. This includes the purchase price, installation cost, transportation cost, legal fees, accounting costs, and possibly other costs over and above the purchase price. As the asset is depreciated, companies keep track of the undepreciated portion of the original capital cost through a UCC account. The UCC is the remaining book value for the assets subject to depreciation for taxation purposes, which may or may not differ from the market or salvage value.

The undepreciated capital cost for each asset is not usually recorded individually within a class; instead, assets in each class are pooled and only one account is maintained for each asset class. The capital cost allowance for a particular asset class is then calculated from the CCA rate for that class and its UCC.

Prior to November 13, 1981, a company was allowed to include in its base for the calculation of CCA the full purchase price of an asset purchased within the year, regardless of when the asset was purchased during the taxation year. Consequently, there was considerable motivation for companies to purchase assets at the end of their fiscal year. Recognizing the substantial tax losses brought about in this manner, the Canadian government changed the rules effective Friday, November 13, 1981. Since that date, only half of the capital cost of acquiring an asset is considered in the CCA in the year of purchase of that asset, while the other half is included in the following year. This is commonly referred to as the “half-year rule” in the CCA system.

To see the effect of this change and to illustrate the UCC account, consider a company that has just purchased a \$1 000 000 piece of equipment. For simplicity, we will assume that this equipment is the only equipment in its class. The CCA rate for the equipment is 20 percent and the company’s tax rate is 50 percent. Table 8.5 shows the

**Table 8.5 UCC Amounts Using Pre-1981 CCA Rules**

| Year | Adjustments to UCC from Purchases and Dispositions | Base UCC Amount for Capital Cost Allowance | Capital Cost Allowance | Remaining UCC | Tax Savings Due to the CCA |
|------|--|--|------------------------|---------------|----------------------------|
| 1    | \$1 000 000  | \$1 000 000                                | \$200 000              | \$800 000     | \$100 000                  |
| 2    | 0  | 800 000                                    | 160 000                | 640 000       | 80 000                     |
| 3    | 0  | 640 000                                    | 128 000                | 512 000       | 64 000                     |
| 4    | 0  | 512 000                                    | 102 400                | 409 600       | 51 200                     |

company's UCC amounts for the first four years of the asset's life, assuming that the purchase was made before the 1981 tax rule change. Table 8.6 shows equivalent figures assuming that the purchase was made after the 1981 tax change.

To explain some of the amounts, we will start with Table 8.5, showing the pre-1981 rules. The UCC at the end of the year in which the asset was bought, prior to claiming the CCA, was the purchase cost of the asset. The CCA rate for the equipment was 20 percent.

Thus the company could claim a capital cost allowance of 20 percent of the \$1 000 000 for the first year, leaving a UCC of \$800 000 at the end of the first year. In the second year, the CCA amount is 20 percent of the UCC from the end of the previous year: 20 percent of \$800 000 = \$160 000. The UCC of the asset thus declined by 20 percent of the current book value each year as the CCA rate is applied to the undepreciated capital cost from the previous year.

Table 8.6 shows what happens to the UCC account if it is assumed that the purchase occurred after the 1981 CCA tax regulation change. In the year of purchase, the full first cost of \$1 000 000 can be added to the UCC account, but only half of that amount is subject to a CCA claim. Thus the CCA amount in the first year is 20 percent of \$500 000, leaving a balance of \$900 000 of undepreciated capital cost. The CCA amount for the second year is then 20 percent of \$900 000, or \$180 000. The remainder of the CCA calculations are computed as usual.

**Table 8.6 UCC Amounts Using Post-1981 CCA Rules**

| Year | Adjustments to UCC from Purchases and Dispositions | Base UCC Amount for Capital Cost Allowance | Capital Cost Allowance | Remaining UCC | Tax Savings Due to the CCA |
|------|--|--|------------------------|---------------|----------------------------|
| 1    | \$1 000 000  | \$500 000                                  | \$100 000              | \$900 000     | \$50 000                   |
| 2    | 0  | 900 000                                    | 180 000                | 720 000       | 90 000                     |
| 3    | 0  | 720 000                                    | 144 000                | 576 000       | 72 000                     |
| 4    | 0  | 576 000                                    | 115 200                | 460 800       | 57 600                     |

Notice that the CCA expenses generate tax savings by reducing taxable income. At a 10 percent interest rate, the present worth of the tax savings using the pre-1981 rules is \$240 079, while that for the post-1981 rules is only \$213 271.

Since the change in the tax law pertaining to the half-year rule in 1981, there still remains an incentive to purchase equipment at the end of the (fiscal) year. However, the incentive has been reduced as the tax effects have been diminished.

The previous example illustrated a simple case in which only one asset was purchased. In fact, a company typically purchases assets over time and disposes of them when they are no longer required. It is important to note that, if an asset is disposed of in the same year that another one in the same CCA class is purchased, the disposal amount (for the class) is subtracted from the purchase amount (for the class) before applying the half-year rule. For any given year, the UCC balance can be calculated as follows:

$$\text{UCC}_{\text{opening}} + \text{additions} - \text{disposals} - \text{CCA} = \text{UCC}_{\text{ending}}$$

Table 8.7 summarizes the effect of the half-year rule under various circumstances.

To illustrate the use of UCC accounts when several assets of the same class are acquired and then disposed of, consider Example 8.5.

**Table 8.7 Summary: The Half-Year Rule**

| Component                                     | Treatment   |
|---|---|
| Purchase                                      | Add only half of the purchase cost of an asset to the base UCC amount for its CCA class in the year of purchase. After the CCA calculation, add the other half to the remaining UCC (note that the second half is not considered an acquisition in the following year). |
| Disposition                                   | Subtract the full amount received for a disposition of an asset from the base UCC amount for its CCA class.   |
| Purchases and Dispositions in class Same year | Subtract total dispositions from total purchases for a CCA. If the remainder is positive, treat it as a purchase. If the remainder is negative, treat it as a disposition.  |

**EXAMPLE****8.5**

Economical Corporation, an injection-moulding firm, is planning to set up business. It will purchase two used injection moulders for \$5000 each in 2012, a new, full-featured moulder for \$20 000 in 2013, and a computer controller for the new moulder for \$5000 in 2017. One used moulder will be salvaged for \$2000 in 2017, and the other for the same amount in 2012. If the CCA rate for all these assets is 20 percent, determine the balance in the UCC account in years 2012 to 2019.

Table 8.8 illustrates the calculations for the UCC balance for Example 8.5. It can be assumed that the original balance is zero, since the company is just starting up. In 2012, purchases totalling \$10 000 were made. However, only half of that amount, \$5000, is used for the CCA calculations because of the half-year rule. At 20 percent, the CCA claim is then \$1000. The UCC account for that class is increased by the full amount of the purchase, so subtracting the \$1000 from the UCC results in a balance of \$9000. In 2013, a purchase of \$20 000 increases the amount subject to the UCC to \$19 000 (since

**Table 8.8 UCC Computations with Several Changes in Asset Holdings**

| Year | Adjustments to UCC from Purchases and Dispositions | Base UCC Amount for Capital Cost Allowance | Capital Cost Allowance | Remaining UCC |
|------|--|--|------------------------|---------------|
| 2012 | \$10 000   | \$ 5 000                                   | \$1 000                | \$ 9 000      |
| 2013 | 20 000   | 19 000                                     | 3 800                  | 25 200        |
| 2014 | 0  | 25 200                                     | 5 040                  | 20 160        |
| 2015 | 0  | 20 160                                     | 4 032                  | 16 128        |
| 2016 | 0  | 16 128                                     | 3 226                  | 12 902        |
| 2017 | 3 000  | 14 402                                     | 2 880                  | 13 022        |
| 2018 | (2 000)  | 11 022                                     | 2 204                  | 8 817         |
| 2019 | 0  | 8 817                                      | 1 763                  | 7 054         |

only half of the cost of the new purchase can be included in 2013), resulting in a CCA claim for that year of \$3800. The UCC balance is calculated as  $\$9000 + \$20\ 000 - \$3800 = \$25\ 200$ . For years 2014 to 2016, the CCA claim is simply 20 percent of the UCC balance for the previous year, since no acquisitions or disposals are made.

In 2017, two transactions occurred. A computer controller is purchased for \$5000 and a moulder is salvaged for \$2000. This results in a net positive adjustment to the UCC account of \$3000. It is this \$3000 that is subject to the half-year rule. The UCC amount for CCA calculations is half of this amount plus the UCC balance from the previous year:  $\$1500 + \$12\ 902 = \$14\ 402$ . The UCC balance at the end of 2017 includes the whole \$3000 amount:  $\$12\ 902 + \$3000 - \$2880 = \$13\ 022$ . The negative adjustment in 2018 is not subject to the half-year rule since the rule only applies to net purchases over the year, so the UCC amount for CCA calculations is \$2000 less than the previous year's balance. In 2019, there are no adjustments to the UCC account other than the CCA claim, leaving a final balance of \$7054. ■

### 8.8.3 The Capital Tax Factor and Capital Salvage Factor

From Tables 8.2 and 8.3, it is clear that the CCA creates tax savings. For example, if an asset with a CCA rate of 20 percent is purchased for \$100 000, this provides a CCA claim of \$10 000 in the year of purchase. With a tax rate of 50 percent, this deduction from income saves \$5000 in taxes that would not have occurred if the \$100 000 had not been spent for the asset in the first place. Therefore, the present worth equivalent of the first cost of the asset is actually less than \$100 000—it is reduced by the present worth of all of the tax savings that result from its depreciation in all future years. In this example, the tax savings for each year of the asset's life are shown in Table 8.9.

$$\begin{aligned} \text{Present worth of tax savings} &= \$5000(P/F,i,1) + \$9000(P/F,i,2) \\ &\quad + \$7200(P/F,i,3) + \$5760(P/F,i,4) \\ &\quad + \$4608(P/F,i,5) + \dots \end{aligned}$$

The present worth of the tax savings essentially reduces the first cost of the investment because making the investment and depreciating it over time brings about tax benefits. The **capital tax factor (CTF)** is a value that summarizes the effect of the benefit of future tax savings due to CCA and allows analysts to take these benefits into account when calculating the present worth equivalent of an asset. The CTF is constant for a given CCA rate, interest rate, and tax rate, and allows the determination of

**Table 8.9 Tax Savings Due to the CCA (50% Tax Rate)**

| Year | Base UCC Amount for Capital Cost Allowance | Capital Cost Allowance | Remaining UCC | Tax Savings Due to the CCA |
|------|--|------------------------|---------------|----------------------------|
| 1    | \$50 000                                   | \$10 000               | \$90 000      | \$5 000                    |
| 2    | 90 000                                     | 18 000                 | 72 000        | 9 000                      |
| 3    | 72 000                                     | 14 400                 | 57 600        | 7 200                      |
| 4    | 57 600                                     | 11 520                 | 46 080        | 5 760                      |
| 5    | 46 080                                     | 9 216                  | 36 864        | 4 608                      |

the present worth independently of the actual first cost of the asset. This makes it a very useful element.

Similarly, when an asset is salvaged or scrapped, there is an ongoing tax effect that can be captured and summarized. However, when an asset is salvaged or scrapped, the effect on the UCC is immediate and not subject to the half-year rule. The equivalent measure in this case is the **capital salvage factor (CSF)**.

The CSF and CTF are derived in Appendix 8A as:

$$\text{CSF} = 1 - \frac{td}{(i+d)} \quad \text{CTF} = 1 - \frac{td(1 + \frac{i}{2})}{(i + d)(1 + i)}$$

where:

$t$  = taxation rate

$d$  = CCA rate

$i$  = after-tax interest rate

The present worth of the first cost (at the time of purchase) of an asset is found by multiplying the first cost by the CTF. This takes into account the tax benefits forever. When an asset is salvaged or scrapped, we need to terminate the remaining stream of tax savings. This is done by applying the CSF. Examples in Section 8.8.4 will clarify the process.

### EXAMPLE

### 8.6

An automobile purchased by Lestev Corporation for \$25 000 has a CCA rate of 30 percent. Lestev is subject to a 43 percent corporate tax rate and the corporate (after-tax) MARR is 12 percent. What is the present worth of the first cost of this automobile, taking into account the future tax savings resulting from depreciation?

The car is an asset being purchased, so the CTF applies. The CTF is calculated as:

$$\begin{aligned} \text{CTF} &= 1 - \frac{(0.43)(0.3)(1 + 0.06)}{(0.12 + 0.3)(1 + 0.12)} \\ &= 0.709311 \end{aligned}$$

The present worth of the first cost of the car is then calculated as:

$$\begin{aligned} \text{PW} &= 0.709311(\$25\,000) \\ &= \$17\,733 \end{aligned}$$

The present worth of the first cost of the car, taking into account all future tax savings due to depreciation, is about \$17 733. The tax benefit due to claiming CCA has effectively reduced the cost of the car from \$25 000 to \$17 733 in terms of present worth. ■

It may seem strange that the effective cost of purchasing an asset is less than its first cost. However, bear in mind that the first cost is not the only impact that the purchase of an asset has on cash flows. The purchase will also likely generate savings. These savings are income, which is also taxed. Taking taxes into account when determining the present worth or annual worth of an asset will affect the present or annual worth positively

because of the tax benefits resulting from future CCA, but also negatively because of the taxation of future savings.

#### 8.8.4 Components of a Complete Tax Calculation

As discussed in Section 8.4, when evaluating projects with the explicit consideration of taxes, it is important to recognize that there is a difference between a before-tax MARR and an after-tax MARR. A before-tax MARR is chosen to reflect the fact that taxes are not explicitly taken into account in the economic calculations, and the after-tax MARR is used when taxes are explicitly taken into account.

Evaluating the economic impact of purchasing a depreciable asset goes beyond the impact of taxes on the first cost. There are two other components in a complete economic analysis. First, we need to assess the tax implications of the savings or additional expenses brought about by the asset over its useful life. Second, when the asset is disposed of, we no longer can take advantage of its CCA and thus must terminate the stream of tax savings resulting from depreciating the asset.

Each of these components has a tax effect that has to be taken into account when doing a cash flow analysis such as determining present worth or annual worth. A summary of the procedure for a present worth computation is shown in Table 8.10.

**Table 8.10 Components of a Complete Present-Worth Tax Calculation**

| Component           | Treatment   |
|---------------------|---|
| First cost          | Multiply by the CTF.                              |
| Savings or expenses | Multiply by $(1 - t)$ . Convert to present worth. |
| Salvage value       | Multiply by the CSF. Convert to present worth.    |

First cost: As presented in Section 8.7, the first cost of an asset purchased after 1981 is reduced by the tax savings due to the CCA. Multiply the first cost by the CTF to find the after-tax first cost.

Savings or expenses: Reduce savings or expenses by the tax rate by multiplying by  $(1 - t)$ . There is an assumption that the company is making a profit, so taxes are paid on all the savings at the rate  $t$ , and expenses will reduce taxes at the rate  $t$ .

Salvage value: Apply the CSF. When an asset is disposed of, the salvage value reduces the UCC for the full amount in the year of disposal (at least in the absence of a corresponding purchase in the same year). The effect of reducing the UCC is the same in magnitude but opposite in sign as increasing the UCC. The CTF has built into it a delay in depreciating half of the value of the asset, whereas the full effect occurs immediately when disposing of an asset. Consequently, the CSF is the one to use at the time an asset is sold.

Note that, technically, when assets are disposed of in a given year, they are netted against any additions for the year before the half-year rule is applied. However, in project analysis, we generally want to evaluate the project independently, at least in preliminary evaluation. When we determine the salvage value, we do not consider the effects of other additions or disposals that the company may be planning for the same year. Nevertheless, it is worth noting that significant tax advantages can be made by properly planning the timing of investment additions and disposals. Our goal at this time is to decide on the merits of the project on a more basic level.

**EXAMPLE****8.7**

The owner of a spring water bottling company in Erbsville has just purchased an automated bottle capper. What is the after-tax present worth of the new automated bottle capper if it costs \$10 000 and saves \$4000 per year over its five-year life? Assume a \$2000 salvage value and a 50 percent tax rate. The after-tax MARR is 12 percent.

A CCA rate is not given in this question. As production equipment, the new bottle capper can be assumed to be in CCA Class 8 with a rate of 20 percent.

The present worth of the first cost must take into account the tax benefits of CCA. The after-tax first cost is

$$PW(\text{first cost}) = -\$10\,000(CTF)$$

where the CTF is calculated as:

$$CTF = 1 - \frac{(0.5)(0.2)(1 + 0.06)}{(0.12 + 0.2)(1 + 0.12)}$$

Therefore, the present worth of the first cost is

$$\begin{aligned} PW(\text{first cost}) &= -\$10\,000(0.70424) \\ &\approx -\$7042 \end{aligned}$$

The annual savings are taxed at 50 percent, so the present worth of the savings is

$$\begin{aligned} PW(\text{annual savings}) &= \$4000(P/A, 12\%, 5)(1 - t) \\ &= \$4000(3.6047)(0.5) \\ &\approx \$7209 \end{aligned}$$

The salvage value is not simply \$2000 five years from now. It reduces the UCC and thus diminishes the tax benefits resulting from the CCA on the original purchase. The after-tax benefits can be determined by applying the CSF:

$$PW(\text{salvage value}) = \$2000(P/F, 12\%, 5)\text{CSF}$$

$$\begin{aligned} \text{CSF} &= 1 - \frac{(0.5)(0.2)}{(0.12 + 0.2)} \\ &= 0.6875 \end{aligned}$$

$$\begin{aligned} PW(\text{salvage value}) &= \$2000(0.56743)(0.6875) \\ &\approx \$780 \end{aligned}$$

Summing the present worths:

$$\begin{aligned} PW &= PW(\text{first cost}) + PW(\text{annual savings}) + PW(\text{salvage value}) \\ &= -\$7042 + \$7209 + \$780 \\ &= \$947 \end{aligned}$$

The present worth after taxes for the new bottle capper is \$947. ■

Example 8.7 illustrated the complete effect of taxes for a present worth analysis. Similar adjustments are made to an annual worth computation, as illustrated in Table 8.11.

**Table 8.11 Components of a Complete Annual-Worth Tax Calculation**

| Component           | Treatment                                     |
|---------------------|---|
| First cost          | Multiply by the CTF. Convert to annual worth. |
| Savings or expenses | Multiply by $(1 - t)$ .                       |
| Salvage value       | Multiply by the CSF. Convert to annual worth. |

**EXAMPLE**

**8.8**

A small device used to test printed circuit boards has a first cost of \$45 000. The tester is expected to reduce labour costs and improve the defect detection rate to bring about savings of \$23 000 per year. Additional operating costs are expected to be \$7300 per year. The salvage value of the tester will be \$5000 in five years. With an after-tax MARR of 12 percent, a CCA rate of 20 percent, and a tax rate of 42 percent, what is the annual worth of the tester, taking into account the effect of taxes?

The basic process for adjusting for tax effects in an annual worth comparison is similar to a present worth analysis. First, we apply the CTF to the first cost and convert it into an annual amount over five years. Next, the annual savings and expenses are multiplied by  $(1 - t)$ . Finally, the salvage value at the end of five years is multiplied by CSF and then converted into an annual amount:

$$\begin{aligned} AW(\text{tester}) = & -\$45\,000(A/P, 12\%, 5)\text{CTF} + (\$23\,000 - \$7300)(1 - t) \\ & + \$5000(A/F, 12\%, 5)\text{CSF} \end{aligned}$$

Using  $d = 0.20$ ,  $t = 0.42$ , and  $i = 0.12$ , we have:

$$\text{CTF} = 0.7516$$

$$\text{CSF} = 0.7375$$

Therefore:

$$\begin{aligned} AW(\text{tester}) = & -\$45\,000(0.27741)(0.7516) + (\$23\,000 - \$7300)(0.58) \\ & + \$5000(0.15741)(0.7375) \\ \cong & \$304 \end{aligned}$$

The annual worth, taking into account taxes, is \$304. ■

As the last two examples show, taking taxes into account for present worth and annual worth analyses is relatively straightforward. IRR computations are a bit more involved, however, as the next example illustrates.

**EXAMPLE**

**8.9**

Find the after-tax IRR for the testing equipment described in Example 8.8.

First, observe that if we solve for  $i$  in  $AW(\text{receipts}) - AW(\text{disbursements}) = 0$ , or  $PW(\text{receipts}) - PW(\text{disbursements}) = 0$ , the resulting rate will be an after-tax IRR, since

the amounts will have been adjusted for taxes. Since Example 8.8 was expressed in terms of annual amounts, we will find the after-tax IRR by solving for  $i$  in  $\text{AW}(\text{receipts}) - \text{AW}(\text{disbursements}) = 0$  using the operations listed in Table 8.11:

$$(\$23\,000 - \$7300)(1 - t) + \$5000(A/F, i, 5)\text{CSF} - \$45\,000(A/P, i, 5)\text{CTF} = 0$$

In order to solve this equation, a trial-and-error approach is necessary because the interest rate,  $i$ , appears in the capital cost tax factors as well as in the compound interest factors. Table 8.12 shows the result of this process obtained through the use of a spreadsheet.

**Table 8.12 Trial-and-Error Process for Finding the After-Tax IRR**



| $i$  | $\text{AW}(\text{receipts}) - \text{AW}(\text{disbursements})$ |
|------|--|
| 0.10 | \$997.54   |
| 0.11 | 651.37   |
| 0.12 | 304.36   |
| 0.13 | -43.62   |
| 0.14 | -392.68  |

From the spreadsheet computations, we can see that the after-tax IRR on the testing equipment is between 12 percent and 13 percent. Additional trial-and-error iterations with the spreadsheet program give an after-tax IRR of 12.87 percent. ■

## REVIEW PROBLEMS

### REVIEW PROBLEM 8.1

Angus and his sister Oona operate a small charter flight service that takes tourists on sightseeing tours over the beautiful Margaree River on Cape Breton Island. At the end of 2009, they had one four-seater plane in the aircraft asset class with a UCC of \$30 000. In 2010, they purchased a second plane for \$50 000. Business was going well in 2011, so they sold the old plane they had in 2009 for \$15 000 and bought a newer version for \$64 000. What was the UCC balance in the aircraft asset class at the end of 2012? The CCA rate for aircraft is 25 percent.

### ANSWER

Table 8.13 shows the fluctuation in the UCC balance over time. At the end of 2009, the UCC for the aircraft asset class was \$30 000. In 2010, half of the capital cost of the airplane purchased in 2010 (\$25 000) contributed to the CCA calculation. The CCA rate of 25 percent gave a CCA amount of \$13 750 and resulted in a UCC balance of \$66 250 at the end of 2010. In 2011, the net positive adjustment to the UCC due to the capital cost of \$64 000 for the new plane and the \$15 000 benefit from the sale of the old plane was \$49 000. Half of this amount, \$24 500, contributed to the CCA calculation. After subtracting the CCA amount of \$22 688 for 2010, the remaining UCC was \$92 563 (\$66 250 + \$49 000 - \$22 688). In 2012, there were no further adjustments to the UCC, and after the CCA was deducted, the closing UCC account balance was \$69 422. ■

**Table 8.13 Summary of UCC Computations for Review Problem 8.1**

| Year | Adjustments to UCC from Purchases or Dispositions | Base UCC Amount for Capital Cost Allowance | CCA Allowance | Remaining UCC |
|------|---|--|---------------|---------------|
| 2009 |   |  |               | \$30 000      |
| 2010 | \$50 000  | \$55 000                                   | \$13 750      | 66 250        |
| 2011 | 49 000  | 90 750                                     | 22 688        | 92 563        |
| 2012 | 0   | 92 563                                     | 23 141        | 69 422        |

**REVIEW PROBLEM 8.2**

David Cosgrove has just started a management consulting firm that he operates out of his home at Paradise Lake. As part of his new business, David is considering buying a new \$30 000 van, which will be used 100 percent of the time for earning business income. He estimates that the expenses associated with operating the van will be \$3000 per year for gas, \$1200 per year for insurance, \$600 annually for parking, and maintenance costs of \$1000 for the first year, rising by \$400 per year thereafter. He expects to keep the van for five years. At the end of this time, he estimates a salvage value of \$6000. The CCA rate for vans is 30 percent.

The alternative for David is to lease the van. With a lease arrangement, he will have to pay for parking, gas, and insurance, but the leasing company will pay for the repairs. The lease costs are \$10 500 per year.

David estimates his after-tax cost of capital to be 12 percent per year and his tax rate is 40 percent. On the basis of an annual worth analysis over the five years, should David buy the van or lease it?

**ANSWER**

The approach will be to find the after-tax annual worth of each alternative. Since the parking, insurance, and gas costs are the same for both alternatives, we can exclude them from the analysis.

The after-tax annual costs of purchasing the van are:

$$\begin{aligned} AW(\text{van}) &= \$30\,000(A/P, 12\%, 5)\text{CTF} - \$6000(A/F, 12\%, 5)\text{CSF} \\ &\quad + [\$1000 + \$400(A/G, 12\%, 5)](1 - t) \end{aligned}$$

We can calculate that:

$$\begin{aligned} \text{CTF} &= 1 - \frac{td(1 + \frac{i}{2})}{(i + d)(1 + i)} \\ &= 1 - \frac{(0.4)(0.3)(1 + \frac{0.12}{2})}{(0.12 + 0.30)(1 + 0.12)} \\ &= 0.7296 \end{aligned}$$

$$\text{CSF} = 1 - \frac{td}{(i + d)}$$

$$= 1 - \frac{(0.4)(0.3)}{(0.12 + 0.30)}$$

$$= 0.71429$$

Thus:

$$\begin{aligned} AW(\text{van}) &= \$30\,000(0.27741)(0.7296) - \$6000(0.15741)(0.71429) \\ &\quad + [\$1000 + \$400(1.7745)](1 - 0.4) \\ &\cong \$6423 \end{aligned}$$

The annual cost of purchasing and operating the van over a five-year period is a little over \$6400.

There is a large difference between buying and leasing. When we lease, we do not have a depreciable asset on which to claim depreciation expenses; we only have lease payment expenses. Therefore, the impact of taxes on the lease expense is simply to multiply the leasing costs by  $(1 - t)$ :

$$\begin{aligned} AW(\text{lease}) &= \$10\,500(1 - 0.4) \\ &= \$10\,500(0.6) \\ &= \$6300 \end{aligned}$$

The after-tax annual cost of leasing is \$6300. It is less expensive to lease the van than it is to buy it, so David should lease the van. It is worth noting that this example is a simplified version of real life, since we have ignored numerous tax rules relating to the eligibility of expenses for automobiles for illustration purposes. ■

### **REVIEW PROBLEM 8.3**

Putco does subcontracting for an electronics firm that assembles printed circuit boards. Business has been good lately, and Putco is thinking of purchasing a new IC chip-placement machine. It has a first cost of \$450 000 and is expected to save the company \$125 000 per year in labour and operating costs compared with the manual system it has now. A similar system that also automates the circuit board loading and unloading process costs \$550 000 and will save about \$155 000 per year. The life of either system is expected to be four years. The salvage value of the \$450 000 machine will be \$180 000, and that of the \$550 000 machine will be \$200 000. Putco uses an after-tax MARR of 9 percent to make decisions about such projects. On the basis of an IRR comparison, which alternative (if either) should it choose? Putco pays taxes at a rate of 40 percent and the CCA rate for the equipment is 20 percent.

### **ANSWER**

Putco has three mutually exclusive alternatives:

1. Do nothing.
2. Buy the chip-placement machine.
3. Buy a similar chip-placement machine with an automated loading and unloading process.

Following the procedure from Chapter 5, the projects are already ordered on the basis of first cost. We therefore begin with the first alternative: The before-tax (and thus the after-tax) IRR of the “do nothing” alternative is 0 percent.

The before-tax IRR on the incremental investment to the second alternative can be found by solving for  $i$  in

$$-\$450\,000 + \$125\,000(P/A,i,4) + \$180\,000(P/F,i,4) = 0$$

By trial and error, we obtain an  $IRR_{\text{before-tax}}$  of 15.92 percent. This gives an approximate  $IRR_{\text{after-tax}}$  of  $0.1592(1 - 0.40) = 0.0944$ , or 9.44 percent. With an after-tax MARR of 9 percent, it would appear that this alternative is acceptable, though a detailed after-tax computation may be in order. We need to solve for  $i$  in:

$$(-\$450\,000)\text{CTF} + \$125\,000(P/A,i,4)(1 - t) + \$180\,000(P/F,i,4)\text{CSF} = 0$$

Doing so gives an  $IRR_{\text{after-tax}}$  of 9.36 percent. Since this exceeds the required after-tax MARR of 9 percent, this alternative becomes the current best. We next find the  $IRR_{\text{after-tax}}$  on the incremental investment required for the third alternative. The  $IRR_{\text{before-tax}}$  is first found by solving for  $i$  in:

$$\begin{aligned} & -(\$550\,000 - \$450\,000) + (\$155\,000 - \$125\,000)(P/A,i,4) \\ & + (\$200\,000 - \$180\,000)(P/F,i,4) = 0 \end{aligned}$$

This gives an  $IRR_{\text{before-tax}}$  of 13.66 percent, or an approximate  $IRR_{\text{after-tax}}$  of 8.2 percent. This is close enough to the required after-tax MARR of 9 percent to warrant a detailed incremental  $IRR_{\text{after-tax}}$  computation. ■

## REVIEW PROBLEM 8.4

David Cosgrove (from Review Problem 8.2) is still thinking over whether to buy a van. Assuming he remains in business for the foreseeable future, he will need a vehicle for transportation indefinitely, whether he owns or leases it. In his original analysis, he assumed that the van would be replaced at the end of five years. Because appearances are important to David, he would not consider keeping a vehicle for longer than five years, but he now recognizes that the economic life of the van may be shorter than that. Assuming that the van depreciates in value by a constant proportion each year, determine how frequently David should replace it. The CCA rate is 30 percent and his tax rate is 40 percent.

### ANSWER

The first step in the solution is to recognize that David is facing a cyclic replacement problem, since it is reasonable to assume that he will always replace each van with one similar to the previous one. We now need to assess the annual cost of replacing a van each year, every two years, and so on, up to replacement every five years. Before proceeding, however, we need to determine the depreciation rate to use so that we can determine the approximate value of the van when it is  $n$  years old for  $n = 1, 2, 3$ , and 4.

Referring back to Chapter 6, we have for the declining-balance method of depreciation:

$$d = 1 - \sqrt[n]{\frac{S}{P}} = 1 - \sqrt[5]{\frac{6000}{30\,000}} = 0.27522$$

Using the formula  $BV_{\text{db}}(n) = P(1 - d)^n$ , we find that the book value of the van at the end of each year is:

| End of Year | Book Value |
|-------------|------------|
| 0           | \$30 000   |
| 1           | 21 743     |
| 2           | 15 759     |
| 3           | 11 422     |
| 4           | 8 278      |
| 5           | 6 000      |

Note that these are book values, not the UCC balances. The book values are estimates of the market value, which is needed to judge when the asset should be replaced. A UCC balance is similar to a book value but is used for calculating the CCA only.

Now the annual worth computations can be done using the CTF values calculated in Review Problem 8.2:

$$\text{AW(replace every year)}$$

$$\begin{aligned} &= \text{AW(capital recovery)} + \text{AW(operating)} \\ &= \$30\,000(A/P, 12\%, 1)\text{CTF} - \$21\,743(A/F, 12\%, 1)\text{CSF} + (\$5800)(1 - t) \\ &= \$12\,463 \end{aligned}$$

$$\text{AW(replace every two years)}$$

$$\begin{aligned} &= \$30\,000(A/P, 12\%, 2)\text{CTF} - \$15\,759(A/F, 12\%, 2)\text{CSF} \\ &\quad + \$5800(1 - t) + \$400(A/F, 12\%, 2)(1 - t) \\ &= \$11\,235 \end{aligned}$$

$$\text{AW(replace every three years)}$$

$$\begin{aligned} &= \$30\,000(A/P, 12\%, 3)\text{CTF} - \$11\,422(A/F, 12\%, 3)\text{CSF} \\ &\quad + \$5800(1 - t) + [\$400(F/P, 12\%, 1) + \$800](A/F, 12\%, 3)(1 - t) \\ &= \$10\,397 \end{aligned}$$

$$\text{AW(replace every four years)}$$

$$\begin{aligned} &= \$30\,000(A/P, 12\%, 4)\text{CTF} - \$8278(A/F, 12\%, 4)\text{CSF} \\ &\quad + \$5800(1 - t) + [\$400(F/P, 12\%, 2) + \$800(F/P, 12\%, 1) \\ &\quad + \$1200](A/F, 12\%, 4)(1 - t) \\ &= \$9775 \end{aligned}$$

$$\text{AW(replace every five years)}$$

$$\begin{aligned} &= \$30\,000(A/P, 12\%, 5)\text{CTF} - \$6000(A/F, 12\%, 5)\text{CSF} \\ &\quad + \$5800(1 - t) + [\$400(F/P, 12\%, 3) + \$800(F/P, 12\%, 2) \\ &\quad + \$1200(F/P, 12\%, 1) + \$1600](A/F, 12\%, 5)(1 - t) \\ &= \$9303 \end{aligned}$$

On the basis of these calculations, it is best for David to replace the van at the end of every five years. Its economic life may be longer than five years, but as far as David is concerned, a five-year-old van has reached the end of its useful life and must be replaced. ■

## S U M M A R Y

Income taxes can have a significant effect on engineering economics decisions. In particular, taxes reduce the effective cost of an asset, the savings generated, and the value of the sale of an asset.

In this chapter, we provided a basic introduction to the Canadian capital cost allowance system and the use of undepreciated capital cost accounts. The CCA rate is a declining-balance rate that is mandated for use in calculating the depreciation expenses for capital assets. These depreciation expenses are then used in determining the amount of taxes owing for the year. Assets are designated as belonging to a particular CCA class. The book values for taxation purposes calculated for all the assets in each class are accumulated into a UCC account.

The future CCA claims that arise from the purchase of an asset are benefits that reduce the after-tax first cost. The CTF permits the quick calculation of the net effect of these benefits, while similarly, the CSF permits the calculation of the net effect of future loss of CCA claims for assets that are sold or scrapped.

It was noted that, for after-tax calculations, an after-tax MARR must be used. After-tax calculations were illustrated for present worth, annual worth, and IRR evaluations. An approximate IRR comparison method was also given.

The review problems at the end of the chapter illustrated how taxes affect present worth and annual worth comparisons, replacement analyses, and internal rate of return computations.

### CLOSE-UP 8.3

#### Tax Rules in Other Countries

The principles of taking tax effects into account when making economic decisions are not dependent on the particular tax rules of different countries, and methods like the approximate after-tax rate of return calculations are applicable anywhere. However, for an accurate determination of a project's viability, the particular rules of the project's jurisdiction must be taken into account. Here is an overview of the tax rules of the United States, the United Kingdom, and Australia:

**United States:** The United States requires that assets be depreciated for tax purposes according to a particular mixture of declining-balance and straight-line methods called MACRS (modified accelerated cost recovery system). Like the Canadian system, different types of assets belong to different classes that determine how quickly the asset depreciates to a zero book value. Similar to the Canadian half-year rule, every asset is assumed to be purchased in the middle of the first year. Salvage value can either be taxable income if it is a larger amount than the book value, or a deductible expense if it is lower than book value.

**United Kingdom:** In the UK system, the values of all assets are added at the time of purchase to a pool similar to the UCC pool under the CCA system, except that there are no classes—everything is pooled together. Each year, 25 percent of the remaining value of the pool—called the written down value (WDV)—is considered to be an expense. There are only a few exceptions, notably buildings, which are allowed to depreciate at only 4 percent per year. When an asset is salvaged, a corresponding adjustment is made to the pool.

**Australia:** Australia has very simple rules. The taxpayer can choose both the life of the asset and the depreciation method. The depreciation method can either be straight-line or declining-balance, and if it is declining-balance, then the rate is 150 percent divided by the life of the asset. First year depreciation expense is pro-rated by the number of days the asset is in use.

## ENGINEERING ECONOMICS IN ACTION, PART 8B

### The Work Report

"So what is this, anyhow?" Clem was looking at the report that Naomi had handed him. "A consulting report?"

"Sorry, chief, it is a bit thick." Naomi looked a little embarrassed. "You see, Terry has to do a work report for his university. It's part of the co-op program. He got interested in the 10-stage die problem and asked me if he could make that study his work report. I said okay, subject to its perhaps being confidential. I didn't expect it to be so thick, either. But he's got a good executive summary at the front."

"Hmm . . ." The room was quiet for a few minutes while Clem read the summary. He then leafed through the remaining parts of the report. "Have you read this through? It looks really quite good."

"I have. He has done a very professional job—er, at least what seems to me to be very professional." Naomi suddenly remembered that she hadn't yet gained her professional engineer's designation. She also hadn't been working at Canadian Widgets much longer than Terry. "I gathered most of the data for him, but he did an excellent job of analyzing it. As you can see from the summary, he set up the replacement problem as a set of mutually exclusive alternatives, involving upgrading the die now or later and even more than once. He did a nice job on the taxes, too."

"Tell me more about how he handled the taxes."

"He was really thorough. I had to hold his hand a bit to make sure he understood how the UCC accounts work, but once he had that, everything else seemed to fall into place. He reduced the purchase price by the benefits of future CCA claims. The installation cost and future savings were reduced by the taxation rate, and the salvage values were reduced for loss of future CCA claims."

"Did he understand about the capital tax factor and the capital salvage factor?"

"Yes, he did."

"Not bad. I think we've got a winner there, Naomi. Let's make sure we get him back for his next work term."

Naomi nodded. "What about his work report, Clem? Should we ask him to keep it confidential?"

Clem laughed. "Well, I think we should, and not just because there are trade secrets in the report. I don't want anyone else knowing what a gem we have in Terry!"

## P R O B L E M S

### A. Key Concepts

- 8.1 Collingwood Caskets generally uses a before-tax MARR of 14 percent. Vincent wants to do a detailed calculation of the cash flows associated with a new planer for the assembly line. What would be an appropriate after-tax MARR for him to use if Collingwood Caskets pays
  - (a) 40 percent corporate taxes?
  - (b) 50 percent corporate taxes?
  - (c) 60 percent corporate taxes?
- 8.2 What is the approximate after-tax IRR on a two-year project for which the first cost is \$12 000, savings are \$5000 in the first year and \$10 000 in the second year, and taxes are at 40 percent?
- 8.3 Waterloo Industries pays 40 percent corporate income taxes and its after-tax MARR is 18 percent. A project has a before-tax IRR of 24 percent. Should the project be approved? What would your decision be if the after-tax MARR were 14 percent?

- 8.4** What is the exact after-tax IRR on a project for which the first cost is \$12 000, savings are \$5000 in the first year and \$10 000 in the second year, taxes are at 40 percent, and the CCA rate is 30 percent?
- 8.5** Identify each of the following according to their CCA class(es) and CCA rate(s):
- A soldering gun costing \$75
  - A garage used to store spare parts
  - A new computer
  - A 100-tonne punch press
  - A crop-dusting attachment for a small airplane
  - An oscilloscope worth exactly \$600
- 8.6** Calculate the CTF and CSF for each of the following:
- Tax rate of 50 percent, CCA of 20 percent, and an after-tax MARR of 9 percent
  - Tax rate of 35 percent, CCA of 30 percent, and an after-tax MARR of 12 percent
  - Tax rate of 55 percent, CCA of 5 percent, and an after-tax MARR of 6 percent
- 8.7** A company's first year's operations can be summarized as follows:
- Revenues: \$110 000
- Expenses (except CCA): \$65 000
- Its capital asset purchases in the first year totalled \$100 000. With a CCA rate of 20 percent and a tax rate of 55 percent, how much income tax did it pay?
- 8.8** A company's second year operations can be summarized as follows:
- Revenues: \$110 000
- Expenses (except CCA): \$65 000
- Its capital asset purchases in the first year totalled \$100 000, with none in the second year. With a CCA rate of 20 percent and a tax rate of 55 percent, how much income tax did it pay?
- 8.9** The UCC balance for a firm's automobile fleet at the end of 2009 was \$10 000. There was one truck in service at this time. At the beginning of 2010, the firm purchased two trucks for a total of \$50 000. At the beginning of 2012, it purchased another truck for \$20 000. At the beginning of 2013, the truck owned in 2009 was sold for \$3000. The CCA rate for automobiles is 30 percent. What was the firm's UCC balance at the end of 2013?
- 8.10** Use a spreadsheet program to create a chart showing how the values of the CTF and the CSF change for after-tax MARRs of 0 percent to 30 percent. Assume a fixed tax rate of 50 percent and a CCA rate of 20 percent.

## B. Applications

- 8.11** Enrique has just completed a detailed analysis of the IRR of a waste-water treatment plant for Montreal Meat Products. The 8.7 percent after-tax IRR he calculated compared favourably with a 5.2 percent after-tax MARR. For reporting to upper management, he wants to present this information as a before-tax IRR. If Montreal Meat

Products pays 53 percent corporate taxes, what figures will Enrique report to upper management?

- 8.12** International Computing Corporation (ICC) is considering updating its automated inventory control system. The supplier has estimated the first cost as \$2 300 000 and the annual savings as \$880 000; after its 10-year life, it will have a salvage value of \$200 000. The tax rate for ICC is 45 percent and the company uses an after-tax MARR of 10 percent. What is the present worth of the new system? Should ICC make this investment?
- 8.13** Go to the Canada Revenue Agency website and search for the T2 Corporation—Income Tax Guide. Find the section in the guide dealing with CCA rates and identify each of the following according to its CCA class(es) and CCA rate(s).
- A heated, rigid-frame greenhouse worth \$57 000
  - A small tractor with attachments for earth-moving and snow removal worth \$24 000
  - New software for inventory control purposes costing \$70 000
  - Advanced data network switching equipment that costs \$56 000
  - Cash register and barcode scanning device used for point-of-sales data collection. Cost is \$12 000 for the cash register and \$200 for the scanning device.
- 8.14** What is the after-tax present worth of a chip placer if it costs \$55 000 and saves \$17 000 per year? After-tax interest is at 10 percent. Assume the device will be sold for a \$1000 salvage value at the end of its six-year life. The CCA rate is 20 percent and the corporate income tax rate is 54 percent.
- 8.15** Quebec Widgets is looking at a \$400 000 digital midget rigid widget gadget (CCA Class 8). The gadget is expected to save \$85 000 per year over its 10-year life, with no scrap value. The company's tax rate is 45 percent, and its after-tax MARR is 15 percent. On the basis of an approximate IRR, should it invest in this gadget?
- 8.16** Canada Widgets is looking at a \$400 000 digital midget rigid widget gadget (CCA Class 8). The gadget is expected to save \$85 000 per year over its 10-year life, with no scrap value. The company's tax rate is 45 percent and its after-tax MARR is 15 percent. On the basis of an exact IRR, should it invest in this gadget?
- 8.17** Go to the Canada Revenue Agency website and find the form Schedule 8 for the T2 Short Corporation Income Tax Return. This is a worksheet for calculating UCC balances. Use the sheet to make the following calculations (separately). In all cases, there are no adjustments (e.g., for GST rebates or for investment tax credits).
- The UCC balance at the end of the previous year is \$10 000. Assets purchased in Class 10 for the current year amount to \$30 000. Find the UCC at the end of the year.
  - The UCC balance at the end of the previous year is \$10 000. Assets purchased in Class 10 for the current year amount to \$20 000. Dispositions were \$5000. Find the UCC at the end of the year.
  - The UCC balance at the end of the previous year for Class 8 was \$20 000. This year, an asset worth \$20 000 was added to Class 8 and another worth \$15 000 was disposed of. What is the year-end UCC for this year?
- 8.18** Churchill Metal Products opened for business in 1999. Over the following years, its transactions for CCA Class 8 assets consisted of the following:

| Date               | Item      | Activity | Amount    |
|--------------------|-----------|----------|-----------|
| March 11, 1999     | Machine 1 | Purchase | \$ 50 000 |
| April 24, 1999     | Machine 2 | Purchase | 150 000   |
| November 3, 2002   | Machine 3 | Purchase | 250 000   |
| November 22, 2002  | Machine 1 | Sale     | 10 000    |
| May 20, 2006       | Machine 4 | Purchase | 60 000    |
| August 3, 2011     | Machine 5 | Purchase | 345 000   |
| September 12, 2012 | Machine 3 | Sale     | 45 000    |

What CCA amount did Churchill Metal Products claim for the 20 percent UCC account in 2013?

- 8.19** What is the total after-tax annual cost of a machine with a first cost of \$45 000 and operating and maintenance costs of \$0.22 per unit produced? It will be sold for \$4500 at the end of five years. Production is 750 units per day, 250 days per year. The CCA rate is 30 percent, the after-tax MARR is 20 percent, and the corporate income tax rate is 40 percent.
- 8.20** Chrétien Brothers Salvage made several equipment purchases in the past decade. Its first asset was a tow truck bought in 2006 for \$25 000. In 2008, a van was purchased for \$14 000. A second tow truck was bought in 2011 for \$28 000, and the first one was sold the following year for \$5000. Using a 30 percent CCA rate (automobiles, trucks, and vans), what was the balance of its UCC account at the end of 2013?
- 8.21** Whitehorse Construction just bought a crane for \$380 000. At a CCA rate of 20 percent, what is the present worth of the crane, taking into account the future benefits resulting from the CCA? Whitehorse has a tax rate of 35 percent and an after-tax MARR of 6 percent.
- 8.22** Hull Hulls is considering the purchase of a 30-tonne hoist. The first cost is expected to be \$230 000. Net savings will be \$35 000 per year over a 12-year life, and it will be salvaged for \$30 000. If the company's after-tax MARR is 8 percent and it is taxed at 45 percent, what is the present worth of this project?
- 8.23** Kanata Konstruktion is considering the purchase of a truck. Over its five-year life, it will provide net revenues of \$15 000 per year at an initial cost of \$65 000 and a salvage value of \$20 000. KK pays 35 percent in taxes, the CCA rate for trucks is 30 percent, and the after-tax MARR is 12 percent. What is the annual cost or worth of this purchase?
- 8.24** A new binder will cost Revelstoke Printing \$17 000, generate net savings of \$3000 per year over a seven-year life, and be salvaged for \$1000. Revelstoke's before-tax MARR is 10 percent, it is taxed at 40 percent, and the binder has a 20 percent CCA rate. What is the company's exact after-tax IRR on this investment? Should the investment be made?
- 8.25** A slitter for sheet sandpaper owned by Abbotsford Abrasives (AA) requires regular maintenance costing \$7500 per year. Every five years it is overhauled at a cost of \$25 000. The original capital cost was \$200 000, and an additional \$25 000 in non-capital expenses was incurred at the time of installation. The machine has an expected life of 20 years and a \$15 000 salvage value, and it will not be overhauled at the end of its life. AA pays taxes at a rate of 45 percent and expects an after-tax rate of return of 10 percent on

investments. Recalling that the CCA rate for production equipment is 20 percent, what is the after-tax annual cost of the slitter?

- 8.26** Roch bought a \$100 000 machine (Machine A) on November 12, 1981. As a CCA Class 8 asset, what was its book value, measured as its contribution to the UCC for that class, at the end of 1991? Roch purchased an identical \$100 000 machine (Machine B) on November 14, 1981. What was its book value at the end of 1991?
- 8.27** A chemical recovery system costs \$30 000 and saves \$5280 each year of its seven-year life. The salvage value is estimated at \$7500. The after-tax MARR is 9 percent and taxes are at 45 percent. What is the net after-tax annual benefit or cost of purchasing the chemical recovery system?
- 8.28** Salim is considering the purchase of a backhoe for his pipeline contracting firm. The machine will cost \$110 000, last six years with a salvage value of \$20 000, and reduce annual maintenance, insurance, and labour costs by \$30 000 per year. The after-tax MARR is 9 percent and Salim's corporate tax rate is 55 percent. What is the exact after-tax IRR for this investment? What is the approximate after-tax IRR? Should Salim buy the backhoe?
- 8.29** Refer to Problem 5.21 on page 157. Grazemont Dairy has a corporate tax rate of 40 percent and the filling machine for the dairy line has a CCA rate of 30 percent. The firm has an after-tax MARR of 10 percent. On the basis of the exact IRR method, determine which alternative Grazemont Dairy should choose.

### C. More Challenging Problems

- 8.30** In 2011, the Sackville Furniture Company bought a new band saw for \$360 000. Aside from depreciation expenses, its yearly expenses totalled \$1 300 000 versus \$1 600 000 in income. How much tax (at 50 percent) would the company have paid in 2011 if it had been permitted to use each of the following depreciation schemes?
- Straight-line, with a life of 10 years and a zero salvage value
  - Straight-line, with a life of five years and a zero salvage value
  - Declining-balance, at 20 percent
  - Declining-balance, at 40 percent
  - Fully expensed that year
- 8.31** Rodney has discovered that, for the past three years, his company has been classifying as CCA Class 8 items costing between \$200 and \$500 that should be in CCA Class 12. If an estimated \$10 000 of assets per year were misclassified, what is the present worth today of the cost of this mistake? Assume that the mistake can only be corrected for assets bought in the future. Rodney's company pays taxes at 50 percent and its after-tax MARR is 9 percent.
- 8.32** CB Electronix needs to expand its capacity. It has two feasible alternatives under consideration. Both alternatives will have essentially infinite lives.

Alternative 1: Construct a new building of 20 000 square metres now. The first cost will be \$2 000 000 and annual maintenance costs will be \$10 000. In addition, the building will need to be painted every 15 years (starting in 15 years) at a cost of \$15 000.

Alternative 2: Construct a new building of 12 500 square metres now and an addition of 7500 square metres in 10 years. The first cost of the 12 500-square-metre building will

be \$1 250 000. The annual maintenance costs will be \$5000 for the first 10 years (i.e., until the addition is built). The 7500-square-metre addition will have a first cost of \$1 000 000. Annual maintenance costs of the renovated building (the original building and the addition) will be \$11 000. The renovated building will cost \$15 000 to repaint every 15 years (starting 15 years after the addition is done).

Given a CCA rate of 5 percent for the buildings, a corporate tax rate of 45 percent, and an after-tax MARR of 15 percent, carry out an annual-worth comparison of the two alternatives. Which is preferred?

*The following Ridgely Custom Metal Products (RCMP) case is used for Problems 8.33 to 8.38. RCMP must purchase a new tube bender. There are three models:*

| Model | First Cost | Economic Life | Yearly Net Savings | Salvage Value |
|-------|------------|---------------|--------------------|---------------|
| T     | \$100 000  | 5 years       | \$50 000           | \$20 000      |
| A     | 150 000    | 5 years       | 60 000             | 30 000        |
| X     | 200 000    | 3 years       | 75 000             | 100 000       |

*RCMP's after-tax MARR is 11 percent and the corporate tax rate is 52 percent. A tube bender is a CCA Class 8 asset.*

- 8.33** Using the present worth method and the least-cost multiple of the service lives, which tube bender should it buy?
- 8.34** RCMP realizes that it can forecast demand for its products for only three years in advance. The salvage value for model T after three years is \$40 000 and for model A is \$80 000. Using the present worth method and a three-year study period, which of the three alternatives is now best?
- 8.35** Using the annual worth method, which tube bender should Ridgely buy?
- 8.36** What is the approximate after-tax IRR for each of the tube benders?
- 8.37** What is the exact after-tax IRR for each of the tube benders?
- 8.38** Using the approximate after-tax IRR comparison method, which of the tube benders should Ridgely buy? (Reminder: You must look at the incremental investments.)
- 8.39** Refer to Problem 5.21 on page 157. Grazemont Dairy has a corporate tax rate of 40 percent and the filling machine for the dairy line has a CCA rate of 30 percent. The company has an after-tax MARR of 10 percent. Using an approximate IRR approach, determine which alternative Grazemont Dairy should choose.
- 8.40** A cash flow sequence has a receipt of \$10 000 today, followed by a disbursement of \$8000 at the end of this year and again next year, and then a receipt of \$5500 three years from now. The MARR is 6 percent. Taxes are at 50 percent, and the disbursements are not depreciable assets and thus can be expensed in the year they occur.
- (a) What is the ERR for this set of cash flows?
- (b) What is the approximate ERR for this set of cash flows?
- (c) Would a project with these cash flows be a good investment?

## MINI-CASE 8.1

### Flat Taxes

As discussed in this chapter, personal income taxes in most developed countries are **progressive**, meaning that the rate of taxation increases at higher income levels. However, in several countries a **flat** (or **proportional**) tax is used. In its basic form, a flat tax requires that a fixed percentage of income (for individuals) or profit (for corporations) be paid as tax. It is simple and said to be fairer for everyone, as well as more lucrative for governments.

Following the fall of communism, Russia and most of the former communist Eastern European countries instituted flat tax regimes. These countries have since had strong economic growth, and many economists credit the flat tax as an important factor in this growth. Other countries that have flat tax include Hong Kong, Saudi Arabia, Nigeria, Uruguay, and the Bahamas.

For individuals, a flat tax is very easy to manage. For most workers, no tax return is required since deductions made by employers on their behalf complete their tax compliance. Also there is usually a minimum income before any tax is payable, so that low income earners are protected.

There are several important consequences of a flat tax on businesses. One is that in most implementations profit is taxed only once, as opposed to some countries in which it is taxed at the company level and then again at the taxpayer level when profits are distributed as dividends or capital gains. Another important consequence for most implementations of a flat tax is that all capital purchases for a company are expensed in the year of purchase. Flat taxes also reduce the administration costs for companies, individuals, and governments.

### Discussion

The concept of flat taxes has generated heated debate in some developed countries. The simplicity and efficiency of flat taxes make them attractive alternatives to bloated tax bureaucracies, massively complex tax rules, and frustrating reporting requirements that are the norm. Also, there is evidence that flat taxes promote compliance and generate more tax revenue than progressive taxes. However, there are those who would be adversely affected by a change to a flat tax, and there is considerable resistance generated whenever a serious effort to institute a flat tax is made. This may be why the only major countries to move to a flat tax in recent years are ones such as Russia and the Eastern European countries that are newly instituting income taxes and consequently are not threatening entrenched interests.

### Questions:

1. Which of the following groups would likely welcome a flat tax? Which would resist it?
  - (a) Accountants
  - (b) Small business owners
  - (c) Individuals with investments in the stock market
  - (d) Government tax department workers
  - (e) Low-wage earners
  - (f) Rich people

- (g) Capital equipment manufacturers
  - (h) Leasing companies
  - (i) Welfare recipients
  - (j) You
2. If a country changed from a tax regime in which corporate profits are taxed twice to a regime in which they are taxed only once, what would likely be the effect on the stock market?
  3. In the long run, would the ability to expense capital equipment fully in the year of purchase dramatically affect a company's investment decisions? If so, do you think society is better off as a consequence of these effects, or worse off?

## Appendix 8A Deriving the Capital Tax Factor

The change in tax laws of November 13, 1981, made the formula for the CTF a bit complex. To derive the CTF formula, it is easiest to look at the situation before the laws were changed.

Before November 13, 1981, the tax benefit that could be obtained for a depreciable asset with a CCA rate  $d$  and a first cost  $P$ , when the company was paying tax at rate  $t$ , was

$Ptd$  for the first year

$Ptd(1 - d)$  for the second year

$Ptd(1 - d)^{N-1}$  for the  $N$ th year

Taking the present worth of each of these benefits and summing gives:

$$\begin{aligned} \text{PW(benefits)} &= Ptd \left( \frac{1}{(1+i)} + \frac{(1-d)}{(1+i)^2} + \cdots + \frac{(1-d)^{N-1}}{(1+i)^N} + \cdots \right) \\ &= \frac{Ptd}{(1+i)} \left( 1 + \frac{(1-d)^1}{(1+i)} + \frac{(1-d)^2}{(1+i)^2} + \cdots + \frac{(1-d)^N}{(1+i)^N} + \cdots \right) \end{aligned}$$

Noting that for  $q < 1$ ,

$$\lim_{n \rightarrow \infty} (1 + q + q^2 + \cdots + q^n) = \frac{1}{1 - q}$$

And

$$\frac{1 - d}{1 + i} < 1$$

Then:

$$\begin{aligned}
 \text{PW(benefit)} &= \frac{Ptd}{(1+i)} \left[ \frac{1}{1 - \frac{(1-d)}{(1+i)}} \right] \\
 &= \frac{Ptd}{(1+i)} \left[ \frac{1}{\frac{(1+i)}{(1+i)} - \frac{(1-d)}{(1+i)}} \right] \\
 &= \frac{Ptd}{(1+i)} \left( \frac{(1+i)}{(i+d)} \right) \\
 &= \frac{Ptd}{(i+d)}
 \end{aligned}$$

If we subtract the present worth of the tax benefits from the first cost, it will give us the present worth of the asset, taking into account all tax benefits from depreciation forever.

$$\begin{aligned}
 \text{PW(asset)} &= P - \frac{Ptd}{(i+d)} \\
 &= P \left( 1 - \frac{td}{(i+d)} \right)
 \end{aligned}$$

The factor

$$\left( 1 - \frac{td}{(i+d)} \right)$$

is now called the capital salvage factor (CSF) and was the formula in use for acquisitions before November 13, 1981.

Now, however, only half of the first cost of an asset can be used in the CCA calculations for the first year. By recognizing that the net effect of current laws is to delay the tax benefits of half of the first cost by one year, the present worth of the benefits is then

$$\begin{aligned}
 \text{PW(benefits)} &= 0.5 \left( \frac{Ptd}{(i+d)} \right) + 0.5 \left( \frac{Ptd}{(i+d)} \right) \left( \frac{1}{(1+i)} \right) \\
 &= 0.5 \frac{Ptd}{(i+d)} \left( 1 + \frac{1}{(1+i)} \right) \\
 &= 0.5 \frac{Ptd}{(i+d)} \left( \frac{(1+i)}{(1+i)} + \frac{1}{(1+i)} \right)
 \end{aligned}$$

$$= 0.5 \frac{Ptd}{(i+d)} \left( \frac{(2+i)}{(1+i)} \right)$$

$$= P \frac{td(1+\frac{i}{2})}{(i+d)(1+i)}$$

And the present worth of the asset itself is

$$PW(\text{asset}) = P - P \frac{td(1+\frac{i}{2})}{(i+d)(1+i)}$$

$$= P \left[ 1 - \frac{td(1+\frac{i}{2})}{(i+d)(1+i)} \right]$$

Thus the CTF is

$$CTF = 1 - \frac{td(1+\frac{i}{2})}{(i+d)(1+i)}$$



# Inflation

# Inflation

**Engineering Economics in Action, Part 9A: The Inflated Expert**

- 9.1** Introduction
- 9.2** Measuring the Inflation Rate
- 9.3** Economic Evaluation With Inflation
  - 9.3.1** Converting Between Real and Current Dollars
- 9.4** The Effect of Correctly Anticipated Inflation
  - 9.4.1** The Effect of Inflation on the MARR
  - 9.4.2** The Effect of Inflation on the IRR
- 9.5** Project Evaluation Methods With Inflation

**Review Problems**

**Summary**

**Engineering Economics in Action, Part 9B: Exploiting Volatility**

**Problems**

**Mini-Case 9.1: Economic Comparison of High Pressure and Conventional Pipelines: Associated Engineering**

**Appendix 9A: Computing a Price Index**

## ENGINEERING ECONOMICS IN ACTION, PART 9A

### The Inflated Expert

Terry left Canadian Widgets to go back for his final term of school. Naomi and Terry had worked through several projects over Terry's last few work terms, and Naomi had been increasingly taking part in projects involving sister companies of Canadian Widgets; all were owned by Global Conglomerate Inc., often referred to as "head office."

"There's a guy from head office to see you, Naomi." It was Carole announcing the expected visitor, Bill Astad. Bill was one of the company's troubleshooters. His current interest concerned a sister company, Mexifab, a maquiladora on the Mexican border with Texas. (A maquiladora is an assembly plant that manufactures finished goods in northern Mexico under special tariff and tax rules.) After a few minutes of socializing, Bill explained the concern.

"It's the variability in the Mexican inflation rate that causes the problems. Mexico gets a new president every six years, and usually, about the time the president changes, the economy goes out of whack. And we can't price everything in dollars or euros. We do some of that, but we are located in Mexico and so we have to use Mexican pesos for a lot of our transactions."

"I understand from Anna Kulkowski that you know something about how to treat problems like that," Bill continued.

Naomi smiled to herself. She had written a memo a few weeks earlier pointing out how Canadian Widgets had been missing some good projects by failing to take advantage of the current very low inflation rates, and suddenly she was the expert!

"Well," she said, "I might be able to help. What you can do is this."

## 9.1

### Introduction

Prices of goods and services bought and sold by individuals and firms change over time. Some prices, like those of agricultural commodities, may change several times a day. Other prices, like those for sugar and paper, change infrequently. While prices for some consumer goods and services occasionally decrease (as with high-tech products), on average it is more typical for prices to increase over time.

**Inflation** is the increase over time in average prices of goods and services. It can also be described as a decrease in the purchasing power of money over time. While most developed countries have experienced inflation in most years since World War II, there have been short periods when average prices in some countries have fallen. A decrease over time in average prices is called **deflation**. It can also be viewed as an increase in the purchasing power of money over time.

Because of inflation or deflation, prices are likely to change over the lives of most engineering projects. These changes will affect cash flows associated with the projects. Engineers may have to take predicted price changes into account during project evaluation to prevent the changes from distorting decisions.

In this chapter, we shall discuss how to incorporate an expectation of inflation into project evaluation. We focus on inflation because it has been the dominant pattern of price changes since the beginning of the twentieth century. The chapter begins with a discussion of how inflation is measured. We then show how to convert cash flows that occur at different points in time into dollars with the same purchasing power. We then consider how inflation affects the MARR, the internal rate of return, and the present worth of a project.

## 9.2 | Measuring the Inflation Rate

The **inflation rate** is the rate of increase in average prices of goods and services over a specified time period, usually a year. If prices of all goods and services moved up and down together, determining the inflation rate would be trivial. If all prices increased by 2 percent over a year, it would be clear that the average inflation rate would also be 2 percent. But prices do not move in perfect synchronization. In any period, some prices will increase, others will fall, and some will remain about the same. For example, candy bars are about 10 times as costly now as they were in the 1960s, but some electronics are about the same price or cheaper.

Because prices do not move in perfect synchronization, a variety of methods have been developed to measure the inflation rate. Most countries' governments track movement of average prices for a number of different collections of goods and services and calculate inflation rates from the changes in prices in these collections over time.

One set of prices typically tracked consists of goods and services bought by consumers. This set forms the basis of the **consumer price index (CPI)**. The CPI for a given period relates the average price of a fixed “basket” of these goods in the given period to the average price of the same basket in a *base period*. For example, Figure 9.1 shows the CPI for Canada for the period 1920 to 2010. The CPI for this chart has the year 2002 as the base year. The base year index is set at 100.

### N E T   V A L U E   9 . 1

#### Statistics Canada

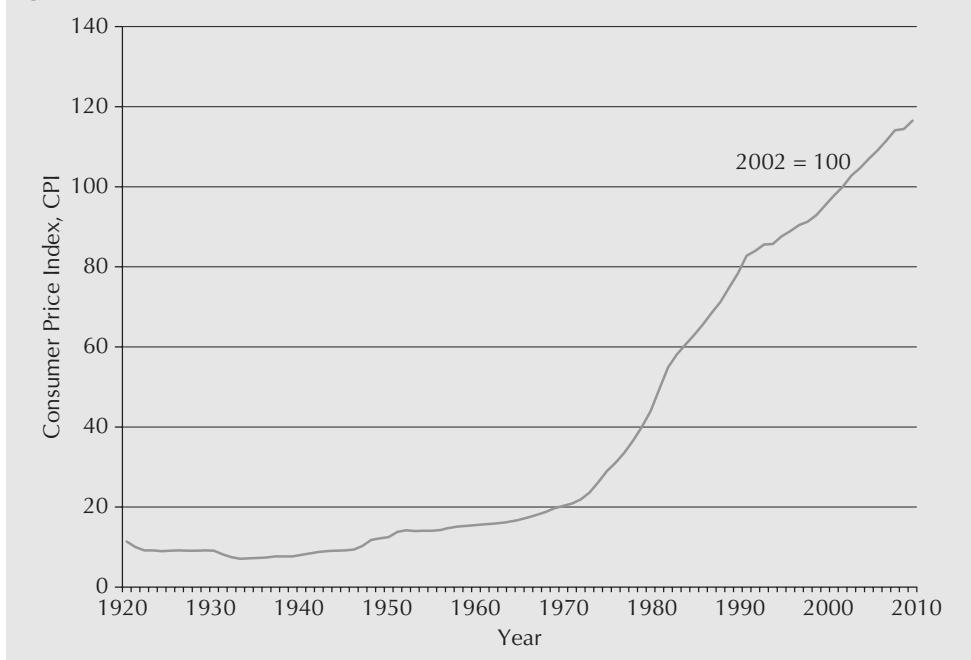
A prime source of Canadian statistics, including the CPI and inflation rates, is Statistics Canada. Figures are readily available from the Statistics Canada website at [www.statcan.gc.ca](http://www.statcan.gc.ca). Four main categories for Canadian statistics are economy, land, people, and state. Each of these categories includes information that can be useful in engineering economics. For example, the economy category includes the latest economic indicators such as gross domestic product (GDP) and personal spending on consumer goods and services; and manufacturing and

construction figures such as the new housing price index, energy supply and demand, and the number of manufacturing shipments. The land category includes information on the environment (air pollution, expenditures on environmental protection by industry, etc.), which may be useful in estimating the impact of environmental factors in engineering projects. In the people category, statistics on population, labour, and employment can be found. Lastly, in the state category, statistics specific to the public sector (e.g., employment, assets and liabilities, and revenue and expenditures) can be obtained.

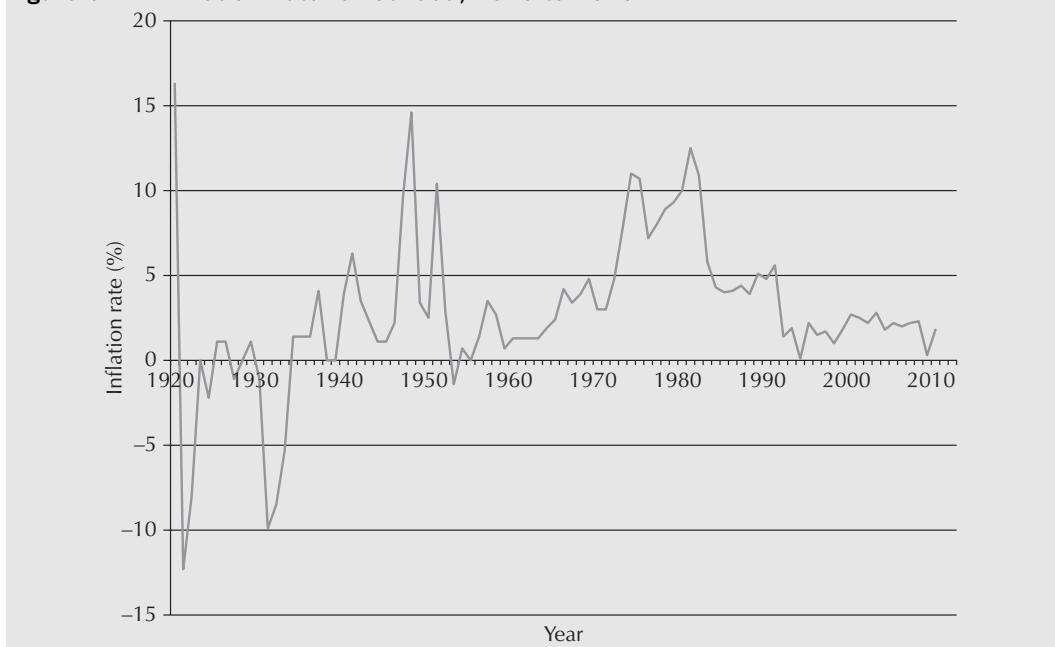
The index for any other year indicates the number of dollars needed in that year to buy the fixed basket of goods that cost \$100 in 2002. Figure 9.1 shows that a basket of goods that cost \$100 in 2002 (the base year) would have cost approximately \$78 in 1988, and slightly more than \$116 in 2010. More information about the CPI can be found on the Statistics Canada website at [www.statcan.gc.ca](http://www.statcan.gc.ca) (see also Net Value 9.1).

An inflation rate can be estimated directly from the CPI by expressing the changes in the CPI as a year-by-year percentage change. This is probably the most commonly used estimate of a national inflation rate. Figure 9.2 shows the national inflation rate for the period from 1920 to 2010 as derived from the CPI quantities in Figure 9.1.

It is important to note that, although the CPI is a commonly accepted inflation index, many different indexes are used to measure inflation. The value of an index depends on

**Figure 9.1 CPI for Canada, 1920 to 2010**

the method used to compute the index and the set of goods and services for which the index measures price changes. To judge whether an index is appropriate for a particular purpose, the analyst should know how the goods and services for which he or she is estimating inflation compare with the set of goods and services used to compute the index. For this reason, we provide Appendix 9A, in which we illustrate the computation of one popularly used index.

**Figure 9.2 Inflation Rate for Canada, 1920 to 2010**

CPI values and inflation rates vary considerably over time, as seen in Figures 9.1 and 9.2, and can be even more extreme. Low expected rates of inflation may be safely ignored, given the typical imprecision of predicted future cash flows. However, when expected inflation is high, it is necessary to include inflation in detailed economic calculations to avoid rejecting good projects.

Throughout the rest of this chapter, we assume that an analyst is able to obtain estimates for expected inflation rates over the life of a project and that project cash flows will change at the same rate as average prices. Consequently, the cash flows for a project can be assumed to increase at approximately the rate of inflation per year.

## 9.3 | Economic Evaluation With Inflation

---

When prices change, the amount of goods a dollar will buy changes too. If prices fall, more goods may be bought for a given number of dollars, and the value of a dollar has risen. If prices rise, fewer goods may be bought for a given number of dollars, and the value of a dollar has fallen.

In project evaluation, we cannot make comparisons of dollar values across time without taking the price changes into account. We want dollars, not for themselves, but for what we can get for them. Workers are not directly interested in the money wages they will earn in a job. They are interested in how many hours of work it will take to cover expenses for their families, or how long it will take them to accumulate enough to make down payments on houses. Similarly, investors want to know if making an investment now will enable them to buy more real goods in the future, and by how much the amount they can buy in the future will increase. To know if an investment will lead to an increase in the amount they can buy in the future, they must take into account expected price changes.

We can take price changes into account in an approximate way by measuring the cash flows associated with a project in monetary units of constant purchasing power called **real dollars** (sometimes called **constant dollars**). This is in contrast to **current dollars** (sometimes called **actual dollars** or **nominal dollars**), which are expressed in the monetary units at the time the cash flows occur.

For example, if a photocopier will cost \$2200 one year from now, the \$2200 represents **current dollars** since that is the amount that would be paid at that time. If inflation is expected to be 10 percent over the year, the \$2200 is equivalent to \$2000 in real dollars. Of course, depending on the currency, one could also use the term “real euros” or “real pounds,” for example.

Real dollars always need to be associated with a particular date (usually a year), called the **base year**. The base year need not be the present; it could be any time. People speak of “2000 dollars” or “1985 dollars” to indicate that real dollars are being used as well as indicating the base year associated with them. Provided that cash flows occurring at different times are converted into real dollars with the same base year, they can be compared fairly in terms of buying power.

### 9.3.1 Converting Between Real and Current Dollars

Converting current dollars in year  $N$  into real dollars in year  $N$  relative to a base year 0 is straightforward, provided that the value of a global price index like the CPI at year  $N$  relative to the base year is available. Let

$C_N$  = current dollars in year  $N$

$R_{0,N}$  = real dollars equivalent to  $C_N$  relative to year 0, the base year

$I_{0,N}$  = the value of a global price index (like the CPI) at year  $N$ , relative to year 0

Then the conversion from current to real dollars is

$$R_{0,N} = \frac{C_N}{I_{0,N}/100} \quad (9.1)$$

Note that in Equation (9.1),  $I_{0,N}$  is divided by 100 to convert it into a fraction because of the convention that a price index is set at 100 for the base year.

Transforming current dollar values into real dollars gives only an approximate offset to the effect of inflation. The reason is that there may be no readily available price index that accurately matches the “basket” of goods and services being evaluated. Despite the fact that available price indexes are approximate, they do provide a reasonable means of converting current cash flows to real cash flows relative to a base year.

An alternative means of converting current dollars to real dollars is available if we have an estimate for the average yearly inflation between now (year 0) and year  $N$ . Let

$C_N$  = current dollars in year  $N$

$R_{0,N}$  = real dollars equivalent to  $C_N$  relative to year 0, the base year

$f$  = the inflation rate per year, assumed to be constant from year 0 to year  $N$

Then the conversion from current dollars in year  $N$  to real dollars in year  $N$  relative to the base year 0 is

$$R_{0,N} = \frac{C_N}{(1+f)^N}$$

When the base year is omitted from the notation for real dollars, it is understood that the current year (year 0) is the base year, as in

$$R_N = \frac{C_N}{(1+f)^N} \quad (9.2)$$

Equation (9.2) can also conveniently be written in terms of the present worth compound interest factor

$$R_N = C_N(P/F,f,N) \quad (9.3)$$

### EXAMPLE

### 9.1

Elliot Weisgerber's income rose from \$40 000 per year in 2010 to \$42 000 in 2013. At the same time the CPI (base year 2000) rose from 113.5 in 2010 to 122.3 in 2013. Was Elliot worse off or better off in 2013 compared with 2010?

We can convert Elliot's current dollar income in 2010 and 2013 into real dollars. This will tell us if his total purchasing power increased or decreased over the period from 2010 to 2013. Since the base year for the CPI is 2000, we will compare his 2010 and 2013 incomes in terms of real 2000 dollars.

His real incomes in 2010 and 2013 in terms of 2000 dollars, using Equation (9.1), were

$$R_{00,10} = 40\,000/1.135 = 35\,242$$

$$R_{00,13} = 42\,000/1.223 = 34\,342$$

Even though Elliot's current dollar income rose between 2010 and 2013, his purchasing power fell, since the real dollar value of his income, according to the CPI, fell about 3 percent. ■

**EXAMPLE****9.2**

The cost of replacing a storage tank one year from now is expected to be €2 000 000. If inflation is assumed to be 5 percent per year, what is the cost of replacing the storage tank in real (today's) euros?

First, note that the €2 000 000 is expressed in current euros one year from today. The cost of replacing the tank in real (today's) euros can be found by letting

$C_1 = 2\,000\,000$  = the current cost one year from the base year (today)

$R_1$  = the real euro cost of the storage tank in one year

$f$  = the inflation rate per year

Then, with Equation (9.2)

$$R_1 = \frac{C_1}{1+f} = \frac{2\,000\,000}{1.05} = €1\,904\,762$$

Alternatively, Equation (9.3) gives

$$R_1 = C_1(P/F, 5\%, 1) = 2\,000\,000 (0.9524) = 1\,904\,762$$

The €2 000 000 current cost is equivalent to €1 904 762 real (today's) euros at the end of one year. ■

**EXAMPLE****9.3**

The cost of replacing a storage tank in 15 years is expected to be €2 000 000. If inflation is assumed to be 5 percent per year, what is the cost of replacing the storage tank 15 years from now in real (today's) euros?

The cost of the tank 15 years from now in real euros can be found by letting

$C_{15} = 2\,000\,000$  = the current cost 15 years from the base year (today)

$R_{15}$  = the real euro cost of the storage tank in 15 years

$f$  = the inflation rate per year

Then, with the use of Equation (9.2), we have

$$R_{15} = \frac{C_{15}}{(1+f)^{15}} = \frac{2\,000\,000}{(1.05)^{15}} = 962\,040$$

Alternatively, Equation (9.3) gives

$$R_{15} = C_{15}(P/F, 5\%, 15) = 2\,000\,000 (0.48102) = 962\,040$$

In 15 years, the storage tank will cost €962 040 in real (today's) euros. Note that this €962 040 is money to be paid 15 years from now. What this means is that the new storage tank can be replaced at a cost that would have the same purchasing power as about €962 040 today.

Now that we have the ability to convert from current to real dollars using an index or an inflation rate, we turn to the question of how inflation affects project evaluation.



## S P R E A D S H E E T S A V V Y

Preparation of supporting documents for an engineering report usually entails printing portions of one or more worksheets or charts for inclusion in an appendix or in the main body of the report. There are many customizations that can be made to a printed Excel spreadsheet to identify the file it belongs to, its page number in sequence, or simply the date the printout was produced. This Spreadsheet Savvy provides some ideas on what some of these options are.

Most of these features can be located on the Page Layout tab of the ribbon. The most basic of these options is to set the size of the paper (e.g., legal versus letter), the orientation of the page (e.g., landscape versus portrait), and the page margins (e.g., wide, normal, or other settings, including custom). The Page Setup dialogue box (click on the small arrow to the bottom right of the Page Layout portion of the ribbon) allows you to set these features, as well as a number of other characteristics of the printed page. One handy option is to add a header or footer to the page (select the Header/Footer tab in the Page Setup dialogue box). Under Custom Header/Footer, you can insert one or more of text, page number, date, time, file path, or name and sheet name. These can be placed on the left or right or centred on the page.

Selected worksheet areas can be scaled to fit the page width or height or scaled to a set percentage of its original size by choosing the Scale to Fit option in the Page Layout tab. These choices are not directly available for chart objects, but scale options might be available under print options, depending on the printer used. Scaling to fit the page is useful when you want to print an area of a worksheet that is just a bit too large to fit onto a page. This option will automatically scale the printed area just enough so that either all the rows or all the columns appear. In addition to setting these scale options, worksheet gridlines and row and column headers can be shown or suppressed in the printed output by selecting the appropriate Sheet Options area on the Page Layout tab.

If you are working with a large worksheet and have data that runs over several pages, the option to print the same header on each page can be helpful. This way, the reader does not lose track of what each column means. To set the print header to appear on each printed page, select Print Titles on the Page Layout tab of the ribbon (under Page Setup). This will prompt you to enter the cell range containing the title to be repeated.

Another useful feature for a large worksheet is to be able to force a page break at a natural location in the data rather than having Excel insert a page break at an inconvenient place. This is done by selecting the row *below the one you want the page break* and then clicking on the Breaks option under the Page Setup section of Page Layout. Page breaks can also be removed one at a time or all at once.

## 9.4

## The Effect of Correctly Anticipated Inflation

The main observation made in this section is that engineers must be aware of potential changes in price levels over the life of a project. We shall see that when future inflation is expected over the life of a project, the MARR needs to be increased. Engineers need to recognize this effect of inflation on the MARR to avoid rejecting good projects.

### 9.4.1 The Effect of Inflation on the MARR

If we expect inflation, the number of current dollars that will be returned in the future does not tell us the value, in terms of purchasing power, of the future cash flow. The purchasing power of the earnings from an investment depends on the *real* dollar value of those earnings.

The **current interest rate** is the stated or observed interest rate based on current dollars. If we wish to earn interest at the current interest rate,  $i$ , on a one-year investment, and we invest  $\$M$ , the investment will yield  $\$M(1 + i)$  at the end of the year. If the inflation rate over the next year is  $f$ , the real value of our cash flow is  $\$M(1 + i)/(1 + f)$ . We can use this to define the *real* interest rate. The **real interest rate**,  $i'$ , is the interest rate that would yield the same number of real dollars in the absence of inflation as the current interest rate yields in the presence of inflation.

$$\begin{aligned} M(1 + i') &= M\left(\frac{1 + i}{1 + f}\right) \\ i' &= \frac{1 + i}{1 + f} - 1 \end{aligned} \quad (9.4)$$

We may see terms like *real rate of return* or *real discount rate*. These are just special cases of the real interest rate.

The definition of the real interest rate can be turned around by asking the following question: If an investor wants a real rate of return,  $i'$ , over the next year, and the inflation rate is expected to be  $f$ , what current interest rate,  $i$ , must be realized to get a real rate of return of  $i'$ ?

The answer can be obtained with some manipulation of the definition of the real interest rate in Equation (9.4):

$$i = (1 + i')(1 + f) - 1 \text{ or, equivalently, } i = i' + f + i'f \quad (9.5)$$

Therefore, an investor who desires a real rate of return  $i'$  and who expects inflation at a rate of  $f$  will require a current interest rate  $i = i' + f + i'f$ . This has implications for the current MARR used in economic analyses. The **current MARR** is the minimum acceptable rate of return when cash flows are expressed in current dollars. If investors expect inflation, they require higher current rates of return on their investments than if inflation were not expected. The current MARR then will be the real MARR plus an upward adjustment that reflects the effect of inflation. The **real MARR** is the minimum acceptable rate of return when cash flows are expressed in real, or constant, dollars.

If we denote the current MARR by  $MARR_C$  and the real MARR by  $MARR_R$ , we have from Equation (9.5)

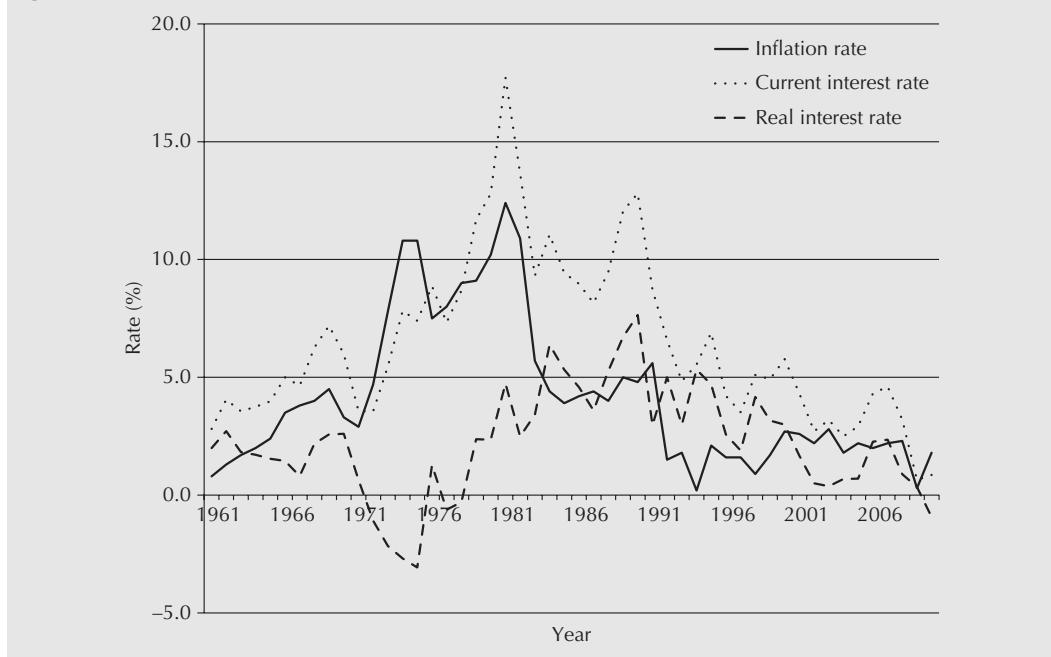
$$MARR_C = MARR_R + f + MARR_R \times f \quad (9.6)$$

Note that if  $MARR_R$  and  $f$  are small, the term  $MARR_R \times f$  may be ignored and  $MARR_C = MARR_R + f$  can be used as a “back of the envelope” approximation.

The real MARR can also be expressed as a function of the current MARR and the expected inflation rate:

$$MARR_R = \frac{1 + MARR_C}{1 + f} - 1 \quad (9.7)$$

Figure 9.3 shows the Canadian experience with inflation, the current prime interest rate, and the real interest rate for the 1961–2010 period. From 1961 to 1971, when

**Figure 9.3 Canadian Inflation Rate and Current and Real Interest Rates 1961–2010**

inflation was moderate and stable, the real interest rate was also stable, except for one blip in 1967. In the 1970s, conditions were very different when inflation exploded. This was due partly to large jumps in energy prices. Real interest rates were negative for the period 1972 to 1975 and 1977 to 1978. In the 1980s and early 1990s, real interest rates were quite high. The rest of the 1990s and early 2000s experienced lower inflation rates and real interest rates.

The high inflation rates of the 1970s were very unusual. Inflation in the range of 2 percent to 4 percent per year is more typical of developed countries like Canada. Averages of real interest rates and current inflation rates over subperiods are shown in Table 9.1.

**Table 9.1 Average Canadian Real and Current Interest Rates**

| Period    | Average<br>Real Interest Rate (%) | Average<br>Current Interest Rate (%) |
|-----------|-----------------------------------|--------------------------------------|
| 1961–1971 | 1.82                              | 2.75                                 |
| 1972–1981 | 0.08                              | 8.47                                 |
| 1982–1991 | 4.84                              | 5.29                                 |
| 1992–2003 | 2.95                              | 1.81                                 |

**EXAMPLE****9.4**

Security Trust is paying 12 percent on one-year guaranteed investment certificates (GICs). The inflation rate is expected to be 5 percent over the next year. What is the real rate of interest? For a \$5000 GIC, what will be the real dollar value of the amount received at the end of the year?

The real interest rate is

$$i' = \frac{1+i}{1+f} - 1 = \frac{1.12}{1.05} - 1 = 0.067, \text{ or } 6.7\%$$

A \$5000 GIC will return \$5600 at the end of the year. The real value of the \$5600 in *today's* dollars is  $\$5600/1.05 = \$5333$ . This is the same as if there were no inflation and the investment earned 6.7 percent interest. ■

**EXAMPLE****9.5**

Susan got a \$1000 present from her aunt on her 16th birthday. She has noticed that Security Trust offers 6.5 percent on one-year guaranteed investment certificates (GICs). Her mother's business newspaper indicates that analysts are predicting an inflation rate of about 3.5 percent for the coming year. Susan's real MARR for such investments is 4 percent. If the analysts are correct, what is Susan's current MARR? Should she invest?

If the analysts are correct, Susan's current MARR is

$$\begin{aligned} \text{MARR}_C &= \text{MARR}_R + f + \text{MARR}_R \times f \\ &= 0.04 + 0.035 + (0.04)(0.035) \\ &= 0.0764 \end{aligned}$$

Susan's current MARR is about 7.64 percent. Since the current interest rate on the GIC is only 6.5 percent, she should not invest in the GIC. ■

### 9.4.2 The Effect of Inflation on the IRR

The effect of expected inflation on the current internal rate of return of a project is similar to the effect of inflation on the current MARR. Suppose that we are considering a  $T$ -year investment. The **current internal rate of return** on the project,  $\text{IRR}_C$ , is the rate of return of the project on the basis of current dollar cash flows. It can be found by solving for  $i^*$  in

$$\sum_{t=0}^T \frac{C_t}{(1_C + i^*)^t} = 0$$

where

$C_t$  = the current cash flow in period  $t$  (receipts – disbursements)

$T$  = the number of time periods

$i^*$  = the actual internal rate of return

Suppose further that a yearly inflation rate  $f$  is expected over the  $T$ -year life of the project. In terms of real dollars (with a base year of the time of the first cost), the current cash flow in period  $t$  can be written as  $C_t = R_t(1 + f)^t$  where  $R_t$  refers to the *real* dollar amount equivalent to the cash flow  $C_t$ . The expression that gives the current internal rate of return can be rewritten as

$$\sum_{t=0}^T \frac{R_t(1+f)^t}{(1+i^*)^t} = 0 \quad (9.8)$$

In contrast, the **real internal rate of return** for the project,  $\text{IRR}_R$ , is the rate of return obtained on the real dollar cash flows associated with the project. It is the solution for  $i'$  in

$$\sum_{t=0}^T \frac{R_t}{(1+i')^t} = 0 \quad (9.9)$$

What is the relationship between  $\text{IRR}_R$  and  $\text{IRR}_C$ ? We have, from Equations (9.4) and (9.5),

$$\frac{1}{1+i'} = \frac{1+f}{1+i} \quad \text{or} \quad i = i' + f + i'f$$

and thus, analogous to Equation (9.5),

$$\text{IRR}_C = \text{IRR}_R + f + \text{IRR}_R \times f \quad (9.10)$$

Or, analogous to Equation (9.4), the real IRR can be expressed in terms of the current IRR and the inflation rate:

$$\text{IRR}_R = \frac{1 + \text{IRR}_C}{1 + f} - 1 \quad (9.11)$$

In summary, the effect of inflation on the IRR is that the current IRR will be the real IRR plus an upward adjustment that reflects the effect of inflation.

### EXAMPLE

### 9.6

Consider a two-year project that has a \$10 000 000 first cost and that is expected to bring about savings of \$15 000 000 at the end of the two years. If inflation is expected to be 5 percent per year and the real MARR is 13 percent, should the project be undertaken? Base your answer on an IRR analysis.

From the information given,  $C_0 = -10 000 000$ ,  $C_2 = 15 000 000$ , and  $f = 0.05$ . The current IRR can be found by solving for  $i$  in

$$C_0 + \frac{C_2}{(1+i)^2} = 0$$

$$10 000 000 = 15 000 000/(1+i)^2$$

which leads to a current IRR of 22.475 percent.

The real IRR is then

$$\begin{aligned} \text{IRR}_R &= \frac{1 + \text{IRR}_C}{1 + f} - 1 \\ &= \frac{1 + 0.22475}{1 + 0.05} - 1 \\ &= 0.1664 \text{ or } 16.64\% \end{aligned}$$

Since the real IRR exceeds the real MARR, the project should be undertaken. ■

In conclusion, the impact of inflation on the current MARR and the current IRR is that both have an adjustment for expected inflation implicitly included in them. The main implication of this observation is that, since both the current MARR and the current IRR increase in the same fashion, any project that was acceptable without inflation remains acceptable when inflation is expected. Any project that was unacceptable remains unacceptable.

## 9.5 | Project Evaluation Methods With Inflation

---

The engineer typically starts a project evaluation with an observed (current) MARR and projections of cash flows. As we have seen, the current MARR has two parts: the real rate of return on investment that investors require to put money into the company, plus an adjustment for the expected rate of inflation. The engineer usually observes only the sum and not the individual parts.

As for the projected cash flows, these are typically based on current prices. Because the projected cash flows are based on prices of the period in which evaluations are being carried out, they are in *real* dollars. They do not incorporate the effect of inflation. In this case, the challenge for the engineer is to correctly analyze the project when he or she has a *current* MARR (which incorporates inflation implicitly) and *real* cash flows (which do not take inflation into account).

Though it is common to do so, the engineer or analyst does not always start out with a *current* MARR and *real* cash flows. The cash flows may already have inflation implicitly factored in (in which case the cash flows are said to be current amounts). To carry out a project evaluation properly, the analyst must know whether inflation has been accounted for already in the MARR and the cash flows or whether it needs to be dealt with explicitly.

As a brief example, consider a one-year project that requires an investment of \$1000 today and yields \$1200 in one year. The current MARR is 25 percent. Whether the project is considered acceptable will depend on whether the \$1200 in one year is understood to be the current cash flow in one year or if it is the real value of the cash flow received in one year.

If the \$1200 is taken to be the current cash flow, the current internal rate of return,  $\text{IRR}_C$ , is found by solving for  $i^*$  in

$$-1000 + \frac{1200}{(1 + i^*)} = 0$$

$$i^* = \text{IRR}_C = 20\%$$

Hence, the project would not be considered economical. However, if the \$1200 is taken to be the real value of the cash flow in one year, and inflation is expected to be 5 percent over the year, then the current internal rate of return is found by solving for  $i^*$  in:

$$-1000 + \frac{1200 (1 + 0.05)}{(1 + i^*)} = 0$$

$$i^* = \text{IRR}_C = 26\%$$

Hence, the project *would* be considered acceptable. As seen in this example, the economic viability of the project may depend on whether the \$1200 in one year has inflation implicitly factored in (i.e., is taken to be the current amount). This is why it is important to know what type of cash flows you are dealing with.

