Example A.11. Determine for what values of z the function $f(z) = 1/z$ is analytic.

Solution. We can deduce the analyticity properties of f as follows. First, we observe that f is a rational function. Then, we recall that a rational function is analytic everywhere except at points where its denominator polynomial becomes zero. Since the denominator polynomial of f only becomes zero at 0, f is analytic everywhere except at 0.

Alternate Solution. To study the analyticity of f, we use Theorem A.3. We express z in Cartesian form as $z = x + jy$. We rewrite f in the form $f(x,y) = v(x,y) + jw(x,y)$ as follows:

$$f(z) = f(x+jy) = \frac{1}{x+jy} = \left(\frac{1}{x+jy}\right)\left(\frac{x-jy}{x-jy}\right) = \frac{x-jy}{x^2+y^2}.$$

Thus, we have that $f(x,y) = v(x,y) + jw(x,y)$, where

$$\begin{aligned} v(x,y) &= \frac{x}{x^2+y^2} = x(x^2+y^2)^{-1} \quad \text{and} \\ w(x,y) &= \frac{-y}{x^2+y^2} = -y(x^2+y^2)^{-1}. \end{aligned}$$

Now, computing the partial derivatives, we obtain

$$\begin{aligned} \frac{\partial v}{\partial x} &= (x^2+y^2)^{-1} + (-1)(x^2+y^2)^{-2}(2x^2) = \frac{-2x^2+(x^2+y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}, \\ \frac{\partial w}{\partial y} &= (-1)(x^2+y^2)^{-1} + (-1)(x^2+y^2)^{-2}(2y)(-y) = \frac{2y^2-(x^2+y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}, \\ \frac{\partial v}{\partial y} &= (-1)(x^2+y^2)^{-2}(2y)x = \frac{-2xy}{(x^2+y^2)^2}, \quad \text{and} \\ \frac{\partial w}{\partial x} &= (-1)(x^2+y^2)^{-2}(2x)(-y) = \frac{2xy}{(x^2+y^2)^2}. \end{aligned}$$

So, we have that, for $z \neq 0$ (i.e., x and y not both zero),

$$\frac{\partial v}{\partial x} = \frac{\partial w}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial x}$$

(i.e., the Cauchy-Riemann equations are satisfied). Therefore, f is analytic everywhere except at 0. ■

A.15 Zeros and Singularities

If a function f is analytic in a domain D and is zero at a point z_0 in D, f is said to have a **zero** at z_0. If, in addition, the first $n-1$ derivatives of f are also zero at z_0 (i.e., $f^{(1)}(z_0) = f^{(2)}(z_0) = \ldots = f^{(n-1)}(z_0) = 0$), f is said to have an ***n*th-order zero** at z_0. An analytic function f is said to have an nth order zero at infinity if the function $g(z) = f(1/z)$ has an nth order zero at 0.

A point at which a function fails to be analytic is called a **singularity**. A singularity may be isolated or nonisolated. If a function f is analytic for z in an annulus $0 < |z - z_0| < r$ but not at z_0, f is said to have an **isolated singularity** at z_0. There are three types of isolated singularities: a removable singularity, an essential singularity, and a pole.

Herein, we are often interested in polynomial and rational functions. Polynomial functions do not have singularities, since such functions are analytic everywhere. In contrast, rational functions can have singularities. In the case of rational functions, we are normally interested in poles (since rational functions cannot have essential singularities and removable singularities are not very interesting).

Consider a rational function f. We can always express such a function in factored form as

$$f(z) = \frac{K(z-a_1)^{\alpha_1}(z-a_2)^{\alpha_2}\cdots(z-a_M)^{\alpha_M}}{(z-b_1)^{\beta_1}(z-b_2)^{\beta_2}\cdots(z-b_N)^{\beta_N}},$$

where K is complex, $a_1, a_2, \ldots, a_M, b_1, b_2, \ldots, b_N$ are distinct complex constants, and $\alpha_1, \alpha_2, \ldots, \alpha_M$ and $\beta_1, \beta_2, \ldots, \beta_N$ are positive integers. One can show that f has poles at $b_1, b_2, \ldots, b_N$ and zeros at $a_1, a_2, \ldots, a_M$. Furthermore, the kth pole (i.e., b_k) is of order β_k, and the kth zero (i.e., a_k) is of order α_k. A pole of first order is said to be **simple**, whereas a pole of order two or greater is said to be **repeated**. A similar terminology can also be applied to zeros (i.e., **simple zero** and **repeated zero**).

Example A.12 (Poles and zeros of a rational function)**.** Find and plot the (finite) poles and zeros of the function

$$f(z) = \frac{z^2(z^2+1)(z-1)}{(z+1)(z^2+3z+2)(z^2+2z+2)}.$$

Solution. We observe that f is a rational function, so we can easily determine the poles and zeros of f from its factored form. We now proceed to factor f. First, we factor z^2+3z+2. To do this, we solve for the roots of $z^2+3z+2=0$ to obtain

$$z = \frac{-3 \pm \sqrt{3^2-4(1)(2)}}{2(1)} = -\tfrac{3}{2} \pm \tfrac{1}{2} = \{-1, -2\}.$$

(For additional information on how to find the roots of a quadratic equation, see Section A.16.) So, we have

$$z^2+3z+2 = (z+1)(z+2).$$

Second, we factor z^2+2z+2. To do this, we solve for the roots of $z^2+2z+2=0$ to obtain

$$z = \frac{-2 \pm \sqrt{2^2-4(1)(2)}}{2(1)} = -1 \pm j = \{-1+j, -1-j\}.$$

So, we have

$$z^2+2z+2 = (z+1-j)(z+1+j).$$

Lastly, we factor z^2+1. Using the well-known factorization for a sum of squares, we obtain

$$z^2+1 = (z+j)(z-j).$$

Combining the above results, we can rewrite f as

$$f(z) = \frac{z^2(z+j)(z-j)(z-1)}{(z+1)^2(z+2)(z+1-j)(z+1+j)}.$$

From this expression, we can trivially deduce that f has:

- first order zeros at 1, j, and $-j$,
- a second order zero at 0,
- first order poles at $-1+j$, $-1-j$, -2, and
- a second order pole at -1.

