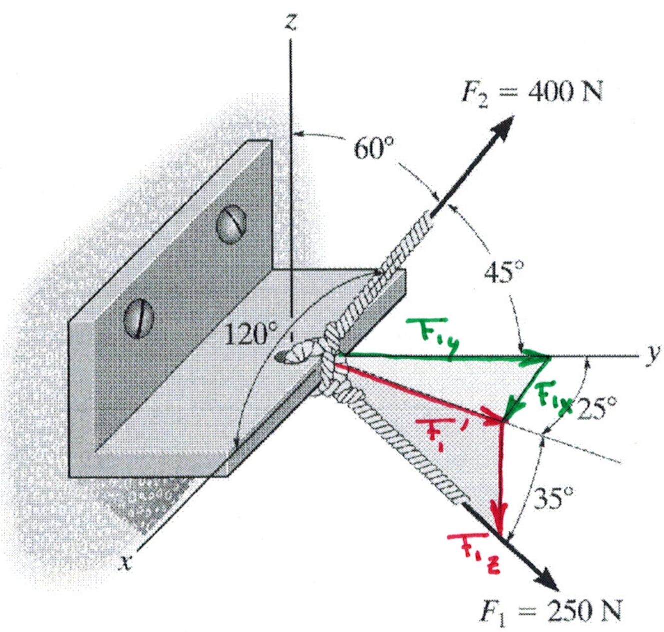
Find the magnitude and direction of the following system of forces.



- Resolve Force F_1

decompose F_1 into F_{1z} and F_1'

$$F_{1z} = -250 \sin 35° = -143.4 \text{ N}$$

$$F_1' = 250 \cos 35 = 204.8 \text{ N}$$

decompose F_1' into F_{1x} and F_{1y}.

$$F_{1x} = 204.8 \sin 25° = 86.6 \text{ N}$$

$$F_{1y} = 204.8 \cos 25° = 185.6 \text{ N}$$

Hence, $F_1 = \{86.6i + 185.6j - 143.4k\}$ N

- Resolve Force F_2

$$F_2 = 400\{\cos 120° i + \cos 45° j + \cos 60°\} \text{ N}$$

$$= \{-200i + 282.8j + 200k\} \text{ N}$$

- Find Resultant

$$F_R = \{(86.6 - 200)i + (185.6 + 282.2)j + (-143.4 + 200)k\} \text{ N}$$

$$= \{-113.4i + 468.4j + 56.6k\} \text{ N}$$

- Find Magnitude and direction angles

$$\|F_R\| = \sqrt{(-113.4)^2 + (468.4)^2 + (56.6)^2}$$

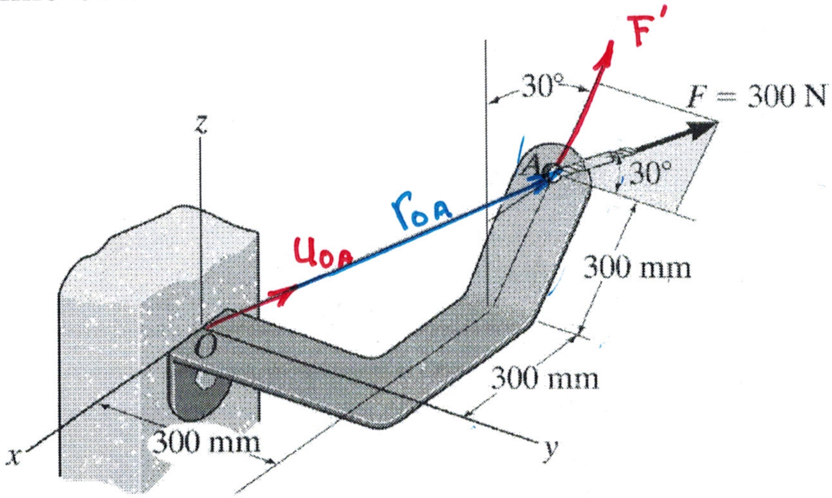
$$= 485.2 \text{ N}$$

$$\alpha = \cos^{-1}\left(\frac{F_{Rx}}{\|F_R\|}\right) = \cos^{-1}\left(\frac{-113.4}{485.2}\right) = 104°$$

$$\beta = \cos^{-1}\left(\frac{F_{Ry}}{\|F_R\|}\right) = \cos^{-1}\left(\frac{468.4}{485.2}\right) = 15.1°$$

$$\gamma = \cos^{-1}\left(\frac{F_{Rz}}{\|F_R\|}\right) = \cos^{-1}\left(\frac{56.6}{485.2}\right) = 83.3°$$

Find the magnitude of the projected component of this force acting along line OA.



Force F lies on the plane of the last portion of the bracket

- Determine the unit vector u_{OA}

$$r_{OA} = \{-0.45i + 0.3j + 0.26k\} \text{ m}$$

$$\|r_{OA}\| = \sqrt{(-0.45)^2 + (0.3)^2 + (0.26)^2} = 0.6 \text{ m}$$

$$u_{OA} = \frac{r_{OA}}{\|r_{OA}\|} = \{-0.75i + 0.5j + 0.433k\}$$

- Resolve force F

$$F' = 300 \sin 30^\circ = 150 \text{ N}$$

$$F = \{-150 \sin 30\, i + 300 \cos 30\, j + 150 \cos 30\, k\} \text{ N}$$

$$= \{-75i + 259.8j + 129.9k\} \text{ N}$$

$$\|F\| = \sqrt{(-75)^2 + 259.8^2 + 129.9^2} = 300 \text{ N} \quad \text{(As expected)}$$

- Dot product

$$\mathbf{F} \cdot \mathbf{r}_{OA} = (-75)(-0.45) + (259.8)(0.3) + (129.9)(0.26)$$
$$= 145.5 \text{ N·m}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{F} \cdot \mathbf{r}_{OA}}{\|\mathbf{F}\| \|\mathbf{r}_{OA}\|}\right) = \cos^{-1}\left(\frac{145.5}{(300)(0.6)}\right) = 36.1^\circ$$

Finally, the magnitude of the projected component of $\mathbf{F}$ along line OA

$$\|\mathbf{F}_{OA}\| = \|\mathbf{F}\| \cos\theta = 300 \cos 36.1 = 242 \text{ N}$$

or

$$\|\mathbf{F}_{OA}\| = \mathbf{F} \cdot \mathbf{u}_{OA} = (-75)(-0.75) + (259.8)(0.5) + (129.9)(0.433)$$
$$= 242 \text{ N}$$

Note we can find vector $\mathbf{F}_{OA}$, the projected component of $\mathbf{F}$ along OA, can be found as

$$\mathbf{F}_{OA} = \|\mathbf{F}_{OA}\| \mathbf{u}_{OA}$$