Example 3.16 (Ideal integrator)**.** Determine whether the system $\mathcal{H}$ is memoryless, where

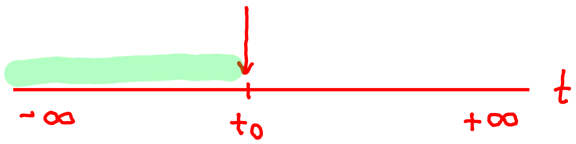
$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Solution. Consider the calculation of $\mathcal{H}x(t)$ at any arbitrary point $t = t_0$. We have

$$\mathcal{H}x(t_0) = \int_{-\infty}^{t_0} x(\tau)d\tau.$$

Thus, $\mathcal{H}x(t_0)$ depends on $x(t)$ for $-\infty < t \leq t_0$. So, $\mathcal{H}x(t_0)$ is dependent on $x(t)$ for some $t \neq t_0$ (e.g., $t_0 - 1$). Therefore, the system has memory (i.e., is not memoryless). ■

consider computation
of output at this point



at what points must
this input be known?

Example 3.19 (Ideal integrator)**.** Determine whether the system $\mathcal{H}$ is causal, where

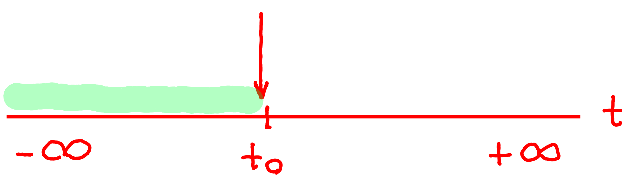
$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Solution. Consider the calculation of $\mathcal{H}x(t_0)$ for arbitrary t_0. We have

$$\mathcal{H}x(t_0) = \int_{-\infty}^{t_0} x(\tau)d\tau.$$

Thus, we can see that $\mathcal{H}x(t_0)$ depends only on $x(t)$ for $-\infty < t \leq t_0$. Since all of the values in this interval are less than or equal to t_0, the system is causal. ■

consider computation
of output at this point



at what points must
input be known?

Example 3.20. Determine whether the system $\mathcal{H}$ is causal, where

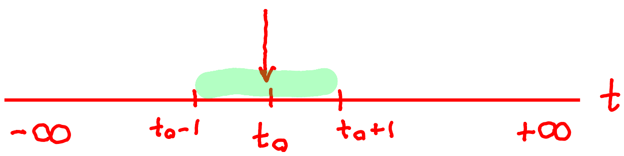
$$\mathcal{H}x(t) = \int_{t-1}^{t+1} x(\tau)d\tau.$$

Solution. Consider the calculation of $\mathcal{H}x(t_0)$ for arbitrary t_0. We have

$$\mathcal{H}x(t_0) = \int_{t_0-1}^{t_0+1} x(\tau)d\tau.$$

Thus, we can see that $\mathcal{H}x(t_0)$ only depends on $x(t)$ for $t_0 - 1 \leq t \leq t_0 + 1$. Since some of the values in this interval are greater than t_0 (e.g., $t_0 + 1$), the system is not causal. ■

consider computation
of output at this point



at which points must
input be known?

Example 3.23. Determine whether the system $\mathcal{H}$ is invertible, where

$$\mathcal{H}x(t) = x(t - t_0)$$

y = Hx

substitute t+t₀ for t

and t_0 is a real constant.

Solution. Let $y = \mathcal{H}x$. By substituting $t + t_0$ for t in $y(t) = x(t - t_0)$, we obtain

$$\begin{aligned} y(t + t_0) &= x(t + t_0 - t_0) \\ &= x(t). \end{aligned}$$

Thus, we have shown that

$$x(t) = y(t + t_0).$$

This, however, is simply the equation of the inverse system $\mathcal{H}^{-1}$. In particular, we have that

$$x(t) = \mathcal{H}^{-1}y(t)$$

where

$$\mathcal{H}^{-1}y(t) = y(t + t_0).$$

Thus, we have found $\mathcal{H}^{-1}$. Therefore, the system $\mathcal{H}$ is invertible. ■