Q:1 (a) $(3+2i)(8-2i) = 24+4+16i-6i = 28+10i$

(b) $\dfrac{1+4i}{3+2i} = \dfrac{1+4i}{3+2i} \times \dfrac{3-2i}{3-2i} = \dfrac{3+8+12i-2i}{9+4} = \dfrac{11}{13} + \dfrac{10}{13}i$

(c) $\overline{2i(4-i)} = \overline{8i+2} = 2-8i$

(d) $e^{\frac{\pi}{6}i} = \cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)$

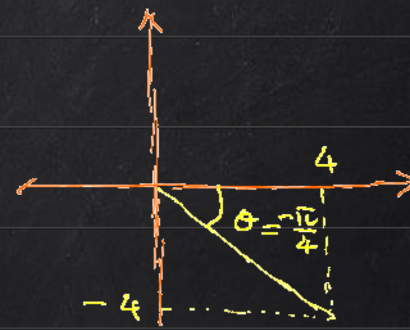
$= \left(\frac{\sqrt{3}}{2}\right) + i\left(\frac{1}{2}\right.$

Q. 2. (a) $z = 4 - 4i$

$$r = \sqrt{4^2 + (-4)^2} = 4\sqrt{2}$$

$$\theta = \tan^{-1}\left(\frac{-4}{4}\right) = -\frac{\pi}{4}$$

$$\Rightarrow \quad z = 4\sqrt{2}\, e^{-i\frac{\pi}{4}}$$

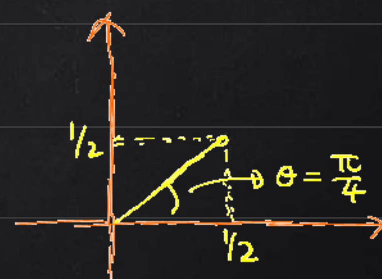


(b) $z = \frac{1}{2} + \frac{1}{2} i$

$$r = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1}\left(\frac{1/2}{1/2}\right) = \frac{\pi}{4}$$

$$\Rightarrow \quad z = \frac{1}{\sqrt{2}}\, e^{i\pi/4}$$

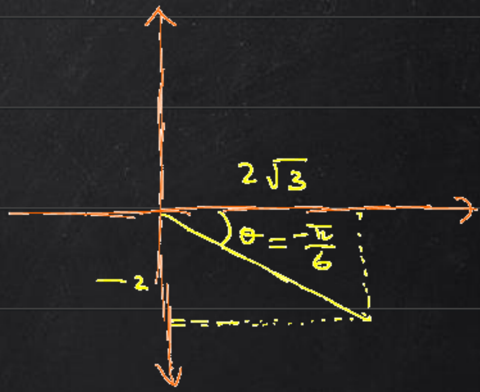


(c). $z = 2\sqrt{3} - 2i$

$$r = \sqrt{4(3) + 4} = 4$$

$$\theta = \tan^{-1}\left(\frac{-2}{2\sqrt{3}}\right) = \frac{-\pi}{6}$$

$$\Rightarrow z = 4e^{-i\pi/6}$$

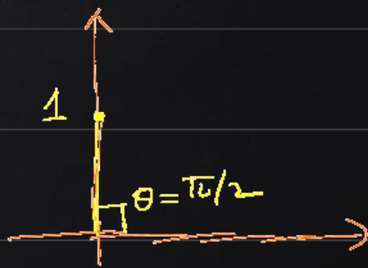


(d). $z = \frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = i$

$$r = \sqrt{0^2 + 1^2} = 1$$

$$\theta = \tan^{-1}\left(\frac{1}{0}\right) = \frac{\pi}{2}$$

$$\Rightarrow z = e^{i\pi/2}$$



Q:3. **(a)** $z = (1-\sqrt{3}\,i)^5$

Getting this to the polar form; $z = \left[2e^{-\frac{\pi}{3}i}\right]^5 = 32e^{-\frac{5\pi}{3}i}$

