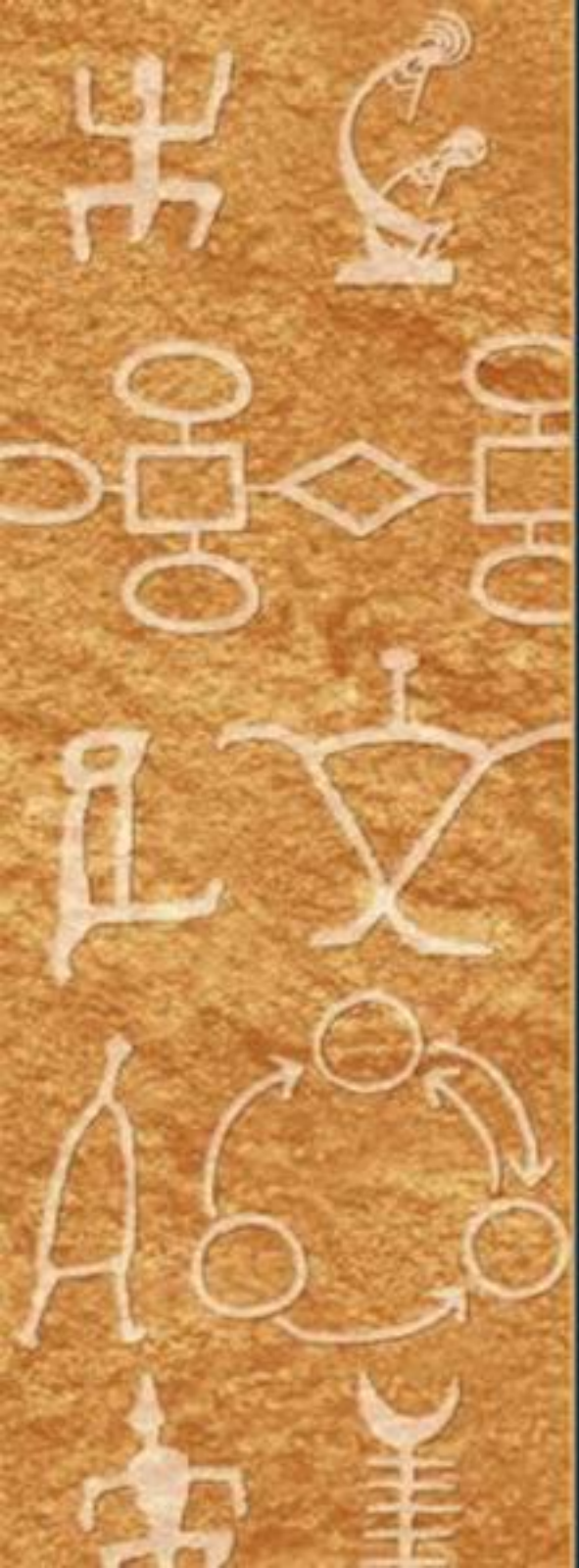

DATABASE SYSTEMS

THE COMPLETE BOOK

SECOND EDITION

Hector Garcia-Molina
Jeffrey D. Ullman
Jennifer Widom

DATABASE SYSTEMS

The Complete Book

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The Complete Book

Second Edition

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Upper Saddle River, NJ 07458

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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-606701-8
978-0-13-606701-6

Pearson Education Ltd., *London*
Pearson Education Australia Pty. Ltd., *Sydney*
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Preface

This book covers the core of the material taught in the database sequence at Stanford. The introductory course, CS145, uses the first twelve chapters, and is designed for all students — those who want to use database systems as well as those who want to get involved in database implementation. The second course, CS245 on database implementation, covers most of the rest of the book. However, some material is covered in more detail in special topics courses. These include CS346 (implementation project), which concentrates on query optimization as in Chapters 15 and 16. Also, CS345A, on data mining and Web mining, covers the material in the last two chapters.

What's New in the Second Edition

After a brief introduction in Chapter 1, we cover relational modeling in Chapters 2–4. Chapter 4 is devoted to high-level modeling. There, in addition to the E/R model, we now cover UML (Unified Modeling Language). We also have moved to Chapter 4 a shorter version of the material on ODL, treating it as a design language for relational database schemas.

The material on functional and multivalued dependencies has been modified and remains in Chapter 3. We have changed our viewpoint, so that a functional dependency is assumed to have a set of attributes on the right. We have also given explicitly certain algorithms, including the "chase," that allow us to manipulate dependencies. We have augmented our discussion of third normal form to include the 3NF synthesis algorithm and to make clear what the tradeoff between 3NF and BCNF is.

Chapter 5 contains the coverage of relational algebra from the previous edition, and is joined by (part of) the treatment of Datalog from the old Chapter 10. The discussion of recursion in Datalog is either moved to the book's Web site or combined with the treatment of recursive SQL in Chapter 10 of this edition.

Chapters 6–10 are devoted to aspects of SQL programming, and they represent a reorganization and augmentation of the earlier book's Chapters 6, 7, 8, and parts of 10. The material on views and indexes has been moved to its own chapter, number 8, and this material has been augmented with a discussion of

important new topics, including materialized views, and automatic selection of indexes.

The new Chapter 9 is based on the old Chapter 8 (embedded SQL). It is introduced by a new section on 3-tier architecture. It also includes an expanded discussion of JDBC and new coverage of PHP.

Chapter 10 collects a number of advanced SQL topics. The discussion of authorization from the old Chapter 8 has been moved here, as has the discussion of recursive SQL from the old Chapter 10. Data cubes, from the old Chapter 20, are now covered here. The rest of the chapter is devoted to the nested-relation model (from the old Chapter 4) and object-relational features of SQL (from the old Chapter 9).

Then, Chapters 11 and 12 cover XML and systems based on XML. Except for material at the end of the old Chapter 4, which has been moved to Chapter 11, this material is all new. Chapter 11 covers modeling; it includes expanded coverage of DTD's, along with new material on XML Schema. Chapter 12 is devoted to programming, and it includes sections on XPath, XQuery, and XSLT.

Chapter 13 begins the study of database implementation. It covers disk storage and the file structures that are built on disks. This chapter is a condensation of material that, in the first edition, occupied Chapters 11 and 12.

Chapter 14 covers index structures, including B-trees, hashing, and structures for multidimensional indexes. This material also condenses two chapters, 13 and 14, from the first edition.

Chapters 15 and 16 cover query execution and query optimization, respectively. They are similar to the old chapters of the same numbers. Chapter 17 covers logging, and Chapter 18 covers concurrency control; these chapters are also similar to the old chapters with the same numbers. Chapter 19 contains additional topics on concurrency: recovery, deadlocks, and long transactions. This material is a subset of the old Chapter 19.

Chapter 20 is on parallel and distributed databases. In addition to material on parallel query execution from the old Chapter 15 and material on distributed locking and commitment from the old Chapter 19, there are several new sections on distributed query execution: the map-reduce framework for parallel computation, peer-to-peer databases and their implementation of distributed hash tables.

Chapter 21 covers information integration. In addition to material on this subject from the old Chapter 20, we have added a section on local-as-view mediators and a section on entity resolution (finding records from several databases that refer to the same entity, e.g., a person).

Chapter 22 is on data mining. Although there was some material on the subject in the old Chapter 20, almost all of this chapter is new. It covers association rules and frequent itemset mining, including both the famous A-Priori Algorithm and certain efficiency improvements. Chapter 22 includes the key techniques of shingling, minhashing, and locality-sensitive hashing for finding similar items in massive databases, e.g., Web pages that quote substantially

from other Web pages. The chapter concludes with a study of clustering, especially for massive datasets.

Chapter 23, all new, addresses two important ways in which the Internet has impacted database technology. First is search engines, where we discuss algorithms for crawling the Web, the well-known PageRank algorithm for evaluating the importance of Web pages, and its extensions. This chapter also covers data-stream-management systems. We discuss the stream data model and SQL language extensions, and conclude with several interesting algorithms for executing queries on streams.

Prerequisites

We have used the book at the "mezzanine" level, in a sequence of courses taken both by undergraduates and by beginning graduate students. The formal prerequisites for the course are Sophomore-level treatments of:

1. Data structures, algorithms, and discrete math, and
2. Software systems, software engineering, and programming languages.

Of this material, it is important that students have at least a rudimentary understanding of such topics as: algebraic expressions and laws, logic, basic data structures, object-oriented programming concepts, and programming environments. However, we believe that adequate background is acquired by the Junior year of a typical computer science program.

Exercises

The book contains extensive exercises, with some for almost every section. We indicate harder exercises or parts of exercises with an exclamation point. The hardest exercises have a double exclamation point.

Support on the World Wide Web

The book's home page is

`http://infolab.stanford.edu/~ullman/dscb.html`

You will find errata as we learn of them, and backup materials, including homeworks, projects, and exams. We shall also make available there the sections from the first edition that have been removed from the second.

In addition, there is an accompanying set of on-line homeworks and programming labs using a technology developed by Gradiance Corp. See the section following the Preface for details about the GOAL system. GOAL service

can be purchased at `http://www.prenhall.com/goal`. Instructors who want to use the system in their classes should contact their Prentice-Hall representative or request instructor authorization through the above Web site.

There is a solutions manual for instructors available at

`http://www.prenhall.com/ullman`

This page also gives you access to GOAL and all book materials.

Acknowledgements

We would like to thank Donald Kossmann for helpful discussions, especially concerning XML and its associated programming systems. Also, Bobbie Cochrane assisted us in understanding trigger semantics for a earlier edition.

A large number of people have helped us, either with the development of this book or its predecessors, or by contacting us with errata in the books and/or other Web-based materials. It is our pleasure to acknowledge them all here.

Marc Abromowitz, Joseph H. Adamski, Brad Adelberg, Gleb Ashimov, Donald Aingworth, Teresa Almeida, Brian Babcock, Bruce Baker, Yunfan Bao, Jonathan Becker, Margaret Benitez, Eberhard Bertsch, Larry Bonham, Phillip Bonnet, David Brokaw, Ed Burns, Alex Butler, Karen Butler, Mike Carey, Christopher Chan, Sudarshan Chawathe.

Also Per Christensen, Ed Chang, Surajit Chaudhuri, Ken Chen, Rada Chirkova, Nitin Chopra, Lewis Church, Jr., Bobbie Cochrane, Michael Cole, Alissa Cooper, Arturo Crespo, Linda DeMichiel, Matthew F. Dennis, Tom Dienstbier, Pearl D'Souza, Oliver Duschka, Xavier Faz, Greg Fichtenholtz, Bart Fisher, Simon Frettloeh, Jarl Friis.

Also John Fry, Chiping Fu, Tracy Fujieda, Prasanna Ganesan, Suzanne Garcia, Mark Gjol, Manish Godara, Seth Goldberg, Jeff Goldblat, Meredith Goldsmith, Luis Gravano, Gerard Guillemette, Himanshu Gupta, Petri Gynther, Zoltan Gyongyi, Jon Heggland, Rafael Hernandez, Masanori Higashihara, Antti Hjelt, Ben Holtzman, Steve Huntsberry.

Also Sajid Hussain, Leonard Jacobson, Thulasiraman Jeyaraman, Dwight Joe, Brian Jorgensen, Mathew P. Johnson, Sameh Kamel, Jawed Karim, Seth Katz, Pedram Keyani, Victor Kimeli, Ed Knorr, Yeong-Ping Koh, David Koller, Gyorgy Kovacs, Phillip Koza, Brian Kulman, Bill Labiosa, Sang Ho Lee, Younghan Lee, Miguel Licona.

Also Olivier Lobry, Chao-Jun Lu, Waynn Lue, John Manz, Arun Marathe, Philip Minami, Le-Wei Mo, Fabian Modoux, Peter Mork, Mark Mortensen, Ramprakash Narayanaswami, Hankyung Na, Mor Naaman, Mayur Naik, Marie Nilsson, Torbjorn Norbye, Chang-Min Oh, Mehul Patel, Soren Peen, Jian Pei.

Also Xiaobo Peng, Bert Porter, Limbek Reka, Prahash Ramanan, Nisheeth Ranjan, Suzanne Rivoire, Ken Ross, Tim Roughgarten, Mema Roussopoulos, Richard Scherl, Loren Shevitz, Shrikrishna Shrin, June Yoshiko Sison,

Man Cho A. So, Elizabeth Stinson, Qi Su, Ed Swierk, Catherine Tornabene, Anders Uhl, Jonathan Ullman, Mayank Upadhyay.

Also Anatoly Varakin, Vassilis Vassalos, Krishna Venuturimilli, Vikram Vijayaraghavan, Terje Viken, Qiang Wang, Steven Whang, Mike Wiacek, Kristian Widjaja, Janet Wu, Sundar Yamunachari, Takeshi Yokukawa, Bing Yu, Min-Sig Yun, Torben Zahle, Sandy Zhang.

The remaining errors are ours, of course.

H. G.-M.
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March, 2008

GOAL

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Instructors should contact their local Pearson Sales Representative for sales and ordering information for the GOAL Student Access Code and textbook value package.

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Table of Contents

DATABASE SYSTEMS

The Complete Book

Chapter 1

The Worlds of Database Systems

Databases today are essential to every business. Whenever you visit a major Web site — Google, Yahoo!, Amazon.com, or thousands of smaller sites that provide information — there is a database behind the scenes serving up the information you request. Corporations maintain all their important records in databases. Databases are likewise found at the core of many scientific investigations. They represent the data gathered by astronomers, by investigators of the human genome, and by biochemists exploring properties of proteins, among many other scientific activities.

The power of databases comes from a body of knowledge and technology that has developed over several decades and is embodied in specialized software called a *database management system*, or *DBMS*, or more colloquially a "database system." A DBMS is a powerful tool for creating and managing large amounts of data efficiently and allowing it to persist over long periods of time, safely. These systems are among the most complex types of software available. In this book, we shall learn how to design databases, how to write programs in the various languages associated with a DBMS, and how to implement the DBMS itself.

1.1 The Evolution of Database Systems

What is a database? In essence a database is nothing more than a collection of information that exists over a long period of time, often many years. In common parlance, the term *database* refers to a collection of data that is managed by a DBMS. The DBMS is expected to:

1. Allow users to create new databases and specify their *schemas* (logical structure of the data), using a specialized *data-definition language*.

2. Give users the ability to *query* the data (a "query" is database lingo for a question about the data) and modify the data, using an appropriate language, often called a *query language* or *data-manipulation language.*

3. Support the storage of very large amounts of data — many terabytes or more — over a long period of time, allowing efficient access to the data for queries and database modifications.

4. Enable *durability*, the recovery of the database in the face of failures, errors of many kinds, or intentional misuse.

5. Control access to data from many users at once, without allowing unexpected interactions among users (called *isolation*) and without actions on the data to be performed partially but not completely (called *atomicity*).

1.1.1 Early Database Management Systems

The first commercial database management systems appeared in the late 1960's. These systems evolved from file systems, which provide some of item (3) above; file systems store data over a long period of time, and they allow the storage of large amounts of data. However, file systems do not generally guarantee that data cannot be lost if it is not backed up, and they don't support efficient access to data items whose location in a particular file is not known.

Further, file systems do not directly support item (2), a query language for the data in files. Their support for (1) — a schema for the data — is limited to the creation of directory structures for files. Item (4) is not always supported by file systems; you can lose data that has not been backed up. Finally, file systems do not satisfy (5). While they allow concurrent access to files by several users or processes, a file system generally will not prevent situations such as two users modifying the same file at about the same time, so the changes made by one user fail to appear in the file.

The first important applications of DBMS's were ones where data was composed of many small items, and many queries or modifications were made. Examples of these applications are:

1. Banking systems: maintaining accounts and making sure that system failures do not cause money to disappear.

2. Airline reservation systems: these, like banking systems, require assurance that data will not be lost, and they must accept very large volumes of small actions by customers.

3. Corporate record keeping: employment and tax records, inventories, sales records, and a great variety of other types of information, much of it critical.

The early DBMS's required the programmer to visualize data much as it was stored. These database systems used several different data models for

describing the structure of the information in a database, chief among them the "hierarchical" or tree-based model and the graph-based "network" model. The latter was standardized in the late 1960's through a report of CODASYL (Committee on Data Systems and Languages).[1]

A problem with these early models and systems was that they did not support high-level query languages. For example, the CODASYL query language had statements that allowed the user to jump from data element to data element, through a graph of pointers among these elements. There was considerable effort needed to write such programs, even for very simple queries.

1.1.2 Relational Database Systems

Following a famous paper written by Ted Codd in 1970,[2] database systems changed significantly. Codd proposed that database systems should present the user with a view of data organized as tables called *relations*. Behind the scenes, there might be a complex data structure that allowed rapid response to a variety of queries. But, unlike the programmers for earlier database systems, the programmer of a relational system would not be concerned with the storage structure. Queries could be expressed in a very high-level language, which greatly increased the efficiency of database programmers. We shall cover the relational model of database systems throughout most of this book. SQL ("Structured Query Language"), the most important query language based on the relational model, is covered extensively.

By 1990, relational database systems were the norm. Yet the database field continues to evolve, and new issues and approaches to the management of data surface regularly. Object-oriented features have infilrated the relational model. Some of the largest databases are organized rather differently from those using relational methodology. In the balance of this section, we shall consider some of the modern trends in database systems.

1.1.3 Smaller and Smaller Systems

Originally, DBMS's were large, expensive software systems running on large computers. The size was necessary, because to store a gigabyte of data required a large computer system. Today, hundreds of gigabytes fit on a single disk, and it is quite feasible to run a DBMS on a personal computer. Thus, database systems based on the relational model have become available for even very small machines, and they are beginning to appear as a common tool for computer applications, much as spreadsheets and word processors did before them.

Another important trend is the use of documents, often tagged using XML (eXtensible Modeling Language). Large collections of small documents can

[1] *CODASYL Data Base Task Group April 1971 Report*, ACM, New York.

[2] Codd, E. F., "A relational model for large shared data banks," *Comm. ACM*, **13**:6, pp. 377–387, 1970.

serve as a database, and the methods of querying and manipulating them are different from those used in relational systems.

1.1.4 Bigger and Bigger Systems

On the other hand, a gigabyte is not that much data any more. Corporate databases routinely store terabytes (10^{12} bytes). Yet there are many databases that store petabytes (10^{15} bytes) of data and serve it all to users. Some important examples:

1. Google holds petabytes of data gleaned from its crawl of the Web. This data is not held in a traditional DBMS, but in specialized structures optimized for search-engine queries.

2. Satellites send down petabytes of information for storage in specialized systems.

3. A picture is actually worth way more than a thousand words. You can store 1000 words in five or six thousand bytes. Storing a picture typically takes much more space. Repositories such as Flickr store millions of pictures and support search of those pictures. Even a database like Amazon's has millions of pictures of products to serve.

4. And if still pictures consume space, movies consume much more. An hour of video requires at least a gigabyte. Sites such as YouTube hold hundreds of thousands, or millions, of movies and make them available easily.

5. Peer-to-peer file-sharing systems use large networks of conventional computers to store and distribute data of various kinds. Although each node in the network may only store a few hundred gigabytes, together the database they embody is enormous.

1.1.5 Information Integration

To a great extent, the old problem of building and maintaining databases has become one of *information integration*: joining the information contained in many related databases into a whole. For example, a large company has many divisions. Each division may have built its own database of products or employee records independently of other divisions. Perhaps some of these divisions used to be independent companies, which naturally had their own way of doing things. These divisions may use different DBMS's and different structures for information. They may use different terms to mean the same thing or the same term to mean different things. To make matters worse, the existence of legacy applications using each of these databases makes it almost impossible to scrap them, ever.

As a result, it has become necessary with increasing frequency to build structures on top of existing databases, with the goal of integrating the information

distributed among them. One popular approach is the creation of *data warehouses*, where information from many legacy databases is copied periodically, with the appropriate translation, to a central database. Another approach is the implementation of a mediator, or "middleware," whose function is to support an integrated model of the data of the various databases, while translating between this model and the actual models used by each database.

