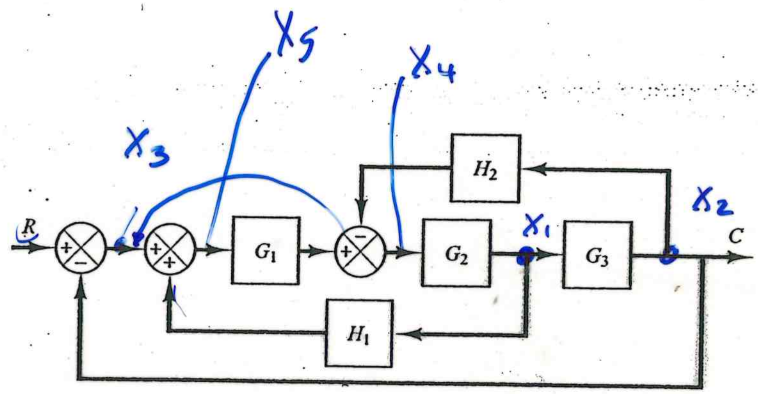
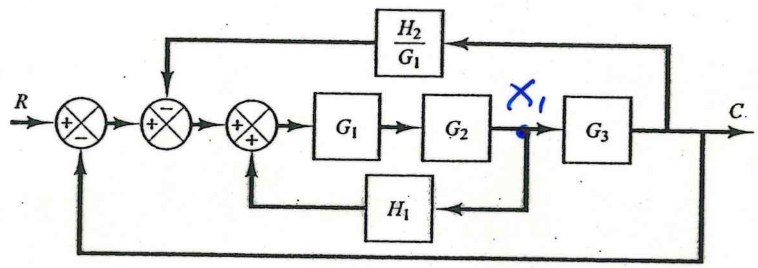
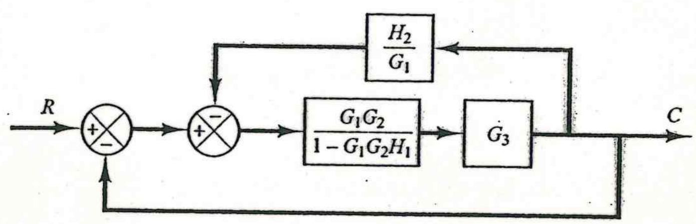
Block Diagram Reduction

$$-G_2\frac{H_2}{G_1}G_1X_2$$

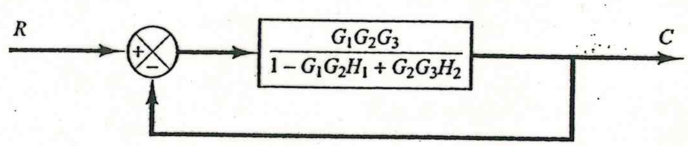
$$X_1 = -G_2H_2X_2 + G_1G_2H_1X_1 + G_1G_2X_3$$

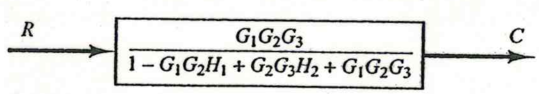
(a)


(b)


(c)


$$\frac{\dfrac{G_1G_2G_3}{1-G_1G_2H_1}}{1+\dfrac{G_1G_2G_3H_2}{(1-G_1G_2H_1)}}$$

(d)


(e)


EXAMPLE 3–13 Consider the system shown in Figure 3–39. A signal flow graph for this system is shown in Figure 3–40. Let us obtain the closed-loop transfer function $C(s)/R(s)$ by use of Mason's gain formula.

In this system there is only one forward path between the input $R(s)$ and the output $C(s)$. The forward path gain is

$$P_1 = G_1G_2G_3$$

A18a

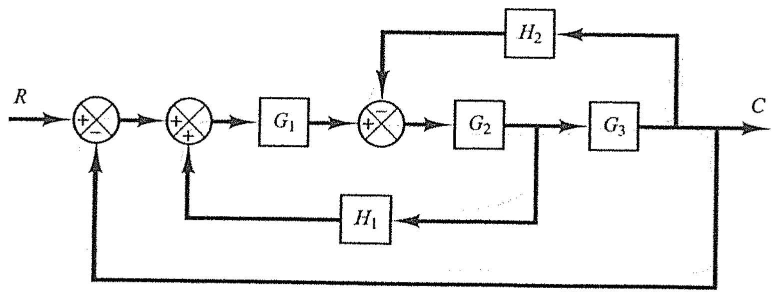


Figure 3–39 Multiple-loop system.

