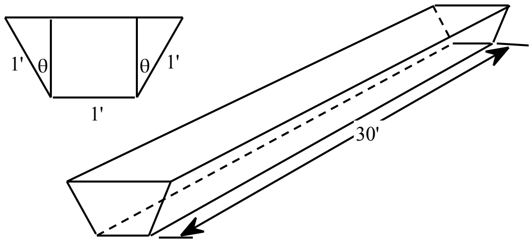
Student: Arfaz Hossain **Date:** 11/14/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 8

The trough in the figure is to be made to the dimensions shown. Only the angle θ can be varied. What value of θ will maximize the trough's volume?



Use the fact that the volume of the trough is the product of the cross section area and the length to express the volume of the trough as a function of θ.

Notice that the cross section area of the trough is the sum of the area of a rectangle and two congruent right triangles. Use the fact the the length of the side adjacent to an angle divided by the hypotenuse is equal to the cosine of the angle.

$$\begin{aligned} A_R &= L \bullet W \\ &= \mathbf{cos}\ \theta \end{aligned}$$

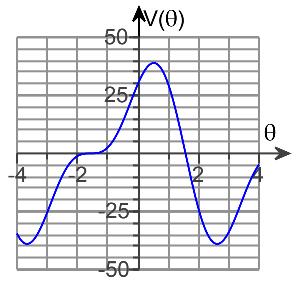
Now determine the area of the two right triangles A_T. Use the fact the the length of the side opposite an angle divided by the hypotenuse is equal to the sine of the angle.

$$\begin{aligned} A_T &= 2\left(\frac{1}{2} \bullet b \bullet h\right) \\ &= \mathbf{sin}\,\theta\,\mathbf{cos}\,\theta \end{aligned}$$

Now write the volume of the trough as a function of θ, as shown below.

$$\begin{aligned} V(\theta) &= 30(\mathbf{cos}\ \theta + \mathbf{sin}\,\theta\,\mathbf{cos}\,\theta) \\ &= 30\,\mathbf{cos}\ \theta + 30\,\mathbf{sin}\,\theta\,\mathbf{cos}\,\theta \end{aligned}$$

Next, graph the function, as shown to the right. Determine the values of θ that make sense in the problem. Since only positive values of volume make sense, the most reasonable interval is from about $\theta = -1.5$ to $\theta = 1.5$.



Notice that in the interval from $\theta = -1.5$ to $\theta = 1.5$, there appears to be a maximum value at about $\theta = \frac{1}{2}$.

Now determine the first derivative of V. Use the product rule.

$$\begin{aligned} V(\theta) &= 30\,\mathbf{cos}\ \theta + 30\,\mathbf{sin}\,\theta\,\mathbf{cos}\,\theta \\ V'(\theta) &= -30\,\mathbf{sin}\ \theta + 30\,\mathbf{cos}^2\theta - 30\,\mathbf{sin}^2\theta \end{aligned}$$

Notice that two of the terms contain sine functions and the third term is squared. Use the trigonometric identity $\mathbf{cos}^2\,\theta = 1 - \mathbf{sin}^2\,\theta$ to simplify the derivative.

Substitute $1 - \mathbf{sin}^2\,\theta$ for $\mathbf{cos}^2\,\theta$ and simplify.

$$\begin{aligned} V'(\theta) &= -30\,\mathbf{sin}\ \theta + 30\,\mathbf{cos}^2\theta - 30\,\mathbf{sin}^2\theta \\ &= -30\,\mathbf{sin}\ \theta + 30\left(1 - \mathbf{sin}^2\,\theta\right) - 30\,\mathbf{sin}^2\,\theta \\ &= -60\,\mathbf{sin}^2\,\theta - 30\,\mathbf{sin}\ \theta + 30 \end{aligned}$$

Now determine where the derivative of the function is zero or fails to exist. Notice that since the derivative does not contain division, it exists for all reals.

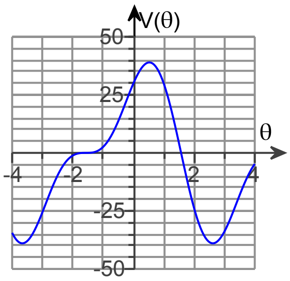
To determine where the derivative of the function is zero, set it equal to zero, divide both sides by the common factor, and factor the quadratic equation.

$$-60\,\mathbf{sin}^2\theta - 30\,\mathbf{sin}\,\theta + 30 = 0$$
$$2\,\mathbf{sin}^2\theta + \mathbf{sin}\,\theta - 1 = 0$$
$$(2\,\mathbf{sin}\,\theta - 1)(\mathbf{sin}\,\theta + 1) = 0$$

Now solve for θ in the approximate interval .

$$2\,\mathbf{sin}\,\theta - 1 = 0 \quad \text{or} \quad \mathbf{sin}\,\theta + 1 = 0$$
$$\mathbf{sin}\,\theta = \frac{1}{2} \qquad \mathbf{sin}\,\theta = -1$$
$$\theta = \frac{\pi}{6} \qquad \theta = -\frac{\pi}{2}$$

Finally, use the graph of the function, shown to the right, to determine which value of θ will maximize the trough's volume. Recall that this value was about $\frac{1}{2}$, which is close to $\frac{\pi}{6}$. Notice that the volume is zero when θ is $-\frac{\pi}{2}$, which is the minimum.



The trough has a maximum volume when the value of θ is $\frac{\pi}{6}$ radians.