Chapter 4 – Force System Resultants

Contents

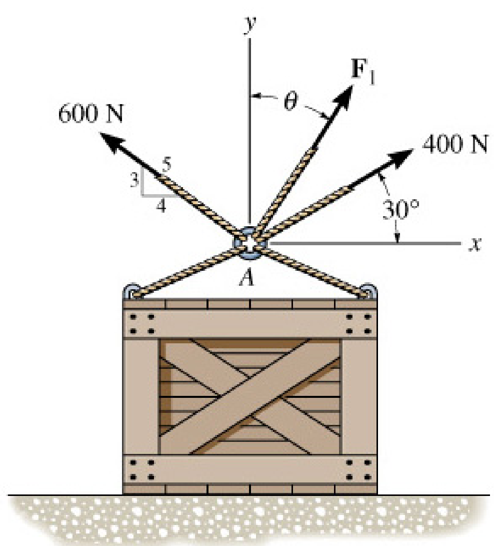
The force applied to the wrench will produce a tendency to produce rotation, this effect is called a moment.

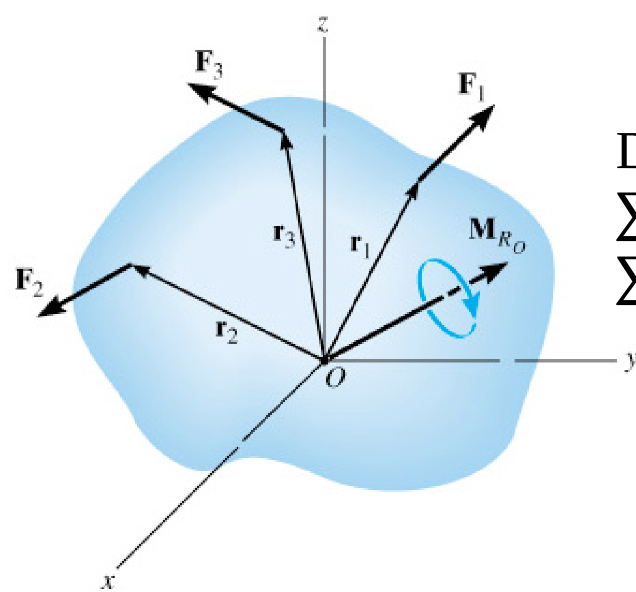
Moment of a Force

Particle vs Rigid Body Equilibrium

The ability to calculate force system resultants will be used in the equilibrium analysis of rigid bodies.

Consider the differences between a particle equilibrium and rigid body equilibrium problem.





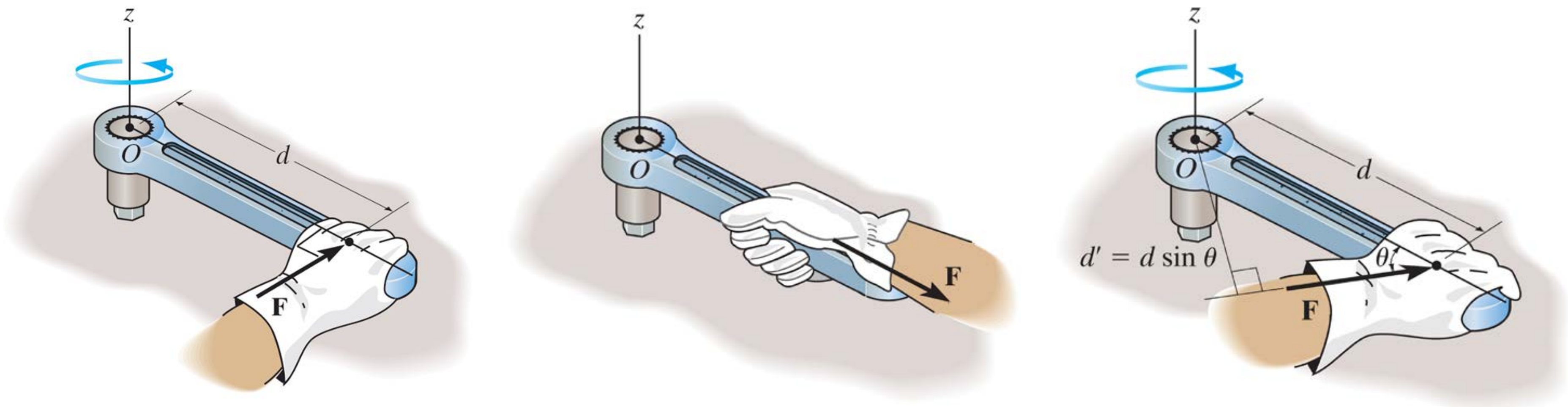
Dynamics
$\sum \mathbf{F} = m\boldsymbol{a}$
$\sum \mathbf{M}_o = I\boldsymbol{\alpha}$

Moment of a Force

Definition of a Moment

When a force is applied to a body, it will produce a tendency to create rotation about a point or axis. This tendency is referred to a ***moment***.

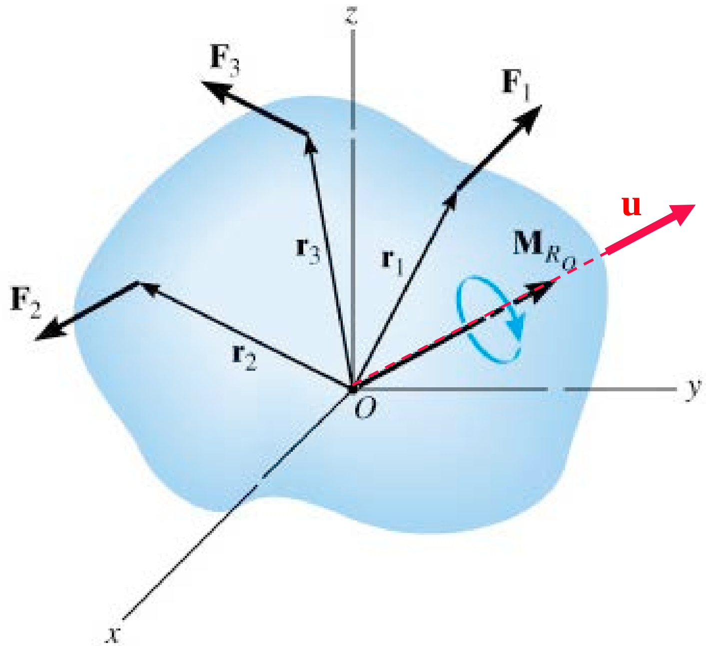
A ***torque*** is a similar concept; however, this term is employed when the applied force affects the change of rate of angular momentum.



Moment of a Force

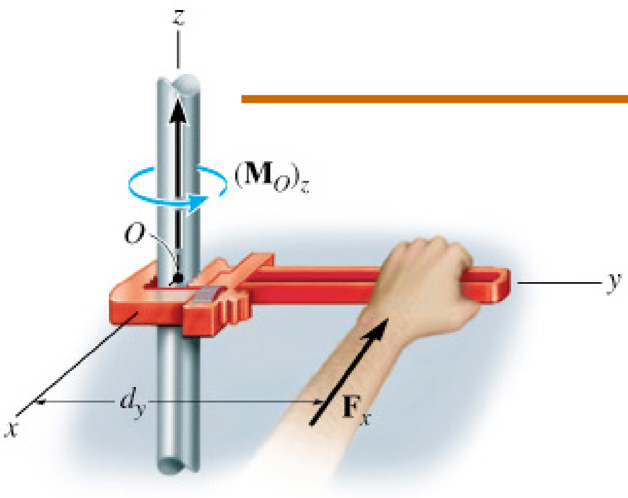
We can exert a pure moment or induce a moment by applying a force to a body. A Moment is a **vector quantity**:

- Magnitude: Strength of the tendency to create rotation.
- Direction (line of action): axis where the body tends to rotate about.
- Sense: Rotation is defined by the Right-Hand Rule (RHR).



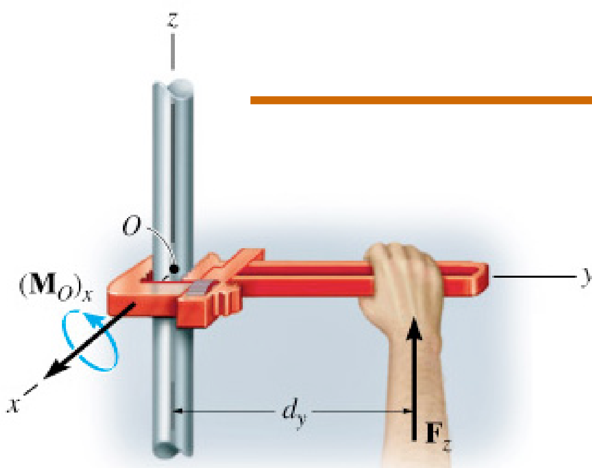
Sign convention with RHR
Clockwise (CW) → negative
Counter-clockwise (CCW) → positive

Moment of a Force



(a)

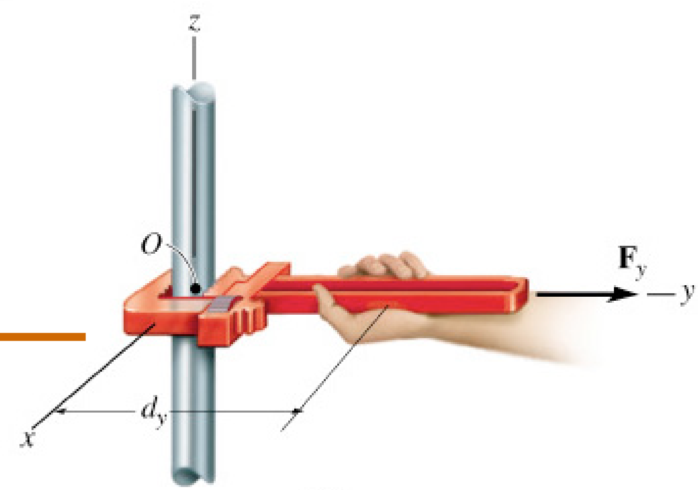
Tendency for the bar to rotate about the *z* axis.



(b)

Tendency for the bar to rotate about the *x* axis.

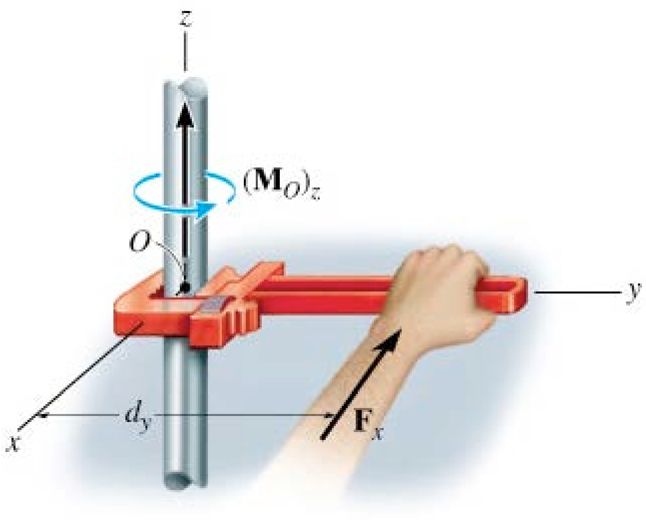
No tendency for the bar to rotate.

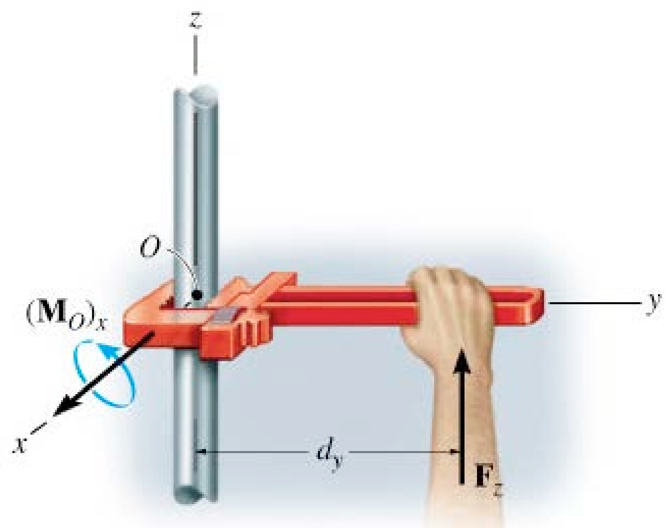


(c)

Moment of a Force

The magnitude of a moment is directly proportional to the magnitude of **F** and the perpendicular distance or ***moment arm d***.





$$\boldsymbol{M}_{\mathbf{0}} = (F_x d_y)\boldsymbol{k} = M_{0z}\boldsymbol{k}$$

$$\boldsymbol{M}_{\mathbf{0}} = (F_z d_y)\boldsymbol{i} = M_{0x}\boldsymbol{i}$$

Moment of a Force

Scalar Formulation (Planar Problems)

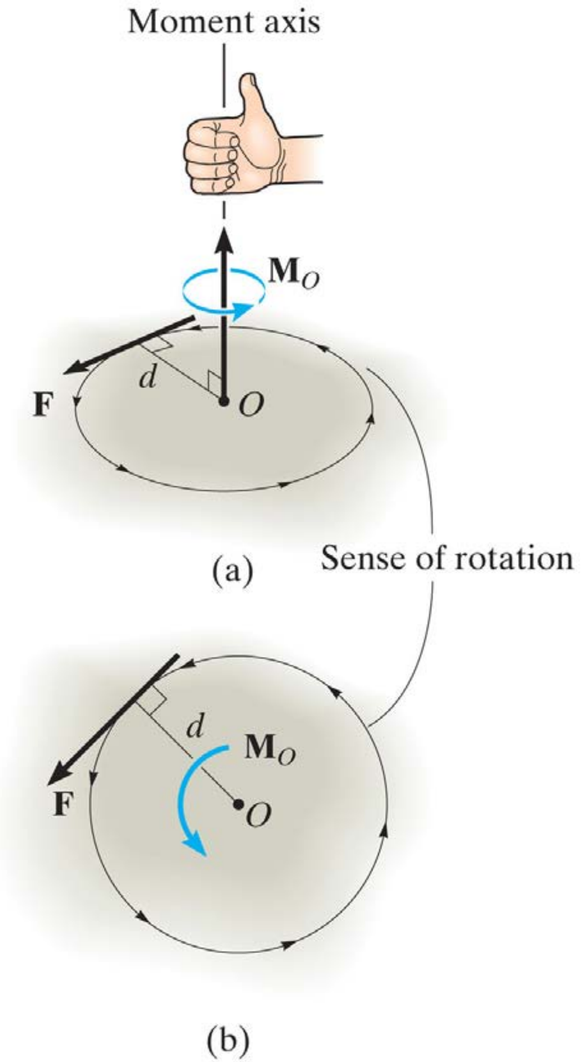
In a 2-D case, the *magnitude* of the moment is

$$M_o = F\,d$$

As shown, *d* is the perpendicular distance from point *O* to the line of action of the force.

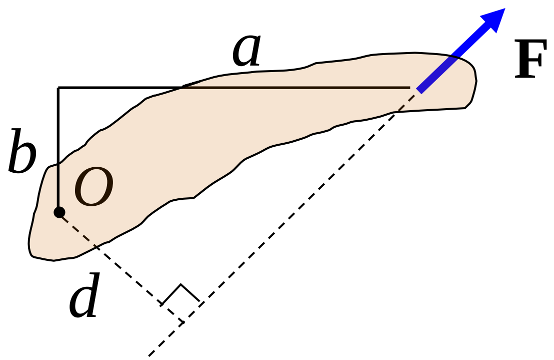
The *direction* of M_O must be along the $\pm$z axis

The *sense* is either clockwise (CW –) or counter-clockwise (CCW +), according with the RHR.

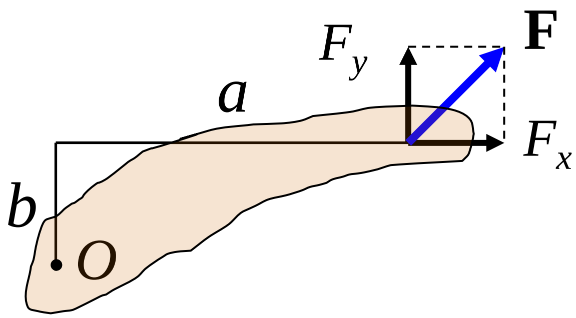


Moment of a Force

Calculation of a Moment



For example, $M_O = F\,d$ and the direction is counter-clockwise. One would have to determine d.



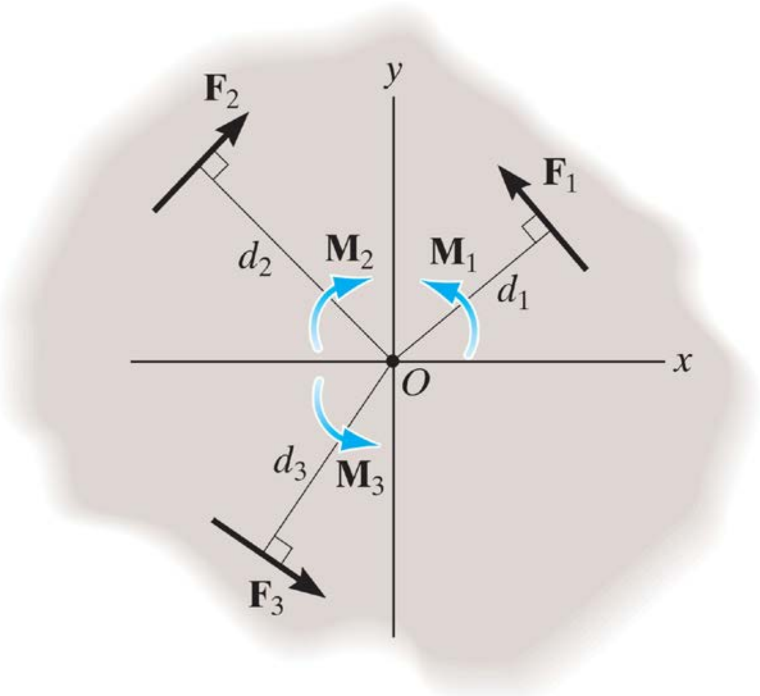
Often it is easier to determine M_O by using the components of **F** as shown.

Then, $M_O = (F_y\,a) - (F_x\,b)$. Note the different signs on the terms! While F_y produces a CCW rotation about O, F_x produces a CW rotation about O.

