List of Listings

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Preface

This book is primarily intended to be used as a text for undergraduate students in engineering (and related) disciplines. The book provides a basic introduction to continuous-time and discrete-time signals and systems. Since many engineering curricula use MATLAB as a teaching tool, the book also includes a detailed introduction to MATLAB as an appendix. This book evolved from a detailed set of lecture notes that the author prepared in order to teach two different undergraduate courses on continuous-time signals and systems at the University of Victoria (Victoria, BC, Canada). In particular, the first version of these lectures notes was developed while the author was teaching ELEC 260 (Signal Analysis) in the Summer 2003 term, with some further content being added to accommodate the teaching of ELEC 255 (System Dynamics) in the Fall 2003 term. Over time, the lecture notes underwent many changes, eventually evolving into a textbook. The earlier versions of this book only covered the continuous-time case, with coverage of the discrete-time case being added later. All of this work ultimately led to the book that you are now reading.

Acknowledgments

I would like to thank my colleague, Dr. Wu-Sheng Lu, for many interesting technical discussions that helped to clarify some of the finer points of the mathematics behind signals and systems. Also, I would like to thank my past students for their feedback regarding earlier revisions of this manuscript. They have helped me to eliminate numerous errors in this manuscript that would have otherwise gone undetected.

Michael Adams
Victoria, BC
2022-12-31

Guidance for Instructors

The theory of continuous-time (CT) and discrete-time (DT) signals and systems is taught in many engineering and related disciplines. The manner in which this material is split across courses (or terms in the case of multi-term courses) can vary dramatically from one institution or program to another. For this reason, this textbook has been organized in such a way as to accommodate the teaching of courses with a wide variety of structures. Some possibilities include (but are not limited to):

- sequential presentation of the CT and DT cases with the CT case covered first (e.g., two single-term courses, where the first covers the CT case and the second covers the DT case);
- sequential presentation of the CT and DT cases with the DT case covered first (e.g., two single-term courses, where the first covers the DT case and the second covers the CT case);
- integrated presentation of both the CT and DT cases (e.g., a one- or two-term course that covers both the CT and DT cases together).

Sequential Presentation of the CT and DT Cases

The sequential presentation of the CT and DT cases with the CT case treated first might, for example, cover material from the textbook as follows:

1. First Course/Term: The CT Case
   (a) Chapter 1 (Introduction) with an emphasis on the material most relevant to the CT case
   (b) if a review of complex analysis is desired: Appendix A (Complex Analysis)
   (c) Chapter 2 (Preliminaries) with an emphasis on the material most relevant to the CT case
   (d) Chapter 3 (Continuous-Time Signals and Systems)
   (e) Chapter 4 (Continuous-Time Linear Time-Invariant Systems)
   (f) Chapter 5 (Continuous-Time Fourier Series)
   (g) Chapter 6 (Continuous-Time Fourier Transform)
   (h) if an introduction to partial fraction expansions is needed: Appendix B (Partial Fraction Expansions)
   (i) Chapter 7 (Laplace Transform)
2. Second Course/Term: The DT Case
   (a) Chapter 1 (Introduction) with an emphasis on the material most relevant to the DT case
   (b) Chapter 2 (Preliminaries) with an emphasis on the material most relevant to the DT case
   (c) Chapter 8 (Discrete-Time Signals and Systems)
   (d) Chapter 9 (Discrete-Time Linear Time-Invariant Systems)
   (e) Chapter 10 (Discrete-Time Fourier Series)
   (f) Chapter 11 (Discrete-Time Fourier Transform)
   (g) Chapter 12 (z Transform)

To cover the DT case first, one should, for the most part, be able to simply swap the order of the two courses described above, since the textbook tries to minimize dependencies on whether the CT case is covered before the DT case. This said, however, there are a small number of dependencies. To resolve these dependencies, the following changes would need to be made. A small amount of material would need to be moved from the course covering the DT case to the course covering the CT case, namely:

- Section 11.17, which covers the relationship between the discrete-time Fourier transform (DTFT) and continuous-time Fourier series (CTFS); and
- Section 11.18, which covers the relationship between the DTFT and continuous-time Fourier transform (CTFT).

