Chapter 6 – Structural Analysis

Contents

The Eiffel Tower is probably the most famous truss structure in the World. It was built between 1887 and 1889, it is 300 m high (without antenna).

Structural Analysis

Structural Analysis

A **structure** is an arrangement of rigid members that are held together by joints.

The analysis of structures will be done in two parts:

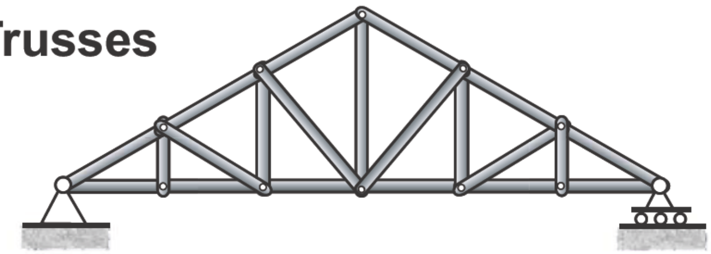
- Analysis of trusses (§6.1 – §6.4).
- Frames and machines. (§6.6).

The static analysis of structures focuses on the interactions between the structural components.

When we break a structure down into components there are several FBD's we can choose from.

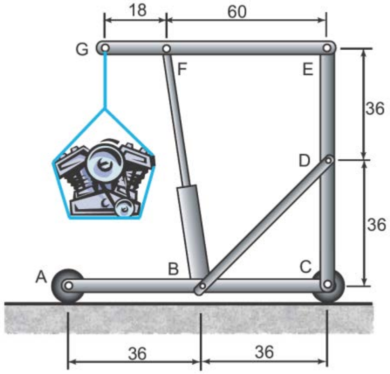
Structural Analysis

Trusses



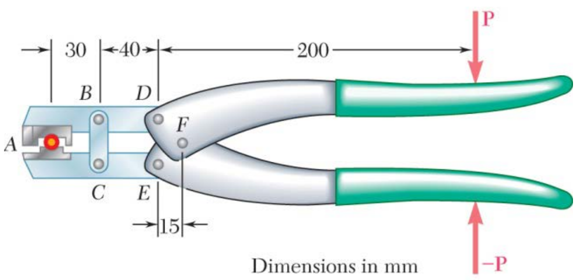
2-force members
(Tension/Compression)

Frames



At least one multi-force members
May include 2-force members

Machine



Like frames but have moving parts and transmit and modify forces

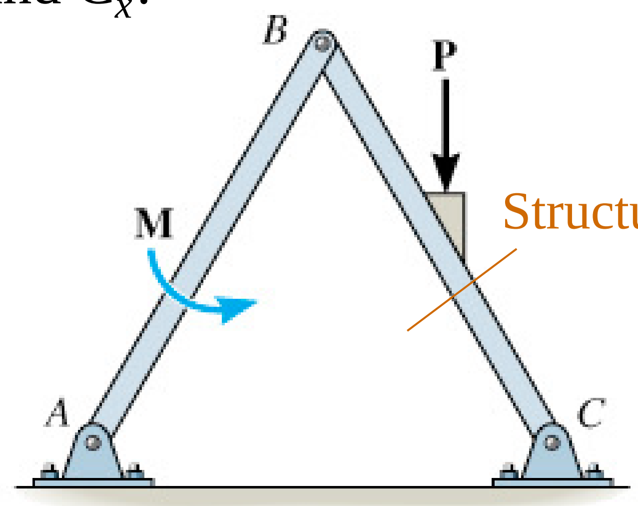
Planar Structures

Planar Structures

Coplanar equations of equilibrium

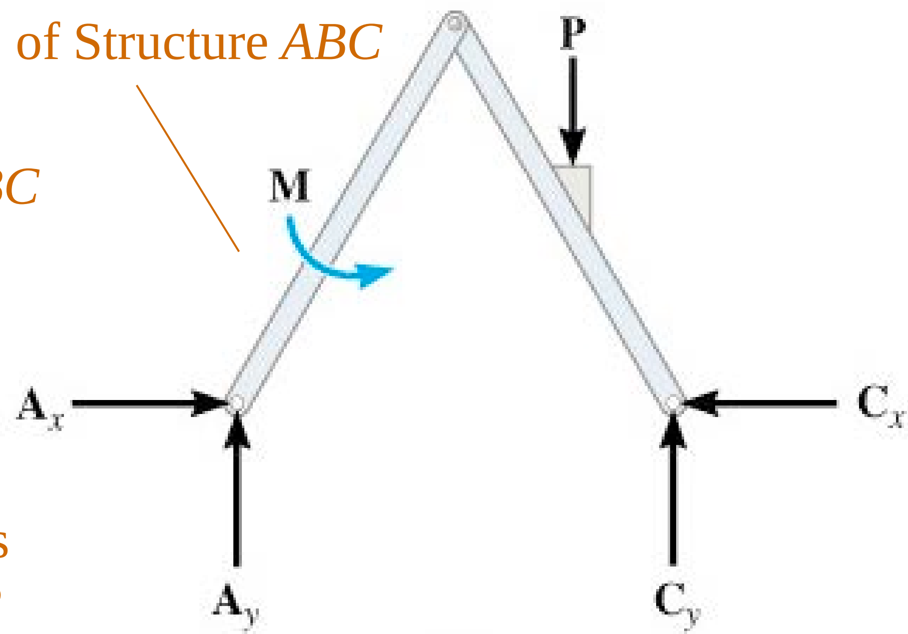
$$\sum F_x = 0 \quad ; \quad \sum F_y = 0 \quad ; \quad \sum M_{Oz} = 0$$

What is the force sustained by the pin at *B*? What about reactions A_x and C_x?



Structure *ABC*

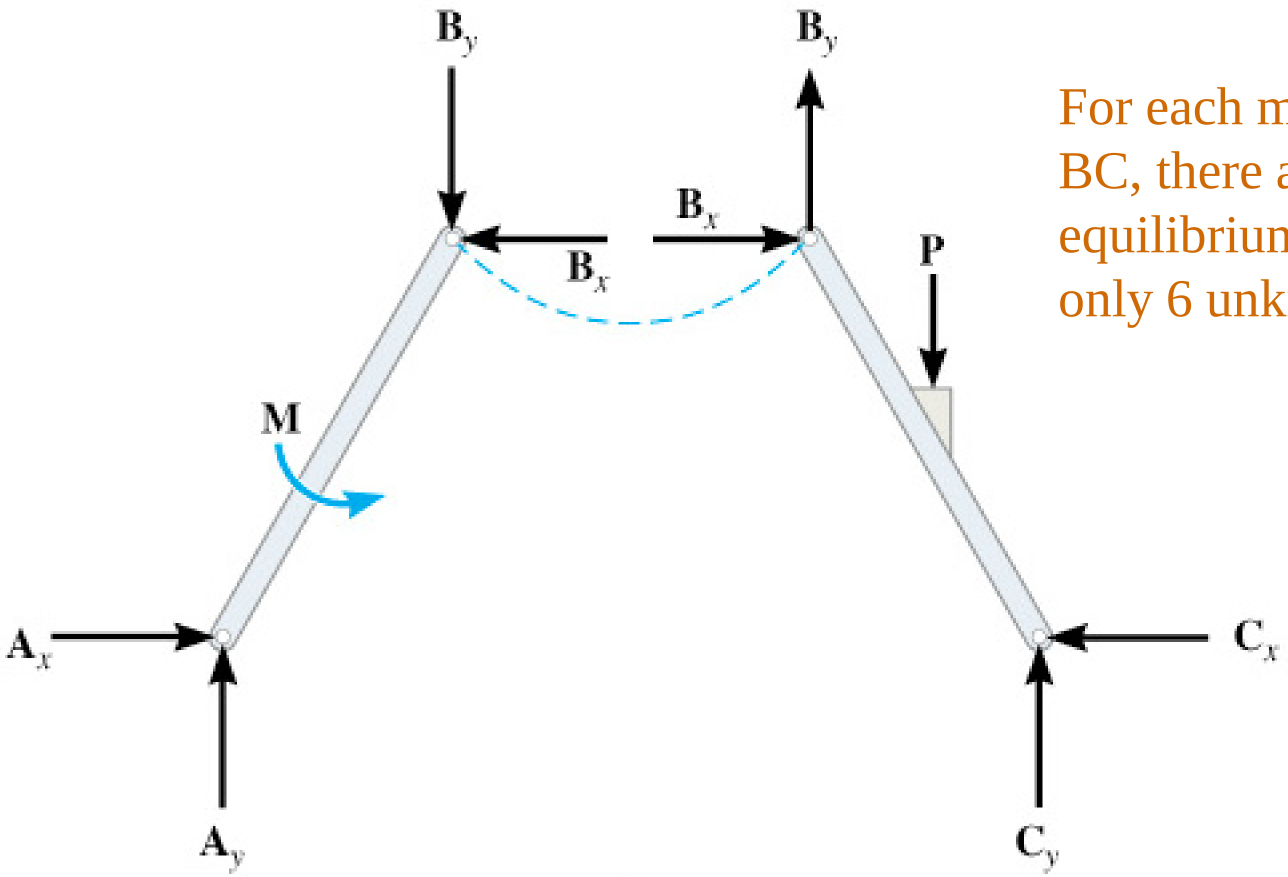
FBD of Structure *ABC*



Three equations and four unknowns, is this a statically indeterminate system?

Planar Structures

If we break down the structure, we can create multiple FBD, one for each member.

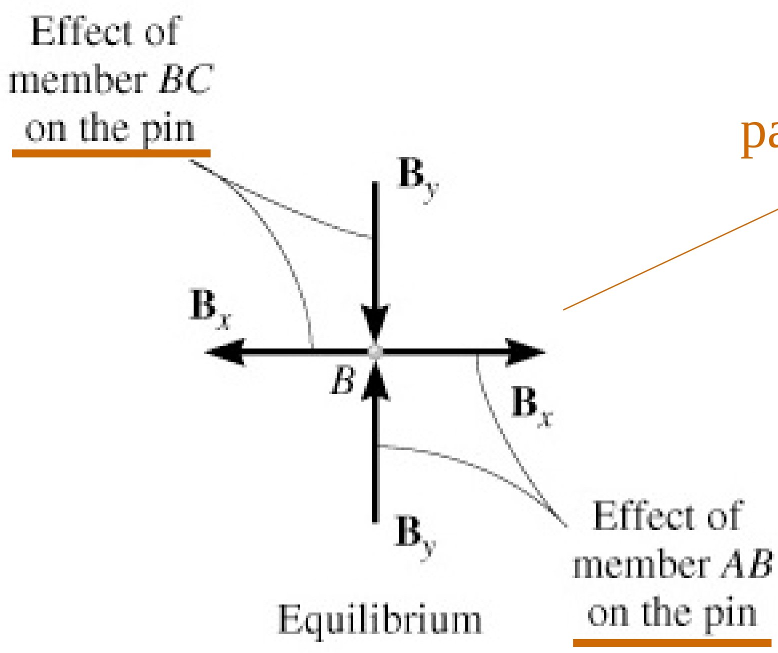


For each member, both AB and BC, there are 3 equations of equilibrium. In total there are only 6 unknowns.

This is a statically determinate system

Planar Structures

In general, provided the structure or machine contains no more supports than is necessary to keep the structure from collapsing, the system is statically determinate.

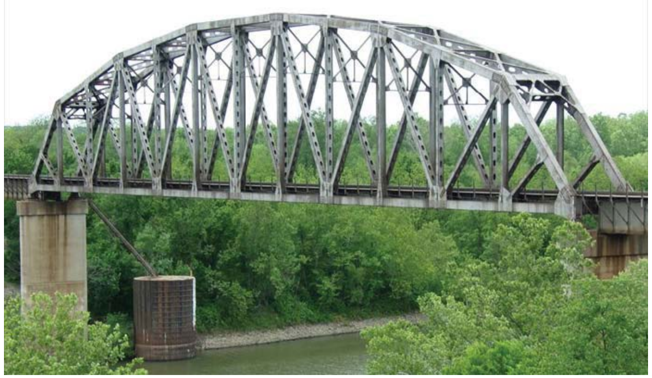


Netwon's 3rd law is paramount in our analysis of structures.

Simple Trusses

Applications

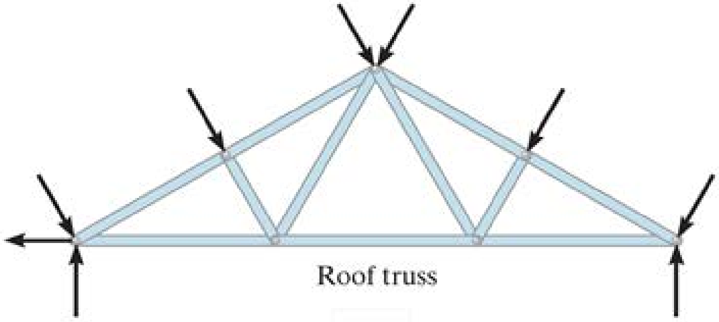
Trusses are commonly used to support roofs, cranes, power towers, and bridges.



Simple Trusses

Trusses

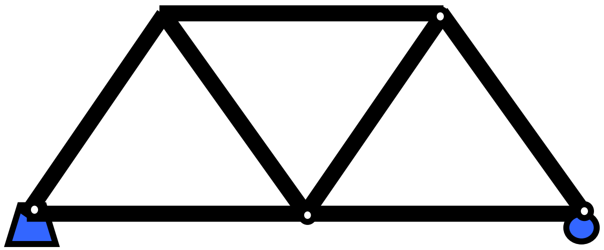
A **truss** is a structure composed of slender members joined together at their end points. If a truss, along with the load that acts on it, lies in a single plane then it is called a planar truss.



A **simple truss** is a planar truss which begins with a *triangular* element and can be expanded by adding two members and a joint. For these trusses, the number of members (m), the number of joints (j), and the number of reactions (r) are related by the equation

$$m = 2j - r.$$

Simple Trusses



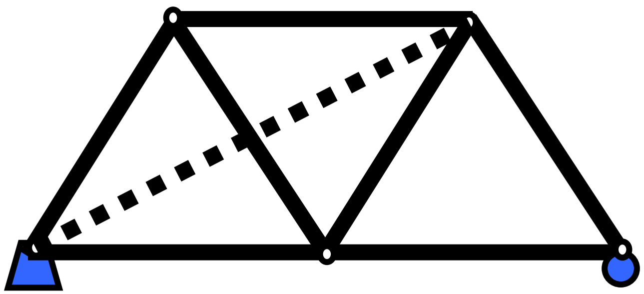
Statically Determinate

$m = 7$

$j = 5$

$r = 3$

$m = 2j - r$



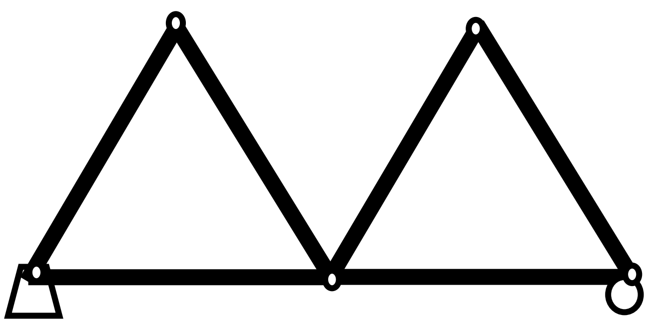
Statically Indeterminate

$m = 8$

$j = 5$

$r = 3$

$m > 2j - r$



Statically Unstable

$m = 6$

$j = 5$

$r = 3$

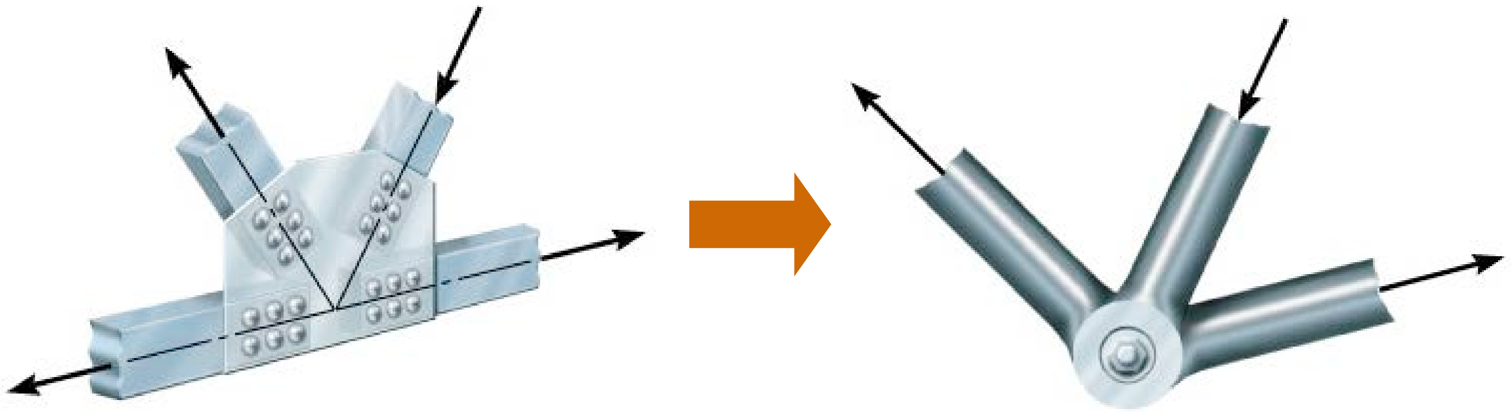
$m < 2j - r$

Simple Trusses

Assumptions for design

In order to design a truss, we need to determine the force that will be developed in each member. Thus, we make the following assumptions

- All external loads are applied at the joints.
- The members are joined together by smooth pins.

