ECON 180 SUMMER 2021: PROJECT 2

DUE June 2nd, 2021 by 11:59 PM VICTORIA, B.C. TIME

Honor Code: I guarantee that this submission is **entirely my own work**. I have **cited any outside sources** in APA or IEEE style. **(You must accept this code to receive a mark.)**

Name or Signature for Honor Code: _________**ANSWER KEY**______________________

Last 3 digits of student number: _________

Please enter your answers in the spaces and tables provided. Your submission must be in either PDF or Microsoft 365 (Word, etc.) format, so Brightspace can read it properly.

Question		Marks
1	a	78
	b	78
	Q1 (Average)	78
2	a	68
	b	5
	c	5
	Q2 (Total)	78
3	a	78
	b	78
	c	78
	Q3 (Average)	78
Q1 to Q3	(Q1+Q2+Q3)/3	78
4	a	3
	b	4
	c	3
	Q4 (Total)	10
Subtotal	(Q1 to Q3)+Q4	88
Communication		6 (doubles if subtotal ≥ 83)
Total		100

I've provided an Excel spreadsheet with this project, but **you don't have to use it**. None of the questions require that you submit it, but you may find it useful as a starting point. If you think the table is confusing, or difficult to use, feel free to ignore it.

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- 4.a: Lecture 6, Lecture 3

Basic Information and Assumptions

- Note: Like programmers, project planners count from 0.
- It is currently Month 0, Year 0.
- Sam will be a student until the end of Year 2 (Month 35)
- To keep things simple, assume Sam has no income and no housing costs while studying[1].
- Sam will work as an engineer for exactly 40 years (480 months).
- Sam will start working as an engineer in Year 3 (Month 36).
- For this project, assume that **Sam's MARR is 2.45% per year** (the current prime lending rate in Canada at the time of writing this project).
- While it's important to calculate the final values asked for in each question, I'm not very interested in the numerical results, which will vary by student: I want to see that you understand how to use discount factors such as (P/A,i,N), (P/F,i,N), etc.
- →When asked to show your work, please write your answer using this functional notation – e.g. $300 x (P/A,6%,45). There's no need to show all your arithmetic – I encourage you to use a math program or Excel to do it for you. It's fine to write $300x(P/A,6%,45) on one line, and $300 x 15.4558 on the next. (I used Excel to determine that (P/A,6%,45) was about 15.4558.)
- While you're not given marks for it, it may be useful to start each question by sketching a cash flow diagram (or two) to help you picture the situations you're studying.
- While it's fine to use 'brute force' (finding the present value, etc. of each cash flow, one time period at a time) to check your solution, this will not receive full marks.

[1] This is an unrealistic assumption, but I'm making it to reduce the work you need to do. In a normal term, I usually ask students to consider tuition costs, food, etc. This is a compressed out-of-major course during a global crisis, and I feel more streamlined projects are appropriate.

Question 1: Housing Costs (Present Value, Annuities)

a. Housing costs in Vancouver and Toronto

i. For reference only, please fill out your baseline values from Project 1. If you did not submit Project 1, you may use the values from the Project 1 answer key.

Not for marks. This will vary by student. Values provided are the same as in the Project 1 key.

City	Baseline Monthly Rent ($)
Vancouver	$2,903
Toronto	$2,226

ii. Calculate the present value (Month 0 value) of the rent in Vancouver and Toronto.

- Monthly rent in each city remains at its baseline level forever. (We'll relax this in later projects.)
- Sam pays rent from Month 36 to Month 515, inclusive. This is a total of 480 months.
- Consider converting Sam's MARR (2.45% per year) into an equivalent % per month.
- Show your work for the Vancouver calculation (no need to show the Toronto calculation, since it would be identical except for the numerical values).
- When showing your work, you must use correct functional notation such as (P/F,10%,12) for full marks. (Think of how problems are presented in the lecture notes).
- Your solution should make use of (P/A,i,N).

Values will, of course, vary by student, and I don't expect you (the TAs) to double-check the math. You're checking for HOW these values were obtained – the method and thought process.

City	Present Value of Rent ($)
Vancouver	$830,965.65
Toronto	$637,178.62

[Show your work for the Vancouver calculation]

The monthly MARR is about .202% per month: $(1+2.45\%)^{1/12} - 1$, obtained by setting $(1+i_{monthly})^{12} = (1+i_{yearly})$ and solving.

