

4 5.1 For each case below, find the Fourier series representation (in complex exponential form) of the function x , explicitly identifying the fundamental period of x and the Fourier series coefficient sequence c .

(a) $x(t) = 1 + \cos(\pi t) + \sin^2(\pi t)$;

(b) $x(t) = \cos(4t) \sin(t)$; and

(c) $x(t) = |\sin(2\pi t)|$. [Hint: $\int e^{ax} \sin(bx) dx = \frac{e^{ax}[a \sin(bx) - b \cos(bx)]}{a^2 + b^2} + C$, where a and b are arbitrary complex and nonzero real constants, respectively.]

4 Answer (a).

We are given the function

$$x(t) = 1 + \cos(\pi t) + \sin^2(\pi t).$$

We can rewrite x in the form of a Fourier series by simple algebraic manipulation as follows:

$$\begin{aligned} x(t) &= 1 + \cos(\pi t) + \sin^2(\pi t) \\ &= 1 + \frac{1}{2}[e^{j\pi t} + e^{-j\pi t}] + \left[\frac{1}{2j}[e^{j\pi t} - e^{-j\pi t}] \right]^2 \\ &= 1 + \frac{1}{2}e^{j\pi t} + \frac{1}{2}e^{-j\pi t} - \frac{1}{4}[e^{j2\pi t} - 2 + e^{-j2\pi t}] \\ &= -\frac{1}{4}e^{-j2\pi t} + \frac{1}{2}e^{-j\pi t} + \frac{3}{2} + \frac{1}{2}e^{j\pi t} - \frac{1}{4}e^{j2\pi t}. \end{aligned}$$

Thus, we have that

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},$$

where $\omega_0 = \pi$ (i.e., $T = 2$) and

$$c_k = \begin{cases} \frac{3}{2} & k = 0 \\ \frac{1}{2} & k = \pm 1 \\ -\frac{1}{4} & k = \pm 2 \\ 0 & \text{otherwise.} \end{cases}$$

4 Answer (c).

We are given the function

$$x(t) = |\sin(2\pi t)|.$$

The function x is periodic with period $T = \frac{1}{2}$ and frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{1/2} = 4\pi$. From the Fourier series

analysis equation, we have

$$\begin{aligned}
 c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\
 &= \frac{1}{1/2} \int_0^{1/2} |\sin(2\pi t)| e^{-jk4\pi t} dt \\
 &= 2 \int_0^{1/2} e^{-j4\pi k t} \sin(2\pi t) dt \\
 &= 2 \left[\frac{e^{-j4\pi k t} [-j4\pi k \sin(2\pi t) - 2\pi \cos(2\pi t)]}{(-j4\pi k)^2 + (2\pi)^2} \right] \Big|_0^{1/2} \\
 &= \frac{2(2\pi)}{-16\pi^2 k^2 + 4\pi^2} \left[e^{-j4\pi k t} [-j2k \sin(2\pi t) - \cos(2\pi t)] \right] \Big|_0^{1/2} \\
 &= \frac{1}{\pi(1-4k^2)} \left[e^{-j4\pi k/2} [-j2k \sin(2\pi/2) - \cos(2\pi/2)] - [-\cos 0] \right] \\
 &= \frac{1}{\pi(1-4k^2)} [e^{-j2\pi k} [-j2k \sin(\pi) - \cos(\pi)] + \cos(0)] \\
 &= \frac{1}{\pi(1-4k^2)} [2] \\
 &= \frac{2}{\pi(1-4k^2)}.
 \end{aligned}$$

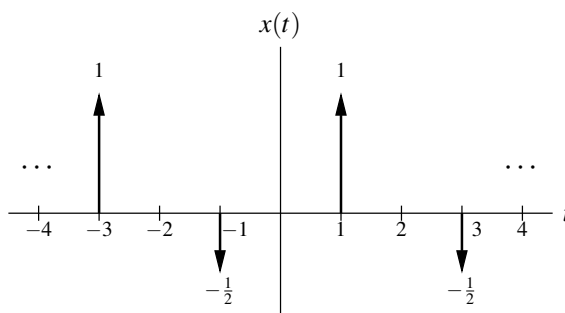
Since the integral table entry given (i.e., for the integral $\int e^{ax} \sin(bx) dx$) is valid for the case of $a = 0$, we did not need to assume that $k \neq 0$ in the above integration. Therefore, the above expression is valid for all k . Thus, we have that

