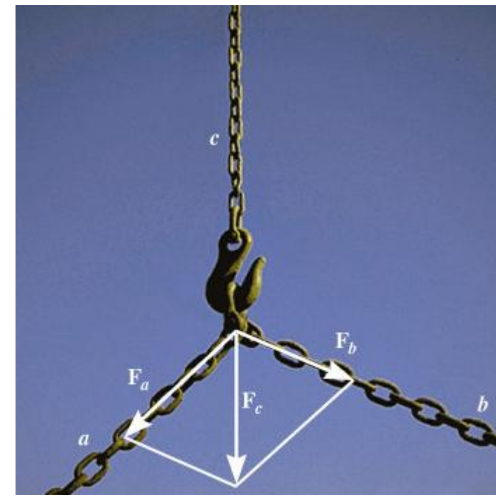
University of Victoria Engineering

Chapter 2 – Force Vectors

Contents

The parallelogram Law must be used to determine the resultant of the two forces acting on the hook



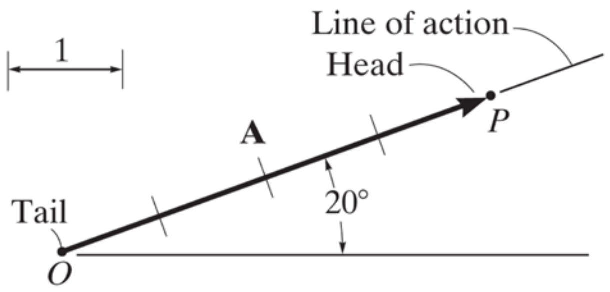
Force Vectors

Scalar and Vectors

Many physical quantities in engineering are measured using either scalars or vectors.

Scalars A scalar is a quantity with positive or negative magnitude. Examples are length, mass, and time.

Vectors A vector is a quantity with magnitude and direction. Vectors are represented by an arrow. Examples are force, position, and moment.

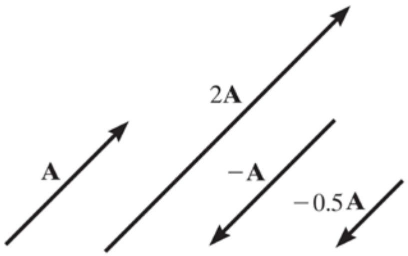


Length of the vector represents its *magnitude*, the angle relative to a fixed axis represents the *direction* of the line of action, the *head* of the arrow indicates the sense

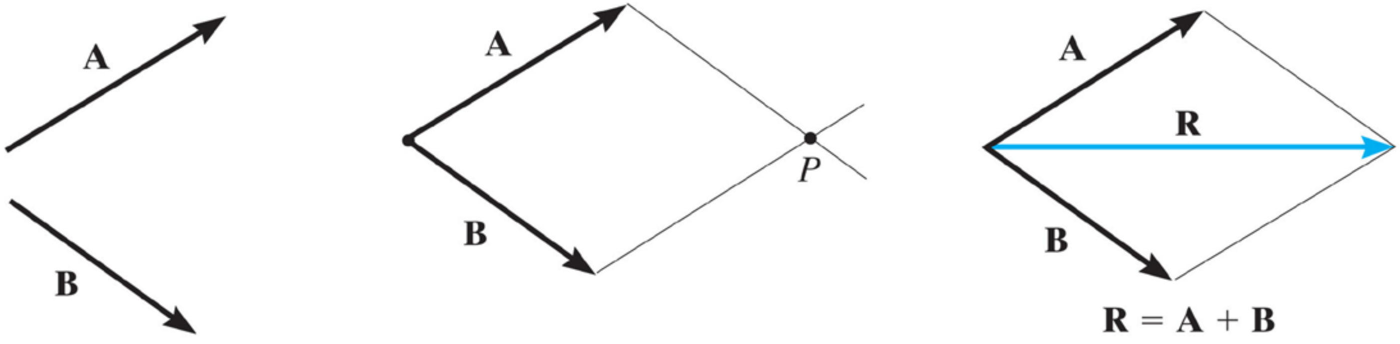
Graphical Representation

Vector Operations

Vector operations (addition and subtraction) must not only account for magnitudes of a vector but how each vector is oriented in space. All vector operations have a geometric interpretation. Multiplying or dividing a vector by a scalar change its magnitude and sense (if scalar is negative).



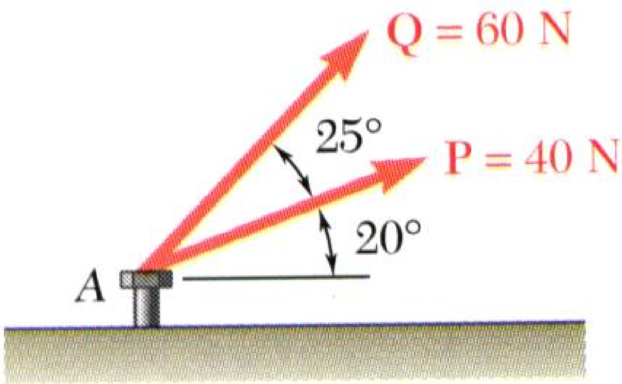
Addition (Parallelogram Law): when adding two vectors **A** and **B**, join the vectors, form a parallelogram by drawing parallel lines at the head of these vectors, find the resultant vector **R**.



Parallelogram law

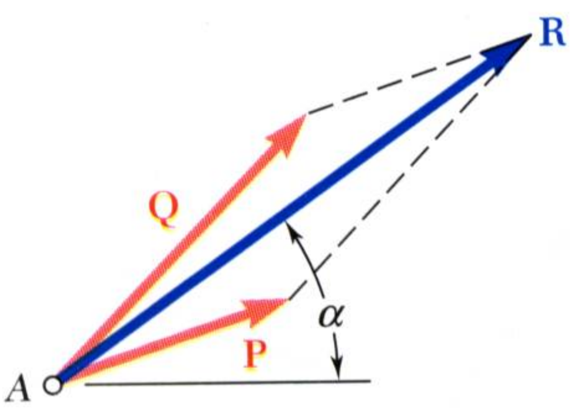
Sample Problem

There are two forces acting on a bolt at A. Determine their resultant using the parallelogram law.



SOLUTION:

In order to construct the parallelogram, we draw the forces as vectors by maintaining their magnitudes proportional, for example **P** = 40N = 4 cm, and **Q** = 60N = 6cm.



Draw a line parallel to **P** at the tip of vector **Q**. Similarly, draw a line parallel to **Q** at the tip of **P**, where these lines intersect yields the resultant **R**. The magnitude and direction of **R** are measured and scaled.

$$\boxed{\mathrm{R} = 9.8cm = 98\mathrm{N} \quad \alpha = 35°}$$

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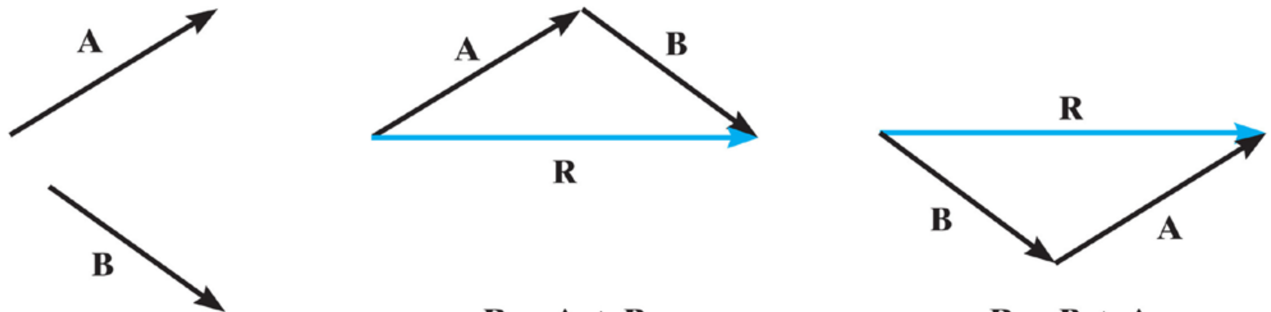
Graphical Representation

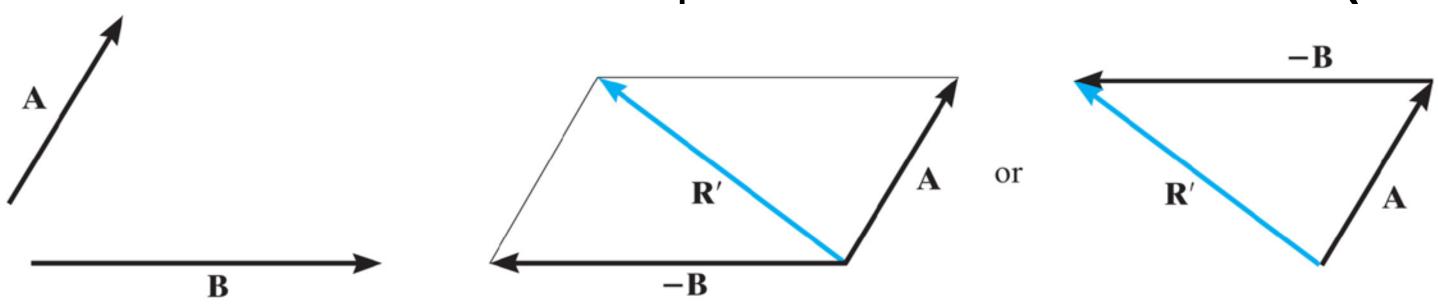
Addition (Triangle Law): Connect vectors head-to-tail (either **A** to **B** or **B** to **A**), find the resultant vector **R**.