The zeros and poles of this function are plotted in Figure A.8. In such plots, the poles and zeros are typically denoted by the symbols “x” and “o”, respectively. ■

Example A.13. Find the (finite) poles and zeros of the function

$$f(z) = \frac{z^3-2}{z^5+4}.$$

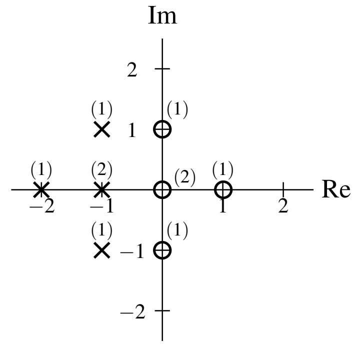

Figure A.8: Plot of the poles and zeros of f (with their orders indicated in parentheses).

Solution. The zeros of f are given by the roots of $z^3-2=0$, which is equivalent to $z^3=2=2e^{j0}$. This equation has three distinct solutions (i.e., the three third roots of 2), namely,

$$\sqrt[3]{2},\quad \sqrt[3]{2}e^{j2\pi/3},\quad\text{and}\quad \sqrt[3]{2}e^{j4\pi/3}.$$

The poles of f are given by the roots of $z^5+4=0$, which is equivalent to $z^5=-4=4e^{-j\pi}$. This equation has five distinct solutions (i.e., the five fifth roots of -4), namely,

$$\begin{aligned}\sqrt[5]{4}e^{j(-\pi+0)/5} &= \sqrt[5]{4}e^{-j\pi/5},\\ \sqrt[5]{4}e^{j(-\pi+2\pi)/5} &= \sqrt[5]{4}e^{j\pi/5},\\ \sqrt[5]{4}e^{j(-\pi+4\pi)/5} &= \sqrt[5]{4}e^{j3\pi/5},\\ \sqrt[5]{4}e^{j(-\pi+6\pi)/5} &= \sqrt[5]{4}e^{j\pi} = -\sqrt[5]{4},\quad\text{and}\\ \sqrt[5]{4}e^{j(-\pi+8\pi)/5} &= \sqrt[5]{4}e^{j7\pi/5}.\end{aligned}$$

■

A.16 Quadratic Formula

Consider the equation $az^2+bz+c=0$, where a, b, c, and z are complex, and $a\neq 0$. The roots of this equation are given by

$$z=\frac{-b\pm\sqrt{b^2-4ac}}{2a}. \tag{A.13}$$

This formula is often useful in factoring quadratic polynomials. For example, from the quadratic formula, we can conclude that the general quadratic az^2+bz+c has the factorization

$$az^2+bz+c=a(z-z_0)(z-z_1),$$

where

$$z_0=\frac{-b-\sqrt{b^2-4ac}}{2a}\quad\text{and}\quad z_1=\frac{-b+\sqrt{b^2-4ac}}{2a}.$$

A.17 Exercises

A.17.1 Exercises Without Answer Key

A.1 Express each complex number given below in Cartesian form.
(a) $2e^{j2\pi/3}$;
(b) $\sqrt{2}e^{j\pi/4}$;
(c) $2e^{j7\pi/6}$; and
(d) $3e^{j\pi/2}$.

A.2 Express each complex number given below in polar form. In each case, plot the value in the complex plane, clearly indicating its magnitude and argument. State the principal value for the argument.
(a) $-\sqrt{3}+j$;
(b) $-\frac{1}{2}-j\frac{\sqrt{3}}{2}$;
(c) $\sqrt{2}-j\sqrt{2}$;
(d) $1+j\sqrt{3}$;
(e) $-1-j\sqrt{3}$; and
(f) $-3+4j$.

A.3 Evaluate each of the expressions below, stating the final result in the specified form. When giving a final result in polar form, state the principal value of the argument.
(a) $2\left(\frac{\sqrt{3}}{2}-j\frac{1}{2}\right)+j\left(\frac{1}{\sqrt{2}}e^{j(-3\pi/4)}\right)$ in Cartesian form;
(b) $\left(\frac{\sqrt{3}}{2}-j\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}e^{j(-3\pi/4)}\right)$ in polar form;
(c) $\left(\frac{\sqrt{3}}{2}-j\frac{1}{2}\right)/(1+j)$ in polar form;
(d) $e^{1+j\pi/4}$ in Cartesian form;
(e) $\left[\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)^*\right]^8$ in polar form;
(f) $(1+j)^{10}$ in Cartesian form;
(g) $\dfrac{1+j}{1-j}$ in polar form;
(h) $\dfrac{1}{1+re^{j\theta}}$ in Cartesian form, where r and θ are real constants and $r \geq 0$; and
(i) $\dfrac{1}{1-re^{j\theta}}$ in Cartesian form, where r and θ are real constants and $r \geq 0$.

A.4 Show that each of the identities below holds, where z, z_1, and z_2 are arbitrary complex numbers and n is an arbitrary integer.
(a) $|z_1/z_2| = |z_1|/|z_2|$ for $z_2 \neq 0$;
(b) $\arg(z_1/z_2) = \arg z_1 - \arg z_2$ for $z_2 \neq 0$;
(c) $z+z^* = 2\operatorname{Re}\{z\}$;
(d) $zz^* = |z|^2$;
(e) $(z_1z_2)^* = z_1^*z_2^*$;
(f) $|z^n| = |z|^n$; and
(g) $\arg(z^n) = n\arg z$.

A.5 For each function f of a real variable given below, find an expression for $|f(\omega)|$ and $\arg f(\omega)$.
(a) $f(\omega) = \dfrac{1}{(1+j\omega)^{10}}$;

(b) $f(\omega) = \dfrac{-2-j\omega}{(3+j\omega)^2}$;
(c) $f(\omega) = \dfrac{2e^{j11\omega}}{(3+j5\omega)^7}$;
(d) $f(\omega) = \dfrac{-5}{(-1-j\omega)^4}$;
(e) $f(\omega) = \dfrac{j\omega^2}{(j\omega-1)^{10}}$; and
(f) $f(\omega) = \dfrac{j\omega-1}{j\omega+1}$.

A.6 Use Euler's relation to show that each of the identities below holds, where θ is an arbitrary real constant.
(a) $\cos\theta = \frac{1}{2}\left[e^{j\theta}+e^{-j\theta}\right]$;
(b) $\sin\theta = \frac{1}{2j}\left[e^{j\theta}-e^{-j\theta}\right]$; and
(c) $\cos^2\theta = \frac{1}{2}[1+\cos(2\theta)]$.

A.7 Consider the functions x_1 and x_2 given by $x_1(t) = e^{jat}+e^{jbt}$ and $x_2(t) = e^{jat}-e^{jbt}$.
(a) Express x_1 in terms of cos.
(b) Express x_2 in terms of sin.

