ELEC 260

MIDTERM EXAM 1

SOLUTIONS

(FALL 2012)

PROBLEM 1A

$$F(s) = \frac{s^2+5s+6}{s^4+8s^3+7s^2} = \frac{s^2+5s+6}{s^2(s^2+8s+7)}$$

$$= \frac{(s+2)(s+3)}{s^2(s+1)(s+7)}$$

zeros:

location	order
-3	1
-2	1

poles:

location	order
-7	1
-1	1
0	2

PROBLEM 1B

$$H(w) = \frac{1}{(jw-1)^{10}}$$

$$\arg H(w) = \arg \frac{1}{(jw-1)^{10}}$$

$$= \arg 1 - \arg (jw-1)^{10}$$

$$= 0 - \arg\left\{\left[\sqrt{w^2+1}\, e^{j[\arctan(-w/1)+\pi]}\right]^{10}\right\}$$

$$= -\arg\left\{\left[\sqrt{w^2+1}\right]^{10} e^{j10[\arctan(-w)+\pi]}\right\}$$

$$= -10\left[\arctan(-w)+\pi\right]$$

$$= -10\pi - 10\arctan(-w)$$

$$= -10\pi + 10\arctan w$$

PROBLEM 2

$$x(t) = \begin{cases} e^t & t<0 \\ 1-t^2 & 0 \le t < 1 \\ \sin \pi t & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases}$$

$$x(t) = e^t u(-t) + [1-t^2][u(t)-u(t-1)] + [\sin \pi t][u(t-1)-u(t-2)]$$

$$= e^t u(-t) + u(t) - u(t-1) - t^2 u(t) + t^2 u(t-1) + [\sin \pi t]u(t-1) - [\sin \pi t]u(t-2)$$

$$= e^t u(-t) + [1-t^2]u(t) + [-1+t^2+\sin \pi t]u(t-1) - [\sin \pi t]u(t-2)$$

PROBLEM 3

$$y(t) = \int_{-\infty}^{t} e^{-(2t-\tau)} x(\tau-3)\, d\tau$$

If $x(t) = \delta(t)$, then $y(t) = h(t)$.

$$h(t) = \int_{-\infty}^{t} e^{-(2t-\tau)} \delta(\tau-3)\, d\tau$$

$$= \int_{-\infty}^{t} e^{-(2t-3)} \delta(\tau-3)\, d\tau$$

$$= e^{-(2t-3)} \int_{-\infty}^{t} \delta(\tau-3)\, d\tau$$

$$= e^{-(2t-3)} u(t-3)$$

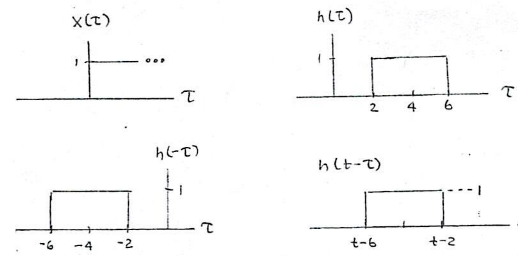
Note that

$$\int_{-\infty}^{t} \delta(\tau-3)\, d\tau = \begin{cases} 1 & t \geq 3 \\ 0 & \text{otherwise} \end{cases}$$

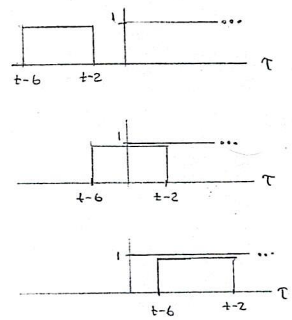
$$= u(t-3)$$

PROBLEM 4

(a)



(b,c)



$t-2 < 0 \Rightarrow \underline{t<2}$

$x * h(t) = 0$

$t-6 < 0$ and $t-2 \geq 0 \Rightarrow$

$t < 6$ and $t \geq 2 \Rightarrow \underline{2 \leq t < 6}$

$x * h(t) = \int_{0}^{t-2} (1)(1)\, d\tau$

$t-6 \geq 0 \Rightarrow \underline{t \geq 6}$

$x * h(t) = \int_{t-6}^{t-2} (1)(1)\, d\tau$

PROBLEM 5

(a) Let $x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y_3(t)$

If for all $a_1, a_2 \in \mathbb{C}$ and all x_1, x_2 the following condition holds, then the system is linear:

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$$

(b)

$$y_1(t) = x_1(t^2)$$

$$y_2(t) = x_2(t^2)$$

$$y_3(t) = [a_1 x_1(\lambda) + a_2 x_2(\lambda)]\Big|_{\lambda=t^2}$$

$$= a_1 x_1(t^2) + a_2 x_2(t^2)$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

Therefore, the system is linear.

