

Solution Key.  
Sept 14

**MATH 100, Fall, 2021**  
**Tutorial #1**  
**PreCalculus and some Calculus**

- Q1 (a) Find the domain and range (expressed in interval notation) of the function

$$f(x) = 2 + \sqrt{9 - x^2}.$$

- (b) Make a rough sketch of the function  $y = 2 - (x + 1)^3$ . Label all axis intercepts.
- Q2 Find the exact (i.e. no decimal approximation!) value of  $\cos(\frac{11\pi}{12})$ :
- (a) What are the exact values of  $\cos \frac{\pi}{4}$ ,  $\sin \frac{\pi}{4}$ ,  $\cos \frac{2\pi}{3}$  and  $\sin \frac{2\pi}{3}$ ?
- (b) What is the exact value of  $\cos(\frac{\pi}{4} + \frac{2\pi}{3})$ ? Explain (in words/equations) how this solves the problem first stated.
- Q3 Sketch the functions  $y = 3^x$  and  $y = 7$ . Solve for  $x$ :  $3^x = 7$ . State the exact value ( $x =$ ) and a three-decimal approximation ( $x \approx$ ) of the solution.
- Q4 Let  $f(x) = x^2 - x + 1$ .
- (a) Write out the expression  $\frac{f(x+h) - f(x)}{h}$  and simplify as much as possible.
- (b) For fixed  $x$ , find the limiting value of the expression in part (a) as  $h \rightarrow 0$ . There are at least two ways to do this. Make sure you see both.
- Q5 The position  $s$  of a particle at time  $t$  is given by  $s(t) = 2t^2 - t^3 + t - 7$ .
- (a) What is the average speed of the particle during the time interval  $-2 \leq t \leq -1$ ?
- (b) What is its instantaneous speed when  $t = 1$ ?

(a) Domain: Need  $9 - x^2 \geq 0 \Leftrightarrow -3 \leq x \leq 3$ .

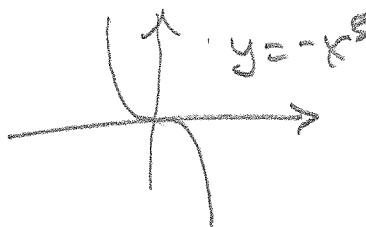
Domain:  $x \in [-3, 3]$  in interval notation.

Range: As  $x$  ranges from  $-3$  to  $3$ ,  $\sqrt{9 - x^2}$  ranges from  $0$  to  $3$  and then back to zero.

$\sqrt{9 - x^2} \in [0, 3]$  so  $f(x)$  ranges from  $2$  to  $5$  inclusive.

Range:  $f(x) \in [2, 5]$ .

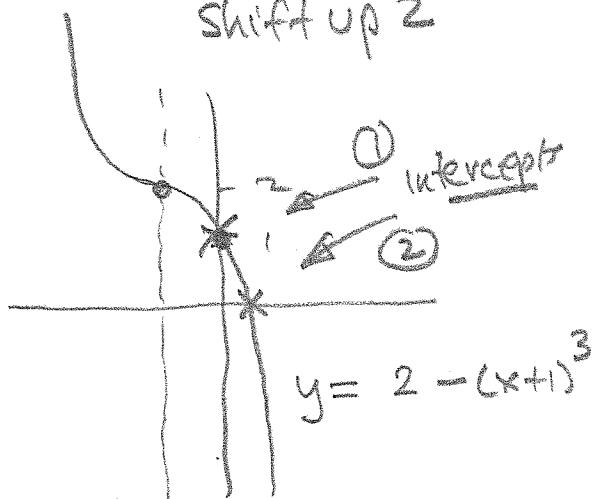
(b) The graph is some sort of cubic.  
First graph  $y = -x^3$ :



For  $y = 2 - (x+1)^3$

shift up 2

shift left 1



Intercepts: ①  $x = 0$   $y = 2 - 1 = 1$

②  $2 - (x+1)^3 = 0$

$\Leftrightarrow (x+1)^3 = 2$

$\Leftrightarrow x = \sqrt[3]{2} - 1 \approx 0.260$

MATH 100, Fall 2021

Tutorial Worksheet

Tutorial Section (T01, T02 etc) \_\_\_\_\_

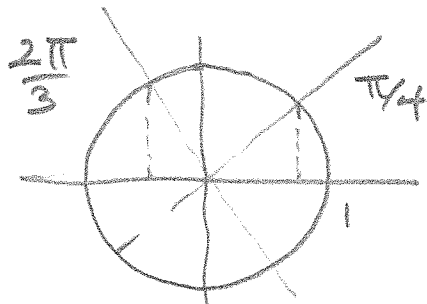
Tutorial Instructor Name: \_\_\_\_\_

Question Number Attempted (Q1, Q2, etc) Q2

Your Name: KEY

Your Student Number: V00 \_\_\_\_\_

Today's Date: \_\_\_\_\_



$$\cos \pi/4 = \frac{\sqrt{2}}{2} ; \sin \pi/4 = \frac{\sqrt{2}}{2}$$

$$\cos \frac{2\pi}{3} = -\frac{1}{2} ; \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$$

Sum Formula.