(b) $z = (1-i)^8$

Getting this to the polar form: $z = \left[\sqrt{2}e^{-\frac{\pi}{4}i}\right]^8 = (\sqrt{2})^8 e^{-\frac{8\pi}{4}i}$

Q:4 **(a)** $z = 32$

$$(32)^{\frac{1}{n}} = (32)^{\frac{1}{n}}\left[\cos\left(\frac{0+2k\pi}{n}\right) + i\sin\left(\frac{0+2k\pi}{n}\right)\right]$$

with $n=5$.

$$(32)^{\frac{1}{5}} = \sqrt[5]{32}\left[\cos\left(\frac{2k\pi}{5}\right) + i\sin\left(\frac{2k\pi}{n}\right)\right]$$

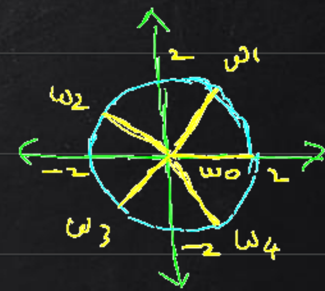
$k = 0$: $\quad \omega_0 = 2\left[\cos(0) + i\sin(0)\right] = 2$

$k = 1$: $\quad \omega_1 = 2\left[\cos(2\pi/5) + i\sin(2\pi/5)\right]$

$k = 2$: $\quad \omega_2 = 2\left[\cos(4\pi/5) + i\sin(4\pi/5)\right]$

$k = 3$: $\quad \omega_3 = 2\left[\cos(6\pi/5) + i\sin(6\pi/5)\right]$

$k = 4$: $\quad \omega_4 = 2\left[\cos(8\pi/5) + i\sin(8\pi/5)\right]$



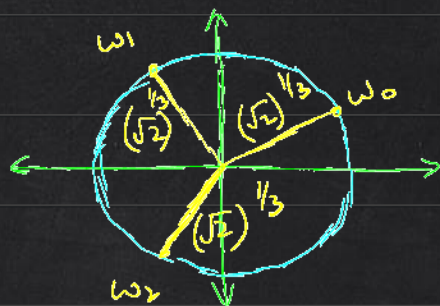
(b) $\quad z = 1+i \quad , \quad r = \sqrt{2}, \quad \theta = \frac{\pi}{4}$

$$(1+i)^{\frac{1}{3}} = (\sqrt{2})^{\frac{1}{3}}\left[\cos\left(\frac{\pi/4 + 2k\pi}{3}\right) + i\sin\left(\frac{\pi/4 + 2k\pi}{3}\right)\right]$$

$k=0$: $\quad \omega_0 = (\sqrt{2})^{\frac{1}{3}}\left[\cos\left(\frac{\pi}{12}\right) + i\sin\left(\frac{\pi}{12}\right)\right]$

$\underline{k=1}$: $\omega_1 = (\sqrt{2})^{1/3}\left[\cos\left(\frac{9\pi}{12}\right) + i\sin\left(\frac{9\pi}{12}\right)\right]$

$\underline{k=2}$: $\omega_2 = (\sqrt{2})^{1/3}\left[\cos\left(\frac{17\pi}{12}\right) + i\sin\left(\frac{17\pi}{12}\right)\right]$



$\underline{Q.5.}$ we have $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\Rightarrow e^{i\theta} = 1 + (i\theta) + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \cdots = 1 + i\theta - \frac{\theta^2}{2!} - \frac{i\theta^3}{3!} + \cdots$

or $e^{-i\theta} = 1 - i\theta - \frac{i\theta^2}{2!} + \frac{i\theta^3}{3!} + \cdots$

$$e^{i\theta} = \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right) = \cos\theta + i\sin\theta$$

$$\left(e^{i\theta} + e^{-i\theta}\right)/2 = \left(2 - 2\theta^2/2! + 2\theta^4/4! - \cdots\right)/2 = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots = \cos\theta.$$