

# Laplace Transform Table and Properties

$$F(s) = \int_0^{\infty} f(t)e^{-st} dt$$

Property/Rule	Mathematical Expression
<b>Linearity</b>	$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bG(s)$
<b>Shifting in Time (t-translation)</b>	$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$
<b>s-shift (Frequency Shift)</b>	$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$
<b>Differentiation (First Derivative)</b>	$\mathcal{L}\{f'(t)\} = sF(s) - f(0^-)$
<b>Differentiation (Second Derivative)</b>	$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0^-) - f'(0^-)$
<b>Differentiation (nth Derivative)</b>	$\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0^-) - \dots - f^{(n-1)}(0^-)$
<b>t-Multiplication</b>	$\mathcal{L}\{tf(t)\} = -F'(s)$
<b>t-Multiplication (nth power)</b>	$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$
<b>Integration of f(t)</b>	$\mathcal{L}\left\{\int_0^t f(\tau) d\tau\right\} = \frac{F(s)}{s}$
<b>Convolution</b>	$\mathcal{L}\{(f*g)(t)\} = F(s)G(s)$ , where $(f*g)(t) = \int_0^t f(\tau)g(t-\tau) d\tau$
<b>Integration with <math>\frac{f(t)}{t}</math></b>	$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(\sigma) d\sigma$

Function $f(t)$	Transform $F(s)$	ROC
1	$\frac{1}{s}$	$\Re(s) > 0$
$e^{at}$	$\frac{1}{s-a}$	$\Re(s) > \Re(a)$
$t$	$\frac{1}{s^2}$	$\Re(s) > 0$
$t^n$	$\frac{n!}{s^{n+1}}$	$\Re(s) > 0$
$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$	$\Re(s) > 0$
$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$	$\Re(s) > 0$
$e^{at} \cos(\omega t)$	$\frac{s-a}{(s-a)^2 + \omega^2}$	$\Re(s) > \Re(a)$
$e^{at} \sin(\omega t)$	$\frac{\omega}{(s-a)^2 + \omega^2}$	$\Re(s) > \Re(a)$
$\delta(t)$	1	All $s$
$\delta(t-a)$	$e^{-as}$	All $s$
$\cosh(kt)$	$\frac{s}{s^2 - k^2}$	$\Re(s) >  k $
$\sinh(kt)$	$\frac{k}{s^2 - k^2}$	$\Re(s) >  k $
$\frac{1}{2\omega^3} (\sin(\omega t) - \omega t \cos(\omega t))$	$\frac{1}{(s^2 + \omega^2)^2}$	$\Re(s) > 0$
$\frac{t}{2\omega} \sin(\omega t)$	$\frac{s}{(s^2 + \omega^2)^2}$	$\Re(s) > 0$
$\frac{1}{2\omega} (\sin(\omega t) + \omega t \cos(\omega t))$	$\frac{s^2}{(s^2 + \omega^2)^2}$	$\Re(s) > 0$
$u(t-a)$	$\frac{e^{-as}}{s}$	$\Re(s) > 0$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	$\Re(s) > \Re(a)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$\Re(s) > 0$
$t^a$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$\Re(s) > 0$
$\cos(\omega t) - \omega t \sin(\omega t)$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	$\Re(s) > 0$