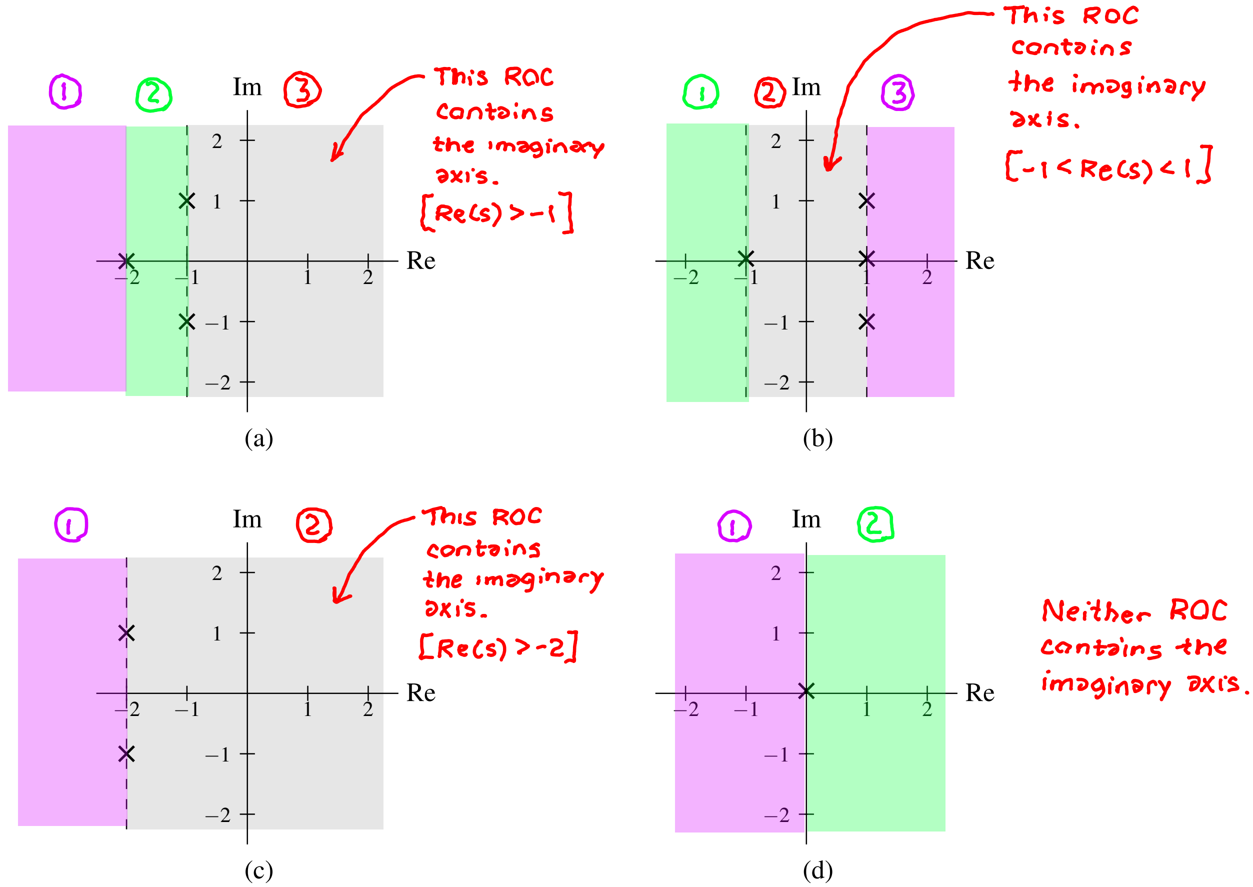


Figure 7.22: Poles and ROCs of the system function H in the (a) first, (b) second, (c) third, and (d) fourth parts of the example.

Example 7.35. Consider the LTI system with system function

$$H(s) = \frac{s+1}{s+2} \quad \text{for } \operatorname{Re}(s) > -2.$$

Determine all possible inverses of this system. Comment on the stability of each of these inverse systems.