$\mathbf{R} = \mathbf{A} + \mathbf{B}$
Triangle rule

$\mathbf{R} = \mathbf{B} + \mathbf{A}$
Triangle rule

Subtraction (Parallelogram or Triangle Law): The difference between two vectors can be expressed as **R' = A – B = A + (– B)**.

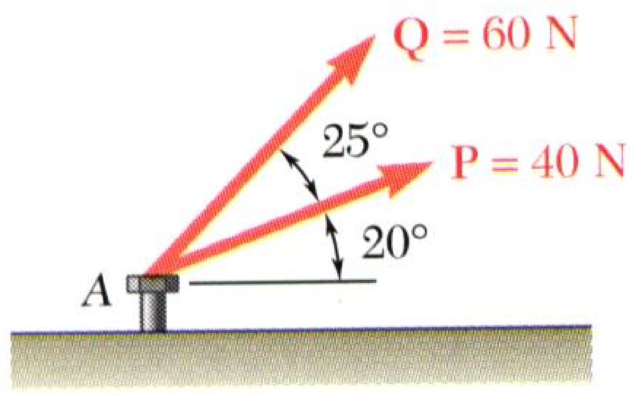


Parallelogram law

Triangle construction

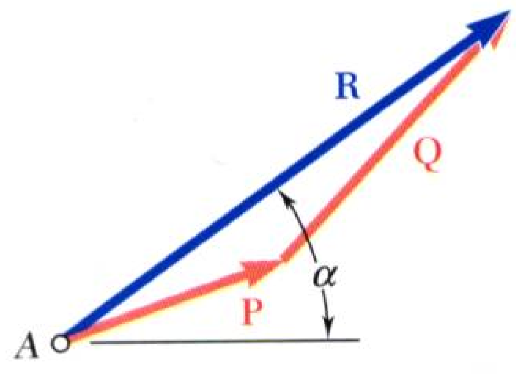
Sample Problem

Determine the resultant of the two forces acting on the bolt using the triangle rule.



SOLUTION:

Draw one force to scale (**P** = 40N = 4 cm) and attach the second vector at its tip, preserving direction and proportion, i.e., **Q** = 60N = 6cm.



Close the loop, forming a triangle. The resultant force is the third side of the triangle. The magnitude and direction of **R** are measured and scaled.

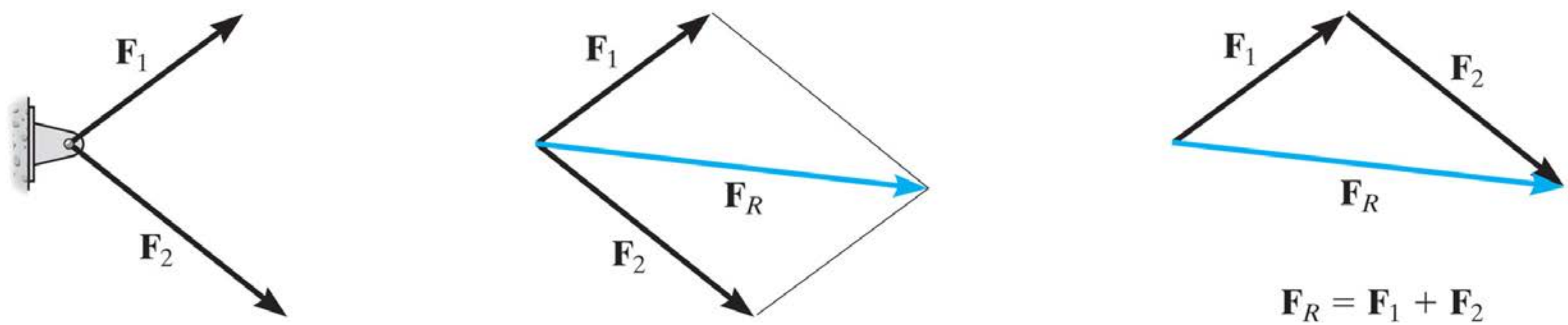
$\boxed{R = 9.8cm = 98N \quad \alpha = 35°}$

Vector Addition of Forces

Vector Addition of Forces

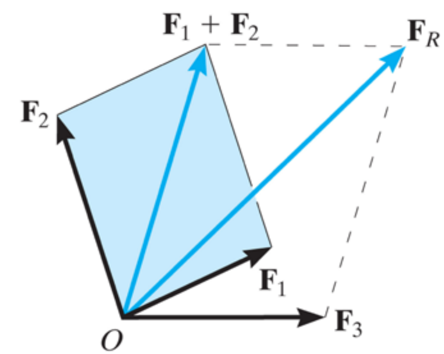
Two common problems are to determine the resultant of a force and to determine the components of a force (relative to some directions).

Finding the Resultant: If two forces act at the same point, the resultant force is the addition of these forces.



When multiple forces are acting, one can find the resultant between two forces and then add it to another force

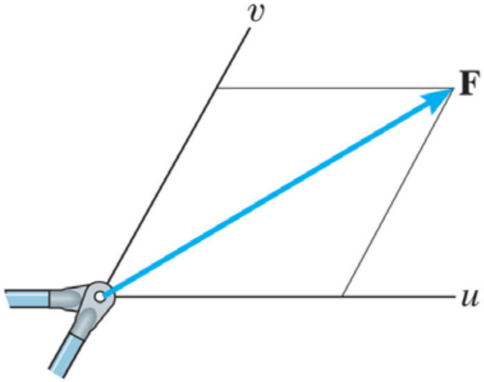
$$\mathbf{F}_R = (\mathbf{F}_1 + \mathbf{F}_2) + \mathbf{F}_3$$

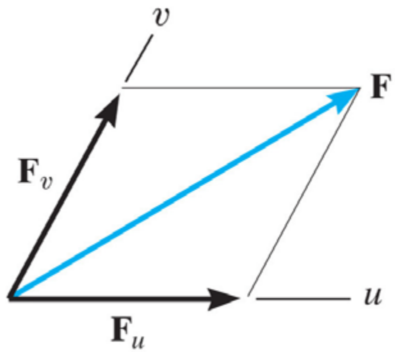


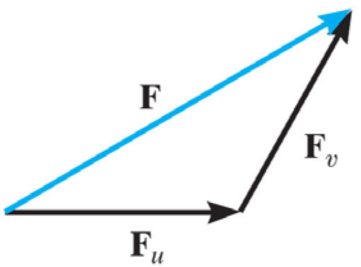
Vector Addition of Forces

Finding the Components of a Force: Sometimes, it is necessary to decompose or resolve a force into two components in order to study the pulling or pushing effect in two specific directions.

This is accomplished by applying the parallelogram or triangle law, as the direction of the parallelogram sides is given by the two directions. $\boldsymbol{F_u}$ is the projection of $\boldsymbol{F}$ along direction $\boldsymbol{u}$ and $\boldsymbol{F_v}$ is the projection of $\boldsymbol{F}$ along the direction $\boldsymbol{v}$.

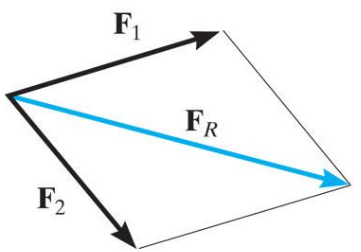




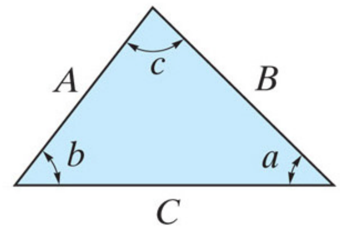


Trigonometric Solution

Procedure for Analysis: Solving the problems graphically is not precise (unless done carefully in CAD), alternatively one can use ***sine*** and ***cosine laws.***



(a)



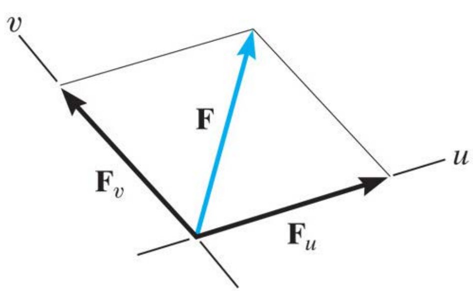
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB\cos c}$$

Sine law:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

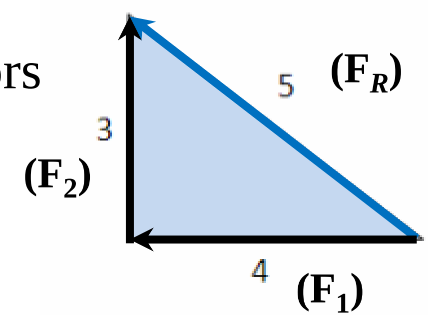
(c)



(b)

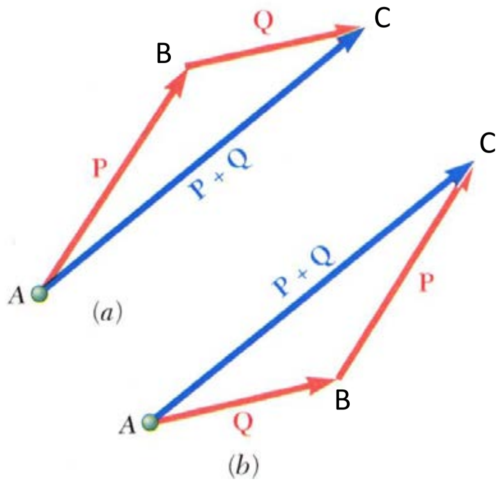
Important: When adding vectors $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$, we are not simply adding magnitudes

$$5 \neq 4 + 3$$



Trigonometric Solution

More accurate results can be obtained using trigonometry.



- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

- Law of sines,

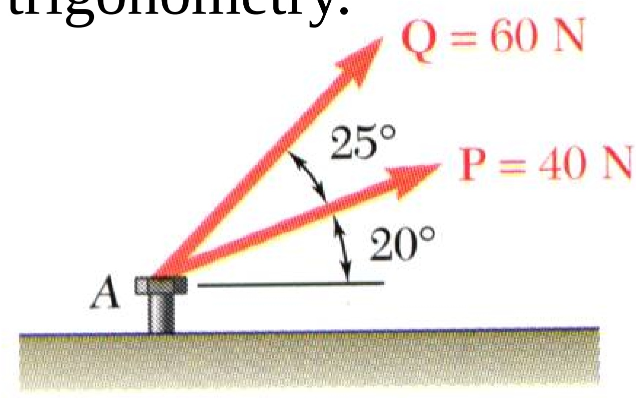
$$(a) \quad \frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{P}$$

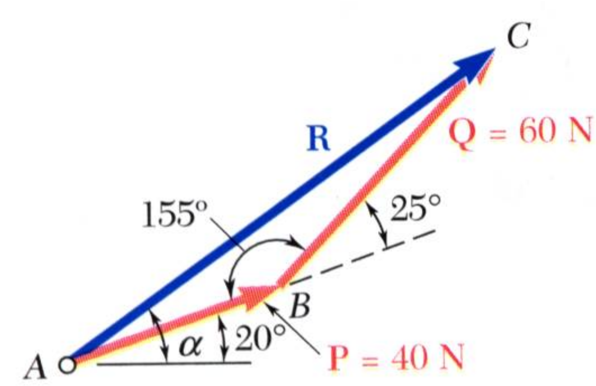
$$(b) \quad \frac{\sin A}{P} = \frac{\sin B}{R} = \frac{\sin C}{Q}$$

where R, P, and Q are the magnitudes of vectors **R**, **P**, and **Q**, respectively.

Sample Problem

Determine the resultant of the two forces acting on the bolt using trigonometry.





SOLUTION:

Use the triangle rule solution for support.

Using Law of Cosines:

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

Using Law of Sines:

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B\frac{Q}{R} = \sin 155° \frac{60N}{97.73N}$$

$$A = 15.04°$$

$$\alpha = 20° + A$$

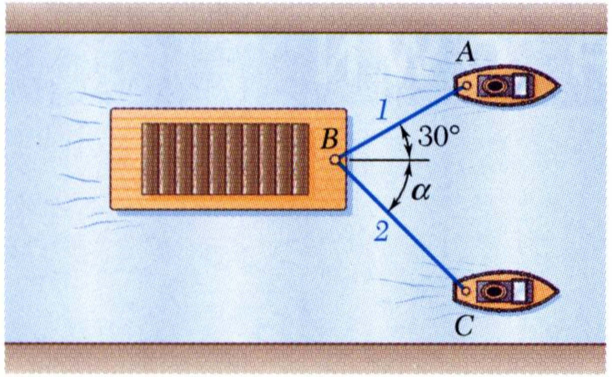
$$\boxed{\alpha = 35.04°}$$

Example

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

a) the tension in each of the ropes for $\alpha = 45^o$, using both graphical and trigonometric solutions

b) the value of α for which the tension in rope 2 is a minimum.



Addition of a System of Coplanar Forces

Reference Frames:

The Laufenburg bridge mistake. Germany used the North Sea and Switzerland the Mediterranean Sea. When they, there was a 54 cm difference.

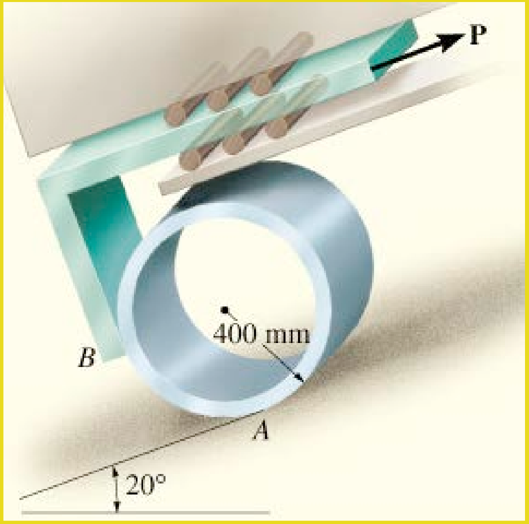
In order to eliminate any potential ambiguity and errors, we must describe vectors relative to the same **reference frame**.

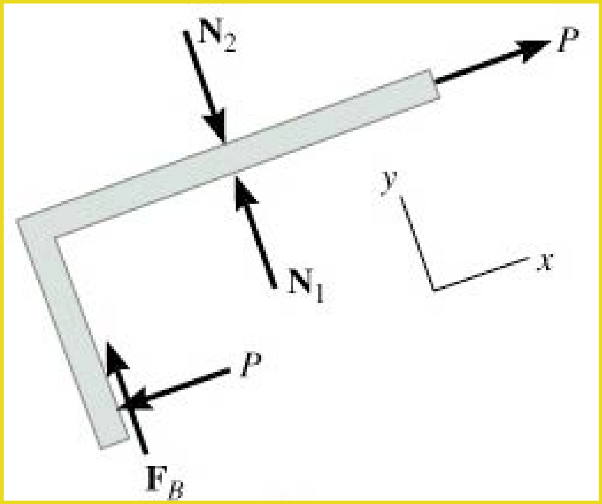
A reference frame is a collection of three axes that span three-dimensional space. In order to use a reference frame, you have to define it. On the plane, the reference frame requires two axes *x* and *y* that are orthogonal to each other.

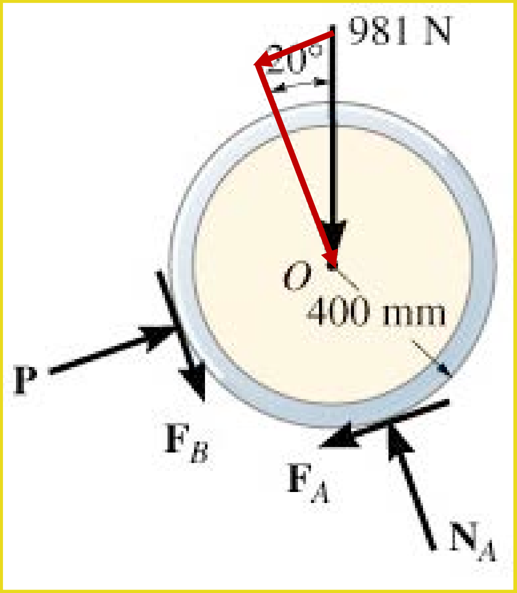
Addition of a System of Coplanar Forces

Every vector problem you complete in ENGR 141 requires that you define a Cartesian frame of reference.

When a reference frame is present, any vector can be broken down into scalar Cartesian components. Each component thus has a ‘+’ and ‘-’ sense, *e.g.*, N_1 is ‘+’ and N_2 is ‘-’ with respect to the *y* direction.

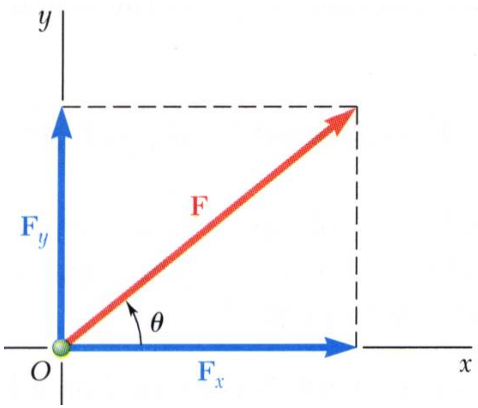






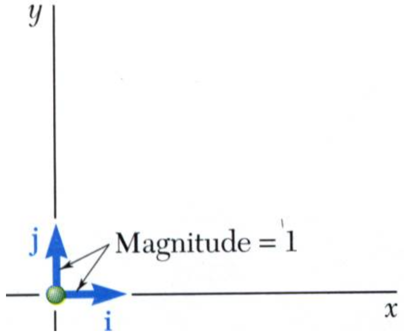
Addition of a System of Coplanar Forces

Scalar Notation:



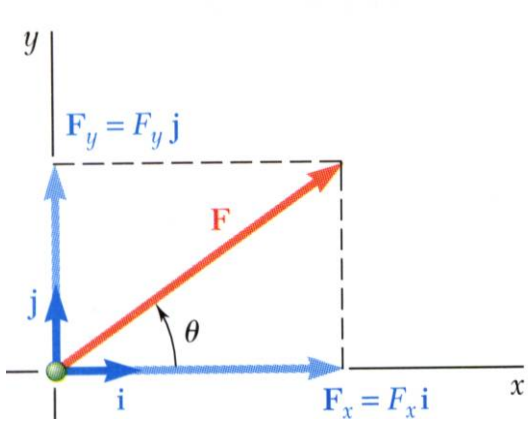
We can decompose a force vector into two perpendicular components so that the resulting parallelogram is a rectangle. $\mathbf{F}_x$ and $\mathbf{F}_y$ are referred to as *rectangular vector components* and

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$



We define perpendicular *unit vectors* **i** and **j** which are colinear to the *x* and *y* axes.

Addition of a System of Coplanar Forces



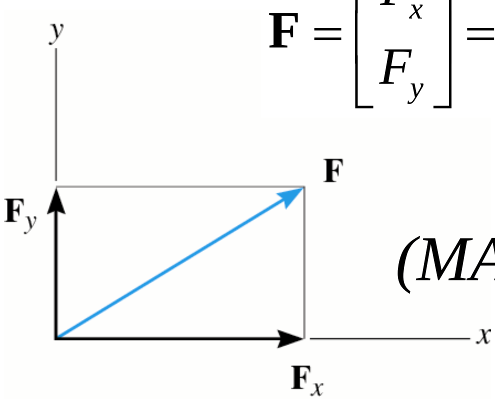
Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

F_x and F_y are the *scalar components* of **F**.