Let A be rent. Sam makes 480 rent payments, with the first one in month 36. We can get a Month 35 value by using A x $(P/A,i_{monthly},480)$, since Ax(P/A,i,N) returns an equivalent single value one time period before the first payment in the original sequence.

To turn that into a Month 0 value, we can use $(P/F,i_{monthly},35)$.

All together:

PV Rent = A x $(P/A,i_{monthly},480)$ x $(P/F,i_{monthly},35)$

For my Vancouver values, this boils down to

$2,903 x (P/A,.2019%,480) x (P/F,.2019%,35) = $830,965.65

b. Housing costs in Edmonton

i. For reference only, please fill out your baseline house price and mortgage % per month from Project 1. If you did not submit Project 1, you may use the value from the Project 1 answer key.

Again, these values will vary by student.

City	Baseline House Price ($)
Edmonton	$360,996

Baseline Mortgage interest	0.225	% per month

ii. Calculate Sam's monthly mortgage payments if they choose to buy a house in Edmonton. **Show your work**, and use correct notation (e.g. (P/A,10%,52)).

- Assume Sam takes out a mortgage for the *entire* value of the house. That means that the only housing costs you need to consider in Edmonton are the mortgage payments. (We'll relax this assumption in later projects.)
- Sam buys the house in Month 36. The net cash flow related to housing in month 36 is zero, because the money coming in from the bank and the money going out to pay for the house are exactly the same, and cancel out.
- Sam's Edmonton mortgage is for 25 years. Sam makes exactly 25 years (300 months) of payments, and the first payment is in Month 37.
- To calculate mortgage payments, you need to split the house price into an equivalent sequence of monthly payments, using (A/P,i,N). The 'i' you use will be the *mortgage interest rate*, **not** Sam's MARR, because you want to find the monthly payments the *bank* thinks are equivalent (worth) the initial cost of the house.
- For example: suppose Alex took out a mortgage and bought a house in Month 0 for $10,000. Suppose that the interest rate on the mortgage is 1% per month, and Alex will pay it off over 10 years (120 months), with the first payment due the month after buying the house – month 1. In that case, the amount of each monthly mortgage payment is given by $10,000 x (A/P,1%,120) = $143.47

This will vary by student. TAs are to assess the method by which the result was obtained.

Sam's monthly mortgage payment ($)	$1,656.09	per month

[Show your work (take an extra page if you need it)]

There are 300 mortgage payments (25 years x 12 months/year).

Per the example, we can find the mortgage payment by using

P x (A/P,$i_{mortgage}$,300)

Where P is the house price, and $i_{mortgage}$ is the monthly mortgage interest rate.

For my values, this is

$360,996 x (A/P,0.225%,300) = $1,656.09

iii. Calculate the present (Month 0) value of the mortgage payments. Show your work.

- For this calculation, you *will* use Sam's MARR of 2.45% per year (or the monthly equivalent).
- You can treat the mortgage payments as a sequence of 300 equal payments, starting in month 37.
- Your solution should make use of (P/A,i,N).

This will vary by student. TAs are to assess the method by which the result was obtained.

City	PV of Mortgage ($)
Edmonton	$346,287.89

[Show your work]

Let M be the monthly mortgage payment. There are 300 payments, the first of which is in Month 37. Using M x (P/A,$i_{monthly}$,300) will therefore return an equivalent Month 36 value (since Ax(P/A,i,N) returns an equivalent single value one time period before the first payment in the original sequence). We need to use (P/F,$i_{monthly}$, 36) to get a Month 0 value.

All together:

PV Mortgage = M x (P/A,$i_{monthly}$,300) x (P/F,$i_{monthly}$,36)

For my values:

PV Mortgage = $1,656.09 x (P/A,.2019%,300) x (P/F,.2019%,36) = $346,287.89

Question 2: Salary (Present Value, Gradients)

a. A bonus and a yearly raise

i. For reference only, please fill out your baseline salaries from Project 1. If you did not submit Project 1, you may use the values from the Project 1 answer key.

