ECE 260

EXAM 3

SOLUTIONS

(FALL 2023)

QUESTION 1

$T = 8, \quad w_0 = \frac{2\pi}{T} = \frac{2\pi}{8} = \frac{\pi}{4}$

$x(t) = 4\delta(t+3) - 2\delta(t+1) + 2\delta(t-1) + 4\delta(t-3) \quad \text{for} \quad -\frac{T}{2} \le t < \frac{T}{2}$

$$C_K = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jkw_0 t}\, dt$$

$$= \frac{1}{8}\int_{-4}^{4} \left[4\delta(t+3) - 2\delta(t+1) + 2\delta(t-1) + 4\delta(t-3)\right] e^{-jk(\pi/4)t}\, dt$$

$$= \frac{1}{2}\int_{-4}^{4} \delta(t+3)\, e^{-jk(\pi/4)t}\, dt - \frac{1}{4}\int_{-4}^{4} \delta(t+1)\, e^{-jk(\pi/4)t}\, dt$$

$$+ \frac{1}{4}\int_{-4}^{4} \delta(t-1)\, e^{-jk(\pi/4)t}\, dt + \frac{1}{2}\int_{-4}^{4} \delta(t-3)\, e^{-jk(\pi/4)t}\, dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \delta(t+3)\, e^{-jk(\pi/4)t}\, dt - \frac{1}{4}\int_{-\infty}^{\infty} \delta(t+1)\, e^{-jk(\pi/4)t}\, dt$$

$$+ \frac{1}{4}\int_{-\infty}^{\infty} \delta(t-1)\, e^{-jk(\pi/4)t}\, dt + \frac{1}{2}\int_{-4}^{4} \delta(t-3)\, e^{-jk(\pi/4)t}\, dt$$

$$= \frac{1}{2} e^{-jk(\pi/4)(-3)} - \frac{1}{4} e^{-jk(\pi/4)(-1)} + \frac{1}{4} e^{-jk(\pi/4)(1)} + \frac{1}{2} e^{-jk(\pi/4)(3)}$$

$$= \frac{1}{2}\left(e^{j3\pi k/4} + e^{-j3\pi k/4}\right) - \frac{1}{4}\left(e^{j\pi k/4} - e^{-j\pi k/4}\right)$$

$$= \frac{1}{2}\left[2\cos\left(\frac{3\pi k}{4}\right)\right] - \frac{1}{4}\left[2j\sin\left(\frac{\pi k}{4}\right)\right]$$

$$= \cos\left(\frac{3\pi k}{4}\right) - \frac{j}{2}\sin\left(\frac{\pi k}{4}\right)$$

QUESTION 2

$x(t) = 1 + 6\cos(4t)$

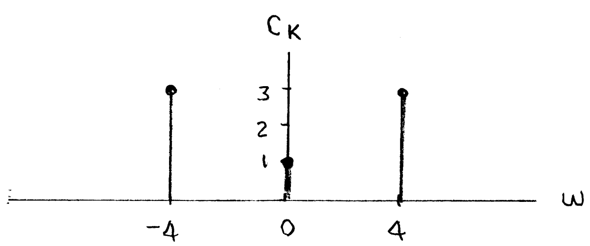
PART (A)

$$T = \frac{2\pi}{4} = \frac{\pi}{2}, \quad \omega_0 = 4$$

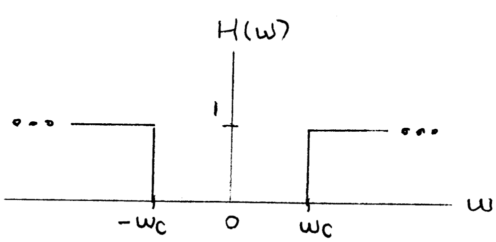
$$x(t) = e^{j0t} + 6\left[\tfrac{1}{2}\left(e^{j4t} + e^{-j4t}\right)\right]$$
$$= e^{j0t} + 3e^{j4t} + 3e^{-j4t}$$

$$C_K = \begin{cases} 3 & K \in \{-1, 1\} \\ 1 & K = 0 \\ 0 & \text{otherwise} \end{cases}$$

PART (B)



PART (C)



A highpass filter with cutoff frequency ω_c such that $\omega_c < 4$ would suffice. More generally, any frequency-selective filter with a frequency response H satisfying $H(0) = 0$ and $H(-4) = H(4) = 1$ would suffice.

