ASTR 101 Lab 5
Date Of Lab: November 27, 2023

**Objective:**
The objective of this lab is to explore and understand Kepler's laws of planetary motion, particularly focusing on the orbits of Earth and Mars in our Solar System. Through a hands-on exercise, students will familiarize themselves with the heliocentric and geocentric coordinate systems used in astronomy. The goal is to replicate Kepler's method of determining the orbit of Mars based on observational data, emphasizing the application of triangulation and graphical techniques. This lab aimed to provide students with practical insights into orbital motion and the positioning of celestial bodies, enhancing their comprehension of the seasonal changes in the sky. We into Kepler's ground-breaking laws of planetary motion formulated around 1609. Kepler, building upon Tycho Brahe's meticulous observations, devised these laws solely from visual data, predating Galileo's telescopic observations. The lab focuses on the orbits of Earth and Mars, emphasizing their positions in both heliocentric and geocentric coordinate systems. The Earth's orbit around the Sun, known as the ecliptic, and the close adherence of planetary orbits to this plane are central themes.

**Discussion:**
The geometric construction of Mars's orbit based on Kepler's triangulation method, using heliocentric and geocentric coordinates, provided valuable insights into the dynamics of planetary motion. The calculated eccentricity of Mars's orbit ($e = 0.1125$) aligns with its slightly elliptical nature, and the derived perihelion and aphelion distances (1.405 AU and 1.694 AU, respectively) showcase the orbital variation. The analysis of distances between Earth and Mars during different positions revealed an average Earth-Mars distance of 1.48 AU. The discussion of Earth and Mars's relative proximity during oppositions, occurring approximately every 26 months, emphasizes the cyclical nature of their orbital alignment. Furthermore, the estimation of closest and farthest approaches based on the given dates adds a temporal dimension to understanding the changing configurations of these neighboring planets.

## Questions and Answers:

**(1) Since this lab exercise relates to Kepler's laws, use any reference, and write the three laws (remember to cite the reference).**

*Answer.* Johannes Kepler published his laws of planetary motion between 1609 and 1619. These laws modified Copernicus' heliocentric theory, replacing circular orbits with elliptical trajectories. Kepler's laws explain the orbits of planets around the Sun and how planetary velocities vary.

Kepler's Three Laws:

### First Law (Law of Ellipses):

*The orbit of every planet is an ellipse with the Sun at one of the two foci.*

**Implications:** The distance between a planet and the Sun changes as the planet moves along its orbit, and the Sun is offset from the center of the planet's orbit.

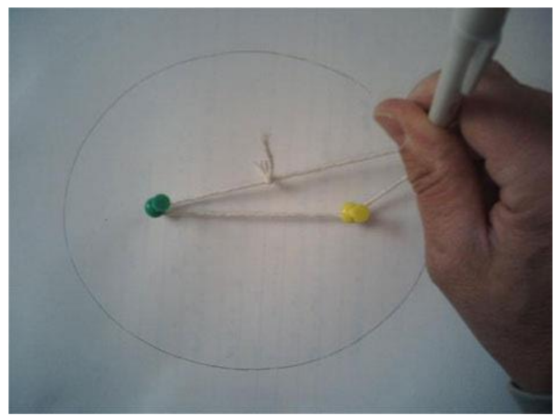
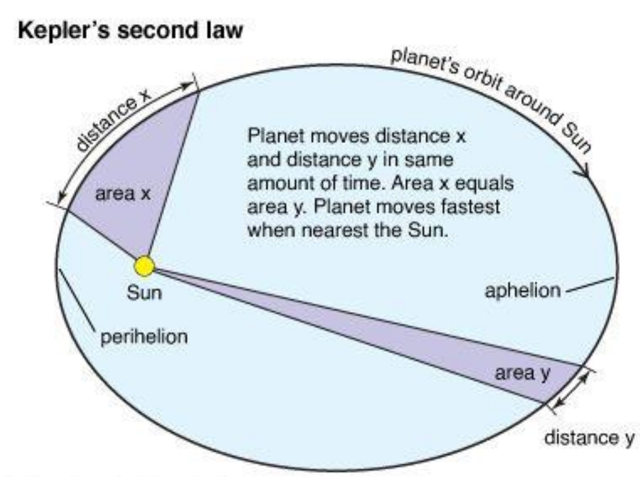


*Figure 1.* Law of Ellipses.




### Second Law (Law of Equal Areas):

*A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.*

**Implications:** Planets move faster when closer to the Sun (perihelion) and slower when farther away (aphelion).

*Figure 2.* Law of Equal Areas.

