ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 10: Net Present Value

Note: <u>Present Worth</u> and <u>Present Value</u> are *synonyms*, and will be used interchangeably in this course. (Both terms are in common use.)

September 29, 2021

Version 1.15 – Fixed a typo on slide 23, deleted slides not covered in class.

Required Reading

- *Engineering Economics*, Chapter 4, Sections 4.2-4.4
- Blue Light Robotics and the NPV (Handout)

Optional Reading on the MARR

- **Minimum Attractive Rate of Return** [Web Page]. (n.d.). Retrieved from http://engineeringandeconomicanalysis.blogspot.ca/2013/12/minimum-attractive-rate-of-return.html
- Heck, F.M. (1961). **The Cost-of-Capital in Economic Studies**. *Transactions of the American Institute of Electrical Engineers*,80(3), 775 – 785. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/4501136
- Hunter, J. (2015, May 26). Power from **Site C dam** ‘dramatically’ more costly than thought: expert. *The Globe and Mail*. https://go-gale-com.ezproxy.library.uvic.ca/ps/i.do?p=CPI&u=uvictoria&id=GALE|A414963241&v=2.1&it=r&sid=summon
- Peace Valley Landowner Association. (2017). Site C Inquiry. https://www.peacevalleyland.com/sitecinquiry
- Prescott, L. (1999). **The minimum acceptable rate of return: engineering economic theory and practice** [University of Alberta Master of Science Thesis]. Retrieved from https://era.library.ualberta.ca/files/47429c047#.WAPMXZMrJBw

Optional Reading on NPV & the environment

- Baker, S. A. & Raboy, D. G. (2018). The misuse of net present value in energy efficiency standards. *Renewable and Sustainable Energy Reviews, 96*, 218-225. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.rser.2018.07.047
- Knoke, T., Gosling, E. & Paul, C. (2020). Use and misuse of the net present value in environmental studies. *Ecological Economics*, *174,* 10664. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.ecolecon.2020.106664
- Zore, Z. et al. (2018). Maximizing the sustainability net present value of renewable energy supply networks. *Chemical Engineering Research and Design, 131*, 245-265. https://doi-org.ezproxy.library.uvic.ca/10.1016/j.cherd.2018.01.035

Learning Objectives

- Understand the significance of a zero, positive or negative present value of a project when evaluated at the MARR.
- Be able to calculate and interpret the present worth of an individual project.
- Be able to calculate and compare present worth for multiple projects which may have different lifetimes.

Relevant Solved Problems I

- From *Engineering Economics*, 6th edition:
- Present Worth of Independent Projects: Example 4.2, Example 4.3, 4.4, 4.5
- Constructing Mutually Exclusive Alternatives (Optional): Review Problem 4.1, 4.1, 4.2, 4.3, 4.21, 4.23
- Present Worth for Mutually Exclusive Projects: Example 4.4, Example 4.6, Example 4.7, Review Problem 4.1, 4.6, 4.7, 4.8, 4.15, 4.16, 4.24, 4.27, 4.28, 4.29, 4.32, 4.33, 4.36.b., 4.42
- Repeated Lives and Study Period: 4.15, 4.16, 4.28, 4.29, 4.32, 4.36.b., 4.33, 4.42

Relevant Solved Problems II

- From Stuart Nielsen's Engineering Economics: The Basics, 2nd edition:
- Present Worth: Example 5-7, Example 10-3, Example 10-6, Example 11-1, Example 11-2, Example 11-6, Example 11-7
- **Note: The chapter 10 questions are benefit-cost ratio questions, but you can solve them as NPV or annual worth questions with just a slight rearrangement (e.g. NPV = PW of Benefits - PW of Costs).**

Notation Dictionary

(Not provided on quiz/final formula sheet)

- AW = Annual Worth
- MARR = Minimum Acceptable Rate of Return
- NPV = Net Present Value
- PW = Present Worth

- Conversion factors are of the form (X/Y,z)
- Read as: X, given Y and z.
- X is the element we want.
- Y is the element we have.
- z represents additional parameters.
- e.g. (P/F,i,N)
- Present Value, given a Future Value at time N and interest rate i.