$$\begin{aligned} \cos \left( \frac{\pi}{4} + \frac{2\pi}{3} \right) &= \cos \pi/4 \cos \frac{2\pi}{3} - \sin \pi/4 \cdot \sin \frac{2\pi}{3} \\ &= \frac{\sqrt{2}}{2} \cdot \left( -\frac{1}{2} \right) - \frac{\sqrt{2}}{2} \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} ( -\sqrt{2} - \sqrt{2}\sqrt{3} ) = \underbrace{-\frac{\sqrt{2}}{4} (1 + \sqrt{3})}_{= -\frac{1 + \sqrt{3}}{2\sqrt{2}}} \end{aligned}$$

Now:  $\cos \left( \frac{11\pi}{12} \right) = \cos \left( \underbrace{\frac{\pi}{4} + \frac{2\pi}{3}}_{= \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}} \right)$

Done

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Tutorial Worksheet

Tutorial Section (T01, T02 etc) \_\_\_\_\_

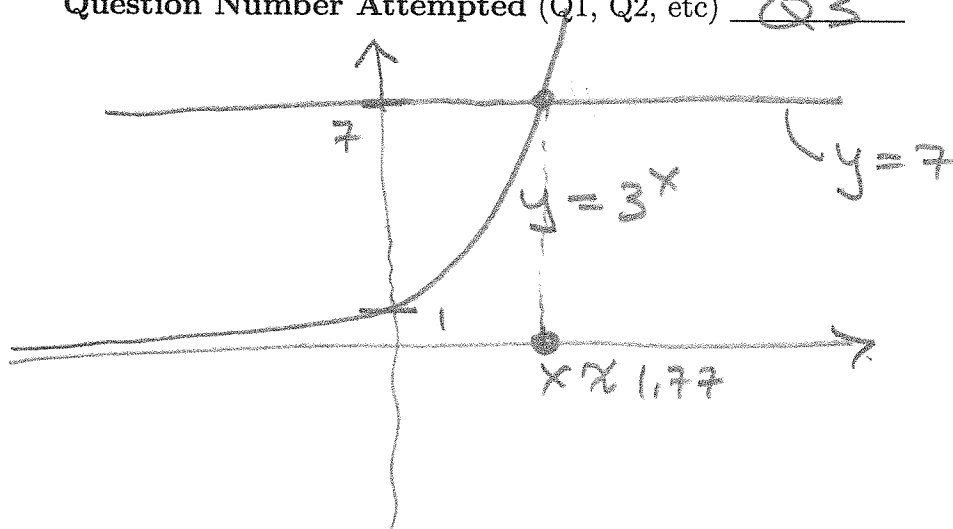
Tutorial Instructor Name: \_\_\_\_\_

Your Name: Key.

Your Student Number: V00 \_\_\_\_\_

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Question Number Attempted (Q1, Q2, etc) Q3



Solve for  $x$  :  $3^x = 7$

$$x \ln 3 = \ln 7$$

$$x = \frac{\ln 7}{\ln 3} \approx 1.771$$

MATH 100, Fall 2021

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Tutorial Section (T01, T02 etc) \_\_\_\_\_

Tutorial Instructor Name: \_\_\_\_\_

Question Number Attempted (Q1, Q2, etc) Q4

Your Name: Key.

Your Student Number: V00 \_\_\_\_\_

Today's Date: \_\_\_\_\_

$$\begin{aligned} (a) \quad & \frac{(x+h)^2 - (x+h) + 1 - (x^2 - x + 1)}{h} (*) \\ &= \frac{1}{h} [\cancel{x^2} + 2xh + h^2 - \cancel{x} - h + 1 - \cancel{x^2} + \cancel{x} - 1] \\ &= \frac{2xh + h^2 - h}{h} = \underline{\underline{2x + h - 1}} \end{aligned}$$

$$(b) \text{ Fix } x, \quad h \rightarrow 0 \text{ expr} \rightarrow \underline{\underline{2x - 1}} \quad \checkmark$$

Also  $(*) =$  derivative quotient for

$$f(x) = x^2 - x + 1$$

$$\lim_{h \rightarrow 0} * = f'(x) = \underline{\underline{2x - 1}} \quad \checkmark$$

MATH 100, Fall 2021

Tutorial Worksheet

Tutorial Section (T01, T02 etc) \_\_\_\_\_

Tutorial Instructor Name: \_\_\_\_\_

Question Number Attempted (Q1, Q2, etc) Q5

Your Name: Key.

Your Student Number: V00 \_\_\_\_\_

Today's Date: \_\_\_\_\_

$$\text{Average velocity: } \frac{S(-1) - S(-2)}{-1 - (-2)} *$$

$$S(-1) = 2 - (-1) + (-1) - 7 = -5$$

$$S(-2) = 8 - (-8) + (-2) - 7 = 7$$

$$* = \frac{-5 - 7}{1} = \boxed{-12}$$

$$\text{average speed} = |-12| = \boxed{12} \checkmark$$

$$\begin{aligned} \text{Instantaneous speed} &= \left| S'(t) \right|_{t=1} \\ &= \left| 4t - 3t^2 + 1 \right|_{t=1} \\ &= \left| 4 - 3 + 1 \right| \\ &= \boxed{2} \checkmark \end{aligned}$$