Moment of a Force

Resultant Moment

For 2D problems, where all the forces lie within the *x*-*y* plane, the resultant moment about a point *O* can be found by finding the algebraic sum of the moments caused by all the forces.



$$M_{R0} = F_1 d_1 - F_2 d_2 + F_3 d_3$$

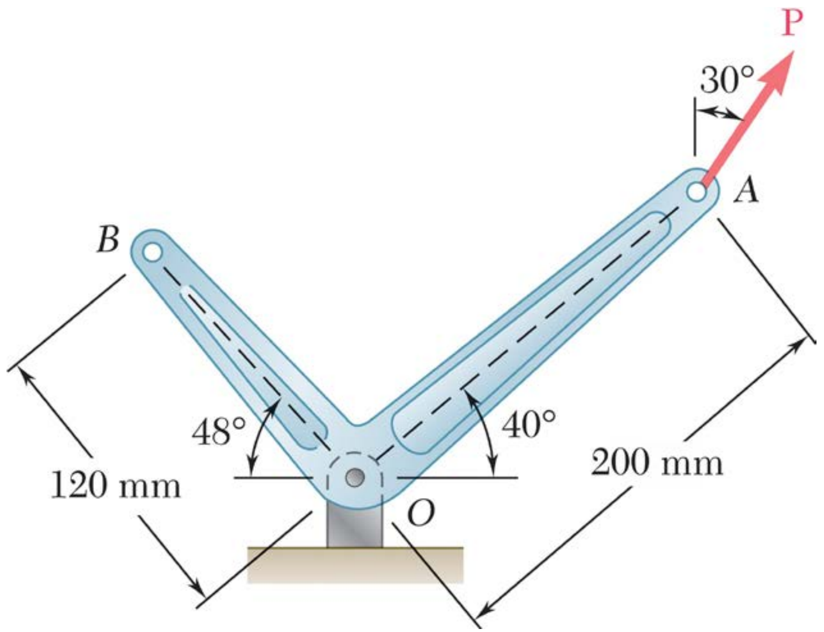
$$\mathbf{M}_{R0} = M_{R0}\boldsymbol{k}$$

Example

A 400-N force **P** is applied at Point A of the bell crank shown.

a) Compute the moment of the force **P** about O by resolving it into components along line OA and in a direction perpendicular to that line.

b) Determine the magnitude and direction of the smallest force **Q** applied at B that has the same moment as **P** about O.

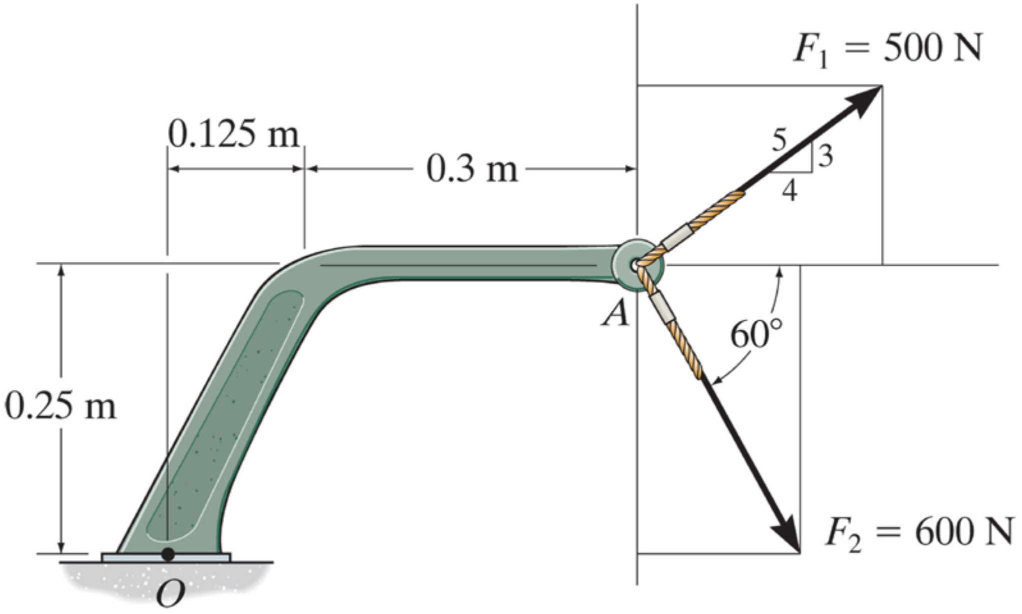


Example

Determine the resultant moment produced by the forces about point O.

a) Use the rectangular component approach

b) Find d using trigonometry



Chapter 4 – Force System Resultants

Contents

The force applied to the wrench will produce a tendency to produce rotation, this effect is called a moment.

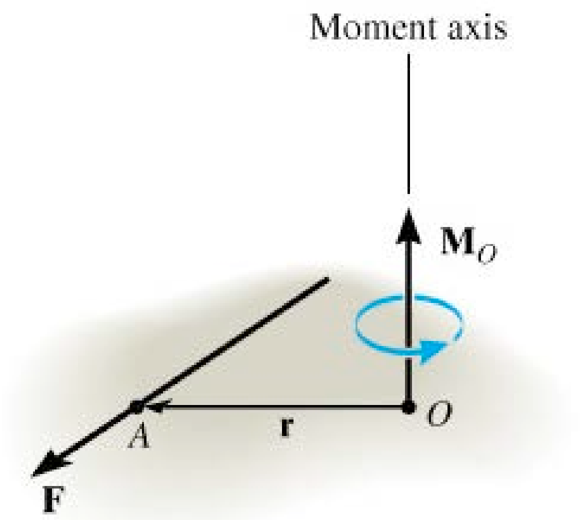
Moment of a Force

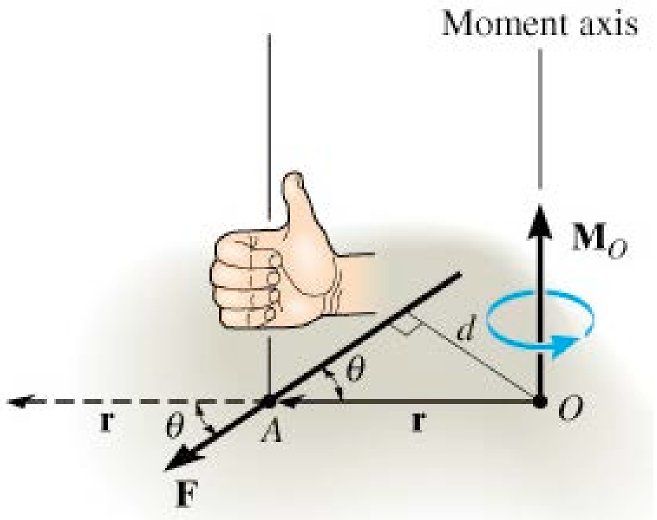
Review Moments of Coplanar Forces

On a plane, magnitude $M_O = F\,d$ and vector $\mathbf{M}_O = M_O\,\boldsymbol{k} = F\,d\,\boldsymbol{k}$.

Assume the case that we want to know $\mathbf{M}_O$, and vectors **F** and **r** are given, where *F* and *r* are their magnitudes. One would have to determine $d = r \sin(\theta)$, and then find the moment

$$\mathbf{M}_O = F\,d\,\boldsymbol{k} = F\,r\,\sin(\theta)\,\boldsymbol{k}.$$





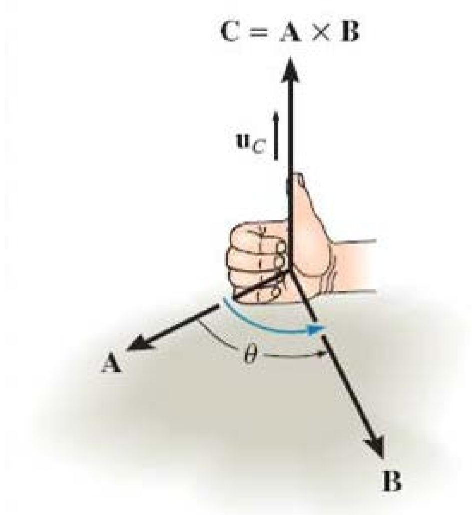
Cross Product

Review Linear Algebra

In general, the cross product of two vectors **A** and **B** results in another vector, **C** , i.e., **C** = **A** × **B**. The magnitude and direction of the resulting vector can be written as

$$\mathbf{C} = \mathbf{A} \times \mathbf{B} = A\,B \sin\theta\, \boldsymbol{u}_C$$

As shown, $\boldsymbol{u}_C$ is the unit vector perpendicular to both A and B vectors (or to the plane containing the A and B vectors).



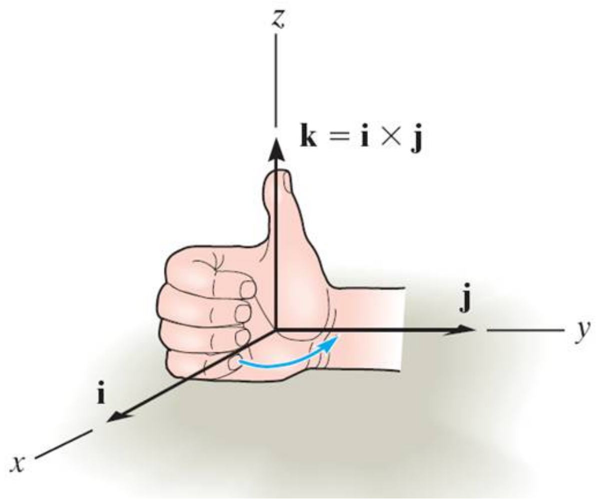
Note from previous slide
$\mathbf{M}_O = F\, r \sin(\theta)\, \boldsymbol{k}$

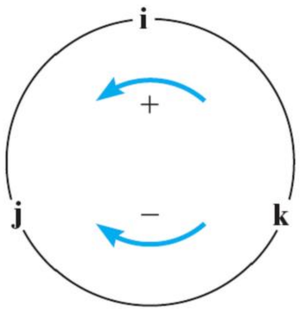
Cross Product

The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$ *(see convention below)*

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$





Cross Product

Also, the cross product can be written as a determinant.

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.

For element **i**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$

For element **j**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$

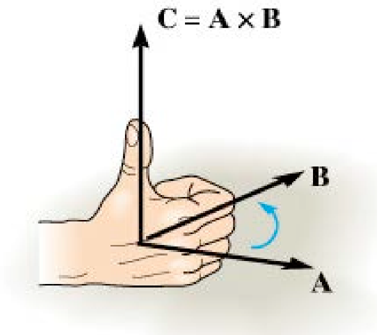
Remember the negative sign

For element **k**: $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$

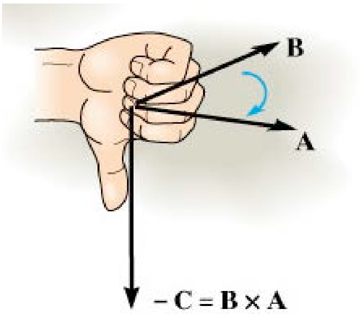
Cross Product

The output of a cross product is a vector.

The direction component of the solution comes entirely from the Right Hand Rule (RHR) and the order of the cross product.



"A cross B"



"B cross A"

Laws of operation:

– The commutative law does NOT hold: $\boldsymbol{A} \times \boldsymbol{B} \neq \boldsymbol{B} \times \boldsymbol{A}$

$$\boldsymbol{A} \times \boldsymbol{B} = -\boldsymbol{B} \times \boldsymbol{A}$$

– Scalar Multiplication: $\alpha(\boldsymbol{A} \times \boldsymbol{B}) = (\alpha\boldsymbol{A}) \times \boldsymbol{B} = \boldsymbol{A} \times (\alpha\boldsymbol{B})$

– Distributive law: $\boldsymbol{A} \times (\boldsymbol{B} + \boldsymbol{D}) = (\boldsymbol{A} \times \boldsymbol{B}) + (\boldsymbol{A} \times \boldsymbol{D})$

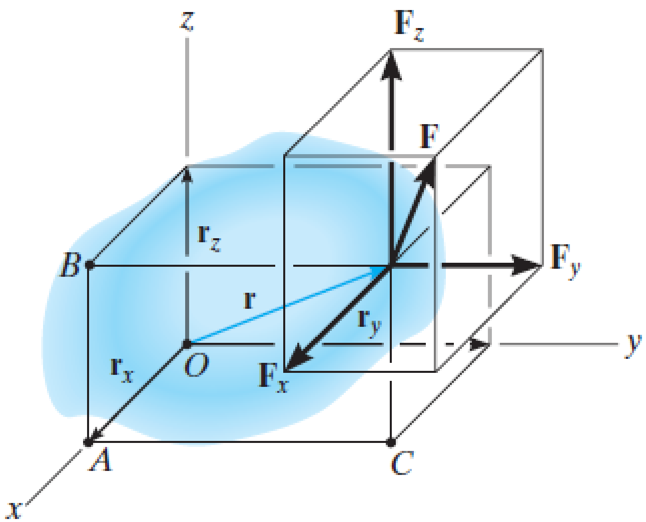
Moment of a Force

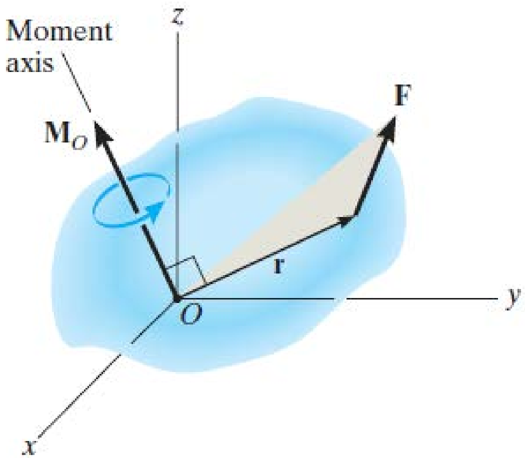
Spatial Moments

Moments in 3-D can be calculated using the **vector cross product**.

Using the vector cross product, $\boxed{\mathbf{M}_O = \mathbf{r} \times \mathbf{F}}$ → *Note the order of the vectors within the cross product.*

where $\mathbf{r}$ is the position vector from point O to any point on the line of action of $\mathbf{F}$.





Moment of a Force

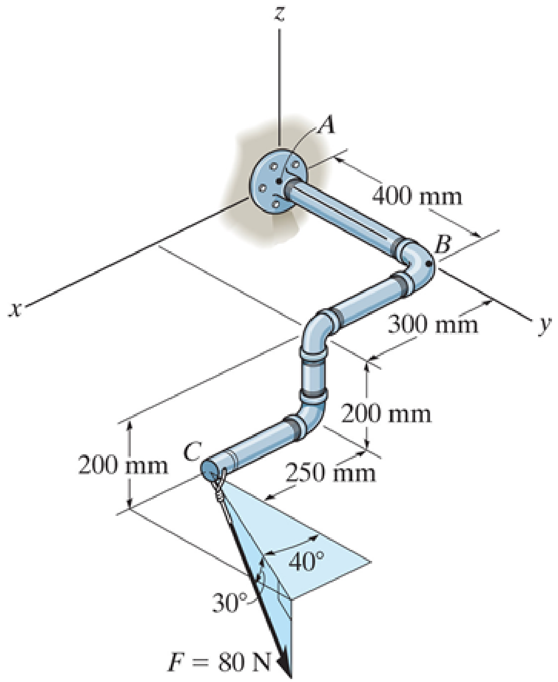
So, using the cross product, a moment can be expressed as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants, we get the moment of a force

$$\mathbf{M_O} = (r_y\ F_z\ - r_z\ F_y)\ \boldsymbol{i}\ -\ (r_x F_z\ -\ r_z F_x\ )\ \boldsymbol{j}\ +\ (r_x F_y\ - r_y F_x)\ \boldsymbol{k}$$

Example



Given: The force and geometry shown.

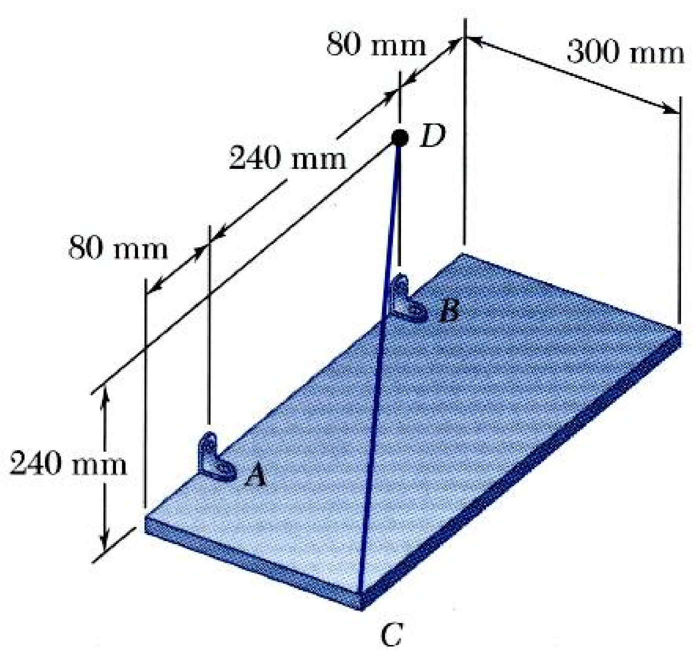
Find: Moment of F about point A.

Plan:

1) Find **F** and $\boldsymbol{r}_{AC}$.

2) Determine $\mathbf{M_A} = \boldsymbol{r}_{AC} \times \mathbf{F}$.

Example



Given: The rectangular plate is supported by the brackets at *A* and *B* and by a wire CD. Knowing that the tension in the wire is 200 N.

Find: The moment about *A* of the force exerted by the wire at *C*.

Plan:

1) Find **F** and ***r***.

2) Determine $\mathbf{M}_O = \boldsymbol{r} \times \mathbf{F}$.

Chapter 4 – Force System Resultants

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The force applied to the wrench will produce a tendency to produce rotation, this effect is called a moment.