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t},$$

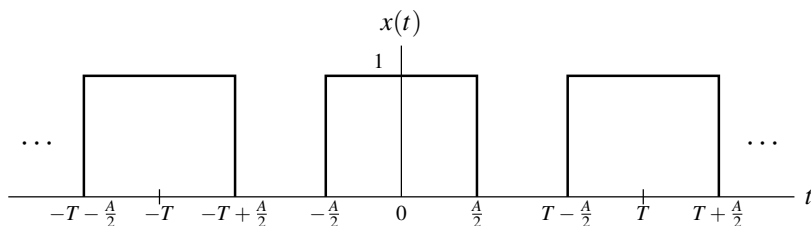
where $\omega_0 = 4\pi$ and

$$c_k = \frac{2}{\pi(1-4k^2)}.$$

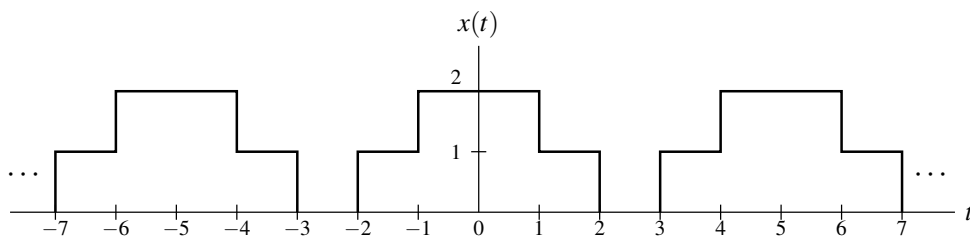
- 4 5.2** For each of the periodic functions shown in the figures below, find the corresponding Fourier series coefficient sequence.



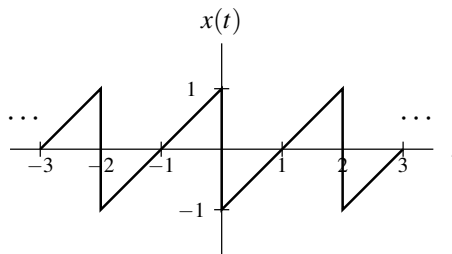
(a)



(b)



(c)



(d)

4 Answer (c).

The function x is periodic with period $T = 5$ and frequency $\omega_0 = \frac{2\pi}{5}$. From the Fourier series analysis equation,

we can write

$$\begin{aligned}
c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\
&= \frac{1}{5} \int_{-5/2}^{5/2} x(t) e^{-j2\pi kt/5} dt \\
&= \frac{1}{5} \left[\int_{-2}^{-1} e^{-j2\pi kt/5} dt + \int_{-1}^1 2e^{-j2\pi kt/5} dt + \int_1^2 e^{-j2\pi kt/5} dt \right] \\
&= \frac{1}{5} \left[\int_{-2}^2 e^{-j2\pi kt/5} dt + \int_{-1}^1 e^{-j2\pi kt/5} dt \right] \\
&= \frac{1}{5} \left[\left[\frac{1}{-j2\pi k/5} e^{-j2\pi kt/5} \right]_{-2}^2 + \left[\frac{1}{-j2\pi k/5} e^{-j2\pi kt/5} \right]_{-1}^1 \right] \quad \text{for } k \neq 0 \\
&= \frac{1}{-j2\pi k} \left[e^{-j2\pi kt/5} \Big|_{-2}^2 + e^{-j2\pi kt/5} \Big|_{-1}^1 \right] \\
&= \frac{1}{-j2\pi k} \left[e^{-j4\pi k/5} - e^{j4\pi k/5} + e^{-j2\pi k/5} - e^{j2\pi k/5} \right] \\
&= \frac{1}{-j2\pi k} [-2j \sin(4\pi k/5) - 2j \sin(2\pi k/5)] \\
&= \frac{1}{\pi k} [\sin(4\pi k/5) + \sin(2\pi k/5)] \\
&= \frac{\sin(4\pi k/5)}{\pi k} + \frac{\sin(2\pi k/5)}{\pi k} \\
&= \frac{4}{5} \text{sinc}(4\pi k/5) + \frac{2}{5} \text{sinc}(2\pi k/5).
\end{aligned}$$

