# Stat 260 Lecture Notes
## Set 11 - The Binomial Distribution

Let's revisit two examples we've seen in previous sets.

**Example (Set 7):** A diagnostic test is correct 95% of the time. Suppose 3 people are independently tested. What is the probability that exactly two of the three receive a correct diagnosis?

**Example (Set 8):** Suppose 25% of people are left-handed. Suppose we independently sample 3 people and count how many are right-handed. What is the probability that exactly two people are right handed?

Both these questions can be solved by using a tree diagram. We want to generalize the solution for these common question setups.

In these questions the random variable $X =$ the number of "successes" in the 3 trials.
In the Set 7 question success = correct diagnosis.
In the Set 8 question success = right-handed.

These are **binomial experiments**. $X$ is a **binomial random variable**.

We have a binomial experiment if:

1. There is a fixed number of trials, $n$.
2. Each trial results in one of two possible outcomes: a success (S) or a failure (F).
3. The trials are independent and the probability of success in each trial is the same.
   $p = P(\text{success in a single trial})$
   $1 - p = P(\text{failure in a single trial})$
4. The random variable $X$ counts the number of successes in $n$ trials.

If $X$ is a binomial random variable we write $X \sim \text{binomial}(n, p)$.

Each trial of a binomial experiment is a **Bernoulli trial** - there are only two outcomes.

The pmf for a binomial random variable $X$ is given by the formula

$$f(x) = P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

where $x = 0, 1, 2, \ldots, n$.

We looked at the pmf of the right handed example in Set 8. Check that this pmf for the binomial distribution also gives the values in the pmf table we found there.

**Example 1:** A carrier of TB has a 10% chance of passing on the disease to strangers. Suppose a carrier is in close contact with 20 strangers in a day.

(a) What is the probability that at least one of the strangers contracts the disease?

(b) If at least one stranger gets the disease, what is the probability that at most three get the disease?

**Rules for the binomial distribution:**

- $E(X) = n \cdot p$
- $V(X) = n \cdot p(1-p)$
- $\sigma_X = \sqrt{n \cdot p(1-p)}$

In Example 1, the expected number of the 20 strangers who contract the disease is:

For the cdf of a binomial random variable $X$ we have tables with results already calculated (see page 1 of the stats table package).

**Example 2:** Suppose we have a binomial experiment with $n = 15$, $p = 0.3$, and the random variable $X$ counts the number of successes. Find:

(a) $P(X \leq 4)$

(b) $P(X < 2)$

(c) $P(X = 5)$

(d) $P(X \leq 7 \mid X \geq 5)$