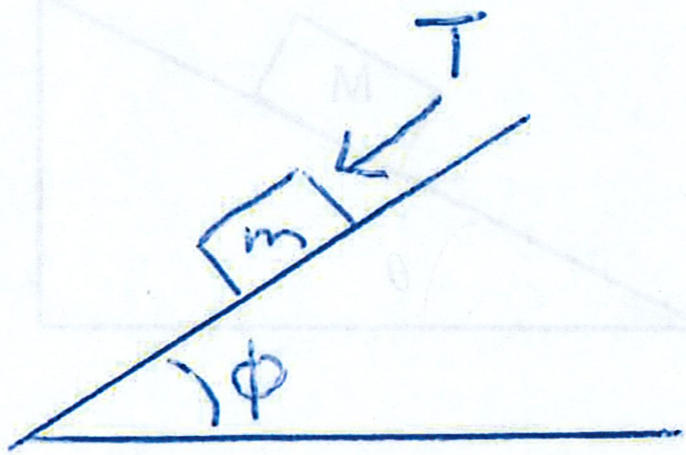
2-6-Example-TransEqII-a

Translational Equilibrium - II

A $5kg$ mass is stationary on a rough slope. The slope makes an angle of $\phi = 25°$ with the horizontal, and the mass is subject to an external force of magnitude $T = 20N$ pushing parallel to the slope and downwards.



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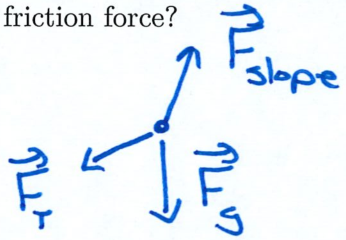
- What is the magnitude of the total force the slope exerts on the mass?
- What is the component of the force the slope exerts at 90° to its plane? In other words, what is the normal force on the mass?
- What is the component of the force the slope exerts along its plane? In other words, what is the friction force?

$\vec{F}_{Net} = 0$



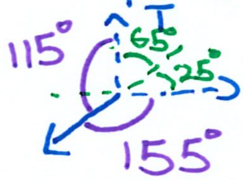
$$\vec{F}_{Net} = 0 = \vec{F}_T + \vec{F}_g + \vec{F}_{slope}$$

$$\vec{F}_{slope} = -\vec{F}_T - \vec{F}_g$$

Magnitude 2 ways

$$\vec{F}_g = -mg\hat{k} = -5kg\, 9.8 N/kg\, \hat{k} = -49N\hat{k}$$

$$\vec{F}_T = 20N\cos155\,\hat{i} + 20N\cos115\,\hat{k}$$



$$= -18.1N\hat{i} - 8.4N\hat{k}$$

$$\vec{F}_{slope} = -(-18.1N\hat{i} - 8.4N\hat{k}) - (-49N\hat{k})$$

$$= 18.1N\hat{i} + 57.4N\hat{k}$$

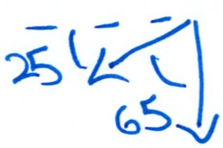
$$|\vec{F}_{slope}| = \sqrt{(18.1N)^2 + (0N)^2 + (57.4N)^2}$$

$$= 60.2N$$

$$|\vec{F}_{slope}|^2 = \vec{F}_{slope}\cdot\vec{F}_{slope} = (-\vec{F}_T - \vec{F}_g)\cdot(-\vec{F}_T - \vec{F}_g)$$

$$= \vec{F}_T\cdot\vec{F}_T + \vec{F}_g\cdot\vec{F}_T + \vec{F}_T\cdot\vec{F}_g + \vec{F}_g\cdot\vec{F}_g$$

