Student: Arfaz Hossain	**Instructor:** Muhammad Awais	**Assignment:** HW-6 [Sections 10.4, 10.5 & 10.6]
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Does the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+11}$ converge absolutely, converge conditionally, or diverge?

A series $\sum a_n$ converges absolutely (is absolutely convergent) if the corresponding series of absolute values, $\sum |a_n|$, converges. If the series converges, but is not absolutely convergent, then the series converges conditionally. Otherwise, the series diverges.

Find the terms of the corresponding series of absolute values.

$$\left|(-1)^{n+1} \frac{n}{n+11}\right| = \frac{n}{n+11}$$

Although there are several tests that could be used to determine if $\sum_{n=1}^{\infty} \frac{n}{n+11}$ converges, this example uses the nth-Term Test. The nth-Term Test states that $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n$ fails to exist or is different from zero.

Apply the nth-Term Test to $\sum_{n=1}^{\infty} \frac{n}{n+11}$. Find the limit of the terms of the sequence.

$$\lim_{n\to\infty} \frac{n}{n+11} = 1$$

Since $\lim_{n\to\infty} \frac{n}{n+11}$ is different from zero, the series $\sum_{n=1}^{\infty} \frac{n}{n+11}$ diverges per the nth-Term Test and the given series does not converge absolutely.

Now, test to see if the given series converges conditionally. To do so, use the nth-Term Test again.

Apply the nth-Term Test to $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+11}$. Find the limit of the terms of the sequence.

$$\lim_{n\to\infty} (-1)^{n+1} \frac{n}{n+11} = \lim_{n\to\infty} (-1)^{n+1}$$

For odd values of n, $(-1)^{n+1} \frac{n}{n+11}$ approaches 1. For even values of n, $(-1)^{n+1} \frac{n}{n+11}$ approaches -1. Therefore, $\lim_{n\to\infty} (-1)^{n+1} \frac{n}{n+11}$ fails to exist, so the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n+11}$ diverges by the nth-Term Test.