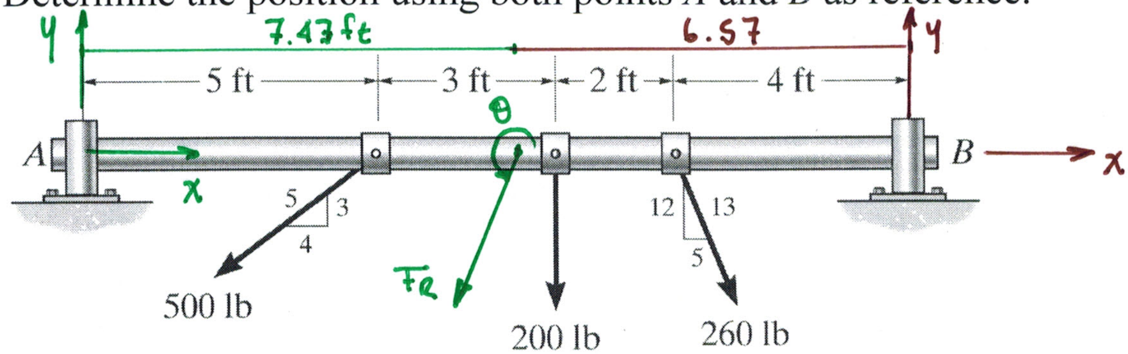
Replace the three forces acting on the shaft by a single resultant force. Determine the position using both points *A* and *B* as reference.



a) Resultant Force

$F_{R_x} = \Sigma F_x \qquad \Sigma F_x = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb}$

$F_{R_y} = \Sigma F_y \qquad \Sigma F_y = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb}$

$\|F_R\| = \sqrt{300^2 + 740^2} = 798.5 \text{ lb}$

$\theta = \tan^{-1}\left(\frac{-740}{-300}\right) = 67.9°$ Note, since denominator is negative we need to add 180°, i.e. $\theta = 247.9$

$M_{R_A} = \Sigma M_A \qquad \Sigma M_A = -500\left(\frac{3}{5}\right)(5) - 200(8) - 260\left(\frac{12}{13}\right)(10)$

(Only y components produce moment about A) $= -5500 \text{ lb}\cdot\text{ft}$

The resultant force F_R that produces a moment of $M_{R_A} = -5500$ lb·f must be placed at an x distance from A.

$$(-740)\,x = -5500 \text{ lb}\cdot\text{ft}$$

$$x = \frac{-5500}{-740} = 7.43 \text{ ft}$$

b) Same Resultant Force

$$\overline{F_R} = \{-300i - 740j\} \text{ lb} \quad \|\overline{F_R}\| = 798.5 \text{ lb} \quad \text{at } \theta = 247.9°$$

$$M_{R_B} = \Sigma M_B \qquad \Sigma M_B = -260\left(\frac{12}{13}\right)(-4) - 200(-6) - 500\left(\frac{3}{5}\right)(-9)$$

$$= 4860 \text{ lb} \cdot \text{ft}$$

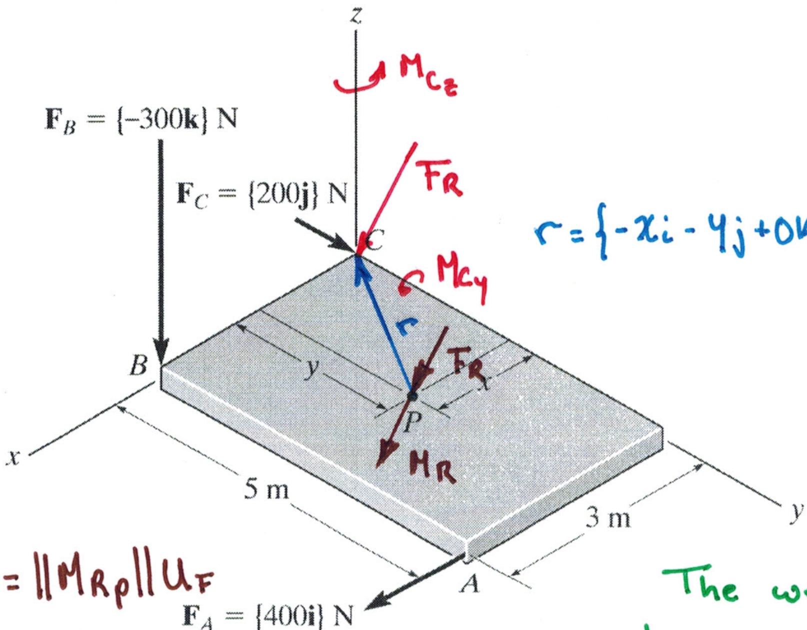
The resultant moment that produces a moment $M_{R_B} = 4860$ lb·ft

must be placed at an x distance from B.

$$(-740)(x) = 4860$$

$$x = \frac{4860}{-740} = -6.57 \text{ ft}$$

Replace the three forces acting on the plate by a wrench. Specify the location where the wrench intersects the x-y plane.



$r = \{-xi - yj + 0k\}$

Force vector:

$\overline{F_R} = \{400i + 200j - 300k\}$ N

$\|F_R\| = \sqrt{400^2 + 200^2 + 300^2} = 538.5$ N

Unit vector:

$u_F = \dfrac{F_R}{\|F_R\|} = \{0.7428i + 0.3714j - 0.5571k\}$

$M_{R_P} = \|M_{R_P}\| u_F$

The wrench is a quantity that combines the force and the moment vectors (collinear).

