Example 7.31. For the LTI system with each system function H below, determine whether the system is causal.

rational
(a) $H(s) = \frac{1}{s+1}$ for $\operatorname{Re}(s) > -1$;
(b) $H(s) = \frac{1}{s^2-1}$ for $-1 < \operatorname{Re}(s) < 1$;

not rational
(c) $H(s) = \frac{e^s}{s+1}$ for $\operatorname{Re}(s) < -1$; and
(d) $H(s) = \frac{e^s}{s+1}$ for $\operatorname{Re}(s) > -1$.

causal $\Rightarrow$ ROC is RHP

if rational : causal $\Leftrightarrow$ ROC is RHP

Solution. (a) The poles of H are plotted in Figure 7.19(a) and the ROC is indicated by the shaded area. The system function H is rational and the ROC is the right-half plane to the right of the rightmost pole. Therefore, the system is causal.

(b) The poles of H are plotted in Figure 7.19(b) and the ROC is indicated by the shaded area. The system function is rational but the ROC is not a right-half plane. Therefore, the system is not causal.

(c) The system function H has a left-half plane ROC. Therefore, h is a left-sided signal. Thus, the system is not causal.

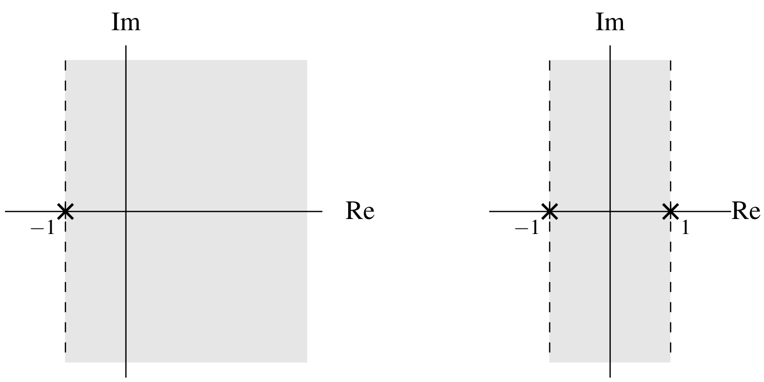
(d) The system function H has a right-half plane ROC but is not rational. Thus, we cannot make any conclusion directly from the system function. Instead, we draw our conclusion from the impulse response h. Taking the inverse Laplace transform of H, we obtain

$$h(t) = e^{-(t+1)}u(t+1).$$

not causal function since $h(t) \neq 0$ for $t \in (-1, 0)$

Thus, the impulse response h is not causal. Therefore, the system is not causal.

H is rational and ROC is RHP $\Rightarrow$ h is causal



(a) (b)

H is rational and ROC is not RHP $\Rightarrow$ h is not causal

Figure 7.19: Pole and ROCs of the rational system functions in the causality example. The cases of the (a) first (b) second system functions.

■

Example 7.32. A LTI system has the system function

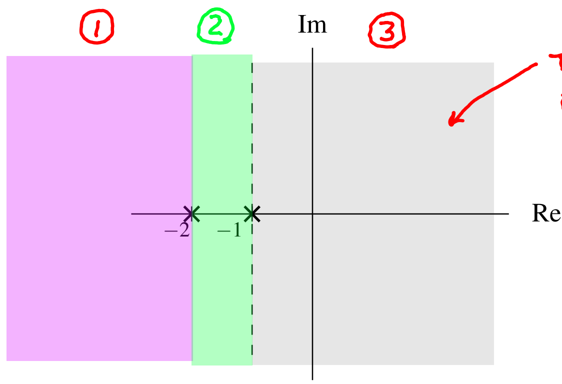
$$H(s) = \frac{1}{(s+1)(s+2)}.$$

Given that the system is BIBO stable, determine the ROC of H.

