4-10-Example-VelAccIV

Velocity and Acceleration - IV

A child kicks a ball at time $t = 0s$. At the instant the ball is kicked it travels at $25\frac{m}{s}$ at an angle of $30°$ above the horizontal. The ball flies through the air and lands on a school roof. The roof is $4m$ above the level where the ball was kicked from.

- When does the ball land on the roof?
- How far is the ball (horizontally) from the point at which it was kicked?
- What is the ball's velocity just before it hits the roof?

Because of projectile motion approx. we know ball's position between $t=0s$ & when lands

$$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 [t - t_0] + \frac{1}{2}\vec{a}_0 [t-t_0]^2$$

starts "on ground" call it origin

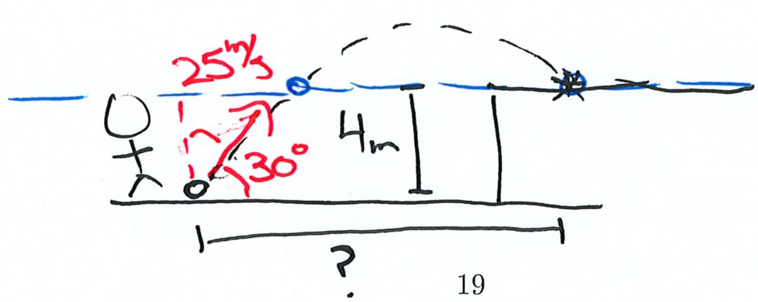
data given

0s

identify $\vec{a} = -g\hat{k}$ with $g = 9.8 m/s^2$ from context

0s

Sketch



$$\vec{r}(t) = 0 + \vec{v}_0\left[t - 0s\right] + \frac{1}{2}\left(-9.8\,m/s^2\,\hat{k}\right)\left[t - 0s\right]^2$$

0 & when lands

$$\vec{v}_0 = 25\,m/s\cos 30\,\hat{\imath} + 25\,m/s\cos 60\,\hat{k}$$

$$\vec{r}(t) = \left(21.65\,m/s\,\hat{\imath} + 12.5\,m/s\,\hat{k}\right)t - \frac{1}{2}9.8\,m/s^2\,t^2\,\hat{k}$$

What do we mean by "landing"

Place ball lands

$$\vec{r}_{land} = x\,\hat{\imath} + 4m\,\hat{k}$$

Find t_{land} so

$$\vec{r}(t_{land}) = \vec{r}_{land}$$

$$t_{land}21.65\,m/s\,\hat{\imath} + 12.5\,m/s\,t_{land}\,\hat{k} - \frac{1}{2}9.8\,m/s^2\,t_{land}^2\,\hat{k} = x\,\hat{\imath} + 4m\,\hat{k}$$

x-components same

$$21.65\,m/s\,t_{land} = x$$

z-components

$$12.5\,m/s\,t_{land} - \frac{1}{2}9.8\,m/s^2\,t_{land}^2 = 4m$$

$$0 = \frac{1}{2} 9.8\,\text{m/s}^2\, t_{land}^2 - 12.5\,\text{m/s}\, t_{land} + 4\,\text{m}$$

$$t_{land} = \frac{12.5\,\text{m/s} \pm \sqrt{(-12.5\,\text{m/s})^2 - 4\,\frac{1}{2}\,9.8\,\text{m/s}^2\, 4\,\text{m}}}{2\left(\frac{1}{2}\right)(9.8\,\text{m/s}^2)}$$

$$= \frac{12.5\,\text{m/s} \pm 11.175\,\text{m/s}}{9.8\,\text{m/s}^2}$$

$$= \underline{2.416\,\text{s}} \text{ or } 0.135\,\text{s}$$

still up

$$x = 21.65\,\text{m/s}\, t_{land}$$

$$= 52.3\,\text{m}$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$

$$= \frac{d}{dt}\left[21.65\,\text{m/s}\, t\, \hat{\imath} + \left(12.5\,\text{m/s}\, t - \frac{1}{2} 9.8\,\text{m/s}^2\, t^2\right)\hat{k}\right]$$

