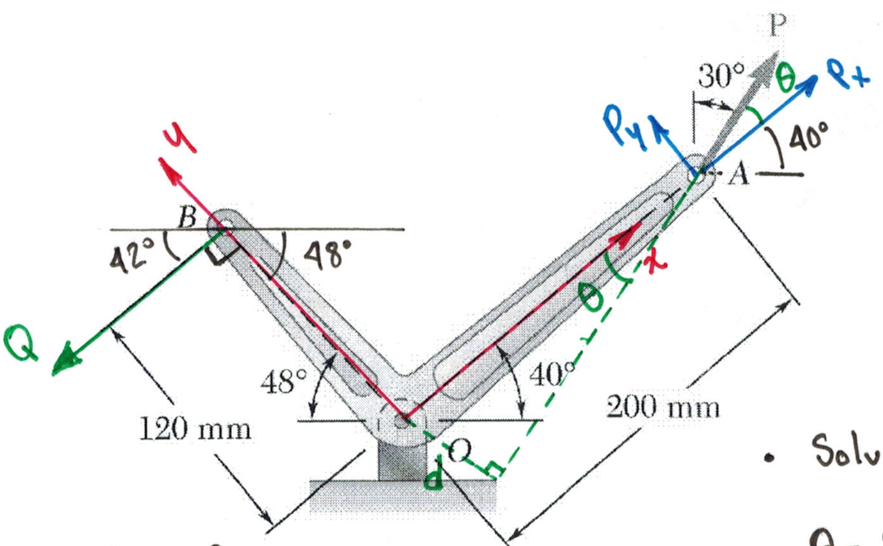
A 400-N force **P** is applied at point *A* of the bell crank shown.

a) Compute the moment of the force **P** about *O* by resolving it into components along line *OA* and in a direction perpendicular to that line.

b) Determine the magnitude and direction of the smallest force **Q** applied at *B* that has the same moment as **P** about *O*.



a) We can orient the reference frame in different ways, all of them will take us to the same solution.

- Solve for angle θ

$\theta = 90 - 30 - 40 = 20°$

- Resolve force P

$P_x = P\cos\theta = 400\cos 20 = 375.9\text{ N}$

$P_y = P\sin\theta = 400\sin 20 = 136.8\text{ N}$

$M_o = P_y \cdot d_x + \cancel{P_x \cdot d_y}^{\,0} = 136.8\,(0.2) = \underline{27.4\text{ N}\cdot\text{m}}$

Note we can also find M_o with the moment arm d (trigonometry)

$d = 0.2\sin\theta = 0.0684$ $\qquad M_o = F \cdot d = 400 \cdot 0.0684 = 27.4\text{ N}\cdot\text{m}$

b) The smallest force must be perpendicular to line OB and Q must also produce a CCW moment

$M_o = Qd \Rightarrow Q = \frac{M_o}{d} = \frac{27.4}{0.12} = 228\text{ N}$

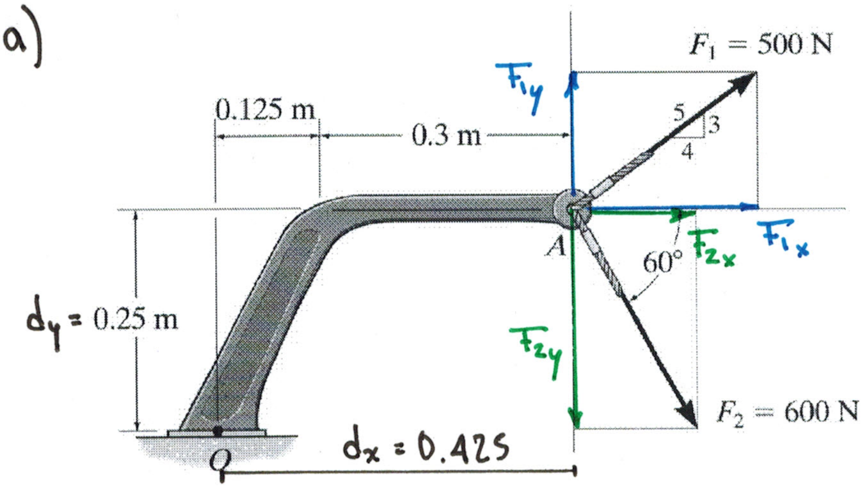
$Q = 228\text{ N} \measuredangle 42°$

Determine the resultant moment produced by the forces about point O.

a) Use the rectangular component approach

b) Find d using trigonometry

a)



Note that while F_{1x}, F_{2x}, and F_{2y} produce a CW (negative) moment about O, F_{1y} produces a CCW moment.