City	Baseline Yearly Salary ($)
Vancouver	$71,000
Toronto	$69,000
Edmonton	$81,000

ii. Calculate the present (Year 0) value of Sam's income, given the following assumptions:

- To keep things simple, assume that Sam gets paid once a year, at the start of the year.
- This means you can use 'years' as your time scale.
- Sam's first salary payment is in year 3
- There are a total of 40 salary payments.
- In year 3, in addition to the year's salary, Sam receives a one-time bonus equal to one fourth of the starting salary.
- Sam's salary increases by 1.5% each year, so that their year 4 salary is 1.015 x their Year 3 salary, etc.
- Show your work for the Vancouver calculation (all other calculations will be the same except for the numbers). Your answer should make use of (P/A,g,i,N).

This will vary by student. TAs are to assess the method by which the result was obtained.

City	PV of Sam's Income ($)
Vancouver	$2,231,616.99
Toronto	$2,168,754.54
Edmonton	$2,545,929.24

[Show your work for the Vancouver calculation]

Bonus aside, Sam's salary is a geometric gradient with a first payment, A, equal to the baseline salary, and a growth rate, g, equal to 1.5% per year.

If we calculate A x (P/A,g,i,N) this will return a single equivalent payment one time period before the first payment in the original sequence. The first salary payment is in Year 3, so Ax(P/A,g,i,N) will return a single equivalent value in Year 3-1 = Year 2. We then need to use (P/F,i,2) to turn that Year 2 value into a present (Year 0) value.

The bonus is a single payment of A/4 in Year 3, so we use (P/F,i,3) to bring it to Year 0.

There are 40 salary payments, so N=40, and our time period is years, so i=2.45% per year.

i = 2.45% = Sam's MARR!

PV Salary = A x (P/A,g,i,40) x (P/F,i,2) + (A/4) x (P/F,i,3)
PV Salary = A x ((P/A,io,40)/(1+g)) x (P/F,i,2) + (A/4) x (P/F,i,3)
where io=(1+i)/(1+g) – 1.

io = (1+2.45%)/(1+1.5%) – 1 = 0.936% per year.

For my Vancouver values,

PV Salary = \$71,000 x (P/A,0.936%,40)/(1+1.5%) x (P/F,2.45%,2) + \$71,000/4 x (P/F,2.45%,3)

PV Salary = \$2,231,616.99

b. From a geometric gradient to an arithmetic gradient.

This question is only worth 5 marks out of the total 78 for question 2.

Suppose that Sam is in Vancouver, but instead of Sam's salary going up by 1.5% each year, it goes up by a constant amount G per year, so Sam's salary in Year 4 is equal to their Year 3 starting salary plus G, etc. Calculate the value of G needed to make the present value of this stream of income equal to the present value of Sam's income that you calculated (for Vancouver) in question 2.a.

- Apart from replacing a yearly raise of 1.5% with a yearly raise of G dollars, everything else is exactly as in question 2.a. This includes the signing bonus of ¼ of the starting salary.
- Show your work. Your answer should make use of (A/G,i,N).

Required value of G: $ ___1,276.66_______________

[Show your work (only need Vancouver values)]

Let A be Sam's starting salary. If Sam is being paid $A in Year 3, then $A+$G in Year 4, then $A+$2G in Year 3, etc., we have an arithmetic gradient of step size G sitting on top of an annuity of magnitude A.

We've dealt with situations like this in class: G x (A/G,i,N) turns the arithmetic gradient into an equivalent annuity with the same timing as the annuity it's sitting on (40 payments, first payment in Year 3).

Total annuity representing Sam's income: 40 total payments, first payment in Year 3, magnitude (A + G x (A/G,i,40)), where A is Sam's starting salary.

We already have the present value of Sam's salary (plus bonus) so we can use that to find out what G is.

(A + G x (A/G,i,40)) x (P/A,i,40) gives us a Year 2 value that is equivalent to Sam's stream of income (minus the bonus). (Recall that Ax(P/A,i,N) returns an equivalent single value one time period before the first payment in the original sequence.) We can multiply this by (P/F,i,2) to get a Year 0 value.

To that we just need to add the present value of the bonus, which is the same as in part a:

A/4 x (P/F,i,3).

All together, PV Salary = (A + G x (A/G,i,40)) x (P/A,i,40) x (P/F,i,2) + (A/4) x (P/F,i,3)

For my Vancouver values:

PV Salary = \$2,231,616.99
A = \$71,000
i=2.45% per year

2,231,616.99 = (71,000 + G x (A/G,2.45%,40)) x (P/A,2.45%,40) x (P/F,2.45%,2) +
+ (71,000/4) x (P/F,2.45%,3)

This is one equation, with one unknown (G). You can solve it analytically or numerically.

