Solution

$\sum_{n=1}^{\infty}\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}$: Radius of convergence is $\frac{1}{8}$, Interval of convergence is $\frac{1}{2}\le x\le\frac{3}{4}$

Steps

$\sum_{n=1}^{\infty}\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}$

Use the Ratio Test to compute the convergence interval

Hide Steps

$\sum_{n=1}^{\infty}\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{(8x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}}}{\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{(8x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}}}{\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\frac{\frac{(8x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}}}{\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}}\right|\right)$

Simplify $\frac{\frac{(8x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}}}{\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}}$: $\frac{n^{\frac{3}{2}}(8x-5)^2}{(n+1)^{\frac{3}{2}}}$

Hide Steps

$\frac{\frac{(8x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}}}{\frac{(8x-5)^{2n+1}}{n^{\frac{3}{2}}}}$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}$

$=\frac{(8x-5)^{2(n+1)+1}n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}(8x-5)^{2n+1}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{(8x-5)^{2(n+1)+1}}{(8x-5)^{2n+1}}=(8x-5)^{2(n+1)+1-(2n+1)}$

$=\frac{n^{\frac{3}{2}}(8x-5)^{2(n+1)+1-(2n+1)}}{(n+1)^{\frac{3}{2}}}$

Add similar elements: $2(n+1)+1-(2n+1)=2$

$=\frac{n^{\frac{3}{2}}(8x-5)^2}{(n+1)^{\frac{3}{2}}}$

$L=\lim_{n\to\infty}\left(\left|\frac{n^{\frac{3}{2}}(8x-5)^2}{(n+1)^{\frac{3}{2}}}\right|\right)$

$L=\left|(8x-5)^2\right|\cdot\lim_{n\to\infty}\left(\left|\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}\right|\right)$

$\lim_{n\to\infty}\left(\left|\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}\right|\right)=1$

Hide Steps

$\lim_{n\to\infty}\left(\left|\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}\right|\right)$

$\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}$ is positive when $n\to\infty$. Therefore $\left|\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}\right|=\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}$

$=\lim_{n\to\infty}\left(\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}\right)$

Simplify $\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}$: $\left(\frac{n}{n+1}\right)^{\frac{3}{2}}$

Hide Steps

$$\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}$$

Apply exponent rule: $\frac{a^c}{b^c}=\left(\frac{a}{b}\right)^c$

$\frac{n^{\frac{3}{2}}}{(n+1)^{\frac{3}{2}}}=\left(\frac{n}{n+1}\right)^{\frac{3}{2}}$

$=\left(\frac{n}{n+1}\right)^{\frac{3}{2}}$

$=\lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^{\frac{3}{2}}\right)$

$\lim_{x\to a}[f(x)]^b=[\lim_{x\to a}f(x)]^b$
With the exception of indeterminate form

$=\left(\lim_{n\to\infty}\left(\frac{n}{n+1}\right)\right)^{\frac{3}{2}}$

Divide by highest denominator power: $\frac{1}{1+\frac{1}{n}}$ *Hide Steps*

$\frac{n}{n+1}$

Divide by n

$=\frac{\frac{n}{n}}{\frac{n}{n}+\frac{1}{n}}$

Refine

$=\frac{1}{1+\frac{1}{n}}$

$=\left(\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)\right)^{\frac{3}{2}}$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right]=\frac{\lim_{x\to a}f(x)}{\lim_{x\to a}g(x)},\quad \lim_{x\to a}g(x)\neq 0$
With the exception of indeterminate form

$=\left(\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}\right)^{\frac{3}{2}}$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=1$ *Hide Steps*

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)$

$\lim_{x\to a}[f(x)\pm g(x)]=\lim_{x\to a}f(x)\pm\lim_{x\to a}g(x)$
With the exception of indeterminate form

$=\lim_{n\to\infty}(1)+\lim_{n\to\infty}\left(\frac{1}{n}\right)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$\lim_{n\to\infty}\left(\frac{1}{n}\right)=0$ *Hide Steps*

