Please read the instructions below and in the Written Assignment 4 assignment on Crowdmark.

For full marks, your work must be neatly written, and contain enough detail that it is clear how you arrived at your solutions. Messy, poorly formatted work will receive deductions, or may not be graded at all.

Talking to your classmates about assigned work is a healthy practice that is encouraged. However, in the end, each person is expected to write their own solutions, in their own words, and in a way that reflects their own understanding. Assignment questions are not to be posted to homework "help" websites.

Late policy: Late assignments will be accepted until the final cutoff of 11:59pm on Sunday February 19th. Solutions submitted within 1 hour of the Friday deadline will have a 5% late penalty automatically applied within Crowdmark. Solutions submitted after 1 hour of the Friday deadline but before the final Sunday cutoff will have a 20% late penalty applied. Solutions submitted after the final Sunday cutoff will be graded for feedback, but marks will not be awarded.

1. **[2 marks]** A drive-up mechanic shop offers a variety of services, including an oil change and a full service tune-up.

   (a) Let the random variable X represent the number of customers that request an oil change in a day. The distribution of X is given in the table below.

x	1	2	3	4	5	6	7	8	9
$f(x)$	0.02	0.04	0.09	0.17	0.21	0.24	0.15	0.05	0.03

   Suppose at least 4 customers request an oil change in a day. What is the probability that 6 or more customers request an oil change that day?

   $$P(X \geq 6 \mid X \geq 3) = \frac{P(X \geq 6 \cap X \geq 2)}{P(X \geq 3)} = \frac{P(X \geq 6)}{P(X \geq 4)}$$

   $$= \frac{0.24 + 0.15 + 0.05 + 0.03}{0.17 + 0.21 + 0.24 + 0.15 + 0.05 + 0.03} = \frac{0.47}{0.85}$$

   $$= 0.5529$$

   (b) Let the random variable Y represent the number of cylinders in a car. The distribution of Y is given in the table below.

y	4	6	8
$f(y)$	0.54	0.29	0.17

   The cost of a full service tune-up depends on the number of cylinders in the car. Suppose the cost of a full service tune-up is calculated by the function $c(Y) = 0.5Y^2 + 4Y + 75$. Find the expected cost of a full service tune-up.

   $$E(Y) = 4(.54) + 6(.29) + 8(.17) = 5.26$$
   $$E(Y^2) = 4^2(.54) + 6^2(.29) + 8^2(.17) = 29.96$$

   $$E(c(Y)) = E(0.5Y^2 + 4Y + 75) = 0.5E(Y^2) + 4E(Y) + 75$$

   $$= 0.5(29.96) + 4(5.26) + 75$$

   $$= 111.02 \qquad \text{Expected cost is } \$111.02.$$

2. **[2 marks]** The quality assurance department at a widget factory conducts a test where they select widgets until a defective one is found. Let the random variable X represent the number of widgets selected until a defective one is found. The cumulative distribution function (cdf) of X is given below:

x	1	2	3	4	5	6	7	8
$F(x)$	0.07	0.16	0.27	0.48	0.61	0.75	0.88	1
$f(x)$	.07	.09	.11	.21	.13	.14	.13	.12

(a) What is the probability that at least 3 widgets and at no more than 7 widgets are selected until a defective one is found?

$$P(3 \leq X \leq 7) = P(X \leq 7) - P(X \leq 2)$$
$$= 0.88 - 0.16$$
$$= 0.72$$

(b) Find the standard deviation for the number of widgets selected until a defective one is found. That is, find σ_X.

$$E(x) = 1(.07) + 2(.09) + 3(.11) + 4(.21) + 5(.13) + 6(.14) + 7(.13) + 8(.12)$$
$$= 4.78$$

$$E(x^2) = 1^2(.07) + 2^2(.09) + 3^2(.11) + 4^2(.21) + 5^2(.13) + 6^2(.14) + 7^2(.13) + 8^2(.12)$$
$$= 27.12$$

$$V(x) = E(x^2) - (E(x))^2 = 27.12 - (4.78)^2 = 4.2716$$

$$\sigma_X = \sqrt{V(x)} = \sqrt{4.2716} = 2.0668$$

3. **[2 marks]** A company that sells merchandise online allows customers to pay with a credit card or with PayPal. The company estimates that 70% of their customers pay with their credit card.

(a) The company looks at a random selection of 13 recent sales. What is the probability that at least two of these sales paid with PayPal?

X = # that used Pay Pal

X is binomial with $n = 13$, $p = 0.3$

$$P(X \geq 2) = 1 - P(x \leq 1) = 1 - P(x=0) - P(X=1)$$
$$= 1 - \binom{13}{0}(.3)^0(.7)^{13} - \binom{13}{1}(.3)^1(.7)^{12}$$
$$= 0.9363$$

or $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.0637 = 0.9363$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\nwarrow$ from stat tables.

(b) The company looks at a random selection of 19 recent sales. What is the probability that between 11 and 13 sales (inclusive) paid with a credit card?

X = # that used credit card

X is binomial with $n = 19$, $p = 0.7$

$$P(11 \leq X \leq 13) = P(X=11) + P(X=12) + P(X=13)$$
$$= \binom{19}{11}(.7)^{11}(.3)^8 + \binom{19}{12}(.7)^{12}(.3)^7 + \binom{19}{13}(.7)^{13}(.3)^6$$
$$= 0.4422$$

from stat tables
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\downarrow\qquad\searrow$

or $P(11 \leq X \leq 13) = P(X \leq 13) - P(X \leq 10) = 0.5261 - 0.0839$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 0.4422$