A.8 Use induction to prove De Moivre's theorem (i.e., Theorem A.2), which states that $(e^{j\theta})^n = e^{j\theta n}$ for all real θ and all integer n.

A.9 Show that each of the following identities hold:
(a) $\displaystyle\sum_{n=\langle N\rangle} e^{j(2\pi/N)kn} = \begin{cases} N & k/N \in \mathbb{Z} \\ 0 & \text{otherwise,} \end{cases}$
where $\sum_{n=\langle N\rangle}$ denotes a summation over a single period of the N-periodic summand (i.e., the expression being summed) [Hint: Use the formula for the sum of a geometric sequence.]; and
(b) $\displaystyle\int_T e^{j(2\pi/T)kt} = \begin{cases} T & k=0 \\ 0 & \text{otherwise,} \end{cases}$
where $\int_T$ denotes integration over a single period of the T-periodic integrand (i.e., the expression being integrated).

A.10 Let $k,N,N_1,N_2 \in \mathbb{Z}$, where $N>0$ and $N_1 \leq N_2$. Show that

$$\sum_{n=N_1}^{N_2} e^{-j(2\pi/N)kn} = \begin{cases} e^{-j\pi(N_1+N_2)k/N}\left[\dfrac{\sin[\pi(N_2-N_1+1)k/N]}{\sin(\pi k/N)}\right] & \frac{k}{N} \notin \mathbb{Z} \\ N_2-N_1+1 & \frac{k}{N} \in \mathbb{Z}. \end{cases}$$

A.11 Determine the points at which each function f given below is: i) continuous, ii) differentiable, and iii) analytic. To deduce the answer, use your knowledge about polynomial and rational functions. Simply state the final answer along with a short justification (i.e., two or three sentences). (In other words, it is not necessary to use the Cauchy-Riemann equations for this problem.)
(a) $f(z) = 3z^3 - jz^2 + z - \pi$;
(b) $f(z) = \dfrac{z-1}{(z^2+3)(z^2+z+1)}$;
(c) $f(z) = \dfrac{z}{z^4-16}$; and
(d) $f(z) = z+2+z^{-1}$.

A.12 Use the Cauchy-Riemann equations to show that the function $f(z) = e^{az}$ is analytic for all z, where a is a real constant and z is complex.

A.13 For each rational function f of a complex variable given below, find the (finite) poles and zeros of f and their orders. Also, plot these poles and zeros in the complex plane.
(a) $f(z) = z^2 + jz + 3$;
(b) $f(z) = z + 3 + 2z^{-1}$;
(c) $f(z) = \dfrac{(z^2+2z+5)(z^2+1)}{(z^2+2z+2)(z^2+3z+2)}$;
(d) $f(z) = \dfrac{z^3-z}{z^2-4}$;
(e) $f(z) = \dfrac{z+\frac{1}{2}}{(z^2+2z+2)(z^2-1)}$; and
(f) $f(z) = \dfrac{z^2(z^2-1)}{(z^2+4z+\frac{17}{4})^2(z^2+2z+2)}$.

A.17.2 Exercises With Answer Key

A.101 Express each of the complex numbers given below in polar form, stating the principal value for the argument. [Note: Table F.1 may be helpful for some parts of this exercise.]
(a) $\sqrt{3} + j$;
(b) $-\frac{\sqrt{3}}{3} + j$; and
(c) $(1+j)^{14}$.

Short Answer. (a) $2e^{j\pi/6}$; (b) $\frac{2\sqrt{3}}{3}e^{j2\pi/3}$; (c) $128e^{-j\pi/2}$

A.102 For each function f of a real variable given below, find an expression for $|f(\omega)|$ and $\arg f(\omega)$.
(a) $f(\omega) = -5 + j\omega$; and
(b) $f(\omega) = \dfrac{(j\omega+1)^7}{(j\omega^2-5)^{10}}$.

Short Answer. (a) $|f(\omega)| = \sqrt{\omega^2+25}$ and $\arg[f(\omega)] = \pi - \arctan(\omega/5)$; (b) $|f(\omega)| = \frac{(\omega^2+1)^{7/2}}{(\omega^4+25)^5}$ and $\arg[f(\omega)] = 7\arctan(\omega) + 10\arctan(\omega^2/5) - 10\pi$

A.103 For each function f given below, find the (finite) poles and zeros of f and their orders.
(a) $f(z) = \dfrac{z-3}{z^5+7z}$.

Short Answer. (a) first-order zero at 3; first-order poles at 0, $\sqrt[4]{7}e^{j\pi/4}$, $\sqrt[4]{7}e^{j3\pi/4}$, $\sqrt[4]{7}e^{-j3\pi/4}$, and $\sqrt[4]{7}e^{-j\pi/4}$.

A.104 A rational function f has a first-order pole at -1, a second-order pole at -2, and a first-order zero at 0. The function is known not to have any other (finite) poles or zeros. If $f(1) = 1$, find f.

Short Answer. $f(z) = \frac{18z}{(z+1)(z+2)^2}$

A.18 MATLAB Exercises

A.201 Consider the rational function

$$f(z) = \frac{64z^4 - 48z^3 - 36z^2 + 27z}{64z^6 - 128z^5 - 112z^4 + 320z^3 - 84z^2 - 72z + 27}.$$

Use the Symbolic Math Toolbox in order to find the (finite) poles and zeros of f and their orders. (Hint: Some of the following functions may be useful: `sym`, `solve`, `factor`, and `pretty`.)

A.202 Use the `roots` function to find the (finite) poles and zeros of the rational function

$$f(z) = \frac{z^4 + 6z^3 + 10z^2 + 8z}{z^9 + 21z^8 + 199z^7 + 1111z^6 + 4007z^5 + 9639z^4 + 15401z^3 + 15689z^2 + 9192z + 2340}.$$

Plot these poles and zeros using the `plot` function.

Appendix B

Partial Fraction Expansions

B.1 Introduction

Sometimes we find it beneficial to be able to express a rational function as a sum of lower-order rational functions. This type of decomposition is known as a partial fraction expansion. Partial fraction expansions are often useful in the calculation of inverse Laplace transforms, inverse z transforms, and inverse (continuous-time and discrete-time) Fourier transforms.

