Example 7.9 (Linearity property of the Laplace transform and pole-zero cancellation)**.** Find the Laplace transform X of the function

$$x = x_1 - x_2,$$

$$\left\{\begin{array}{l} x_1(t) = e^{-t}u(t) \\ x_2(t) = e^{-t}u(t) - e^{-2t}u(t) \end{array}\right.$$

where x_1 and x_2 are as defined in the previous example.

Solution. From the previous example, we know that

$$\left.\begin{array}{ll} (1) & X_1(s) = \frac{1}{s+1} \quad \text{for } \operatorname{Re}(s) > -1 \quad \text{and} \\ (2) & X_2(s) = \frac{1}{(s+1)(s+2)} \quad \text{for } \operatorname{Re}(s) > -1. \end{array}\right\} \text{from LT table}$$

From the definition of X, we have

$$\begin{aligned}
X(s) &= \mathcal{L}\{x_1 - x_2\}(s) && \text{linearity} \\
&= X_1(s) - X_2(s) && \text{substituting expressions for } X_1 \text{ and } X_2 \text{ in (1) and (2)} \\
&= \frac{1}{s+1} - \frac{1}{(s+1)(s+2)} && \text{common denominator} \\
&= \frac{s+2-1}{(s+1)(s+2)} && \text{simplify numerator} \\
\text{pole-zero cancellation} \quad &= \frac{s+1}{(s+1)(s+2)} && \text{cancel common factor of } s+1 \\
&= \frac{1}{s+2}.
\end{aligned}$$

Now, we must determine the ROC of X. We know that the ROC of X must at least contain the intersection of the ROCs of X_1 and X_2. Therefore, the ROC must contain $\operatorname{Re}(s) > -1$. Since X is rational, we also know that the ROC must be bounded by poles or extend to infinity. Since X has only one pole and this pole is at -2, the ROC must also include $-2 < \operatorname{Re}(s) < -1$. Therefore, the ROC of X is $\operatorname{Re}(s) > -2$. In effect, the pole at -1 has been cancelled by a zero at the same location. As a result, the ROC of X is larger than the intersection of the ROCs of X_1 and X_2. The various ROCs are illustrated in Figure 7.10. So, in conclusion, we have

$$X(s) = \frac{1}{s+2} \quad \text{for } \operatorname{Re}(s) > -2. \qquad \blacksquare$$

