# CSC 320 Midterm 1 Practice Questions

**1**) Let $\Sigma = 0, 1$ be an alphabet, and let $L$ be a language over $\Sigma$. Circle every true statement.

(a) $\Sigma$ is countable
(b) $\Sigma^*$ is counbable
(c) $L \subseteq \Sigma^*$ and $L$ is countable
(d) $\mathcal{P}(\Sigma^*)$ is countable
(e) $L^+$ is countable

**2**) Let $R$ be a regular expression. Circle every true statement.

(a) $R \cup \emptyset = \emptyset$
(b) $R\emptyset = R$
(c) There exists a DFA $M$ with $L(M) = L(R)$
(d) There exists an NFA $M$ with $L(M) = L(R)$
(e) There exists a DFA $M$ with $L(M) = R$

**3**) Let $\Sigma = a, b, c, d$ and let $R = (c \cup d)^* d(a \cup ab)^*$. Select every true statement about $L(R)$.

(a) If $w \in L(R)$ then $|w| > 0$
(b) If $w \in L(R)$ then $|w| > 1$
(c) If $w \in L(R)$ then $w$ contains at least one $d$
(d) If $w \in L(R)$ and if $w$ contains an $a$, then $w$ contains at least one $d$ somewhere after the occurrence of $a$

**4**) Is the language $\{110, 101\}$ a regular language? Explain.

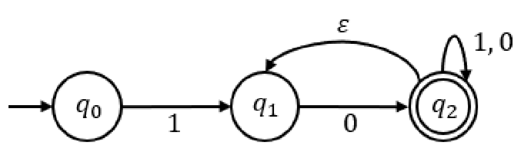
Yes. We can write a regular expression to describe the language:

$110 \cup 101$

**5**) Can a subset of a non-regular language be a regular language? Explain.

Yes. The empty language $\emptyset$ is a subset of every language, including non-regular languages, and the empty language is a regular language.
Therefore, a subset of a non-regular language can be a regular language.

**6**) Consider the following state diagram for finite automaton $M$:



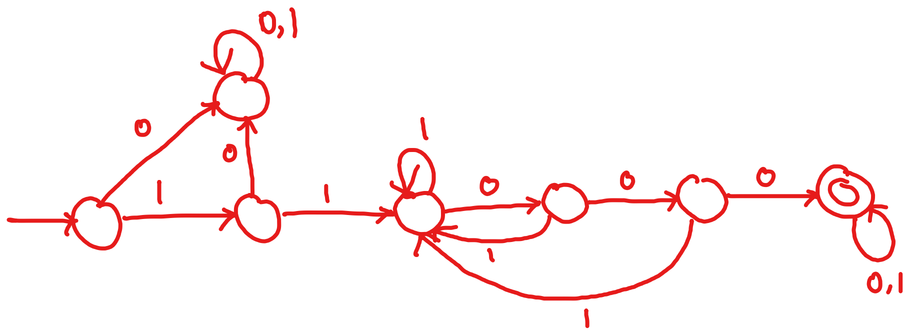

Describe the language recognized by $M$ using your own words and a regular expression.

- Strings containing 0 and 1 which begin with 10

- $10(0 \cup 1)^*$

**7**) Consider the language $L = \{w \in \{0,1\}^* \mid w \text{ starts with 11 and contains 000 as a substring}\}$.

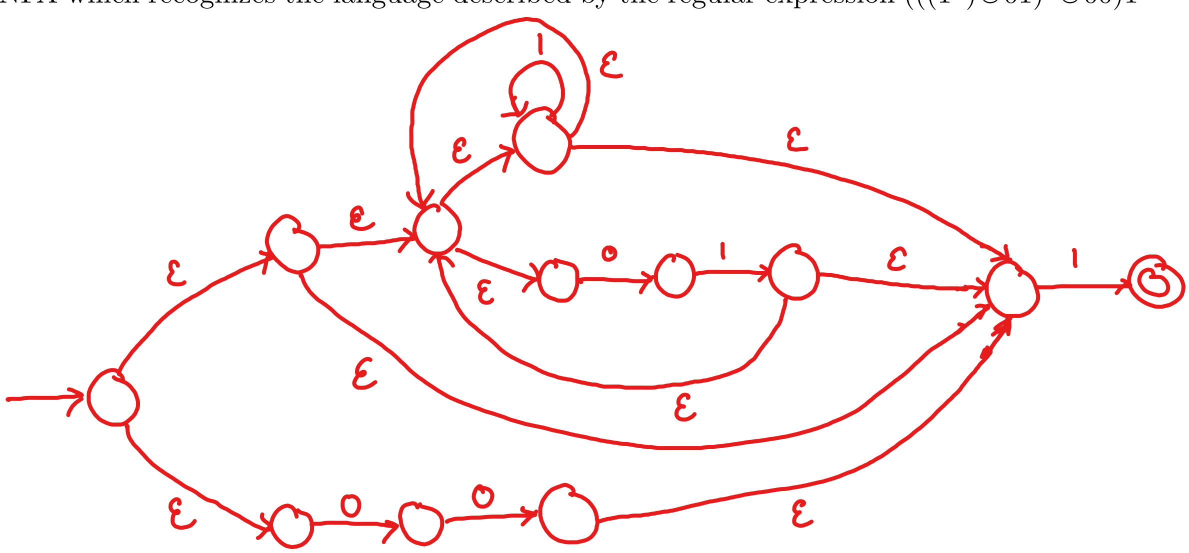
(a) Construct a DFA which recognizes $L$



