Exercise 4.105

L Answer (a). We are asked to express h in terms of h_1 and h_2.

We are given a LTI system $\mathcal{H}$ with impulse response h, where

$$\mathcal{H}x = \tfrac{1}{2}\mathcal{H}_1(4x) - \tfrac{1}{2}\mathcal{H}_2(4x) \quad (1)$$

and each LTI system $\mathcal{H}_k$ has impulse response h_k. From the preceding equation, we have

$$\begin{aligned}
\mathcal{H}x &= \tfrac{1}{2}\mathcal{H}_1(4x) - \tfrac{1}{2}\mathcal{H}_2(4x) && \text{from (1)}\\
&= \tfrac{1}{2}[(4x) * h_1] - \tfrac{1}{2}[(4x) * h_2] && \mathcal{H}_k x = x * h_k \text{ (since } \mathcal{H}_k \text{ is LTI)}\\
&= 2(x * h_1) - 2(x * h_2) && (af) * g = a(f*g)\\
&= x * (2h_1) + x * (-2h_2) && a(f*g) = f*(ag)\\
&= x * (2h_1 - 2h_2) && \text{distributive property of convolution}\\
&= x * [2(h_1 - h_2)]. && \text{factor out 2}
\end{aligned}$$

Therefore, we conclude $h = 2(h_1 - h_2)$. (h since $\mathcal{H}$ is LTI)

$$h = 2(h_1 - h_2).$$

