University of Victoria
Exam 1
Fall 2021

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 10971), A02 (CRN 10972)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: DRAKE
Given Name(s): STEPHEN
Student Number: V00 717907

16 / 25

This examination paper has **8 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

Question 1.

Consider the function f given by $f(z) = \frac{(z^2-2z)^2}{(z^2+2z+1)^4}$, where z is complex.

(A) Find the (finite) poles and zeros of f as well as their corresponding orders. **Show all of your work** and **do not skip any steps** in your answer. [3 marks]

$$f(z) = \frac{(z(z-2))^2}{((z+1)^2)^4} = \frac{z^2(z-2)^2}{(z+1)^8}$$

zeros: $z=2$ (order 2)
$z=0$ (order 0) 2
$z=-1$ (order 8)

$\frac{2.5}{3}$

(B) Determine the points at which f is analytic. **[1 mark]**

Since f is a rational function, it is analytic everywhere except where the denominator is 0, ie $z=-1$

$\frac{1}{1}$

Question 2. Using the MATLAB programming language, write a function called `myfunc` that takes a single real value x as a parameter and returns the real value y, where

$$y = \begin{cases} \sum_{n=0}^{50} (-1)^n x^n & \text{if } -1 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Your code must **use proper indentation** and **must not exceed 15 lines** in length. Be sure to use correct syntax in your answer, since syntax clearly matters here. **[2 marks]**

Line #	Line of Code
1	function myfunc :
2	if((x ≥ 1) || (x ≤ -1))
3	return 0 ;
4	else
5	for n=0 : 1 : 50 y=0;
6	y = y + (-1) .^ (n) . x .^(n);
7	
8	
9	
10	
11	
12	
13	
14	
15	y=0

0.5/2

??

Question 3. Consider the (single-input single-output) system associated with the operator $\mathscr{H}$.

(A) State, in mathematical terms, the condition that must be satisfied for $\mathscr{H}$ to be linear. You must use **operator notation** (e.g., do not use arrow notation). You must **fully define all quantities** (e.g., variables and constants) appearing in your answer and be specific about **what values they can take**. Failure to do so will likely result in a mark of zero on this question. **[2 marks]**

$\mathcal{H}$ is linear

$\Longleftrightarrow$

$\mathcal{H}\{a_1x_1 + a_2x_2\}(t) = a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t)$

a_1, a_2 are scalar values, $\forall a_1, a_2 \in \mathbb{R}$

t is the independent variable

× x_1, x_2 are real functions $f_1, f_2 \in \mathbb{R}^{\mathbb{R}}$

$\mathcal{H}$ is system operator $\mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$

$\frac{1.5}{2}$

(B) Suppose now that $\mathscr{H}x(t) = 2x(t) - 1$. **Using the condition stated in your answer to part (a)** of this question, determine whether $\mathscr{H}$ is linear. **[2 marks]**

$\mathcal{H}\{a_1x_1 + a_2x_2\}(t) = 2(a_1x_1 + a_2x_2)(t) - 1$

$= 2a_1x_1(t) + 2a_2x_2(t) - 1$

$\neq a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t) = a_1(2x_1(t) - 1) + a_2(2x_2(t) - 1)$

$= 2a_1x_1(t) - a_1 + 2a_2x_2(t) - a_2$

Since $a_1 + a_2 \neq 1 \ \forall a_1, a_2$

$\Rightarrow$ Non-linear

✓

$\frac{2}{2}$

stion 4. Consider the function x, given by

$$x(t) = \underbrace{\int_{-\infty}^{t+1} e^{-\tau}\delta(\tau)d\tau}_{x_1} + \underbrace{\int_{0}^{20} \tan(\tau)\delta(\tau+1)d\tau}_{x_2} + \underbrace{\int_{-\infty}^{\infty} t^2\sin(\tau)\delta\left(\tau-\tfrac{\pi}{2}\right)d\tau}_{x_3}.$$