I decided to solve it numerically with Excel, just because I found that to be slightly faster and more easily checked for accuracy.

I had Excel calculate the PV above using a trial value of G, which I initialized to \$100.
I then had Excel calculate the difference between that calculated PV(G) and the PV of Vancouver salary from the previous part of the question.
Finally, I used GoalSeek to ask Excel to set that difference equal to zero by changing the trial value of G.

After doing this, Excel returned a value of G=\$1,276.66.

The spreadsheet includes a brute force calculation showing the answer is correct.

c. A more realistic payment scheme

This question is only worth 5 marks out of the total 78 for question 2.

Suppose that Sam is in Vancouver. The situation is exactly as in question 2.a, except that Sam is paid once a month, instead of once a year. Sam's salary still goes up by 1.5% a year, starting with the first salary payment each year. In the first year of work, Sam's monthly salary is equal to 1/12 of the baseline yearly salary. In the second year of work, Sam's monthly salary is equal to 1.015 times their Year 1 monthly salary, and so on. The sign-on bonus of ¼ of Sam's starting yearly salary is still paid all at once.

Timing notes: Assume that the bonus is paid at the start of month 36, and the first salary payment is at the start of month 37. (The bonus gets Sam through the first month, and at the start of the next month, Sam gets paid for the work done the previous month. There are a total of 40x12 = 480 salary payments.)

Calculate the present value of Sam's income[2] and show your work. You must use (P/A,g,i,N) in your answer. **(Hint: You'll also want to use (P/A,i,N), and you may find a use for both the 'per year' and 'per month' versions of the MARR.)**

Present value of Sam's income: $ __________________

[Show your work (only need Vancouver values)]

Consider two years of Sam's salary, one after the other. If Sam's salary is M per month in the first year, it's (1+1.5%) x M per month in the second year.

If we multiply M x (P/A,$i_{monthly}$,12), we get a single equivalent payment in December of the previous year, to Sam's salary for the year in which they were paid M per month.

Doing the same the following year would give us M x (1.5%) x (P/A,$i_{monthly}$,12)

This is true for ANY two years in which Sam is earning income. Each 'annual summary' payment is

In this way, we see that Sam's salary can be represented by an equivalent geometric gradient, with years as the time period. Letting A be Sam's baseline yearly salary, the first payment is (A/12) x (P/A,$i_{monthly}$,12), and each payment after that is 1.5% higher than the previous one, so g=1.5%.

[2] This will NOT be the same as in 2.a, because the time distribution of payments is different. Getting $12,000 now is not the same as getting $1,000 a month, starting today.

The timing's a bit inconvenient, though. These yearly payments are actually in December of the year before the salaries they're summarizing. We should multiply by (F/P,$i_{monthly}$,1) to get an equivalent 'start of year' value.

(If you're confused about why I'm doing this – remember that Ax(P/A,i,N) returns a single equivalent value one time period before the first payment in the original sequence. Our 'sequence' is monthly salary payments, with the first being in January, and January – 1 month = December of the previous year.)

The modified first payment after this change is A x (P/A,$i_{monthly}$,12) x (F/P,$i_{monthly}$,1).

Multiplying this by (P/A,g,iyearly,40) gives us an equivalent Year 2 value, since the first of these yearly summary payments is in Year 3. We then multiply by (P/F,iyearly,2) to get a present value.

Then we just have to add the bonus, which is exactly as before: A/4 x (P/F,iyearly,3).

(Note to TAs: no marks off if the only 'error' is forgetting to add the bonus, though this may contribute to a lower mark if there are other errors, as well. The same goes for the one-month adjustment. The main concept I'm checking for here is that student's realized that the yearly 'summaries' of the monthly salary payments constitute a geometric gradient on their own.)

All together:

A x (P/A,$i_{monthly}$,12) x (F/P,$i_{monthly}$,1) x (P/A,1,5%,i_{yearly},40) x (P/F,i_{yearly},2) + A/4 x (P/F,i_{yearly},3)

For my Vancouver values, this works out to \$2,207,231.87.

Question 3: Project Comparisons

a. Net Present Value Comparisons

Note: This question is intentionally very simple.

