Chapter 3 – Equilibrium of a Particle

Contents

When the load is lifted at constant speed, or is just suspended, then it is in a state of equilibrium.

Equilibrium of a Particle

Recalling the Definition of a Particle

As engineers, we make assumptions to simplify the application of the theory, for example neglecting small deformations on a rigid body or concentrating forces at one point.

We can make idealizations to simplify the application of the theory

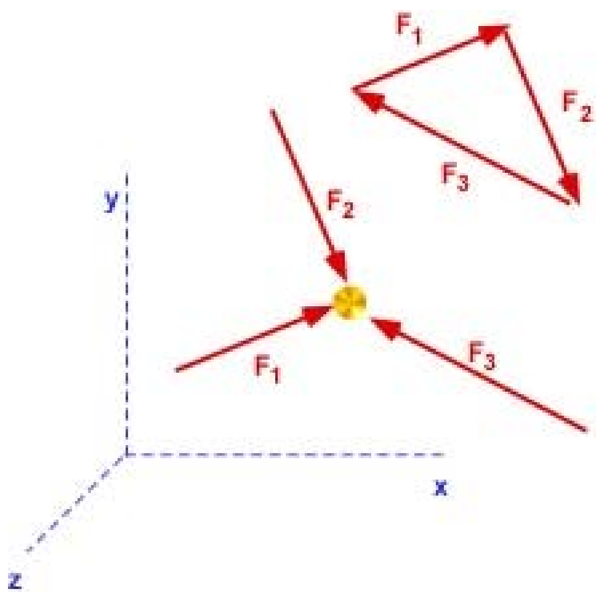
Particle – A particle has a mass, but the size can be neglected.
Some bodies can be idealized as particles. These are bodies where we are not interested about their rotation. The Earth can be considered as a particle moving around its orbit, with all its mass concentrated at the centre of mass.

Rigid Body – A rigid body is a combination of a large number of particles that remain at a fixed distance from one another.
Here the effect of rotations is critical.

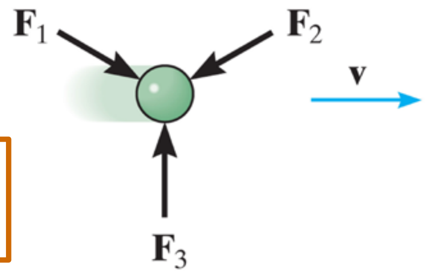
Equilibrium of a Particle

Condition for the Equilibrium of a Particle

When the resultant of all forces acting on a particle is zero, the particle is in ***equilibrium***, i.e., there is no unbalanced force.



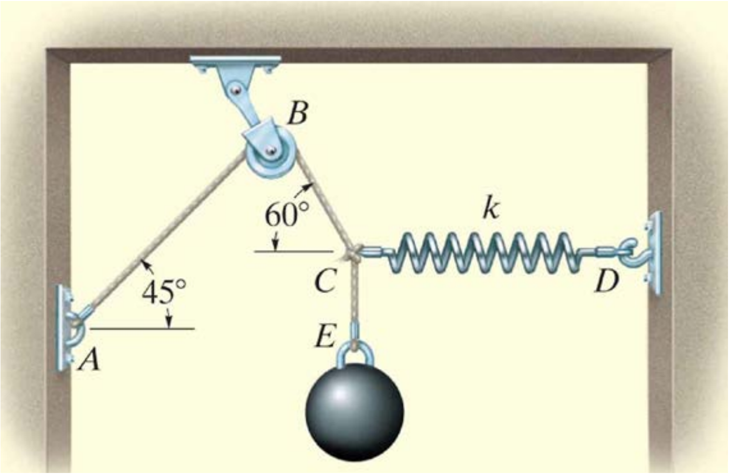
Recalling Newton's First Law:
A particle will remain at rest or moving in a straight line with a constant velocity if it is NOT acted upon by an unbalanced external force

