Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

Find the series' radius of convergence.

$$\sum_{n=1}^{\infty} \frac{n!}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n} x^n$$

A power series about $x = a$ is a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots + c_n(x-a)^n + \cdots$, in which the center a and the coefficients $c_0, c_1, c_2, \ldots, c_n, \ldots$ are constants.

Note that this is a power series with $c_n = \frac{n!}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}$ and $a = 0$.

R is called the radius of convergence of the power series, and the interval of radius R centered at $x = a$ is called the interval of convergence. At points x with $|x-a| < R$, the series converges absolutely. If the series converges for all values of x, it is said that its radius of convergence is infinite. If it converges only at $x = a$, it is said that its radius of convergence is zero.

Next apply the ratio test to the series $\sum |u_n|$, where u_n is the nth term of the power series. Determine $\left|\frac{u_{n+1}}{u_n}\right|$.

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{\frac{(n+1)! \cdot x^{n+1}}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9(n+1)}}{\frac{n! \cdot x^n}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}}\right| \qquad \text{Substitute.}$$

$$= \left|\frac{(n+1)! \cdot x^{n+1}}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9(n+1)} \cdot \frac{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}{n! \cdot x^n}\right| \qquad \text{Invert and multiply.}$$

Now simplify the expression.

$$\left|\frac{u_{n+1}}{u_n}\right| = \left|\frac{(n+1)! \cdot x^{n+1}}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9(n+1)} \cdot \frac{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}{n! \cdot x^n}\right|$$

$$= \frac{(n+1)! \cdot |x|^{n+1}}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9(n+1)} \cdot \frac{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}{n! \cdot |x|^n}$$

$$= \frac{(n+1) \cdot n! \cdot |x|^n \cdot |x|}{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9(n+1)} \cdot \frac{9 \cdot 18 \cdot 27 \cdot \cdots \cdot 9n}{n! \cdot |x|^n} \qquad \text{Write } |x|^{n+1} \text{ as } |x|^n \cdot |x| \text{ and } (n+1)! \text{ as } (n+1) \cdot n!.$$

$$= \frac{1}{9}|x| \qquad \text{Simplify.}$$

The series converges absolutely for $\frac{1}{9}|x| < 1$.

Thus, the series converges absolutely for $|x| < 9$.

The series converges for all x in $|x| < 9$ or $-9 < x < 9$.

Therefore, the radius of convergence is 9.