

Exercise 5.10

R Answer (c).

Let ω_0 denote the fundamental frequency of x . So, $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$. We have

$$\begin{aligned}
 c_k &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk\omega_0 t} dt = \frac{1}{4} \int_{-2}^2 e^{-2|t|} e^{-jk(\pi/2)t} dt \\
 &= \frac{1}{4} \left[\int_{-2}^0 e^{2t} e^{-jk(\pi/2)t} dt + \int_0^2 e^{-2t} e^{-jk(\pi/2)t} dt \right] = \frac{1}{4} \left[\int_{-2}^0 e^{(2-jk\pi/2)t} dt + \int_0^2 e^{(-2-jk\pi/2)t} dt \right] \\
 &= \frac{1}{4} \left(\left[\frac{e^{(2-jk\pi/2)t}}{2 - \frac{j\pi}{2}k} \right] \Big|_{-2}^0 + \left[\frac{e^{(-2-jk\pi/2)t}}{-2 - \frac{j\pi}{2}k} \right] \Big|_0^2 \right) \\
 &= \frac{1}{4} \left[\left(\frac{1}{2 - \frac{j\pi}{2}k} \right) (1 - e^{(2-jk\pi/2)(-2)}) + \left(\frac{1}{-2 - \frac{j\pi}{2}k} \right) (e^{(-2-jk\pi/2)(2)} - 1) \right] \\
 &= \frac{1}{4} \left[\left(\frac{1}{2 - \frac{j\pi}{2}k} \right) (1 - e^{-4+jk\pi}) + \left(\frac{1}{-2 - \frac{j\pi}{2}k} \right) (e^{-4-jk\pi} - 1) \right] \\
 &= \frac{1}{4} \left[\frac{1 - e^{-4}(-1)^k}{2 - \frac{j\pi}{2}k} + \frac{1 - e^{-4}(-1)^k}{2 + \frac{j\pi}{2}k} \right] = \frac{1}{4} [1 - e^{-4}(-1)^k] \left[\frac{1}{2 - \frac{j\pi}{2}k} + \frac{1}{2 + \frac{j\pi}{2}k} \right] \\
 &= \frac{1}{4} [1 - e^{-4}(-1)^k] \left[\frac{2 + \frac{j\pi}{2}k + 2 - \frac{j\pi}{2}k}{4 + \frac{\pi^2}{4}k^2} \right] = \frac{1 - e^{-4}(-1)^k}{4 + \frac{\pi^2}{4}k^2} \\
 &= \frac{4 [1 - e^{-4}(-1)^k]}{16 + \pi^2 k^2}.
 \end{aligned}$$

Exercise 5.11

R Answer (b).

Clearly, x is periodic with period $T = \frac{1}{2}$. So, x has the Fourier series representation

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk(2\pi/T)t}$$

From the Fourier series analysis equation, we have

$$\begin{aligned} c_k &= \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk(2\pi/T)t} dt \\ &= \frac{1}{1/2} \int_{-1/4}^{1/4} \delta(t) e^{-jk4\pi t} dt \\ &= 2 \int_{-\infty}^{\infty} \delta(t) e^{-jk4\pi t} dt \\ &= 2 \left[e^{-jk4\pi t} \right] \Big|_{t=0} \\ &= 2. \end{aligned}$$

Since the system is LTI, we have

$$\begin{aligned} y(t) &= \sum_{k=-\infty}^{\infty} H\left(\frac{2\pi}{T}k\right) c_k e^{jk(2\pi/T)t} \\ &= \sum_{k=-\infty}^{\infty} H(4\pi k) c_k e^{j4\pi k t} \\ &= H(-4\pi) c_{-1} e^{j4\pi(-1)t} + H(0) c_0 + H(4\pi) c_1 e^{j4\pi(1)t} \\ &= (1)(2) e^{-j4\pi t} + (1)(2) + (1)(2) e^{j4\pi t} \\ &= 2e^{-j4\pi t} + 2e^{j4\pi t} + 2 \\ &= 2(e^{j4\pi t} + e^{-j4\pi t}) + 2 \\ &= 4\cos(4\pi t) + 2. \end{aligned}$$

$$\text{rect}(t) = \begin{cases} 1 & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

Exercise 5.12

R Answer (b).

To begin, we observe that the function x satisfies the Dirichlet conditions. Therefore, at each point t_a of discontinuity of x , we have $y(t_a) = \frac{1}{2} [x(t_a^-) + x(t_a^+)]$. Thus, we have

$$\begin{aligned} y(0) &= \frac{1}{2} [x(0^-) + x(0^+)] \\ &= \frac{1}{2} [-25 + 1] \\ &= \frac{1}{2} [-24] \\ &= -12 \quad \text{and} \\ y(2) &= \frac{1}{2} [x(2^-) + x(2^+)] \\ &= \frac{1}{2} [e^2 + (-2^2)] \\ &= \frac{1}{2} [e^2 - 4] \\ &= \frac{e^2 - 4}{2}. \end{aligned}$$

Exercise 5.13

R Answer (b).

We are given

$$c_k = \frac{4jk + 4}{(jk - 1)^2} \quad \text{and} \quad T = 4.$$

First, we compute the magnitude spectrum of x . We have

$$\begin{aligned} |c_k| &= \left| \frac{4jk + 4}{(jk - 1)^2} \right| = \frac{|4jk + 4|}{|(jk - 1)^2|} = \frac{4|jk + 1|}{(\sqrt{k^2 + 1})^2} = \frac{4\sqrt{k^2 + 1}}{(\sqrt{k^2 + 1})^2} \\ &= \frac{4}{\sqrt{k^2 + 1}}. \end{aligned}$$

Next, we compute the phase spectrum of x . We have

$$\begin{aligned} \arg c_k &= \arg \left[\frac{4jk + 4}{(jk - 1)^2} \right] = \arg(4jk + 4) - \arg[(jk - 1)^2] \\ &= \arctan\left(\frac{4k}{4}\right) - 2 \left[\arctan\left(\frac{k}{-1}\right) + \pi \right] \\ &= \arctan(k) - 2 \arctan(-k) - 2\pi = \arctan(k) + 2 \arctan(k) - 2\pi \\ &= 3 \arctan(k) - 2\pi. \end{aligned}$$

Since the argument is not uniquely determined, in the most general case, we have

$$\arg c_k = 3 \arctan(k) + 2\pi\ell$$

for all integer ℓ .

Exercise 5.14

R Answer (d).

We are given the Fourier series coefficient sequence c , where

$$c_k = j \operatorname{sgn}(k) e^{-|3k|}.$$

To begin, we observe that sgn is an odd function. So, we have

$$\begin{aligned} c_{-k} &= j \operatorname{sgn}(-k) e^{-|-3k|} \\ &= j[-\operatorname{sgn}(k)] e^{-|3k|} \\ &= -j \operatorname{sgn}(k) e^{-|3k|} \\ &= -c_k. \end{aligned}$$

Therefore, c is odd. (Or, alternatively, the sequence c is the product of the odd sequence $v_1(k) = \operatorname{sgn}(k)$ and the even sequence $v_2(k) = j e^{-|3k|}$. Since the product of an odd sequence and an even sequence is an odd sequence, c is odd.) Since c is purely imaginary and the conjugate of a purely imaginary number is its negative, c being odd implies that c is conjugate symmetric. Since c is conjugate symmetric, x is real. Therefore, we conclude that x is real and odd.