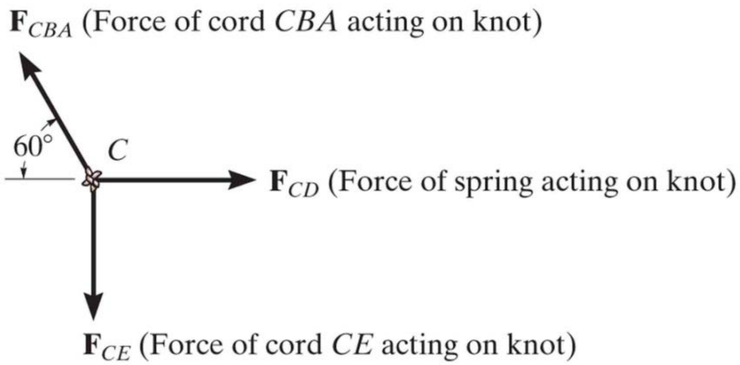


$$\boxed{\mathbf{F}_R = \sum \mathbf{F}_i = \mathbf{0}}$$

Free Body Diagrams

The **Free Body Diagram (FBD)** is a tool to help identify the external forces acting on particles inside a more complex system.



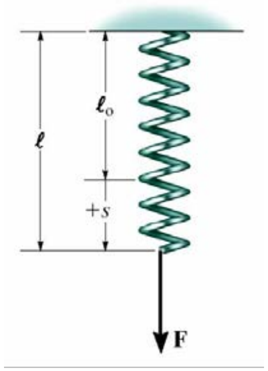


Free Body Diagrams

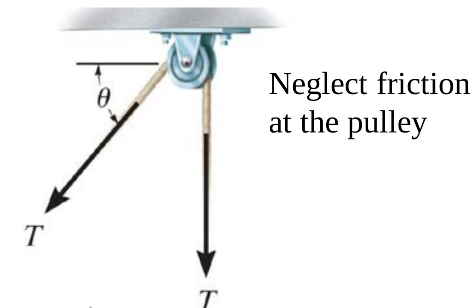
Supports that often appear while solving particle equilibrium problems

Springs The length of a linearly elastic spring the spring will change in direct proportion to the force acting on it. The change in distance ***s*** depends on the *stiffness* ***k*** of the spring.

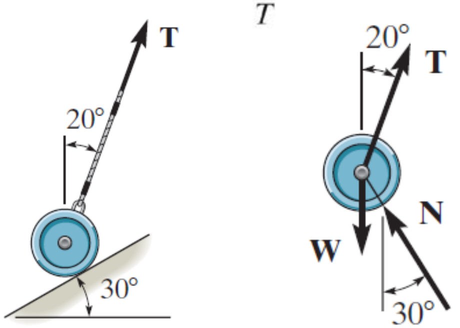
$$\boxed{F = ks}$$



Cables and Pulleys Cables will be assumed to have negligible weight and cannot stretch. They can only support tension force in the direction of the cable. The tension must have a constant magnitude.



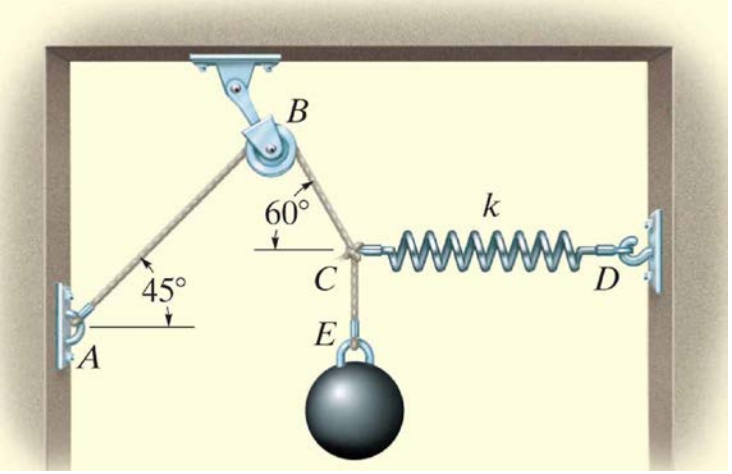
Smooth Contact If an object rests on a smooth surface, then the surface will exert a force on the object that is normal to the surface.

