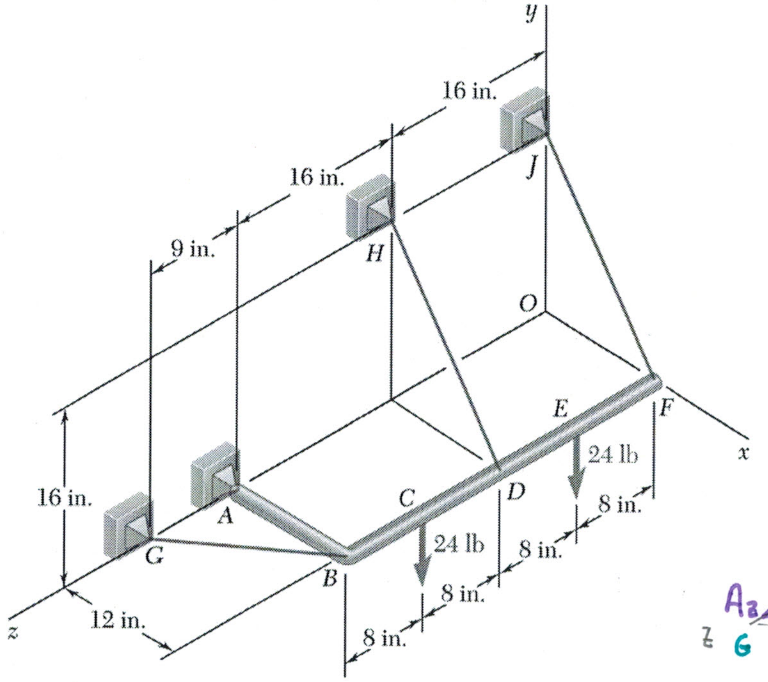
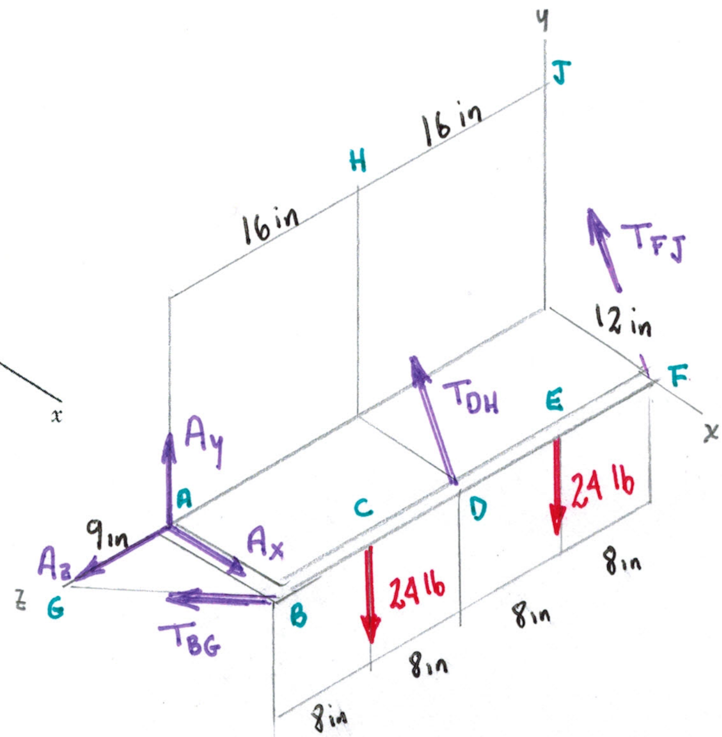
The bent rod is supported by a ball-and-socket joint at A and by three cables. Determine the tension in each cable and the reaction at A.





Resolve Forces

$$T_{BG} = \|T_{BG}\| \hat{u}_{BG} = \|T_{BG}\| \frac{r_{BG}}{\|r_{BG}\|} = \|T_{BG}\| \left\{ \frac{-12\hat{i} + 0\hat{j} + 9\hat{k}}{15} \right\} = \|T_{BG}\| \left\{ -0.8\hat{i} + 0\hat{j} + 0.6\hat{k} \right\}$$

$$T_{DH} = \|T_{DH}\| \hat{u}_{DH} = \|T_{DH}\| \frac{r_{DH}}{\|r_{DH}\|} = \|T_{DH}\| \left\{ \frac{-12\hat{i} + 16\hat{j} + 0\hat{k}}{20} \right\} = \|T_{DH}\| \left\{ -0.6\hat{i} + 0.8\hat{j} + 0\hat{k} \right\}$$

$$T_{FJ} = \|T_{FJ}\| \hat{u}_{FJ} = \|T_{FJ}\| \frac{r_{FJ}}{\|r_{FJ}\|} = \|T_{FJ}\| \left\{ -0.6\hat{i} + 0.8\hat{j} + 0\hat{k} \right\}$$

