Electronic Circuits

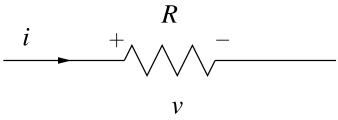
- An **electronic circuit** is a network of one or more interconnected circuit elements.
- The three most basic types of circuit elements are:
  1. resistors;
  2. inductors; and
  3. capacitors.
- Two fundamental quantities of interest in electronic circuits are current and voltage.
- **Current** is the rate at which electric charge flows through some part of a circuit, such as a circuit element, and is measured in units of amperes (A).
- **Voltage** is the difference in electric potential between two points in a circuit, such as across a circuit element, and is measured in units of volts (V).
- Voltage is essentially a force that makes electric charge (or current) flow.

Resistors

- A **resistor** is a circuit element that opposes the flow of current.
- A resistor is characterized by an equation of the form

$$v(t) = Ri(t) \quad \left(\text{or equivalently, } i(t) = \tfrac{1}{R}v(t)\right),$$

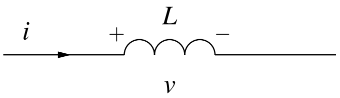
  where R is a nonnegative real constant, and v and i respectively denote the voltage across and current through the resistor as a function of time.
- As a matter of terminology, the quantity R is known as the **resistance** of the resistor.
- Resistance is measured in units of ohms (Ω).
- In circuit diagrams, a resistor is denoted by the symbol shown below.



- An **inductor** is a circuit element that converts an electric current into a magnetic field and vice versa.
- An inductor uses the energy stored in a magnetic field in order to ***oppose changes in current*** (through the inductor).
- An inductor is characterized by an equation of the form

$$v(t) = L\tfrac{d}{dt}i(t) \quad \text{(or equivalently, } i(t) = \tfrac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau\text{)},$$

  where L is a nonnegative real constant, and v and i respectively denote the voltage across and current through the inductor as a function of time.
- As a matter of terminology, the quantity L is known as the **inductance** of the inductor.
- Inductance is measured in units of henrys (H).
- In circuit diagrams, an inductor is denoted by the symbol shown below.

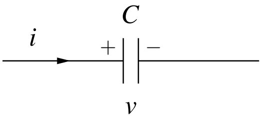


Capacitors

- A **capacitor** is a circuit element that stores electric charge.
- A capacitor uses the energy stored in an electric field in order to ***oppose changes in voltage*** (across the capacitor).
- A capacitor is characterized by an equation of the form

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \quad \text{(or equivalently, } i(t) = C\tfrac{d}{dt}v(t)\text{)},$$

  where C is a nonnegative real constant, and v and i respectively denote the voltage across and current through the capacitor as a function of time.
- As a matter of terminology, the quantity C is known as the **capacitance** of the capacitor.
- Capacitance is measured in units of farads (F).
- In circuit diagrams, a capacitor is denoted by the symbol shown below.



- The Laplace transform is a very useful tool for circuit analysis.
- The utility of the Laplace transform is partly due to the fact that the ***differential/integral*** equations that describe inductors and capacitors are much simpler to express in the Laplace domain than in the time domain.
- Let v and i denote the voltage across and current through a circuit element, and let V and I denote the Laplace transforms of v and i, respectively.
- In the Laplace domain, the equations characterizing a resistor, an inductor, and a capacitor respectively become:

$$V(s) = RI(s) \quad \text{(or equivalently, } I(s) = \tfrac{1}{R}V(s)\text{);}$$
$$V(s) = sLI(s) \quad \text{(or equivalently, } I(s) = \tfrac{1}{sL}V(s)\text{);} \quad \text{and}$$
$$V(s) = \tfrac{1}{sC}I(s) \quad \text{(or equivalently, } I(s) = sCV(s)\text{).}$$

- Note the absence of differentiation and integration in the above equations for an inductor and a capacitor.

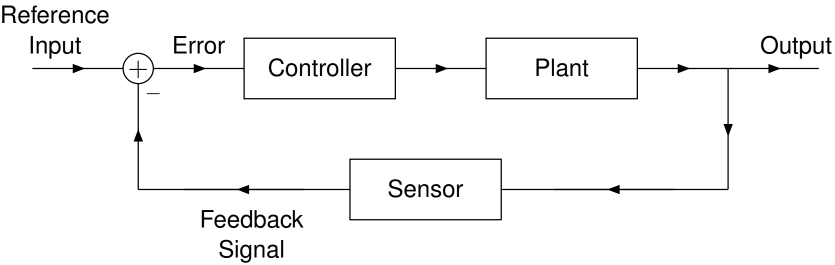
Example 7.38

Section 7.7

Application: Design and Analysis of Control Systems

Control Systems

- A control system manages the behavior of one or more other systems with some specific goal.
- Typically, the goal is to force one or more physical quantities to assume particular desired values, where such quantities might include: positions, velocities, accelerations, forces, torques, temperatures, or pressures.
- The ***desired*** values of the quantities being controlled are collectively viewed as the ***input*** of the control system.
- The ***actual*** values of the quantities being controlled are collectively viewed as the ***output*** of the control system.
- A control system whose behavior is not influenced by the actual values of the quantities being controlled is called an **open loop** (or **non-feedback**) system.
- A control system whose behavior is influenced by the actual values of the quantities being controlled is called a **closed loop** (or **feedback**) system.
- An example of a simple control system would be a thermostat system, which controls the temperature in a room or building.



