Symbolab Symbolab

Solution

$\sum_{n=0}^{\infty}\frac{n(x+2)^n}{3^n}$: Radius of convergence is 3, Interval of convergence is $-5<x<1$

Steps

$\sum_{n=0}^{\infty}\frac{n(x+2)^n}{3^n}$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=0}^{\infty}\frac{n(x+2)^n}{3^n}$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{(n+1)(x+2)^{(n+1)}}{3^{(n+1)}}}{\frac{n(x+2)^n}{3^n}}\right|$$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{(n+1)(x+2)^{(n+1)}}{3^{(n+1)}}}{\frac{n(x+2)^n}{3^n}}\right|\right)$ *Hide Steps*

$$L=\lim_{n\to\infty}\left(\left|\frac{\frac{(n+1)(x+2)^{(n+1)}}{3^{(n+1)}}}{\frac{n(x+2)^n}{3^n}}\right|\right)$$

Simplify $\frac{\frac{(n+1)(x+2)^{(n+1)}}{3^{(n+1)}}}{\frac{n(x+2)^n}{3^n}}$: $\frac{(n+1)(x+2)}{3n}$ *Hide Steps*

$$\frac{\frac{(n+1)(x+2)^{n+1}}{3^{n+1}}}{\frac{n(x+2)^n}{3^n}}$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$$=\frac{(n+1)(x+2)^{n+1}\cdot 3^n}{3^{n+1}n(x+2)^n}$$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$$\frac{(x+2)^{n+1}}{(x+2)^n}=(x+2)^{n+1-n}$$

$$=\frac{3^n(n+1)(x+2)^{n-n+1}}{3^{n+1}n}$$

Add similar elements: $n+1-n=1$

$$=\frac{3^n(n+1)(x+2)}{3^{n+1}n}$$

Apply exponent rule: $\frac{x^a}{x^b}=\frac{1}{x^{b-a}}$

$$\frac{3^n}{3^{n+1}}=\frac{1}{3^{n+1-n}}$$

$$=\frac{(n+1)(x+2)}{3^{n-n+1}n}$$

Add similar elements: $n+1-n=1$

$$=\frac{(n+1)(x+2)}{3n}$$

$$L=\lim_{n\to\infty}\left(\left|\frac{(n+1)(x+2)}{3n}\right|\right)$$

$$L=\left|\frac{x+2}{3}\right|\cdot\lim_{n\to\infty}\left(\left|\frac{n+1}{n}\right|\right)$$

$\lim_{n\to\infty}\left(\left|\frac{n+1}{n}\right|\right)=1$ *Hide Steps*

$$\lim_{n\to\infty}\left(\left|\frac{n+1}{n}\right|\right)$$

$\frac{n+1}{n}$ is positive when $n \to \infty$. Therefore $\left|\frac{n+1}{n}\right| = \frac{n+1}{n}$

$= \lim_{n\to\infty}\left(\frac{n+1}{n}\right)$

Divide by highest denominator power: $1 + \frac{1}{n}$ *Hide Steps*

$\frac{n+1}{n}$

Divide by n

$= \frac{\frac{n}{n} + \frac{1}{n}}{\frac{n}{n}}$

Refine

$= 1 + \frac{1}{n}$

$= \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)$

$\lim_{x\to a}\left[f(x) \pm g(x)\right] = \lim_{x\to a}f(x) \pm \lim_{x\to a}g(x)$
With the exception of indeterminate form

$= \lim_{n\to\infty}(1) + \lim_{n\to\infty}\left(\frac{1}{n}\right)$

$\lim_{n\to\infty}(1) = 1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c = c$

$= 1$

$\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$ *Hide Steps*

$\lim_{n\to\infty}\left(\frac{1}{n}\right)$

Apply Infinity Property: $\lim_{x\to\infty}\left(\frac{c}{x^a}\right) = 0$

$= 0$

$= 1 + 0$

Simplify

$= 1$

$L = \left|\frac{x+2}{3}\right| \cdot 1$

Simplify

$L = \frac{|x+2|}{3}$

$L = \frac{|x+2|}{3}$

The power series converges for $L < 1$

$\frac{|x+2|}{3} < 1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$\frac{|x+2|}{3} < 1: \quad |x+2| < 3$ *Hide Steps*