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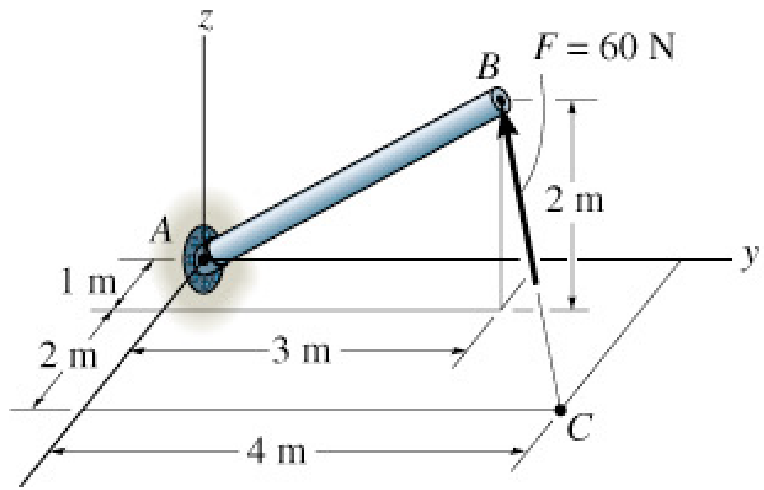
Moment of a Force

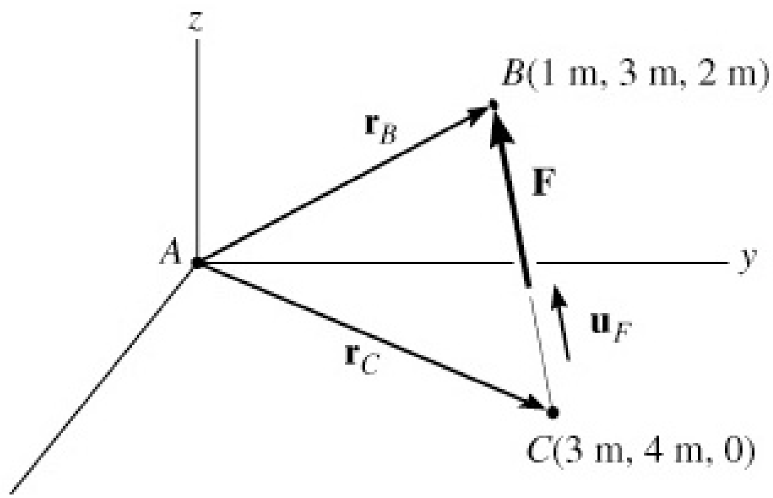
Principle of Transmissibility

A force vector acting on a body can be *slid* along its line of action without changing:

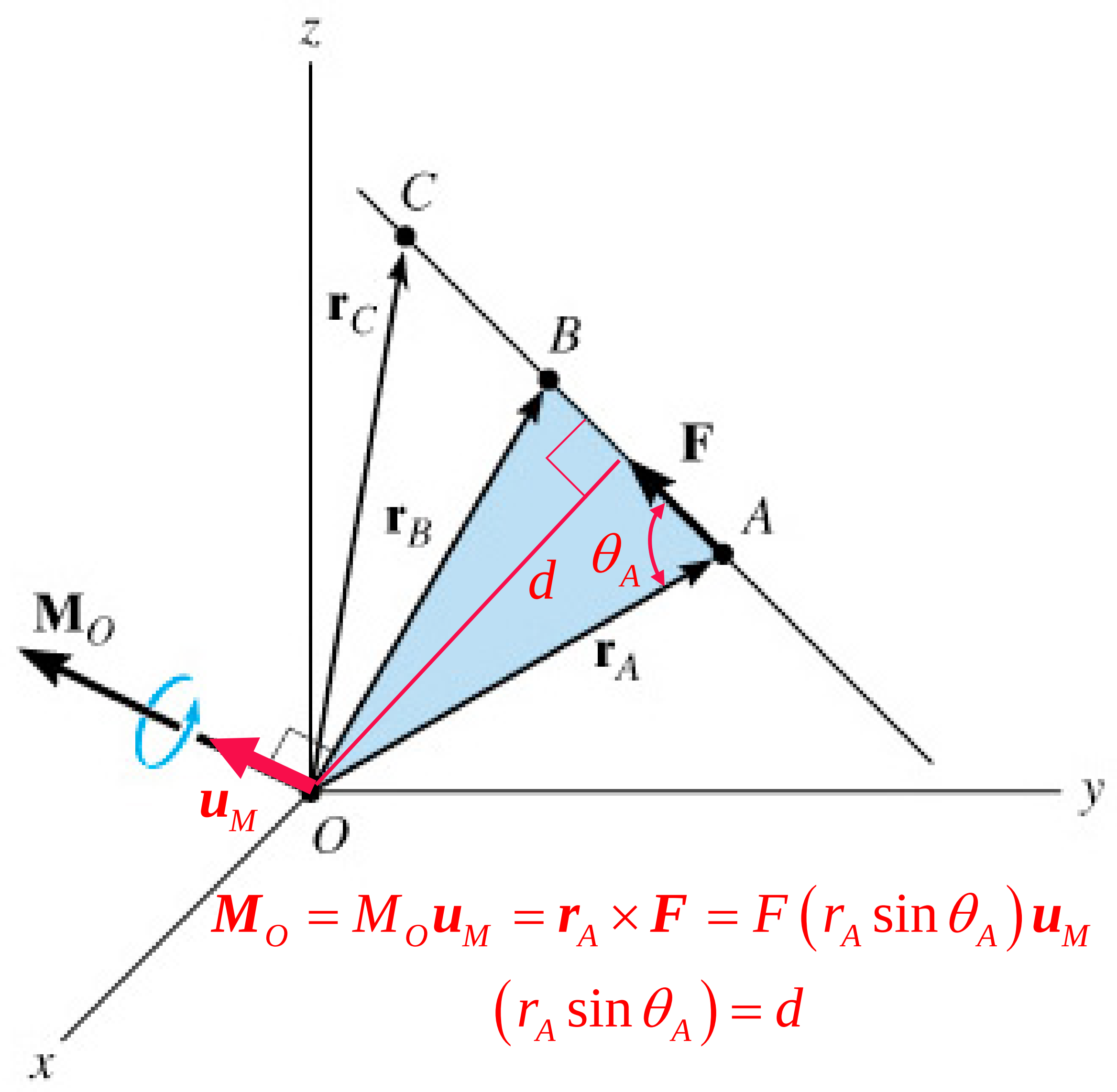
- its tendency to produce translation of the body.
- Its tendency to cause rotation (its moment about a reference point).

Forces are *sliding* vectors.





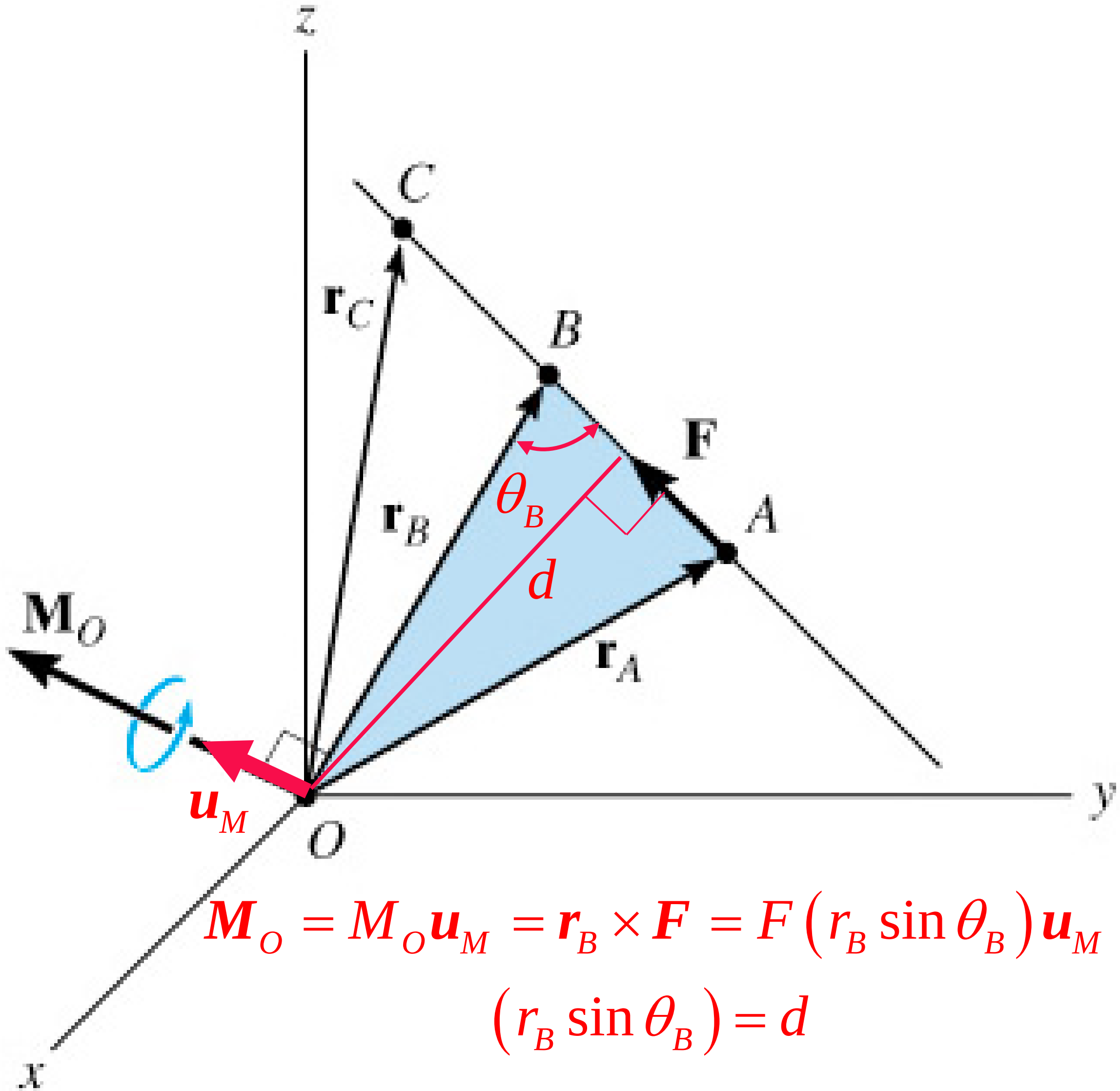
Moment of a Force



$$\boldsymbol{M}_O = M_O \boldsymbol{u}_M = \boldsymbol{r}_A \times \boldsymbol{F} = F\left(r_A \sin\theta_A\right)\boldsymbol{u}_M$$

$$\left(r_A \sin\theta_A\right) = d$$

Moment of a Force



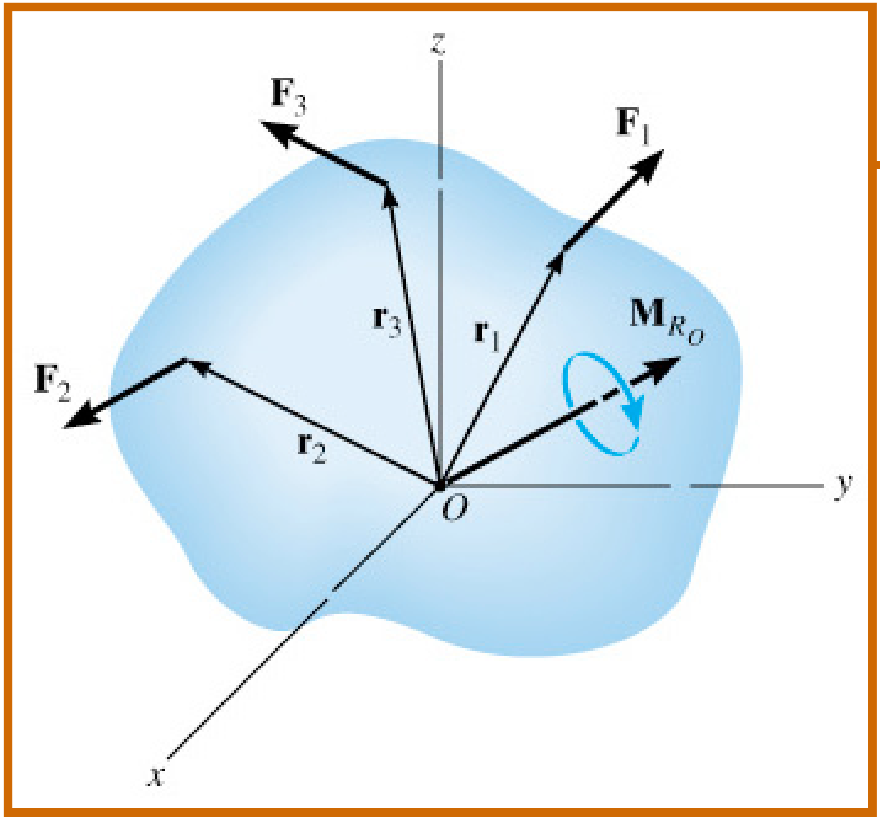
$$\boldsymbol{M}_O = M_O \boldsymbol{u}_M = \boldsymbol{r}_B \times \boldsymbol{F} = F\left(r_B \sin\theta_B\right)\boldsymbol{u}_M$$

$$\left(r_B \sin\theta_B\right) = d$$

Moment of a Force

Resultant Moments

A resultant moment refers to an accumulation of *rotation tendencies*.



$$\mathbf{M}_{R_O} = \sum_{m=1}^{3} (\mathbf{r}_m \times \mathbf{F}_m)$$

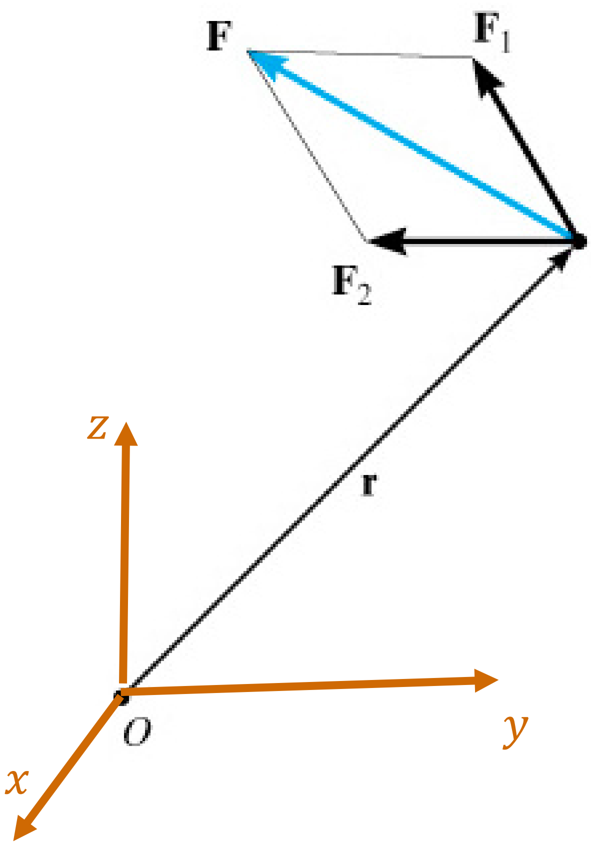
Or in general…

$$\mathbf{M}_{R_O} = \sum_{m=1}^{N} (\mathbf{r}_m \times \mathbf{F}_m)$$

$N \equiv$ the number of forces in the system.

Moment of a Force

Principle of Moments (*Varignon's Theorem*).



The calculation of a moment of a force can be found by summing the moments of the force components.

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$
$$= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = (\mathbf{r} \times \mathbf{F}_1) + (\mathbf{r} \times \mathbf{F}_2)$$

Useful with coplanar systems of forces.

Moment of a Force

Remarks

Once we have calculated a moment vector we have.

- An indication of the tendency for rotation (from the magnitude of the moment).
- An indication of the axis about which the rotation will tend to occur.

Often, a system will contain interconnections (or joints) that can (or cannot) sustain a moment.

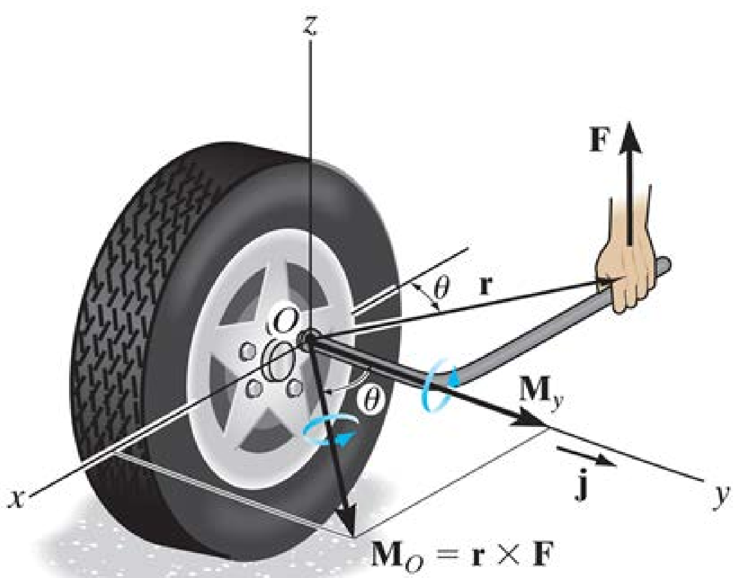
- We need to check whether there is any tendency for rotation about that particular joint.
- A pin/revolute joint is a simple example.

Moment of a Force about Axis

Moment of a force about a specified axis

In some occasions, we are interested in determining the moment of a force about a specific axis (not only about a point).

For example, in the figure below the moment about point *O* results in a vector that lies on the *xy* plane, however only the moment about axis *y* has an effect on screwing the nut.

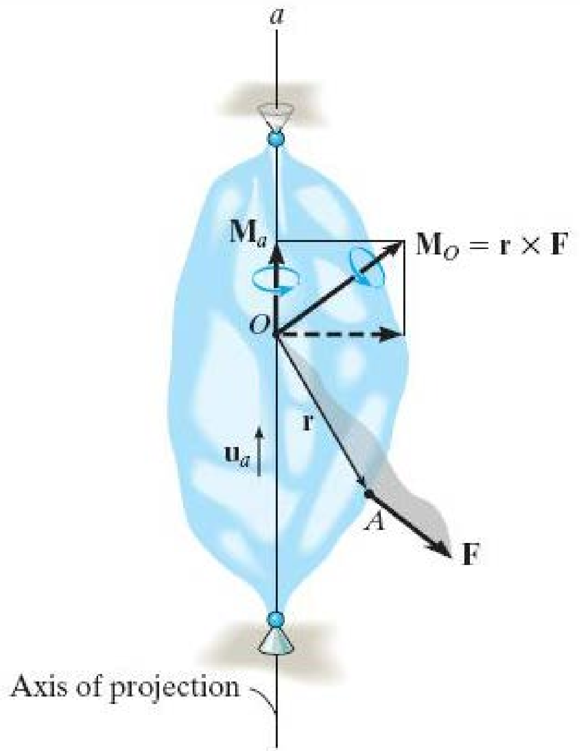


The M_y component can be found as

$$M_y = F_z\,(d_x) = F\,(r \cos\theta)$$

However, finding specific components for inclined **F** and **r** vectors is not trivial.

Moment of a Force about Axis



In this section our goal is to find the moment of **F** (the tendency to rotate the body) about the *a*-axis.

First compute the moment of **F** about any *arbitrary* point *O* that lies on the *a*-axis using the cross product.

