ECE 260

EXAM 3

SOLUTIONS

(SUMMER 2020)

QUESTION 1

$$x(t) = 5e^{3t} \text{ for } t \in [0,5)$$

$$T = 5, \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{5}$$

$$c_k = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{5}\int_0^5 5e^{3t} e^{-jk(2\pi/5)t}\, dt$$

$$= \int_0^5 e^{3t - jk2\pi t/5}\, dt$$

$$= \int_0^5 e^{t(3-jk2\pi/5)}\, dt$$

$$= \frac{1}{3-jk2\pi/5}\, e^{t(3-jk2\pi/5)}\Big|_0^5$$

$$= \frac{5}{15-j2\pi k}\left[e^{15-j2\pi k} - 1\right]$$

$$= \frac{5}{15-j2\pi k}\left[e^{15}(1)^k - 1\right]$$

$$= \frac{5(e^{15}-1)}{15-j2\pi k}$$

QUESTION 2

```
function y = myfunc(x)
    if x >= 0
        y = 0;
        for k = 0 : 99
            y = y + exp(k * x) * cos(k * x);
        end
    else
        y = exp(x);
    end
end
```

QUESTION 3

The function x satisfies the Dirichlet conditions.

Since x is discontinuous at 0 and 2, we have

$$y(0) = \tfrac{1}{2}\left[x(0^-) + x(0^+)\right]$$
$$= \tfrac{1}{2}[-25 + 1]$$
$$= \tfrac{1}{2}[-24]$$
$$= -12$$

$$y(2) = \tfrac{1}{2}\left[x(2^-) + x(2^+)\right]$$
$$= \tfrac{1}{2}\left[e^2 + (-2^2)\right]$$
$$= \tfrac{1}{2}\left[e^2 - 4\right]$$
$$= \frac{e^2 - 4}{2}$$

QUESTION 4

$$c_k = \frac{-1}{(2+j\pi k)^2}, \quad T = 2, \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$$

magnitude spectrum is $|c_k|$

phase spectrum is $\arg c_k$

$$|c_k| = \left| \frac{-1}{(2+j\pi k)^2} \right| = \frac{|-1|}{|(2+j\pi k)^2|} = \frac{1}{\left(\sqrt{2^2+(\pi k)^2}\right)^2}$$

$$= \frac{1}{\left(\sqrt{4+\pi^2 k^2}\right)^2} = \frac{1}{|4+\pi^2 k^2|} = \frac{1}{4+\pi^2 k^2}$$

$$\arg c_k = \arg\left[\frac{-1}{(2+j\pi k)^2}\right] = \arg(-1) - \arg\left[(2+j\pi k)^2\right]$$

$$= \pi - 2\arg\left[2+j\pi k\right] = \pi - 2\arctan\left(\frac{\pi k}{2}\right)$$

[or more generally, $\arg c_k = (2n+1)\pi - 2\arctan\left(\frac{\pi k}{2}\right), \ n \in \mathbb{Z}$]

QUESTION 5

$$x(t) = 4 + 3\cos(t) + 2\cos(3t)$$

$$= 4 + 3\left[\tfrac{1}{2}\left(e^{jt} + e^{-jt}\right)\right] + 2\left[\tfrac{1}{2}\left(e^{j3t} + e^{-j3t}\right)\right]$$

$$= 4 + \tfrac{3}{2}e^{jt} + \tfrac{3}{2}e^{-jt} + e^{j3t} + e^{-j3t}$$

Since the system is LTI and $e^{j\omega t}$ is an eigenfunction of the system with eigenvalue $H(\omega)$, we have

$$y(t) = H(0)[4] + H(1)\left[\tfrac{3}{2}e^{jt}\right] + H(-1)\left[\tfrac{3}{2}e^{-jt}\right] + H(3)\left[e^{j3t}\right] + H(-3)\left[e^{-j3t}\right]$$

$$= 0 + (1)\left(\tfrac{3}{2}e^{jt}\right) + (-1)\left(\tfrac{3}{2}e^{-jt}\right) + (1)\left(e^{j3t}\right) + (-1)\left(e^{-j3t}\right)$$

$$= \tfrac{3}{2}e^{jt} - \tfrac{3}{2}e^{-jt} + e^{j3t} - e^{-j3t}$$

$$= \tfrac{3}{2}\left[e^{jt} - e^{-jt}\right] + \left[e^{j3t} - e^{-j3t}\right]$$

$$= \tfrac{3}{2}\left[2j\sin(t)\right] + 2j\sin(3t)$$

$$= 3j\sin(t) + 2j\sin(3t)$$