

7.41	RC network. . . . .	312
7.42	RLC network. . . . .	313
8.1	Example of time shifting. . . . .	330
8.2	Example of time reversal. . . . .	330
8.3	Downsampling example. (a) Original sequence $x$ . (b) Result obtained by 2-fold downsampling of $x$ . . . . .	331
8.4	Upsampling example. (a) Original sequence $x$ . (b) Result obtained by 2-fold upsampling of $x$ . . . . .	332
8.5	Sequences for Example 8.3. . . . .	337
8.6	Examples of sequences with various sidedness properties. A sequence that is (a) left sided but not right sided, (b) right sided but not left sided, (c) finite duration, and (d) two sided. . . . .	338
8.7	The sequence $x$ from Example 8.5. . . . .	340
8.8	Example of a real-sinusoidal sequence. . . . .	341
8.9	The effect of increasing the frequency of a real sinusoidal sequence. A plot of $x(n) = \cos(\Omega n)$ for $\Omega$ having each of the values (a) $\frac{0\pi}{8} = 0$ , (b) $\frac{1\pi}{8} = \frac{\pi}{8}$ , (c) $\frac{2\pi}{8} = \frac{\pi}{4}$ , (d) $\frac{4\pi}{8} = \frac{\pi}{2}$ , (e) $\frac{8\pi}{8} = \pi$ , (f) $\frac{12\pi}{8} = \frac{3\pi}{2}$ , (g) $\frac{14\pi}{8} = \frac{7\pi}{4}$ , (h) $\frac{15\pi}{8}$ , and (i) $\frac{16\pi}{8} = 2\pi$ . . . . .	342
8.10	Examples of real exponential sequences. (a) $ a  > 1, a > 0$ [ $a = \frac{5}{4}; c = 1$ ]; (b) $ a  < 1, a > 0$ [ $a = \frac{4}{5}; c = 1$ ]; (c) $ a  = 1, a > 0$ [ $a = 1; c = 1$ ]; (d) $ a  > 1, a < 0$ [ $a = -\frac{5}{4}; c = 1$ ]; (e) $ a  < 1, a < 0$ [ $a = -\frac{4}{5}; c = 1$ ]; and (f) $ a  = 1, a < 0$ [ $a = -1; c = 1$ ]. . . . .	344
8.11	Example of complex sinusoidal sequence $x(n) = e^{j(2\pi/7)n}$ . The (a) real and (b) imaginary parts of $x$ . . . . .	345
8.12	Various mode of behavior for the real and imaginary parts of a complex exponential sequence. (a) $ a  > 1$ ; (b) $ a  < 1$ ; and (c) $ a  = 1$ . . . . .	346
8.13	The unit-step sequence. . . . .	347
8.14	The rectangular sequence. . . . .	347
8.15	The unit-impulse sequence. . . . .	349
8.16	Representing a piecewise-linear sequence using unit-step sequences. (a) The sequence $x$ . (b), (c), and (d) Three sequences whose sum is $x$ . . . . .	351
8.17	Block diagram of system. . . . .	351
8.18	Interconnection of systems. The (a) series interconnection and (b) parallel interconnection of the systems $\mathcal{H}_1$ and $\mathcal{H}_2$ . . . . .	352
8.19	Systems that are equivalent (assuming $\mathcal{H}^{-1}$ exists). (a) First and (b) second system. . . . .	355
8.20	Systems that are equivalent if $\mathcal{H}$ is time invariant (i.e., $\mathcal{H}$ commutes with $\mathcal{S}_{n_0}$ ). (a) A system that first time shifts by $n_0$ and then applies $\mathcal{H}$ (i.e., $y = \mathcal{H}\mathcal{S}_{n_0}x$ ); and (b) a system that first applies $\mathcal{H}$ and then time shifts by $n_0$ (i.e., $y = \mathcal{S}_{n_0}\mathcal{H}x$ ). . . . .	358
8.21	Systems that are equivalent if $\mathcal{H}$ is additive (i.e., $\mathcal{H}$ commutes with addition). (a) A system that first performs addition and then applies $\mathcal{H}$ (i.e., $y = \mathcal{H}(x_1 + x_2)$ ); and (b) a system that first applies $\mathcal{H}$ and then performs addition (i.e., $y = \mathcal{H}x_1 + \mathcal{H}x_2$ ). . . . .	360
8.22	Systems that are equivalent if $\mathcal{H}$ is homogeneous (i.e., $\mathcal{H}$ commutes with scalar multiplication). (a) A system that first performs scalar multiplication and then applies $\mathcal{H}$ (i.e., $y = \mathcal{H}(ax)$ ); and (b) a system that first applies $\mathcal{H}$ and then performs scalar multiplication (i.e., $y = a\mathcal{H}x$ ). . . . .	360
8.23	Systems that are equivalent if $\mathcal{H}$ is linear (i.e., $\mathcal{H}$ commutes with linear combinations). (a) A system that first computes a linear combination and then applies $\mathcal{H}$ (i.e., $y = \mathcal{H}(a_1x_1 + a_2x_2)$ ); and (b) a system that first applies $\mathcal{H}$ and then computes a linear combination (i.e., $y = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$ ). . . . .	360
9.1	Plots for Example 9.1. Plots of (a) $x(k)$ , (b) $h(k)$ , and (c) $h(n-k)$ versus $k$ . . . . .	372
9.2	Plots for Example 9.2. Plots of (a) $x(k)$ , (b) $h(k)$ , and (c) $h(n-k)$ versus $k$ . . . . .	375
9.3	The sequence $x*h$ for Example 9.2. . . . .	375
9.4	Plots for Example 9.4. Plots of (a) $x(k)$ , (b) $h(k)$ , and (c) $h(n-k)$ versus $k$ . . . . .	377
9.5	Block diagram representation of discrete-time LTI system with input $x$ , output $y$ , and impulse response $h$ . . . . .	384
9.6	Equivalences for the series interconnection of discrete-time LTI systems. The (a) first and (b) second equivalences. . . . .	385
9.7	Equivalence for the parallel interconnection of discrete-time LTI systems. . . . .	385

