Vectors - IV

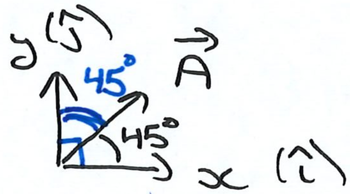
$\vec{A}$ is $5m$ long and makes an angle of 45° measured counterclockwise in the xy plane from $\hat{\imath}$. $\vec{B} = 3m\hat{\imath} - 5m\hat{\jmath}$.

- What is the dot product between $\vec{A}$ and $\vec{B}$?
- What is the angle between $\vec{A}$ and $\vec{B}$?

① Write $\vec{A}$ in components, use rule.

② Write $\vec{B}$ as magnitude & direction do geometry Find angle, use other rule.

Way 1



$$A_x = |\vec{A}|\cos 45° = 3.54m$$

$$A_y = |\vec{A}|\cos 45° = 3.54m$$

$$\vec{A} = 3.54m\hat{\imath} + 3.54m\hat{\jmath} + 0m\hat{k}$$

$$\vec{B} = 3m\hat{\imath} - 5m\hat{\jmath} + 0m\hat{k}$$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$

$$= (3.54m)3m + (3.54m)(-5m) + (0m)^2$$

$$\boxed{= -7.07m^2}$$

Angle between $\vec{A}$ & $\vec{B}$

$$\underbrace{\vec{A}\cdot\vec{B}}_{\text{calculated}} = |\vec{A}||\vec{B}|\cos\Theta$$

calculated   know   can calculate   want

$$-7.07\,m^2 = (5m)\sqrt{(3m)^2+(-5m)^2+(0m)^2}\cos\Theta$$

$$-0.243 = \cos\Theta$$

$$\boxed{104^\circ = \Theta}$$

Way 2

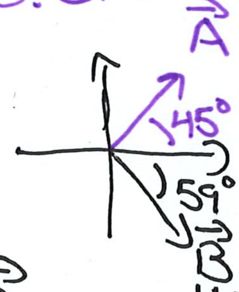
What is $|\vec{B}| = \sqrt{B_x^2+B_y^2+B_z^2} = 5.83m$

What is angle? ← $\vec{B}$ & $\hat{i}$

$$B_x = |\vec{B}|\cos\Theta$$

$$\Theta = 59^\circ$$

$$\frac{3m}{5.83m} = \cos\Theta = 0.514$$



Angle btw $\vec{A}$ & $\vec{B}$ = 104°

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\Theta = (5m)(5.83m)\cos 104^\circ$$

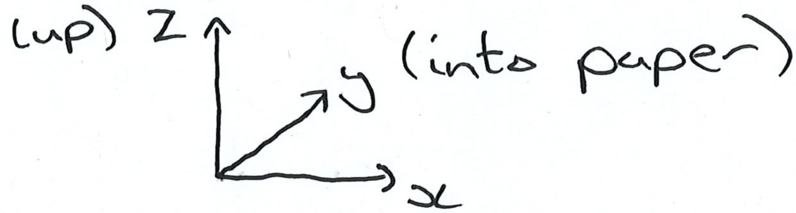
$$= -7.07$$

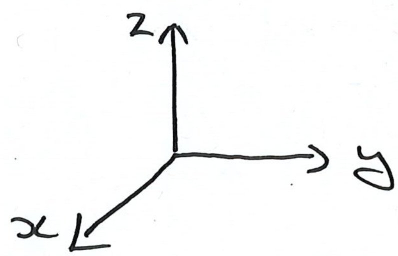
Coordinate Systems:

So far:

x, y, z coordinates

$\hat{i}$ $\hat{j}$ $\hat{k}$ all at 90° to each other



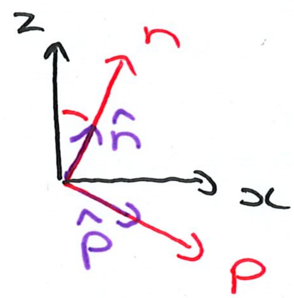


"Right-handed" ← meaning when we talk about "cross products"

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

Nothing special about x, y, z as coordinates.

Any set of three unit vectors all at 90° to each other can form "basis" for coordinate system



$$\vec{A} = A_p \hat{p} + A_y \hat{j} + A_n \hat{n}$$

$$A_p = \vec{A} \cdot \hat{p}$$

Cross product.

