STAT260 Spring 2023 – R Assignment 2

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Part 1 A radioactive object emits particles according to a Poisson process at an average rate of 5.5 particles per second. We observe the object for a total of 6.5 seconds.

(a) [**1 mark**] What is the probability that no more than 40 particles will be emitted during this interval?

(b) [**1 mark**] What is the probability that exactly 38 particles will be emitted during this interval?

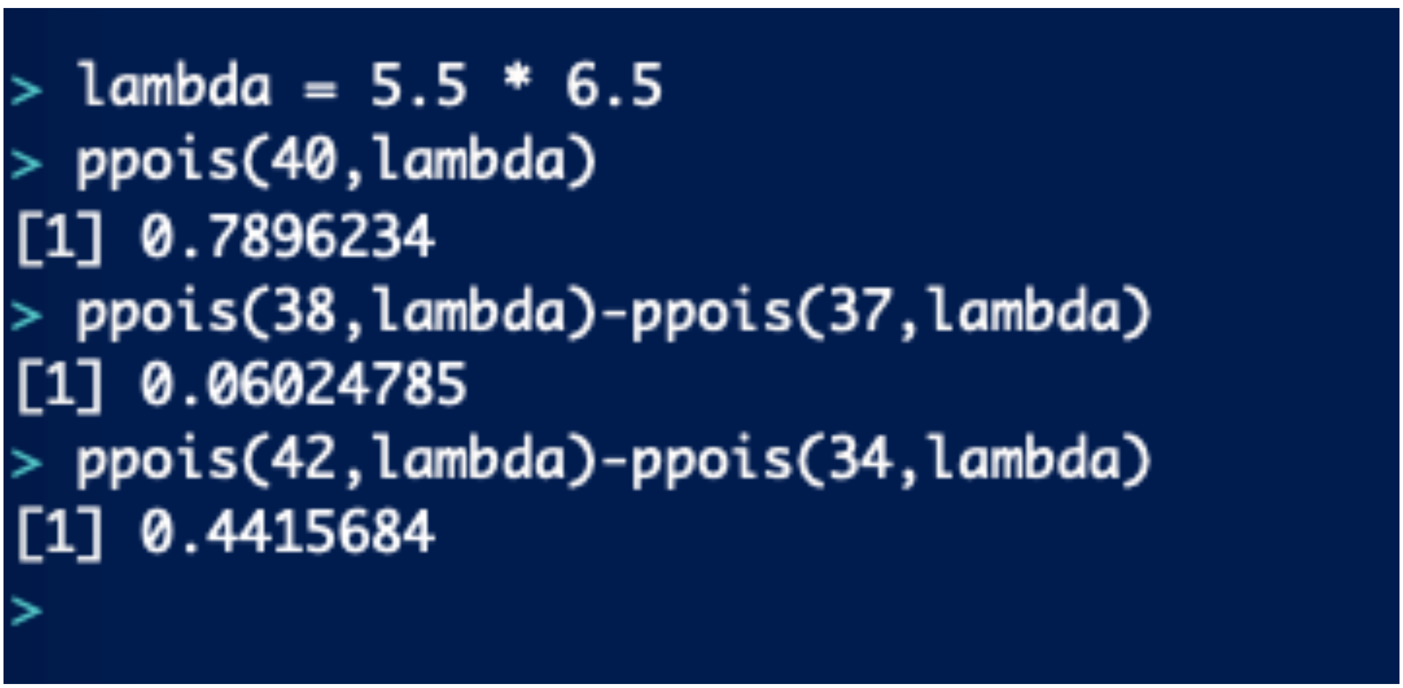
(c) [**2 marks**] Suppose it is known that at least 34 particles will be emitted during this interval. What is the probability that no more than 42 particles will be emitted during this interval?

*lambda, λ = 5.5 * 6.5 = 35.75*

(a) 0.7896234, *ppois(40,lambda)*

(b) 0.06024785, *ppois(38,lambda)-ppois(37,lambda)*

(c) 0.4415684, *ppois(42,lambda)-ppois(34,lambda)*



```
> lambda = 5.5 * 6.5
> ppois(40,lambda)
[1] 0.7896234
> ppois(38,lambda)-ppois(37,lambda)
[1] 0.06024785
> ppois(42,lambda)-ppois(34,lambda)
[1] 0.4415684
>
```

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Part 2 A manufacturer of ceramic blades estimates that 0.81% of all blades produced are too brittle to use. Suppose we take a random sample of 145 blades and test them for brittleness. We want to find the probability that at least 3 blades will be too brittle to use.