Fi a fully-simplified expression for $x(t)$. **Show all of your work** and **do not skip any steps** in your answer. **[4 marks]**

$x(t) = 1, \; t \geq -1$ (Filters at $\tau = 0$)

$0, \; t < -1$

α

$x(t) = 0$ ✓ ($\delta(\tau+1)$ is 0 for all $\tau \in [0, 20]$

$_3(t) = t^2 \sin\left(\frac{\pi}{2}\right)$ (Filters at $\tau = \frac{\pi}{2}$)

$= t^2$ ✓

$\frac{2}{4}$

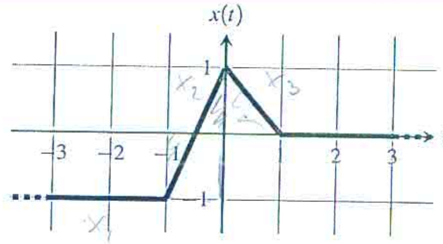
$X(t) = x_1(t) + x_2(t) + x_3(t)$

$= 1 + t^2, \; t \geq -1$

$= t^2, \; t < -1$

Question 5.

Consider the function x shown in the figure. Write an expression for $x(t)$ that consists of only a single case and is valid for all t. [Note that $x(t) = -1$ for all $t < -1$ and $x(t) = 0$ for all $t > 1$.] Show all of your work and do not skip any steps in your answer. [4 marks]



$x(t) = x_1(t) + x_2(t) + x_3(t)$

$x_1(t) = (-1)(1 - u(t+1))$

$x_2(t) = (2x + 1)(u(t+1) - u(t))$

$x_3(t) = (-x + 1)(u(t) - u(t-1))$

$\Rightarrow x(t) = (-1)(1 - u(t+1)) + (2x+1)(u(t+1) - u(t)) + (-x+1)(u(t) - u(t$ ✓

(~~t~~) ✓

$\frac{4}{4}$

Question 6.

A function x (of a real variable) has the following properties:

1. $x(t) = t - 1$ for $0 \le t \le 1$;
2. the function v is causal, where $v(t) = x(t-1)$; and
3. the function w is odd, where $w(t) = x(t) + 1$.

$v(t) = 0, \; t < 0$

Find $x(t)$ for all t. You must **make clear** how you arrived at your answer. **Show all of your work** and **do not skip any steps** in your answer. **[5 marks]**

2.) v is causal $\Rightarrow v(t) = 0, \; t < 0$ ✓

$\Rightarrow x(t-1) = 0, \; t < 0$ ✓

$\boxed{\Rightarrow x(t) = 0, \; t < -1}$

3.) w is odd $\Rightarrow w(t) = -w(-t)$ ✓

$\Rightarrow x(t) + 1 = -(x(-t) + 1)$

$\Rightarrow x(t) = t, \; -1 \le t \le 0$ ✗

$x(t) = (t-1)(u(t) - u(t-1))$

$+ (t)(u(t+1 - u(t)))$ ✗

$\frac{0.5}{5}$

Question 7.

A system $\mathscr{H}$ is defined by the equation $\mathscr{H}x(t) = \mathscr{D}x(t)$, where $\mathscr{D}$ denotes the derivative operator (e.g., for $y(t) = t^2$, $\mathscr{D}y(t) = 2t$). For each function x given below, determine if x is an eigenfunction of $\mathscr{H}$, and if it is, state its corresponding eigenvalue.
(a) $x(t) = \cos(t)$; and
(b) $x(t) = \pi$.
Show all of your work and **do not skip any steps** in your answer. **[2 marks]**

$\mathcal{H}x = \lambda x$

a) $\mathcal{H}x(t) = -\sin(t) = x(t) \cdot (-\tan(t))$, since $-\tan(t)$ is not constant
x is not an eigenfunction

b) $\mathcal{H}x(t) = 0 = 0\,x(t) \Rightarrow x$ is an eigenfunction
with corresponding eigenvalue 0.

2/2

END