University of Victoria
Exam 4
Fall 2023

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 11010), A02 (CRN 11011)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ______________________________

Given Name(s): ______________________________

Student Number: _______________

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

Question 1. A function x has the Fourier transform X as given by

$$X(\omega) = \begin{cases} -1 & -2 \leq \omega \leq -1 \\ 1 & 1 \leq \omega \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

Without explicitly finding x, determine whether x has each of the following properties: (a) real; (b) finite energy; (c) finite duration. In the case of each property, you must **explicitly state** any conditions being tested. **Do not skip any steps** in your solution and **show all of your work**. **[3 marks]**

Question 2. Using properties of the Fourier transform, find the Fourier transform Y of the function y in terms of the Fourier transform X of the function x, where $y(t) = e^{-j3t}(t/2)x(t/2)$. The final answer for y must be **fully simplified**, **including the evaluation of derivatives**. (**Hint**: A solution that does not require the use of the chain rule is possible.) You must use a **systematic method**, **show all of your work**, and you **must not skip any steps**. A correct final answer with an incorrect or incomplete justification (or one that cannot be **clearly understood** by the marker) may receive **zero marks**. **[6 marks]**

Question 3. A LTI electronic circuit has input v_1, output v_2, and frequency response $H(\omega) = \frac{\omega^2}{\omega^2 - 2j\omega - 1}$. In the remainder of this question, let V_1 and V_2 denote the Fourier transforms of v_1 and v_2, respectively.

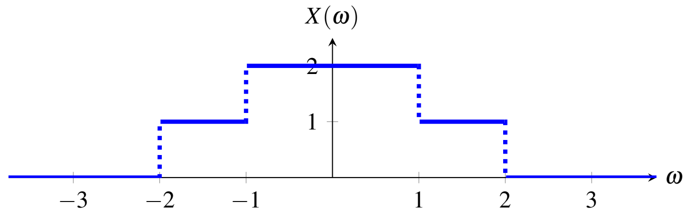
(A) Find **fully simplified** expression for the magnitude response of the circuit. [**Hint**: A fully simplfied answer will not contain any square root operations.] **[3 marks]**

QUESTION 3 CONTINUED

(B) Determine the type of ideal frequency-selective filter that the circuit best approximates. Your answer must be **fully justified**. (A correct final answer with no justification will receive **zero marks**.) **[1 mark]**

Question 4.

Consider the function x with the frequency spectrum X as shown in the figure.



(A) Determine the lowest rate at which x can be sampled such that aliasing does not occur. **Do not skip any steps** in your solution and **show all of your work**. Your answer **must be justified** (e.g., any relevant formulas/identities/theorems must be clearly identified). **[1 mark]**

(B) Consider the function $y(t) = x(t)\cos(8t)$. Find and plot the frequency spectrum Y of y. Determine the lowest rate at which y can be sampled in order to avoid aliasing. **Do not skip any steps** in your solution and **show all of your work**. Your answer **must be justified**. **[3 marks]**

QUESTION 4 CONTINUED

(C) Let s denote the signal obtained by (impulse) sampling the signal x with the sampling frequency $\omega_s = 2$; and let r denote the signal obtained by applying bandlimited interpolation (i.e., lowpass filtering) to s. In other words, we have

$$s(t) = x(t)p(t) \text{ where } p(t) = \sum_{k=-\infty}^{\infty} \delta\left(t - \tfrac{2\pi}{\omega_s}k\right); \quad \text{and} \quad r(t) = s * h(t) \text{ where } h(t) = \operatorname{sinc}\left(\tfrac{\omega_s t}{2}\right).$$

Let S and R denote the frequency spectra of s and r, respectively. Plot S and R. (Tables/formulas may be used to reduce the amount of work in your answer.) **[3 marks]**

Question 5. Consider a LTI system with input x, output y, and impulse response $h(t) = e^{-3t}u(t)$. **Using the Fourier transform**, find a **fully simplified** formula for y in the case that $x(t) = e^{-3t}\cos(2t)u(t)$. **Do not skip any steps** in your solution and **show all of your work**. **[5 marks]**

END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$$
$$c_k = \tfrac{1}{T}\int_T x(t)e^{-j(2\pi/T)kt}\,dt$$
$$S(\omega) = \tfrac{\omega_s}{2\pi}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

$$\mathscr{F}x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$
$$\mathscr{F}^{-1}X(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$
$$H(\omega) = \tfrac{2\pi}{\omega_s}\operatorname{rect}\left(\tfrac{\omega}{\omega_s}\right)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$
$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$
$$a_k = \tfrac{1}{T}X_T(k\omega_0)$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
8	$\operatorname{rect}\left(\frac{t}{T}\right)$	$\lvert T\rvert\operatorname{sinc}\left(\frac{T\omega}{2}\right)$
9	$\operatorname{sinc}(Bt)$	$\frac{\pi}{\lvert B\rvert}\operatorname{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$
12	$e^{-at}\cos(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
14	$e^{at}u(-t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a-j\omega}$