Stat 260 Lecture Notes
Set 10 - Variance, and Expected Value and Variance Rules

The *variance* of a r.v. X with pmf $f(x)$ is

$$\begin{aligned} V(X) = \sigma_X^2 = \sigma^2 &= E((X-\mu)^2) \\ &= \sum_{\text{all } x} (x-\mu)^2 \cdot f(x) \\ &= \sum_{\text{all } x} (x-\mu)^2 \cdot P(X=x) \end{aligned}$$

Example 1: The discrete random variable X has pmf as follows:

x	25	45	65
$f(x)$	1/2	1/3	1/6

Find the variance of X. That is, find $V(X)$.

There is a shortcut formula to calculate $V(X)$.

$$\begin{aligned} V(X) &= E(X^2) - (E(X))^2 \\ &= E(X^2) - \mu^2 \\ &= \left(\sum_{\text{all } x} x^2 \cdot f(x) \right) - \mu^2 \end{aligned}$$

Example 2: Calculate $V(X)$ from Example 1 again, but use the shortcut formula.

Recall: standard deviation $= \sqrt{\text{variance}}$.

The *standard deviation* of r.v. X is $\sigma_X = \sigma = \sqrt{\sigma_X^2} = \sqrt{V(X)}$.

Rules for Expected Value:

- for a constant c, $E(X + c) = E(X) + c$
- $E(c) = c$
- $E(cX) = c \cdot E(X)$

Putting these together we get the rule

$$E(aX + b) = a \cdot E(X) + b$$

where a and b are constants.

Rules for Variance and Standard Deviation:

- for a constant c, $V(X + c) = V(X)$
- $V(c) = 0$
- $V(cX) = c^2 \cdot V(X)$
- $\sigma_{X+c} = \sigma_X$
- $\sigma_c = 0$
- $\sigma_{cX} = |c| \cdot \sigma_X$

Putting these together we get the rules

$$V(aX + b) = V(aX) = a^2 V(X)$$

$$\sigma_{aX+b} = \sigma_{aX} = |a| \cdot \sigma_X$$

where a and b are constants.

Rule: For random variables X and Y, we have that
$E(X + Y) = E(X) + E(Y)$.

Rule: For random variables X and Y that are **independent**, we have that $V(X + Y) = V(X) + V(Y)$.

Example 3: Say $X_1, X_2, \ldots, X_n$ are all random variables with expected value μ.

Look at $\overline{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$.

Find $E(\overline{X})$.

Therefore we have the rule that $E(\overline{X}) = \mu_X = \mu$. Following similar arguments (and using a couple extra assumptions) we can show the rules that $V(\overline{X}) = \dfrac{\sigma_X^2}{n}$ and $\sigma_{\overline{X}} = \dfrac{\sigma_X}{\sqrt{n}}$. (Remember for notation: $V(X) = \sigma_X^2$ and $\sigma_X = \sigma$.)

We will use these rules lots in our later Sets!