1.2 Overview of a Database Management System

In Fig. 1.1 we see an outline of a complete DBMS. Single boxes represent system components, while double boxes represent in-memory data structures. The solid lines indicate control and data flow, while dashed lines indicate data flow only. Since the diagram is complicated, we shall consider the details in several stages. First, at the top, we suggest that there are two distinct sources of commands to the DBMS:

1. Conventional users and application programs that ask for data or modify data.
2. A *database administrator*: a person or persons responsible for the structure or *schema* of the database.

1.2.1 Data-Definition Language Commands

The second kind of command is the simpler to process, and we show its trail beginning at the upper right side of Fig. 1.1. For example, the database administrator, or *DBA*, for a university registrar's database might decide that there should be a table or relation with columns for a student, a course the student has taken, and a grade for that student in that course. The DBA might also decide that the only allowable grades are A, B, C, D, and F. This structure and constraint information is all part of the schema of the database. It is shown in Fig. 1.1 as entered by the DBA, who needs special authority to execute schema-altering commands, since these can have profound effects on the database. These schema-altering data-definition language (DDL) commands are parsed by a DDL processor and passed to the execution engine, which then goes through the index/file/record manager to alter the *metadata*, that is, the schema information for the database.

1.2.2 Overview of Query Processing

The great majority of interactions with the DBMS follow the path on the left side of Fig. 1.1. A user or an application program initiates some action, using the data-manipulation language (DML). This command does not affect the schema of the database, but may affect the content of the database (if the

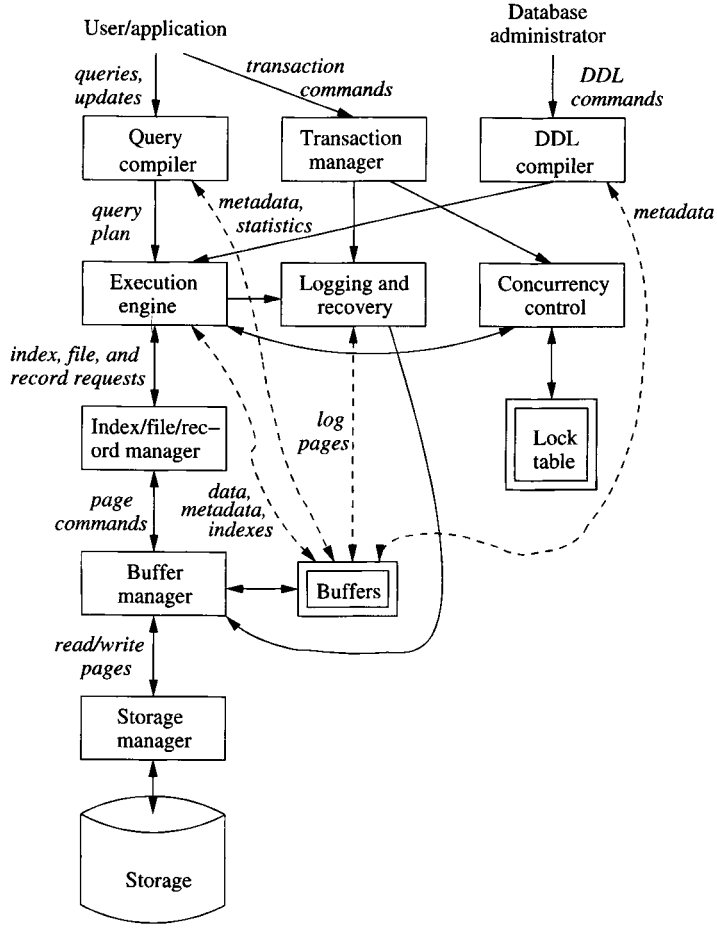

Figure 1.1: Database management system components

action is a modification command) or will extract data from the database (if the action is a query). DML statements are handled by two separate subsystems, as follows.

Answering the Query

The query is parsed and optimized by a *query compiler*. The resulting *query plan*, or sequence of actions the DBMS will perform to answer the query, is passed to the *execution engine*. The execution engine issues a sequence of requests for small pieces of data, typically records or tuples of a relation, to a resource manager that knows about *data files* (holding relations), the format and size of records in those files, and *index files*, which help find elements of data files quickly.

The requests for data are passed to the *buffer manager*. The buffer manager's task is to bring appropriate portions of the data from secondary storage (disk) where it is kept permanently, to the main-memory buffers. Normally, the page or "disk block" is the unit of transfer between buffers and disk.

The buffer manager communicates with a storage manager to get data from disk. The storage manager might involve operating-system commands, but more typically, the DBMS issues commands directly to the disk controller.

Transaction Processing

Queries and other DML actions are grouped into *transactions*, which are units that must be executed atomically and in isolation from one another. Any query or modification action can be a transaction by itself. In addition, the execution of transactions must be *durable*, meaning that the effect of any completed transaction must be preserved even if the system fails in some way right after completion of the transaction. We divide the transaction processor into two major parts:

1. A *concurrency-control manager*, or *scheduler*, responsible for assuring atomicity and isolation of transactions, and

2. A *logging and recovery manager*, responsible for the durability of transactions.

1.2.3 Storage and Buffer Management

The data of a database normally resides in secondary storage; in today's computer systems "secondary storage" generally means magnetic disk. However, to perform any useful operation on data, that data must be in main memory. It is the job of the *storage manager* to control the placement of data on disk and its movement between disk and main memory.

In a simple database system, the storage manager might be nothing more than the file system of the underlying operating system. However, for efficiency

purposes, DBMS's normally control storage on the disk directly, at least under some circumstances. The *storage manager* keeps track of the location of files on the disk and obtains the block or blocks containing a file on request from the buffer manager.

The *buffer manager* is responsible for partitioning the available main memory into *buffers*, which are page-sized regions into which disk blocks can be transferred. Thus, all DBMS components that need information from the disk will interact with the buffers and the buffer manager, either directly or through the execution engine. The kinds of information that various components may need include:

1. *Data*: the contents of the database itself.
2. *Metadata*: the database schema that describes the structure of, and constraints on, the database.
3. *Log Records*: information about recent changes to the database; these support durability of the database.
4. *Statistics*: information gathered and stored by the DBMS about data properties such as the sizes of, and values in, various relations or other components of the database.
5. *Indexes*: data structures that support efficient access to the data.

1.2.4 Transaction Processing

It is normal to group one or more database operations into a *transaction*, which is a unit of work that must be executed atomically and in apparent isolation from other transactions. In addition, a DBMS offers the guarantee of durability: that the work of a completed transaction will never be lost. The *transaction manager* therefore accepts *transaction commands* from an application, which tell the transaction manager when transactions begin and end, as well as information about the expectations of the application (some may not wish to require atomicity, for example). The transaction processor performs the following tasks:

1. *Logging*: In order to assure durability, every change in the database is logged separately on disk. The *log manager* follows one of several policies designed to assure that no matter when a system failure or "crash" occurs, a *recovery manager* will be able to examine the log of changes and restore the database to some consistent state. The log manager initially writes the log in buffers and negotiates with the buffer manager to make sure that buffers are written to disk (where data can survive a crash) at appropriate times.
2. *Concurrency control*: Transactions must appear to execute in isolation. But in most systems, there will in truth be many transactions executing

The ACID Properties of Transactions

Properly implemented transactions are commonly said to meet the "ACID test," where:

- "A" stands for "atomicity," the all-or-nothing execution of transactions.
- "I" stands for "isolation," the fact that each transaction must appear to be executed as if no other transaction is executing at the same time.
- "D" stands for "durability," the condition that the effect on the database of a transaction must never be lost, once the transaction has completed.

The remaining letter, "C," stands for "consistency." That is, all databases have consistency constraints, or expectations about relationships among data elements (e.g., account balances may not be negative after a transaction finishes). Transactions are expected to preserve the consistency of the database.

at once. Thus, the scheduler (concurrency-control manager) must assure that the individual actions of multiple transactions are executed in such an order that the net effect is the same as if the transactions had in fact executed in their entirety, one-at-a-time. A typical scheduler does its work by maintaining *locks* on certain pieces of the database. These locks prevent two transactions from accessing the same piece of data in ways that interact badly. Locks are generally stored in a main-memory *lock table*, as suggested by Fig. 1.1. The scheduler affects the execution of queries and other database operations by forbidding the execution engine from accessing locked parts of the database.

3. *Deadlock resolution*: As transactions compete for resources through the locks that the scheduler grants, they can get into a situation where none can proceed because each needs something another transaction has. The transaction manager has the responsibility to intervene and cancel ("roll-back" or "abort") one or more transactions to let the others proceed.

1.2.5 The Query Processor

The portion of the DBMS that most affects the performance that the user sees is the *query processor*. In Fig. 1.1 the query processor is represented by two components:

1. The *query compiler*, which translates the query into an internal form called a *query plan*. The latter is a sequence of operations to be performed on the data. Often the operations in a query plan are implementations of "relational algebra" operations, which are discussed in Section 2.4. The query compiler consists of three major units:
   (a) A *query parser*, which builds a tree structure from the textual form of the query.
   (b) A *query preprocessor*, which performs semantic checks on the query (e.g., making sure all relations mentioned by the query actually exist), and performing some tree transformations to turn the parse tree into a tree of algebraic operators representing the initial query plan.
   (c) A *query optimizer*, which transforms the initial query plan into the best available sequence of operations on the actual data.

   The query compiler uses metadata and statistics about the data to decide which sequence of operations is likely to be the fastest. For example, the existence of an *index*, which is a specialized data structure that facilitates access to data, given values for one or more components of that data, can make one plan much faster than another.
2. The *execution engine*, which has the responsibility for executing each of the steps in the chosen query plan. The execution engine interacts with most of the other components of the DBMS, either directly or through the buffers. It must get the data from the database into buffers in order to manipulate that data. It needs to interact with the scheduler to avoid accessing data that is locked, and with the log manager to make sure that all database changes are properly logged.

1.3 Outline of Database-System Studies

We divide the study of databases into five parts. This section is an outline of what to expect in each of these units.

Part I: Relational Database Modeling

The relational model is essential for a study of database systems. After examining the basic concepts, we delve into the theory of relational databases. That study includes *functional dependencies*, a formal way of stating that one kind of data is uniquely determined by another. It also includes *normalization*, the process whereby functional dependencies and other formal dependencies are used to improve the design of a relational database.

We also consider high-level design notations. These mechanisms include the Entity-Relationship (E/R) model, Unified Modeling Language (UML), and Object Definition Language (ODL). Their purpose is to allow informal exploration of design issues before we implement the design using a relational DBMS.

Part II: Relational Database Programming

We then take up the matter of how relational databases are queried and modified. After an introduction to abstract programming languages based on algebra and logic (Relational Algebra and Datalog, respectively), we turn our attention to the standard language for relational databases: SQL. We study both the basics and important special topics, including constraint specifications and triggers (active database elements), indexes and other structures to enhance performance, forming SQL into transactions, and security and privacy of data in SQL.

We also discuss how SQL is used in complete systems. It is typical to combine SQL with a conventional or *host* language and to pass data between the database and the conventional program via SQL calls. We discuss a number of ways to make this connection, including embedded SQL, Persistent Stored Modules (PSM), Call-Level Interface (CLI), Java Database Interconnectivity (JDBC), and PHP.

Part III: Semistructured Data Modeling and Programming

The pervasiveness of the Web has put a premium on the management of hierarchically structured data, because the standards for the Web are based on nested, tagged elements (*semistructured data*). We introduce XML and its schema-defining notations: Document Type Definitions (DTD) and XML Schema. We also examine three query languages for XML: XPATH, XQuery, and Extensible Stylesheet Language Transform (XSLT).

Part IV: Database System Implementation

We begin with a study of *storage management*: how disk-based storage can be organized to allow efficient access to data. We explain the commonly used B-tree, a balanced tree of disk blocks and other specialized schemes for managing multidimensional data.

We then turn our attention to *query processing*. There are two parts to this study. First, we need to learn *query execution*: the algorithms used to implement the operations from which queries are built. Since data is typically on disk, the algorithms are somewhat different from what one would expect were they to study the same problems but assuming that data were in main memory. The second step is *query compiling*. Here, we study how to select an efficient query plan from among all the possible ways in which a given query can be executed.

Then, we study *transaction processing*. There are several threads to follow. One concerns *logging*: maintaining reliable records of what the DBMS is doing, in order to allow *recovery* in the event of a crash. Another thread is *scheduling*: controlling the order of events in transactions to assure the ACID properties. We also consider how to deal with deadlocks, and the modifications to our algorithms that are needed when a transaction is *distributed* over many independent

sites.

Part V: Modern Database System Issues

In this part, we take up a number of the ways in which database-system technology is relevant beyond the realm of conventional, relational DBMS's. We consider how *search engines* work, and the specialized data structures that make their operation possible. We look at information integration, and methodologies for making databases share their data seamlessly. *Data mining* is a study that includes a number of interesting and important algorithms for processing large amounts of data in complex ways. *Data-stream systems* deal with data that arrives at the system continuously, and whose queries are answered continuously and in a timely fashion. *Peer-to-peer systems* present many challenges for management of distributed data held by independent hosts.

1.4 References for Chapter 1

Today, on-line searchable bibliographies cover essentially all recent papers concerning database systems. Thus, in this book, we shall not try to be exhaustive in our citations, but rather shall mention only the papers of historical importance and major secondary sources or useful surveys. A searchable index of database research papers was constructed by Michael Ley [5], and has recently been expanded to include references from many fields. Alf-Christian Achilles maintains a searchable directory of many indexes relevant to the database field [3].

While many prototype implementations of database systems contributed to the technology of the field, two of the most widely known are the System R project at IBM Almaden Research Center [4] and the INGRES project at Berkeley [7]. Each was an early relational system and helped establish this type of system as the dominant database technology. Many of the research papers that shaped the database field are found in [6].

The 2003 "Lowell report" [1] is the most recent in a series of reports on database-system research and directions. It also has references to earlier reports of this type.

You can find more about the theory of database systems than is covered here from [2] and [8].

1. S. Abiteboul et al., "The Lowell database research self-assessment," *Comm. ACM* **48**:5 (2005), pp. 111–118. `http://research.microsoft.com/~gray/lowell/LowellDatabaseResearchSelfAssessment.htm`

2. S. Abiteboul, R. Hull, and V. Vianu, *Foundations of Databases*, Addison-Wesley, Reading, MA, 1995.

3. `http://liinwww.ira.uka.de/bibliography/Database` .

4. M. M. Astrahan et al., "System R: a relational approach to database management," *ACM Trans. on Database Systems* **1**:2, pp. 97–137, 1976.

5. `http://www.informatik.uni-trier.de/~ley/db/index.html` . A mirror site is found at `http://www.acm.org/sigmod/dblp/db/index.html` .

6. M. Stonebraker and J. M. Hellerstein (eds.), *Readings in Database Systems*, Morgan-Kaufmann, San Francisco, 1998.

7. M. Stonebraker, E. Wong, P. Kreps, and G. Held, "The design and implementation of INGRES," *ACM Trans. on Database Systems* **1**:3, pp. 189–222, 1976.

8. J. D. Ullman, *Principles of Database and Knowledge-Base Systems, Volumes I and II*, Computer Science Press, New York, 1988, 1989.

Part I

Relational Database Modeling

Chapter 2

The Relational Model of Data

This chapter introduces the most important model of data: the two-dimensional table, or "relation." We begin with an overview of data models in general. We give the basic terminology for relations and show how the model can be used to represent typical forms of data. We then introduce a portion of the language SQL — that part used to declare relations and their structure. The chapter closes with an introduction to relational algebra. We see how this notation serves as both a query language — the aspect of a data model that enables us to ask questions about the data — and as a constraint language — the aspect of a data model that lets us restrict the data in the database in various ways.

2.1 An Overview of Data Models

The notion of a "data model" is one of the most fundamental in the study of database systems. In this brief summary of the concept, we define some basic terminology and mention the most important data models.

2.1.1 What is a Data Model?

A *data model* is a notation for describing data or information. The description generally consists of three parts:

1. *Structure of the data.* You may be familiar with tools in programming languages such as C or Java for describing the structure of the data used by a program: arrays and structures ("structs") or objects, for example. The data structures used to implement data in the computer are sometimes referred to, in discussions of database systems, as a *physical data model*, although in fact they are far removed from the gates and electrons that truly serve as the physical implementation of the data. In the database

world, data models are at a somewhat higher level than data structures, and are sometimes referred to as a *conceptual model* to emphasize the difference in level. We shall see examples shortly.

2. *Operations on the data.* In programming languages, operations on the data are generally anything that can be programmed. In database data models, there is usually a limited set of operations that can be performed. We are generally allowed to perform a limited set of *queries* (operations that retrieve information) and *modifications* (operations that change the database). This limitation is not a weakness, but a strength. By limiting operations, it is possible for programmers to describe database operations at a very high level, yet have the database management system implement the operations efficiently. In comparison, it is generally impossible to optimize programs in conventional languages like C, to the extent that an inefficient algorithm (e.g., bubblesort) is replaced by a more efficient one (e.g., quicksort).

3. *Constraints on the data.* Database data models usually have a way to describe limitations on what the data can be. These constraints can range from the simple (e.g., "a day of the week is an integer between 1 and 7" or "a movie has at most one title") to some very complex limitations that we shall discuss in Sections 7.4 and 7.5.

2.1.2 Important Data Models

Today, the two data models of preeminent importance for database systems are:

1. The relational model, including object-relational extensions.

2. The semistructured-data model, including XML and related standards.

The first, which is present in all commercial database management systems, is the subject of this chapter. The semistructured model, of which XML is the primary manifestation, is an added feature of most relational DBMS's, and appears in a number of other contexts as well. We turn to this data model starting in Chapter 11.

2.1.3 The Relational Model in Brief

The relational model is based on tables, of which Fig. 2.1 is an example. We shall discuss this model beginning in Section 2.2. This relation, or table, describes movies: their title, the year in which they were made, their length in minutes, and the genre of the movie. We show three particular movies, but you should imagine that there are many more rows to this table — one row for each movie ever made, perhaps.

The structure portion of the relational model might appear to resemble an array of structs in C, where the column headers are the field names, and each

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

Figure 2.1: An example relation

of the rows represent the values of one struct in the array. However, it must be emphasized that this physical implementation is only one possible way the table could be implemented in physical data structures. In fact, it is not the normal way to represent relations, and a large portion of the study of database systems addresses the right ways to implement such tables. Much of the distinction comes from the scale of relations — they are not normally implemented as main-memory structures, and their proper physical implementation must take into account the need to access relations of very large size that are resident on disk.

The operations normally associated with the relational model form the "relational algebra," which we discuss beginning in Section 2.4. These operations are table-oriented. As an example, we can ask for all those rows of a relation that have a certain value in a certain column. For example, we can ask of the table in Fig. 2.1 for all the rows where the genre is "comedy."

The constraint portion of the relational data model will be touched upon briefly in Section 2.5 and covered in more detail in Chapter 7. However, as a brief sample of what kinds of constraints are generally used, we could decide that there is a fixed list of genres for movies, and that the last column of every row must have a value that is on this list. Or we might decide (incorrectly, it turns out) that there could never be two movies with the same title, and constrain the table so that no two rows could have the same string in the first component.

2.1.4 The Semistructured Model in Brief

Semistructured data resembles trees or graphs, rather than tables or arrays. The principal manifestation of this viewpoint today is XML, a way to represent data by hierarchically nested tagged elements. The tags, similar to those used in HTML, define the role played by different pieces of data, much as the column headers do in the relational model. For example, the same data as in Fig. 2.1 might appear in an XML "document" as in Fig. 2.2.

The operations on semistructured data usually involve following paths in the implied tree from an element to one or more of its nested subelements, then to subelements nested within those, and so on. For example, starting at the outer `<Movies>` element (the entire document in Fig. 2.2), we might move to each of its nested `<Movie>` elements, each delimited by the tag `<Movie>` and matching `</Movie>` tag, and from each `<Movie>` element to its nested `<Genre>`

```
<Movies>
    <Movie title="Gone With the Wind">
        <Year>1939</Year>
        <Length>231</Length>
        <Genre>drama</Genre>
    </Movie>
    <Movie title="Star Wars">
        <Year>1977</Year>
        <Length>124</Length>
        <Genre>sciFi</Genre>
    </Movie>
    <Movie title="Wayne's World">
        <Year>1992</Year>
        <Length>95</Length>
        <Genre>comedy</Genre>
    </Movie>
</Movies>
```

Figure 2.2: Movie data as XML

element, to see which movies belong to the "comedy" genre.

