5.1 (a) $x(t) = 1 + \cos \pi t + \sin^2 \pi t$

$= 1 + \cos \pi t + \frac{1}{2}(1 - \cos 2\pi t)$

$= 1 + \cos \pi t + \frac{1}{2} - \frac{1}{2}\cos(\pi t 2)$

$= \left(\frac{3}{2}\right) + \frac{e^{j\pi t} + e^{-j\pi t}}{2} - \frac{1}{2} \cdot \frac{e^{j2\pi t} + e^{-j2\pi t}}{2}$

$\therefore x(t) = \frac{3}{2} + \frac{1}{2}e^{j\pi t} + \frac{1}{2}e^{-j\pi t} - \frac{1}{4}e^{j2\pi t} - \frac{1}{4}e^{-j2\pi t}$ —— $\boxed{\text{Equation 1}}$

period of $\cos \pi t = \frac{2\pi}{\pi} = \boxed{2 = T_1}$

" " $\cos 2\pi t = 2\pi / 2\pi = \boxed{1 = T_2}$

period of $x(t) = \text{LCM}(T_1, T_2)$

$= \text{LCM}(2, 1)$

$= 2$

So fundamental period = 2 seconds

fundamental frequency $= 2\pi/2 = \pi$ rad/seconds

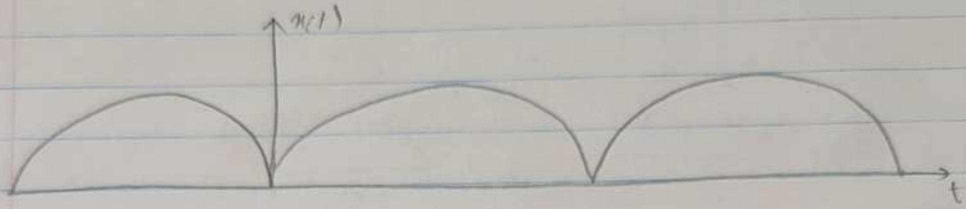
Now, $x(t) = \sum c_k e^{jk\omega_0 t}$ (Fourier series equation)

Comparing w/ equation 1, we get

$c_0 = \frac{3}{2}$, $c_1 = c_{-1} = \frac{1}{2}$

$c_2 = c_{-2} = -\frac{1}{4}$

(c) $x(t) = |\sin 2\pi t|$



T, Time period $= \frac{1}{2}\left(\frac{2\pi}{2\pi}\right) = \frac{1}{2}$ second

ω_0, Frequency $= 2\pi/0.5 = 4\pi$

$$c_n = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= 2\int_0^T (\sin 2\pi t) \cdot e^{-j\omega_0 n t}\, dt$$

$$= 2\int_0^{0.5} e^{-jn\omega_0 t}(\sin 2\pi t)\, dt$$

$$= 2 \times \left[\frac{e^{-jn\omega_0 t} \cdot \{(\sin 2\pi t)(-jn\omega_0) - (2\pi \cos 2\pi t)\}}{(-jn\omega_0)^2 + (2\pi)^2}\right]_0^{0.5}$$

$$= 2\left[\frac{\left(e^{-jn\omega_0 \frac{1}{2}}\right)(0 + 2\pi)}{-n^2\omega_0^2 + 4\pi^2} - \frac{(-2\pi)}{-n^2\omega_0^2 + 4\pi^2}\right]$$

$$= 2\left[\frac{e^{-jn\frac{1}{2}(4\pi)}\, 2\pi}{-16\pi^2 n^2 + 4\pi^2} + \frac{2\pi}{-16\pi^2 n^2 + 4\pi^2}\right] \quad \omega_0 = 4\pi$$

$$= 2\left[\frac{e^{-jn2(\pi)}\, 2\pi + 2\pi}{4\pi^2(1 - 4n^2)}\right] =$$

$$c_n = \frac{4\pi}{4\pi^2(1-4n^2)}\left(e^{-j2\pi n}+1\right)$$

$$= \frac{1}{\pi(1-4n^2)}(1+1) = \frac{2}{\pi(1-4n^2)}$$

$$c_n = \frac{2}{\pi(1-4n^2)}$$

$$c_0 = \frac{2}{\pi}$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn(4\pi)t}$$

$$= \frac{2}{\pi} + \frac{2}{\pi(1-4)} \cdot e^{j4\pi t} + \frac{2}{\pi(1-4)} e^{-j4\pi t} + \sum_{-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} e^{jn4\pi t}$$

