UNIVERSITY OF VICTORIA

FINAL EXAMINATIONS –December 2023

ECE 360 – CONTROL THEORY AND SYSTEMS I

SECTION A01 CRN: 11032

TO BE ANSWERED IN BOOKLETS

DURATION: 3 hours

INSTRUCTOR: Dr. P. Agathoklis

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS 5 PAGES, INCLUDING THIS COVER PAGE AND AN ATTACHED FIGURE.

FOUR (4) PAGES OF HANDWRITTEN NOTES AND PHOTOCOPIES OF LAPLACE TRANSFORMS ARE PERMITTED.

DETACH PAGE 5 FROM THE EXAMINATION PAPER AND HAND IT IN WITH YOUR ANSWER BOOKLET.

(6) 4. a) Sketch the root-locus of a system with the following open-loop transfer function:

$$G(s) = \frac{K(s+1)}{(s^2-4s+13)}$$

b) Discuss the transient response of the closed loop system when K goes from 0 to infinity.

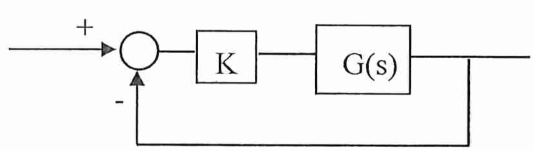
c) Discuss the steady state response of the closed loop system for unit step, unit ramp and unit parabola inputs.

(6) 5. Sketch the Bode and Nyquist plots of

i) $G_1(s) = \frac{1}{s(s+2)(s+5)}$

ii) $G_2(s) = \frac{s-1}{s^2(s+5)}$

(4) 6. Consider the Bode and Nyquist plots of the systems in question 5.
Determine the stability of the two closed loop systems given by:



where $G(s)$ is equal to $G_1(s)$ and $G_2(s)$ respectively

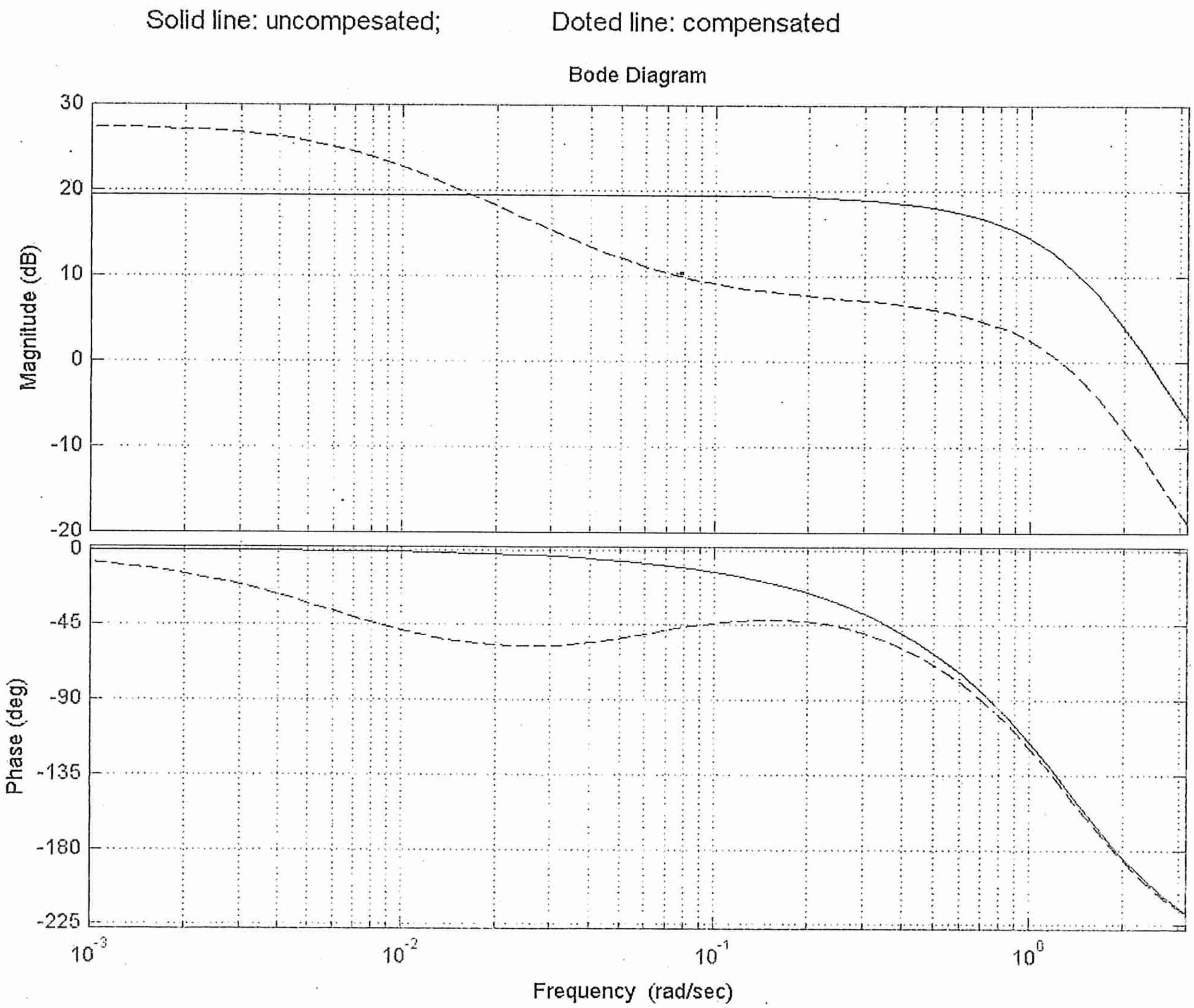
Justify your answers.