Constraints on the structure of data in this model often involve the data type of values associated with a tag. For instance, are the values associated with the `<Length>` tag integers or can they be arbitrary character strings? Other constraints determine which tags can appear nested within which other tags. For example, must each `<Movie>` element have a `<Length>` element nested within it? What other tags, besides those shown in Fig. 2.2 might be used within a `<Movie>` element? Can there be more than one genre for a movie? These and other matters will be taken up in Section 11.2.

2.1.5 Other Data Models

There are many other models that are, or have been, associated with DBMS's. A modern trend is to add object-oriented features to the relational model. There are two effects of object-orientation on relations:

1. Values can have structure, rather than being elementary types such as integer or strings, as they were in Fig. 2.1.

2. Relations can have associated methods.

In a sense, these extensions, called the *object-relational* model, are analogous to the way structs in C were extended to objects in C++. We shall introduce the object-relational model in Section 10.3.

There are even database models of the purely object-oriented kind. In these, the relation is no longer the principal data-structuring concept, but becomes only one option among many structures. We discuss an object-oriented database model in Section 4.9.

There are several other models that were used in some of the earlier DBMS's, but that have now fallen out of use. The *hierarchical model* was, like semistructured data, a tree-oriented model. Its drawback was that unlike more modern models, it really operated at the physical level, which made it impossible for programmers to write code at a conveniently high level. Another such model was the *network model*, which was a graph-oriented, physical-level model. In truth, both the hierarchical model and today's semistructured models, allow full graph structures, and do not limit us strictly to trees. However, the generality of graphs was built directly into the network model, rather than favoring trees as these other models do.

2.1.6 Comparison of Modeling Approaches

Even from our brief example, it appears that semistructured models have more flexibility than relations. This difference becomes even more apparent when we discuss, as we shall, how full graph structures are embedded into tree-like, semistructured models. Nevertheless, the relational model is still preferred in DBMS's, and we should understand why. A brief argument follows.

Because databases are large, efficiency of access to data and efficiency of modifications to that data are of great importance. Also very important is ease of use — the productivity of programmers who use the data. Surprisingly, both goals can be achieved with a model, particularly the relational model, that:

1. Provides a simple, limited approach to structuring data, yet is reasonably versatile, so anything can be modeled.

2. Provides a limited, yet useful, collection of operations on data.

Together, these limitations turn into features. They allow us to implement languages, such as SQL, that enable the programmer to express their wishes at a very high level. A few lines of SQL can do the work of thousands of lines of C, or hundreds of lines of the code that had to be written to access data under earlier models such as network or hierarchical. Yet the short SQL programs, because they use a strongly limited sets of operations, can be optimized to run as fast, or faster than the code written in alternative languages.

2.2 Basics of the Relational Model

The relational model gives us a single way to represent data: as a two-dimensional table called a *relation*. Figure 2.1, which we copy here as Fig. 2.3, is an example of a relation, which we shall call `Movies`. The rows each represent a

movie, and the columns each represent a property of movies. In this section, we shall introduce the most important terminology regarding relations, and illustrate them with the `Movies` relation.

title	*year*	*length*	*genre*
`Gone With the Wind`	1939	231	`drama`
`Star Wars`	1977	124	`sciFi`
`Wayne's World`	1992	95	`comedy`

Figure 2.3: The relation `Movies`

2.2.1 Attributes

The columns of a relation are named by *attributes*; in Fig. 2.3 the attributes are `title`, `year`, `length`, and `genre`. Attributes appear at the tops of the columns. Usually, an attribute describes the meaning of entries in the column below. For instance, the column with attribute `length` holds the length, in minutes, of each movie.

2.2.2 Schemas

The name of a relation and the set of attributes for a relation is called the *schema* for that relation. We show the schema for the relation with the relation name followed by a parenthesized list of its attributes. Thus, the schema for relation `Movies` of Fig. 2.3 is

```
Movies(title, year, length, genre)
```

The attributes in a relation schema are a set, not a list. However, in order to talk about relations we often must specify a "standard" order for the attributes. Thus, whenever we introduce a relation schema with a list of attributes, as above, we shall take this ordering to be the standard order whenever we display the relation or any of its rows.

In the relational model, a database consists of one or more relations. The set of schemas for the relations of a database is called a *relational database schema*, or just a *database schema*.

2.2.3 Tuples

The rows of a relation, other than the header row containing the attribute names, are called *tuples*. A tuple has one *component* for each attribute of the relation. For instance, the first of the three tuples in Fig. 2.3 has the four components `Gone With the Wind`, `1939`, `231`, and `drama` for attributes `title`, `year`, `length`, and `genre`, respectively. When we wish to write a tuple

Conventions for Relations and Attributes

We shall generally follow the convention that relation names begin with a capital letter, and attribute names begin with a lower-case letter. However, later in this book we shall talk of relations in the abstract, where the names of attributes do not matter. In that case, we shall use single capital letters for both relations and attributes, e.g., $R(A, B, C)$ for a generic relation with three attributes.

in isolation, not as part of a relation, we normally use commas to separate components, and we use parentheses to surround the tuple. For example,

(`Gone With the Wind`, 1939, 231, `drama`)

is the first tuple of Fig. 2.3. Notice that when a tuple appears in isolation, the attributes do not appear, so some indication of the relation to which the tuple belongs must be given. We shall always use the order in which the attributes were listed in the relation schema.

2.2.4 Domains

The relational model requires that each component of each tuple be atomic; that is, it must be of some elementary type such as integer or string. It is not permitted for a value to be a record structure, set, list, array, or any other type that reasonably can have its values broken into smaller components.

It is further assumed that associated with each attribute of a relation is a *domain*, that is, a particular elementary type. The components of any tuple of the relation must have, in each component, a value that belongs to the domain of the corresponding column. For example, tuples of the `Movies` relation of Fig. 2.3 must have a first component that is a string, second and third components that are integers, and a fourth component whose value is a string.

It is possible to include the domain, or data type, for each attribute in a relation schema. We shall do so by appending a colon and a type after attributes. For example, we could represent the schema for the `Movies` relation as:

```
Movies(title:string, year:integer, length:integer, genre:string)
```

2.2.5 Equivalent Representations of a Relation

Relations are sets of tuples, not lists of tuples. Thus the order in which the tuples of a relation are presented is immaterial. For example, we can list the three tuples of Fig. 2.3 in any of their six possible orders, and the relation is "the same" as Fig. 2.3.

Moreover, we can reorder the attributes of the relation as we choose, without changing the relation. However, when we reorder the relation schema, we must be careful to remember that the attributes are column headers. Thus, when we change the order of the attributes, we also change the order of their columns. When the columns move, the components of tuples change their order as well. The result is that each tuple has its components permuted in the same way as the attributes are permuted.

For example, Fig. 2.4 shows one of the many relations that could be obtained from Fig. 2.3 by permuting rows and columns. These two relations are considered "the same." More precisely, these two tables are different presentations of the same relation.

year	*genre*	*title*	*length*
1977	sciFi	Star Wars	124
1992	comedy	Wayne's World	95
1939	drama	Gone With the Wind	231

Figure 2.4: Another presentation of the relation Movies

2.2.6 Relation Instances

A relation about movies is not static; rather, relations change over time. We expect to insert tuples for new movies, as these appear. We also expect changes to existing tuples if we get revised or corrected information about a movie, and perhaps deletion of tuples for movies that are expelled from the database for some reason.

It is less common for the schema of a relation to change. However, there are situations where we might want to add or delete attributes. Schema changes, while possible in commercial database systems, can be very expensive, because each of perhaps millions of tuples needs to be rewritten to add or delete components. Also, if we add an attribute, it may be difficult or even impossible to generate appropriate values for the new component in the existing tuples.

We shall call a set of tuples for a given relation an *instance* of that relation. For example, the three tuples shown in Fig. 2.3 form an instance of relation Movies. Presumably, the relation Movies has changed over time and will continue to change over time. For instance, in 1990, Movies did not contain the tuple for Wayne's World. However, a conventional database system maintains only one version of any relation: the set of tuples that are in the relation "now." This instance of the relation is called the *current instance.*[1]

[1]Databases that maintain historical versions of data as it existed in past times are called *temporal databases.*

2.2.7 Keys of Relations

There are many constraints on relations that the relational model allows us to place on database schemas. We shall defer much of the discussion of constraints until Chapter 7. However, one kind of constraint is so fundamental that we shall introduce it here: *key* constraints. A set of attributes forms a *key* for a relation if we do not allow two tuples in a relation instance to have the same values in all the attributes of the key.

Example 2.1: We can declare that the relation `Movies` has a key consisting of the two attributes `title` and `year`. That is, we don't believe there could ever be two movies that had both the same title and the same year. Notice that `title` by itself does not form a key, since sometimes "remakes" of a movie appear. For example, there are three movies named *King Kong*, each made in a different year. It should also be obvious that `year` by itself is not a key, since there are usually many movies made in the same year. □

We indicate the attribute or attributes that form a key for a relation by underlining the key attribute(s). For instance, the `Movies` relation could have its schema written as:

`Movies(`<u>`title`</u>`, `<u>`year`</u>`, length, genre)`

Remember that the statement that a set of attributes forms a key for a relation is a statement about all possible instances of the relation, not a statement about a single instance. For example, looking only at the tiny relation of Fig. 2.3, we might imagine that `genre` by itself forms a key, since we do not see two tuples that agree on the value of their `genre` components. However, we can easily imagine that if the relation instance contained more movies, there would be many dramas, many comedies, and so on. Thus, there would be distinct tuples that agreed on the `genre` component. As a consequence, it would be incorrect to assert that `genre` is a key for the relation `Movies`.

While we might be sure that `title` and `year` can serve as a key for `Movies`, many real-world databases use artificial keys, doubting that it is safe to make any assumption about the values of attributes outside their control. For example, companies generally assign employee ID's to all employees, and these ID's are carefully chosen to be unique numbers. One purpose of these ID's is to make sure that in the company database each employee can be distinguished from all others, even if there are several employees with the same name. Thus, the employee-ID attribute can serve as a key for a relation about employees.

In US corporations, it is normal for every employee to have a Social-Security number. If the database has an attribute that is the Social-Security number, then this attribute can also serve as a key for employees. Note that there is nothing wrong with there being several choices of key, as there would be for employees having both employee ID's and Social-Security numbers.

The idea of creating an attribute whose purpose is to serve as a key is quite widespread. In addition to employee ID's, we find student ID's to distinguish

students in a university. We find drivers' license numbers and automobile registration numbers to distinguish drivers and automobiles, respectively. You undoubtedly can find more examples of attributes created for the primary purpose of serving as keys.

```
Movies(
    title:string,
    year:integer,
    length:integer,
    genre:string,
    studioName:string,
    producerC#:integer
)
MovieStar(
    name:string,
    address:string,
    gender:char,
    birthdate:date
)
StarsIn(
    movieTitle:string,
    movieYear:integer,
    starName:string
)
MovieExec(
    name:string,
    address:string,
    cert#:integer,
    netWorth:integer
)
Studio(
    name:string,
    address:string,
    presC#:integer
)
```

Figure 2.5: Example database schema about movies

2.2.8 An Example Database Schema

We shall close this section with an example of a complete database schema. The topic is movies, and it builds on the relation `Movies` that has appeared so far in examples. The database schema is shown in Fig. 2.5. Here are the things we need to know to understand the intention of this schema.

`Movies`

This relation is an extension of the example relation we have been discussing so far. Remember that its key is `title` and `year` together. We have added two new attributes; `studioName` tells us the studio that owns the movie, and `producerC#` is an integer that represents the producer of the movie in a way that we shall discuss when we talk about the relation `MovieExec` below.

`MovieStar`

This relation tells us something about stars. The key is `name`, the name of the movie star. It is not usual to assume names of persons are unique and therefore suitable as a key. However, movie stars are different; one would never take a name that some other movie star had used. Thus, we shall use the convenient fiction that movie-star names are unique. A more conventional approach would be to invent a serial number of some sort, like social-security numbers, so that we could assign each individual a unique number and use that attribute as the key. We take that approach for movie executives, as we shall see. Another interesting point about the `MovieStar` relation is that we see two new data types. The gender can be a single character, M or F. Also, birthdate is of type "date," which might be a character string of a special form.

`StarsIn`

This relation connects movies to the stars of that movie, and likewise connects a star to the movies in which they appeared. Notice that movies are represented by the key for `Movies` — the title and year — although we have chosen different attribute names to emphasize that attributes `movieTitle` and `movieYear` represent the movie. Likewise, stars are represented by the key for `MovieStar`, with the attribute called `starName`. Finally, notice that all three attributes are necessary to form a key. It is perfectly reasonable to suppose that relation `StarsIn` could have two distinct tuples that agree in any two of the three attributes. For instance, a star might appear in two movies in one year, giving rise to two tuples that agreed in `movieYear` and `starName`, but disagreed in `movieTitle`.

`MovieExec`

This relation tells us about movie executives. It contains their name, address, and networth as data about the executive. However, for a key we have invented "certificate numbers" for all movie executives, including producers (as appear in the relation `Movies`) and studio presidents (as appear in the relation `Studio`, below). These are integers; a different one is assigned to each executive.

acctNo	*type*	*balance*
12345	savings	12000
23456	checking	1000
34567	savings	25

The relation `Accounts`

firstName	*lastName*	*idNo*	*account*
Robbie	Banks	901-222	12345
Lena	Hand	805-333	12345
Lena	Hand	805-333	23456

The relation `Customers`

Figure 2.6: Two relations of a banking database

`Studio`

This relation tells about movie studios. We rely on no two studios having the same name, and therefore use `name` as the key. The other attributes are the address of the studio and the certificate number for the president of the studio. We assume that the studio president is surely a movie executive and therefore appears in `MovieExec`.

2.2.9 Exercises for Section 2.2

Exercise 2.2.1: In Fig. 2.6 are instances of two relations that might constitute part of a banking database. Indicate the following:

a) The attributes of each relation.

b) The tuples of each relation.

c) The components of one tuple from each relation.

d) The relation schema for each relation.

e) The database schema.

f) A suitable domain for each attribute.

g) Another equivalent way to present each relation.

Exercise 2.2.2: In Section 2.2.7 we suggested that there are many examples of attributes that are created for the purpose of serving as keys of relations. Give some additional examples.

!! **Exercise 2.2.3:** How many different ways (considering orders of tuples and attributes) are there to represent a relation instance if that instance has:

a) Three attributes and three tuples, like the relation `Accounts` of Fig. 2.6?

b) Four attributes and five tuples?

c) n attributes and m tuples?

2.3 Defining a Relation Schema in SQL

SQL (pronounced "sequel") is the principal language used to describe and manipulate relational databases. There is a current standard for SQL, called SQL-99. Most commercial database management systems implement something similar, but not identical to, the standard. There are two aspects to SQL:

1. The *Data-Definition* sublanguage for declaring database schemas and

2. The *Data-Manipulation* sublanguage for *querying* (asking questions about) databases and for modifying the database.

The distinction between these two sublanguages is found in most languages; e.g., C or Java have portions that declare data and other portions that are executable code. These correspond to data-definition and data-manipulation, respectively.

In this section we shall begin a discussion of the data-definition portion of SQL. There is more on the subject in Chapter 7, especially the matter of constraints on data. The data-manipulation portion is covered extensively in Chapter 6.

2.3.1 Relations in SQL

SQL makes a distinction between three kinds of relations:

1. Stored relations, which are called *tables*. These are the kind of relation we deal with ordinarily — a relation that exists in the database and that can be modified by changing its tuples, as well as queried.

2. *Views*, which are relations defined by a computation. These relations are not stored, but are constructed, in whole or in part, when needed. They are the subject of Section 8.1.

3. Temporary tables, which are constructed by the SQL language processor when it performs its job of executing queries and data modifications. These relations are then thrown away and not stored.

In this section, we shall learn how to declare tables. We do not treat the declaration and definition of views here, and temporary tables are never declared. The SQL `CREATE TABLE` statement declares the schema for a stored relation. It gives a name for the table, its attributes, and their data types. It also allows us to declare a key, or even several keys, for a relation. There are many other features to the `CREATE TABLE` statement, including many forms of constraints that can be declared, and the declaration of *indexes* (data structures that speed up many operations on the table) but we shall leave those for the appropriate time.

2.3.2 Data Types

To begin, let us introduce the primitive data types that are supported by SQL systems. All attributes must have a data type.

1. Character strings of fixed or varying length. The type `CHAR(n)` denotes a fixed-length string of up to n characters. `VARCHAR(n)` also denotes a string of up to n characters. The difference is implementation-dependent; typically `CHAR` implies that short strings are padded to make n characters, while `VARCHAR` implies that an endmarker or string-length is used. SQL permits reasonable coercions between values of character-string types. Normally, a string is padded by trailing blanks if it becomes the value of a component that is a fixed-length string of greater length. For example, the string `'foo'`,[2] if it became the value of a component for an attribute of type `CHAR(5)`, would assume the value `'foo '` (with two blanks following the second o).

2. Bit strings of fixed or varying length. These strings are analogous to fixed and varying-length character strings, but their values are strings of bits rather than characters. The type `BIT(n)` denotes bit strings of length n, while `BIT VARYING(n)` denotes bit strings of length up to n.

3. The type `BOOLEAN` denotes an attribute whose value is logical. The possible values of such an attribute are `TRUE`, `FALSE`, and — although it would surprise George Boole — `UNKNOWN`.

4. The type `INT` or `INTEGER` (these names are synonyms) denotes typical integer values. The type `SHORTINT` also denotes integers, but the number of bits permitted may be less, depending on the implementation (as with the types `int` and `short int` in C).

[2] Notice that in SQL, strings are surrounded by single-quotes, not double-quotes as in many other programming languages.

Dates and Times in SQL

Different SQL implementations may provide many different representations for dates and times, but the following is the SQL standard representation. A date value is the keyword `DATE` followed by a quoted string of a special form. For example, `DATE '1948-05-14'` follows the required form. The first four characters are digits representing the year. Then come a hyphen and two digits representing the month. Finally there is another hyphen and two digits representing the day. Note that single-digit months and days are padded with a leading 0.

A time value is the keyword `TIME` and a quoted string. This string has two digits for the hour, on the military (24-hour) clock. Then come a colon, two digits for the minute, another colon, and two digits for the second. If fractions of a second are desired, we may continue with a decimal point and as many significant digits as we like. For instance, `TIME '15:00:02.5'` represents the time at which all students will have left a class that ends at 3 PM: two and a half seconds past three o'clock.

5. Floating-point numbers can be represented in a variety of ways. We may use the type `FLOAT` or `REAL` (these are synonyms) for typical floating-point numbers. A higher precision can be obtained with the type `DOUBLE PRECISION`; again the distinction between these types is as in C. SQL also has types that are real numbers with a fixed decimal point. For example, `DECIMAL(n,d)` allows values that consist of n decimal digits, with the decimal point assumed to be d positions from the right. Thus, 0123.45 is a possible value of type `DECIMAL(6,2)`. `NUMERIC` is almost a synonym for `DECIMAL`, although there are possible implementation-dependent differences.

6. Dates and times can be represented by the data types `DATE` and `TIME`, respectively (see the box on "Dates and Times in SQL"). These values are essentially character strings of a special form. We may, in fact, coerce dates and times to string types, and we may do the reverse if the string "makes sense" as a date or time.

2.3.3 Simple Table Declarations

The simplest form of declaration of a relation schema consists of the keywords `CREATE TABLE` followed by the name of the relation and a parenthesized, comma-separated list of the attribute names and their types.

