Lecture Slides for Signals and Systems

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NEXT SLIDE: Course Introduction

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Other Textbooks and Lecture Slides by the Author I

1. M. D. Adams, *Lecture Slides for Programming in C++ (Version 2019-02-04)*, University of Victoria, Victoria, BC, Canada, Feb. 2019, xxii + 2516 slides, ISBN 978-1-55058-640-4 (print), ISBN 978-1-55058-641-1 (PDF). Available from Google Books, Google Play Books, University of Victoria Bookstore, and author's web site http://www.ece.uvic.ca/~mdadams/cppbook.
2. M. D. Adams, *Multiresolution Signal and Geometry Processing: Filter Banks, Wavelets, and Subdivision (Version 2013-09-26)*, University of Victoria, Victoria, BC, Canada, Sept. 2013, xxxviii + 538 pages, ISBN 978-1-55058-507-0 (print), ISBN 978-1-55058-508-7 (PDF). Available from Google Books, Google Play Books, University of Victoria Bookstore, and author's web site http://www.ece.uvic.ca/~mdadams/waveletbook.

Other Textbooks and Lecture Slides by the Author II

3 M. D. Adams, *Lecture Slides for Multiresolution Signal and Geometry Processing (Version 2015-02-03)*, University of Victoria, Victoria, BC, Canada, Feb. 2015, xi + 587 slides, ISBN 978-1-55058-535-3 (print), ISBN 978-1-55058-536-0 (PDF). Available from Google Books, Google Play Books, University of Victoria Bookstore, and author's web site http://www.ece.uvic.ca/~mdadams/waveletbook.

4 M. D. Adams, *Continuous-Time Signals and Systems*, Edition 2.0, University of Victoria, Victoria, BC, Canada, Feb. 2020, xxx + 366 pages, ISBN 978-1-55058-657-2 (print), ISBN 978-1-55058-658-9 (PDF). Available from Google Books, Google Play Books, University of Victoria Bookstore, and author's web site http://www.ece.uvic.ca/~mdadams/sigsysbook.

Part 0

Preface

About These Lecture Slides

- This document constitutes a detailed set of lecture slides on signals and systems, covering both the continuous-time and discrete-time cases.
- These slides are organized in such a way as to facilitate the teaching of a course that covers: only the continuous-time case, or only the discrete-time case, or both the continuous-time and discrete-time cases.
- To teach a course on only the continuous-time case, these slides can be used in conjunction with the following textbook:
  - M. D. Adams, *Continuous-Time Signals and Systems*, Edition 2.0, University of Victoria, Victoria, BC, Canada, Feb. 2020. xxx + 366 pages. ISBN 978-1-55058-657-2 (print), ISBN 978-1-55058-658-9 (PDF). Available online from http://www.ece.uvic.ca/~mdadams/sigsysbook.
- The author is currently in the process of developing a new textbook that covers both the continuous-time and discrete-time cases. These lecture slides are also intended for use with this new textbook, when it becomes available.

Typesetting Conventions

- In a definition, the term being defined is often typeset in a font **like this**.
- To emphasize particular words, the words are typeset in a font ***like this***.

Part 1

Introduction

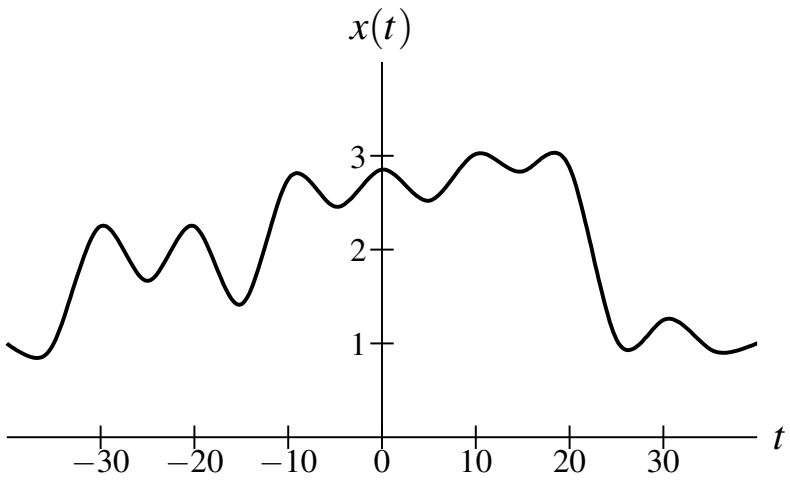
Signals

- A **signal** is a function of one or more variables that conveys information about some (usually physical) phenomenon.
- For a function f, in the expression $f(t_1, t_2, \ldots, t_n)$, each of the $\{t_k\}$ is called an **independent variable**, while the function value itself is referred to as a **dependent variable**.
- Some examples of signals include:
  - a voltage or current in an electronic circuit
  - the position, velocity, or acceleration of an object
  - a force or torque in a mechanical system
  - a flow rate of a liquid or gas in a chemical process
  - a digital image, digital video, or digital audio
  - a stock market index

