Example 3.2. Let $x_1(t) = \sin(\pi t)$ and $x_2(t) = \sin t$. Determine whether the function $y = x_1 + x_2$ is periodic.

Solution. Denote the fundamental periods of x_1 and x_2 as T_1 and T_2, respectively. We then have

$$T_1 = \frac{2\pi}{\pi} = 2 \quad \text{and} \quad T_2 = \frac{2\pi}{1} = 2\pi.$$

Here, we used the fact that the fundamental period of $\sin(\alpha t)$ is $\frac{2\pi}{|\alpha|}$. Thus, we have

$$\frac{T_1}{T_2} = \frac{2}{2\pi} = \frac{1}{\pi}.$$

Since π is an irrational number, $\frac{T_1}{T_2}$ is not rational. Therefore, y is not periodic. ■

Example 3.4. Let $x_1(t) = \cos(6\pi t)$ and $x_2(t) = \sin(30\pi t)$. Determine if the function $y = x_1 + x_2$ is periodic, and if it is, find its fundamental period.

Solution. Let T_1 and T_2 denote the fundamental periods of x_1 and x_2, respectively. We have

$$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3} \quad \text{and} \quad T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}.$$

Thus, we have

$$\frac{T_1}{T_2} = \left(\frac{1}{3}\right)/\left(\frac{1}{15}\right) = \frac{15}{3} = \frac{5}{1}.$$

5 and 1 are coprime

Since $\frac{T_1}{T_2}$ is a rational number, y is periodic. Let T denote the fundamental period of y. Since 5 and 1 are coprime, we have

$$T = 1T_1 = 5T_2 = \frac{1}{3}.$$ ■

$$\frac{T_1}{T_2} = \frac{p}{q}$$

cross multiplication pattern (p,q coprime)

Example 3.8 (Sifting property example)**.** Evaluate the integral

$$\int_{-\infty}^{\infty} [\sin t]\delta(t - \pi/4)dt.$$

Solution. Using the sifting property of the unit impulse function, we have

$$\begin{aligned}\int_{-\infty}^{\infty} [\sin t]\delta(t - \pi/4)dt &= \sin\left(\tfrac{\pi}{4}\right) \quad = \sin t \Big|_{t=\pi/4} \\ &= \tfrac{1}{\sqrt{2}}.\end{aligned}$$

■

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$$

in this example, $x(t) = \sin t$ and $t_0 = \frac{\pi}{4}$

Example 3.9 (Sifting property example)**.** Evaluate the integral

does not have form of sifting property due to "4"

$$\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(\underbrace{4t}_{\tau}-1)dt.$$

sifting property

Solution. First, we observe that the integral to be evaluated does not quite have the same form as (3.24). So, we need to perform a change of variable. Let $\tau = 4t$ so that $t = \tau/4$ and $dt = d\tau/4$. Performing the change of variable, we obtain

change of variable

$$\begin{aligned}\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(4t-1)dt &= \int_{-\infty}^{\infty} \tfrac{1}{4}[\sin(2\pi\tau/4)]\delta(\tau-1)d\tau \\ &= \int_{-\infty}^{\infty} \underbrace{\left[\tfrac{1}{4}\sin(\pi\tau/2)\right]}_{x(\tau)}\delta(\tau-1)d\tau.\end{aligned}$$

Now the integral has the desired form, and we can use the sifting property of the unit-impulse function to write

$$\begin{aligned}\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(4t-1)dt &= \left[\tfrac{1}{4}\sin(\pi\tau/2)\right]\big|_{\tau=1} \\ &= \tfrac{1}{4}\sin(\pi/2) \\ &= \tfrac{1}{4}.\end{aligned}$$

■

Sifting property

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t_0)\,d\tau = x(t_0)$$

in this example, $x(\tau) = \frac{1}{4}\sin\left(\frac{\pi}{2}\tau\right)$ and $t_0 = 1$