CSC 225

Algorithms and Data Structures I

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ECS 516

Sorting

- **Sorting definition.** The process of ordering a sequence of objects according to some linear order.
- **Total versus partial order.** If any two elements in a set are comparable, then the set can be totally ordered otherwise partially ordered.
- Total order
  - 9 9 14 17 86
  - Coady Müller Stege Storey Thomo
- Partial order (Topological sort)
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Partial Order

Partial Order

A relation, $\leq$, on a set A is called a partial order if $\leq$ is

i. **Reflexive:** For all $k \in A, k \leq k$.

ii. **Antisymmetric:** For all $k_1, k_2 \in A$, if $k_1 \leq k_2$ and $k_2 \leq k_1$, then $k_1 = k_2$.

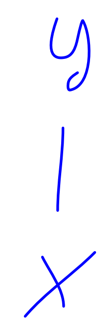
iii. **Transitive:** For all $k_1, k_2, k_3 \in A$, if $k_1 \leq k_2$ and $k_2 \leq k_3$, then $k_1 \leq k_3$.

Partially Ordered Sets (Posets)

Partially Ordered Set

Let A be a set and $\leq$ a relation on A. The pair $(A, \leq)$ is called a <u>partially ordered set</u> (or <u>poset</u>) if $\leq$ on A is a partial order.

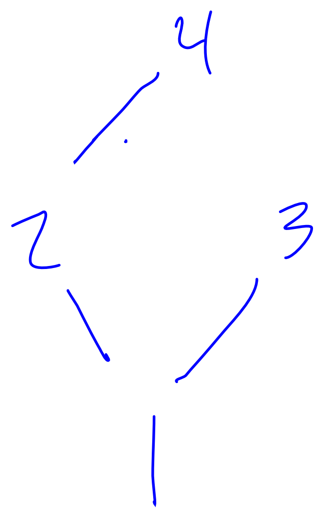
Hasse Diagram



If $\leq$ is a partial order on A, we construct a <u>Hasse diagram</u> for $\leq$ on A by connecting x "up" to y if and only if $x \leq y$ and there are no other $z \in A$ such that $x \leq z$ and $z \leq y$.

Example 1

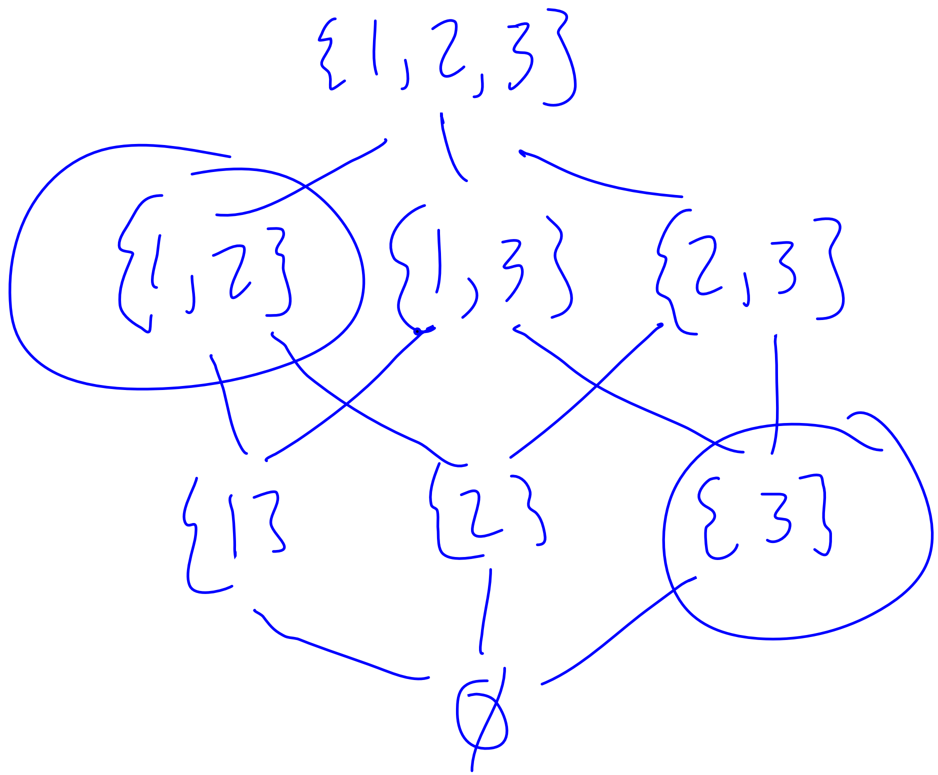
Let $A = \{1,2,3,4\}$ and define $\leq$ on A by $x \leq y$ if $x, y \in A$ and $x|y$. Draw a Hasse diagram for $\leq$.



Example 2

Consider the power set, $P(A)$, where $A = \{1,2,3\}$. Draw the Hasse diagram to illustrate the subset relation. $\preceq = \subseteq$

$P(A)$



Total Order

Total Order

If $(A, \leq)$ is a poset, it is a total order if for all $k_1, k_2 \in A$, either $k_1 \leq k_2$ or $k_2 \leq k_1$.

Examples

- Letters of the alphabet via lexicographic order.
- The set of real numbers by $\leq$ or $\geq$ relations.

Computational Problem: **Sorting**

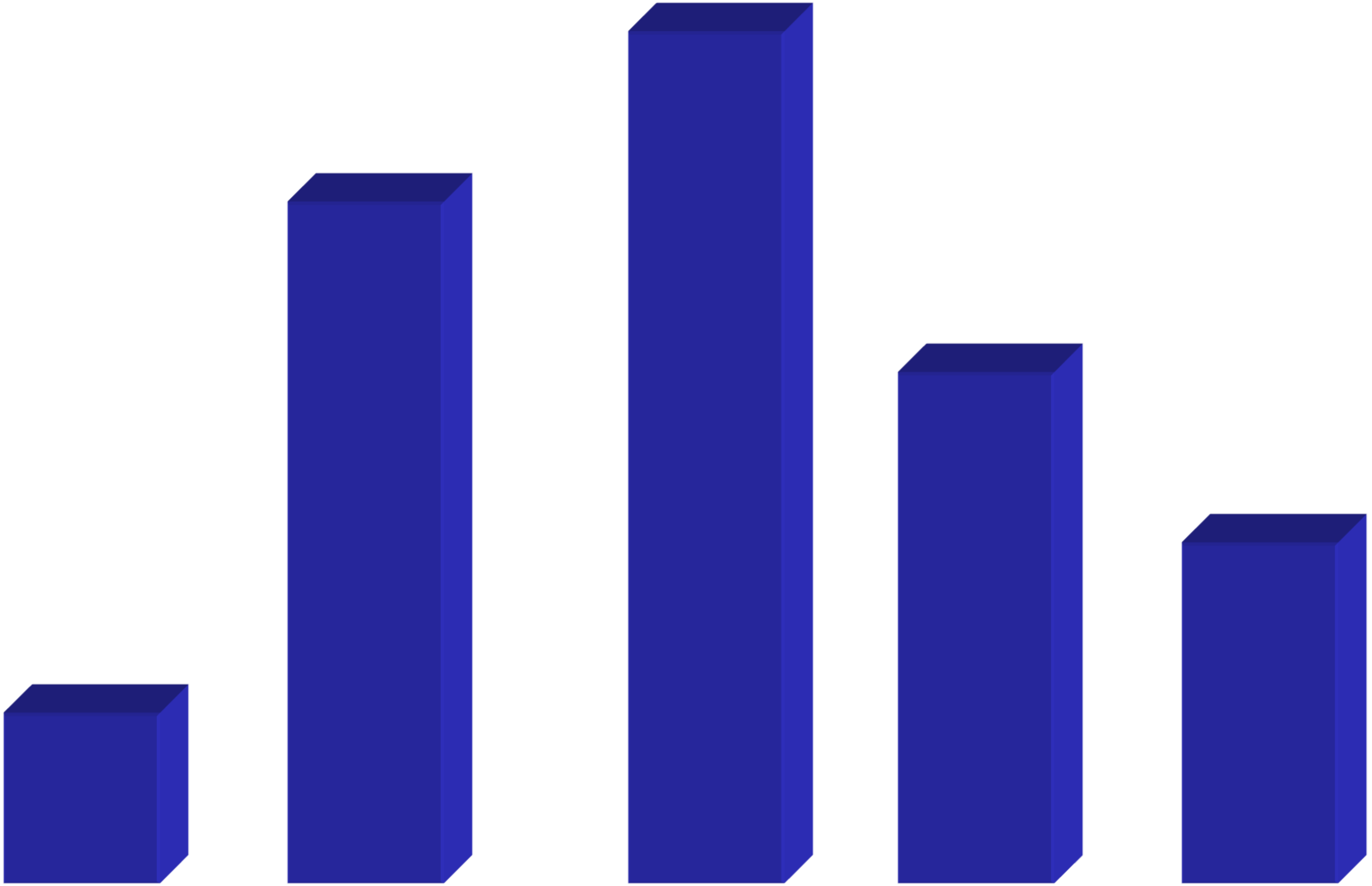
Input: A collection of *n* objects (stored in a data structure) and a comparator defining a total order on these objects

Output: Produce a linear ordered representation (ascending or descending) of these objects

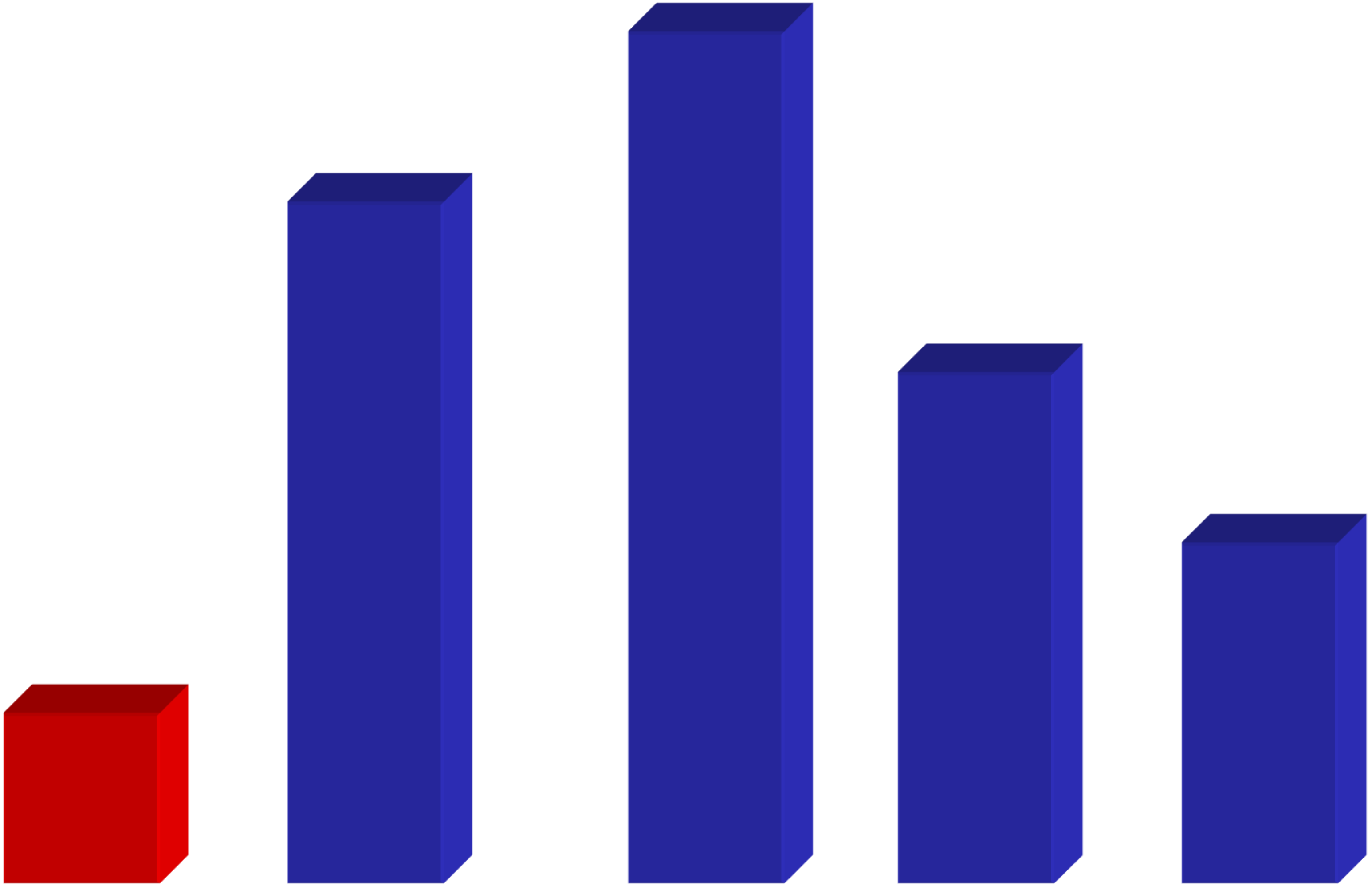
Algorithm Design Technique
Brute Force

- **Brute force** is a straightforward approach to solving a problem, usually directly based on the problem statement and definitions of the concepts involved.
- Simplest algorithm design technique
- Does not usually produce the most elegant or most efficient algorithms
- Examples of the Brute Force technique
  - Selection sort
  - Bubble sort
  - Insertion sort

Selection Sort Animation



Selection Sort Animation



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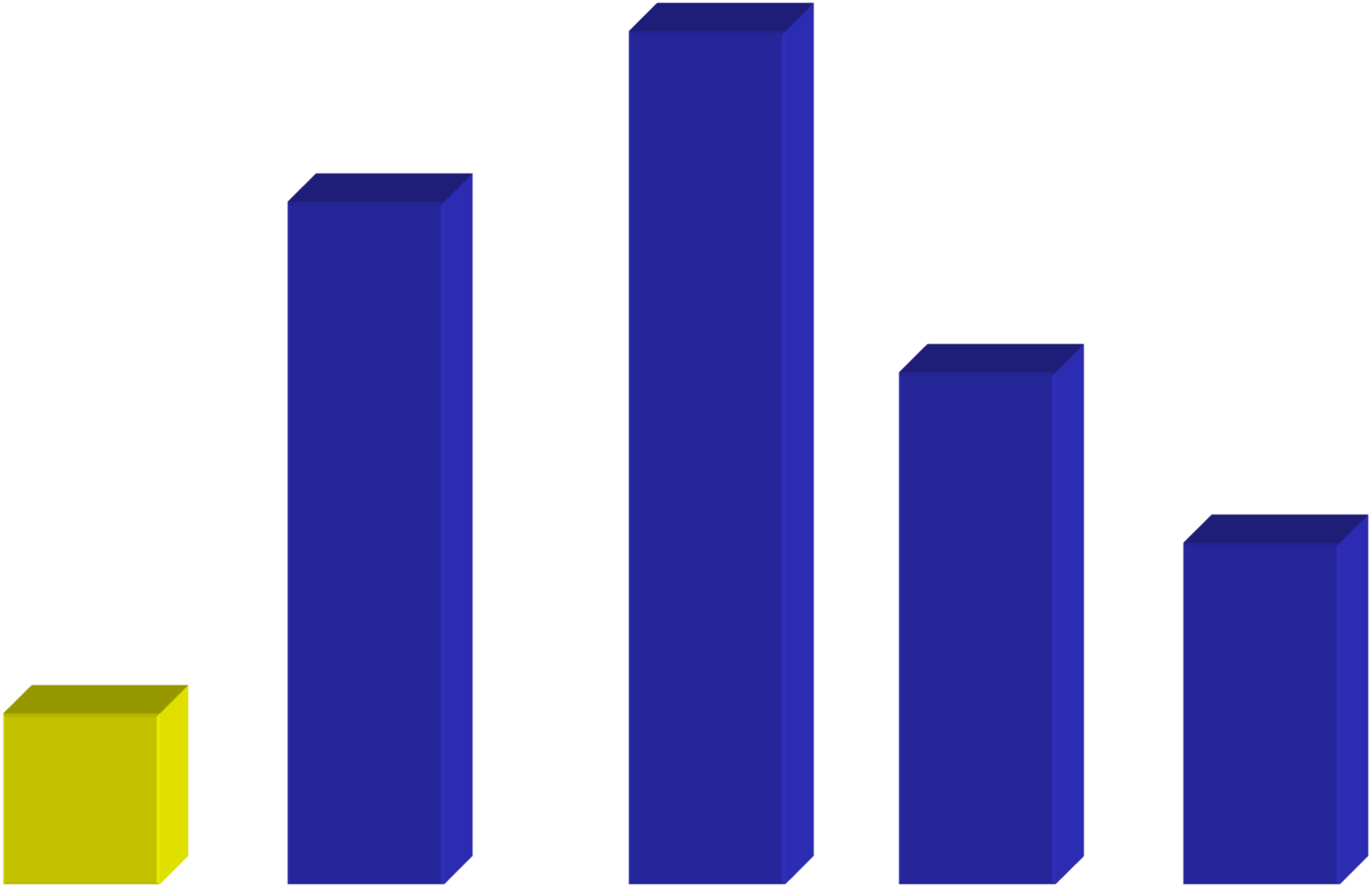

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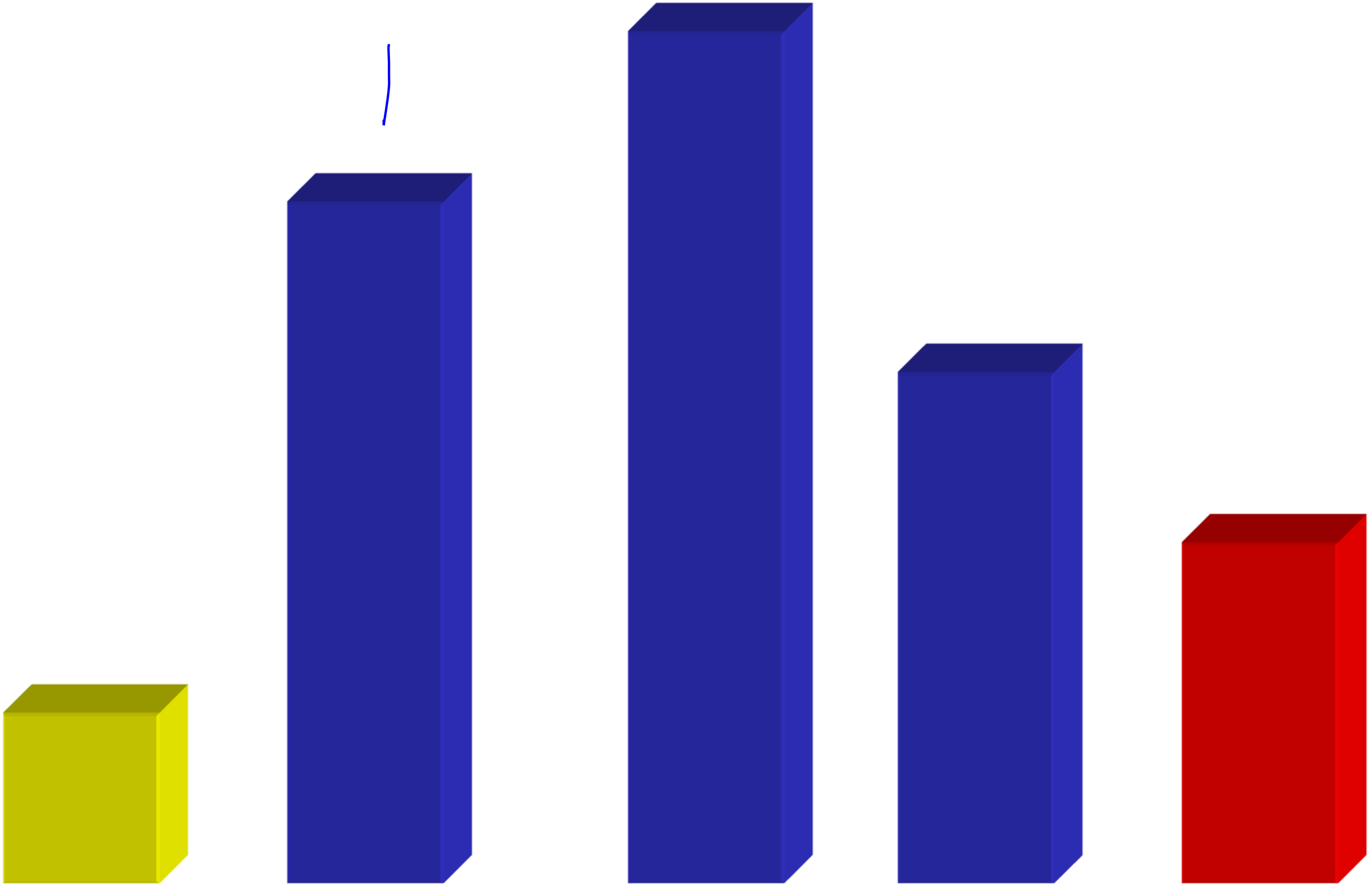