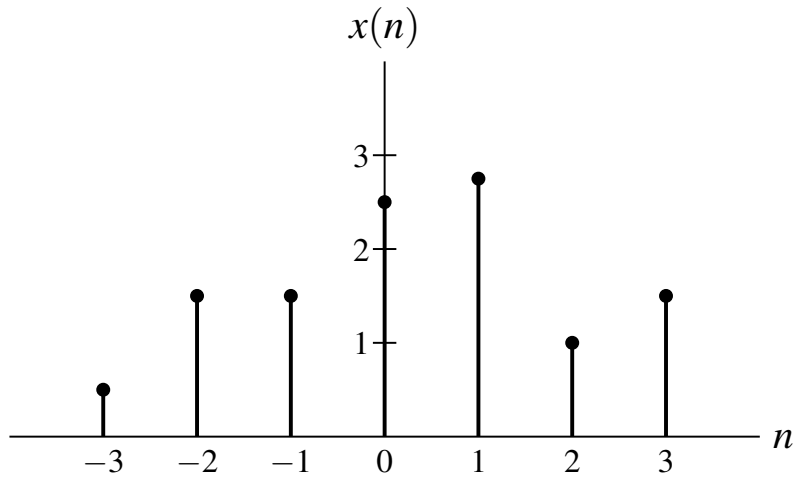
Classification of Signals

- Number of independent variables (i.e., dimensionality):
  - A signal with ***one*** independent variable is said to be **one dimensional** (e.g., audio).
  - A signal with ***more than one*** independent variable is said to be **multi-dimensional** (e.g., image).
- Continuous or discrete independent variables:
  - A signal with ***continuous*** independent variables is said to be **continuous time (CT)** (e.g., voltage waveform).
  - A signal with ***discrete*** independent variables is said to be **discrete time (DT)** (e.g., stock market index).
- Continuous or discrete dependent variable:
  - A signal with a ***continuous*** dependent variable is said to be **continuous valued** (e.g., voltage waveform).
  - A signal with a ***discrete*** dependent variable is said to be **discrete valued** (e.g., digital image).
- A ***continuous-valued CT*** signal is said to be **analog** (e.g., voltage waveform).
- A ***discrete-valued DT*** signal is said to be **digital** (e.g., digital audio).

