Exercise 2.101

R Answer (a).

First, we compute $\mathcal{D}x$ to obtain

$$\begin{aligned}\mathcal{D}x(t) &= \tfrac{d}{dt}(t^2+2t+1) \\ &= 2t+2.\end{aligned}$$

So, we have

$$\begin{aligned}\mathcal{D}x(3t) &= 2(3t)+2 \\ &= 6t+2.\end{aligned}$$

Exercise 2.101

R Answer (b).

First, we give a name v to the anonymous function represented by $x(3\cdot)$. That is, we define $v(t) = x(3t)$. So, we have

$$\begin{aligned} v(t) &= x(3t) \\ &= (3t)^2 + 2(3t) + 1 \\ &= 9t^2 + 6t + 1. \end{aligned}$$

So, we have

$$\begin{aligned} y(t) &= \mathcal{D}\{x(3\cdot)\}(t) \\ &= \mathcal{D}v(t) \\ &= \tfrac{d}{dt}(9t^2 + 6t + 1) \\ &= 18t + 6. \end{aligned}$$

Exercise 3.102

R Answer (c).

Let $x_1(t) = 7\cos(35t+3)$ and $x_2(t) = 5\sin(15t-2)$. Let T_1 and T_2 denote the fundamental periods of x_1 and x_2, respectively. We have that

$$T_1 = \frac{2\pi}{35}, \quad T_2 = \frac{2\pi}{15}, \quad \text{and} \quad \frac{T_1}{T_2} = \frac{\left(\frac{2\pi}{35}\right)}{\left(\frac{2\pi}{15}\right)} = \frac{2\pi}{35}\left(\frac{15}{2\pi}\right) = \frac{15}{35} = \frac{3}{7}.$$

Since $\frac{T_1}{T_2}$ is rational, x is periodic. We have that

$$T = 7T_1 = 3T_2 = 7\left(\tfrac{2\pi}{35}\right) = \tfrac{2\pi}{5}.$$

Exercise 3.103

R **Answer (j).**

We are told that the function x is such that:

1. $x(t) = 2 - t$ for $0 \le t < 1$;
2. the function $v(t) = x(t) - 2$ is causal; and
3. the function $w(t) = x(t+1)$ is odd.

First, we consider the consequences of v being causal. From the fact that $v(t) = x(t) - 2$ is causal, we have

$$\begin{aligned} v(t) &= 0 \text{ for all } t < 0 \Rightarrow \\ x(t) - 2 &= 0 \text{ for all } t < 0 \Rightarrow \\ x(t) &= 2 \text{ for all } t < 0. \end{aligned}$$

Now, we consider the consequences of w being odd. Since $w(t) = x(t+1)$ we have

$$x(t) = w(t-1).$$

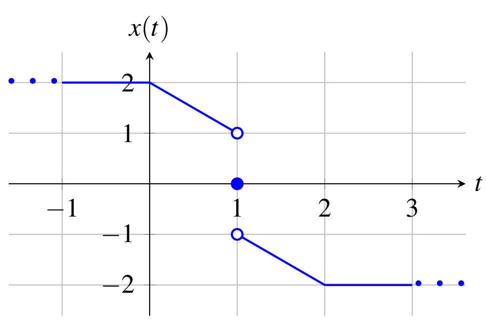
(i.e., x is w shifted to the right by 1). Thus, since w is odd and x is w shifted right by 1, x has odd symmetry about 1. Therefore, $x(1) = 0$. Next, we determine $x(t)$ for $1 < t \le 2$. This can be deduced either graphically or algebraically. Since a graphical approach is easier, we will use this approach here. (An algebraic approach is presented at the end of this solution.) With a graphical approach, we can simply visualize the consequences of the symmetry in x from a graph of $x(t)$ for $0 \le t < 1$. (See the part of the plot of $x(t)$ below for $0 \le t < 1$, which is known from the information given in the problem statement.) This allows us to deduce that

$$x(t) = -t \text{ for } 1 < t \le 2.$$

Combining the results from above, we conclude

$$x(t) = \begin{cases} 2 & t < 0 \\ 2 - t & 0 \le t < 1 \\ 0 & t = 1 \\ -t & 1 < t \le 2 \\ -2 & t > 2. \end{cases}$$

A plot of x is shown in the figure below.



