**Example 6.3** (Fourier transform of the rectangular function). Find the Fourier transform X of the function

$$x(t) = \text{rect} t$$
.  $\begin{bmatrix} \text{rect } t = \begin{cases} 1 & \text{lt} < \frac{t}{2} \\ 0 & \text{otherwise} \end{bmatrix}$ 

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \operatorname{rect}(t) e^{-j\omega t} dt$$

Solution. From the definition of the Fourier transform, we can write  $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \qquad X(\omega) = \int_{-\infty}^{\infty} \operatorname{rect}(t) e^{-j\omega t} dt.$  Substitute given function x into Fourier transform analysis equation

From the definition of the rectangular function, we can simplify this equation to obtain change limits since

$$X(\omega) = \int_{-1/2}^{1/2} \operatorname{rect}(t) e^{-j\omega t} dt$$
$$= \int_{-1/2}^{1/2} e^{-j\omega t} dt.$$

Evaluating the integral and simplifying, we have

$$X(\omega) = \int_{-1/2}^{1/2} \operatorname{rect}(t) e^{-j\omega t} dt$$

$$= \int_{-1/2}^{1/2} e^{-j\omega t} dt.$$

$$= \int_{-1/2}^{1/2} e^{-j\omega t} dt.$$

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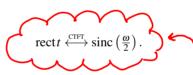
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Thus, we have shown that



Note: This is why the sinc function is of great importance.