ECON 180

Introduction to Principles of Microeconomics and Financial Project Evaluation

Lecture 34: The Critical Path

November 30, 2021

Version 1

Required Reading and Viewing

- Emmanuel, J. (2017, July 17). Project Scheduling – PERT/CPM | Finding the Critical Path [Video File]. https://youtu.be/-TDh-5n90vk
- Levy, F.K., Thompson, G.L. & Wiest, J.D. (1961). The ABCs of the Critical Path Method, *Harvard Business Review*, September 1963. http://ezproxy.library.uvic.ca/login?url=http://search.ebscohost.com/login.aspx?direct=true&db=bth&AN=6770388&site=ehost-live&scope=site
  - **Easy to understand overview of critical paths and crashing. (Note: Click the 'PDF Full Text' link on the left of the page for the full article.)**
  - Alternate (non-Uvic) link: https://hbr.org/1963/09/the-abcs-of-the-critical-path-method

Recommended Reading

- *Engineering Economics* Chapter 11, Sections 11.3.3.1 and 11.3.3.2
- …especially Example 11.5 starting on p. 403

Case Studies: Critical Path I

- Batson, R. G. (1987). Critical path acceleration and simulation in aircraft technology planning. *IEEE Transactions on Engineering Management*, EM-34(4), 244 – 251. Retrievedn from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6499014
- **Non-technical report of an early application of CPM by Lockheed Martin.**
- Tang, P. (2011). Using Schedule Simulation Approaches to Reduce Greenhouse Gas Emissions in Highway Construction Project. *Proceedings of the 2011 Winter Simulation Conference (WSC)*, 805-815. Retrieved from https://ieeexplore-ieee-org.ezproxy.library.uvic.ca/document/6147807
- **Highway Construction Example**
- Kelley, J. E., Jr. (1961). Critical-Path Planning and Scheduling: Mathematical Basis. *Operations Research*, 9(3), 296-320. Retrieved from https://www-jstor-org.ezproxy.library.uvic.ca/stable/167563
- **Activity on Arc Crashing Example. Since we use Activity on Node notation, this article may be challenging to parse.**

Case studies: Critical Path II

- Cho, S. M. et al. (2021). Design of Very High-Speed Pipeline FIR Filter Through Precise Critical Path Analysis. *IEEE Access, 9.* https://ieeexplore.ieee.org/abstract/document/9361650
- Doostali, S., Babamir, S. M. & Eini, Maryam. (2021). CP-PGWO: multi-objective workflow scheduling for cloud computing using critical path. *Cluster Computing, 24,* 3607-3627. https://doi.org/10.1007/s10586-021-03351-y
- Kenney, C. (2021). Agile Improvements to Critical Path Method. *Acquisition Management,* SYM-AM-21-081. https://dair.nps.edu/handle/123456789/4388

Learning Objectives

- To be able to create and interpret an AoN or CPM diagram in the context of project planning.
- To be able to find the critical path of a project.
- To be able to use a CPM diagram to determine which activities to crash, and by how much, in order to shorten a project's completion time.
- To be able to calculate the cost of crashing a project.

Relevant Solved Problems

- From *Engineering Economics,* 6th ed.
- Critical Path Calculation: Example 11.4, Example 11.5 (first part), Review Problem 11.4, 11.17, 11.18, 11.19, 11.20,11.24 (b) and (c), 11.31, 11.36 (c), 11.38 (b), 11.39, 11.40 (c)
- Crashing Activities: 11.21, 11.22, 11.23, 11.24 (f), 11.32, 11.33, 11.41

ESSENTIALS (14 slides)

Critical Path Method (CPM)

- Uses directed node diagrams to illustrate the interdependence between activities.
- Two styles of node diagrams:
  - Activity on arc: nodes mark start and end time of activities, activities are arrows between nodes. Less common (but be aware it exists).
  - **Activity on node**: nodes represent activities, arrows between nodes indicate dependencies. More common.
- The critical path is the set of activities that must be completed exactly as planned to keep the project on schedule.
- There is slack if and only if an activity is NOT on the critical path.

