ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 23: One-Sided Sensitivity Analysis

October 29, 2021

Version 1.

Required Reading

- Eschenbach, T. G. (1992). Spiderplots versus Tornado Diagrams for Sensitivity Analysis. *Interfaces*, 22(6), 40-46. Retrieved from https://www-jstor-org.ezproxy.library.uvic.ca/stable/25061678
  - A short and important discussion **of sensitivity (spider) plots and tornado diagrams**. Includes a worked example.

Recommended Reading

- *Engineering Economics*, 12.1-12.3
- Eschenbach, T. G. & McKeague, L. S. (1989). Exposition on Using Graphs for Sensitivity Analysis. *The Engineering Economist*, 34(4), pp. 315 – 333. Retrieved from http://dx.doi.org/10.1080/00137918908902996
- **A more detailed discussion of the graphs and what they mean. Note that their 'tornado graph' is turned on its side compared to ours.**
- MBA Charts. (n.d.) Tornado Charts [Web Page]. Retrieved from https://web.archive.org/web/20170618203132/http://mbacharts.com/2015/12/23/tornado-chart/
- **A step-by-step guide to creating tornado charts in Excel. (Archived version from WaybackMachine, since the page no longer exists.)**
- Woodill, A. J., Nakamoto, S. T., Kawabata, A. M. & Leung, P. (2017). To Spray or Not to Spray: A Decision Analysis of Coffee Borer in Hawaii. *Insects*, 8(4), 116. Retrieved from http://dx.doi.org/10.3390/insects8040116
- **An excellent, easy-to-read article that ties together scenario analysis, sensitivity graphs and decision trees.**

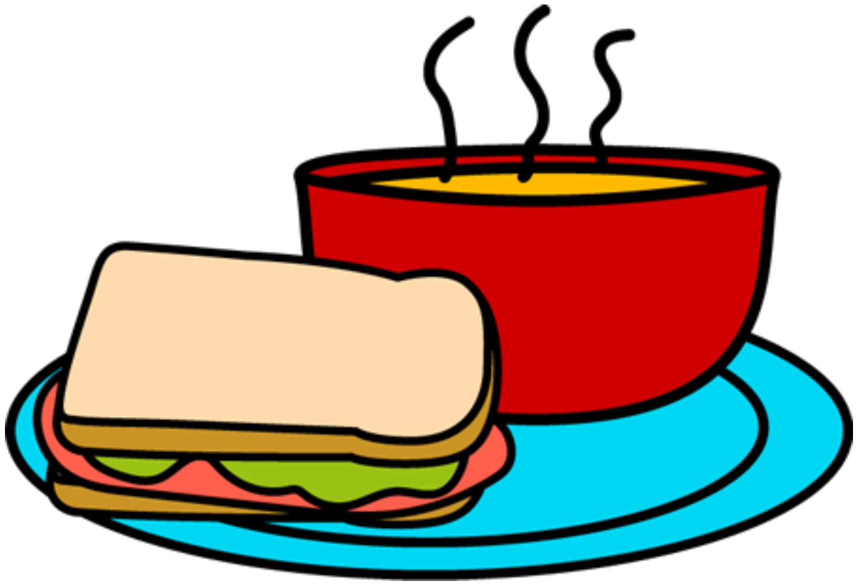
Case Studies: Scenarios, Spider & Tornado Plots

- Chiţescu, C. L., Nicolau, A. I., Römkens, P. & Van Der Fels-Klerx, H. J. (2014). Quantitative modelling to estimate the transfer of pharmaceuticals through the food production system. *Journal of Environmental Science and Health, Part B*, 49(7), 457-467. Retrieved from http://search.ebscohost.com.ezproxy.library.uvic.ca/login.aspx?direct=true&db=mnh&AN=24813980&site=ehost-live&scope=site
  - **Published Spider Plot Example**
- Ghazali, Z. & Majid, M.A.A. (2011). Engineering economic analysis for waste heat boilers: A case of an integrated petrochemical complex in Malaysia. *2011 IEEE Colloquium on Humanities, Science and Engineering (CHUSER)*, pp. 311-316. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6163740
  - **Waste Heat Boiler Example**
- Redfield, C.R., Nores, M., Barnett, S. & Schweinhart, L. (2006). The High/Scope Perry Preschool Program: Cost-Benefit Analysis Using Data from the Age-40 Followup. *The Journal of Human Resources*, 41(1), pp. 162 – 190. Retrieved from https://www-jstor-org.ezproxy.library.uvic.ca/stable/40057261
  - **Published Scenario Table Example**
- Wang, S. J. et al. (2003). A Cost-Benefit Analysis of Electronic Records in Primary Care. *The American Journal of Medicine*, 114(5), pp. 397 – 403. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1016/S0002-9343(03)00057-3
  - **Published Tornado Diagram Example**

ESSENTIALS (20 slides)

The Cost of Soup & a Sandwich

- **So far, we've assumed we know 'the' value of parameters. What if we don't?**
- The local lunch counter makes delicious food, and their chef prepares a different menu every day. The menu varies with the chef's mood and what's available at the local farmer's market.
- You plan to buy a soup & sandwich for lunch.
- You expect the soup to cost $4…
- …but it could be as high as $5 or as low as $2.
- You expect the sandwich to cost $7…
- ...but it could be as high as $9 or as low as $3.
- What's the total cost of lunch (soup & sandwich)?