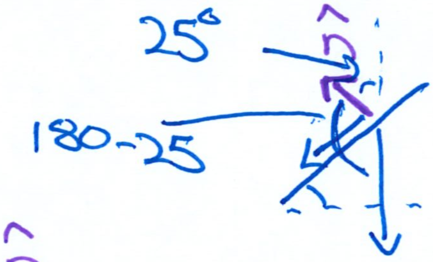
$$= (20N)^2 + (49N)^2 + 2\vec{F}_g\cdot\vec{F}_T$$



$$= (20N)^2 + (49N)^2 + 2(49N)(20N)\cos65$$

$$= (60.2N)^2$$

Normal Force



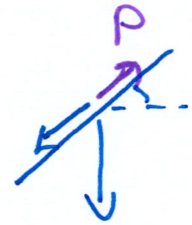
$$\vec{F}_{slope} = -\vec{F}_g - \vec{F}_T$$

$$\vec{F}_{slope} \cdot \hat{n} = -\vec{F}_g \cdot \hat{n} - \vec{F}_T \cdot \hat{n}$$

$$|\vec{F}_N| = -49N\cos 155 - 20N\cos 90$$

"normal force"

$$|\vec{F}_N| = \overset{44.4}{\cancel{40.8}}N$$



Friction

$$\vec{F}_{slope} = -\vec{F}_g - \vec{F}_T$$

$$\vec{F}_{slope} \cdot \hat{p} = -\vec{F}_g \cdot \hat{p} - \vec{F}_T \cdot \hat{p}$$

$$|\vec{F}_f| = -49N\cos 115 - 20N\cos 180$$

"friction force"

$$|\vec{F}_f| = 40.7N$$

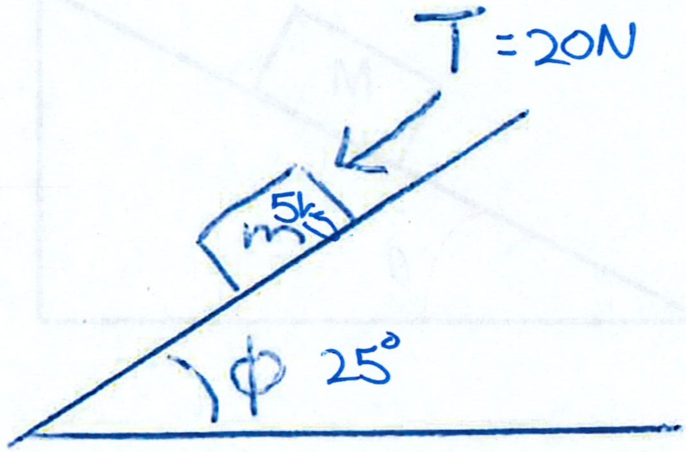
$$\vec{F}_{slope} = |\vec{F}_N|\hat{n} + |\vec{F}_f|\hat{p}$$

$$|\vec{F}_{slope}| = \sqrt{(44.4N)^2 + (40.7N)^2} = 60.2N$$

2-7-Example-TransEqII-b

Translational Equilibrium - II

A $5kg$ mass is stationary on a rough slope. The slope makes an angle of $\phi = 25°$ with the horizontal, and the mass is subject to an external force of magnitude $T = 20N$ pushing parallel to the slope and downwards.



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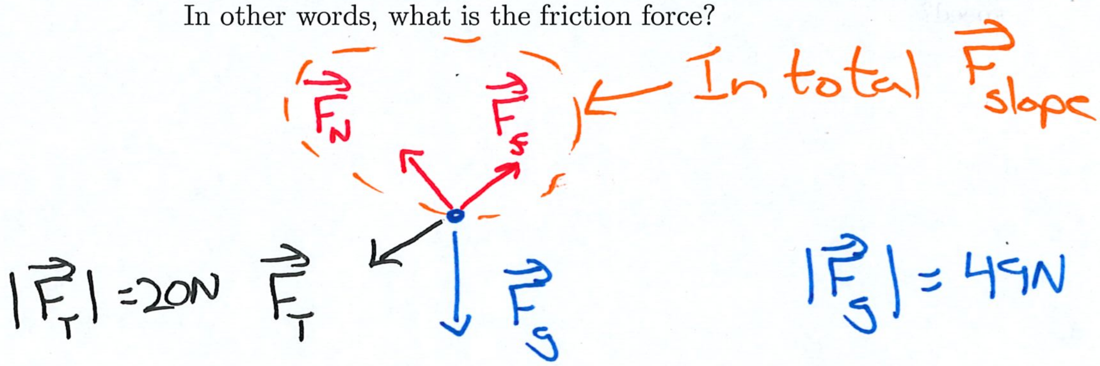
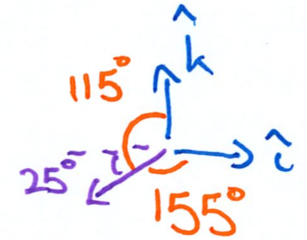


