Student: Arfaz Hossain **Date:** 12/05/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 10

Find the derivative.

$$\frac{d}{dx}\int_{1}^{\sin x} 14t^{13}\,dt$$

a. by evaluating the integral and differentiating the result.

b. by differentiating the integral directly.

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a. To find $\frac{d}{dx}\int_{1}^{\sin x} 14t^{13}\,dt$, first evaluate the integral $\int_{1}^{\sin x} 14t^{13}\,dt$.

Use Fundamental Theorem of Calculus Part 2 because it describes how to evaluate definite integrals without having to calculate limits of Riemann sums.

The Fundamental Theorem of Calculus, Part 2, states that if f is continuous over [a,b] and F is any antiderivative of f on [a,b], then $\int_{a}^{b} f(x)\,dx = F(b) - F(a)$.

To evaluate the definite integral, first find the antiderivative of $14t^{13}$. As Fundamental Theorem of Calculus Part 2 requires any antiderivative of the function, do not include constant of integration.

$$\frac{d}{dx}\int_{1}^{\sin x} 14t^{13}\,dt = \frac{d}{dx}\left[t^{14}\right]_{1}^{\sin x}$$

Now substitute the limits of integration.

$$\begin{aligned}\frac{d}{dx}\left[t^{14}\right]_{1}^{\sin x} &= \frac{d}{dx}\left((\sin x)^{14} - 1^{14}\right)\\ &= \frac{d}{dx}\left(\sin^{14} x - 1\right) \qquad \text{Simplify.}\end{aligned}$$

To simplify $\frac{d}{dx}\left(\sin^{14} x - 1\right)$ further, use the the Difference Rule because the expression $\sin^{14} x - 1$ is a difference of differentiable functions.

Apply the Difference Rule.

$$\frac{d}{dx}\left(\sin^{14} x - 1\right) = \frac{d}{dx}\left(\sin^{14} x\right) - \frac{d}{dx}(1)$$

To differentiate $\sin^{14} x$, use the Power Chain Rule because $\sin x$ is a differentiable function of x and $\sin^{14} x$ can be written as $(\sin x)^{14}$.

The Power Chain Rule states that if n is any real number and f is a power function, $f(u) = u^n$, $\frac{d}{dx}\left(u^n\right) = nu^{n-1}\frac{du}{dx}$.

Let $u = \sin x$ and apply the Power Chain Rule.

$$\begin{aligned}\frac{d}{dx}\left(\sin^{14} x\right) &= \frac{d}{du}\left(u^{14}\right) \cdot \frac{d}{dx}(\sin x)\\ &= 14u^{13} \cdot \frac{d}{dx}(\sin x) \qquad \text{Find } \frac{d}{du}\left(u^{14}\right).\end{aligned}$$

$= 14u^{13} \cdot (\cos x)$ Find $\frac{d}{dx}(\sin x)$.

Finally, replace u with **sin** x.

$$\frac{d}{dx}\left(\sin^{14} x\right) = 14u^{13} \cdot (\cos x)$$
$$= 14 \sin^{13} x \cos x$$

Substitute the value of $\frac{d}{dx}\left(\sin^{14} x\right)$ into the expression $\frac{d}{dx}\left(\sin^{14} x\right) - \frac{d}{dx}(1)$.

$$\frac{d}{dx}\left(\sin^{14} x\right) - \frac{d}{dx}(1) = 14 \sin^{13} x \cos x - \frac{d}{dx}(1)$$

Now find $\frac{d}{dx}(1)$. Note that $\frac{d}{dx}(c) = 0$, where c is any constant number.

$$14 \sin^{13} x \cos x - \frac{d}{dx}(1) = 14 \sin^{13} x \cos x - 0$$
$$= 14 \sin^{13} x \cos x$$ Simplify.

Thus, by evaluating the integral and differentiating the result, $\frac{d}{dx}\int_1^{\sin x} 14t^{13}\,dt = 14 \sin^{13} x \cos x$.

b. To differentiate the integral directly, use Part 1 of the Fundamental Theorem of Calculus.

The Fundamental Theorem of Calculus, Part 1, states that if f is continuous on [a,b], then $F(x) = \int_a^x f(t)\,dt$ is continuous on [a,b] and differentiable on (a,b) and its derivative is f(x). That is, $F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.

To find $\frac{d}{dx}\int_1^{\sin x} 14t^{13}\,dt$ using Fundamental Theorem of Calculus, Part 1, use the Chain Rule with u = **sin** x because the upper limit of integration is **sin** x.

So, apply the chain rule $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

$$\frac{d}{dx}\int_1^{\sin x} 14t^{13}\,dt = \frac{d}{du}\int_1^{u} 14t^{13}\,dt \cdot \frac{d}{dx}(\sin x)$$

Use Part 1 of the Fundamental Theorem of Calculus $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ to find $\frac{d}{du}\int_1^u 14t^{13}\,dt$.

$$\frac{d}{du}\int_1^{u} 14t^{13}\,dt \cdot \frac{d}{dx}(\sin x) = 14u^{13} \cdot \frac{d}{dx}(\sin x)$$
$$= 14u^{13}(\cos x)$$ Find $\frac{d}{dx}(\sin x)$.

Finally, replace u with **sin** x.

$$14u^{13}(\mathbf{cos}\ x) = 14\ \mathbf{sin}^{13} x\ \mathbf{cos}\ x$$

Thus, by differentiating the integral directly, $\dfrac{d}{dx}\displaystyle\int_{1}^{\mathbf{sin}\ x} 14t^{13}\ dt = 14\ \mathbf{sin}^{13} x\ \mathbf{cos}\ x$.