CHAPTER 5

B-5-1. Time constant = 0.25 min. The steady-state error is 2.5 degrees.

B-5-2.

Rise time = 2.42 sec

Peak time = 3.63 sec

Maximum overshoot = 0.163

Settling time = 8 sec (2 % criterion)

B-5-3. The maximum overshoot of 5% corresponds to $\zeta = 0.69$. Hence

$$\omega_n = \frac{2}{\zeta} = \frac{2}{0.69} = 2.90 \text{ rad/sec}$$

B-5-4. When the mass m is set into motion by a unit-impulse force, the system equation becomes

$$m\ddot{x} + kx = \delta(t)$$

Define another impulse force to stop the motion as A δ (t − T), where A is the undetermined magnitude of the impulse force and t = T is the undetermined instant that this impulse is to be given to the system. Then, the equation for the system when the two impulse forces are given is

$$m\ddot{x} + kx = \delta(t) + A\delta(t - T), \quad x(0-) = 0, \ \dot{x}(0-) = 0$$

The Laplace transform of this last equation gives

$$(ms^2+k)X(s) = 1 + Ae^{-sT}$$

Solving for X(s),

$$X(s) = \frac{1}{ms^2+k} + \frac{Ae^{-sT}}{ms^2+k}$$

$$= \frac{1}{\sqrt{km}} \frac{\sqrt{\frac{k}{m}}}{s^2+\frac{k}{m}} + \frac{A}{\sqrt{km}} \frac{\sqrt{\frac{k}{m}}\, e^{-sT}}{s^2+\frac{k}{m}}$$

The inverse Laplace transform of X(s) is

$$x(t) = \frac{1}{\sqrt{km}} \sin\sqrt{\frac{k}{m}}\,t + \frac{A}{\sqrt{km}} \left[\sin\sqrt{\frac{k}{m}}(t-T)\right] 1(t-T)$$

If the motion of the mass m is to be stopped at t = T, then x(t) must be identically zero for $t \geqslant T$.

Notice that x(t) can be made identically zero for $t \geqslant T$ if we choose

$$A = 1, \quad T = \frac{\pi}{\sqrt{\frac{k}{m}}}, \quad \frac{3\pi}{\sqrt{\frac{k}{m}}}, \quad \frac{5\pi}{\sqrt{\frac{k}{m}}}, \ldots$$

Thus, the motion of the mass m can be stopped by another impulse force, such as

$$\delta\left(t-\frac{\pi}{\sqrt{\frac{k}{m}}}\right), \quad \delta\left(t-\frac{3\pi}{\sqrt{\frac{k}{m}}}\right), \quad \delta\left(t-\frac{5\pi}{\sqrt{\frac{k}{m}}}\right), \ldots$$

B-5-5. For a unit-impulse input

$$c(t) = -te^{-t} + 2e^{-t} \quad (t \geqslant 0)$$

For a unit-step input

$$c(t) = 1 + te^{-t} - e^{-t} \quad (t \geqslant 0)$$

B-5-6.

$$\frac{x_1}{x_2} = \frac{e^{-\zeta\omega_n t_1}}{e^{-\zeta\omega_n(t_1+T)}} = \frac{1}{e^{-\zeta\omega_n T}} = e^{\zeta\omega_n T}$$

$$\frac{x_1}{x_n} = \frac{1}{e^{-\zeta\omega_n(n-1)T}} = e^{(n-1)\zeta\omega_n T}$$

$$\text{Logarithmic decrement} = \ln\frac{x_1}{x_2} = \frac{1}{n-1}\ln\frac{x_1}{x_n}$$

$$= \zeta\omega_n T = \zeta\omega_n\frac{2\pi}{\omega_d}$$

$$= \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Define

$$\frac{1}{n-1}\ln\frac{x_1}{x_n} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \Delta$$

Then

$$4\pi^2\zeta^2 = \Delta^2(1-\zeta^2)$$

or

$$\zeta^2 = \frac{\Delta^2}{4\pi^2+\Delta^2}$$

Thus

$$\zeta = \frac{\Delta}{\sqrt{4\pi^2+\Delta^2}}$$

$$= \frac{\left(\frac{1}{n-1}\right)\left(\ln\frac{x_1}{x_n}\right)}{\sqrt{4\pi^2+\left(\frac{1}{n-1}\right)^2\left(\ln\frac{x_1}{x_n}\right)^2}}$$