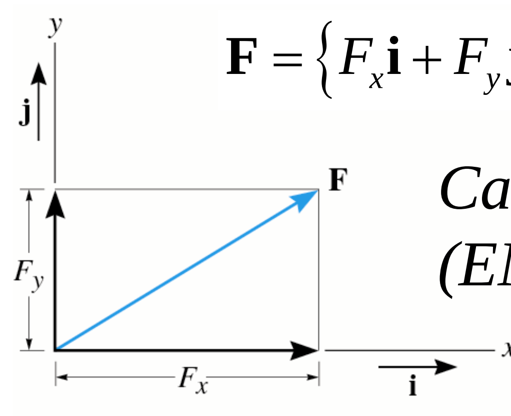
Vector Notation:

$$\mathbf{F} = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} F\cos\theta \\ F\sin\theta \end{bmatrix}$$



(MATH 110)

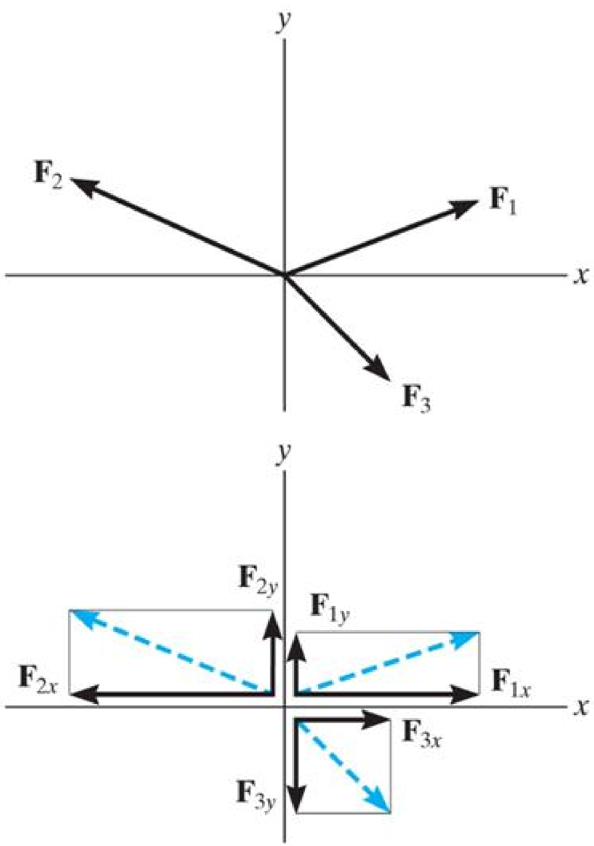
$$\mathbf{F} = \left\{F_x\mathbf{i} + F_y\mathbf{j}\right\} = \left\{F_x\vec{i} + F_y\vec{j}\right\}$$



Cartesian Notation (ENGR 141)

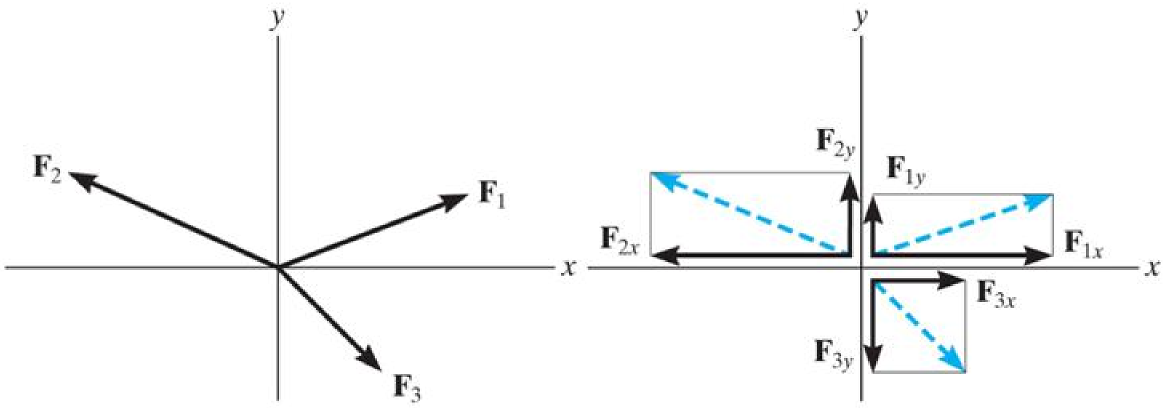
Addition of a System of Coplanar Forces

Coplanar Force Resultant:



- Step 1 is to resolve each force into its components.
- Step 2 is to add all the x-components together, followed by adding all the y-components together. These two totals are the x and y-components of the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

Addition of a System of Coplanar Forces



Break the three vectors into components, then add them.

$$\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$$

$$= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$$

$$= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$$

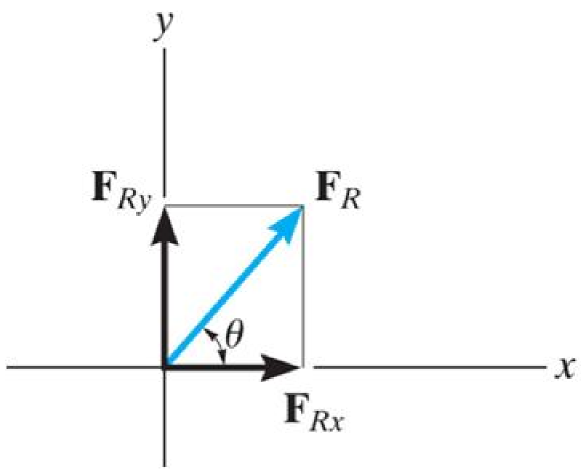
$$= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$$

Addition of a System of Coplanar Forces

The magnitude of the resultant $\mathbf{F_R}$ is found using the Pythagorean theorem and the direction (θ) is obtained using trigonometry.

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

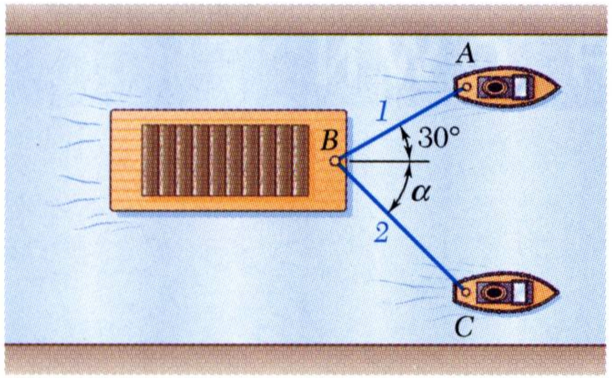


Important: If the denominator (F_{Rx}) is negative, you must add 180° to the resulting angle.

Example

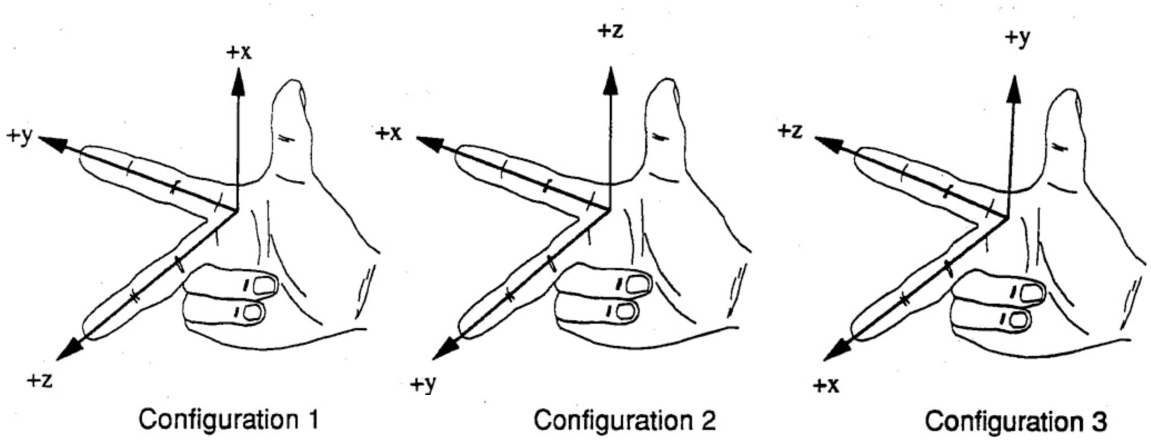
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

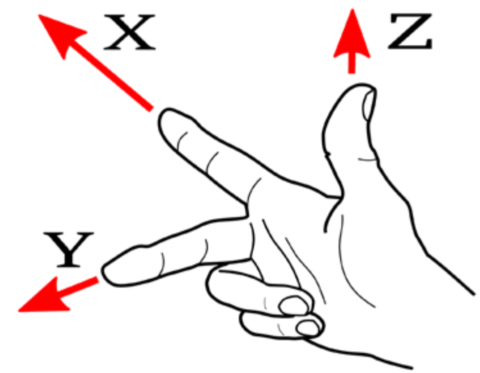
c) the tension in each of the ropes for $\alpha = 45^o$, using rectangular coordinates



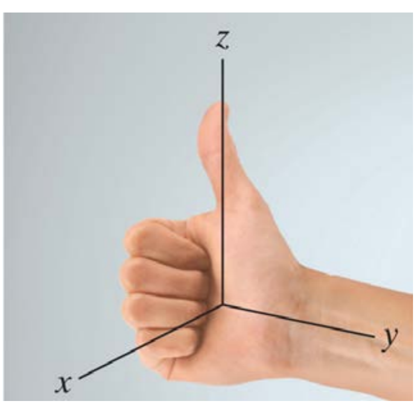
Cartesian Vectors

Right-Handed Coordinate System



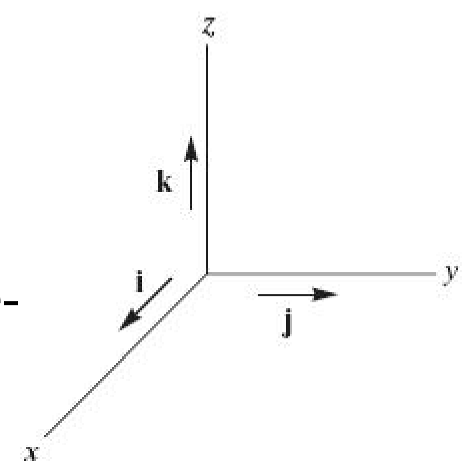


There is not a unique configuration, my preference is Configuration 2.