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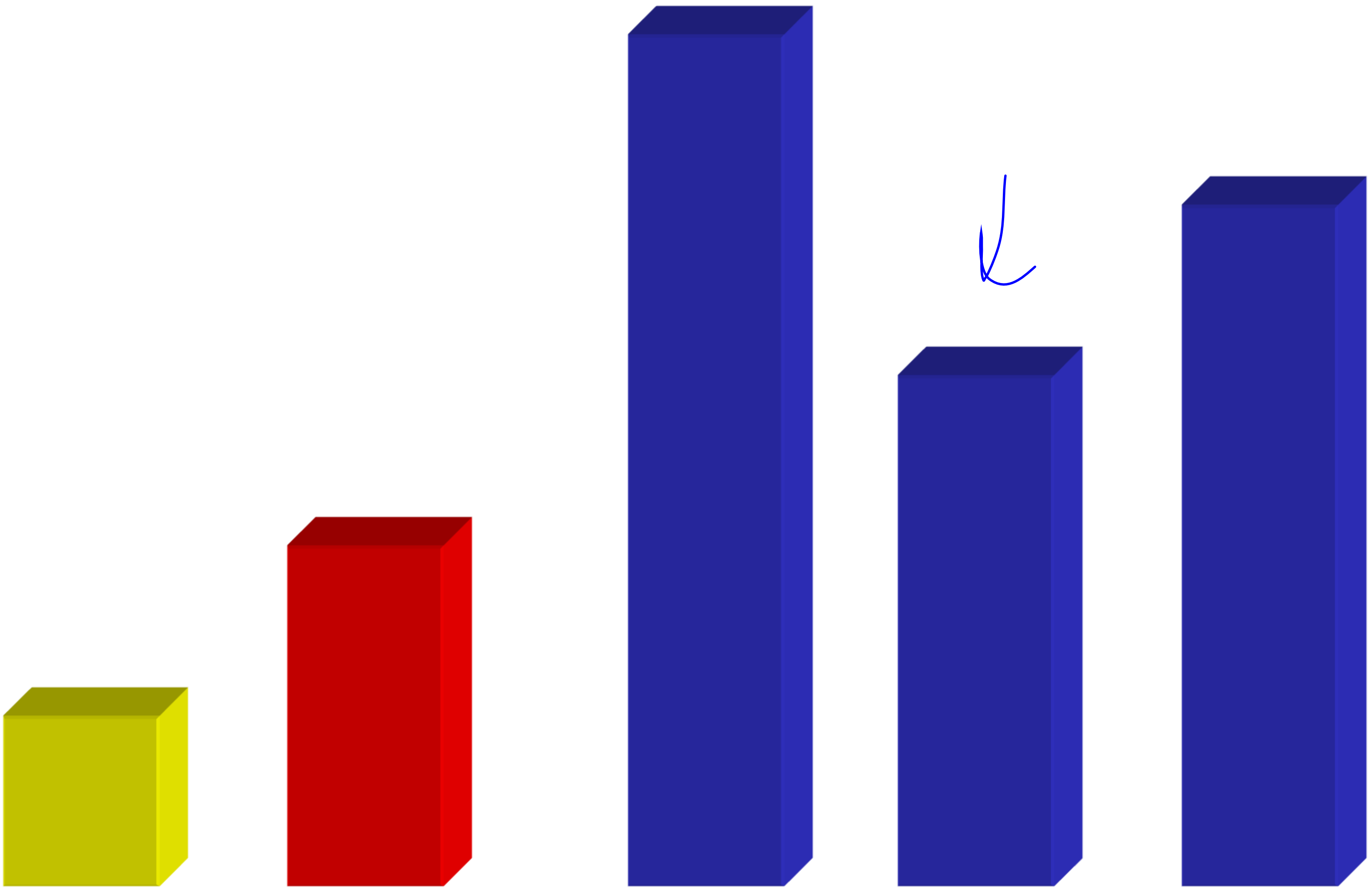

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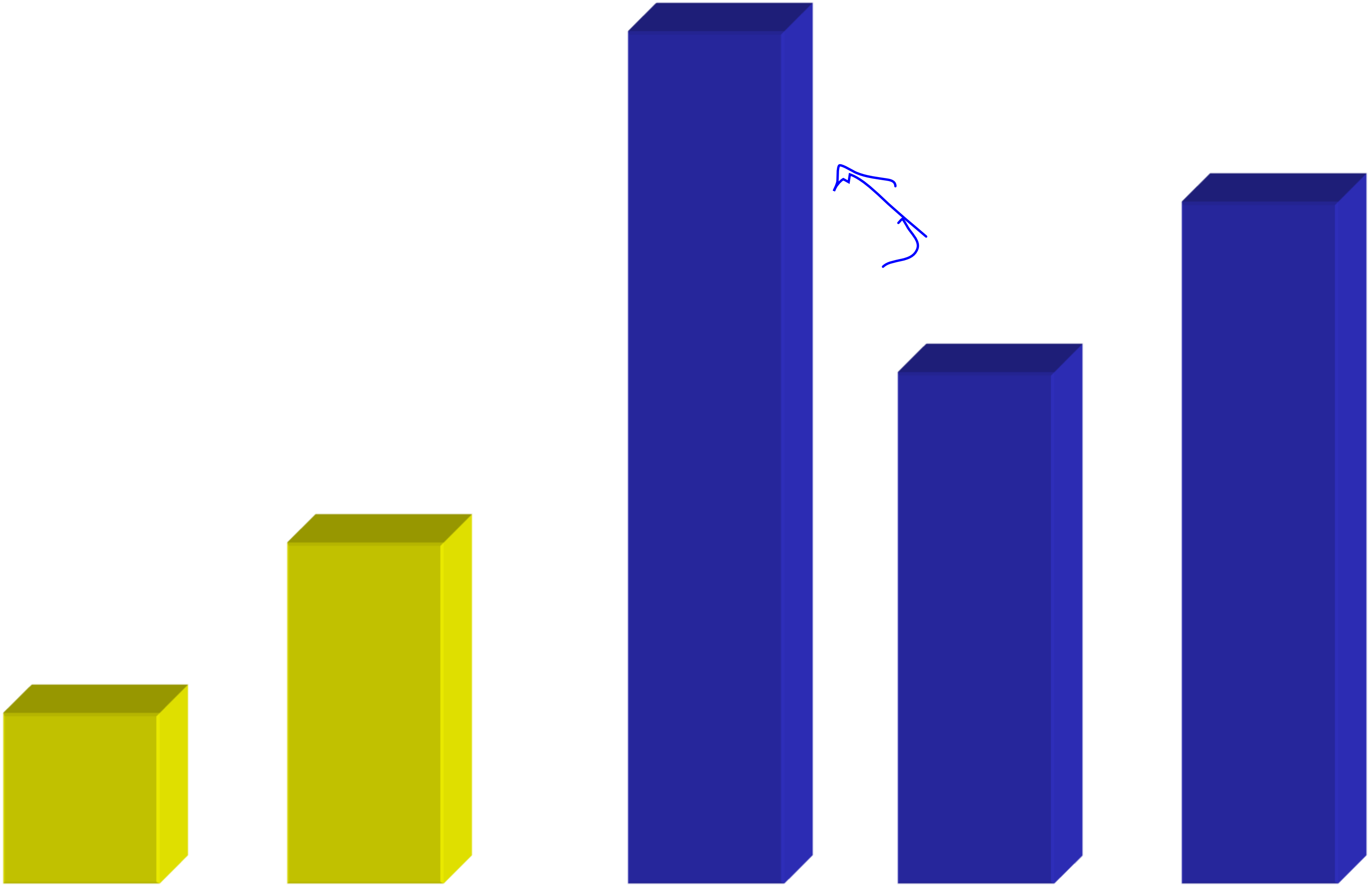

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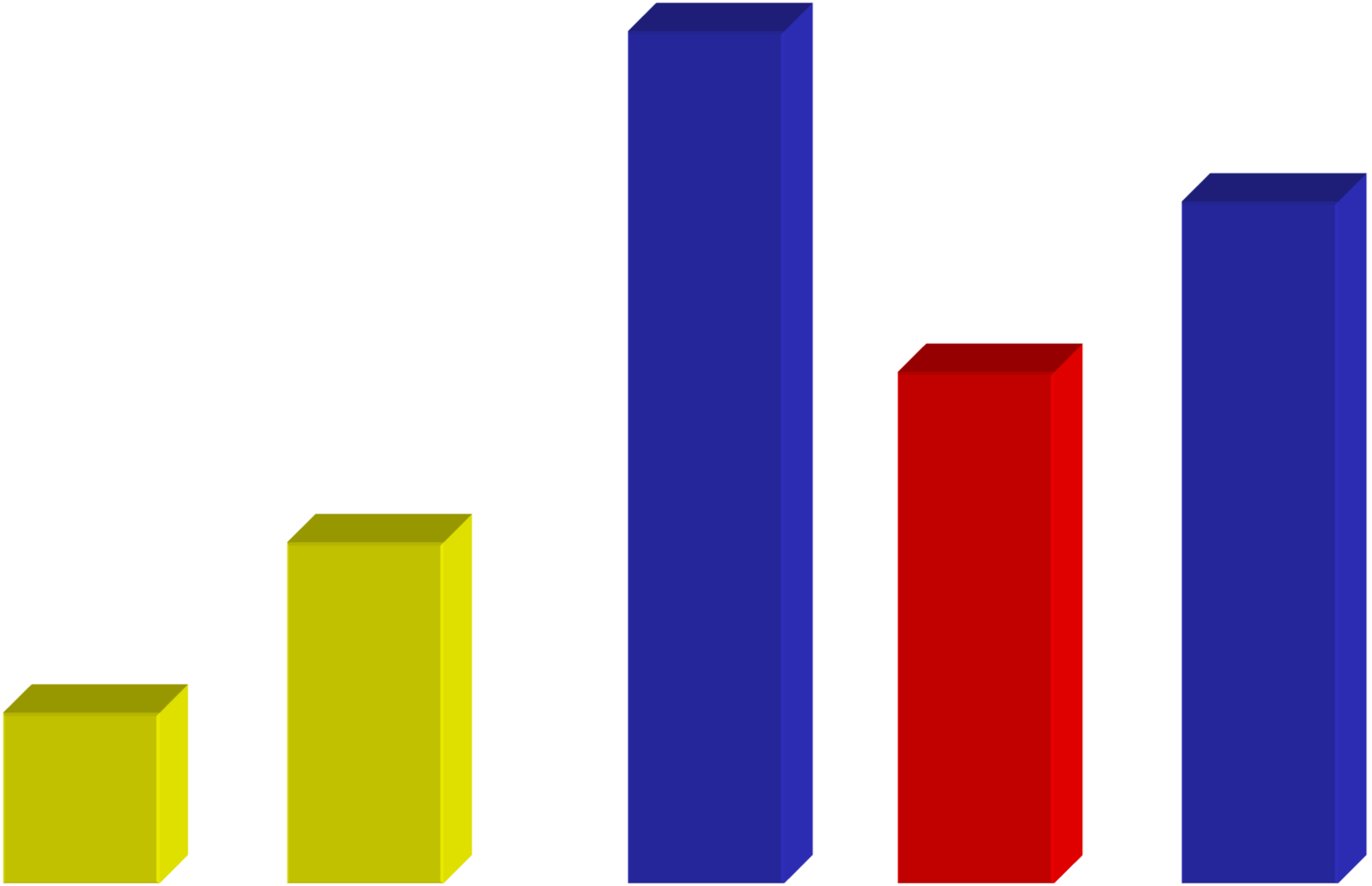

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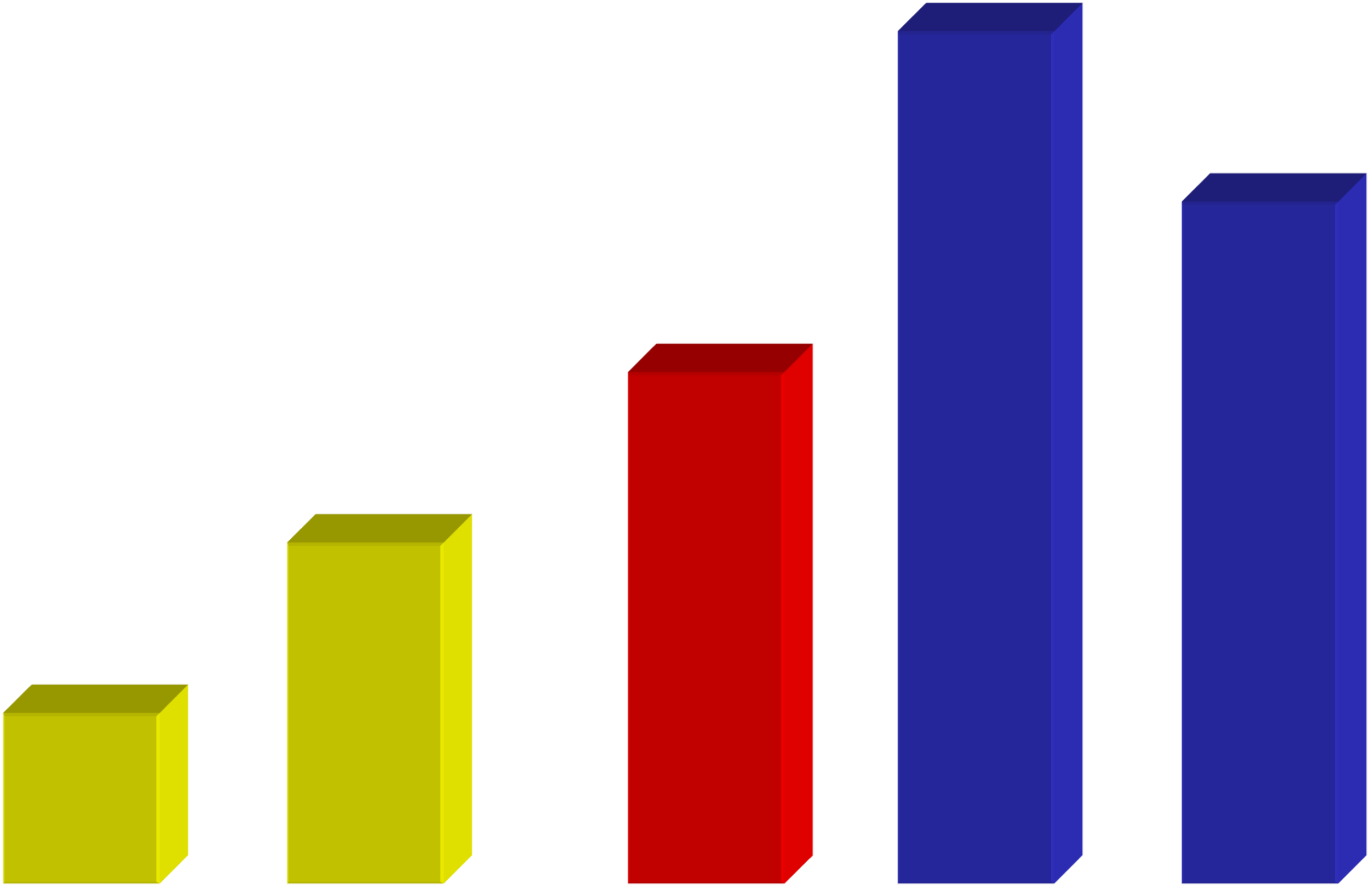

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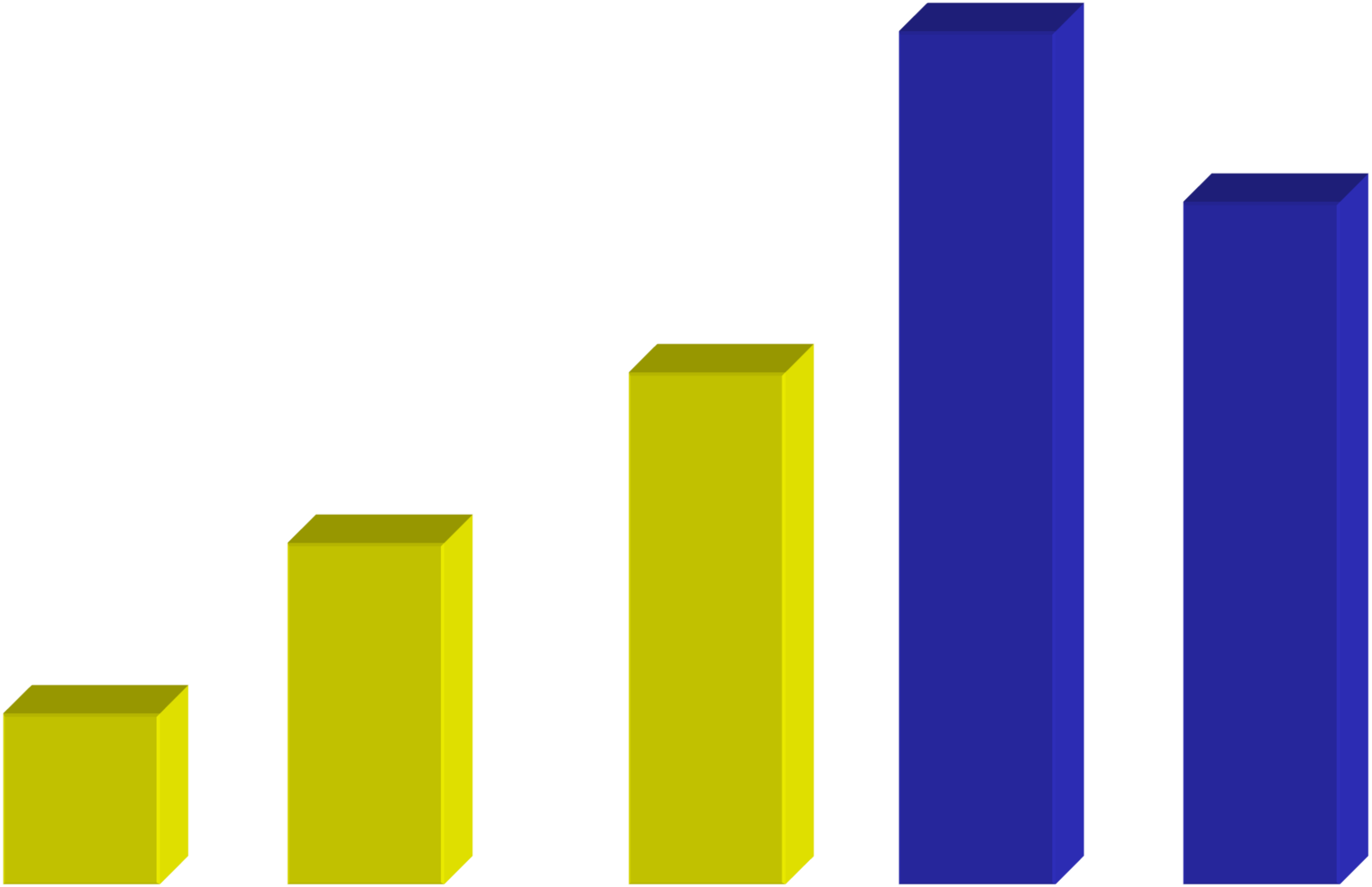

