5 The Genius of Kepler (*or*) Mars Orbit

Date of lab: ______________________

Report due date & time: ______________________

5.1 Objective

This lab focusses on Kepler's laws of planetary motion, which he formulated based entirely on observations of the Sun and the planets. This exercise aims to familiarize you with the orbits of the planets in the Solar System, and thus enable you to understand the seasonal changes in the positions of the Sun as well as the planets in the sky as seen from Earth. The lab also introduces you to two important coordinate systems, the *heliocentric*, and the *geocentric* systems used by astronomers to locate the positions of solar system objects, and teaches you to use them with graphical methods.

This lab exercise corresponds to the course material presented in Chapter 6 of the book, and is intended to help you understand the concepts of orbital motion in a planetary system, such as our Solar System.

5.2 Introduction

5.2.1 Kepler's laws

Kepler formulated the first two of his three laws of planetary motion circa 1609. He based these empirical laws on the careful analysis of observations made by the Danish astronomer, Tycho Brahe of Mars and other planets over many years. All these were just visual observations using available measuring devices, since they even preceded Galileo's observations of the skies with the newly invented *telescope*. That Kepler was able to devise these laws and use them to calculate the orbital parameters of Mars and other planets just from these observations is a clear mark of his true genius. Kepler, along with Copernicus and Galileo, played a leading role in changing the *geocentric* view of the solar system, indeed of the Universe, held till then, and proved the correct *heliocentric* model.

More information about the history of these observations are given in the supplementary document, *Orbit of Mars* by Dr. Owen Gingerich (Harvard U) available on Brightspace. We will redo Kepler's brilliant analysis of the Orbit of Mars, following the method described in this document. However, unlike the lab exercise given there, we will be using modern observations provided by the Jet Propulsion Laboratory (`http://ssd.jpl.nasa.gov/horizons.cgi`).

5.2.2 Planetary Motion

The Earth orbits the Sun with a period of one year. However, as the Earth goes around the Sun, to an observer on Earth, the Sun appears to 'travel' in the sky against the fixed background of distant stars (= *the celestial sphere*) during the course of the year. The Sun's annual path across the stars defines the great circle called *the ecliptic*. This represents the plane of the Earth's orbit (= *the orbital plane*) projected into space. It is this Earth centred view of the Sun and other solar system objects that gave rise to the (incorrect) geocentric view held in early times.

Along with the Sun, all the bright planets too are observed to move in paths which closely follow the ecliptic. Therefore it follows that all their orbits lie approximately in the plane of the ecliptic.The shape of the orbits of all planets, with the exception of Mercury, are very nearly circular. Planet orbits are in reality ellipses but with very small eccentricities; only Mercury's orbit is clearly elliptical.

For this lab, we will focus on the orbits of Earth and Mars. Following Kepler's method, we will determine the orbit of Mars based entirely on measurements made from Earth. For this, we will assume that the orbit of Earth is circular. Since the orbits of planets lie on a single plane, the ecliptic, we can draw them on a flat sheet of paper. We will draw a top-down scale "map" of part of the Solar System showing the orbits of Earth and Mars, then make measurements on this map to place the planets on this chart. We will measure their positions using two different coordinate systems, the *heliocentric*, the *geocentric* used by astronomers to locate solar system objects. These coordinates can be used to point a telescope at the planets, and determine which planets will be visible and where to find them at any time during the course of the year. Brief descriptions of these coordinates are given below, while the textbook provides more complete details. Note that the *equatorial* system, measuring positions in Right Ascension and Declination, is the more commonly used system in astronomy for distant celestial objects.

5.2.3 Heliocentric coordinates

Derived from the Greek word, *helios* for the Sun, the heliocentric coordinate system places the Sun at the origin. It is used mainly for solar system objects only. In this three-dimensional *spherical* coordinate system, along with the radial distances, angular positions east-west are measured by *longitude*, and north-south by *latitude*. The heliocentric longitudes are measured counterclockwise from 0 to 360°, while latitudes run from 0 at the ecliptic to +90° North, and to -90° South. The heliocentric longitude is defined with respect to the First Point of Aries as 0°. The First Point of Aries is the point in the celestial sphere directly behind the Sun at noon on the day of the vernal equinox. This point coincides with the constellation, Aries, and is thus called the First

Point of Aries, and is given the zodiacal symbol Υ.

5.2.4 Geocentric coordinates

Geo, meaning Earth, is the root of the name of this coordinate system, which places Earth at the origin. This coordinate system perhaps dates back to a time when astronomers viewed Earth as the center of the (very limited) Universe known then. Other than this change of origin, this three-dimensional *spherical* coordinate system is defined similar to the heliocentric system, and measures longitudes from the First Point of Aries.

5.2.5 Triangulation

The key principle used by Kepler was to determine the positions of Mars using *triangulation*. Triangulation is a technique which permits the determination of the distance to a far away object using its direction as seen from two different vantages. To determine the distance of Mars from Earth, Kepler determined its direction using geocentric coordinates from two different positions of Earth's orbit. However, since Mars too is moving, he had to wait till it was in the same position during two consecutive orbits. From earlier observations, Kepler knew that Mars's orbital period is 687 days. So by using the measured directions of Mars exactly 687 days apart, Kepler could use triangulation to determine its distance. Kepler repeated these measurements at different positions of Mars in its orbit, and thus successfully constructed the overall geometry of its orbit. Very ingenious!

For this lab exercise, we will be focussing on the heliocentric longitudes of Earth and the geocentric positions of Mars over the course of several years. Using a geometrical method based on triangulation, we will determine the orbit of Mars.

5.3 Equipment

For this lab exercise, you will only need a ruled sheet of paper, and basic geometrical drawing instruments, a compass, protractor, and a ruler.

