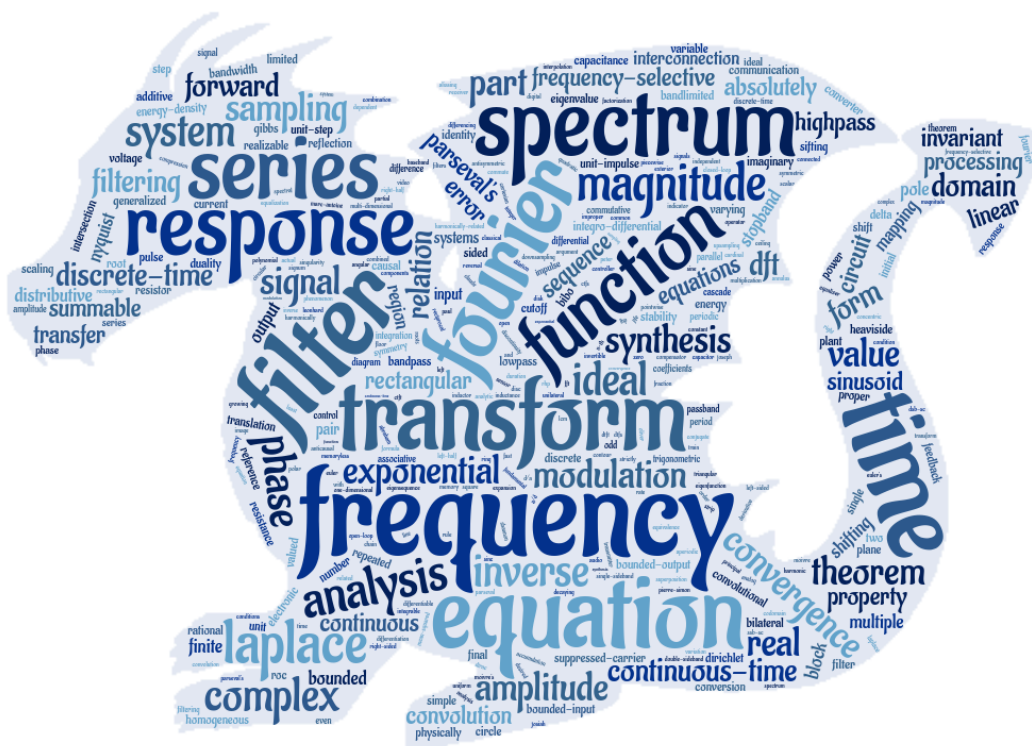


Signals and Systems

Edition 5.0



Michael D. Adams



To obtain the **most recent version** of this book (with functional hyperlinks) or for additional information and resources related to this book (such as lecture slides, **video lectures**, and errata), please visit:

<https://www.ece.uvic.ca/~mdadams/sigsysbook>

If you like this book, **please consider posting a review** of it at:

<https://play.google.com/store/search?q=ISBN:9781990707001&c=books> or
<https://books.google.com/books?vid=ISBN9781990707001>



youtube.com/iamcanadian1867



github.com/mdadams



@mdadams16

Signals and Systems

Edition 5.0



Michael D. Adams

Department of Electrical and Computer Engineering

University of Victoria

Victoria, British Columbia, Canada

The author has taken care in the preparation of this book, but makes no expressed or implied warranty of any kind and assumes no responsibility for errors or omissions. No liability is assumed for incidental or consequential damages in connection with or arising out of the use of the information or programs contained herein.

Copyright © 2012, 2013, 2020, 2022 Michael D. Adams

This book is licensed under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 Unported (CC BY-NC-ND 3.0) License. A copy of this license can be found in the section titled “License” on page **xxxi** of this book. For a simple explanation of the rights granted by this license, see:

<https://creativecommons.org/licenses/by-nc-nd/3.0/>

MATLAB is a registered trademark of The MathWorks, Inc.

Image Processing Toolbox, Optimization Toolbox, Symbolic Math Toolbox, Signal Processing Toolbox, and Wavelet Toolbox are registered trademarks of The MathWorks, Inc.

UNIX and X Window System are registered trademarks of The Open Group.

Linux is a registered trademark of Linus Torvalds.

Windows is a registered trademark of Microsoft Corporation.

macOS is a registered trademark of Apple Inc.

Chrome OS is a registered trademark of Google LLC.

Fedora is a registered trademark of Red Hat, Inc.

Ubuntu is a registered trademark of Canonical Ltd.

The YouTube logo is a registered trademark of Google, Inc.

The GitHub logo is a registered trademark of GitHub, Inc.

The Twitter logo is a registered trademark of Twitter, Inc.

This book was typeset with \LaTeX .

ISBN 978-1-990707-00-1 (PDF)

To my students, past, present, and future

Contents

License	xxxi
Preface	xxxvii
Acknowledgments	xxxvii
Guidance for Instructors	xxxviii
About the Author	xli
Other Works by the Author	xlili
1 Introduction	1
1.1 Signals and Systems	1
1.2 Signals	1
1.2.1 Classification of Signals	1
1.2.2 Notation and Graphical Representation of Signals	2
1.2.3 Examples of Signals	2
1.3 Systems	2
1.3.1 Classification of Systems	4
1.3.2 Examples of Systems	4
1.4 Why Study Signals and Systems?	4
1.5 Overview of This Book	6
2 Preliminaries	7
2.1 Overview	7
2.2 Sets	7
2.3 Mappings	8
2.4 Functions	9
2.5 Sequences	10
2.6 Remarks on Abuse of Notation	11
2.7 Dot Notation for Functions and Sequences	12
2.8 System Operators	13
2.9 Transforms	13
2.10 Basic Signal Properties	14
2.10.1 Symmetry of Functions and Sequences	14
2.10.2 Periodicity of Functions and Sequences	15
2.11 Exercises	17
2.11.1 Exercises Without Answer Key	17
2.11.2 Exercises With Answer Key	17