Remarks on algebraic approach. As mentioned above, the formula for $x(t)$ for $1 < t \le 2$ can also be deduced algebraically (instead of graphically). Now, we will perform this deduction using an algebraic approach. Since x has odd symmetry about 1, we know that

$$\begin{aligned} x(1+t) &= -x(1-t) \text{ for all } t \quad \Rightarrow \\ x(t) &= -x(1-(t-1)) \text{ for all } t \quad \Rightarrow \\ x(t) &= -x(2-t) \text{ for all } t. \end{aligned}$$

Substituting into the preceding equation for the case that $0 \leq t < 1$, we have

$$\begin{gathered}
x(t) = -x(2-t) \text{ for } 0 \leq t < 1 \quad \Rightarrow \\
x(t) = -[2-(2-t)] \text{ for } 0 \leq 2-t < 1 \quad \Rightarrow \\
x(t) = -2+2-t \text{ for } 0 \leq 2-t < 1 \quad \Rightarrow \\
x(t) = -t \text{ for } 1 < t \leq 2.
\end{gathered}$$

(Above, we used that fact that $0 \leq 2-t < 1 \Leftrightarrow 0 \leq 2-t$ and $2-t < 1 \Leftrightarrow t \leq 2$ and $1 < t \Leftrightarrow 1 < t \leq 2$.)

Exercise 3.105

R Answer (e).

Let $y(t)$ denote the value of the given expression for a particular value of t. So, we are given

$$y(t) = \int_t^{\infty} (\tau^2+1)\delta(\tau-2)d\tau.$$

From the equivalence property of the δ function, we have

$$\begin{aligned} y(t) &= \int_t^{\infty} \left[\tau^2+1\right]\Big|_{\tau=2} \delta(\tau-2)d\tau \\ &= \int_t^{\infty} (2^2+1)\delta(\tau-2)d\tau \\ &= \int_t^{\infty} 5\delta(\tau-2)d\tau \\ &= 5\int_t^{\infty} \delta(\tau-2)d\tau. \end{aligned}$$

Since $\delta(\cdot-2)$ has an area of 1 concentrated at the point 2, we have

$$\begin{aligned} y(t) &= \begin{cases} 5(1) & t \leq 2 \\ 5(0) & \text{otherwise} \end{cases} \\ &= \begin{cases} 5 & t \leq 2 \\ 0 & \text{otherwise} \end{cases} \\ &= 5u(2-t). \end{aligned}$$

Exercise 3.106

R Answer (b).

We are given the function

$$x(t) = \begin{cases} -e^{t+1} & t < -1 \\ t & -1 \leq t < 1 \\ (t-2)^2 & 1 \leq t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

We have

$$\begin{aligned} x(t) &= -e^{t+1}[u(t-[-\infty]) - u(t+1)] + t[u(t+1) - u(t-1)] + (t-2)^2[u(t-1) - u(t-2)] \\ &= -e^{t+1}[1 - u(t+1)] + t[u(t+1) - u(t-1)] + (t-2)^2[u(t-1) - u(t-2)] \\ &= -e^{t+1} + (t + e^{t+1})u(t+1) + [(t-2)^2 - t]u(t-1) - (t-2)^2 u(t-2) \\ &= -e^{t+1} + (t + e^{t+1})u(t+1) + [t^2 - 5t + 4]u(t-1) - (t-2)^2 u(t-2) \\ &= -e^{t+1} + (t + e^{t+1})u(t+1) + (t-1)(t-4)u(t-1) - (t-2)^2 u(t-2). \end{aligned}$$

Exercise 3.110

R Answer (a).

A system $\mathcal{H}$ is said to be BIBO stable if $\mathcal{H}x$ is bounded for every bounded function x. We have

$$\mathcal{H}x(t) = u(t)x(t).$$

Assume that $|x(t)| \leq A < \infty$ (i.e., x is bounded). Then, we need to show that this implies that $\mathcal{H}x$ is bounded. Taking the magnitude of both sides of the system equation, we have

$$\begin{aligned} |\mathcal{H}x(t)| &= |u(t)x(t)| \\ &= |u(t)|\,|x(t)|. \end{aligned}$$

Replacing the expressions $|u(t)|$ and $|x(t)|$ in the preceding equation by their upper bounds (of 1 and A, respectively), we obtain the inequality

$$|\mathcal{H}x(t)| \leq 1 \cdot A = A.$$

Thus, $|\mathcal{H}x(t)| \leq A < \infty$ (i.e., $\mathcal{H}x$ is bounded). Since the boundedness of x implies the boundedness of $\mathcal{H}x$, the system is BIBO stable.

Exercise 3.114

R Answer (b).

We have

$$\mathcal{H}x_1(t) = \int_{-\infty}^{t} x_1(\tau)d\tau = \tfrac{1}{2}e^{2t} = \tfrac{1}{2}x_1(t) \quad \text{and}$$

$$\mathcal{H}x_2(t) = \int_{-\infty}^{t} x_2(\tau)d\tau = \begin{cases} e^t & t < 0 \\ 1 & \text{otherwise.} \end{cases}$$

Therefore, x_1 is an eigenfunction with eigenvalue $\frac{1}{2}$ and x_2 is not an eigenfunction.