Student: Arfaz Hossain **Date:** 11/07/21	**Instructor:** UVIC Math **Course:** MATH 100 (A01, A02, A03) Fall 2021 **Assignment:** Assignment 7

Answer the following questions about the function whose derivative is $f'(x) = (x-3)e^{-2x}$.
a. What are the critical points of f?
b. On what open intervals is f increasing or decreasing?
c. At what points, if any, does f assume local maximum and minimum values?

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a. An interior point of the domain of a function f where f' is zero or undefined is a critical point of f.

Set $f'(x) = 0$ and solve.

$$\begin{aligned} f'(x) &= 0 \\ (x-3)e^{-2x} &= 0 \\ x &= 3 \end{aligned}$$

Notice that both $x-3$ and e^{-2x} are defined for all real numbers, so there are no values of x where the derivative $f'(x)$ is undefined.

Thus, the critical point of f is $x = 3$.

b. To determine on what open intervals f is increasing or decreasing, use the critical point to subdivide the domain into nonoverlapping open intervals in which f' is either positive or negative.

The open intervals are $(-\infty,3)$ and $(3,\infty)$.

Suppose that f is continuous on [a,b] and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

Determine the sign of f' by evaluating f' at a convenient point in each interval. For the interval $(-\infty,3)$, evaluate $f'(-1)$, rounding to four decimal places.

$$\begin{aligned} f'(-1) &= (-1-3)e^{-2(-1)} \\ &\approx -29.5562 \end{aligned}$$

So, the function is decreasing on the open interval $(-\infty,3)$.

Now, for the interval $(3,\infty)$, evaluate $f'(4)$, rounding to four decimal places.

$$\begin{aligned} f'(4) &= (4-3)e^{-2(4)} \\ &\approx 0.0003 \end{aligned}$$

So, the function is increasing on the open interval $(3,\infty)$.

Thus, the function f is decreasing on the open interval $(-\infty,3)$, and it is increasing on the open interval $(3,\infty)$.

c. To determine the location of local maxima and minima, suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if f' changes from negative to positive at c, then f has a local minimum at c. If f' changes from positive to negative at c, then f has a local maximum at c.

Recall that the sign of f' changes from negative to positive at $x = 3$. So f has a local minimum at $x = 3$.

Therefore, since there are no other critical points, the function f has a local minimum at $x = 3$ and no local maximum.