$n=0$

$n=1$

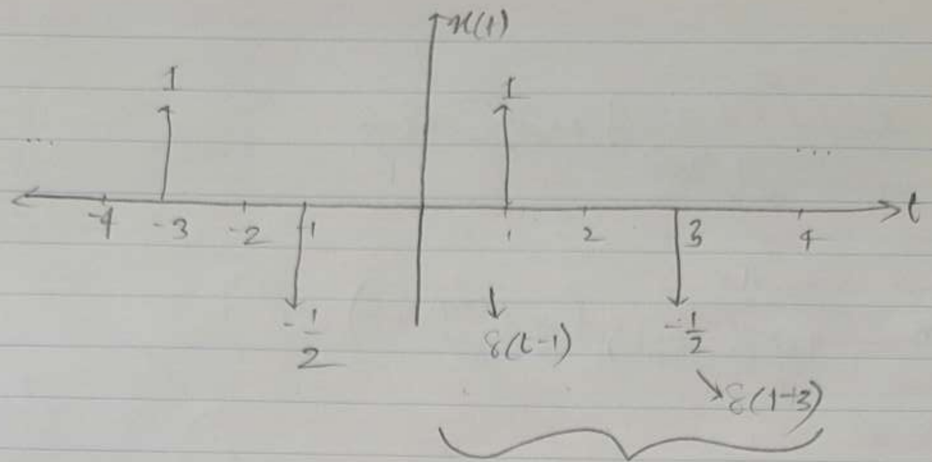
$n=-1$

all otheres except $n \neq 0, 1, -1$

(Ans)

52

(a)



(0,4)

We know that

$$c_n = \frac{1}{T}\int_0^{T} x(t)\, e^{-jn\omega_0 t}\, dt \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$= \frac{1}{4}\int_0^4 \left[x(t)\right] e^{-jn\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^4 \left[\delta(t-1) - \frac{1}{2}(\delta-3)\right] e^{-jn\frac{\pi}{2}t}\, dt$$

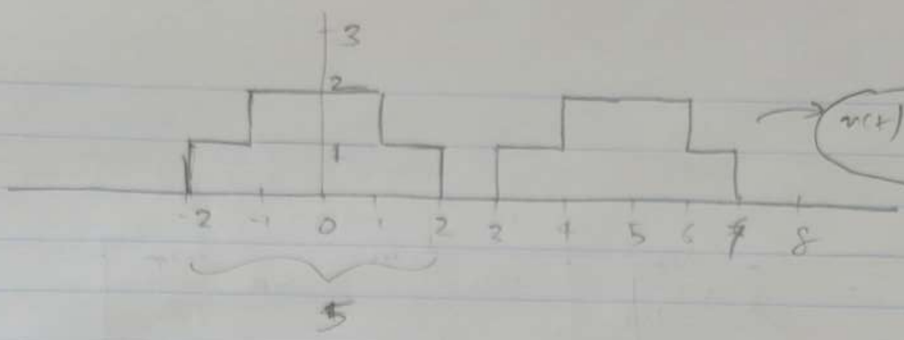
$$= \frac{1}{4}\left[e^{-jn\frac{\pi}{2}(1)} - \frac{1}{2}e^{-jn\frac{\pi}{2}(3)}\right]$$

$$= \frac{1}{4}\left[\cos\frac{n\pi}{2} - j\sin\frac{n\pi}{2} - \frac{1}{2}\cos\frac{n\pi}{2} - \frac{1}{2}j\sin\frac{n\pi}{2}\right]$$

$$= \frac{1}{4}\left[\frac{1}{2}\cos\left(\frac{n\pi}{2}\right) - \frac{3}{2}j\sin\left(\frac{n\pi}{2}\right)\right]$$

$$c_n = \frac{1}{8}\left[\cos\left(\frac{n\pi}{2}\right) - 3j\sin\left(\frac{n\pi}{2}\right)\right]$$

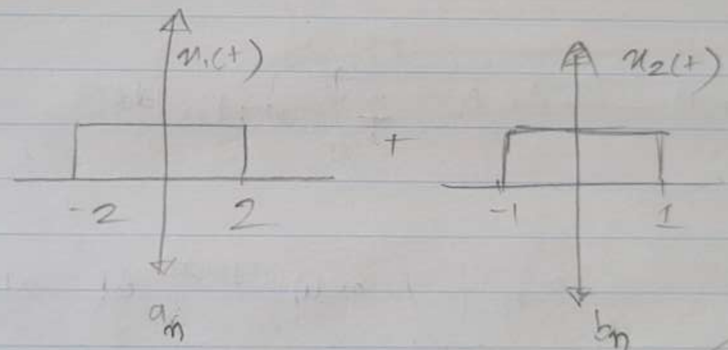
(c)



$$x(t) = \underbrace{x_1(t)}_{a_n} + \underbrace{x_2(t)}_{b_n}$$

$$c_n = \frac{1}{T}\int_T x(t)\, e^{-j\omega_0 n t}\, dt$$

$x(t)$ is the sum of 2 rectangular periodic signals



$= x(t)$

So, $x(t) \xrightarrow[\text{Conversion}]{\text{Fourier Series}} c_n = a_n + b_n$