Before we find the coordinates of P (location where wrench intersects plate), we can find the moment about point C.

$$\Sigma M_C = \{0i + \underbrace{900}_{300(3)}j - \underbrace{2000}_{(400)(5)}k\} \text{ N.m}$$

Now we translate the force vector and moment vector to point P.

We know that M_R has the same direction as $\overline{F_R}$

$$M_{R_P} = \Sigma M_C + \Sigma(r \times F_R) = \{0i + 900j - 2000k\} + \begin{vmatrix} i & j & k \\ -x & -y & 0 \\ 400 & 200 & -300 \end{vmatrix}$$

$$= \{0i + 900j - 2000k\} + \{300yi - 300xj + (-200x + 400y)k\}$$

$\|M_{R_P}\| u_x = 300y$

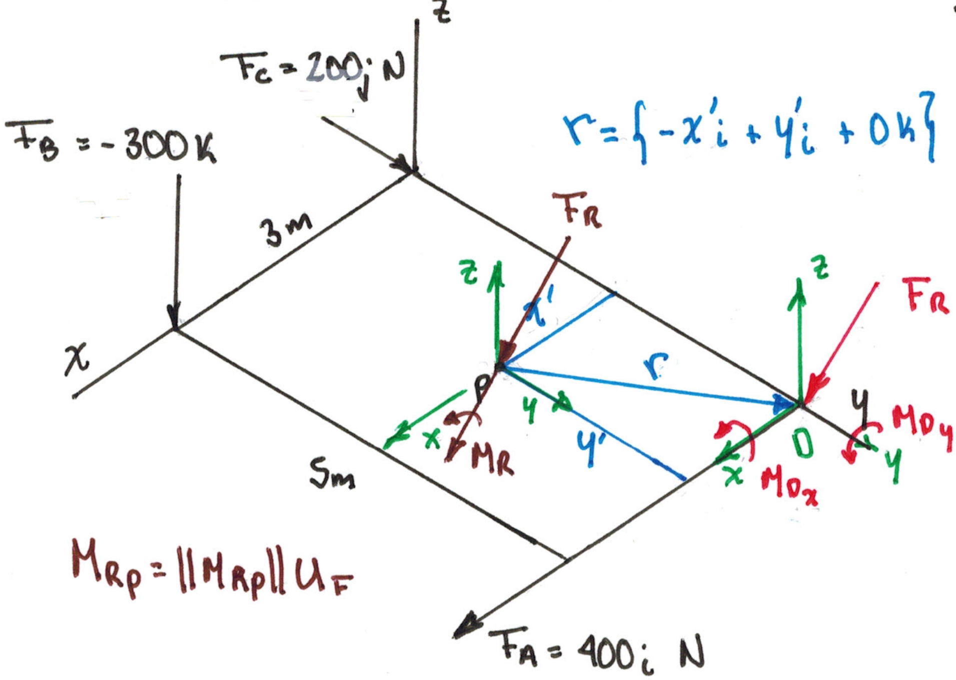
$\|M_{R_P}\| u_y = 900 - 300x$

$\|M_{R_P}\| u_z = -2000 + 400y - 200x$

$$\Rightarrow \begin{bmatrix} 0 & -300 & 0.7428 \\ 300 & 0 & 0.3714 \\ 200 & -400 & -0.5571 \end{bmatrix} \begin{bmatrix} x \\ y \\ \|M_{R_P}\| \end{bmatrix} = \begin{bmatrix} 0 \\ 900 \\ -2000 \end{bmatrix}$$

$x = 1.2069$ m, $y = 3.5862$ m, $\|M_{R_P}\| = 1{,}448.4$ N.m

Note, we could have selected any other point to calculate moments



$M_{Rp} = \|M_{Rp}\| u_F$

- Moment about D

Only F_B creates a moment about point D

$$\Sigma M_D = \{300(5)i + 300(3)j + 0k\}$$
$$= \{1500i + 900j + 0k\}$$

Now, we translate F_R and M_D to point P

$$M_{Rp} = \Sigma M_D + \Sigma(r \times F_R) = \{1500i + 900j + 0k\} + \begin{vmatrix} i & j & k \\ -x' & y' & 0 \\ 400 & 200 & -300 \end{vmatrix}$$

$$= \{1500i + 900j + 0k\} + \{-300y'i - 300x'j - (200x' + 400y')k\}$$

$$\|M_{Rp}\| u_x = 1500 - 300y'$$
$$\|M_{Rp}\| u_y = -300x' + 900$$
$$\|M_{Rp}\| u_z = -200x' - 400y'$$

$\Longrightarrow$

$$300y' + \|M_{Rp}\| u_x = 1500$$
$$300x' + \|M_{Rp}\| u_y = 900$$
$$200x' + 400y' + \|M_{Rp}\| u_z = 0$$

$$\begin{bmatrix} 0 & 300 & 0.7428 \\ 300 & 0 & 0.3714 \\ 200 & 400 & -0.5571 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ \|M_{Rp}\| \end{bmatrix} = \begin{bmatrix} 1500 \\ 900 \\ 0 \end{bmatrix}$$

$x' = 1.2069$ m

$y' = 1.4138$

$\|M_{Rp}\| = 1,448.4$ N.m

where $y = 5 - y' = 5 - 1.4138 = 3.5862$ m