ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 34: From Gantt Charts to the Critical Path

November 26, 2021

Version 1

Learning Objectives

- Be able to create and interpret a Gantt chart from a WBS and duration/predecessor information.
- Be able to create and interpret an AoN diagram from predecessor information.
- To be able to find the critical path of a project.

Recommended Reading & Viewing

- ProjectManager. (2016, March 11). Gantt Charts, Simplified – Project Management Training. https://youtu.be/cGkHjby1xKM
- Emmanuel, J. (2017, July 17). Project Scheduling – PERT/CPM | Finding the Critical Path [Video File]. https://youtu.be/-TDh-5n90vk
- Levy, F.K., Thompson, G.L. & Wiest, J.D. (1961). The ABCs of the Critical Path Method, *Harvard Business Review*, September 1963. http://ezproxy.library.uvic.ca/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=bth&AN=6770388&site=ehost-live&scope=site

Optional Reading: Gantt Charts & AoN

- Kelley, J. E., Jr. (1961). Critical-Path Planning and Scheduling: Mathematical Basis. *Operations Research*, 9(3), 296-320. Retrieved from https://www-jstor-org.ezproxy.library.uvic.ca/stable/167563
  - **AoA Example, Source for Lecture AoN example.**
- Lee, S. & Shetsova, O. A. (2019). Optimization of the Technology Transfer Process Using Gantt Charts and Critical Path Analysis Flow Diagrams: Case Study of the Korean Automobile Industry. *Processes, 7*(12), 917. https://doi-org.ezproxy.library.uvic.ca/10.3390/pr7120917
- Potoradi, J. et al. (2002). Using Simulation-Based Scheduling to Maximize Demand Fulfillment in a Semiconductor Assembly Facility. *Proceedings of the 2002 Winter Simulation Conference*, 1857-1861. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/1166479
  - **Using Gantt Diagrams for Semiconductor Production Planning.**

History of the Gantt Chart

- Robles, V. D. (2018). Visualizing Certainty: What the Cultural History of the Gantt Chart Teaches Technical and Professional Communicators about Management. *Technical Communication Quarterly*, *27*(4), 300-321. https://doi-org.ezproxy.library.uvic.ca/10.1080/10572252.2018.1520025
- Wilson, J. M. (2003). Gantt charts: A centenary appreciation. *European Journal of Operational Research*, 149(2), 430-437. Retrieved from https://doi-org.ezproxy.library.uvic.ca/10.1016/S0377-2217(02)00769-5
  - **History of Gantt diagrams.**
- Wren, D. A. (2015). Implementing the Gantt chart in Europe and Britain: the contributions of Wallace Clark. *Journal of Management History*, *21*(3), 309-327. https://www-emerald-com.ezproxy.library.uvic.ca/insight/content/doi/10.1108/JMH-09-2014-0163/full/html

Case Studies: Critical Path I

- Batson, R. G. (1987). Critical path acceleration and simulation in aircraft technology planning. *IEEE Transactions on Engineering Management*, EM-34(4), 244 – 251. Retrievedn from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6499014
- **Non-technical report of an early application of CPM by Lockheed Martin.**
- Tang, P. (2011). Using Schedule Simulation Approaches to Reduce Greenhouse Gas Emissions in Highway Construction Project. *Proceedings of the 2011 Winter Simulation Conference (WSC)*, 805-815. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6147807
- **Highway Construction Example**
- Kelley, J. E., Jr. (1961). Critical-Path Planning and Scheduling: Mathematical Basis. *Operations Research*, 9(3), 296-320. Retrieved from https://www-jstor-org.ezproxy.library.uvic.ca/stable/167563
- **Activity on Arc Crashing Example. Since we use Activity on Node notation, this article may be challenging to parse.**

Case studies: Critical Path II

- Cho, S. M. et al. (2021). Design of Very High-Speed Pipeline FIR Filter Through Precise Critical Path Analysis. *IEEE Access, 9.* https://ieeexplore.ieee.org/abstract/document/9361650
- Doostali, S., Babamir, S. M. & Eini, Maryam. (2021). CP-PGWO: multi-objective workflow scheduling for cloud computing using critical path. *Cluster Computing, 24,* 3607-3627. https://doi.org/10.1007/s10586-021-03351-y
- Kenney, C. (2021). Agile Improvements to Critical Path Method. *Acquisition Management,* SYM-AM-21-081. https://dair.nps.edu/handle/123456789/4388

Relevant solved problems

- From *Engineering Economics,* 6th ed.
- Gantt Charts: Example 11.2, Review Problem 11.3, 11.9, 11.10, 11.11, 11.12, 11.30, 11.35, 11.37, 11.40.a., 11.41 (first part)
- AoN Diagrams: Example 11.3, Review Problem 11.2, 11.13, 11.14, 11.15(b), 11.16(b), 11.24(a), 11.31(a), 11.36(a), 11.38(a), 11.39 (first part), 11.40(b)
- Critical Path Calculation: Example 11.4, Example 11.5 (first part), Review Problem 11.4, 11.17, 11.18, 11.19, 11.20,11.24 (b) and (c), 11.31, 11.36 (c), 11.38 (b), 11.39, 11.40 (c)

ESSENTIALS (23 slides)

(*Lee et al.*)

Name	Duration
⊟REQMS	9.75 days
⊟Use Case 1: Edit Requirements	9.75 days
⊟Implementation	7 days
Service Module	3 days
GUI	4 days
Web Interface	4 days
⊟Test	4.75 days
Integration Testing	2 days
System Testing	1.75 days
⊟Use Case 2: Edit Use Cases	10 days
⊟Implementation	8 days
Service Module	5 days
GUI	3 days
Web Interface	3 days
⊟Test	5 days
Integration Testing	1 day
System Testing	4 days

Something doesn't add up…

$\mathbf{7 = 3 + 4 + 4}$?

Work in parallel?

$\mathbf{4.75 = 2 + 1.75}$?

Lead time?

$\mathbf{8 = 5 + 3 + 3}$?

Work in parallel?

