STAT 260 Spring 2023: Assignment 6

Due: Friday March 17th BEFORE 11:59pm PT to Crowdmark

Please read the instructions below and in the Written Assignment 6 assignment on Crowdmark.

For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. Messy, poorly formatted work will receive deductions, or may not be graded at all.

Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding. Assignment questions are not to be posted to homework "help" websites.

Late policy: Late assignments will be accepted until the final cutoff of 11:59pm on Sunday March 19th. Solutions submitted within 1 hour of the Friday deadline will have a 5% late penalty automatically applied within Crowdmark. Solutions submitted after 1 hour of the Friday deadline but before the final Sunday cutoff will have a 20% late penalty applied. Solutions submitted after the final Sunday cutoff will be graded for feedback, but marks will not be awarded.

1. **[1 mark]** A technical support centre at a university has determined that 14% of all tech support requests require more than one technician to answer the request.
   If 600 random tech support requests are reviewed, find the probability that at least 95 requests require more than one technician. Use the normal approximation to the binomial distribution with the continuity correction. Be sure to confirm that the normal distribution is an appropriate approximation to use here.

$np = (600)(.14) = 84 \geq 5$

$n(1-p) = (600)(.86) = 516 \geq 5$

X = # of requests that require more than one technician

$\mu = np = (600)(.14) = 84$

$\sigma = \sqrt{np(1-p)} = \sqrt{600(.14)(.86)} = \sqrt{72.24}$

$$\underset{\text{binomial}}{P(X \geq 95)} = 1 - P(X \leq 94) \approx \underset{\text{normal}}{1 - P(X \leq 94.5)}$$

$$= 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{94.5-84}{\sqrt{72.24}}\right)$$

$$= 1 - P(Z \leq 1.24)$$

$$= 1 - 0.8925$$

$$= 0.1075$$

2. The lifetime of a laser diode is normally distributed with a mean of 42,000 hours and a standard deviation of 6100 hours.

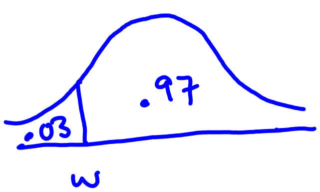
(a) [**1 mark**] Suppose a randomly selected laser diode has lasted at least 37,000 hours. What is the probability that the diode will last at most 50,000 hours?

$$P(X \le 50000 \mid X \ge 37000) = \frac{P(37000 \le X \le 50000)}{P(X \ge 37000)}$$

$$= \frac{P\left(\frac{37000-42000}{6100} \le \frac{X-\mu}{\sigma} \le \frac{50000-42000}{6100}\right)}{P\left(\frac{X-\mu}{\sigma} \ge \frac{37000-42000}{6100}\right)} = \frac{P(-0.82 \le Z \le 1.31)}{P(Z \ge -0.82)}$$

$$= \frac{P(Z \le 1.31) - P(Z \le -0.82)}{1 - P(Z \le -0.82)} = \frac{0.9049 - 0.2061}{1 - 0.2061} = \frac{0.6988}{0.7939} = 0.8802$$

(b) [**1 mark**] The manufacturer of the diode wants to advertise the lifetime w as the amount so that 97% of diodes will last longer than this amount. What is the lifetime w that the manufacturer should advertise?



$$Z = -1.88$$

$$Z = \frac{X-\mu}{\sigma}$$

$$-1.88 = \frac{w - 42000}{6100}$$

$$w = (-1.88)(6100) + 42000$$

$$w = 30532$$

(c) [**1 mark**] Suppose that 9 diodes are selected randomly and the lifetime of the diodes is independent. What is the probability that exactly 7 of the diodes will last longer than 35,500 hours?

$$p = P(X > 35500) = P\left(Z > \frac{35500 - 42000}{6100}\right) = P(Z > -1.07)$$

$$= 1 - P(Z \le -1.07) = 1 - 0.1423 = 0.8577$$

y = # of diodes that last longer than 35500 hours

binomial $n = 9$ $p = 0.8577$

$$P(y=4) = \binom{9}{7}(.8577)^7(.1423)^2 = 0.2489$$

3. **[1 mark]** The lifespan, in years, of a particular brand of photocopier is known to be gamma distributed with $\alpha = 2$ and a mean of 12 years. Determine the probability that the printer has a lifespan of at most 15 years. If integration is used below, all steps must be shown.

$\alpha = 2 \quad \Gamma(2) = (2-1)! = 1! = 1$

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} = \frac{x e^{-x/6}}{6^2(1)} = \frac{x e^{-x/6}}{36}$$

$E(X) = \alpha\beta = 2\beta = 12 \Rightarrow \beta = 6$

$$P(X \le 15) = \int_0^{15} \frac{x e^{-x/6}}{36}\, dx$$

$\int u\,dv = uv - \int v\,du$

$u = x \quad dv = e^{-x/6}\,dx$

$du = dx \quad v = -6e^{-x/6}$

$$= \frac{1}{36}\left[-6xe^{-x/6}\Big|_0^{15} - \int_0^{15} -6e^{-x/6}\,dx\right]$$

$$= \frac{1}{36}\left[-6xe^{-x/6} - 36e^{-x/6}\right]\Big|_0^{15}$$

$$= \frac{1}{36}\left[\left(-6(15)e^{-15/6} - 36e^{-15/6}\right) - \left(0 - 36e^0\right)\right]$$

$$= \frac{1}{36}\left[36 - 126e^{-5/2}\right] = 1 - \frac{7}{2}e^{-5/2}$$

$= 0.7127$

4. **[1 mark]** On an online shopping website, users are asked to complete a customer service questionnarie when checking out. Suppose the time it takes, in minutes, to complete the questionnaire is exponentially distributed with a mean of 2.5 minutes. For a random selection of 10 customers who complete the questionnaire, what is the probability that exactly 3 of the customers took longer than 3 minutes to complete the questionnaire?

$E(X) = 2.5 = 1/\lambda \Rightarrow \lambda = 2/5$

$P(X > 3) = 1 - (1 - e^{-2/5(3)}) = e^{-6/5} = 0.3012$

Y = # of customers that took longer than 3 minutes

binomial $\quad n = 10 \quad p = 0.3012$

$P(Y=3) = \binom{10}{3}(.3012)^3(.6988)^7 = 0.2668$