$T = 5$ seconds

$\omega_0 = 2\pi/T = 2\pi/5$ rad/sec

$$\Rightarrow c_n = \frac{1}{5}\int_{-2}^{2} e^{-jn\left(\frac{2}{5}\pi\right)t}\, dt + \frac{1}{5}\int_{-1}^{1} e^{-jn\left(\frac{2}{5}\pi\right)t}\, dt$$

$$= \frac{1}{5}\left[\frac{e^{-jn\frac{2\pi}{5}t}}{-jn\left(\frac{2\pi}{5}\right)}\right]_{-2}^{2} + \frac{1}{5}\left[\frac{e^{-jn\left(\frac{2\pi}{5}\right)t}}{jn\left(\frac{2\pi}{5}\right)}\right]_{-1}^{1}$$

$$= \frac{1}{5}\left[\frac{2j\sin\left(\frac{4n\pi}{5}\right)}{jn\left(\frac{2\pi}{5}\right)} + \frac{2j\sin\left(\frac{2n\pi}{5}\right)}{jn\left(\frac{2\pi}{5}\right)}\right]$$

$$c_n = \frac{2}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{2}\pi n}\left[\sin\frac{4\pi n}{5} + \sin\frac{2\pi n}{5}\right]$$

$$= \frac{1}{\pi n}\left[\sin\frac{4\pi n}{5} + \sin\frac{2\pi n}{5}\right]$$

<u>56 b</u>

$$x(t) \xrightarrow{\text{F-Series}} c_k = \frac{1}{T}\int_T x(t)\, e^{-jw_0 n t}$$

$$\text{let } x_{(\omega)} = \frac{1}{T}\int_T x(u)\, e^{-jw_0 n t}\, dt \quad \text{at } u = t - t_0$$

Time shifted right

$t = p + t_0$

$$= \frac{1}{T}\int_T x(t - t_0)\, e^{-jw_0 n (u + t_0)}\, du$$

$$= \frac{e^{-jn w_0 (t_0)}}{T}\underbrace{\int_T x(u)\, e^{-jw_0 n u}}$$

$$= e^{-jn w_0 (t_0)} \cdot x(w)$$

$$\therefore x(t - t_0) = x(u) = e^{-jn w_0 (t_0)}\, x(w)$$

$$-x\left(t-\frac{T}{2}\right) \longrightarrow -e^{-j\omega_0 n\left(\frac{T}{2}\right)} X(\omega)$$

$$\text{or } -x\left(t-\frac{T}{2}\right) = -e^{-j\left(\frac{2\pi}{T}\right)n\left(\frac{T}{2}\right)} x(\omega)$$

$$\text{or } -x\left(t-\frac{T}{2}\right) = -e^{-j\pi n}\, x(\omega)$$

$$e^{-j\pi n} = \left[\cos\pi n - j\sin\pi n\right]$$

at $n=1$, $(\cos\pi n - j\sin\pi n) = -1$

$\rightarrow n=2 \longrightarrow 1$

$\rightarrow n=3 \longrightarrow -1$

$\rightarrow n=4 \longrightarrow 1$

⋮

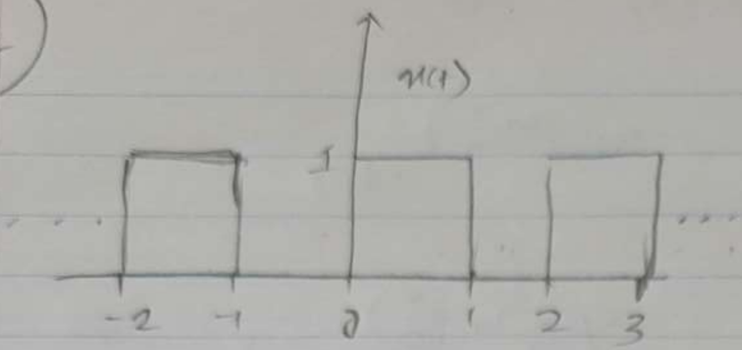
all odd n satisfies the time shifted values

n must be 1, 3, 5, 7 .... all odd values

So, $x(t)$ is odd harmonic

(Ans)