Clipart Kid

The general case

- We have a value function, V()
- V() is a function of various parameters $x_i = x_1, x_2, x_3 \ldots, x_n$
- The value of each x_i is not certain. Instead, it varies within a range.
- We are interested in knowing the numerical value of V()
- For ease of intuition, instead of writing $V(x_1, \ldots, x_n)$, I will write V(a,b,c), but this does *not* mean that the value function can only depend on three parameters! (It just avoids my having to use potentially confusion logic notation for sets excluding an element, etc.)
- In our simple example, $V(x_1, \ldots, x_n) = \text{LunchCost}(P_{soup}, P_{sandwich})$

Sensitivity Analysis to the rescue!

- Sensitivity analysis studies how changes in ONE parameter (e.g. the cost of soup) affects a value of interest (cost of lunch).
- It lets us figure out what happens to our results if our input values 'aren't quite right'.
- Sensitivity analysis is a topic worthy of a course on its own…
- In this introductory lecture, I'll focus on three types of sensitivity analyses that are often found in published papers:
- Scenario Analysis, Tornado Diagrams and Spider Plots (also called Sensitivity Graphs).

Minimum, Maximum and Baseline Values

	Minimum	Maximum	Baseline
Soup	\$2	\$5	\$4
Sandwich	\$3	\$9	\$7

- The baseline values are the ones we think are ‘about right’, or usual.
- They are NOT always the average of the Min and Max.
- The ’Minimum’ and ’Maximum’ values above are hard limits.
- The values won’t be above the Maximum, or below the Minimum values.
- How do we deal with this uncertainty?

The general case

- Each parameter in our value function can vary within a range.
- This range is between the minimum and maximum values.
- $\mathrm{x_i} \in \left[\mathrm{x_i^{min}}, \mathrm{x_i^{max}}\right]$
- In addition to the minimum and maximum values, we also have *baseline* values for each parameter, $\mathrm{x_i^{base}}$.
- The baseline value is our 'main' value – the one we'd use in our analysis if we could ignore uncertainty.
- "I think it'll be about 11 degrees today, but it could get as cold as 6 or as hot as 12" → Baseline = 11, Min = 6, Max = 12

A first attempt: Scenario Analysis

- It's common to report baseline, worst case and best case scenarios.
- **Baseline**: What we expect the outcome to be.
- **Worst Case**: what's the worst outcome possible, given our limits?
- **Best Case**: what's the best outcome possible, given our limits?
- This has a few advantages:
- Only need 3 sets of numbers (min, max, baseline)
- These numbers are 'easy' to obtain (not so easy to narrow the range)
- Easy to interpret

The general case

- Assume our value function is such that a higher V() is better than a lower V(). (Not necessarily the case: V() as in our 'cost of lunch' example, V() could be something you want to minimize.)
- If a higher V() is better than a lower V(), and V() = V(a,b,c)
- Baseline Scenario: $V(a^{base},b^{base},c^{base})$
- Best case scenario: The maximum value of V(a,b,c), *given* the limits of a,b and c. (e.g. $a_{min} \leqq a \leqq a_{max}$)
- Worst case scenario: The minimum value of V(a,b,c), *given* the limits of a,b and c. (e.g. $a_{min} \leqq a \leqq a_{max}$)

For our example

	Low	High	Baseline	Worst Case	Best Case
Soup	$2	$5	**$4**	**$5**	**$2**
Sandwich	$3	$9	**$7**	**$9**	**$3**
$ for Lunch			**$11**	**$14**	**$5**

A second attempt: tornado graphs

- Our value of interest is the cost of Lunch.
- Lunch could be as cheap as $5, or as expensive as $14.
- It’s important to know which parameters our value of interest is most sensitive to (so we can be sure to get it right, or focus on changing it).
- One way to do this is to run a scenario analysis where we change only ONE parameter at a time.
- We keep all other values at baseline, and let our parameter of interest take on its highest and lowest possible values.
- We then draw a bar from the lowest to the highest value achieved this way, and add a vertical line in the middle to remind us where the baseline is.

Example: Allowing Soup to change

- Lunch Cost = Soup + Sandwich
- For the baseline value of Sandwich ($7):
- Lunch Cost = Soup + 7
- Soup can range from $2 to $5 (baseline is $4):
- $2 + $7 = $9
- $5 + $7 = $12
- We can represent this as a bar from $9 to $12:

