Unit 2: Short Truth Tables

The problem with truth tables, even partial truth tables, is that they become excessively long when our argument has more than a few letters. Say that our argument has seven letters. That would mean 2 x 2 x 2 x 2 x 2 x 2 x 2 = 128 lines in our truth table! So we need to come up with a way to figure out if arguments with many letters are valid or invalid. Short truth tables enable us to do just that.

With partial truth tables we were able to ignore some of the lines because there was no way that that line would give us information that would be relevant for an invalid argument. That is, we were looking for a row that had all true premises and a false conclusion. If we completed the column for the conclusion, then any row that had a true conclusion was a waste of time to complete. We need the conclusion to be false. Similarly for the premises. If we completed the column for a premise and there was a row where the premise was false, then there's no point in continuing on with that line. We need our premises to be true.

So, with a short truth table we cut the work down even further by only concerning ourselves with the combination that we are looking for.

Before we get to short truth tables, see if we can figure out the following question.

What truth values for A, B would make the following false?

(A v B) -> ~B

(A & ~B) -> (B v A)

With short truth tables we focus our attention on the specific line(s) in a truth table that show an argument to be invalid.

That is, lines where the premises are all true and the conclusion is false.

<u>Step One</u>

Take the argument and write it out horizontally.

~A → (B v C)
<u>~B______</u>
C → A

~A → (B v C) / ~B // C → A

<u>Step Two</u>

Write T under the major operator of each premises and F under the major operator of the conclusion.

~A → (B v C) / ~B // C → A

<u>Step Three</u>

Assign values to each letter to make the statements have the required truth value.

~A → (B v C) / ~B // C → A

T T F

Sub the truth value of the letters into the other sentences.

$$\sim A \rightarrow (B \lor C) \;/\; \sim B \;//\; C \rightarrow A$$

	F	F
T	T	F

If these values make the final sentence the value that it is supposed to be then you have found a truth assignment that makes the premises true and the conclusion false.

$A \rightarrow (B \lor C)$
$B \rightarrow D$
$\underline{A}$
$\sim C \rightarrow D$

$$A \rightarrow (B \lor C) \;/\; B \rightarrow D \;/\; A \;//\; \sim C \rightarrow D$$

If you stumble across a contradiction (e.g. premise must be T and F) then you were unable to find a truth assignment to make the premises T and the conclusion F.

If there are no other possible combinations for the truth values of the letters then the argument is VALID.

$\sim A \rightarrow B$ / $B \rightarrow A$ / $A \rightarrow \sim B$ // $A \,\&\, \sim B$

The problem with the previous question is that it is not clear where we should start. Remember, we always want to start with the sentence that FORCES us to pick certain values for the letters. The premises in this case all have →s. There's lots of different ways that an arrow can be true (i.e. T → T, F → T, F → F). And for the conclusion, there's lots of different ways that an & can be false (i.e. T & F, F & T, F & F). So, where do we start?

In cases like these, where it isn't clear where you should start, pick the easiest sentence and write down all the possible combinations. Then try each combination out on its own.

For instance, if we picked the premise B → A, then we would write the options T → T, F → T, F → F underneath that premise and try each one. If we try one option and when we figure out the rest of the values it gives us an invalid line, then we're done. We know the argument is invalid. But if we try an option and it fails (we get a contradiction like we need a premise to be true but it turns out to be false) then we have to try the other options. If all the options fail, then we know that the argument is valid.

$\sim A \rightarrow B$ / $B \rightarrow A$ / $A \rightarrow \sim B$ // $A \& \sim B$

Need:	T		T	T	F
Try 1:		T	T		
Try 2:		F	T		
Try 3:		F	F		

Use the short truth table method to determine if the following are valid or invalid.

$K \rightarrow (R \vee M)$
$\underline{K \& \sim R}$
$M \rightarrow \sim K$

$(N \vee C) \rightarrow E$
$N \rightarrow \sim(C \vee H)$
$\underline{H \rightarrow E}$
$C \rightarrow H$

$K \rightarrow (R \vee M)$ / $K \& {\sim}R$ // $M \rightarrow {\sim}K$

$(N \vee C) \rightarrow E$ / $N \rightarrow {\sim}(C \vee H)$ / $H \rightarrow E$ // $C \rightarrow H$

$G \rightarrow H$	$(M \vee N) \rightarrow O$
$H \rightarrow I$	$O \rightarrow (N \vee P)$
$\sim J \rightarrow G$	$M \rightarrow (\sim Q \rightarrow N)$
$\underline{\sim I}$	$\underline{(Q \rightarrow M) \rightarrow \sim P}$
J	$N \rightarrow O$

$G \rightarrow H \;/\; H \rightarrow I \;/\; \sim J \rightarrow G \;/\; \sim I \;//\; J$

$(M \vee N) \rightarrow O \;/\; O \rightarrow (N \vee P) \;/\; M \rightarrow (\sim Q \rightarrow N) \;/\; (Q \rightarrow M) \rightarrow \sim P \;//\; N \rightarrow O$