# CHAPTER 7

# Solutions to Chapter-End Problems

## A. Key Concepts

## Reasons for Replacement:

- 7.1 Probably not. If new, almost identical, printers are selling for \$5000, CCC surely won't get nearly as much as that for their year-old model. Since they use the printer for only special printing jobs, it likely has incurred little wear. The \$20 000 is a sunk cost which will not have any bearing on a decision to replace it.
- 7.2 No. The overhaul is a periodic cost which increases the cost of keeping the defender for this year only. Evelyn should look at the EAC for the defender for a two or three year period to see if the challenger is still cheaper on an annual basis.

## Capital Costs and Other Costs:

- 7.3 Only a), c) and h) are capital costs.
  - b), e) g) and possibly i) are operating and maintenance costs
  - d), f) and i) are installation costs
- **7.4** Only the data collection nodes, server and software would be included for calculating capital cost. The other costs are installation costs.

Capital cost = 
$$[(4500 \times 5) + 6000 + (1190 \times 5) + 1950] \times 0.3 = 10920$$

The capital cost in the first year is \$10 920.

#### Economic Life:

**7.5** As an example calculation, for Item 1, year 4:

Salvage value = 
$$10\ 000(1-0.2)^4 = 4096$$
  
Capital cost =  $(12\ 000 - 4096)(A/P, 8\%, 4) + 4069(0.08) = 2714$   
Operating cost =  $300 + 300(A/G, 8\%, 4) = 300 + 300(1.4040) = 721.19$   
Total cost =  $2714 + 721.2 = 3435.2$ 

As can be seen from the spreadsheets below, Item 1 has an economic life of 8 years, Item 2 an economic life of 18 years, and Item 3 an economic life of 3 years.

Item 1:		EAC	EAC	EAC

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Year	Salvage	Installation	Operating	Capital	Operating	Total
0	10000	2000				
1	8000		300	4960	300	5260
2	6400		600	3652	444	4097
3	5120		900	3079	585	3664
4	4096		1200	2714	721	3435
5	3277		1500	2447	854	3301
6	2621		1800	2238	983	3221
7	2097		2100	2070	1108	3178
8	1678		2400	1930	1230	3160
9	1342		2700	1813	1347	3161
10	1074		3000	1714	1461	3176

Item 2:				EAC	EAC	EAC
Year	Salvage	Installation	Operating	Capital	Operating	Total
0	20000	2000				
1	16000		200	7760	200	7960
2	12800		400	6183	296	6479
3	10240		600	5382	390	5772
4	8192		800	4824	481	5305
5	6554		1000	4393	569	4962
6	5243		1200	4044	655	4700
7	4194		1400	3756	739	4494
8	3355		1600	3513	820	4333
9	2684		1800	3307	898	4205
10	2147		2000	3130	974	4105
11	1718		2200	2978	1048	4026
12	1374		2400	2847	1119	3966
13	1100		2600	2732	1188	3920
14	880		2800	2632	1255	3887
15	704		3000	2544	1319	3863
16	563		3200	2467	1381	3848
17	450		3400	2399	1441	3839
18	360		3600	2338	1498	3836
19	288		3800	2284	1554	3838
20	231		4000	2236	1607	3843

Item 3:				EAC	EAC	EAC
Year	Salvage	Installation	Operating	Capital	Operating	Total
0	30000	3000				
1	24000		2000	11640	2000	13640
2	19200		4000	9275	2962	12236
3	15360		6000	8074	3897	11971
4	12288		8000	7236	4808	12044
5	9830		10000	6589	5693	12282
6	7864		12000	6066	6553	12619

# **7.6 (a)** The capital and operating costs of the bottle capper are as follows:

			EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	40000				
1	35000	3000	15400	3000	18400
2	30000	4000	12475	3472	15947
3	25000	5000	11327	3925	15252
4	20000	6000	10631	4359	14990
5	15000	7000	10122	4775	14897
6	10000	8000	9713	5172	14885
7	5000	9000	9365	5551	14916
8	0	10000	9059	5913	14972

For example, equivalent annual costs for the capper if it is kept 2 years are:

Book value of capper after 2 years =  $40\ 000 - 2(40\ 000 - 0)/8 = 30\ 000$ 

Notice that the installation costs are included in the original price paid (P), but not in the depreciable value of the asset.

EAC(capital costs)

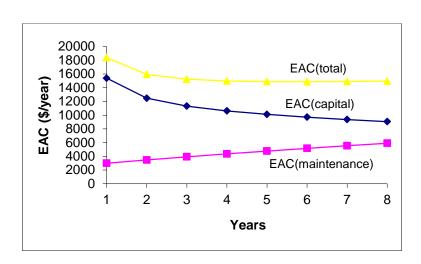
$$= (40\ 000 + 5000 - 30\ 000)(A/P, 12\%, 2) + 30\ 000(0.12) = 12\ 475$$

EAC(maintenance costs)

= 
$$[3000(P/F, 12\%, 1) + 4000(P/F, 12\%, 2)](A/P, 12\%, 2) = 3472$$

$$EAC(total) = 12475 + 3472 = 15947$$

(b)



(c) The economic life of the bottle capper is 6 years.

Defender and Challenger are Identical:

**7.7** The economic life of the pump is 8 years.

Years between	EAC	EAC	EAC
replacement	Capital	Operating	Total
1	4100	500	4600
2	3290	643	3933
3	2876	781	3657
4	2588	914	3502
5	2365	1043	3408
6	2186	1167	3353
7	2038	1286	3325
8	1915	1401	3317
9	1811	1512	3323
10	1723	1618	3340
11	1647	1719	3366
12	1582	1817	3399

Example calculation:

Salvage value after 10 years can be calculated as:  $10\ 000(1-0.2)^{10} = 1074$ 

EAC(capital costs)

$$= (10\ 000 + 1000 - 1074)(A/P, 10\%, 10) + 1074(0.1)$$

= 9926(0.16275) + 107.4 = 1722.83

EAC(operating costs)

$$= 500 + 300(A/G, 10\%, 10) = 500 + 300(3.7255) = 1617.64$$

EAC(total) = 3340.47

**7.8** Assumption: the new pump can be repeated.

(a) Example calculation: Old pump annual costs this year are EAC = (1000 - 1000)(A/P, 10%, 1) + 1000(0.1) + 2500 = 2600

	EAC	EAC	EAC
Year	Capital	Operating	Total
1	100	2500	2600
2	100	2690	2790
3	100	2875	2975
4	100	3052	3152
5	100	3224	3324
6	100	3389	3489
7	100	3549	3649

Until the fifth year, the old pump will have lower average costs than the new pump will have at its economic life (\$3317). Replace the old pump after four years.

**(b)** For the first year:

$$EAC = (1000 - 1000)(A/P, 10\%, 1) + 1000(0.1) + 3500 = 3600$$

The old pump should be replaced immediately.

One Year Principle:

- **7.9** No the capital costs are high compared to the operating and maintenance costs
- 7.10 No the operating and maintenance costs are decreasing
- **7.11** No the operating and maintenance costs do not have an even pattern from year to year
- **7.12** Technically, no the operating and maintenance costs do not have an even pattern from year to year. However, on the assumption that since the asset is inaccessible, replacement would only actually be considered on a

4-year basis, one could use the 4-year period (with an approximately \$16 000 maintenance cost) as an evaluation basis, instead of one-year, and the approach would be valid.

## **B.** Applications

**7.13** This requires calculating the service life of the car.

```
EAC (capital, first year)
    = [(15\ 000 - (0.7)(15\ 000)](A/P, 8\%, 1) + (0.7)(15\ 000)(0.08)
    = 4500(1.08) + 10500(0.08) = 5700
EAC(operating, first year) = 0
EAC(total, first year) = $5700
EAC (capital, two years)
    = [15\ 000 - (0.7^2)(15\ 000)](A/P, 8\%, 2) + (0.7^2)(15\ 000)(0.08)
    = 7650(0.56077) + 7350(0.08) = 4877.89
EAC(operating, two years) = 0
EAC(total, two years) = $4877.89
EAC (capital, three years)
   = [15\ 000 - (0.7^3)(15\ 000)](A/P, 8\%, 3) + (0.7^3)(15\ 000)(0.08)
    = 9855(0.38803) + 5145(0.08) = 4235.64
EAC(operating, three years) = 2500(A/F, 8\%, 3) = 2000(0.30803) = 770.08
EAC(total, three years) = $5005.72
Gerry should get a new car every two years.
```

```
7.14 EAC (capital, first year)
           = [15\ 000 - (0.7)(15\ 000)](A/P, 8\%, 1) + (0.7)(15\ 000)(0.08)
           = 4500 (1.08) + 10500(0.08) = 5700
       EAC(operating, first year) = 0
       EAC(total, first year) = $5700
       EAC (capital, two years)
          = [15\ 000 - (0.7^2)(15\ 000)](A/P, 8\%, 2) + (0.7^2)(15\ 000)(0.08)
           = 7650(0.56077) + 7350(0.08) = 4877.89
       EAC(operating, two years) = 0
       EAC(total, two years) = $4877.89
       EAC (capital, three years)
           = [15\ 000 - (0.7^3)(15\ 000)](A/P, 8\%, 3) + (0.7^3)(15\ 000)(0.08)
           = 9855(0.38803) + 5145(0.08) = 4235.64
       EAC(operating, three year) = 1500(A/F, 8\%, 3) = 1500(0.30803) = 462.05
       EAC(total, three years) = $4697.67
```

EAC (capital, four years)

=  $[15\ 000 - (0.7^4)(15\ 000)](A/P, 8\%, 4) + (0.7^4)(15\ 000)(0.08)$ 

= 11398.50(0.30192) + 3601.50(0.08) = 3729.56

EAC(operating, four year)

= [1500(F/P, 8%, 1) + (1500)(1.5)](A/F, 8%, 4)

= [1500(1.08) + 2250](0.22192) = 1098.50

EAC(total, four years) = \$4828.06

Gerry should now get a new car every three years.

