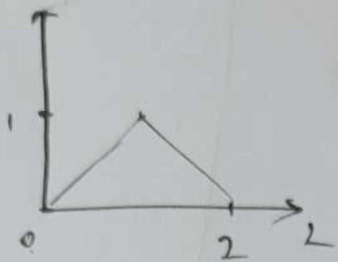
Arifaz Hossain

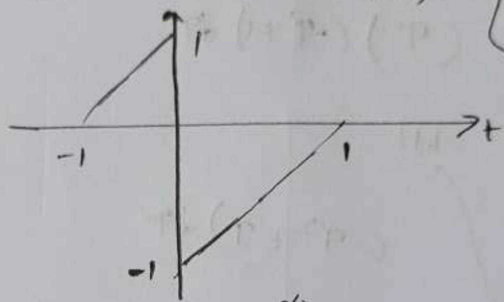
V00984826

(4) (1) e



$$x(t) = \begin{cases} 1 & ; \ 0 \le t \le 1 \\ -t+2 & ; \ 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} t+1 & ; \ -1 \le t \le 0 \\ t-1 & ; \ 0 \le t < 1 \\ 0 & ; \ \text{otherwise} \end{cases}$$



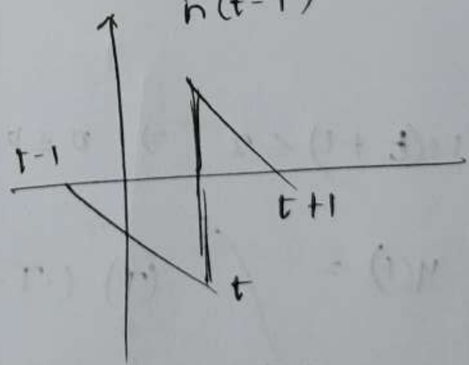
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

For $\boxed{t+1 < 0} \Rightarrow \boxed{t < -1}$

there is no overlap between

$x(\tau)$ and $h(t-\tau)$

$\Rightarrow y(t) = 0$



For $(t+1)$ between 0 and 1

$0 \le (t+1) < 1$

$\Rightarrow -1 \le t < 0$

$$y(t) = \int_0^{t+1} (\tau)(-\tau+1)\, d\tau$$

$$= \int_0^{t+1} (-\tau^2+\tau)\, d\tau$$

$$= \left[-\frac{\tau^3}{3} + \frac{\tau^2}{2} \right]_0^{t+1}$$

$$= \frac{-(t+1)^3}{3} + \frac{(t+1)^2}{2}$$

For $1 \le (t+1) \le 2 \Rightarrow 0 \le t < 1$

$$y(t) = \int_{t-1}^{t} (\tau)(-\tau-1)\, d\tau + \int_t^{t+1} (-\tau+2)(-\tau-1)\, d\tau$$

$$= \int_{t-1}^{t} (-\tau^2-\tau)\, d\tau + \int_t^{t+1} (\tau^2-\tau-2)\, d\tau$$

$$= \left[-\frac{\tau^3}{3} - \frac{\tau^2}{2} \right]^{t} + \left[\frac{\tau^3}{3} - \frac{\tau^2}{2} - 2\tau \right]_{t}^{t+1}$$

$$= \frac{-t^3}{3} - \frac{t^2}{2} + \frac{(t-1)^3}{3} + \frac{(t-1)^2}{2} - \frac{t^3}{3} + 2t - \frac{(t+1)^2}{2} - 2(t+1) + \frac{(t+1)^3}{3}$$

$$= -\frac{2t^3}{3} + \frac{(t-1)^3}{3} + \frac{(t+1)^3}{3} + \frac{(t-1)^2}{2} - \frac{(t+1)^2}{2} - 2$$

