Average Forces

7-6-Theory-
Average Force

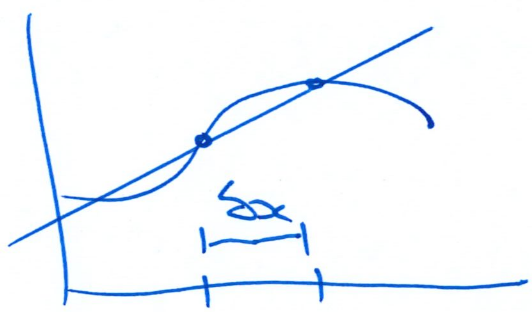
2nd $\vec{a} = \frac{\vec{F}_{net}}{m}$

$$\vec{F}_{net} = m\vec{a}$$
$$= m\frac{d}{dt}\vec{v}$$
$$= \frac{d}{dt}(m\vec{v})$$

$$\vec{F}_{net} = \frac{d}{dt}\vec{p}$$

"The net force on an object gives us the time rate of change of object's momentum"

How to calculate an average force?



$$\frac{d}{dx}f(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$\frac{d}{dt}\vec{p}(t) = \lim_{\Delta t \to 0} \frac{\vec{p}(t+\Delta t) - \vec{p}(t)}{\Delta t}$$

to get the average force you simply don't do this limit

$$\vec{F}_{net,avg} = \frac{\Delta \vec{p}}{\Delta t}$$

$$\frac{\text{change in } \vec{p}}{\text{how long it took}}$$

Momentum - III

A ball of mass $3kg$ travels with velocity $10\frac{m}{s}\hat{\imath}$ on a horizontal frictionless surface. A second ball of mass $4kg$ travels with velocity $-9\frac{m}{s}\hat{\imath}$. The balls collide; they interact for a period of $0.2s$.

After the collision the $3kg$ ball travels at $-8\frac{m}{s}\hat{\jmath}$.

- ✓ What is the speed of $4kg$ ball after the collision?
- ✓ What angle does the $4kg$ ball's velocity make with the positive x-axis (with $\hat{\imath}$)?
- ✓ What is the change in the momentum of the $4kg$ ball during the interaction?
- What was the average force on the $4kg$ ball?

$$\vec{P}_{before} = \vec{P}_{after}$$

$$\underbrace{\vec{P}_{3,b}}_{\text{know}} + \underbrace{\vec{P}_{4,b}}_{\text{know}} = \underbrace{\vec{P}_{3,a}}_{\text{know}} + \vec{P}_{4,a}$$

$$\vec{P}_{4,a} = \vec{P}_{4,b} + \underbrace{\vec{P}_{3,b} - \vec{P}_{3,a}}_{-\Delta\vec{P}_3}$$

$$= 4kg(-9\,m/s\,\hat{\imath}) + 3kg\,10\,m/s\,\hat{\imath} - 3kg(-8\,m/s\,\hat{\jmath})$$

$$= -6\,kg\,m/s\,\hat{\imath} + 24\,kg\,m/s\,\hat{\jmath}$$

1 $\vec{p}_{4,a} = m_4 \vec{v}_a = -6\,\mathrm{kg\,m/s}\,\hat{i} + 24\,\mathrm{kg\,m/s}\,\hat{j}$

$$\vec{v}_{4,a} = -1.5\,\mathrm{m/s}\,\hat{i} + 6\,\mathrm{m/s}\,\hat{j}$$

$$|\vec{v}_{4,a}| = 6.185\,\mathrm{m/s}$$

$$\cos\Theta = \frac{v_{4,a,x}}{|\vec{v}_{4,a}|} = \frac{-1.5\,\mathrm{m/s}}{6.185\,\mathrm{m/s}} = -0.2425$$

$$\Theta = 104^\circ$$

$$\Delta\vec{p}_4 = \vec{p}_{4,a} - \vec{p}_{4,b}$$

$$= -6\,\mathrm{kg\,m/s}\,\hat{i} + 24\,\mathrm{kg\,m/s}\,\hat{j} - (-36\,\mathrm{kg\,m/s}\,\hat{i})$$

$$= 30\,\mathrm{kg\,m/s}\,\hat{i} + 24\,\mathrm{kg\,m/s}\,\hat{j}$$

$$\Delta\vec{p}_4 = -\Delta\vec{p}_3 = -\left(3\,\mathrm{kg}(-8\,\mathrm{m/s}\,\hat{j}) - 3\,\mathrm{kg}(10\,\mathrm{m/s}\,\hat{i})\right)$$

$$= 30\,\mathrm{kg\,m/s}\,\hat{i} + 24\,\mathrm{kg\,m/s}\,\hat{j}$$

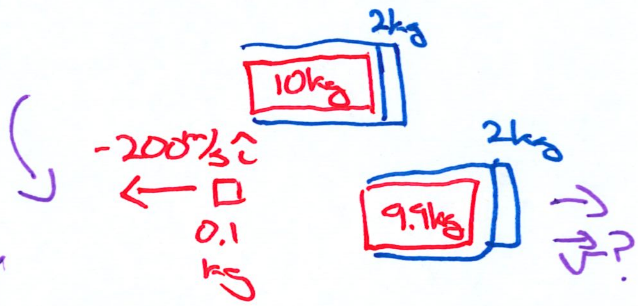
$$\vec{F}_{avg,4} = \frac{\Delta\vec{p}_4}{0.2\,\mathrm{s}} = 150\,\mathrm{N}\,\hat{i} + 120\,\mathrm{N}\,\hat{j}$$