Also, a small amount of material would need to be moved from the course covering the CT case to the course covering the DT case, namely:

- Section 3.5.11, which introduces the delta function.

Integrated Presentation of the CT and DT Cases

The integrated presentation of the CT and DT cases might, for example, cover material from the textbook as follows:

1. First Term
   (a) Chapter 1 (Introduction)
   (b) if a review of complex analysis is desired: Appendix A (Complex Analysis)
   (c) Chapter 2 (Preliminaries)
   (d) Chapter 3 (Continuous-Time Signals and Systems)
   (e) Chapter 8 (Discrete-Time Signals and Systems)
   (f) Chapter 4 (Continuous-Time Linear Time-Invariant Systems)
   (g) Chapter 9 (Discrete-Time Linear Time-Invariant Systems)
   (h) Chapter 5 (Continuous-Time Fourier Series)
   (i) Chapter 10 (Discrete-Time Fourier Series)
2. Second Term
   (a) Chapter 6 (Continuous-Time Fourier Transform)
   (b) Chapter 11 (Discrete-Time Fourier Transform)
   (c) if an introduction to partial fraction expansions is needed: Appendix B (Partial Fraction Expansions)
   (d) Chapter 7 (Laplace Transform)
   (e) Chapter 12 (z Transform)

Video Lectures

Video lectures are available for some of the material covered in this textbook. These video lectures are likely to be helpful to instructors, either for planning their own courses or for using as additional reference material for their students. More information on these video lectures can be found in Appendix G.

Lecture Slides

This textbook is intended to be used in conjunction with the following set of lecture slides:

- M. D. Adams, *Lecture Slides for Signals and Systems*, Edition 5.0, Dec. 2022, ISBN 978-1-990707-02-5 (PDF). Available from Google Books, Google Play Books, and author's web site `https://www.ece.uvic.ca/~mdadams/sigsysbook`.

Textbook Web Site

This textbook has an associated web site whose URL is:

- `https://www.ece.uvic.ca/~mdadams/sigsysbook`

To obtain the most recent version of this textbook (with functional hyperlinks) or for additional information and resources related to this textbook (such as lecture slides, video lectures, and errata), please visit this site.

Companion Git Repository

This textbook has an associated Git repository containing some source code and other supplemental files. The URL for this repository is:

- `https://github.com/mdadams/sigsysbook_companion.git`

About the Author

Michael Adams received the B.A.Sc. degree in computer engineering from the University of Waterloo, Waterloo, ON, Canada in 1993, the M.A.Sc. degree in electrical engineering from the University of Victoria, Victoria, BC, Canada in 1998, and the Ph.D. degree in electrical engineering from the University of British Columbia, Vancouver, BC, Canada in 2002. From 1993 to 1995, Michael was a member of technical staff at Bell-Northern Research in Ottawa, ON, Canada, where he developed real-time software for fiber-optic telecommunication systems. Since 2003, Michael has been on the faculty of the Department of Electrical and Computer Engineering at the University of Victoria, Victoria, BC, Canada, first as an Assistant Professor and currently as an Associate Professor.

Michael is the recipient of a Natural Sciences and Engineering Research Council (of Canada) Postgraduate Scholarship. He has served as a voting member of the Canadian Delegation to ISO/IEC JTC 1/SC 29 (i.e., Coding of Audio, Picture, Multimedia and Hypermedia Information), and been an active participant in the JPEG-2000 standardization effort, serving as co-editor of the JPEG-2000 Part-5 standard and principal author of one of the first JPEG-2000 implementations (i.e., JasPer). His research interests include software, signal processing, image/video/audio processing and coding, multiresolution signal processing (e.g., filter banks and wavelets), geometry processing, and data compression.

