Solution

$\sum_{n=0}^{\infty}\frac{x^n}{n\sqrt{n}\,11^n}$: Radius of convergence is 11, Interval of convergence is $-11\le x\le 11$

Steps

$\sum_{n=0}^{\infty}\frac{x^n}{n\sqrt{n}\cdot 11^n}$

Use the Ratio Test to compute the convergence interval *Hide Steps*

$\sum_{n=0}^{\infty}\frac{x^n}{n\sqrt{n}\cdot 11^n}$

Series Ratio Test:

If there exists an N so that for all $n\ge N$, $a_n\ne 0$ and $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=L$:

If $L<1$, then $\sum a_n$ converges

If $L>1$, then $\sum a_n$ diverges

If $L=1$, then the test is inconclusive

$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{\frac{x^{(n+1)}}{(n+1)\sqrt{n+1}\,11^{(n+1)}}}{\frac{x^n}{n\sqrt{n}\,11^n}}\right|$$

Compute $L=\lim_{n\to\infty}\left(\left|\frac{\frac{x^{(n+1)}}{(n+1)\sqrt{n+1}\,11^{(n+1)}}}{\frac{x^n}{n\sqrt{n}\,11^n}}\right|\right)$ *Hide Steps*

$$L=\lim_{n\to\infty}\left(\left|\frac{\frac{x^{(n+1)}}{(n+1)\sqrt{n+1}\cdot 11^{(n+1)}}}{\frac{x^n}{n\sqrt{n}\cdot 11^n}}\right|\right)$$

Show Steps

Simplify $\frac{\frac{x^{(n+1)}}{(n+1)\sqrt{n+1}\cdot 11^{(n+1)}}}{\frac{x^n}{n\sqrt{n}\cdot 11^n}}$: $\frac{n\sqrt{n}\,x}{11(n+1)\sqrt{n+1}}$

$$L=\lim_{n\to\infty}\left(\left|\frac{n\sqrt{n}\,x}{11(n+1)\sqrt{n+1}}\right|\right)$$

$$L=\left|\frac{x}{11}\right|\cdot\lim_{n\to\infty}\left(\left|\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}\right|\right)$$

$\lim_{n\to\infty}\left(\left|\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}\right|\right)=1$ *Hide Steps*

$$\lim_{n\to\infty}\left(\left|\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}\right|\right)$$

$\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}$ is positive when $n\to\infty$. Therefore $\left|\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}\right|=\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}$

$$=\lim_{n\to\infty}\left(\frac{n\sqrt{n}}{(n+1)\sqrt{n+1}}\right)$$

Apply L'Hopital's Rule *Hide Steps*

L'Hopital Theorem:

For $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$, if $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{0}{0}$ or $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\frac{\pm\infty}{\pm\infty}$, then

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)=\lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$$

Test L'Hopital condition: $\frac{\infty}{\infty}$ *Hide Steps*

$\lim_{n\to\infty}\left(n\sqrt{n}\right)=\infty$ *Show Steps*

$\lim_{n\to\infty}\left((n+1)\sqrt{n+1}\right)=\infty$ *Show Steps*

Meets L'hopital condition of: $\frac{\infty}{\infty}$

$$=\lim_{n\to\infty}\left(\frac{\left(n\sqrt{n}\right)'}{\left((n+1)\sqrt{n+1}\right)'}\right)$$

$$= \lim_{n\to\infty}\left(\frac{(n\sqrt{n})'}{((n+1)\sqrt{n+1})'}\right)$$

$(n\sqrt{n})' = \frac{3\sqrt{n}}{2}$ Hide Steps

$\frac{d}{dn}(n\sqrt{n})$

Simplify $n\sqrt{n}$: $n^{\frac{3}{2}}$ Show Steps

$= \frac{d}{dn}\left(n^{\frac{3}{2}}\right)$

Apply the Power Rule: $\frac{d}{dx}(x^a) = a \cdot x^{a-1}$

$= \frac{3}{2}n^{\frac{3}{2}-1}$

Simplify $\frac{3}{2}n^{\frac{3}{2}-1}$: $\frac{3\sqrt{n}}{2}$ Show Steps

$= \frac{3\sqrt{n}}{2}$

$((n+1)\sqrt{n+1})' = \frac{3\sqrt{n+1}}{2}$ Hide Steps

$\frac{d}{dn}((n+1)\sqrt{n+1})$

Simplify $(n+1)\sqrt{n+1}$: $(n+1)^{\frac{3}{2}}$ Show Steps

$= \frac{d}{dn}\left((n+1)^{\frac{3}{2}}\right)$

Apply the chain rule: $\frac{3\sqrt{n+1}}{2}\frac{d}{dn}(n+1)$ Show Steps

$= \frac{3\sqrt{n+1}}{2}\frac{d}{dn}(n+1)$

$\frac{d}{dn}(n+1) = 1$ Show Steps

$= \frac{3\sqrt{n+1}}{2} \cdot 1$

Simplify

$= \frac{3\sqrt{n+1}}{2}$

$$= \lim_{n\to\infty}\left(\frac{\frac{3\sqrt{n}}{2}}{\frac{3\sqrt{n+1}}{2}}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\frac{3\sqrt{n}}{2}}{\frac{3\sqrt{n+1}}{2}}\right)$$