Graphical Representation of Signals



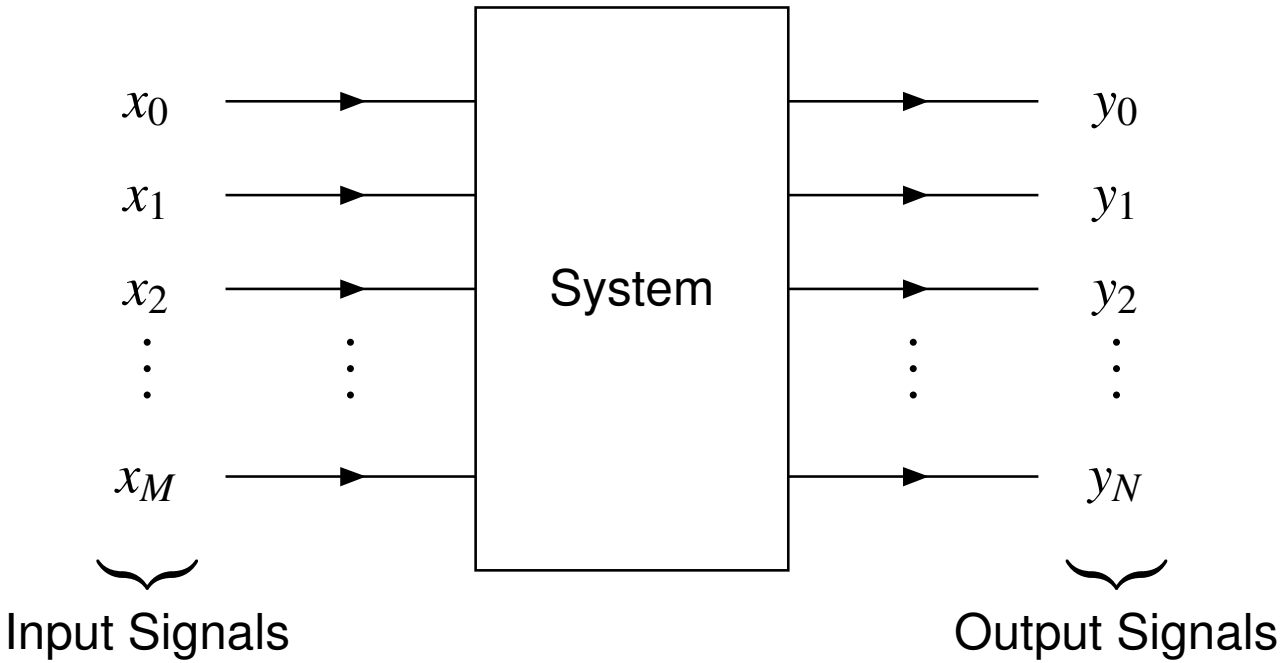
Continuous-Time (CT) Signal



Discrete-Time (DT) Signal

Systems

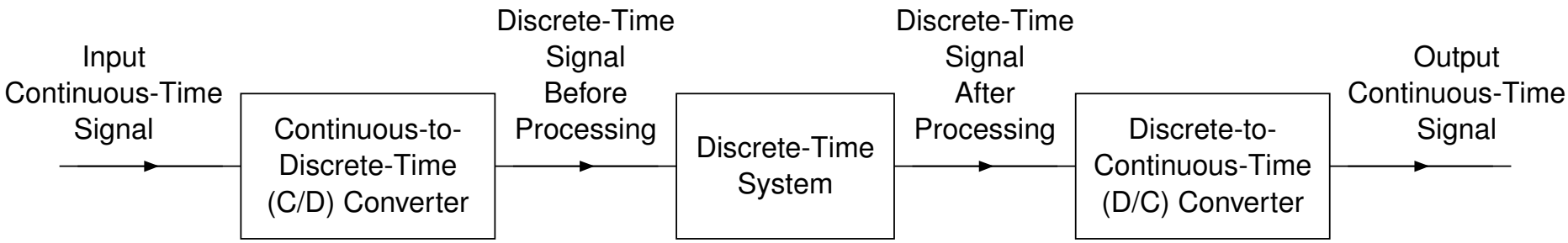
- A **system** is an entity that processes one or more input signals in order to produce one or more output signals.



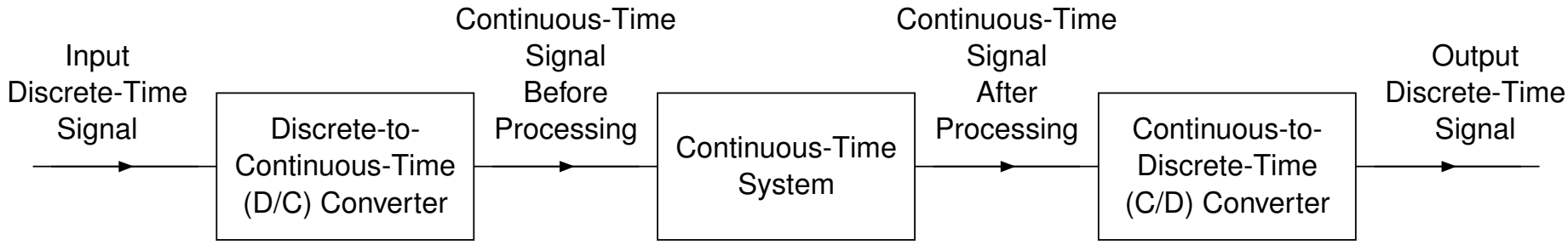
Classification of Systems

- Number of inputs:
  - A system with ***one*** input is said to be **single input (SI)**.
  - A system with ***more than one*** input is said to be **multiple input (MI)**.
- Number of outputs:
  - A system with ***one*** output is said to be **single output (SO)**.
  - A system with ***more than one*** output is said to be **multiple output (MO)**.
- Types of signals processed:
  - A system can be classified in terms of the ***types of signals*** that it processes.
  - Consequently, terms such as the following (which describe signals) can also be used to describe systems:
    - one-dimensional and multi-dimensional,
    - continuous-time (CT) and discrete-time (DT), and
    - analog and digital.
  - For example, a continuous-time (CT) system processes CT signals and a discrete-time (DT) system processes DT signals.