I	Continuous-Time Signals and Systems	19
3	Continuous-Time Signals and Systems	21
3.1	Overview	21
3.2	Transformations of the Independent Variable	21
3.2.1	Time Shifting (Translation)	21
3.2.2	Time Reversal (Reflection)	21
3.2.3	Time Compression/Expansion (Dilation)	23
3.2.4	Time Scaling (Dilation/Reflection)	23
3.2.5	Combining Time Shifting and Time Scaling	24
3.2.6	Two Perspectives on Independent-Variable Transformations	25
3.3	Transformations of the Dependent Variable	26
3.3.1	Amplitude Shifting	26
3.3.2	Amplitude Scaling	26
3.3.3	Combining Amplitude Shifting and Scaling	26
3.4	Properties of Functions	28
3.4.1	Remarks on Symmetry	28
3.4.2	Remarks on Periodicity	29
3.4.3	Support of Functions	31
3.4.4	Bounded Functions	31
3.4.5	Signal Energy and Power	32
3.4.6	Examples	32
3.5	Elementary Functions	34
3.5.1	Real Sinusoidal Functions	34
3.5.2	Complex Exponential Functions	34
3.5.2.1	Real Exponential Functions	35
3.5.2.2	Complex Sinusoidal Functions	35
3.5.2.3	General Complex Exponential Functions	36
3.5.3	Relationship Between Complex Exponential and Real Sinusoidal Functions	38
3.5.4	Unit-Step Function	38
3.5.5	Signum Function	38
3.5.6	Rectangular Function	39
3.5.7	Indicator Function	41
3.5.8	Triangular Function	41
3.5.9	Cardinal Sine Function	41
3.5.10	Rounding-Related Functions	42
3.5.11	Delta Function	42
3.6	Representation of Arbitrary Functions Using Elementary Functions	47
3.7	Continuous-Time Systems	50
3.7.1	Block Diagram Representation	51
3.7.2	Interconnection of Systems	51
3.8	Properties of Systems	51
3.8.1	Memory	51
3.8.2	Causality	52
3.8.3	Invertibility	54
3.8.4	Bounded-Input Bounded-Output (BIBO) Stability	55
3.8.5	Time Invariance	57
3.8.6	Linearity	59
3.8.7	Eigenfunctions	63
3.9	Exercises	64
3.9.1	Exercises Without Answer Key	64
3.9.2	Exercises With Answer Key	69

4	Continuous-Time Linear Time-Invariant Systems	75
4.1	Introduction	75
4.2	Continuous-Time Convolution	75
4.3	Properties of Convolution	83
4.4	Periodic Convolution	87
4.5	Characterizing LTI Systems and Convolution	87
4.6	Step Response of LTI Systems	89
4.7	Block Diagram Representation of Continuous-Time LTI Systems	91
4.8	Interconnection of Continuous-Time LTI Systems	91
4.9	Properties of Continuous-Time LTI Systems	93
4.9.1	Memory	93
4.9.2	Causality	94
4.9.3	Invertibility	95
4.9.4	BIBO Stability	98
4.10	Eigenfunctions of Continuous-Time LTI Systems	101
4.11	Exercises	105
4.11.1	Exercises Without Answer Key	105
4.11.2	Exercises With Answer Key	109
5	Continuous-Time Fourier Series	115
5.1	Introduction	115
5.2	Definition of Continuous-Time Fourier Series	115
5.3	Determining the Fourier Series Representation of a Continuous-Time Periodic Function	116
5.4	Convergence of Continuous-Time Fourier Series	122
5.5	Properties of Continuous-Time Fourier Series	124
5.5.1	Linearity	124
5.5.2	Time Shifting (Translation)	127
5.5.3	Frequency Shifting (Modulation)	127
5.5.4	Time Reversal (Reflection)	128
5.5.5	Conjugation	129
5.5.6	Periodic Convolution	129
5.5.7	Multiplication	130
5.5.8	Parseval's Relation	131
5.5.9	Even and Odd Symmetry	132
5.5.10	Real Functions	132
5.6	Fourier Series and Frequency Spectra	134
5.7	Fourier Series and LTI Systems	139
5.8	Filtering	142
5.9	Exercises	146
5.9.1	Exercises Without Answer Key	146
5.9.2	Exercises With Answer Key	148
5.10	MATLAB Exercises	152
6	Continuous-Time Fourier Transform	153
6.1	Introduction	153
6.2	Development of the Continuous-Time Fourier Transform for Aperiodic Functions	153
6.3	Generalized Fourier Transform	154
6.4	Definition of the Continuous-Time Fourier Transform	157
6.5	Remarks on Notation Involving the Fourier Transform	159
6.6	Convergence of the Continuous-Time Fourier Transform	161
6.7	Properties of the Continuous-Time Fourier Transform	163
6.7.1	Linearity	163

6.7.2	Time-Domain Shifting (Translation)	164
6.7.3	Frequency-Domain Shifting (Modulation)	165
6.7.4	Time- and Frequency-Domain Scaling (Dilation)	166
6.7.5	Conjugation	167
6.7.6	Duality	168
6.7.7	Time-Domain Convolution	169
6.7.8	Time-Domain Multiplication	171
6.7.9	Time-Domain Differentiation	172
6.7.10	Frequency-Domain Differentiation	173
6.7.11	Time-Domain Integration	174
6.7.12	Parseval's Relation	175
6.7.13	Even/Odd Symmetry	177
6.7.14	Real Functions	178
6.8	Continuous-Time Fourier Transform of Periodic Functions	180
6.9	More Fourier Transforms	182
6.10	Frequency Spectra of Functions	187
6.11	Bandwidth of Functions	189
6.12	Energy-Density Spectra	191
6.13	Characterizing LTI Systems Using the Fourier Transform	192
6.13.1	Unwrapped Phase	193
6.13.2	Magnitude and Phase Distortion	194
6.14	Interconnection of LTI Systems	196
6.15	LTI Systems and Differential Equations	199
6.16	Filtering	201
6.17	Equalization	207
6.18	Circuit Analysis	208
6.19	Amplitude Modulation	211
6.19.1	Modulation With a Complex Sinusoid	211
6.19.2	DSB/SC Amplitude Modulation	213
6.19.3	SSB/SC Amplitude Modulation	215
6.20	Sampling and Interpolation	217
6.20.1	Sampling	220
6.20.2	Interpolation and Reconstruction of a Function From Its Samples	222
6.20.3	Sampling Theorem	224
6.21	Exercises	227
6.21.1	Exercises Without Answer Key	227
6.21.2	Exercises With Answer Key	233
6.22	MATLAB Exercises	239
7	Laplace Transform	241
7.1	Introduction	241
7.2	Motivation Behind the Laplace Transform	241
7.3	Definition of the Laplace Transform	241
7.4	Remarks on Notation Involving the Laplace Transform	242
7.5	Relationship Between Laplace Transform and Continuous-Time Fourier Transform	244
7.6	Laplace Transform Examples	245
7.7	Region of Convergence for the Laplace Transform	248
7.8	Properties of the Laplace Transform	256
7.8.1	Linearity	256
7.8.2	Time-Domain Shifting	259
7.8.3	Laplace-Domain Shifting	261
7.8.4	Time-Domain/Laplace-Domain Scaling	262