9.8	System interconnection example. . . . .	386
9.9	System in cascade with its inverse. . . . .	389
9.10	Feedback system with input $x$ and output $y$ . . . . .	390
10.1	Frequency spectrum of $x$ . (a) Magnitude spectrum and (b) phase spectrum. . . . .	427
10.2	Frequency spectrum of $x$ . . . . .	427
10.3	Magnitude spectrum of input sequence $x$ . . . . .	431
10.4	Magnitude spectrum of output sequence $y$ . . . . .	431
10.5	Frequency responses of (a) ideal lowpass, (b) ideal highpass, and (c) ideal bandpass filters. . . . .	434
10.6	Frequency spectra of the (a) input sequence $x$ and (b) output sequence $y$ . . . . .	437
11.1	A plot of $e^{- n /10}$ versus $n$ . . . . .	448
11.2	Spectra for downsampling example. (a) Spectrum $X$ of $x$ . (b) First summation $Y_0$ in expression for $Y$ . (c) Second summation $Y_1$ in expression for $Y$ . (d) Spectrum $Y$ of $y$ . . . . .	458
11.3	Frequency spectra. The frequency spectra (a) $X_1$ and (b) $X_2$ . . . . .	472
11.4	The 16-periodic sequence $x$ . . . . .	473
11.5	The 7-periodic sequence $x$ . . . . .	478
11.6	Frequency spectrum $X$ of the sequence $x$ . (a) Magnitude spectrum and (b) phase spectrum. . . . .	484
11.7	Frequency spectrum $X$ of the sequence $x$ . . . . .	484
11.8	Frequency spectrum $X$ of the sequence $x$ . (a) Magnitude spectrum and (b) phase spectrum. . . . .	486
11.9	Example of the Fourier transform $X$ of a sequence $x$ that is bandlimited to frequencies in $[-B, B]$ . . . . .	486
11.10	Time-domain view of a LTI system with input $x$ , output $y$ , and impulse response $h$ . . . . .	489
11.11	Frequency-domain view of a LTI system with input spectrum $X$ , output spectrum $Y$ , and frequency response $H$ . . . . .	489
11.12	Frequency response of example system. . . . .	490
11.13	Unwrapped phase example. (a) The phase function restricted such that its range is in $(-\pi, \pi]$ and (b) the corresponding unwrapped phase. . . . .	491
11.14	Equivalence involving frequency responses and the series interconnection of LTI systems. . . . .	493
11.15	Equivalence involving frequency responses and the parallel interconnection of LTI systems. . . . .	493
11.16	Frequency responses of (a) ideal lowpass, (b) ideal highpass, and (c) ideal bandpass filters. . . . .	496
11.17	Frequency responses of each of the (a) first, (b) second, and (c) third systems from the example. . . . .	498
11.18	Frequency spectra for the lowpass filtering example. (a) Frequency spectrum $X$ of the input $x$ . (b) Frequency response $H$ of the system. (c) Frequency spectrum $Y$ of the output $y$ . . . . .	500
11.19	Frequency spectra for the bandpass filtering example. (a) Frequency spectrum $X$ of the input $x$ . (b) Frequency response $H$ of the system. (c) Frequency spectrum $Y$ of the output $y$ . . . . .	502
11.20	The sampled DT Fourier transform obtained from the DFT when $N = 4$ . (a) Magnitude spectrum. (b) Phase spectrum. . . . .	507
11.21	The sampled DT Fourier transform obtained from the DFT when $N = 8$ . (a) Magnitude spectrum. (b) Phase spectrum. . . . .	507
11.22	The sampled DT Fourier transform obtained from the DFT when $N = 16$ . (a) Magnitude spectrum. (b) Phase spectrum. . . . .	508
11.23	The sampled DT Fourier transform obtained from the DFT when $N = 64$ . (a) Magnitude spectrum. (b) Phase spectrum. . . . .	508
12.1	A plot of $e^{- n /10}$ versus $n$ . . . . .	517
12.2	Examples of various types of sets. (a) A disk with center 0 and radius $r$ ; (b) an annulus with center 0, inner radius $r_0$ , and outer radius $r_1$ ; and (c) an exterior of a circle with center 0 and radius $r$ . . . . .	522
12.3	Example of set intersection. The sets (a) $R_1$ and (b) $R_2$ ; and (c) their intersection $R_1 \cap R_2$ . . . . .	523
12.4	Example of multiplying a set by a scalar. (a) The set $R$ . (b) The set $2R$ . . . . .	524
12.5	Example of the reciprocal of a set. (a) The set $R$ ; and its reciprocal $R^{-1}$ . . . . .	525
12.6	Examples of sets that would be either valid or invalid as the ROC of a $z$ transform. . . . .	525
12.7	Examples of sets that would be either valid or invalid as the ROC of a rational $z$ transform. . . . .	526

