Math 211 202101
Complex Numbers Worksheet

The goal of this document is to help you become comfortable performing operations with complex numbers. You'll find a bunch of questions on the first page; the answers are after that (with no commentary). Let Joseph know if you find errors.

1. **Complex Arithmetic:**

   (a) Compute $(3+5i)+(-2+7i)$.
   (b) Compute $3(1-3i)+i(2+i)$.
   (c) Compute $(3+2i)(4-3i)$.
   (d) Compute $(\sqrt{2}+\sqrt{3}i)(\sqrt{2}-\sqrt{3}i)$.
   (e) Compute $(2-i)^{-1}$.
   (f) Compute $(1+i)/(2+3i)$.

2. **Complex Conjugation and the Modulus:**

   (a) Compute $\overline{2+3i}$.
   (b) Compute $\overline{(5+4i)+(-3-i)}$.
   (c) Compute $\overline{(3+4i)(1-i)}$.
   (d) Compute $\overline{\overline{1+i}}$.
   (e) Compute $(3-5i)\overline{(3-5i)}$.
   (f) Compute $\left|2+\sqrt{5}i\right|$.
   (g) Compute $|(1+i)(2-3i)|$.
   (h) Compute $|3-i|^2-(3-i)\overline{(3-i)}$.

3. **Polar and Exponential Form:**

   (a) Convert $2+2i$ into polar form.
   (b) Convert $-3i$ into polar form.
   (c) Convert $-1-\sqrt{3}i$ into exp. form.
   (d) Compute $2(\cos(3\pi/4)+i\sin(3\pi/4))$.
   (e) Compute $e^{i\pi/2+\log_e(5)}$.
   (f) Compute $e^{3-i}e^{4+i}$.

4. **Roots of Polynomials:**

   (a) Find the roots of z^3-1.
   (b) Find the roots of z^3+8.
   (c) Find the roots of $z^4+6z^3+9z^2$.
   (d) Find the roots of $z^2+3iz-3$.

5. **$\mathbb{C}$ as $\mathbb{R}^2$ / Geometry:**

   (a) Draw the points $\pm 2 \pm 3i$.
   (b) Draw the set defined by $|z|=1$.
   (c) Draw the set defined by $\mathrm{Re}(z)=\frac{1}{2}$.
   (d) Draw the set defined by $\mathrm{Im}(z)=\pi$.
   (e) Draw the set $\left\{z \in \mathbb{C} \ : \ \mathrm{Arg}(z)=-\frac{\pi}{3}\right\}$.
   (f) Draw the set $\{z \in \mathbb{C} \ : \ z=\bar{z} \text{ or } z=-\bar{z}\}$.

Answers:

1. **Complex Arithmetic:**

   (a) $1 + 12i$

   (b) $2 - 7i$

   (c) $18 - i$

   (d) 5

   (e) $\frac{1}{5}(2 + i) = \frac{2}{5} + \frac{1}{5}i$

   (f) $\frac{1}{13}(5 - i) = \frac{5}{13} - \frac{1}{13}i$

2. **Complex Conjugation and the Modulus:**

   (a) $2 - 3i$

   (b) $2 - 3i$

   (c) $7 - i = \overline{(3 + 4i)}\ \overline{(1 - i)}$

   (d) $1 + i$

   (e) $|3 - 5i|^2 = 34$

   (f) $3 = \sqrt{2^2 + (\sqrt{5})^2}$

   (g) $\sqrt{26} = |1 + i|\,|2 - 3i|$

   (h) 0 ($|z|^2 = z\bar{z}$ for all $z \in \mathbb{C}$)

3. **Polar and Exponential Form:**

   (a) $2\sqrt{2}(\cos(\pi/4) + i\sin(\pi/4))$

   (b) $3(\cos(-\pi/2) + i\sin(-\pi/2))$

   (c) $e^{\log_e(2) - 2i\pi/3} = 2e^{-2i\pi/3}$

   (d) $-\sqrt{2} + \sqrt{2}i$

   (e) $5e^{i\pi/2} = 5i$

   (f) $e^{3-i}e^{4+i} = e^{7+0} = e^7$

4. **Roots of Polynomials:**

   (a) $1, e^{2i\pi/3}, e^{-2i\pi/3}$

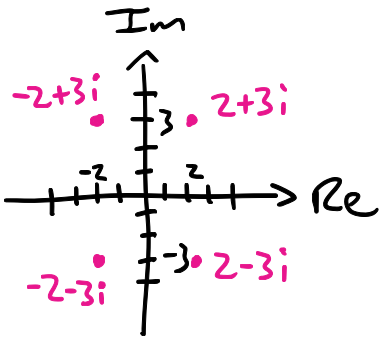
   (b) $-2, 2e^{i\pi/3}, 2e^{-i\pi/3}$

   (c) 0 (twice), -3 (twice)

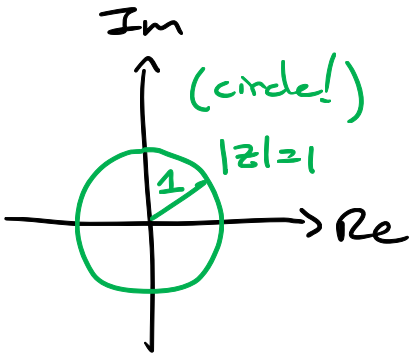
   (d) $\frac{\sqrt{3}}{2} - \frac{3}{2}i, -\frac{\sqrt{3}}{2} - \frac{3}{2}i$

5. $\mathbb{C}$ **as** $\mathbb{R}^2$ **/ Geometry:**

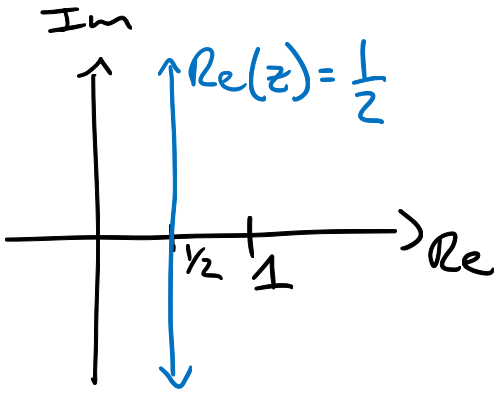
(a)



(b)



(c)



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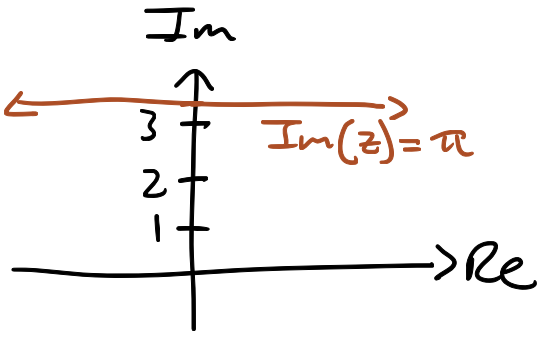
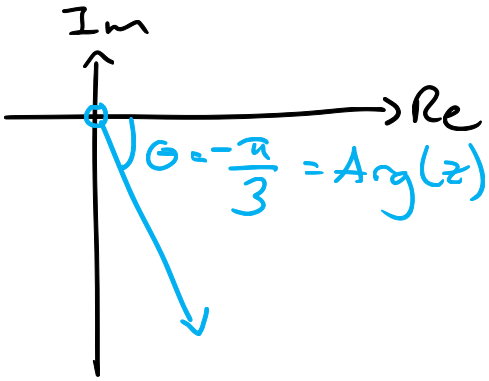


(e)



(f)