Bilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain	ROC
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$	$R+\mathrm{Re}\{s_0\}$
Time/Laplace-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Time-Domain Convolution	$x_1(t)*x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time-Domain Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathrm{Re}\{s\}>0\}$

Property	
Initial Value Theorem	$x(0^+)=\lim\limits_{s\to\infty}sX(s)$
Final Value Theorem	$\lim\limits_{t\to\infty}x(t)=\lim\limits_{s\to 0}sX(s)$

Unilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$
Time/Laplace-Domain Scaling	$x(at)$, $a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1(t)*x_2(t)$, $x_1(t)$ and $x_2(t)$ are causal	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)-x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_{0^-}^{t}x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property	
Initial Value Theorem	$x(0^+)=\lim\limits_{s\to\infty}sX(s)$
Final Value Theorem	$\lim\limits_{t\to\infty}x(t)=\lim\limits_{s\to 0}sX(s)$

Bilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}>0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}<0$
4	$t^nu(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}>0$
5	$-t^nu(-t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}<0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}>-a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}<-a$
8	$t^ne^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}>-a$
9	$-t^ne^{-at}u(-t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}<-a$
10	$[\cos\omega_0t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
11	$[\sin\omega_0t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
12	$[e^{-at}\cos\omega_0t]u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$
13	$[e^{-at}\sin\omega_0t]u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$

Unilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos\omega_0t$	$\frac{s}{s^2+\omega_0^2}$
7	$\sin\omega_0t$	$\frac{\omega_0}{s^2+\omega_0^2}$
8	$e^{-at}\cos\omega_0t$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
9	$e^{-at}\sin\omega_0t$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

Unit:
Complex Analysis

Example A.10. Determine for what values of z the function $f(z) = z^2$ is analytic.

Solution. First, we observe that f is a polynomial function. Then, we recall that polynomial functions are analytic everywhere. Therefore, f is analytic everywhere.

Example A.11. Determine for what values of z the function $f(z) = 1/z$ is analytic.

Solution. We can deduce the analyticity properties of f as follows. First, we observe that f is a rational function. Then, we recall that a rational function is analytic everywhere except at points where its denominator polynomial becomes zero. Since the denominator polynomial of f only becomes zero at 0, f is analytic everywhere except at 0.