Exercise 5.106

L **Answer (g).**

We are given the Fourier series coefficient sequence c, where

$$c_k = \begin{cases} j\sin\left(\frac{\pi}{2}k\right) & k \in [-32\,..\,32] \\ 0 & \text{otherwise.} \end{cases} \quad 1$$

To begin, we determine if c has either even or odd symmetry. For $k \in [-32\,..\,32]$, we have

$$\begin{aligned} c_k &= j\sin\left(\tfrac{\pi}{2}k\right) && \text{from 1 for } k \in [-32\,..\,32] \\ &= -j\sin\left[\tfrac{\pi}{2}(-k)\right] && \text{sin is odd} \\ &= -c_{-k}. && \text{from 1 for } k \in [-32\,..\,32] \end{aligned}$$

For $k \notin [-32\,..\,32]$, we trivially have (since $c_k = 0$ and $0 = -0$) that

$$c_k = -c_{-k}. \quad \text{from 1 for } k \notin [-32\,..\,32]$$

Therefore, c is odd. Next, we can see (by inspection) that c is purely imaginary (i.e., $\operatorname{Re}(c_k) = 0$ for all k). Since c is purely imaginary, $c_k^* = -c_k$ for all k, or equivalently,

$$c_k = -c_k^*. \quad 2$$

Furthermore, since $c_k = -c_k^*$ for all k, c being odd implies c is conjugate symmetric. In other words, we have

$$\begin{aligned} & c_k = -c_{-k} && \text{c is odd} \\ \Rightarrow\; & -c_k^* = -c_{-k} && \text{substitute 2} \\ \Rightarrow\; & c_k^* = c_{-k} && \text{multiply both sides by -1} \\ \Rightarrow\; & c_k = c_{-k}^*. && \text{conjugate both sides} \end{aligned}$$

Therefore, c is conjugate symmetric. Since c is conjugate symmetric, x is real. Since c is odd, x is odd. Therefore, we conclude that x is real and odd.

Copyright © 2012–2023 Michael D. Adams