Right-Handed Configuration
from the textbook

Unit vectors in a three-dimensional space



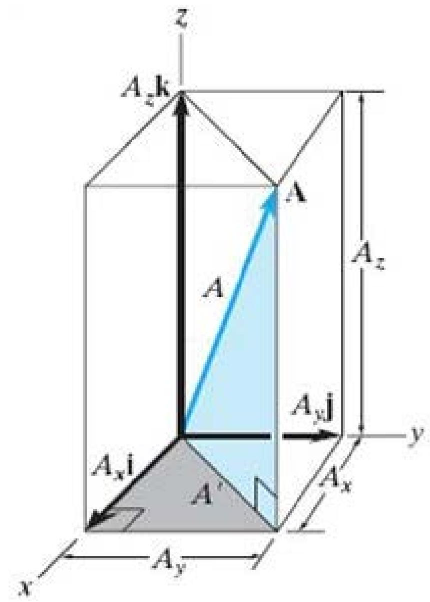
Cartesian Vectors

Cartesian Vector Representation

The vector A can be defined as

$$\mathbf{A} = (\mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z)\ \text{m}$$

$$\mathbf{A} = (A_x\, \boldsymbol{i} + A_y\, \boldsymbol{j} + A_z\, \boldsymbol{k})\ \text{m}$$



Magnitude of a Cartesian Vector

The magnitude of vector **A** is defined as

$$A = (A_x^2 + A_y^2 + A_z^2)^{½}$$

Cartesian Vectors

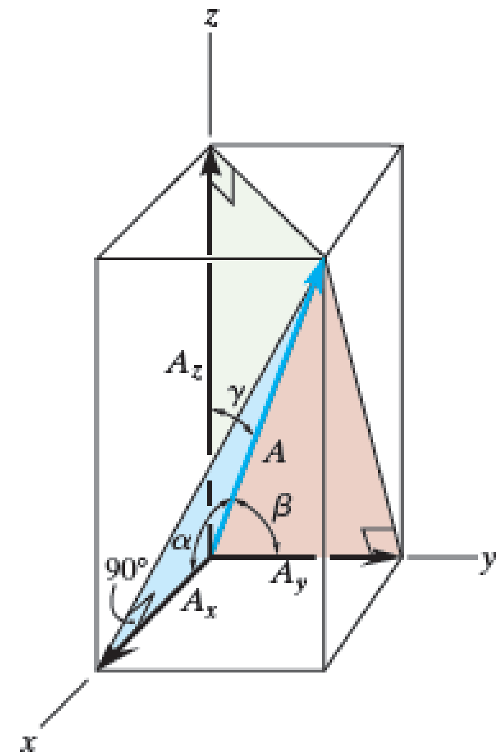
Coordinate Direction Angles

The direction or orientation of vector **A** is defined by the angles α, β, and γ.

These angles are measured between the vector and the positive X, Y and Z axes.

Using trigonometry, “direction cosines” are found using

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$



These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Cartesian Vectors

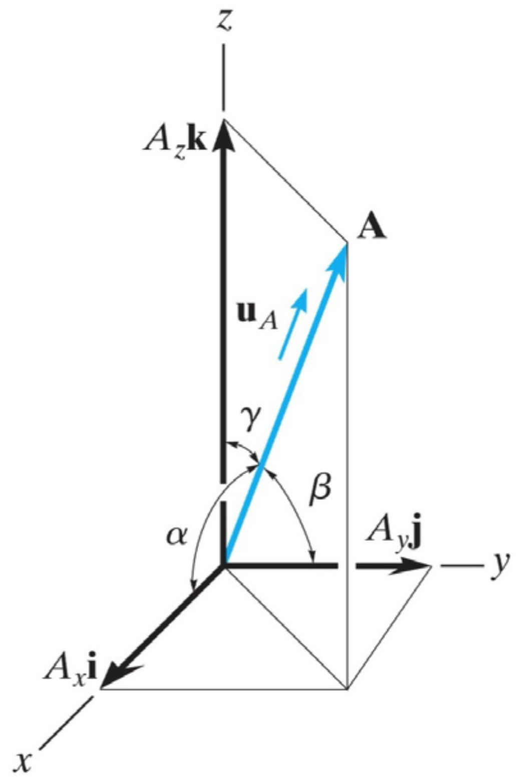
Coordinate Direction Angles

The unit vector $\mathbf{u}_A$ is defined as:

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}}$$

or in terms of direction cosines:

$$\mathbf{u}_A = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$$



Hence,

$$\mathbf{A} = A\mathbf{u}_A$$

$$= A\cos\alpha\, \boldsymbol{i} + A\cos\beta\, \boldsymbol{j} + A\cos\gamma\, \boldsymbol{k}$$

$$= A_x\, \boldsymbol{i} + A_y\, \boldsymbol{j} + A_z\, \boldsymbol{k}$$

Addition of Cartesian Vectors

Resultant Cartesian Vector

Once individual vectors are written in the Cartesian form, one can add the components to determine the resultant.

$$\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}}$$

Example:

$$\boldsymbol{F}_1 = \{50\boldsymbol{i} - 100\boldsymbol{j} + 100\boldsymbol{k}\}\ \text{lb}$$

$$\boldsymbol{F}_2 = \{0\boldsymbol{i} + 60\boldsymbol{j} + 80\boldsymbol{k}\}\ \text{lb}$$

$$\boldsymbol{F}_\mathrm{R} = \boldsymbol{F}_1 + \boldsymbol{F}_2$$

$$= \{(50+0)\boldsymbol{i} + (-100+60)\boldsymbol{j} + (100+80)\boldsymbol{k}\}$$

In General:

$$\boldsymbol{F}_R = \boldsymbol{F}_1 + \boldsymbol{F}_2 + \cdots + \boldsymbol{F}_N$$

$$\boldsymbol{F}_R = \{F_{Rx}\boldsymbol{i} + F_{Ry}\boldsymbol{j} + F_{Rz}\boldsymbol{k}\}$$

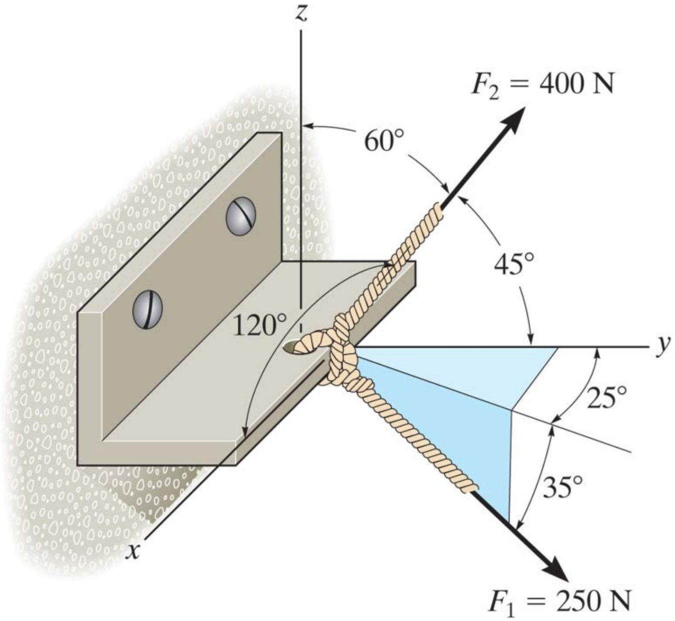
$$F_{Rx} = F_{1x} + F_{2x} + \cdots + F_{Nx}$$

$$F_{Ry} = F_{1y} + F_{2y} + \cdots + F_{Ny}$$

$$F_{Rz} = F_{1z} + F_{2z} + \cdots + F_{Nz}$$

Example

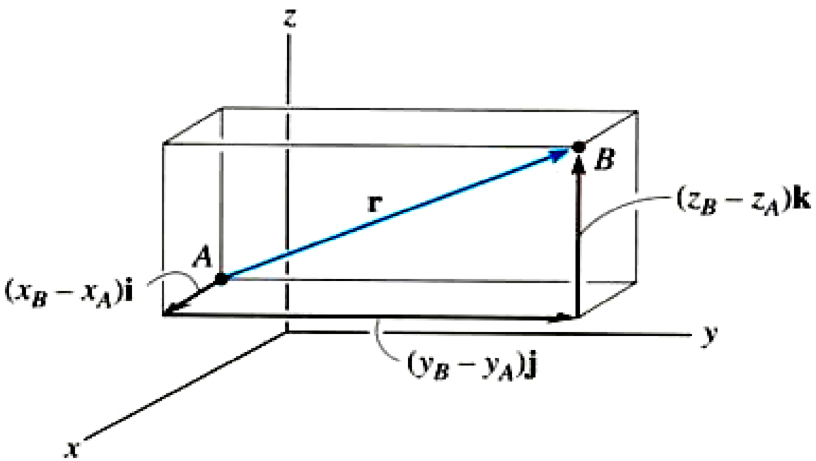
Find the magnitude and direction of the following system of forces.