Simplify $\frac{\frac{3\sqrt{n}}{2}}{\frac{3\sqrt{n+1}}{2}}$: $\sqrt{\frac{n}{n+1}}$ Hide Steps

$\frac{\frac{3\sqrt{n}}{2}}{\frac{3\sqrt{n+1}}{2}}$

Divide fractions: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}$

$= \frac{3\sqrt{n} \cdot 2}{2 \cdot 3\sqrt{n+1}}$

Cancel the common factor: 3

$= \frac{\sqrt{n} \cdot 2}{2\sqrt{n+1}}$

Cancel the common factor: 2

$= \frac{\sqrt{n}}{\sqrt{n+1}}$

Combine same powers : $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$

$= \sqrt{\frac{n}{n+1}}$

$$= \lim_{n\to\infty}\left(\sqrt{\frac{n}{n+1}}\right)$$

$\lim_{x\to a}[f(x)]^b = [\lim_{x\to a} f(x)]^b$
With the exception of indeterminate form

$= \sqrt{\lim_{n\to\infty}\left(\frac{n}{n+1}\right)}$

Divide by highest denominator power: $\frac{1}{1+\frac{1}{n}}$ Show Steps

$= \sqrt{\lim_{n\to\infty}\left(\frac{1}{1+\frac{1}{n}}\right)}$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \quad \lim_{x\to a} g(x) \neq 0$
With the exception of indeterminate form

$= \sqrt{\frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}}$

$\lim_{n\to\infty}(1) = 1$ Show Steps

$\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1$ Show Steps

$= \sqrt{\frac{1}{1}}$

Simplify

$= 1$

$L = \left|\frac{x}{11}\right| \cdot 1$

Simplify

$L = \frac{|x|}{11}$

$L = \frac{|x|}{11}$

The power series converges for $L < 1$

$\frac{|x|}{11} < 1$

Find the radius of convergence Hide Steps

To find radius of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for $|x-a|$

$\frac{|x|}{11} < 1: \quad |x| < 11$ Show Steps

Therefore

Radius of convergence is 11

Radius of convergence is 11

Find the interval of convergence Hide Steps

To find the interval of convergence of a power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ solve for x

$\frac{|x|}{11} < 1 \quad : \quad -11 < x < 11$ Show Steps

$-11 < x < 11$

Check the interval end points: $x = -11$:converges, $x = 11$:converges Hide Steps

For $x = -11, \sum_{n=0}^{\infty} \frac{(-11)^n}{n\sqrt{n}\,11^n}$: converges Hide Steps

$\sum_{n=1}^{\infty} \frac{(-11)^n}{n\sqrt{n} \cdot 11^n}$

$(-11)^n = 11^n(-1)^n$

$= \sum_{n=1}^{\infty} \frac{11^n(-1)^n}{n\sqrt{n} \cdot 11^n}$

Cancel the common factor: 11^n

$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

Apply Alternating Series Test: converges Hide Steps

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$

Alternating Series Test:

Suppose that for a_n, there exists an N so that for all $n \geq N$

1. a_n is positive and monotone decreasing
2. $\lim_{n\to\infty} a_n = 0$

Then the alternating series $\sum(-1)^n a_n$ and $\sum(-1)^{n-1} a_n$ both converge

$a_n = \frac{1}{n\sqrt{n}}$

a_n is positive and monotone decreasing from N = 1

$\lim_{n\to\infty}\left(\frac{1}{n\sqrt{n}}\right) = 0$ *Hide Steps*

$\lim_{n\to\infty}\left(\frac{1}{n\sqrt{n}}\right)$

$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}, \quad \lim_{x\to a} g(x) \neq 0$

With the exception of indeterminate form

$= \frac{\lim_{n\to\infty}(1)}{\lim_{n\to\infty}(n\sqrt{n})}$

$\lim_{n\to\infty}(1) = 1$ *Show Steps*

$\lim_{n\to\infty}(n\sqrt{n}) = \infty$ *Show Steps*

$= \frac{1}{\infty}$

Apply Infinity Property: $\frac{c}{\infty} = 0$

$= 0$

By the alternating series test criteria

$=$ converges

$=$ converges

For $x = 11$, $\sum_{n=0}^{\infty}\frac{11^n}{n\sqrt{n}\,11^n}$: converges *Hide Steps*

$\sum_{n=0}^{\infty}\frac{11^n}{n\sqrt{n}\cdot 11^n}$

Refine

$= \sum_{n=0}^{\infty}\frac{1}{n\sqrt{n}}$

Simplify $n\sqrt{n}$: $n^{\frac{3}{2}}$ *Show Steps*

$= \sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

Apply p – Series Test: converges *Hide Steps*

$\sum_{n=1}^{\infty}\frac{1}{n^{\frac{3}{2}}}$

p – Series Test:

If the series is of the form $\sum_{n=1}^{\infty}\frac{1}{n^p}$, where $p > 0$

If $p > 1$, then the p – series converges
If $0 < p \le 1$, then the p – series diverges

$p = \frac{3}{2}$, $p > 1$, by the p – Series test criteria

$=$ converges

$=$ converges

$x = -11$:converges, $x = 11$:converges

Therefore

Interval of convergence is $-11 \le x \le 11$

Interval of convergence is $-11 \le x \le 11$

Radius of convergence is 11, Interval of convergence is $-11 \le x \le 11$