B-5-7. For the system shown in Figure 5-74(b), we have

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + (1 + 10K_h)s + 10}$$

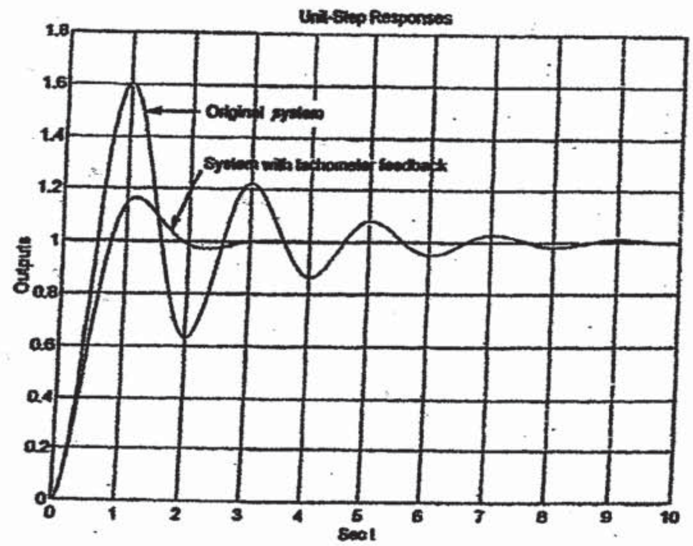
Noting that $2\zeta\omega_n = 1 + 10K_h$, $\omega_n^2 = 10$, $\zeta = 0.5$, we obtain

$$1 + 10K_h = 2 \times 0.5 \times \sqrt{10} = \sqrt{10}$$

Hence

$$K_h = \frac{\sqrt{10} - 1}{10} = 0.216$$

The unit-step response curves of both systems are shown below.



Note that for the original system

$$\frac{E_1(s)}{R(s)} = \frac{R(s) - C_1(s)}{R(s)}$$

$$= \frac{s^2 + s}{s^2 + s + 10}$$

For the tachometer-feedback system

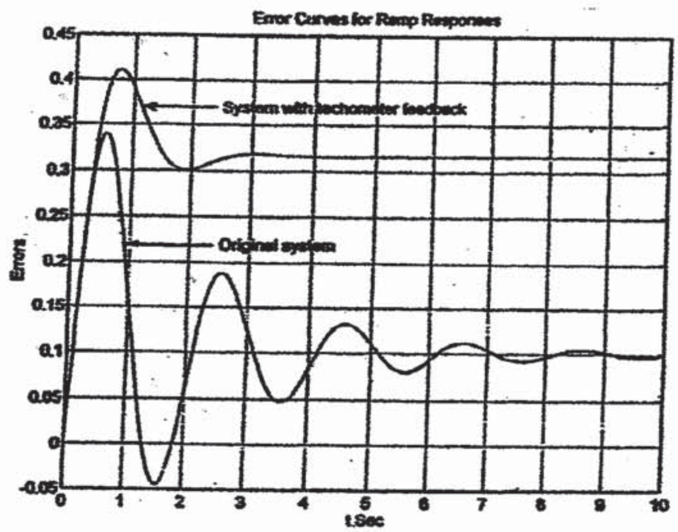
$$\frac{E_2(s)}{R(s)} = \frac{R(s) - C_2(s)}{R(s)} = \frac{s^2 + 3.16s}{s^2 + 3.16s + 10}$$

For the unit-ramp input, we have

$$E_1(s) = \left(\frac{s^2+s}{s^2+s+10}\,\frac{1}{s}\right)\frac{1}{s} = \frac{s^2+s}{s^3+s^2+10s}\,\frac{1}{s}$$

$$E_2(s) = \left(\frac{s^2+3.16s}{s^2+3.16s+10}\,\frac{1}{s}\right)\frac{1}{s} = \frac{s^2+3.16s}{s^3+3.16s^2+10s}\,\frac{1}{s}$$

The error versus time curves [$e_1(t)$ versus t and $e_2(t)$ versus t] are shown below.



B-5-8. For the given system we have

$$\frac{C(s)}{R(s)} = \frac{K}{s^2 + 2s + Kks + K}$$

Note that

$$K = \omega_n^2 = 4^2 = 16$$

Since

$$2\zeta\omega_n = 2 + Kk$$

we obtain

$$2 \times 0.7 \times 4 = 2 + Kk = 2 + 16k$$

Thus

$$k = 0.225$$

B-5-9.

