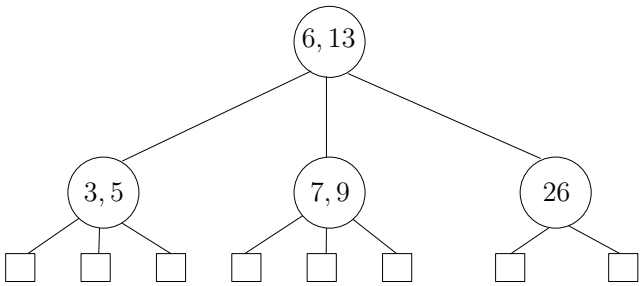
CSC 225: Lab 8

1. Look at the $2-3$ tree in Figure 1. What is the corresponding red-black tree?

Figure 1: A $2-3$ tree of height 2.



2. Find a sequence of insertions that would construct this $2-3$ tree.

3. Consider the insertion sequence 3, 5, 7, 6, 13, 9, 26. Draw the final tree? Is it the $2-3$ tree in Figure 1?

4. Look at the red-black tree in Figure 2. Is it a valid red-black tree? How can you check? If it is not a valid red-black tree reconstruct the tree with the same keys in such a way that it is a valid red-black tree.

Figure 2: A red-black tree of black height 2.

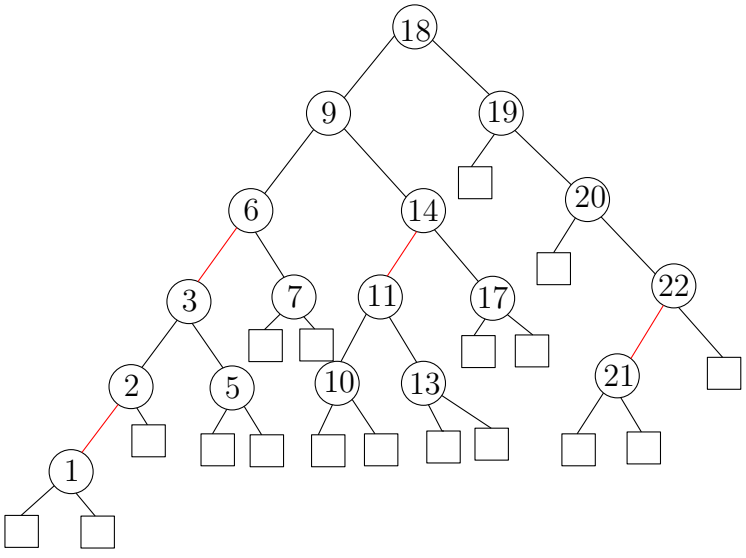
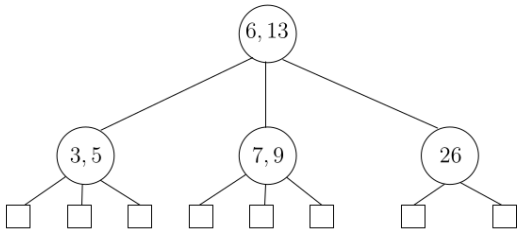
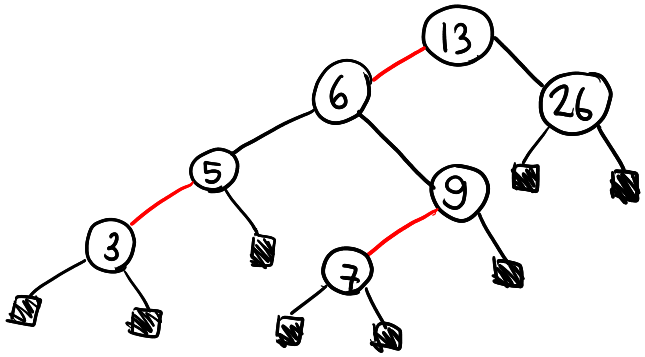