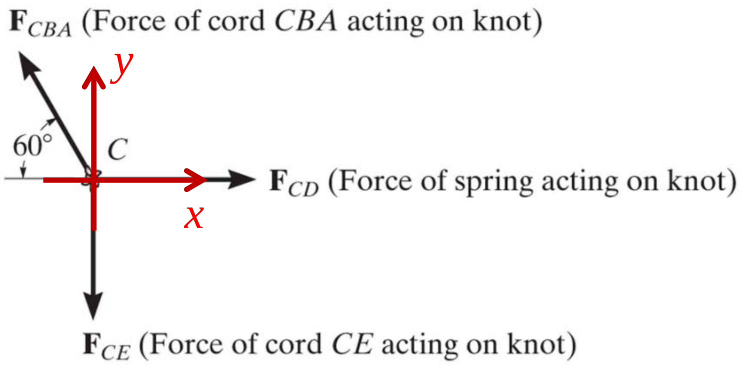


Free Body Diagrams

Creating the Free Body Diagram

- Identify the particle of interest.
- Choose and draw the frame of reference.
- Cut the particle free from its surroundings and draw its outlined shape within the frame of reference.
- Draw all the forces that are acting on the particle.
- Indicate any known magnitudes and/or orientations of the forces.

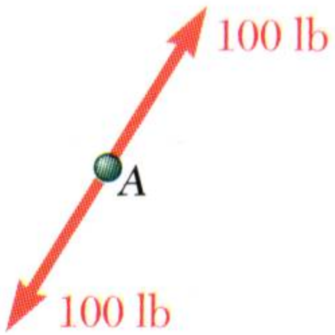




Equilibrium of a Particle

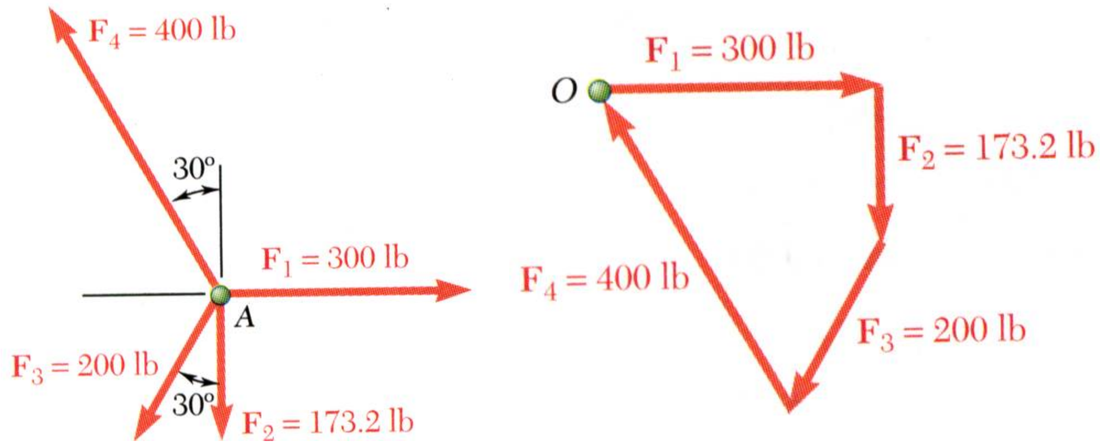
Thus, the resultant of all the forces must equal zero.

Two forces:



- equal magnitude
- same line of action
- opposite sense

Multiple Forces:



- graphical solution: closed polygon
- algebraic solution

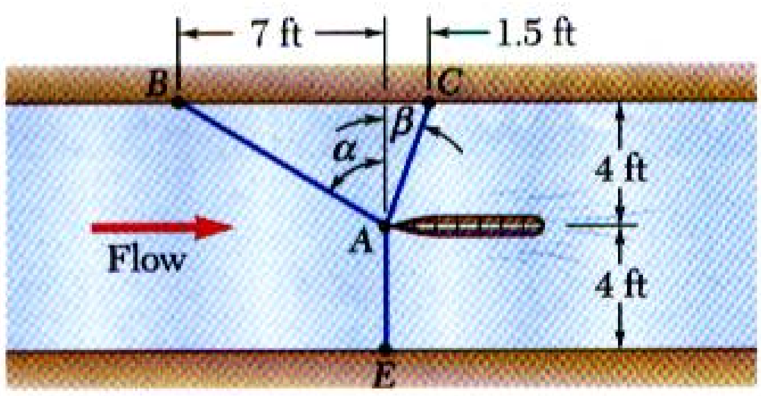
$$\mathbf{F}_R = \sum \mathbf{F}_i = \left\{\sum F_x \boldsymbol{i} + \sum F_y \boldsymbol{j}\right\} = \mathbf{0}$$

$$\sum F_x = 0 \ \text{ and } \ \sum F_y = 0$$

Example

Given: The drag of a prototype sailboat hull is being tested. Three cables are used to align its bow on the channel centerline. For a given speed, the tension in cables AB and AE are 40 lb and 60 lb, respectively.