- **input**: ***desired value*** of the quantity to be controlled
- **output**: ***actual value*** of the quantity to be controlled
- **error**: ***difference*** between the desired and actual values
- **plant**: system to be controlled
- **sensor**: device used to measure the actual output
- **controller**: device that monitors the error and changes the input of the plant with the goal of forcing the error to zero

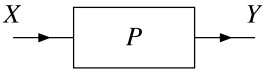
Stability Analysis of Feedback Systems

- Often, we want to ensure that a system is BIBO stable.
- The BIBO stability property is more easily characterized in the Laplace domain than in the time domain.
- Therefore, the Laplace domain is extremely useful for the stability analysis of systems.

Example 7.40 Exercise 7.30

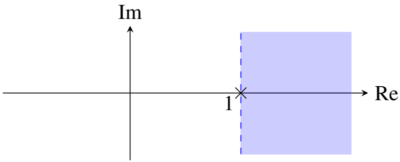
Stabilization Example: Unstable Plant

- causal LTI plant:



$$P(s) = \frac{10}{s-1}$$

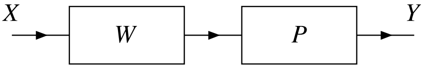
- ROC of P:



- system is not BIBO stable

Stabilization Example: Using Pole-Zero Cancellation

- system formed by series interconnection of plant and causal LTI compensator:

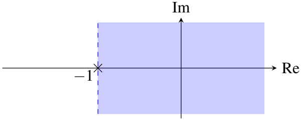


$$P(s) = \frac{10}{s-1}, \quad W(s) = \frac{s-1}{10(s+1)}$$

- system function H of overall system:

$$H(s) = W(s)P(s) = \left(\frac{s-1}{10(s+1)}\right)\left(\frac{10}{s-1}\right) = \frac{1}{s+1}$$

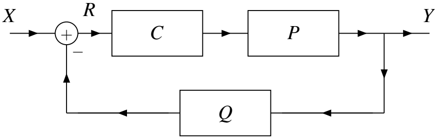
- ROC of H:



- overall system is BIBO stable

Stabilization Example: Using Feedback (1)

- feedback system (with causal LTI compensator and sensor):

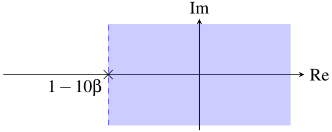


$$P(s) = \frac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$

- system function H of feedback system:

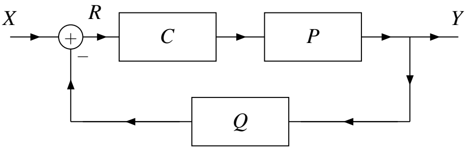
$$H(s) = \frac{C(s)P(s)}{1+C(s)P(s)Q(s)} = \frac{10\beta}{s-(1-10\beta)}$$

- ROC of H:



- feedback system is BIBO stable if and only if $1-10\beta < 0$ or equivalently $\beta > \frac{1}{10}$

Stabilization Example: Using Feedback (2)



$$R(s) = X(s) - Q(s)Y(s)$$
$$Y(s) = C(s)P(s)R(s)$$

$$\begin{aligned} Y(s) &= C(s)P(s)R(s) \\ &= C(s)P(s)[X(s) - Q(s)Y(s)] \\ &= C(s)P(s)X(s) - C(s)P(s)Q(s)Y(s) \end{aligned}$$

$$[1 + C(s)P(s)Q(s)]Y(s) = C(s)P(s)X(s)$$
$$H(s) = \frac{Y(s)}{X(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)Q(s)}$$

Stabilization Example: Using Feedback (3)

$$P(s) = \tfrac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$

$$\begin{aligned}
H(s) &= \frac{C(s)P(s)}{1+C(s)P(s)Q(s)} \\
&= \frac{\beta(\frac{10}{s-1})}{1+\beta(\frac{10}{s-1})(1)} \\
&= \frac{10\beta}{s-1+10\beta} \\
&= \frac{10\beta}{s-(1-10\beta)}
\end{aligned}$$

Remarks on Stabilization Via Pole-Zero Cancellation

- Pole-zero cancellation is not achievable in practice, and therefore it cannot be used to stabilize real-world systems.
- The theoretical models used to represent real-world systems are only approximations due to many factors, including the following:
  - Determining the system function of a system involves measurement, which always has some error.
  - A system cannot be built with such precision that it will have exactly some prescribed system function.
  - The system function of most systems will vary at least slightly with changes in the physical environment.
  - Although a LTI model is used to represent a system, the likely reality is that the system is not exactly LTI, which introduces error.
- Due to approximation error, the effective poles and zeros of the system function will only be approximately where they are expected to be.
- Since pole-zero cancellation requires that a pole and zero be placed at exactly the same location, any error will prevent this cancellation from being achieved.