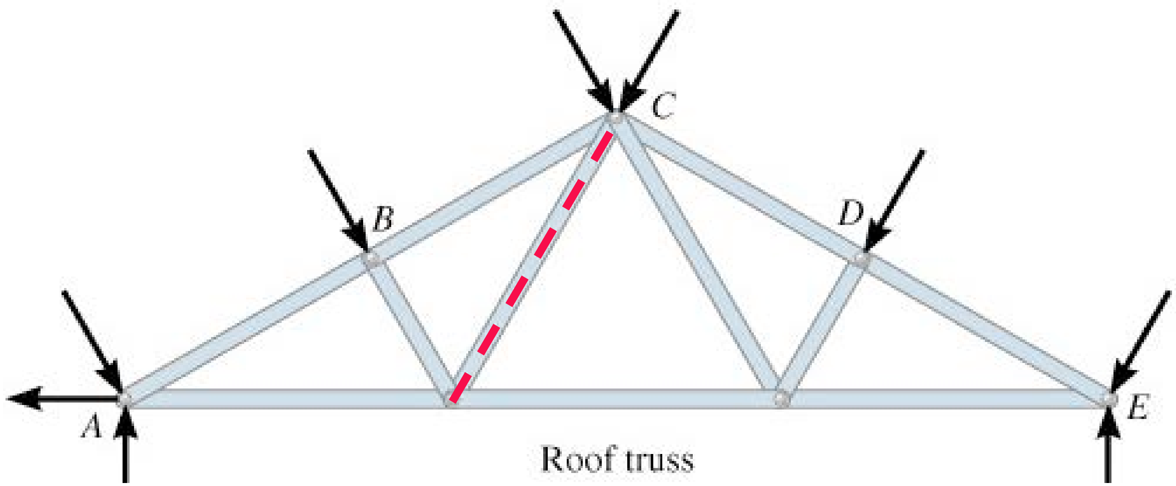


- The weights of the members are very small when compared to the external loads being applied.

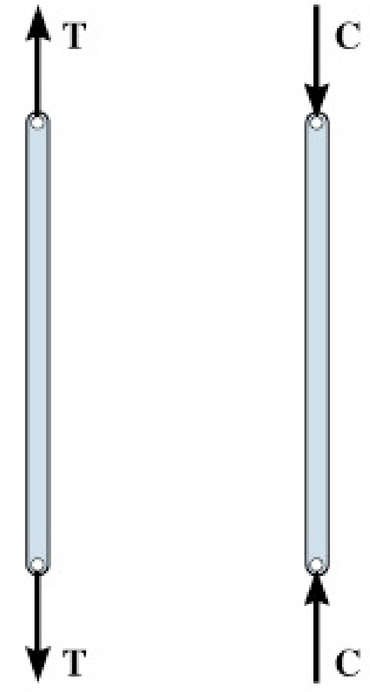
Simple Trusses

Truss Members

Since the weight of the members is the significantly smaller than the forces acting on it, it can be neglected. Thus, each member acts as a two-force member. They are loaded in either tension or compression.

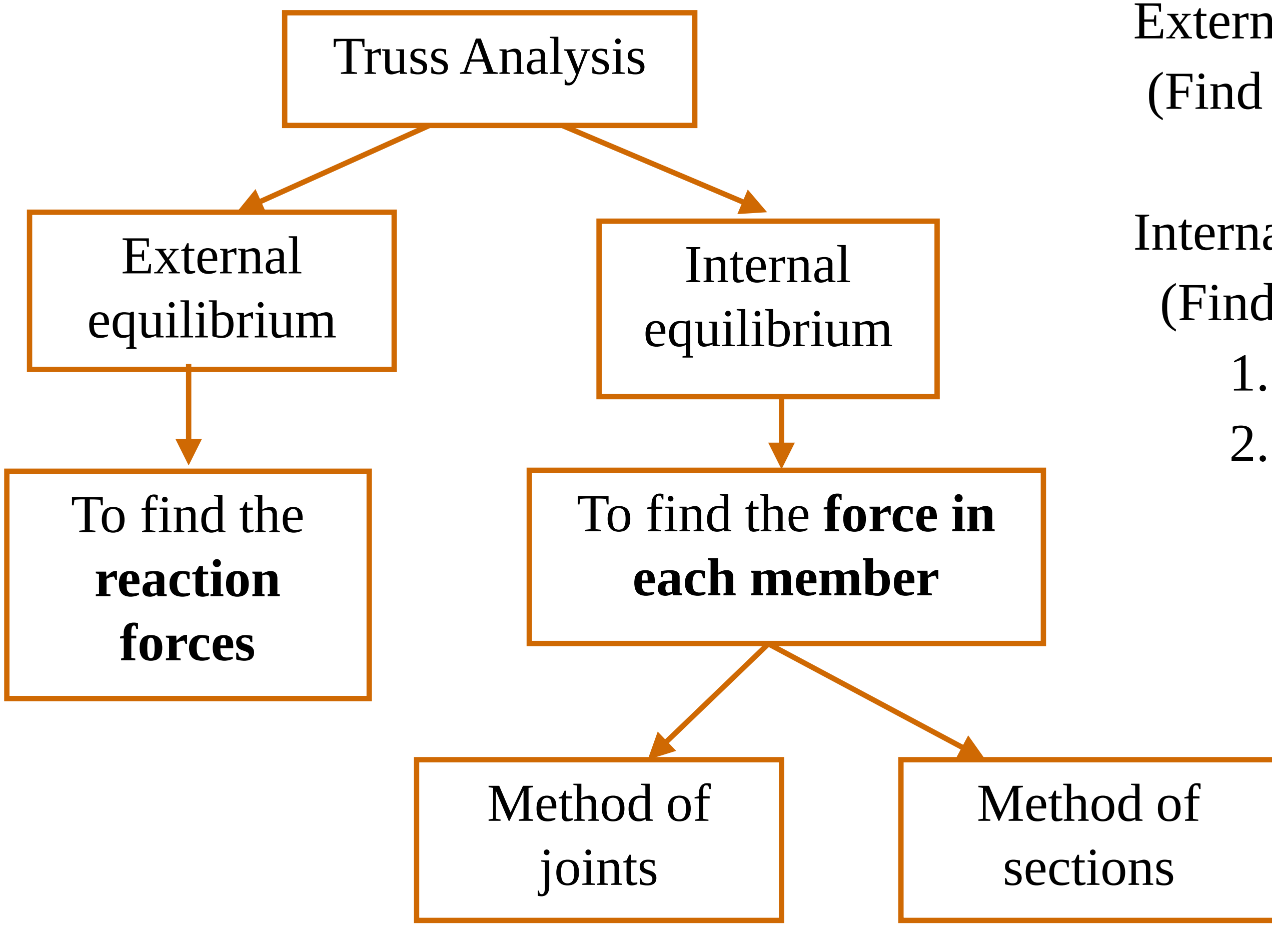


Roof truss



Tension   Compression

Truss Analysis



External Equilibrium
(Find Reaction Forces)

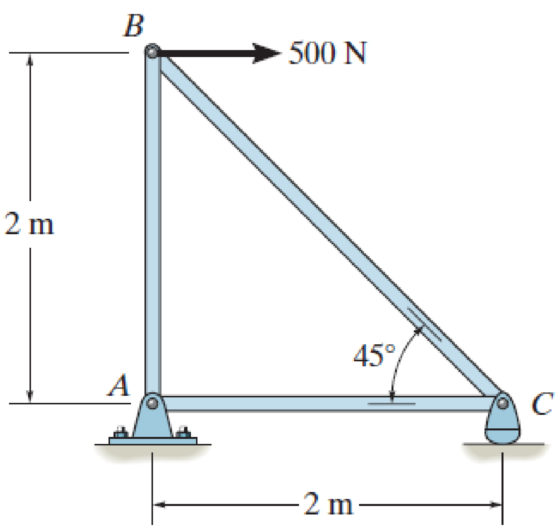
Internal Equilibrium
(Find Force in each Member)
1. Method of Joints
2. Method of Sections

Method of Joints

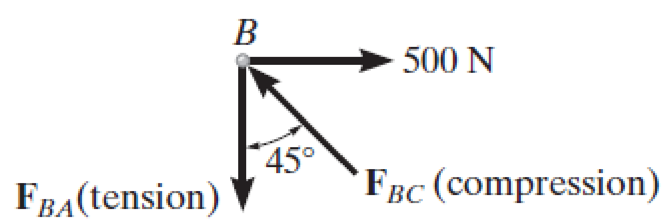
Truss Members

When using the method of joints to solve for the forces in truss members, the equilibrium of a joint (pin) is considered, i.e., a particle.

All forces acting at the joint are shown in a FBD. This includes all external forces (including support reactions) as well as the forces acting in the members. Equations of equilibrium for coplanar systems are used to solve for the unknown forces acting at the joints.



$$\sum F_x = 0 \qquad \sum F_y = 0$$



Method of Joints

Steps for Analysis

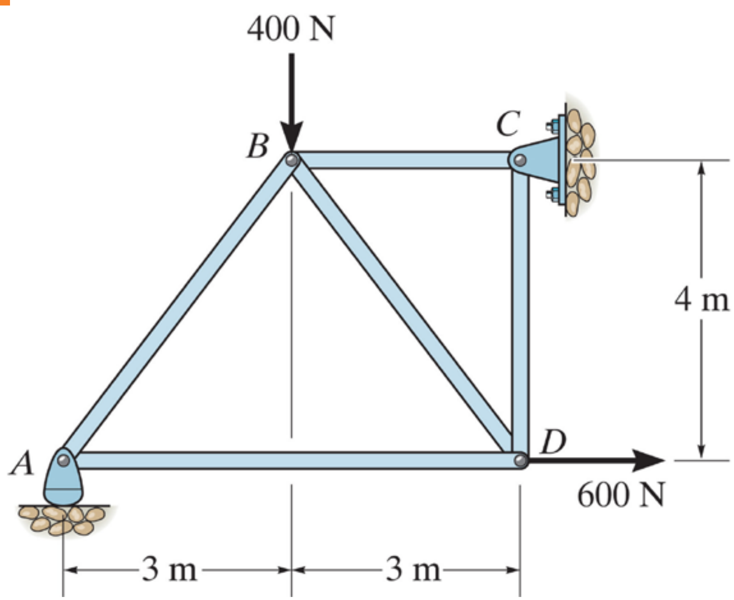
1. If the truss's support reactions are not given, draw a FBD of the entire truss and determine the support reactions.

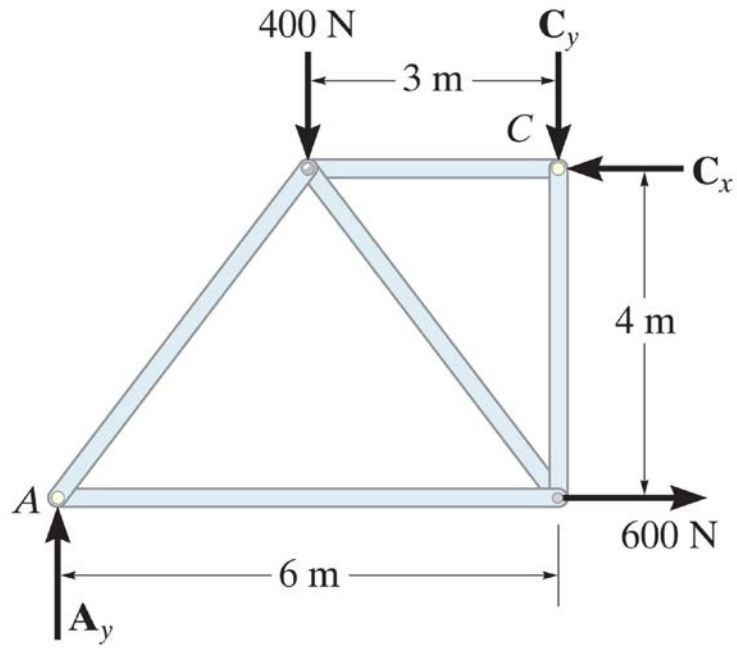
2. Draw the free-body diagram of a joint with one or two unknowns. Assume that all unknown member forces act in tension. Draw all the force vectors from the joint outwards.

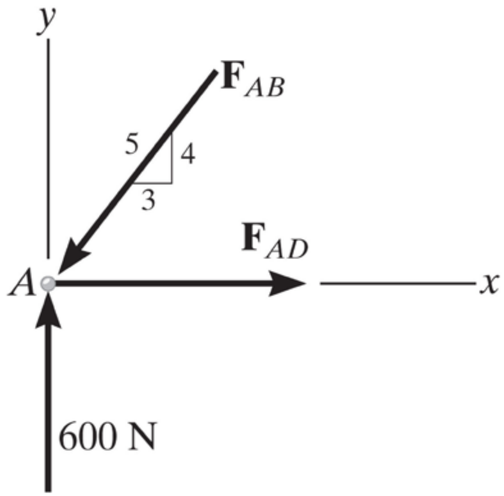
3. Apply the scalar equations of equilibrium, $\sum F_x = 0$ and $\sum F_y = 0$ to determine the unknown(s). Based on Newton's 3rd law, if the answer is positive, then the assumed direction (tension) is correct, otherwise the member is in compression.

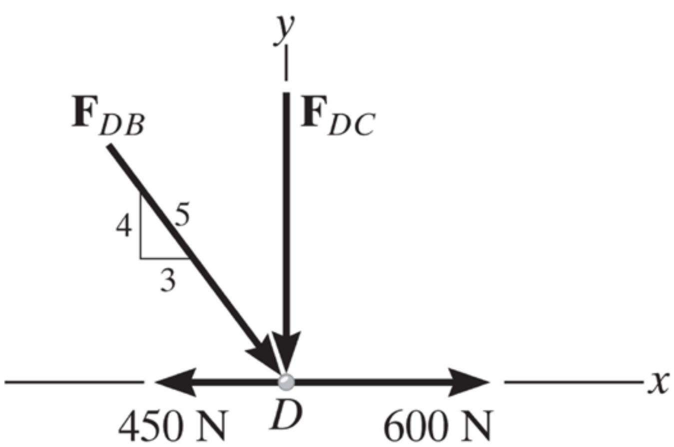
4. Repeat steps 2 and 3 at each joint in succession until all the required forces are determined.