## Third Law (Harmonic Law):

*The ratio of the square of an object's orbital period to the cube of the semi-major axis of its orbit is constant for all objects orbiting the same primary.*

Kepler enunciated this law to relate planetary orbits to the "music of the spheres" and expressed it in terms of musical notation.

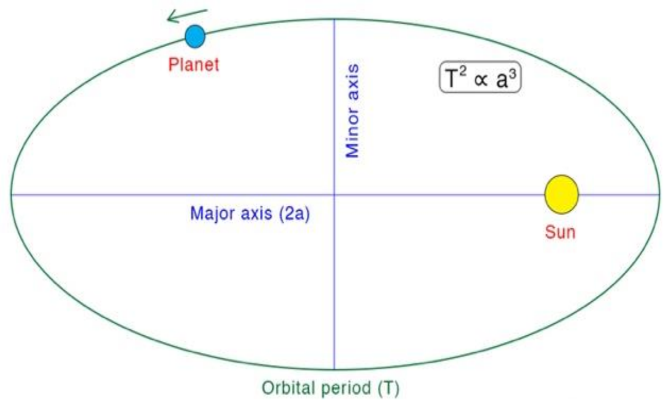



*Figure 3.* Harmonic Law.

**Additional Information:** Kepler's laws were inferred from calculations of Mars' orbit and later applied to other bodies in the Solar System. Isaac Newton showed in 1687 that Kepler's laws result from his own laws of motion and law of universal gravitation. Kepler's first law describes elliptical orbits, and the eccentricity of an ellipse is related to the distance between its foci. Kepler's second law implies changes in a planet's velocity along its orbit, with faster motion near perihelion and slower near aphelion. Kepler's third law establishes a mathematical relationship between a planet's orbital period and the semi-major axis of its orbit.

**(2) In the geometrical construction, we used the radius of the circle to represent the average distance between Earth and the Sun. This is a scale of 5cm = 1 AU. Using this scale, calculate the average distance (in AU) from the Sun to Mars. Calculate the distances to al six positions, then take the average.**

*Answer.* The distances from each intersecting points (Mars) and the Sun are as follows:

| Mars Points | Distance from Sun (in cm) | Distance from Sun (in AU) | Average (in AU) |
|---|---|---|---|
| A | 6.8 | 1.36 | $\frac{1.36+1.06+1.72+1.66+1.74+1.34}{6}$ = 1.48 AU |
| B | 5.3 | 1.06 | |
| C | 8.6 | 1.72 | |
| D | 8.3 | 1.66 | |
| E | 8.7 | 1.74 | |
| F | 6.7 | 1.34 | |

**(3) The eccentricity of Mars's orbit is defined as the ratio of the focal length to the semi-major axis. Calculate this based on your measurements.**

*Answer.*

Largest Diameter, D = 15.5 cm.
Semi-major axis = (Aphelion + Perihelion distances in cm) / 2 = (8.473778 + 7.026221) cm / 2 = 7.75 cm
Measured Focal Length = 0.872 cm

Eccentricity, e = 0.872 cm / 7.75 cm = 0.1125

**(4) What are the perihelion and aphelion distances of Mars in AU?**

*Answer.*

Aphelion distance = 8.473778 cm = (8.473778 cm / 5 cm) = 1.694 AU
Perihelion distance = 7.026221 cm = (7.026221 cm / 5 cm) = 1.405 AU

**(5) Based on the orbital periods of Earth and Mars, how often are the two planets closer than usual to each other? Discuss the reasons.**

*Answer.* To find out how often Earth and Mars are closer to each other than usual, we need to consider their orbital periods and positions in their respective orbits. The orbital period of Mars is about 687 days, which means that it takes Mars about 687 Earth days to complete one orbit around the Sun. On the other hand, Earth's orbital period is about 365 days. Because of this difference in orbital periods, the relative positions of Earth and Mars vary over time, and there are specific points in their orbits where they are closer or farther apart.

During opposition, Earth and Mars are aligned with the Sun, and Mars is relatively close to Earth. The opposition of Mars occurs approximately every 26 months or, 780 Earth days, the time it takes for a planet to return to the same relative position with respect to Earth and the Sun. While opposition brings Mars closer to Earth, not every opposition results in the closest possible approach due to the elliptical shape of Mars' orbit. The eccentricity of Mars' orbit is found to be e = 0.1125, indicating that the orbit is slightly elliptical.

