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Assignment: Practice Questions for
 Sections 6.3 & 7.2 [Not for

Solve the differential equation.

$$y^4 \frac{dy}{dx} = 6x^5 y^5 - 18x^5$$

A differential equation $\frac{dy}{dx} = f(x,y)$ is separable if f can be expressed as a product of a function of x and a function of y . The differential equation then has the form $\frac{dy}{dx} = g(x)H(y)$, or $\frac{dy}{dx} = \frac{g(x)}{h(y)}$, where $h(y) = \frac{1}{H(y)}$.

First factor the right side of the equation.

$$\begin{aligned} y^4 \frac{dy}{dx} &= 6x^5 y^5 - 18x^5 \\ &= 6x^5 (y^5 - 3) \end{aligned}$$

Notice that the right side of the equation is now a product of a function of x and a function of y . Since the term multiplying $\frac{dy}{dx}$ on the left side of the equation is a function only of y , the equation is separable.

Rewrite the equation in its differential form, $h(y)dy = g(x)dx$.

$$\begin{aligned} y^4 \frac{dy}{dx} &= 6x^5 (y^5 - 3) \\ \frac{y^4}{y^5 - 3} dy &= 6x^5 dx \end{aligned}$$

Now integrate both sides of the equation.

$$\begin{aligned} \frac{y^4}{y^5 - 3} dy &= 6x^5 dx \\ \int \frac{y^4}{y^5 - 3} dy &= \int 6x^5 dx \end{aligned}$$

Begin by integrating the left side. Use the Substitution Rule to integrate $\int \frac{y^4}{y^5 - 3} dy$. Use $u = y^5 - 3$.

Find du .

$$du = 5y^4 dy$$

Thus $du = 5y^4 dy$, or $dy = \frac{du}{5y^4}$. Rewrite the integrand in terms of u .

$$\int \frac{y^4}{y^5 - 3} dy = \int \frac{1}{5u} du$$

Now integrate.

$$\int \frac{1}{5u} du = \frac{1}{5} \ln |u| + C_1$$

Use $u = y^5 - 3$ to rewrite the result in terms of y .

$$\frac{1}{5} \ln |u| + C_1 = \frac{1}{5} \ln |y^5 - 3|$$

Now integrate the right side of the equation.

$$\frac{1}{5} \ln |y^5 - 3| + C_1 = \int 6x^5 \, dx$$

$$\frac{1}{5} \ln |y^5 - 3| + C_1 = x^6 + C_2$$

Thus, $\frac{1}{5} \ln |y^5 - 3| + C_1 = x^6 + C_2$, which defines y implicitly as a function of x . Combine the constants of integration C_1 and C_2 as C .

$$\frac{1}{5} \ln |y^5 - 3| = x^6 + C$$

Thus, solving the differential equation $y^4 \frac{dy}{dx} = 6x^5 y^5 - 18x^5$ gives $\frac{1}{5} \ln |y^5 - 3| = x^6 + C$.