Exercise 4.109

L **Answer (c).**

We are given a LTI system with system function

$$H(s) = \frac{1}{s+1} \quad \text{for } s \in \mathbb{C} \text{ such that } \operatorname{Re}(s) > -1. \qquad (1)$$

Furthermore, we are given

$$x(t) = 2\cos(t).$$

Euler

Rewriting x in terms of eigenfunctions of LTI systems, we obtain

$$\begin{aligned} x(t) &= 2\left[\tfrac{1}{2}\left(e^{jt} + e^{-jt}\right)\right] \\ &= e^{jt} + e^{-jt}. \end{aligned}$$

simplify

Since the system is LTI, the response y of the system to the input x is given by

from linearity of system and eigenfunction property (eigenvalues obtained from (1))

$$(2) \quad y(t) = H(j)e^{jt} + H(-j)e^{-jt}.$$

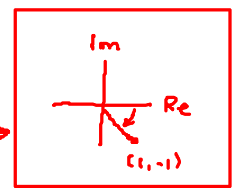
Now, we compute $H(j)$ and $H(-j)$. We have

$$\begin{aligned} H(j) &= \frac{1}{1+j} \\ &= \frac{1-j}{2} \\ &= \tfrac{1}{2} - j\tfrac{1}{2} \\ &= \sqrt{\tfrac{1}{2}}\, e^{j\arctan(-1)} \\ &= \tfrac{1}{\sqrt{2}} e^{-j\pi/4} \quad \text{and} \end{aligned}$$

multiply by $\frac{1-j}{1-j}$

write in Cartesian form

convert to polar form



$\arctan(-1) = -\frac{\pi}{4}$

$$\begin{aligned} H(-j) &= \frac{1}{1-j} \\ &= H(j)^* \\ &= \tfrac{1}{\sqrt{2}} e^{j\pi/4}. \end{aligned}$$

by comparison with $H(j)$

conjugate already computed value for $H(j)$

So, we have

$$\begin{aligned} y(t) &= \tfrac{1}{\sqrt{2}} e^{-j\pi/4} e^{jt} + \tfrac{1}{\sqrt{2}} e^{j\pi/4} e^{-jt} \\ &= \tfrac{1}{\sqrt{2}}\left[e^{j(t-\pi/4)} + e^{-j(t-\pi/4)}\right] \\ &= \tfrac{1}{\sqrt{2}}\left[2\cos(t-\pi/4)\right] \\ &= \sqrt{2}\cos\left(t - \tfrac{\pi}{4}\right). \end{aligned}$$

substitute computed eigenvalues into (2)

factor

Euler [i.e., $e^{j\theta} + e^{-j\theta} = 2\cos\theta$]

multiply constants

