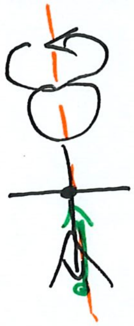
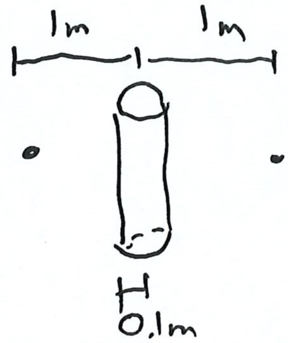
8-9-Example-AM4

Angular Momentum - IV

A skater is rotating with their arms out at a rate of two times per second. They then draw in their arms. What is their final rotation rate?

Model the skater as a vertical cylinder of radius $0.1m$ with a moment of inertia of $0.8kgm^2$, and that their hands are $0.5kg$ at the end of $1m$ long massless arms.





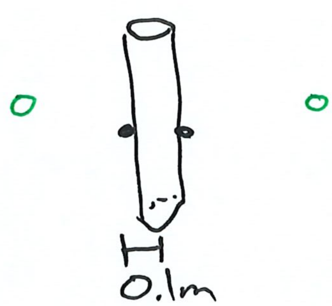
$$I_{total, before} = I_{cyl} + 2I_{hand}$$

$$= 0.8kgm^2 + 2(0.5kg)(1m)^2$$

$$= 1.8 kgm^2$$

$$\vec{L} = \left(\frac{d\theta}{dt}\right)_i (1.8kgm^2)\hat{k}$$

As arms drawn in I changed



$$I_{aster} = I_{cyl} + 2I_{hands}$$

$$= 0.8\,kgm^2 + 2(0.5\,kg)(0.1\,m)^2$$

$$= 0.81\,kgm^2$$

$$\vec{L} = \left(\frac{d\theta}{dt}\right)_f (0.81\,kgm^2)\hat{k}$$

$$\left(\frac{d\theta}{dt}\right)_i (1.8\,kgm^2)\hat{k} = \left(\frac{d\theta}{dt}\right)_f (0.81\,kgm^2)\hat{k}$$

2 rev/s

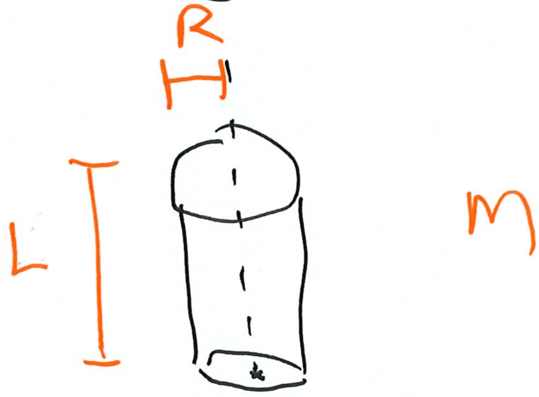
$4\pi\ ^1/_s$

$$(4\pi\ ^1/_s)(1.8\,kgm^2) = \left(\frac{d\theta}{dt}\right)_f (0.81\,kgm^2)$$

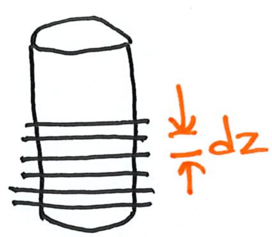
$$27.9\,s^{-1} = \left(\frac{d\theta}{dt}\right)_f$$

$$4.44\ ^{rev}/_s = \left(\frac{d\theta}{dt}\right)_f$$

The way to find I for a uniform cylinder around its center

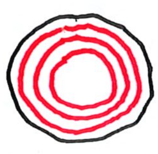


- Break up into lots of bits
- Find m for each bit
- Find $|\vec{r}|^2$ " " "
- Add together.