5.4 Procedure

We will follow Kepler's procedure, as described in Dr. Gingerich's lab exercise. In the centre of the paper, draw a circle to represent Earth's orbit. Use a radius of 5cm, which leaves enough room on the paper to draw the orbit of Mars as well. Place the centre of the circle on one of the ruled lines. Then you can use the line to mark the First Point of Aries, which will form the origin of the heliocentric longitudes. You will measure the given longitudes going counter clockwise from the direction of the First Point of Aries.

Table 1: Orbital positions of Earth and Mars.

Position	Date	HL_E	GL_M
		[degrees]	[degrees]
A	2001-Oct-10	17	290
A	2005-Jul-15	293	21
B	2009-Aug-11	318	79
B	2011-Jun-29	277	64
C	2004-Apr-13	203	73
C	2006-Mar-01	160	63
D	2010-Mar-28	187	125
D	2013-Dec-31	99	191
E	2006-Oct-16	22	203
E	2010-Jul-21	298	175
F	2005-Mar-22	181	303
F	2008-Dec-25	93	268

Heliocentric longitude of Earth (HL_E), and corresponding geocentric longitude of Mars (GL_M) given in degrees for a series of dates in the range 2001 to 2013. Both longitudes are measured counterclockwise from the direction of the First Point of Aries. These positions are given in pairs, as indicated in the first column. Each pair is used to triangulate for the corresponding position of Mars.

Table 1 provides the heliocentric locations of Earth, and the corresponding geocentric longitudes of Mars for a series of dates in the period 2001 to 2013. These positions are given in pairs and each pair may be used to triangulate for the corresponding position of Mars. Since the dates in each pair are separated by 687 days (or a multiple thereof), which is the orbital period of Mars, that planet would be at the same position in its orbits on both dates. However, since this difference of 687 days is not an even multiple of the orbital period of Earth, that planet would be at two different locations in its orbit on those two dates. Hence we may apply triangulation from these two positions of Earth to locate the position of Mars.

For each pair of positions, first use the given heliocentric longitudes of Earth to locate its positions in its orbit. Use the protractor to measure the angles given in Table 1. Place the protractor so its centre coincides with the position of the Sun (= centre of the circle). Measure heliocentric longitudes counterclockwise from the First Point of Aries. At each measured angle, draw a line from the centre of the circle to intersect the circle. This is the location of Earth on that date. Now at each of these locations of Earth, draw a line parallel to the First Point of Aries. Then using the protractor, measure the geocentric longitude of Mars, again going counterclockwise from the direction of the First Point of Aries. At each measured angle, draw a line in that direction. Where these two lines intersect is the position of Mars in its orbit. Do this construction for all six pairs of points. Your TA will demonstrate this construction for one pair of points. You can then complete it for the remaining pairs of points.

If the construction is done properly, this will provide fairly evenly spaced positions of Mars in its orbit. Draw a smooth closed curve passing through these points. Careful freehand drawing is fine. However, if you wish, you are welcome to attempt the construction technique outlined in Dr. Gingerich's document. You have thus determined the orbit of Mars based only on these measurements!

Now, measure the distances between all pairs of these six locations of Mars. The longest distance may be taken to be the major axis of Mars's orbit. Connect the two points at the greatest distance with a line. You will notice that the line passes through the position of the Sun, which lies at the centre of Earth's orbit.

Locate the midpoint of the major axis of Mars's orbit, and measure the *semi-major distance* in centimeters. Measure the distance between the Sun and the midpoint of Mars's major axis. This is called the *focal length.* Also, measure the distances between the Sun and the position of Mars at either end of the major axis. The position closer to the Sun is called *perihelion* and the farther point is called the *aphelion.*

Questions

(1) Since this lab exercise relates to Kepler's laws, use any reference and write the three laws (remember to cite the reference).

(2) In the geometrical construction, we used the radius of the circle to represent the average distance between Earth and the Sun. This is a scale of 5cm = 1 AU. Using this scale, calculate the average distance (in AU) from the Sun to Mars. Calculate the distances to all six positions, then take the average.

(3) The *eccentricity* of Mars's orbit is defined as the ratio of the focal length to the semi-major axis. Calculate this based on your measurements.

(4) What are the perihelion and aphelion distances of Mars in AU?

(5) Based on the orbital periods of Earth and Mars, how often are the two planets closer than usual to each other? Discuss the reasons.

(6) Based on the given dates of the observations, estimate in which month Earth and Mars would be at *closest* approach? For a *superior* planet like Mars, when this happens, Earth and Mars will be on the same side of the Sun. This is called *Opposition*

(7) Using a similar rationale as the previous question, estimate the month when the two planets with be *farthest* from each other? This happens when Mars and Earth are on opposite sides of the Sun, and this is called *conjuction*.

(8) Kepler's third law relates the orbital period (in days) to the semi-major axis distance (in AU) of all planets. Since you now know these quantities for Earth and Mars, calculate the ratio for each planet using Kepler's third law (remember to use the correct exponents for the period and semi-major axis). Discuss how well the values of the ratios for Earth and Mars match each other, thus justifying the prediction of Kepler's law.

Discussion Points

For the discussion section of your report, elaborate on these following points.

- Using some reliable source, find what the accepted values are for the various distances you calculated in the questions. Discuss each of them in turn, highlighting how well they match and possible reasons for any discrepancies.
- Earth's orbit is slightly elliptical with a very small eccentricity of 0.0167. However, we assumed it to be circular. Discuss if and how your results may be affected by the assumption.
- Using online search, look up when Earth and Mars were last at *closest* approach. Discuss how well the month in which this occurred matches with the prediction you made in Question *(7)*. When will the closest approach occur next? Discuss the implications.