ASSIGNMENT 2B

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3.2 Part B

Regular Problems

- 3.22 c [representations using unit-step function]
- 3.24 d g [memoryless]
- 3.25 b f [causal]
- 3.26 b e [invertible]
- 3.27 d e [BIBO stable]
- 3.28 b d [time invariant]
- 3.29 b e [linear]
- 3.33 b [eigenfunctions]

MATLAB Problems

- D.102 [temperature conversion, looping]
- D.107 a b c [write unit-step function]

3.22. c

$$x(t) = \begin{cases} 4t+4 & ; \; -1 \le t < -\frac{1}{2} \\ 4t^2 & ; \; -\frac{1}{2} \le t < \frac{1}{2} \\ -4t+4 & ; \; \frac{1}{2} \le t < 1 \\ 0 & ; \; \text{otherwise} \end{cases}$$

we've to express them as unit function

$$x(t) = \left[(4t+4)\,u(t-(-1)) - (4t+4)\,u\left(t-\left(-\tfrac{1}{2}\right)\right) \right]$$
$$+ \left[4t^2\,u\left(t-\left(-\tfrac{1}{2}\right)\right) - 4t^2\,u\left(t-\tfrac{1}{2}\right) \right]$$
$$+ \left[(-4t+4)\,u\left(t-\tfrac{1}{2}\right) - (4t+4)\,u(t-1) \right]$$

$$= (4t+4)\,u(t+1) - (4t+4)\,u\left(t+\tfrac{1}{2}\right) + 4t^2 u\left(t+\tfrac{1}{2}\right)$$
$$- 4t^2 u\left(t-\tfrac{1}{2}\right) + (-4t+4)\,u\left(t-\tfrac{1}{2}\right) - (-4t+4)\,u(t-1)$$

$$= (4t+4)\,u(t+1) + (-4t-4+4t^2)\,u\left(t+\tfrac{1}{2}\right)$$
$$+ (-4t^2-4t+4)\,u\left(t-\tfrac{1}{2}\right) + (4t-4)\,u(t-1)$$

$$= 4(t+1)\,u(t-1) + 4(t^2-t-1)\,u\left(t+\tfrac{1}{2}\right) + 4(1-t-t^2)\,u\left(t-\tfrac{1}{2}\right)$$
$$+ 4(t-1)\,u(t-1)$$

3.24 $\cancel{g}$

$$H\{x(t)\} = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} x(t)\,\delta(t-\tau)\,d\tau$$

$$= x(t)\int_{-\infty}^{\infty} \delta(t-\tau)\,d\tau$$

$$= x(t)$$

$\therefore$ The system is memoryless.

3.24 d $\quad H x(t) = \int_{t}^{\infty} x(\tau)\,d\tau$

putting $\tau = 2w$

$d\tau = 2dw$

$$Hx(t) = 2\int_{-\infty}^{t} x(2w)\,dw$$

Here since scaling is present, the system is true for ~~values~~ multiple values. So, the system is meaningless.

3.25 (b) $\mathcal{H}x(t) = \text{Even } x(t)$

$$= \frac{x(t) + x(-t)}{2}$$

Letting $t = 2$, $\mathcal{H}x(2) = \frac{x(2) + x(-2)}{2}$

Letting $t = -2$, $\mathcal{H}x(-2) = \frac{x(-2) + x(2)}{2}$

Here, $x(-2)$ is a present value but $x(2)$ is a future value. Non-Causal system.

PLEASE TURN OVER

3.26 e.

$$Hx(t) = x^2(t)$$

$$Hx(t) = (\pm x(t))^2$$

putting different values in $+x(t)$ & $-x(t)$

we are gonna get the same outputs.

$\therefore$ The system isn't inverse.

(f) $$Hx(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau$$

$$Hx(t) = \Big[x(\tau)u(t-\tau) \Big]_{-\infty}^{\infty}$$

$$= \Big[x(\infty)u(\infty-2) \Big] - \Big[x(-\infty)u(-\infty-2) \Big]$$

$x(\infty)$ and $u(\infty \cdot 2)$ present value
$x(-\infty)$ and $u(-\infty-2)$ past values
} it's a causal system.

3.26 (b)

$\mathcal{H}x(t) = e^{x(t)}$ where x is a real function.

let $x(t) = u(t)$

$\Rightarrow H(u(t)) = e^{u(t)}$

Leting $x(t) = -u(t)$

$H(-u(t)) = e^{-u(t)}$

Taking $u(t)$ we get different outputs for different inputs.

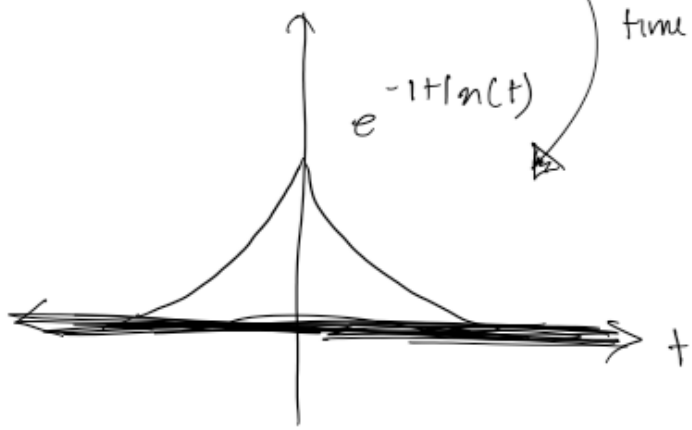
$\therefore$ The system is invertible.