Find: Determine the drag force and the tension in cable AC.



Three-Dimensional Force Systems

Equation of Equilibrium

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero $(\Sigma\, \mathbf{F} = 0)$.

This equation can be written in terms of its x, y and z components.

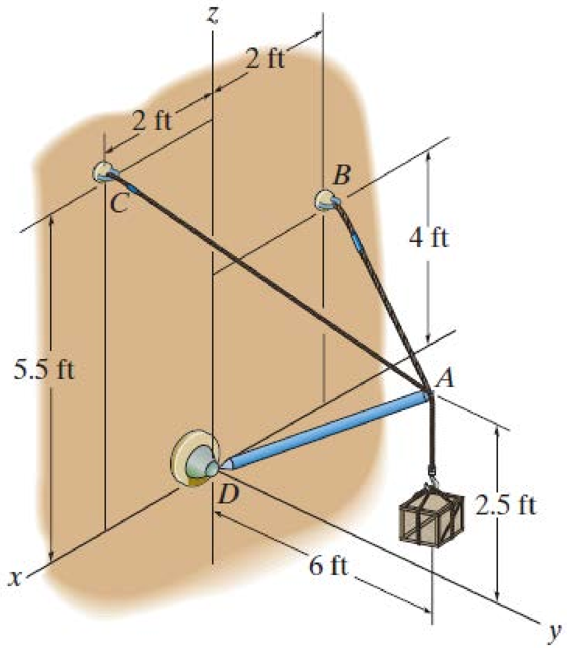
$$(\Sigma\, F_x)\, \boldsymbol{i} + (\Sigma\, F_y)\, \boldsymbol{j} + (\Sigma\, F_z)\, \boldsymbol{k} = 0$$

This vector equation will be satisfied only when

$$\boxed{\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0}$$

These equations are the **three scalar equations of equilibrium**. They are valid for any point in equilibrium and allow you to solve for up to three unknowns.

Example



Given: A 400 lb crate, as shown, is in equilibrium and supported by two cables and a strut *AD*.

Find: Magnitude of the tension in each of the cables and the force developed along strut *AD*.

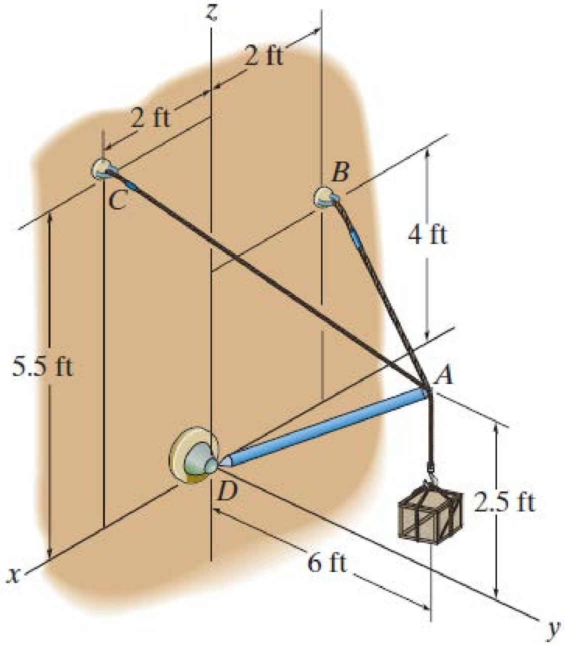
Plan:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B , F_C , and F_D.

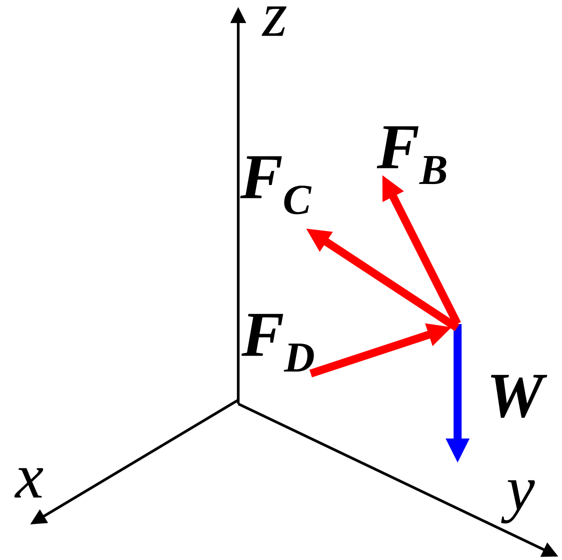
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns