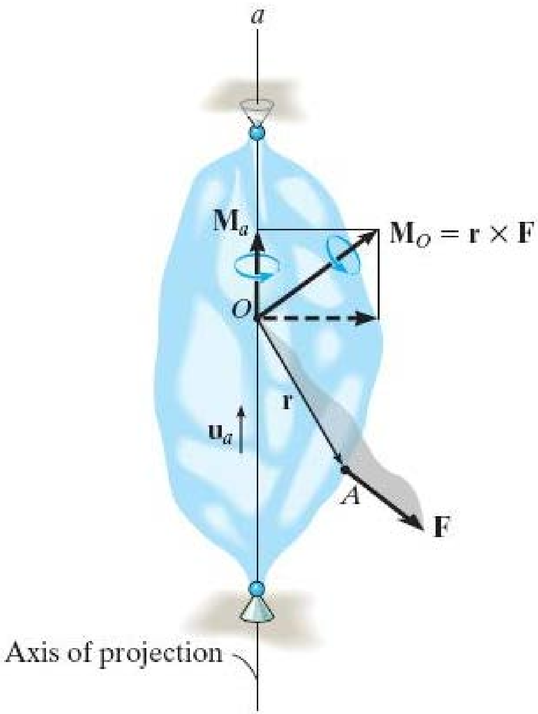
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

Now, find the component of $\mathbf{M}_O$ along the *a*-axis using the dot product (this is a scalar, the magnitude of $\mathbf{M}_O$ along the *a*-axis).

$$M_a = \mathbf{u}_a \cdot \mathbf{M}_O$$

The moment vector about the *a*-axis is obtained as $\mathbf{M}_a = M_a \, \mathbf{u}_a$

Moment of a Force about Axis



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

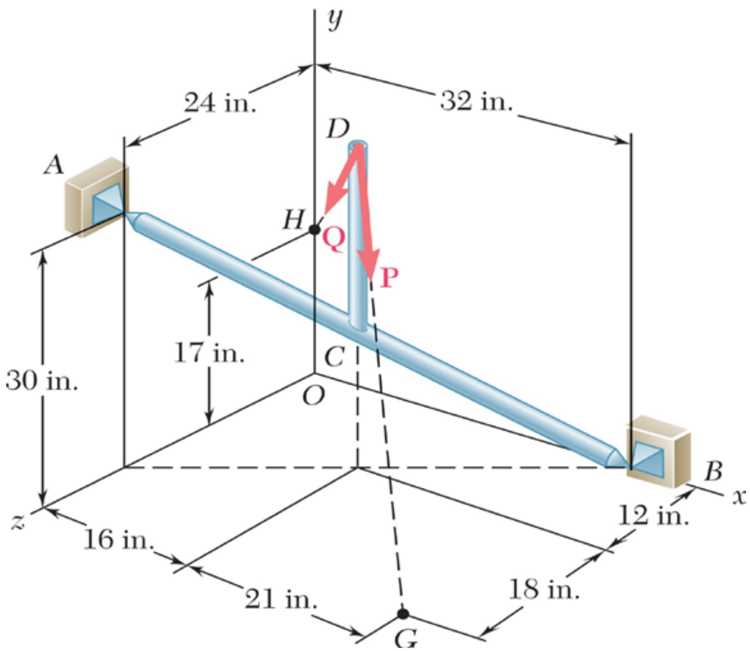
The above equation is also called the *triple scalar product*.

$\mathbf{u}_a$ represents the unit vector along the *a*-axis,

$\mathbf{r}$ is the position vector from any point on the *a*-axis to any point on the line of action of the force, and

$\mathbf{F}$ is the force vector.

Example



Given: In the midpoint of a 50-in. rod *AB*, there is a vertical rod *CD* that measures 23-in.

Find: The moment about *AB* of the 235-lb force **P**.

Plan:

1) Find **P**, **u** and **r**.

2) Determine $\mathbf{M}_{\mathbf{AB}} = \mathbf{u} \bullet (\mathbf{r} \times \mathbf{P})$.

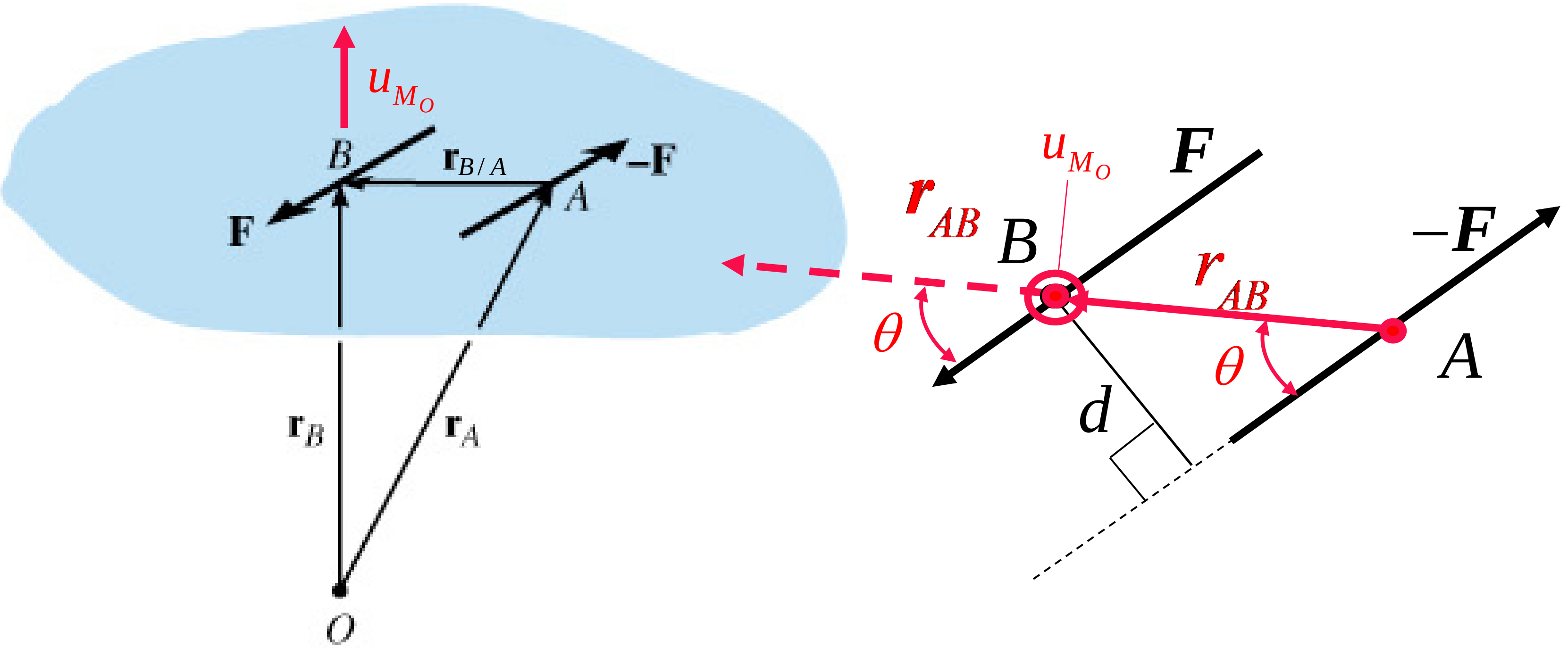
Moment of a Couple

Moment of a Couple

When you grip a vehicle's steering wheel with both hands and turn, a couple moment is applied to the wheel.

A "force couple" is a pair of forces of equal magnitude that point in opposing directions but are *not collinear*.

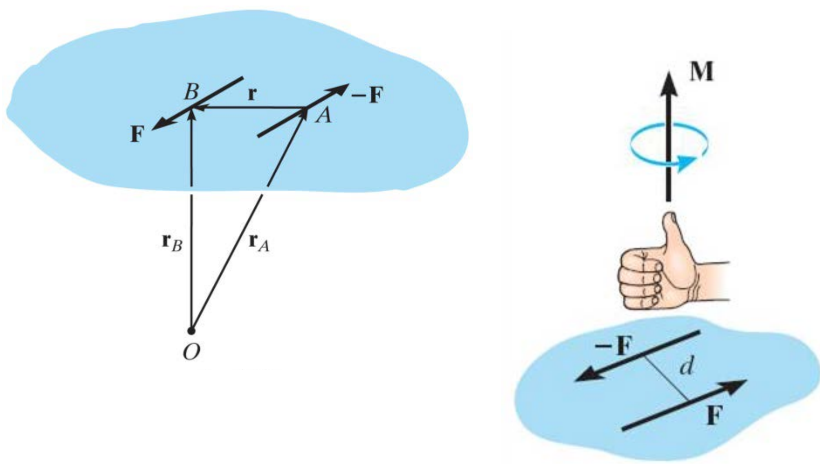
Moment of a Couple



$$\mathbf{M}_O = (\mathbf{r}_A \times (-\mathbf{F})) + (\mathbf{r}_B \times \mathbf{F})$$

$$= (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F} = \mathbf{r}_{AB} \times \mathbf{F} = (F\, r_{AB} sin\theta)\boldsymbol{u}_{M_O} = (Fd)\boldsymbol{u}_{M_O}$$

Moment of a Couple



A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance *d*.

The moment of a couple is defined as

$M_O = F\,d$ (using a scalar analysis) or as

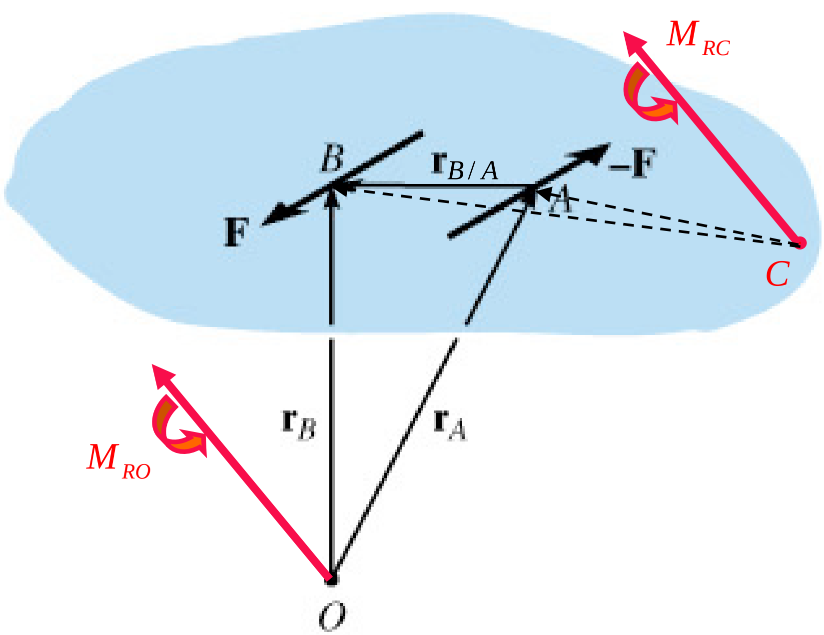
$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$ (using a vector analysis).

Here, **r** is any position vector from the line of action of **-F** to the line of action of **F**.

Moment of a Couple

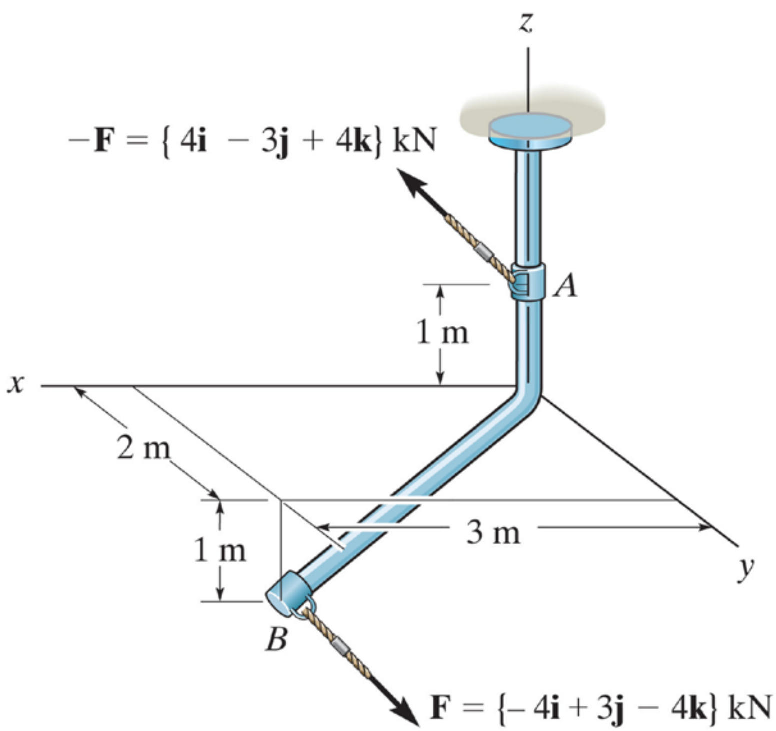
The moment of a couple depends only on the distance between the forces, the moment of a couple is a **free vector**. It can be moved anywhere on the body and have the same effect

It does not depend on the reference point *O*. Moments due to couples can be added together using the same rules as adding any vectors.



Example

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?



Chapter 4 – Force System Resultants

Contents

The force applied to the wrench will produce a tendency to produce rotation, this effect is called a moment.

Please refrain from uploading course materials onto online sharing platforms, such as Course Hero, OneClass or equivalent sharing platforms.

Simplification – Force/Couple System

Definition of Equivalent System

Objective: take all of the forces and couple moments acting on a body and replace them with a simple equivalent system of loads.

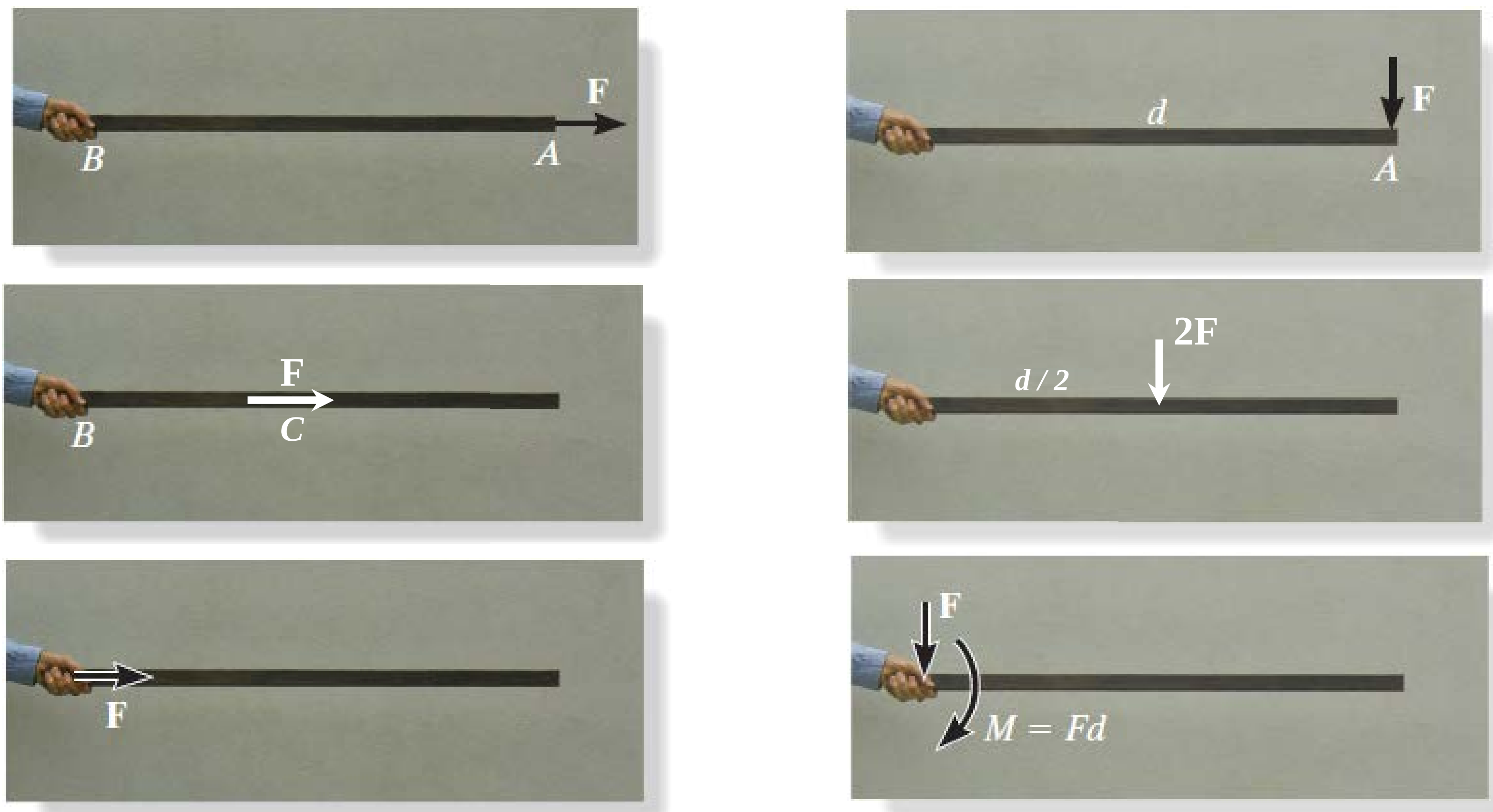
Equivalent system is a single force and/or moment vector that produces the same external effects.

External effects are the tendencies for the body to translate and rotate if free to move.

This process is referred to as “reduction to an equivalent system.”

Simplification – Force/Couple System

Assume you are holding a stick, what is the resultant effect on your hand if the force is applied as follows?



Simplification – Force/Couple System

Some notes on equivalent systems

The original loading and the equivalent loading have to produce the same tendency for translation and rotation.

Tendency for translation is caused by overall force.

Tendency for rotation is caused by overall moment.

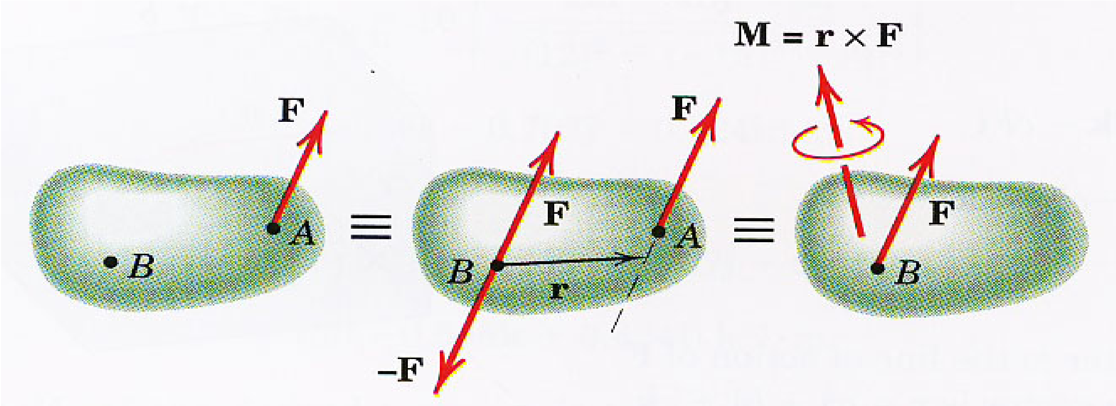
To calculate moment of a force we need a reference point.