ESSENTIALS (17 slides)

A note on notation

- ‘Present Worth’ and ‘Present Value’ are synonyms. They are both in common use, and are used interchangeably in this course.
- Present Worth (Value) is a general term, in that you can talk about the present worth of a single cash flow element, or all the cash flows in a project, or just the positive cash flows into a project, or just the negative cash flows.
- **Net** Present Value (Worth) is more specific: when you say you are finding the Net Present Value (NPV) of a project, you’re specifying that you’re finding the present value of the positive flows minus (net of) the present value of the negative flows, and not leaving anything out.
- All apples are fruits, but not all fruits are apples: the NPV is a present value, but not all present values are NET present values.

Wait, don’t we already know this?

- Why are we having a lecture on present value? We know this!
- Give us a cash flow, and we can break it up into elements…
- ...calculate the present worth of each element...
- ...add them all up, and boom!
- That’s all we need, right? To compare two projects, just pick the one with the higher present value?

- Answer: Kind of. There are a few things to watch out for (unequal lifetimes)
- Also: develop intuition regarding NPV & Profit.

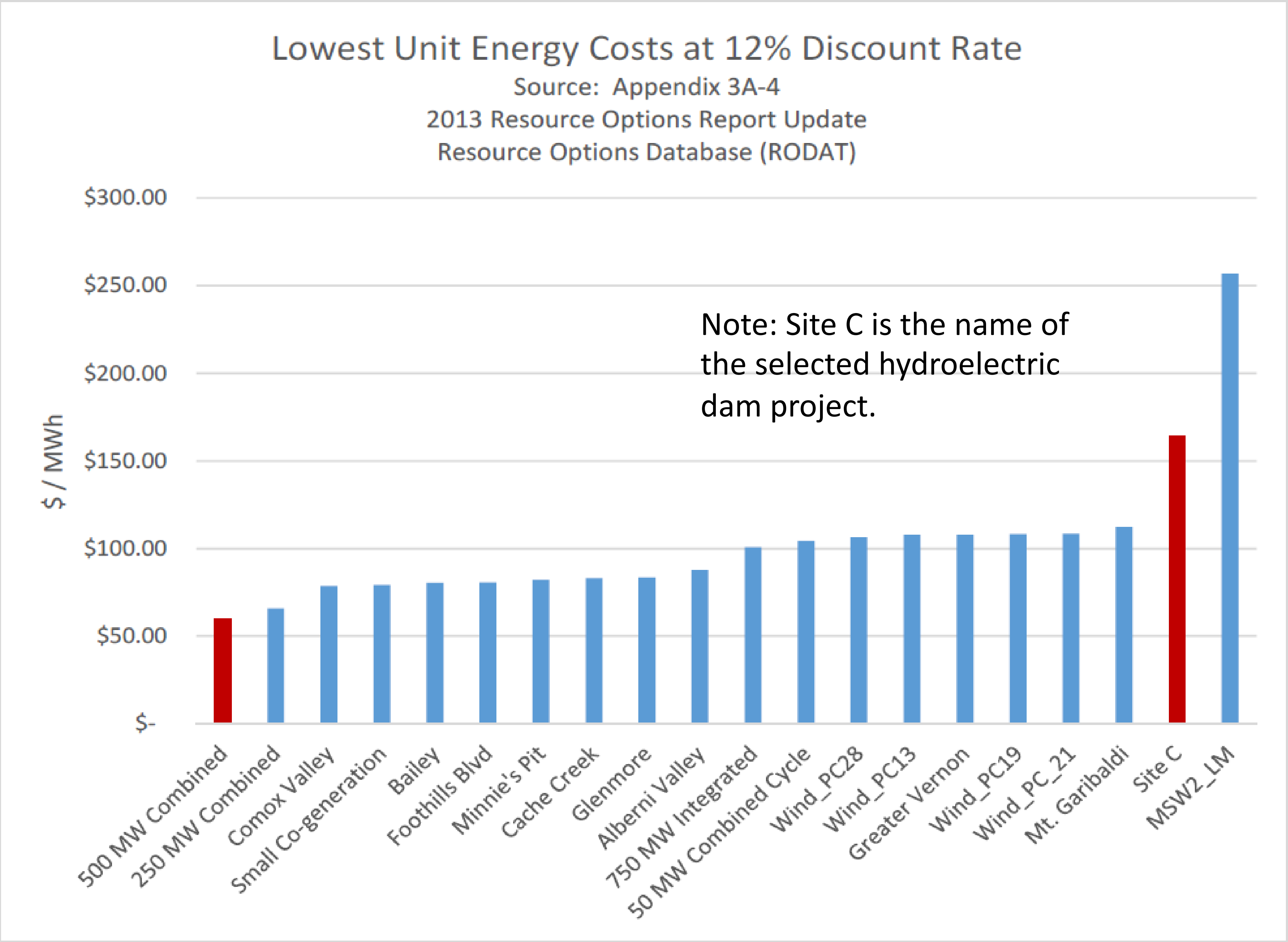
Reminder: Minimum Acceptable Rate of Return (MARR)

