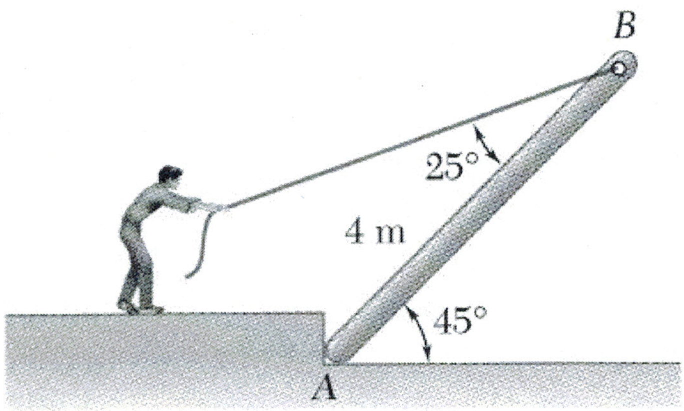
A man raises a 10 kg joist, of length 4 m, by pulling on a rope. Find the tension in the rope and the reaction at *A*.

a) Using a graphical method

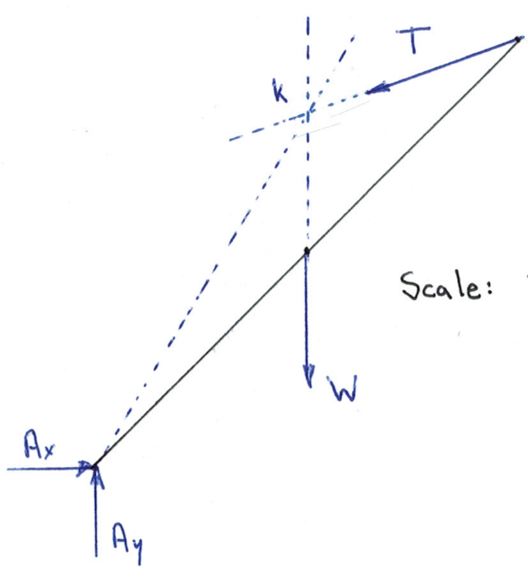
b) Using an analytical method



a) Draw a FBD of the system.

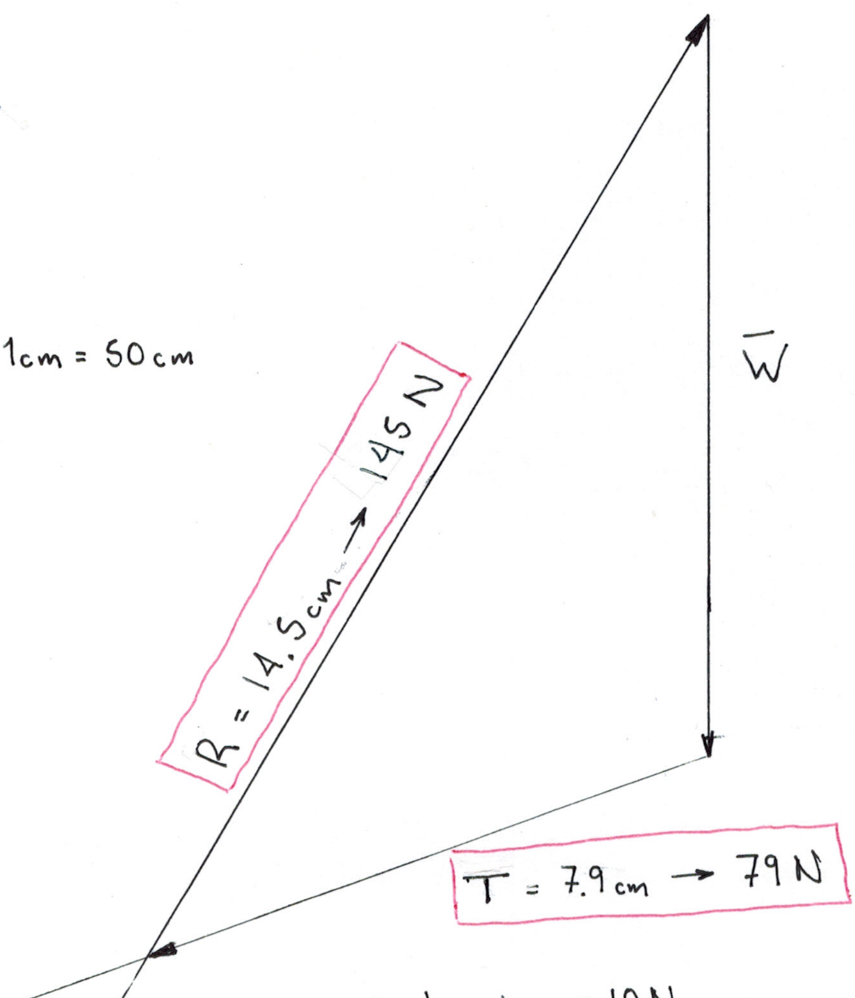
Since there are three forces acting on the joist we can consider it as a three-force body

Intersect the lines of the known force directions (W and T). The reaction force R must also intersect that point.



