ECE 260 EXAM 1 SOLUTIONS

QUESTION 1

(a)

$$f(z)=\frac{z^2-4}{(z^2+4)^3}=\frac{(z+2)(z-2)}{[(z+2j)(z-2j)]^3}=\frac{(z+2)(z-2)}{(z+2j)^3(z-2j)^3}$$

first order zeros at -2 and 2
third order poles at -2j and 2j

(b)

a rational function is analytic everywhere except at its poles.
therefore, $f$ is analytic everywhere except at $-2j$ and $2j$.

QUESTION 2

(a)

$x(t)=t\,\delta(t)=[t\,\delta(t)]\big|_{t=0}=0$
[by equivalence property of $\delta$ function]

(b)

$x(t)=\int_0^4 \delta(\tau+3)\tan(\tau)\,d\tau=0$
[since integrand is identically zero]

(c)

$x(t)=\int_{-\pi}^{\pi}\delta(\tau-\pi/2)u(\tau)\,d\tau=[u(\tau)]\big|_{\tau=\pi/2}=1$
[by sifting property of $\delta$ function]

(d)

$x(t)=1$ if $t+1\le 0$ and $x(t)=0$ otherwise
[since all area of 1 concentrated at origin]
since $t+1\le 0 \Rightarrow t\le -1$, we have

$$x(t) = \begin{cases} 1 & t\le -1 \\ 0 & \text{otherwise} \end{cases} = u(-t-1)$$

QUESTION 3

$$\begin{aligned} x(t) &= (1)[u(t-[-\infty])-u(t+1)]+(-t)[u(t+1)-u(t)]+(t)[u(t)-u(t-1)]+(1)[u(t-1)-u(t-\infty)] \\ &= 1-u(t+1)+(-t)[u(t+1)-u(t)]+(t)[u(t)-u(t-1)]+u(t-1) \end{aligned}$$

alternatively, this can be rewritten as

$x(t)=1+(-1-t)u(t+1)+(2t)u(t)+(1-t)u(t-1)$

QUESTION 4

$x$ is causal $\Rightarrow x(t)=0$ for $t<0$.
$v_1$ is anticausal $\Rightarrow v_1(t)=0$ for $t>0 \Rightarrow x(t)=0$ for $t>2$.
$v_2$ is even $\Rightarrow v_2(t)=v_2(-t) \Rightarrow x(t+1)=x(-t+1) \Rightarrow x(t)=x(2-t)=(2-t)-1=1-t$.
therefore, we conclude that

$$x(t)=\begin{cases} 1-t & 0\le t<1 \\ t-1 & 1\le t\le 2 \\ 0 & \text{otherwise}. \end{cases}$$

QUESTION 5

expressing $x_3$ in terms of $x_1$ and $x_2$, we have
$x_3(t)=2x_1(t)+2x_1(t-1)+x_2(t-1)$.
since the system is LTI, we have that the input $a\,x_i(t-b)$ yields the output $a\,y_i(t-b)$.
therefore, we have (from equation above)
$y_3(t)=2y_1(t)+2y_1(t-1)+y_2(t-1)$.