Stat 260 Lecture Notes
Set 21 - The Central Limit Theorem

independence

Rule (from Set 20): Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from a distribution that is normal with mean μ and standard deviation σ. Then $\overline{X}$ is normal with mean μ and standard deviation $\sigma/\sqrt{n}$.

Furthermore $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ is standard normal.

$$Z = \frac{\text{r.v} - \text{expected value}}{\text{st. dev. of r.v}}$$

Central Limit Theorem (CLT): Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from *any* distribution with mean μ and standard deviation σ. For a large n ($n \geq 30$), then $\overline{X}$ is approximately normal with mean μ and standard deviation $\sigma/\sqrt{n}$.

Furthermore $Z = \dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ is standard normal.

Rule: Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from *any* distribution with mean μ and unknown standard deviation. (i.e. We don't know σ, so we use s instead.) For a large n ($n \geq 30$), then $\overline{X}$ is approximately normal with mean μ and standard deviation $s/\sqrt{n}$.

Furthermore $Z = \dfrac{\overline{X} - \mu}{s/\sqrt{n}}$ is standard normal.

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \frac{\overline{X} - \mu}{s/\sqrt{n}}$$

When we don't know σ

Example 1: Suppose the height of students in a statistics class has a mean of $\mu = 156cm$ and a standard deviation of $\sigma = 10cm$. What is the probability that the mean height for a random 50 students is between $152cm$ and $155cm$?

$\uparrow \bar{X}$ $\uparrow n = 50 \geq 30$

So use C.L.T. $\therefore$ $\bar{X}$ is normal

$$Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$$

$$P(152 \leq \bar{X} \leq 155)$$

$$= P\left(\frac{152-156}{10/\sqrt{50}} \leq \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \leq \frac{155-156}{10/\sqrt{50}}\right)$$

$$= P(-2.83 \leq Z \leq -0.71)$$

$$= P(Z \leq -0.71) - P(Z \leq -2.83)$$

$$= 0.2389 - 0.0023$$

$$= 0.2366$$