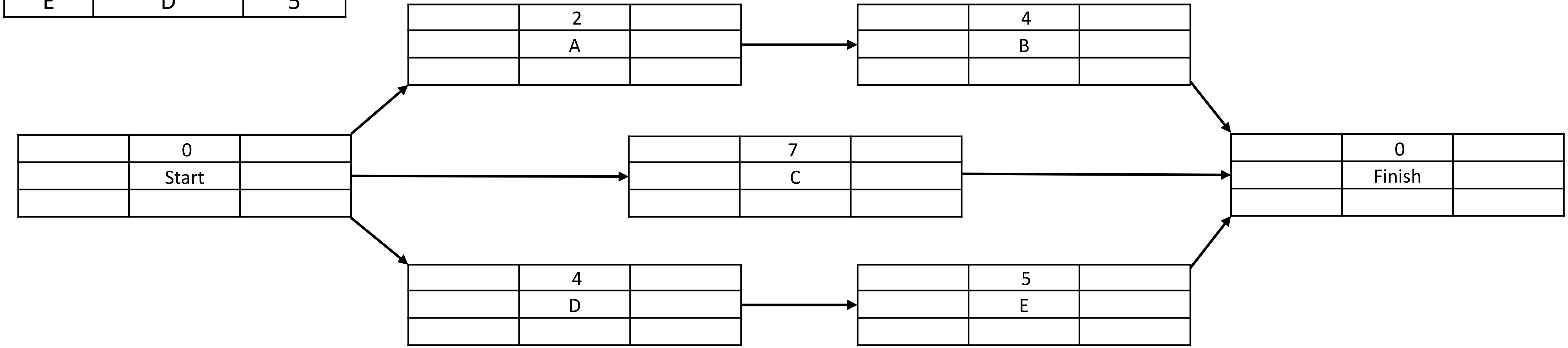
How to find the critical path

- Step 1: Find the earliest possible completion time for the project.
  - Determine the earliest possible start (ES) and finish (EF) times for each activity.
  - EF = ES + T, where T is the shortest length of time the activity can take.
  - Tip: The ES/EF will be the Start/Finish times in your Gantt chart!
- Step 2: Taking the completion time from Step 1 as given…
  - Work backwards and find the latest time each activity can start and finish (LS and LF).
  - LF = MIN(LS of successor activities)
- Step 3: Calculate (LF – EF) for each activity. This is the slack time of that activity. An activity is on the critical path if and only if this value is zero.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Drawing the AoN Diagram

ES	T	EF
	ID	
LS	Slack	LF

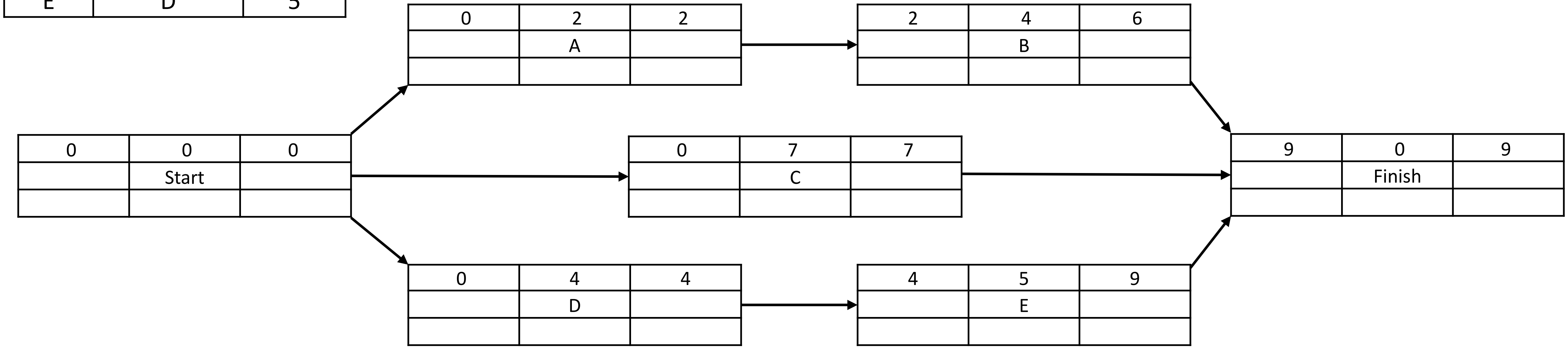


- For the network diagram & subsequent analysis, I’ll be using grid notation inspired by practiging engineers.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Forward Pass