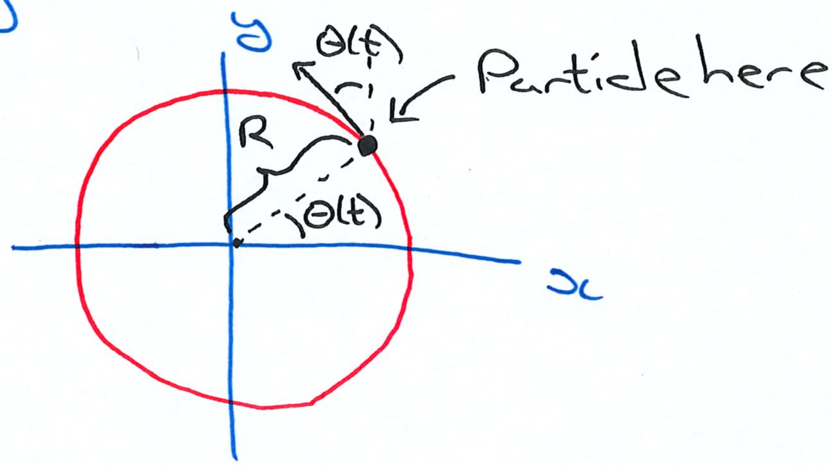
$$= \underbrace{21.65\,\text{m/s}}_{\text{x-comp constant}}\, \hat{\imath} + \underbrace{\left(12.5\,\text{m/s} - 9.8\,\text{m/s}^2\, t\right)}_{\text{vertical changes}}\hat{k}\,\Big]$$

$$\vec{v}(2.416\,\text{s}) = 21.65\,\text{m/s}\, \hat{\imath} + (-11.18\,\text{m/s})\hat{k}$$

Circular Motion

What are $\vec{v}(t)$, $\underbrace{\vec{a}(t)}_{\text{accel changes!}}$ for circular motion

Diagram



$$\vec{r}(t) = R\cos\underbrace{\theta(t)}_{\text{angle changes in time}}\hat{i} + R\sin\theta(t)\hat{j}$$

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t)$$
$$= \frac{d}{dt}\left[R\cos\theta(t)\hat{i} + R\sin\theta(t)\hat{j}\right]$$

$$\frac{d}{dx}\sin x = \cos x \; ; \; \frac{d}{dx}\cos x = -\sin x$$

Chain rule: $\frac{d}{dx} \underline{f}(\underline{g(x)}) = f'(g(x))g'(x)$

$$\frac{d}{dt} \underline{\cos}\underline{\theta(t)} = -\sin\theta(t)\left(\frac{d}{dt}\theta(t)\right)$$

$$\frac{d}{dt}\sin\theta(t) = \cos\theta(t)\left(\frac{d}{dt}\theta(t)\right)$$

$$\vec{v}(t) = -R\sin\theta\left(\frac{d}{dt}\theta(t)\right)\hat{i} + R\cos\theta(t)\left(\frac{d}{dt}\theta(t)\right)\hat{j}$$

$$= \underbrace{R\left(\frac{d}{dt}\theta(t)\right)}_{\text{related to magnitude}}\underbrace{\left[-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}\right]}_{\text{a unit vector direction}}$$

$$|\vec{v}(t)|^2 = \left[R^2\left(\frac{d}{dt}\theta(t)\right)^2\right]1$$

$$|\vec{v}(t)| = R\left|\frac{d}{dt}\theta(t)\right|$$

Grade 12
"$v = R\omega$"

Velocity vector is tangent to (90° to) circular path.

$$\vec{v}(t)\cdot\vec{r}(t) = 0$$

What is $\vec{a}(t)$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt} \text{"speed" "direction"}$$

Product rule

$$\frac{d}{dx}(f(x)g(x))$$

$$= f'(x)g(x) + f(x)g'(x)$$

$$= \left(\frac{d}{dt}\text{speed}\right)\text{"direction"}$$

$$+ \text{"speed"}\left(\frac{d}{dt}\text{direction}\right)$$

Acceleration is either a change in speed or direction or both.

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt}\, R\left(\frac{d}{dt}\theta(t)\right)\left[-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}\right]$$

$$= R\left(\frac{d^2}{dt^2}\theta(t)\right)\left[-\sin\theta(t)\hat{i} + \cos\theta(t)\hat{j}\right]$$

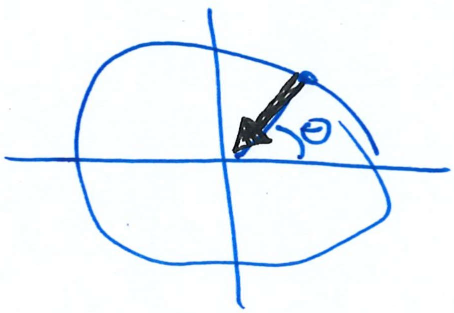
$$+ R\left(\frac{d}{dt}\theta(t)\right)\left[-\cos\theta(t)\left(\frac{d}{dt}\theta(t)\right)\hat{i} - \sin\theta(t)\left(\frac{d}{dt}\theta(t)\right)\hat{j}\right]$$

$$= R\left(\frac{d^2}{dt^2}\Theta(t)\right)\left[-\sin\Theta(t)\hat{i} + \cos\Theta(t)\hat{j}\right]$$