12.8	Examples of sets that would be either valid or invalid as the ROC of the $z$ transform of a finite-duration sequence. . . . .	526
12.9	Examples of sets that would be either valid or invalid as the ROC of the $z$ transform of a sequence that is right sided but not left sided. . . . .	527
12.10	Examples of sets that would be either valid or invalid as the ROC of the $z$ transform of a sequence that is left sided but not right sided. . . . .	527
12.11	Examples of sets that would be either valid or invalid as the ROC of the $z$ transform of a two-sided sequence. . . . .	527
12.12	Examples of sets that would be either valid or invalid as the ROC of a rational $z$ transform of a left/right-sided sequence. . . . .	528
12.13	Examples of sets that would not be a valid ROC of a $z$ transform. . . . .	529
12.14	ROCs for example. The (a) first, (b) second, and (c) third possible ROCs for $X$ . . . . .	530
12.15	ROCs for the linearity example. The (a) ROC of $X_1$ , (b) ROC of $X_2$ , (c) ROC associated with the intersection of the ROCs of $X_1$ and $X_2$ , and (d) ROC of $X$ . . . . .	532
12.16	ROCs for the linearity example. The (a) ROC of $X_1$ , (b) ROC of $X_2$ , (c) ROC associated with the intersection of the ROCs of $X_1$ and $X_2$ , and (d) ROC of $X$ . . . . .	534
12.17	ROCs for complex modulation. The ROC of the $z$ transform of the sequence (a) before and (b) after scaling. . . . .	537
12.18	ROCs for time reversal. The ROC of the $z$ transform of the sequence (a) before and (b) after time reversal. . . . .	539
12.19	Time-domain view of a LTI system with input $x$ , output $y$ , and impulse response $h$ . . . . .	563
12.20	$z$ -domain view of a LTI system with input $z$ transform $X$ , output $z$ transform $Y$ , and system function $H$ . . . . .	563
12.21	Equivalence involving system functions and the series interconnection of LTI systems. . . . .	564
12.22	Equivalence involving system functions and the parallel interconnection of LTI systems. . . . .	564
12.23	Poles and ROCs of the rational system functions in the causality example. The cases of part (a), (b), (c), and (d). . . . .	566
12.24	ROC for example. . . . .	568
12.25	The poles and ROC of the system function. . . . .	569
12.26	Poles and ROCs of the system function $H$ in the (a) first, (b) second, (c) third, and (d) fourth parts of the example. . . . .	570
12.27	Feedback system. . . . .	574
A.1	Graphical representation of a complex number. . . . .	594
A.2	Representations of complex numbers. The (a) Cartesian and (b) polar forms. . . . .	595
A.3	Conjugate of complex number. . . . .	596
A.4	Example of converting complex numbers from Cartesian to polar form. The case of the (a) first and (b) second part of the example. . . . .	601
A.5	Circle about $z_0$ with radius $r$ . . . . .	603
A.6	Open disk of radius $r$ about $z_0$ . . . . .	603
A.7	Open annulus about $z_0$ with inner radius $r_1$ and outer radius $r_2$ . . . . .	603
A.8	Plot of the poles and zeros of $f$ (with their orders indicated in parentheses). . . . .	607
D.1	Plot from example. . . . .	646
D.2	Plot from example. . . . .	647
D.3	Plot from example. . . . .	648
D.4	Plot from example. . . . .	649
D.5	The frequency response of the filter as produced by the <code>freqs</code> function. . . . .	655
D.6	The frequency response of the filter as produced by the <code>myfreqs</code> function. . . . .	657
D.7	The impulse and step responses of the system obtained from the code example. . . . .	658
D.8	The frequency response of the Butterworth lowpass filter obtained from the code example. . . . .	659
D.9	The frequency response of the Bessel lowpass filter obtained from the code example. . . . .	660
D.10	The frequency response of the filter as produced by the <code>freqz</code> function. . . . .	661

D.11	The frequency response of the filter as produced by the <code>myfreqz</code> function. . . . .	662
D.12	The impulse and step responses of the system obtained from the code example. . . . .	664
D.13	The frequency response of the Chebyshev type-II lowpass filter obtained from the code example. . .	665
D.14	The frequency response of the linear-phase FIR bandpass filter obtained from the code example. . . .	666



# List of Listings

D.1	mysinc.m . . . . .	642
D.2	myfactorial.m . . . . .	643
D.3	mysum.m . . . . .	643
D.4	mysum2.m . . . . .	644
D.5	myfreqs.m . . . . .	655
D.6	Computing and plotting the impulse and step responses . . . . .	657
D.7	Butterworth lowpass filter design . . . . .	658
D.8	Bessel lowpass filter design . . . . .	658
D.9	myfreqz.m . . . . .	660
D.10	Computing and plotting the impulse and step responses . . . . .	663
D.11	Chebyshev type-II lowpass filter design . . . . .	663
D.12	Linear-phase FIR bandpass filter design . . . . .	664



# License

This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 Unported (CC BY-NC-ND 3.0) License. A copy of this license is provided below. For a simple explanation of the rights granted by this license, see:

<https://creativecommons.org/licenses/by-nc-nd/3.0/>

## Creative Commons Attribution-NonCommercial-NoDerivs 3.0 Unported License

Creative Commons Legal Code

Attribution-NonCommercial-NoDerivs 3.0 Unported

CREATIVE COMMONS CORPORATION IS NOT A LAW FIRM AND DOES NOT PROVIDE LEGAL SERVICES. DISTRIBUTION OF THIS LICENSE DOES NOT CREATE AN ATTORNEY-CLIENT RELATIONSHIP. CREATIVE COMMONS PROVIDES THIS INFORMATION ON AN "AS-IS" BASIS. CREATIVE COMMONS MAKES NO WARRANTIES REGARDING THE INFORMATION PROVIDED, AND DISCLAIMS LIABILITY FOR DAMAGES RESULTING FROM ITS USE.

License

THE WORK (AS DEFINED BELOW) IS PROVIDED UNDER THE TERMS OF THIS CREATIVE COMMONS PUBLIC LICENSE ("CCPL" OR "LICENSE"). THE WORK IS PROTECTED BY COPYRIGHT AND/OR OTHER APPLICABLE LAW. ANY USE OF THE WORK OTHER THAN AS AUTHORIZED UNDER THIS LICENSE OR COPYRIGHT LAW IS PROHIBITED.

BY EXERCISING ANY RIGHTS TO THE WORK PROVIDED HERE, YOU ACCEPT AND AGREE TO BE BOUND BY THE TERMS OF THIS LICENSE. TO THE EXTENT THIS LICENSE MAY BE CONSIDERED TO BE A CONTRACT, THE LICENSOR GRANTS YOU THE RIGHTS CONTAINED HERE IN CONSIDERATION OF YOUR ACCEPTANCE OF SUCH TERMS AND CONDITIONS.

### 1. Definitions

- a. "Adaptation" means a work based upon the Work, or upon the Work and other pre-existing works, such as a translation, adaptation, derivative work, arrangement of music or other alterations of a literary or artistic work, or phonogram or performance and includes cinematographic adaptations or any other form in which the Work may be recast, transformed, or adapted including in any form recognizably derived from the original, except that a work that constitutes a Collection will not be considered an Adaptation for the purpose of this License. For the avoidance of doubt, where the Work is a musical work, performance or phonogram, the synchronization of the Work in timed-relation with a moving image ("synching") will be considered an Adaptation for the purpose of this License.
- b. "Collection" means a collection of literary or artistic works, such as encyclopedias and anthologies, or performances, phonograms or broadcasts, or other works or subject matter other than works listed



in Section 1(f) below, which, by reason of the selection and arrangement of their contents, constitute intellectual creations, in which the Work is included in its entirety in unmodified form along with one or more other contributions, each constituting separate and independent works in themselves, which together are assembled into a collective whole. A work that constitutes a Collection will not be considered an Adaptation (as defined above) for the purposes of this License.