Name: ______________________________ **Student No.:** __________________

Figure for Question 7

Solid Line: System **without** compensator

Dashed Line: System **with** compensator



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(4) 1) $\ddot{x}(t)+6\dot{x}(t)+8x(t)=u(t)$

$u(t)=\begin{cases} e^{-2t} & 0\le t\le 1 \\ 0 & \end{cases}$

$X(s)=\dfrac{U(s)}{s^2+6s+8}$

$u(t)=e^{-2t}(u(t)-u(t-1))$

$=e^{-2t}u(t)-e^{-2}e^{-2(t-1)}u(t-1)$

$U(s)=\dfrac{1}{s+2}(1-\underbrace{e^{-2}e^{-s}}_{e^{-(s+2)}})$ (1)

$X(s)=\dfrac{(1-e^{-(s+2)})}{(s+2)(s+2)(s+4)}$ (1)

$A=\dfrac{1}{(s+2)^2}\Big|_{s=-4}=\dfrac{1}{2^2}=0.25$

$B_1=\dfrac{1}{s+4}\Big|_{s=-2}=\dfrac{1}{2}=0.5$ (1)

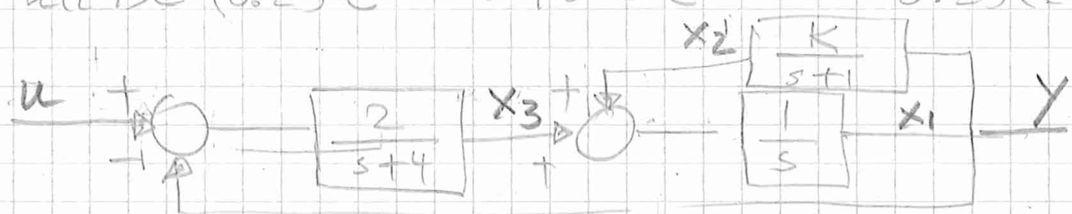
$B_2=\dfrac{d}{ds}\left(\dfrac{1}{s+4}\right)\Big|_{s=-2}=\left(\dfrac{-1}{(s+4)^2}\right)\Big|_{s=-2}=\dfrac{-1}{2^2}=-0.25$