In the above derivation, we assumed that $k \neq 0$. So, now we must consider the case of $k = 0$. From the Fourier series analysis equation, we have

$$\begin{aligned}
c_0 &= \frac{1}{T} \int_T x(t) dt \\
&= \frac{1}{5} \int_{-5/2}^{5/2} x(t) dt \\
&= \frac{1}{5} \left[\int_{-2}^{-1} dt + \int_{-1}^1 2dt + \int_1^2 dt \right] \\
&= \frac{1}{5} [1 + 4 + 1] \\
&= \frac{6}{5}.
\end{aligned}$$

Therefore, we have that

$$\begin{aligned}
c_k &= \begin{cases} \frac{6}{5} & k = 0 \\ \frac{4}{5} \text{sinc}(4\pi k/5) + \frac{2}{5} \text{sinc}(2\pi k/5) & \text{otherwise} \end{cases} \\
&= \frac{4}{5} \text{sinc}(4\pi k/5) + \frac{2}{5} \text{sinc}(2\pi k/5).
\end{aligned}$$

The first few coefficients are approximately as follows:

$$c_0 = 1.2, \quad c_1 = c_{-1} \approx 0.489828, \quad \text{and} \quad c_2 = c_{-2} \approx -0.057816.$$

4 5.3 Find the Fourier series coefficient sequence c of each periodic function x given below with fundamental period T .

(a) $x(t) = 2\delta(t-3) + 2\delta(t-5) + \delta(t-7) - \delta(t-9) + 3\delta(t-12)$ and $T = 16$; express c in terms of sin and cos to whatever extent is possible; and

(b) $x(t) = \delta(t) + 6\delta(t-1) + 6\delta(t-2)$ and $T = 3$; express c in terms of sin and cos to whatever extent is possible.

4 Answer (a).

We are given the T -periodic function x , where

$$x(t) = 2\delta(t-3) + 2\delta(t-5) + \delta(t-7) - \delta(t-9) + 3\delta(t-12) \quad \text{and} \quad T = 16.$$

From the Fourier series analysis equation, we have

$$\begin{aligned} c_k &= \frac{1}{T} \int_0^T x(t) e^{-j(2\pi/T)kt} dt \\ &= \frac{1}{16} \int_0^{16} [2\delta(t-3) + 2\delta(t-5) + \delta(t-7) - \delta(t-9) + 3\delta(t-12)] e^{-j(2\pi/16)kt} dt \\ &= \frac{1}{16} \int_{-\infty}^{\infty} [2\delta(t-3) + 2\delta(t-5) + \delta(t-7) - \delta(t-9) + 3\delta(t-12)] e^{-j(\pi/8)kt} dt \\ &= \frac{1}{16} \left[\int_{-\infty}^{\infty} 2\delta(t-3) e^{-j(\pi/8)kt} dt + \int_{-\infty}^{\infty} 2\delta(t-5) e^{-j(\pi/8)kt} dt + \int_{-\infty}^{\infty} \delta(t-7) e^{-j(\pi/8)kt} dt \right. \\ &\quad \left. - \int_{-\infty}^{\infty} \delta(t-9) e^{-j(\pi/8)kt} dt + \int_{-\infty}^{\infty} 3\delta(t-12) e^{-j(\pi/8)kt} dt \right] \\ &= \frac{1}{16} \left[2e^{-j(\pi/8)k(3)} + 2e^{-j(\pi/8)k(5)} + e^{-j(\pi/8)k(7)} - e^{-j(\pi/8)k(9)} + 3e^{-j(\pi/8)k(12)} \right] \\ &= \frac{1}{16} \left[2e^{-j(3\pi/8)k} + 2e^{-j(5\pi/8)k} + e^{-j(7\pi/8)k} - e^{-j(9\pi/8)k} + 3e^{-j(3\pi/2)k} \right] \\ &= \frac{1}{16} \left[2e^{-j(4\pi/8)k} \left(e^{j(\pi/8)k} + e^{-j(\pi/8)k} \right) + e^{-j\pi k} \left(e^{j(\pi/8)k} - e^{-j(\pi/8)k} \right) + 3e^{-j(3\pi/2)k} \right] \\ &= \frac{1}{16} \left[2(-j)^k \left[2\cos\left(\frac{\pi}{8}k\right) \right] + (-1)^k \left[2j\sin\left(\frac{\pi}{8}k\right) \right] + 3j^k \right] \\ &= \frac{1}{16} \left[4(-j)^k \cos\left(\frac{\pi}{8}k\right) + 2j(-1)^k \sin\left(\frac{\pi}{8}k\right) + 3j^k \right] \\ &= \frac{1}{4}(-j)^k \cos\left(\frac{\pi}{8}k\right) + \frac{j}{8}(-1)^k \sin\left(\frac{\pi}{8}k\right) + \frac{3}{16}j^k. \end{aligned}$$

