$$c(t) = \cos(\omega_c t)$$
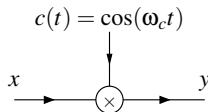
$x \longrightarrow \otimes \longrightarrow y$

$$y(t) = \cos(\omega_c t)x(t)$$

take FT

$$X = \mathcal{F}x, \quad Y = \mathcal{F}y$$

use modulation property
not multiplication property!

$$
\begin{aligned}
Y(\omega) &= \mathcal{F}\{\cos(\omega_c t)x(t)\}(\omega) \\
&= \mathcal{F}\left\{\tfrac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)x(t)\right\}(\omega) \\
&= \tfrac{1}{2}\left[\mathcal{F}\{e^{j\omega_c t}x(t)\}(\omega) + \mathcal{F}\{e^{-j\omega_c t}x(t)\}(\omega)\right] \\
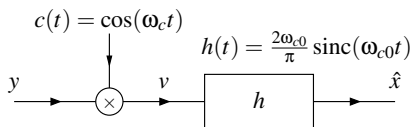&= \tfrac{1}{2}\left[X(\omega - \omega_c) + X(\omega + \omega_c)\right]
\end{aligned}
$$

Euler

linearity property

modulation property

$$c(t) = \cos(\omega_c t)$$

$$h(t) = \frac{2\omega_{c0}}{\pi} \operatorname{sinc}(\omega_{c0} t)$$

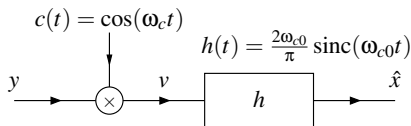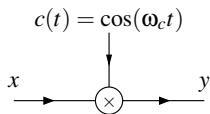$$y \longrightarrow \otimes \xrightarrow{v} \boxed{h} \xrightarrow{\hat{x}}$$

$$v(t) = \cos(\omega_c t) y(t), \quad h(t) = \frac{2\omega_{c0}}{\pi} \operatorname{sinc}(\omega_{c0} t), \quad \hat{x}(t) = v * h(t)$$

① ② ③

$$Y = \mathcal{F}y, \quad V = \mathcal{F}v, \quad H = \mathcal{F}h, \quad \hat{X} = \mathcal{F}\hat{x}$$

use modulation property, not multiplication property !!!

$$V(\omega) = \mathcal{F}\{\cos(\omega_c t) y(t)\}(\omega) \quad \text{FT of ①}$$

$$= \mathcal{F}\left\{ \frac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) y(t) \right\}(\omega) \quad \text{Euler}$$

$$= \frac{1}{2}\left[\mathcal{F}\left\{e^{j\omega_c t} y(t)\right\}(\omega) + \mathcal{F}\left\{e^{-j\omega_c t} y(t)\right\}(\omega) \right] \quad \text{linearity property}$$

$$= \frac{1}{2}\left[Y(\omega - \omega_c) + Y(\omega + \omega_c) \right] \quad \text{modulation property}$$

linearity

$$H(\omega) = \mathcal{F}\left\{ \frac{2\omega_{c0}}{\pi} \operatorname{sinc}(\omega_{c0} t) \right\}(\omega) \quad \text{FT of ②}$$

from FT table

$$\frac{B}{\pi}\operatorname{sinc}(Bt) \xleftrightarrow{\text{FT}} \operatorname{rect}\left(\frac{w}{2B}\right)$$

$$= 2\operatorname{rect}\left(\frac{\omega}{2\omega_{c0}} \right)$$

$$\hat{X}(\omega) = H(\omega)V(\omega) \quad \text{FT of ③ using convolution property}$$

DSB-SC AM: Complete System

$$c(t) = \cos(\omega_c t)$$

$$x \xrightarrow{\quad} \otimes \xrightarrow{\quad} y$$

$$c(t) = \cos(\omega_c t) \qquad h(t) = \frac{2\omega_{c0}}{\pi} \operatorname{sinc}(\omega_{c0} t)$$

$$y \xrightarrow{\quad} \otimes \xrightarrow{\ v\ } \boxed{h} \xrightarrow{\quad} \hat{x}$$

① $\quad Y(\omega) = \frac{1}{2}\left[X(\omega - \omega_c) + X(\omega + \omega_c)\right]$ ← *from result for transmitter*

② $\quad V(\omega) = \frac{1}{2}\left[Y(\omega - \omega_c) + Y(\omega + \omega_c)\right]$ ← *from result for receiver*

$\qquad = \frac{1}{2}\left[\frac{1}{2}\left[X([\omega - \omega_c] - \omega_c) + X([\omega - \omega_c] + \omega_c)\right] + \right.$ ← *substitute* ①

$\qquad\qquad \left. \frac{1}{2}\left[X([\omega + \omega_c] - \omega_c) + X([\omega + \omega_c] + \omega_c)\right]\right]$

$\qquad = \frac{1}{2}X(\omega) + \frac{1}{4}X(\omega - 2\omega_c) + \frac{1}{4}X(\omega + 2\omega_c)$ ← *Simplify*

③ $\quad \hat{X}(\omega) = H(\omega)V(\omega)$ ← *from result for receiver*

$\qquad = H(\omega)\left[\frac{1}{2}X(\omega) + \frac{1}{4}X(\omega - 2\omega_c) + \frac{1}{4}X(\omega + 2\omega_c)\right]$ ← *substitute* ②

$\qquad = \frac{1}{2}H(\omega)X(\omega) + \frac{1}{4}H(\omega)X(\omega - 2\omega_c) + \frac{1}{4}H(\omega)X(\omega + 2\omega_c)$ ← *multiply*

$\qquad = \frac{1}{2}\left[2X(\omega)\right] + \frac{1}{4}(0) + \frac{1}{4}(0)$ ← $X(\omega-2\omega_c)=0$ and $X(\omega+2\omega_c)=0$ when $H(\omega)\neq 0$ (since $\omega_b < \omega_{c0} < 2\omega_c - \omega_b$)

$\qquad = X(\omega)$ ← *Simplify*