7.8.5	Conjugation	265
7.8.6	Time-Domain Convolution	266
7.8.7	Time-Domain Differentiation	267
7.8.8	Laplace-Domain Differentiation	268
7.8.9	Time-Domain Integration	269
7.8.10	Initial and Final Value Theorems	270
7.9	More Laplace Transform Examples	272
7.10	Determination of the Inverse Laplace Transform	280
7.11	Characterizing LTI Systems Using the Laplace Transform	285
7.12	Interconnection of LTI Systems	286
7.13	System Function and System Properties	287
7.13.1	Causality	287
7.13.2	BIBO Stability	288
7.13.3	Invertibility	291
7.14	LTI Systems and Differential Equations	294
7.15	Circuit Analysis	295
7.16	Stability Analysis	298
7.16.1	Feedback Control Systems	302
7.17	Unilateral Laplace Transform	305
7.18	Solving Differential Equations Using the Unilateral Laplace Transform	308
7.19	Exercises	315
7.19.1	Exercises Without Answer Key	315
7.19.2	Exercises With Answer Key	319
7.20	MATLAB Exercises	326
II	Discrete-Time Signals and Systems	327
8	Discrete-Time Signals and Systems	329
8.1	Overview	329
8.2	Transformations of the Independent Variable	329
8.2.1	Time Shifting (Translation)	329
8.2.2	Time Reversal (Reflection)	329
8.2.3	Downsampling	331
8.2.4	Upsampling (Time Expansion)	331
8.2.5	Combined Independent-Variable Transformations	331
8.2.6	Two Perspectives on Independent-Variable Transformations	332
8.3	Properties of Sequences	333
8.3.1	Remarks on Symmetry	333
8.3.2	Remarks on Periodicity	334
8.3.3	Support of Sequences	336
8.3.4	Bounded Sequences	338
8.3.5	Signal Energy	339
8.3.6	Examples	339
8.4	Elementary Sequences	340
8.4.1	Real Sinusoidal Sequences	340
8.4.2	Complex Exponential Sequences	343
8.4.2.1	Real Exponential Sequences	343
8.4.2.2	Complex Sinusoidal Sequences	343
8.4.2.3	General Complex Exponential Sequences	346
8.4.3	Relationship Between Complex Exponentials and Real Sinusoids	346
8.4.4	Unit-Step Sequence	347

8.4.5	Unit-Rectangular Pulse	347
8.4.6	Unit-Impulse Sequence	348
8.5	Representing Arbitrary Sequences Using Elementary Sequences	349
8.6	Discrete-Time Systems	350
8.6.1	Block Diagram Representation	352
8.6.2	Interconnection of Systems	352
8.7	Properties of Systems	352
8.7.1	Memory	352
8.7.2	Causality	353
8.7.3	Invertibility	354
8.7.4	BIBO Stability	356
8.7.5	Time Invariance	358
8.7.6	Linearity	359
8.7.7	Eigensequences	363
8.8	Exercises	365
8.8.1	Exercises Without Answer Key	365
8.8.2	Exercises With Answer Key	368
9	Discrete-Time Linear Time-Invariant Systems	369
9.1	Introduction	369
9.2	Discrete-Time Convolution	369
9.3	Properties of Convolution	376
9.4	Periodic Convolution	380
9.5	Characterizing LTI Systems and Convolution	381
9.6	Unit Step Response of LTI Systems	383
9.7	Block Diagram Representation of Discrete-Time LTI Systems	384
9.8	Interconnection of Discrete-Time LTI Systems	384
9.9	Properties of Discrete-Time LTI Systems	386
9.9.1	Memory	386
9.9.2	Causality	387
9.9.3	Invertibility	388
9.9.4	BIBO Stability	391
9.10	Eigensequences of Discrete-Time LTI Systems	393
9.11	Exercises	397
9.11.1	Exercises Without Answer Key	397
9.11.2	Exercises With Answer Key	400
9.12	MATLAB Exercises	400
10	Discrete-Time Fourier Series	401
10.1	Introduction	401
10.2	Definition of Discrete-Time Fourier Series	401
10.3	Determining the Fourier-Series Representation of a Sequence	402
10.4	Comments on Convergence of Discrete-Time Fourier Series	411
10.5	Properties of Discrete-Time Fourier Series	411
10.5.1	Linearity	411
10.5.2	Translation (Time Shifting)	412
10.5.3	Modulation (Frequency Shifting)	412
10.5.4	Reflection (Time Reversal)	413
10.5.5	Conjugation	414
10.5.6	Duality	414
10.5.7	Periodic Convolution	415
10.5.8	Multiplication	416

10.5.9 Parseval's Relation	417
10.5.10 Even/Odd Symmetry	418
10.5.11 Real Sequences	418
10.6 Discrete Fourier Transform (DFT)	421
10.7 Fourier Series and Frequency Spectra	423
10.8 Fourier Series and LTI Systems	428
10.9 Filtering	433
10.10 Exercises	438
10.10.1 Exercises Without Answer Key	438
10.10.2 Exercises With Answer Key	440
10.11 MATLAB Exercises	440
11 Discrete-Time Fourier Transform	441
11.1 Introduction	441
11.2 Development of the Discrete-Time Fourier Transform for Aperiodic Sequences	441
11.3 Generalized Fourier Transform	443
11.4 Definition of the Discrete-Time Fourier Transform	444
11.5 Remarks on Notation Involving the Fourier Transform	447
11.6 Convergence Issues Associated with the Discrete-Time Fourier Transform	449
11.7 Properties of the Discrete-Time Fourier Transform	449
11.7.1 Periodicity	450
11.7.2 Linearity	450
11.7.3 Translation (Time Shifting)	451
11.7.4 Modulation (Frequency-Domain Shifting)	452
11.7.5 Conjugation	453
11.7.6 Time Reversal	454
11.7.7 Upsampling	455
11.7.8 Downsampling	456
11.7.9 Convolution	457
11.7.10 Multiplication	460
11.7.11 Frequency-Domain Differentiation	462
11.7.12 Differencing	463
11.7.13 Accumulation	464
11.7.14 Parseval's Relation	465
11.7.15 Even/Odd Symmetry	466
11.7.16 Real Sequences	468
11.8 Discrete-Time Fourier Transform of Periodic Sequences	470
11.9 More Fourier Transforms	473
11.10 Frequency Spectra of Sequences	482
11.11 Bandwidth of Sequences	486
11.12 Energy-Density Spectra	487
11.13 Characterizing LTI Systems Using the Fourier Transform	488
11.13.1 Unwrapped Phase	490
11.13.2 Magnitude and Phase Distortion	490
11.14 Interconnection of LTI Systems	492
11.15 LTI Systems and Difference Equations	493
11.16 Filtering	495
11.17 Relationship Between DT Fourier Transform and CT Fourier Series	503
11.18 Relationship Between DT and CT Fourier Transforms	505
11.19 Relationship Between DT Fourier Transform and DFT	505
11.20 Exercises	509
11.20.1 Exercises Without Answer Key	509