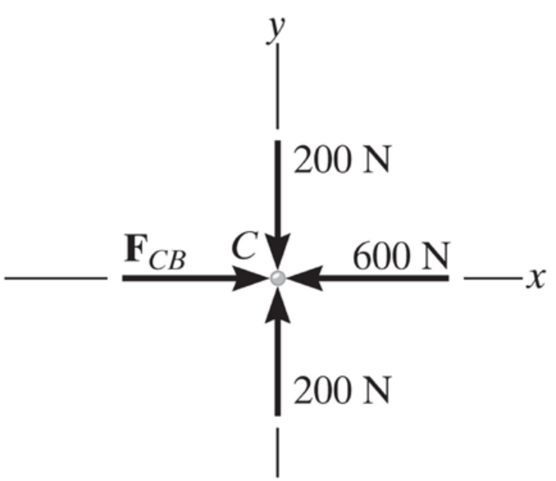
Method of Joints



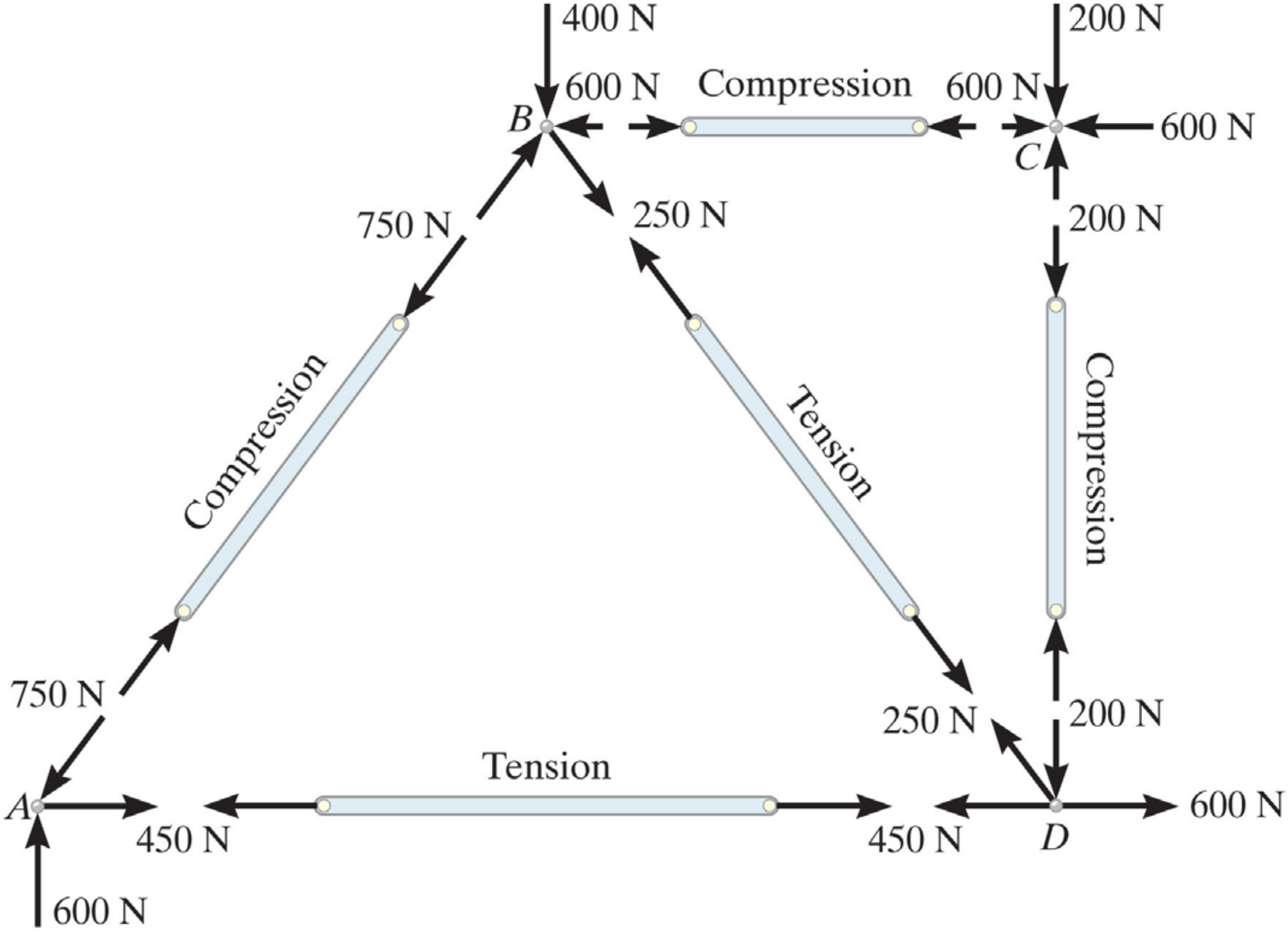








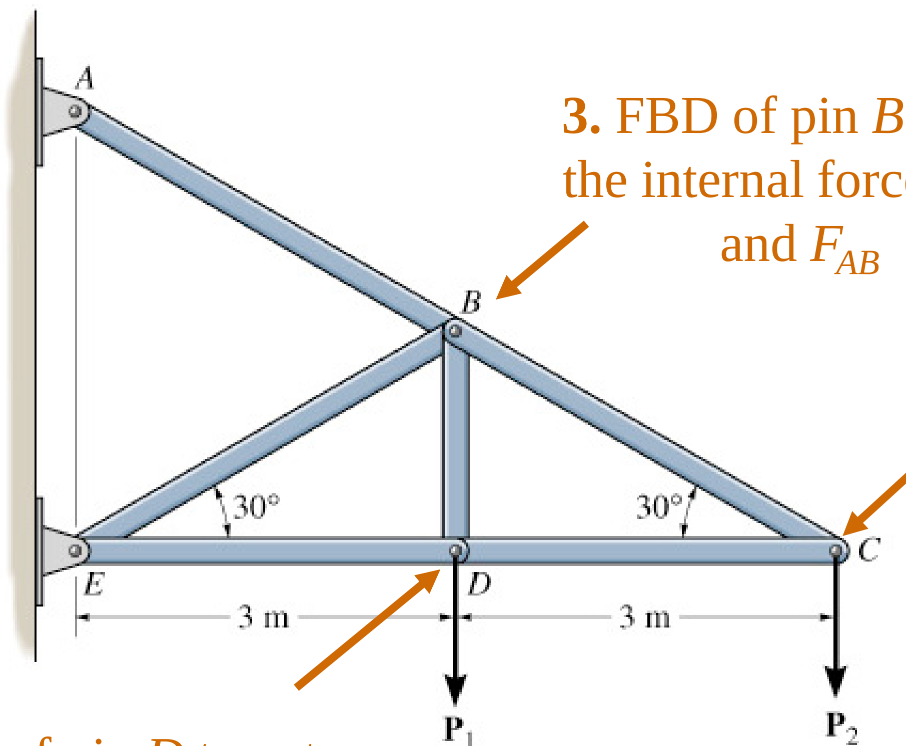
Method of Joints



Note Newton's 3rd Law when members are in tension and in compression. A member in tension will have the arrow at the joint pointing outwards.

Method of Joints

When using the method of joints, always start at a joint having at most two unknown forces. In this way, application of $\sum F_x = 0$ and $\sum F_y = 0$ yields two equations. Say we want to find the reactions at pin *B*.



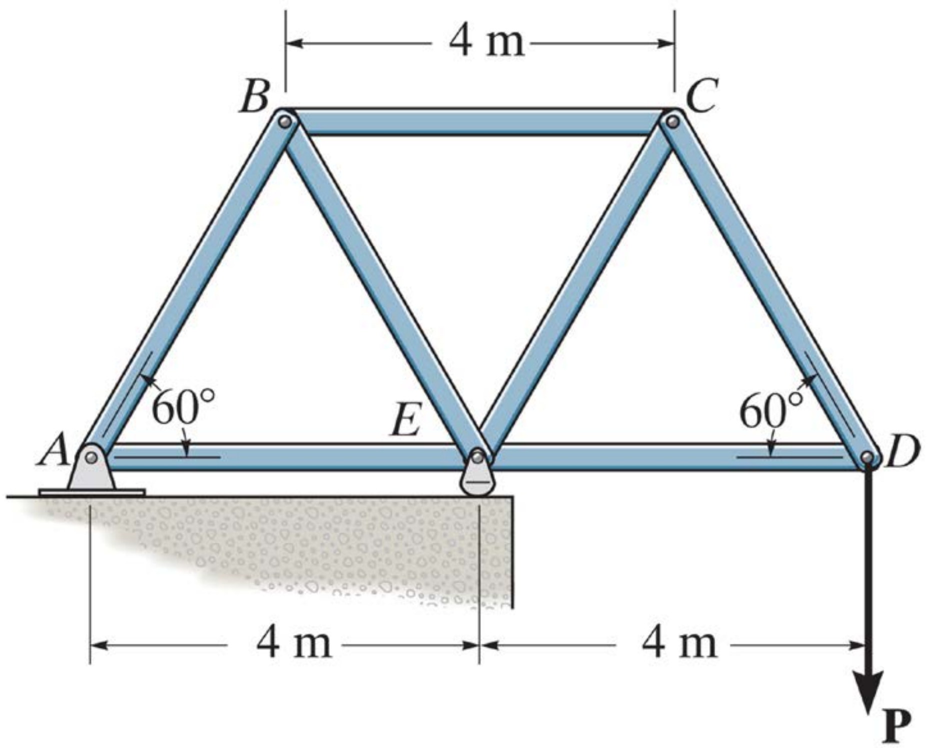
3. FBD of pin *B* to get the internal forces F_{BE} and F_{AB}

1. FBD of pin *C* to get the internal forces F_{BC} and F_{CD}

2. FBD of pin *D* to get F_{DE} and F_{BD}

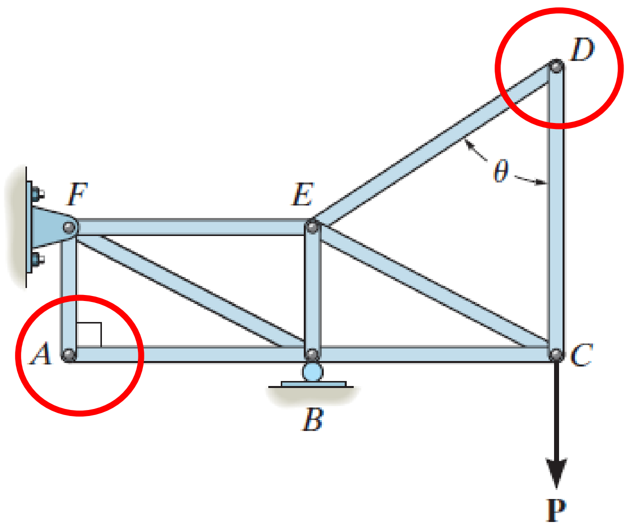
Example

If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force **P** that can be supported at joint *D*.



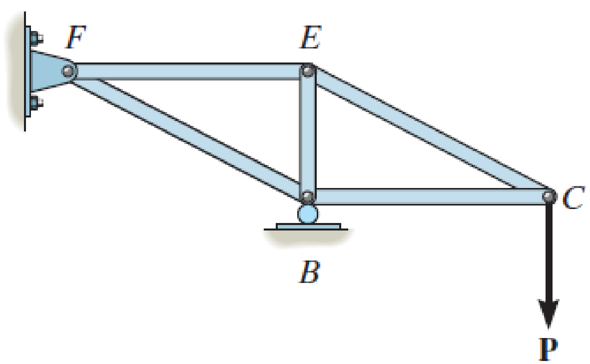
Zero-Force Members

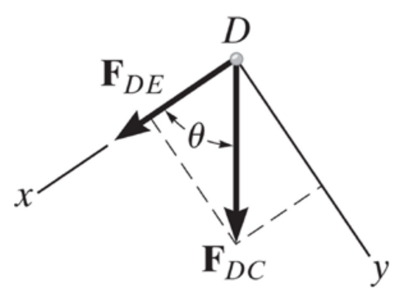
Case 1

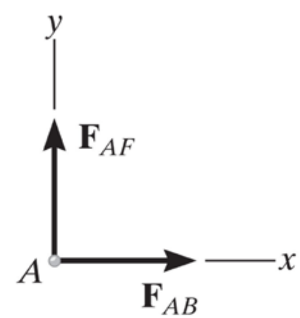


If a joint has only two non-collinear members and there is no external load or support reaction at that joint, then those two members are zero-force members.

In this example members DE, DC, AF, and AB are zero force members. If we apply the equations of equilibrium to joints D and A, these forces must be zero.



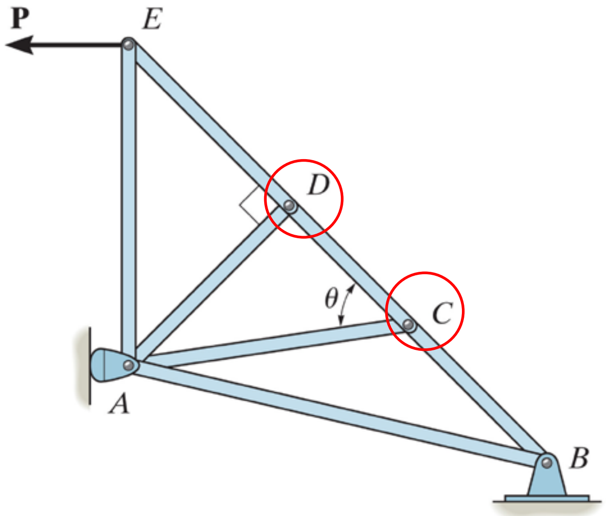




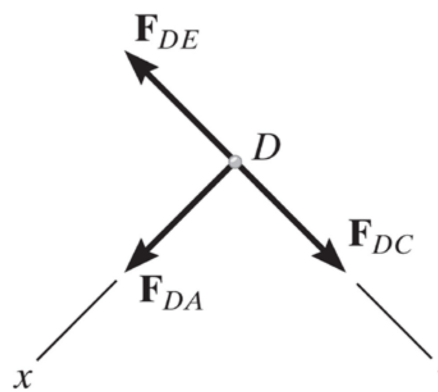
Zero-force members can be removed when analyzing the truss.

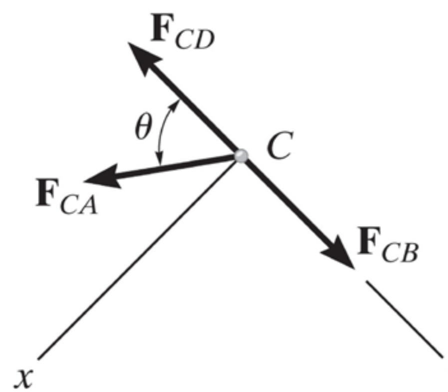
Zero-Force Members

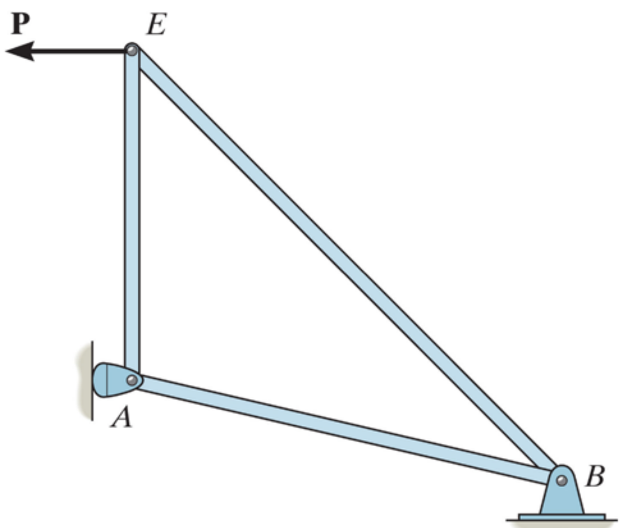
Case 2



If three members form a truss joint, and two members are collinear and there is no external load or reaction at that joint, then the third non-collinear member is a zero force member.



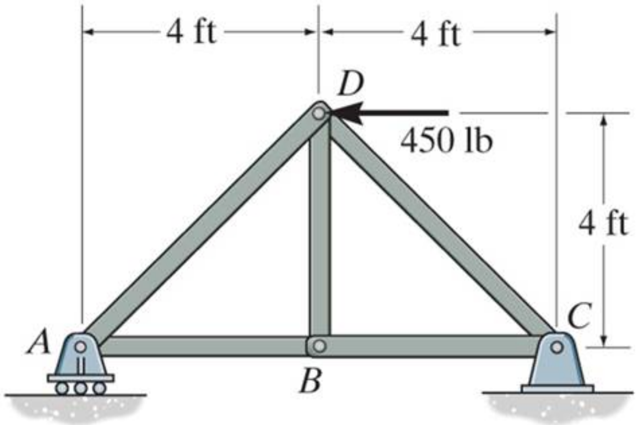




One can also remove the zero-force member.

Zero-force members are used to increase stability and rigidity of the truss (buckling), and to provide support for various loading conditions.

Sample Problem



Given: Loads as shown on the truss

Find: The forces in each member of the truss.

Plan:

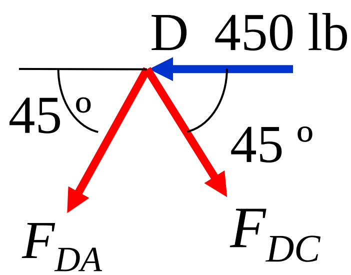
a) Check if there are any zero-force members.

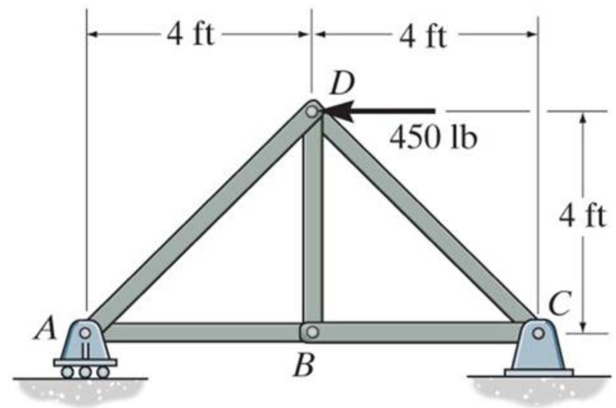
b) First analyze pin D and then pin A

a) Note that member BD is zero-force member. $F_{BD} = 0$

Sample Problem

Joint *D* is now subjected to two unknown member forces.





$$\sum F_x = 0 \qquad -450 + F_{DC}\cos 45^\circ - F_{AD}\cos 45^\circ = 0$$

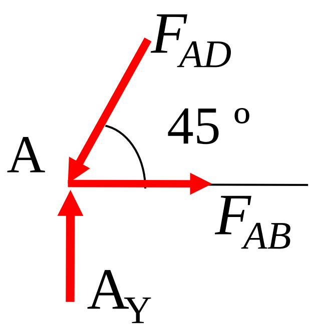
$$\sum F_y = 0 \qquad -F_{DC}\sin 45^\circ - F_{DA}\sin 45^\circ = 0$$

Solve system of equations. $F_{DC} = 318 \text{ lb}$ and $F_{DA} = -318 \text{ lb}$

$$\boxed{\begin{aligned} F_{DC} &= 318 \text{ lb (T)} \\ F_{DA} &= 318 \text{ lb (C)} \end{aligned}}$$

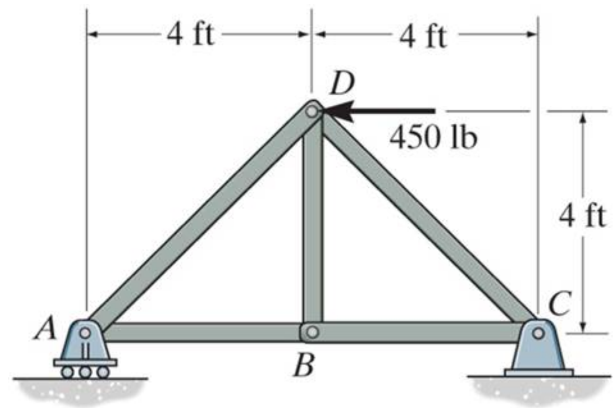
Sample Problem

Joint *A* is now subjected to only one unknown member force and one reaction.



