Stat 260 Lecture Notes
Set 5 - Introduction to Probability

Probability is used to express the likelihood that some event will or will not occur. We measure probability on a scale from 0 to 1, where 0 indicates that it is impossible for the event to occur and 1 indicates that the event is guaranteed to occur.

For an event A, we denote the probability that A will occur by $P(A)$.

Axioms of Probability:

- For any event A, $P(A) \geq 0$.
- $P(\mathcal{S}) = 1$, where $\mathcal{S}$ represents the sample space.
- If events A_1, A_2, A_3, $\ldots$, are mutually exclusive (disjoint), then $P(\bigcup_{i=1}^{\infty} A_i) = P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) + P(A_2) + P(A_3) + \ldots$.

In the last axiom, the fact that the events are disjoint is important. Below we will see how to find the probability of the union when the events are not disjoint.

Notice that an event A and its complement $\overline{A}$ are disjoint and together they comprise all of the sample space $\mathcal{S}$. Therefore we can say that $P(A \cup \overline{A}) = P(A) + P(\overline{A}) = P(\mathcal{S}) = 1$.

Rule: $P(A) + P(\overline{A}) = 1$.

This is often useful in the form $P(A) = 1 - P(\overline{A})$.

Theorem: General Addition Rule

for two sets:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

for three sets:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Example 1: Suppose that when Michelle is on the computer the only activities she does are:

- event E_1: marking R assignments, occurs with probability p
- event E_2: prepping lectures, 8 times as likely as R assignments
- event E_3: answering email, 3 times as likely as R assignments
- event E_4: updating course website, 2 times as likely as R assignments
- event E_5: creating new assignments, 6 times as likely as R assignments

Assume Michelle can only do one activity at a time (i.e. all events are mutually exclusive). A student visits Michelle at a random time and finds that she is working on the computer. What is the probability that she is not marking R assignments?

Example 2: In a colony of 160 rabbits

- 104 are grey
- 105 have straight ears
- 126 have short fur
- 90 have short fur & are grey
- 80 have straight ears & short fur
- 149 are grey **or** have straight ears
- 5 have none of these qualities

What is the probability that a randomly selected rabbit from the colony has all three qualities?
What is the probability that a randomly selected rabbit is grey but has neither of the other two qualities? (i.e. "just grey")