11.20.2 Exercises With Answer Key	513
11.21 MATLAB Exercises	513
12 z Transform	515
12.1 Introduction	515
12.2 Motivation Behind the z Transform	515
12.3 Definition of z Transform	515
12.4 Remarks on Notation Involving the z Transform	516
12.5 Relationship Between z Transform and Discrete-Time Fourier Transform	518
12.6 z Transform Examples	519
12.7 Region of Convergence for the z Transform	522
12.8 Properties of the z Transform	529
12.8.1 Linearity	529
12.8.2 Translation (Time Shifting)	533
12.8.3 Complex Modulation (z-Domain Scaling)	536
12.8.4 Conjugation	538
12.8.5 Time Reversal	538
12.8.6 Upsampling (Time Expansion)	540
12.8.7 Downsampling	541
12.8.8 Convolution	544
12.8.9 z-Domain Differentiation	545
12.8.10 Differencing	546
12.8.11 Accumulation	547
12.8.12 Initial and Final Value Theorems	548
12.9 More z Transform Examples	550
12.10 Determination of the Inverse z Transform	554
12.10.1 Partial Fraction Expansions	554
12.10.2 Laurent-Polynomial and Power-Series Expansions	560
12.11 Characterizing LTI Systems Using the z Transform	563
12.12 Interconnection of LTI Systems	563
12.13 System Function and System Properties	564
12.13.1 Causality	564
12.13.2 BIBO Stability	565
12.13.3 Invertibility	571
12.14 LTI Systems and Difference Equations	572
12.15 Stability Analysis	574
12.16 Unilateral z Transform	575
12.17 Solving Difference Equations Using the Unilateral z Transform	578
12.18 Exercises	586
12.18.1 Exercises Without Answer Key	586
12.18.2 Exercises With Answer Key	589
12.19 MATLAB Exercises	589
III Appendices	591
A Complex Analysis	593
A.1 Introduction	593
A.2 Complex Numbers	593
A.3 Representations of Complex Numbers	594
A.4 Arithmetic Operations	595
A.4.1 Conjugation	595

A.4.2	Addition	595
A.4.3	Multiplication	596
A.4.4	Division	597
A.4.5	Properties of the Magnitude and Argument	598
A.5	Arithmetic Properties of Complex Numbers	598
A.5.1	Commutative Property	598
A.5.2	Associative Property	598
A.5.3	Distributive Property	599
A.6	Roots of Complex Numbers	599
A.7	Euler's Relation and De Moivre's Theorem	599
A.8	Conversion Between Cartesian and Polar Form	600
A.9	Complex Functions	601
A.10	Circles, Disks, and Annuli	602
A.11	Limit	602
A.12	Continuity	603
A.13	Differentiability	603
A.14	Analyticity	604
A.15	Zeros and Singularities	605
A.16	Quadratic Formula	607
A.17	Exercises	608
A.17.1	Exercises Without Answer Key	608
A.17.2	Exercises With Answer Key	610
A.18	MATLAB Exercises	611
B	Partial Fraction Expansions	613
B.1	Introduction	613
B.2	Partial Fraction Expansions	613
B.3	Exercises	617
B.3.1	Exercises Without Answer Key	617
B.3.2	Exercises With Answer Key	617
B.4	MATLAB Exercises	617
C	Solution of Constant-Coefficient Linear Differential Equations	619
C.1	Overview	619
C.2	Constant-Coefficient Linear Differential Equations	619
C.3	Solution of Homogeneous Equations	619
C.4	Particular Solution of Nonhomogeneous Equations	621
C.5	General Solution of Nonhomogeneous Equations	623
C.6	Exercises	627
C.6.1	Exercises Without Answer Key	627
C.6.2	Exercises With Answer Key	627
C.7	MATLAB Exercises	627
D	MATLAB	629
D.1	Introduction	629
D.2	Octave	629
D.3	Invoking MATLAB	629
D.3.1	UNIX	629
D.3.2	Microsoft Windows	630
D.4	Command Line Editor	630
D.5	MATLAB Basics	630
D.5.1	Identifiers	630

D.5.2	Basic Functionality	631
D.6	Arrays	633
D.6.1	Arrays with Equally-Spaced Elements	634
D.6.2	Array Subscripting	634
D.6.3	Other Array Functions	634
D.7	Scripts	634
D.8	Relational and Logical Operators	637
D.9	Operator Precedence	638
D.10	Control Flow	638
D.10.1	If-Elseif-Else	639
D.10.2	Switch	639
D.10.3	For	640
D.10.4	While	641
D.10.5	Break and Continue	641
D.11	Functions	642
D.12	Graphing	644
D.13	Printing	649
D.14	Symbolic Math Toolbox	649
D.14.1	Symbolic Objects	650
D.14.2	Creating Symbolic Objects	650
D.14.3	Manipulating Symbolic Objects	650
D.14.4	Plotting Symbolic Expressions	653
D.15	Signal Processing	654
D.15.1	Continuous-Time Signal Processing	654
D.15.1.1	Frequency Responses	654
D.15.1.2	Impulse and Step Responses	656
D.15.1.3	Filter Design	657
D.15.2	Discrete-Time Signal Processing	659
D.15.2.1	Frequency Responses	659
D.15.2.2	Impulse and Step Responses	662
D.15.2.3	Filter Design	663
D.16	Miscellany	664
D.17	Exercises	667
E	Additional Exercises	671
E.1	Overview	671
E.2	Continuous-Time Signals and Systems	671
E.3	Discrete-Time Signals and Systems	672
F	Miscellaneous Information	673
F.1	Overview	673
F.2	Combinatorial Formulas	673
F.3	Derivatives	673
F.4	Integrals	674
F.5	Arithmetic and Geometric Sequences	674
F.6	Taylor/Maclaurin Series	675
F.7	Other Formulas for Sums	675
F.8	Trigonometric Identities	675
F.9	Exact Trigonometric Function Values	676
F.10	Miscellany	676

