Symbolab

Solution

$\sum_{n=0}^{\infty}\frac{8}{7}\left(\frac{x}{7}\right)^n$: Radius of convergence is 7, Interval of convergence is $-7<x<7$

Steps

$\sum_{n=0}^{\infty}\frac{8}{7}\left(\frac{x}{7}\right)^n$

Use the Ratio Test to compute the convergence interval

Hide Steps

$\sum_{n=0}^{\infty}\frac{8}{7}\left(\frac{x}{7}\right)^n$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{8}{7}\left(\frac{x}{7}\right)^{(n+1)}}{\frac{8}{7}\left(\frac{x}{7}\right)^n}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{8}{7}\left(\frac{x}{7}\right)^{(n+1)}}{\frac{8}{7}\left(\frac{x}{7}\right)^n}\right|\right)$

Hide Steps

$L=\lim_{n\to\infty}\left(\left|\frac{\frac{8}{7}\left(\frac{x}{7}\right)^{(n+1)}}{\frac{8}{7}\left(\frac{x}{7}\right)^n}\right|\right)$

Simplify $\frac{\frac{8}{7}\left(\frac{x}{7}\right)^{(n+1)}}{\frac{8}{7}\left(\frac{x}{7}\right)^n}$: $\frac{x}{7}$

Hide Steps

$\frac{\frac{8}{7}\left(\frac{x}{7}\right)^{n+1}}{\frac{8}{7}\left(\frac{x}{7}\right)^n}$

Multiply $\frac{8}{7}\left(\frac{x}{7}\right)^n$: $\frac{8x^n}{7^{n+1}}$

Hide Steps

$\frac{8}{7}\left(\frac{x}{7}\right)^n$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{8\left(\frac{x}{7}\right)^n}{7}$

$\left(\frac{x}{7}\right)^n=\frac{x^n}{7^n}$

Hide Steps

$\left(\frac{x}{7}\right)^n$

Apply exponent rule: $\left(\frac{a}{b}\right)^c=\frac{a^c}{b^c}$

$=\frac{x^n}{7^n}$

$=\frac{8\cdot\frac{x^n}{7^n}}{7}$

Multiply $8\cdot\frac{x^n}{7^n}$: $\frac{8x^n}{7^n}$

Hide Steps

$8\cdot\frac{x^n}{7^n}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\cdot b}{c}$

$=\frac{x^n\cdot 8}{7^n}$

$=\frac{\frac{8x^n}{7^n}}{7}$

Apply the fraction rule: $\frac{\frac{b}{c}}{a}=\frac{b}{c\cdot a}$

$=\frac{x^n\cdot 8}{7^n\cdot 7}$

$7^n\cdot 7=7^{n+1}$

Hide Steps

$7^n\cdot 7$

Apply exponent rule: $a^b\cdot a^c=a^{b+c}$

$7^n \cdot 7 = 7^{n+1}$

$= 7^{n+1}$

$= \frac{8x^n}{7^{n+1}}$

$= \frac{\frac{8}{7}\left(\frac{x}{7}\right)^{n+1}}{\frac{8x^n}{7^{n+1}}}$

Multiply $\frac{8}{7}\left(\frac{x}{7}\right)^{n+1}$: $\frac{8x^{n+1}}{7^{n+2}}$ *Hide Steps*

$\frac{8}{7}\left(\frac{x}{7}\right)^{n+1}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{8\left(\frac{x}{7}\right)^{n+1}}{7}$

$\left(\frac{x}{7}\right)^{n+1} = \frac{x^{n+1}}{7^{n+1}}$ *Hide Steps*

$\left(\frac{x}{7}\right)^{n+1}$

Apply exponent rule: $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$

$= \frac{x^{n+1}}{7^{n+1}}$

$= \frac{8 \cdot \frac{x^{n+1}}{7^{n+1}}}{7}$

Multiply $8 \cdot \frac{x^{n+1}}{7^{n+1}}$: $\frac{8x^{n+1}}{7^{n+1}}$ *Hide Steps*

