Exercise 5.1

L Answer (b).

We are given the function

$$x(t) = \cos(4t)\sin(t).$$

We can rewrite x in the form of a Fourier series by simple algebraic manipulation. We have

$$\begin{aligned} x(t) &= \cos(4t)\sin(t) \\ &= \tfrac{1}{2}\left(e^{j4t} + e^{-j4t}\right)\tfrac{1}{2j}\left(e^{jt} - e^{-jt}\right) \\ &= -\tfrac{j}{4}\left(e^{j4t} + e^{-j4t}\right)\left(e^{jt} - e^{-jt}\right) \\ &= -\tfrac{j}{4}\left(e^{j5t} - e^{j3t} + e^{-j3t} - e^{-j5t}\right) \\ &= -\tfrac{j}{4}e^{j5t} + \tfrac{j}{4}e^{j3t} - \tfrac{j}{4}e^{-j3t} + \tfrac{j}{4}e^{-j5t}. \end{aligned}$$

Euler

multiply constants

factor and sort terms by exponent

multiply / expand

Thus, we have

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},$$

where $\omega_0 = 1$ (i.e., $T = \frac{2\pi}{\omega_0} = 2\pi$) and

$$c_k = \begin{cases} -\frac{j}{4} & k \in \{-3, 5\} \\ \frac{j}{4} & k \in \{-5, 3\} \\ 0 & \text{otherwise.} \end{cases}$$