We need a better tool for scheduling…

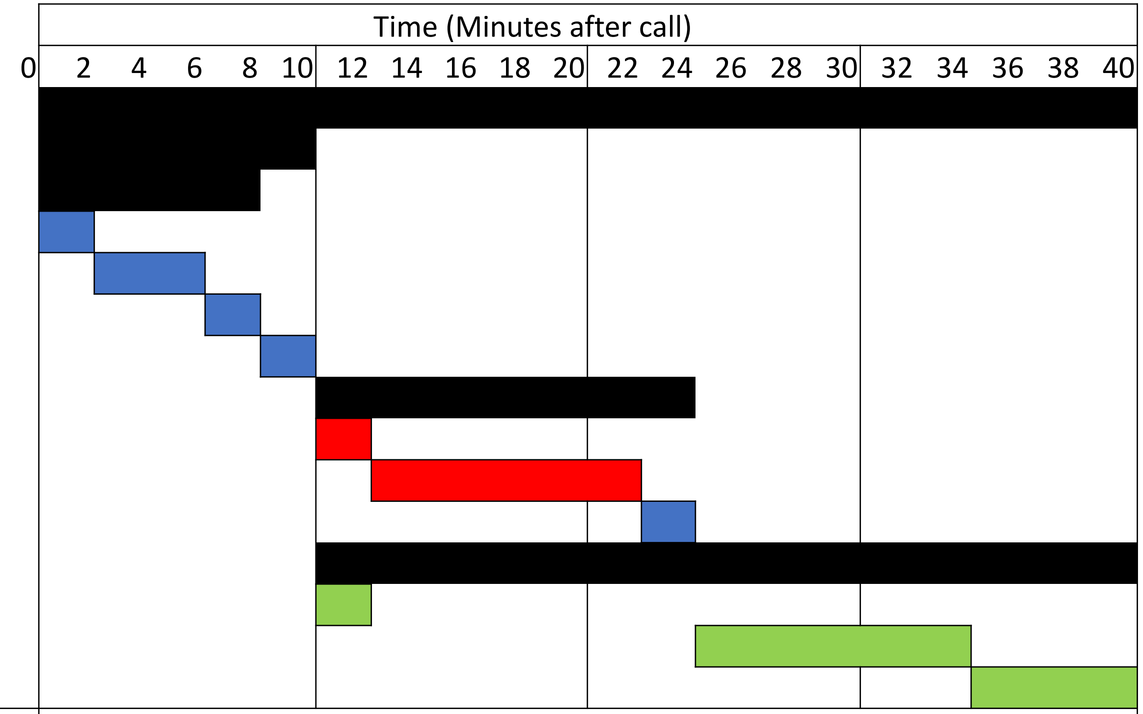
From work to work schedule: Gantt Charts

- A WBS is excellent for clearing up responsibilities, establishing hierarchies and making sure there are no gaps or wasteful overlap in the work plan.
- It’s not that great at illustrating the order in which activities take place, how they’re scheduled or when ‘crunch times’ are.
- It’s also difficult to calculate project (or summary task) duration from a WBS when tasks can take place in parallel.
- For this, we use a **Gantt Chart**: a bar chart where each bar illustrates when a work package or work on a deliverable starts and ends.
- The chart is named after Henry Gantt, a WWI mechanical engineer who famously used it to schedule production and track progress in an ammunition plant.
- Good news: if you’ve already created a WBS and dictionary, the Gantt chart can be tacked on to the right.
- We’ll take a peek at an example, then talk about the details.

First, with all the needed information…

ID	WBS Code	Title	Responsibility	Duration (Minutes)	Predecessors
1	**1**	**Complete Pizza Phone Order**		*40*	
2	**1.1**	**Process Order**		*10*	
3	**1.1.1**	**Confirm Order**		*8*	
4	1.1.1.1	Greet Customer	Front of House	2	-
5	1.1.1.2	Take down order	Front of House	4	4
6	1.1.1.3	Verbally verify order	Front of House	2	5
7	1.1.2	Send order to kitchen	Front of House	2	3
8	**1.2**	**Ready pizza for delivery**		*14*	
9	1.2.1	Verbally confirm order	Kitchen	2	7
10	1.2.2	Bake Pizza	Kitchen	10	9
11	1.2.3	Box Pizza	Front of House	2	10
12	**1.3**	**Complete Delivery**		*30*	
13	1.3.1	Confirm address with POS	Delivery Person	2	7
14	1.3.2	Deliver pizza to address	Delivery Person	10	8
15	1.3.3	Process payment	Delivery Person	6	14

WBS

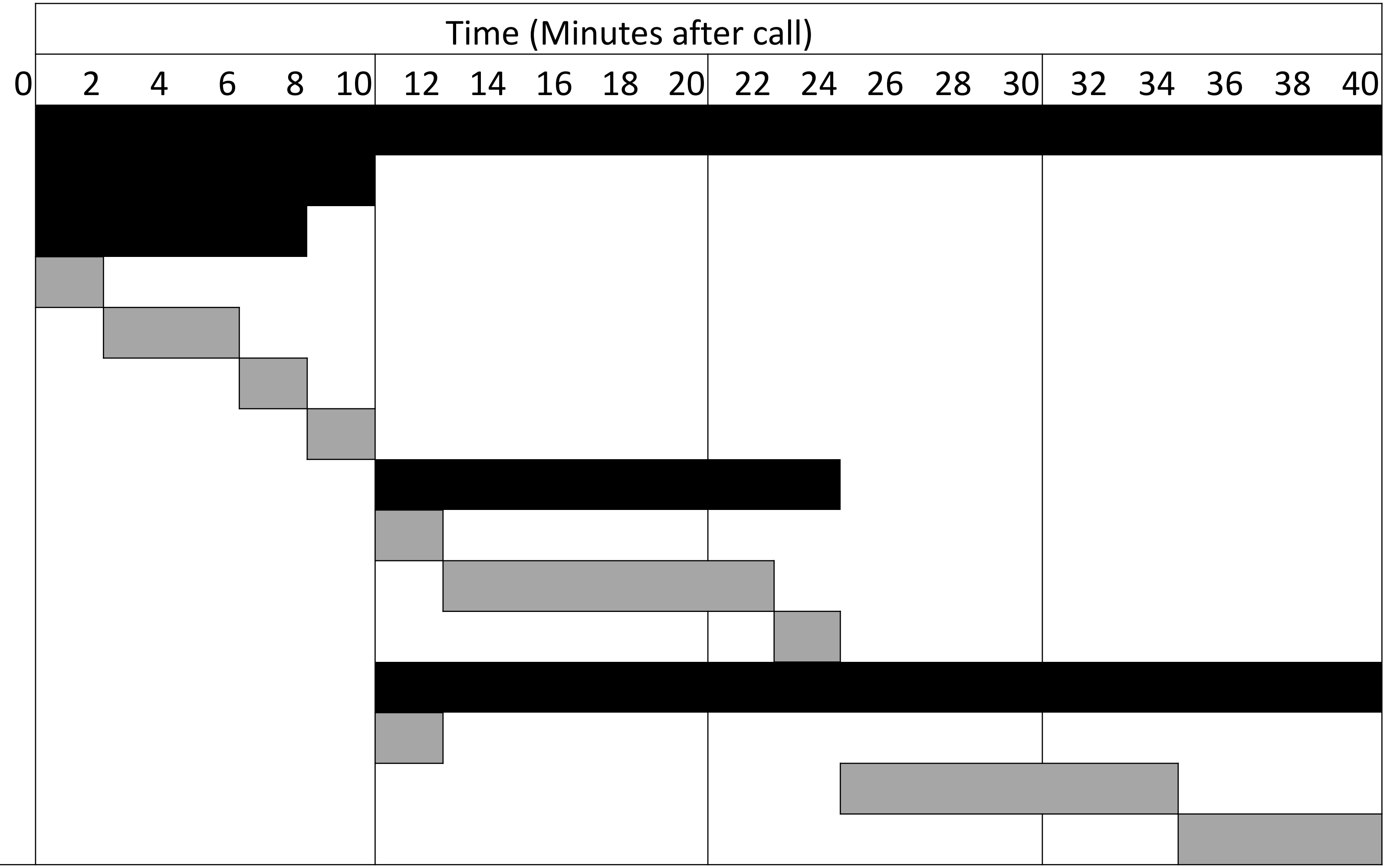


Gantt Chart

- Bar length = task duration, endpoints = start/stop times.
- Bars for work packages are color-coded according to responsibility (optional).
- Work on a task is assumed to start immediately after all predecessors have finished.
- Bars for summary activities are black by convention. Note they extend from the start of the earliest component task, to the end of the latest, and may include periods with no activity.

