Sample Test 2b: Sets 11 to 20

1. The main office of a business receives phone calls according to a Poisson process, at an average rate of 3 calls per 10 minutes. What is the probability that over a 30 minute interval there will be *at least* 10 calls received?

2. The time until the fire department is called to put out a fire is known to be exponentially distributed, with the average time being 2.1 days. What is probability that the next time they are called to put out a fire will be at some point between 3.1 and 5.2 days from now?

   Questions 3 and 4 refer to the following scenario: In a particular type of nuclear reactor, the water temperature at the core (in degrees Celsius) is known to normally distributed, with a mean of 398.1 and a standard deviation of 4.3.

3. If I were to take the water temperature at the core, what is the probability that I would find a water temperature between $396°C$ and $400°C$?

4. Find the value k such that only 3% of temperature readings at the core are **less** than k.

   Questions 5 and 6 refer to the following scenario: Suppose that X is a continuous random variable, with the following pdf:

$$f(x) = \begin{cases} \dfrac{25}{3}x^{-3}, & \text{if } 2 \leq x \leq 10 \\ 0, & \text{otherwise} \end{cases}$$

5. Calculate $P(3 \leq X \leq 5)$.

6. Calculate $E(X)$.

   Questions 7 and 8 refer to the following scenario: In a library, very old documents have been stored on microfilm. The probability that any single microfilm will be too damaged to read is 0.1, independently of all other microfilms.

7. Suppose that 12 microfilms are selected at random. What is the probability that no more than three of these will be too damaged to read?

8. Suppose that 200 microfilms are selected at random. Use an *appropriate approximation* to find the probability that no more than 25 of the microfilms will be too damaged to read.

9. The continuous random variable X has the following cumulative distribution function:

$$F(x) = \begin{cases} 1 - e^{-27x^3}, & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the 95^{th} percentile of X.

10. Let X and Y be jointly distributed discrete random variables with the following joint pmf.

$f(x,y)$		y		
		0	1	2
	0	0.1	0.1	0.2
x	1	0.2	0.1	0.3

(a) Calculate $Cov(X, Y)$.

(b) Only using your answer from part (a), can you conclude that X and Y are **not** independent? Why?

11. In each box of noodles with cheese sauce, there is a bag of noodles, and a bag of sauce. The mass of the bag of noodles is normally distributed, with a mean of 210 grams and a standard deviation of 5 grams. The mass of the bag of sauce is normally distributed, with a mean of 20 grams and a standard deviation of 1 gram. Suppose the masses of the noodles and the sauce are independent of each other.

What is the probability that the **total** mass of a box of noodles with cheese sauce (one bag of noodles and one bag of sauce) will be no more than 220 grams?

Answers:

1. 0.4126 2. 0.1144 3. 0.3579 4. 390.016
5. $8/27 \approx 0.2963$ 6. $10/3 \approx 3.3333$ 7. 0.9744 8. ≈ 0.9032
9. ≈ 0.4805 10(a). $Cov(X, Y) = 0.7 - (0.6)(1.2) = -0.02$
10(b). Since $Cov(X, Y) \neq 0$, then X and Y must not be independent.

11. $X_{noodle} \sim N(\mu = 210, \sigma = 5)$, $X_{sauce} \sim N(\mu = 20, \sigma = 1)$.
Let $T = X_{noodle} + X_{sauce}$
$T \sim N(\mu = 230, \sigma = \sqrt{26})$
$P(T \leq 220) = P(Z \leq -1.96) = 0.0250$