Symbolab

Solution

$\sum_{n=1}^{\infty}(-1)^n(x+7)^{n-1}$: Radius of convergence is 1, Interval of convergence is $-8<x<-6$

Steps

$\sum_{n=1}^{\infty}(-1)^n(x+7)^{n-1}$

Use the Ratio Test to compute the convergence interval — Hide Steps

$\sum_{n=1}^{\infty}(-1)^n(x+7)^{n-1}$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{(-1)^{(n+1)}(x+7)^{(n+1)-1}}{(-1)^n(x+7)^{n-1}}\right|$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{(-1)^{(n+1)}(x+7)^{(n+1)-1}}{(-1)^n(x+7)^{n-1}}\right|\right)$ — Hide Steps

$L=\lim_{n\to\infty}\left(\left|\frac{(-1)^{(n+1)}(x+7)^{(n+1)-1}}{(-1)^n(x+7)^{n-1}}\right|\right)$

Simplify $\frac{(-1)^{(n+1)}(x+7)^{(n+1)-1}}{(-1)^n(x+7)^{n-1}}$: $-x-7$ — Hide Steps

$\frac{(-1)^{n+1}(x+7)^{(n+1)-1}}{(-1)^n(x+7)^{n-1}}$

Remove parentheses: $(a)=a$

$=\frac{(-1)^{n+1}(x+7)^{n+1-1}}{(-1)^n(x+7)^{n-1}}$

$1-1=0$

$=\frac{(-1)^{n+1}(x+7)^{n}}{(-1)^n(x+7)^{n-1}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{(-1)^{n+1}}{(-1)^n}=(-1)^{n+1-n}$

$=\frac{(-1)^{n-n+1}(x+7)^n}{(x+7)^{n-1}}$

Add similar elements: $n+1-n=1$

$=\frac{(-1)(x+7)^n}{(x+7)^{n-1}}$

Apply exponent rule: $\frac{x^a}{x^b}=x^{a-b}$

$\frac{(x+7)^n}{(x+7)^{n-1}}=(x+7)^{n-(n-1)}$

$=(-1)(x+7)^{n-(n-1)}$

Add similar elements: $n-(n-1)=1$

$=(-1)(x+7)$

Refine

$=-(x+7)$

Distribute parentheses

$=-(x)-(7)$

Apply minus – plus rules

$+(-a)=-a$

$=-x-7$

$L=\lim_{n\to\infty}(|-x-7|)$

$L=|-x-7|\cdot\lim_{n\to\infty}(1)$

$\lim_{n\to\infty}(1)=1$ — Hide Steps

$\lim_{n\to\infty}(1)$

$\lim_{x\to a}c=c$

$=1$

$L=|-x-7|\cdot 1$

Simplify

$L=|x+7|$

$L=|x+7|$

The power series converges for $L<1$

$|x+7|<1$

Find the radius of convergence *Hide Steps*

To find radius of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for $|x-a|$

$|x+7|<1$

Therefore

Radius of convergence is 1

Radius of convergence is 1

Find the interval of convergence *Hide Steps*

To find the interval of convergence of a power series $\sum_{n=0}^{\infty}c_n(x-a)^n$ solve for x