Using the same assumptions as in Question 1 and Question 2.a, calculate the Net Present Value of living & working in Vancouver, Toronto & Edmonton. (This is the present value of income minus the present value of housing costs.) This calculation can be done very quickly by referring back to your answers for Question 1 and Question 2.a.

There's no need to show your work, unless you want to[3], because I've given you the formula:

NPV = PV of Income – PV of Housing

Different students will have different values here.

City	PV Income	PV Housing	NPV
Vancouver	$2,231,616.99	$830,965.65	**$1,400,651.33**
Toronto	$2,168,754.54	$637,178.62	**$1,531,575.91**
Edmonton	$2,545,929.24	$346,287.89	**$2,199,641.34**

Based on your calculations, which is the preferred city for Sam to work in? Briefly explain your reasoning.

Preferred City: ____Edmonton (Will vary by student)____________________

Reasoning:

Edmonton has the highest NPV (and the projects all have the same lifetime, so it's legitimate to compare NPVS).

[3] In fact, if you use the companion spreadsheet, the relevant calculations are automated for you.

b. Working at the family restaurant

Suppose that Sam is considering an alternative to graduating and working as an engineer. In particular, they are considering dropping out of school immediately, and working for one year at their family's restaurant.

If Sam works at their family's restaurant for a year:

- They will quit school immediately (Time 0)
- They will be paid \$1,250 twice a month (24 times per year)
- Sam's family pays them in advance of the work, so the first payment will be now (Time 0), the second payment will be one half-month from now, the third payment will be two half-months from now, etc.
- They will not have any housing costs, as they will live at home and their family will cover room and board.
- For the purposes of this project, Sam will not have any other costs[4].

Calculate the NPV of this one-year (24 half-month) project. Show your work. Things to keep in mind: it will be useful to calculate the 'per half-month' version of the MARR, and the first payment is *in the present*.

NPV of working at the family restaurant for one year (24 half-months): \$___29,654.79____

Show your work (using appropriate notation, e.g. \$50x(P/A,12%,60)):

One 'present value' payment at Time 0, and then an annuity of 23 payments from time 1 to 23, when time is measured in half-months.

There are 24 half-months in a year, so $(1+i_{half\text{-}monthly})^{24} = (1+i_{year})$
$i_{half\text{-}monthly} = (1+2.45\%)^{(1/24)} - 1$

NPV = \$1,250 + \$1,250 x $(P/A, i_{half\text{-}monthly}, 23)$ = \$29,654.79

[4] A simplifying assumption to make things easier for you. We're ignoring food, unpaid student loans, etc.

c. Repeated Lifetimes

Assume that, if Sam stays in school, they will live and work in Vancouver after graduation. Use a repeated lifetimes, net present value approach to determine which project is preferred: the 43-year project of going to school then working in Vancouver, or the one-year project of working at the family restaurant.

Note that you already have the (43-year) NPV of living and working in Vancouver from part a., and the (1 year) NPV of working for the family restaurant from part b. There's no need to recalculate them. You can refer to them as NPV_{Van} and NPV_{rest}, and substitute the appropriate numerical values when you have to.

Preferred Project: ___**Living and working in Vancouver (may vary by student)**__

Show your work (using appropriate notation, e.g. $50x(P/A,12%,60)):

I sent out an announcement pointing out that the '43 years' consists of Year 0 and Years 1-42. If students extend the timing to include Year 43, or only include Years 0 to 41 (for a total of 42) **that's a minor issue that should not, on its own, lead to a lower mark** (though it may contribute to one, ***slightly***, if there are other errors, as well).

That's true for any other minor timing issues (determining the correct 'N'). The main thing I'm checking for is that students know that they need to 'match lifetimes', and repeat projects with different lifetimes until their least common multiple of lifetimes is reached.

Once the repeated lifetime NPVs are found, then it's just a matter of comparing them and picking the larger one.

In this case, we have a one-year project and a 43-year project. There are a few ways of repeating the one-year project to match lifetimes, but they should all be equivalent.