- c. "Distribute" means to make available to the public the original and copies of the Work through sale or other transfer of ownership.
  - d. "Licensor" means the individual, individuals, entity or entities that offer(s) the Work under the terms of this License.
  - e. "Original Author" means, in the case of a literary or artistic work, the individual, individuals, entity or entities who created the Work or if no individual or entity can be identified, the publisher; and in addition (i) in the case of a performance the actors, singers, musicians, dancers, and other persons who act, sing, deliver, declaim, play in, interpret or otherwise perform literary or artistic works or expressions of folklore; (ii) in the case of a phonogram the producer being the person or legal entity who first fixes the sounds of a performance or other sounds; and, (iii) in the case of broadcasts, the organization that transmits the broadcast.
  - f. "Work" means the literary and/or artistic work offered under the terms of this License including without limitation any production in the literary, scientific and artistic domain, whatever may be the mode or form of its expression including digital form, such as a book, pamphlet and other writing; a lecture, address, sermon or other work of the same nature; a dramatic or dramatico-musical work; a choreographic work or entertainment in dumb show; a musical composition with or without words; a cinematographic work to which are assimilated works expressed by a process analogous to cinematography; a work of drawing, painting, architecture, sculpture, engraving or lithography; a photographic work to which are assimilated works expressed by a process analogous to photography; a work of applied art; an illustration, map, plan, sketch or three-dimensional work relative to geography, topography, architecture or science; a performance; a broadcast; a phonogram; a compilation of data to the extent it is protected as a copyrightable work; or a work performed by a variety or circus performer to the extent it is not otherwise considered a literary or artistic work.
  - g. "You" means an individual or entity exercising rights under this License who has not previously violated the terms of this License with respect to the Work, or who has received express permission from the Licensor to exercise rights under this License despite a previous violation.
  - h. "Publicly Perform" means to perform public recitations of the Work and to communicate to the public those public recitations, by any means or process, including by wire or wireless means or public digital performances; to make available to the public Works in such a way that members of the public may access these Works from a place and at a place individually chosen by them; to perform the Work to the public by any means or process and the communication to the public of the performances of the Work, including by public digital performance; to broadcast and rebroadcast the Work by any means including signs, sounds or images.
  - i. "Reproduce" means to make copies of the Work by any means including without limitation by sound or visual recordings and the right of fixation and reproducing fixations of the Work, including storage of a protected performance or phonogram in digital form or other electronic medium.
2. Fair Dealing Rights. Nothing in this License is intended to reduce, limit, or restrict any uses free from copyright or rights arising from limitations or exceptions that are provided for in connection with the copyright protection under copyright law or other applicable laws.
3. License Grant. Subject to the terms and conditions of this License, Licensor hereby grants You a worldwide, royalty-free, non-exclusive, perpetual (for the duration of the applicable copyright) license to

exercise the rights in the Work as stated below:

- a. to Reproduce the Work, to incorporate the Work into one or more Collections, and to Reproduce the Work as incorporated in the Collections; and,
- b. to Distribute and Publicly Perform the Work including as incorporated in Collections.

The above rights may be exercised in all media and formats whether now known or hereafter devised. The above rights include the right to make such modifications as are technically necessary to exercise the rights in other media and formats, but otherwise you have no rights to make Adaptations. Subject to 8(f), all rights not expressly granted by Licensor are hereby reserved, including but not limited to the rights set forth in Section 4(d).

4. Restrictions. The license granted in Section 3 above is expressly made subject to and limited by the following restrictions:

- a. You may Distribute or Publicly Perform the Work only under the terms of this License. You must include a copy of, or the Uniform Resource Identifier (URI) for, this License with every copy of the Work You Distribute or Publicly Perform. You may not offer or impose any terms on the Work that restrict the terms of this License or the ability of the recipient of the Work to exercise the rights granted to that recipient under the terms of the License. You may not sublicense the Work. You must keep intact all notices that refer to this License and to the disclaimer of warranties with every copy of the Work You Distribute or Publicly Perform. When You Distribute or Publicly Perform the Work, You may not impose any effective technological measures on the Work that restrict the ability of a recipient of the Work from You to exercise the rights granted to that recipient under the terms of the License. This Section 4(a) applies to the Work as incorporated in a Collection, but this does not require the Collection apart from the Work itself to be made subject to the terms of this License. If You create a Collection, upon notice from any Licensor You must, to the extent practicable, remove from the Collection any credit as required by Section 4(c), as requested.
- b. You may not exercise any of the rights granted to You in Section 3 above in any manner that is primarily intended for or directed toward commercial advantage or private monetary compensation. The exchange of the Work for other copyrighted works by means of digital file-sharing or otherwise shall not be considered to be intended for or directed toward commercial advantage or private monetary compensation, provided there is no payment of any monetary compensation in connection with the exchange of copyrighted works.
- c. If You Distribute, or Publicly Perform the Work or Collections, You must, unless a request has been made pursuant to Section 4(a), keep intact all copyright notices for the Work and provide, reasonable to the medium or means You are utilizing: (i) the name of the Original Author (or pseudonym, if applicable) if supplied, and/or if the Original Author and/or Licensor designate another party or parties (e.g., a sponsor institute, publishing entity, journal) for attribution ("Attribution Parties") in Licensor's copyright notice, terms of service or by other reasonable means, the name of such party or parties; (ii) the title of the Work if supplied; (iii) to the extent reasonably practicable, the URI, if any, that Licensor specifies to be associated with the Work, unless such URI does not refer to the copyright notice or licensing information for the Work. The credit required by this Section 4(c) may be implemented in any reasonable manner; provided, however, that in the case of a Collection, at a minimum such credit will appear, if a credit for all contributing authors of Collection appears, then as part of these credits and in a manner at least as prominent as the credits for the other contributing authors. For the avoidance of doubt, You may only use the credit required by this Section for the purpose of attribution in the manner set out above and, by exercising Your rights under this License, You may not implicitly or explicitly assert or imply any connection with, sponsorship or endorsement by the Original Author,

Licensors and/or Attribution Parties, as appropriate, of You or Your use of the Work, without the separate, express prior written permission of the Original Author, Licensors and/or Attribution Parties.

d. For the avoidance of doubt:

- i. Non-waivable Compulsory License Schemes. In those jurisdictions in which the right to collect royalties through any statutory or compulsory licensing scheme cannot be waived, the Licensor reserves the exclusive right to collect such royalties for any exercise by You of the rights granted under this License;
  - ii. Waivable Compulsory License Schemes. In those jurisdictions in which the right to collect royalties through any statutory or compulsory licensing scheme can be waived, the Licensor reserves the exclusive right to collect such royalties for any exercise by You of the rights granted under this License if Your exercise of such rights is for a purpose or use which is otherwise than noncommercial as permitted under Section 4(b) and otherwise waives the right to collect royalties through any statutory or compulsory licensing scheme; and,
  - iii. Voluntary License Schemes. The Licensor reserves the right to collect royalties, whether individually or, in the event that the Licensor is a member of a collecting society that administers voluntary licensing schemes, via that society, from any exercise by You of the rights granted under this License that is for a purpose or use which is otherwise than noncommercial as permitted under Section 4(b).
- e. Except as otherwise agreed in writing by the Licensor or as may be otherwise permitted by applicable law, if You Reproduce, Distribute or Publicly Perform the Work either by itself or as part of any Collections, You must not distort, mutilate, modify or take other derogatory action in relation to the Work which would be prejudicial to the Original Author's honor or reputation.

