Assignment 8

ECE 360

V00984826

B-7-9

A system with the open-loop transfer function

$$G(s)H(s) = \frac{K}{s^2(T_1 s + 1)}$$

is inherently unstable. This system can be stabilized by adding derivative control. Sketch the polar plots for the open-loop transfer function with and without derivative control.

S-7-9

The given open-loop transfer function is:

$$G(s)H(s) = \frac{K}{s^2(T_1 s + 1)}$$

This system is unstable because of the double pole at the origin. Introducing derivative control modifies the transfer function as follows:

$$G(s)H(s) = \frac{K(T_2 s + 1)}{s^2(T_1 s + 1)}, \quad (T_2 > T_1 > 0)$$

Below are the Nyquist plots for the two cases:

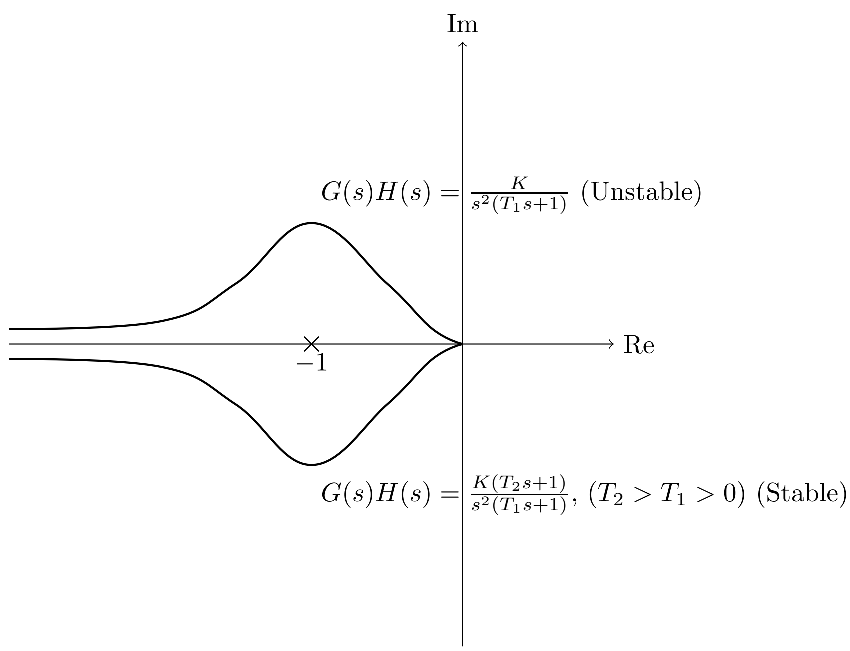
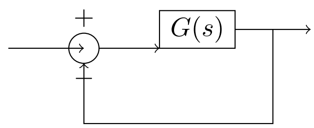


Figure 1: Nyquist plots for systems with and without derivative control

B-7-16

Consider the closed-loop system shown below. $G(s)$ has no poles in the right-half s plane.



If the Nyquist plot of $G(s)$ is as shown in Figure 7-158(a), is this system stable?
If the Nyquist plot is as shown in Figure 7-158(b), is this system stable?

S-7-16

Let's analyze this system step by step using the Nyquist stability criterion:

1) First, recall that for a system with no poles in the right-half plane, the system is stable if and only if the Nyquist plot does not enclose the $-1 + j0$ point.

2) For Figure 7-158(a): - The Nyquist plot does not enclose the $-1 + j0$ point - The plot passes near but does not encircle the critical point - Therefore, according to the Nyquist stability criterion, the system is stable

3) For Figure 7-158(b): - The Nyquist plot makes one complete counterclockwise encirclement of the $-1 + j0$ point - According to the Nyquist criterion, this means the closed-loop system has one pole in the right-half plane - Therefore, the system is unstable

Therefore: - For case (a): The system is stable - For case (b): The system is unstable

B-7-23

Consider the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{as + 1}{s^2}$$

Determine the value of a so that the phase margin is 45.

S-7-23

For this unity-feedback system, we need to analyze the frequency response to find the value of a that gives a phase margin of 45.