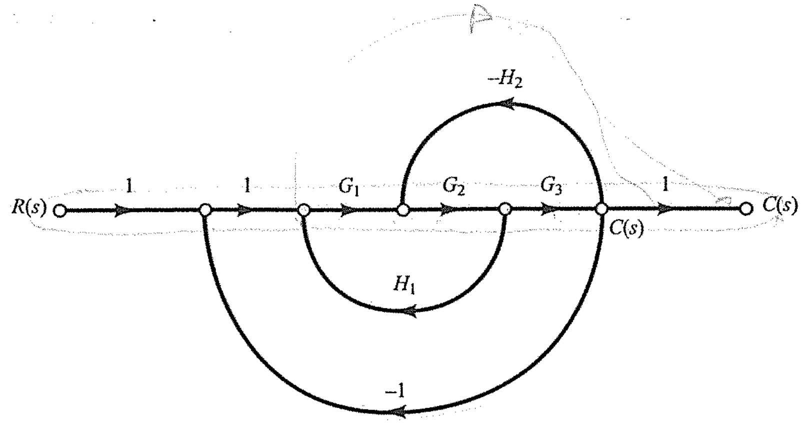


Figure 3–40 Signal flow graph for the system in Figure 3–39.

From Figure 3–40, we see that there are three individual loops. The gains of these loops are

$$L_1 = G_1G_2H_1$$
$$L_2 = -G_2G_3H_2$$
$$L_3 = -G_1G_2G_3$$

Note that since all three loops have a common branch, there are no nontouching loops. Hence, the determinant Δ is given by

$$\begin{aligned}\Delta &= 1 - (L_1 + L_2 + L_3)\\ &= 1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3\end{aligned}$$

The cofactor Δ_1 of the determinant along the forward path connecting the input node and output node is obtained from Δ by removing the loops that touch this path. Since path P_1 touches all three loops, we obtain

$$\Delta_1 = 1$$

Therefore, the overall gain between the input $R(s)$ and the output $C(s)$, or the closed-loop transfer function, is given by

$$\begin{aligned}\frac{C(s)}{R(s)} = P &= \frac{P_1\Delta_1}{\Delta}\\ &= \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}\end{aligned}$$

which is the same as the closed-loop transfer function obtained by block diagram reduction. Mason's gain formula thus gives the overall gain $C(s)/R(s)$ without a reduction of the graph.

A 286

EXAMPLE 3–14 Consider the system shown in Figure 3–41. Obtain the closed-loop transfer function $C(s)/R(s)$ by use of Mason's gain formula.

In this system, there are three forward paths between the input $R(s)$ and the output $C(s)$. The forward path gains are

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

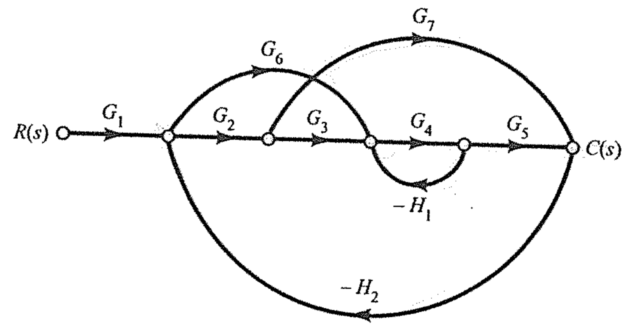


Figure 3–41
Signal flow graph for a system.

There are four individual loops, The gains of these loops are

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

Loop L_1 does not touch loop L_2. Hence, the determinant Δ is given by

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2 \tag{3–82}$$

The cofactor Δ_1, is obtained from Δ by removing the loops that touch path P_1. Therefore, by removing L_1, L_2, L_3, L_4, and L_1L_2 from Equation (3–82), we obtain

$$\Delta_1 = 1$$

Similarly, the cofactor Δ_2 is

$$\Delta_2 = 1$$

The cofactor Δ_3 is obtained by removing L_2, L_3, L_4, and L_1L_2 from Equation (3–82), giving

$$\Delta_3 = 1 - L_1$$

The closed-loop transfer function $C(s)/R(s)$ is then

$$\frac{C(s)}{R(s)} = P = \frac{1}{\Delta}(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3)$$

$$= \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7(1 + G_4H_1)}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_4H_1G_2G_7H_2}$$