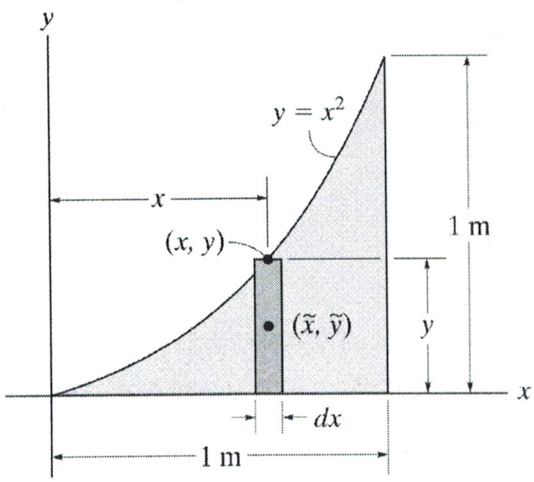
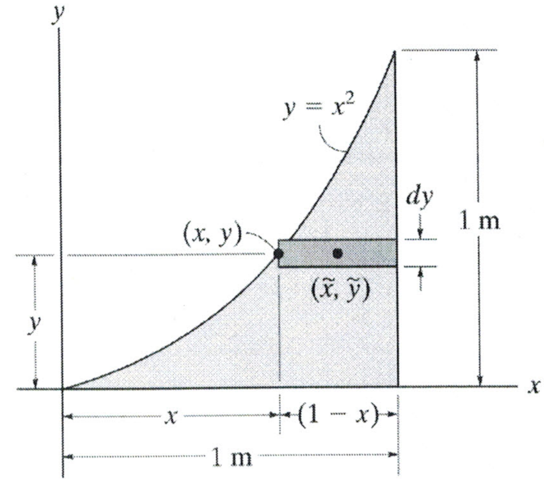
Find the centroid of the area shown below





$dA = y\,dx$ $\qquad y = x^2$

$\tilde{x} = x \qquad \tilde{y} = \frac{y}{2}$

$dA = (1-x)\,dy \qquad x = \sqrt{y}$

$\tilde{y} = y \qquad \tilde{x} = x + \frac{(1-x)}{2} = \frac{1+x}{2}$

Solution I

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^1 x\,y\,dx}{\int_0^1 y\,dx} = \frac{\int_0^1 x^3\,dx}{\int_0^1 x^2\,dx} = \frac{\frac{1}{4}x^4\big|_0^1}{\frac{1}{3}x^3\big|_0^1} = \frac{3}{4} = 0.75\,m$$

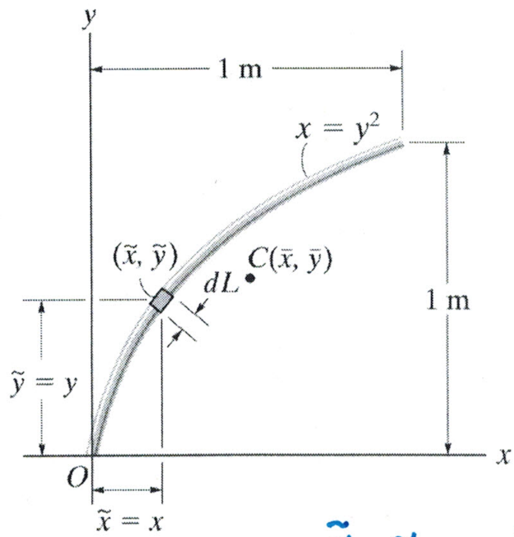
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^1 \frac{y}{2}\,y\,dx}{\int_0^1 y\,dx} = \frac{\int_0^1 \frac{1}{2}x^4\,dx}{\int_0^1 x^2\,dx} = \frac{\frac{1}{10}x^5\big|_0^1}{\frac{1}{3}x^3\big|_0^1} = \frac{3}{10} = 0.3\,m$$