The magnitude and phase of $G(j\omega)$ are:

$$|G(j\omega)| = \frac{\sqrt{a^2\omega^2 + 1}}{\omega^2}$$

$$\angle G(j\omega) = \tan^{-1}(a\omega) - 180$$

At the gain crossover frequency ω_1 (where $|G(j\omega_1)| = 1$), we require:

$$\frac{\sqrt{a^2\omega_1^2 + 1}}{\omega_1^2} = 1$$

$$\tan^{-1}(a\omega_1) - 180 = -135 \quad \text{(for 45 phase margin)}$$

From these conditions, we can write:

$$a^2\omega_1^2 + 1 = \omega_1^4$$

$$a\omega_1 = 1$$

Solving these equations:

$$a = \left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} = 0.841$$

Therefore, the value of a that gives a phase margin of 45 is 0.841.

B-7-26

Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K}{s(s^2+s+4)} = \frac{0.25K}{s(0.25s^2+0.25s+1)}$$

The quadratic term in the denominator has the undamped natural frequency of 2 rad/sec and the damping ratio of 0.25. Determine the value of the gain K such that the phase margin is 50. What is the gain margin with this gain K?

S-7-26

Let's solve this step by step:

1) At frequency ω_1 corresponding to -130 phase (for 50 phase margin):

$$\angle G(j\omega_1) = -1j\omega_1 - [1 - 0.25\omega_1^2 + j0.25\omega_1] = -130$$

$$= -90 - \tan^{-1}\frac{0.25\omega_1}{1-0.25\omega_1^2} = -130$$

2) Solving this equation, we find $\omega_1 = 1.491$ rad/sec

3) At this frequency, the magnitude must be unity:

$$|G(j1.491)| = \left|\frac{0.25K}{(j1.491)(-0.555+j0.3725+1)}\right| = 0.2890K = 1$$

4) Setting $|G(j1.491)| = 0.2890K = 1$, we find:

$$K = 3.46$$

5) Note that the phase crossover frequency is at $\omega = 2$ rad/sec:

$$\angle G(j2) = -\angle j2 - \angle(-0.25x2^2 + 0.25xj2 + 1) = -90 - 90 = -180$$

6) The magnitude $|G(j2)|$ with $K = 3.46$ becomes:

$$|G(j2)| = \left|\frac{0.865}{(j2)(-1+0.5j+1)}\right| = 0.865 = -1.26 \text{ dB}$$

Therefore: - The required gain $K = 3.46$ - The gain margin is 1.26 dB

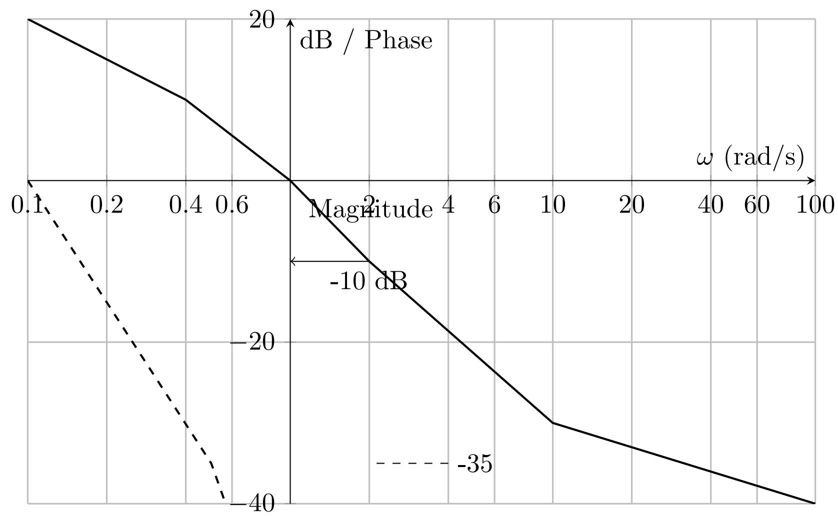


Figure 2: Bode plot with magnitude and phase.