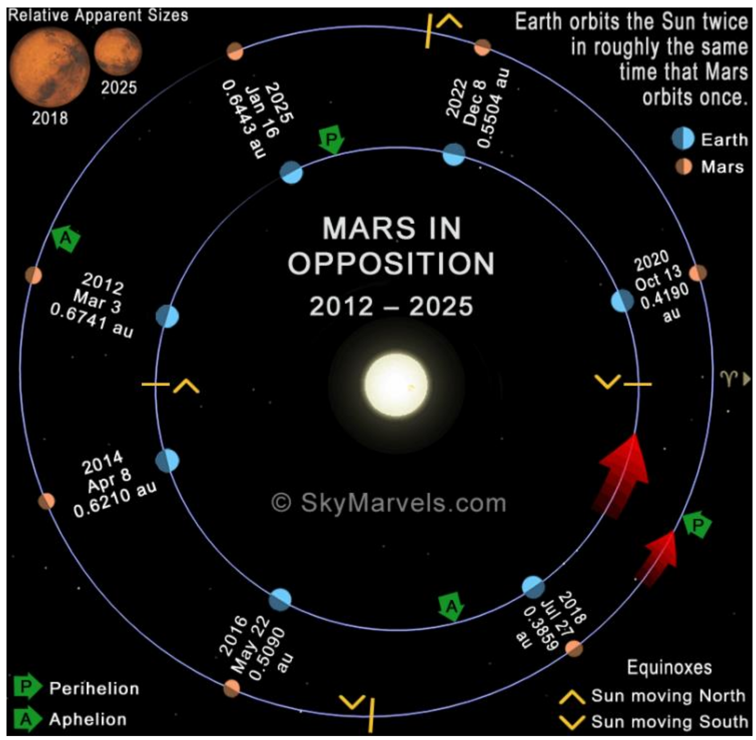



*Figure 4*. Mars Oppositions

From the figure 4, we can see that during perihelion, Mars is at its closest point to the Sun, and since Earth is also aligned with the Sun on the opposite side (opposition), it is the closest among all the Mars and Earth Oppositions. Earth and Mars are closer than usual during oppositions that coincide with Mars being near perihelion. These events happen approximately every 26 months, but the actual closeness can vary depending on the specific orbital positions of both planets during that opposition.

**(6) Based on the given dates of the observations, estimate ni which month Earth and Mars would be at closest approach? For a superior planet like Mars, when this happens, Earth and Mars wil be on the same side of the Sun.**

Solution. At Point B Earth and Mars, So 2011 June 29.

**(7) Using a similar rationale as the previous question, estimate the month when the two planets with eb farthest from each other? This happens when Mars and Earth are on opposite sides of the Sun, and this is called conjunction.**

Solution. At Point D Earth and Mars, 2013 December 31

**Procedure:**

By drawing a circle at the center of the paper to represent Earth's orbit, with a scaled radius of 5cm. The circle's center aligns with a ruled line marking the First Point of Aries, serving as the origin for heliocentric longitudes. Utilizing provided heliocentric longitudes of Earth and corresponding geocentric longitudes of Mars for various dates, students use a protractor to measure angles counter clockwise from the First Point of Aries. Lines drawn from the center of the circle through these angles represent Earth's positions. Parallel lines to the First Point of Aries are drawn through these Earth positions, and using the protractor, geocentric longitudes of Mars are measured. The intersection of these lines determines Mars' positions, allowing the construction of its orbit for six pairs of points. The resulting evenly spaced positions are connected to form a smooth curve, representing Mars' orbit. Distances between these points help identify the major axis, and measurements from the Sun to the major axis's midpoint provide the semi-major distance. Perihelion and aphelion positions are determined, completing the determination of Mars' orbit solely from observational measurements.

**Conclusion:**

We replicated Kepler's methodology to determine the orbit of Mars, providing a practical understanding of Kepler's laws of planetary motion. The geometrical construction using heliocentric and geocentric coordinates allowed for the visualization of Mars's elliptical orbit, with the calculated eccentricity and distances providing quantitative insights. The application of Kepler's laws, specifically during oppositions and conjunctions, illustrated the cyclical patterns in Earth-Mars relative positions. The hands-on approach not only deepened comprehension of celestial mechanics but also highlighted the significance of observational data in advancing our understanding of the solar system.

**References:**

[1] J. Kepler, "Kepler's laws of planetary motion," Wikipedia. [Online]. Available: https://en.wikipedia.org/wiki/Kepler%27s_laws_of_planetary_motion. [Accessed: November 28, 2023].

[2] "Kepler's Laws of Planetary Motion," HyperPhysics, Georgia State University. [Online]. Available: http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html. [Accessed: November 28, 2023].