CSc 225, Spring 2006

Algorithms and Data Structures

Final Examination

April 25, 2006

University of Victoria

Student ID	*Name*

INSTRUCTIONS

- Duration: 3 hours
- All answers are to be written on the examination paper provided.
- There are **8** questions on **16** pages.
- Marks for each question are indicated on the question sheet.
- This is a closed book, closed notes, no calculator exam. **No aids are allowed.**

QUESTION 1 (24 MARKS)		QUESTION 5 (16 MARKS)	
QUESTION 2 (20 MARKS)		QUESTION 6 (25 MARKS)	
QUESTION 3 (20 MARKS)		QUESTION 7 (20 MARKS)	
QUESTION 4 (15 MARKS)		QUESTION 8 (10 MARKS)	
TOTAL (150 MARKS)			

Question 1. (24 marks) For each of the following statements decide whether it is true (**T**) or false (**F**). Circle the correct answer.

1. Quicksort, Mergesort, Insertion Sort, and Heapsort are all algorithms that solve the same computational problem. **T** **F**

2. Efficient algorithm-design techniques are called data structures. **T** **F**

3. Mergesort is a dynamic programming algorithm. **T** **F**

4. Heapsort is a more space efficient sorting algorithm than Tree Selection. **T** **F**

5. The convex hull for a point set P in the Euclidean plane is the smallest concave polygon enclosing all the points in P. **T** **F**

6. The expected worst-case running time of a randomized algorithm can be expressed using asymptotic notation. **T** **F**

7. Consider the following recurrence equation. Assume that a and b are positive constants.

$$T(n) = \begin{cases} a & (n = 1) \\ 2T\left(\frac{n}{2}\right) + bn & n > 1 \end{cases}.$$

   a. $T(n)$ describes the worst-case running time of Quicksort. **T** **F**

   b. $T(n)$ describes the worst-case running time of Insertion Sort. **T** **F**

   c. $T(n)$ describes the worst-case running time of Mergesort. **T** **F**

   d. $T(n)$ describes the expected running time of randomized Quicksort. **T** **F**

8. $17x^2$ is $O(x^2)$. **T** **F**

9. $17x^2$ is $o(x^2)$. **T** **F**

10. $17x^2$ is $\omega(x^2)$. **T** **F**

11. $17x^2$ is $\Omega(x^2)$. **T** **F**

12. $17x^2$ is $\Theta(x^2)$. **T** **F**

13. $\Omega(n \log n)$ is a lower bound for sorting in a comparison based sorting model. **T** **F**

14. The Graham Scan to compute the convex hull for a given point set P in the Euclidean plane, $|P| = n$, has a worst case time complexity of $O(n \log h)$. Here, h denotes the number of convex hull points. **T** **F**

15. The following data structures are binary search trees.

    a. Red-Black trees **T** **F**

    b. Heaps **T** **F**

    c. Binary trees **T** **F**

    d. AVL trees **T** **F**

    e. Union-Find data structure **T** **F**

16. Algorithm `LinearSelect` selects the k^{th} smallest element from a sequence of n numbers in time $O(n)$. **T** **F**

17. An equivalence relation satisfies the following properties: transitivity, anti-symmetry and reflexivity. **T** **F**

Question 2. (20 marks, 5 each) The algorithm SelectionSort can be considered a greedy algorithm.

a) Describe the algorithm SelectionSort in pseudo-code.

b) Describe the loop-invariant of the loop in your algorithm.

c) Explain the main characteristics of a greedy algorithm.

d) Argue why SelectionSort is a greedy algorithm.

Question 3 (5+5+10 = 20 marks). Consider the following recursive algorithm.

Algorithm recursion(n)
    if $n \leq 1$ **then return** n
    else return n(recursion($n-1$))

a) Describe the worst-case running time of "Algorithm recursion" in form of a recurrence equation.

b) Compute Big-Oh of your recurrence equation.

c) Order the following functions according to their growth rates. Indicate the direction of growth.

1) $14n$
2) 177^{17}
3) $17\, n \log n^2$
4) $n \log n^3$
5) $n^2 \log n$
6) 1^{17n}
7) $n^3 \log n^2$
8) $n!$
9) 2^n
10) $1^{n \log n}$

Question 4 (15 marks, 3 marks each). Define each of the following terms.

