Chapter 8 – Friction

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Characteristics of Dry Friction

Friction is defined as a force of resistance acting on a body which prevents or resists the slipping of a body relative to a second body.

Experiments show that frictional forces act tangent (parallel) to the contacting surface in a direction opposing the relative motion or tendency for motion.

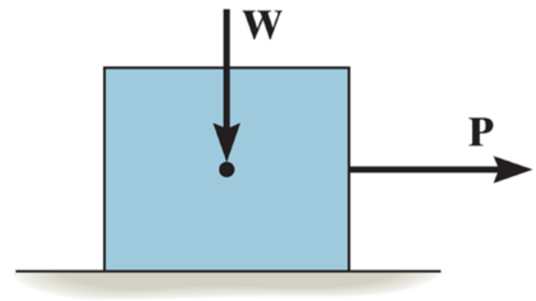
Dry Coulomb friction is a force created between two dry, rigid surfaces that acts against (impending) motion.

Friction plays an important role in many machines: Wedges, power screws, flat belts, etc.

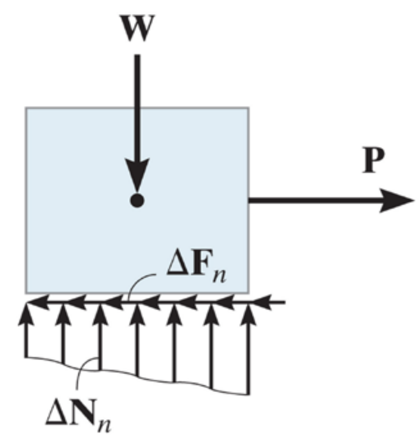
Characteristics of Dry Friction

Theory of Dry Friction

Let a rigid body of weight W be pulled by a force P horizontally.

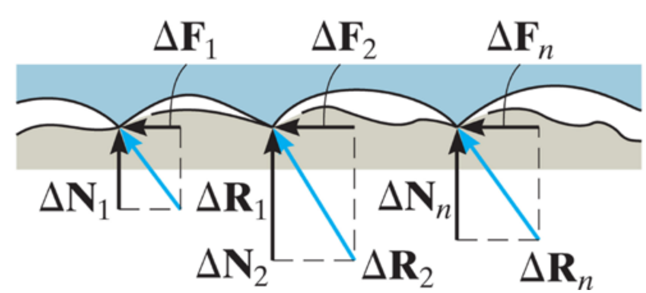


The rough surface exerts an uneven distribution of both normal ΔN_n and frictional force ΔF_n.



To be in equilibrium, the normal forces must balance the weight, and the frictional forces prevent applied force P from moving the block.

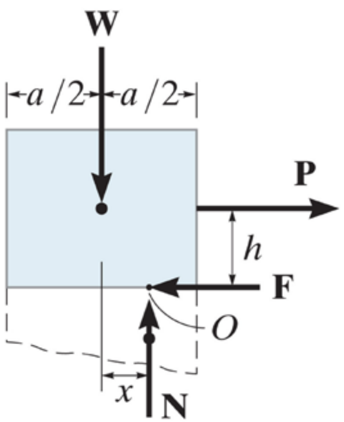
Microscopic irregularities between the two surfaces develop these reactive forces.



Characteristics of Dry Friction

The effect of the distributed normal and frictional forces yield the resultants **N** and **F**, respectively. Therefore, in order to be in equilibrium

$$\mathbf{W} = \mathbf{N} \qquad \text{and} \qquad \mathbf{F} = \mathbf{P}$$



Resultant normal and frictional forces

Note that the normal force acts at a distance x relative to the line of action of **W**.

This separation is necessary in order to balance the tipping effect caused by the separation between **P** and **F**. The moment equilibrium of the force couples requires

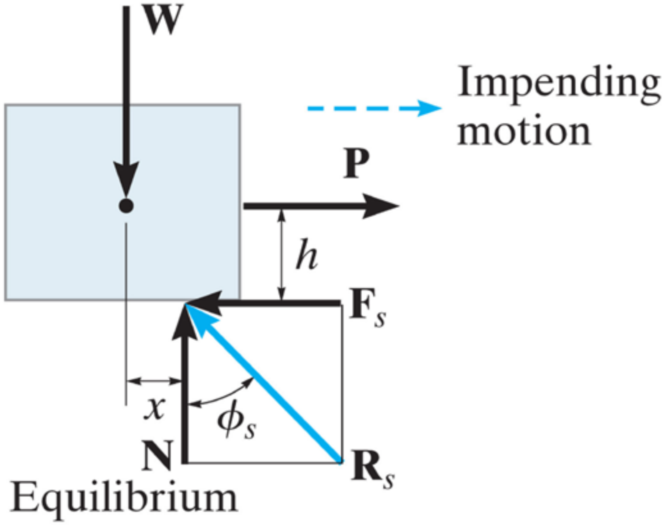
$$Wx = Ph$$

Characteristics of Dry Friction

Impending Motion

There is a limit in which an arbitrary force **P** cannot be balanced by **F**, and consequently the block would slip.

It has been shown experimentally that the *static frictional force* is proportional to the resultant normal force.



$$\boxed{F \le \mu_s N}$$

$F \equiv$ frictional force (N)

$N \equiv$ Normal contact force (N)

$\mu_s \equiv$ Coefficient of static friction.

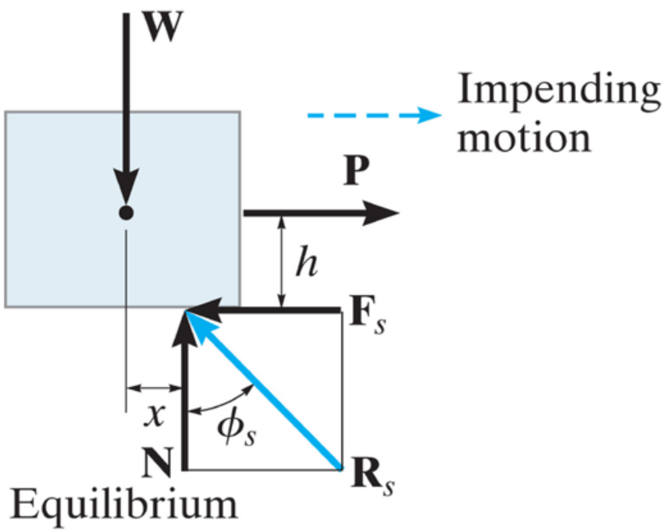
Dry Friction – *Coulomb friction*

Characteristics of Dry Friction

When the *limiting static friction* develops, we know the line of action of the overall reaction force in the contact plane.

The limiting static frictional force exists when the block is on the verge of sliding/translating.

Right up until this point, the force F acts against the impending motion and balances the applied force P.



$$\boxed{F_S = \mu_S N}$$

$F_S \equiv$ limiting frictional force (N)

$$\mu_S = \frac{F_S}{N} = \tan(\phi_S)$$

$$\therefore \phi_S = \tan^{-1}(\mu_S)$$

angle of static friction

Characteristics of Dry Friction

An engineer relies on test data to determine coefficients friction for combinations of surface types. These coefficients are found experimentally.

Table 8–1 Typical Values for μ_s	
Contact Materials	Coefficient of Static Friction (μ_s)
Metal on ice	0.03–0.05
Wood on wood	0.30–0.70
Leather on wood	0.20–0.50
Leather on metal	0.30–0.60
Copper on copper	0.74–1.21

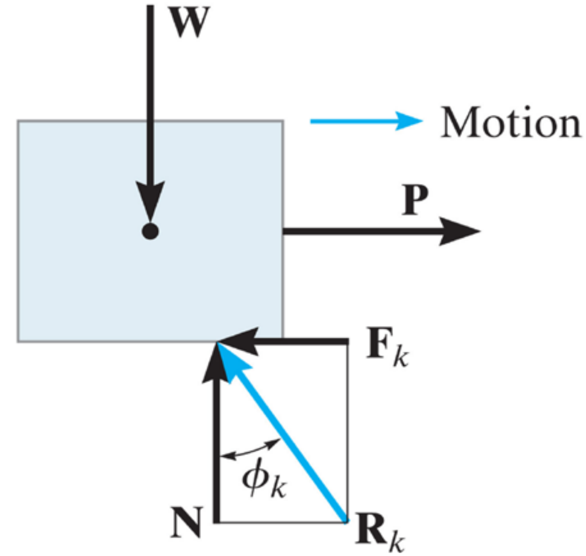
Characteristics of Dry Friction

Motion

There is a limit in which an arbitrary force **P** cannot be balanced by **F**, and consequently the block would slip.

The frictional force will drop to a smaller value called *kinetic frictional force*.