Example 2.2: The relation `Movies` with the schema given in Fig. 2.5 can be declared as in Fig. 2.7. The title is declared as a string of (up to) 100 characters.

```
CREATE TABLE Movies (
    title       CHAR(100),
    year        INT,
    length      INT,
    genre       CHAR(10),
    studioName  CHAR(30),
    producerC#  INT
);
```

Figure 2.7: SQL declaration of the table `Movies`

The year and length attributes are each integers, and the genre is a string of (up to) 10 characters. The decision to allow up to 100 characters for a title is arbitrary, but we don't want to limit the lengths of titles too strongly, or long titles would be truncated to fit. We have assumed that 10 characters are enough to represent a genre of movie; again, that is an arbitrary choice, one we could regret if we had a genre with a long name. Likewise, we have chosen 30 characters as sufficient for the studio name. The certificate number for the producer of the movie is another integer. □

Example 2.3: Figure 2.8 is a SQL declaration of the relation `MovieStar` from Fig. 2.5. It illustrates some new options for data types. The name of this table is `MovieStar`, and it has four attributes. The first two attributes, `name` and `address`, have each been declared to be character strings. However, with the name, we have made the decision to use a fixed-length string of 30 characters, padding a name out with blanks at the end if necessary and truncating a name to 30 characters if it is longer. In contrast, we have declared addresses to be variable-length character strings of up to 255 characters.[3] It is not clear that these two choices are the best possible, but we use them to illustrate the two major kinds of string data types.

```
CREATE TABLE MovieStar (
    name      CHAR(30),
    address   VARCHAR(255),
    gender    CHAR(1),
    birthdate DATE
);
```

Figure 2.8: Declaring the relation schema for the `MovieStar` relation

[3]The number 255 is not the result of some weird notion of what typical addresses look like. A single byte can store integers between 0 and 255, so it is possible to represent a varying-length character string of up to 255 bytes by a single byte for the count of characters plus the bytes to store the string itself. Commercial systems generally support longer varying-length strings, however.

The gender attribute has values that are a single letter, M or F. Thus, we can safely use a single character as the type of this attribute. Finally, the birthdate attribute naturally deserves the data type DATE. □

2.3.4 Modifying Relation Schemas

We now know how to declare a table. But what if we need to change the schema of the table after it has been in use for a long time and has many tuples in its current instance? We can remove the entire table, including all of its current tuples, or we could change the schema by adding or deleting attributes.

We can delete a relation R by the SQL statement:

```
DROP TABLE R;
```

Relation R is no longer part of the database schema, and we can no longer access any of its tuples.

More frequently than we would drop a relation that is part of a long-lived database, we may need to modify the schema of an existing relation. These modifications are done by a statement that begins with the keywords ALTER TABLE and the name of the relation. We then have several options, the most important of which are

1. ADD followed by an attribute name and its data type.

2. DROP followed by an attribute name.

Example 2.4: Thus, for instance, we could modify the MovieStar relation by adding an attribute phone with:

```
ALTER TABLE MovieStar ADD phone CHAR(16);
```

As a result, the MovieStar schema now has five attributes: the four mentioned in Fig. 2.8 and the attribute phone, which is a fixed-length string of 16 bytes. In the actual relation, tuples would all have components for phone, but we know of no phone numbers to put there. Thus, the value of each of these components is set to the special *null value*, NULL. In Section 2.3.5, we shall see how it is possible to choose another "default" value to be used instead of NULL for unknown values.

As another example, the ALTER TABLE statement:

```
ALTER TABLE MovieStar DROP birthdate;
```

deletes the birthdate attribute. As a result, the schema for MovieStar no longer has that attribute, and all tuples of the current MovieStar instance have the component for birthdate deleted. □

2.3.5 Default Values

When we create or modify tuples, we sometimes do not have values for all components. For instance, we mentioned in Example 2.4 that when we add a column to a relation schema, the existing tuples do not have a known value, and it was suggested that NULL could be used in place of a "real" value. However, there are times when we would prefer to use another choice of *default* value, the value that appears in a column if no other value is known.

In general, any place we declare an attribute and its data type, we may add the keyword DEFAULT and an appropriate value. That value is either NULL or a constant. Certain other values that are provided by the system, such as the current time, may also be options.

Example 2.5: Let us consider Example 2.3. We might wish to use the character ? as the default for an unknown gender, and we might also wish to use the earliest possible date, DATE '0000-00-00' for an unknown birthdate. We could replace the declarations of gender and birthdate in Fig. 2.8 by:

```
gender CHAR(1) DEFAULT '?',
birthdate DATE DEFAULT DATE '0000-00-00'
```

As another example, we could have declared the default value for new attribute phone to be 'unlisted' when we added this attribute in Example 2.4. In that case,

```
ALTER TABLE MovieStar ADD phone CHAR(16) DEFAULT 'unlisted';
```

would be the appropriate ALTER TABLE statement. □

2.3.6 Declaring Keys

There are two ways to declare an attribute or set of attributes to be a key in the CREATE TABLE statement that defines a stored relation.

1. We may declare one attribute to be a key when that attribute is listed in the relation schema.
2. We may add to the list of items declared in the schema (which so far have only been attributes) an additional declaration that says a particular attribute or set of attributes forms the key.

If the key consists of more than one attribute, we have to use method (2). If the key is a single attribute, either method may be used.

There are two declarations that may be used to indicate keyness:

a) PRIMARY KEY, or

b) UNIQUE.

The effect of declaring a set of attributes S to be a key for relation R either using PRIMARY KEY or UNIQUE is the following:

- Two tuples in R cannot agree on all of the attributes in set S, unless one of them is NULL. Any attempt to insert or update a tuple that violates this rule causes the DBMS to reject the action that caused the violation.

In addition, if PRIMARY KEY is used, then attributes in S are not allowed to have NULL as a value for their components. Again, any attempt to violate this rule is rejected by the system. NULL is permitted if the set S is declared UNIQUE, however. A DBMS may make other distinctions between the two terms, if it wishes.

Example 2.6: Let us reconsider the schema for relation MovieStar. Since no star would use the name of another star, we shall assume that name by itself forms a key for this relation. Thus, we can add this fact to the line declaring name. Figure 2.9 is a revision of Fig. 2.8 that reflects this change. We could also substitute UNIQUE for PRIMARY KEY in this declaration. If we did so, then two or more tuples could have NULL as the value of name, but there could be no other duplicate values for this attribute.

```
CREATE TABLE MovieStar (
   name CHAR(30) PRIMARY KEY,
   address VARCHAR(255),
   gender CHAR(1),
   birthdate DATE
);
```

Figure 2.9: Making name the key

Alternatively, we can use a separate definition of the key. The resulting schema declaration would look like Fig. 2.10. Again, UNIQUE could replace PRIMARY KEY. □

```
CREATE TABLE MovieStar (
   name CHAR(30),
   address VARCHAR(255),
   gender CHAR(1),
   birthdate DATE,
   PRIMARY KEY (name)
);
```

Figure 2.10: A separate declaration of the key

Example 2.7: In Example 2.6, the form of either Fig. 2.9 or Fig. 2.10 is acceptable, because the key is a single attribute. However, in a situation where the key has more than one attribute, we must use the style of Fig. 2.10. For instance, the relation `Movie`, whose key is the pair of attributes `title` and `year`, must be declared as in Fig. 2.11. However, as usual, `UNIQUE` is an option to replace `PRIMARY KEY`. □

```
CREATE TABLE Movies (
    title       CHAR(100),
    year        INT,
    length      INT,
    genre       CHAR(10),
    studioName  CHAR(30),
    producerC#  INT,
    PRIMARY KEY (title, year)
);
```

Figure 2.11: Making `title` and `year` be the key of `Movies`

2.3.7 Exercises for Section 2.3

Exercise 2.3.1: In this exercise we introduce one of our running examples of a relational database schema. The database schema consists of four relations, whose schemas are:

```
Product(maker, model, type)
PC(model, speed, ram, hd, price)
Laptop(model, speed, ram, hd, screen, price)
Printer(model, color, type, price)
```

The `Product` relation gives the manufacturer, model number and type (PC, laptop, or printer) of various products. We assume for convenience that model numbers are unique over all manufacturers and product types; that assumption is not realistic, and a real database would include a code for the manufacturer as part of the model number. The `PC` relation gives for each model number that is a PC the speed (of the processor, in gigahertz), the amount of RAM (in megabytes), the size of the hard disk (in gigabytes), and the price. The `Laptop` relation is similar, except that the screen size (in inches) is also included. The `Printer` relation records for each printer model whether the printer produces color output (true, if so), the process type (laser or ink-jet, typically), and the price.

Write the following declarations:

a) A suitable schema for relation `Product`.

b) A suitable schema for relation PC.

c) A suitable schema for relation Laptop.

d) A suitable schema for relation Printer.

e) An alteration to your Printer schema from (d) to delete the attribute color.

f) An alteration to your Laptop schema from (c) to add the attribute od (optical-disk type, e.g., cd or dvd). Let the default value for this attribute be 'none' if the laptop does not have an optical disk.

Exercise 2.3.2: This exercise introduces another running example, concerning World War II capital ships. It involves the following relations:

```
Classes(class, type, country, numGuns, bore, displacement)
Ships(name, class, launched)
Battles(name, date)
Outcomes(ship, battle, result)
```

Ships are built in "classes" from the same design, and the class is usually named for the first ship of that class. The relation Classes records the name of the class, the type ('bb' for battleship or 'bc' for battlecruiser), the country that built the ship, the number of main guns, the bore (diameter of the gun barrel, in inches) of the main guns, and the displacement (weight, in tons). Relation Ships records the name of the ship, the name of its class, and the year in which the ship was launched. Relation Battles gives the name and date of battles involving these ships, and relation Outcomes gives the result (sunk, damaged, or ok) for each ship in each battle.

Write the following declarations:

a) A suitable schema for relation Classes.

b) A suitable schema for relation Ships.

c) A suitable schema for relation Battles.

d) A suitable schema for relation Outcomes.

e) An alteration to your Classes relation from (a) to delete the attribute bore.

f) An alteration to your Ships relation from (b) to include the attribute yard giving the shipyard where the ship was built.

2.4 An Algebraic Query Language

In this section, we introduce the data-manipulation aspect of the relational model. Recall that a data model is not just structure; it needs a way to query the data and to modify the data. To begin our study of operations on relations, we shall learn about a special algebra, called *relational algebra*, that consists of some simple but powerful ways to construct new relations from given relations. When the given relations are stored data, then the constructed relations can be answers to queries about this data.

Relational algebra is not used today as a query language in commercial DBMS's, although some of the early prototypes did use this algebra directly. Rather, the "real" query language, SQL, incorporates relational algebra at its center, and many SQL programs are really "syntactically sugared" expressions of relational algebra. Further, when a DBMS processes queries, the first thing that happens to a SQL query is that it gets translated into relational algebra or a very similar internal representation. Thus, there are several good reasons to start out learning this algebra.

2.4.1 Why Do We Need a Special Query Language?

Before introducing the operations of relational algebra, one should ask why, or whether, we need a new kind of programming languages for databases. Won't conventional languages like C or Java suffice to ask and answer any computable question about relations? After all, we can represent a tuple of a relation by a struct (in C) or an object (in Java), and we can represent relations by arrays of these elements.

The surprising answer is that relational algebra is useful because it is *less* powerful than C or Java. That is, there are computations one can perform in any conventional language that one cannot perform in relational algebra. An example is: determine whether the number of tuples in a relation is even or odd. By limiting what we can say or do in our query language, we get two huge rewards — ease of programming and the ability of the compiler to produce highly optimized code — that we discussed in Section 2.1.6.

2.4.2 What is an Algebra?

An algebra, in general, consists of operators and atomic operands. For instance, in the algebra of arithmetic, the atomic operands are variables like x and constants like 15. The operators are the usual arithmetic ones: addition, subtraction, multiplication, and division. Any algebra allows us to build *expressions* by applying operators to atomic operands and/or other expressions of the algebra. Usually, parentheses are needed to group operators and their operands. For instance, in arithmetic we have expressions such as $(x + y) * z$ or $\big((x + 7)/(y - 3)\big) + x$.

Relational algebra is another example of an algebra. Its atomic operands are:

1. Variables that stand for relations.
2. Constants, which are finite relations.

We shall next see the operators of relational algebra.

2.4.3 Overview of Relational Algebra

The operations of the traditional relational algebra fall into four broad classes:

a) The usual set operations — union, intersection, and difference — applied to relations.

b) Operations that remove parts of a relation: "selection" eliminates some rows (tuples), and "projection" eliminates some columns.

c) Operations that combine the tuples of two relations, including "Cartesian product," which pairs the tuples of two relations in all possible ways, and various kinds of "join" operations, which selectively pair tuples from two relations.

d) An operation called "renaming" that does not affect the tuples of a relation, but changes the relation schema, i.e., the names of the attributes and/or the name of the relation itself.

We generally shall refer to expressions of relational algebra as *queries.*

2.4.4 Set Operations on Relations

The three most common operations on sets are union, intersection, and difference. We assume the reader is familiar with these operations, which are defined as follows on arbitrary sets R and S:

- $R \cup S$, the *union* of R and S, is the set of elements that are in R or S or both. An element appears only once in the union even if it is present in both R and S.
- $R \cap S$, the *intersection* of R and S, is the set of elements that are in both R and S.
- $R - S$, the *difference* of R and S, is the set of elements that are in R but not in S. Note that $R - S$ is different from $S - R$; the latter is the set of elements that are in S but not in R.

When we apply these operations to relations, we need to put some conditions on R and S:

1. R and S must have schemas with identical sets of attributes, and the types (domains) for each attribute must be the same in R and S.

2. Before we compute the set-theoretic union, intersection, or difference of sets of tuples, the columns of R and S must be ordered so that the order of attributes is the same for both relations.

Sometimes we would like to take the union, intersection, or difference of relations that have the same number of attributes, with corresponding domains, but that use different names for their attributes. If so, we may use the renaming operator to be discussed in Section 2.4.11 to change the schema of one or both relations and give them the same set of attributes.

name	*address*	*gender*	*birthdate*
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
Mark Hamill	456 Oak Rd., Brentwood	M	8/8/88

Relation R

name	*address*	*gender*	*birthdate*
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
Harrison Ford	789 Palm Dr., Beverly Hills	M	7/7/77

Relation S

Figure 2.12: Two relations

Example 2.8: Suppose we have the two relations R and S, whose schemas are both that of relation MovieStar Section 2.2.8. Current instances of R and S are shown in Fig. 2.12. Then the union $R \cup S$ is

name	*address*	*gender*	*birthdate*
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99
Mark Hamill	456 Oak Rd., Brentwood	M	8/8/88
Harrison Ford	789 Palm Dr., Beverly Hills	M	7/7/77

Note that the two tuples for Carrie Fisher from the two relations appear only once in the result.

The intersection $R \cap S$ is

name	*address*	*gender*	*birthdate*
Carrie Fisher	123 Maple St., Hollywood	F	9/9/99

Now, only the Carrie Fisher tuple appears, because only it is in both relations.

The difference $R - S$ is

name	*address*	*gender*	*birthdate*
Mark Hamill	456 Oak Rd., Brentwood	M	8/8/88

That is, the Fisher and Hamill tuples appear in R and thus are candidates for $R - S$. However, the Fisher tuple also appears in S and so is not in $R - S$. □

2.4.5 Projection

The *projection* operator is used to produce from a relation R a new relation that has only some of R's columns. The value of expression $\pi_{A_1,A_2,\ldots,A_n}(R)$ is a relation that has only the columns for attributes $A_1, A_2, \ldots, A_n$ of R. The schema for the resulting value is the set of attributes $\{A_1, A_2, \ldots, A_n\}$, which we conventionally show in the order listed.

title	*year*	*length*	*genre*	*studioName*	*producerC#*
Star Wars	1977	124	sciFi	Fox	12345
Galaxy Quest	1999	104	comedy	DreamWorks	67890
Wayne's World	1992	95	comedy	Paramount	99999

Figure 2.13: The relation Movies

Example 2.9: Consider the relation Movies with the relation schema described in Section 2.2.8. An instance of this relation is shown in Fig. 2.13. We can project this relation onto the first three attributes with the expression:

$$\pi_{title,year,length}(\texttt{Movies})$$

The resulting relation is

title	*year*	*length*
Star Wars	1977	124
Galaxy Quest	1999	104
Wayne's World	1992	95

As another example, we can project onto the attribute genre with the expression $\pi_{genre}(\texttt{Movies})$. The result is the single-column relation

genre
sciFi
comedy

Notice that there are only two tuples in the resulting relation, since the last two tuples of Fig. 2.13 have the same value in their component for attribute genre, and in the relational algebra of sets, duplicate tuples are always eliminated. □

A Note About Data Quality :-)

While we have endeavored to make example data as accurate as possible, we have used bogus values for addresses and other personal information about movie stars, in order to protect the privacy of members of the acting profession, many of whom are shy individuals who shun publicity.

2.4.6 Selection

The *selection* operator, applied to a relation R, produces a new relation with a subset of R's tuples. The tuples in the resulting relation are those that satisfy some condition C that involves the attributes of R. We denote this operation $\sigma_C(R)$. The schema for the resulting relation is the same as R's schema, and we conventionally show the attributes in the same order as we use for R.

C is a conditional expression of the type with which we are familiar from conventional programming languages; for example, conditional expressions follow the keyword `if` in programming languages such as C or Java. The only difference is that the operands in condition C are either constants or attributes of R. We apply C to each tuple t of R by substituting, for each attribute A appearing in condition C, the component of t for attribute A. If after substituting for each attribute of C the condition C is true, then t is one of the tuples that appear in the result of $\sigma_C(R)$; otherwise t is not in the result.

Example 2.10: Let the relation `Movies` be as in Fig. 2.13. Then the value of expression $\sigma_{length \geq 100}(\texttt{Movies})$ is

title	*year*	*length*	*genre*	*studioName*	*producerC#*
Star Wars	1977	124	sciFi	Fox	12345
Galaxy Quest	1999	104	comedy	DreamWorks	67890

The first tuple satisfies the condition $length \geq 100$ because when we substitute for *length* the value 124 found in the component of the first tuple for attribute `length`, the condition becomes $124 \geq 100$. The latter condition is true, so we accept the first tuple. The same argument explains why the second tuple of Fig. 2.13 is in the result.

The third tuple has a `length` component 95. Thus, when we substitute for *length* we get the condition $95 \geq 100$, which is false. Hence the last tuple of Fig. 2.13 is not in the result. □

Example 2.11: Suppose we want the set of tuples in the relation `Movies` that represent Fox movies at least 100 minutes long. We can get these tuples with a more complicated condition, involving the `AND` of two subconditions. The expression is

$$\sigma_{length \geq 100 \texttt{ AND } studioName = \texttt{'Fox'}}(\texttt{Movies})$$

The tuple

title	*year*	*length*	*genre*	*studioName*	*producerC#*
Star Wars	1977	124	sciFi	Fox	12345

is the only one in the resulting relation. □

2.4.7 Cartesian Product

The *Cartesian product* (or *cross-product*, or just *product*) of two sets R and S is the set of pairs that can be formed by choosing the first element of the pair to be any element of R and the second any element of S. This product is denoted $R \times S$. When R and S are relations, the product is essentially the same. However, since the members of R and S are tuples, usually consisting of more than one component, the result of pairing a tuple from R with a tuple from S is a longer tuple, with one component for each of the components of the constituent tuples. By convention, the components from R (the left operand) precede the components from S in the attribute order for the result.

The relation schema for the resulting relation is the union of the schemas for R and S. However, if R and S should happen to have some attributes in common, then we need to invent new names for at least one of each pair of identical attributes. To disambiguate an attribute A that is in the schemas of both R and S, we use $R.A$ for the attribute from R and $S.A$ for the attribute from S.

Example 2.12: For conciseness, let us use an abstract example that illustrates the product operation. Let relations R and S have the schemas and tuples shown in Fig. 2.14(a) and (b). Then the product $R \times S$ consists of the six tuples shown in Fig. 2.14(c). Note how we have paired each of the two tuples of R with each of the three tuples of S. Since B is an attribute of both schemas, we have used $R.B$ and $S.B$ in the schema for $R \times S$. The other attributes are unambiguous, and their names appear in the resulting schema unchanged. □

2.4.8 Natural Joins

More often than we want to take the product of two relations, we find a need to *join* them by pairing only those tuples that match in some way. The simplest sort of match is the *natural join* of two relations R and S, denoted $R \bowtie S$, in which we pair only those tuples from R and S that agree in whatever attributes are common to the schemas of R and S. More precisely, let $A_1, A_2, \ldots, A_n$ be all the attributes that are in both the schema of R and the schema of S. Then a tuple r from R and a tuple s from S are successfully paired if and only if r and s agree on each of the attributes $A_1, A_2, \ldots, A_n$.