- There are 43 x 12 x 2 = 1,032 half-months in the 43 years. Repeating the project would mean that Sam is paid $1,250 in Half-Month 0, and then 1,031 times from Half-Month 1 to Half-Month 1,031. NPV = $1,250 + $1,250 x (P/A,$i_{half\text{-}monthly}$,1031)
- We have the NPV for the first round of the project. Call this NPVr, for restaurant. Repeating it a total of 43 times would be like getting NPVr today (Time 0), and then getting NPVr once a year from Year 1 to Year 42. NPV = NPVr + NPVr x (P/A,i_{yearly},42)
- We can get a bit ahead of ourselves and calculate the equivalent annual worth of the one-year project. We have the present value, NPVr, and it's a one-year project, so to get the equivalent annuity[5] we use NPVr x (A/P,i_{yearly},1). Once we have that annual worth, find the PV of paying that from Year 1 to Year 43: (NPVr x (A/P,i_{yearly},1)) x (P/A,i_{yearly},43).
- About the '43' above – yes, it's a bit confusing! But recall that the Year 0 payments were summarized by a Year 1 value when we calculated the annuity, so it stands to reason that the Year 42 payments would be summarized by a Year 43 value.
- In the companion spreadsheet, I've calculated this repeated lifetimes NPV in all of the three ways listed above, and it works out to $802,102.57. If this is greater than the Vancouver NPV, then the restaurant is preferred. Otherwise studying & working as an engineer is preferred.
- For my values, the Vancouver NPV as $1,400,651.33, so it is the preferred project, as this is greater than the repeated-lifetimes PV of the one-year restaurant project.

[5] Even though we already have a single payment summarizing the year of half-monthly payments, that is NOT the 'annual worth'; it's the present value. Recall that the definition of an annuity includes a 'rest' period with no payments before the first payment of the annuity. This is easier to understand if you think of the payments as 'end of year' payments, and that the end of the first year is basically the start of the second. In any case, it means that for a one-year project, the 'Annual Worth' is a Year 1 value: (A/P,i,1) = (F/P,i,1).

d. Annual Worth Comparisons

We’re again comparing living & working in Vancouver for 43 years to working in the family restaurant for 1 year, but this time we’re using an Annual Worth comparison.

Note that you already have the (43-year) NPV of living and working in Vancouver from part a., and the (1 year) NPV of working for the family restaurant from part b. There’s no need to recalculate them. You can refer to them as NPV_{Van} and NPV_{rest}, and substitute the appropriate numerical values when you have to.

i. Calculate the annual worth of living and working in Vancouver for 43 years. Show your work, using appropriate notation.

Annual worth: $__53,052.63_____ (Will vary by student)

Show your work (using appropriate notation, e.g. $50x(P/A,12%,60)):

Let the present value of the 43-year living and working in Vancouver project be NPVv. The annual worth is NPVv x (P/A,2.45%,43). (Again, if students use 42 instead of 43, that’s a minor issue that should not lead, on its own, to a lower mark. To see why it needs to be 43, and not 42, note that our ‘restaurant’ project takes place exclusively during Year 0 and is still considered a one-year project, not an instant one.)

ii. Calculate the annual worth of working in the family restaurant for 1 year. Show your work, using appropriate notation.

Annual worth: $___30,381.33_______

Let the present value of the 1-year restaurant project be NPVr. The annual worth is NPVr x (P/A,2.45%,1).

iii.

Based on your calculations in part i. and part ii., which is the preferred project? Briefly explain your reasoning.

Preferred Project: ___Living & Working in Vancouver _ (May vary by student)

Reasoning:

The project with the higher annual worth is the preferred project.

4. (Challenge) Replacement Decisions

And now for something completely different. This question is based on a published paper:

Al-Chalabi, H., Lundberg, J., Alireza, A. & Jonsson, A. (2015). Case Study: Model for Economic Lifetime of Drilling Machines in the Swedish Mining Industry. *The Engineering Economist, 60*(2), 138-154. https://doi-org.ezproxy.library.uvic.ca/10.1080/0013791X.2014.952466

You may find it useful to read the paper. I have simplified the case study slightly for this project. In particular, I've approximated their Lorentzian cost equations by geometric sequences calibrated to their estimates for months 75 – 120, to make it easier to use the DCFA taught in ECON 180.

Sam has been hired by a Swedish mining company to determine how often they should replace their drilling machines. Currently, they are replacing them every 120 months, but recently it's been suggested that they could save money by choosing a slightly different replacement period.