- 7.15 (a) No the capital costs are low but there is an uneven pattern of costs
  - **(b)** Yes the capital costs are low and there is an even pattern of costs
  - (c) Yes the capital costs are low and there is an even pattern of costs
  - (d) Yes the capital costs are low and there is an even pattern of costs
  - (e) No there is an even pattern of costs but the capital costs are **not** low
- **7.16** (a) The capital and operating costs of the roller conveyor are as follows:

			EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	100000				
1	75000	6000	57000	6000	63000
2	56250	7200	42357	6571	48929
3	42188	15640	35508	9311	44819
4	31641	10368	31039	9539	40578
5	23730	12442	27769	10014	37783
6	17798	21930	25246	11559	36805
7	13348	17916	23242	12229	35470
8	10011	21499	21618	13039	34657
9	7508	32799	20284	14495	34778
10	5631	30959	19176	15528	34704
11	4224	37150	18248	16694	34942
12	3168	51581	17463	18326	35789
13	2376	53497	16797	19760	36557
14	1782	64196	16226	21348	37574
15	1336	84035	15735	23321	39056

For example, equivalent annual costs for the conveyor if it is kept 2 years:

Book value of conveyor after 2 years =  $100\ 000(1-0.25)^2 = 56\ 250$ 

Notice that the installation costs are included in the original price paid (P), but not in the depreciable value of the asset.

EAC(capital costs)

$$= (100\ 000 + 20\ 000 - 56\ 250)(A/P, 10\%, 2) + 56\ 250(0.10) = 42\ 357$$

EAC(maintenance costs)

$$= [6000(P/F, 10\%, 1) + 7200(P/F, 10\%, 2)](A/P, 10\%, 2) = 6571$$

$$EAC(total) = 42 357 + 6571 = 48 928$$

70000 60000 40000 30000 EAC(total) 20000 10000 EAC(capital) 0 1 3 5 7 9 11 13 15

- **(c)** The economic life of the conveyor is 8 years.
- **7.17** First, the EAC of keeping the robot as is for the duration of the contract is found by the EAC(total costs) for an additional life of 5 years.

**Years** 

Historical costs are no longer relevant for the computation of the EAC(capital costs) and, hence, they do not include the original installation costs of the robot.

3 year old Defe	ender:				
Additional			EAC	EAC	EAC
years	Salvage	Maintenance	Capital	Maintenance	Total
0	153600				
1	122880	50000	53760	50000	103760
2	98304	55000	48759	52326	101085
3	78643	60500	44626	54680	99305
4	62915	66550	41201	57057	98258
5	50332	73205	38356	59452	97808

From the computations we can see that the EAC(total costs) of keeping the robot as is over the next 5 years is \$97 808.

If the robot is moved, \$25 000 moving cost is incurred, but the operating and maintenance costs are reduced for the 5-year duration of the contract. For this option, the EAC(total costs) is minimized by keeping the moved robot for the duration of the contract. The minimum EAC(total costs) of the challenger over the five years is \$93 376. The robot should be moved.

Challenger (M	ove Robot):				
Additional			EAC	EAC	EAC
years	Salvage	Maintenance	Capital	Maintenance	Total
0	153600				
1	122880	40000	82510	40000	122510
2	98304	44000	64137	41860	105997
3	78643	48400	55575	43744	99319
4	62915	53240	49958	45645	95603
5	50332	58564	45814	47561	93376

Since we know that the robot should be moved, the question is when? The cost of keeping the defender one more year is \$103 760 (see table on defender). For two years, the cost is \$101 085. We see that there is no number of additional years in the 5-year horizon for which the defender is less expensive to keep. Therefore, the robot should be moved immediately.

- **7.18** In general, the options available to BB are to:
  - 1) Replace the current robot at the end of its economic or useful life with a stream of new robots, or
  - 2) Move the robot and then replace it with a stream of new robots at the end of the moved robot's new economic life.
  - (a) The EAC(total costs) for keeping the existing robot for the rest of its useful life is shown in the table below. The table shows that the remaining economic life is 5 years for a minimum EAC(total costs) of \$97 808.

3 year old Defe	ender:				
Additional			EAC	EAC	EAC
years	Salvage	Maintenance	Capital	Maintenance	Total
0	153600				
1	122880	50000	53760	50000	103760
2	98304	55000	48759	52326	101085
3	78643	60500	44626	54680	99305
4	62915	66550	41201	57057	98258
5	50332	73205	38356	59452	97808
6	40265	80526	35987	61859	97846
7	32212	88578	34009	64274	98282
8	25770	97436	32352	66689	99042
9	20616	107179	30962	69102	100064

**(b)** The EAC(total costs) of moving the robot and operating it to the end of its economic life is \$91 275 (another 8 years).

Moving the ex	isting robot (c	hallenger):			
Additional		-	EAC	EAC	EAC
years	Salvage	Maintenance	Capital	Maintenance	Total
0	153600				
1	122880	40000	82510	40000	122510
2	98304	44000	64137	41860	105997
3	78643	48400	55575	43744	99319
4	62915	53240	49958	45645	95603
5	50332	58564	45814	47561	93376
6	40265	64420	42593	49487	92080
7	32212	70862	40018	51419	91437
8	25770	77949	37924	53352	91275
9	20616	85744	36202	55281	91483

**(c)** The EAC(total costs) of purchasing and installing a new robot are below.

Defender, when new:			EAC	EAC	EAC
Year Salvage		Maintenance	Capital	Maintenance	Total
0	300000				
1	240000	40000	162500	40000	202500
2	192000	40000	125988	40000	165988

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3	153600	40000	109059	40000	149059
4	122880	40000	97984	40000	137984
5	98304	44000	89830	40593	130424
6	78643	48400	83499	41485	124984
7	62915	53240	78441	42547	120988
8	50332	58564	74331	43714	118045
9	40265	64420	70952	44948	115900
10	32212	70862	68152	46224	114376
11	25770	77949	65816	47527	113343
12	20616	85744	63857	48845	112702

(d) If a new robot is purchased, it should be kept for its full useful life of 12 years, as this is its economic life with EAC(total costs) = \$112 702. For BB, the EAC of installing a new robot is higher than the EAC for both continuing with the current robot or moving the existing robot and using them to the end of their useful lives. BB should avoid replacement for as long as possible, keeping their costs as low as possible in the meantime. This reduces to the question of whether or not BB should move the current robot, and if so, when.

Since the economic life of the current robot has an EAC(total costs) of \$97 808 and the economic life of the moved robot has an EAC(total costs) of \$91 275, the robot should be moved. We now need to answer the question of when it should be moved.

Looking at the cost of keeping defender one more, two more, etc. years, we see that there is no additional life for the defender for which the EAC of that time is less than the EAC associated with the economic life of the challenger (moving the robot). Hence, the current robot should be moved immediately. Furthermore, it should be kept in operation until the end of its useful life.

**7.19** First, it is worth noting that there is a challenger available, and moreover, even if there was not, the furnace is necessary and should not be abandoned. No information is given about subsequent challengers, so it is reasonable to assume that subsequent challengers will be the same as the current challenger.

We do not know the maintenance costs for the gas furnace after the 5-year guarantee period. However, it's reasonable to assume that the economic life of a furnace is longer than 5 years, so that the annual costs over a 5-year study will be no less than the annual costs over the economic life.

The salvage value of the gas furnace at the end of five years can be calculated as:

$$BV_{s1}(5) = 4500 - 5(4500 - 500)/10 = 4500 - 5(400) = 2500$$

The capital cost for the gas furnace is calculated as follows (using the capital recovery formula):

$$A_{gas} = (5000 - 2500)(A/P, 10\%, 5) + 2500(0.1)$$
  
= 2500(0.2638) + 250 = 909.50

Since there are no maintenance costs, the total cost for the new furnace will be about \$910 per year. The energy savings of \$500 will be included as an operating cost for the old furnace.

Capital cost for the old furnace for the next year is:

$$A_{cap} = (500 - 500)(A/P, 10\%, 1) + 500(0.1) = 50$$

Note that it is not zero since \$500 is being tied up in the old furnace.

Operating costs will be \$300 for the maintenance contract, \$200 for parts, and \$500 in extra energy costs.

$$P_{o&m} = 200 + 300 + 500 = 1000$$

Total cost for the next year will be \$1050 for the defender.

By observation, the annual costs for the defender will only increase since the parts charge increases every year and the other cost remain the same per year.

Since the annual costs for the new furnace are more than \$140 less than the old one, even if the new furnace is replaced every 5 years, Nico should replace his furnace immediately with a new gas furnace.

**7.20** (a) We can approximate the cost of using the car by getting the annual cost and dividing by 12.

This implies that all costs are incurred at the end of each year. This will be an understatement of the monthly car cost since, in fact, some of the costs are incurred during the year. If we got the car costs on a monthly basis, we would be overstating the car costs since some of the car costs are incurred less frequently than once a month. The following table shows annual cost for the car.

End of			EAC	EAC	EAC	Annual
year	Salvage	Maintenance	Capital	Maintenance	Total	Cost/12
4	5900					
5	5015	2670	1475	2670	4145	345.42
6	4263	2870	1370	2765	4135	344.58

A conservative estimate of car costs is much higher than the cost of a bus pass.