Other Works by the Author

Some other open-access textbooks and slide decks by the author of this book include:

1. M. D. Adams, *Lecture Slides for Signals and Systems*, Edition 5.0, Dec. 2022, ISBN 978-1-990707-02-5 (PDF). Available from Google Books, Google Play Books, and author's web site https://www.ece.uvic.ca/~mdadams/sigsysbook.

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4. M. D. Adams, *Multiresolution Signal and Geometry Processing: Filter Banks, Wavelets, and Subdivision (Version 2013-09-26)*, University of Victoria, Victoria, BC, Canada, Sept. 2013, ISBN 978-1-55058-507-0 (print), ISBN 978-1-55058-508-7 (PDF). Available from Google Books, Google Play Books, and author's web site https://www.ece.uvic.ca/~mdadams/waveletbook.

5. M. D. Adams, *Lecture Slides for Multiresolution Signal and Geometry Processing (Version 2015-02-03)*, University of Victoria, Victoria, BC, Canada, Feb. 2015, ISBN 978-1-55058-535-3 (print), ISBN 978-1-55058-536-0 (PDF). Available from Google Books, Google Play Books, and author's web site https://www.ece.uvic.ca/~mdadams/waveletbook.

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Chapter 1

Introduction

1.1 Signals and Systems

Mathematics has a very broad scope, encompassing many areas such as: linear algebra, calculus, probability and statistics, geometry, differential equations, and numerical methods. For engineers, however, an area of mathematics of particular importance is the one that pertains to signals and systems (which is, loosely speaking, the branch of mathematics known as functional analysis). It is this area of mathematics that is the focus of this book. Before we can treat this topic in any meaningful way, however, we must first explain precisely what signals and systems are. This is what we do next.

1.2 Signals

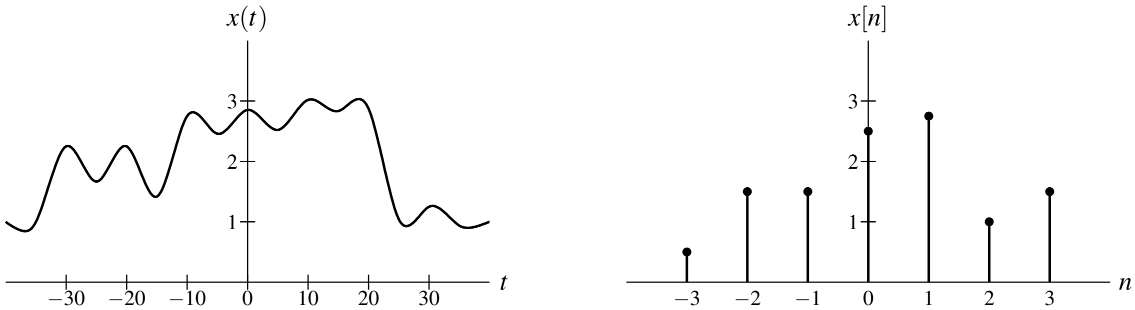
A **signal** is a function of one or more variables that conveys information about some (usually physical) phenomenon. Some examples of signals include:

- a human voice
- a voltage in an electronic circuit
- the temperature of a room controlled by a thermostat system
- the position, velocity, and acceleration of an aircraft
- the acceleration measured by an accelerometer in a cell phone
- the force measured by a force sensor in a robotic system
- the electromagnetic waves used to transmit information in wireless computer networks
- a digitized photograph
- a digitized music recording
- the evolution of a stock market index over time

1.2.1 Classification of Signals

Signals can be classified based on the number of independent variables with which they are associated. A signal that is a function of only one variable is said to be **one dimensional**. Similarly, a signal that is a function of two or more variables is said to be **multi-dimensional**. Human speech is an example of a one-dimensional signal. In this case, we have a signal associated with fluctuations in air pressure as a function of time. An example of a two-dimensional signal is a monochromatic image. In this case, we have a signal that corresponds to a measure of light intensity as a function of horizontal and vertical position.