B.2 Partial Fraction Expansions

Suppose that we have a rational function

$$f(z) = \frac{\alpha_m z^m + \alpha_{m-1} z^{m-1} + \ldots + \alpha_1 z + \alpha_0}{\beta_n z^n + \beta_{n-1} z^{n-1} + \ldots + \beta_1 z + \beta_0},$$

where m and n are nonnegative integers. Such a function is said to be **strictly proper** if $m < n$ (i.e., the order of the numerator polynomial is strictly less than the order of the denominator polynomial). We can always write a rational function as the sum of a polynomial function and a strictly proper rational function. This can be accomplished through polynomial long division. In what follows, we consider partial fraction expansions of strictly proper rational functions.

Consider a rational function f of the form

$$f(z) = \frac{a_m z^m + a_{m-1} z^{m-1} + \ldots + a_1 z + a_0}{z^n + b_{n-1} z^{n-1} + \ldots + b_1 z + b_0},$$

where $a_0, a_1, \ldots, a_{m-1}, b_0, b_1, \ldots, b_{n-1}$ are complex constants, m and n are nonnegative integers, and $m < n$ (i.e., f is strictly proper). Let us denote the polynomial in the denominator of the above expression for f as d. We can always factor d as

$$d(z) = (z - p_1)(z - p_2) \cdots (z - p_n),$$

where the p_k are the roots of d.

First, let us suppose that the roots of d are distinct (i.e., the p_k are distinct). In this case, f can be expanded as

$$f(z) = \frac{A_1}{z - p_1} + \frac{A_2}{z - p_2} + \ldots + \frac{A_{n-1}}{z - p_{n-1}} + \frac{A_n}{z - p_n},$$

where

$$A_k = \left. (z - p_k) f(z) \right|_{z = p_k}.$$

To see why the preceding formula for A_k is correct, we simply evaluate $(z-p_k)f(z)|_{z=p_k}$. We have that

$$\begin{aligned}(z-p_k)f(z)|_{z=p_k} &= \left[\frac{A_1(z-p_k)}{z-p_1}+\frac{A_2(z-p_k)}{z-p_2}+\ldots+\frac{A_{k-1}(z-p_k)}{z-p_{k-1}}+A_k+\right.\\ &\qquad \left.\left.\frac{A_{k+1}(z-p_k)}{z-p_{k+1}}+\ldots+\frac{A_n(z-p_k)}{z-p_n}\right]\right|_{z=p_k}\\ &= A_k\end{aligned}$$

Now, let us suppose that the roots of d are not distinct. In this case, we can factor d as

$$d(z)=(z-p_1)^{q_1}(z-p_2)^{q_2}\cdots(z-p_P)^{q_P}.$$

One can show that, in this case, f has a partial fraction expansion of the form

$$\begin{aligned}f(z) &= \left[\frac{A_{1,1}}{z-p_1}+\frac{A_{1,2}}{(z-p_1)^2}+\ldots+\frac{A_{1,q_1}}{(z-p_1)^{q_1}}\right]\\ &\quad+\left[\frac{A_{2,1}}{z-p_2}+\ldots+\frac{A_{2,q_2}}{(z-p_2)^{q_2}}\right]\\ &\quad+\ldots+\left[\frac{A_{P,1}}{z-p_P}+\ldots+\frac{A_{P,q_P}}{(z-p_P)^{q_P}}\right],\end{aligned}$$

where

$$A_{k,\ell}=\frac{1}{(q_k-\ell)!}\left.\left[\left[\tfrac{d}{dz}\right]^{q_k-\ell}\left[(z-p_k)^{q_k}f(z)\right]\right]\right|_{z=p_k}.$$

Note that the q_kth-order pole p_k contributes q_k terms to the partial fraction expansion.

Example B.1 (Simple pole)**.** Find the partial fraction expansion of the function

$$f(z)=\frac{3}{z^2+3z+2}.$$

Solution. First, we rewrite f with the denominator polynomial factored to obtain

$$f(z)=\frac{3}{(z+1)(z+2)}.$$

From this, we know that f has a partial fraction expansion of the form

$$f(z)=\frac{A_1}{z+1}+\frac{A_2}{z+2},$$

where A_1 and A_2 are constants to be determined. Now, we calculate A_1 and A_2 as follows:

$$\begin{aligned}A_1 &= (z+1)f(z)|_{z=-1}\\ &= \left.\frac{3}{z+2}\right|_{z=-1}\\ &= 3 \quad \text{and}\\ A_2 &= (z+2)f(z)|_{z=-2}\\ &= \left.\frac{3}{z+1}\right|_{z=-2}\\ &= -3.\end{aligned}$$

Thus, the partial fraction expansion of f is given by

$$f(z)=\frac{3}{z+1}-\frac{3}{z+2}.$$

■

Example B.2 (Repeated pole)**.** Find the partial fraction expansion of the function

$$f(z) = \frac{4z+8}{(z+1)^2(z+3)}.$$

Solution. Since f has a repeated pole, we know that f has a partial fraction expansion of the form

$$f(z) = \frac{A_{1,1}}{z+1} + \frac{A_{1,2}}{(z+1)^2} + \frac{A_{2,1}}{z+3}.$$

where $A_{1,1}, A_{1,2}$, and $A_{2,1}$ are constants to be determined. To calculate these constants, we proceed as follows:

$$\begin{aligned}
A_{1,1} &= \frac{1}{(2-1)!} \left[\left(\tfrac{d}{dz}\right)^{2-1} \left[(z+1)^2 f(z)\right] \right]\Big|_{z=-1} \\
&= \frac{1}{1!} \left[\tfrac{d}{dz} \left[(z+1)^2 f(z)\right]\right]\Big|_{z=-1} \\
&= \left[\tfrac{d}{dz} \left(\frac{4z+8}{z+3} \right) \right]\Big|_{z=-1} \\
&= \left[4(z+3)^{-1} + (-1)(z+3)^{-2}(4z+8)\right]\Big|_{z=-1} \\
&= \left[\frac{4}{(z+3)^2} \right]\Big|_{z=-1} \\
&= \tfrac{4}{4} \\
&= 1, \\
A_{1,2} &= \frac{1}{(2-2)!} \left[\left(\tfrac{d}{dz}\right)^{2-2} \left[(z+1)^2 f(z)\right] \right]\Big|_{z=-1} \\
&= \frac{1}{0!} \left[(z+1)^2 f(z)\right]\Big|_{z=-1} \\
&= \left[\frac{4z+8}{z+3} \right]\Big|_{z=-1} \\
&= \tfrac{4}{2} \\
&= 2, \quad \text{and} \\
A_{2,1} &= (z+3)f(z)|_{z=-3} \\
&= \frac{4z+8}{(z+1)^2}\Big|_{z=-3} \\
&= \tfrac{-4}{4} \\
&= -1.
\end{aligned}$$