**(b)** There are many benefits of having a car that are not reflected in this calculation. These may more than offset the higher costs of having a car. Therefore, we cannot say which option is better on the basis of the information given.

## **7.21** (a) The required spreadsheet is as follows:

Defender, v	when new:		EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	100000				
1	85000	15000	30000	15000	45000
2	72250	20000	27907	17326	45233
3	61413	25000	26112	19536	45648
4	52201	30000	24573	21631	46204
5	44371	45000	23251	25097	48348
6	37715	20000	22115	24515	46630
7	32058	25000	21139	24559	45698
8	27249	30000	20300	24955	45255
9	23162	35000	19578	25554	45131
10	19687	50000	18956	26758	45713
11	16734	25000	18420	26685	45105
12	14224	30000	17958	26800	44757
13	12091	35000	17559	27038	44598
14	10277	40000	17215	27358	44574
15	8735	45000	16918	27729	44647

Note that the EAC(total costs) is no longer convex over the life of the turbine. The lowest total costs occur with a replacement interval of 14 years for an EAC(total costs) of \$44 574.

**(b)** A spreadsheet similar to that of part (a), except with a 6-year overhaul cycle gives:

Defender, v	when new (6-ye	ar overhaul cycle):			
			EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	100000				
1	85000	15000	30000	15000	45000
2	72250	20000	27907	17326	45233
3	61413	25000	26112	19536	45648
4	52201	30000	24573	21631	46204
5	44371	35000	23251	23614	46865
6	37715	50000	22115	26628	48744
7	32058	20000	21139	26029	47169
8	27249	25000	20300	25954	46254
9	23162	30000	19578	26195	45773
10	19687	35000	18956	26629	45585
11	16734	40000	18420	27178	45598
12	14224	55000	17958	28137	46095
13	12091	25000	17559	28046	45605
14	10277	30000	17215	28094	45310
15	8735	35000	16918	28240	45158

The minimum EAC is with a 15-year life, but the total EAC is \$45 158, more than that for the 5-year overhaul cycle. Ener-G should not use a 6-year overhaul cycle.

# **7.22 (a)** The present worth of costs for the defender are shown in the following table.

Defender:					
Additional			PW	PW	PW
years	Salvage	Operating	Capital	Operating	Total
0	1200				
1	600	20000	664	17857	18521
2	300	20500	961	34200	35160
3	150	21013	1093	49156	50249

**(b)** The challenger's costs for three years are shown in the following table.

Challenger:			PW	PW	PW
Year	Salvage	Operating	Capital	Operating	Total
0	20000				
1	14000	13875	12500	12388	24888
2	9800	14361	17188	23837	41024
3	6860	14863	20117	34416	54533

(c) We want to find the salvage value, S, such that

PW(defender cost)

= PW(challenger operating cost) + challenger(first cost) – S(P/F, 12%, 3)

Solving this we get: S = \$12879

Alternate method: We find the difference between the present worths of the two projects, and project it to the end of three years:

$$(54\ 533.15 - 50\ 249.13)(F/P,\ 12\%,\ 3) = (4284.02)(1.404928) = 6018.74$$

Since the challenger has the higher PW of costs, adding \$6018.74 to its salvage value after three years will make its PW equal to the defender's:

$$6018.74 + 6860.00 = 12878.74$$

The challenger's salvage value would have to be \$12 879 to equalize the two projects' present worths.

**7.23 (a)** The economic life of the challenger is shown in the following table. We see that the economic life is 11 years. The minimum equivalent annual cost is about \$20 144 per year.

Challenger	:		PW	PW	PW	EAC
Year	Salvage	Operating	Capital	Operating	Total	Total
0	20000					
1	14000	13875	12500	12388	24888	27875

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2	9800	14361	17188	23837	41024	24274
3	6860	14863	20117	34416	54533	22705
4	4802	15383	21948	44192	66141	21776
5	3361	15922	23093	53227	76320	21172
6	2353	16479	23808	61576	85384	20768
7	1647	17056	24255	69291	93546	20498
8	1153	17653	24534	76421	100955	20323
9	807	18271	24709	83009	107718	20216
10	565	18910	24818	89098	113916	20161
11	395	19572	24886	94724	119611	20144
12	277	20257	24929	99924	124853	20156

**(b)** We need to know the equivalent annual cost over its economic life for a new copy of the defender to decide whether we should replace the current defender with the challenger or with a new copy of the defender. The economic life and the equivalent annual cost for a new copy of the defender are shown below.

Defender	when new:		PW	PW	PW	EAC
Year	Salvage	Operating	Capital	Operating	Total	Total
0	15000					
1	9846	17250	8709	15402	24110	27004
2	6464	17681	12347	29497	41845	24759
3	4243	18123	14480	42397	56877	23681
4	2785	18576	15730	54203	69933	23024
5	1828	19041	16463	65007	81469	22600
6	1200	19517	16892	74895	91787	22325
7	600	20005	17229	83944	101172	22169
8	300	20505	17379	92225	109604	22064
9	150	21017	17446	99804	117250	22005
10	150	21543	17452	106741	124192	21980
11	150	22081	17457	113089	130545	21986
12	150	22633	17461	118898	136359	22013
13	150	23199	17466	124215	141680	22056

We see that new copies of the defender have economic lives of 10 years. The equivalent annual cost over this life is about \$21 980. This is greater than the cost for a challenger. Therefore, we should replace the current defender with a challenger.

**(c)** The current defender should be replaced immediately. The equivalent annual costs for any feasible life are greater than the minimum equivalent annual costs.

Defender at 6 Years Old:						
Additional			PW	PW	PW	EAC
years	Salvage	Operating	Capital	Operating	Total	Total
0	1200					
1	600	20000	664	17857	18521	20744
2	300	20500	961	34200	35160	20804
3	150	21013	1093	49156	50249	20921

## **7.24** (a) There are 10 alternatives:

additional years	for Y years	5562	challenger 0	average 5562
for X	challenger	EAC defender	EAC	Weighted
Keep defender	Then use			

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8	1	5497	5700	5520
7	2	5439	5495	5451
6	3	5391	5349	5377
5	4	5359	5249	5310
4	5	5349	5186	5258
3	6	5359	5150	5220
2	7	5407	5137	5197
1	8	5500	5140	5180
0	9	0	5156	5156

- **(b)** The alternative which has the least weighted average of equivalent annual costs is to replace the defender immediately and use the challenger for 9 years.
- **7.25** Note that it is obvious that the propane model is inherently better. First find the EAC for the service life of the challenger.

New propa	ne model:		EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	58000				
1	40000	10000	22640	10000	32640
2	32000	11200	17140	10577	27717
3	25600	12544	14620	11183	25803
4	20480	14049	12966	11819	24785
5	16384	15735	11734	12487	24220
6	13107	17623	10760	13187	23946
7	10486	19738	9965	13921	23886
8	8389	22107	9304	14691	23995
9	6711	24760	8747	15497	24244

The economic life for the new forklifts is 7 years at an EAC of about \$23 886. Compare this with the cost of the electric fork lifts for the next year:

2 year old Defei	nder:				
Additional			EAC	EAC	EAC
years	Salvage	Maintenance	Capital	Maintenance	Total
0	10000				
1	8000	20000	2800	20000	22800
2	6400	22400	2531	21154	23685
3	5120	25088	2303	22366	24669
4	4096	28099	2110	23638	25748
5	3277	31470	1946	24973	26919
6	2621	35247	1806	26374	28179
7	2097	39476	1686	27842	29528
8	1678	44214	1582	29381	30964
9	1342	49519	1493	30994	32487

EAC(capital) = 
$$(10\ 000 - 10\ 000 \times 0.8)(A/P, 8\%, 1) + (10\ 000 \times 0.8)(0.08)$$

= 2000(1.08) + 8000(0.08) = 2800

EAC(oper.) = 20 000EAC(total) = 22 800

The electric fork lifts' average cost will not exceed that of the propane fork lifts until the third year. Conclusion: Keep the electric fork lifts, at least for another year.

**7.26** The trade-in allowance of \$14 000 should be treated as a \$4000 reduction in the price of the new propane forklift. The modified costs are:

New propane model:			EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	54000				
1	40000	10000	18320	10000	28320
2	32000	11200	14897	10577	25474
3	25600	12544	13068	11183	24251
4	20480	14049	11759	11819	23578
5	16384	15735	10732	12487	23218
6	13107	17623	9894	13187	23081
7	10486	19738	9197	13921	23118
8	8389	22107	8608	14691	23299
9	6711	24760	8107	15497	23604

Since the costs of keeping the electric forklift for an additional year is \$22 800, less than the minimum EAC(total costs) for the propane model, Chatham Automotive should not replace the electric forklifts now.

## **7.27** (a) Sample calculations:

EAC(capital, 2 additional years)

= (P - S)(A/P, 10%, 2) + Si

 $= (33\ 000 - 25\ 555.20)(0.57619) + 25\ 555.20(0.1) \approx 6845.00$ 

EAC(operating, 2 additional years)

= [3400(F/P, 10%, 1) + 3900](A/F, 10%, 2)

 $= [3400(1.1) + 3900](0.47619) \cong 3638$ 

The summary of the calculation results:

Year	Salvage value	Op/Maint cost	Capital EAC	Operating EAC	Total EAC
0	33 000				
1	29 040	3400	7260	3400	10 660
2	25 555	3900	6845	3638	10 483
3	22 488	4400	6476	3868	10 344

A new joint former has to be less than about \$10 344.

