Exercise 63

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Explanation

Verified

Step 1

First we solve the interval wher the curve is beneath the (y=0) axis ($\left[-\frac{\pi}{4}, 0\right]$)

Result=Area between (y=0) and the curve + Area of rectangle=

Rectangle area =$\sqrt{2} \cdot \frac{\pi}{4} = \frac{\pi\sqrt{2}}{4}$

Note the rectangle, bounded by lines

$y = \sqrt{2}$

$y = 0$

$\theta = 0$

$\theta = -\frac{\pi}{4}$

Calculate the area.

Step 2

$-\int_{-\frac{\pi}{4}}^{0} \sec\theta\tan\theta d\theta = [-\sec\theta]_{-\frac{\pi}{4}}^{0}$ =

=$(-\sec 0) - \left(-\sec\left(-\frac{\pi}{4}\right)\right) = \sqrt{2} - 1$

The area between the curve and the (y=0) axis on $\left[-\frac{\pi}{4}, 0\right]$...

... is a definite integral for a curve UNDER the axis (minus sign) ...

(table of given antideratives, FTS)

Step 3

Area of the shaded region on $\left[-\frac{\pi}{4}, 0\right]$ is

$$\frac{\pi\sqrt{2}}{4} + \left(\sqrt{2} - 1\right)$$

Step 4

NOW, FOR THE OTHER INTERVAL, $\left[0, \frac{\pi}{4}\right]$

Rectangle bounds:

$\theta = \frac{\pi}{4}, \theta = 0,$

$$y = \sqrt{2}, y = 0$$

Step 5

Area of rectangle=

$$\sqrt{2} \cdot \frac{\pi}{4} = \frac{\pi\sqrt{2}}{4}$$

Step 6

$\int_0^{\frac{\pi}{4}} \sec\theta\tan\theta d\theta = [\sec\theta]_0^{\frac{\pi}{4}}$ =

= $\sec\frac{\pi}{4} - \sec 0 = \sqrt{2} - 1$

Area underneath the curve.

Step 7

The shaded area on $\left[0, \frac{\pi}{4}\right]$

= $\frac{\pi\sqrt{2}}{4} - \left(\sqrt{2} - 1\right)$

Step 8

TOTAL AREA:

$$\left(\frac{\pi\sqrt{2}}{4} + \sqrt{2} - 1\right) + \left(\frac{\pi\sqrt{2}}{4} - \sqrt{2} + 1\right) = \frac{\pi\sqrt{2}}{2}$$

is the sum of the left shaded region and the right shaded region...

Result

$$\frac{\pi\sqrt{2}}{2}$$