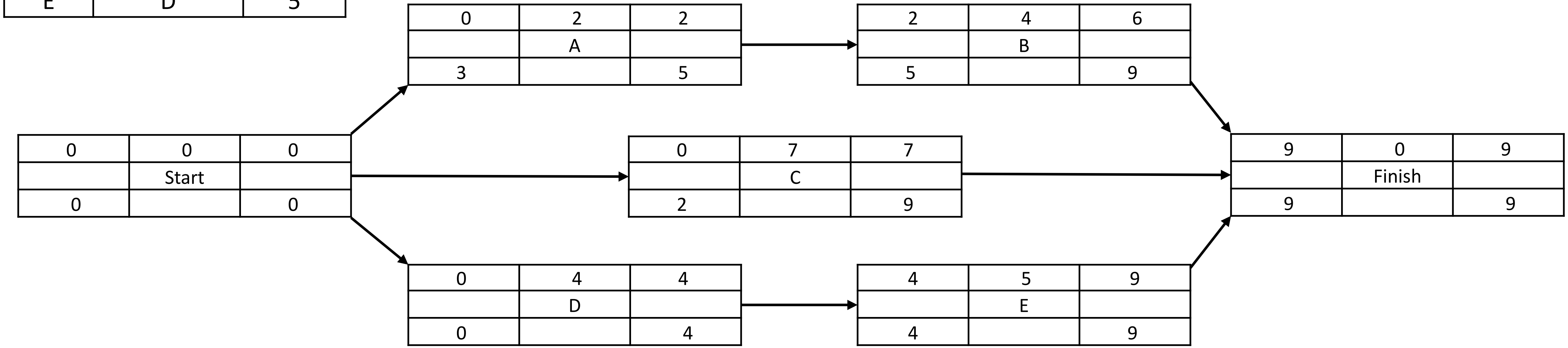
ES	T	EF
	ID	
LS	Slack	LF



Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Backward Pass

ES	T	EF
	ID	
LS	Slack	LF



- For the backward pass, we start by assuming the project finishes at its earliest possible finish time (9 in this case).
- We use that as the Finish milestone's latest possible finish time and work backward, seeing how late each activity can start and still have the project finish on time.

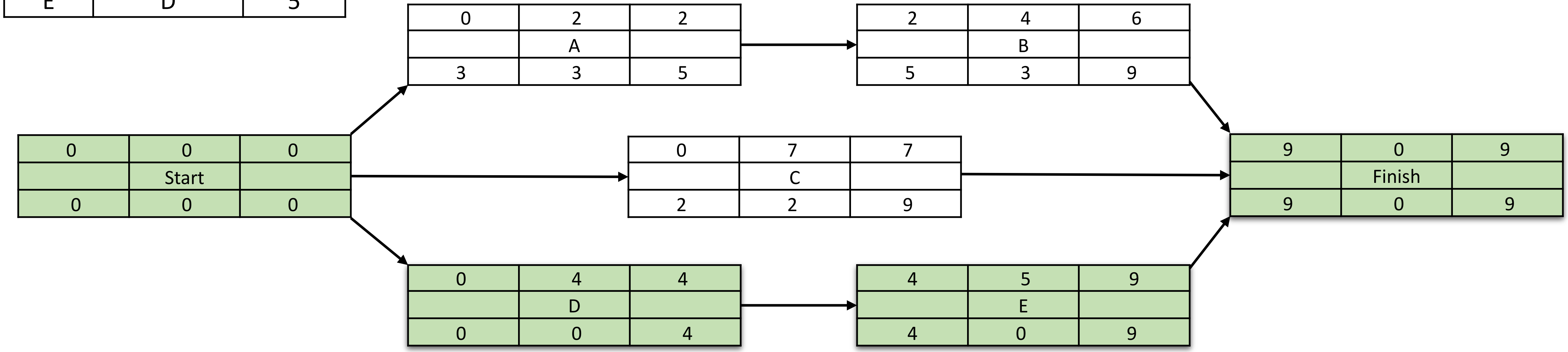
Slack Time & the Critical Path

- The value (LF – EF) calculated in Step 3 is the slack mentioned earlier.
- For project planners, it's nice to know what activities have a built-in buffer…
- Meanwhile, knowing what activities DON'T have a buffer (i.e. are on the critical path) is also very useful.
- These are the activities that drive project completion time. If a project must be completed ahead of schedule (by paying overtime, etc.) these are the activities to focus resources on.

Activity	Predecessors	Duration
A	-	2
B	A	4
C	-	7
D	-	4
E	D	5

Slack & Critical Path

ES	T	EF
	ID	
LS	Slack	LF



- Slack is the difference between early and late start & finish times
- i.e. Slack = (LS-ES) or (LF-EF) [They're off by T, so both give the same result.]
- Activities are on the critical path if and only if their slack is zero. I've shaded these green for convenience.
- The Start and Finish milestones will always be on the critical path.

Implications for our motivating example

- Last lecture, we saw that in the early 2000s, at least one large software company was responding to scheduling pressures by having *everyone* work overtime for long periods…
- CPM shows this is probably not required, or a good idea.

- Suppose we DO want to reduce project completion time in a rigorous, efficient manner.
- How do we go about it?