Recall

$F_{AD} = -318$ lb



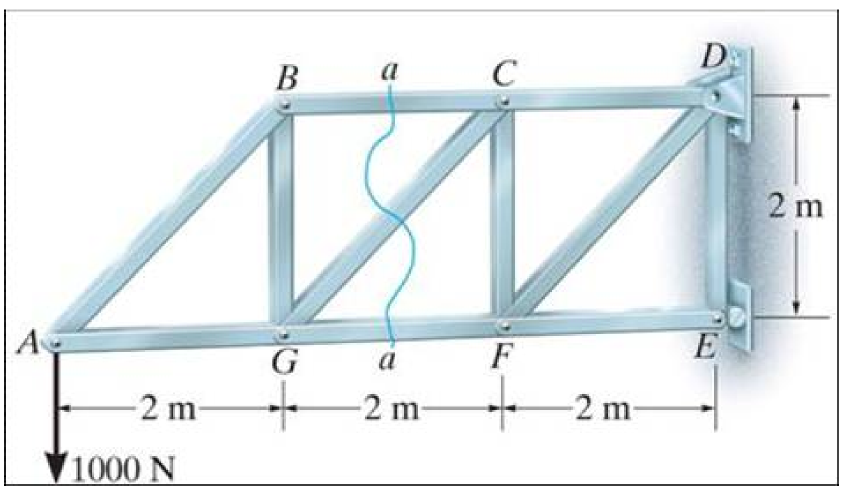
$$\sum F_X = F_{AB} + (-318)\cos 45° = 0; \qquad F_{AB} = 225 \text{ lb (T)}$$

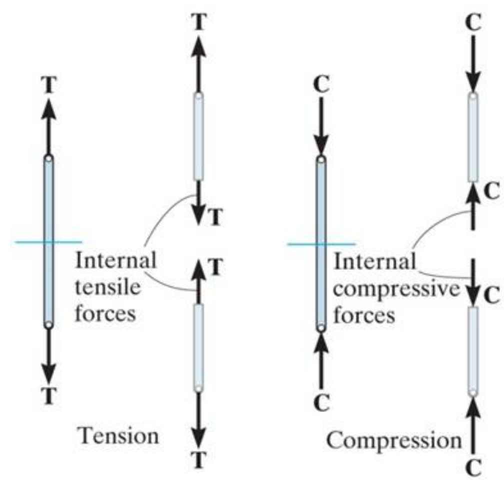
Method of Sections

Truss Members

While the method of joints requires us to determine the internal forces in succession, the method of sections allows us to determine the internal forces of specific members.

In the method of sections, a truss is divided into two parts by taking an imaginary "cut" (shown here as a-a) through the truss. The internal forces at the cut members also will be either tensile or compressive.





Method of Sections

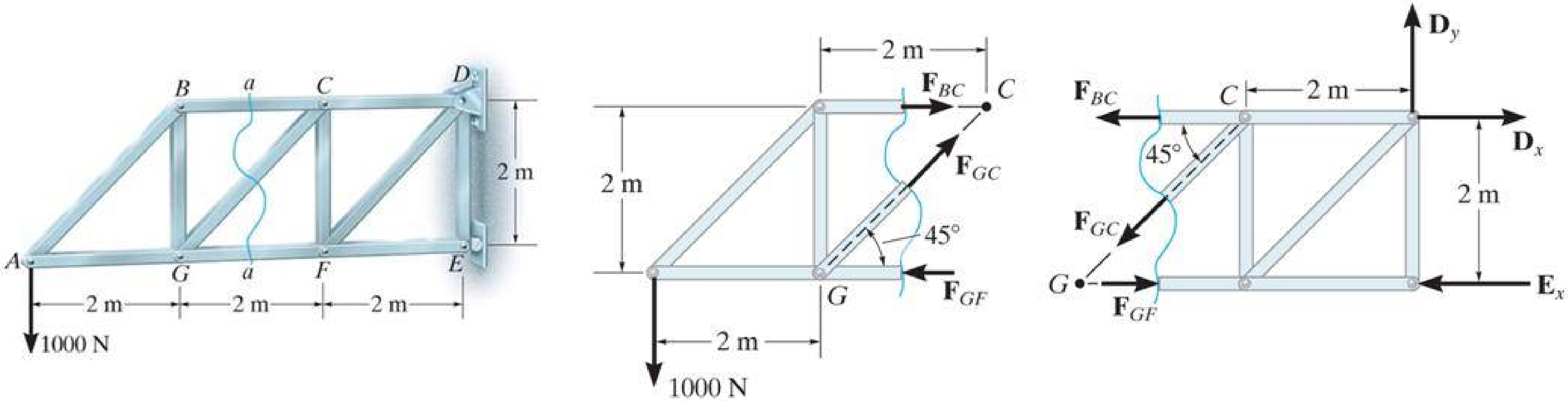
Steps for Analysis

1. Decide how to *cut* the truss. This is based on: a) the forces that you want to determine and b) There are no more that three unknowns.

2. Decide which *section* of the cut truss is easier to work with (minimize the number of external reactions that need to be calculated).

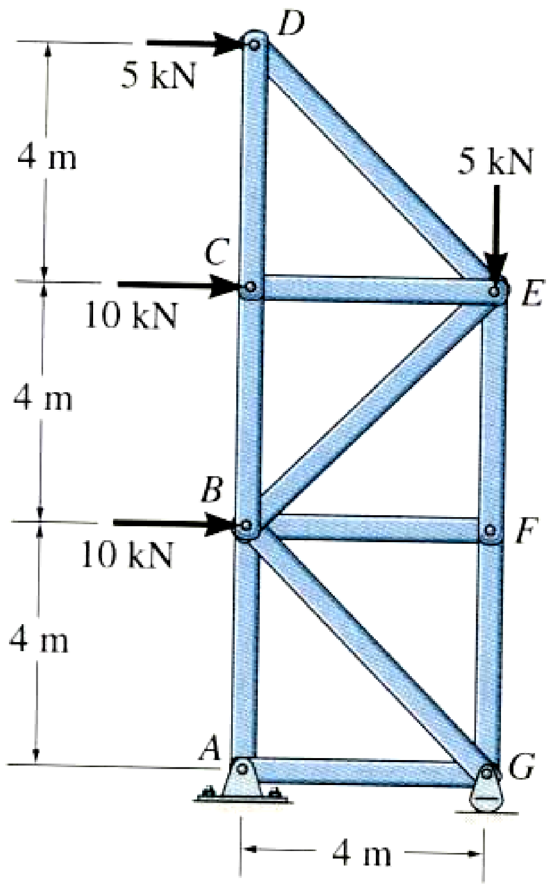
3. Determine any necessary support reactions of the entire truss.

4. Draw the FBD of the section. Include the internal unknown forces at the cut members in tension. Upon solving, (+) tension or (-) compression.

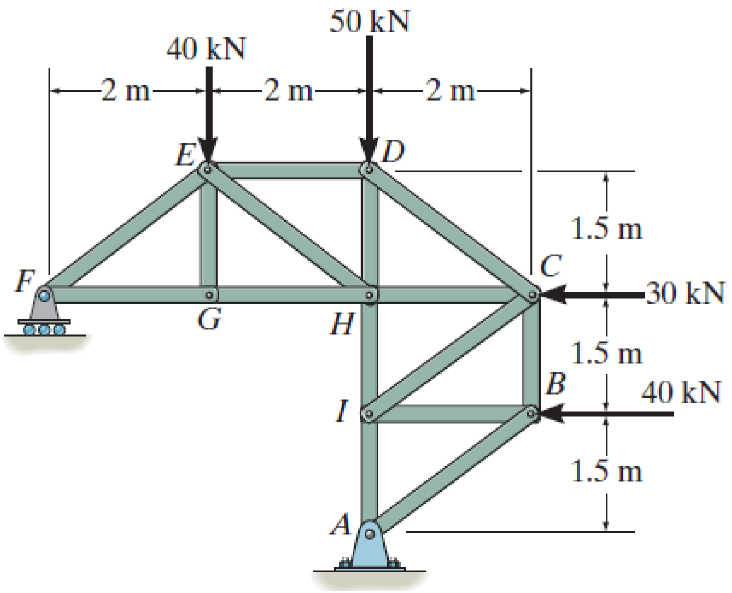


Example

Determine the force in members BC, BE, and EF from the truss shown below.



Example



Given: Loads as shown on the truss.

Find: The forces in members ED, EH, and GH.

Chapter 6 – Structural Analysis

Contents

The Eiffel Tower is probably the most famous truss structure in the World. It was built between 1887 and 1889, it is 300 m high (without antenna).

Please refrain from uploading course materials onto online sharing platforms, such as Course Hero, OneClass or equivalent sharing platforms.

Frames and Machines

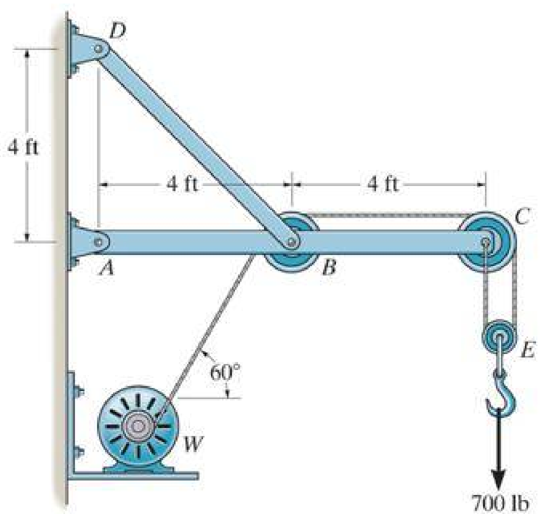
Structural Analysis

Frames and machines are two common types of structures that have **at least one multi-force member**.

Recall that trusses have nothing but two-force members.

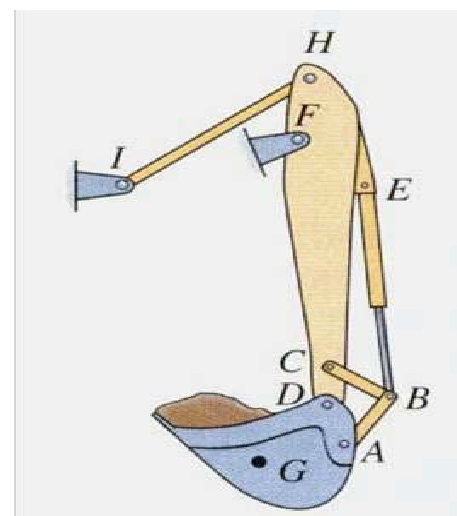
Frames: stationary structures that bear external load.

Machines: contain moving parts and are designed to alter the effect of forces.

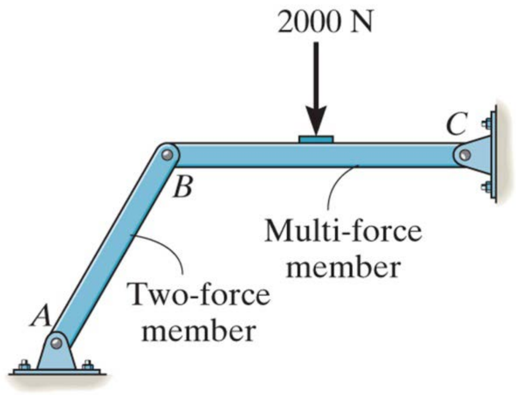


← Frame

Machine →



Map the Solution of a Problem

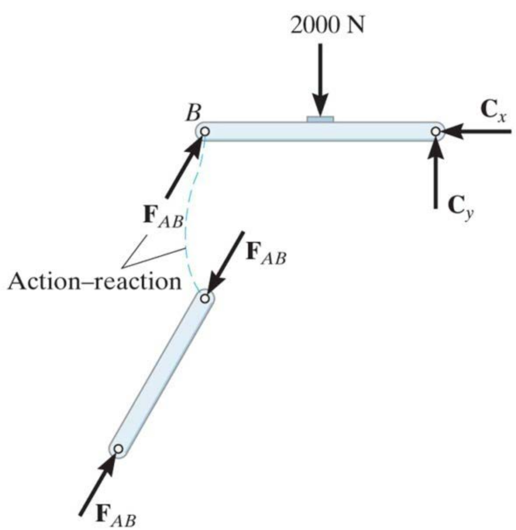


1. Visually separate the frame or machine in individual members and joints. Draw FBD for support.

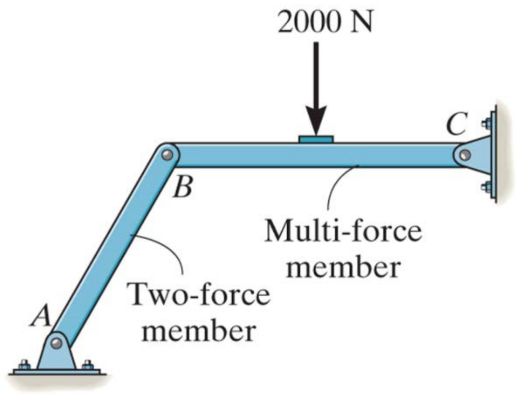
Identify any two-force members,

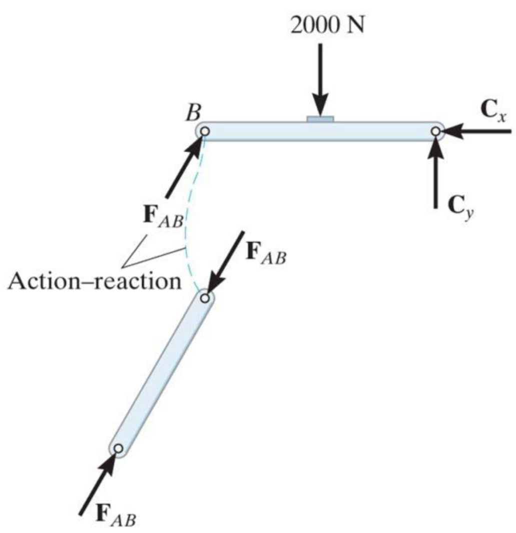
Note that forces on contacting surfaces (usually between a pin and a member) are equal and opposite, and,

For a joint with more than two members or an external force, it is advisable to draw a FBD of the pin.



Map the Solution of a Problem





2. Develop a strategy (path) to apply the equations of equilibrium to solve for the unknowns.

   For complex problems, map your path backwards (from output to input force)

   Look for ways to form single equations and single unknowns.

   Problems are going to be challenging since there could be several unknowns. Establish the path before you start writing down equations

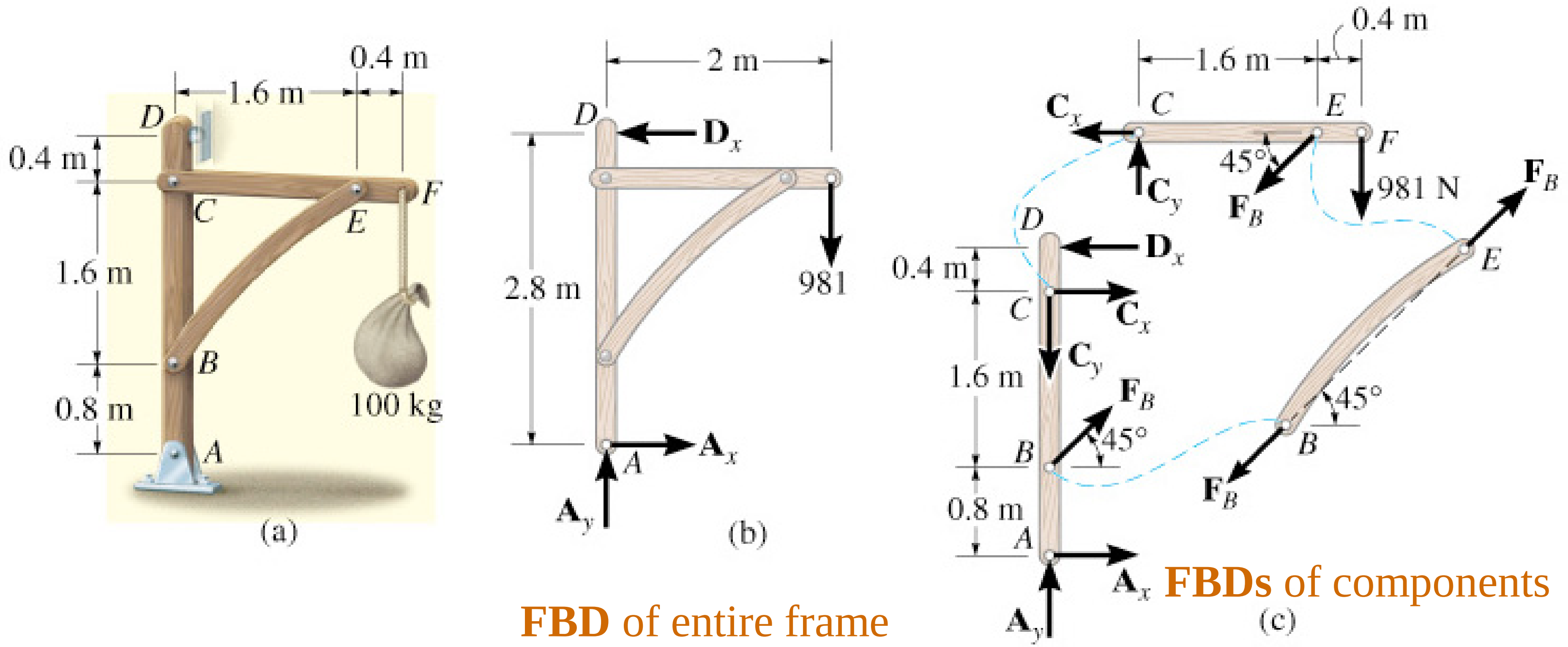
3. Write the equations of equilibrium.