- **(b)** Yes, replace the old joint former with the new one since the EAC for the economic life of the new one is lower than any of the EAC's for the old one. Replace it now.
- **(c)** Since the EAC for the old asset appears to be decreasing, there may be a period for which its EAC is lower than that of the challenger. It is necessary to check if replacement should occur more than three years from now.
- **7.28** Reading from the first table, the challenger's economic life is 5 years, with

an EAC of \$39 452.

			EAC	EAC	EAC
Year	Salvage	O&M	Capital	O&M	Total
0	90000				
1	72000	12000	31500	12000	43500
2	57600	14400	28570	13116	41686
3	46040	17280	26159	14315	40475
4	36864	20736	24141	15601	39742
5	29491	24883	22474	16978	39452
6	23593	29860	21086	18449	39536
7	18874	35832	19927	20020	39947
8	15099	42998	18957	21694	40651
9	12080	51598	18142	23476	41618

## For the defender:

			EAC	EAC	EAC
Year	Salvage	O&M	Capital	O&M	Total
0	49000				
1	36500	17000	19850	17000	36850
2	19875	21320	20897	19009	39906
3	15656	26806	16952	21255	38207
4	6742	33774	15813	23762	39575

Keep the defender for 3 more years before replacing it by the challenger. Note that the defender becomes undesirable over a 2-year period, but for a 3 year period is again preferred.

## 7.29 Example Calculations (row 2):

$$EAC(Capital\ Cost) = (40\ 970\ -\ 22\ 310)(A/P,\ 12\%,\ 2) + (22\ 310)(0.12)$$

EAC(Operating & Maintenance)

= [16 500(P/F, 12%, 1) + 20 690(P/F, 12%, 2)](A/P, 12%, 2)

 $EAC(Overhaul) = 14\,000(A/P, 12\%, 2)$ 

Life	Salvage	Operating&	EAC	EAC	EAC	EAC
(years)	Value	Maintenance	Op&M	Capital	Overhaul	Total
0	55000				14000	
1	40970	16500	14348	16500	16100	54880
2	22310	20690	15645	18449	8612	41885
3	17574	26013	17104	20627	6132	31469
4	7568	32775	18739	23060	4904	32604

The old cutter should be overhauled since the EAC corresponding to the economic life is less with an overhaul (\$31 469) than without one (\$38 207). The new cutter should replace the old after 4 years since the EAC for the old cutter after 4 years is still less (\$32 604) than the EAC corresponding to the economic life for the new cutter (\$39 452).

## **7.30** Overhaul in 3 years:

Life	Salvage	Operating&	EAC	EAC	EAC	EAC
(years)	Value	Maintenance	Op&M	Capital	Overhaul	Total

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0	10000					
1	8000	500	500	4100		4600
2	6400	800	643	3290		3933
3	5120	1100	781	2876		3657
4	4096	500	720	2588	237	3545
5	3277	800	733	2365	198	3297
6	2621	1100	781	2186	173	3139
7	2097	1400	846	2038	154	3039
8	1678	1700	921	1915	141	2977
9	1342	2000	1000	1811	130	2942
10	1074	2300	1082	1723	122	2927
11	859	2600	1164	1647	116	2927
12	687	2900	1245	1582	110	2938
13	550	3200	1325	1526	106	2957

## Overhaul in 5 years:

Life	Salvage	Operating&	EAC	EAC	EAC	EAC
(years)	Value	Maintenance	Op&M	Capital	Overhaul	Total
0	10000					
1	8000	500	500	4100		4600
2	6400	800	643	3290		3933
3	5120	1100	781	2876		3657
4	4096	1400	914	2588		3502
5	3277	1700	1043	2365		3408
6	2621	500	973	2186	143	3301
7	2097	800	954	2038	128	3120
8	1678	1100	967	1915	116	2999
9	1342	1400	999	1811	108	2918
10	1074	1700	1043	1723	101	2867
11	859	2000	1095	1647	96	2838
12	687	2300	1151	1582	91	2824
13	550	2600	1210	1526	87	2824
14	434	2900	1271	1477	84	2832
15	352	3200	1331	1435	81	2848

Example Calculations (row 5 for overhaul in 3 years):

```
EAC(Capital Cost)
```

= 
$$(10\ 000 + 1000 - 10\ 000 \times 0.8^{5})(A/P, 10\%, 6) + (10\ 000 \times 0.8^{5})(0.1)$$

EAC(Operating & Maintenance)

= [500(P/F, 10%, 1) + 800(P/F, 10%, 2)]

+ 1100(P/F, 10%, 3) + 500(P/F, 10%, 4) + 800(P/F, 10%, 5)](A/P, 10%, 5)

EAC(Overhaul) = 1000(P/F, 10%, 4)(A/P, 10%, 5)

The water pump should be overhauled since the EAC corresponding to the economic life is less with an overhaul (\$2927 or \$2824) than without one (\$3317). Further, it should be overhauled in 5 years.

## C. More Challenging Problems

**7.31** The economic life is when the slopes of the two cost functions are equal. The slopes are the derivatives of the cost functions.

$$500n(2^{n-1}) = 10000n(0.8)^{n-1}$$

$$2^{n-1} = 20(0.8)^{n-1}$$

$$(n-1)\ln(2) = \ln(20) + (n-1)\ln(0.8)$$

$$\ln 2 = \ln(20)/(n-1) + \ln(0.8)$$

$$n = \ln(20)/[\ln(2) - \ln(0.8)] + 1 = 4.27$$

The economic life of the asset is about 4.27 years.

- **7.32** (a) The economic life is 14 years.
  - (b) The equivalent annual cost over 14 years is \$8138 per year.

Costs and	Costs and Salvage Values for Various Lives: MARR = 0.12						
	PW	PW	EAC	EAC	EAC		
Year	Capital	Operating	Capital	Operating	Total		
0	5000						
1	10714	2679	12000	3000	15000		
2	14796	5250	8755	3106	11861		
3	17711	7717	7374	3213	10587		
4	19794	10086	6517	3321	9837		
5	21281	12359	5904	3429	9332		
6	22344	14541	5435	3537	8971		
7	23103	16635	5062	3645	8707		
8	23645	18646	4760	3753	8513		
9	24032	20575	4510	3861	8372		
10	24309	22427	4302	3969	8271		
11	24506	24204	4127	4076	8204		
12	24647	25910	3979	4183	8162		
13	24748	27548	3853	4289	8141		
14	24820	29120	3745	4393	8138		
15	24871	30628	3652	4497	8149		
16	24908	32076	3572	4599	8171		
17	24934	33466	3502	4701	8203		

- (c) The economic life is now only 12 years.
- (d) The equivalent annual cost is now only \$7152.

	PW	PW	EAC	EAC	EAC
Year	Capital	Operating	Capital	Operating	Total
0	5000				
1	9762	2857	10250	3000	13250
2	13390	5782	7201	3110	10311
3	16154	8777	5932	3223	9155
4	18260	11843	5150	3340	8490
5	19865	14982	4588	3461	8049
6	21088	18196	4155	3585	7740
7	22019	21487	3805	3713	7519
8	22729	24855	3517	3846	7362
9	23270	28304	3274	3982	7256
10	23682	31835	3067	4123	7190
11	23996	35450	2889	4268	7157
12	24235	39152	2734	4417	7152
13	24417	42941	2599	4571	7171
14	24556	46821	2481	4730	7211
15	24662	50792	2376	4893	7269
16	24742	54859	2283	5062	7345
17	24804	59022	2200	5235	7435
18	24850	63285	2126	5414	7540

- **(e)** Decreasing the MARR reduces the capital cost, thus decreasing the total costs. Also, since the capital costs decrease with increased life, and are generally smaller with smaller MARR, they will form a minimum total cost sooner; thus the economic life will be shorter.
- **7.33** Note that in the table below, the true salvage value at time 0 is actually \$25 000.

The spreadsheet uses the \$23 000 shown as the initial cost in the capital cost formula. The \$23 000 is composed of the \$25 000 minus the \$2000 net bonus. The salvage values for year 1 onwards are based on the true initial salvage value of \$25 000, not the \$23 000.

New equip	ment:		EAC	EAC	EAC
Year	Salvage	Maintenance	Capital	Maintenance	Total
0	23000				
1	20000	6000	4840	6000	10840
2	16000	6600	5205	6288	11494
3	12800	7260	4982	6588	11570
4	10240	7986	4672	6898	11570
5	8192	8785	4364	7220	11584
6	6554	9663	4082	7553	11635
7	5243	10629	3830	7898	11728
8	4194	11692	3608	8254	11862
9	3355	12862	3413	8623	12036

Note that there is a minimum cost of \$10 840 at year 1. This comes about due to the trade-in bonus of \$2000 that only occurs in the first year. A more credible service life is 3 or 4 years, with an annual cost of \$11 570.

The capital costs of using the defender for one more year are:

EAC(capital, 1 more year) = 
$$(10\ 000 - 8\ 000)(A/P, 8\%, 1) + 8000(0.08) = 2000(1.08) + 640 = 2800$$

The operating and maintenance costs are 9000 for a total:

EAC(keep defender 1 more year) = 
$$2800 + 9000 = 11800$$

This is higher than the EAC of the challenger at its economic life (either 3 or 4 years) with an EAC of 11 570. Now we ask the question if there is a life for the defender which will have a lower EAC than the challenger at its economic life.

