Stat 260 Lecture Notes
Set 22 - Confidence Intervals

Recall: A **point estimate** is a single valued statistic used to estimate a population parameter. $\overline{x}$ is a point estimate for μ.

The downside of using a point estimate such as $\overline{x}$ is that we don't know how accurate our estimate is. Another method of estimation is to give a range of possible values - an **interval estimate**.

A **confidence interval (CI)** for μ is an interval $[L, U]$ which gives an estimate for the population mean μ with some degree of certainty.

A 95% confidence interval for μ has $P(L \leq \mu \leq U) = 0.95$
A 99% confidence interval for μ has $P(L \leq \mu \leq U) = 0.99$.

We find the numerical values for L and U by sampling. From each round of sampling we get an estimated range of values that μ might be contained in.

Interpretation: For a 95% CI, in the long run 95% of the CIs we create by sampling will actually contain μ.

How do we find L and U?

A $(1-\alpha)\%$ CI for μ is

$$[L, U] = \left[\overline{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \quad \overline{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right].$$

Sometimes we write this as

$$\overline{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \qquad \text{or} \qquad \overline{x} \pm z_{\alpha/2} \cdot \frac{s}{\sqrt{n}}.$$

When can we use this formula?

Whenever $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}}$ or $\dfrac{\overline{X} - \mu}{s/\sqrt{n}}$ is normally distributed:

- $X_1, X_2, \ldots, X_n$ from a normal distribution and we know σ
- $X_1, X_2, \ldots, X_n$ from any distribution and n is big ($n \geq 30$) and we know σ
- $X_1, X_2, \ldots, X_n$ from any distribution and n is big ($n \geq 30$) and we don't know σ (so we use the estimate s instead)

Example 1: $X_1, X_2, \ldots, X_{15}$ from a normal distribution with $\overline{x} = 147.33$ and $\sigma = 40$. Find a 95% confidence interval for μ and find a 99% confidence interval for μ.

Example 2: How does changing σ, n, and the confidence level $(1-\alpha)\%$ affect the CI?

$\overline{x}$	σ	n	confidence level	$\alpha/2$	$z_{\alpha/2}$	$[L, U]$
147.33	40	15	95%	.025	1.96	$[127.09, 167.57]$
147.33	40	**50**	95%	.025	1.96	$[136.25, 158.42]$
147.33	40	15	**99%**	.005	2.575	$[120.74, 173.92]$
147.33	**80**	15	95%	.025	1.96	$[106.85, 187.82]$

Example 3: How did we get the formula

$$[L, U] = \left[\overline{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \quad \overline{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right] \quad ?$$

Look at the 95% confidence interval, where we have $z_{\alpha/2} = z_{0.025} = 1.96$. We can say

$$P(\overline{x} - 1.96 \cdot \frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{x} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}) = 0.95$$

but once we fill in values for $\overline{x}$, σ, and $\sqrt{n}$ we can no longer use this probability.

So for example we can't say $P(127.09 \leq \mu \leq 167.57) = 0.95$ with our confidence interval $[127.09, 167.58]$ because either μ is in this interval, or it isn't. (There are no variables here so there is no chance on where the value of μ sits. The probability is 1 if the value is in the interval, or 0 if it isn't.)