Example 6.16 (Time-domain differentiation property). Find the Fourier transform X of the function

$$x(t) = \frac{d}{dt}\delta(t).$$

Solution. Taking the Fourier transform of both sides of the given equation for x yields

$$X(\omega) = \left(\mathcal{F}\left\{\frac{d}{dt}\delta(t)\right\}\right)(\omega).$$

Using the time-domain differentiation property of the Fourier transform, we can write

$$X(\omega) = \left(\mathcal{F}\left\{\frac{d}{dt}\delta(t)\right\}\right)(\omega) \qquad \text{← from definition of } x$$
$$= j\omega\mathcal{F}\delta(\omega). \qquad \text{← time-domain differentiation property}$$

Evaluating the Fourier transform of δ using Table 6.2, we obtain

$$F\delta(w) = 1$$

$$X(\omega) = j\omega(1)$$
$$= j\omega. \qquad \blacksquare$$