

Exercise 6.3

L Answer (b).

We are asked to find the Fourier transform X of

$$x(t) = e^{-j5t} \underbrace{u(t+2)}_{v(t)}.$$

Defining the function

$$v(t) = u(t+2), \quad \textcircled{1}$$

we have that

$$x(t) = e^{-j5t} v(t). \quad \textcircled{2}$$

Let V and U denote the Fourier transforms of v and u , respectively. Taking the Fourier transforms of the preceding equations for x and v and well as the Fourier transform of u , we obtain

$$\begin{aligned} \textcircled{3} \quad X(\omega) &= V(\omega+5), && \leftarrow \text{FT of } \textcircled{2} \text{ (modulation)} \\ \textcircled{4} \quad V(\omega) &= e^{j2\omega} U(\omega), && \leftarrow \text{FT of } \textcircled{1} \text{ (time shift)} \text{ and} \\ \textcircled{5} \quad U(\omega) &= \pi\delta(\omega) + \frac{1}{j\omega}. && \leftarrow \text{FT of } u \text{ from table} \end{aligned}$$

Combining the above results, we have

$$\begin{aligned} X(\omega) &= V(\omega+5) && \leftarrow \textcircled{3} \\ &= e^{j2(\omega+5)} U(\omega+5) && \leftarrow \text{substitute } \textcircled{4} \text{ for } V \\ &= e^{j2(\omega+5)} \left[\pi\delta(\omega+5) + \frac{1}{j(\omega+5)} \right] && \leftarrow \text{substitute } \textcircled{5} \text{ for } U \\ &= \pi e^{j2(\omega+5)} \delta(\omega+5) + \frac{e^{j2(\omega+5)}}{j(\omega+5)} && \leftarrow \text{multiply} \\ &= \pi\delta(\omega+5) + \frac{e^{j2(\omega+5)}}{j(\omega+5)}. && \leftarrow \text{equivalence property} \end{aligned}$$