Symbolab

Solution

$\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(x-8)^n$: Radius of convergence is 3, Interval of convergence is $5<x<11$

Steps

$\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(x-8)^n$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(x-8)^n$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\left(-\frac{1}{3}\right)^{(n+1)}(x-8)^{(n+1)}}{\left(-\frac{1}{3}\right)^n(x-8)^n}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\left(-\frac{1}{3}\right)^{(n+1)}(x-8)^{(n+1)}}{\left(-\frac{1}{3}\right)^n(x-8)^n}\right|\right)$ *Hide Steps*

$L=\lim_{n\to\infty}\left(\left|\frac{\left(-\frac{1}{3}\right)^{(n+1)}(x-8)^{(n+1)}}{\left(-\frac{1}{3}\right)^n(x-8)^n}\right|\right)$

Simplify $\frac{\left(-\frac{1}{3}\right)^{(n+1)}(x-8)^{(n+1)}}{\left(-\frac{1}{3}\right)^n(x-8)^n}$: $-\frac{1}{3}(x-8)$ *Hide Steps*

$\frac{\left(-\frac{1}{3}\right)^{n+1}(x-8)^{n+1}}{\left(-\frac{1}{3}\right)^n(x-8)^n}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{\left(-\frac{1}{3}\right)^{n+1}}{\left(-\frac{1}{3}\right)^n}=\left(-\frac{1}{3}\right)^{n+1-n}$

$=\frac{\left(-\frac{1}{3}\right)^{n-n+1}(x-8)^{n+1}}{(x-8)^n}$

Add similar elements: $n+1-n=1$

$=\frac{\left(-\frac{1}{3}\right)(x-8)^{n+1}}{(x-8)^n}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{(x-8)^{n+1}}{(x-8)^n}=(x-8)^{n+1-n}$

$=\left(-\frac{1}{3}\right)(x-8)^{n-n+1}$

Add similar elements: $n+1-n=1$

$=\left(-\frac{1}{3}\right)(x-8)$

Remove parentheses: $(-a)=-a$

$=-\frac{1}{3}(x-8)$

$L=\lim_{n\to\infty}\left(\left|-\frac{1}{3}(x-8)\right|\right)$

$L=\left|-\frac{1}{3}(x-8)\right|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$L=\left|-\frac{1}{3}(x-8)\right|\cdot 1$

Simplify

$L=\frac{1}{3}|x-8|$

$L=\frac{1}{3}|x-8|$

The power series converges for $L<1$

$\frac{1}{3}|x-8|<1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$\frac{1}{3}|x-8|<1: \quad |x-8|<3$ Hide Steps

$\frac{1}{3}|x-8|<1$

Multiply both sides by 3

$3\cdot\frac{1}{3}|x-8|<1\cdot 3$

Simplify

$|x-8|<3$

Therefore

Radius of convergence is 3

Radius of convergence is 3

Find the interval of convergence Hide Steps

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for x

$\frac{1}{3}|x-8|<1 \quad : \quad 5<x<11$ Hide Steps

$\frac{1}{3}|x-8|<1$

Multiply both sides by 3

$3\cdot\frac{1}{3}|x-8|<1\cdot 3$

Simplify

$|x-8|<3$

Apply absolute rule: If $|u|<a, a>0$ then $-a<u<a$

$-3<x-8<3$

$x-8>-3 \quad$ and $\quad x-8<3$ Hide Steps

$x-8>-3 \quad$ and $\quad x-8<3$

$x-8>-3 \quad : \quad x>5$ Hide Steps

$x-8>-3$

Add 8 to both sides

$x-8+8>-3+8$

Simplify

$x>5$

$x-8<3 \quad : \quad x<11$ Hide Steps

$x-8<3$

Add 8 to both sides

$x-8+8<3+8$

Simplify

$x<11$

Combine the intervals

$x>5 \quad$ and $\quad x<11$

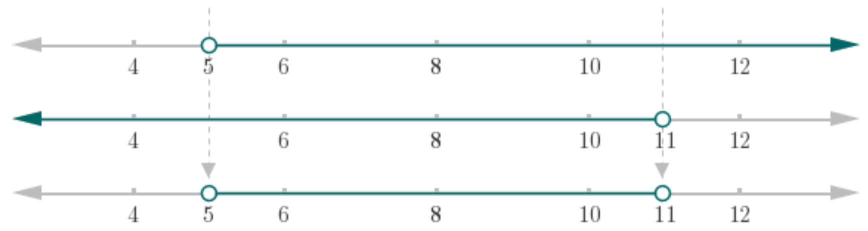
$x>5 \quad$ and $\quad x<11$

Merge Overlapping Intervals Hide Steps

The intersection of two intervals is the set of numbers which are in both intervals

$x>5 \quad$ and $\quad x<11$

$5<x<11$



$5<x<11$

$5<x<11$

Check the interval end points: $\quad x=5$:diverges, $x=11$:diverges Hide Steps

For $x=5, \sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(5-8)^n$: diverges Hide Steps

$\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(5-8)^n$

Refine

$=\sum_{n=0}^{\infty}(-3)^n\left(-\frac{1}{3}\right)^n$

$(-3)^n=3^n(-1)^n$

$=\sum_{n=0}^{\infty}3^n(-1)^n\left(-\frac{1}{3}\right)^n$

Apply Series Divergence Test: diverges Hide Steps

$\sum_{n=0}^{\infty}3^n(-1)^n\left(-\frac{1}{3}\right)^n$

Series Divergence Test:

If $\lim_{n\to\infty}a_n\neq 0$ then $\sum a_n$ diverges

$\lim_{n\to\infty}\left(3^n(-1)^n\left(-\frac{1}{3}\right)^n\right)=1$ Hide Steps