If the engineer has an estimate of inflation, there are two equivalent ways to carry out a project evaluation properly. The first is to work with current values for cash flows and current interest rates. The second is to work with real values for cash flows and real interest rates. *The two methods should not be mixed.*

These two methods of dealing with expected inflation, as well as two incorrect methods, are shown in Table 9.2.

The engineer must have a forecast of the inflation rate over the life of the project in order to adjust the MARR or cash flows for inflation. The best source of such forecasts may be the estimates of experts. Financial publications regularly report such predictions for relatively short periods of up to one year. Because there is evidence that even the short-term estimates are not totally reliable, and estimates for longer periods will necessarily be imprecise, it is

**Table 9.2 Methods of Incorporating Inflation Into Project Evaluation**

|   |
|---|
| <b>1. Real MARR and Real Cash Flows</b>   |
| The real MARR does not include the effect of expected inflation.<br>Cash flows are determined by today's prices.<br><b>Correct</b>                                      |
| <b>2. Current MARR and Current Cash Flows</b>   |
| The current MARR includes the effect of anticipated inflation.<br>Cash flows include increases due to inflation.<br><b>Correct</b>                                      |
| <b>3. Current MARR and Real Cash Flows</b>  |
| The current MARR includes the effect of anticipated inflation.<br>Cash flows are determined by today's prices.<br><b>Incorrect:</b> Biased against investments          |
| <b>4. Real MARR and Current Cash Flows</b>  |
| The real MARR does not include the effect of expected inflation.<br>Cash flows include increases due to inflation.<br><b>Incorrect:</b> Biased in favour of investments |

good practice to determine a range of possible inflation values for both long- and short-term projects. The engineer should test for sensitivity of the decision to values in the range. The subject of sensitivity analysis is addressed more fully in Chapter 12. Close-Up 9.1 discusses atypical patterns of price changes that may be specific to certain industries.

| CLOSE-UP 9.1   | Relative Price Changes |
|--|------------------------|
| Engineers usually expect prices associated with a project to move with the general inflation rate. However, there are situations in which it makes sense to expect prices associated with a project to move differently from the average. This can happen when there are atypical forces affecting either the supply of or the demand for the goods.   |                        |
| For example, reductions in the availability of logs in North America caused a decrease in the supply of wood for construction, furniture, and pulp and paper in the 1980s. Average wood prices more than doubled between 1986 and 1995. This was about twice the increase in the CPI over that period. Since then, average wood prices have been decreasing while the CPI continues to increase. Another example is the price of computers. Product development and increases in productivity have led to increases in the supply of computers. This, in turn, has led to reductions in the relative price of computing power. |                        |
| Changes in the relative prices of the goods sold by a specific industry will generally not have a noticeable effect on a MARR because investors are concerned with the overall purchasing power of the dollars they receive from an investment. Changes in the relative prices of the goods of one industry will not have much effect on investors' abilities to buy what they want.   |                        |
| Because the relative price changes will not affect the MARR, the analyst must incorporate expected relative price changes directly into the expected cash flows associated with a project. If the rate of relative price change is expected to be constant over the life of the project, this can be done using a geometric gradient to present worth conversion factor.   |                        |

**EXAMPLE****9.7**

Jagdeep can put his money into an investment that will pay him \$1000 a year for the next four years and \$10 000 at the end of the fifth year. Inflation is expected to be 5 percent over the next five years. Jagdeep's real MARR is 8 percent. What is the present worth of this investment?

The present worth may be obtained with real dollar cash flows and a real MARR or with current dollar cash flows and a current MARR.

The first solution approach will be to use real dollar and  $MARR_R$ . The real dollar cash flows in terms of today's dollars are

$$\begin{aligned} R_1, R_2, R_3, R_4, R_5 &= \frac{C_1}{(1+f)}, \frac{C_2}{(1+f)^2}, \frac{C_3}{(1+f)^3}, \frac{C_4}{(1+f)^4}, \frac{C_5}{(1+f)^5} \\ &= \frac{1000}{(1.05)}, \frac{1000}{(1.05)^2}, \frac{1000}{(1.05)^3}, \frac{1000}{(1.05)^4}, \frac{10\,000}{(1.05)^5} \end{aligned}$$

The present worth of the real cash flows, discounted by  $MARR_R = 8$  percent, is

$$\begin{aligned} PW &= \frac{1000}{(1.05)(1.08)} + \frac{1000}{(1.05)^2(1.08)^2} + \frac{1000}{(1.05)^3(1.08)^3} \\ &\quad + \frac{1000}{(1.05)^4(1.08)^4} + \frac{10\,000}{(1.05)^5(1.08)^5} \\ &\approx 8282 \end{aligned}$$

The present worth of Jagdeep's investment is about \$8282.

Alternatively, the present worth can be found in terms of current dollars and  $MARR_C$ :

$$\begin{aligned} PW &= \frac{1000}{(1+MARR_C)} + \frac{1000}{(1+MARR_C)^2} + \frac{1000}{(1+MARR_C)^3} \\ &\quad + \frac{1000}{(1+MARR_C)^4} + \frac{10\,000}{(1+MARR_C)^5} \end{aligned}$$

where

$$MARR_C = MARR_R + f + MARR_R \times f$$

Note that this is the sum of a four-period annuity with equal payments of \$1000 for four years and a single payment of \$10 000 in period 5. With this observation, the present worth computation can be simplified by the use of compound interest formulas:

$$PW = 1000(P/A, MARR_C, 4) + 10\,000(P/F, MARR_C, 5)$$

With a real MARR of 8 percent and an inflation rate of 5 percent, the current MARR is then

$$\begin{aligned} MARR_C &= MARR_R + f + MARR_R \times f \\ &= 0.08 + 0.05 + (0.08)(0.05) \\ &= 0.134 \end{aligned}$$

and the present worth of Jagdeep's investment is

$$\begin{aligned} \text{PW} &= 1000(P/A, 13.4\%, 4) + 10\,000(P/F, 13.4\%, 5) \\ &\approx 8282 \end{aligned}$$

The present worth of Jagdeep's investment is about \$8282, as was obtained through the use of the real MARR and a conversion from current to real dollars. ■

Though there are two distinct means of correctly adjusting for inflation in project analysis, the norm for engineering analysis is to make comparisons with the current MARR. One reason this is done has to do with how a MARR is chosen. As discussed in Chapter 3, the MARR is based on, among other things, the cost of capital. Since lenders and investors recognize the need to have a return on their investments higher than the expected inflation rate, they will lend to or invest in companies only at a rate that exceeds the inflation rate. In other words, the cost of capital already has inflation included. A MARR based on this cost of capital already includes, to some extent, inflation.

Consequently, if inflation is fairly static (even if it is high), a *current* dollar MARR is sensible. On the other hand, if changes in inflation are foreseen, or if sensitivity analysis specifically for inflation is desired, it may be wise to set a *real* dollar MARR and recognize an inflation rate explicitly in the analysis.

### EXAMPLE

### 9.8

Lethbridge Communications is considering an investment in plastic moulding equipment for its product casings. The project involves \$150 000 in first costs and is expected to generate net savings (in current dollars) of \$65 000 per year over its three-year life. The company forecasts an inflation rate of 15 percent over the next year, and then inflation of 10 percent per year thereafter. Its real dollar MARR is 5 percent. Should this project be accepted on the basis of an IRR analysis?

In this problem, the inflation rate is not constant over the life of the project, so it is easiest to consider the cash flows for each year separately and to work in real dollars. First, as shown in Table 9.3, the current cash flows are converted into real cash flows.

**Table 9.3 Converting From Current to Real Dollars for Lethbridge Communications**

| Year | Current Dollars | Real Dollars   |
|------|-----------------|--|
| 0    | -\$150 000      | -\$150 000   |
| 1    | 65 000          | $65\,000(P/F, 15\%, 1)$<br>$= 65\,000(0.86957)$                        |
| 2    | 65 000          | $65\,000(P/F, 15\%, 1)(P/F, 10\%, 1)$<br>$= 65\,000(0.86957)(0.90909)$ |
| 3    | 65 000          | $65\,000(P/F, 15\%, 1)(P/F, 10\%, 2)$<br>$= 65\,000(0.86957)(0.82645)$ |

Then, the real IRR can be found by solving for  $i'$  in

$$56\,522(P/F,i',1) + 51\,384(P/F,i',2) + 46\,713(P/F,i',3) = 150\,000$$

At  $i' = 1\%$ , LHS (left-hand side) = 151 673

At  $i' = 2\%$ , LHS = 148 821

The real IRR is between 1 percent and 2 percent. This is less than the real dollar MARR of 5 percent, so the project should not be undertaken. ■

**EXAMPLE****9.9**

Glasgow Resources has been offered a contract to sell land to the government at the end of 20 years. The contract states that Glasgow will get \$500 000 after 20 years from today, with no costs or benefits in the intervening years. A financial analyst for the firm believes that the inflation rate will be 4 percent for the next two years, rise to 15 percent for the succeeding 10 years, and then go down to 10 percent, where it will stay forever. Glasgow's real dollar MARR is 10 percent. What is the present worth of the contract?

In this case, it is easiest to proceed by calculating the current dollar MARR for each of the different inflation periods:

$$\begin{aligned} \text{MARR}_C, \text{years } 13 \text{ to } 20 &= 0.10 + 0.10 + (0.10)(0.10) \\ &= 0.21 \text{ or } 21\% \end{aligned}$$

$$\begin{aligned} \text{MARR}_C, \text{years } 3 \text{ to } 12 &= 0.10 + 0.15 + (0.10)(0.15) \\ &= 0.265 \text{ or } 26.5\% \end{aligned}$$

$$\begin{aligned} \text{MARR}_C, \text{years } 0 \text{ to } 2 &= 0.10 + 0.04 + (0.10)(0.04) \\ &= 0.144 \text{ or } 14.4\% \end{aligned}$$

With the individual MARRs, the present worth of the \$500 000 for each of years 12, 2, and 0 can be found:

$$\text{PW}(\text{year } 12) = 500\,000(P/F, 21\%, 8) = 500\,000 \times 1/(1.21)^8 = 108\,815$$

$$\text{PW}(\text{year } 2) = 108\,815 (P/F, 26.5\%, 10) = 108\,815 \times 1/(1.265)^{10} = 10\,370$$

$$\text{PW}(\text{year } 0) = 10\,370 (P/F, 14.4\%, 2) = 10\,370 \times 1/(1.144)^2 = 7924$$

The present worth of the contract is approximately \$7924. ■

**EXAMPLE****9.10**

Bildmet is an extruder of aluminum shapes used in construction. It is experiencing a high scrap rate of 5 percent. The manager, Greta Kehl, estimates that reprocessing scrap costs about \$0.30 per kilogram. The high scrap rate is due partly to operator error. Ms. Kehl believes that a short training course for the operator would reduce the scrap rate to about 4 percent. The course would cost about \$1100. Bildmet is now working with a before-tax actual MARR of 22 percent. Past experience suggests that operators quit their jobs after about five years; the correct time horizon for the retraining project is therefore five years. The data pertaining to the training course are summarized in Table 9.4. Should Bildmet retrain its operator?

**Table 9.4 Training Course Data**

|                                   |         |
|-----------------------------------|---------|
| Output (kilograms/year)           | 125 000 |
| Scrap (kilograms/year)            | 6250    |
| Reprocessing cost (\$/kilogram)   | 0.30    |
| Scrap cost (\$/year)              | 1875    |
| Savings due to training (\$/year) | 375     |
| First cost of training (\$)       | 1100    |
| Inflation rate (%/year)           | 5       |
| Current MARR (%/year)             | 22      |

First, note that the current MARR  $i = 22$  percent incorporates an estimate by investors of inflation of  $f = 5$  percent per year over the next five years. If this estimate of future inflation is correct, Ms. Kehl needs to make an adjustment to take inflation into account. Either the projected annual saving from reduced scrap needs to be increased by the 5 percent rate of inflation, or she needs to reduce the MARR to its real value. We will illustrate the first approach with current cash flows and the current MARR.

Increasing savings to take inflation into account leads to projected (current) savings as shown in Table 9.5.

For example, using Equation (9.2), the expected saving in year 3 is  $\$375(1 + f)^3 = \$434.11$ . The present worth of the savings in year 3 is  $\$434.11/(1 + i)^3 = \$239.07$ . Note that this assumes that the savings begin in year 1.

The present worth of the savings over the five-year time frame, when discounted at the current MARR of 22 percent, is \$1222. This makes the project viable since its cost is \$1100.

We note that the same result could have been reached by working with the real MARR and the constant cost savings of \$375 per year. MARR<sub>R</sub> is given by

$$\text{MARR}_R = 1 + \frac{\text{MARR}_C}{1 + f} - 1 = \frac{1.22}{1.05} - 1 = 0.1619$$

**Table 9.5 Savings Due to the Training Course**

| Year | Current Savings | Present Worth |
|------|-----------------|---------------|
| 1    | \$393.75        | \$322.75      |
| 2    | 413.44          | 277.77        |
| 3    | 434.11          | 239.07        |
| 4    | 455.81          | 205.75        |
| 5    | 478.61          | 177.08        |

The present worth of the real stream of returns, when these are discounted by the real MARR, is given by

$$PW = 375(P/A, 16.19\%, 5) \cong 1222$$

which is the same result as the one obtained with the current MARR and current cash flows. ■

## REVIEW PROBLEMS

### **REVIEW PROBLEM 9.1**

Athabaska Engineering was paid \$100 000 to manage a construction project in 1990. How much would the same job have cost in 2010 if the average annual inflation rate between 1990 and 2010 was 5 percent?

#### **ANSWER**

The compound amount factor can be used to calculate the value of 100 000 1990 dollars in 2010 dollars, using the inflation rate as an interest rate:

$$\begin{aligned} \text{2010 dollars} &= 100\,000(F/P, 5\%, 20) \\ &= 100\,000(2.6533) \\ &= 265\,330 \end{aligned}$$

The same job would have cost about \$265 330 in 2010 dollars. ■

### **REVIEW PROBLEM 9.2**

A computerized course drop-and-add program is being developed for a local community college. It will cost \$300 000 to develop and is expected to save \$50 000 per year in administrative costs over its 10-year life. If inflation is expected to be 4 percent per year for the next 10 years and a real MARR of 5 percent is required, should the project be adopted?

#### **ANSWER**

First, assuming that \$50 000 in administrative costs are current dollars, we can calculate the current IRR for the project. The current IRR is the solution for  $i$  in

$$300\,000 = 50\,000(P/A, i, 10)$$

$$(P/A, i, 10) = 6$$

$$\text{For } i = 11\%, (P/A, i, 10) = 5.8892$$

$$\text{For } i = 10\%, (P/A, i, 10) = 6.1445$$

The current IRR of 10.55 percent is found by interpolating between these two points. We then convert the current IRR into a real IRR to determine if the project is viable:

$$IRR_R = \frac{1 + IRR_C}{1 + f} - 1 = \frac{1.1055}{1.04} - 1 = 0.06298 \text{ or } 6.3\%$$

Since the real IRR of 6.3 percent exceeds the real MARR of 5 percent, the project should be undertaken. ■

### **REVIEW PROBLEM 9.3**

Robert is considering purchasing a bond with a face value of \$5000 and a coupon rate of 8 percent, due in 10 years. Inflation is expected to be 5 percent over the next 10 years. Robert's real MARR is 10 percent, compounded semiannually. What is the present worth of this bond to Robert?

**ANSWER**

This problem can be done with either real interest and real cash flows or current interest and current cash flows. It is somewhat easier to work with current cash flows, so we must first convert the real interest rate given to a current interest rate.

Robert's annual real MARR is  $(1 + 0.10/2)^2 - 1 = 0.1025$ . (Recall that the 10 percent is a nominal rate, compounded semiannually.)

If annual inflation is 5 percent, Robert's current *annual* MARR is

$$\begin{aligned} \text{MARR}_C &= \text{MARR}_R + f + \text{MARR}_R \times f \\ &= 0.1025 + 0.05 + (0.1025)(0.05) \\ &= 0.15763 \text{ or } 15.763\% \end{aligned}$$

The present worth of the \$5000 Robert will get in 10 years is then

$$\begin{aligned} \text{PW} &= 5000(P/F, \text{MARR}_C, 10) \\ &= 5000(0.23138) \cong 1157 \end{aligned}$$

Next, the bond pays an annuity of  $\$5000 \times 0.08/2 = \$200$  every six months. To convert the annuity payments to their present worth, we need a current six-month MARR. This can be obtained with a six-month inflation rate and Robert's six-month real MARR of 10 percent/2 = 5 percent. With  $f = 5$  percent per annum, the inflation rate per six-month period can be calculated with

$$\begin{aligned} f_{12} &= (1 + f_6)^2 - 1 \\ f_6 &= (1 + f_{12})^{1/2} - 1 \\ &= (1 + 0.05)^{1/2} - 1 = 0.0247 \text{ or } 2.47\% \end{aligned}$$

The current MARR per six-month period is then given by

$$\begin{aligned} \text{MARR}_C &= \text{MARR}_R + f + \text{MARR}_R \times f = 0.05 + 0.0247 + (0.05)(0.0247) \\ &= 0.07593 \text{ or } 7.593\% \end{aligned}$$

The present worth of the dividend payments is

$$\begin{aligned} \text{PW}(\text{dividends}) &= 200(P/A, 7.59\%, 20) \\ &= 200(10.125) \\ &= 2025 \end{aligned}$$

Finally,

$$\begin{aligned} \text{PW}(\text{bond}) &= 1157 + 2025 \\ &= 3182 \end{aligned}$$

The present worth of the bond is \$3182. ■

**REVIEW PROBLEM 9.4**

Trimfit, a manufacturer of automobile interior trim, is considering the addition of a new product to its line. Data concerning the project are given below. Should Trimfit accept the project?

| New Product Line Information      |            |
|-----------------------------------|------------|
| First cost (\$)                   | 11 500 000 |
| Planned output (units/year)       | 275 000    |
| Current MARR                      | 20%        |
| Range of possible inflation rates | 0% to 4%   |
| Study period                      | 10 years   |

| Current-Year Prices (\$/unit) |       |
|-------------------------------|-------|
| Raw materials                 | 16.00 |
| Labour                        | 6.25  |
| Product sales price           | 32.00 |

### ANSWER

First, we note that the expected net revenue per unit (not counting amortization of first costs) is  $\$9.75 = \$32 - \$16 - \$6.25$ . The project is potentially viable.

In doing the project evaluation, we can proceed with either current or real dollars. Since we do not know what the inflation rate will be, the easiest way to account for inflation is to keep all prices in real dollars and adjust the current MARR to a real MARR by using values for the inflation rate within the potential range given. The project can then be evaluated with one of the standard methods. Since many of the figures are given in terms of annual amounts, an annual worth analysis will be carried out. Inflation rates of 0 percent, 1 percent, and 4 percent will be used. The results are shown in Table 9.6.

In Table 9.6, the annual worth of the project depends on the inflation rate assumed. Since the current MARR of 20 percent implicitly includes anticipated inflation, different

**Table 9.6 Annual Worth Computations for Trimfit**

| Annual Worth Comparisons for Various Inflation Rates |           |                          |                             |                       |                                     |
|--|-----------|--------------------------|-----------------------------|-----------------------|-------------------------------------|
| Inflation Rate per Year                              | Real MARR | Fixed Cost per Year (\$) | Variable Cost per Year (\$) | Revenue per Year (\$) | Annual Worth (Profit) per Year (\$) |
| 0%   | 20.00%    | 2 743 012                | 6 118 750                   | 8 800 000             | − 61 762                            |
| 1%   | 18.81%    | 2 633 122                | 6 118 750                   | 8 800 000             | 48 128                              |
| 4%   | 15.38%    | 2 325 083                | 6 118 750                   | 8 800 000             | 356 167                             |

trial inflation rates imply different values for the real MARR. For example, at 1 percent inflation, the real MARR implied is

$$\text{MARR}_R = \frac{1 + \text{MARR}_C}{1 + f} - 1 = \frac{1.20}{1.01} - 1 = 0.1881 \text{ or } 18.81\%$$

The fixed cost per year is obtained by finding the annual amount over 10 years equivalent to the first cost when the appropriate real MARR is used. For example, with 1 percent inflation, the fixed cost per year is

$$A = P(A/P, \text{MARR}_R, 10) = 11\,500\,000 \left( \frac{0.1881 (1.1881)^{10}}{(1.1881)^{10} - 1} \right) \\ = 2\,633\,122$$

Next, the variable cost per year is the sum of the raw material cost and the labour cost per unit multiplied by the total expected output per year—that is,  $\$22.25 \times 275\,000 = \$6\,118\,750$ . Revenue per year is the sales price multiplied by the expected output:  $\$32 \times 275\,000 = \$8\,800\,000$ . Notice that the variable cost and the revenue are the same for all three values of the inflation rate. This is because they are given in real dollars.

Finally, the annual worth of the project is determined by the revenue per year less the fixed and variable costs per year. The annual worth is negative for zero inflation, but is positive for both 1 percent and 4 percent inflation rates. Since periods of at least 10 years in which there has been zero inflation have been rare in the twentieth and twenty-first centuries, it is probably safe to assume that there will be some inflation over the life of the project. Therefore, the project appears to be acceptable, since its annual worth will be positive if inflation is at least 1 percent. ■

## SUMMARY

In this chapter, the concept of inflation was introduced and we considered the impact that inflation has on project evaluation. We began by discussing methods of measuring inflation. The main result here was that there are many possible measures, all of which are only approximate.

The concept of current cash flows and interest rates and real cash flows and interest rates was introduced. Current dollars are in currency at the time of payment or receipt, while real dollars are constant over time and are expressed with respect to a base year. Compound amount factors can be used to convert single payments between real and current dollars.

Most of the chapter was concerned with the effect of correctly anticipated inflation on project evaluation and on how to incorporate inflation into project evaluation correctly. We showed that, where engineers have no reason to believe project prices will behave differently from average prices, project decisions are the same with or without correctly anticipated inflation. Finally, we pointed out that predicting inflation is very difficult. This implies that engineers should work with ranges of values for possible future inflation rates. The engineer should test for sensitivity of decisions to possible inflation rates.

## ENGINEERING ECONOMICS IN ACTION, PART 9B

### Exploiting Volatility

Bill Astad of head office had been asking Naomi about how to deal with the variable inflation rates experienced by a sister company in Mexico. “Okay, Naomi, let’s see if I have this straight. For long-term projects of, say, six years or more, it makes sense to use a single inflation figure—the average rate. I can just add that to the real MARR to get a current MARR. Boy, it’s easy to get confused between the real and the current. But I do understand the principle.”

“And the short-term projects?” Naomi prompted.

“For the short-term ones, it makes more sense to break them up into time periods. For each period, select a ‘best guess’ inflation rate and do a stepwise calculation from period to period. So the inflation rate in the middle of the presidential cycle would be relatively low, while near the changeover time it would be a higher estimate. Of course, the actual values used would depend on the political and economic situation at the time the decision is made. I understand that one, too, but it is complicated.”

“I agree,” said Naomi. “I guess we’re lucky things are more predictable here.”

“We are,” Bill replied. “On the other hand, if we can make good decisions in spite of a volatile economy in Mexico, Mexifab may have an advantage over its competitors. Thanks for your help, Naomi.”

## P R O B L E M S

### A. Key Concepts

- 9.1 Which of the following are real and which are current?
  - (a) Allyson has been promised a \$10 000 inheritance when her uncle Bill dies.
  - (b) Bette’s auto insurance will pay the cost of a new windshield if her current one breaks.
  - (c) Cory’s meal allowance while he is in university is \$2000 per term.
  - (d) Dieter’s company promises that its prices will always be the same as they were in 1975.
  - (e) Engworth will construct a house for Zolda, and Zolda will pay Engworth \$150 000 when the house is finished.
  - (f) Fran’s current salary is \$3000 per month.
  - (g) Greta’s retirement plan will pay her \$1500 per month, adjusted for the cost of living.
  
- 9.2 Find the real amounts (with today as the base year) corresponding to the current amounts shown below for a 4 percent inflation rate.
  - (a) \$400 three years from now
  - (b) €400 three years ago
  - (c) \$10 next year
  - (d) \$350 983 ten years from now
  - (e) £1 one thousand years ago
  - (f) \$1 000 000 000 three hundred years from now

- 9.3** Find the present worth today in real value corresponding to the current values shown below for a 4 percent inflation rate and a 4 percent interest rate.
- (a) \$400 three years from now
  - (b) €400 three years ago
  - (c) \$10 next year
  - (d) \$350 983 ten years from now
  - (e) £1 one thousand years ago
  - (f) \$1 000 000 000 three hundred years from now
- 9.4** An investment pays \$10 000 in five years.
- (a) If inflation is 10 percent per year, what is the real value of the \$10 000 in today's dollars?
  - (b) If inflation is 10 percent and the real MARR is 10 percent, what is the present worth?
  - (c) What current dollar MARR is equivalent to a 10 percent real MARR when inflation is 10 percent?
  - (d) Compute the present worth using the current dollar MARR from part (c).
- 9.5** An annuity pays \$1000 per year for 10 years. Inflation is 6 percent per year.
- (a) If the real MARR is 8 percent, what is the current dollar MARR?
  - (b) Using the current dollar MARR from part (a), calculate the present worth of the annuity.
- 9.6** An annuity pays \$1000 per year for 12 years. Inflation is 6 percent per year. The annuity costs \$7500 now.
- (a) What is the current dollar internal rate of return?
  - (b) What is the real internal rate of return?
- 9.7** A bond pays \$10 000 per year for the next 10 years. The bond costs \$90 000 now. Inflation is expected to be 5 percent over the next 10 years.
- (a) What is the current dollar internal rate of return?
  - (b) What is the real internal rate of return?
- 9.8** Inflation is expected to average about 4 percent over the next 50 years. How much would we expect to pay 50 years from now for each of the following?
- (a) \$1.59 hamburger
  - (b) \$15 000 automobile
  - (c) \$180 000 house
- 9.9** The average person now has assets totalling \$38 000. If the average real wealth per person remains the same and if inflation averages 5 percent in the future, when will the average person become a millionaire?

## B. Applications

- 9.10** How much is the present worth of \$10 000 ten years from now under each of the following patterns of inflation, if interest is at 5 percent? On the basis of your answers, is it generally reasonable to use an average inflation rate in economic calculations?
- Inflation is 4 percent.
  - Inflation is 0 percent for five years, and then 8 percent for five years.
  - Inflation is 8 percent for five years, and then 0 percent for five years.
  - Inflation is 6 percent for five years and then 2 percent for five years.
  - Inflation is 0 percent for nine years and then 40 percent for one year.
- 9.11** The current dollar MARR for Jungle Products Ltd. of Parador is 300 percent. The inflation rate in Parador is 250 percent. What is the company's real MARR?
- 9.12** Krystyna has a long-term consulting contract with an insurance company that guarantees her \$25 000 per year for five years. She believes inflation will be 3 percent this year and 5 percent next year, and then will stay at 10 percent indefinitely. Krystyna's real MARR is 12 percent. What is the present worth of this contract?
- 9.13** I have a bond that will pay me \$2000 every year for the next 30 years. My first payment will be a year from today. I expect inflation to average 3 percent over the next 30 years. My real MARR is 10 percent. What is the present worth of this bond?
- 9.14** Ken will receive a \$15 000 annual payment from a family trust. This will continue until Ken is 30; he is now 20. Inflation averages 4 percent and Ken's real MARR is 8 percent. If the first payment is a year from now and a total of 10 payments are to be made, what is the present worth of his remaining income from the trust?
- 9.15** Inflation in Russistan currently averages 40 percent per month. It is expected to diminish to 20 percent per month following the presidential elections 12 months from now. The Russistan Oil Company (ROC) has just signed an agreement with the Global Petroleum Group for the sale of future shipments. The ROC will receive 500 million rubles per month over the next two years, and also 500 million rubles per month indexed to inflation (i.e., real rubles). If the ROC has a real MARR of 1.5 percent per month, what is the total present worth of this contract?
- 9.16** The widget industry maintains a price index for a standard collection of widgets. The base year was 2002 until 2012, when the index was recomputed with 2012 as the base year. The following data concerning prices for the years 2010 to 2013 are available:

| Year | Price Index 2002 Base | Price Index 2012 Base |
|------|-----------------------|-----------------------|
| 2010 | 125                   | N/A                   |
| 2011 | 127                   | N/A                   |
| 2012 | 130                   | 100                   |
| 2013 | N/A                   | 110                   |

What was the percentage increase in prices of widgets between 2010 and 2013?

- 9.17** A group of farmers in Inverness is considering building an irrigation system from a water supply in some nearby mountains. They want to build a concrete reservoir with a steel pipe system. The first cost would be \$200 000 with (current) annual maintenance costs of \$2000. They expect the irrigation system will bring them \$22 000 per year in

additional (current) revenues due to better crop production. Their real dollar MARR is 4 percent and they anticipate inflation to be 3 percent per year. Assume that the reservoir will have a 20-year life.

- (a) Using the current cash flows, find the current IRR on this project.
- (b) What is the current MARR?
- (c) Should they invest?

**9.18** Refer to Problem 9.17.

- (a) Convert the current cash flows into real cash flows.
- (b) Find the present worth of the project using the real MARR.
- (c) Should they invest?

**9.19** Bosco Consulting is considering a potential contract with the Upper Sobonian government to advise it on exploration for oil in Upper Sobonia. Bosco would make an investment of 1 500 000 Sobonian zerts to set up a Sobonian office in 2015. The Upper Sobonian government would pay Bosco 300 000 zerts in 2016. In the years 2017 to 2022, the current zerts value of the payments would increase at the rate of inflation in Upper Sobonia. The following data are available concerning the project:

| <b>Investment in Upper Sobonia</b>             |           |
|--|-----------|
| Expected Sobonian inflation rate (2015–2022)   | 15%/year  |
| Expected inflation rate (2015–2022)            | 3%/year   |
| Value of Sobonian zerts in 2015                | \$0.25    |
| Expected decline in value of zerts (2015–2022) | 10%/year  |
| First cost in 2015 (zerts)                     | 1 500 000 |
| Cash flows in 2016–2022 (real 2016 zerts)      | 300 000   |
| Bosco's current dollar MARR                    | 22%       |

- (a) Construct a table with the following items:

- Real (2015) zert cash flows
- Current zert cash flows
- Current dollar cash flows
- Real dollar cash flows

- (b) What is the present worth in 2015 dollars of this project?

**9.20** Johnson Products now buys a certain part for its chain saws. The managers are considering the production of the part in-house. They can install a production system that would have a life of five years with no salvage value. They believe that over the next five years the real price of purchased parts will remain fixed. They expect the real price of labour and other inputs to production to rise over the next five years. Further information about the situation is in the table below.

- (a) Assume inflation is 2 percent per year in the first year of operation. What will be the current dollar cost of labour for in-house production in the second year?

|  |         |
|--|---------|
| Annual cost of purchase (\$/year)                        | 750 000 |
| Expected real change in cost of purchase                 | 0%      |
| Expected real change in labour cost                      | 4%      |
| Expected real change in other operating costs            | 2%      |
| Labour cost/unit (first year of operation) (\$)          | 10.5    |
| Other operating cost/unit (first year of operation) (\$) | 9       |
| In-house first cost (\$)                                 | 200 000 |
| Use rate (units/year)                                    | 25 000  |
| Current dollar MARR                                      | 20%     |
| Study period (years)                                     | 5       |

- (b) Assume inflation is 2 percent per year in the first two years of operation. What will be the current dollar cost of other operating inputs for in-house production in the third year?
- (c) Assume that inflation averages 2 percent per year over the five-year life of the project. What is the present worth of costs for purchase and for in-house production?
- 9.21** Metcan Ltd.'s smelter produces its own electric power. The plant's power capacity exceeds its current requirements. Metcan has been offered a contract to sell excess power to a nearby utility company. Metcan would supply the utility company with 17 500 megawatt-hours per year (MWh/a) for 10 years. The contract would specify a price of \$22.75 per megawatt-hour for the first year of supply, which would rise by 1 percent per year after this. This is independent of the current rate of inflation over the 10 years.

Metcan would incur a first cost to connect its plant to the utility system. There would also be operating costs attributable to the contract. Metcan believes these costs would track the current inflation rate. The terms of the contract and Metcan's costs are shown in the tables below.

| <b>Metcan Sale of Power</b>    |             |
|--------------------------------|-------------|
| Output price in 2014 (\$/MWh)  | 22.75       |
| Price adjustment (2015–2023)   | 1% per year |
| Power to be supplied (MWh/a)   | 17 500      |
| Contract length                | 10 years    |
| <b>Metcan's Costs</b>          |             |
| First cost in 2013 (\$)        | 175 000     |
| Operating cost in 2008 (\$)    | 332 500     |
| Current dollar MARR before tax | 20%         |

- (a) Find the present worth of the contract under the assumption that there is no inflation over the life of the contract.
- (b) Find the present worth of the contract under four assumptions: inflation is (i) 1 percent per year, (ii) 2 percent per year, (iii) 3 percent per year, and (iv) 4 percent per year.
- (c) Should Metcan accept the contract?

- 9.22** Free Wheels has a plant that assembles bicycles. The plant now has a small cafeteria for the workers, but the kitchen equipment is in need of substantial overhaul. Free Wheels has been offered a contract by Besteats to supply food to the workers. The particulars of the situation are shown in the table. Should Free Wheels continue with the in-house food service or contract the service to Besteats?

| Food Service: In-House Versus Contract             |        |
|--|--------|
| Food service labour (hours/year)                   | 6000   |
| Wage rate (real, time 1, \$/hour)                  | 7.5    |
| Overhead cost (real, time 1, \$/year)              | 18 000 |
| Kitchen equipment first cost (current, time 0, \$) | 25 000 |
| Contract cost, years 1 to 3 (current \$)           | 55 000 |
| Contract cost, years 4 to 6 (current \$)           | 63 700 |
| Current dollar MARR                                | 22%    |
| Expected annual inflation rate                     | 5%     |
| Study period (years)                               | 6      |

### C. More Challenging Problems

- 9.23** Go to the Statistics Canada website ([www.statcan.gc.ca](http://www.statcan.gc.ca)).
- What is the current base year for the CPI? What was the trend over the last two years?
  - Summarize, if the information is given, what the commodities in the CPI basket are, the relative importance of the commodities in the CPI basket, how the CPI basket is updated, and how prices are collected for the CPI.
  - Which of the subcategories (e.g., food, transportation, etc.) appears to be contributing the most to the overall CPI index?
  - Are there regions of the country with a different CPI than other regions? Why?
- 9.24** Go to the Statistics Canada website ([www.statcan.gc.ca](http://www.statcan.gc.ca)).
- Find or calculate the most recent inflation rate. What has been the trend in the inflation rate over the last two years?
  - How does the current inflation rate compare with the interest rates offered at banks and other financial institutions for savings accounts? Can you estimate the real interest rate available for your savings?
  - How does inflation in Canada compare with inflation elsewhere? Find a country with particularly low inflation (or high deflation) and a country with particularly high inflation.
- 9.25** Bildkit, a building products company, is considering an agreement with a distributor in the foreign country of Maloria to supply kits for constructing houses in Maloria. Sales would start next year. The expected receipts from the sale of the kits next year is 30 000 000 Malorian yen. The number of units sold is expected to grow by 10 percent per year over the life of the contract. The actual yen price is expected to grow at the rate of Malorian inflation.

There will be a first cost for Bildkit. As well, there will be operating costs over the life of the contract. Operating cost per unit will be constant in real dollars over the life of the contract. Since the number of units sold will rise by 10 percent per year, real operating costs will rise by 10 percent per year. Actual operating costs per unit will rise at the rate of inflation in Bildkit's country.

The value of the Malorian yen is expected to increase over the life of the contract. Data concerning the proposed contract are shown in the table below.

| <b>Bildkit in Maloria</b>               |            |  |         |
|---|------------|--|---------|
| Receipts in first year<br>(current yen) | 30 000 000 | First cost now<br>(current \$)                               | 200 000 |
| Growth of receipts<br>(real yen)        | 10%/year   | Operating cost in<br>first year of<br>operation (current \$) | 350 000 |
| Malorian inflation rate                 | 1%/year    | Inflation rate in<br>Bildkit's country                       | 3%      |
| Value of yen year 0 (\$)                | 0.015      | Current dollar MARR  | 22%     |
| Rate of increase in value<br>of yen     | 2%         | Study period   | 8 years |

(a) What is the present worth of receipts in dollars?

(b) What is the present worth of the cash outflows in dollars?

**9.26** Leftway Information Systems is considering a contract with the Iberian government to supply consulting services over a five-year period. The following real Iberian pound cash flows are expected:

| <b>Cash Flows in Year 2010 Iberian Pounds</b> |           |
|---|-----------|
| First cost                                    | 1 800 000 |
| Net revenue 2014 to 2018                      | 550 000   |

Further information is in the table below:

|  |       |
|--|-------|
| Expected Iberian inflation rate                                      | 10%   |
| Value of Iberian pound in 2013 (\$)                                  | 1.25  |
| Expected annual rate of decline in the value of the<br>Iberian pound | 5%    |
| Expected inflation rate in Leftway's country                         | 2.50% |
| Leftway's real MARR  | 15%   |

- (a) What is the real Iberian pound internal rate of return on this project? (*Hint:* Leftway's country can be ignored in answering this question.)
- (b) What is the current pound internal rate of return? (*Hint:* Leftway's country can be ignored in answering this question.)
- (c) Use the internal rate of return in dollars to decide if Leftway should accept the proposed contract.

**9.27** Sonar warning devices are being purchased by the St. James department store chain to help trucks back up at store loading docks. The total cost of purchase and installation is \$220 000. There are two types of saving from the system. Faster turnaround time at the congested loading docks will save \$50 000 per year in today's dollars. Reduced damage to the loading docks will save \$30 000 per year in today's dollars. St. James has an observed current dollar MARR of 18 percent. The sonar system has a life of four years. Its scrap value in today's dollars is \$20 000. The inflation rate is expected to be 6 percent per year over the next four years.

- (a) What is St. James's real MARR?
- (b) What is the real internal rate of return? (This is most easily done with a spreadsheet.)
- (c) Compute the current internal rate of return using Equation (9.10).
- (d) Compute the current internal rate of return from the current dollar cash flows. (This is most easily done with a spreadsheet.)
- (e) What is the present worth of the system?

**9.28** Lifewear, a manufacturer of women's sports clothes, is considering adding a line of skirts and jackets. The production would take place in a part of its factory that is now not being used. The first output would be available in time for the 2015 fall season. The following information is available:

| <b>New Product Line Information</b>      |            |
|--|------------|
| First cost in 2014 (\$)                  | 15 500 000 |
| Planned output (units/year)              | 325 000    |
| Observed, current dollar MARR before tax | 0.25       |
| Study period                             | 6 years    |
| <b>Year 2014 Prices (\$/unit)</b>        |            |
| Materials                                | 12         |
| Labour                                   | 7.75       |
| Output                                   | 35         |

- (a) What is the real internal rate of return?
- (b) What inflation rate will make the real MARR equal to the real internal rate of return?
- (c) Calculate the present worth of the project under three possible future inflation rates. Assume that the inflation rate will be 1 percent, 2 percent, or 3 percent per year.
- (d) Decide if Lifewear should add this new line of skirts and jackets. Explain your answer.

- 9.29** Century Foods, a producer of frozen meat products, is considering a new plant near Essen, Germany, for its sausage rolls and frozen meat pies. The company has estimates of production cost and selling prices in the first year. It expects the real value of operating costs per unit to fall because of improved operating methods. It also expects competitive pressures to cause the real value of product prices to fall. The following data are available:

| <b>Century Food Plant Data</b>      |               |
|-------------------------------------|---------------|
| Output price in 2015 (€/box)        | 22            |
| Operating cost in 2015 (€/box)      | 15.5          |
| Planned output rate (boxes/year)    | 275 000       |
| Fall in real output price           | 1.5% per year |
| Fall in real operating cost per box | 1.0% per year |
| First cost in 2014 (€)              | 7 500 000     |
| Study period                        | 10 years      |
| Current euro MARR before tax        | 20%           |

- (a) Assume that there is zero inflation. What is the present worth, in 2014, of the project?
- (b) Assume that there is zero inflation. What is the internal rate of return? (This is most easily done with a spreadsheet.)
- (c) At what inflation rate would the current euro internal rate of return equal 20 percent?
- (d) Should Century Foods build the new plant? Explain your answer.
- 9.30** Clarkwood is a wood products manufacturer. Its managers are considering a modification to the production line that would enable an increase in output. One of Clarkwood's concerns is that the price of wood is rising more rapidly than inflation. The managers expect that because of this the operating cost per unit will rise at a rate 4 percent higher than the rate of inflation. That is, if the rate of inflation is  $f$ , Clarkwood's operating cost will rise at the rate  $f_c = 1.04(1 + f) - 1$ . However, competitive pressures from plastics will prevent the prices of Clarkwood's products from rising more than 1 percent above the inflation rate. The particulars of the project are shown in the table below.

| <b>Clarkwood's Project</b>                  |                    |
|---|--------------------|
| Output price in 2014 (\$/unit)              | 30                 |
| Price increase                              | 2% above inflation |
| Operating cost in 2014 (\$/unit)            | 24                 |
| Operating cost increase                     | 4% above inflation |
| Expected output due to project (units/year) | 50 000             |
| First cost in 2013 (\$)                     | 900 000            |
| Observed current dollar MARR                | 0.25               |
| Time horizon (years)                        | 10                 |

- (a) Find the present worth of the project under the assumption of zero inflation.
- (b) Find the present worth of the project under these assumptions: the expected inflation is (i) 1 percent per year, and (ii) 2 percent per year.
- (c) Should Clarkwood accept the project?
- 9.31** Smooth-Top is a manufacturer of desktops. It is considering an increase of capacity. Consulting engineers have submitted two routes to accomplish this: (1) install a new production line that would produce wooden desktops finished with hardwood veneer and (2) install a new production line that would produce wooden desktops finished with simulated wood made from hard plastic.

Smooth-Top is concerned about the price of hardwood veneer. It believes the price of veneer will rise over the next 10 years. However, it believes the price of veneer-finished desktops will rise by less than the rate at which the price of veneer rises. Information about the two potential projects is in the following table.

- (a) Compute the present worth of each option under the assumption that the real price of hardwood-finished desktops and real cost of hardwood veneer do not change (rather than as stated in the table). Assume zero inflation.

| Smooth-Top Desktop Project                     |           |
|--|-----------|
| Plastic-finish real price and real cost change | 0%        |
| Veneer-finish expected real price change       | 1%        |
| Veneer-finish expected real cost change        | 5%        |
| Wood cost/unit (\$)                            | 12.5      |
| Plastic cost/unit (\$)                         | 9         |
| Wood price/unit (\$)                           | 32        |
| Plastic price/unit (\$)                        | 26        |
| Wood first cost (\$)                           | 2 050 000 |
| Plastic first cost (\$)                        | 2 700 000 |
| Wood output rate (units/year)                  | 30 000    |
| Plastic output rate (units/year)               | 45 000    |
| Study period                                   | 10 years  |
| Current dollar MARR                            | 25%       |

- (b) Compute the present worth of each option under the assumption that the real price of hardwood-finished desktops and the real value of hardwood veneer desktop operating costs increase as indicated in the table. Assume that inflation is expected to be 2 percent over the study period.

**9.32** Belmont Grocers has a distribution centre in Winnipeg. The manual materials-handling system at the centre has deteriorated to the point that it must be either replaced or substantially refurbished. Replacement with an automated system would cost about \$240 000. Refurbishing the manual system would cost about \$50 000. In either case, capital expenditures would take place this year. Operating either the new system or the refurbished system would begin next year. It is expected that either the new system or the refurbished system will operate for 10 years with no further capital expenditures. Belmont is concerned that labour costs in Winnipeg may rise in real terms over the next 10 years. The range of increases in real terms that appears possible is from 4 percent to 7 percent per year. Inflation rates between 2 percent and 4 percent are expected over the next 10 years. Complete data on the two alternatives are given in the table that follows.

- (a) Find the total costs per unit for each of the two alternatives under the assumption of zero inflation and no increase in costs for the manual system.
- (b) Make a recommendation about which alternative to adopt. Base the recommendation on the present worth of costs for the two systems under various assumptions concerning inflation and the rate of change in the real operating cost of the manual system. Explain your recommendation.

| Materials Handling Data                                      |          |
|--|----------|
| Automated expected real operating cost change                | 0%       |
| Manual expected real operating cost change                   | 4% to 7% |
| Manual operating cost/unit (first year of operation) (\$)    | 10.5     |
| Automated operating cost/unit (first year of operation) (\$) | 9        |
| Manual first cost (\$)                                       | 50 000   |
| Automated first cost (\$)                                    | 240 000  |
| Output rate (units/year)                                     | 15 000   |
| Current dollar MARR  | 20%      |
| Study period   | 10 years |
| Possible inflation rates                                     | 2% to 4% |

**9.33** The United Gum Workers have a cost-of-living clause in their contract with Mont-Gum-Ery Foods. The contract is for two years. The contract states that, if the inflation rate in the first year exceeds 1 percent, wages in the second year will increase by the inflation rate of the first year. Does this clause increase or decrease risk? Explain.

**9.34** In 10 years, Mid-Atlantic Corp. will be investing \$200 000 either in Columbo or in Avalon. The exchange rate between the Columboan dollar and the Avalonian pound is fixed at \$2 = £1. Dollars and pounds can be exchanged at no cost at any time.

If the \$200 000 is invested in Columbo, each dollar invested will return \$0.30 per year for each of the following five years. If the \$200 000 is converted to pounds and invested in Avalon, each pound invested will return £0.24 per year for the following seven years.

Columbo is subject to ongoing average inflation of 4 percent, while inflation in Avalon averages 2 percent. If the real MARR for Mid-Atlantic is 10 percent, which investment is preferred? How much money should Mid-Atlantic set aside now (invested at the MARR) to ensure that it has enough money to make the investment in 10 years?

## MINI-CASE 9.1

### **Economic Comparison of High Pressure and Conventional Pipelines: Associated Engineering**

Associated Engineering conducted an evaluation of sources of water supply for a municipality. One of the considerations was the choice of high-pressure or conventional pipelines for transmitting treated water to the municipality from a distant water source.

Conventional pipelines, most often made of concrete, have a limited maximum tensile strength, which for analysis purposes was taken to be 200 pounds per square inch (psi). High-pressure pipe, made of steel, can withstand up to 60 000 psi, although the pipe examined by Associated Engineering had a strength of 42 000 psi.

The major advantage of the steel pipe is that fewer pumping stations are needed than with the concrete pipe. The distance to be pumped is 85 kilometres; this requires either one pumping station for high-pressure pipe or six pumping stations for concrete pipe.

Each pipeline type was analyzed over a range of pipeline diameters ranging from 24" to 72". Construction costs included the pipe, pumping stations, and a reception reservoir, with the time of the cost taken to be the commissioning date of 2025. Operating and maintenance costs starting in 2026 were included, and administration, engineering fees, contingencies, and taxes were also accounted for.

The best alternative was chosen on the basis of a present worth comparison with a 4 percent discount rate. In the analysis, real 1993 dollars were used and an inflation rate of 2 percent was assumed for the period of study. The result was that a 360-diameter high-pressure pipeline was economically best at a present cost \$7.5 million lower than for the best conventional pipeline.

#### **Discussion**

Estimating future inflation is difficult. The average inflation in many developed countries over the past 50 years has been about 4 percent, but there have been periods of several years when it has been 2 percent to 2.5 percent. For some other periods, inflation has averaged over 10 percent. Historically, many countries have experienced hyperinflation (extreme inflation) or even deflation. How can we estimate future inflation?

One way is simply to assume that inflation will remain at the current value. This is probably wrong; as has been seen, inflation typically changes over time. However, there are factors that are controlling the inflation rate. Lacking knowledge of any reason why these controlling factors might change, the current rate seems to be a reasonable choice.

A second approach is to use the long-term average. Knowing that inflation will change over time suggests that the long-term average is a good choice even if inflation is lower or higher than the average right now. After all, those controlling factors have changed in the past and are likely to change again.

A third way is to take into account the controlling factors for inflation. These include government policy: A government committed to social welfare is likely to induce more inflation than one committed to fiscal responsibility. Trends in business and consumer behaviour affect inflation: Large labour-contract increases presage inflation, as does high consumer borrowing. Social trends like the aging of the baby boomers also have an effect on inflation.

Understanding the effect of the controlling factors for inflation in detail is very difficult. So usually decision makers make a broad judgment based on both the current inflation rate and the historical average, and perhaps informed by a general understanding of the contributing factors.

### Questions

1. How significant would the difference have been to the savings of the high-pressure pipeline if an inflation rate of 4 percent had been used instead? Assume the only difference between the concrete- and steel-pipe systems was the capital cost, expended in 2025. Would the decision be any different? Could it be different for any assumed inflation rate?
2. Design two cash flow structures for projects that start in 2025, such that the present worth in the current year at a discount rate of 4 percent is higher for one project than the other at an inflation rate of 2 percent, but lower at an inflation rate of 4 percent. Is there a significant opportunity to control the best choice in a decision situation by selecting the appropriate inflation rate?
3. Why would the analysts have chosen to separate the inflation rate from the discount rate for this problem rather than combining them into a current dollar discount rate? Do you think the analysts estimated the current dollar cost of the alternatives in 2025, or would they have used the real costs?

## Appendix 9A Computing a Price Index

We can represent changes in average prices over time with a **price index**. A price index relates the average price of a given set of goods in some time period to the average price of the same set of goods in another period. Commonly used price indexes work with weighted averages because simple averages do not reflect the differences in importance of the various goods and services in which we are interested.

Many different ways of weighting changes in prices may be used, and each method leads to a different price index. We shall discuss only the most commonly used index, the **Laspeyres price index**. It can be explained as follows.

Suppose there are  $n$  goods in which we are interested. We want to represent their prices at a time,  $t_1$ , relative to a **base period**,  $t_0$ —the period from which the expenditure shares are calculated.

The prices of the  $n$  goods at times  $t_0$  and  $t_1$  are denoted by  $p_{01}, p_{02}, \dots, p_{0n}$  and  $p_{11}, p_{12}, \dots, p_{1n}$ . The quantities of the  $n$  goods purchased at  $t_0$  are denoted by  $q_{01}, q_{02}, \dots, q_{0n}$ . The share,  $s_{0j}$ , of good  $j$  in the total expenditure for the period,  $t_0$ , is defined as

$$s_{0j} = \frac{p_{0j}q_{0j}}{p_{01}q_{01} + p_{02}q_{02} + \dots + p_{0n}q_{0n}}$$

Note that

$$\sum_{j=1}^n s_{0j} = 1$$

A Laspeyres price index,  $\pi_{01}$ , is defined as a weighted average of relative prices.

$$\pi_{01} = \left( \frac{p_{11}}{p_{01}} s_{01} + \frac{p_{12}}{p_{02}} s_{02} + \dots + \frac{p_{1n}}{p_{0n}} s_{0n} \right) \times 100$$

The term in the brackets is a weighted average because the weights (the expenditure shares in the base period) sum to one. The relative prices are the prices of the individual goods in period  $t_1$  relative to the base period,  $t_0$ . The weighted average is multiplied by 100 to put the index in percentage terms.

**EXAMPLE****9A.1**

A student uses four foods for hamburgers: (1) ground beef, (2) hamburger buns, (3) onions, and (4) breath mints. Suppose that, in one year, the price of ground beef fell by 10 percent, the price of buns fell by 1 percent, the price of onions rose by 5 percent, and the price of breath mints rose by 50 percent.

The price and quantity data for the student's hamburger are shown in Table 9A.1.

**Table 9A.1 Price and Quantity Data for Hamburger**

|                  | Quantity at $t_0$ | Price at $t_0$ (\$) | Price at $t_1$ (\$) |
|------------------|-------------------|---------------------|---------------------|
| Ground beef (kg) | 0.25              | 3.5/kg              | 3.15/kg             |
| Buns             | 1                 | 0.40                | 0.396               |
| Onions           | 1                 | 0.20                | 0.21                |
| Breath mints     | 1                 | 0.10                | 0.15                |

The Laspeyres price index is calculated in four steps:

1. Compute the base period expenditure for each ingredient.
2. Compute the share of each ingredient in the total base period expenditure.
3. Compute the relative price for each ingredient.
4. Use the shares to form a weighted average of the relative prices.

These computations are shown in Table 9A.2.

**Table 9A.2 The Laspeyres Price Index Calculation**

|                  | Price at $t_0$ (\$) | Share at $t_0$ | Relative Price | Weighted Relative Price |
|------------------|---------------------|----------------|----------------|-------------------------|
| Ground beef (kg) | 0.875               | 0.556          | 0.900          | 0.500                   |
| Buns             | 0.400               | 0.254          | 0.990          | 0.251                   |
| Onions           | 0.200               | 0.127          | 1.050          | 0.133                   |
| Breath mints     | 0.100               | 0.063          | 1.500          | 0.095                   |
| Sums             | 1.575               | 1.000          |                | 0.980                   |

As an example of the computations, the price of the ground beef per hamburger at  $t_0$  is found by multiplying the price per kilogram by the weight of the hamburger used:

$\$3.50/\text{kg} \times 0.25 \text{ kg} = \$0.875$ . Similar computations for each of the other ingredients lead to a total cost of  $\$1.575$  per hamburger. The ground beef then represents a share of  $0.875/1.575 = 0.556$  of the total cost. The relative price for the hamburger is  $3.15/3.5 = 0.9$  and thus the weighted relative price is  $0.556 \times 0.9 = 0.50$ . Similar computations for the other ingredients lead to a total weighted average of 0.98. After multiplying by 100, the Laspeyres price index is 98 (it is understood that this is a percentage). Therefore, the cost of the hamburger ingredients at  $t_1$  was 2 percent lower than in the base period. ■

Governments compile many Laspeyres price indexes. The consumer price index (CPI) is a Laspeyres price index in which the weights are the shares of urban consumers' budgets in the base year. Another well-known Laspeyres price index is the gross national product (GNP) deflator. For the GNP deflator, the weights are the shares of total output in the base year.

The CPI and the GNP deflator are global indexes in that they represent an economy-wide set of prices. As well, Laspeyres price indexes can be calculated by sector. For example, there are price indexes for durable consumer goods, for exports, and for investment by businesses. It is up to the analyst to know the composition of the different indexes and to decide which is best for his or her purposes.

**EXAMPLE****9A.2**

We can classify consumer goods and services into four classes: durable goods, semi-durable goods, non-durable goods, and services. Assume the classes had the following prices in 2005 and 2012:

| Category     | Price (\$) |       |
|--------------|------------|-------|
|              | 2005       | 2012  |
| Durable      | 2.421      | 2.818 |
| Semi-durable | 2.849      | 3.715 |
| Non-durable  | 4.926      | 6.404 |
| Services     | 4.608      | 6.263 |

Quantities in 2005 were

| Quantity in 2005 (Units) |        |
|--------------------------|--------|
| Durable                  | 21.304 |
| Semi-durable             | 11.315 |
| Non-durable              | 19.159 |
| Services                 | 31.422 |

Find the Laspeyres price index for 2012 with 2005 as a base. We first calculate the relative prices:

| Category     | Price (\$) |       |                |
|--------------|------------|-------|----------------|
|              | 2005       | 2012  | Relative Price |
| Durable      | 2.421      | 2.818 | 1.164          |
| Semi-durable | 2.849      | 3.715 | 1.304          |
| Non-durable  | 4.926      | 6.404 | 1.300          |
| Services     | 4.608      | 6.263 | 1.359          |

We next determine expenditure shares in 2005:

| Category     | Expenditure | Share  |
|--------------|-------------|--------|
| Durable      | 51.583      | 0.1597 |
| Semi-durable | 32.235      | 0.0998 |
| Non-durable  | 94.381      | 0.2922 |
| Services     | 144.801     | 0.4483 |
| Total        | 323.000     |        |

We then multiply the relative prices by the shares and sum. We get the index by multiplying the sum by 100. For example, the term for durable goods is given by  $1.164(0.1597) = 0.186$ .

|              | Index |
|--------------|-------|
| Durable      | 0.186 |
| Semi-durable | 0.130 |
| Non-durable  | 0.380 |
| Services     | 0.609 |
| Total        | 1.305 |

This gives a Laspeyres price index of 130.5. ■



# Public Sector Decision Making

**Engineering Economics in Action, Part 10A:** New Challenges Down Under

## 10.1 Introduction

## 10.2 Market Failure

### 10.2.1 Market Failure Defined

### 10.2.2 Remedies for Market Failure

## 10.3 Decision Making in the Public Sector

### 10.3.1 The Point of View Used for Project Evaluation

### 10.3.2 Identifying and Measuring the Costs of Public Projects

### 10.3.3 Identifying and Measuring the Benefits of Public Projects

### 10.3.4 Benefit–Cost Ratios

### 10.3.5 The MARR in the Public Sector

## Review Problems

## Summary

**Engineering Economics in Action, Part 10B:** Look at It Broadly

## Problems

## Mini-Case 10.1: Emissions Trading

## ENGINEERING ECONOMICS IN ACTION, PART 10A

### New Challenges Down Under

“Hi, Naomi. How’s it going down under?” Naomi could easily imagine Bill with his feet up on his desk, leaning back in his chair, the telephone wedged against his ear. Naomi was in Melbourne, Australia, checking into plans for how Melbourne Manufacturing, a sister company of Canadian Widgets, was dealing with a change to the fuel tax credit system recently introduced by the Australian government. The fuel tax credit system provided rebates to Melbourne Manufacturing provided that its vehicles satisfied specific emissions-control-related environmental criteria. Melbourne Manufacturing had an extensive distribution system in Eastern Australia that used a large fleet of trucks, so the fuel tax rebates were a significant issue. The fuel tax credit system had been in place for some time, but what was new was that in order to claim more than \$3 million in credits, Melbourne Manufacturing had to be a member of the Greenhouse Challenge Plus program.

Naomi answered, “Melbourne is great. I can look out my window and see the rowing sculls in the Yarra River, and I got a chance to go to St. Kilda Beach on the weekend. Not bad for late February!” She continued, “Things look pretty good at Melbourne Manufacturing but we’re up against some tough decisions. Melbourne Manufacturing has an ongoing program of truck replacement—it’s spent over \$5 million in the past few years on updating the fleet. Some of this was because of government emissions regulations, but mostly it was just part of its regular replacement program.” She paused. “But now it looks like we might have to make some pretty big investments.”

Carbon dioxide, unburned hydrocarbons, and nitrous oxides are the main gaseous pollutants produced by the combustion process in gasoline and diesel-powered vehicles. Carbon dioxide is a greenhouse gas that traps the Earth’s heat and contributes to global warming. Nitrous oxides contribute to the formation of ozone, a lung and eye irritant, and are a major source of acid rain. In 2000, Australia had agreed to use the United Nations Economic Commission for Europe targets to reduce emissions, and had introduced measures to either require or encourage emissions reductions. “So far, Melbourne Manufacturing has been able to comply with government restrictions just by purchasing fuel efficient trucks.”

“So, what’s the problem, then?” Bill interjected.

“Well,” Naomi began, “new legislation requires Melbourne Manufacturing to be a member of the Greenhouse Challenge Plus program if it wants to claim more than \$3 million in fuel tax credits in a financial year. It’s easily beyond that amount. The problem is, to become a member of Greenhouse Challenge Plus, it has to measure and monitor greenhouse gas emissions and report annually to the government. That’s not a minor proposition—it means revamping all kinds of procedures and processes. It’s going to be expensive, and it’s not clear that the extra tax credits are going to be enough to make it worth doing.”

“Well, maybe there is more to it,” Bill responded. “Being required to monitor greenhouse gas emissions might uncover opportunities for reducing waste and saving energy. Let’s explore things further. I’ll try to get some thoughts from the people here. See if you can get more information from the folks there. I’ll set up a meeting for us with Anna Kulkowski when you are back. I suspect she may have some helpful ideas, too.”

## 10.1

### Introduction

All organizations—public or private—and the engineers who work for them need to take into account the effects of what they do on society as a whole. Consider these two examples.

1. Lead has been known since ancient times. It is a malleable, corrosion-resistant element with a low melting point. The ancient Romans used lead to make water pipes, some of which are still in use today. Today, lead is used as a liner for tanks

that hold corrosive liquids and as a coating on wires to prevent corrosion. Its density makes it useful as a shield against X-ray and gamma-ray radiation.

Lead alloys are also widely used. Solder, an alloy of lead and tin, has a relatively low melting point and is used, for example, to join electrical components to circuit boards. Other lead alloys are used to reduce friction in bearings. Lead compounds are used in glass, rubber, and paint. The lead makes the paint more vibrant in colour, more weather resistant, and faster drying.

Unfortunately, this useful element is also toxic. Lead causes a variety of adverse effects, such as vomiting, learning and behavioural problems, seizures, and death. Children six years old and under are most at risk of permanent brain damage. Lead-contaminated household dust and exposure to deteriorating lead-based paints are the major sources of exposure in children in developed countries, while air pollution in countries where leaded gas is still used has been a leading cause of poisoning in children. Most adult exposure to lead comes from occupational hazards in lead-related industries such as smelting and battery production or is due to work with lead-based paints.

The toxicity of lead was recognized as early as 200 B.C., but it was not until the late 1800s and early 1900s that medical studies of children exposed to lead-based paints prompted some countries to ban them. In 1909, France, Belgium, and Austria were among the earliest to ban lead in paint. Many other European countries followed suit quickly. Canada adopted a voluntary phase-out in 1972 and a complete ban in 2006. The United States banned lead paint in 1978.

Governments in countries all over the world have introduced regulations on the use of lead in manufactured products. Since the early 1900s, regulations that limit workers' exposure to hazardous workplace substances—such as lead—have also been introduced. Unfortunately, there are no global standards for these limits or global regulations regarding the use of lead in consumer products. This poses a growing issue to companies that subcontract work to other countries. Mattel, one of the world's largest toy producers, had three major product recalls in 2007 and removed almost 21 million toys from shelves worldwide due to lead paint having been used on the toys. A class action lawsuit has also been filed demanding that Mattel pay for blood lead-level tests for children whose toys were recalled.

2. The extraction of coal and the production of steel was once the mainstay of the economy in Nova Scotia. The centre of the steel prosperity, Sydney, is now the site of one of North America's largest environmental disasters—the Sydney Tar Ponds. The contaminants in the tar ponds are mainly the result of steel-making operations—the core of which is the coke oven. As coal is heated in a coke oven, tar and gases are separated off from the desired coke. Over more than 80 years, these toxic wastes were discharged into an estuary of Sydney Harbour in an area known as Muggah Creek. The water in the area has been seriously contaminated with arsenic, lead, benzene, and other toxins such as PCBs (polychlorinated biphenyls), many of which are believed to cause cancer. Though concerns about air and ground pollution had been evident for decades, little money was spent to reduce the large volumes of water and air pollution. The steel mills, owned privately until the 1960s, were finally closed in 2000 after 40 years of operation by Sydney Steel (SYSCO), a provincial Crown corporation.

Residents say that when it rains, puddles in the area turn fluorescent green. Others who live near the ponds report orange ooze in their basements, massive headaches, nosebleeds, and serious breathing problems. Sydney has one of the highest rates of cancer, birth defects, and miscarriages in Canada. In March 2004,

residents filed a \$1 billion lawsuit, the largest class-action suit to date in Canada, for damages allegedly caused by years of pollution from the SYSCO operations. In May 2004, the federal and provincial governments agreed to commit \$400 million to assist with the cleanup. It remains to be seen what the outcome of the lawsuit will be and how the cleanup operation will remediate the long-polluted site.

The lead-poisoning and Sydney Tar Ponds examples illustrate a phenomenon that has important implications for engineers. It is not enough to produce goods and services at a cost that customers are willing to pay. Engineers must also pay attention to broader social values. This is because the market prices that guide most production decisions may not reflect all the social benefits and costs of engineering decisions adequately. Where markets fail to reflect all social benefits and costs, society uses other means of attaining social values.

Up to this point in our coverage of engineering economics, we have been concerned with project evaluation methods for private sector organizations. The private sector's frame of reference is profitability of the firm itself—a well-understood objective. To be profitable, firms sell goods and services to customers with the goal of realizing a return on investment. The focus in project evaluation is usually on the impact it has on the financial well-being of the firm, even if the impact of the project may have effects on individuals or groups external to the firm. For example, building a new pulp mill will provide additional production capacity, but it also may have an impact on local water quality. The firm may limit its economic analysis to the direct impact of the plant on its own profitability. This would include, for example, the construction and operating costs of the mill and the additional revenues associated with expanded production capacity. It may not consider the effect its emissions may have on the local fisheries.

When we consider the broader social context, the concept of profitability is extremely difficult to define because the frame of reference is society at large. This is particularly true for projects where there may not be an open market in which customers have free choice over what products they buy. Costs and benefits for these projects can be difficult to identify and value in a quantifiable manner. For example, if Transport Canada were to consider construction of a new highway to improve traffic flow in a congested area, the analysis should take into account, in addition to the costs of construction, the benefits and costs to society at large. This includes assigning a value to social benefits such as increased safety and reduced travel time for business and recreational travellers. It also includes assigning values to social costs such as the inconvenience to owners due to loss of land and houses that lie on the highway route and increased noise to nearby houses.

In this chapter, we look at the social aspects of engineering decision making. First, we consider the reasons markets may fail in such areas as the environment and health. We also consider different methods that society may use to correct these failures. Second, we consider decision making in the public sector. Here we will be concerned mainly with government projects or government-supported projects.



## S P R E A D S H E E T   S A V V Y

Inserting spreadsheet charts and tables into reports and presentations is a good way to communicate the work you have done. While the charts and tables provide the core of the analysis, the report (or presentation) provides the commentary leading up to the analysis and conclusions stemming from it. There are many ways to insert spreadsheet tables and charts into a document or presentation. This Spreadsheet Savvy outlines some of the alternatives and the pros and cons of each.

All the alternatives begin by selecting the cells or chart to be copied. On the Home tab, under Clipboard, select the Copy option. Alternatively, a keyboard shortcut is to press the Ctrl key and, while holding it down, press the C key (often written as Ctrl-C). When you go to paste the copied material, there are a number of options available. They will depend on the application into which you are pasting the copied material, so some of the more common options will be outlined here.

In Microsoft products such as Word and PowerPoint, the options available appear under the Clipboard section of the Home tab on the ribbon. By selecting the paste icon, several other small icons will appear, indicating the basic paste options. Several special formats are available as well, under Paste special. There are three primary options: (1) Paste as a bitmap or picture, 2) paste as text for worksheet tables (either formatted or unformatted), and 3) paste as an embedded object. Pasting as a bitmap or picture will preserve the formatting of the table or object and it can be resized or moved in its destination, but the amount of subsequent editing you can do is limited. For worksheet tables, pasting as text or formatted text is less useful, as any table formatting will be lost. Pasting as an embedded object has some advantages, however, as the actual spreadsheet will be embedded into the document or presentation. This means that if you need to make changes to the copied table or chart, you can make them without having to recopy the material. By right-clicking on the pasted object, the Excel editing tools will appear so that you can make the necessary changes, save them, and return to the document or presentation. While this approach provides flexibility, it does come at a slight cost in terms of the size and processing speed of the saved document. Unless the document has numerous embedded spreadsheet objects, the performance degradation will not be large.

If you have done a number of copy operations and you want to see what items are on the clipboard, or if you want to paste an item you selected to copy earlier, click on the small arrow in the Clipboard section of the Home tab. A list of items on the clipboard will appear. Any one of them can be retrieved for pasting elsewhere or deleted from the clipboard.

Aside from using the Clipboard section of the ribbon, a copied item can be pasted using Ctrl-V, which is achieved by holding down the Ctrl key and pressing V. This will paste the copied material into its new location. On the lower right-hand side of the copied material, there will be a small tab where you can then select the paste option you wish. This is a quick method if you have many copy and paste operations to perform.

## 10.2 Market Failure

A **market** is a group of buyers and sellers linked by trade in a particular product or service. The prices in a market that guide most production decisions usually reflect all the social benefits and costs of engineering decisions adequately. This is not always true, however. When prices do not reflect all social benefits and costs of a decision, we say that there has been *market failure*. When this occurs, society will seek a means of correcting the failure. In this section, we define market failure and give examples of its effects. Then we discuss a number of ways in which society seeks remedies for market failure.

### 10.2.1 Market Failure Defined

Most decisions in the private sector lead to market behaviour that has desirable outcomes. This is because these decisions affect mainly those people who are party to those decisions. Since people can generally freely choose to participate in markets, it is reasonable to assume that the individuals who participate must somehow benefit by their actions. In most cases, this is the end of the story. In an **efficient market**, decisions are made so that it is impossible to find a way for at least one person to be better off and no person to be worse off.

In some cases, however, decisions have important effects on people who are not party to the decisions. In these cases, it is possible that the gains to the decision makers, and any others who might benefit from the decisions, are less than the losses imposed on those who are affected by the decisions. Such situations are clearly undesirable. These decisions are instances of market failure. **Market failure** occurs when a market, left on its own, fails to make decisions in which resources are allocated efficiently. When decisions are made in which aggregate benefits to all persons who benefit from the decision are less than aggregate costs imposed on persons who bear the costs that result from the decision, the decision is inefficient and hence market failure has occurred. Market failure can occur if the decision maker does not correctly take into account the gains and losses imposed on others by the consequences of a decision.

There are several reasons for market failure. First, there may be no market through which those affected by the decision can induce the decision maker to take their situations into account. However, losses may exceed gains even when there is a market, if the market has insufficient information about the gains and losses resulting from decisions. Market failure can also occur whenever a single buyer or seller (a monopolist) can influence prices or output. Market failure can even occur when someone decides *not* to do something that would create benefits to others. The market would fail if the cost of creating the benefits was less than the value of the benefits.

Acid rain is an example of the effects of market failure. The burning of high-sulphur coal by thermal-electric power plants is believed to be one of the causes of acid rain. These plants could burn low-sulphur fuels, but they do not, partly because low-sulphur fuels are more expensive than high-sulphur fuels. If a market existed through which those affected by acid rain could buy a reduction in power plant sulphur emissions, they could try to make a deal with the power plants. They would be able to offer the power plants enough to offset the higher costs of low-sulphur fuel and still come out ahead. But there is no such market. The reason for this is that there is no single private individual or group whose loss from acid rain is large enough to make it worthwhile to offer the power plants payment to reduce sulphur emissions. It would require a large number of those affected by acid rain to form a coalition to make the offer. There are markets for electric power and for coal. However, they do not lead to socially desirable decisions about the use of power or coal. This is because the market prices for power and coal do not reflect the costs related to acid rain. If the prices for power and high-sulphur coal reflected these costs, less power would be used, and less of it would be made with high-sulphur coal. Both would reduce acid rain.

The damage to the residents of Sydney, Nova Scotia, due to air pollution and toxic waste is another example of the effects of market failure. The market failed to take into account the health and environmental costs in the price of steel. In the early years of steel production, there was little information about the deleterious health effects of air pollution and of the toxic coke oven by-products. However, as early as the 1960s the government started to exert pressure on the Dominion Steel and Coal Company (DOSCO), then operating the steel mill, to reduce its emissions. DOSCO balked, however, indicating that the estimated cost would put it out of business. Not long after, DOSCO announced its intention to close the steel mills due to its inability to compete in international markets. This led the local residents to lobby DOSCO and the provincial government to keep the mills open. They claimed that closing the mill would have a devastating effect on the local economy. SYSCO then took over the operation of the steel mill. SYSCO continued to contribute to the pollution problem, and it was only in 1982, when contamination in the local lobster catch forced the fisheries to close, that the effects of market failure really started to become evident. The loss to the fishermen certainly had not been factored into the costs of running the steel mills. Throughout the 1980s, other costs associated with the steel mills, unaccounted for in the price of

steel, began to become more evident. Health problems such as birth defects, cancer, severe headaches, and lung disease in local residents were the subject of numerous scientific studies. The government spent \$250 million between 1980 and 2000 in several unsuccessful attempts to clean up the toxic wastes. Finally, a decision was made in 2000 to close the steel mill as it became evident that the true costs of the mill far exceeded the benefits to the steel mill and to the local economy. Estimates range from \$400 million to almost \$1 billion.

We can see how market failure has caused socially undesirable outcomes such as acid rain and contaminated industrial sites. When markets fail, as in cases such as these, society will seek to remedy these problems through a variety of mechanisms. These remedies are the subject of the next section.

### 10.2.2 Remedies for Market Failure

There are three main formal methods of eliminating or reducing the impact of market failure:

1. Policy instruments used by the government
2. The ability of persons or companies adversely affected by the actions of others to seek compensation in the courts
3. Government provision of goods and services

We shall discuss the first two methods in this section. Government provision of goods and services is discussed in the next section under decision making in the public sector.

The first and most common means of trying to deter or reduce the effects of market failure is the use of what are commonly called **policy instruments**—government-imposed rules intended to modify behaviour.

One class of policy instruments includes *regulations* such as standards, bans, permits, or quotas. The regulations are backed by penalties for non-compliance. Most countries have regulations concerning such widely differing areas as product labelling, automobile emissions, and the use of bodies of water to dispose of waste. A challenge associated with developing regulations is that they may be inefficient. For example, suppose that we wish to improve the quality of a lake by reducing the amount of effluents dumped into it. These effluents may contain material with excessive biological oxygen demand (BOD). A typical regulation to control dumping would require all those who dump to meet the same BOD standards. But the costs of meeting these standards are likely to differ among the producers of the effluent. To attain a regulated reduction in BOD in their effluent, some producers will have to make expensive changes to their procedures, while others can respond with a lower cost. The most efficient way to obtain the reduction in BOD in the lake is to have those with low effluent-cleaning costs make the greatest reduction in BOD. Uniform regulation is not likely to do this. A common means of implementing regulations so that they are reasonably efficient is the use of a tradeable permit system. These permits allow producers emissions (as an example) up to a certain level, and if the producer is able to find ways to reduce pollution, it can sell its “spare” emissions capacity to others that find achieving such pollution reduction to be more difficult. Mini-Case 10.1 at the end of this chapter provides some additional information about such systems.

A second class of policy instruments meant to overcome market failure is *monetary incentives* or *deterrents* to induce desired behaviour. Monetary incentives or deterrents, often more efficient than regulations, may be subsidies or special tax treatments. For example, referring back to effluent dumping, there could be subsidies for the installation of equipment that would reduce the amount of effluent produced. In this way, producers

for whom the cost of reducing BOD is low will do so since this would be cheaper than paying fees. By setting an appropriate subsidy, the desired reduction can be attained. The Australian fuel excise tax rebate noted in the opening vignette is another example of a monetary incentive.

Policy instruments are constantly being evaluated and modified. The reason is that market failure is a complex issue and policy makers may not fully understand its causes or the implications of implementing a given policy. Nonetheless, their use is widespread and they can be highly effective in mitigating the effects of market failure.

A second formal means of reducing the effects of market failure is *litigation*. In the past several decades, the use of courts as a means of reducing the health and safety effects of market failure has grown all over the world, particularly in the United States. Most countries have established legal regulations by which regular sellers of a product implicitly guarantee that the product is fit for reasonable use. Where the cost of reducing a risk in the use of their product is less than the objectively estimated expected loss, sellers are supposed to reduce the risk. Moreover, these sellers are held legally responsible for having expertise in the production and use of the products. It is not enough for sellers to say they did not know that use of the product was risky. Sellers are supposed to make reasonable efforts to determine potential risks in the use of the products they sell.

While the name by which these regulations are known varies by country—*Uniform Commercial Code* (United States), *Trade Practices Act* (Australia), *Consumer Code* (Britain, Canada)—they all fall into the general category of consumer law. The development of consumer law has been made more complex in recent years due to the prevalence of global markets where buyers and sellers are in different countries, each with different regulations.

The third formal method of reducing the effects of market failure is *government provision of goods and services*. This provision may be direct, as in the case of police services, or indirect, as in the case of health care given by physicians. Market failure is remedied when public sector analysts take into account all parties affected by a decision through a comprehensive assessment of total costs and benefits of a decision. Health care provision, transportation, municipal services, and electric and gas utilities are some examples of goods and services provided by the public sector. Each service requires numerous economic decisions to be made in the best interests of the public. This is of sufficient importance for us to devote a separate section of this chapter to the topic.

In addition to formal methods for dealing with market failure, *informal methods* can be used. Groups of individuals, whether formally organized or not, can exert pressure on companies or government to change their policies with respect to a specific issue. Direct pressure tactics can include lobbying companies, politicians, or the legal system itself. Demonstrations and boycotts are a peaceful means of social protest, but pressure tactics such as civil disobedience or criminal acts have also been used in attempts to change policy or practice. Another informal method is information dissemination to industry and to the general public, encouraging or discouraging certain behaviours.

## 10.3 | Decision Making in the Public Sector

This section is devoted to the decision-making process for public provision of goods and services. Public (government) production generally occurs mainly in two classes of goods and services. The first class includes those services for which there is no market because it is not practical to require people to pay for the service. Police and fire protection,

defence, and the maintenance of city streets are examples of government services that it is not practical for users to pay for.

The second class includes those services for which scale economies make it inefficient to have more than one provider. Where there is only a single provider of a service, there is no market competition to enforce efficiency and low prices. There is a danger that the single provider, called a *monopolist*, will charge excessive prices and/or be inefficient. To ameliorate this potential problem, governments are often the provider.

For example, local deliveries of natural gas and electric power are situations where economies of scale are important enough that having more than one provider is not efficient. These services may be provided by publicly owned monopolies. For example, Canada Post is the government-owned national postal service for Canada. An alternative to government provision of services where there is one provider is for the government to monitor and regulate the performance of a private monopolist. For example, Ontario Power Generation (OPG) is a private Ontario-based company with primary business in the generation and sale of electricity to customers in Ontario and interconnected markets.

Privatization of traditionally government-run functions has been a growing trend worldwide in the past several decades. Proponents of privatization argue that the private sector is more efficient than the government in its operations. Those against privatization argue that private organizations will not take into account all the social costs and benefits of their actions and thus contribute to the potential market failure. Close-Up 10.1 describes **public-private partnership (P3)** projects, in which private sector companies invest in public sector projects and recoup their investments through user-based tolls, fees, or tariffs.

In this section, we consider **benefit-cost analysis (BCA)**, a method of project evaluation widely used in the public sector. BCA provides a general framework for assessing the gains and losses associated with alternative projects when a broad societal view is necessary. This method is commonly used for public projects because the government is responsible for applying public resources in a manner that balances the costs and benefits of a project in such a way as to produce the greatest overall social benefit. BCA can also be used for project evaluation in the private sector where the broader impact on society must be taken into account.

The starting point for a BCA is a clear understanding of what alternative courses of action, or projects, are being considered. Once these are established, there are two main sets of issues to resolve. The first is to identify and measure the benefits associated with each project, and to clearly understand to whom these benefits accrue. The second is to identify and measure the costs associated with each project, and to establish who pays the costs. Once these issues are resolved, then the projects can be fairly evaluated by a variety of comparison methods. Benefit-cost ratios are a commonly used comparison method for public sector projects, though the methods of Chapters 4 and 5 are also applicable.

### 10.3.1 The Point of View Used for Project Evaluation

In carrying out a benefit-cost analysis, it is important to clearly establish what point of view is to be taken and to use this point of view consistently. The basic questions to address are “Who will benefit from the project?” and “Who will pay for the project?” By identifying these two points of view clearly at the outset of the evaluation, confusion about what to include and what not to include can be reduced. The point of view defines who is “in” and who is “out.”

**CLOSE-UP 10.1****Public–Private Partnerships**

Traditionally, large-scale public infrastructure projects—such as roads, bridges, power plants, and public utilities—have been financed, built, and maintained by government. In recent years, many countries have moved increasingly to public–private partnerships.

A public–private partnership, also known as a PPP or P3, is a contractual agreement between the government and a private organization to provide a service or product to society. The private partner may be a single company or a consortium of several companies. The public sector typically maintains an oversight and quality-assessment role, while the private sector is responsible for provision of the service or project.

Numerous categories of P3s exist. Each is categorized on the basis of the extent of public and private sector involvement and the allocation of financial responsibility between the two. The most popular forms are the BOOT (build–own–operate–transfer) and BOT (build–operate–transfer) models, which are used most frequently to build government facilities such as schools, hospitals, bridges, and roads. In another category of PPP, the government sells an asset and then leases it back from the private partner.

Some high-profile examples of PPPs in Canada are the Confederation Bridge construction between Nova Scotia and Prince Edward Island, and Ontario's 407 ETR toll highway.

PPPs are not without problems, however. Critics worry that privatization may not bring about more efficient operations and that the traditionally profit-oriented private sector may not take social good into account in its decision making. In addition, the private sector is not willing to use interest rates as low as the government typically uses, thus making the projects appear more expensive than when they are undertaken by the government.

Generally, members of society are the users and beneficiaries of project services, and the government—the sponsor—pays for the project. The evaluation will thus take into account the impact of the project on both the users and the sponsors. For example, if a government is considering a highway improvement project to reduce traffic congestion and to improve safety, the point of view taken would focus on the impact of the project on the government's finances and on the benefits to the users most affected by the highway improvement. Therefore the analysis would include government costs such as construction and maintenance expenses. Savings (reduction in costs) to the government could include new property tax income if land values were to go up due to increased economic activity in the area. Social benefits could include factors such as reduced travel time, reduced vehicle operating costs, and increased safety for users of the goods and services provided by the project. Social costs (also called disbenefits) include the costs of travel delays and disruption during the construction period, increased traffic noise in nearby neighbourhoods, and possibly negative environmental impacts, all of which can affect users. Careful definition of the point of view taken ensures that all the effects of the project are taken into account.

One of the challenges associated with the analysis of public projects is that there may be several reasonable points of view, each with a different set of users or sponsors to be included in the analysis. Consider the decision to construct a dam in an agricultural area in order to provide hydroelectric power and irrigation. An analyst providing advice on such a project would need to consider the impact of relocating families from the flooded areas as well as the increased productivity and income to farmers due to improved irrigation. Aside

## NET VALUE 10.1

### **Canadian Benefit–Cost Analysis Guides**

Various branches of the Canadian government make benefit–cost analysis guides available to assist analysts and managers with the task of evaluating public projects. Here are two useful ones.

*Canadian Cost–Benefit Analysis Guide* (Treasury Board of Canada): This guide provides a framework for submissions to the Treasury Board of Canada for projects that have major social, economic, or environmental impacts. The guide indicates the steps to be taken in a thorough benefit–cost analysis and sets out best practices at each stage. It also provides insights into identifying and measuring costs and benefits associated with public projects. One section provides examples of estimating values when no market prices exist.

Among the examples given are how to estimate the value of travel-time savings, job creation, and health and safety. The guide can be found at [www.tbs-sct.gc.ca/ri-qr/documents/g1-l1d/analys/analys-eng.pdf](http://www.tbs-sct.gc.ca/ri-qr/documents/g1-l1d/analys/analys-eng.pdf).

*Guide to Benefit–Cost Analysis in Transport Canada* (Transport Canada): This guide provides a framework for evaluating alternative projects when they have a transportation focus. It also gives several illustrative examples of benefit–cost analysis applied to airport refurbishment, the acquisition of depth-sounding catamarans in the St. Lawrence River, and consolidation of maintenance work centres for air navigation electronic systems. The guide can be found at [www.tc.gc.ca/media/documents/corporate-services/bca.pdf](http://www.tc.gc.ca/media/documents/corporate-services/bca.pdf).

from the costs and benefits to the local community, the analyst would need to consider the potential impact on the wider environment due to the potential of moving from coal-fired electricity production to hydroelectricity. The point of view chosen delineates whose costs and benefits are to be taken into account in a benefit–cost analysis. The fact that several points of view may be taken can lead to ambiguity and controversy over the results. Nonetheless, BCA is a common framework for analysis.

Another important factor to consider in project evaluation is the frame of reference for measuring the impacts of the project. The analysis should consider costs and benefits associated with the project based on the difference between what would occur *with* the project versus what would occur *without* the project. In other words, we concern ourselves with the marginal benefits and costs associated with the project so that its impact is fairly measured.

### **10.3.2 Identifying and Measuring the Costs of Public Projects**

Once we have a clear view of who will benefit from a project (the users) and who will pay for a project (the sponsor), the next step in a BCA is to identify and measure the various costs and benefits associated with the project. The fundamental questions are “In what ways will users benefit from the project, and by how much will they benefit?” and “What costs will be incurred by the sponsor, and how much will these costs be?”

This section deals with identifying and measuring the costs of public projects to the project sponsor. For the most part, these costs are straightforward to identify and measure. This is not the case with benefits, which will be covered in the next section.

The sponsor costs broadly include all of the resources, goods, and services required to develop, implement, and maintain a project or program. These can be generally classified into the initial capital costs and ongoing operating and administration costs. If the project creates savings, these are deducted from costs to produce a net cost to the sponsor.

For the highway project mentioned in previous sections, the following is a classification of some of the sponsor's costs and savings:

#### **Costs to the Sponsor**

- Construction costs
- Operating and maintenance costs
- Administrative costs

#### **Savings to the Sponsor**

- Increased tax revenues due to higher land values

Some costs can be *directly* attributed to a project, while others may be stimulated indirectly by the project. These are referred to as *direct* and *indirect* costs, respectively. For example, in the above project, construction and operating costs are direct costs, and increased tax revenues are indirect savings.

While identifying and valuing the costs of a public project can be relatively straightforward, the same cannot generally be said about identifying and valuing social benefits.

### **10.3.3 Identifying and Measuring the Benefits of Public Projects**

This section deals with identifying and measuring the benefits associated with public projects. We begin by providing an overview of the process, and then indicate some of the challenges associated with this portion of a benefit–cost analysis.

The benefits of a project to society include the value of all goods and services that result from the project or program. Generally, the benefits of a public project will be positive. However, some of the effects of a project may be negative. These are referred to as social costs, and are subtracted from the benefits to obtain a net measure of social benefits.

For the example of a highway improvement project, the following is a classification of some of the social benefits and social costs:

#### **Social Benefits**

- Reduced travel time for business and recreational users
- Increased safety
- Reduced vehicle operating costs for business and recreational users

#### **Social Costs**

- Increased noise and air pollution
- Disruption to the local environment
- Disruption of traffic flows or business transactions during construction
- Loss of business elsewhere due to traffic rerouting onto the new highway

As with sponsor costs, some of the social benefits or social costs can be *directly* attributed to a project, while others may be stimulated indirectly. These are referred to as *direct* and *indirect* benefits, respectively. For example, direct benefits of an improved highway may be decreased travel time and increased safety. Indirect costs could include lost business to other areas if traffic is rerouted from these areas onto the new highway. A thorough benefit–cost analysis should always include the direct benefits and will include indirect benefits if they have an important effect on the overall project.

The task of identifying and measuring the social benefits and costs of public projects can be challenging. The challenge arises because the benefits may not be reflected in the monetary flows of the project. We are concerned with the real effects of a project, but the cash flows may or may not reflect all these real effects. For example, in the highway improvement project, there are cash flows for the wages of the workers who construct the road. This is a real cost to the government, and the wages reflect these costs well. In

contrast, consider the intangible social cost of disruption of traffic during road construction. These costs are not reflected in the cash flows of a project, but are nonetheless an important cost of putting the road in place.

Beyond the challenge of identifying certain costs and benefits is that of assigning a value or measure to each. For goods and services that are distributed through markets, we have prices to measure values. Unfortunately, these prices may be distorted and need adjustment prior to use in an economic evaluation. We will assume the use of market values, since the process of adjusting distorted prices is beyond the scope of this text. Valuation is relatively straightforward for these items. Many public projects create or use goods and services for which there is no market in which prices reflect values. These intangible, non-market goods and services are challenging to value.

Consider the highway improvement project mentioned earlier. Obvious sponsor costs are the labour, materials, and equipment used for the project. These costs can be relatively easy to estimate, as there are markets for these goods and services through which appropriate prices have been established. However, there are several intangible social costs and benefits that are somewhat more difficult to measure. First, there are the social costs associated with temporary travel-time disruptions due to construction. Measuring the cost of traffic disruption during the work requires valuing the time delays incurred by car passengers and trucks. There are approximations to these delay costs based on earnings per hour of passengers and the hourly cost of running trucks. The approximation for the value of travellers' time is based on the idea that a person who can earn, for example, \$35 an hour working should be willing to pay \$35 an hour for time saved travelling to work. The disruption costs may be large enough that they make it more efficient to have the work done at night, despite the fact that this will raise the explicit construction cost. The intangible social benefits of reduced travel time once the project is completed can also be factored into the benefit-cost analysis in a similar way.

An intangible social benefit associated with many public projects is improved health and safety. A common method for assessing health and safety benefits is to estimate the reduction in the number and type of injuries expected with a project or program and then to put a value on these injuries. A variety of methods exist, but one approach is to use lost wages, treatment costs, or insurance claims as the basis for estimating the value of certain injuries. Estimates of the value of a human life using this method range from several hundred thousand to several million dollars.

Other examples of intangible items commonly valued in a benefit-cost analysis are the value of noise abatement and the value of the environment. Two basic methods have been developed to value intangible social benefits and costs. The first is *contingent valuation*. This method uses surveys to ask members of the public what they would be willing to pay for the good or service in question. For example, members of the public may be asked how much they would be willing to pay for each minute of reduced travel time due to a highway improvement. Contingent valuation has the benefit of being a direct approach to valuation, but has the drawback that individuals may not provide accurate or truthful responses.

The second commonly used method for assessing the value of non-market goods and services is called the *hedonic price* method. This method involves deducing indirectly individuals' valuations based on their behaviour in other markets for goods and services. For example, the value of noise pollution due to a busy road may be estimated by examining the selling prices of houses near other busy roads and comparing these prices to those of similar houses away from busy roads. The value of the noise pollution cannot be higher than the difference in prices; otherwise, the owners would move. The advantage of this method is that members of the public reveal their true preferences through their behaviour. The disadvantage of the hedonic price method is that it requires reasonably sophisticated knowledge.

The following example provides further illustration of how the measurement of costs and benefits without market prices can be challenging, and also how these challenges can be dealt with.

**EXAMPLE****10.1**

Consider the construction of a bridge across a narrow part of a lake that gives access to a park. The major benefit of the bridge will be reduced travel time to get to the park from a nearby urban centre. This will lower the cost of camping trips at the park. As well, more people are expected to use the park because of the lower cost per visit. How can these benefits be measured?

Data concerning the number of week-long visits and their costs are shown in Table 10.1.