Project Crash

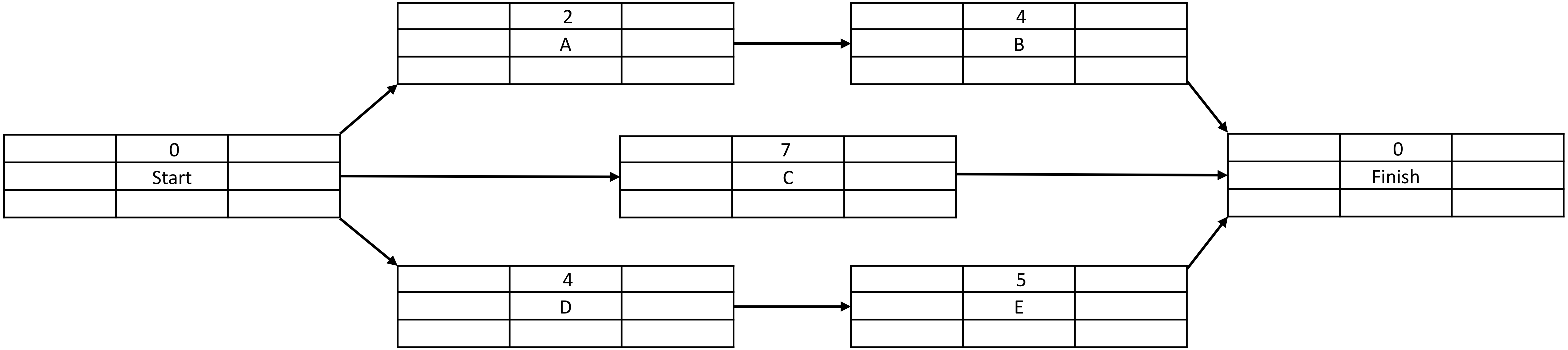
- GIVEN a critical path, a planner can make an informed tradeoff between reduced completion time and extra cost.
- Implicit: best-practice WBS and accurate cost/time estimates.
- Need Crashed and Uncrashed duration and cost data.
- To crash:
  - Identify which critical path activities are the least expensive to hasten.
  - Shorten those activities by the max amount or **until another critical path is created**.
  - Repeat until done or unable to continue.
- We’ll be looking at that in detail next lecture.
- For today, we’ll just look at a bird’s-eye view.

A closer look at multiple critical paths.

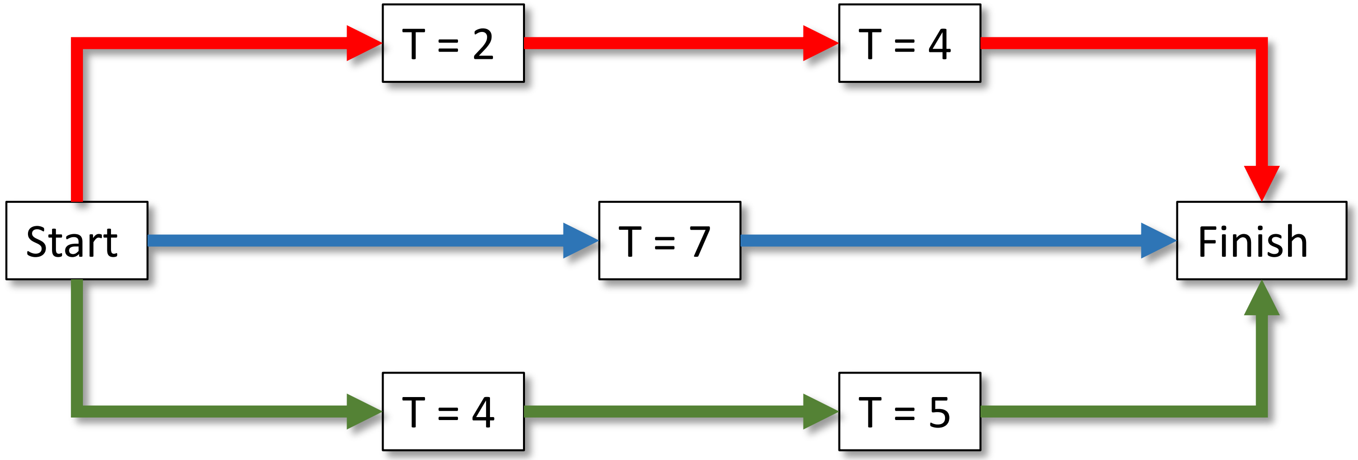
- Students (and project planners!) are often confused about what causes a new critical path to form.
- The following very simple example should clear things up.
- In this example, we start with one critical path, but gradually transform the project into one in with three critical paths.
- Tip: A critical path is a path from the start to the finish of the project. If there's a gap in the path, or if your start and finish nodes aren't critical, something's wrong! Check your work.