Assembling the Tornado

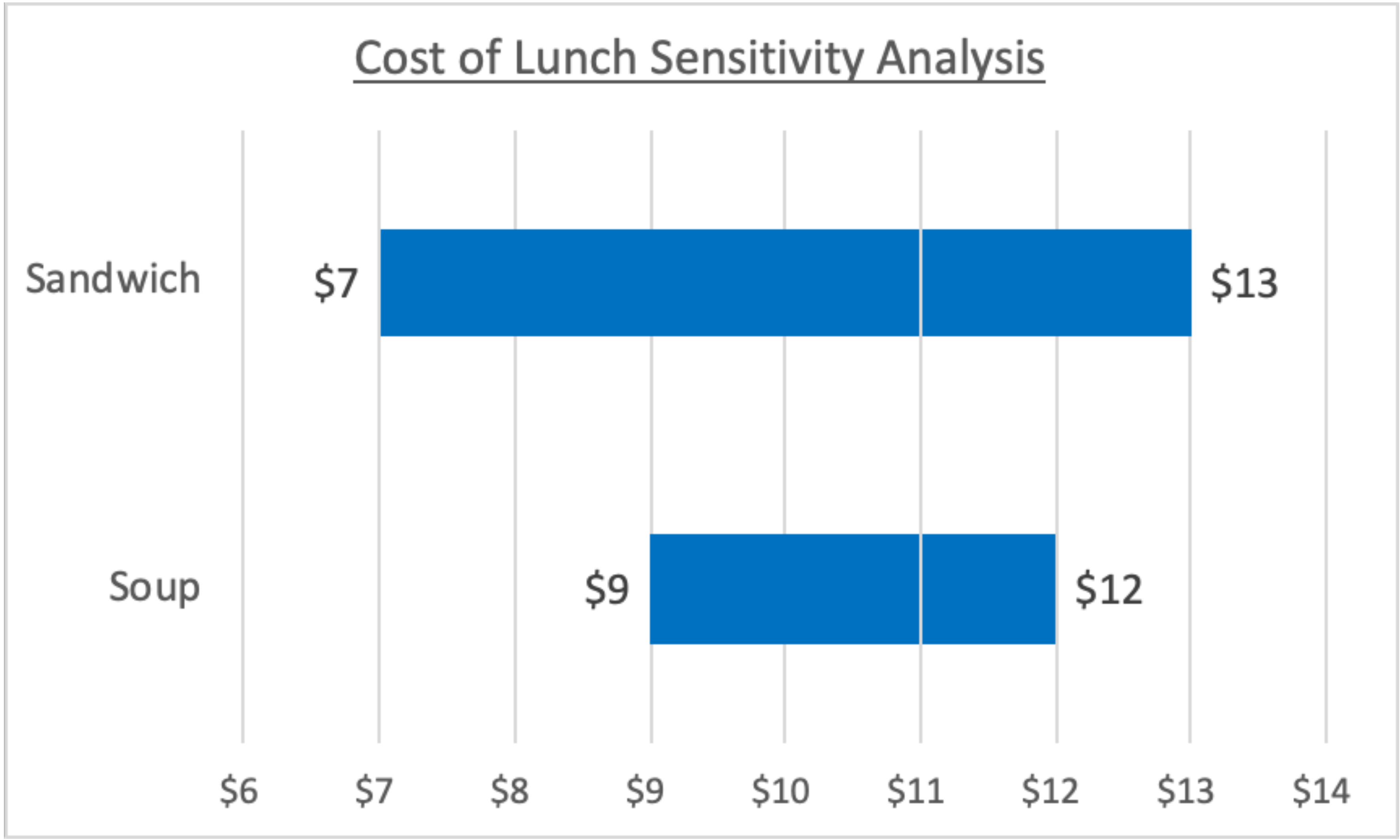
- Sandwich ranges from \$3 to \$9. Keeping Soup at baseline (\$4), this can make lunch cost vary from \$7 (= 4 + 3) to \$13 (= 4 + 9)

Tornado Diagram Lunch Cost Values			
If we vary…	Low	High	Bar Width
Soup	\$9	\$12	\$3
Sandwich	\$7	\$13	\$6

By convention, the widest bars are at the top, and the smallest at the bottom. This puts more sensitive parameters at top, and makes the diagram easier to interpret.

It’s also what gives it the ‘tornado’ shape!

Using Excel



- For more information, see the companion spreadsheet.

The general case

- Each tornado ‘bar’ is drawn between two values of V().
- At these values, one parameter is set at its minimum and maximum levels, and the rest are kept at baseline.
- For V(a,b,c):
- ‘a’ bar: Connects $V(a_{min},b_{base},c_{base})$ and $V(a_{max},b_{base},c_{base})$
- ‘b’ bar: Connects $V(a_{base},b_{min},c_{base})$ and $V(a_{base},b_{max},c_{base})$
- ‘c’ bar: Connects $V(a_{base},b_{base},c_{min})$ and $V(a_{base},b_{base},c_{max})$
- Arrange the bars so the widest bars are on top and the narrowest on the bottom.

The next step: spider (sensitivity) graphs

- A tornado graph shows, one parameter at a time, the lowest and highest possible outcomes (plus the baseline).
- What if we want to know *how* we get from the baseline to the extreme outcomes? Is it slow? Fast? Linear? Exponential?
- To find out, we repeat the tornado-type calculations (hold all but one value at baseline), but we calculate the value of interest for lots of values in between the minimum and the maximum.
- Problem: If we want to have all the relevant plots on one graph, we'll need a common unit of measurement. (e.g. Discount rates are in %, Costs are in $).
- It's standard to use '% deviation from baseline'.

For our example…

$$\text{\% Deviation from Baseline: } \frac{\text{Value}}{\text{Baseline Value}} - 1$$

- Soup: Min 2, Max 5, Baseline 4.
- 2/4 = 0.5, 5/4 = 1.25
- →Soup ranges from 50% of baseline to 125% of baseline.
- i.e. -50% to + 25% deviation from baseline. (How far we are from baseline.)
- Sandwich: Min 3, Max 9, Baseline 7.
- 3/7 = 0.43, 9/7 = 1.29
- →Sandwich ranges from 43% of baseline to 129% of baseline
- i.e. -57% (.43 – 1) to +29% deviation from baseline.
- Our 'deviation from baseline' axis needs to extend at least from -57% to +29%. (To use round numbers, let's make it go from -60% to +30%).

So, what are we plotting?