Solution. The system function H_{inv} of the inverse system is given by

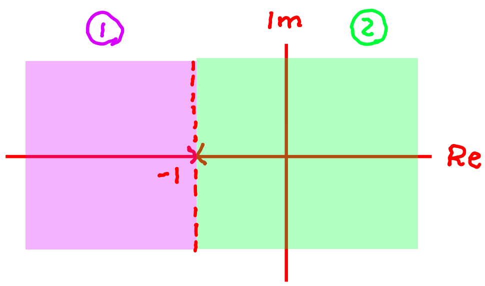
$$H_{\mathsf{inv}}(s) = \frac{1}{H(s)} = \frac{s+2}{s+1}.$$

Two ROCs are possible for H_{inv}:

i) $\operatorname{Re}(s) < -1$ and

ii) $\operatorname{Re}(s) > -1$.

Each ROC is associated with a distinct inverse system. The first ROC is associated with an unstable system since this ROC does not include the imaginary axis. The second ROC is associated with a stable system, since this ROC includes the entire imaginary axis. ■



region 1 does not contain the imaginary axis and therefore corresponds to an unstable system

region 2 contains the imaginary axis and therefore corresponds to a stable system

Example 7.36 (Differential equation to system function)**.** A LTI system with input x and output y is characterized by the differential equation

$$y''(t) + \tfrac{D}{M}y'(t) + \tfrac{K}{M}y(t) = x(t),$$

where D, K, and M are positive real constants, and the prime symbol is used to denote derivative. Find the system function H of this system.

Solution. Taking the Laplace transform of the given differential equation, we obtain

$$s^2Y(s) + \tfrac{D}{M}sY(s) + \tfrac{K}{M}Y(s) = X(s).$$

taking LT using
time-domain differentiation
property

Rearranging the terms and factoring, we have

$$\left(s^2 + \tfrac{D}{M}s + \tfrac{K}{M}\right)Y(s) = X(s).$$

rearrange terms and factor

Dividing both sides by $\left(s^2 + \tfrac{D}{M}s + \tfrac{K}{M}\right)X(s)$, we obtain

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + \frac{D}{M}s + \frac{K}{M}}.$$

divide both sides by
$\left(s^2 + \frac{D}{M}s + \frac{K}{M}\right)X(s)$

Thus, H is given by

$$H(s) = \frac{1}{s^2 + \frac{D}{M}s + \frac{K}{M}}.$$

$Y(s) = X(s)H(s) \Rightarrow$
$H(s) = \dfrac{Y(s)}{X(s)}$

■

Example 7.37 (System function to differential equation)**.** A LTI system with input x and output y has the system function

$$H(s) = \frac{s}{s+R/L},$$

where L and R are positive real constants. Find the differential equation that characterizes this system.

Solution. Let X and Y denote the Laplace transforms of x and y, respectively. To begin, we have

system is LTI

$$\begin{aligned} Y(s) &= H(s)X(s) \\ &= \left(\frac{s}{s+R/L}\right)X(s). \end{aligned}$$

substitute given H

multiply both sides by $(s+R/L)$

Rearranging this equation, we obtain

$$\begin{aligned} &(s+\tfrac{R}{L})Y(s) = sX(s) \\ \Rightarrow\quad & sY(s)+\tfrac{R}{L}Y(s) = sX(s). \end{aligned}$$

simplify

take inverse LT (and use linearity)

Taking the inverse Laplace transform of both sides of this equation (by using the linearity and time-differentiation properties of the Laplace transform), we have

$$\begin{aligned} &\mathcal{L}^{-1}\{sY(s)\}(t)+\tfrac{R}{L}\mathcal{L}^{-1}Y(t) = \mathcal{L}^{-1}\{sX(s)\}(t) \\ \Rightarrow\quad & \tfrac{d}{dt}y(t)+\tfrac{R}{L}y(t) = \tfrac{d}{dt}x(t). \end{aligned}$$

time-domain differentiation property

Therefore, the system is characterized by the differential equation

$$\tfrac{d}{dt}y(t)+\tfrac{R}{L}y(t) = \tfrac{d}{dt}x(t).$$

■