G	Video Lectures	679
G.1	Introduction	679
G.2	2020-05 ECE 260 Video Lectures	679
G.2.1	Video-Lecture Catalog	680
G.2.1.1	Introduction	680
G.2.1.2	Complex Analysis	680
G.2.1.3	Preliminaries — Introduction	681
G.2.1.4	Preliminaries — Functions, Sequences, System Operators, and Transforms	681
G.2.1.5	Preliminaries — Signal Properties	681
G.2.1.6	CT Signals and Systems — Introduction	681
G.2.1.7	CT Signals and Systems — Independent/Dependent-Variable Transformations	682
G.2.1.8	CT Signals and Systems — Function Properties	682
G.2.1.9	CT Signals and Systems — Elementary Functions	682
G.2.1.10	CT Signals and Systems — Systems	683
G.2.1.11	CT Signals and Systems — System Properties	683
G.2.1.12	CT LTI Systems — Introduction	684
G.2.1.13	CT LTI Systems — Convolution	684
G.2.1.14	CT LTI Systems — Convolution and LTI Systems	684
G.2.1.15	CT LTI Systems — Properties of LTI Systems	684
G.2.1.16	Interlude	685
G.2.1.17	CT Fourier Series — Introduction	685
G.2.1.18	CT Fourier Series — Fourier Series	685
G.2.1.19	CT Fourier Series — Convergence Properties of Fourier Series	685
G.2.1.20	CT Fourier Series — Properties of Fourier Series	686
G.2.1.21	CT Fourier Series — Fourier Series and Frequency Spectra	686
G.2.1.22	CT Fourier Series — Fourier Series and LTI Systems	686
G.2.1.23	CT Fourier Transform — Introduction	686
G.2.1.24	CT Fourier Transform — Fourier Transform	687
G.2.1.25	CT Fourier Transform — Convergence Properties	687
G.2.1.26	CT Fourier Transform — Properties of the Fourier Transform	687
G.2.1.27	CT Fourier Transform — Fourier Transform of Periodic Functions	688
G.2.1.28	CT Fourier Transform — Fourier Transform and Frequency Spectra of Functions	688
G.2.1.29	CT Fourier Transform — Fourier Transform and LTI Systems	688
G.2.1.30	CT Fourier Transform — Application: Filtering	689
G.2.1.31	CT Fourier Transform — Application: Circuit Analysis	689
G.2.1.32	CT Fourier Transform — Application: Amplitude Modulation	689
G.2.1.33	CT Fourier Transform — Application: Sampling and Interpolation	689
G.2.1.34	Partial Fraction Expansions (PFEs)	690
G.2.1.35	Laplace Transform — Introduction	690
G.2.1.36	Laplace Transform — Laplace Transform	690
G.2.1.37	Laplace Transform — Region of Convergence	691
G.2.1.38	Laplace Transform — Properties of the Laplace Transform	691
G.2.1.39	Laplace Transform — Determination of Inverse Laplace Transform	692
G.2.1.40	Laplace Transform — Laplace Transform and LTI Systems	692
G.2.1.41	Laplace Transform — Application: Circuit Analysis	692
G.2.1.42	Laplace Transform — Application: Design and Analysis of Control Systems	692
G.2.1.43	Laplace Transform — Unilateral Laplace Transform	693
	Index	697

List of Tables

2.1	Examples of dot notation for functions and sequences. Examples for (a) functions and (b) sequences.	12
2.2	Examples of transforms	14
5.1	Properties of CT Fourier series	135
6.1	Properties of the CT Fourier transform	179
6.2	Transform pairs for the CT Fourier transform	183
7.1	Properties of the (bilateral) Laplace transform	273
7.2	Transform pairs for the (bilateral) Laplace transform	274
7.3	Properties of the unilateral Laplace transform	307
7.4	Transform pairs for the unilateral Laplace transform	308
9.1	Convolution computation for Example 9.3	376
9.2	Convolution computation for Example 9.5	377
9.3	Convolution computation for Example 9.7	383
10.1	Properties of DT Fourier series	422
10.2	Properties of the Discrete Fourier Transform	424
11.1	Properties of the DT Fourier transform	469
11.2	Transform pairs for the DT Fourier transform	474
12.1	Relationship between the sidedness properties of x and the ROC of $X = \mathcal{Z}x$	528
12.2	Properties of the (bilateral) z transform	551
12.3	Transform pairs for the (bilateral) z transform	552
12.4	Properties of the unilateral z transform	577
12.5	Transform pairs for the unilateral z transform	578
C.1	Forms for the particular solution	622
D.1	Keys for command-line editing	630
D.2	Predefined variables	631
D.3	Operators	631
D.4	Elementary math functions	632
D.5	Other math-related functions	632
D.6	Exponential and logarithmic functions	632
D.7	Trigonometric functions	632
D.8	Other math functions	632
D.9	Radix conversion functions	633
D.10	Array size functions	634
D.11	Examples of abbreviated forms of vectors	634

D.12	Array subscripting examples	635
D.13	Special matrix/vector functions	635
D.14	Basic array manipulation functions	635
D.15	Examples of expressions involving relational operators	637
D.16	Relational operators	637
D.17	Logical operators	637
D.18	Relational and logical functions	638
D.19	Operator precedence	638
D.20	Special predefined function variables	643
D.21	Basic plotting functions	644
D.22	Other graphing functions/commands	645
D.23	Line styles	645
D.24	Line colors	645
D.25	Marker styles	645
D.26	Graph annotation functions	645
D.27	Special symbols for annotation text	646
D.28	Some functions related to signal processing	654
D.29	Miscellaneous functions/commands	665
F.1	Exact values of various trigonometric functions for certain special angles	677

