MATHEMATICS 101 Midterm #1 September 27, 2012

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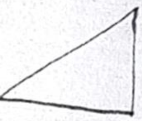
Your section no.: A08

Total marks: 30. Please be sure to show sufficient work to justify your answers. As you have only 50 minutes to do this test, it would be a good idea to read the paper through once quickly and identify which questions you want to do first. You can do rough work on the backs of the pages; it does not have to be copied onto the front of the page. **AS STATED CLEARLY IN THE COURSE OUTLINE, ON ALL EXAMINATIONS THE ONLY ACCEPTABLE CALCULATOR IS THE SHARP EL-510R. DO NOT BRING ANY OTHER CALCULATOR TO ANY EXAMINATION. RECALL THAT YOUR CALCULATOR MUST BE IN RADIAN MODE FOR CALCULUS PROBLEMS.**

1. Find the following derivatives and evaluate each of them at $x = 0.2$. Use your calculator to express the result as a number which is accurate to at least three decimal places.

[3] (a) $\frac{d}{dx} \arctan \frac{x}{2}$

$\int \frac{1}{x^2+1} = \tan^{-1}(x)$? $\therefore \frac{d}{dx} \tan^{-1}\left(\frac{1}{2}x\right)$

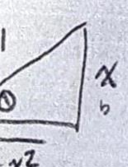


$$= \frac{1}{\frac{1}{4}x^2+1} \cdot \frac{1}{2} = \boxed{\left(\frac{1}{2}x^2+2\right)^{-1}} \ (?)$$

[3] (b) $\frac{d}{dx} \cos(\arcsin x)$. Hint: you can use the chain rule, or you can simplify the function first and then take its derivative. SOHCAHTOA $a^2+b^2=c^2$

$\sin\theta = \frac{O}{H}$ $\sin^{-1}(\theta) = x$

$\cos\theta = \sqrt{1-x^2}$



$$\frac{d}{dx}\sqrt{1-x^2} = \frac{d}{dx}(1-x^2)^{1/2} = \frac{1}{2}(1-x^2)^{-1/2} \cdot -2x$$

$$\boxed{= -x(1-x^2)^{-1/2}}$$

$$\text{or} = \frac{-x}{\sqrt{1-x^2}}$$

2. [5] Find the arc length of the curve $y = \frac{4}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 6$. Use your calculator to express the result as a number which is accurate to at least three decimal places.

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx \qquad y = \frac{4}{3}x^{3/2} \to y' = \frac{3}{2}\cdot\frac{4}{3}x^{1/2} = 2x^{1/2}$$

$$(2\sqrt{x})^2 = 4x$$

$$L = \int_0^6 \sqrt{1 + 4x}\, dx \to \int_0^6 (1+4x)^{1/2} = \frac{2}{12}(1+4x)^{3/2} = \frac{1}{6}(1+4x)^{3/2}$$

$$L = \left[\frac{1}{6}(1+4x)^{3/2}\right]_0^6 = \left[\frac{125}{6}\right] - \left[\frac{1}{6}\right] = \frac{124}{6} = \boxed{20.66667 \text{ units}}$$

3. [5] Find the exact volume of the solid generated by revolving the region bounded by the curves $y = 12 - x^2$, $y = 3$, about the x-axis. Now use your calculator to express the exact volume as a number which is accurate to at least three decimal places.

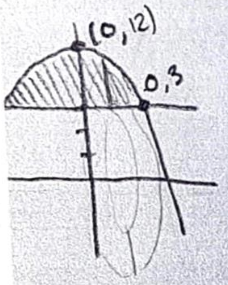
$$V = \pi \int_a^b [f(x)]^2\, dx \to V = \pi r^2 h$$

$$(9 - x^2)(9 - x^2)$$

$$81 - 18x^2 + x^4$$

$$y = -x^2 + 12$$

$$V = \pi \int_a^b [f(x) - 3]^2\, dx$$



$$3 = 12 - x^2$$
$$-9 = -x^2$$
$$9 = x^2$$
$$x = \pm 3$$

$$V = 2\pi \int_0^3 (12 - x^2 - 3)^2\, dx$$

$$= 2\pi \int_0^3 (9 - x^2)^2\, dx$$

$$= 2\pi \int_0^3 x^4 - 18x^2 + 81\, dx$$

$$= 2\pi \left[\frac{1}{5}x^5 - \frac{18}{3}x^3 + 81x\right]_0^3$$

$$= 2\pi\left(\frac{243}{5} - \frac{486}{3} + 243\right)$$

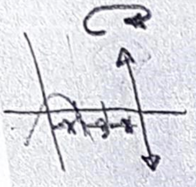
$$\boxed{\approx 814.3008 \text{ units}^3}$$

4. [5] Find the exact volume of the solid generated by revolving the region bounded by the curves $y = x - x^3$ $(0 \le x \le 1)$ and $y = 0$ about the line $x = 3$. Now use your calculator to express the exact volume as a number which is accurate to at least three decimal places.

$y = x - x^3$
$= x(1-x^2)$

$V = \int_a^b (2\pi x) f(x)\,dx \rightarrow 2\pi \int_0^1 (3-x)(x-x^3)\,dx$

$3x - 3x^2 - x^2 + x^4$
$= x^4 - 4x^2 + 3x$



$V = 2\pi \int_0^1 x^4 - 4x^2 + 3x$

$V = 2\pi \left[\frac{1}{5}x^5 - \frac{4}{3}x^3 + \frac{3}{2}x^2\right]_0^1 = 2\pi\left(\frac{1}{5} - \frac{4}{3} + \frac{3}{2}\right)$

$\boxed{\approx 2.30383 \text{ units}^3}$

5. [4] Find the definite integral

$$\int_0^1 \frac{1}{x^2 + 2x + 1}\,dx$$

4

$x^2 + 2x + 1$

$(x+1)(x+1)$
$x^2 + 2x + 1$

$(x+1)^2$

$= \int_0^1 (x+1)^{-2} = \left[-(x+1)^{-1}\right]_0^1$ or $\left.\frac{-1}{x+1}\right|_0^1$

$= \left[\frac{-1}{2}\right] - [-1] = \boxed{\frac{1}{2}}$

sin csc
cos sec
tan cot

6. [5] Evaluate the definite integral

5

$$\int_{-2}^{2} \frac{dx}{\sqrt{x^2+1}}$$

Use your calculator to express the result as a number which is accurate to at least three decimal places. Hint: let $x = \sinh u$. You will also need to use the "2nd F arc hyp" buttons on your calculator.

$\int_{-2}^{2} \frac{1}{\sqrt{x^2+1}}\,dx \rightarrow$ let $x = \sinh(u)$; $dx = \cosh(u)\,du$

$u = \sinh^{-1}(x)$

$\int_{-2}^{2} \frac{\cosh(u)\,du}{\sqrt{\sinh^2 u + 1}}$; $\sinh^2 u + 1 = \cosh^2 u$; $\sqrt{\cosh^2 u} = \cosh x$

$= \int_{-2}^{2} \frac{\cosh(u)\,du}{\cosh(u)} = \int_{-2}^{2} 1\,du = \left[u\right]_{-2}^{2} = \left[\sinh^{-1}(x)\right]_{-2}^{2}$

$= (1.443635) - (-1.443635) \approx \boxed{2.88727}$

$3.63 - (-3.63)$