Therefore, an equivalent system is specific to a chosen reference point.

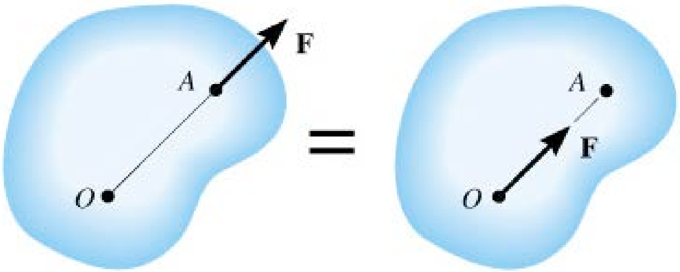
Simplification – Force/Couple System

There are two basic operations that occur when reducing a loading to its simplest equivalent:

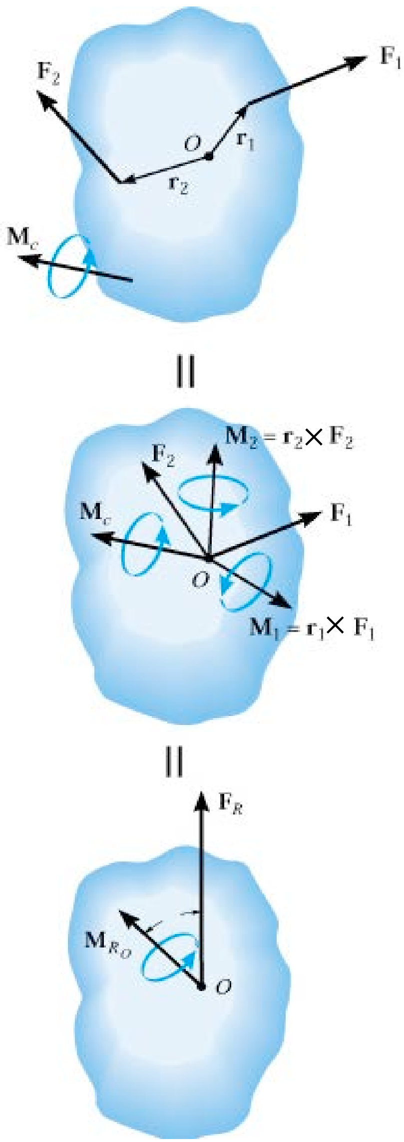
A force is moved off of its line of action to get to the reference point.



A force is slid along its line of action to get to a reference point:



Simplification – Force/Couple System



Step 1: Identify the reference point and locate the forces relative to this reference point *O*.

Step 2: Move the force vectors to the reference point *O*. Account for moments of the forces at the reference point *O*.

Step 3: Move the couple moment vectors (free vectors) to the reference point *O*.

Step 4: Sum the forces to produce a single resultant force. Add the moments of the forces and couple moments to get the resultant moment vector.

Simplification – Force/Couple System

In terms of Cartesian vector operations.

J = number of forces originally applied.

N = number of force couples originally applied.

$$\mathbf{F}_R = \sum_{j=1}^{J} \mathbf{F}_j$$

$$\mathbf{M}_{R_O} = \sum_{n=1}^{N} \mathbf{M}_{C\,n} + \sum_{j=1}^{J} (\mathbf{r}_j \times \mathbf{F}_j) = \sum_{n=1}^{N} \mathbf{M}_{C\,n} + \sum_{j=1}^{J} \mathbf{M}_{O\,j}$$

Original couple moments applied to the body.

Moments of the original forces applied to the body.

Simplification – Force/Couple System

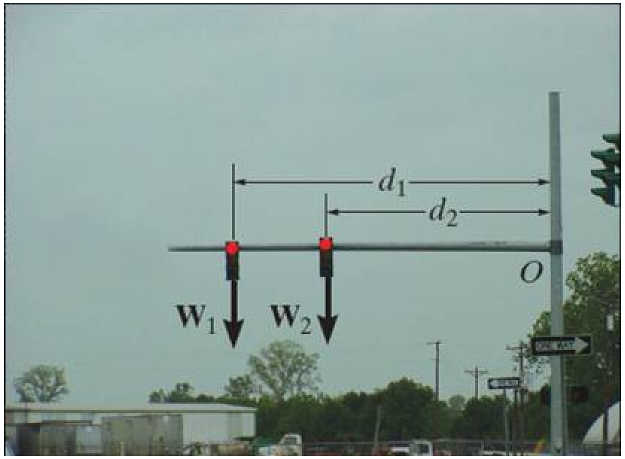
For coplanar system of forces, these two vector equations reduce to 3 scalars.

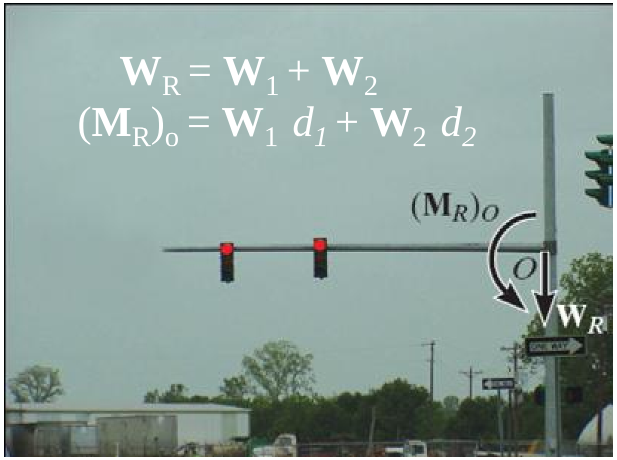
$$F_{Rx} = \sum_{j=1}^{J} F_{jx} \qquad F_{Ry} = \sum_{j=1}^{J} F_{jy}$$

x and *y* components of the resultant force calculation.

$$M_{RO} = \sum_{n=1}^{N} M_{Cn} + \sum_{j=1}^{J} M_{Oj}$$

z component of the resultant moment calculation.





Simplification – Force/Couple System

Further Reduction of Force and Couple System

There are times when it is possible to simplify a loading beyond a single force and moment vector. In such cases, a reference point can be chosen so that *the equivalent system has a single force vector*.

There are four situations for which this is assured.

a) The resultant force and moment are perpendicular.

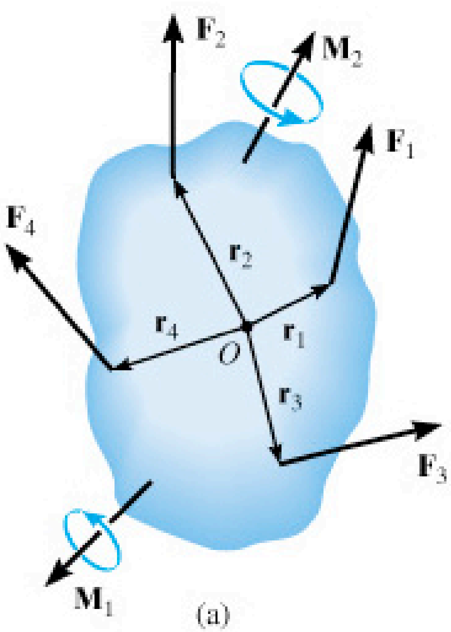
b) Concurrent force systems.

c) Coplanar force systems.

d) Parallel force systems.

Simplification – Force/Couple System

Resultant force and moment are perpendicular (spatial systems)



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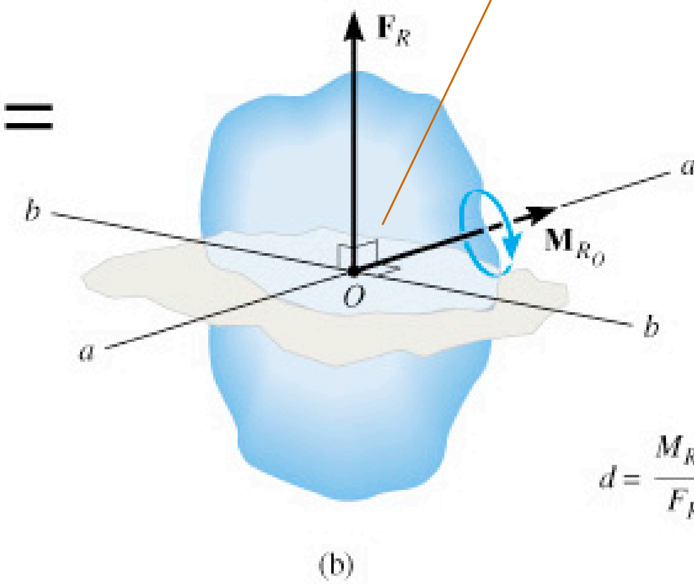
Special case – the resultant force and moment are perpendicular.



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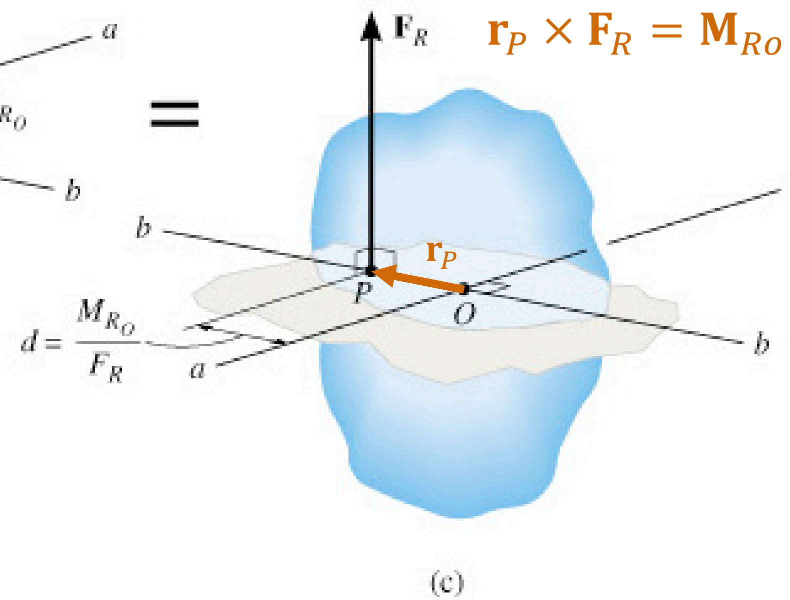
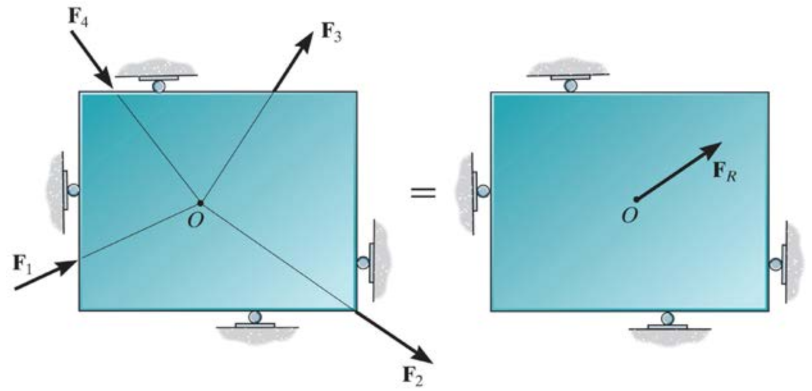
$$\mathbf{r}_P \times \mathbf{F}_R = \mathbf{M}_{Ro}$$



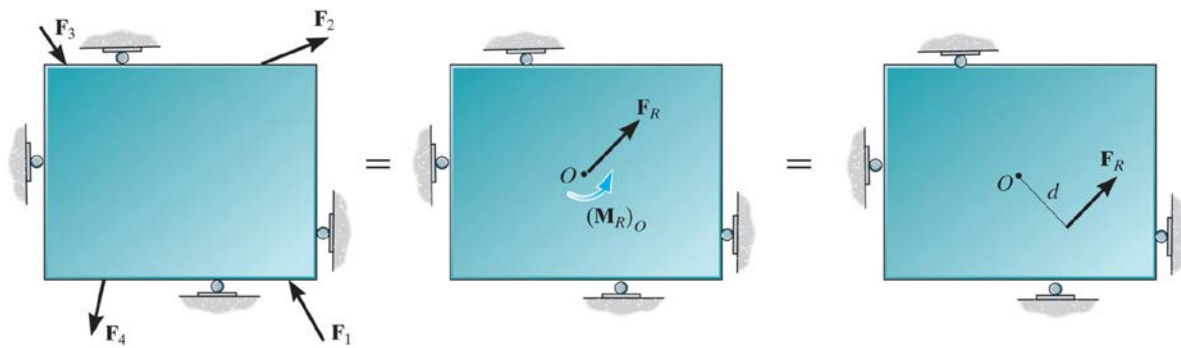
No further simplification can be made if the resulting force and moment are not orthogonal.

Simplification – Force/Couple System

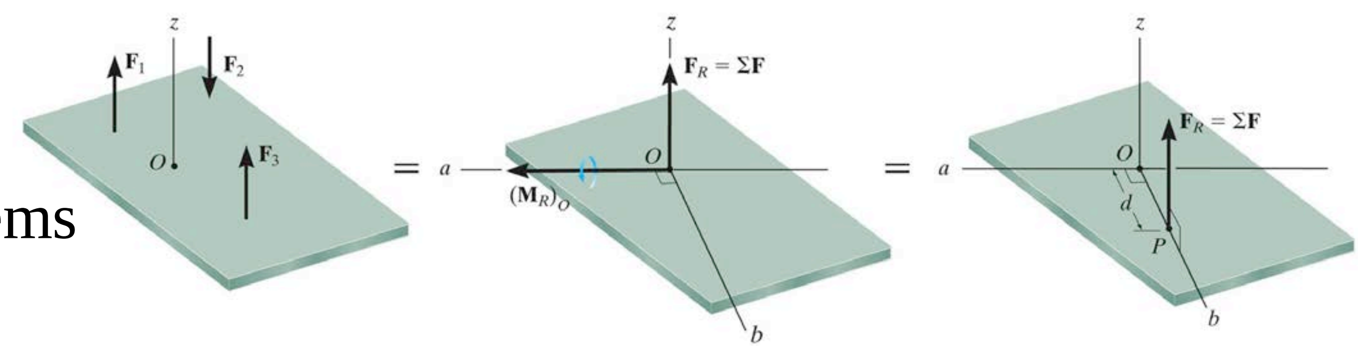
Concurrent force systems



Coplanar force systems



Parallel force systems



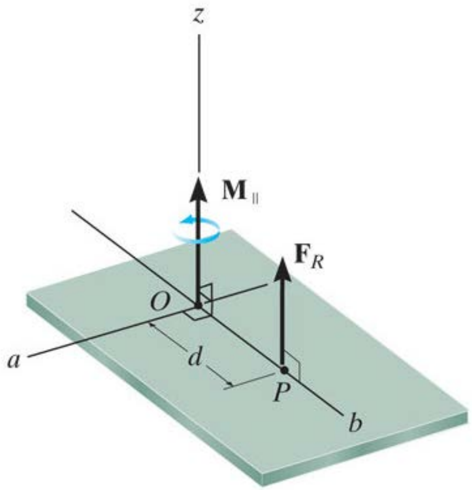
Simplification – Force/Couple System

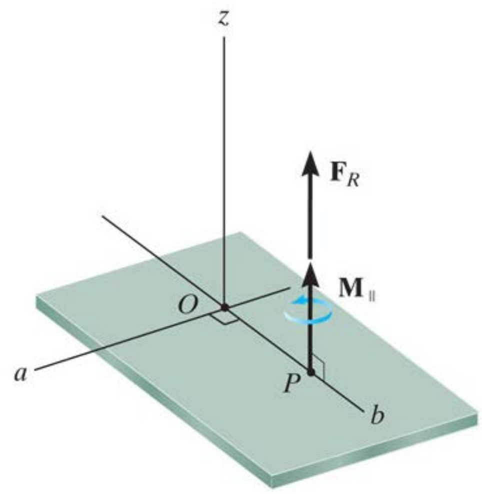
Reduction to a Wrench

In the most general cases we cannot reduce down to a single force.

We define a new quantity called a ***wrench***.

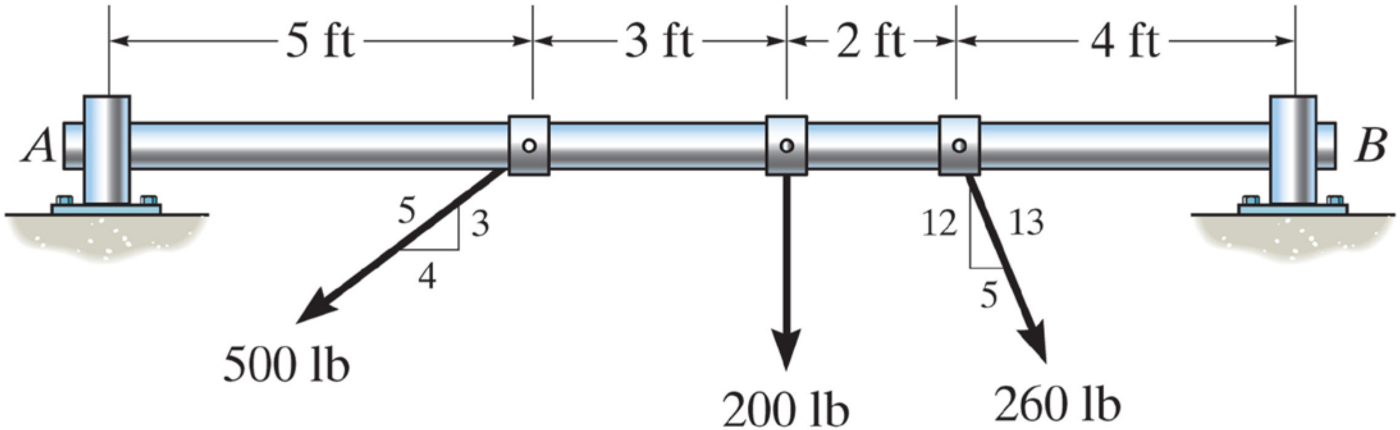
The wrench is a combination of a force vector and a parallel moment vector.