4 5.7 A periodic function x with period T and Fourier series coefficient sequence c is said to be odd harmonic if $c_k = 0$ for all even k .

(a) Show that if x is odd harmonic, then $x(t) = -x(t - \frac{T}{2})$ for all t .

(b) Show that if $x(t) = -x(t - \frac{T}{2})$ for all t , then x is odd harmonic.

4 Answer (a,b).

Using the Fourier series synthesis equation, we can write

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}. \quad (5.1)$$

Substituting $t - \frac{T}{2}$ for t in the preceding equation, we obtain

$$\begin{aligned} x(t - \frac{T}{2}) &= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0(t-T/2)} = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} e^{-jk\omega_0 T/2} = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} e^{-j\pi k} \\ &= \sum_{k=-\infty}^{\infty} (-1)^k c_k e^{jk\omega_0 t}. \end{aligned}$$

(Note that, in simplifying the above equation, we used the fact that $\frac{T}{2} = \frac{\pi}{\omega_0}$ and $e^{-j\pi k} = (-1)^k$.) Thus, we have

$$x(t - \frac{T}{2}) = \sum_{k=-\infty}^{\infty} (-1)^k c_k e^{jk\omega_0 t}. \quad (5.2)$$

Using (5.1) and (5.2), we can write

$$\begin{aligned} x(t) &= -x(t - \frac{T}{2}) \\ \Leftrightarrow \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} &= - \sum_{k=-\infty}^{\infty} (-1)^k c_k e^{jk\omega_0 t} \\ \Leftrightarrow \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} &= \sum_{k=-\infty}^{\infty} (-1)^{k+1} c_k e^{jk\omega_0 t} \\ \Leftrightarrow c_k &= (-1)^{k+1} c_k = \begin{cases} c_k & k \text{ odd} \\ -c_k & k \text{ even} \end{cases} \\ \Leftrightarrow c_k &= 0 \text{ for even } k. \end{aligned}$$

Thus, we have shown that x is odd harmonic if and only if $x(t) = -x(t - \frac{T}{2})$ for all t .

4 Answer (b[alternative]).

ALTERNATIVE SOLUTION. From the Fourier series analysis equation, we have

$$\begin{aligned} c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\ &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt + \int_{T/2}^T x(t) e^{-jk\omega_0 t} dt \right]. \end{aligned}$$

Now, we employ a change a variable in the second integral. Let $\lambda = t + T/2$ so that $t = \lambda - T/2$ and $d\lambda = dt$. Applying this change of variable, we obtain

$$\begin{aligned} c_k &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt + \int_T^{3T/2} x(\lambda - \frac{T}{2}) e^{-jk\omega_0(\lambda - T/2)} d\lambda \right] \\ &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt + \int_T^{3T/2} -x(\lambda) e^{-jk\omega_0(\lambda - T/2)} d\lambda \right] \\ &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt - \int_T^{3T/2} x(\lambda) e^{jk\omega_0 T/2} e^{-jk\omega_0 \lambda} d\lambda \right] \\ &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt - e^{jk\pi} \int_T^{3T/2} x(\lambda) e^{-jk\omega_0 \lambda} d\lambda \right]. \end{aligned}$$

