Exercise 6.10

L Answer (b).

We are asked to find the frequency spectrum of x, where

$$x(t) = \text{sinc}\left(\tfrac{1}{200}t - \tfrac{1}{200}\right).$$

x is sinc function that is shifted by 1/200 and then time scaled by 1/200

We begin by rewriting x as

$$x(t) = \text{sinc}\left[\tfrac{1}{200}(t-1)\right].$$

x is sinc function that is time scaled by 1/200 and the shifted by 1

Defining the function $v_1(t) = \text{sinc}(t/200)$, we have

$$x(t) = v_1(t-1). \quad (2)$$

Let X and V_1 denote the Fourier transforms of x and v_1, respectively. Taking the Fourier transform of the above equations for x and v_1, we obtain

$$X(\omega) = e^{-j\omega}V_1(\omega) \quad (3) \text{ and}$$

FT of (2) (time shift)

$$V_1(\omega) = \tfrac{\pi}{1/200}\,\text{rect}\left(\tfrac{1}{2(1/200)}\omega\right) = 200\pi\,\text{rect}(100\omega). \quad (4)$$

FT of (1) (from table)

simplify

Combining the above results, we obtain

$$\begin{aligned} X(\omega) &= e^{-j\omega}V_1(\omega) \quad (5) \\ &= e^{-j\omega}[200\pi\,\text{rect}(100\omega)] \\ &= 200\pi e^{-j\omega}\,\text{rect}(100\omega). \end{aligned}$$

(3) substitute (4) for V_1

reorder factors

The magnitude spectrum of x is given by

$$\begin{aligned} |X(\omega)| &= |200\pi e^{-j\omega}\,\text{rect}(100\omega)| \\ &= 200\pi\,\text{rect}(100\omega) \\ &= \begin{cases} 200\pi & |\omega| < \frac{1}{200} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

take magnitude of (5)

$|z_1 z_2| = |z_1||z_2|$

definition of rect

The phase spectrum of x is given by

$$\begin{aligned} \arg X(\omega) &= \arg[200\pi e^{-j\omega}\,\text{rect}(100\omega)] \\ &= \arg(e^{-j\omega}) \\ &= -\omega. \end{aligned}$$

take argument of (5)

$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

arg of polar form

The magnitude and phase spectra are plotted below.