- Threshold return on projects.
- Projects earning less than the MARR are not accepted:
  - Money could be invested more profitably elsewhere (and earn the MARR)
- This sees the MARR as an *internal* opportunity cost…

- We can also see it as an *external* opportunity cost: the rate of return needed to convince investors to invest. “Why should we invest in your project when others in the economy are offering us the MARR?”

Getting it right is important (Heck)

- Under-shooting or over-shooting the MARR can have drastic consequences.
- Recently, BC was accused of using a 5% MARR to evaluate energy projects when 12% may have been more appropriate…
- Hydroelectric dam (high initial cost, low operating costs) looked good, Mixed natural gas plant (low initial cost, high fuel and operating costs) looked bad.
- A lower MARR makes future costs look 'bigger' (since (P/F,i,N) = $(1 + \mathrm{i})^{-\mathrm{N}}$)
- →A too-low MARR would favor projects with front-loaded costs.


Lowest Unit Energy Costs at 12% Discount Rate
Source: Appendix 3A-4
2013 Resource Options Report Update
Resource Options Database (RODAT)
$300.00
$250.00
$200.00
$150.00
$100.00
$50.00
$-
$ / MWh
Note: Site C is the name of the selected hydroelectric dam project.
500 MW Combined
250 MW Combined
Comox Valley
Small Co-generation
Bailey
Foothills Blvd
Minnie's Pit
Cache Creek
Glenmore
Alberni Valley
750 MW Integrated
50 MW Combined Cycle
Wind_PC28
Wind_PC13
Greater Vernon
Wind_PC19
Wind_PC_21
Mt. Garibaldi
Site C
MSW2_LM

A simple numerical example

Year	0	1	2	3	4	5	6	7	8	9	10
Dam Cost	60	1	1	1	1	1	1	1	1	1	1
Mixed Plant Cost	1	10	10	10	10	10	10	10	10	10	10

- Mimics the real-world situation:
- Dam: High first costs, low operating costs
- Mixed Plant: Low first costs, high operating We'll look at the present worth of costs for interest rates of 5% and 12%.

The present value of each alternative's costs

- Dam cost: 60 in Year 0, 1 per year from Year 1 to Year 10.
- PV = 60 + 1 x (P/A,i,10)
- PV (i = 5%) = 60 + 7.72 = 67.72
- PV(i = 12%) = 60 + 5.65 = 65.65
- Mixed Plant Cost: 1 in Year 0, 10 per year from Year 1 to Year 10.
- PV = 1 + 10 x (P/A,i,10)
- PV(i = 5%) = 1 + 10 x 7.72 = 1 + 77.2 = 78.2 (More than Dam @ 5%)
- PV(i = 12%) = 1 + 10 x 5.65 = 1 + 56.5 = 57.5 (Less than Dam @ 12%)

NPV and Independent Projects

- NPV = Present value of ALL cash flows in a project.
- For each project, two options: do, or do not (credit to Yoda)
- ‘Do not’ → money can be invested elsewhere and earn the MARR
- NPV of money invested at the MARR = 0
- Suppose you calculate the PW of the ‘Do’ option using the MARR as the discount (interest) rate.
- If NPV > 0, the project is worthwhile (better than ‘Do Not’)
- If NPV < 0, the project is inferior to available alternatives
- If NPV = 0, we say it’s marginally acceptable (but in real world applications, the existence of uncertainty means a project with PW = 0 usually won’t be taken on.)

Profit and Present Worth

- Remember we can think of the MARR as the cost of capital.
- **The Net Present Value (NPV) of a project (evaluated at the MARR) is its *profit*: what is earned over and above the cost of using the resources it needs. If the NPV is negative, it is a *loss*.**
- **(Recall: NPV = the present value of the positive cash flows in the project, net of the negative ones, the total present value.)**
- This also means if you *have* to choose between (mutually exclusive) projects, it's valid to pick the one with the greater Net Present Value…
- …as long as all projects have the same lifetimes.
- (More on that later.)

