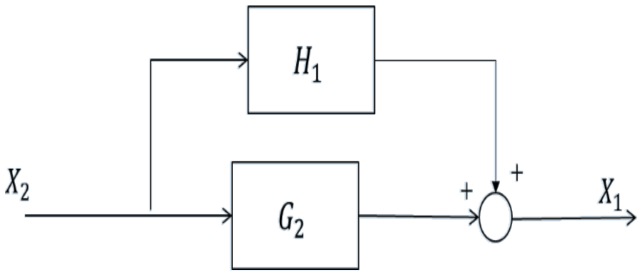
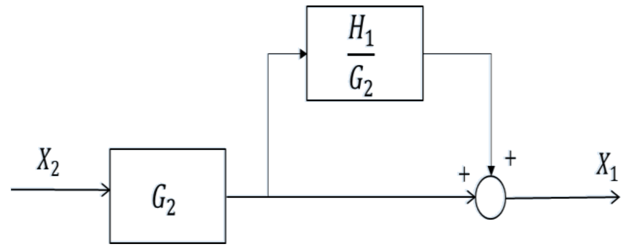
<u>B-2-3</u>

Using the fact that



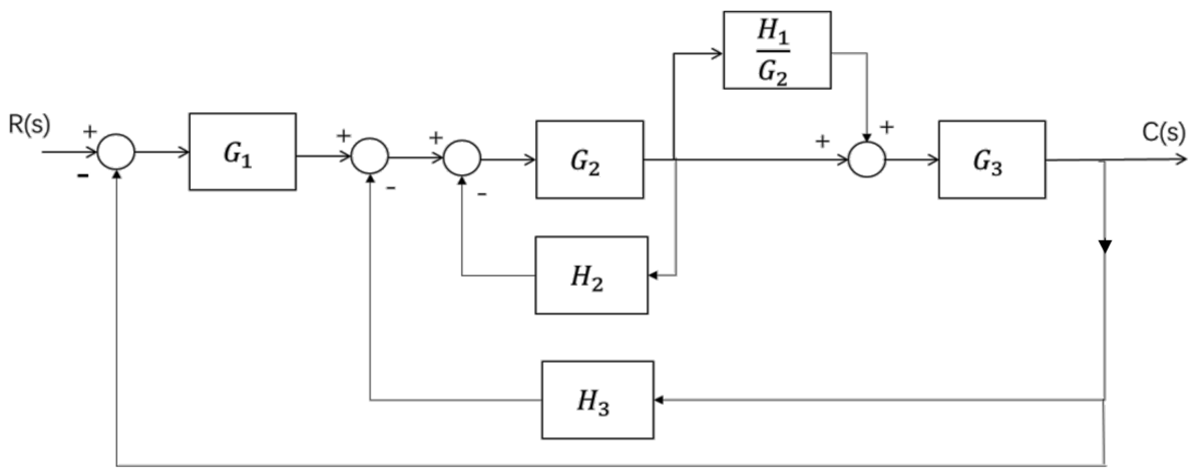
&



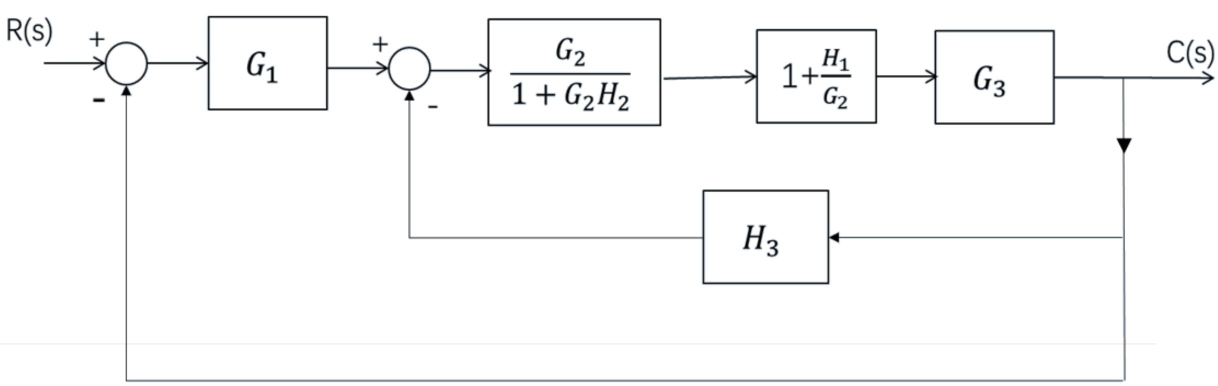
$X_1 = H_1X_2 + G_2X_2$

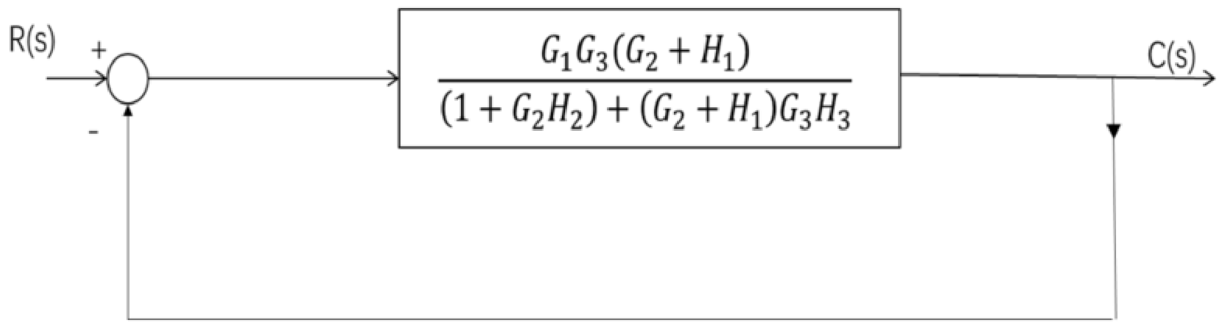
$X_1 = \frac{H_1}{\cancel{G_2}}\cancel{G_2}X_2 + G_2X_2$

equivalently, Fig. 2-31 can be modified to



and,





$$=> \frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_3H_1}{1 + G_2H_2 + G_2G_3H_3 + G_3H_1H_3 + G_1G_2G_3 + G_1G_3H_1}$$

