**Example 3.2.** Let  $x_1(t) = \sin(\pi t)$  and  $x_2(t) = \sin t$ . Determine whether the function  $y = x_1 + x_2$  is periodic.

Solution. Denote the fundamental periods of  $x_1$  and  $x_2$  as  $T_1$  and  $T_2$ , respectively. We then have

$$T_1 = \frac{2\pi}{\pi} = 2$$
 and  $T_2 = \frac{2\pi}{1} = 2\pi$ .

Here, we used the fact that the fundamental period of  $\sin(\alpha t)$  is  $\frac{2\pi}{|\alpha|}$ . Thus, we have

$$\frac{T_1}{T_2} = \frac{2}{2\pi} = \frac{1}{\pi}.$$

Since  $\pi$  is an irrational number,  $\frac{T_1}{T_2}$  is not rational. Therefore, y is not periodic.

**Example 3.4.** Let  $x_1(t) = \cos(6\pi t)$  and  $x_2(t) = \sin(30\pi t)$ . Determine if the function  $y = x_1 + x_2$  is periodic, and if it is, find its fundamental period.

Solution. Let  $T_1$  and  $T_2$  denote the fundamental periods of  $x_1$  and  $x_2$ , respectively. We have

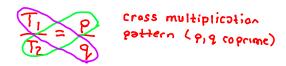
$$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$$
 and  $T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}$ .

Thus, we have

$$\frac{T_1}{T_2} = (\frac{1}{3})/(\frac{1}{15}) = \frac{15}{3} = \frac{5}{1}$$
 5 and 1 are coprime

Since  $\frac{T_1}{T_2}$  is a rational number, y is periodic. Let T denote the fundamental period of y. Since 5 and 1 are coprime, we have

$$T = 1T_1 = 5T_2 = \frac{1}{3}$$
.



**Example 3.8** (Sifting property example). Evaluate the integral

$$\int_{-\infty}^{\infty} [\sin t] \delta(t - \pi/4) dt.$$

Solution. Using the sifting property of the unit impulse function, we have

in this example,  $x(t) = \sin t$  and  $t_0 = \frac{\pi}{4}$ 

Example 3.9 (Sifting property example). Evaluate the integral does not have form of Sifting property due to "4" 
$$\int_{-\infty}^{\infty} [\sin(2\pi t)] \delta(4t-1) dt.$$

$$\int_{-\infty}^{\infty} [\sin(2\pi t)] \delta(4t-1) dt$$

Solution. First, we observe that the integral to be evaluated does not quite have the same form as (3.24). So, we need to perform a change of variable. Let  $\tau = 4t$  so that  $t = \tau/4$  and  $dt = d\tau/4$ . Performing the change of variable, we obtain

$$\int_{-\infty}^{\infty} [\sin(2\pi t)] \delta(4t-1) dt = \int_{-\infty}^{\infty} \frac{1}{4} [\sin(2\pi \tau/4)] \delta(\tau-1) d\tau$$

$$= \int_{-\infty}^{\infty} \left[ \frac{1}{4} \sin(\pi \tau/2) \right] \delta(\tau-1) d\tau.$$
Now the integral has the desired form, and we can use the sifting property of the unit-impulse function to write

$$\int_{-\infty}^{\infty} [\sin(2\pi t)] \delta(4t-1) dt = \left[\frac{1}{4}\sin(\pi\tau/2)\right]_{\tau=1}$$

$$= \frac{1}{4}\sin(\pi/2)$$

$$= \frac{1}{4}.$$
In this example,  $\chi(T) = \frac{1}{4}\sin(\frac{\pi}{2}T)$  and  $t_0 = \frac{1}{4}\sin(\frac{\pi}{2}T)$