1. (1 point) Find an antiderivative to this function: $f(x) = \cos^3(x)$

(A) $\cos^4(x)$ (B) $\sin^3(x) - \sin(x)$ (C) $\frac{\cos^4(x)}{4}$ (D) $\sin^3(x)$

(E) $3\cos^2(x)\sin(x)$ (F) $\sin(x) - \frac{1}{3}\sin^3(x)$ (G) $3\cos^2(x)$ (H) None of those

$= \cos^2 x \cos x = (1-\sin^2 x)(\cos x) = \int \cos x - \int \sin^2 x \cos x$

let $u = \sin x$

$du = \cos x\,dx$

$= \sin x - \int u^2 du = \sin x - \frac{u^3}{3} = \sin x - \frac{\sin^3 x}{3}$

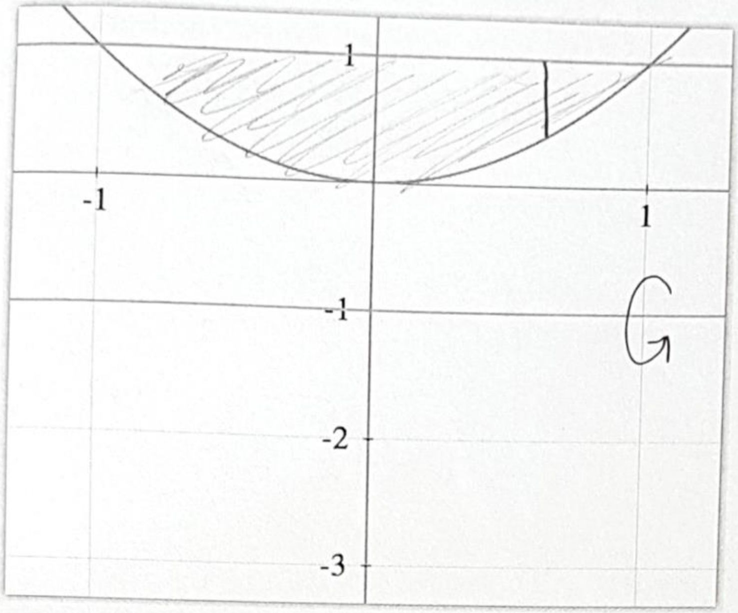
2. (1 point) Find an antiderivative to this function: $g(x) = \sin(3x)\cos(x)$

(A) $\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$ (B) $\frac{1}{4}\cos(2x) + \frac{1}{8}\sin(4x)$ (C) $\frac{1}{2}\cos(2x) - \frac{1}{4}\cos(4x)$

(D) $\frac{1}{2}\cos(2x) + \frac{1}{4}\sin(4x)$ (E) $\frac{1}{4}\sin(2x) - \frac{1}{8}\cos(4x)$ (F) $\frac{1}{4}\sin(2x) + \frac{1}{8}\cos(4x)$

(G) $-\frac{1}{4}\cos(2x) - \frac{1}{8}\cos(4x)$ (H) $\frac{1}{4}\cos(2x) + \frac{1}{8}\cos(4x)$ (I) None of those

Last minute guess



The next three questions (#3 – #5) are related to the following solid of the revolution. A solid is obtained by rotating the region bounded by the curves

$y = x^2$ and $y = 1$ about the line $y = -1$.

Set-up the formula for calculation of the volume of this solid using method of "washers", in the given x/y orientation (see graph on this page)

3. (1 point) What is the formula for the outer radius R_{out} used for calculation of the volume using "washers"?