Why is the NPV at the MARR Profit/Loss?

- Your friend makes you the following offer: pay \$1,000 today and get \$1,001 twenty years from now. Should you accept this offer?
- Suppose your MARR is 1% a year, because that's what you can get on the best available other use of your money, a 20-year \$1,000 Canadian government savings bond.
- To buy the bond today (Year 0), you need to pay \$1,000. This is a *negative* cash flow of -\$1,000 in Year 0.
- When you cash in that bond, you'll have \$1,000 x $(1.01)^{20}$ = . This is a *positive* future cash flow in Year 20.

Calculating the NPV

- Pay \$1,000 today, get \$1,000 x $(1 + 1\%)^{20}$ in 20 years.
- NPV = -1,000 + \$1,000 x $(1.01)^{20}$ x (P/F,1%,20)
- NPV = -\$1,000 + \$1,000 x $(1.01)^{20}/\ (1.01)^{20}$
- NPV = \$1,000 - \$1,000 = 0
- By definition, evaluated at an interest rate of 1%, the present worth of that bond purchase is zero: the present worth of the \$1,000 x $(1.01)^{20}$ is \$1,000, which is exactly the price you pay for the bond.

What this implies

- If the NPV of your friend’s project evaluated at the MARR is less than 0, that means you can do better with your fallback investment, and you’d take a loss if you accepted your friend’s offer.
- If the NPV is greater than zero, you make a profit over your default investment.
- The <u>size</u> of that profit or loss (compared to the fallback project) is exactly equal to the present worth of the project evaluated at the MARR.

In plain English

- You've calculated the NPV of a new project, and it's $NPV.
- The discount rate used was your MARR, derived from your fallback investment.
- **If you stuck with your fallback project, you would have had to spend *all the resources used in that project, PLUS the NPV, to do as well as you're doing with the new project*.**
- NPV > 0? Your fallback investment can't do as well with the same resources.
- NPV < 0? You could have done just as well, using less resources.
- Example: A project costs $1,000 today and pays $1,375 tomorrow.
- Your MARR is 10% per day. Put $1 in your fallback project, get $1.10 tomorrow.
- NPV of the new project = -$1,000 + $1,375/(1+10%) = $250
- The best your fallback project can do if you put $1,000 in it today, is to give you $1,000 x (1+10%) = $1,100 tomorrow.
- If you want $1,375 tomorrow, you're going to have to put in not just that $1,000, but an additional $250: ($1,000 + $250) x (1+10%) = $1,375

In our simple example…

- Your friend's investment is NOT a good deal, given a MARR of 1%:
- $1,001 <<< $1,000 x $(1.01)^{20}$.
- How much, exactly, would you be losing?
- The NPV of your friend's offer is equal to the present worth of the $1,001 in 20 years, minus the $1,000 you have to pay today.
- The net present value (NPV) is
- NPV = $1,001 x (P/F,1%,20) - $1,000
- NPV = $1,001 / $(1 + 1\%)^{20}$ - $1,000 = -$179.64

Another way of looking at it…

For home use: We'll zoom over these slides in the lecture.

- Still not convinced?
- If you buy the bond, at the end of 20 years you'll get \$1,220.19.
- (That's \$1,000 x $(1.01)^{20}$.)
- Your friend will give you \$1,001.00 at the end of 20 years.
- The difference between these two values is \$219.19.
- At 1% interest, how much would you need to put aside in order to have that \$219.19 in 20 years?
- The answer is \$179.64 (rounded to the nearest cent).
- \$179.64 x $(1.01)^{20}$ = \$219.19
- Therefore, accepting your friend's offer is equivalent to giving up \$179.64 today, if you could save that money at 1% interest.

Summary: Net Present Value (NPV)

- NPV is the present value of the cash flows representing the project, evaluated at the MARR.
- NPV is equal to the profit from the project, compared to the next best use of the money. If NPV < 0, there is negative profit; a loss.
- *The investment is worthwhile if and only if* NPV ≥ 0

AFTER HOURS

- More intuition on the MARR (6 slides)

Additional intuition on the MARR and NPV

- Having a strong, correct intuition about the MARR and NPV is important, so I've provided it from a few different perspectives in the next few slides.
- Suppose you have \$100, and there are two things you can do with it:
- Option A: Put it into your savings account for a year, just like you always do. Your savings account pays you 2% interest per year.
- Option B: Put it into an investment. The investment will take your \$100 today, and will pay you \$110 one year from now.
- In this intentionally simple example, the choice is clear: pick Option B.
- It gives you \$110 one year from now, instead of \$102.
- Equivalently, option B works as if it were a bank account that offers you 10% interest, instead of the usual 2%.
- Most projects will be much more complicated than this, but it will help to clear up concepts if we use this very simple example to illustrate NPV.

