Sample Test Two: Sets 11 to 20

1. The probability that a caller to a help-line will be satisfied with their service is 0.7, independently of all other callers. Suppose that 20 callers to the help-line are selected at random. What is the probability that at least 11 but no more than 17 customers are satisfied with the service they receive?

Questions 2, 3, and 4 refer to the following scenario:

The mass of white lab rats is known to be normally distributed with a mean of $\mu = 20$ grams and a standard deviation of $\sigma = 1.6$ grams.

2. What is the probability that a randomly selected white lab rat will have a mass exceeding 22 grams?

3. Find the mass α such that 11.5% of all white lab rats have a mass below α.

4. If a randomly selected rat has a mass of at least $16\ g$, what is the probability that its mass will be no more than $22\ g$?

Questions 5 and 6 refer to the following scenario:

Consider the following joint probability function of X and Y.

		y		
$f(x,y)$		0	1	2
x	0	0.4	0.1	0.2
	5	0.1	0.1	0.1

5. Find $P(Y \geq 1 | X = 5)$.

6. Find $Cov(X, Y)$.

Questions 7 and 8 refer to the following scenario:

The lifespans of light-emitting diodes are known to be exponentially distributed, with a mean lifespan of 12 years.

7. Find the probability that a randomly selected light-emitting diode will have a lifespan of at least 10 years.

8. What is the 75th percentile? That is, what is the value of α such that 75% of all diodes have a lifespan less than α?

9. The number of customers that pass through a grocery-checkout has a Poisson distribution, with an average of 4 customers every half-hour. Suppose we observe the checkout for one hour. What is the probability that no more than the expected number of customers will pass through the checkout?

10. Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 2x^{-3} & 1 \le x < \infty \\ 0 & \text{otherwise} \end{cases}$$

(a) Calculate $P(2 \le X \le 3)$

(b) Calculate $E(X)$.

11. Suppose that X_1, X_2, X_3 are independent normal random variables with means $10, 11, 15$ (respectively), and standard deviations $2, 5, 1$ (respectively). Let $Y = 3X_1 + X_2 - 2X_3$. Calculate $P(10 \le Y \le 11.5)$.

Answers:

1. 0.91655
2. 0.1056
3. 18.08
4. ≈ 0.89374
5. 2/3
6. 0.3
7. ≈ 0.4346
8. ≈ 16.63553233
9. 0.5925
10.(a) 5/36
10.(b) 2
11. 0.0717