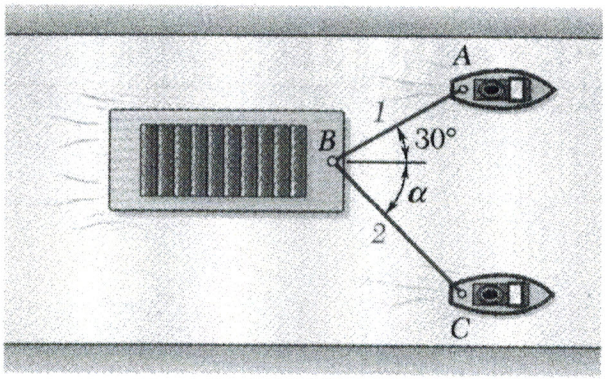
A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lb_f directed along the axis of the barge, determine

a) the tension in each of the ropes for $\alpha = 45°$, using a graphical and trigonometric solution.

b) the value of α for which the tension in rope 2 is a minimum

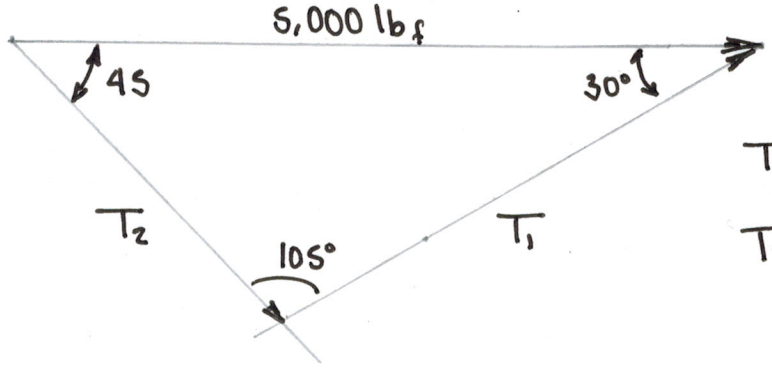


a)

Graphical Solution

Let 5,000 lb_f = 10 cm

∴ Scale 1 cm = 500 lb_f



T_1 = 7.3 cm = 3,650 lb_f

T_2 = 5.2 cm = 2,600 lb_f

Trigonometric Solution

Law of sines

$$\frac{5000}{\sin 105°} = \frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°}$$

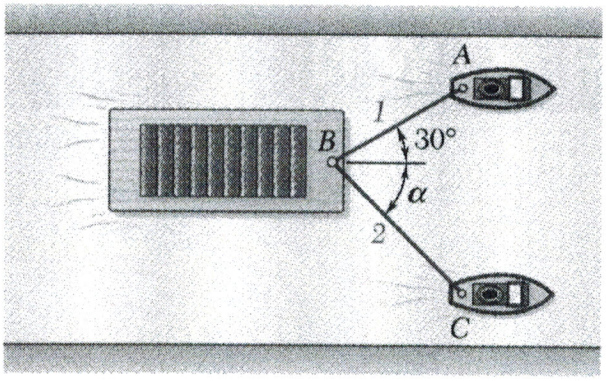
$$T_1 = 5{,}000\left(\frac{\sin 45°}{\sin 105°}\right) = 3{,}660\ lb_f$$

$$T_2 = 5{,}000\left(\frac{\sin 30°}{\sin 105°}\right) = 2{,}588\ lb_f$$

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lb_f directed along the axis of the barge, determine