## 5. Representations, Warranties and Disclaimer

UNLESS OTHERWISE MUTUALLY AGREED BY THE PARTIES IN WRITING, LICENSOR OFFERS THE WORK AS-IS AND MAKES NO REPRESENTATIONS OR WARRANTIES OF ANY KIND CONCERNING THE WORK, EXPRESS, IMPLIED, STATUTORY OR OTHERWISE, INCLUDING, WITHOUT LIMITATION, WARRANTIES OF TITLE, MERCHANTIBILITY, FITNESS FOR A PARTICULAR PURPOSE, NON-INFRINGEMENT, OR THE ABSENCE OF LATENT OR OTHER DEFECTS, ACCURACY, OR THE PRESENCE OF ABSENCE OF ERRORS, WHETHER OR NOT DISCOVERABLE. SOME JURISDICTIONS DO NOT ALLOW THE EXCLUSION OF IMPLIED WARRANTIES, SO SUCH EXCLUSION MAY NOT APPLY TO YOU.

6. Limitation on Liability. EXCEPT TO THE EXTENT REQUIRED BY APPLICABLE LAW, IN NO EVENT WILL LICENSOR BE LIABLE TO YOU ON ANY LEGAL THEORY FOR ANY SPECIAL, INCIDENTAL, CONSEQUENTIAL, PUNITIVE OR EXEMPLARY DAMAGES ARISING OUT OF THIS LICENSE OR THE USE OF THE WORK, EVEN IF LICENSOR HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

## 7. Termination

- a. This License and the rights granted hereunder will terminate automatically upon any breach by You of the terms of this License. Individuals or entities who have received Collections from You under this License, however, will not have their licenses terminated provided such individuals or entities remain in full compliance with those licenses. Sections 1, 2, 5, 6, 7, and 8 will survive any termination of this License.
- b. Subject to the above terms and conditions, the license granted here is perpetual (for the duration of the applicable copyright in the Work). Notwithstanding the above, Licensor reserves the right to release the Work under different license terms or to stop distributing the Work at any time; provided, however that any such election will not serve to withdraw this License (or any other license that has been, or is required to be, granted under the terms of this License), and this License will continue in full force and effect unless terminated as stated above.

## 8. Miscellaneous

- a. Each time You Distribute or Publicly Perform the Work or a Collection, the Licensor offers to the recipient a license to the Work on the same terms and conditions as the license granted to You under this License.
- b. If any provision of this License is invalid or unenforceable under applicable law, it shall not affect the validity or enforceability of the remainder of the terms of this License, and without further action by the parties to this agreement, such provision shall be reformed to the minimum extent necessary to make such provision valid and enforceable.
- c. No term or provision of this License shall be deemed waived and no breach consented to unless such waiver or consent shall be in writing and signed by the party to be charged with such waiver or consent.
- d. This License constitutes the entire agreement between the parties with respect to the Work licensed here. There are no understandings, agreements or representations with respect to the Work not specified here. Licensor shall not be bound by any additional provisions that may appear in any communication from You. This License may not be modified without the mutual written agreement of the Licensor and You.
- e. The rights granted under, and the subject matter referenced, in this License were drafted utilizing the terminology of the Berne Convention for the Protection of Literary and Artistic Works (as amended on September 28, 1979), the Rome Convention of 1961, the WIPO Copyright Treaty of 1996, the WIPO Performances and Phonograms Treaty of 1996 and the Universal Copyright Convention (as revised on July 24, 1971). These rights and subject matter take effect in the relevant jurisdiction in which the License terms are sought to be enforced according to the corresponding provisions of the implementation of those treaty provisions in the applicable national law. If the standard suite of rights granted under applicable copyright law includes additional rights not granted under this License, such additional rights are deemed to be included in the License; this License is not intended to restrict the license of any rights under applicable law.

## Creative Commons Notice

Creative Commons is not a party to this License, and makes no warranty whatsoever in connection with the Work. Creative Commons will not be liable to You or any party on any legal theory for any damages whatsoever, including without limitation any general, special, incidental or consequential damages arising in connection to this license. Notwithstanding the foregoing two (2) sentences, if Creative Commons has expressly identified itself as the Licensor hereunder, it shall have all rights and obligations of Licensor.

Except for the limited purpose of indicating to the public that the Work is licensed under the CCPL, Creative Commons does not authorize the use by either party of the trademark "Creative Commons" or any related trademark or logo of Creative Commons without the prior written consent of Creative Commons. Any permitted use will be in compliance with Creative Commons' then-current trademark usage guidelines, as may be published on its website or otherwise made available upon request from time to time. For the avoidance of doubt, this trademark restriction does not form part of this License.

Creative Commons may be contacted at <https://creativecommons.org/>.



# Preface

This book is primarily intended to be used as a text for undergraduate students in engineering (and related) disciplines. The book provides a basic introduction to continuous-time and discrete-time signals and systems. Since many engineering curricula use MATLAB as a teaching tool, the book also includes a detailed introduction to MATLAB as an appendix. This book evolved from a detailed set of lecture notes that the author prepared in order to teach two different undergraduate courses on continuous-time signals and systems at the University of Victoria (Victoria, BC, Canada). In particular, the first version of these lectures notes was developed while the author was teaching ELEC 260 (Signal Analysis) in the Summer 2003 term, with some further content being added to accommodate the teaching of ELEC 255 (System Dynamics) in the Fall 2003 term. Over time, the lecture notes underwent many changes, eventually evolving into a textbook. The earlier versions of this book only covered the continuous-time case, with coverage of the discrete-time case being added later. All of this work ultimately led to the book that you are now reading.

