201909 Math 122 Quiz #6

This is a take-home quiz. It is due no later than 2:30 PM on Wednesday, December 4. Late quizzes will not be accepted except in documented cases of illness, emergency, accident, or affliction.

There are several ways your solutions can be submitted: (1) to Gary in DTB A442 between 1 PM and 2:30 PM; and (2) in the assignment drop box on the right hand side of the receptionist's window in the Math and Stats Office, DTB A425.

Please write "MATH 122 A01 – Gary" on the top of your answer paper.

This quiz is to be done individually. You may consult any pre-existing resources on the course page or elsewhere. Any form of collaboration or communication between persons is not permitted.

There are 5 questions with marks as shown, and a total of 15 marks available. For each question, it is necessary to show clearly organized work in order to receive full or partial credit. Answers must be written in your own words in a way that reflects your own understanding.

---

1. [2] Is the function $f : \mathbb{Q} \setminus \{0\} \to \mathbb{Q}$ defined by $f(x) = \frac{1}{x} + 2$ onto? Why or why not?

2. [3] Let $A = \{1, 2, 3, 4, 5, 6\}$, and $f : A \to A$ be the function given in the table below.

x	1	2	3	4	5	6
$f(x)$	3	5	6	2	4	1

   (a) Explain why f is invertible.
   (b) Is it true that $f^{-1} = f \circ f$? Why or why not?

3. [2] Let $\mathcal{R}$ be a relation on $\{1, 2, 3\}$ which is reflexive, antisymmetric, and transitive. Suppose $(2, 1), (1, 3) \in \mathcal{R}$. Write $\mathcal{R}$ as a set of ordered pairs. If there are ordered pairs not in your set, justify why they are not in $\mathcal{R}$.

4. [4] Let $\mathcal{R}$ be the relation on $A = \{20, 21, \ldots, 57\}$ defined by $x\mathcal{R}y \Leftrightarrow$ the product of the digits in the decimal representation of x equals the product of the digits in the decimal representation of y. For example, $(44, 28) \in \mathcal{R}$ because $4 \times 4 = 2 \times 8$.

   (a) Prove that $\mathcal{R}$ is an equivalence relation.
   (b) How many different equivalence classes are there? Explain.

5. [4] Answer each question **True** or **False**. In each case give a brief justification for your answer.

   (a) There exist functions $f : \{1, 2\} \to \{1, 2, 3\}$ and $g : \{1, 2, 3\} \to \{1, 2\}$ such that $g \circ f$ is 1-1 but g is not 1-1.
   (b) The set $A = \{\lfloor x \rfloor : x \in \mathbb{R}\}$ is countable.
   (c) The set $\mathbb{Q} \cup (0, 1)$ is uncountable, where $(0, 1)$ denotes the open interval of real numbers x such that $0 < x < 1$.
   (d) Every uncountable set has a countable subset.