Frames

Identify the number of forces acting at every member of the frame

Draw Free Body Diagrams and apply Newton's 3rd Law.

The position of the frame members do not change.

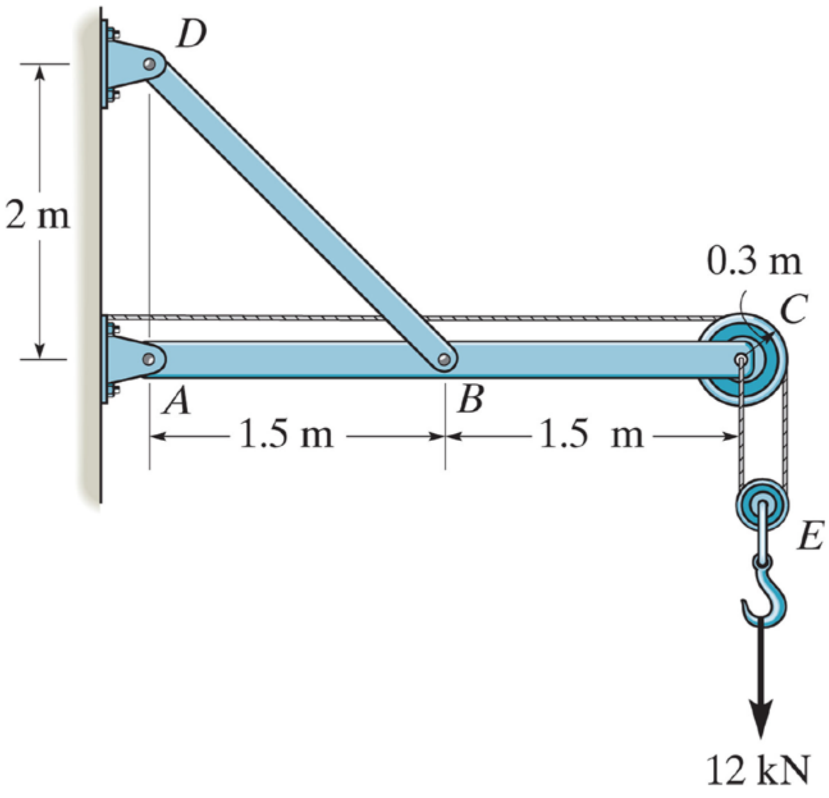


FBD of entire frame

FBDs of components

Example

Determine the horizontal and vertical components of force at pins *A* and *D*.

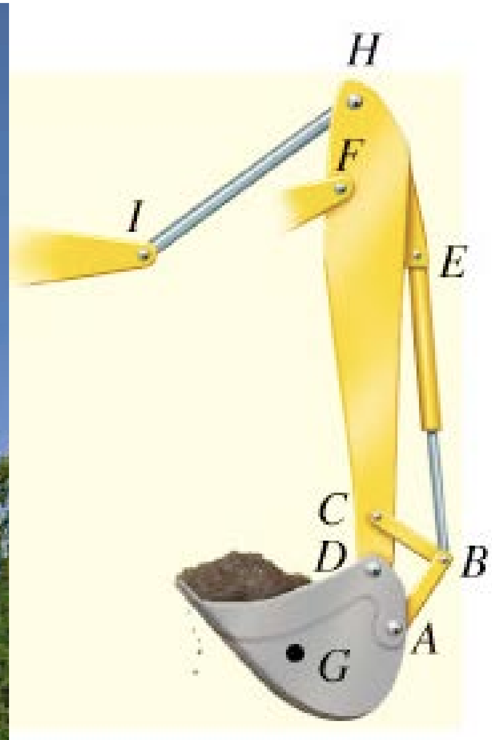


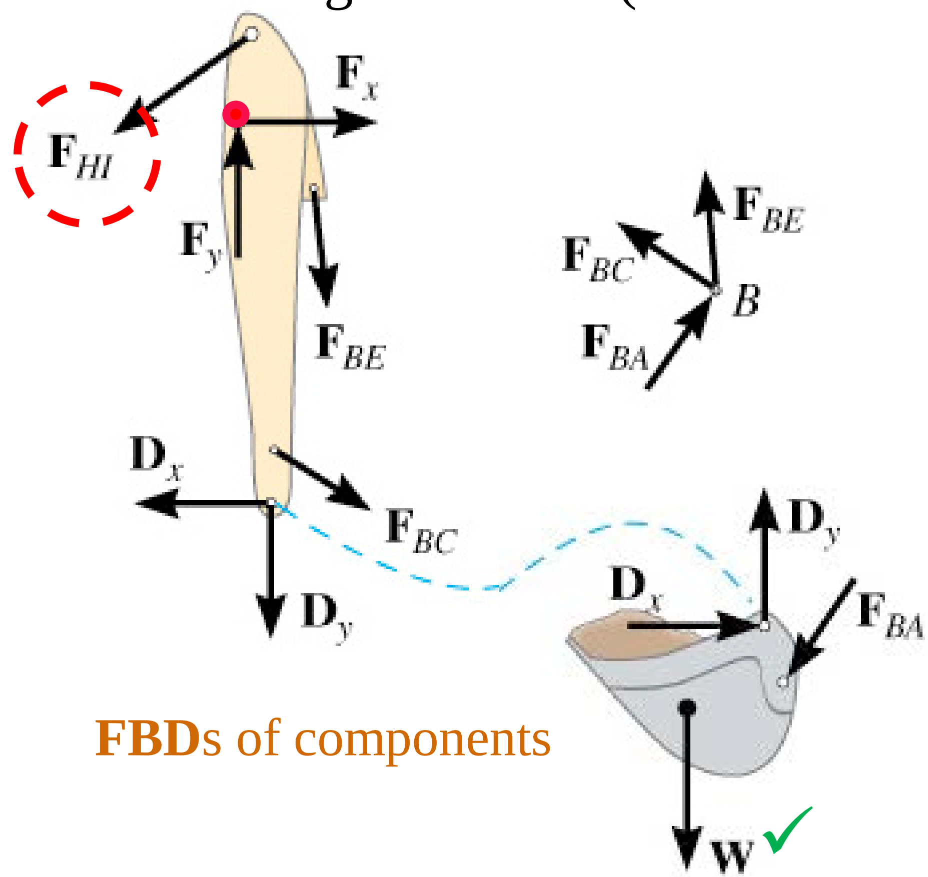
Machines

Identify the number of forces acting at every member

Draw Free Body Diagrams and apply Newton's 3rd Law.

As the members change position, the forces change direction (one analysis for each posture)

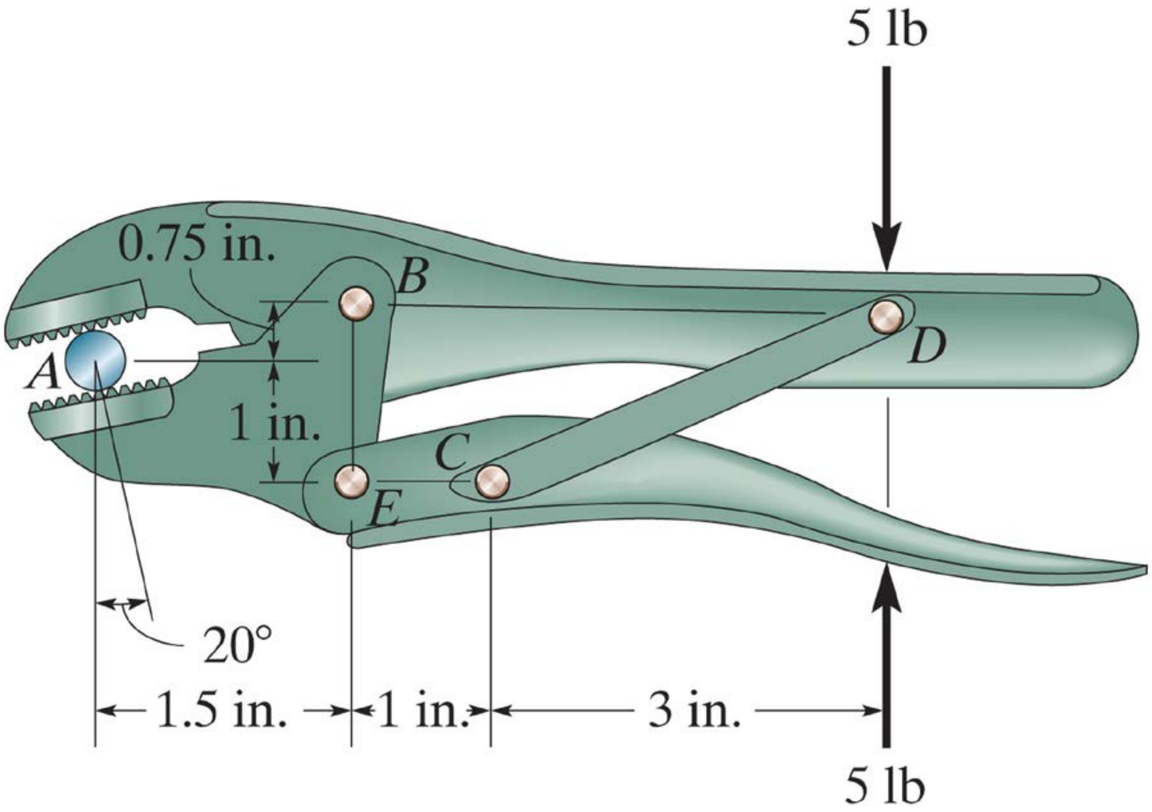




FBDs of components

Example

A 5-lb force is applied to the handles of the vise grip. Determine the compressive force developed on the smooth bolt shank *A* at the jaws.

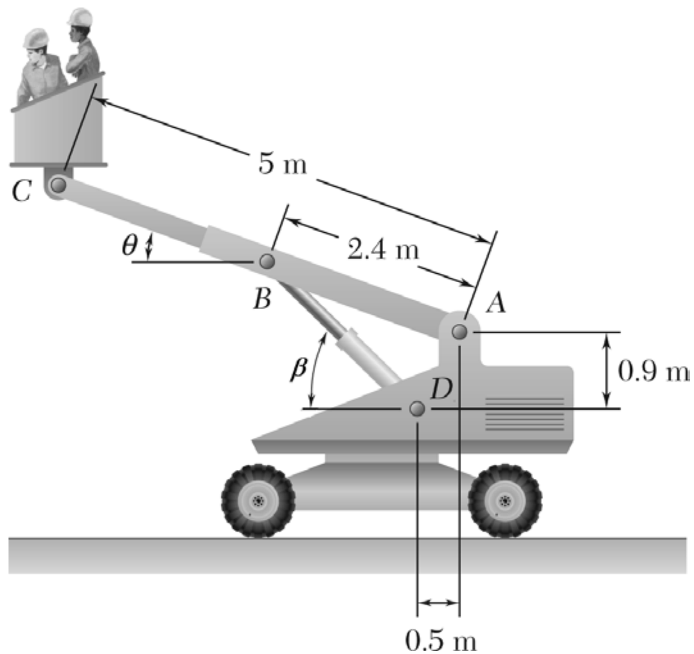


Example

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The combined mass of workers and platform is 200 kg and a centre of gravity is located directly above *C*. For the position when $\theta = 20°$ and $\beta = 44.43°$, find

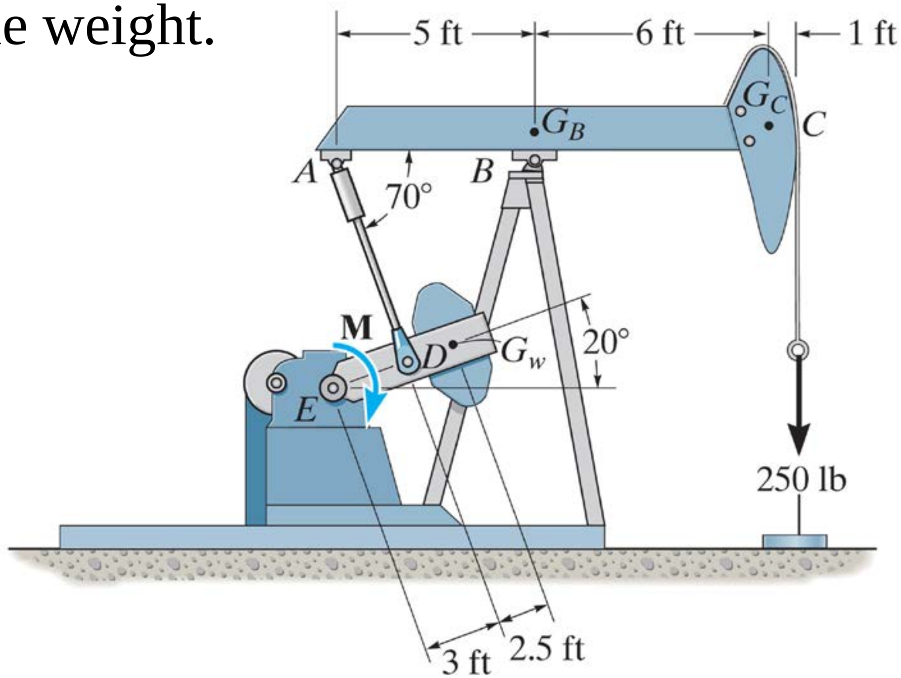
a) The force exerted at B by the single hydraulic cylinder BD

b) The force exerted on the supporting carriage at A



Example

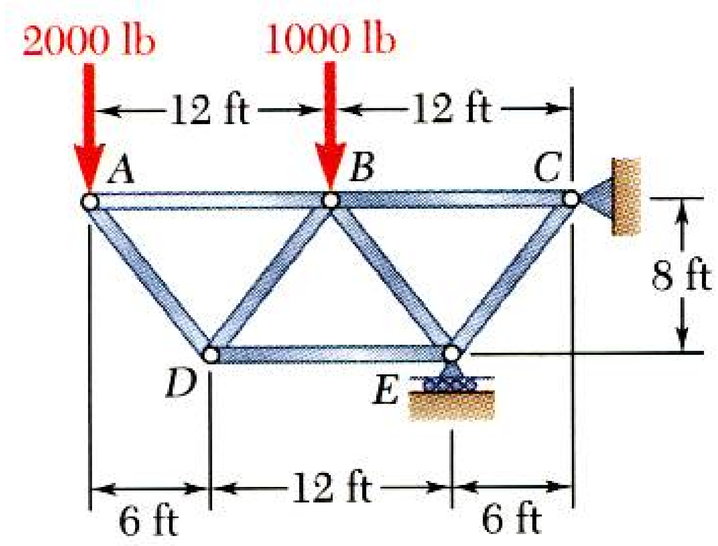
The pumping unit is used to recover oil. When the walking beam *ABC* is horizontal, the force acting in the wireline at the well head is 250 lb. Determine the torque **M** which must be exerted by the motor in order to overcome this load. The horse-head *C* weighs 60 lb and has a center of gravity at G_C. The walking beam *ABC* has a weight of 130 lb and a center of gravity at G_B, and the counterweight has a weight of 200 lb and a center of gravity at G_W. The pitman, *D*, is pin connected at its ends and has negligible weight.



Sample Problems for Students to Review

Chapter 6

Sample Problem (§ 6.2)



Given: Loads as shown on the truss

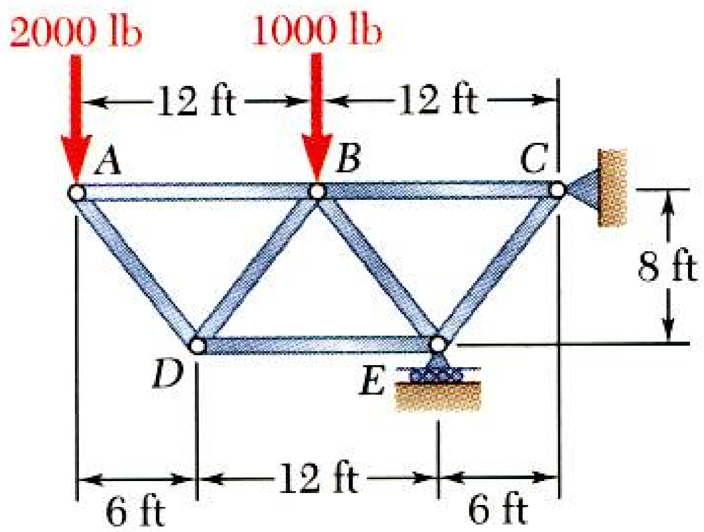
Find: The forces in each member of the truss.