Section 7.8

Unilateral Laplace Transform

- The **unilateral Laplace transform** of the function x, denoted $\mathcal{L}_{\mathrm{u}}x$ or X, is defined as

$$\mathcal{L}_{\mathrm{u}}x(s) = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt.$$

- The unilateral Laplace transform is related to the bilateral Laplace transform as follows:

$$\mathcal{L}_{\mathrm{u}}x(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt = \int_{-\infty}^{\infty} x(t)u(t)e^{-st}dt = \mathcal{L}\{xu\}(s).$$

- In other words, the unilateral Laplace transform of the function x is simply the bilateral Laplace transform of the function xu.
- Since $\mathcal{L}_{\mathrm{u}}x = \mathcal{L}\{xu\}$ and xu is always a ***right-sided*** function, the ROC associated with $\mathcal{L}_{\mathrm{u}}x$ is always either a ***RHP*** or the ***entire complex plane***.
- For this reason, we often ***do not explicitly indicate the ROC*** when working with the unilateral Laplace transform.

- With the unilateral Laplace transform, the same inverse transform equation is used as in the bilateral case.
- The unilateral Laplace transform is ***only invertible for causal functions***.
- In particular, we have

$$\begin{aligned}\mathcal{L}_{\mathrm{u}}^{-1}\{\mathcal{L}_{\mathrm{u}}x\}(t) &= \mathcal{L}_{\mathrm{u}}^{-1}\{\mathcal{L}\{xu\}\}(t) \\ &= \mathcal{L}^{-1}\{\mathcal{L}\{xu\}\}(t) \\ &= x(t)u(t) \\ &= \begin{cases} x(t) & t \geq 0 \\ 0 & t < 0. \end{cases}\end{aligned}$$

- For a noncausal function x, we can only recover $x(t)$ for $t \geq 0$.

Unilateral Versus Bilateral Laplace Transform

- Due to the close relationship between the unilateral and bilateral Laplace transforms, these two transforms have some similarities in their properties.
- Since these two transforms are not identical, however, their properties differ in some cases, often in subtle ways.
- In the unilateral case, we have that:
  1. the time-domain convolution property has the additional requirement that the functions being convolved must be ***causal***;
  2. the time/Laplace-domain scaling property has the additional constraint that the scaling factor must be ***positive***;
  3. the time-domain differentiation property has an ***extra term*** in the expression for $\mathcal{L}_{\mathrm{u}}\{\mathcal{D}x\}(t)$, where $\mathcal{D}$ denotes the derivative operator (namely, $-x(0^-)$);
  4. the time-domain integration property has a ***different lower limit*** in the time-domain integral (namely, 0^- instead of $-\infty$); and
  5. the time-domain shifting property ***does not hold*** (except in special circumstances).

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$
Time/Laplace-Domain Scaling	$x(at)$, $a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1 * x_2(t)$, x_1 and x_2 are causal	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)-x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_{0^-}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property	
Initial Value Theorem	$x(0^+)=\lim_{s\to\infty} sX(s)$
Final Value Theorem	$\lim_{t\to\infty} x(t)=\lim_{s\to 0} sX(s)$

Unilateral Laplace Transform Pairs

Pair	$x(t),\ t \geq 0$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$
7	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
8	$e^{-at}\cos(\omega_0 t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
9	$e^{-at}\sin(\omega_0 t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$

- Many systems of interest in engineering applications can be characterized by constant-coefficient linear differential equations.
- One common use of the unilateral Laplace transform is in solving constant-coefficient linear differential equations with nonzero initial conditions.

Part 8

Complex Analysis

- A **complex number** is a number of the form $z = x + jy$ where x and y are real numbers and j is the constant defined by $j^2 = -1$ (i.e., $j = \sqrt{-1}$).
- The **Cartesian form** of the complex number z expresses z in the form

$$z = x + jy,$$

where x and y are real numbers. The quantities x and y are called the **real part** and **imaginary part** of z, and are denoted as $\operatorname{Re} z$ and $\operatorname{Im} z$, respectively.

- The **polar form** of the complex number z expresses z in the form

$$z = r(\cos\theta + j\sin\theta) \quad \text{or equivalently} \quad z = re^{j\theta},$$

where r and θ are real numbers and $r \geq 0$. The quantities r and θ are called the **magnitude** and **argument** of z, and are denoted as $|z|$ and $\arg z$, respectively. [Note: $e^{j\theta} = \cos\theta + j\sin\theta$.]

- Since $e^{j\theta} = e^{j(\theta+2\pi k)}$ for all real θ and all integer k, the argument of a complex number is only uniquely determined to within an additive multiple of 2π.
- The **principal argument** of a complex number z, denoted $\operatorname{Arg} z$, is the particular value θ of $\arg z$ that satisfies $-\pi < \theta \leq \pi$.
- The principal argument of a complex number (excluding zero) is ***unique***.