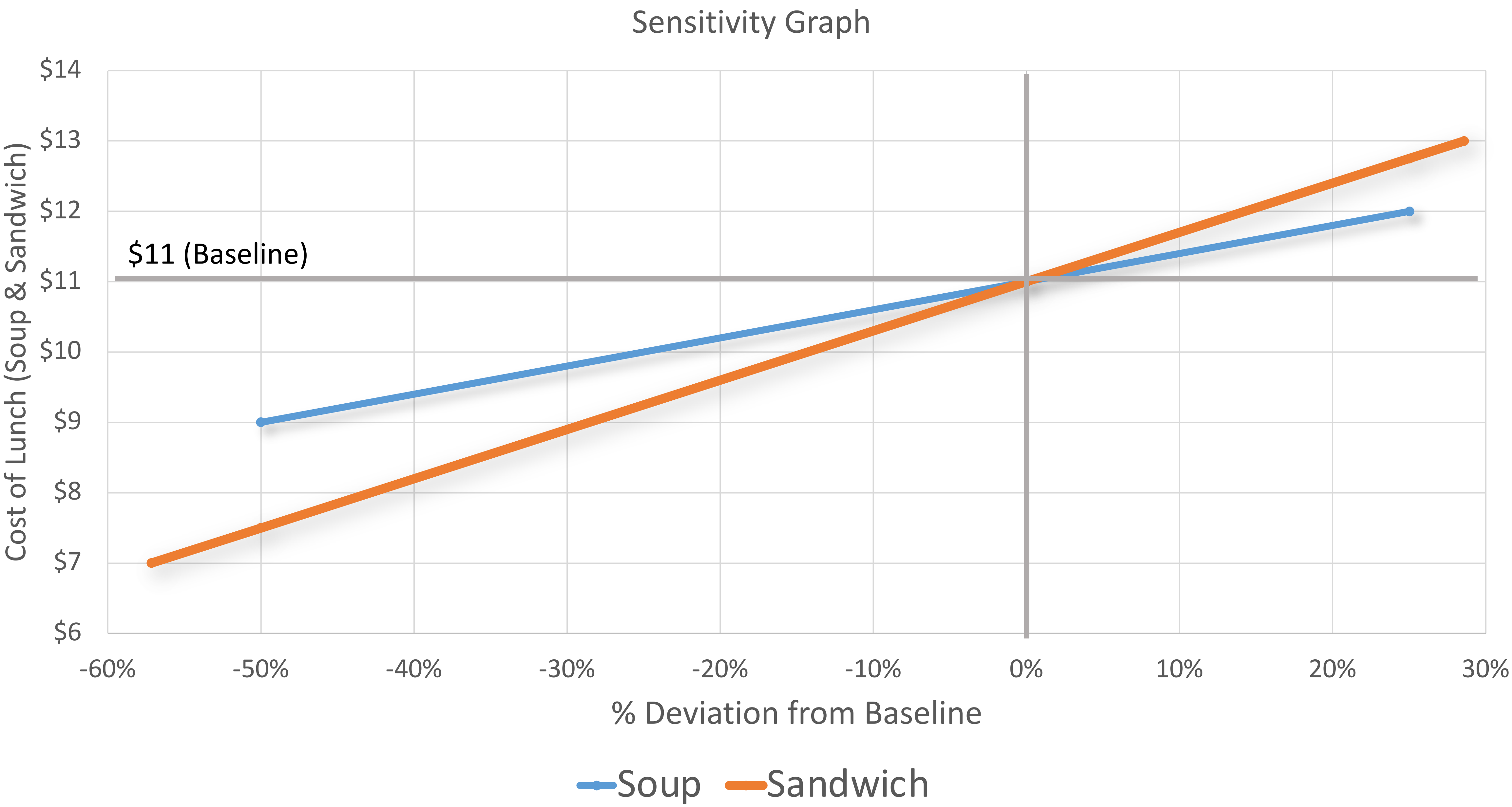
i. Take the equation for your value of interest, and one at a time, replace each of your parameters by (1 + d) x (Parameter Baseline Value)

- For Soup:
- Lunch Cost = \$4 x (1 + d) + \$7, d = -50% to +25%
- (d = deviation from baseline)
- For Sandwich:
- Lunch cost = \$4 + \$7 x (1 + d), d = -57% to +29%

ii. 'd' stands for % deviation from baseline, so plot each of the functions you created in step i. from the minimum 'd' for the parameter being varied, to the maximum 'd' for the parameter being varied. No more, and no less.