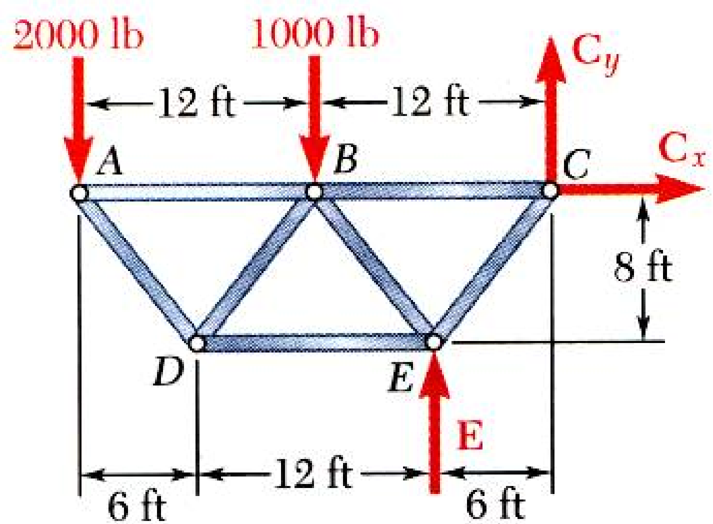
Plan:

a) Determine reaction forces at *E* and *C*.

b) Begin analysis where there are two unknowns, e.g., joint A.

c) In succession, determine unknown member forces at joints.





$$\sum M_C = 0$$
$$= (2000\text{ lb})(24\text{ ft}) + (1000\text{ lb})(12\text{ ft}) - E(6\text{ ft})$$

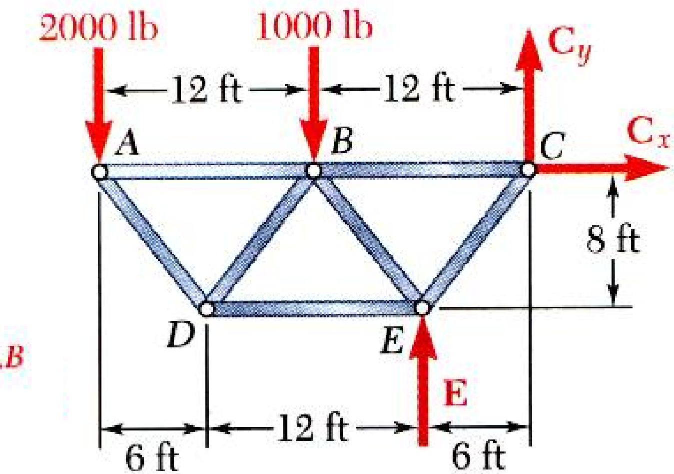
$$\boxed{E = 10{,}000\text{ lb} \uparrow}$$

$$\sum F_x = 0 = C_x$$

$$\boxed{C_x = 0}$$

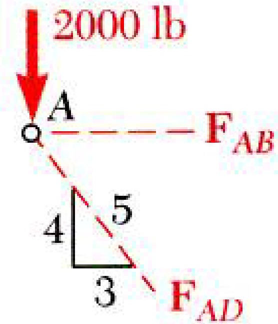
$$\sum F_y = 0 = -2000\text{ lb} - 1000\text{ lb} + 10{,}000\text{ lb} + C_y$$

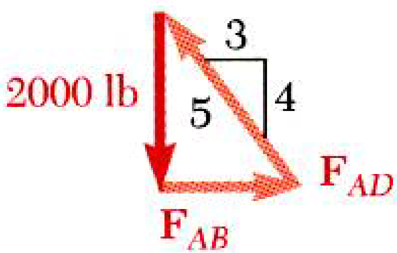
$$\boxed{C_y = 7000\text{ lb} \downarrow}$$



Joint *A* is subjected to only two unknown member forces.

Graphical Solution:



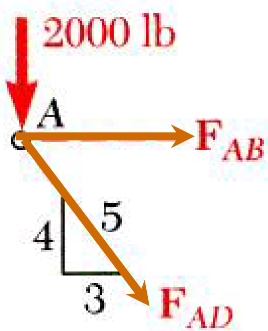


$$F_{AD} = \frac{(5)2000}{4}$$

$$F_{AB} = \frac{(3)2000}{4}$$

$$\boxed{\begin{aligned} F_{AB} &= 1500 \text{ lb } \ T \\ F_{AD} &= 2500 \text{ lb } \ C \end{aligned}}$$

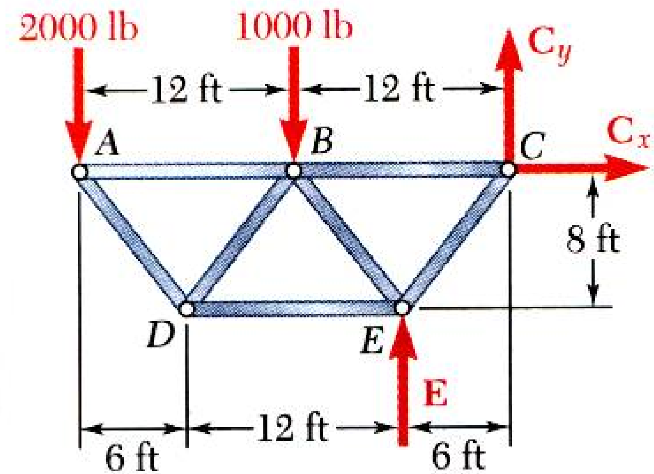
Analytical Solution:



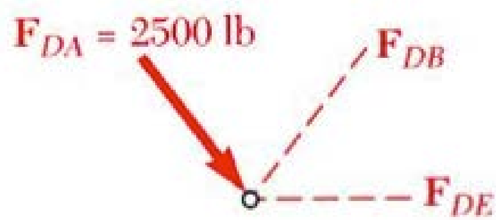
$$\sum F_y = 0 \qquad -2000 - \frac{4}{5}F_{AD} = 0$$

$$\sum F_x = 0 \qquad \frac{3}{5}F_{AD} + F_{AB} = 0$$

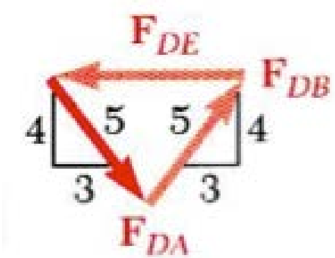
$$\boxed{\begin{aligned} F_{AD} &= -2500 \text{ lb} \\ F_{AB} &= 1500 \text{ lb} \end{aligned}}$$



Joint *D* is now subjected to two unknown member forces.



Graphical Solution:

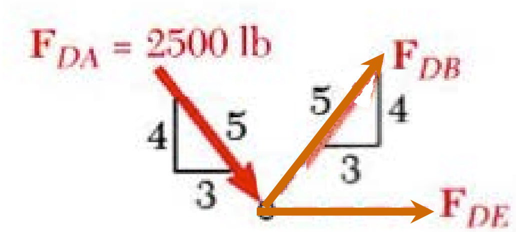


$$F_{DB} = F_{DA}$$

$$F_{DE} = 2\left(\frac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 2500 \text{ lb } \ T \\ F_{DE} &= 3000 \text{ lb } \ C \end{aligned}}$$

Analytical Solution:

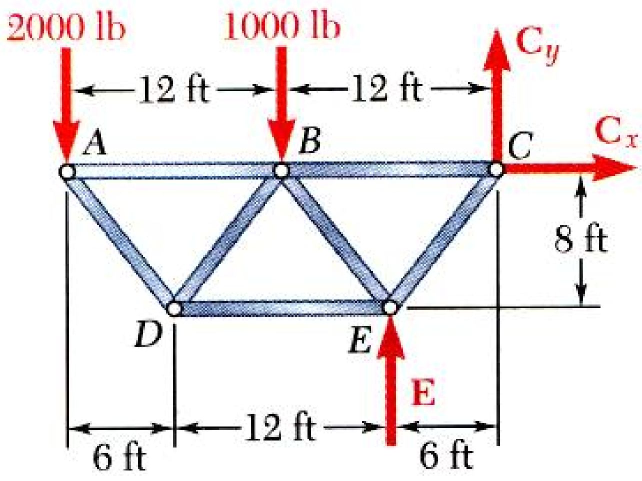


$$\sum F_y = 0 \qquad -2500\frac{4}{5} + \frac{4}{5}F_{DB} = 0$$

$$\sum F_x = 0 \qquad \frac{3}{5}2500 + \frac{3}{5}F_{DB} + F_{DE} = 0$$

$$\boxed{\begin{aligned} F_{DB} &= 2500 \text{ lb} \\ F_{DE} &= -3000 \text{ lb} \end{aligned}}$$

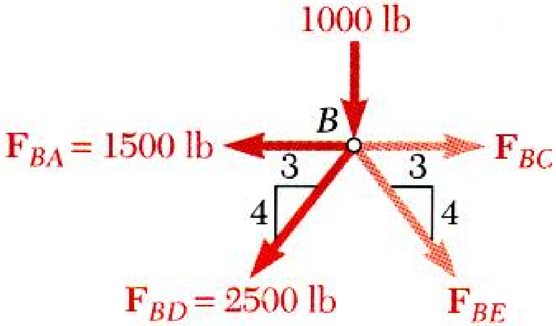
Joint B is now subjected to two unknown member forces.



Analytical Solution:

Recall:

$F_{AB} = 1500 \text{ lb } T$
$F_{DB} = 2500 \text{ lb } T$



$$\sum F_y = 0 = -1000 - \frac{4}{5}(2500) - \frac{4}{5}F_{BE}$$

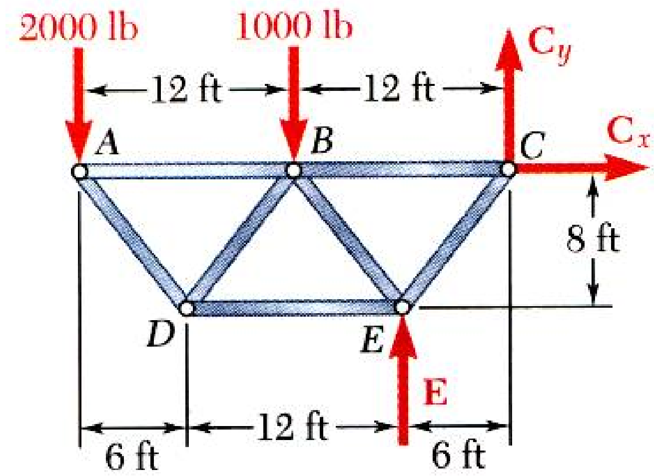
$$F_{BE} = -3750 \text{ lb}$$

$$\boxed{F_{BE} = 3750 \text{ lb } C}$$

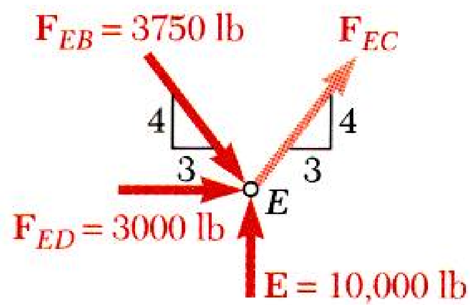
$$\sum F_x = 0 = F_{BC} - 1500 - \frac{3}{5}(2500) - \frac{3}{5}(3750)$$

$$F_{BC} = +5250 \text{ lb}$$

$$\boxed{F_{BC} = 5250 \text{ lb } T}$$



Joint *E* is now subjected to one unknown member force.



Recall:

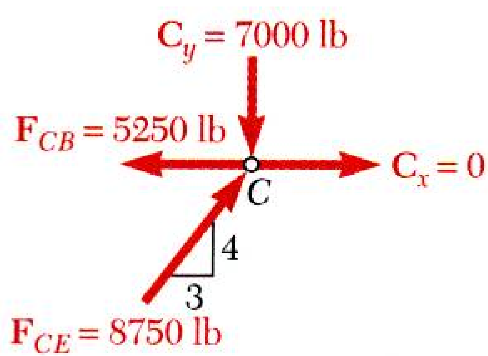
$F_{EB} = 3750 \text{ lb } C$
$F_{ED} = 3000 \text{ lb } C$
$E = 10{,}000 \text{ lb } \uparrow$

$$\sum F_x = 0 = \frac{3}{5}F_{EC} + 3000 + \frac{3}{5}(3750) \qquad F_{EC} = -8750 \text{ lb} \qquad \boxed{F_{EC} = 8750 \text{ lb } C}$$

Joint *C*, all forces are known. We can still check conditions of equilibrium.

Recall:

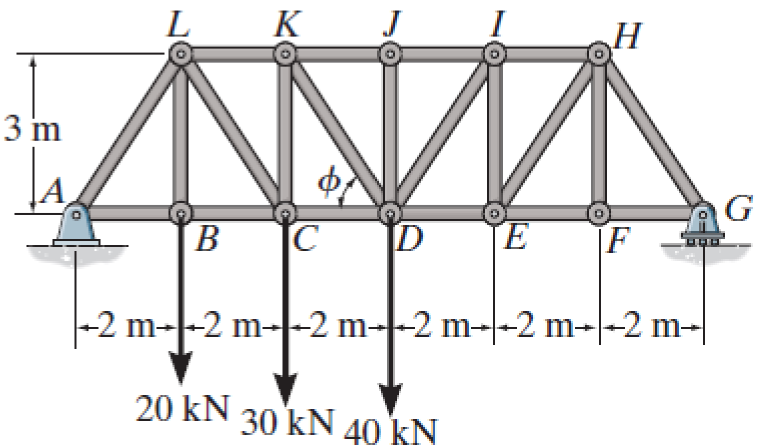
$C_x = 0$
$C_y = 7000 \text{ lb } \downarrow$
$F_{EC} = 8750 \text{ lb } C$
$F_{BC} = 5250 \text{ lb } T$



$$\sum F_x = -5250 + \frac{3}{5}(8750) = 0 \quad \text{(checks)}$$

$$\sum F_y = -7000 + \frac{4}{5}(8750) = 0 \quad \text{(checks)}$$

Sample Problem (§ 6.4)



Given: Loads as shown on the truss.

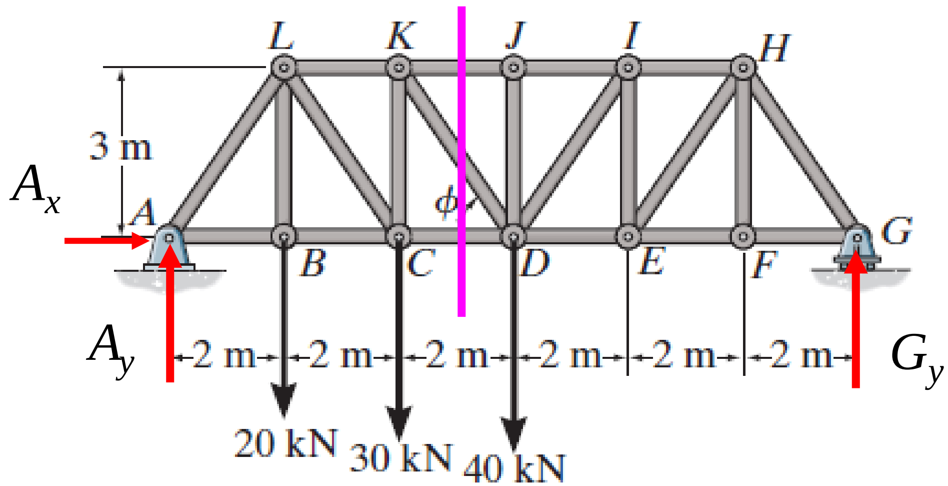
Find: The force in members KJ, KD, and CD.

Plan:

a) Take a cut through members KJ, KD and CD.

b) Select one of the two sections and solve for support reactions.

c) Apply the equations of equilibrium to find the forces in KJ, KD and CD.