For $2 \le (t+1) < 3 \Rightarrow 1 \le t < 2$

$$y(t) = \int_{t-1}^{2} (-\tau+2)(-\tau-1)\, d\tau = \int_{t-1}^{2} (\tau^2 - \tau - 2)\, d\tau$$

$$= \left[\frac{\tau^3}{3} - \frac{\tau^2}{2} - 2\tau \right]_{t-1}^{2}$$

$$= -\frac{(t-1)^3}{3} + \frac{(t-1)^2}{2} + 2(t-1) + \frac{8}{3} - 2 - 4$$

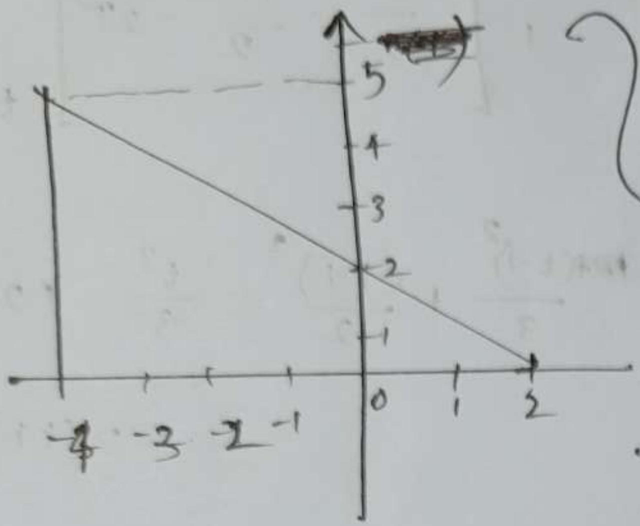
$$= -\frac{(t-1)^3}{3} + \frac{(t-1)^2}{2} + 2t - \frac{16}{3}$$

for $(t+1) \geq 3 \Rightarrow t \geq 2$

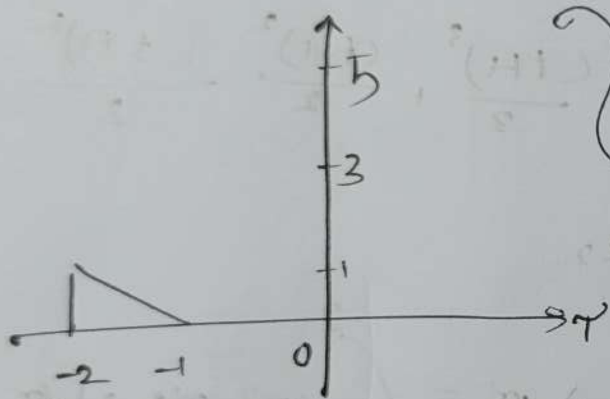
$y(t) = 0$

Therefore,
$$y(t) = \begin{cases} 0 \,;\ (t < -1) \\ \left[-\frac{(t+1)^3}{3} + \frac{(t+1)^2}{2} \right] ;\ (-1 \le t < 0) \\ \left[\frac{-2t^3}{3} + \frac{(t-1)^3}{3} + \frac{(t+1)^3}{3} + \frac{(t-1)^2}{2} - \frac{(t+1)^2}{2} - 2 \right] ;\ (0 \le t < 1) \\ \left[-\frac{(t-1)^3}{3} + \frac{(t+1)^2}{2} + 2t - 16/3 \right] ;\ (1 \le t < 2) \\ 0 \,;\ (t \geq 2) \end{cases}$$

(2)

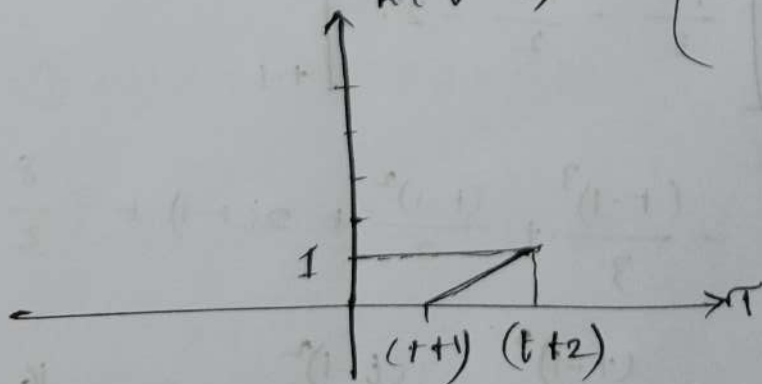


$$x(\tau) = \begin{cases} -\tau + 2 \; ; & -3 \leq \tau \leq 2 \\ 0 \; ; & \text{otherwise} \end{cases}$$



