Stat 260 Lecture Notes
Set 8 - Random Variables

A **random variable (r.v.)** (usually we denote it by X) is a function or a rule that assigns a number to each outcome of the experiment. Back in Set 1 we saw that random variables can be discrete or continuous.

The **probability mass function (pmf)**, or **probability distribution**, is a table, formula, or graph that describes the possible values of the r.v. and the probability that each value will occur.

Think of the pmf as a function f.

$$f(x) = P(X = x)$$

A pmf for a a discrete r.v. X must meet the requirements:

1. $f(x) = P(X = x)$ is defined for all values of x.
2. $f(x) = P(X = x) \geq 0$ for all values of x.
3. $\sum_{\text{all} x} f(x) = \sum_{\text{all} x} P(X = x) = 1$ (the sum of all probabilities is 1).

Continuous probability distributions are studied in a later Set.

Example 1: Dominant writing hands.

Suppose 25% of people are left-handed. Suppose we independently sample 3 people and count how many are right handed.

Let the r.v. X be the number of right-handed people in the 3 sampled. The possible values of X are $0, 1, 2, 3$.

After some work we can find the pmf:

x	0	1	2	3
$f(x) = P(X = x)$	0.015625	0.140625	0.421875	0.421875

Notice that all probabilities are ≥ 0 and that

$$\sum_x f(x) = 0.015625 + 0.140625 + 0.421875 + 0.421875 = 1.$$

(a) Find $P(X = 2)$.

(b) Find $P(X \geq 1)$.

(c) Find $P(X \leq 2 \mid X \geq 1)$.

The **cumulative distribution function** (cdf) of a r.v. X is defined as $F(x) = P(X \leq x)$.
So for a value c, $F(c) = P(X \leq c) = \sum_{x \leq c} P(X = x) = \sum_{x \leq c} f(x)$.

Example 2: Find the cdf for the dominant writing hand example.

Example 3: The cdf for an experiment is given below. Find the pmf.

x	$F(x)$
0	0.15
1	0.38
2	0.74
3	0.92
4	0.98
5	1

Example 4: Using the distribution from Example 3, find:

(a) $P(X = 2)$

(b) $P(X \geq 3)$

(c) $P(1 < X \leq 4)$

Rules for discrete r.v.s:

- $P(X \geq x) = 1 - P(X < x)$
- $P(X > x) = 1 - P(X \leq x)$
- $P(a \leq X \leq b) = P(X \leq b) - P(X < a)$