$\frac{|x+2|}{3} < 1$

Multiply both sides by 3

$\frac{3|x+2|}{3} < 1 \cdot 3$

Simplify

$|x+2| < 3$

Therefore

Radius of convergence is 3

Radius of convergence is 3

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for x

$\frac{|x+2|}{3} < 1 \quad : \quad -5 < x < 1$ *Hide Steps*

$\frac{|x+2|}{3} < 1$

Multiply both sides by 3

$\frac{3|x+2|}{3} < 1 \cdot 3$

Simplify

$|x+2| < 3$

Apply absolute rule: If $|u| < a, a > 0$ then $-a < u < a$

$-3 < x+2 < 3$

$x+2 > -3$ and $x+2 < 3$ *Hide Steps*

$x+2 > -3$ and $x+2 < 3$

$x+2 > -3 \quad : \quad x > -5$ *Hide Steps*

$x+2 > -3$

Subtract 2 from both sides

$x+2-2 > -3-2$

Simplify

$x > -5$

$x+2 < 3 \quad : \quad x < 1$ *Hide Steps*

$x+2 < 3$

Subtract 2 from both sides

$x+2-2 < 3-2$

Simplify

$x < 1$

Combine the intervals

$x > -5$ and $x < 1$

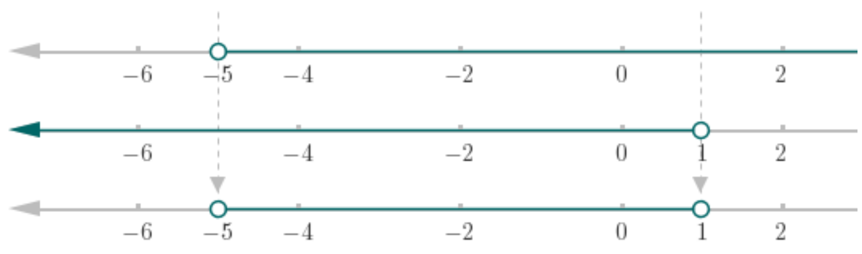
$x > -5$ and $x < 1$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x > -5$ and $x < 1$

$-5 < x < 1$



$-5 < x < 1$

$-5 < x < 1$

Check the interval end points: $x = -5$:diverges, $x = 1$:diverges *Hide Steps*

For $x = -5$, $\sum_{n=0}^{\infty} \frac{n((-5)+2)^n}{3^n}$: diverges *Hide Steps*

$\sum_{n=0}^{\infty} \frac{n((-5)+2)^n}{3^n}$

Refine

$= \sum_{n=0}^{\infty} \frac{(-3)^n n}{3^n}$

$(-3)^n = 3^n(-1)^n$

$= \sum_{n=0}^{\infty} \frac{3^n(-1)^n n}{3^n}$

Cancel the common factor: 3^n

$= \sum_{n=0}^{\infty} (-1)^n n$

Apply Series Divergence Test: diverges *Hide Steps*

$\sum_{n=0}^{\infty} (-1)^n n$

Series Divergence Test:

If $\lim_{n\to\infty} a_n \neq 0$ then $\sum a_n$ diverges

$\lim_{n\to\infty} \left((-1)^n n\right)$ = diverges *Hide Steps*

$\lim_{n\to\infty}\left((-1)^n n\right)$

Apply Limit Divergence Criterion: diverges

Hide Steps

$\lim_{n\to\infty}\left((-1)^n n\right)$

Limit Divergence Criterion Test:

If two sequences exist, $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ with

$x_n \neq c$ and $y_n \neq c$

$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c$

$\lim_{n\to\infty} f(x_n) \neq \lim_{n\to\infty} f(y_n)$

Then $\lim_{x\to c} f(x)$ does not exist

$c=\infty$, $x_n = 2k$, $y_n = 2k+1$

$\lim_{k\to\infty}(2k)=\infty$

Hide Steps

$\lim_{k\to\infty}(2k)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty, a>0,$
n is odd
$a=2, n=1$

