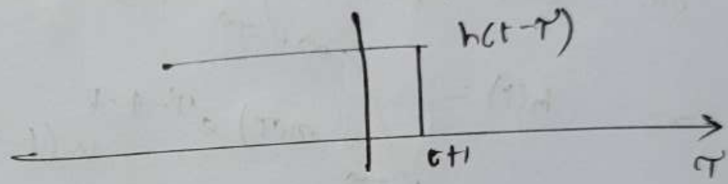
Arifa Hossain V00984826

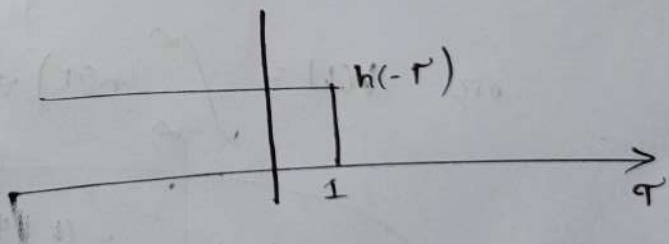
4.11 (a) $y_m(t) = \int_{-\infty}^{t+1} x(\tau)\, d\tau$ } Integral convolution states that $y_m(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$

comparing we get, $\boxed{h(t-\tau)=1}$ for $-\infty < \tau \leq (t+1)$

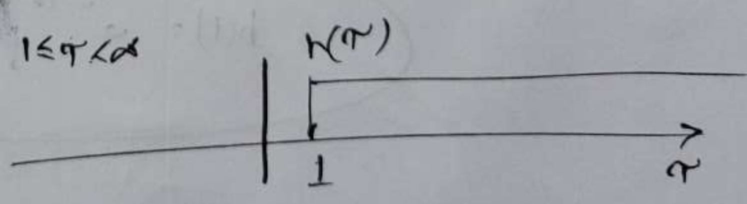


putting $t=0$,

$\boxed{h(-\tau)=1}$ for $-\infty < \tau \leq 1$



$\boxed{h(\tau)=1}$ for $1 \leq \tau < \infty$



Time reversal

$\therefore \boxed{h(\tau) = u(t-1)}$. impulse response

(b) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau+5) \cdot e^{\tau-t+1} u(t-\tau-2)\, d\tau$

~~$h(t) = \int_{-\infty}^{\infty} \delta(\tau+5) \cdot e^{t(\tau-t+1)} u(t-\tau-2)\, d\tau$~~

or, $h(t) = \mathcal{H}x(t) = \int_{-\infty}^{\infty} x(u)\, e^{u-5+1-t} u(t-u+5-2)\, du$

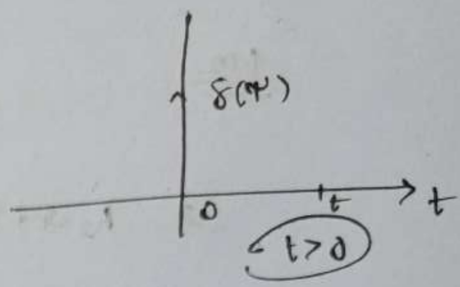
or, $h(t) = \int_{-\infty}^{\infty} x(\tau)\, e^{\tau-4-t} u(t-\tau+3)\, d\tau$

~~$h(t) = \int_{-\infty}^{\infty} x(\tau)\, e^{(\tau-4-t)} u($~~

or, $h(t) = \int_{-\infty}^{\infty} x(\tau)\, e^{-(t+4-\tau)} u(t+3-\tau)\, d\tau$

$h(t) = e^{-(t+4)} u(t+3)$

(c) $x_{tr}(t) = \int_{-\infty}^{t} u(\tau)\, v(t-\tau)\, d\tau$



$$\cancel{y(t) = x_{tr}(t) = \int_{-\infty}^{\infty} x(\tau)\, u(t-\tau)\, v(t-\tau)\, d\tau}$$

$$\cancel{h(t) = v(t)u(t)}$$

$$h(t) = \int_{-\infty}^{t} \left(\delta(\tau) \cdot v(t-\tau)\right) d\tau$$

$$= \int_{-\infty}^{t} \left[\delta(\tau)\, v(t-\tau)\right]\Big|_{\tau=0} d\tau$$