Solution II

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^1 \left(\frac{1+x}{2}\right)(1-x)\,dy}{\int_0^1 (1-x)\,dy} = \frac{\frac{1}{2}\int_0^1 (1-y)\,dy}{\int_0^1 (1-\sqrt{y})\,dy} = \frac{\frac{1}{2}\left(y-\frac{1}{2}y^2\right)\big|_0^1}{y-\frac{2}{3}y^{3/2}\big|_0^1} = \frac{\frac{1}{4}}{\frac{1}{3}} = 0.75\,m$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^1 y(1-x)\,dy}{\int_0^1 (1-x)\,dy} = \frac{\int_0^1 y(1-\sqrt{y})\,dy}{\int_0^1 (1-\sqrt{y})\,dy} = \frac{\frac{1}{2}y^2-\frac{2}{5}y^{5/2}\big|_0^1}{y-\frac{2}{3}y^{3/2}\big|_0^1} = \frac{\frac{1}{10}}{\frac{1}{3}} = 0.3\,m$$

Find the centroid of the curved rod



Using Pythogoras

$$dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\right) dy$$

since $x = y^2$, $\dfrac{dx}{dy} = \dfrac{d(y^2)}{dy} = 2y$

thus $dL = \left(\sqrt{(2y)^2 + 1}\right) dy$

$\tilde{x} = x$ and $\tilde{y} = y$

Integration

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{\int_0^1 x\sqrt{4y^2 + 1}\, dy}{\int_0^1 \sqrt{4y^2 + 1}\, dy} = \frac{\int_0^1 y^2\sqrt{4y^2 + 1}\, dy}{\int_0^1 \sqrt{4y^2 + 1}\, dy} = \frac{0.6063}{1.479} = 0.41\,m$$

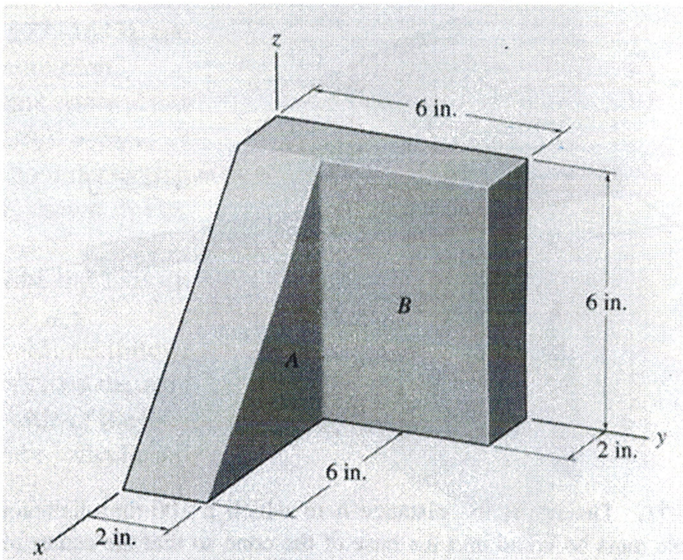
$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{\int_0^1 y\sqrt{4y^2 + 1}\, dy}{\int_0^1 \sqrt{4y^2 + 1}\, dy} = \frac{0.8484}{1.479} = 0.574\,m$$

Two blocks of different materials are assembled as shown.

The densities of the materials are:

$$\rho_A = 150 \text{ lb / ft}^3 \quad \text{and} \quad \rho_B = 400 \text{ lb / ft}^3.$$

Determine the centre of gravity of this assembly.



Weight $W = \rho V$

$W_A = 150\left(\frac{2\times6\times6}{2}\right)\left(\frac{1}{12^3}\right) = 3.125$ lb

$W_B = 400\left(6\times6\times2\right)\left(\frac{1}{12^3}\right) = 16.67$ lb

Segment	W (lb)	$\bar{x}$ (in)	$\bar{y}$ (in)	$\bar{z}$ (in)	$W\bar{x}$	$W\bar{y}$	$W\bar{z}$
A	3.125	4	1	2	12.5	3.125	6.25
B	16.67	1	3	3	16.67	50	50
Σ	19.79				29.17	53.125	56.25

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{29.17}{19.79} = 1.47 \text{ in}$$

$$\bar{y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{53.125}{19.79} = 2.68 \text{ in}$$

$$\bar{z} = \frac{\Sigma \bar{z} W}{\Sigma W} = \frac{56.25}{19.79} = 2.84 \text{ in}$$

Coordinates of the centre of gravity