Experiments show that the kinetic friction force is proportional to the magnitude of the normal force, by a factor μ_k the coefficient of kinetic friction, which is about 25% less then μ_s.



$$\boxed{F_K = N\mu_K}$$

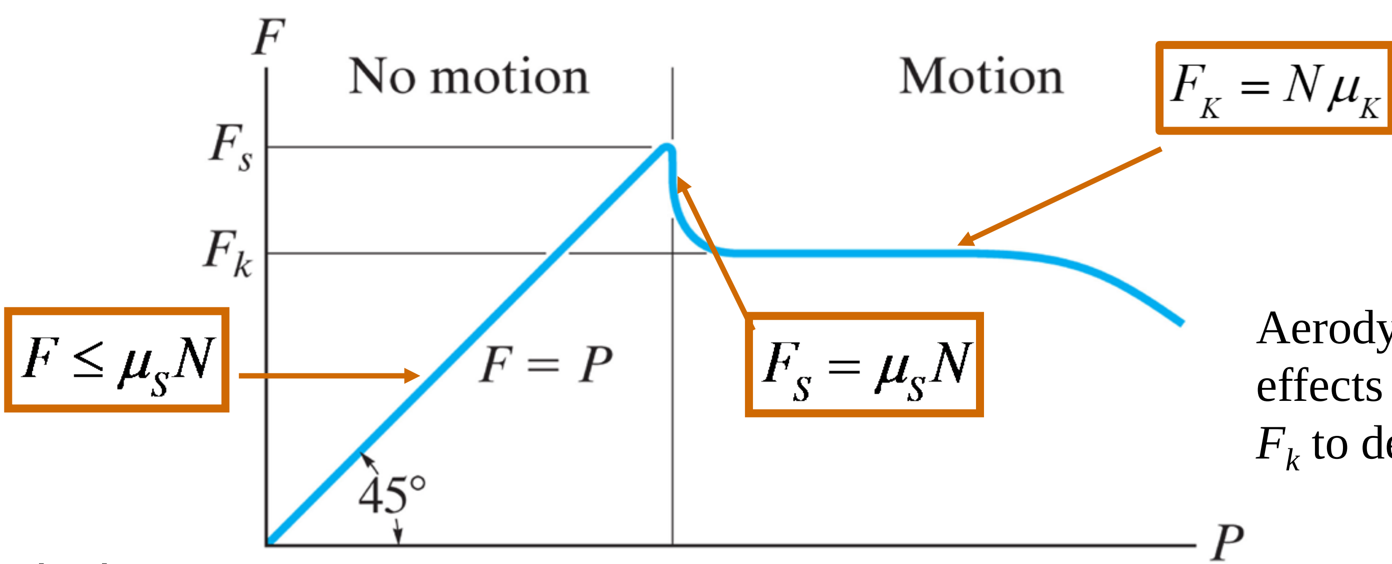
$$\phi_K = \tan^{-1}\left(\frac{F_K}{N}\right) = \tan^{-1}(\mu_K)$$

$$\phi_K < \phi_S$$

Characteristics of Dry Friction

The friction force *may be less* than the maximum friction force $F \leq \mu_s N$, just because the object is not moving, do not assume the friction force is at its maximum.

- F is the static frictional force (static equilibrium)
- F_s is the limiting static frictional force (static equilibrium)
- F_k is the kinetic frictional force (sliding)



Aerodynamic effects will cause F_k to decrease.

Problems involving Dry Friction

There are three classes of friction problems. Distinction between the three is made by careful observation of the system FBDs.

> Equilibrium problems: Frictional force does not reach the limit F_S. Solve for the actual frictional force F.
>
> Impending motion at all points: The limiting static force, F_S, or the kinetic friction force, F_K, are applied at all contact points.
>
> Impending motion at some points: The limiting static force, F_S, or the kinetic friction force, F_K, are applied at some contact points.

Note that "impending motion" could be one of several types.

A problem involving motion has several possible equilibrium states. We must identify the "correct" one.

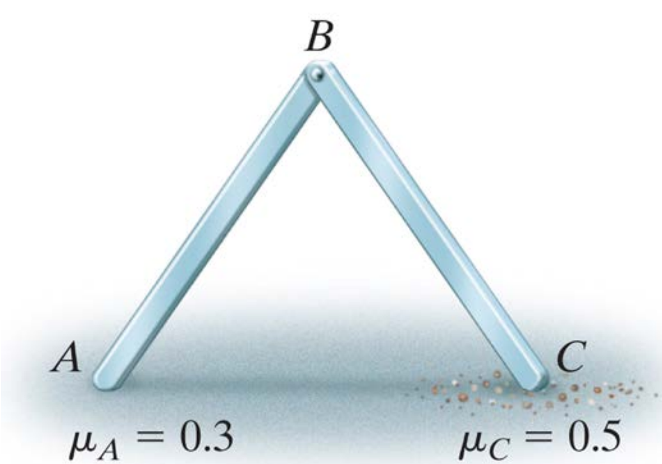
Problems involving Dry Friction

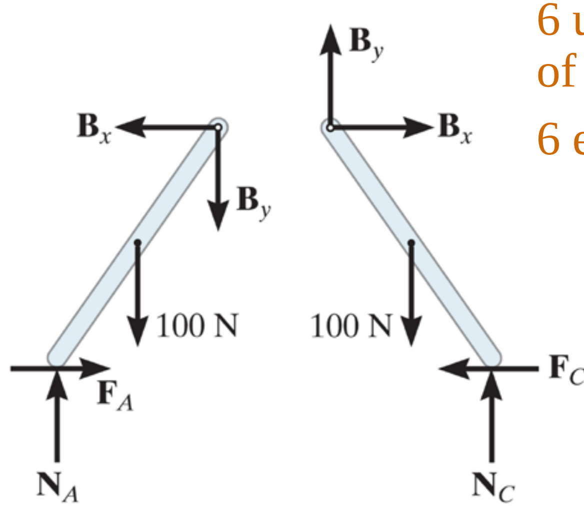
Case 1. Equilibrium problems: Frictional force does not reach the limit F_S. Solve for the actual frictional force F.

The number of unknowns must equal the number of equations

Once the frictional forces are determined, the inequality $F_s \leq \mu_s N$. must be checked, if violated, slipping occurs.

For example, find frictions at A and C, if each bar weighs 100N.





6 unknowns in the original FBD of the frame components.

6 equilibrium equations

- Solve for the required friction forces.
- Check to see that limits on static friction are not exceeded.

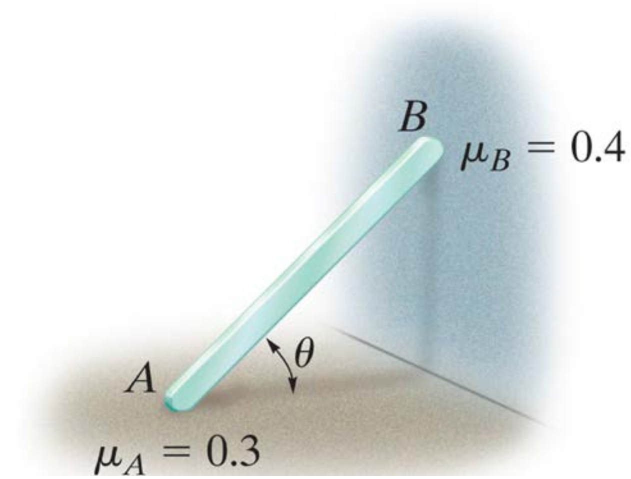
$F_A \leq \mu_A N_A$ and $F_C \leq \mu_C N_C$

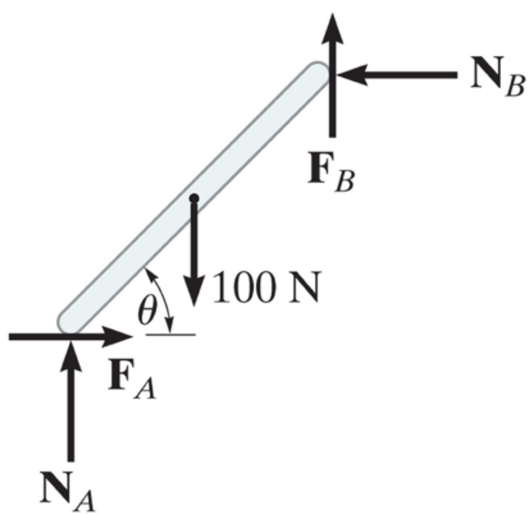
Problems involving Dry Friction

Case 2. Impending motion at all points: The limiting static force, F_S, or the kinetic friction force, F_K, are applied at all contact points.

The number of unknowns must equal the number of equations plus the total number of available frictional forces $F = \mu N$. If motion is impending $F_s = \mu_s N$, if body is slipping $F_k = \mu_k N$.

Example, find smallest angle θ at which the 100N bar can be placed against the wall without slipping.