Scale: 1cm = 50 cm

Once the direction of the forces is known, draw a scaled vector of $W = 98.1\,N$ (9.81 cm). Draw lines of the other two forces. Measure and scale

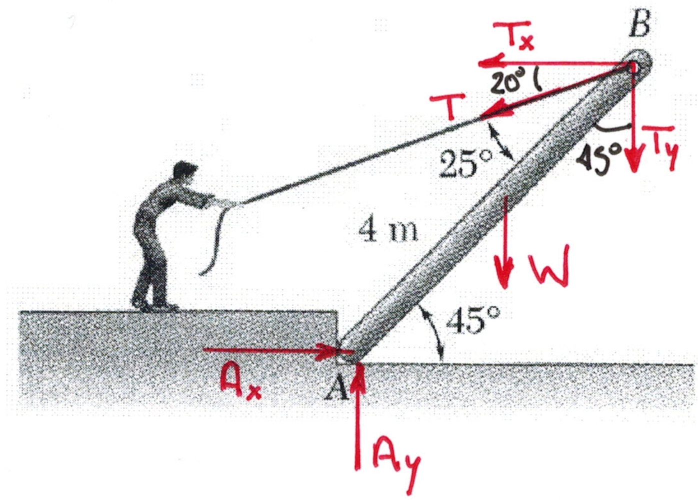


Scale: 1cm = 10 N

A man raises a 10 kg joist, of length 4 m, by pulling on a rope. Find the tension in the rope and the reaction at A.

a) Using the graphical solution

b) Using rectangular components



$T_x = \|T\| \cos(20)\, i$

$T_y = \|T\| \sin(20)\, j$

$W = mg = 10(9.81) = 98.1\ N$

$\Sigma M_A = 0$

$$T_x (4 \sin 45) - T_y (4 \cos 45) - W(2 \cos 45) = 0$$

$$\|T\| \cos 20 (4 \sin 45) - \|T\| \sin 20 (4 \cos 45) - 98.1(2 \cos 45) = 0$$

$$\|T\| = \frac{98.1 (2 \cos 45)}{\cos 20(4 \sin 45) - \sin 20(4 \cos 45)} = 82.07\ N$$

$\Sigma F_x = 0$

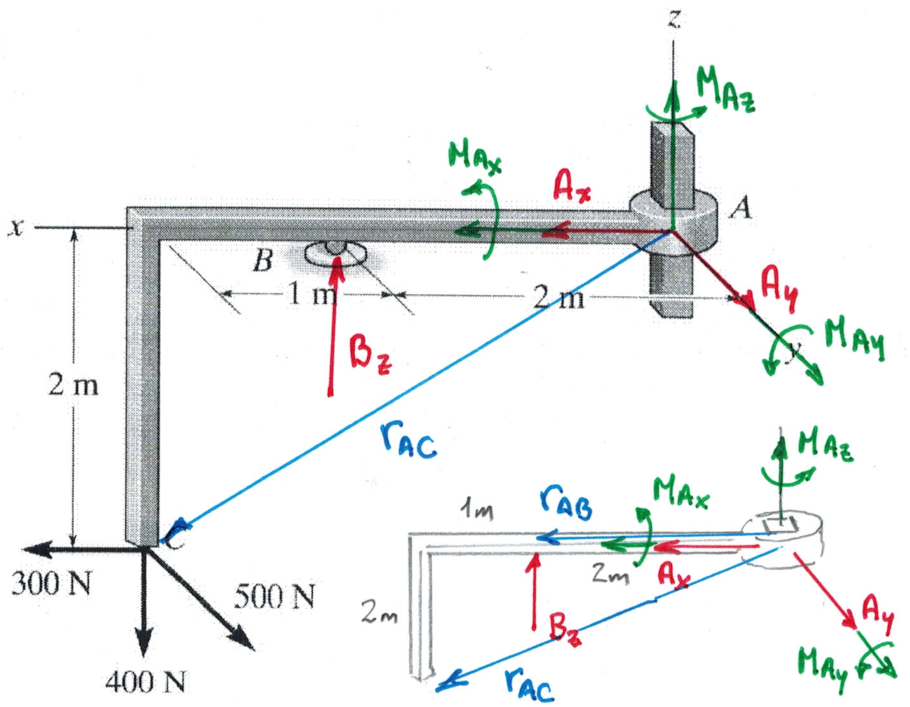
$$A_x - T_x = 0 \qquad A_x = \|T\| \cos(20) = 77.12\ N$$

$\Sigma F_y = 0$

$$A_y - T_y - W = 0 \qquad A_y = \|T\| \sin(20) + 98.1 = 126.17\ N$$

$$A = \sqrt{A_x^2 + A_y^2} = 147.9\ N \qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = 58.6°$$