$|x+7|<1 \quad : \quad -8<x<-6$ *Hide Steps*

$|x+7|<1$

Apply absolute rule: If $|u|\ <\ a,\ a>0$ then $-a\ <\ u\ <\ a$

$-1<x+7<1$

$x+7>-1$ and $x+7<1$ *Hide Steps*

$x+7>-1$ and $x+7<1$

$x+7>-1 \quad : \quad x>-8$ *Hide Steps*

$x+7>-1$

Subtract 7 from both sides

$x+7-7>-1-7$

Simplify

$x>-8$

$x+7<1 \quad : \quad x<-6$ *Hide Steps*

$x+7<1$

Subtract 7 from both sides

$x+7-7<1-7$

Simplify

$x<-6$

Combine the intervals

$x>-8$ and $x<-6$

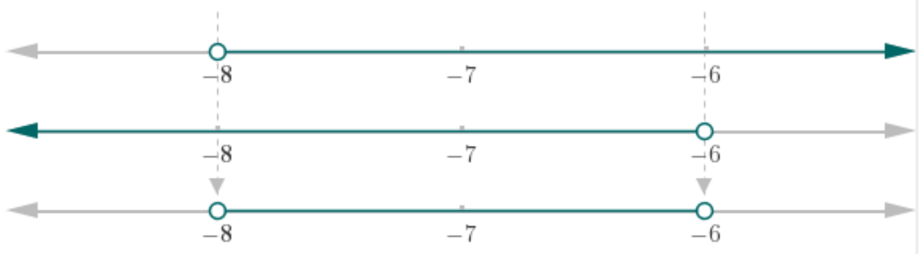
$x>-8$ and $x<-6$

Merge Overlapping Intervals *Hide Steps*

The intersection of two intervals is the set of numbers which are in both intervals

$x>-8$ and $x<-6$

$-8<x<-6$



$-8<x<-6$

$-8<x<-6$

Check the interval end points: $x=-8$:diverges, $x=-6$:diverges *Hide Steps*

For $x=-8$, $\sum_{n=1}^{\infty}(-1)^n((-8)+7)^{n-1}$: diverges *Hide Steps*

$\sum_{n=1}^{\infty}(-1)^n((-8)+7)^{n-1}$

Refine

$=\sum_{n=1}^{\infty}(-1)^{2n-1}$

Simplify $(-1)^{2n-1}$: $(-1)^{2n}(-1)^{-1}$ *Hide Steps*

$(-1)^{2n-1}$

Apply exponent rule: $a^{b+c}=a^b\cdot a^c$

$(-1)^{2n-1} = (-1)^{2n}(-1)^{-1}$

$= (-1)^{2n}(-1)^{-1}$

$= \sum_{n=1}^{\infty}(-1)^{2n}(-1)^{-1}$

Apply the constant multiplication rule: $\sum c \cdot a_n = c \cdot \sum a_n$

$= (-1)^{-1} \cdot \sum_{n=1}^{\infty}(-1)^{2n}$

$(-1)^{-1} = -1$ *Hide Steps*

$(-1)^{-1}$

Apply exponent rule: $a^{-1} = \frac{1}{a}$

$= \frac{1}{-1}$

Apply the fraction rule: $\frac{a}{-b} = -\frac{a}{b}$

$= -\frac{1}{1}$

Apply the fraction rule: $\frac{a}{1} = a$

$\frac{1}{1} = 1$

$= -1$

$= (-1) \cdot \sum_{n=1}^{\infty}(-1)^{2n}$

$(-1)^{2n} = 1$ *Hide Steps*

$(-1)^{2n}$

Apply exponent rule: $(-a)^n = a^n$, if n is even

$(-1)^{2n} = 1^{2n}$

$= 1^{2n}$

Apply rule $1^a = 1$

$= 1$

$= (-1) \cdot \sum_{n=1}^{\infty} 1$

Every infinite sum of a non – zero constant diverges

$= (-1)$diverges

$=$ diverges

For $x = -6, \sum_{n=1}^{\infty}(-1)^n((-6)+7)^{n-1}$: diverges *Hide Steps*

$\sum_{n=1}^{\infty}(-1)^n((-6)+7)^{n-1}$

Refine

$= \sum_{n=1}^{\infty}(-1)^n$

$\sum_{n=k}^{\infty}(a_n) = \sum_{n=0}^{\infty}(a_n) - a_0 - a_1 - \dots a_{k-1}$

$= \sum_{n=0}^{\infty}(-1)^n - (-1)^0$

$\sum_{n=0}^{\infty}(-1)^n =$ diverges *Hide Steps*

$\sum_{n=0}^{\infty}(-1)^n$

Apply Series Geometric Test: diverges *Hide Steps*

$\sum_{n=0}^{\infty}(-1)^n$

Geometric Series:

If the series is of the form $\sum_{n=0}^{\infty} r^n$

If $|r| < 1$, then the geometric series converges to $\frac{1}{1-r}$

If $|r| \geq 1$, then the geometric series diverges

$r = -1, |r| = 1 \geq 1$, by the geometric test criteria

$=$ diverges

$=$ diverges

If part of the expression diverges, then entire expression diverges

$=$ diverges

$x = -8$:diverges, $x = -6$:diverges

Therefore

Interval of convergence is $-8 < x < -6$

Interval of convergence is $-8 < x < -6$

Radius of convergence is 1, Interval of convergence is $-8 < x < -6$