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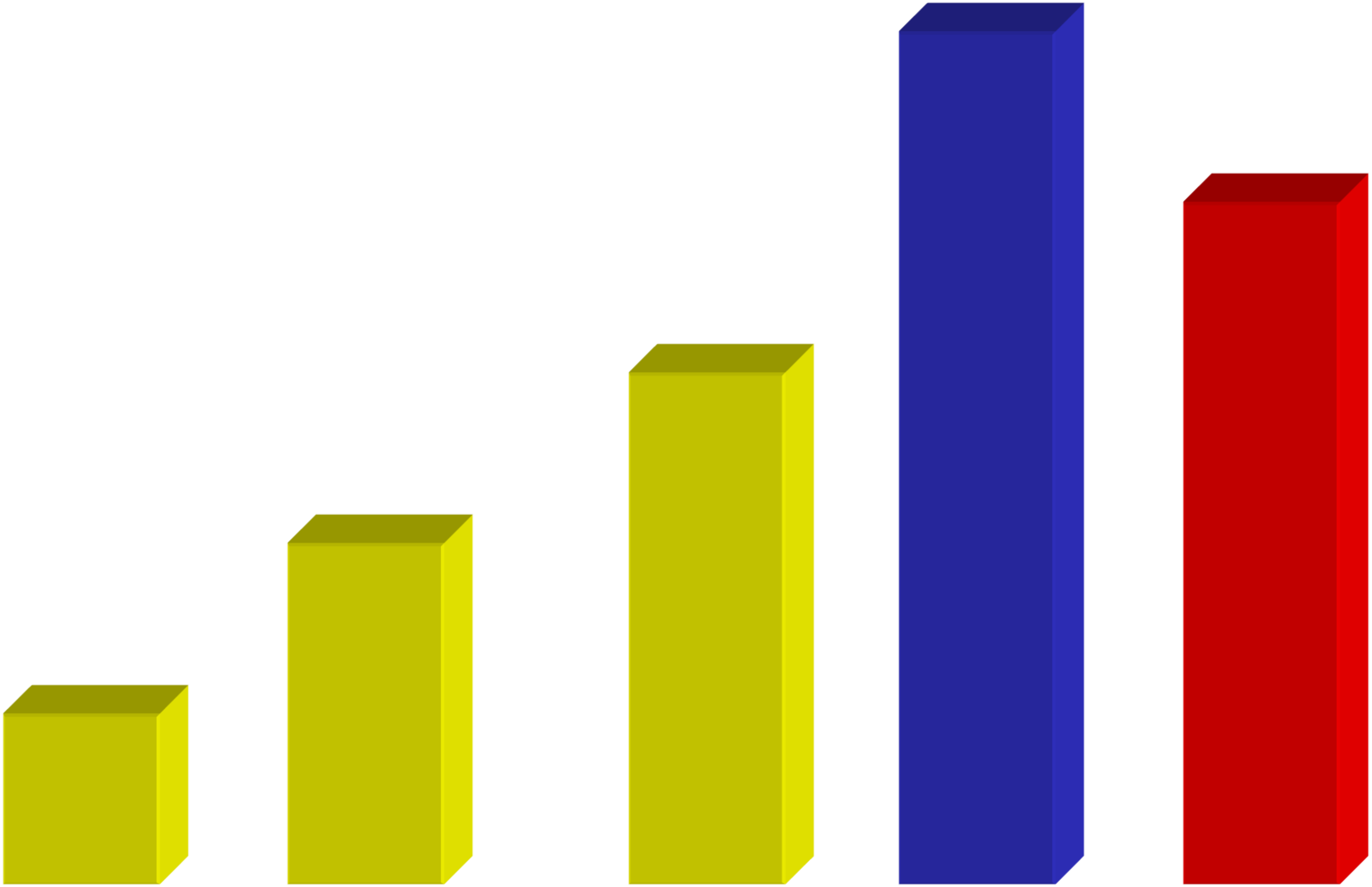

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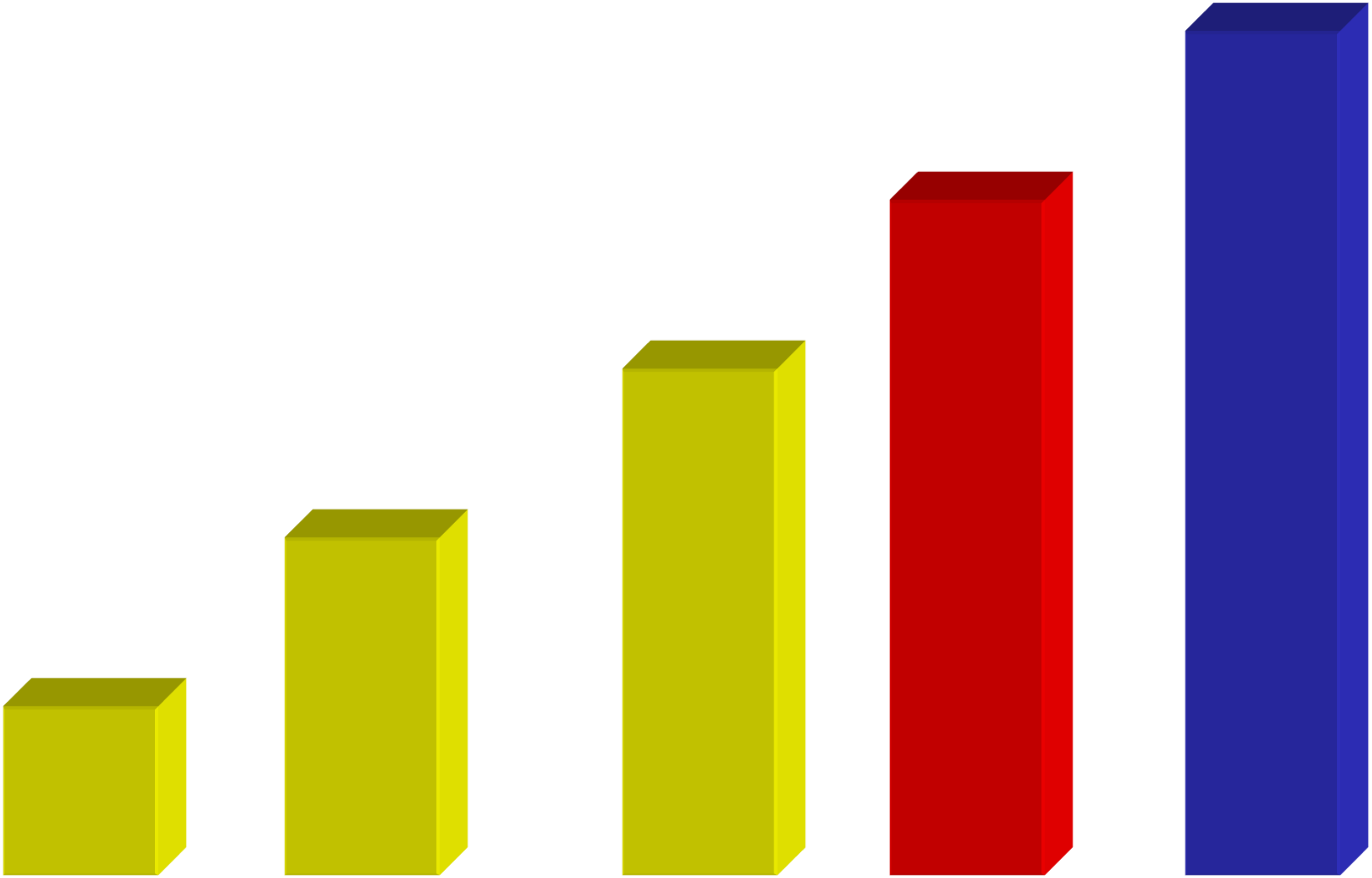

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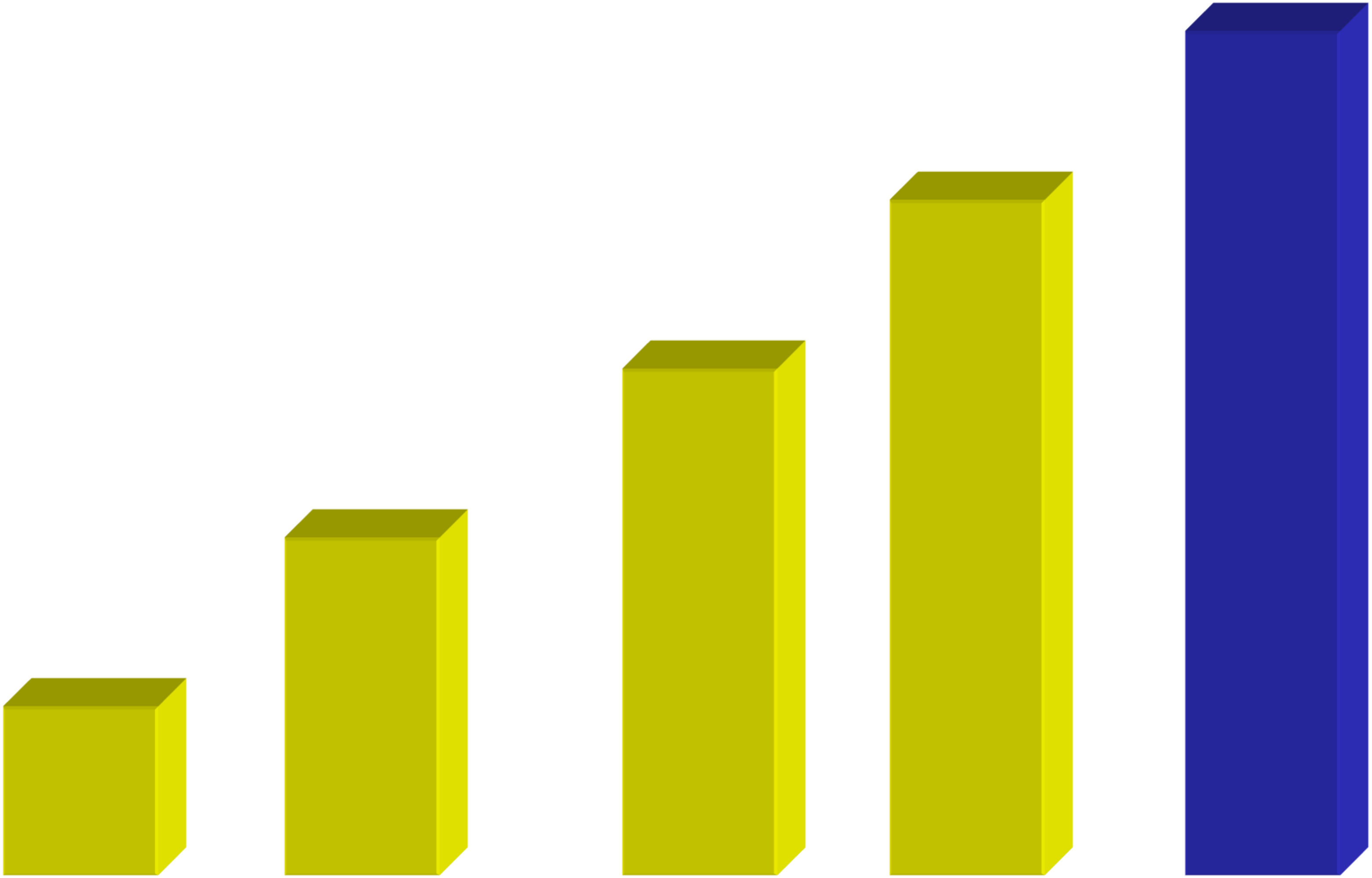

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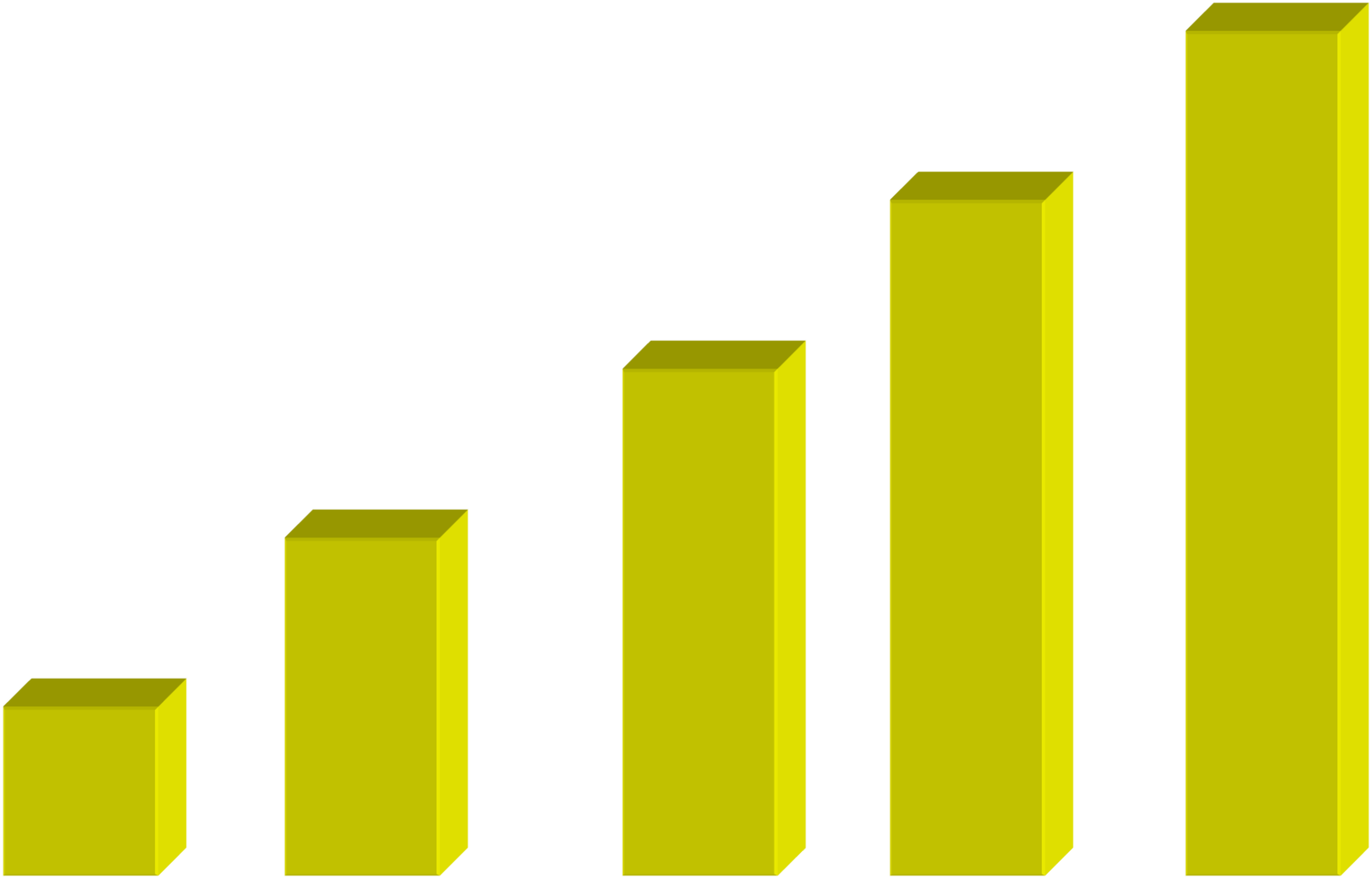


What is the Worst-case Running Time of Selection Sort?

```
Algorithm selectionSort(A,n):
  Input: Array A of size n
  Output: Array A sorted

  for k ← 0 to n−2 do
     min ← k
     for j ← k+1 to n−1 do
          if A[j] < A[min] then
               min ← j
          end
     end
     swap(A[k], A[min])
  end
end
```

$$\sum_{k=0}^{n-2} \sum_{j=k+1}^{n-1} 1$$

What is the Worst-case Running Time of Selection Sort?

$$\sum_{k=0}^{n-2}\sum_{j=k+1}^{n-1} 1 = \sum_{k=0}^{n-2}\Big((n-1)-(k+1)+1\Big)$$

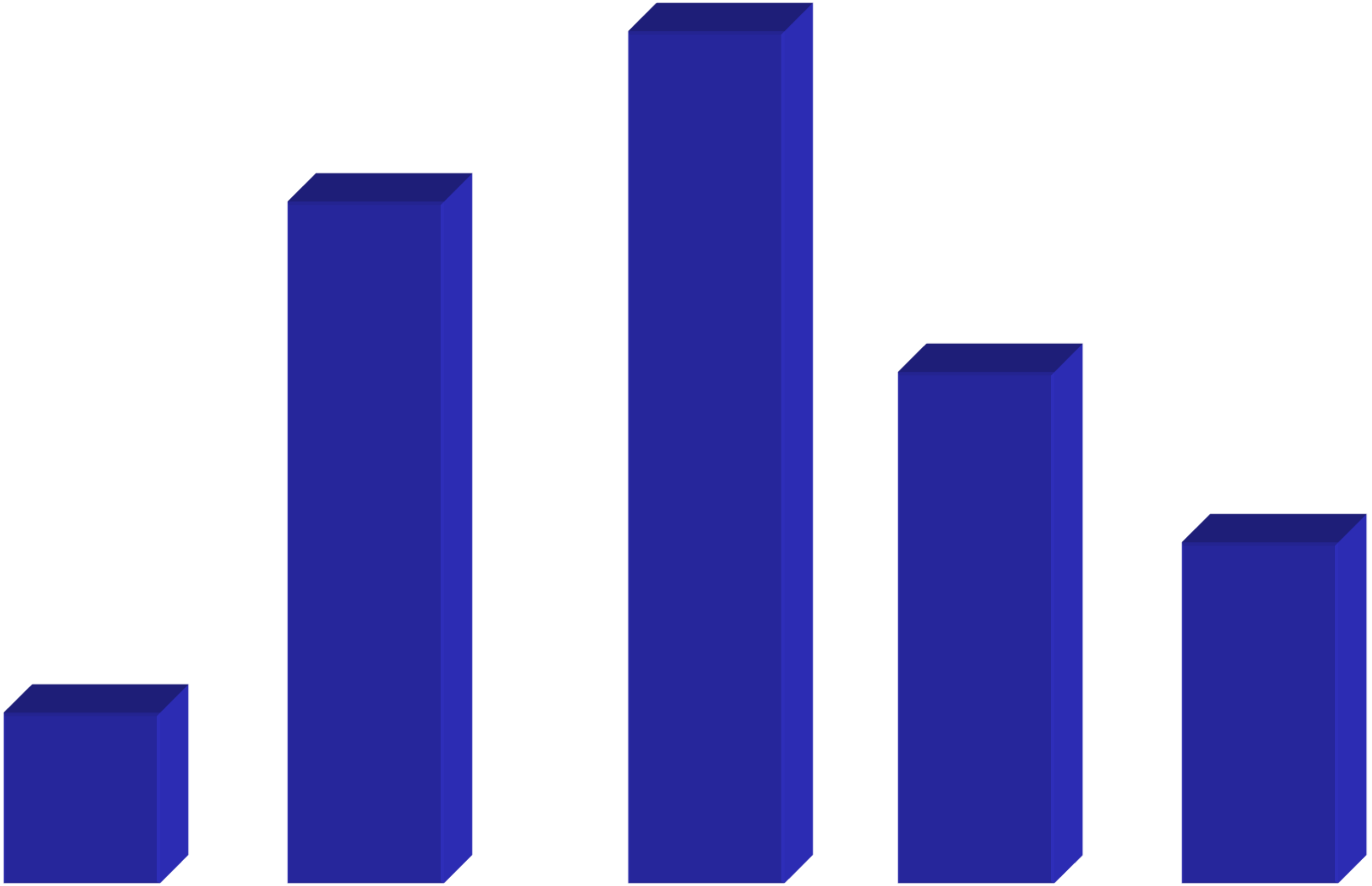
$$= \sum_{k=0}^{n-2}\left(n-k-1\right)$$