(a) [**1 mark**] Find the exact probability that at least 3 blades will be too brittle to use.

(b) [**1 mark**] Use an appropriate approximation to find the approximate probability that at least 3 blades will be too brittle to use.

(a) *At least 3 blades* $= P(X \geq 3)$
$= 1 - P(X < 3) = 1 - P(X \leq 2)$
= 1-pbinom(2,145,0.0081)
= 0.1143122

(b) Mean = μ = 145 * 0.0081 = 1.1745
Standard Deviation, $\sigma = \sqrt{np(1-p)}$ = 1.079345427

At least 3 blades $= P(X \geq 3)$
$= 1 - P(X < 3)$
$= 1 - P(X < (3-0.5))$
$= 1 - P(X < 2.5)$
= 1-pnorm(2.5,1.1745,1.079345427)
= 0.1097124

```
> 1-pbinom(2,145,0.0081)
[1] 0.1143122
> 1-pnorm(2.5,1.1745,1.079345427)
[1] 0.1097124
> |
```

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Part 3 The fracture toughness (in $MPa\sqrt{m}$) of a particular steel alloy is known to be normally distributed with a mean of 29.2 and a standard deviation of 2.17. We select one sample of this alloy at random and measure its fracture toughness.

(a) [**1 mark**] What is the probability that the fracture toughness will be between 24.8 and 31.5?

(b) [**1 mark**] What is the probability that the fracture toughness will be at least 28.2?

(c) [**2 marks**] Given that the fracture toughness is at least 26, what is the probability that the fracture toughness will be no more than 32.1?

(a) 0.8341087
(b) 0.3224605
(c) 0.9024481

```
> mean = 29.2
> stdev = 2.17
> pnorm(31.5,mean,stdev)-pnorm(24.8,mean,stdev)
[1] 0.8341087
> pnorm(28.2,mean,stdev)
[1] 0.3224605
> (pnorm(32.1,mean,stdev)-pnorm(26,mean,stdev))/(1-pnorm(26,mean,stdev))
[1] 0.9024481
>
```

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Part 4 The purpose of this question is to help you visualize the normal approximation to the binomial distribution which have seen in Set 16.

(a) [**1 mark**] Let $X \sim binomial(n = 85, p = 0.32)$. Create a vector called **simulation.data** which contains a simulation for 3700 values for X. (i.e. Simulate 3700 experiments, each being binomial with $n = 85$ and $p = 0.32$.) Provide a copy of the R command which you used to create this vector. You do **not** need to copy the 3700 values you generated.

(b) [**2 marks**] Create a histogram of **simulation.data** and copy it and your line of R code into your assignment. Your histogram should have an appropriate title and an appropriate label on the x-axis. Comment on the shape of the histogram. (We are looking for a single phrase here to describe the histogram. It should be a shape we've discussed recently.)

(c) [**2 marks**] Calculate the sample mean of **simulation.data**. Copy the command used, and the output. How close is your sample mean to what you would expect? (Hint: We have discussed the expected value of the sample mean $\overline{X}$. We have also discussed the expected value of a binomial random variable X.)

(a) R-Code: simulation.data = rbinom (3700,85,0.32)

(b) R-Code: hist(simulation.data, main="Histogram of Random Numbers from Binomial (85,0.32)",xlab="Random Numbers") {image in the next page}

(c) Sample mean is very close to population mean and this is what we expected because as sample size increases the sample mean tends to get close to the population mean.

mean (simulation.data) = 27.14189

Population Mean = E(X) = n × p = 85 × 0.32 = 27.2 ≅ 27.14189

```
> simulation.data = rbinom (3700,85,0.32)
> hist(simulation.data, main="Histogram of Random Numbers from Binomial (85,0.32)",xlab="Random Numbers")
> mean(simulation.data)
[1] 27.14189
> 85*0.32
[1] 27.2
>
```