$8 \cdot \frac{x^{n+1}}{7^{n+1}}$

Multiply fractions: $a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$

$= \frac{x^{n+1} \cdot 8}{7^{n+1}}$

$= \frac{\frac{8x^{n+1}}{7^{n+1}}}{7}$

Apply the fraction rule: $\frac{\frac{b}{c}}{a} = \frac{b}{c \cdot a}$

$= \frac{x^{n+1} \cdot 8}{7^{n+1} \cdot 7}$

$7^{n+1} \cdot 7 = 7^{n+2}$ *Hide Steps*

$7^{n+1} \cdot 7$

Apply exponent rule: $a^b \cdot a^c = a^{b+c}$

$7^{n+1} \cdot 7 = 7^{n+1+1}$

$= 7^{n+1+1}$

Add the numbers: $1 + 1 = 2$

$= 7^{n+2}$

$= \frac{8x^{n+1}}{7^{n+2}}$

$= \frac{\frac{8x^{n+1}}{7^{n+2}}}{\frac{8x^n}{7^{n+1}}}$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$

$= \frac{x^{n+1} \cdot 8 \cdot 7^{n+1}}{7^{n+2}x^n \cdot 8}$

Cancel the common factor: 8

$= \frac{x^{n+1} \cdot 7^{n+1}}{7^{n+2}x^n}$

Apply exponent rule: $\frac{x^a}{x^b} = x^{a-b}$

$\frac{x^{n+1}}{x^n} = x^{n+1-n}$

$= \frac{7^{n+1}x^{n-n+1}}{7^{n+2}}$

Add similar elements: $n+1-n=1$

$=\frac{7^{n+1}x}{7^{n+2}}$

Apply exponent rule: $\frac{x^a}{x^b}=\frac{1}{x^{b-a}}$

$\frac{7^{n+1}}{7^{n+2}}=\frac{1}{7^{n+2-(n+1)}}$

$=\frac{x}{7^{n+2-(n+1)}}$

Add similar elements: $n+2-(n+1)=1$

$=\frac{x}{7}$

$L=\lim_{n\to\infty}\left(\left|\frac{x}{7}\right|\right)$

$L=\left|\frac{x}{7}\right|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$ *Hide Steps*

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$L=\left|\frac{x}{7}\right|\cdot 1$

Simplify

$L=\frac{|x|}{7}$

$L=\frac{|x|}{7}$

The power series converges for $L<1$

$\frac{|x|}{7}<1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$\frac{|x|}{7}<1:\quad |x|<7$ *Hide Steps*

$\frac{|x|}{7}<1$

Multiply both sides by 7

$\frac{7|x|}{7}<1\cdot 7$

Simplify

$|x|<7$

Therefore

Radius of convergence is 7

Radius of convergence is 7

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for x

$\frac{|x|}{7}<1\quad:\quad -7<x<7$ *Hide Steps*

$\frac{|x|}{7}<1$

Multiply both sides by 7

$\frac{7|x|}{7}<1\cdot 7$

Simplify

$|x|<7$

Apply absolute rule: If $|u|<a,\ a>0$ then $-a<u<a$

$-7<x<7$

$-7<x<7$

Check the interval end points: $x=-7$:diverges, $x=7$:diverges *Hide Steps*

For $x=-7$, $\sum_{n=0}^{\infty}\frac{8}{7}\left(\frac{(-7)}{7}\right)^n$: diverges *Hide Steps*

$\sum_{n=0}^{\infty}\frac{8}{7}\left(\frac{(-7)}{7}\right)^n$

Refine

$=\sum_{n=0}^{\infty}\frac{8}{7}(-1)^n$

Apply the constant multiplication rule: $\sum c \cdot a_n = c \cdot \sum a_n$

$= \frac{8}{7} \cdot \sum_{n=0}^{\infty} (-1)^n$

Apply Series Geometric Test: diverges

Hide Steps

$\sum_{n=0}^{\infty} (-1)^n$

Geometric Series:

If the series is of the form $\sum_{n=0}^{\infty} r^n$

If $|r| < 1$, then the geometric series converges to $\frac{1}{1-r}$

If $|r| \geq 1$, then the geometric series diverges

$r = -1, |r| = 1 \geq 1,$ by the geometric test criteria

$=$ diverges

$= \frac{8}{7}$diverges

$=$ diverges

For $x = 7, \sum_{n=0}^{\infty} \frac{8}{7}\left(\frac{7}{7}\right)^n$: diverges

Hide Steps

$\sum_{n=0}^{\infty} \frac{8}{7}\left(\frac{7}{7}\right)^n$

Refine

$= \sum_{n=0}^{\infty} \frac{8}{7}$

Every infinite sum of a non – zero constant diverges

$=$ diverges

$x = -7$:diverges, $x = 7$:diverges

Therefore

Interval of convergence is $-7 < x < 7$

Interval of convergence is $-7 < x < 7$

Radius of convergence is 7, Interval of convergence is $-7 < x < 7$