$X(s)=\left(\dfrac{0.25}{s+4}+\dfrac{0.5}{s+2}-\dfrac{0.25}{(s+2)^2}\right)(1-e^{-(s+2)})$ (1)

$x(t)=\left(0.25e^{-4t}+0.5e^{-2t}-0.25te^{-2t}\right)u(t)$

$u(t-1)e^{-2}\left(0.25e^{-4(t-1)}+0.5e^{-2(t-1)}-0.25(t-1)e^{2(t-1)}\right)$

(5) 2)



$G_1(s)=\dfrac{\frac{1}{s}}{1-\frac{K}{s(s+1)}}=\dfrac{(s+1)}{(s^2+s-K)}$

$G(s)=\dfrac{\frac{2G_1(s)}{s+4}}{1+\frac{2G_1(s)}{s+4}}=\dfrac{2(s+1)}{s^3+s^2-Ks+4s^2+4s-4K+2s+2}$

$=\dfrac{2s+2}{s^3+\underset{a_1}{5}s^2+s\underset{a_2}{(6-K)}+\underset{a_3}{(2-4K)}}$

$b_3=b_2=2$
$b_1=b_0=0$

(3)

$\dfrac{2(s+1)}{(s+4)(s^2+s-K)+2(s+1)}$

2

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 0 & -(2-4K) \\ 1 & 0 & -(6-K) \\ 0 & 1 & -5 \end{bmatrix} \underline{x} + \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} u \quad Y = [0, 0, 1]\, \underline{X}$$

(2)

Other option

$$X_1(s) = \frac{1}{s}\left(X_3(s) + X_2(s)\right) \qquad \dot{X}_1 = X_2 + X_3$$

$$X_2(s) = \frac{K}{s+1} X_1(s) \qquad \dot{X}_2 = -X_2 + K X_1$$

$$X_3(s) = \frac{2}{s+4}\left(U(s) - X_1(s)\right) \qquad \dot{X}_3 = -4X_3 - 2X_1 + 2u$$

$$\dot{\underline{X}} = \begin{bmatrix} 0 & 1 & 1 \\ K & -1 & 0 \\ -2 & 0 & -4 \end{bmatrix} \underline{X} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u \quad Y = [1, 0, 0]\, \underline{X}$$

$\beta = 0$
$\beta_1 = 2 = b_1 - a_1\beta_0$
$\beta_2 = -8 = b_2 - a_1\beta_1 - a_2\beta_0$

(4) 3.

$$G_{cl} = \frac{G(s)}{1 + G(s)} = \frac{K}{P(s)} \qquad P(s) = s^3 + 6s^2 + 5s + (K-12)$$

(1)

$$\begin{array}{lll} s^3 & 1 & 5 \\ s^2 & 6 & K-12 \\ s^1 & \frac{42-K}{6} & \\ s^0 & K-12 & \end{array}$$

(1)

$$b_1 = \frac{K-12-30}{-6} = \frac{42-K}{6}$$

$42 - K > 0 \rightarrow K < 42$

$K - 12 > 0 \rightarrow K > 12$

$12 < K < 42$ (2)

for closed-loop stability

③

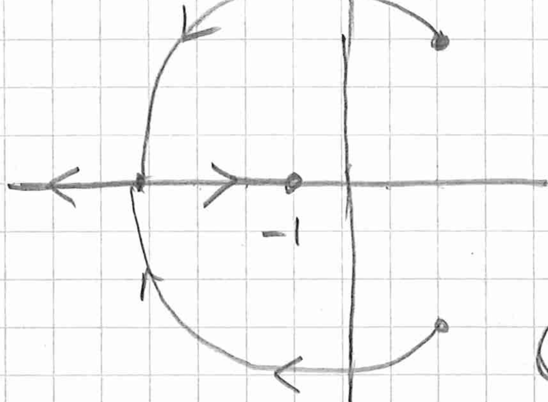
(6) 4. $G(s) = \dfrac{K(s+1)}{s^2-4s+13}$ poles $= 2 \pm j3$
zeros $= -1$