**Table 10.1 Average Cost per Visit and Number of Visits per Year**

|                            | Without Bridge | With Bridge |
|----------------------------|----------------|-------------|
| Travel cost per visit (\$) | 140.00         | 87.50       |
| Use of equipment (\$)      | 50.00          | 50.00       |
| Food cost per visit (\$)   | 100.00         | 100.00      |
| Total cost per visit (\$)  | 290.00         | 237.50      |
| Number of visits/year      | 8000           | 11 000      |

First, the reduction in cost for the 8000 visits that would have been made even without the bridge creates a straightforward benefit:

$$\text{Travel cost saving on 8000 visits} = (140 - 87.50) \times 8000 = 420\,000$$

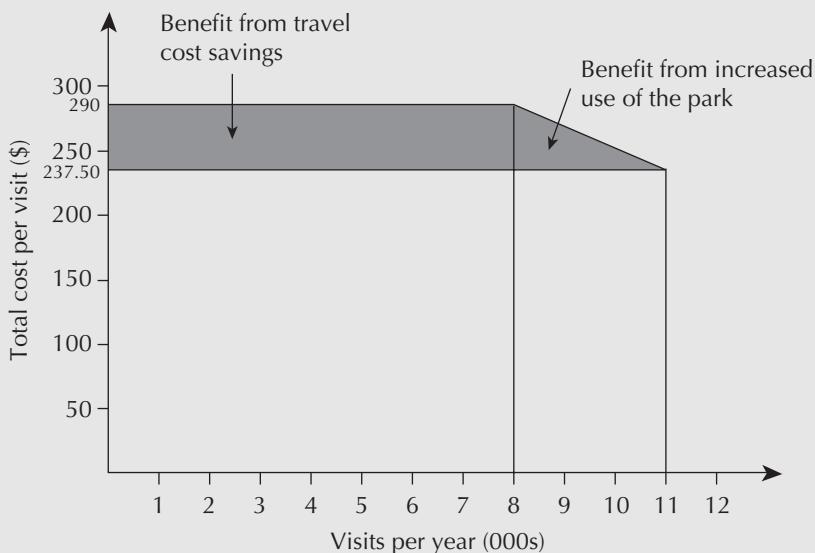
There is a benefit of \$420 000 per year from reduced travel cost on the 8000 visits that would have been made even without the bridge.

Next, we see that the number of visits to the park is expected to rise from 8000 per year to 11 000 per year. But how much is this worth? We do not have prices for park visits, but we do have data that enable estimates of actual costs to campers. These costs may be used to infer the value of visits to campers.

We see that before the bridge, the cost of a week-long park visit, including travel and other costs, averaged \$290.

It is reasonable to assume that a week spent camping was worth at least \$290 to anyone who incurred that cost. The average cost of a week-long visit to a park would fall from \$290 per visit to \$237.50 per visit if the bridge were built. We are concerned with the value of the incremental 3000 visits per year. Clearly, none of these visits would be made if the cost were \$290 per trip. And each of them is worth at least \$237.50 or else the trip would not have been taken. The standard approximation in cases like this is halfway between the highest and lowest possible values. This gives an aggregate benefit of the increased use of the park of

$$\frac{(290.00 + 237.50)}{2} \times 3000 = 791\,250$$

**Figure 10.1 Benefits From Bridge**

Therefore, the value of the incremental 3000 visits per year is estimated as approximately \$791 000 per year. However, there is also a cost of \$237.50 per visit. The net benefit of the incremental 3000 visits is therefore

$$791\,250 - 237.50(3000) = 78\,750$$

The total value of benefits of the bridge is the sum of the reduced travel cost plus increased use:

$$420\,000 + 78\,750 = 498\,750$$

The total value of the benefits yielded by the bridge is almost \$500 000 per year (the shaded area shown in Figure 10.1). These benefits must then be weighed against the costs of the bridge. ■

#### 10.3.4 Benefit–Cost Ratios

Once we have identified and obtained measures for the costs and benefits associated with a public project, we can evaluate its economic viability. The same comparison methods that are used for private sector projects are appropriate for government sector projects. That is, we can use the present worth, annual worth, and internal-rate-of-return performance measures in the same ways in both the private and government sectors. It is important to emphasize this, because other methods based on ratios of benefits to costs have been used frequently in government project evaluations, almost to the exclusion of present worth, annual worth, and internal-rate-of-return methods. Because of the prevalent use of these ratios, this section is devoted to a discussion of several benefit–cost ratios that are commonly used in public sector decision making. We then point out several problems associated with the

use of benefit–cost ratios so that the reader is aware of them and understands the correct way of using them.

Benefit–cost ratios can be based on either the present worths or the annual worths of benefits and costs of projects. The resulting ratios are equivalent in that they lead to the same decisions. We shall discuss the ratios in terms of present worths, but the reader should be aware that everything we say about ratios based on present worths applies to ratios based on annual worths.

The conventional **benefit–cost ratio (BCR)** is given by the ratio of the present worth of net users' benefits (social benefits less social costs) to the present worth of the net sponsors' costs for a project. That is,

$$\text{BCR} = \frac{\text{PW(users' benefits)}}{\text{PW(sponsors' costs)}}$$

A modified **benefit–cost ratio**, also in common use and denoted BCRM, is given by the ratio of the present worth of net benefits minus the present worth of operating costs to the present worth of capital costs—that is,

$$\text{BCRM} = \frac{\text{PW(users' benefits)} - \text{PW(sponsors' operating costs)}}{\text{PW(sponsors' capital costs)}}$$

Though the modified benefit–cost ratio is less used than the conventional benefit–cost ratio, it has the advantage that it provides a measure of the net gain per dollar invested by the project sponsor.

In general, a project is considered desirable if its benefit–cost ratio exceeds one—that is, its benefits exceed its costs.

### EXAMPLE

### 10.2

The town of Elmira, Ontario, has limited parking for cars near its main shopping street. The town plans to pave a lot for parking near King Street. The main beneficiaries will be the merchants and their customers. The present worth of expected benefits is \$3 000 000. The cost to the town of buying the lot, clearing, paving, and painting is expected to be \$500 000. The present worth of expected maintenance costs over the lifetime of the project is \$50 000. In the period in which the lot is being cleared and paved, there will be some disbenefits to the local merchants and customers due to disruption of traffic in the King Street area. The present worth of the disruption is expected to be about \$75 000. What is the benefit–cost ratio?

$$\text{BCR} = \frac{\text{PW(users' benefits)}}{\text{PW(sponsors' costs)}} = \frac{3\,000\,000 - 75\,000}{500\,000 + 50\,000} = 5.3$$

We can see that the benefit–cost ratio exceeds one, and thus the proposal is economically justified. ■

### EXAMPLE

### 10.3

A fire department in a medium-sized city is considering a new dispatch system for its firefighting equipment responding to calls. The system would select routes, taking into account recently updated traffic conditions. This would reduce response times. An

indirect effect would be a reduction in required fire-fighting equipment. The present worths of benefits and costs are:

*Users' Benefits*

$$PW(\text{benefits}) = \$37\,500\,000$$

*Sponsors' Costs*

$$PW(\text{operating costs}) = \$3\,750\,000$$

$$PW(\text{capital costs for the city}) = \$13\,500\,000$$

$$PW(\text{reduced equipment requirements}) = \$2\,250\,000$$

What is the benefit–cost ratio for the dispatch system? What is the modified benefit–cost ratio for the dispatch system? Is the project economically justifiable?

$$BCR = \frac{37\,500\,000}{(13\,500\,000 + 3\,750\,000 - 2\,250\,000)} = 2.50$$

$$BCRM = \frac{(37\,500\,000 - 3\,750\,000)}{(13\,500\,000 - 2\,250\,000)} = 3.00$$

We see for this example that both the conventional and modified cost-benefit ratio are greater than one, indicating that the project, under either criterion, is economically justified. ■

Examples 10.2 and 10.3 demonstrate the basic use of benefit–cost ratios. It is worth noting that if a conventional benefit–cost ratio is greater (less) than one, it follows that the modified benefit–cost ratio will be greater (less) than one. This means that the two types of benefit–cost ratios will lead to the same decision. We present both because the reader may encounter either ratio.

For *independent* projects, then, the following decision rule may be used: *Accept all projects with a benefit–cost ratio greater than one*. In other words, accept a project if

$$BCR = \frac{PW(\text{users' benefits})}{PW(\text{sponsors' costs})} > 1, \text{ or}$$

$$BCRM = \frac{PW(\text{users' benefits}) - PW(\text{sponsors' operating costs})}{PW(\text{sponsors' capital costs})} > 1$$

This rule, using either the benefit–cost ratio or the modified benefit–cost ratio, is equivalent to the rule that all projects with a present worth of benefits greater than the present worth of costs should be accepted. This is the same rule that was presented in Chapter 4, which accepts a project if its present worth is positive. This is easily shown. If

$$\frac{PW(\text{users' benefits})}{PW(\text{sponsors' costs})} > 1$$

it follows that

$$PW(\text{users' benefits}) > PW(\text{sponsors' costs})$$

which is equivalent to

$$\text{PW(users' benefits)} - \text{PW(sponsors' costs)} > 0$$

Also recall that, as shown in Chapter 5, the present/annual worth method and the internal rate of return method give the same conclusion if an independent project has a unique IRR.

To use benefit–cost ratios to choose among *mutually exclusive* projects, we must evaluate the increment between projects, just as we did in Chapter 5 with the internal rate of return method. Suppose we have two mutually exclusive projects, X and Y, with present worths of benefits,  $B_X$  and  $B_Y$ , and present worths of costs,  $C_X$  and  $C_Y$ . We first check to see if the individual benefit–cost ratios are greater than one. We discard a project with a benefit–cost ratio of less than one. If both projects have benefit–cost ratios greater than one, we rank the projects in ascending order by the present worths of costs. Suppose  $C_X \geq C_Y$ . We then form the ratio of the differences in benefits and costs,

$$\text{BCR}(X - Y) = \frac{B_X - B_Y}{C_X - C_Y}$$

If this ratio is greater than one, we choose project X. If it is less than one, we choose project Y. If  $C_X = C_Y$  (in which case the ratio is undefined), we choose the project with the greater present worth of benefits. If  $B_X = B_Y$ , we choose the project with the lower present worth of costs.

This rule is the same as comparing two mutually exclusive projects using the internal rate of return that was presented in Chapter 5. In order to choose a project, we saw in Chapter 5 that not only the IRR of the individual project but also the IRR of the incremental investment must exceed the MARR. This rule is also the same as choosing the project with the largest present worth, as presented in Chapter 4, provided the present worth is positive.

The following example, concerning two mutually exclusive projects, summarizes our discussion of benefit–cost ratios and illustrates their use.

### EXAMPLE

### 10.4

A medium-sized city is considering increasing its airport capacity. At the current airport, flights are frequently delayed and congestion at the terminal has limited the number of flights. Two mutually exclusive alternatives are being considered. Alternative A is to construct a new airport 65 kilometres from the city. Alternative B is to enlarge and otherwise upgrade the current airport, which is only 15 kilometres from the city. The advantage of a new airport is that there are essentially no limits on its size. The disadvantage is that it will require travellers to spend additional travel time getting to and from the airport.

There are two disadvantages in upgrading and enlarging the current airport. One disadvantage is that the increase in size is limited by existing development. The second disadvantage is that the noise of airplanes in a new flight path to the current airport will reduce the value of homes near that flight path. A thousand homes will be affected, with the average loss in value about \$25 000. Such losses are large enough that it is not reasonable to expect that the owners' losses will be offset by gains elsewhere. If the city wishes to ensure that their losses are offset, it must compensate these owners. Note that such compensation would not be an additional social cost. It would merely be a transfer from taxpayers (through the government) to the affected owners. Benefit and cost data are shown in Table 10.2.

The city will use a MARR of 10 percent and a 10-year time horizon for this project. What are the benefit–cost ratios for the two alternatives? Which alternative should be accepted?

**Table 10.2 Airport Benefits and Costs**

| Effect                       | Alternative A<br>New Airport<br>(millions of \$) | Alternative B<br>Current Airport<br>(millions of \$) |
|------------------------------|--|--|
| Improved service/year        | 55   | 28.5   |
| Increased travel cost/year   | 15   | 0  |
| Cost of highway improvements | 50   | 10   |
| Construction costs           | 150  | 115  |
| Reduced value of houses      | 0  | 25   |

Before we start the computation of benefit–cost ratios, we note that the comparison can be done on an annual worth or a present worth basis; we have elected the latter, and thus we need to convert all values to present worths. Two effects are shown on an annual basis: improved service, which occurs under both alternatives, and increased travel cost, which appears only with the new airport. We get the present worths (in millions of \$) by multiplying the relevant terms by the series present worth factor:

$$\begin{aligned} \text{PW(improved service of A)} &= 55(P/A, 10\%, 10) \\ &= 55(6.1446) \\ &= 337.95 \end{aligned}$$

$$\text{PW(improved service of B)} = 28.5(6.1446) = 175.10$$

$$\text{PW(increased travel cost of A)} = 15(6.1446) = 92.17$$

The remainder of the costs are already in terms of present worth.

The benefit–cost ratio for alternative A is

$$\text{BCR(alternative A)} = \frac{337.95}{(150 + 50 + 92.17)} \approx 1.16$$

and for alternative B is

$$\text{BCR(alternative B)} = \frac{175.10}{(10 + 115 + 25)} = 1.1673$$

First, we note that both benefit–cost ratios exceed one. Both alternatives are viable. Since they are mutually exclusive, we must choose one of the two. To decide which alternative is better, we must compute the benefit–cost ratio of the incremental investment between the alternatives. We use the alternative with the smallest present worth of costs as the starting point (alternative B), and compute the benefit–cost ratio associated with the marginal investment:

$$\begin{aligned} \text{BCR}(A - B) &= (B_A - B_B)/(C_A - C_B) \\ &= \frac{(337.95 - 92.17 - 175.10)}{(50 + 150) - (10 + 115 + 25)} = 1.4136 \end{aligned}$$

The ratio of the benefits of the new airport minus the benefits of the current airport modification over the difference in their costs is greater than one. We interpret this to mean that the benefit–cost ratio ranks the new airport ahead of the current airport.

We also note that the same ranking would result if we were to compare the present worths of the two alternatives. The present worth of the new airport (in millions of \$) is

$$PW(\text{alternative A}) = 337.95 - 92.17 - 150 - 50 = 45.78$$

The present worth of modifying the current airport (in millions of \$) is

$$PW(\text{alternative B}) = 175.10 - 10 - 115 - 25 = 25.10$$

In addition to the quantifiable aspects of each of the two airports, recall that the new airport is preferred in terms of the unmeasured value of space for future growth. As well, the new airport does not entail the difficulties associated with compensating home owners for loss in value of their homes. Together with the benefit–cost ratios (or equivalently, the present worths), this means that the city should build the new airport. ■

A word of caution is needed before we conclude this section. There may be some ambiguity in how to correctly calculate benefit–cost ratios, either conventional or modified. The reason is that for some projects, it may not be clear whether certain positive effects of projects are benefits to the public or reductions in cost to the government, or whether certain negative effects are disbenefits to the public or increases in costs to the government. As a result, benefit–cost ratios for a given project may not be unique. This lack of uniqueness means that a *comparison of the benefit–cost ratios of two projects is meaningless*. It is for this reason that we need to exercise caution when comparing projects with the benefit–cost ratio. To avoid this ambiguity, many experts recommend the use of present worth or annual worth for comparisons rather than benefit–cost ratios. Despite this recommendation, benefit–cost ratios are still used extensively for project comparisons, and since they are still in use we present readers with enough material to properly construct and interpret them.

To illustrate the ambiguity in constructing benefit–cost ratios, consider Example 10.4. The reduction in the value of residential properties could have been treated as a disbenefit to the public instead of a cost to the government for compensating homeowners. As a result, there are two reasonable benefit–cost ratios for alternative B. The second is

$$BCR_2(\text{alternative B}) = \frac{175.10 - 25}{(10 + 115)} = 1.2008$$

The lack of uniqueness of benefit–cost ratios does not mean that the ratios cannot be used, but it does mean that some care needs to be given to their correct application. Comparison methods based on benefit–cost ratios remain valid because the comparison methods depend only on whether the benefit–cost ratio is less than or greater than one. Whether it is greater or less than one does not depend upon how positive and negative effects are classified. This is clearly illustrated in the following example.

**EXAMPLE****10.5**

A certain project has present worth of benefits,  $B$ , and present worth of costs,  $C$ . As well, there is a positive effect with a present worth of  $d$ ; the analyst is unsure of whether

this positive effect is a benefit or a reduction in cost. There are two possible benefit–cost ratios,

$$\text{BCR}_1 = \frac{B + d}{C}$$

and

$$\text{BCR}_2 = \frac{B}{C - d}$$

For a ratio to exceed one, the numerator must be greater than the denominator. This means that, for  $\text{BCR}_1$ ,

$$\text{BCR}_1 = \frac{B + d}{C} > 1 \Leftrightarrow B + d > C \Leftrightarrow B > C - d$$

But this is the same as

$$\frac{B}{C - d} = \text{BCR}_2 > 1$$

Consequently, any BCR that is greater than one or less than one will be so regardless of whether any positive effects are treated as positive benefits or as negative costs. A similar analysis would show that the choice of classification of a negative effect as a cost or as a reduction in benefits does not affect whether the benefit–cost ratio is greater or less than one. ■

To conclude this section, we can note several things about the evaluation of public sector projects. First, the comparison methods developed in Chapters 4 and 5 are fully applicable to public sector projects. Second, the reader may encounter the use of benefit–cost ratios for public projects, despite the fact that benefit–cost ratios can be ambiguous. Next, the reader should be wary of decisions based on the absolute magnitude of benefit–cost ratios, as these ratios may be changed by reclassification of some of the positive effects of projects as benefits or as reductions in cost, or by reclassification of some of the negative effects as reductions in benefits or increases in costs. However, since the classifications do not affect whether a ratio is greater or less than one, it is possible to use benefit–cost ratios to reach the same conclusions as those reached using the methods of Chapters 4 and 5.

### 10.3.5 The MARR in the Public Sector

There are significant differences between the private sector and public (government) sector of society with respect to investment. As observed in the previous sections, public sector organizations provide a mechanism for resources to be allocated to projects believed beneficial to society in general. These include projects for which scale economies make it inefficient to have more than one supplier, and projects in markets that would otherwise suffer market failure. The government also regulates markets and collects and redistributes taxes toward the goal of maximizing social benefits. Typical projects include health, safety, education programs, cultural development, and infrastructure development. Profits generated by public projects are not taxed.

Private institutions and individuals, in contrast to public institutions, are more concerned with generating wealth (profits) and are taxed by the government on this wealth. We would therefore expect the MARR for a public institution to be lower than that of a

private institution, because the latter has a substantial extra expense acting to reduce its profits.

In evaluating public sector projects, the MARR is used in the same way as in evaluating private sector projects—it captures the time value of money. In the private sector, the MARR expresses the minimum rate of return required on projects, taking into account that those profits will ultimately be taxed. The MARR used for public projects, often called the *social discount rate*, reflects the more general investment goal of maximizing social benefits. Of course, as with private projects, public projects will sometimes be chosen on the basis of lowest present cost (or the equivalent) rather than highest present worth.

There is substantial debate as to what an appropriate social discount rate should be. Several viewpoints can shed some light on this issue.

The first is simply that the MARR should be the interest rate on capital borrowed to fund a project. In the public sector, funds are typically raised by issuing government bonds. Hence, the current bond rate might be an appropriate MARR to use. The second is that the MARR used to evaluate public projects should take into account that government spending on public projects consumes capital that might otherwise be used by taxpayers for private purposes. This viewpoint says that funds taken away from individuals and private organizations in the form of taxes might otherwise be used to fund private projects. This line of thinking might lead to a social discount rate that is the same as the MARR for the taxpayers. The two viewpoints are lower and upper bounds, respectively, to what can be seen as reasonable MARRs to apply when evaluating public projects.

In practice, a range of MARRs is used for evaluating public projects so that the impact of these differing points of view can be understood. The Treasury Board of Canada recommends a range of 3 percent to 7 percent for projects with a life of more than 30 years, and 8 percent to 12 percent otherwise.

## REVIEW PROBLEMS

### **REVIEW PROBLEM 10.1**

This review problem is adapted from an example in the Canadian Treasury Board's *Benefit-Cost Analysis Guide* (1976).

There are periodic floods in the spring and drought conditions in the summer that cause losses in a 15 000-square-kilometre Prairie river basin that has a population of 50 000 people. The area is mostly farmland, but there are several towns. Several flood control and irrigation alternatives are being considered:

1. Dam the river to provide flood control, irrigation, and recreation.
2. Dam the river to provide flood control and irrigation without recreation.
3. Control flooding with a joint Canada–United States water control project on the river.
4. Develop alternative land uses that would not be affected by flooding.

The constraints faced by the government are the following:

1. The project must not reduce arable land.
2. Joint Canada–United States projects are subject to delays caused by legal and political obstacles.

3. Damming of the river in the United States will cause damage to wildlife refuges.
4. The target date for completion is three years.

Taking into account the constraints, alternatives 3 and 4 above can be eliminated, leaving two:

1. Construct a dam for flood control, irrigation, and recreation.
2. Construct a dam for flood control and irrigation only.

A number of assumptions were made with respect to the dam and the recreational facilities:

1. An earthen dam will have a 50-year useful life.
2. Population and demand for recreational facilities will grow by 3.25 percent per year.
3. A three-year planning and construction period is reasonable for the dam.
4. Operating and maintenance costs for the dam will be constant in real dollars.
5. Recreational facilities will be constructed in year 2.
6. It will be necessary to replace the recreational facilities every 10 years. This will occur in years 12, 22, 32, and 42. Replacement costs will be constant in real dollars.
7. Operating and maintenance costs for the recreational facilities will be constant in real dollars.
8. The real dollar opportunity cost of funds used for this project is estimated to be in the range of 5 percent to 15 percent.

The benefits and costs of the two projects are shown in Tables 10.3 and 10.4.

Notice that the benefits and costs are estimated averages. For example, the value of reduced flood damages will vary from year to year, depending on such factors as rainfall and snowmelt. It is not possible to predict actual values for a 50-year period.

- (a) What is the present worth of building the dam only? What is the benefit–cost ratio? What is the modified benefit–cost ratio? Use 10 percent as the MARR.
- (b) What is the present worth of building the dam plus the recreational facilities? Use 10 percent as the MARR.
- (c) What is the benefit–cost ratio for building the dam and recreation facilities together? What is the modified benefit–cost ratio?

**Table 10.3 Estimated Average Benefits of the Two Projects**

| Year | Flood Damage Reduction | Irrigation Benefits | Recreation Benefits |
|------|------------------------|---------------------|---------------------|
| 0    | \$ 0                   | \$ 0                | \$ 0                |
| 1    | 0                      | 0                   | 0                   |
| 2    | 0                      | 0                   | 0                   |
| 3    | 182 510                | 200 000             | 27 600              |
| 4    | 182 510                | 200 000             | 28 497              |
| :    | :                      | :                   | :                   |
| 52   | 182 510                | 200 000             | 132 288             |

**Table 10.4 Estimated Average Costs of the Two Projects**

| Year | Dam Construction | Operating and Maintenance Dam | Recreation Construction | Operating and Maintenance Recreation |
|------|------------------|-------------------------------|-------------------------|--------------------------------------|
| 0    | \$ 300 000       | \$ 0                          | \$ 0                    | \$ 0                                 |
| 1    | 750 000          | 0                             | 0                       | 0                                    |
| 2    | 1 500 000        | 0                             | 50 000                  | 0                                    |
| 3    | 0                | 30 000                        | 0                       | 15 000                               |
| 4    | 0                | 30 000                        | 0                       | 15 000                               |
| :    | :                | :                             | :                       | :                                    |
| 11   | 0                | 30 000                        | 0                       | 15 000                               |
| 12   | 0                | 30 000                        | 20 000                  | 15 000                               |
| 13   | 0                | 30 000                        | 0                       | 15 000                               |
| :    | :                | :                             | :                       | :                                    |
| 21   | 0                | 30 000                        | 0                       | 15 000                               |
| 22   | 0                | 30 000                        | 20 000                  | 15 000                               |
| 23   | 0                | 30 000                        | 0                       | 15 000                               |
| :    | :                | :                             | :                       | :                                    |
| 31   | 0                | 30 000                        | 0                       | 15 000                               |
| 32   | 0                | 30 000                        | 20 000                  | 15 000                               |
| 33   | 0                | 30 000                        | 0                       | 15 000                               |
| :    | :                | :                             | :                       | :                                    |
| 41   | 0                | 30 000                        | 0                       | 15 000                               |
| 42   | 0                | 30 000                        | 20 000                  | 15 000                               |
| 43   | 0                | 30 000                        | 0                       | 15 000                               |
| :    | :                | :                             | :                       | :                                    |
| 52   | 0                | 30 000                        | 0                       | 15 000                               |

- (d) Which project, 1 or 2, is preferred, on the basis of your benefit–cost analysis? Use 10 percent as the MARR.

### ANSWER

- (a) We need to determine the present worth of benefits and costs of the dam alone. There are two benefits from the dam alone. They are those resulting from reduced flood damage and those associated with the benefits of irrigation. Both are approximated as annuities that start in year 3. We get the present worths of these benefits by multiplying the annual benefits by the series present worth factor and the present worth factor. The present worth of benefits resulting from reduced flood damage is

$$\begin{aligned}
 \text{PW}(\text{flood control}) &= 182\ 510(P/A, 10\%, 50)(P/F, 10\%, 2) \\
 &= \frac{182\ 510(9.99148)}{(1.1)^2} \\
 &= 1\ 495\ 498
 \end{aligned}$$

Similar computations give the present worth of irrigation as

$$PW(\text{irrigation}) = 1\,638\,812$$

There are two costs for the dam: capital costs that are incurred at time 0 and over years 1 and 2, and operating and maintenance costs that are approximated as an annuity that begins in year 3. Capital costs are given by

$$\begin{aligned} PW(\text{dam, capital cost}) &= 300\,000 + 750\,000(P/F, 10\%, 1) \\ &\quad + 1\,500\,000(P/F, 10\%, 2) \\ &= 2\,221\,487 \end{aligned}$$

The present worth of operating and maintenance costs is obtained in the same way as the present worths of flood control and irrigation benefits. The result is

$$PW(\text{operating and maintenance}) = 245\,822$$

The present worth of the dam alone is

$$\begin{aligned} PW(\text{dam}) &= 1\,495\,498 + 1\,638\,812 - (2\,221\,488 + 245\,822) \\ &= 667\,001 \end{aligned}$$

The benefit–cost ratio for the dam is

$$BCR(\text{dam}) = \frac{1\,495\,498 + 1\,638\,812}{2\,221\,488 + 245\,822} = 1.27$$

The modified benefit–cost ratio is given by

$$BCRM(\text{dam}) = \frac{1\,495\,498 + 1\,638\,812 - 245\,822}{2\,221\,488} = 1.30$$

The present worth of the dam alone is positive, and both benefit–cost ratios are greater than one. The dam alone appears to be economically viable.

- (b) We already have the present worths of benefits and costs for the dam alone. Therefore, we need only compute the present worths of benefits and costs for the recreation facilities. The capital costs for the recreation facilities consist of five outlays, in years 2, 12, 22, 32, and 42. The present worth of capital costs for the recreational facilities is given by

$$\begin{aligned} PW(\text{recreation facilities, capital cost}) &= 50\,000(P/F, 10\%, 2) \\ &\quad + 20\,000[(P/F, 10\%, 12) + (P/F, 10\%, 22) \\ &\quad \quad + (P/F, 10\%, 32) + (P/F, 10\%, 42)] \\ &= 51\,464 \end{aligned}$$

Operating and maintenance costs are estimated as an annuity that starts in year 3. The computation is the same as that of similar annuities that were used for the benefits and operating and maintenance costs of the dam. Thus, the present worth of recreation operating and maintenance costs is

$$PW(\text{recreation facilities, operating and maintenance}) = 122\,911$$

To obtain the present worth of the benefits from recreation, we need to define a growth-adjusted interest rate with  $i = 10$  percent and  $g = 3.25$  percent per year.

$$i^o = \frac{1+i}{1+g} - 1 = \frac{1+0.10}{1+0.0325} - 1 = 0.0653$$

We then use this to get the present worth geometric gradient series factor,

$$\begin{aligned}(P/A, g, i, N) &= \left( \frac{(1+i^o)^N - 1}{i^o(1+i^o)^N} \right) \left( \frac{1}{1+g} \right) \\ &= \left( \frac{(1.0653)^{50} - 1}{0.0653(1.0653)^{50}} \right) \left( \frac{1}{1.0325} \right) = 14.19\end{aligned}$$

To bring this to the end of year 0, we multiply by  $(P/F, 10\%, 2)$ . We then multiply by the initial value to get the present worth of recreation benefits.

$$\begin{aligned}\text{PW(recreation benefits)} &= [(P/A, 3.25\%, 10\%, N)(27\,600) (P/F, 10\%, 2)] \\ &= 14.19(27\,600)(1.1)^{-2} \\ &= 323\,679\end{aligned}$$

Another way to get this result is to use a spreadsheet. First, a column that contains the benefits in each year is created. The benefits start at \$27 600 in the third year and grow at 3.25 percent each year. Each benefit is then multiplied by the appropriate present worth factor in another column to obtain its present worth. The individual present worths are then summed to obtain the overall total of \$323 679. Some of the spreadsheet computations are shown in Table 10.5.

The total present worth of the recreation facilities is

$$\begin{aligned}\text{PW(total recreation facilities)} &= 323\,679 - 51\,464 - 122\,911 \\ &= 149\,304\end{aligned}$$

The present worth of the dam plus recreation facilities is obtained by adding the present worth of the dam alone to the present worth of the recreation facility. The final result is given by

$$\begin{aligned}\text{PW(dam and recreation facility)} &= 667\,001 + 149\,304 \\ &= 816\,305\end{aligned}$$

**Table 10.5 Spreadsheet Computations**



| Year     | Recreation Benefits | PW of Recreation Benefits |
|----------|---------------------|---------------------------|
| 0        | \$ 0.00             | \$ 0.00                   |
| 1        | 0.00                | 0.00                      |
| 2        | 0.00                | 0.00                      |
| 3        | 28 600.00           | 20 736.29                 |
| 4        | 28 497.00           | 19 463.83                 |
| 5        | 29 423.15           | 18 269.46                 |
| 6        | 30 379.40           | 17 148.38                 |
| :        | :                   | :                         |
| 52       | 132 288.48          | 931.33                    |
| Total PW |                     | 323 678.84                |

In present worth terms, the present worth of the dam *and* recreation facility exceeds that of the dam alone, so the dam and the recreation facility should be chosen.

- (c) The benefit–cost ratio for the dam and recreation facilities together is

$$\text{BCR}(\text{dam and recreation})$$

$$= \frac{(1\ 495\ 498 + 1\ 638\ 812 + 323\ 679)}{(2\ 221\ 488 + 245\ 822 + 51\ 464 + 122\ 911)} = 1.31$$

The modified benefit–cost ratio is

$$\text{BCRM}(\text{dam and recreation})$$

$$= \frac{(1\ 495\ 498 + 1\ 638\ 812 + 323\ 679 - 245\ 822 - 122\ 911)}{(2\ 221\ 488 + 51\ 464)}$$

$$= 1.36$$

The dam and recreation facilities project appears to be viable, since the benefit–cost ratios are greater than one.

- (d) On the basis of benefit–cost ratios, which of the two projects, project 1 or project 2, should be chosen? The dam and recreation facility is more costly, so the correct benefit–cost ratio for comparing the two is

$$\text{BCR} = \frac{\text{Benefits}(\text{dam and recreation facility}) - \text{Benefits}(\text{dam})}{\text{Costs}(\text{dam and recreation facility}) - \text{Costs}(\text{dam})}$$

$$= \frac{323\ 679}{(51\ 464 + 122\ 911)} = 1.86$$

The ratio exceeds one, and hence the dam and recreation facilities project should be chosen. This is consistent with the original present worth computations. ■

## SUMMARY

Chapter 10 concerns decision making in the public sector. We started by considering why markets may fail to lead to efficient decisions. We presented three formal methods by which society seeks to remedy market failure, one of which is to have production by government. Next, we laid out three issues in decision making about government production. First, we saw that the identification and measurement of costs and benefits in the public sector are more difficult than in the private sector. Identification may be difficult because there may not be cash flows that reflect the costs or benefits. Measurement may be difficult because there are no prices to indicate values. Second, we discussed the use of benefit–cost ratios in public sector project evaluation. While it is possible to use benefit–cost ratios so as to give the same conclusions as those obtained using the comparison methods discussed in Chapters 4 and 5, care must be taken because there may be ambiguity in the way in which

costs and benefits are classified. Third and last, we considered the MARR for government sector investments. The result of this discussion is that it is wise to use a range of MARRs corresponding to the opportunity cost of funds used in the public sector.

## ENGINEERING ECONOMICS IN ACTION, PART 10B

### Look at It Broadly

"How was your trip, Naomi?" Anna Kulkowski asked as she walked into the conference room. Bill and Naomi were already there waiting.

"Well, I'm here," responded Naomi, "but my jets are still lagging."

"You'll get used to it." Anna looked at Bill. "So, what's it going to cost to implement the Greenhouse Challenge Plus program for the folks down under?"

"Naomi got some additional information from the people at Melbourne Manufacturing," Bill answered. "They did some checking around with a couple of freight companies based in the same region. Apparently it does not cost anything, directly, to join the Greenhouse Challenge Plus program, but there are lots of costs. As it turns out, there are lots of benefits as well. The big positive is that the program's focus is on reducing greenhouse gas emissions overall, which is much broader than just reducing fuel emissions from our truck fleet."

Naomi continued, "It does require Melbourne Manufacturing to do an energy audit and progress report each year, but there is quite a bit of information available on the internet about this, and energy reducing ideas, too." She paused. "So things are not as bad as I first told Bill at all. Once we found out how broad the program actually is, lots of possibilities opened up. Melbourne Manufacturing has not focused much on making the warehouse and offices more energy efficient yet, so the company can benefit quite a bit from simple things like more efficient light bulbs and computer monitors, or setting the air conditioning at a higher temperature. The office staff had an impromptu focus group discussion the day before I left that was really productive. A lot of no-cost ideas came up. One of the newest engineers even suggested that she might be able to increase the amount shipped per truckload by some clever delivery route redesign. That could reduce fuel consumption without changing our service levels or delivery volumes."

"That all makes sense," Anna said. "It's a combination of zero-cost initiatives and some changes that will need Melbourne Manufacturing to make some relatively small investments. Once the company is able to project the savings due to energy reduction, it should be off to a good start."

"Yes," Naomi said. "By the time I left, the company had started to look into some of the government incentive plans for investing in energy efficient office equipment and to estimate the potential savings. Based on reports from other companies that have implemented some of these changes, it should be able to be a lot greener and save money doing it."

"And to think," Naomi mused, "that this all started with trying to make sure that Melbourne Manufacturing would be able to claim its fuel tax credits. Who would have thought it could have such a broad impact on operations?"

"That's true, Naomi," Anna commented. "Good work!"

## PROBLEMS

### A. Key Concepts

- 10.1** It is common for cities in cold countries to provide snowplow service for public roads. The major benefit of such services is to allow the convenient movement of vehicles over public roads following snow accumulation, at a cost of the snowplow (capital and operating) and driver. Are there other costs and benefits? List all you can think of, along with how they could be measured.

- 10.2** Most provinces and states provide travel information kiosks alongside major highways just across the border from neighbouring provinces and states. These kiosks provide maps and brochures on attractions, and some will make hotel and campground reservations. There are obvious costs, such as staffing costs and building capital and maintenance costs. What other costs and benefits are associated with this government service? How can these costs and benefits be measured?

- 10.3** The following data are available for a project:

|  |              |
|--|--------------|
| Present worth of benefits                        | \$17 000 000 |
| Present worth of operating and maintenance costs | \$5 000 000  |
| Present worth of capital costs                   | \$6 000 000  |

- (a) Find the benefit–cost ratio.
- (b) Find the modified benefit–cost ratio.

- 10.4** The following data are available for two mutually exclusive projects:

|                                     | Project A    | Project B    |
|-------------------------------------|--------------|--------------|
| PW(benefits)                        | \$19 000 000 | \$15 000 000 |
| PW(operating and maintenance costs) | 5 000 000    | 8 000 000    |
| PW(capital cost)                    | 5 000 000    | 1 000 000    |

- (a) Compute the benefit–cost ratios for both projects.
- (b) Compute the modified benefit–cost ratios for both projects.
- (c) Compute the benefit–cost ratio for the increment between the projects.
- (d) Compute the present worths of the two projects.
- (e) Which is the preferred project? Explain.

- 10.5** The following data are available for two mutually exclusive projects:

|                                     | Project A   | Project B   |
|-------------------------------------|-------------|-------------|
| PW(benefits)                        | £17 000 000 | £17 000 000 |
| PW(operating and maintenance costs) | 5 000 000   | 11 000 000  |
| PW(capital cost)                    | 6 000 000   | 1 000 000   |

- (a) Compute the benefit–cost ratios for both projects.
- (b) Compute the modified benefit–cost ratios for both projects.
- (c) Compute the benefit–cost ratio for the increment between the projects.
- (d) Compute the present worths of the two projects.
- (e) Which is the preferred project? Explain.

- 10.6** The following data are available for two mutually exclusive projects:

- (a) Compute the benefit–cost ratios for both projects.
- (b) Compute the modified benefit–cost ratios for both projects.

|                                     | Project A    | Project B    |
|-------------------------------------|--------------|--------------|
| PW(benefits)                        | \$17 000 000 | \$15 000 000 |
| PW(operating and maintenance costs) | 5 000 000    | 8 000 000    |
| PW(capital cost)                    | 6 000 000    | 3 000 000    |

- (c) Compute the benefit–cost ratio for the increment between the projects.
- (d) Compute the present worths of the two projects.
- (e) Which is the preferred project? Explain.
- 10.7** In the summer of 2004, the city of Kitchener, Ontario, often experienced the worst air pollution in Canada, affecting the health of hundreds of thousands of people. Explain how air pollution is an example of market failure. Give an example of how each of the four remedies for market failure listed in Subsection 10.2.2 might be applied to the case of air pollution.
- 10.8** The East Coast fishing industry has been devastated in recent years because of overfishing. Tens of thousands of jobs have been lost in Nova Scotia and Newfoundland as quotas have been cut drastically in an attempt to save fish such as cod, haddock, and whiting from extinction. How is overfishing an example of market failure? Give an example of how each of the four remedies for market failure listed in Subsection 10.2.2 might be applied to the case of overfishing.
- 10.9** Consider these situations in which cutting of trees is relevant:
1. The Brown family owns a view house in North Vancouver. Their neighbours across the street, the Smith family, have trees on their lot that are obstructing the Browns' view of the Burrard Inlet. The Browns are the only ones affected by the Smiths' trees. The Browns have asked the Smiths to top their trees, but the Smiths refuse to do so. The Browns have also offered to pay the Smiths for the topping and an additional \$500 to cover any loss they might feel because their tall trees were topped. The Smiths still refuse.
  2. The Brown family, the Green family, the White family, and the Blue family own view houses in North Vancouver. Their neighbours across the street, the Smith family, have trees on their lot that are obstructing everyone's view of Matilda Bay. The Browns, Greens, Whites, and Blues have asked the Smiths to top their trees, but the Smiths refuse to do so.
  3. Timber companies in British Columbia sometimes use clear-cutting on old-growth forests. Environmentalists have asked the companies to change this practice because it leads to reduced biodiversity.
- Why is there no market failure in the first situation involving the Smiths and the Browns? Why may there be market failure in the situation with several families? Why is there market failure in the third situation involving the timber companies?
- 10.10** Technical changes in electricity supply and information transmission have made it efficient for consumers of both services to be served by suppliers using different technologies and operating in different locations. Does this increase or decrease the need for government regulation in these industries?
- 10.11** What determines the MARR used on government-funded projects?
- 10.12** How will a decrease in the tax rate on investment income affect the MARR used for evaluating government-funded projects?

**10.13** How does an expectation of inflation affect the MARR for public sector projects?

**B. Applications**

**10.14** There are three petrochemical plants, A, B, and C, in Sarnia, Ontario. The three plants produce Good Stuff. Unfortunately, they also dump Bad Stuff into Ontario air. Data concerning their outputs of Stuff are shown below:

| Outputs |               |                  |
|---------|---------------|------------------|
|         | Good (kg/day) | Bad/Good (cL/kg) |
| Plant A | 17 000        | 10               |
| Plant B | 11 000        | 15               |
| Plant C | 8000          | 18               |

The city wants to reduce the dumping of Bad Stuff by 150 000 centilitres per day. Costs for reducing the concentration of Bad Stuff in output are shown below.

The city is considering two methods: (1) Require all plants to meet the performance level of the best-practice plant, plant A, which is 10 centilitres of Bad Stuff per kilogram of Good Stuff. (2) Impose a tax of \$0.20/centilitre of Bad Stuff dumped.

| Incremental Cost of Reducing Bad Stuff/Good Stuff (\$/cL/kg) |      |       |       |       |       |       |      |       |
|--|------|-------|-------|-------|-------|-------|------|-------|
| Reduction<br>(cL/kg)   | 1    | 2     | 3     | 4     | 5     | 6     | 7    | 8     |
| Plant A  | 0.02 | 0.032 | 0.048 | 0.1   | 0.18  | 0.26  | 0.38 | 0.57  |
| Plant B  | 0.06 | 0.06  | 0.063 | 0.068 | 0.075 | 0.193 | 0.27 | 0.405 |
| Plant C  | 0.25 | 0.25  | 0.25  | 0.25  | 0.25  | 0.25  | 0.25 | 0.375 |

- (a) What will be the reduction in dumping of Bad Stuff under the best-practice regulation? What will be the cost of this reduction?
- (b) Under the tax, how much Bad Stuff will be dumped from the three plants combined? What will be each plant's reduction in dumping per kilogram of Good Stuff?
- (c) How much will the reduction of dumping cost for each company under the tax?

**10.15** An electric utility company is considering a re-engineering of a major hydroelectric facility. The project would yield greater capacity and lower cost per kilowatt-hour of power. As a result of the project, the price of power would be reduced. This is expected to increase the quantity of power demanded. The following data are available:

| Effect of Reduced Price of Power |            |
|----------------------------------|------------|
| Current price (\$/kWh)           | 0.07       |
| Current consumption (kWh/year)   | 9 000 000  |
| New price (\$/kWh)               | 0.05       |
| Expected consumption (kWh/year)  | 12 250 000 |

What is the annual benefit to consumers of power from this project?

**10.16** Brisbane and Johnsonburg are two towns separated by the Wind River. Traffic between them crosses the river by a ferry run by the Johnsonburg Ferry Company, which charges a toll. The two towns are considering building a bridge somewhat upstream from the ferry crossing; there would be no toll on the bridge. Travel time between the towns would be about the same with the bridge as with the ferry because of the bridge's upstream location. The following information is available concerning the crossing.

| Ferry/Bridge Information                 |        |
|--|--------|
| Ferry crossings (number/year)            | 60 000 |
| Average cost of ferry trip (\$/crossing) | 1      |
| Ferry fare (\$/crossing)                 | 1.5    |
| Bridge toll (\$/crossing)                | 0      |
| Expected bridge crossings (number/year)  | 90 000 |
| EAC of bridge (\$/year)                  | 85 000 |

Note that all data are on an annual basis. The cost of the bridge is given as the equivalent annual cost of capital and operating costs. We assume that all bridge costs are independent of use—that is, there are no costs that are due to use of the bridge. The average cost per crossing of the ferry includes capital cost and operating cost.

- (a) If the bridge were built, what would be the annual benefits to travellers?
- (b) How much would the owners of the Johnsonburg Ferry Company lose if the bridge were built?
- (c) What would be the effect on taxpayers if the bridge were built? (Assume that Johnsonburg Ferry pays no taxes.)
- (d) What would be the net social gains or losses if the bridge were built? Take into account the effects on travellers, Johnsonburg Ferry owners, and taxpayers.
- (e) Would the net social gains or losses be improved if there were a toll for crossing the bridge?

**10.17** In order to encourage the use of carpooling, the city of Edmonton is considering putting carpool parking lots near major transportation arteries into downtown Edmonton. Each parking lot allows commuters to meet in separate cars, park all but one, and proceed in one car to a joint destination. Studies estimate that an average of 200 cars will be parked at each lot on weekdays, with one-quarter of that number on weekends. The average commuting distance from the parking lots is 75 kilometres, and the marginal cost of driving an average car is \$0.50 per kilometre.

- (a) If it is assumed that all the cars that are parked in the lot would otherwise have been driven to work, how much will be saved by this commuter parking lot per year? Assume an interest rate of 0.
- (b) How could you find out how many would have been parked somewhere else?
- (c) How could you calculate, per parking lot, the benefit to all drivers of having fewer cars on the road?

**10.18** A medium-sized city (population 250 000, 45 000 families) is considering introducing a recycling program. The program would require residents to separate newspaper, cardboard, and cans from their regular waste so that it could be collected weekly, sorted, and sold, rather than being put into the local landfill site. The program would also require households to separate “wet,” or compostable, waste, which they would then be responsible for composting. The city would provide free composting units to the households. What kinds of potential costs and benefits can you identify for this project? How might this information be gathered?

**10.19** A four-day school week has been advocated as a means of reducing education costs. School days would be longer so as to maintain the same number of hours per week as under the current five-day-a-week system. The main expected cost savings are in school cleaning and maintenance and in school bus operation. The main effects that are anticipated are

- (a) Reduced school cleaning and maintenance
- (b) Reduced use of school buses on the off-day
- (c) Reduced driving to school by parents, students, and staff on the off-day
- (d) Reduced public transportation use on the off-day
- (e) Increased performance of part-time work by some high school students and school staff for the off-day
- (f) Reduced absences of students and staff; this is mainly because some required personal activities could be scheduled for the off-day
- (g) Reduced subsidized school lunch requirements
- (h) Increased need for day care on the off-day for working parents; about a third of elementary schools could be opened for day care, and the costs would be covered by fees
- (i) Reduced learning by elementary students is possible because of their limited attention spans
- (j) Reduced school taxes

Which of these effects are benefits? Who receives the benefits? Which are costs? Who bears the costs? Which are neither costs nor benefits?

**10.20** A new suburban development project is being planned to the west of Monterrey, Mexico. There is now a two-lane road from the site of the development along the base of the mountain *Cerro de la Silla* (Saddle Hill). The new development will require additional road capacity. Two alternatives are being considered. The first is to upgrade the existing road to four wide modern lanes. The second is to build a new four-lane highway east through Saddle Hill tunnel. The following data are available concerning the two routes:

| Route            | Distance | First Cost<br>(pesos) | Operating and<br>Maintenance<br>Costs per Year (pesos) |
|------------------|----------|-----------------------|--|
| Base of mountain | 20 km    | 210 million           | 900 000  |
| Tunnel           | 10 km    | 450 million           | 1 300 000  |

The planning period is 40 years for both routes. The MARR is 10 percent. Cars will travel at 100 kilometres per hour along either route. Operating cost for the cars is expected to be about 2.50 pesos per kilometre along either route. About 400 000 trips per year are expected on either route.

- (a) Which route should be built? Use only the data given. Use annual worth to make your decision.
- (b) What important benefit of the Saddle Hill tunnel route has been left out of the analysis?
- (c) Do you need more information about travellers to determine if the benefit that has been left out of the analysis from part (b) would change the recommendation? Explain.
- (d) How would the possibility of collecting a toll on the tunnel route affect your recommendation?

- 10.21** The Principality of Upper Pigovia has just one export, pig crackling. The crackling is produced in two plants, Old Gloria and New Gloria. Both plants give off a delightful, mouth-watering odour while in operation. This odour has created a health problem. The citizens' appetites are huge, and, consequently, so are the citizens. Princess Piglet has decreed that the daily emission of odour from the two plants must be reduced by 7000 Odour Units (OU) per day. The Princess prides herself on her even-handedness. The decree specifies that each plant is to reduce its emission by 3500 OU per day. The following table shows the incremental cost in Upper Pigovian dollars (U\$) per 1000 OU of attaining various levels of odour reduction in the two plants. To help in interpreting the table, note that the cost for Old Gloria to remove 3000 OU per day would be  $2(U\$25) + U\$30 = U\$80$ .

| Quantity<br>of Odour<br>Removed<br>(1000 OU/Day) | Incremental Cost of Removing Odour in \$U/1000 OU |        |        |        |
|--|---|--------|--------|--------|
|  | 0 to 2  | Over 2 | Over 4 | Over 5 |
| Old  | 25  | 30     | 40     | 50     |
| New  | 20  | 20     | 25     | 30     |

- (a) What is the cost per day of implementing Princess Piglet's decree?
- (b) Can you suggest a tax scheme that will yield the same reduction in odour emission as the decree at a lower cost?
- (c) Can you suggest a subsidy scheme that will yield the same reduction in odour emission as the decree, but at a lower cost?

- 10.22** The transportation department is considering the construction of a bridge over a narrow point in a lake. Traffic now goes around the lake. The bridge will save 30 kilometres in travel distance. Three alternatives are being considered: (1) Do nothing, (2) build the bridge, and (3) build the bridge and charge a toll. If the bridge is built, the present road will be maintained, but its use will decline. One effect of the reduced use of the present

| Costs and Benefits  | Do Nothing<br>(000s \$) | Bridge With Toll<br>(000s \$) | Bridge Without Toll<br>(000s \$) |
|---|-------------------------|-------------------------------|----------------------------------|
| First cost  | 0                       | 184 000                       | 184 000                          |
| Annual road and/or bridge operating and maintenance costs | 640                     | 320                           | 40                               |
| Annual vehicle operating cost                             | 13 200                  | 400                           | 400                              |
| Annual driver and passenger time cost                     | 10 000                  | 2000                          | 2000                             |
| Annual accident cost                                      | 2000                    | 40                            | 40                               |
| Annual revenue lost by roadside businesses                | 0                       | 4000                          | 4000                             |
| Annual toll revenues                                      | N/A                     | 4800                          | N/A                              |

road is a loss of revenue by businesses along that road. Available data, in thousands of dollars, is given in the table above.

- (a) Identify social benefits and costs of the bridge.
- (b) Which of the costs and revenues in the table are neither social benefits nor social costs?
- (c) The table makes an implicit assumption about the effect of the toll on bridge traffic. This assumption is probably incorrect. What is the assumption?
- (d) How would you expect the toll to affect benefits and costs? Explain.

**10.23** An example concerning the effect of a flood control project is found in the benefit–cost analysis chapter of an imaginary engineering economics text. The benefits of the project are stated as:

| Benefit   | Cost              |
|---|-------------------|
| Prevented losses due to floods in the Amahata River Basin       | \$48 000 000/year |
| Annual worth of increased land value in the Amahata River Basin | \$4 800 000       |

Comment on these two items.

**10.24** A province in Canada is considering the construction of a bridge. The bridge would cross a narrow part of a lake near a provincial park. The major benefit of the bridge would be reduced travel time to a campsite from a nearby urban centre. This lowers the cost of camping trips at the park. As well, an increase in the number of visits resulting from the lower cost per visit is expected.

Data concerning the number of week-long visits and their costs are shown below:

| Inputs                | Number of Visits and Average Cost per Visit to Park |             |
|-----------------------|---|-------------|
|                       | Without Bridge                                      | With Bridge |
| Travel cost (\$)      | 140   | 87.5        |
| Use of equipment (\$) | 50  | 50          |
| Food (\$)             | 100   | 100         |
| Total (\$)            | 290   | 237.5       |
| Number of visits/year | 8000  | 11 000      |

The following data are available as well:

1. The bridge will take one year to build.
2. The bridge will have a 25-year life once it is completed. This means that the time horizon for computations is 26 years.
3. Construction cost for the bridge is \$3 750 000. Assume that this cost is incurred at the beginning of year 1.
4. Annual operating and maintenance costs for the bridge are given by

$$\$7500 + 0.25q$$

where \$7500 is the fixed operating and maintenance cost per year and  $q$  is the number of crossings.

5. Operating and maintenance costs are incurred at the end of each year over which the bridge is in operation. This is at the ends of years 2, 3, ..., 26.
6. The MARR is 10 percent.

(Notice that the annual benefits for this project were computed as part of the discussion of Example 10.1.)

- (a) Compute the present worth of the project.
- (b) Compute the benefit–cost ratio.
- (c) Compute the modified benefit–cost ratio.

- 10.25** The city of Calgary has a new subdivision, Paradise Mountain, at its outskirts. The city wants to encourage the growth of Paradise Mountain by improving transportation between Paradise Mountain and the centre of Calgary. Two alternatives are being considered: (1) new buses on the route between Paradise Mountain and Calgary and (2) improvement of the road between Paradise Mountain and Calgary.

Both projects will have as their main benefit improved transportation between Paradise Mountain and Calgary. Rather than measuring the value of this benefit directly to the city, engineers have estimated the benefit in terms of an increase in the value of land in Paradise Mountain. That is, potential residents are expected to show their evaluations of the present worth of improved access to the town centre by their willingness to pay more for homes in Paradise Mountain.

The road improvement will entail construction cost and increased operating and maintenance costs. As well, the improved road will require construction of a parking garage in the centre of Calgary. The new buses will have a first cost as well as operating and maintenance costs. Information about the two alternatives is shown below.

|                                     | Road Improvement | New Buses   |
|-------------------------------------|------------------|-------------|
| First cost                          | \$15 000 000     | \$4 500 000 |
| PW (operating and maintenance cost) | 5 000 000        | 12 000 000  |
| Parking garage cost                 | 4 000 000        |             |
| Estimated increased land value      | 26 000 000       | 18 000 000  |

- (a) Compute the benefit–cost ratio of both alternatives. Is each individually viable?
- (b) Using an incremental benefit–cost ratio approach, which of the two alternatives should be chosen?
- (c) Compute the present worths of the two alternatives. Compare the decision based on present worths with the decisions based on benefit–cost ratios.

**10.26** A local government is considering a new two-lane road through a mountainous area. The new road would improve access to a city from farms on the other side of the mountains. The improved access would permit farmers to switch from grains to perishable soft fruits that would be either frozen at an existing plant near the city or sold in the city. Two routes are being considered. Route A is more roundabout. Even though the speed on route B would be less than that on route A, the trip on route B would take less time. Almost all vehicles using either road would go over the full length of the road. A department of transport engineer has produced information shown in the table below. The government uses a 10 percent MARR for road projects. The road will take one year to build. The government uses a 21-year time horizon for this project, since it is not known what the market for perishable crops will be in the distant future. Comment on the engineer’s list of benefits; there may be a couple of errors.

| Costs and Benefits of the New Road           |           |           |
|--|-----------|-----------|
|  | Route A   | Route B   |
| <i>Properties</i>                            |           |           |
| Distance (km)                                | 24        | 16        |
| Construction cost (\$)                       | 53 400    | 75 000    |
| Operating and maintenance cost per year (\$) | 60        | 45        |
| Resurfacing after 10 years of use (\$)       | 3 100     | 2 350     |
| <i>Road Use</i>                              |           |           |
| Number of vehicles per year                  | 1 000 000 | 1 200 000 |
| Vehicle cost per kilometre (\$)              | 0.3       | 0.3       |
| Speed (km/h)                                 | 100       | 80        |
| Value of time per vehicle hour (\$)          | 15        | 15        |
| <i>Benefits</i>                              |           |           |
| Increased crop value per year (\$)           | 13 500    | 18 000    |
| Increased land value (\$)                    | 104 485   | 139 313   |
| Increased tax collections per year (\$)      | 811.21    | 1 081.61  |

**10.27** The environmental protection agency of a county would like to preserve a piece of land as a wilderness area. The owner of the land will lease the land to the county for a 20-year period for the sum of \$1 750 000, payable immediately. The protection agency estimates that the land will generate benefits of \$150 000 per year, but it will forgo \$20 000 per year in taxes. Assume that the MARR for the county is 5 percent.

- (a) Calculate a BCR using annual worth and classify the forgone taxes as a cost to the government.
- (b) Repeat part (a), but consider the forgone taxes a reduction in benefits.
- (c) Using your results from part (b), determine the most the county would be willing to pay for the land (within \$10 000) if it accepts projects with a BCM of one or more.

**10.28** The data processing centre of an insurance company has been troubled recently by the increasing incidence of repetitive strain injuries in the workplace. Health and safety consultants have recommended to management that it invest in upgrading computer desks and chairs at a cost of \$500 000. They advise that this would reduce the number and severity of medical costs by \$70 000 per year and that productivity losses and sick leaves could be reduced by a further \$80 000 per year. The furniture has a life of eight years with zero scrap value. The city uses a MARR of 9 percent. Should the centre purchase the furniture? Use a benefit–cost analysis.

**10.29** Several new big-box stores have created additional congestion at an intersection in north Barrie, Ontario. City engineers have recommended the addition of a turn lane, a computer-controlled signal, and sidewalks, at an estimated cost of \$1.5 million. The annual maintenance costs at the new intersection will be \$8000, but users will save \$50 000 per year due to reduced waiting time. In addition, accidents are expected to decline, representing a property and medical savings of \$175 000 per year. The renovation is expected to handle traffic adequately over a 10-year period. The city uses a MARR of 5 percent.

- (a) What is the BCR of this project?
- (b) What is the BCRM of the project?
- (c) Comment on whether the project should be done.

**10.30** There are two beef packing plants, A and B, in the city of Calgary, Alberta. Both plants put partially treated liquid waste into the Bow River. The two plants together dump over 33 000 kilograms of BOD<sub>5</sub> per day. (BOD<sub>5</sub> is the amount of oxygen used by microorganisms over five days to decompose the waste.) This is more than half the total BOD<sub>5</sub> dumped into the river at this location. The city wants to reduce the BOD<sub>5</sub> dumped by the two plants by 10 000 kilograms per day.

The following data are available concerning the two plants:

| Outputs of the Two Plants |            |                              |
|---------------------------|------------|------------------------------|
|                           | Steers/Day | BOD <sub>5</sub> /Steer (kg) |
| Plant A                   | 20 000     | 1.0                          |
| Plant B                   | 9 000      | 1.5                          |

The costs of making reductions in BOD5 per steer in dollars are shown below:

| Reduction (kg/steer) | Incremental Cost of Reducing BOD<br>(dollars/kg/steer) |     |     |     |      |      |      |
|----------------------|--|-----|-----|-----|------|------|------|
|                      | 0.1  | 0.2 | 0.3 | 0.4 | 0.5  | 0.6  | 0.7  |
| Plant A              | 1.5  | 2.4 | 3.6 | 7.5 | 13.5 | 19.5 | 28.5 |
| Plant B              | 4.5  | 4.5 | 4.5 | 4.5 | 4.5  | 10.5 | 13.5 |

For example, to reduce the BOD5 of plant A by 0.25 kilograms per steer, the cost is calculated as

$$(0.1 \times 1.5) + (0.1 \times 2.4) + (0.05 \times 3.6) = \$0.57/\text{steer}$$

The council is considering three methods of inducing the plants to reduce their BOD5 dumping: (1) a regulation that limits BOD5 dumping to 0.81 kilograms/steer, (2) a tax of \$4.8/kilogram of BOD5 dumped, and (3) a subsidy paid by the town to the plants of \$4.8/kilogram reduction from their current levels in BOD5 dumped.

- (a) Verify that, if both plants reduce their BOD5 dumping to 0.81 kilograms/steer, there will be a 10 000 kilograms/day reduction in BOD5 dumped. What will this cost?
- (b) Under a tax of \$4.8/kilogram, how much BOD5 will plant A dump? How much will plant B dump? (Assume that outputs of steers would not be affected by the tax.) Verify that this will lead to more than a 10 000 kilograms/day reduction in BOD5. What will this cost?
- (c) Under a subsidy of \$4.8/kilogram reduction in BOD5, how much will plant A dump? How much will plant B dump? Verify that this will lead to more than a 10 000 kilograms/day reduction in BOD5. What will this cost?
- (d) Explain why the tax and subsidy schemes lead to the same behaviour by the meat packing plants.
- (e) Explain why the tax and subsidy schemes have lower costs for the company than the regulation.

## C. More Challenging Problems

- 10.31** Consider the bridge project of Problem 10.24. We assumed there would be no toll for crossing. Now suppose the province is considering a toll of \$7 per round trip over the bridge. It estimates that, if the toll is charged, the number of park visits will rise to only 10 600 per year instead of 11 000.

- (a) Compute the present worth of the project if the toll is charged.
- (b) Why is the present worth of the project reduced by the toll?

- 10.32** Consider Problem 10.30. We saw that an effluent tax enabled the same reduction in BOD5 as a regulation, but with a lower cost. We did not consider the distribution of this cost.

- (a) How much tax does plant A pay under the tax of \$20 per kilogram dumped? How much does plant B pay?
- (b) What are the two total effluent costs (tax plus cleaning cost) for plant A? For plant B?

- (c) Compare these costs with the costs under regulation.
- (d) Suppose that the city used the proceeds of the tax to provide benefits that had equal value to each plant. How great would these benefits have to be to ensure that both plants would be better off with the combination of tax and benefits?
- 10.33** The recreation department for Southampton, Ontario, is trying to decide how to develop a piece of land. It has narrowed the choices down to tennis courts or a swimming pool. The swimming pool will cost \$2.5 million to construct and will cost \$300 000 per year to operate, but will bring benefits of \$475 000 per year over its 25-year expected life. Tennis courts would cost \$200 000 to build, cost \$20 000 per year to operate, and bring \$60 000 per year in benefits over their eight-year life. Both projects are assumed to have a salvage value of zero. The appropriate MARR is 5 percent.
- (a) Which project is preferable? Use a BCR and an annual worth approach.
- (b) Which project is preferable? Use a BCRM and an annual worth approach.
- 10.34** Consider the road project in Problem 10.26. (*Note:* Correct for the errors mentioned.)
- (a) Compute a benefit–cost ratio for route A with road use costs counted as a cost.
- (b) Compute a benefit–cost ratio for route A with road use costs counted as a reduction in benefits.
- (c) In what way are the two benefit–cost ratios consistent, even though the numerical values differ?
- (d) Make a recommendation as to what the province should do regarding these two roads. Explain your answer briefly.
- 10.35** Find the net present worths for the dam and the dam plus recreation facilities considered in Review Problem 10.1. Use a MARR of 15 percent. Make a recommendation as to which option should be adopted.
- 10.36** The department of road transport and highways has \$660 000 000 that it can commit to highway safety projects. The goal is to maximize the total life-years saved per dollar. The potential projects are: (1) flashing lights at 10 railroad crossings; (2) flashing lights and gates at the same 10 railroad crossings; (3) widening the roadway on an existing rural bridge from 3 m to 3.5 m; (4) widening the roadway on a second and third rural bridge from 3 m to 3.5 m; (5) reducing the density of utility poles on rural roads from 30 to 15 poles per kilometre; and (6) building runaway lanes for trucks on steep downhill grades.

| Highway Safety Projects |                           |                         |                              |
|-------------------------|---------------------------|-------------------------|------------------------------|
|                         |                           | Total Cost<br>(000s \$) | Life-Years<br>Saved per Year |
| 1                       | Flashing lights           | 18 000                  | 14                           |
| 2                       | Flashing lights and gates | 30 000                  | 20                           |
| 3                       | Widening bridge #1        | 48 000                  | 14                           |
| 4                       | Widening bridge #2        | 28 000                  | 10                           |
| 5                       | Widening bridge #3        | 44 000                  | 18                           |
| 6                       | Pole density reduction    | 120 000                 | 96                           |
| 7                       | Runaway lane #1           | 240 000                 | 206                          |
| 8                       | Runaway lane #2           | 240 000                 | 156                          |

The data for the flashing-lights and flashing-lights-with-gates projects reflect the costs and benefits for the entire set of 10 crossings. Portions of the projects may also be completed for individual crossings at proportional reductions in costs and savings. At any single site, the lights and lights-with-gates projects are mutually exclusive. Any fraction of the reduction of utility pole density project can be carried out. Data concerning costs and safety effects of the projects are shown on the preceding page. (A life-year saved is one year of additional life for one person.)

Advise the department of transportation how best to commit the \$660 000 000. Assume that the money must be used to increase highway safety.

- 10.37** A department of transportation is considering widening lanes on major highways from 6 m to 7.5 m. The objective is to reduce the accident rate. Accidents have both material and human costs. Data for highway section XYZ are found below.

- (a) Compute the present worth of costs of lane-widening.
- (b) Compute the present worth of savings of non-human accident costs.
- (c) What minimum value for a serious personal injury would justify the project?

| Lane Widening on Section XYZ                                |           |
|---|-----------|
| Accidents per 100 000 000 vehicle-kilometres in 6 m lanes   | 150       |
| Accidents per 100 000 000 vehicle-kilometres in 7.5 m lanes | 90        |
| Serious personal injuries per accident                      | 10%       |
| Average non-human cost per accident (\$)                    | 2500      |
| Annual road use (vehicles)                                  | 7 500 000 |
| First cost per kilometre (\$)                               | 175 000   |
| Operating and maintenance costs per kilometre/year (\$)     | 7500      |
| Project life (years)  | 25        |
| MARR  | 10%       |

- 10.38** The Chinese government is considering three new flood control projects on the Huaihe River. Projects A and B consist of permanent dikes. Project C is a small dam. The dam will have recreation and irrigation benefits as well as the flood control benefits. Facts about the three projects are shown in the following table. Each project has a life of 25 years. The MARR is 10 percent.

|   | Project A | Project B | Project C |
|---|-----------|-----------|-----------|
| First cost (millions of yuan)                             | 200       | 256       | 416       |
| Annual benefits (millions of yuan)                        | 26.4      | 33.6      | 56.8      |
| Annual operating and maintenance costs (millions of yuan) | 2.4       | 2.4       | 4.0       |

- (a) Use present worth to choose the best project.
- (b) Compute the benefit–cost ratios for the three projects.
- (c) Use incremental benefit–cost to choose the best project.

- (d) It is possible that the dam (project C) will cause some wildlife damage. This damage might be prevented by an additional expenditure of 24 million yuan when the dam is constructed. Does this change the choice of best project? Compute the benefit–cost ratio of project C assuming that the additional 24 million yuan is an addition to the first cost.

- 10.39** A highway department needs to upgrade a 20-kilometre rural road to accommodate increased traffic. There will be 5000 cars per day in the first year after upgrading. The number of cars will increase each year by 200 cars per day for the next 20 years. A modern two-lane road will be adequate for current traffic levels, but it will be inadequate after about 10 years. At that time, a four-lane road will be required. A four-lane road will permit greater speed even at the current traffic level.

Two alternatives are being considered. One proposal is for a modern two-lane road now, followed by addition of lanes to make a four-lane road after 10 years. The second alternative is for a four-lane road now. The planning horizon for both alternatives is 20 years. Costs for the alternatives are shown in the following table. All values are in real dollars.

|   | Two Lanes | Four Lanes |
|---|-----------|------------|
| First cost (millions of \$)   | 20        | 32         |
| Adding lanes after 10 years (millions of \$)                        | 18        | 0          |
| Annual operating and maintenance costs<br>(years 1–10) (in \$000s)  | 30        | 40         |
| Annual operating and maintenance costs<br>(years 11–20) (in \$000s) | 40        | 40         |

The average value of travel time for each car is estimated at \$40 an hour. Speeds will be as shown below.

- (a) Which alternative has a greater present worth if the MARR is 5 percent?
- (b) Which alternative has a greater present worth if the MARR is 20 percent?
- (c) Suppose the real after-tax rate of return on government savings bonds is 5 percent and the real before-tax rate of return on investment in the private sector is 20 percent. Use your answers to parts (a) and (b) to decide which alternative is better. Explain your answer.

| Years    | Two Lanes | Four Lanes |
|----------|-----------|------------|
| 1 to 5   | 80 km/h   | 100 km/h   |
| 6 to 10  | 70 km/h   | 100 km/h   |
| 11 to 20 | N/A       | 100 km/h   |

- 10.40** A small town needs to upgrade the town dump to meet environment standards. Two alternatives are being considered. Alternative A has a first cost of \$420 000 and annual operating and maintenance costs of \$52 500. Alternative B has a first cost of \$315 000 and annual operating and maintenance costs of \$74 000. Both alternatives have 15-year

lives with no salvage. An increase in dumping fees for households and businesses in the town is expected to yield \$50 000 per year.

- (a) Which alternative has the lower cost if the MARR is 5 percent? Use annual worth.
- (b) Which alternative has the lower cost if the MARR is 20 percent? Use annual worth.
- (c) The after-tax return on government savings bonds is 5 percent. The average rate of return before taxes in private sector investment is 20 percent. Which alternative should be chosen?

- 10.41** Consider the following situation. The production and use of a certain pesticide poses both health and environmental risks to the workers who make the pesticide, those who use it, and the environment. The government regulates certain aspects of its production and distribution and limits its use and release into the environment. These control measures require the manufacturer to pay considerable monitoring and compliance costs and require users and transport companies to take expensive precautions around its use and transport.

The government is considering a total ban on the production and use of the pesticide. In evaluating the social costs and benefits of the ban, you recognize that the manufacturer, distributors, and users will save substantial amounts because they will no longer need to monitor and control the production and use of the pesticide.

- (a) In general terms, what do you consider to be the social benefits of the ban? What are the social costs?
- (b) In computing the overall social costs of the ban, is it reasonable to deduct from other social costs the savings arising from not having to monitor and control the pesticide's production and use? Why or why not?

## MINI-CASE 10.1

### Emissions Trading

Fossil fuel-burning electric power plants in the United States and Canada produce air pollution as a by-product of their operation. The pollutants produced by these plants are linked to high levels of acid rain in the northeastern United States and the Canadian provinces of Ontario and Quebec, as well as a growing number of "smog" days in Ontario. The primary pollutants associated with acid rain are nitrogen oxide and sulphur dioxide ( $\text{NO}_x, \text{SO}_2$ ). Acid rain is linked to increased fish morbidity rates, destruction of forests, and building decay. Overall increases in smog levels have led to a host of respiratory illnesses such as asthma and lung cancer in the northeastern United States and Ontario.

Air quality is a public good. Though there are markets for fossil fuels and electricity, they fail to take into account the public costs of smog, acid rain, and climate change. Government regulation in the form of an emissions trading system has been used as a mechanism to produce more socially desirable outcomes.

Emissions trading refers to a market-based approach to controlling pollution in which companies can buy and sell allowances for emitting specified pollutants. Alternatively, companies can get credits for reducing emissions. In such trading systems, a central agency—typically a government—sets goals or limits on the allowable

volume of pollutants that can be emitted. Companies must hold sufficient allowances for that volume. Those that reduce their emissions can sell their allowances and those that have emissions above their allowable limit must purchase credits from those that pollute less.

Since 1995, the US Environmental Protection Agency (EPA) has used an emissions trading system to permit plants to adopt the most cost-efficient approach to reduce their sulphur dioxide emissions. The EPA sets caps, or limits, on allowable total emissions levels, and then issues a limited number of permits that are auctioned off annually. These permits can be traded or saved. In 2003, a permit for the atmospheric release of one tonne of sulphur dioxide cost approximately US \$170. By 2004, the cost was \$225 per tonne, and by 2006, it was more than \$1500 per tonne. In 2001, Ontario introduced a “Cap, Credit, and Trade” emission trading system for controlling sulphur dioxide.

Emissions trading systems apply to controlling other pollutants. The recent rise in temperature of the earth’s atmosphere, global warming, is understood to be the result of higher concentrations of certain greenhouse gases in our atmosphere. The Kyoto Protocol is a 1997 international agreement that went into effect in 2005. It commits developed countries to reducing their emissions of certain greenhouse gases to below 1990 levels by 2012. The Kyoto Protocol uses an emissions trading scheme to coordinate reductions in emissions. In conjunction with the Kyoto accord, the European Union (EU) has introduced a mandatory carbon trading program to control emissions of carbon dioxide. In the United States, which is the only major country that has not ratified the Kyoto accord, a number of trading schemes for carbon dioxide and other pollutants are underway or planned. These systems are other examples of a growing trend for countries around the world to implement emissions trading systems so that the overall levels of greenhouse gases can be reduced in coming years. These trading systems are not without their implementation challenges, however, as the discussion questions that follow indicate.

Sources: “EPA’s Clean Air Markets—Acid Rain Program,” Environmental Protection Agency site, [www.epa.gov/airmarkets/progregs/arp/index.html](http://www.epa.gov/airmarkets/progregs/arp/index.html); “Ontario Emissions Trading Registry Introduction,” Ontario Ministry of the Environment site, [www.ene.gov.on.ca/envision/air/etr/index.htm](http://www.ene.gov.on.ca/envision/air/etr/index.htm); United Nations Framework Convention on Climate Change, [www.unfccc.int/2860.php](http://www.unfccc.int/2860.php); “European Union Emission Trading Scheme,” European Commission site, [ec.europa.eu/environment/climat/emission.htm](http://ec.europa.eu/environment/climat/emission.htm); “Emissions Trading Analysis,” Australian Greenhouse Office site, [www.greenhouse.gov.au/emissionstrading/index.html](http://www.greenhouse.gov.au/emissionstrading/index.html). All accessed November 10, 2007.

## Discussion

Despite the early successes of these systems, some issues remain. First, these regulations may be successful in reducing overall emission levels, but they may not address local “hot spots.” Second, the global targets themselves may not be sufficient to truly reflect the social costs of acid rain or greenhouse gases. Finally, there may be more efficient ways of distributing the allowances so that the overall cost of operating the system is lowered.

Government regulatory programs such as the emissions trading systems are established to remediate the impact of market failure. These programs are intended to act as an incentive to allow plants to adopt the most cost-efficient means of emissions reductions. But despite the best intentions of the government, individuals or companies can exploit poorly designed programs for their own gain. At the best of times, such as with SO<sub>2</sub> or carbon emission controls, the inherent process of establishing the system incurs inefficiencies. On the other hand, the benefits to society of correcting situations of market failure can nevertheless be enormous.

**Questions**

1. What are the implications of having too high a level of carbon allowances in an emissions trading system?
2. What problem could arise if an emissions trading system allowed companies to sell carbon credits to other companies at a mutually acceptable price? Why would an open-market auction be preferable?
3. Emissions trading schemes require some form of enforcement. One kind of enforcement uses regulators that can fine companies that do not have sufficient allowances for their emissions. What are potential shortcomings of this form of enforcement?
4. An alternative to an emissions trading system is an emissions tax. Some economists argue that a tax would be simpler to implement, would have more predictable costs, and would be less prone to corruption. Can you see any disadvantages to an emissions tax?



# Project Management

Engineering Economics in Action, Part 11A: Filling a Vacuum

## 11.1 Introduction

## 11.2 Project Management Lifecycle

- 11.2.1 Initiation
- 11.2.2 Planning
- 11.2.3 Execution
- 11.2.4 Monitoring and Controlling
- 11.2.5 Closure

## 11.3 Project Management Tools

- 11.3.1 Work Breakdown Structure
- 11.3.2 Gantt Charts
- 11.3.3 The Critical Path Method
  - 11.3.3.1 Project Scheduling and the Critical Path Method
  - 11.3.3.2 Project Crashing and Time–Cost Tradeoffs

## Review Problems

## Summary

Engineering Economics in Action, Part 11B: A Crash Course

## Problems

## Mini-Case 11.1: No Longer LEEDing

## ENGINEERING ECONOMICS IN ACTION, PART 11A

### Filling a Vacuum

"I have something new for you, Naomi. It's going to require some imagination and disciplined thinking at the same time." Anna's tone indicated to Naomi that something interesting was coming.

"As you may know, Canadian Widgets has been working toward getting into consumer products for some time," Anna continued. "An opportunity has come up to cooperate on a new vacuum cleaner with Powerluxe. It has some potential designs for a vacuum with electronic sensing capabilities that will sense carpet height and density. The vacuum will then adjust the power and the angle of the head to optimize cleaning on a continuous basis. Our role in this would be to design the manufacturing system and to do the actual manufacturing for North America. Sound interesting so far?"

"Yes, Ms. Kulkowski," Naomi answered. She couldn't help being respectful in front of Anna, who was Canadian Widgets's president, among other things. "What would my role be?"

"For one thing, you will be working with Bill Astad from head office. I think you know him." Naomi did; she had given Bill some advice on handling variable rates of inflation a few weeks earlier. "This is a big project, and it's going to need careful management if it's going to be successful. Bill's great on the marketing side, but I want you to help keep things organized."

Naomi thought about this for a second. A quick worry went through her head that maybe Anna meant she would be doing document management or scheduling Bill's appointments. Her concerns only lasted for a second before it all clicked into place. "So are you looking for something to simply make sure we know what needs to be done and when, or something more rigorous so that we can meet a deadline at minimum costs?"

Naomi meant it as a simple question, but the half-smile that appeared on Anna's face let her know that she had passed some sort of test. "Absolutely the latter. This is a big project, and everybody has a different role. I want you to make sure Bill has everything he needs to make good decisions. Do you think you can do that?"

"I know I can!"

## 11.1

### Introduction

This textbook deals primarily with the financial evaluation of engineering projects. The meaning of the term "project" in this context is an investment alternative or an activity that must be undertaken or one that may have benefits that outweigh its costs. In this chapter, however, we look at how economic evaluation fits within the larger picture of managing the lifecycle of more complex endeavours, also called projects, from initial conception through to completion.

In this chapter, a project is defined as a temporary activity that is carried out to deliver a specific product or service. It has a beginning and an end. Resources are assigned to the project while it is ongoing and then reallocated to other activities when it has been completed. Projects can be short and straightforward, such as moving a small piece of equipment, or large and complex, such as the construction of the Confederation Bridge.

**Project management** is mainly concerned with the processes of planning such a project, organizing the necessary resources, leading the individuals responsible for carrying out the project, monitoring its progress, and implementing its deliverables. The challenge of project management is to bring about the goals and objectives of a project within the constraints of time, scope, and budget. The larger the project, or the more complex it is or uncertain its environment, the more necessary and challenging is the task of project management. There are many project management methods available to assist project managers, a number of which are introduced in this chapter.

## 11.2 | Project Management Lifecycle

---

Regardless of scope or complexity, all projects go through a similar sequence of stages during their life. Dividing the work into stages helps the project manager plan and organize resources for each stage, to gauge whether the project goals are being achieved, and to justify decisions to continue, modify, or end the project. The phases are initiation, planning, execution, monitoring and controlling, and closure.

All projects start with an idea for a product, a service, or a change (such as process improvement) that will produce an identifiable benefit. In the **initiation stage**, the nature and scope of a project are defined. This is normally documented and communicated to others through a project *terms of reference* document, also known as a *project charter*. The project charter describes what the project will accomplish.

Following the initiation stage is the **planning stage**. In this stage, the project is broken down into smaller modules, the sequence of activities is defined, and the resources needed for each are identified. This information is recorded in a project plan document. Careful project planning is an important component of managing project risk—uncertain events or outcomes can have an impact on success.

In the **execution stage**, work is carried out to deliver the product, service, or desired outcome. Most of the effort and cost related to the project is expended at this stage. Once the work has begun, the project manager is also responsible for ongoing control of the project. In the **monitoring and control stage**, regular progress reports and constant assessment of issues and risks are carried out. The project manager should be regularly comparing the project plan document with its actual progress so that the expected benefits will be realized.

Finally, when the work on the project comes to an end, the project has reached the **closure stage**. At this stage, the project manager should ensure that the client is satisfied with the outcome and agrees that the work is done. The project manager should also review and document lessons learned so that future projects can benefit from this information.

It is important to note that not all projects will pass through every stage, since some will be terminated along the way. Some projects will pass through the planning, execution, and monitoring and controlling stages several times if unexpected events require that it “go back to the drawing board.”

Each industry may use different names to describe the stages of a project lifecycle and will have different issues that need to be managed carefully. It is therefore important to become acquainted with industry-specific practices and conventions. For example, in the oil and gas industry, projects involving gas fields, pipelines, and natural gas plants can be worth billions of dollars and extend over 20 years or more. For a project of this scale, a key process that needs to be managed carefully is the Environmental and Social Health Impact Assessment. Making sure that the approval requirements are mapped out early and carefully, and that the project plan allows sufficient time and budget for the approvals process, can help avoid later delays that could cost millions of dollars.

While every industry will have its own set of project management issues that need to be carefully addressed, overall project management processes are very similar across industries, no matter what the stages are called and what issues tend to dominate. This chapter provides only an overview. More resources on industry-specific project management topics can be found through professional groups such as the Project Management Institute and in general purpose or industry-specific textbooks on project management. There is also a large amount of information about project management on the internet.

### 11.2.1 Initiation

All projects start with an idea for a product, a service, or a change that will produce benefits for a group or organization. When an idea gets to the point where management is considering its implementation, a sponsor is identified. The **project sponsor** is normally a person or group that has the resources to complete the project and acts as the project champion, leading the process of developing a project charter document. The charter defines the project in such a way that it can be understood by a broad audience, yet it must provide enough information so that an appropriate project manager can be chosen. The project charter should include:

- Background: What are the reasons for creating the project, and who will benefit from it?
- Objectives: What are the key project goals? Are they expressed in specific and measurable terms?
- Methodology: What methodology will be taken to complete the project?
- Justification: Is the project feasible? Does it meet an identifiable need? Do its benefits outweigh the costs? This section of the project charter may refer to other documents such as a feasibility study or needs analysis.
- Requirements: What are the required features and functions of the product or service (or desired outcome)?
- Constraints: What are the restrictions that limit the project in terms of time, budget, and requirements?
- Assumptions: What major project-related factors are considered to be true?
- Risks: What uncertain events or outcomes could have an effect on one or more objectives of the project? What impact might each have? Can the risks be controlled?
- Deliverables: What outcomes from the project are necessary to achieve the stated objectives?

The **project manager** is assigned the responsibility of fulfilling the objectives of the project. He or she is in charge of all aspects of the project, including the development of a plan, getting the project started, keeping it on track, and regularly reporting to the sponsor, management, and any other parties that have an interest (i.e., the stakeholders) in how the project is progressing.

When the project sponsor and management agree on the contents of the charter and have chosen an appropriate manager, this constitutes the formal start of the project.

### 11.2.2 Planning

Once the project has been approved to go into the planning phase, the project manager is responsible for developing a detailed project plan based on the project charter. The primary purpose of the **project plan** is to estimate the time, cost, and resources needed to complete the project as it is set out in the project charter. A thorough project plan can increase the likelihood that a project will be successful. It includes a number of components:

- The project scope: What is the overall goal of the project? What is the justification for the project? What are its deliverables? What are the product's or service's success criteria?
- The project team: Who will be responsible for carrying out the project? What skill sets are needed?
- A work breakdown structure: What is the nature and scope of each activity in the project? How are the activities related? Who will be responsible for each task?

- A project schedule: How much time will each task take? In what sequence will tasks be done? When will each start and finish? What are the major milestones?
- A budget: What are the major cost drivers? What will be the costs, and when will they occur?
- A risk plan: What are the major risks associated with the project? What impact could each risk have? Can these risks be controlled? If so, how?
- A communications plan: What aspects of the project should be communicated? When and to whom should project updates be given?

The project plan will contain all the information the project manager will need to determine whether the project should continue to the next phase. Part of this decision will be based on a detailed economic evaluation, now possible because the resources needed for the project have been identified. The methods discussed in Chapters 4 through 10 are appropriate tools for the economic evaluation. The project manager is responsible for communicating the results of the economic evaluation to the sponsor and to management. If the evaluation indicates that the project is economically justified, the sponsor usually approves a decision to move into the execution stage.

### 11.2.3 Execution

In this stage, the actual work of the project is done. Execution involves initiating the processes defined in the project plan and carrying through with them so that the project's objectives are achieved. In this stage, the project manager is responsible for selecting a project team and then coordinating the people and resources necessary to produce the project deliverables. The deliverables are the outputs of the project as defined initially in the project charter and then identified in more detail in the project plan.

### 11.2.4 Monitoring and Controlling

The processes of monitoring and controlling take place throughout the execution stage. By monitoring and controlling a project, the project manager tracks progress so that potential problems can be identified quickly and action taken to correct them. The project manager measures progress against the project plan, and any changes or corrective actions that are taken are aimed at returning to the plan or modifying it in light of new circumstances. Monitoring and controlling activities include carrying out the following activities regularly as work on the project is completed:

- Reviewing the project activities: What is the status of each activity?
- Monitoring the project constraints: Where should we be (compared to the project plan) in terms of project costs, time, and scope?
- Identifying corrective actions: Are there problems or issues with the project? If so, what can be done to get back on track? Does the plan need to be revised?
- Implementing changes: What is the impact of plan changes? How can the changes be managed?

This phase of a project concludes when the project's goals and objectives have been met. Note that a project could also be terminated prior to coming to a conclusion if priorities change, funds no longer exist, or the project is not meeting its objectives. If goals and objectives have been met, the project moves on to its final stage: closure.

### 11.2.5 Closure

Closure occurs when all activities defined in the project charter document have been completed, the project is signed off with the project sponsor, and the project team has been disbanded. A project *closure report* is a helpful tool that documents the conclusion of the project and contains an assessment of how well the project performed against its original goals. It is also a worthwhile document in which to note lessons learned during the project as a learning mechanism for future projects. Finally, the project closure report documents that the sponsor has accepted the final deliverables. The project manager is responsible for developing the project closure report, managing the signoff process, and disbanding the project team.

## 11.3 Project Management Tools

There is a wide range of tools available to a project manager aimed at supporting all stages of effective project management. For example, there are tools to help structure and sequence the manager's work and to provide guidance as to what issues need to be addressed at each stage. The tools covered in this section focus primarily on those used in planning a project, as this is the phase in which information gathered is used to analyze project viability.

We will look at three tools: the work breakdown structure (WBS), the Gantt chart, and the critical path method (CPM). The WBS is used to decompose a project into smaller, more manageable activities; the Gantt chart is used to sequence these activities; and CPM is used to schedule and control project activities.

### 11.3.1 Work Breakdown Structure

A **work breakdown structure** (WBS) is used to organize all the work to be done in a project by decomposing the project into successively smaller units. At the lowest level, each unit of work or “**work package**” represents a specific project deliverable. Constructing a WBS helps identify the activities involved in a project and the resources that are necessary to complete each of them. It is also a good tool for recognizing and avoiding potential duplication of activities. As well as being a planning tool, a WBS can be updated over the course of a project, as necessary, to record changes to the plan.

A WBS can be represented in a variety of ways. Two common depictions are a diagram that looks much like an organizational chart, and an indented list. In the diagram representation, the topmost level represents the project as a whole. Each successive layer below represents a finer level of detail in the WBS. In an indented list, each successive layer is indented from the layer above. The indented list format is commonly used for complex projects as it can more easily accommodate many thousands of items.

#### EXAMPLE

#### 11.1

This example has to do with construction of a simple beam bridge. A beam bridge consists of a horizontal beam supported at each end by an abutment. A simple example of a beam bridge is a log or wooden plank placed across a stream. If the beam is long enough, it may need intermediate supports called piers.

In this example, the beam is made from a slab of reinforced concrete, hence the name concrete slab bridge. The main components of the slab are its substructure, a

superstructure, and the deck. The construction project of building the slab beam bridge has six primary activities, as illustrated in Figure 11.1. They are setting up the site services; preparing the site; building the substructure, superstructure, and deck; and then site cleanup.

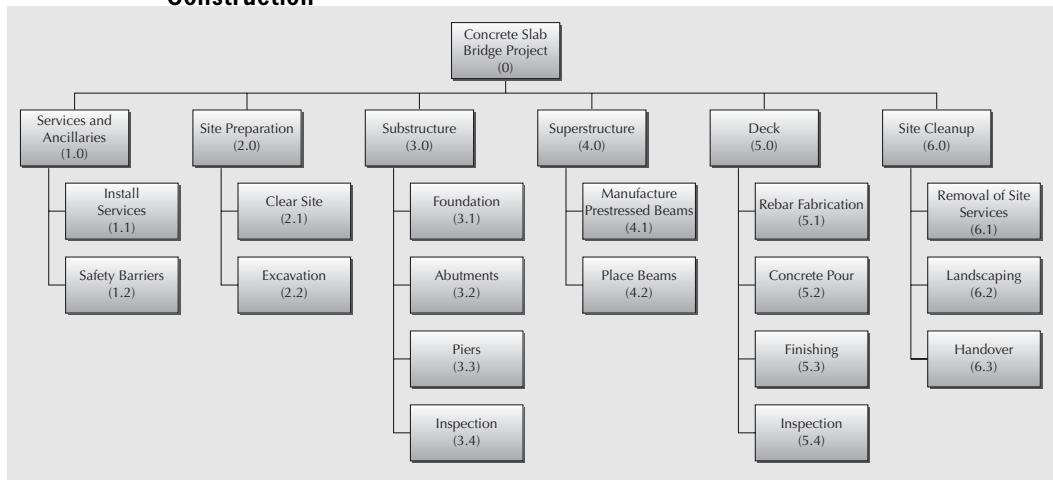
Figure 11.1 depicts the first two levels of the WBS; Table 11.1 shows an indented list view of the same WBS. Note that a numbering scheme is used to identify the successive layers of the WBS, whether in diagram form or an indented list. The numbering scheme is useful for identifying specific components and subcomponents of the work.

In a real WBS, there would be many other list layers below the two shown that would define more detailed work elements. At the lowest level of a WBS, it is possible to quantify the precise resources needed to complete the work. For example, under services, the lowest level of the WBS would quantify the number and capacity of electrical generators needed to provide power to the site or the precise type of washroom facilities necessary for the work crew. Under “Landscaping,” there would be a specification of the volume and quality of topsoil and the numbers, size, and types of plants to be placed on the banks of the abutments. Similarly, the foundation and abutments work components would specify the quality and volume of concrete needed for each.

Once a project manager has constructed a WBS down to its lowest level, the overall costs for the project can be “rolled up” from the bottom layer to arrive at a well-considered estimate for the total project cost.

There are several best practices to observe when constructing a WBS. First, items in the WBS should represent distinct project deliverables, separate bundles, or work packages that can be allocated to one or more team members or subcontracted. Next, the items in the WBS should be mutually exclusive and collectively exhaustive. That is, the WBS should represent 100 percent of the work—there should be no duplication or overlap of work among work packages. This makes it more straightforward to assign responsibility for each item and to construct cost estimates for the project. In terms of the level of detail, a common guideline is the “80 hour rule,” which suggests that work packages at the lowest level of the WBS should be no larger than 80 hours in duration.

**Figure 11.1 Chart View of Work Breakdown Structure for Concrete Slab Bridge Construction**



**Table 11.1 Indented List View of Work Breakdown Structure (WBS)**

| Work Level | Work Element                  | Expected Duration (days) |
|------------|-------------------------------|--------------------------|
| 0.0        | Concrete Slab Bridge Project  |                          |
| 1.0        | Services and Ancillaries      |                          |
| 1.1        | Install Services              | 7                        |
| 1.2        | Safety Barriers               | 7                        |
| 2.0        | Site Preparation              |                          |
| 2.1        | Clear Site                    | 7                        |
| 2.2        | Excavation                    | 14                       |
| 3.0        | Substructure                  |                          |
| 3.1        | Foundation                    | 7                        |
| 3.2        | Abutments                     | 14                       |
| 3.3        | Piers                         | 30                       |
| 3.4        | Inspection                    | 1                        |
| 4.0        | Superstructure                |                          |
| 4.1        | Manufacture Prestressed Beams | 42                       |
| 4.2        | Place Beams                   | 21                       |
| 5.0        | Deck                          |                          |
| 5.1        | Rebar Fabrication             | 21                       |
| 5.2        | Concrete Pour                 | 14                       |
| 5.3        | Finishing                     | 14                       |
| 5.4        | Inspection                    | 1                        |
| 6.0        | Site Cleanup                  |                          |
| 6.1        | Removal of Site Services      | 14                       |
| 6.2        | Landscaping                   | 7                        |
| 6.3        | Handover                      | 1                        |

When a WBS has been constructed according to the above practices, the items at the lowest level can be used to estimate resources needed for the project. Because these items represent 100 percent of the project and are mutually exclusive, we can sum across items to produce project totals for budget, time, and other resources. In this way, the WBS forms the basis for cost estimation and assigning work responsibilities. A WBS can also be used as a basis for project scheduling. ■

### 11.3.2 Gantt Charts

A Gantt chart is a graph or bar chart that depicts the timing and sequence of project activities. It was first developed by the pioneering industrial engineer Henry Gantt to help plan and control production of ammunition during World War I. It is one of the simplest and most fundamental project scheduling tools available.

