CSC 225 FALL 2022
ALGORITHMS AND DATA STRUCTURES I
ASSIGNMENT 2 - WRITTEN
UNIVERSITY OF VICTORIA

1. [4 marks] Count the number of assignments and comparisons for each of the following algorithms.

   a) **Algorithm** Loop1(n):

   ```
   p ← 1
   for i ← 1 to n² + 1 do
           p ← p · i
   ```

   b) **Algorithm** Loop2(n):

   ```
   s ← 0
   for i ← 1 to n² + 1 do
           for j ← 1 to i do
                   s ← s + i
   ```

2. [4 marks] Describe a recursive algorithm for finding both the minimum and maximum elements in an array A of n elements. Your method should return a pair (a, b) where a is the minimum element and b is the maximum element. Count the assignments (including returns) and comparisons in order to derive a recurrence equation for the worst-case runtime of your algorithm.

3. a) [2 marks] Consider the following recurrence equation, defining a function T(n):

$$T(n) = \begin{cases} 1, & \text{if } n = 1 \\ T(n-1) + n, & \text{if } n \geq 2 \end{cases}$$

   Show by induction that $T(n) = n(n+1)/2$.

   b) [2 marks] Consider the following recurrence equation, defining a function T(n):

$$T(n) = \begin{cases} 1, & \text{if } n = 0 \\ T(n-1) + 2^n, & \text{if } n \geq 1 \end{cases}$$

   Show by induction that $T(n) = 2^{n+1} - 1$.

4. [4 marks] Using the definition of Big-Oh, prove the following statements are true:
   a) $3n^2 - 100n + 6$ is $O(n^2)$.
   b) $2n^3 + n\sqrt{n}$ is $O(n^3)$.
   c) $3n \log n + 2n\sqrt{n}$ is $O(n^{1.5})$.
   d) $(x + y)^2$ is $O(x^2 + y^2)$ where $x, y > 0$.

5. [4 marks] Order the following list of functions by their big-Oh notation. Group together (for example, by underlining) those functions that are big-Theta of one another. (No justification needed).
Note: $\log n = \log_2 n$ unless otherwise stated.

n	2^n	$n \log n$	$\ln n$
$\log n$	$\sqrt{n}$	e^n	$n^2 + \log n$
2^{n-1}	$\log \log n$	n^3	$(\log n)^2$
$n^{1.375}$	$n - n^3 + 7n^5$	n^2	$n!$