Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

(a) Find the series' radius and interval of convergence. Find the values of x for which the series converges **(b)** absolutely and **(c)** conditionally.

$$\sum_{n=0}^{\infty} \frac{(x-6)^n}{8^n}$$

(a) Find the radius and interval of convergence.

Use the ratio test to find the interval on which the series converges. Begin by finding $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|$.

$$\begin{aligned}\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| &= \lim_{n\to\infty}\left|\frac{(x-6)^{n+1}/8^{n+1}}{(x-6)^n/8^n}\right| \\ &= \left|\frac{x-6}{8}\right|\end{aligned}$$

The series will converge whenever $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \frac{|x-6|}{8} < 1$. To find the interval of convergence, solve this inequality for x.

Since 8 is positive, $\left|\frac{x-6}{8}\right|$ can be rewritten as $\frac{|x-6|}{8}$.

To find the interval of convergence, solve the inequality $\frac{|x-6|}{8} < 1$ for x. Each possible solution corresponds to one of the interval's bounds.

$$\frac{|x-6|}{8} < 1$$

$$|x-6| < 8 \qquad \text{Multiply both sides by 8.}$$

$$-8 < x-6 < 8$$

$$-2 < x < 14$$

Next, determine whether or not the series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{8^n}$ converges at the two endpoints, $x = -2$ and $x = 14$. Take the limit of the series at each point, starting with $x = -2$. If the nth term does not approach zero, the series diverges.

$\lim_{n\to\infty} \frac{(-2-6)^n}{8^n}$ does not exist.

Therefore, the series diverges at $x = -2$.

Do the same for $x = 14$.

$$\lim_{n\to\infty} \frac{(14-6)^n}{8^n} = 1$$

Therefore, the series diverges at $x = 14$.

Thus, the interval of convergence for the series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{8^n}$ is $-2 < x < 14$. Notice that the endpoints are not included in the interval since the series diverges at each of these endpoints.

To find the radius of convergence, R, calculate the distance between the point on which the series is centered and one of the interval's endpoints. Determine the point, a, on which the series is centered. Use $\left|\frac{x-a}{R}\right| = \left|\frac{x-6}{8}\right|$, where R is the radius of convergence and a is the center point.

$R = 8$

(b) Find the values of x for which the series converges absolutely.

A power series converges absolutely for x along an interval $|x-a| < R$, where R is the radius of convergence. Given that the series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{8^n}$ has the ratio $\left|\frac{x-6}{8}\right|$, find the interval along which the series converges absolutely.

$$\begin{aligned} |x-6| &< 8 \\ -8 < x-6 &< 8 \\ -2 < x &< 14 \end{aligned}$$

Note that 8 is positive.

The series $\sum_{n=0}^{\infty} \frac{(x-6)^n}{8^n}$ converges absolutely along the interval $-2 < x < 14$.

(c) Find the values of x for which the series converges conditionally.

By definition, a series converges conditionally along any interval in which it converges, but does not do so absolutely. The series converges along the interval $-2 < x < 14$ and converges absolutely along the same interval. As such, the series never converges conditionally.