R **2.4** In each case below, find a fully-simplified expression for the function y.
(a) $y(t) = \mathcal{D}x(3t)$, where $x(t) = t^2 + 2t + 1$ and $\mathcal{D}$ denotes the derivative operator;
(b) $y(t) = \mathcal{D}\{x(3\cdot)\}(t)$, where $x(t) = t^2 + 2t + 1$ and $\mathcal{D}$ denotes the derivative operator;
(c) $y(t) = \mathcal{H}x(t-3)$, where $\mathcal{H}x(t) = tx(t)$ and $x(t) = 2t + 1$; and
(d) $y(t) = \mathcal{H}\{x(\cdot - 3)\}(t)$, where $\mathcal{H}x(t) = tx(t)$ and $x(t) = 2t + 1$.

Short Answer. (a) $y(t) = 6t + 2$; (b) $y(t) = 18t + 6$; (c) $y(t) = 2t^2 - 11t + 15$; (d) $y(t) = 2t^2 - 5t$

R **Answer (a).**

First, we compute $\mathcal{D}x$ to obtain

$$\begin{aligned}\mathcal{D}x(t) &= \tfrac{d}{dt}(t^2 + 2t + 1)\\ &= 2t + 2.\end{aligned}$$

So, we have

$$\begin{aligned}\mathcal{D}x(3t) &= 2(3t) + 2\\ &= 6t + 2.\end{aligned}$$

R **Answer (b).**

First, we give a name v to the anonymous function represented by $x(3\cdot)$. That is, we define $v(t) = x(3t)$. So, we have

$$\begin{aligned}v(t) &= x(3t)\\ &= (3t)^2 + 2(3t) + 1\\ &= 9t^2 + 6t + 1.\end{aligned}$$

So, we have

$$\begin{aligned}y(t) &= \mathcal{D}\{x(3\cdot)\}(t)\\ &= \mathcal{D}v(t)\\ &= \tfrac{d}{dt}(9t^2 + 6t + 1)\\ &= 18t + 6.\end{aligned}$$

R **3.35** For each function x given below, determine whether x is periodic, and if it is, find its fundamental period T.
(a) $x(t) = 3\cos(\sqrt{2}t) + 7\cos(2t)$;
(b) $x(t) = [3\cos(2t)]^3$; and
(c) $x(t) = 7\cos(35t+3) + 5\sin(15t-2)$.

Short Answer. (a) not periodic; (b) π-periodic; (c) $\left(\frac{2\pi}{5}\right)$-periodic

R **Answer (c).**

Let $x_1(t) = 7\cos(35t+3)$ and $x_2(t) = 5\sin(15t-2)$. Let T_1 and T_2 denote the fundamental periods of x_1 and x_2, respectively. We have that

$$T_1 = \frac{2\pi}{35}, \quad T_2 = \frac{2\pi}{15}, \quad \text{and} \quad \frac{T_1}{T_2} = \frac{\left(\frac{2\pi}{35}\right)}{\left(\frac{2\pi}{15}\right)} = \frac{2\pi}{35}\left(\frac{15}{2\pi}\right) = \frac{15}{35} = \frac{3}{7}.$$

Since $\frac{T_1}{T_2}$ is rational, x is periodic. We have that $T = 7T_1 = 3T_2 = 7(\frac{2\pi}{35}) = \frac{2\pi}{5}$.

R 3.36 For each case below, for the function x (of a real variable) having the properties stated, find $x(t)$ for all t.

(a) The function x is such that:

- $x(t) = 1 - t$ for $0 \leq t \leq 1$;
- the function w is even, where $w(t) = x(t+1)$; and
- the function v is causal, where $v(t) = x(t) - 1$.

(b) The function x is such that:

- $x(t) = e^{t+4}$ for $t < -4$;
- $x(t) = a$ for $-4 \leq t \leq -2$, where a is a real constant; and
- the function $v(t) = x(t-3)$ is odd.