$$= \left[\int_{-\infty}^{t} \delta(\tau)\, d\tau\right] v(t)$$

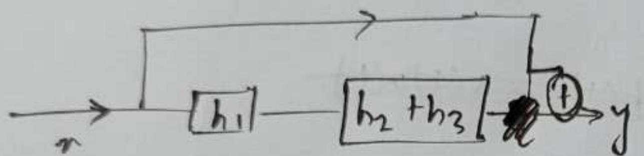
$$= v(t) \cdot 1 \quad \boxed{t>0}$$

$$h(t) = v(t) \cdot u(t)$$

4.12

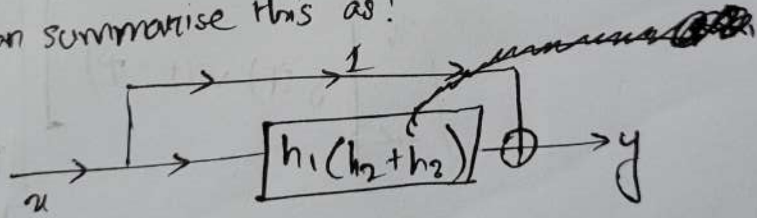
(a) h_2 and h_3 are in parallel

hence their equivalent is $h_2 + h_3$

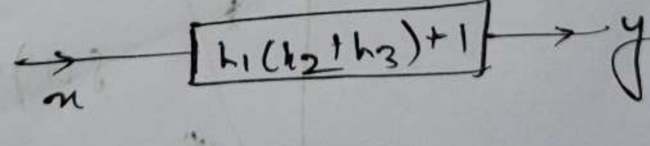


h_1 and $(h_2 + h_3)$ are in cascaded, equivalent product: $h_1 * (h_2 + h_3)$

so, we can summarise this as:



or,



overall ~~[illegible]~~ impulse response is given by $h(t) = h_1 * (h_2 + h_3) + 1$

$= h_1 h_2 + h_1 h_3 + 1$

(b) $h_1(t) = \delta(t+1)$, $h_2(t) = \delta(t)$ & $h_3(t) \Rightarrow \delta(t)$.

$$h_1(t) = h(t) \otimes [h_2(t) + h_3(t)] + 1$$

$$h(t) = \delta(t+1) \otimes [\delta(t) + \delta(t)] + 1$$

$$h(t) = 2[\delta(t+1) \otimes \delta(t)] + 1$$

$$h(t) \Rightarrow 2\delta(t+1) + 1$$

$$\left\{ \delta(t-t_1) \otimes \delta(t-t_2) = \delta(t-t_1-t_2) \right.$$

4.13 $x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$$x \rightarrow \boxed{h_1} \rightarrow \boxed{h_2} \rightarrow y$$

$$TF = \frac{Y(s)}{X(s)} \quad \therefore y(t) = h_1 h_2 \, x(t)$$

(b) $h_1(t) = \delta(t+1)$; $h_2(t) = \delta(t+1)$

$$\Rightarrow h_1(t) h_2(t) = \delta(t+1)\,\delta(t+1)$$

$$\Rightarrow h_1(t) h_2(t) = \delta(t+2)$$

$$\Rightarrow H_1(S) H_2(S) = e^{2S}$$

$$\Rightarrow Y(s) = e^{2s}\left(\frac{1}{S}\right) \qquad y(t) = u(t+2)$$

c) $h_1(t) = e^{-3t} u(t)$

$h_2(t) = \delta(t)$

$x(t) = u(t)$

$h_1(t)$ & $h_2(t)$ are cascaded : therefore

$$y(t) = h_1(t) * h_2(t) \times x(t)$$

Convolution in time domain = multiplication in frequency domain

Taking laplace in both sides:

$$y(s) = h_1(s) \cdot h_2(s) \cdot x(s)$$

$$\Rightarrow y(s) = \frac{1}{s+3} \cdot 1 \cdot \frac{1}{s}$$

$$\Rightarrow y(s) = \frac{1}{s(s+3)}$$

$h_1(s) = \frac{1}{s+3}$

$h_2(s) = 1$

$X(s) = \frac{1}{s}$

Applying partial fraction,

$$y(s) = \frac{1/3}{s} \cdot \frac{1/3}{s+3}$$

Taking inverse laplace $\boxed{y(t) = \frac{1}{3} u(t) - \frac{1}{3} e^{-3t}}$

4.14 An LTI system is causal if $\underline{h(t) = 0}$ for $\underline{t < 0}$ —— I

An LTI system is memoryless if ~~h(t) = k δ(t)~~ —— II

$h(t) = 0$ for all $t \neq 0$

(a) $h(t) = (t+1)\,u(t-1) = \begin{cases} (t+1), & t \geq 1 \\ 0, & t < 1 \end{cases}$ $\left(\text{as, } u(t-1) = 1 \text{ for } t \geq 1 \right)$

$h(t)$ satisfies I $\Rightarrow$ causal system

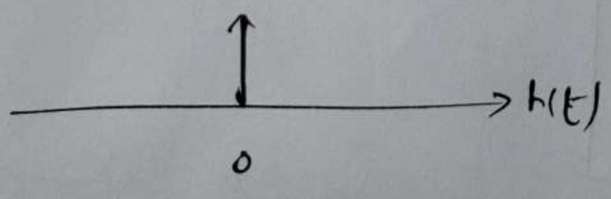
$h(t)$ doesn't satisfy II, so system is not memoryless

$\therefore$ $h(t)$ is causal and got some memory.

(f) $h(t) = e^{-3|t|} = \begin{cases} e^{-3t}, & t > 0 \\ e^{3t}, & t < 0 \end{cases}$ $\left(\text{as, } |t| = t, \ t > 0; \ = -t, \ t < 0 \right)$

$h(t)$ doesn't satisfy I and II, therefore the system is non causal & non-memoryless

(g) $h(t) = 3\,\delta(t) = \begin{cases} 3, & t = 0 \\ 0, & t \neq 0 \end{cases}$



System satisfies both (I) and (II), therefore it is causal and memoryless

4.15

(a) $h(t) = e^{at} u(-t)$

An LTI system w/ transfer functions h(t) is stable when $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ i.e converges.

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} |e^{at}|\, u(-t)\,dt$$

a is positive and real. e^{at} is positive for all values from $-\infty$ to 0. This is why we can write $|e^{at}|$ instead of e^{at}.

$$= \int_{-\infty}^{0} |e^{at}|\,dt = \left[\frac{e^{at}}{a}\right]_{-\infty}^{0}$$

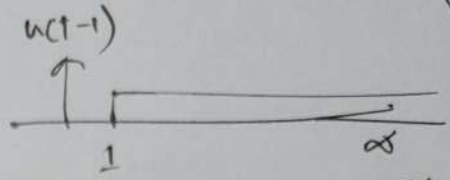
$$= \frac{1}{a}\left(e^{0} - e^{-\infty}\right) = \frac{1}{a}\left(1 - \frac{1}{\infty}\right) = \frac{1}{a}(1) = 1/a$$

$\therefore \frac{1}{a} < \infty$ so system is stable

$$e^{a(-\infty)} = e^{-a(\infty)} = e^{-\infty} = 0$$

$$e^{a(0)} = e^{0} = 1$$

(b) $h(t) = \frac{1}{t}u(t-1) = \int_{-\infty}^{\infty} |h(t)|\,dt$



changing limit to $\int_1^\infty$ from graph

$$= \int_{-\infty}^{\infty} \left|\frac{1}{t}\cdot u(t-1)\right| dt$$

$$= \int_{1}^{\infty} \left|\frac{1}{t}\right| dt$$

$$= \int_{1}^{\infty} \frac{1}{t}\, dt$$

$$= \Big[\ln(t)\Big]_1^{\infty} = \ln\infty - \ln 1 = \ln\infty = \infty$$

Since the integral doesn't converge, the system is not BIBO stable

4.16

Two systems $h_1(t)$ & $h_2(t)$ are inverses of each other, if $h_1(t) * h_2(t) = \delta(t)$

given $h_1(t) = \frac{1}{2}\delta(t-1)$

$h_2(t) = 2(\delta(t-1)$

Now, $h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\tau) h_2(t-\tau) d\tau$

$= \int_{-\infty}^{\infty} \frac{1}{2}\delta(\tau-1) \times 2\delta(t-\tau+1) d\tau$

