Sample Final 2

Instructions:

1. Questions 1 to 30 are short answer questions. On the real final exam, put your answer in the box provided. An example is given in question 1. **You must show some work for each question in order to receive any marks.**
2. Questions 31 and 32 are full-answer questions. For full-answer questions, marks will be deducted for incomplete or poorly presented solutions.
3. Space will be provided for you to work out your answers on the actual exam.

Duration: You should be able to complete this midterm within 3 hours. If you cannot, this means that more practice is still needed.

Questions 1, and 2 refer to the following scenario: The following measurements of weight (in grams) have been recorded for a random sample of 10 forty-day-old rats of a certain strain:

131, 116, 94, 129, 140, 103, 106, 101, 126, 147

1. Compute the sample mean minus the sample median, $\bar{x} - \tilde{x}$.

Answer:

2. Compute the sample standard deviation s for these data.
3. A certain municipality has two fire trucks operating independently. The probability that the older truck is available when needed is .70, while the probability that the newer truck is available when needed is .90. What is the probability that at least one of these two trucks is available when needed?

Questions 4, 5 and 6 refer to the following scenario: A certain kind of passenger car comes equipped with either an automatic (A) or a manual (M) transmission, and the car is available in one of three different models. The following table gives proportions of car sales for the various categories.

		Model		
		VE	CD	LE
Transmission	A	.32	.18	.25
Type	M	.16	.06	.03

4. What is the probability that the next car sold of this kind is a VE model or has a manual transmission?

5. Suppose a car of this kind has just been sold and it is not an LE model. Given this information, what is the probability that this car has an automatic transmission?

6. What is the probability that the next two cars sold (independently) of this kind are both of the same model?

Questions 7 and 8 refer to the following scenario: Three technicians regularly make repairs when breakdowns occur on an automated production line. Janet, who services 35% of the breakdowns, makes an incomplete repair 1 time in 20; Tom, who services 50% of the breakdowns, makes an incomplete repair 1 time in 10; and Peter, who services 15% of the breakdowns, makes an incomplete repair 1 time in 8.

7. What is the probability that the initial repair of a randomly chosen breakdown is incomplete?

8. If the initial repair of a randomly chosen breakdown is incomplete, what is the probability that this initial repair was made by Janet?

9. An engineering firm is faced with the task of preparing a proposal for a research contract. The cost of preparing the proposal is \$5000, and the probabilities for a potential gross profit of \$50,000, \$30,000, \$10,000, or \$5000 are .2, .4, .3, and .1, respectively, provided the proposal is accepted. If the probability is .5 that the firms proposal will be accepted, what is its expected net profit? (Here, net profit = gross profit minus the cost of preparing the proposal.)

Questions 10 and 11 refer to the following scenario: A food processor claims that at most 10% their jars of instant coffee contain less coffee than stated on the label. To test the food processor's claim, 20 jars of their instant coffee are randomly selected and the contents are weighed. The food processor's claim will be accepted if 3 or fewer of the 20 jars are under-filled. Otherwise, the claim will be rejected.

10. What is the probability that the food processors claim will be accepted when the actual percentage of under-filled jars of instant coffee is 25%?

11. What is the probability that the food processors claim will be rejected when the actual percentage of under-filled jars of instant coffee is 10%?

12. In the inspection of tin plate produced by a continuous electrolytic process, 1 imperfection is spotted every 5 minutes on the average. Find the probability of spotting more than two imperfections in 15 minutes.

Questions 13, 14 and 15 refer to the following scenario: Suppose the cumulative distribution function (cdf) of random variable X is given by:

$$F(x) = \begin{cases} 1 - (1+x)^{-2} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

13. Find $P(1 < X < 2)$.

14. Find the median, $\tilde{\mu}$, of the probability distribution of X.

15. Find the probability density function of X evaluated at 1, i.e. find $f(1)$.

Questions 16, 17 and 18 refer to the following scenario: A certain company rates its product on reliability, X, and customer satisfaction, Y. Suppose the joint pmf of X and Y is given by the following table.

		y			
		0	1	2	3
	0	.10	.06	.03	.01
x	1	.05	.15	.40	.20

16. Compute $P(Y \geq 2|X = 1)$.

17. Compute $E(X + Y)$.

18. Here the standard deviation of X is 0.40 and the standard deviation of Y is $\sqrt{0.93}$. Find the correlation coefficient ρ for X and Y.

