Example 4.7. Consider the system with input x, output y, and impulse response h as shown in Figure 4.9. Each subsystem in the block diagram is LTI and labelled with its impulse response. Find h.

Solution. From the left half of the block diagram, we can write   To begin, we label all signals in Figure 4.9.

$$① \qquad v(t) = x(t) + x * h_1(t) + x * h_2(t)$$
$$= x * \underline{\delta(t)} + x * h_1(t) + x * h_2(t)$$
$$= (x * [\delta + h_1 + h_2])(t).$$

δ is convolutional identity

distributive property

Similarly, from the right half of the block diagram, we can write

$$y(t) = v * h_3(t). \qquad ②$$

Substituting the expression for v into the preceding equation we obtain

$$y(t) = v * h_3(t)$$
$$= (x * [\delta + h_1 + h_2]) * h_3(t)$$
$$= x * [\underline{h_3 + h_1 * h_3 + h_2 * h_3}](t).$$

from ②

substituting ①
for v

distributive and associative properties and convolutional identity

h

Thus, the impulse response h of the overall system is

$$h(t) = h_3(t) + h_1 * h_3(t) + h_2 * h_3(t). \qquad ■$$

Recall that, for any LTI system with input x,
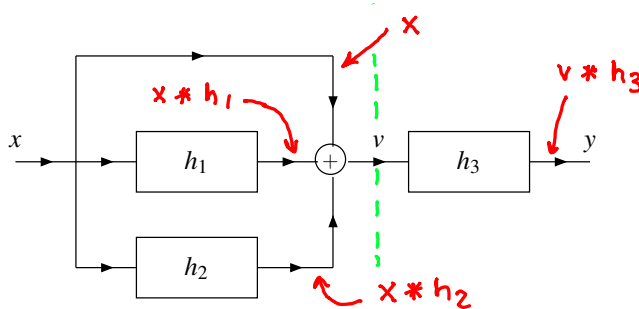output y, and impulse response h, y = x * h.

x

*x * h₁*

*v * h₃*

*x * h₂*

Figure 4.9: System interconnection example.