# Unit 2: Truth Tables

Now that we know how to translate English sentences into symbolic logic, we can use these sentences in truth tables to determine the validity of an argument pattern. First, let's summarize what we know about the truth values of each of the logical operators.

| P | Q | P & Q | P ∨ Q | P → Q | ~P |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

You are going to need to know these values well. You may want to keep the above chart by your side as you complete the work on truth tables. One way to remember these values is to focus on the "special cases". For instance, in the "&" column, there is only one time when the "&" is true, that's when both sides are true. So for a conjunction, the sentence will only be true when both sides are true, otherwise the sentence will be false. You don't need to memorize all the different scenarios for when the conjunction is false, it's just all the other cases than T & T.

The special case for the "∨" is the bottom row where F ∨ F is F. All other combinations make the ∨ true.

For the "→" the special case is the second row where T → F is false. Remember, the → is only false when you do your part (the antecedent is true) and I fail to do mine (consequent is false). So, all the other cases will be true.

If you remember these three special cases then you will automatically know all the other entries.

Let's review how we determine the truth value of a longer sentence.

We have already looked at finding the truth value of a complex sentence when we are given the truth value of the atomic sentences.

E.g. $A \ \& \ {\sim}(X \rightarrow B)$ when A, B are true, X is false

We have determined the truth value of this sentence when A and B are true and X is false but what about all the other possible combinations? For example, what is the truth value of the sentence when all of the letters are true? Or when all of the letters are false?

In a truth table, we consider all the possible combinations of truth values for all of the letters and then find the truth values of our sentences for each combination.

The first step in making a truth table is to figure out how many rows you are going to need. There is an easy way to determine this. For each atomic letter in your sentences, multiply by two. So, in our sentence above, we have three letters, A, B and X. So the number of rows that we'll need in our truth table is 2 x 2 x 2 = 8. If we have four letters, we'll need 2 x 2 x 2 x 2 = 16 rows and so on.

Once you know how many rows that you'll need, you need to fill out all the possible combinations of truth values for the atomic letters. There is a specific way that I want you to do this (so all of our answers will look the same).

Now we are going to determine the truth value for complex sentences given all possible combinations of truth values for the parts.

1) Write each atomic letter in a right-hand column (in alphabetical order).
2) There will be 2 x 2 x 2….. n times rows for n atomic letters. E.g. 2 letters need 4 rows, 3 letters need 8 rows, 4 letters need 16 rows.
3) Fill in all the possible combinations for the truth values of the atomic letters.

a) In the first column, the first half of the rows are T and the second half are F.

b) In the second column, for the rows where the first column has T, take the top half and mark T, the bottom half will be F. Repeat for rows where first column was F.

c) Repeat pattern for all other columns.

| A | B | X | A & ~(X → B) |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

Notice that below we have used one column for each step that we must follow to figure out the truth value of the entire sentence. First we figure out the value of the →**.** Then we figure out the ~ and finally, the &. Once you get good at working with truth tables, you'll want to do all the same work in only one column.

Try 1

| A | B | X | X → B | ~(X → B) | A & ~(X → B) |
|---|---|---|---|---|---|
| T | T | T | | | |
| T | T | F | | | |
| T | F | T | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | T | F | | | |
| F | F | T | | | |
| F | F | F | | | |

Try 2

| A | B | X | $A \;\&\; \sim (X \to B)$ |
|---|---|---|---|
| T | T | T | |
| T | T | F | |
| T | F | T | |
| T | F | F | |
| F | T | T | |
| F | T | F | |
| F | F | T | |
| F | F | F | |

Truth Tables for Arguments

An argument is **valid** if it is impossible for its premises to be true and the conclusion false.

The truth table for a valid argument will never have a row where the premises are true and the conclusion is false.

If such a row is found, then the argument is **invalid**.

| P | Q | Premise | Premise | Conclusion |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

| P | Q | Premise | Premise | Conclusion |
|---|---|---|---|---|
| | | | | |
| | | | | |
| | | | | |
| | | | | |

E.g.

If fossil fuel combustion continues at its present rate (C), then a greenhouse effect will occur (G). If a greenhouse effect occurs (G), then world temperatures will rise (W). Therefore, if fossil fuel combustion continues at its present rate (C), then world temperatures will rise (W).

First, symbolize the argument.

