

**MATHEMATICS 100 (Sections A01-A03),**  
**October 19, 20XX**  
Time: 90 minutes

Last name: \_\_\_\_\_

StudentID: V00\_\_\_\_\_

First name: \_\_\_\_\_

Lecture section number: A\_\_\_\_\_

Problem #	1 – 12	13	14	TOTAL
Points (max)	24	5	5	34
Score				

- The only calculators allowed on any examination are the models that start with Sharp EL-510R.
- Place your student card on the table now, face up.
- Before starting your test enter your Name (Last, First) and Student Number on the top of this page. Find your Scantron multiple-choice bubble sheet. Write your Name, Course and Date at the right side of your multiple-choice sheet now. In “Form” field bubble “A”, leave “Special” empty.
- Code your student number with bubbles on the multiple-choice sheet now.
- This test consists of 14 questions and has 10 pages (including this cover, the **Blank** page on the last page).
  - Questions 1 through 12 are multiple-choice, where each question will be marked as CORRECT or INCORRECT (full marks only). Write your work and answer in this exam paper in the provided space. **Clearly circle your final answer among the multiple choices next to the question.** You need to show your work for all answers, as we may disallow any answer which is not properly justified. For the numerical answers, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices. Transfer your multiple choice answer to the Scantron bubble sheet.
  - Question 13 and 14 are long-answer questions and can have partial marks. Write your detailed solutions in space provided in this booklet.
- Pens, pencils and erasers are permitted at your desk. If you have a pencil case it must be stored with your belongings in the front of the room.
- If you have finished working on your paper with less than 10 minutes before the end of the examination, please close your paper and **remain seated** until the test time is completed. It is important to minimize the disruptions in the room. At the end of the 120-minute test, turn-in this booklet and your bubble sheet to the invigilator(s).

1. (1 point) Evaluate  $\lim_{x \rightarrow 2} \frac{2x^2 - 3x}{x + 2}$ .

- (A)  $-\infty$       (B)  $-14$       (C)  $-2$       (D)  $-\frac{1}{2}$       (E)  $0$
- (F)  $\frac{1}{2}$       (G)  $2$       (H)  $4$       (I)  $14$       (J)  $+\infty$

The next two questions refer to the following experiment.

A biologist grows bacteria on a culture plate. The area infected (in  $mm^2$ ) at time  $t$  (in hours) is denoted  $A(t)$ . She records her findings in the following table.

t	0	5	10	15	20	25
A(t)	1	12	25	62	107	263

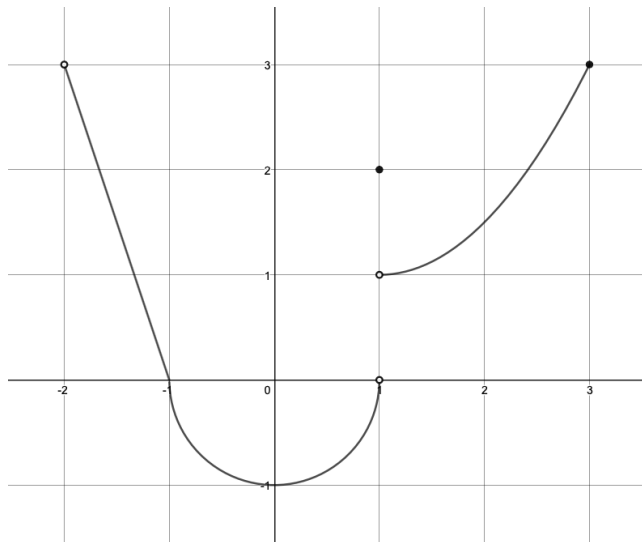
2. (1 point) Find the average rate of growth of infected area during the time interval  $[5, 20]$ .

(A)  $-12.5$       (B)  $-10.5$       (C)  $-6.3$       (D)  $-5.3$       (E)  $4.0$   
(F)  $5.0$       (G)  $6.3$       (H)  $10.5$       (I)  $12.5$       (J)  $20.5$

3. (1 point) Assume the growth function  $A(t)$  is continuous. Find an equation for the secant line to the graph  $y = A(t)$  that passes through the points  $(10, 25)$  and  $(20, 107)$ .

(A)  $y = \frac{19}{3}$       (B)  $y = \frac{82}{10}$       (C)  $y = \frac{82}{10}x - 57$       (D)  $y = \frac{19}{3}x + \frac{131}{3}$   
(E)  $x = \frac{19}{3}$       (F)  $x = \frac{82}{10}$       (G)  $y = \frac{82}{10}x + 107$       (H)  $y = \frac{19}{3}x - \frac{59}{3}$   
(I) Not enough information to find a secant line      (J) None of those

The next four questions refer to the following graph of  $y = f(x)$ .