$\lim_{n\to\infty}\left(\frac{1}{n}\right)$

Apply Infinity Property: $\lim_{x\to\infty}\left(\frac{c}{x^a}\right)=0$

$=0$

$=1+0$

Simplify

$=1$

$=\left(\frac{1}{1}\right)^{\frac{3}{2}}$

Simplify $\left(\frac{1}{1}\right)^{\frac{3}{2}}$: 1 *Hide Steps*

$\left(\frac{1}{1}\right)^{\frac{3}{2}}$

Apply rule $\frac{a}{1}=a$

$\frac{1}{1}=1$

$= 1^{\frac{3}{2}}$

$1^{\frac{3}{2}} = 1 \cdot 1^{\frac{1}{2}}$ *Hide Steps*

$1^{\frac{3}{2}}$

$1^{\frac{3}{2}} = 1^{1+\frac{1}{2}}$

$= 1^{1+\frac{1}{2}}$

Apply exponent rule: $x^{a+b} = x^a x^b$

$= 1^1 \cdot 1^{\frac{1}{2}}$

Refine

$= 1 \cdot 1^{\frac{1}{2}}$

$= 1 \cdot 1^{\frac{1}{2}}$

Multiply: $1 \cdot 1^{\frac{1}{2}} = 1^{\frac{1}{2}}$

$= 1^{\frac{1}{2}}$

Apply rule $1^a = 1$

$= 1$

$= 1$

$L = \left|(8x-5)^2\right| \cdot 1$

Simplify

$L = |8x-5|^2$

$L = |8x-5|^2$

The power series converges for $L < 1$

$|8x-5|^2 < 1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for $|x-a|$

$|8x-5|^2 < 1: \quad \left|x - \frac{5}{8}\right| < \frac{1}{8}$ *Hide Steps*

$|8x-5|^2 < 1$

Take the square root of both sides of an inequality

$\sqrt{|8x-5|^2} < \sqrt{1}$

Simplify

$|8x-5| < 1$

Divide both sides by 8

$\frac{|8x-5|}{8} < \frac{1}{8}$

Simplify

$\left|x - \frac{5}{8}\right| < \frac{1}{8}$

Therefore

Radius of convergence is $\frac{1}{8}$

Radius of convergence is $\frac{1}{8}$

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$|8x-5|^2 < 1 \quad : \quad \frac{1}{2} < x < \frac{3}{4}$ *Hide Steps*

$|8x-5|^2 < 1$

Find positive and negative intervals *Hide Steps*

Find intervals for $|8x-5|$

$8x - 5 \geq 0 : x \geq \frac{5}{8}, \quad |8x-5| = 8x-5$ *Hide Steps*

$8x - 5 \geq 0 \quad : \quad x \geq \frac{5}{8}$ *Hide Steps*

$8x - 5 \geq 0$

Add 5 to both sides

$8x - 5 + 5 \geq 0 + 5$

Simplify

$8x \geq 5$

Divide both sides by 8

$\frac{8x}{8} \ge \frac{5}{8}$

Simplify

$x \ge \frac{5}{8}$

Rewrite $|8x-5|$ for $8x-5 \ge 0$: $|8x-5| = 8x-5$ *Hide Steps*

Apply absolute rule: If $u \ge 0$ then $|u| = u$

$|8x-5| = 8x-5$

$8x-5<0: x<\frac{5}{8}, \quad |8x-5| = -(8x-5)$ *Hide Steps*

$8x-5<0 \quad : \quad x<\frac{5}{8}$ *Hide Steps*

$8x-5<0$

Add 5 to both sides

$8x-5+5<0+5$

Simplify

$8x<5$

Divide both sides by 8

$\frac{8x}{8}<\frac{5}{8}$

Simplify

$x<\frac{5}{8}$

Rewrite $|8x-5|$ for $8x-5<0$: $|8x-5| = -(8x-5)$ *Hide Steps*

Apply absolute rule: If $u<0$ then $|u| = -u$

$|8x-5| = -(8x-5)$

Identify the intervals:

