Student: Arfaz Hossain	**Instructor:** Muhammad Awais	**Assignment:** Practice Questions for Sections 11.4 & 11.5 [Not f
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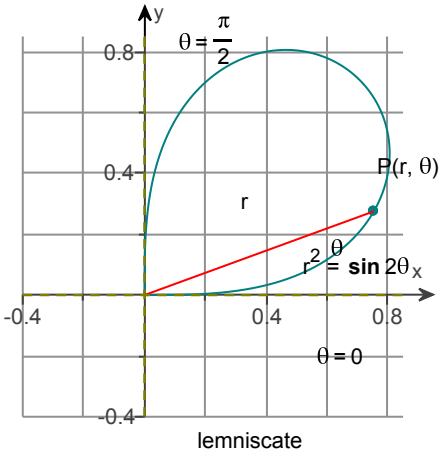
Find the area inside one loop of the lemniscate $r^2 = \sin 2\theta$.

Sketch the loop to determine the limits of integration. The point P(r, θ) traces the curve once, counterclockwise as θ runs from α to β.



The area inside one loop of the lemniscate is given by $A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 d\theta$.

The loops starts at $\theta = 0$ and closes at $\theta = \frac{\pi}{2}$.

Substituting for α and β, the area inside one loop of the lemniscate is given by $A = \int_{0}^{\pi/2} \frac{1}{2} r^2 d\theta$.

Eliminate the variable, r, to evaluate the integral. Substitute for r^2 in the integral formula for the area and integrate.

$$A = \int_{0}^{\pi/2} \frac{1}{2} r^2 d\theta$$

$$= \int_{0}^{\pi/2} \frac{1}{2} (\sin 2\theta) d\theta$$

$$= \frac{1}{2} \left[-\frac{\cos 2\theta}{2} \right]_{0}^{\pi/2}$$

Evaluate at each limit, using $\cos \pi = -1$ and $\cos 0 = 1$.

$$A = \frac{1}{2} \left[-\frac{\cos 2\theta}{2} \right]_{0}^{\pi/2}$$

$$= \frac{1}{2} \left(\left(\frac{1}{2} \right) - \left(-\frac{1}{2} \right) \right)$$

Simplify to obtain the area inside one loop of the lemniscate.

$$A = \frac{1}{2} \left(\left(\frac{1}{2} \right) - \left(-\frac{1}{2} \right) \right)$$

$$= \frac{1}{2}$$