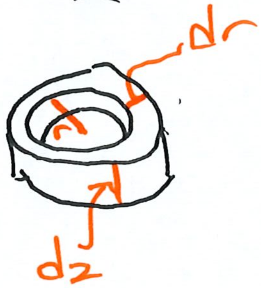
break cylinder into disks
Each of length dz

⇒ Whole bunch of disks, each dz thick



break disk into rings
Each ring has radius r
& thickness dr

For the ring, radius r, thickness dr
height dz



Volume $(2\pi r)\,dr\,dz$

The mass of ring will be (density)(Volume)

$$\text{density} = \frac{\text{total mass}}{\text{total volume}}$$

$$\frac{M}{\pi R^2 L}$$

Mass of ring $= dm = \dfrac{M}{\pi R^2 L}\, 2\pi r\,dr\,dz$

I for ring $\quad dI = dm\,|\vec{r}|^2$

$$= dm\, r^2$$

$$= \frac{2M}{R^2 L}\, r\,dr\,dz\, r^2$$

$$= \frac{2M}{R^2 L}\, r^3\,dr\,dz$$

$$I = \int_{r=0}^{R}\int_{z=0}^{L} \frac{2m}{R^2L} r^3\,dr\,dz$$

$$= \int_{r=0}^{R} \frac{2m}{R^2L} r^3\,dr \; z\Big|_0^L$$

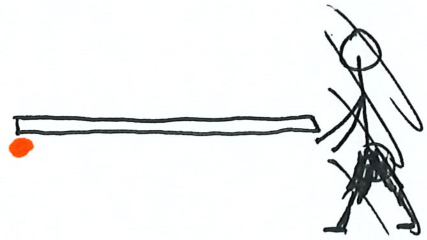
$$= \int_{r=0}^{R} \frac{2m}{R^2\cancel{L}} r^3\,dr\,\cancel{L}$$

$$= \frac{2m}{R^2}\,\frac{1}{4} r^4\Big|_0^R$$

$$= \frac{2m}{R^2}\,\frac{1}{4} R^4$$

$$= \frac{1}{2} mR^2$$

Example



Bar, mass M, length L held horizontally at rest

Scale at one end, person holds other end.

Person lets go.

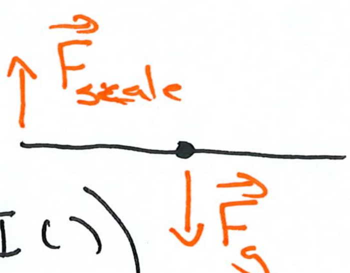
What is reading on the scale?

Useful fact: $I = \frac{1}{12} M L^2$ around center of mass.

Know $\vec{\tau}_{net} = \frac{d}{dt} \vec{L}$

$$\vec{F}_{net} = m\vec{a} = \frac{d}{dt}\vec{p}$$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_{scale}$$



$$\vec{\tau}_{net} = \frac{d}{dt}\left(\frac{d\theta}{dt} I ( ) \right)$$

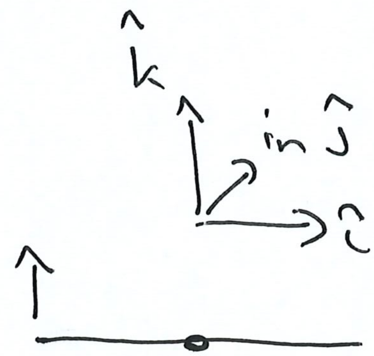
If CM of bar is origin

$$\vec{\tau}_{net} = \vec{\tau}_g + \vec{\tau}_{scale}$$

$$= 0 \times (-mg\hat{k}) + \left(-\frac{L}{2}\hat{\imath}\right) \times \left(|\vec{F}_{scale}|\hat{k}\right)$$

$$= \frac{L}{2}|\vec{F}_{scale}|\,\hat{\jmath}$$

$$= \frac{d^2\theta}{dt^2} I \,\hat{\jmath}$$

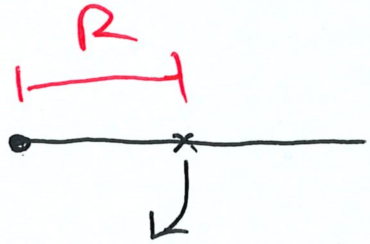


$$\boxed{\frac{d^2\theta}{dt^2} I = \frac{L}{2}|\vec{F}_{scale}|}$$

$$-Mg\hat{k} + |\vec{F}_{scale}|\hat{k} = M\frac{d^2}{dt^2}z(t)\,\hat{k}$$

$$\boxed{-Mg + |\vec{F}_{scale}| = M\frac{d^2}{dt^2}z(t)}$$

Need eqn relating $\frac{d^2\theta}{dt^2}$ & $\frac{d^2z}{dt^2}$



like center travels in a circle

For moving in a circle

$$\vec{a} = \frac{|\vec{v}|^2}{R}(\text{to center}) + \overset{L/2}{\cancel{R}}\,\frac{d^2\theta}{dt^2}(\text{in } \vec{v})$$