$$h(\tau) = \begin{cases} -\tau - 1 \; ; & -2 \leq \tau \leq -1 \\ 0 \; ; & \text{otherwise} \end{cases}$$

$$h(t-\tau) = \begin{cases} \tau - (t+1) \; ; & (t+1) \leq \tau \leq (t+2) \\ 0 \; ; & \text{otherwise} \end{cases}$$



For $(t+2) \leq -3 \Rightarrow t < -5$

$y(t) = 0$ as there is no overlap between $x(\tau)$ and $h(t-\tau)$

For $-3 \leq (t+2) \leq -2 \Rightarrow -5 \leq t \leq -4$

$$y(t) = \int_{-3}^{t+2} (-\tau+2).(\tau-1).d\tau$$

$$= \int_{-3}^{t+2} (-\tau^2+3\tau-2)\, d\tau$$

$$= \left[\frac{-\tau^3}{3} + \frac{3\tau^2}{2} - 2\tau\right]_{-3}^{t+2}$$

$$= \frac{-(t+2)^3}{3} + \frac{3(t+2)^2}{2} - 2(t+2)$$

$$- \frac{27}{3} - \frac{27}{2} - 6$$

$$= \frac{-(t+2)^3}{3} + \frac{3(t+2)^2}{2} - 2t - \frac{195}{6}$$

For $-2 \le (t+2) < 2, \Rightarrow -4 \le t < 0$

$$y(t) = \int_{t+1}^{t+2} (-\tau + 2) \cdot (+\tau - 1)\, d\tau$$

$$= \int_{t+1}^{t+2} (-\tau^2 + 3\tau - 2)\, d\tau$$

$$= \left[-\frac{\tau^3}{3} + \frac{3\tau^2}{2} - 2\tau \right]_{t+1}^{t+2}$$

$$= -\frac{(t+2)^3}{3} + \frac{(t+1)^3}{3} + \frac{3(t+2)^2}{2}$$

$$- \frac{3(t+1)^2}{2} - 2$$

For $2 \le (t+2) < 3 \Rightarrow 0 \le t < 1$

$$y(t) = \int_{t+1}^{2} (-\tau+2)(\tau-1)\,d\tau = \int_{t+1}^{2} (-\tau^2+3\tau-2)\,d\tau$$

$$= \left[-\frac{\tau^3}{3} + \frac{3\tau^2}{2} - 2\tau \right]_{t+1}^{2}$$

$$= \frac{(t+1)^3}{3} - \frac{3(t+1)^2}{2} + 2(t+1) - \frac{8}{3} + 6 - 4$$

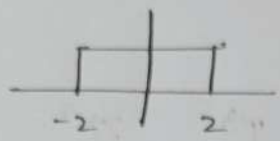
$$= \frac{(t+1)^3}{3} - 3\frac{(t+1)^2}{2} + 2t + \frac{4}{3}$$

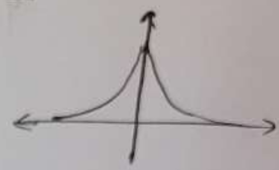
For $t+2 \geq 3 \Rightarrow t \geq 1$

$$y(t) = 0$$

$$\therefore y(t) = \begin{cases} 0 ; (t < -5) \\ -\frac{(t+2)^3}{3} + \frac{3(t+2)^2}{2} - 2t - \frac{105}{6} ; (-5 \le t < -4) \\ -\frac{(t+2)^3}{3} + \frac{(t+1)^3}{3} + \frac{3(t+2)^2}{2} - \frac{3(t+1)^2}{2} - 2 ; (-4 \le t < 0) \\ \frac{(t+1)^3}{3} - \frac{3(t+1)^2}{2} + 2t + \frac{4}{3} ; 0 \le t < 1 \\ 0 ; (t \geq 1) \end{cases}$$

