Quiz 6 Sol'n ideas

1a) $122^{122} \equiv 2^{122} \pmod{10}$

Since $2^5 \equiv 2 \pmod{10}$ we have

$$2^{122} \equiv 2^2\, 2^{120} \equiv 2^2 (2^5)^{20} \equiv 2^2\, 2^{20} \equiv 2^2 (2^5)^4$$
$$\equiv 2^6 \equiv 2^1 2^5 \equiv 2^2 \equiv 4$$

$\therefore$ TRUE

b) We have $k \equiv -5 \equiv 2 \pmod 7$

$$\therefore\ 4k^3 + 6k \equiv 4 \cdot 8 + 6 \cdot 2$$
$$\equiv 4 \cdot 1 - 2 \equiv 2 \pmod 7$$

$\therefore$ TRUE

c) The possible values of $x^2 \pmod 4$ are 0 & 1.

$x^2 \equiv 0 \pmod 4 \iff x$ is even

$\therefore$ TRUE

d) $4 \cdot 25 + 6 \cdot 15^5 - 8 \equiv 4 \cdot 0 + 6 \cdot 0 - 8 \equiv 2 \pmod 5$

$\therefore$ FALSE

2a) Suppose $f(x_1) = f(x_2)$

Then $\frac{3}{2}x_1 - \frac{7}{3} = \frac{3}{2}x^2 - \frac{7}{3}$, so $x_1 = x_2$

$\therefore f$ is 1-1

Take any $y \in \mathbb{Q}$.

Then $f(x) = y \iff \frac{3}{2}x - \frac{7}{3} = y$

$$\iff x = \frac{2}{3}\left(y + \frac{7}{3}\right)$$

If $y \in \mathbb{Q}$ then so is $\frac{2}{3}\left(y + \frac{7}{3}\right)$,

$\therefore f$ is onto.

2b) Yes. It is 1-1 and onto.

3 a) Reflexive: Let $x \in A$. Since the 2^{nd} digit of x equals the 2^{nd} digit of x, we have $x \sim x$ $\therefore \sim$ is reflexive.

Symmetric: Let $x, y \in A$ and suppose $x \sim y$. Then the 2^{nd} digit of x equals the 2^{nd} digit of y
$\therefore$ " " " " y " " " " " x
$\therefore y \sim x$ $\therefore \sim$ is symmetric

Transitive: Let $x, y, z \in A$ and suppose $x \sim y$ and $y \sim z$. Then x & y have the same 2^{nd} digit, as do y & x
$\therefore x$ & z have the same 2^{nd} digit
$\therefore x \sim z$ $\therefore \sim$ is transitive

$\therefore \sim$ is an equivalence relation

b) There is one equivalence class for each possible 2^{nd} digit. Since each of $0, 1, \ldots, 9$ occurs as the 2^{nd} digit of a number in A there are <u>10</u> distinct equivalence classes

4 a) $(1,2) \in R \Rightarrow (2,1) \in R$ by symmetry
$(1,2), (2,1) \in R \Rightarrow (1,1) \in R$ by transitivity
$\therefore$ <u>TRUE</u>

b) We have $(1,3) \in R$ by transitivity, and $(3,1) \in R$ by anti-symmetry
$\therefore$ <u>TRUE</u>

c) $R = \{(4,4)\}$ has the given property but is not reflexive as $(1,1)$, $(2,2)$ & $(3,3)$ are all missing from R
∴ FALSE

d) The same R as in (c) above has those properties ∴ TRUE