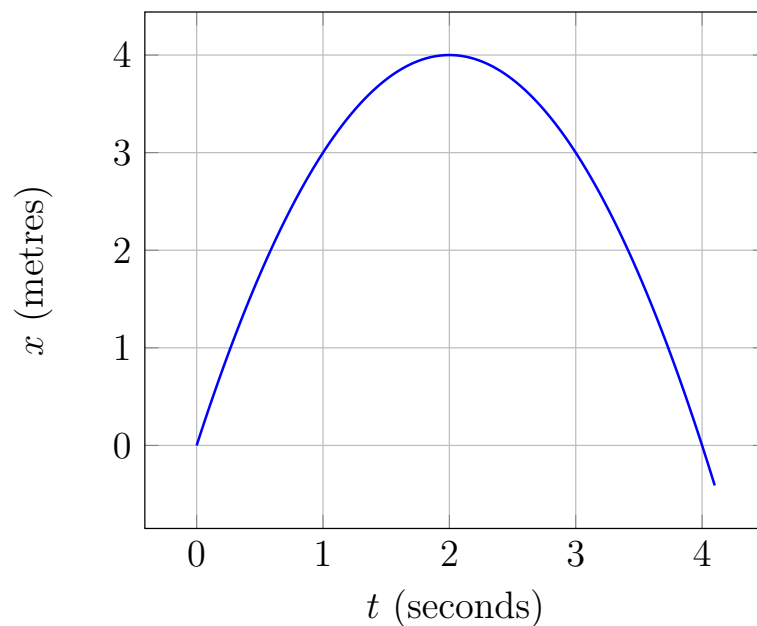


MATH 100, Fall, 2021
Tutorial #2
Rates of change and limits

Q1 The altitude of a ball on an alien planet thrown into the air at time 0 with initial velocity 4 is given by

$$x(t) = 4t - t^2$$

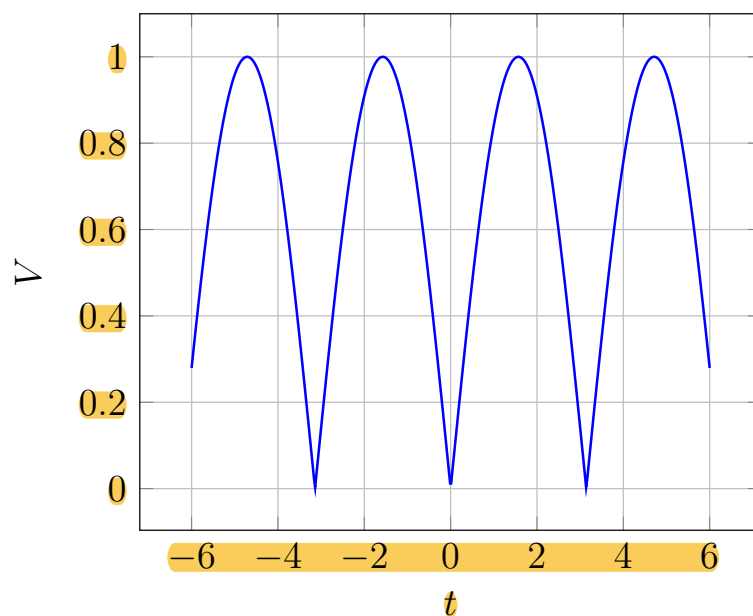


1. Calculate the average speed (with units!) of the ball over the time interval $0 \leq t \leq 2$.
2. What is the total distance travelled from time 0 to 4?
3. Calculate the instantaneous speed at time 3?
4. At what time(s) is the instantaneous speed 0?

Q2 Compute each of the following limits, or explain why it does not exist:

1. $\lim_{x \rightarrow \frac{2}{\pi}} x^2 \sin\left(\frac{1}{x}\right)$
2. $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$

Q3 The voltage of a rectified power supply at time t is modeled by $V(t) = |\sin(t)|$. A graph of V over the interval $[-6, 6]$ is shown below.



1. Find an equation of the tangent line to the graph of the function V at $t = \frac{1}{2}\pi$.
2. Compute $\lim_{h \rightarrow 0^+} \frac{V(h) - V(0)}{h}$ and $\lim_{h \rightarrow 0^-} \frac{V(h) - V(0)}{h}$. Does the graph of V have a slope at 0? Use your calculations to explain your answer.

Q4 Let m be a fixed real number. Define f by

$$f(x) = \begin{cases} x^2 + x & \text{if } x < 0, \\ mx & \text{if } x \geq 0 \end{cases}.$$

1. Which values of m make f continuous at $x = 0$?
2. Compute $\lim_{h \rightarrow 0^+} \frac{f(h) - f(0)}{h}$ and $\lim_{h \rightarrow 0^-} \frac{f(h) - f(0)}{h}$.
3. Find all values of m that make f differentiable at $x = 0$.

Q5 We know from Example 11, p73 of the textbook that the functions $\sin x$, $\cos x$ are continuous at $x = 0$.

1. Write down precisely what this statement means mathematically.
2. For fixed $x \in \mathbb{R}$ calculate $\lim_{h \rightarrow 0} \sin(x + h)$. Hint: a trig identity may be helpful here.
3. Show precisely how to use 2. to derive continuity of \sin for every $x \in \mathbb{R}$.