(A) $1 - x^2$ (B) $x - 1$ (C) $\cancel{1 - 1 = 0}$

(D) $x^2 + 1$ (E) $x + 1$ (F) $1 - (-1) = 2$

(G) $x^2 - 1$ (H) $1 - x$ (I) None of those

$r_{out} =$

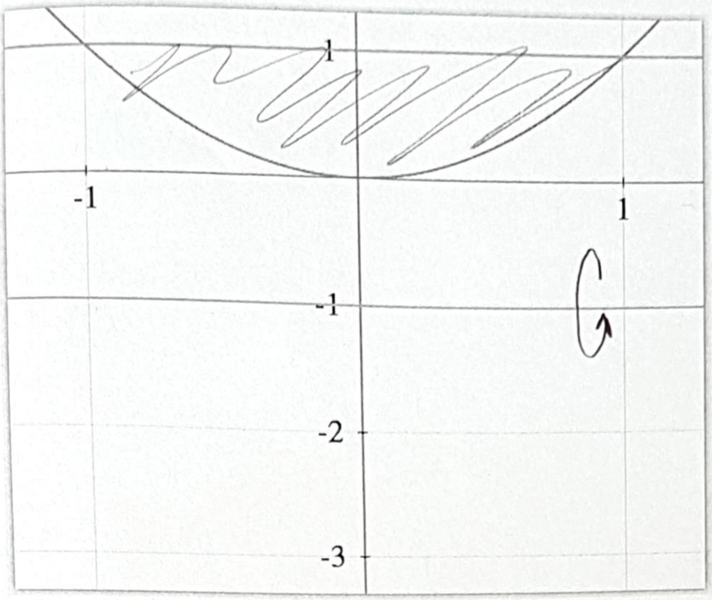
4. (1 point) What is the formula for the inner radius R_{in} used for calculation of the volume using "washers"?

(A) $1 - x^2$ (B) $x - 1$ (C) $1 - 1 = 0$

(D) $x^2 + 1$ (E) $x + 1$ (F) $1 - (-1) = 2$

(G) $x^2 - 1$ (H) $1 - x$ (I) None of those

$1+x^2$



A solid is obtained by rotating the region bounded by the curves
$y = x^2$ and $y = 1$ about the line $y = -1$.

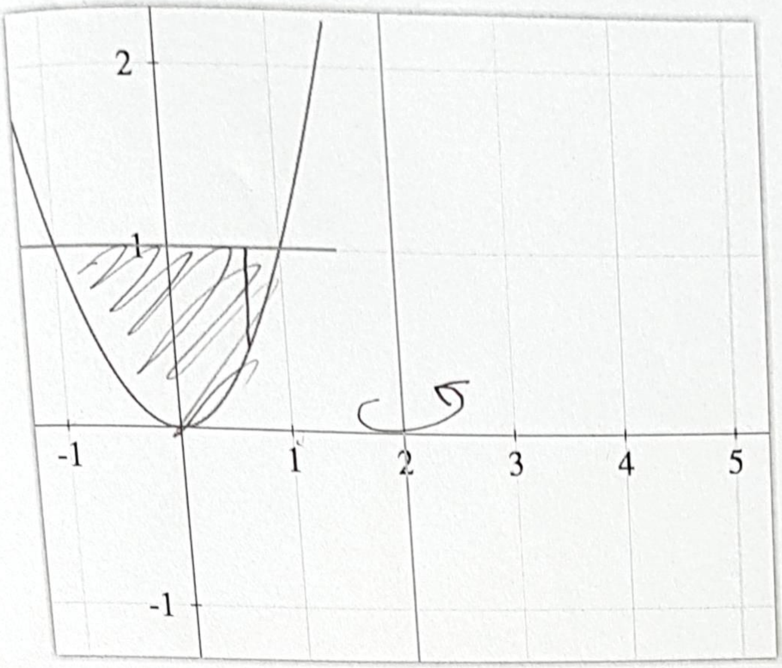
5. (1 point) What is the formula for the volume of this solid using method of "washers"?

(A) $\int_{-1}^{1} 2\pi R_{out} R_{in}\, dx$ (B) $\int_{-1}^{1} \pi(R_{out}^2 - R_{in}^2)\, dx$ (C) $\int_{-1}^{1} \pi(R_{out} - R_{in})^2\, dx$

(D) $\int_{-1}^{0} 2\pi R_{out} R_{in}\, dx$ (E) $\int_{-1}^{0} \pi(R_{out}^2 - R_{in}^2)\, dx$ (F) $\int_{-1}^{0} \pi(R_{out} - R_{in})^2\, dx$

(G) $\int_{0}^{1} 2\pi R_{out} R_{in}\, dx$ (H) $\int_{0}^{1} \pi(R_{out}^2 - R_{in}^2)\, dx$ (I) None of those

$$\int_{-1}^{1} \pi \left(R_2^2 - R_1^2\right)$$



A **new solid** is obtained by rotating region bounded by the curves
$y = x^2$ and $y = 1$ about the line $x = 2$.
Set-up the formula for calculation of the volume of this solid using "cylindrical shells", in the given x/y orientation (see graph on this page)

6. (1 point) What is the formula for the radius R of the cylindrical shells?