A more usual format

ID	WBS Code	Title
1	**1**	**Complete Pizza Phone Order**
2	**1.1**	**Process Order**
3	**1.1.1**	**Confirm Order**
4	1.1.1.1	Greet Customer
5	1.1.1.2	Take down order
6	1.1.1.3	Verbally verify order
7	1.1.2	Send order to kitchen
8	**1.2**	**Ready pizza for delivery**
9	1.2.1	Verbally confirm order
10	1.2.2	Bake Pizza
11	1.2.3	Box Pizza
12	**1.3**	**Complete Delivery**
13	1.3.1	Confirm address with POS
14	1.3.2	Deliver pizza to address
15	1.3.3	Process payment



Creating a Gantt Chart, step by step

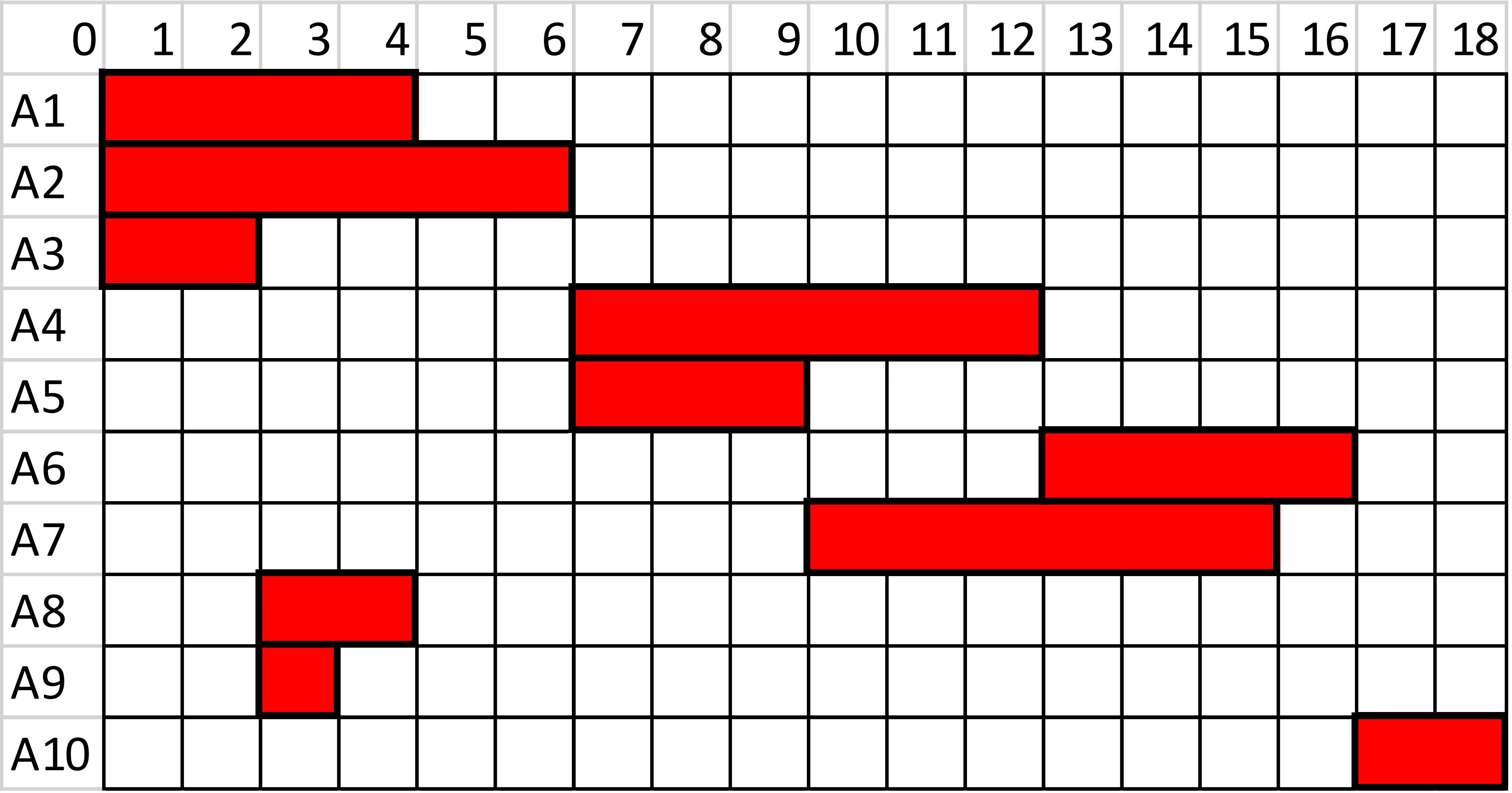
- Start by having the abbreviated dictionary information available.
- For now, ignore summary activities – the ones in **bold text**. *Let N = 2*.
- Step 1: From the remaining tasks, find the ones with no predecessors. On their row of the chart, draw a bar starting at '0' and ending at the time corresponding to their duration.
- Step N: Next, find those tasks that list only Step 1 to Step (N – 1) tasks as predecessors. On their rows, draw a bar with a length equal to the task duration, and with a starting point equal to the latest finish time among their predecessors.
- $N \rightarrow N + 1$. Iterate until no more non-summary tasks are left.
- Finally, draw bars for the summary activities. These bars should start at the earliest component task start time, and end at the latest component task end time. Color these bars black to distinguish them from the rest.

Problem 11.11 from the textbook

Activity	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Predecessor	-	-	-	A1,A2	A1,A2	A4	A5	A3	A3	A6,A7,A8,A9
Time Taken	4	6	2	6	3	4	6	2	1	2
Start	0	0	0	6	6	12	9	2	2	16
Stop	4	6	2	12	9	16	15	4	3	18

- A1,A2 and A3 have no predecessors, so their start time is T=0, and their stop time is 0 + Time Taken.
- A4 and A5 have A1 and A2 as predecessors, so their start time is the higher of the stop times for A1 and A2 – both A1 and A2 need to finish before A4 and A5 can start.
- Similarly, the start time for A10 is the maximum of the stop times for (A6,A7,A8,A9).
- The lowest possible completion time for this project is 18 time periods, since that’s the highest stop time.

Gantt chart for Problem 11.11



What does this buy us?

- Gantt Charts are THE essential tool for scheduling work.
- They are very good at clearing up what tasks can take place simultaneously.
- →They provide a good first estimate of the duration of a project.
- Without the chart, our pizza delivery may have been thought to take 42 minutes instead of 40, but in creating the chart we see that the 2-minute ‘checking the address’ task can take place at the same time as another task, and therefore does not extend the project duration.
- One of the Gantt Chart’s greatest strengths is showing at a glance when a lot of work takes place at once: a column whose rows are mostly filled represents crunch time.
- In particular, if 20 (non-summary) rows are filled in a column, and you only have 15 staff members, this could be a problem.

How do we deal with crunch or overloading?

- If staff are overloaded, we'd like to reschedule some of the work…
- …ideally, without changing the duration of the entire project.
- How can we tell which (if any) activities can be safely rescheduled?
- On the flip side, suppose we find ourselves behind schedule. How can we get the project back on track?
- A common approach in the software industry is to make *everyone* work overtime, all the time, but this is wasteful:
- If some activities can be extended without affecting project duration, that means crunching those same activities won't reduce the project completion time.
- A WBS and Gantt chart aren't enough to answer these questions, but next lecture we'll use them to develop a technique that *is*: **Critical Path Management** (CPM).