List of Figures

1.1	Graphical representations of (a) continuous-time and (b) discrete-time signals.	2
1.2	Segment of digitized human speech.	3
1.3	A monochromatic image.	3
1.4	System with one or more inputs and one or more outputs.	3
1.5	A simple RC network.	4
1.6	Signal processing systems. (a) Processing a continuous-time signal with a discrete-time system. (b) Processing a discrete-time signal with a continuous-time system.	5
1.7	Communication system.	5
1.8	Feedback control system.	5
2.1	The mapping f	9
2.2	Example of an even function.	14
2.3	Example of an even sequence.	14
2.4	Example of an odd function.	15
2.5	Example of an odd sequence.	15
2.6	Example of a periodic function (with a fundamental period of T).	16
2.7	Example of a periodic sequence (with a fundamental period of 4).	16
3.1	Example of time shifting. (a) The function x ; and the result of applying a time-shifting transformation to x with a shift of (b) 1 and (c) -1	22
3.2	Example of time reversal. (a) The function x ; and (b) the result of applying a time-reversal transformation to x	22
3.3	Example of time compression/expansion. (a) The function x ; and the result of applying a time compression/expansion transformation to x with a scaling factor of (b) 2 and (c) $\frac{1}{2}$	23
3.4	Example of time scaling. (a) The function x ; and the result of applying a time-scaling transformation to x with a scaling factor of (b) 2, (c) $\frac{1}{2}$, and (d) -1	24
3.5	Two different interpretations of a combined time-shifting and time-scaling transformation. (a) Original function. Results obtained by shifting followed by scaling: (b) intermediate result and (c) final result. Results obtained by scaling followed by shifting: (d) intermediate result and (e) final result.	25
3.6	Example of amplitude shifting. (a) The function x ; and the result obtained by applying an amplitude-shifting transformation to x with a shifting value of -2	26
3.7	Example of amplitude scaling. (a) The function x ; and the result of applying an amplitude-scaling transformation to x with a scaling factor of (b) 2, (c) $\frac{1}{2}$, and (d) -2	27
3.8	Examples of functions with various sidedness properties. A function that is (a) left sided but not right sided, (b) right sided but not left sided, (c) finite duration, and (d) two sided.	32
3.9	The function x from Example 3.6.	34
3.10	Real sinusoidal function.	34
3.11	Real exponential function for (a) $\lambda > 0$, (b) $\lambda = 0$, and (c) $\lambda < 0$	35
3.12	Complex sinusoidal function. (a) Real and (b) imaginary parts.	36
3.13	The complex sinusoidal function $x(t) = e^{j\omega t}$ for (a) $\omega = 2\pi$ and (b) $\omega = -2\pi$	37

3.14	Real part of a general complex exponential function for (a) $\sigma > 0$, (b) $\sigma = 0$, and (c) $\sigma < 0$.	38
3.15	Unit-step function.	39
3.16	Signum function.	39
3.17	Rectangular function.	39
3.18	Using the rectangular function to extract one period of a periodic function x . (a) The function x . (b) A time-scaled rectangular function v . (c) The product of x and v .	40
3.19	Triangular function.	41
3.20	Delta function.	43
3.21	Scaled and time-shifted delta function.	43
3.22	The rectangular-pulse function g_ε .	43
3.23	Plot of g_ε for several values of ε .	44
3.24	Plot of d_ε for several values of ε .	45
3.25	Graphical interpretation of equivalence property. (a) A function x ; (b) a time-shifted delta function; and (c) the product of the these two functions.	46
3.26	Representing the rectangular function using unit-step functions. (a) A shifted unit-step function, (b) another shifted unit-step function, and (c) their difference (which is the rectangular function).	48
3.27	Representing a piecewise-linear function using unit-step functions. (a) The function x . (b), (c), and (d) Three functions whose sum is x .	49
3.28	Representing a piecewise-polynomial function using unit-step functions. (a) The function x ; and (b), (c), and (d) three functions whose sum is x .	50
3.29	Representing a periodic function using unit-step functions. (a) The periodic function x ; and (b) a function v that consists of a single period of x .	50
3.30	Block diagram of system.	51
3.31	Interconnection of systems. (a) Series interconnection of the systems \mathcal{H}_1 and \mathcal{H}_2 . (b) Parallel interconnection of the systems \mathcal{H}_1 and \mathcal{H}_2 .	51
3.32	Systems that are equivalent (assuming \mathcal{H}^{-1} exists). (a) First and (b) second system.	54
3.33	Systems that are equivalent if \mathcal{H} is time invariant (i.e., \mathcal{H} commutes with \mathcal{S}_{t_0}). (a) A system that first time shifts by t_0 and then applies \mathcal{H} (i.e., $y = \mathcal{H}\mathcal{S}_{t_0}x$); and (b) a system that first applies \mathcal{H} and then time shifts by t_0 (i.e., $y = \mathcal{S}_{t_0}\mathcal{H}x$).	57
3.34	Systems that are equivalent if \mathcal{H} is additive (i.e., \mathcal{H} commutes with addition). (a) A system that first performs addition and then applies \mathcal{H} (i.e., $y = \mathcal{H}(x_1 + x_2)$); and (b) a system that first applies \mathcal{H} and then performs addition (i.e., $y = \mathcal{H}x_1 + \mathcal{H}x_2$).	59
3.35	Systems that are equivalent if \mathcal{H} is homogeneous (i.e., \mathcal{H} commutes with scalar multiplication). (a) A system that first performs scalar multiplication and then applies \mathcal{H} (i.e., $y = \mathcal{H}(ax)$); and (b) a system that first applies \mathcal{H} and then performs scalar multiplication (i.e., $y = a\mathcal{H}x$).	59
3.36	Systems that are equivalent if \mathcal{H} is linear (i.e., \mathcal{H} commutes with linear combinations). (a) A system that first computes a linear combination and then applies \mathcal{H} (i.e., $y = \mathcal{H}(a_1x_1 + a_2x_2)$); and (b) a system that first applies \mathcal{H} and then computes a linear combination (i.e., $y = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$).	60
4.1	Evaluation of the convolution $x * h$. (a) The function x ; (b) the function h ; plots of (c) $h(-\tau)$ and (d) $h(t - \tau)$ versus τ ; the functions associated with the product in the convolution integral for (e) $t < -1$, (f) $-1 \leq t < 0$, (g) $0 \leq t < 1$, and (h) $t \geq 1$; and (i) the convolution result $x * h$.	78
4.2	Evaluation of the convolution $x * h$. (a) The function x ; (b) the function h ; plots of (c) $h(-\tau)$ and (d) $h(t - \tau)$ versus τ ; the functions associated with the product in the convolution integral for (e) $t < 0$, (f) $0 \leq t < 1$, (g) $1 \leq t < 2$, and (h) $t \geq 2$; and (i) the convolution result $x * h$.	80
4.3	Evaluation of the convolution $x * h$. (a) The function x ; (b) the function h ; plots of (c) $h(-\tau)$ and (d) $h(t - \tau)$ versus τ ; the functions associated with the product in the convolution integral for (e) $t < 0$, (f) $0 \leq t < 1$, (g) $1 \leq t < 2$, (h) $2 \leq t < 3$, and (i) $t \geq 3$; and (j) the convolution result $x * h$.	82
4.4	Evaluation of the convolution $x * h$. (a) The function x ; (b) the function h ; plots of (c) $h(-\tau)$ and (d) $h(t - \tau)$ versus τ ; and the functions associated with the product in the convolution integral for (e) $t < 0$ and (f) $t > 0$.	84