5 unknowns in the original FBD

3 equilibrium equations

2 statical frictional equations

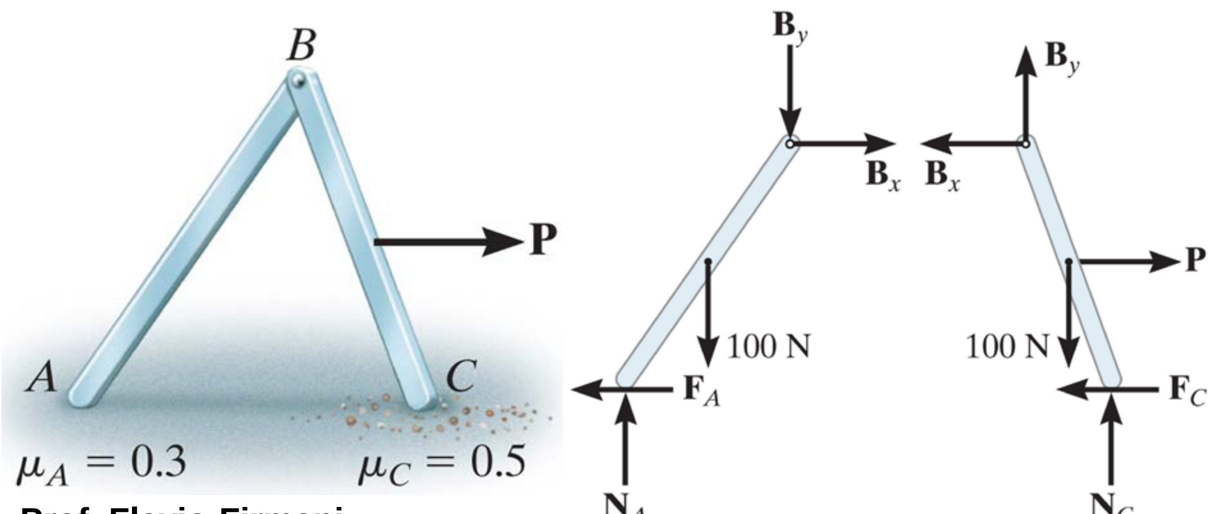
$F_A = \mu_A N_A$ and $F_B = \mu_B N_B$

Problems involving Dry Friction

Case 3. Impending motion at some contact points: The limiting static force, F_S, or the kinetic friction force, F_K, are applied at some contact points.

The number of unknowns will be less than the number of equations plus the total number of available frictional forces or conditional equations for tipping. Several possibilities of motion exist, so it is necessary to determine the type of motion that actually occurs.

Determine force P needed to cause movement in the 100N bars.



7 unknowns (including P)

6 equilibrium equations

1 statical frictional equations

$F_A = \mu_A N_A$ and $F_C \le \mu_C N_C$

or

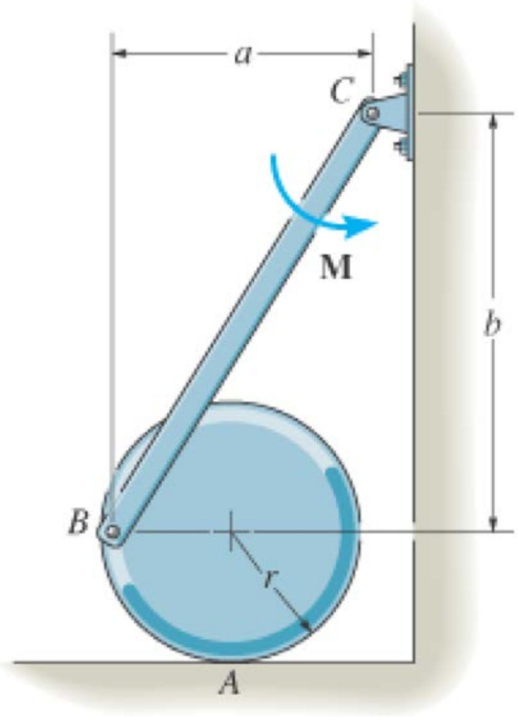
$F_C = \mu_C N_C$ and $F_A \le \mu_A N_A$

Calculate both cases and pick *smaller* P.

Example

The disk of mass m_o rests on the surface for which the coefficient of static friction is μ_A. Determine the friction force at A.

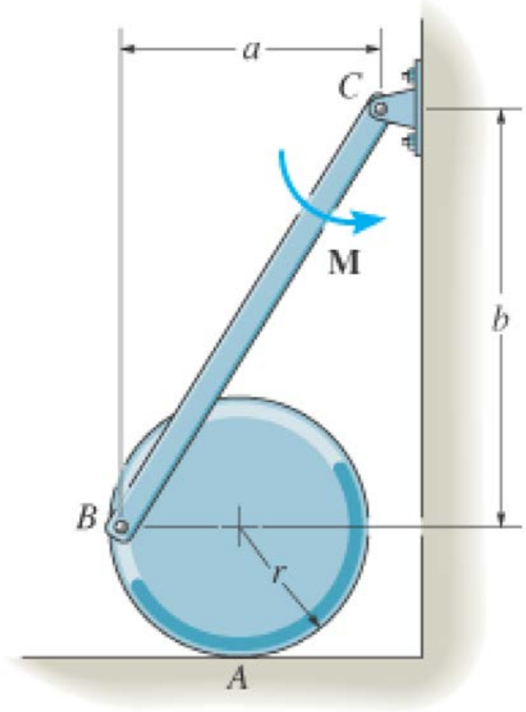
$$M = 50 \text{ Nm}$$
$$m_o = 45 \text{ kg}$$
$$\mu_A = 0.15$$
$$a = 300 \text{ mm}$$
$$b = 400 \text{ mm}$$
$$r = 125 \text{ mm}$$



Example

The disk of mass m_o rests on the surface for which the coefficient of static friction is μ_A. Determine the magnitude of the moment M needed to cause the disc to spin.

$$m_o = 45 \text{ kg}$$
$$\mu_A = 0.15$$
$$a = 300 \text{ mm}$$
$$b = 400 \text{ mm}$$
$$r = 125 \text{ mm}$$

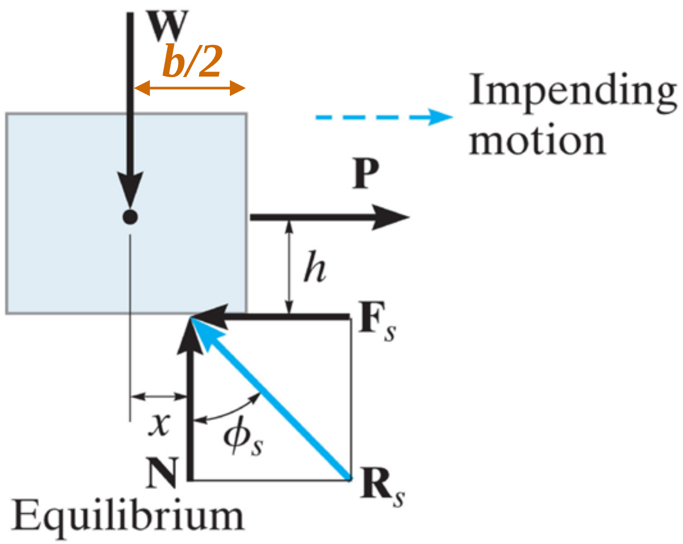


Problems involving Dry Friction

Tipping vs Sliding

There are two predominant "types" of motion that develop in our systems.

- Tipping: the location of the friction force moves to the point $x = b/2$ due to the moment of P about the contact point.
- Sliding: the friction force F reaches the static limit F_S while $x < b/2$



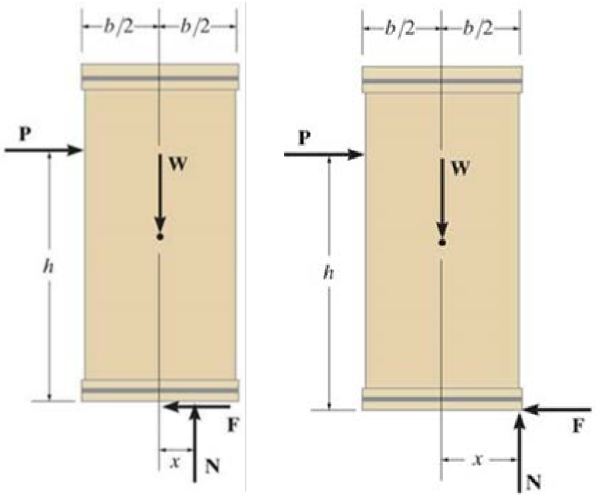
Problems involving Dry Friction

For a given **W** and *h* of the box, how can we determine if the block will slide or tip first?

There four unknowns (**F**, **N**, *x*, and **P**) and only the three equations of equilibrium.

The fourth equations will be an *assumption*, i.e., the limiting static frictional force (slip) or the moment about corner (tipping).

Solve for the unknowns using the four equations and finally check if the *assumption* was correct.