19. If a certain type of hard drive has exponentially distributed lifetime with mean $\mu = \frac{1}{\lambda} = 4$ years, what is the probability that two randomly chosen hard drives of this type will **both** last more than 5 years?

Questions 20 and 21 refer to the following scenario: Extruded plastic rods are automatically cut into nominal lengths of 150 mm. Actual lengths are normally distributed with mean = 150 mm and standard deviation = 0.8 mm.

20. What proportion of the rods are within tolerance limits of 149 mm to 151 mm?

21. To what value does the standard deviation need to be reduced if 95% of the rods must be within tolerance limits of 149 mm to 151 mm?

22. Suppose the weight of a certain type of empty crate is normally distributed with mean 50 kg and standard deviation 2 kg. When this type of crate is filled with its cargo, the gross weight (crate plus cargo) has mean 250 kg and standard deviation 6 kg. Assume the weight of the empty crate and the weight of its cargo are independent, normally distributed random variables. What is the probability that the crates cargo weighs less than 195 kg?

Questions 23 and 24 refer to the following scenario: Suppose a randomly chosen 4-litre can of a certain kind of paint covers, on the average, 55 square metres with a standard deviation of 4 square metres.

23. What is the probability that the sample total area covered by a random sample of 50 of these 4-litre cans will be larger than 2770 square metres? (Note: $T_0 = \Sigma X_i > 2770$ if and only if $\bar{X} > 55.4$.)

24. There is a 20% chance that the sample total area covered by a random sample of 50 of these 4-litre cans will be smaller than t_0 square metres? Find t_0. (Note: $T_0 < t_0$ if and only if $\bar{X} < t_0/50$.)

25. Suppose X_1, X_2, X_3 is a random sample of size $n = 3$ from a population distribution having unknown mean μ and unknown standard deviation σ. Consider the following three estimators for μ.

$$U = \frac{1}{2}X_1 + \frac{1}{2}X_2 + \frac{1}{2}X_3; \qquad W = \frac{7}{18}X_1 + \frac{4}{18}X_2 + \frac{7}{18}X_3; \qquad Y = \frac{6}{18}X_1 + \frac{4}{18}X_2 + \frac{8}{18}X_3.$$

Which of the following statements are true?

i The expected value of U is μ.

ii The variance of W is less than the variance of Y, i.e. $Var(W) < Var(Y)$.

iii The variance of W is less than the variance of U, i.e. $Var(W) < Var(U)$.

26. Consider the following 10 determinations of the daily emission (in metric tons) of sulphur oxides from a certain industrial plant: 15.8, 26.4, 17.3, 11.2, 23.9, 14.8, 18.7, 13.9, 9.0, 13.2. Assuming these data constitute an observed random sample from a normal population distribution, compute the upper limit of a 99% confidence interval for the true mean daily emission of sulphur oxides from this industrial plant.

Questions 27 and 28 refer to the following scenario: Compressive strength was measured on random samples of 6 specimens each of aluminum alloy A and aluminum alloy B undergoing development for the next generation of aircraft. Assume the sampled populations are normally distributed. The data are summarized in the following table.

Alloy A	$m = 6$	$\bar{x} = 70.7$	$s_1 = 2.2$
Alloy B	$n = 6$	$\bar{y} = 76.1$	$s_2 = 2.4$

27. Compute the lower limit of a 90% confidence interval for the difference in mean strength, Alloy A minus Alloy B, i.e. $\mu_1 - \mu_2$.

28. Use these data as a pilot study to determine the common sample size n (m = n) needed to estimate $\mu_1 - \mu_2$ to within 0.5 with 95% confidence (i.e. 95% CI of length 1).