(c) The function x is such that:

- the function $v(t) = x(t) + 1$ is causal;
- the function $w(t) = x(-t) - 1$ is causal; and
- $x(0) = 0$.

(d) The function x is such that:

- the function $x(t) = 1$ for $2 < t \leq 3$;
- the function $v_1(t) = x(t+1)$ is causal;
- the function $v_2(t) = x(t+3)$ is anticausal; and
- the function $v_3(t) = x(t+2)$ is odd.

(e) The function x is such that:

- the function $x(t) = t - 1$ for $1 \leq t \leq 2$;
- x is causal;
- the function $v_1(t) = x(t+2)$ is anticausal; and
- the function $v_2(t) = x(t+1)$ is even.

(f) The function x is such that:

- $\operatorname{Even} x(t) = t$ for $t \leq 0$; and
- $\operatorname{Odd} x(t) = t^2$ for $t > 0$.

(g) The function x is such that:

- the function $v(t) = x(t+2)$ is conjugate symmetric; and
- $x(t) = j[u(t-3) - u(t-5)]$ for $t \geq 2$.

(h) The function x is such that:

- the function $v(t) = x(t) - 1$ is causal; and
- x is odd.

(i) The function x is such that:

- $\operatorname{Re} x(t) = t$ for $t \geq 0$;
- $\operatorname{Im} x(t) = t^2$ for $t < 0$; and
- x is conjugate symmetric.

Short Answer.

(a) $x(t) = \begin{cases} 1 - t & 0 \leq t \leq 1 \\ t - 1 & 1 < t \leq 2 \\ 1 & \text{otherwise;} \end{cases}$

(b) $x(t) = \begin{cases} e^{t+4} & t < -4 \\ 0 & -4 \leq t \leq -2 \\ -e^{-t-2} & t > -2; \end{cases}$

(c) $x(t) = \operatorname{sgn}(t)$;

(d) $x(t) = \begin{cases} -1 & 1 < t < 2 \\ 1 & 2 < t < 3 \\ 0 & \text{otherwise;} \end{cases}$

(e) $x(t) = \begin{cases} 1-t & 0 \le t < 1 \\ t-1 & 1 \le t \le 2 \\ 0 & \text{otherwise;} \end{cases}$

(f) $x(t) = (t^2 - t)\operatorname{sgn}(t)$;

(g) $x(t) = \begin{cases} -j & -1 < t \le 1 \\ j & 3 \le t < 5 \\ 0 & \text{otherwise;} \end{cases}$

(h) $x(t) = -\operatorname{sgn}(t)$; and

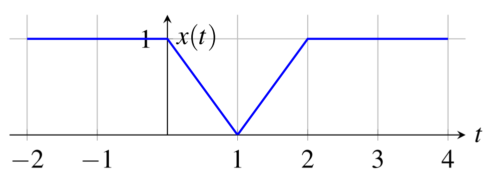
(i) $x(t) = (t - jt^2)\operatorname{sgn}(t)$.

R **Answer (a).**

Trivially, we have that $x(t) = 1 - t$ for $0 \le t \le 1$. Since $v(t) = x(t) - 1$ is causal, we have

$$\begin{aligned} & v(t) = 0 \text{ for all } t < 0 \\ \Rightarrow \quad & x(t) - 1 = 0 \text{ for all } t < 0 \\ \Rightarrow \quad & x(t) = 1 \text{ for all } t < 0. \end{aligned}$$

Since $w(t) = x(t+1)$ is even, x shifted leftwards by 1 unit is even (i.e., x is symmetric about the point 1). Combining the above observations, we conclude that x is the piecewise-linear function shown below.