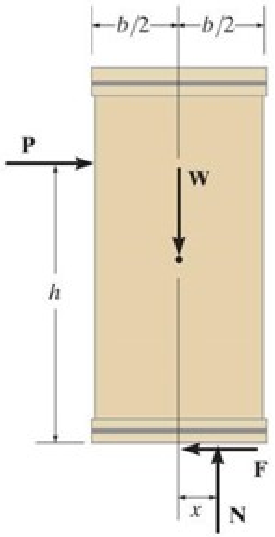
Problems involving Dry Friction

Assume: Slipping occurs

Known: $\mathbf{F} = \mu_s \mathbf{N}$

Solve: x, $\mathbf{P}$, and $\mathbf{N}$

Check if: $0 \leq x \leq b/2$

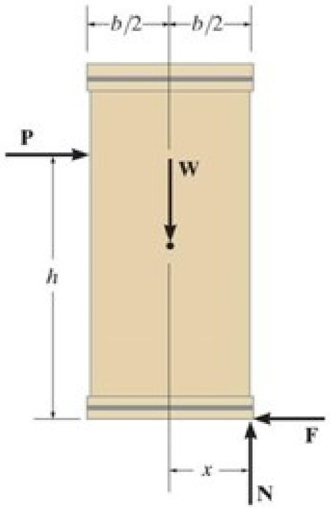


Assume: Tipping occurs

Known: $x = b/2$

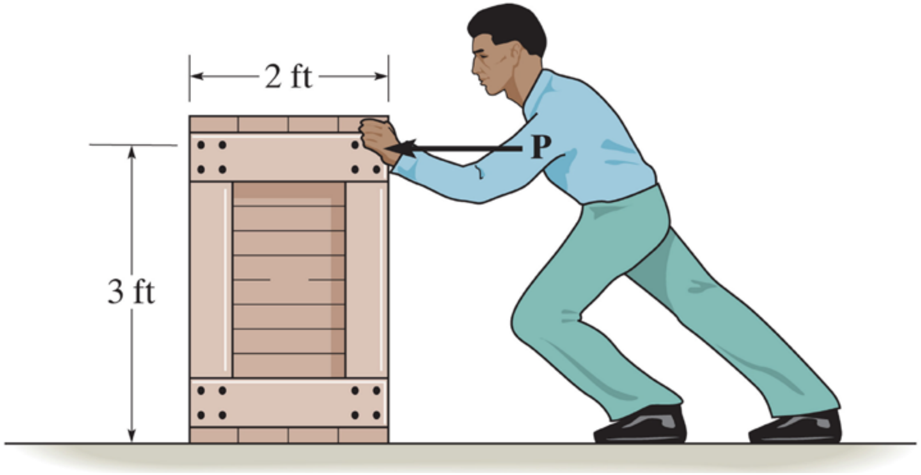
Solve: $\mathbf{P}$, $\mathbf{N}$, and $\mathbf{F}$

Check if : $\mathbf{F} \leq \mu_s \mathbf{N}$



Example

Determine the smallest force *P* that must be applied in order to cause the 150-lb uniform crate to move. The coefficent of static friction between the crate and the floor is $\mu_s = 0.5$.



Chapter 8 – Friction

Contents

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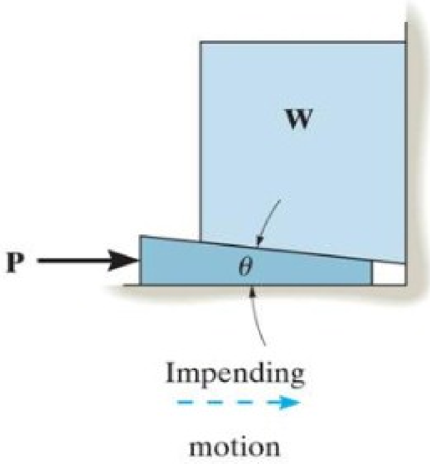
Wedges

A wedge is a simple machine that is used to transform an applied force into much larger forces.

Wedges are used to adjust the elevation or provide stability for heavy objects such as in this large steel pipe.



A wedge is a simple machine in which a small force P is used to lift a large weight W. If P = 0, and there is no motion and only friction holds block, the wedge is referred to as self locking.

Wedges

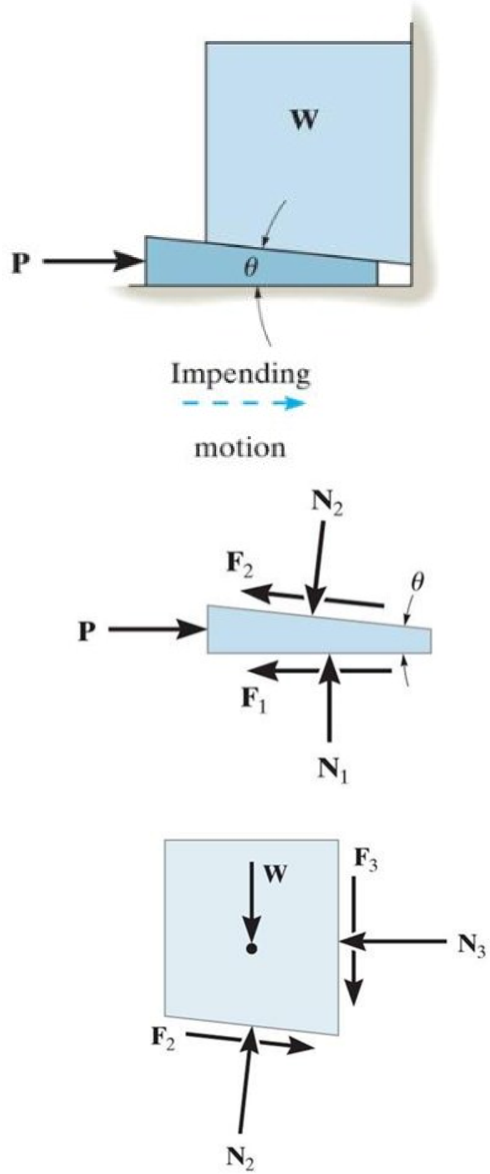
To determine the force required to push the wedge in or out, it is necessary to draw FBDs of the wedge and object on top of it. Note that:

a) the friction forces are always in the direction opposite to the motion, or impending motion, of the wedge;

b) the friction forces are along the contacting surfaces;

c) the normal forces are perpendicular to the contacting surfaces.

d) between the wedge and the object, the forces are equal in magnitude and opposite in direction to those on the wedge;

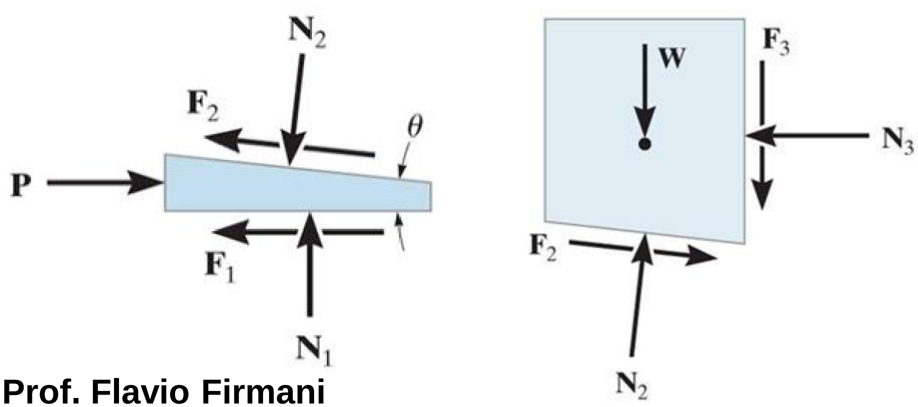


Wedges

The equations of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ are applied to the wedge and object. The equation of equilibrium of moments is generally not required as we are not concerned if the objects are rotating (points of application of normal forces).

We will consider all the points of contact to have impending motion $F = \mu_S N$.

First, analyze the FBD in which the number of unknowns are less than or equal to the number of equations of equilibrium and frictional equations.



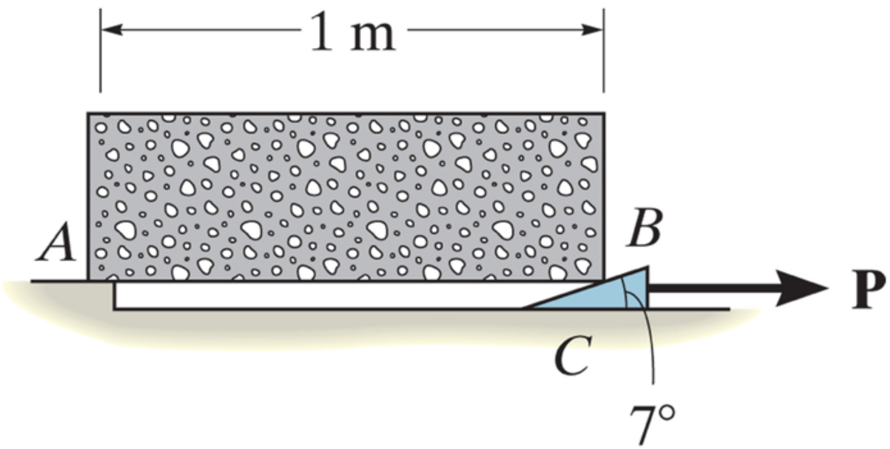
Find P needed to lift W

7 unknowns: P, N_1, N_2, N_3, F_1, F_2, F_3.

7 equations: Two sets of $\Sigma F_x = 0$ and $\Sigma F_y = 0$ equations, one for each FBD, and three equations of impending motion $F_i = \mu_i N_i$, for $i = 1,2,3$.

Example

The uniform stone shown below has a mass of 500 kg and is held in the horizontal position using a wedge at B. If the coefficient of static friction is $\mu_s = 0.3$ at the surfaces of contact, determine the minimum force P needed to remove the wedge. Assume that the stone does not slip at A.

