Symbolab

Solution

$\sum_{n=0}^{\infty}x^{n}$: Radius of convergence is 1, Interval of convergence is $-1<x<1$

Steps

$\sum_{n=0}^{\infty}x^{n}$

Use the Root Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}x^{n}$

Series Root Test:

If $\lim_{n\to\infty}|a_n|^{\frac{1}{n}}=L$, and:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|a_n^{\frac{1}{n}}\right|=\left|x^{n\frac{1}{n}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|x^{n\frac{1}{n}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\left(x^{n}\right)^{\frac{1}{n}}\right|\right)$

Simplify $\left(x^{n}\right)^{\frac{1}{n}}$: x

Show Steps

$L=\lim_{n\to\infty}\left(|x|\right)$

$L=|x|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$

Show Steps

$L=|x|\cdot 1$

Simplify

$L=|x|$

$L=|x|$

The power series converges for $L<1$

$|x|<1$

Find the radius of convergence

Hide Steps

To find radius of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for $|x-a|$

$|x| < 1$

Therefore

Radius of convergence is 1

Radius of convergence is 1

Find the interval of convergence

Hide Steps

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$|x| < 1 \quad : \quad -1 < x < 1$

Show Steps

$-1 < x < 1$

Check the interval end points: $x = -1$:diverges, $x = 1$:diverges

Show Steps

Therefore

Interval of convergence is $-1 < x < 1$

Interval of convergence is $-1 < x < 1$

Radius of convergence is 1, Interval of convergence is $-1 < x < 1$