Simplifying things a bit

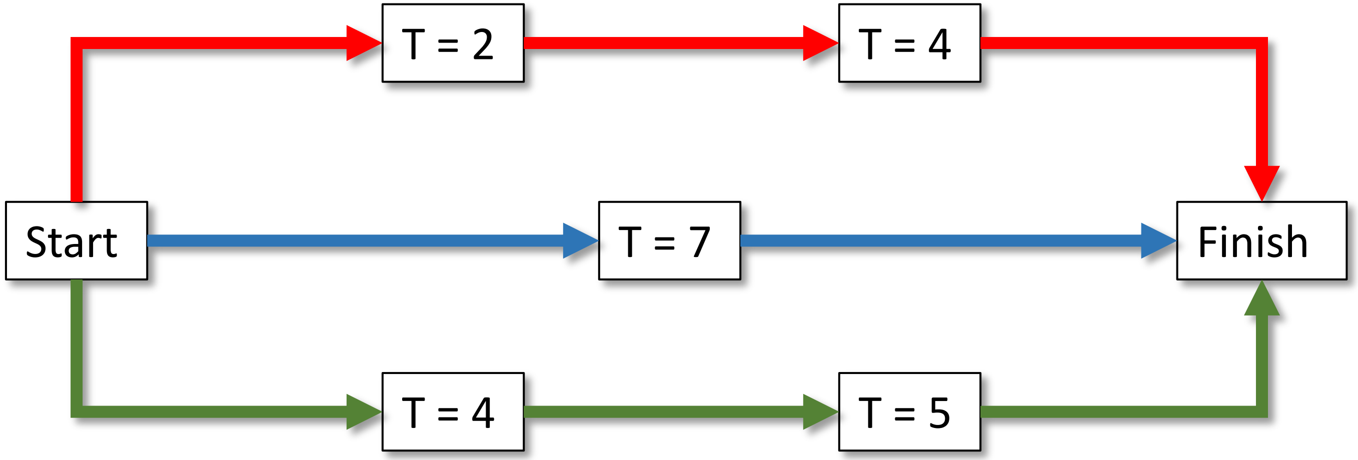
From our original AoN diagram…



To a schematic version

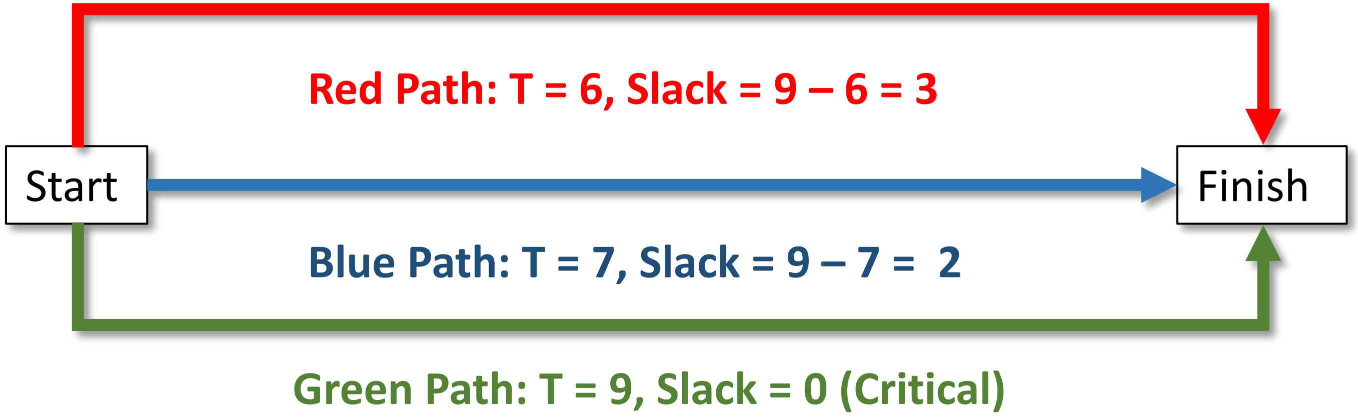


The beginning: three paths, one critical.



The **Green Path** takes 9 time units to finish.