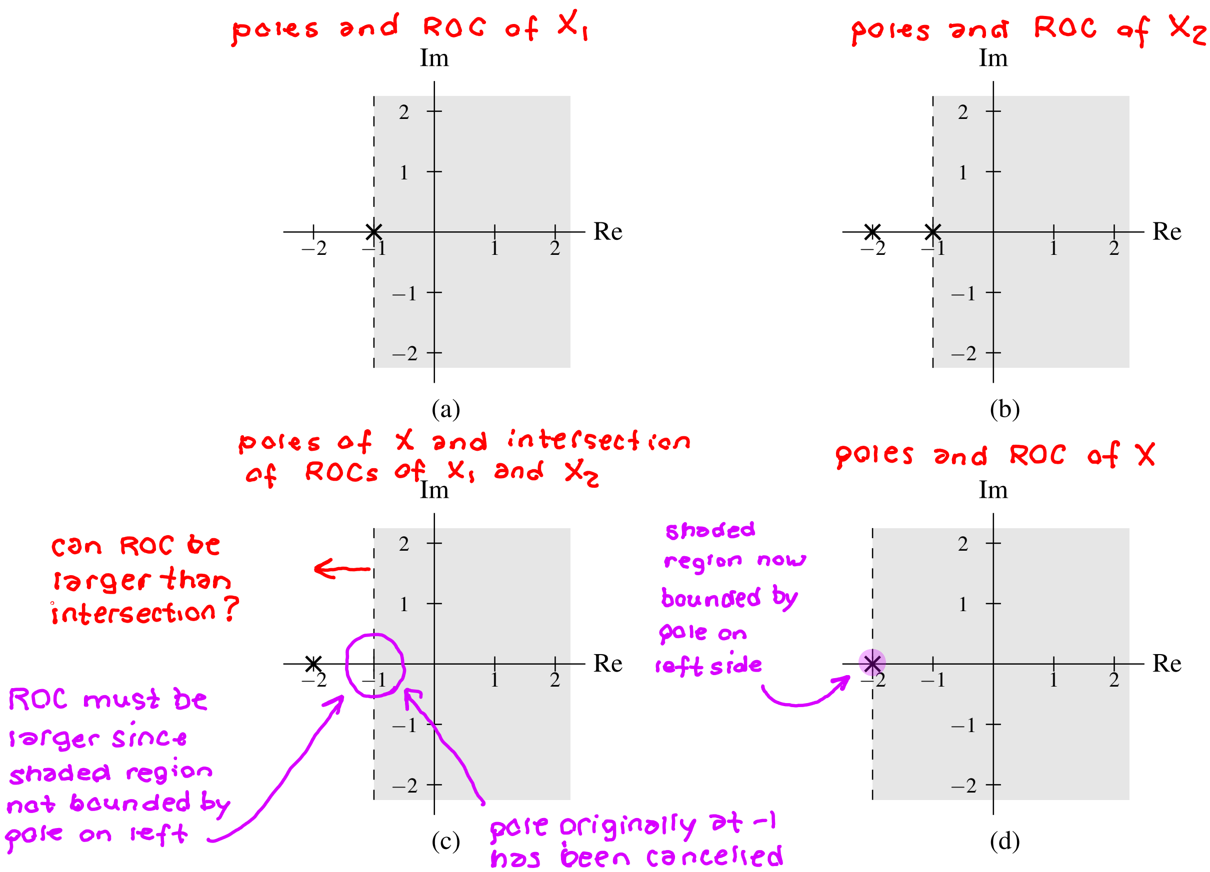


Figure 7.10: ROCs for the linearity example. The (a) ROC of X_1, (b) ROC of X_2, (c) ROC associated with the intersection of the ROCs of X_1 and X_2, and (d) ROC of X.

Example 7.10 (Time-domain shifting property)**.** Find the Laplace transform X of

table of LT pairs

$$x(t) = u(t-1).$$

Solution. From Table 7.2, we know that

$$u(t) \overset{\text{LT}}{\longleftrightarrow} 1/s \text{ for } \operatorname{Re}(s) > 0.$$

from LT table

Using the time-domain shifting property, we can deduce

multiply by e^{-s}

ROC unchanged

shift by 1

$$x(t) = u(t-1) \overset{\text{LT}}{\longleftrightarrow} X(s) = e^{-s}\left(\tfrac{1}{s}\right) \text{ for } \operatorname{Re}(s) > 0.$$

Therefore, we have

$$X(s) = \frac{e^{-s}}{s} \text{ for } \operatorname{Re}(s) > 0. \qquad \blacksquare$$

Example 7.11 (Laplace-domain shifting property)**.** Using only the properties of the Laplace transform and the transform pair

$$e^{-|t|} \stackrel{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \operatorname{Re}(s) < 1,$$

find the Laplace transform X of

$$x(t) = e^{5t}e^{-|t|}.$$

Solution. We are given

$$e^{-|t|} \stackrel{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \operatorname{Re}(s) < 1.$$

shift ROC by 5

Using the Laplace-domain shifting property, we can deduce

shift s by 5

multiply by e^{5t}

$$x(t) = e^{5t}e^{-|t|} \stackrel{\text{LT}}{\longleftrightarrow} X(s) = \frac{2}{1-(s-5)^2} \quad \text{for } \underbrace{-1+5}_{4} < \operatorname{Re}(s) < \underbrace{1+5}_{6},$$

Thus, we have

$$X(s) = \frac{2}{1-(s-5)^2} \quad \text{for } 4 < \operatorname{Re}(s) < 6.$$

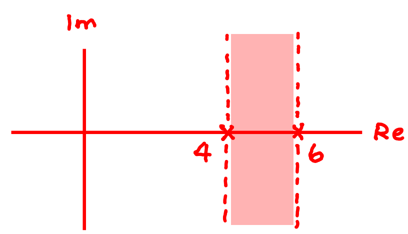
not strictly necessary except to check answer

Rewriting X in factored form, we have

$$X(s) = \frac{2}{1-(s-5)^2} = \frac{2}{1-(s^2-10s+25)} = \frac{2}{-s^2+10s-24} = \frac{-2}{s^2-10s+24} = \frac{-2}{(s-6)(s-4)}.$$

Therefore, we have

$$X(s) = \frac{-2}{(s-4)(s-6)} \quad \text{for } 4 < \operatorname{Re}(s) < 6. \qquad \blacksquare$$



sanity check:
are stated algebraic expression and stated ROC self consistent?
yes, ROC bounded by poles