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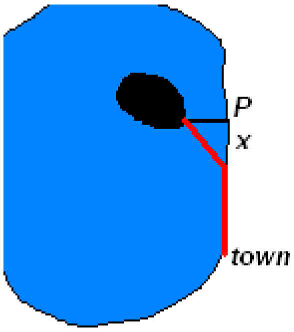
| **Student:** Arfaz Hossain<br>**Date:** 11/14/21 | **Instructor:** UVIC Math<br>**Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 8 |

A small island is 3 miles from the nearest point P on the straight shoreline of a large lake. If a woman on the island can row a boat 3 miles per hour and can walk 4 miles per hour, where should the boat be landed in order to arrive at a town 16 miles down the shore from P in the least time?

Begin by drawing a sketch of the situation.




The distance between P and town is 16 miles.
The distance between the island and P is 3 miles.
The woman must paddle along the the red path until the shore. Then she must walk the remainder of the red path.
The distance between P and the landing point on the shore is x.

She must row a distance of $\sqrt{x^2+9}$.

She must walk a distance of $16-x$.

For uniform motion problems, recall that distance equals rate times time, or $d=rt$. Solve this equation for t.

$$t=\frac{d}{r}$$

The time traveled over water plus the time traveled over land can be written

$\frac{\sqrt{x^2+9}}{3}+\frac{16-x}{4}$ hours.

This is the function that will be minimized. The domain for the function is $0 \le x \le 16$.

In order to find critical points, we need to find the derivative, $T'(x)$. $T=\frac{\sqrt{x^2+9}}{3}+\frac{16-x}{4}$.

The derivative is $\frac{x}{3\sqrt{x^2+9}}-\frac{1}{4}$.

Rewrite $T'(x)$ as a single rational expression.

$$\begin{aligned} T'(x) &= \frac{x}{3\sqrt{x^2+9}}-\frac{1}{4} \\ &= \frac{4x-3\sqrt{x^2+9}}{12\sqrt{x^2+9}} \quad \text{Find a common denominator.} \end{aligned}$$

To find critical points, we need to find the values of the variable that make the derivative $T'(x)$either 0 or undefined. Note that the only values of the variable that make the rational expression undefined are the values that make the denominator equal to 0, but this can't happen here since the radicand is always positive.

Note that the only values of the variable that make the rational expression equal to 0 are the values that make the numerator equal to 0. Determine the values of x that make the numerator equal to 0.

$$\begin{aligned} 4x-3\sqrt{x^2+9} &= 0 \\ -3\sqrt{x^2+9} &= -4x \qquad \text{Subtract 4x from both sides.} \end{aligned}$$

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| $9\left(x^2+9\right)$ | $= 16x^2$ | Square both sides. |
| $9x^2+81$ | $= 16x^2$ | Use the distributive property. |
| $-7x^2$ | $= -81$ | Combine like terms. |
| $x^2$ | $= \frac{81}{7}$ | Take the principal square root of both sides, since the domain excludes negative numbers. |
| $x$ | $= \frac{9}{\sqrt{7}}$ | Take the square root of both sides. |

Thus, the critical points are the endpoints 0 and 16, and the stationary point, $\frac{9}{\sqrt{7}}$.

To find out which is the minimum amount of time, substitute the critical points in for x in the function, $T = \frac{\sqrt{x^2+9}}{3} + \frac{16-x}{4}$.

$$T(0) = \frac{\sqrt{0^2+9}}{3} + \frac{16-0}{4} = 5$$

$$T\left(\frac{9}{\sqrt{7}}\right) = \frac{\sqrt{\left(\frac{9}{\sqrt{7}}\right)^2+9}}{3} + \frac{16-\left(\frac{9}{\sqrt{7}}\right)}{4} = 4.661$$

$$T(16) = \frac{\sqrt{16^2+9}}{3} + \frac{16-16}{4} = 5.426$$

The minimum time occurs at the stationary point. Therefore, she should land the boat $\frac{9}{\sqrt{7}}$ miles down the shore from point P.