$$= n\sum_{k=0}^{n-2}1 - \sum_{k=0}^{n-2}k - \sum_{k=0}^{n-2}1 \qquad \in O(n^2)$$

$$= n\big((n-2)+1\big) - \frac{(n-2)(n-1)}{2} - \big((n-2)+1\big)$$

$$= n^2 - n - \frac{n^2-3n+2}{2} - n + 1 = \frac{n^2}{2} - \frac{n}{2}$$

Bubble Sort



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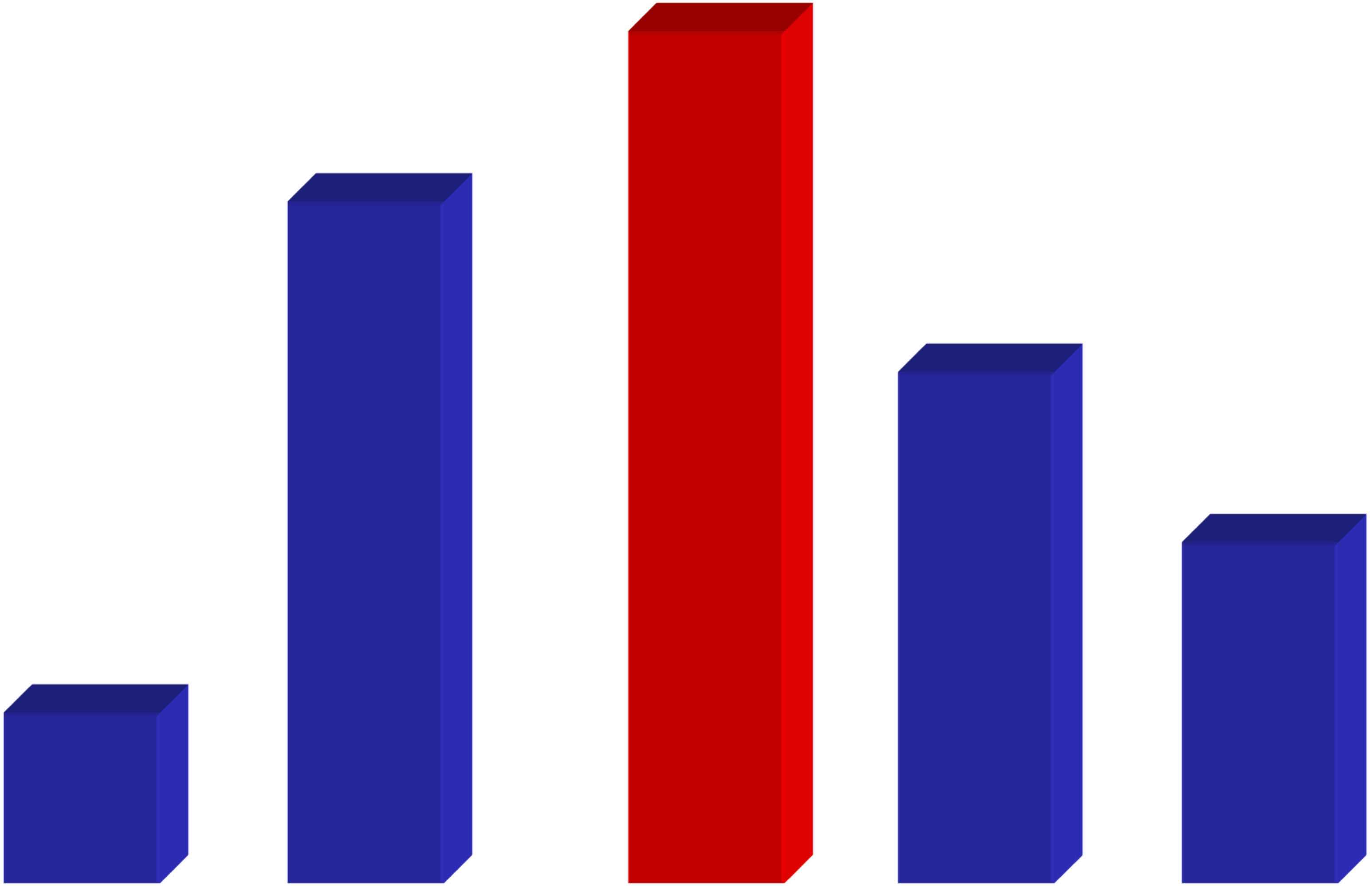

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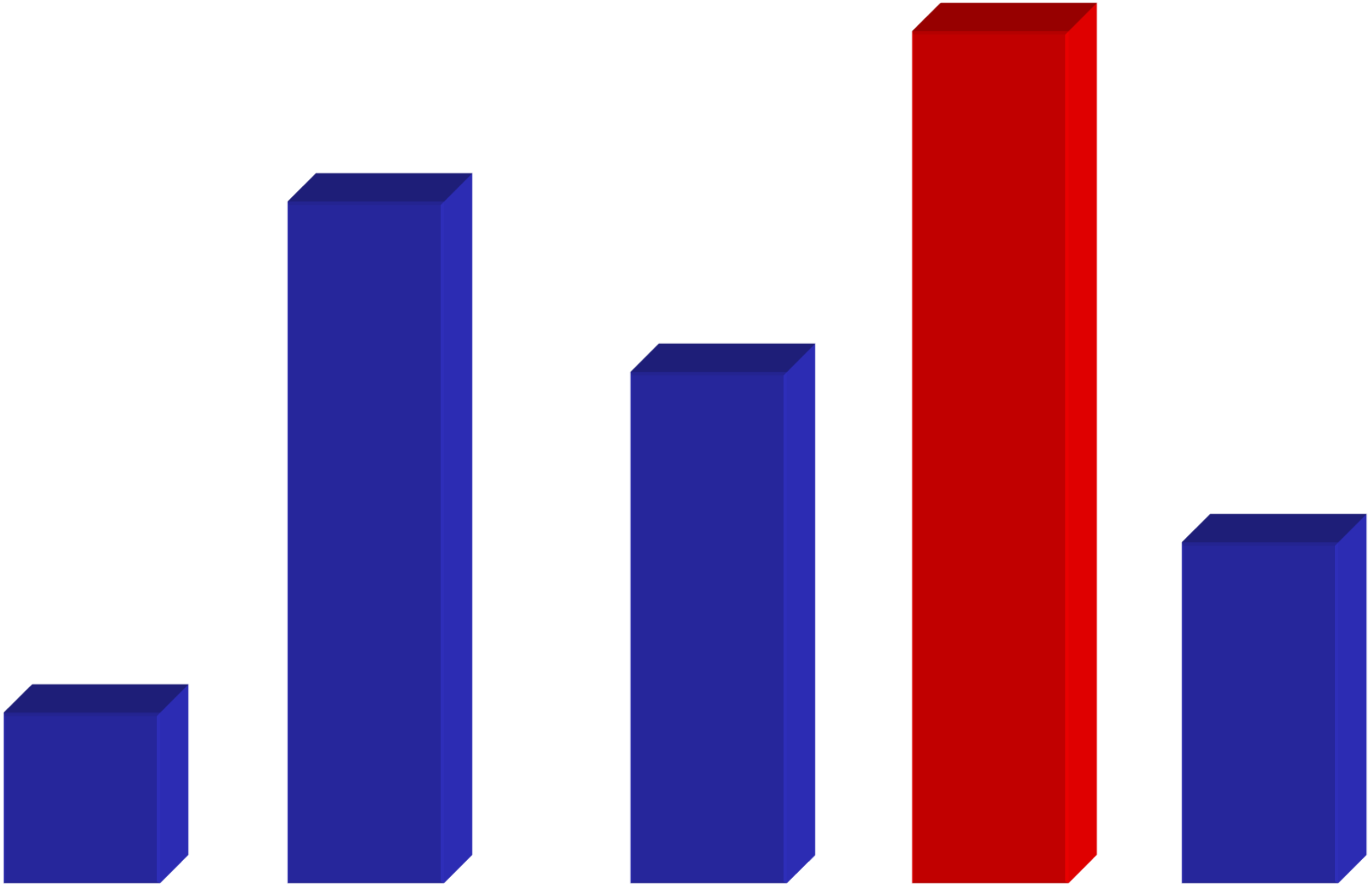

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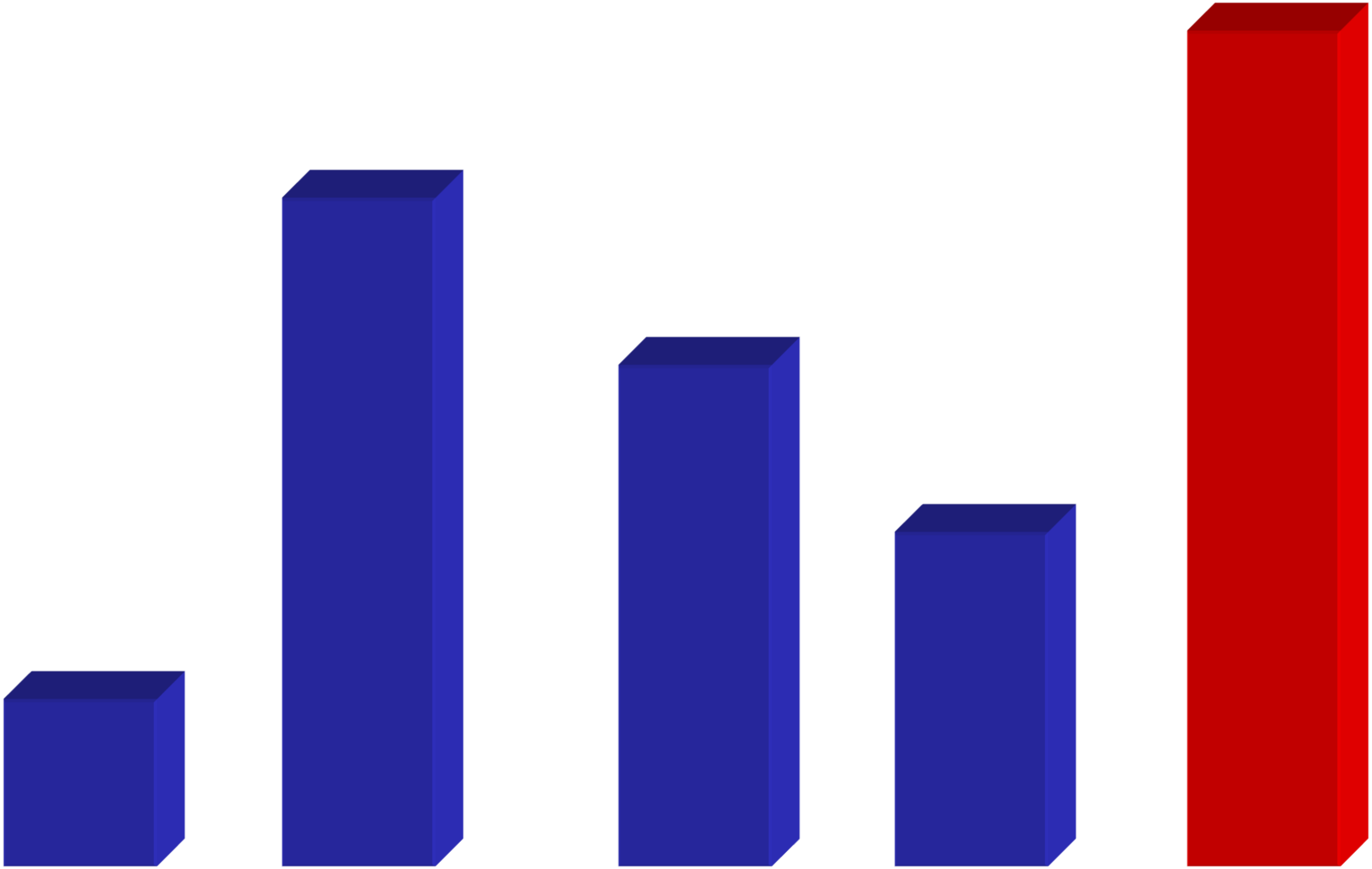

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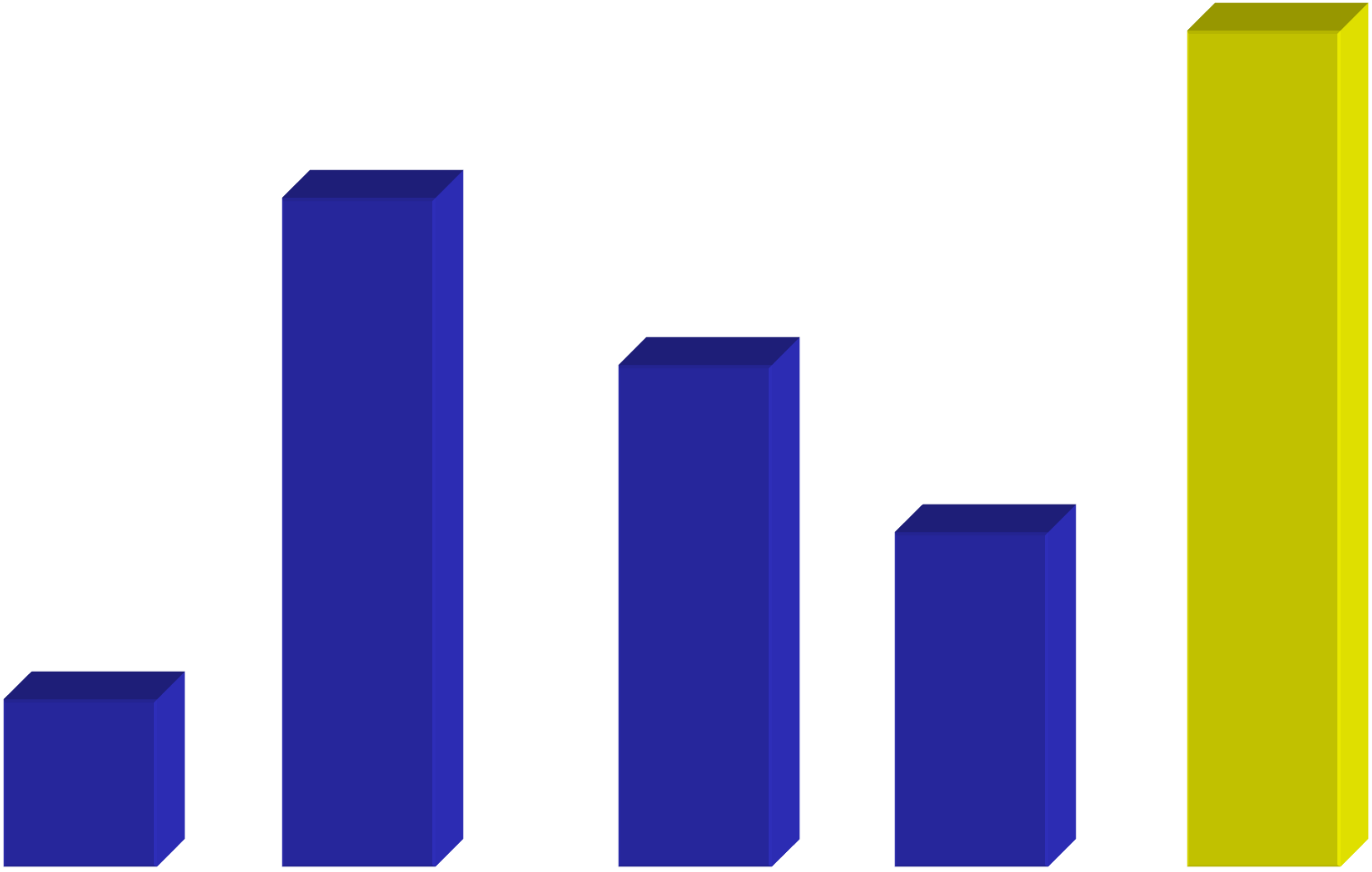

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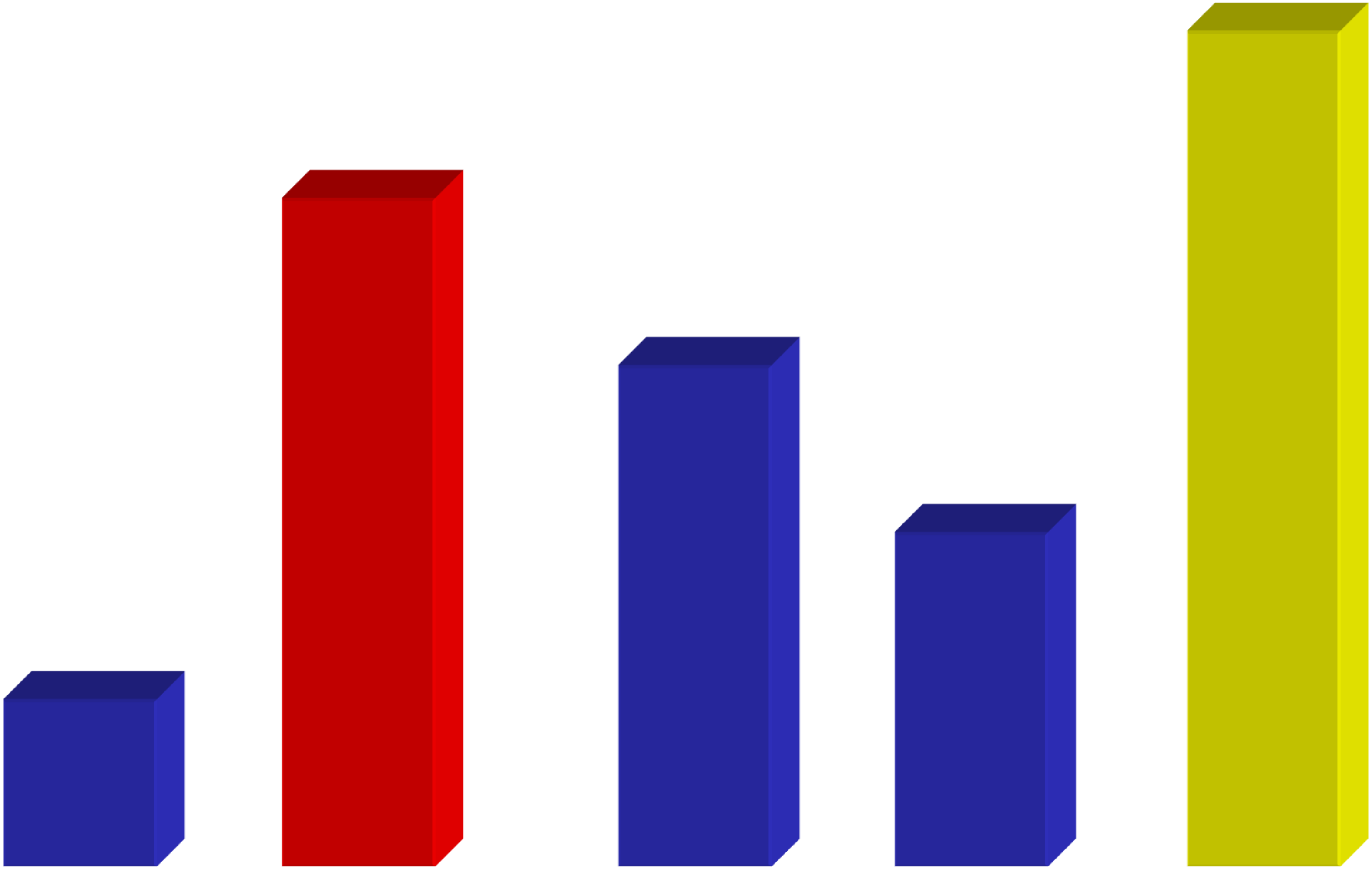