For this simple truss, we can select either side. Analyzing the entire truss for the reactions at A, we get

$$\Sigma\, F_x = A_x = 0$$

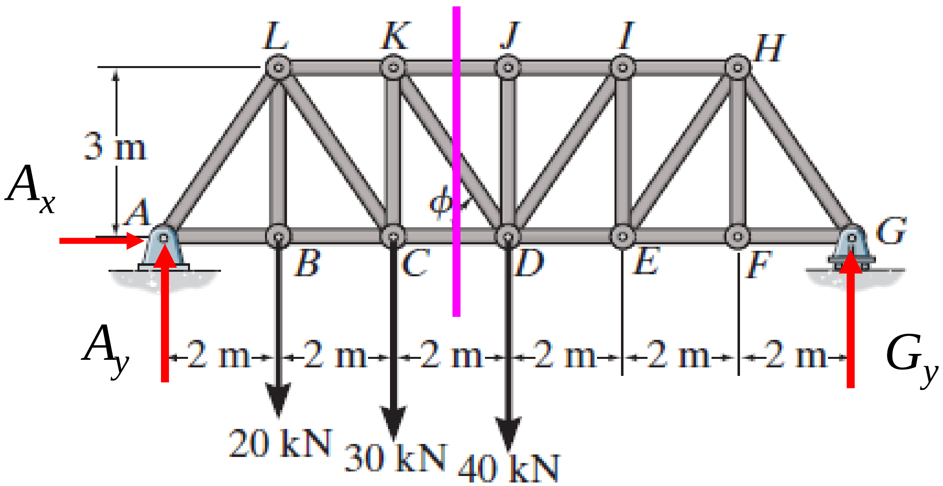
$$\boxed{A_x = 0}$$

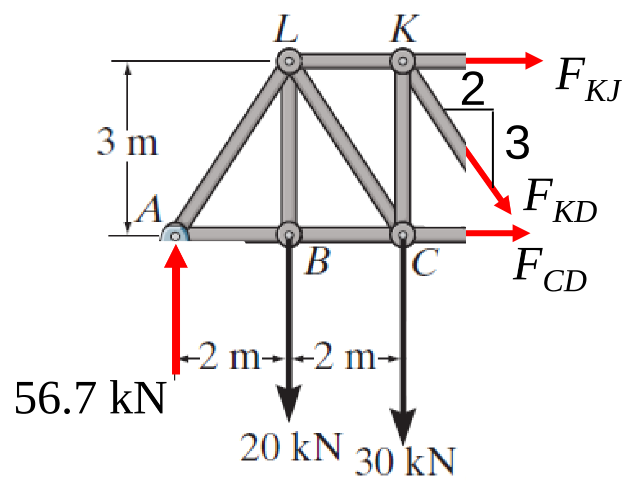
A moment equation about G to find A_y results in:

$$\Sigma\ \ M_G = A_y\,(12) - 20\,(10) - 30\,(8) - \ 40\,(6) = 0;$$

$$\boxed{A_y = 56.7\ kN}$$

$$A_y = \ 56.7\text{ kN}$$





Now we take moments about point *D*.

$$\sum M_D = -56.7\,(6) + 20\,(4) + 30\,(2) - F_{KJ}\,(3) = 0$$

$$F_{KJ} = -66.7 \text{ kN}$$

$$\boxed{F_{KJ} = 66.7\ kN\ C}$$

$$\sum F_y = 56.7 - 20 - 30 - (3/\sqrt{13})\,F_{KD} = 0;$$

$$F_{KD} = 8.05 \text{ kN}$$

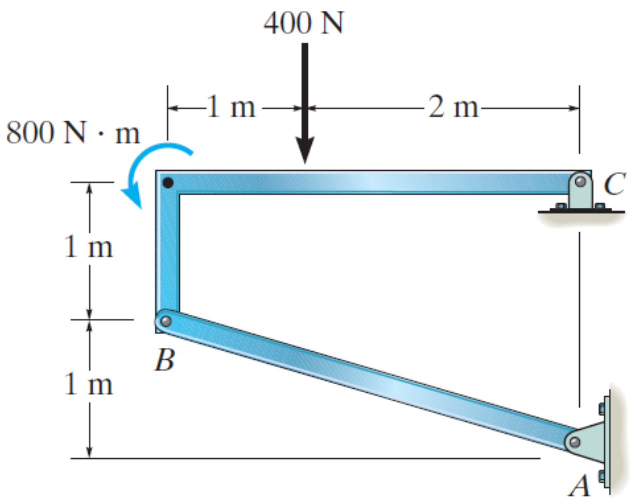
$$\boxed{F_{KD} = 8.05\ kN\ T}$$

$$\sum F_x = (-66.7) + (2/\sqrt{13})\,(\,8.05\,) + F_{CD} = 0;$$

$$F_{CD} = 62.2 \text{ kN}$$

$$\boxed{F_{CD} = 62.2\ kN\ T}$$

Sample Problem (§ 6.6)



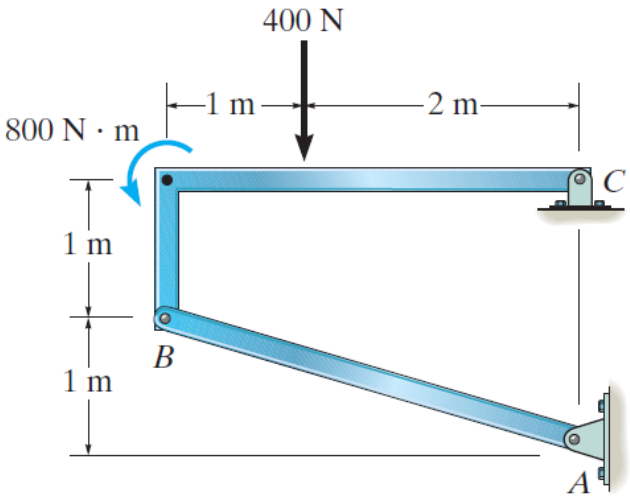
Given: The frame supports an external load and moment as shown.

Find: The horizontal and vertical components of the pin reactions at C and the magnitude of reaction at B.

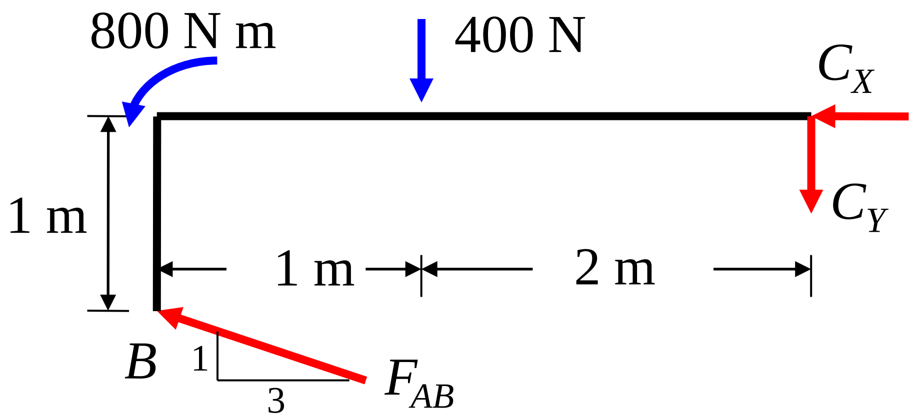
Plan:

Draw a FBD of frame member BC.

Apply the equations of equilibrium and solve for the unknowns at C and B.



FBD of member BC



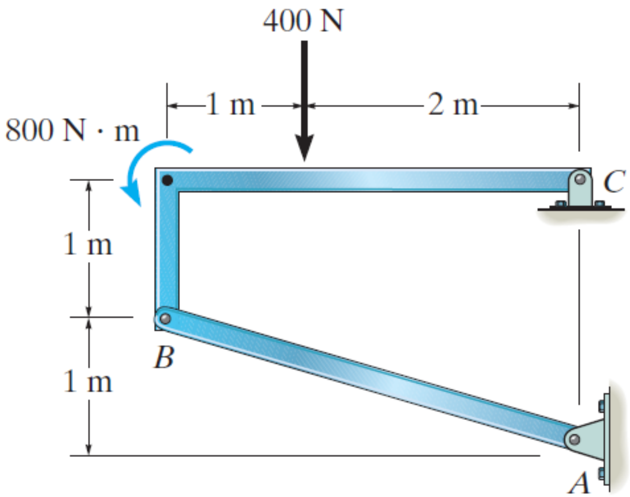
Note member *AB* is a *two-force* member.

Equations of Equilibrium:

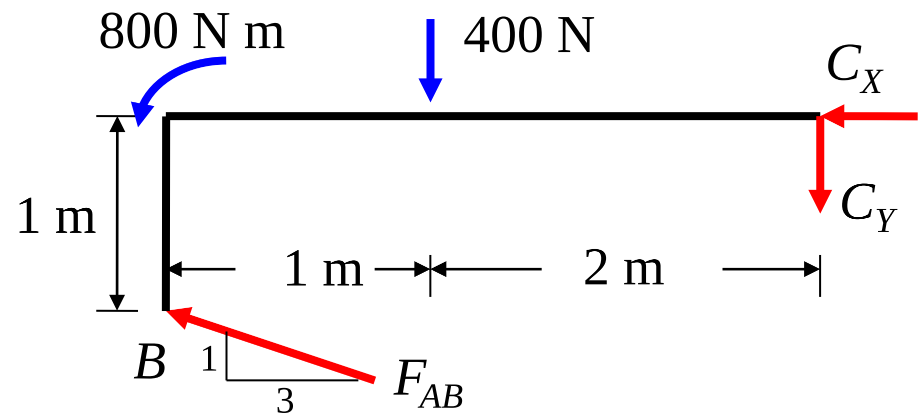
Start with $\sum M_C$ since it yields one unknown.

$$\sum M_C = -F_{AB}\,(3/\sqrt{10})\,(1) - F_{AB}\,(1/\sqrt{10})\,(3) + 800 + 400\,(2) = 0$$

$$F_{AB} = 843.3 = 843 \text{ N}$$



FBD of member BC



Now use the x and y-direction Equations of Equilibrium:

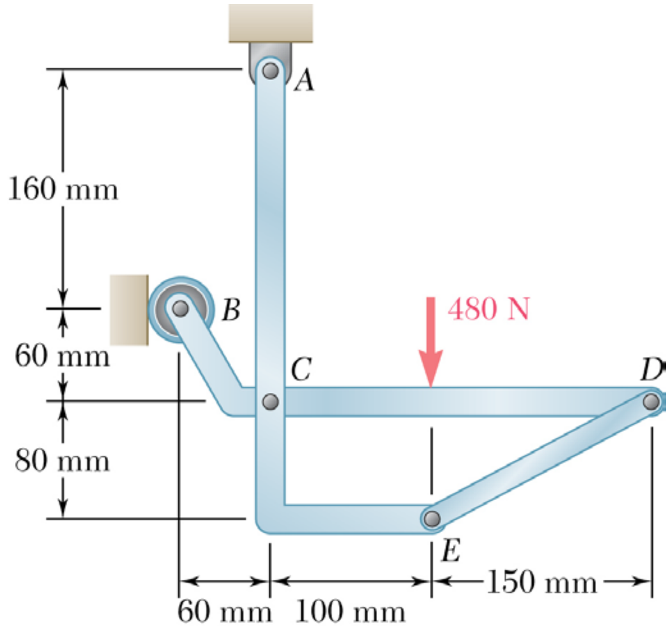
$$\sum F_X = -C_X - 843.3\,(3/\sqrt{10}) = 0$$

$$C_X = -800\text{ N} = 800\text{ N} \rightarrow$$

$$\sum F_Y = -C_Y + 843.3\,(1/\sqrt{10}) - 400 = 0$$

$$C_Y = -133\text{ N} = 133\text{ N} \uparrow$$

Sample Problem (§ 6.4)



Given: The frame supports an external load as shown.

Find: The force in link *DE* and the components of the force exerted at *C* on member *BCD*

Plan:

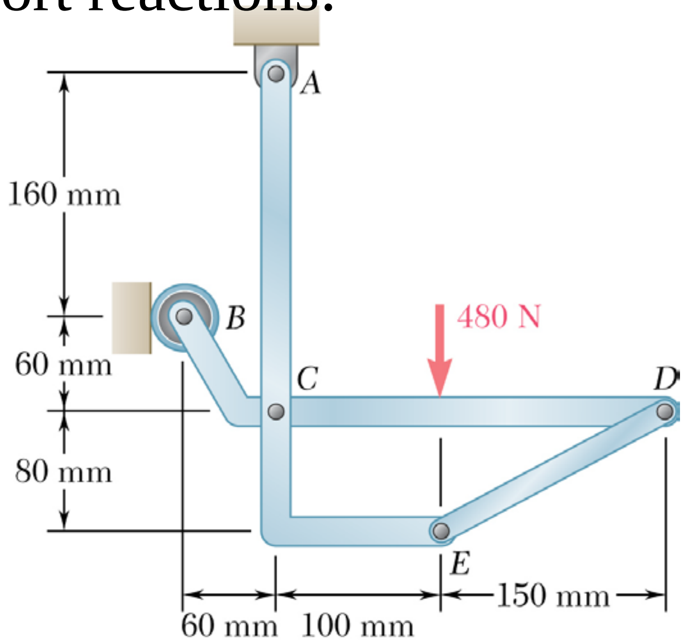
Draw a FBD and find support reactions.

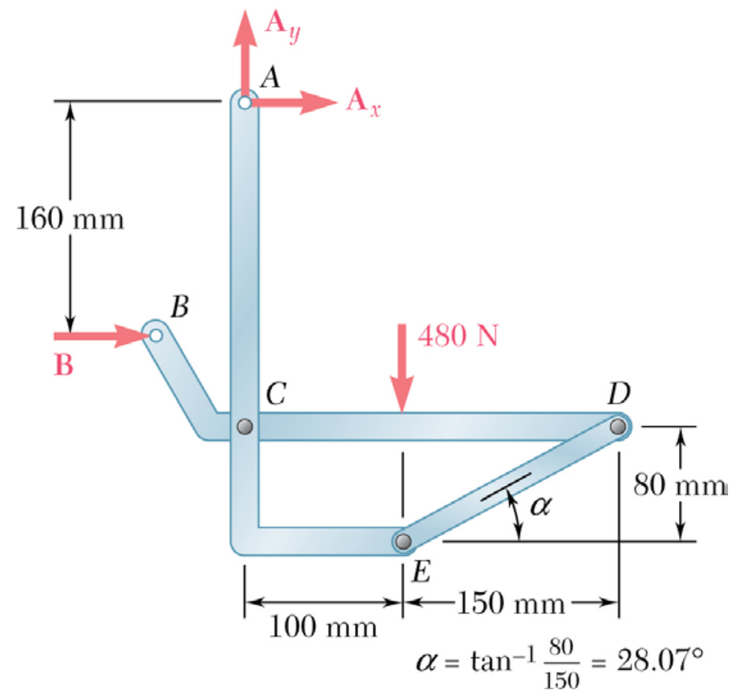
Draw a FBD of frame member BCD

Apply the equations of equilibrium. Solve for unknowns at C and D.

Check results using frame member ACE

Support reactions:





$$\sum F_y = 0 = A_y - 480\text{ N}$$

$$\boxed{A_y = 480\text{ N} \ \uparrow}$$

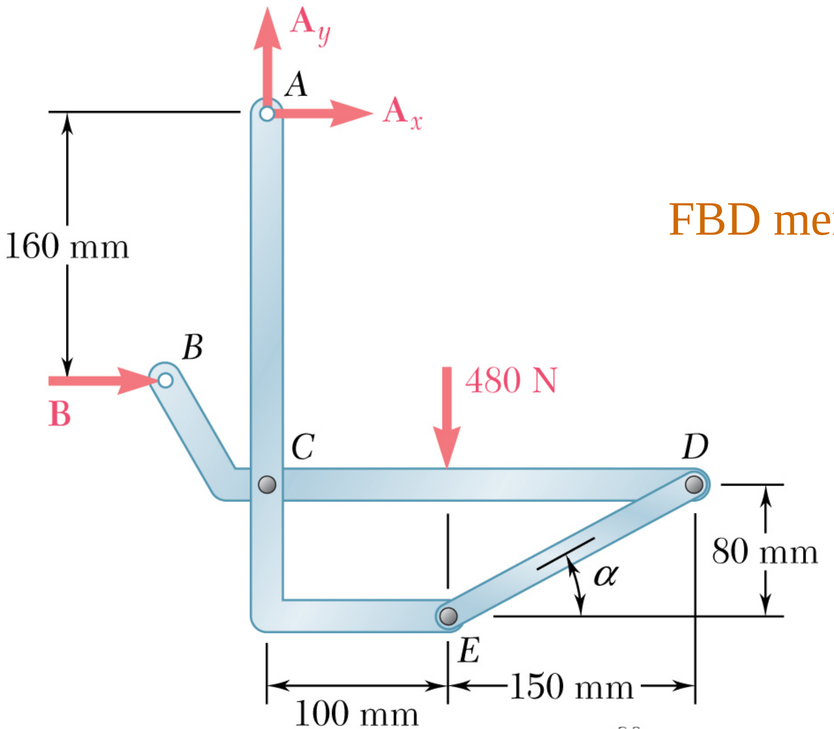
$$\sum M_A = 0 = -(480\text{ N})(100\text{ mm}) + B(160\text{ mm})$$

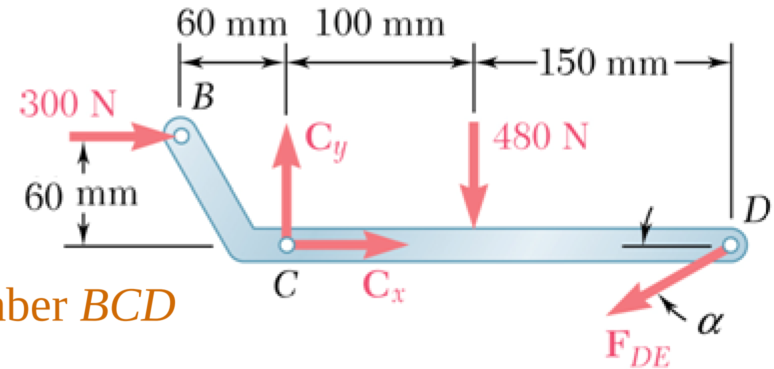
$$\boxed{B = 300\text{ N} \ \rightarrow}$$

$$\sum F_x = 0 = B + A_x$$

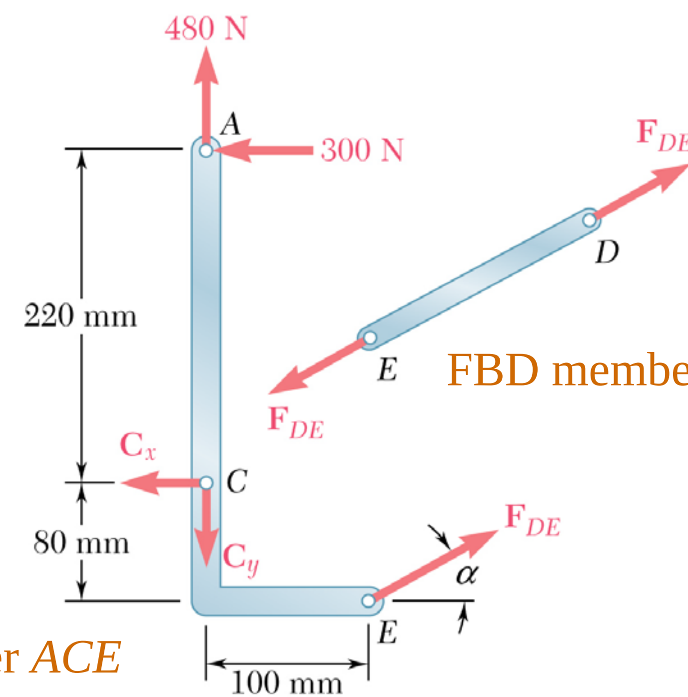
$$\boxed{A_x = -300\text{ N} \ \leftarrow}$$