A Gantt chart consists of a list of the work items and a diagram indicating when each item starts and ends. The WBS is a good basis to start a Gantt chart, as it estimates the time required to carry out each task but not the schedule of work. A Gantt chart indicates the sequence of activities required to complete a project and includes project milestones such as review and inspection. Once the overall schedule is set up, a project manager can use a Gantt chart to monitor project progress.

**EXAMPLE****11.2**

A Gantt chart for the concrete slab bridge project from Example 11.1 is shown in Figure 11.2. The interpretation of this chart is straightforward. Each activity is a row, and the bars or diamonds in the calendar portion graphically indicate the start and finish of the activities (bars) or the specific date of the event (diamonds). For example, Activity 2.2, Site Excavation, is a short bar covering the two-week period from April 23 to May 6. Activity 5.4, Inspection, is an event of short duration occurring on September 17 and is represented as a diamond.

**Figure 11.2 Gantt Chart for Concrete Slab Bridge Construction**

| Number | Task                          | Start     | End       | 2012  |     |      |      |        |           |         |
|--------|-------------------------------|-----------|-----------|-------|-----|------|------|--------|-----------|---------|
|        |                               |           |           | April | May | June | July | August | September | October |
| 1      | Services and Ancillaries      | 2/4/2012  | 15/4/2012 |       |     |      |      |        |           |         |
| 1.1    | Install Site Services         | 2/4/2012  | 8/4/2012  | ■     |     |      |      |        |           |         |
| 1.2    | Safety Barriers               | 9/4/2012  | 15/4/2012 | ■     |     |      |      |        |           |         |
| 2      | Site Preparation              | 16/4/2012 | 6/5/2012  |       |     |      |      |        |           |         |
| 2.1    | Clear Site                    | 16/4/2012 | 22/4/2012 | ■     |     |      |      |        |           |         |
| 2.2    | Site Excavation               | 23/4/2012 | 6/5/2012  |       | ■   |      |      |        |           |         |
| 3      | Substructure                  | 7/5/2012  | 10/6/2012 |       |     |      |      |        |           |         |
| 3.1    | Foundation                    | 21/5/2012 | 27/5/2012 |       |     | ■    |      |        |           |         |
| 3.2    | Abutments                     | 28/5/2012 | 10/6/2012 |       |     | ■    |      |        |           |         |
| 3.3    | Piers                         | 7/5/2012  | 5/6/2012  |       | ■   | ■    |      |        |           |         |
| 4      | Superstructure                | 2/4/2012  | 22/7/2012 |       |     |      |      |        |           |         |
| 4.1    | Manufacture Prestressed Beams | 2/4/2012  | 13/5/2012 | ■     | ■   |      |      |        |           |         |
| 4.2    | Place Beams                   | 2/7/2012  | 22/7/2012 |       |     |      | ■    |        |           |         |
| 5      | Deck                          | 23/7/2012 | 17/9/2012 |       |     |      |      |        |           |         |
| 5.1    | Rebar Fabrication/Placement   | 23/7/2012 | 12/8/2012 |       |     |      | ■    |        |           |         |
| 5.2    | Concrete Pour (slab)          | 13/8/2012 | 2/9/2012  |       |     |      |      | ■      |           |         |
| 5.3    | Finishing                     | 3/9/2012  | 16/9/2012 |       |     |      |      |        | ■         |         |
| 5.4    | Inspection                    | 17/9/2012 | 17/9/2012 |       |     |      |      |        | ◆         |         |
| 6      | Site Cleanup                  | 18/9/2012 | 8/10/2012 |       |     |      |      |        |           |         |
| 6.1    | Removal of Site Services      | 18/9/2012 | 1/10/2012 |       |     |      |      |        | ■         |         |
| 6.2    | Landscaping                   | 2/10/2012 | 8/10/2012 |       |     |      |      |        |           | ■       |
| 6.3    | Handover                      | 8/10/2012 | 8/10/2012 |       |     |      |      |        |           | ◆       |

A Gantt chart is an excellent tool for determining precedence among tasks. For example, Activity 2.2, Site Excavation, cannot begin until Activity 2.1, Clear Site, has been completed, while Activity 4.1, Manufacture Prestressed Beams, can be initiated early and at the same time as other activities. There are a large number of precedence relationships in the concrete slab bridge example, which makes the Gantt chart particularly useful.

Precedence relationships are reasonably straightforward to keep track of for small projects, but they may be difficult to manage and communicate for larger projects. This shortcoming reduces the effectiveness of Gantt charts as a planning and control tool for

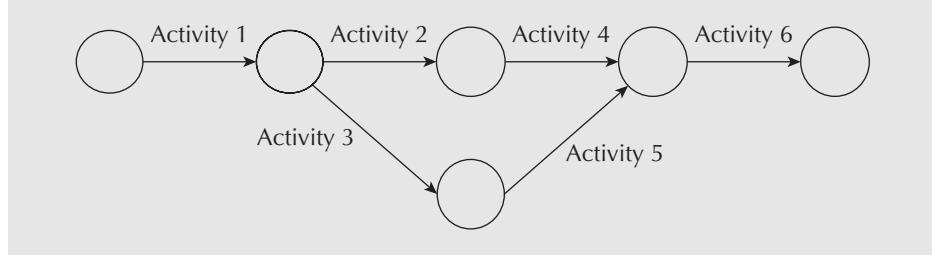
larger projects and led to the development of more sophisticated methods for scheduling, such as the **Critical Path Method (CPM)** and the **Project Evaluation and Review Technique (PERT)**. These methods are the subject of the next section. ■

### 11.3.3 The Critical Path Method

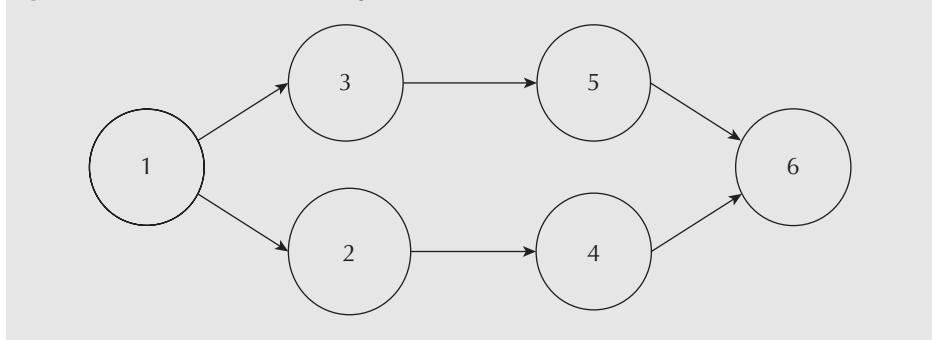
Managers responsible for planning and controlling large projects have found CPM and PERT to be very helpful. Both are based on a network representation that is able to capture the interdependence of activities not well communicated through a Gantt chart. Although these methods were developed independently, they serve the same general purpose and have many similarities. CPM was initially developed for projects with certain task times that can be crashed, or shortened, through the use of additional resources and offers a method to make cost and time tradeoffs for a project. In contrast, PERT was developed to plan and control projects where task times are assumed to be uncertain. Since their initial use in the 1950s and 1960s, these two methods have evolved. They may be referred to together as PERT/CPM, and the combination draws on the strengths of each. This chapter will focus primarily on the CPM.

In both CPM and PERT, project activities are represented by a network of nodes and arcs. In the “**activity on arc**,” or **AOA**, representation, arcs represent activities and nodes are used to capture interrelationships between activities, as illustrated in Figure 11.3. Figure 11.3 depicts a project with six activities where Activity 1 precedes Activities 2 and 3, Activity 4 follows Activity 2, and Activities 4 and 5 must be completed prior to the start of Activity 6. Activity 1 is represented by an arc connecting two nodes that mark the start and end of Activity 1. Once Activity 1 has been completed, Activities 2 and 3 can begin. Thus the node that represents the end of Activity 1 also represents the start of Activities 2 and 3. Activities 2 and 3 are drawn as two arcs coming from the node that marks the end of Activity 1. Next, since Activity 4 can start once Activity 2 has been completed, an arc denoting Activity 4 is drawn coming out of the node that indicates the end of Activity 2. Similarly, an arc denoting Activity 5 is drawn out of the node marking the end of Activity 3. Finally, since Activity 6 can begin only when both Activities 4 and 5 are done, the nodes marking the end of Activities 4 and 5 are joined together into one node denoting the event that both are completed. Thus Activity 6 is represented as an arc emanating from a single node that represents the end of Activities 4 and 5 (or the start of Activity 6).

**Figure 11.3 AOA Network Representation**



The second style of modelling a project uses an “**activity on node**” (**AON**) representation, as shown in Figure 11.4. Figure 11.4 depicts the same project as shown in Figure 11.3 with the nodes representing activities rather than the arcs. In this approach, two concurrent activities that can start only after a common predecessor has been completed are represented by two arcs coming out of the predecessor. Activities 2 and 3 can begin (independently) when Activity 1 has completed. Activity 4 can be started after

**Figure 11.4 AON Network Representation**

Activity 2 has been finished. Activity 5 can begin once Activity 3 is done. Finally, Activity 6 must wait until both Activities 4 and 5 are finished, which is represented by arcs drawn from Activities 4 and 5 into the node for Activity 6. Example 11.3 demonstrates the differences between the AOA and AON network representations.

**EXAMPLE****11.3**

Consider a project in which an internet merchant is installing an automated picking line in its distribution centre. The first step is to prepare and send out a request for proposals (RFP) from vendors. Once vendors have submitted proposals, the winning vendor will be chosen and the system components ordered (vertical shelving, lights, roller conveyor, computer systems). The new layout of the centre must be designed and the current layout changed. Once this has been completed and the new picking line system components are in, the equipment can be installed, the computers can be installed, and the software can be configured. Once both the equipment and computer systems are in place, they can be tested. Finally, the users will be trained. Details about these activities are summarized in Table 11.2. Note that the “Predecessor Activities” column refers to *immediate* predecessors only, not all predecessors. You have been asked to develop a CPM network for the project. Having just learned about the differences between AOA and AON representations, you have decided to draw both diagrams.

**Table 11.2 Activities for New Picking Line Project**

| Activity | Description        | Expected Completion Time (weeks) | Predecessor Activities |
|----------|--------------------|----------------------------------|------------------------|
| 1        | Obtain RFPs        | 6                                | —                      |
| 2        | Select vendor      | 4                                | 1                      |
| 3        | Order equipment    | 6                                | 2                      |
| 4        | Design new layout  | 7                                | 2                      |
| 5        | Implement layout   | 9                                | 4                      |
| 6        | Install computers  | 3                                | 3,5                    |
| 7        | Configure software | 3                                | 6                      |
| 8        | Install equipment  | 5                                | 3,5                    |
| 9        | Test system        | 3                                | 7,8                    |
| 10       | Train users        | 2                                | 9                      |

Figure 11.5 shows an AOA representation of the project presented in Table 11.2. The activity “Select Vendor” follows “Send Out RFP” in sequence. Since “Order System” and “Design Layout” can only start once “Select Vendor” has been completed, both arcs come out of the node representing the end of the “Select Vendor” activity. “Implement Layout” follows “Design Layout.” Once both “Order System” and “Implement Layout” have been completed, “Install Computers” and “Install Equipment” can begin. Thus a single node represents the end of “Order System” and “Implement Layout,” as well as the start of “Install Computers” and “Install Equipment.” “Configure Software” follows “Install Computers,” and “Test System” begins once “Install Equipment” and “Configure Software” are completed. Once “Test System” has been completed, “Train Users” can begin.

**Figure 11.5 AOA Network for Picking Line Project**

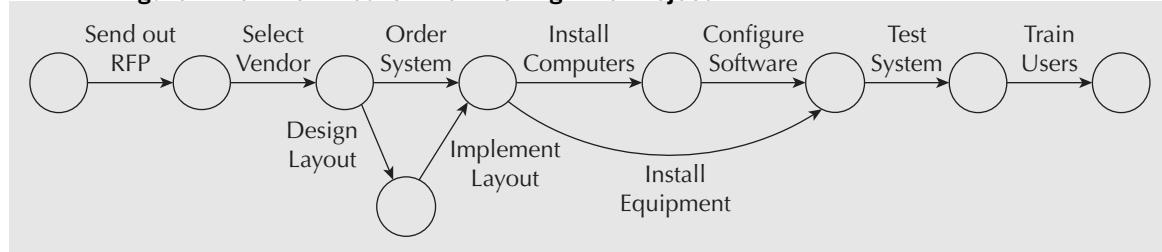
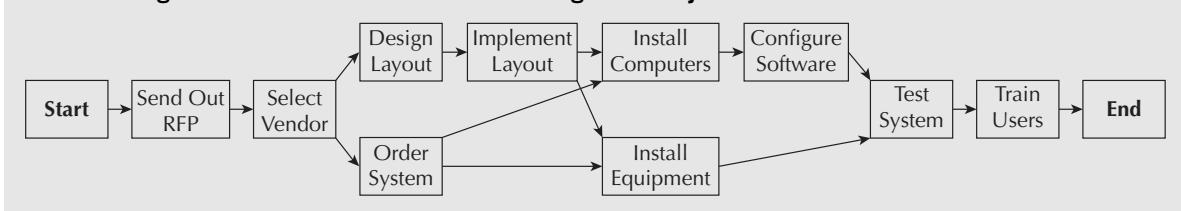


Figure 11.6 shows the AON network for the same project. Starting from the left in the AON diagram, “Send Out RFP” precedes the activity “Select Vendor.” Once “Select Vendor” has been completed, both “Design Layout” and “Order System” activities can begin. This is depicted by two arcs out of the “Select Vendor” node. The two arcs entering “Install Equipment” indicate that both “Implement Layout” and “Order System” must be completed before this activity can begin.

**Figure 11.6 AON Network for Picking Line Project**



While both AOA and AON representations for project activities are equally valid, modern practice is to use AON networks because they seem to be a more natural way of representing project activities. For this reason, the rest of the chapter will focus primarily on AON networks.

In summary, there have been several network-based methodologies developed to assist managers with project management, including PERT and CPM. How activities are represented in PERT or CPM can take on two formats: AON and AOA. In this chapter, we focus on the CPM and use the AON network representation. We now turn our attention to project scheduling.



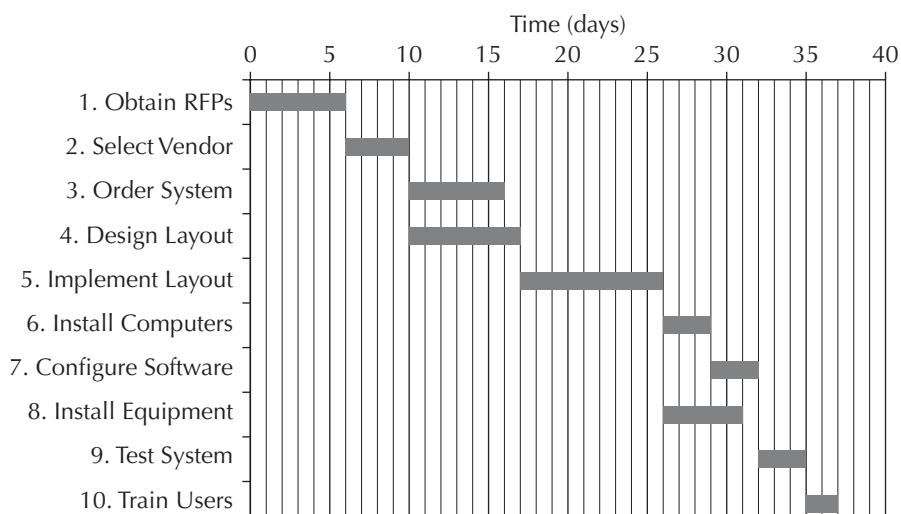
## S P R E A D S H E E T   S A V V Y

Specialized project management software such as Microsoft Office Project offer many features that general packages like Excel do not provide. However, there can be a fairly substantial learning curve associated with such packages. For simple projects, familiar tools such as spreadsheets can be quite sufficient for project management purposes. Spreadsheets are a natural tool for composing work breakdown structures and can be used to make simple Gantt charts. This Spreadsheet Savvy demonstrates how to use Excel to construct a Gantt chart for the project from Example 11.3.

The first step is to enter the list of activities, along with their start dates and duration, as shown in the screenshot:

|    | A                     | B     | C               | D |
|----|-----------------------|-------|-----------------|---|
| 1  |                       |       |                 |   |
| 2  | Activity              | Start | Duration(weeks) |   |
| 3  | 1. Obtain RFPs        | 0     | 6               |   |
| 4  | 2. Select Vendor      | 6     | 4               |   |
| 5  | 3. Order System       | 10    | 6               |   |
| 6  | 4. Design Layout      | 10    | 7               |   |
| 7  | 5. Implement Layout   | 17    | 9               |   |
| 8  | 6. Install Computers  | 26    | 3               |   |
| 9  | 7. Configure Software | 29    | 3               |   |
| 10 | 8. Install Equipment  | 26    | 5               |   |
| 11 | 9. Test System        | 32    | 3               |   |
| 12 | 10. Train Users       | 35    | 2               |   |

Next, select cells B2:D12 and insert a stacked bar chart. The activities will be listed in reverse order and the stacked lines will need to be edited before it will look like a Gantt chart. First, select the vertical axis and right-click. Select Categories in Reverse Order from the axis options. This will list the activities in ascending order. To show only the duration of the activities, right-click in the blue area (the default colour) on one of the bars and select No Fill for the bar option. This will cause only the activity time to appear, starting at the correct time on the horizontal axis. Next, to show more detail on the timing, right-click on the horizontal axis (now displayed along the top) and select Add Minor Gridlines. To show a frame around the chart, right-click somewhere in the plot area, and under Format Plot Area, select the solid line option under Border Colour. A title can be added by clicking somewhere on the chart and selecting Axis Titles from the Chart Layout options. The Gantt chart now looks like this:



### 11.3.3.1 Project Scheduling and the Critical Path Method

Constructing a network representation of a project that captures all the precedence relationships among activities enables the project manager to develop a project schedule and to identify the **critical path**. The critical path is the set of activities that must be completed exactly as planned in order to keep the project on schedule. There is no slack or “wiggle room” for completing the activities on the critical path, as delaying any one of them will cause a delay in the completion time of the entire project. At the very least, being able to identify the activities that are on the critical path will help a manager focus on the key tasks that affect the project completion date.

Finding the critical path involves making two “passes” through the CPM network diagram. In the first pass, the earliest possible start and finish times for each activity are determined. This will provide the manager with the earliest feasible completion time for the project. The second pass through the network goes in reverse, starting with the completion time established in the first pass and computing the *latest* time each activity can start and finish. This identifies the activities that can start (and finish) later than the dates found in the first pass without delaying the project. The activities for which the earliest start time and the latest start time are the same are on the critical path. That is, there is no slack available in their start or finish times. They must be completed as scheduled or the project will be delayed.

In order to keep track of the start and finish times, some additional notation is needed. The duration of each activity is denoted by  $T$  and the earliest start and finish times are denoted by  $ES$  and  $EF$ , respectively. Similarly, the latest start and finish times are written as  $LS$  and  $LF$ , respectively.

#### EXAMPLE

#### 11.4

Find the critical path for the distribution picking line problem of Example 11.3.

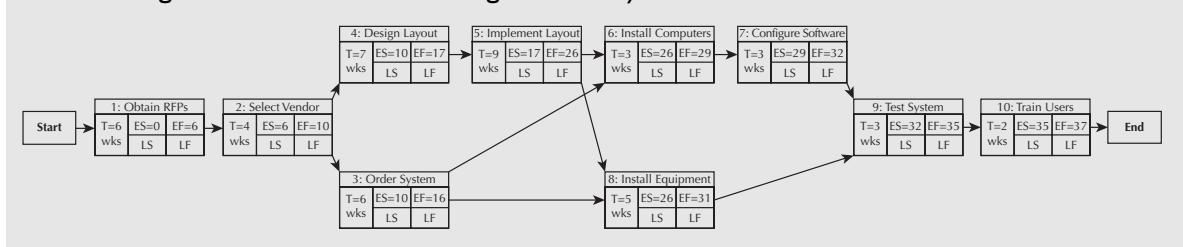
Figure 11.7 shows the result of the first pass through the CPM network. Each node gives the name of the activity, its completion time ( $T$ ), and its earliest start ( $ES$ ) and earliest finish ( $EF$ ) times. The project is assumed to start at time  $T = 0$ . Therefore,  $ES = 0$  for activity “Obtain RFPs,” and with an expected duration of  $T = 6$  weeks, the earliest time it can be completed is  $0 + 6 = 6$  weeks. In general, the earliest finish time for an activity is computed by:

$$EF = ES + T$$

For Activity 1 (Obtain RFPs), the  $EF$  is:

$$EF = ES + T = 0 + 6 = 6 \text{ weeks}$$

**Figure 11.7 First Pass Through the PERT/CPM Network**



Continuing on, the earliest start time for “Select Vendor” is  $ES = 6$  weeks, since it follows the conclusion of “Send out RFPs.” Since four weeks is needed to select a vendor,

$EF = ES + T = 6 + 4 = 10$  weeks. This makes  $ES = 10$  weeks for both of the “Design Layout” and “Order System” activities. The earliest start for “Install Computers” must take into account the earliest finish times of both “Implement Layout” and “Order System.” Since “Implement Layout” is expected to finish in 26 weeks, the earliest start time for “Install Computers” is also 26 weeks. In general, the earliest start time for an activity is the *latest* earliest finish time of its immediate predecessors:

$$ES = \text{Max}(EF \text{ of immediate predecessors})$$

For “Install Computers”:

$$ES = \text{Max}(EF \text{ of Implement Layout, Order Systems}) = \text{Max}(26, 16) = 26 \text{ weeks}$$

And

$$EF = ES + T = 26 + 3 = 29 \text{ weeks.}$$

For “Install Equipment”:

$$ES = \text{Max}(EF \text{ of Implement Layout, Order System}) = \text{Max}(16, 26) = 26 \text{ weeks}$$

And

$$EF = ES + T = 26 + 5 = 31 \text{ weeks}$$

And for “Test System”:

$$ES = \text{Max}(EF \text{ of Configure Software, Install Equipment}) = \text{Max}(31, 32) = 32 \text{ weeks}$$

And

$$EF = ES + T = 32 + 3 = 35 \text{ weeks}$$

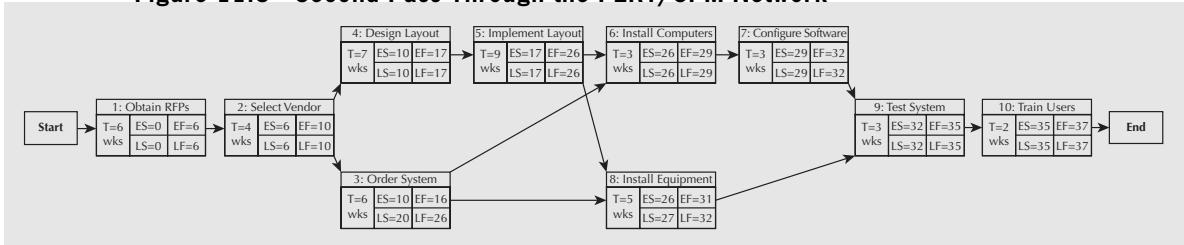
Finally, the earliest possible completion time for the entire project is the earliest finish date for the last activity of “Train Users”:

$$EF = ES + T = 35 + 2 = 37 \text{ weeks}$$

The earliest project completion time is useful information for the project manager as it conveys the shortest possible time the project will take to complete if every task proceeds according to plan. The manager would also like to know the critical path—the set of activities that must be completed without deviation from the plan in order to avoid the delay of the entire project.

Finding the critical path requires a second pass through the network, this time starting from the earliest finish date for the entire project—in this case, 37 weeks. The second pass through the network determines how *late* each activity can start and finish without delaying this completion date. Figure 11.8 shows the latest start (LS) and latest

**Figure 11.8 Second Pass Through the PERT/CPM Network**



finish (LF) times for each activity. Working backward, the “Train Users” activity is assigned a latest finish date of 37 weeks (LF = 37). Given a two-week duration, “Train Users” will have a latest start time of LS = 35 weeks:

$$LS = LF - T = 37 - 2 = 35 \text{ weeks}$$

Working further back through the activities, “Test System” has LF = 35 weeks, so its LS = LF - T = 35 - 3 = 32 weeks. This means that LF = 32 weeks for each of “Configure Software” and “Install Equipment.” Similarly, “Install Computers” has LF = 29 weeks, and LS = LF - T = 29 - 3 = 26 weeks. But what is the latest finish time for the activity “Implement Layout”? Both “Install Equipment” and “Install Computers” depend on the completion of “Implement Layout.” The latest start time for “Install Computers” is LS = 26 weeks, and for “Install Equipment” is LS = 27 weeks. Unless “Implement Layout” completes by week 26, “Install Computers” will not be able to start on time without making the project late. In general, the latest finish time for any activity is the *most constraining*—in other words, the earliest of the LS times for its successor activities.

$$LF = \text{Min}(LS \text{ of successor activities})$$

With this logic, the latest finish time for “Order System” is

$$\begin{aligned} LF &= \text{Min}(LS \text{ of Install Computers, Install Equipment}) \\ &= \text{Min}(26, 27) = 26 \text{ weeks} \end{aligned}$$

and

$$LS = LF - T = 26 - 6 = 20 \text{ weeks.}$$

Working further back, the latest finish time for “Select Vendor” is

$$LF = \text{Min}(LS \text{ of Design Layout, Order System}) = \text{Min}(10, 20) = 10 \text{ weeks,}$$

and

$$LS = LF - T = 10 - 4 = 6 \text{ weeks.}$$

And finally, for “Obtain RFPs,” LF = 6 weeks and LS = 0 weeks.

With this second pass completed, the critical path can now be determined. The critical path is the longest path in terms of time through the project network, and the activities on it are called critical activities. They are easily recognized as those for which ES = LS and EF = LF. The critical path for the distribution picking line project is Activities 1–2–4–5–6–7–9–10, and the length of the critical path is 37 weeks.

Activities not on the critical path have ES < LS and EF < LF. For example, the “Order System” task has ES = 10, LS = 20, EF = 16, and LF = 26 weeks. As long as this task begins and ends within the window of weeks 10 to 26, the rest of the project will not be delayed. This scheduling flexibility is referred to as slack, S, and is defined by:

$$S = LS - ES = LF - EF$$

Knowing that there are 10 weeks of slack for the “Order System” activity is useful for the project manager. If, for some reason, the “Order Computers” task gets started late, or if the order takes longer than six weeks, the project manager knows that these dates can slip by up to 10 weeks without concern that the rest of the project will be delayed. The task “Install Equipment” has slack of one week:

$$S = LS - ES = 27 - 26 = 1 \text{ week}$$

or

$$S = LF - EF = 32 - 31 = 1 \text{ week}$$

Activities on the critical path have  $S = 0$ . The slacks for each activity of the picking line project are summarized in Table 11.3.

**Table 11.3 Picking Line Slack Computations**

| Activity | Time, T<br>(weeks) | ES<br>(weeks) | EF<br>(weeks) | LS<br>(weeks) | LF<br>(weeks) | Slack, S<br>(weeks) |
|----------|--------------------|---------------|---------------|---------------|---------------|---------------------|
| 1        | 6                  | 0             | 6             | 0             | 6             | 0                   |
| 2        | 4                  | 6             | 10            | 6             | 10            | 0                   |
| 3        | 6                  | 10            | 16            | 20            | 26            | 10                  |
| 4        | 7                  | 10            | 17            | 10            | 17            | 0                   |
| 5        | 9                  | 17            | 26            | 17            | 26            | 0                   |
| 6        | 3                  | 26            | 29            | 26            | 29            | 0                   |
| 7        | 3                  | 29            | 32            | 29            | 32            | 0                   |
| 8        | 5                  | 26            | 31            | 27            | 32            | 1                   |
| 9        | 3                  | 32            | 35            | 32            | 35            | 0                   |
| 10       | 2                  | 35            | 37            | 35            | 37            | 0                   |

There are a variety of software packages available that can help managers with project management activities such as creating Gantt charts, CPM networks, and finding the critical path. ■

### 11.3.3.2 Project Crashing and Time–Cost Tradeoffs

One of the strengths of the CPM method is that it provides managers with a mechanism for examining the possibility of reducing overall project completion time through the application of additional resources. This is referred to as **crashing** a project. Since resources such as overtime, extra equipment, and additional staff generally result in higher costs, project managers need to make a tradeoff between the extra costs associated with crashing and the reduced project completion time. Example 11.5 illustrates this approach.

#### EXAMPLE

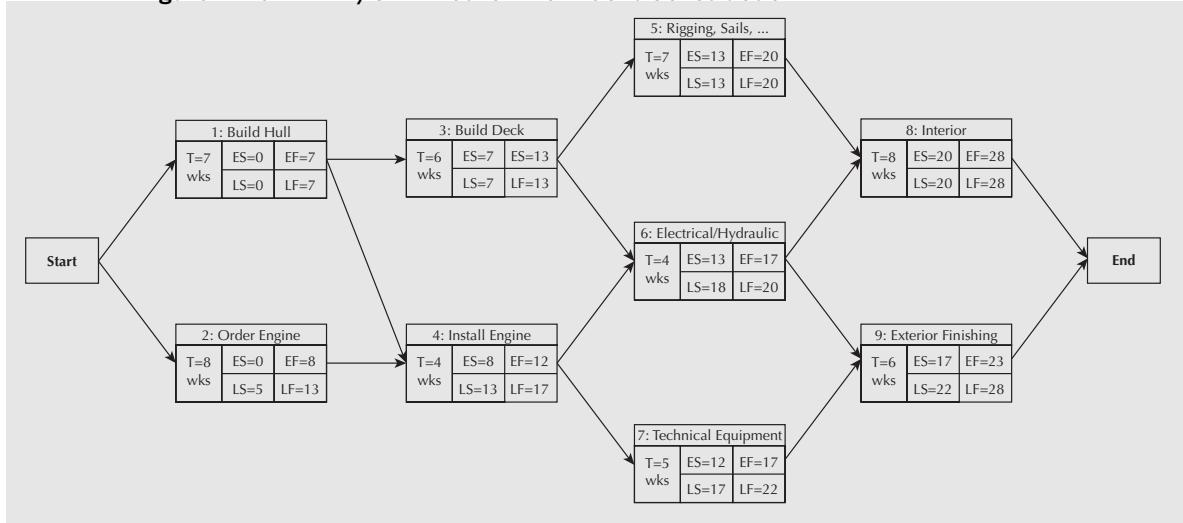
#### 11.5

You are responsible for managing the project of building a yacht. The project has been broken down into a series of activities and the expected duration of each activity as well as the precedence relationships have been established. This information is summarized in Table 11.4. The overall budget for the project is \$4 000 000. Based on a CPM analysis, you have planned for a completion time of 28 weeks. The individual you are building the yacht for is particularly anxious to use it and has asked how much it would cost to speed up the project so that it could be finished five weeks early. Since you have considerable experience with similar projects, you have been able to assemble data on how much each activity in the schedule can be crashed and how much it will cost. This information is also summarized in Table 11.4.

**Table 11.4 Yacht Construction Crashing Data**

| Activity                            | Predecessors | Time (weeks) |         | Cost (\$) |           |           |
|-------------------------------------|--------------|--------------|---------|-----------|-----------|-----------|
|                                     |              | Normal       | Crashed | Normal    | Crashed   | Cost/Week |
| 1 Build Hull                        |              | 7            | 4       | 655 000   | 775 000   | 40 000    |
| 2 Order Engine                      |              | 8            | 4       | 125 000   | 205 000   | 20 000    |
| 3 Build Deck                        | 1            | 6            | 4       | 275 000   | 415 000   | 70 000    |
| 4 Install Engine                    | 2            | 4            | 3       | 50 000    | 130 000   | 80 000    |
| 5 Rigging, Sails,<br>Deck Equipment | 3            | 7            | 6       | 545 000   | 630 000   | 85 000    |
| 6 Electrical/Hydraulic              | 3,4          | 4            | 3       | 275 000   | 360 000   | 85 000    |
| 7 Technical<br>Equipment            | 3,4          | 5            | 4       | 725 000   | 790 000   | 65 000    |
| 8 Interior                          | 5,6          | 8            | 5       | 900 000   | 1 305 000 | 135 000   |
| 9 Exterior Finishing                | 7            | 6            | 4       | 450 000   | 590 000   | 70 000    |
|                                     |              |              |         | 4 000 000 | 5 200 000 |           |

The procedure for determining the critical path is the same as that presented earlier in the chapter. Making these computations, the CPM network with a project completion time of 28 weeks is shown in Figure 11.9. The critical path associated with the 28-week completion time is 1–3–5–8. Activities 2, 4, 7, and 9 have a slack of five weeks each, and Activity 6 has a slack of three weeks.

**Figure 11.9 PERT/CPM Network for Yacht Construction**

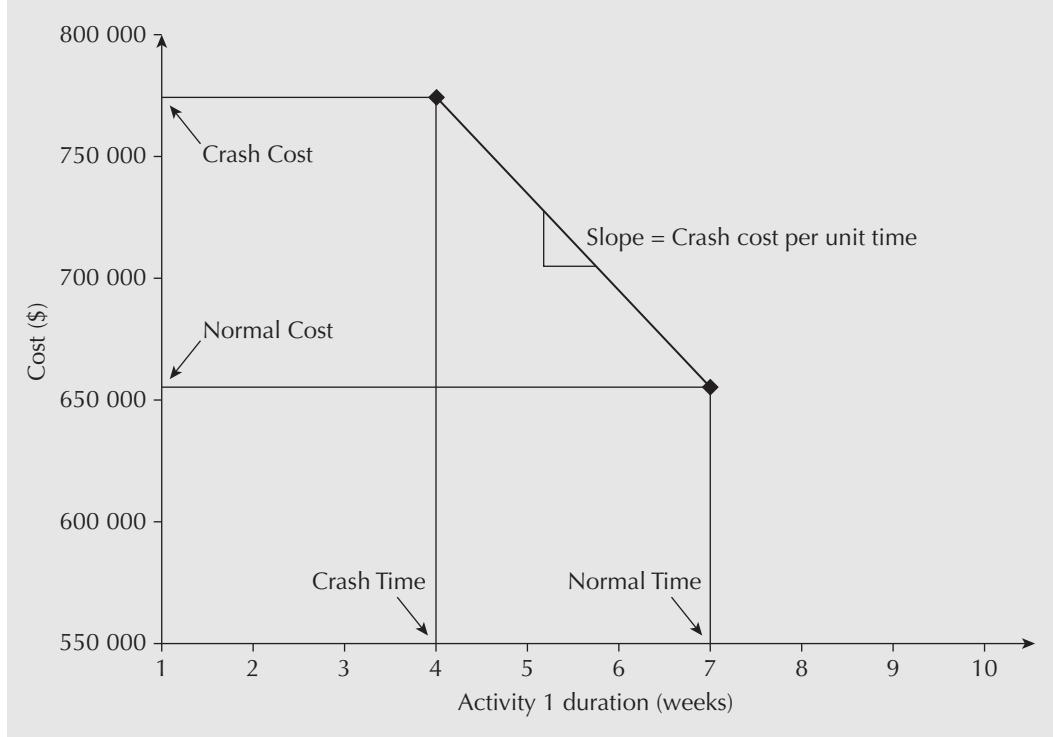
In order to deliver the yacht by an earlier date, one or more of the activities on the critical path must be crashed. The crashing procedure identifies activities that are the least expensive to shorten, then shortens each either by its maximum amount or until another critical path is created by the crashing. This is repeated until the desired reduction in the project completion time is achieved or until no further crashing can be done.

To determine which activities to crash, a crash cost per unit time is computed for each activity by using the crash cost and crash time information as follows:

$$\text{crash cost per unit time} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

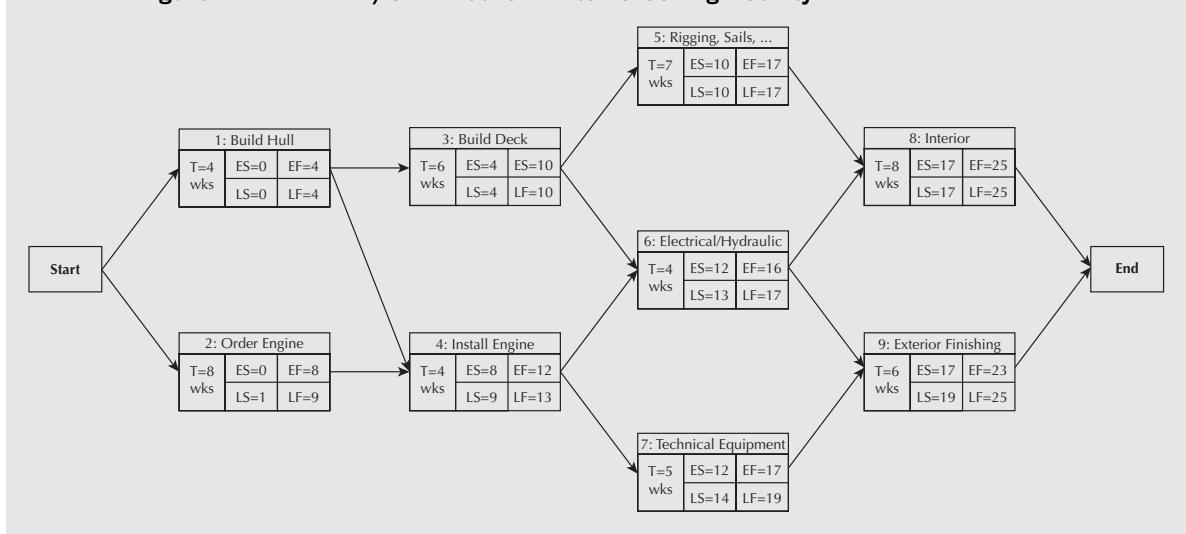
This computation assumes that the additional cost to crash an activity is linear in the amount of time the activity is reduced, as illustrated for Activity 1 in Figure 11.10. What this means is that any fraction of the activity crash time is possible, and that the cost of crashing will be proportional to that fraction. Although it is an approximation, it is nonetheless useful for this purpose.

**Figure 11.10 Crashing Time–Cost Tradeoffs**



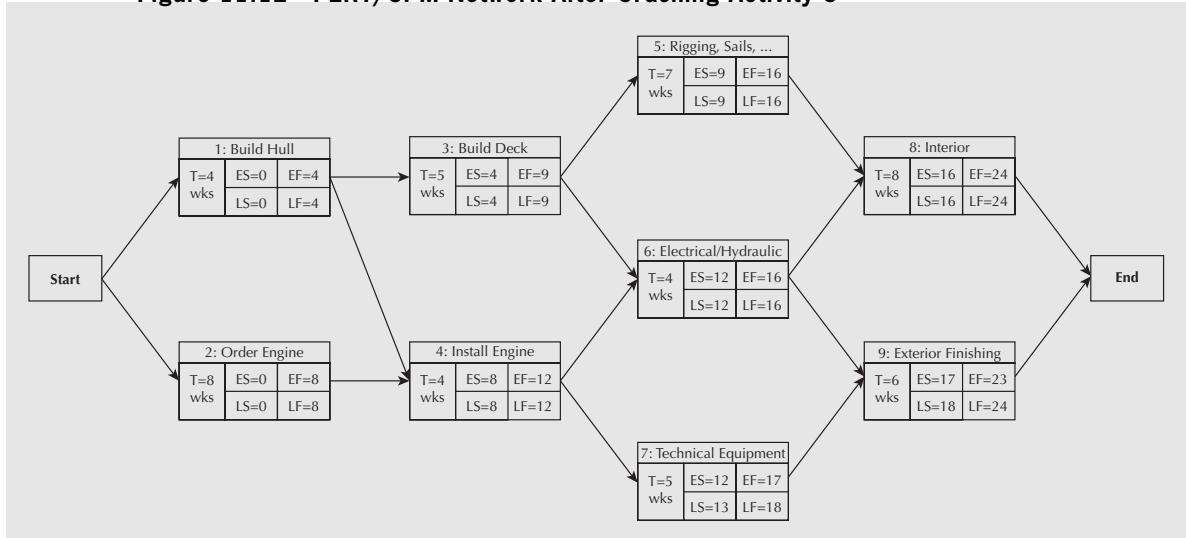
The crashing process begins by looking at the crash cost per unit time of the critical path activities. The critical path is 1-3-5-8. The project can be shortened only by reducing one or more of the activities on the critical path. But which will be the least expensive? Table 11.4 provides the crash cost per week for all activities (rightmost column). Of those on the critical path, the least expensive to crash is Activity 1, “Build Hull,” at a cost per week of \$40,000. This activity can be crashed by up to three weeks, but care must be taken in deciding how much to crash Activity 1. In reducing its duration, another set of activities may become critical, making further reductions unnecessary and costly. Inspection of the CPM diagram in Figure 11.9 indicates that Activity 1 can be fully crashed without creating a new critical path. The total cost of crashing Activity 1 will be \$120,000—three weeks of crashing at \$40,000 per week.

Having crashed Activity 1 to four weeks, the CPM diagram must be updated to reflect the new schedule. The updated ES, EF, LS, and LF times are shown in Figure 11.11.

**Figure 11.11 PERT/CPM Network After Crashing Activity 1**

The new project completion time is 25 weeks and the critical path is still 1–3–5–8. Notice that the slack in Activities 2, 4, and 6 is one week, and is two weeks for Activities 7 and 9. This is informative because it indicates how much potential for further crashing there is before one or more of these activities will become critical.

To reduce the project duration further, additional crashing is needed. Returning to Table 11.4, the next least expensive activity on the critical path that still has crashing potential is Activity 3, “Build Deck.” This task has a crash cost per week of \$70 000 and can be crashed from its normal duration of six weeks down to four weeks. However, its duration can only be reduced by one week before a second critical path, which includes Activities 2, 4, and 6, becomes active. Figure 11.12 shows the result of crashing Activity 3 by one week. There are two critical paths, 1–3–5–8 and 2–4–6–8, both with a duration of 24 weeks.

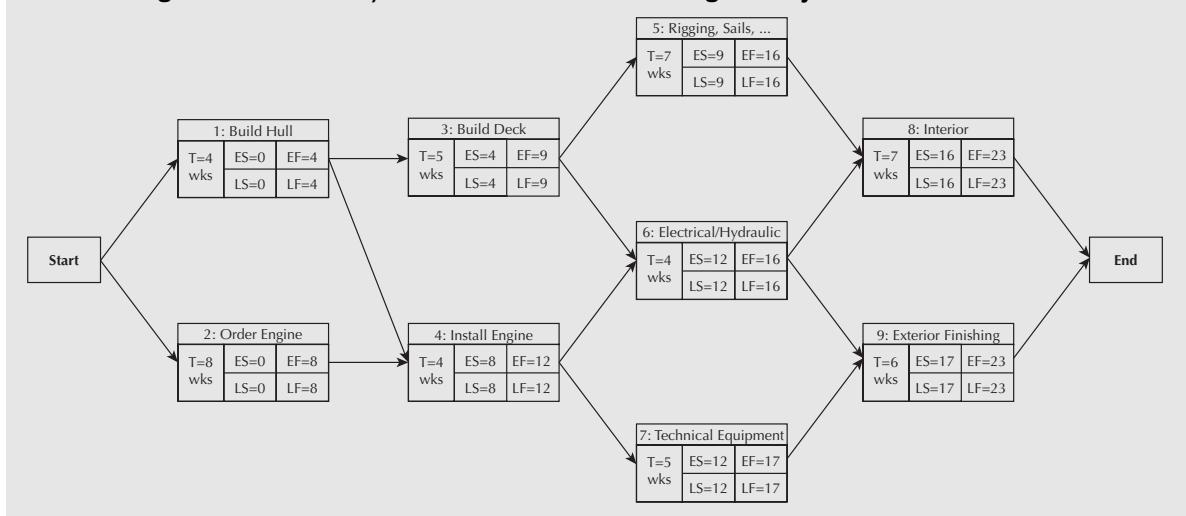
**Figure 11.12 PERT/CPM Network After Crashing Activity 3**

When two paths are simultaneously critical, activities on both paths must be crashed in order to reduce the project completion time. The question then becomes which combination of activities will reduce the project completion time at the least total cost per unit time. Careful inspection of the network in Figure 11.12 reveals the following alternatives:

1. Reduce Activities 3 and 4 by one week each for a total cost of \$70 000 + \$80 000 = \$150 000.
2. Reduce Activities 5 and 6 by one week each for a total cost of \$85 000 + \$85 000 = \$170 000.
3. Reduce Activity 8 by one week for a total cost of \$135 000.

The reason why only one activity needs crashing for the third alternative is that Activity 8 is common to both critical paths. Despite the high individual cost of crashing this activity (\$135 000), it is still the least expensive option to reduce the duration of the project by an additional week. Making this change to the CPM network, as shown in Figure 11.13, reveals that all the activities are now critical—none have remaining slack.

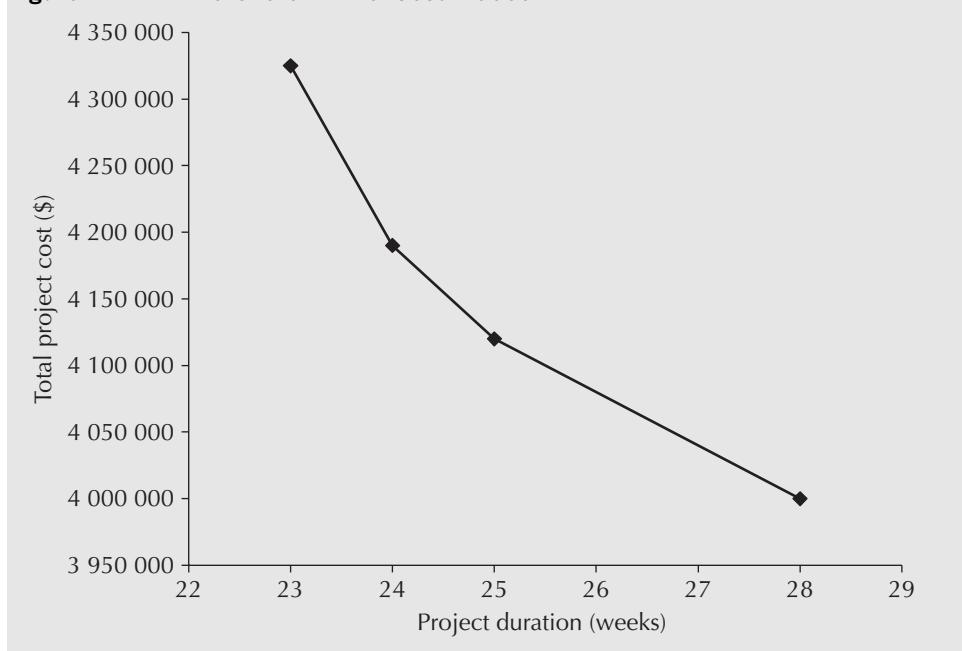
**Figure 11.13 PERT/CPM Network After Crashing Activity 8**



The project completion time is now 23 weeks, which meets the client's request to have the yacht ready five weeks earlier than the originally scheduled 28 weeks. Table 11.5 summarizes the crashing steps taken to attain a completion date of 23 weeks. Figure 11.14 provides a graphical representation of the sequence of crashing decisions showing that the cost to crash each additional week increases in accordance with the number of weeks the project is crashed.

**Table 11.5 Summary of Project Crashing Steps**

| Step | Activity Crashed | Crashing Amount (weeks) | Crashing Cost (\$) | Project Completion Time (weeks) | Project Completion Cost (\$) |
|------|------------------|-------------------------|--------------------|---------------------------------|------------------------------|
| 0    |                  |                         |                    | 28                              | 4 000 000                    |
| 1    | 1                | 3                       | 120 000            | 25                              | 4 120 000                    |
| 2    | 3                | 1                       | 70 000             | 24                              | 4 190 000                    |
| 3    | 8                | 1                       | 135 000            | 23                              | 4 325 000                    |

**Figure 11.14** The Overall Time–Cost Tradeoff

This example has demonstrated the basic approach for crashing a project. For a small CPM project, this trial-and-error approach is quite effective. However, it does require that great care be taken in revising the ES, EF, LS, and LF data. For larger networks, this approach very quickly becomes unmanageable and more mathematical approaches are necessary. Linear programming, a highly popular operations research methodology, can be used to find the optimal set of crashing decisions subject to the constraints imposed by task sequencing and time. This topic is beyond our text, but it is commonly found in introductory operations research or management science courses. ■

## REVIEW PROBLEMS

The review questions pertain to Project X, which concerns the development of controlling software for a new electric vehicle. The main activities of Project X are shown in Table 11.6.

**Table 11.6** Project X Information

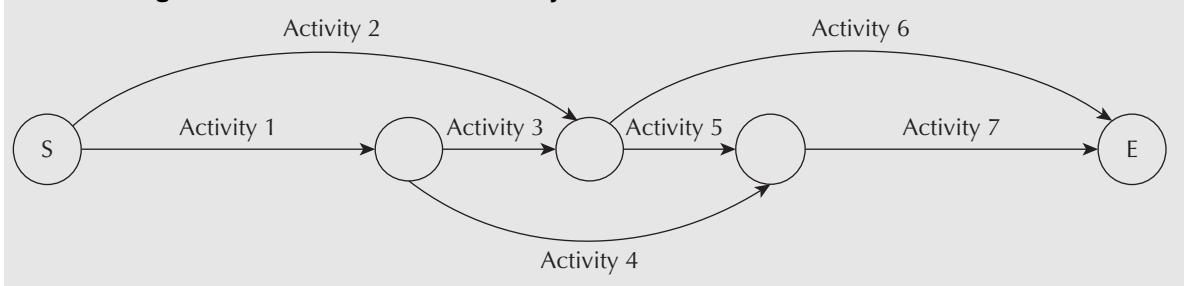
| Activity         | 1 | 2 | 3 | 4 | 5   | 6   | 7   |
|------------------|---|---|---|---|-----|-----|-----|
| Predecessor      | — | — | 1 | 1 | 2,3 | 2,3 | 4,5 |
| Duration(months) | 1 | 6 | 2 | 6 | 3   | 4   | 6   |

### REVIEW PROBLEM 11.1

Construct an AOA network representation for Project X.

**ANSWER**

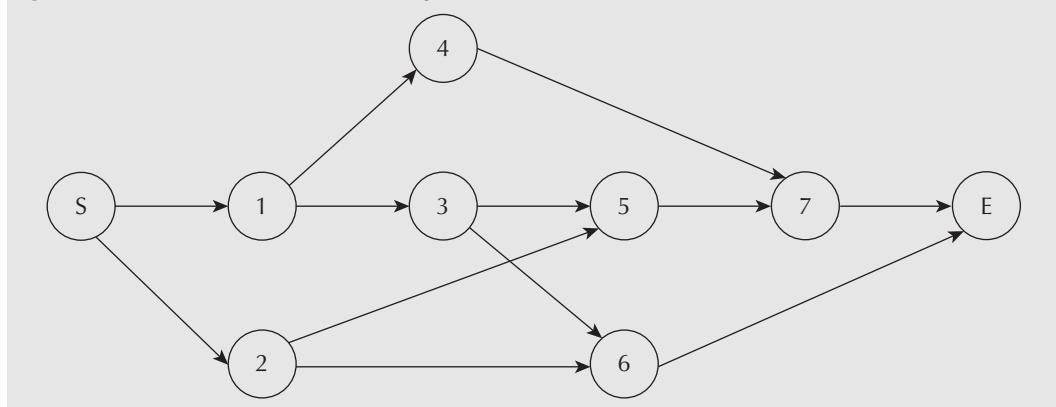
Figure 11.15 shows the required network. Activities 1 and 2 have no predecessors, so they can begin in parallel. In an AOA network, this can be denoted by two arcs, each with a start and end node. To indicate the start of the project, two dummy arcs have been introduced, which emanate from the node marked “S” (for start). Activities 3 and 4 follow Activity 1—this is represented by two arcs coming out of the node that marks the end of Activity 1. Activities 5 and 6 have two immediate predecessors, Activities 2 and 3. In order to capture this, a single node is used to represent both the conclusion of Activities 2 and 3 and the start of Activities 5 and 6. Next, Activity 7 can begin when both Activities 4 and 5 are finished. The end of Activities 4 and 5 are represented by a single node that also denotes the start of Activity 7. To denote the end of the project, a dummy activity is drawn from the end-node for Activity 7 to a node marked “E” for end.

**Figure 11.15 AOA Network for Project X****REVIEW PROBLEM 11.2**

Construct an AON network representation for Project X.

**ANSWER**

Figure 11.16 shows the required network. To denote the start and end of the project, two nodes marked “S” and “E” have been introduced. Activities 1 and 2 can begin in parallel, so two nodes, 1 and 2, are drawn with separate arcs into each. Activities 3 and 4 follow Activity 1, which is captured by drawing two nodes, 3 and 4, each with separate arcs

**Figure 11.16 AON Network for Project X**

emanating from the node representing Activity 1. Next, Activities 5 and 6 follow Activity 2, which is represented as two arcs coming out of the node for Activity 2. Activities 5 and 6 also follow Activity 3, so two arcs from Activity 3 must be drawn to connect to Activities 5 and 6. Activity 7 cannot start until both Activity 4 and Activity 5 are completed, so two arcs into the node for Activity 7 are needed—one for Activity 4 and one for Activity 5. Once Activities 6 and 7 are completed, the project is finished. To indicate this, two arcs are drawn from nodes 6 and 7 to the node “E,” which denotes the end of the project.

### REVIEW PROBLEM 11.3

- (a) Construct a Gantt chart for Project X, assuming that each activity starts at the earliest possible time.
- (b) How much can Activity 3 be delayed without delaying the entire project? How much can Activity 2 be delayed?

### ANSWER

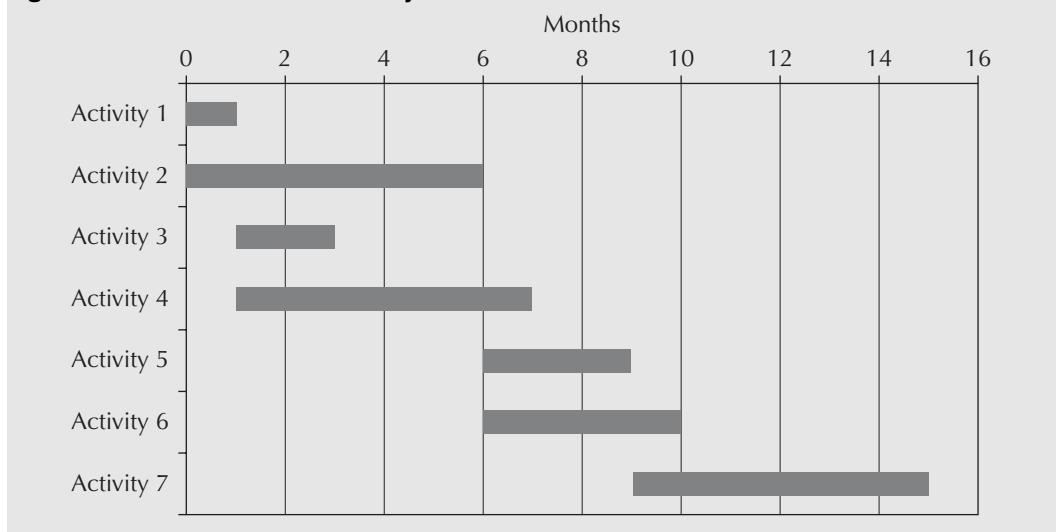
- (a) The earliest start times can be computed using the duration of each activity and the precedence constraints (see Table 11.7). From this information, the Gantt chart shown in Figure 11.17 can be constructed.

**Table 11.7 Earliest Start and Finish Times for Project X Activities**

|            | Earliest Start | Duration (months) | Earliest Finish |
|------------|----------------|-------------------|-----------------|
| Activity 1 | 0              | 1                 | 1               |
| Activity 2 | 0              | 6                 | 6               |
| Activity 3 | 1              | 2                 | 3               |
| Activity 4 | 1              | 6                 | 7               |
| Activity 5 | 6              | 3                 | 9               |
| Activity 6 | 6              | 4                 | 10              |
| Activity 7 | 9              | 6                 | 15              |

Activities 1 and 2 can begin immediately, at time “0.” Activities 3 and 4 can begin once Activity 1 is done, so they can begin in one month. Activities 5 and 6 can begin once both Activities 2 and 3 are done. Activity 2 completes in month 6, and Activity 3 in month 2. This means that Activities 5 and 6 must wait for the later time to start, which is month 6. Activity 7 must wait for both Activity 4 and Activity 5 to complete. Activity 4 will be done by month 7, and Activity 5 by month 9, so Activity 7 can begin in month 9. The overall project should take 15 months.

- (b) Activity 3 can be delayed by three months without delaying the project. This can be seen by visual inspection of the Gantt chart. If Activity 2 is delayed, however, the entire project will be delayed. This is because Activities 5 and 7, which follow Activity 2, have no slack in their schedule if the 15-month timeline is to be achieved.

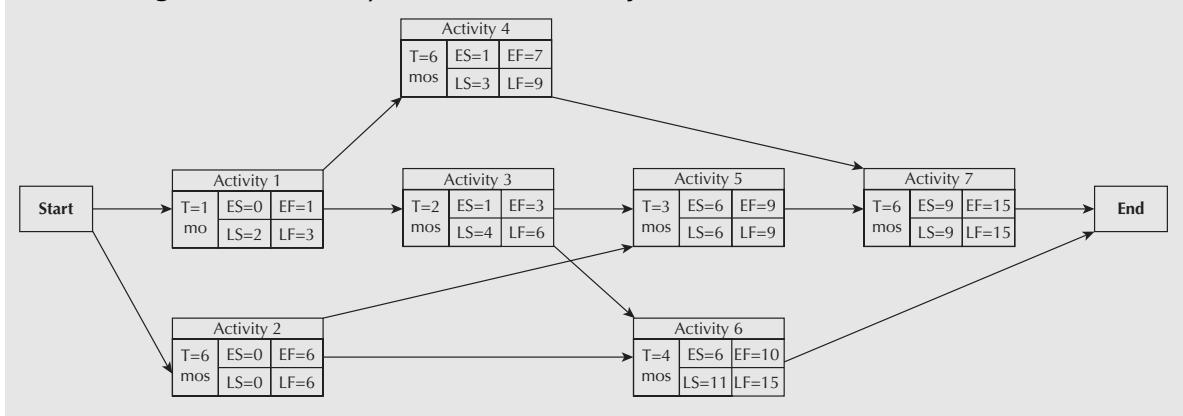
**Figure 11.17 Gantt Chart for Project X****REVIEW PROBLEM 11.4**

- (a) Compute the ES, EF, LS, and LF times for each activity using a CPM network.
- (b) How long do you expect the project to take?
- (c) What is the critical path?
- (d) Which activities have slack? How much slack does each have?

**ANSWER**

- (a) The CPM network is shown in Figure 11.18.

Starting with the AON network representation, we can compute the ES and EF times for each activity. Activities 1 and 2 can begin immediately (ES = 0 for both). Thus Activity 1 has an earliest finish time of one month, EF = 1, and Activity 2 has EF = 6. Following the completion of Activity 1, Activities 3 and 4 can begin immediately. This makes ES = 1 and EF = 3 (months) for Activity 3, and ES = 1 and EF = 7 (months) for Activity 4. Activity 5 must wait until both Activities 2 and 3 are completed, so its earliest start time is the maximum of the earliest finish times of Activities 2 and 3. This is six months (Activity 2). Therefore ES = 6 and EF = 9 for Activity 5. Similarly, ES = 6 and EF = 10 months for

**Figure 11.18 PERT/CPM Network for Project X**

Activity 6. Next, Activity 7 can begin once Activities 4 and 5 are completed. The largest of the earliest completion times for Activities 4 and 5 is nine months (Activity 5), so Activity 7 has  $ES = 9$  and  $EF = 15$ . The project concludes when both Activities 6 and 7 are done. Thus the earliest the project can be completed is in 15 months.

Working backward from the 15-month completion date, the LS and LF times can be computed. Activity 7 must be completed by month 15 and thus start by month 9 in order to make the 15-month schedule, so  $LS = 9$  and  $LF = 15$ . Activity 6 has  $LF = 15$  and therefore  $LS = 11$ . Activities 4 and 5 precede Activity 7, so in order for Activity 7 to start by  $LS = 9$ , both Activities 4 and 5 must be completed by this time. Therefore, for Activity 4,  $LF = 9$  and  $LS = 3$ . Similarly, for Activity 5,  $LF = 9$  and  $LS = 6$ . Activities 2 and 3 must both be done for Activities 5 and 6 to start, so we must consider Activities 5 and 6 in determining the LF times for Activities 2 and 3. For Activity 3, it must be completed so that neither Activity 5 nor Activity 6 are delayed. In other words, its LF time is the smallest of the LS times for Activities 5 and 6. This makes  $LF = 6$  and  $LS = 4$  for Activity 3. By the same logic,  $LF = 6$  and  $LS = 0$  for Activity 2. The LF time for Activity 1 is the smallest LS time for Activities 4 and 3— $LF = 3$  and  $LS = 2$  months for Activity 1. This concludes computation of the ES, EF, LS, and LF times for all the activities.

- (b) The project is expected to take 15 months to complete.
- (c) The critical path is Activities 2–5–7. These are the activities in the CPM diagram that have  $ES = LS$  and  $EF = LS$ .
- (d) Activity 1 has two months of slack, which is seen by  $LS - ES = LF - EF = 2$ . By similar computations, Activity 3 has three months of slack, Activity 4 has two months, and Activity 6 has five months of slack.

## SUMMARY

This chapter has provided an overview of project management lifecycle processes and an introduction to several widely used planning and scheduling tools. The work breakdown structure, the Gantt chart, and the CPM network are fundamental tools for managing a project of any size. Understanding how these tools fit into the overall lifecycle of a project is valuable in itself. These tools are also used to develop cash flow diagrams, as they indicate what work is to be done, what resources are necessary to carry out the work, and when the resources will be used. This chapter therefore helps to answer the question of how the cash flows used in an economic analysis are obtained.

## ENGINEERING ECONOMICS IN ACTION, PART 11B

### A Crash Course

"So in order to meet our deadline of next December, we have to do the product design in parallel with setting up the production process? How can we do that? Don't we have to know what the product design is before we set up production?" Bill asked.

Bill and Naomi were poring over the activity network for the Adaptamatic vacuum project. Adaptamatic wasn't necessarily the final name for the product, but the name that had started as a bit of whimsy had stuck. "We can, Bill," Naomi responded. "There are some things we don't know yet about the design that depend on the market research"—she pointed to a different arc in the network—"but that's really small stuff. We can make a quick

start on the production line long before those details are pinned down. And that takes the production line off the critical path, saving us lots of time and money!"

Bill nodded. "Okay, I get it. What about . . ."

Just then, there was a knock on Bill's office door. Clem opened it right away and stuck his head in. "Don't get too comfortable, guys. We just had a change in plans. That December deadline? Forget it. Think June! I guess the bigshots are worried that those guys over at Erie Gadgets have some sort of similar thing under study. Well, that's all I had to tell you. Work hard." Clem winked and disappeared as quickly as he had arrived.

Bill looked at Naomi. "Okay, now I really get it. This is a crash, right?"

Naomi smiled. "Right, and here's what we do . . ."

## PROBLEMS

### A. Key Concepts

**11.1** What is the purpose of a project charter? Why is it important?

**11.2** What is the purpose of project planning? Why is it important?

**11.3** Organize the following project management tasks in the order in which they would take place in a project:

- (a) Estimate costs
- (b) Track project expenses
- (c) Define deliverables
- (d) Make progress reports to stakeholders
- (e) Identify project activities
- (f) Deliverables accepted by client
- (g) Identify stakeholders
- (h) Approval of project plan
- (i) Develop schedule
- (j) Form project team
- (k) Manage schedule changes

**11.4** For each of the following activities, identify the stage in the project management process in which they would take place. If there are several possible stages, indicate your reasoning.

- (a) Project funding is approved
- (b) Paying a bill and allocating its cost to the project
- (c) Documenting an unexpected complication in the project for future reference
- (d) A discussion of whether a programming team has worked together before on a project
- (e) Looking into whether your company has done a project like this before
- (f) Identifying who should be on the project steering committee and how often it should meet

- (g) Selecting a contractor to lay optical cabling
- (h) Looking up industry standard costs for materials
- (i) A meeting to discuss the implications of late delivery for a key piece of equipment needed to complete a project activity
- (j) Establishing the project team members
- (k) Pouring concrete footings

**11.5** What is the purpose of the work breakdown structure (WBS)? Why is it a useful tool for project management?

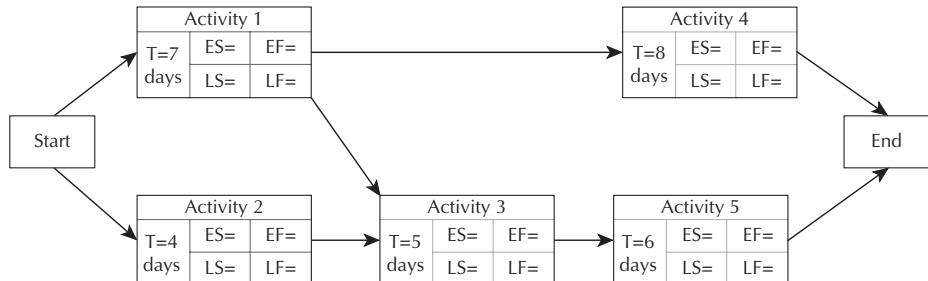
**11.6** You are in charge of organizing an alumni reunion next year and have started a WBS that has five major work groupings: food, guests, facilities, reunion event, and supervision. Organize the work packages (tasks) below into the appropriate work grouping:

|                  |                 |                    |                |              |
|------------------|-----------------|--------------------|----------------|--------------|
| Menu Planning    | Door Prizes     | Business Card Draw | Utensils       | Decorations  |
| Budget           | Purchase Orders | Caterer Selection  | Bill Payment   | Coordination |
| Invitations/RSVP | Coat Check      | Name Tags          | Seating/Tables | Guest List   |

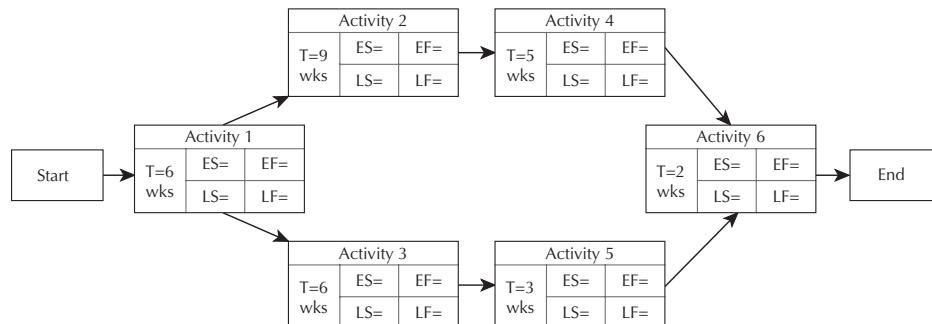
**11.7** Why is it important that the work packages in a WBS not overlap? What could happen if they do overlap?

**11.8** Think of a recent project you have worked on. Construct a first draft of a WBS for the project. Review the WBS by asking whether you have followed each of the guidelines for constructing work breakdown structures and revise the first draft accordingly, identifying what changes you made and why.

**11.9** For the project depicted below, construct a Gantt chart. How long do you expect the project to take to complete?



**11.10** For the project depicted below, construct a Gantt chart. How long do you expect the project to take to complete?



**11.11** For the project described below:

- (a) Construct a Gantt chart assuming each activity starts at the earliest time possible.

| Activity        | 1 | 2 | 3 | 4   | 5   | 6 | 7 | 8 | 9 | 10      |
|-----------------|---|---|---|-----|-----|---|---|---|---|---------|
| Predecessor     | — | — | — | 1,2 | 1,2 | 4 | 5 | 3 | 3 | 6,7,8,9 |
| Duration(weeks) | 4 | 6 | 2 | 6   | 3   | 4 | 6 | 2 | 1 | 2       |

- (b) How long will the project take to complete?

- 11.12** The project from Problem 11.11 is in week 5. You have been advised that Activity 2 is going to take one week longer than expected. Construct a revised Gantt chart for the project. Which other activities have been affected? Will the project be delayed?
- 11.13** Construct an activity on node network representation for the project described in the table below:

| Activity    | 1 | 2 | 3 | 4     | 5 |
|-------------|---|---|---|-------|---|
| Predecessor | — | — | 1 | 1,2,5 | 2 |

- 11.14** Construct an activity on node network representation for the project described in the table below:

| Activity    | 1 | 2 | 3 | 4   | 5 | 6   | 7 | 8 | 9   | 10  |
|-------------|---|---|---|-----|---|-----|---|---|-----|-----|
| Predecessor | — | — | 1 | 2,3 | 4 | 4,5 | 3 | 7 | 5,6 | 8,9 |

- 11.15** Refer to the project described below.

| Activity        | 1 | 2 | 3 | 4   | 5   | 6 | 7   | 8   | 9   |
|-----------------|---|---|---|-----|-----|---|-----|-----|-----|
| Predecessor     | — | — | 1 | 1,2 | 1,2 | 3 | 4,5 | 5,7 | 6,8 |
| Duration (days) | 3 | 2 | 4 | 4   | 2   | 4 | 8   | 3   | 1   |

- (a) Construct an AOA network representation for the project.  
 (b) Construct an AON network representation for the project.

- 11.16** Refer to Problem 11.11.

- (a) Construct an AOA network representation for the project.  
 (b) Construct an AON network representation for the project.

- 11.17** Refer to Problem 11.9.

- (a) Compute the ES, EF, LS, and LF times for the project.  
 (b) Explain how the ES for Activity 4 is obtained.  
 (c) Explain how the LF time for Activity 3 is obtained.  
 (d) What is the critical path for the project?  
 (e) What is its expected completion time?

**11.18** Refer to Problem 11.10.

- (a) Compute the ES, EF, LS, and LF times for the network.
- (b) Explain how the ES for Activity 3 is obtained.
- (c) Explain how the LF time for Activity 1 is obtained.
- (d) What is the critical path for the project?
- (e) When do you expect the project to complete?

**11.19** Refer to Problem 11.15.

- (a) Compute the ES, EF, LS, and LF times for each activity using a CPM network.
- (b) How long do you expect the project to take?
- (c) What is the critical path?
- (d) Which activities have slack? How much slack does each have?

**11.20** Refer to Problem 11.11.

- (a) Compute the ES, EF, LS, and LF times for each activity using a CPM network.
- (b) How long do you expect the project to take to complete?
- (c) What is the critical path?
- (d) Which activities have slack? How much slack does each have?

**11.21** Construct an example of a set of project activities:

- (a) Where crashing Activity 1 by three days will shorten the project completion time by three days.
- (b) Where crashing Activity 1 by three days will not shorten the project.

**11.22** Construct an example of a set of project activities:

- (a) Where crashing Activity 1 by seven weeks will reduce the slack for Activity 2 by seven weeks.
- (b) Where crashing Activity 1 by seven weeks will reduce the slack for Activity 2 by three weeks.

**11.23** You have crashing information for a project that needs to be finished one week earlier than it is currently expected to complete.

| Activity | Normal Duration (weeks) | Crashed Duration (weeks) | Normal Cost (\$000s) | Crash Cost (\$000s) | Slack (weeks) |
|----------|-------------------------|--------------------------|----------------------|---------------------|---------------|
| 1        | 4                       | 2                        | 50                   | 30                  | 1             |
| 2        | 7                       | 4                        | 70                   | 25                  | 0             |
| 3        | 12                      | 8                        | 120                  | 100                 | 2             |
| 4        | 5                       | 4                        | 40                   | 20                  | 0             |

- (a) Which activity should be crashed?
- (b) What will be the additional cost to the project?

**11.24** For the project below:

| Activity | Predecessor | Normal Duration (days) | Crashed Duration (days) | Normal Cost (\$000s) | Crash Cost (\$000s) |
|----------|-------------|------------------------|-------------------------|----------------------|---------------------|
| 1        | —           | 6                      | 4                       | 8                    | 14                  |
| 2        | 1           | 1                      | 1                       | 4                    | 4                   |
| 3        | —           | 8                      | 4                       | 8                    | 24                  |
| 4        | 3           | 5                      | 3                       | 10                   | 24                  |
| 5        | 2,3         | 9                      | 5                       | 10                   | 18                  |
| 6        | 4,5         | 12                     | 6                       | 20                   | 36                  |
| 7        | 6           | 3                      | 2                       | 10                   | 18                  |

- (a) Construct a CPM network representation
- (b) Determine the ES, EF, LS, and LF times for the project.
- (c) What is the critical path?
- (d) How long will the project take to complete?
- (e) What is the normal cost of the project?
- (f) What activity or activities should you crash to reduce the duration of the project by one day?

## B. Applications

- 11.25** (a) What is the purpose of project monitoring and controlling? Why are these activities important?  
 (b) What is the purpose of project closure? Why is it important?
- 11.26** Search the internet to find examples of project management stages specific to three different engineering application areas (e.g., construction, software development, process improvement, or other types of engineering projects).
  - (a) For each example, classify the project stages into initiation, planning, execution, monitoring and controlling, and closure.
  - (b) Give three examples of resources needed for each type of project.
  - (c) Provide two examples of risks that might cause each project to be delayed or terminated. Discuss briefly how you might reduce the likelihood or impact of each of the risks.
- 11.27** Consider the project of moving from your family home into residence to attend university. Write a brief description outlining the activities involved at each stage of this project.
- 11.28** Consider the project of preparing for a one-month trip to a set of destinations of your choice. Write a brief description outlining the activities involved at each stage of this project.

- 11.29** Construct a WBS for Problem 11.27. Estimate the time required for each work package.  
**11.30** Construct a Gantt chart for the project.

| Activity        | 1 | 2 | 3 | 4   | 5   | 6 | 7   | 8   | 9   |
|-----------------|---|---|---|-----|-----|---|-----|-----|-----|
| Predecessor     | – | – | 1 | 1,2 | 1,2 | 3 | 4,5 | 5,7 | 6,8 |
| Duration (days) | 3 | 2 | 4 | 4   | 2   | 4 | 8   | 3   | 1   |

- (b)** How long do you expect the project to take to complete?  
**(c)** If Activity 6 is delayed by three days, will the project be delayed?
- 11.31** Below is a set of activities involved with redesigning a product and its packaging.

| Activity                                   | Duration (weeks) | Predecessor(s) |
|--|------------------|----------------|
| 1. Redesign product                        | 6                | –              |
| 2. Design new packaging                    | 2                | –              |
| 3. Obtain materials for redesigned product | 3                | 1              |
| 4. Obtain materials for new packaging      | 2                | 2              |
| 5. Make redesigned product                 | 4                | 3              |
| 6. Make new packaging                      | 1                | 4              |
| 7. Package redesigned product              | 1                | 5,6            |
| 8. Test-market redesigned product          | 6                | 7              |
| 9. Revise redesigned product               | 3                | 8              |
| 10. Revise new packaging                   | 1                | 8              |
| 11. Launch revised product and packaging   | 1                | 9,10           |

- (a)** Construct a CPM network for the project.  
**(b)** Compute the ES, EF, LS, and LF times for each activity.  
**(c)** What is the critical path for the project? How long will it take to complete?
- 11.32** Refer to Problem 11.31. You have been asked to investigate the feasibility of shortening the project by two weeks and have assembled some information about crashing times and crashing costs.
- (a)** What activities should be crashed, and by how much, in order to achieve a reduction of two weeks in the completion time of the project? What will be the increase in the project cost?

| Activity                                   | Normal Duration (weeks) | Shortest Duration (weeks) | Cost to Attain Shortest Duration (\$000s) |
|--|-------------------------|---------------------------|---|
| 1. Redesign product                        | 6                       | 4                         | 30  |
| 2. Design new packaging                    | 2                       | 1                         | 12.5                                      |
| 3. Obtain materials for redesigned product | 3                       | 2                         | 8   |
| 4. Obtain materials for new packaging      | 2                       | 2.5                       | 7   |
| 5. Make redesigned product                 | 4                       | 1.5                       | 13  |
| 6. Make new packaging                      | 1                       | 0.5                       | 6   |
| 7. Package redesigned product              | 1                       | 0.5                       | 6   |
| 8. Test-market redesigned product          | 6                       | 4                         | 22  |
| 9. Revise redesigned product               | 3                       | 2                         | 15  |
| 10. Revise new packaging                   | 1                       | 0.5                       | 7.25                                      |
| 11. Launch revised product and packaging   | 1                       | 1                         | N/A                                       |

**11.33** Refer back to Problem 11.24.

- (a) What activities would you crash in order to shorten the project duration by two days? What is the revised project cost?
- (b) What activities would you crash in order to shorten the project duration by three days? What is the revised project cost?

### C. More Challenging Problems

**11.34** For the projects identified in Problem 11.26, identify two different types of events that could put each project at risk. For each risk, suggest means by which the likelihood or impact of the risk can be mitigated.

Problems 11.35 and 11.36 pertain to the project below for building a house.

| Activity | Description       | Predecessors | Duration (days) |
|----------|-------------------|--------------|-----------------|
| 1        | Excavation        | —            | 7               |
| 2        | Foundation        | 1            | 6               |
| 3        | Frame, roof       | 2            | 14              |
| 4        | Order cabinets    | —            | 32              |
| 5        | Electrical wiring | 3            | 10              |
| 6        | Plumbing          | 3            | 9               |

|    |   |       |    |
|----|---|-------|----|
| 7  | Windows                                 | 3     | 9  |
| 8  | Ductwork (heating/cooling)              | 3     | 5  |
| 9  | Roofing, gutters, downspouts            | 3     | 7  |
| 10 | Furnace/AC                              | 8     | 4  |
| 11 | Insulation                              | 5,6,7 | 3  |
| 12 | Exterior finish                         | 7,9   | 11 |
| 13 | Wallboard, tape, and mud                | 11    | 10 |
| 14 | Driveway, walks, landscaping            | 12    | 6  |
| 15 | Cabinets (kitchen and bath), appliances | 4,13  | 5  |
| 16 | Painting/flooring/trim/fixtures         | 15    | 14 |

**11.35** Construct a Gantt chart for the project. How long is the project expected to take?

**11.36 (a)** Construct an AON CPM network for this project.

**(b)** How long will the house take to construct?

**(c)** What is the critical path?

**(d)** You have just heard news that the window supplier has gone on strike. The windows were supposed to be ready by day 27 of the project, but will probably be delayed by 21 days. How will this change the expected completion date of the house?

Problems 11.37, 11.38, and 11.39 pertain to the following software development project:

| Activity                       | Duration (days) | Predecessors |
|--------------------------------|-----------------|--------------|
| 1. High level analysis         | 7               |              |
| 2. Hardware selection          | 2               | 1            |
| 3. Hardware installation       | 14              | 2            |
| 4. Design of core modules      | 14              | 1            |
| 5. Design of auxiliary modules | 14              | 4            |
| 6. Program core modules        | 9               | 4            |
| 7. Program auxiliary modules   | 21              | 5            |
| 8. QA of core modules          | 7               | 6            |
| 9. QA of auxiliary modules     | 7               | 7            |
| 10. Core module training       | 1               | 3,8          |
| 11. Develop accounting reports | 7               | 5            |
| 12. Develop management reports | 7               | 5            |
| 13. Develop query interface    | 7               | 11,12        |
| 14. Develop query interface    | 7               | 9,10,13      |

- 11.37** Develop a Gantt chart for the project. What is the expected project completion time?
- 11.38 (a)** Develop a network representation for this project.
- (b)** What is the critical path? How long do you expect the project to take?
- 11.39** You have been advised that the quality assurance specialist assigned to this project can only work on one major activity at a time. How would you change the network diagram to reflect this change? Does it affect the critical path? How does it affect the rest of the project activities?
- 11.40** Canadian Widgets has recently developed plans to launch a new item in its product line. The activities associated with the launch of the new product are outlined in the table below.

| Activity                      | Duration (days) | Predecessors |
|-------------------------------|-----------------|--------------|
| 1. Finalize new product plans | 4               | —            |
| 2. Complete drawings          | 12              | 1            |
| 3. Write specifications       | 8               | 1            |
| 4. Market assessment          | 9               | 1            |
| 5. Develop prototype          | 17              | 2,3          |
| 6. Develop marketing strategy | 12              | 4            |
| 7. Purchase raw materials     | 14              | 5            |
| 8. Set up equipment           | 10              | 5            |
| 9. Produce test batch         | 7               | 7,8          |
| 10. Launch marketing campaign | 1               | 6            |
| 11. Evaluate test batch       | 7               | 9            |
| 12. Test-market feedback      | 21              | 9            |
| 13. Engineering changes       | 7               | 11,12        |
| 14. Full-scale production     | 3               | 6,13         |

- (a)** Construct a Gantt chart for the purposes of scheduling the earliest time each activity can start.
- (b)** Construct a CPM network for the project. How long do you expect it to take to begin full-scale production?
- (c)** What are the critical path activities?
- 11.41** Refer to Problem 11.32. What is the minimum time in which the project can be completed? What is the crashing cost?

## MINI-CASE 11.1

### No Longer LEEDing

On March 30, 2011, the *Nanaimo Daily News* reported that the Nanaimo City Council had changed the existing environmental and energy standards for public buildings in the city. In 2006, the city had adopted a policy requiring any building of 500 square metres or more to meet the Leadership in Energy and Environmental Design (LEED) standard of silver or better. However, the new rules require this standard only for buildings of 900 square metres or more. Moreover, for a building of any size, City Council can now choose to opt out of the requirement altogether.

The reason given in the article was that the certification process adds as much as 20 percent to the design and project management of a building. “We’re going to continue to build to LEED standards, and preferably gold, which is the policy of the city,” said Councillor Fred Pattje. “But only in certain circumstances of large buildings will we apply for certification because it can be a very expensive undertaking.”

Additional information on this issue can be found at [www.bclocalnews.com/vancouver\\_island\\_central/nanaimonewsbulletin/news/118826404.html](http://www.bclocalnews.com/vancouver_island_central/nanaimonewsbulletin/news/118826404.html).

### Discussion

The Leadership in Energy and Environmental Design (LEED) certification process was developed in 2002 by the U.S. Green Building Council ([www.usgbc.org](http://www.usgbc.org)) as a way to ensure that the highest level of environmental design and engineering is used when a building is constructed. It is now recognized as the international standard for environmentally responsible building construction. The key element of LEED is a set of technical criteria that have been developed in an open and transparent manner, with more than 20 000 professionals contributing to the process through a system of committees representing a wide range of interest groups.

If a building is LEED Certified, it means that the building has conformed to the recommended technical criteria to a sufficient degree. This is determined by providing documentation to the appropriate organization—in Canada, this is the Canada Green Building Council ([www.cagbc.org](http://www.cagbc.org)). The application is assessed through a system of points (out of 100) that are assigned for adherence to principles such as energy savings, water efficiency, CO<sub>2</sub> emissions, indoor environmental quality, and stewardship of resources. There are several levels of certification, depending on the number of points accumulated, with the most thorough one being the “gold” standard with 80 points or more.

In recent years, LEED certification has also been an important contributor to commercial branding and political respectability. On the commercial side, society values environmental responsibility, and, to a certain degree, is willing to pay for it. For example, the Toyota Prius is a popular car even though the fuel savings do not justify the higher purchase price of the vehicle. Consequently, a company can either reinforce a “green” brand or offset the negative aspects of a polluting or wasteful product by publicly embracing LEED certification in new construction. Similarly, politicians can demonstrate their social awareness and potentially avoid controversy from politically savvy environmentalists by requiring LEED certification for new public buildings.

### Questions

1. For politicians to backtrack on an issue such as environmental responsibility is obviously very unusual. Was it just the money, or do you think there was more to it?

2. The LEED standards are developed with a clear idea of the tradeoffs a company or government must make between responsible financial management and environmental stewardship. Nanaimo is a particularly green city in a particularly green country. What went wrong?
3. What proportion of the overall cost of a project should be spent on project management? Why?
4. Project management and green practices save money because they use resources more effectively. Why would LEED certification add costs, rather than reduce them?



# Dealing With Uncertainty and Risk

**Engineering Economics in Action, Part 12A:** Trees from Another Planet

- 12.1** Introduction
- 12.2** Sensitivity Graphs
- 12.3** Break-Even Analysis
  - 12.3.1** Break-Even Analysis for a Single Project
  - 12.3.2** Break-Even Analysis for Multiple Projects
- 12.4** Basic Concepts of Probability
- 12.5** Structuring Decisions With Decision Trees

## Review Problems

## Summary

**Engineering Economics in Action, Part 12B:** Chances Are Good

## Problems

### Mini-Case 12.1: China Steel Australia Limited

### Appendix 12A: Decision Matrices Problems

**Engineering Economics in Action, Part 12C:** Moving On

## ENGINEERING ECONOMICS IN ACTION, PART 12A

### Trees from Another Planet

Bill Astad and Naomi were working through the market demand figures provided by Powerluxe for the new Adaptamatic vacuum. They had to determine if the project was really going to be viable and make money for Canadian Widgets.

"These figures are pretty ambiguous," Bill said. "We have three approaches: a set of opinions taken from focus groups and surveys of customers, the same thing from dealers and distributors, and an analysis of trends in a set of parallel products such as fuzzy-logic appliances." Marketing was Bill's area of expertise, so Naomi was listening carefully.

"What we should do," Bill continued, "is come up with three scenarios: low demand, expected demand, and high demand. If we behave according to expected demand and the true demand is low, we will lose money because our capital investments won't be recouped as fast and we may have passed up other opportunities. Similarly, if the demand is high, we will lose by having to pay overtime, paying for contracting out, or dropping customers. But if we make money in all three cases, there really isn't much of a problem."

Naomi nodded. "But we also might have the Erie Gadget competitive product. That's going to mess things up too."

"Yeah, that's right," Bill replied, a bit gloomily. "We've got to get this sorted out soon—this is on the critical path!" Bill was showing off with this last statement, so Naomi gave him the smile he expected, but his point was real. They had to figure out quickly if there was a real opportunity for Canadian Widgets. Everything else was waiting for this.

"I think we can do this, Bill. The key word is *trees*."

Bill looked at Naomi as if she were from another planet. "Trees! Trees?"

## 12.1 | Introduction

To this point in our coverage of engineering economics, we have assumed that project parameters such as prices, interest rates, and the magnitude and timing of cash flows have values that are known with certainty. In fact, many of these values are estimates and are subject to some uncertainty. Since the results of an evaluation can be influenced by variations in uncertain parameters, it is important to know how *sensitive* the outcome is to variations in these parameters.

There are several reasons why there may be uncertainty in estimating project parameter values. Technological change can unexpectedly shorten the life of a product or piece of equipment. A change in the number of competing firms may affect sales volume or market share or the life of a product. In addition, the general economic environment may affect inflation and interest rates and overall activity levels within an industry. All these factors may result in cash flows different from what was expected in both timing and size, or in other changes to the parameters of an evaluation.

Making decisions under uncertainty is challenging because the overall impact of uncertainty on project evaluation may initially not be well understood. **Sensitivity analysis** is an approach to project evaluation that can be used to gain a better understanding of how uncertainty affects the outcome of the evaluation by examining how sensitive the outcome is to changes in the uncertain parameters. It can help an engineer decide whether it is worthwhile to get more accurate data as part of a more detailed evaluation, or whether it may be necessary to control or limit project uncertainties. Sometimes we

have enough information to characterize uncertain outcomes according to their chance or likelihood. This converts a problem from one of uncertainty to one of *risk*.

Decisions made under risk are those where the analyst can characterize a possible range of future outcomes and has available an estimate of the probability of each outcome. The term *risk* is also often used to refer to the probability distribution of outcomes associated with a project, or the probability of an undesirable outcome (this last definition of risk is not utilized in this text, although it is commonly used in the financial literature). Knowledge of the probability distribution of outcomes often permits us to draw more authoritative conclusions than we can draw using sensitivity analysis alone.

Economic analyses are not complete unless we try to assess the potential effects of project uncertainties on the outcomes of the evaluations. Because parameter estimates can be so hard to determine, analysts usually consider a range of possible values for uncertain components of a project. There is then naturally a range of values for present worth, annual worth, or whatever the relevant performance measure is. In this way, the analyst can get a better understanding of the range of possible outcomes and can make better decisions.

In this chapter, we will first look at the use of *sensitivity graphs*. Sensitivity graphs illustrate the sensitivity of a particular measure (e.g., present worth or annual worth) to one-at-a-time changes in the uncertain parameters of a project. Sensitivity graphs can reveal key parameters that have a significant impact on the performance measures of interest, and hence we should be particularly careful to get good estimates for these key parameters.

Next, in order to understand risk, we review some of the basic principles of probability. This allows us to introduce *decision trees*. Decision trees allow probabilistic information about the outcomes of a decision to be structured so that the best choice can be made.

Sensitivity graphs and decision trees are tools that allow us to enrich an economic analysis to help deal with the uncertainties and risks of the real world. However, they are simply two of a large number of methods, each with its own strengths and weaknesses. A good economic analysis should include a method of dealing with uncertainty and risk that is appropriate to the specifics of that particular problem.

## 12.2 | Sensitivity Graphs

Sensitivity graphs are used to assess the effect of one-at-a-time changes in key parameter values of a project on an economic performance measure. We usually begin with a “base case” where all the estimated parameter values are used to evaluate the present worth, annual worth, or IRR of a project, whatever the appropriate measure is. We then vary parameters above and below the base case one at a time, *holding all other parameters fixed*. A graph of the changes in a performance measure brought about by these one-at-a-time parameter changes is called a **sensitivity graph**. From the graph, the analyst can see which parameters have a significant impact on the performance measure and which do not.

### EXAMPLE

### 12.1

Cogenesis Corporation is replacing its current steam plant with a six-megawatt cogeneration plant that will produce both steam and electric power for operations. The new plant will use wood as a source of fuel, which will eliminate the need for Cogenesis to purchase a large amount of electric power from a public utility. To move to the new system, Cogenesis will have to integrate a new turbogenerator and cooling tower with

its current system. The estimated first cost of the equipment and installation is \$3 000 000, though there is some uncertainty surrounding this estimate. The plant is expected to have a 20-year life and no scrap value at the end of this life. In addition to the first cost, the turbogenerator will require an overhaul with an estimated cost of \$35 000 at the end of years 4, 8, 12, and 16. The cooling tower will need an overhaul at the end of 10 years. This is expected to cost \$17 000.

