STAT 260 Spring 2023: Assignment 8

Due: Friday March 31st BEFORE 11:59pm PT to Crowdmark

Please read the instructions below and in the Written Assignment 6 assignment on Crowdmark.

For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. Messy, poorly formatted work will receive deductions, or may not be graded at all.

Questions must be answered using the methods covered in our lectures and using the Stat 260 distribution tables.

Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding. Assignment questions are not to be posted to homework "help" websites.

Late policy: Late assignments will be accepted until the final cutoff of 11:59pm on Sunday April 2nd. Solutions submitted within 1 hour of the Friday deadline will have a 5% late penalty automatically applied within Crowdmark. Solutions submitted after 1 hour of the Friday deadline but before the final Sunday cutoff will have a 20% late penalty applied. Solutions submitted after the final Sunday cutoff will be graded for feedback, but marks will not be awarded.

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1. A study of strength properties was conducted on an experimental mix of concrete which uses special binders. The flexural strength (the ability to resist bending) for a random sample of 10 concrete beams using the experimental mix was measured. The results (measured in the units of MPa) are recorded below:

   7.7 9.1 6.3 7.0 10.5 8.4 11.6 9.7 10.3 5.9

   $\bar{x} = 8.65$

   $s = 1.9127$

   If needed, you may assume that flexural strength is normally distributed.

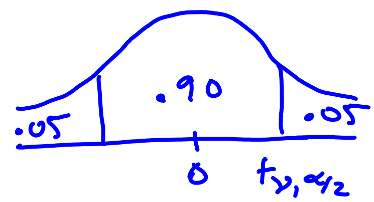
   (a) **[1 mark]** Calculate a 90% confidence interval for μ, the true mean flexural strength of the experimental concrete mix.

$n = 10$ is small, using s, Xs are normal $\Rightarrow$ $\bar{X}$ is T distributed with $df = \nu = n-1 = 9$

$$\bar{x} \pm t_{\nu,\alpha/2} \cdot s/\sqrt{n}$$

$$= 8.65 \pm 1.833 \cdot 1.9127/\sqrt{10}$$

$$= [7.5413, 9.7587]$$



$= t_{9,.05}$

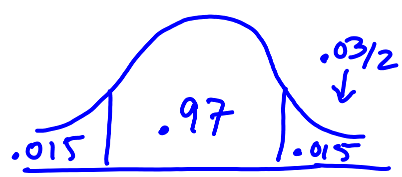
$= 1.833$

   (b) **[0.5 marks]** Using the sample above as a pilot study, determine the sample size needed to estimate the true mean flexural strength of the experimental concrete mix to within 0.3 MPa with 97% confidence.

$$d = z_{\alpha/2} \cdot s/\sqrt{n}$$

$$n = \left(\frac{z_{\alpha/2} \cdot s}{d}\right)^2$$

$$= \left(\frac{2.17 \cdot 1.9127}{0.3}\right)^2 = (13.8350)^2 = 191.41$$



$z_{\alpha/2} = z_{.015}$

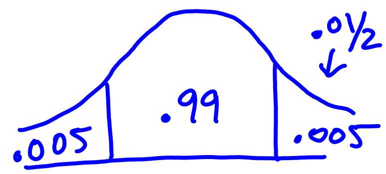
$= 2.17$

round up, so $n = 192$

2. A town is attempting to measure its unemployment rate (the proportion of residents that are unemployed). A random sample of 325 residents is taken and it is found that 14 of them are currently unemployed.

(a) **[1 mark]** Calculate a 99% confidence interval for p, the true proportion of town residents who are unemployed.

$n = 325 \quad \hat{p} = 14/325$



$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$z_{\alpha/2} = z_{.005} = 2.576$

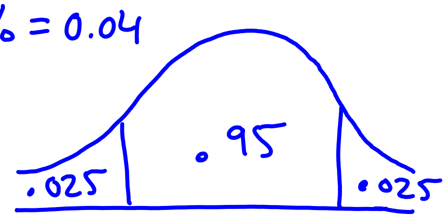
(or 2.575 from z table)

$$= 14/325 \pm 2.576 \sqrt{\frac{14/325\,(311/325)}{325}}$$

$$= [\,0.0141,\ 0.0721\,]$$

(b) **[0.5 marks]** The town has determined that the sampling methods for their sample of 325 residents may have been biased, and so they do not want to use this measurement in further calculations. Without using this sample, determine the sample size needed to estimate p, the true proportion of residents who are unemployed, to within 4% with 95% confidence.

no $\hat{p}$, so use $\hat{p} = 1/2 = 0.5$, $d = 4\% = 0.04$



$$d = z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$z_{\alpha/2} = z_{.025} = 1.96$

$$n = \left(\frac{z_{\alpha/2}\sqrt{\hat{p}(1-\hat{p})}}{d}\right)^2 = \left(\frac{1.96\sqrt{0.5(0.5)}}{.04}\right)^2$$

$$= (24.5)^2 = 600.25$$

round up, so $n = 601$

3. A spectrophotometer is used for measuring the concentration of CO gas in a controlled gas. The control gas is manufactured to have a mean CO concentration of 70 ppm, so if the spectrophotometer returns a mean measurement of the control gas that differs from this amount, it will require recalibration. Suppose a random sample of 9 measurements of the control gas returned the following readings (in ppm):

$\bar{x} = 73.8889$

85 76 72 66 81 73 68 70 74

$s = 6.0713$

If needed, you may assume that the CO concentration measurements are normally distributed.

(a) **[1 mark]** Define the parameter of interest using the correct notation. Then, state the null and alternative hypotheses for this study.

testing μ - true mean CO concentration

$H_0: \mu = 70$

$H_1: \mu \neq 70$

(b) **[0.5 marks]** Calculate the observed value of the test statistic. State the distribution (and degrees of freedom if needed) it follows.

$n=9$ is small, using s, Xs are normal $\Rightarrow \bar{X}$ is T distributed with $df = \nu = n-1 = 8$

$$T_{obs} = \frac{\bar{x}-\mu}{s/\sqrt{n}} = \frac{73.8889-70}{6.0713/\sqrt{9}} = 1.922$$

(c) **[0.5 marks]** Compute the p-value or provide a range of appropriate values for the p-value.

$p\text{-value} = P(T_8 < -1.922) + P(T_8 > 1.922) = 2 \cdot P(T_8 > 1.922)$

$2(0.025) < p\text{-value} < 2(0.05)$

$.05 < p\text{-value} < .10$

(d) **[1 mark]** Using the significance level $\alpha = 0.05$, state your conclusions about if the spectrophotometer needs calibration.

Since p-value $> .05 \Rightarrow p$-value is big $\Rightarrow$ keep H_0.

We conclude there is not enough evidence to say that $\mu \neq 70$, so the spectrophotometer does not need calibration.