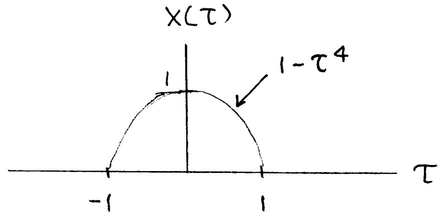
ECE 260

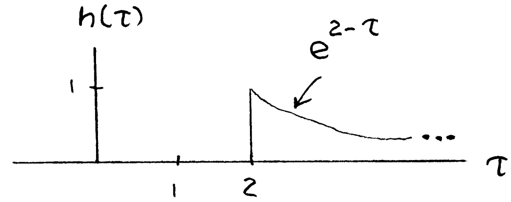
EXAM 2

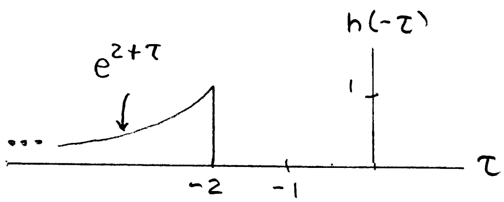
SOLUTIONS

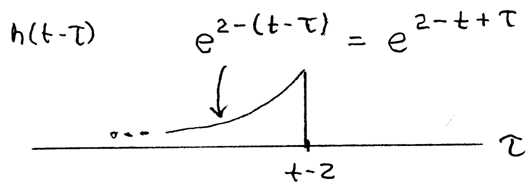
(FALL 2022)

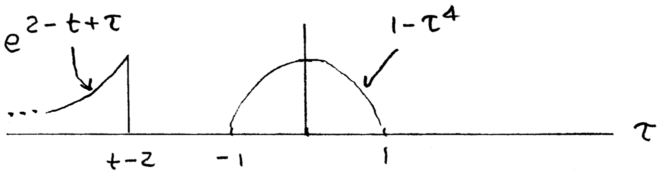
QUESTION 1





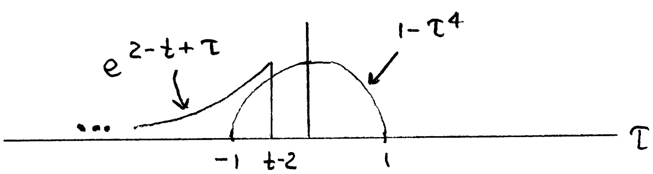






for $t-2<-1 \Rightarrow t<1$

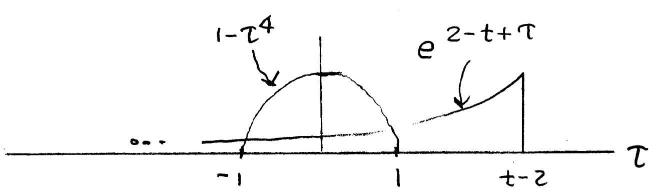
$x*h(t) = 0$



for $-1 \leq t-2 < 1 \Rightarrow t \geq 1$ and $t<3 \Rightarrow$

$1 \leq t < 3$

$$x*h(t) = \int_{-1}^{t-2} (1-\tau^4)\, e^{2-t+\tau}\, d\tau$$



for $t-2 \geq 1 \Rightarrow t \geq 3$

$$x*h(t) = \int_{-1}^{1} (1-\tau^4)\, e^{2-t+\tau}\, d\tau$$

QUESTION 2

$$\mathcal{H}x(t) = \int_{-\infty}^{t+3} e^{2\tau-2t}\, x(\tau)\, d\tau$$

$$h(t) = \mathcal{H}\delta(t)$$

$$= \int_{-\infty}^{t+3} e^{2\tau-2t}\, \delta(\tau)\, d\tau$$

$$= \int_{-\infty}^{t+3} \left[e^{2\tau-2t}\right]\Big|_{\tau=0} \delta(\tau)\, d\tau$$

$$= \int_{-\infty}^{t+3} e^{-2t}\, \delta(\tau)\, d\tau$$

$$= e^{-2t} \int_{-\infty}^{t+3} \delta(\tau)\, d\tau$$

$$= \begin{cases} e^{-2t} & t+3 \geqslant 0 \Rightarrow t \geqslant -3 \\ 0 & \text{otherwise} \end{cases}$$

$$= e^{-2t}\, u(t+3)$$

QUESTION 3

(a) $h = \mathcal{H}\delta$

$= \delta + \mathcal{H}_2 \mathcal{H}_1 \delta$

$= \delta + \mathcal{H}_2 h_1$

$= \delta + h_1 * h_2$

$\mathcal{H}_1 x = x * h_1$, δ is convolutional identity

since $\mathcal{H}_1$ is LTI

$\mathcal{H}_2 x = x * h_2$

since $\mathcal{H}_2$ is LTI

(b) $h(t) = \delta(t) + h_1 * h_2(t)$

$= \delta(t) + \int_{-\infty}^{\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau$

$= \delta(t) + \int_{-\infty}^{\infty} \delta(\tau-3)\, u(t-\tau-2)\, d\tau$

$= \delta(t) + \left[u(t-\tau-2)\right]\Big|_{\tau=3}$

$= \delta(t) + u(t-5)$

sifting property

QUESTION 4

$H(s) = s^2$ for all $s \in \mathbb{C}$ and $x(t) = 7 + e^{-5t} + 4\cos(3t)$

$$\begin{aligned}
x(t) &= 7 + e^{-5t} + 4\cos(3t) \\
&= 7e^{0t} + e^{-5t} + 4\left[\tfrac{1}{2}\left(e^{j3t} + e^{-j3t}\right)\right] \\
&= 7e^{0t} + e^{-5t} + 2e^{j3t} + 2e^{-j3t}
\end{aligned}$$

Since the system is LTI, we have

$$\begin{aligned}
y(t) &= H(0)\,e^{0t} + H(-5)\,e^{-5t} + H(j3)\left[2e^{j3t}\right] + H(-j3)\left[2e^{-j3t}\right] \\
&= H(0)\,e^{0t} + H(-5)\,e^{-5t} + 2H(j3)\,e^{j3t} + 2H(-j3)\,e^{-j3t} \\
&= 25e^{-5t} + 2(-9)\,e^{j3t} + 2(-9)\,e^{-j3t} \\
&= 25e^{-5t} - 18e^{j3t} - 18e^{-j3t} \\
&= 25e^{-5t} - 18\left(e^{j3t} + e^{-j3t}\right) \\
&= 25e^{-5t} - 18\left(2\cos(3t)\right) \\
&= 25e^{-5t} - 36\cos(3t)
\end{aligned}$$

QUESTION 5

(a) A LTI System with impulse response h is BIBO stable if and only if

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty .$$

(b) $\int_{-\infty}^{\infty} |h(t)|\, dt$

$$= \int_{-\infty}^{\infty} |e^{t-2} u(2-t)|\, dt$$

$$= \int_{-\infty}^{2} |e^{t-2}|\, dt$$

$$= \int_{-\infty}^{2} e^{t-2}\, dt$$

$$= \left[e^{t-2} \right] \Big|_{-\infty}^{2}$$

$$= e^{0} - e^{-\infty}$$

$$= 1$$

$$< \infty$$

Therefore, the system is BIBO stable.