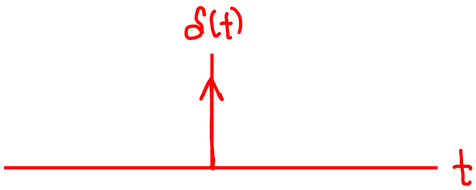
Example 4.9. Consider the LTI system with the impulse response h given by

$$h(t) = \delta(t).$$

Determine whether this system has memory.

Solution. Clearly, h is only nonzero at the origin. This follows immediately from the definition of the unit-impulse function δ. Therefore, the system is memoryless (i.e., does not have memory). ■



memoryless $\Longleftrightarrow$ $h(t) = 0$ for all $t \neq 0$

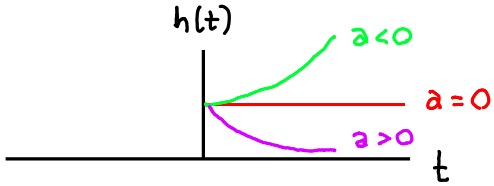
Example 4.10. Consider the LTI system with impulse response h given by

$$h(t) = e^{-at}u(t),$$

where a is a real constant. Determine whether this system is causal.

Solution. Clearly, $h(t) = 0$ for $t < 0$ (due to the $u(t)$ factor in the expression for $h(t)$). Therefore, the system is causal. ■

this is true regardless of a



causal $\iff$ $h(t)=0$ for all $t<0$

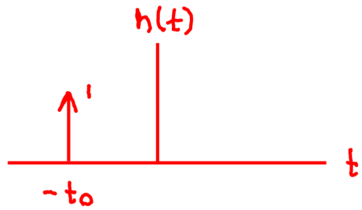
Example 4.11. Consider the LTI system with impulse response h given by

$$h(t) = \delta(t + t_0),$$

$t_0 > 0$

where t_0 is a strictly positive real constant. Determine whether this system is causal.

Solution. From the definition of δ, we can easily deduce that $h(t) = 0$ except at $t = -t_0$. Since $-t_0 < 0$, the system is not causal. ■



causal : $h(t) = 0$ for all $t < 0$

Example 4.12. Consider the LTI system $\mathcal{H}$ with impulse response h given by

$$h(t) = A\delta(t - t_0),$$

where A and t_0 are real constants and $A \neq 0$. Determine if $\mathcal{H}$ is invertible, and if it is, find the impulse response h_{inv} of the system $\mathcal{H}^{-1}$.

Solution. If the system $\mathcal{H}^{-1}$ exists, its impulse response h_{inv} is given by the solution to the equation

$$h * h_{\text{inv}} = \delta. \tag{4.34}$$

$\mathcal{H}$ is invertible if and only if a solution for h_{inv} exists

So, let us attempt to solve this equation for h_{inv}. Substituting the given function h into (4.34) and using straightforward algebraic manipulation, we can write

$$\begin{aligned}
& h * h_{\text{inv}}(t) = \delta(t) \\
\Rightarrow\ & \int_{-\infty}^{\infty} h(\tau) h_{\text{inv}}(t - \tau) d\tau = \delta(t) \\
\Rightarrow\ & \int_{-\infty}^{\infty} A\delta(\tau - t_0) h_{\text{inv}}(t - \tau) d\tau = \delta(t) \\
\Rightarrow\ & \int_{-\infty}^{\infty} \delta(\tau - t_0) \underline{h_{\text{inv}}(t - \tau)} d\tau = \tfrac{1}{A}\delta(t).
\end{aligned}$$

definition of convolution

substitute given function h

divide both sides by $A \neq 0$

Using the sifting property of the unit-impulse function, we can simplify the integral expression in the preceding equation to obtain

$h_{\text{inv}}(t - \tau)\big|_{\tau = t_0} = \frac{1}{A}\delta(t)$ sifting property

$$h_{\text{inv}}(t - t_0) = \tfrac{1}{A}\delta(t). \tag{4.35}$$

Substituting $t + t_0$ for t in the preceding equation yields

$$\begin{aligned}
h_{\text{inv}}([t + t_0] - t_0) = \tfrac{1}{A}\delta(t + t_0) \quad &\Leftrightarrow \\
h_{\text{inv}}(t) = \tfrac{1}{A}\delta(t + t_0).&
\end{aligned}$$

impulse response of inverse system

Since $A \neq 0$, the function h_{inv} is always well defined. Thus, $\mathcal{H}^{-1}$ exists and consequently $\mathcal{H}$ is invertible. ■