$x<\frac{5}{8}, \ x \ge \frac{5}{8}$

	$x<\frac{5}{8}$	$x \ge \frac{5}{8}$
$\lvert 8x-5\rvert$	$-$	$+$

$x<\frac{5}{8}, \ x \ge \frac{5}{8}$

$x<\frac{5}{8}, \ x \ge \frac{5}{8}$

Solve the inequality for each interval *Hide Steps*

$x<\frac{5}{8}, \ x \ge \frac{5}{8}$

For $x<\frac{5}{8}$: $\frac{1}{2}<x<\frac{5}{8}$ *Hide Steps*

For $x<\frac{5}{8}$ rewrite $|8x-5|^2<1$ as $(-(8x-5))^2<1$

$(-(8x-5))^2<1 \quad : \quad \frac{1}{2}<x<\frac{3}{4}$ *Hide Steps*

$(-(8x-5))^2<1$

For $u^n<a$, if n is even then $-\sqrt[n]{a}<u<\sqrt[n]{a}$

$-1<-(8x-5)<1$

If $a<u<b$ then $a<u$ and $u<b$

$-1<-(8x-5)$ and $-(8x-5)<1$

$-1<-(8x-5) \quad : \quad x<\frac{3}{4}$ *Hide Steps*

$-1<-(8x-5)$

Switch sides

$-(8x-5)>-1$

Multiply both sides by -1 (reverse the inequality)

$(-(8x-5))(-1)<(-1)(-1)$

Simplify

$8x-5<1$

Add 5 to both sides

$8x-5+5<1+5$

Simplify

$8x<6$

Divide both sides by 8

$\frac{8x}{8}<\frac{6}{8}$

Simplify

$x<\frac{3}{4}$

$-(8x-5)<1 \quad : \quad x>\frac{1}{2}$ *Hide Steps*

$-(8x-5)<1$

Multiply both sides by -1 (reverse the inequality)

$(-(8x-5))(-1)>1\cdot(-1)$

Simplify

$8x-5>-1$

Add 5 to both sides

$8x-5+5>-1+5$

Simplify

$8x>4$

Divide both sides by 8

$\frac{8x}{8}>\frac{4}{8}$

Simplify

$x>\frac{1}{2}$

Combine the intervals

$x<\frac{3}{4}$ and $x>\frac{1}{2}$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x<\frac{3}{4}$ and $x>\frac{1}{2}$

$\frac{1}{2}<x<\frac{3}{4}$

$\frac{1}{2}<x<\frac{3}{4}$

Combine the intervals

$\frac{1}{2}<x<\frac{3}{4}$ and $x<\frac{5}{8}$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$\frac{1}{2}<x<\frac{3}{4}$ and $x<\frac{5}{8}$

$\frac{1}{2}<x<\frac{5}{8}$

$\frac{1}{2}<x<\frac{5}{8}$

For $x\ge\frac{5}{8}$: $\frac{5}{8}\le x<\frac{3}{4}$ *Hide Steps*

For $x\ge\frac{5}{8}$ rewrite $|8x-5|^2<1$ as $(8x-5)^2<1$

$(8x-5)^2<1 \quad : \quad \frac{1}{2}<x<\frac{3}{4}$ *Hide Steps*

$(8x-5)^2<1$

For $u^n<a$, if n is even then $-\sqrt[n]{a}<u<\sqrt[n]{a}$

$-1<8x-5<1$

If $a<u<b$ then $a<u$ and $u<b$

$-1<8x-5$ and $8x-5<1$

$-1<8x-5 \quad : \quad x>\frac{1}{2}$ Hide Steps

$-1<8x-5$

Switch sides

$8x-5>-1$

Add 5 to both sides

$8x-5+5>-1+5$

Simplify

$8x>4$

Divide both sides by 8

$\frac{8x}{8}>\frac{4}{8}$

Simplify

$x>\frac{1}{2}$

$8x-5<1 \quad : \quad x<\frac{3}{4}$ Hide Steps

$8x-5<1$

Add 5 to both sides

$8x-5+5<1+5$

Simplify

$8x<6$

Divide both sides by 8

$\frac{8x}{8}<\frac{6}{8}$

Simplify

$x<\frac{3}{4}$

Combine the intervals

$x>\frac{1}{2}$ and $x<\frac{3}{4}$

Merge Overlapping Intervals Hide Steps

The intersection of two intervals is the set of numbers which are in both intervals

$x>\frac{1}{2}$ and $x<\frac{3}{4}$

$\frac{1}{2}<x<\frac{3}{4}$

$\frac{1}{2}<x<\frac{3}{4}$

Combine the intervals

$\frac{1}{2}<x<\frac{3}{4}$ and $x\geq\frac{5}{8}$

Merge Overlapping Intervals Hide Steps

The intersection of two intervals is the set of numbers which are in both intervals