If the tuples r and s are successfully paired in the join $R \bowtie S$, then the result of the pairing is a tuple, called the *joined tuple*, with one component for each of the attributes in the union of the schemas of R and S. The joined tuple

A	B
1	2
3	4

(a) Relation R

B	C	D
2	5	6
4	7	8
9	10	11

(b) Relation S

A	$R.B$	$S.B$	C	D
1	2	2	5	6
1	2	4	7	8
1	2	9	10	11
3	4	2	5	6
3	4	4	7	8
3	4	9	10	11

(c) Result $R \times S$

Figure 2.14: Two relations and their Cartesian product

agrees with tuple r in each attribute in the schema of R, and it agrees with s in each attribute in the schema of S. Since r and s are successfully paired, the joined tuple is able to agree with both these tuples on the attributes they have in common. The construction of the joined tuple is suggested by Fig. 2.15. However, the order of the attributes need not be that convenient; the attributes of R and S can appear in any order.

Example 2.13: The natural join of the relations R and S from Fig. 2.14(a) and (b) is

A	B	C	D
1	2	5	6
3	4	7	8

The only attribute common to R and S is B. Thus, to pair successfully, tuples need only to agree in their B components. If so, the resulting tuple has components for attributes A (from R), B (from either R or S), C (from S), and D (from S).

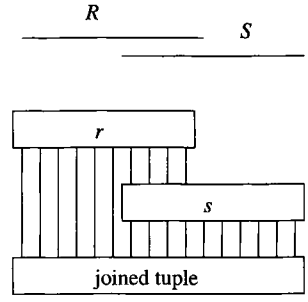

Figure 2.15: Joining tuples

In this example, the first tuple of R successfully pairs with only the first tuple of S; they share the value 2 on their common attribute B. This pairing yields the first tuple of the result: $(1, 2, 5, 6)$. The second tuple of R pairs successfully only with the second tuple of S, and the pairing yields $(3, 4, 7, 8)$. Note that the third tuple of S does not pair with any tuple of R and thus has no effect on the result of $R \bowtie S$. A tuple that fails to pair with any tuple of the other relation in a join is said to be a *dangling tuple.* □

Example 2.14: The previous example does not illustrate all the possibilities inherent in the natural join operator. For example, no tuple paired successfully with more than one tuple, and there was only one attribute in common to the two relation schemas. In Fig. 2.16 we see two other relations, U and V, that share two attributes between their schemas: B and C. We also show an instance in which one tuple joins with several tuples.

For tuples to pair successfully, they must agree in both the B and C components. Thus, the first tuple of U joins with the first two tuples of V, while the second and third tuples of U join with the third tuple of V. The result of these four pairings is shown in Fig. 2.16(c). □

2.4.9 Theta-Joins

The natural join forces us to pair tuples using one specific condition. While this way, equating shared attributes, is the most common basis on which relations are joined, it is sometimes desirable to pair tuples from two relations on some other basis. For that purpose, we have a related notation called the *theta-join.* Historically, the "theta" refers to an arbitrary condition, which we shall represent by C rather than θ.

The notation for a theta-join of relations R and S based on condition C is $R \bowtie_C S$. The result of this operation is constructed as follows:

1. Take the product of R and S.

2. Select from the product only those tuples that satisfy the condition C.

A	B	C
1	2	3
6	7	8
9	7	8

(a) Relation U

B	C	D
2	3	4
2	3	5
7	8	10

(b) Relation V

A	B	C	D
1	2	3	4
1	2	3	5
6	7	8	10
9	7	8	10

(c) Result $U \bowtie V$

Figure 2.16: Natural join of relations

As with the product operation, the schema for the result is the union of the schemas of R and S, with "R." or "S." prefixed to attributes if necessary to indicate from which schema the attribute came.

Example 2.15: Consider the operation $U \bowtie_{A<D} V$, where U and V are the relations from Fig. 2.16(a) and (b). We must consider all nine pairs of tuples, one from each relation, and see whether the A component from the U-tuple is less than the D component of the V-tuple. The first tuple of U, with an A component of 1, successfully pairs with each of the tuples from V. However, the second and third tuples from U, with A components of 6 and 9, respectively, pair successfully with only the last tuple of V. Thus, the result has only five tuples, constructed from the five successful pairings. This relation is shown in Fig. 2.17. □

Notice that the schema for the result in Fig. 2.17 consists of all six attributes, with U and V prefixed to their respective occurrences of attributes B and C to distinguish them. Thus, the theta-join contrasts with natural join, since in the latter common attributes are merged into one copy. Of course it makes sense to

A	$U.B$	$U.C$	$V.B$	$V.C$	D
1	2	3	2	3	4
1	2	3	2	3	5
1	2	3	7	8	10
6	7	8	7	8	10
9	7	8	7	8	10

Figure 2.17: Result of $U \bowtie_{A<D} V$

do so in the case of the natural join, since tuples don't pair unless they agree in their common attributes. In the case of a theta-join, there is no guarantee that compared attributes will agree in the result, since they may not be compared with $=$.

Example 2.16: Here is a theta-join on the same relations U and V that has a more complex condition:

$$U \bowtie_{A<D \text{ AND } U.B \neq V.B} V$$

That is, we require for successful pairing not only that the A component of the U-tuple be less than the D component of the V-tuple, but that the two tuples disagree on their respective B components. The tuple

A	$U.B$	$U.C$	$V.B$	$V.C$	D
1	2	3	7	8	10

is the only one to satisfy both conditions, so this relation is the result of the theta-join above. □

2.4.10 Combining Operations to Form Queries

If all we could do was to write single operations on one or two relations as queries, then relational algebra would not be nearly as useful as it is. However, relational algebra, like all algebras, allows us to form expressions of arbitrary complexity by applying operations to the result of other operations.

One can construct expressions of relational algebra by applying operators to subexpressions, using parentheses when necessary to indicate grouping of operands. It is also possible to represent expressions as expression trees; the latter often are easier for us to read, although they are less convenient as a machine-readable notation.

Example 2.17: Suppose we want to know, from our running `Movies` relation, "What are the titles and years of movies made by Fox that are at least 100 minutes long?" One way to compute the answer to this query is:

1. Select those `Movies` tuples that have $length \geq 100$.

2. Select those `Movies` tuples that have *studioName* = `'Fox'`.

3. Compute the intersection of (1) and (2).

4. Project the relation from (3) onto attributes `title` and `year`.

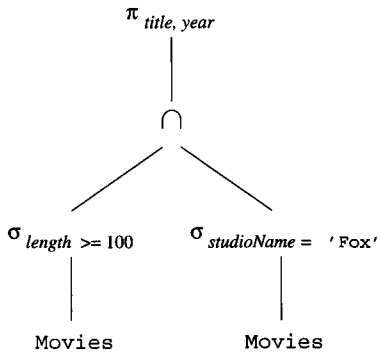

Figure 2.18: Expression tree for a relational algebra expression

In Fig. 2.18 we see the above steps represented as an expression tree. Expression trees are evaluated bottom-up by applying the operator at an interior node to the arguments, which are the results of its children. By proceeding bottom-up, we know that the arguments will be available when we need them. The two selection nodes correspond to steps (1) and (2). The intersection node corresponds to step (3), and the projection node is step (4).

Alternatively, we could represent the same expression in a conventional, linear notation, with parentheses. The formula

$$\pi_{title,year}\Big(\sigma_{length\geq 100}(\texttt{Movies}) \cap \sigma_{studioName=\texttt{'Fox'}}(\texttt{Movies})\Big)$$

represents the same expression.

Incidentally, there is often more than one relational algebra expression that represents the same computation. For instance, the above query could also be written by replacing the intersection by logical `AND` within a single selection operation. That is,

$$\pi_{title,year}\Big(\sigma_{length\geq 100 \texttt{ AND } studioName=\texttt{'Fox'}}(\texttt{Movies})\Big)$$

is an equivalent form of the query. □

Equivalent Expressions and Query Optimization

All database systems have a query-answering system, and many of them are based on a language that is similar in expressive power to relational algebra. Thus, the query asked by a user may have many *equivalent expressions* (expressions that produce the same answer whenever they are given the same relations as operands), and some of these may be much more quickly evaluated. An important job of the query "optimizer" discussed briefly in Section 1.2.5 is to replace one expression of relational algebra by an equivalent expression that is more efficiently evaluated.

2.4.11 Naming and Renaming

In order to control the names of the attributes used for relations that are constructed by applying relational-algebra operations, it is often convenient to use an operator that explicitly renames relations. We shall use the operator $\rho_{S(A_1,A_2,\ldots,A_n)}(R)$ to rename a relation R. The resulting relation has exactly the same tuples as R, but the name of the relation is S. Moreover, the attributes of the result relation S are named $A_1, A_2, \ldots, A_n$, in order from the left. If we only want to change the name of the relation to S and leave the attributes as they are in R, we can just say $\rho_S(R)$.

Example 2.18: In Example 2.12 we took the product of two relations R and S from Fig. 2.14(a) and (b) and used the convention that when an attribute appears in both operands, it is renamed by prefixing the relation name to it. Suppose, however, that we do not wish to call the two versions of B by names $R.B$ and $S.B$; rather we want to continue to use the name B for the attribute that comes from R, and we want to use X as the name of the attribute B coming from S. We can rename the attributes of S so the first is called X. The result of the expression $\rho_{S(X,C,D)}(S)$ is a relation named S that looks just like the relation S from Fig. 2.14, but its first column has attribute X instead of B.

A	B	X	C	D
1	2	2	5	6
1	2	4	7	8
1	2	9	10	11
3	4	2	5	6
3	4	4	7	8
3	4	9	10	11

Figure 2.19: $R \times \rho_{S(X,C,D)}(S)$

When we take the product of R with this new relation, there is no conflict of names among the attributes, so no further renaming is done. That is, the result of the expression $R \times \rho_{S(X,C,D)}(S)$ is the relation $R \times S$ from Fig. 2.14(c), except that the five columns are labeled A, B, X, C, and D, from the left. This relation is shown in Fig. 2.19.

As an alternative, we could take the product without renaming, as we did in Example 2.12, and then rename the result. The expression

$$\rho_{RS(A,B,X,C,D)}(R \times S)$$

yields the same relation as in Fig. 2.19, with the same set of attributes. But this relation has a name, RS, while the result relation in Fig. 2.19 has no name. □

2.4.12 Relationships Among Operations

Some of the operations that we have described in Section 2.4 can be expressed in terms of other relational-algebra operations. For example, intersection can be expressed in terms of set difference:

$$R \cap S = R - (R - S)$$

That is, if R and S are any two relations with the same schema, the intersection of R and S can be computed by first subtracting S from R to form a relation T consisting of all those tuples in R but not S. We then subtract T from R, leaving only those tuples of R that are also in S.

The two forms of join are also expressible in terms of other operations. Theta-join can be expressed by product and selection:

$$R \bowtie_C S = \sigma_C(R \times S)$$

The natural join of R and S can be expressed by starting with the product $R \times S$. We then apply the selection operator with a condition C of the form

$$R.A_1 = S.A_1 \text{ AND } R.A_2 = S.A_2 \text{ AND} \cdots \text{AND } R.A_n = S.A_n$$

where $A_1, A_2, \ldots, A_n$ are all the attributes appearing in the schemas of both R and S. Finally, we must project out one copy of each of the equated attributes. Let L be the list of attributes in the schema of R followed by those attributes in the schema of S that are not also in the schema of R. Then

$$R \bowtie S = \pi_L\Big(\sigma_C(R \times S)\Big)$$

Example 2.19: The natural join of the relations U and V from Fig. 2.16 can be written in terms of product, selection, and projection as:

$$\pi_{A,U.B,U.C,D}\Big(\sigma_{U.B=V.B \text{ AND } U.C=V.C}(U \times V)\Big)$$

That is, we take the product $U \times V$. Then we select for equality between each pair of attributes with the same name — B and C in this example. Finally, we project onto all the attributes except one of the B's and one of the C's; we have chosen to eliminate the attributes of V whose names also appear in the schema of U.

For another example, the theta-join of Example 2.16 can be written

$$\sigma_{A<D \text{ AND } U.B \neq V.B}(U \times V)$$

That is, we take the product of the relations U and V and then apply the condition that appeared in the theta-join. □

The rewriting rules mentioned in this section are the only "redundancies" among the operations that we have introduced. The six remaining operations — union, difference, selection, projection, product, and renaming — form an independent set, none of which can be written in terms of the other five.

2.4.13 A Linear Notation for Algebraic Expressions

In Section 2.4.10 we used an expression tree to represent a complex expression of relational algebra. An alternative is to invent names for the temporary relations that correspond to the interior nodes of the tree and write a sequence of assignments that create a value for each. The order of the assignments is flexible, as long as the children of a node N have had their values created before we attempt to create the value for N itself.

The notation we shall use for assignment statements is:

1. A relation name and parenthesized list of attributes for that relation. The name `Answer` will be used conventionally for the result of the final step; i.e., the name of the relation at the root of the expression tree.

2. The assignment symbol `:=`.

3. Any algebraic expression on the right. We can choose to use only one operator per assignment, in which case each interior node of the tree gets its own assignment statement. However, it is also permissible to combine several algebraic operations in one right side, if it is convenient to do so.

Example 2.20: Consider the tree of Fig. 2.18. One possible sequence of assignments to evaluate this expression is:

$$\begin{aligned}
&\texttt{R(t,y,l,i,s,p)} := \sigma_{length \geq 100}(\texttt{Movies}) \\
&\texttt{S(t,y,l,i,s,p)} := \sigma_{studioName=\text{'Fox'}}(\texttt{Movies}) \\
&\texttt{T(t,y,l,i,s,p)} := \texttt{R} \cap \texttt{S} \\
&\texttt{Answer(title, year)} := \pi_{t,y}(\texttt{T})
\end{aligned}$$

The first step computes the relation of the interior node labeled $\sigma_{length \geq 100}$ in Fig. 2.18, and the second step computes the node labeled $\sigma_{studioName='Fox'}$. Notice that we get renaming "for free," since we can use any attributes and relation name we wish for the left side of an assignment. The last two steps compute the intersection and the projection in the obvious way.

It is also permissible to combine some of the steps. For instance, we could combine the last two steps and write:

$$\texttt{R(t,y,l,i,s,p)} := \sigma_{length \geq 100}(\texttt{Movies})$$
$$\texttt{S(t,y,l,i,s,p)} := \sigma_{studioName='Fox'}(\texttt{Movies})$$
$$\texttt{Answer(title, year)} := \pi_{t,y}(\texttt{R} \cap \texttt{S})$$

We could even substitute for R and S in the last line and write the entire expression in one line. □

2.4.14 Exercises for Section 2.4

Exercise 2.4.1: This exercise builds upon the products schema of Exercise 2.3.1. Recall that the database schema consists of four relations, whose schemas are:

```
Product(maker, model, type)
PC(model, speed, ram, hd, price)
Laptop(model, speed, ram, hd, screen, price)
Printer(model, color, type, price)
```

Some sample data for the relation `Product` is shown in Fig. 2.20. Sample data for the other three relations is shown in Fig. 2.21. Manufacturers and model numbers have been "sanitized," but the data is typical of products on sale at the beginning of 2007.

Write expressions of relational algebra to answer the following queries. You may use the linear notation of Section 2.4.13 if you wish. For the data of Figs. 2.20 and 2.21, show the result of your query. However, your answer should work for arbitrary data, not just the data of these figures.

a) What PC models have a speed of at least 3.00?

b) Which manufacturers make laptops with a hard disk of at least 100GB?

c) Find the model number and price of all products (of any type) made by manufacturer B.

d) Find the model numbers of all color laser printers.

e) Find those manufacturers that sell Laptops, but not PC's.

! f) Find those hard-disk sizes that occur in two or more PC's.

maker	*model*	*type*
A	1001	pc
A	1002	pc
A	1003	pc
A	2004	laptop
A	2005	laptop
A	2006	laptop
B	1004	pc
B	1005	pc
B	1006	pc
B	2007	laptop
C	1007	pc
D	1008	pc
D	1009	pc
D	1010	pc
D	3004	printer
D	3005	printer
E	1011	pc
E	1012	pc
E	1013	pc
E	2001	laptop
E	2002	laptop
E	2003	laptop
E	3001	printer
E	3002	printer
E	3003	printer
F	2008	laptop
F	2009	laptop
G	2010	laptop
H	3006	printer
H	3007	printer

Figure 2.20: Sample data for Product

model	*speed*	*ram*	*hd*	*price*
1001	2.66	1024	250	2114
1002	2.10	512	250	995
1003	1.42	512	80	478
1004	2.80	1024	250	649
1005	3.20	512	250	630
1006	3.20	1024	320	1049
1007	2.20	1024	200	510
1008	2.20	2048	250	770
1009	2.00	1024	250	650
1010	2.80	2048	300	770
1011	1.86	2048	160	959
1012	2.80	1024	160	649
1013	3.06	512	80	529

(a) Sample data for relation PC

model	*speed*	*ram*	*hd*	*screen*	*price*
2001	2.00	2048	240	20.1	3673
2002	1.73	1024	80	17.0	949
2003	1.80	512	60	15.4	549
2004	2.00	512	60	13.3	1150
2005	2.16	1024	120	17.0	2500
2006	2.00	2048	80	15.4	1700
2007	1.83	1024	120	13.3	1429
2008	1.60	1024	100	15.4	900
2009	1.60	512	80	14.1	680
2010	2.00	2048	160	15.4	2300

(b) Sample data for relation Laptop

model	*color*	*type*	*price*
3001	true	ink-jet	99
3002	false	laser	239
3003	true	laser	899
3004	true	ink-jet	120
3005	false	laser	120
3006	true	ink-jet	100
3007	true	laser	200

(c) Sample data for relation Printer

Figure 2.21: Sample data for relations of Exercise 2.4.1

! g) Find those pairs of PC models that have both the same speed and RAM. A pair should be listed only once; e.g., list (i, j) but not (j, i).

!! h) Find those manufacturers of at least two different computers (PC's or laptops) with speeds of at least 2.80.

!! i) Find the manufacturer(s) of the computer (PC or laptop) with the highest available speed.

!! j) Find the manufacturers of PC's with at least three different speeds.

!! k) Find the manufacturers who sell exactly three different models of PC.

Exercise 2.4.2: Draw expression trees for each of your expressions of Exercise 2.4.1.

Exercise 2.4.3: This exercise builds upon Exercise 2.3.2 concerning World War II capital ships. Recall it involves the following relations:

```
Classes(class, type, country, numGuns, bore, displacement)
Ships(name, class, launched)
Battles(name, date)
Outcomes(ship, battle, result)
```

Figures 2.22 and 2.23 give some sample data for these four relations.[4] Note that, unlike the data for Exercise 2.4.1, there are some "dangling tuples" in this data, e.g., ships mentioned in `Outcomes` that are not mentioned in `Ships`.

Write expressions of relational algebra to answer the following queries. You may use the linear notation of Section 2.4.13 if you wish. For the data of Figs. 2.22 and 2.23, show the result of your query. However, your answer should work for arbitrary data, not just the data of these figures.

a) Give the class names and countries of the classes that carried guns of at least 16-inch bore.

b) Find the ships launched prior to 1921.

c) Find the ships sunk in the battle of the Denmark Strait.

d) The treaty of Washington in 1921 prohibited capital ships heavier than 35,000 tons. List the ships that violated the treaty of Washington.

e) List the name, displacement, and number of guns of the ships engaged in the battle of Guadalcanal.

f) List all the capital ships mentioned in the database. (Remember that all these ships may not appear in the `Ships` relation.)