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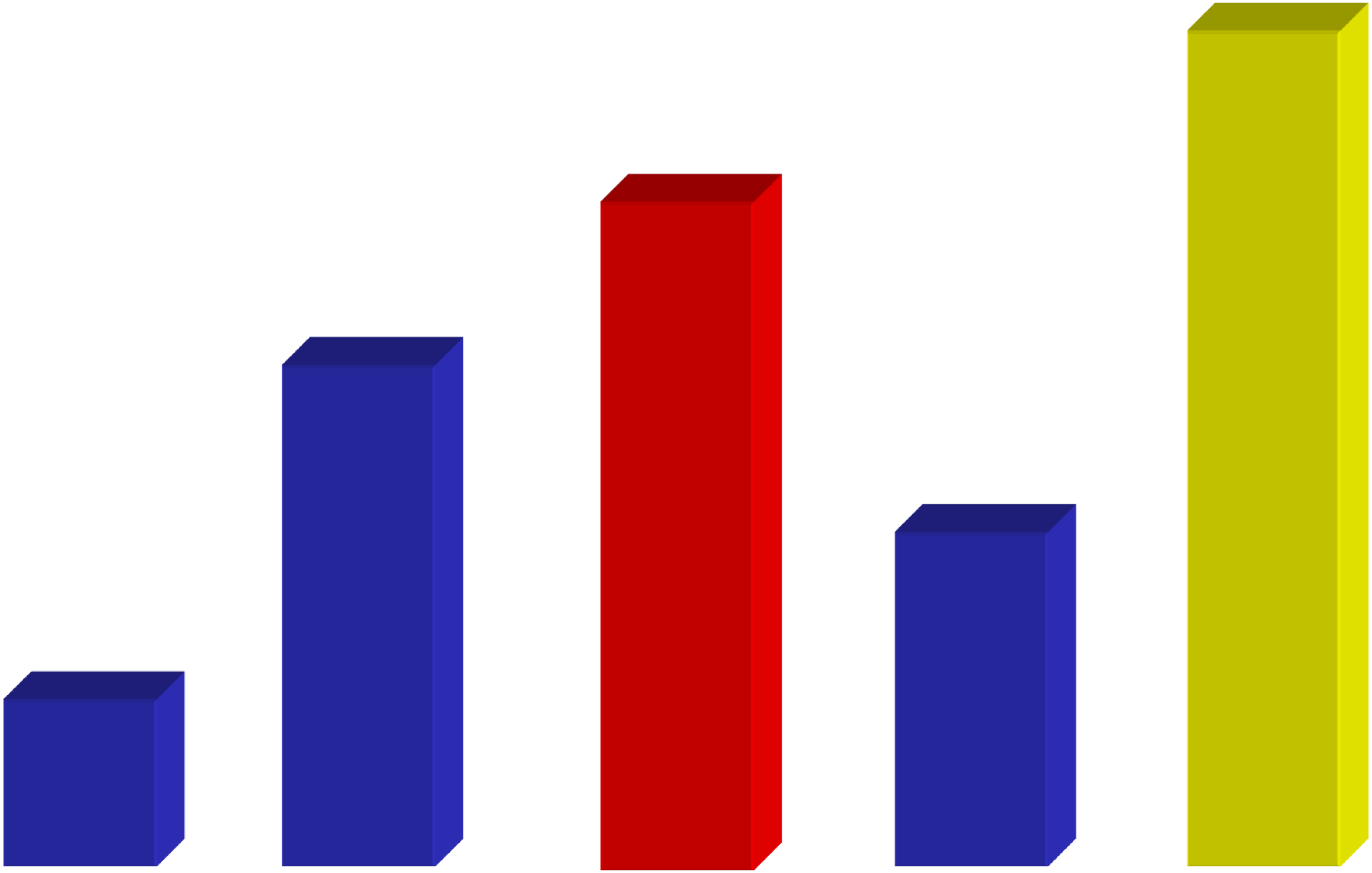

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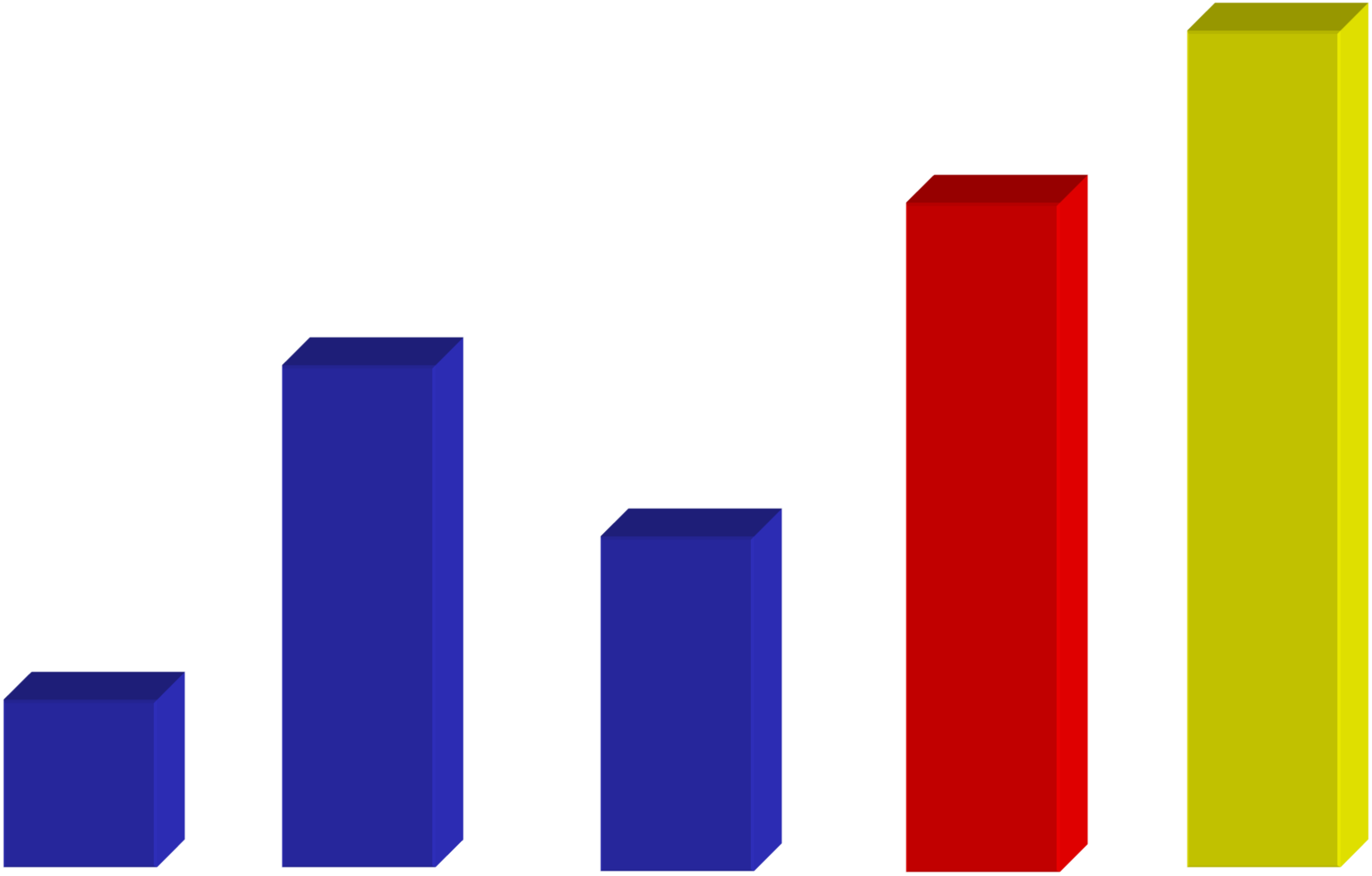

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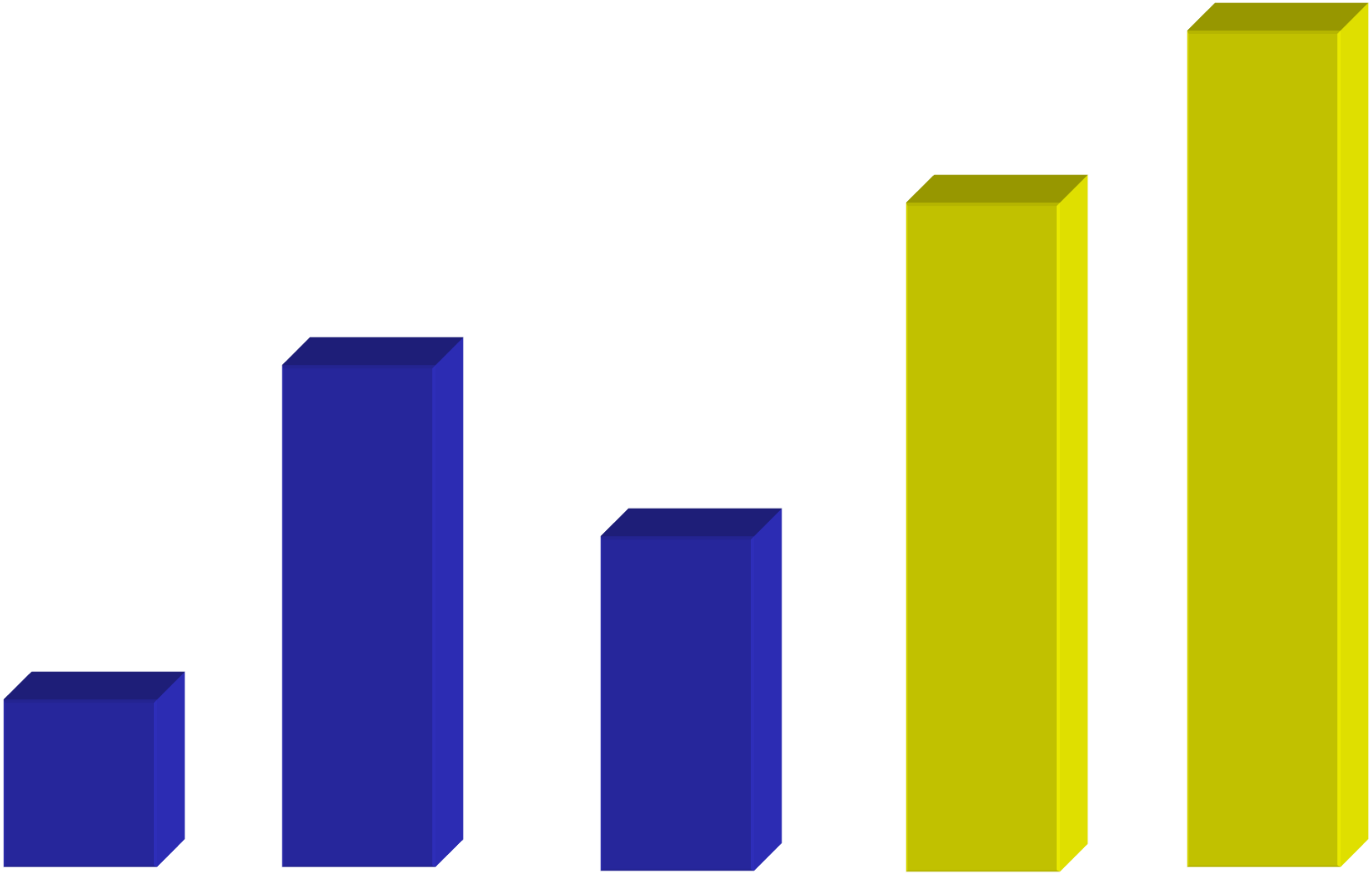

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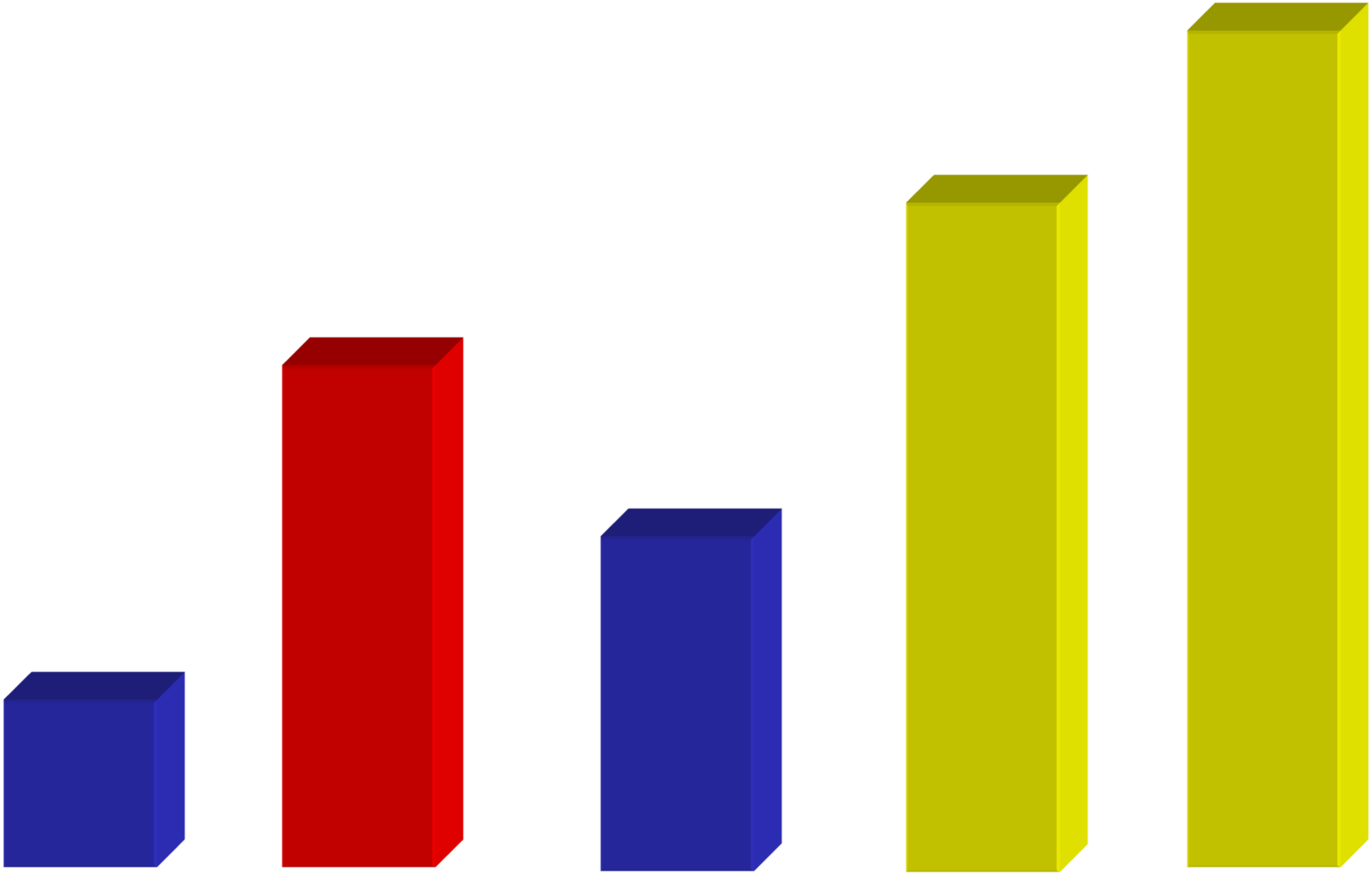

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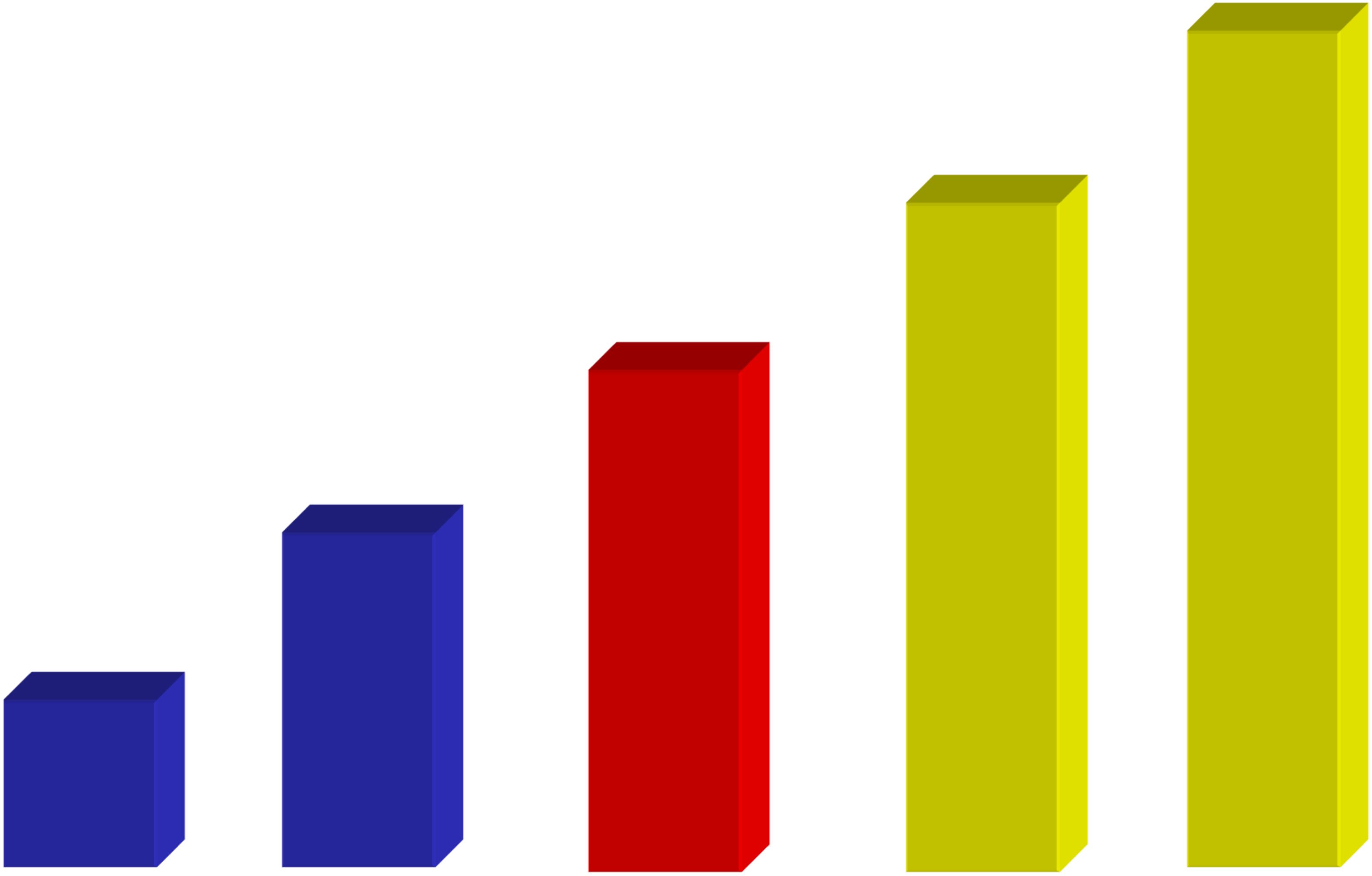