There are six unknowns and six equations, $\Sigma F = 0$ and $\Sigma M = 0$. If we take moments about A, there will be three equations in three unknowns.

$$\sum M_A = 0$$

External forces

$$(r_{AB} \times T_{BG}) + (r_{AD} \times T_{DH}) + (r_{AF} \times T_{FJ}) + (r_{AC} \times -24\hat{j}) + (r_{AF} \times -24\hat{j}) = 0$$

$$\begin{vmatrix} i & j & k \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} \|T_{BG}\| + \begin{vmatrix} i & j & k \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} \|T_{DH}\| + \begin{vmatrix} i & j & k \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} \|T_{FJ}\| + \begin{vmatrix} i & j & k \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

$\hat{i}$ components $\quad 12.8\|T_{DH}\| + 25.6\|T_{FJ}\| - 192 - 576 = 0$

$\hat{j}$ components $\quad -7.2\|T_{BG}\| + 9.6\|T_{DH}\| + 19.2\|T_{FJ}\| = 0$

$\hat{k}$ components $\quad 9.6\|T_{DH}\| + 9.6\|T_{FJ}\| - 288 - 288 = 0$

In matrix form

$$\begin{bmatrix} 0 & 12.8 & 25.6 \\ -7.2 & 9.6 & 19.2 \\ 0 & 9.6 & 9.6 \end{bmatrix} \begin{bmatrix} \|T_{BG}\| \\ \|T_{DH}\| \\ \|T_{FJ}\| \end{bmatrix} = \begin{bmatrix} 768 \\ 0 \\ 576 \end{bmatrix} \Longrightarrow \begin{array}{l} \|T_{BG}\| = 80 \text{ lb} \\ \|T_{DH}\| = 60 \text{ lb} \\ \|T_{FJ}\| = 0 \text{ lb} \end{array}$$

$\sum F_x = 0$

$$A_x - 0.8\|T_{BG}\| - 0.6\|T_{DH}\| - 0.6\|T_{FJ}\| = 0$$

$$A_x - 0.8(80) - 0.6(60) - 0.6(0) = 0 \quad \Longrightarrow \quad A_x = 100 \text{ lb}$$

$\sum F_y = 0$

$$A_y - 24 + \|T_{DH}\|(0.8) - 24 + \|T_{FJ}\|(0.8) = 0$$

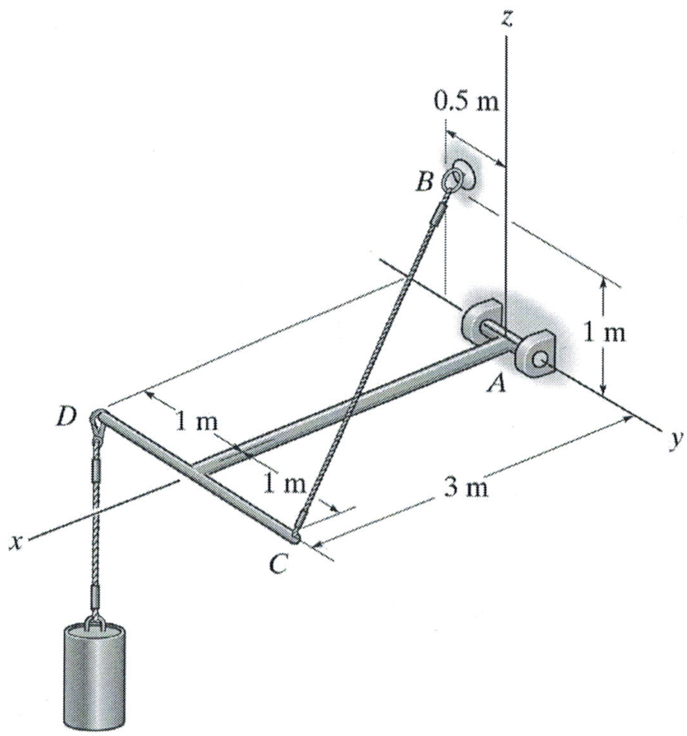
$$A_y - 24 + 60(0.8) - 24 + (0)(0.8) = 0 \quad \Longrightarrow \quad A_y = 0 \text{ lb}$$