(b) Given $x(t) = e^{-|t|}$ and $h(t) = \text{rect}\left[\frac{1}{2}\left(t-\frac{1}{2}\right)\right]$

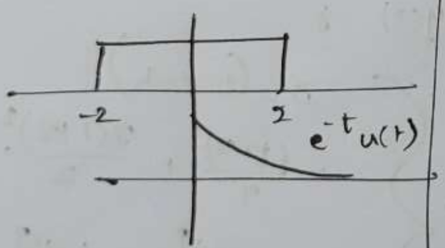
$h(t) \rightarrow$ 

; $x(t) =$ 

$$x(t) * h(t) = h(t) * e^{-t}u(t) + h(t) * e^{t}u(-t)$$

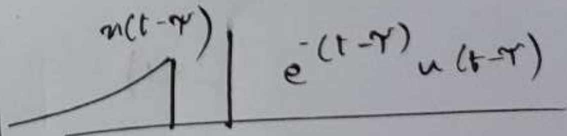
$$\left[x(t) = e^{-t}u(t) + e^{t}u(-t)\right]$$

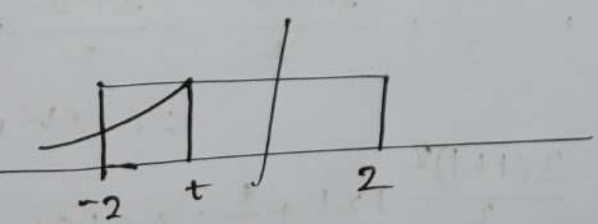
Now, ① $h(t) * e^{-t}u(t)$



replacing t and shifting by τ

$t \rightarrow (t-\tau)$





By definition,

$$x(t) * h(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{t} h(\tau)\, x(t-\tau)\, d\tau$$

for this case convolution becomes

$$\int_{-2}^{t} h(\tau)\,x(t-\tau)\,d\tau$$

[limits are -2 to t because it is non zero product in that interval]

substituting $h(\tau) = 1$

$$x(t-\tau) = e^{-(t-\tau)}u(t-\tau)$$

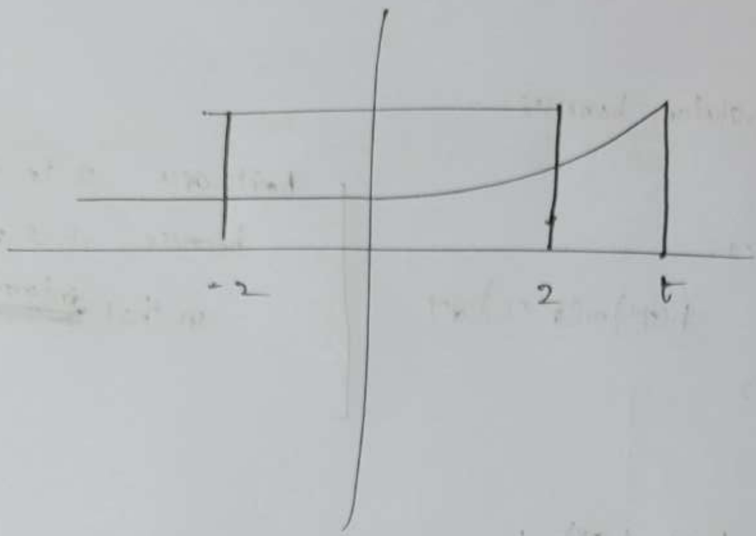
$$\int_{-2}^{t} e^{-t} e^{\tau}\,d\tau$$