The MARR and NPV

- The MARR is the return from the next best thing you could be doing with your resources.
- In this case, the obvious choice for a MARR is the 2%/year interest you get from your bank.
- That’s your fallback alternative: it’s always available, so any other project had better give you at LEAST that much, or you’ll just put your money in the bank instead.
- When we calculate the present value of a future cash flow using the MARR as an interest rate, we’re asking, ‘how much money would I have to set aside today, at the MARR, to have that cash flow in the future?’.

Plugging in numbers

- Consider Option A, where you give up $100 today to have $102 one year from today.
- What is the present value of $102 evaluated at the MARR?
- It's $102/(1 + 2%) = $102/1.02 = $100.
- The present value of $102, one year from now, evaluated at MARR = 2%/year, is $100.
- In other words, at the MARR of 2%/year, you would have to give up $100 today to have $102 one year from now.
- The NET present value (NPV) of Option A must therefore be $0:
- NPV = Present value of -$100 today + Present value of $102 in 1 year
- NPV = -$100 + $100 = $0
- This tells us that this investment is giving you EXACTLY what you pay in (in present value terms).

Now consider option B…

- In Option B, where you give up \$100 today to have \$110 one year from today.
- What is the present value of \$110 evaluated at the MARR?
- It’s \$110/(1 + 2%) = \$102/1.02 = \$107.84 (to the nearest cent).
- The present value of \$110, one year from now, evaluated at MARR = 2%/year, is \$107.84.
- In other words, at the MARR of 2%/year, you would have to give up \$107.84 today to have \$110 one year from now.
- That’s MORE than the \$100 you’re giving up today. The NET present value (NPV) of Option B must therefore be greater than \$0:
- NPV = Present value of -\$100 today + Present value of \$110 in 1 year
- NPV = -\$100 + \$107.84 = \$7.84
- This tells us that this investment is BETTER than the fallback investment that provided your MARR.
- In order to be as well off one year from now with your fallback activity (putting the money in the bank) as you are with this investment, you would have to put in \$107.84.
- Taking this investment instead of putting your \$100 in the bank is as IF you had been given an extra \$7.84 today, and then put all your money in the bank.
- That \$7.84 is the *profit* from Option B (compared to your fallback activity providing the MARR, option A).

About that $7.84…

- Options A and B cost the same amount today.
- In Option A, you get $102 one year from now.
- In Option B, you get $110 one year from now.
- The difference between options B and A is an extra $8, one year from now, in Option B.
- The present value of $8 evaluated at the MARR is $8/(1+2%) = $7.84.
- That is the same number we got when we calculated the NPV of Option B at the MARR.
- →The NPV evaluated at the MARR gives you the present value of the profit (or loss) of the project you're looking at, *compared* to the project from which the MARR is derived.
- That's why we say it's the present value of profits (or losses, if negative):
- Your money wouldn't be sitting idle if you didn't invest in option B; you need to calculate these profits compared to what you would have earned otherwise.
- (If you still have trouble seeing this, imagine that options A and B paid off immediately instead of a year from now. Option A offers $2 today, Option B offers $10 today. It's not right to say you profited to the tune of $10 by choosing option B, since if you hadn't chosen option B, you would have chosen A, and that gives you $2, not $10. The profit from Option B is only $8 compared to the alternative.)

What does NPV = $100 mean?

- The NPV is the present value of the project, evaluated at the MARR.
- Your MARR represents the next-best use of your resources: a fallback project, or your cost of borrowing, etc.
- If the NPV is $100, it means that the return on the project is just as good as if you had taken all the resources used by the project, plus an additional $100 today, and used them all in the activity from which your MARR is derived (earning your fallback return, paying off your debts, etc.).
- In that sense, the NPV represents the profit from the project, *compared to using those resources in the activity from which your MARR is derived*.
- A negative NPV, say, -$100, means that the return from the project is as if you had put all the project's resources, *minus* $100 (in our example), into the activity from which the MARR is derived.