R **3.37** Simplify each of the following expressions:

(a) $\dfrac{(\omega^2+1)\delta(\omega-1)}{\omega^2+9}$;

(b) $\dfrac{\sin(k\omega)\delta(\omega)}{\omega}$;

(c) $\int_{-\infty}^{\infty} e^{t-1}\cos[\frac{\pi}{2}(t-5)]\delta(t-3)dt$;

(d) $\int_{-\infty}^{\infty} \delta(2t-3)\sin(\pi t)dt$;

(e) $\int_{t}^{\infty}(\tau^2+1)\delta(\tau-2)d\tau$;

(f) $\int_{-4}^{4} e^{-\tau}\cos(\tau)\delta\left(\tau-\frac{\pi}{3}\right)d\tau+\int_{-2}^{2}\tau^2\cos(\tau)\delta(\tau-\pi)d\tau$; and

(g) $(t^2+1)^4 e^{-t}\operatorname{sinc}(t)\delta(t-\pi)$.

Short Answer. (a) $\frac{1}{5}\delta(\omega-1)$; (b) $k\delta(\omega)$; (c) $-e^2$; (d) $-\frac{1}{2}$; (e) $5u(2-t)$; (f) $\frac{1}{2}e^{-\pi/3}$; (g) 0

R **Answer (e).**

We have

$$\begin{aligned}
\int_{t}^{\infty}(\tau^2+1)\delta(\tau-2)d\tau &= \int_{t}^{\infty}(2^2+1)\delta(\tau-2)d\tau \\
&= 5\int_{t}^{\infty}\delta(\tau-2)d\tau \\
&= \begin{cases} 5 & t\leq 2 \\ 0 & \text{otherwise} \end{cases} \\
&= 5u(2-t).
\end{aligned}$$

R **3.38** For each function x given below, find a single expression for x (i.e., an expression that does not involve multiple cases). If the expression for x consists of a finite number of terms, group similar unit-step function terms together in the expression for x.
(a) $x(t) = 1 - t^2$ for $-1 \leq t < 1$ and $x(t) = x(t-2)$ for all t;
(b) $x(t) = \begin{cases} -e^{t+1} & t < -1 \\ t & -1 \leq t < 1 \\ (t-2)^2 & 1 \leq t < 2 \\ 0 & \text{otherwise;} \end{cases}$ and
(c) $x(t) = \begin{cases} (t/\pi+1)^2 & t < -\pi \\ \cos(t/2) & -\pi \leq t \leq \pi \\ (t/\pi-1)^2 & t > \pi. \end{cases}$

Short Answer.
(a) $x(t) = \sum_{k=-\infty}^{\infty}(-t^2 + 4kt - 4k^2 + 1)[u(t-2k+1) - u(t-2k-1)]$;
(b) $x(t) = -e^{t+1} + (t + e^{t+1})u(t+1) + (t-1)(t-4)u(t-1) - (t-2)^2u(t-2)$;
(c) $x(t) = (t/\pi+1)^2 + [\cos(t/2) - (t/\pi+1)^2]u(t+\pi) + [(t/\pi-1)^2 - \cos(t/2)]u(t-\pi)$.

R **Answer (b).**

We have

$$\begin{aligned}
x(t) &= -e^{t+1}[u(t-[-\infty]) - u(t+1)] + t[u(t+1) - u(t-1)] + (t-2)^2[u(t-1) - u(t-2)] \\
&= -e^{t+1}[1 - u(t+1)] + t[u(t+1) - u(t-1)] + (t-2)^2[u(t-1) - u(t-2)] \\
&= -e^{t+1} + (t + e^{t+1})u(t+1) + [(t-2)^2 - t]u(t-1) - (t-2)^2u(t-2) \\
&= -e^{t+1} + (t + e^{t+1})u(t+1) + [t^2 - 5t + 4]u(t-1) - (t-2)^2u(t-2) \\
&= -e^{t+1} + (t + e^{t+1})u(t+1) + (t-1)(t-4)u(t-1) - (t-2)^2u(t-2).
\end{aligned}$$

R **3.42** Determine whether each system $\mathcal{H}$ given below is BIBO stable.
(a) $\mathcal{H}x(t) = u(t)x(t)$;
(b) $\mathcal{H}x(t) = \ln x(t)$;
(c) $\mathcal{H}x(t) = e^{x(t)}$;
(d) $\mathcal{H}x(t) = e^{t}x(t)$;
(e) $\mathcal{H}x(t) = \cos[x(t)]$;
(f) $\mathcal{H}x(t) = x * x(t)$, where $f * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$;
(g) $\mathcal{H}x(t) = 3x(3t+3)$; and
(h) $\mathcal{H}x(t) = 2x(t)+1$.