Critical Path Method (CPM)

- Uses directed node diagrams to illustrate the interdependence between activities.
- Two styles of node diagrams:
  - Activity on arc: nodes mark start and end time of activities, activities are arrows between nodes. Less common (but be aware it exists).
  - **Activity on node**: nodes represent activities, arrows between nodes indicate dependencies. More common.
- The <u>critical path</u> is the set of activities that must be completed exactly as planned to keep the project on schedule.
- There is <u>slack</u> if and only if an activity is NOT on the critical path.

The two representations (from the text)

Figure 11.3 AOA Network Representation

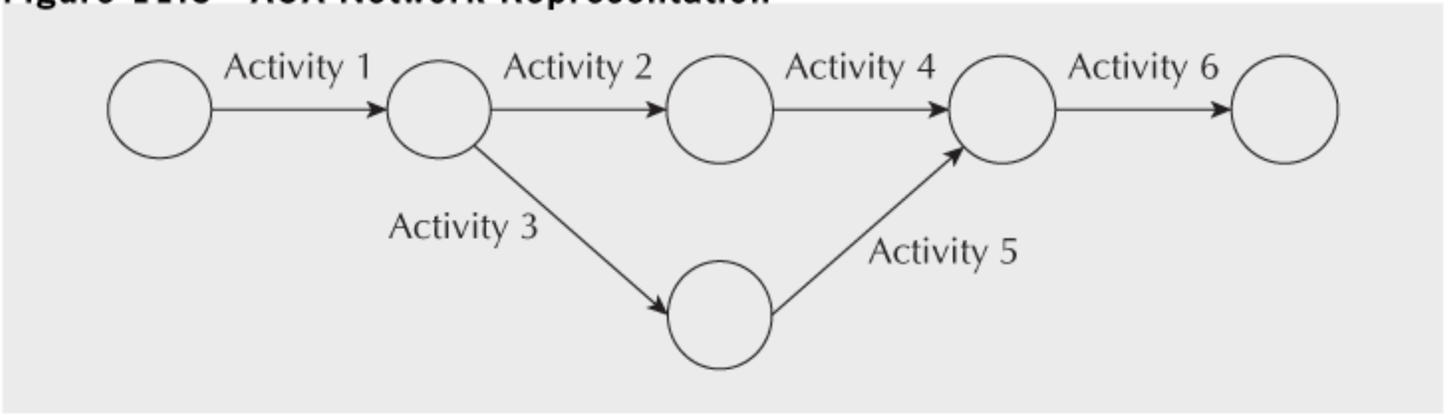
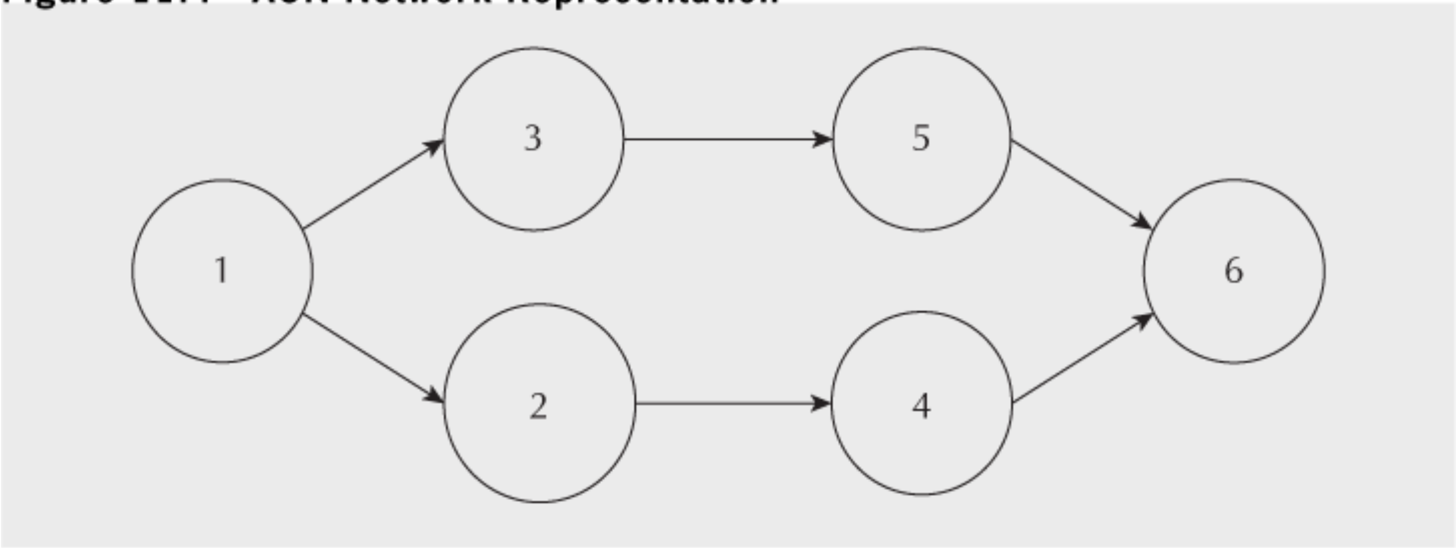


Figure 11.4 AON Network Representation



How to create an AoN diagram from WBS and predecessor/duration information

- 1. Go through your WBS and check if any tasks list a summary activity as a predecessor. If so, convert that predecessor into the equivalent non-summary task (the latest-finishing work package in the summary task.)
- 2. Delete all summary tasks.
- 3. Create a ‘Start’ milestone task. Place it at the left of your diagram.
- 4. Connect any tasks with no predecessors to the Start milestone.
- 5. Iteratively add tasks to the right of the network as you introduce their predecessors. Connect predecessors and successors with arrows.
- 6. Once no tasks are left, Create a ‘Finish’ milestone task. Place it to the right of your diagram.
- 7. Connect any tasks with no successors to the End milestone.

Example

WBS Code	Task ID	Predecessors	Duration	Start	Stop
1	**A**		**31**	**0**	**31**
1.1	**B**		**22**	**0**	**22**
1.1.1	**D**		**10**	**0**	**10**
1.1.1.1	I	-	1	0	1
1.1.1.2	J	K	3	7	10
1.1.2	E	D	5	10	15
1.1.3	F	E	7	15	22
1.2	**C**		**31**	**0**	**31**
1.2.1	G	F	9	22	31
1.2.2	**H**		**26**	**0**	**26**
1.2.2.1	K	-	7	0	7
1.2.2.2	L	-	5	0	5
1.2.2.3	**M**		**4**	**22**	**26**
1.2.2.3.1	N	F,K	3	22	25
1.2.2.3.2	P	N	1	25	26

