Example 3.24. Determine whether the system $\mathcal{H}$ is invertible, where

$$\mathcal{H}x(t) = \sin[x(t)].$$

Solution. Consider an input of the form $x(t) = 2\pi k$ where k is an arbitrary integer. The response $\mathcal{H}x$ to such an input is given by

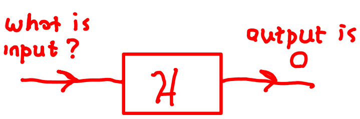
①

$$\begin{aligned}\mathcal{H}x(t) &= \sin[x(t)] \\ &= \sin 2\pi k \\ &= 0.\end{aligned}$$

substitute ①

sin function is zero at all integer multiples of π

Thus, we have found an infinite number of distinct inputs (i.e., $x(t) = 2\pi k$ for $k = 0, \pm 1, \pm 2, \ldots$) that all result in the same output. Therefore, the system is not invertible. ■



we don't know what the input is. input could be $x(t) = 0$ or $x(t) = 2\pi$ or $x(t) = -2\pi$ or ...

Example 3.27 (Ideal integrator)**.** Determine whether the system $\mathcal{H}$ is BIBO stable, where

$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Solution. Suppose that we choose the input $x = u$ (where u denotes the unit-step function). Clearly, u is bounded (i.e., $|u(t)| \leq 1$ for all t). Calculating the response $\mathcal{H}x$ to this input, we have

$$\begin{aligned}\mathcal{H}x(t) &= \int_{-\infty}^{t} u(\tau)d\tau \\ &= \int_{0}^{t} d\tau \\ &= [\tau]|_0^t \\ &= t.\end{aligned}$$

$u(\tau) = 0$ for $\tau < 0$

From this result, however, we can see that as $t \to \infty$, $\mathcal{H}x(t) \to \infty$. Thus, the output $\mathcal{H}x$ is unbounded for the bounded input x. Therefore, the system is not BIBO stable. ■

A system $\mathcal{H}$ is said to be BIBO stable if, for every bounded function x, $\mathcal{H}x$ is bounded. That is,

$$|x(t)| \leq A < \infty \text{ for all } t \implies |\mathcal{H}x(t)| \leq B < \infty \text{ for all } t.$$

To show that a system is not BIBO stable, we simply need to find a counterexample (i.e., an example of a bounded input that yields an unbounded output).

Example 3.28 (Squarer)**.** Determine whether the system $\mathcal{H}$ is BIBO stable, where

$$\mathcal{H}x(t) = x^2(t).$$

Solution. Suppose that the input x is bounded such that (for all t)

$$|x(t)| \le A,$$

where A is a finite real constant. Squaring both sides of the inequality, we obtain

$$|x(t)|^2 \le A^2.$$

Interchanging the order of the squaring and magnitude operations on the left-hand side of the inequality, we have

$$|x^2(t)| \le A^2.$$

Using the fact that $\mathcal{H}x(t) = x^2(t)$, we can write

$$|\mathcal{H}x(t)| \le A^2.$$

Since A is finite, A^2 is also finite. Thus, we have that $\mathcal{H}x$ is bounded (i.e., $|\mathcal{H}x(t)| \le A^2 < \infty$ for all t). Therefore, the system is BIBO stable. ■

squaring a finite number always yields a finite result

A system $\mathcal{H}$ is said to be BIBO stable if, for every bounded function x, $\mathcal{H}x$ is bounded. That is,

$|x(t)| \le A < \infty$ for all t $\Longrightarrow$ $|y(t)| \le B < \infty$ for all t.

To show a system is BIBO stable, we must show that every bounded input produces a bounded output.

Example 3.32. Determine whether the system $\mathcal{H}$ is time invariant, where

$$\mathcal{H}x(t) = \sin[x(t)]. \quad (1)$$

Solution. Let (2) $x'(t) = x(t - t_0)$, where t_0 is an arbitrary real constant. From the definition of $\mathcal{H}$, we can easily deduce that

equal for all x and all t_0

$$\mathcal{H}x(t - t_0) = \sin[x(t - t_0)] \quad \text{and}$$

by substituting $t-t_0$ for t in (1)

$$\begin{aligned}\mathcal{H}x'(t) &= \sin x'(t) \\ &= \sin[x(t - t_0)].\end{aligned}$$

from definition of $\mathcal{H}$ in (1)

from definition of x' in (2)

Since $\mathcal{H}x(t - t_0) = \mathcal{H}x'(t)$ for all x and t_0, the system is time invariant. ■

A system $\mathcal{H}$ is said to be time invariant if, for every function x and every real constant t_0, the following condition holds:

$$\mathcal{H}x(t-t_0) = \mathcal{H}x'(t) \text{ for all } t, \text{ where } x'(t) = x(t-t_0)$$