Please upload your solutions to the Math 122 A01 Crowdmark page. Solutions are due Monday March 7th by 11:59pm (PT).

For this assignment, everyone will submit their own copy of the solutions.

You may write your solutions on paper, with a tablet, or typed on computer. Note that in Crowdmark we grade work question by question, so each submission area needs a solution in order for us to view all your work.

Math 122 In-Class Assignment 8 - Solutions

Consider the sequence recursively defined by $a_0 = 1$, $a_n = 3a_{n-1} + 2$ for $n \geq 1$.

(a) Write the first terms a_1, a_2, a_3, a_4, a_5 in a way that will help you see the pattern of how the terms are created. That is, calculating just the final numerical value will not help you here - leave your answer in the form of an unsimplified sum of terms.

(b) Based on your answer to (a), write a guess for a formula for a_n, $n \geq 0$, that depends only on n (i.e. give an explicit form, not a recursive definition). Simplify your formula as best you can, don't just leave it as one big summation. (Only write your guess for a_n, you do not need to write an induction proof on this assignment.)

Solutions:

(a)
- $a_1 = 3a_0 + 2 = 3(1) + 2$
- $a_2 = 3a_1 + 2 = 3[3(1) + 2] + 2 = 3^2(1) + 3(2) + 2$
- $a_3 = 3a_2 + 2 = 3[3^2(1) + 3(2) + 2] + 2 = 3^3(1) + 3^2(2) + 3(2) + 2$
- $a_4 = 3a_3 + 2 = 3[3^3(1) + 3^2(2) + 3(2) + 2] + 2 = 3^4(1) + 3^3(2) + 3^2(2) + 3(2) + 2$
- $a_5 = 3a_4 + 2 = 3[3^4(1) + 3^3(2) + 3^2(2) + 3(2) + 2] + 2 = 3^5(1) + 3^4(2) + 3^3(2) + 3^2(2) + 3(2) + 2$

(b) Guess:

$$
\begin{aligned}
a_n &= 3^n + 3^{n-1}(2) + 3^{n-2}(2) + \ldots + 3^2(2) + 3(2) + 2 \\
&= 3^n + 3^{n-1}(2) + 3^{n-2}(2) + \ldots + 3^2(2) + 3(2) + 1(2) \\
&= 3^n + 2[3^{n-1} + 3^{n-2} + \ldots + 3^2 + 3 + 1] \\
&= 3^n + 2\left(\frac{3^n - 1}{3 - 1}\right) \\
&= 4^n + 2\left(\frac{3^n - 1}{2}\right) \\
&= 3^n + (3^n - 1) \\
&= 2 \cdot 3^n - 1
\end{aligned}
$$