MATHEMATICS 101 (all sections), Spring 2012,
Midterm # 1.
January 20th, 2012 — Happy birthday, Federico Fellini!
Time: 2 hours

- **As stated in the course outline, the only calculator we allow on any examination is the Sharp EL-510R.**
- This test consists of 8 questions (numbered 2 through 9) and has 7 pages (including this cover). All questions are worth the same and all are multiple choice. Enter all your answers on the bubble sheet.
- For questions with numerical answers, the exact answer may not be among the options. In that case, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices.
- Before starting your test enter your name, student number, and section number on the bubble sheet, as follows:
  - Under "Identification", write the last 6 digits of your UVic student number. For example, if your student number is "V00123456", then write "123456". Then fill in the bubbles below.
  - Under "Course & Section" write only the two digits of your section. For example, if your section is "A09", then write "09". Then fill in the bubbles below.
  - Under "Name" write your last and first name. **Write your last (family) name first, followed by your first (given) name.** Then fill in the bubbles below.
- At the end of your test, turn in both this paper and the green bubble sheet.
- Enter "J" as your answer to Question 1 now.

1. Enter "J" as your answer to Question 1 now.

2. Evaluate $\int_0^3 \pi(2-(y+1)^{1/2})dy$.

A) 20.0 B) 10.0 C) 8.0 D) 4.0 E) 2.0
F) 1.0 G) 0.5 H) 0.2 I) 0.1 J) 0.0

$\pi \int_0^3 [2-(y+1)^{1/2}]dy$

$= \pi \left(2y - \frac{2}{3}(y+1)^{3/2}\right)\Big|_0^3$

$= \pi \left(6 - \frac{2}{3}(4)^{3/2}\right)$

$= \pi \left(6 - \frac{2}{3}(8)\right) = \pi\left(6 - \frac{16}{3}\right)$

$\pi\left(\frac{18}{3} - \frac{16}{3}\right) = \frac{2\pi}{3}$

3. Evaluate $\int_0^{0.5} \frac{x}{\sqrt{1-x^2}}dx$.

A) 0.5 B) 0.2 C) 0.1 D) 0.05 E) 0
F) -0.05 G) -0.1 H) -0.2 I) -0.5 J) -1

let $1-x^2 = u$

$\frac{du}{dx} = -2x$

$\frac{1}{2}\int_1^{0.75} \frac{1}{\sqrt{u}}$

$-\frac{1}{2}\int_1^{0.75} u^{-1/2}$

$= -\frac{1}{2}\left(2u^{1/2}\right)\Big|_1^{0.75}$

$= -\frac{1}{2}\left(2(0.75)^{1/2} - 2(1)\right)$

$= 0.133,$

4. Evaluate $\int_1^3 \frac{[\ln(z)]^2}{z}\,dz$.

$A)$ 3.0	$B)$ 1.0	$C)$ 0.7	$D)$ 0.4	$E)$ 0.2
$F)$ 0.1	$G)$ 0.04	$H)$ 0.02	$I)$ 0.01	$J)$ 0

let $\ln(z) = u$ $\quad \frac{du}{dz} = \frac{1}{z}$

$$= \int_{x}^{xx} u^2\,du$$

$$= \left(\frac{u^3}{3}\right)\Big|_{x}^{xx} = \left(\frac{(\ln(z))^3}{3}\right)\Big|_1^3 = \frac{(\ln 3)^3}{3} - \frac{(\ln(1))^3}{3}$$

$$= 0.441489653 - 0$$

$$= 0.44$$

$\frac{(\ln z)^3}{3}$

5. Set up, but do not evaluate, the integral that gives the arc length of the curve $y = e^{3x}$ from $(0,1)$ to $(2, e^6)$.

$A)$ $\int_1^{e^6} \sqrt{1+9e^{6x}}\,dx$	$B)$ $\int_0^{2} \sqrt{1+9e^{6x}}\,dx$	$C)$ $\int_1^{e^6} \sqrt{1+e^{6x}}\,dx$
$D)$ $\int_0^{2} \sqrt{1+3e^{3x}}\,dx$	$E)$ $\int_1^{e^6} \sqrt{1+3e^{3x}}\,dx$	$F)$ $\int_0^{2} \sqrt{1+e^{3x}}\,dx$
$G)$ $\int_0^{2} \sqrt{1+e^{6x}/9}\,dx$	$H)$ $\int_1^{e^6} \sqrt{1+e^{3x}/3}\,dx$	$I)$ $\int_0^{2} \sqrt{1+9e^{9x}}\,dx$