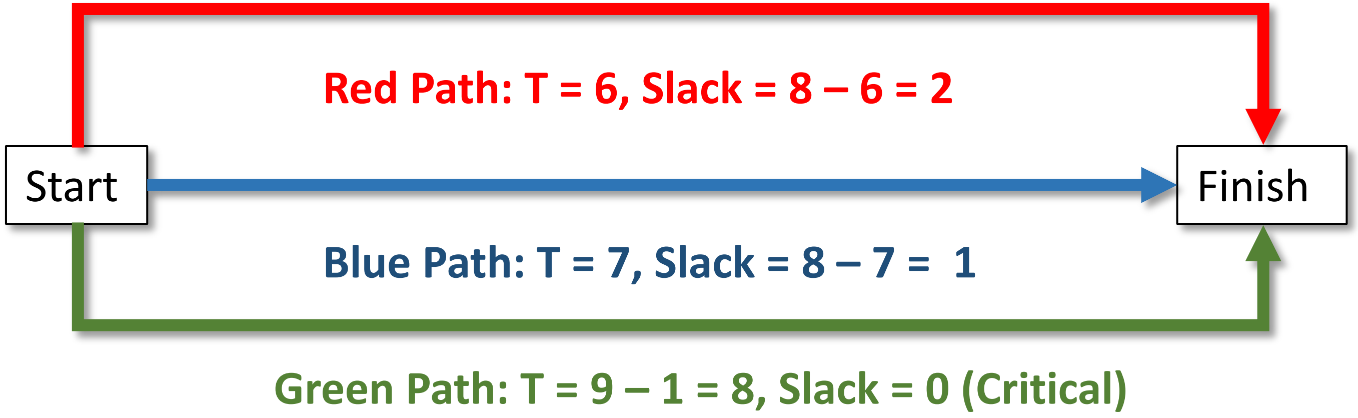
Start → Finish
T = 9



Since the other paths are shorter, it's the **Green Path** that determines completion time (i.e. is critical)

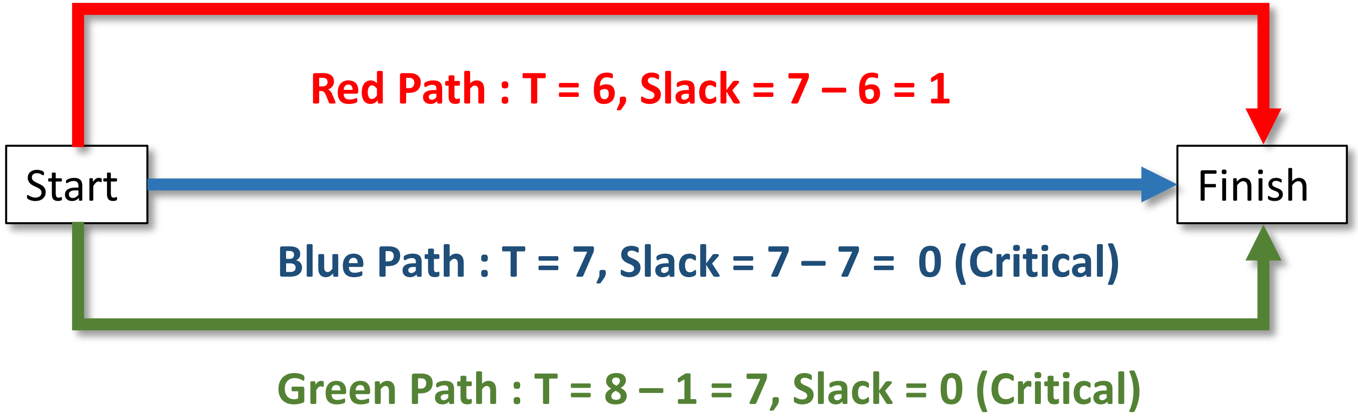
Crashing first by 1, then by 2 time periods.

Start → Finish
T = 8



We need to Crash the **Green Path** to see a reduction in completion time.

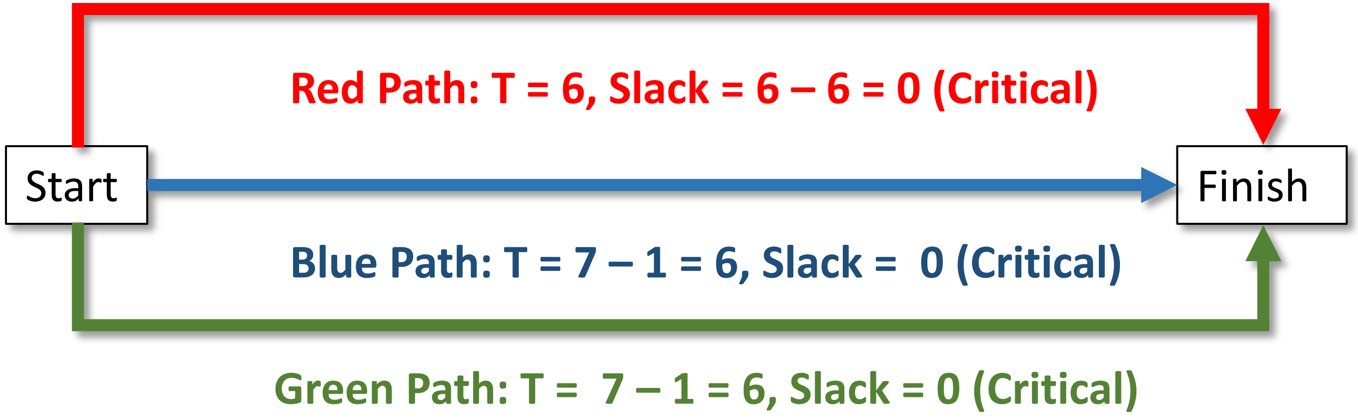
Start → Finish
T = 7



If we crash by *two* time periods, the **Blue Path** becomes critical because its slack is used up.