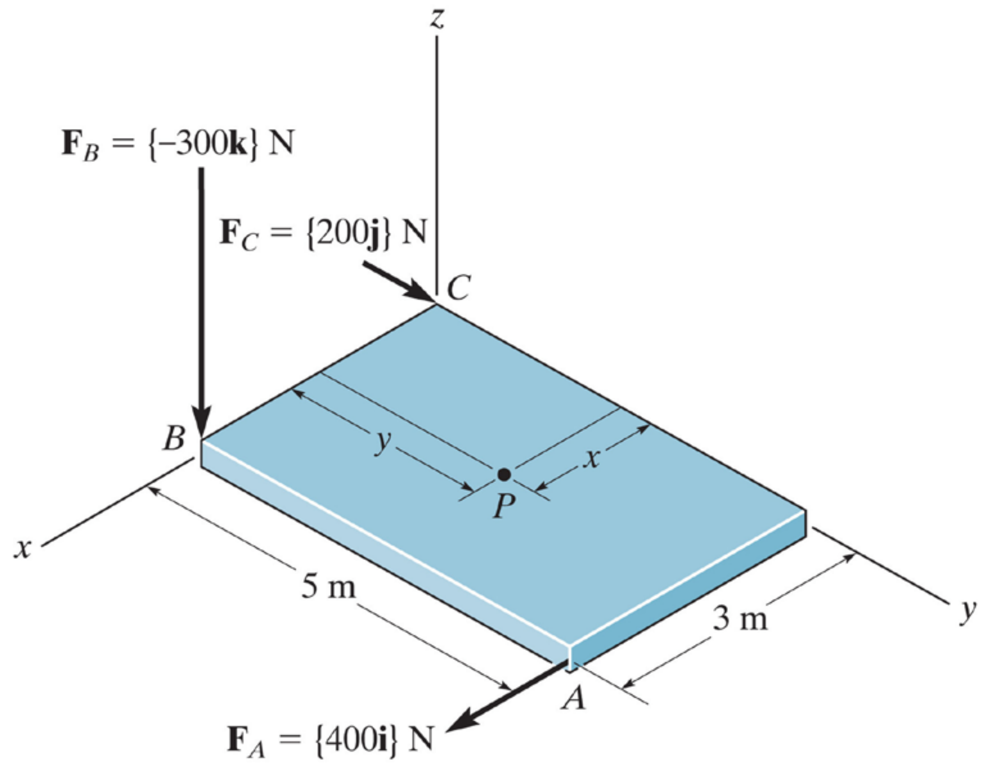
Example

Replace the three forces acting on the shaft by a single resultant force. Determine the position using both points *A* and *B* as reference.



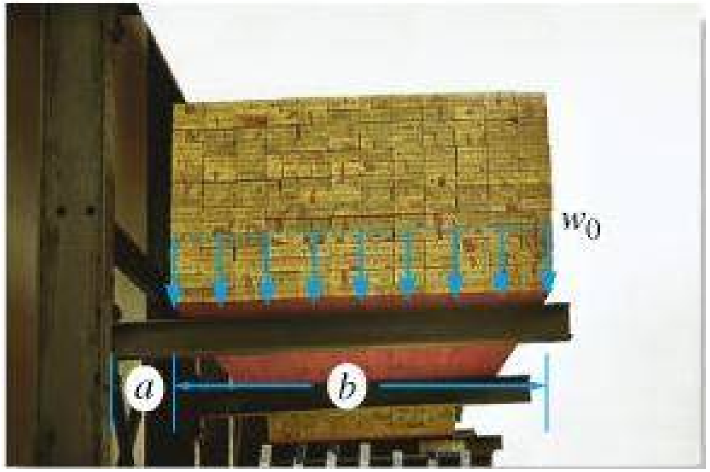
Example

Replace the three forces acting on the plate by a wrench. Specify the location where the wrench intersects the *x*-*y* plane.



Reduction of a Simple Distributed Loading

Sometimes a body is subjected to a loading that is distributed over its surface



A bundle (bunk) of 2” x 4” boards is stored. The lumber places a distributed load (due to the weight of the wood) on the beams.

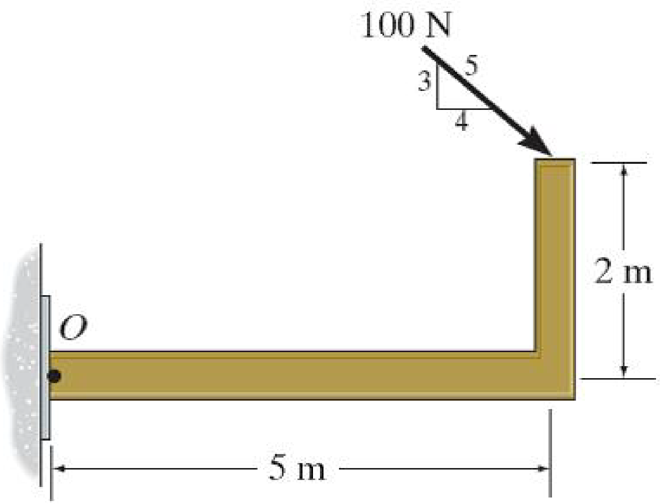
The roof of these houses are supporting a distributed load of snow.

We will cover this section later in the course. We will learn how to reduce this distributed load to a single force

Sample Problems for Students to Review

Chapter 4

Sample Problem (§ 4.1)



Given: A 100 N force is applied to the frame.

Find: The moment of the force at point O using

a) rectangular components

b) $M_O = F\,d$

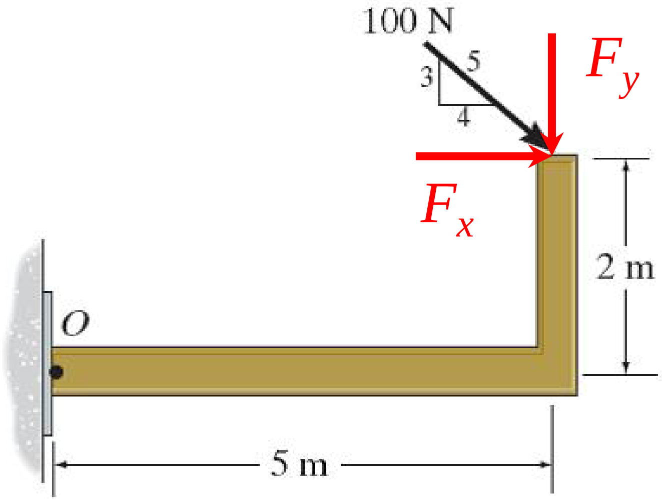
Plan:

1) Resolve the 100 N force along x and y-axes. Find M_O using a scalar analysis of two force components and then add those two moments together.

2) Find d using trigonometry and then apply $M_O = F\,d$

Solution:

a) Find rectangular components



$$+\uparrow F_y = -100\,(3/5)\ N$$

$$+\rightarrow F_x = 100\,(4/5)\ N$$

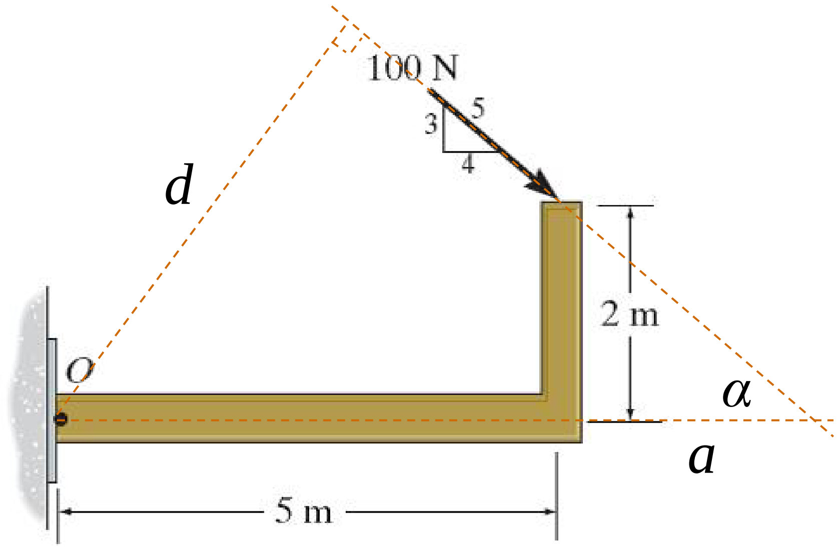
Evaluate the moments caused by each force component and add them together

$$+\circlearrowleft M_O = \{-100\,(3/5)\cdot(5) - (100)(4/5)\cdot(2)\}\ N\cdot m$$

$$= -460\ N\cdot m \quad \text{or} \quad 460\ N\cdot m\ (\text{CW})$$

Solution:

b) Finding d and $M_O = F\,d$



Similar triangles,

$$\frac{3}{4} = \frac{2}{a} \rightarrow a = \frac{8}{3}$$

Find α using trigonometry, $\alpha = tan^{-1}\left(\frac{3}{4}\right) = 36.9^{\circ}$

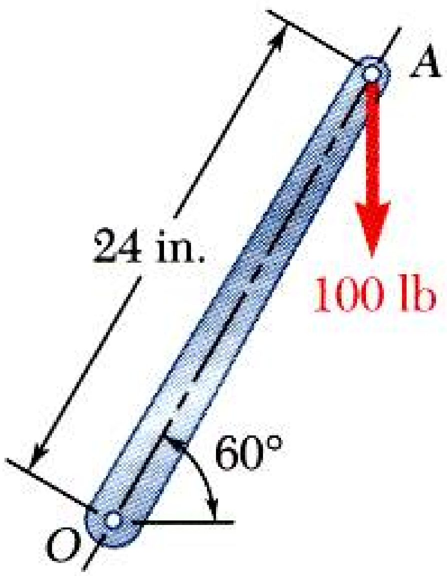
Find moment arm, $d = (5 + a)sin\alpha = 4.6\ m$

Determine M_O, $M_O = F\,d = 100\ (4.6) = 460\ N{\cdot}m$

Sample Problem (§ 4.1)

Given: A 100 lb force is applied to the lever.

Find: a) The moment of the force at the shaft point *O*.



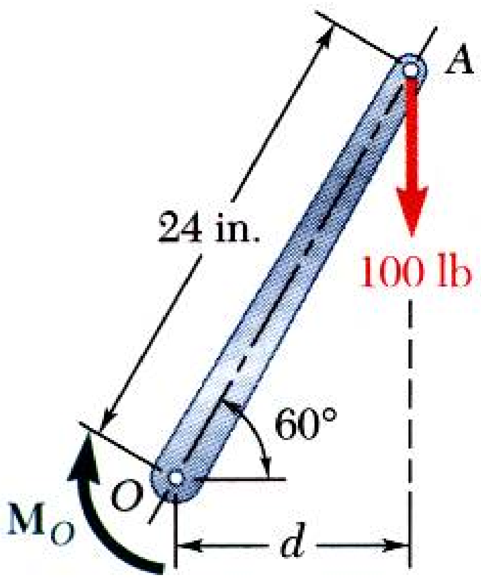
b) horizontal force at *A* which creates the same moment,

c) smallest force at A which produces the same moment,

d) location for a 240-lb vertical force to produce the same moment,

e) whether any of the forces from b, c, and d is equivalent to the original force.

a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.



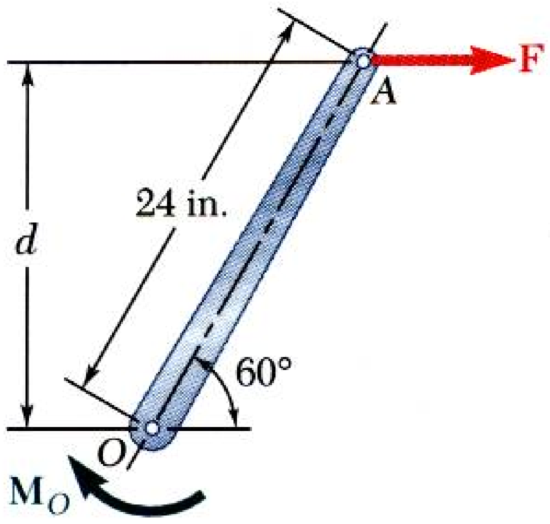
$$M_O = Fd$$

$$d = (24\,\text{in.})\cos 60° = 12\,\text{in.}$$

$$M_O = (100\,\text{lb})(12\,\text{in.})$$

$$\boxed{M_O = 1200\,\text{lb}\cdot\text{in}} \quad \text{CW}$$

b) Horizontal force at A that produces the same moment (1200lb CW):



$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$

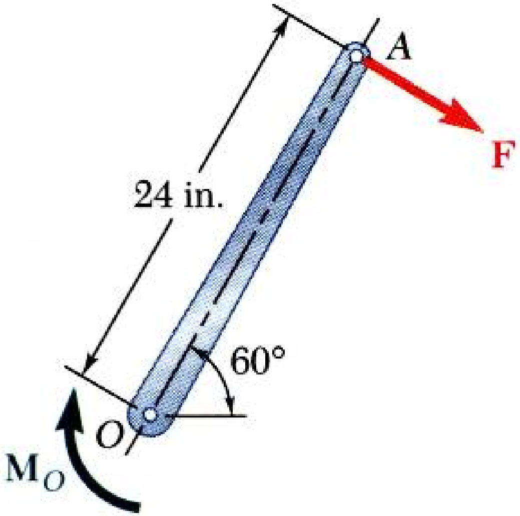
$$M_O = Fd$$

$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$

$$F = \frac{1200\text{ lb}\cdot\text{in.}}{20.8\text{ in.}}$$

$$\boxed{F = 57.7\text{ lb}}$$

c) The smallest force *A* to produce the same moment occurs when the perpendicular distance is a maximum or when *F* is perpendicular to *OA*.



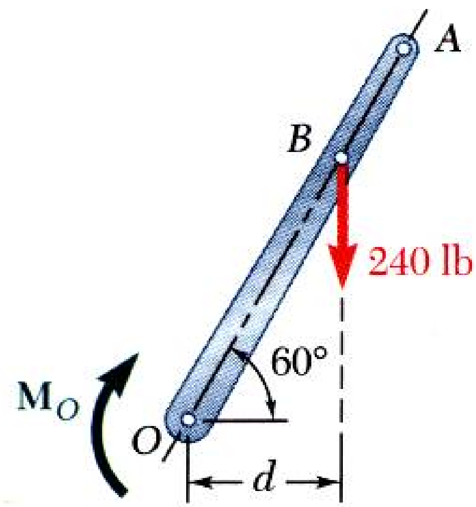
$$M_O = Fd$$

$$1200\,\text{lb}\cdot\text{in.} = F(24\,\text{in.})$$

$$F = \frac{1200\,\text{lb}\cdot\text{in.}}{24\,\text{in.}}$$

$$\boxed{F = 50\,\text{lb}}$$

d) To determine the point of application of a 240 lb vertical force to produce the same moment,



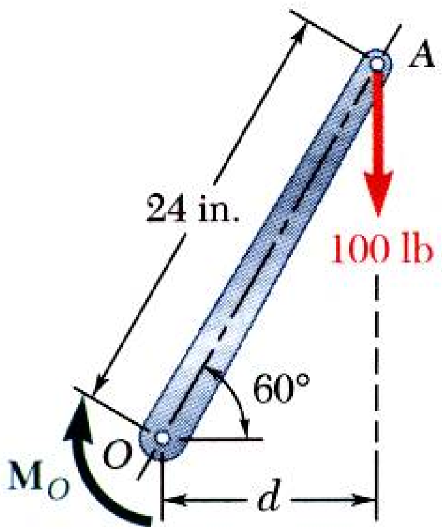
$$M_O = Fd$$

$$1200\ \text{lb}\cdot\text{in.} = (240\ \text{lb})d$$

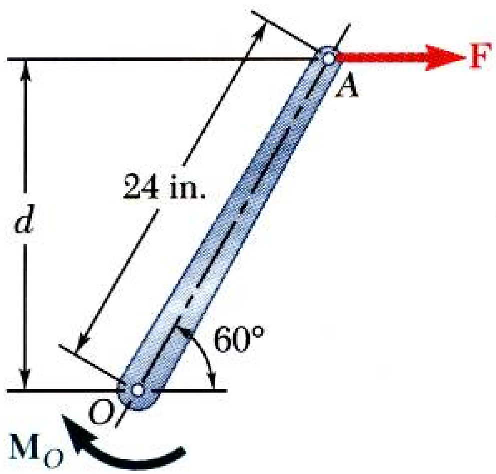
$$d = \frac{1200\ \text{lb}\cdot\text{in.}}{240\ \text{lb}} = 5\ \text{in.}$$

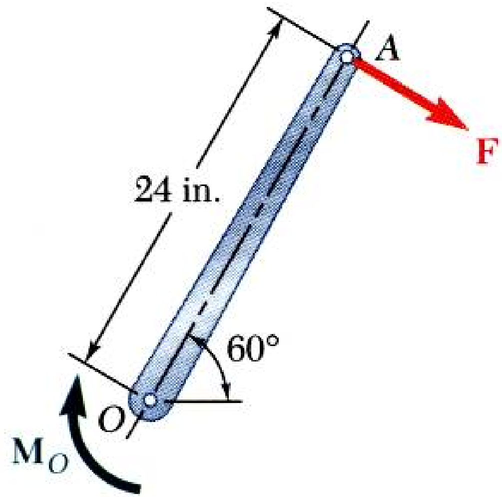
$$OB\cos 60° = 5\ \text{in.}$$

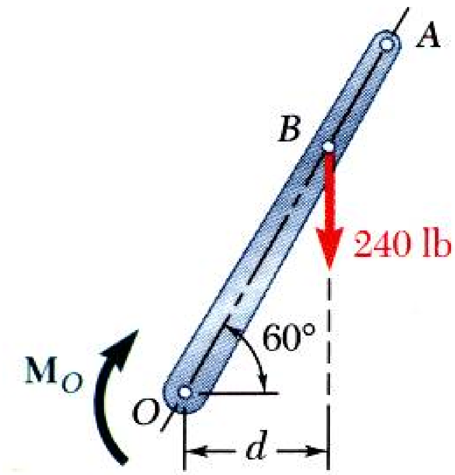
$$\boxed{OB = 10\ \text{in.}}$$



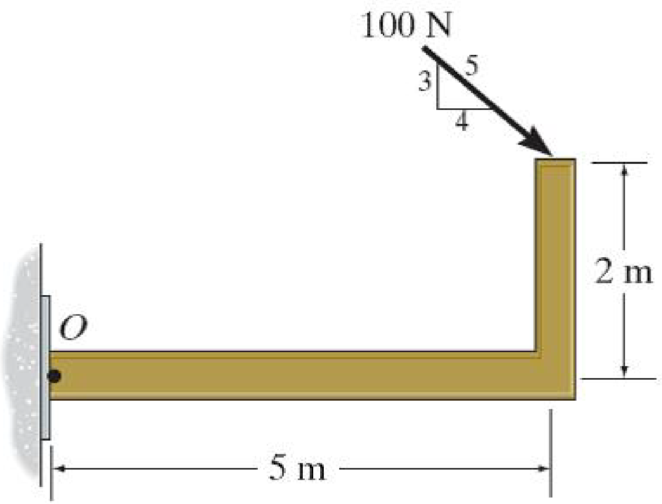
e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.