Example



FBD of Point A



W = weight of crate = **-** 400 ***k*** lb

$$\mathbf{F}_{\boldsymbol{B}} = F_B(\boldsymbol{r}_{\boldsymbol{AB}}/r_{AB}) = F_B\{(-2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 1.5\,\boldsymbol{k}) / (6.5)\}\ \text{lb}$$

$$\mathbf{F}_{\boldsymbol{C}} = F_C(\boldsymbol{r}_{\boldsymbol{AC}}/r_{AC}) = F_C\{(2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 3\,\boldsymbol{k}) / (7)\}\text{lb}$$

$$\mathbf{F}_{\boldsymbol{D}} = F_D(\boldsymbol{r}_{\boldsymbol{AD}}/r_{AD}) = F_D\{(6\,\boldsymbol{j} + 2.5\,\boldsymbol{k}) / (6.5)\}\text{lb}$$

Example

The particle A is in equilibrium, hence

$$\mathbf{F_B} + \mathbf{F_C} + \mathbf{F_D} + \mathbf{W} = \mathbf{0}$$

Equate the respective ***i***, ***j***, ***k*** components to zero

$$\sum F_x = -(4/13)\,F_B + (2/7)\,F_C = 0 \qquad (1)$$

$$\sum F_y = -(12/13)\,F_B - (6/7)\,F_C + (12/13)\,F_D = 0 \qquad (2)$$

$$\sum F_z = (3/13)\,F_B + (3/7)\,F_C + (5/13)\,F_D - 400 = 0 \qquad (3)$$

Solve the three simultaneous equations of the form **Ax = b**

$F_B = 274$ lb

$F_C = 295$ lb

$F_D = 547$ lb

$$\begin{bmatrix} -4/13 & 2/7 & 0 \\ -12/13 & -6/7 & 12/13 \\ 3/13 & 3/7 & 5/13 \end{bmatrix} \begin{bmatrix} F_B \\ F_C \\ F_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 400 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Matlab: `x = A\b`

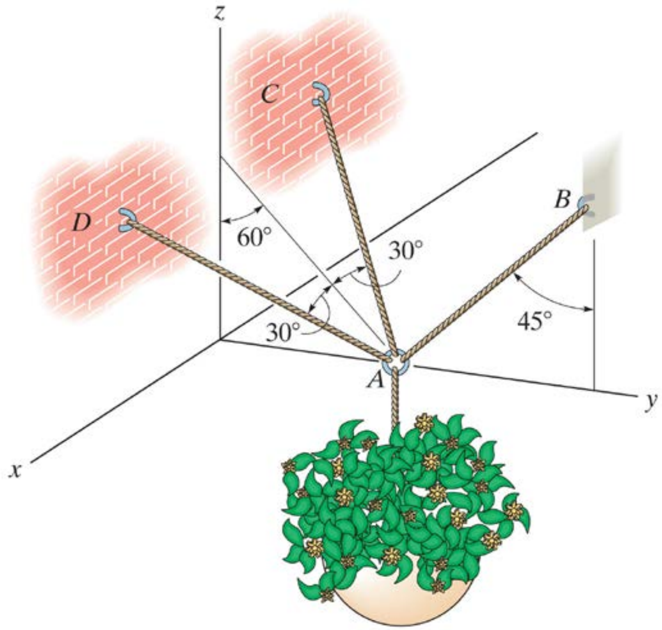
Example

$$\begin{bmatrix} -4/13 & 2/7 & 0 \\ -12/13 & -6/7 & 12/13 \\ 3/13 & 3/7 & 5/13 \end{bmatrix} \begin{bmatrix} F_B \\ F_C \\ F_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 400 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

Matlab: `x = A\b`

Example



Given: The 25 kg flowerpot is supported at *A* by three cords.

Find: The tension in each of the cords for equilibrium.

