Stat 260 Lecture Notes
Set 7 - Independent and Mutually Exclusive Events

Idea: Knowing extra information can change the probability of an outcome. This is what we saw with conditional probabilities. Here we look at the case when knowing the extra information does not change the probability of the outcome. This is the idea of having *independent events*.

Definition: Events A and B are *independent* when $P(A|B) = P(A)$ (or when $P(B|A) = P(B)$).

Using this definition along with the conditional probability formula we arrive at the alternate definition that events A and B are independent exactly when

$$P(A \cap B) = P(A) \cdot P(B)$$

Definition: Events A and B are *mutually exclusive* if they cannot occur at the same time (i.e. there is no overlap between A and B).

In the mathematical sense, we have that A and B are mutually exclusive when $P(A \cap B) = 0$ (i.e. when having both event A and B occur together is impossible).

Example 1: Revisiting the clear coating and rust example.

A maufacturer wants to see if using a clear coating of paint on their product is connected to if the product rusts.

	rust present (R)	no rust present ($\overline{R}$)	
clear coating used (C)	0.03	0.12	
no clear coating used ($\overline{C}$)	0.17	0.68	

Is having rust present independent of using the clear coating?

Rule: If events A and B are independent, then $\overline{A}$ and B are independent too (and A and $\overline{B}$ are independent, and also $\overline{A}$ and $\overline{B}$ are independent).

Is using the clear coating independent of not using the clear coating?

Careful! "Mutually exclusive" and "independent" are not the same thing. Here "clear coating" and "no clear coating" are mutually exclusive (since they are disjoint), but they are not independent.

The rule that if A and B are independent then

$$P(A \cap B) = P(A) \cdot P(B)$$

can be extended to more than two events.

Rule: If events $E_1, E_2, E_3, \ldots, E_n$ are independent events, then

$$P(E_1 \cap E_2 \cap E_3 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdot P(E_3) \cdot \cdots \cdot P(E_n).$$

Note: When $n \geq 3$ this rule does not work the other way around. That is, just because you have $P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \cdot \cdots \cdot P(E_n)$ does not guarantee that the events E_1, E_2, $\ldots$, E_n are all independent. (To guarantee independence you would have to do this formula check on all pairs, triples, quadruples, etc.)

(So the independence formula is an "if and only if" statement for two sets, and just an "if" statement for three or more sets.)

Example 2: A diagnostic test is correct 95% of the time. Suppose 3 people are independently tested. What is the probability that exactly two of the three receive a correct diagnosis?

What is the probability that none of the three receive a correct diagnosis?

What is the probability that at least one of the three receive a correct diagnosis?