Solution. Clearly, the system function H is rational with poles at -1 and -2. Therefore, only three possibilities exist for the ROC:

i) $\mathrm{Re}(s) < -2$,
ii) $-2 < \mathrm{Re}(s) < -1$, and
iii) $\mathrm{Re}(s) > -1$.

In order for the system to be stable, however, the ROC of H must include the entire imaginary axis. Therefore, the ROC must be $\mathrm{Re}(s) > -1$. This ROC is illustrated in Figure 7.20.



This ROC contains the imaginary axis.

Figure 7.20: ROC for example.

■

Example 7.33. A LTI system is causal and has the system function

$$H(s) = \frac{1}{(s+2)(s^2+2s+2)}.$$

Determine whether this system is BIBO stable.

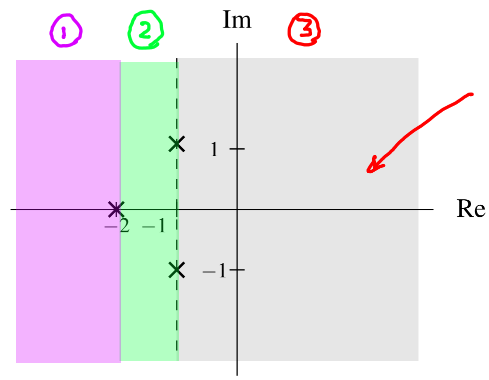
Solution. We begin by factoring H to obtain

$$H(s) = \frac{1}{(s+2)(s+1-j)(s+1+j)}.$$

(Using the quadratic formula, one can confirm that $s^2+2s+2=0$ has roots at $s=-1\pm j$.) Thus, H has poles at -2, $-1+j$, and $-1-j$. The poles are plotted in Figure 7.21. Since the system is causal and all of the poles of H are in the left half of the plane, the system is stable.

Since causal, ROC of H is RHP

Three possibilities exist for the ROC of H as shown.



This ROC is RHP.
This ROC contains imaginary axis.

Figure 7.21: Poles of the system function.

■

Example 7.34. For each LTI system with system function H given below, determine the ROC of H that corresponds to a BIBO stable system.

(a) $H(s) = \dfrac{s(s-1)}{(s+2)(s+1+j)(s+1-j)}$;

(b) $H(s) = \dfrac{s}{(s+1)(s-1)(s-1-j)(s-1+j)}$;

(c) $H(s) = \dfrac{(s+j)(s-j)}{(s+2-j)(s+2+j)}$; and

(d) $H(s) = \dfrac{s-1}{s}$.

all rational functions

Solution. (a) The function H has poles at -2, $-1+j$, and $-1-j$. The poles are shown in Figure 7.22(a). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only three distinct ROCs are possible:

i) $\mathrm{Re}(s) < -2$,
ii) $-2 < \mathrm{Re}(s) < -1$, and
iii) $\mathrm{Re}(s) > -1$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $\mathrm{Re}(s) > -1$. This is the shaded region in the Figure 7.22(a).

(b) The function H has poles at -1, 1, $1+j$, and $1-j$. The poles are shown in Figure 7.22(b). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only three distinct ROCs are possible:

i) $\mathrm{Re}(s) < -1$,
ii) $-1 < \mathrm{Re}(s) < 1$, and
iii) $\mathrm{Re}(s) > 1$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $-1 < \mathrm{Re}(s) < 1$. This is the shaded region in Figure 7.22(b).

(c) The function H has poles at $-2+j$ and $-2-j$. The poles are shown in Figure 7.22(c). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only two distinct ROCs are possible:

i) $\mathrm{Re}(s) < -2$ and
ii) $\mathrm{Re}(s) > -2$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $\mathrm{Re}(s) > -2$. This is the shaded region in Figure 7.22(c).

(d) The function H has a pole at 0. The pole is shown in Figure 7.22(d). Since H is rational, it cannot converge at 0 (which is a pole of H). Consequently, the ROC can never include the entire imaginary axis. Therefore, the system function H can never be associated with a stable system. ■