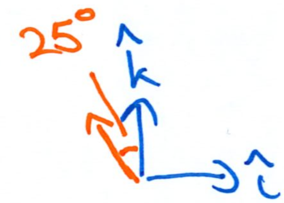
Plan: - Write each force as components $\hat{x}, \hat{z}$

- Algebra it.

$$\vec{F}_g = -49\,N\,\hat{k}$$

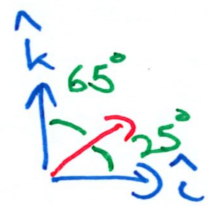


$$\vec{F}_T = 20\cos155\,\hat{\imath} + 20N\cos115\,\hat{k}$$
$$= -18.1N\,\hat{\imath} - 8.5N\,\hat{k}$$



$$\vec{F}_N = |\vec{F}_N|\underbrace{\cos115}_{-\sin25}\,\hat{\imath} + |\vec{F}_N|\cos25\,\hat{k}$$

$$= -\underline{|\vec{F}_N|}\,\underset{0.423}{\sin25}\,\hat{\imath} + |\vec{F}_N|\,\overset{0.906}{\cos25}\,\hat{k}$$



$$\vec{F}_f = \underline{|\vec{F}_f|}\cos25\,\hat{\imath} + |\vec{F}_f|\underbrace{\cos65}_{\sin25}\,\hat{k}$$
$$= |\vec{F}_f|\cos25\,\hat{\imath} + |\vec{F}_f|\sin25\,\hat{k}$$

$$\vec{F}_{Net} = 0 = \vec{F}_g + \vec{F}_T + \vec{F}_N + \vec{F}_f$$
$$= (-49N\,\hat{k}) + (-18.1N\,\hat{\imath} - 8.5N\,\hat{k})$$
$$+ (-|\vec{F}_N|\sin25\,\hat{\imath} + |\vec{F}_N|\cos25\,\hat{k})$$
$$+ (|\vec{F}_f|\cos25\,\hat{\imath} + |\vec{F}_f|\sin25\,\hat{k})$$

x-component

$$0 = -18.1N - |\vec{F}_N| \sin 25 + |\vec{F}_f| \cos 25$$

z-component

$$0 = -49N - 8.5N + |\vec{F}_N| \cos 25 + |\vec{F}_f| \sin 25$$

$$|\vec{F}_f| = \frac{18.1N}{\cos 25} + |\vec{F}_N| \frac{\sin 25}{\cos 25}$$

$$0 = -57.5N + |\vec{F}_N| \cos 25 + \left(\frac{18.1N}{\cos 25} + |\vec{F}_N| \frac{\sin 25}{\cos 25}\right) \sin 25$$

$$0 = -49N + |\vec{F}_N| \cos 25 + |\vec{F}_N| \frac{\sin^2 25}{\cos 25}$$

$$49N \cos 25 = |\vec{F}_N| \cos^2 25 + |\vec{F}_N| \sin^2 25$$

$$|\vec{F}_N| = 49N \cos 25 \qquad (mg\cos\theta)$$

$$= 44.4N$$

$$0 = -18.1N - (49N\cos 25) \sin 25 + |\vec{F}_f| \cos 25$$

$$= -36.89N + |\vec{F}_f| \cos 25$$

$$|\vec{F}_f| = 40.7N$$

Friction:

Observed in demo that in scenario

If T small, mass stationary

There is some T at which mass starts to slide.

Smallest T for slide was bigger if m was bigger.

Express mathematically

$$|\vec{F}_{f(s)}| \leq \mu_s |\vec{F}_N|$$

force of static friction

coefficient of static friction depends on materials

Whatever it has to be as long as below max.

When moving

$$|\vec{F}_{f(k)}| = \mu_k |\vec{F}_N|$$

force of kinetic friction

opposite direction to motion.