[4]Source: J. N. Westwood, *Fighting Ships of World War II*, Follett Publishing, Chicago, 1975 and R. C. Stern, *US Battleships in Action*, Squadron/Signal Publications, Carrollton, TX, 1980.

class	*type*	*country*	*numGuns*	*bore*	*displacement*
Bismarck	bb	Germany	8	15	42000
Iowa	bb	USA	9	16	46000
Kongo	bc	Japan	8	14	32000
North Carolina	bb	USA	9	16	37000
Renown	bc	Gt. Britain	6	15	32000
Revenge	bb	Gt. Britain	8	15	29000
Tennessee	bb	USA	12	14	32000
Yamato	bb	Japan	9	18	65000

(a) Sample data for relation Classes

name	*date*
Denmark Strait	5/24-27/41
Guadalcanal	11/15/42
North Cape	12/26/43
Surigao Strait	10/25/44

(b) Sample data for relation Battles

ship	*battle*	*result*
Arizona	Pearl Harbor	sunk
Bismarck	Denmark Strait	sunk
California	Surigao Strait	ok
Duke of York	North Cape	ok
Fuso	Surigao Strait	sunk
Hood	Denmark Strait	sunk
King George V	Denmark Strait	ok
Kirishima	Guadalcanal	sunk
Prince of Wales	Denmark Strait	damaged
Rodney	Denmark Strait	ok
Scharnhorst	North Cape	sunk
South Dakota	Guadalcanal	damaged
Tennessee	Surigao Strait	ok
Washington	Guadalcanal	ok
West Virginia	Surigao Strait	ok
Yamashiro	Surigao Strait	sunk

(c) Sample data for relation Outcomes

Figure 2.22: Data for Exercise 2.4.3

name	*class*	*launched*
California	Tennessee	1921
Haruna	Kongo	1915
Hiei	Kongo	1914
Iowa	Iowa	1943
Kirishima	Kongo	1915
Kongo	Kongo	1913
Missouri	Iowa	1944
Musashi	Yamato	1942
New Jersey	Iowa	1943
North Carolina	North Carolina	1941
Ramillies	Revenge	1917
Renown	Renown	1916
Repulse	Renown	1916
Resolution	Revenge	1916
Revenge	Revenge	1916
Royal Oak	Revenge	1916
Royal Sovereign	Revenge	1916
Tennessee	Tennessee	1920
Washington	North Carolina	1941
Wisconsin	Iowa	1944
Yamato	Yamato	1941

Figure 2.23: Sample data for relation Ships

! g) Find the classes that had only one ship as a member of that class.

! h) Find those countries that had both battleships and battlecruisers.

! i) Find those ships that "lived to fight another day"; they were damaged in one battle, but later fought in another.

Exercise 2.4.4: Draw expression trees for each of your expressions of Exercise 2.4.3.

Exercise 2.4.5: What is the difference between the natural join $R \bowtie S$ and the theta-join $R \bowtie_C S$ where the condition C is that $R.A = S.A$ for each attribute A appearing in the schemas of both R and S?

! **Exercise 2.4.6:** An operator on relations is said to be *monotone* if whenever we add a tuple to one of its arguments, the result contains all the tuples that it contained before adding the tuple, plus perhaps more tuples. Which of the operators described in this section are monotone? For each, either explain why it is monotone or give an example showing it is not.

! **Exercise 2.4.7:** Suppose relations R and S have n tuples and m tuples, respectively. Give the minimum and maximum numbers of tuples that the results of the following expressions can have.

a) $R \cup S$.

b) $R \bowtie S$.

c) $\sigma_C(R) \times S$, for some condition C.

d) $\pi_L(R) - S$, for some list of attributes L.

! **Exercise 2.4.8:** The *semijoin* of relations R and S, written $R \ltimes S$, is the set of tuples t in R such that there is at least one tuple in S that agrees with t in all attributes that R and S have in common. Give three different expressions of relational algebra that are equivalent to $R \ltimes S$.

! **Exercise 2.4.9:** The *antisemijoin* $R \overline{\ltimes} S$ is the set of tuples t in R that do *not* agree with any tuple of S in the attributes common to R and S. Give an expression of relational algebra equivalent to $R \overline{\ltimes} S$.

!! **Exercise 2.4.10:** Let R be a relation with schema

$$(A_1, A_2, \ldots, A_n, B_1, B_2, \ldots, B_m)$$

and let S be a relation with schema $(B_1, B_2, \ldots, B_m)$; that is, the attributes of S are a subset of the attributes of R. The *quotient* of R and S, denoted $R \div S$, is the set of tuples t over attributes $A_1, A_2, \ldots, A_n$ (i.e., the attributes of R that are not attributes of S) such that for every tuple s in S, the tuple ts, consisting of the components of t for $A_1, A_2, \ldots, A_n$ and the components of s for $B_1, B_2, \ldots, B_m$, is a member of R. Give an expression of relational algebra, using the operators we have defined previously in this section, that is equivalent to $R \div S$.

2.5 Constraints on Relations

We now take up the third important aspect of a data model: the ability to restrict the data that may be stored in a database. So far, we have seen only one kind of constraint, the requirement that an attribute or attributes form a key (Section 2.3.6). These and many other kinds of constraints can be expressed in relational algebra. In this section, we show how to express both key constraints and "referential-integrity" constraints; the latter require that a value appearing in one column of one relation also appear in some other column of the same or a different relation. In Chapter 7, we see how SQL database systems can enforce the same sorts of constraints as we can express in relational algebra.

2.5.1 Relational Algebra as a Constraint Language

There are two ways in which we can use expressions of relational algebra to express constraints.

1. If R is an expression of relational algebra, then $R = \emptyset$ is a constraint that says "The value of R must be empty," or equivalently "There are no tuples in the result of R."

2. If R and S are expressions of relational algebra, then $R \subseteq S$ is a constraint that says "Every tuple in the result of R must also be in the result of S." Of course the result of S may contain additional tuples not produced by R.

These ways of expressing constraints are actually equivalent in what they can express, but sometimes one or the other is clearer or more succinct. That is, the constraint $R \subseteq S$ could just as well have been written $R - S = \emptyset$. To see why, notice that if every tuple in R is also in S, then surely $R - S$ is empty. Conversely, if $R - S$ contains no tuples, then every tuple in R must be in S (or else it would be in $R - S$).

On the other hand, a constraint of the first form, $R = \emptyset$, could just as well have been written $R \subseteq \emptyset$. Technically, $\emptyset$ is not an expression of relational algebra, but since there are expressions that evaluate to $\emptyset$, such as $R - R$, there is no harm in using $\emptyset$ as a relational-algebra expression.

In the following sections, we shall see how to express significant constraints in one of these two styles. As we shall see in Chapter 7, it is the first style — equal-to-the-emptyset — that is most commonly used in SQL programming. However, as shown above, we are free to think in terms of set-containment if we wish and later convert our constraint to the equal-to-the-emptyset style.

2.5.2 Referential Integrity Constraints

A common kind of constraint, called a *referential integrity constraint*, asserts that a value appearing in one context also appears in another, related context. For example, in our movies database, should we see a `StarsIn` tuple that has person p in the `starName` component, we would expect that p appears as the name of some star in the `MovieStar` relation. If not, then we would question whether the listed "star" really was a star.

In general, if we have any value v as the component in attribute A of some tuple in one relation R, then because of our design intentions we may expect that v will appear in a particular component (say for attribute B) of some tuple of another relation S. We can express this integrity constraint in relational algebra as $\pi_A(R) \subseteq \pi_B(S)$, or equivalently, $\pi_A(R) - \pi_B(S) = \emptyset$.

Example 2.21: Consider the two relations from our running movie database:

```
Movies(title, year, length, genre, studioName, producerC#)
MovieExec(name, address, cert#, netWorth)
```

We might reasonably assume that the producer of every movie would have to appear in the `MovieExec` relation. If not, there is something wrong, and we would at least want a system implementing a relational database to inform us that we had a movie with a producer of which the database had no knowledge.

To be more precise, the `producerC#` component of each `Movies` tuple must also appear in the `cert#` component of some `MovieExec` tuple. Since executives are uniquely identified by their certificate numbers, we would thus be assured that the movie's producer is found among the movie executives. We can express this constraint by the set-containment

$$\pi_{producerC\#}(\texttt{Movies}) \subseteq \pi_{cert\#}(\texttt{MovieExec})$$

The value of the expression on the left is the set of all certificate numbers appearing in `producerC#` components of `Movies` tuples. Likewise, the expression on the right's value is the set of all certificates in the `cert#` component of `MovieExec` tuples. Our constraint says that every certificate in the former set must also be in the latter set. □

Example 2.22: We can similarly express a referential integrity constraint where the "value" involved is represented by more than one attribute. For instance, we may want to assert that any movie mentioned in the relation

```
StarsIn(movieTitle, movieYear, starName)
```

also appears in the relation

```
Movies(title, year, length, genre, studioName, producerC#)
```

Movies are represented in both relations by title-year pairs, because we agreed that one of these attributes alone was not sufficient to identify a movie. The constraint

$$\pi_{movieTitle,\ movieYear}(\texttt{StarsIn}) \subseteq \pi_{title,\ year}(\texttt{Movies})$$

expresses this referential integrity constraint by comparing the title-year pairs produced by projecting both relations onto the appropriate lists of components. □

2.5.3 Key Constraints

The same constraint notation allows us to express far more than referential integrity. Here, we shall see how we can express algebraically the constraint that a certain attribute or set of attributes is a key for a relation.

Example 2.23: Recall that `name` is the key for relation

```
MovieStar(name, address, gender, birthdate)
```

That is, no two tuples agree on the name component. We shall express algebraically one of several implications of this constraint: that if two tuples agree on name, then they must also agree on address. Note that in fact these "two" tuples, which agree on the key name, must be the same tuple and therefore certainly agree in all attributes.

The idea is that if we construct all pairs of MovieStar tuples (t_1, t_2), we must not find a pair that agree in the name component and disagree in the address component. To construct the pairs we use a Cartesian product, and to search for pairs that violate the condition we use a selection. We then assert the constraint by equating the result to $\emptyset$.

To begin, since we are taking the product of a relation with itself, we need to rename at least one copy, in order to have names for the attributes of the product. For succinctness, let us use two new names, MS1 and MS2, to refer to the MovieStar relation. Then the requirement can be expressed by the algebraic constraint:

$$\sigma_{MS1.name=MS2.name \text{ AND } MS1.address \neq MS2.address}(\texttt{MS1} \times \texttt{MS2}) = \emptyset$$

In the above, MS1 in the product MS1 × MS2 is shorthand for the renaming:

$$\rho_{MS1(name,address,gender,birthdate)}(\texttt{MovieStar})$$

and MS2 is a similar renaming of MovieStar. □

2.5.4 Additional Constraint Examples

There are many other kinds of constraints that we can express in relational algebra and that are useful for restricting database contents. A large family of constraints involve the permitted values in a context. For example, the fact that each attribute has a type constrains the values of that attribute. Often the constraint is quite straightforward, such as "integers only" or "character strings of length up to 30." Other times we want the values that may appear in an attribute to be restricted to a small enumerated set of values. Other times, there are complex limitations on the values that may appear. We shall give two examples, one of a simple *domain constraint* for an attribute, and the second a more complicated restriction.

Example 2.24: Suppose we wish to specify that the only legal values for the gender attribute of MovieStar are 'F' and 'M'. We can express this constraint algebraically by:

$$\sigma_{gender \neq \text{'F'} \text{ AND } gender \neq \text{'M'}}(\texttt{MovieStar}) = \emptyset$$

That is, the set of tuples in MovieStar whose gender component is equal to neither 'F' nor 'M' is empty. □

Example 2.25: Suppose we wish to require that one must have a net worth of at least \$10,000,000 to be the president of a movie studio. We can express this constraint algebraically as follows. First, we need to theta-join the two relations

```
MovieExec(name, address, cert#, netWorth)
Studio(name, address, presC#)
```

using the condition that `presC#` from `Studio` and `cert#` from `MovieExec` are equal. That join combines pairs of tuples consisting of a studio and an executive, such that the executive is the president of the studio. If we select from this relation those tuples where the net worth is less than ten million, we have a set that, according to our constraint, must be empty. Thus, we may express the constraint as:

$$\sigma_{netWorth<10000000}(\texttt{Studio} \bowtie_{presC\#=cert\#} \texttt{MovieExec}) = \emptyset$$

An alternative way to express the same constraint is to compare the set of certificates that represent studio presidents with the set of certificates that represent executives with a net worth of at least \$10,000,000; the former must be a subset of the latter. The containment

$$\pi_{presC\#}(\texttt{Studio}) \subseteq \pi_{cert\#}\Big(\sigma_{netWorth\geq 10000000}(\texttt{MovieExec})\Big)$$

expresses the above idea. □

2.5.5 Exercises for Section 2.5

Exercise 2.5.1: Express the following constraints about the relations of Exercise 2.3.1, reproduced here:

```
Product(maker, model, type)
PC(model, speed, ram, hd, price)
Laptop(model, speed, ram, hd, screen, price)
Printer(model, color, type, price)
```

You may write your constraints either as containments or by equating an expression to the empty set. For the data of Exercise 2.4.1, indicate any violations to your constraints.

a) A PC with a processor speed less than 2.00 must not sell for more than \$500.

b) A laptop with a screen size less than 15.4 inches must have at least a 100 gigabyte hard disk or sell for less than \$1000.

! c) No manufacturer of PC's may also make laptops.

!! d) A manufacturer of a PC must also make a laptop with at least as great a processor speed.

! e) If a laptop has a larger main memory than a PC, then the laptop must also have a higher price than the PC.

Exercise 2.5.2: Express the following constraints in relational algebra. The constraints are based on the relations of Exercise 2.3.2:

```
Classes(class, type, country, numGuns, bore, displacement)
Ships(name, class, launched)
Battles(name, date)
Outcomes(ship, battle, result)
```

You may write your constraints either as containments or by equating an expression to the empty set. For the data of Exercise 2.4.3, indicate any violations to your constraints.

a) No class of ships may have guns with larger than 16-inch bore.

b) If a class of ships has more than 9 guns, then their bore must be no larger than 14 inches.

! c) No class may have more than 2 ships.

! d) No country may have both battleships and battlecruisers.

!! e) No ship with more than 9 guns may be in a battle with a ship having fewer than 9 guns that was sunk.

! **Exercise 2.5.3:** Suppose R and S are two relations. Let C be the referential integrity constraint that says: whenever R has a tuple with some values $v_1, v_2, \ldots, v_n$ in particular attributes $A_1, A_2, \ldots, A_n$, there must be a tuple of S that has the same values $v_1, v_2, \ldots, v_n$ in particular attributes $B_1, B_2, \ldots, B_n$. Show how to express constraint C in relational algebra.

! **Exercise 2.5.4:** Another algebraic way to express a constraint is $E_1 = E_2$, where both E_1 and E_2 are relational-algebra expressions. Can this form of constraint express more than the two forms we discussed in this section?

2.6 Summary of Chapter 2

✦ *Data Models*: A data model is a notation for describing the structure of the data in a database, along with the constraints on that data. The data model also normally provides a notation for describing operations on that data: queries and data modifications.

- *Relational Model*: Relations are tables representing information. Columns are headed by attributes; each attribute has an associated domain, or data type. Rows are called tuples, and a tuple has one component for each attribute of the relation.
- *Schemas*: A relation name, together with the attributes of that relation and their types, form the relation schema. A collection of relation schemas forms a database schema. Particular data for a relation or collection of relations is called an instance of that relation schema or database schema.
- *Keys*: An important type of constraint on relations is the assertion that an attribute or set of attributes forms a key for the relation. No two tuples of a relation can agree on all attributes of the key, although they can agree on some of the key attributes.
- *Semistructured Data Model*: In this model, data is organized in a tree or graph structure. XML is an important example of a semistructured data model.
- *SQL*: The language SQL is the principal query language for relational database systems. The current standard is called SQL-99. Commercial systems generally vary from this standard but adhere to much of it.
- *Data Definition*: SQL has statements to declare elements of a database schema. The `CREATE TABLE` statement allows us to declare the schema for stored relations (called tables), specifying the attributes, their types, default values, and keys.
- *Altering Schemas*: We can change parts of the database schema with an `ALTER` statement. These changes include adding and removing attributes from relation schemas and changing the default value associated with an attribute. We may also use a `DROP` statement to completely eliminate relations or other schema elements.
- *Relational Algebra*: This algebra underlies most query languages for the relational model. Its principal operators are union, intersection, difference, selection, projection, Cartesian product, natural join, theta-join, and renaming.
- *Selection and Projection*: The selection operator produces a result consisting of all tuples of the argument relation that satisfy the selection condition. Projection removes undesired columns from the argument relation to produce the result.
- *Joins*: We join two relations by comparing tuples, one from each relation. In a natural join, we splice together those pairs of tuples that agree on all attributes common to the two relations. In a theta-join, pairs of tuples are concatenated if they meet a selection condition associated with the theta-join.

- *Constraints in Relational Algebra*: Many common kinds of constraints can be expressed as the containment of one relational algebra expression in another, or as the equality of a relational algebra expression to the empty set.

2.7 References for Chapter 2

The classic paper by Codd on the relational model is [1]. This paper introduces relational algebra, as well. The use of relational algebra to describe constraints is from [2]. References for SQL are given in the bibliographic notes for Chapter 6.

The semistructured data model is from [3]. XML is a standard developed by the World-Wide-Web Consortium. The home page for information about XML is [4].

1. E. F. Codd, "A relational model for large shared data banks," *Comm. ACM* **13**:6, pp. 377–387, 1970.

2. J.-M. Nicolas, "Logic for improving integrity checking in relational databases," *Acta Informatica* **18**:3, pp. 227–253, 1982.

3. Y. Papakonstantinou, H. Garcia-Molina, and J. Widom, "Object exchange across heterogeneous information sources," *IEEE Intl. Conf. on Data Engineering*, pp. 251–260, March 1995.

4. World-Wide-Web Consortium, `http://www.w3.org/XML/`

Chapter 3

Design Theory for Relational Databases

There are many ways we could go about designing a relational database schema for an application. In Chapter 4 we shall see several high-level notations for describing the structure of data and the ways in which these high-level designs can be converted into relations. We can also examine the requirements for a database and define relations directly, without going through a high-level intermediate stage. Whatever approach we use, it is common for an initial relational schema to have room for improvement, especially by eliminating redundancy. Often, the problems with a schema involve trying to combine too much into one relation.

Fortunately, there is a well developed theory for relational databases: "dependencies," their implications for what makes a good relational database schema, and what we can do about a schema if it has flaws. In this chapter, we first identify the problems that are caused in some relation schemas by the presence of certain dependencies; these problems are referred to as "anomalies."

Our discussion starts with "functional dependencies," a generalization of the idea of a key for a relation. We then use the notion of functional dependencies to define normal forms for relation schemas. The impact of this theory, called "normalization," is that we decompose relations into two or more relations when that will remove anomalies. Next, we introduce "multivalued dependencies," which intuitively represent a condition where one or more attributes of a relation are independent from one or more other attributes. These dependencies also lead to normal forms and decomposition of relations to eliminate redundancy.

3.1 Functional Dependencies

There is a design theory for relations that lets us examine a design carefully and make improvements based on a few simple principles. The theory begins by

having us state the constraints that apply to the relation. The most common constraint is the "functional dependency," a statement of a type that generalizes the idea of a key for a relation, which we introduced in Section 2.5.3. Later in this chapter, we shall see how this theory gives us simple tools to improve our designs by the process of "decomposition" of relations: the replacement of one relation by several, whose sets of attributes together include all the attributes of the original.

3.1.1 Definition of Functional Dependency

A *functional dependency* (FD) on a relation R is a statement of the form "If two tuples of R agree on all of the attributes $A_1, A_2, \ldots, A_n$ (i.e., the tuples have the same values in their respective components for each of these attributes), then they must also agree on all of another list of attributes $B_1, B_2, \ldots, B_m$. We write this FD formally as $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and say that

"$A_1, A_2, \ldots, A_n$ functionally determine $B_1, B_2, \ldots, B_m$"

Figure 3.1 suggests what this FD tells us about any two tuples t and u in the relation R. However, the A's and B's can be anywhere; it is not necessary for the A's and B's to appear consecutively or for the A's to precede the B's.

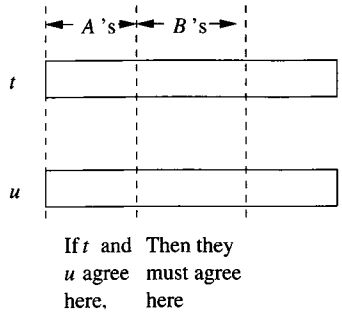

Figure 3.1: The effect of a functional dependency on two tuples.