$$\int_0^2 \sqrt{1+9e^{6x}}\,dx$$

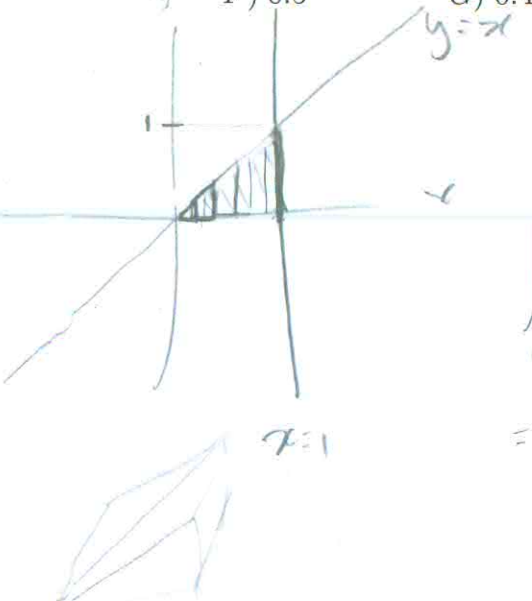
$f(x) = e^{3x}$

$f'(x) = 3e^{3x}$

$[f'(x)]^2 = (3e^{3x})^2 = 3e^{6x}$

6. The base of a certain solid is the triangular region in the xy-plane bounded by the line $y = x$, the x-axis and the line $x = 1$. Find the volume of the solid if each cross-section perpendicular to the x-axis is a square.

$A)$ 1.0	$B)$ 0.9	$C)$ 0.8	$D)$ 0.7	$E)$ 0.6
$F)$ 0.5	$G)$ 0.4	$H)$ 0.3	$I)$ 0.2	$J)$ 0.1



$V = bhl$

$b = h$
l changes

$V = \int [f(x)]^2$

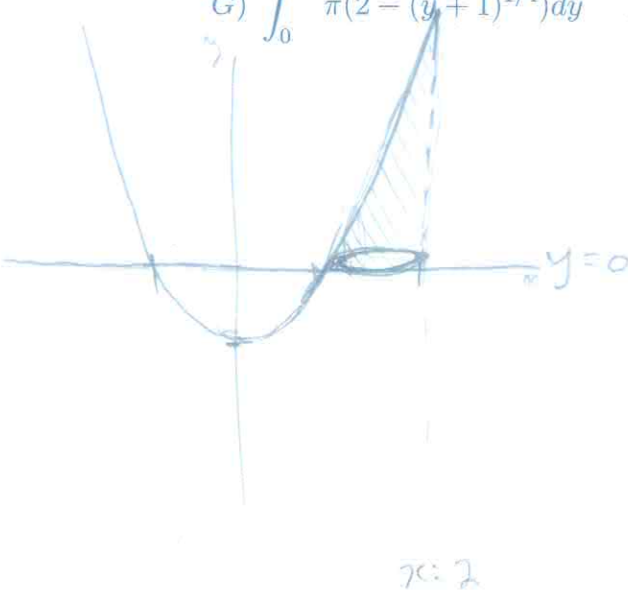
$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

7. The region bounded by the curves $y = x^4 - 1$, $y = 0$ and $x = 2$ is revolved around the y-axis. Using the method of **discs and washers** set up but do not evaluate the integral to find the volume of the resulting solid. $(x^2-1)(x^2+1)$ $(x-1)(x$

$A)\ \int_0^{15} \pi((x^4-1)^2)dx$ $\quad B)\ \int_1^{2} \pi((x^4-1)^2)dx$ $\quad C)\ \int_0^{15} 2\pi y(y+1)^{1/4}dy$

$D)\ \int_1^{2} \pi(4-(y+1)^{1/2})dy$ $\quad E)\ \int_0^{15} \pi(4-(x^4-1)^2)dx$ $\quad F)\ \int_1^{2} \pi(4-(x^4-1)^2)dx$

$G)\ \int_0^{15} \pi(2-(y+1)^{1/4})dy$ $\quad H)\ \int_1^{2} \pi(2-(y+1)^{1/4})dy$ $\quad I)\ \int_0^{15} \pi(4-(y+1)^{1/2})dy$



$\int \pi r^2$ $\qquad r = 2 - f(y)$

$r =$

$\int \pi\left((2)^2 - \left((y+1)^{1/4}\right)^2\right)$

$= \int_0^{15} \pi\left(4 - (y+1)^{1/2}\right)$

$\sqrt[4]{y+1} = x$

The region bounded by the curves $y = 4(x - x^2)$ and $y = 1 - x$ is revolved around the line $x = 2$. Using the method of **cylindrical shells** set up but do not evaluate the integral to find the volume of the resulting solid.