Takes 2 vectors, gives a third vector.

How this happens:

- Two (non-parallel vector) define a plan.
- Plane in 3D has only one direction at 90° to it
- Up to ± (change in direction) the normal vector to plane defines cross product.

$$\vec{A}\times\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\times(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

Symbol for cross product

$$= (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

$$\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$$

Properties:

- Direction of $\vec{A}\times\vec{B}$ at 90° to $\vec{A}$ and 90° to $\vec{B}$, in sense by "Right-hand rule"
  - Fingers of R hand along $\vec{A}$
  - Curl fingers to $\vec{B}$
  - Thumb points in $\vec{A}\times\vec{B}$
- $|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

angle between $\vec{A}$ & $\vec{B}$

$$\hat{\imath}\times\hat{\jmath} = \hat{k}$$

$$\hat{\imath}\times\hat{\imath} = 0$$
$$\hat{\imath}\times\hat{\jmath} = \hat{k}$$
$$\hat{\imath}\times\hat{k} = -\hat{\jmath}$$
$$\hat{\jmath}\times\hat{\imath} = -\hat{k}$$
$$\hat{\jmath}\times\hat{\jmath} = 0$$
$$\hat{\jmath}\times\hat{k} = \hat{\imath}$$
$$\hat{k}\times\hat{\imath} = \hat{\jmath}$$
$$\hat{k}\times\hat{\jmath} = -\hat{\imath}$$
$$\hat{k}\times\hat{k} = 0$$

Linear algebra

$$\vec{A} \times \vec{B} = \det \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix}$$

Vectors - V

$\vec{A} = 3m\hat{\imath} + 2m\hat{\jmath} - 4m\hat{k}$ and $\vec{B} = 2m\hat{\imath} - 3m\hat{\jmath} - 1m\hat{k}$.

$\overline{A_x}$ $\overline{A_y}$ $\overline{A_z}$ $\quad$ $\overline{B_x}$ $\overline{B_y}$ $\overline{B_z}$

- What are the x, y, and z components of $\vec{A} \times \vec{B}$?
- What is the magnitude of $\vec{A} \times \vec{B}$?
- What is the angle between $\vec{A}$ and $\vec{A} \times \vec{B}$?

$$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= (A_yB_z - A_zB_y)\hat{\imath}$$

$$+ (A_zB_x - A_xB_z)\hat{\jmath}$$

$$+ (A_xB_y - A_yB_x)\hat{k}$$

$$= (2m(-1m) - (-4m)(-3m))\hat{\imath}$$

$$+ ((-4m)2m - (3m)(-1m))\hat{\jmath}$$

$$+ (3m(-3m) - (2m)2m)\hat{k}$$

$$= \underline{(-14m^2)}\hat{\imath} + (-5m^2)\hat{\jmath}$$

$$+ (-13m^2)\hat{k}$$

x-comp of $\vec{A} \times \vec{B}$

$$|\vec{A}\times\vec{B}| = \sqrt{(-14m^2)^2+(-5m^2)^2+(-13m^2)^2}$$

$$= 19.748m^2$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\Theta$$

$$\sqrt{(3m)^2+(2m)^2+(-4m)^2}$$

$$5.385m$$

$$\sqrt{(2m)^2+(-3m)^2+(-1m)^2}$$

$$3.742m$$

angle

$$\vec{A}\cdot\vec{B} = A_xB_x+A_yB_y+A_zB_z = |\vec{A}||\vec{B}|\cos\Theta$$

$$(3m)(2m)+(2m)(-3m)+(-4m)(-1m) = |\vec{A}||\vec{B}|\cos\Theta$$

$$4m^2 = (5.385m)(3.742m)\cos\Theta$$

$$0.1985 = \cos\Theta$$

$$\Theta = 78.55^\circ$$

$$19.748m^2 = 5.385m(3.742m)\sin 78.55$$

$$= 19.75m^2$$

$$\vec{A}\cdot(\vec{A}\times\vec{B}) = |\vec{A}||\vec{A}\times\vec{B}|\cancel{\sin\Theta}\cos\overset{90^\circ}{\Theta} = 0$$

expect

$$= (3m)(-14m^2)+2m(-5m^2)+(-4m)(-13m^2)$$

$$= 0$$