| | |
|---|---|
| **Student:** Arfaz Hossain<br>**Date:** 11/14/21 | **Instructor:** UVIC Math<br>**Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 8 |

Find the smallest perimeter and the dimensions for a rectangle with an area of 121 $in^2$.

With $\ell$ representing the length and w representing the width of a rectangle, the perimeter, P, and area, A, are $P = 2\ell + 2w$ and $A = \ell w$.

Substitute 121 for A, and solve for the variable $\ell$.

$$\ell = \frac{A}{w} = \frac{121}{w}$$

Substitute the expression for $\ell$ into the formula for perimeter.

$$P = 2\frac{121}{w} + 2w$$

Find the derivative with respect to w.

$$\frac{dP}{dw} = -\frac{242}{w^2} + 2$$

Set the derivative equal to zero, and solve.

$$-\frac{242}{w^2} + 2 = 0$$

$$w = \pm 11$$

Since a rectangle dimension cannot be negative, discard $-11$ and test 11.

The second derivative of the perimeter function $P = \frac{242}{w} + 2w$ is $\frac{d^2P}{dw^2} = \frac{484}{w^3}$.

Since $\frac{484}{w} > 0$ when w is positive, the perimeter $\frac{242}{w} + 2w$ for $w = 11$ is the smallest perimeter.

Substituting $w = 11$ into the area formula, $121 = \ell w$, the length is 11 in.

The smallest perimeter is $2(11) + 2(11) = 44$ in.

The dimensions of the rectangle of smallest perimeter with area 121 $in^2$ are 11 in by 11 in.