Exercise 5.105

L **Answer (d).**

We are given

$$1 \qquad c_k = \left(\frac{e^{j3k}}{j2k-1}\right)^2 \quad \text{and} \quad T = 2.$$

First, we compute the magnitude spectrum of x. We have

$$\begin{aligned}
|c_k| &= \left|\left(\frac{e^{j3k}}{j2k-1}\right)^2\right| && \text{take magnitude of both sides of 1}\\
&= \left|\frac{e^{j6k}}{(j2k-1)^2}\right| && \text{square}\\
&= \frac{1}{|j2k-1|^2} && \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}\\
&= \frac{1}{(\sqrt{4k^2+1})^2} && \text{take magnitude}\\
&= \frac{1}{4k^2+1}. && \text{cancel square and square root } (4k^2+1>0)
\end{aligned}$$

Next, we compute the phase spectrum of x. We have

$$\begin{aligned}
\arg c_k &= \arg\left[\left(\frac{e^{j3k}}{j2k-1}\right)^2\right] && \text{take argument of both sides of 1}\\
&= \arg\left[\frac{e^{j6k}}{(j2k-1)^2}\right] && \text{square}\\
&= \arg[e^{j6k}] - \arg[(j2k-1)^2] && \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2\\
&= 6k - 2\arg(j2k-1) && \arg e^{j\theta} = \theta\\
&= 6k - 2\left[\arctan\left(\tfrac{2k}{-1}\right) + \pi\right] && \operatorname{Re}(j2k-1) = -1\\
&= 6k - 2\arctan(-2k) - 2\pi && \text{simplify}\\
&= 6k + 2\arctan(2k) - 2\pi. && \text{arctan is odd}
\end{aligned}$$

(In the above simplification, we used the fact that arctan is odd.) Since the argument is not uniquely determined, in the most general case, we have

$$\arg c_k = 6k + 2\arctan(2k) + 2\pi\ell$$

for all integer ℓ.