The cogeneration system is expected to have higher annual operating and maintenance costs than the current system, and will require the use of chemicals to treat the water used in the new plant. These incremental costs are estimated to be \$65 000 per year. The incremental annual costs of wood fuel are estimated to be \$375 000. The cogeneration plant will save Cogenesis from having to purchase 40 000 000 kilowatt-hours of electricity per year at \$0.025 per kilowatt-hour, an annual savings of \$1 000 000. Cogenesis uses a MARR of 12 percent. What is the present worth of the incremental investment in the cogeneration plant? What is the impact of a 5 percent and 10 percent increase and decrease in each of the parameters of the problem?

PW(cogeneration plant)

$$\begin{aligned}
 &= -3 000 000 - (65 000 + 375 000 - 1 000 000) (P/A, 12\%, 20) \\
 &\quad - 17 000 (P/F, 12\%, 10) \\
 &\quad - 35 000 [(P/F, 12\%, 4) + (P/F, 12\%, 8) + (P/F, 12\%, 12) \\
 &\quad + (P/F, 12\%, 16)] \\
 &= 1 126 343
 \end{aligned}$$

The present worth of the incremental investment is \$1 126 343. On the basis of this assessment, the project appears to be economically viable.

In order to better understand the situation, analysts for Cogenesis have also completed some sensitivity graphs that indicate how sensitive the present worth is to changes in some of the parameters. In particular, they feel that some of the cash flows may turn out to be different from their estimates, and they would like to get a feel for what impact these errors may have on the evaluation of the cogeneration plant. To investigate, they have labelled their current estimates the “base case” and have generated other cash flow estimates that are 5 percent and 10 percent above and below the base case for each major cash flow category. These are summarized in Table 12.1.

For example, the initial investment may be more than the estimate of \$3 000 000 if the company runs into unforeseen difficulties in the installation. Or the savings in electricity costs may be overestimated if the cost per kilowatt-hour drops in the future. The analysts would like to get a better understanding of which of these changes would have the greatest impact on the evaluation of the plant.

To keep the illustration simple, we will consider changes to the initial investment; annual chemical, operations, and maintenance costs; the MARR; and the savings in electrical costs. Each of these is varied one at a time, leaving all other cash flow estimates at the base case values. For example, if the initial investment is 10 percent below the initial estimate of \$3 000 000, and all other estimates are as in the base case, the present worth of the project will be \$1 426 343 (see the first row of Table 12.2, under -10 percent). Similarly, if the first cost is 10 percent more than the original estimate, the present worth drops to \$826 343.

Interest rate uncertainty will almost always be present in an economic analysis. If Cogenesis’s MARR increases by 10 percent (with all other parameters at their base case values), the present worth of the project drops to \$835 115, about the same impact as if the first cost ended up being 10 percent more than expected. Other variations are shown in

**Table 12.1 Summary Data for Example 12.1**

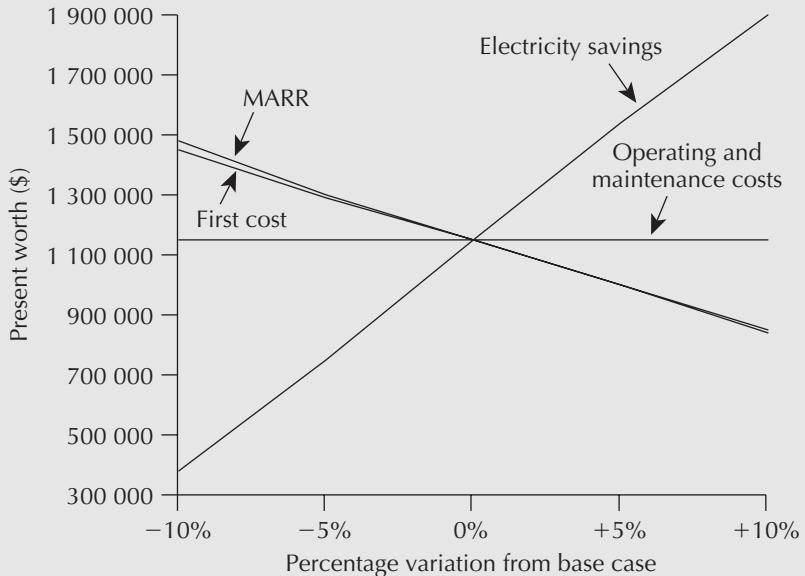
| Cost Category  | -10%        | -5%         | Base Case   | +5%         | +10%        |
|--|-------------|-------------|-------------|-------------|-------------|
| Initial investment   | \$2 700 000 | \$2 850 000 | \$3 000 000 | \$3 150 000 | \$3 300 000 |
| Annual chemical,<br>operations, and<br>maintenance costs         | 58 500      | 61 750      | 65 000      | 68 250      | 71 500      |
| Cooling tower<br>overhaul<br>(after 10 years)                    | 15 300      | 16 150      | 17 000      | 17 850      | 18 700      |
| Turbogenerator<br>overhauls<br>(after 4, 8, 12,<br>and 16 years) | 31 500      | 33 250      | 35 000      | 36 750      | 38 500      |
| Annual wood costs  | 337 500     | 356 250     | 375 000     | 393 750     | 412 500     |
| Annual savings<br>in electricity costs                           | 900 000     | 950 000     | 1 000 000   | 1 050 000   | 1 100 000   |
| MARR   | 0.108       | 0.114       | 0.12        | 0.126       | 0.132       |

Table 12.2. A sensitivity graph, shown in Figure 12.1, illustrates the impact of one-at-a-time parameter variations on the present worth.

Small changes in the annual chemical, operations, and maintenance costs do not have much of an impact on the present worth of the project, as can be seen from Table 12.2 and Figure 12.1. What appears to have the greatest impact on the viability of the project is the savings in electricity costs. A 10 percent drop in the savings causes the present worth of the project to drop to about one-third of the base case estimate. This change could occur because of a drop in electricity rates or a drop in demand.

**Table 12.2 Present Worth of Variations from Base Case in Example 12.1**

| Cost Category  | -10%        | -5%         | Base Case   | +5%        | +10%       |
|--|-------------|-------------|-------------|------------|------------|
| Initial investment   | \$1 426 343 | \$1 276 343 | \$1 126 343 | \$ 976 343 | \$ 826 343 |
| Annual chemical,<br>operations, and<br>maintenance costs         | 1 174 894   | 1 150 619   | 1 126 343   | 1 102 067  | 1 077 792  |
| Cooling tower<br>overhaul<br>(after 10 years)                    | 1 126 890   | 1 126 617   | 1 126 343   | 1 126 069  | 1 125 796  |
| Turbogenerator<br>overhauls<br>(after 4, 8, 12,<br>and 16 years) | 1 131 450   | 1 128 897   | 1 126 343   | 1 123 789  | 1 121 236  |
| Annual wood costs  | 1 406 447   | 1 266 395   | 1 126 343   | 986 291    | 846 239    |
| Savings in<br>electricity costs                                  | 379 399     | 752 871     | 1 126 343   | 1 499 815  | 1 873 287  |
| MARR   | 1 456 693   | 1 286 224   | 1 126 343   | 976 224    | 835 115    |

**Figure 12.1 Sensitivity Graph for Example 12.1**

Alternatively, the present worth of the project increases to almost \$1 900 000 if the savings are higher than anticipated. This could, once again, occur because of a change in either rates or demand for power. Clearly, if Cogenesis is to expend effort in getting better forecasts, it should be for energy consumption and power rates.

One final point about this example should be noted. If management feels that, individually, the cash flow estimates will fall within the  $\pm 10$  percent range, the investment looks economically viable (i.e., yields a positive present worth) and the company should go ahead with it. ■

As we can see from Example 12.1, the benefit of a sensitivity graph is that it can be used to select key parameters in an economic analysis. It is easy to understand and communicates a lot of information in a single diagram. There are, however, several shortcomings of sensitivity graphs. First, they are valid only over the range of parameter values in the graph. The impact of parameter variations outside the range considered may not be simply a linear extrapolation of the lines in the graph. If you need to assess the impact of greater variations, the computations should be redone. Second, and probably the greatest drawback of sensitivity graphs, is that they do not consider the possible interaction between two or more parameters. You cannot simply "add up" the impacts of individual changes when several parameters are varied, producing an interaction effect.

## 12.3 | Break-Even Analysis

In this section, we cover a second type of sensitivity analysis called break-even analysis. Once again, we are trying to answer the question of what impact changes (or errors) in parameter estimates will have on the economic performance measures we use in our analyses, or on a decision made on the basis of an economic performance measure. In general, **break-even analysis** is the process of varying a parameter of a problem and determining what parameter value causes the performance measure to reach some threshold or "break-even" value. In Example 12.1, we saw that an increase in the MARR

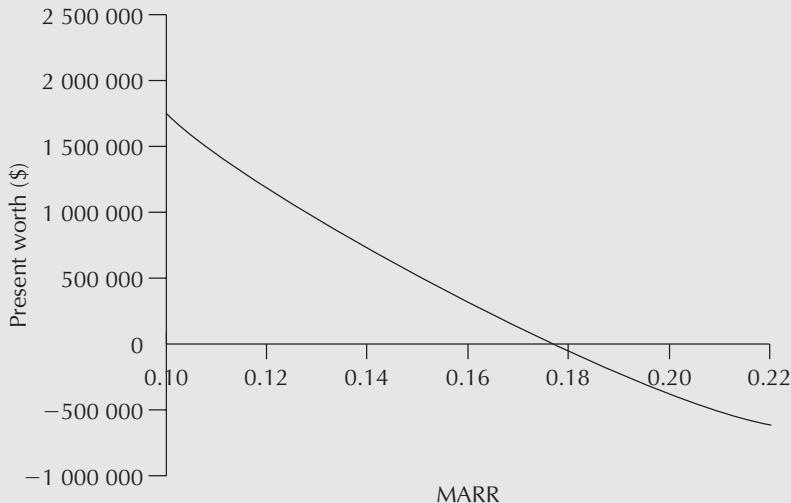
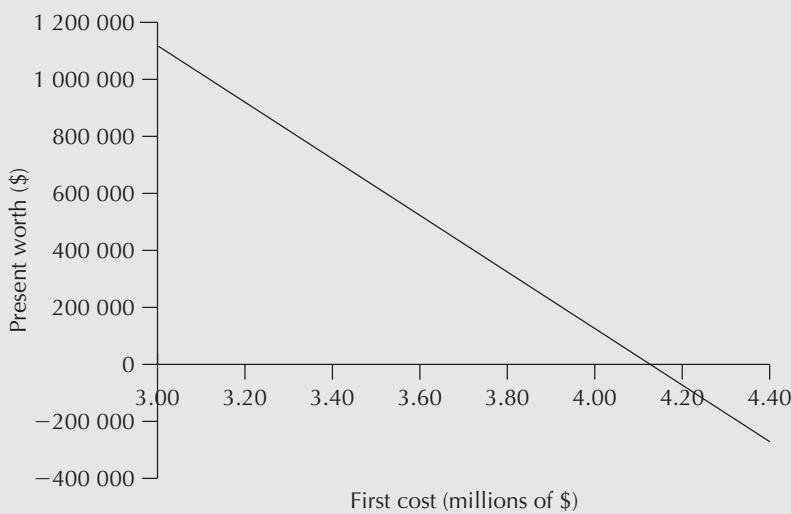
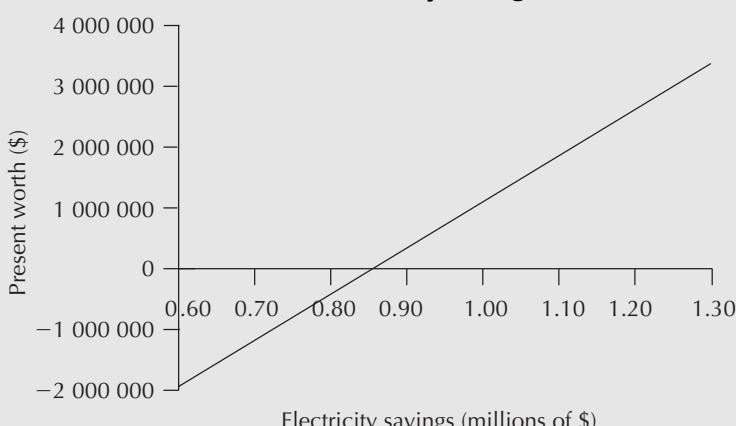
caused the present worth of the cogeneration plant to decrease. If the MARR were to increase sufficiently, the project might have a zero present worth. A break-even analysis could answer the question “What MARR will result in a zero present worth?” This analysis would be particularly useful if Cogenesis were uncertain about the MARR and wanted to find a threshold MARR above which the project would not be viable. Other such break-even questions could be posed for the cogeneration problem to try to get a better understanding of the impact of changes in parameter values on the economic analysis.

Break-even analysis can also be used in the comparison of two or more projects. We have already seen in Chapter 4 that the best choice among mutually exclusive alternatives may depend on the interest rate, the production level, or a variety of other problem parameters. Break-even analysis applied to multiple projects can answer questions like “Over what range of interest rates is project A the best choice?” or “For what output level are we indifferent between two projects?” Notice that we are varying a single parameter in two or more projects and asking when the performance measure for the projects meets some threshold or break-even point. The point of doing this analysis is to try to get a better understanding of how sensitive a decision is to changes in the parameters of the problem.

### 12.3.1 Break-Even Analysis for a Single Project

In this section, we show how break-even analysis can be applied to a single project to illustrate how sensitive a project evaluation is to changes in project parameters. We will continue with Example 12.1 to expand upon the information provided by the sensitivity graphs.

| EXAMPLE   | 12.2 |
|---|------|
| <p>Having completed the sensitivity graph in Example 12.1, management recognizes that the present worth of the cogeneration plant is quite sensitive to the savings in electricity costs, the MARR, and the initial costs. Since there is some uncertainty about these estimates, the company wants to explore further the impact of changes in these parameters on the viability of the project. You are to carry out a break-even analysis for each of these parameters to find out what range of values results in a viable project (i.e., <math>PW &gt; 0</math>) and to determine the “break-even” parameter values that make the present worth of the project zero. You are also to construct a graph to illustrate the present worth of the project as a function of each parameter.</p> <p>First, Figure 12.2 shows the present worth of the project as a function of the MARR. It shows that the break-even MARR is 17.73 percent. In other words, the project has a positive present worth for any MARR less than 17.73 percent (all other parameters fixed) and a negative present worth for a MARR more than 17.73 percent. Notice that the break-even interest rate is, in fact, the IRR for the project.</p> <p>A similar break-even chart for the first cost, Figure 12.3, shows that the first cost can be as high as \$4 126 350 before the present worth declines to zero. Assuming that all other cost estimates are accurate, the project will be viable as long as the first cost is below this break-even amount. One issue management should assess is the likelihood that the first cost will exceed \$4 126 350.</p> <p>Finally, a break-even chart for the savings in electrical power costs is shown in Figure 12.4. We have already seen from the sensitivity graph that the viability of the project is very sensitive to the savings in electricity produced by the cogeneration plant.</p> |      |

**Figure 12.2 Break-Even Chart for the MARR****Figure 12.3 Break-Even Chart for First Cost****Figure 12.4 Break-Even Chart for Electricity Savings**

Provided that the annual savings are above \$849 207, the project is viable. Below this break-even level, the present worth of the project is negative. If the actual saving in electrical power costs is likely to be much below the estimate, this will put the project's viability at risk. Given the particular sensitivity of the present worth to the savings, it may be worthwhile to spend additional time looking into the two factors that make up these savings: the cost per kilowatt-hour and the total kilowatt-hours of demand provided for by the new plant. ■

Break-even analysis done for a single project expands upon the information that sensitivity graphs provide. It has the advantage that it is easy to apply and allows us to determine the range of values for a parameter within which the project is viable or some other criteria are met. It can provide us with break-even parameter values that give an indication of how much a parameter can change from its original estimate before the project's viability becomes a concern. Graphical presentation of the break-even analysis, as in Figures 12.2, 12.3, and 12.4, summarizes the information in an easily understood way.

### 12.3.2 Break-Even Analysis for Multiple Projects

In the previous section, we saw how break-even analysis can be applied to a single project in order to understand more clearly the impact of changes in parameter values on the evaluation of the project. This analysis may influence a decision on whether the project should be undertaken. When there is a choice among several projects, be they independent or mutually exclusive, the basic question remains the same. We are concerned with the impact that changes in problem parameters have on the relevant economic performance measure, and, ultimately, on the decision made with respect to the projects. With one project, we are concerned with whether the project should be undertaken and how changes in parameter values affect this decision. With multiple projects, we are concerned about how changes in parameter values affect which project or projects are chosen.

For multiple independent projects, assuming that there are sufficient funds to finance all projects, break-even analysis can be carried out on each project independently, as was done for a single project in the previous section. This will lead to insights into how robust a decision is under changes in the parameters.

For mutually exclusive projects, the best choice will seldom stand out as clearly superior from all points of view. Even if we have narrowed down the choices, it is still likely that the best choice may depend on a particular interest rate, level of output, or first cost. A break-even comparison can reveal the range over which each alternative is preferred and can show the break-even points where we are indifferent between two projects. Break-even analysis will provide a decision maker with further information about each of the projects and how they relate to one another when parameters change.

#### EXAMPLE

#### 12.3

Westmount Waxworks (see Problem 4.26 on page 115) is considering buying a new wax melter for its line of replicas of statues of government leaders. Westmount has two choices of suppliers: Finedetail and Simplicity. The proposals are as follows:

|               | Finedetail Wax Melter       | Simplicity Wax Melter       |
|---------------|-----------------------------|-----------------------------|
| Expected life | 7 years                     | 10 years                    |
| First cost    | \$200 000                   | \$350 000                   |
| Maintenance   | \$10 000/year + \$0.05/unit | \$20 000/year + \$0.01/unit |
| Labour        | \$1.25/unit                 | \$0.50/unit                 |
| Other costs   | \$6 500/year + \$0.95/unit  | \$15 500/year + \$0.55/unit |
| Salvage value | \$5 000                     | \$20 000                    |

The marketing manager has indicated that sales have averaged 50 000 units per year over the past five years. In addition to this information, management thinks that it will sell about 30 000 replicas per year if there is stability in world governments. If the world becomes very unsettled so that there are frequent overturns of governments, sales may be as high as 200 000 units per year. There is also some uncertainty about the “other costs” of the Simplicity wax melter. These include energy costs and an allowance for scrap. Though the costs are estimated to be \$0.55 per unit, the Simplicity model is a new technology, and the costs may be as low as \$0.45 per unit or as high as \$0.75 per unit. Westmount Waxworks would like to carry out a break-even analysis on the sales volume and on the “other costs” of the Simplicity wax melter. It wants to know which the preferred supplier would be as sales vary from 30 000 per year to 200 000 per year. The company also wishes to know which is the preferred supplier if the “other costs” per unit for the Simplicity model are as low as \$0.45 per unit or as high as \$0.75 per unit. Westmount Waxworks uses an after-tax MARR of 15 percent for equipment projects. It pays taxes in Canada with a tax rate of 40 percent. The CCA rate for such equipment is 30 percent.

Assuming that the “other costs” of the Simplicity wax melter are \$0.55 per unit, a break-even chart that shows the present worth of the projects as a function of sales levels can give much insight into the supplier selection. Table 12.3 gives the annual cost of each of the two alternatives, and Figure 12.5 shows the break-even chart for sales level. A sample computation for the Finedetail wax melter at the 60 000 sales level is

$$\begin{aligned}
 AW(\text{Finedetail}) &= CTF(200\,000)(A/P, 15\%, 7) \\
 &\quad - CSF(5000)(A/F, 15\%, 7) \\
 &\quad + (1 - t)[10\,000 + 6500 + (0.05 + 1.25 + 0.95)(\text{sales level})] \\
 &= 0.75073(200\,000)(0.24036) - 0.73333(5000)(0.09036) \\
 &\quad + (1 - 0.4)[16\,500 + 2.25(60\,000)] \\
 &= \$126\,658
 \end{aligned}$$

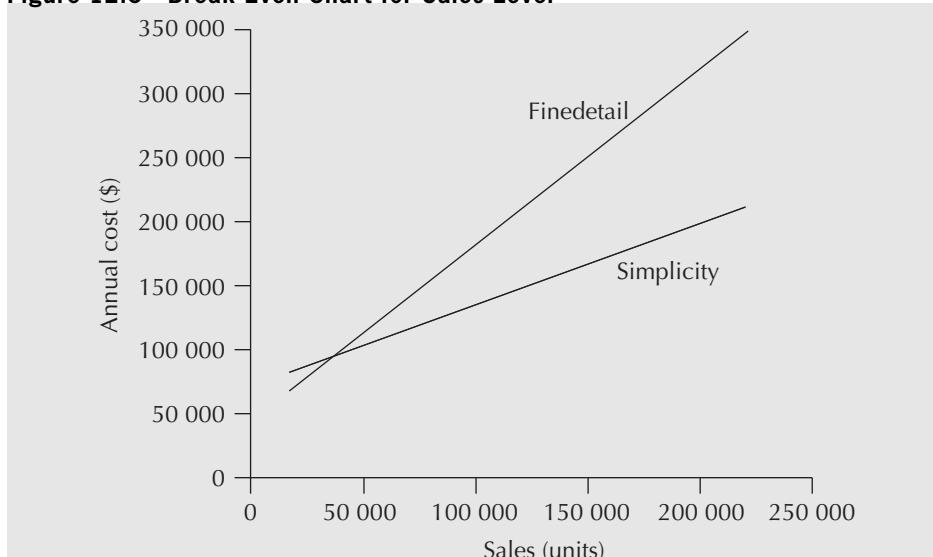
where

$$CSF = 1 - \frac{td}{(i + d)} = 1 - \frac{(0.4)(0.3)}{(0.15 + 0.30)} = 0.73333$$

$$CSF = 1 - \frac{td(1 + \frac{i}{2})}{(i + d)(1 + i)} = 1 - \frac{(0.4)(0.3)(1 + 0.075)}{(0.15 + 0.30)(1 + 0.15)}$$

**Table 12.3 Annual Cost as a Function of Sales**

| Sales<br>(Units) | Annual Costs (\$) |            |
|------------------|-------------------|------------|
|                  | Finedetail        | Simplicity |
| 20 000           | 72 658            | 85 651     |
| 60 000           | 126 658           | 111 091    |
| 100 000          | 180 658           | 136 531    |
| 140 000          | 234 658           | 161 971    |
| 180 000          | 288 658           | 187 411    |
| 220 000          | 342 658           | 212 851    |

**Figure 12.5 Break-Even Chart for Sales Level**

If sales are 30 000 units per year, the Finedetail wax melter is slightly preferred to the Simplicity melter. At a sales level of 200 000 units per year, the preference is for the Simplicity wax melter. Interpolation of the amounts in Table 12.3 indicates that the break-even sales level is 38 199 units. That is to say, for sales below 38 199 per year, Finedetail is preferred, and Simplicity is preferred for sales levels of 38 199 units and above.

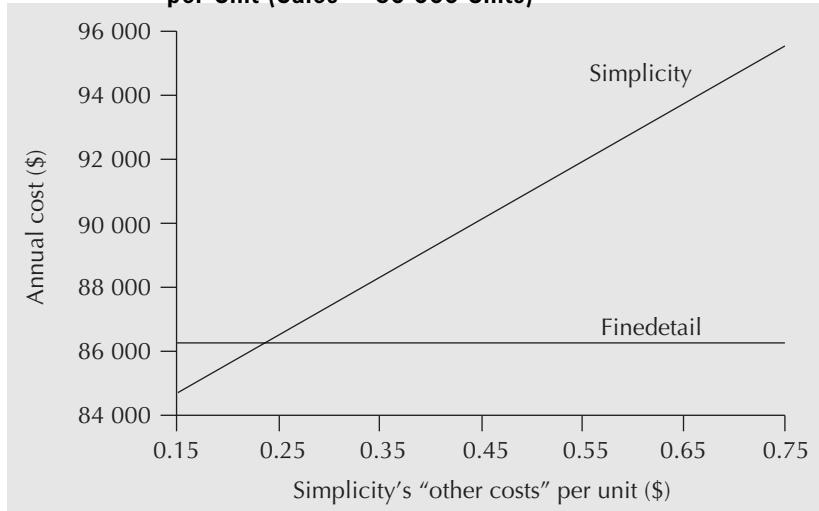
Since 30 000 units per year is the lowest sales will likely be, and sales have averaged 50 000 units per year over the past five years, it appears that the Simplicity wax melter would be the preferred choice, assuming that its "other costs" per unit are \$0.55. The robustness of this decision may be affected by the other types of costs, such as maintenance and labour, of the Simplicity melter.

To assess the sensitivity of the choice of wax melter to the variable other costs of Simplicity, a break-even analysis similar to that for sales level can be carried out. We can vary the "other costs" from the estimate of \$0.45 per unit to \$0.75 per unit and observe the effect on the preferred wax melter. Table 12.4 gives the annual costs for the two wax melters as a function of the "other costs" of the Simplicity model for sales levels of 30 000, 50 000, and 200 000 units per year. In each case, we see that the best choice is not sensitive to the "other costs" of the Simplicity wax melter. In fact, for a sales level of 30 000 units

**Table 12.4 Annual Cost as a Function of Simplicity's Other Costs per Unit**

| Other Costs per Unit (\$) | Sales = 30 000 Units/Year |            | Sales = 50 000 Units/Year |            | Sales = 200 000 Units/Year |            |
|---------------------------|---------------------------|------------|---------------------------|------------|----------------------------|------------|
|                           | Finedetail                | Simplicity | Finedetail                | Simplicity | Finedetail                 | Simplicity |
| 0.45                      | 86 158                    | 90 211     | 113 158                   | 101 731    | 315 658                    | 188 131    |
| 0.55                      | 86 158                    | 92 011     | 113 158                   | 104 731    | 315 658                    | 200 131    |
| 0.65                      | 86 158                    | 93 811     | 113 158                   | 107 731    | 315 658                    | 212 131    |
| 0.75                      | 86 158                    | 95 611     | 113 158                   | 110 731    | 315 658                    | 224 131    |

per year, the break-even “other costs” are less than \$0.25, as shown in Figure 12.6. This means that the other costs per unit would have to be lower than \$0.25 for the best choice to change from Finedetail to Simplicity. For a sales level of 200 000 per year, the break-even “other costs” are much higher, at \$1.51 per unit, and for a sales level of 50 000 units per year the break-even cost per unit is \$0.83. For both of the latter sales levels, the Simplicity model is preferred.

**Figure 12.6 Break-Even Chart for Simplicity's Other Costs per Unit (Sales = 30 000 Units)**

Having done the break-even analysis for both sales level and “other costs” per unit for the Simplicity wax melter, it would appear that the Simplicity model is the better choice if sales are at all likely to exceed the break-even sales level of 38 199. Historically, sales have exceeded this amount. Even if sales in a particular year fall below the break-even level, the Simplicity wax melter does not have annual costs far in excess of those of the Finedetail model, so the decision would appear to be robust with respect to possible sales levels. Similarly, the decision is not sensitive to the other costs per unit of the Simplicity wax melter.

We have seen in this section that break-even analysis for either a single project or multiple projects is a simple tool and that it can be used to extract insights from a

modest amount of data. It communicates threshold (break-even) parameter values where preference changes from one alternative to another or where a project changes from being economically justified to not justified. Break-even analysis is a popular means of assessing the impact of errors or changes in parameter values on an economic performance measure or a decision. The main disadvantage of break-even analysis is that it cannot easily capture interdependencies among variables. Although we can vary one or two parameters at a time and graph the results, more complicated analyses are not often feasible. For example, an engineer may wish to incorporate into the analysis the impact of several outcomes for uncertain future events.

One tool used by engineers to structure and make decisions when the alternatives involve risk is a decision tree. A decision tree decomposes a problem clearly into its decision alternatives and uncertain events. Before we introduce the use of decision trees, though, we need to review some basic concepts of probability.

## 12.4 Basic Concepts of Probability

---

Suppose you are concerned with the market success of a new fuel-cell technology, but you are uncertain about its outcome. It could be that the fuel cell fails to gain market acceptance, it may produce adequate sales, or it could gain market dominance. In the terminology of decision analysis, “market success” is referred to as a *random variable*. A **random variable** is a parameter or variable that can take on a number of possible outcomes. Only one of these outcomes will eventually occur, but which one will occur is unknown at the time a decision is being made. For example, if market success is considered to be a random variable, then the set of possible outcomes are failure to gain market acceptance, adequate sales, or market dominance. To construct a model of this uncertainty, you will need to know the probability associated with each possible outcome. This is accomplished through a probability distribution function.

Consider a random variable  $X$  (e.g., the market success of a fuel cell) that can take on  $m$  discrete outcomes  $x_1, x_2, \dots, x_m$ . If these events are mutually exclusive (if one occurs, another cannot) and collectively exhaustive (one of them must occur), a **probability distribution function**  $p(x)$  is a set of numerical measures  $p(x_i)$  such that:

$$0 \leq p(x_i) \leq 1 \quad \text{for } i = 1, \dots, m$$

and

$$\sum_{i=1}^m p(x_i) = 1$$

with the intuitive interpretation that the higher  $p(x_i)$ , the more likely it is that  $x_i$  will occur.

The first statement above says that the **probability** associated with each outcome must be positive and must be between 0 and 1 (inclusive). Intuitively, this means that any outcome cannot have a chance of occurring less than 0 percent or more than 100 percent. The second statement above says that since the outcomes are mutually exclusive and collectively exhaustive, only one of the outcomes will occur, and one *must* occur.

Over the years, various views on probability have emerged. Each is appropriate in different circumstances. Close-Up 12.1 summarizes different views on probability and when they are useful.

**CLOSE-UP 12.1**

## Views on Probability

*Classical or symmetric probability:* This was the first view of probability and relies on games of chance such as dice, where the outcomes are equally likely. For example, if there are  $m$  possible outcomes for an uncertain event, since only one can and must occur, and each is assumed to be equally likely, then the chance of each occurring is  $1/m$ . For example, the probability that a coin toss will result in “heads” is  $1/2$  because there are two sides and each is equally likely.

*Relative frequency:* The outcome of an random event  $E$  is observed over a large number of experiments,  $N$ . If the number of times the outcome  $e_i$  occurs is  $n_i$ , then we can estimate the probability of event  $p(e_i)$  by  $n_i/N$ . More formally, the relative frequency view on probability says that  $p(e_i) = \lim_{N \rightarrow \infty} n_i / N$ . An example of this is flipping a coin 1000 times and discovering that it lands on its edge five times in 1000. An estimate of the probability of the coin landing on its edge is then  $5/1000 = 0.005$ .

*Subjective probability:* Subjective or personal probability is an attempt to deal with unique events that cannot be repeated and hence can't be given a frequency interpretation. In rough terms, subjective probability can be interpreted as the odds one would personally give in betting on an event, or it may be a matter of human judgment and intuition as formed by physical relationships and experimental results. An example of this is a person who judges that the chance of winning a coin toss with one of the authors of this text is very low, say  $1/1000$ , because, in that person's experience, the authors usually cheat.

*Axiomatic probability:* One of the problems associated with defining probabilities is that the definition of probability requires using probability itself. To get around this circular logic, axiomatic probability makes no attempt to define probability, but simply states the rules or axioms it follows. Other properties can then be derived from the basic axioms.

Each of the above methods may be correct, given the circumstances. When the physics of a process suggest a clear judgment of probability, the classical approach makes sense. Where formal experimentation is possible, the relative frequency method may be justified. In many real-world cases, subjective probability supported by historical information and other data is frequently used.

When the number of outcomes for a random variable  $X$  is discrete,  $p(x)$  is referred to as a **discrete probability distribution function** (PDF). Examples of discrete random variables are the number of good items in a batch of 100 tested products, the number of car accidents at an intersection each year, the number of days since the last plant shutdown, or the number of bugs found in software testing. Whether  $p(x)$  is estimated using the classical, the relative frequency, or the subjective approach, the same terminology for the probability distribution function,  $p(x)$ , is used.

Various symbols may be used to define a random variable. The normal convention is to capitalize the symbol used for the random variable, and to use subscript lowercase letters to denote its various outcomes. For example, for a discrete random variable  $X$ , outcomes are denoted by  $x_1, x_2, x_3, \dots$  and its probability distribution function is  $p(x)$ . The probability that  $X$  takes on the value  $x_1$  is written as  $\Pr(X = x_1) = p(x_1)$ .

**EXAMPLE****12.4**

Suppose that you are testing solder joints on a printed circuit board and that you are interested in determining the probability distribution function for the random variable  $X$ —the number of open joints in three tested joints (to keep it simple). Prior to testing, you don't know how many open joints there will be. Since there are three joints and each will be either open or closed,  $X$  is a discrete random variable that can take on four possible values:  $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$ , and  $x_4 = 3$ .

Note that there are eight distinct test result sequences that can occur. Denoting the result of a single test with O for open and C for closed, the set of possible test results are (O,O,O), (O,O,C), (O,C,O), (C,O,O), (O,C,C), (C,C,O), (C,O,C), and (C,C,C). We must look through the set of individual test results to see which corresponds to  $x_1 = 0$ ,  $x_2 = 1$ ,  $x_3 = 2$ , and  $x_4 = 3$  open solder joints.

You know from previous data collection efforts that the result of a single test is uncertain. The probability that a single tested joint will be open is 20 percent. In other words, the outcome of the test is a random variable, say,  $Y$ , where the result of a single test can have two outcomes:  $y_1 = O$  = open and  $y_2 = C$  = closed, and  $\Pr(Y = O) = p(O) = 0.2$  and  $\Pr(Y = C) = p(C) = 0.8$ . Further, suppose it is reasonable to assume that the quality of a solder joint does not change from joint to joint (i.e., the test results are independent of one another). Then the probability that a test sequence results in three open joints is calculated by

$$\Pr(X = 3) = p(x_4) = p(O) \times p(O) \times p(O) = (0.2) \times (0.2) \times (0.2) = 0.008$$

Similar calculations, shown in Table 12.5, yield the probabilities for each of the eight possible test sequences.

Finally, the probability distribution function of  $X$ , the number of “open” joints in the three tests, is

$$\Pr(X = 0) = p(x_1) = 0.512$$

$$\Pr(X = 1) = p(x_2) = 0.384$$

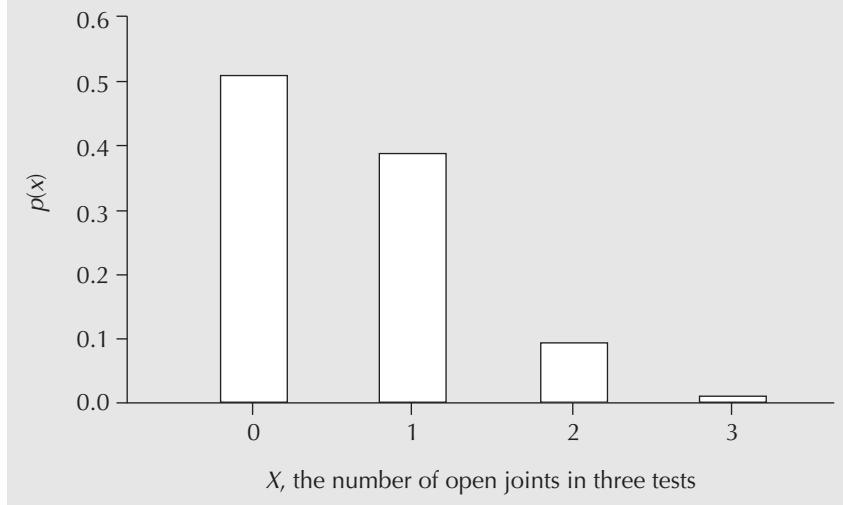
$$\Pr(X = 2) = p(x_3) = 0.096$$

$$\Pr(X = 3) = p(x_4) = 0.008$$

Note that the two important properties of probabilities hold:  $p(x_i) \geq 0$  for all  $i$ , and  $\sum p(x_i) = 1$ .

**Table 12.5 Probability Corresponding to the Outcomes of the Solder Joint Testing With a 20% Chance of an Open Joint**

| Test Sequence | Number of “Opens” | Probability                         |
|---------------|-------------------|-------------------------------------|
| (O,O,O)       | 3                 | $0.008 = 0.2 \times 0.2 \times 0.2$ |
| (O,O,C)       | 2                 | $0.032 = 0.2 \times 0.2 \times 0.8$ |
| (O,C,O)       | 2                 | $0.032 = 0.2 \times 0.8 \times 0.2$ |
| (C,O,O)       | 2                 | $0.032 = 0.8 \times 0.2 \times 0.2$ |
| (O,C,C)       | 1                 | $0.128 = 0.2 \times 0.8 \times 0.8$ |
| (C,C,O)       | 1                 | $0.128 = 0.8 \times 0.8 \times 0.2$ |
| (C,O,C)       | 1                 | $0.128 = 0.8 \times 0.2 \times 0.8$ |
| (C,C,C)       | 0                 | $0.512 = 0.8 \times 0.8 \times 0.8$ |

**Figure 12.7 Probability Distribution for the Solder Joint Example**

It is often useful to display a probability distribution function in graphical format. Such a graph is referred to as a histogram. Figure 12.7 shows the probability distribution function associated with the solder joint testing results.

In contrast to a discrete random variable, a continuous random variable can take on any real value over a defined interval. For example, daily demand for drinking water in a municipality might be anywhere between 10 million litres and 200 million litres. The actual amount consumed—the outcome—is a continuous random variable with a minimum value of 10 million litres and a maximum value of 200 million litres.

In this chapter, we focus on applications of discrete random variables in engineering economics analysis. We do not use continuous random variables because proper treatment requires more advanced mathematical concepts such as differential and integral calculus. Also, continuous random variables can be well approximated as discrete random variables by grouping the possible output values into a number of categories or ranges. For example, rather than treating demand for drinking water as a continuous random variable, demand could be characterized as high, medium, or low. Figure 12.8 shows an example of a probability distribution associated with future demand for water, approximated by a discrete random variable denoted by  $D$ .

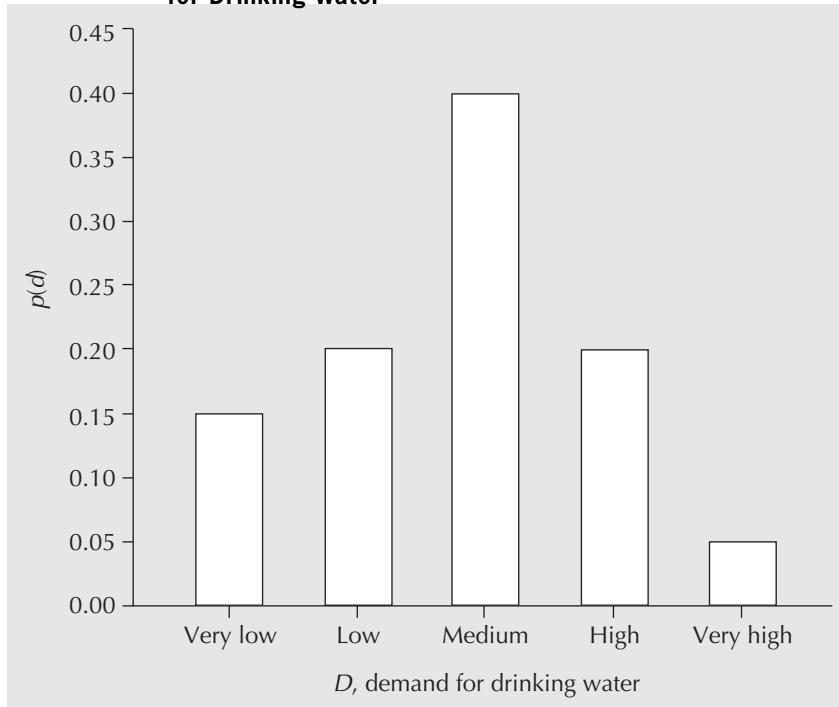
The probability distribution of a random variable contains a great deal of information that can be useful for decision-making purposes. However, certain summary statistics are often used to capture an overall picture rather than working with the entire distribution. One particularly useful summary statistic is the **expected value**, or **mean**, of a random variable. The expected value of a discrete random variable  $X$ ,  $E(X)$ , which can take on values  $x_1, x_2, \dots, x_m$ , is defined as follows:

$$E(X) = \sum_{i=1}^m x_i p(x_i)$$

You will no doubt observe that computing the expected value of a random variable is much like computing the centre of mass for an object. The expected value is simply the centre of the probability “mass.”

How are random variables, probability distributions, and expected values relevant to engineering economics? In conducting engineering economic studies, costs and benefits

**Figure 12.8 Probability Distribution Function of the Demand for Drinking Water**



can be influenced by several uncertain factors. Often, an engineer does not know the outcome of a project or may not know with certainty the actual value of one or more parameters important to a project. The life of a product may shorten or lengthen unexpectedly due to market forces, the maintenance costs may be difficult to state with certainty, a car may have safety features that may increase or decrease the liability for the vendor, or product demand may be more or less than anticipated. In each case, if the engineer can determine the range of outcomes and their associated probabilities, this information can present a much richer view for the decision-making process. Random variables and their probability distributions are, in fact, the building blocks for many tools that are useful in decision making under risk.

Examples 12.5 and 12.6 illustrate how the expected value and probability distribution information may be used in a decision-making context.

### EXAMPLE

### 12.5

Recall from Example 12.3 that the management of Westmount Waxworks had some uncertainty about the future sales levels of its line of statues of government leaders. Expert opinion helped it assess the probability of the pessimistic, expected, and optimistic sales scenarios. Management thinks that the probability that sales will be 50 000 per year for the next few years is roughly 50 percent and that the pessimistic and optimistic scenarios have probabilities of 20 percent and 30 percent, respectively. Table 12.6 reproduces the annual cost information for the two wax melters, Finedetail and Simplicity. On the basis of expected annual costs, which is the best choice?

**Table 12.6 Annual Cost Information for the Finedetail and Simplicity Wax Melters**

| Scenario    | Annual Cost for Finedetail | Annual Cost for Simplicity | Probability |
|-------------|----------------------------|----------------------------|-------------|
| Pessimistic | \$ 85 314                  | \$ 94 381                  | 0.2         |
| Expected    | 112 314                    | 103 501                    | 0.5         |
| Optimistic  | 314 814                    | 186 901                    | 0.3         |

The sales level can be represented by a discrete random variable,  $X$ . The possible values for  $X$  are:  $x_1$  = pessimistic,  $x_2$  = expected, and  $x_3$  = optimistic.

The expected annual cost of the Finedetail wax melter is:

$$E(\text{Finedetail, annual cost})$$

$$\begin{aligned} &= (85 314)p(x_1) + (112 314)p(x_2) + (314 814)p(x_3) \\ &= (85 314)(0.2) + (112 314)(0.5) + (314 814)(0.3) \\ &= 167 663 \end{aligned}$$

The expected annual cost of the Simplicity wax melter is:

$$E(\text{Simplicity, annual cost})$$

$$\begin{aligned} &= (94 381)p(x_1) + (103 501)p(x_2) + (186 901)p(x_3) \\ &= (94 381)(0.2) + (103 501)(0.3) + (186 901)(0.4) \\ &= 126 697 \end{aligned}$$

The expected annual cost of the Simplicity wax melter is lower than that of the Finedetail melter. Hence, the Simplicity melter is preferred. ■

**EXAMPLE****12.6**

Regional Express is a small courier service company. At the central office, all parcels from the surrounding area are collected, sorted, and distributed to the appropriate destinations. Regional Express is considering the purchase of a new computerized sorting device for its central office. The device is so new—in fact, it is still under continuous improvement—that its maximum capacity is somewhat uncertain at the present time. The company has been told that the possible capacity can be 40 000, 60 000, or 80 000 parcels per month, regardless of the size of the parcels. It has estimated the probabilities corresponding to the three capacity levels. Table 12.7 shows this information. What is the expected capacity level for the new sorting device? Regional Express is growing steadily, so such a computerized sorting device will be a necessity in the future. However, if Regional Express currently deals with an average of 50 000 parcels per month, should it seriously consider purchasing the device now or should it wait?

If the discrete random variable  $X$  denotes the capacity of the device, then the expected capacity level  $E(X)$  is

**Table 12.7 Probability Distribution Function for Capacity Levels of the New Sorting Device**

| <i>i</i> | Capacity Level<br>(Parcels/Month) | <i>p</i> ( <i>x<sub>i</sub></i> ) |
|----------|-----------------------------------|-----------------------------------|
| 1        | 40 000                            | 0.3                               |
| 2        | 60 000                            | 0.6                               |
| 3        | 80 000                            | 0.1                               |

$$\begin{aligned}
 E(X) &= (40\,000)p(x_1) + (60\,000)p(x_2) + (80\,000)p(x_3) \\
 &= (40\,000)(0.3) + (60\,000)(0.6) + (80\,000)(0.1) \\
 &= 56\,000 \text{ parcels per month}
 \end{aligned}$$

The expected capacity level exceeds the average monthly demand of 50 000 parcels per month, so according to the expected value analysis alone, Regional Express should consider buying the sorting device now. However, by studying the probability distribution, we see that there is a 30 percent probability that the capacity level may fall below 50 000 parcels per month. Perhaps Regional Express should include this information in its decision making, and ask itself whether a 30 percent chance of not meeting its demand is too risky or costly if it decides to purchase the sorting device. ■

In summary of this section, engineers can be faced with a variety of uncertain events in project evaluation. When the outcomes of each event can be characterized by a probability distribution, this greatly enhances the analyst's ability to develop a deeper understanding of the risks associated with various decisions. This section provided an introduction to random variables and probability distributions as a starting point for an analysis of project risk.

## 12.5 Structuring Decisions With Decision Trees

Many different types of uncertainties exist in decision making. When an economic analysis becomes complex due to these uncertainties, formal analysis methods can help in several ways. First, formal methods can help by providing a means of decomposing a problem and structuring it clearly. Second, formal methods can help by suggesting a variety of decision criteria to help with the process of selecting a preferred course of action. This section provides an introduction to decision trees, which are a graphical means of structuring a decision-making situation where the uncertainties can be characterized by probability distributions. Other means of structuring decisions such as influence diagrams are available, but are beyond the scope of our coverage.

Decision trees help decompose and structure problems characterized by a sequence of one or more decisions and event outcomes. For example, a judgment about the chance of a thunderstorm tomorrow will affect your decision to plan for a picnic tomorrow afternoon. Similarly, the success or failure of a new product may largely depend on future demand for the product. As another example, a decision on the replacement interval for an asset relies on an assessment of its economic life, which can be highly uncertain if the equipment employs an emerging technology.

When a decision is influenced by outcomes of one or more random events, the decision maker must anticipate what those outcomes might be as part of the process of analysis. This

section presents a useful tool for structuring such problems, called a decision tree. It is particularly suited to decisions and events that have a natural sequence in time or space.

A **decision tree** is a graphical representation of the logical structure of a decision problem in terms of the sequence of decisions to be made and outcomes of chance events. It provides a mechanism to decompose a large and complex problem into a sequence of small and essential components. In this way, a decision tree clarifies the options a decision maker has and provides a framework with which to deal with the risk involved.

Example 12.7 introduces the overall approach to constructing a decision tree. A detailed explanation of the components and structure of the decision tree is included in the example.

### EXAMPLE

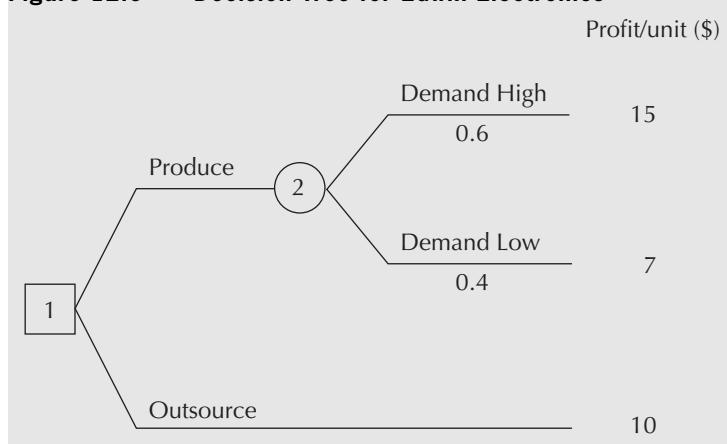
### 12.7

Edwin Electronics (EE) has a factory for assembling TVs. One of the key components is the TV screen. EE does not currently produce TV screens onsite; they are outsourced to a supplier elsewhere. Recently, EE's industrial engineering team asked if it should continue outsourcing the TV screens or produce them in-house. The team realized that it was important to consider the uncertainty in demand for the company's TVs. If the future demand is low, outsourcing seems to be the reasonable option in order to save production costs. On the other hand, if the demand is high, then it may be worthwhile to produce the screens onsite, thus getting economies of scale. EE's engineers analyzed the effect of the demand uncertainty in their decision making. They represented their decision problem in a graphical manner with a decision tree. Figure 12.9 represents EE's decision tree.

There are four main components in a decision tree: decision nodes, chance nodes, branches, and leaves. A decision node represents a decision to be made by the decision maker and is denoted by a square in the tree diagram. In Figure 12.9, the single square node represents the decision to produce or outsource TV screens (node 1). A chance node represents an event whose outcome is uncertain, but which has to be considered during decision making. The outcome of a chance node is a discrete random variable, as it has a number of distinct outcomes and each outcome has an associated probability. The circle in the diagram denotes a chance node. The chance node in Figure 12.9 represents the uncertain demand for TV screens (node 2). The branches of a tree are the lines connecting nodes depicting the sequence of possible decisions and chance events. Finally, the leaves indicate the values, or payoffs, associated with each branch of the decision tree.

A decision tree grows from left to right and usually begins with a decision node. The leftmost decision node represents an immediate decision faced by the decision maker.

**Figure 12.9 Decision Tree for Edwin Electronics**



The branches extending from a decision node represent the decision options available for the decision maker at that node, whereas the branches extending from a chance node represent the possible outcomes of the chance event. Each branch extending from a chance node has an associated probability. In Edwin Electronics' case, the two decision options, to produce or to outsource, are represented by the two branches extending from the decision node. The two branches from the chance node indicate that the future demand may be high or low. It is important in decision making that all branches out of a node, whether a decision node or chance node, constitute a set of mutually exclusive and collectively exhaustive consequences. In other words, when a decision is made, exactly one option is taken, or when uncertainty is resolved, exactly one outcome occurs as a result.

Whenever a chance node follows a decision node, as in Figure 12.9, it implies that the decision maker must anticipate the outcome of future uncertain events in decision making. On the other hand, when a decision node follows a chance node, it implies that a decision must be made assuming that a particular outcome of a chance event has occurred. Finally, the rightmost branches lead to the leaves of the decision tree, indicating all possible outcomes of the overall decision situation represented by the tree. Each leaf has an associated valuation, referred to as a "payoff"; quite typically, the payoff is a monetary value. Edwin Electronics uses profit per TV unit as its performance measure. ■

The decision tree for Edwin Electronics from Figure 12.9 can be modified to show more complex decision situations.

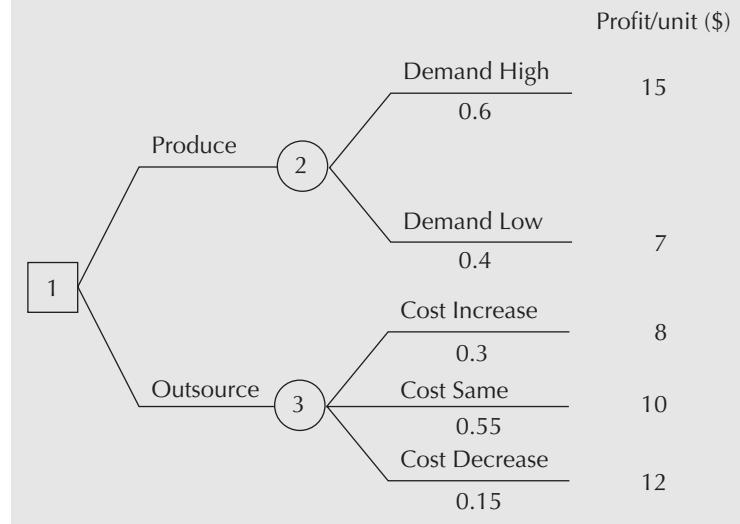
### EXAMPLE

#### 12.8

The EE engineering team from Example 12.7 has realized that the cost per TV screen may vary in the future, especially since EE is subject to purchasing conditions set by the supplier. How does this affect the decision tree?

Figure 12.10 includes the additional uncertainty in the TV screen cost charged by the supplier. The cost may increase, remain the same, or decrease in the future, as shown at node 3.

**Figure 12.10 Edwin Electronics' Modified Decision Tree 1**

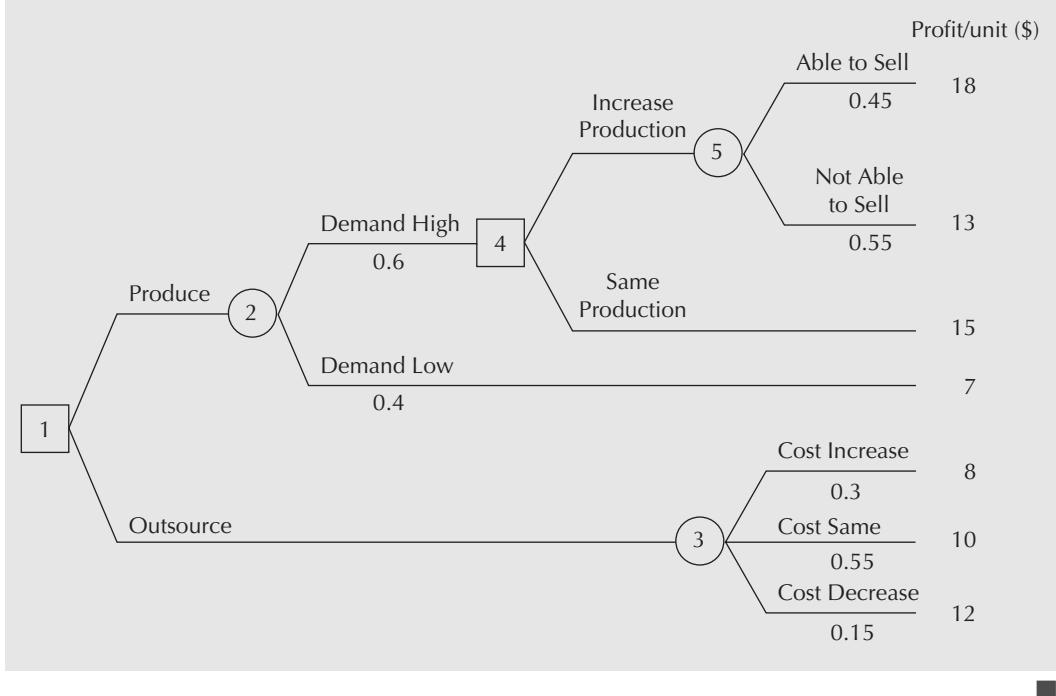


**EXAMPLE****12.9**

The EE engineering team then considered increasing in-house production of the TV screens if the demand is high. This raises an additional uncertainty about the ability of the market to absorb the increased production. How does this change the decision tree?

Figure 12.11 further modifies Figure 12.10 to include a new decision component and a chance event (see nodes 4 and 5). The decision component indicates the choice of whether to increase production. If production is increased, a new chance node captures the uncertainty about Edwin's ability to sell the excess production.

**Figure 12.11 Edwin Electronics' Modified Decision Tree 2**



One criterion for selecting among alternatives when there are risky outcomes is to pick the alternative that has the highest expected value, EV. If the units of value associated with the rightmost branches of a decision tree are measured in dollars, then this criterion may be referred to as *expected monetary value*, or EMV.

When a decision problem has been structured with a decision tree, finding the expected value of a particular decision is obtained by a procedure referred to as “rolling back” the decision tree. The procedure is as follows.

1. *Structure the problem:* Develop a decision tree representing the decision situation in question.
2. *Rollback:* Execute the **rollback procedure** (also known as **backward induction**) on the decision tree from right to left as follows:
  - (a) At each chance node, compute the expected value of the possible outcomes. The resulting expected value becomes the value associated with the chance node and the branch on the left of that node (if there is one).

- (b) At each decision node, select the option with the best expected value (best may be highest value or lowest cost depending on the context). The best expected value becomes the value associated with the decision node and the branch on the left of that node (if there is one). For the option(s) not selected at this time, indicate their termination by a double-slash (//) on the corresponding branch.
- (c) Continue rolling back until the leftmost node is reached.
3. *Conclusion:* The expected value associated with the final node is the expected value of the overall decision. Tracing forward (left to right), the non-terminated decision options indicate the set of recommended decisions at each subsequent node.

| N E T      V A L U E      1 2 . 1   |
|---|
| <p><b>Decision Tree Software</b></p> <p>Constructing decision trees is awkward for a variety of reasons. For one thing, they are a combination of diagrams, words, and numbers, so standard software is of limited value. They can be extensive, so a single sheet of paper may not be large enough to hold the whole tree. Mathematical errors can be made. Also, making changes to a tree that has already been constructed is very difficult.</p> <p>All of these issues are easily solved by using an application program designed for building and managing decision trees. There are many on the market, and a simple internet search for “decision tree software” will bring up a variety of choices. Some examples include: TreeAge Software (<a href="http://www.treeage.com">www.treeage.com</a>), Vanguard Software (<a href="http://www.vanguard.com">www.vanguard.com</a>), Insight Tree (<a href="http://www.insighttree.com">www.insighttree.com</a>), and Palisades Precision Tree (<a href="http://www.palisade.com">www.palisade.com</a>). Most vendors provide a trial version at no charge so that if you are building decision trees only occasionally (like for a course at school!), you can have all the functionality needed for developing high-quality decision trees without having to invest in purchasing software.</p> |

**EXAMPLE****12.10**

Carry out a decision tree analysis on Edwin Electronics’ modified tree in Figure 12.10 using the expected value criterion.

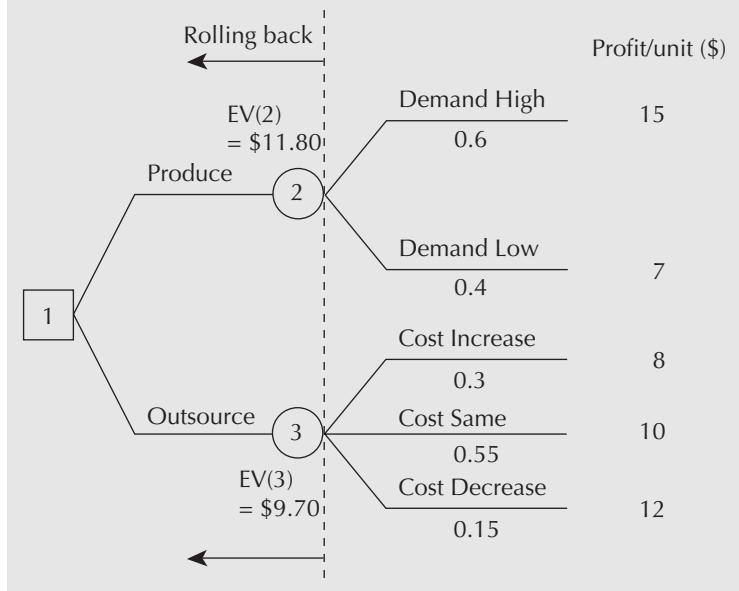
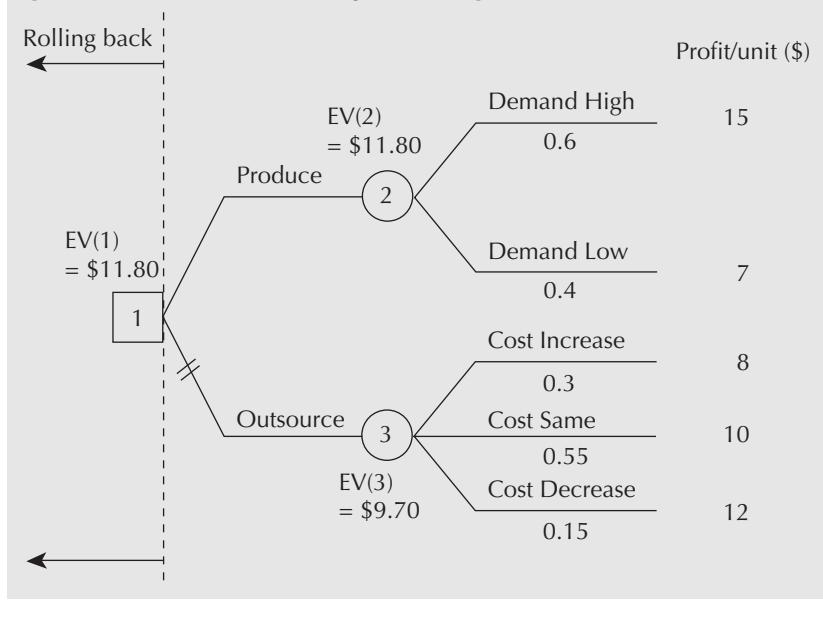
Since the decision tree is already provided, step 1 is complete. The rollback procedure described in step 2 has two phases in this case. First, the tree is rolled back to each of the chance nodes as in step 2(a) (phase 1). The expected values at nodes 2 and 3 are computed as follows:

$$EV(2) = 0.6(15) + 0.4(7) = 11.80$$

$$EV(3) = 0.3(8) + 0.55(10) + 0.15(12) = 9.70$$

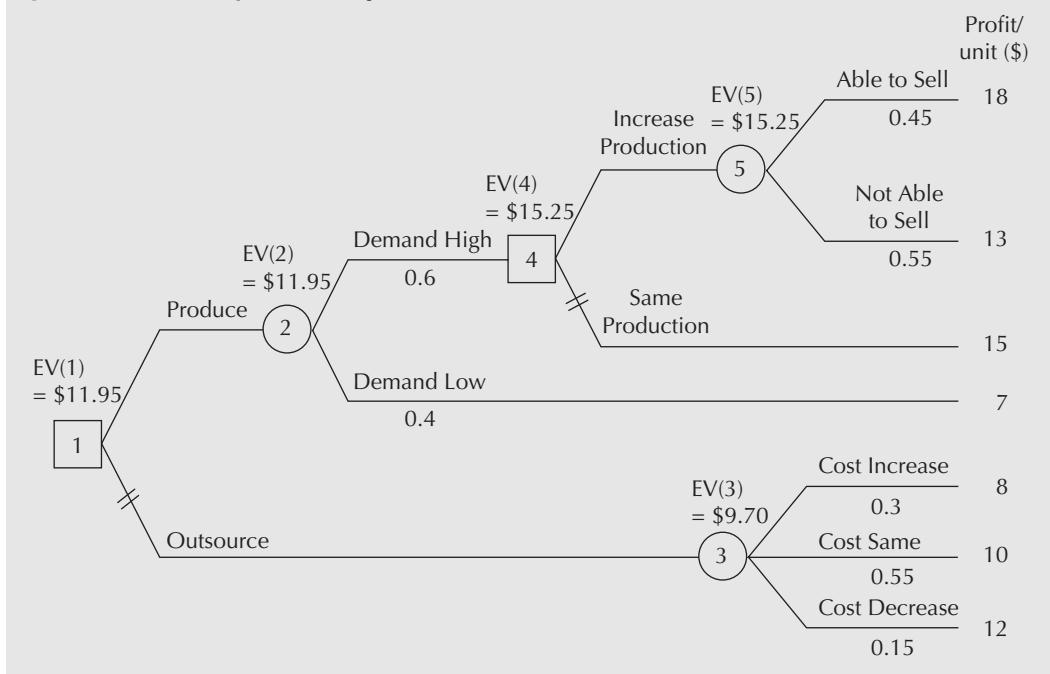
Figure 12.12 shows the rollback so far.

Next, the tree is further rolled back to the decision node as in step 2(b) (phase 2). The expected value at node 1 is then  $EV(1) = \$11.80$ , which is equal to  $EV(2)$  since  $EV(2)$  is higher than  $EV(3)$ . Figure 12.13 shows this result. As for step 3, the following conclusion is made: The expected value of the overall decision is \$11.80 per unit and the recommended decision is to produce TV screens in-house.

**Figure 12.12 Phase 1 of Step 2: Rolling Back to the Chance Nodes****Figure 12.13 Phase 2 of Step 2: Rolling Back to the Decision Nodes****EXAMPLE****12.11**

Perform decision tree analysis on Edwin Electronics' second modified tree in Figure 12.11.

The result of this analysis is shown in Figure 12.14. The overall expected profit for this tree is \$11.95 per unit. The recommended decision is to produce TV screens in-house, and if the demand is high, the production level should be increased.

**Figure 12.14 Completed Analysis for EE's Modified Decision Tree 2**

## REVIEW PROBLEMS

The following case is the basis of Review Problems 12.1 and 12.2.

Betteryet Insurance Inc. is considering two independent energy efficiency improvement projects. Each has a lifetime of 10 years and will have a scrap value of zero at the end of this time. Betteryet can afford to do both if both are economically justified. The first project involves installing high-efficiency motors in the air-conditioning system. High-efficiency units use about 7 percent less electricity than the current motors, which represents annual savings of 70 000 kilowatt-hours. They cost \$28 000 to purchase and install and will require maintenance costs of \$700 annually.

The second project involves installing a heat exchange unit in the current ventilation system. During the winter, the heat exchange unit transfers heat from warm room air to the cold ventilation air before the air is sent back into the building. This will save about 2 250 000 cubic feet of natural gas per year. In the summer, the heat exchange unit removes heat from the hot ventilation air before it is added to the cooler room air for recirculation. This saves about 29 000 kilowatt-hours of electricity annually. Each heat exchange unit costs \$40 000 to purchase and install and annual maintenance costs are \$3200.

Betteryet Insurance would like to evaluate the two projects, but there is some uncertainty surrounding what the electricity and natural gas prices will be over the life of the project. Current prices are \$0.07 per kilowatt-hour for electricity and \$3.50 per thousand cubic feet of natural gas, but some changes are anticipated. The company uses a MARR of 10 percent.

**REVIEW PROBLEM 12.1**

Construct a sensitivity graph to determine the effect that a 5 percent and 10 percent drop or increase in the cost of electricity and the cost of natural gas would have upon the present worth of each project.

**ANSWER**

Table 12.8 gives the costs of electricity and natural gas with 5 percent and 10 percent increases and decreases from the base case of \$0.07 per kilowatt-hour for electricity and \$3.50 per 1000 cubic feet of natural gas. The table also shows the present worths of the two energy efficiency projects as the costs vary. A sample calculation for the heat exchange unit with base case costs is

$$\begin{aligned} \text{PW(Heat exchanger)} &= -40\,000 + (P/A, 10\%, 10) \\ &\quad \times [29\,000(0.07) + 2250(3.50) - 3200] \\ &\approx 1199 \end{aligned}$$

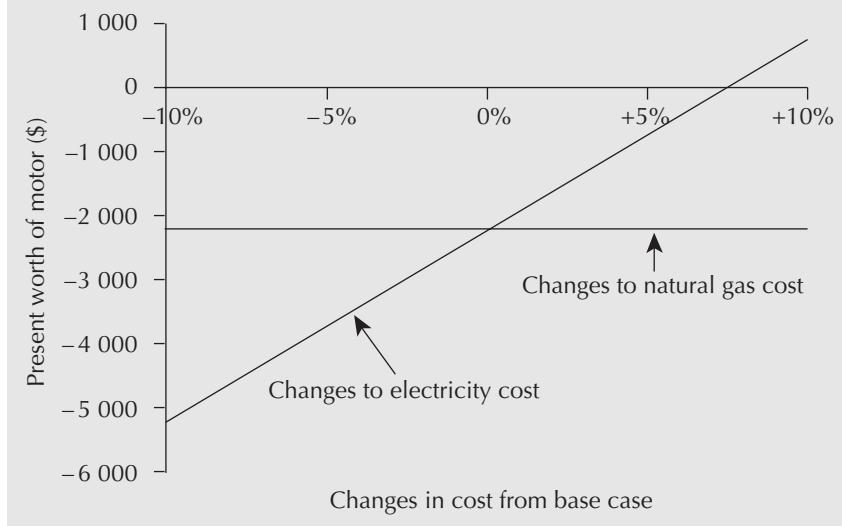
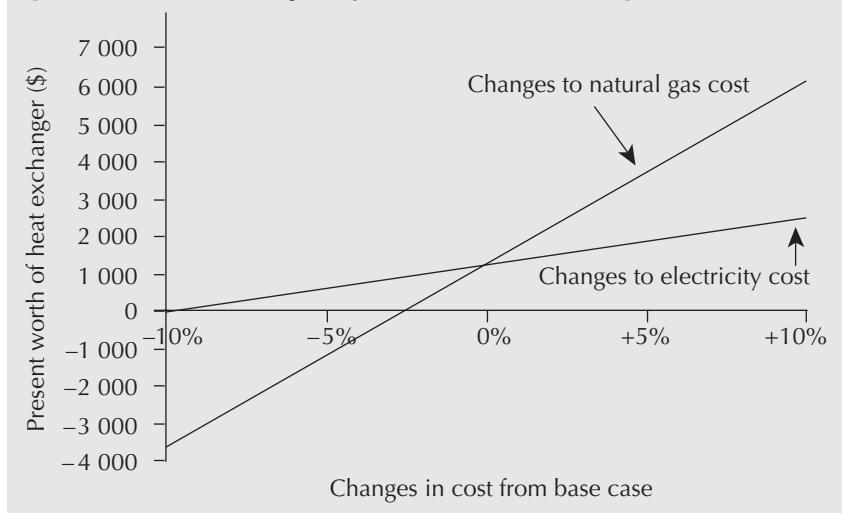
Figure 12.15 is a sensitivity graph for the high-efficiency motor. It graphically illustrates the effect of changes in the costs of electricity and natural gas on the present worth of a motor. The high-efficiency motor is not economically viable at the current prices for electricity and gas. Only if there is an increase of almost 10 percent in electricity costs for the life of the project will the motor produce sufficient savings for the project to have a positive present worth.

Figure 12.16 is the sensitivity graph for the heat exchange unit. The heat exchange unit has a positive present worth for the current prices, but the present worth is quite sensitive to the price of natural gas. A drop in the price of natural gas in the range of only 2 percent to 3 percent (reading from the graph) will cause the project to have a negative present worth. ■

**Table 12.8 Costs Used as the Basis of the Sensitivity Graph for Review Problem 12.1**



|   | -10%  | -5%    | 0%    | +5%    | +10%  |
|---|-------|--------|-------|--------|-------|
| Cost of electricity (\$/kWh)                | 0.063 | 0.0665 | 0.07  | 0.0735 | 0.077 |
| Cost of natural gas<br>(\$/1000 cubic feet) | 3.15  | 3.325  | 3.5   | 3.675  | 3.85  |
| PW of high-efficiency motor (\$)            |       |        |       |        |       |
| With changes to<br>electricity costs        | -5204 | -3698  | -2193 | -687   | 818   |
| With changes to<br>natural gas cost         | -2193 | -2193  | -2193 | -2193  | -2193 |
| PW of heat exchanger (\$)                   |       |        |       |        |       |
| With changes to<br>electricity costs        | -48   | 576    | 1199  | 1823   | 2447  |
| With changes to<br>natural gas cost         | -3640 | -1220  | 1199  | 3619   | 6038  |

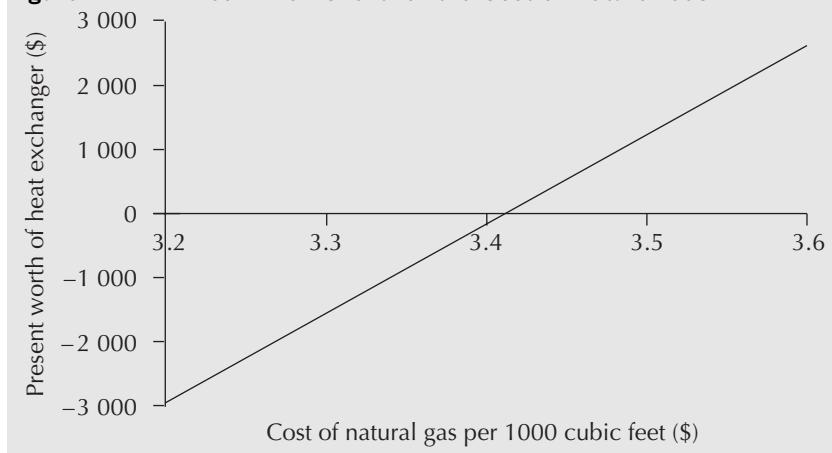
**Figure 12.15 Sensitivity Graph for the High-Efficiency Motor****Figure 12.16 Sensitivity Graph for the Heat Exchanger**

### REVIEW PROBLEM 12.2

Refer to Review Problem 12.2. How much of a drop in the cost of natural gas will result in the heat exchange unit's having a present worth of zero? Construct a break-even graph to illustrate this break-even cost.

### ANSWER

By varying the cost of natural gas from the base case, the break-even graph shown in Figure 12.17 can be constructed. The break-even cost of natural gas is \$3.41 per 1000 cubic feet, which is not much below the current price for gas. Betteryet Insurance should probably look more seriously into forecasts of natural gas prices for the life of the heat exchange unit. ■

**Figure 12.17 Break-Even Chart for the Cost of Natural Gas****REVIEW PROBLEM 12.3**

Power Tech is a North America-based company that specializes in building power-surge protection devices. Power Tech has been focusing its efforts on the North American market until now. Recently, a deal with a Chinese manufacturing company has surfaced. If Power Tech decides to become partners with this manufacturing company, its market will expand to include Asia. It is, however, concerned with the uncertainty associated with possible changes in North American demand and Asian demand. From studying the current economy, Power Tech feels that the chance of no change or an increase in demand in North America over the next three years is 60 percent and the chance of demand decrease is 40 percent. After discussions with its potential partners in China, Power Tech estimates that Asian demand may increase (or remain the same) with a probability of 30 percent and decrease with a probability of 70 percent over the next three years. Power Tech has estimated the revenue increase that can be expected under different scenarios if it establishes the partnership; this information is shown in Table 12.9.

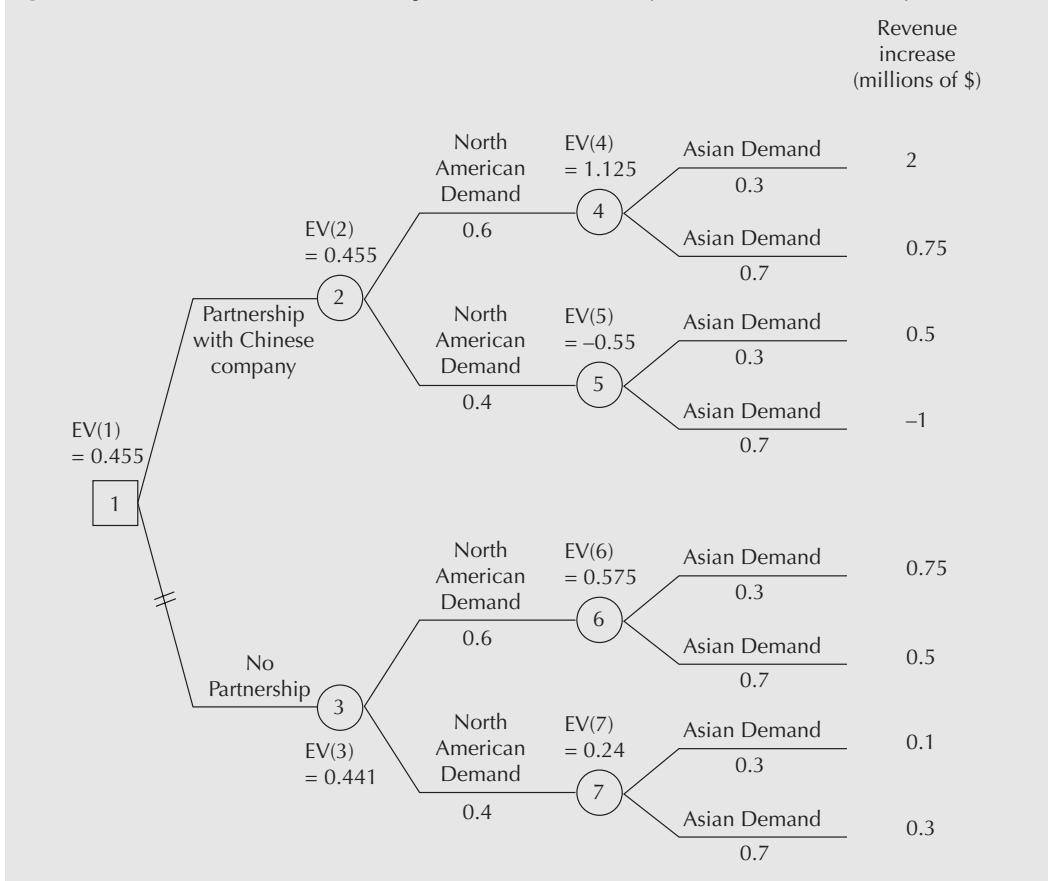
Conduct a decision tree analysis for Power Tech and make a recommendation regarding the partnership with the Chinese company.

**Table 12.9 Expected Revenue Increase for Power Tech**

|                                     | North American Demand | Asian Demand | Revenue Increase (millions of \$) |
|-------------------------------------|-----------------------|--------------|-----------------------------------|
| Partnership with Chinese company    | increase              | increase     | 2                                 |
|                                     | increase              | decrease     | 0.75                              |
|                                     | decrease              | increase     | 0.5                               |
|                                     | decrease              | decrease     | -1                                |
| No partnership with Chinese company | increase              | increase     | 0.75                              |
|                                     | increase              | decrease     | 0.5                               |
|                                     | decrease              | increase     | 0.1                               |
|                                     | decrease              | decrease     | 0.3                               |

**ANSWER**

The result of the analysis is shown in Figure 12.18. The expected value calculations at each chance node are shown below.

**Figure 12.18 Decision Tree Analysis for Power Tech (Review Problem 12.3)**

The first phase of rollback (in millions of dollars):

$$EV(4) = 0.3(2) + 0.7(0.75) = 1.125$$

$$EV(5) = 0.3(0.5) + 0.7(-1) = -0.55$$

$$EV(6) = 0.3(0.75) + 0.7(0.5) = 0.575$$

$$EV(7) = 0.3(0.1) + 0.7(0.3) = 0.24$$

The second phase of rollback (in millions of dollars):

$$EV(2) = 0.6(1.125) + 0.4(-0.55) = 0.455$$

$$EV(3) = 0.6(0.575) + 0.4(0.24) = 0.441$$

According to the expected value criterion, the partnership with the Chinese company is recommended, since the expected value for forming the partnership is higher than for not forming it. However, Power Tech should also note that these expected values have only a marginal difference. It is perhaps wise to collect more information regarding other aspects of this proposed partnership. ■

#### REVIEW PROBLEM 12.4

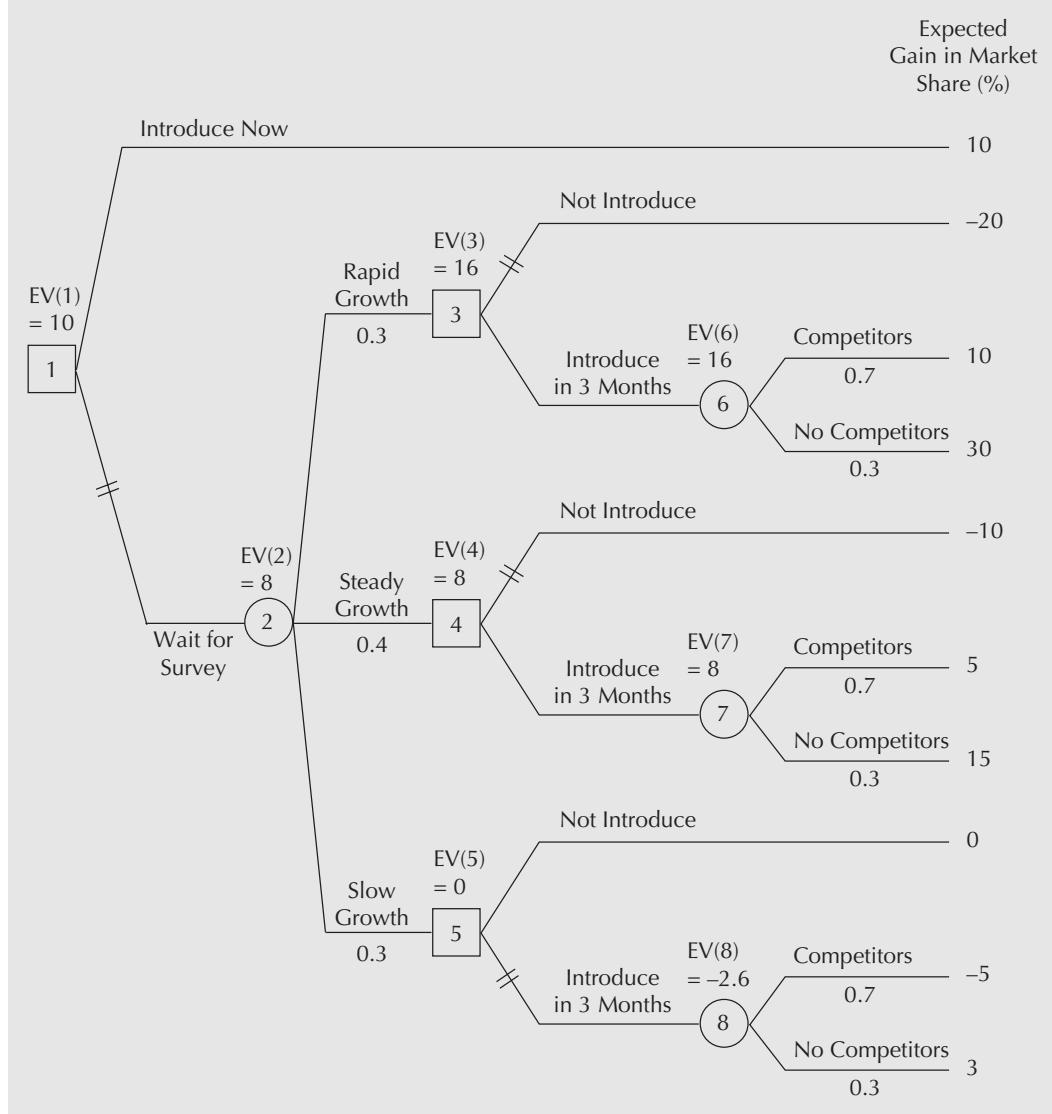
A telephone company called LOTell thinks the introduction of a new internet service package for rural residential customers would give it an advantage over its competitors. However, a survey of the potential growth of rural internet home users would take at least three months. LOTell has two options at present: first, to introduce the internet package without the survey result in order to make sure that no competitors are present

at the time of market entry, and second, to wait for the survey result in order to minimize the risk of failing to attract enough customers. If LOTell decides to wait for the survey result, there are three possible outcomes: the market growth is rapid (30 percent probability), steady (40 percent), or slow (30 percent). Depending on the survey result, LOTell may decide to introduce or not introduce the new internet service. If it decides to launch the new service after the survey, which is three months from now, then there is a 70 percent chance that the competitors will come up with a similar service package. What decision will result in the highest expected market share for LOTell?

### ANSWER

A decision tree for the problem is shown in Figure 12.19, which also shows the results of the analysis. The recommended decision is to introduce the internet service now because it produces a higher expected gain in market share compared to waiting for the survey results. ■

**Figure 12.19 Decision Tree Analysis for LOTell**



## S U M M A R Y

In this chapter, we considered three basic methods used by analysts to better understand the effect that uncertainties and risks in estimated cash flows have on economic decisions. The first was the use of sensitivity graphs. Sensitivity graphs illustrate the sensitivity of a particular measure (e.g., present worth or annual worth) to changes in one or more of the parameters of a project. The second method was the use of break-even analysis for evaluating both individual projects and comparisons among projects. Finally, decision trees were introduced. A decision tree is a graphical representation of the logical structure of a decision problem in terms of the sequence of decisions and outcomes of chance events. An analysis based on decision trees can therefore account for the sequential nature of decisions.

## ENGINEERING ECONOMICS IN ACTION, PART 12B

### Chances Are Good

“So let me get this straight,” Clem started in, a few days later. “You structured this as a tree. We—Canadian Widgets—have the first decision of whether to proceed with development of the Adaptamatic at all. Then the next node represents the probability that the Adaptamatic will have a competitor. Then there is our choice of going into production or not. Then there are the customers, which we represent as chance nodes, who can have low, medium, and high demand. Finally, there are the outcomes, each of which has a dollar value associated with it. Is that right?”

“Perfect,” Naomi said.

“And the dollar values came from . . . ?”

“Well, some reasonable assumptions about the marketplace,” Naomi replied. “We had enough information to quantify three levels of demand. We figured we had as much consumer acceptance as any competitor because of our association with Powerluxe, so when a competitive product was in the marketplace, we assumed we would have the same market share as anyone else.”

“What about the probabilities?” Clem continued.

“That was harder,” Bill answered. Naomi couldn’t help but smile. A few days ago, she didn’t know anything about marketing and Bill didn’t know about decision trees, but now she was answering the questions about marketing and he was answering the ones about decision trees! Go figure.

“We had no hard information on probabilities,” Bill continued. “We set it all up as a spreadsheet so that we could adjust the probabilities freely. First we put in our best guesses—subjective probabilities. We talked to Prabha in marketing and to a bunch of people at Powerluxe, and refined that down. Finally, we tested it for a whole range of possibilities. Bottom line: Chances are good that we will make a killing.”

Clem looked at the decision tree and at the table Bill gave him that reported the expected value of proceeding with the Adaptamatic development under various assumptions. “Are you sure this is right? How come we make more money if we have a competitor than if we don’t?”

“Good observation, Clem.” Bill was beaming. “That’s interesting, isn’t it? That’s what’s called ‘building the category’ in marketing. The principle is that when you have a new product, you have to spend a lot of resources educating the consumer. If you have to do that yourself, it’s really costly. But a competitor can work for you by taking a share of those education costs and actually making it cheaper for you. That’s what happens here.”



"It was the decision tree approach that revealed this dynamic in this case," Bill went on. "By ourselves, we could lose money if demand was low, even though the expected value was fairly high. However, with a competitor on the market, we can almost guarantee making money because the efforts of our competitor reduce our marketing costs and expand the market at the same time. Combine them both in the decision tree and we have a high expected value with almost no risk. Cool, eh?"

"Really cool." Clem glanced at Naomi with an appraising look. "Speaking of chances—chances are good that Anna's going to like this. Nice job."

## PROBLEMS

### A. Key Concepts

- 12.1** Identify possible parameters that are involved in economic analysis for the following situations:
- Buying new equipment
  - Supplying products to a foreign country with a high inflation rate
- 12.2** For the following examples of parameters, how would you assign a reasonable base case and a range of variation so that you can carry out sensitivity analysis? Assign specific numerical figures wherever you can.
- Canada's inflation rate
  - U.S. dollar exchange rate for Canadian dollars
  - Expected annual savings from a new piece of equipment similar to the one you already have
  - Expected annual revenue from an internet-based business
  - Salvage value of a personal computer
- 12.3** Which sensitivity analysis method may be appropriate for analyzing the following uncertain situations?
- Corral Cartage leases trucks to service its shipping contracts. Larger trucks have cheaper operating costs if there is sufficient business, but are more expensive if they are not full. Corral Cartage is not certain about future demand.
  - Joan runs a dog kennel. She is considering installing a heating system for the interior runs that will allow her to operate all year. Joan is not sure how much the annual heating expenses will be.
  - Pushpa runs a one-person company producing custom paints for hobbyists. She is considering buying printing equipment to produce her own labels. However, she is not sure if she will have enough orders in the future to justify the purchase of the new equipment.
  - Lemuel is an engineer working for the electric company. He is estimating the total cost for building transmission lines from a distant nuclear plant to new industrial parks north of the city. Lemuel is uncertain about the construction cost (per kilometre) of transmission lines.

- (e) Thanh's company is growing very quickly and has a hard time meeting its orders. An opportunity to purchase additional production equipment has arisen. She is not certain if the company will continue to grow at the same rate in the future, and she is not even certain how long the growth may last.
- 12.4** The Kelowna Go-Kart Klub has decided to build a clubhouse and track several years from now. The club needs to accumulate \$50 000 by setting aside a uniform amount at the end of every year. It believes it is possible to set aside \$7000 every year at 10 percent interest. It wishes to know how many years it will take to save \$50 000 and how sensitive this result is to a 5 percent and a 10 percent increase or decrease in the amount saved per year and in the interest rate. Construct a sensitivity graph to illustrate the situation.
- 12.5** A chemical plant is considering installing a new water purification system that costs \$21 500. The expected service life of the system is 10 years and the salvage value is computed using the declining-balance method with a depreciation rate of 20 percent. The operating and maintenance costs are estimated to be \$5 per hour of operation. The expected savings are \$10 per operating hour.
- Find the annual worth of the new water purification system if the current operating hours are 1500 per year on average. The MARR is 10 percent.
  - What is the break-even level of operating hours? Construct a graph showing the annual worth for various levels of operating hours.
- 12.6** Fantastic Footwear can invest in one of two different automated clicker cutters. The first, A, has a \$100 000 first cost. A similar one, B, with many extra features, has a first cost of \$400 000. A will save \$50 000 per year over the cutter now in use. B will save between \$120 000 and \$150 000 per year. Each clicker cutter will last five years and have a zero scrap value.
- If the MARR is 10 percent, and B will save \$150 000 per year, which alternative is better?
  - B will save between \$120 000 and \$150 000 per year. Determine the IRR for the incremental investment from A to B for this range, in increments of \$5000. Plot savings of B versus the IRR of the incremental investment. Over what range of savings per year is your answer from part (a) valid? What are the break-even savings for alternative B such that below this amount, A is preferred and above this amount, B is preferred?
- 12.7** Ridgely Custom Metal Products (RCMP) must purchase a new tube bender. It is considering two alternatives that have the following characteristics:

|                | Model T   | Model A   |
|----------------|-----------|-----------|
| First cost     | \$100 000 | \$150 000 |
| Economic life  | 5 years   | 5 years   |
| Yearly savings | \$50 000  | \$62 000  |
| Salvage value  | \$20 000  | \$30 000  |

Construct a break-even graph showing the present worth of each alternative as a function of interest rates between 6 percent and 20 percent. Which is the preferred choice at

8 percent interest? Which is the preferred choice at 16 percent interest? What is the break-even interest rate?

