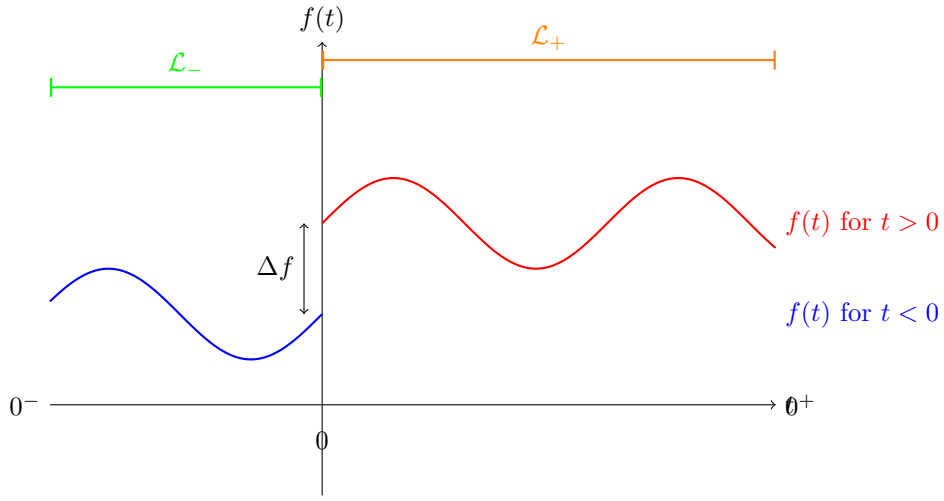
Comparison of $\mathcal{L}_+$ and $\mathcal{L}_-$ Laplace Transforms



$$\mathcal{L}_-[f(t)] = \mathcal{L}_+[f(t)] + \int_{0^-}^{0^+} f(t)e^{-st}dt$$
$$\mathcal{L}_-[f(t)] = \mathcal{L}_+[f(t)] \text{ iff } \int_{0^-}^{0^+} f(t)e^{-st}dt = 0$$

Explanation:

- The blue curve represents $f(t)$ for $t < 0$, and the red curve for $t > 0$.
- $\mathcal{L}_-$ includes the integral from 0^- to ∞ (green and orange regions).
- $\mathcal{L}_+$ only includes the integral from 0^+ to ∞ (orange region).
- The difference Δf at $t = 0$ contributes to the integral $\int_{0^-}^{0^+} f(t)e^{-st}dt$.
- If this integral is zero (i.e., no jump discontinuity at $t = 0$), then $\mathcal{L}_-[f(t)] = \mathcal{L}_+[f(t)]$.