$\lim_{n\to\infty}\left(3^n(-1)^n\left(-\frac{1}{3}\right)^n\right)$

Apply exponent rule: $a^n\cdot b^n=(a\cdot b)^n$

$3^n(-1)^n\left(-\frac{1}{3}\right)^n=\left(3(-1)\left(-\frac{1}{3}\right)\right)^n$

$=\lim_{n\to\infty}\left(\left(3(-1)\left(-\frac{1}{3}\right)\right)^n\right)$

$\left(3(-1)\left(-\frac{1}{3}\right)\right)^n=1$ Hide Steps

$\left(3(-1)\left(-\frac{1}{3}\right)\right)^n$

Remove parentheses: $(-a)=-a,\ -(-a)=a$

$=\left(3\cdot 1\cdot\frac{1}{3}\right)^n$

Multiply $3\cdot 1\cdot\frac{1}{3}$: $1\cdot 1$ Hide Steps

$3\cdot 1\cdot\frac{1}{3}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=1\cdot\frac{1\cdot 3}{3}$

Cancel the common factor: 3

$=1\cdot 1$

$=(1\cdot 1)^n$

Multiply the numbers: $1\cdot 1=1$

$=1^n$

Apply rule $1^a=1$

$=1$

$=\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

By the divergence test criteria

$=$ diverges

$=$ diverges

For $x=11$, $\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(11-8)^n$: diverges Hide Steps

$\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n(11-8)^n$

Refine

$=\sum_{n=0}^{\infty}3^n\left(-\frac{1}{3}\right)^n$

Apply Series Divergence Test: diverges Hide Steps

$\sum_{n=0}^{\infty}3^n\left(-\frac{1}{3}\right)^n$

Series Divergence Test:

If $\lim_{n\to\infty}a_n\neq 0$ then $\sum a_n$ diverges

$\lim_{n\to\infty}\left(3^n\left(-\frac{1}{3}\right)^n\right)=$ diverges Hide Steps

$\lim_{n\to\infty}\left(3^n\left(-\frac{1}{3}\right)^n\right)$

Apply exponent rule: $a^n\cdot b^n=(a\cdot b)^n$

$3^n\left(-\frac{1}{3}\right)^n=\left(3\left(-\frac{1}{3}\right)\right)^n$

$=\lim_{n\to\infty}\left(\left(3\left(-\frac{1}{3}\right)\right)^n\right)$

Simplify $\left(3\left(-\frac{1}{3}\right)\right)^n$: $(-1)^n$ Hide Steps

$\left(3\left(-\frac{1}{3}\right)\right)^n$

Remove parentheses: $(-a)=-a$

$=\left(-3\cdot\frac{1}{3}\right)^n$

Multiply $-3\cdot\frac{1}{3}$: -1 Hide Steps

$-3\cdot\frac{1}{3}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$= -\frac{1\cdot 3}{3}$

Cancel the common factor: 3

$= -1$

$= (-1)^n$

$= \lim_{n\to\infty}\left((-1)^n\right)$

Apply Limit Divergence Criterion: diverges

Hide Steps

$\lim_{n\to\infty}\left((-1)^n\right)$

Limit Divergence Criterion Test:

If two sequences exist, $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ with
$x_n \neq c$ and $y_n \neq c$
$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c$
$\lim_{n\to\infty} f(x_n) \neq \lim_{n\to\infty} f(y_n)$
Then $\lim_{x\to c} f(x)$ does not exist

$c = \infty$, $x_n = 2k$, $y_n = 2k+1$

$\lim_{k\to\infty}(2k) = \infty$

Hide Steps

$\lim_{k\to\infty}(2k)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n + \cdots + bx + c\right) = \infty$, $a>0$, n is odd
$a=2, n=1$

$=\infty$

$\lim_{k\to\infty}(2k+1) = \infty$

Hide Steps

$\lim_{k\to\infty}(2k+1)$

Apply Infinity Property: $\lim_{x\to\infty}\left(ax^n + \cdots + bx + c\right) = \infty$, $a>0$, n is odd
$a=2, n=1$

$=\infty$

$\lim_{n\to\infty} x_n = \lim_{n\to\infty} y_n = c = \infty$

$\lim_{k\to\infty}\left((-1)^{2k}\right) = 1$

Hide Steps

$\lim_{k\to\infty}\left((-1)^{2k}\right)$

$(-1)^{2k} = 1, \forall k\in\mathbb{Z}$

$= \lim_{k\to\infty}(1)$

$\lim_{k\to\infty}(1) = 1$

Hide Steps

$\lim_{k\to\infty}(1)$

$\lim_{x\to a} c = c$

$=1$

$=1$

$\lim_{k\to\infty}\left((-1)^{(2k+1)}\right) = -1$

Hide Steps

$\lim_{k\to\infty}\left((-1)^{(2k+1)}\right)$

$(-1)^{(2k+1)} = (-1), \forall k\in\mathbb{Z}$

$= \lim_{k\to\infty}(-1)$

$\lim_{k\to\infty}(-1) = -1$

Hide Steps

$\lim_{k\to\infty}(-1)$

$\lim_{x\to a} c = c$

$= -1$

$= -1$

$\lim_{n\to\infty} f(x_n) \neq \lim_{n\to\infty} f(y_n)$

Therefore $\lim_{n\to\infty}\left((-1)^n\right)$ is divergent at $n\to\infty$

= diverges

= diverges

By the divergence test criteria

= diverges

= diverges

$x=5$:diverges, $x=11$:diverges

Therefore

Interval of convergence is $5<x<11$

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Radius of convergence is 3, Interval of convergence is $5<x<11$