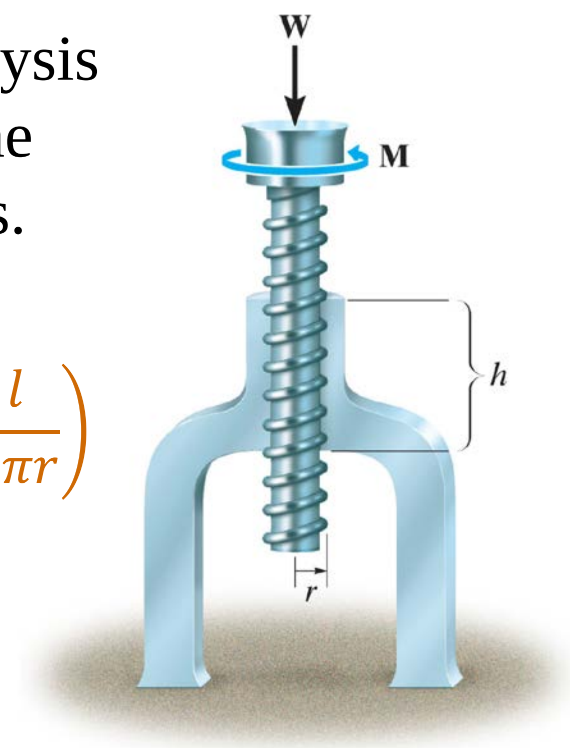


Frictional Forces on Screws

Screws are usually used as fasteners, but in some cases, they can be used to transmit power or motion, e.g., lead screws (linear actuators).

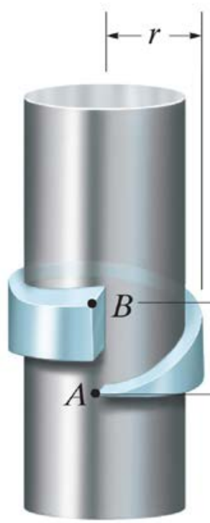
Square threaded screws are commonly used for this purpose.

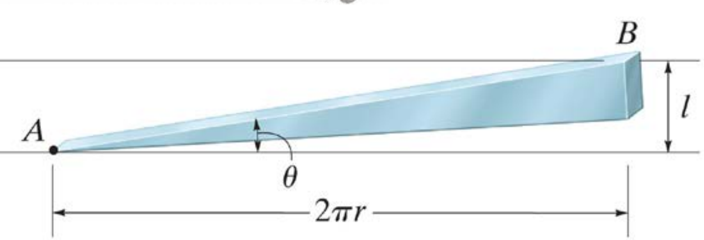
We apply an analogy to an inclined plane analysis to relate the moment applied to the screw to the resultant force the power screw exerts/sustains.



Lead angle

$$\theta = tan^{-1}\left(\frac{l}{2\pi r}\right)$$

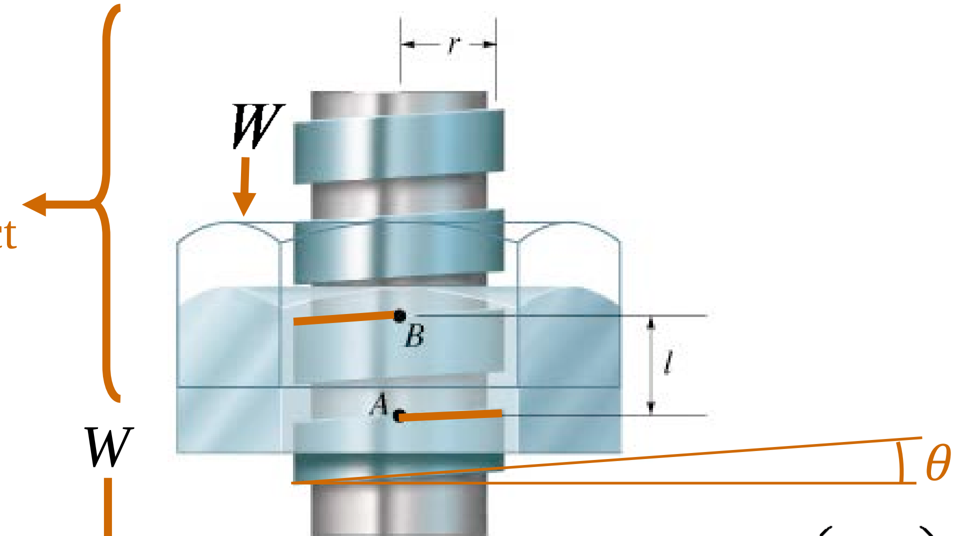




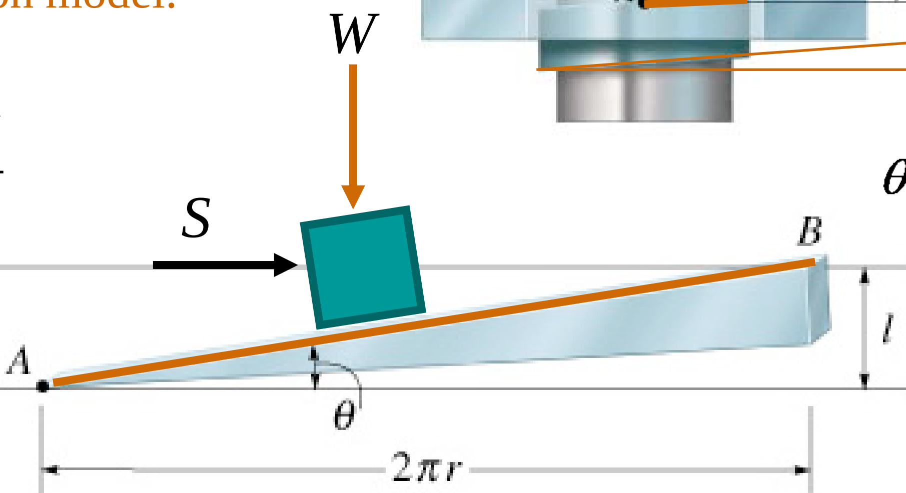
Frictional Forces on Screws

$$M = Sr$$

The nut rests on the screw teeth. The area of contact is not important when using a dry friction model.



$$S = \frac{M}{r}$$



$$\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$$

r = mean screw radius.
l = lead of the screw.

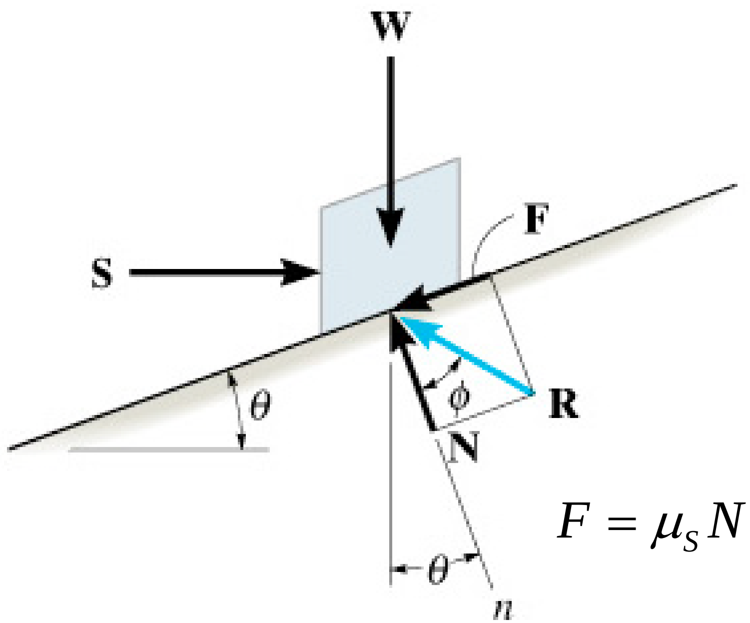
Frictional Forces on Screws

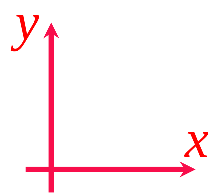
The type of motion that is impending defines the method of analysis.

- Case 1: "Upward" screw motion (fighting the sustained force **W**, *i.e.*, tightening).
- Case 2: "Downward" screw motion (going with the sustained load **W**, *i.e.*, loosening).
  - Smooth teeth: The screw is not self locking. Not much friction develops between screw teeth and the nut, and the applied load **W** can cause rotation (**W** drives the block down the ramp itself).
  - Rough teeth: The screw is self locking. A motive force S must be applied to get the block to slide down the ramp (**W** is insufficient to drive the block down the ramp itself).

Frictional Forces on Screws

Case 1: "Upward" screw motion (tightening).





$$S = \frac{M}{r}$$

$$\sum F_x = 0 = S - R\sin(\theta + \phi)$$

$$\sum F_y = 0 = -W + R\cos(\theta + \phi)$$

$$F = \mu_S N$$

$$\boxed{M = rW\tan(\theta + \phi)}$$

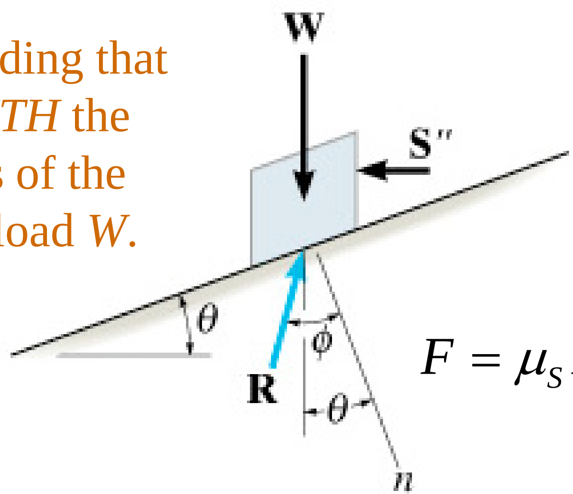
Angle of static friction $\phi = \tan^{-1}(\mu_S)$

Lead angle $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$

Frictional Forces on Screws

Case 2: "Downward" screw motion (loosening) – screw is self-locking.