$= \int_{-\infty}^{\infty} \delta(\tau-1)\,\delta(t-\tau+1)\, d\tau$

Using sifting property, i.e., $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t)\Big|_{t=t_0}$

Here, $h_1(t) * h_2(t) = \int_{-\infty}^{\infty} \delta(\tau-1)\,\underbrace{\delta(t-\tau+1)}_{f(t)}\, d\tau$

$= f(t)\Big|_{t=1}$

$\boxed{h_1(t) * h_2(t) = \text{Unit Impulse} = \delta(t)}$

$= \delta(t-\tau+1)\Big|_{\tau=1}$

Hence $h_1(t)$ and $h_2(t)$ are both inverses.

$= \delta(t-1+1) = \delta(t)$

~~4.17 (a) $H(s) = \frac{1}{s+1} \Rightarrow \frac{1}{jw+1} = H(jw)$~~

~~at $w=0$ $H(jw) = 1$~~

~~at $w=3$ $H(jw) = \frac{1}{1+3j} = 0.3162$~~

4.17 (a) $H(s) = \frac{1}{s+1}$, $x(t) = 10 + 4(\cos 3t) + 2\sin 3t$

$$X(s) = 10 + \frac{4s}{s^2+9} + \frac{2\times 5}{s^2+25}$$

$$Y(s) = X(s)H(s)$$

$$= \left(10 + \frac{4s}{s^2+9} + \frac{10}{s^2+25}\right) + \frac{1}{s+1}$$

$$= \frac{10}{s+1} + \frac{4s}{(s+1)(s^2+9)} + \frac{10}{(s+1)(s^2+25)}$$

$$\frac{4s}{(s+1)(s^2+9)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+9} = \frac{As^2 + 9A + Bs^2 + Bs + Cs + C}{(s+1)(s^2+9)}$$

$A+B = 0$; $-B = +A$

$9A + C = 0$

$C = -9A$ → $B - 9A = 4$

$B + C = 4$

$B + 9B = 4$

$B = 2/5$

$A = \frac{-2}{5}$; $C = \frac{+18}{5}$

$$= \frac{-2/5}{s+1} + \frac{2/5\, s}{s^2+9} + \frac{18/5}{s^2+9}$$

$$\frac{10}{(s+1)(s^2+25)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+25}$$

$$= \frac{As^2+25A+Bs^2+Bs+Cs+C}{(s+1)(s^2+25)}$$

$A+B=0$
$\Rightarrow A=-B$

$B+C=0$
$\Rightarrow B=-C$
$\Rightarrow -A=-C$
$\Rightarrow A=C$

$25A+C=10$
$\Rightarrow 26A=10$
$\Rightarrow A=\frac{5}{13}=C$

$B=-\frac{5}{13}$

$$= \frac{5/13}{s+1} + \frac{-5/13\,s}{s^2+25} + \frac{5/13}{s^2+25}$$

$$y(s) = \frac{10}{s+1} + \frac{-2/5}{s+1} + \frac{2/5\,s}{s^2+9} + \frac{18/5}{s^2+9} + \frac{5/13}{s+1} - \frac{5/13\,s}{s^2+25} + \frac{5/13}{s^2+25}$$

$$= \frac{13.98}{s+1} + \frac{2/5\,s}{s^2+9} + \frac{18/5}{s^2+9} - \frac{5/13\,s}{s^2+25} + \frac{5/13}{s^2+25}$$

$$y(t) = 13.98e^{-t} + \frac{2}{5}\cos 3t + \frac{6}{5}\sin 3t - \frac{5}{13}\cos 5t + \frac{1}{13}\sin 5t$$

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Ass3A.m

```
close all
clear all

iterations = 100;
angle = 90;
drawpattern(iterations, angle)

iterations = 100;
angle = 89;
drawpattern(iterations, angle)

iterations = 100;
angle = 144;
drawpattern(iterations, angle)

iterations = 100;
angle = 154;
drawpattern(iterations, angle)

function drawpattern (i, angle)
    iterations=i;
    modAngle = (angle*(pi/180));
    p = [0 0]';
    x = p';

    for i = 1 : 1 :(i - 1)
        p = p+[ cos(modAngle) sin(modAngle); -sin(modAngle) cos(modAngle) ]^(i-1)*[i 0]';
        x = [x; p'];
    end

    plot(x(:,1),x(:,2))
    axis('equal');
    title(['i = ', int2str(iterations), ' and \theta = ', num2str(modAngle*(180/pi))])
    set(gca, "linewidth", 1, "fontsize", 10);
end
```

