Symbolab

Solution

Check convergence of $\sum_{n=1}^{\infty}\frac{n^{\sqrt{2}}}{14^n}$: converges

Steps

$\sum_{n=1}^{\infty}\frac{n^{\sqrt{2}}}{14^n}$

Apply Series Ratio Test: converges

Hide Steps

$\sum_{n=1}^{\infty}\frac{n^{\sqrt{2}}}{14^n}$

Series Ratio Test:

If there exists an N so that for all $n \ge N$, $a_n \ne 0$ and $\lim_{n\to\infty}|\frac{a_{n+1}}{a_n}| = L$:

If $L < 1$, then $\sum a_n$ converges

If $L > 1$, then $\sum a_n$ diverges

If $L = 1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(n+1)^{\sqrt{2}}}{14^{(n+1)}}}{\frac{n^{\sqrt{2}}}{14^n}}\right|$$

Hide Steps

Simplify $\left|\frac{\frac{(n+1)^{\sqrt{2}}}{14^{(n+1)}}}{\frac{n^{\sqrt{2}}}{14^n}}\right|$: $\frac{\left|(n+1)^{\sqrt{2}}\right|}{14\left|n^{\sqrt{2}}\right|}$

$$= \left|\frac{\frac{(n+1)^{\sqrt{2}}}{14^{n+1}}}{\frac{n^{\sqrt{2}}}{14^n}}\right|$$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$

$$= \left|\frac{(n+1)^{\sqrt{2}} \cdot 14^n}{14^{n+1} n^{\sqrt{2}}}\right|$$

Apply exponent rule: $\frac{x^a}{x^b} = \frac{1}{x^{b-a}}$

$\frac{14^n}{14^{n+1}}=\frac{1}{14^{n+1-n}}$

$=\frac{(n+1)^{\sqrt{2}}}{14^{n-n+1}n^{\sqrt{2}}}$

Add similar elements: $n+1-n=1$

$=\left|\frac{(n+1)^{\sqrt{2}}}{14n^{\sqrt{2}}}\right|$

Apply absolute rule: $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$=\frac{\left|(n+1)^{\sqrt{2}}\right|}{\left|14n^{\sqrt{2}}\right|}$

Apply absolute rule: $|ax| = a|x|, a \geq 0$

$\left|14n^{\sqrt{2}}\right|=14\left|n^{\sqrt{2}}\right|$

$=\frac{\left|(n+1)^{\sqrt{2}}\right|}{14\left|n^{\sqrt{2}}\right|}$

$\lim_{n\to\infty}\left(\frac{\left|(n+1)^{\sqrt{2}}\right|}{14\left|n^{\sqrt{2}}\right|}\right)=\frac{1}{14}$ *Hide Steps*

$\lim_{n\to\infty}\left(\frac{\left|(n+1)^{\sqrt{2}}\right|}{14\left|n^{\sqrt{2}}\right|}\right)$

$(n+1)^{\sqrt{2}}$ is positive when $n\to\infty$. Therefore $\left|(n+1)^{\sqrt{2}}\right|=(n+1)^{\sqrt{2}}$

$=\lim_{n\to\infty}\left(\frac{(n+1)^{\sqrt{2}}}{14\left|n^{\sqrt{2}}\right|}\right)$

$n^{\sqrt{2}}$ is positive when $n\to\infty$. Therefore $\left|n^{\sqrt{2}}\right|=n^{\sqrt{2}}$

$=\lim_{n\to\infty}\left(\frac{(n+1)^{\sqrt{2}}}{14n^{\sqrt{2}}}\right)$

$\lim_{x\to a}[c\cdot f(x)]=c\cdot\lim_{x\to a}f(x)$

$=\frac{1}{14}\cdot\lim_{n\to\infty}\left(\frac{(n+1)^{\sqrt{2}}}{n^{\sqrt{2}}}\right)$

Simplify $\frac{(n+1)^{\sqrt{2}}}{n^{\sqrt{2}}}$: $\left(\frac{n+1}{n}\right)^{\sqrt{2}}$ *Show Steps*

$=\frac{1}{14}\cdot\lim_{n\to\infty}\left(\left(\frac{n+1}{n}\right)^{\sqrt{2}}\right)$

$\lim_{x\to a}[f(x)]^b = \left[\lim_{x\to a} f(x)\right]^b$
With the exception of indeterminate form

$$= \frac{1}{14}\left(\lim_{n\to\infty}\left(\frac{n+1}{n}\right)\right)^{\sqrt{2}}$$

Divide by highest denominator power: $1+\frac{1}{n}$ Show Steps

$$= \frac{1}{14}\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\right)^{\sqrt{2}}$$

$\lim_{x\to a}[f(x)\pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
With the exception of indeterminate form

$$= \frac{1}{14}\left(\lim_{n\to\infty}(1)+\lim_{n\to\infty}\left(\frac{1}{n}\right)\right)^{\sqrt{2}}$$

$\lim_{n\to\infty}(1) = 1$ Show Steps

$\lim_{n\to\infty}\left(\frac{1}{n}\right) = 0$ Show Steps

$$= \frac{1}{14}(1+0)^{\sqrt{2}}$$

Simplify

$$= \frac{1}{14}$$

$L<1$, by the ratio test

$=$ converges

$=$ converges