Taking the inverse Fourier transform of both sides of this equation, we obtain

taking inverse FT

$$\begin{aligned} v_2(t) &= \mathcal{F}^{-1}\left\{\frac{2L}{R+j\omega L}\right\}(t) \\ &= \mathcal{F}^{-1}\left\{\frac{2}{R/L+j\omega}\right\}(t) \\ &= 2\mathcal{F}^{-1}\left\{\frac{1}{R/L+j\omega}\right\}(t). \end{aligned}$$

divide numerator and denominator by L

linearity

Using Table 6.2, we can simplify to obtain

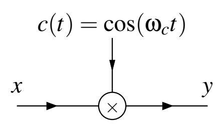
from FT table

$e^{-at}u(t) \overset{FT}{\longleftrightarrow} \frac{1}{a+j\omega}$

$$v_2(t) = 2e^{-(R/L)t}u(t).$$

Thus, we have found the response v_2 to the input $v_1(t) = \operatorname{sgn} t$. ■

DSB-SC AM: Transmitter



$$y(t) = \cos(\omega_c t)x(t)$$

take FT

$$X = \mathcal{F}x, \quad Y = \mathcal{F}y$$

use modulation property
not multiplication property!

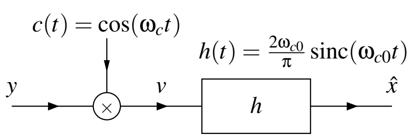
$$\begin{aligned}
Y(\omega) &= \mathcal{F}\{\cos(\omega_c t)x(t)\}(\omega) \\
&= \mathcal{F}\left\{\tfrac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)x(t)\right\}(\omega) \\
&= \tfrac{1}{2}\left[\mathcal{F}\{e^{j\omega_c t}x(t)\}(\omega) + \mathcal{F}\{e^{-j\omega_c t}x(t)\}(\omega)\right] \\
&= \tfrac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]
\end{aligned}$$

Euler

linearity property

modulation property

DSB-SC AM: Receiver



$$v(t) = \cos(\omega_c t) y(t), \quad h(t) = \frac{2\omega_{c0}}{\pi}\operatorname{sinc}(\omega_{c0} t), \quad \hat{x}(t) = v * h(t)$$

(1) (2) (3)

use modulation property, not multiplication property!!!

$$Y = \mathcal{F}y, \quad V = \mathcal{F}v, \quad H = \mathcal{F}h, \quad \hat{X} = \mathcal{F}\hat{x}$$

$$\begin{aligned} V(\omega) &= \mathcal{F}\{\cos(\omega_c t) y(t)\}(\omega) \\ &= \mathcal{F}\left\{\tfrac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right) y(t)\right\}(\omega) \\ &= \tfrac{1}{2}\left[\mathcal{F}\left\{e^{j\omega_c t} y(t)\right\}(\omega) + \mathcal{F}\left\{e^{-j\omega_c t} y(t)\right\}(\omega)\right] \\ &= \tfrac{1}{2}\left[Y(\omega - \omega_c) + Y(\omega + \omega_c)\right] \end{aligned}$$

FT of (1)

Euler

linearity property

modulation property

linearity

$$\begin{aligned} H(\omega) &= \mathcal{F}\left\{\frac{2\omega_{c0}}{\pi}\operatorname{sinc}(\omega_{c0} t)\right\}(\omega) \\ &= 2\operatorname{rect}\left(\frac{\omega}{2\omega_{c0}}\right) \end{aligned}$$

FT of (2)

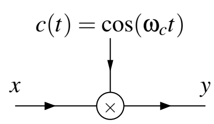
from FT table

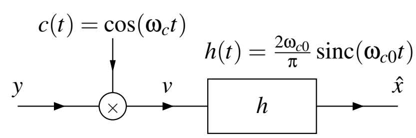
$\frac{B}{\pi}\operatorname{sinc}(Bt) \overset{FT}{\longleftrightarrow} \operatorname{rect}\left(\frac{\omega}{2B}\right)$

$$\hat{X}(\omega) = H(\omega)V(\omega)$$

FT of (3) using convolution property

DSB-SC AM: Complete System





① $Y(\omega) = \frac{1}{2}\left[X(\omega-\omega_c) + X(\omega+\omega_c)\right]$ ← from result for transmitter

② $V(\omega) = \frac{1}{2}\left[Y(\omega-\omega_c) + Y(\omega+\omega_c)\right]$ ← from result for receiver

$$\begin{aligned} V(\omega) &= \tfrac{1}{2}\left[Y(\omega-\omega_c) + Y(\omega+\omega_c)\right] \\ &= \tfrac{1}{2}\left[\tfrac{1}{2}\left[X([\omega-\omega_c]-\omega_c) + X([\omega-\omega_c]+\omega_c)\right] + \right. \\ &\qquad \left. \tfrac{1}{2}\left[X([\omega+\omega_c]-\omega_c) + X([\omega+\omega_c]+\omega_c)\right]\right] \\ &= \tfrac{1}{2}X(\omega) + \tfrac{1}{4}X(\omega-2\omega_c) + \tfrac{1}{4}X(\omega+2\omega_c) \end{aligned}$$

substitute ①

simplify

③ $\hat{X}(\omega) = H(\omega)V(\omega)$ ← from result for receiver

$$\begin{aligned} \hat{X}(\omega) &= H(\omega)V(\omega) \\ &= H(\omega)\left[\tfrac{1}{2}X(\omega) + \tfrac{1}{4}X(\omega-2\omega_c) + \tfrac{1}{4}X(\omega+2\omega_c)\right] \\ &= \tfrac{1}{2}H(\omega)X(\omega) + \tfrac{1}{4}H(\omega)X(\omega-2\omega_c) + \tfrac{1}{4}H(\omega)X(\omega+2\omega_c) \\ &= \tfrac{1}{2}\left[2X(\omega)\right] + \tfrac{1}{4}(0) + \tfrac{1}{4}(0) \\ &= X(\omega) \end{aligned}$$

substitute ②

multiply

$X(\omega-2\omega_c)=0$ and $X(\omega+2\omega_c)=0$ when $H(\omega)\neq 0$ (since $\omega_b < \omega_{c0} < 2\omega_c - \omega_b$)

simplify