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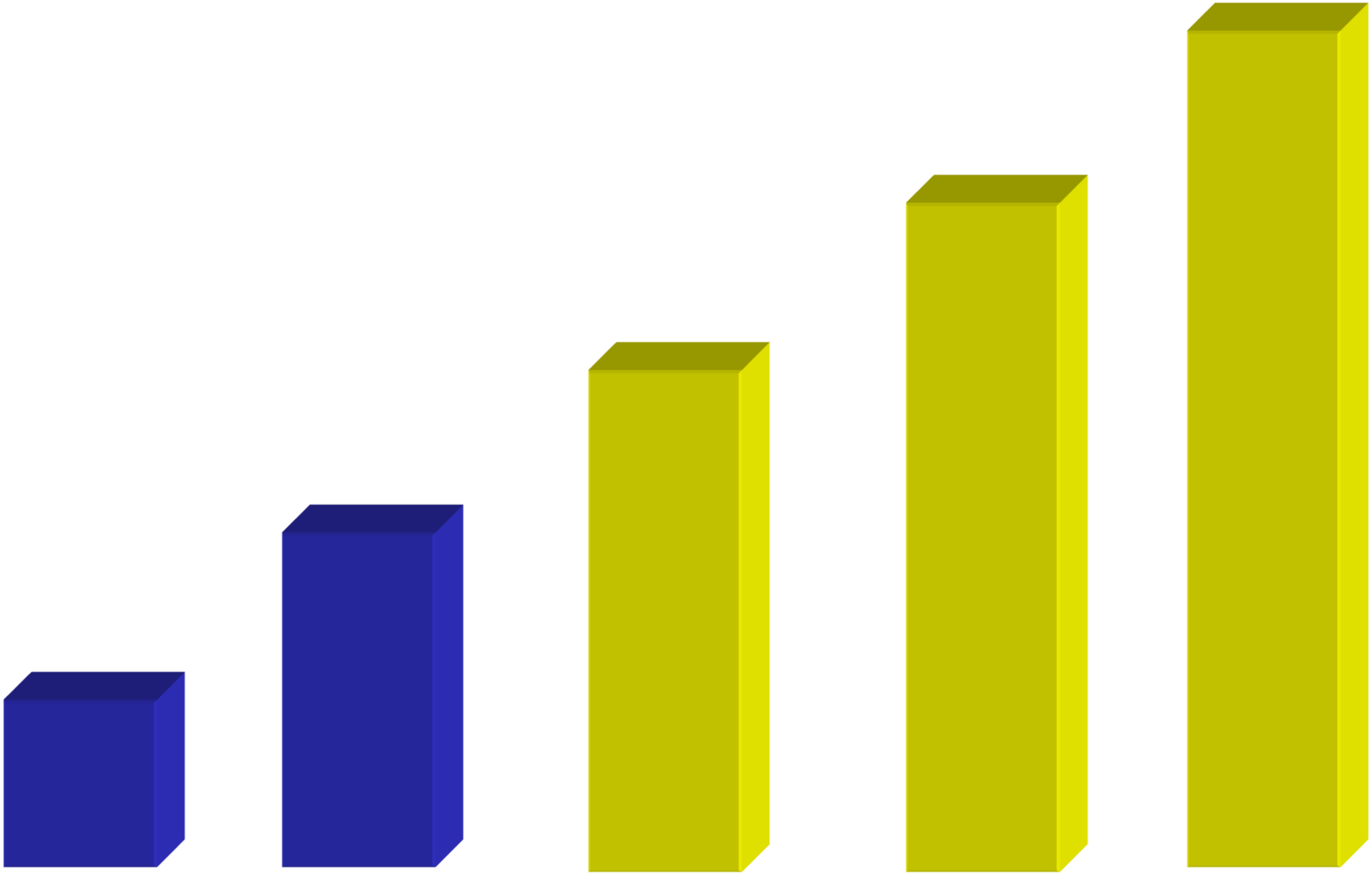

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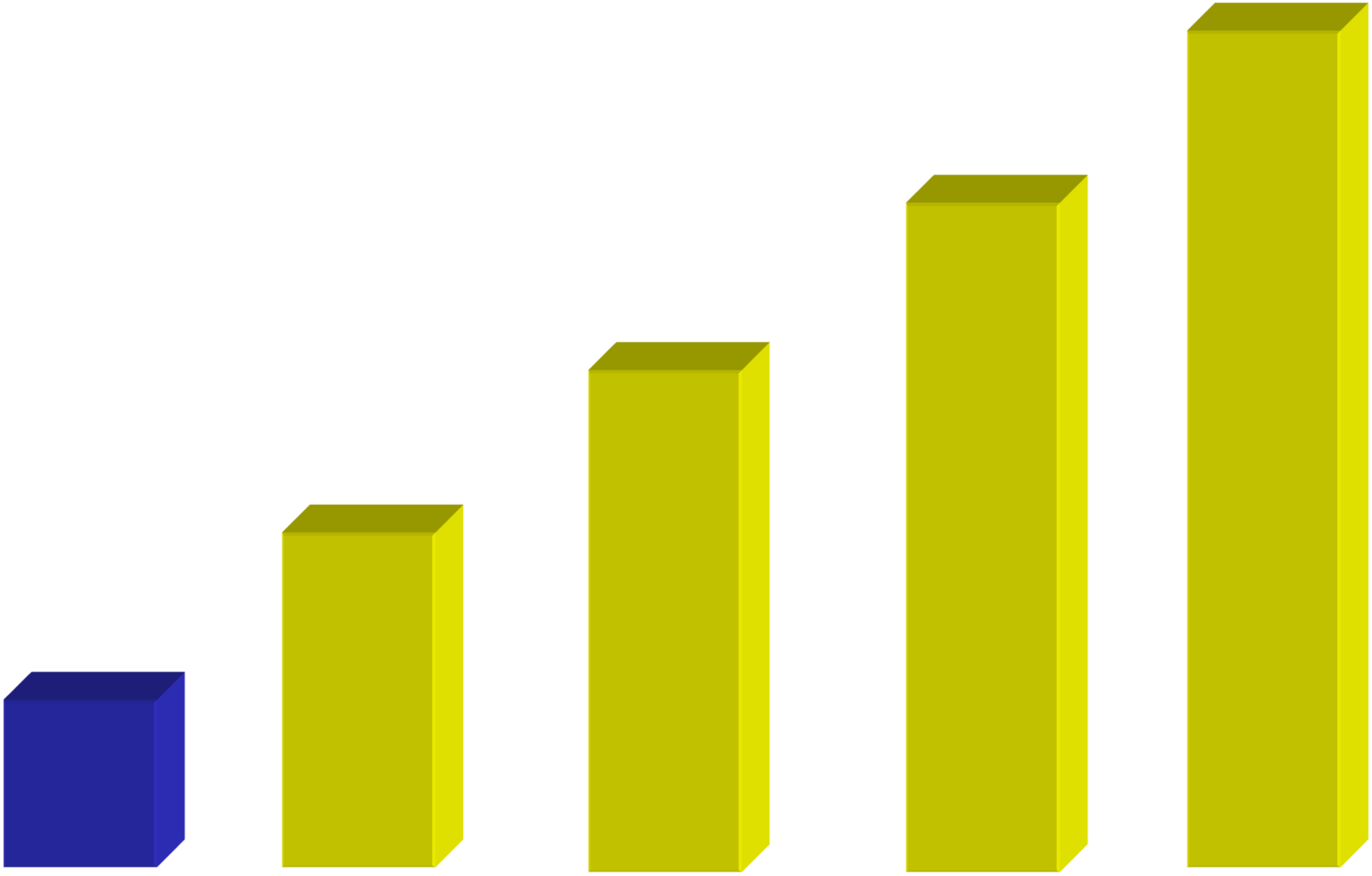

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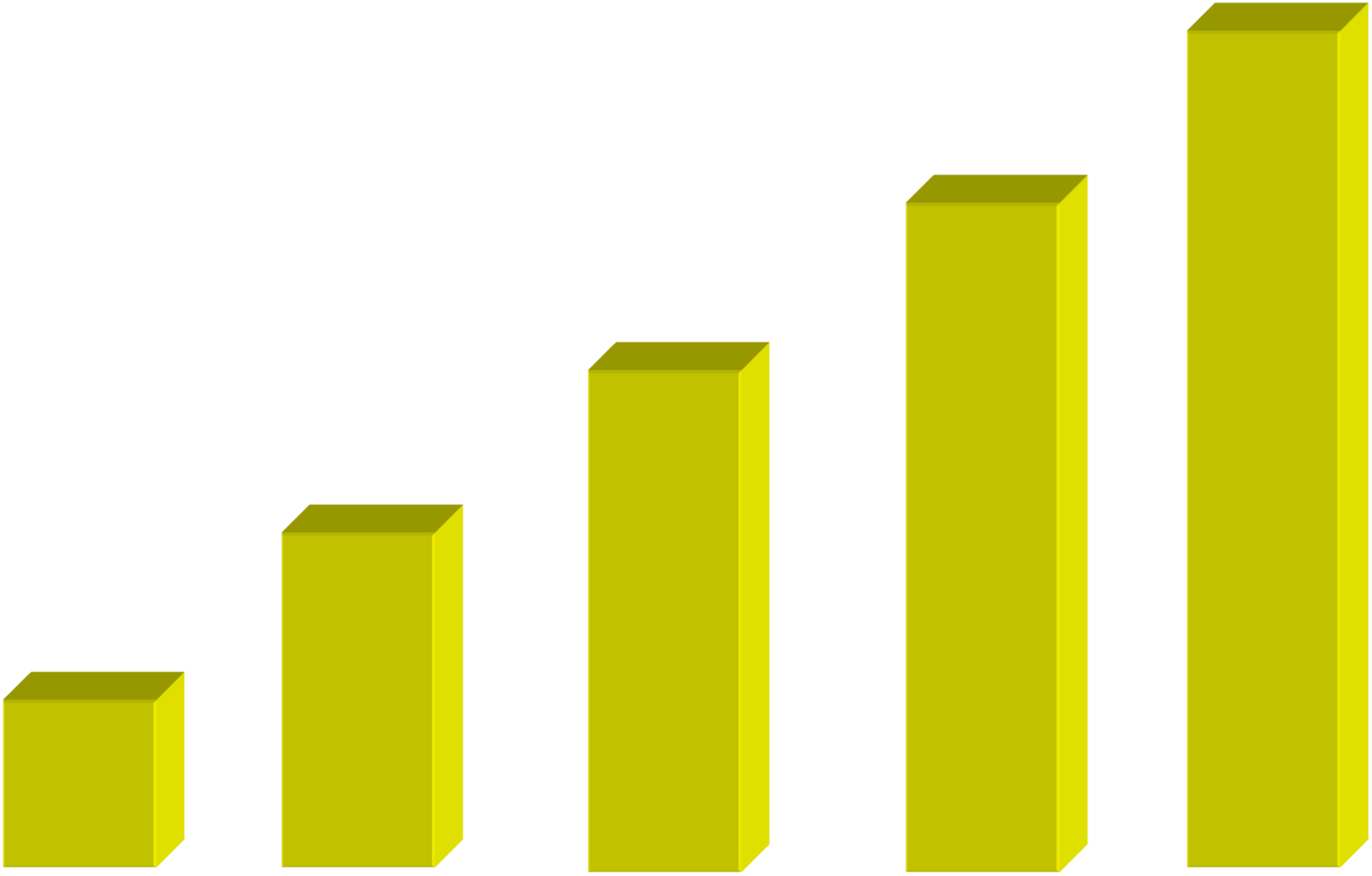


What is the Worst-case Running Time of Bubble Sort?

Algorithm bubbleSort(A,n):
  Input: Array A of size n
  Output: Array A sorted

  for $k \leftarrow 0$ **to** $n-2$ **do**
    for $j \leftarrow 0$ **to** $n-2-k$ **do**
      if $A[j+1] < A[j]$ **then**
        swap($A[j]$, $A[j+1]$)
      end
    end
  end
end

$$\sum_{k=0}^{n-2} \sum_{j=0}^{n-2-k} 1$$

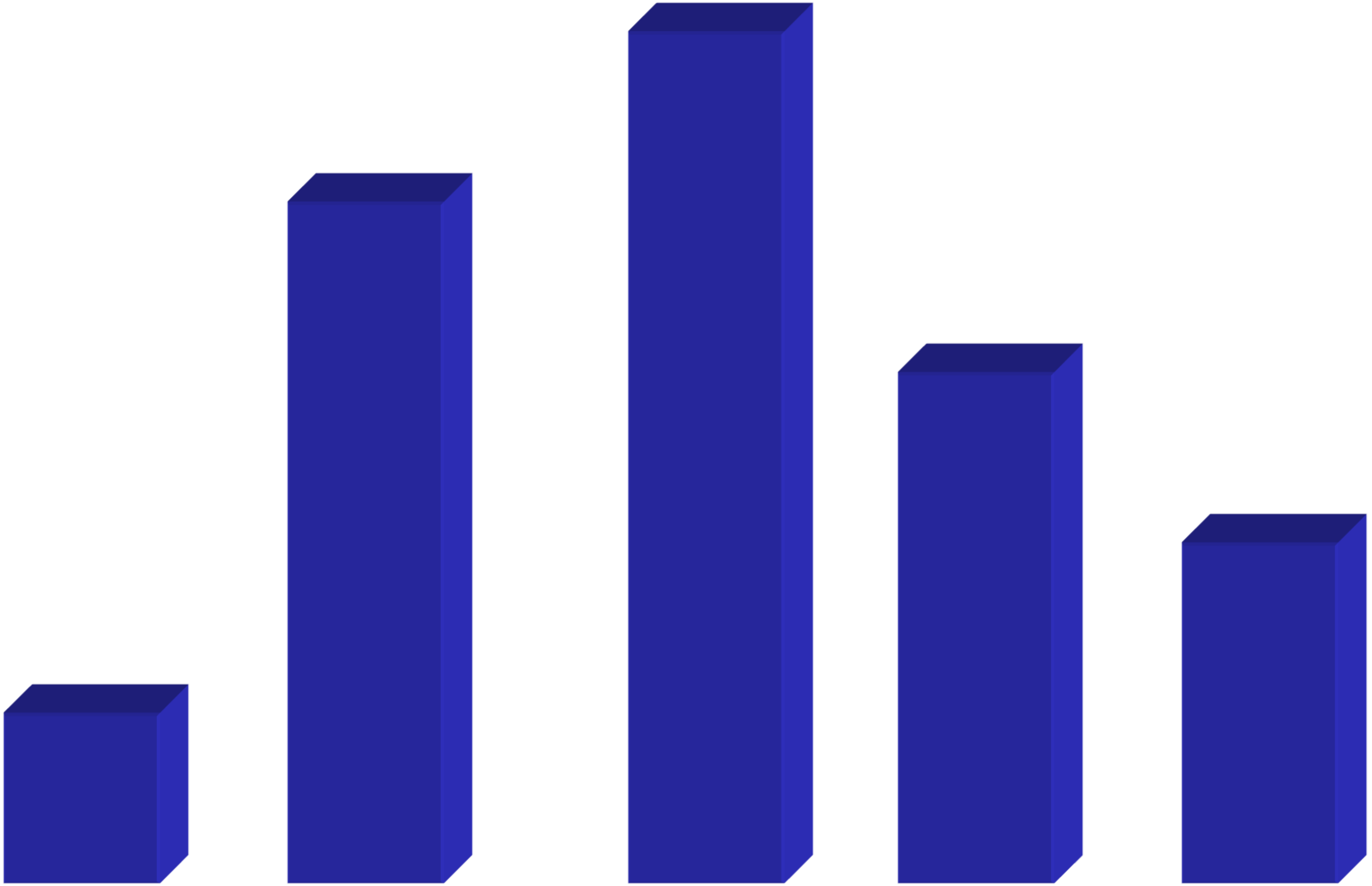
B.C. $\in O(n)$

What is the Worst-case Running Time of Bubble Sort?

$$\sum_{k=0}^{n-2} \sum_{j=0}^{n-2-k} 1 = \sum_{k=0}^{n-2} \left((n-2-k) + 1 \right)$$

$$= \sum_{k=0}^{n-2} n - 1 - k = \frac{n^2 - n}{2} \in O(n^2)$$

Insertion Sort



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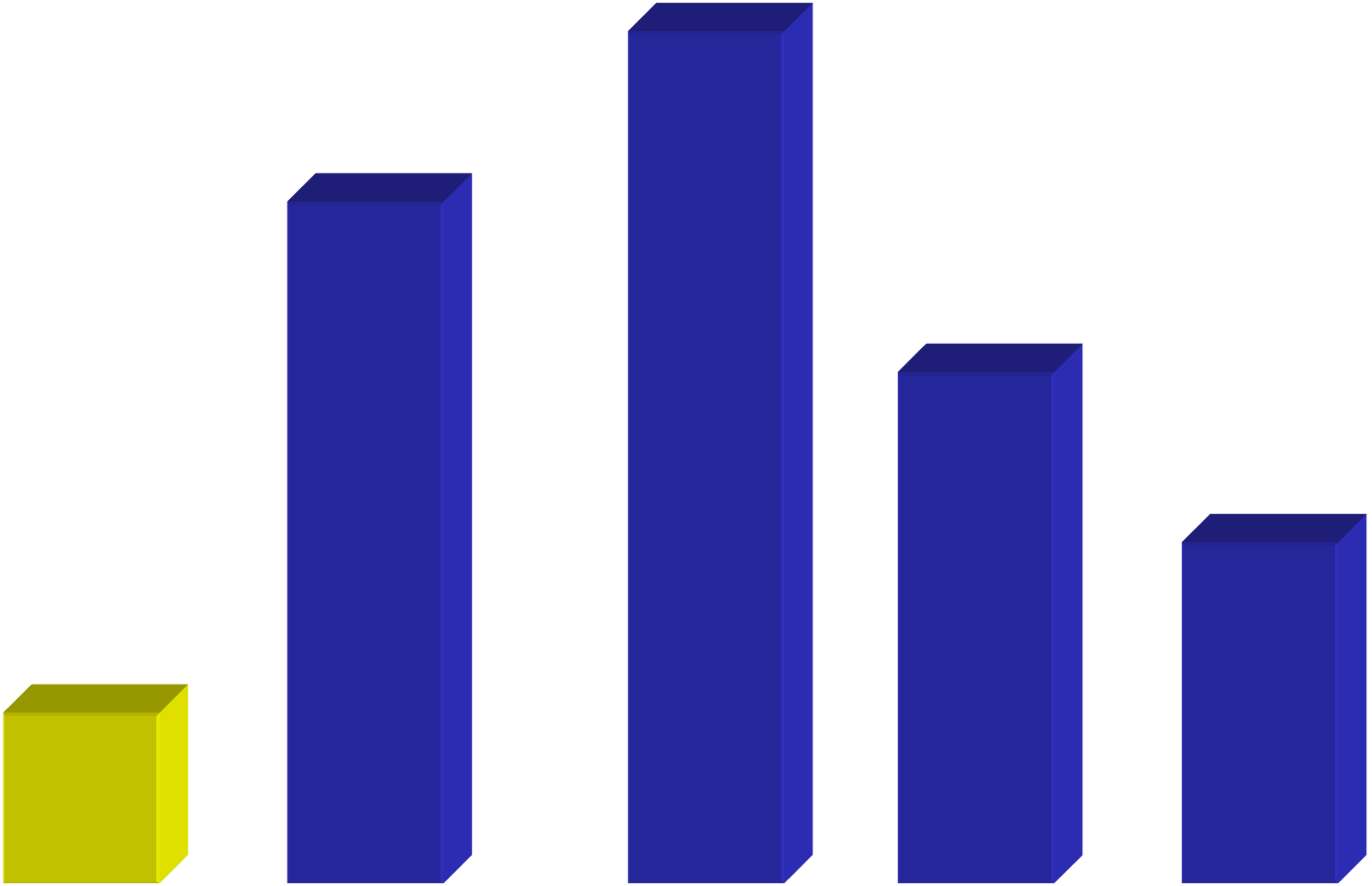

