Rotational Equilibrium

$\vec{\tau}_{net} = 0 \quad \vec{F}_{net} = 0$

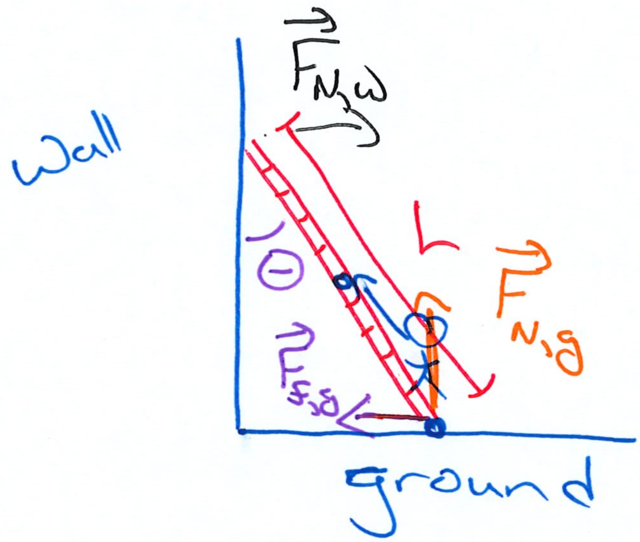
Putting together

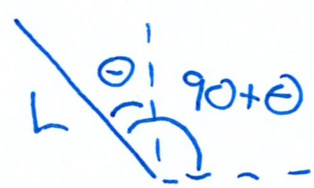
Ladder mass M, uniform bar of length L

Placed against a smooth wall

Coef. of static friction $\mu$ between ladder & ground.

What are angles where ladder won't slide?



Forces on ladder

- Normal force by ground
- Friction by ground
- Gravity on ladder
- Normal force by wall

Smooth wall → No friction by wall.

Ladder in equilibrium

$$0 = \underset{-\hat{k}}{\vec{F}_g} + \underset{\hat{i}}{\vec{F}_{N,w}} + \underset{\hat{k}}{\vec{F}_{N,g}} + \underset{-\hat{i}}{\vec{F}_{f,g}}$$

$x$-component

$$|\vec{F}_{N,w}| = |\vec{F}_{f,g}|$$

$z$-component

$$\underbrace{|\vec{F}_{N,g}|}_{\text{derived}} = \underset{\text{know}}{Mg}$$

$\vec{\tau}_{Net} = 0$ pick contact point as origin

$\vec{F}_{N,g}$: $\vec{r}_{N,g} = 0$ , $\vec{F}_{N,g} = |\vec{F}_{N,g}|\hat{k}$

$\vec{F}_{f,g}$: $\vec{r}_{f,g} = 0$ $\vec{F}_{f,g} = -|\vec{F}_{f,g}|\hat{i}$

$\vec{F}_{N,w}$: $\vec{r}_{N,w} = L\cos(90+\theta)\hat{i} + L\cos\theta\hat{k}$

$= -L\sin\theta\,\hat{i} + L\cos\theta\,\hat{k}$

$\vec{F}_{N,w} = |\vec{F}_{N,w}|\hat{i}$

$\vec{F}_g$: $\vec{r}_g = -\frac{L}{2}\sin\theta\,\hat{i} + \frac{L}{2}\cos\theta\,\hat{k}$

$\vec{F}_g = -Mg\hat{k}$

$$\vec{\tau}_{Net} = 0 + 0 + (-L\sin\theta\hat{i} + L\cos\theta\hat{k}) \times (|\vec{F}_{N,w}|\hat{i})$$
$$+ \left(-\frac{L}{2}\sin\theta\hat{i} + \frac{L}{2}\cos\theta\hat{k}\right) \times (-Mg\hat{k})$$

$$0 = L\cos\theta\,|\vec{F}_{N,w}|\,\hat{j} \cancel{+} - \frac{L}{2}\sin\theta\,Mg\,\hat{j}$$

y-component

$$0 = L\cos\theta\,|\vec{F}_{N,w}| - \frac{L}{2}\sin\theta\,Mg$$

$|\vec{F}_{N,w}| = \frac{1}{2}\tan\theta\, Mg$ Must be for $\vec{\tau}_{Net} = 0$