Thus, the partial fraction expansion of f is given by

$$f(z) = \frac{1}{z+1} + \frac{2}{(z+1)^2} - \frac{1}{z+3}. \qquad \blacksquare$$

Example B.3 (Improper rational function)**.** Find the partial fraction expansion of the function

$$f(z) = \frac{2z^3+9z^2-z+2}{z^2+3z+2}.$$

Solution. Since f is not strictly proper, we must rewrite f as the sum of a polynomial function and a strictly proper rational function. Using polynomial long division, we have

$$\begin{array}{r|rrrr} & & & 2z & +\ 3 \\ \cline{2-5} z^2+3z+2 & 2z^3 & +\ 9z^2 & -\ z & +\ 2 \\ & 2z^3 & +\ 6z^2 & +\ 4z & \\ \cline{2-5} & & 3z^2 & -\ 5z & +\ 2 \\ & & 3z^2 & +\ 9z & +\ 6 \\ \cline{3-5} & & & -\ 14z & -\ 4. \end{array}$$

Thus, we have

$$f(z) = 2z+3+g(z),$$

where

$$g(z) = \frac{-14z-4}{z^2+3z+2} = \frac{-14z-4}{(z+2)(z+1)}.$$

Now, we find a partial fraction expansion of g. Such an expansion is of the form

$$g(z) = \frac{A_1}{z+1} + \frac{A_2}{z+2}.$$

Solving for the expansion coefficients, we have

$$\begin{aligned} A_1 &= (z+1)g(z)|_{z=-1} \\ &= \left.\frac{-14z-4}{z+2}\right|_{z=-1} \\ &= 10 \quad \text{and} \\ A_2 &= (z+2)g(z)|_{z=-2} \\ &= \left.\frac{-14z-4}{z+1}\right|_{z=-2} \\ &= -24. \end{aligned}$$

Thus, g has the expansion

$$g(z) = \frac{10}{z+1} - \frac{24}{z+2}.$$

Thus, we can decompose f using a partial fraction expansion as

$$f(z) = 2z+3+\frac{10}{z+1} - \frac{24}{z+2}.$$

■

B.3 Exercises

B.3.1 Exercises Without Answer Key

B.1 Find the partial fraction expansion of each function f given below.

(a) $f(z) = \dfrac{-z^2+2z+7}{4z^3+24z^2+44z+24}$;

(b) $f(z) = \dfrac{-16z-10}{8z^2+6z+1}$;

(c) $f(z) = \dfrac{7z+26}{z^2+7z+10}$;

(d) $f(z) = \dfrac{-2z^2+5}{z^3+4z^2+5z+2}$;

(e) $f(z) = \dfrac{2z^2+15z+21}{z^2+4z+3}$; and

(f) $f(z) = \dfrac{4z^3+36z^2+103z+95}{(z+1)(z+3)^3}$.

B.3.2 Exercises With Answer Key

B.101 Find the partial fraction expansion of each function f given below.

(a) $f(z) = \dfrac{2}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$; and

(b) $f(z) = \dfrac{\frac{1}{10}}{\left(1-\frac{9}{10}z^{-1}\right)\left(1-z^{-1}\right)}$.

Short Answer. (a) $f(z) = \frac{-2}{1-\frac{1}{4}z^{-1}} + \frac{4}{1-\frac{1}{2}z^{-1}}$; (b) $f(z) = \frac{-9/10}{1-\frac{9}{10}z^{-1}} + \frac{1}{1-z^{-1}}$

B.4 MATLAB Exercises

B.201 Use MATLAB to find a partial fraction expansion for each function f given below. [Hint: The `partfrac` function may be helpful.]

(a) $f(z) = \dfrac{1}{z^2+3z+1}$.

Appendix C

Solution of Constant-Coefficient Linear Differential Equations

C.1 Overview

Many systems of practical interest can be represented using linear differential equations with constant coefficients. For this reason, we are interested in solution techniques for such equations. This appendix briefly introduces time-domain methods for solving constant-coefficient linear differential equations.

C.2 Constant-Coefficient Linear Differential Equations

An Nth-order linear differential equation with constant coefficients has the general form

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = \sum_{k=0}^{M} a_k \left(\tfrac{d}{dt}\right)^k x(t),$$

where the a_k and b_k are constants. If the right-hand side of the above equation is identically equal to zero, the equation is said to be **homogeneous**. Otherwise, the equation is said to be **nonhomogeneous**. Depending on whether the above equation is homogeneous or nonhomogeneous, the solution method differs slightly.

C.3 Solution of Homogeneous Equations

First, we consider the solution of homogeneous equations. In this case, we have an equation of the form

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = 0. \tag{C.1}$$

Let us define the quantity

$$\phi(s) \triangleq \sum_{k=0}^{N} b_k s^k.$$

Then, we refer to

$$\phi(s) = 0$$

as the characteristic (or auxiliary) equation of (C.1). The solution of (C.1) depends on the roots of the characteristic equation, as specified by the theorem below.

Theorem C.1. *Suppose that $\phi(s) = 0$ is the characteristic equation associated with the homogeneous linear differential equation*

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = 0.$$

If $\phi(s) = 0$ has a real root p of multiplicity k, then a solution of the differential equation is

$$(a_0 + a_1 t + \ldots + a_{k-1} t^{k-1}) e^{pt}.$$

If $\phi(s) = 0$ has a pair of complex conjugate roots $\sigma \pm j\omega$, each root being of multiplicity k, then a solution of the differential equation is

$$e^{\sigma t} \left[(a_0 + a_1 t + \ldots + a_{k-1} t^{k-1}) \cos(\omega t) + (b_0 + b_1 t + \ldots + b_{k-1} t^{k-1}) \sin(\omega t) \right].$$

A general solution of the differential equation is obtained by taking a linear combination of the solutions obtained by considering all roots of $\phi(s) = 0$.

From the above theorem, we can see that, in order to solve an equation of the form (C.1), we begin by finding the roots of the corresponding characteristic equation. Then, we find a solution associated with each distinct root (or pair of complex conjugate roots). Finally, the general solution is formed by taking a linear combination of all of these individual solutions.