ignoring $u(t-\tau)$

$$= e^{-t}\left[e^{\tau}\right]_{-2}^{t}$$

$$= e^{-t}\left[e^{t} - e^{-2}\right]$$

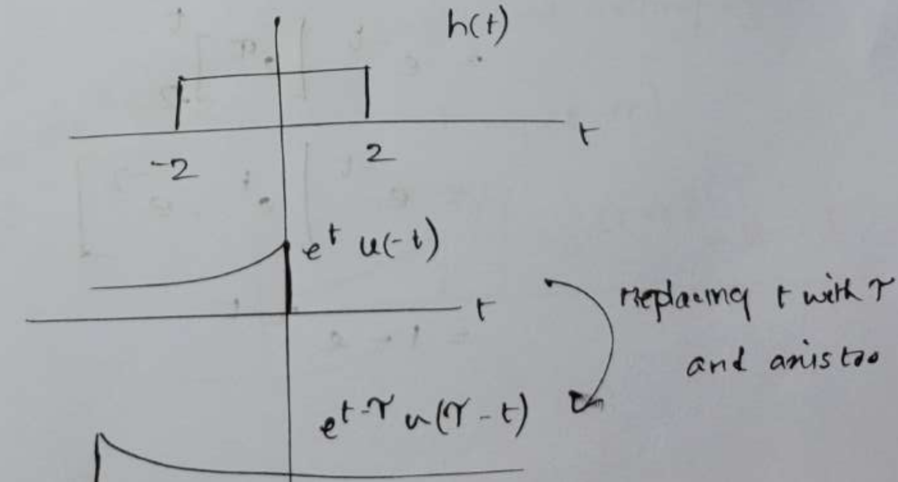
$$= 1 - e^{-2-t}$$

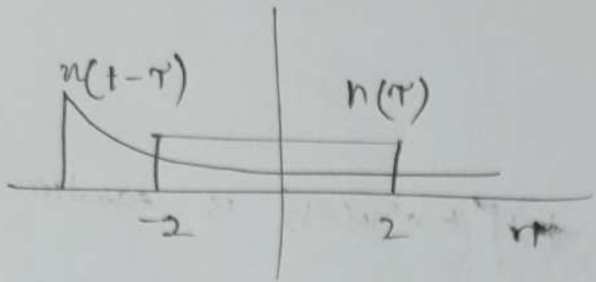


convolution of $h(t) * e^{-t}u(t)$ is ① + ②

$$= 1 - 2e^{-2-t} + e^{2-t} \quad \text{——— ①}$$

The second part is : $h(t) * e^{t}u(-t)$





case (i)

limits of this convolution: -2 to 2

$$= \int_{-2}^{2} h(\tau)\, x(t-\tau)\, d\tau$$

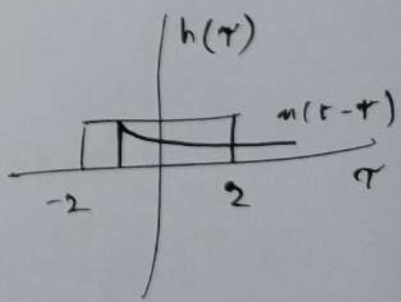
$$= \int_{-2}^{2} e^{t} e^{-\tau}\, d\tau = \frac{e^{t}}{-1}\left[e^{-\tau}\right]_{-2}^{2}$$

$$= -e^{t}\left[e^{-2} - e^{2}\right]$$

$$= e^{2+t} - e^{t-2} \quad \text{—— (3)}$$

---

limits of convolution: t to 2



$$= \int_{t}^{2} e^{t} \cdot e^{-\tau}\, d\tau$$

$$= \frac{e^{t}}{-1}\left[e^{-\tau}\right]_{t}^{2}$$

$$= -e^{t}\left[e^{-2} - e^{-t}\right] = 1 - e^{t-2} \quad \text{—— (4)}$$

convolution of $h(t)$ + $e^{t}u(-t)$ is equation 3 + 4

$$= 1 + e^{2+t} - 2e^{t-2} \longrightarrow (11)$$

$$h(t) * x(t) = \mathrm{I} + \mathrm{II}$$

$$= 1 - 2e^{-2-t} + e^{2-t} + 1 + e^{2+t} - 2e^{t-2}$$

$$= 2 - 2e^{-2-t} + e^{2-t} + e^{2+t} - 2e^{t-2}$$

4.5. From definition:

$$v(t) = \int_{-\infty}^{\infty} x(-\tau - b)\, h(\tau + at)\, d\tau$$

convolution of two functions given by:

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t - \tau)\, d\tau \qquad \text{(i)}$$

$$y(at - b) = \int_{-\infty}^{\infty} x(\tau)\, h(at - b - \tau)\, d\tau \qquad \text{(ii)}$$

$$v(t) = \int_{-\infty}^{\infty} x(-\tau - b)\, h(\tau + at)\, d\tau$$

putting $-\tau - b = \tau'$

$\Rightarrow -d\tau = d\tau'$

$$= -\int_{\infty}^{-\infty} x(\tau')\, h(at - b - \tau')\, d\tau'$$

$$= \int_{-\infty}^{\infty} x(\tau')\, h(at - b - \tau')\, d\tau'$$

$= y(at - b)$; using (ii)

$\therefore \boxed{v(t) = y(at - b)}$

a and b are real constants

4.6. (a)

From definition of convolution,

$$y(t) = x * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$

Supposing x is periodic w/ period T, we have $x(t) = x(t+T)$. Rewriting above equation, we get

$$y(t) = \int_{-\infty}^{\infty} x(t+T)\, h(t-\tau)\, d\tau$$

Let $T + \tau = a$

or, $\tau = a - T$

or, $d\tau = da$

$$y(t) = \int_{-\infty}^{\infty} x(a)\, h(t-\tau)\, da$$

$$= \int_{-\infty}^{\infty} x(a)\, h(t - a + T)\, da$$

$$= \int_{-\infty}^{\infty} x(a)\, h((t+T) - a)\, da$$

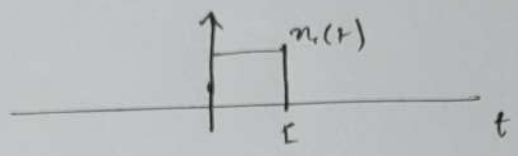
$$= x * h(t+T) = y(t+T)$$

Therefore y is periodic with T period.

49

~~Expressing $x_2(t)$ in terms of $x_1(t)$~~

given $x_1(t) = u(t) - u(t-1)$



Now, $x_1(t) \longrightarrow \boxed{\text{system}} \longrightarrow y_1(t)$

~~and $x_2(t)$ is given as~~

Expressing $x_2(t)$ in terms of $x_1(t)$ as

$$x_2(t) = x_1(t) - 2x_1(t-1) + x_1(t+1) + 2x_1(t+2)$$

Since the system is LTI $\longrightarrow$

LTI System:

$x_1(t) \xleftrightarrow{S} y_1(t)$

$x_1(t-1) \xleftrightarrow{S} y_1(t-1)$

$-2x_1(t-1) \longleftrightarrow -2y_1(t$

Similarly,

$x_1(t+1) \xleftrightarrow{S} y_1(t+1)$

& $2x_1(t+2) \xleftrightarrow{S} 2y_1(t+2)$

therefore,

$x_2(t) \longrightarrow \boxed{\begin{matrix}\text{System} \\ \text{LTI}\end{matrix}} \longrightarrow y_2(t)$

$x_1(t) - 2x_1(t-1) + x_1(t+1) + 2x_1(t+2)$

$y_2(t) = y_1(t) + y_1(t+1) - 2y_1(t-1) + 2y_1(t+2)$ $\longrightarrow y_2(t) \longrightarrow$ Answer.

somefuncAs3A.m

```
w= -10:0.01:10;
f = (j * w + 1).^(-1);

subplot(2, 1, 1);
plot(w, abs(f));
xlabel('omega');
ylabel('|F(omega)|');

subplot(2, 1, 2);
plot(w, unwrap(angle(f)));
xlabel('omega');
ylabel('arg F(omega)');
```

