| | | |
|---|---|---|
| **Student:** Arfaz Hossain<br>**Date:** 02/28/22 | **Instructor:** Muhammad Awais<br>**Course:** Math 101 A04 Spring 2022 | **Assignment:** Practice Questions for Sections 6.3 & 7.2 [Not for |

Use the arc length formula to find the length of the line segment $y = 5 - 3x$, $0 \le x \le 5$.

---

Notice that the equation of the curve is defined by expressing y as a function of x.

If $f'$ is continuous on [a, b], then the length (arc length) of the curve $y = f(x)$ from the point $A = (a,f(a))$ to the point $B = (b,f(b))$ is as given below.

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx = \int_a^b \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\,dx$$

Begin by finding the derivative $\frac{dy}{dx}$ of the given function $y = 5 - 3x$.

$$\frac{dy}{dx} = -3$$

The derivative is continuous on [0, 5]. Use the above formula to find the length of the given line segment. Find $\left(\frac{dy}{dx}\right)^2$.

$$\frac{dy}{dx} = -3$$

$$\left(\frac{dy}{dx}\right)^2 = 9 \qquad \text{Square both sides.}$$

Add 1 to both sides of the equation and simplify.

$$1 + \left(\frac{dy}{dx}\right)^2 = 10$$

Substitute the values of a and b, and the expression for $1 + \left(\frac{dy}{dx}\right)^2$, into the formula.

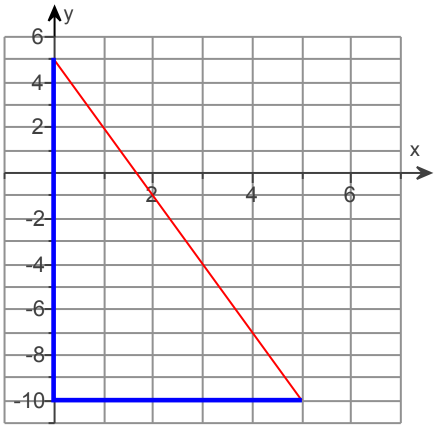
$$L = \int_a^b \sqrt{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}\,dx$$

$$= \int_0^5 \sqrt{10}\,dx$$

Now integrate to find the length of the line segment.

$$\int_0^5 \sqrt{10}\,dx = 5\sqrt{10}$$

Now check the answer by finding the length of the segment as the hypotenuse of a right triangle using the Pythagorean theorem. The graph of the line segment $y = 5 - 3x$, $0 \le x \le 5$ is shown in red to the right.




$$(\text{hypotenuse})^2 = (15)^2 + (5)^2$$
$$(\text{hypotenuse})^2 = 250$$
$$\text{hypotenuse} = 5\sqrt{10}$$

The answer checks. Therefore, the length of the line segment is $5\sqrt{10}$.