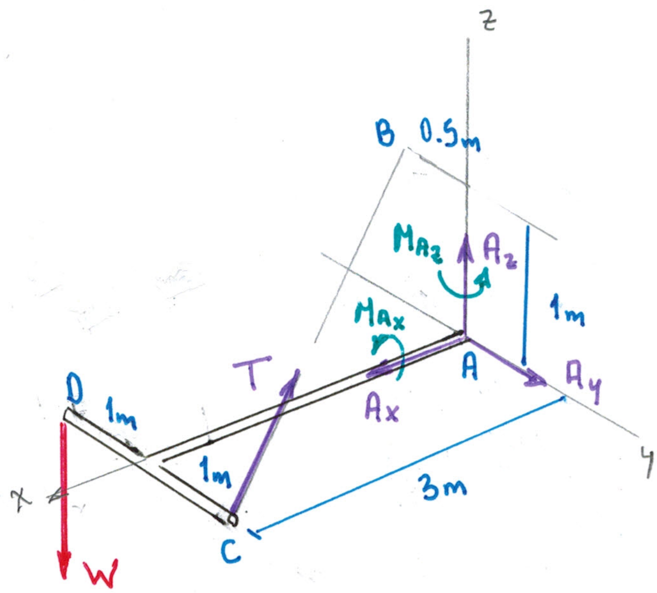
$\sum F_z = 0$

$$A_z + \|T_{BG}\|(0.6) = 0$$

$$A_z + (80)(0.6) = 0 \quad \Longrightarrow \quad A_z = -48 \text{ lb}$$

The member is supported by a pin at A and cable BC. Determine the components of reaction at these supports if the cylinder has a mass of 40 kg.





Resolve Forces and Moments

$$T = \|T\| u_{CB} = \|T\| \frac{r_{CB}}{\|r_{CB}\|} = \|T\| \left\{ -\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k} \right\}$$

$$W = -40(9.81)\hat{k} = -392.4\hat{k} \quad \text{or} \quad W = \{0\hat{i} + 0\hat{j} - 392.4\hat{k}\}$$

$$M_A = \{M_{Ax}\hat{i} + 0\hat{j} + M_{Az}\hat{k}\}$$

$$F_A = \{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\}$$

There are six equations in six unknowns. If we take moments about A, there will be three equations in three unknowns $\|T\|$, M_{Ax} and M_{Az}

$$\sum M_A = 0$$

$$(r_{AC} \times T) + (r_{AD} \times W) + M_A = 0$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ -6/7 & -3/7 & 2/7 \end{vmatrix} \|T\| + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 0 & 0 & -392.4 \end{vmatrix} + M_{Ax}\hat{i} + M_{Az}\hat{k} = 0$$

$\hat{i}$ component $\quad \frac{2}{7}\|T\| + 392.4 + M_{Ax} = 0 \quad (1)$

$\hat{j}$ component $\quad -\frac{6}{7}\|T\| + 1177.2 = 0 \quad (2)$

$\hat{k}$ component $\quad \left(-\frac{9}{7} + \frac{6}{7}\right)\|T\| + M_{Az} = 0 \quad (3)$

From (2) $\quad \|T\| = 1373.4\ N$

From (1) $\quad M_{Ax} = -784.8\ N \cdot m$

From (3) $\quad M_{Az} = 589\ N \cdot m$

$$\sum F = 0$$

$$\left(-\frac{6}{7}\|T\| + A_x\right)\hat{i} + \left(-\frac{3}{7}\|T\| + A_y\right)\hat{j} + \left(\frac{2}{7}\|T\| - 392.4 + A_z\right)\hat{k} = 0$$

$\hat{i}$ component $\quad -\frac{6}{7}(1373.4) + A_x = 0 \quad A_x = 1177.2\ N$

$\hat{j}$ component $\quad -\frac{3}{7}(1373.4) + A_y = 0 \quad A_y = 588.6\ N$

$\hat{k}$ component $\quad \frac{2}{7}(1373.4) - 392.4 + A_z = 0 \quad A_z = 0\ N$