Student: Arfaz Hossain **Date:** 12/05/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 10

Express the solution of the initial value problem in terms of an integral.

$$\frac{dy}{dx} = \sqrt{2 + x^2}, \ y(2) = -3$$

Find the function $y = F(x)$ with derivative $f(x) = \frac{dy}{dx} = \sqrt{2 + x^2}$ that satisfies the condition that $F(2) = -3$.

According to the first part of the fundamental theorem of calculus, if f is continuous on [a,b] then $F(x) = \int_a^x f(t)\,dt$ is continuous on [a,b] and differentiable on (a,b) and the following equation is true.

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Since $F(2) = -3$, then $F(2) = \int_2^2 \sqrt{2 + t^2}\,dt - 3 = -3$ because $\int_2^2 \sqrt{2 + t^2}\,dt = 0$.

Thus, the solution of the initial value problem $\frac{dy}{dx} = \sqrt{2 + x^2}$, $y(2) = -3$ in terms of an integral with variable t is shown below.

$$y = \int_2^x \sqrt{2 + t^2}\,dt - 3$$