Inverse of the system is $H(x(t)) = e^{x(t)}$

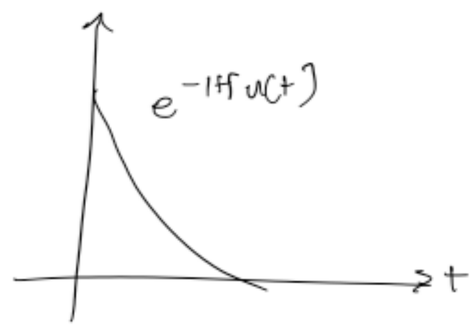
$\Rightarrow x(t) = \log_e \mathcal{H}(x(t))$

3.27 (d) $\mathcal{H}x(t) = e^{-|t|}x(t)$

time graph of this system.



Letting $x(t) = u(t)$ we get



For bounded input, the system ~~is~~ has bounded output $\rightarrow$ stable

$$\left| e^{-|t|} x(t) \right| < m \quad \text{for some } m \text{ which is finite}$$

for all t.

$\therefore$ System is stable.

$$\int_{-\infty}^{\infty} |u(x(t))| \, dt < \infty$$

The area is also finite.

$\therefore$ The system is BIBO stable

ⓔ $y(t) = \left(\dfrac{1}{t-1} \right) x(t)$

we can see that at $t=1$, the system goes to ∞

$\therefore$ Not Bounded $\therefore$ Not Stable

Also, $\displaystyle\int_{-\infty}^{\infty} \left[\left| \frac{1}{t-1} \right| x(t) \right] dt \not< \infty$

since it's not bounded.

$\therefore$ System is not BIBO stable

3.28. (b) If $x(t)$ = Even $x(t)$

$$= \frac{x(t) + x(-t)}{2} = \frac{x(t)}{2} + \frac{x(-t)}{2}$$

Delay in input and checking the output,

$x(t) \rightarrow x(-t)$

$x(t - t_0) \rightarrow x(-t - t_0)$ —— (i)

Delay in output,

$t = (t - t_0)$

output $= x(-(t - t_0)) = x(-t + t_0)$ — (ii)

(i) $\neq$ (ii)

So, Even $x(t) \longrightarrow$ Time variant

(d) If $x(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$ [h is an arbitrary fixed function]

Delay in input and checking output

$$x(t-t_0) \longrightarrow \int_{-\infty}^{\infty} \left[x(\tau - t_0) \cdot h(t-\tau) d\tau \right] \quad \text{(i)}$$

Delay in output,

$$\text{If } x(t-t_0) \longrightarrow \int_{-\infty}^{\infty} x(\tau) h(t - t_0 - \tau) \quad \text{(ii)}$$

(i) $\int_{-\infty}^{\infty} x(\tau - t_0) h(t-\tau) d\tau$

Let $\tau - t_0 = \lambda$ | when $\tau \to \infty$ $\lambda \to +\infty$

$\Rightarrow \tau = \lambda + t_0$ | when $\tau \to -\infty$ $\lambda \to -\infty$

$$\therefore \int_{-\infty}^{\infty} x(\tau - t_1)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(\lambda)\, h(t - t_0 - \lambda)\, d\lambda \quad \text{(11)}$$

So, $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$ = Time Invariant

3.20 (b) Considering $\mathcal{H}[x(t)] = y(t)$

Given system $y(t) = e^{x(t)}$. Let $x(t) = a_1 x_1(t) + a_2 x_2(t)$

$y_1(t) = e^{x_1(t)}$, $y_2(t) = e^{x_2(t)}$

$y(t) = [y_1(t)]^{a_1} [y_2(t)]^{a_2}$ $\therefore$ Doesn't follow superposition principal

Since $y(t) \neq a_1 y_1(t) + a_2 y_2(t)$

the system isn't linear.

(e) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

all integral functions are linear

$$\mathcal{H}x_1(t) = \int_{-\infty}^{\infty} x_1(\tau)\, h(t-\tau)\, d\tau \quad — (1)$$

$$\mathcal{H}x_2(t) = \int_{-\infty}^{\infty} x_2(\tau)\, h(t-\tau)\, d\tau \quad — (2)$$

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} \left[a_1 x_1(\tau) + a_2 x_2(\tau)\right] h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} a_1 x_1(\tau)\, h(t-\tau)\, d\tau + \int_{-\infty}^{\infty} a_2 x_2(\tau)\, h(t-\tau)\, d\tau$$

$$= a_1 \int_{-\infty}^{\infty} x_1(\tau)\, h(t-\tau)\, d\tau + a_2 \int_{-\infty}^{\infty} x_2(\tau)\, h(t-\tau)\, d\tau$$

$$\therefore \mathcal{H}x(t) = a_1 \mathcal{H}x_1(t) + a_2 \mathcal{H}x_2(t)$$

which follows principal of superposition

$\therefore$ The system is linear.

