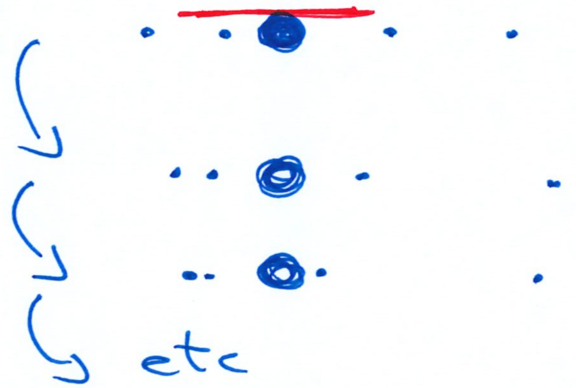
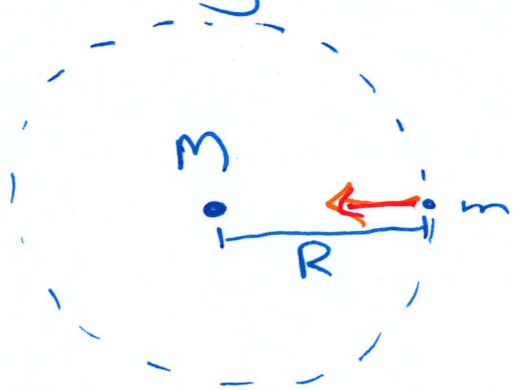
Newtonian gravity & orbits



We found mass of Jupiter by watching moons orbit!



$$\vec{F}_{on\,m} = \frac{GMm}{R^2} \text{ (towards M)}$$

$$\frac{\vec{F}_{on\,m}}{m} = \frac{|\vec{v}|^2}{R} \text{ (into center)}$$

$$\frac{GM\cancel{m}}{R^{\cancel{2}}} = \frac{\cancel{m}|\vec{v}|^2}{\cancel{R}}$$

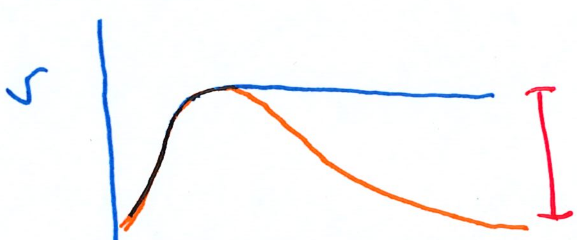
$$\frac{GM}{R} = |\vec{v}|^2$$

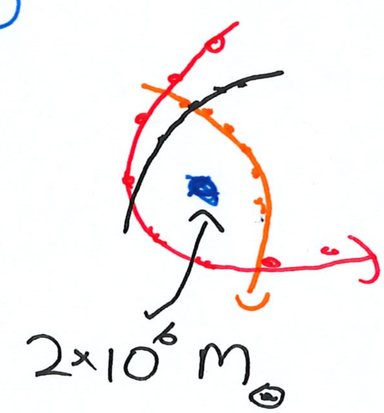
measure period of orbit T

$$|\vec{v}| = \frac{2\pi R}{T}$$

$$\frac{GM}{R} = \left(\frac{2\pi R}{T}\right)^2$$

$$T^2 = \frac{(2\pi)^2}{GM} R^3$$





Coulomb force

Can be attractive or repulsive

Charge can be positive or negative

positive repels positives

" attract negative

negative repels negative.

~~magnitu~~ respective charges

$$\vec{F}_{onAbyB} = \frac{1}{4\pi\varepsilon_0} \frac{q_A q_B}{|\vec{r}_A - \vec{r}_B|^2} \frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$$

unit vector B to A

$$\frac{1}{4\pi\varepsilon_0} \approx 9.0\times10^{9} \frac{Nm^2}{C^2}$$

$(\text{separation})^2$

Newtonian gravity

$$\vec{F}_{onAbyB} = -G \frac{m_A m_B}{|\vec{r}_A - \vec{r}_B|^2} \frac{\vec{r}_A - \vec{r}_B}{|\vec{r}_A - \vec{r}_B|}$$

Forces - III

A $3\times10^{-3}C$ charge is at $-2m\hat{\imath}+3m\hat{\jmath}$. A $5\times10^{-3}C$ charge is at $4m\hat{\imath}+3m\hat{\jmath}$. A $-4\times10^{-3}C$ charge is at $-4m\hat{\imath}+1m\hat{\jmath}$.

- * What is the Coulomb force on the first charge by the second charge?
- What is the total Coulomb force on the first charge?

