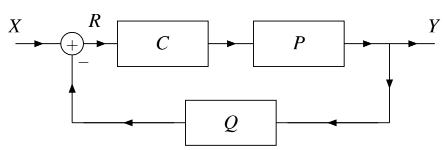
Stabilization Example: Using Feedback (1)

- feedback system (with causal LTI compensator and sensor):



$$P(s) = \frac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$

- system function H of feedback system: substitute given C, P, and Q and simplify

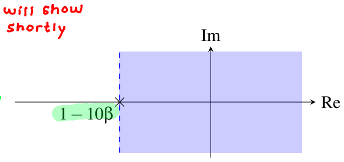
$$H(s) = \frac{C(s)P(s)}{1+C(s)P(s)Q(s)} = \frac{10\beta}{s-(1-10\beta)}$$

we will show this shortly

has pole at $1-10\beta$

- ROC of H:

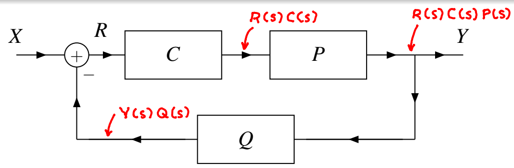
ROC contains imaginary axis if $1-10\beta<0$



ROC is RHP since system is causal

- feedback system is BIBO stable if and only if $1 - 10\beta < 0$ or equivalently $\beta > \frac{1}{10}$

Stabilization Example: Using Feedback (2)



① $R(s) = X(s) - Q(s)Y(s)$ ← equation for adder

② $Y(s) = C(s)P(s)R(s)$ ← equation for output

$$
\begin{aligned}
Y(s) &= C(s)P(s)R(s) \quad \text{from ②} \\
&= C(s)P(s)[X(s) - Q(s)Y(s)] \quad \text{substituting formula for R from ①} \\
&= C(s)P(s)X(s) - C(s)P(s)Q(s)Y(s) \quad \text{multiply}
\end{aligned}
$$

$$[1 + C(s)P(s)Q(s)]Y(s) = C(s)P(s)X(s)$$

move terms containing Y to the left-hand side and factor

$$H(s) = \frac{Y(s)}{X(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)Q(s)}$$

divide both sides by $X(s)[1+C(s)P(s)Q(s)]$

$Y(s) = X(s)H(s)$

Stabilization Example: Using Feedback (3)

$$P(s) = \tfrac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$ given

$$H(s) = \frac{C(s)P(s)}{1 + C(s)P(s)Q(s)}$$ result from previous slide

substitute given C, P, and Q

$$= \frac{\beta(\frac{10}{s-1})}{1 + \beta(\frac{10}{s-1})(1)}$$

multiply by $\frac{s-1}{s-1}$

$$= \frac{10\beta}{s - 1 + 10\beta}$$

rewrite to explicitly show pole

$$= \frac{10\beta}{s - (1 - 10\beta)}$$

pole at $1-10\beta$

Remarks on Stabilization Via Pole-Zero Cancellation

- Pole-zero cancellation is not achievable in practice, and therefore it cannot be used to stabilize real-world systems.
- The theoretical models used to represent real-world systems are only approximations due to many factors, including the following:
  - Determining the system function of a system involves measurement, which always has some error.
  - A system cannot be built with such precision that it will have exactly some prescribed system function.
  - The system function of most systems will vary at least slightly with changes in the physical environment.
  - Although a LTI model is used to represent a system, the likely reality is that the system is not exactly LTI, which introduces error.
- Due to approximation error, the effective poles and zeros of the system function will only be approximately where they are expected to be.
- Since pole-zero cancellation requires that a pole and zero be placed at exactly the same location, any error will prevent this cancellation from being achieved.