A signal can also be classified on the basis of whether it is a function of continuous or discrete variables. A signal that is a function of continuous variables (e.g., a real variable) is said to be **continuous time**. Similarly, a signal that is a function of discrete variables (e.g., an integer variable) is said to be **discrete time**. Although the independent variable need not represent time, for matters of convenience, much of the terminology is chosen as if this were so.



(a) (b)

Figure 1.1: Graphical representations of (a) continuous-time and (b) discrete-time signals.

For example, a digital image (which consists of a rectangular array of pixels) would be referred to as a discrete-time signal, even though the independent variables (i.e., horizontal and vertical position) do not actually correspond to time.

If a signal is a function of discrete variables (i.e., discrete-time) and the value of the function itself is also discrete, the signal is said to be **digital**. Similarly, if a signal is a function of continuous variables, and the value of the function itself is also continuous, the signal is said to be **analog**.

Many phenomena in our physical world can be described in terms of continuous-time signals. Some examples of continuous-time signals include: voltage or current waveforms in an electronic circuit; electrocardiograms, speech, and music recordings; position, velocity, and acceleration of a moving body; forces and torques in a mechanical system; and flow rates of liquids or gases in a chemical process. Any signals processed by digital computers (or other digital devices) are discrete-time in nature. Some examples of discrete-time signals include digital video, digital photographs, and digital audio data.

A discrete-time signal may be inherently discrete or correspond to a sampled version of a continuous-time signal. An example of the former would be a signal corresponding to the Dow Jones Industrial Average stock market index (which is only defined on daily intervals), while an example of the latter would be the sampled version of a (continuous-time) speech signal.

1.2.2 Notation and Graphical Representation of Signals

In the case of discrete-time signals, we sometimes refer to the signal as a **sequence**. The nth element of a sequence x is denoted as either $x(n)$ or x_n. Figure 1.1 shows how continuous-time and discrete-time signals are represented graphically.

1.2.3 Examples of Signals

A number of examples of signals have been suggested previously. Here, we provide some graphical representations of signals for illustrative purposes. Figure 1.2 depicts a digitized speech signal. Figure 1.3 shows an example of a monochromatic image. In this case, the signal represents light intensity as a function of two variables (i.e., horizontal and vertical position).

1.3 Systems

A **system** is an entity that processes one or more input signals in order to produce one or more output signals, as shown in Figure 1.4. Such an entity is represented mathematically by a system of one or more equations.

In a communication system, the input might represent the message to be sent, and the output might represent the received message. In a robotics system, the input might represent the desired position of the end effector (e.g., gripper), while the output could represent the actual position.

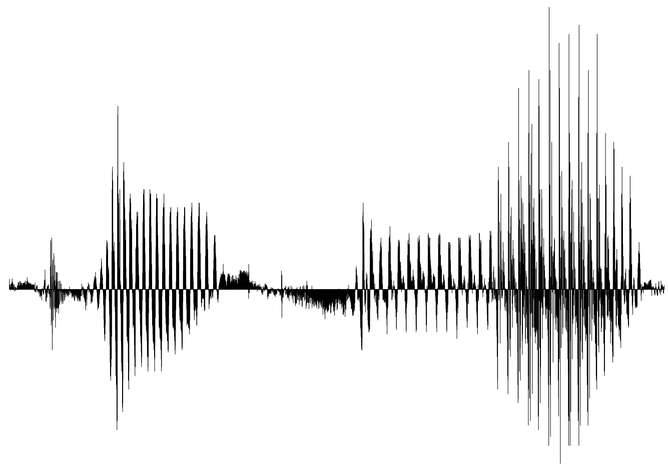


Figure 1.2: Segment of digitized human speech.

Figure 1.3: A monochromatic image.