- Find $\vec{F}_{on 1 by 2}$ A
- Find $\vec{F}_{on 1 by 3}$
- Add

B

A

$$\vec{F}_{on A by B} = \frac{1}{4\pi\varepsilon_0}\frac{q_A q_B}{|\vec{r}_A-\vec{r}_B|^2}\frac{\vec{r}_A-\vec{r}_B}{|\vec{r}_A-\vec{r}_B|}$$

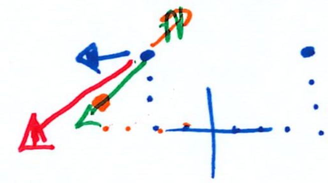
$$q_A = 3\times10^{-3}C \quad \vec{r}_A = -2m\hat{\imath}+3m\hat{\jmath}$$

$$q_B = 5\times10^{-3}C \quad \vec{r}_B = 4m\hat{\imath}+3m\hat{\jmath}$$

$$\vec{r}_A-\vec{r}_B = (-2m\hat{\imath}+3m\hat{\jmath})-(4m\hat{\imath}+3m\hat{\jmath})$$
$$= -6m\hat{\imath}$$

$$|\vec{r}_A-\vec{r}_B| = 6m$$

$$\vec{F}_{on\,1^{st}\,by\,2^{nd}} = 9\times10^{9}\,\frac{Nm^2}{C^2}\,\frac{3\times10^{-3}C\;5\times10^{-3}C}{(6m)^2}\,\frac{(-6m\,\hat{i})}{6m}$$



* $$= -3.75\times10^{3}\,N\,\hat{i}$$

—

$$q_A = 3\times10^{-3}C \qquad \vec{r}_A = -2m\,\hat{i} + 3m\,\hat{j}$$

$$q_B = -4\times10^{-3}C \qquad \vec{r}_B = -4m\,\hat{i} + 1m\,\hat{j}$$

$$\vec{r}_A - \vec{r}_B = (-2m\,\hat{i} + 3m\,\hat{j}) - (-4m\,\hat{i} + 1m\,\hat{j})$$

$$= 2m\,\hat{i} + 2m\,\hat{j}$$

$$|\vec{r}_A - \vec{r}_B| = \sqrt{(2m)^2 + (2m)^2} = 2.83m$$

$$\vec{F}_{on\,1^{st}\,by\,3^{rd}} = 9\times10^{9}\,\frac{Nm^2}{C^2}\,\frac{3\times10^{-3}C\,(-4\times10^{-3}C)}{(2.83m)^2}\,\frac{2m\,\hat{i} + 2m\,\hat{j}}{2.83m}$$

* $$= -9.54\times10^{3}\,N\,\hat{i} - 9.54\times10^{3}\,N\,\hat{j}$$

$$\vec{F}_{total\,on\,1^{st}} = -1.33\times10^{4}\,N\,\hat{i} - 9.54\times10^{3}\,N\,\hat{j}$$

Lorentz Force & Magnetism

Describe in terms of a "magnetic field" ($\vec{B}$)

Can express Coulomb force in terms of "Electric field"

$\cdot\, q_1 \vec{r}_1$

$\vec{r}$

$Q \bullet$

$\cdot\, q_2 \vec{r}_2$

$\vdots$

$\cdot\, q_{793257}$

$\vec{r}_{793257}$

$$\vec{F}_{net,Q} = \vec{F}_{1 on Q} + \vec{F}_{2 on Q} + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 Q}{|\vec{r}-\vec{r}_1|^2} (\text{vector}) + \frac{1}{4\pi\varepsilon_0} \frac{q_2 Q}{|\vec{r}-\vec{r}_2|^2} (\quad) + \ldots$$

$$= Q \left[\frac{1}{4\pi\varepsilon_0} \left(\begin{array}{c} \text{something depends on } \vec{r} \\ \text{and all } q_i\text{'s } \vec{r}_i\text{'s} \end{array} \right) \right]$$

