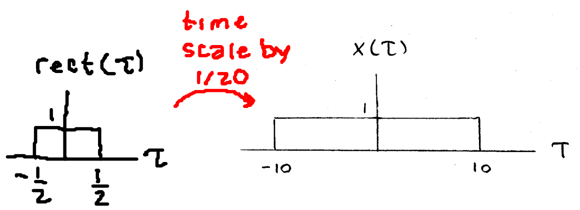
Exercise 4.101

L Answer (k).

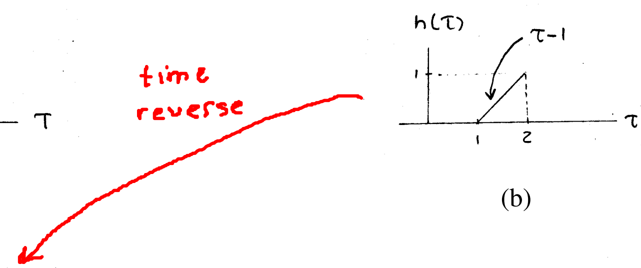
We need to compute $x * h$, where

$$x(t) = \mathrm{rect}(t/20) \quad \text{and} \quad h(t) = \begin{cases} t-1 & 1 \leq t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

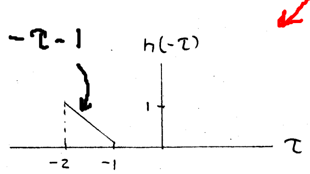
To assist in the convolution computation, we first plot $x(\tau)$ and $h(t-\tau)$ versus τ as shown below in Figures (a) and (d), respectively. (Figures (b) and (c) show intermediate results obtained in the determination of Figure (d).)



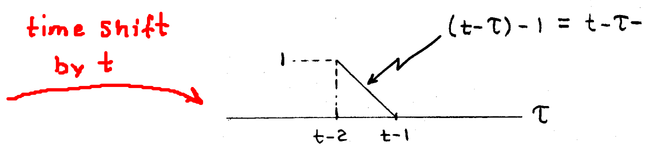
(a)



(b)

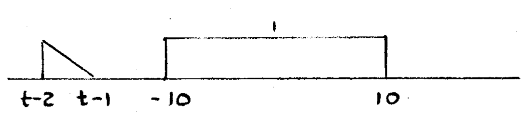


(c)

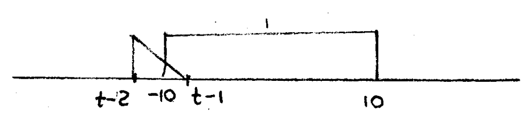


(d)

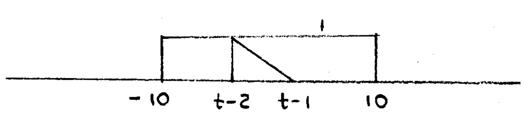
From the above plots, we can deduce that there are five cases (i.e., intervals of t) to be considered, which correspond to the scenarios shown in the graphs below.



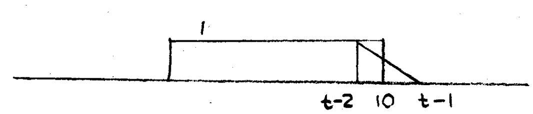
(e)



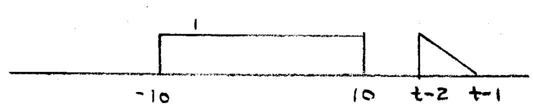
(f)



(g)



(h)



(i)

From Figure (e), for $t < -9$ (i.e., $t-1 < -10$), we have $x * h(t) = 0$. From Figure (f), for $-9 \le t < -8$ (i.e., $t-1 \ge -10$ and $t-2 < -10$), we have

$$x * h(t) = \int_{-10}^{t-1} (1)(t-\tau-1)d\tau.$$

From Figure (g), for $-8 \le t < 11$ (i.e., $t-2 \ge -10$ and $t-1 < 10$), we have

$$x * h(t) = \int_{t-2}^{t-1} (1)(t-\tau-1)d\tau.$$

From Figure (h), for $11 \le t < 12$ (i.e., $t-1 \ge 10$ and $t-2 < 10$), we have

$$x * h(t) = \int_{t-2}^{10} (1)(t-\tau-1)d\tau.$$

From Figure (i), for $t \ge 12$ (i.e., $t-2 \ge 10$), we have $x * h(t) = 0$. Combining the above results, we have that

$$x * h(t) = \begin{cases} \int_{-10}^{t-1}(t-\tau-1)d\tau & -9 \le t < -8 \\ \int_{t-2}^{t-1}(t-\tau-1)d\tau & -8 \le t < 11 \\ \int_{t-2}^{10}(t-\tau-1)d\tau & 11 \le t < 12 \\ 0 & \text{otherwise.} \end{cases}$$