Short Answer. (a) BIBO stable; (b) not BIBO stable; (c) BIBO stable; (d) not BIBO stable; (e) BIBO stable (if x is real valued or complex valued); (f) not BIBO stable; (g) BIBO stable; (h) BIBO stable

R **Answer (a).**

We have

$$\mathcal{H}x(t) = u(t)x(t).$$

Assume that $|x(t)| \leq A < \infty$ (i.e., x is bounded). Then, we need to show that this implies that $\mathcal{H}x$ is bounded. Taking the magnitude of both sides of the system equation, we have

$$\begin{aligned} |\mathcal{H}x(t)| &= |u(t)x(t)| \\ &= |u(t)|\,|x(t)|\,. \end{aligned}$$

Replacing the expressions $|u(t)|$ and $|x(t)|$ in the preceding equation by their upper bounds (of 1 and A, respectively), we obtain the inequality

$$|\mathcal{H}x(t)| \leq 1 \cdot A = A.$$

Thus, $|\mathcal{H}x(t)| \leq A < \infty$ (i.e., $\mathcal{H}x$ is bounded). Since the boundedness of x implies the boundedness of $\mathcal{H}x$, the system is BIBO stable.

R **3.46** For each system $\mathcal{H}$ and the functions $\{x_k\}$ given below, determine if each of the x_k is an eigenfunction of $\mathcal{H}$, and if it is, also state the corresponding eigenvalue.
(a) $\mathcal{H}x(t) = \mathcal{D}^2 x(t)$, $x_1(t) = \cos t$, $x_2(t) = \sin t$, and $x_3(t) = 42$, where $\mathcal{D}$ denotes the derivative operator;
(b) $\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau$, $x_1(t) = e^{2t}$, and $x_2(t) = e^t u(-t)$;
(c) $\mathcal{H}x(t) = t^2\mathcal{D}^2 x(t) + t\mathcal{D}x(t)$ and $x_1(t) = t^k$, where k is an integer constant such that $k \geq 2$, and $\mathcal{D}$ denotes the derivative operator; and
(d) $\mathcal{H}x(t) = u(t)x(t)$, $x_1(t) = 0$, $x_2(t) = 1$, $x_3(t) = u(t+1)$, and $x_4(t) = u(t-1)$.

Short Answer. (a) x_1 is an eigenfunction with eigenvalue -1, x_2 is an eigenfunction with eigenvalue -1, x_3 is an eigenfunction with eigenvalue 0; (b) x_1 is an eigenfunction with eigenvalue $\frac{1}{2}$, x_2 is not an eigenfunction; (c) x_1 is an eigenfunction with eigenvalue k^2; (d) x_1 is an eigenfunction with eigenvalue 0; x_2 is not an eigenfunction; x_3 is not an eigenfunction; x_4 is an eigenfunction with eigenvalue 1

R **Answer (b).**

We have

$$\mathcal{H}x_1(t) = \int_{-\infty}^{t} x_1(\tau)d\tau = \tfrac{1}{2}e^{2t} = \tfrac{1}{2}x_1(t) \quad \text{and}$$
$$\mathcal{H}x_2(t) = \int_{-\infty}^{t} x_2(\tau)d\tau = \begin{cases} e^t & t < 0 \\ 1 & \text{otherwise.} \end{cases}$$

Therefore, x_1 is an eigenfunction with eigenvalue $\frac{1}{2}$ and x_2 is not an eigenfunction.