Exercise 6.8

L **Answer.**

We are asked to compute $h_1 * h_2$, where

$$h_1(t) = 2000\,\mathrm{sinc}(2000\pi t) \quad \text{and} \quad h_2(t) = \delta(t) - 1000\,\mathrm{sinc}(1000\pi t).$$

Let H_1 and H_2 denote the Fourier transforms of h_1 and h_2, respectively. Using the time-domain convolution property of the Fourier transform, we have

convolution property

$$\mathcal{F}\{h_1 * h_2\}(\omega) = H_1(\omega)H_2(\omega).$$

Computing H_1, we obtain

from given h_1

to facilitate FT table use

from FT table

simplify

1
$$\begin{aligned} H_1(\omega) &= \mathcal{F}\{2000\,\mathrm{sinc}(2000\pi t)\}(\omega) \\ &= \mathcal{F}\left\{\tfrac{2000\pi}{\pi}\,\mathrm{sinc}(2000\pi t)\right\}(\omega) \\ &= \mathrm{rect}\left[\frac{\omega}{2(2000\pi)}\right] \\ &= \mathrm{rect}\left(\frac{\omega}{4000\pi}\right). \end{aligned}$$

Computing H_2, we obtain

from given h_2

linearity of FT

rewrite to facilitate FT table use

from FT table

simplify

2
$$\begin{aligned} H_2(\omega) &= \mathcal{F}\{\delta(t) - 1000\,\mathrm{sinc}(1000\pi t)\}(\omega) \\ &= \mathcal{F}\delta(\omega) - \mathcal{F}\{1000\,\mathrm{sinc}(1000\pi t)\}(\omega) \\ &= \mathcal{F}\delta(\omega) - \mathcal{F}\left\{\tfrac{1000\pi}{\pi}\,\mathrm{sinc}(1000\pi t)\right\}(\omega) \\ &= 1 - \mathrm{rect}\left[\frac{\omega}{2(1000\pi)}\right] \\ &= 1 - \mathrm{rect}\left(\frac{\omega}{2000\pi}\right). \end{aligned}$$

Thus, we can write

convolution property

substitute 1 and 2

multiply

product of two time scaled rectangular pulses is narrower pulse

3
$$\begin{aligned} \mathcal{F}\{h_1 * h_2\}(\omega) &= H_1(\omega)H_2(\omega) \\ &= \mathrm{rect}\left(\frac{\omega}{4000\pi}\right)\left[1 - \mathrm{rect}\left(\frac{\omega}{2000\pi}\right)\right] \\ &= \mathrm{rect}\left(\frac{\omega}{4000\pi}\right) - \mathrm{rect}\left(\frac{\omega}{2000\pi}\right)\mathrm{rect}\left(\frac{\omega}{4000\pi}\right) \\ &= \mathrm{rect}\left(\frac{\omega}{4000\pi}\right) - \mathrm{rect}\left(\frac{\omega}{2000\pi}\right). \end{aligned}$$

Taking the inverse Fourier transform of the preceding equation, we have

take inverse FT of 3

linearity of FT

rewrite to facilitate FT table use

from table

cancel π's

$$\begin{aligned} h_1 * h_2(t) &= \mathcal{F}^{-1}\left\{\mathrm{rect}\left(\frac{\omega}{4000\pi}\right) - \mathrm{rect}\left(\frac{\omega}{2000\pi}\right)\right\}(t) \\ &= \mathcal{F}^{-1}\left\{\mathrm{rect}\left(\frac{\omega}{4000\pi}\right)\right\}(t) - \mathcal{F}^{-1}\left\{\mathrm{rect}\left(\frac{\omega}{2000\pi}\right)\right\}(t) \\ &= \mathcal{F}^{-1}\left\{\mathrm{rect}\left[\frac{\omega}{2(2000\pi)}\right]\right\}(t) - \mathcal{F}^{-1}\left\{\mathrm{rect}\left[\frac{\omega}{2(1000\pi)}\right]\right\}(t) \\ &= \tfrac{2000\pi}{\pi}\,\mathrm{sinc}(2000\pi t) - \tfrac{1000\pi}{\pi}\,\mathrm{sinc}(1000\pi t) \\ &= 2000\,\mathrm{sinc}(2000\pi t) - 1000\,\mathrm{sinc}(1000\pi t). \end{aligned}$$