Example C.1. Find the general solution to the differential equation

$$y''(t) + 4y'(t) + 5y(t) = 0.$$

Solution. The given differential equation has the characteristic equation

$$s^2 + 4s + 5 = 0.$$

Solving for the roots of the characteristic equation yields

$$\begin{aligned} s &= \frac{-4 \pm \sqrt{4^2 - 4(5)}}{2} \\ &= -2 \pm \tfrac{1}{2}\sqrt{-4} \\ &= -2 \pm j. \end{aligned}$$

Thus, we have one pair of complex conjugate roots (i.e., $-2 \pm j$), each root being of multiplicity 1. Therefore, the general solution to the given equation is of the form

$$y(t) = e^{-2t} (a_1 \cos t + b_1 \sin t). \qquad \blacksquare$$

Example C.2. Find the general solution to the differential equation

$$y''(t) + 5y'(t) + 6y(t) = 0.$$

Solution. The given differential equation has the characteristic equation

$$s^2 + 5s + 6 = 0,$$

which can be factored as

$$(s+2)(s+3) = 0.$$

Clearly, the characteristic equation has the roots -2 and -3, each of multiplicity 1. Therefore, the general solution of the given equation is of the form

$$y(t) = a_1 e^{-3t} + a_2 e^{-2t}. \qquad \blacksquare$$

Example C.3. Find the general solution to the differential equation

$$y''(t)+2y'(t)+y(t)=0.$$

Solution. The given differential equation has the characteristic equation

$$s^2+2s+1=0,$$

which can be factored as

$$(s+1)^2=0.$$

Clearly, the characteristic equation has the root -1 of multiplicity 2. Therefore, the general solution to the given equation is of the form

$$y(t)=(a_0+a_1t)e^{-t}.$$ ■

C.4 Particular Solution of Nonhomogeneous Equations

So far, we have only considered the solution of homogeneous equations. Now, we consider the nonhomogeneous case. In the nonhomogeneous case, we have an equation of the form

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = f(t). \tag{C.2}$$

As it turns out, in order to find a general solution to the above equation, we must first find a particular solution.

To find a particular solution, we must consider the form of the function f. Suppose that a particular solution to (C.2) is given by the function y_p. Since y_p is a solution to (C.2), when we substitute y_p into (C.2), the left-hand side of (C.2) must equal f. Therefore, y_p and its derivatives must be comprised of terms that resemble the terms of f. Thus, by examining f, we can deduce a general expression for y_p containing one or more unknown coefficients. We, then, solve for these unknown coefficients. This solution technique is sometimes referred to as the **method of undetermined coefficients**.

Table C.1 shows the general form of y_p that should be nominally used in the case of several frequently encountered forms for f. There is, however, one caveat. The general expression chosen for y_p must not include any terms that are linearly dependent on terms in the solution to the corresponding complementary equation. If linearly dependent terms exist in our preliminary choice of y_p, we must replace each term $v(t)$ with $t^m v(t)$ where m is the smallest positive integer eliminating this linear dependence. To illustrate how this solution process works in more detail, we will now consider a few example problems.

Example C.4. Find a particular solution of the differential equation

$$y''(t)+4y'(t)+5y(t)=5t^2+3t+8.$$

Solution. We begin by considering the form of the function on the right-hand side of the given differential equation. Since terms in t^2, t^1, and t^0 yield terms in t^2, t^1, and t^0 when substituted into the left-hand side of the given equation, we deduce that a particular solution is of the form

$$y_\mathsf{p}(t)=a_2t^2+a_1t+a_0. \tag{C.3}$$

Table C.1: Forms for the particular solution

$f(t)$	$y_p(t)$
$c_0 + c_1 t + \ldots + c_n t^n$	$p_0 + p_1 t + \ldots + p_n t^n$
ce^{at}	pe^{at}
$c \sin bt$ or $c \cos bt$	$p \sin bt + q \cos bt$
$(c_0 + c_1 t + \ldots + c_n t^n)e^{at}$	$(p_0 + p_1 t + \ldots + p_n t^n)e^{at}$
$(c_0 + c_1 t + \ldots + c_n t^n)\cos bt$ or $(c_0 + c_1 t + \ldots + c_n t^n)\sin bt$	$(p_0 + p_1 t + \ldots + p_n t^n)\cos bt + (q_0 + q_1 t + \ldots + q_n t^n)\sin bt$
$(c_0 + c_1 t + \ldots + c_n t^n)e^{at}\cos bt$ or $(c_0 + c_1 t + \ldots + c_n t^n)e^{at}\sin bt$	$(p_0 + p_1 t + \ldots + p_n t^n)e^{at}\cos bt + (q_0 + q_1 t + \ldots + q_n t^n)e^{at}\sin bt$

Differentiating y_p, we obtain

$$y_p'(t) = 2a_2 t + a_1 \quad \text{and}$$
$$y_p''(t) = 2a_2.$$

Substituting y_p and its derivatives into the given differential equation yields

$$\begin{aligned} & 2a_2 + 4[2a_2 t + a_1] + 5\left[a_2 t^2 + a_1 t + a_0\right] = 5t^2 + 3t + 8 \\ \Rightarrow\quad & 2a_2 + 8a_2 t + 4a_1 + 5a_2 t^2 + 5a_1 t + 5a_0 = 5t^2 + 3t + 8 \\ \Rightarrow\quad & [5a_2]t^2 + [8a_2 + 5a_1]t + [2a_2 + 4a_1 + 5a_0] = 5t^2 + 3t + 8. \end{aligned}$$

Comparing the left- and right-hand sides of the above equation, we see that

$$5a_2 = 5,$$
$$8a_2 + 5a_1 = 3, \quad \text{and}$$
$$2a_2 + 4a_1 + 5a_0 = 8.$$

Solving this system of equations yields $a_0 = 2$, $a_1 = -1$, and $a_2 = 1$. Therefore, from (C.3), the given differential equation has the particular solution

$$y_p(t) = t^2 - t + 2.$$ ■

Example C.5. Find a particular solution of the differential equation

$$y''(t) + 4y'(t) + 5y(t) = e^{-2t}.$$

Solution. We begin by considering the form of the function on the right-hand side of the given differential equation. Since terms in e^{-2t} yield terms in e^{-2t} when substituted into the left-hand side of the equation, we deduce that the particular solution y_p is of the form

$$y_p(t) = ae^{-2t}.$$

Differentiating y_p, we obtain

$$y_p'(t) = -2ae^{-2t} \quad \text{and}$$
$$y_p''(t) = 4ae^{-2t}.$$

Substituting y_p and its derivatives into the given differential equation yields

$$\begin{aligned}
& 4ae^{-2t} + 4\left[-2ae^{-2t}\right] + 5\left[ae^{-2t}\right] = e^{-2t} \\
\Rightarrow\quad & 4ae^{-2t} - 8ae^{-2t} + 5ae^{-2t} = e^{-2t} \\
\Rightarrow\quad & ae^{-2t} = e^{-2t}.
\end{aligned}$$