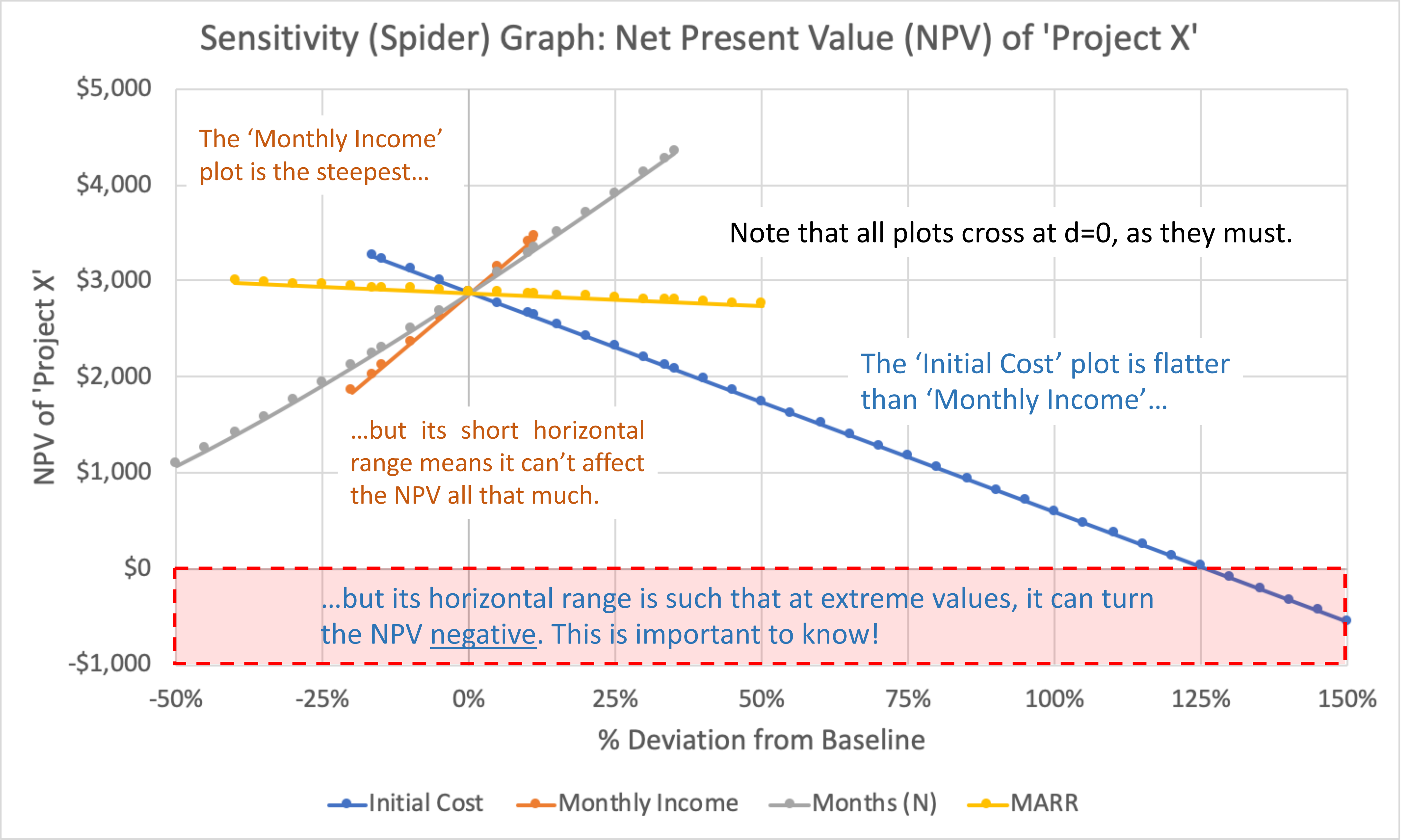
At 0% deviation from baseline, all your functions must cross (have the same value), since at that point, (1 + d) = (1 + 0) = 1. That means all parameters are at baseline levels for all functions plotted. **If they don't cross at d = 0, you did something wrong**.

Sensitivity (Spider) Graph		
d	Soup	Sandwich
-57%		\$7.00
-50%	\$9.00	\$7.50
0%	\$11.00	\$11.00
25%	\$12.00	\$12.75
29%		\$13.00


Sensitivity Graph
$14
$13
$12
$11
$10
$9
$8
$7
$6
Cost of Lunch (Soup & Sandwich)
$11 (Baseline)
-60%
-50%
-40%
-30%
-20%
-10%
0%
10%
20%
30%
% Deviation from Baseline
Soup
Sandwich

A few things to notice

- All the plots cross when deviation = 0%. By definition, at that point all values are their baseline values.
- The steeper a plot is, the more sensitive the project is to that parameter.
- Reasoning: a small deviation from baseline will lead to a larger change in the value of interest.
- In our example, the cost of lunch is more sensitive, for any given % deviation from baseline, to the cost of a sandwich.
- In General: Steep = more sensitive, Flat = less sensitive
- BUT remember that there are *limits* to how much a parameter can influence a value of interest: our plots only extend from the minimum deviation from baseline to the maximum deviation from baseline, and we could have a very steep plot that does not extend very far because the maximum and minimum are very close together.
- We need also keep track of which plots/parameters actually have the capacity to change our recommendation regarding the project.
- Consider a project we'll call 'Project X'. (The details don't matter; we just want to look at the spider plot.)



Sensitivity (Spider) Graph: Net Present Value (NPV) of 'Project X'
$5,000
$4,000
$3,000
$2,000
$1,000
$0
-$1,000
NPV of 'Project X'
-50%
-25%
0%
25%
50%
75%
100%
125%
150%
% Deviation from Baseline
Initial Cost
Monthly Income
Months (N)
MARR
The ‘Monthly Income’ plot is the steepest…
Note that all plots cross at d=0, as they must.
The ‘Initial Cost’ plot is flatter than ‘Monthly Income’…
…but its short horizontal range means it can’t affect the NPV all that much.
…but its horizontal range is such that at extreme values, it can turn the NPV negative. This is important to know!