Understanding that M acts *WITH* the tendencies of the sustained load *W*.



$$F = \mu_S N$$

$$S'' = \frac{M}{r}$$

$$\sum F_x = 0 = -S'' + R\sin(\phi - \theta)$$

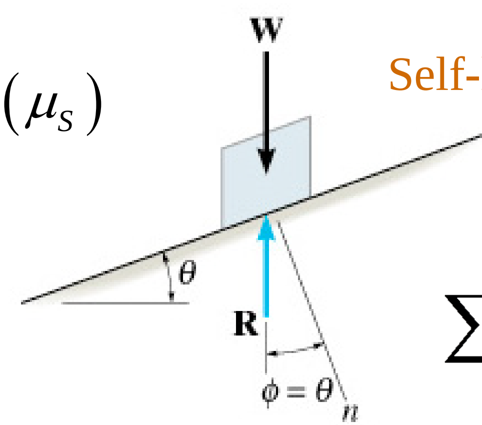
$$\sum F_y = 0 = -W + R\cos(\phi - \theta)$$

$$\boxed{M = rW\tan(\phi - \theta)}$$

Self locking $\phi > \theta$

Angle of static friction $\phi = \tan^{-1}(\mu_S)$

Lead angle $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$



Self-locked screw (limit) $W = R$

$$M = 0$$

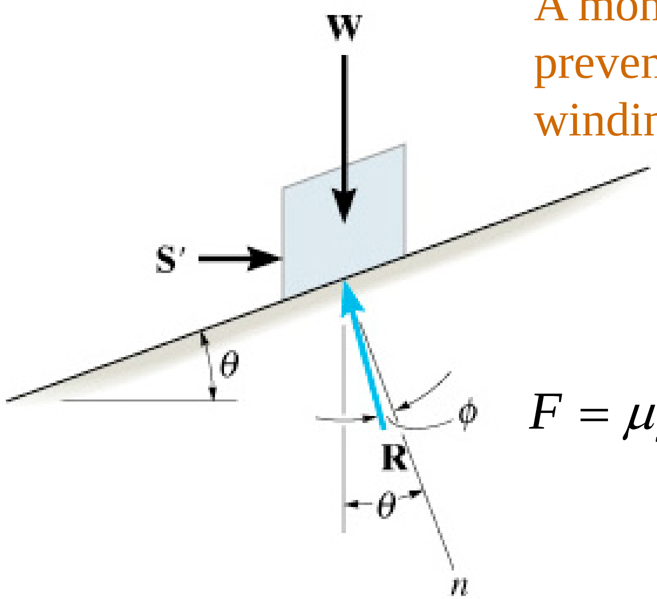
$$\sum F_x = 0 = R\sin(\phi - \theta)$$

$$\sum F_y = 0 = -W + R\cos(\phi - \theta)$$

Frictional Forces on Screws

Case 3: "Downward" screw motion (loosening) – screw is not self-locking.

A moment is needed to prevent screw from winding downward .



$$F = \mu_S N$$

$$S' = \frac{M}{r}$$

$$\sum F_x = 0 = S' - R\sin(\theta - \phi)$$

$$\sum F_y = 0 = -W + R\cos(\theta - \phi)$$

$$\boxed{M = rW\tan(\theta - \phi)}$$

Not Self locking $\theta > \phi$

Angle of static friction $\phi = \tan^{-1}(\mu_S)$

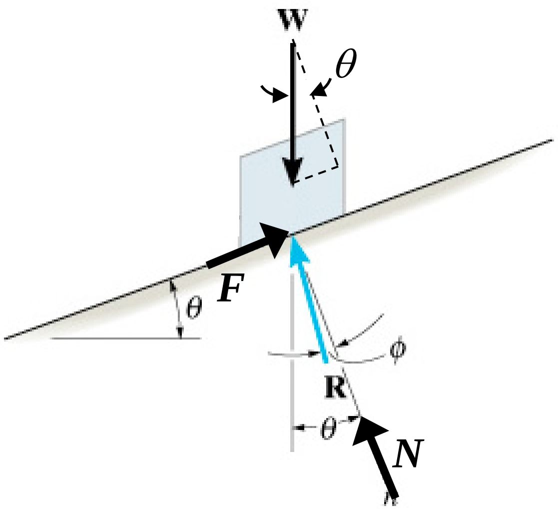
Lead angle $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$

Understanding that M acts *AGAINST* the tendencies of the sustained load *W*.

Frictional Forces on Screws

Self – Locking Screw Conditions

A screw is self-locking if it remains in place under any axial load **W** when the moment M is removed.

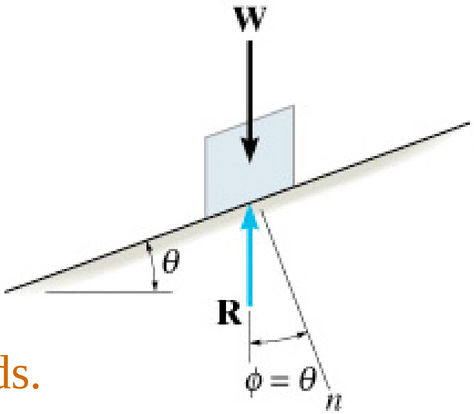


$$W\sin(\theta) < \mu_S N \quad , \quad N = W\cos(\theta)$$

$$\tan(\theta) < \mu_S \Rightarrow \tan(\theta) < \tan(\phi)$$

Condition for Self-locking:

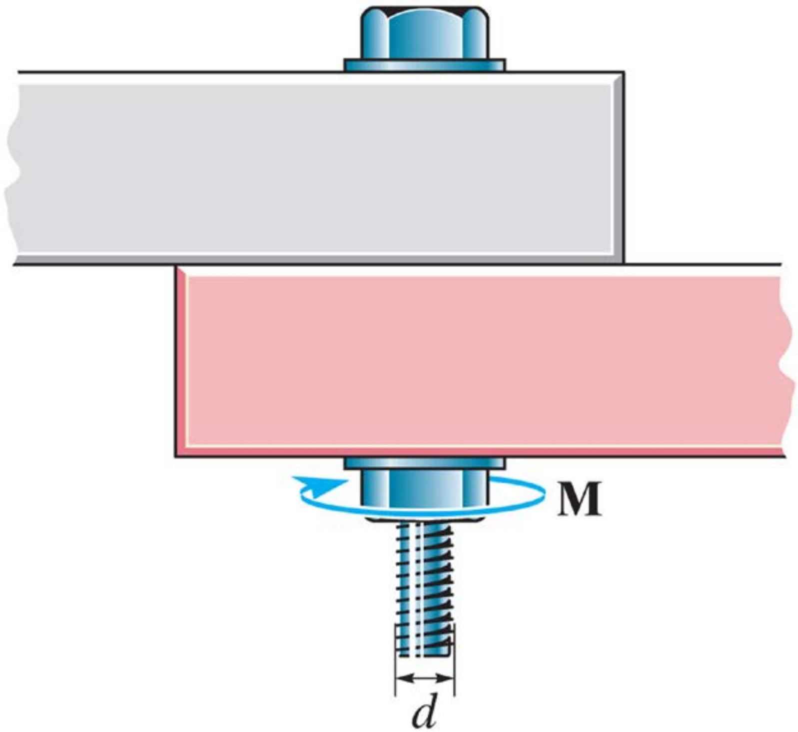
$$\boxed{\tan(\theta) < \mu_S \quad or \quad \theta < \phi}$$



Self – locking screw on the verge of rotating downwards.

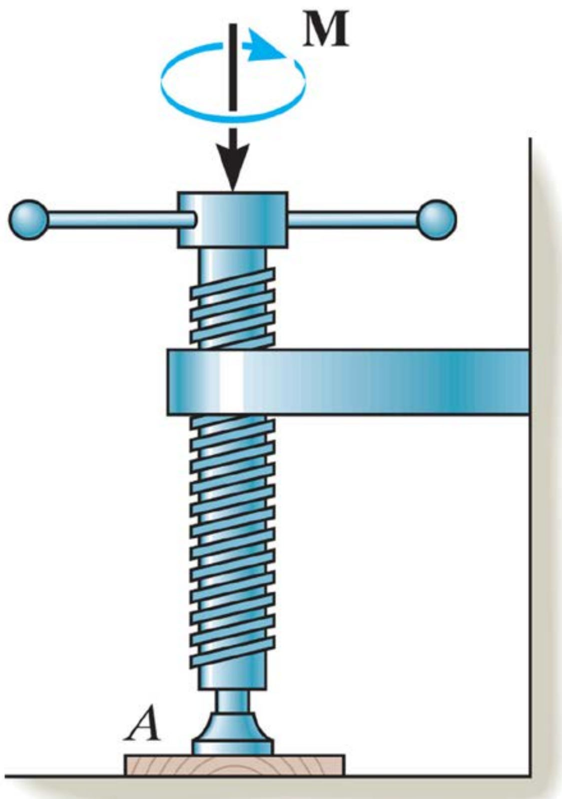
Example

The square-threaded bolt is used to join two plates together. If the bolt has a mean diameter of d = 20 mm and a lead of l = 3 mm, determine the smallest torque **M** required to loosen the bolt if the tension in the bolt is T = 40 kN. The coefficient of static friction between the threads and the bolt is $\mu_s = 0.15$.