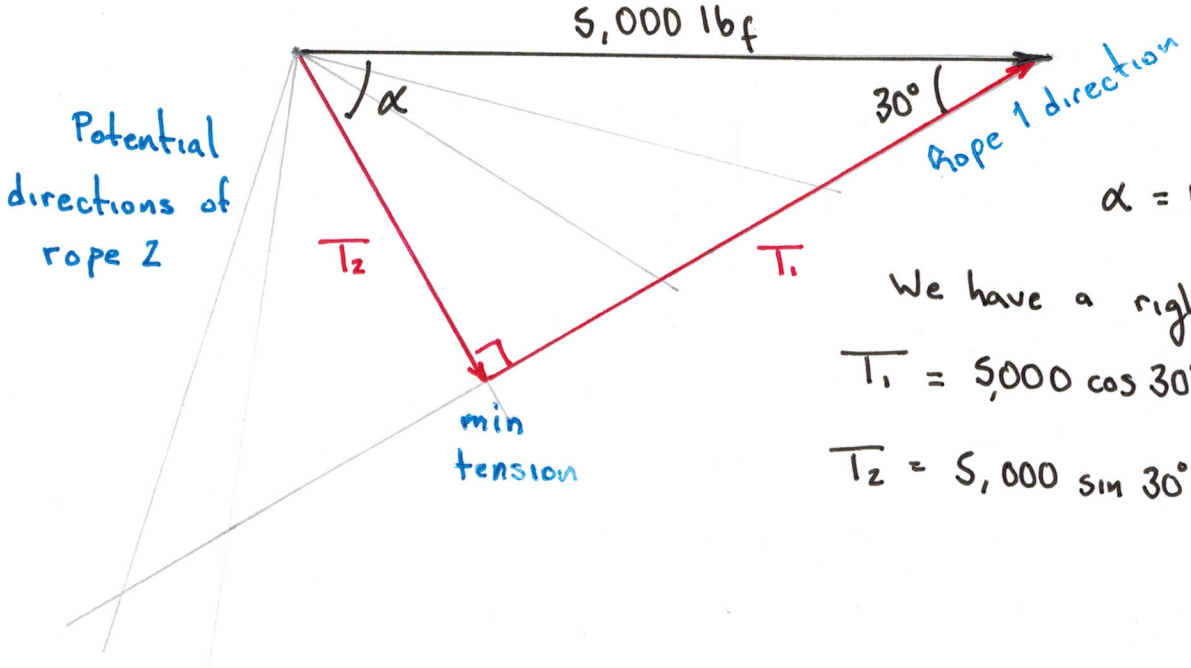
a) the tension in each of the ropes for $\alpha = 45°$, using a graphical and trigonometric solution.

b) the value of α for which the tension in rope 2 is a minimum



Scale 1cm = 500 lb_f

b)



$\alpha = 180 - 90 - 30 = 60$

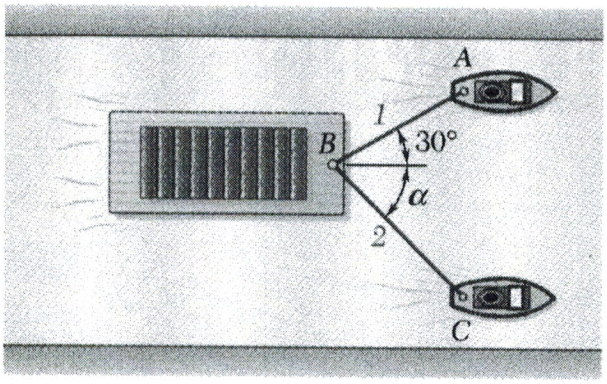
We have a right triangle

$T_1 = 5{,}000 \cos 30° = 4{,}330\ lb_f$

$T_2 = 5{,}000 \sin 30° = 2{,}500\ lb_f$

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lb_f directed along the axis of the barge, determine

c) the tension in each of the ropes for $\alpha = 45°$, using rectangular components.



$\overline{\overline{T}}$ → vector force

$\overline{T}$ → magnitude of force

Decompose forces from tugboats

$$\overline{\overline{T_1}} = T_1 \cos 30\,\hat{i} + T_1 \sin 30\,\hat{j}$$

$$\overline{\overline{T_2}} = T_2 \cos 45\,\hat{i} - T_2 \sin 45\,\hat{j}$$

$$\overline{\overline{F}} = \overline{\overline{T_1}} + \overline{\overline{T_2}} \qquad \text{where} \quad \overline{\overline{F}} = 5000\,\hat{i}\,(lb) \quad \text{Resultant}$$

$$5000\,\hat{i} = T_1 \cos 30\,\hat{i} + T_1 \sin 30\,\hat{j} + T_2 \cos 45\,\hat{i} - T_2 \sin 45\,\hat{j}$$

$\hat{i}$ component $\quad 5000 = T_1 \cos 30 + T_2 \cos 45 \quad (1)$

$\hat{j}$ component $\quad 0 = T_1 \sin 30 - T_2 \sin 45 \quad (2)$

$$T_1 = T_2 \left(\frac{\sin 45}{\sin 30}\right)$$

Sub (2) in (1)

$$5000 = T_2 \left(\frac{\sin 45}{\sin 30}\right) \cos 30 + T_2 \cos 45$$

$$T_2 = \frac{5000}{\left(\frac{\sin 45}{\sin 30}\right)\cos 30 + \cos 45} = 2,588.2$$

$T_2 = 2,590$ lb

$$T_1 = T_2 \left(\frac{\sin 45}{\sin 30}\right) = 2,588.2 \left(\frac{\sin 45}{\sin 30}\right) = 3,660.2$$

$T_1 = 3,660$ lb