The general case

- For V(a,b,c)
- ‘a’ line: $V((1+d)a_{base},b_{base},c_{base})$ for $\left(\frac{a_{min}}{a_{base}} - 1\right) \leq d \leq \left(\frac{a_{max}}{a_{base}} - 1\right)$
- ‘b’ line: $V(a_{base},(1+d)b_{base},c_{base})$ for $\left(\frac{b_{min}}{b_{base}} - 1\right) \leq d \leq \left(\frac{b_{max}}{b_{base}} - 1\right)$
- ‘c’ line: $V(a_{base}, b_{base},(1+d)c_{base})$ for $\left(\frac{c_{min}}{c_{base}} - 1\right) \leq d \leq \left(\frac{c_{max}}{c_{base}} - 1\right)$

AFTER HOURS

- Spider plot cheat sheet (1 slide)
- Published examples (6 slides)

The short version: Sensitivity (Spider) Plots

- Plot how the value of interest changes if a parameter changes from its baseline value.
- Horizontal axis: % variation from baseline. Usually goes from a -% to a +%, with the baseline of 0% in the middle of the horizontal axis.
- **Do not extend plots beyond their allowed ranges! (VERY common mistake.)**
- Vertical axis: Value of interest. (BCR, NPV, IRR, etc.)
- Plots: constants are changed only one at a time. There's a separate plot to represent changes brought about by variations in each constant.
- **Plots should only extend from the minimum to the maximum values of their parameters (after putting them in terms of deviation from baseline).**
- The **steeper** a plot, the more sensitive the value of interest is to variations in that constant.
- Things to watch out for: check to see if any plot dips into the '**danger zone**' (NPV > 0, etc.). These plots represent constants that are *very* important to get right, since getting them wrong means the valuation of the project (worthwhile or not worthwhile?) might change.
- **Do not extend plots beyond their allowed ranges! (Common mistake in published papers.)**
- **Fun activity: run a Google Image Search for 'sensitivity analysis spider plot' and see how many of the spider plots make this mistake.**

A scenario table (van Genugten *et al*. 2003)

Table 3. Hospitalizations in the scenario analysis of influenza pandemic[a]

Scenario	No. of hospitalizations		
	Base case	Gross attack rate 10%	Gross attack rate 50%
Nonintervention	10,186	3,395	16,977
Influenza vaccination			
Total population	3,847	1,282	6,412
Risk groups	3,968	1,223	6,614
Pneumococcal vaccination	7,008	2,326	11,679
Neuraminidase inhibitors	5,093	1,698	8,489

[a]Assuming gross attack rates of 10% and 50%.

Tornado chart information (Wang et al. 2003)

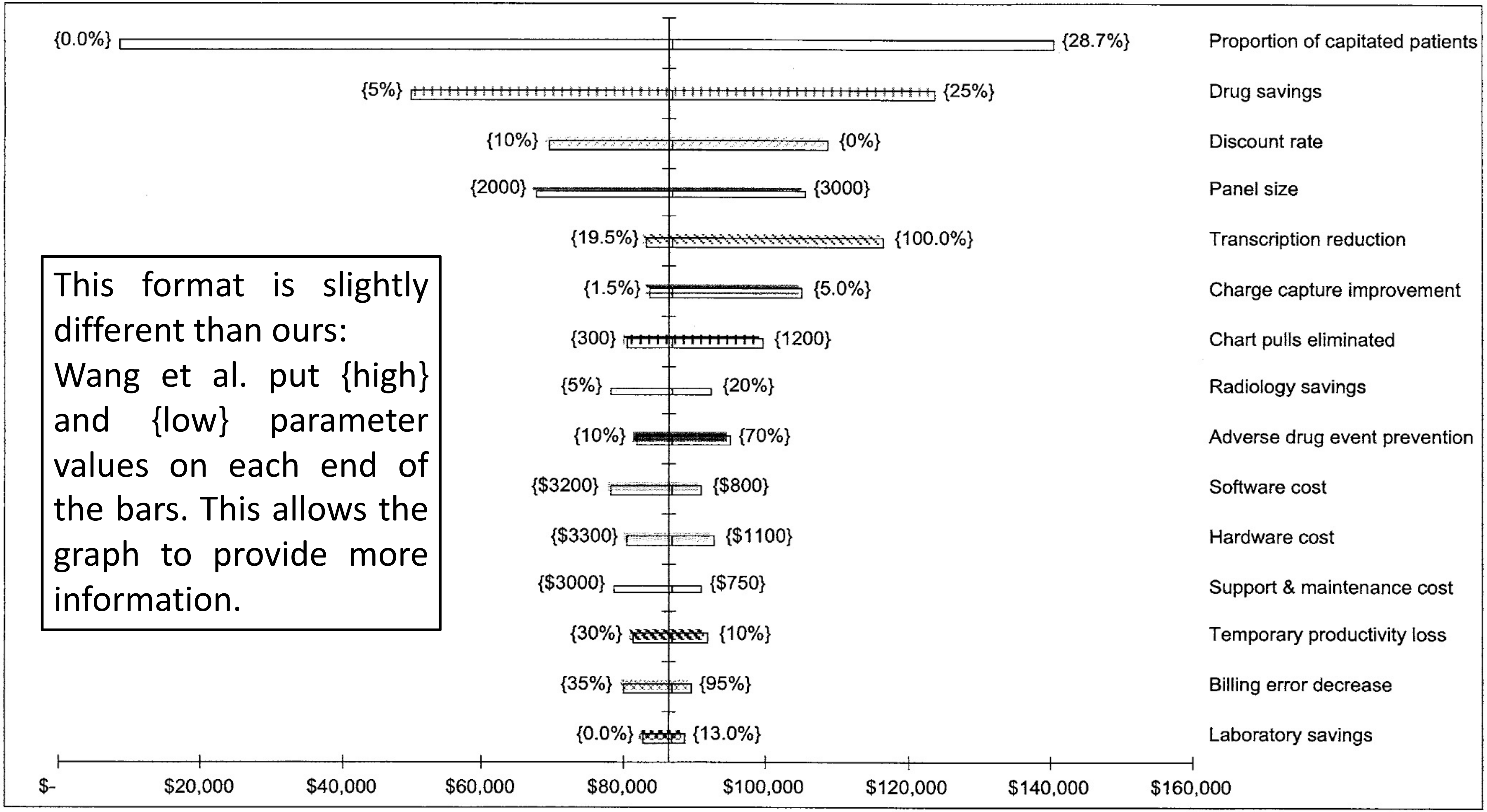
Table 2. Annual Expenditures Per Provider (in 2002 U.S. Dollars) before Electronic Medical Record System Implementation and Expected Savings after Implementation

