Continuous-Time Signals and Systems
Annotated Lecture Examples
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Formulas and Tables

Useful Formulae and Other Information

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

$$\mathscr{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$\mathscr{F}^{-1}\{X(\omega)\} = x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$

$$a_k = \tfrac{1}{T}X_T(k\omega_0)$$

$$\mathscr{L}\{x(t)\} = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$$

$$\mathscr{L}^{-1}\{X(s)\} = x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds$$

$$\mathscr{U}\mathscr{L}\{x(t)\} = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \tfrac{1}{2}\left[e^{j\theta} + e^{-j\theta}\right]$$

$$\sin\theta = \tfrac{1}{2j}\left[e^{j\theta} - e^{-j\theta}\right]$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0
$\frac{5\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\frac{3\pi}{2}$	0	-1
$\frac{7\pi}{4}$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$

$$A_k = (v - p_k)F(v)\big|_{v=p_k}$$

$$A_{kl} = \frac{1}{(q_k - l)!}\left[\frac{d^{q_k-l}}{dv^{q_k-l}}\left[(v-p_k)^{q_k}F(v)\right]\right]\Bigg|_{v=p_k}$$

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$Ax(t)+By(t)$	Aa_k+Bb_k
Time-Domain Shifting	$x(t-t_0)$	$e^{-jk\omega_0 t_0}a_k$
Time Reversal	$x(-t)$	a_{-k}

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(\omega)+a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1(t)*x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega)*X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega)+\pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2\,d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
6	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
7	$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
8	$\mathrm{rect}(t/T)$	$\lvert T\rvert\,\mathrm{sinc}(T\omega/2)$
9	$\frac{\lvert B\rvert}{\pi}\,\mathrm{sinc}\,Bt$	$\mathrm{rect}\,\frac{\omega}{2B}$
10	$e^{-at}u(t)$, $\mathrm{Re}\{a\}>0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\mathrm{Re}\{a\}>0$	$\frac{(n-1)!}{(a+j\omega)^n}$