4.5	Evaluation of the convolution $x * h$. (a) The function x ; (b) the function h ; plots of (c) $h(-\tau)$ and (d) $h(t - \tau)$ versus τ ; the functions associated with the product in the convolution integral for (e) $t < 0$, (f) $0 \leq t < 1$, (g) $1 \leq t < 2$, and (h) $t \geq 2$; and (i) the convolution result $x * h$	90
4.6	Block diagram representation of continuous-time LTI system with input x , output y , and impulse response h	91
4.7	Equivalences for the series interconnection of continuous-time LTI systems. The (a) first and (b) second equivalences.	92
4.8	Equivalence for the parallel interconnection of continuous-time LTI systems.	93
4.9	System interconnection example.	93
4.10	System in cascade with its inverse.	96
4.11	Feedback system with input x and output y	97
5.1	Periodic square wave.	117
5.2	Periodic impulse train.	118
5.3	Periodic impulse train.	119
5.4	Periodic function x	123
5.5	Examples of functions that violate the Dirichlet conditions. (a) A function that is not absolutely integrable over a single period. (b) A function that has an infinite number of maxima and minima over a single period. (c) A function that has an infinite number of discontinuities over a single period.	125
5.6	Gibbs phenomenon. The Fourier series for the periodic square wave truncated after the N th harmonic components for (a) $N = 3$, (b) $N = 7$, (c) $N = 11$, and (d) $N = 101$	126
5.7	Approximation of the Fourier series for the function x . (a) The function x . (b) The approximation obtained by taking the 4 terms in the Fourier series with the largest magnitude coefficients. (c) The approximation obtained by taking the 4 terms in the Fourier series with the smallest magnitude (nonzero) coefficients.	138
5.8	Frequency spectrum of the periodic square wave. (a) Magnitude spectrum and (b) phase spectrum.	139
5.9	Frequency spectrum for the periodic impulse train. (a) Magnitude spectrum and (b) phase spectrum.	140
5.10	Frequency responses of (a) ideal lowpass, (b) ideal highpass, and (c) ideal bandpass filters.	143
5.11	Frequency spectra of the (a) input function x and (b) output function y	145
6.1	An example of the functions used in the derivation of the Fourier transform representation, where $T_1 > \frac{T}{2}$. (a) An aperiodic function x ; (b) the function x_T ; and (c) the T -periodic function \tilde{x}	155
6.2	An example of the functions used in the derivation of the Fourier transform representation, where $T_1 < \frac{T}{2}$. (a) An aperiodic function x ; (b) the function x_T ; and (c) the T -periodic function \tilde{x}	156
6.3	Integral obtained in the derivation of the Fourier transform representation.	156
6.4	A plot of $e^{- t }$ versus t	160
6.5	Function x	162
6.6	Frequency spectra. The frequency spectra (a) X_1 and (b) X_2	181
6.7	Periodic function x	182
6.8	Frequency spectrum of the amplitude-scaled time-shifted signum function x . (a) Magnitude spectrum and (b) phase spectrum of x	188
6.9	Frequency spectrum of the time-scaled sinc function x	189
6.10	Frequency spectrum of the time-shifted signum function. (a) Magnitude spectrum and (b) phase spectrum of x	190
6.11	Bandwidth of a function x with the Fourier transform X	190
6.12	Time-domain view of a LTI system with input x , output y , and impulse response h	193
6.13	Frequency-domain view of a LTI system with input spectrum X , output spectrum Y , and frequency response H	193
6.14	Frequency response of example system.	193
6.15	Unwrapped phase example. (a) The phase function restricted such that its range is in $(-\pi, \pi]$ and (b) the corresponding unwrapped phase.	194