PROBLEM 6

```
function y = prodOfNonzero(x)

numRows = size(x, 1);
numCols = size(x, 2);

y = 1;

for row = 1 : numRows
    for col = 1 : numCols
        if x(row, col) ~= 0
            y = y * x(row, col);
        end
    end
end
```

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PROBLEM 1.

(A) Let $F(s) = \dfrac{s^2+5s+6}{s^4+8s^3+7s^2}$, where s is complex. Find the poles and zeros of the function $F(s)$ and determine the order of each pole and zero. **[3 marks]**

ZEROS: $s^2 + 5s + 6$
$= (s+3)(s+2)$

$$\boxed{F(s) = \frac{(s+3)(s+2)}{s^2(s-1)(s-7)}}$$

POLES: $s^4 + 8s^3 + 7s^2 = s^2(s^2 + 8s + 7)$

$z = \dfrac{-8 \pm \sqrt{64-28}}{2}$ (36)

$= \dfrac{-8+6}{2}$ & $\dfrac{-8-6}{2}$

DERP!!

∴ Function F(s) has 1ST order zeros at −2 & −3. ✓ Has 1ST order poles at 1 & 7 as well as 2ND order pole at 0. ✓

✓

$\frac{3}{3}$

(B) Let $H(\omega) = \dfrac{1}{(j\omega-1)^{10}}$, where ω is real. Find a fully simplified expression for $\arg H(\omega)$. **[2 marks]**

$\frac{0}{2}$

$H(\omega) = (j\omega - 1)^{-10}$

$= \left[e^{j(\omega-1)}\right]^{-10}$

$= \left(\cos(\omega-1) + j\sin(\omega-1)\right)^{-10}$

X

$\arg H(\omega) = \arctan\left(\dfrac{\sin(\omega-1)}{\cos(\omega-1)}\right)$

PROBLEM 2. Suppose that we have the signal $x(t)$ given by

$$x(t) = \begin{cases} e^t & t < 0 \\ 1 - t^2 & 0 \le t < 1 \\ \sin \pi t & 1 \le t < 2 \\ 0 & t \ge 2. \end{cases}$$

Use unit-step functions to find a single expression for $x(t)$ that is valid for all t. When stating your final answer, you must group together terms having the same unit-step function factor. **[3 marks]**

$v_1(t) = e^t [u(t)]$ ← ONLY MISTAKE

$v_2(t) = (1-t^2)(u(t) - u(t-1)) = u(t) - t^2u(t) - u(t-1) + t^2u(t-1)$

$v_3(t) = (\sin \pi t)[u(t-1) - u(t-2)] = (\sin \pi t)u(t-1) - (\sin \pi t)u(t-2)$

0/3

$V_4(t) = \varnothing$

$x(t) = v_1(t) + v_2(t) + v_3(t) + \cancel{v_4(t)}^{0}$

$= e^t u(t) + u(t) - t^2u(t) - u(t-1) + t^2u(t-1) + (\sin \pi t)u(t-1) - (\sin \pi t)u(t-2)$

$$\boxed{x(t) = u(t)[e^t + 1 - t^2] + u(t-1)[t^2 + \sin \pi t - 1] + u(t-2)[\sin \pi t]}$$

PROBLEM 3. A LTI system with input $x(t)$ and output $y(t)$ is characterized by the equation $y(t) = \int_{-\infty}^{t} e^{-(2t-\tau)} x(\tau - 3) d\tau$. Find the impulse response $h(t)$ of the system. **[3 marks]**

$h(t) =$

$$y(t) = \int_{-\infty}^{t} e^{-(2t-\tau)} x(\tau-3) d\tau$$

$$\neq \int_{-\infty}^{t} e^{-(2t-3)} x(\tau-3) d\tau$$

$$u(t) = \begin{cases} 1 & t>3 \\ 0 & \text{otherwis} \end{cases}$$

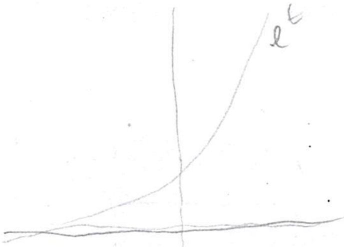
$$= \int_{3}^{\infty} e^{-(2t-3)} d\tau$$