QUESTION 3

```
function x = func2(t)
    x = (t >= -10 & t < 0) .* (t .* sin(pi * t) ./ (t .^ 2 + 1)) ...
      + (t >= 1 & t < 10) .* (2 * sin(4 * pi * t) ./ (t + 1) .^ 2);
end
```

QUESTION 4

PART (A)

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[\delta(t) - \frac{10}{\pi}\,\mathrm{sinc}(10t)\right] e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt - \int_{-\infty}^{\infty} \frac{10}{\pi}\,\mathrm{sinc}(10t)\, e^{-j\omega t}\, dt$$

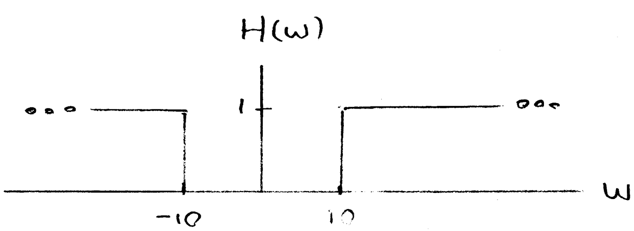
$$= \left[e^{-j\omega t}\right]\Big|_{t=0} - \frac{10}{\pi}\int_{-\infty}^{\infty} \mathrm{sinc}(10t)\, e^{-j\omega t}\, dt$$

$$= 1 - \frac{10}{\pi}\left[\frac{\pi}{10}\,\mathrm{rect}\left(\frac{\omega}{20}\right)\right]$$

$$= 1 - \mathrm{rect}\left(\frac{\omega}{20}\right)$$

$$= \begin{cases} 1 & |\omega| \geq 10 \\ 0 & \text{otherwise} \end{cases}$$

PART (B)



From the plot of H, we can see that the system corresponds to an ideal highpass filter with a cutoff frequency of 10.

QUESTION 5

$H(\omega) = j\omega e^{j2\omega}$

$x(t) = 1 + 4\sin(3t)$

$$x(t) = 1 + 4\left[\frac{1}{j2}\left(e^{j3t} - e^{-j3t}\right)\right]$$
$$= 1 + \frac{2}{j} e^{j3t} - \frac{2}{j} e^{-j3t}$$
$$= 1 - j2\, e^{j3t} + 2j\, e^{-j3t}$$

$$y(t) = \mathcal{H}x(t)$$
$$= \mathcal{H}\{ e^{j0\cdot} - j2\, e^{j3\cdot} + 2j\, e^{-j3\cdot} \}(t)$$
$$= \mathcal{H}\{ e^{j0\cdot} \}(t) - j2\, \mathcal{H}\{ e^{j3\cdot} \}(t) + 2j\, \mathcal{H}\{ e^{-j3\cdot} \}(t)$$
$$= H(0)\, e^{j0t} - j2\, H(3)\, e^{j3t} + 2j\, H(-3)\, e^{-j3t}$$
$$= -j2\left(j3\, e^{j6}\right) e^{j3t} + 2j\left(-j3\, e^{-j6}\right) e^{-j3t}$$
$$= 6\, e^{j(3t+6)} + 6\, e^{-j(3t+6)}$$
$$= 6\left(e^{j(3t+6)} + e^{-j(3t+6)}\right)$$
$$= 6\left[2\cos(3t+6)\right]$$
$$= 12\cos(3t+6)$$
$$= 12\cos\left[3(t+2)\right]$$