	Annual Expenditures before Implementation		Expected Savings after Implementation		
	Amount	Reference	Base Case Estimated Savings	Sensitivity Analysis (Range)	Reference
Payer independent					
Chart pulls	$5 (per chart)	*	600 charts	300–1200	*
Transcription	$9600	*	28%	20%–100%	*,32
Capitated patients					
Adverse drug events	$6500	33–36	34%	10%–70%	‡
Drug utilization	$109,000	†	15%	5%–25%	‡
Laboratory utilization	$27,600	†	8.8%	0–13%	37–39
Radiology utilization	$59,100	†	14%	5%–20%	‡
Fee-for-service patients					
Charge capture	$383,100	†	2% (increase)	1.5%–5%	25,40
Billing errors	$9700	†	78%	35%–95%	‡

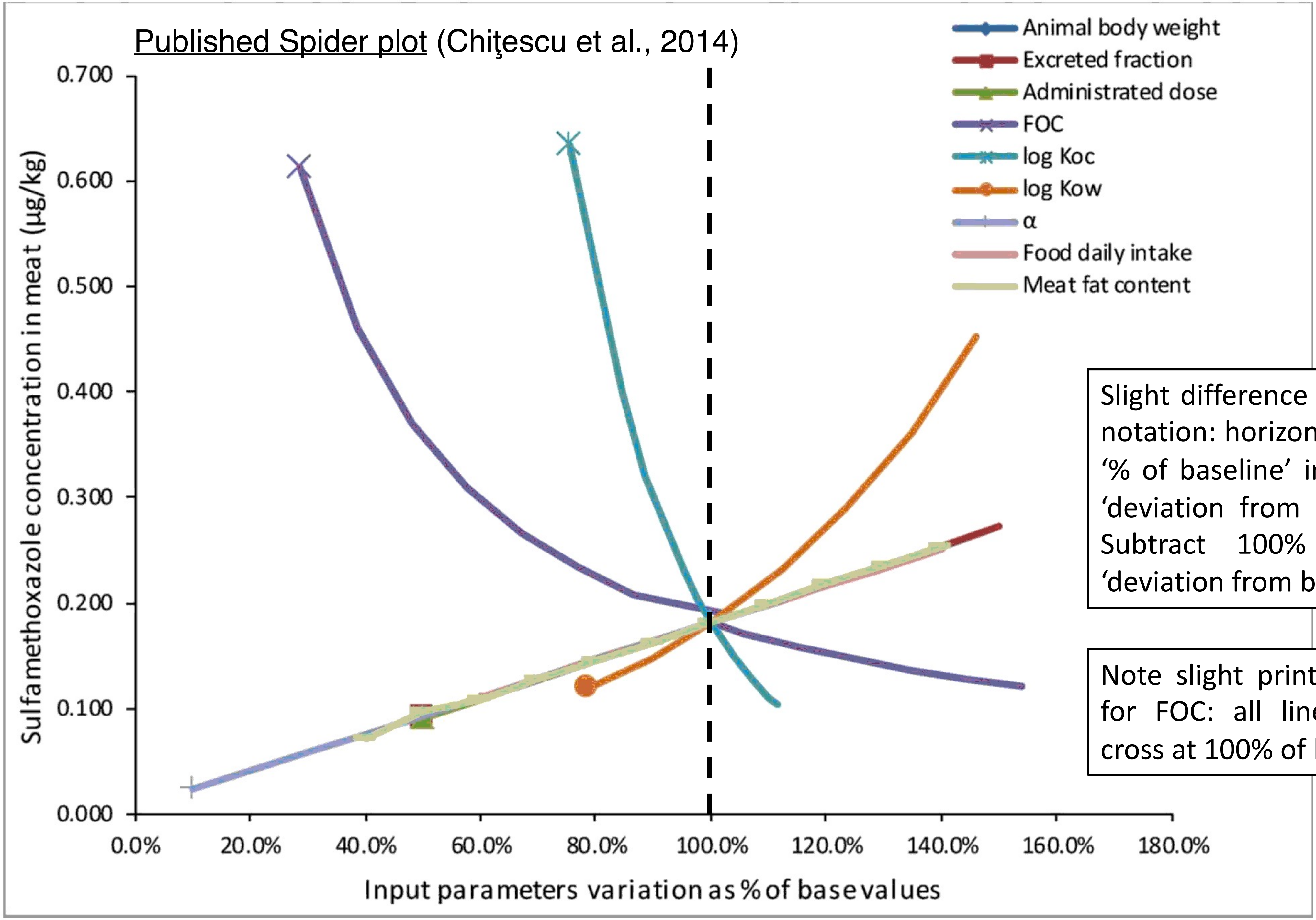
* Primary data from the Partners HealthCare Electronic Medical Record System, Boston, Massachusetts.