Sample Problem (§ 4.3)



Given: A 100 N force is applied to the frame.

Find: The moment of the force at point *O* using

c) using cross products

Plan:

Resolve the 100 N force along x and y-axes. Find M_O using cross products.

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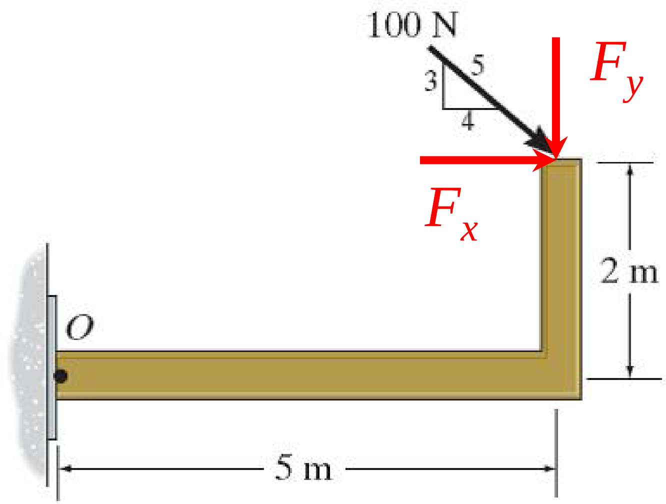
Solution:

a) Find rectangular components



$$+\uparrow F_y = -100\,(3/5)\ N$$

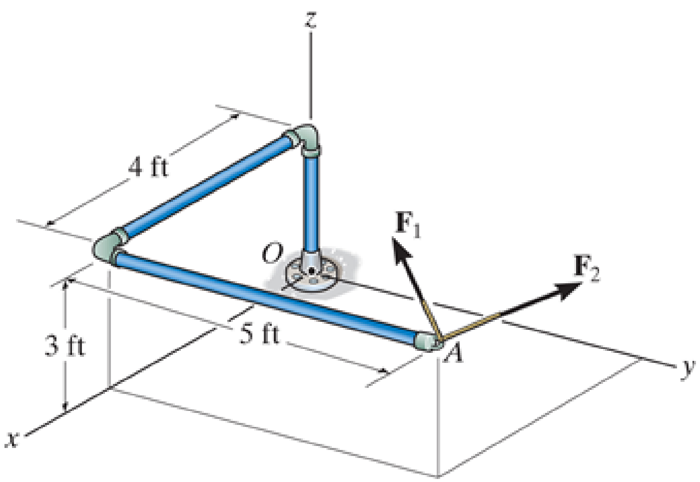
$$+\rightarrow F_x = 100\,(4/5)\ N$$

Important: we do not have to figure out the sense of the moment as we did it before. You need to preserve the signs of the force components

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = \begin{bmatrix}5\\2\\0\end{bmatrix}\times\begin{bmatrix}100(4/5)\\-100(3/5)\\0\end{bmatrix} = \begin{bmatrix}0\\0\\(5)(-100)(3/5)-(2)(100)(4/5)\end{bmatrix}$$

$$\mathbf{M}_O = \mathbf{r}\times\mathbf{F} = -460\,\boldsymbol{k}\,N\cdot m \quad or \quad \mathbf{M}_O = 460\,\boldsymbol{k}N\cdot m\ \text{(CW)}$$

Sample Problem (§ 4.3)



Given: $\mathbf{F_1}=\{100\,\boldsymbol{i} - 120\,\boldsymbol{j} + 75\,\boldsymbol{k}\}$lb

$\mathbf{F_2}=\{-200\,\boldsymbol{i} +250\,\boldsymbol{j} + 100\,\boldsymbol{k}\}$lb

Find: Resultant moment by the forces about point O.

Plan:

1) Find $\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$ and $\boldsymbol{r}_{OA}$.

2) Determine $\mathbf{M_O} = \boldsymbol{r}_{OA} \times \mathbf{F}$.

Solution:

$\mathbf{F_1}=\{100\ \boldsymbol{i} - 120\ \boldsymbol{j} + 75\ \boldsymbol{k}\}\text{lb}$

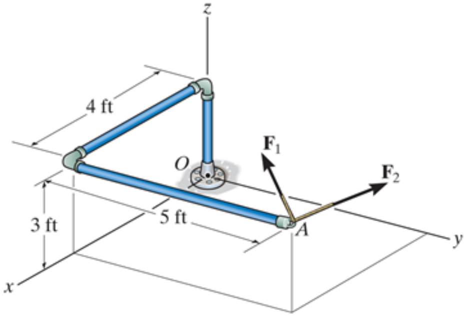
$\mathbf{F_2}=\{-200\ \boldsymbol{i} + 250\ \boldsymbol{j} + 100\ \boldsymbol{k}\}\text{lb}$

First, find the resultant force vector **F**

$$\mathbf{F} = \mathbf{F_1} + \mathbf{F_2}$$

$$= \{ (100 - 200)\ \boldsymbol{i} + (-120 + 250)\ \boldsymbol{j} + (75 + 100)\ \boldsymbol{k}\}\ \text{lb}$$

$$= \{-100\ \boldsymbol{i} + 130\ \boldsymbol{j} + 175\ \boldsymbol{k}\}\ \text{lb}$$



Find the position vector $\boldsymbol{r_{OA}}$

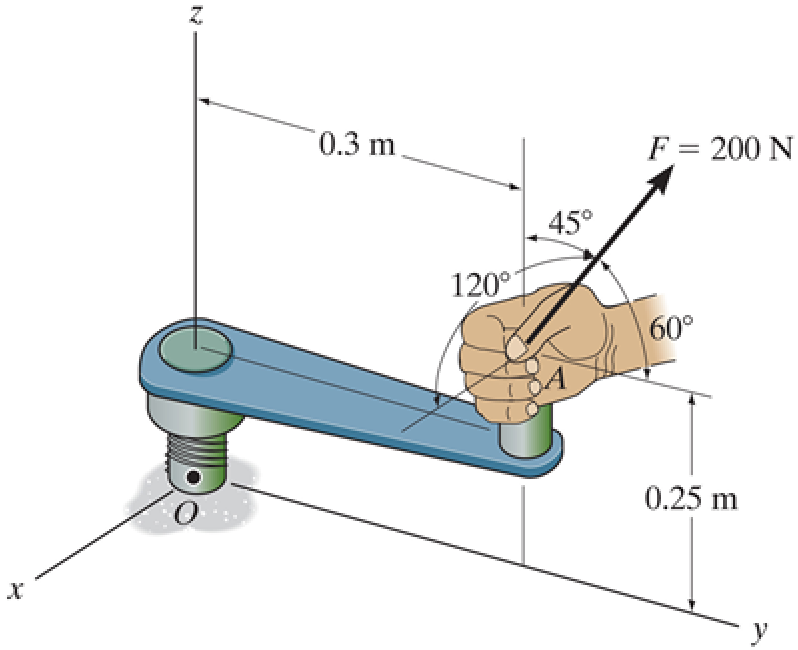
$$\boldsymbol{r_{OA}} = \{4\ \boldsymbol{i} + 5\ \boldsymbol{j} + 3\ \boldsymbol{k}\}\ \text{ft}$$

Then find the moment by using the vector cross product.

$$\mathbf{M}_{\boldsymbol{O}} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 4 & 5 & 3 \\ -100 & 130 & 175 \end{vmatrix} = [\{5(175) - 3(130)\}\ \boldsymbol{i} - \{4(175) - 3(-100)\}\ \boldsymbol{j} + \{4(130) - 5(-100)\}\ \boldsymbol{k}]\ \text{ft}\cdot\text{lb}$$

$$= \{485\ \boldsymbol{i} - 1000\ \boldsymbol{j} + 1020\ \boldsymbol{k}\}\ \text{ft}\cdot\text{lb}$$

Sample Problem (§ 4.5)



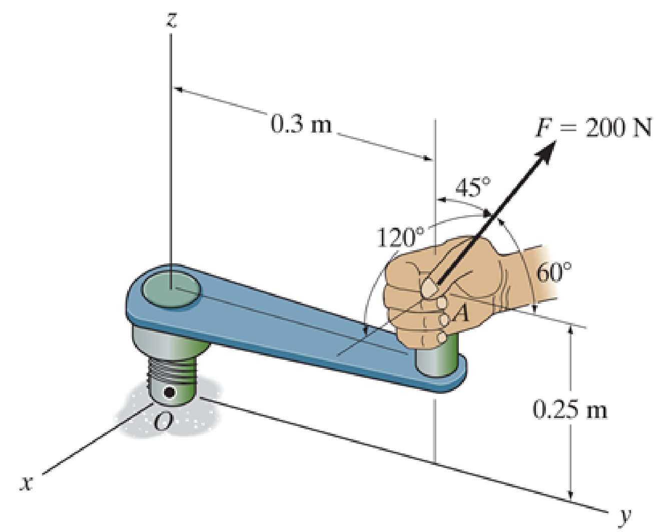
Given: A force is applied to the tool as shown.

Find: The magnitude of the moment of this force about the x axis and about the z axis

Plan:

1) Use $M_x = \mathbf{u}_x \cdot (\mathbf{r} \times \mathbf{F})$ and $M_z = \mathbf{u}_z \cdot (\mathbf{r} \times \mathbf{F})$
2) Find the force **F** and the position vector ***r***
3) Define the axes of rotation $\mathbf{u}_x$ and $\mathbf{u}_z$.

Solve for M_x: $M_x = \mathbf{u}_x \cdot (\mathbf{r} \times \mathbf{F})$



$\mathbf{u}_x = 1\,\boldsymbol{i}$

$\mathbf{r}_{OA} = \{0\,\boldsymbol{i} + 0.3\,\boldsymbol{j} + 0.25\,\boldsymbol{k}\}$ m

$\mathbf{F} = 200\,(\cos 120\,\boldsymbol{i} + \cos 60\,\boldsymbol{j} + \cos 45\,\boldsymbol{k})$ N

$= \{-100\,\boldsymbol{i} + 100\,\boldsymbol{j} + 141.4\,\boldsymbol{k}\}$ N

Now find $M_x = \mathbf{u}_x \bullet (\mathbf{r}_{OA} \times \mathbf{F})$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1\{0.3\,(141.4) - 0.25\,(100)\}\ \text{N}\cdot\text{m}$$

$$M_x = 17.4\ \text{N}\cdot\text{m (CCW)}$$

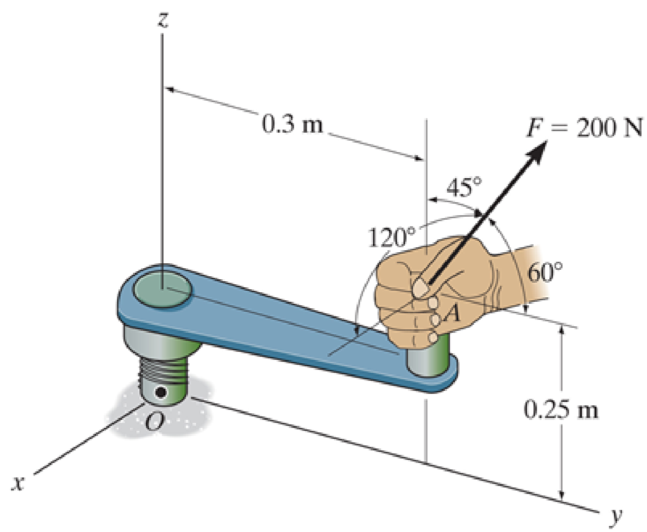
Solve for M_z: $M_z = \mathbf{u}_z \cdot (\mathbf{r} \times \mathbf{F})$

$\mathbf{u}_z = 1\,\boldsymbol{k}$

$\mathbf{r}_{OA} = \{0\,\boldsymbol{i} + 0.3\,\boldsymbol{j} + 0.25\,\boldsymbol{k}\}$ m

$\mathbf{F} = 200\,(\cos 120\,\boldsymbol{i} + \cos 60\,\boldsymbol{j} + \cos 45\,\boldsymbol{k})$ N

$= \{-100\,\boldsymbol{i} + 100\,\boldsymbol{j} + 141.4\,\boldsymbol{k}\}$ N



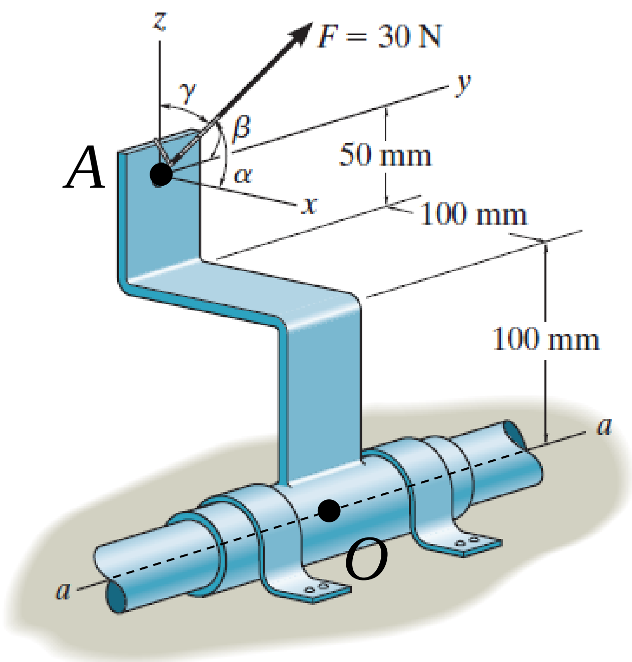
Now find $M_z = \mathbf{u}_z \bullet (\mathbf{r}_{OA} \times \mathbf{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0 & 0.3 & 0.25 \\ -100 & 100 & 141.4 \end{vmatrix} = 1\{0\,(100) - 0.3\,(-100)\}\ \text{N·m}$$

$M_z = 30$ N·m (CCW)

Note: $\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \{17.43\,\boldsymbol{i} - 25\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N·m

Sample Problem (§ 4.5)



Given: The force of $F = 30$ N acts on the bracket. $\alpha = 60°$, $\beta = 60°$, $\gamma = 45°$.

Find: The moment of **F** about the a-a axis.

Plan:

1) Find $\mathbf{u}_a$ and $\mathbf{r}_{OA}$

2) Find **F** in Cartesian vector form.

3) Calculate $M_a = \mathbf{u}_a \bullet (\mathbf{r}_{OA} \times \mathbf{F})$.

Solution:

Find $\mathbf{u}_a$ and $\mathbf{r}_{OA}$

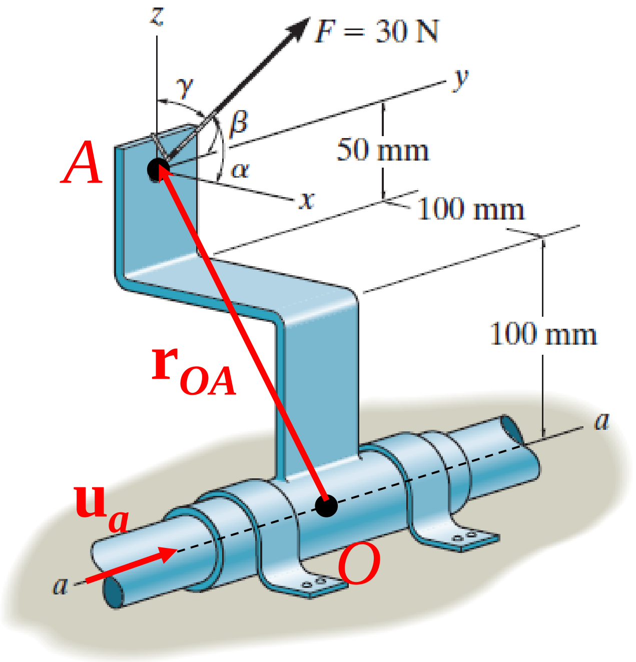
$\boldsymbol{u_a} = \boldsymbol{j}$

$\boldsymbol{r_{OA}} = \{-0.1\, \boldsymbol{i} + 0.15\, \boldsymbol{k}\}$ m

Find **F** in Cartesian vector form.

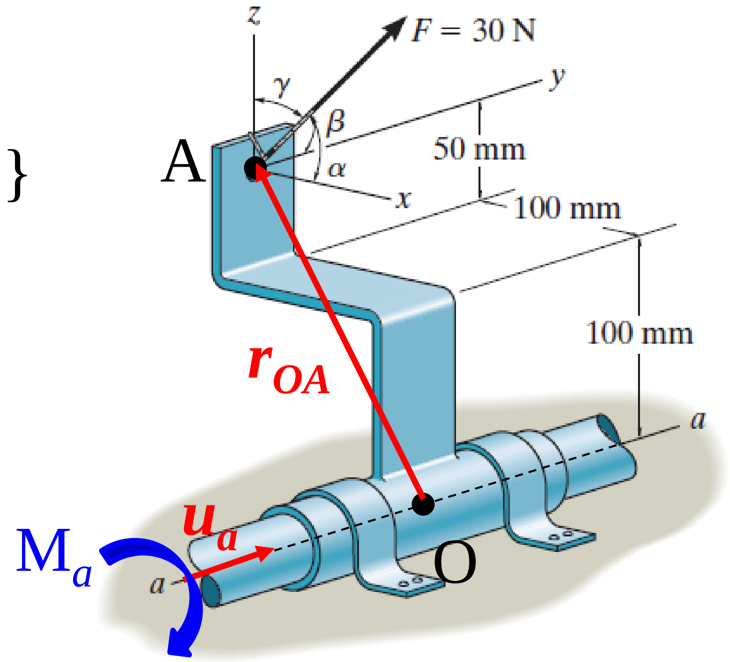
$\boldsymbol{F} = 30\, \{\cos 60°\, \boldsymbol{i} + \cos 60°\, \boldsymbol{j} + \cos 45°\, \boldsymbol{k}\}$ N

$\boldsymbol{F} = \{\, 15\, \boldsymbol{i} + 15\, \boldsymbol{j} + 21.21\, \boldsymbol{k}\}$ N



Now find the triple scalar product, $M_a = \mathbf{u_a} \bullet (\mathbf{r_{OA}} \times \mathbf{F})$

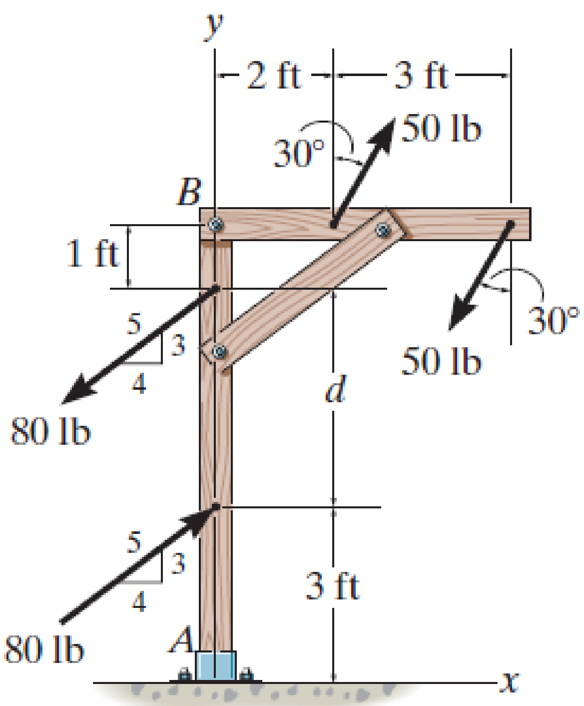
$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} \text{ N·m}$$



$$M_a = -1 \{-0.1\,(21.21) - 0.15\,(15)\}$$

$$= 4.37 \text{ N·m (CCW)}$$

Sample Problem (§ 4.6)



Given: Two couples act on the frame with the geometry shown and d = 4 ft.