Now, we rename the dummy variable of integration in the second integral from λ to t . This yields

$$\begin{aligned} c_k &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt - e^{jk\pi} \int_T^{3T/2} x(t) e^{-jk\omega_0 t} dt \right] \\ &= \frac{1}{T} \left[\int_0^{T/2} x(t) e^{-jk\omega_0 t} dt - (-1)^k \int_0^{T/2} x(t) e^{-jk\omega_0 t} dt \right] \\ &= \frac{1}{T} \left[(1 - (-1)^k) \int_0^{T/2} x(t) e^{-jk\omega_0 t} dt \right] \\ &= \begin{cases} \frac{2}{T} \int_0^{T/2} x(t) e^{-jk\omega_0 t} dt & k \text{ odd} \\ 0 & k \text{ even.} \end{cases} \end{aligned}$$

Therefore, $c_k = 0$ for even k .

4 Answer (b[alternative]).

ALTERNATIVE SOLUTION. From the Fourier series analysis equation, we have

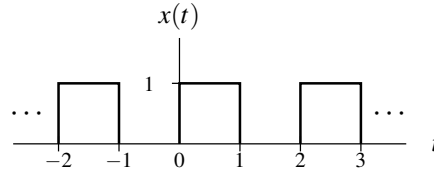
$$\begin{aligned} c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\ &= \frac{1}{T} \int_T (-x(t - T/2)) e^{-jk\omega_0 t} dt \\ &= -\frac{1}{T} \int_T x(t - T/2) e^{-jk\omega_0 t} dt \\ &= -\frac{1}{T} \int_{\alpha}^{\alpha+T} x(t - T/2) e^{-jk\omega_0 t} dt. \end{aligned}$$

Now, we employ a change of variable. Let $v = t - T/2$ so that $t = v + T/2$ and $dv = dt$. Applying the change of variable, we obtain

$$\begin{aligned} c_k &= -\frac{1}{T} \int_{\alpha - T/2}^{\alpha + T/2} x(v) e^{-jk\omega_0(v + T/2)} dv \\ &= -\frac{1}{T} \int_T x(v) e^{-jk\omega_0 v} e^{-jk\omega_0 T/2} dv \\ &= -\frac{1}{T} \int_T x(v) e^{-jk\omega_0 v} e^{-jk(2\pi/2)} dv \\ &= (-1)^k \left(-\frac{1}{T} \int_T x(v) e^{-jk\omega_0 v} dv \right) \\ &= (-1)^k (-c_k) \\ &= (-1)^{k+1} c_k. \end{aligned}$$

So, we have that $c_k = (-1)^{k+1}c_k$. If k is even, then $c_k = -c_k$. This implies, however, that $c_k = 0$. Therefore, for even k , we have that $c_k = 0$.

- 4 5.9** Find the Fourier series coefficient sequence c of the periodic function x shown in the figure below. Plot the frequency spectrum of x , including the first five harmonics.



4 Answer.

The signal x is periodic with period $T = 2$ and frequency $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$. From the Fourier series analysis equation, we have

$$\begin{aligned}
 c_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\
 &= \frac{1}{2} \int_0^2 x(t) e^{-j\pi k t} dt \\
 &= \frac{1}{2} \int_0^1 e^{-j\pi k t} dt \\
 &= \frac{1}{2} \left[\frac{1}{-j\pi k} e^{-j\pi k t} \right]_0^1 \quad \text{for } k \neq 0 \\
 &= \frac{-1}{j2\pi k} \left[e^{-j\pi k} \right]_0^1 \\
 &= \frac{1}{j2\pi k} \left[1 - e^{-j\pi k} \right] \\
 &= \frac{1}{j2\pi k} \left[1 - (-1)^k \right] \\
 &= \begin{cases} -\frac{j}{\pi k} & k \text{ odd} \\ 0 & k \text{ even, } k \neq 0 \end{cases}
 \end{aligned}$$