The EAC of capital costs for keeping the defender two more years is:

EAC(capital, keep 2 more years) = 
$$(10\ 000 - 6400)(A/P, 8\%, 2) + 6400(0.08) = 2531$$

The EAC of operating and maintenance costs are:

EAC(op. and maint., keep 2 more years) = 9000 + 0.1×9000(P/F, 8%, 2)(A/P, 8%, 2) = 9000 + 900(0.85734)(0.56077) = 9433

The total EAC for keeping the defender for two more years is then: EAC(keep 2 more years) = 2531 + 9433 = 11964

Given that capital costs are going down at a slower rate than the operating and maintenance costs are increasing, the costs of keeping the defender for three more years, four more years, etc., will only increase. Thus, there is no additional life for keeping the defender such that the EAC of its costs will be lower than the challenger at its economic life. Hence, we should replace immediately.

## **7.34** (a) There are 22 possible combinations. They are shown in the table.

	Year	1	2	3	4	5	6	7	8	9	10
1	defender challenger 1	Х	Х	Х	Х	Х	.,	.,	.,	.,	.,
2	challenger 2 defender challenger 1 challenger 2	Х	Х	Х	Х	х	X X	X	X X	X X	X
3	defender challenger 1 challenger 2	Х	Х	Х	х	x	x	x	x	X	x
4	defender challenger 1 challenger 2	Х	х	х	x	х	x	x	х	х	х
5	defender challenger 1 challenger 2	х	х	х	х	х	х	х	х	х	х
6	defender challenger 1 challenger 2	Х	х	х	х	х	х	х	х	х	х
:											

13	defender	Х									
	challenger 1		Х								
	challenger 2			Х	Х	Х	Х	Х	Х	Х	Х
14	defender										
	challenger 1	Х	Х	Х	Х	Х	Х	Х	Х	Х	Х
	challenger 2										
15	defender										
	challenger 1	Х	Х	Х	Х	Х	Х	Х	Х	Х	
	challenger 2										х
÷											
22	defender										
	challenger 1	х	Х								
	challenger 2			Χ	Х	Χ	Х	Х	Χ	Х	Х

**(b)** The best combination is number 4. Wait two years and install the second challenger.

The defender has lower costs than the first challenger over the next year. See the first row of the table labelled "Defender Now" and the row for the minimum EAC in the 7th year for the first challenger. Therefore, all combinations starting with number 14, that have the first challenger replacing the defender now, are eliminated.

The costs for the defender in the 2nd year are higher than the costs for the first challenger for a life of at least 3 years. This means that we must consider combinations in which we replace the defender with the first challenger after one year. We would need to keep the 1st challenger for at least 3 years for this to be worthwhile. This eliminates combinations 12 and 13 that have the 1st challenger replace the defender in the 2nd year and being used for 1 or 2 years only.

We next consider combinations that have the 1st challenger for at least 3 years. We shall also compare these with combination 4 that does not include the 1st challenger. The present worth of combination 11 that has the 1st challenger for 3 years starting in the 2nd year is given by:

PW(cost for defender for 1 year)

- + PW(costs for 1st challenger for 3 years)(P/F, 10%, 1)
- + PW(costs for 2nd challenger for 6 years)(P/F, 10%, 4)

The present worth of combination 10 that has the 1st challenger for 4 years is obtained in a similar manner. The present worth of combination 4 that has the second challenger replace the defender after two years is obtained similarly. The present worths of costs for combinations 4, 10 and 11 are:

PW(costs for combination 4) = \$126 004 PW(costs for combination 10) = \$138 629 PW(costs for combination 11) = \$135 660

We see that keeping the 1st challenger for 4 years has higher cost than keeping it for 3 years. We infer from this that we need not compute the present worth of costs that have the 1st challenger last longer than 3 years. As well, going directly to the 2nd challenger after 2 years has lower cost than either combination involving the 1st challenger. It is apparent that we need not consider combinations that include the first challenger.

We must consider combinations that have the second challenger come in after more than 2 years. We see that the second challenger has lower costs than any incremental years for the defender after 2 years. See the tables below. We conclude, therefore, that combination 4 has the lowest cost.

Defender Now:						
Additional			PW	PW	PW	EAC
Years	Salvage	Operating	Capital	Operating	Total	Total
0	4000					
1	3000	20000	1273	18182	19455	21400
2	2000	25000	2347	38843	41190	23733
3	1000	30000	3249	61382	64631	25989
4	500	35000	3658	85288	88946	28060
5	500	40000	3690	110125	113814	30024

Defender's Co	st after 1 More \	ear:				
Additional			PW	PW	PW	EAC
Years	Salvage	Operating	Capital	Operating	Total	Total
0	3000					
1	2000	25000	1182	22727	23909	26300
2	1000	30000	2174	47521	49694	28633
3	500	35000	2624	73817	76441	30738
4	500	40000	2658	101137	103796	32745

Defender after	2 More Years:					
Additional			PW	PW	PW	EAC
Years	Salvage	Operating	Capital	Operating	Total	Total
0	2000					
1	1000	30000	1091	27273	28364	31200
2	500	35000	1587	56198	57785	33295
3	500	40000	1624	86251	87875	35336

First Cha	llenger:		PW	PW	PW	EAC
Year	Salvage	Operating	Capital	Operating	Total	Total
0	25000					
1	20000	16800	11818	15273	27091	29800
2	16000	17640	16777	29851	46628	26867
3	12800	18522	20383	43767	64150	25796
4	10240	19448	23006	57050	80056	25255
5	8192	20421	24913	69730	94643	24967
6	6554	21442	26301	81833	108134	24828
7	5243	22514	27310	93386	120696	24792
8	4194	23639	28043	104414	132457	24828
9	3355	24821	28577	114941	143518	24920
10	2684	26062	28965	124989	153954	25055

Second Ch	allenger:		PW	PW	PW	EAC
Year	Salvage	Operating	Capital	Operating	Total	Total
0	25000					
1	20000	12000	11818	10909	22727	25000
2	16000	12600	16777	21322	38099	21952
3	12800	13230	20383	31262	51645	20767
4	10240	13892	23006	40750	63756	20113
5	8192	14586	24913	49807	74721	19711
6	6554	15315	26301	58452	84753	19460
7	5243	16081	27310	66704	94014	19311
8	4194	16885	28043	74581	102625	19236
9	3355	17729	28577	82100	110677	19218
10	2684	18616	28965	89278	118243	19243

**7.35 (a)** The costs of keeping the car for the additional years are as follows. If you do not paint the car, you should keep it an additional 5 years before replacing it. The EAC(total costs) is \$5880.

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Car, when two	years old:				
Additional			EAC	EAC	EAC
years	Salvage	O&M	Capital	O&M	Total
0	12000				
1	9000	2100	4440	2100	6540
2	6750	2520	3916	2298	6215
3	5063	3024	3496	2513	6009
4	3797	3629	3156	2747	5903
5	2848	4355	2881	3000	5880
6	2136	5225	2656	3274	5930
7	1602	6271	2471	3571	6042
8	1201	7525	2318	3892	6210

**(b)** The costs for painting the car after 3 years:

Car with painting and salvage value increased by \$2435:							
Additional			EAC	EAC	EAC		
years	Salvage	O&M	Capital	O&M	Total		
0	12000						
1	9000	2100	4440	2100	6540		
2	6750	2520	3916	2298	6215		
3	7498	5024	2774	3106	5880		
4	5623	3629	2774	3215	5990		
5	4217	4355	2665	3395	6060		
6	3163	5225	2529	3620	6149		
7	2372	6271	2394	3883	6277		
8	1779	7525	2271	4179	6450		

In the above table, the operating and maintenance costs in year 3 include the \$2000 cost of painting the car, and the salvage value has been increased by \$2435, then falling by 25% each year thereafter. With the car's value increased by \$2435, the minimum EAC(total costs) is \$5880 and the economic life of the car is 3 more years. If the painting increases the car's value by more than \$2435, the EAC(total costs) will fall below \$5880 and painting the car will be the preferred choice. In addition to this, rather than keeping the car an additional 5 years, it will be sold at the end of 3 years.

**7.36** Considering periods of 18 months in length, it can be seen that computers of equal power cost the following:

Period	Cost
-2	\$320 000
-1	\$160 000
0	\$80 000
1	\$40 000
2	\$20 000

Consequently, the current computer has a salvage value of

 $320\ 000(0.5^3) = $40\ 000$ 

There are four obvious replacement plans available:

- 1) Not replace the computer but keep it another 3 years
- 2) Replace the computer now, and keep the replacement for 3 years
- 3) To replace the computer in 18 months
- 4) Replace the computer now and in 18 months

In fact there are an infinite number of possible replacement plans since replacement could be considered at any point in time over the next 3 years. However, the fact that we are assuming that new models are being released every 18 months makes it natural to consider this limited set.

Plan 1: Do not replace the computer but keep it another 3 years

Salvage value after 3 years:  $S = 40\ 000(1 - 0.5)^3 = $5\ 000$ 

Capital cost:

$$P = 40\ 000 - 5000(P/F, 12\%, 3) = 40\ 000 - 5000(0.71178) = $36\ 441$$

Maintenance costs:

#### Year 1

Accumulated depreciation =  $320\ 000 - (40\ 000 \times 0.5) = 300\ 000$ Maintenance cost =  $300\ 000(0.1) = 30\ 000$ 

#### Year 2

Accumulated depreciation =  $320\ 000 - (40\ 000 \times 0.5^2) = 310\ 000$ Maintenance cost =  $310\ 000(0.1) = 31\ 000$ 

#### Year 3

Accumulated depreciation =  $320\ 000 - (40\ 000 \times 0.5^3) = 315\ 000$ Maintenance cost =  $315\ 000(0.1) = 31\ 500$ 

```
PW(costs, plan 1)
= 36 441 + 30 000(P/F, 12%, 1) + 31 000(P/F, 12%, 2)
+ 31 500(P/F, 12%, 3)
= 36 441 + 30 000(0.89286) + 31 000(0.79719) + 31 500(0.71178)
= 110 361
```

The present cost for retaining the defender for three more years is about \$110 000.