If we can be sure every instance of a relation R will be one in which a given FD is true, then we say that R *satisfies* the FD. It is important to remember that when we say that R satisfies an FD f, we are asserting a constraint on R, not just saying something about one particular instance of R.

It is common for the right side of an FD to be a single attribute. In fact, we shall see that the one functional dependency $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ is equivalent to the set of FD's:

$$\begin{aligned} &A_1A_2 \cdots A_n \rightarrow B_1 \\ &A_1A_2 \cdots A_n \rightarrow B_2 \\ &\cdots \\ &A_1A_2 \cdots A_n \rightarrow B_m \end{aligned}$$

title	*year*	*length*	*genre*	*studioName*	*starName*
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	drama	MGM	Vivien Leigh
Wayne's World	1992	95	comedy	Paramount	Dana Carvey
Wayne's World	1992	95	comedy	Paramount	Mike Meyers

Figure 3.2: An instance of the relation Movies1(title, year, length, genre, studioName, starName)

Example 3.1: Let us consider the relation

Movies1(title, year, length, genre, studioName, starName)

an instance of which is shown in Fig. 3.2. While related to our running Movies relation, it has additional attributes, which is why we call it "Movies1" instead of "Movies." Notice that this relation tries to "do too much." It holds information that in our running database schema was attributed to three different relations: Movies, Studio, and StarsIn. As we shall see, the schema for Movies1 is not a good design. But to see what is wrong with the design, we must first determine the functional dependencies that hold for the relation. We claim that the following FD holds:

title year → length genre studioName

Informally, this FD says that if two tuples have the same value in their title components, and they also have the same value in their year components, then these two tuples must also have the same values in their length components, the same values in their genre components, and the same values in their studioName components. This assertion makes sense, since we believe that it is not possible for there to be two movies released in the same year with the same title (although there could be movies of the same title released in different years). This point was discussed in Example 2.1. Thus, we expect that given a title and year, there is a unique movie. Therefore, there is a unique length for the movie, a unique genre, and a unique studio.

On the other hand, we observe that the statement

title year → starName

is false; it is not a functional dependency. Given a movie, it is entirely possible that there is more than one star for the movie listed in our database. Notice that even had we been lazy and only listed one star for *Star Wars* and one star for *Wayne's World* (just as we only listed one of the many stars for *Gone With the Wind*), this FD would not suddenly become true for the relation Movies1.

The reason is that the FD says something about all possible instances of the relation, not about one of its instances. The fact that we *could* have an instance with multiple stars for a movie rules out the possibility that title and year functionally determine starName. □

3.1.2 Keys of Relations

We say a set of one or more attributes $\{A_1, A_2, \ldots, A_n\}$ is a *key* for a relation R if:

1. Those attributes functionally determine all other attributes of the relation. That is, it is impossible for two distinct tuples of R to agree on all of $A_1, A_2, \ldots, A_n$.
2. No proper subset of $\{A_1, A_2, \ldots, A_n\}$ functionally determines all other attributes of R; i.e., a key must be *minimal*.

When a key consists of a single attribute A, we often say that A (rather than $\{A\}$) is a key.

Example 3.2: Attributes {`title`, `year`, `starName`} form a key for the relation `Movies1` of Fig. 3.2. First, we must show that they functionally determine all the other attributes. That is, suppose two tuples agree on these three attributes: `title`, `year`, and `starName`. Because they agree on `title` and `year`, they must agree on the other attributes — `length`, `genre`, and `studioName` — as we discussed in Example 3.1. Thus, two different tuples cannot agree on all of `title`, `year`, and `starName`; they would in fact be the same tuple.

Now, we must argue that no proper subset of {`title`, `year`, `starName`} functionally determines all other attributes. To see why, begin by observing that `title` and `year` do not determine `starName`, because many movies have more than one star. Thus, {`title`, `year`} is not a key.

{`year`, `starName`} is not a key because we could have a star in two movies in the same year; therefore

`year starName` → `title`

is not an FD. Also, we claim that {`title`, `starName`} is not a key, because two movies with the same title, made in different years, occasionally have a star in common.[1] □

Sometimes a relation has more than one key. If so, it is common to designate one of the keys as the *primary key*. In commercial database systems, the choice of primary key can influence some implementation issues such as how the relation is stored on disk. However, the theory of FD's gives no special role to "primary keys."

[1]Since we asserted in an earlier book that there were no known examples of this phenomenon, several people have shown us we were wrong. It's an interesting challenge to discover stars that appeared in two versions of the same movie.

What Is "Functional" About Functional Dependencies?

$A_1A_2\cdots A_n \rightarrow B$ is called a "functional" dependency because in principle there is a function that takes a list of values, one for each of attributes $A_1, A_2, \ldots, A_n$ and produces a unique value (or no value at all) for B. For instance, in the `Movies1` relation, we can imagine a function that takes a string like `"Star Wars"` and an integer like 1977 and produces the unique value of `length`, namely 124, that appears in the relation `Movies1`. However, this function is not the usual sort of function that we meet in mathematics, because there is no way to compute it from first principles. That is, we cannot perform some operations on strings like `"Star Wars"` and integers like 1977 and come up with the correct length. Rather, the function is only computed by lookup in the relation. We look for a tuple with the given `title` and `year` values and see what value that tuple has for `length`.

3.1.3 Superkeys

A set of attributes that contains a key is called a *superkey*, short for "superset of a key." Thus, every key is a superkey. However, some superkeys are not (minimal) keys. Note that every superkey satisfies the first condition of a key: it functionally determines all other attributes of the relation. However, a superkey need not satisfy the second condition: minimality.

Example 3.3: In the relation of Example 3.2, there are many superkeys. Not only is the key

{`title`, `year`, `starName`}

a superkey, but any superset of this set of attributes, such as

{`title`, `year`, `starName`, `length`, `studioName`}

is a superkey. □

3.1.4 Exercises for Section 3.1

Exercise 3.1.1: Consider a relation about people in the United States, including their name, Social Security number, street address, city, state, ZIP code, area code, and phone number (7 digits). What FD's would you expect to hold? What are the keys for the relation? To answer this question, you need to know something about the way these numbers are assigned. For instance, can an area

Other Key Terminology

In some books and articles one finds different terminology regarding keys. One can find the term "key" used the way we have used the term "superkey," that is, a set of attributes that functionally determine all the attributes, with no requirement of minimality. These sources typically use the term "candidate key" for a key that is minimal — that is, a "key" in the sense we use the term.

code straddle two states? Can a ZIP code straddle two area codes? Can two people have the same Social Security number? Can they have the same address or phone number?

Exercise 3.1.2: Consider a relation representing the present position of molecules in a closed container. The attributes are an ID for the molecule, the x, y, and z coordinates of the molecule, and its velocity in the x, y, and z dimensions. What FD's would you expect to hold? What are the keys?

!! **Exercise 3.1.3:** Suppose R is a relation with attributes $A_1, A_2, \ldots, A_n$. As a function of n, tell how many superkeys R has, if:

a) The only key is A_1.

b) The only keys are A_1 and A_2.

c) The only keys are $\{A_1, A_2\}$ and $\{A_3, A_4\}$.

d) The only keys are $\{A_1, A_2\}$ and $\{A_1, A_3\}$.

3.2 Rules About Functional Dependencies

In this section, we shall learn how to *reason* about FD's. That is, suppose we are told of a set of FD's that a relation satisfies. Often, we can deduce that the relation must satisfy certain other FD's. This ability to discover additional FD's is essential when we discuss the design of good relation schemas in Section 3.3.

3.2.1 Reasoning About Functional Dependencies

Let us begin with a motivating example that will show us how we can infer a functional dependency from other given FD's.

Example 3.4: If we are told that a relation $R(A, B, C)$ satisfies the FD's $A \to B$ and $B \to C$, then we can deduce that R also satisfies the FD $A \to C$. How does that reasoning go? To prove that $A \to C$, we must consider two tuples of R that agree on A and prove they also agree on C.

Let the tuples agreeing on attribute A be (a, b_1, c_1) and (a, b_2, c_2). Since R satisfies $A \to B$, and these tuples agree on A, they must also agree on B. That is, $b_1 = b_2$, and the tuples are really (a, b, c_1) and (a, b, c_2), where b is both b_1 and b_2. Similarly, since R satisfies $B \to C$, and the tuples agree on B, they agree on C. Thus, $c_1 = c_2$; i.e., the tuples *do* agree on C. We have proved that any two tuples of R that agree on A also agree on C, and that is the FD $A \to C$. □

FD's often can be presented in several different ways, without changing the set of legal instances of the relation. We say:

- Two sets of FD's S and T are *equivalent* if the set of relation instances satisfying S is exactly the same as the set of relation instances satisfying T.
- More generally, a set of FD's S *follows* from a set of FD's T if every relation instance that satisfies all the FD's in T also satisfies all the FD's in S.

Note then that two sets of FD's S and T are equivalent if and only if S follows from T, and T follows from S.

In this section we shall see several useful rules about FD's. In general, these rules let us replace one set of FD's by an equivalent set, or to add to a set of FD's others that follow from the original set. An example is the *transitive rule* that lets us follow chains of FD's, as in Example 3.4. We shall also give an algorithm for answering the general question of whether one FD follows from one or more other FD's.

3.2.2 The Splitting/Combining Rule

Recall that in Section 3.1.1 we commented that the FD:

$$A_1 A_2 \cdots A_n \to B_1 B_2 \cdots B_m$$

was equivalent to the set of FD's:

$$A_1 A_2 \cdots A_n \to B_1, \quad A_1 A_2 \cdots A_n \to B_2, \ldots, A_1 A_2 \cdots A_n \to B_m$$

That is, we may split attributes on the right side so that only one attribute appears on the right of each FD. Likewise, we can replace a collection of FD's having a common left side by a single FD with the same left side and all the right sides combined into one set of attributes. In either event, the new set of FD's is equivalent to the old. The equivalence noted above can be used in two ways.

- We can replace an FD $A_1 A_2 \cdots A_n \to B_1 B_2 \cdots B_m$ by a set of FD's $A_1 A_2 \cdots A_n \to B_i$ for $i = 1, 2, \ldots, m$. This transformation we call the *splitting rule.*

- We can replace a set of FD's $A_1A_2\cdots A_n \rightarrow B_i$ for $i = 1, 2, \ldots, m$ by the single FD $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$. We call this transformation the *combining rule.*

Example 3.5: In Example 3.1 the set of FD's:

```
title year → length
title year → genre
title year → studioName
```

is equivalent to the single FD:

```
title year → length genre studioName
```

that we asserted there. □

The reason the splitting and combining rules are true should be obvious. Suppose we have two tuples that agree in $A_1, A_2, \ldots, A_n$. As a single FD, we would assert "then the tuples must agree in all of $B_1, B_2, \ldots, B_m$." As individual FD's, we assert "then the tuples agree in B_1, and they agree in B_2, and,..., and they agree in B_m." These two conclusions say exactly the same thing.

One might imagine that splitting could be applied to the left sides of FD's as well as to right sides. However, there is no splitting rule for left sides, as the following example shows.

Example 3.6: Consider one of the FD's such as:

```
title year → length
```

for the relation `Movies1` in Example 3.1. If we try to split the left side into

```
title → length
year → length
```

then we get two false FD's. That is, `title` does not functionally determine `length`, since there can be several movies with the same title (e.g., *King Kong*) but of different lengths. Similarly, `year` does not functionally determine `length`, because there are certainly movies of different lengths made in any one year. □

3.2.3 Trivial Functional Dependencies

A constraint of any kind on a relation is said to be *trivial* if it holds for every instance of the relation, regardless of what other constraints are assumed. When the constraints are FD's, it is easy to tell whether an FD is trivial. They are the FD's $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ such that

$$\{B_1, B_2, \ldots, B_m\} \subseteq \{A_1, A_2, \ldots, A_n\}$$

That is, a trivial FD has a right side that is a subset of its left side. For example,

```
title year → title
```

is a trivial FD, as is

```
title → title
```

Every trivial FD holds in every relation, since it says that "two tuples that agree in all of $A_1, A_2, \ldots, A_n$ agree in a subset of them." Thus, we may assume any trivial FD, without having to justify it on the basis of what FD's are asserted for the relation.

There is an intermediate situation in which some, but not all, of the attributes on the right side of an FD are also on the left. This FD is not trivial, but it can be simplifed by removing from the right side of an FD those attributes that appear on the left. That is:

- The FD $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ is equivalent to

  $$A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$$

  where the C's are all those B's that are not also A's.

We call this rule, illustrated in Fig. 3.3, the *trivial-dependency rule.*

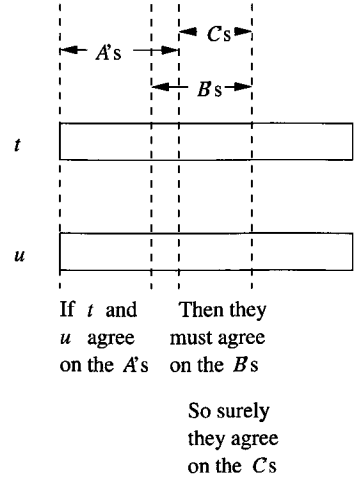

Figure 3.3: The trivial-dependency rule

3.2.4 Computing the Closure of Attributes

Before proceeding to other rules, we shall give a general principle from which all true rules follow. Suppose $\{A_1, A_2, \ldots, A_n\}$ is a set of attributes and S

is a set of FD's. The *closure* of $\{A_1, A_2, \ldots, A_n\}$ under the FD's in S is the set of attributes B such that every relation that satisfies all the FD's in set S also satisfies $A_1A_2\cdots A_n \rightarrow B$. That is, $A_1A_2\cdots A_n \rightarrow B$ follows from the FD's of S. We denote the closure of a set of attributes $A_1A_2\cdots A_n$ by $\{A_1, A_2, \ldots, A_n\}^+$. Note that $A_1, A_2, \ldots, A_n$ are always in $\{A_1, A_2, \ldots, A_n\}^+$ because the FD $A_1A_2\cdots A_n \rightarrow A_i$ is trivial when i is one of $1, 2, \ldots, n$.

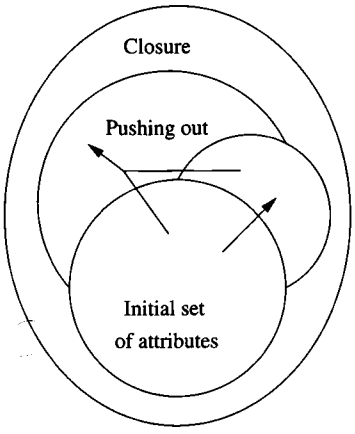

Figure 3.4: Computing the closure of a set of attributes

Figure 3.4 illustrates the closure process. Starting with the given set of attributes, we repeatedly expand the set by adding the right sides of FD's as soon as we have included their left sides. Eventually, we cannot expand the set any further, and the resulting set is the closure. More precisely:

Algorithm 3.7: Closure of a Set of Attributes.

INPUT: A set of attributes $\{A_1, A_2, \ldots, A_n\}$ and a set of FD's S.

OUTPUT: The closure $\{A_1, A_2, \ldots, A_n\}^+$.

1. If necessary, split the FD's of S, so each FD in S has a single attribute on the right.

2. Let X be a set of attributes that eventually will become the closure. Initialize X to be $\{A_1, A_2, \ldots, A_n\}$.

3. Repeatedly search for some FD

$$B_1B_2\cdots B_m \rightarrow C$$

such that all of $B_1, B_2, \ldots, B_m$ are in the set of attributes X, but C is not. Add C to the set X and repeat the search. Since X can only grow, and the number of attributes of any relation schema must be finite, eventually nothing more can be added to X, and this step ends.

4. The set X, after no more attributes can be added to it, is the correct value of $\{A_1, A_2, \ldots, A_n\}^+$.

□

Example 3.8: Let us consider a relation with attributes A, B, C, D, E, and F. Suppose that this relation has the FD's $AB \to C$, $BC \to AD$, $D \to E$, and $CF \to B$. What is the closure of $\{A, B\}$, that is, $\{A, B\}^+$?

First, split $BC \to AD$ into $BC \to A$ and $BC \to D$. Then, start with $X = \{A, B\}$. First, notice that both attributes on the left side of FD $AB \to C$ are in X, so we may add the attribute C, which is on the right side of that FD. Thus, after one iteration of Step 3, X becomes $\{A, B, C\}$.

Next, we see that the left sides of $BC \to A$ and $BC \to D$ are now contained in X, so we may add to X the attributes A and D. A is already there, but D is not, so X next becomes $\{A, B, C, D\}$. At this point, we may use the FD $D \to E$ to add E to X, which is now $\{A, B, C, D, E\}$. No more changes to X are possible. In particular, the FD $CF \to B$ can not be used, because its left side never becomes contained in X. Thus, $\{A, B\}^+ = \{A, B, C, D, E\}$. □

By computing the closure of any set of attributes, we can test whether any given FD $A_1A_2 \cdots A_n \to B$ follows from a set of FD's S. First compute $\{A_1, A_2, \ldots, A_n\}^+$ using the set of FD's S. If B is in $\{A_1, A_2, \ldots, A_n\}^+$, then $A_1A_2 \cdots A_n \to B$ does follow from S, and if B is not in $\{A_1, A_2, \ldots, A_n\}^+$, then this FD does not follow from S. More generally, $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ follows from set of FD's S if and only if all of $B_1, B_2, \ldots, B_m$ are in

$$\{A_1, A_2, \ldots, A_n\}^+$$

Example 3.9: Consider the relation and FD's of Example 3.8. Suppose we wish to test whether $AB \to D$ follows from these FD's. We compute $\{A, B\}^+$, which is $\{A, B, C, D, E\}$, as we saw in that example. Since D is a member of the closure, we conclude that $AB \to D$ does follow.

On the other hand, consider the FD $D \to A$. To test whether this FD follows from the given FD's, first compute $\{D\}^+$. To do so, we start with $X = \{D\}$. We can use the FD $D \to E$ to add E to the set X. However, then we are stuck. We cannot find any other FD whose left side is contained in $X = \{D, E\}$, so $\{D\}^+ = \{D, E\}$. Since A is not a member of $\{D, E\}$, we conclude that $D \to A$ does not follow. □

3.2.5 Why the Closure Algorithm Works

In this section, we shall show why Algorithm 3.7 correctly decides whether or not an FD $A_1A_2 \cdots A_n \to B$ follows from a given set of FD's S. There are two parts to the proof:

1. We must prove that Algorithm 3.7 does not claim too much. That is, we must show that if $A_1A_2 \cdots A_n \to B$ is asserted by the closure test (i.e.,

B is in $\{A_1, A_2, \ldots, A_n\}^+$), then $A_1A_2 \cdots A_n \to B$ holds in any relation that satisfies all the FD's in S.

2. We must prove that Algorithm 3.7 does not fail to discover a FD that truly follows from the set of FD's S.

Why the Closure Algorithm Claims only True FD's

We can prove by induction on the number of times that we apply the growing operation of Step 3 that for every attribute D in X, the FD $A_1A_2 \cdots A_n \to D$ holds. That is, every relation R satisfying all of the FD's in S also satisfies $A_1A_2 \cdots A_n \to D$.

BASIS: The basis case is when there are zero steps. Then D must be one of $A_1, A_2, \ldots, A_n$, and surely $A_1A_2 \cdots A_n \to D$ holds in any relation, because it is a trivial FD.

INDUCTION: For the induction, suppose D was added when we used the FD $B_1B_2 \cdots B_m \to D$ of S. We know by the inductive hypothesis that R satisfies $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$. Now, suppose two tuples of R agree on all of $A_1, A_2, \ldots, A_n$. Then since R satisfies $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$, the two tuples must agree on all of $B_1, B_2, \ldots, B_m$. Since R satisfies $B_1B_2 \cdots B_m \to D$, we also know these two tuples agree on D. Thus, R satisfies $A_1A_2 \cdots A_n \to D$.