$=\infty$

$\lim_{k\to\infty}(2k+1)=\infty$

Hide Steps

$\lim_{k\to\infty}(2k+1)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty, a>0,$
n is odd
$a=2, n=1$

$=\infty$

$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c = \infty$

$\lim_{k\to\infty}\left((-1)^{2k}\cdot 2k\right)=\infty$

Hide Steps

$\lim_{k\to\infty}\left((-1)^{2k}\cdot 2k\right)$

$(-1)^{2k}=1, \forall\, k\in\mathbb{Z}$

$=\lim_{k\to\infty}(1\cdot 2k)$

$\lim_{k\to\infty}(1\cdot 2k)=\infty$

Hide Steps

$\lim_{k\to\infty}(1\cdot 2k)$

$\lim_{x\to a}[c\cdot f(x)] = c\cdot\lim_{x\to a} f(x)$

$=1\cdot 2\cdot\lim_{k\to\infty}(k)$

Simplify

$=2\cdot\lim_{k\to\infty}(k)$

Apply the common limit: $\lim_{k\to\infty}(k)=\infty$

$=2\cdot\infty$

Apply Infinity Property: $c\cdot\infty=\infty$

$=\infty$

$=\infty$

$\lim_{k\to\infty}\left((-1)^{(2k+1)}(2k+1)\right) = -\infty$

Hide Steps

$\lim_{k\to\infty}\left((-1)^{(2k+1)}(2k+1)\right)$

$(-1)^{(2k+1)}=(-1), \forall\, k\in\mathbb{Z}$

$=\lim_{k\to\infty}((-1)(2k+1))$

$\lim_{k\to\infty}((-1)(2k+1)) = -\infty$

Hide Steps

$\lim_{k\to\infty}((-1)(2k+1))$

$\lim_{x\to a}[c\cdot f(x)] = c\cdot\lim_{x\to a} f(x)$

$=-\lim_{k\to\infty}(2k+1)$

$\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
With the exception of indeterminate form

$=-\left(\lim_{k\to\infty}(2k)+\lim_{k\to\infty}(1)\right)$

$\lim_{k\to\infty}(2k)=\infty$

Hide Steps

$\lim_{k\to\infty}(2k)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n+\cdots+bx+c\right)=\infty$,
$a>0$, n is odd
$a=2,\ n=1$

$=\infty$

$\lim_{k\to\infty}(1)=1$ *Hide Steps*

$\lim_{k\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$=-(\infty+1)$

Simplify $-(\infty+1)$: $-\infty$ *Hide Steps*

$-(\infty+1)$

$\infty+1=\infty$ *Hide Steps*

$\infty+1$

Apply Infinity Property: $\infty+c=\infty$

$=\infty$

$=-(\infty)$

Remove parentheses: $(a)=a$

$=-\infty$

$=-\infty$

$=-\infty$

$\lim_{n\to\infty}f(x_n)\neq\lim_{n\to\infty}f(y_n)$

Therefore $\lim_{n\to\infty}\left((-1)^n n\right)$ is divergent at $n\to\infty$

= diverges

= diverges

By the divergence test criteria

= diverges

= diverges

For $x=1$, $\sum_{n=0}^{\infty}\frac{n(1+2)^n}{3^n}$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\frac{n(1+2)^n}{3^n}$

Refine

$=\sum_{n=0}^{\infty}n$

Apply Series Divergence Test: diverges *Hide Steps*

$\sum_{n=0}^{\infty}n$

Series Divergence Test:

If $\lim_{n\to\infty}a_n\neq0$ then $\sum a_n$ diverges

$\lim_{n\to\infty}(n)=\infty$ *Hide Steps*

$\lim_{n\to\infty}(n)$

Apply the common limit: $\lim_{n\to\infty}(n)=\infty$

$=\infty$

By the divergence test criteria

= diverges

= diverges

$x=-5$:diverges, $x=1$:diverges

Therefore

Interval of convergence is $-5<x<1$

Interval of convergence is $-5<x<1$

Radius of convergence is 3, Interval of convergence is $-5<x<1$