Exercise 4.104

L Answer (c).

Let $\mathcal{S}_{t_0}$ denote an operator that shifts a function by t_0 (i.e., $\mathcal{S}_{t_0}x(t) = x(t-t_0)$ for all t). From the graphs of x_1 and x_2, we can see that

See A and B

$$x_2 = \mathcal{S}_{-2}x_1 + \mathcal{S}_{-1}x_1 + \mathcal{S}_0 x_1. \quad (1)$$

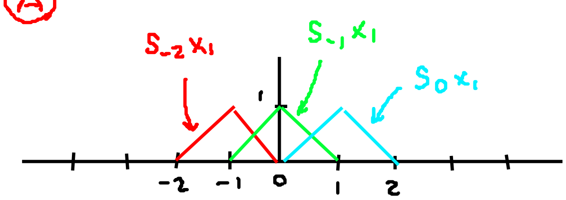
So, we have

$$\begin{aligned}
y_2 &= \mathcal{H}x_2 && \text{from definition of } \mathcal{H} \\
&= \mathcal{H}(\mathcal{S}_{-2}x_1 + \mathcal{S}_{-1}x_1 + \mathcal{S}_0 x_1) && \text{substitute (1)} \\
&= \mathcal{H}(\mathcal{S}_{-2}x_1) + \mathcal{H}(\mathcal{S}_{-1}x_1) + \mathcal{H}(\mathcal{S}_0 x_1) && \text{linearity of } \mathcal{H} \\
&= \mathcal{S}_{-2}\mathcal{H}x_1 + \mathcal{S}_{-1}\mathcal{H}x_1 + \mathcal{S}_0\mathcal{H}x_1 && \text{time invariance of } \mathcal{H} \\
&= \mathcal{S}_{-2}y_1 + \mathcal{S}_{-1}y_1 + \mathcal{S}_0 y_1. && \text{definition of } \mathcal{H}
\end{aligned}$$

Equivalently, using non-operator notation, we have that the function y_2 is given by the equation

$$y_2(t) = y_1(t+2) + y_1(t+1) + y_1(t). \quad \text{definition of } S_{t_0}$$

A



B