Plan 2: replace the computer now, and keep the replacement for 3 years

Cost of new computer now = \$80 000

Salvage value after 3 years:  $S = 80\ 000 \times (1 - 0.5)^3 = $10\ 000$ 

## Capital cost:

Purchase price including installation = 80 000(1.15) = 92 000 Salvage value of old computer = 40 000

#### Maintenance costs:

#### Year 1

Accumulated depreciation =  $80\ 000 - (80\ 000 \times 0.5) = 40\ 000$ Maintenance cost =  $40\ 000(0.1) = 4000$ 

#### Year 2

Accumulated depreciation =  $80\ 000 - (80\ 000 \times 0.5^2) = 60\ 000$ Maintenance cost =  $60\ 000(0.1) = 6000$ 

#### Year 3

Accumulated depreciation =  $80\ 000 - (80\ 000 \times 0.5^3) = 70\ 000$ Maintenance cost =  $70\ 000(0.1) = 7000$ 

The present cost for replacing the defender now and keeping the challenger for three years is about \$58 600.

#### Plan 3: Replace the current computer in 18 months

In order to carry out calculations for a computer bought in 18 months, it is necessary to determine a MARR and a depreciation rate for an 18-month period.

$$MARR_{18} = (1 + MARR_{12})^{1.5} - 1 = 1.12^{1.5} - 1 = 18.53\%$$

#### Depreciation rate:

One way to calculate the depreciation rate for 18 months is to recognize that after three years or two 18-month periods, the book value must be the same, i.e., for some book value at time 0, P:

$$P(1 - d_{18})^2 = P(1 - d_{12})^3$$

$$(1 - d_{18})^2 = (1 - d_{12})^3$$
  
 $d_{18} = 1 - (1 - d_{12})^{3/2} = 1 - (1 - 0.5)^{3/2} = 0.646$ 

Salvage value of current computer in 18 months:  $40\ 000(1-0.646) = \$14\ 142$ 

Cost of new computer in 18 months = \$40 000

Salvage value of new computer after 18 months (three years from now):  $40\ 000(1-0.646) = \$14\ 142$ 

Capital cost of current computer:

$$P_{cur} = 40\ 000 - 14\ 142(P/F, 18.53\%, 1)$$
  
= 40\ 000 - 14\ 142(1/1.1853) = 28\ 069

Capital cost of new computer as present worth, noting installation cost of \$40 000×0.15 =\$6 000:

$$P_{\text{new}} = [46\ 000 - 14\ 142(P/F, 18.53\%, 1)](P/F, 18.53, 1)$$
  
=  $[46\ 000 - 14\ 142(1/1.1853)](1/1.1853) = 28\ 743$ 

Total capital cost:  $P = 28\ 069 + 28\ 743 = \$56\ 812$ 

Maintenance costs:

#### Period 1

Accumulated depreciation =  $320\ 000 - 320\ 000(1 - 0.646)^3 = 305\ 804$ Maintenance cost =  $305\ 804(0.15) = 45\ 871$ 

#### Period 2

Accumulated depreciation =  $40\ 000 - (40\ 000 \times 0.646) = 25\ 840$ Maintenance cost =  $25\ 840(0.15) = 3876$ 

PW(costs; plan 3) = 56 812 + 45 871(P/F, 18.53%, 1) + 3876(P/F, 18.53%, 2) = 56 812 + 45 871(1/1.1853) + 3876(1/1.1853<sup>2</sup>) = 98 271

The present cost for replacing the defender 18 months from now is about \$98 300.

Plan 4: Replace the computer now and in 18 months

Cost of first new computer today = \$80 000

Salvage value of first new computer after 18 months  $= 80\ 000(1 - 0.646) = $28\ 284$ 

Cost of second new computer in 18 months = \$40 000

Salvage value of second new computer after 18 months (three years from now) =  $40\ 000(1-0.646)$  = \$14 142

Capital cost of first new computer as present worth, noting installation cost of  $\$80\ 000\times0.15 = \$12\ 000$ :

$$P_1 = (96\ 000 - 40\ 000) - 28\ 284(P/F, 18.53\%, 1)$$
  
= 56\ 000 - 28\ 284(1/1.1853) = 32\ 138

Capital cost of second new computer as present worth, noting installation cost of \$40 000×0.15= \$6000:

$$P_2 = [46\ 000 - 14\ 142(P/F, 18.53\%, 1)](P/F, 18.53, 1)$$
  
=  $[46\ 000 - 14\ 142(1/1.1853)](1/1.1853) = 28\ 743$ 

Total capital cost: P = 32 138 + 28 743 = 60 881

Maintenance costs:

#### Period 1

Accumulated depreciation =  $80\ 000 - 80\ 000(1-0.646) = 51\ 680$ Maintenance cost =  $51\ 680(0.15) = 7752$ 

#### Period 2

Accumulated depreciation =  $40\ 000 - (40\ 000 \times 0.646) = 25\ 840$ Maintenance cost:  $25\ 840(0.15) = 3876$ 

```
PW(costs, plan 4)
= 60 881 + 7752(P/F, 18.53%, 1) + 3876(P/F, 18.53%, 2)
= 100 881 + 7752(1/1.1853) + 3876(1/1.1853<sup>2</sup>)
= 70 180
```

The present cost for replacing the defender immediately and 18 months from now is about \$70 000.

A summary of these alternative replacement decisions and their respective costs is shown in the following table. It can be seen that the best choice is to replace the current computer, and keep the replacement for three years (plan 2).

Plan PW(capital cost)	PW(maintenance)	PW(total)
-----------------------	-----------------	-----------

Chapter 7 - Replacement Decisions

1	36 441	73 920	110 361
2	44 882	13 718	58 600
3	56 812	41 488	98 300
4	60 881	9 299	70 180

## **7.37** (a) There are 8 feasible combinations:

	Truck kept for	r between time poin	t a and point b: de	noted by (a, b)
Truck	Comb.1	Comb.2	Comb.3	Comb.4
#1	(0, 5)	(0, 5)	(0, 4)	(0, 4)
#2	(2, 5)	(2, 4)	(4, 5)	(4, 5)
#3	(4, 5)	(4, 5)	(2, 5)	(2, 4)
#4		(4, 5)	(4, 5)	(4, 5)
#5				(4, 5)
#6				
EAC	57 149	83 210	83 819	109 880

	Truck kept for	between time poin	t a and point b: der	noted by (a, b)
Truck	Comb.5	Comb.6	Comb.7	Comb.8
#1	(0, 2)	(0, 2)	(0, 2)	(0, 2)
#2	(2, 5)	(2, 5)	(2, 4)	(2, 4)
#3	(2, 5)	(2, 4)	(4, 5)	(4, 5)
#4	(4, 5)	(4, 5)	(2, 5)	(2, 4)
#5		(4, 5)	(4, 5)	(4, 5)
#6		. ,	. ,	(4, 5)
EAC	77 180	103 242	103 242	129 303

## **Example Calculations:**

$$EAC(0, 5) = 30\ 000(A/P, 12\%, 5) + 7200$$

= 
$$(30\ 000 - 30\ 000 \times 0.6^4)(A/P,\ 12\%,\ 4) + 0.12(30\ 000 \times 0.6^4) + 7200$$

EAC(0, 2)

= 
$$(30\ 000 - 30\ 000 \times 0.6^2)(A/P, 12\%, 2) + 0.12(30\ 000 \times 0.6^2) + 7200$$

$$EAC(2, 5) = [30\ 000(A/P, 12\%, 3) + 7200](P/F, 12\%, 2)$$

EAC(2, 4)

= 
$$[(30\ 000 - 30\ 000 \times 0.6^2)(A/P, 12\%, 2) + 0.12(30\ 000 \times 0.6^2) + 7200](P/F, 12\%, 2)$$

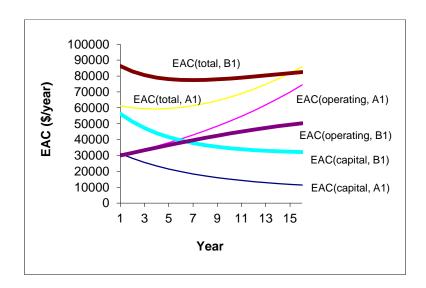
$$EAC(4, 5) = [30\ 000(A/P, 12\%, 1) + 7200](P/F, 12\%, 4)$$

- **(b)** Combination 1 has the smallest EAC, and hence, the best purchase/replacement option (buy a truck at time 0, time 2, and time 4, and once purchased, keep each truck until the end of 5 years).
- **7.38** (a) The economic lives are shown in the following table.

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		A1			B1	
	EAC	EAC	EAC	EAC	EAC	EAC
Year	Capital	Operating	Total	Capital	Operating	Total
1	31250	30000	61250	56250	30000	86250
2	28201	31829	60030	51250	31667	82917
3	25600	33777	59376	47250	33320	80570
4	23372	35851	59223	44050	34949	78999
5	21459	38059	59518	41490	36546	78036
6	19810	40412	60221	39442	38102	77544
7	18383	42918	61301	37804	39609	77412
8	17144	45589	62733	36493	41061	77554
9	16065	48435	64500	35444	42453	77897
10	15121	51469	66590	34605	43780	78386
11	14293	54703	68996	33934	45040	78975
12	13564	58152	71716	33397	46231	79629
13	12919	61830	74749	32968	47352	80320
14	12347	65753	78100	32624	48403	81027
15	11839	69937	81776	32350	49384	81734
16	11385	74401	85786	32130	50298	82427

(b)



(c) We see that the economic life is longer when the MARR is higher. Intuitively the reason for this is as follows: We are assuming that the assets under consideration are parts of sequences of repeated identical assets. Replacement of productive assets in a situation of repeated identical assets is done only to avoid rising operating costs from ageing assets. Where the MARR is high the effect of distant increases in operating costs are felt less than where the MARR is low. Therefore, we replace assets less frequently when the MARR is high.