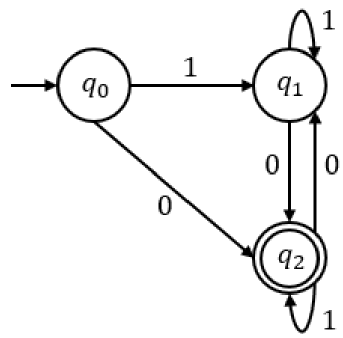

(b) Write a regular expression which describes $L$

$11(0 \cup 1)^* 000 (0 \cup 1)^*$

**8**) Create an NFA which recognizes the language described by the regular expression $(((1^*) \cup 01)^* \cup 00)1$

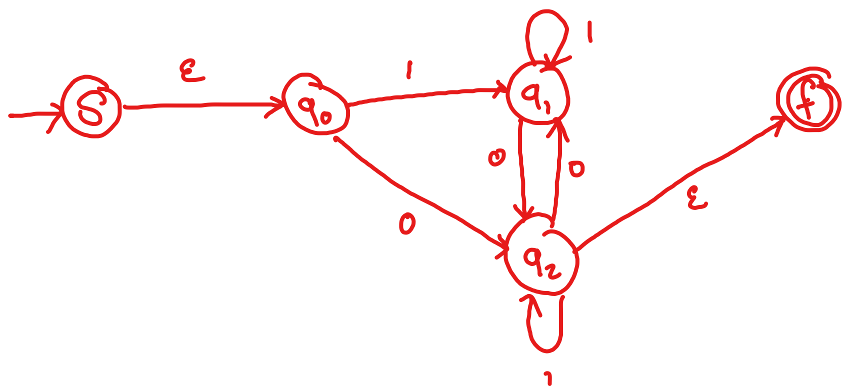


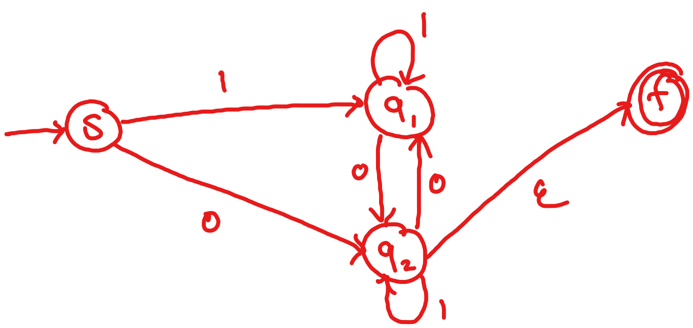
**9**) Consider the following DFA $D$:

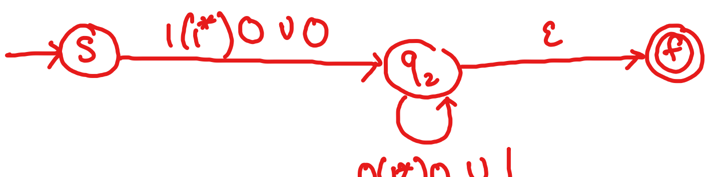



Convert the DFA $D$ to a regular expression. Show your work by drawing the state diagram for the corresponding GNFA and the state diagram after removing states $q_0$, $q_1$, and $q_2$. Remove the states in lexicographic order.

GNFA





$$\boxed{(1(1^*)0 \cup 0)(0(1^*)0 \cup 1)^*}$$

**10**) Prove that the language $L = \{0^n 1^{n+1} 0^{n+1} 0^n \mid n \geq 0\}$ is non-regular using the pumping lemma.

Assume for a contradiction that L is regular.

Let p be the pumping length given by the pumping lemma.

Choose $s = 0^p 1^{p+1} 0^{p+1} 0^p$

Since $s \in L$ and $|s| \geq p$, we can rewrite $s = xyz$ such that

1. $|y| > 0$ $(y \neq \varepsilon)$
2. $|xy| \leq p$
3. $xy^i z \in L$ for all $i \geq 0$

By property 2, $xy$ consists of 0's

By property 1, y is a nonempty substring of 0's

Consider $xy^2z$ (or $xy^0z$). The resulting string is not in the language since it is not of form $0^n 1^{n+1} 0^{n+1} 0^n$, which violates property 3 of the pumping lemma.

Hence, we cannot rewrite $s = xyz$ such that all the properties of the PL hold, which is a contradiction.

Therefore, L is not regular.