## Acknowledgments

I would like to thank my colleague, Dr. Wu-Sheng Lu, for many interesting technical discussions that helped to clarify some of the finer points of the mathematics behind signals and systems. Also, I would like to thank my past students for their feedback regarding earlier revisions of this manuscript. They have helped me to eliminate numerous errors in this manuscript that would have otherwise gone undetected.

Michael Adams  
Victoria, BC  
2022-12-31

## Guidance for Instructors

The theory of continuous-time (CT) and discrete-time (DT) signals and systems is taught in many engineering and related disciplines. The manner in which this material is split across courses (or terms in the case of multi-term courses) can vary dramatically from one institution or program to another. For this reason, this textbook has been organized in such a way as to accommodate the teaching of courses with a wide variety of structures. Some possibilities include (but are not limited to):

- sequential presentation of the CT and DT cases with the CT case covered first (e.g., two single-term courses, where the first covers the CT case and the second covers the DT case);
- sequential presentation of the CT and DT cases with the DT case covered first (e.g., two single-term courses, where the first covers the DT case and the second covers the CT case);
- integrated presentation of both the CT and DT cases (e.g., a one- or two-term course that covers both the CT and DT cases together).

## Sequential Presentation of the CT and DT Cases

The sequential presentation of the CT and DT cases with the CT case treated first might, for example, cover material from the textbook as follows:

1. First Course/Term: The CT Case
  - (a) Chapter 1 (Introduction) with an emphasis on the material most relevant to the CT case
  - (b) if a review of complex analysis is desired: Appendix A (Complex Analysis)
  - (c) Chapter 2 (Preliminaries) with an emphasis on the material most relevant to the CT case
  - (d) Chapter 3 (Continuous-Time Signals and Systems)
  - (e) Chapter 4 (Continuous-Time Linear Time-Invariant Systems)
  - (f) Chapter 5 (Continuous-Time Fourier Series)
  - (g) Chapter 6 (Continuous-Time Fourier Transform)
  - (h) if an introduction to partial fraction expansions is needed: Appendix B (Partial Fraction Expansions)
  - (i) Chapter 7 (Laplace Transform)
2. Second Course/Term: The DT Case
  - (a) Chapter 1 (Introduction) with an emphasis on the material most relevant to the DT case
  - (b) Chapter 2 (Preliminaries) with an emphasis on the material most relevant to the DT case
  - (c) Chapter 8 (Discrete-Time Signals and Systems)
  - (d) Chapter 9 (Discrete-Time Linear Time-Invariant Systems)
  - (e) Chapter 10 (Discrete-Time Fourier Series)
  - (f) Chapter 11 (Discrete-Time Fourier Transform)
  - (g) Chapter 12 (z Transform)

To cover the DT case first, one should, for the most part, be able to simply swap the order of the two courses described above, since the textbook tries to minimize dependencies on whether the CT case is covered before the DT case. This said, however, there are a small number of dependencies. To resolve these dependencies, the following changes would need to be made. A small amount of material would need to be moved from the course covering the DT case to the course covering the CT case, namely:

- Section 11.17, which covers the relationship between the discrete-time Fourier transform (DTFT) and continuous-time Fourier series (CTFS); and
- Section 11.18, which covers the relationship between the DTFT and continuous-time Fourier transform (CTFT).

Also, a small amount of material would need to be moved from the course covering the CT case to the course covering the DT case, namely:

- Section 3.5.11, which introduces the delta function.

## Integrated Presentation of the CT and DT Cases

The integrated presentation of the CT and DT cases might, for example, cover material from the textbook as follows:

## 1. First Term

- (a) Chapter 1 (Introduction)
- (b) if a review of complex analysis is desired: Appendix A (Complex Analysis)
- (c) Chapter 2 (Preliminaries)
- (d) Chapter 3 (Continuous-Time Signals and Systems)
- (e) Chapter 8 (Discrete-Time Signals and Systems)
- (f) Chapter 4 (Continuous-Time Linear Time-Invariant Systems)
- (g) Chapter 9 (Discrete-Time Linear Time-Invariant Systems)
- (h) Chapter 5 (Continuous-Time Fourier Series)
- (i) Chapter 10 (Discrete-Time Fourier Series)

## 2. Second Term

- (a) Chapter 6 (Continuous-Time Fourier Transform)
- (b) Chapter 11 (Discrete-Time Fourier Transform)
- (c) if an introduction to partial fraction expansions is needed: Appendix B (Partial Fraction Expansions)
- (d) Chapter 7 (Laplace Transform)
- (e) Chapter 12 (z Transform)

## Video Lectures

Video lectures are available for some of the material covered in this textbook. These video lectures are likely to be helpful to instructors, either for planning their own courses or for using as additional reference material for their students. More information on these video lectures can be found in Appendix G.

## Lecture Slides

This textbook is intended to be used in conjunction with the following set of lecture slides:

- M. D. Adams, *Lecture Slides for Signals and Systems*, Edition 5.0, Dec. 2022, ISBN 978-1-990707-02-5 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/sigsysbook>.

## Textbook Web Site

This textbook has an associated web site whose URL is:

- <https://www.ece.uvic.ca/~mdadams/sigsysbook>

To obtain the most recent version of this textbook (with functional hyperlinks) or for additional information and resources related to this textbook (such as lecture slides, video lectures, and errata), please visit this site.

## Companion Git Repository

This textbook has an associated Git repository containing some source code and other supplemental files. The URL for this repository is:

- [https://github.com/mdadams/sigsysbook\\_companion.git](https://github.com/mdadams/sigsysbook_companion.git)





## About the Author



Michael Adams received the B.A.Sc. degree in computer engineering from the University of Waterloo, Waterloo, ON, Canada in 1993, the M.A.Sc. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada in 1998, and the Ph.D. degree in electrical engineering from the University of British Columbia, Vancouver, BC, Canada in 2002. From 1993 to 1995, Michael was a member of technical staff at Bell-Northern Research in Ottawa, ON, Canada, where he developed real-time software for fiber-optic telecommunication systems. Since 2003, Michael has been on the faculty of the Department of Electrical and Computer Engineering at the University of Victoria, Victoria, BC, Canada, first as an Assistant Professor and currently as an Associate Professor.

