University of Victoria
Exam 4
Fall 2024

Course Name:	**ECE260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 10960), A02 (CRN 10961)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ________________________________

Given Name(s): ________________________________

Student Number: ________________

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 26

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

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Do not write on this page unless instructed to do so.

Question 1. A LTI system has the frequency response $H(\omega) = \dfrac{\omega^2 - 2j\omega}{(j\omega - 1)^2}$.

(A) Find a **fully-simplified** expression for the magnitude response of the system. **[2 marks]**

(B) Determine the type of frequency-selective filter that this system best approximates. Your answer must be **fully justified**. A correct final answer that is not justified will receive **zero marks**. **[2 marks]**

Question 2. In this question, we consider the application of an audio cassette player. An audio tape is played in an audio cassette player to produce an audio signal x that is bandlimited to frequencies ω in the range $-4000\pi < \omega < 4000\pi$. In **every part of this question**, the following comments apply: 1) you must **show all steps** in your solution; 2) your answer must be **fully justified** (e.g., any identities, formulas, or theorems being used must be explicitly stated) and the logic leading to your final answer must be **clear**; 3) **zero marks** will be given for a correct final answer that has a missing or incorrect justification.

(A) Determine the minimum rate at which x can be sampled such that x can be exactly reconstructed from its samples. **[1 mark]**

(B) A mechanical engineering student, with too much spare time on their hands, modifies the audio cassette player so that it plays a tape at a factor of 8 slower than the standard speed to produce the audio signal $v(t) = x(t/8)$. Determine the minimum rate at which v can be sampled such that v can be exactly reconstructed from its samples. **[2 marks]**

Question 2(b) Continued

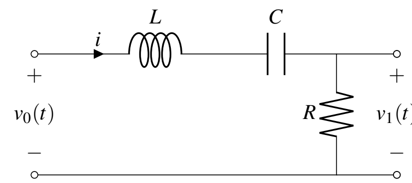
(C) Not to be outdone by a mechanical engineering student, an electrical engineering student, with considerably more spare time on their hands, builds an AM radio transmitter to broadcast the audio signal x to their classmates. The output y of the AM transmitter is given by $y(t) = x(t)\cos(8000\pi t)$. Determine the minimum rate at which y can be sampled such that y can be exactly reconstructed from its samples. **[3 marks]**

Question 3. A LTI system with the input x and output y has the frequency response $H(\omega) = \frac{\omega+7}{3\omega^3-2\omega^2+1}$. Find the differential equation that characterizes this system. You may use $\mathscr{D}$ to denote the derivative operator in your answer. **[5 marks]**

Question 4. Using the Fourier transform pair $e^{-|t|} \overset{\text{CTFT}}{\longleftrightarrow} \frac{2}{\omega^2+1}$ and properties of the Fourier transform, find the Fourier transform X of the function $x(t) = e^{-|2t-3|}$. You must use a **systematic method**, **show all of your work**, and you **must not skip any steps**. A correct final answer with an incorrect or incomplete justification may receive zero marks. **[6 marks]**

Question 5.

Consider the LTI resistor-inductor-capacitor (RLC) circuit with input v_0 and output v_1, as shown in the figure. This circuit can be shown to be characterized by the equations: $i(t) = 4v_1(t)$ and $\mathscr{D}v_0(t) = 4\mathscr{D}^2 i(t) + 2i(t) + \mathscr{D}v_1(t)$, where $\mathscr{D}$ is the derivative operator. Find the frequency response H of the circuit. **Do not skip any steps** and **show all of your work**. **[5 marks]**



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Useful Formulae and Other Information

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
$$\cos(\theta) = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin(\theta) = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$
$$\operatorname{sinc}(t) = \sin(t)/t$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$$
$$c_k = \tfrac{1}{T}\int_T x(t)e^{-j(2\pi/T)kt}\,dt$$
$$S(\omega) = \tfrac{\omega_s}{2\pi}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

$$\mathscr{F}x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$
$$\mathscr{F}^{-1}X(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$
$$H(\omega) = \tfrac{2\pi}{\omega_s}\operatorname{rect}\left(\tfrac{\omega}{\omega_s}\right)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$
$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$
$$a_k = \tfrac{1}{T}X_T(k\omega_0)$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\,d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$
8	$\operatorname{rect}\left(\frac{t}{T}\right)$	$\lvert T\rvert \operatorname{sinc}\left(\frac{T\omega}{2}\right)$
9	$\operatorname{sinc}(Bt)$	$\frac{\pi}{\lvert B\rvert}\operatorname{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$
12	$e^{-at}\cos(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
14	$e^{at}u(-t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a-j\omega}$