Unit 2: Short Truth Tables
Practice Assignment

Use a short truth table to determine if the following arguments are valid or invalid. Answers below.

1. ~(R & S) → A
   (B & ~L) ∨ (A → C)
   ~C → S

2. W & ~(D ∨ P)
   (Q → W) → (D ∨ B)
   ~(B → P)

3. (G & H) → J
   ~B ∨ (C & G)
   ~(B → M)
   H & B

4. ~M ∨ (B & H)
   ~N → M
   (H ∨ N) & (M → B)

1.

~(R & S) → A / (B & ~L) ∨ (A → C) // ~C → S

         T   T   F

We need to start off with a sentence that forces us to pick certain values for the letters. So, we won't start with either of the premises since there are lots of ways that an → or an ∨ can be true. There is only way that an → can be false so we'll start with the conclusion.

~(R & S) → A / (B & ~L) ∨ (A → C) // ~C → S

                                             F

                                     T   F

        T   T   F

                                   (1) - C = F, S = F

~(R & S) → A / (B & ~L) ∨ (A → C) // ~C → S

         F   F   F

                                     T   F

        T   T   F

We don't have enough information to figure out the second premise so we'll start to work on the first premise.

~(R & S) → A / (B & ~L) ∨ (A → C) // ~C → S

          F   F   F

        F   T   F

   T   T

         T   T   F

        (2) A = T Note: Since when S is false, the & is automatically false, we are not committed to a value for R at this point.

~(R & S) → A / (B & ~L) ∨ (A → C) // ~C → S

                                  F

          F   T T   T F   F

       F   T   F   T   F

  T   T

      T   T   F

(3) B = T, L = F

We were able to make the premises true while the conclusion is false, so the argument is

INVALID when :

A	B	C	L	R	S
T	T	F	F	T/F	F

Note: Because, at no point were we forced into picking a value for R, R can be either true or false. If we had done the truth table for this argument (all 64 lines) we would have found two lines that showed the argument to be invalid. These lines would have the values above with R as true and the same values with R as false.

2.

        W & ~(D ∨ P) / (Q → W) → (D ∨ B) //   ~(B → P)

          T   T   F

For this question we have to start off with the first premise. The second premise has many options to be true and the conclusion has many options to be false.

        W & ~(D ∨ P) / (Q → W) → (D ∨ B) //   ~(B → P)
        T   F   F
             F
         T
       T   T   F
(1) W = T, D = F, P = F

        W & ~(D ∨ P) / (Q → W) → (D ∨ B) //   ~(B → P)
        T   F   F   T   F   F
             F
         T
       T   T   F

At this point we can work on the remaining premise or the conclusion. I will start with the remaining premise.

W & ~(D ∨ P) / (Q → W) → (D ∨ B) //   ~(B → P)
T   F   F   T   F   T   F
      F   T   T
   T
   T   T   F
              (2) B = T

Since the W = T makes the first → true, the second → must be T → T. We already know that D is false, so for that ∨ to be true, B must be true. We are not committed to a value for Q.

And now sub B= T into the conclusion.

W & ~(D ∨ P) / (Q → W) → (D ∨ B) //   ~(B → P)
T   F   F   T   F   T   T   F
      F   T   T   F
   T
   T   T   **T !!!**

When we substitute B = T into the conclusion, that makes the conclusion true. But we need the conclusion to be false. We have been unable to make all the premises true and the conclusion false at the same time. The argument is VALID.

3.
        (G & H) → J / ~B ∨ (C & G) / ~(B → M) // H & B

            T   T   T   F

We cannot start with the first two premises since there are many ways for an → and ∨ to be true. Also, we cannot start with the conclusion because there are many ways for an $ to be false. So, we must start off with the third premise.

        (G & H) → J / ~B ∨ (C & G) / ~(B → M) // H & B
                                           T   F
                                              F
            T   T   T   F
                        (1) B = T, M = F

        (G & H) → J / ~B ∨ (C & G) / ~(B → M) // H & B
                         T   T   F   T
                                               F
            T   T   T   F

Now that we know the value of B, we have enough information to work on the second premise and also the conclusion. I'll start with the second premise.

$(G \& H) \rightarrow J \ / \ \sim B \lor (C \& G) \ / \ \sim(B \rightarrow M) \ // \ H \& B$

                    T   T   T   T   F   F   T

              F   T   F

      T   T   T   F

        (2) C= T, G = T   (3) H = F

We still have to make sure that these values work in the first premise.

$(G \& H) \rightarrow J \ / \ \sim B \lor (C \& G) \ / \ \sim(B \rightarrow M) \ // \ H \& B$

   T   F   T   T   T   T   F   F   T

     F   F   T   F

       T   T   T   F

(4) First premise is true. We are not committed to a value for J. J can be true or false.

We successfully made the premises true and the conclusion false. This argument is invalid when:

B	C	G	H	J	M
T	T	T	F	T/F	F

4.

$\sim M \lor (B \& H) \ / \ \sim N \rightarrow M \ // \ (H \lor N) \& (M \rightarrow B)$

       T   T   F

In some ways this is the most difficult question in this practice assignment. It is difficult because it is not clear where we should start. There are many ways that the premises can be true (with an $\lor$ and an $\rightarrow$). There are also many ways that the conclusion, with its &, can be false.

In cases like these, choose the sentence that is easiest to figure out and write down all the possibilities to consider. The $\rightarrow$ can be true in three different ways.

$\sim M \lor (B \& H) \ / \ \sim N \rightarrow M \ // \ (H \lor N) \& (M \rightarrow B)$

       T   T   F

Try 1:   T   T

Try 2:   F   T

Try 3:   F   F

We must work on each try separately.

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
                           F
Try 1:   T   T
        T   T   F
                    (1) N = F, M = T

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
     T   T T   F   T   F   T
     F   T
Try 1:   T   T
          T   T   F

(2) B = T, H = T

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
     T   T T   F   T   T F   T   T
     F   T   T   T
Try 1:   T   T

          T   T   **T!!**

But when we substitute those values into the conclusion, the conclusion becomes true (and we need it to be false). We were unable to make Try 1 invalid. Before we can determine the argument's validity, we need to work on the other trys.

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
                           T
Try 2:   F   T
          T   T   F
                    (1) N = T, M = T

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
     T   T T   T   T T   T   T
Try 2: F   T   F   T   T   T
          T   T   **T!!**

(2) B = T, H = T   (3) Conclusion is T but we need it to be F.

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
                  T

Try 3:   F   F

          T   T   F

                (1) N = T, M = F

~M ∨ (B & H) / ~N → M // (H ∨ N) & (M → B)
  F   T   T   F
                                T   T

Try 3:   F   F

          T   T   **T!!**

                          (2) The conclusion is T but we need it to be F.

Since all three trys were unsuccessful, the argument is VALID.