Michael is the recipient of a Natural Sciences and Engineering Research Council (of Canada) Postgraduate Scholarship. He has served as a voting member of the Canadian Delegation to ISO/IEC JTC 1/SC 29 (i.e., Coding of Audio, Picture, Multimedia and Hypermedia Information), and been an active participant in the JPEG-2000 standardization effort, serving as co-editor of the JPEG-2000 Part-5 standard and principal author of one of the first JPEG-2000 implementations (i.e., JasPer). His research interests include software, signal processing, image/video/audio processing and coding, multiresolution signal processing (e.g., filter banks and wavelets), geometry processing, and data compression.



## Other Works by the Author

Some other open-access textbooks and slide decks by the author of this book include:

1. M. D. Adams, *Lecture Slides for Signals and Systems*, Edition 5.0, Dec. 2022, ISBN 978-1-990707-02-5 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/sigsysbook>.
2. M. D. Adams, *Exercises for Programming in C++ (Version 2021-04-01)*, Apr. 2021, ISBN 978-0-9879197-5-5 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/cppbook>.
3. M. D. Adams, *Lecture Slides for Programming in C++ (Version 2021-04-01)*, Apr. 2021, ISBN 978-0-9879197-4-8 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/cppbook>.
4. M. D. Adams, *Multiresolution Signal and Geometry Processing: Filter Banks, Wavelets, and Subdivision (Version 2013-09-26)*, University of Victoria, Victoria, BC, Canada, Sept. 2013, ISBN 978-1-55058-507-0 (print), ISBN 978-1-55058-508-7 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/waveletbook>.
5. M. D. Adams, *Lecture Slides for Multiresolution Signal and Geometry Processing (Version 2015-02-03)*, University of Victoria, Victoria, BC, Canada, Feb. 2015, ISBN 978-1-55058-535-3 (print), ISBN 978-1-55058-536-0 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/waveletbook>.
6. M. D. Adams, *Lecture Slides for Linux System Programming*, Edition 0.0, Dec. 2022, ISBN 978-1-990707-03-2 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/cppbook>.
7. M. D. Adams, *Lecture Slides for the Clang Libraries*, Edition 0.0, Dec. 2022, ISBN 978-1-990707-04-9 (PDF). Available from Google Books, Google Play Books, and author's web site <https://www.ece.uvic.ca/~mdadams/cppbook>.



# Chapter 1

## Introduction

### 1.1 Signals and Systems

Mathematics has a very broad scope, encompassing many areas such as: linear algebra, calculus, probability and statistics, geometry, differential equations, and numerical methods. For engineers, however, an area of mathematics of particular importance is the one that pertains to signals and systems (which is, loosely speaking, the branch of mathematics known as functional analysis). It is this area of mathematics that is the focus of this book. Before we can treat this topic in any meaningful way, however, we must first explain precisely what signals and systems are. This is what we do next.

### 1.2 Signals

A **signal** is a function of one or more variables that conveys information about some (usually physical) phenomenon. Some examples of signals include:

- a human voice
- a voltage in an electronic circuit
- the temperature of a room controlled by a thermostat system
- the position, velocity, and acceleration of an aircraft
- the acceleration measured by an accelerometer in a cell phone
- the force measured by a force sensor in a robotic system
- the electromagnetic waves used to transmit information in wireless computer networks
- a digitized photograph
- a digitized music recording
- the evolution of a stock market index over time

#### 1.2.1 Classification of Signals

Signals can be classified based on the number of independent variables with which they are associated. A signal that is a function of only one variable is said to be **one dimensional**. Similarly, a signal that is a function of two or more variables is said to be **multi-dimensional**. Human speech is an example of a one-dimensional signal. In this case, we have a signal associated with fluctuations in air pressure as a function of time. An example of a two-dimensional signal is a monochromatic image. In this case, we have a signal that corresponds to a measure of light intensity as a function of horizontal and vertical position.

A signal can also be classified on the basis of whether it is a function of continuous or discrete variables. A signal that is a function of continuous variables (e.g., a real variable) is said to be **continuous time**. Similarly, a signal that is a function of discrete variables (e.g., an integer variable) is said to be **discrete time**. Although the independent variable need not represent time, for matters of convenience, much of the terminology is chosen as if this were so.

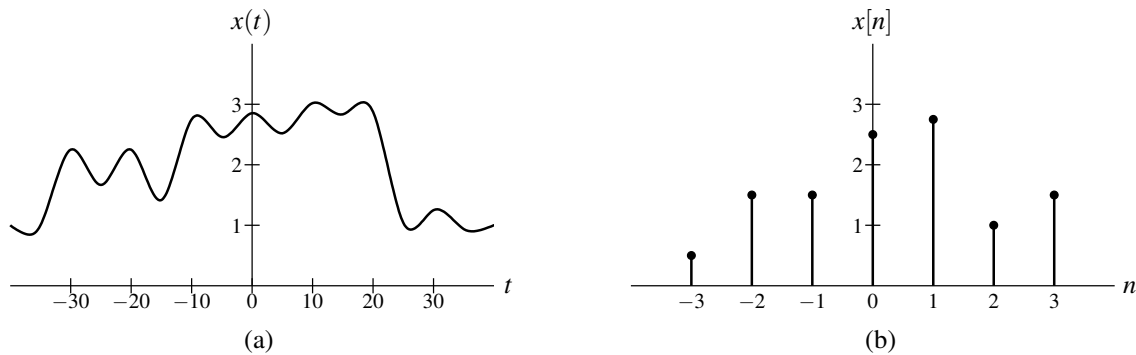


Figure 1.1: Graphical representations of (a) continuous-time and (b) discrete-time signals.

For example, a digital image (which consists of a rectangular array of pixels) would be referred to as a discrete-time signal, even though the independent variables (i.e., horizontal and vertical position) do not actually correspond to time.

If a signal is a function of discrete variables (i.e., discrete-time) and the value of the function itself is also discrete, the signal is said to be **digital**. Similarly, if a signal is a function of continuous variables, and the value of the function itself is also continuous, the signal is said to be **analog**.

Many phenomena in our physical world can be described in terms of continuous-time signals. Some examples of continuous-time signals include: voltage or current waveforms in an electronic circuit; electrocardiograms, speech, and music recordings; position, velocity, and acceleration of a moving body; forces and torques in a mechanical system; and flow rates of liquids or gases in a chemical process. Any signals processed by digital computers (or other digital devices) are discrete-time in nature. Some examples of discrete-time signals include digital video, digital photographs, and digital audio data.