$\frac{1}{2}<x<\frac{3}{4}$ and $x\geq\frac{5}{8}$

$\frac{5}{8}\leq x<\frac{3}{4}$

$\frac{5}{8}\leq x<\frac{3}{4}$

Combine the intervals

$\frac{1}{2}<x<\frac{5}{8}$ or $\frac{5}{8}\leq x<\frac{3}{4}$

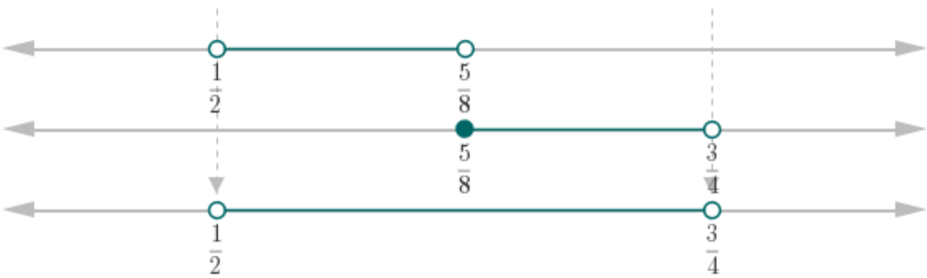
$\frac{1}{2}<x<\frac{5}{8}$ or $\frac{5}{8}\le x<\frac{3}{4}$

Merge Overlapping Intervals *Hide Steps*

The union of two intervals is the set of numbers which are in either interval

$\frac{1}{2}<x<\frac{5}{8}$ or $\frac{5}{8}\le x<\frac{3}{4}$

$\frac{1}{2}<x<\frac{3}{4}$



The union of two intervals is the set of numbers which are in either interval

$\frac{1}{2}<x<\frac{3}{4}$ or $x=\frac{1}{2}$

$\frac{1}{2}\le x<\frac{3}{4}$

The union of two intervals is the set of numbers which are in either interval

$\frac{1}{2}\le x<\frac{3}{4}$ or $x=\frac{3}{4}$

$\frac{1}{2}\le x\le\frac{3}{4}$

$\frac{1}{2}\le x\le\frac{3}{4}$

$\frac{1}{2}<x<\frac{3}{4}$

Check the interval end points: $x=\frac{1}{2}$:converges, $x=\frac{3}{4}$:converges *Hide Steps*

For $x=\frac{1}{2}$, $\sum_{n=1}^{\infty}\frac{\left(8\left(\frac{1}{2}\right)-5\right)^{2n+1}}{n^{\frac{3}{2}}}$: converges *Hide Steps*

$\sum_{n=1}^{\infty}\frac{\left(8\left(\frac{1}{2}\right)-5\right)^{2n+1}}{n^{\frac{3}{2}}}$

Refine

$=\sum_{n=1}^{\infty}-\frac{1}{n^{\frac{3}{2}}}$

Apply the constant multiplication rule: $\sum c\cdot a_n=c\cdot\sum a_n$

$=-\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

Apply p − Series Test: converges *Hide Steps*

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

p − Series Test:

If the series is of the form $\sum_{n=1}^{\infty}\frac{1}{n^{p}}$, where $p>0$

If $p>1$, then the p − series converges

If $0<p\le 1$, then the p − series diverges

$p=\frac{3}{2},\ p>1$, by the p − Series test criteria

= converges

= −converges

= converges

Hide Steps

For $x=\frac{3}{4}$, $\sum_{n=1}^{\infty}\frac{\left(8\left(\frac{3}{4}\right)-5\right)^{2n+1}}{n^{\frac{3}{2}}}$: converges

$\sum_{n=1}^{\infty}\frac{\left(8\left(\frac{3}{4}\right)-5\right)^{2n+1}}{n^{\frac{3}{2}}}$

Refine

$=\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

Apply p – Series Test: converges

Hide Steps

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

p – Series Test:

If the series is of the form $\sum_{n=1}^{\infty}\frac{1}{n^p}$, where $p>0$

If $p>1$, then the p – series converges
If $0<p\le 1$, then the p – series diverges

$p=\frac{3}{2}$, $p>1$, by the p – Series test criteria

$=$ converges

$=$ converges

$x=\frac{1}{2}$:converges, $x=\frac{3}{4}$:converges

Therefore

Interval of convergence is $\frac{1}{2}\le x\le\frac{3}{4}$

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Radius of convergence is $\frac{1}{8}$, Interval of convergence is $\frac{1}{2}\le x\le\frac{3}{4}$