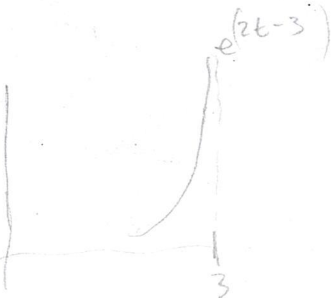
$$= e^{-(2t-3)} \Big|_{3}^{\infty}$$

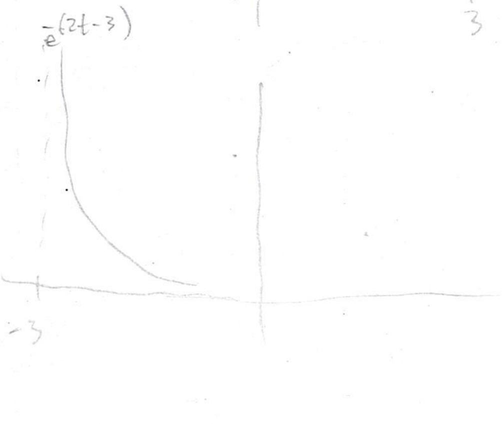
$$= \cancel{e^{-\infty}} - e^{-3}$$

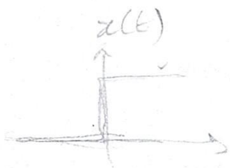
$$\boxed{h(t) = 3 - e^{-3}}$$

$\frac{0}{3}$





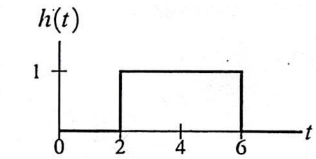




$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

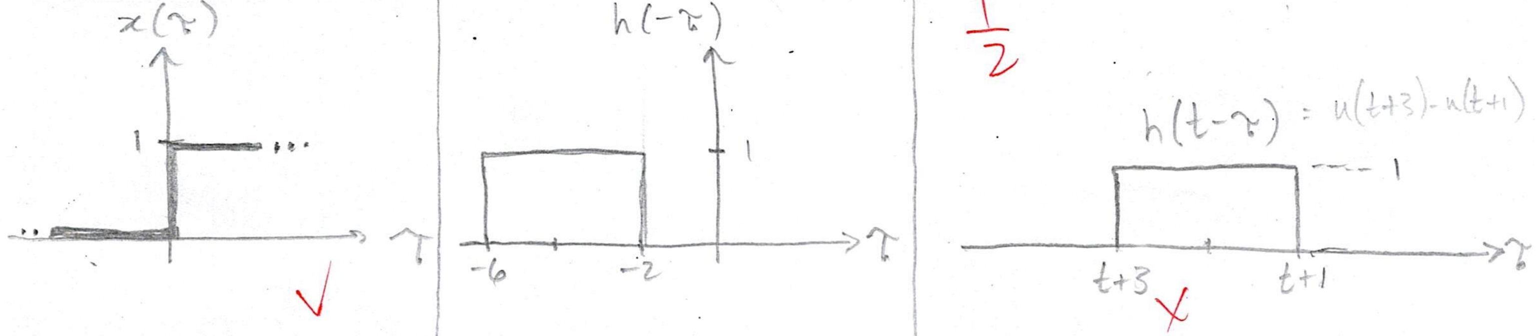
PROBLEM 4.

Using graphical methods, compute the convolution $y(t) = x(t) * h(t)$ where $\boxed{x(t) = u(t)}$ and $h(t)$ is as shown in the figure to the right. **[8 marks]**



(A) Plot $x(\tau)$ and $h(t-\tau)$ versus τ. Be very careful to plot these graphs correctly. If you make a mistake here, all of your subsequent work will be completely wrong, and you will lose a very substantial number of marks as a result. **[2 marks]**

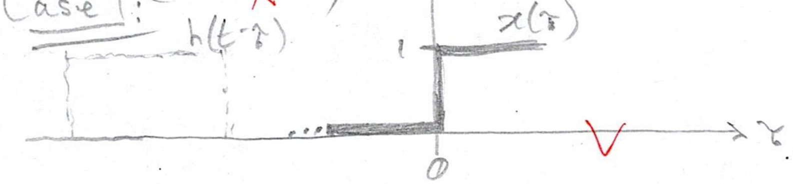
$\frac{1}{2}$



(B) For each of the cases (i.e., ranges for t) to be considered in the computation of the convolution result $y(t)$, carefully sketch and fully label the graph that includes both $x(\tau)$ and $h(t-\tau)$ plotted versus τ, and also indicate the corresponding range for t. **[3 marks]**