Using Mason's formula, from Fig. 2-31 we have:

Forward paths : $P_1 = G_1G_2G_3, \quad P_2 = G_1H_1G_3$

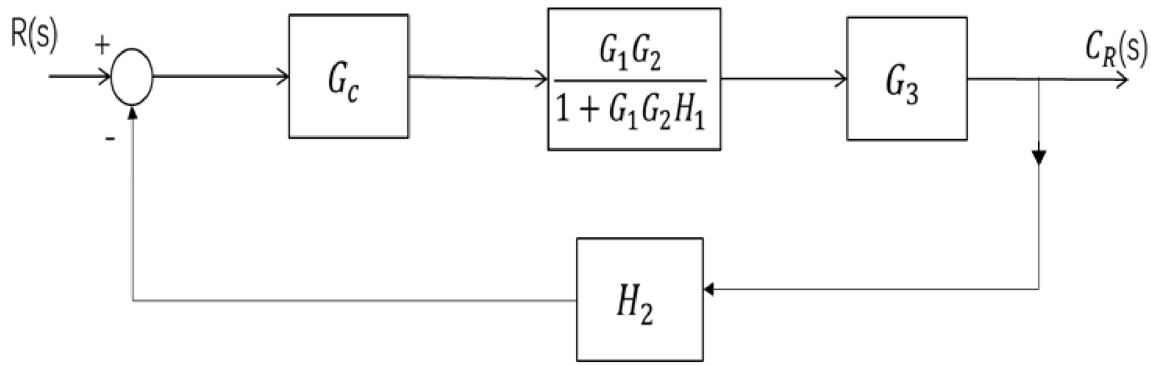
Loops : $L_1 = -G_2H_2, \quad L_2 = -G_2G_3H_3, \quad L_3 = -H_1G_3H_3$
$L_4 = -G_1G_2G_3, \quad L_5 = -G_1H_1G_3$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) \qquad \Delta_1 = 1 \quad \Delta_2 = 1$$

$$\frac{C(s)}{R(s)} = \frac{P_1 + P_2}{\Delta} = \frac{G_1G_2G_3 + G_1H_1G_3}{1 + G_2H_2 + G_2G_3H_3 + H_1G_3H_3 + G_1G_2G_3 + G_1H_1G_3}$$