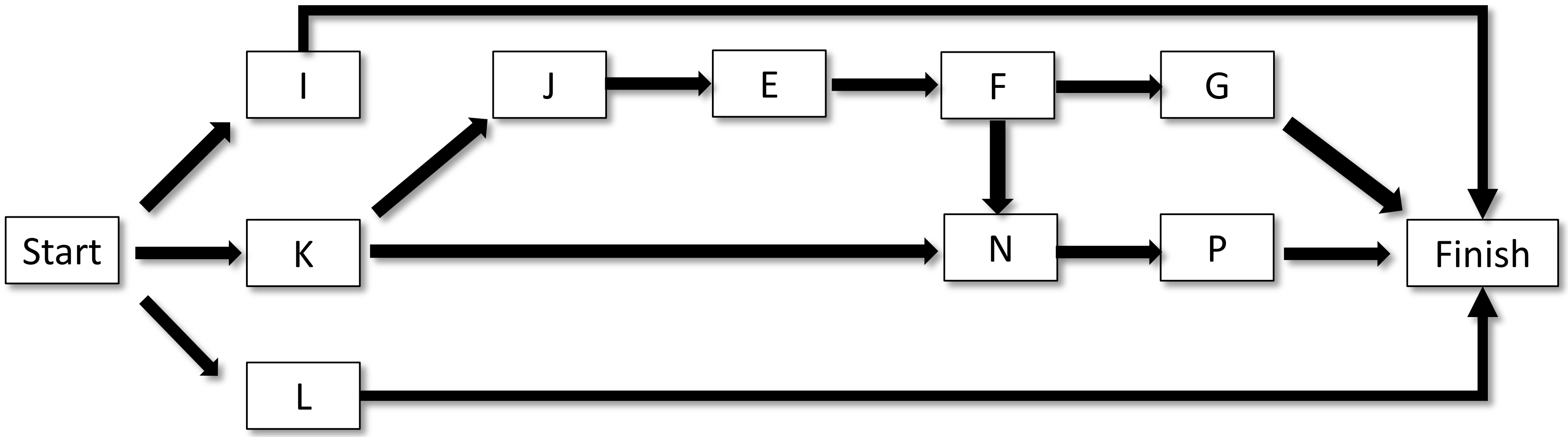
This is the same information we use to create a Gantt chart. Cells in green were calculated, not given.

The only summary task predecessor is D, which is a predecessor of Task E.

The latest-finishing component of summary task D is J, so a predecessor task D translates into a predecessor task J.

Building the Network

Task ID	Pred.
I	-
J	K
E	J
F	E
G	F
K	-
L	-
N	F,K
P	N



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- There is slack if and only if an activity is NOT on the critical path.

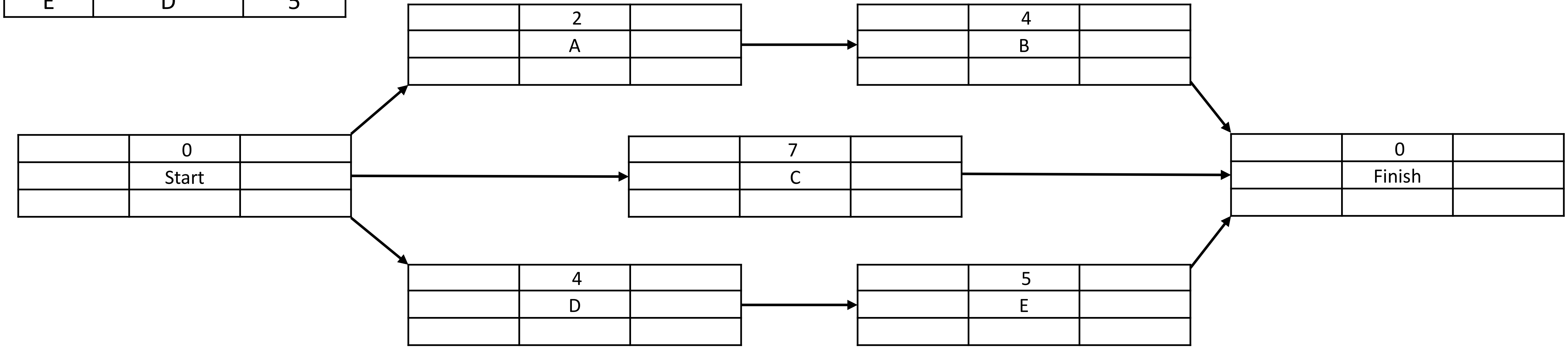
How to find the critical path

- Step 1: Find the earliest possible completion time for the project.
  - Determine the earliest possible start (ES) and finish (EF) times for each activity.
  - EF = ES + T, where T is the shortest length of time the activity can take.
  - Tip: The ES/EF will be the Start/Finish times in your Gantt chart!
- Step 2: Taking the completion time from Step 1 as given…
  - Work backwards and find the latest time each activity can start and finish (LS and LF).
  - LF = MIN(LS of successor activities)
- Step 3: Calculate (LF – EF) for each activity. This is the slack time of that activity. An activity is on the critical path if and only if this value is zero.

Drawing the AoN Diagram

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

ES	T	EF
	ID	
LS	Slack	LF

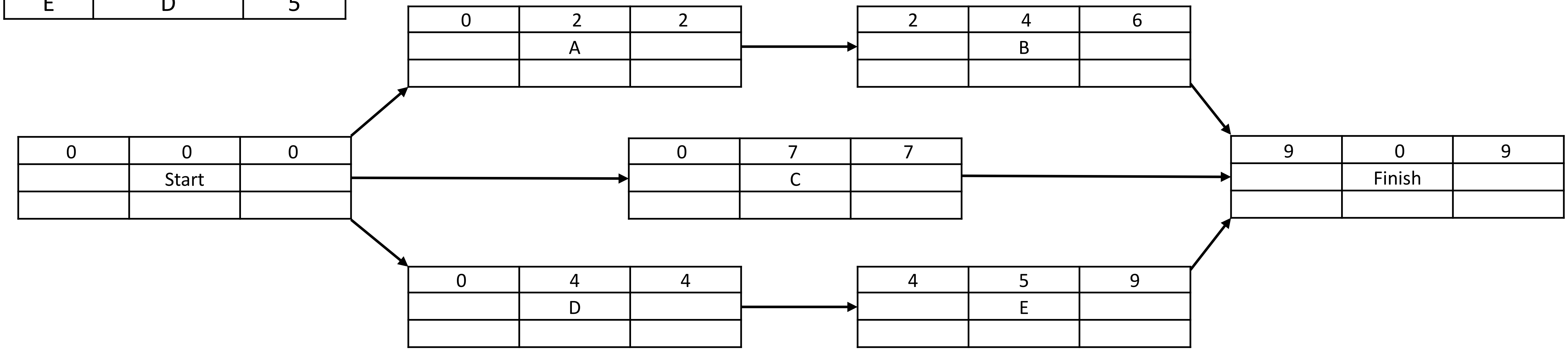


- For the network diagram & subsequent analysis, I'll be using grid notation inspired by practicing engineers.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Forward Pass