Since we assumed that $k \neq 0$ in the derivation above, we must now consider the case of $k = 0$. From the Fourier series analysis equation, we have

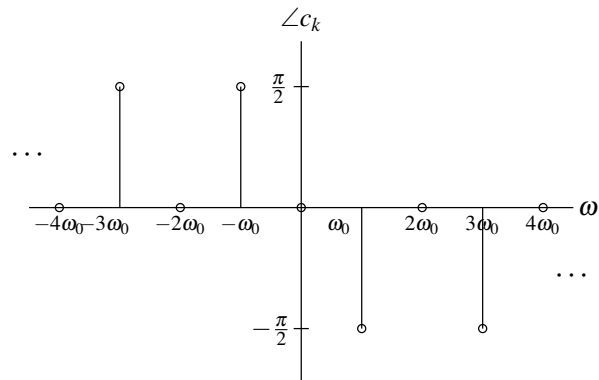
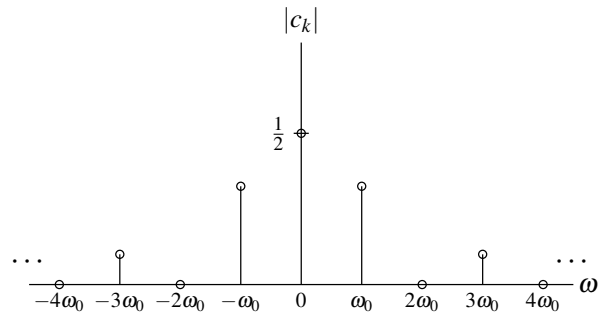
$$\begin{aligned}
 c_0 &= \frac{1}{T} \int_T x(t) dt \\
 &= \frac{1}{2} \int_0^2 x(t) dt \\
 &= \frac{1}{2} \int_0^1 dt \\
 &= \frac{1}{2} [t]_0^1 \\
 &= \frac{1}{2}.
 \end{aligned}$$

Thus, we have

$$c_k = \begin{cases} \frac{1}{2} & k = 0 \\ -\frac{j}{\pi k} & k \text{ odd} \\ 0 & k \text{ even, } k \neq 0 \end{cases}$$

Calculating the first several Fourier series coefficients yields the following:

k	$ c_k $	$\arg c_k$
0	$\frac{1}{2}$	0
1	$\frac{1}{\pi}$	$-\frac{\pi}{2}$
2	0	0
3	$\frac{1}{3\pi}$	$-\frac{\pi}{2}$
4	0	0
5	$\frac{1}{5\pi}$	$-\frac{\pi}{2}$



4 5.10 Consider a LTI system with frequency response

$$H(\omega) = \begin{cases} 1 & |\omega| \geq 5 \\ 0 & \text{otherwise.} \end{cases}$$

Using frequency-domain methods, find the output y of the system if the input x is given by

$$x(t) = 1 + 2\cos(2t) + 2\cos(4t) + \frac{1}{2}\cos(6t).$$

4 Answer.

We begin by finding the Fourier series representation of x . Using Euler's relation, we can rewrite x as

$$\begin{aligned} x(t) &= 1 + 2\cos(2t) + 2\cos(4t) + \frac{1}{2}\cos(6t) \\ &= 1 + 2\left[\frac{1}{2}(e^{j2t} + e^{-j2t})\right] + 2\left[\frac{1}{2}(e^{j4t} + e^{-j4t})\right] + \frac{1}{2}\left[\frac{1}{2}(e^{j6t} + e^{-j6t})\right] \\ &= 1 + e^{j2t} + e^{-j2t} + e^{j4t} + e^{-j4t} + \frac{1}{4}e^{j6t} + \frac{1}{4}e^{-j6t}. \end{aligned}$$

Thus, we have that the Fourier series representation of x is given by

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

where $\omega_0 = 2$ and

$$a_k = \begin{cases} 1 & k = 0 \\ 1 & k = \pm 1 \\ 1 & k = \pm 2 \\ \frac{1}{4} & k = \pm 3 \\ 0 & \text{otherwise.} \end{cases}$$

