Exercise 5.8

L Answer (a).

Given that the function x has the Fourier series coefficient sequence c, we are asked to find the Fourier series coefficient sequence c' of the function x', where

$$x' = \mathrm{Even}\{x\}. \qquad 1$$

We know that x has the Fourier series representation

since x is periodic

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}. \qquad 2$$

From the definition of x' and the preceding equation, we have

$$\begin{aligned}
x'(t) &= \mathrm{Even}\{x\}(t) && \text{from 1} \\
&= \tfrac{1}{2}[x(t) + x(-t)] && \text{definition of even function} \\
&= \tfrac{1}{2}\left[\sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt} + \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)k(-t)}\right] && \text{substitute 2} \\
&= \tfrac{1}{2}\left[\sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt} + \sum_{k=-\infty}^{\infty} c_k e^{-j(2\pi/T)kt}\right] && \text{move minus sign in 2nd term} \\
&= \tfrac{1}{2}\left[\sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt} + \sum_{\ell=-\infty}^{\infty} c_{-\ell} e^{j(2\pi/T)\ell t}\right] && \text{change of variable in 2nd summation let } \ell = -k \\
&= \tfrac{1}{2}\sum_{k=-\infty}^{\infty} [c_k + c_{-k}] e^{j(2\pi/T)kt} && \text{factor} \\
&= \sum_{k=-\infty}^{\infty} \tfrac{1}{2}[c_k + c_{-k}] e^{j(2\pi/T)kt} && \text{pull } \tfrac{1}{2} \text{ inside summation} \\
&= \sum_{k=-\infty}^{\infty} \underbrace{\mathrm{Even}\{c\}(k)}_{\text{Fourier series coefficient}} e^{j(2\pi/T)kt}. && \text{definition of even sequence}
\end{aligned}$$

Therefore, x' has the Fourier series representation

$$x'(t) = \sum_{k=-\infty}^{\infty} c'(k) e^{j(2\pi/T)kt},$$

where $c' = \mathrm{Even}\{c\}$.