5.8



Time period, $T = 2$

Frequency $\omega = 2\pi/2 = \pi$

$$c_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{2}\int_0^{2} x(t)\, e^{-jn\left(\frac{2\pi}{2}\right)t}\, dt$$

$$= \frac{1}{2}\int_0^{1} 1 \cdot e^{-jnt}\, dt$$

$$\cancel{= \left[e^{-jnt}\right.} = \left[\frac{e^{-jnt}}{-2jn}\right]_0^1$$

$$= \frac{j}{2n}\left[e^{-jn} - 1\right]$$

$$c_0 = \frac{\text{Area under 1 Time period}}{\text{Time period}} = \frac{1\times 1}{2} = 0.5$$

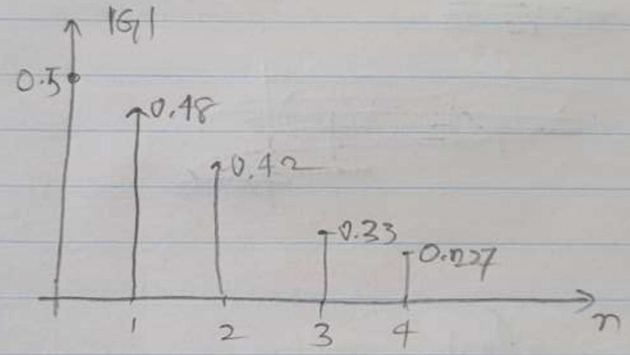
$c_1 = \frac{j}{2}\left(e^{-j1} - 1\right) = (-0.23 - 0.42j)\,j = 0.42 - 0.23j$

$= 0.48 \angle -0.50$ radian

$c_2 = \frac{j}{4}\left(e^{-j2} - 1\right) = (0.23 - 0.35j) = 0.42 \angle -0.99$ radian

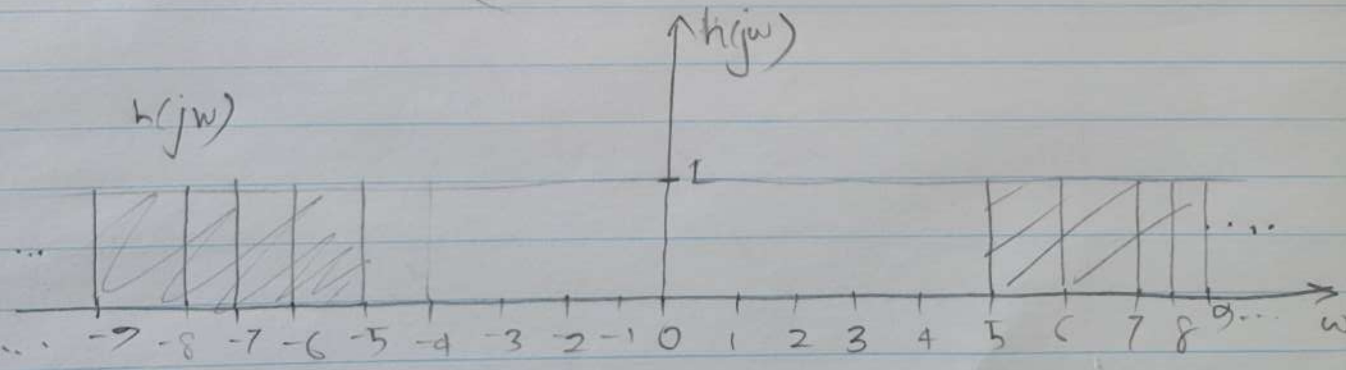
$c_3 = \frac{j}{6}\left(e^{-j3} - 1\right) = (0.33) \angle -1.5$ radian

$c_4 = \frac{j}{8}\left(e^{-j4} - 1\right) = 0.227 \angle -2$ radian



5.9 Given,

$$h(j\omega) = \begin{cases} 1 & \text{if } |\omega| \geq 5 \\ 0 & \text{else} \end{cases}$$



$x(t) \longrightarrow \boxed{h(j\omega)} \longrightarrow y(t)$

$x(t) = 1 + 2(\cos(2t)) + 2\cos(4t) + \frac{1}{2}(\cos(6t))$

at $\omega_{0_1} = 2$, $\omega_{0_2} = 0$ and $\omega_{0_3} = 4$ ~~$H(j\omega)$~~ $\omega_{0_4} = 6$

$\therefore y(t) = \frac{1}{2}\cos(6t)$ cos only $\frac{1}{2}(\cos 6t)$ passes through LTI system having frequency response $h(j\omega)$

~~$x(t)$~~

~~$\Rightarrow y(t) = h(j\omega)\Big|_{\omega=6}$~~

(Ans)