ENGR 141 – Engineering Mechanics

Midterm Exam (Spring 2023)

Name: Solutions

Student Number: _____

Section (A01 or A02): _____

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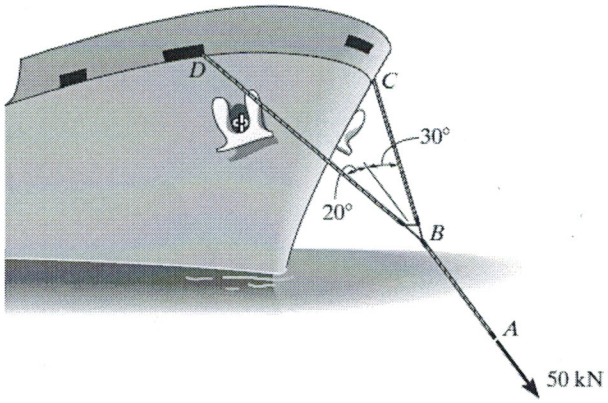
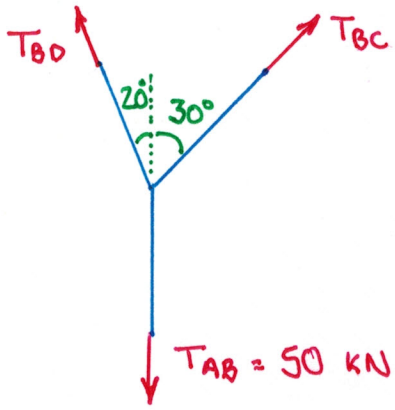
Date: March 13, 2023
Duration: 1hr 30min

There are 4 questions and 4 pages in the examination. Exam is worth 30 pts.

Q1) A tugboat exerts a force of 50kN to the towing pendant *AB*, as the ship moves with a constant velocity of 10 km/hr. Determine the force in each of the bridles *BD* and *BC*. (4 pts)



The three ropes are contained on the same plane.



Equilibrium of a particle

$\Sigma F_x = 0$

$$-T_{BD}\sin 20 + T_{BC}\sin 30 = 0$$

$$T_{BC} = \frac{\sin 20}{\sin 30} T_{BD} \quad \ldots (1)$$

$\Sigma F_y = 0$

$$T_{BD}\cos 20 + T_{BC}\cos 30 - 50 = 0$$

Sub (1)

$$T_{BD}\left(\cos 20 + \frac{\sin 20}{\sin 30}\cos 30\right) = 50$$

$$T_{BD} = \frac{50}{1.532} = 32.635 \text{ kN}$$

$T_{BD} = 32.6$ kN

Sub T_{BD} in (1)

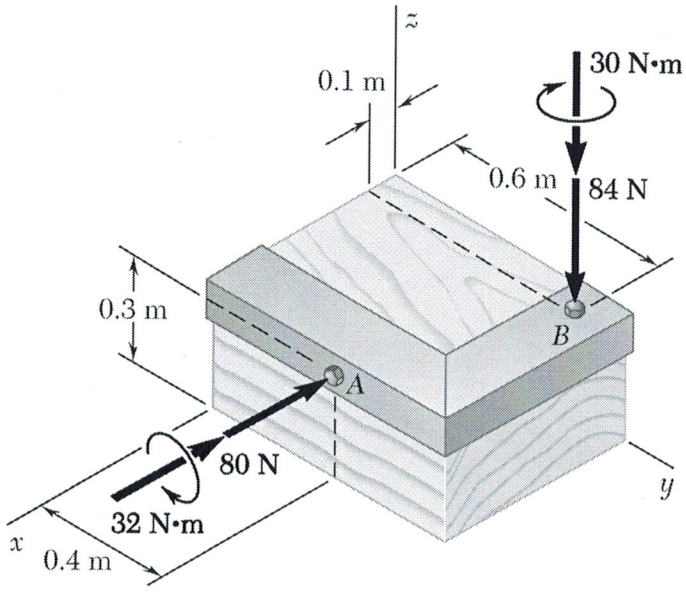
$$T_{BC} = \frac{\sin 20}{\sin 30}(32.6) = 22.32 \text{ kN}$$

$T_{BC} = 22.3$ kN

Q2) Two screws are tightened at A and B by applying the shown wrenches.