Example

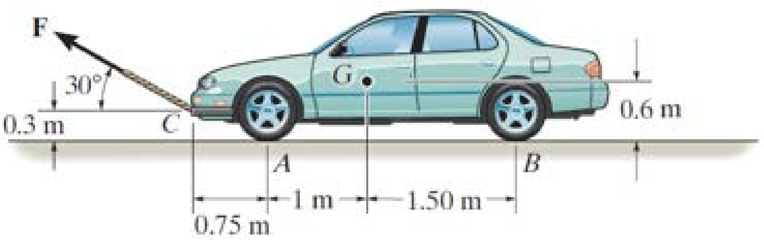
Determine the clamping force on the board A if the screw is tightened with a torque of M = 8 N·m. The square threaded screw has a mean radius of r =10 mm and a lead of l = 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



Sample Problems for Students to Review

Chapter 8

Sample Problem (§ 8.2)



Given: Car weight = 2000 kg and $\mu_s = 0.3$.

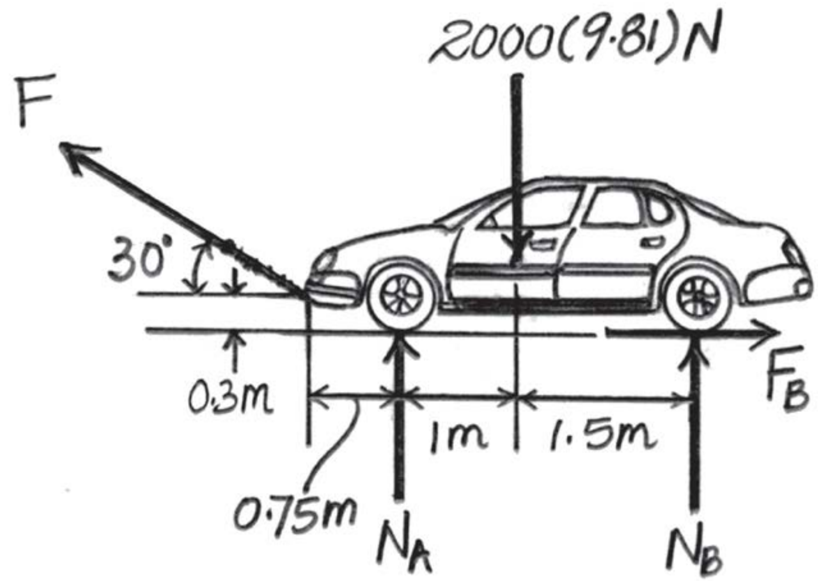
Find: The smallest magnitude of **F** required to move the car if the back brakes are locked and the front wheels are free to roll.

Plan:

Draw a FBD of the car and identify the unknowns.

Apply equations of equilibrium plus friction equation from the rear wheels, to solve for the unknowns.

The front wheels are free to spin and the rear wheels are locked. Although there is friction in the front wheel, this friction has no contribution to the motion of the car, it simply produces a moment that spins the front wheel. Therefore, there are four unknowns: F, N_A, N_B, and F_B.



Equations of Equilibrium:

$$\Sigma\ F_X = F_B - F(\cos 30°) = 0 \qquad (1)$$

$$\Sigma\ F_Y = N_A + N_B + F(\sin 30°) - 19620 = 0 \qquad (2)$$

$$\Sigma\ M_A = F\cos 30°(0.3) - F\sin 30°\,(0.75) + N_B(2.5) - 19620(1) = 0 \qquad (3)$$

Assume that the rear wheels are on the verge of slip. Thus

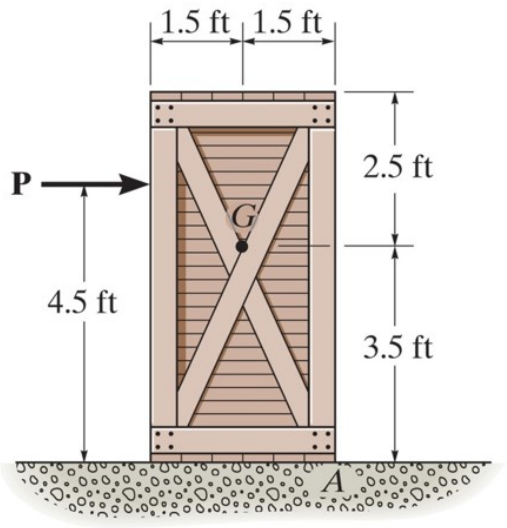
$$F_B = \mu_s N_B = 0.3 N_B \qquad (4)$$

This leads to a system of four equations in four unknowns.

Solving Equations (1) to (4),

$F = 2762$ N, $N_A = 10263$ N, $N_B = 7975$ N, $F_B = 2393$ N.

Sample Problem (§ 8.2)



Given: Crate weight = 250 lb and $\mu_s = 0.4$.

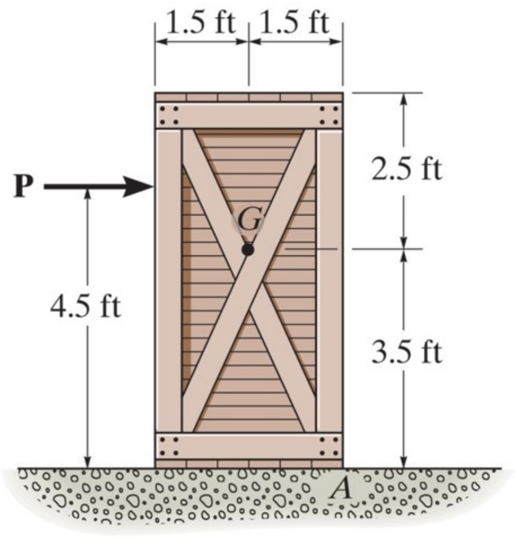
Find: The maximum force **P** that can be applied without causing movement of the crate.

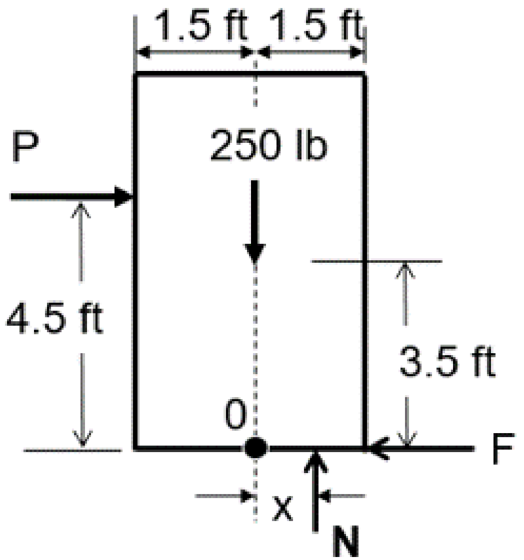
Plan:

Draw a FBD of the box and identify the unknowns.

Assume that the crate slips (friction assumption).

Apply equations of equilibrium plus friction equation (assumption), to solve for the unknowns. Check if assumption is correct.





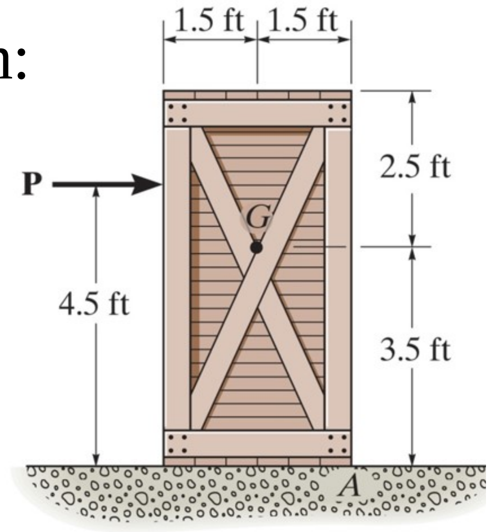
FBD of the crate

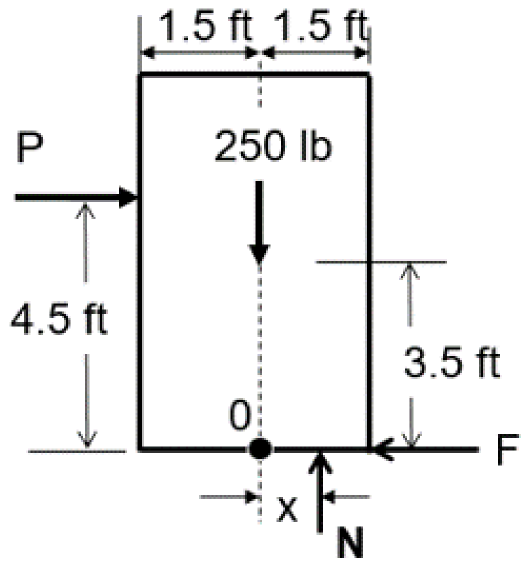
There are four unknowns: **P**, **N**, **F** and *x*.