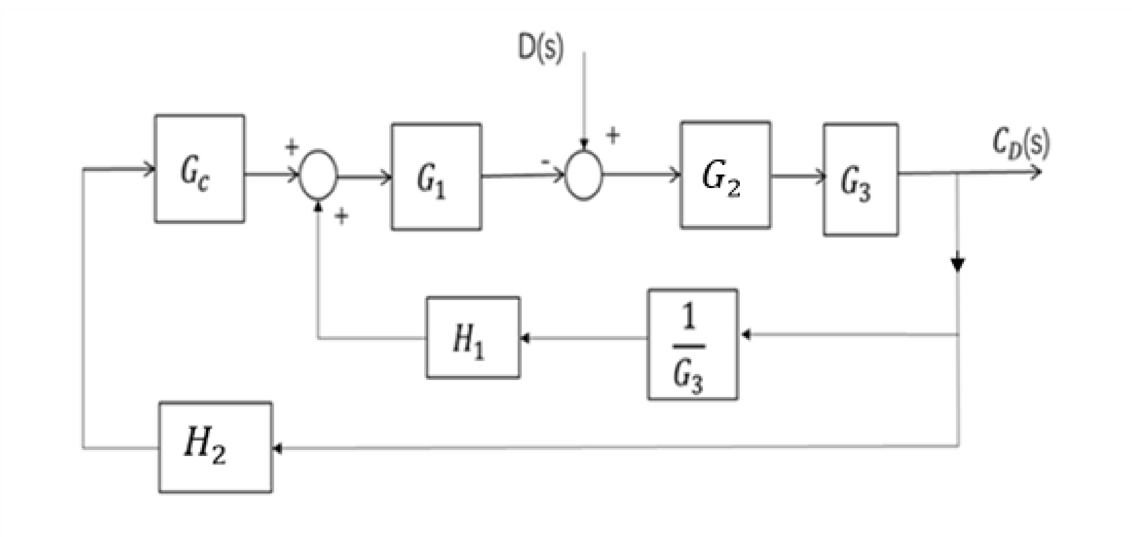
B-2-7

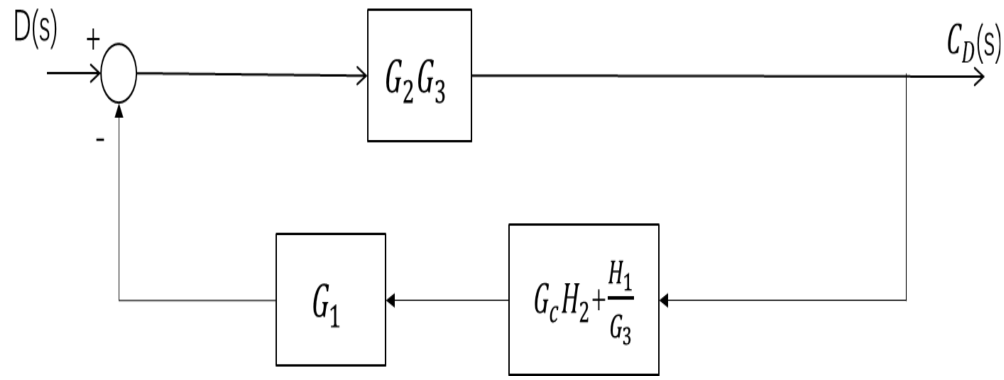
The block diagram of the system in Fig. 2-34 for D(s) = 0 would be:



$$=> \frac{C_R(s)}{R(s)} = \frac{\frac{G_CG_1G_2G_3}{1+G_1G_2H_1}}{1+\frac{G_CG_1G_2G_3H_2}{1+G_1G_2H_1}} = \frac{G_CG_1G_2G_3}{1+G_1G_2H_1+G_CG_1G_2G_3H_2}$$

The block diagram of system shown in Fig. 2-34 for R(s) = 0, would be:





$$=> \frac{C_D(s)}{D(s)} = \frac{G_2G_3}{1 + G_2G_3G_1(G_CH_2 + \frac{H_1}{G_3})} = \frac{G_2G_3}{1 + G_1G_2G_3G_CH_2 + G_1G_2H_1}$$

B-2-9

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

by defining, $\begin{cases} x_1 = y \\ x_2 = \dot{y}, \\ x_3 = \ddot{y} \end{cases}$ we have : $\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = \dddot{y} = -3x_3 - 2x_2 + u \end{cases}$

$$=> \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \qquad y = [\,1\ \ 0\ \ 0\,] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B-2-11

$$A_m = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix},\ b_m = \begin{bmatrix} 2 \\ 5 \end{bmatrix},\ c_m = [\,1\ \ 2\,]$$

$$G(s) = c_m(sI_m - A_m)^{-1}b_m = [\,1\ \ 2\,]\begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}^{-1}\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= [\,1\ \ 2\,]\frac{1}{(s+5)(s+1)+3}\begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}\begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \frac{1}{s^2+6s+8}[\,1\ \ 2\,]\begin{bmatrix} 2s-3 \\ 5s+31 \end{bmatrix}$$

$$= \frac{12s+59}{s^2+6s+8}$$