a) Replace the two wrenches by an equivalent resultant force and a couple moment at the origin of the reference frame. (3 pts)

b) Replace the two wrenches by a single equivalent wrench. Determine the location where the axis of the wrench intersects the xy plane and the magnitude of the resulting couple moment. (8 pts)



a) Resultant force

$$F_R = \{-80\,\hat{i} + 0\,\hat{j} - 84\,\hat{k}\}\ N$$

Couple moment at O

$$\Sigma M_O = \Sigma M_C + \Sigma r \times F = M_{R_O}$$

$$M_{R_O} = -32\,\hat{i} - 30\,\hat{k} - 84(0.6)\,\hat{i} + 84(0.1)\,\hat{j} - 80(0.3)\,\hat{j} + 80(0.4)\,\hat{k}$$

$$M_{R_O} = -82.4\,\hat{i} - 15.6\,\hat{j} + 2\,\hat{k}$$

b) On a wrench, moment must have same direction as the force

$$\hat{u}_{F_R} = \frac{F_R}{\|F_R\|} = \frac{\{-80\,\hat{i} + 0\,\hat{j} - 84\,\hat{k}\}}{116} = \{-0.6897\,\hat{i} + 0\,\hat{j} - 0.7241\,\hat{k}\}$$

$M_{R_P} = \Sigma M_C + \Sigma r \times F_R$, where $r = \{-x, -y, 0\}$ coordinates of wrench location on xy plane

$$\|M_{R_P}\|\,\hat{u}_{F_R} = M_{R_O} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -x & -y & 0 \\ -80 & 0 & -84 \end{vmatrix} = \{-82.4\,\hat{i} - 15.6\,\hat{j} + 2\,\hat{k}\} + \{84y\,\hat{i} - 84x\,\hat{j} - 80y\,\hat{k}\}$$

$$\|M_{R_P}\|\{-0.6897\,\hat{i} + 0\,\hat{j} - 0.7241\,\hat{k}\} = \{(-82.4 + 84y)\,\hat{i} + (-15.6 - 84x)\,\hat{j} + (2 - 80y)\,\hat{k}\}$$

From $\hat{j}$: $0 = -15.6 - 84x$ $\quad x = 0.1857$ m

From $\hat{i}$ & $\hat{k}$: $(\|M_{R_P}\|(-0.6897) = -82.4 + 84y)\,80$

Eliminate y: $(\|M_{R_P}\|(-0.7241) = 2 - 80y)\,84$

$$-116\,\|M_{R_P}\| = -6592 + 168$$

$$\|M_{R_P}\| = \frac{-6424}{-116} = 55.37\ N\cdot m$$

$$y = \frac{(\|M_{R_P}\|(-0.7241) - 2)}{-80}$$

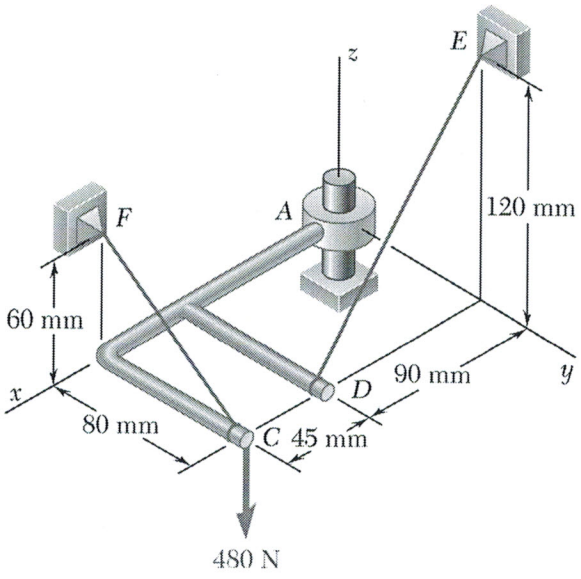
$\|M_{R_P}\| = 55.4\ N\cdot m$

$= -0.5263$ m

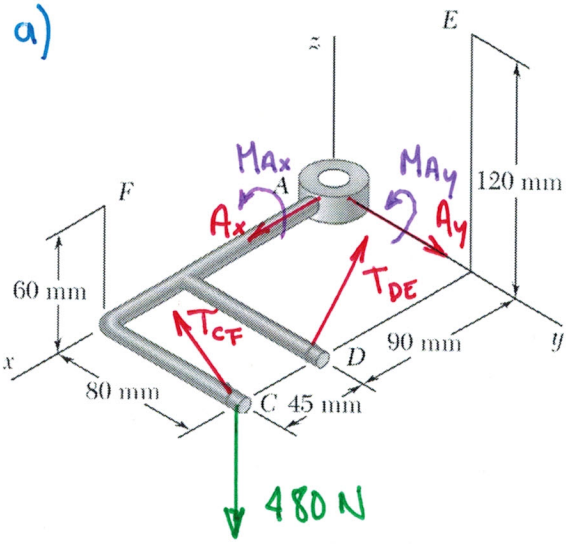
$y = -0.526$ m

Q3) A load of 480N is applied to an assembly ACD which is welded to a collar that freely rotates and translates along pin *A*. The assembly is also supported by two cables *CF* and *DE*.