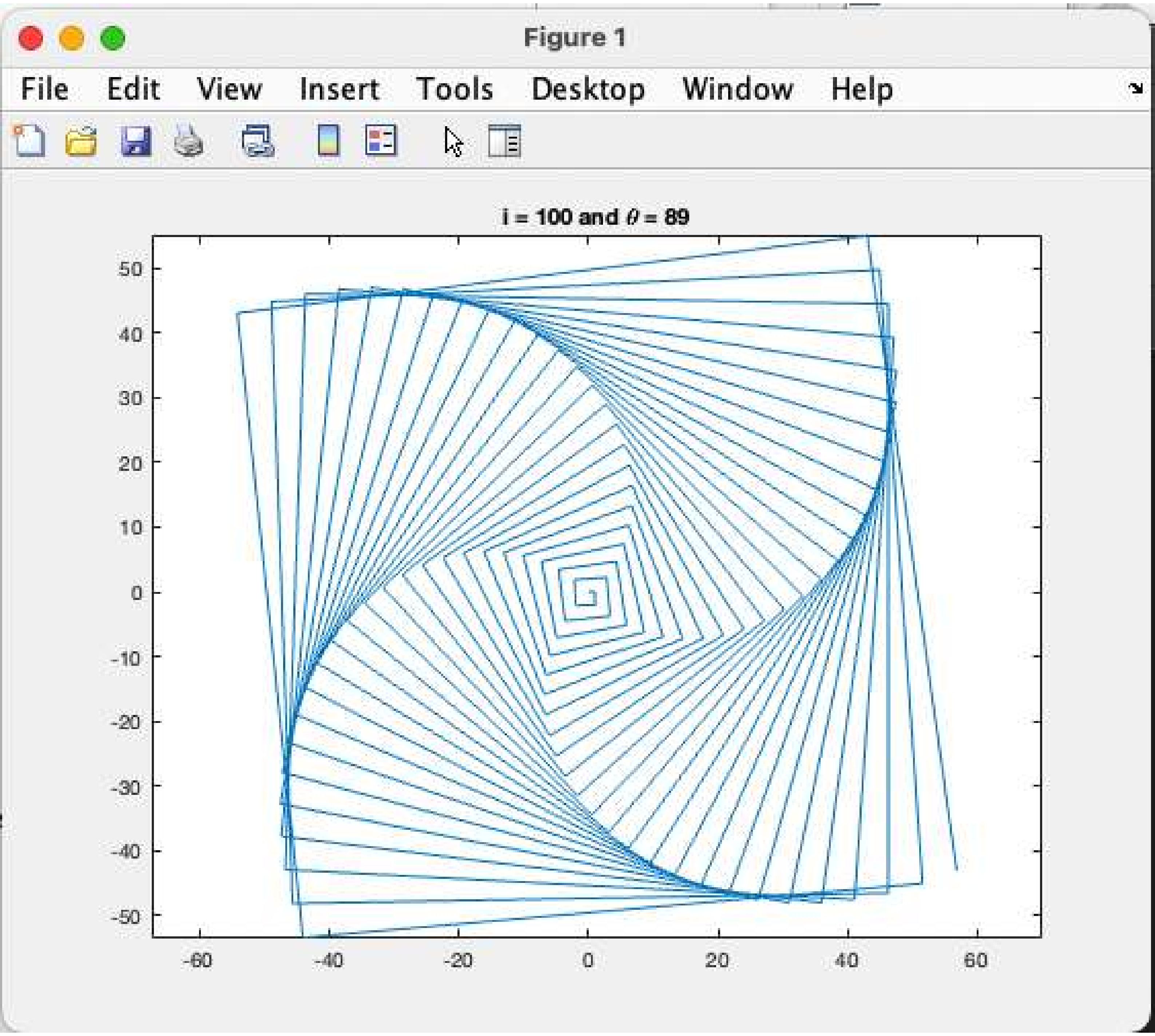
Figure 1
File Edit View Insert Tools Desktop Window Help
i = 100 and θ = 89
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-40
-20
0
20
40
60