4. (1 point) Determine the *domain* of the function  $f$ .

- (A)  $(-\infty, +\infty)$  (B)  $(-2, 3)$  (C)  $[-2, 3)$  (D)  $(-2, 1) \cup (1, 3)$   
 (E)  $[-\infty, +\infty]$  (F)  $[-2, 3]$  (G)  $(-2, 3]$  (H)  $(-2, 1) \cup (1, 3]$   
 (I) None of those

5. (1 point) Evaluate  $\lim_{x \rightarrow 1^-} f(x)$

- (A)  $-1$  (B)  $0$  (C)  $1$  (D)  $0$  and  $1$   
 (E)  $0, 1$  and  $2$  (F)  $2$  (G)  $3$  (H) Does not exist  
 (I) None of those

6. (1 point) Evaluate  $\lim_{x \rightarrow 0} f(x^2 + 1)$

- (A)  $-1$  (B)  $0$  (C)  $1$  (D)  $0$  and  $1$   
 (E)  $0, 1$  and  $2$  (F)  $2$  (G)  $3$  (H) Does not exist  
 (I) None of those

7. (1 point) Evaluate  $f'\left(-\frac{3}{2}\right)$

- (A)  $-4.0$  (B)  $-3.0$  (C)  $-1.5$  (D)  $-1.0$  (E)  $0.0$   
 (F)  $1.0$  (G)  $1.5$  (H)  $3.0$  (I)  $4.0$  (J) Does not exist

8. (1 point) Evaluate  $\lim_{h \rightarrow 0} \frac{(x+h)^{100} - x^{100}}{h}$ .

Hint: think “derivative quotient” or definition of “derivative of function”.

- (A) 0      (B)  $x^{100}$       (C)  $100x$       (D)  $99x^{100}$       (E) Does not exist
- (F) 1      (G)  $h^{100}$       (H)  $\frac{x}{100}$       (I)  $100x^{99}$       (J) None of those

9. (1 point) Find  $\frac{d^{715}}{dx^{715}}(\sin x)$ , the 715<sup>th</sup> derivative of the sine function evaluated at  $x = \pi/3$ .

(A)  $-\frac{\sqrt{2}}{2}$       (B)  $-\frac{1}{2}$       (C)  $-\frac{\sqrt{3}}{2}$       (D)  $-1$       (E)  $0$

(F)  $+\frac{\sqrt{2}}{2}$       (G)  $+\frac{1}{2}$       (H)  $+\frac{\sqrt{3}}{2}$       (I)  $+1$       (J) Does not exist

10. (1 point) Suppose

$$f(x) = \begin{cases} \frac{x^3-4x}{|x-2|} & \text{for } x < 2 \\ kx & \text{for } x \geq 2 \end{cases}$$

Find (if possible) a value for the constant  $k$  so that  $f$  is continuous at all points of  $(-\infty, \infty)$ .

(A)  $-5$       (B)  $-4$       (C)  $-3$       (D)  $-2$       (E)  $-1$

(F)  $1$       (G)  $2$       (H)  $3$       (I)  $4$       (J) Such  $k$  does not exist

11. (1 point) A particle moves along the  $x$ -axis with position at time  $t$  given by  $x(t) = te^t$ .

Find the particle's **acceleration** when  $t = \frac{1}{2}$ .

- (A) 1.5      (B) 2.0      (C) 2.5      (D) 3.0      (E) 3.5  
(F) 4.0      (G) 4.5      (H) 5.0      (I) 5.5      (J) 6.0

12. (1 point) Determine which of the given intervals contains at least one root of the function

$$f(x) = x^3 - 7x + 9.$$

- (A)  $(-4, -3)$       (B)  $(-3, -2)$       (C)  $(-2, -1)$       (D)  $(-1, 0)$       (E) None of those

13. Let  $f$  be given by

$$f(x) = \begin{cases} 4\sqrt{x} & \text{when } 0 \leq x \leq 1 \\ x^2 + 3 & \text{when } x > 1 \end{cases}$$

a) (3 points) Show directly from the definition of differentiability that  $f'(1)$  exists and find its value. No credit for will be given for solutions that do not work directly from the definition of derivative.

b) (2 points)] Is  $f$  continuous at  $x = 1$ ? Explain.



14. (5 points) For  $f(x) = \frac{2x^2 + 7x + 5}{x^2 + 3x + 2}$  find all asymptotes, if there are any.

Show all your work justifying your conclusion about existence of each asymptote.

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