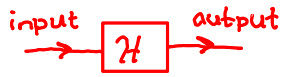
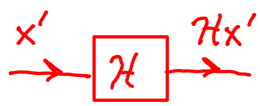
Example 2.7. For a system operator $\mathcal{H}$, function x', and real number t, the expression $\mathcal{H}x'(t)$ denotes result of taking the function y produced as the output of the system $\mathcal{H}$ when the input is the function x' and then evaluating y at t. ■

$\mathcal{H}$ is a system.



$\underbrace{\mathcal{H}x'}_{\text{function}}$ is the output of the system $\mathcal{H}$ when the input is $\underbrace{x'}_{\text{function}}$.



Since $\mathcal{H}x'$ is a function, we can evaluate it at a point such as t.

$$\overbrace{\underbrace{\mathcal{H}x'}_{\text{function}}\underbrace{(t)}_{\substack{\text{point at}\\\text{which}\\\text{function is}\\\text{evaluated}}}}^{\text{number}}$$

Unit:
CT Signals and Systems

3.3 Suppose that we have two functions x and y related as

$$y(t) = x(at - b),$$

where a and b are real constants and $a \neq 0$.
(a) Show that y can be formed by first time shifting x by b and then time scaling the result by a.
(b) Show that y can also be formed by first time scaling x by a and then time shifting the result by $\frac{b}{a}$.

When working with time transformed functions, always give each transformed function a name

Answer (a). (shift then scale)

Let f denote the result of time shifting x by b. So, by definition, we have

$$f(t) = x(t - b). \quad ①$$

Let g denote the result of time scaling f by a. So, by definition, we have

$$g(t) = f(at).$$

Substituting the above formula for f into the equation for g, we obtain

$$\begin{aligned} g(t) &= f(at) \\ &= x(at - b) \\ &= y(t). \end{aligned}$$

substituting ①

Therefore, y can be formed in the manner specified in the problem statement.

Answer (b). (scale then shift)

Let f denote the result of time scaling x by a. So, by definition, we have

$$f(t) = x(at).$$

Let g denote the result of time shifting f by $\frac{b}{a}$. So, by definition, we have

$$g(t) = f\left(t - \tfrac{b}{a}\right).$$

Substituting the above formula for f into the equation for g, we obtain

$$\begin{aligned} g(t) &= f\left(t - \tfrac{b}{a}\right) \\ &= x\left(a\left[t - \tfrac{b}{a}\right]\right) \\ &= x(at - b) \\ &= y(t). \end{aligned}$$

substituting ①

Therefore, y can be formed in the manner specified in the problem statement.

Theorem 3.1 (Decomposition of function into even and odd parts)**.** *Any arbitrary function x can be uniquely represented as the sum of the form*

$$x(t) = x_e(t) + x_o(t), \tag{3.7}$$

where x_e and x_o are even and odd, respectively, and given by

$$x_e(t) = \tfrac{1}{2}[x(t) + x(-t)] \quad \textit{and} \tag{3.8}$$

$$x_o(t) = \tfrac{1}{2}[x(t) - x(-t)]. \tag{3.9}$$

As a matter of terminology, x_e is called the ***even part*** *of x and is denoted* $\operatorname{Even}\{x\}$*, and x_o is called the* ***odd part*** *of x and is denoted* $\operatorname{Odd}\{x\}$.

Partial *Proof.* From (3.8) and (3.9), we can easily confirm that $x_e + x_o = x$ as follows:

$$\begin{aligned} x_e(t) + x_o(t) &= \overbrace{\tfrac{1}{2}[x(t) + x(-t)]}^{x_e} + \overbrace{\tfrac{1}{2}[x(t) - x(-t)]}^{x_o} \\ &= \tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t) + \tfrac{1}{2}x(t) - \tfrac{1}{2}x(-t) \\ &= x(t). \end{aligned}$$

from the definition of xe and xo

x(-t) terms cancel

Furthermore, we can easily verify that x_e is even and x_o is odd. From the definition of x_e in (3.8), we have

$$\begin{aligned} x_e(-t) &= \tfrac{1}{2}[x(-t) + x(-[-t])] \\ &= \tfrac{1}{2}[x(t) + x(-t)] \\ &= x_e(t). \end{aligned}$$

substitute -t for t in definition of xe

even

Thus, x_e is even. From the definition of x_o in (3.9), we have

$$\begin{aligned} x_o(-t) &= \tfrac{1}{2}[x(-t) - x(-[-t])] \\ &= \tfrac{1}{2}[-x(t) + x(-t)] \\ &= -x_o(t). \end{aligned}$$

substitute -t for t in definition of xo

odd

Thus, x_o is odd.