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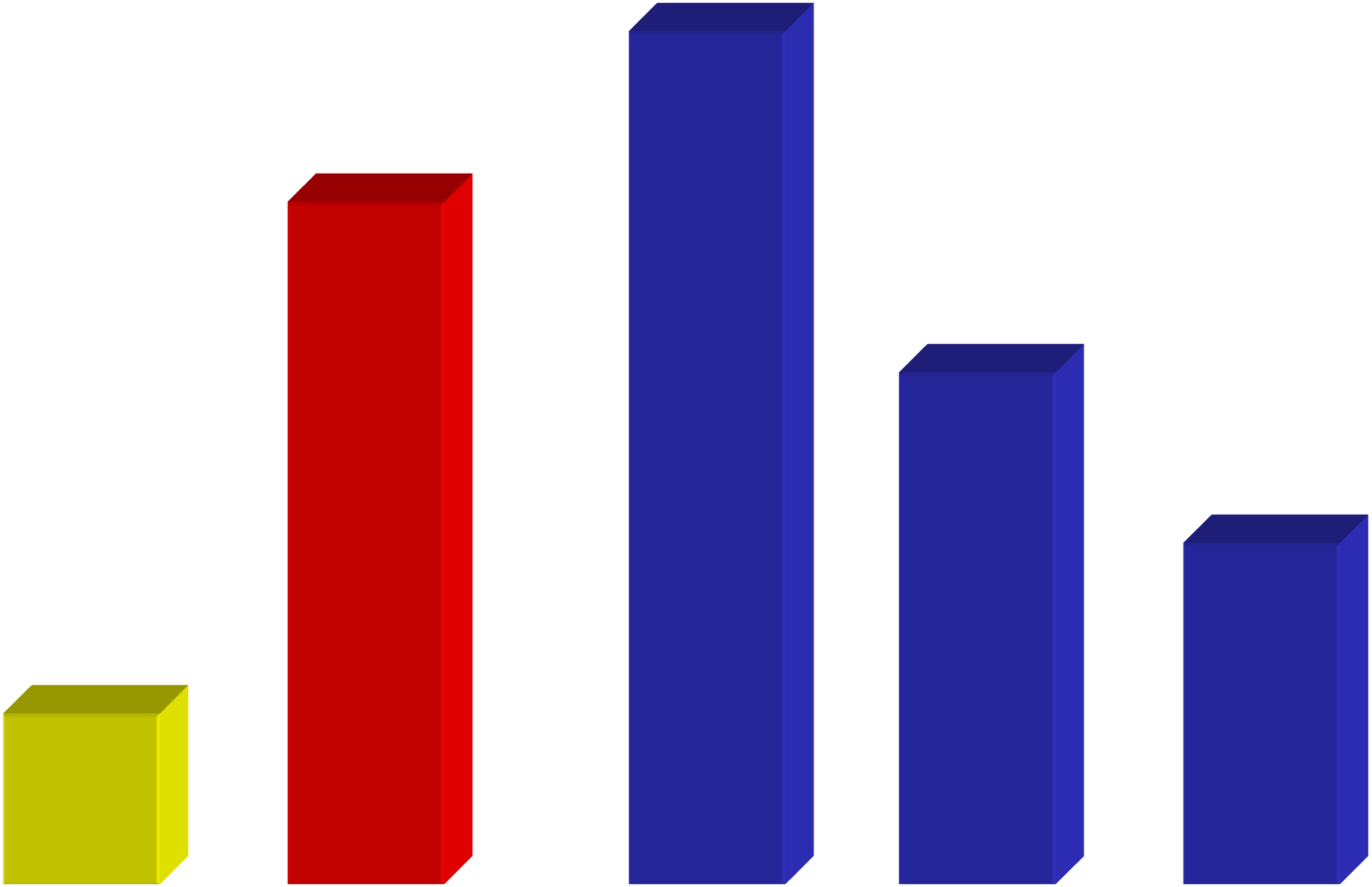

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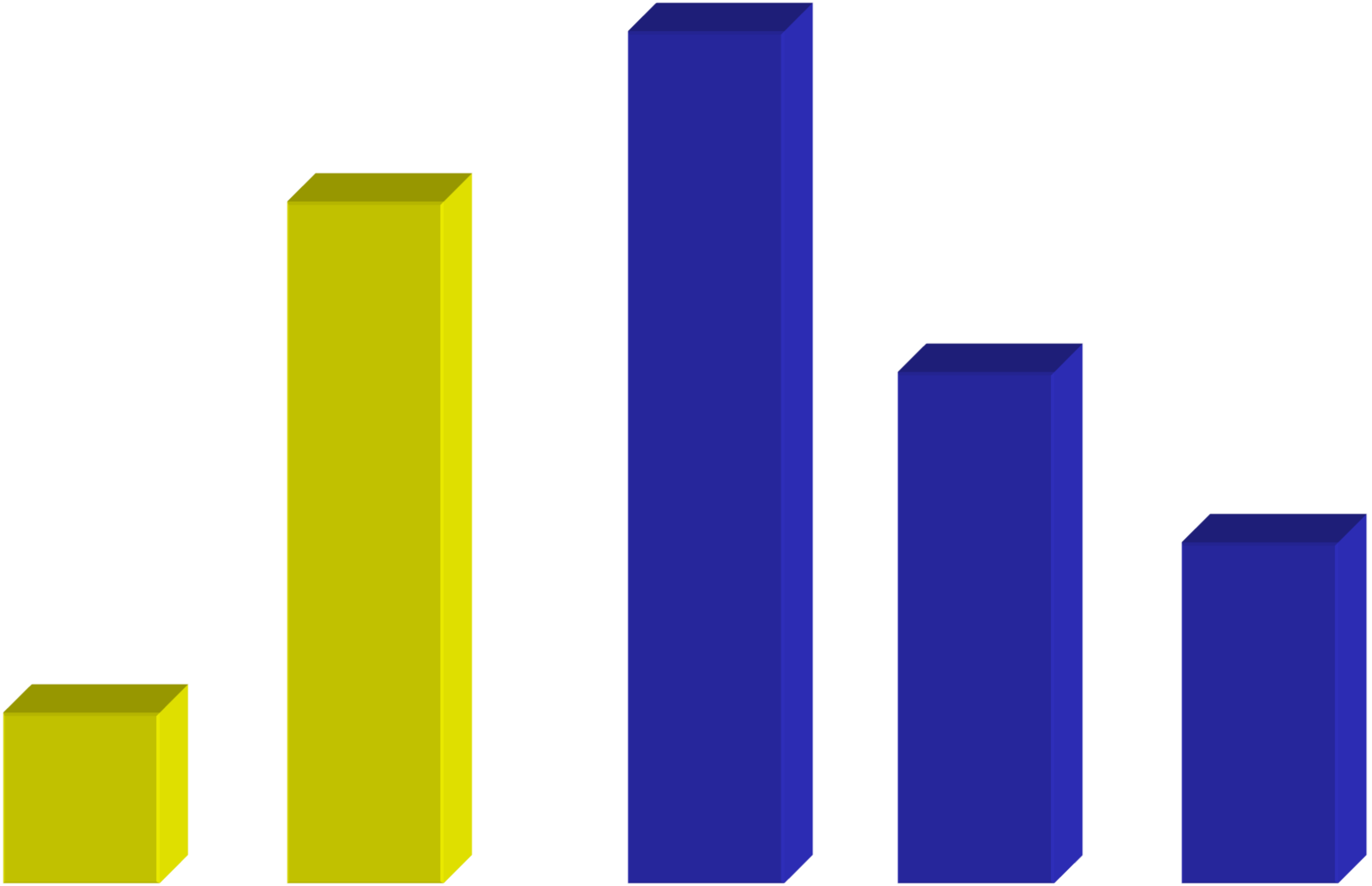

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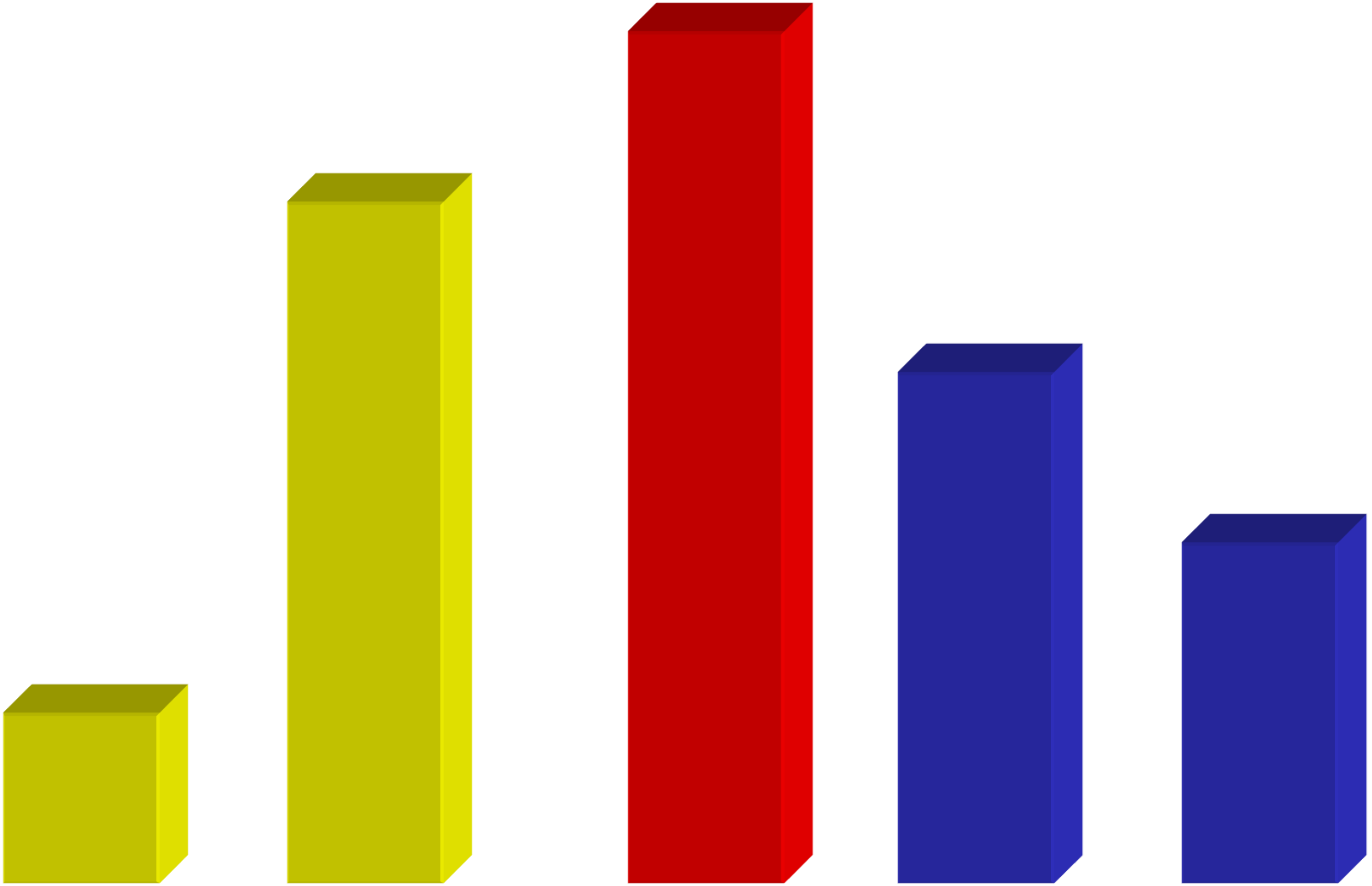

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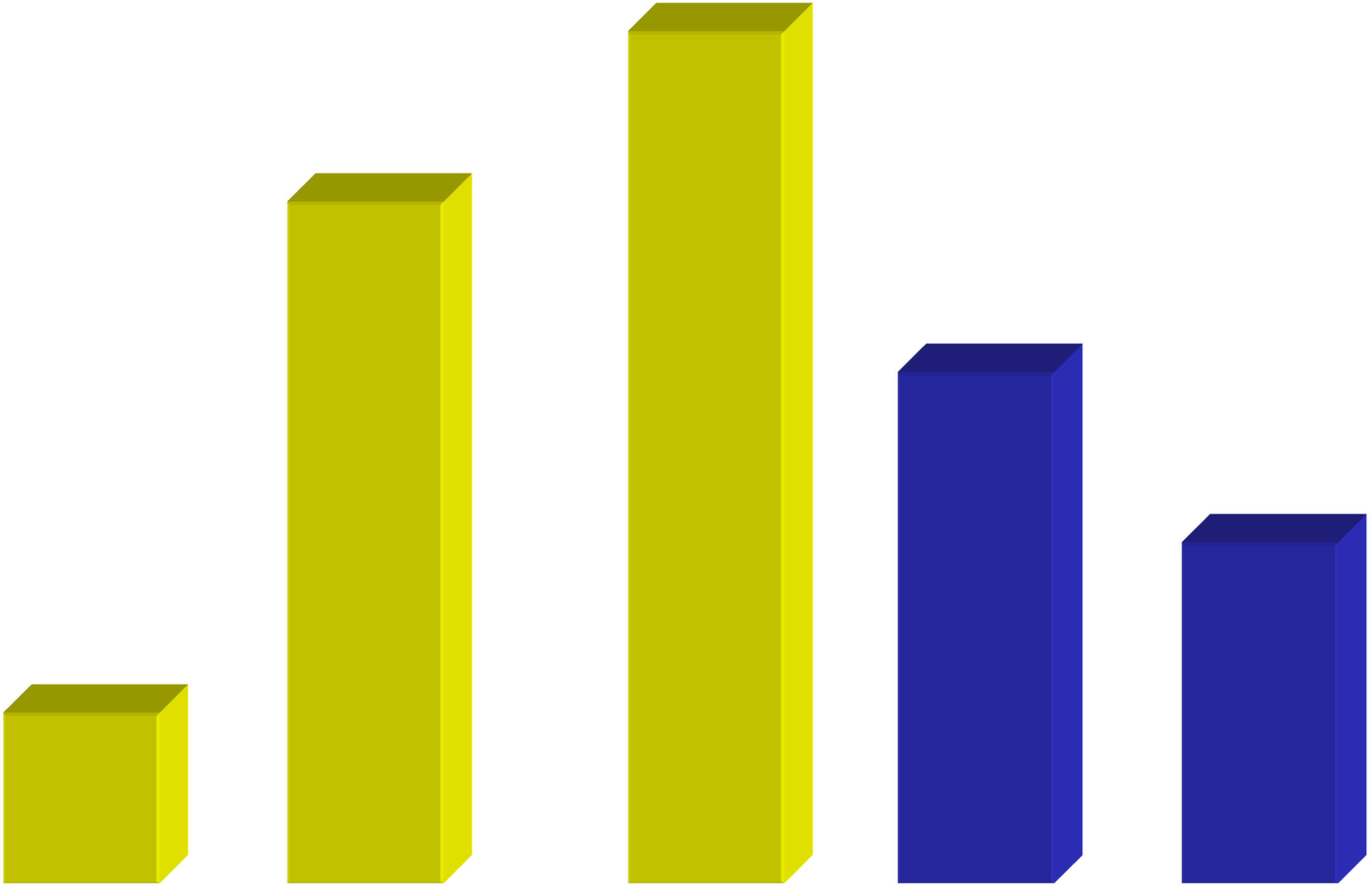

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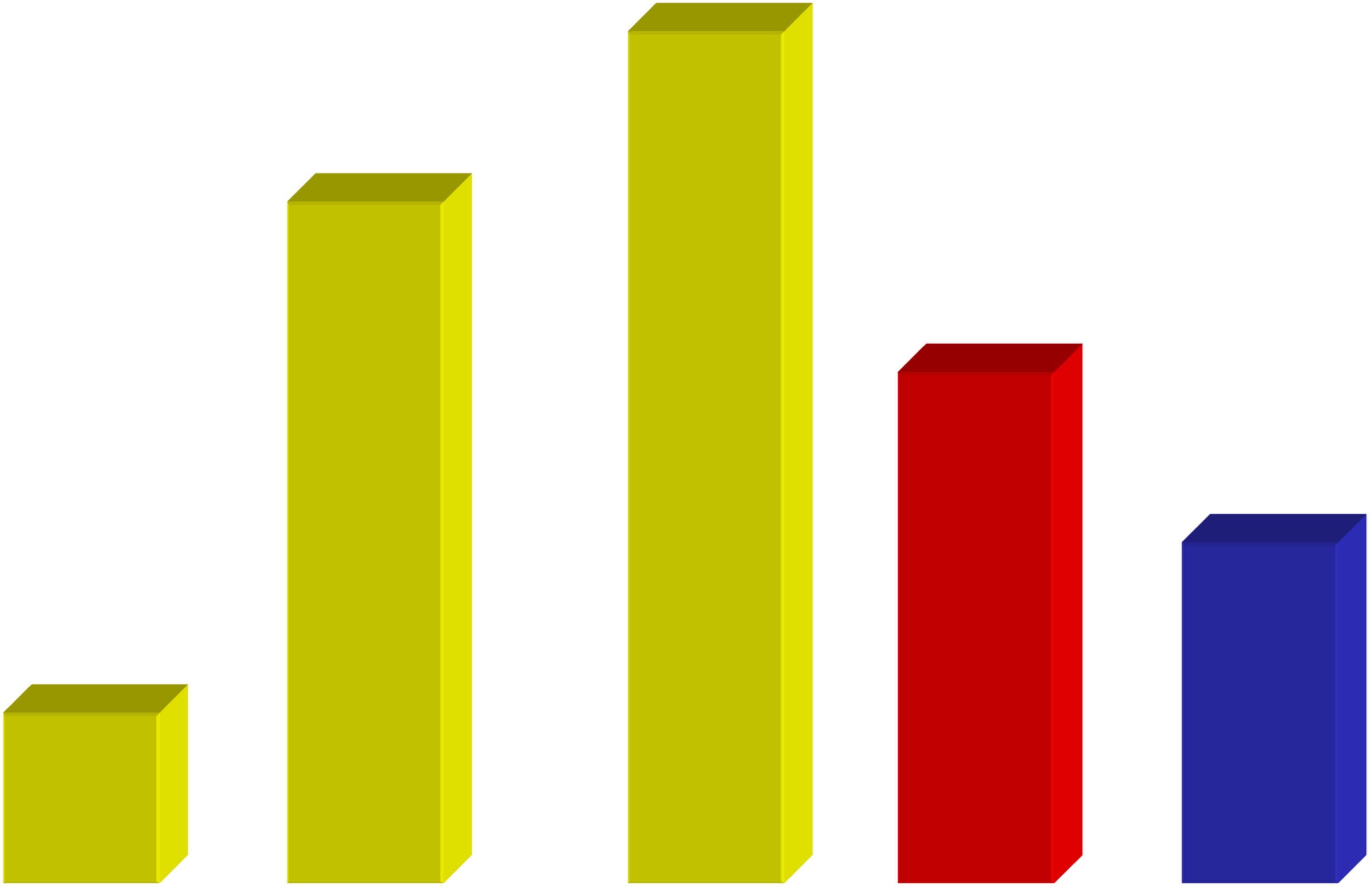