Free Body Diagrams:





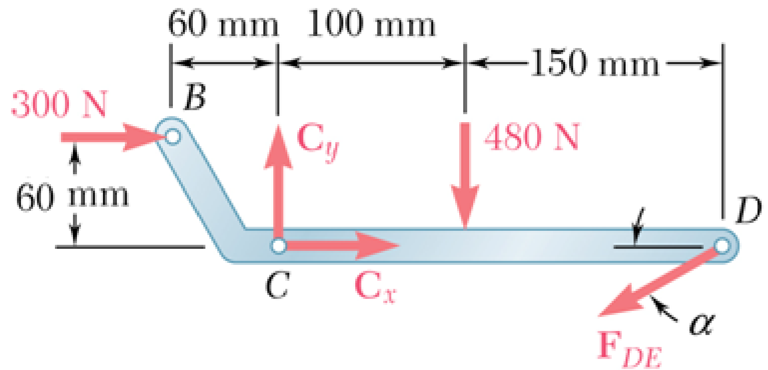
FBD member *BCD*



FBD member *DE*

FBD member *ACE*

FBD member *BCD*:



$$\alpha = \tan^{-1}\frac{80}{150} = 28.07°$$

$$\sum M_C = 0 = -(F_{DE}\sin\alpha)(250\text{ mm}) - (300\text{ N})(60\text{ mm}) - (480\text{ N})(100\text{ mm})$$

$$F_{DE} = -561\text{ N}$$

$$\boxed{F_{DE} = 561\text{ N } \ C}$$

$$\sum F_x = 0 = C_x - F_{DE}\cos\alpha + 300\text{ N}$$

$$0 = C_x - (-561\text{ N})\cos\alpha + 300\text{ N}$$

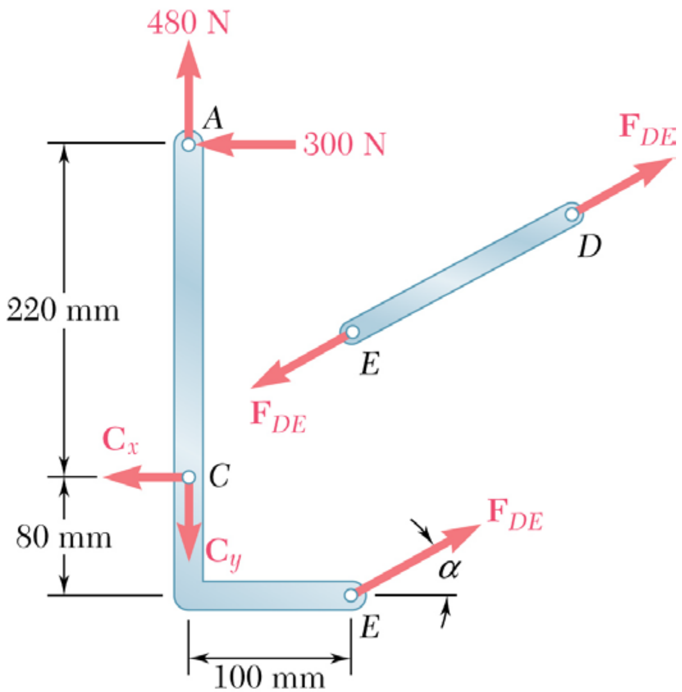
$$\boxed{C_x = -795\text{ N}}$$

$$\sum F_y = 0 = C_y - F_{DE}\sin\alpha - 480\text{ N}$$

$$0 = C_y - (-561\text{ N})\sin\alpha$$

$$\boxed{C_y = -216\text{ N}}$$

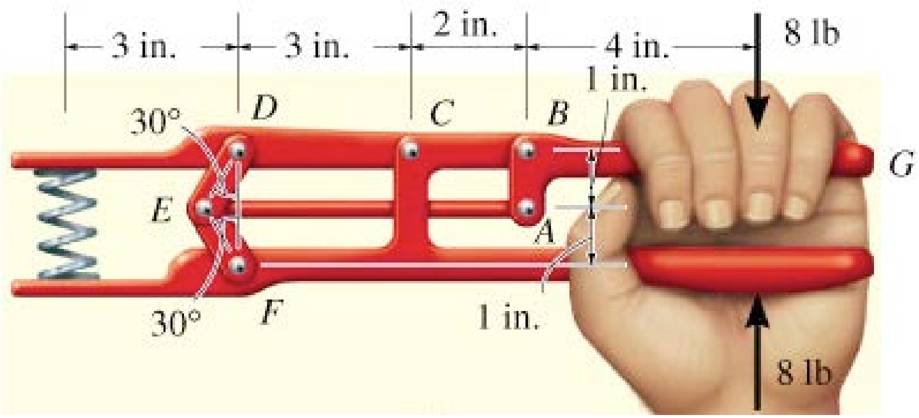
FBD member *ACE* (check):



$$\sum M_A = (F_{DE}\cos\alpha)(300\text{ mm}) + (F_{DE}\sin\alpha)(100\text{ mm}) - C_x(220\text{ mm})$$
$$= (-561\cos\alpha)(300\text{ mm}) + (-561\sin\alpha)(100\text{ mm}) - (-795)(220\text{ mm}) = 0$$

(checks)

Sample Problem (§ 6.6)



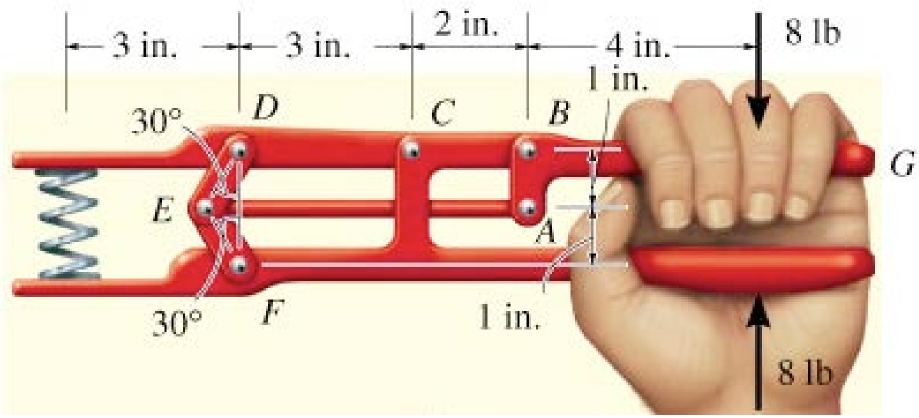
Given: Hand exerts a force of 8 lb on the grip of the spring compressor shown.

Find: Determine the force in the spring necessary to ensure equilibrium of the mechanism.

Plan:

Work your way backwards, from the spring force to the input force and develop the FBD of the members

Apply the equations of equilibrium and solve for the unknowns



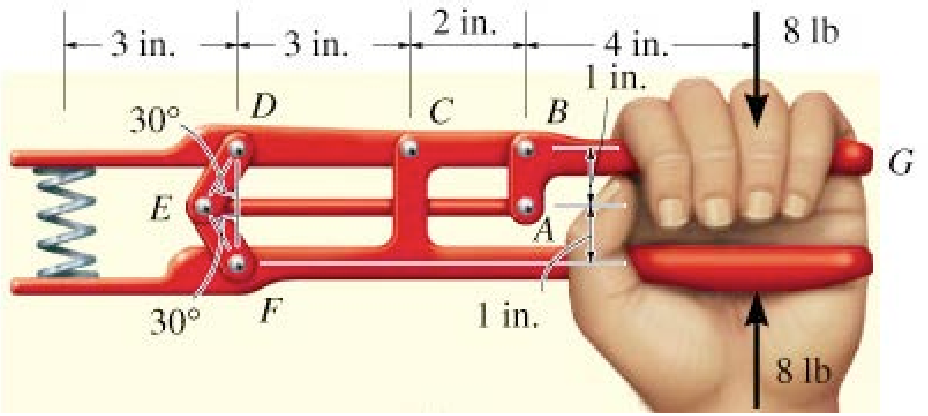
Two members are in contact with the spring.

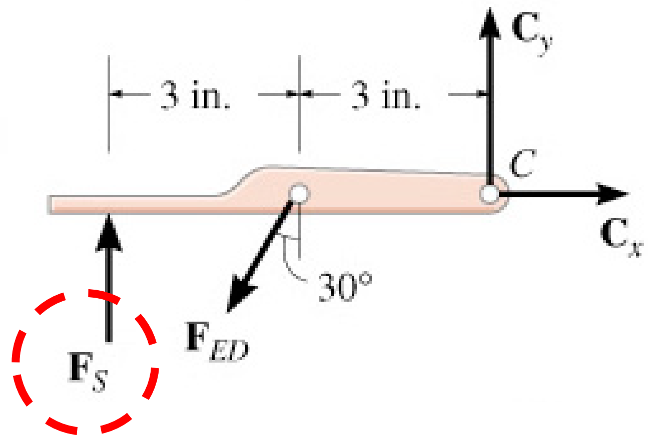
- The upper member is pinned at *D* and *C*.

  Member DE is a two-force member (one unknown at D), pin C has two reactions (two unknows). Total 3 unknowns + Fs

- The lower member is pinned at *F*, *C*, *B*.

  Member EF is a two-force member (one unknown at F), pin C has two reactions (two unknows), and pin B has two reactions. Total 5 unknowns + Fs.



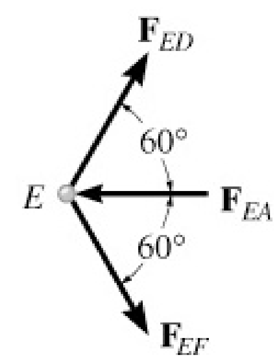


Draw the FBD of the upper member.

- There are four unknowns, our goal is to reach the 8 lb input.

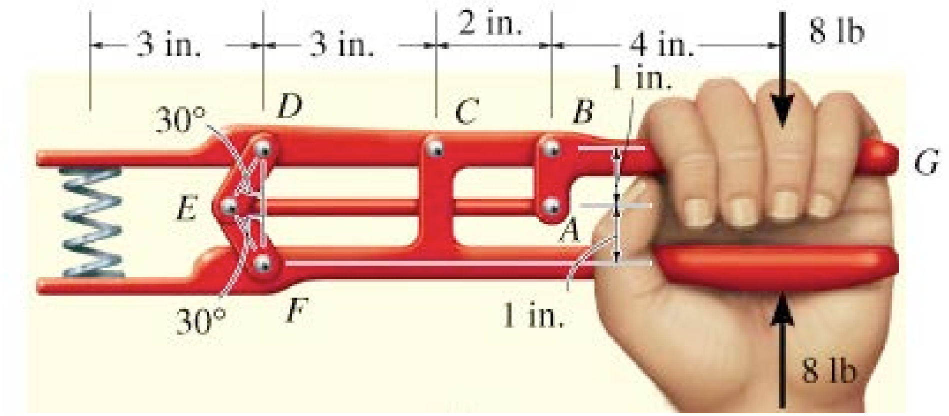
  If we took moments about C and F_{ED} was known, we could solve for F_S.

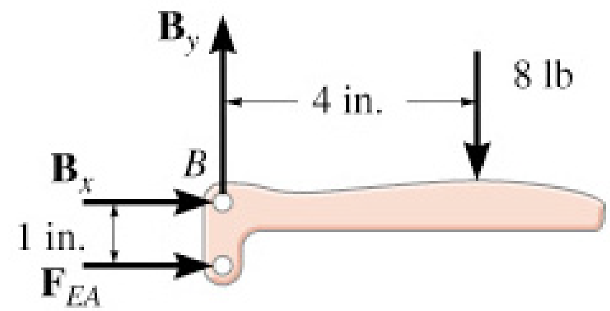
Find F_{ED}, draw the FBD of pin E.



- There are three unknowns.

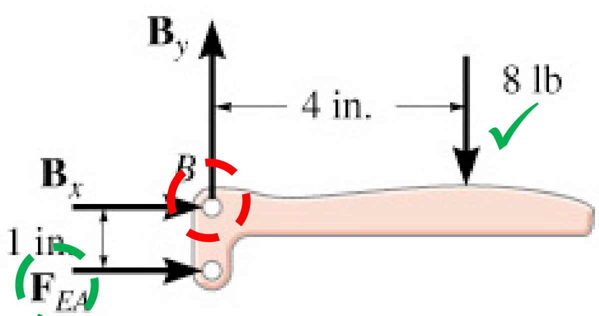
  Since F_{EA} has a horizontal component only, $F_{EDy} – F_{EFy} = 0$.

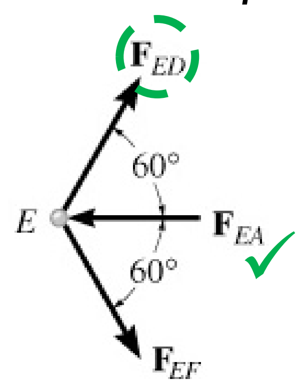


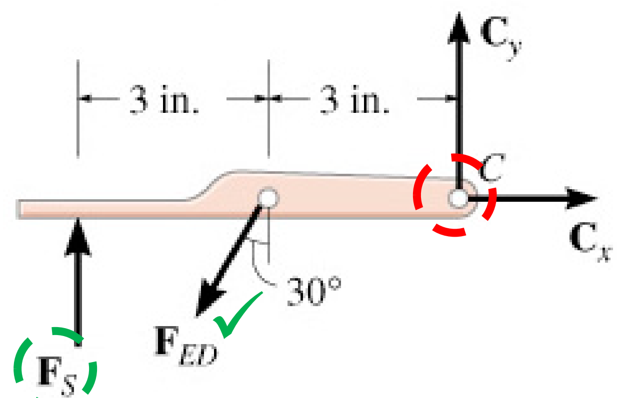


Given that member *EA* is a two-force member. We move to the handle, or member *ABG* and draw FBD.

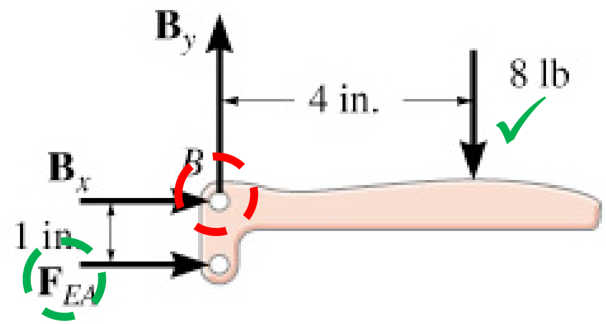
If we took moments about B we can determine F_{EA} and move towards was known, we could solve for F_S.







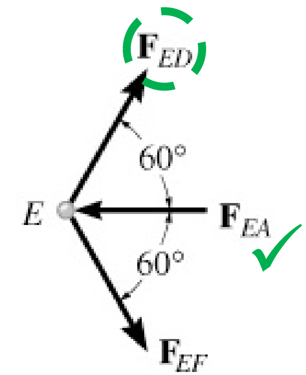
Now that we recognize the path, we can determine the forces.



$\Sigma M_B = 0$

F_{EA} (1 in) – 8 lb (4 in) = 0

F_{EA} = 32 lb (C)



$\Sigma F_y = 0$

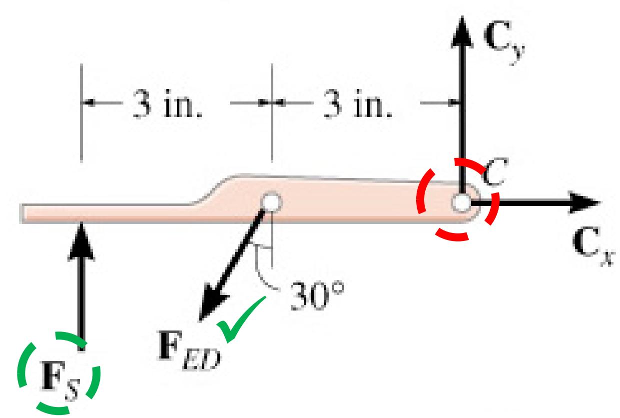
$F_{ED} \sin 60^o - F_{EF} \sin 60^o = 0$

$F_{ED} = F_{EF}$

$\Sigma F_x = 0$

$2 F_{ED} \cos 60 - 32 \text{ lb} = 0$

F_{ED} = 32 lb



$\Sigma M_C = 0$

$- F_S \text{ (6 in)} + 32 \cos 30^o \text{ lb (3 in)} = 0$

F_S = 13.9 lb