† From the Department of Finance, Brigham and Women's Hospital, Partners HealthCare System.

‡ Expert panel consensus.



This format is slightly different than ours: Wang et al. put {high} and {low} parameter values on each end of the bars. This allows the graph to provide more information.

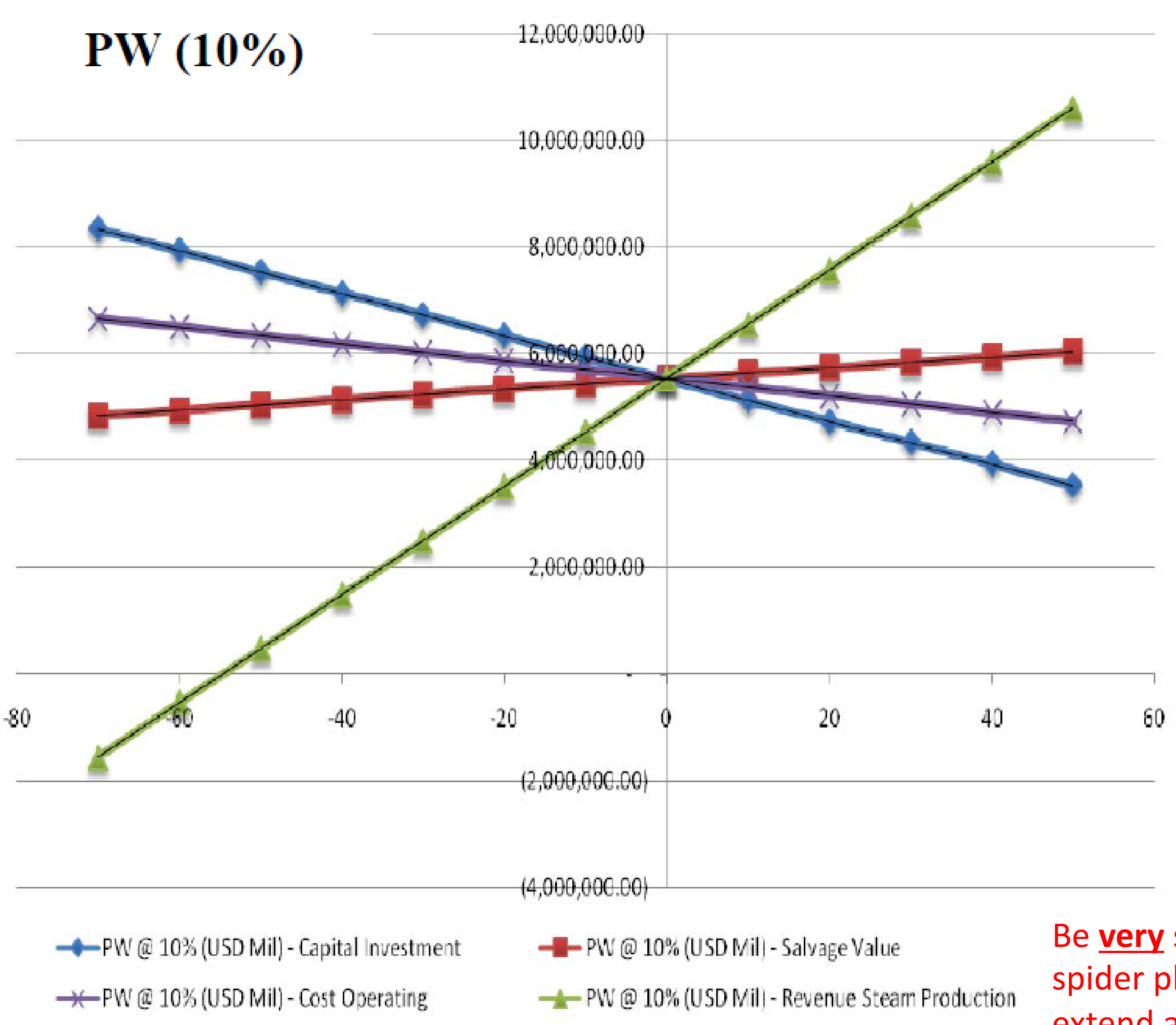


Slight difference from our notation: horizontal axis is '% of baseline' instead of 'deviation from baseline'. Subtract 100% to get 'deviation from baseline'.

Note slight printing error for FOC: all lines MUST cross at 100% of baseline.

Waste Heat Boilers in Malaysia

- For a second real-world spider plot example, we'll look at a sensitivity graph from an investment proposal for an integrated petrochemical complex in Malaysia (published in 2011 by Ghazali et al.).
- The proposal was to replace two waste heat boilers with new models that have great technical advantages, but also huge capital costs.
- Because of the cost, it was imperative that the evaluation be correct. In addition to all the usual evaluations a sensitivity analysis was run to find which factors the project was most sensitive to mis-estimation of.



Capital Investment
Operating Costs
Salvage Value
Revenue
Vertical axis: PW at 10% MARR

Note the (likely) error: All plots have exactly the same horizontal range!

This implies one of three things…

Either this plot is so ‘**zoomed in**’ that we can’t see any of the limits being reached…

…or they made exactly the same **assumption**, in terms of d, about the min/max parameter values (e.g. ‘Assume the minimum value for all parameters is -70% deviation from baseline)…

…or (most likely) they **ignored** the maxima and minima of each parameter, and just plotted every function between the two endpoints they chose.

None of these options is ‘good’!

Be **very** suspicious when you see spider plots where all the lines extend across the whole x-range!