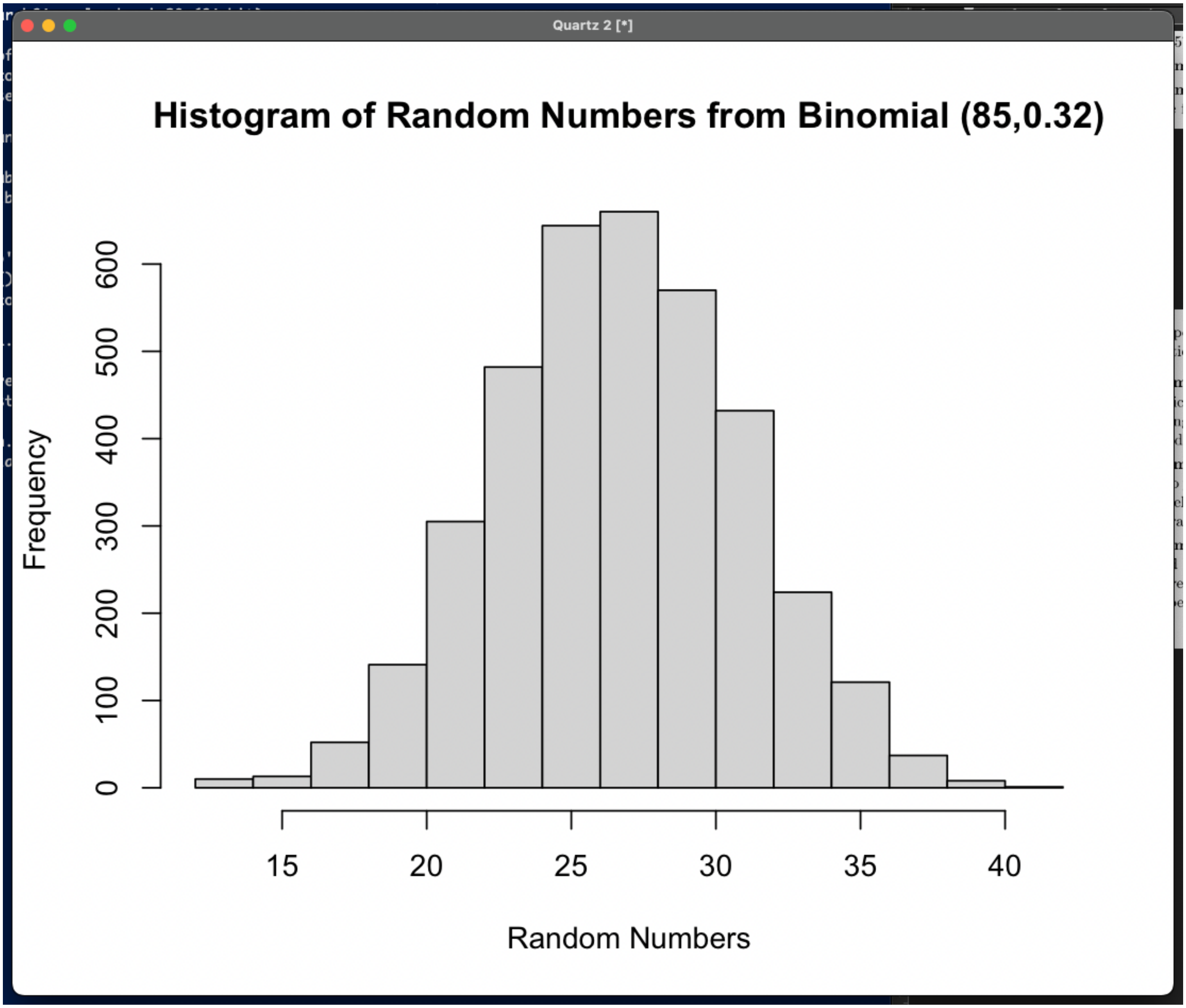
Quartz 2 [*]
Histogram of Random Numbers from Binomial (85,0.32)
Frequency
0
100
200
300
400
500
600
15
20
25
30
35
40
Random Numbers

Figure: 4(c)