$$\frac{d^2 z}{dt^2}\hat{k} = 0 + \overset{L/2}{\cancel{R}}\,\frac{d^2\theta}{dt^2}(-\hat{k})$$

$$\boxed{\frac{d^2 z}{dt^2} = -\overset{L/2}{\cancel{R}}\,\frac{d^2\theta}{dt^2}}$$

$$-mg + \frac{2I}{L}\frac{d^2\theta}{dt^2} = m\frac{d^2}{dt^2}z$$

$$-mg + \frac{2I}{L}\frac{d^2\theta}{dt^2} = m\left(-\frac{L}{2}\frac{d^2\theta}{dt^2}\right)$$

$$\frac{2I}{L}\frac{d^2\theta}{dt^2} + \frac{mL}{2}\left(\frac{d^2\theta}{dt^2}\right) = mg$$

$$\frac{d^2\theta}{dt^2} = \frac{Mg}{\frac{2I}{L} + \frac{ML}{2}}$$

$$I \frac{d^2\theta}{dt^2} = \frac{L}{2} |\vec{F}_{scale}|$$

$$\frac{I2}{L}\left(\frac{Mg}{\frac{2I}{L} + \frac{ML}{2}}\right) = |\vec{F}_{scale}|$$

$$\frac{IMg}{I + ML^2/4} = |\vec{F}_{scale}|$$

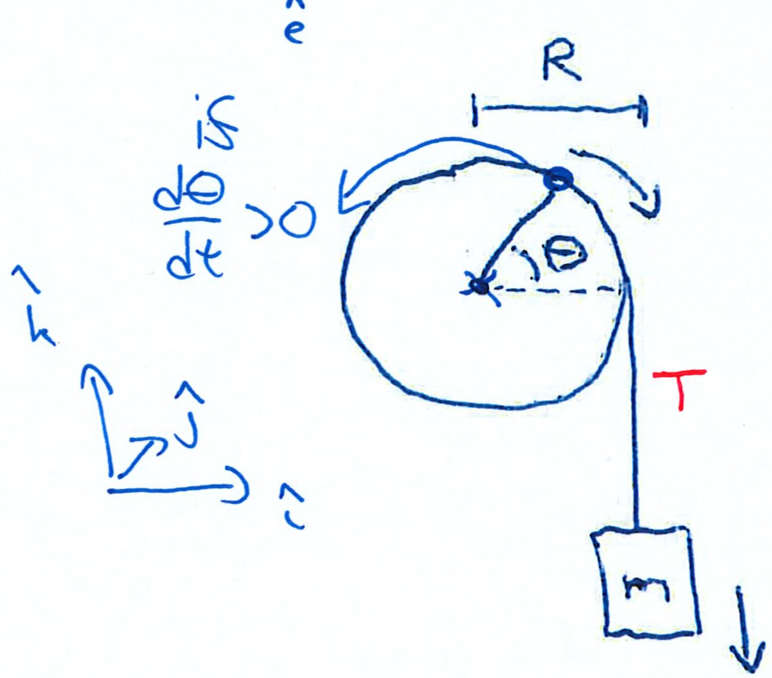
$$\frac{\frac{1}{12}ML^2 Mg}{\frac{1}{12}ML^2 + \frac{ML^2}{4}} = |\vec{F}_{scale}|$$

$$\frac{1}{4}Mg = |\vec{F}_{scale}|$$

8-11-Example-AM5

Angular Momentum - V

A disk of radius $R = 0.5m$ and moment of inertia $I = 10kgm^2$ has a light rope wrappd around it. That rope is attached to a mass $m = 2kg$.



The rope does not slip, and the disk is free to rotate without friction.

disk rotates/not move

- What is the acceleration of the mass m?
- What is the *angular acceleration* of the disk?
- What is the tension in the rope?