A) $\int_0^1 2\pi(x-2)[4(x-x^2)-(1-x)]dx$

B) $\int_0^1 2\pi x[4(x-x^2)-(1-x)]dx$

C) $\int_0^1 2\pi(2-x)[4(x-x^2)-(1-x)]dx$

D) $\int_0^1 2\pi[4(x-x^2)-(1-x)]dx$

E) $\int_{1/4}^1 2\pi(x-2)[4(x-x^2)-(1-x)]dx$

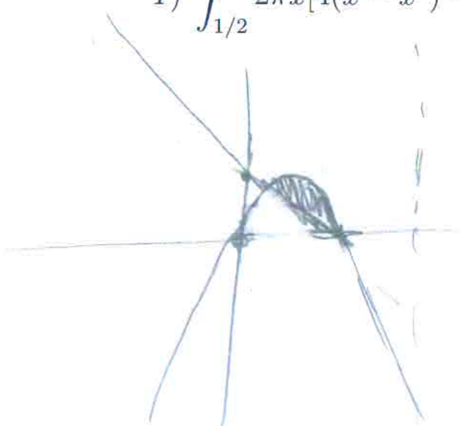
F) $\int_{1/4}^1 2\pi x[4(x-x^2)-(1-x)]dx$

G) $\int_{1/4}^1 2\pi(2-x)[4(x-x^2)-(1-x)]dx$

H) $\int_{1/2}^1 2\pi(x-2)[4(x-x^2)-(1-x)]dx$

I) $\int_{1/2}^1 2\pi x[4(x-x^2)-(1-x)]dx$

J) $\int_{1/2}^1 2\pi(2-x)[4(x-x^2)-(1-x)]dx$



$4(x-x^2$

$4x(1-x)=0$

$x=0,1$

$-4x^2+4x=x+1$

$4x^2-5x+1=0$

$4x^2-4x-x+1=0$

$4x(x-1)-1(x-1)=0$

$(4x-1)(x-1)=0$

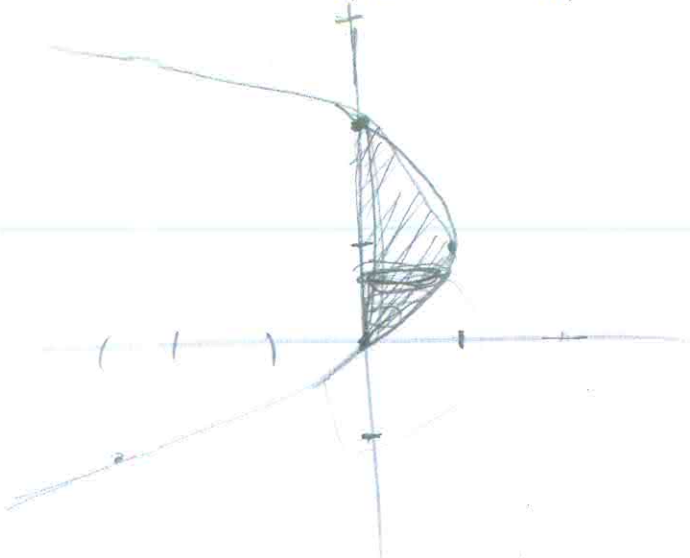
$x=1, 1/4$

$r=(2-x)$

$\int_0^1 2\pi(2-x)(4(x-x^2)-(1-x))$

9. Find the volume of the solid generated when the region enclosed by $x = 2y - y^2$ and the y-axis is revolved around the x-axis.

$A)$ 9	$B)$ 8	$C)$ 7	$D)$ 6	$E)$ 5
$F)$ 4	$G)$ 3	$H)$ 2	$I)$ 1	$J)$ 0.5



$y(2-y) = x$

$= \int 2\pi r h$

$= \int 2\pi$

$h = (2y - y^2)$

$r = y$

$\int_0^2 2\pi (2y - y^2) y$

$= 2\pi \int_0^2 2y^2 - y^3$

$= 2\pi \left(\frac{2y^3}{3} - \frac{y^4}{4} \right) \Big|_0^2$

$= 2\pi \left(\frac{2(2)^3}{3} - \frac{(2)^4}{4} \right)$

$= 2\pi \left(\frac{2(8)}{3} - \frac{16}{4} \right) = 2\pi \left(\frac{16}{3} - 4 \right)$

$= 2\pi \left(\frac{16}{3} - \frac{12}{3} \right) = 2\pi \left(\frac{4}{3} \right) = \frac{8\pi}{3}$

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$2\pi r h$

$r = 1+x$

$h = f(x)$

$\pi \int_0^{15} \left(4 - (y+1)^{1/2}\right)$

$\pi \left(4y - \frac{2}{3}(y+1)^{3/2}\right)\Big|_0^{15}$

$\pi \left(4(15) - \frac{2}{3}(16)^{3/2}\right) - \left(-\frac{2}{3}\right)$

$= \pi \left(60 - \frac{128}{3} + \frac{2}{3}\right.$

$\frac{52\pi}{3} =$

$2\pi \int_1^2 (1+x)(x^4-1)$

$2\pi \int_1^2 \left(x^4 - 1 + x^5 - x\right)$

$2\pi \left(\frac{x^5}{5} - x + \frac{x^6}{6} - \frac{x^2}{2}\right)\Big|_1^2$

$2\pi \left(\frac{(2)^5}{5} - 2 + \frac{(2)^6}{6} - \frac{(2)^2}{2}\right) - \left(\frac{1}{5} - 1 + \frac{1}{6} - \frac{1}{2}\right)$

$2\pi \left(\quad 38.\overline{666}\right.$

250