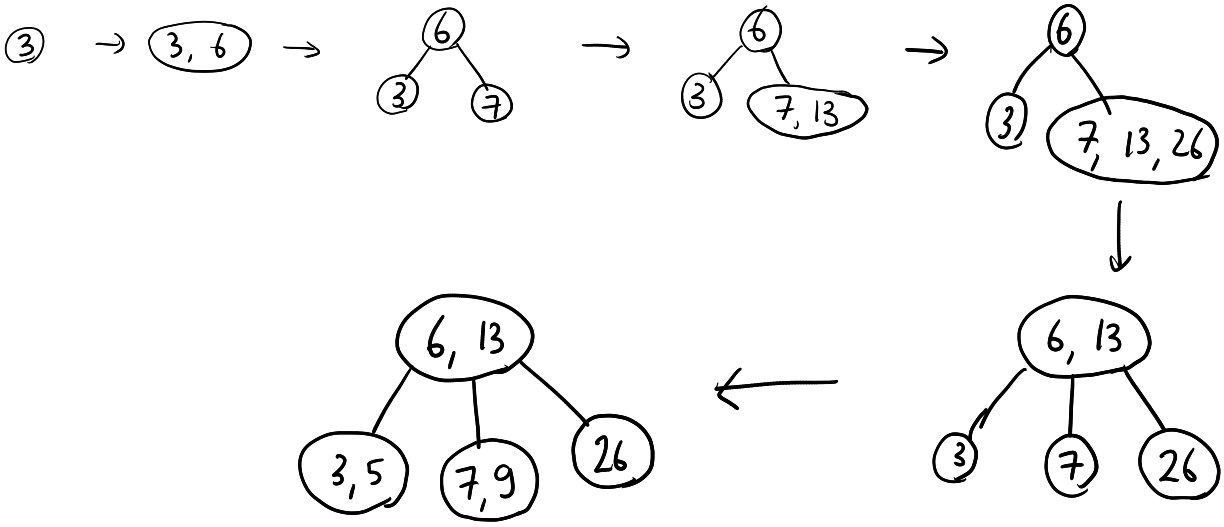
Figure 1: A 2 – 3 tree of height 2.



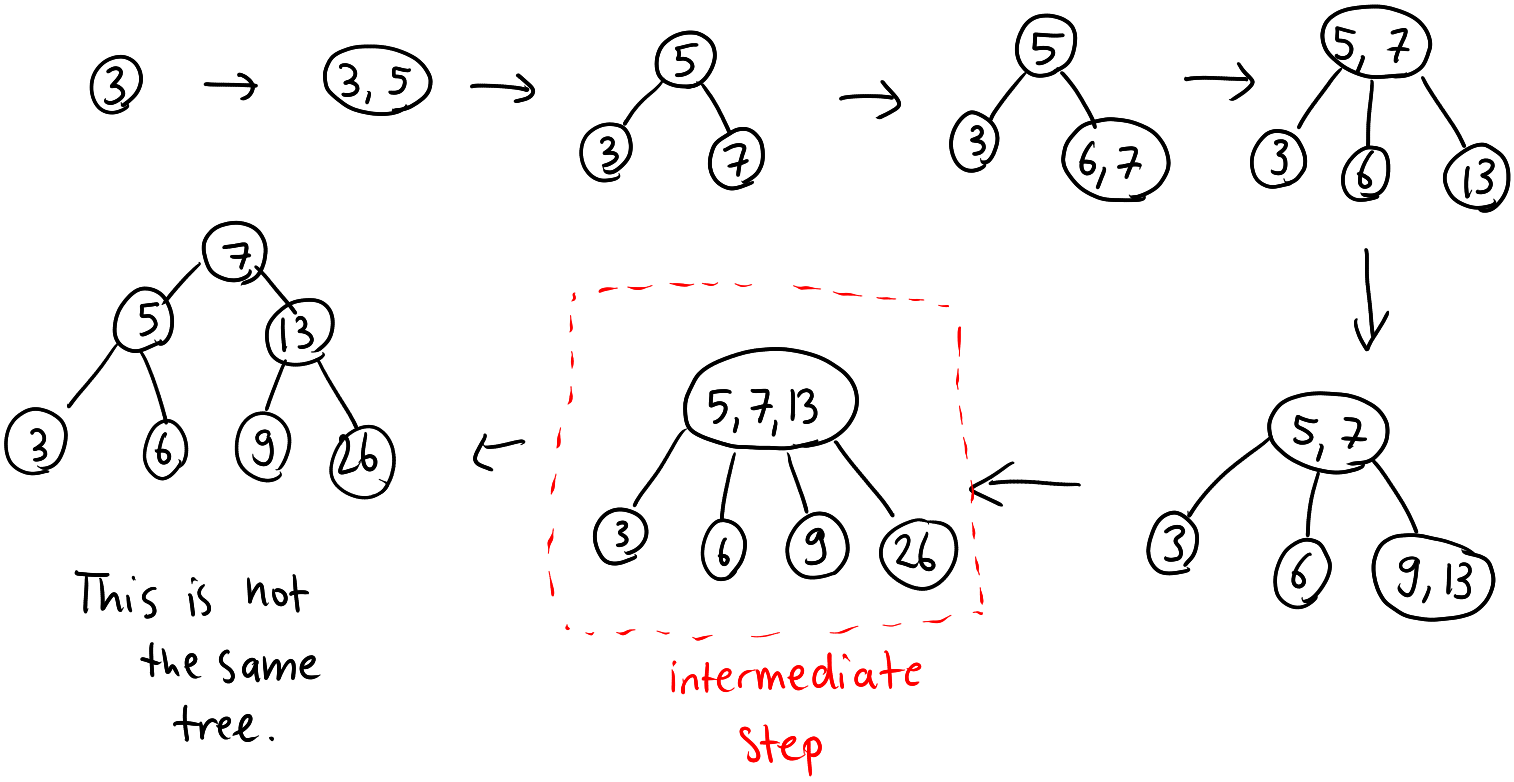
① The corresponding red-black tree separates all the 3-nodes using left-leaning red links.