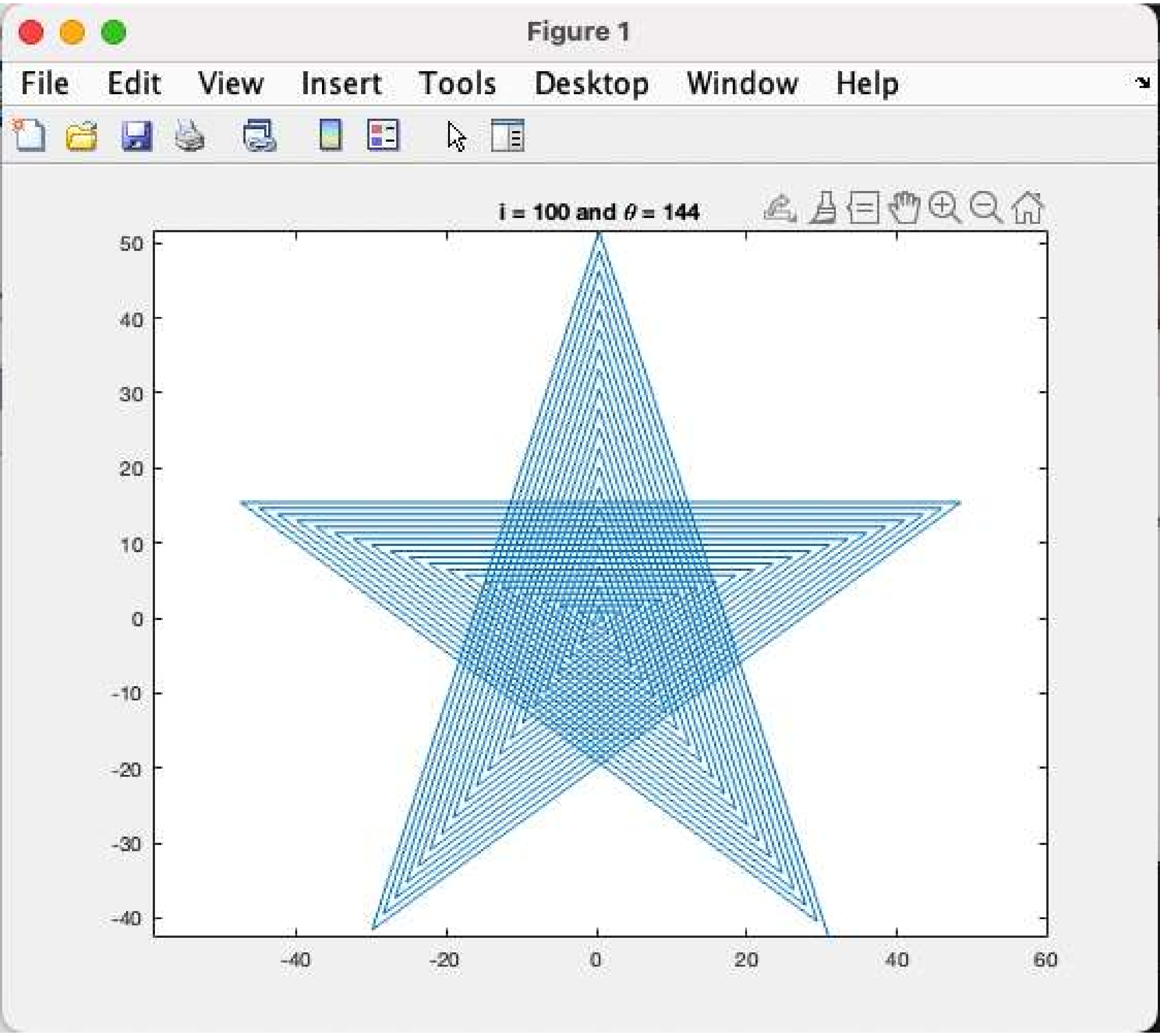
Figure 1
File Edit View Insert Tools Desktop Window Help
i = 100 and θ = 144
50
40
30
20
10
0
-10
-20
-30
-40
-40
-20
0
20
40
60

