Math 101, Fall 2012
Assignment 2

Due at the **beginning** of tutorial, Thursday, September 20, 2012.
No late assignments will be accepted.

We won't award full marks for answers without an explanation or for messy work. We encourage you to talk to each other about assignment problems, but you must each write up your own solutions. Important note regarding plagiarism of homework solutions: experience shows that for every mark a person picks up by doing this, they lose anywhere from five to ten marks on the midterms and the final examination because they do not understand the material. So, it is entirely self-policing.

Questions 0 to 4 — maximum score: 20 points: 17/20

1 0. (1 point) Fill in the boxes below and staple this page to your written solutions for questions 1–4.

Section Number	A08
Last Name	Banks ✓
First Name	Daniel
Student number	V00 759012

1. (4 points) Find the volume of the solid generated by revolving the region bounded by the curves $y = 2x - x^2$ and $y = 0$ about the y-axis.

2. (5 points) Find the exact volume of the solid generated by revolving the region bounded by

$$y = \sqrt{x - 2}$$

and

$$y = 0$$

and

$$x = 6$$

about the line

$$y = 4.$$

Now, use your calculator to express the exact volume as a number which is accurate to at least three decimal places.

3. (5 points) The region under the graph of

$$y = \sin x \qquad \int_0^\pi \sin(x)\,dx$$

and above

$$y = 0$$

between

$$x = 0$$

and

$$x = \pi$$

is rotated about the x-axis. Find the resulting volume exactly. You may use the trig identity

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x).$$

Now use your calculator to express the exact volume as a number which is accurate to at least three decimal places.

4. (5 points) Find the arc length of the curve

$$\approx 1.1752$$

$$y = \frac{1}{2}(e^x + e^{-x})$$

$$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$$

from

$$x = 0$$

to

$$\frac{d}{dx}\cosh(x) = \sinh(x)$$

$$x = 1.$$

Now use your calculator to express the exact ~~volume~~ arc length as a number which is accurate to at least three decimal places.

$$A = \int \sqrt{1 + \sinh^2(x)}$$

$$\frac{e^1 - \frac{1}{e^1}}{2}$$

$$= \int \sqrt{\cosh^2(x)}$$

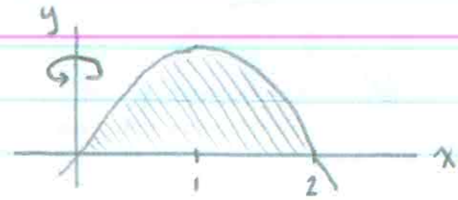
$$= \int \cosh(x) = \sinh(x)$$

Assignment #2

① $y = 2x - x^2 \qquad y = 0$

4/4

$y = x(2-x)$



$$V = 2\pi \int_a^b x\, f(x)\, dx$$

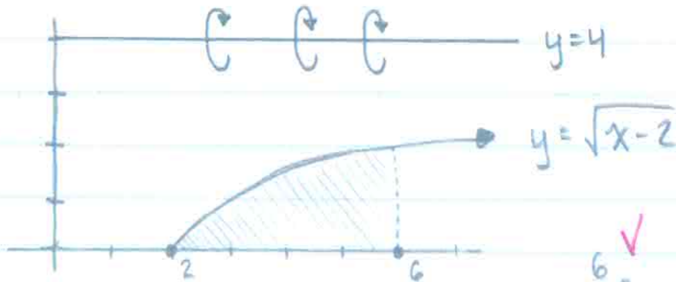
$$V = 2\pi \int_0^2 x(2x - x^2)\, dx = 2\pi \int_0^2 2x^2 - x^3\, dx$$

$$V = 2\pi \left[\frac{2}{3}x^3 - \frac{1}{4}x^4 \right]_0^2 = 2\pi\left[\frac{16}{3} - 4\right] - 2\pi[0]$$

$$V = 2\pi\left(\frac{4}{3}\right) = \frac{8\pi}{3} \approx \boxed{8.37758 \text{ units}^3}$$

②

2/5



★ Area is between: $y = \sqrt{x-2}$, $y = 0$ & $x = 6$ rotated about $y = 4$

ⓐ $V = \pi \int_a^b [f(x)]^2\, dx \rightarrow V = \pi \int_2^6 \left[\sqrt{x-2} - 4\right]^2 dx$ ✗

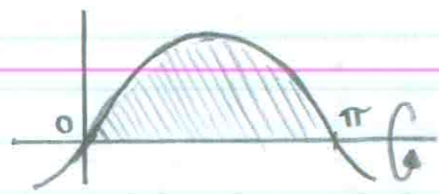
$$= \pi \int_2^6 \left[(\sqrt{x-2} - 4)(\sqrt{x-2} - 4)\right] dx = \pi \int_2^6 \left(x - 8(x-2)^{1/2} + 14\right) dx$$

$$= \pi \left[\frac{1}{2}x^2 - \frac{16}{3}(x-2)^{3/2} + 14x \right]_2^6$$

$$= \pi\left[(18) - \left(\frac{128}{3}\right) + (84)\right] - \pi\left[(2) - (0) + (28)\right] = \boxed{\frac{88\pi}{3}}$$

$$\frac{88\pi}{3} \approx 92.15338 \text{ units}^3$$ ✗

5/5 ③



$y = \sin(x),\ y = 0,\ x = 0\ \&\ x = \pi$

$$V = \pi \int_a^b [f(x)]^2\, dx = \pi \int_0^\pi [\sin(x)]^2\, dx$$

$$V = \pi \int_0^\pi \frac{1}{2} - \frac{1}{2}\cos(2x)\ dx = \frac{\pi}{2}\int_0^\pi 1 - \cos(2x)\, dx$$

$$V = \frac{\pi}{2}\left[x - \frac{1}{2}\sin(2x)\right]_0^\pi = \frac{\pi}{2}[\pi] = \boxed{\pi^2/2} \approx 4.9348$$

5/5 ④ $f(x) = \frac{1}{2}(e^x + e^{-x})$ on $[0,1]$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$$

if $f(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$ then $f'(x) = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

$$L = \int_0^1 \sqrt{1 + (\tfrac{1}{2}e^x - \tfrac{1}{2}e^{-x})^2}\ dx$$ ★ $\frac{1}{2}e^x - \frac{1}{2}e^{-x} = \sinh(x)$

$$L = \int_0^1 \sqrt{1 + \sinh^2(x)}\ dx$$

★identity: $\cosh^2(x) - \sinh^2(x) = 1$

$$L = \int_0^1 \sqrt{\cosh^2(x)}\ dx = \int_0^1 \cosh(x)\, dx = [\sinh(x)]_0^1$$

★ $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ ∴ $\frac{1}{2}\left(e - \frac{1}{e}\right) = 1.175201\ldots$

$$\boxed{V = 1.1752\,\text{units}^3}$$