Exercise 6.22

L **Answer (a).**

Consider the squarer-based system in Figure (b). From the system block diagram, we have

$$\underbrace{v_1(t) = x(t) + \cos(\omega_c t)}_{1} \quad \text{and} \quad \underbrace{v_2(t) = v_1^2(t)}_{2}.$$

We can rewrite v_2 as

$$\begin{aligned}
v_2(t) &= v_1^2(t) \qquad 2 \\
&= [x(t) + \cos(\omega_c t)]^2 \qquad \text{substitute } 1 \\
&= x^2(t) + [2\cos(\omega_c t)]x(t) + \cos^2(\omega_c t) \qquad \text{square} \\
&= x^2(t) + \left[e^{j\omega_c t} + e^{-j\omega_c t}\right]x(t) + \left[\tfrac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})\right]^2 \qquad \text{Euler} \\
&= x^2(t) + e^{j\omega_c t}x(t) + e^{-j\omega_c t}x(t) + \tfrac{1}{4}\left[e^{j2\omega_c t} + 2 + e^{-j2\omega_c t}\right] \qquad \text{square} \\
&= x^2(t) + e^{j\omega_c t}x(t) + e^{-j\omega_c t}x(t) + \tfrac{1}{4}e^{j2\omega_c t} + \tfrac{1}{2} + \tfrac{1}{4}e^{-j2\omega_c t}. \qquad \text{multiply}
\end{aligned}$$

Taking the Fourier transform of v_2, we obtain

$$\begin{aligned}
V_2(\omega) &= \mathcal{F}\{x^2(\cdot)\}(\omega) + \mathcal{F}\{e^{j\omega_c(\cdot)}x(\cdot)\}(\omega) + \mathcal{F}\{e^{-j\omega_c(\cdot)}x(\cdot)\}(\omega) + \tfrac{1}{4}\mathcal{F}\{e^{j2\omega_c(\cdot)}\}(\omega) + \tfrac{1}{2}\mathcal{F}\{1\}(\omega) \qquad \text{take FT using linearity} \\
&\quad + \tfrac{1}{4}\mathcal{F}\{e^{-j2\omega_c(\cdot)}\}(\omega) \\
&= \mathcal{F}\{x^2\}(\omega) + \mathcal{F}\{x\}(\omega - \omega_c) + \mathcal{F}\{x\}(\omega + \omega_c) + \tfrac{1}{4}\mathcal{F}\{1\}(\omega - 2\omega_c) + \tfrac{1}{2}\mathcal{F}\{1\}(\omega) + \tfrac{1}{4}\mathcal{F}\{1\}(\omega + 2\omega_c) \qquad \text{modulation} \\
&= \tfrac{1}{2\pi}X * X(\omega) + X(\omega - \omega_c) + X(\omega + \omega_c) + \tfrac{1}{4}[2\pi\delta(\omega - 2\omega_c)] + \tfrac{1}{2}[2\pi\delta(\omega)] + \tfrac{1}{4}[2\pi\delta(\omega + 2\omega_c)]. \qquad \text{multiplication and FT table} \\
&= \tfrac{1}{2\pi}X * X(\omega) + X(\omega - \omega_c) + X(\omega + \omega_c) + \tfrac{\pi}{2}\delta(\omega - 2\omega_c) + \pi\delta(\omega) + \tfrac{\pi}{2}\delta(\omega + 2\omega_c). \quad 3 \qquad \text{multiply constants}
\end{aligned}$$

In passing, we note that the input spectrum X and output spectrum Y of the DSB/SC AM transmitter in Figure (a) are related by

$$Y(\omega) = \tfrac{1}{2}X(\omega - \omega_c) + \tfrac{1}{2}X(\omega + \omega_c).$$

(This formula is obtained by taking the Fourier transform of $y(t) = \frac{1}{2}\left[e^{j\omega_c t} + e^{-j\omega_c t}\right]x(t)$.) So, $\frac{1}{2}V_2(\omega)$ contains the two terms from the preceding formula for $Y(\omega)$ plus some extraneous terms. In particular, we have

$$\tfrac{1}{2}V_2(\omega) = \underbrace{\tfrac{1}{4\pi}X * X(\omega)} + \tfrac{1}{2}X(\omega - \omega_c) + \tfrac{1}{2}X(\omega + \omega_c) + \underbrace{\tfrac{\pi}{4}\delta(\omega - 2\omega_c)} + \underbrace{\tfrac{\pi}{2}\delta(\omega)} + \underbrace{\tfrac{\pi}{4}\delta(\omega + 2\omega_c)}. \qquad \text{from 3 multiplied by } \tfrac{1}{2}$$

$\boxed{\text{To obtain the desired AM modulated signal, we do not want the terms marked by frownie faces.}}$

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