Sample Midterm 3A

Instructions:

1. Questions 1 to 7 are short answer questions. Put your answer in the box.
2. Question 8 is a full-answer question. For full-answer questions, marks will be deducted for incomplete or poorly presented solutions.
3. Space will be provided for you to work out your answers on the actual midterm.

Duration: You should be able to complete this midterm within 45 minutes. If you cannot, this means that more practice is still needed.

Questions 1 [redacted] refer to the following scenario:

In a factory which produces a particular electronic component, they wish to estimate p, the proportion of components which are flawed. A random sample of 200 components is taken, and it is found that 15 of the components are flawed.

1. Find the upper confidence limit of an 85% confidence interval for p.

Answer:

[redacted]

[redacted]

3. The lifespans of light-emitting diodes are known to be exponentially distributed, with a mean lifespan of 12 years. Suppose that 45 diodes are selected at random. What is the probability that the average lifespan for this sample is between 11.5 and 12.3 years?

Answer:

Questions 4 and 5 refer to the following scenario:

Let p denote the proportion of elderly people with artificial hips. In an earlier study, the estimated proportion of the number of elderly people with artificial hips was found to be 11%.

4. Using the earlier study's estimate, find the sample size that we would need in order to create a 95% confidence interval for the proportion of elderly people with artificial hips which has width 0.10.

Answer:

5. **Without** using the earlier study's estimate, find the *maximum* sample size needed to create a 93% confidence interval for the proportion of elderly people with artificial hips which has a margin of error of 0.04?

Answer:

Questions 6 and 7 refer to the following scenario:

Let μ denote the mean height of Andean flamingos. The heights are known to be normally distributed. In a sample of 6 flamingos, we find an average height of 100 cm with a standard deviation of 3.5 cm.

6. What is the upper confidence limit of an 80% confidence interval for μ?

Answer:

7. Using this data as a pilot study, estimate the sample size needed to construct a 99% confidence interval to estimate μ within 0.5 cm.

Answer:

8. We take a random sample of 10 eggs and measure their fat content. It is found that for this sample the average fat content is 5.5 g with a standard deviation of 0.4 g. We are interested in determining if the mean fat content of eggs is greater than 5 g.

   (a) Define the parameter of interest, and state the hypotheses in terms of the parameter.

   (b) State the test statistic you will use. What distribution (including degrees of freedom, if appropriate) you will use to calculate the p-value.

   (c) Find the observed value of the test statistic.

   (d) Compute (or bracket) the p-value within the accuracy of the tables.

   (e) What level of evidence against H_0 do you find?

   (f) If we were testing our hypothesis at the level $\alpha = 0.01$, would we reject H_0? Explain why or why not. In order to receive a mark, you **must** provide a (correct) explanation.

Answers:

1. 0.1018
2. ≈ 0.1052
3. 0.1778
4. ≈ 150.44, so 151 people.
5. ≈ 511.89, so 512 people.
6. ≈ 102.109
7. ≈ 324.9, so 325 flamingos.

8(a). Let μ be the mean fat content of eggs.

$H_0 : \mu = 5$ (or $\mu \leq 5$), $H_1 : \mu > 5$.

8(b). Test statistic $\dfrac{\overline{x} - \mu}{s/\sqrt{n}}$ The distribution is t_9.

8(c). Observed value: ≈ 3.9528.
8(d). 0.0005 <p-value< 0.0025
8(e). There is **very strong** evidence against H_0.
8(f). We reject H_0 since the p-value is less than α.