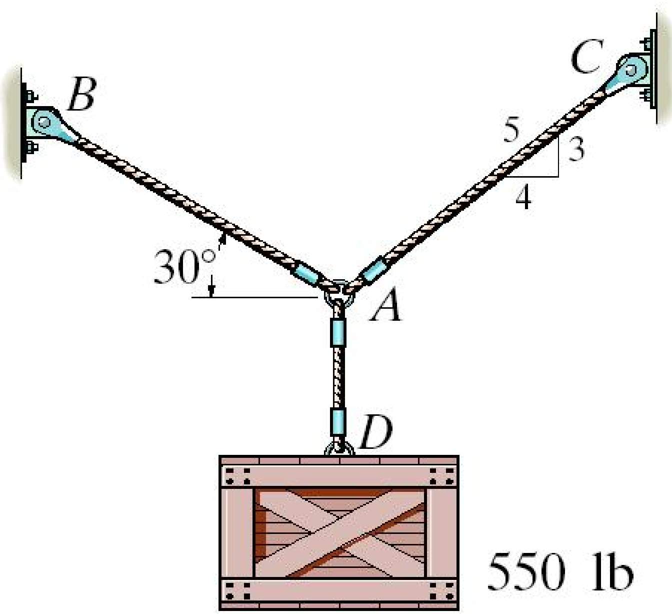
Plan:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes of the cords.

2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns

Sample Problem (§ 3.3)



Given: The box weighs 550 lb and geometry is as shown.

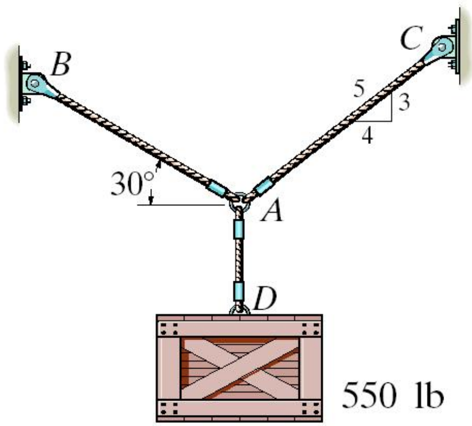
Find: The forces in the ropes AB and AC.

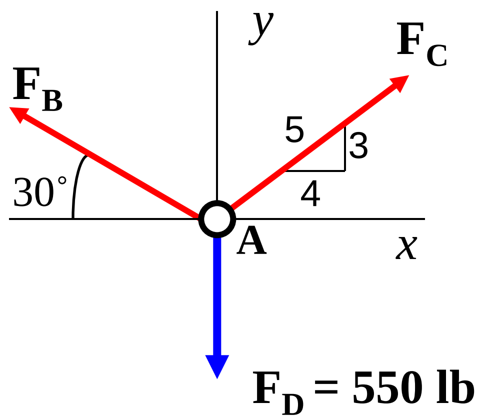
Plan:

1. Draw a FBD for point A.

2. Apply the equations of equilibrium to solve for the forces in ropes AB and AC.

FBD at point A





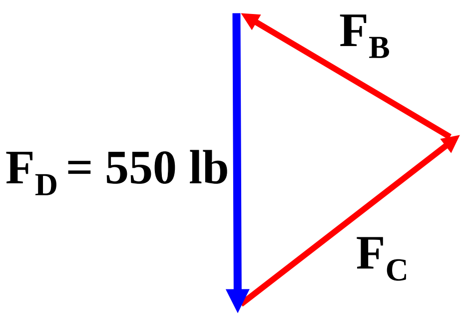
Apply the scalar equations of equilibrium at A, we get;

$$+\rightarrow \sum F_x = -F_B \cos 30° + F_C\,(4/5) = 0$$

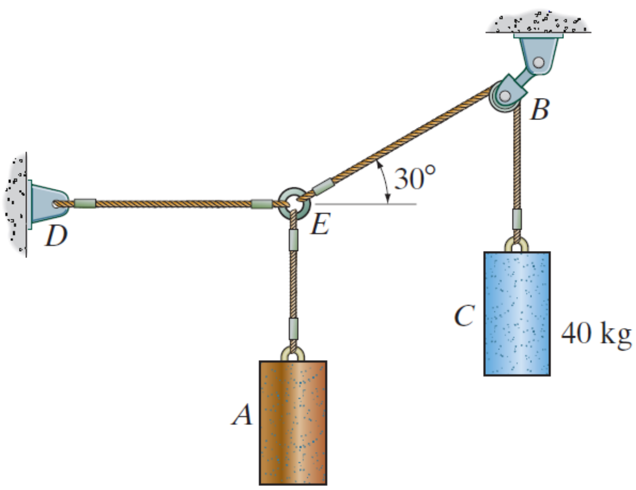
$$+\uparrow \sum F_y = F_B \sin 30° + F_C\,(3/5) - 550 \text{ lb} = 0$$

