Math 122 In-Class Assignment 7 - Solutions

1. You are in the foreign country of Mathlandia, which uses a \$3 coin and a \$7 coin in its currency system. Using PMI, show that for all $n \geq 12$ you can pay exact change for an item that costs \$n, using only \$3 coins and \$7 coins.

Solution:
Here our statement is $S(n)$: the value n can be written as a sum of 3s and/or 7s.

Basis:

- $n = 12$: We can write $12 = 3 + 3 + 3 + 3$ so 12 can be written as the sum of 3s and/or 7s.
- $n = 13$: We can write $13 = 3 + 3 + 7$ so 13 can be written as the sum of 3s and/or 7s.
- $n = 14$: We can write $14 = 7 + 7$ so 14 can be written as the sum of 3s and/or 7s.

Induction Hypothesis: Assume each of $12, 13, 14, \ldots, k$ can be written as a sum of 3s and/or 7s. We know $k \geq 14$.

Induction Step: Look at $n = k + 1$. We need to show that $k + 1$ can be written as a sum of 3s and/or 7s. Notice that $k + 1 = (k + 1 - 3) + 3 = (k - 2) + 3$. Therefore if $k - 2$ can be written as a sum of 3s and/or 7s, adding 3 more on will give $k + 1$ as a sum of 3s and/or 7s. Notice that we have $k \geq 14$ in our induction hypothesis, so $k - 2 \geq 12$ (and this is still included in our induction hypothesis). By the induction hypothesis we can say that $k - 2$ can be written as the sum of 3s and/or 7s. Taking this sum and adding on and extra 3 gives $k - 2 + 3 = k + 1$ as a sum of 3s and/or 7s.

Conclusion: By the (strong form of the) Principal of Mathematical Induction, we can say that n can be written as the sum of 3s and/or 7s for all $n \geq 12$.