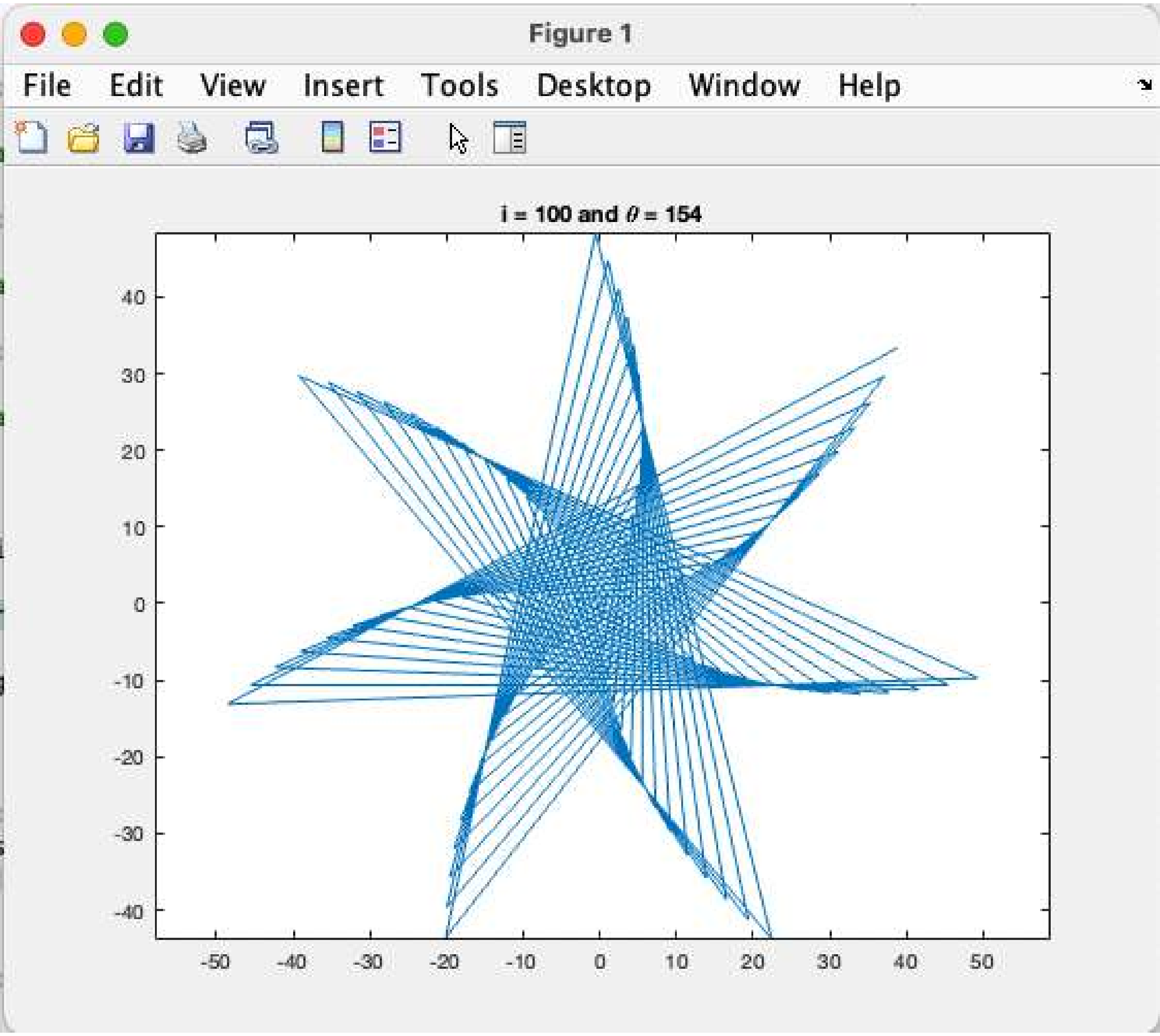
Figure 1
File Edit View Insert Tools Desktop Window Help
i = 100 and θ = 154
40
30
20
10
0
-10
-20
-30
-40
-50 -40 -30 -20 -10 0 10 20 30 40 50