3.33 (b) A function $x(t)$ is called an eigenfunction of a system $\mathcal{H}x$ if $\mathcal{H}x(t)$ is a scalar multiple of $x(t)$ that is if $\boxed{\mathcal{H}x(t) = \lambda x(t)}$ where λ is a scalar constant. This scalar constant λ is called an eigenvalue corresponding to the eigenfunction $x(t)$.

Given, $\mathcal{H}x(t) = \mathcal{D}x(t)$ where $\mathcal{D}$ is the derivative operator.

$\boxed{x_1(t) = e^{at}}$ $\boxed{x_2 = e^{at_2}}$ $\boxed{x_3 t = 42}$

$$\mathcal{H}x_1(t) = \mathcal{D}x_1(t) = \frac{d}{dt}(x_1(t))$$

$$= \frac{d}{dt}(e^{at})$$

$$= ae^{at} = ax_1(t)$$

$\mathcal{H}x_1(t) = ax_1(t)$ where a is a scalar

Therefore, $x_1(t)$ is an eigenfunction of $\mathcal{H}$ and the corresponding eigenvalue is a.

$$\mathcal{H}x_2(t) = Dx_2(t) = \frac{d}{dt}\left(x_2(t)\right)$$

$$= \frac{d}{dt}\left(e^{at^2}\right)$$

$$= e^{at^2} \cdot \frac{d}{dt}\left(at^2\right)$$

$$= 2ate^{at^2}$$ which is

not a scalar multiple of $x_2(t) = e^{at^2}$

So, $x_2(t)$ is not an eigenfunction of $\mathcal{H}$.

$$\mathcal{H}x_3(t) = Dx_3(t) = \frac{d}{dt}\left(x_3(t)\right) = \frac{d}{dt}(42)$$

$$= 0 = 0(42)$$

$$= 0.\, x_3(t)$$

$\therefore \mathcal{H}x_3(t) = 0.\, x_3(t)$, where 0 is a scalar.

Therefore $x_3(t)$ is an **eigenfunction** of $\mathcal{H}$ and the corresponding eigenvalue is 0.

D.102 Let T_C, T_F, and T_K denote the temperature measured in units of Celsius, Fahrenheit, and Kelvin, respectively. Then, these quantities are related by

$$T_F = \tfrac{9}{5}T_C + 32 \quad \text{and}$$
$$T_K = T_C + 273.15.$$

Write a program that generates a temperature conversion table. The first column of the table should contain the temperature in Celsius. The second and third columns should contain the corresponding temperatures in units of Fahrenheit and Kelvin, respectively. The table should have entries for temperatures in Celsius from -50 to 50 in steps of 10.

Editor – /Users/arfazhussain/Documents/TempTable.m

TempTable.m | unitstep.m | +

```
% MATLAB PROGRAM

function TempTable ()
    fprintf("\tC\tF\tK\n");
    for i = -50:10:50
        cel = i;
        fahren = (cel*(9/5)) + 32;
        kelv = cel + 273.15;
        %fprintf("%.2f%cC | %.2f%cF | %.2f%cK\n",cel, "°", fahren, "°", kelv, "°");
        disp([cel',fahren', kelv]);
    end
end
```

Command Window

```
>> TempTable
        C        F        K
  -50.0000  -58.0000  223.1500

  -40.0000  -40.0000  233.1500

  -30.0000  -22.0000  243.1500

  -20.0000   -4.0000  253.1500

  -10.0000   14.0000  263.1500

         0   32.0000  273.1500

   10.0000   50.0000  283.1500

   20.0000   68.0000  293.1500

   30.0000   86.0000  303.1500

   40.0000  104.0000  313.1500

   50.0000  122.0000  323.1500
```

D.107 (a) Write a function called `unitstep` that takes a single real argument t and returns $u(t)$, where

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Modify the function from part (a) so that it takes a single vector argument $t = [t_1\ t_2 \cdots t_n]^T$ (where $n \geq 1$ and $t_1, t_2, \ldots, t_n$ are real) and returns the vector $[u(t_1)\ u(t_2) \ldots u(t_n)]^T$. Your solution must employ a looping construct (e.g., a for loop).

(c) With some ingenuity, part (b) of this exercise can be solved using only two lines of code, without the need for any looping construct. Find such a solution. [Hint: In MATLAB, to what value does an expression like "`[-2 -1 0 1 2] >= 0`" evaluate?]

(A)

Editor – /Users/arfazhussain/Documents/MATLAB/unitstep.m

TempTable.m | unitstep.m | +

```matlab
function unitstep (t)
    if t>=0
        t=1;
        fprintf("1\n");
    else
        t=0;
        fprintf("0\n");
    end
end
```

```
>> unitstep(5)
1
>> unitstep(0)
1
>> unitstep(-2472)
0
```

(B) ~ (C) ~

END OF ASSIGNMENT 2B