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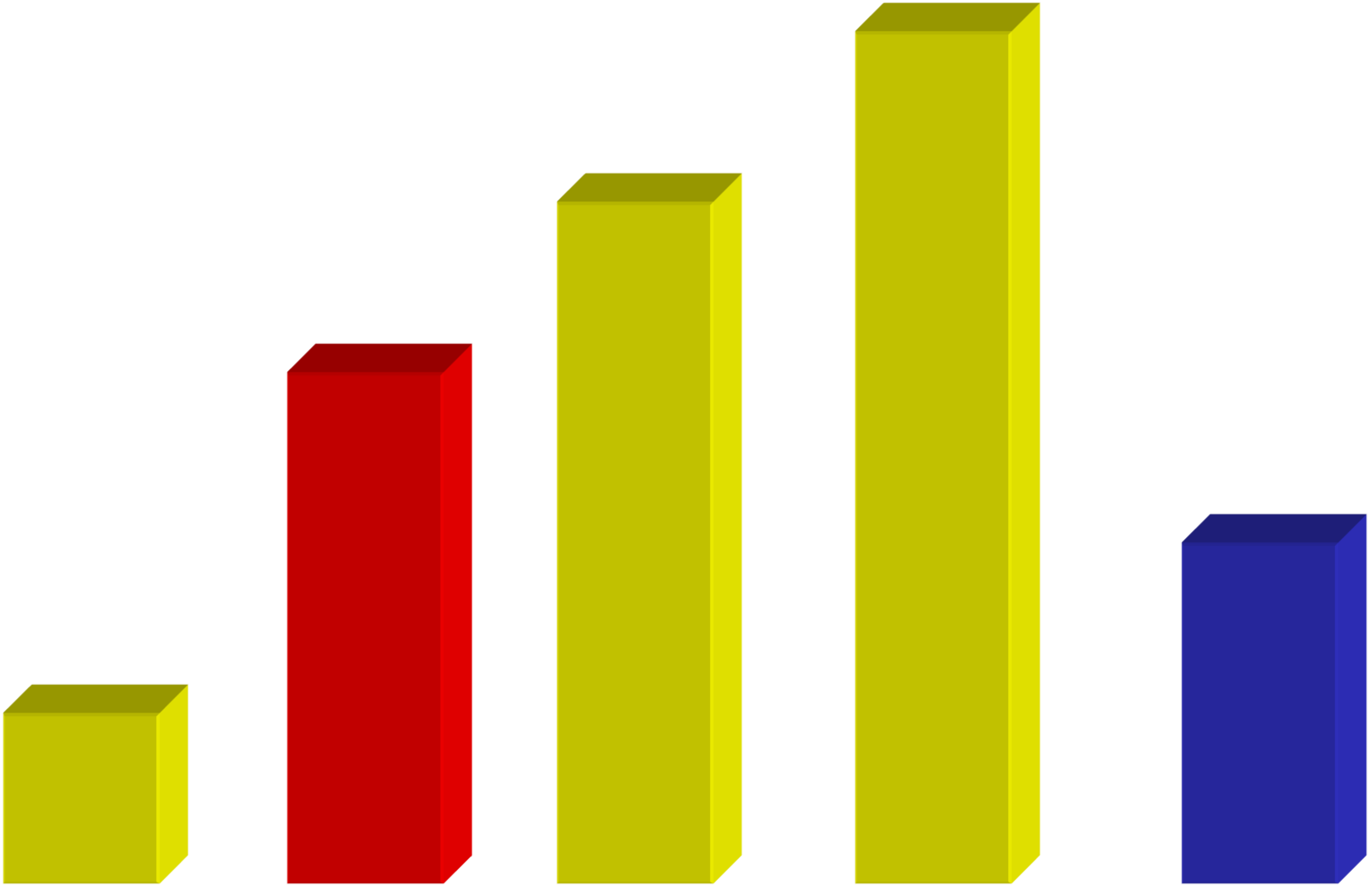

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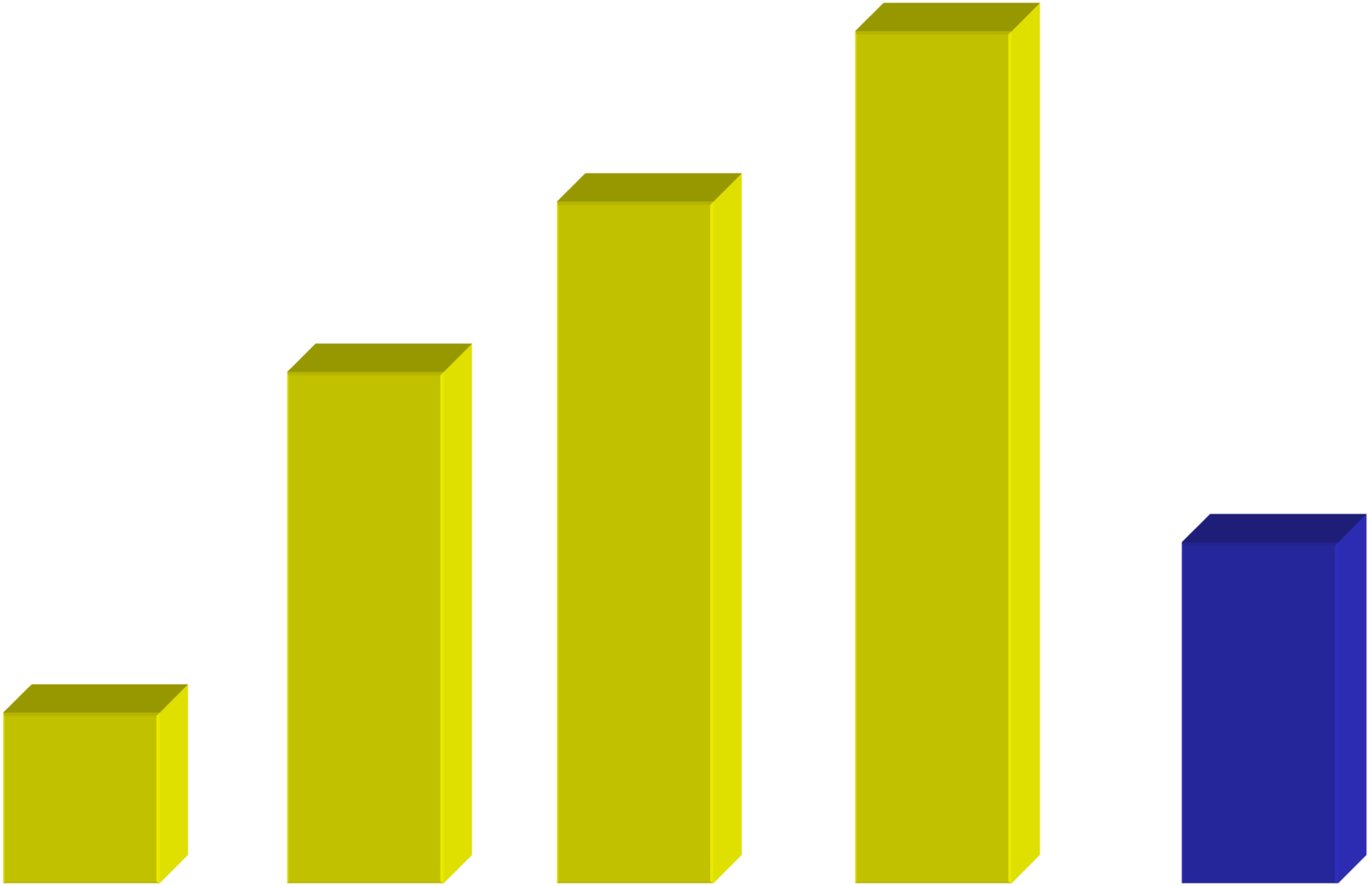

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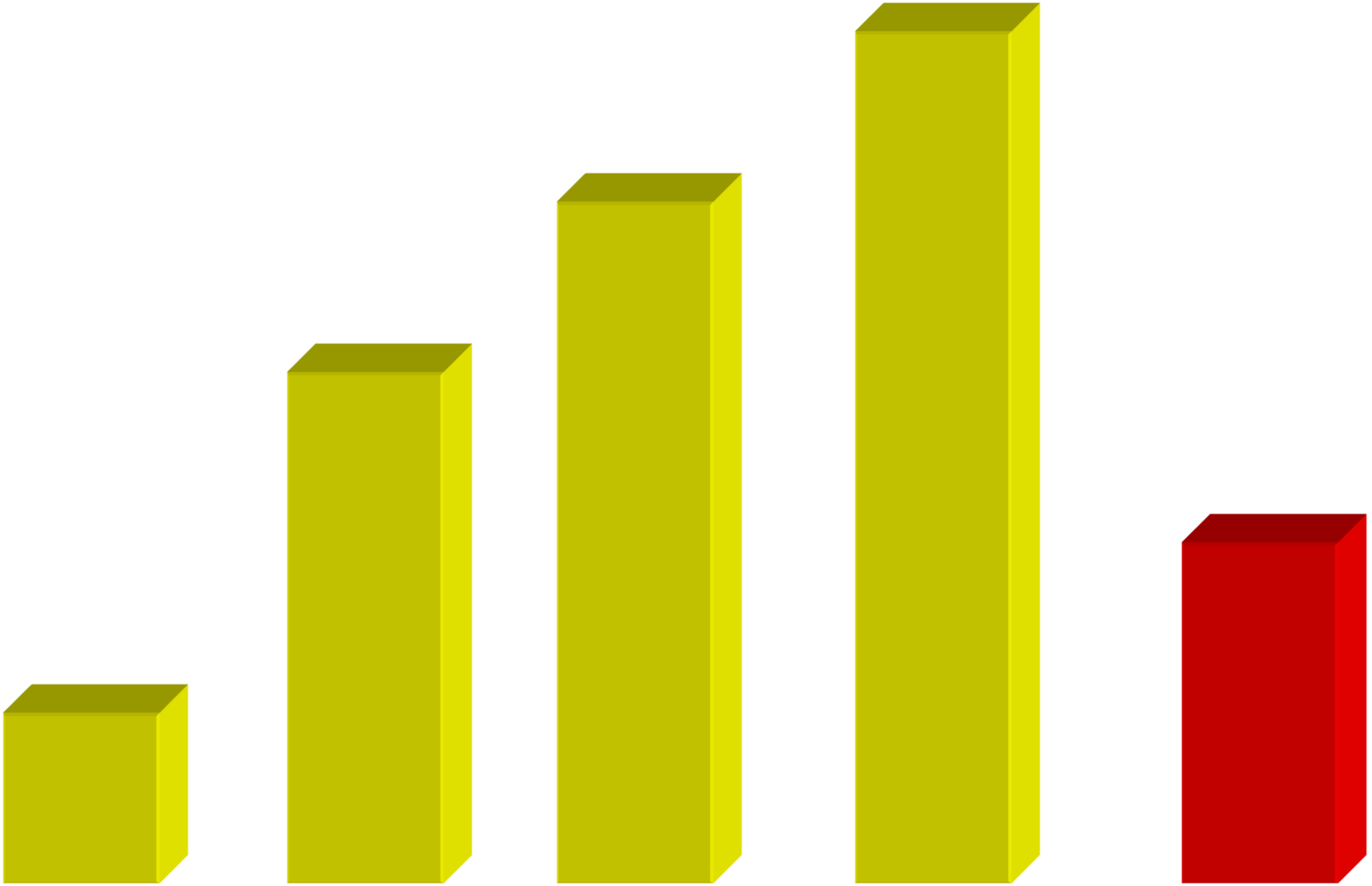

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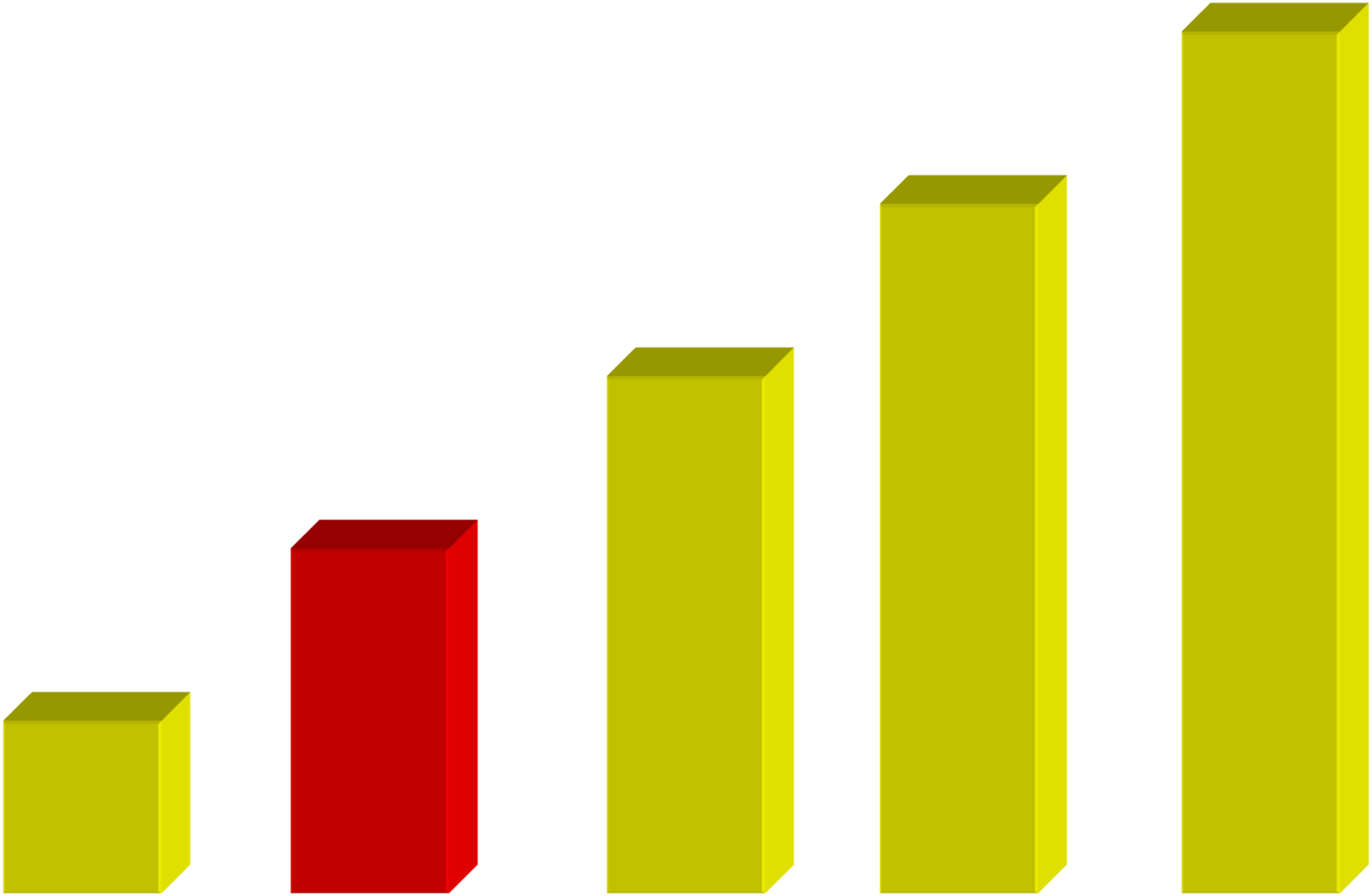

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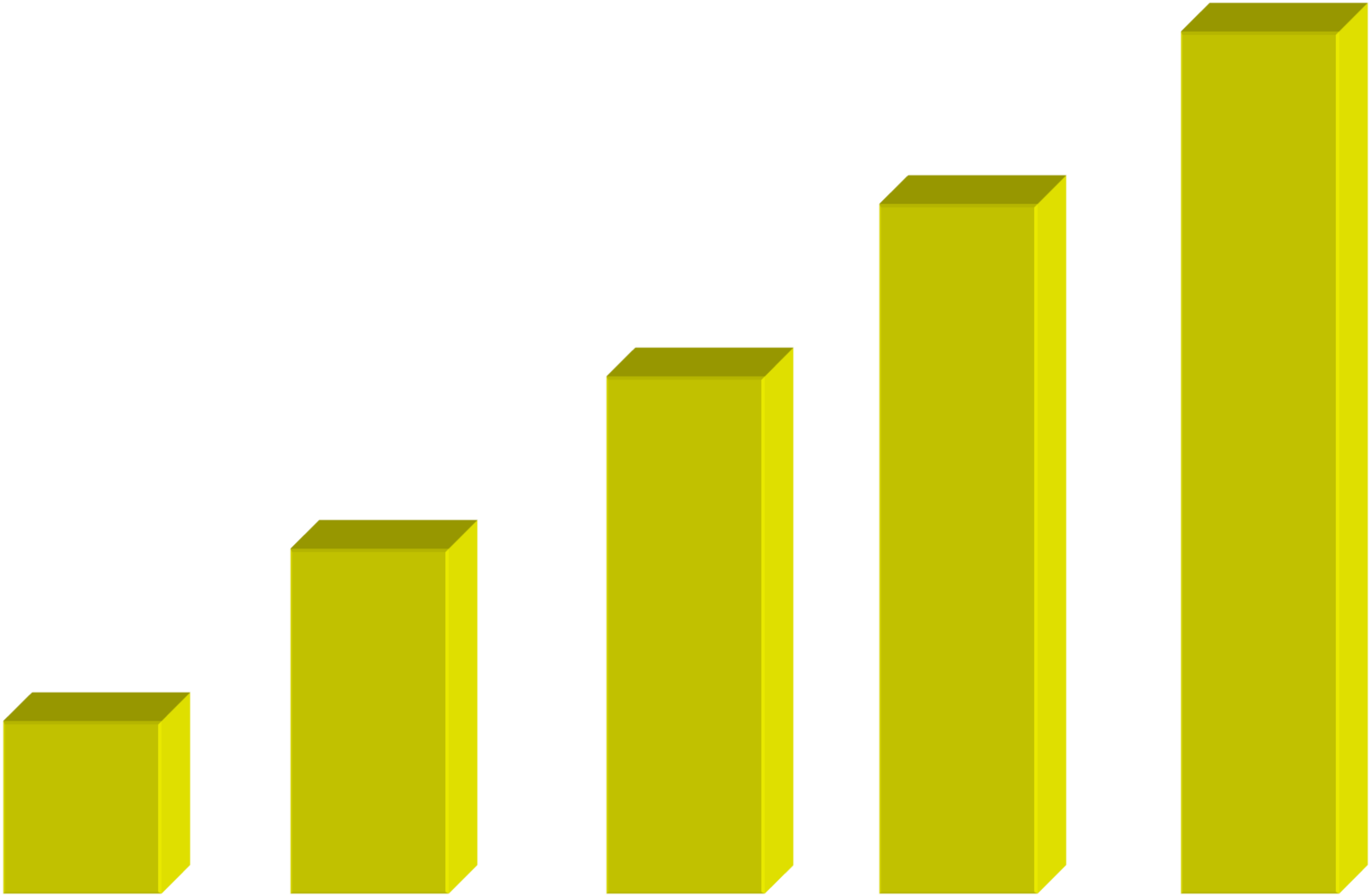

What is the Worst-case Running Time of Insertion Sort?

```
Algorithm insertionSort(A,n):
  Input: Array A of size n
  Output: Array A sorted

  for k ← 1 to n-1 do
      val ← A[k]
      j ← k-1
      while j ≥ 0 and A[j] > val do
          A[j+1] ← A[j]
          j ← j - 1
      end
      A[j+1] = val
  end
end
```

What is the Worst-case Running Time of Insertion Sort?

Dancing Sorts

- Selection sort - https://www.youtube.com/watch?v=Ns4TPTC8whw

- Bubble sort - https://www.youtube.com/watch?v=lyZQPjUT5B4

- Insertion sort - https://www.youtube.com/watch?v=ROalU379l3U