Unit 2: Propositional Logic
Translations

One of the topics that we covered in Unit 1 was the counter example method. We used this method to help us prove that a deductive argument was invalid. However, we found that this method was not useful if the argument pattern happened to be valid. In Unit 2 we will use a method that will allow us to determine whether a deductive argument pattern is valid or invalid. We will be able to decide the validity of any argument pattern by using something called a truth table. But before we can learn to use truth tables we must first learn how to take English statements and translate them into logical symbols in a system of logic called 'propositional logic'.

Translations

Validity and Propositional Forms

It is often easy to see *that* one claim follows from another, but to explain *why* can be difficult.

What makes an argument form valid?

A *proposition* is a singular statement.
E.g. "John is tall".

Propositions are made into compound statements using *connectives*.

John is tall *and* Harry is short.

What makes the statement "John is tall and Harry is short" true? Well, the truth of the statement depends both on the truth of the smaller sentences and the logical behavior of the word "and". "And" is called a logical connective. It determines the truth value of a compound sentence in a very specific way. Other logical connectives are words like "or", "if … then" and "not".

The first logical connective we will consider will be the conjunction. These are words like “and” and “but”. We will symbolize the conjunction with the symbol ‘&’ (called an ‘ampersand’). Other textbooks may use a dot “·” or a wedge “^” for the conjunction. If you have trouble drawing the “&” then you can try something like this “ ٩ ”.

Conjunction (&)

What truth conditions govern this connective?

P	Q	P & Q
T	T	
T	F	
F	T	
F	F	

The chart shown above is called a truth table. Notice that under the individual letters all of the possible combinations of true and false sentences are considered. That is, the top line considers what happens when both sentences are true, the bottom line when both sentences are false.

To determine the values under the “&” we just need to consider how any two English statements behave. Let’s consider the sentences “John is tall” and “Harry is short”. The sentence “John is tall and Harry is short” will only be true when both sides of the conjunction are true. All other combinations will be false.

Now let’s look at how we will translate English sentences that have an “and” into symbolic logic.

The truth values of “and” do not depend on what propositions we use.

P & Q	Roses are red and violets are blue.
P	Roses are red and violets are blue.
P & Q	Roses are red and roses are red.
P & P	Roses are red and roses are red.
P & P	Roses are red and violets are blue.
P & P	Roses are red.

“and” is not always used to connect two distinct sentences.

Steffi Graf and Monica Seles are tennis players.

Steffi Graf and Monica Seles are playing each other.

Steffi and Monica are playing tennis.

Jack, Jill and Wendle went up the hill.

Jack and Jill went up the hill and fell down the hill.

Jack went up the hill but Jill went down the hill.

Although Wendle went up the hill, Jack went down.

Jane speaks both French and English.

Someone who speaks both French and English is bilingual.

The next connective to consider is the disjunction. The symbol “$\vee$” will be used to replace words like “or”. In the truth table below, consider what values the smaller sentences “John will win” and “Harry will win” will need to have in order for the disjunction to be true.

<u>Disjunction ($\vee$)</u>

John will win or Harry will win.
P v Q

P	Q	P v Q
T	T	
T	F	
F	T	
F	F	

Inclusive ‘or’ (T v T = T)

E.g. I can bring the salad or the dessert to dinner. (Still okay to bring both.)

S v D

Exclusive ‘or’ (T v T = F)

E.g. I will graduate with a B.Sc. or a BA in psychology. (Not both)

(S v A) & ~(S & A)

Jack or Jill went up the hill but Wendle went down.

Henry will go to the picnic unless it rains. (H, R)

The next connective is a bit different from the previous two. The negation (i.e. "not") does not connect two sentences together but rather, applies only to the sentence that follows. Since the connector does not join two sentences, the truth table for the negation will only need two rows. The symbol we use for the negation is "~" (known as a tilde). The tilde applies to whatever directly follows the symbol.

~A & B In this sentence the "~" only applies to the A.

~(A & B) In this sentence the "~" applies to the whole bracket (A & B).

Negation (~)

P	~P
T	
F	

John is not clever.

Nobody owns Mars.

John is unlucky.

Mary hit a home run and a triple.

Dogs don't like bumblebees.

Neither John nor Mary likes gooseberries.

Not both O'Toole and Singh will be elected.

Either Ted or John will bring the dessert but not both.

John and Mary are married but not to each other.

The final connective that we will consider is the conditional or if … then statements. We will use the symbol "→" with the antecedent of the if … then statement (i.e. the part after the 'if') always going in front of the arrow.

Conditionals (→)

If ________ then __________.
Antecedent consequent

If you do your homework then I'll buy you a beer. $H \rightarrow B$

H	B	H → B
T	T	
T	F	
F	T	
F	F	

The conditional will only be false when the antecedent is true and the conclusion is false.

If it rains then I'll bring an umbrella.

If it doesn't rain then I won't bring an umbrella.

I'll go to the movies if I get my essay finished.

If Juan misses the game, we'll lose for sure.

Provided I get the loan from the bank, I'll buy the car.

I get a cold whenever I stay up late.

Only If and Unless

I'll go to the party if I'm invited.

I'll go to the party only if I'm invited.

To see the difference consider what happens when I'm not invited.

If I'm not invited then I won't go to the party.

P if Q. $Q \rightarrow P$
P only if Q. $P \rightarrow Q$

A student may receive BC student loans only if the student attends a BC school.

I will buy the car unless I lose my job.

C = I will but the car.
L = I lose my job.

C unless L   C v L

$\sim L \rightarrow C$

Now that we know how all the logical connectives behave (i.e. see their truth tables), we can now figure out the truth values of more complicated statements.

We can now construct propositions whose truth value depends on the truth value of its parts.

Let A, B, C be true. X, Y, Z are false.

~X v Y   ~(X v Y)

$\sim(A \rightarrow \sim(Z \vee X))$

$A \vee ((\sim B \;\&\; C) \vee \sim(\sim B \rightarrow \sim(Z \rightarrow B)))$