② 3 6 7 13 26 5 9

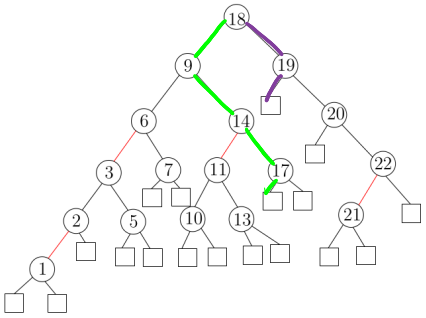


3. Consider the insertion sequence 3, 5, 7, 6, 13, 9, 26. Draw the final tree? Is it the 2 – 3 tree in Figure 1?



4. Look at the red-black tree in Figure 2. Is it a valid red-black tree? How can you check? If it is not a valid red-black tree reconstruct the tree with the same keys in such a way that it is a valid red-black tree.

Figure 2: A red-black tree of black height 2.

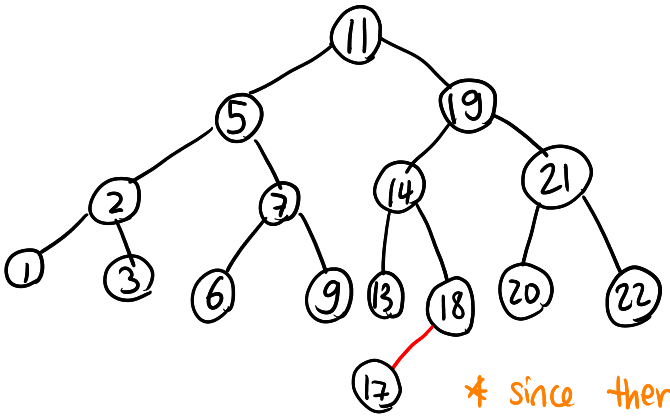


a) This is not a valid red-black tree. The number of black edges to get to any leaf node should be the same.

But the green path has 4 black edges while the purple path only has 2.

b) The keys are 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 17, 18, 19, 20, 21, 22.

There are many ways to restructure the tree, but the easiest way is to simply make a regular Binary Search Tree, since all BSTs are also red-black trees.



* since there is an odd number of elements, there will be one red link to ensure all paths from root to any leaf have the same number of black edges.