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Find the length of the spiral, $r = 5\theta^2$, $0 \le \theta \le \sqrt{21}$.

If $r = f(\theta)$ has a continuous first derivative for $\alpha \le \theta \le \beta$ and if the point $P(r,\theta)$ traces the curve $r = f(\theta)$ exactly once as θ runs from α to β, then the length of the curve is $L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$. For this problem, it is given that $\beta = \sqrt{21}$ and $\alpha = 0$.

For $r = 5\theta^2$, $\frac{dr}{d\theta} = 10\theta$.

$$L = \int_0^{\sqrt{21}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\sqrt{21}} \sqrt{\left(5\theta^2\right)^2 + (10\theta)^2}\, d\theta$$ Each squared term has factor of $(5\theta)^2$.

$$= \int_0^{\sqrt{21}} 5\theta\sqrt{\theta^2 + 4}\, d\theta$$

Using $\int f(v)d\theta = \int \frac{f(v)}{\frac{dv}{d\theta}} dv$, substitute $v = \theta^2 + 4$, so that $\frac{dv}{d\theta} = 2\theta$.

Also, if $\theta = 0$, then $v = 4$, and if $\theta = \sqrt{21}$, then $v = 25$.

The length, L, transforms into the following integral.

$$L = \int_0^{\sqrt{21}} 5\theta\sqrt{\theta^2 + 4}\, d\theta$$

$$= \int_4^{25} \frac{5\theta\sqrt{v}}{2\theta} dv$$

$$= \frac{5}{2}\int_4^{25} \sqrt{v}\, dv$$

Integrate.

$$L = \frac{5}{2}\int_4^{25} \sqrt{v}\, dv$$

$$= \frac{5}{2}\left[\frac{2}{3}v^{3/2}\right]_4^{25}$$, using $\int v^n dv = \frac{v^{n+1}}{n+1}$

$$= \frac{5}{3}\left(25^{3/2} - 4^{3/2}\right)$$

Simplify to solve for the length.

$$
\begin{aligned}
L &= \frac{5}{3}\left(25^{3/2} - 4^{3/2}\right) \\
&= \frac{5}{3}\left(5^{3} - 2^{3}\right) \\
&= \frac{585}{3}
\end{aligned}
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