Position Vectors

Position Vectors

A position vector is defined as a fixed vector that locates a point in space relative to another point.



Consider two points, A and B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively. The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\mathbf{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Important. Always subtract the "tail" coordinates from the "tip" coordinates!

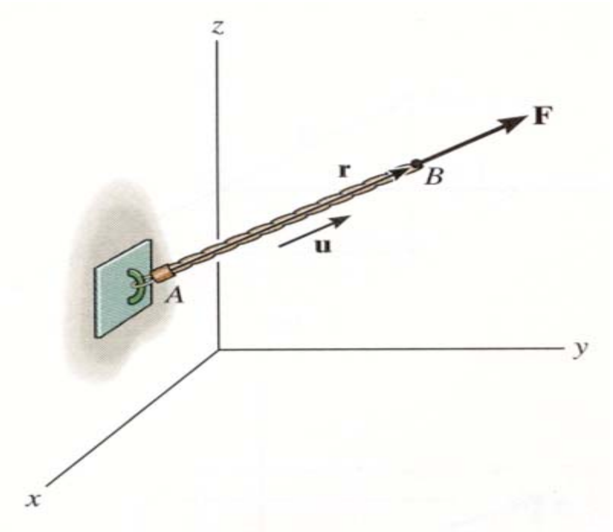
Force Vectors Directed along Lines

If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude.

a) Find the position vector, $\mathbf{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $u_{AB} = (\mathbf{r}_{AB}/r_{\mathrm{AB}})$.

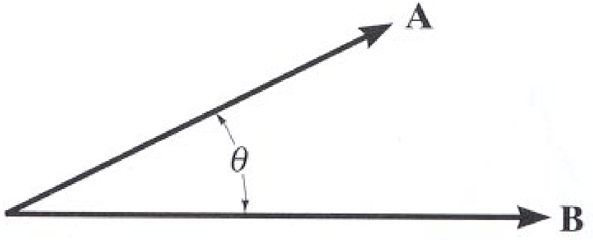
c) Multiply the unit vector by the magnitude of the force, $\mathbf{F} = F\ u_{AB}$.



Dot Product

The dot product of vectors **A** and **B** is defined as $\mathbf{A} \cdot \mathbf{B} = A\,B \cos\theta$.

The angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.



Examples: By definition, $\boldsymbol{i} \cdot \boldsymbol{j} = 0$

$$\boldsymbol{i} \cdot \boldsymbol{i} = 1$$

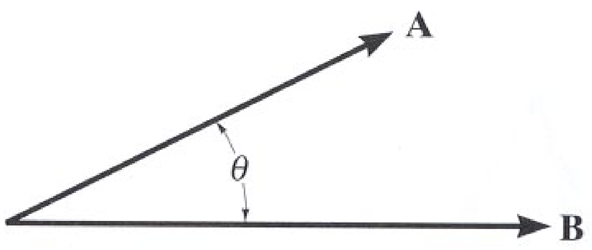
$$\mathbf{A} \cdot \mathbf{B} = (A_x\,\boldsymbol{i} + A_y\,\boldsymbol{j} + A_z\,\boldsymbol{k}) \cdot (B_x\,\boldsymbol{i} + B_y\,\boldsymbol{j} + B_z\,\boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

Dot Product

Relative orientation:

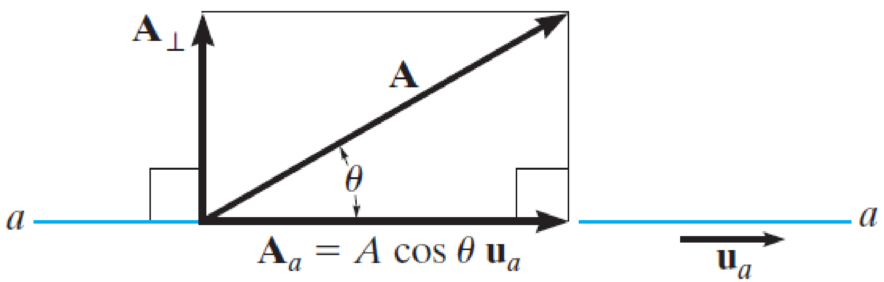
$$\theta = cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$

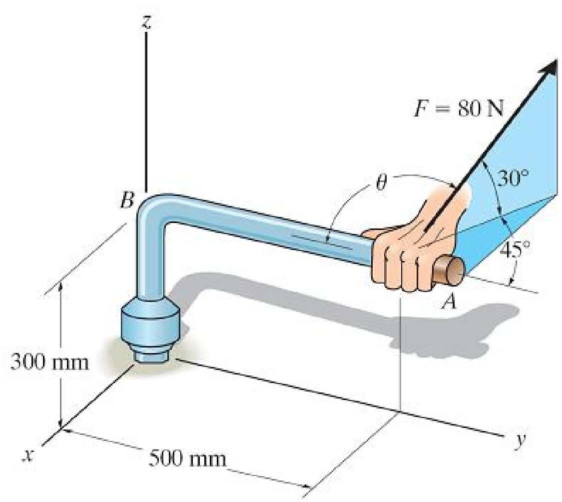


Vector decomposition:

$$\boldsymbol{A}_{\parallel} = (\boldsymbol{A}\cdot\boldsymbol{u})\boldsymbol{u}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{\parallel} = \boldsymbol{A} - (\boldsymbol{A}\cdot\boldsymbol{u})\boldsymbol{u}$$

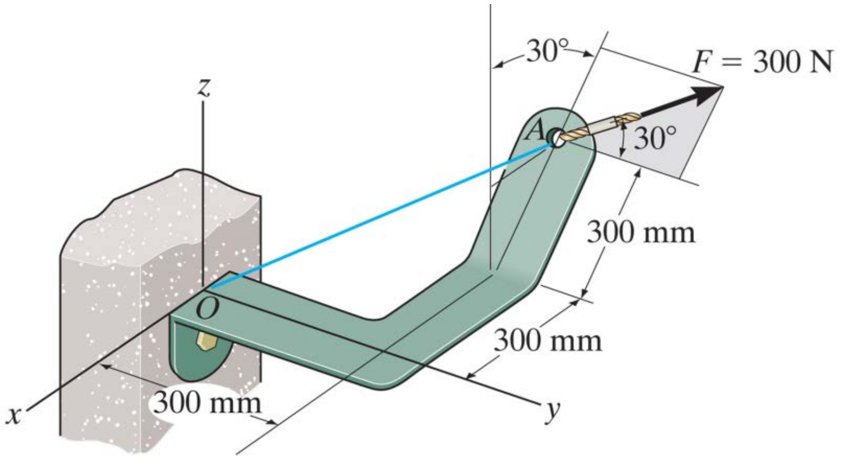




For the force ***F*** applied to the wrench at Point A, what component of it actually helps turn the bolt (i.e., the force component acting perpendicular to arm AB of the pipe)?

Example

Find the magnitude of the projected component of this force acting along line OA.



Summary of the Chapter

Summary of the different Methods:

Graphical solution (2D problems)

- Parallelogram Law – Simple concept but inaccurate.
- Triangle Rule – Simple concept but inaccurate.

2D Analytical Solution

- Trigonometry (sine and cosine laws) – Simple solution, visual representation, limited to 2D problems
- Rectangular Components ($\mathbf{F}_x$ and $\mathbf{F}_y$) – Methodical solution, same implementation for Cartesian vectors (3D problems)

Summary of the Chapter

Summary of the different Methods:

3D Analytical Solution

- Cartesian Vectors – Use this method if the force vector is located at the origin of the reference frame. Direction is defined with direction cosines (angles of force vector given).
- Force Vectors Directed along a Line – Use this method if the force is aligned along a line, which can be defined by position vectors. Determine unit vector ***u***, and find force as **F** = F ***u*** (coordinates of the line where force acts is given).
- Dot Product – This method is useful when the we try to determine the component of a force along a line. Say **F** and ***r*** are known, find projection of **F** along ***r*** (force and line are given).

Sample Problem (§ 2.4)

Find the magnitude and angle of the resultant force

- Resolve the forces into components.