$|\vec{F}_{f,g}| = |\vec{F}_{N,w}|$ From $\vec{F}_{Net} = 0$

$|\vec{F}_{f,g}| = \frac{1}{2}\tan\theta\, Mg$

$|\vec{F}_{N,g}| = Mg$ From z-comp $\vec{F}_{Net} = 0$

by how friction works

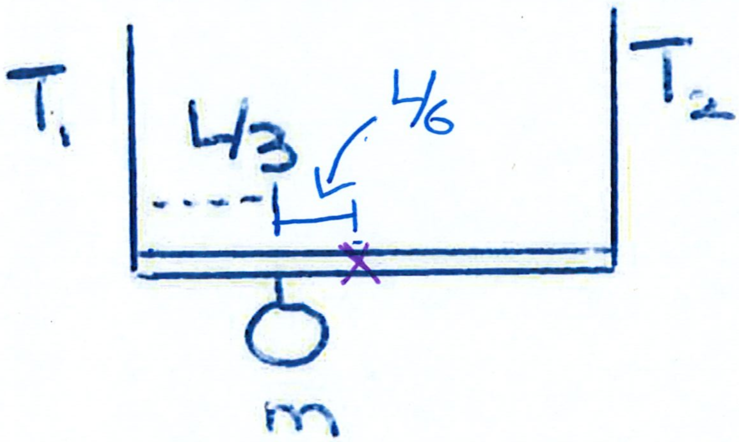
$|\vec{F}_{f,g}| \leq \mu|\vec{F}_{N,g}|$

$\frac{1}{2}\tan\theta\, Mg \leq \mu Mg$

$\tan\theta \leq 2\mu$

**Rotational Equilibrium - IV**

A $12kg$ bar of length $L = 3m$ is held horizontally by two vertical ropes, one at each end. A $6kg$ mass is located $\frac{L}{3} = 1m$ from the left end of the bar.




- What is the tension $T_1$ in the left-hand rope?
- What is the tension $T_2$ in the right-hand rope?

Pick center of mass of bar as origin

Rope1 $\vec{r}_1 = -\frac{L}{2}\hat{i}$ $\vec{F}_1 = T_1\hat{k}$

Rope2 $\vec{r}_2 = \frac{L}{2}\hat{i}$ $\vec{F}_2 = T_2\hat{k}$

12kg

gravity $\vec{r}_g = 0$ $\vec{F}_g = -M_{bar}g\hat{k}$

suspended mass $\vec{r}_{sm} = -\frac{L}{6}\hat{i}$ $\vec{F}_{sm} = -mg\hat{k}$

$$\vec{\tau}_{net} = 0$$

$$= \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_g + \vec{\tau}_{sm}$$

$$= \left(-\frac{L}{2}\hat{i}\right)\times\left(T_1\hat{k}\right) + \left(\frac{L}{2}\hat{i}\right)\times\left(T_2\hat{k}\right)$$

$$+ (0)\times\left(-M_{bar}g\hat{k}\right) + \left(-\frac{L}{6}\hat{i}\right)\times\left(-mg\hat{k}\right)$$

$$= \frac{L}{2}T_1\hat{j} + \frac{L}{2}T_2(-\hat{j}) + 0 + \frac{L}{6}mg(-\hat{j})$$

y-component of expression

$$0 = \frac{\cancel{L}}{\cancel{2}}T_1 - \frac{\cancel{L}}{\cancel{2}}T_2 - \frac{\cancel{L}}{\cancel{6}\,3}mg$$

or

$$\boxed{0 = T_1 - T_2 - \frac{1}{3}mg}$$

$$\vec{F}_{net} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_g + \vec{F}_{sm}$$

$$= T_1\hat{k} + T_2\hat{k} - M_{bar}g\hat{k} - mg\hat{k}$$

z-component

$$\boxed{0 = T_1 + T_2 - M_{bar}g - mg}$$

$$T_1 = T_2 + \frac{1}{3}mg$$

$$0 = \left(T_2 + \frac{1}{3}mg\right) + T_2 - M_{bar}g - mg$$

$$0 = 2T_2 - M_{bar}g - \frac{2}{3}mg$$

$$T_2 = \frac{M_{bar}g}{2} + \frac{1}{3}mg = 78.4N$$

$$0 = T_1 - (78.4N) - \frac{1}{3}mg$$

$$T_1 = 98N$$

Total mass: 18kg $\rightarrow |\vec{F}_{g\ total}| = 176.4N$

$T_1 + T_2$ = same

Why we're writing $\vec{\tau}$ as a vector.

In Physics 12 taught
"ccw torque" = "cw torques"

implicit in examples so far

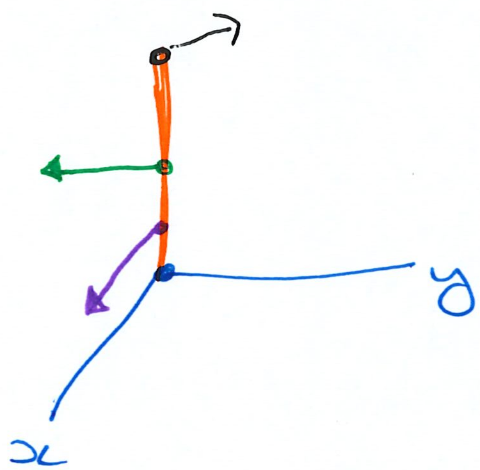
If all forces & $\vec{r}$'s are "in a 2d plane" then there are only two directions $\vec{\tau}$ can go: into or out of surface

In 3d, have to take account of ~~all~~ $\vec{\tau}$ as a vector

**Rotational Equilibrium - V**

A $6m$-long pole is held upright by three ropes pulling horizontally. The bottom of the pole is at the origin and touching the ground. One rope is attached at $1m$ up from the ground and it pulls with a force of $200N\hat{\imath}$. A second rope is attached $3m$ up from the ground and it pulls with a force of $-150N\hat{\jmath}$. The third rope is attached at the top.

