MATH 100, Fall 2021
Tutorial Worksheet
Tutorial Section (T01, T02 etc)________
Tutorial Instructor Name:________________
Question Number Attempted (Q1, Q2, etc) Q1

Your Name: KEY
Your Student Number: V00 000001
Today's Date: Sept 16.

1. Average speed: $\dfrac{x(2)-x(0)}{2-0} = \dfrac{4-0}{2} = 2$

   Units are m/sec.

2. 8 metres. $(4\uparrow, 4\downarrow)$. from graph.

3. We could only estimate from graph. Exact calc:

$$V(3) = \lim_{h\to 0} \frac{[4(3+h) - (3+h)^2] - [12-9]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{12} + 4h - (\cancel{9} + 6h + h^2) - \cancel{12} + \cancel{9}}{h}$$

$$= \lim_{h\to 0} \frac{-2h - h^2}{h} = \lim_{h\to 0} \frac{-2-h}{1} = -2$$

<u>Speed</u> $= |v| = |-2| = 2$ metres/sec.

<u>Accepted</u>: $V(3) = x'(3) = 4 - 2t \Big|_{t=3} = -2$
Speed $= |v| = |-2| = 2$. m/sec.

4. Want speed $= 0 \iff |v| = 0 \iff v = 0$
$\iff x'(t) = 0$
$\iff 4 - 2t = 0$
$\iff \boxed{t = 2}$. time $t = 2$

<u>Note</u>: From the graph, $t=2$ is one time. From the calc, $t=2$ is the <u>only</u> time.

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Tutorial Worksheet
Tutorial Section (A01, A02 etc)________
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Question Number Attempted (Q1, Q2, etc) Q2

Your Name: Key
Your Student Number: V00__________
Today's Date:______________

1) Since $\lim\limits_{x \to 2/\pi} \frac{1}{x} = \pi/2$

and $\lim\limits_{x \to 2/\pi} \sin\left(\frac{1}{x}\right) = \sin(\pi/2) = 1$,

then $\lim\limits_{x \to 2/\pi} x^2 \sin\left(\frac{1}{x}\right) = \lim\limits_{x \to 2/\pi} x^2 \lim\limits_{x \to 2/\pi} \sin\left(\frac{1}{x}\right)$

$= \left(2/\pi\right)^2 \cdot 1 = \frac{4}{\pi^2}$

2) Since limit is $x \to 0$ we cannot do the same as in 1).

Note $-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$ and so

for $x > 0$, $-x \leq x\cos\left(\frac{1}{x}\right) \leq x$

for $x < 0$ $-x \geq x\cos\left(\frac{1}{x}\right) \geq x$

$\lim\limits_{x \to 0^+} x\cos\left(\frac{1}{x}\right) = 0$ by the sandwich thm

$\lim\limits_{x \to 0^-} x\cos\left(\frac{1}{x}\right) = 0$ " " " "

Limits are equal, so $\lim\limits_{x \to 0} x\cos\left(\frac{1}{x}\right) = 0$

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Tutorial Worksheet
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Question Number Attempted (Q1, Q2, etc) Q3

Your Name: KEY
Your Student Number: V00__________
Today's Date:________________

1. For t near $\pi/2$, $|\sin t| = \sin t$ and

$\sin(\pi/2) = 1 \quad \left[\frac{d}{dt}\sin t\right](\pi/2) = \cos(\pi/2) = 0$

Eq^n of tangent is $\boxed{y = 1}$ [horizontal tangent]

2. ✱ $\lim\limits_{h\to 0^+} \dfrac{|\sin h| - |\sin 0|}{h} = \lim\limits_{h\to 0^+} \dfrac{\sin h - 0}{h}$

$= \lim\limits_{h\to 0^+} \dfrac{\sin h}{h} = \boxed{1}$

✱ $\lim\limits_{h\to 0^-} \dfrac{|\sin h| - |\sin 0|}{h} = \lim\limits_{h\to 0^-} \dfrac{-\sin h - 0}{h}$

$= \lim\limits_{h\to 0^-} \dfrac{-\sin h}{h} = \boxed{-1}$

Both one-sided limits exist, but they are different. Therefore there is no tangent slope at $x = 0$.

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Tutorial Worksheet
Tutorial Section (A01, A02 etc)________
Tutorial Instructor Name:________________________
Question Number Attempted (Q1, Q2, etc) Q 4

Your Name: GEORGE LEE
Your Student Number: V00__________
Today's Date:__________

1. For f to be continuous at 0, we check:

* $f(0) = 0$

* Then need $\lim_{x\to 0} f(x) = 0$: Sufficient to check both the signed limits agree and are 0:

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x^2+x) = 0+0=0, \quad \text{and}$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} mx = m\cdot 0 = 0.$$

Therefore : All m values result in a continuous function at 0

2. $$\boxed{\begin{aligned}&\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^+} \frac{mh-0}{h} = \lim_{h\to 0^+} m = m, \text{ whereas} \\ &\lim_{h\to 0^-} \frac{f(h)-f(0)}{h} = \lim_{h\to 0^-} \frac{h^2+h-0}{h} = \lim_{h\to 0} (h+1) = 1\end{aligned}}$$

3: $\lim_{h\to 0} \frac{f(h)-f(0)}{h} = f'(0)$ exists if and only if the signed limits from (2.) agree, ie: $\lim_{h\to 0^+} \frac{f(h)-f(0)}{h} = \boxed{m=1} = \lim_{h\to 0^-} \frac{f(h)-f(0)}{h}$

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Question Number Attempted (Q1, Q2, etc) Q5

(1) The functions $\sin(x)$ and $\cos(x)$ are continuous at 0 if

$$\lim_{x \to 0} \sin(x) = \sin(0)$$

and $\lim_{x \to 0} \cos(x) = \cos(0)$.

(2) $\lim_{h \to 0} \sin(x+h) = \lim_{h \to 0} \sin(x)\cos(h) + \cos(x)\sin(h)$

$= \sin(x) \lim_{h \to 0} \cos(h) + \cos(x) \lim_{h \to 0} \sin(h)$

by (1) $= \sin(x) \cdot 1 + \cos(x) \cdot 0$

$= \sin(x)$.

(3) $\lim_{x \to c} \sin(x) = \sin(c)$, ~~$\lim_{x \to c} \sin(x) - \sin(c) = 0$~~.

$\lim_{x \to c} \sin x$ is the same as $\lim_{x-c \to 0} \sin((x-c)+c)$

$= \lim_{x-c \to 0} \sin(x-c)\cos(c) + \cos(x-c)\sin c$

$= \sin(0)\cos(c) + \cos(0)\sin(c)$

$= \sin(c)$.