Solve the above equations, we get;

$F_B = 478$ lb $\nwarrow$ and $F_C = 518$ lb $\nearrow$



Sample Problem (§ 3.3)



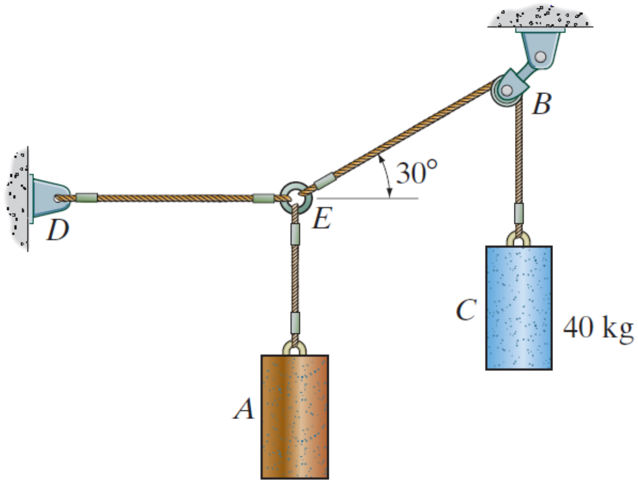
Given: The mass of cylinder C is 40 kg and geometry is as shown.

Find: The tensions in cables DE, EA, and EB.

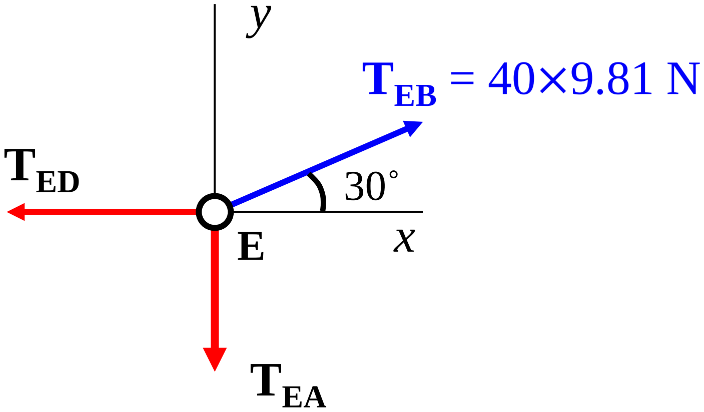
Plan:

1. Draw a FBD for point E.

2. Apply the E-of-E to solve for the forces in cables DE, EA, and EB.



FBD at point E



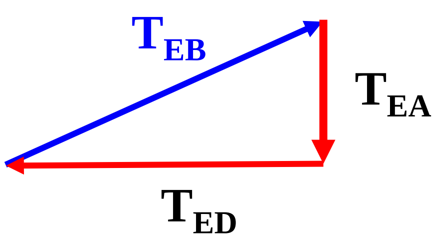
Applying the scalar equations of equilibrium at E, we get;

$$+\rightarrow \sum F_x = -T_{ED} + (40\times 9.81)\cos 30° = 0$$

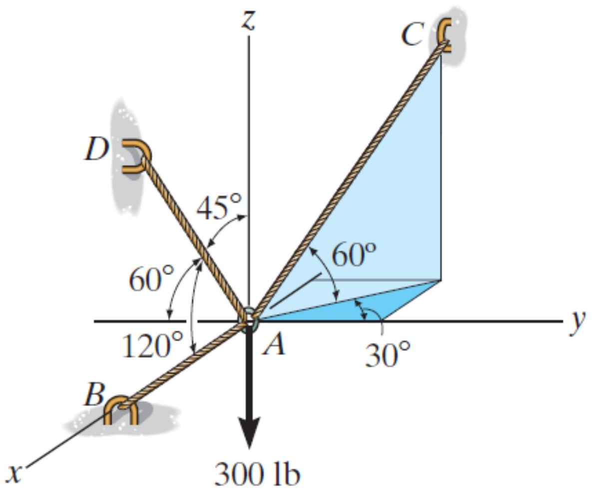
$$+\uparrow \sum F_y = (40\times 9.81)\sin 30° - T_{EA} = 0$$

Solving the above equations, we get;

$T_{ED} = 340\text{ N} \leftarrow$ and $T_{EA} = 196\text{ N} \downarrow$



Sample Problem (§ 3.4)



Given: The four forces and geometry shown.