(A) $1 - x^2$ (B) $2 - x$ (C) $2 - x^2$

(D) $x^2 + 1$ (E) $2 + x$ (F) 2

(G) $x^2 - 1$ (H) $x - 2$ (I) None of those

$r = 2 - x$ ✓ $\int dx$

7. (2 points) Calculate $\int \sqrt{9-x^2}\,dx$.

(A) $\frac{2}{3}(9-x^2)^{3/2}+C$ (B) $\frac{9}{2}\arcsin\left(\frac{x}{3}\right)+\frac{\sqrt{9-x^2}}{2x}+C$ (C) $\frac{1}{2\sqrt{9-x^2}}+C$

(D) $\frac{-x}{\sqrt{9-x^2}}+C$ (E) $\frac{9}{2}\arcsin\left(\frac{x}{3}\right)+\frac{x}{2}\sqrt{9-x^2}+C$ (F) None of those

let $x = 3\cos\theta$

$dx = -3\sin\theta\,d\theta$

~~let $x = 3\sec\theta$~~

~~$dx = 3\sec\theta\tan\theta\,d\theta$~~

$$\int \sqrt{9-9\cos^2\theta}\,(-3\sin\theta)\,d\theta$$

~~$\int \sqrt{9-9\sec^2\theta}\,(3\sec\theta\tan\theta)\,d\theta$~~

~~$\int \sqrt{9(1-\sec^2\theta)}\,(3\sec\theta\tan\theta)\,d\theta$~~

~~$\int 3\sqrt{\tan}$~~

$$= \int 3\sqrt{1-\cos^2\theta}\,(-3\sin\theta)\,d\theta$$

$$= \int -3\sqrt{\sin^2\theta}\,(3\sin\theta)$$

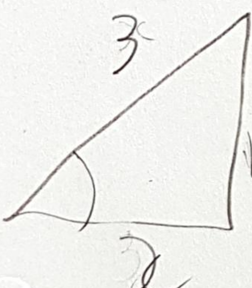
$$= \int -9\sin^2\theta\,d\theta$$

$$= \int -9\sin^2\theta\,d\theta$$

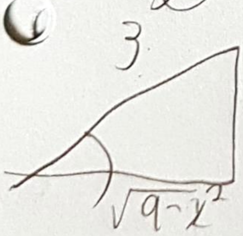
$\sec = \frac{1}{\cos} = \frac{hyp}{adj}$

$$= -9\int \frac{1-\cos 2\theta}{2}\,d\theta$$

$= -\frac{9}{2}$



$$\sqrt{9-x^2} = -9\int \frac{1}{2}\,d\theta + 9\int \frac{\cos 2\theta}{2}\,d\theta$$



$$= \frac{-9}{2}\theta + \frac{9\sin(2\theta)}{4} = \frac{-9}{2}\arccos\left(\frac{x}{3}\right) + \frac{9}{4}\,\frac{2\sqrt{9-x^2}}{3}$$

$= \frac{3}{2}$

8. (2 points) Assuming that the proper rational function $\dfrac{f(x)}{g(x)}$ has $(x^2+16)^2$ in its denominator, give all the associated terms in the partial fractions expansion of $\dfrac{f(x)}{g(x)}$.

(A) $\dfrac{Ax+B}{x^2+16}+\dfrac{(Cx+D)^2}{(x^2+16)^2}$ (B) $\dfrac{Ax+B}{(x^2+16)^2}$ (C) $\dfrac{Ax+B}{x^2+16}+\dfrac{Cx+D}{(x^2+16)^2}$

(D) $\dfrac{A}{x^2+16}+\dfrac{Bx+C}{(x^2+16)^2}$ (E) $\dfrac{A}{(x^2+16)^2}$ (F) $\dfrac{A}{x^2+16}+\dfrac{B}{(x^2+16)^2}$

(G) $\dfrac{Ax+B}{x^2+16}+\dfrac{Cx^3+Dx^2+Ex+F}{(x^2+16)^2}$ (H) $\dfrac{Ax^2+Bx+C}{(x^2+16)^2}$ (I) None of those

$$(x^2+16)^2 = \frac{Ax+B}{(x^2+16)} + \frac{Cx+D}{(x^2+16)^2}$$

9. (2 points) Compute $\int_1^e \frac{dx}{x\,(\ln(x))^{2/3}}$, if the integral converges.