- 12.8** Julia must choose between two different designs for a safety enclosure. Model A has a life of three years, has a first cost of \$8000, and requires maintenance of \$1000 per year. She believes that a salvage value can be estimated for model A using a depreciation rate of between 30 percent and 40 percent and declining-balance depreciation. Model B will last four years, has a first cost of \$10 000, and has maintenance costs of \$700 per year. A salvage value for model B can be estimated using straight-line depreciation and the knowledge that after one year the salvage value will be \$7500. Interest is at 11 percent. Which of the two models would you suggest Julia choose? What break-even depreciation rate for model A will make her indifferent between the two models? Construct a sensitivity graph showing the break-even depreciation rate.
- 12.9** An investment has possible rates of return of 7 percent, 10 percent, and 15 percent over five years. The probabilities of attaining these rates, estimated on the basis of the current economy, are 0.65, 0.25, and 0.1, respectively. If you have \$10 000 to invest, what is the expected rate of return from this investment?
- 12.10** Rockies Adventure Wear, Inc., sells athletic and outdoor clothing through catalogue sales. Its managers want to upgrade their order-processing centre so that they have less chance of losing customers by putting them on hold. The upgrade may result in a processing capacity of 30, 40, 50, or 60 calls per hour with the probabilities of 0.2, 0.4, 0.3, and 0.1, respectively. Market research indicates that the average number of calls that Rockies may receive is 50 per hour. How many customers are expected to be lost per hour due to the lack of processing capacity?
- 12.11** Power Tech builds power-surge protection devices. One of the components, a plastic moulded cover, can be produced by two automated machines, A1 and X1000. Each machine produces a number of defects with probabilities shown in the following table.

| A1                             |             | X1000                          |             |
|--------------------------------|-------------|--------------------------------|-------------|
| No. of Defects<br>(out of 100) | Probability | No. of Defects<br>(out of 100) | Probability |
| 0                              | 0.3         | 0                              | 0.25        |
| 1                              | 0.28        | 1                              | 0.33        |
| 2                              | 0.15        | 2                              | 0.26        |
| 3                              | 0.15        | 3                              | 0.1         |
| 4                              | 0.1         | 4                              | 0.05        |
| 5                              | 0.02        | 5                              | 0.01        |

Which machine is better with regard to the expected number of defective products?

- 12.12** Lightning City is famous for having many thunderstorms during the summer months (from June to August). One of the CB Electronix factories is located in Lightning City. It has collected information, shown in the table that follows, regarding the number of blackouts caused by lightning.

| Number of Blackouts<br>(per Month) | Probability<br>(Summer Months) | Probability<br>(Non-Summer Months) |
|------------------------------------|--------------------------------|------------------------------------|
| 0                                  | 0                              | 0.45                               |
| 1                                  | 0.4                            | 0.4                                |
| 2                                  | 0.25                           | 0.15                               |
| 3                                  | 0.2                            | 0                                  |
| 4                                  | 0.1                            | 0                                  |
| 5                                  | 0.05                           | 0                                  |

For the first three blackouts in a month, the cost due to suspended manufacturing is \$800 per blackout. For the fourth and fifth blackouts, the cost increases to \$1500 per blackout. A local insurance company offers protection against lightning-related expenses. The monthly payment is \$500 for complete annual coverage. Assume that the number of blackouts in any month is independent of those in any other month. What is the expected cost related to blackouts over the summer months? Over the non-summer months? Should CB consider purchasing the insurance policy?

- 12.13** Randall at Churchill Circuits (CC) has just received an emergency order for one of CC's special-purpose circuit boards. Five are in stock at the moment. However, when they were tested last week, two were defective but were mixed up with the three good ones. There is not enough time to retest the boards before shipment to the customer. Randall can either choose one of the five boards at random to ship to the customer or he can obtain a proven non-defective one from another plant. If the customer gets a bad board, the total incremental cost to CC is \$10 000. The incremental cost to CC of getting the board from another plant is \$5000.

- (a) What is the chance that the customer gets a bad board if Randall sends one of the five in stock?
- (b) What is the expected value of the decision to send the customer one of the five boards in stock?
- (c) Draw a decision tree for Randall's decision. On the basis of EV, what should he do?

- 12.14** St. Jacobs Cheese Factory (SJCF) is getting ready for a busy tourist season. SJCF wants to either increase production or produce the same amount as last year, depending on the demand level for the coming season. SJCF estimates the probabilities for high, medium, and low demands to be 0.4, 0.35, and 0.25, respectively, on the basis of the number of tourists forecasted by the local recreational bureau. If SJCF increases production, the expected profits corresponding to high, medium, and low demands are \$750 000, \$350 000, and \$100 000, respectively. If SJCF does not increase production, the expected profits are \$500 000, \$400 000, and \$200 000, respectively. Construct a decision tree for SJCF. On the basis of EV, what should SJCF do?

- 12.15** LOTell, a telephone company, has two options for its new internet service package: It can introduce a combined rate for the residential phone line and the internet access or it can offer various add-on internet service rates in addition to the regular phone rate. LOTell can only afford to introduce one of the packages at this point. The expected gain in market share by introducing the internet service would likely differ for different market growth rates. LOTell has estimated that if it introduces the combined rate, it would gain 30 percent, 15 percent, and 3 percent of the market share with rapid, steady,

or slow market growth. If it introduces the add-on rates, it gains 15 percent, 10 percent, and 5 percent of the market share with rapid, steady, or slow market growth. Construct a decision tree for LOTell. If it wishes to maximize expected market share growth, which package should LOTell introduce to the market now?

## B. Applications

- 12.16** A new software package is expected to improve productivity at Suretown Insurance. However, because of training and implementation costs, savings are not expected to occur until the third year of operation. Annual savings of approximately \$10 000 are expected, increasing by about \$1000 per year for the following five years. After this time (eight years from implementation), the software will be abandoned with no scrap value. Construct a sensitivity graph showing what would happen to the present worth of the software with 7.5 percent and 15 percent increases and decreases in the interest rate, the \$10 000 base savings, and the \$1000 savings gradient. MARR is 15 percent.
- 12.17** A regional municipality is studying a water supply plan for the area to the end of the year 2050. To satisfy the water demand, one suggestion is to construct a pipeline from a distant lake. It is now the end of 2010. Construction would start in the year 2015 (five years from now) and take five years to complete at a cost of \$20 million per year. Annual maintenance and repair costs are expected to be \$2 million and will start the year following project completion (all costs are based on current estimates). From a predicted inflation rate of 3 percent per year, and the real MARR, city engineers have determined that a MARR of 7 percent per year is appropriate. Assume that all cash flows take place at the end of the year and that there is no salvage value at the end of 2055.
- (a) Find the present worth of the project.
  - (b) Construct a sensitivity graph showing the effects of 5 percent and 10 percent increases and decreases in the construction costs, maintenance costs, and inflation rate. To which is the present worth most sensitive?
- 12.18** The city of Brandon is installing a new swimming pool in the downtown recreation centre. One design being considered is a reinforced concrete pool that will cost \$6 000 000 to install. Thereafter, the inner surface of the pool will need to be refinished and painted every 10 years at a cost of \$40 000 per refinishing. Assuming that the pool will have essentially an infinite life, what is the present worth of the costs associated with the pool design? The city uses a MARR of 5 percent. If the installation costs, refinishing costs, and MARR are subject to 5 percent or 10 percent increases or decreases, how is the present worth affected? To which parameter is the present worth most sensitive?
- 12.19** You and two friends are thinking about setting up a grocery delivery service for local residents to finance your last two years at university. In order to start up the business, you will need to purchase a car. You have found a used car that costs \$6000 and you expect to be able to sell it for \$3000 at the end of two years. Insurance costs are \$600 for each six months of operation, starting now. Advertising costs (e.g., flyers, newspaper advertisements) are estimated to be \$100 per month, but these might vary as much as 20 percent above or below the \$100, depending on the intensity of your advertising. The big questions you have now are how many customers you will have and how much of a service fee to charge per delivery. You estimate that you will have 300 deliveries every month, and are thinking of setting a \$2-per-delivery fee, payable at the end of each month. The interest rate over the two-year period is expected to be 8 percent per year, compounded monthly, but may be 20 percent above or below this figure.

Using equivalent monthly worth, construct a sensitivity graph showing how sensitive the monthly worth of this project will be to the interest rate, advertising costs, and

the number of deliveries you make each month. To which parameter is the equivalent monthly worth most sensitive?

- 12.20** Timely Testing (TT) does subcontracting work for printed circuit board manufacturers. It performs a variety of specialized functional tests on the assembled circuit boards. TT is considering buying a new probing device that will assist the technicians in diagnosing functional defects in the printed circuit boards. Two vendors have given the company quotes on first costs and expected operating costs over the life of its equipment.

|                   | Vendor A                    | Vendor B                    |
|-------------------|-----------------------------|-----------------------------|
| Expected life     | 7 years                     | 10 years                    |
| First cost        | \$200 000                   | \$350 000                   |
| Maintenance costs | \$10 000/year + \$0.05/unit | \$20 000/year + \$0.01/unit |
| Labour costs      | \$1.25/unit                 | \$0.50/unit                 |
| Other costs       | \$6 500/year + \$0.95/unit  | \$15 000/year + \$0.55/unit |
| Salvage value     | \$5 000                     | \$20 000                    |

Production levels vary for TT. They may be as low as 20 000 boards per year or as high as 200 000 boards per year if a contract currently under negotiation comes through. TT expects, however, that production quantities will be about 50 000 boards. Timely Testing uses a MARR of 15 percent for equipment projects and will be using an annual worth comparison for the two devices.

Timely Testing is aware that the equipment vendors have given estimates only for costs. In particular, TT would like to know how sensitive the annual worth of each device is to the first cost, annual fixed costs (maintenance + other), variable costs (maintenance + labour + other), and the salvage value.

- (a) Construct a sensitivity graph for vendor A's device, showing the effects of 5 percent and 10 percent decreases and increases in the first cost, annual fixed costs, variable costs, and the salvage value. Assume an annual production level of 50 000 units.
- (b) Construct a sensitivity graph for vendor B's device, showing the effects of 5 percent and 10 percent decreases and increases in the first cost, annual fixed costs, variable costs, and the salvage value. Assume an annual production level of 50 000 units.

- 12.21** Refer back to Problem 12.19.

- (a) Assuming base case figures for advertising costs and interest rates, what is the break-even number of deliveries per month? Construct a graph showing the break-even number.
- (b) Assuming base case figures for advertising costs and number of deliveries per month, what is the break-even interest rate? Construct a graph illustrating the break-even interest rate.

- 12.22** Merry Metalworks would like to implement a local area network (LAN) for file transfer, email, and database access throughout its facility. Two feasible network topologies have been identified, which it has labelled alternative A and alternative B. The three main components of costs for the network are (1) initial hardware and installation costs, (2) initial software development costs, and (3) software and hardware maintenance costs. The installation and hardware costs for both systems are somewhat uncertain, as prices for the components are changing and Merry Metalworks is not

sure of the installation costs for the LAN hardware. The costs for each alternative are summarized below.

|   | Alternative A | Alternative B |
|---|---------------|---------------|
| Initial hardware and installation costs (\$): |               |               |
| Optimistic estimate                           | 70 000        | 86 000        |
| Average estimate                              | 92 500        | 105 500       |
| Pessimistic estimate                          | 115 000       | 125 000       |
| Initial software cost (\$)                    | 138 750       | 158 250       |
| Annual maintenance costs (\$)                 | 9250          | 10 550        |
| Annual benefits (\$):                         |               |               |
| Optimistic estimate                           | 80 000        | 94 000        |
| Average estimate                              | 65 000        | 74 000        |
| Pessimistic estimate                          | 50 000        | 54 000        |

Benefits from the LAN are increased productivity because of faster file transfer times, reduced data redundancy, and improved data accuracy because of the database access. The benefits were difficult to quantify and are stated below as only a range of possible values and an average.

Merry Metalworks uses a 15 percent MARR and has established a 10-year study period for this decision. It wishes to compare the projects on the basis of annual worth.

- (a) Construct a sensitivity graph for alternative A. For the base case, use the average values for the initial hardware cost and the annual benefits. Each graph should indicate the effect of a 5 percent and a 10 percent drop or increase in the initial hardware cost and the annual benefits. Which of the two factors most affects the annual worth of alternative A?
  - (b) Construct a sensitivity graph for alternative B. For the base case, use the average values for the initial hardware cost and the annual benefits. Each graph should indicate the effect of a 5 percent and a 10 percent drop or increase in the initial hardware cost and the annual benefits. Which of the two factors most affects the annual worth of alternative B?
- 12.23** Refer back to Problem 12.4. Members of the Go-Kart Klub do not wish to wait for more than five years to build their clubhouse. They have decided to start a fundraising campaign to increase their ability to save each year between \$7000 and whatever is necessary to have \$50 000 saved in five years. Construct a table and a graph that illustrate how the number of years they must wait depends on the amount they save each year. What additional funds per year will allow them to save \$50 000 in five years? Use a 10 percent interest rate.
- 12.24** The Bountiful Bread Company produces home bread-making machines. Currently, it pays a custom moulder \$0.19 per piece (not including material costs) for the clear plastic face on the control panel. Demand for the bread-makers is forecast to be 200 000 machines per year, but there is some uncertainty surrounding this estimate. Bountiful is considering installing a plastic moulding system to produce the parts itself. The moulder costs \$20 000 plus \$7000 to install, and has an expected life of six years. Operating and maintenance costs are expected to be \$30 000 in the first year and to rise at the rate of 5 percent per year. Bountiful estimates its capital costs using a declining-balance depreciation model with a rate of 40 percent, and uses a MARR of 15 percent for such investments.

Determine the total equivalent annual cost of the new moulder. What is the cost per unit, assuming that production is 200 000 units per year? Also, determine the break-even production quantity. That is, what is the production quantity below which it is better to continue to purchase parts and above which it is better to purchase the moulder and make the parts in-house?

- 12.25** Tenspeed Trucking (TT) is considering the purchase of a new \$65 000 truck. The truck is expected to generate revenues between \$12 000 and \$22 000 each year, and will have a salvage value of \$20 000 at the end of its five-year life. TT pays taxes at the rate of 35 percent. TT is located in Canada and the CCA rate for trucks is 30 percent. TT's after-tax MARR is 12 percent. Find the annual worth of the truck if the annual revenues are \$12 000, and for each \$1000 revenue increment up to \$22 000. What is the break-even annual revenue? Provide a graph to illustrate the break-even annual revenue.
- 12.26** A new bottle-capping machine costs \$45 000, including \$5000 for installation. Operating and maintenance costs are expected to be \$3000 for the first year, increasing by \$1000 each year thereafter. The salvage value is calculated by straight-line depreciation where a value of zero is assumed at the end of the service life.
- Construct a spreadsheet that computes the equivalent annual cost (EAC) for the bottle capper. What is the economic life if the expected service life is 6, 7, 8, 9, or 10 years? Interest is 12 percent.
  - How sensitive is the economic life to the different length of service life? Construct a sensitivity graph to illustrate this point.
- 12.27** Sam is considering buying a new lawnmower. He has a choice between a "Lawn Guy" model or a Bargain Joe's "Clip Job" model. Sam has a MARR of 5 percent. The mowers' salvage values at the end of their respective service lives are zero. Sam has collected the following information about the two mowers.

|                    | Lawn Guy | Clip Job |
|--------------------|----------|----------|
| First cost         | \$350    | \$120    |
| Life               | 10 years | 4 years  |
| Annual gas         | \$60     | \$40     |
| Annual maintenance | \$30     | \$60     |

Although Sam has estimated the maintenance costs of the Clip Job at \$60, he has heard that the machines have had highly variable maintenance costs. One friend claimed that her Clip Job had maintenance costs comparable to those of the Lawn Guy, but another said the maintenance costs could be as high as \$80 per year. Construct a table that shows the annual worth of the Clip Job for annual maintenance costs varying from \$30 per year to \$80 per year. What Clip Job maintenance costs would make Sam indifferent about the choice between the two mowers, on the basis of annual worth? Construct a graph showing the break-even maintenance costs. Which mower would you recommend to Sam?

- 12.28** Ganesh is considering buying a \$24 000 car. After five years, he thinks he will be able to sell the car for \$8000, but this is just an estimate that he is not certain about. He is confident that gas will cost \$2000 per year, insurance \$800 per year, and parking \$600 per year, and that maintenance costs for the first year will be \$1000, rising by \$400 per year thereafter.

The alternative is for Ganesh to take taxis everywhere. This will cost an estimated \$7000 per year. If he has no car, Ganesh will rent a car for the family vacation every year at a total (year-end) cost of \$1000. Ganesh values money at 11 percent annual interest. If the salvage value of the car is \$8000, should he buy the car? Base your answer on annual worth. Determine the annual worth of the car for a variety of salvage values so that you can help Ganesh decide whether this uncertainty will affect his decision. At what break-even salvage value will he be indifferent between taking taxis and buying a car? Construct a break-even graph showing the annual worth of both alternatives as a function of the salvage value of the car. What advice would you give Ganesh?

- 12.29** A new wave-soldering machine is expected to generate monthly savings of either \$800 000, \$1 000 000, \$1 200 000, or \$1 400 000 over the next two years. The manager is not sure about the likelihood of the four savings scenarios, so she assumes that they are equally likely. What is the present worth of the expected monthly savings? Use a MARR of 12 percent, compounded monthly, for this problem.
- 12.30** Regional Express is a small courier service. By introducing a new computerized tracking device, it anticipates some increase in revenue, currently estimated at \$2.75 per parcel. The possible new revenue ranges from \$2.95 to \$5.00 per parcel, with probabilities shown in the table below. Assuming that Regional's monthly capacity is 60 000 parcels and the monthly operating and maintenance costs are \$8000, what is the present worth of the expected revenue over 12 months? Regional's MARR is 12 percent, compounded monthly.

|                           |        |        |        |        |        |
|---------------------------|--------|--------|--------|--------|--------|
| <b>Revenue per parcel</b> | \$2.95 | \$3.25 | \$3.50 | \$4.00 | \$5.00 |
| <b>Probability</b>        | 0.1    | 0.35   | 0.3    | 0.15   | 0.1    |

- 12.31** Katrina is thinking about buying an automobile. She figures her monthly payments will be €90 for insurance, €30 for gas, and €20 for general maintenance. The car she would like to buy may last for four, five, or six years before a major repair, with probabilities of 0.4, 0.4, and 0.2, respectively. Calculate the present worth of the monthly expenses over the expected life of the car (before a major repair). Katrina's MARR is 10 percent, compounded monthly.
- 12.32** Pharma-Excel is a pharmaceutical company. It is currently studying the feasibility of a research project that involves improvement of vitamin C pills. To examine the optimistic, expected, and pessimistic scenarios for this project, it gathered the data shown below. What is the expected annual cost of the vitamin C project? Assume Pharma-Excel's MARR is 15 percent. Note that the lead time is different for each scenario.

|  | Optimistic | Expected  | Pessimistic |
|--|------------|-----------|-------------|
| Research and development costs<br>(at the end of research) | \$75 000   | \$240 000 | \$500 000   |
| Lead time to production (years)                            | 1          | 2         | 3           |
| Probability  | 0.15       | 0.5       | 0.35        |

- 12.33** Mega City Hospital is selling lottery tickets. All proceeds go to its cancer research program. Each ticket costs \$100, but the campaign catchphrase promises a 1-in-1000

chance of winning the first prize. The first prize is a “dream” house, which is worth \$250 000. On the basis of decision tree analysis, is buying a ticket worthwhile?

- 12.34** See Problem 12.33. Determine the price of a ticket so that not buying a ticket is the preferred option and determine the chance of winning so that not buying a ticket is the preferred option.

- 12.35** Bockville Brackets (BB) uses a robot for welding small brackets onto car-frame assemblies. BB’s R&D team is proposing a new design for the welding robot. The new design should provide substantial savings to BB by increasing efficiency in the robot’s mobility. However, the new design is based on the latest technology, and there is some uncertainty associated with the performance level of the robot. The R&D team estimates that the new robot may exhibit high, medium, and low performance levels with the probabilities of 0.35, 0.55, and 0.05 respectively. The annual savings corresponding to high, medium, and low performance levels are \$500 000, \$250 000, and \$150 000 respectively. The development cost of the new robot is \$550 000.

- (a) On the basis of a five-year study period, what is the present worth of the new robot for each performance scenario? Assume BB’s MARR is 12 percent.
- (b) Construct a decision tree. On the basis of EV, should BB approve the development of a new robot?

- 12.36** Refer to Review Problem 12.3 on page 451. Power Tech is still considering the partnership with the Chinese manufacturing company. The analysis in Review Problem 12.3 has shown that the partnership is recommended (by a marginal difference in the expected revenue increase between the two options). Power Tech now wants to further examine the possible shipping delay and quality control problems associated with the partnership. Power Tech estimates that shipping may be delayed 40 percent of the time due to the distance. Independently of the shipping problem, there may be a quality problem 25 percent of the time due to communication difficulties and lack of close supervision by Power Tech. The payoff information is estimated as shown below. Develop a decision tree for Power Tech’s shipping and quality control problems and analyze it. On the basis of EV, what is the recommendation regarding the possible partnership?

| Shipping Problem  | Quality Problem    | Gain in Annual Profit |
|-------------------|--------------------|-----------------------|
| No shipping delay | Acceptable quality | \$ 200 000            |
| No shipping delay | Poor quality       | 25 000                |
| Shipping delay    | Acceptable quality | 100 000               |
| Shipping delay    | Poor quality       | -100 000              |

- 12.37** Refer to Problem 12.10. Rockies Adventure Wear, Inc. has upgraded its order-processing centre in order to improve the processing speed and customer access rate. Before completely switching to the upgraded system, Rockies has an option of testing it. The test will cost Rockies \$50 000, which includes the testing cost and loss of business due to shutting down the business for a half-day. If Rockies does not test the system, there is a 55 percent chance of severe failure (\$150 000 repair and loss of business costs), a 35 percent chance of minor failure (\$35 000 repair and loss of business costs), and a 10 percent chance of no failure. If Rockies tests the system, the result can be favourable with the probability of 0.34, which requires no modification, and not favourable with the probability of 0.66. If the test result is not favourable, Rockies has two options: minor modification and major modification. The minor modification costs \$5000 and the major modification costs

\$30 000. After the minor modification, there is still a 15 percent chance of severe failure (\$150 000 costs), a 45 percent chance of minor failure (\$35 000 costs), and a 40 percent chance of no failure. Finally, after the major modification, there is still a 5 percent chance of severe failure, a 30 percent chance of minor failure, and a 65 percent chance of no failure. What is the recommended action for Rockies, using a decision tree analysis?

### More Challenging Problems

**12.38** Refer back to Problem 12.22 in which Merry Metalworks is considering two LAN alternatives.

- (a) For alternative A, by how much will the installation cost have to rise before the annual worth becomes zero? In other words, what is the break-even installation cost? Is the break-even level within or above the range of likely values Merry Metalworks has specified?
- (b) What is the break-even annual benefit for alternative A? Use the average installation costs. Is the break-even level within or above the range of likely values Merry Metalworks has specified?

**12.39** Repeat Problem 12.38 for alternative B.

**12.40** Refer back to Timely Testing, Problem 12.20.

- (a) TT charges \$3.25 per board tested. Assuming that costs are as in vendor A's estimates, what production level per year would allow TT to break even if it selects vendor A's equipment? That is, for what production level would annual revenues equal annual costs? Construct a graph showing total revenues and total costs for various production levels, and indicate on it the break-even production level.
- (b) Repeat part (a) for vendor B's equipment.

**12.41** Your neighbour, Kelly Strome, is trying to make a decision about his growing home-based copying business. He needs to acquire colour copiers able to handle maps and other large documents. He is looking at one set of copiers that will cost \$15 000 to purchase. If he purchases the equipment, he will need to buy a maintenance contract that will cost \$1000 for the first year, rising by \$400 per year afterward. He intends to keep the copiers for five years, and expects to salvage them for \$2500. The CCA rate for office equipment is 20 percent.

Rather than buy the copiers, Kelly could lease them for \$5500 per year with no maintenance fee. His business volume has varied over the past few years, and his tax rate has varied from a low of 20 percent to a high of 40 percent. Kelly's current cost of capital is 8 percent. Kelly has asked you for some help in deciding what to do. He wants to know whether he should lease or buy the copiers and, moreover, he wants to know the impact of his tax rate on the decision. Evaluate both alternatives for him for a variety of tax rates between 20 percent and 40 percent so that you can advise him confidently. What do you advise?

**12.42** Western Insurance wants to introduce a new accounting software package for its human resources department. A small-scale version is sufficient and economical if the number of employees is less than 50. A large-scale version is effective for managing 80 employees or more. All relevant information on the two packages is shown in the following table. Western Insurance's business is growing, and the number of employees has increased from 10 to 40 in the past three years. Construct a graph showing the annual worth of the two software packages as a function of the number of employees ranging from 40 to 100. On the basis of break-even analysis, which accounting package is a better choice for Western Insurance? MARR is 12 percent.

| Parameter   |   | Small-Scale       | Large-Scale       |
|---|---|-------------------|-------------------|
| First cost (\$)   |   | 6000              | 10 000            |
| Training cost at the time of installation (\$)  |   | 1500              | 3500              |
| Service life (years)  |   | 5                 | 5                 |
| Salvage value   |   | 0                 | 0                 |
| Expected annual savings<br>(\$ per employee) if the<br>average number of employees<br>over the next 5 years is: | Less than 50<br>Between 50<br>and 80<br>Greater than 80 | 200<br>170<br>120 | 250<br>300<br>400 |

**12.43** Refer to Problem 12.32. As a part of Pharma-Excel's feasibility study, it wants to include information on the acceptance attitude of the public toward the new vitamin C product. Regardless of the optimistic, expected, and pessimistic scenarios for research and development, there is a chance the general public may not feel comfortable with the new product because it is based on a new technology. It estimates that the likelihood of the public accepting the product (and purchasing it) is 33.3 percent and not accepting it is 66.7 percent. The expected annual profit after the research is \$1 000 000 if the public accepts the new product and \$200 000 if the public does not accept it.

(a) Calculate the annual worth for all possible combinations of three R&D scenarios (optimistic, expected, and pessimistic) and two scenarios for public reaction (accept or not accept). Pharma-Excel's MARR is 15 percent.

(b) Using the annual worth information as the payoff information, build a decision tree for Pharma's problem. Should it proceed with the development of this new vitamin C product?

**12.44** Baby Bear Beads (BBB) found itself confronting a decision problem when a packaging line suffered a major breakdown. Ross, the manager of maintenance, Rita, the plant manager, and Ravi, the company president, met to discuss the problem.

Ross reported that the current line could be repaired, but the cost and result were uncertain. He estimated that for \$40 000, there was a 75 percent chance the line would be as good as new. Otherwise, an extra \$100 000 would have to be spent to achieve the same result.

Rita's studies suggested that for \$90 000, the whole line might be replaced by a new piece of equipment. However, there was a 40 percent chance an extra \$20 000 might be required to modify downstream operations to accept a slightly different package size.

Ravi, who had reviewed his sales projections, revealed that there was a 30 percent chance the packaging line would no longer be required anyway, but that this wouldn't be known until after a replacement decision was made. Rita then pointed out that there was an 80 percent chance the new equipment she proposed could easily be adapted to other purposes, so that the investment, including the modifications to downstream operations, could be completely recovered even if the line was no longer needed. On the other hand, the repaired packaging line would have to be scrapped with essentially no recovery of the costs.

The present worth of the benefit of having the line running is \$150 000. Use decision tree analysis to determine what BBB should do about the packaging line.

**12.45** Refer to Review Problem 12.3 on page 451. Power Tech feels comfortable about the probability estimate regarding the change in North American demand. However, it would like to examine the probability estimate for Asian demand more carefully. Perform sensitivity analysis on the probability that Asian demand increases. Try the following values, {0.1, 0.2, 0.3, 0.4, 0.5}, in which 0.3 is the base case value. Analyze the result and give a revised recommendation as to Power Tech's possible partnership.

**12.46** Refer to Review Problem 12.4 on page 452. LOTell is happy with the decision recommendation suggested by the previous decision tree analysis considering information on market growth. However, LOTell feels that the uncertainty in market growth is the most important factor in the overall decision regarding the introduction of the internet service package. Hence, it wishes to examine the sensitivity of the probability estimates for the market growth. Answer the following questions on the basis of the decision tree developed for Review Problem 12.4.

- (a) Let  $p_1$  be the probability of rapid market growth,  $p_2$  be the probability of steady growth, and  $p_3$  be the probability of slow market growth. Express the expected value at Node 2, EV(2), in terms of  $p_1$ ,  $p_2$ , and  $p_3$ .
- (b) If  $\text{EV}(2) < 10$ , then the option to introduce the package now is preferred. Using the expression of  $\text{EV}(2)$  that was developed in part (a), graph all possible values of  $p_1$  and  $p_2$  that lead to the decision to introduce the package now. (You will see that  $p_3$  from part (a) is not involved.) What can you observe from the graph regarding the values of  $p_1$  and  $p_2$ ?

**12.47** SoftWaire Inc., a United States-based software development company, is contemplating outsourcing a portion of its operations to Bangalore, India. The primary attractions for SoftWaire are the lower salaries in India and the availability of English-speaking engineers. The average salary of the 20 people it is thinking of replacing with an offshore operation is US\$75 000 per year, while the salary of a comparable engineer in India is 600 000 rupees. With an exchange rate of 40 rupees per dollar, the salary costs in India are about 20 percent of what they are in the United States. This could bring substantial savings to SoftWaire; however, there appear to be numerous uncertainties associated with outsourcing. You have been asked to conduct an analysis so that SoftWaire Inc. can better understand the potential benefits and risks of the project.

Based on your initial data gathering, you have found the following information.

1. The salaries of software developers in India are indeed lower than those in the United States; however, demand from other local and international employers is strong and is causing salaries to increase at a rate of 10 percent–15 percent per year, with some forecasts as high as 20 percent per year. By comparison, American salaries are expected to grow at a rate of roughly 5 percent per year.
2. Due to the high level of competition for good software developers in India, there is a high turnover rate in the industry. Estimates run from 20 percent to 30 percent. The American market is stable by comparison.
3. Most foreign companies use the services of a recruiter to find suitable employees. Recruiters typically charge a fee of about 30 percent of the salary of the successful employee to solicit applications, screen candidates, and present alternatives for final selection.
4. Training costs for new employees in their first year can be up to 70 percent of their salary for education in best practices and procedures used by the employer. Their

productivity is typically about 50 percent of a trained employee's due to the learning process.

5. For development outsourcing to be successful, other firms report that they spend considerable time ensuring that software specifications are precise, that quality assurance procedures are clearly spelled out and followed, and that developers adhere to best practices. The cost of this additional project management has been estimated to add 10 percent–20 percent to salary costs.
6. Periodic problems with infrastructure and cultural differences have been reported to reduce overall development productivity to about 70 percent of that in the United States.
7. Setting up an outsourced team can entail considerable start-up costs. Travel, recruiter selection, facilities selection, and equipment installation for a team of 20 can range from \$300 000 to \$500 000.
8. SoftWaire would lay off 20 software engineers at the start of the project. This will cost one year's salary per person.
9. Office space lease costs will be roughly the same whether SoftWaire outsources or not.
10. SoftWaire uses a MARR of 25 percent.
11. The American dollar has fluctuated against the Indian rupee in recent years, and there is some concern that the rupee may strengthen against the dollar by as much as 10 percent in coming years.

You have decided to set up a spreadsheet to help with your analysis. From the information above, you plan to use the following as variables in a sensitivity analysis: (1) salary growth rates for both American and overseas software developers, (2) turnover rate of overseas employees, (3) overseas recruiting costs, as a percent of salary, (4) training costs, as a percent of salary, (5) productivity rate of an employee in training, (6) productivity rate of a trained employee, (7) additional project management costs, as a percent of salary, (8) SoftWaire's MARR, (9) the exchange rate between the rupee and the American dollar (currently 40 rupees per dollar), and (10) start-up costs.

- (a) Construct a spreadsheet with the above variables to project the costs of outsourcing for a study period of 10 years. Start with the basic projected salaries, and adjust for recruiting and training costs, then for training and long-term productivity costs, and finally for project management, start-up, and layoff costs. Assume that the overseas developers are hired at time zero. When a range of values has been given for a variable, use the midpoint as the base case.
- (b) Determine the present worth of outsourcing.
- (c) Determine the present worth of insourcing—that is, using the United States-based developers.
- (d) What is the present worth of the benefits of outsourcing?
- (e) Using the spreadsheet, conduct a sensitivity analysis for each of the uncertain factors, varying each factor by 10 percent to determine which has the greatest impact on the present worth of the benefits of outsourcing.
- (f) What factors have the greatest impact on the present worth of the benefits of outsourcing? What would you advise at this point?
- (g) At what exchange rate would the present worth of outsourcing be zero?

- (h) At what productivity rate would the present worth of outsourcing be zero?
- (i) Drawing on specific examples from this problem, describe some of the drawbacks of sensitivity analysis when making the decision to outsource or not.

## MINI-CASE 12.1

### **China Steel Australia Limited**

Stainless steel is an alloy of iron, chromium, and other metals. It has valuable uses for applications where strength and resistance to corrosion are important. Uses include kitchen cookware, surgical tools, building structures, aircraft, food processing equipment, and automotive components.

There are numerous grades of stainless steel, depending on the needs of the particular application. They vary by the alloying elements and the consequent crystalline structure of the steel. About 70 percent of stainless steel production, on a global basis, is *austenitic* stainless steel, which is composed of a maximum of 0.15 percent carbon and a minimum of 18 percent chromium, along with a significant percentage of nickel. A typical grade used for household flatware has 18 percent chromium and 10 percent nickel, for example.

In recent years, the cost and availability of nickel have created a bottleneck for stainless steel producers. There are a limited number of nickel ore deposits, and the majority of deposits consist of *laterite* nickel ore, which is generally not used for producing refined nickel because of its low nickel content.

China Steel has a plant in Linyi, Shandong Province, China, that produces nickel pig iron from laterite nickel ore. This nickel pig iron contains 5 percent–11 percent nickel, which can substitute for refined nickel in the production of austenitic stainless steel. The growth of Chinese manufacturing has created very strong demand for stainless steel in general, and for China Steel's nickel pig iron in particular. The Linyi plant is running at full capacity and has offtake commitments in place for the next five years. There is currently a substantial demand for nickel pig iron that cannot be met. Consequently, China Steel wants to expand its plant to increase capacity in order to take advantage of this opportunity.

In February 2008, China Steel released a prospectus seeking investors to fund its expansion plans. A prospectus is a document that lays out an investment opportunity according to specific rules legislated to ensure that potential investors are treated fairly. The prospectus in this case sought AUS\$15 000 000 (Australian dollars) to provide a foundation for plant expansion by setting up a structure that would facilitate access to capital markets in Australia and elsewhere.

In order to demonstrate the economic viability of the company, the prospectus included a forecast of earnings for 2008. The forecast was based on expected costs and revenues, barring unusual deviations in the economic and competitive environment. Under these conditions, the profit after income tax expenditures for 2008 was estimated to be AUS\$12 304 000.

However, to enrich this estimate, the prospectus also shows the effect of changes in key assumptions on the estimated profit. These changes are shown in Table 11.9.

#### **Discussion**

Predicting the future can never be done with complete assurance because everything is uncertain. We can only make our best guess at the future, recognizing that things might turn out differently than we thought.

**Table 11.9 Sensitivity Analysis for China Steel Australia**

| Parameter                            | Variation  | Increase (\$) | Decrease (\$) |
|--------------------------------------|------------|---------------|---------------|
| Revenue                              | 10% change | 2 105 000     | -2 105 000    |
| Cost of sales                        | 10% change | -1 185 000    | 1 185 000     |
| Administrative expenses              | 10% change | -102 000      | 102 000       |
| Corporate overheads                  | 10% change | -51 000       | 51 000        |
| Cost of borrowing                    | 1%         | 442 000       | -461 000      |
| Australian dollar/yuan exchange rate | 10% change | -745 000      | 909 000       |

Engineering design often assumes that the world is much simpler than it really is. When an engineer designs a roof truss, for example, he or she often assumes that the lumber making up the truss will behave in a standard, predictable manner. Similarly, the engineer who designs a circuit will assume that the electrical components will behave according to their nominal values. But lumber is a natural product, and individual pieces will be weaker or stronger than expected. Electrical components, similarly, will have actual values and behaviour different, in general, from their nominal values and mathematical models.

Good engineers understand this and design accordingly. The truss builder specifies a certain grade of lumber or makes sure that redundant support is built into the design. The circuit designer similarly specifies the tolerances of significant components, or designs the circuit in a robust way.

The role of sensitivity analysis in economic studies is exactly the same. We don't know the cash flow of a project exactly, just as we don't know the behaviour of a piece of wood or a circuit component exactly. We want to design the project to control the uncertainty of the economic elements as well as the physical ones. In the China Steel Australia prospectus, the expected future was insufficient to adequately explain what a prospective investor might expect in terms of financial performance. This needed to be augmented by explaining the effects of variations on this future view.

### Questions

1. Most of the changes made by China Steel were for plus or minus 10 percent. Does this make sense? If not, what amounts would have been a better choice? Why was the variation on the cost of borrowing only 1 percent?
2. It is clear that a 10 percent change in revenue has the biggest effect on profit. Does it necessarily mean that this is also the biggest risk to China Steel's viability as a company?
3. If you were considering investing in China Steel, what would you do to help reduce the uncertainty of the investment?
4. Derek has been assigned the task of designing a parking facility for an insurance company. He must keep in mind a number of different issues, including land acquisition costs, building costs (if a parking building is required), expected usage, fee method (monthly fees, hourly fees, or in-and-out fees), whether the company will subsidize the facility in part or completely, etc. His boss is particularly concerned about reducing the uncertainty of the future cash flows associated with the project. How would you advise Derek?

## Appendix 12A Decision Matrices

Most of this book has been concerned with making decisions based on a single economic measure such as present worth, annual worth, or internal rate of return. This is natural, since many of the decisions that are made by an individual, and most that are made by businesses, have the financial impact of a project as a primary consideration. However, rarely are costs and benefits the only consideration in evaluating a project. Sometimes other considerations are paramount.

For decisions made by and for an individual, cost may be relatively unimportant. One individual may buy vegetables on the basis of their freshness, regardless of the cost. A dress or suit may be purchased because it is fashionable or attractive. A car may be chosen for its comfort and not its cost.

Traditionally, firms were different from individuals in this way. It was felt that all decisions for a firm *should* be made on the basis of the costs and benefits as measured in money (even if they sometimes were not, in practice), since the firm's survival depended solely on being financially competitive.

Society has changed, however. Companies now make decisions that apparently involve factors that are very difficult to measure in monetary terms. Money spent by firms on charities and good causes provides a benefit in image that is very hard to quantify. Resource companies that demonstrate a concern for the environment incur costs with no clear financial benefit. Companies that provide benefits for employees beyond statute or collective agreement norms gain something that is hard to measure.

The fact that firms are making decisions on the basis of criteria other than only money most individuals would hail as a good thing. It seems to be a good thing for the companies, too, since those that do so tend to be successful. However, it can make the process of decision making more difficult, because there is no longer a single measure of value.

Money has the convenient feature that, in general, more is better. For example, of several mutually exclusive projects (of identical service lives), the one with the highest present worth is the best choice. People prefer a higher salary to a lower one. However, if there are reasons to make a choice other than just the cost, things get somewhat more difficult. For example, which is better: the project with the higher present worth that involves clear-cutting a forest, or the one with lower present worth that preserves the forest? Does a high salary compensate for working for a company that does business with a totalitarian government?

Although such considerations have had particular influence in recent years, the problem of including both qualitative *and* quantitative criteria in engineering decisions has always been present. This leads to the question of how a decision maker deals with multiple objectives, be they quantitative or qualitative.

One approach to choosing the best alternative is to put numerical weights on the criteria. For example, if cost were most important, it would have a high weight, while a less important criterion might be given a low weight. If criteria are evaluated according to a scale that can be used directly as a measure of preference, then the weights and preference measures can be combined mathematically to determine a best alternative.

In a **decision matrix**, the rows of the matrix represent the criteria and the columns the alternatives. There is an extra column for the weights of the criteria. The cells of the matrix (other than the criteria weights) contain an evaluation of the alternatives on a scale from 0 to 10, where 0 is worst and 10 is best. The weights are chosen so that they sum to 10.

The following algorithm can be used:

1. Give a weight to each criterion to express its relative importance: the higher the weight, the more important the criterion. Choose the weight values so that they sum to 10.
2. For each alternative, give a rating from 0 to 10 of how well it meets each criterion. A rating of 0 is given to the worst possible fulfillment of the criterion and 10 to the best possible.
3. For each alternative, multiply each rating by the corresponding criterion weight, and sum to give an overall score.
4. The alternative with the highest score is best. The value of the score can be interpreted as the percentage of an ideal solution achieved by the alternative being evaluated.
5. Carry out some sensitivity analysis with respect to weights or rating estimates to verify the indicated decision or to determine under which conditions different choices are made.

### Example 12A.1

Skiven is evaluating surveillance cameras for a security system. The criteria he is taking into account, in order of importance for him, are low-light performance, picture clarity, weight, and price. The details for six models are shown in Table 12A.1. Which model is best?

In order to follow the steps given above, we need to determine the criteria weights. It is usually fairly easy for a decision maker to determine which criteria are more important than others, but generally more difficult to specify particular weights. There exist many formal methods for establishing such weights in a rigorous way, but in practice, estimating weights on the basis of careful consideration or a discussion with the decision maker is sufficient. Recall that a sensitivity analysis forms part of the overall decision process, and this compensates somewhat for the imprecision of the weights.

Skiven suggests that weights of 1, 1.5, 3.5, and 4 for price, weight, picture clarity, and low-light performance, respectively, are appropriate weights for this problem. These weights are listed as the second column of Table 12A.2.

**Table 12A.1 Set of Surveillance Camera Alternatives**

| Camera | Price (\$) | Weight (Grams) | Picture Clarity (0–10 Scale) | Low-Light Performance (0–10 Scale) |
|--------|------------|----------------|------------------------------|------------------------------------|
| 1      | 230        | 900            | 3                            | 6                                  |
| 2      | 243        | 640            | 5                            | 4                                  |
| 4      | 313        | 433            | 5                            | 7                                  |
| 6      | 415        | 330            | 6                            | 6                                  |
| 7      | 418        | 552            | 7                            | 5                                  |
| 10     | 765        | 255            | 9                            | 5                                  |

**Table 12A.2 Decision Matrix for Example 12A.1**

| Criterion             | Criterion Weight | Alternatives |      |      |      |      |      |
|-----------------------|------------------|--------------|------|------|------|------|------|
|                       |                  | 1            | 2    | 4    | 6    | 7    | 10   |
| Price                 | 1.0              | 10.0         | 9.8  | 8.4  | 6.5  | 6.5  | 0.0  |
| Weight                | 1.5              | 0.0          | 4.0  | 7.2  | 8.8  | 5.4  | 10.0 |
| Clarity               | 3.5              | 3.0          | 5.0  | 5.0  | 6.0  | 7.0  | 9.0  |
| Low-light performance | 4.0              | 6.0          | 4.0  | 7.0  | 6.0  | 5.0  | 5.0  |
| Score                 | 10.0             | 44.5         | 49.3 | 64.8 | 64.8 | 59.1 | 66.5 |

The ratings for each alternative for picture clarity and low-light performance are already on a scale from 0 to 10, so those ratings can be used directly. To select ratings for the price and weight, two different measures could be used.

1. *Normalization:* The rating  $r$  for the least preferred alternative ( $\alpha$ ) is 0 and the most preferred ( $\beta$ ) is 10. For each remaining measure ( $\gamma$ ) the rating  $r$  can be determined as

$$r = 10 \times \frac{\gamma - \alpha}{\beta - \alpha}$$

For this problem, the rating of alternative 6 for price would be

$$r_{6,price} = 10 \times \frac{415 - 765}{230 - 765} = 6.54$$

The advantage of normalization is that it provides a mathematical basis for the rating evaluations. One disadvantage is that the rating may not reflect the value as perceived by the decision maker. A second disadvantage is that it may overrate the best alternative and underrate the worst, since these are set to the extreme values. A third disadvantage is that the addition or deletion of a single alternative (the one with the highest or lowest evaluation for a criterion) will change the entire set of ratings.

2. *Subjective evaluation:* Ask the decision maker to rate the alternatives on the 0 to 10 scale. For example, asked to rate alternative 6 for cost, Skiven might give it a 7. The advantages of subjective evaluation include the fact that it is relatively immune to changes in the alternative set, and that it may be more accurate since it includes perceptions of worth that cannot be directly calculated from the criteria measures. Its main disadvantage is that people often make mistakes and give inconsistent evaluations.

For the ratings shown in Table 12A.2, the normalization process was used. The overall score is then calculated by summing for each alternative the rating for a criterion multiplied by the weighting for that criterion. From Table 12A.2, the total score for alternative 1 is calculated as

$$1 \times 10 + 1.5 \times 0 + 3.5 \times 3 + 4 \times 6 = 44.5$$

It can be seen in Table 12A.2 that the highest score is for alternative 10. This means essentially that the greatest total benefit is achieved if alternative 10 is taken.

Also note that a “perfect” alternative—that is, one that rated 10 on every criterion—would have a total score of 100. Thus the 66.5 score for alternative 10 means that it is

only about 66.5 percent of the score of a perfect alternative. The practice of making weights sum to 10 and rating the alternatives on a scale from 0 to 10 is done specifically so that the resulting score can be interpreted as a percentage of the ideal; if this is not desired, any relative weights or rating scale can be used.

Alternative 10 is the best choice for the particular weights and ratings given, but some sensitivity analysis should be done to verify its robustness. There are several ways to do this sensitivity analysis, but the most sensible is to vary the weights of the criteria to see how the results change. This is easy to do when a spreadsheet is being used to calculate the scores.

**Table 12A.3 Sensitivity Analysis for the Surveillance Camera**

| Criterion             | Criterion Weights  |             |      |      |      |      |             |
|-----------------------|--------------------|-------------|------|------|------|------|-------------|
| Alternative           | Alternative Scores |             |      |      |      |      |             |
| Price                 | 1                  | 1           | 1    | 1    | 2    | 2.5  | 1           |
| Weight                | 1.5                | 2           | 1    | 2    | 2    | 2.5  | 1           |
| Picture clarity       | 3.5                | 3           | 3    | 2    | 2    | 2.5  | 4           |
| Low-light performance | 4                  | 4           | 5    | 5    | 4    | 2.5  | 4           |
| Camera 1              | 44.5               | 43.0        | 49.0 | 46.0 | 50.0 | 47.5 | 46.0        |
| Camera 2              | 49.3               | 48.8        | 48.8 | 47.8 | 53.6 | 57.0 | 49.8        |
| Camera 4              | 64.8               | 65.9        | 65.7 | 67.9 | 69.4 | 67.2 | 63.7        |
| Camera 6              | 64.8               | 66.2        | 63.4 | 66.2 | 66.8 | 68.4 | 63.4        |
| Camera 7              | 59.1               | 58.3        | 57.9 | 56.3 | 57.8 | 59.7 | 59.9        |
| Camera 10             | <b>66.5</b>        | <b>67.7</b> | 62.0 | 63.6 | 58.0 | 60.0 | <b>66.0</b> |

Table 12A.3 shows a range of criteria weights and the corresponding alternative scores. It can be seen that cameras 4 and 6 can also be identified as best in some of the criteria weight possibilities. For the final recommendation, it may be necessary to review these results with Skiven to let him determine which of the weight possibilities are most appropriate for him. ■

As has been seen in Example 12A.1, the decision matrix approach structures information about multiple objectives of the problem. An additive utility model permits the calculation of an overall score for each alternative. A comparison of the scores permits the best one to be selected. Doing a sensitivity analysis may reveal promising alternatives from relatively small changes in the alternative weight assumptions.

## PROBLEMS

**12A.1** A large city's transit commission is considering building a new subway line. Twelve alternatives are being considered. All the alternatives are shown in the tables, along with their criteria values. The relevant criteria are:

**C1** Population and jobs served per kilometre

**C2** Projected daily traffic per kilometre

**C3** Capital cost per kilometre (in millions of dollars)

**C4** IRR

**C5** Structural effect on urbanization

It is desirable to have high population served, high traffic, low capital cost per kilometre, and a high IRR. Criterion C5 concerns the benefits for urban growth caused by the subway location and is measured on a scale from 0 to 10, with the higher values being preferred.

#### Alternative Subway Lines

|    | A      | B      | C      | D      | E      | F      | G      |
|----|--------|--------|--------|--------|--------|--------|--------|
| C1 | 81 900 | 36 500 | 31 800 | 28 200 | 31 100 | 16 100 | 11 500 |
| C2 | 25 500 | 10 300 | 11 600 | 7 400  | 10 500 | 3 500  | 7 100  |
| C3 | 65     | 13.2   | 45     | 30     | 35     | 10     | 29     |
| C4 | 8.6    | 3      | 6.3    | 6      | 14.1   | 11.8   | 4.5    |
| C5 | 3      | 6      | 7      | 10     | 6      | 4      | 4      |

- (a) Establish ratings for a decision matrix for each of the subway line alternatives through normalization. The weights of the five criteria are C1: 1.5, C2: 2, C3: 2.5, C4: 3, C5: 1. Construct a decision matrix and determine the best subway route.
- (b) If the criteria weights were C1: 2, C2: 2, C3: 2, C4: 2, C5: 2, would the recommended alternative be different?

**12A.2** Francis has several job opportunities for his co-op work term. He would like a job with good pay that is close to home, contributes to his engineering studies, and is with a smaller company.

| Job                     | Criteria |      |         |      |
|-------------------------|----------|------|---------|------|
|                         | Pay      | Home | Studies | Size |
| 1. Spinoff Consulting   | 1700     | 2    | 3       | 5    |
| 2. Nub Automotive       | 1600     | 5    | 3       | 500  |
| 3. Soutel               | 2200     | 80   | 4       | 150  |
| 4. Turbine Hydro        | 1800     | 100  | 3       | 3000 |
| 5. Fitzsimon Associates | 1700     | 100  | 1       | 20   |
| 6. General Auto         | 2000     | 150  | 2       | 2500 |
| 7. Ring Casper          | 2200     | 250  | 5       | 300  |
| 8. Jones Mines          | 2700     | 500  | 3       | 20   |
| 9. Resources, Inc.      | 2700     | 2000 | 2       | 40   |

Pay: Monthly salary in dollars

Home: Distance from home in kilometres

Studies: Contribution to engineering studies, 0 = none, 5 = a lot

Size: Number of employees at that location

- (a) Some of these alternatives can immediately be removed from further consideration. For example, the job with Nub Automotive (Job 2) is worse in all respects than the one with Spinoff Consulting (Job 1). It is said that Job 1 dominates Job 2. Only four of these 10 alternatives cannot be dominated by another. Which are the four jobs that Francis should consider further?

- (b) Francis feels that the following weights can represent the importance of his four criteria:

|          |     |
|----------|-----|
| Pay:     | 4.0 |
| Home:    | 2.5 |
| Studies: | 2.0 |
| Size:    | 1.5 |

Using normalization to establish the ratings, which job is best?

## ENGINEERING ECONOMICS IN ACTION, PART 12C

### Moving On

Three weeks later, Naomi was seated in Anna Kulkowski's office. The Powerluxe project report had been submitted two days before.

"Naomi, the work you and Bill did and your report are first-rate," Anna said. "We're going to finalize our negotiations with Powerluxe to bring the Adaptamatic line of vacuum cleaners to market. I had no idea anybody could make such clear recommendations on such a complex problem. Congratulations on a good job."

Naomi thought back on all the people who had helped her in her almost two years at Canadian Widgets—how Clem had taught her practical problem solving, how Dave had shown her the ropes, and how Terry had helped her realize the benefits of attention to detail. Bill had shown her how the real world mixes engineering with marketing, business, and government. Anna, too, had shown her how to manage people. "Thank you very much, Ms. Kulkowski," Naomi responded, a small break in her voice betraying her emotion.

"Do you enjoy this kind of work?" Anna asked.

"Yes, I do," Naomi replied. "It's exciting to see how engineering relates to everything else."

"I think we have a new long-term assignment for you," Anna said. "This is just the first step for Canadian Widgets in developing new products. We have a first-rate team of engineers. We want to make better use of them. Ed Burns is going to head up a product development group. He read your report on the Adaptamatic line of vacuum cleaners and was quite impressed. He and I would like you in that development group. We need someone with analytic skills who can manage projects and who can understand the marketing side, too. What do you say?"

"I'm in!" Naomi had a big grin on her face.

A few days later, Naomi answered the phone.

"Hey, Naomi, it's Terry. I hear you got promoted!"

"Hi, Terry. Nice to hear from you. Well, the money's about the same, but it sure will be interesting. How are things with you?" Naomi had fond memories of working with Terry.

"Well, I graduate next month and I have a job. Guess where?"

Naomi knew exactly where; Clem had told her about the interviews. It wasn't really fair to the other candidates—Clem had decided to hire Terry as soon as he applied. "Here? Really?"

# Compound Interest Factors for Discrete Compounding, Discrete Cash Flows

$i = 0.5\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |           |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|-----------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |           |
| N                      | (F/P,i,N)            | (P/F,i,N)           | (A/F,i,N)             | (F/A,i,N)               | (A/P,i,N)                   | (P/A,i,N)                         | (A/G,i,N) |
| 1                      | 1.0050               | 0.99502             | 1.0000                | 1.0000                  | 1.0050                      | 0.99502                           | 0.00000   |
| 2                      | 1.0100               | 0.99007             | 0.49875               | 2.0050                  | 0.50375                     | 1.9851                            | 0.49875   |
| 3                      | 1.0151               | 0.98515             | 0.33167               | 3.0150                  | 0.33667                     | 2.9702                            | 0.99667   |
| 4                      | 1.0202               | 0.98025             | 0.24813               | 4.0301                  | 0.25313                     | 3.9505                            | 1.4938    |
| 5                      | 1.0253               | 0.97537             | 0.19801               | 5.0503                  | 0.20301                     | 4.9259                            | 1.9900    |
| 6                      | 1.0304               | 0.97052             | 0.16460               | 6.0755                  | 0.16960                     | 5.8964                            | 2.4855    |
| 7                      | 1.0355               | 0.96569             | 0.14073               | 7.1059                  | 0.14573                     | 6.8621                            | 2.9801    |
| 8                      | 1.0407               | 0.96089             | 0.12283               | 8.1414                  | 0.12783                     | 7.8230                            | 3.4738    |
| 9                      | 1.0459               | 0.95610             | 0.10891               | 9.1821                  | 0.11391                     | 8.7791                            | 3.9668    |
| 10                     | 1.0511               | 0.95135             | 0.09777               | 10.228                  | 0.10277                     | 9.7304                            | 4.4589    |
| 11                     | 1.0564               | 0.94661             | 0.08866               | 11.279                  | 0.09366                     | 10.677                            | 4.9501    |
| 12                     | 1.0617               | 0.94191             | 0.08107               | 12.336                  | 0.08607                     | 11.619                            | 5.4406    |
| 13                     | 1.0670               | 0.93722             | 0.07464               | 13.397                  | 0.07964                     | 12.556                            | 5.9302    |
| 14                     | 1.0723               | 0.93256             | 0.06914               | 14.464                  | 0.07414                     | 13.489                            | 6.4190    |
| 15                     | 1.0777               | 0.92792             | 0.06436               | 15.537                  | 0.06936                     | 14.417                            | 6.9069    |
| 16                     | 1.0831               | 0.92330             | 0.06019               | 16.614                  | 0.06519                     | 15.340                            | 7.3940    |
| 17                     | 1.0885               | 0.91871             | 0.05651               | 17.697                  | 0.06151                     | 16.259                            | 7.8803    |
| 18                     | 1.0939               | 0.91414             | 0.05323               | 18.786                  | 0.05823                     | 17.173                            | 8.3658    |
| 19                     | 1.0994               | 0.90959             | 0.05030               | 19.880                  | 0.05530                     | 18.082                            | 8.8504    |
| 20                     | 1.1049               | 0.90506             | 0.04767               | 20.979                  | 0.05267                     | 18.987                            | 9.3342    |
| 21                     | 1.1104               | 0.90056             | 0.04528               | 22.084                  | 0.05028                     | 19.888                            | 9.8172    |
| 22                     | 1.1160               | 0.89608             | 0.04311               | 23.194                  | 0.04811                     | 20.784                            | 10.299    |
| 23                     | 1.1216               | 0.89162             | 0.04113               | 24.310                  | 0.04613                     | 21.676                            | 10.781    |
| 24                     | 1.1272               | 0.88719             | 0.03932               | 25.432                  | 0.04432                     | 22.563                            | 11.261    |
| 25                     | 1.1328               | 0.88277             | 0.03765               | 26.559                  | 0.04265                     | 23.446                            | 11.741    |
| 26                     | 1.1385               | 0.87838             | 0.03611               | 27.692                  | 0.04111                     | 24.324                            | 12.220    |
| 27                     | 1.1442               | 0.87401             | 0.03469               | 28.830                  | 0.03969                     | 25.198                            | 12.698    |
| 28                     | 1.1499               | 0.86966             | 0.03336               | 29.975                  | 0.03836                     | 26.068                            | 13.175    |
| 29                     | 1.1556               | 0.86533             | 0.03213               | 31.124                  | 0.03713                     | 26.933                            | 13.651    |
| 30                     | 1.1614               | 0.86103             | 0.03098               | 32.280                  | 0.03598                     | 27.794                            | 14.126    |
| 31                     | 1.1672               | 0.85675             | 0.02990               | 33.441                  | 0.03490                     | 28.651                            | 14.601    |
| 32                     | 1.1730               | 0.85248             | 0.02889               | 34.609                  | 0.03389                     | 29.503                            | 15.075    |
| 33                     | 1.1789               | 0.84824             | 0.02795               | 35.782                  | 0.03295                     | 30.352                            | 15.548    |
| 34                     | 1.1848               | 0.84402             | 0.02706               | 36.961                  | 0.03206                     | 31.196                            | 16.020    |
| 35                     | 1.1907               | 0.83982             | 0.02622               | 38.145                  | 0.03122                     | 32.035                            | 16.492    |
| 40                     | 1.2208               | 0.81914             | 0.02265               | 44.159                  | 0.02765                     | 36.172                            | 18.836    |
| 45                     | 1.2516               | 0.79896             | 0.01987               | 50.324                  | 0.02487                     | 40.207                            | 21.159    |
| 50                     | 1.2832               | 0.77929             | 0.01765               | 56.645                  | 0.02265                     | 44.143                            | 23.462    |
| 55                     | 1.3156               | 0.76009             | 0.01584               | 63.126                  | 0.02084                     | 47.981                            | 25.745    |
| 60                     | 1.3489               | 0.74137             | 0.01433               | 69.770                  | 0.01933                     | 51.726                            | 28.006    |
| 65                     | 1.3829               | 0.72311             | 0.01306               | 76.582                  | 0.01806                     | 55.377                            | 30.247    |
| 70                     | 1.4178               | 0.70530             | 0.01197               | 83.566                  | 0.01697                     | 58.939                            | 32.468    |
| 75                     | 1.4536               | 0.68793             | 0.01102               | 90.727                  | 0.01602                     | 62.414                            | 34.668    |
| 80                     | 1.4903               | 0.67099             | 0.01020               | 98.068                  | 0.01520                     | 65.802                            | 36.847    |
| 85                     | 1.5280               | 0.65446             | 0.00947               | 105.59                  | 0.01447                     | 69.108                            | 39.006    |
| 90                     | 1.5666               | 0.63834             | 0.00883               | 113.31                  | 0.01383                     | 72.331                            | 41.145    |
| 95                     | 1.6061               | 0.62262             | 0.00825               | 121.22                  | 0.01325                     | 75.476                            | 43.263    |
| 100                    | 1.6467               | 0.60729             | 0.00773               | 129.33                  | 0.01273                     | 78.543                            | 45.361    |

*i* = 1%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT |                        | UNIFORM SERIES       |                     |                       |                         | Arithmetic Gradient Series Factor |           |
|----------------|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------------|-----------|
|                | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor |                                   |           |
| N              | (F/P,i,N)              | (P/F,i,N)            | (A/F,i,N)           | (F/A,i,N)             | (A/P,i,N)               | (P/A,i,N)                         | (A/G,i,N) |
| 1              | 1.0100                 | 0.99010              | 1.0000              | 1.0000                | 1.0100                  | 0.99010                           | 0.00000   |
| 2              | 1.0201                 | 0.98030              | 0.49751             | 2.0100                | 0.50751                 | 1.9704                            | 0.49751   |
| 3              | 1.0303                 | 0.97059              | 0.33002             | 3.0301                | 0.34002                 | 2.9410                            | 0.99337   |
| 4              | 1.0406                 | 0.96098              | 0.24628             | 4.0604                | 0.25628                 | 3.9020                            | 1.4876    |
| 5              | 1.0510                 | 0.95147              | 0.19604             | 5.1010                | 0.20604                 | 4.8534                            | 1.9801    |
| 6              | 1.0615                 | 0.94205              | 0.16255             | 6.1520                | 0.17255                 | 5.7955                            | 2.4710    |
| 7              | 1.0721                 | 0.93272              | 0.13863             | 7.2135                | 0.14863                 | 6.7282                            | 2.9602    |
| 8              | 1.0829                 | 0.92348              | 0.12069             | 8.2857                | 0.13069                 | 7.6517                            | 3.4478    |
| 9              | 1.0937                 | 0.91434              | 0.10674             | 9.3685                | 0.11674                 | 8.5660                            | 3.9337    |
| 10             | 1.1046                 | 0.90529              | 0.09558             | 10.462                | 0.10558                 | 9.4713                            | 4.4179    |
| 11             | 1.1157                 | 0.89632              | 0.08645             | 11.567                | 0.09645                 | 10.368                            | 4.9005    |
| 12             | 1.1268                 | 0.88745              | 0.07885             | 12.683                | 0.08885                 | 11.255                            | 5.3815    |
| 13             | 1.1381                 | 0.87866              | 0.07241             | 13.809                | 0.08241                 | 12.134                            | 5.8607    |
| 14             | 1.1495                 | 0.86996              | 0.06690             | 14.947                | 0.07690                 | 13.004                            | 6.3384    |
| 15             | 1.1610                 | 0.86135              | 0.06212             | 16.097                | 0.07212                 | 13.865                            | 6.8143    |
| 16             | 1.1726                 | 0.85282              | 0.05794             | 17.258                | 0.06794                 | 14.718                            | 7.2886    |
| 17             | 1.1843                 | 0.84438              | 0.05426             | 18.430                | 0.06426                 | 15.562                            | 7.7613    |
| 18             | 1.1961                 | 0.83602              | 0.05098             | 19.615                | 0.06098                 | 16.398                            | 8.2323    |
| 19             | 1.2081                 | 0.82774              | 0.04805             | 20.811                | 0.05805                 | 17.226                            | 8.7017    |
| 20             | 1.2202                 | 0.81954              | 0.04542             | 22.019                | 0.05542                 | 18.046                            | 9.1694    |
| 21             | 1.2324                 | 0.81143              | 0.04303             | 23.239                | 0.05303                 | 18.857                            | 9.6354    |
| 22             | 1.2447                 | 0.80340              | 0.04086             | 24.472                | 0.05086                 | 19.660                            | 10.100    |
| 23             | 1.2572                 | 0.79544              | 0.03889             | 25.716                | 0.04889                 | 20.456                            | 10.563    |
| 24             | 1.2697                 | 0.78757              | 0.03707             | 26.973                | 0.04707                 | 21.243                            | 11.024    |
| 25             | 1.2824                 | 0.77977              | 0.03541             | 28.243                | 0.04541                 | 22.023                            | 11.483    |
| 26             | 1.2953                 | 0.77205              | 0.03387             | 29.526                | 0.04387                 | 22.795                            | 11.941    |
| 27             | 1.3082                 | 0.76440              | 0.03245             | 30.821                | 0.04245                 | 23.560                            | 12.397    |
| 28             | 1.3213                 | 0.75684              | 0.03112             | 32.129                | 0.04112                 | 24.316                            | 12.852    |
| 29             | 1.3345                 | 0.74934              | 0.02990             | 33.450                | 0.03990                 | 25.066                            | 13.304    |
| 30             | 1.3478                 | 0.74192              | 0.02875             | 34.785                | 0.03875                 | 25.808                            | 13.756    |
| 31             | 1.3613                 | 0.73458              | 0.02768             | 36.133                | 0.03768                 | 26.542                            | 14.205    |
| 32             | 1.3749                 | 0.72730              | 0.02667             | 37.494                | 0.03667                 | 27.270                            | 14.653    |
| 33             | 1.3887                 | 0.72010              | 0.02573             | 38.869                | 0.03573                 | 27.990                            | 15.099    |
| 34             | 1.4026                 | 0.71297              | 0.02484             | 40.258                | 0.03484                 | 28.703                            | 15.544    |
| 35             | 1.4166                 | 0.70591              | 0.02400             | 41.660                | 0.03400                 | 29.409                            | 15.987    |
| 40             | 1.4889                 | 0.67165              | 0.02046             | 48.886                | 0.03046                 | 32.835                            | 18.178    |
| 45             | 1.5648                 | 0.63905              | 0.01771             | 56.481                | 0.02771                 | 36.095                            | 20.327    |
| 50             | 1.6446                 | 0.60804              | 0.01551             | 64.463                | 0.02551                 | 39.196                            | 22.436    |
| 55             | 1.7285                 | 0.57853              | 0.01373             | 72.852                | 0.02373                 | 42.147                            | 24.505    |
| 60             | 1.8167                 | 0.55045              | 0.01224             | 81.670                | 0.02224                 | 44.955                            | 26.533    |
| 65             | 1.9094                 | 0.52373              | 0.01100             | 90.937                | 0.02100                 | 47.627                            | 28.522    |
| 70             | 2.0068                 | 0.49831              | 0.00993             | 100.68                | 0.01993                 | 50.169                            | 30.470    |
| 75             | 2.1091                 | 0.47413              | 0.00902             | 110.91                | 0.01902                 | 52.587                            | 32.379    |
| 80             | 2.2167                 | 0.45112              | 0.00822             | 121.67                | 0.01822                 | 54.888                            | 34.249    |
| 85             | 2.3298                 | 0.42922              | 0.00752             | 132.98                | 0.01752                 | 57.078                            | 36.080    |
| 90             | 2.4486                 | 0.40839              | 0.00690             | 144.86                | 0.01690                 | 59.161                            | 37.872    |
| 95             | 2.5735                 | 0.38857              | 0.00636             | 157.35                | 0.01636                 | 61.143                            | 39.626    |
| 100            | 2.7048                 | 0.36971              | 0.00587             | 170.48                | 0.01587                 | 63.029                            | 41.343    |

$i = 1.5\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT               |                            | UNIFORM SERIES            |                             |                               |                                   | Arithmetic<br>Gradient<br>Series<br>Factor |           |
|------------------------------|----------------------------|---------------------------|-----------------------------|-------------------------------|-----------------------------------|--|-----------|
| Compound<br>Amount<br>Factor | Present<br>Worth<br>Factor | Sinking<br>Fund<br>Factor | Uniform<br>Series<br>Factor | Capital<br>Recovery<br>Factor | Series<br>Present<br>Worth Factor |  |           |
| N                            | (F/P,i,N)                  | (P/F,i,N)                 | (A/F,i,N)                   | (F/A,i,N)                     | (A/P,i,N)                         | (P/A,i,N)                                  | (A/G,i,N) |
| 1                            | 1.0150                     | 0.98522                   | 1.0000                      | 1.0000                        | 1.0150                            | 0.98522                                    | 0.00000   |
| 2                            | 1.0302                     | 0.97066                   | 0.49628                     | 2.0150                        | 0.51128                           | 1.9559                                     | 0.49628   |
| 3                            | 1.0457                     | 0.95632                   | 0.32838                     | 3.0452                        | 0.34338                           | 2.9122                                     | 0.99007   |
| 4                            | 1.0614                     | 0.94218                   | 0.24444                     | 4.0909                        | 0.25944                           | 3.8544                                     | 1.4814    |
| 5                            | 1.0773                     | 0.92826                   | 0.19409                     | 5.1523                        | 0.20909                           | 4.7826                                     | 1.9702    |
| 6                            | 1.0934                     | 0.91454                   | 0.16053                     | 6.2296                        | 0.17553                           | 5.6972                                     | 2.4566    |
| 7                            | 1.1098                     | 0.90103                   | 0.13656                     | 7.3230                        | 0.15156                           | 6.5982                                     | 2.9405    |
| 8                            | 1.1265                     | 0.88771                   | 0.11858                     | 8.4328                        | 0.13358                           | 7.4859                                     | 3.4219    |
| 9                            | 1.1434                     | 0.87459                   | 0.10461                     | 9.5593                        | 0.11961                           | 8.3605                                     | 3.9008    |
| 10                           | 1.1605                     | 0.86167                   | 0.09343                     | 10.703                        | 0.10843                           | 9.2222                                     | 4.3772    |
| 11                           | 1.1779                     | 0.84893                   | 0.08429                     | 11.863                        | 0.09929                           | 10.071                                     | 4.8512    |
| 12                           | 1.1956                     | 0.83639                   | 0.07668                     | 13.041                        | 0.09168                           | 10.908                                     | 5.3227    |
| 13                           | 1.2136                     | 0.82403                   | 0.07024                     | 14.237                        | 0.08524                           | 11.732                                     | 5.7917    |
| 14                           | 1.2318                     | 0.81185                   | 0.06472                     | 15.450                        | 0.07972                           | 12.543                                     | 6.2582    |
| 15                           | 1.2502                     | 0.79985                   | 0.05994                     | 16.682                        | 0.07494                           | 13.343                                     | 6.7223    |
| 16                           | 1.2690                     | 0.78803                   | 0.05577                     | 17.932                        | 0.07077                           | 14.131                                     | 7.1839    |
| 17                           | 1.2880                     | 0.77639                   | 0.05208                     | 19.201                        | 0.06708                           | 14.908                                     | 7.6431    |
| 18                           | 1.3073                     | 0.76491                   | 0.04881                     | 20.489                        | 0.06381                           | 15.673                                     | 8.0997    |
| 19                           | 1.3270                     | 0.75361                   | 0.04588                     | 21.797                        | 0.06088                           | 16.426                                     | 8.5539    |
| 20                           | 1.3469                     | 0.74247                   | 0.04325                     | 23.124                        | 0.05825                           | 17.169                                     | 9.0057    |
| 21                           | 1.3671                     | 0.73150                   | 0.04087                     | 24.471                        | 0.05587                           | 17.900                                     | 9.4550    |
| 22                           | 1.3876                     | 0.72069                   | 0.03870                     | 25.838                        | 0.05370                           | 18.621                                     | 9.9018    |
| 23                           | 1.4084                     | 0.71004                   | 0.03673                     | 27.225                        | 0.05173                           | 19.331                                     | 10.346    |
| 24                           | 1.4295                     | 0.69954                   | 0.03492                     | 28.634                        | 0.04992                           | 20.030                                     | 10.788    |
| 25                           | 1.4509                     | 0.68921                   | 0.03326                     | 30.063                        | 0.04826                           | 20.720                                     | 11.228    |
| 26                           | 1.4727                     | 0.67902                   | 0.03173                     | 31.514                        | 0.04673                           | 21.399                                     | 11.665    |
| 27                           | 1.4948                     | 0.66899                   | 0.03032                     | 32.987                        | 0.04532                           | 22.068                                     | 12.099    |
| 28                           | 1.5172                     | 0.65910                   | 0.02900                     | 34.481                        | 0.04400                           | 22.727                                     | 12.531    |
| 29                           | 1.5400                     | 0.64936                   | 0.02778                     | 35.999                        | 0.04278                           | 23.376                                     | 12.961    |
| 30                           | 1.5631                     | 0.63976                   | 0.02664                     | 37.539                        | 0.04164                           | 24.016                                     | 13.388    |
| 31                           | 1.5865                     | 0.63031                   | 0.02557                     | 39.102                        | 0.04057                           | 24.646                                     | 13.813    |
| 32                           | 1.6103                     | 0.62099                   | 0.02458                     | 40.688                        | 0.03958                           | 25.267                                     | 14.236    |
| 33                           | 1.6345                     | 0.61182                   | 0.02364                     | 42.299                        | 0.03864                           | 25.879                                     | 14.656    |
| 34                           | 1.6590                     | 0.60277                   | 0.02276                     | 43.933                        | 0.03776                           | 26.482                                     | 15.073    |
| 35                           | 1.6839                     | 0.59387                   | 0.02193                     | 45.592                        | 0.03693                           | 27.076                                     | 15.488    |
| 40                           | 1.8140                     | 0.55126                   | 0.01843                     | 54.268                        | 0.03343                           | 29.916                                     | 17.528    |
| 45                           | 1.9542                     | 0.51171                   | 0.01572                     | 63.614                        | 0.03072                           | 32.552                                     | 19.507    |
| 50                           | 2.1052                     | 0.47500                   | 0.01357                     | 73.683                        | 0.02857                           | 35.000                                     | 21.428    |
| 55                           | 2.2679                     | 0.44093                   | 0.01183                     | 84.530                        | 0.02683                           | 37.271                                     | 23.289    |
| 60                           | 2.4432                     | 0.40930                   | 0.01039                     | 96.215                        | 0.02539                           | 39.380                                     | 25.093    |
| 65                           | 2.6320                     | 0.37993                   | 0.00919                     | 108.80                        | 0.02419                           | 41.338                                     | 26.839    |
| 70                           | 2.8355                     | 0.35268                   | 0.00817                     | 122.36                        | 0.02317                           | 43.155                                     | 28.529    |
| 75                           | 3.0546                     | 0.32738                   | 0.00730                     | 136.97                        | 0.02230                           | 44.842                                     | 30.163    |
| 80                           | 3.2907                     | 0.30389                   | 0.00655                     | 152.71                        | 0.02155                           | 46.407                                     | 31.742    |
| 85                           | 3.5450                     | 0.28209                   | 0.00589                     | 169.67                        | 0.02089                           | 47.861                                     | 33.268    |
| 90                           | 3.8189                     | 0.26185                   | 0.00532                     | 187.93                        | 0.02032                           | 49.210                                     | 34.740    |
| 95                           | 4.1141                     | 0.24307                   | 0.00482                     | 207.61                        | 0.01982                           | 50.462                                     | 36.160    |
| 100                          | 4.4320                     | 0.22563                   | 0.00437                     | 228.80                        | 0.01937                           | 51.625                                     | 37.530    |

*i* = 2%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.0200               | 0.98039             | 1.0000                | 1.0000                  | 1.0200                      | 0.98039                           | 0.00000            |
| 2                      | 1.0404               | 0.96117             | 0.49505               | 2.0200                  | 0.51505                     | 1.9416                            | 0.49505            |
| 3                      | 1.0612               | 0.94232             | 0.32675               | 3.0604                  | 0.34675                     | 2.8839                            | 0.98680            |
| 4                      | 1.0824               | 0.92385             | 0.24262               | 4.1216                  | 0.26262                     | 3.8077                            | 1.4752             |
| 5                      | 1.1041               | 0.90573             | 0.19216               | 5.2040                  | 0.21216                     | 4.7135                            | 1.9604             |
| 6                      | 1.1262               | 0.88797             | 0.15853               | 6.3081                  | 0.17853                     | 5.6014                            | 2.4423             |
| 7                      | 1.1487               | 0.87056             | 0.13451               | 7.4343                  | 0.15451                     | 6.4720                            | 2.9208             |
| 8                      | 1.1717               | 0.85349             | 0.11651               | 8.5830                  | 0.13651                     | 7.3255                            | 3.3961             |
| 9                      | 1.1951               | 0.83676             | 0.10252               | 9.7546                  | 0.12252                     | 8.1622                            | 3.8681             |
| 10                     | 1.2190               | 0.82035             | 0.09133               | 10.950                  | 0.11133                     | 8.9826                            | 4.3367             |
| 11                     | 1.2434               | 0.80426             | 0.08218               | 12.169                  | 0.10218                     | 9.787                             | 4.8021             |
| 12                     | 1.2682               | 0.78849             | 0.07456               | 13.412                  | 0.09456                     | 10.575                            | 5.2642             |
| 13                     | 1.2936               | 0.77303             | 0.06812               | 14.680                  | 0.08812                     | 11.348                            | 5.7231             |
| 14                     | 1.3195               | 0.75788             | 0.06260               | 15.974                  | 0.08260                     | 12.106                            | 6.1786             |
| 15                     | 1.3459               | 0.74301             | 0.05783               | 17.293                  | 0.07783                     | 12.849                            | 6.6309             |
| 16                     | 1.3728               | 0.72845             | 0.05365               | 18.639                  | 0.07365                     | 13.578                            | 7.0799             |
| 17                     | 1.4002               | 0.71416             | 0.04997               | 20.012                  | 0.06997                     | 14.292                            | 7.5256             |
| 18                     | 1.4282               | 0.70016             | 0.04670               | 21.412                  | 0.06670                     | 14.992                            | 7.9681             |
| 19                     | 1.4568               | 0.68643             | 0.04378               | 22.841                  | 0.06378                     | 15.678                            | 8.4073             |
| 20                     | 1.4859               | 0.67297             | 0.04116               | 24.297                  | 0.06116                     | 16.351                            | 8.8433             |
| 21                     | 1.5157               | 0.65978             | 0.03878               | 25.783                  | 0.05878                     | 17.011                            | 9.2760             |
| 22                     | 1.5460               | 0.64684             | 0.03663               | 27.299                  | 0.05663                     | 17.658                            | 9.7050             |
| 23                     | 1.5769               | 0.63416             | 0.03467               | 28.845                  | 0.05467                     | 18.292                            | 10.1320            |
| 24                     | 1.6084               | 0.62172             | 0.03287               | 30.422                  | 0.05287                     | 18.914                            | 10.5550            |
| 25                     | 1.6406               | 0.60953             | 0.03122               | 32.030                  | 0.05122                     | 19.523                            | 10.9740            |
| 26                     | 1.6734               | 0.59758             | 0.02970               | 33.671                  | 0.04970                     | 20.121                            | 11.391             |
| 27                     | 1.7069               | 0.58586             | 0.02829               | 35.344                  | 0.04829                     | 20.707                            | 11.804             |
| 28                     | 1.7410               | 0.57437             | 0.02699               | 37.051                  | 0.04699                     | 21.281                            | 12.214             |
| 29                     | 1.7758               | 0.56311             | 0.02578               | 38.792                  | 0.04578                     | 21.844                            | 12.621             |
| 30                     | 1.8114               | 0.55207             | 0.02465               | 40.568                  | 0.04465                     | 22.396                            | 13.025             |
| 31                     | 1.8476               | 0.54125             | 0.02360               | 42.379                  | 0.04360                     | 22.938                            | 13.426             |
| 32                     | 1.8845               | 0.53063             | 0.02261               | 44.227                  | 0.04261                     | 23.468                            | 13.823             |
| 33                     | 1.9222               | 0.52023             | 0.02169               | 46.112                  | 0.04169                     | 23.989                            | 14.217             |
| 34                     | 1.9607               | 0.51003             | 0.02082               | 48.034                  | 0.04082                     | 24.499                            | 14.608             |
| 35                     | 1.9999               | 0.50003             | 0.02000               | 49.994                  | 0.04000                     | 24.999                            | 14.996             |
| 40                     | 2.2080               | 0.45289             | 0.01656               | 60.402                  | 0.03656                     | 27.355                            | 16.889             |
| 45                     | 2.4379               | 0.41020             | 0.01391               | 71.893                  | 0.03391                     | 29.490                            | 18.703             |
| 50                     | 2.6916               | 0.37153             | 0.01182               | 84.579                  | 0.03182                     | 31.424                            | 20.442             |
| 55                     | 2.9717               | 0.33650             | 0.01014               | 98.587                  | 0.03014                     | 33.175                            | 22.106             |
| 60                     | 3.2810               | 0.30478             | 0.00877               | 114.05                  | 0.02877                     | 34.761                            | 23.696             |
| 65                     | 3.6225               | 0.27605             | 0.00763               | 131.13                  | 0.02763                     | 36.197                            | 25.215             |
| 70                     | 3.9996               | 0.25003             | 0.00667               | 149.98                  | 0.02667                     | 37.499                            | 26.663             |
| 75                     | 4.4158               | 0.22646             | 0.00586               | 170.79                  | 0.02586                     | 38.677                            | 28.043             |
| 80                     | 4.8754               | 0.20511             | 0.00516               | 193.77                  | 0.02516                     | 39.745                            | 29.357             |
| 85                     | 5.3829               | 0.18577             | 0.00456               | 219.14                  | 0.02456                     | 40.711                            | 30.606             |
| 90                     | 5.9431               | 0.16826             | 0.00405               | 247.16                  | 0.02405                     | 41.587                            | 31.793             |
| 95                     | 6.5617               | 0.15240             | 0.00360               | 278.08                  | 0.02360                     | 42.380                            | 32.919             |
| 100                    | 7.2446               | 0.13803             | 0.00320               | 312.23                  | 0.02320                     | 43.098                            | 33.986             |

$i = 3\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT               |                            | UNIFORM SERIES            |                             |                               |                                   | Arithmetic<br>Gradient<br>Series<br>Factor |           |
|------------------------------|----------------------------|---------------------------|-----------------------------|-------------------------------|-----------------------------------|--|-----------|
| Compound<br>Amount<br>Factor | Present<br>Worth<br>Factor | Sinking<br>Fund<br>Factor | Uniform<br>Series<br>Factor | Capital<br>Recovery<br>Factor | Series<br>Present<br>Worth Factor |  |           |
| N                            | (F/P,i,N)                  | (P/F,i,N)                 | (A/F,i,N)                   | (F/A,i,N)                     | (A/P,i,N)                         | (P/A,i,N)                                  | (A/G,i,N) |
| 1                            | 1.0300                     | 0.97087                   | 1.0000                      | 1.0000                        | 1.0300                            | 0.97087                                    | 0.00000   |
| 2                            | 1.0609                     | 0.94260                   | 0.49261                     | 2.0300                        | 0.52261                           | 1.9135                                     | 0.49261   |
| 3                            | 1.0927                     | 0.91514                   | 0.32353                     | 3.0909                        | 0.35353                           | 2.8286                                     | 0.98030   |
| 4                            | 1.1255                     | 0.88849                   | 0.23903                     | 4.1836                        | 0.26903                           | 3.7171                                     | 1.4631    |
| 5                            | 1.1593                     | 0.86261                   | 0.18835                     | 5.3091                        | 0.21835                           | 4.5797                                     | 1.9409    |
| 6                            | 1.1941                     | 0.83748                   | 0.15460                     | 6.4684                        | 0.18460                           | 5.4172                                     | 2.4138    |
| 7                            | 1.2299                     | 0.81309                   | 0.13051                     | 7.6625                        | 0.16051                           | 6.2303                                     | 2.8819    |
| 8                            | 1.2668                     | 0.78941                   | 0.11246                     | 8.8923                        | 0.14246                           | 7.0197                                     | 3.3450    |
| 9                            | 1.3048                     | 0.76642                   | 0.09843                     | 10.159                        | 0.12843                           | 7.7861                                     | 3.8032    |
| 10                           | 1.3439                     | 0.74409                   | 0.08723                     | 11.464                        | 0.11723                           | 8.5302                                     | 4.2565    |
| 11                           | 1.3842                     | 0.72242                   | 0.07808                     | 12.808                        | 0.10808                           | 9.2526                                     | 4.7049    |
| 12                           | 1.4258                     | 0.70138                   | 0.07046                     | 14.192                        | 0.10046                           | 9.9540                                     | 5.1485    |
| 13                           | 1.4685                     | 0.68095                   | 0.06403                     | 15.618                        | 0.09403                           | 10.635                                     | 5.5872    |
| 14                           | 1.5126                     | 0.66112                   | 0.05853                     | 17.086                        | 0.08853                           | 11.296                                     | 6.0210    |
| 15                           | 1.5580                     | 0.64186                   | 0.05377                     | 18.599                        | 0.08377                           | 11.938                                     | 6.4500    |
| 16                           | 1.6047                     | 0.62317                   | 0.04961                     | 20.157                        | 0.07961                           | 12.561                                     | 6.8742    |
| 17                           | 1.6528                     | 0.60502                   | 0.04595                     | 21.762                        | 0.07595                           | 13.166                                     | 7.2936    |
| 18                           | 1.7024                     | 0.58739                   | 0.04271                     | 23.414                        | 0.07271                           | 13.754                                     | 7.7081    |
| 19                           | 1.7535                     | 0.57029                   | 0.03981                     | 25.117                        | 0.06981                           | 14.324                                     | 8.1179    |
| 20                           | 1.8061                     | 0.55368                   | 0.03722                     | 26.870                        | 0.06722                           | 14.877                                     | 8.5229    |
| 21                           | 1.8603                     | 0.53755                   | 0.03487                     | 28.676                        | 0.06487                           | 15.415                                     | 8.9231    |
| 22                           | 1.9161                     | 0.52189                   | 0.03275                     | 30.537                        | 0.06275                           | 15.937                                     | 9.3186    |
| 23                           | 1.9736                     | 0.50669                   | 0.03081                     | 32.453                        | 0.06081                           | 16.444                                     | 9.7093    |
| 24                           | 2.0328                     | 0.49193                   | 0.02905                     | 34.426                        | 0.05905                           | 16.936                                     | 10.095    |
| 25                           | 2.0938                     | 0.47761                   | 0.02743                     | 36.459                        | 0.05743                           | 17.413                                     | 10.477    |
| 26                           | 2.1566                     | 0.46369                   | 0.02594                     | 38.553                        | 0.05594                           | 17.877                                     | 10.853    |
| 27                           | 2.2213                     | 0.45019                   | 0.02456                     | 40.710                        | 0.05456                           | 18.327                                     | 11.226    |
| 28                           | 2.2879                     | 0.43708                   | 0.02329                     | 42.931                        | 0.05329                           | 18.764                                     | 11.593    |
| 29                           | 2.3566                     | 0.42435                   | 0.02211                     | 45.219                        | 0.05211                           | 19.188                                     | 11.956    |
| 30                           | 2.4273                     | 0.41199                   | 0.02102                     | 47.575                        | 0.05102                           | 19.600                                     | 12.314    |
| 31                           | 2.5001                     | 0.39999                   | 0.02000                     | 50.003                        | 0.05000                           | 20.000                                     | 12.668    |
| 32                           | 2.5751                     | 0.38834                   | 0.01905                     | 52.503                        | 0.04905                           | 20.389                                     | 13.017    |
| 33                           | 2.6523                     | 0.37703                   | 0.01816                     | 55.078                        | 0.04816                           | 20.766                                     | 13.362    |
| 34                           | 2.7319                     | 0.36604                   | 0.01732                     | 57.730                        | 0.04732                           | 21.132                                     | 13.702    |
| 35                           | 2.8139                     | 0.35538                   | 0.01654                     | 60.462                        | 0.04654                           | 21.487                                     | 14.037    |
| 40                           | 3.2620                     | 0.30656                   | 0.01326                     | 75.401                        | 0.04326                           | 23.115                                     | 15.650    |
| 45                           | 3.7816                     | 0.26444                   | 0.01079                     | 92.720                        | 0.04079                           | 24.519                                     | 17.156    |
| 50                           | 4.3839                     | 0.22811                   | 0.00887                     | 112.80                        | 0.03887                           | 25.730                                     | 18.558    |
| 55                           | 5.0821                     | 0.19677                   | 0.00735                     | 136.07                        | 0.03735                           | 26.774                                     | 19.860    |
| 60                           | 5.8916                     | 0.16973                   | 0.00613                     | 163.05                        | 0.03613                           | 27.676                                     | 21.067    |
| 65                           | 6.8300                     | 0.14641                   | 0.00515                     | 194.33                        | 0.03515                           | 28.453                                     | 22.184    |
| 70                           | 7.9178                     | 0.12630                   | 0.00434                     | 230.59                        | 0.03434                           | 29.123                                     | 23.215    |
| 75                           | 9.1789                     | 0.10895                   | 0.00367                     | 272.63                        | 0.03367                           | 29.702                                     | 24.163    |
| 80                           | 10.641                     | 0.09398                   | 0.00311                     | 321.36                        | 0.03311                           | 30.201                                     | 25.035    |
| 85                           | 12.336                     | 0.08107                   | 0.00265                     | 377.86                        | 0.03265                           | 30.631                                     | 25.835    |
| 90                           | 14.300                     | 0.06993                   | 0.00226                     | 443.35                        | 0.03226                           | 31.002                                     | 26.567    |
| 95                           | 16.578                     | 0.06032                   | 0.00193                     | 519.27                        | 0.03193                           | 31.323                                     | 27.235    |
| 100                          | 19.219                     | 0.05203                   | 0.00165                     | 607.29                        | 0.03165                           | 31.599                                     | 27.844    |

