Student: Arfaz Hossain **Date:** 03/07/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]

Use any method to determine if the series converges or diverges. Give reasons for your answer.

$$\sum_{n=1}^{\infty} \frac{(-13)^n}{n^3 9^n}$$

Since the given series does not have a common ratio, it is not a geometric series. Consider the Ratio Test now.

Let $\sum a_n$ be any series and suppose that $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \rho$. The Ratio Test states that the series converges absolutely if $\rho < 1$ and diverges if $\rho > 1$ or ρ is infinite. The test is inconclusive if $\rho = 1$.

Using the terms of the series and substituting into the limit for the Ratio Test, gives the following limit. Simplify the expression to find the limit. Invert and multiply.

$$\lim_{n\to\infty}\left|\frac{\dfrac{(-13)^{n+1}}{(n+1)^3 9^{n+1}}}{\dfrac{(-13)^n}{n^3 9^n}}\right| = \lim_{n\to\infty}\left|\frac{(-13)^{n+1}}{(n+1)^3 9^{n+1}} \cdot \frac{n^3 9^n}{(-13)^n}\right|$$

Use the law of exponents $\frac{a^m}{a^n} = a^{m-n}$ to simplify.

$$\lim_{n\to\infty}\left|\frac{(-13)^{n+1}}{(n+1)^3 9^{n+1}} \cdot \frac{n^3 9^n}{(-13)^n}\right| = \lim_{n\to\infty}\left|-\frac{13}{9}\left(\frac{n}{n+1}\right)^3\right|$$

Next divide the numerator and denominator of the fraction by n and find the limit.

$$\lim_{n\to\infty}\left|-\frac{13}{9}\left(\frac{n}{n+1}\right)^3\right| = \lim_{n\to\infty}\left|-\frac{13}{9}\left(\frac{1}{1+\dfrac{1}{n}}\right)^3\right|$$

$$= \frac{13}{9}$$

Thus, $\rho = \frac{13}{9}$. This indicates that the series diverges.

Since the limit is greater than 1, the series $\sum_{n=1}^{\infty} \frac{(-13)^n}{n^3 9^n}$ diverges by the Ratio Test.