$\frac{1}{3}$

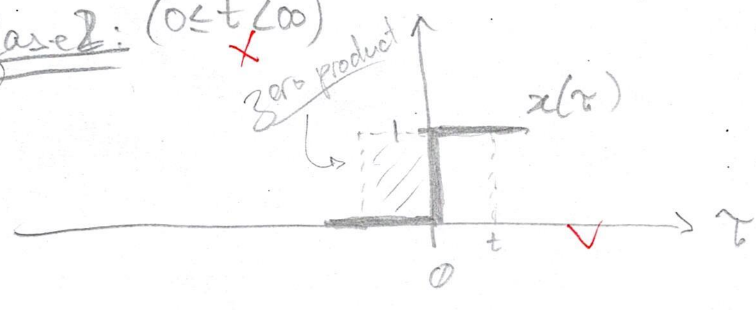
Case 1: $(-\infty \leq t < 0)$



$\int_{-\infty}^{0^-} x(\tau)\,h(t-\tau)\,d\tau = x(t) * h(t)$

$= 0$

Case 2: $(0 \leq t < \infty)$

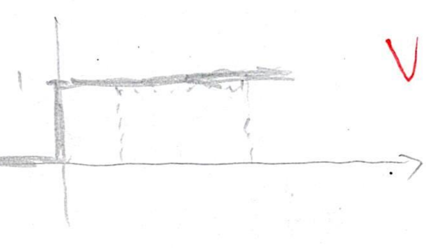


$\int_0^t x(\tau)\,h(t-\tau)\,d\tau +$

$\int_t^{\infty} x(\tau)\,h(t-\tau)\,d\tau$

Case 3: (is Same as Case 2) S.T.

$\int_0^{\infty} = \int_0^t + \int_t^{\infty}$



(C) Use the graphs from part (b) to determine the convolution result $y(t)$. You may state your final answer in terms of integrals, but the expressions appearing in the integrals must be simplified as much as possible without actually integrating. For example, you should not have any unit-step functions appearing in any of the integrals. **[3 marks]**

$$y(t) = x(t) * h(t) = \int x(\tau)\, h(t-\tau)\, d\tau$$

$\frac{0}{3}$

$$= \int_0^t \cancel{u(\tau)} \left[\cancel{u(\tau+3)} - \cancel{u(\tau+1)} \right] d\tau$$

$u(t) = \begin{cases} 1 & \forall t > 0 \\ 0 ; & \text{otherwise} \end{cases}$

$$= \tau \Big|_0^t \checkmark$$

$$= t$$

PROBLEM 5. Suppose that we have a system $\mathscr{H}$ with input $x(t)$ and output $y(t)$.

(A) Clearly state, in mathematical terms, the condition that must be satisfied in order for the system $\mathscr{H}$ to be linear. Be sure to define all quantities such as variables, functions, and constants. Otherwise, you will receive zero marks. Be careful with the notation that you choose to employ. If, for example, you confuse arrows and equal signs in your solution, you will probably receive zero marks. **[2 marks]**

$y(t) = a_1 x_1(t) + a_2 x_2(t) \ldots + a_n x_n(t)$

$\frac{0}{2}$

where
a_n is constant
$x_n(t)$ is continuous function
t is independent variable

(B) Suppose now that the system $\mathscr{H}$ is characterized by the equation $y(t) = x(t^2)$. Using the condition stated in part (a), determine whether this system is linear. **[2 marks]**

$\frac{0}{2}$

PROBLEM 6. Using the MATLAB programming language, write a function called `prodOfNonzero` that takes a matrix (containing at least one element) as input and returns the product of all of the nonzero elements in the matrix. For example, `prodOfNonzero([2 3 4])` returns 24 and `prodOfNonzero([2 3 0])` returns 6. Your code must not call any other functions, except **`size`**. Be sure to use correct syntax in your answer, since syntax clearly matters here. [Note that **`size`**`(x, 1)` and **`size`**`(x, 2)` return the number of rows and colums in x, respectively.] **[2 marks]**

$\frac{0}{2}$

END

This page was intentionally left blank to accommodate duplex printing.

Do not write on this page unless instructed to do so.