Find: The tension developed in cables *AB*, *AC*, and *AD*.

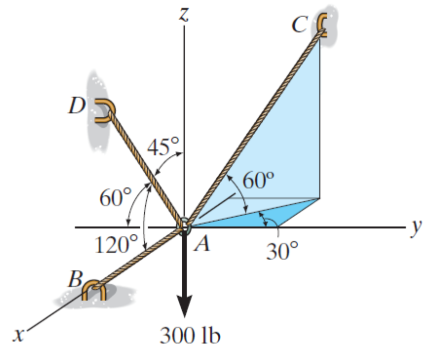
Plan:

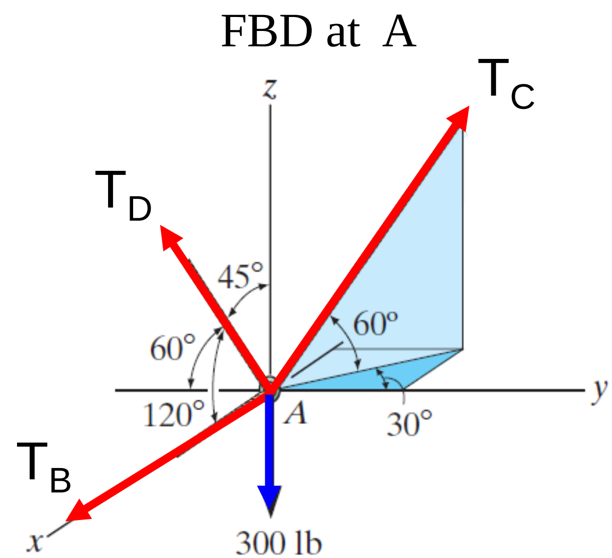
1) Draw a FBD of particle *A*.

2) Write the unknown cable forces T_B, T_C , and T_D in Cartesian vector form.

3) Apply the three equilibrium equations to solve for the tension in cables.

Draw FBD at *A* and Resolve each force





$$\mathbf{T_B} = T_B\ \boldsymbol{i}$$

$$\mathbf{T_C} = -(T_C \cos 60°) \sin 30°\ \boldsymbol{i} + (T_C \cos 60°) \cos 30°\ \boldsymbol{j} + T_C \sin 60°\ \boldsymbol{k}$$

$$\mathbf{T_C} = T_C\,(-0.25\ \boldsymbol{i}\ +0.433\ \boldsymbol{j}\ +0.866\ \boldsymbol{k}\ )$$

$$\mathbf{T_D} = T_D \cos 120°\ \boldsymbol{i} + T_D \cos 120°\ \boldsymbol{j} + T_D \cos 45°\ \boldsymbol{k}$$

$$\mathbf{T_D} = T_D\,(-\,0.5\ \boldsymbol{i} - 0.5\ \boldsymbol{j} + 0.7071\ \boldsymbol{k}\ )$$

$$\boldsymbol{W} = -300\ \boldsymbol{k}$$

Apply equations of equilibrium:

$$\Sigma \mathbf{F}_{\boldsymbol{R}} = 0 = T_B\,\boldsymbol{i} + T_C\,(-\,0.25\,\boldsymbol{i} + 0.433\,\boldsymbol{j} + 0.866\,\boldsymbol{k}\,) + T_D\,(-\,0.5\,\boldsymbol{i} - 0.5\,\boldsymbol{j} + 0.7071\,\boldsymbol{k}\,) - 300\,\boldsymbol{k}$$

Equate the respective ***i***, ***j***, ***k*** components to zero,

$$\Sigma F_x = T_B - 0.25\,T_C - 0.5\,T_D = 0 \qquad (1)$$

$$\Sigma F_y = 0.433\,T_C - 0.5\,T_D = 0 \qquad (2)$$

$$\Sigma F_z = 0.866\,T_C + 0.7071\,T_D - 300 = 0 \qquad (3)$$

This is a system of three linear equations in three unknowns

Use eqs. (2) and (3), to solve for T_C and T_D i.e.,

$$T_C = 203 \text{ lb and } T_D = 176 \text{ lb}$$

Substitute T_C and T_D in eq. (1), to find T_B

$$T_B = 139 \text{ lb}$$