Student: Arfaz Hossain **Date:** 03/14/22	**Instructor:** Muhammad Awais **Course:** Math 101 A04 Spring 2022	**Assignment:** HW-7 [Sections 10.7 & 10.8]

The series below converges to **sec** x for $-\pi/2 < x < \pi/2$. Complete parts (a) and (b).

$$\sec x = 1 + \frac{x^2}{2} + \frac{5}{24}x^4 + \frac{61}{720}x^6 + \frac{277}{8064}x^8 + \cdots$$

a. Find the first five terms of a power series for $\ln|\sec x + \tan x|$. For what values of x should the series converge?

Recall that the antiderivative of **sec** x is $\ln|\sec x + \tan x| + C$. Use the term-by-term integration theorem to find the first five terms of a power series for $\ln|\sec x + \tan x|$. This theorem states that if $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ converges for $a - R < x < a + R$ $(R > 0)$, then $\sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1}$ converges for $a - R < x < a + R$ and $\int f(x)\,dx = \sum_{n=0}^{\infty} c_n \frac{(x-a)^{n+1}}{n+1} + C$ for $a - R < x < a + R$.

Integrate the series for **sec** x term by term.

$$\ln|\sec x + \tan x| = \int_0^x \sec t\,dt$$

$$\ln|\sec x + \tan x| = \int_0^x 1 + \frac{t^2}{2} + \frac{5}{24}t^4 + \frac{61}{720}t^6 + \frac{277}{8064}t^8 + \cdots \qquad \text{Substitute.}$$

$$\ln|\sec x + \tan x| = t + \frac{t^3}{6} + \frac{t^5}{24} + \frac{61}{5040}t^7 + \frac{277}{72{,}576}t^9 + \cdots \Big]_0^x \qquad \text{Integrate.}$$

$$\ln|\sec x + \tan x| = x + \frac{x^3}{6} + \frac{x^5}{24} + \frac{61}{5040}x^7 + \frac{277}{72{,}576}x^9 + \cdots \qquad \text{Evaluate.}$$

Thus, the first five terms of a power series for $\ln|\sec x + \tan x|$ are shown below. From the term-by-term integration theorem, this series converges for $-\pi/2 < x < \pi/2$.

$$\ln|\sec x + \tan x| = x + \frac{x^3}{6} + \frac{x^5}{24} + \frac{61}{5040}x^7 + \frac{277}{72{,}576}x^9 + \cdots$$

b. Find the first four terms of a series for **sec** x **tan** x. For what values of x should the series converge?

Recall that the derivative of **sec** x is **sec** x **tan** x. Use the term-by-term differentiation theorem to find the first four terms of a series for **sec** x **tan** x. This theorem states that if $\sum c_n(x-a)^n$ has radius of convergence $R > 0$, it defines a function $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ on the interval $a - R < x < a + R$. This function f has derivatives of all orders inside the interval, and the derivatives are obtained by differentiating the original series term by term. The first order derivative is shown below.

$$f'(x) = \sum_{n=1}^{\infty} nc_n(x-a)^{n-1}$$

This derived series converges at every point of the interval $a - R < x < a + R$.

Differentiate the given series for **sec** x term by term.

$$\begin{aligned}
\textbf{sec}\,x\,\textbf{tan}\,x &= \frac{d}{dx}\,\textbf{sec}\,x \\
&= \frac{d}{dx}\left[1+\frac{x^2}{2}+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\cdots\right] && \text{Substitute.} \\
&= x+\frac{5}{6}x^3+\frac{61}{120}x^5+\frac{277}{1008}x^7+\cdots && \text{Differentiate and simplify.}
\end{aligned}$$

Thus, the first four terms of a series for **sec** x **tan** x are shown below. From the term-by-term differentiation theorem, the series converges at every point of the interval $-\pi/2 < x < \pi/2$.

$$\textbf{sec}\,x\,\textbf{tan}\,x = x+\frac{5}{6}x^3+\frac{61}{120}x^5+\frac{277}{1008}x^7+\cdots$$