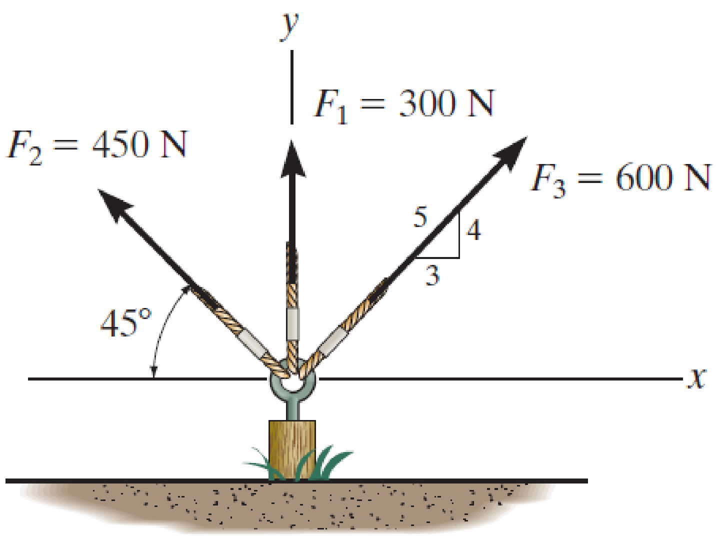
$\mathbf{F_1} = \{0\, \boldsymbol{i} + 300\, \boldsymbol{j}\ \}$ N

$\mathbf{F_2} = \{-\,450 \cos(45°)\, \boldsymbol{i} + 450 \sin(45°)\, \boldsymbol{j}\ \}$ N

$= \{-\,318.2\, \boldsymbol{i} + 318.2\, \boldsymbol{j}\ \}$ N

$\mathbf{F_3} = \{\ (3/5)\ 600\, \boldsymbol{i} + (4/5)\ 600\, \boldsymbol{j}\ \}$ N

$= \{\ 360\, \boldsymbol{i} + 480\, \boldsymbol{j}\ \}$ N



- Sum up all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components

$\mathbf{F_R} = \{\ (0 - 318.2 + 360)\, \boldsymbol{i} + (300 + 318.2 + 480)\, \boldsymbol{j}\ \}$ N $= \{\ 41.80\, \boldsymbol{i} + 1098\, \boldsymbol{j}\ \}$ N

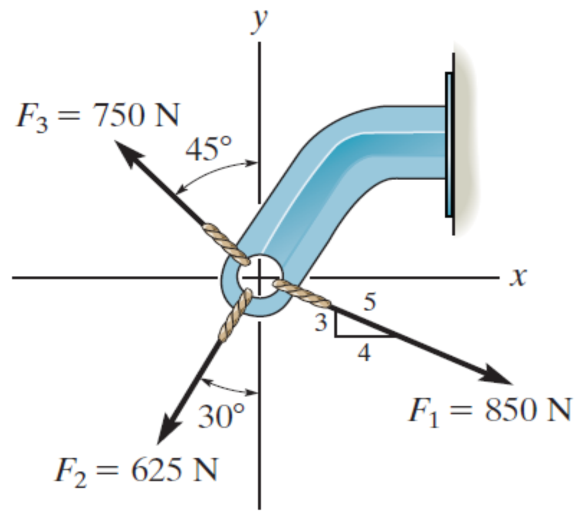
- Find magnitude and direction:

$F_R = ((41.80)^2 + (1098)^2)^{1/2} =$ **1099 N** $\qquad \phi = \tan^{-1}(1098/41.80) =$ **87.8°**

Sample Problem (§ 2.4)

Find the magnitude and angle of the resultant force

- Resolve the forces into components.



$\mathbf{F_1}$ = {850 (4/5) $\boldsymbol{i}$ – 850 (3/5) $\boldsymbol{j}$ } N

= { 680 $\boldsymbol{i}$ – 510 $\boldsymbol{j}$ } N

$\mathbf{F_2}$ = {- 625 sin (30°) $\boldsymbol{i}$ – 625 cos (30°) $\boldsymbol{j}$ } N

= {- 312.5 $\boldsymbol{i}$ – 541.3 $\boldsymbol{j}$ } N

$\mathbf{F_3}$ = {-750 sin (45°) $\boldsymbol{i}$ + 750 cos (45°) $\boldsymbol{j}$ } N

{- 530.3 $\boldsymbol{i}$ + 530.3 $\boldsymbol{j}$ } N

- Sum up all the $\boldsymbol{i}$ and $\boldsymbol{j}$ components

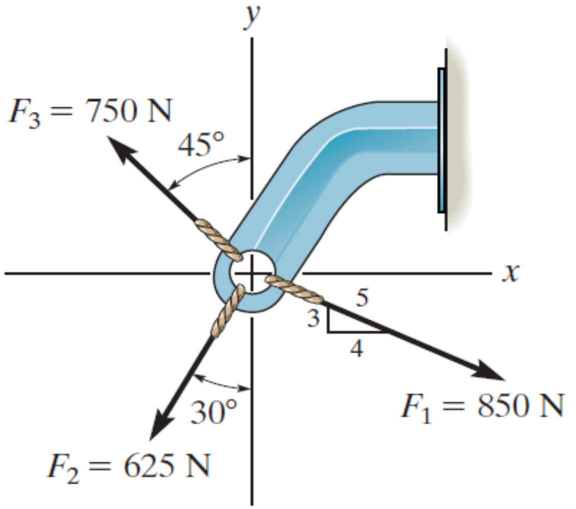
$\mathbf{F_R}$ = { (680 – 312.5 – 530.3) $\boldsymbol{i}$ + (–510 – 541.3 + 530.3) $\boldsymbol{j}$ }N

= {– 162.8 $\boldsymbol{i}$ – 520.9 $\boldsymbol{j}$ } N

Recalling resultant

$$\mathbf{F_R} = \{\ (680 - 312.5 - 530.3)\ \boldsymbol{i} + (-510 - 541.3 + 530.3)\ \boldsymbol{j}\ \}\text{N}$$

$$= \{-\ 162.8\ \boldsymbol{i} - 520.9\ \boldsymbol{j}\ \}\ \text{N}$$



- Find magnitude and direction:

$$F_R = ((-\ 162.8)^2 + (-\ 520.9)^2)^{½} = \underline{\mathbf{546\ N}}$$

$$\phi = \tan^{-1}(\ -520.9\ /\ -162.8\ ) = 72.6°$$

Warning. That angle does not make sense. Note that the denominator (F_{Rx}) is negative, we need to add 180°.

$$\phi = \tan^{-1}(\ -520.9\ /\ -162.8\ ) + 180 = \underline{\mathbf{252.6°}}$$

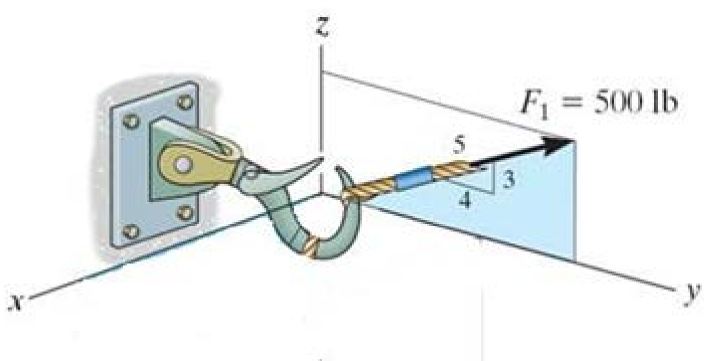
Sample Problem (§ 2.6)

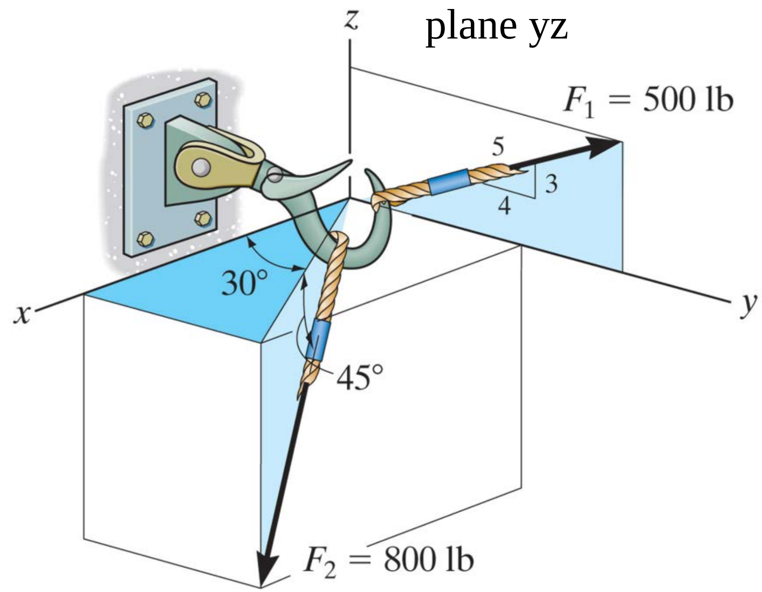
Force $\mathbf{F_1}$ is only contained on the plane yz

Find the resultant in the Cartesian form

- Resolve the forces into components.

Resolve force $\mathbf{F_1}$.