A discrete-time signal may be inherently discrete or correspond to a sampled version of a continuous-time signal. An example of the former would be a signal corresponding to the Dow Jones Industrial Average stock market index (which is only defined on daily intervals), while an example of the latter would be the sampled version of a (continuous-time) speech signal.

## 1.2.2 Notation and Graphical Representation of Signals

In the case of discrete-time signals, we sometimes refer to the signal as a **sequence**. The  $n$ th element of a sequence  $x$  is denoted as either  $x(n)$  or  $x_n$ . Figure 1.1 shows how continuous-time and discrete-time signals are represented graphically.

## 1.2.3 Examples of Signals

A number of examples of signals have been suggested previously. Here, we provide some graphical representations of signals for illustrative purposes. Figure 1.2 depicts a digitized speech signal. Figure 1.3 shows an example of a monochromatic image. In this case, the signal represents light intensity as a function of two variables (i.e., horizontal and vertical position).

## 1.3 Systems

A **system** is an entity that processes one or more input signals in order to produce one or more output signals, as shown in Figure 1.4. Such an entity is represented mathematically by a system of one or more equations.

In a communication system, the input might represent the message to be sent, and the output might represent the received message. In a robotics system, the input might represent the desired position of the end effector (e.g., gripper), while the output could represent the actual position.



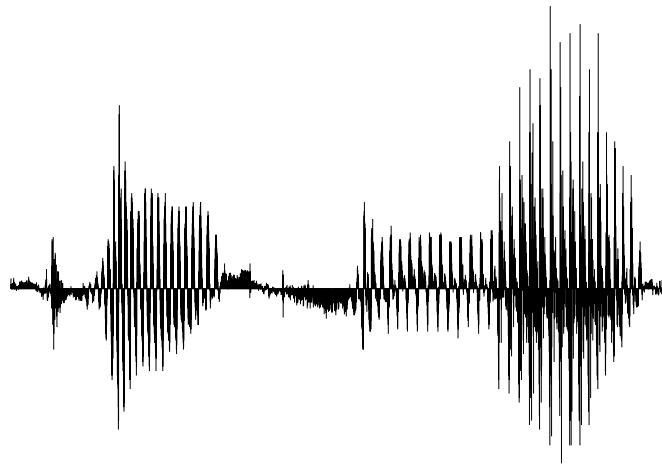


Figure 1.2: Segment of digitized human speech.



Figure 1.3: A monochromatic image.

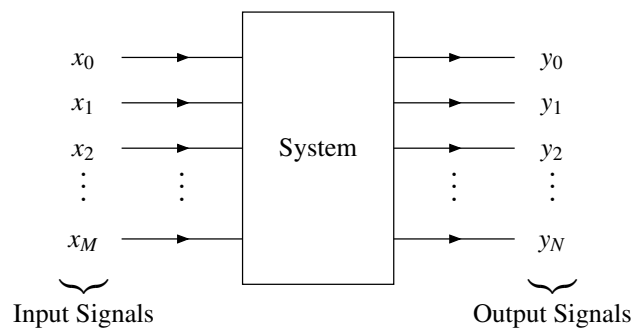


Figure 1.4: System with one or more inputs and one or more outputs.



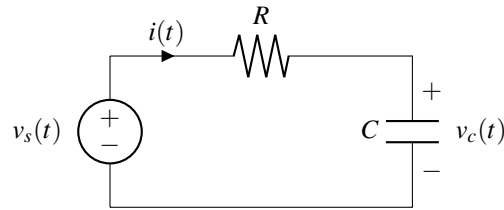


Figure 1.5: A simple RC network.

### 1.3.1 Classification of Systems

A system can be classified based on the number of inputs and outputs it has. A system with only one input is described as **single input**, while a system with multiple inputs is described as **multi-input**. Similarly, a system with only one output is said to be **single output**, while a system with multiple outputs is said to be **multi-output**. Two commonly occurring types of systems are single-input single-output (SISO) and multi-input multi-output (MIMO).

A system can also be classified based on the types of signals with which it interacts. A system that deals with continuous-time signals is called a **continuous-time system**. Similarly, a system that deals with discrete-time signals is said to be a **discrete-time system**. A system that handles both continuous- and discrete-time signals, is sometimes referred to as a **hybrid system** (or sampled-data system). Similarly, systems that deal with digital signals are referred to as **digital**, while systems that handle analog signals are referred to as **analog**. If a system interacts with one-dimensional signals, the system is referred to as **one-dimensional**. Likewise, if a system handles multi-dimensional signals, the system is said to be **multi-dimensional**.

### 1.3.2 Examples of Systems

Systems can manipulate signals in many different ways and serve many useful purposes. Sometimes systems serve to extract information from their input signals. For example, in the case of speech signals, systems can be used in order to perform speaker identification or voice recognition. A system might analyze electrocardiogram signals in order to detect heart abnormalities. Amplification and noise reduction are other functionalities that systems could offer.

One very basic system is the resistor-capacitor (RC) network shown in Figure 1.5. Here, the input would be the source voltage  $v_s$  and the output would be the capacitor voltage  $v_c$ .

Consider the signal-processing systems shown in Figure 1.6. The system in Figure 1.6(a) uses a discrete-time system (such as a digital computer) to process a continuous-time signal. The system in Figure 1.6(b) uses a continuous-time system (such as an analog computer) to process a discrete-time signal. The first of these types of systems is ubiquitous in the world today.

Consider the communication system shown in Figure 1.7. This system takes a message at one location and reproduces this message at another location. In this case, the system input is the message to be sent, and the output is the estimate of the original message. Usually, we want the message reproduced at the receiver to be as close as possible to the original message sent by the transmitter.

A system of the general form shown in Figure 1.8 frequently appears in control applications. Often, in such applications, we would like an output to track some reference input as closely as possible. Consider, for example, a robotics application. The reference input might represent the desired position of the end effector, while the output represents the actual position.

## 1.4 Why Study Signals and Systems?

As can be seen from the earlier examples, there are many practical applications in which we need to develop systems that manipulate signals. In order to do this, we need a formal mathematical framework for the study of such systems. Such a framework can be used to guide the design of new systems as well as to analyze the behavior of already existing systems. Over time, the complexity of systems designed by engineers has continued to grow. Today, most systems of practical interest are highly complex. For this reason, a formal mathematical framework to guide the design of