$$= Q \vec{E}(\vec{r})$$

$\frac{N}{C}$

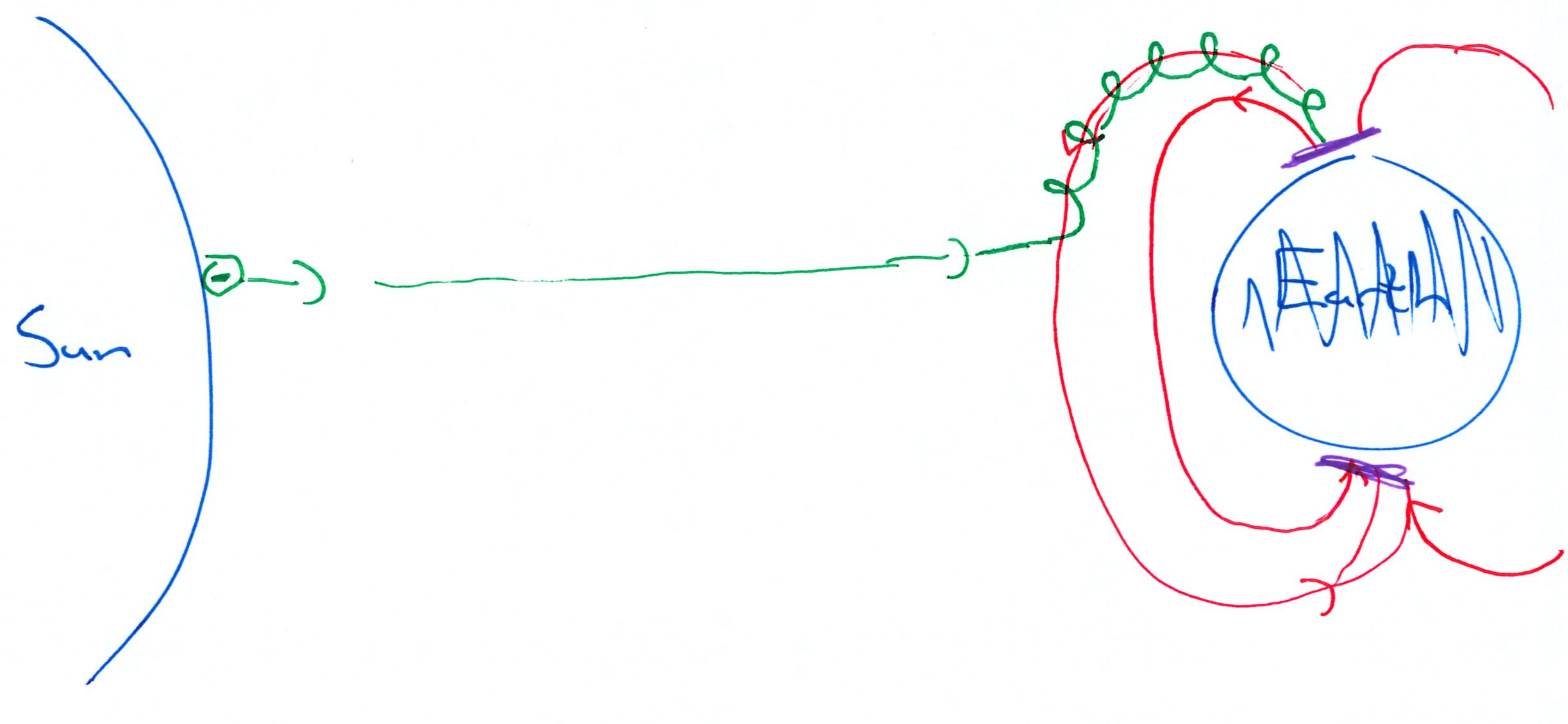
describes magnitude & direction of $\vec{F}$ on Q at $\vec{r}$

Similarly

$\vec{B}$ ← based on positions, charges and velocities of all other charges

Charged particle moving ~~nea~~ in a $\vec{B}$

$$\vec{F} = q\vec{v} \times \vec{B}$$



Forces - IV

A $3.0kg$ mass with a charge of $-2 \times 10^{-3}C$ is moving at a velocity of $200\frac{m}{s}\hat{\imath} + 300\frac{m}{s}\hat{\jmath}$ in a region where the magnetic field is $1.2T\hat{\jmath}$.

What is the acceleration of the mass?

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$N \quad C\frac{m}{s} \quad \frac{N}{C\,m/s} \sim T$$

$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{q\vec{v} \times \vec{B}}{m}$$

$$= \frac{(-2\times10^{-3}C)}{3kg}\left(200\,m/s\,\hat{\imath} + 300\,m/s\,\hat{\jmath}\right) \times (1.2T\hat{\jmath})$$

$$= \frac{-2\times10^{-3}C}{3kg}\left(\underbrace{240\,T\,m/s}_{N/C}\,\hat{k} + 0\right)$$

$$= -0.16\,m/s^2\,\hat{k}$$