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| **Student:** Arfaz Hossain<br>**Date:** 02/28/22 | **Instructor:** Muhammad Awais<br>**Course:** Math 101 A04 Spring 2022 | **Assignment:** HW-5 [Sections 10.1, 10.2 & 10.3] |

Write out the first four terms of the series to show how the series starts. Then find the sum of the series or show that it diverges.

$$\sum_{n=0}^{\infty}\left(\frac{10}{3^n}+\frac{4}{5^n}\right)$$

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To find the first four terms of the series, substitute the first four values for n into the summation. Notice the first value of n is 0.

Substitute $n=0$ into $\frac{10}{3^n}+\frac{4}{5^n}$.

$$\begin{aligned} a_0 &= \frac{10}{3^0}+\frac{4}{5^0} \\ &= 10+4 \end{aligned}$$

Thus, $a_0 = 10+4$. Although $(10+4)=14$, to better show the series, leave it written as $(10+4)$. Now find $a_1$ by substituting 1 into the expression.

$$\begin{aligned} a_1 &= \frac{10}{3^1}+\frac{4}{5^1} \\ &= \frac{10}{3}+\frac{4}{5} \end{aligned}$$

Continue this process to find $a_2$ and $a_3$. The first four terms of the series are shown below.

$$(10+4)+\left(\frac{10}{3}+\frac{4}{5}\right)+\left(\frac{10}{9}+\frac{4}{25}\right)+\left(\frac{10}{27}+\frac{4}{125}\right)+\cdots$$

If $\sum a_n = A$ and $\sum b_n = B$ are convergent series, then the Sum Rule states that $\sum\left(a_n+b_n\right)=\sum a_n+\sum b_n = A+B$.

Notice that the series $\sum_{n=0}^{\infty}\frac{10}{3^n}$ is a geometric series. Recall that for geometric series, if $|r|<1$, then $\sum_{n=1}^{\infty} ar^{n-1}=\frac{a}{1-r}$. If $|r|>1$, then the series diverges.

The value of r in the geometric series $\sum_{n=0}^{\infty}\frac{10}{3^n}$ is $\frac{1}{3}$.

Therefore, the series $\sum_{n=0}^{\infty}\frac{10}{3^n}$ converges.

Notice that the series $\sum_{n=0}^{\infty}\frac{4}{5^n}$ is a geometric series. Use the same rule as above to determine if the sum converges.

The value of r in the geometric series $\sum_{n=0}^{\infty}\frac{4}{5^n}$ is $\frac{1}{5}$.

Therefore, the series $\sum_{n=0}^{\infty}\frac{4}{5^n}$ converges.

Thus, $\sum_{n=0}^{\infty}\left(\frac{10}{3^n}+\frac{4}{5^n}\right)=\sum_{n=0}^{\infty}\frac{10}{3^n}+\sum_{n=0}^{\infty}\frac{4}{5^n}$.

Recall that for a geometric series, $\sum_{n=1}^{\infty} ar^{n-1}=\frac{a}{1-r}$. In the geometric series $\sum_{n=0}^{\infty}\frac{10}{3^n}$, the value of r was already found to be $\frac{1}{3}$. The value of a in the series is 10.

Now simplify $\frac{a}{1-r}$.

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{10}{3^n} &= \frac{a}{1-r}\\ &= \frac{10}{1-1/3}\\ &= 15 \qquad \text{Simplify.}\end{aligned}$$

For the geometric series $\sum_{n=0}^{\infty}\frac{4}{5^n}$, r was already found to be $\frac{1}{5}$. The value of a in the series is 4.

Now simplify $\frac{a}{1-r}$.

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{4}{5^n} &= \frac{a}{1-r}\\ &= \frac{4}{1-1/5}\\ &= 5 \qquad \text{Simplify.}\end{aligned}$$

Now add the sums of the two series.

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{10}{3^n}+\sum_{n=0}^{\infty}\frac{4}{5^n} &= 15+5\\ &= 20\end{aligned}$$

Thus, $\sum_{n=0}^{\infty}\left(\frac{10}{3^n}+\frac{4}{5^n}\right)=20$.