Since the system is LTI, we know that the output y has the form

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t},$$

where $b_k = a_k H(k\omega_0)$. Using the results from above, we can calculate the b_k as follows:

$$\begin{aligned} b_0 &= a_0 H([0][2]) = 0, \\ b_1 &= a_1 H([1][2]) = 0, \\ b_{-1} &= a_{-1} H([-1][2]) = 0, \\ b_2 &= a_2 H([2][2]) = 0, \\ b_{-2} &= a_{-2} H([-2][2]) = 0, \\ b_3 &= a_3 H([3][2]) = \frac{1}{4}(1) = \frac{1}{4}, \quad \text{and} \\ b_{-3} &= a_{-3} H([-3][2]) = \frac{1}{4}(1) = \frac{1}{4}. \end{aligned}$$

Thus, we have

$$b_k = \begin{cases} \frac{1}{4} & k = \pm 3 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the output y is given by

$$\begin{aligned}y(t) &= \frac{1}{4}e^{-j6t} + \frac{1}{4}e^{j6t} \\&= \frac{1}{4}[e^{-j6t} + e^{j6t}] \\&= \frac{1}{4}[2\cos(6t)] \\&= \frac{1}{2}\cos(6t).\end{aligned}$$

- 4 5.201** Consider the periodic function x shown in Figure B of Exercise 5.2, where $T = 1$ and $A = \frac{1}{2}$. We can show that x has the Fourier series representation

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

where $c_k = \frac{1}{2} \text{sinc}\left(\frac{\pi k}{2}\right)$ and $\omega_0 = 2\pi$. Let $\hat{x}_N(t)$ denote the above infinite series truncated after the N th harmonic component. That is,

$$\hat{x}_N(t) = \sum_{k=-N}^N c_k e^{jk\omega_0 t}.$$

- (a) Use MATLAB to plot $\hat{x}_N(t)$ for $N = 1, 5, 10, 50, 100$. You should see that as N increases, \hat{x}_N converges to x . [Hint: You may find the `sym`, `symsum`, `subs`, and `ezplot` functions useful for this problem. Note that the MATLAB `sinc` function does not compute the sinc function as defined herein. Instead, the MATLAB `sinc` function computes the normalized sinc function as defined by (3.21).]
- (b) By examining the graphs obtained in part (a), answer the following: As $N \rightarrow \infty$, does \hat{x}_N converge to x uniformly (i.e., at the same rate everywhere)? If not, where is the rate of convergence slower?
- (c) The function x is not continuous everywhere. For example, x has a discontinuity at $\frac{1}{4}$. As $N \rightarrow \infty$, to what value does \hat{x}_N appear to converge at this point? Again, deduce your answer from the graphs obtained in part (a).

4 Answer (a,b,c).

- (a) The graphs necessary in this problem can be generated using the code given below.