- What is the x-component of the force exerted by the third rope?
- What is the y-component of the force exeerted by the third rope?
- What angle does the third rope make with $\hat{\imath}$ and $\hat{\jmath}$?
- What is the horizontal part of the force that the ground exerts on the pole?



Forces on pole

| | | |
|---|---|---|
| Rope1 | $\vec{r}_1 = 1\text{m}\,\hat{k}$ | $\vec{F}_1 = 200\text{N}\,\hat{i}$ |
| Rope2 | $\vec{r}_2 = 3\text{m}\,\hat{k}$ | $\vec{F}_2 = -150\text{N}\,\hat{j}$ |
| Rope3 | $\vec{r}_3 = 6\text{m}\,\hat{k}$ | $\vec{F}_3 = \underline{F_{3x}}\,\hat{i} + \underline{F_{3y}}\,\hat{j}$ |
| gravity | $\vec{r}_g = 3\text{m}\,\hat{k}$ | $\vec{F}_g = -mg\hat{k}$ |
| Normal | $\vec{r}_N = 0$ | $\vec{F}_N = ?$ |
| * Friction | $\vec{r}_f = 0$ | $\vec{F}_f = F_{fx}\,\hat{i} + F_{fy}\,\hat{j}$ |

$$\vec{\tau}_{net} = 0$$

$$0 = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + \vec{\tau}_g + \vec{\tau}_N + \vec{\tau}_f$$

$$= \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 + 0 + 0 + 0$$

$$= 1\text{m}\,\hat{k} \times 200\text{N}\,\hat{i} + 3\text{m}\,\hat{k} \times (-150\text{N}\,\hat{j})$$
$$+ 6\text{m}\,\hat{k} \times (F_{3x}\,\hat{i} + F_{3y}\,\hat{j})$$

$$0 = 200\text{Nm}\,\hat{j} + 450\text{Nm}\,\hat{i}$$
$$+ 6\text{m}\,F_{3x}\,\hat{j} - 6\text{m}\,F_{3y}\,\hat{i}$$

x-component

$$0 = 450\,\text{Nm} - 6\,\text{m}\,F_{3y}$$

1 $F_{3y} = 75\,\text{N}$

y-comp.

$$0 = 200\,\text{Nm} + 6\,\text{m}\,F_{3x}$$

1 $F_{3x} = -33.3\,\text{N}$

Angle with $\hat{i}$

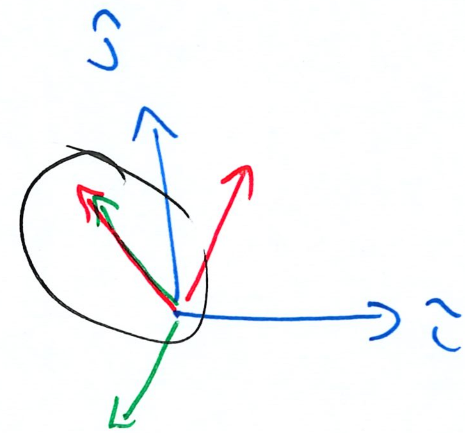
$$\vec{F}_3 \cdot \hat{i} = |\vec{F}_3| \cos\theta$$

$$F_{3x} = \sqrt{F_{3x}^2 + F_{3y}^2}\,\cos\theta$$

$$-33.3\,\text{N} = 82.1\,\text{N} \cos\theta$$

$$\cos\theta = -0.406$$

$$\theta = 114^\circ$$




Angle with $\hat{j}$

$$\vec{F}_3 \cdot \hat{j} = |\vec{F}_3| \cos\theta$$

$$F_{3y} = \sqrt{F_{3x}^2 + F_{3y}^2}\,\cos\theta$$

$$75\,\text{N} = 82.1\,\text{N} \cos\theta$$

$$\cos\theta = 0.914$$

$$\theta = 24^\circ$$

$$\vec{F}_{Net} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_g + \vec{F}_N + \vec{F}_f$$

$$= 200N\hat{i} + (-150N\hat{j}) + (-33.3N\hat{i} + 75N\hat{j})$$

$$+ (\text{only in } \hat{k}) + (F_{fx}\hat{i} + F_{fy}\hat{j})$$

x-component

$$0 = 200N - 33.3N + F_{fx}$$

$$F_{fx} = -167N$$

y-component

$$0 = -150N + 75N + F_{fy}$$

$$F_{fy} = 75N$$

$$|\vec{F}_f| = 183N$$