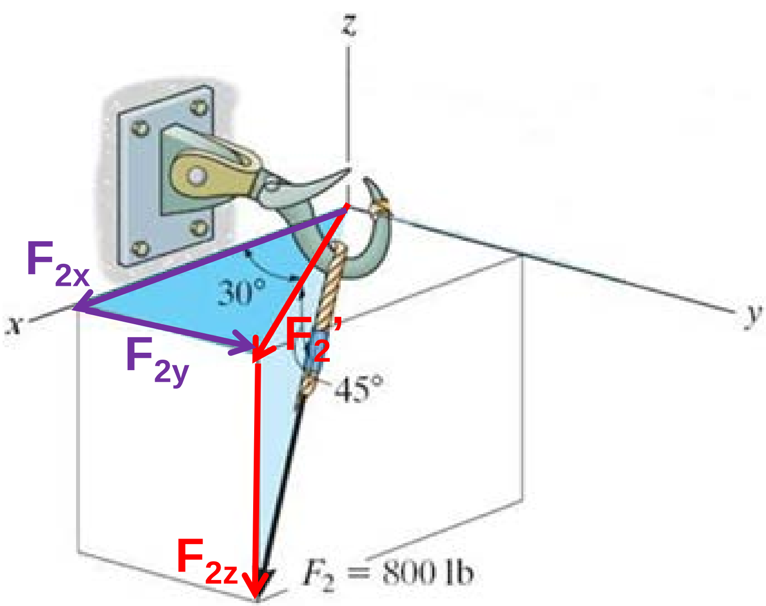
$F_x = 0 = 0$ lb

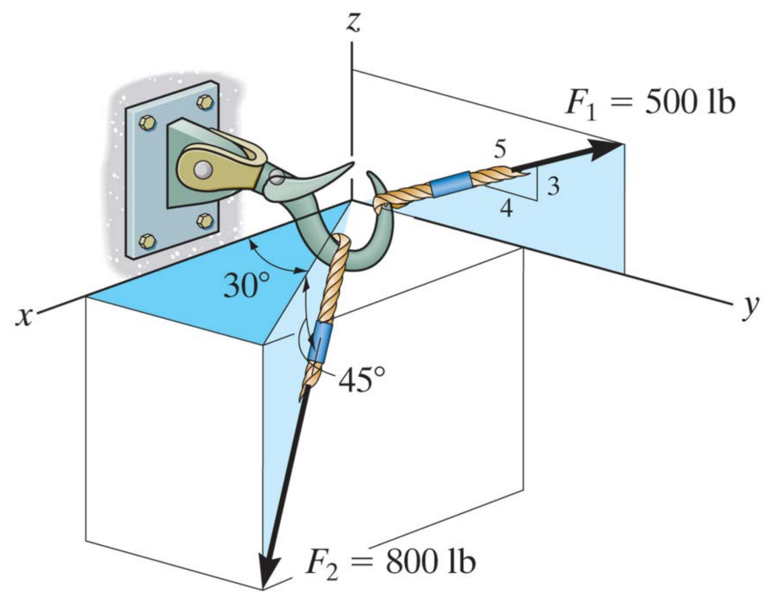
$F_y = 500\ (4/5) = 400$ lb

$F_z = 500\ (3/5) = 300$ lb

$\mathbf{F_1} = \{0\,\boldsymbol{i} + 400\ \boldsymbol{j} + 300\,\boldsymbol{k}\}$ lb

Resolve force $\mathbf{F_2}$.





F_{2z} = -800 sin 45° = – 565.7 lb

F_2' = 800 cos 45° = 565.7 lb

F_2' can be further resolved as,

F_{2x} = 565.7 cos 30° = 489.9 lb

F_{2y} = 565.7 sin 30° = 282.8 lb

Thus, we can write:

$$\mathbf{F_2} = \{489.9\,\boldsymbol{i} + 282.8\,\boldsymbol{j} - 565.7\,\boldsymbol{k}\}\text{ lb}$$

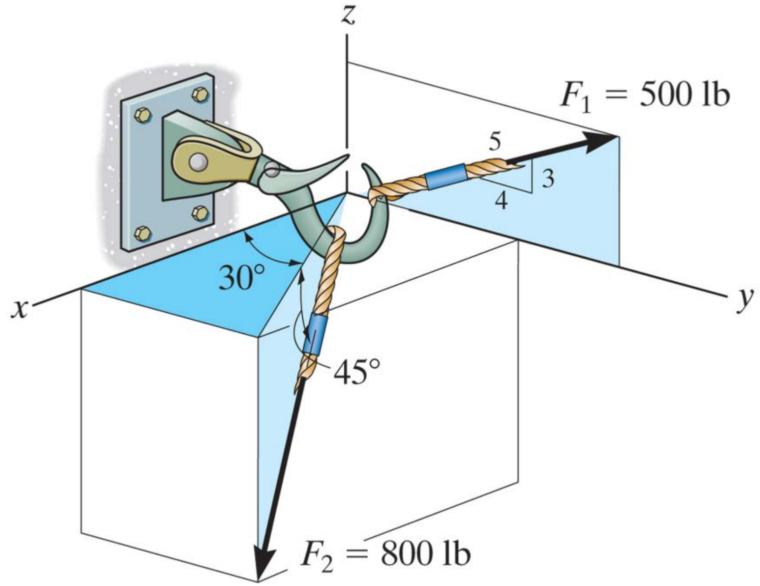
Finally, we find the resultant in the Cartesian form

Since $\mathbf{F_R} = \mathbf{F_1} + \mathbf{F_2}$ and

$\mathbf{F_1} = \{0\,\boldsymbol{i} + 400\,\boldsymbol{j} + 300\,\boldsymbol{k}\}$ lb

$\mathbf{F_2} = \{489.9\,\boldsymbol{i} + 282.8\,\boldsymbol{j} - 565.7\,\boldsymbol{k}\}$ lb

$\mathbf{F_R} = \{ 490\,\boldsymbol{i} + 683\,\boldsymbol{j} - 266\,\boldsymbol{k} \}$ lb



Sample Problem (§ 2.7)

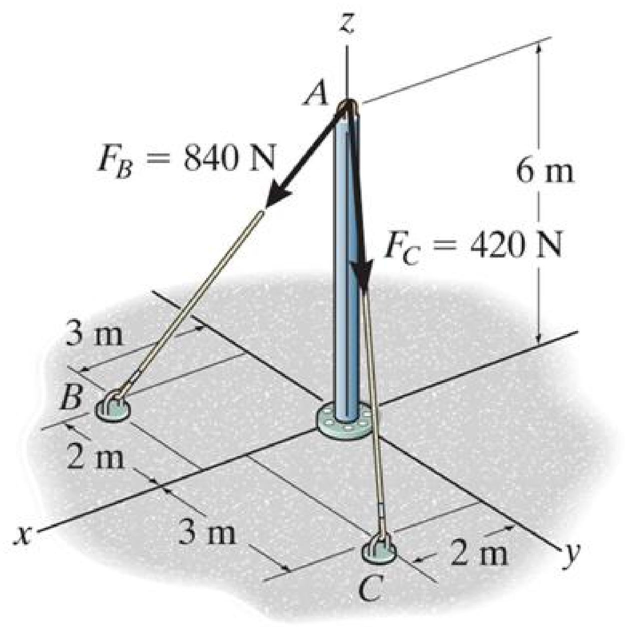
Find the force $\mathbf{F}_{AC}$ in the Cartesian vector form.

From the figure we can determine the position vector $\boldsymbol{r}_{AC}$

$\boldsymbol{r}_{AC} = \{2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ m.

(We could also find $\boldsymbol{r}_{AC}$ by subtracting the coordinates of A from the coordinates of C.)

$$r_{AC} = \{2^2 + 3^2 + (-6)^2\}^{1/2} = 7 \text{ m}$$



Since $\boldsymbol{u}_{AC} = \boldsymbol{r}_{AC}/r_{AC}$ and $\boldsymbol{F}_{AC} = 420\,\boldsymbol{u}_{AC} = 420\,(\boldsymbol{r}_{AC}/r_{AC})$, then

$$\boldsymbol{F}_{AC} = 420\{(2\,\boldsymbol{i} + 3\,\boldsymbol{j} - 6\,\boldsymbol{k})/7\}\text{ N} = \{120\,\boldsymbol{i} + 180\,\boldsymbol{j} - 360\,\boldsymbol{k}\}\text{ N}$$

Sample Problem (§ 2.8)

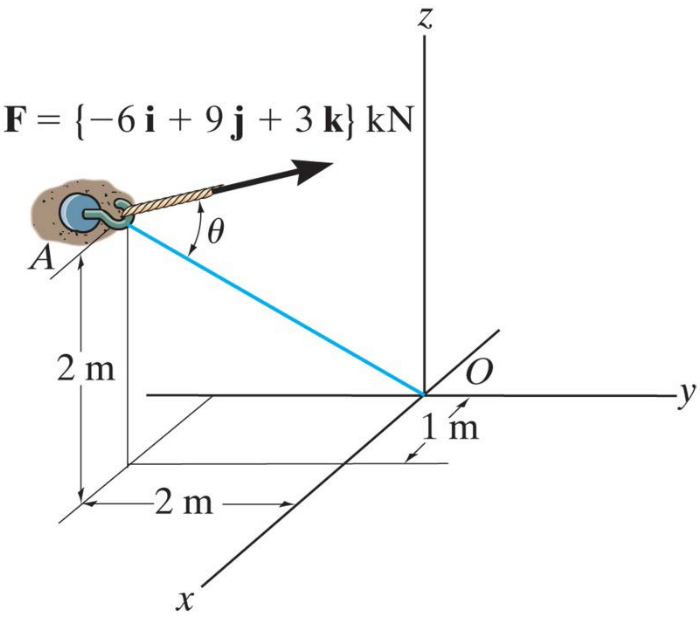
Find the angle between the force vector and the line AO, and the magnitude of the projection of the force along the line AO.

$\boldsymbol{r}_{AO} = \{-1\,\boldsymbol{i} + 2\,\boldsymbol{j} - 2\,\boldsymbol{k}\}$ m

$r_{\mathrm{AO}} = \{(-1)^2 + 2^2 + (-2)^2\}^{1/2} = 3$ m

$\boldsymbol{F} = \{-6\,\boldsymbol{i} + 9\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ kN

$F = \{(-6)^2 + 9^2 + 3^2\}^{1/2} = 11.22$ kN



$\mathbf{F} \cdot \boldsymbol{r}_{AO} = (-6)(-1) + (9)(2) + (3)(-2) = 18$ kN·m

$\theta = \cos^{-1}\{(\mathbf{F} \cdot \boldsymbol{r}_{AO})/(F\, r_{\mathrm{AO}})\}$

$\theta = \cos^{-1}\{18 / (11.22 \times 3)\} =$ **57.67°**

Finally, the magnitude of the projection of F along the line AO is determined.

$$\boldsymbol{u}_{AO} = \boldsymbol{r}_{AO} / \mathrm{r}_{\mathrm{AO}} = (-1/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} + (-2/3)\,\boldsymbol{k}$$

$$F_{\mathrm{AO}} = \boldsymbol{F} \cdot \boldsymbol{u}_{AO} = (-6)(-1/3) + (9)(2/3) + (3)(-2/3) = \mathbf{6.00\ kN}$$

or: $F_{\mathrm{AO}} = F \cos\theta = 11.22 \cos(57.67°) = \mathbf{6.00\ kN}$