(1)

$\gamma = \pm \dfrac{180}{1}(2k+1) = \pm 180^\circ$

$\underbrace{(2s-4)(s+1)}_{2s^2-4s+2s-4} - (s^2-4s+13) = s^2+2s-17$

$s_1 = -5.24$, ~~$s_2 = +3.24$~~



(2)

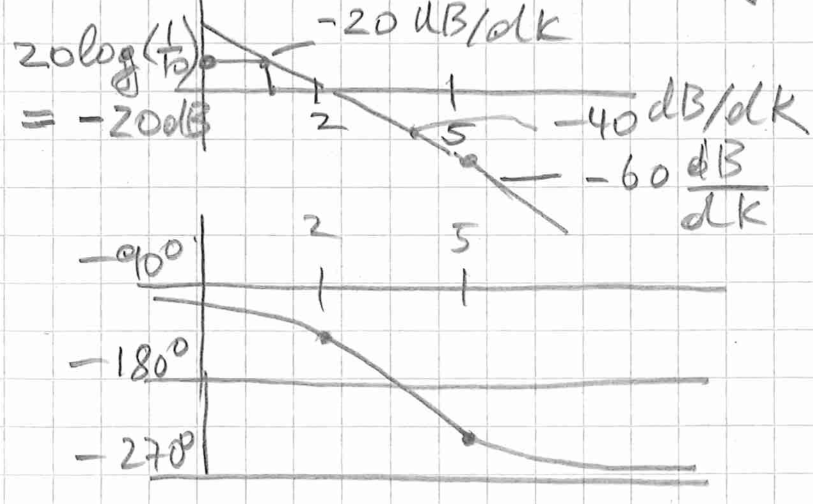
(b) • starts unstable
• underdamped
faster
(2) overshoot ↓
• critticaly damped
• overdamped
• dominant pole
slower.

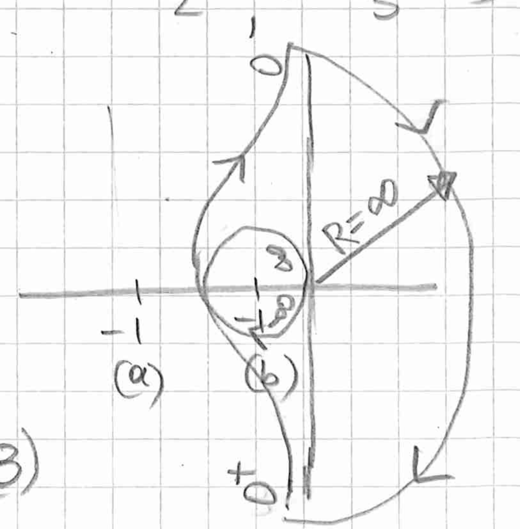
c) system of type 0
(1) step: $K_p = G(0) = \dfrac{K}{13}$ $e_{ss} = \dfrac{1}{1+\frac{K}{13}} = \dfrac{13}{13+K}$

ramp and parabola $e_{ss} = \infty$

(6) 5.

