Exercise 6.115

Answer (a,b,c,d,e).

The solution to this problem follows trivially from the following properties of the Fourier transform:

$$x \text{ is real} \Leftrightarrow X(\omega) = X^*(-\omega) \Leftrightarrow |X(\omega)| \text{ is even and } \arg X(\omega) \text{ is odd;}$$
$$x \text{ is even} \Leftrightarrow X \text{ is even;}$$
$$x \text{ is odd} \Leftrightarrow X \text{ is odd;}$$
$$x \text{ is finite energy} \Leftrightarrow X \text{ is finite energy;}$$
$$x \text{ cannot be both finite duration and have finite bandwidth;} \quad \text{and}$$
$$x \text{ is periodic} \Leftrightarrow X(\omega) = \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0).$$

Exercise 6.115

L **Answer (d).**

We are given the function

$$X(\omega) = \delta(\omega+3) + \delta(\omega+1) + \delta(\omega-1) + \delta(\omega-3).$$

We can make the following observations:

- Since X is conjugate symmetric, x is real.
- Since X is even, x is even.
- Since X is of the form $\sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$ (where $\omega_0 = 1$), x is periodic.
- Since X is bandlimited, x cannot be finite duration.
- Since X is periodic (and not the zero function), x is not finite energy.

Therefore, we conclude that x is real, even, and periodic, but not finite duration and not finite energy.