The situation is as follows:

- The machines cost $6,000 in month 0.
- Once the company is done with the machines, it sells them.
- Immediately (less than one second) after purchase, the machine's resale value falls to 90% of its purchase price. This is the 'driving the car off the lot' depreciation familiar to car owners.
- After this immediate drop in value, the machines lose d% of their resale value each month. If the resale value is $V in month N, it's (1 – d%)xV in month N+1.
- Sam's company contact isn't sure what the value of d is, but they know that the machine's resale value is $50 in month 50.
- The machine is subject to ongoing operating and maintenance costs.
- Operating costs grow by 4.043% per month.
- Maintenance costs grow by 3.895% per month.
- Sam's company contact only has operating & maintenance values for month 75 on hand. In month 75, operating costs for the month are $9.89, and maintenance costs for the month are $24.20.
- The mining company's MARR is 10% per **year**[6].

Help Sam out by doing the following:

[6] The company in the original paper is Boliden, and they still use a 10% MARR: https://www.boliden.com/investor-relations/financials/financial-targets

a. Write the EAC equation using appropriate notation

(3 marks) Write the EAC(N) equation for the drilling machine using DCFA notation (e.g. (P/F,6%,N)). You will probably want to use (P/A,g,i,N) and (A/P,i,N).

There are a number of ways to approach this. The way I'll do it is to find an equation for the total costs of the project as a function of N, then multiply that by (A/P,i,N). Despite the 'A' in EAC standing for 'annual', in this case the relevant time period is months.

Present Value Calculations:

- MARR: 10% per year, which works out to about 0.797% per month.
- Initial cost: Already in present value terms. $6,000.
- Operating costs: A geometric gradient with g = 4.043%. The present value is Ax(P/A,g,i,N). We need to find A, the first cost in the sequence, which in this case is a Month 1 cost. We know the Month 75 cost is $9.89. If the Month 1 cost is A, since costs grow at 4.043% per month, the Month 75 cost (74 months later) should be Ax$(1+4.043\%)^{74}$. Solving this for A, we find A = $\$9.89/1.04043^{74}$ = $0.53 (approx.)
    - PV Operating Costs: $0.53 x (P/A,4.043%,0.797%,N)
- Maintenance costs: A geometric gradient with g = 3.895%. The present value is Ax(P/A,g,i,N). We need to find A, the first cost in the sequence, which in this case is a Month 1 cost. We know the Month 75 cost is $24.20. If the Month 1 cost is A, since costs grow at 3.895% per month, the Month 75 cost (74 months later) should be Ax$(1+3.895\%)^{74}$. Solving this for A, we find A = $\$24.20/1.03895^{74}$ = $1.43 (approx.)
    - PV Operating Costs: $1.43 x (P/A,3.895%,0.797%,N)
- Resale Value: The machine loses 10% of its resale value immediately after purchase. So, right after purchase, the resale value is 90% x $6,000 = $5,400. After that, it loses a constant d% of its resale value each month, so that its resale value in Month N is $5,400 x $(1-d)^N$. We know that the resale value is $50 in month 50. $\$5,400 \times (1-d)^{50} = \$50 \rightarrow 1-d = (50/5400)^{1/50} \rightarrow d = 1 - (50/5400)^{1/50}$ = 8.94% per month (approx.). Since the resale takes place in Month N, the resale income is a 'negative cost' at Time N.
    - PV Resale Income: -$5,400 x $(1 - 8.94\%)^N$ x (P/F,0.797%,N)
- Total Present Value of Costs: $6,000 + $0.53 x (P/A,4.043%,0.797%,N) + $1.43 x (P/A,3.895%,0.797%,N) – $5,400 x $(1 - 8.94\%)^N$ x (P/F,0.797%,N)
- EAC(N): ($6,000 + $0.53 x (P/A,4.043%,0.797%,N) + $1.43 x (P/A,3.895%,0.797%,N) – $5,400 x $(1 - 8.94\%)^N$ x (P/F,0.797%,N)) x (A/P,0.797%,N)

b. Calculate the economic lifetime

(4 marks) Use your answer for part a. to calculate the economic lifetime of the drilling machine to the nearest month. Back up your answer (explain your reasoning) with one or more of a graph[7], a table or a step-by-step analytical solution. **(I recommend a graph and a table, if you're comfortable with Excel.)**

Economic lifetime: ___104___ months (It's fine for students to be off by a month or two due to rounding error.)