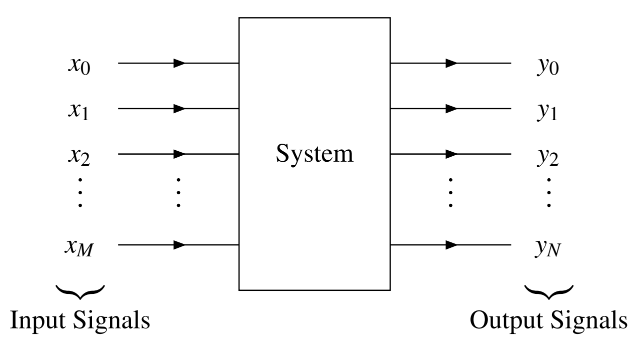


Figure 1.4: System with one or more inputs and one or more outputs.

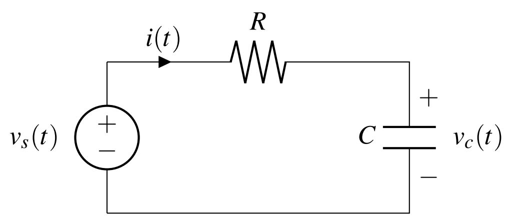


Figure 1.5: A simple RC network.

1.3.1 Classification of Systems

A system can be classified based on the number of inputs and outputs it has. A system with only one input is described as **single input**, while a system with multiple inputs is described as **multi-input**. Similarly, a system with only one output is said to be **single output**, while a system with multiple outputs is said to be **multi-output**. Two commonly occurring types of systems are single-input single-output (SISO) and multi-input multi-output (MIMO).

A system can also be classified based on the types of signals with which it interacts. A system that deals with continuous-time signals is called a **continuous-time system**. Similarly, a system that deals with discrete-time signals is said to be a **discrete-time system**. A system that handles both continuous- and discrete-time signals, is sometimes referred to as a **hybrid system** (or sampled-data system). Similarly, systems that deal with digital signals are referred to as **digital**, while systems that handle analog signals are referred to as **analog**. If a system interacts with one-dimensional signals, the system is referred to as **one-dimensional**. Likewise, if a system handles multi-dimensional signals, the system is said to be **multi-dimensional**.

1.3.2 Examples of Systems

Systems can manipulate signals in many different ways and serve many useful purposes. Sometimes systems serve to extract information from their input signals. For example, in the case of speech signals, systems can be used in order to perform speaker identification or voice recognition. A system might analyze electrocardiogram signals in order to detect heart abnormalities. Amplification and noise reduction are other functionalities that systems could offer.

One very basic system is the resistor-capacitor (RC) network shown in Figure 1.5. Here, the input would be the source voltage v_s and the output would be the capacitor voltage v_c.

Consider the signal-processing systems shown in Figure 1.6. The system in Figure 1.6(a) uses a discrete-time system (such as a digital computer) to process a continuous-time signal. The system in Figure 1.6(b) uses a continuous-time system (such as an analog computer) to process a discrete-time signal. The first of these types of systems is ubiquitous in the world today.

Consider the communication system shown in Figure 1.7. This system takes a message at one location and reproduces this message at another location. In this case, the system input is the message to be sent, and the output is the estimate of the original message. Usually, we want the message reproduced at the receiver to be as close as possible to the original message sent by the transmitter.

A system of the general form shown in Figure 1.8 frequently appears in control applications. Often, in such applications, we would like an output to track some reference input as closely as possible. Consider, for example, a robotics application. The reference input might represent the desired position of the end effector, while the output represents the actual position.

1.4 Why Study Signals and Systems?

As can be seen from the earlier examples, there are many practical applications in which we need to develop systems that manipulate signals. In order to do this, we need a formal mathematical framework for the study of such systems. Such a framework can be used to guide the design of new systems as well as to analyze the behavior of already existing systems. Over time, the complexity of systems designed by engineers has continued to grow. Today, most systems of practical interest are highly complex. For this reason, a formal mathematical framework to guide the design of

