

# Math 101 Tutorial Worksheet 4

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There is no associated quiz due on BrightSpace for this tutorial worksheet

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1. Determine if given improper integrals converge or diverge.

(a)  $\int_2^{\infty} \frac{x}{x^2 + \sin(x)} dx$

(b)  $\int_{-1}^{\infty} \frac{1}{\sqrt{x^4 + 1}} dx$

(c)  $\int_1^{\infty} \frac{1}{x + \cos^2(x)} dx$

(d)  $\int_0^{\infty} \frac{x \cos^2(x)}{e^x} dx$

(e)  $\int_0^1 \tan(2\pi x) dx$

(f)  $\int_{-\infty}^{+\infty} x e^{-x^2} dx$

2. Consider a solid that lies between planes perpendicular to the  $x$ -axis at  $x = -\frac{\pi}{3}$  and  $x = \frac{\pi}{3}$ . The cross-sections perpendicular to the  $x$ -axis are:

- (a) circular disks with diameters running from the curve  $y = \tan(x)$  to the curve  $y = \sec(x)$   
(b) squares whose bases run from the curve  $y = \tan(x)$  to the curve  $y = \sec(x)$ .

Now, do the following, both for part(a) and part(b) separately:

- Sketch both functions  $y = \tan(x)$  to the curve  $y = \sec(x)$  on the same grid.
- Sketch the solid and its typical cross-section using information provided in the exercise.
- What is the thickness of the slice:  $\Delta x$  or  $\Delta y$  ? Explain your conclusion.
- Find a formula for the area of a typical cross section,  $A(x)$  or  $A(y)$ , depending on your conclusion in bullet 3.
- What are the limits of integration? Justify.
- Complete the integration to find the volume of the solid.