MATH 100, Fall, 2021
Tutorial #6
Linear Approximation and Optimization Part 1

Q1. You know the formula for the volume V and surface area S of a sphere of radius $r > 0$. Develop in your group a formula for surface area in terms of volume: $S = S(V)$.

> Assignment:
> Use linear approximation to estimate the change in surface area ΔS (m^2) when a sphere of radius $r = 8$ (m) has its volume increased by $\Delta V = 0.3$ (m^3).

Q2. For each of the following scenarios, find an example (i.e. a **function**, its **domain** and its **graph**) that satisfies the given conditions. First discuss in your group what kind of graphs might work, then try to come up with simple functions and domains that have those graphs.

a) A continuous function on a domain D that has an absolute maximum, but no absolute minimum.

b) A continuous function on a domain D that has no local maximum or local minimum.

Q3. For each of the following scenarios, find an example (i..e. a **function**, its **domain** and its **graph**) that satisfies the given conditions. First discuss in your group what kind of graphs might work, then try to come up with simple functions and domains that have those graphs.

a) A continuous function on a domain D that has a local minimum, but no absolute minimum.

b) A discontinuous function on a domain $[a, b]$ that has both absolute maximum and absolute minimum.

Q4. During a 2 hour car trip, at some point your speedometer will read exactly the same value as your average speed over the duration of the trip. Discuss this fact and interpret in terms of the mean value theorem in calculus.

> Assignment:
> Suppose you take a trip, with distance travelled up to time t (hours) given by $d(t) = t^3 + 2t^2$ km. You travel from $t = 0$ to $t = 2$ hours. What is your average speed over the duration of this trip? Find a time $t \in (0, 2)$ when your instantaneous speed equals this average speed.

Q5. The SQUASHEM waste management company is designing a new compactor machine that takes a large cube of waste and compresses it into a small cube. Years of experience has shown that the volume of a cube of waste can be compressed at a rate of at most 10% per second or it will overheat and (worse case scenario) explode. At what percentage rate should the new design squash the side dimensions of the cube to keep within this safety margin?

Assignment:
Using variables V for volume and x for side length of the cube (at time t in seconds) derive an limit for the percentage rate decrease per second of the edge length ($\frac{\frac{dx}{dt}}{x} \times 100$), so that the percentage rate per second of volume decrease ($\frac{\frac{dV}{dt}}{V} \times 100$) remains safe.

Hint: The problem can be solved as a related rates problem, but there is also a nice solution using logarithmic differentiation.