Exercise 4.109

Answer (e).

We are given a LTI system $\mathcal{H}$ with system function

$$H(s) = \frac{1}{e^s(s+4)} \quad \text{for } s \in \mathbb{C} \text{ such that } \operatorname{Re}(s) > -4.$$

Furthermore, we are given

$$x(t) = 11 + 7e^{-2t} + 5e^{-3t}.$$

From the linearity of $\mathcal{H}$, we have

$$\begin{aligned}
y(t) &= \mathcal{H}x(t) \\
&= \mathcal{H}\left\{11e^{0\cdot} + 7e^{-2\cdot} + 5e^{-3\cdot}\right\}(t) \\
&= 11\mathcal{H}\left\{e^{0\cdot}\right\}(t) + 7\mathcal{H}\left\{e^{-2\cdot}\right\}(t) + 5\mathcal{H}\left\{e^{-3\cdot}\right\}(t)
\end{aligned}$$

From the eigenfunction properties of $\mathcal{H}$, we have

$$\begin{aligned}
y(t) &= 11H(0)e^{0t} + 7H(-2)e^{-2t} + 5H(-3)e^{-3t} \\
&= 11\left(\tfrac{1}{4}\right) + 7\left[\frac{1}{e^{-2}(-2+4)}\right]e^{-2t} + 5\left[\frac{1}{e^{-3}(-3+4)}\right]e^{-3t} \\
&= \tfrac{11}{4} + 7\left[\frac{1}{2e^{-2}}\right]e^{-2t} + 5\left[\frac{1}{e^{-3}}\right]e^{-3t} \\
&= \tfrac{11}{4} + \tfrac{7}{2}e^{2}e^{-2t} + 5e^{3}e^{-3t} \\
&= \tfrac{11}{4} + \tfrac{7}{2}e^{2(1-t)} + 5e^{3(1-t)}.
\end{aligned}$$