Comparing the left- and right-hand sides of the above equation, we have that $a = 1$. Therefore, the given differential equation has the particular solution

$$y_\mathsf{p}(t) = e^{-2t}.$$

■

Example C.6. Find a particular solution of the differential equation

$$y''(t) + 4y'(t) + 5y(t) = \sin t.$$

Solution. To begin, we examine the form of the function of the right-hand side of the given differential equation. Since terms in $\sin t$ yield terms in $\sin t$ and $\cos t$ when substituted into the left-hand side of the given equation, we deduce that the particular solution y_p is of the form

$$y_\mathsf{p}(t) = a_1 \cos t + a_2 \sin t.$$

Differentiating y_p, we obtain

$$\begin{aligned}
y_\mathsf{p}'(t) &= -a_1 \sin t + a_2 \cos t \quad \text{and} \\
y_\mathsf{p}''(t) &= -a_1 \cos t - a_2 \sin t.
\end{aligned}$$

Substituting y_p and its derivatives into the given differential equation yields

$$\begin{aligned}
& [-a_1 \cos t - a_2 \sin t] + 4[-a_1 \sin t + a_2 \cos t] + 5[a_1 \cos t + a_2 \sin t] = \sin t \\
\Rightarrow\quad & [-a_1 + 4a_2 + 5a_1] \cos t + [-a_2 - 4a_1 + 5a_2] \sin t = \sin t \\
\Rightarrow\quad & [4a_1 + 4a_2] \cos t + [4a_2 - 4a_1] \sin t = \sin t.
\end{aligned}$$

Comparing the left- and right-hand sides of the above equation, we have that

$$\begin{aligned}
4a_1 + 4a_2 &= 0 \quad \text{and} \\
4a_2 - 4a_1 &= 1.
\end{aligned}$$

Solving this system of equations yields $a_1 = -\frac{1}{8}$ and $a_2 = \frac{1}{8}$. Therefore, the given differential equation has the particular solution

$$y_\mathsf{p}(t) = -\tfrac{1}{8} \cos t + \tfrac{1}{8} \sin t.$$

■

C.5 General Solution of Nonhomogeneous Equations

With every nonhomogeneous constant-coefficient linear differential equation

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = \sum_{k=0}^{M} a_k \left(\tfrac{d}{dt}\right)^k x(t),$$

we can associate a homogeneous equation

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = 0$$

called the **complementary equation**. The complementary equation is formed by simply setting the function x (and its derivatives) to zero in the original equation.

As it turns out, in order to find the solution of a nonhomogeneous equation, we must find a particular solution to the given equation and also a general solution to its complementary equation. This process is more precisely specified by the theorem below.

Theorem C.2. *A general solution of the linear differential equation*

$$\sum_{k=0}^{N} b_k \left(\tfrac{d}{dt}\right)^k y(t) = \sum_{k=0}^{M} a_k \left(\tfrac{d}{dt}\right)^k x(t)$$

has the form

$$y(t) = y_{\mathsf{c}}(t) + y_{\mathsf{p}}(t),$$

where y_{c} is a general solution of the associated complementary equation and y_{p} is any particular solution of the given equation.

Example C.7. Consider the differential equation

$$y''(t) + 2y'(t) + 2y(t) = -2t + 4.$$

(a) Find the general solution of this equation.
(b) Find the solution if $y(0) = 1$ and $y'(0) = 0$.

Solution. (a) First, we need to find the general solution y_{c} of the corresponding complementary equation

$$y''(t) + 2y'(t) + 2y(t) = 0.$$

This equation has the characteristic equation

$$s^2 + 2s + 2 = 0.$$

Solving for the roots of this equation, we have

$$\begin{aligned} s &= \frac{-2 \pm \sqrt{2^2 - 4(2)}}{2} \\ &= -1 \pm j. \end{aligned}$$

Therefore, the characteristic equation has a pair of complex conjugate roots $-1 \pm j$, each root being of multiplicity 1. From this, we know that the complementary equation has a general solution of the form

$$y_{\mathsf{c}}(t) = e^{-t}(a_1 \cos t + b_1 \sin t). \tag{C.4}$$

Now, we must find a particular solution y_{p} of the given differential equation. We consider the form of the function on the right-hand side of the given equation. Since terms in t^1 and t^0 yield terms in t^1 and t^0 when substituted into the left-hand side of the equation, we deduce that y_{p} is of the form

$$y_{\mathsf{p}}(t) = c_1 t + c_0.$$

Differentiating y_p, we obtain

$$y'_\mathsf{p}(t) = c_1 \quad \text{and}$$
$$y''_\mathsf{p}(t) = 0.$$

Substituting y_p and its derivatives into the given differential equation yields

$$\begin{aligned} &2c_1 + 2[c_1 t + c_0] = -2t + 4 \\ \Rightarrow \quad &[2c_1]t + [2c_1 + 2c_0] = -2t + 4. \end{aligned}$$

Comparing the left- and right-hand sides of the above equation, we have

$$2c_1 = -2 \quad \text{and}$$
$$2c_1 + 2c_0 = 4.$$

Solving this system of equations yields $c_0 = 3$ and $c_1 = -1$. Therefore, the particular solution is given by

$$y_\mathsf{p}(t) = -t + 3. \tag{C.5}$$

Combining the results of (C.4) and (C.5), we conclude that the given equation has the general solution

$$\begin{aligned} y(t) &= y_\mathsf{c}(t) + y_\mathsf{p}(t) \\ &= e^{-t}(a_1 \cos t + b_1 \sin t) - t + 3. \end{aligned}$$

(b) We compute the derivative of y as

$$y'(t) = -e^{-t}(a_1 \cos t + b_1 \sin t) + e^{-t}(-a_1 \sin t + b_1 \cos t) - 1.$$

From the given initial conditions, we have

$$0 = -a_1 + b_1 - 1 \quad \text{and}$$
$$1 = a_1 + 3.$$

Solving for a_1 and b_1 yields $a_1 = -2$ and $b_1 = -1$. Therefore, we have

$$\begin{aligned} y(t) &= e^{-t}(-2\cos t - \sin t) - t + 3 \\ &= -e^{-t}(2\cos t + \sin t) - t + 3. \end{aligned}$$

■

Example C.8. Consider the differential equation

$$y''(t) + 3y'(t) + 2y(t) = e^{-t}.$$

(a) Find the general solution of this equation.
(b) Determine the solution if $y(0) = -1$ and $y'(0) = 1$.