Hanging mass:

$$\frac{\vec{F}_{net}}{m} = \vec{a} = \frac{d^2}{dt^2}\left(R\hat{i} + z\hat{k}\right) = \left(\frac{d^2}{dt^2}z\right)\hat{k}$$

$$\frac{-mg\hat{k} + T\hat{k}}{m} = -g\hat{k} + \frac{T}{m}\hat{k} = \left(\frac{d^2}{dt^2}z\right)\hat{k}$$

$$\boxed{-g + \frac{T}{m} = \frac{d^2}{dt^2}z} \quad \#1$$

Disk

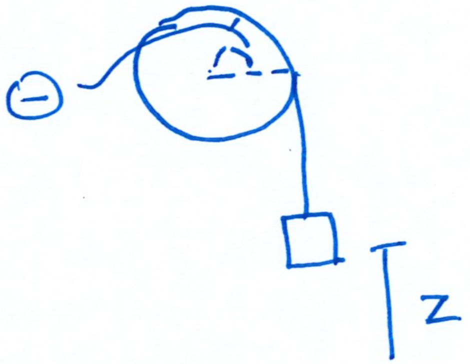
$$\vec{\tau}_{net} = \frac{d}{dt}\vec{L}$$

$$(R\hat{i}) \times (-T\hat{k}) = \frac{d}{dt}\left[\frac{d\theta}{dt} I \left(-\hat{j}\right)\right]$$

this gives right dir for $\vec{L}$ when $d\theta/dt > 0$

$$RT\hat{j} = -\frac{d^2\theta}{dt^2} I \hat{j}$$

$$\boxed{RT = -\frac{d^2\theta}{dt^2} I} \quad \#2$$



$$\frac{d\theta}{dt} > 0 \qquad \frac{dz}{dt} > 0$$

$$R\frac{d\theta}{dt} = \frac{dz}{dt}$$

$$\boxed{R\frac{d^2\theta}{dt^2} = \frac{d^2z}{dt^2}}$$

#3

$$-g + \frac{T}{m} = \frac{d^2}{dt^2}z$$

$$RT = -\frac{d^2\theta}{dt^2} I$$

$$R\frac{d^2\theta}{dt^2} = \frac{d^2z}{dt^2}$$

$$-g + \frac{T}{m} = R\frac{d^2\theta}{dt^2}$$

$$RT = -\frac{d^2\theta}{dt^2} I$$

$$-g + \frac{1}{m}\left(-\frac{d^2\theta}{dt^2}\frac{I}{R}\right) = R\frac{d^2\theta}{dt^2}$$

$$-g = \left(\frac{I}{mR} + R\right)\frac{d^2\theta}{dt^2}$$

$R = 0.5\,\mathrm{m}$
$I = 10\,\mathrm{kg\,m^2}$
$m = 2\,\mathrm{kg}$

$$\frac{-g}{\frac{I}{mR} + R} = \frac{d^2\theta}{dt^2}$$

$$\frac{-gm}{\frac{I}{R} + Rm} = \frac{d^2\theta}{dt^2} = -\frac{9.8\,\mathrm{N/kg}\;2\,\mathrm{kg}}{\frac{10\,\mathrm{kg\,m^2}}{0.5\,\mathrm{m}} + 2\,\mathrm{kg}\;0.5\,\mathrm{m}} = -0.93\,\mathrm{m/s^2}$$

$$\frac{d^2 z}{dt^2} = R\frac{d^2\theta}{dt^2} = R\left(\frac{-gm}{\frac{I}{R} + Rm}\right)$$

$$= \frac{-gm}{\left(\frac{I}{R^2} + m\right)} = \frac{-9.8\,\mathrm{N/kg}\;2\,\mathrm{kg}}{\left(\frac{10\,\mathrm{kg\,m^2}}{(0.5\,\mathrm{m})^2} + 2\,\mathrm{kg}\right)} = -0.47\,\mathrm{m/s^2}$$

$$T = -\frac{I}{R}\frac{d^2\theta}{dt^2}$$

$$= -\frac{I}{R}\left(\frac{-gm}{\frac{I}{R} + mR}\right)$$

$$= \frac{gIm}{I + mR^2} = \frac{9.8\,\mathrm{N/kg}\;10\,\mathrm{kg\,m^2}\;2\,\mathrm{kg}}{10\,\mathrm{kg\,m^2} + 2\,\mathrm{kg}\,(0.5\,\mathrm{m})^2}$$

$$= 18.7\,\mathrm{N}$$