a) Red-Black tree

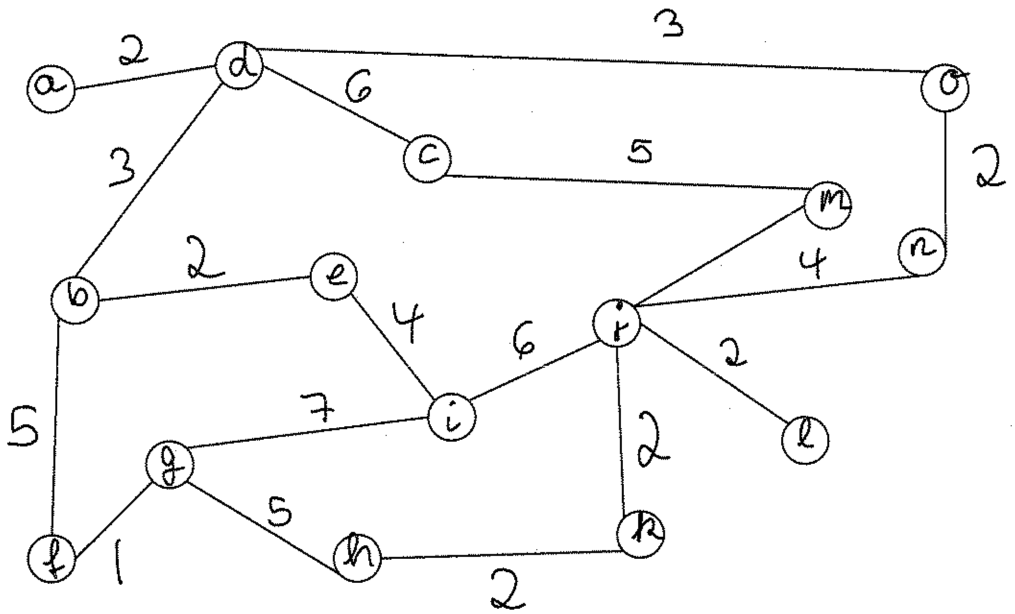
b) Backtracking

c) Priority Queue

d) Biconnected Component

e) Longest Common Subsequence

Question 5 (2+7+7 = 16 marks). Consider the graph G below.

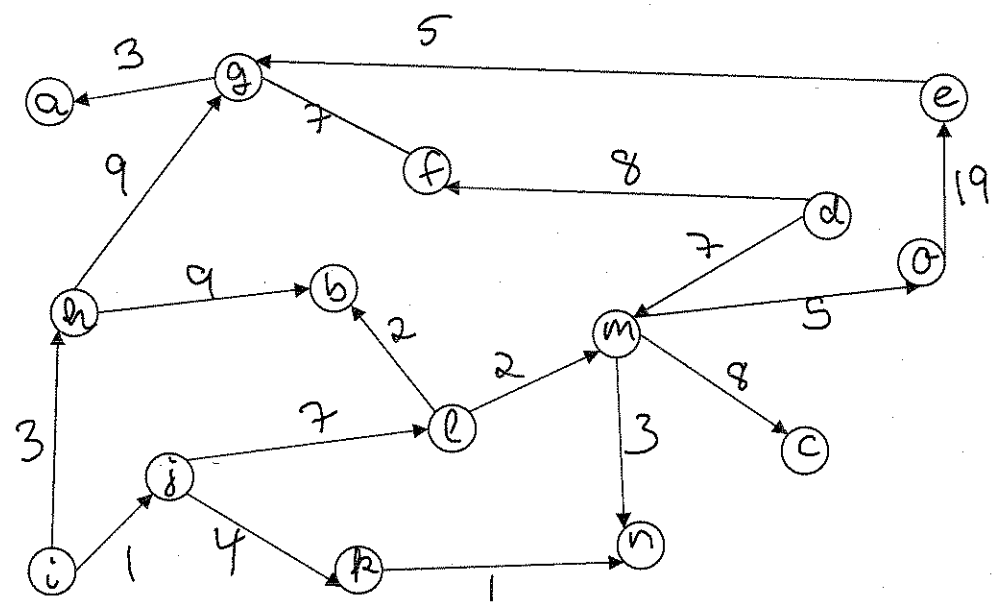


a) How many biconnected components does G contain?

b) Compute a minimum spanning tree of G using Kruskal's algorithm. Report the edges in the same order as discovered by the algorithm.

c) Compute all shortest paths in G from vertex c to every other vertex in G using Dijkstra's single source shortest path algorithm. Report the vertices and their distances in the same order as computed in the algorithm.

Question 6 (6+2+7+10 = 25 marks). Consider the digraph G below.



a) Report the transitive closure for G.

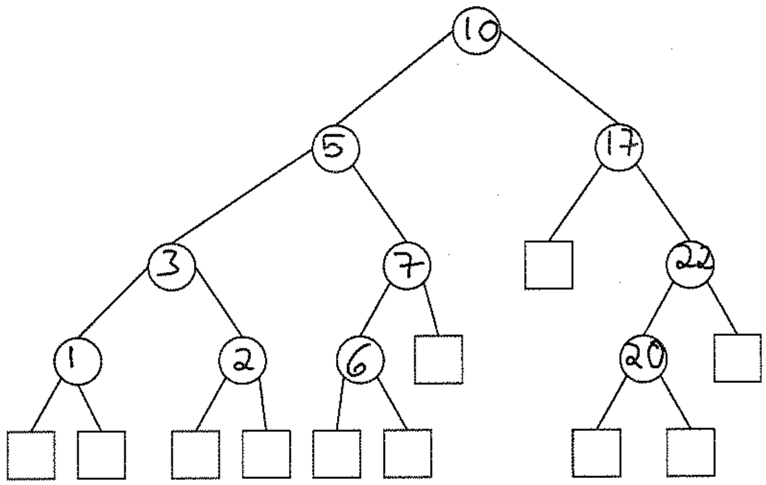
b) What is the running time of Floyd-Warshall's algorithms for computing the transitive closure of a graph containing n vertices and m edges?

c) Is it possible to compute a topological ordering for G? Argue convincingly!

d) Describe, in pseudo-code, how to compute the single source shortest path problem for an acyclic digraph in time $O(n+m)$.

Question 7. (20 marks)

a) The following tree is the resulting tree after a binary-deletion in an AVL-tree. Finish the deletion process for this AVL tree: Restructure the tree such that the resulting tree is an AVL tree. Indicate each step, and comment your steps carefully, such that the grader can reconstruct what happened!



b) Prove that the height of an AVL tree for n elements is $O(\log n)$.

Question 8 (10 marks). Describe in your own words how to build up a heap for n given elements in linear time. Argue why your described algorithm indeed does not take more than time $O(n)$!

(* the end *)