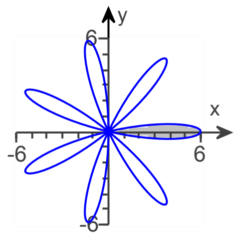
Student: Arfaz Hossain **Date:** 04/20/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** Practice Questions for Sections 11.4 & 11.5 [Not f

Find the area of the region inside one leaf of the seven-leaved rose $r = 6\,\mathbf{cos}\,7\theta$.

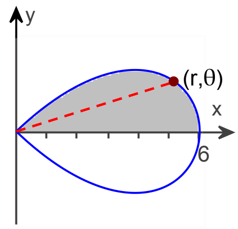


Notice that the graph of $r = 6\,\mathbf{cos}\,7\theta$ has seven leaves and is symmetric about the x-axis. For simplicity, find the area of half of the leaf that lies on the x-axis and then double the result.

Let R be the region bounded by the graph of $r = f(\theta)$, between $\theta = \alpha$ and $\theta = \beta$, where f is continuous and $f(\theta) \geq 0$ on $[\alpha, \beta]$. Then the area of R is given by the formula below.

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

Apply the formula to the shaded region shown in the graph to the right, that is, the upper half of the leaf that lies on the x-axis. This region is generated as the dashed line shown in the graph sweeps through some interval $[\alpha, \beta]$. Note that α and β are also the limits of integration used in the formula.



The curve bounding the half-leaf starts when the curve intersects the x-axis, that is, when $\theta = 0$. Thus, the lower limit of integration is $\alpha = 0$.

The curve bounding the half-leaf ends when the curve intersects the origin, that is, when $r = 6\,\mathbf{cos}\,7\theta = 0$. Since solving for θ results in more than one possible value, use symmetry. Note that the complete rose curve is generated on the interval $0 \leq \theta \leq \pi$. Since there are seven symmetric leaves, divide π by the number of half-leaves in the entire rose. Thus, the upper limit of integration is $\beta = \frac{\pi}{14}$.

Apply the area formula to the upper half of the leaf that lies on the x-axis. Substitute $\alpha = 0$, $\beta = \frac{\pi}{14}$, and $r = 6\,\mathbf{cos}\,7\theta$.

$$\int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta = \int_{0}^{\pi/14} \frac{1}{2}(6\,\mathbf{cos}\,(7\theta))^2 \, d\theta$$

Double this amount to find the area of the entire leaf and simplify.

$$\text{Area of leaf} = 2 \cdot \int_{0}^{\pi/14} \frac{1}{2}(6\,\mathbf{cos}\,(7\theta))^2 \, d\theta$$

$$\text{Area of leaf} = \int_{0}^{\pi/14} 36\,\mathbf{cos}^2(7\theta) \, d\theta$$ Cancel common factors and square $f(\theta)$.

Now complete the integration.

$$\text{Area of leaf} = \int_0^{\pi/14} 36\,\cos^2(7\theta)\,d\theta$$

$$\text{Area of leaf} = 36\int_0^{\pi/14} \left(\frac{1}{2} + \frac{\cos(14\theta)}{2}\right) d\theta$$

Use the fact that $\cos^2 u = \dfrac{1+\cos 2u}{2}$.

$$\text{Area of leaf} = 36\left(\frac{\theta}{2} + \frac{\sin(14\theta)}{28}\right)\Bigg|_0^{\pi/14}$$

Integrate.

Evaluate the result.

$$\text{Area of leaf} = 36\left(\frac{\theta}{2} + \frac{\sin(14\theta)}{28}\right)\Bigg|_0^{\pi/14}$$

$$\text{Area of leaf} = 36\left(\frac{\frac{\pi}{14}}{2} + 0 - \left(\frac{0}{2} + 0\right)\right)$$

$$\text{Area of leaf} = \frac{9\pi}{7}$$

Therefore, the area inside one leaf is $\dfrac{9\pi}{7}$.