Exercise 88

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Thomas' Calculus Early Transcendentals

ISBN: 9780134439020

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Explanation

Verified

Step 1

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$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Find $f'(0)$.

Step 2

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$$\begin{aligned} f'(0) &= \lim_{h\to 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h\to 0} \frac{e^{-\frac{1}{h^2}} - 0}{h} && \bullet \text{ Substitution} \\ &= \lim_{h\to 0} \frac{\frac{1}{h}}{e^{\frac{1}{h^2}}} && \bullet \diamondsuit e^{-a} = \frac{1}{e^a} \\ &\longrightarrow \frac{\infty}{\infty} && \bullet \text{ Indeterminate form} \end{aligned}$$

Substitute $f(h) = e^{-\frac{1}{h^2}}$ for $h \neq 0$ and evaluate limit.

Step 3

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$$t(h) = \frac{1}{h}$$
$$g(h) = e^{\frac{1}{h^2}}$$

$\mathit{Step1}$: Identify $t(h)$ and $g(x)$ where $f(h) = \frac{t(h)}{g(h)}$

Step 4

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$$t'(h) = -\frac{1}{h^2}$$
$$\begin{aligned} g'(h) &= e^{\frac{1}{h^2}} \cdot \frac{d}{dh}\left[\frac{1}{h^2}\right] && \bullet \frac{d}{dh}\left[e^{q(h)}\right] = e^{q(h)} \cdot q'(h) \\ &= e^{\frac{1}{h^2}} \cdot -\frac{2}{h^3} \end{aligned}$$

$\mathit{Step2}$: Find derivatives $t'(h)$ and $g'(h)$

Step 5

$$\lim_{h\to 0} = \lim_{h\to 0} \frac{\frac{1}{h}}{e^{\frac{1}{h^2}}} \qquad \bullet \diamondsuit$$

$$= \lim_{h\to 0} \frac{-\frac{1}{h^2}}{e^{\frac{1}{h^2}} \cdot -\frac{2}{h^3}} \qquad \bullet \text{ Substitution of derivatives}$$

$$= \lim_{h\to 0} \frac{h}{2} e^{-\frac{1}{h^2}} \qquad \bullet\, e^{-\frac{1}{h^2}} \longrightarrow 0 \text{ as } h \to 0$$

$$= 0$$

$Step3$: Apply L'Hopitals Rule

Result

0

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