6.16	Importance of phase information in images. The (a) potatohead and (b) hongkong images. (c) The potatohead image after having its magnitude spectrum replaced with the magnitude spectrum of the hongkong image. (d) The potatohead image after having its phase spectrum replaced with the phase spectrum of the hongkong image.	197
6.17	Importance of phase information in images. The (a) potatohead and (b) random images. (c) The potatohead image after having its magnitude spectrum replaced with the magnitude spectrum of the random image. (d) The potatohead image after having its phase spectrum replaced with the phase spectrum of the random image.	198
6.18	Equivalences involving frequency responses and the series interconnection of LTI systems. The (a) first and (b) second equivalences.	198
6.19	Equivalence involving frequency responses and the parallel interconnection of LTI systems.	199
6.20	Frequency responses of (a) ideal lowpass, (b) ideal highpass, and (c) ideal bandpass filters.	202
6.21	Frequency responses of each of the (a) first, (b) second, and (c) third systems from the example.	204
6.22	Frequency spectra for the lowpass filtering example. (a) Frequency spectrum of the input x . (b) Frequency response of the system. (c) Frequency spectrum of the output y	206
6.23	Frequency spectra for bandpass filtering example. (a) Frequency spectrum of the input x . (b) Frequency response of the system. (c) Frequency spectrum of the output y	207
6.24	Equalization example. (a) Original system. (b) New system with equalization.	208
6.25	System from example that employs equalization.	208
6.26	Basic electrical components. (a) Resistor, (b) inductor, and (c) capacitor.	209
6.27	Simple RC network.	210
6.28	Simple communication system. (a) Transmitter and (b) receiver.	212
6.29	Frequency spectra for modulation with a complex sinusoid. (a) Spectrum of the transmitter input. (b) Spectrum of the complex sinusoid used in the transmitter. (c) Spectrum of the complex sinusoid used in the receiver. (d) Spectrum of the transmitted signal. (e) Spectrum of the receiver output.	212
6.30	DSB/SC amplitude modulation system. (a) Transmitter and (b) receiver.	214
6.31	Signal spectra for DSB/SC amplitude modulation. (a) Spectrum of the transmitter input. (b) Spectrum of the sinusoidal function used in the transmitter and receiver. (c) Frequency response of the filter in the receiver. (d) Spectrum of the transmitted signal. (e) Spectrum of the multiplier output in the receiver. (f) Spectrum of the receiver output.	216
6.32	SSB/SC amplitude modulation system. (a) Transmitter and (b) receiver.	216
6.33	Signal spectra for SSB/SC amplitude modulation. (a) Spectrum of the transmitter input. (b) Spectrum of the sinusoid used in the transmitter and receiver. (c) Frequency response of the filter in the transmitter. (d) Frequency response of the filter in the receiver. (e) Spectrum of the multiplier output in the transmitter. (f) Spectrum of the transmitted signal. (g) Spectrum of the multiplier output in the receiver. (h) Spectrum of the receiver output.	218
6.34	Ideal C/D converter with input function x and output sequence y	219
6.35	Example of periodic sampling. (a) The function x to be sampled and (b) the sequence y produced by sampling x with a sampling period of 10.	219
6.36	Ideal D/C converter with input sequence y and output function \hat{x}	219
6.37	Model of ideal C/D converter with input function x and output sequence y	220
6.38	An example of the various signals involved in the sampling process for a sampling period of T . (a) The function x to be sampled. (b) The sampling function p . (c) The impulse-modulated function s . (d) The sequence y produced by sampling.	221
6.39	Effect of impulse-train sampling on the frequency spectrum. (a) Spectrum of the function x being sampled. (b) Spectrum of s in the absence of aliasing. (c) Spectrum of s in the presence of aliasing.	223
6.40	Model of ideal D/C converter with input sequence y and output function \hat{x}	223
6.41	Frequency spectrum of the function x	225
6.42	Frequency spectrum of the function x_1	226
7.1	A plot of $e^{- t }$ versus t	243
7.2	Region of convergence for the case that (a) $a > 0$ and (b) $a < 0$	246

7.3	Region of convergence for the case that (a) $a > 0$ and (b) $a < 0$.	247
7.4	Examples of LHPs and RHPs. An example of a LHP in the case that (a) $a < 0$ and (b) $a > 0$. An example of a RHP in the case that (c) $a < 0$ and (d) $a > 0$.	249
7.5	Example of set intersection. The sets (a) R_1 and (b) R_2 ; and (c) their intersection $R_1 \cap R_2$.	250
7.6	Example of adding a scalar to a set. (a) The set R . (b) The set $R + 1$.	251
7.7	Example of multiplying a set by a scalar. (a) The set R . The sets (b) $2R$ and (c) $-2R$.	252
7.8	Examples of sets that would be either valid or invalid as the ROC of a Laplace transform.	252
7.9	Examples of sets that would be either valid or invalid as the ROC of a rational Laplace transform.	253
7.10	Examples of sets that would be either valid or invalid as the ROC of a Laplace transform of a finite-duration function.	253
7.11	Examples of sets that would be either valid or invalid as the ROC of the Laplace transform of a function that is right sided but not left sided.	254
7.12	Examples of sets that would be either valid or invalid as the ROC of the Laplace transform of a function that is left sided but not right sided.	254
7.13	Examples of sets that would be either valid or invalid as the ROC of the Laplace transform of a two-sided function.	254
7.14	Examples of sets that would be either valid or invalid as the ROC of a rational Laplace transform of a left/right-sided function.	255
7.15	Relationship between the sidedness properties of x and the ROC of $X = \mathcal{L}x$.	255
7.16	Examples of sets that would not be a valid ROC of a Laplace transform.	255
7.17	ROCs for example. The (a) first, (b) second, (c) third, and (d) fourth possible ROCs for X .	256
7.18	ROCs for the linearity example. The (a) ROC of X_1 , (b) ROC of X_2 , (c) ROC associated with the intersection of the ROCs of X_1 and X_2 , and (d) ROC of X .	258
7.19	ROCs for the linearity example. The (a) ROC of X_1 , (b) ROC of X_2 , (c) ROC associated with the intersection of the ROCs of X_1 and X_2 , and (d) ROC of X .	260
7.20	Regions of convergence for Laplace-domain shifting. (a) Before shift. (b) After shift.	262
7.21	Regions of convergence for time-domain/Laplace-domain scaling. (a) Before scaling. After scaling for (b) $a > 0$ and (c) $a < 0$.	264
7.22	Function for the Laplace transform example.	278
7.23	Function for the Laplace transform example.	279
7.24	The poles and possible ROCs for the rational expressions (a) $\frac{1}{s-2}$; and (b) $\frac{1}{s+1}$.	281
7.25	The poles and possible ROCs for the rational expressions (a) $\frac{1}{s+1}$ and $\frac{1}{(s+1)^2}$; and (b) $\frac{1}{s+2}$.	282
7.26	Time-domain view of a LTI system with input x , output y , and impulse response h .	286
7.27	Laplace-domain view of a LTI system with input Laplace transform X , output Laplace transform Y , and system function H .	286
7.28	Equivalences involving system functions and the series interconnection of LTI systems. The (a) first and (b) second equivalences.	286
7.29	Equivalence involving system functions and the parallel interconnection of LTI systems.	287
7.30	Pole and ROCs of the rational system functions in the causality example. The cases of the (a) first (b) second system functions.	288
7.31	ROC for example.	290
7.32	Poles of the system function.	290
7.33	Poles and ROCs of the system function H in the (a) first, (b) second, (c) third, and (d) fourth parts of the example.	292
7.34	Basic electrical components. (a) Resistor, (b) inductor, and (c) capacitor.	296
7.35	Simple RC network.	297
7.36	Feedback system.	299
7.37	Feedback system.	300
7.38	Feedback control system.	302
7.39	Plant.	303
7.40	Two configurations for stabilizing the unstable plant. (a) Simple cascade system and (b) feedback control system.	303