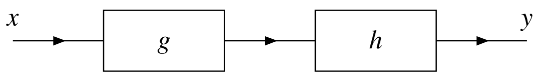
Example 6.39 (Communication channel equalization)**.** Consider a LTI communication channel with frequency response

$$H(\omega) = \tfrac{1}{3+j\omega}.$$

Unfortunately, this channel has the undesirable effect of attenuating higher frequencies. Find the frequency response G of an equalizer that when connected in series with the communication channel yields an ideal (i.e., distortionless) channel. The new system with equalization is shown in Figure 6.24, where g and h denote the inverse Fourier transforms of G and H, respectively.

distortionless channel



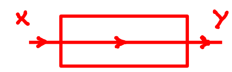


Figure 6.24: System from example that employs equalization.

Solution. An ideal communication channel has a frequency response equal to one for all frequencies. Consequently, we want $H(\omega)G(\omega) = 1$ or equivalently $G(\omega) = 1/H(\omega)$. Thus, we conclude that

rearrange

$$\begin{aligned} G(\omega) &= \frac{1}{H(\omega)} \\ &= \frac{1}{\left(\frac{1}{3+j\omega}\right)} \\ &= 3 + j\omega. \end{aligned}$$

substitute given H

simplify

■

Unit:
Partial Fraction Expansions

Example B.1 (Simple pole)**.** Find the partial fraction expansion of the function

$$f(z) = \frac{3}{z^2+3z+2}.$$

strictly proper

Solution. First, we rewrite f with the denominator polynomial factored to obtain

$$f(z) = \frac{3}{(z+1)(z+2)}.$$

simple (i.e., 1st order) poles at −1 and −2

From this, we know that f has a partial fraction expansion of the form

$$f(z) = \frac{A_1}{z+1} + \frac{A_2}{z+2}, \quad (1)$$

where A_1 and A_2 are constants to be determined. Now, we calculate A_1 and A_2 as follows:

$$\left.\begin{aligned}
A_1 &= (z+1)f(z)|_{z=-1} \\
&= \left.\frac{3}{z+2}\right|_{z=-1} \\
&= 3 \quad \text{and} \\
A_2 &= (z+2)f(z)|_{z=-2} \\
&= \left.\frac{3}{z+1}\right|_{z=-2} \\
&= -3.
\end{aligned}\right\} (2)$$

Thus, the partial fraction expansion of f is given by

from (1) and (2)

$$f(z) = \frac{3}{z+1} - \frac{3}{z+2}.$$

■

Example B.2 (Repeated pole). Find the partial fraction expansion of the function

$$f(z) = \frac{4z+8}{(z+1)^2(z+3)}.$$

strictly proper with 2nd order pole at −1 and 1st order pole at −3

Solution. Since f has a repeated pole, we know that f has a partial fraction expansion of the form

terms contributed by pole at −1

$$f(z) = \boxed{\frac{A_{1,1}}{z+1} + \frac{A_{1,2}}{(z+1)^2}} + \boxed{\frac{A_{2,1}}{z+3}}. \quad (1)$$

term contributed by pole at −3

where $A_{1,1}$, $A_{1,2}$, and $A_{2,1}$ are constants to be determined. To calculate these constants, we proceed as follows:

coefficient number

pole order

$$\begin{aligned}
A_{1,1} &= \frac{1}{(2-1)!}\left[\left(\tfrac{d}{dz}\right)^{2-1}\left[(z+1)^2 f(z)\right]\right]\Big|_{z=-1} \\
&= \frac{1}{1!}\left[\tfrac{d}{dz}\left[(z+1)^2 f(z)\right]\right]\Big|_{z=-1} \\
&= \left[\tfrac{d}{dz}\left(\frac{4z+8}{z+3}\right)\right]\Big|_{z=-1} \\
&= \left[4(z+3)^{-1} + (-1)(z+3)^{-2}(4z+8)\right]\Big|_{z=-1} \\
&= \left[\frac{4}{(z+3)^2}\right]\Big|_{z=-1} \\
&= \tfrac{4}{4} \\
&= 1,
\end{aligned}$$

formula for case of repeated pole

substitute for f

differentiate

$$\begin{aligned}
A_{1,2} &= \frac{1}{(2-2)!}\left[\left(\tfrac{d}{dz}\right)^{2-2}\left[(z+1)^2 f(z)\right]\right]\Big|_{z=-1} \\
&= \frac{1}{0!}\left[(z+1)^2 f(z)\right]\Big|_{z=-1} \\
&= \left[\frac{4z+8}{z+3}\right]\Big|_{z=-1} \\
&= \tfrac{4}{2} \\
&= 2, \quad \text{and}
\end{aligned}$$

formula for case of repeated pole

$$\begin{aligned}
A_{2,1} &= (z+3)f(z)\big|_{z=-3} \\
&= \frac{4z+8}{(z+1)^2}\Big|_{z=-3} \\
&= \tfrac{-4}{4} \\
&= -1.
\end{aligned}$$

formula for case of simple pole

substitute for f

Thus, the partial fraction expansion of f is given by

$$f(z) = \frac{1}{z+1} + \frac{2}{(z+1)^2} - \frac{1}{z+3}. \qquad \blacksquare$$

substitute computed coefficients into (1)