University of Victoria
Exam 3
Fall 2023

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 11010), A02 (CRN 11011)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ________________________________

Given Name(s): ________________________________

Student Number: ________________

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

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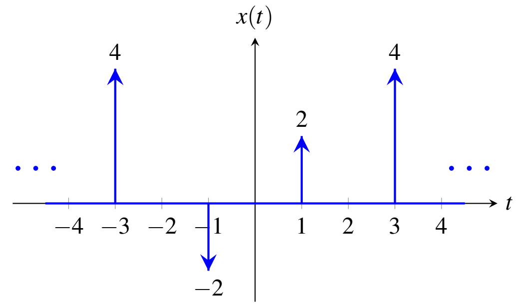
Do not write on this page unless instructed to do so.

Question 1.
Consider the periodic function x with fundamental period $T = 8$ and fundamental frequency ω_0 shown in the figure. Using the Fourier series analysis equation, find the Fourier series coefficient sequence c for the function x. Your solution **must consider the single period of** $x(t)$ **for** $-\frac{T}{2} \leq t < \frac{T}{2}$. Your final answer must be **fully simplified**. You must express your final answer **in terms of** cos **and/or** sin to whatever extent is possible. **[6 marks]**



Question 2. Consider the periodic signal $x(t) = 1 + 6\cos(4t)$. Let T and ω_0 denote the fundamental period and fundamental frequency of x, respectively.

(A) Find the Fourier series coefficient sequence c for x. Also, state the values for T and ω_0 in your answer. **Show all of your work** and **do not skip any steps**. **[2 marks]**

QUESTION 2 CONTINUED

(B) Plot the frequency spectrum of x. **[1 mark]**

(C) Suppose now that x corresponds to a noise-corrupted version of the signal $v(t) = 6\cos(4t)$, which has been distorted by adding to it the noise signal $n(t) = 1$. (In other words, in the formula for $x(t)$ in part (a), the first term represents noise (i.e., n), and the second term represents a noise-free information signal (i.e., v). Sketch and fully label the frequency response H of an ideal frequency-selective filter that will remove the noise n from x. **[1 mark]**

Question 3. Write a MATLAB function called `func2` that takes an $m \times n$ matrix t and returns a matrix x of the same dimensions where $x_{i,j} = f(t_{i,j})$ and

$$f(t) = \begin{cases} \dfrac{t\sin(\pi t)}{t^2+1} & -10 \le t < 0 \\ \dfrac{2\sin(4\pi t)}{(t+1)^2} & 1 \le t < 10 \\ 0 & \text{otherwise.} \end{cases}$$

Note that $x_{i,j}$ and $t_{i,j}$ denote the (i,j)th element of x and t, respectively. Your code is **not permitted** to use any conditional statements (such as **`if`** statements) or looping constructs (such as **`for`** or **`while`** statements). Recall that, in MATLAB, an ellipsis (i.e., "`...`") **must be used** to continue a statement onto the next line. Your code must **use proper indentation** and **must not be excessively long**. Be sure to **use correct syntax** in your answer, since syntax clearly matters here. **[3 marks]**

Line #	Line of Code
1	
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Question 4. A LTI system $\mathscr{H}$ has the impulse response $h(t) = \delta(t) - \frac{10}{\pi}\operatorname{sinc}(10t)$.

(A) Find a **fully-simplified** formula for the frequency response H of the system $\mathscr{H}$. **Show all of your work** and **do not skip any steps**. [Note that $\int_{-\infty}^{\infty} \operatorname{sinc}(at)e^{-j\omega t}dt = \frac{\pi}{|a|}\operatorname{rect}\left(\frac{\omega}{2a}\right)$ for all nonzero real a.] **[5 marks]**

(B) Identify the type of frequency-selective filter that the system $\mathscr{H}$ best approximates, and state any relevant parameters for that type of filter. The rationale for your final answer **must be made clear**. **[1 mark]**

Question 5. Let $\mathscr{H}$ denote the LTI system with the frequency response $H(\omega) = j\omega e^{j2\omega}$. Let x be the (periodic) function $x(t) = 1 + 4\sin(3t)$; and let $y = \mathscr{H}x$. Find a **fully-simplified** formula for y that is expressed **in terms of** cos **and** sin to whatever extent is possible. **Show all of your work** and **do not skip any steps**. **[6 marks]**

END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

θ Degrees	θ Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
135	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
180	π	0	-1	0
225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
270	$\frac{3\pi}{2}$	-1	0	undefined
315	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$$
$$c_k = \frac{1}{T}\int_T x(t)e^{-j(2\pi/T)kt}\,dt$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$

Property	
Even Symmetry	x is even $\Leftrightarrow$ a is even
Odd Symmetry	x is odd $\Leftrightarrow$ a is odd
Real / Conjugate Symmetry	x is real $\Leftrightarrow$ a is conjugate symmetric

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}\,dt$$