*i* = 4%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.0400               | 0.96154             | 1.0000                | 1.0000                  | 1.0400                      | 0.96154                           | 0.00000            |
| 2                      | 1.0816               | 0.92456             | 0.49020               | 2.0400                  | 0.53020                     | 1.8861                            | 0.49020            |
| 3                      | 1.1249               | 0.88900             | 0.32035               | 3.1216                  | 0.36035                     | 2.7751                            | 0.97386            |
| 4                      | 1.1699               | 0.85480             | 0.23549               | 4.2465                  | 0.27549                     | 3.6299                            | 1.4510             |
| 5                      | 1.2167               | 0.82193             | 0.18463               | 5.4163                  | 0.22463                     | 4.4518                            | 1.9216             |
| 6                      | 1.2653               | 0.79031             | 0.15076               | 6.6330                  | 0.19076                     | 5.2421                            | 2.3857             |
| 7                      | 1.3159               | 0.75992             | 0.12661               | 7.8983                  | 0.16661                     | 6.0021                            | 2.8433             |
| 8                      | 1.3686               | 0.73069             | 0.10853               | 9.2142                  | 0.14853                     | 6.7327                            | 3.2944             |
| 9                      | 1.4233               | 0.70259             | 0.09449               | 10.583                  | 0.13449                     | 7.4353                            | 3.7391             |
| 10                     | 1.4802               | 0.67556             | 0.08329               | 12.006                  | 0.12329                     | 8.1109                            | 4.1773             |
| 11                     | 1.5395               | 0.64958             | 0.07415               | 13.486                  | 0.11415                     | 8.7605                            | 4.6090             |
| 12                     | 1.6010               | 0.62460             | 0.06655               | 15.026                  | 0.10655                     | 9.3851                            | 5.0343             |
| 13                     | 1.6651               | 0.60057             | 0.06014               | 16.627                  | 0.10014                     | 9.9856                            | 5.4533             |
| 14                     | 1.7317               | 0.57748             | 0.05467               | 18.292                  | 0.09467                     | 10.563                            | 5.8659             |
| 15                     | 1.8009               | 0.55526             | 0.04994               | 20.024                  | 0.08994                     | 11.118                            | 6.2721             |
| 16                     | 1.8730               | 0.53391             | 0.04582               | 21.825                  | 0.08582                     | 11.652                            | 6.6720             |
| 17                     | 1.9479               | 0.51337             | 0.04220               | 23.698                  | 0.08220                     | 12.166                            | 7.0656             |
| 18                     | 2.0258               | 0.49363             | 0.03899               | 25.645                  | 0.07899                     | 12.659                            | 7.4530             |
| 19                     | 2.1068               | 0.47464             | 0.03614               | 27.671                  | 0.07614                     | 13.134                            | 7.8342             |
| 20                     | 2.1911               | 0.45639             | 0.03358               | 29.778                  | 0.07358                     | 13.590                            | 8.2091             |
| 21                     | 2.2788               | 0.43883             | 0.03128               | 31.969                  | 0.07128                     | 14.029                            | 8.5779             |
| 22                     | 2.3699               | 0.42196             | 0.02920               | 34.248                  | 0.06920                     | 14.451                            | 8.9407             |
| 23                     | 2.4647               | 0.40573             | 0.02731               | 36.618                  | 0.06731                     | 14.857                            | 9.2973             |
| 24                     | 2.5633               | 0.39012             | 0.02559               | 39.083                  | 0.06559                     | 15.247                            | 9.6479             |
| 25                     | 2.6658               | 0.37512             | 0.02401               | 41.646                  | 0.06401                     | 15.622                            | 9.9925             |
| 26                     | 2.7725               | 0.36069             | 0.02257               | 44.312                  | 0.06257                     | 15.983                            | 10.331             |
| 27                     | 2.8834               | 0.34682             | 0.02124               | 47.084                  | 0.06124                     | 16.330                            | 10.664             |
| 28                     | 2.9987               | 0.33348             | 0.02001               | 49.968                  | 0.06001                     | 16.663                            | 10.991             |
| 29                     | 3.1187               | 0.32065             | 0.01888               | 52.966                  | 0.05888                     | 16.984                            | 11.312             |
| 30                     | 3.2434               | 0.30832             | 0.01783               | 56.085                  | 0.05783                     | 17.292                            | 11.627             |
| 31                     | 3.3731               | 0.29646             | 0.01686               | 59.328                  | 0.05686                     | 17.588                            | 11.937             |
| 32                     | 3.5081               | 0.28506             | 0.01595               | 62.701                  | 0.05595                     | 17.874                            | 12.241             |
| 33                     | 3.6484               | 0.27409             | 0.01510               | 66.210                  | 0.05510                     | 18.148                            | 12.540             |
| 34                     | 3.7943               | 0.26355             | 0.01431               | 69.858                  | 0.05431                     | 18.411                            | 12.832             |
| 35                     | 3.9461               | 0.25342             | 0.01358               | 73.652                  | 0.05358                     | 18.665                            | 13.120             |
| 40                     | 4.8010               | 0.20829             | 0.01052               | 95.026                  | 0.05052                     | 19.793                            | 14.477             |
| 45                     | 5.8412               | 0.17120             | 0.00826               | 121.03                  | 0.04826                     | 20.720                            | 15.705             |
| 50                     | 7.1067               | 0.14071             | 0.00655               | 152.67                  | 0.04655                     | 21.482                            | 16.812             |
| 55                     | 8.6464               | 0.11566             | 0.00523               | 191.16                  | 0.04523                     | 22.109                            | 17.807             |
| 60                     | 10.520               | 0.09506             | 0.00420               | 237.99                  | 0.04420                     | 22.623                            | 18.697             |
| 65                     | 12.799               | 0.07813             | 0.00339               | 294.97                  | 0.04339                     | 23.047                            | 19.491             |
| 70                     | 15.572               | 0.06422             | 0.00275               | 364.29                  | 0.04275                     | 23.395                            | 20.196             |
| 75                     | 18.945               | 0.05278             | 0.00223               | 448.63                  | 0.04223                     | 23.680                            | 20.821             |
| 80                     | 23.050               | 0.04338             | 0.00181               | 551.24                  | 0.04181                     | 23.915                            | 21.372             |
| 85                     | 28.044               | 0.03566             | 0.00148               | 676.09                  | 0.04148                     | 24.109                            | 21.857             |
| 90                     | 34.119               | 0.02931             | 0.00121               | 827.98                  | 0.04121                     | 24.267                            | 22.283             |
| 95                     | 41.511               | 0.02409             | 0.00099               | 1012.8                  | 0.04099                     | 24.398                            | 22.655             |
| 100                    | 50.505               | 0.01980             | 0.00081               | 1237.6                  | 0.04081                     | 24.505                            | 22.980             |

$i = 5\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |           |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|-----------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |           |
| N                      | (F/P,i,N)            | (P/F,i,N)           | (A/F,i,N)             | (F/A,i,N)               | (A/P,i,N)                   | (P/A,i,N)                         | (A/G,i,N) |
| 1                      | 1.0500               | 0.95238             | 1.0000                | 1.0000                  | 1.0500                      | 0.95238                           | 0.00000   |
| 2                      | 1.1025               | 0.90703             | 0.48780               | 2.0500                  | 0.53780                     | 1.8594                            | 0.48780   |
| 3                      | 1.1576               | 0.86384             | 0.31721               | 3.1525                  | 0.36721                     | 2.7232                            | 0.96749   |
| 4                      | 1.2155               | 0.82270             | 0.23201               | 4.3101                  | 0.28201                     | 3.5460                            | 1.4391    |
| 5                      | 1.2763               | 0.78353             | 0.18097               | 5.5256                  | 0.23097                     | 4.3295                            | 1.9025    |
| 6                      | 1.3401               | 0.74622             | 0.14702               | 6.8019                  | 0.19702                     | 5.0757                            | 2.3579    |
| 7                      | 1.4071               | 0.71068             | 0.12282               | 8.1420                  | 0.17282                     | 5.7864                            | 2.8052    |
| 8                      | 1.4775               | 0.67684             | 0.10472               | 9.5491                  | 0.15472                     | 6.4632                            | 3.2445    |
| 9                      | 1.5513               | 0.64461             | 0.09069               | 11.027                  | 0.14069                     | 7.1078                            | 3.6758    |
| 10                     | 1.6289               | 0.61391             | 0.07950               | 12.578                  | 0.12950                     | 7.7217                            | 4.0991    |
| 11                     | 1.7103               | 0.58468             | 0.07039               | 14.207                  | 0.12039                     | 8.3064                            | 4.5144    |
| 12                     | 1.7959               | 0.55684             | 0.06283               | 15.917                  | 0.11283                     | 8.8633                            | 4.9219    |
| 13                     | 1.8856               | 0.53032             | 0.05646               | 17.713                  | 0.10646                     | 9.3936                            | 5.3215    |
| 14                     | 1.9799               | 0.50507             | 0.05102               | 19.599                  | 0.10102                     | 9.8986                            | 5.7133    |
| 15                     | 2.0789               | 0.48102             | 0.04634               | 21.579                  | 0.09634                     | 10.380                            | 6.0973    |
| 16                     | 2.1829               | 0.45811             | 0.04227               | 23.657                  | 0.09227                     | 10.838                            | 6.4736    |
| 17                     | 2.2920               | 0.43630             | 0.03870               | 25.840                  | 0.08870                     | 11.274                            | 6.8423    |
| 18                     | 2.4066               | 0.41552             | 0.03555               | 28.132                  | 0.08555                     | 11.690                            | 7.2034    |
| 19                     | 2.5270               | 0.39573             | 0.03275               | 30.539                  | 0.08275                     | 12.085                            | 7.5569    |
| 20                     | 2.6533               | 0.37689             | 0.03024               | 33.066                  | 0.08024                     | 12.462                            | 7.9030    |
| 21                     | 2.7860               | 0.35894             | 0.02800               | 35.719                  | 0.07800                     | 12.821                            | 8.2416    |
| 22                     | 2.9253               | 0.34185             | 0.02597               | 38.505                  | 0.07597                     | 13.163                            | 8.5730    |
| 23                     | 3.0715               | 0.32557             | 0.02414               | 41.430                  | 0.07414                     | 13.489                            | 8.8971    |
| 24                     | 3.2251               | 0.31007             | 0.02247               | 44.502                  | 0.07247                     | 13.799                            | 9.2140    |
| 25                     | 3.3864               | 0.29530             | 0.02095               | 47.727                  | 0.07095                     | 14.094                            | 9.5238    |
| 26                     | 3.5557               | 0.28124             | 0.01956               | 51.113                  | 0.06956                     | 14.375                            | 9.8266    |
| 27                     | 3.7335               | 0.26785             | 0.01829               | 54.669                  | 0.06829                     | 14.643                            | 10.122    |
| 28                     | 3.9201               | 0.25509             | 0.01712               | 58.403                  | 0.06712                     | 14.898                            | 10.411    |
| 29                     | 4.1161               | 0.24295             | 0.01605               | 62.323                  | 0.06605                     | 15.141                            | 10.694    |
| 30                     | 4.3219               | 0.23138             | 0.01505               | 66.439                  | 0.06505                     | 15.372                            | 10.969    |
| 31                     | 4.5380               | 0.22036             | 0.01413               | 70.761                  | 0.06413                     | 15.593                            | 11.238    |
| 32                     | 4.7649               | 0.20987             | 0.01328               | 75.299                  | 0.06328                     | 15.803                            | 11.501    |
| 33                     | 5.0032               | 0.19987             | 0.01249               | 80.064                  | 0.06249                     | 16.003                            | 11.757    |
| 34                     | 5.2533               | 0.19035             | 0.01176               | 85.067                  | 0.06176                     | 16.193                            | 12.006    |
| 35                     | 5.5160               | 0.18129             | 0.01107               | 90.320                  | 0.06107                     | 16.374                            | 12.250    |
| 40                     | 7.0400               | 0.14205             | 0.00828               | 120.80                  | 0.05828                     | 17.159                            | 13.377    |
| 45                     | 8.9850               | 0.11130             | 0.00626               | 159.70                  | 0.05626                     | 17.774                            | 14.364    |
| 50                     | 11.467               | 0.08720             | 0.00478               | 209.35                  | 0.05478                     | 18.256                            | 15.223    |
| 55                     | 14.636               | 0.06833             | 0.00367               | 272.71                  | 0.05367                     | 18.633                            | 15.966    |
| 60                     | 18.679               | 0.05354             | 0.00283               | 353.58                  | 0.05283                     | 18.929                            | 16.606    |
| 65                     | 23.840               | 0.04195             | 0.00219               | 456.80                  | 0.05219                     | 19.161                            | 17.154    |
| 70                     | 30.426               | 0.03287             | 0.00170               | 588.53                  | 0.05170                     | 19.343                            | 17.621    |
| 75                     | 38.833               | 0.02575             | 0.00132               | 756.65                  | 0.05132                     | 19.485                            | 18.018    |
| 80                     | 49.561               | 0.02018             | 0.00103               | 971.23                  | 0.05103                     | 19.596                            | 18.353    |
| 85                     | 63.254               | 0.01581             | 0.00080               | 1245.1                  | 0.05080                     | 19.684                            | 18.635    |
| 90                     | 80.730               | 0.01239             | 0.00063               | 1594.6                  | 0.05063                     | 19.752                            | 18.871    |
| 95                     | 103.03               | 0.00971             | 0.00049               | 2040.7                  | 0.05049                     | 19.806                            | 19.069    |
| 100                    | 131.50               | 0.00760             | 0.00038               | 2610.0                  | 0.05038                     | 19.848                            | 19.234    |

*i = 6%*

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |           |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|-----------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |           |
| N                      | (F/P,i,N)            | (P/F,i,N)           | (A/F,i,N)             | (F/A,i,N)               | (A/P,i,N)                   | (P/A,i,N)                         | (A/G,i,N) |
| 1                      | 1.0600               | 0.94340             | 1.0000                | 1.0000                  | 1.0600                      | 0.94340                           | 0.00000   |
| 2                      | 1.1236               | 0.89000             | 0.48544               | 2.0600                  | 0.54544                     | 1.8334                            | 0.48544   |
| 3                      | 1.1910               | 0.83962             | 0.31411               | 3.1836                  | 0.37411                     | 2.6730                            | 0.96118   |
| 4                      | 1.2625               | 0.79209             | 0.22859               | 4.3746                  | 0.28859                     | 3.4651                            | 1.4272    |
| 5                      | 1.3382               | 0.74726             | 0.17740               | 5.6371                  | 0.23740                     | 4.2124                            | 1.8836    |
| 6                      | 1.4185               | 0.70496             | 0.14336               | 6.9753                  | 0.20336                     | 4.9173                            | 2.3304    |
| 7                      | 1.5036               | 0.66506             | 0.11914               | 8.3938                  | 0.17914                     | 5.5824                            | 2.7676    |
| 8                      | 1.5938               | 0.62741             | 0.10104               | 9.8975                  | 0.16104                     | 6.2098                            | 3.1952    |
| 9                      | 1.6895               | 0.59190             | 0.08702               | 11.491                  | 0.14702                     | 6.8017                            | 3.6133    |
| 10                     | 1.7908               | 0.55839             | 0.07587               | 13.181                  | 0.13587                     | 7.3601                            | 4.0220    |
| 11                     | 1.8983               | 0.52679             | 0.06679               | 14.972                  | 0.12679                     | 7.8869                            | 4.4213    |
| 12                     | 2.0122               | 0.49697             | 0.05928               | 16.870                  | 0.11928                     | 8.3838                            | 4.8113    |
| 13                     | 2.1329               | 0.46884             | 0.05296               | 18.882                  | 0.11296                     | 8.8527                            | 5.1920    |
| 14                     | 2.2609               | 0.44230             | 0.04758               | 21.015                  | 0.10758                     | 9.2950                            | 5.5635    |
| 15                     | 2.3966               | 0.41727             | 0.04296               | 23.276                  | 0.10296                     | 9.7122                            | 5.9260    |
| 16                     | 2.5404               | 0.39365             | 0.03895               | 25.673                  | 0.09895                     | 10.106                            | 6.2794    |
| 17                     | 2.6928               | 0.37136             | 0.03544               | 28.213                  | 0.09544                     | 10.477                            | 6.6240    |
| 18                     | 2.8543               | 0.35034             | 0.03236               | 30.906                  | 0.09236                     | 10.828                            | 6.9597    |
| 19                     | 3.0256               | 0.33051             | 0.02962               | 33.760                  | 0.08962                     | 11.158                            | 7.2867    |
| 20                     | 3.2071               | 0.31180             | 0.02718               | 36.786                  | 0.08718                     | 11.470                            | 7.6051    |
| 21                     | 3.3996               | 0.29416             | 0.02500               | 39.993                  | 0.08500                     | 11.764                            | 7.9151    |
| 22                     | 3.6035               | 0.27751             | 0.02305               | 43.392                  | 0.08305                     | 12.042                            | 8.2166    |
| 23                     | 3.8197               | 0.26180             | 0.02128               | 46.996                  | 0.08128                     | 12.303                            | 8.5099    |
| 24                     | 4.0489               | 0.24698             | 0.01968               | 50.816                  | 0.07968                     | 12.550                            | 8.7951    |
| 25                     | 4.2919               | 0.23300             | 0.01823               | 54.865                  | 0.07823                     | 12.783                            | 9.0722    |
| 26                     | 4.5494               | 0.21981             | 0.01690               | 59.156                  | 0.07690                     | 13.003                            | 9.3414    |
| 27                     | 4.8223               | 0.20737             | 0.01570               | 63.706                  | 0.07570                     | 13.211                            | 9.6029    |
| 28                     | 5.1117               | 0.19563             | 0.01459               | 68.528                  | 0.07459                     | 13.406                            | 9.8568    |
| 29                     | 5.4184               | 0.18456             | 0.01358               | 73.640                  | 0.07358                     | 13.591                            | 10.103    |
| 30                     | 5.7435               | 0.17411             | 0.01265               | 79.058                  | 0.07265                     | 13.765                            | 10.342    |
| 31                     | 6.0881               | 0.16425             | 0.01179               | 84.802                  | 0.07179                     | 13.929                            | 10.574    |
| 32                     | 6.4534               | 0.15496             | 0.01100               | 90.890                  | 0.07100                     | 14.084                            | 10.799    |
| 33                     | 6.8406               | 0.14619             | 0.01027               | 97.343                  | 0.07027                     | 14.230                            | 11.017    |
| 34                     | 7.2510               | 0.13791             | 0.00960               | 104.18                  | 0.06960                     | 14.368                            | 11.228    |
| 35                     | 7.6861               | 0.13011             | 0.00897               | 111.43                  | 0.06897                     | 14.498                            | 11.432    |
| 40                     | 10.286               | 0.09722             | 0.00646               | 154.76                  | 0.06646                     | 15.046                            | 12.359    |
| 45                     | 13.765               | 0.07265             | 0.00470               | 212.74                  | 0.06470                     | 15.456                            | 13.141    |
| 50                     | 18.420               | 0.05429             | 0.00344               | 290.34                  | 0.06344                     | 15.762                            | 13.796    |
| 55                     | 24.650               | 0.04057             | 0.00254               | 394.17                  | 0.06254                     | 15.991                            | 14.341    |
| 60                     | 32.988               | 0.03031             | 0.00188               | 533.13                  | 0.06188                     | 16.161                            | 14.791    |
| 65                     | 44.145               | 0.02265             | 0.00139               | 719.08                  | 0.06139                     | 16.289                            | 15.160    |
| 70                     | 59.076               | 0.01693             | 0.00103               | 967.93                  | 0.06103                     | 16.385                            | 15.461    |
| 75                     | 79.057               | 0.01265             | 0.00077               | 1300.9                  | 0.06077                     | 16.456                            | 15.706    |
| 80                     | 105.80               | 0.00945             | 0.00057               | 1746.6                  | 0.06057                     | 16.509                            | 15.903    |
| 85                     | 141.58               | 0.00706             | 0.00043               | 2343.0                  | 0.06043                     | 16.549                            | 16.062    |
| 90                     | 189.46               | 0.00528             | 0.00032               | 3141.1                  | 0.06032                     | 16.579                            | 16.189    |
| 95                     | 253.55               | 0.00394             | 0.00024               | 4209.1                  | 0.06024                     | 16.601                            | 16.290    |
| 100                    | 339.30               | 0.00295             | 0.00018               | 5638.4                  | 0.06018                     | 16.618                            | 16.371    |

$i = 7\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT               |                            | UNIFORM SERIES            |                             |                               |                                   |           | Arithmetic<br>Gradient<br>Series<br>Factor |
|------------------------------|----------------------------|---------------------------|-----------------------------|-------------------------------|-----------------------------------|-----------|--|
| Compound<br>Amount<br>Factor | Present<br>Worth<br>Factor | Sinking<br>Fund<br>Factor | Uniform<br>Series<br>Factor | Capital<br>Recovery<br>Factor | Series<br>Present<br>Worth Factor |           |  |
| N                            | (F/P,i,N)                  | (P/F,i,N)                 | (A/F,i,N)                   | (F/A,i,N)                     | (A/P,i,N)                         | (P/A,i,N) | (A/G,i,N)                                  |
| 1                            | 1.0700                     | 0.93458                   | 1.0000                      | 1.0000                        | 1.0700                            | 0.93458   | 0.00000                                    |
| 2                            | 1.1449                     | 0.87344                   | 0.48309                     | 2.0700                        | 0.55309                           | 1.8080    | 0.48309                                    |
| 3                            | 1.2250                     | 0.81630                   | 0.31105                     | 3.2149                        | 0.38105                           | 2.6243    | 0.95493                                    |
| 4                            | 1.3108                     | 0.76290                   | 0.22523                     | 4.4399                        | 0.29523                           | 3.3872    | 1.4155                                     |
| 5                            | 1.4026                     | 0.71299                   | 0.17389                     | 5.7507                        | 0.24389                           | 4.1002    | 1.8650                                     |
| 6                            | 1.5007                     | 0.66634                   | 0.13980                     | 7.1533                        | 0.20980                           | 4.7665    | 2.3032                                     |
| 7                            | 1.6058                     | 0.62275                   | 0.11555                     | 8.6540                        | 0.18555                           | 5.3893    | 2.7304                                     |
| 8                            | 1.7182                     | 0.58201                   | 0.09747                     | 10.260                        | 0.16747                           | 5.9713    | 3.1465                                     |
| 9                            | 1.8385                     | 0.54393                   | 0.08349                     | 11.978                        | 0.15349                           | 6.5152    | 3.5517                                     |
| 10                           | 1.9672                     | 0.50835                   | 0.07238                     | 13.816                        | 0.14238                           | 7.0236    | 3.9461                                     |
| 11                           | 2.1049                     | 0.47509                   | 0.06336                     | 15.784                        | 0.13336                           | 7.4987    | 4.3296                                     |
| 12                           | 2.2522                     | 0.44401                   | 0.05590                     | 17.888                        | 0.12590                           | 7.9427    | 4.7025                                     |
| 13                           | 2.4098                     | 0.41496                   | 0.04965                     | 20.141                        | 0.11965                           | 8.3577    | 5.0648                                     |
| 14                           | 2.5785                     | 0.38782                   | 0.04434                     | 22.550                        | 0.11434                           | 8.7455    | 5.4167                                     |
| 15                           | 2.7590                     | 0.36245                   | 0.03979                     | 25.129                        | 0.10979                           | 9.1079    | 5.7583                                     |
| 16                           | 2.9522                     | 0.33873                   | 0.03586                     | 27.888                        | 0.10586                           | 9.4466    | 6.0897                                     |
| 17                           | 3.1588                     | 0.31657                   | 0.03243                     | 30.840                        | 0.10243                           | 9.7632    | 6.4110                                     |
| 18                           | 3.3799                     | 0.29586                   | 0.02941                     | 33.999                        | 0.09941                           | 10.059    | 6.7225                                     |
| 19                           | 3.6165                     | 0.27651                   | 0.02675                     | 37.379                        | 0.09675                           | 10.336    | 7.0242                                     |
| 20                           | 3.8697                     | 0.25842                   | 0.02439                     | 40.995                        | 0.09439                           | 10.594    | 7.3163                                     |
| 21                           | 4.1406                     | 0.24151                   | 0.02229                     | 44.865                        | 0.09229                           | 10.836    | 7.5990                                     |
| 22                           | 4.4304                     | 0.22571                   | 0.02041                     | 49.006                        | 0.09041                           | 11.061    | 7.8725                                     |
| 23                           | 4.7405                     | 0.21095                   | 0.01871                     | 53.436                        | 0.08871                           | 11.272    | 8.1369                                     |
| 24                           | 5.0724                     | 0.19715                   | 0.01719                     | 58.177                        | 0.08719                           | 11.469    | 8.3923                                     |
| 25                           | 5.4274                     | 0.18425                   | 0.01581                     | 63.249                        | 0.08581                           | 11.654    | 8.6391                                     |
| 26                           | 5.8074                     | 0.17220                   | 0.01456                     | 68.676                        | 0.08456                           | 11.826    | 8.8773                                     |
| 27                           | 6.2139                     | 0.16093                   | 0.01343                     | 74.484                        | 0.08343                           | 11.987    | 9.1072                                     |
| 28                           | 6.6488                     | 0.15040                   | 0.01239                     | 80.698                        | 0.08239                           | 12.137    | 9.3289                                     |
| 29                           | 7.1143                     | 0.14056                   | 0.01145                     | 87.347                        | 0.08145                           | 12.278    | 9.5427                                     |
| 30                           | 7.6123                     | 0.13137                   | 0.01059                     | 94.461                        | 0.08059                           | 12.409    | 9.7487                                     |
| 31                           | 8.1451                     | 0.12277                   | 0.00980                     | 102.07                        | 0.07980                           | 12.532    | 9.9471                                     |
| 32                           | 8.7153                     | 0.11474                   | 0.00907                     | 110.22                        | 0.07907                           | 12.647    | 10.138                                     |
| 33                           | 9.3253                     | 0.10723                   | 0.00841                     | 118.93                        | 0.07841                           | 12.754    | 10.322                                     |
| 34                           | 9.9781                     | 0.10022                   | 0.00780                     | 128.26                        | 0.07780                           | 12.854    | 10.499                                     |
| 35                           | 10.677                     | 0.09366                   | 0.00723                     | 138.24                        | 0.07723                           | 12.948    | 10.669                                     |
| 40                           | 14.974                     | 0.06678                   | 0.00501                     | 199.64                        | 0.07501                           | 13.332    | 11.423                                     |
| 45                           | 21.002                     | 0.04761                   | 0.00350                     | 285.75                        | 0.07350                           | 13.606    | 12.036                                     |
| 50                           | 29.457                     | 0.03395                   | 0.00246                     | 406.53                        | 0.07246                           | 13.801    | 12.529                                     |
| 55                           | 41.315                     | 0.02420                   | 0.00174                     | 575.93                        | 0.07174                           | 13.940    | 12.921                                     |
| 60                           | 57.946                     | 0.01726                   | 0.00123                     | 813.52                        | 0.07123                           | 14.039    | 13.232                                     |
| 65                           | 81.273                     | 0.01230                   | 0.00087                     | 1146.8                        | 0.07087                           | 14.110    | 13.476                                     |
| 70                           | 113.99                     | 0.00877                   | 0.00062                     | 1614.1                        | 0.07062                           | 14.160    | 13.666                                     |
| 75                           | 159.88                     | 0.00625                   | 0.00044                     | 2269.7                        | 0.07044                           | 14.196    | 13.814                                     |
| 80                           | 224.23                     | 0.00446                   | 0.00031                     | 3189.1                        | 0.07031                           | 14.222    | 13.927                                     |
| 85                           | 314.50                     | 0.00318                   | 0.00022                     | 4478.6                        | 0.07022                           | 14.240    | 14.015                                     |
| 90                           | 441.10                     | 0.00227                   | 0.00016                     | 6287.2                        | 0.07016                           | 14.253    | 14.081                                     |
| 95                           | 618.67                     | 0.00162                   | 0.00011                     | 8823.9                        | 0.07011                           | 14.263    | 14.132                                     |
| 100                          | 867.72                     | 0.00115                   | 0.00008                     | 12 382.0                      | 0.07008                           | 14.269    | 14.170                                     |

*i* = 8%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.0800               | 0.92593             | 1.0000                | 1.0000                  | 1.0800                      | 0.92593                           | 0.00000            |
| 2                      | 1.1664               | 0.85734             | 0.48077               | 2.0800                  | 0.56077                     | 1.7833                            | 0.48077            |
| 3                      | 1.2597               | 0.79383             | 0.30803               | 3.2464                  | 0.38803                     | 2.5771                            | 0.94874            |
| 4                      | 1.3605               | 0.73503             | 0.22192               | 4.5061                  | 0.30192                     | 3.3121                            | 1.4040             |
| 5                      | 1.4693               | 0.68058             | 0.17046               | 5.8666                  | 0.25046                     | 3.9927                            | 1.8465             |
| 6                      | 1.5869               | 0.63017             | 0.13632               | 7.3359                  | 0.21632                     | 4.6229                            | 2.2763             |
| 7                      | 1.7138               | 0.58349             | 0.11207               | 8.9228                  | 0.19207                     | 5.2064                            | 2.6937             |
| 8                      | 1.8509               | 0.54027             | 0.09401               | 10.637                  | 0.17401                     | 5.7466                            | 3.0985             |
| 9                      | 1.9990               | 0.50025             | 0.08008               | 12.488                  | 0.16008                     | 6.2469                            | 3.4910             |
| 10                     | 2.1589               | 0.46319             | 0.06903               | 14.487                  | 0.14903                     | 6.7101                            | 3.8713             |
| 11                     | 2.3316               | 0.42888             | 0.06008               | 16.645                  | 0.14008                     | 7.1390                            | 4.2395             |
| 12                     | 2.5182               | 0.39711             | 0.05270               | 18.977                  | 0.13270                     | 7.5361                            | 4.5957             |
| 13                     | 2.7196               | 0.36770             | 0.04652               | 21.495                  | 0.12652                     | 7.9038                            | 4.9402             |
| 14                     | 2.9372               | 0.34046             | 0.04130               | 24.215                  | 0.12130                     | 8.2442                            | 5.2731             |
| 15                     | 3.1722               | 0.31524             | 0.03683               | 27.152                  | 0.11683                     | 8.5595                            | 5.5945             |
| 16                     | 3.4259               | 0.29189             | 0.03298               | 30.324                  | 0.11298                     | 8.8514                            | 5.9046             |
| 17                     | 3.7000               | 0.27027             | 0.02963               | 33.750                  | 0.10963                     | 9.1216                            | 6.2037             |
| 18                     | 3.9960               | 0.25025             | 0.02670               | 37.450                  | 0.10670                     | 9.3719                            | 6.4920             |
| 19                     | 4.3157               | 0.23171             | 0.02413               | 41.446                  | 0.10413                     | 9.6036                            | 6.7697             |
| 20                     | 4.6610               | 0.21455             | 0.02185               | 45.762                  | 0.10185                     | 9.8181                            | 7.0369             |
| 21                     | 5.0338               | 0.19866             | 0.01983               | 50.423                  | 0.09983                     | 10.017                            | 7.2940             |
| 22                     | 5.4365               | 0.18394             | 0.01803               | 55.457                  | 0.09803                     | 10.201                            | 7.5412             |
| 23                     | 5.8715               | 0.17032             | 0.01642               | 60.893                  | 0.09642                     | 10.371                            | 7.7786             |
| 24                     | 6.3412               | 0.15770             | 0.01498               | 66.765                  | 0.09498                     | 10.529                            | 8.0066             |
| 25                     | 6.8485               | 0.14602             | 0.01368               | 73.106                  | 0.09368                     | 10.675                            | 8.2254             |
| 26                     | 7.3964               | 0.13520             | 0.01251               | 79.954                  | 0.09251                     | 10.810                            | 8.4352             |
| 27                     | 7.9881               | 0.12519             | 0.01145               | 87.351                  | 0.09145                     | 10.935                            | 8.6363             |
| 28                     | 8.6271               | 0.11591             | 0.01049               | 95.339                  | 0.09049                     | 11.051                            | 8.8289             |
| 29                     | 9.3173               | 0.10733             | 0.00962               | 103.97                  | 0.08962                     | 11.158                            | 9.0133             |
| 30                     | 10.063               | 0.09938             | 0.00883               | 113.28                  | 0.08883                     | 11.258                            | 9.1897             |
| 31                     | 10.868               | 0.09202             | 0.00811               | 123.35                  | 0.08811                     | 11.350                            | 9.3584             |
| 32                     | 11.737               | 0.08520             | 0.00745               | 134.21                  | 0.08745                     | 11.435                            | 9.5197             |
| 33                     | 12.676               | 0.07889             | 0.00685               | 145.95                  | 0.08685                     | 11.514                            | 9.6737             |
| 34                     | 13.690               | 0.07305             | 0.00630               | 158.63                  | 0.08630                     | 11.587                            | 9.8208             |
| 35                     | 14.785               | 0.06763             | 0.00580               | 172.32                  | 0.08580                     | 11.655                            | 9.9611             |
| 40                     | 21.725               | 0.04603             | 0.00386               | 259.06                  | 0.08386                     | 11.925                            | 10.570             |
| 45                     | 31.920               | 0.03133             | 0.00259               | 386.51                  | 0.08259                     | 12.108                            | 11.045             |
| 50                     | 46.902               | 0.02132             | 0.00174               | 573.77                  | 0.08174                     | 12.233                            | 11.411             |
| 55                     | 68.914               | 0.01451             | 0.00118               | 848.92                  | 0.08118                     | 12.319                            | 11.690             |
| 60                     | 101.26               | 0.00988             | 0.00080               | 1253.2                  | 0.08080                     | 12.377                            | 11.902             |
| 65                     | 148.78               | 0.00672             | 0.00054               | 1847.2                  | 0.08054                     | 12.416                            | 12.060             |
| 70                     | 218.61               | 0.00457             | 0.00037               | 2720.1                  | 0.08037                     | 12.443                            | 12.178             |
| 75                     | 321.20               | 0.00311             | 0.00025               | 4002.6                  | 0.08025                     | 12.461                            | 12.266             |
| 80                     | 471.95               | 0.00212             | 0.00017               | 5886.9                  | 0.08017                     | 12.474                            | 12.330             |
| 85                     | 693.46               | 0.00144             | 0.00012               | 8655.7                  | 0.08012                     | 12.482                            | 12.377             |
| 90                     | 1018.9               | 0.00098             | 0.00008               | 12724.0                 | 0.08008                     | 12.488                            | 12.412             |
| 95                     | 1497.1               | 0.00067             | 0.00005               | 18702.0                 | 0.08005                     | 12.492                            | 12.437             |
| 100                    | 2199.8               | 0.00045             | 0.00004               | 27485.0                 | 0.08004                     | 12.494                            | 12.455             |

$i = 9\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             |           | Arithmetic Gradient Series Factor |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------|-----------------------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |           |                                   |
| N                      | (F/P,i,N)            | (P/F,i,N)           | (A/F,i,N)             | (F/A,i,N)               | (A/P,i,N)                   | (P/A,i,N) | (A/G,i,N)                         |
| 1                      | 1.0900               | 0.91743             | 1.0000                | 1.0000                  | 1.0900                      | 0.91743   | 0.00000                           |
| 2                      | 1.1881               | 0.84168             | 0.47847               | 2.0900                  | 0.56847                     | 1.7591    | 0.47847                           |
| 3                      | 1.2950               | 0.77218             | 0.30505               | 3.2781                  | 0.39505                     | 2.5313    | 0.94262                           |
| 4                      | 1.4116               | 0.70843             | 0.21867               | 4.5731                  | 0.30867                     | 3.2397    | 1.3925                            |
| 5                      | 1.5386               | 0.64993             | 0.16709               | 5.9847                  | 0.25709                     | 3.8897    | 1.8282                            |
| 6                      | 1.6771               | 0.59627             | 0.13292               | 7.5233                  | 0.22292                     | 4.4859    | 2.2498                            |
| 7                      | 1.8280               | 0.54703             | 0.10869               | 9.2004                  | 0.19869                     | 5.0330    | 2.6574                            |
| 8                      | 1.9926               | 0.50187             | 0.09067               | 11.028                  | 0.18067                     | 5.5348    | 3.0512                            |
| 9                      | 2.1719               | 0.46043             | 0.07680               | 13.021                  | 0.16680                     | 5.9952    | 3.4312                            |
| 10                     | 2.3674               | 0.42241             | 0.06582               | 15.193                  | 0.15582                     | 6.4177    | 3.7978                            |
| 11                     | 2.5804               | 0.38753             | 0.05695               | 17.560                  | 0.14695                     | 6.8052    | 4.1510                            |
| 12                     | 2.8127               | 0.35553             | 0.04965               | 20.141                  | 0.13965                     | 7.1607    | 4.4910                            |
| 13                     | 3.0658               | 0.32618             | 0.04357               | 22.953                  | 0.13357                     | 7.4869    | 4.8182                            |
| 14                     | 3.3417               | 0.29925             | 0.03843               | 26.019                  | 0.12843                     | 7.7862    | 5.1326                            |
| 15                     | 3.6425               | 0.27454             | 0.03406               | 29.361                  | 0.12406                     | 8.0607    | 5.4346                            |
| 16                     | 3.9703               | 0.25187             | 0.03030               | 33.003                  | 0.12030                     | 8.3126    | 5.7245                            |
| 17                     | 4.3276               | 0.23107             | 0.02705               | 36.974                  | 0.11705                     | 8.5436    | 6.0024                            |
| 18                     | 4.7171               | 0.21199             | 0.02421               | 41.301                  | 0.11421                     | 8.7556    | 6.2687                            |
| 19                     | 5.1417               | 0.19449             | 0.02173               | 46.018                  | 0.11173                     | 8.9501    | 6.5236                            |
| 20                     | 5.6044               | 0.17843             | 0.01955               | 51.160                  | 0.10955                     | 9.1285    | 6.7674                            |
| 21                     | 6.1088               | 0.16370             | 0.01762               | 56.765                  | 0.10762                     | 9.2922    | 7.0006                            |
| 22                     | 6.6586               | 0.15018             | 0.01590               | 62.873                  | 0.10590                     | 9.4424    | 7.2232                            |
| 23                     | 7.2579               | 0.13778             | 0.01438               | 69.532                  | 0.10438                     | 9.5802    | 7.4357                            |
| 24                     | 7.9111               | 0.12640             | 0.01302               | 76.790                  | 0.10302                     | 9.7066    | 7.6384                            |
| 25                     | 8.6231               | 0.11597             | 0.01181               | 84.701                  | 0.10181                     | 9.8226    | 7.8316                            |
| 26                     | 9.3992               | 0.10639             | 0.01072               | 93.324                  | 0.10072                     | 9.9290    | 8.0156                            |
| 27                     | 10.245               | 0.09761             | 0.00973               | 102.72                  | 0.09973                     | 10.027    | 8.1906                            |
| 28                     | 11.167               | 0.08955             | 0.00885               | 112.97                  | 0.09885                     | 10.116    | 8.3571                            |
| 29                     | 12.172               | 0.08215             | 0.00806               | 124.14                  | 0.09806                     | 10.198    | 8.5154                            |
| 30                     | 13.268               | 0.07537             | 0.00734               | 136.31                  | 0.09734                     | 10.274    | 8.6657                            |
| 31                     | 14.462               | 0.06915             | 0.00669               | 149.58                  | 0.09669                     | 10.343    | 8.8083                            |
| 32                     | 15.763               | 0.06344             | 0.00610               | 164.04                  | 0.09610                     | 10.406    | 8.9436                            |
| 33                     | 17.182               | 0.05820             | 0.00556               | 179.80                  | 0.09556                     | 10.464    | 9.0718                            |
| 34                     | 18.728               | 0.05339             | 0.00508               | 196.98                  | 0.09508                     | 10.518    | 9.1933                            |
| 35                     | 20.414               | 0.04899             | 0.00464               | 215.71                  | 0.09464                     | 10.567    | 9.3083                            |
| 40                     | 31.409               | 0.03184             | 0.00296               | 337.88                  | 0.09296                     | 10.757    | 9.7957                            |
| 45                     | 48.327               | 0.02069             | 0.00190               | 525.86                  | 0.09190                     | 10.881    | 10.160                            |
| 50                     | 74.358               | 0.01345             | 0.00123               | 815.08                  | 0.09123                     | 10.962    | 10.430                            |
| 55                     | 114.41               | 0.00874             | 0.00079               | 1260.1                  | 0.09079                     | 11.014    | 10.626                            |
| 60                     | 176.03               | 0.00568             | 0.00051               | 1944.8                  | 0.09051                     | 11.048    | 10.768                            |
| 65                     | 270.85               | 0.00369             | 0.00033               | 2998.3                  | 0.09033                     | 11.070    | 10.870                            |
| 70                     | 416.73               | 0.00240             | 0.00022               | 4619.2                  | 0.09022                     | 11.084    | 10.943                            |
| 75                     | 641.19               | 0.00156             | 0.00014               | 7113.2                  | 0.09014                     | 11.094    | 10.994                            |
| 80                     | 986.55               | 0.00101             | 0.00009               | 10951.0                 | 0.09009                     | 11.100    | 11.030                            |
| 85                     | 1517.9               | 0.00066             | 0.00006               | 16855.0                 | 0.09006                     | 11.104    | 11.055                            |
| 90                     | 2335.5               | 0.00043             | 0.00004               | 25939.0                 | 0.09004                     | 11.106    | 11.073                            |
| 95                     | 3593.5               | 0.00028             | 0.00003               | 39917.0                 | 0.09003                     | 11.108    | 11.085                            |
| 100                    | 5529.0               | 0.00018             | 0.00002               | 61423.0                 | 0.09002                     | 11.109    | 11.093                            |

*i* = 10%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT |                        | UNIFORM SERIES       |                     |                       |                         | Arithmetic Gradient Series Factor |           |
|----------------|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------------|-----------|
|                | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor |                                   |           |
| N              | (F/P,i,N)              | (P/F,i,N)            | (A/F,i,N)           | (F/A,i,N)             | (A/P,i,N)               | (P/A,i,N)                         | (A/G,i,N) |
| 1              | 1.1000                 | 0.90909              | 1.0000              | 1.0000                | 1.1000                  | 0.90909                           | 0.00000   |
| 2              | 1.2100                 | 0.82645              | 0.47619             | 2.1000                | 0.57619                 | 1.7355                            | 0.47619   |
| 3              | 1.3310                 | 0.75131              | 0.30211             | 3.3100                | 0.40211                 | 2.4869                            | 0.93656   |
| 4              | 1.4641                 | 0.68301              | 0.21547             | 4.6410                | 0.31547                 | 3.1699                            | 1.3812    |
| 5              | 1.6105                 | 0.62092              | 0.16380             | 6.1051                | 0.26380                 | 3.7908                            | 1.8101    |
| 6              | 1.7716                 | 0.56447              | 0.12961             | 7.7156                | 0.22961                 | 4.3553                            | 2.2236    |
| 7              | 1.9487                 | 0.51316              | 0.10541             | 9.4872                | 0.20541                 | 4.8684                            | 2.6216    |
| 8              | 2.1436                 | 0.46651              | 0.08744             | 11.436                | 0.18744                 | 5.3349                            | 3.0045    |
| 9              | 2.3579                 | 0.42410              | 0.07364             | 13.579                | 0.17364                 | 5.7590                            | 3.3724    |
| 10             | 2.5937                 | 0.38554              | 0.06275             | 15.937                | 0.16275                 | 6.1446                            | 3.7255    |
| 11             | 2.8531                 | 0.35049              | 0.05396             | 18.531                | 0.15396                 | 6.4951                            | 4.0641    |
| 12             | 3.1384                 | 0.31863              | 0.04676             | 21.384                | 0.14676                 | 6.8137                            | 4.3884    |
| 13             | 3.4523                 | 0.28966              | 0.04078             | 24.523                | 0.14078                 | 7.1034                            | 4.6988    |
| 14             | 3.7975                 | 0.26333              | 0.03575             | 27.975                | 0.13575                 | 7.3667                            | 4.9955    |
| 15             | 4.1772                 | 0.23939              | 0.03147             | 31.772                | 0.13147                 | 7.6061                            | 5.2789    |
| 16             | 4.5950                 | 0.21763              | 0.02782             | 35.950                | 0.12782                 | 7.8237                            | 5.5493    |
| 17             | 5.0545                 | 0.19784              | 0.02466             | 40.545                | 0.12466                 | 8.0216                            | 5.8071    |
| 18             | 5.5599                 | 0.17986              | 0.02193             | 45.599                | 0.12193                 | 8.2014                            | 6.0526    |
| 19             | 6.1159                 | 0.16351              | 0.01955             | 51.159                | 0.11955                 | 8.3649                            | 6.2861    |
| 20             | 6.7275                 | 0.14864              | 0.01746             | 57.275                | 0.11746                 | 8.5136                            | 6.5081    |
| 21             | 7.4002                 | 0.13513              | 0.01562             | 64.002                | 0.11562                 | 8.6487                            | 6.7189    |
| 22             | 8.1403                 | 0.12285              | 0.01401             | 71.403                | 0.11401                 | 8.7715                            | 6.9189    |
| 23             | 8.9543                 | 0.11168              | 0.01257             | 79.543                | 0.11257                 | 8.8832                            | 7.1085    |
| 24             | 9.8497                 | 0.10153              | 0.01130             | 88.497                | 0.11130                 | 8.9847                            | 7.2881    |
| 25             | 10.835                 | 0.09230              | 0.01017             | 98.347                | 0.11017                 | 9.0770                            | 7.4580    |
| 26             | 11.918                 | 0.08391              | 0.00916             | 109.18                | 0.10916                 | 9.1609                            | 7.6186    |
| 27             | 13.110                 | 0.07628              | 0.00826             | 121.10                | 0.10826                 | 9.2372                            | 7.7704    |
| 28             | 14.421                 | 0.06934              | 0.00745             | 134.21                | 0.10745                 | 9.3066                            | 7.9137    |
| 29             | 15.863                 | 0.06304              | 0.00673             | 148.63                | 0.10673                 | 9.3696                            | 8.0489    |
| 30             | 17.449                 | 0.05731              | 0.00608             | 164.49                | 0.10608                 | 9.4269                            | 8.1762    |
| 31             | 19.194                 | 0.05210              | 0.00550             | 181.94                | 0.10550                 | 9.4790                            | 8.2962    |
| 32             | 21.114                 | 0.04736              | 0.00497             | 201.14                | 0.10497                 | 9.5264                            | 8.4091    |
| 33             | 23.225                 | 0.04306              | 0.00450             | 222.25                | 0.10450                 | 9.5694                            | 8.5152    |
| 34             | 25.548                 | 0.03914              | 0.00407             | 245.48                | 0.10407                 | 9.6086                            | 8.6149    |
| 35             | 28.102                 | 0.03558              | 0.00369             | 271.02                | 0.10369                 | 9.6442                            | 8.7086    |
| 40             | 45.259                 | 0.02209              | 0.00226             | 442.59                | 0.10226                 | 9.7791                            | 9.0962    |
| 45             | 72.890                 | 0.01372              | 0.00139             | 718.90                | 0.10139                 | 9.8628                            | 9.3740    |
| 50             | 117.39                 | 0.00852              | 0.00086             | 1163.9                | 0.10086                 | 9.9148                            | 9.5704    |
| 55             | 189.06                 | 0.00529              | 0.00053             | 1880.6                | 0.10053                 | 9.9471                            | 9.7075    |
| 60             | 304.48                 | 0.00328              | 0.00033             | 3034.8                | 0.10033                 | 9.9672                            | 9.8023    |
| 65             | 490.37                 | 0.00204              | 0.00020             | 4893.7                | 0.10020                 | 9.9796                            | 9.8672    |
| 70             | 789.75                 | 0.00127              | 0.00013             | 7887.5                | 0.10013                 | 9.9873                            | 9.9113    |
| 75             | 1271.9                 | 0.00079              | 0.00008             | 12709.0               | 0.10008                 | 9.9921                            | 9.9410    |

$i = 11\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT               |                            | UNIFORM SERIES            |                             |                               |                                   |           | Arithmetic<br>Gradient<br>Series<br>Factor |
|------------------------------|----------------------------|---------------------------|-----------------------------|-------------------------------|-----------------------------------|-----------|--|
| Compound<br>Amount<br>Factor | Present<br>Worth<br>Factor | Sinking<br>Fund<br>Factor | Uniform<br>Series<br>Factor | Capital<br>Recovery<br>Factor | Series<br>Present<br>Worth Factor | (A/G,i,N) |  |
| N                            | (F/P,i,N)                  | (P/F,i,N)                 | (A/F,i,N)                   | (F/A,i,N)                     | (A/P,i,N)                         | (P/A,i,N) | (A/G,i,N)                                  |
| 1                            | 1.1100                     | 0.90090                   | 1.0000                      | 1.0000                        | 1.1100                            | 0.90090   | 0.00000                                    |
| 2                            | 1.2321                     | 0.81162                   | 0.47393                     | 2.1100                        | 0.58393                           | 1.7125    | 0.47393                                    |
| 3                            | 1.3676                     | 0.73119                   | 0.29921                     | 3.3421                        | 0.40921                           | 2.4437    | 0.93055                                    |
| 4                            | 1.5181                     | 0.65873                   | 0.21233                     | 4.7097                        | 0.32233                           | 3.1024    | 1.3700                                     |
| 5                            | 1.6851                     | 0.59345                   | 0.16057                     | 6.2278                        | 0.27057                           | 3.6959    | 1.7923                                     |
| 6                            | 1.8704                     | 0.53464                   | 0.12638                     | 7.9129                        | 0.23638                           | 4.2305    | 2.1976                                     |
| 7                            | 2.0762                     | 0.48166                   | 0.10222                     | 9.783                         | 0.21222                           | 4.7122    | 2.5863                                     |
| 8                            | 2.3045                     | 0.43393                   | 0.08432                     | 11.859                        | 0.19432                           | 5.1461    | 2.9585                                     |
| 9                            | 2.5580                     | 0.39092                   | 0.07060                     | 14.164                        | 0.18060                           | 5.5370    | 3.3144                                     |
| 10                           | 2.8394                     | 0.35218                   | 0.05980                     | 16.722                        | 0.16980                           | 5.8892    | 3.6544                                     |
| 11                           | 3.1518                     | 0.31728                   | 0.05112                     | 19.561                        | 0.16112                           | 6.2065    | 3.9788                                     |
| 12                           | 3.4985                     | 0.28584                   | 0.04403                     | 22.713                        | 0.15403                           | 6.4924    | 4.2879                                     |
| 13                           | 3.8833                     | 0.25751                   | 0.03815                     | 26.212                        | 0.14815                           | 6.7499    | 4.5822                                     |
| 14                           | 4.3104                     | 0.23199                   | 0.03323                     | 30.095                        | 0.14323                           | 6.9819    | 4.8619                                     |
| 15                           | 4.7846                     | 0.20900                   | 0.02907                     | 34.405                        | 0.13907                           | 7.1909    | 5.1275                                     |
| 16                           | 5.3109                     | 0.18829                   | 0.02552                     | 39.190                        | 0.13552                           | 7.3792    | 5.3794                                     |
| 17                           | 5.8951                     | 0.16963                   | 0.02247                     | 44.501                        | 0.13247                           | 7.5488    | 5.6180                                     |
| 18                           | 6.5436                     | 0.15282                   | 0.01984                     | 50.396                        | 0.12984                           | 7.7016    | 5.8439                                     |
| 19                           | 7.2633                     | 0.13768                   | 0.01756                     | 56.939                        | 0.12756                           | 7.8393    | 6.0574                                     |
| 20                           | 8.0623                     | 0.12403                   | 0.01558                     | 64.203                        | 0.12558                           | 7.9633    | 6.2590                                     |
| 21                           | 8.949                      | 0.11174                   | 0.01384                     | 72.265                        | 0.12384                           | 8.0751    | 6.4491                                     |
| 22                           | 9.934                      | 0.10067                   | 0.01231                     | 81.214                        | 0.12231                           | 8.1757    | 6.6283                                     |
| 23                           | 11.026                     | 0.09069                   | 0.01097                     | 91.15                         | 0.12097                           | 8.2664    | 6.7969                                     |
| 24                           | 12.239                     | 0.08170                   | 0.00979                     | 102.17                        | 0.11979                           | 8.3481    | 6.9555                                     |
| 25                           | 13.585                     | 0.07361                   | 0.00874                     | 114.41                        | 0.11874                           | 8.4217    | 7.1045                                     |
| 26                           | 15.080                     | 0.06631                   | 0.00781                     | 128.00                        | 0.11781                           | 8.4881    | 7.2443                                     |
| 27                           | 16.739                     | 0.05974                   | 0.00699                     | 143.08                        | 0.11699                           | 8.5478    | 7.3754                                     |
| 28                           | 18.580                     | 0.05382                   | 0.00626                     | 159.82                        | 0.11626                           | 8.6016    | 7.4982                                     |
| 29                           | 20.624                     | 0.04849                   | 0.00561                     | 178.40                        | 0.11561                           | 8.6501    | 7.6131                                     |
| 30                           | 22.892                     | 0.04368                   | 0.00502                     | 199.02                        | 0.11502                           | 8.6938    | 7.7206                                     |
| 31                           | 25.410                     | 0.03935                   | 0.00451                     | 221.91                        | 0.11451                           | 8.7331    | 7.8210                                     |
| 32                           | 28.206                     | 0.03545                   | 0.00404                     | 247.32                        | 0.11404                           | 8.7686    | 7.9147                                     |
| 33                           | 31.308                     | 0.03194                   | 0.00363                     | 275.53                        | 0.11363                           | 8.8005    | 8.0021                                     |
| 34                           | 34.752                     | 0.02878                   | 0.00326                     | 306.84                        | 0.11326                           | 8.8293    | 8.0836                                     |
| 35                           | 38.575                     | 0.02592                   | 0.00293                     | 341.59                        | 0.11293                           | 8.8552    | 8.1594                                     |
| 40                           | 65.001                     | 0.01538                   | 0.00172                     | 581.83                        | 0.11172                           | 8.9511    | 8.4659                                     |
| 45                           | 109.53                     | 0.00913                   | 0.00101                     | 986.6                         | 0.11101                           | 9.0079    | 8.6763                                     |
| 50                           | 184.56                     | 0.00542                   | 0.00060                     | 1668.8                        | 0.11060                           | 9.0417    | 8.8185                                     |
| 55                           | 311.00                     | 0.00322                   | 0.00035                     | 2818.2                        | 0.11035                           | 9.0617    | 8.9135                                     |

*i* = 12%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT |                             | UNIFORM SERIES              |                             |                             |                             | Arithmetic Gradient Series Factor |                             |
|----------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------------|-----------------------------|
|                | Compound Amount Factor      | Present Worth Factor        | Sinking Fund Factor         | Uniform Series Factor       | Capital Recovery Factor     | Series Present Worth Factor       |                             |
| <i>N</i>       | (F/P, <i>i</i> , <i>N</i> ) | (P/F, <i>i</i> , <i>N</i> ) | (A/F, <i>i</i> , <i>N</i> ) | (F/A, <i>i</i> , <i>N</i> ) | (A/P, <i>i</i> , <i>N</i> ) | (P/A, <i>i</i> , <i>N</i> )       | (A/G, <i>i</i> , <i>N</i> ) |
| 1              | 1.1200                      | 0.89286                     | 1.0000                      | 1.0000                      | 1.1200                      | 0.89286                           | 0.00000                     |
| 2              | 1.2544                      | 0.79719                     | 0.47170                     | 2.1200                      | 0.59170                     | 1.6901                            | 0.47170                     |
| 3              | 1.4049                      | 0.71178                     | 0.29635                     | 3.3744                      | 0.41635                     | 2.4018                            | 0.92461                     |
| 4              | 1.5735                      | 0.63552                     | 0.20923                     | 4.7793                      | 0.32923                     | 3.0373                            | 1.3589                      |
| 5              | 1.7623                      | 0.56743                     | 0.15741                     | 6.3528                      | 0.27741                     | 3.6048                            | 1.7746                      |
| 6              | 1.9738                      | 0.50663                     | 0.12323                     | 8.1152                      | 0.24323                     | 4.1114                            | 2.1720                      |
| 7              | 2.2107                      | 0.45235                     | 0.09912                     | 10.089                      | 0.21912                     | 4.5638                            | 2.5515                      |
| 8              | 2.4760                      | 0.40388                     | 0.08130                     | 12.300                      | 0.20130                     | 4.9676                            | 2.9131                      |
| 9              | 2.7731                      | 0.36061                     | 0.06768                     | 14.776                      | 0.18768                     | 5.3282                            | 3.2574                      |
| 10             | 3.1058                      | 0.32197                     | 0.05698                     | 17.549                      | 0.17698                     | 5.6502                            | 3.5847                      |
| 11             | 3.4785                      | 0.28748                     | 0.04842                     | 20.655                      | 0.16842                     | 5.9377                            | 3.8953                      |
| 12             | 3.8960                      | 0.25668                     | 0.04144                     | 24.133                      | 0.16144                     | 6.1944                            | 4.1897                      |
| 13             | 4.3635                      | 0.22917                     | 0.03568                     | 28.029                      | 0.15568                     | 6.4235                            | 4.4683                      |
| 14             | 4.8871                      | 0.20462                     | 0.03087                     | 32.393                      | 0.15087                     | 6.6282                            | 4.7317                      |
| 15             | 5.4736                      | 0.18270                     | 0.02682                     | 37.280                      | 0.14682                     | 6.8109                            | 4.9803                      |
| 16             | 6.1304                      | 0.16312                     | 0.02339                     | 42.753                      | 0.14339                     | 6.9740                            | 5.2147                      |
| 17             | 6.8660                      | 0.14564                     | 0.02046                     | 48.884                      | 0.14046                     | 7.1196                            | 5.4353                      |
| 18             | 7.6900                      | 0.13004                     | 0.01794                     | 55.750                      | 0.13794                     | 7.2497                            | 5.6427                      |
| 19             | 8.6128                      | 0.11611                     | 0.01576                     | 63.440                      | 0.13576                     | 7.3658                            | 5.8375                      |
| 20             | 9.6463                      | 0.10367                     | 0.01388                     | 72.052                      | 0.13388                     | 7.4694                            | 6.0202                      |
| 21             | 10.804                      | 0.09256                     | 0.01224                     | 81.699                      | 0.13224                     | 7.5620                            | 6.1913                      |
| 22             | 12.100                      | 0.08264                     | 0.01081                     | 92.503                      | 0.13081                     | 7.6446                            | 6.3514                      |
| 23             | 13.552                      | 0.07379                     | 0.00956                     | 104.60                      | 0.12956                     | 7.7184                            | 6.5010                      |
| 24             | 15.179                      | 0.06588                     | 0.00846                     | 118.16                      | 0.12846                     | 7.7843                            | 6.6406                      |
| 25             | 17.000                      | 0.05882                     | 0.00750                     | 133.33                      | 0.12750                     | 7.8431                            | 6.7708                      |
| 26             | 19.040                      | 0.05252                     | 0.00665                     | 150.33                      | 0.12665                     | 7.8957                            | 6.8921                      |
| 27             | 21.325                      | 0.04689                     | 0.00590                     | 169.37                      | 0.12590                     | 7.9426                            | 7.0049                      |
| 28             | 23.884                      | 0.04187                     | 0.00524                     | 190.70                      | 0.12524                     | 7.9844                            | 7.1098                      |
| 29             | 26.750                      | 0.03738                     | 0.00466                     | 214.58                      | 0.12466                     | 8.0218                            | 7.2071                      |
| 30             | 29.960                      | 0.03338                     | 0.00414                     | 241.33                      | 0.12414                     | 8.0552                            | 7.2974                      |
| 31             | 33.555                      | 0.02980                     | 0.00369                     | 271.29                      | 0.12369                     | 8.0850                            | 7.3811                      |
| 32             | 37.582                      | 0.02661                     | 0.00328                     | 304.85                      | 0.12328                     | 8.1116                            | 7.4586                      |
| 33             | 42.092                      | 0.02376                     | 0.00292                     | 342.43                      | 0.12292                     | 8.1354                            | 7.5302                      |
| 34             | 47.143                      | 0.02121                     | 0.00260                     | 384.52                      | 0.12260                     | 8.1566                            | 7.5965                      |
| 35             | 52.800                      | 0.01894                     | 0.00232                     | 431.66                      | 0.12232                     | 8.1755                            | 7.6577                      |
| 40             | 93.051                      | 0.01075                     | 0.00130                     | 767.09                      | 0.12130                     | 8.2438                            | 7.8988                      |
| 45             | 163.99                      | 0.00610                     | 0.00074                     | 1358.2                      | 0.12074                     | 8.2825                            | 8.0572                      |
| 50             | 289.00                      | 0.00346                     | 0.00042                     | 2400.0                      | 0.12042                     | 8.3045                            | 8.1597                      |
| 55             | 509.32                      | 0.00196                     | 0.00024                     | 4236.0                      | 0.12024                     | 8.3170                            | 8.2251                      |

*i* = 13%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             |                    | Arithmetic Gradient Series Factor |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|--------------------|-----------------------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                    |                                   |
| <i>N</i>               | (F/P, <i>i,N</i> )   | (P/F, <i>i,N</i> )  | (A/F, <i>i,N</i> )    | (F/A, <i>i,N</i> )      | (A/P, <i>i,N</i> )          | (P/A, <i>i,N</i> ) | (A/G, <i>i,N</i> )                |
| 1                      | 1.1300               | 0.88496             | 1.0000                | 1.0000                  | 1.1300                      | 0.88496            | 0.00000                           |
| 2                      | 1.2769               | 0.78315             | 0.46948               | 2.1300                  | 0.59948                     | 1.6681             | 0.46948                           |
| 3                      | 1.4429               | 0.69305             | 0.29352               | 3.4069                  | 0.42352                     | 2.3612             | 0.91872                           |
| 4                      | 1.6305               | 0.61332             | 0.20619               | 4.8498                  | 0.33619                     | 2.9745             | 1.3479                            |
| 5                      | 1.8424               | 0.54276             | 0.15431               | 6.4803                  | 0.28431                     | 3.5172             | 1.7571                            |
| 6                      | 2.0820               | 0.48032             | 0.12015               | 8.3227                  | 0.25015                     | 3.9975             | 2.1468                            |
| 7                      | 2.3526               | 0.42506             | 0.09611               | 10.405                  | 0.22611                     | 4.4226             | 2.5171                            |
| 8                      | 2.6584               | 0.37616             | 0.07839               | 12.757                  | 0.20839                     | 4.7988             | 2.8685                            |
| 9                      | 3.0040               | 0.33288             | 0.06487               | 15.416                  | 0.19487                     | 5.1317             | 3.2014                            |
| 10                     | 3.3946               | 0.29459             | 0.05429               | 18.420                  | 0.18429                     | 5.4262             | 3.5162                            |
| 11                     | 3.8359               | 0.26070             | 0.04584               | 21.814                  | 0.17584                     | 5.6869             | 3.8134                            |
| 12                     | 4.3345               | 0.23071             | 0.03899               | 25.650                  | 0.16899                     | 5.9176             | 4.0936                            |
| 13                     | 4.8980               | 0.20416             | 0.03335               | 29.985                  | 0.16335                     | 6.1218             | 4.3573                            |
| 14                     | 5.5348               | 0.18068             | 0.02867               | 34.883                  | 0.15867                     | 6.3025             | 4.6050                            |
| 15                     | 6.2543               | 0.15989             | 0.02474               | 40.417                  | 0.15474                     | 6.4624             | 4.8375                            |
| 16                     | 7.0673               | 0.14150             | 0.02143               | 46.672                  | 0.15143                     | 6.6039             | 5.0552                            |
| 17                     | 7.9861               | 0.12522             | 0.01861               | 53.739                  | 0.14861                     | 6.7291             | 5.2589                            |
| 18                     | 9.0243               | 0.11081             | 0.01620               | 61.725                  | 0.14620                     | 6.8399             | 5.4491                            |
| 19                     | 10.1197              | 0.09806             | 0.01413               | 70.749                  | 0.14413                     | 6.9380             | 5.6265                            |
| 20                     | 11.523               | 0.08678             | 0.01235               | 80.947                  | 0.14235                     | 7.0248             | 5.7917                            |
| 21                     | 13.021               | 0.07680             | 0.01081               | 92.470                  | 0.14081                     | 7.1016             | 5.9454                            |
| 22                     | 14.714               | 0.06796             | 0.00948               | 105.49                  | 0.13948                     | 7.1695             | 6.0881                            |
| 23                     | 16.627               | 0.06014             | 0.00832               | 120.20                  | 0.13832                     | 7.2297             | 6.2205                            |
| 24                     | 18.788               | 0.05323             | 0.00731               | 136.83                  | 0.13731                     | 7.2829             | 6.3431                            |
| 25                     | 21.231               | 0.04710             | 0.00643               | 155.62                  | 0.13643                     | 7.3300             | 6.4566                            |
| 26                     | 23.991               | 0.04168             | 0.00565               | 176.85                  | 0.13565                     | 7.3717             | 6.5614                            |
| 27                     | 27.109               | 0.03689             | 0.00498               | 200.84                  | 0.13498                     | 7.4086             | 6.6582                            |
| 28                     | 30.633               | 0.03264             | 0.00439               | 227.95                  | 0.13439                     | 7.4412             | 6.7474                            |
| 29                     | 34.616               | 0.02889             | 0.00387               | 258.58                  | 0.13387                     | 7.4701             | 6.8296                            |
| 30                     | 39.116               | 0.02557             | 0.00341               | 293.20                  | 0.13341                     | 7.4957             | 6.9052                            |
| 31                     | 44.201               | 0.02262             | 0.00301               | 332.32                  | 0.13301                     | 7.5183             | 6.9747                            |
| 32                     | 49.947               | 0.02002             | 0.00266               | 376.52                  | 0.13266                     | 7.5383             | 7.0385                            |
| 33                     | 56.440               | 0.01772             | 0.00234               | 426.46                  | 0.13234                     | 7.5560             | 7.0971                            |
| 34                     | 63.777               | 0.01568             | 0.00207               | 482.90                  | 0.13207                     | 7.5717             | 7.1507                            |
| 35                     | 72.069               | 0.01388             | 0.00183               | 546.68                  | 0.13183                     | 7.5856             | 7.1998                            |
| 40                     | 132.78               | 0.00753             | 0.00099               | 1013.7                  | 0.13099                     | 7.6344             | 7.3888                            |
| 45                     | 244.64               | 0.00409             | 0.00053               | 1874.2                  | 0.13053                     | 7.6609             | 7.5076                            |
| 50                     | 450.74               | 0.00222             | 0.00029               | 3459.5                  | 0.13029                     | 7.6752             | 7.5811                            |
| 55                     | 830.45               | 0.00120             | 0.00016               | 6380.4                  | 0.13016                     | 7.6830             | 7.6260                            |

*i* = 14%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.1400               | 0.87719             | 1.0000                | 1.0000                  | 1.1400                      | 0.87719                           | 0.00000            |
| 2                      | 1.2996               | 0.76947             | 0.46729               | 2.1400                  | 0.60729                     | 1.6467                            | 0.46729            |
| 3                      | 1.4815               | 0.67497             | 0.29073               | 3.4396                  | 0.43073                     | 2.3216                            | 0.91290            |
| 4                      | 1.6890               | 0.59208             | 0.20320               | 4.9211                  | 0.34320                     | 2.9137                            | 1.3370             |
| 5                      | 1.9254               | 0.51937             | 0.15128               | 6.6101                  | 0.29128                     | 3.4331                            | 1.7399             |
| 6                      | 2.1950               | 0.45559             | 0.11716               | 8.5355                  | 0.25716                     | 3.8887                            | 2.1218             |
| 7                      | 2.5023               | 0.39964             | 0.09319               | 10.730                  | 0.23319                     | 4.2883                            | 2.4832             |
| 8                      | 2.8526               | 0.35056             | 0.07557               | 13.233                  | 0.21557                     | 4.6389                            | 2.8246             |
| 9                      | 3.2519               | 0.30751             | 0.06217               | 16.085                  | 0.20217                     | 4.9464                            | 3.1463             |
| 10                     | 3.7072               | 0.26974             | 0.05171               | 19.337                  | 0.19171                     | 5.2161                            | 3.4490             |
| 11                     | 4.2262               | 0.23662             | 0.04339               | 23.045                  | 0.18339                     | 5.4527                            | 3.7333             |
| 12                     | 4.8179               | 0.20756             | 0.03667               | 27.271                  | 0.17667                     | 5.6603                            | 3.9998             |
| 13                     | 5.4924               | 0.18207             | 0.03116               | 32.089                  | 0.17116                     | 5.8424                            | 4.2491             |
| 14                     | 6.2613               | 0.15971             | 0.02661               | 37.581                  | 0.16661                     | 6.0021                            | 4.4819             |
| 15                     | 7.1379               | 0.14010             | 0.02281               | 43.842                  | 0.16281                     | 6.1422                            | 4.6990             |
| 16                     | 8.1372               | 0.12289             | 0.01962               | 50.980                  | 0.15962                     | 6.2651                            | 4.9011             |
| 17                     | 9.2765               | 0.10780             | 0.01692               | 59.118                  | 0.15692                     | 6.3729                            | 5.0888             |
| 18                     | 10.575               | 0.09456             | 0.01462               | 68.394                  | 0.15462                     | 6.4674                            | 5.2630             |
| 19                     | 12.056               | 0.08295             | 0.01266               | 78.969                  | 0.15266                     | 6.5504                            | 5.4243             |
| 20                     | 13.743               | 0.07276             | 0.01099               | 91.025                  | 0.15099                     | 6.6231                            | 5.5734             |
| 21                     | 15.668               | 0.06383             | 0.00954               | 104.77                  | 0.14954                     | 6.6870                            | 5.7111             |
| 22                     | 17.861               | 0.05599             | 0.00830               | 120.44                  | 0.14830                     | 6.7429                            | 5.8381             |
| 23                     | 20.362               | 0.04911             | 0.00723               | 138.30                  | 0.14723                     | 6.7921                            | 5.9549             |
| 24                     | 23.212               | 0.04308             | 0.00630               | 158.66                  | 0.14630                     | 6.8351                            | 6.0624             |
| 25                     | 26.462               | 0.03779             | 0.00550               | 181.87                  | 0.14550                     | 6.8729                            | 6.1610             |
| 26                     | 30.167               | 0.03315             | 0.00480               | 208.33                  | 0.14480                     | 6.9061                            | 6.2514             |
| 27                     | 34.390               | 0.02908             | 0.00419               | 238.50                  | 0.14419                     | 6.9352                            | 6.3342             |
| 28                     | 39.204               | 0.02551             | 0.00366               | 272.89                  | 0.14366                     | 6.9607                            | 6.4100             |
| 29                     | 44.693               | 0.02237             | 0.00320               | 312.09                  | 0.14320                     | 6.9830                            | 6.4791             |
| 30                     | 50.950               | 0.01963             | 0.00280               | 356.79                  | 0.14280                     | 7.0027                            | 6.5423             |
| 31                     | 58.083               | 0.01722             | 0.00245               | 407.74                  | 0.14245                     | 7.0199                            | 6.5998             |
| 32                     | 66.215               | 0.01510             | 0.00215               | 465.82                  | 0.14215                     | 7.0350                            | 6.6522             |
| 33                     | 75.485               | 0.01325             | 0.00188               | 532.04                  | 0.14188                     | 7.0482                            | 6.6998             |
| 34                     | 86.053               | 0.01162             | 0.00165               | 607.52                  | 0.14165                     | 7.0599                            | 6.7431             |
| 35                     | 98.100               | 0.01019             | 0.00144               | 693.57                  | 0.14144                     | 7.0700                            | 6.7824             |
| 40                     | 188.88               | 0.00529             | 0.00075               | 1342.0                  | 0.14075                     | 7.1050                            | 6.9300             |
| 45                     | 363.68               | 0.00275             | 0.00039               | 2590.6                  | 0.14039                     | 7.1232                            | 7.0188             |
| 50                     | 700.23               | 0.00143             | 0.00020               | 4994.5                  | 0.14020                     | 7.1327                            | 7.0714             |
| 55                     | 1348.2               | 0.00074             | 0.00010               | 9623.1                  | 0.14010                     | 7.1376                            | 7.1020             |

*i* = 15%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT               |                            | UNIFORM SERIES            |                             |                               |                                   | Arithmetic<br>Gradient<br>Series<br>Factor |                    |
|------------------------------|----------------------------|---------------------------|-----------------------------|-------------------------------|-----------------------------------|--|--------------------|
| Compound<br>Amount<br>Factor | Present<br>Worth<br>Factor | Sinking<br>Fund<br>Factor | Uniform<br>Series<br>Factor | Capital<br>Recovery<br>Factor | Series<br>Present<br>Worth Factor |  |                    |
| <i>N</i>                     | (F/P, <i>i,N</i> )         | (P/F, <i>i,N</i> )        | (A/F, <i>i,N</i> )          | (F/A, <i>i,N</i> )            | (A/P, <i>i,N</i> )                | (P/A, <i>i,N</i> )                         | (A/G, <i>i,N</i> ) |
| 1                            | 1.1500                     | 0.86957                   | 1.0000                      | 1.0000                        | 1.1500                            | 0.86957                                    | 0.00000            |
| 2                            | 1.3225                     | 0.75614                   | 0.46512                     | 2.1500                        | 0.61512                           | 1.6257                                     | 0.46512            |
| 3                            | 1.5209                     | 0.65752                   | 0.28798                     | 3.4725                        | 0.43798                           | 2.2832                                     | 0.90713            |
| 4                            | 1.7490                     | 0.57175                   | 0.20027                     | 4.9934                        | 0.35027                           | 2.8550                                     | 1.3263             |
| 5                            | 2.0114                     | 0.49718                   | 0.14832                     | 6.7424                        | 0.29832                           | 3.3522                                     | 1.7228             |
| 6                            | 2.3131                     | 0.43233                   | 0.11424                     | 8.7537                        | 0.26424                           | 3.7845                                     | 2.0972             |
| 7                            | 2.6600                     | 0.37594                   | 0.09036                     | 11.067                        | 0.24036                           | 4.1604                                     | 2.4498             |
| 8                            | 3.0590                     | 0.32690                   | 0.07285                     | 13.727                        | 0.22285                           | 4.4873                                     | 2.7813             |
| 9                            | 3.5179                     | 0.28426                   | 0.05957                     | 16.786                        | 0.20957                           | 4.7716                                     | 3.0922             |
| 10                           | 4.0456                     | 0.24718                   | 0.04925                     | 20.304                        | 0.19925                           | 5.0188                                     | 3.3832             |
| 11                           | 4.6524                     | 0.21494                   | 0.04107                     | 24.349                        | 0.19107                           | 5.2337                                     | 3.6549             |
| 12                           | 5.3503                     | 0.18691                   | 0.03448                     | 29.002                        | 0.18448                           | 5.4206                                     | 3.9082             |
| 13                           | 6.1528                     | 0.16253                   | 0.02911                     | 34.352                        | 0.17911                           | 5.5831                                     | 4.1438             |
| 14                           | 7.0757                     | 0.14133                   | 0.02469                     | 40.505                        | 0.17469                           | 5.7245                                     | 4.3624             |
| 15                           | 8.1371                     | 0.12289                   | 0.02102                     | 47.580                        | 0.17102                           | 5.8474                                     | 4.5650             |
| 16                           | 9.3576                     | 0.10686                   | 0.01795                     | 55.717                        | 0.16795                           | 5.9542                                     | 4.7522             |
| 17                           | 10.761                     | 0.09293                   | 0.01537                     | 65.075                        | 0.16537                           | 6.0472                                     | 4.9251             |
| 18                           | 12.375                     | 0.08081                   | 0.01319                     | 75.836                        | 0.16319                           | 6.1280                                     | 5.0843             |
| 19                           | 14.232                     | 0.07027                   | 0.01134                     | 88.212                        | 0.16134                           | 6.1982                                     | 5.2307             |
| 20                           | 16.367                     | 0.06110                   | 0.00976                     | 102.44                        | 0.15976                           | 6.2593                                     | 5.3651             |
| 21                           | 18.822                     | 0.05313                   | 0.00842                     | 118.81                        | 0.15842                           | 6.3125                                     | 5.4883             |
| 22                           | 21.645                     | 0.04620                   | 0.00727                     | 137.63                        | 0.15727                           | 6.3587                                     | 5.6010             |
| 23                           | 24.891                     | 0.04017                   | 0.00628                     | 159.28                        | 0.15628                           | 6.3988                                     | 5.7040             |
| 24                           | 28.625                     | 0.03493                   | 0.00543                     | 184.17                        | 0.15543                           | 6.4338                                     | 5.7979             |
| 25                           | 32.919                     | 0.03038                   | 0.00470                     | 212.79                        | 0.15470                           | 6.4641                                     | 5.8834             |
| 26                           | 37.857                     | 0.02642                   | 0.00407                     | 245.71                        | 0.15407                           | 6.4906                                     | 5.9612             |
| 27                           | 43.535                     | 0.02297                   | 0.00353                     | 283.57                        | 0.15353                           | 6.5135                                     | 6.0319             |
| 28                           | 50.066                     | 0.01997                   | 0.00306                     | 327.10                        | 0.15306                           | 6.5335                                     | 6.0960             |
| 29                           | 57.575                     | 0.01737                   | 0.00265                     | 377.17                        | 0.15265                           | 6.5509                                     | 6.1541             |
| 30                           | 66.212                     | 0.01510                   | 0.00230                     | 434.75                        | 0.15230                           | 6.5660                                     | 6.2066             |
| 31                           | 76.144                     | 0.01313                   | 0.00200                     | 500.96                        | 0.15200                           | 6.5791                                     | 6.2541             |
| 32                           | 87.565                     | 0.01142                   | 0.00173                     | 577.10                        | 0.15173                           | 6.5905                                     | 6.2970             |
| 33                           | 100.70                     | 0.00993                   | 0.00150                     | 664.67                        | 0.15150                           | 6.6005                                     | 6.3357             |
| 34                           | 115.80                     | 0.00864                   | 0.00131                     | 765.37                        | 0.15131                           | 6.6091                                     | 6.3705             |
| 35                           | 133.18                     | 0.00751                   | 0.00113                     | 881.17                        | 0.15113                           | 6.6166                                     | 6.4019             |
| 40                           | 267.86                     | 0.00373                   | 0.00056                     | 1779.1                        | 0.15056                           | 6.6418                                     | 6.5168             |
| 45                           | 538.77                     | 0.00186                   | 0.00028                     | 3585.1                        | 0.15028                           | 6.6543                                     | 6.5830             |
| 50                           | 1083.7                     | 0.00092                   | 0.00014                     | 7217.7                        | 0.15014                           | 6.6605                                     | 6.6205             |
| 55                           | 2179.6                     | 0.00046                   | 0.00007                     | 14524.0                       | 0.15007                           | 6.6636                                     | 6.6414             |

*i* = 20%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.2000               | 0.83333             | 1.0000                | 1.0000                  | 1.2000                      | 0.83333                           | 0.00000            |
| 2                      | 1.4400               | 0.69444             | 0.45455               | 2.2000                  | 0.65455                     | 1.5278                            | 0.45455            |
| 3                      | 1.7280               | 0.57870             | 0.27473               | 3.6400                  | 0.47473                     | 2.1065                            | 0.87912            |
| 4                      | 2.0736               | 0.48225             | 0.18629               | 5.3680                  | 0.38629                     | 2.5887                            | 1.2742             |
| 5                      | 2.4883               | 0.40188             | 0.13438               | 7.4416                  | 0.33438                     | 2.9906                            | 1.6405             |
| 6                      | 2.9860               | 0.33490             | 0.10071               | 9.9299                  | 0.30071                     | 3.3255                            | 1.9788             |
| 7                      | 3.5832               | 0.27908             | 0.07742               | 12.916                  | 0.27742                     | 3.6046                            | 2.2902             |
| 8                      | 4.2998               | 0.23257             | 0.06061               | 16.499                  | 0.26061                     | 3.8372                            | 2.5756             |
| 9                      | 5.1598               | 0.19381             | 0.04808               | 20.799                  | 0.24808                     | 4.0310                            | 2.8364             |
| 10                     | 6.1917               | 0.16151             | 0.03852               | 25.959                  | 0.23852                     | 4.1925                            | 3.0739             |
| 11                     | 7.4301               | 0.13459             | 0.03110               | 32.150                  | 0.23110                     | 4.3271                            | 3.2893             |
| 12                     | 8.9161               | 0.11216             | 0.02526               | 39.581                  | 0.22526                     | 4.4392                            | 3.4841             |
| 13                     | 10.699               | 0.09346             | 0.02062               | 48.497                  | 0.22062                     | 4.5327                            | 3.6597             |
| 14                     | 12.839               | 0.07789             | 0.01689               | 59.196                  | 0.21689                     | 4.6106                            | 3.8175             |
| 15                     | 15.407               | 0.06491             | 0.01388               | 72.035                  | 0.21388                     | 4.6755                            | 3.9588             |
| 16                     | 18.488               | 0.05409             | 0.01144               | 87.442                  | 0.21144                     | 4.7296                            | 4.0851             |
| 17                     | 22.186               | 0.04507             | 0.00944               | 105.93                  | 0.20944                     | 4.7746                            | 4.1976             |
| 18                     | 26.623               | 0.03756             | 0.00781               | 128.12                  | 0.20781                     | 4.8122                            | 4.2975             |
| 19                     | 31.948               | 0.03130             | 0.00646               | 154.74                  | 0.20646                     | 4.8435                            | 4.3861             |
| 20                     | 38.338               | 0.02608             | 0.00536               | 186.69                  | 0.20536                     | 4.8696                            | 4.4643             |
| 21                     | 46.005               | 0.02174             | 0.00444               | 225.03                  | 0.20444                     | 4.8913                            | 4.5334             |
| 22                     | 55.206               | 0.01811             | 0.00369               | 271.03                  | 0.20369                     | 4.9094                            | 4.5941             |
| 23                     | 66.247               | 0.01509             | 0.00307               | 326.24                  | 0.20307                     | 4.9245                            | 4.6475             |
| 24                     | 79.497               | 0.01258             | 0.00255               | 392.48                  | 0.20255                     | 4.9371                            | 4.6943             |
| 25                     | 95.396               | 0.01048             | 0.00212               | 471.98                  | 0.20212                     | 4.9476                            | 4.7352             |
| 26                     | 114.48               | 0.00874             | 0.00176               | 567.38                  | 0.20176                     | 4.9563                            | 4.7709             |
| 27                     | 137.37               | 0.00728             | 0.00147               | 681.85                  | 0.20147                     | 4.9636                            | 4.8020             |
| 28                     | 164.84               | 0.00607             | 0.00122               | 819.22                  | 0.20122                     | 4.9697                            | 4.8291             |
| 29                     | 197.81               | 0.00506             | 0.00102               | 984.07                  | 0.20102                     | 4.9747                            | 4.8527             |
| 30                     | 237.38               | 0.00421             | 0.00085               | 1181.9                  | 0.20085                     | 4.9789                            | 4.8731             |
| 31                     | 284.85               | 0.00351             | 0.00070               | 1419.3                  | 0.20070                     | 4.9824                            | 4.8908             |
| 32                     | 341.82               | 0.00293             | 0.00059               | 1704.1                  | 0.20059                     | 4.9854                            | 4.9061             |
| 33                     | 410.19               | 0.00244             | 0.00049               | 2045.9                  | 0.20049                     | 4.9878                            | 4.9194             |
| 34                     | 492.22               | 0.00203             | 0.00041               | 2456.1                  | 0.20041                     | 4.9898                            | 4.9308             |
| 35                     | 590.67               | 0.00169             | 0.00034               | 2948.3                  | 0.20034                     | 4.9915                            | 4.9406             |

$i = 25\%$ 

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |           |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|-----------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |           |
| N                      | (F/P,i,N)            | (P/F,i,N)           | (A/F,i,N)             | (F/A,i,N)               | (A/P,i,N)                   | (P/A,i,N)                         | (A/G,i,N) |
| 1                      | 1.2500               | 0.80000             | 1.0000                | 1.0000                  | 1.2500                      | 0.80000                           | 0.00000   |
| 2                      | 1.5625               | 0.64000             | 0.44444               | 2.2500                  | 0.69444                     | 1.4400                            | 0.44444   |
| 3                      | 1.9531               | 0.51200             | 0.26230               | 3.8125                  | 0.51230                     | 1.9520                            | 0.85246   |
| 4                      | 2.4414               | 0.40960             | 0.17344               | 5.7656                  | 0.42344                     | 2.3616                            | 1.2249    |
| 5                      | 3.0518               | 0.32768             | 0.12185               | 8.2070                  | 0.37185                     | 2.6893                            | 1.5631    |
| 6                      | 3.8147               | 0.26214             | 0.08882               | 11.259                  | 0.33882                     | 2.9514                            | 1.8683    |
| 7                      | 4.7684               | 0.20972             | 0.06634               | 15.073                  | 0.31634                     | 3.1611                            | 2.1424    |
| 8                      | 5.9605               | 0.16777             | 0.05040               | 19.842                  | 0.30040                     | 3.3289                            | 2.3872    |
| 9                      | 7.4506               | 0.13422             | 0.03876               | 25.802                  | 0.28876                     | 3.4631                            | 2.6048    |
| 10                     | 9.3132               | 0.10737             | 0.03007               | 33.253                  | 0.28007                     | 3.5705                            | 2.7971    |
| 11                     | 11.642               | 0.08590             | 0.02349               | 42.566                  | 0.27349                     | 3.6564                            | 2.9663    |
| 12                     | 14.552               | 0.06872             | 0.01845               | 54.208                  | 0.26845                     | 3.7251                            | 3.1145    |
| 13                     | 18.190               | 0.05498             | 0.01454               | 68.760                  | 0.26454                     | 3.7801                            | 3.2437    |
| 14                     | 22.737               | 0.04398             | 0.01150               | 86.949                  | 0.26150                     | 3.8241                            | 3.3559    |
| 15                     | 28.422               | 0.03518             | 0.00912               | 109.69                  | 0.25912                     | 3.8593                            | 3.4530    |
| 16                     | 35.527               | 0.02815             | 0.00724               | 138.11                  | 0.25724                     | 3.8874                            | 3.5366    |
| 17                     | 44.409               | 0.02252             | 0.00576               | 173.64                  | 0.25576                     | 3.9099                            | 3.6084    |
| 18                     | 55.511               | 0.01801             | 0.00459               | 218.04                  | 0.25459                     | 3.9279                            | 3.6698    |
| 19                     | 69.389               | 0.01441             | 0.00366               | 273.56                  | 0.25366                     | 3.9424                            | 3.7222    |
| 20                     | 86.736               | 0.01153             | 0.00292               | 342.94                  | 0.25292                     | 3.9539                            | 3.7667    |
| 21                     | 108.42               | 0.00922             | 0.00233               | 429.68                  | 0.25233                     | 3.9631                            | 3.8045    |
| 22                     | 135.53               | 0.00738             | 0.00186               | 538.10                  | 0.25186                     | 3.9705                            | 3.8365    |
| 23                     | 169.41               | 0.00590             | 0.00148               | 673.63                  | 0.25148                     | 3.9764                            | 3.8634    |
| 24                     | 211.76               | 0.00472             | 0.00119               | 843.03                  | 0.25119                     | 3.9811                            | 3.8861    |
| 25                     | 264.70               | 0.00378             | 0.00095               | 1054.8                  | 0.25095                     | 3.9849                            | 3.9052    |
| 26                     | 330.87               | 0.00302             | 0.00076               | 1319.5                  | 0.25076                     | 3.9879                            | 3.9212    |
| 27                     | 413.59               | 0.00242             | 0.00061               | 1650.4                  | 0.25061                     | 3.9903                            | 3.9346    |
| 28                     | 516.99               | 0.00193             | 0.00048               | 2064.0                  | 0.25048                     | 3.9923                            | 3.9457    |
| 29                     | 646.23               | 0.00155             | 0.00039               | 2580.9                  | 0.25039                     | 3.9938                            | 3.9551    |
| 30                     | 807.79               | 0.00124             | 0.00031               | 3227.2                  | 0.25031                     | 3.9950                            | 3.9628    |
| 31                     | 1009.7               | 0.00099             | 0.00025               | 4035.0                  | 0.25025                     | 3.9960                            | 3.9693    |
| 32                     | 1262.2               | 0.00079             | 0.00020               | 5044.7                  | 0.25020                     | 3.9968                            | 3.9746    |
| 33                     | 1577.7               | 0.00063             | 0.00016               | 6306.9                  | 0.25016                     | 3.9975                            | 3.9791    |
| 34                     | 1972.2               | 0.00051             | 0.00013               | 7884.6                  | 0.25013                     | 3.9980                            | 3.9828    |
| 35                     | 2465.2               | 0.00041             | 0.00010               | 9856.8                  | 0.25010                     | 3.9984                            | 3.9858    |

*i* = 30%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT |                        | UNIFORM SERIES       |                     |                       |                         | Arithmetic Gradient Series Factor |           |
|----------------|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------------|-----------|
|                | Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor |                                   |           |
| N              | (F/P,i,N)              | (P/F,i,N)            | (A/F,i,N)           | (F/A,i,N)             | (A/P,i,N)               | (P/A,i,N)                         | (A/G,i,N) |
| 1              | 1.3000                 | 0.76923              | 1.0000              | 1.0000                | 1.3000                  | 0.76923                           | 0.00000   |
| 2              | 1.6900                 | 0.59172              | 0.43478             | 2.3000                | 0.73478                 | 1.3609                            | 0.43478   |
| 3              | 2.1970                 | 0.45517              | 0.25063             | 3.9900                | 0.55063                 | 1.8161                            | 0.82707   |
| 4              | 2.8561                 | 0.35013              | 0.16163             | 6.1870                | 0.46163                 | 2.1662                            | 1.1783    |
| 5              | 3.7129                 | 0.26933              | 0.11058             | 9.0431                | 0.41058                 | 2.4356                            | 1.4903    |
| 6              | 4.8268                 | 0.20718              | 0.07839             | 12.756                | 0.37839                 | 2.6427                            | 1.7654    |
| 7              | 6.2749                 | 0.15937              | 0.05687             | 17.583                | 0.35687                 | 2.8021                            | 2.0063    |
| 8              | 8.1573                 | 0.12259              | 0.04192             | 23.858                | 0.34192                 | 2.9247                            | 2.2156    |
| 9              | 10.604                 | 0.09430              | 0.03124             | 32.015                | 0.33124                 | 3.0190                            | 2.3963    |
| 10             | 13.786                 | 0.07254              | 0.02346             | 42.619                | 0.32346                 | 3.0915                            | 2.5512    |
| 11             | 17.922                 | 0.05580              | 0.01773             | 56.405                | 0.31773                 | 3.1473                            | 2.6833    |
| 12             | 23.298                 | 0.04292              | 0.01345             | 74.327                | 0.31345                 | 3.1903                            | 2.7952    |
| 13             | 30.288                 | 0.03302              | 0.01024             | 97.625                | 0.31024                 | 3.2233                            | 2.8895    |
| 14             | 39.374                 | 0.02540              | 0.00782             | 127.91                | 0.30782                 | 3.2487                            | 2.9685    |
| 15             | 51.186                 | 0.01954              | 0.00598             | 167.29                | 0.30598                 | 3.2682                            | 3.0344    |
| 16             | 66.542                 | 0.01503              | 0.00458             | 218.47                | 0.30458                 | 3.2832                            | 3.0892    |
| 17             | 86.504                 | 0.01156              | 0.00351             | 285.01                | 0.30351                 | 3.2948                            | 3.1345    |
| 18             | 112.46                 | 0.00889              | 0.00269             | 371.52                | 0.30269                 | 3.3037                            | 3.1718    |
| 19             | 146.19                 | 0.00684              | 0.00207             | 483.97                | 0.30207                 | 3.3105                            | 3.2025    |
| 20             | 190.05                 | 0.00526              | 0.00159             | 630.17                | 0.30159                 | 3.3158                            | 3.2275    |
| 21             | 247.06                 | 0.00405              | 0.00122             | 820.22                | 0.30122                 | 3.3198                            | 3.2480    |
| 22             | 321.18                 | 0.00311              | 0.00094             | 1067.3                | 0.30094                 | 3.3230                            | 3.2646    |
| 23             | 417.54                 | 0.00239              | 0.00072             | 1388.5                | 0.30072                 | 3.3254                            | 3.2781    |
| 24             | 542.80                 | 0.00184              | 0.00055             | 1806.0                | 0.30055                 | 3.3272                            | 3.2890    |
| 25             | 705.64                 | 0.00142              | 0.00043             | 2348.8                | 0.30043                 | 3.3286                            | 3.2979    |
| 26             | 917.33                 | 0.00109              | 0.00033             | 3054.4                | 0.30033                 | 3.3297                            | 3.3050    |
| 27             | 1192.5                 | 0.00084              | 0.00025             | 3971.8                | 0.30025                 | 3.3305                            | 3.3107    |
| 28             | 1550.3                 | 0.00065              | 0.00019             | 5164.3                | 0.30019                 | 3.3312                            | 3.3153    |
| 29             | 2015.4                 | 0.00050              | 0.00015             | 6714.6                | 0.30015                 | 3.3317                            | 3.3189    |
| 30             | 2620.0                 | 0.00038              | 0.00011             | 8730.0                | 0.30011                 | 3.3321                            | 3.3219    |
| 31             | 3406.0                 | 0.00029              | 0.00009             | 11 350.0              | 0.30009                 | 3.3324                            | 3.3242    |
| 32             | 4427.8                 | 0.00023              | 0.00007             | 14 756.0              | 0.30007                 | 3.3326                            | 3.3261    |
| 33             | 5756.1                 | 0.00017              | 0.00005             | 19 184.0              | 0.30005                 | 3.3328                            | 3.3276    |
| 34             | 7483.0                 | 0.00013              | 0.00004             | 24 940.0              | 0.30004                 | 3.3329                            | 3.3288    |
| 35             | 9727.9                 | 0.00010              | 0.00003             | 32 423.0              | 0.30003                 | 3.3330                            | 3.3297    |

