Vectors

Encode ideas:

- Direction (Which way)
- Magnitude (How much)

$$\vec{A} \stackrel{\text{is made of}}{=} A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

"unit vector in the x-direction"

x-component of $\vec{A}$

"The vector named A"

$$= \sum (\text{components})(\text{unit vectors})$$

"how much" "which way"

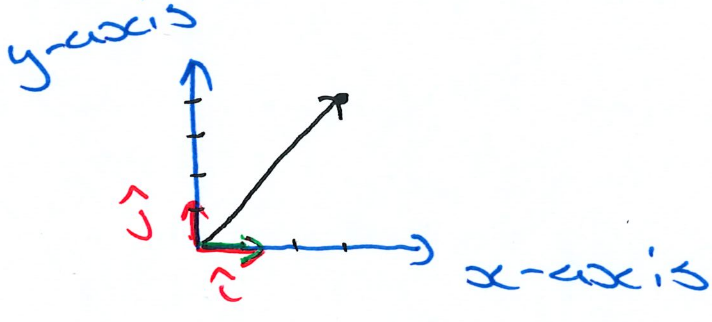
May have seen:

$$\vec{A} = (A_x, A_y, A_z)$$

$$\vec{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Not using b/c they are cumbersome

Consider a vector with $A_z = 0$



$\vec{A} = 3\hat{i} + 4\hat{j} + 0\hat{k}$ (3,4,0)

If you know the components

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

magnitude of $\vec{A}$

$$|\vec{A}| = \sqrt{3^2 + 4^2 + 0^2} = 5$$

To measure direction, use angles

angle between x-axis ($\hat{i}$) & $\vec{A}$

General rule: $A_x = |\vec{A}|\cos\theta$

$A_y = |\vec{A}| \cos \Theta^*$ ← angle between $\vec{A}$ & y-axis

To find Θ: $\cos\Theta = \frac{A_x}{|\vec{A}|} = \frac{3}{5} \Rightarrow \Theta = 53^\circ$

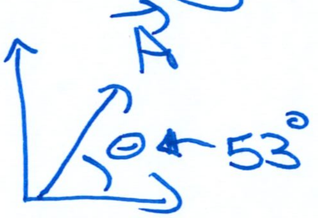
$\vec{A}$ & $\hat{i}$

What is Θ^* with y-axis

$$\cos\Theta^* = \frac{A_y}{|\vec{A}|} = \frac{4}{5} \Rightarrow \Theta^* = 37^\circ$$

Add to 90°

You may have seen:



$A_x = |\vec{A}| \cos\Theta$

$A_y = |\vec{A}| \sin\Theta$

Useful for vectors in 2D

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$A_n = |\vec{A}|\cos\theta$$

x, y, z

angle between $\vec{A}$ and $\hat{i}, \hat{j}, \hat{k}$

1-3-Example
-Vectors I

Vectors - I

The vector $\vec{A} = -3m\hat{\imath} + 4m\hat{\jmath}$. $+ 0m\hat{k}$

- What is the length of $\vec{A}$? ie what is $\left|\vec{A}\right|$?
- What is the angle between $\vec{A}$ and the positive x-axis ($\hat{\imath}$)? (range 0°-180°)
- What is the angle between $\vec{A}$ and the positive y-axis ($\hat{\jmath}$)?

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$A_x = -3m$

$A_y = 4m$

$A_z = 0m$

$$|\vec{A}| = \sqrt{(-3m)^2 + (4m)^2 + (0m)^2}$$

$$= \sqrt{25m^2}$$

$$= 5m$$

$$A_n = |\vec{A}|\cos\Theta$$

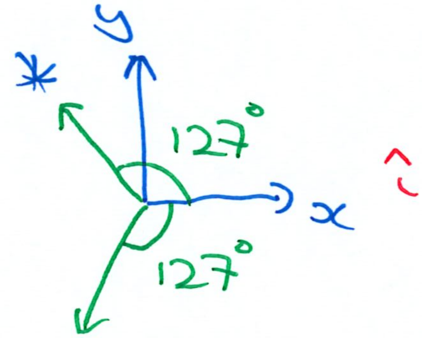
angle between $\vec{A}$
and unit vector
in the n^{th} direction

$$A_x = |\vec{A}|\cos\Theta$$

$$-3m = 5m\cos\Theta$$

$$\cos\Theta = -0.6$$

$$\Theta = 127°$$

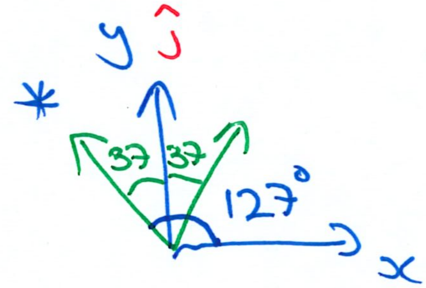


$$A_y = |\vec{A}|\cos\Theta$$ ← angle from $\vec{A}$ to $\hat{j}$

$$4m = 5m\cos\Theta$$

$$\cos\Theta = 0.8$$

$$\Theta = 37°$$



Vectors - II

The vector $\vec{A}$ makes an angle of 30° with the positive x-axis, and an angle of 120° with the positive y-axis. The length of $\vec{A}$ is $5m$.

- What is the x-component of $\vec{A}$? ie what is A_x?
- What is the y-component of $\vec{A}$? ie what is A_y?

$A_n = |\vec{A}|\cos\theta$

want ?

angle between $\vec{A}$ and unit vector in direction ($\hat{i}, \hat{j}, \hat{k}$ for x, y, z)

$|\vec{A}| = 5m$

$\vec{A}$ & $\hat{i}$

$$A_x = |\vec{A}|\cos\theta = 5m\cos 30 = 4.33m$$

$\vec{A}$ & $\hat{j}$

$$A_y = |\vec{A}|\cos\theta = 5m\cos 120 = -2.5m$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$= 4.33_m\,\hat{i} + (-2.5_m)\hat{j} + A_z\hat{k}$$

$\hat{j}$

What is A_z?

0_m!

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$5m = \sqrt{(4.33m)^2 + (-2.5m)^2 + A_z^2}$$

$$25m^2 = (4.33m)^2 + (-2.5m)^2 + A_z^2$$

$$0m^2 = A_z^2$$