(A) e^{-1} (B) $e^{-1}-1$ (C) $e^{-1}+1$ (D) The integral diverges

(E) 1 (F) 2 (G) 3 (H) None of those

$$\int \frac{dx}{x(\ln x)^{2/3}}$$

let $u = \ln x$

$du = \frac{1}{x}dx$

$x\,du = dx$

$$\int \frac{1}{u^{2/3}}$$

$$\int u^{-2/3} = \left(\frac{3}{1}\right)u^{1/3} = 3(\ln x)^{1/3}\Big|_1^e$$

$$= 3(\ln e)^{1/3} - 3(\ln 1)^{1/3}$$

$$= 3 - 0$$

$$\boxed{= 3}$$

a

10. (2 points) Determine whether or not $\int_{-\infty}^{0} \frac{1+e^{w}}{e^{-w}}\, dw$ converges, giving appropriate (and correct) justification.

(A) Converges, by Limit Comparison to $\int_{-\infty}^{0} e^{w}\, dw$.

(B) Diverges, by Limit Comparison to $\int_{-\infty}^{0} e^{w}\, dw$.

(C) ~~Converges, by Limit Comparison to~~ $\cancel{\int_{-\infty}^{0} e^{-w}\, dw}$.

(D) Diverges, by Direct Comparison to $\int_{-\infty}^{0} e^{-w}\, dw$.

(E) Converges, by Direct Comparison to $\int_{-\infty}^{0} e^{2w}\, dw$.

(F) Diverges, by Direct Comparison to $\int_{-\infty}^{0} 1+e^{-w}\, dw$.

(G) Diverges, by Limit Comparison to $\int_{-\infty}^{0} 1\, dw$.

(H) None of those

$$= \left(\frac{1+\frac{1}{LN}}{e^{\infty}}\right) LN \quad = \frac{1+0}{LN} \approx 0$$

$$= \frac{1}{e^{\infty}} + \frac{1}{e^{2\infty}}$$

$$= \frac{\frac{1+e^{-\infty}}{e^{\infty}}}{e^{\infty}}$$

$$= \frac{1+0}{e^{\infty}} \cdot e^{\infty}$$

$$= 1$$

$$\int_{-a}^{0} e^{-w} = -e^{-w}\Big|_{a}^{0}$$

$a \to -\infty$

$$= -e^{0} + e^{-a}$$

$$= -1 + \infty$$

MATHEMATICS 101 (Sections A01-A05)
Formula sheet, Spring 2018
Midterms and Final examinations.

Table of Integrals

1. $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + C, (u < a)$

2. $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$

3. $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C, (u > a)$

4. $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \sinh^{-1}\left(\frac{u}{a}\right) + C, (a > 0)$

5. $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}\left(\frac{u}{a}\right) + C, (u > a > 0)$

6. $\displaystyle\int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| < 1 \\ \frac{1}{a}\coth^{-1}\left(\frac{u}{a}\right) + C, & \text{if } \left|\frac{u}{a}\right| > 1 \end{cases}$

7. $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\text{sech}^{-1}\left(\frac{u}{a}\right) + C, (a > u > 0)$

8. $\displaystyle\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\text{csch}^{-1}\left|\frac{u}{a}\right| + C, (u > 0)$

9. $\displaystyle\int \sec u\, du = \ln|\sec u + \tan u| + C$

10. $\displaystyle\int \csc u\, du = -\ln|\csc u + \cot u| + C$

Trigonmetric and Hyperbolic Identities

1. $\cos^2(\theta) + \sin^2(\theta) = 1$
2. $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
3. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
4. $\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$
5. $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$
6. $\sin(A)\sin(B) = \frac{1}{2}\cos(A-B) - \frac{1}{2}\cos(A+B)$
7. $\cos(A)\cos(B) = \frac{1}{2}\cos(A-B) + \frac{1}{2}\cos(A+B)$
8. $\sin(A)\cos(B) = \frac{1}{2}\sin(A-B) + \frac{1}{2}\sin(A+B)$
9. $\cosh^2(x) - \sinh^2(x) = 1$
10. $\sinh(2x) = 2\sinh(x)\cosh(x)$
11. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
12. $\text{sech}^{-1}(x) = \cosh^{-1}\left(\frac{1}{x}\right)$
13. $\text{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$
14. $\coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$

$1 + \tan^2\theta = \sec^2\theta$