MATH 100, Fall 2021
Tutorial Worksheet
Tutorial Section (T01, T02 etc) T12
Tutorial Instructor Name: Marcus Xie
Question Number Attempted (Q1, Q2, etc) Q4

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Today's Date:

Question 4.1

Given, $\int\int \dfrac{\cot^{-1} x}{x^2}\, dx = k \ln(x^2+1) \stackrel{\text{\tiny x}}{} \ln x - \dfrac{\cot^{-1} x}{x} + c$

Here, $\dfrac{d}{dx}\left(k \ln(x^2+1) - \ln x - \dfrac{\cot^{-1} x}{x} + c \right) = \dfrac{d}{dx}\left[\int \left(\dfrac{\cot^{-1} x}{x^2}\right) dx \right] = \dfrac{\cot^{-1} x}{x^2}$

$\Rightarrow k \cdot \dfrac{1}{x^2+1}(2x + 0) - \dfrac{1}{x} - \left[\dfrac{-1}{(1+x^2)} \cdot \dfrac{1}{x} + \cot^{-1} x \cdot -\dfrac{1}{x^2} + 0 \right] = \dfrac{\cot^{-1} x}{x^2}$

$\Rightarrow \dfrac{2kx}{x^2+1} - \dfrac{1}{x} + \dfrac{1}{x(1+x^2)} + \dfrac{\cot^{-1} x}{x^2} = \dfrac{\cot^{-1} x}{x^2}$

$\Rightarrow \dfrac{2kx}{x^2+1} = \dfrac{1}{x} - \dfrac{1}{x(1+x^2)} = \dfrac{1+x^2 - 1}{x(1+x^2)} = \dfrac{x^2}{x(1+x^2)}$

$= \dfrac{x}{(1+x^2)}$

$\Rightarrow \dfrac{2kx}{(x^2+1)} = \dfrac{x}{(1+x^2)} \Rightarrow 2kx = x$

$\Rightarrow 2k = 1$

$\Rightarrow k = \dfrac{1}{2}$

Answer: $k = \dfrac{1}{2}$

Question 9.2

Given $\quad y(1) = \ln 2$, $\quad k = \frac{1}{2}$

$y(1) \Rightarrow k \ln(x^2+1) - \ln x - \dfrac{\cot^{-1} x}{x} + c \qquad = \ln 2$

$\Rightarrow \dfrac{1}{2} \ln(1^2+1) - \ln 1 - \dfrac{\cot^{-1} 1}{1} + c \qquad = \ln 2$

$\Rightarrow \dfrac{\ln 2}{2} - \ln 1 - \cot^{-1} 1 + c = \ln 2$

$\Rightarrow \dfrac{\ln 2}{2} - 0 - \dfrac{2\cot^{-1} 1}{2} - \dfrac{2\ln 2}{2} = -c$

$\Rightarrow c = \dfrac{2\ln 2 - \ln 2 + 2\cot^{-1} 1}{2} = \dfrac{\ln 2 + 2\cot^{-1} 1}{2}$

$\qquad\qquad\qquad = \dfrac{\ln 2}{2} + \cot^{-1} 1$

$\qquad\qquad\qquad = \dfrac{\ln 2}{2} + \dfrac{\pi}{4}$

(Ans)