Solution. First, we need to find the general solution y_c of the corresponding complementary equation

$$y''(t) + 3y'(t) + 2y(t) = 0.$$

This equation has the characteristic equation

$$s^2 + 3s + 2 = 0,$$

which can be factored as

$$(s+2)(s+1) = 0.$$

Thus, the characteristic equation has roots at -2 and -1, each of multiplicity 1. From this, we can deduce that

$$y_c(t) = a_1 e^{-t} + a_2 e^{-2t}. \tag{C.6}$$

Now, we need to find a particular solution y_p of the given differential equation. Since y_c contains a term with e^{-t}, we deduce that y_p is of the form

$$y_p(t) = cte^{-t}.$$

Differentiating y_p, we obtain

$$\begin{aligned} y_p'(t) &= ce^{-t} - cte^{-t} \quad \text{and} \\ y_p''(t) &= -ce^{-t} - c(e^{-t} - te^{-t}) = -2ce^{-t} + cte^{-t}. \end{aligned}$$

Substituting y_p and its derivatives into the given differential equation yields

$$\begin{aligned} & (-2ce^{-t} + cte^{-t}) + 3(ce^{-t} - cte^{-t}) + 2cte^{-t} = e^{-t} \\ \Rightarrow \quad & (-2c + 3c)e^{-t} + (c - 3c + 2c)te^{-t} = e^{-t} \\ \Rightarrow \quad & ce^{-t} = e^{-t}. \end{aligned}$$

Comparing the left- and right-hand sides of this equation, we conclude $c = 1$. Therefore, we have

$$y_p(t) = te^{-t}. \tag{C.7}$$

Combining the results from (C.6) and (C.7), we have

$$\begin{aligned} y(t) &= y_c(t) + y_p(t) \\ &= a_1 e^{-t} + a_2 e^{-2t} + te^{-t}. \end{aligned}$$

(b) We compute the derivative of y as

$$y'(t) = -a_1 e^{-t} - 2a_2 e^{-2t} + e^{-t} - te^{-t}.$$

From the given initial conditions, we have

$$\begin{aligned} 1 &= -a_1 - 2a_2 + 1 \quad \text{and} \\ -1 &= a_1 + a_2. \end{aligned}$$

Solving for a_1 and a_2 yields

$$\begin{aligned} a_1 &= -2 \quad \text{and} \\ a_2 &= 1. \end{aligned}$$

Therefore, we have that

$$y(t) = -2e^{-t} + e^{-2t} + te^{-t}. \qquad \blacksquare$$

C.6 Exercises

C.6.1 Exercises Without Answer Key

C.1 Find the general solution to each of the differential equations below.
(a) $8y''(t)+6y'(t)+y(t)=0$;
(b) $y'''(t)+5y''(t)+17y'(t)+13y(t)=0$ [Hint: One root of the characteristic equation is -1.];
(c) $y''(t)+9y'(t)+20y(t)=0$; and
(d) $y''(t)+2y'(t)+y(t)=0$.

C.2 Find a particular solution to each of the differential equations below. In each case, the solution y_c of the corresponding complementary equation is given.
(a) $y''(t)+3y'(t)+2y(t)=t^2$ with $y_c(t)=c_1e^{-t}+c_2e^{-2t}$;
(b) $y''(t)+3y'(t)+2y(t)=e^{-3t}+t$ with $y_c(t)=c_1e^{-t}+c_2e^{-2t}$;
(c) $y''(t)+4y'(t)+3y(t)=e^{-t}$ with $y_c(t)=c_1e^{-t}+c_2e^{-3t}$; and
(d) $y''(t)+2y'(t)+y(t)=\sin t$ with $y_c(t)=c_0e^{-t}+c_1te^{-t}$.

C.3 Consider the differential equation

$$y''(t)+3y'(t)+2y(t)=t+1.$$

(a) Find the general solution to this equation.
(b) Determine the solution in the case that $y(0)=-\frac{1}{4}$ and $y'(0)=-\frac{1}{2}$.

C.4 Consider the differential equation

$$y''(t)+5y'(t)+6y(t)=2e^{-3t}.$$

(a) Find the general solution of this equation.
(b) Determine the solution in the case that $y(0)=0$ and $y'(0)=1$.

C.5 Find the general solution to each of the differential equations below.
(a) $y''(t)+7y'(t)+12y(t)=6t^2-5t+18$;
(b) $y''(t)+7y'(t)+12y(t)=e^{-3t}$;
(c) $y''(t)+4y'(t)+8y(t)=e^{-t}$; and
(d) $y''(t)+2y'(y)+5y(t)=1+e^{-t}$.

C.6.2 Exercises With Answer Key

Currently, there are no exercises available with an answer key.

C.7 MATLAB Exercises

C.201 Use the `dsolve` function in MATLAB to solve each of the differential equations in Exercise C.1.

C.202 Use the `dsolve` function in MATLAB to solve Exercise C.4.

Appendix D

MATLAB

D.1 Introduction

MATLAB is a software tool that is useful for solving a wide variety of problems arising in engineering applications. The MATLAB software is a product of a company called MathWorks. Extensive information on this software (including detailed guides and manuals) is available from the company's web site (`https://www.mathworks.com`). A number of helpful books on MATLAB are also available, such as [1, 2, 3, 4, 5, 6, 10, 16]. In this appendix, a reasonably detailed introduction to MATLAB is also provided.

D.2 Octave

Although MATLAB is very powerful, it is a commercial software product. Therefore, MATLAB is not free. Fortunately, an open-source MATLAB-like software package is available called Octave. Octave is available for download from its official web site `https://www.octave.org`. This software is included in several major Linux distributions (e.g., Fedora and Ubuntu). As well, Octave is also available for the Cygwin environment under Microsoft Windows. (For more details about Cygwin, see `https://www.cygwin.com`.)

D.3 Invoking MATLAB

On most UNIX systems, the MATLAB software is started by invoking the `matlab` command.

D.3.1 UNIX

The MATLAB software is invoked using a command line of the form:

`matlab` *[options]*

The `matlab` command supports a number of options including the following:

`-help` or `-h`
: Display information on MATLAB options.

`-nodisplay`
: Disable all graphical output. The MATLAB desktop will not be started.

`-nojvm`
: Disable all Java support by not starting the Java virtual machine. In particular, the MATLAB desktop will not be started.