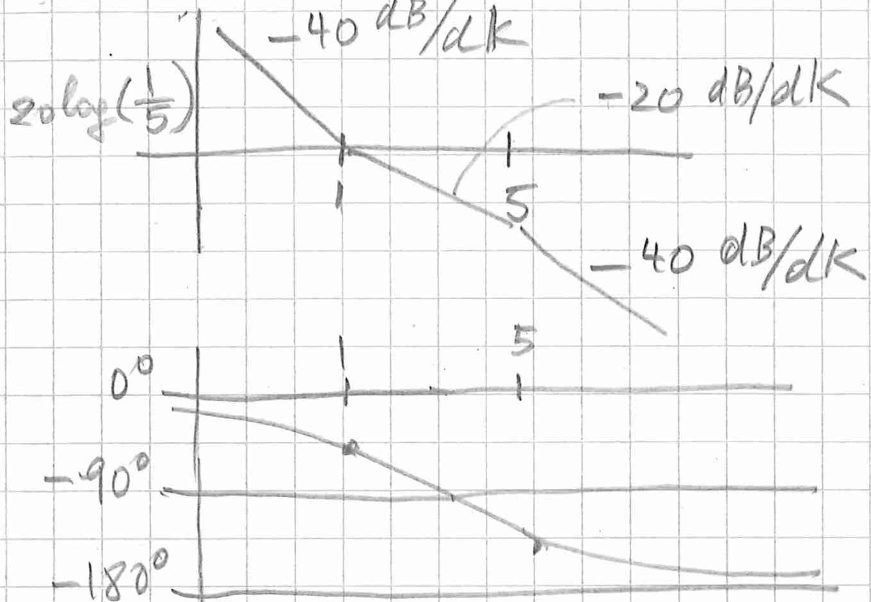
$G_1(jw) = \dfrac{1}{jw(jw+2)(jw+5)} = \dfrac{\frac{1}{10}}{jw\left(\frac{jw}{2}+1\right)\left(\frac{jw}{5}+1\right)}$

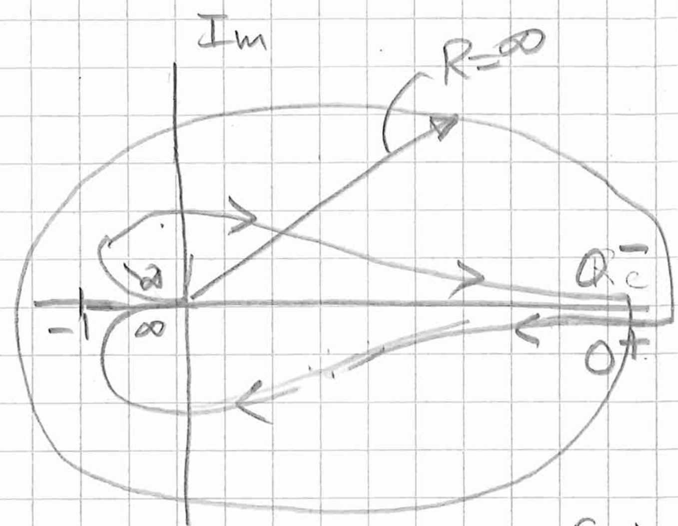




(3)

$$G_2(j\omega) = \frac{(j\omega - 1)}{(j\omega)^2 (j\omega + 5)} = \frac{\frac{1}{5}(j\omega - 1)}{(j\omega)^2 \left(\frac{j\omega}{5} + 1\right)}$$





(3)

(4) 6. (2) i) $P=0$ a) K small $N=0$ $Z=0$ stable

b) K large $N=2$ $Z=2$ unstable

(2) ii) $P=0$ for $K>0$ $N=1$ $Z=1$ unstable.

(6) 7. <u>Uncompensated</u>

(a) ph.m $\approx -22°$ (b) closed-loop unstable

g.m ≈ -5 dB

(c) : type 0 $K_p \approx 20$ db $20 \log K_p = 20$ dB $K_p = 10$

<u>Compensated</u> (a) lag (b) ph m $\approx 40°$ g.m ≈ 6 dB

(c) The compensator stabilized the system, ph.m and g.m are positive. The 40° phasemargin indicates high overshoot. The gain at low frequencies has been increased, $K_p \uparrow$, less steady-state error.

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Figure for Question 7

Solid Line: System **without** compensator

Dashed Line: System **with** compensator