Student: Arfaz Hossain **Date:** 10/10/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 4

It takes 24 hours to drain a storage tank by opening the valve at the bottom. The depth y of fluid in the tank t hours after the valve is opened is given by the formula $y = 12\left(1 - \frac{t}{24}\right)^2$ m. Complete parts (a) through (c) below.

a. Find the rate $\frac{dy}{dt}$ (in meters per hour) at which the tank is draining at time t.

The rate of change of depth is the derivative of the depth with respect to time.

Determine $\frac{dy}{dt}$.

$$\frac{dy}{dt} = \frac{t}{24} - 1$$

b. When is the fluid level in the tank falling fastest? Slowest? What are the values of $\frac{dy}{dt}$ at these times?

Since the rate of change of depth is the derivative of the depth with respect to time, the fluid level in the tank is falling fastest when $\frac{dy}{dt}$ is at a minimum, and it is falling slowest when $\frac{dy}{dt}$ is at a maximum.

The draining process begins at $t = 0$.

The draining process ends at $t = 24$.

Consider the velocity function, $\frac{dy}{dt} = \frac{t}{24} - 1$. Notice that it is a linear function.

The maximum and minimum of a linear function on a given interval are at the endpoints of that interval.

So the maximum and minimum points of $\frac{dy}{dt}$ occur, in some order, when $t = 0$ and $t = 24$. Evaluate the function at $t = 0$.

$$\frac{0}{24} - 1 = -1$$

Evaluate the function at $t = 24$.

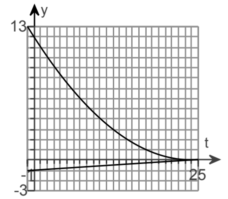
$$\frac{24}{24} - 1 = 0$$

Thus, the fluid level is falling slowest at $t = 24$ h. The value of $\frac{dy}{dt}$ at this time is 0 m / h. The fluid level is falling fastest at $t = 0$ h. The value of $\frac{dy}{dt}$ at this time is -1 m / h.

c. Graph y and $\frac{dy}{dt}$ together and discuss the behavior of y in relation to the signs and values of $\frac{dy}{dt}$.

Use technology to graph the two functions.

Recall that $\frac{dy}{dt} = \frac{t}{24} - 1$. The functions are graphed to the right.



The graph of y goes down when traced from left to right, so the values of y decrease from left to right.

Notice that $\frac{dy}{dt}$ increases toward 0 from left to right. Therefore, y decreases at a decreasing rate from left to right.