Exercise A.5

L Answer (d).

We are given the function

$$f(\omega) = \frac{-5}{(-1-j\omega)^4}.$$

First, we compute the magnitude of $f(\omega)$ to obtain

take magnitude of both sides

$$|f(\omega)| = \left|\frac{-5}{(-1-j\omega)^4}\right|$$

$|z_1/z_2| = |z_1|/|z_2|$

$$= \frac{|-5|}{|(-1-j\omega)^4|}$$

$|z^n| = |z|^n$ and $|-5| = 5$

$$= \frac{5}{|-1-j\omega|^4}$$

definition of magnitude

$$= \frac{5}{\left(\sqrt{1+\omega^2}\right)^4}$$

combine powers

$$= \frac{5}{(1+\omega^2)^2}.$$

Next, we calculate the argument of $f(\omega)$ as

take argument of both sides

$$\arg f(\omega) = \arg\left[\frac{-5}{(-1-j\omega)^4}\right]$$

$\arg(z_1/z_2) = \arg z_1 - \arg z_2$

$$= \arg(-5) - \arg[(-1-j\omega)^4]$$

$\arg(z^n) = n \arg z$

$$= \pi - 4\arg(-1-j\omega)$$

use arctan for argument

$$= \pi - 4(\arctan(\omega) + \pi)$$

simplify

$$\begin{aligned}
&= \pi - 4(\pi + \arctan(\omega)) \\
&= \pi - 4\pi - 4\arctan(\omega) \\
&= -3\pi - 4\arctan(\omega).
\end{aligned}$$

Since the argument is not uniquely determined, in the most general case, we have

$$\arg f(\omega) = -4\arctan(\omega) + (2k+1)\pi$$

for all integer k.