29. A certain change in a manufacturing procedure for component parts is being considered. Samples are taken using both the existing and the new procedure in order to determine if the new procedure results in an improvement. If 75 of 1500 items from the existing procedure were found to be defective and 80 of 2000 items from the new procedure were found to be

defective, find the upper limit of a 95% confidence interval for the true difference, $p_1 - p_2$, where p_1 and p_2 are the true proportions of defectives for the existing and new procedures, respectively

30. A machine is producing metal pieces that are cylindrical in shape. A random sample of 60 pieces taken from this machine's production yields a sample mean diameter of 102 mm and a sample standard deviation of 4 mm. Find the lower limit of an 97% confidence interval for the true mean diameter μ of pieces produced by this machine.

31. LONG ANSWER QUESTION: In a comparison study of a standard (87 octane) versus premium (92 octane) blend of unleaded gasoline, the litres per 100 kilometres for each blend was recorded for five compact automobiles. Let μ_1 denote the true mean litres per 100 kilometres fuel consumption rating using standard gasoline, and let μ_2 denote the true mean litres per 100 kilometres fuel consumption rating using premium gasoline. If the standard blend is less fuel efficient (i.e. gives higher mean L/100km) than the premium blend, then $\mu_D = \mu_1 - \mu_2$ will be positive. Before we run the experiment, we believe that $\mu_D > 0$.

	Car				
	1	2	3	4	5
Standard	8.74	7.45	8.60	9.05	8.25
Premium	8.49	7.59	8.30	8.78	7.92

(a) Define the population parameter(s) of interest.
(b) State the null and alternative hypotheses in terms of the parameter.
(c) State the test statistic you will use. What distribution (including degrees of freedom, if appropriate) will you use to calculate the p-value.
(d) Find the observed value of the test statistic.
(e) Compute (or bracket) the p-value within the accuracy of the tables.
(f) What level of evidence against H_0 do you find?

32. LONG ANSWER QUESTION: A builder claims that heat pumps are installed in 70% of all homes being constructed today in a certain city. To test the builders claim, a random sample of 100 new homes in the city is taken, and 60 of these new homes are found to have heat pumps installed. Do these data provide substantial evidence that the true proportion p of new homes having heat pumps installed differs from the builders claim?

(a) Define the population parameter(s) of interest.
(b) State the null and alternative hypotheses in terms of the parameter.
(c) State the test statistic you will use. What distribution (including degrees of freedom, if appropriate) will you use to calculate the p-value.
(d) Find the observed value of the test statistic.
(e) Compute (or bracket) the p-value within the accuracy of the tables.
(f) What level of evidence against H_0 do you find?

Answers :

1. -1.7	2. 17.95	3. 0.97
4. 0.57	5. 0.6944	6. 0.3664
7. 0.08625	8. 0.2029	9. $7750
10. 0.225	11. 0.133	12. 0.577
13. 0.1389	14. 0.4142	15. 0.25
16. 0.75	17. 2.50	18. 0.4926
19. 0.082	20. 0.7888	21. 0.51
22. 0.1894	23. 0.2389	24. 2726.24
25. ii and iii	26. 21.98	27. -7.81
28. 163	29. 0.0240	30. 100.9

31.(a) μ_D = true mean difference, standard minus premium litres per 100 km fuel consumption
(b) $H_0 : \mu_D = 0$ and $H_1 : \mu_D > 0$
(c) $T = \frac{\bar{D}-0}{s_D/\sqrt{n_d}} \sim t_{(n_d-1)}$
(d) $t_{obs} = 2.33$
(e) $p-value = P(T_4 \geq 2.33)$, $.025 < p-value < .05$
(f) There is strong evidence against the null hypothesis.

32.(a) p=the true proportion of new homes having heat pumps installed
(b)$H_0 : p = 0.7 = p_0$, $H_1 : p \neq 0.7$.
(c) Test statistic $\dfrac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}} \sim Normal(0,1)$, since $n\hat{p}$ and $n(1-\hat{p}) \geq 5$.
(d) Observed test statistic -2.18.
(e) $p-value = 2 * P(Z \leq -2.18) = 0.0292$
(f) There is **strong** evidence against H_0.