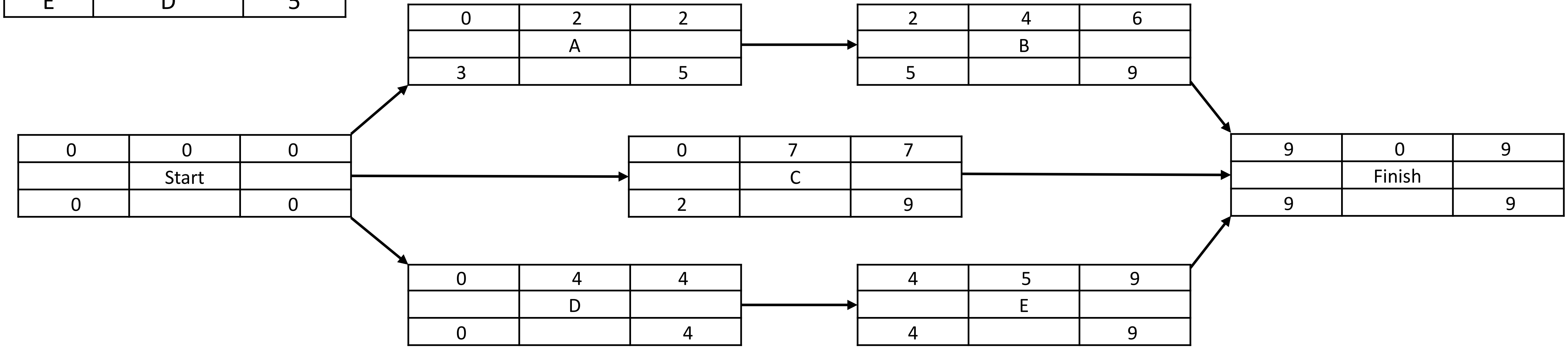
ES	T	EF
	ID	
LS	Slack	LF



Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Backward Pass

ES	T	EF
	ID	
LS	Slack	LF



- For the backward pass, we start by assuming the project finishes at its earliest possible finish time (9 in this case).
- We use that as the Finish milestone's latest possible finish time and work backward, seeing how late each activity can start and still have the project finish on time.

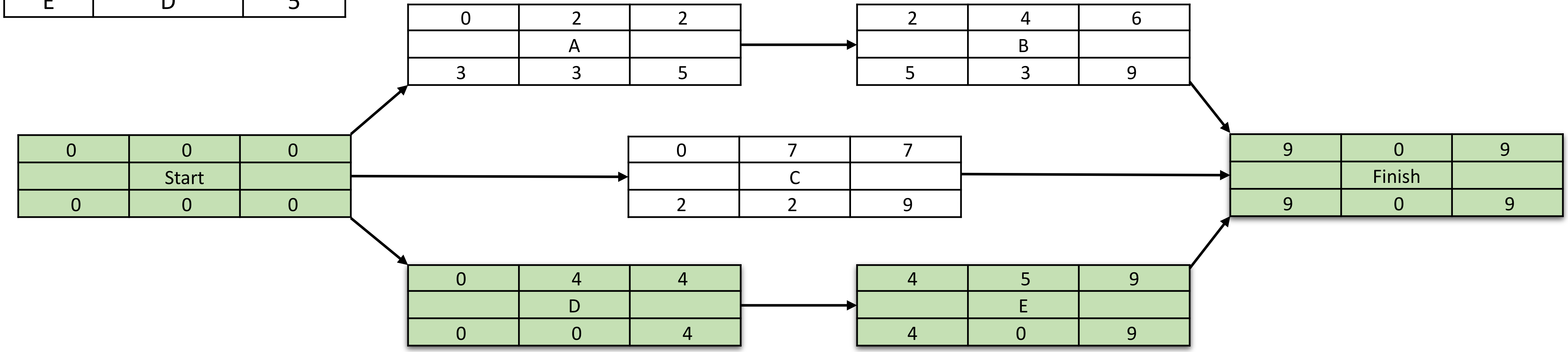
Slack Time & the Critical Path

- The value (LF – EF) calculated in Step 3 is the slack mentioned earlier.
- For project planners, it's nice to know what activities have a built-in buffer…
- Meanwhile, knowing what activities DON'T have a buffer (i.e. are on the critical path) is also very useful.
- These are the activities that drive project completion time. If a project must be completed ahead of schedule (by paying overtime, etc.) these are the activities to focus resources on.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Slack & Critical Path

ES	T	EF
	ID	
LS	Slack	LF



- Slack is the difference between early and late start & finish times
- i.e. Slack = (LS-ES) or (LF-EF) [They're off by T, so both give the same result.]
- Activities are on the critical path if and only if their slack is zero. I've shaded these green for convenience.
- The Start and Finish milestones will always be on the critical path.

Implications for our motivating example

- Last lecture, we saw that in the early 2000s, at least one large software company was responding to scheduling pressures by having *everyone* work overtime for long periods…
- CPM shows this is probably not required, or a good idea.

- Suppose we DO want to reduce project completion time in a rigorous, efficient manner.
- How do we go about it?

Project Crash

- GIVEN a critical path, a planner can make an informed tradeoff between reduced completion time and extra cost.
- Implicit: best-practice WBS and accurate cost/time estimates.
- Need Crashed and Uncrashed duration and cost data.
- To crash:
  - Identify which critical path activities are the least expensive to hasten.
  - Shorten those activities by the max amount or **until another critical path is created**.
  - Repeat until done or unable to continue.
- We'll be looking at that in detail next lecture.

AFTER HOURS

- Published examples (6 slides)

Recent Real-world use in a Semiconductor Plant (Potodari et al.)