a) Draw the free body diagram of the assembly (use right drawing). (2 pts)
b) Determine the tension in each of the two cables. (7 pts)



a)



b) $\Sigma M_z = 0$

$$-(0.135)T_{CF}\left(\frac{80}{100}\right) + (0.08)T_{DE}\left(\frac{90}{150}\right) = 0$$

$$T_{CF} = \left(\frac{0.08}{0.135}\right)\left(\frac{100}{80}\right)\left(\frac{90}{150}\right)T_{DE} \qquad T_{CF} = 0.444\,T_{DE} \quad \ldots(1)$$

$\Sigma F_z = 0$

$$T_{CF}\left(\frac{60}{100}\right) + T_{DE}\left(\frac{120}{150}\right) - 480 = 0$$

Sub (1) $\quad T_{DE}\left(0.444\left(\frac{60}{100}\right) + \left(\frac{120}{150}\right)\right) = 480 \qquad T_{DE} = \frac{480}{1.067} = 450\ N$

$T_{DE} = 450\ N$

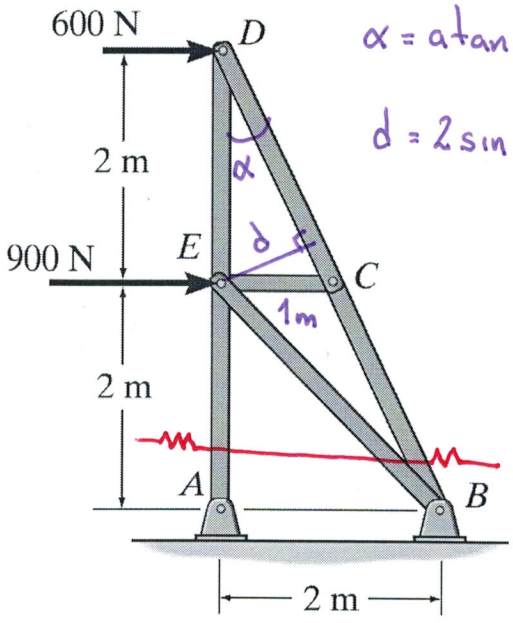
Sub T_{DE} in (1)

$$T_{CF} = 0.444(450) = 200\ N$$

$T_{CF} = 200\ N$

Q4) For the truss shown below,

a) Indicate if there is a zero-force member (1 pt)
b) Determine the forces at members *BC* and *AE* and state whether these members are in tension or in compression. Draw any required free-body diagrams. (5 pts)

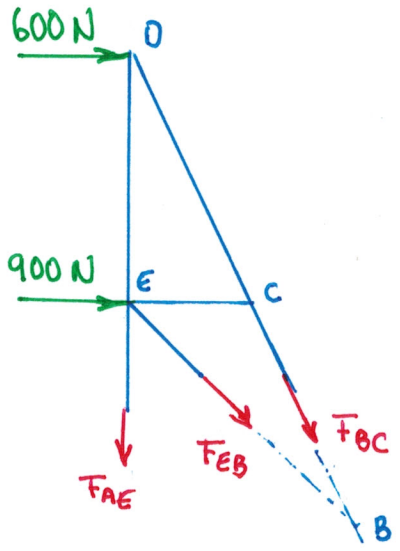


$\alpha = \text{atan}\left(\frac{1}{2}\right) = 26.56°$

$d = 2\sin\alpha = 0.8944$

a) Member EC is a zero-force member based on joint C.

b)



$\Sigma M_E = 0$

$-600(2) - F_{BC}(d) = 0$

$F_{BC} = \frac{1200}{(-0.8944)} = -1341.6\ N$ $\quad F_{BC} = 1342\ N\ (C)$

$\Sigma M_B = 0$

$-600(4) - 900(2) + F_{AE}(2) = 0$

$F_{AE} = \frac{2400 + 1800}{2} = 2100\ N$

$F_{AE} = 2100\ N\ (T)$