Find: The resultant couple moment

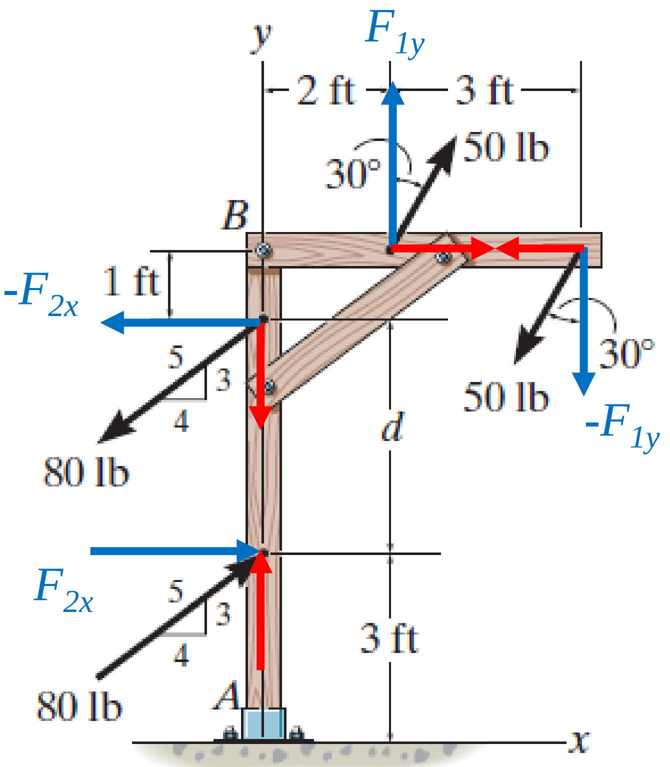
a) decomposing the forces in x and y components

b) Finding distances between forces

Plan:

1) Resolve the forces in x and y-directions or find distance between forces

2) Add these two couples to find the resultant couple.



Note the couple of forces of 50 lb creates a negative moment and the couple of forces of 80 lb creates a positive moment

a) Finding components of the forces.

The decomposed forces have two components that cancel each other out (red) and two components that are separated by a known distance (blue).

F_{1y} = 50 cos(30) = 43.3 lb

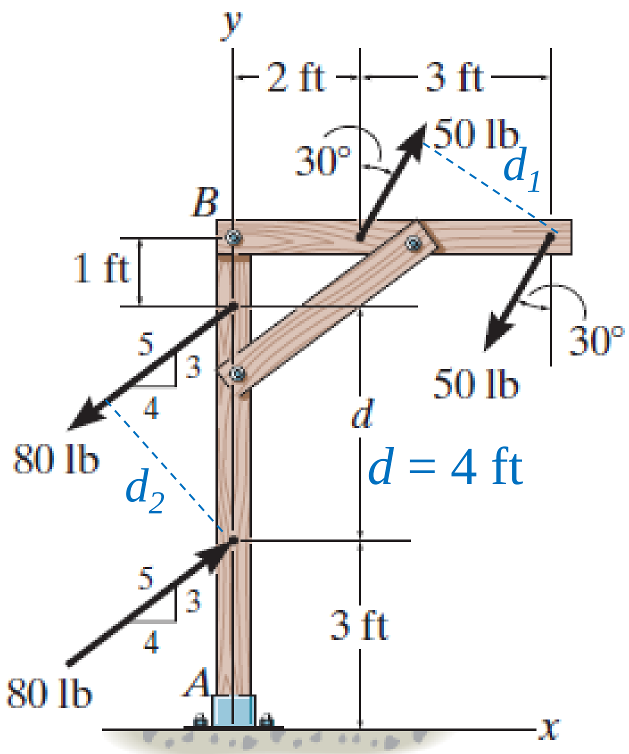
F_{2x} = 80 (4/5) = 64 lb

$$\sum M = \sum (F\ d)$$

$$= -(43.3 \times 3) + 64 \times 4$$

$$= -129.9 + 256 = 126.1 \text{ lb·ft (CCW)}$$

b) Finding d_1 and d_2.



$$d_1 = 3\cos(30) = 2.598 \text{ ft}$$

$$d_2 = d\,(4/5) = 3.2 \text{ ft}$$

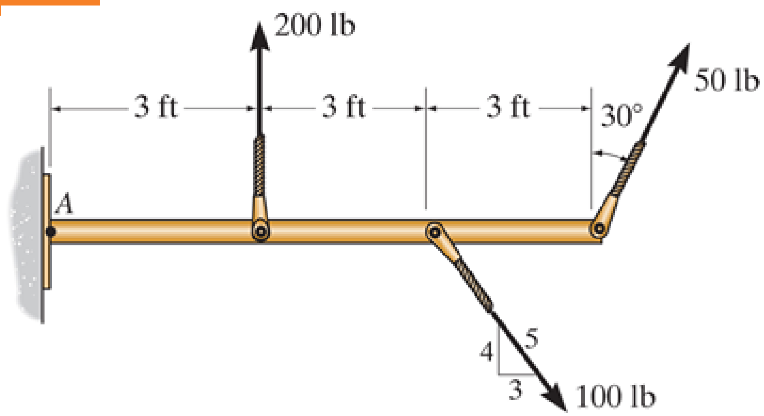
$$\sum M = \sum (F\,d)$$

$$= -(50 \times 2.598) + 80 \times 3.2$$

$$= -129.9 + 256 = 126.1 \text{ lb·ft} \quad \text{(CCW)}$$

Note the couple of forces of 50 lb creates a negative moment and the couple of forces of 80 lb creates a positive moment

Sample Problem (§ 4.7-8)



Given: A 2-D force system with geometry as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location measured from A.

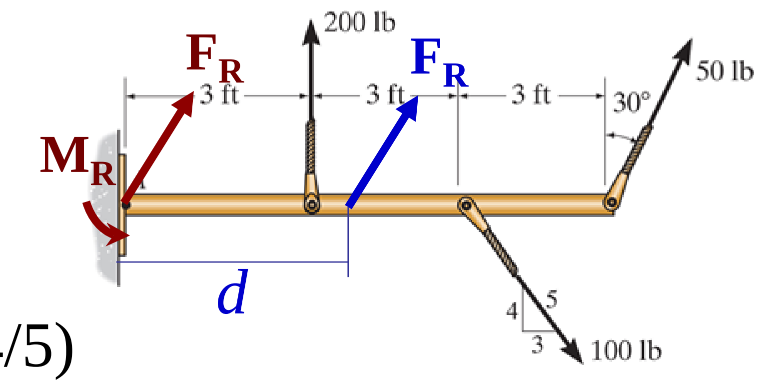
Plan:

1) Sum all the *x* and *y* components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force component to *A*.

3) Shift F_{RA} to a distance *d* such that $d = M_{RA}/F_{Ry}$

Find F_{Rx}, F_{Ry}, and M_{RA}



$F_{Rx} = \Sigma F_x = 50(\sin 30) + 100(3/5)$
$= 85$ lb

$F_{Ry} = \Sigma F_y = 200 + 50(\cos 30) - 100(4/5)$
$= 163.3$ lb

$M_{RA} = \Sigma M_R = 200\,(3) + 50\,(\cos 30)\,(9)$
$- 100\,(4/5)\,6 = 509.7$ lb·ft CCW

$F_R = (\,85^2 + 163.3^2\,)^{1/2} = 184$ lb

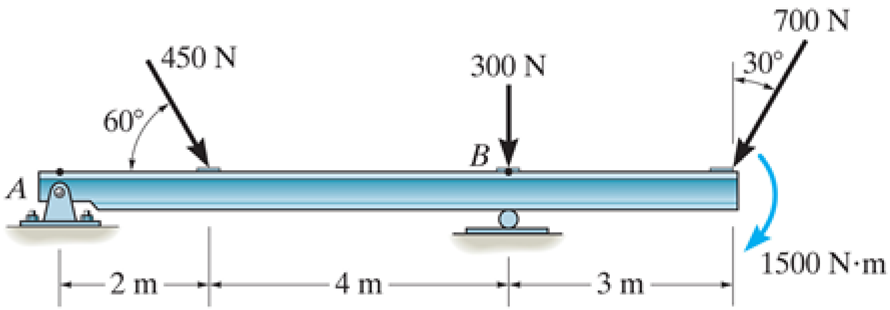
$\angle\ \theta = \tan^{-1}(\,163.3/85\,) = 62.5°$

$M_{RA} = 509.7$ lb·ft CCW

The equivalent single force F_R can be located at a distance d measured from A.

$d = M_{RA}/F_{Ry} = 509.7 / 163.3 = 3.12$ ft

Sample Problem (§ 4.7-8)



Given: A 2-D force and couple system as shown.

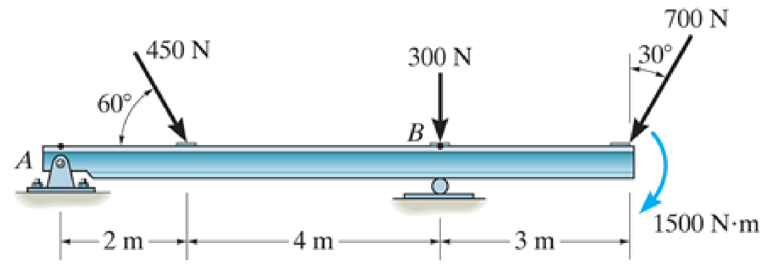
Find: The equivalent resultant force and couple moment acting at *A*.

Plan:

1) Sum all the *x* and *y* components of the two forces to find $\mathbf{F}_{\mathrm{R}A}$.

2) Find the resultant moment $M_{\mathrm{R}A}$, resulting from moving each force to *A* and adding the 1500 N·m free moment.

Summing the force components:



$\Sigma F_x = 450\ (\cos 60) - 700\ (\sin 30)$

$= -125\ \text{N}$

$\Sigma F_y = -450\ (\sin 60)\ - 300 - 700\ (\cos 30)$

$= -1296\ \text{N}$

Now find the magnitude and direction of the resultant.

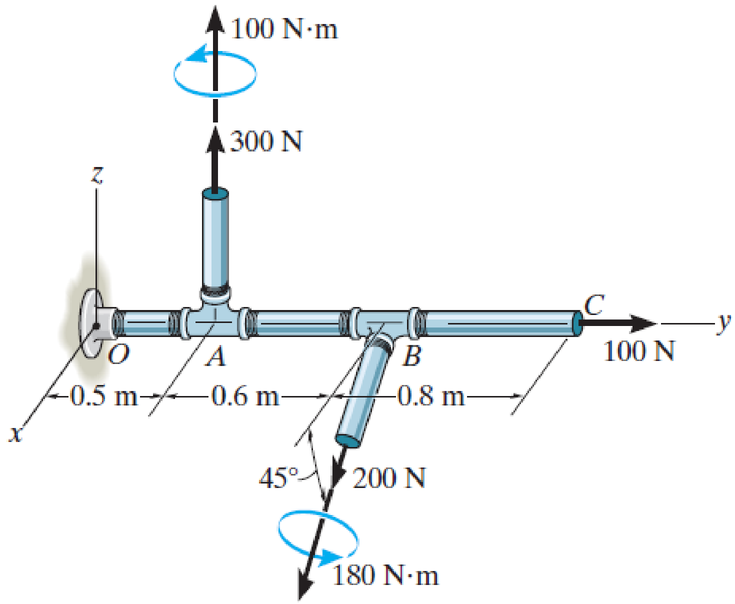
$F_{RA} = (125^2 + 1296^2)^{1/2} = 1302\ \text{N}$ and $\theta = \tan^{-1}(-1296\ /-125)$

$= 84.5°$

$M_{RA} = -450\ (\sin 60)\ (2) - 300\ (6) - 700\ (\cos 30)\ (9) - 1500$

$= -9535\ \text{N·m} = 9535\ \text{N·m CW}$

Sample Problem (§ 4.7-8)



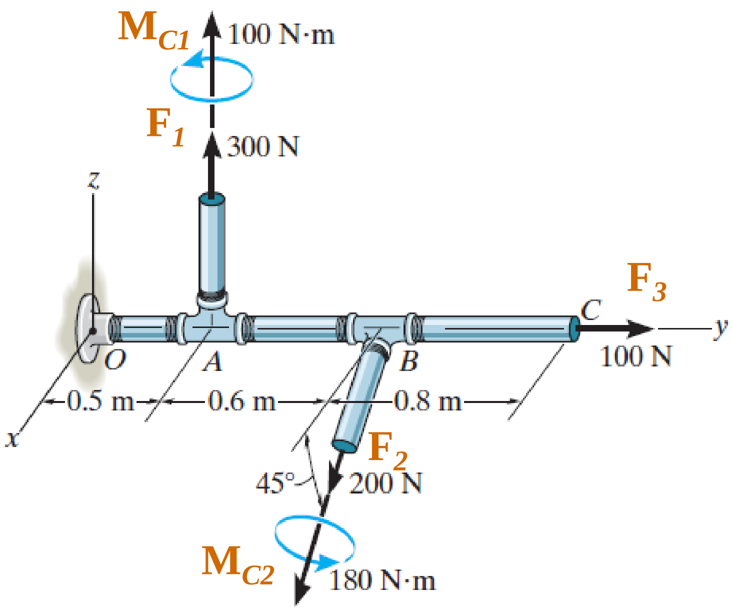
Given: Forces and couple moments are applied to the pipe.

Find: An equivalent resultant force and couple moment at point *O*.

Plan:

1) Find $\mathbf{F}_{RO} = \Sigma\, \mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

2) Find $\mathbf{M}_{RO} = \Sigma\, \mathbf{M}_C + \Sigma\, (\, \mathbf{r}_i \times \mathbf{F}_i\,)$



Position vectors (wrt *O*):

$\mathbf{r}_1 = \{0.5\,\boldsymbol{j}\}$ m

$\mathbf{r}_2 = \{1.1\,\boldsymbol{j}\}$ m

$\mathbf{r}_3 = \{1.9\,\boldsymbol{j}\}$ m

Force vectors:

$\mathbf{F}_1 = \{300\,\boldsymbol{k}\}$ N

$\mathbf{F}_2 = 200\{\cos 45°\,\boldsymbol{i} - \sin 45°\,\boldsymbol{k}\}$ N

$= \{141.4\,\boldsymbol{i} - 141.4\,\boldsymbol{k}\}$ N

$\mathbf{F}_3 = \{100\,\boldsymbol{j}\}$ N

Free couple moments are:

$\mathbf{M}_{C1} = \{100\,\boldsymbol{k}\}$ N·m

$\mathbf{M}_{C2} = 180\{\cos 45°\,\boldsymbol{i} - \sin 45°\boldsymbol{k}\}$N·m

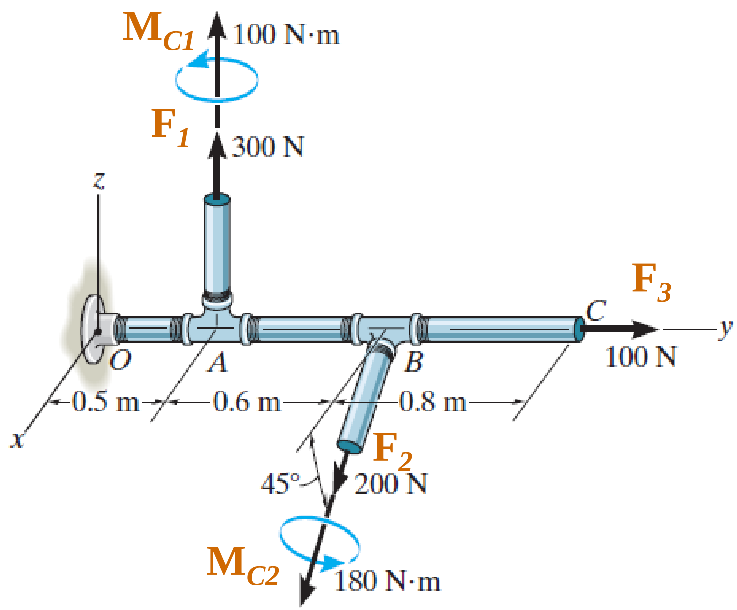
$= \{127.3\,\boldsymbol{i} - 127.3\boldsymbol{k}\}$N·m

Resultant force and couple moment at point O:

$$\mathbf{F}_R = \Sigma\,\mathbf{F}_i = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= \{300\,\boldsymbol{k}\} + \{141.4\,\boldsymbol{i} - 141.4\,\boldsymbol{k}\} + \{100\,\boldsymbol{j}\}$$

$$\mathbf{F}_R = \{141\,\boldsymbol{i} + 100\,\boldsymbol{j} + 158.6\,\boldsymbol{k}\}\ \text{N}$$



$$\mathbf{M}_{RO} = \Sigma\,\mathbf{M}_C + \Sigma\,(\mathbf{r}_i \times \mathbf{F}_i)$$

$$\mathbf{M}_{RO} = \{100\,\boldsymbol{k}\} + \{127.3\,\boldsymbol{i} - 127.3\boldsymbol{k}\}$$

$$+ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.1 & 0 \\ 141.4 & 0 & -141.4 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & 1.9 & 0 \\ 0 & 100 & 0 \end{vmatrix}$$

$$\mathbf{M}_{RO} = \{122\,\boldsymbol{i} - 183\,\boldsymbol{k}\}\ \text{N·m}$$