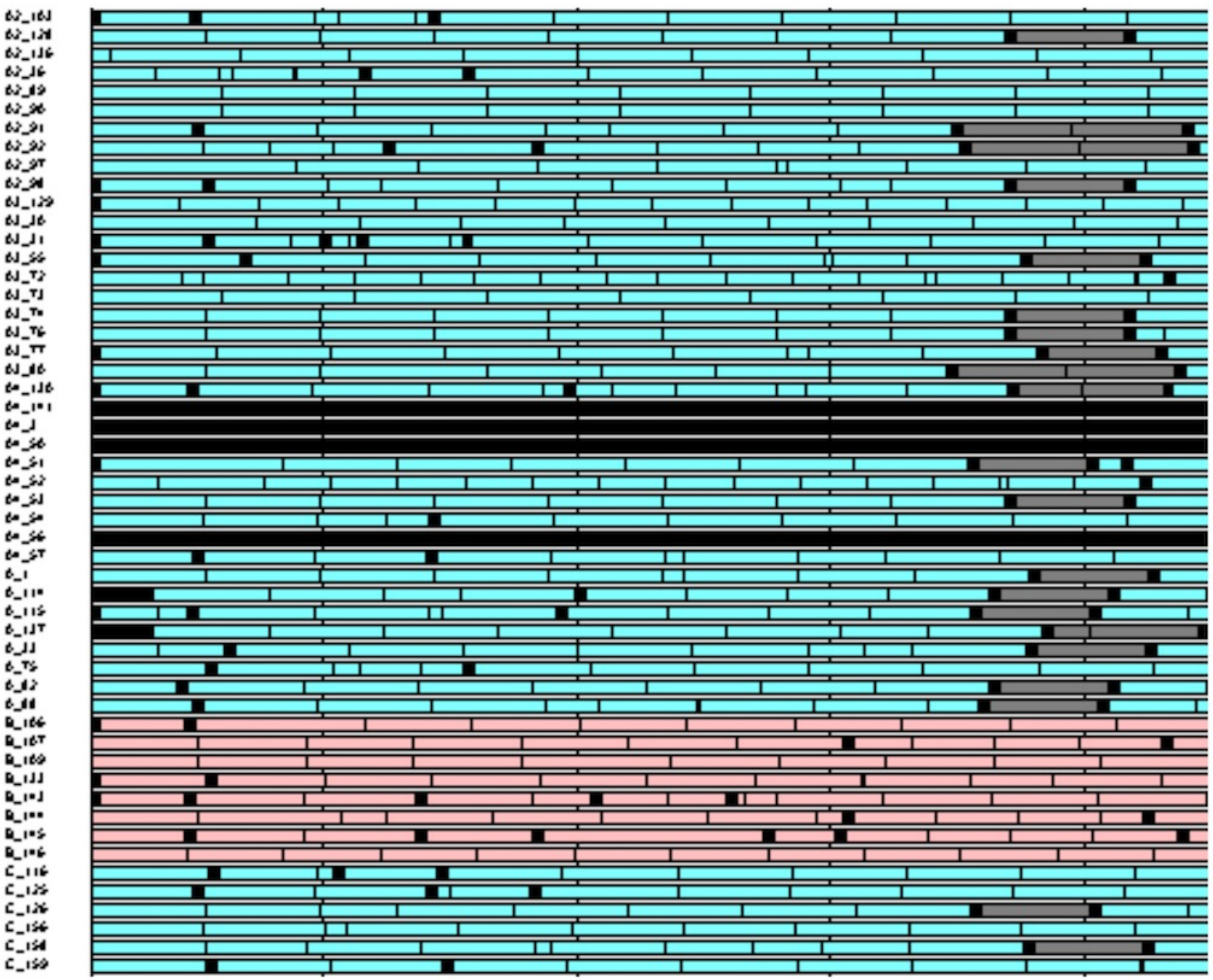


Figure 4: Gantt Chart for Base Schedule

- "[A] large number of products are scheduled to run in parallel"
- "The schedule is generated by a simulation engine and used to control the machines at execution time"
- "This methodology is providing significant improvements in the factory's tardiness performance."

Maintenance for a typical 1950s Chemical Processing Plant (Kelley, 1961)

Activity	Description	Predecessors
Del	Delivery	-
Lead	Lead Time	-
A	Develop Material List	Lead
B	Deactivate Old Line	Del,A
C	Erect Scaffold	A
D	Remove Scaffold	I,M
E	Procure Pipe	A
F	Prefab Pipe Sections	E
G	Place New Pipe	L,J,F
H	Weld Pipe	K,G
I	Fit-up Pipe and Valves	H
J	Procure Valves	A
K	Place Valves	L,J
L	Remove Old Pipe and Valves	B,C
M	Insulate Pipe	H
N	Pressure Test	I
P	Clean-Up and Start-Up	D,N

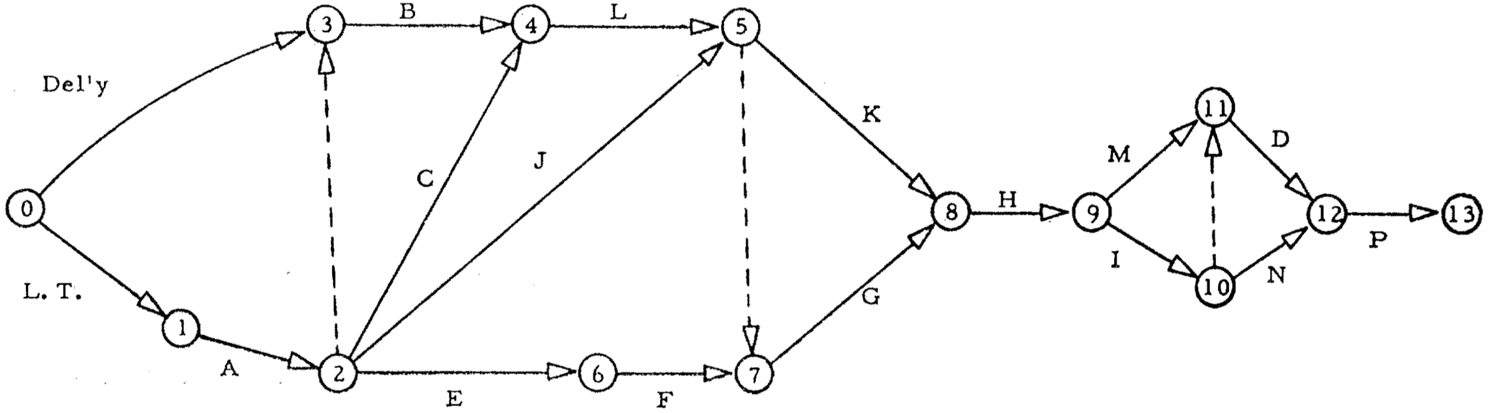


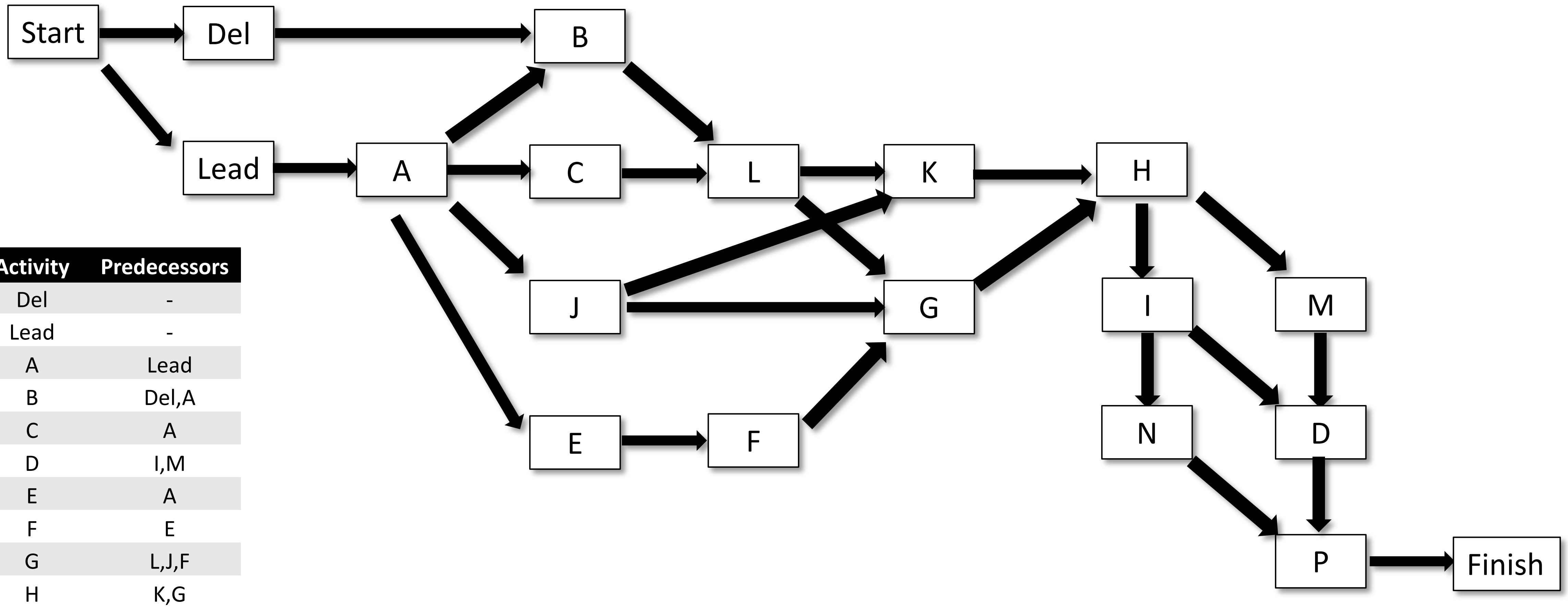
Figure 5

An Activity-on-Arc Diagram. AoN version (by instructor) found on a later slide.