Crashing first by 3, then by 4 time periods.

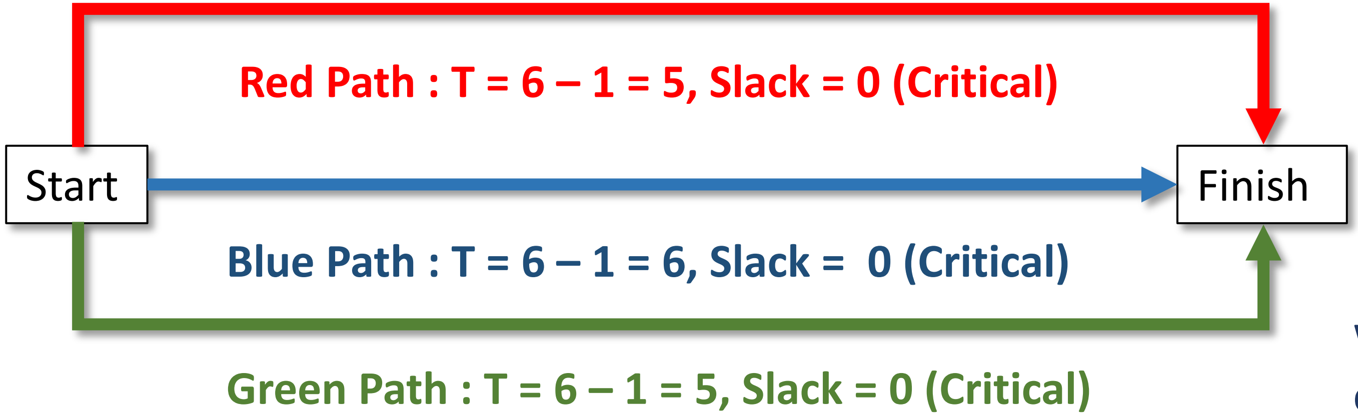
Start → Finish
T = 6



Crashing one path isn't enough – we need to crash *both* the **Green** and **Blue** paths to see a reduction in Finish time.

The end result is that the **Red Path**'s slack is used up, and also becomes critical.

Start → Finish
T = 5



To crash by *four* time periods, we need to crash *all three* paths by 1 time unit.