Listing 5.1: main.m

```
clear all

syms k t;

% Define a function that is one at the origin and zero elsewhere.
delta = @(t) 1 - abs(heaviside(-t) - heaviside(t));

% Define the sinc function in a manner that avoids division by zero when
% the function is evaluated at the origin.
mysinc = @(t) (sin(t) + delta(t)) / (t + delta(t));

w = 2 * pi;

for n = [1 5 10 50 100]

    % Sum the appropriate number of terms.
    f = symsum(0.5 * mysinc(pi / 2 * k) * exp(j * k * w * t), k, -n, n);

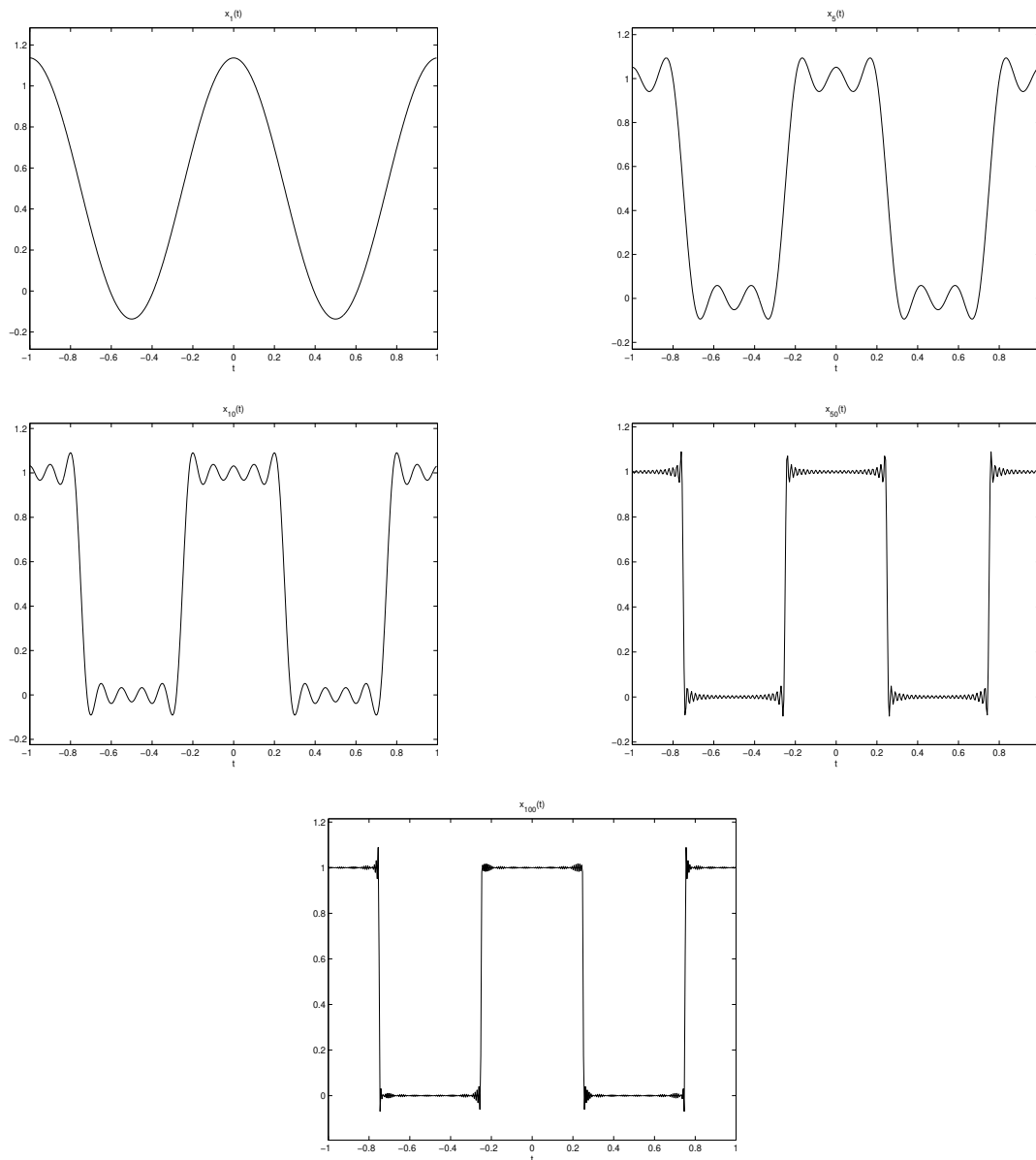
    % Plot the result.
    ezplot(f, [-1 1]);
    title(sprintf('x_{%d}(t)', n));

    % Pause for a moment so that the graph does not disappear too quickly.
    pause(1);

    % Print the graph to a file.
    eval(sprintf('print -dps data/sqrwav_%d.ps', n));
```

end

Using the above code, we obtain the graphs given below.



(b) The function $\hat{x}_N(t)$ does not converge to $x(t)$ uniformly (i.e., at the same rate everywhere). The rate of convergence is (relatively) lower at/near the points of discontinuity of $x(t)$.

(c) At the point of discontinuity of $x(t)$ located at $t = \frac{1}{4}$, the function $\hat{x}_N(t)$ appears to converge to the average of the left and right limits of $x(t)$ at that point, namely the value of $\frac{1}{2}$.