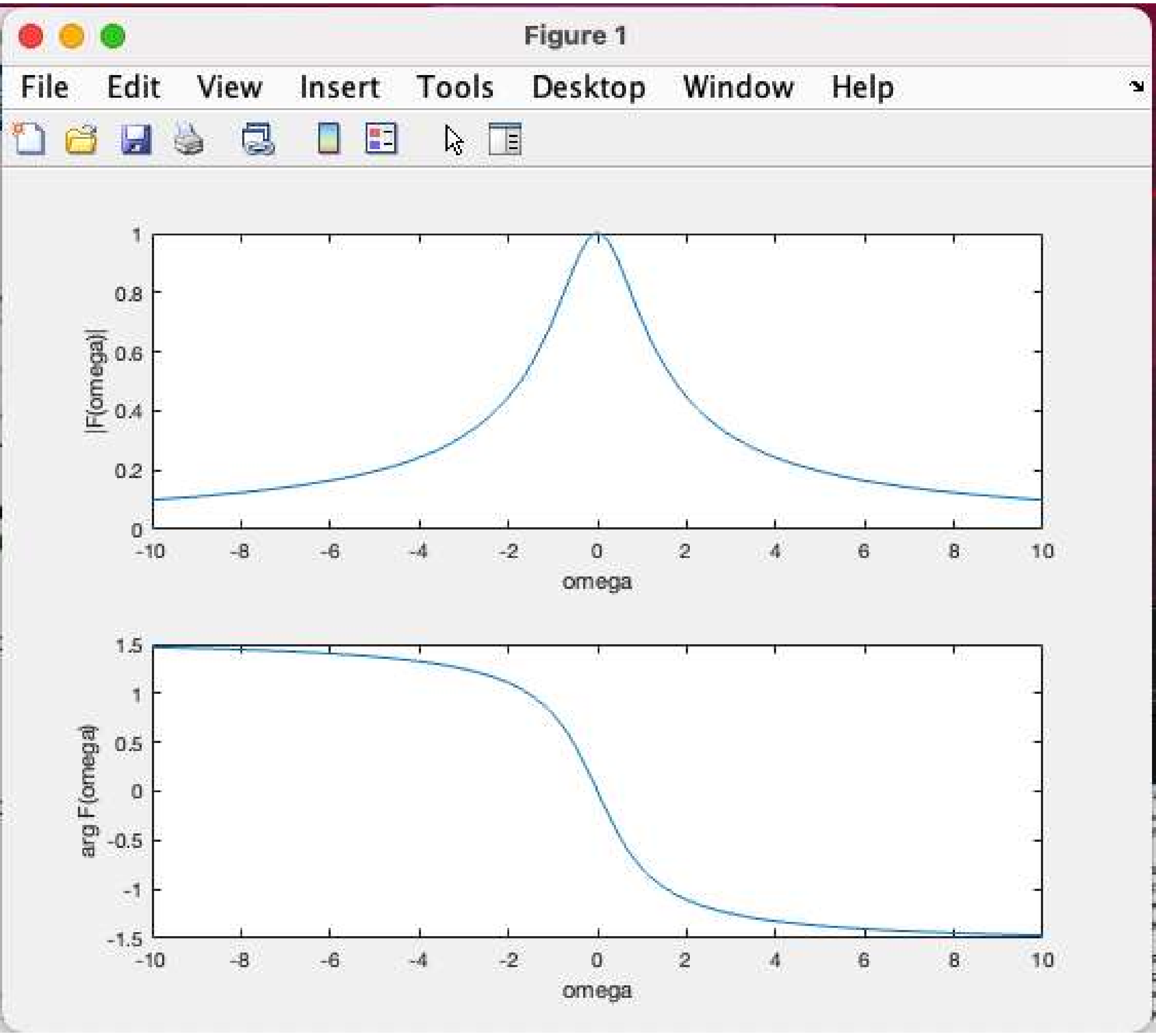


Figure 1
File Edit View Insert Tools Desktop Window Help
|F(omega)|
omega
arg F(omega)
omega
1
0.8
0.6
0.4
0.2
0
1.5
1
0.5
0
-0.5
-1
-1.5
-10 -8 -6 -4 -2 0 2 4 6 8 10
-10 -8 -6 -4 -2 0 2 4 6 8 10