The member is supported by a square rod which fits loosely through the smooth square hole of the attached collar at *A* and by a roller at *B*. Determine the components of reaction at these supports when the member is subjected to the loading shown.



$$F = \{300i + 500j - 400k\}\ N$$

$$F_A = \{A_x i + A_y j\}\ N$$

$$F_B = B_z k\ N$$

$$M_A = \{M_{A_x} i + M_{A_y} j + M_{A_z} k\}$$

$$r_{AC} = \{3i - 2k\}$$

$$r_{AB} = 2i$$

$\Sigma F = 0$ $\qquad F_A + F_B + F = 0$

$$(300 + A_x)i + (500 + A_y)j + (-400 + B_z)k = 0$$

$A_x = -300\ N$

$A_y = -500\ N$

$B_z = 400\ N$

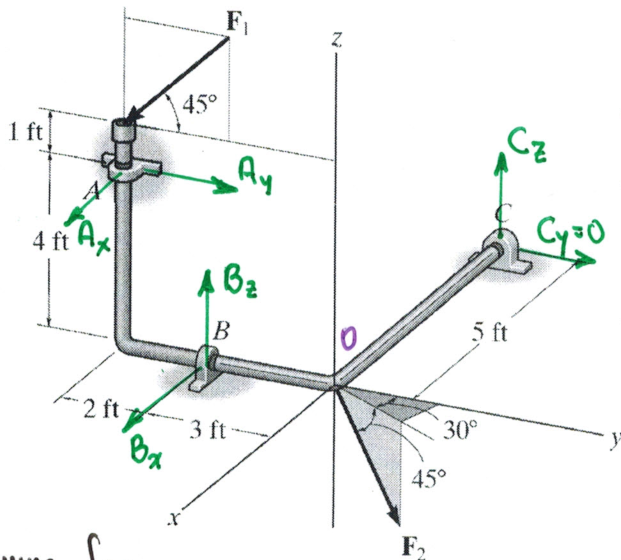
$\Sigma M_A = 0$ $\qquad M_A + r_{AB} \times F_B + r_{AC} \times F = 0$

$$M_{A_x} i + M_{A_y} j + M_{A_z} k + \begin{vmatrix} i & j & k \\ 2 & 0 & 0 \\ 0 & 0 & 400 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 0 & -2 \\ 300 & 500 & -400 \end{vmatrix} = 0$$

$$(M_{A_x} + 1000)i + (M_{A_y} - 800 + 600)j + (M_{A_z} + 1500)k = 0$$

$M_{A_x} = -1000\ N \cdot m \qquad M_{A_y} = 200\ N \cdot m \qquad M_{A_z} = -1500\ N \cdot m$

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.



A single journal bering will develop two reaction forces and two reaction moments; however, with three bearings aligned in different directions, there will be two forces in each direction, say Ax and Bx, that constrain the tendency of rotation about z. Thus, in total, no moment couples are developed

Resolving forces

$$F_1 = \{-300\cos 45\,\hat{j} - 300\sin 45\,\hat{k}\} = \{0\hat{i} - 212.1\hat{j} - 212.1\hat{k}\}$$

$$F_2 = \{\|F_2\|\cos 45^\circ \sin 30^\circ\,\hat{i} + \|F_2\|\cos 45\cos 30\,\hat{j} - \|F_2\|\sin 45\,k\} = \|F_2\|\{0.3536\hat{i} + 0.6124\hat{j} - 0.7071$$

Applying eqs. of equilibrium

Moments about O

$\Sigma F_x = 0 \quad A_x + B_x + 0.3536\|F_2\| = 0$

$\Sigma F_y = 0 \quad A_y + 0.6124\|F_2\| - 212.1 = 0$

$\Sigma F_z = 0 \quad B_z + C_z - 0.7071\|F_2\| - 212.1 = 0$

$\Sigma M_x = 0 \quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0 \quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0 \quad A_x(5) + B_x(3) = 0$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0.3536 \\ 0 & 1 & 0 & 0 & 0 & 0.6124 \\ 0 & 0 & 0 & 1 & 1 & -0.7071 \\ 0 & -4 & 0 & -3 & 0 & 0 \\ 4 & 0 & 0 & 0 & 5 & 0 \\ 5 & 0 & 3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ B_x \\ B_z \\ C_z \\ \|F_2\| \end{bmatrix} = \begin{bmatrix} 0 \\ 212.1 \\ 212.1 \\ -2121 \\ 0 \\ 0 \end{bmatrix}$$

Solve $Ax = b \Rightarrow x = A^{-1}b = \underbrace{A\backslash b}_{\text{Matlab}}$

$A_x = 357$ lb

$A_y = -200$ lb

$B_x = -595$ lb

$B_y = 974$ lb

$C_z = -286$ lb

$\|F_2\| = 674$ lb