Activity	Predecessors
Del	-
Lead	-
A	Lead
B	Del,A
C	A
D	I,M
E	A
F	E
G	L,J,F
H	K,G
I	H
J	A
K	L,J
L	B,C
M	H
N	I
P	D,N

Try this at home!
(*Solution on the next slide*)

Image: DuPont Rocky Flats Chemical Processing Plant, 1954 (Public Domain)

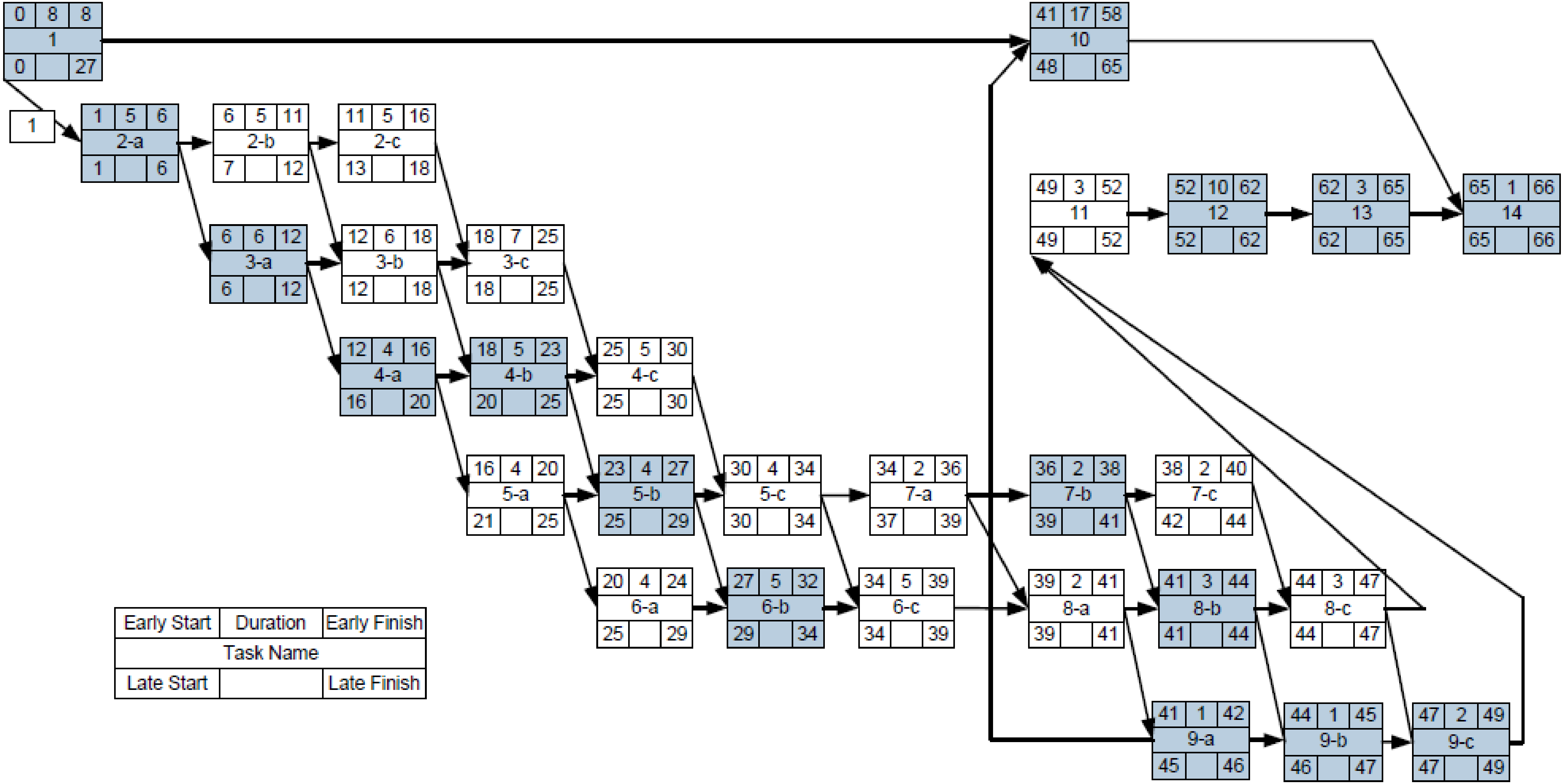


Activity	Predecessors
Del	-
Lead	-
A	Lead
B	Del,A
C	A
D	I,M
E	A
F	E
G	L,J,F
H	K,G
I	H
J	A
K	L,J
L	B,C
M	H
N	I
P	D,N

AoN Diagram for the Chemical Plant Maintenance Project in Kelley (1961)

Just for fun… What does this sort of thing look like in real life?

Activity ID	Activity Description	Duration	Precedence	Constraints
1	Strip Topsoil	8 days	Begin	
2	Remove Concrete Pavement	15 days	1	1 day after the beginning of Activity 1 (soft)
3	Grade Subbase	19 days	2	2 days after the beginning of Activity 2
4	Install Drainage	14 days	2,3	6 days after the beginning of Activity 3
5	Place OGDC Mainline	12 days	2,3,4	2 days after the beginning of Activity 4
6	Pave E.B. Mainline	14 days	5	1 day after the beginning of Activity 5
7	Place OGDC Ramps and Gaps	6 days	4,5,6	7 days after the beginning of Activity 6(soft); after the completion of Activity 5
8	Pave E.B. Gaps and Ramps	8 days	7	0 days after the beginning of Activity 7
9	Place Gravel Shoulder	4 days	8	3 days after the beginning of Activity 8
10	Slope Grading and Restoration E.B.	17 days	9	1 day after the beginning of Activity 9(soft)
11	Stripe to Open Pavement E.B.	3 days	9	0 days after the completion of Activity 9
12	Relocate Barrier Wall	10 days	11	0 days after the completion of Activity 11
13	Re-stripe W.B.	3 days	12	0 days after the completion of Activity 12
14	All Lanes Open	1 day	12,13	0 days after the completion of Activity 1, Activity 13 and Activity 10



0 8 8 1 0 27
1
1 5 6 2-a 1 6
6 5 11 2-b 7 12
11 5 16 2-c 13 18
6 6 12 3-a 6 12
12 6 18 3-b 12 18
18 7 25 3-c 18 25
12 4 16 4-a 16 20
18 5 23 4-b 20 25
25 5 30 4-c 25 30
16 4 20 5-a 21 25
23 4 27 5-b 25 29
30 4 34 5-c 30 34
20 4 24 6-a 25 29
27 5 32 6-b 29 34
34 5 39 6-c 34 39
34 2 36 7-a 37 39
36 2 38 7-b 39 41
38 2 40 7-c 42 44
39 2 41 8-a 39 41
41 3 44 8-b 41 44
44 3 47 8-c 44 47
41 1 42 9-a 45 46
44 1 45 9-b 46 47
47 2 49 9-c 47 49
41 17 58 10 48 65
49 3 52 11 49 52
52 10 62 12 52 62
62 3 65 13 62 65
65 1 66 14 65 66
Early Start
Duration
Early Finish
Task Name
Late Start
Late Finish