Exercise 4.105

L **Answer (a).**

We are given a LTI system $\mathcal{H}$ with impulse response h, where

$$\mathcal{H}x = \tfrac{1}{2}\mathcal{H}_1(4x) - \tfrac{1}{2}\mathcal{H}_2(4x)$$

and each LTI system $\mathcal{H}_k$ has impulse response h_k.

FIRST SOLUTION. From the given equation for $\mathcal{H}$, we have

$$\mathcal{H}\delta = \tfrac{1}{2}\mathcal{H}_1(4\delta) - \tfrac{1}{2}\mathcal{H}_2(4\delta).$$

From the linearity of $\mathcal{H}$, we have

$$\begin{aligned}\mathcal{H}\delta &= \tfrac{1}{2}(4)\mathcal{H}_1\delta - \tfrac{1}{2}(4)\mathcal{H}_2\delta \\ &= 2\mathcal{H}_1\delta - 2\mathcal{H}_2\delta.\end{aligned}$$

From the definition of h_1 and h_2, we have

$$\mathcal{H}\delta = 2h_1 - 2h_2.$$

SECOND SOLUTION. From the given equation for $\mathcal{H}$, we have

$$\mathcal{H}x = \tfrac{1}{2}\mathcal{H}_1(4x) - \tfrac{1}{2}\mathcal{H}_2(4x).$$

Since $\mathcal{H}x = x * h$, $\mathcal{H}_1 x = x * h_1$, and $\mathcal{H}_2 x = x * h_2$ (due to each of $\mathcal{H}$, $\mathcal{H}_1$, and $\mathcal{H}_2$ being LTI), we have

$$\begin{aligned}x * h &= \tfrac{1}{2}[(4x) * h_1] - \tfrac{1}{2}[(4x) * h_2] \\ &= 2(x * h_1) - 2(x * h_2) \\ &= x * (2h_1) + x * (-2h_2) \\ &= x * (2h_1 - 2h_2) \\ &= x * [2(h_1 - h_2)].\end{aligned}$$

By comparing the left- and right-hand sides of the preceding equation, we conclude

$$h = 2(h_1 - h_2).$$

