202201 Math 122 [A01] Quiz #2

February 3rd, 2022

Name: Solutions #V00: ____________

This test has 2 pages and 6 questions. There are 15 marks available. The time limit is 25 minutes. Math and Stats standard calculators are allowed, but not needed. Except where explicitly noted, it is necessary to show clearly organized work in order to receive full or partial credit. Use the back of the pages for rough or extra work.

1. [2] Indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

   F (a) $(p \vee q) \Rightarrow (p \wedge q)$ $(p \vee q) \rightarrow (p \wedge q)$ is not a tautology

   T (b) If $a \Rightarrow b$, then b can be false.   If $a \rightarrow b$ is a tautology, b can be false.

   F (c) In a valid argument, at least one premise must be true.   Can have false premises.

   T (d) The dual of $p \rightarrow q$ is $\neg p \wedge q$. $p \rightarrow q \Leftrightarrow \neg p \vee q$, dual is $\neg p \wedge q$

2. [3] Use the Laws of Logic to show that $p \rightarrow (q \vee r) \Leftrightarrow (p \wedge \neg q) \rightarrow r$.

$p \rightarrow (q \vee r)$

$\Leftrightarrow \neg p \vee (q \vee r)$ known LE

$\Leftrightarrow (\neg p \vee q) \vee r$ Associative

$\Leftrightarrow \neg(\neg p \vee q) \rightarrow r$ known LE

$\Leftrightarrow (\neg\neg p \wedge \neg q) \rightarrow r$ De Morgan's

$\Leftrightarrow (p \wedge \neg q) \rightarrow r$ Double Negation

3. [2] Give a counterexample to show that the argument below is not valid.

$$\begin{array}{rc} & a \vee \neg b \\ & a \rightarrow c \\ \hline \therefore & c \end{array}$$

Want values so conclusion is false but premises are true.
So want c to be false.
Also want $a \rightarrow c$ to be true, so a is false since c is false.
Also want $a \vee \neg b$ to be true, so $\neg b$ is true since a is false, so b is false.

The counterexample a: 0, b: 0, c: 0 shows that the argument is not valid since the premises are true and the conclusion is false.

4. [3] Use the Rules of Inference to show that the argument below is valid.

$$\begin{array}{rc} & \neg(r \wedge s) \\ & \neg s \to p \\ & r \\ \hline \therefore & p \end{array}$$

Steps	Reasons
1) $\neg(r \wedge s)$	premise
2) $\neg s \to p$	premise
3) r	premise
4) $\neg r \vee \neg s$	1), known LE
5) $r \to \neg s$	4) known LE
6) $r \to p$	2), 5) Chain Rule
7) p	3), 6) Modus Ponens

5. [3] Suppose the universe consists only of the integers 2 and 3. Write the following statement without quantifiers, using $\wedge$ and $\vee$, and then determine the truth value of the statement.

$$\exists x, \forall y, (x \geq y) \to (x = 2)$$

When $x=2$: statement is $\forall y\ (2 \geq y) \to (2=2)$

$\big((2\geq 2) \to (2=2)\big) \wedge \big((2\geq 3) \to (2=2)\big)$

$\therefore$ The statement is true.

When $x=3$: statement is $\forall y\ (3 \geq y) \to (3=2)$

$\big((3\geq 2) \to (3=2)\big) \wedge \big((3\geq 3) \to (3=2)\big)$

$$\underbrace{\Big[\underbrace{\big((2\geq 2) \to (2=2)\big)}_{1 \quad 1 \;\to\; 1} \wedge \underbrace{\big((2\geq 3) \to (2=2)\big)}_{0 \to 1 \;\to\; 1}\Big]}_{1} \vee \underbrace{\Big[\underbrace{\big((3\geq 2) \to (3=2)\big)}_{1 \to 0 \;\to\; 0} \wedge \underbrace{\big((3\geq 3) \to (3=2)\big)}_{1 \to 0 \;\to\; 0}\Big]}_{0}$$

6. [2] Indicate whether each statement is **True (T)** or **False (F)**. No reasons are necessary.

F (a) For the universe of real numbers, $\exists x, (x^2 = 2) \wedge (x \text{ is rational})$. $\sqrt{2}, -\sqrt{2}$ are not rational

T (b) For the open statements $p(x)$ and $q(x)$ with the universe of the integers, if $\forall x, p(x) \to q(x)$ is true, then $\exists x, p(x) \to q(x)$ is true.

T (c) The negation of $\exists x, p(x) \to q(x)$ is $\forall x, p(x) \wedge \neg q(x)$.
$p \to q \Leftrightarrow \neg p \vee q$
$\neg(p \to q) \Leftrightarrow \neg(\neg p \vee q) \Leftrightarrow p \wedge \neg q$

F (d) The contrapositive of $\forall x, p(x) \to q(x)$ is $\exists x, \neg q(x) \to \neg p(x)$.
quantifier doesn't change in contrapositive