7-8-Example-MomentumIV

Momentum - IV

A rocket consists of a $2kgmass$ shaped in such a way that it encloses $10kg$ of fuel. The fuel is burned in a series of bursts, with each burst expelling $0.1kg$ of fuel with a velocity $-200m/s\hat{\imath}$ measured relative to the rocket.

- * What is the velocity of the rocket after there have been three fuel emissions?
- How would you calculate the velocity of the rocket after all the fuel has been emitted?
- If the fuel were emitted continuously, what would the final velocity of the rocket be?



Conservation of momentum

before $\vec{P}_{total} = 0$

After $0 = \vec{P}_{total} = (-200\,m/s\,\hat{\imath})(0.1kg) + 11.9kg\,\vec{v}$

$\vec{v} = 1.68\,m/s\,\hat{\imath}$

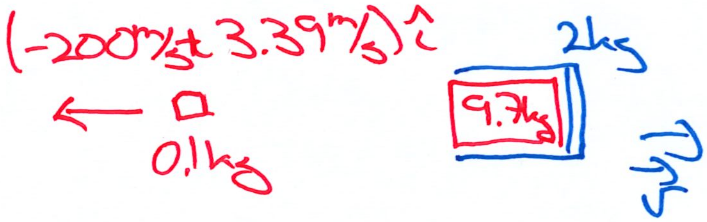
$$\vec{P}_{before} = (11.9\,kg)(1.68\,m/s\,\hat{i})$$

$$\vec{P}_{after} = (0.1\,kg)(-200\,m/s\,\hat{i} + 1.68\,m/s\,\hat{i}) + 11.8\,kg\,\vec{v}$$

$$11.8\,kg\,1.68\,m/s\,\hat{i} = (0.1\,kg)(-200\,m/s\,\hat{i}) + 11.8\,kg\,\vec{v}$$

$$40\,kg\,m/s\,\hat{i} = 11.8\,kg\,\vec{v}$$

$$\vec{v} = 3.39\,m/s\,\hat{i}$$



$$\vec{P}_{before} = 11.8\,kg(3.39\,m/s\,\hat{i})$$

$$\vec{P}_{after} = (0.1\,kg)(-200\,m/s + 3.39\,m/s)\hat{i} + 11.7\,kg\,\vec{v}$$

$$11.7\,kg(3.39\,m/s)\hat{i} = (0.1\,kg)(-200\,m/s)\hat{i} + 11.7\,kg\,\vec{v}$$

$$59.6\,kg\,m/s\,\hat{i} = 11.7\,kg\,\vec{v}$$

$$\boxed{\vec{v} = 5.1\,m/s\,\hat{i}}$$

Starts out with mass M, going at $\vec{v}_i$

$$M\vec{v}_i = \Delta m(-200\,\mathrm{m/s}\,\hat{\imath} + \vec{v}_i) + (M-\Delta m)\vec{v}_f$$

$$(M-\Delta m)\vec{v}_i = \Delta m(-200\,\mathrm{m/s}\,\hat{\imath}) + (M-\Delta m)\vec{v}_f$$

$$\underbrace{\vec{v}_f - \vec{v}_i}_{\text{change in } \vec{v} \text{ during one step}} = \frac{\Delta m}{M-\Delta m}\,200\,\mathrm{m/s}\,\hat{\imath}$$

add all $\Delta\vec{v}$'s

$$\sum \text{all } \Delta\vec{v}\text{'s} = \left(\frac{0.1\,\mathrm{kg}}{11.9\,\mathrm{kg}}\,200\,\mathrm{m/s}\,\hat{\imath} + \frac{0.1\,\mathrm{kg}}{11.8\,\mathrm{kg}}(200\,\mathrm{m/s}\,\hat{\imath}) + \frac{0.1\,\mathrm{kg}}{11.7\,\mathrm{kg}}\,200\,\mathrm{m/s}\,\hat{\imath}\right) + \ldots$$

$$= \sum_{n=1}^{100} \frac{0.1\,\mathrm{kg}}{12\,\mathrm{kg} - 0.1\,\mathrm{kg}\,n}\,200\,\mathrm{m/s}\,\hat{\imath}$$

Continuously: This looks like Riemann sum

$$d\vec{v} = \frac{dM}{M}\,200\,\mathrm{m/s}\,\hat{\imath}$$

$$\int d\vec{v} = \int \frac{dM}{M}\,200\,\mathrm{m/s}\,\hat{\imath}$$

$$= 200\,\mathrm{m/s}\,\hat{\imath}\,\ln(12/2)$$

$$\vec{v}_f = 358\,\mathrm{m/s}\,\hat{\imath}$$