*i* = 40%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| <i>N</i>               | (F/P, <i>i,N</i> )   | (P/F, <i>i,N</i> )  | (A/F, <i>i,N</i> )    | (F/A, <i>i,N</i> )      | (A/P, <i>i,N</i> )          | (P/A, <i>i,N</i> )                | (A/G, <i>i,N</i> ) |
| 1                      | 1.4000               | 0.71429             | 1.0000                | 1.0000                  | 1.4000                      | 0.71429                           | 0.00000            |
| 2                      | 1.9600               | 0.51020             | 0.41667               | 2.4000                  | 0.81667                     | 1.2245                            | 0.41667            |
| 3                      | 2.7440               | 0.36443             | 0.22936               | 4.3600                  | 0.62936                     | 1.5889                            | 0.77982            |
| 4                      | 3.8416               | 0.26031             | 0.14077               | 7.1040                  | 0.54077                     | 1.8492                            | 1.0923             |
| 5                      | 5.3782               | 0.18593             | 0.09136               | 10.946                  | 0.49136                     | 2.0352                            | 1.3580             |
| 6                      | 7.5295               | 0.13281             | 0.06126               | 16.324                  | 0.46126                     | 2.1680                            | 1.5811             |
| 7                      | 10.541               | 0.09486             | 0.04192               | 23.853                  | 0.44192                     | 2.2628                            | 1.7664             |
| 8                      | 14.758               | 0.06776             | 0.02907               | 34.395                  | 0.42907                     | 2.3306                            | 1.9185             |
| 9                      | 20.661               | 0.04840             | 0.02034               | 49.153                  | 0.42034                     | 2.3790                            | 2.0422             |
| 10                     | 28.925               | 0.03457             | 0.01432               | 69.814                  | 0.41432                     | 2.4136                            | 2.1419             |
| 11                     | 40.496               | 0.02469             | 0.01013               | 98.739                  | 0.41013                     | 2.4383                            | 2.2215             |
| 12                     | 56.694               | 0.01764             | 0.00718               | 139.23                  | 0.40718                     | 2.4559                            | 2.2845             |
| 13                     | 79.371               | 0.01260             | 0.00510               | 195.93                  | 0.40510                     | 2.4685                            | 2.3341             |
| 14                     | 111.12               | 0.00900             | 0.00363               | 275.30                  | 0.40363                     | 2.4775                            | 2.3729             |
| 15                     | 155.57               | 0.00643             | 0.00259               | 386.42                  | 0.40259                     | 2.4839                            | 2.4030             |
| 16                     | 217.80               | 0.00459             | 0.00185               | 541.99                  | 0.40185                     | 2.4885                            | 2.4262             |
| 17                     | 304.91               | 0.00328             | 0.00132               | 759.78                  | 0.40132                     | 2.4918                            | 2.4441             |
| 18                     | 426.88               | 0.00234             | 0.00094               | 1064.70                 | 0.40094                     | 2.4941                            | 2.4577             |
| 19                     | 597.63               | 0.00167             | 0.00067               | 1491.58                 | 0.40067                     | 2.4958                            | 2.4682             |
| 20                     | 836.68               | 0.00120             | 0.00048               | 2089.21                 | 0.40048                     | 2.4970                            | 2.4761             |
| 21                     | 1171.36              | 0.00085             | 0.00034               | 2925.89                 | 0.40034                     | 2.4979                            | 2.4821             |
| 22                     | 1639.90              | 0.00061             | 0.00024               | 4097.24                 | 0.40024                     | 2.4985                            | 2.4866             |
| 23                     | 2295.86              | 0.00044             | 0.00017               | 5737.14                 | 0.40017                     | 2.4989                            | 2.4900             |
| 24                     | 3214.20              | 0.00031             | 0.00012               | 8033.00                 | 0.40012                     | 2.4992                            | 2.4925             |
| 25                     | 4499.88              | 0.00022             | 0.00009               | 11 247.0                | 0.40009                     | 2.4994                            | 2.4944             |
| 26                     | 6299.83              | 0.00016             | 0.00006               | 15 747.0                | 0.40006                     | 2.4996                            | 2.4959             |
| 27                     | 8819.76              | 0.00011             | 0.00005               | 22 047.0                | 0.40005                     | 2.4997                            | 2.4969             |
| 28                     | 12 348.0             | 0.00008             | 0.00003               | 30 867.0                | 0.40003                     | 2.4998                            | 2.4977             |
| 29                     | 17 287.0             | 0.00006             | 0.00002               | 43 214.0                | 0.40002                     | 2.4999                            | 2.4983             |
| 30                     | 24 201.0             | 0.00004             | 0.00002               | 60 501.0                | 0.40002                     | 2.4999                            | 2.4988             |
| 31                     | 33 882.0             | 0.00003             | 0.00001               | 84 703.0                | 0.40001                     | 2.4999                            | 2.4991             |
| 32                     | 47 435.0             | 0.00002             | 0.00001               | 118 585.0               | 0.40001                     | 2.4999                            | 2.4993             |
| 33                     | 66 409.0             | 0.00002             | 0.00001               | 166 019.0               | 0.40001                     | 2.5000                            | 2.4995             |
| 34                     | 92 972.0             | 0.00001             | 0.00000               | 232 428.0               | 0.40000                     | 2.5000                            | 2.4996             |
| 35                     | 130 161.0            | 0.00001             | 0.00000               | 325 400.0               | 0.40000                     | 2.5000                            | 2.4997             |

*i* = 50%

## Discrete Compounding, Discrete Cash Flows

| SINGLE PAYMENT         |                      | UNIFORM SERIES      |                       |                         |                             | Arithmetic Gradient Series Factor |                    |
|------------------------|----------------------|---------------------|-----------------------|-------------------------|-----------------------------|-----------------------------------|--------------------|
| Compound Amount Factor | Present Worth Factor | Sinking Fund Factor | Uniform Series Factor | Capital Recovery Factor | Series Present Worth Factor |                                   |                    |
| N                      | (F/P, <i>i</i> ,N)   | (P/F, <i>i</i> ,N)  | (A/F, <i>i</i> ,N)    | (F/A, <i>i</i> ,N)      | (A/P, <i>i</i> ,N)          | (P/A, <i>i</i> ,N)                | (A/G, <i>i</i> ,N) |
| 1                      | 1.5000               | 0.66667             | 1.0000                | 1.0000                  | 1.5000                      | 0.66667                           | 0.00000            |
| 2                      | 2.2500               | 0.44444             | 0.40000               | 2.5000                  | 0.90000                     | 1.1111                            | 0.40000            |
| 3                      | 3.3750               | 0.29630             | 0.21053               | 4.7500                  | 0.71053                     | 1.4074                            | 0.73684            |
| 4                      | 5.0625               | 0.19753             | 0.12308               | 8.1250                  | 0.62308                     | 1.6049                            | 1.0154             |
| 5                      | 7.5938               | 0.13169             | 0.07583               | 13.1875                 | 0.57583                     | 1.7366                            | 1.2417             |
| 6                      | 11.3906              | 0.08779             | 0.04812               | 20.781                  | 0.54812                     | 1.8244                            | 1.4226             |
| 7                      | 17.0859              | 0.05853             | 0.03108               | 32.172                  | 0.53108                     | 1.8829                            | 1.5648             |
| 8                      | 25.6289              | 0.03902             | 0.02030               | 49.258                  | 0.52030                     | 1.9220                            | 1.6752             |
| 9                      | 38.443               | 0.02601             | 0.01335               | 74.887                  | 0.51335                     | 1.9480                            | 1.7596             |
| 10                     | 57.665               | 0.01734             | 0.00882               | 113.330                 | 0.50882                     | 1.9653                            | 1.8235             |
| 11                     | 86.498               | 0.01156             | 0.00585               | 170.995                 | 0.50585                     | 1.9769                            | 1.8713             |
| 12                     | 129.746              | 0.00771             | 0.00388               | 257.493                 | 0.50388                     | 1.9846                            | 1.9068             |
| 13                     | 194.620              | 0.00514             | 0.00258               | 387.239                 | 0.50258                     | 1.9897                            | 1.9329             |
| 14                     | 291.929              | 0.00343             | 0.00172               | 581.86                  | 0.50172                     | 1.9931                            | 1.9519             |
| 15                     | 437.894              | 0.00228             | 0.00114               | 873.79                  | 0.50114                     | 1.9954                            | 1.9657             |
| 16                     | 656.841              | 0.00152             | 0.00076               | 1311.68                 | 0.50076                     | 1.9970                            | 1.9756             |
| 17                     | 985.261              | 0.00101             | 0.00051               | 1968.52                 | 0.50051                     | 1.9980                            | 1.9827             |
| 18                     | 1477.89              | 0.00068             | 0.00034               | 2953.78                 | 0.50034                     | 1.9986                            | 1.9878             |
| 19                     | 2216.84              | 0.00045             | 0.00023               | 4431.68                 | 0.50023                     | 1.9991                            | 1.9914             |
| 20                     | 3325.26              | 0.00030             | 0.00015               | 6648.51                 | 0.50015                     | 1.9994                            | 1.9940             |
| 21                     | 4987.89              | 0.00020             | 0.00010               | 9973.77                 | 0.50010                     | 1.9996                            | 1.9958             |
| 22                     | 7481.83              | 0.00013             | 0.00007               | 14962.0                 | 0.50007                     | 1.9997                            | 1.9971             |
| 23                     | 11 223.0             | 0.00009             | 0.00004               | 22 443.0                | 0.50004                     | 1.9998                            | 1.9980             |
| 24                     | 16 834.0             | 0.00006             | 0.00003               | 33 666.0                | 0.50003                     | 1.9999                            | 1.9986             |
| 25                     | 25 251.0             | 0.00004             | 0.00002               | 50 500.0                | 0.50002                     | 1.9999                            | 1.9990             |
| 26                     | 37 877.0             | 0.00003             | 0.00001               | 75 752.0                | 0.50001                     | 1.9999                            | 1.9993             |
| 27                     | 56 815.0             | 0.00002             | 0.00001               | 113 628.0               | 0.50001                     | 2.0000                            | 1.9995             |
| 28                     | 85 223.0             | 0.00001             | 0.00001               | 170 443.0               | 0.50001                     | 2.0000                            | 1.9997             |
| 29                     | 127 834.0            | 0.00001             | 0.00000               | 255 666.0               | 0.50000                     | 2.0000                            | 1.9998             |
| 30                     | 191 751.0            | 0.00001             | 0.00000               | 383 500.0               | 0.50000                     | 2.0000                            | 1.9998             |
| 31                     | 287 627.0            | 0.00000             | 0.00000               | 575 251.0               | 0.50000                     | 2.0000                            | 1.9999             |
| 32                     | 431 440.0            | 0.00000             | 0.00000               | 862 878.0               | 0.50000                     | 2.0000                            | 1.9999             |
| 33                     | 647 160.0            | 0.00000             | 0.00000               | 1 294 318.0             | 0.50000                     | 2.0000                            | 1.9999             |
| 34                     | 970 740.0            | 0.00000             | 0.00000               | 1 941 477.0             | 0.50000                     | 2.0000                            | 2.0000             |
| 35                     | 1 456 110.0          | 0.00000             | 0.00000               | 2 912 217.0             | 0.50000                     | 2.0000                            | 2.0000             |

*This page intentionally left blank*

# Answers to Selected Problems

**CHAPTER 1****CHAPTER 2**

- 2.1** \$120  
**2.3** \$5000  
**2.6 (a)** \$1210  
**2.8** 18.75 percent  
**2.13** 8 percent  
**2.15** \$29 719  
**2.17 (a)** Five years  
**2.19 (a)** \$6728  
**2.21 (a)** 26.6 percent  
**2.23** 5 percent  
**2.26**  $i_e \text{ continuous} = 8.318$  percent  
**2.27**  $i_e \text{ weekly} = 5.65$  percent  
**2.29** 0.5 percent  
**2.30** \$2140  
**2.34 (a)** 105 months  
**2.36** \$665 270  
**2.38** Brand 2 about \$51 less  
**2.41** Decisional equivalence holds  
**2.43 (b)** Lost \$60 by locking in

**CHAPTER 3**

- 3.3** \$317.22  
**3.6** £74 790  
**3.7** \$18 466 per year  
**3.10** £3.98  
**3.11** \$35 981.09  
**3.14** \$94.13  
**3.16** \$3 086 287  
**3.17** \$257 143  
**3.20** 162.5 MW  
**3.21** 11.7 percent  
**3.22** 5.8 years  
**3.23** 20.3 percent

**3.26 (b)** \$26.44 per week**3.30** \$34 616**3.32** \$6678**3.33** \$74 514**3.35** No,  $P = \$3\ 670\ 261$ **3.37** \$85.9 million**3.38** €122 316**3.40** \$211 000**3.41** No more than \$504**3.42** Up to \$4587**3.46** \$8 013 275**3.49** 27 months**CHAPTER 4**

- 4.3** A, AB, BC, ABC, BCD  
**4.7** Second offer,  $PW = \$137\ 000$   
**4.8** Earthen dam,  $PW = \$396\ 038$   
**4.11** Hydraulic press,  $AW = \$24\ 716$   
**4.12** No,  $AC(\text{car}) = \$10\ 126$   
**4.13** \$1344  
**4.19** 3.71 years  
**4.21** BC, CD  
**4.24 (a)** 18.1 percent  
**4.27** Plastic liner,  $PW = \$1\ 100\ 000$   
**4.28** XJ3,  $PW = -\$6565$   
**4.30** About 20 percent  
**4.32 (a)** T,  $PW = \$96\ 664$   
**4.34** T,  $AW = \$26\ 154$   
**4.36 (a)** Only B,  $AW = \$15\ 746$   
**4.38** Curtains, payback period = 1.67 years  
**4.41** No,  $PW = -\$16$   
**4.45** Landfill site,  $AW = \$125\ 351$

**CHAPTER 5****5.2** Payback period**5.4** IRR or present worth

- 5.6** Payback period  
**5.8** Annual worth  
**5.10 (a)** 9.2 percent  
**5.12** 12.4 percent  
**5.13** B and D  
**5.16** E  
**5.18 (b)** 2.46 percent  
**5.19 (b)** 7.58 percent  
**5.21** New machine  
**5.22** Approximate ERR > 25 percent  
**5.27** A and C  
**5.29** B  
**5.31** 21.7 percent  
**5.33** 3.19 percent  
**5.35 (c)** Clip Job  
**5.36** Used refrigerator  
**5.38 (b)** 2  
**5.41** \$119

**CHAPTER 6**

- 6.1 (a)** Functional loss  
**6.4** Use-related loss  
**6.6** Time-related loss  
**6.7 (b)** \$7714  
**6.9 (b)** \$5806  
**6.12** For 2013, total assets = \$3900, total equity = \$1350, net income = \$450  
**6.13** Equity ratio = 0.296  
**6.15** e.g., current = 0.9, equity = 0.39, ROA = 2 percent  
**6.17** 12 percent  
**6.19** A, PW = -\$22 321  
**6.21 (a)** \$312.50  
**6.23** Fryer 1, AW = \$37 668  
**6.24** Total assets = \$46 500, total equity = \$15 450, net income = \$2700

- 6.27** \$25 000  
**6.29** Acid test = 0.9  
**6.31** Current assets, fixed assets, depreciation, income taxes, net income (operations), and net income

**CHAPTER 7**

- 7.2** No  
**7.4** \$10 920  
**7.6 (c)** Six years  
**7.8 (a)** Replace old pump after four years  
**7.14** Every three years  
**7.17** Move robot immediately  
**7.19** Yes  
**7.20 (a)** Yes  
**7.22 (c)** \$12 879  
**7.26** No  
**7.28** Challenger EAC is \$39 452 with an economic life of five years  
**7.29** Overhaul old cutter and replace after four years  
**7.31** 4.7 years  
**7.33** Replace immediately  
**7.34 (b)** Defender two years, followed by Challenger 2  
**7.36** Replace the computer now, and keep the replacement for three years  
**7.38 (a)** A1 = 4 years, B1 = 7 years  
**7.40 (a)** A3 = 12 years, B3 = 4 years

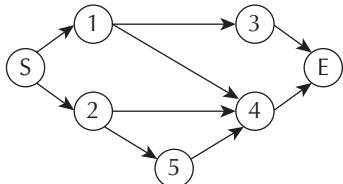
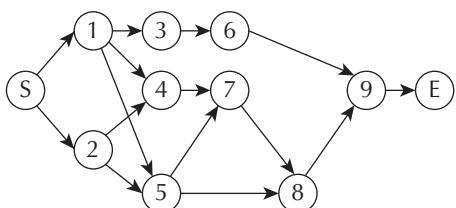
**CHAPTER 8**

- 8.1 (b)** The approximate after-tax MARR is 7 percent.  
**8.4**  $i^* = 6.226$  percent  
**8.6 (a)** CSF = 0.6552  
CTF = 0.6694  
**8.8** Taxes paid = \$14 850

- 8.12** PW = \$1 540 133. ICC should make this investment.
- 8.14** The after-tax present worth of the chip placer is -\$1683.
- 8.16**  $i^* = 10.1087$  percent. Canadian Widgets should not invest.
- 8.18** \$51 929
- 8.20** \$12 962
- 8.22** PW = \$5656. The project should not be done.
- 8.24** MARR<sub>after-tax</sub>  $\cong$  6.0 percent. The purchase should not be made.
- 8.26** \$10 737 for A and \$12 080 for B.
- 8.28** IRR<sub>after-tax</sub>  $\cong$  8.44 percent; IRR<sub>before-tax</sub> = 18.77 percent. The backhoe should not be purchased.
- 8.30 (a)** About \$132 000
- 8.32** Alternative 1 AW = \$274 113; alternative 2 AW = \$204 562. Select alternative 2.
- 8.34** P<sub>T'</sub> = 9974; P<sub>A'</sub> = 7084; P<sub>X'</sub> = 340. Model T has the highest present worth.
- 8.36** IRR<sub>T-after-tax</sub>  $\cong$  20.69 percent, IRR<sub>A-after-tax</sub>  $\cong$  15.07, IRR<sub>X-after-tax</sub>  $\cong$  11.67 percent
- 8.38** IRR<sub>T-after-tax</sub>  $\cong$  20.69%; IRR<sub>A-T after-tax</sub>  $\cong$  2.58%, IRR<sub>X-T after-tax</sub>  $\cong$  5.98%. Model T should be chosen.
- 8.40** ERR = a) 4.88 percent

**CHAPTER 9**

- 9.1 (a)** Current
- 9.3 (a)** \$292
- 9.5 (a)** 14.5 percent
- 9.7 (b)** -2.9 percent
- 9.9** 67 years
- 9.11** 14.3 percent
- 9.13** \$14 683
- 9.15** 11 216 million rubles
- 9.17 (a)** 7.75 percent
- 9.19 (b)** -\$125 532

**9.20 (b)** \$243 547**9.26 (a)** About 16 percent**9.29 (c)** 2.13 percent**9.30 (b)** \$998 for f = 2 percent**CHAPTER 10****10.3 (a)** 1.55**10.3 (b)** 2**10.5 (c)** 0**10.15** \$212 500**10.17 (a)** \$8 580 000 per year**10.19 (d)** \$1333 per day**10.21 (a)** River route**10.25 (b)** 1.07**10.27 (a)** Tennis courts, BCR = 1.18**10.29** Yes, BCR = 1.66**10.31 (a)** \$1599 per day**10.33 (a)** 1.08, 1.1**10.34 (a)** 0.75**10.37 (c)** \$34 510**10.39 (a)** 2 lanes**CHAPTER 11****11.9** 22 weeks**11.11 (b)** 18 weeks**11.13****11.15 (b)**

**11.17** ES = 15 weeks. The critical path is activities 1–2–4–6.

**11.19** 19 days. The critical path is activities 1–4–7–8–9.

## CHAPTER 12

**12.3 (a)** Break-even analysis for multiple projects

**12.5 (a)** \$4146

**12.5 (b)** Break-even operating hours = 671

**12.7** Break-even interest rate 11.2 percent

**12.9** \$855

**12.13 (c)** E(send stock) = 4000

**12.15** E(combined rate) = 15.9 percent

**12.16** PW(base) = \$34 617

**12.18** PW(base) = \$6 636 000, most sensitive to the first cost

**12.20 (a)** AW(base) = \$176 620, most sensitive to the first cost

**12.21 (a)** 179 deliveries per month

**12.25** \$17 535

**12.27** Break-even maintenance cost \$61.50

**12.29** \$23 367 000

**12.31** €6384

**12.33** E(buy ticket) = 150

**12.36** E(partnership) = \$113 750

**12.38 (a)** \$141 045, outside of the likely values

**12.40 (b)** 47 375 boards per year

**12.41** Lease if tax rate is below 27 percent

**12.44 (a)** AW(optimistic, public accept) = \$925 000

**12.44 (b)** E(new product) = \$107 975

*This page intentionally left blank*

# Glossary

**acid-test ratio:** The ratio of quick assets to current liabilities. Quick assets are cash, accounts receivable, and marketable securities—those current assets considered to be highly *liquid*. The acid-test ratio is also known as the quick ratio. 182

**activity on arc (AOA)**

**representation:** In a PERT/CPM network, the use of arcs to represent activities and nodes to capture interrelationships between activities. 396

**activity on node (AON)**

**representation:** In a PERT/CMP network, the use of nodes to represent activities and arcs to capture interrelationships between activities. 396

**actual dollars:** See **current dollars.** 308

**amortization period:** The duration over which a loan is repaid. It is used to compute periodic loan payment amounts. 56

**analogous project:** Similar project to one being undertaken that is used in estimating costs. 208

**annual worth method:** Comparing alternatives by converting all cash flows to a uniform series—i.e., an annuity. 87

**annuity:** A series of uniform-sized receipts or disbursements that start at the end of the first period and continue over a number,  $N$ , of regularly spaced time intervals. 47

**annuity due:** An annuity whose first of  $N$  receipts or disbursements is immediate, at time zero, rather than at the end of the first period. 54

**arithmetic gradient series:** A series of receipts or disbursements that start at the end of the first period and then increase by a constant amount from period to period. 47

**arithmetic gradient to annuity**

**conversion factor:** Denoted by  $(A/G, i, N)$ , gives the value of an annuity,  $A$ , that is equivalent to an arithmetic gradient series where the

constant increase in receipts or disbursements is  $G$  per period, the interest rate is  $i$ , and the number of periods is  $N$ . 60

**asset-management ratios:**

Financial ratios that assess how efficiently a firm is using its assets. Asset management ratios are also known as efficiency ratios. Inventory turnover is an example. 184

**assets:** The economic resources owned by an enterprise. 174

**average costs:** In an engineering project, total costs divided by output level. 208

**backward induction:** See **rollback procedure.** 445

**balance sheet:** A financial statement that gives a snapshot of an enterprise's financial position at a particular point in time, normally the last day of an accounting period. 173

**base period:** A particular date associated with *real dollars* that is used as a reference point for price changes; also the period from which the expenditure shares are calculated in a Laspeyres price index. 338

**base year:** The year on which real dollars are based. 308

**benefit-cost analysis (BCA):** A method of project evaluation widely used in the public sector that provides a general framework for assessing the gains and losses associated with alternative projects when a broad societal view is necessary. 350

**benefit-cost ratio (BCR):** The ratio of the present worth of net users' benefits (social benefits less social costs) to the present worth of the net sponsors' costs for a project. That is,  $BCR = PW(\text{users' benefits})/PW(\text{sponsors' costs})$ . 357

**bond:** An investment that provides an annuity and a future value in return for a cost today. It has a "par" or "face" value, which is the amount for which it can be redeemed after a

certain period of time. It also has a "coupon rate," meaning that the bearer is paid an annuity, usually semi-annually, calculated as a percentage of the face value. 58

**book value:** The depreciated value of an asset for accounting purposes, as calculated with a depreciation model. 165

**break-even analysis:** The process of varying a parameter of a problem and determining what parameter value causes the performance measure to reach some threshold or "break-even" value. 429

**capacity:** The ability to produce, often measured in units of production per time period. 221

**capital cost:** The depreciation expense incurred by the difference between what is paid for the assets required for a particular capacity and what the assets could be resold for some time after purchase. 221

**capital cost allowance (CCA):** The maximum depreciation expense allowed for tax purposes on all assets belonging to an asset class. 277

**capital cost allowance (CCA) asset class:** A categorization of assets for which a specified CCA rate is used to compute CCA. Numerous CCA asset classes exist in the CCA system. 279

**capital cost allowance (CCA) rate:** The maximum depreciation rate allowed for assets in a designated asset class within the CCA system. 279

**capital cost allowance (CCA) system:** The system established by the Canadian government whereby the amount and timing of depreciation expenses on capital assets is controlled. 276

**capital expense:** The expenditure associated with the purchase of a long-term depreciable asset. 265

**capitalized value:** The present worth of an infinitely long series of uniform cash flows. 67

**capital recovery factor:** Denoted by  $(A/P,i,N)$ , gives the value,  $A$ , of the periodic payments or receipts that is equivalent to a present amount,  $P$ , when the interest rate is  $i$  and the number of periods is  $N$ . 52

**capital recovery formula:** A formula that can be used to calculate the savings necessary to justify a capital purchase based on first cost and salvage value. 53

**capital salvage factor (CSF):** A factor that summarizes the effect of the loss of future tax savings when an asset is scrapped or sold. 283

**capital tax factor (CTF):** A factor that summarizes the effect of the future benefit of tax savings due to the CCA. 284

**cash flow diagram:** A chart that summarizes the timing and magnitude of cash flows as they occur over time. The X axis represents time, measured in periods, and the Y axis represents the size and direction of the cash flows. Individual cash flows are indicated by arrows pointing up (positive cash flows, or receipts) or down (negative cash flows, or disbursements). 29

**challenger:** A potential replacement for an existing asset. See **defender**. 219

**closure stage:** In project management, the stage at which work on the project has come to an end. 389

**comparison methods:** Methods of evaluating and comparing projects, such as present worth, annual worth, payback, and IRR. 87

**compound amount factor:** Denoted by  $(F/P,i,N)$ , gives the future amount,  $F$ , that is equivalent to a present amount,  $P$ , when the interest rate is  $i$  and the number of periods is  $N$ . 48

**compound interest:** The standard method of computing interest where interest accumulated in one interest period is added to the principal amount used to calculate interest in the next period. 23

### compound interest factors:

Functions that define the mathematical equivalence of certain common cash flow patterns. 47

**compounding period:** The interest period used with the compound interest method of computing interest. 23

**constant dollars:** See **real dollars**. 308

**consumer price index (CPI):** The CPI relates the average price of a standard set of goods and services in some base period to the average price of the same set of goods and services in another period.

Currently, Statistics Canada uses a base year of 2002 for the CPI. 306

### continuous compounding:

Compounding of interest that occurs continuously over time; i.e., as the length of the compounding period tends toward zero. 28

**continuous models:** Models that assume that all cash flows and all compounding of cash flows occur continuously over time. 47

**cost driver:** A parameter that can be used in a simple function to estimate costs for a project. 210

**cost estimations relationships (CERs):** Parametric models that relate various project design characteristics to aggregate costs. 208

**cost object:** A project, department, activity, or product for which cost data are tracked. 208

**cost of capital:** The minimum rate of return required to induce investors to invest in a business. 91

**cost principle of accounting:** A principle of accounting that states that assets are to be valued on the basis of their cost as opposed to market or other values. 180

**corporation:** A business owned by shareholders. 176

**crashing:** Reducing overall project completion time through the application of additional resources. 403

**critical path method (CPM):** A network representation that is able to capture the interdependence of activities in a project with a network consisting of nodes and arcs. 396

**critical path:** The set of activities that must be completed exactly as planned in order to keep the project on schedule. 400

**current assets:** Cash and other assets that could be converted to cash within a relatively short period of time, usually a year or less. 174

**current dollars:** Monetary units at the time of payment. Also called **actual dollars**. 308

**current interest rate:** The stated, or observed, interest rate based on current dollars. If the real interest rate is  $i'$  and the inflation rate is  $f$ , the current interest rate  $i$  is found by:  $i = i' + f + i'f$ . 312

**current internal rate of return (IRR<sub>C</sub>):** The internal rate of return on a project based on current dollar cash flows associated with the project; also the real internal rate of return that has been adjusted upward to include the effect of inflation. 314

**current liabilities:** Liabilities that are due within some short period of time, usually a year or less. 175

**current MARR:** The minimum acceptable rate of return for *current dollar* cash flows. It is the real MARR adjusted upward for inflation. 312

**current ratio:** The ratio of all current assets to all current liabilities. It is also known as the **working capital ratio**. 181

**decisional equivalence:** Decisional equivalence is a consequence of indifference on the part of a decision maker among available choices. 32

**decision matrix:** A multi-attribute utility theory (MAUT) method in which the rows of a matrix represent criteria, and the columns alternatives. There is an extra column for the weights of the criteria. The cells of the matrix (other than the criteria weights)

contain an evaluation of the alternatives. 471

**decision tree:** A graphical representation of the logical structure of a decision problem in terms of a sequence of decisions and chance events. 443

**declining-balance method of depreciation:** A method of modelling depreciation where the loss in value of an asset in a period is assumed to be a constant proportion of the asset's current value. 168

**defender:** An existing asset being assessed for possible replacement. See **challenger.** 219

**deflation:** The decrease, over time, in average prices. It can also be described as the increase in the purchasing power of money over time. 305

**debt-management ratios:** See **leverage ratios.** 183

**depreciation:** The loss in value of a capital asset. 165

**direct costs:** In an engineering project, costs that can be directly traced back to a **cost object.** 208

**discrete models:** Models that assume all cash flows and all compounding of cash flows occur at the ends of conventionally defined periods like months or years. 47

**discrete probability distribution function:** A **probability distribution function** in which the random variable is discrete. 437

**economic life:** The service life of an asset that minimizes its total cost of use. 223

**earnings before interest and taxes (EBIT):** A measure of a company's operating profit (revenues less operating expenses), which results from making sales and controlling operating expenses. 178

**effective interest rate:** The actual but not usually stated interest rate, found by converting a given interest rate (with an arbitrary compounding period, normally less than a year) to

an equivalent interest rate, with a one-year compounding period. 26

**efficiency ratios:** See **asset-management ratios.** 184

**efficient market:** A market is efficient when decisions are made so that it is impossible to find a way for at least one person to be better off and no person to be worse off. 346

**engineering economics:** Science that deals with techniques of quantitative analysis useful for selecting a preferable alternative from several technically viable ones. 3

**equity ratio:** A financial ratio that is the ratio of total owners' equity to total assets. The smaller this ratio is, the more dependent the firm is on debt for its operations and the higher are the risks the company faces. 183

**equivalence:** A condition that exists when the value of a cost at one time is equivalent to the value of the related benefit at a different time. 32

**equivalent annual cost (EAC):** An annuity that is mathematically equivalent to a more complex set of cash flows. 218

**execution stage:** In project management, the stage in which work is done to deliver the product, service, or desired outcome. 389

**expected value:** A summary statistic of a random variable that gives its mean or average value. Also known as the **mean.** 439

**expensed:** Term applied to an asset with a CCA rate of 100 percent. For all intents and purposes, this is the same as treating the cost of the asset as an operating cost rather than a capital cost. 279

**expenses:** Either real costs associated with performing a corporation's business or a portion of the capital expense for an asset. 265

**external rate of return (ERR):** The rate of return on a project where any cash flows that are not invested in the project are assumed

to earn interest at a predetermined rate (such as the MARR). 140

**extraordinary item:** A gain or loss that does not typically result from a company's normal business activities and is therefore not a recurring item. 186

**financial accounting:** The process of recording and organizing the financial data of a business. The data cover both flows over time, like revenues and expenses, and levels, like an enterprise's resources and the claims on those resources, at a given date. 172

**financial market:** Market for the exchange of capital and credit in the economy. 177

**financial ratio analysis:** Comparison of a firm's financial ratios with ratios computed for the same firm from previous financial statements and with industry standard ratios. 180

**financial ratios:** Ratios between key amounts taken from the financial statements of a firm. They give an analyst a framework for answering questions about the firm's liquidity, asset management, leverage, and profitability. 180

**fixed costs:** Costs that remain the same, regardless of actual units of production. 207

**future worth:** See the definition of **interest rate.** 21

**future worth method:** Comparing alternatives by taking all cash flows to future worth. 95

**Gantt chart:** A graph or bar chart that depicts the timing and sequence of project activities. 395

**geometric gradient series:** A set of disbursements or receipts that change by a constant *proportion* from one period to the next in a sequence of periods. 47

**geometric gradient to present worth conversion factor:** Denoted by  $(P/A,g,i,N)$ , gives the present worth,  $P$ , that is equivalent to a

geometric gradient series where the base receipt or disbursement is  $A$ , and where the rate of growth is  $g$ , the interest rate is  $i$ , and the number of periods is  $N$ . 61

**growth-adjusted interest rate,  $i^*$ :**  $i^* = (1 + i)/(1 + g)$  so that  $1/(1 + i^*) = (1 + i)/(1 + g)$ , where  $i$  is the interest rate and  $g$  is the growth rate. The growth-adjusted interest rate is used in computing the geometric gradient to present worth conversion factor. 63

**income statement:** A financial statement that summarizes an enterprise's revenues and expenses over a specified accounting period. 178

**independent projects:** Two projects are independent if the expected costs and the expected benefits of each of the projects do not depend on whether the other one is chosen. 88

**indirect costs:** In an engineering project, costs that cannot be traced easily back to a particular **cost object**. 208

**inflation:** The increase, over time, in average prices of goods and services. It can also be described as the decrease in the purchasing power of money over time. 305

**inflation rate:** The rate of increase in average prices of goods and services over a specified time period, usually a year; also, the rate of decrease in purchasing power of money over a specified time period, usually a year. 306

**initiation stage:** In project management, the stage of a project in which its nature and scope are defined. 389

**installation costs:** Costs of acquiring capacity (excluding the purchase cost) that may include disruption of production, training of workers, and perhaps a reorganization of other production. 221

**interest:** The compensation for giving up the use of money. 20

**interest period:** The base unit of time over which an interest rate is quoted. The interest period is referred to as the compounding period when compound interest is used. 21

**interest rate:** If the right to  $P$  at the beginning of a time period exchanges for the right to  $F$  at the end of the period, where  $F = P(1 + i)$ ,  $i$  is the interest rate per time period. In this definition,  $P$  is called the *present worth* of  $F$ , and  $F$  is called the *future worth* of  $P$ . 21

**internal rate of return (IRR):** The interest rate,  $i^*$ , such that, when all cash flows associated with a project are discounted at  $i^*$ , the present worth of the cash inflows equals the present worth of the cash outflows. 127

**inventory-turnover ratio:** A financial ratio that captures the number of times that a firm's inventories are replaced (or turned over) per year. It provides a measure of whether the firm has more or less inventory than normal. 184

**Laspeyres price index:** A commonly used price index that measures weighted average changes in prices of a set of goods and services over time as compared with the prices in a base period. The weights are the expenditure shares in the base period. The weights are then converted to percentages by multiplying by 100. 338

**leverage ratios:** Financial ratios that provide information about how liquid a firm is, or how well it is able to meet its current obligations. Also known as **debt-management ratios**. 183

**liabilities:** Claims, other than those of the owners, on a business's assets. 175

**liquidity ratio:** A financial ratio that evaluates the ability of a business to meet its current liability obligations. The current ratio and quick ratio are two examples of liquidity ratios. 181

**long-term assets:** Assets that are not expected to be converted to cash

in the short term, usually taken to be one year. 174

**long-term liabilities:** Liabilities that are not expected to draw on the business's current assets. 175

**management accounting:** The process of analyzing and recording the costs and benefits of the various activities of an enterprise. The goal of management accounting is to provide managers with information to help in decision making. 172

**marginal costs:** In an engineering project, total variable cost per unit. 208

**market:** A group of buyers and sellers linked by trade in a particular product or service. 346

**market equivalence:** The ability to exchange one cash flow for another at zero cost. 32

**market failure:** Condition in which output or consumption decisions are made in which aggregate benefits to all persons who benefit from the decision are less than aggregate costs imposed on persons who bear costs that result from the decision. 32

**market value:** Usually taken as the actual value an asset can be sold for in an open market. 165

**mathematical equivalence:** An equivalence of cash flows due to the mathematical relationship between time and money. 32

**mean:** See **expected value**. 439

**minimum acceptable rate of return (MARR):** An interest rate that must be earned for any project to be accepted. 90

**modified benefit-cost ratio:** The ratio of the present worth (or annual worth) of benefits minus the present worth (or annual worth) of operating costs to the present worth (or annual worth) of capital costs—that is,  $BCRM = [PW(\text{users' benefits}) - PW(\text{sponsors' operating costs})]/PW(\text{sponsors' capital costs})$ . 357

**monitoring and control stage:** In project management, the stage in

which regular progress reports and constant assessment of issues and risks are carried out. 389

**mutually exclusive projects:**

Projects are mutually exclusive if, in the process of choosing one, all the other alternatives are excluded. 88

**net cash flow:** The difference between cash inflows and outflows for the period. The net cash flow,  $A_t$ , is given by  $A_t = R_t - D_t$ , where  $R_t$  is cash inflow in period  $t$ , and  $D_t$  is cash disbursed in period  $t$ . 138

**net-profit ratio:** See **return-on-assets (ROA) ratio.** 186

**nominal interest rate:** The conventional method of stating the annual interest rate. It is calculated by multiplying the interest rate per compounding period by the number of compounding periods per year. 26

**one year principle:** This principle states that if the EAC(capital costs) for the defender are small compared to the EAC(operating costs), and the yearly operating costs are monotonically increasing, the economic life of a defender is one year and its total EAC is the cost of using the defender for one more year. 227

**operating and maintenance costs:** Ongoing costs to operate and maintain an asset over its useful life. Includes costs such as electricity, gasoline, parts, repair, and insurance. 221

**owners' equity:** The interest of the owner or owners of a firm in its assets. 176

**partnership:** A business owned by two or more owners (partners). 176

**par value:** The price per share set by a firm at the time the shares are originally issued. 176

**payback period:** The period of time it takes for an investment to be recouped when the interest rate is assumed to be zero. 101

**payback period method:** A method used for comparing alternatives by

comparing the periods of time required for the investments to pay for themselves. 87

**performance measures:** Calculated values that allow conclusions to be drawn from data. 180

**planning stage:** In project management, the stage in which the project is broken down into smaller modules. 389

**policy instrument:** A remedy for market failure in which the government imposes rules intended to modify behaviour of business organizations. 348

**present worth:** See **interest rate.** 21

**present worth factor:** Denoted by  $(P/F,i,N)$ , gives the present amount,  $P$ , that is equivalent to a future amount,  $F$ , when the interest rate is  $i$  and the number of periods is  $N$ . 49

**present worth method:**

Comparing alternatives by taking all cash flows to present worth. 87

**price index:** A number, usually a percentage, that relates prices of a given set of goods and services in some period,  $t_1$ , to the prices of the same set of goods and services in another period,  $t_0$ . 338

**probability:** The limit of a long-run proportion or relative frequency; see Close-Up 12.1 for other views of probability. 436

**probability distribution function:**

A set of numerical measures (probabilities) associated with outcomes of a random variable. Also known as a **risk profile.** 436

**profitability ratios:** Financial ratios that give evidence of how productively assets have been employed in producing a profit. Return on total assets (or net-profit ratio) is an example of a profitability ratio. 186

**program evaluation and review technique (PERT):** A network representation that is able to capture the interdependence of

activities in a project by a network consisting of nodes and arcs. 396

**progressive tax rate:** A taxation rate that increases as the income level increases. 265

**project:** 1) An investment opportunity. 2) A temporary activity that is carried out to deliver a specific product or service. 87

**project balance:** If a project has a sequence of net cash flows  $A_0, A_1, A_2, \dots, A_T$ , and the interest rate is  $i'$ , there are  $T + 1$  project balances,  $B_0, B_1, \dots, B_T$ , one at the end of each period  $t$ ,  $t = 0, 1, \dots, T$ . A project balance,  $B_t$ , is the cumulative future value of all cash flows up to the end of period  $t$ , compounded at the rate  $i'$ . 138

**project charter:** In project management, a reference document that describes what a project will accomplish. 389

**project closure report:** A document that reports the conclusion of a project and contains an assessment of how well the project performed against its original goals. 392

**project management:** The processes of planning a project, organizing the necessary resources, leading the individuals responsible for carrying out the project, monitoring the project, and implementing its deliverables. 388

**project manager:** A person who has been assigned the responsibility of fulfilling the objectives of a project. 390

**project plan:** A document that defines the sequence of activities and the resources needed for each module of a project. 390

**project sponsor:** A person (or group) who has the resources to complete a project. The project sponsor acts as the project champion and leads the process of developing a project charter document. 390

**public-private partnership (P3):** A cooperative arrangement between the private and public sectors for the provision of infrastructure or services. 350

**quick ratio:** See **acid-test ratio.** 182

**random variable:** A random variable is a parameter or variable that can take on a number of possible outcomes. The probability of each outcome is given by its probability distribution. 436

**real dollars:** Monetary units of constant purchasing power. 308

**real interest rate:** The interest rate,  $i'$ , is the interest rate that would yield the same number of real dollars in the absence of inflation as the current interest rate yields in the presence of inflation at the rate  $f$ . It is given by  $i' = (1 + i)/(1 + f) - 1$ . 312

**real internal rate of return:** The internal rate of return on a project based on real dollar cash flows associated with the project. 314

**real MARR:** The minimum acceptable rate of return when cash flows are expressed in real, or constant, dollars. 312

**related but not mutually exclusive projects:** For pairs of projects in this category, the expected costs and benefits of one project depend on whether the other one is chosen. 88

**repeated lives:** Used for comparing alternatives with different service lives, based on the assumption that alternatives can be repeated in the future, with the same costs and benefits, as often as necessary. The life of each alternative is repeated until a common total time period is reached for all alternatives. 97

**replacement:** An asset may be replaced if there is a cheaper way to get the service the asset provides or if the service provided by the asset is no longer adequate. 220

**retained earnings:** The cumulative sum of earnings from normal

operations, in addition to gains (or losses) from transactions such as the sale of plant assets or investments that have been reinvested in the business—i.e., not paid out as dividends. 176

**retire:** To remove an asset from use without replacement. 220

**return-on-assets (ROA) ratio:** A financial ratio that captures how productively assets have been employed in producing a profit. It is also known as the **net-profit ratio.** 186

**return-on-equity (ROE) ratio:** A financial ratio that captures how much profit a company has earned in comparison to the amount of capital that the owners have tied up in the company. 186

**risk profile:** See **probability distribution function.** 436

**rollback procedure:** A procedure in decision tree analysis that computes an expected value at each chance node and selects a preferred alternative at each decision node; also known as **backward induction.** 445

**salvage value:** Either the actual value of an asset at the end of its useful life (when it is sold) or an estimate of the salvage value calculated using a depreciation model. 165

**scrap value:** Either the actual value of an asset at the end of its physical life (when it is broken up for the material value of its parts) or an estimate of the scrap value calculated using a depreciation model. 165

**sensitivity analysis:** Methods that assess the sensitivity of an economic measure to uncertainties in estimates in the various parameters of a problem. 425

**sensitivity graph:** A graph of the changes in a performance measure, holding all other variables fixed. 426

**series present worth factor:** Denoted by  $(P/A,i,N)$ , gives the present amount,  $P$ , that is equivalent to an annuity,  $A$ , when the interest

rate is  $i$  and the number of periods is  $N$ . 52

**simple interest:** A method of computing interest where interest earned during an interest period is not added to the principal amount used to calculate interest in the next period. Simple interest is rarely used, except as a method of calculating approximate interest. 23

**simple investment:** A project that consists of one or more cash outflows at the beginning, followed only by one or more cash inflows. 142

**sinking fund:** Interest-bearing account into which regular deposits are made in order to accumulate some amount. 51

**sinking fund factor:** Denoted by  $(A/F,i,N)$ , gives the size,  $A$ , of a repeated receipt or disbursement that is equivalent to a future amount,  $F$ , when the interest rate is  $i$  and the number of periods is  $N$ . 51

**small business deduction:** A tax deduction that reduces the effective tax rate for a small Canadian company to less than 20 percent. 267

**sole proprietorship:** A business owned by one person. 176

**specialist company:** A firm that concentrates on manufacturing a limited range of very specialized products. 220

**statement of changes in financial position:** A financial statement that shows the amounts of cash generated by a company's operations and by other sources, and the amounts of cash used for investments and other non-operating disbursements. 178

**straight-line depreciation:** A method of modelling depreciation that assumes that the rate of loss in value of an asset is constant over its useful life. 166

**study period:** A period of time over which alternative projects are compared. 97

**sunk costs:** Costs that were incurred in the past and are no longer relevant in replacement decisions. 230

**tax credits:** Real or nominal costs that are not taxed or are taxed at a reduced rate. 265

**total costs:** In an engineering project, the sum of **fixed** and **variable costs**. 208

**trend analysis:** A form of financial analysis that traces the financial ratios of a firm over several accounting periods. 180

#### **undepreciated capital cost**

**(UCC):** The remaining book value of assets subject to depreciation for taxation purposes. For any given

year, the UCC balance can be calculated as follows:  $\text{UCC}_{\text{opening}} + \text{additions} - \text{disposals} - \text{CCA} = \text{UCC}_{\text{ending}}$ . 280

#### **uniform series compound amount factor:**

Denoted by  $(F/A,i,N)$ , gives the future value,  $F$ , that is equivalent to a series of equal-sized receipts or disbursements,  $A$ , when the interest rate is  $i$  and the number of periods is  $N$ . 52

**variable costs:** Costs that change depending on the number of units produced. 207

#### **weighted average cost of capital:**

A weighted average of the costs of borrowing and of selling shares.

The weights are the fractions of total capital that come from the different sources. 124

#### **work breakdown structure**

**(WBS):** A tool used to organize all the work to be done in a project by decomposing the project into smaller units. 392

**working capital:** The difference between total current assets and total current liabilities. 181

**working capital ratio:** See **current ratio**. 181

**work package:** The lowest level of a WBS—a specific project deliverable. 392

*This page intentionally left blank*

# Photo Credits

**Chapter 1:** © iStockphoto.com/David Jones

**Chapter 2:** © Feng Yu/Shutterstock

**Chapter 3:** © ArtmannWitte/Shutterstock

**Chapter 4:** © iStockphoto.com/Gunnar Pippel

**Chapter 5:** © iStockphoto/Thinkstock

**Chapter 6:** © iStockphoto.com/blackred

**Chapter 7:** © iStockphoto/Thinkstock

**Chapter 8:** © Hemera/Thinkstock

**Chapter 9:** © iStockphoto/José Luis Gutiérrez

**Chapter 10:** © iStockphoto/JamesReillyWilson

**Chapter 11:** © iStockphoto/Andrew Houghton

**Chapter 12:** © iStockphoto/Alan Crosthwaite

*This page intentionally left blank*

# Index

- A**
- abstractions, 6–8
  - accounting
    - cost accounting, 172–173
    - cost principle of accounting, 180
    - depreciation accounting. *See* depreciation
    - financial accounting. *See* financial accounting
    - management accounting, 172–173
  - acid rain, 347, 384–386
  - acid-test ratio, 182–183, 187 $t$
  - across variables, 173
  - activity on arc (AOA), 396–397, 397 $f$ , 398, 398 $f$
  - actual dollars, 308
  - after-tax IRR, 274–276, 287–288, 288 $t$
  - after-tax MARR, 268–269, 285
  - air pollution, 384–386
  - Aktiengesellschaft (AG), 176
  - Alberta
    - Canadian Oil Sands, 79–81
    - professional engineering association, 11
  - amortization period, 56
  - analogous projects, 208
  - annual cost, 91
  - annual worth method, 87, 94–96, 218
    - advantages and disadvantages, 147–149, 147 $t$
    - equivalence of rate of return and present/annual worth methods, 143–147
    - vs. internal rate of return (IRR), 143–149
    - vs. present worth method, 94–95
    - projects having the same life, 94
    - vs. rate of return, 143–149
    - spreadsheets, 101
    - tax calculations, 273–274, 285–288, 287 $t$
  - annuity, 47
    - arithmetic gradient series with base annuity, 59 $f$
    - capital recovery factor, 52
    - compound interest factors, 51–58
    - non-standard annuities, 65–67 over  $N$  periods, 52 $f$
    - series present worth factor, 52
    - sinking fund factor, 51–52
    - sum of, plus arithmetic gradient, 59
    - uniform series compound amount factor, 52
  - annuity due, 54
  - answers to selected problems, 500–504
  - Argentina, corporate tax rate in, 266 $t$
  - arithmetic gradient series, 47
    - with base annuity, 59 $f$
    - conversion factor, 58–61
    - defined, 58–59
    - of disbursements, 59 $f$
    - of receipts, 59 $f$
    - sum of annuity plus arithmetic gradient, 59
  - arithmetic gradient to annuity conversion factor, 60, 71 $t$ , 84
  - asset-management ratios, 184–186
  - assets
    - on balance sheet, 174
    - book value, 165
    - cash flow relationship with financial markets, 177 $f$
    - challenger, 219
    - current assets, 174
    - defender, 219
    - defined, 174
    - economic life, 218, 223, 224
    - existing asset, 230
    - fixed assets, 174–175
    - long-term assets, 174–175
    - market value, 165
    - non-current assets, 174–175
    - replacement of. *See* replacement decisions
    - retirement, 220–221
    - salvage value, 97, 165, 222, 272, 272 $f$
    - scrap value, 165, 222, 272
    - value of, and depreciation, 165–166
  - ASTM International, 209
  - Australia
    - corporate tax rate in, 266 $t$
    - tax rules, 293
    - Trade Practices Act*, 349
  - Austria, corporate tax rate in, 266 $t$
  - average costs, 208
  - axiomatic probability, 437
- B**
- backward induction, 445
  - Bahamas, flat tax regime in, 300
  - balance sheet, 173–178, 175 $t$ , 182 $f$ , 184 $f$ 
    - as across variable, 173
    - assets, 174
    - current assets, 174
    - defined, 173
    - long-term assets, 174–175
    - owners' equity, 175–177
  - Bank of Canada, 20
  - Bank of Canada Act*, 20
  - base period, 306, 338
  - base year, 308
  - before-tax MARR, 268–269, 285
  - benefit-cost analysis (BCA)
    - benefit-cost ratios, 356–362
    - benefits of public projects, identification and measurement of, 353–356
    - Canadian benefit-cost analysis guides, 352
    - costs of public projects, identification and measurement of, 352–353
    - defined, 350
    - modified benefit-cost ratio (BCRM), 357–358
    - point of view, 350–352
  - benefit-cost ratios, 356–362
  - biological oxygen demand (BOD), 348
  - bonds, 58, 181
  - book value, 165
    - under declining-balance depreciation, 168 $f$
    - under straight-line depreciation, 167 $f$
  - BOOT (build-own-operate-transfer), 351
  - BOT (build-operate-transfer), 351
  - branches (decision trees), 443
  - Brazil, corporate tax rate in, 266 $t$

break-even analysis, 429–436  
 defined, 429  
 multiple projects, 432–436  
 single project, 430–432  
 break-even chart, 431f, 434f, 435f  
 British Columbia professional engineering association, 11  
 business ownership, types of, 176

**C****Canada**

*Consumer Code*, 349  
 corporate tax rate, 266t  
 tax rules in, 276–288

Canada Post, 350

Canada Revenue Agency website, 276  
*Canadian Cost-Benefit Analysis Guide*, 352

Canadian Engineering Accreditation Board (CEAB), 11  
 Canadian Oil Sands, 79–81  
 Canadian Securities Administrators, 181  
 capacity, 221  
 capacity factor, 210  
 capacity-factored model, 210  
 capital  
   cost of capital. *See* cost of capital  
   sources of, and company size, 124

capital cost allowance, 270, 276–288  
 capital salvage factor, 283–285  
 capital tax factor, 283–285  
 CCA classes, 279, 279t  
 CCA rates, 279, 279f, 279t  
 half-year rule, 280–283, 282t  
 undepreciated capital cost, 280–283, 280t, 281t

capital costs. *See* cost of capital  
 capital expenses, 265  
 capital good, 33  
 capital recovery factor, 52, 53, 71t, 83  
 capital recovery formula, 52, 71t  
 capital salvage factor, 283–285  
 capital tax factor, 283–285, 301–303  
 capitalized value, 67  
 capitalized value formula, 67, 71t  
 car payment calculators, 95  
 cash flow analysis

  compound amount factor, 48  
 compound interest factors for annuities, 51–58  
 compound interest factors for discrete compounding, 47–48

compound interest factors for single disbursements or receipts, 48–51, 48f  
 conversion factor for arithmetic gradient series, 58–61  
 conversion factor for geometric gradient series, 61–64  
 introduction, 46  
 non-standard annuities and gradients, 65–67  
 present worth computations when  $N \rightarrow 7$ , 67  
 timing of cash flows and modelling, 46–47  
 cash flow diagrams, 29–30, 29f, 31f  
 cash flow patterns, 47, 50–51, 54  
 cash flow statement, 178  
 cash flows  
   *see also* cash flow analysis  
   assets and financial markets, cash flow 177f, 177  
   cash flow diagrams, 29–30, 29f, 31f  
   cash flow patterns, 47, 50–51, 54  
   cash inflow, 87  
   cash outflow, 87  
   discrete cash flows, 478–499  
   net cash flow, 138  
   taxation, effect of, 269–272  
   timing of, 46–47  
 cash inflow, 87  
 cash outflow, 87  
 CCA classes, 279, 279t  
 CCA rates, 279, 279f, 279t  
 challenger, 219  
   different from defender, challenger does not repeat, 234–235  
   different from defender, challenger repeats indefinitely, 227–234  
   identical defender and challenger, 222–227  
 chance nodes, 443, 447f  
 charts, 267–268, 345–346  
 Chevron, 79  
 China  
   corporate tax rate in, 266t  
   industrialization of, 80  
   national oil companies of, 79  
 civil engineer, 2  
 classical probability, 437  
 Clipboard, 346  
 closure stage, 389, 392  
 coal, 344–345

company size, and sources of capital, 124  
 comparison methods  
   *see also* project evaluation  
   advantages and disadvantages of various methods, 147–149, 147t  
 annual worth method, 87, 94–96, 218  
 assumptions, 87–88  
 discounted payback period, 103  
 external rate of return (ERR), 140–143  
 future worth method, 95  
 internal rate of return (IRR).  
   *See* internal rate of return (IRR)  
 introduction, 86–88, 126  
 minimum acceptable rate of return (MARR), 90–91  
 online project comparison resources, 139  
 payback period method, 87, 101–104  
 present worth method, 87, 91–94, 218  
 projects with equal lives, 94, 129  
 projects with unequal lives, 97–101  
 relations among projects, 88–90, 90f  
 repeated lives, 97  
 study period, 97  
 compound amount factor, 48, 49, 71t, 81  
 compound interest, 22–25  
 compound interest factors. *See* compound interest factors  
   computations, 22t, 23t  
   defined, 23  
   vs. simple interest, 25  
   at 24% per year for 20 years, 25f  
 compound interest factors, 46  
   for annuities, 51–58  
   arithmetic gradient to annuity conversion factor, 60, 71t, 84  
 bonds, 58  
 capital recovery factor, 52, 71t, 83  
 cash flow patterns, 47, 54  
 compound amount factor, 48, 49, 71t, 81  
 defined, 47

- derivation of discrete compound interest factors, 81–84
- discrete compounding, 47–48
- discrete compounding, discrete cash flows, 478–499
- equivalent Excel computations, 72t
- geometric gradient to present worth conversion factor, 61, 71t, 84
- present worth factor, 49, 57, 71t, 82
- series present worth factor, 52, 57, 71t, 83–84
- for single disbursements or receipts, 48–51, 48f
- sinking fund factor, 51–52, 71t, 82
- spreadsheets, 142–143
- uniform series compound amount factor, 52, 71t, 82–83
- compounding period, 23
- Confederation Bridge, 351
- constant dollars, 308
- Consumer Code* (Britain, Canada), 349
- consumer price index (CPI), 306–308, 307f, 340
- continuous compounding, 28, 29f
- continuous models, 47
- conversion factor
- arithmetic gradient series, 58–61
  - geometric gradient series, 61–64
- conversion from subcontracted to in-house production, 229–230
- corporate income taxes
- see also* income taxes
  - capital expenses, 265
  - corporate tax rates, 265–268, 266t
  - expenses, 265
  - vs. personal income taxes, 265
  - small company tax rules, 267
- corporate stocks, 181
- corporate tax rates, 265–268, 266t
- corporation, 176
- cost accounting, 172–173
- cost-benefit tradeoffs, 19
- cost concepts, 207–208
- cost estimating relationships (CERs), 208–209, 210–211
- cost estimation
- adjustments, 209–210
  - classification of cost estimates, 208–209
  - cost concepts, 207–208
  - cost estimating relationships (CERs), 208–209, 210–211
  - indicative estimates, 208
  - inflation, adjustments for, 209–210
  - introduction, 207
  - location, adjustments for, 209–210
  - methods, 208–209
  - order of magnitude estimation, 208, 211t
- cost estimation relationships, 208–209
- cost object, 208
- cost of capital
- defined, 122
  - example, 123t
  - and minimum acceptable rate of return (MARR), 122–124
  - non-monotonic, 231–233
  - replacement decisions, 221–222
  - and risk, 122–123
  - weighted average cost of capital, 124
- cost principle of accounting, 180
- costs
- annual cost, 91
  - average costs, 208
  - capital. *See* cost of capital
  - direct costs, 208
  - equivalent annual cost (EAC), 218, 223t, 224–225, 226
  - first costs, 87, 269–271, 271f, 431f
  - fixed costs, 207, 221–222
  - indirect costs, 208, 211–212
  - installation costs, 221
  - marginal costs, 208
  - non-monotonic capital or operating costs, 231–233
  - operating and maintenance costs, 221, 231–233
  - present cost, 91
  - social costs, 351, 353
  - sunk costs, 230–231
  - total costs, 208
  - variable costs, 207–208, 222
- coupon rate, 58
- crashing a project, 403–408, 407t
- credit cards, and students, 44
- critical path, 400
- critical path method (CPM), 396–398
- project crashing, 403–408, 407t
  - project scheduling, 400–403
  - time-cost tradeoffs, 403–408, 405f, 408f
- current assets, 174
- current dollars, 308–311
- current interest rate, 312, 313f, 313t
- current internal rate of return, 314
- current liabilities, 175
- current MARR, 312, 316
- current ratio, 181–182, 187t
- D**
- debt, 122
- debt-management ratios, 183–184
- decision making, 4–5
- decision pyramid, 4, 4f
  - engineering decision making. *See* engineering decision making
- decision matrix, 471–474
- decision nodes, 443, 447f
- decision pyramid, 4, 4f
- decision tree software, 446
- decision trees, 442–448, 443f, 444f, 445f, 448f
- decisional equivalence, 32–33
- declining-balance depreciation, 166, 168–172, 168f
- defender, 219
- different from challenger, challenger does not repeat, 234–235
  - different from challenger, challenger repeats indefinitely, 227–234
  - identical defender and challenger, 222–227
- deflation, 305
- depreciation
- declining-balance depreciation, 166, 168–172, 168f
  - defined, 165
  - double-declining-balance, 166
  - effect of, 172
  - as expense, 277
  - functional loss, 165
  - introduction, 164
  - methods of, 166f
  - 150%-declining-balance, 166
  - reasons for depreciation, 165
  - straight-line method of depreciation, 166–168, 167f
  - sum-of-the-years'-digits, 166

depreciation (*Continued*)  
 and taxes, 270  
 time-related physical loss, 165  
 units-of-production, 166  
 use-related physical loss, 165  
 value of an asset, 165–166  
 deterrents, 348–349  
 digits in a calculation, 12  
 direct costs, 208  
 disbenefits, 351  
 disbursements, 47  
 arithmetic gradient series, 59f  
 single disbursements, 48–51, 48f  
 discounted payback period, 103  
 discrete cash flows, 478–499  
 discrete compound interest factors.  
*See* compound interest factors  
 discrete compounding, 47–48, 478–499  
 discrete models, 47, 71t  
 discrete probability distribution function, 437  
 Dominion Steel and Coal Company (DOSCO), 347  
 double-declining-balance, 166  
 Dun & Bradstreet, 181

**E**  
 earnings before interest and taxes (EBIT), 178  
 Eastern Europe, flat taxes in, 300  
 economic life, 218, 223, 224  
 economies of scale, 210  
 effective interest rate, 25–27, 44  
 efficiency ratios, 184–186  
 efficient market, 346  
 emissions trading, 384–386  
 engineering decision making, 2–3  
   cost-benefit tradeoffs, 19  
   decision matrix, 471–474  
   ethical component, 9–10  
   investment opportunities, 87  
     *see also* projects  
   public sector. *See* public sector decision making  
   replacement decisions. *See* replacement decisions  
 engineering economics, 3–4  
   defined, 3  
   importance of, 5  
   modelling process, 6–7, 7f  
 Engineers Canada, 11  
 equity, 175–177  
 equity investment, 122–123  
 equity ratio, 183–184, 187t

equivalence, 32–34  
 assumptions used, 34  
 decisional equivalence, 32–33  
 defined, 32  
 market equivalence, 32, 33–34  
 mathematical equivalence, 32, 46  
 equivalent annual cost (EAC), 218–219, 218t, 223t, 224–225, 226  
 established buildings, 34  
 estimates, 11  
   cost estimation. *See* cost estimation  
   cost principle of accounting, 180  
   estimated values in financial statements, 180  
   growth rates, 62  
   salvage value, 222  
   scrap value, 222  
 ethics, 9–10  
 European Union, 385  
 Excel. *See* spreadsheet software  
 execution stage, 389, 391  
 existing asset, 230  
 expected monetary value (EMV), 445  
 expected value, 439–442, 445  
 expenses, 265  
 external rate of return (ERR), 140–143  
 extraordinary items, 186  
 ExxonMobil, 79

**F**  
 face value, 58  
 financial accounting, 172  
   across variables, 173  
   balance sheet, 173–178, 175t  
   estimated values in financial statements, 180  
   financial ratio analysis, 180–181  
   financial ratios, 180, 181–188  
   income statement, 178–179, 179f  
   introduction, 164  
   performance of a firm, 173  
   through variables, 173, 174f  
*Financial and Taxation Statistics for Enterprises*, 180–181  
 financial market, 177f  
 financial ratio analysis, 180–181  
 financial ratios, 181–188  
   acid-test ratio, 182–183, 187t

asset-management ratios, 184–186  
 current ratio, 181–182, 187t  
 debt-management ratios, 183–184  
 defined, 180  
 efficiency ratios, 184–186  
 equity ratio, 183–184, 187t  
 financial ratio analysis, 180–181  
 inventory-turnover ratio, 184–186, 187t  
 leverage ratios, 183–184  
 liquidity ratios, 181–183  
 net-profit ratio, 186  
 profitability ratio, 186  
 quick ratio, 182–183, 187t  
 return-on-assets (ROA) ratio, 186, 187t  
 return-on-equity (ROE) ratio, 186, 187t  
 summary of, 187t  
 working capital ratio, 181, 187t  
 financial statements, 173  
   balance sheet, 173–178, 175t, 182f, 184f  
   cash flow statement, 178  
   estimated values, 180  
   funds statement, 178  
   income statement, 173, 174f, 178–179, 179f, 183f, 185f  
   statement of changes in financial position, 178  
   statement of sources and uses of funds, 178  
 finished goods inventory, 180  
 firm performance, 173  
 first costs, 87, 269–271, 271f, 431f  
 fixed assets, 174–175  
 fixed costs, 207, 221–222  
 flat taxes, 300  
 formulas, 24  
 407 ETR toll highway, 351  
 France, corporate tax rate in, 266t  
 functional loss, 165  
 funds statement, 178  
 future value, 20, 20f  
 future worth, 20  
 future worth method, 95

**G**  
 Galore Creek Project, 161–162  
 Gantt, Henry, 394  
 Gantt charts, 394–396, 399  
 Garrison Diversion Unit, North Dakota, 3  
 general partnership, 176

geometric gradient series, 47  
 conversion factor, 61–64  
 defined, 61  
 estimate of growth rates, 62  
 modelling with, 62  
 present worth, 62  
 for receipts with negative growth, 62f  
 for receipts with positive growth, 61f  
 geometric gradient to present worth conversion factor, 61, 71t, 84  
 Germany, corporate tax rate in, 266t  
 GIGO rule (“garbage in, garbage out”), 11  
 glossary of terms, 505–511  
 GNP deflator, 340  
 Google, 139  
 government provision of goods and services, 349  
 gradients  
   arithmetic gradient series. *See* arithmetic gradient series  
   geometric gradient series. *See* geometric gradient series  
   non-standard gradients, 65–67  
 Gray-Donald, James, 261  
 gross domestic product (GDP), 306  
 gross national product (GNP)  
   deflator, 340  
 growth adjusted interest rate, 63  
 growth rates, estimate of, 62  
*Guide to Benefit-Cost Analysis in Transport Canada*, 352

**H**  
 half-year rule, 280–283, 282t  
 hedonic price method, 354  
 Hong Kong, flat tax regime in, 300

**I**  
 identical defender and challenger, 222–227  
 in-house production, 229–230  
 incentives, 266, 348–349  
 income statement, 173, 174f, 178–179, 179f, 183f, 185f  
 income taxes  
   after-tax IRR, 274–276, 287–288, 288t  
   after-tax MARR, 268–269, 285  
   annual worth tax calculations, 273–274, 285–288, 287t  
   before-tax MARR, 268–269, 285

capital cost allowance, 270, 276–288  
 capital expenses, 265  
 capital salvage factor, 283–285  
 capital tax factor, 283–285, 301–303  
 cash flows, effect on, 269–272  
 components of complete tax calculation, 285–288, 285t, 287t  
 corporate tax rates, 265–268, 266t  
 depreciation methods, 270  
 expenses, 265  
 first costs, 269–271, 271f  
 flat taxes, 300  
 half-year rule, 280–283, 282t  
 impact of, 264  
 internal rate of return tax calculations, 274–276  
 introduction, 264  
 personal *vs.* corporate income taxes, 265  
 present worth tax calculations, 273–274, 285–288, 285t  
 progressive tax rate, 265  
 progressive taxes, 300  
 proportional taxes, 300  
 salvage value, 272, 272f  
 savings, effect on, 271, 272f  
 scrap value, 272  
 small company tax rules, 267  
 tax benefit factor, 301–303  
 tax credits, 265  
 tax relief, as incentive, 266  
 tax rules in Canada, 276–288  
 tax rules in other countries, 293  
 undepreciated capital cost, 280–283, 280t, 281t  
 incremental investment, 132, 133, 134, 135–136  
 independent projects, 88  
   benefit-cost ratio, 358  
   internal rate of return (IRR), 129–131  
   marginally acceptable, 92  
   present worth comparisons, 91–93  
 India, corporate tax rate in, 266t  
 indicative estimates, 208  
 indirect costs, 208, 211–212  
 industrial equipment, 53  
*Industry Handbook*, 181  
 industry standards, 180

*Industry Surveys*, 181  
 inflation  
   adjustments for, 209–210  
   consumer price index (CPI), 306–308, 307f  
   correctly anticipated inflation, effect of, 311–315  
   current MARR, 316  
   defined, 305  
   deflation, 305  
   economic evaluation with inflation, 308–311  
   internal rate of return (IRR), effect on, 314–315  
   introduction, 305  
   minimum acceptable rate of return (MARR), effect on, 312–314  
   project evaluation methods, 316–321  
   real dollars, 308–311  
   relative price changes, 317  
 inflation rate, 306–308, 307f, 313f  
 initiation stage, 389, 390  
 Insight Tree, 446  
 installation costs, 221  
 intangible social benefit, 354  
 interest, 19–22  
   compound interest, 22–25, 22t  
   continuous compounding, 28, 29f  
   defined, 20  
   lending and borrowing of money, 21  
   physical basis, 21–22  
   simple interest, 22–25  
 interest period, 21, 22, 23  
 interest rate uncertainty, 427  
 interest rates, 19–22  
   borrowing *vs.* lending, 33  
   current interest rate, 312, 313f, 313t  
   effective interest rate, 25–27, 44  
   growth adjusted interest rate, 63  
   mortgages, 56  
   nominal interest rate, 25–27, 44  
   overnight rates, 20  
   prime rate, 20  
   real interest rate, 312, 313f, 313t

internal rate of return (IRR), 126–129  
 advantages and disadvantages, 147–149, 147<sup>t</sup>  
 after-tax IRR, 274–276, 287–288, 288<sup>t</sup>  
*vs.* annual worth method, 143–149  
 comparisons, 129–143  
 current best alternative, 135  
 current internal rate of return, 314  
 defined, 127  
 equivalence of rate of return and present/annual worth methods, 143–147  
 Excel IRR function, 142  
 incremental investment, 132, 133, 134, 135–136  
 independent projects, 129–131  
 inflation, effect of, 314–315  
 internal rate of return (IRR), 137<sup>f</sup>  
 linear interpolation, 131  
 meaning of, 126–127  
 multiple IRRs, 136–140  
 mutually exclusive projects, 132–136  
 negative IRR, 127  
 positive IRR, 127  
*vs.* present worth method, 143–149  
 real internal rate of return, 314–315  
 tax calculations, 274–276  
 trial and errorinternal rate of return (IRR)  
 130trial and error, 127

Internet  
 Canada Revenue Agency website, 276  
 Canadian benefit-cost analysis guides, 352  
 car payment calculators, 95  
 decision tree software, 446  
 professional engineering association websites, 11  
 project comparison resources, 139  
 as research tool, 62  
 salvage values, estimating, 222  
 scrap values, estimating, 222  
 Statistics Canada, 306  
 inventory-turnover ratio, 184–186, 187<sup>t</sup>  
 investment opportunity. *See* comparison methods; projects

Ireland, corporate tax rate in, 266<sup>t</sup>  
 IRR function, 127, 142  
 Israel, corporate tax rate in, 266<sup>t</sup>

**J**  
 Japan, national oil companies of, 79

**K**  
 Korea, national oil companies of, 79  
 Kyoto Protocol, 385

**L**  
 large companies, 34  
 Laspeyres price index, 338–341  
 lead, 343–344  
 Leadership in Energy and Environmental Design (LEED), 422–423  
 least common multiple of lives of alternatives, 97  
 leaves (decision trees), 443  
 LEED certification, 422–423  
 legal liability, 176  
 leverage ratios, 183–184  
 liabilities  
 on balance sheet, 175  
 current liabilities, 175  
 defined, 175  
 long-term liabilities, 175  
 light bulbs, 261–262  
 limited company, 176  
 limited liability, 176  
 limited partnership, 176  
 linear interpolation, 56, 57<sup>f</sup>, 131  
 liquidity ratios, 181–183  
 litigation, 349  
 long-term assets, 174–175  
 long-term liabilities, 175

**M**  
 management accounting, 172–173  
 Manitoba professional engineering association, 11  
 manufacturing process, 6  
 marginal costs, 208  
 market, 346  
 market equivalence, 32, 33–34  
 market failure  
 defined, 346–348  
 government provision of goods and services, 349  
 informal methods, 349  
 litigation, 349  
 policy instruments, 348–349  
 remedies for, 348–349

market success, 436  
 market value, 165  
 mathematical equivalence, 32, 46  
 maturity, 56  
 mean, 439  
 mini-cases  
 Associated Engineering, 337–338  
 Canadian Oil Sands, 79–81  
 China Steel Australia Limited, 469–470  
 emissions trading, 384–386  
 flat taxes, 300  
 Galore Creek Project, 161–162  
 Leadership in Energy and Environmental Design (LEED), 422–423  
 pipelines, 337–338  
 R. v. Syncrude Canada Ltd., 16–17  
 Research in Motion (RIM), 205–207  
 Rockwell International, 121–122  
 Sears Canada, 261–262  
 student credit cards, 44

minimum acceptable rate of return (MARR), 90–91  
 after-tax MARR, 268–269, 285  
 before-tax MARR, 268–269, 285  
 break-even chart, 431<sup>f</sup>  
 and cost of capital, 122–124  
 current MARR, 312, 316  
 and external rate of return (ERR), 141  
 inflation, effect of, 312–314  
 and internal rate of return (IRR), 129  
 present worth of money invested, 92  
 present worth of mutually exclusive projects, 94  
 in public sector, 362–363  
 real MARR, 312  
 and relative price changes, 317

modelling  
 continuous models, 47  
 discrete models, 47  
 with geometric gradient series, 62  
 modelling process, 6–7, 7<sup>f</sup>  
 and timing of cash flows, 46–47

modified benefit-cost ratio (BCRM), 357–358

monetary incentives or deterrents, 348–349

money, 471  
 monitoring and control stage, 389, 391  
 monopolist, 350  
 moral questions, 9–10  
 mortgages, 56  
 multiple IRRs, 136–140  
 multiple projects, and break-even analysis, 432–436  
 mutually exclusive projects, 88  
     benefit-cost ratio, 359  
     internal rate of return (IRR), 132–136, 137*f*  
 present *vs.* annual cost, 91  
 present worth, as function of interest rate, 145*f*  
 present worth comparisons, 94  
 when service lives are the same, 94

**N**  
 naamloze vennootschap (NV), 176  
 Nanaimo City, 422–423  
 negative IRR, 127  
 net cash flow, 138  
 net present value (NPV)  
     spreadsheets, 101  
 net-profit ratio, 186  
 New Brunswick professional engineering association, 11  
 Newfoundland and Labrador professional engineering association, 11  
 Nigeria, flat tax regime in, 300  
 nominal dollars, 308  
 nominal interest rate, 25–27, 44  
 non-current assets, 174–175  
 non-monotonic capital or operating costs, 231–233  
 non-standard annuities, 65–67  
 non-standard gradients, 65–67  
 normalization, 473  
 North West Territories professional engineering association, 11  
 Nova Scotia  
     professional engineering association, 11  
     Sydney Tar Ponds, 344–345, 347–348  
 Nunavut professional engineering association, 11

**O**

oil industry projects, 79–81  
 150%-declining-balance, 166  
 one year principle, 227–229

Ontario Power Generation (OPG), 350  
 Ontario professional engineering association, 11  
 operating and maintenance costs, 221, 231–233  
 order of magnitude estimation, 208, 211*t*  
 organizational change, 220  
 overnight rates, 20  
 owners' equity, 175–177

**P**

P3. *See* public-private partnership (P3)  
 Palisades Precision Tree, 446  
 par value, 58, 176  
 parametric models, 208–209  
 partnership, 176  
 Pattje, Fred, 422  
 payback period, 101, 102, 104*f*  
 payback period method, 87, 101–104  
 performance measures, 180  
 performance of a firm, 173  
 personal income taxes, 265  
 personal probability, 437  
 pipelines, 337–338  
 planning stage, 389, 390–391  
 plant and equipment, 180  
 PMT functions, 101  
 policy instruments, 348–349  
 politics, 4  
 pollution, 384–386  
 position statement. *See* balance sheet  
 positive IRR, 127  
 power factor, 210  
 power-sizing model, 210  
 PowerPoint, 346  
 PPP. *See* public-private partnership (P3)  
 predictions, 11  
 preferences, 4  
 present cost, 91  
 present worth, 20, 20*f*  
     *see also* present worth method computations when  $N \rightarrow 7$ , 67  
     as function of interest rate, for simple investment, 144*f*  
     as function of interest rate for two simple, mutually exclusive projects, 145*f*  
     geometric gradient series, 62  
     as measure of profit, 93  
     money invested at MARR, 92

projects with unequal lives, 97–100  
 present worth factor, 49, 57, 71*t*, 82  
 present worth method, 87, 218  
     advantages and disadvantages, 147–149, 147*t*  
     *vs.* annual worth method, 94–95  
     equivalence of rate of return and present/annual worth methods, 143–147  
 independent projects, 91–93  
     *vs.* internal rate of return (IRR), 143–149  
 mutually exclusive projects, 94  
     *vs.* rate of return, 143–149  
 spreadsheets, 101  
 tax calculations, 273–274, 285–288, 285*t*  
 price index, 338–341  
 prime rate, 20  
 Prince Edward Island professional engineering association, 11  
 principal amount, 20  
 privatization, 350  
 probability, 436–442  
     axiomatic probability, 437  
     classical probability, 437  
     discrete probability distribution function, 437  
     expected value, 439–442  
     personal probability, 437  
     probability distribution function, 436, 439, 439*f*  
     random variable, 436, 439  
     relative frequency, 437  
     subjective probability, 437  
     symmetric probability, 437  
     views on, 436–437  
 probability distribution function, 436, 439, 439*f*  
 professional engineering associations, 11  
 profit, present worth as measure of, 93  
 profitability ratio, 186  
 progressive tax rate, 265  
 progressive taxes, 300  
 project charter, 389  
 project crashing, 403–408, 407*t*  
 project evaluation  
     *see also* comparison methods  
     inflation, 316–321  
     point of view, 350–352  
     sensitivity analysis, 425  
         *see also* sensitivity analysis

- project evaluation and review technique (PERT), 396–398  
*see also* critical path method (CPM)
- project management defined, 388
- project management lifecycle, 389–392
- project management tools, 392–408
- spreadsheet software, 399
- project management lifecycle, 389–392
- closure stage, 389, 392
- execution stage, 389, 391
- initiation stage, 389, 390
- monitoring and control stage, 389, 391
- planning stage, 389, 390–391
- project charter, 389
- terms of reference document, 389
- project management tools, 392–408
- activity on arc (AOA), 396–397, 397f, 398, 398f
- critical path method (CPM), 396–398, 400–403
- Gantt charts, 394–396, 399
- project crashing, 403–408, 407t
- project evaluation and review technique (PERT), 396–398
- project scheduling, and CPM, 400–403
- time-cost tradeoffs, 403–408, 405f, 408f
- work breakdown structure, 392–394, 394t, 395, 399
- project scheduling, 400
- projects, 87
- analogous projects, 208
- comparison methods. *See* comparison methods
- with equal lives, 94, 129
- independent, 88  
*see also* independent projects
- minimum acceptable rate of return (MARR), 90–91
- mutually exclusive, 88  
*see also* mutually exclusive projects
- related but not mutually exclusive, 88
- relations among projects, 88–90, 90f
- simple investments, 142, 142f, 144f
- with unequal lives, 97–101
- proportional taxes, 300
- public limited company (PLC), 176
- public-private partnership (P3), 350, 351
- public sector decision making
- benefit-cost analysis (BCA), 350–362
- benefit-cost ratios, 356–362
- benefits of public projects, identification and measurement of, 353–356
- classes of goods and services, 349–350
- costs of public projects, identification and measurement of, 352–353
- minimum acceptable rate of return (MARR), 362–363
- modified benefit-cost ratio (BCRM), 357–358
- point of view used for project evaluation, 350–352
- public-private partnership (P3), 350, 351
- social discount rate, 363
- Q**
- quantitative analyses, 11
- Quebec professional engineering association, 11
- quick ratio, 182–183, 187t
- R**
- R. v. Syncrude Canada Ltd., 16–17
- random variable, 436, 439
- rate of return
- vs.* annual worth methods, 143–149
- external rate of return (ERR), 140–143
- internal rate of return (IRR). *See* internal rate of return (IRR)
- vs.* present worth methods, 143–149
- ratios. *See* financial ratios
- real dollars, 308–311
- real interest rate, 312, 313f, 313t
- real internal rate of return, 314–315
- real MARR, 312
- receipts
- arithmetic gradient series, 59f
- geometric gradient series for receipts with negative growth, 62f
- geometric gradient series for receipts with positive growth, 61f
- single receipts, 48–51, 48f
- reducing-balance depreciation. *See* declining-balance depreciation
- regulations, 348, 349
- related but not mutually exclusive, 88
- relations among projects, 88–90, 90f
- relative frequency, 437
- relative price changes, 317
- repeated lives, 97
- replacement decisions
- capital costs, 221–222
- challenger, 219, 222–235
- challenger different from defender, challenger does not repeat, 234–235
- challenger different from defender, challenger repeats indefinitely, 227–234
- conversion from subcontracted to in-house production, 229–230
- cost components for certain older assets, 228f
- cost components for replacement studies, 223f
- defender, 219, 222–235
- equivalent annual cost (EAC), 218–219, 218t, 223t, 224–225, 226
- existing asset, 230
- fixed costs, 221–222
- identical defender and challenger, 222–227
- installation costs, 221
- introduction, 216–217
- non-monotonic capital or operating costs, 231–233
- one year principle, 227–229
- operating and maintenance costs, 221
- other costs, 221–222
- reasons for replacement, 220–221
- replacement example, 217–219
- replacement period, 218
- sunk costs, 230–231
- variable costs, 222
- replacement period, 218
- request for proposals (RFP), 397
- Research in Motion (RIM), 205–207

retained earnings, 176–177  
 retirement, 220–221  
 return-on-assets (ROA) ratio, 186, 187<sup>t</sup>  
 return-on-equity (ROE) ratio, 186, 187<sup>t</sup>  
 risk  
     and cost of capital, 122–123  
     decision trees, 442–448, 443<sup>f</sup>, 444<sup>f</sup>, 445<sup>f</sup>, 448<sup>f</sup>  
     decisions made under risk, 426  
 robust, 12  
 Rockwell International, 121–122  
 rollback procedure, 445  
 Royal Dutch Shell, 79  
 Russia, flat tax regime in, 300

**S**  
 salvage value, 97, 165, 222, 272, 272<sup>f</sup>  
 Saskatchewan professional engineering association, 11  
 Saudi Arabia, flat tax regime in, 300  
 savings, 271, 272<sup>f</sup>, 351  
 scrap value, 165, 222, 272  
 Sears Canada, 261–262  
 securities, 181  
 Securities Exchange Commission (SEC), 181  
 securities regulators, 181  
 sensitivity analysis, 11–12, 425  
     break-even analysis, 429–436  
 sensitivity graphs, 426–429  
 series present worth factor, 52, 57, 71<sup>t</sup>, 83–84  
 service life, 97  
 Shell, 79  
 simple interest, 22–25  
     vs. compound interest, 25  
     defined, 23  
     at 24% per year for 20 years, 25<sup>f</sup>  
 simple investments, 142, 142<sup>f</sup>, 144<sup>f</sup>  
 single disbursements or receipts, 48–51, 48<sup>f</sup>  
 single project, break-even analysis, 430–432  
 sinking fund factor, 51–52, 71<sup>t</sup>, 82  
 small company tax rules, 267  
 social benefits, 353, 354  
 social costs, 351, 353  
 social discount rate, 363  
 société anonyme (SA), 176  
 sole proprietorship, 176  
 South Africa, corporate tax rate in, 266<sup>t</sup>

specialist companies, 220  
 spreadsheet software  
     annual worth comparisons, 101  
     built-in functions, 50–51, 170  
     cash flow patterns, 50–51  
     charts, 267–268, 345–346  
     complex calculations, 8  
     compound interest factors, 72<sup>t</sup>, 142–143  
     depreciation calculations, 170  
     formatting features, 226–227  
     formulas, value of, 24  
     Gantt charts, 399  
     interest rates, 24  
     IRR function, 127, 142  
     normal use, 8  
     NPV function, 101  
     parentheses, 24  
     PMT function, 101  
     present worth comparisons, 101  
     printouts, 311  
     project management, 399  
     tables, 345–346  
     work breakdown structure, 399  
 Standard & Poor's, 181  
 statement of changes in financial position, 178  
 statement of sources and uses of funds, 178  
 Statistics Canada, 62, 180, 306  
 steel, 344–345  
 straight-line method of depreciation, 166–168, 167<sup>f</sup>  
 student credit cards, 44  
 study period, 97  
 subcontracted production, 229–230  
 subjective evaluation, 473  
 subjective probability, 437  
 sum-of-the-years'-digits, 166  
 Suncor, 79  
 sunk costs, 230–231  
 Sydney Steel (SYSCO), 344–345, 347  
 Sydney Tar Ponds, 344–345, 347–348  
 symmetric probability, 437  
 Syncrude Canada Ltd., 16–17, 79

**T**  
 tables, 345–346  
 tax benefit factor, 301–303  
 tax credits, 265  
 tax rates, 265–268, 266<sup>t</sup>  
 taxation. *See* income taxes

technological change, 220  
 term (of mortgage), 56  
 terms of reference document, 389  
 through variables, 173, 174<sup>f</sup>  
 time-cost tradeoffs, 403–408, 405<sup>f</sup>, 408<sup>f</sup>  
 time-related physical loss, 165  
 time value of money  
     cash flow diagrams, 29–30, 29<sup>f</sup>, 31<sup>f</sup>  
     compound interest, 22–25, 22<sup>t</sup>  
     continuous compounding, 28, 29<sup>f</sup>  
     effective interest rate, 25–27  
     equivalence, 32–34  
     interest and interest rates, 19–22  
     nominal interest rate, 25–27  
     simple interest, 22–25  
 total costs, 208  
 Toyota Prius, 422  
*Trade Practices Act* (Australia), 349  
 Transport Canada, 352  
 Treasury Board of Canada, 209, 352  
 TreeAge Software, 446  
 trend analysis, 180

**U**  
 uncertainty, 11–12, 425  
     assessment of, 426  
     interest rate uncertainty, 427  
     probability, 436–442  
     sensitivity analysis. *See* sensitivity analysis  
     sensitivity graphs, 426–429  
 undepreciated capital cost, 280–283, 280<sup>t</sup>, 281<sup>t</sup>  
 unequal lives, 97–101  
*Uniform Commercial Code* (U.S.), 349  
 uniform series compound amount factor, 52, 71<sup>t</sup>, 82–83  
 unit model, 210  
 United Kingdom  
     *Consumer Code*, 349  
     corporate tax rate in, 266<sup>t</sup>  
     tax rules, 293  
 United States  
     corporate tax rate in, 266<sup>t</sup>  
     and Kyoto Protocol, 385  
     tax rules, 293  
     *Uniform Commercial Code*, 349  
 units of production, 165  
 units-of-production depreciation, 166  
 unlimited liability, 176  
 Uruguay, flat tax regime in, 300

|  |                                       |  |
|--|---------------------------------------|--|
| U.S. Census Bureau, 181                          | variable costs, 207–208, 222          | Word, 346  |
| U.S. Green Building Council, 422                 | variables                             | work breakdown structure,<br>392–394, 394 <i>t</i> , 395,<br>399 |
| US Environmental Protection<br>Agency (EPA), 385 | across variables, 173                 | working capital, 181   |
| use-related physical loss, 165                   | random variable, 436, 439             | working capital ratio, 181,<br>187 <i>t</i>                      |
| <b>V</b>   | through variables, 173, 174 <i>f</i>  | write off capital investment,<br>277                             |
| value of an asset, 165–166                       |                                       |  |
| Vanguard Software, 446                           | weighted average cost of capital, 124 |  |
|  | Wikipedia, 139                        |  |

# List of Formulas

---

**After-tax IRR:**

$$\text{IRR}_{\text{after-tax}} \cong \text{IRR}_{\text{before-tax}} \times (1 - t)$$

**After-tax MARR:**

$$\text{MARR}_{\text{after-tax}} \cong \text{MARR}_{\text{before-tax}} \times (1 - t)$$

**Benefit-Cost Ratio:**

$$\text{BCR} = \frac{\text{PW}(\text{benefits})}{\text{PW}(\text{costs})}$$

**Book Value, Declining-Balance:**

$$BV_{db}(n) = P(1 - d)^n$$

**Book Value, Straight-Line:**

$$BV_{sl}(n) = P - n \left( \frac{P - S}{N} \right)$$

**Capital Tax Factor:**

$$\text{CTF} = 1 - \frac{td(1 + i/2)}{(i + d)(1 + i)}$$

**Capital Salvage Factor:**

$$\text{CSF} = 1 - \frac{td}{(i + d)}$$

**Capitalized Value:**

$$P = \frac{A}{i}$$

**Capital Recovery Formula:**

$$A = (P - S)(A/P, i, N) + Si$$

**Compound Interest:**

$$F = P(1 + i)^N$$

**Compound Interest Factors:**

- **Compound Amount Factor**  
 $(F/P, i, N) = (1 + i)^N$
- **Present Worth Factor**

$$(P/F, i, N) = \frac{1}{(1 + i)^N}$$

**Sinking Fund Factor**

$$(A/F, i, N) = \frac{i}{(1 + i)^N - 1}$$

**Uniform Series Compound Amount Factor**

$$(F/A, i, N) = \frac{(1 + i)^N - 1}{i}$$

**Capital Recovery Factor**

$$(A/P, i, N) = \frac{i(1 + i)^N}{(1 + i)^N - 1}$$

**Series Present Worth Factor**

$$(P/A, i, N) = \frac{(1 + i)^N - 1}{i(1 + i)^N}$$

**Arithmetic Gradient to Annuity Conversion Factor**

$$(A/G, i, N) = \frac{1}{i} - \frac{N}{(1 + i)^N - 1}$$

**Geometric Gradient Series to Present Worth Conversion Factor**

$$(P/A, g, i, N) = \frac{(P/A, i^*, N)}{1 + g}$$

$$(P/A, g, i, N) = \left( \frac{(1 + i^*)^N - 1}{i^*(1 + i^*)^N} \right) \frac{1}{1 + g}$$

$$i^* = \frac{1 + i}{1 + g} - 1$$

**Depreciation Amount, Straight Line:**

$$D_{sl}(n) = \frac{P - S}{N}$$

**Depreciation Amount, Declining Balance:**

$$D_{db}(n) = BV_{db}(n - 1) \times d$$

### Depreciation Rate:

$$d = 1 - \sqrt[n]{\frac{S}{P}}$$

### Effective Interest Rate:

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1 \text{ or}$$

$$i_e = (1 + i)^m - 1$$

### Effective Interest Rate for Continuous Compounding:

$$i_e = e^r - 1$$

### Expected Value of the Discrete Random Variable:

$$E(X) = \sum x_i p(x_i)$$

### Financial Ratios:

- Acid test ratio =  $\frac{\text{Quick assets}}{\text{Current liabilities}}$
- Current ratio =  $\frac{\text{Current assets}}{\text{Current liabilities}}$
- Equity ratio =  $\frac{\text{Total equity}}{\text{Total assets}}$
- Inventory turnover =  $\frac{\text{Sales}}{\text{Inventories}}$
- Return on total assets =  $\frac{\text{Profits after taxes}}{\text{Total assets}}$

### Growth-Adjusted Interest Rate:

$$i^* = \frac{1 + i}{1 + g} - 1$$

### Internal Rate of Return:

$$\sum_{t=0}^T \frac{(R_t - D_t)}{(1 + i^*)^t} = 0 \text{ or}$$

$$\sum_{t=0}^T \frac{R_t}{(1 + i^*)^t} = \sum_{t=0}^T \frac{D_t}{(1 + i^*)^t}$$

### Linear Interpolation:

$$x^* = x_1 + (x_2 - x_1) \left[ \frac{y^* - y_1}{y_2 - y_1} \right]$$

### Modified Benefit-Cost Ratio:

$$BCRM = \frac{PW(\text{benefits}) - PW(\text{operating costs})}{PW(\text{capital costs})}$$

### Payback Period:

$$\text{Payback period} = \frac{\text{First cost}}{\text{Annual savings}}$$

### Real Dollars:

$$R_{0,N} = \frac{A_N}{I_{0,N}/100}$$

$$R_N = \frac{A_N}{(1 + f)^N}$$

$$R_N = A_N (P/F, f, N)$$

### Real MARR:

$$MARR_R = \frac{1 + MARR_C}{1 + f} - 1$$

### Real Interest Rate:

$$i' = \frac{1 + i}{1 + f} - 1$$

### Real IRR:

$$IRR_R = \frac{1 + IRR_C}{1 + f} - 1$$

### Simple Interest Amount:

$$I_s = PiN$$