A more formal explanation is as follows: The economic life occurs at the point where the rate of increase in the equivalent annual operating cost equals the absolute value of the rate of decrease in the equivalent annual capital cost. We see from the diagram that the curves for the equivalent annual operating costs for the two values of the MARR diverge. The curve for MARR = 5% becomes much steeper than the curve for MARR = 25%.

The curves for the equivalent annual capital costs are essentially parallel. That is, the slopes of the curves for the equivalent annual capital costs are insensitive to the MARR. This means that equality in absolute value for the slopes of the curves for equivalent annual operating cost and equivalent annual capital cost occurs farther to the left in the diagram for MARR = 5% than for MARR = 25%.

We can see that the curves for the two equivalent annual capital costs are essentially parallel by noting that the difference between them for one-year life is the same as the difference between their asymptotic values. The formula for the equivalent annual capital cost is

$$EAC = (P - S)(A/P, i, N) + Si$$

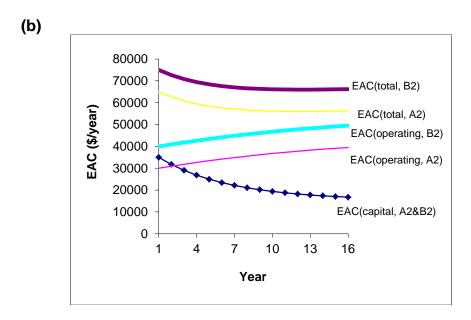
For N = 1, this reduces to: EAC = 
$$(P - S)(1 + i) + Si$$

The difference between the equivalent annual capital costs for two different values of MARR, i and i', is simply P(i - i').

Notice that the asymptotic value of capital recovery factor as the number of periods goes to infinity is i. We see that the asymptotic difference between the equivalent annual capital costs for two different values of the MARR is also P(i-i').

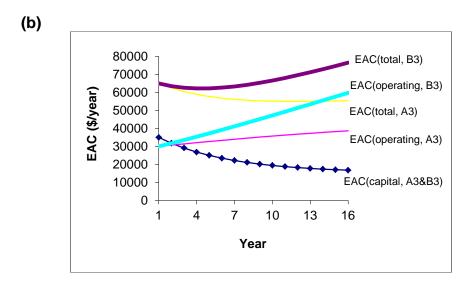
7.39 (a) The economic lives are shown in the following table. Notice that there is only one column for the equivalent annual capital cost. This is because the equivalent annual capital cost is the same for both assets.

		A2		B2	
	EAC	EAC	EAC	EAC	EAC
Year	Capital	Operating	Total	Operating	Total
1	35000	30000	65000	40000	75000
2	31744	30930	62674	40930	72674
3	29053	31814	60868	41814	70868
4	26824	32653	59476	42653	69476
5	24972	33446	58417	43446	68417
6	23429	34194	57623	44194	67623
7	22141	34900	57041	44900	67041
8	21063	35563	56625	45563	66625
9	20158	36184	56342	46184	66342
10	19396	36766	56163	46766	66163
11	18754	37310	56064	47310	66064
12	18211	37816	56028	47816	66028
13	17751	38288	56039	48288	66039
14	17360	38725	56085	48725	66085
15	17028	39130	56158	49130	66158
16	16744	39504	56249	49504	66249



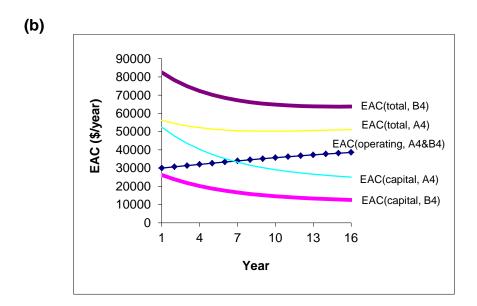
- **(c)** We see that the economic life is unaffected by the level of operating cost. Only the rate of change in operating cost affects the economic life.
- 7.40 (a) The economic lives are shown in the following table. Notice that there is only one column for the equivalent annual capital cost. This is because the equivalent annual capital cost is the same for both assets.

		A3		B3	
	EAC	EAC	EAC	EAC	EAC
Year	Capital	Operating	Total	Operating	Total
1	35000	30000	65000	30000	65000
2	31744	30698	62442	31744	63488
3	29053	31382	60436	33537	62590
4	26824	32052	58876	35375	62198
5	24972	32707	57678	37255	62227
6	23429	33345	56774	39175	62604
7	22141	33964	56105	41131	63272
8	21063	34565	55628	43119	64182
9	20158	35147	55304	45136	65294
10	19396	35708	55104	47178	66574
11	18754	36248	55002	49241	67995
12	18211	36767	54979	51322	69533
13	17751	37266	55017	53417	71168
14	17360	37742	55102	55523	72883
15	17028	38197	55225	57636	74664
16	16744	38631	55375	59752	76497



- **(c)** We see that the economic life is shorter for Asset B3 than for Asset A3. This is because the more rapid increase in operating cost makes more frequent investment in replacing ageing assets worthwhile.
- 7.41 (a) The economic lives are shown in the following table. Notice that there is only one column for the equivalent annual operating cost. This is because the equivalent annual operating cost is the same for both assets.

		A4		B4	
	EAC	EAC	EAC	EAC	EAC
Year	Operating	Capital	Total	Capital	Total
1	30000	26250	56250	52500	82500
2	30698	23808	54506	47616	78314
3	31382	21790	53172	43580	74962
4	32052	20118	52170	40235	72288
5	32707	18729	51436	37457	70164
6	33345	17572	50916	35144	68488
7	33964	16606	50570	33212	67176
8	34565	15797	50362	31594	66159
9	35147	15118	50265	30237	65383
10	35708	14547	50255	29095	64802
11	36248	14066	50314	28131	64379
12	36767	13658	50426	27317	64084
13	37266	13313	50579	26627	63892
14	37742	13020	50762	26040	63782
15	38197	12771	50968	25542	63739
16	38631	12558	51189	25116	63747



**(c)** We see that the economic life is longer where the level of capital cost is higher. This is because a greater savings in operating cost is necessary to pay for a higher investment cost.

## **7.42** (a) The economic life for this asset is shown in the following table.

				PW	PW	PW	PW	EAC
Year	Salv.	Oper.	Overhaul	Market cost	Overhaul	Capital	Oper.	Total
	15000							
1	12000	2000		30000		30000	1667	38000
2	9600	2200		33333		33333	3194	23909
3	7680	2420		35556		35556	4595	19060
4	7500	2662	2500	36383	1206	37589	5879	16791
5	6000	2000		37589	1206	38794	6682	15207
6	4800	2200		38392	1206	39598	7419	14138
7	3840	2420		38928	1206	40134	8095	13380
8	4500	2662	32500	38953	8764	47718	8714	14707
9	3600	2000		39302	8764	48066	9101	14182
10	2880	2800		39535	8764	48299	9553	13799
11	2304	3920		39690	8764	48454	10081	13528
12	2000	5488	17500	39776	10727	50503	10697	13786
13	1200	4000		39888	10727	50615	11070	13609
14	720	8000		39944	10727	50671	11694	13526
15	432	16000		39972	10727	50699	12732	13567

In this table the term market cost refers to the cost due to the decline in the value of the asset. Capital cost is the sum of market cost and overhaul cost.

**(b)** We first show costs for an 8 year old defender.

Defende	Defender that is 8 years old:								
			PW	PW	PW	PW	PW	EAC	
Year	Salv.	Oper.	Overhaul	Market	Overhaul	Capital	Oper.	Total	Total

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1	3600	2000		1500		1500	1667	3167	3800
2	2880	2800		2500		2500	3611	6111	4000
3	2304	3920		3167		3167	5880	9046	4295
4	2000	5488	17500	3535	8439	11975	8526	20501	7919
5	1200	4000		4018	8439	12457	10134	22591	7554
6	720	8000		4259	8439	12698	12813	25511	7671
7	432	16000		4379	8439	12819	17278	30097	8350

The present worth of cost for Plan A is given by

PW(Plan A)

- = PW(3 additional years for defender)
- + PW(7 year life for first replacement)(P/F, 20%, 3)
- + PW(1 year life for second replacement)(P/F, 20%, 10)

The present worth of cost for Plan B is obtained similarly. The numerical values for these present worths are:

PW(Plan A cost) = \$42 071PW(Plan B cost) = \$60 359

- (c) It is necessary to take into account the age of equipment at the end of the 11 years because future costs after the 11 years are affected by the equipment's age. The effect of the equipment's age on future costs may not be reflected in the salvage value. The salvage value depends on the value of the equipment outside the current user's operation. Other potential users would have to incur costs to remove the equipment from its current situation. As well, if they were to use the equipment in production elsewhere, they would incur installation costs. They might also have to modify the equipment to their own special needs. Therefore, the salvage value is not likely to reflect all of the contributions the equipment can make to the current user's operation. Since the equipment will be only one year old under Plan A and 4 years old under Plan B, there is no doubt that Plan A is better than Plan B.
- **7.43** The present worths of costs for the new punch press and the three old ones are shown below.