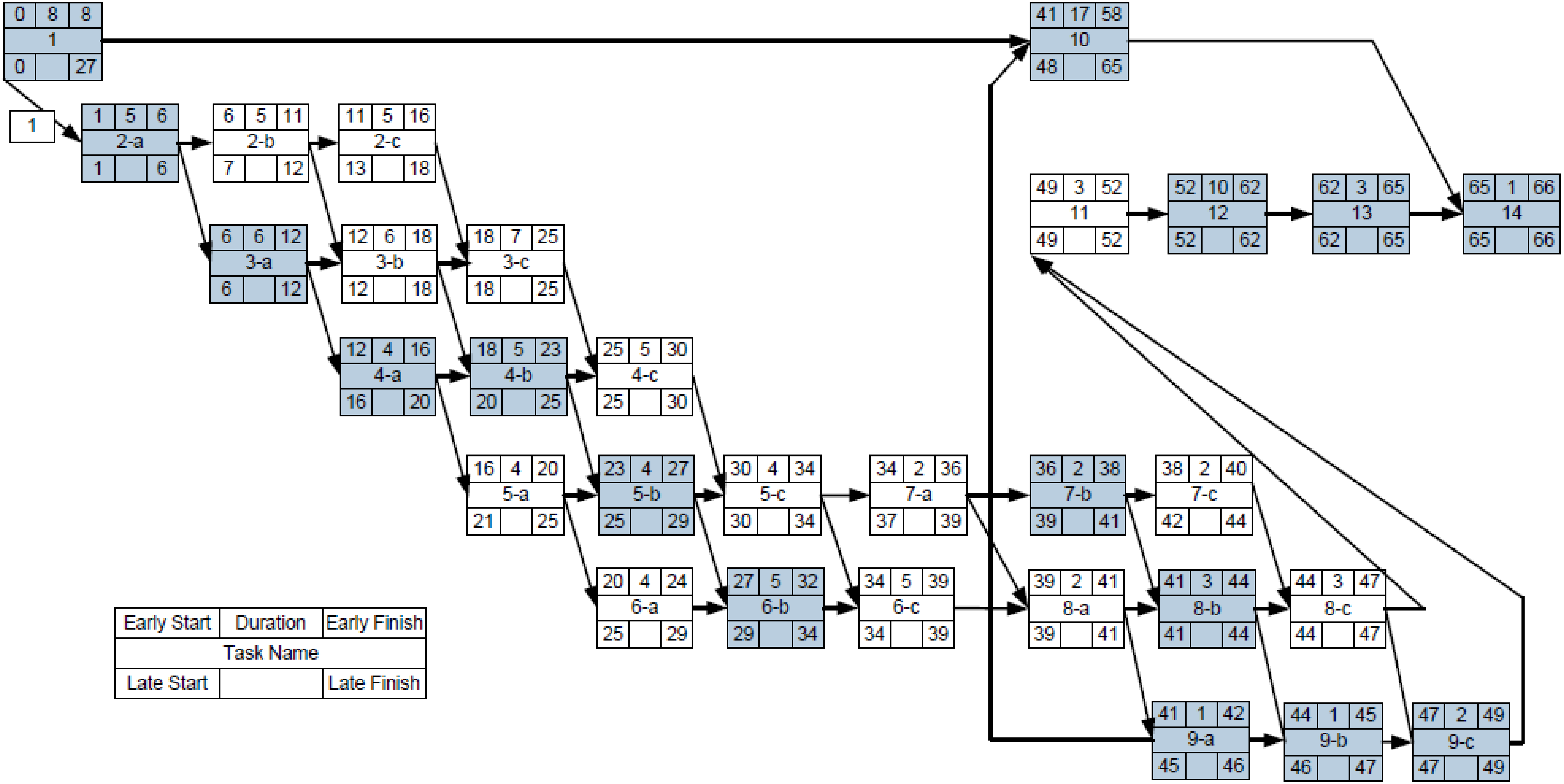
Without ever explicitly mentioning costs, we've obtained intuition into why schedule-shortening becomes very expensive very quickly!

AFTER HOURS

- Published example (2 slides)

Just for fun… What does this sort of thing look like in real life?

Activity ID	Activity Description	Duration	Precedence	Constraints
1	Strip Topsoil	8 days	Begin	
2	Remove Concrete Pavement	15 days	1	1 day after the beginning of Activity 1 (soft)
3	Grade Subbase	19 days	2	2 days after the beginning of Activity 2
4	Install Drainage	14 days	2,3	6 days after the beginning of Activity 3
5	Place OGDC Mainline	12 days	2,3,4	2 days after the beginning of Activity 4
6	Pave E.B. Mainline	14 days	5	1 day after the beginning of Activity 5
7	Place OGDC Ramps and Gaps	6 days	4,5,6	7 days after the beginning of Activity 6(soft); after the completion of Activity 5
8	Pave E.B. Gaps and Ramps	8 days	7	0 days after the beginning of Activity 7
9	Place Gravel Shoulder	4 days	8	3 days after the beginning of Activity 8
10	Slope Grading and Restoration E.B.	17 days	9	1 day after the beginning of Activity 9(soft)
11	Stripe to Open Pavement E.B.	3 days	9	0 days after the completion of Activity 9
12	Relocate Barrier Wall	10 days	11	0 days after the completion of Activity 11
13	Re-stripe W.B.	3 days	12	0 days after the completion of Activity 12
14	All Lanes Open	1 day	12,13	0 days after the completion of Activity 1, Activity 13 and Activity 10


0 8 8
1
0 27
1
1 5 6
2-a
1 6
6 5 11
2-b
7 12
11 5 16
2-c
13 18
6 6 12
3-a
6 12
12 6 18
3-b
12 18
18 7 25
3-c
18 25
12 4 16
4-a
16 20
18 5 23
4-b
20 25
25 5 30
4-c
25 30
16 4 20
5-a
21 25
23 4 27
5-b
25 29
30 4 34
5-c
30 34
20 4 24
6-a
25 29
27 5 32
6-b
29 34
34 5 39
6-c
34 39
34 2 36
7-a
37 39
36 2 38
7-b
39 41
38 2 40
7-c
42 44
39 2 41
8-a
39 41
41 3 44
8-b
41 44
44 3 47
8-c
44 47
41 1 42
9-a
45 46
44 1 45
9-b
46 47
47 2 49
9-c
47 49
41 17 58
10
48 65
49 3 52
11
49 52
52 10 62
12
52 62
62 3 65
13
62 65
65 1 66
14
65 66
Early Start
Duration
Early Finish
Task Name
Late Start
Late Finish