4 **5.18** Find the Fourier series coefficient sequence c of each periodic function x given below with fundamental period T.

(a) $x(t) = 2\delta(t-3) + 2\delta(t-5) + \delta(t-7) - \delta(t-9) + 3\delta(t-12)$ and $T = 16$; express c in terms of sin and cos to whatever extent is possible.

Exercise 5.18

4 Answer (a).

From the Fourier series analysis equation, we have

$$
\begin{aligned}
c_k &= \tfrac{1}{T}\int_0^T x(t)e^{-j(2\pi/T)kt}dt \\
&= \tfrac{1}{16}\int_0^{16} [2\delta(t-3)+2\delta(t-5)+\delta(t-7)-\delta(t-9)+3\delta(t-12)]\,e^{-j(2\pi/16)kt}dt \\
&= \tfrac{1}{16}\int_{-\infty}^{\infty} [2\delta(t-3)+2\delta(t-5)+\delta(t-7)-\delta(t-9)+3\delta(t-12)]\,e^{-j(\pi/8)kt}dt \\
&= \tfrac{1}{16}\left[\int_{-\infty}^{\infty} 2\delta(t-3)e^{-j(\pi/8)kt}dt+\int_{-\infty}^{\infty} 2\delta(t-5)e^{-j(\pi/8)kt}dt+\int_{-\infty}^{\infty} \delta(t-7)e^{-j(\pi/8)kt}dt\right. \\
&\qquad \left.-\int_{-\infty}^{\infty} \delta(t-9)e^{-j(\pi/8)kt}dt+\int_{-\infty}^{\infty} 3\delta(t-12)e^{-j(\pi/8)kt}dt\right] \\
&= \tfrac{1}{16}\left[2e^{-j(\pi/8)k(3)}+2e^{-j(\pi/8)k(5)}+e^{-j(\pi/8)k(7)}-e^{-j(\pi/8)k(9)}+3e^{-j(\pi/8)k(12)}\right] \\
&= \tfrac{1}{16}\left[2e^{-j(3\pi/8)k}+2e^{-j(5\pi/8)k}+e^{-j(7\pi/8)k}-e^{-j(9\pi/8)k}+3e^{-j(3\pi/2)k}\right] \\
&= \tfrac{1}{16}\left[2e^{-j(4\pi/8)k}\left(e^{j(\pi/8)k}+e^{-j(\pi/8)k}\right)+e^{-j\pi k}\left(e^{j(\pi/8)k}-e^{-j(\pi/8)k}\right)+3e^{-j(3\pi/2)k}\right] \\
&= \tfrac{1}{16}\left[2(-j)^k\left[2\cos\left(\tfrac{\pi}{8}k\right)\right]+(-1)^k\left[2j\sin\left(\tfrac{\pi}{8}k\right)\right]+3j^k\right] \\
&= \tfrac{1}{16}\left[4(-j)^k\cos\left(\tfrac{\pi}{8}k\right)+2j(-1)^k\sin\left(\tfrac{\pi}{8}k\right)+3j^k\right] \\
&= \tfrac{1}{4}(-j)^k\cos\left(\tfrac{\pi}{8}k\right)+\tfrac{j}{8}(-1)^k\sin\left(\tfrac{\pi}{8}k\right)+\tfrac{3}{16}j^k.
\end{aligned}
$$