Stat 260 Lecture Notes
Set 4 - Basic Set Theory

The **sample space** $\mathcal{S}$ of a random experiment is a list of all the possible outcomes of the experiment.

Example 1: The students in our Stat 260 class are asked to report what month they were born in.

A **simple event** is a single outcome from the sample space.

e.g. person is born in March

An **event** is a set of one or more simple events.

e.g. the event $E = \{\text{Jan}, \text{Feb}, \text{Mar}\}$ = person is born in the first three months of the year

There are multiple ways to assign probabilities to events:

- **classical approach:** used when possible outcomes are equally likely

$$P(A) = \frac{\#A}{\#\mathcal{S}} = \frac{\#\text{ of ways event } A \text{ can occur}}{\#\text{ of outcomes in } \mathcal{S}}$$

- **frequency approach:** counts the number of times the event occurs in many, many observations

$$P(A) = \lim_{N \to \infty} \left(\frac{\#\text{ of occurrences of } A}{N} \right)$$

In all models, the probability of an event is the sum of its simple events.

Example 2: Calculate P(born in the first three months) using the classical approach.

Tree diagrams help us list all possible outcomes in the sample space.

Example 3: Look at the experiment where we flip a fair coin 3 times and we record the sequence of heads and tails.

The **union** of events A and B, denoted $A \cup B$, is read as "A or B". The set $A \cup B =$ outcomes in A or B or in both.

The **intersection** of events A and B, denoted $A \cap B$, is read as "A and B". The set $A \cap B =$ outcomes that are in both A and B.

In other words, the list of events in $A \cup B$ is the lists from A and B combined together as one larger list. The list of events in $A \cap B$ is the overlap in the lists from A and B.

Sometimes we write AB as a shorter version of $A \cap B$.

The event $\emptyset$ is the **empty event** / **impossible event**.

Rule: $P(\emptyset) = 0$.

The events A and B are **mutually exclusive** (or **disjoint**) if $A \cap B = \emptyset$. (That is, there is no overlap between events A and B.)

So if A and B are mutually exclusive, then $P(A \cap B) = P(\emptyset) = 0$.

The **complement** of an event A, denoted $\overline{A}$, is the set of all outcomes in $\mathcal{S}$ that are not in A.

Venn Diagrams help us picture probabilities.

Rule: DeMorgan's Laws

- $\overline{A \cup B} = \overline{A} \cap \overline{B}$
- $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Note: the phrase "nor" can be translated as "and not". For example "neither A nor B" is the same as "not A and not B".