- Resolve forces

$F_{1x} = \frac{4}{5} 500 = 400$ N $\qquad$ $F_{1y} = \frac{3}{5} 500 = 300$ N

$F_{2x} = 600 \cos 60 = 300$ N $\qquad$ $F_{2y} = 600 \sin 60 = 519.6$ N

- Evaluate M_o

$$M_o = F_{1y} \cdot d_x - F_{1x} d_y - F_{2y} \cdot d_x - F_{2x} \cdot d_y$$

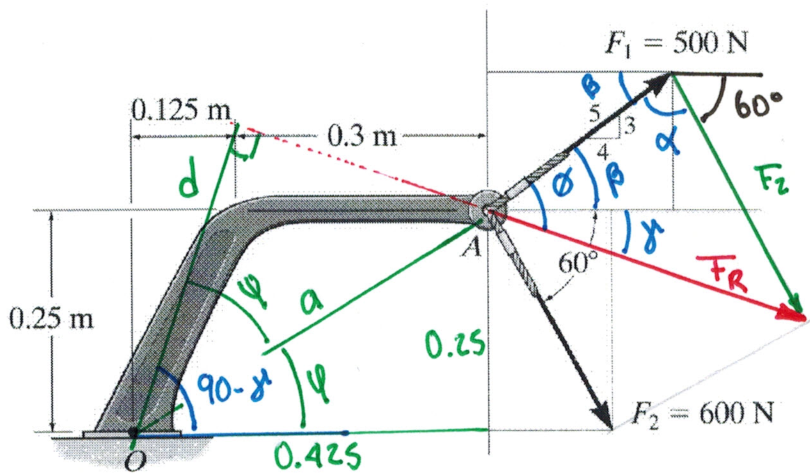
$$= 300(0.425) - 400(0.25) - 519.6(0.425) - 300(0.25) = -268.3 \text{ N}\cdot\text{m}$$

$M_o = 268.3$ N·m ↻ $\qquad$ or $\qquad$ $M_o = 268.3$ N·m (CW)

Determine the resultant moment produced by the forces about point O.

a) Use the rectangular component approach

b) Find d using trigonometry



Since these two forces act at the same point, we can determine the resultant of the two forces and moltiply it by the moment arm d.

From the above figure

$\beta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9°$ $\qquad$ $\alpha = 180 - \beta - 60° = 83.13°$

Law of cosines

$\|F_R\| = \sqrt{500^2 + 600^2 - 2(500)(600)\cos\alpha} = 733.64 \text{ N}$

Law of sines

$\frac{\sin ø}{F_2} = \frac{\sin\alpha}{F_R}$ $\qquad$ $\sin ø = \frac{F_2}{F_R}\sin\alpha = \frac{600}{733.64}\sin 83.13 = 0.8237$

$ø = 54.3°$

Since $ø = \beta + \gamma$ $\qquad$ $\gamma = ø - \beta = 54.3 - 36.9 = 17.4°$

The moment arm is found

$a = \sqrt{0.425^2 + 0.25^2} = 0.493\text{m}$ $\qquad$ $\psi = \tan^{-1}\left(\frac{0.25}{0.425}\right) = 30.47°$

From the figure $\varphi = 90 - \gamma - \psi = 90 - 17.4 - 30.47 = 42.14°$

$d = a\cos\varphi = 0.493\cos 42.14 = 0.366\text{m}$ $\qquad$ $M_O = F_R \cdot d = 733.64(0.366)$

$= 268.2 \text{ N}\cdot\text{m}$

(same as part a)