"along $\vec{v}$"

$$+ R\left(\frac{d}{dt}\Theta(t)\right)^2\left[-\cos\Theta(t)\hat{i} - \sin\Theta(t)\hat{j}\right]$$

towards center of circle



For circular motion

$$\vec{a} = \left(\begin{array}{c}\text{change of}\\ \text{speed}\end{array}\right)(\text{along velocity})$$

$$+ R\left(\frac{d}{dt}\Theta(t)\right)^2 \left(\begin{array}{c}\text{towards center}\\ \text{of circle}\end{array}\right)$$

$$\frac{|\vec{v}|^2}{R}$$

4-12-Example-VelAccV

Velocity and Acceleration - V

A mass is moving in uniform circular motion with a position given by

$$\vec{r}(t) = 3m\cos\left(2s^{-1}t\right)\hat{\imath} - 3m\sin\left(2s^{-1}t\right)\hat{\jmath} \tag{2}$$

- What is the velocity at $t = 2s$?
- What is the acceleration at $t = 2s$?
- What is the magnitude of the velocity?
- What is the magnitude of the acceleration?

$$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) \qquad \Theta(t) = 2s^{-1}t$$

$$= 3m\left(\frac{d}{dt}\Theta(t)\right)\left[-\sin\Theta(t)\hat{\imath} - \cos\Theta(t)\hat{\jmath}\right]$$

$$= 3m(2s^{-1})\left[-\sin\Theta(t)\hat{\imath} - \cos\Theta(t)\hat{\jmath}\right]$$

$$= \underbrace{6\,m/s}_{\text{speed constant}}\left[-\sin\Theta(t)\hat{\imath} - \cos\Theta(t)\hat{\jmath}\right]$$

$$\Theta(2s) = 2s^{-1}(2s) = 4$$

$$\vec{v}(2s) = 6\,m/s\left[-\sin 4\,\hat{\imath} - \cos 4\,\hat{\jmath}\right]$$

$$= 6\,m/s\left[0.7568\hat{\imath} + 0.6536\hat{\jmath}\right]$$

$$= 4.54\,m/s\,\hat{\imath} + 3.92\,m/s\,\hat{\jmath}$$

$$\vec{a}(t) = \frac{d}{dt}\vec{v}(t)$$

$$= \frac{d}{dt}\, 6\,\text{m/s}\left[-\sin\theta(t)\,\hat{i} - \cos\theta(t)\,\hat{j}\right]$$

$$= 6\,\text{m/s}\left[-\cos\theta(t)\left(\frac{d}{dt}\theta(t)\right)\hat{i} + \sin\theta(t)\left(\frac{d}{dt}\theta(t)\right)\hat{j}\right]$$

$$= 6\,\text{m/s}\left(\frac{d}{dt}\theta(t)\right)\left[-\cos\theta(t)\,\hat{i} + \sin\theta(t)\,\hat{j}\right]$$

$$= 12\,\text{m/s}^2\left[-\cos\theta(t)\,\hat{i} + \sin\theta(t)\,\hat{j}\right]$$

$$\vec{a}(2s) = 12\,\text{m/s}^2\left[-\cos 4\,\hat{i} + \sin 4\,\hat{j}\right]$$

$$= 12\,\text{m/s}^2\left[0.6536\,\hat{i} - 0.7568\,\hat{j}\right]$$

$$= 7.84\,\text{m/s}^2\,\hat{i} - 9.08\,\text{m/s}^2\,\hat{j}$$

is this $\frac{|\vec{v}|^2}{R} = \frac{(6\,\text{m/s})^2}{3\text{m}} = 12\,\text{m/s}^2$

Velocity and Acceleration - VI

A mass is moving in a circle of radius $4m$ centered at the origin..

At time $t = 0s$ the mass is on the positive y-axis, moving in the negative x-direction. The mass's speed is $8\frac{m}{s}$ and it is slowing down with $\frac{d}{dt}|\vec{v}| = -2\frac{m}{s^2}$.

- What is the x-component of the acceleration of this mass?
- What is the y-component of its acceleration?
- What is the magnitude of the total acceleration of the mass?

$\hat{v}$ $\quad \hat{v} \cdot \hat{c} = 0$

$$\vec{a} = \left(\frac{d}{dt}|\vec{v}|\right)(\text{along } \vec{v})$$

$\hat{c}$

$$+ \left(R\left(\frac{d\theta}{dt}\right)^2\right)(\text{towards center})$$

$$\frac{|\vec{v}|^2}{R}$$

in dir of $\vec{v}$

$$= \left(-2\frac{m}{s^2}\right)(-\hat{\imath})$$

$$+ \left(\frac{(8\,m/s)^2}{4m}\right)(-\hat{\jmath})$$

$$= \underbrace{2\,m/s^2\,\hat{\imath}}\ \underbrace{-\ 16\,m/s^2\,\hat{\jmath}}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} = 16.125\,m/s^2$$