sin csc
cos sec
tan cot

Math 101, Fall 2012
Assignment 1 — Practice problems

These questions are for your practice only — they are not to be handed in. However, one of these questions, or a slight variant, will appear on the first midterm. Also, they provide necessary practice in using the Sharp EL-510R calculator.

Find the derivative of each of the following functions at the given point.

1. $f(t) = \dfrac{t^2 + t^3 - 1}{t^4}$ at point $t = 0.2$

   Answer: $f'(0.2) = 12,225$

   $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'g - fg'}{g^2}$

2. $y = \dfrac{1}{3\sqrt{x}} + \dfrac{1}{4}$ at point $x = 2$.

   $[f(x)g(x)]' = f'g + fg'$

   Answer: $y'(2) = -0.05892556$

3. $y = \pi^{2x}$ at point $x = 3$.

   $\frac{d}{dx} a^x = a^x \ln(a)$

   Answer: $y'(3) = 2201.06188$

4. $f(z) = \ln(3)z^2 + \ln(4)e^z$ at point $z = -2$.

   Answer: $f'(-2) = -4.206834615$

5. $f(\theta) = 4^{\sqrt{\theta}}$ at point $\theta = 16$.

   Answer: $f'(16) = 44.36141956$

6. $z = \tan\left(\dfrac{1}{\sqrt{t}}\right)$ at point $t = 4$.

   $\tan'(x) = \sec^2(x)$

   Answer: $z'(4) = -0.08115290$

7. $f(\theta) = \ln(\cos(\theta))$ at point $\theta = \pi/4$.

   $\frac{d}{dx} \ln(f(x)) = \frac{f'(x)}{f(x)}$

   Answer: $f'(\pi/4) = -1$

8. $f(x) = \ln(\ln(1 + e^{3x}))$ at point $x = 1/3$.

   Answer: $f'(1/3) = 1.6700218$

9. $g(z) = \dfrac{1}{\ln(z)}$ at point $z = 2$.

   Answer: $g'(2) = -1.040684$

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$0.04 + 0.008$

① $f(t) = t^2 + t^3 - 1 / t^4$ $\quad f'(t) = \dfrac{(2t + 3t^2)(t^4) - (t^2 + t^3 - 1)(4t^3)}{t^8}$

$= \dfrac{2t^5 + 3t^6 - 4t^5 - 4t^6 + 4t^3}{t^8} \rightarrow \dfrac{t^3(2t^2 + 3t^3 - 4t^2 - 4t^3 + 4)}{t^8}$

$\dfrac{-2t^2 - t^3 + 4}{t^5} \rightarrow (-0.08) - (0.008) + 4 / 0.00032$

$= 3.912 / 0.00032 = 12225$

② $y = \frac{1}{3} x^{-1/2} + \frac{1}{4}$ $\quad y' = -\frac{1}{6} x^{-3/2}$ $\quad y'(2) = -0.058925565$

③ $y = \pi^{2x}$ $\quad \therefore \; y' = \pi^{2x} \ln(\pi) \cdot 2$

$y'(3) = 2\pi^6 \cdot \ln(\pi) = 1.1447 \cdot 1922.778 = 2201.06$

④ $f(z) = \underset{f}{\ln(3)}\, \underset{g}{z^2} + \underset{f}{\ln(4)}\, \underset{g}{e^z}$

$f'(z) = \ln(3)\, 2z + \ln(4)\, e^z$

$= z \ln(9) + e^z \ln(4)$

$= (-4.3944) + (0.1876) = -4.206\ldots$

⑤ $f(\theta) = 4^{\theta^{1/2}}$ $\quad f'(\theta) = 4^{\sqrt{\theta}} \ln(4) \cdot \frac{1}{2} \theta^{-1/2} \rightarrow \dfrac{4^{\sqrt{\theta}} \ln(4)}{2\sqrt{\theta}}$

$f'(16) = 4^4 \cdot \ln(4) \cdot \frac{1}{8} = 44.3614\ldots$

⑥ $z = \tan(t^{-1/2}) \rightarrow z' = \sec^2(t^{-1/2}) \cdot (-\frac{1}{2} t^{-3/2})$

$z'(4) = \dfrac{1}{\cos^2(\frac{1}{2})} \cdot \dfrac{-1}{16} = -0.08115\ldots$

⑦ $f(\theta) = \ln(\cos(\theta)) \rightarrow f'(\theta) = \dfrac{-\sin\theta}{\cos\theta} = -\tan(\theta)$

☆ $-\tan(\pi/4) = -1$

Assignment 1 cont.

(10) $\int 3\cos(\phi) + 3\phi^{1/2} \quad d\phi = 3\sin(\phi) + 2\phi^{3/2}\Big]_{\pi/5}^{\pi/3}$

$= [(2.598)+(2.143)] - [(1.763)+(0.996)] = 1.982\ldots$

(11) $\int \frac{x^2+x+1}{x}\,dx \rightarrow \int x + 1 + \frac{1}{x} = \left[\frac{1}{2}x^2 + x + \ln(x)\right]_{0.1}^{2}$

$= [(2)+(2)+(0.693)] - [(0.005)+(0.1)+(-2.3026)] = 6.89\ldots$

(12) $\int \frac{1}{\cos^2(x)} \rightarrow \int \sec^2(x) = \tan(x)\Big]_{\pi/4}^{3\pi/4}$

$= (-1)-(1) = -2$

(13) $\int e^{\sin(x)} \cos x\,dx = e^{\sin(x)}\Big]_{0}^{\pi/2}$ $\quad \sin(\pi/2)=1 \quad \sin(0)=0$

$= (e^1)-(e^0) = 1.71828\ldots$

(14) $\int (\cos\theta+5)^7 \sin\theta \rightarrow -\frac{1}{8}(\cos\theta+5)^8\Big]_{0}^{\pi}$

$= (-8192)-(-209952) = 201760$

(15) $\int \frac{1}{\sqrt{4-x}}\,dx$ let $u = 4-x \therefore du = -1 \rightarrow -\int u^{-1/2}\,du$

$\rightarrow -\left[2u^{1/2}\right]_{-12}^{0} \rightarrow -\left[2(4-x)^{1/2}\right]_{-12}^{0} \rightarrow [-4]-[-8] = 4$

(16) $\int x e^{-x^2}\,dx \rightarrow$ let $u = -x^2 \therefore du = -2x$ & $x = \frac{-1}{2}du$

$\rightarrow \frac{-1}{2}\int e^u\,du = \frac{-1}{2}\left[e^u\right]_{6}^{0.6} \rightarrow \frac{-1}{2}\left[e^{-x^2}\right]_{0}^{0.6}$

$= \left(\frac{e^{-0.36}}{-2}\right) + \frac{1}{2} = 0.15116\ldots$