Exercise 6.3

L Answer (h).

We are asked to find the Fourier transform X of

$$x(t) = \cos(5t-2).$$

x is cos function that has been shifted by 2 and then time scaled by 5

FIRST SOLUTION (WHICH IS SLIGHTLY LONGER). We begin by rewriting x as

significantly

$$\textcircled{1}\quad x(t) = v_2(5t),$$

time scale by 5

where

$$\textcircled{2}\quad v_2(t) = v_1(t-2) \quad \text{and}$$
$$\textcircled{3}\quad v_1(t) = \cos(t).$$

shift by 2

start with cos

Taking the Fourier transform of both sides of the above equations yields

$$\textcircled{4}\quad V_1(\omega) = \pi[\delta(\omega-1)+\delta(\omega+1)],$$
$$\textcircled{5}\quad V_2(\omega) = e^{-j2\omega}V_1(\omega), \quad \text{and}$$
$$\textcircled{6}\quad X(\omega) = \tfrac{1}{5}V_2(\omega/5).$$

FT of ③ (from table)

FT of ② (time shift)

FT of ① (time scaling)

Combining the above results, we have

$$\begin{aligned}
X(\omega) &= \tfrac{1}{5}V_2(\omega/5) \\
&= \tfrac{1}{5}e^{-j2(\omega/5)}V_1(\omega/5) \\
&= \tfrac{1}{5}e^{-j2(\omega/5)}\pi[\delta(\omega/5-1)+\delta(\omega/5+1)] \\
&= \tfrac{\pi}{5}e^{-j2\omega/5}\left[\delta\left(\tfrac{1}{5}[\omega-5]\right)+\delta\left(\tfrac{1}{5}[\omega+5]\right)\right] \\
&= \tfrac{\pi}{5}e^{-j2\omega/5}[5\delta(\omega-5)+5\delta(\omega+5)] \\
&= \pi e^{-j2\omega/5}[\delta(\omega-5)+\delta(\omega+5)].
\end{aligned}$$

⑥ substitute ⑤ for V_2

substitute ④ for V_1

rewrite with time scaling first

time scaling property of δ

cancel 5's

significantly shorter (percentage-wise)

SECOND SOLUTION (WHICH IS SLIGHTLY SHORTER). We begin by rewriting x as

?

$$\begin{aligned}
x(t) &= \cos(5t-2) \\
&= \cos\left[5\left(t-\tfrac{2}{5}\right)\right].
\end{aligned}$$

x is cos function that has been time scaled by 5 and then shifted by 2/5

Defining ① $v_1(t) = \cos(5t)$, we can further rewrite x as

$$\textcircled{2}\quad x(t) = v_1\left(t-\tfrac{2}{5}\right).$$

Taking the Fourier transform of the above equations for x and v_1, we obtain

$$\textcircled{3}\quad X(\omega) = e^{-j(2/5)\omega}V_1(\omega) \quad \text{and}$$
$$\textcircled{4}\quad V_1(\omega) = \pi[\delta(\omega-5)+\delta(\omega+5)].$$

FT of ② (time shifting)

FT of ① (from table)

Combining the above equations for X and V_1, we obtain

$$\begin{aligned}
X(\omega) &= e^{-j2\omega/5}V_1(\omega) \\
&= e^{-j2\omega/5}\pi[\delta(\omega-5)+\delta(\omega+5)] \\
&= \pi e^{-j2\omega/5}[\delta(\omega-5)+\delta(\omega+5)].
\end{aligned}$$

③ substitute ④ for V_1

write π first