Then, construct a truth table.

| C | G | W | $C \rightarrow G$ | $G \rightarrow W$ | $C \rightarrow W$ |
|---|---|---|---|---|---|
| T | T | T | | | |
| T | T | F | | | |
| T | F | T | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | T | F | | | |
| F | F | T | | | |
| F | F | F | | | |

If high school graduates are deficient in reading, they will not be able to compete in the modern world. If high school graduates are deficient in writing, they will not be able to compete in the modern world. Therefore, if high school graduates are deficient in reading, then they are deficient in writing.

$$\begin{array}{l} R \rightarrow {\sim}C \\ \underline{W \rightarrow {\sim}C} \\ R \rightarrow W \end{array}$$

| C | R | W | R → ~C | W → ~C | R → W |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

We can cut down the work in the truth table if we concentrate on the special cases. For instance, in the example above, the major operator for R → ~C is the arrow. We know that the arrow is only false when it is T → F. That means that R → ~C will only be false when R is true and C is true (so ~C will be false). R is true and C is true on the first two rows. So we put a F under the → in the first two rows. Well, if that is the only times that R → ~C will be false, then all the other rows in that column must be T and we quickly fill those in.

$B \rightarrow \sim(C \,\&\, D)$

$\underline{D \vee \sim B}$

$\sim C \rightarrow (D \,\&\, B)$

$\sim A \vee R$

$\sim(N \,\&\, \sim C)$

$R \rightarrow C$

$\underline{C \rightarrow \sim N}$

$A \vee C$

| B | C | D | $B \rightarrow \sim (C \,\&\, D)$ | $D \vee \sim B$ | $\sim C \rightarrow ( D \,\&\, B )$ |
|---|---|---|---|---|---|
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |
| | | | | | |

| A | C | N | R | ~A v R | ~(N & ~C) | R → C | C → ~N | A v C |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |

## Partial Truth Tables

We need to figure out a way to cut down the work needed to do when we are filling out the truth tables. One way to cut down the work is to only do the rows in the truth table that have some chance of being invalid. All other rows are a waste of time to complete.

When we are using a truth table to determine whether or not an argument is valid we are only interested if there are lines where the premises are all true and the conclusion is false.

| A | B | Premise | Premise | Conclusion |
|---|---|---|---|---|
| | | T | F | F |
| | | T | T | F |

All other lines can be ignored.

| A | B | C | A & C | B → C | ~B v A |
|---|---|---|---|---|---|
| T | T | T | | | |
| T | T | F | | | |
| T | F | T | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | T | F | | | |
| F | F | T | | | |
| F | F | F | | | |

| S | T | U | T → ~(S v U) | ~(T & U) | S → U |
|---|---|---|---|---|---|
| T | T | T | | | |
| T | T | F | | | |
| T | F | T | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | T | F | | | |
| F | F | T | | | |
| F | F | F | | | |

How do we know which column we should fill out first? Well, it actually doesn't matter. You can pick any of the columns and it will cut down some of the work. Just keep in mind which rows will still have some chance of being invalid. Those are the ones you need to complete. So, if you choose to do the conclusion, you'll need to continue working on the rows where the conclusion was false. Any row where the conclusion was true does not need to be completed. (We are interested in rows where the premises are true and the conclusion is false.) Alternatively, if you work on a premise column then you'll have to complete the row where that premise was true.

If you want to cut out most of the work, you'll have to think a bit about which column will be the best choice. We must consider how the major operator behaves if we want to pick the best column. For instance, in the example above, the major operator in the first premise is an →**.** If we were to fill out this column we should expect to get a lot of T's because the → is often true. Since that column is a premise, every time I come across a T, I'm going to have to continue on with that row (because I'm interested in rows where the premises are true). So maybe completing the first column will not cut down my work that much. Let's consider the second premise. It is the ~ of an &. An &

is mostly false (look back to our truth table for &) so the ~ of an & will be mostly true. Again, we'll have a premise here where most of the values are true so I will have to complete all of those rows. Look now at the major operator for the conclusion. Here we have an →. The arrow is mostly true but because this column is the conclusion I can ignore all the rows where the conclusion is true. I'm only interested in rows where the conclusion is false. (I'm looking for T T F). So, if I do this last column, I'll get lots of T's and I can just ignore all those rows. In this case, the conclusion is the best choice to do first.

> If you are going to do a partial truth table, you must do at least one column entirely.

B → ~(C & D)<br>
<u>D v ~B</u><br>
~C → (D & B)

~A v R<br>
~(N & ~C)<br>
R → C<br>
<u>C → ~N</u><br>
A v C

You can try this question again as a partial truth table.

| B | C | D | B → ~(C & D) | D v ~B | ~C → (D & B) |
|---|---|---|---|---|---|
| T | T | T | | | |
| T | T | F | | | |
| T | F | T | | | |
| T | F | F | | | |
| F | T | T | | | |
| F | T | F | | | |
| F | F | T | | | |
| F | F | F | | | |

| A | C | N | R | ~A v R | ~(N & ~C) | R → C | C → ~N | A v C |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |