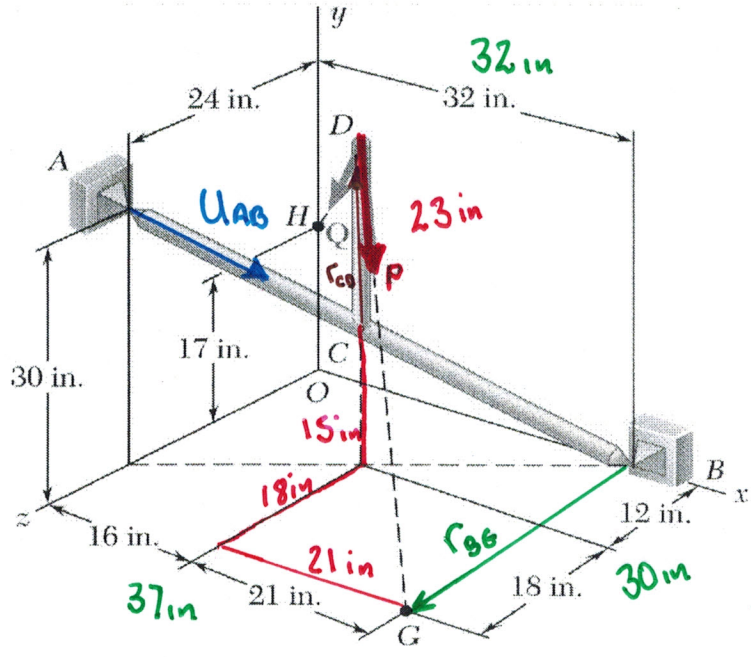
In the midpoint of a 50-in. rod *AB*, there is a vertical rod *CD* that measures 23-in. Determine the moment about *AB* of the 235-lb force **P**.



The moment about an axis is obtained using the triple scalar product

$$M_{AB} = u_{AB} \cdot (r_{BG} \times P)$$

P force vector

r_{BG} any point of axis AB to any point of force P

u_{AB} unit vector of axis AB

- Find force vector

$$P = \|P\| u_{DG} = \|P\| \frac{r_{DG}}{\|r_{DG}\|}$$

$$= 235 \frac{\{21i - 38j + 18k\}}{47}$$

$$= \{105i - 190j + 90k\} \text{ lb}$$

$$r_{DG} = \{21i - 38j + 18k\} \text{ in}$$

$$\|r_{DG}\| = \sqrt{21^2 + 38^2 + 18^2} = 47 \text{ in}$$

- Find Position vector r_{BG}

$$r_{BG} = \{(37 - 32)i + 0j + 30k\} = \{5i + 0j + 30k\} \text{ in}$$

- Unit vector u_{AB}

$$r_{AB} = \{(32 - 0)i + (0 - 30)j + (0 - 24)k\} = \{32i - 30j - 24k\} \text{ in}$$

$$\|r_{AB}\| = \sqrt{32^2 + 30^2 + 24^2} = 50 \text{ in}$$

$$u_{AB} = \frac{r_{AB}}{\|r_{AB}\|} = \{0.64i - 0.6j - 0.48k\}$$

Apply the triple scalar product

$$M_{AB} = u_{AB} \cdot (r_{BG} \times P) = \left| \begin{bmatrix} 0.64 & -0.6 & -0.48 \\ 5 & 0 & 30 \\ 105 & -190 & 90 \end{bmatrix} \right|$$

$$M_{AB} = 0.64\,(0(90) - 30(-190)) - (-0.6)(5(90) - (30)(105)) + (-0.48)(5(-190) - 0(105$$

$$= 2484 \text{ lb} \cdot \text{in}$$

$$M_{AB} = 207 \text{ lb} \cdot \text{ft}$$

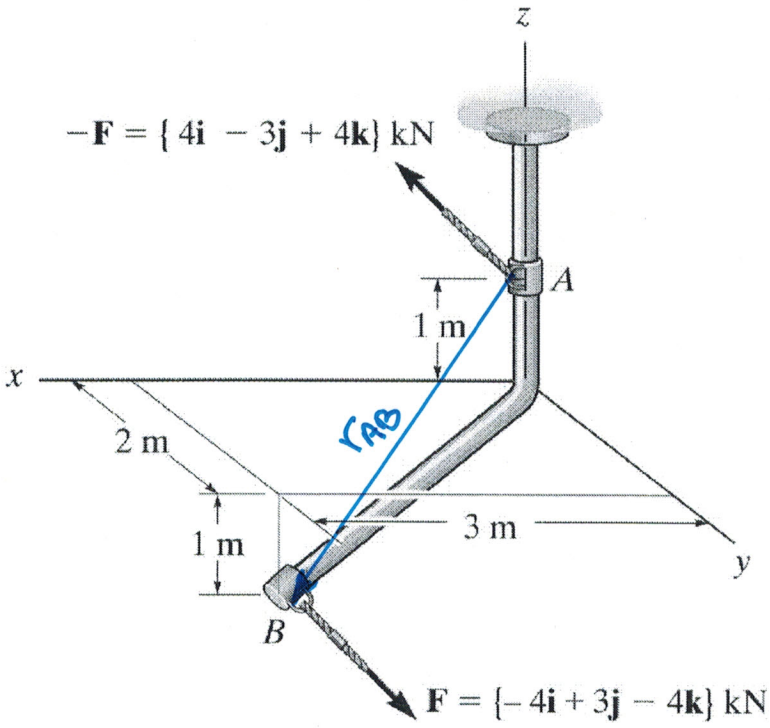
Note that if we use another position vector from a point along axis AB to a point along the line of action of P the result will be the same. Consider vector $r_{CD} = \{0\hat{i} + 23\hat{j} + 0\hat{k}\}$

$$M_{AB} = u_{AB} \cdot (r_{CD} \times P) = \left| \begin{bmatrix} 0.64 & -0.6 & -0.48 \\ 0 & 23 & 0 \\ 105 & -190 & 90 \end{bmatrix} \right|$$

$$= 23\left((0.64)(90) - (-0.48)(105)\right) = 2,484 \text{ lb} \cdot \text{in}$$

Same as before

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?



Coordinates

A(0, 0, 1)

B(3, 2, −1)

$r_{AB} = \overline{AB} = \{3i + 2j - 2k\}$ m

Determine the couple moment

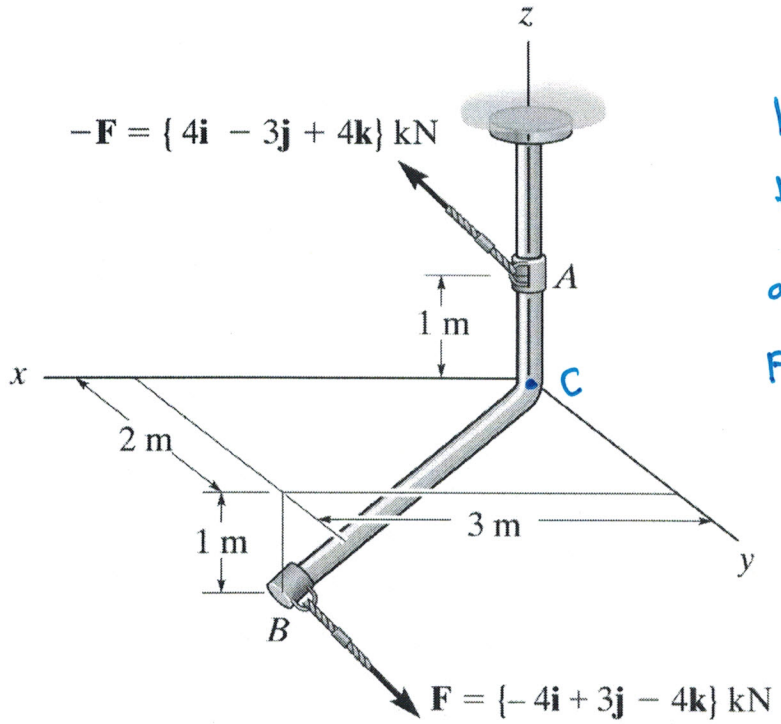
$$M_C = r_{AB} \times F = \begin{vmatrix} i & j & k \\ 3 & 2 & -2 \\ -4 & 3 & -4 \end{vmatrix} = \{-2i + 20j + 17k\} \text{ kN·m}$$

M_C: moment of a couple

Magnitude

$$\|M_C\| = \sqrt{2^2 + 20^2 + 17^2} = 26.3 \text{ kN·m}$$

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?



In this example, we will treat the forces independently, and find the moment about points A, B, and C.

$r_{AB} = \{3i + 2j - 2k\}$

$r_{BA} = \{-3i - 2j + 2k\}$

$r_{CA} = \{0i + 0j + 1k\}$

$r_{CB} = \{3i + 2j - 1k\}$

$$M_A = r_{AB} \times F = \begin{vmatrix} i & j & k \\ 3 & 2 & -2 \\ -4 & 3 & -4 \end{vmatrix} = \{-2i + 20j + 17k\}$$

same

Force $-F$ passes through A, so it does not produce a moment about A.

$$M_B = r_{BA} \times (-F) = \begin{vmatrix} i & j & k \\ -3 & -2 & 2 \\ 4 & -3 & 4 \end{vmatrix} = \{-2i + 20j + 17k\}$$

same

F passes through point B, no moment

$$M_C = r_{CA} \times (-F) + r_{CB} \times (F) = \begin{vmatrix} i & j & k \\ 0 & 0 & 1 \\ 4 & -3 & 4 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 3 & 2 & -1 \\ -4 & 3 & -4 \end{vmatrix}$$

$$= \{3i + 4j + 0k\} + \{-5i + 16j + 17k\} = \{-2i + 20j + 17k\}$$

same

The moment of a couple is a free vector, it depends only on the distance between the two forces.