Automatic	Punch Press:		PW	PW	PW
Year	Salvage	Operating	Capital	Operating	Total
0	125 000				
1	100 000	25 000	145 000	20 000	165 000
2	80 000	23 750	173 800	35 200	209 000
3	64 000	22 563	192 232	46 752	238 984
4	51 200	21 434	204 028	55 532	259 560
5	40 960	20 363	211 578	62 204	273 782

Hand-Fed Presses:						
Additional			PW	PW	PW	PW
years	Salvage	Operating	Capital	Operating	Total	3 Presses
0	10 000					
1	9 000	25 000	2 800	20 000	22 800	68 400
2	8 000	25 000	4 880	36 000	40 880	122 640
3	7 000	25 000	6 416	48 800	55 216	165 648
4	6 000	25 000	7 542	59 040	66 582	199 747
5	5 000	25 000	8 362	67 232	75 594	226 781

We see that using the existing presses appears to cost about £47 000 less than using a new press for 5 years. This means that, if the new automatic press is to have lower costs than the 3 manual presses, the salvage value of the new press after 5 years must exceed the estimated £40 960.

The value required for equality of costs is given by

Alternate method: The salvage value required for equality of costs is found by first taking the difference in project present worths forward 5 years, to the point in time of the final salvage value. This difference is added to the current salvage value estimate for the automatic press:

**7.44 a)** There is no relationship to the intersection of the lines and the lowest total cost. One's eye is misled to that conclusion by the shape of the lines intersecting. The two points would only correspond by coincidence. More precisely, if the function for capital costs is C(n), where n is the number of periods, and the function for operating costs is O(n), the minimum total

costs occurs when 
$$\frac{d}{dn} (C(n) + O(n)) = 0$$
, or equivalently

$$\frac{d}{dn}C(n)+\frac{d}{dn}O(n)=0$$
 . The cost lines intersect at n\* such that  $C(n^*)=O(n^*)$  and the point of intersection will also be the minimum total cost when

$$\frac{d}{dn}C(n^*) = \frac{d}{dn}O(n^*)$$
 . Thus only when the slopes of the two lines are equal and opposite in sign at the same time that their values are equal will the intersection of the lines and the lowest total cost coincide.

**b)** The situation in Table 7.2 reflects the replacement of a defender with a challenger that is not identical. If an identical challenger had been

## Chapter 7 - Replacement Decisions

available, it would have replaced the defender at the time of lowest total cost as illustrated in Table 7.1. Also, any non-identical challengers to the defender in the past must have had a minimum EAC greater than the current costs of the defender. Consequently this situation will persist until a successful challenger is found. This circumstance can happen in a variety of situations such as escalating machinery costs, regulatory changes, technology changes, etc.

# Notes for Case-in-Point 7.1

**1-4)** All of these issues are difficult to address and the answers depend on the views of the individual student.

# Notes for Mini-Case 7.1

- 1) It would make economic sense to replace the bulbs immediately only if the cost of one more day of operating the existing bulb was greater than the net cost per day of a new bulb over its service life
- The savings per year for Sears amounts to about  $$0.65 \times 16\ 000\ 000 =$   $$10\ 400\ 000$ , or about 10,400,000/130,000 = \$80 per bulb. This is clearly an economic issue as well as a publicity issue.
- 3) All of these reasons could be sensible

## Solutions to All Additional Problems

Note: Solutions to odd-numbered problems are provided on the Student CD-ROM.

#### **7S.1**

The \$26 000 cost of the old machine is a sunk cost, and thus irrelevant to the analysis.

The present cost of switching to the new machine is

```
PC = 11\ 000 - 8000 - 1000(P/F, 0.2, 5) - 2000(P/A, 0.2, 5)
= 3000 - 1000(0.4019) - 2000(2.9906)
= -3383
```

Thus, the company will save \$3383 by replacing the old machine. So they should do it.

## **7S.2**

```
Retiring the machine now has a present value of £10 000.
Retiring it in a year's time has a present value of:
PV_1 = (5000+6000)(P/F,0.15,1) = 11\ 000(0.8696) = £9566
```

```
Retiring it in two years time has a present value of:

PV_2 = 5000(P/F, 0.15, 1) + (4000+4000)(P/F, 0.15, 2)
```

```
= 5000(P/P, 0.15, 1) + (4000+4000)(P/P, 0.15, 2)= 5000(0.8696) + 8000(0.7561) = £10 397
```

Retiring it in three years has a present value of:

```
PV_3 = 5000(P/F,0.15,1) + 4000(P/F,0.15,2) + (3000+2000) (P/F,0.15,3)
= 5000(0.8696) + 4000(0.7561) + 5000(0.6575) = £10 660
```

And waiting until the end of the fourth year has a present value of:

```
PV_4 = 5000(P/F,0.15,1) + 4000(P/F,0.15,2) + 3000(P/F,0.15,3) + 2000(P/F,0.15,4)
```

```
= 5000(0.8696) + 4000(0.7561) + 3000(0.6575) + 2000(0.5717) = £10
```

We conclude that the machine should be retired at the end of the third year.

#### **7S.3**

The first thing to do is to see if it is worth replacing one truck now.

The cost of *not* replacing the truck is the present value of the repairs needed to keep it running; this is R 10.000(P/A,0.1,0.2,10).

We evaluate this using the methods given in Chapter 3. First, we calculate the growth-adjusted interest rate,  $i^0$ :

 $i^0 = (1+0.2) / (1+0.1) - 1 = 0.091$ 

Then (P/A,0.1,0.2,10)=(P/A,0.091,10)/(1+0.1)=5.8086

So the present cost of keeping an old truck running for ten years is R 58 086.

Buying a new truck has a present cost of:

R 100 000 – 100 000(0.9) $^{10}$ (P/F,0.2,10) = R 100 000(1-0.0563)=R 94 370

So we can conclude that it is not worth replacing one truck this year. And if it is not worth replacing one truck when our MARR is 20%, it is certainly not worth borrowing money at 30% to replace all of them.

We next ask whether it will be worth replacing an old truck at any time in the next ten years. We set up a spreadsheet (**7S\_3.xls**) to compare the cost of maintaining versus replacing. In developing this spreadsheet, there are a couple of points to note. First, assume the cost figures given are incurred at the beginning of the year in which we make the replacement decision. So we can compare costs within the same row, but not across rows. Secondly, note that the cost of maintaining a truck which has N years of life remaining is R 10 000(1.1<sup>(10-N)</sup>)(P/A,0.1,0.2,N). We must include the factor of 1.1<sup>(10-N)</sup> to take account of the increase in maintenance costs between the present and the decision year.

Our conclusion is that the company should wait to replace the trucks until they are worn out. It can set aside R 100 000 a year in a sinking fund so that it will have the capital to do this.

Years	Maintenance	Cost of one new	
Remaining	Costs	truck	
10	58 086	94 369	
9	59 710	92 492	
8	60 657	89 989	
7	60 695	86 652	
6	59 530	82 202	
5	56 801	76 270	
4	52 062	68 359	
3	44 764	57 813	
2	34 234	43 750	
1	19 648	25 000	

Consider a study period of five years. Suppose that the salvage value of the workstations drops at the same rate as their cost.

The present cost of keeping your current equipment another N years is  $\forall 1\ 000\ 000\ (P/A, 0.2, 0.2, N)$ 

The present cost of bringing in low-end workstations after N years is  $\$45\ 000\ 000((0.6^N)(P/F,0.2,N) - (0.6^5)(P/F,0.2,5))$ 

The present cost of bringing in high-end workstations after N years is  $\forall 7 500 000((0.7^N)(P/F,0.2,N) - (0.7^5)(P/F,0.2,5)) - 1 000 000(P/A,0.2,(5-N))(P/F,0.2,N)$ 

The present cost of starting to rent workstations after N years is  $\forall 1\ 000\ 000(P/A,0.2,(5-N))(P/F,0.2,N)$ 

The present value of keeping your current equipment *N* years, then switching to one of the three alternatives, is calculated in the accompanying spreadsheet, **7S\_4.xls**. All figures on the spreadsheet are in thousands of won. Comparing the last three columns, we see that the best strategy is to wait two years, then replace your old machines with high-end workstations.

				Low-	High	
N	(P/F, 0.2, N)	1000(P/A,0.2,0.2,N)	(P/A, 0.2, (5-N))	Cost	Cost	Rent
0	1.0000	0	2.9906	4844	4003	2991
1	0.8333	833	2.5887	3177	2544	2991
2	0.6944	1667	2.1065	2760	2249	3130
3	0.5787	2500	1.5278	2969	2598	3384
4	0.4823	3333	0.8333	3490	3293	3735
5	0.4019	4167	0.0000	4167	4167	4167

#### **7S.5**

The present value of getting rid of the video game machine after *N* years is an increased profit of Rs 500 per year, extending indefinitely into the future, less the cost of getting rid of the machine. This is

$$(500/0.1 - 200)(P/F,0.1,N) = 4800(P/F,0.1,N)$$

The present value of keeping the machine for *N* more years is the income it provides, less the cost of repairs. We represent the income as an annuity of Rs 1200 plus an arithmetic gradient of –Rs 100. This is

$$(1200 - 100(A/G, 0.1, N)) (P/A, 0.1, N) - 500(P/A, 0.1, 0.1, N)$$
  
=  $(1200 - 100(A/G, 0.1, N)) (P/A, 0.1, N) - 500N/1.1$ 

The Rs 5000 paid for the machine is a sunk cost that does not come into the calculation.

We add these two amounts together and plot their value against *N* in the attached spreadsheet, **7S\_5.xls**.

Mr Patel's initial analysis shows he should get rid of the machine in two years, but if he includes the money brought in by extra soda sales, he should keep it for another six years.