First, let us assume the crate slips. Then the friction equation is $\mathbf{F} = \mu_s \mathbf{N} = 0.4\mathbf{N}$

Apply equations of equilibrium:

$$\Sigma F_X = P - 0.4\,N = 0$$
$$\Sigma F_Y = N - 250 = 0$$





FBD of the crate

Solving these two equations:

N = 250 lb and P = 100 lb

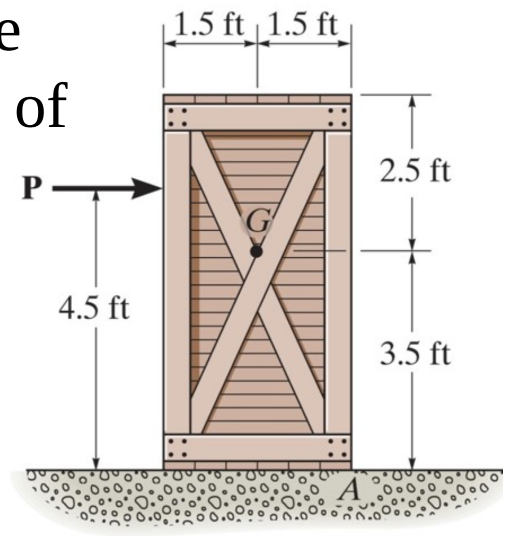
and

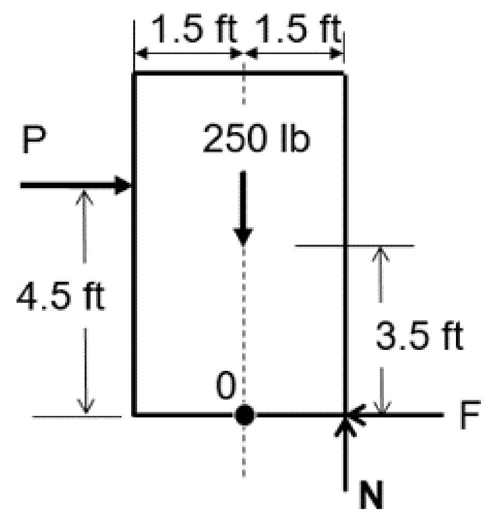
$$\Sigma M_O = -P(4.5) + N(x) = -100\,(4.5) + 250\,(x) = 0$$

Therefore, x = 1.8 ft

Given that **N** must act outside the base of the crate, $x > b/2 = 1.5$, there is no slipping, the crate is tips over. Incorrect assumption!

Since tipping occurs, this is the correct FBD, **N** acts at the end of the base of the crate.





FBD of the crate

$$\Sigma F_X = P - F = 0$$

$$\Sigma F_Y = N - 250 = 0$$

These two equations give:

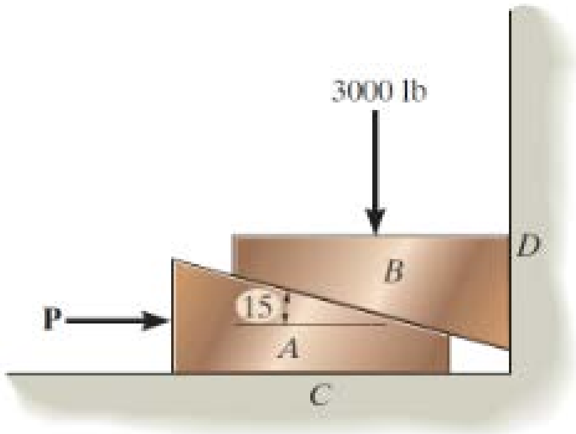
$$N = 250 \text{ lb}$$

$$\Sigma M_O = -P\,(4.5) + 250\,(1.5) = 0$$

Therefore, $P = 83.3 \text{ lb}$

Final check $P = 83.3 \text{ lb} < \mu_s N = 100 \text{ lb}$

Sample Problem (§ 8.3)



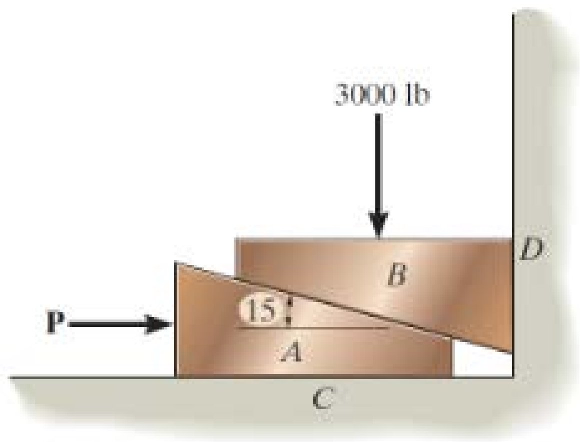
Given: The 3000-lb load is applied to wedge B. Assume the coefficients of static friction be $\mu_{ACs} = 0.3, \mu_{BDs} = 0.3$, and $\mu_{ABs} = 0.4$.

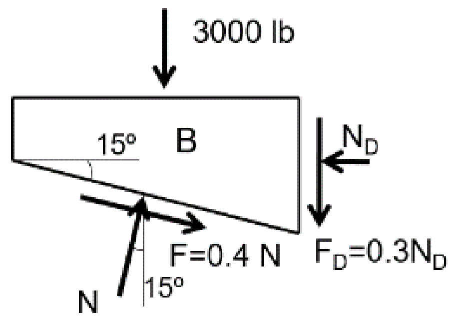
Find: The smallest force P needed to lift the 3000-lb load.

Plan:

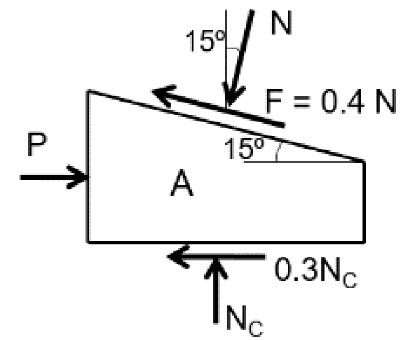
Draw FBDs of wedge A and wedge B.

Apply the equation of equilibrium to wedge B, and then to wedge A.





FBD of Wedge B



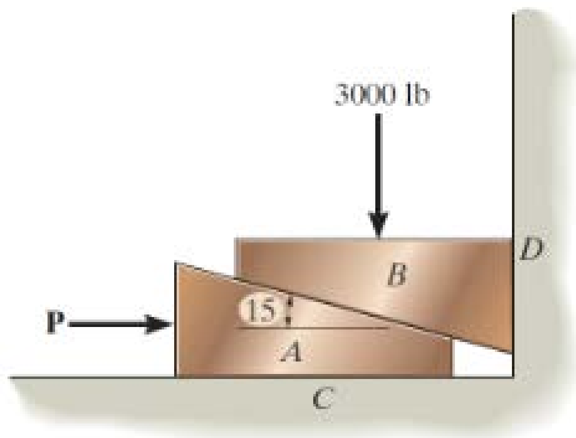
FBD of Wedge A

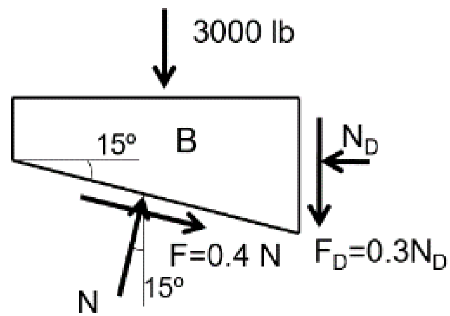
Apply the equations of equilibrium to wedge B. There are four unknowns and four equations. After subbing $F = \mu_S N$, there are two equations and two unknowns, N and N_D.

$$\Sigma F_X = N \sin 15° + 0.4\, N \cos 15° - N_D = 0$$
$$\Sigma F_Y = N \cos 15° - 0.4\, N \sin 15° - 0.3\, N_D - 3000 = 0$$

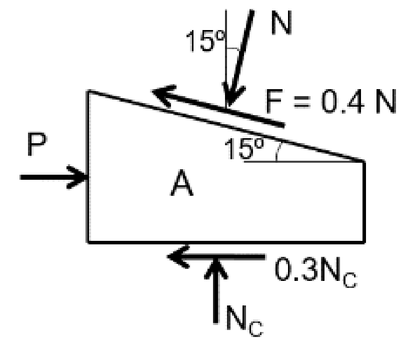
Solving these two equations yields

$$N = 4485 \text{ lb}, \qquad N_D = 2894 \text{ lb}$$





FBD of Wedge B



FBD of Wedge A

Apply the equations of equilibrium to wedge A. After subbing $F = \mu_S N$, there are two equations and two unknowns, P and N_C.

$$\Sigma F_Y = N_C + 0.4\,(4485)\sin 15° - 4485\cos 15° = 0$$

$$N_C = 3868 \text{ lb}$$

$$\Sigma F_X = P - 0.3\,(3868) - 4485\sin 15° - 1794\cos 15° = 0$$

$$P = 4054 \text{ lb}$$