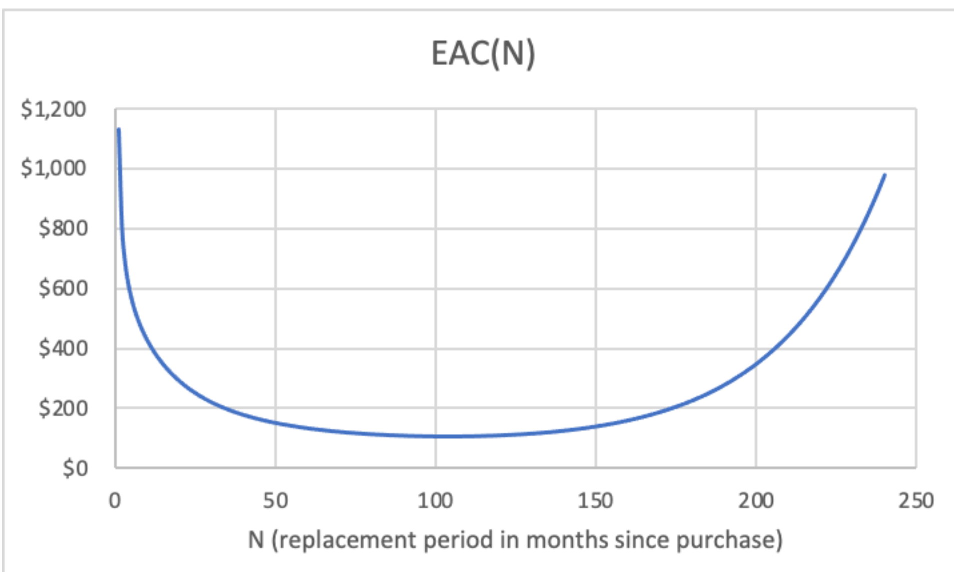
Work backing up your answer:

I've done this two ways in the companion spreadsheet.

Way 1: Less elegant, but arguably easier. I just had Excel calculate what the appropriate costs were for each month, and then find and add up present values as appropriate, month by month. Once I had a total present value for a given N, I used (A/P,i,N) to find the EAC. I then calculated this value for N=1 to 240, and found the minimum EAC.

Way 2: I implemented the equations from part a. in Excel, then had it calculate the EAC for N=1 to 240.

In both cases, I ended up with the same result: an economic lifetime of 104 months, with a corresponding EAC of $105.69 per month[8].



[7] If you are submitting a graph made in Excel, do NOT just copy-paste it from Excel into Word, as this can cause issues with file dependencies, displaying in Brightspace, etc. After copying the graph in Excel, go to Word and choose Edit → Paste Special. From there, pick a standard image format (jpg, gif, png, pdf, etc.).
[8] If students calculated the *annual* worth of this, it's $105.69 x (P/A,0.797%,12) = $1,204.93.

c. Find out how much money the company is losing

(3 marks) Sam's company contact wants to know how much money they are losing per machine each year by replacing the machines every 120 months, instead of at the economic lifetime. Calculate this amount, and show your work. (If we wanted the amount lost per *month*, it would just be the difference between EAC(120 months) and the EAC at the economic lifetime.)

Money lost each year: __$43.53__ (Student numbers may be a bit off due to rounding error, and that's fine.)

[Show your work, using appropriate notation]

From the tables created for part b.:

Economic lifetime = 104 months

EAC(104) = $105.69
EAC(120) = $109.16
Difference = $3.47 per month

The firm's MARR is 10% per year, or about 0.797% per month, so over a year this works out to:

$3.47 x (F/A,0.797%,12) = $43.53

This is not the only way to calculate this. Students could also calculate the present value of one year's worth of the difference, and then use (A/P,10%,1) to find the annual worth.

PV of 12 months of difference: $3.47 x (P/A,0.797%,12) = $39.57
Annual worth of above (1 year): $39.57 x (A/P,10%,1) = $43.53

If students obtained the correct result to within rounding error (annual worth calculation, etc.), then give them full marks. If students gave the present value of 12 months of the difference ($38.57 to within rounding error) with a correct calculation & explanation, then give them full marks, since while it's not the intended answer, it's a legitimate interpretation of 'how much money is being lost per year'. (Only two marks if it's clear they believe they calculated the annual worth, when in fact they have the present value of one year's worth of the difference.) If students just multiplied the difference by 12 ($41.64 within rounding error) and it's clear they had no intention of adjusting for the time value of money using the firm's MARR, give them 1 mark.