Signal Processing Systems

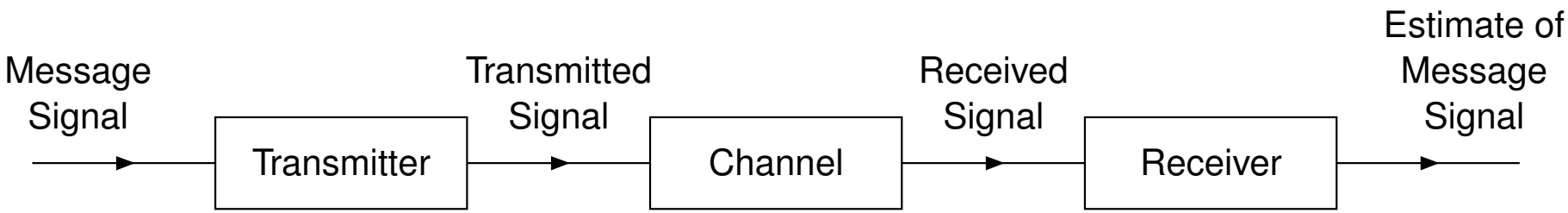


Processing a Continuous-Time Signal With a Discrete-Time System



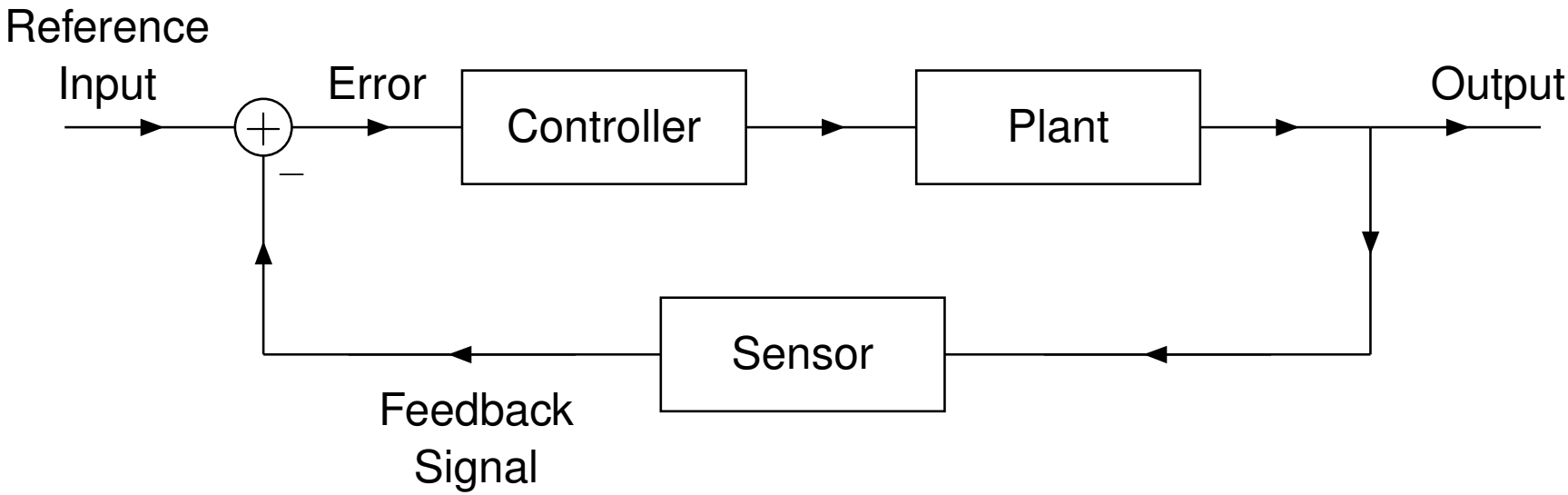
Processing a Discrete-Time Signal With a Continuous-Time System

Communication Systems



General Structure of a Communication System

Control Systems



General Structure of a Feedback Control System

Why Study Signals and Systems?

- Engineers build systems that process/manipulate signals.
- We need a formal mathematical framework for the study of such systems.
- Such a framework is necessary in order to ensure that a system will meet the required specifications (e.g., performance and safety).
- If a system fails to meet the required specifications or fails to work altogether, negative consequences usually ensue.
- When a system fails to operate as expected, the consequences can sometimes be catastrophic.