Why the Closure Algorithm Discovers All True FD's

Suppose $A_1A_2 \cdots A_n \to B$ were an FD that Algorithm 3.7 says does not follow from set S. That is, the closure of $\{A_1, A_2, \ldots, A_n\}$ using set of FD's S does not include B. We must show that FD $A_1A_2 \cdots A_n \to B$ really doesn't follow from S. That is, we must show that there is at least one relation instance that satisfies all the FD's in S, and yet does not satisfy $A_1A_2 \cdots A_n \to B$.

This instance I is actually quite simple to construct; it is shown in Fig. 3.5. I has only two tuples: t and s. The two tuples agree in all the attributes of $\{A_1, A_2, \ldots, A_n\}^+$, and they disagree in all the other attributes. We must show first that I satisfies all the FD's of S, and then that it does not satisfy $A_1A_2 \cdots A_n \to B$.

	$\{A_1, A_2, \ldots, A_n\}^+$	Other Attributes
t:	1 1 1 $\cdots$ 1 1	0 0 0 $\cdots$ 0 0
s:	1 1 1 $\cdots$ 1 1	1 1 1 $\cdots$ 1 1

Figure 3.5: An instance I satisfying S but not $A_1A_2 \cdots A_n \to B$

Suppose there were some FD $C_1C_2 \cdots C_k \to D$ in set S (after splitting right sides) that instance I does not satisfy. Since I has only two tuples, t and s, those must be the two tuples that violate $C_1C_2 \cdots C_k \to D$. That is, t and s agree in all the attributes of $\{C_1, C_2, \ldots, C_k\}$, yet disagree on D. If we

examine Fig. 3.5 we see that all of $C_1, C_2, \ldots, C_k$ must be among the attributes of $\{A_1, A_2, \ldots, A_n\}^+$, because those are the only attributes on which t and s agree. Likewise, D must be among the other attributes, because only on those attributes do t and s disagree.

But then we did not compute the closure correctly. $C_1C_2\cdots C_k \rightarrow D$ should have been applied when X was $\{A_1, A_2, \ldots, A_n\}$ to add D to X. We conclude that $C_1C_2\cdots C_k \rightarrow D$ cannot exist; i.e., instance I satisfies S.

Second, we must show that I does not satisfy $A_1A_2\cdots A_n \rightarrow B$. However, this part is easy. Surely, $A_1, A_2, \ldots, A_n$ are among the attributes on which t and s agree. Also, we know that B is not in $\{A_1, A_2, \ldots, A_n\}^+$, so B is one of the attributes on which t and s disagree. Thus, I does not satisfy $A_1A_2\cdots A_n \rightarrow B$. We conclude that Algorithm 3.7 asserts neither too few nor too many FD's; it asserts exactly those FD's that do follow from S.

3.2.6 The Transitive Rule

The transitive rule lets us cascade two FD's, and generalizes the observation of Example 3.4.

- If $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ and $B_1B_2\cdots B_m \rightarrow C_1C_2\cdots C_k$ hold in relation R, then $A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$ also holds in R.

If some of the C's are among the A's, we may eliminate them from the right side by the trivial-dependencies rule.

To see why the transitive rule holds, apply the test of Section 3.2.4. To test whether $A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$ holds, we need to compute the closure $\{A_1, A_2, \ldots, A_n\}^+$ with respect to the two given FD's.

The FD $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ tells us that all of $B_1, B_2, \ldots, B_m$ are in $\{A_1, A_2, \ldots, A_n\}^+$. Then, we can use the FD $B_1B_2\cdots B_m \rightarrow C_1C_2\cdots C_k$ to add $C_1, C_2, \ldots, C_k$ to $\{A_1, A_2, \ldots, A_n\}^+$. Since all the C's are in

$$\{A_1, A_2, \ldots, A_n\}^+$$

we conclude that $A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$ holds for any relation that satisfies both $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ and $B_1B_2\cdots B_m \rightarrow C_1C_2\cdots C_k$.

Example 3.10: Here is another version of the `Movies` relation that includes both the studio of the movie and some information about that studio.

title	*year*	*length*	*genre*	*studioName*	*studioAddr*
`Star Wars`	1977	124	`sciFi`	`Fox`	`Hollywood`
`Eight Below`	2005	120	`drama`	`Disney`	`Buena Vista`
`Wayne's World`	1992	95	`comedy`	`Paramount`	`Hollywood`

Two of the FD's that we might reasonably claim to hold are:

`title year` $\rightarrow$ `studioName`
`studioName` $\rightarrow$ `studioAddr`

Closures and Keys

Notice that $\{A_1, A_2, \ldots, A_n\}^+$ is the set of all attributes of a relation if and only if $A_1, A_2, \ldots, A_n$ is a superkey for the relation. For only then does $A_1, A_2, \ldots, A_n$ functionally determine all the other attributes. We can test if $A_1, A_2, \ldots, A_n$ is a key for a relation by checking first that $\{A_1, A_2, \ldots, A_n\}^+$ is all attributes, and then checking that, for no set X formed by removing one attribute from $\{A_1, A_2, \ldots, A_n\}$, is X^+ the set of all attributes.

The first is justified because there can be only one movie with a given title and year, and there is only one studio that owns a given movie. The second is justified because studios have unique addresses.

The transitive rule allows us to combine the two FD's above to get a new FD:

```
title year → studioAddr
```

This FD says that a title and year (i.e., a movie) determines an address — the address of the studio owning the movie. □

3.2.7 Closing Sets of Functional Dependencies

Sometimes we have a choice of which FD's we use to represent the full set of FD's for a relation. If we are given a set of FD's S (such as the FD's that hold in a given relation), then any set of FD's equivalent to S is said to be a *basis* for S. To avoid some of the explosion of possible bases, we shall limit ourselves to considering only bases whose FD's have singleton right sides. If we have any basis, we can apply the splitting rule to make the right sides be singletons. A *minimal basis* for a relation is a basis B that satisfies three conditions:

1. All the FD's in B have singleton right sides.
2. If any FD is removed from B, the result is no longer a basis.
3. If for any FD in B we remove one or more attributes from the left side of F, the result is no longer a basis.

Notice that no trivial FD can be in a minimal basis, because it could be removed by rule (2).

Example 3.11: Consider a relation $R(A, B, C)$ such that each attribute functionally determines the other two attributes. The full set of derived FD's thus includes six FD's with one attribute on the left and one on the right; $A \to B$, $A \to C$, $B \to A$, $B \to C$, $C \to A$, and $C \to B$. It also includes the three

A Complete Set of Inference Rules

If we want to know whether one FD follows from some given FD's, the closure computation of Section 3.2.4 will always serve. However, it is interesting to know that there is a set of rules, called *Armstrong's axioms*, from which it is possible to derive any FD that follows from a given set. These axioms are:

1. *Reflexivity.* If $\{B_1, B_2, \ldots, B_m\} \subseteq \{A_1, A_2, \ldots, A_n\}$, then $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$. These are what we have called trivial FD's.

2. *Augmentation.* If $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$, then

   $$A_1A_2\cdots A_nC_1C_2\cdots C_k \rightarrow B_1B_2\cdots B_mC_1C_2\cdots C_k$$

   for any set of attributes $C_1, C_2, \ldots, C_k$. Since some of the C's may also be A's or B's or both, we should eliminate from the left side duplicate attributes and do the same for the right side.

3. *Transitivity.* If

   $$A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m \text{ and } B_1B_2\cdots B_m \rightarrow C_1C_2\cdots C_k$$

   then $A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$.

nontrivial FD's with two attributes on the left: $AB \rightarrow C$, $AC \rightarrow B$, and $BC \rightarrow A$. There are also FD's with more than one attribute on the right, such as $A \rightarrow BC$, and trivial FD's such as $A \rightarrow A$.

Relation R and its FD's have several minimal bases. One is

$$\{A \rightarrow B,\ B \rightarrow A,\ B \rightarrow C,\ C \rightarrow B\}$$

Another is $\{A \rightarrow B,\ B \rightarrow C,\ C \rightarrow A\}$. There are several other minimal bases for R, and we leave their discovery as an exercise. □

3.2.8 Projecting Functional Dependencies

When we study design of relation schemas, we shall also have need to answer the following question about FD's. Suppose we have a relation R with set of FD's S, and we project R by computing $R_1 = \pi_L(R)$, for some list of attributes R. What FD's hold in R_1?

The answer is obtained in principle by computing the *projection of functional dependencies* S, which is all FD's that:

a) Follow from S, and

b) Involve only attributes of R_1.

Since there may be a large number of such FD's, and many of them may be redundant (i.e., they follow from other such FD's), we are free to simplify that set of FD's if we wish. However, in general, the calculation of the FD's for R_1 is exponential in the number of attributes of R_1. The simple algorithm is summarized below.

Algorithm 3.12: Projecting a Set of Functional Dependencies.

INPUT: A relation R and a second relation R_1 computed by the projection $R_1 = \pi_L(R)$. Also, a set of FD's S that hold in R.

OUTPUT: The set of FD's that hold in R_1.

METHOD:

1. Let T be the eventual output set of FD's. Initially, T is empty.
2. For each set of attributes X that is a subset of the attributes of R_1, compute X^+. This computation is performed with respect to the set of FD's S, and may involve attributes that are in the schema of R but not R_1. Add to T all nontrivial FD's $X \to A$ such that A is both in X^+ and an attribute of R_1.
3. Now, T is a basis for the FD's that hold in R_1, but may not be a minimal basis. We may construct a minimal basis by modifying T as follows:
   (a) If there is an FD F in T that follows from the other FD's in T, remove F from T.
   (b) Let $Y \to B$ be an FD in T, with at least two attributes in Y, and let Z be Y with one of its attributes removed. If $Z \to B$ follows from the FD's in T (including $Y \to B$), then replace $Y \to B$ by $Z \to B$.
   (c) Repeat the above steps in all possible ways until no more changes to T can be made.

□

Example 3.13: Suppose $R(A, B, C, D)$ has FD's $A \to B$, $B \to C$, and $C \to D$. Suppose also that we wish to project out the attribute B, leaving a relation $R_1(A, C, D)$. In principle, to find the FD's for R_1, we need to take the closure of all eight subsets of $\{A, C, D\}$, using the full set of FD's, including those involving B. However, there are some obvious simplifications we can make.

- Closing the empty set and the set of all attributes cannot yield a nontrivial FD.

- If we already know that the closure of some set X is all attributes, then we cannot discover any new FD's by closing supersets of X.

Thus, we may start with the closures of the singleton sets, and then move on to the doubleton sets if necessary. For each closure of a set X, we add the FD $X \to E$ for each attribute E that is in X^+ and in the schema of R_1, but not in X.

First, $\{A\}^+ = \{A, B, C, D\}$. Thus, $A \to C$ and $A \to D$ hold in R_1. Note that $A \to B$ is true in R, but makes no sense in R_1 because B is not an attribute of R_1.

Next, we consider $\{C\}^+ = \{C, D\}$, from which we get the additional FD $C \to D$ for R_1. Since $\{D\}^+ = \{D\}$, we can add no more FD's, and are done with the singletons.

Since $\{A\}^+$ includes all attributes of R_1, there is no point in considering any superset of $\{A\}$. The reason is that whatever FD we could discover, for instance $AC \to D$, follows from an FD with only A on the left side: $A \to D$ in this case. Thus, the only doubleton whose closure we need to take is $\{C, D\}^+ = \{C, D\}$. This observation allows us to add nothing. We are done with the closures, and the FD's we have discovered are $A \to C$, $A \to D$, and $C \to D$.

If we wish, we can observe that $A \to D$ follows from the other two by transitivity. Therefore a simpler, equivalent set of FD's for R_1 is $A \to C$ and $C \to D$. This set is, in fact, a minimal basis for the FD's of R_1. □

3.2.9 Exercises for Section 3.2

Exercise 3.2.1: Consider a relation with schema $R(A, B, C, D)$ and FD's $AB \to C$, $C \to D$, and $D \to A$.

a) What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attributes on the right side.

b) What are all the keys of R?

c) What are all the superkeys for R that are not keys?

Exercise 3.2.2: Repeat Exercise 3.2.1 for the following schemas and sets of FD's:

i) $S(A, B, C, D)$ with FD's $A \to B$, $B \to C$, and $B \to D$.

ii) $T(A, B, C, D)$ with FD's $AB \to C$, $BC \to D$, $CD \to A$, and $AD \to B$.

iii) $U(A, B, C, D)$ with FD's $A \to B$, $B \to C$, $C \to D$, and $D \to A$.

Exercise 3.2.3: Show that the following rules hold, by using the closure test of Section 3.2.4.

a) *Augmenting left sides.* If $A_1 A_2 \cdots A_n \to B$ is an FD, and C is another attribute, then $A_1 A_2 \cdots A_n C \to B$ follows.

b) *Full augmentation.* If $A_1A_2\cdots A_n \to B$ is an FD, and C is another attribute, then $A_1A_2\cdots A_nC \to BC$ follows. Note: from this rule, the "augmentation" rule mentioned in the box of Section 3.2.7 on "A Complete Set of Inference Rules" can easily be proved.

c) *Pseudotransitivity.* Suppose FD's $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and

$$C_1C_2\cdots C_k \to D$$

hold, and the B's are each among the C's. Then

$$A_1A_2\cdots A_nE_1E_2\cdots E_j \to D$$

holds, where the E's are all those of the C's that are not found among the B's.

d) *Addition.* If FD's $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and

$$C_1C_2\cdots C_k \to D_1D_2\cdots D_j$$

hold, then FD $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_mD_1D_2\cdots D_j$ also holds. In the above, we should remove one copy of any attribute that appears among both the A's and C's or among both the B's and D's.

! **Exercise 3.2.4:** Show that each of the following are *not* valid rules about FD's by giving example relations that satisfy the given FD's (following the "if") but not the FD that allegedly follows (after the "then").

a) If $A \to B$ then $B \to A$.

b) If $AB \to C$ and $A \to C$, then $B \to C$.

c) If $AB \to C$, then $A \to C$ or $B \to C$.

! **Exercise 3.2.5:** Show that if a relation has no attribute that is functionally determined by all the other attributes, then the relation has no nontrivial FD's at all.

! **Exercise 3.2.6:** Let X and Y be sets of attributes. Show that if $X \subseteq Y$, then $X^+ \subseteq Y^+$, where the closures are taken with respect to the same set of FD's.

! **Exercise 3.2.7:** Prove that $(X^+)^+ = X^+$.

!! **Exercise 3.2.8:** We say a set of attributes X is *closed* (with respect to a given set of FD's) if $X^+ = X$. Consider a relation with schema $R(A, B, C, D)$ and an unknown set of FD's. If we are told which sets of attributes are closed, we can discover the FD's. What are the FD's if:

a) All sets of the four attributes are closed.

b) The only closed sets are $\emptyset$ and $\{A, B, C, D\}$.

c) The closed sets are $\emptyset$, {A,B}, and $\{A, B, C, D\}$.

! **Exercise 3.2.9:** Find all the minimal bases for the FD's and relation of Example 3.11.

! **Exercise 3.2.10:** Suppose we have relation $R(A, B, C, D, E)$, with some set of FD's, and we wish to project those FD's onto relation $S(A, B, C)$. Give the FD's that hold in S if the FD's for R are:

a) $AB \to DE$, $C \to E$, $D \to C$, and $E \to A$.

b) $A \to D$, $BD \to E$, $AC \to E$, and $DE \to B$.

c) $AB \to D$, $AC \to E$, $BC \to D$, $D \to A$, and $E \to B$.

d) $A \to B$, $B \to C$, $C \to D$, $D \to E$, and $E \to A$.

In each case, it is sufficient to give a minimal basis for the full set of FD's of S.

!! **Exercise 3.2.11:** Show that if an FD F follows from some given FD's, then we can prove F from the given FD's using Armstrong's axioms (defined in the box "A Complete Set of Inference Rules" in Section 3.2.7). *Hint*: Examine Algorithm 3.7 and show how each step of that algorithm can be mimicked by inferring some FD's by Armstrong's axioms.

3.3 Design of Relational Database Schemas

Careless selection of a relational database schema can lead to redundancy and related anomalies. For instance, consider the relation in Fig. 3.2, which we reproduce here as Fig. 3.6. Notice that the length and genre for *Star Wars* and *Wayne's World* are each repeated, once for each star of the movie. The repetition of this information is redundant. It also introduces the potential for several kinds of errors, as we shall see.

In this section, we shall tackle the problem of design of good relation schemas in the following stages:

1. We first explore in more detail the problems that arise when our schema is poorly designed.
2. Then, we introduce the idea of "decomposition," breaking a relation schema (set of attributes) into two smaller schemas.
3. Next, we introduce "Boyce-Codd normal form," or "BCNF," a condition on a relation schema that eliminates these problems.
4. These points are tied together when we explain how to assure the BCNF condition by decomposing relation schemas.

title	*year*	*length*	*genre*	*studioName*	*starName*
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	drama	MGM	Vivien Leigh
Wayne's World	1992	95	comedy	Paramount	Dana Carvey
Wayne's World	1992	95	comedy	Paramount	Mike Meyers

Figure 3.6: The relation `Movies1` exhibiting anomalies

3.3.1 Anomalies

Problems such as redundancy that occur when we try to cram too much into a single relation are called *anomalies*. The principal kinds of anomalies that we encounter are:

1. *Redundancy.* Information may be repeated unnecessarily in several tuples. Examples are the length and genre for movies in Fig. 3.6.

2. *Update Anomalies.* We may change information in one tuple but leave the same information unchanged in another. For example, if we found that *Star Wars* is really 125 minutes long, we might carelessly change the length in the first tuple of Fig. 3.6 but not in the second or third tuples. You might argue that one should never be so careless, but it is possible to redesign relation `Movies1` so that the risk of such mistakes does not exist.

3. *Deletion Anomalies.* If a set of values becomes empty, we may lose other information as a side effect. For example, should we delete Vivien Leigh from the set of stars of *Gone With the Wind*, then we have no more stars for that movie in the database. The last tuple for *Gone With the Wind* in the relation `Movies1` would disappear, and with it information that it is 231 minutes long and a drama.

3.3.2 Decomposing Relations

The accepted way to eliminate these anomalies is to *decompose* relations. Decomposition of R involves splitting the attributes of R to make the schemas of two new relations. After describing the decomposition process, we shall show how to pick a decomposition that eliminates anomalies.

Given a relation $R(A_1, A_2, \ldots, A_n)$, we may *decompose* R into two relations $S(B_1, B_2, \ldots, B_m)$ and $T(C_1, C_2, \ldots, C_k)$ such that:

1. $\{A_1, A_2, \ldots, A_n\} = \{B_1, B_2, \ldots, B_m\} \cup \{C_1, C_2, \ldots, C_k\}$.

2. $S = \pi_{B_1, B_2, \ldots, B_m}(R)$.

3. $T = \pi_{C_1, C_2, \ldots, C_k}(R)$.

Example 3.14: Let us decompose the `Movies1` relation of Fig. 3.6. Our choice, whose merit will be seen in Section 3.3.3, is to use:

1. A relation called `Movies2`, whose schema is all the attributes except for `starName`.
2. A relation called `Movies3`, whose schema consists of the attributes `title`, `year`, and `starName`.

The projection of `Movies1` onto these two new schemas is shown in Fig, 3.7. □

title	*year*	*length*	*genre*	*studioName*
Star Wars	1977	124	sciFi	Fox
Gone With the Wind	1939	231	drama	MGM
Wayne's World	1992	95	comedy	Paramount

(b) The relation `Movies2`.

title	*year*	*starName*
Star Wars	1977	Carrie Fisher
Star Wars	1977	Mark Hamill
Star Wars	1977	Harrison Ford
Gone With the Wind	1939	Vivien Leigh
Wayne's World	1992	Dana Carvey
Wayne's World	1992	Mike Meyers

(b) The relation `Movies3`.

Figure 3.7: Projections of relation `Movies1`

Notice how this decomposition eliminates the anomalies we mentioned in Section 3.3.1. The redundancy has been eliminated; for example, the length of each film appears only once, in relation `Movies2`. The risk of an update anomaly is gone. For instance, since we only have to change the length of *Star Wars* in one tuple of `Movies2`, we cannot wind up with two different lengths for that movie.

Finally, the risk of a deletion anomaly is gone. If we delete all the stars for *Gone With the Wind*, say, that deletion makes the movie disappear from `Movies3`. But all the other information about the movie can still be found in `Movies2`.