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8 + 16k)s + 16}$$

From the characteristic polynomial, we find

$$\omega_n = 4, \quad 2\zeta\omega_n = 2 \times 0.5 \times 4 = 0.8 + 16k$$

Hence

$$k = 0.2$$

The rise time t_r is obtained from

$$t_r = \frac{\pi - \beta}{\omega_d}$$

Since

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 4\sqrt{1 - 0.25} = 3.46$$

$$\beta = \sin^{-1} \frac{\omega_d}{\omega_n} = \sin^{-1} 0.866 = \frac{\pi}{3}$$

we have

$$t_r = \frac{\pi - \frac{1}{3}\pi}{3.46} = 0.605 \text{ sec}$$

The peak time t_p is obtained as

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{3.46} = 0.907 \text{ sec}$$

The maximum overshoot M_p is

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.5 \times 3.14}{\sqrt{1-0.25}}} = e^{-1.814} = 0.163$$

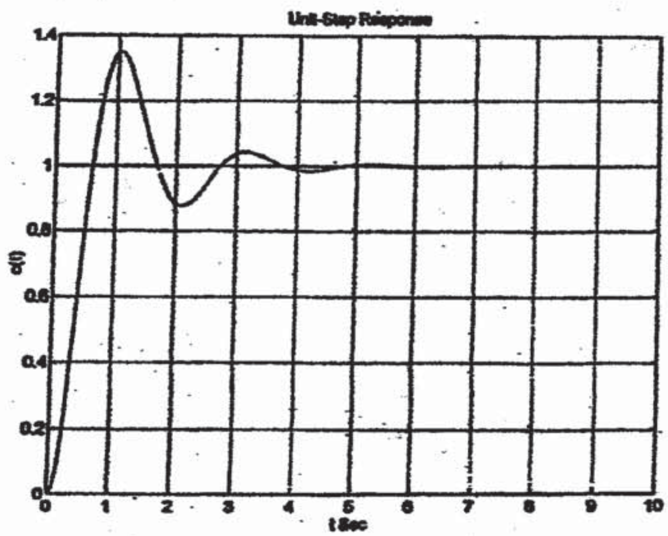
The settling time t_s is

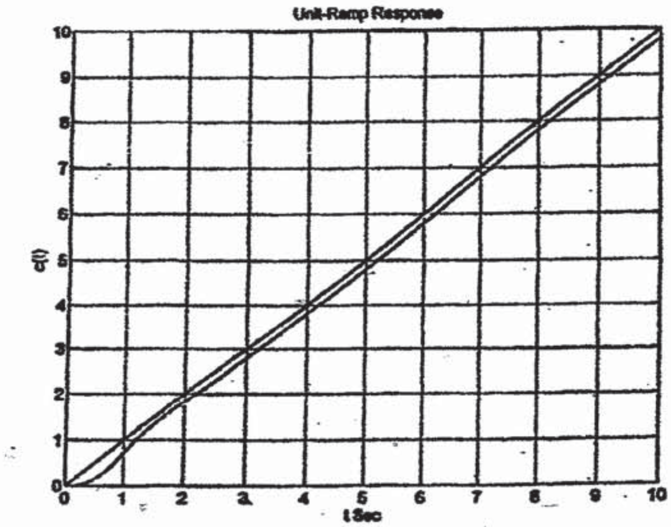
$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 4} = 2 \text{ sec}$$

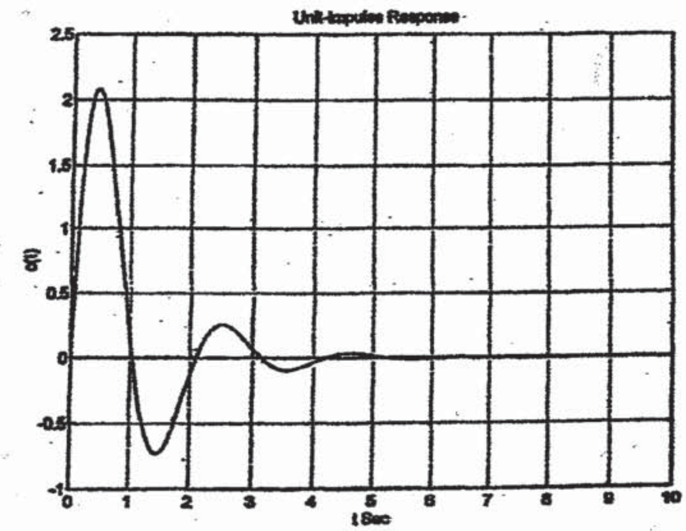
B-5-10. A MATLAB program to obtain the unit-step response, unit-ramp response, and unit-impulse response of the given system is shown on the next page.

```
% ***** Unit-step response *****

num = [0  0  10];
den = [1  2  10];
t = 0:0.02:10;
step(num,den,t)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('c(t)')

% ***** Unit-ramp response *****

numr = [0  0 0  10];
denr = [1  2  10  0];
c = step(numr,denr,t);
plot(t,c,'-',t,t,'--')
grid
title('Unit-Ramp Response')
xlabel('t Sec')
ylabel('c(t)')

% ***** Unit-impulse response *****

impulse(num,den,t)
grid
title('Unit-Impulse Response')
xlabel('t Sec')
ylabel('c(t)')
```

The unit-step response curve is shown below. The unit-ramp response curve and unit-impulse response curve are shown on the next page.

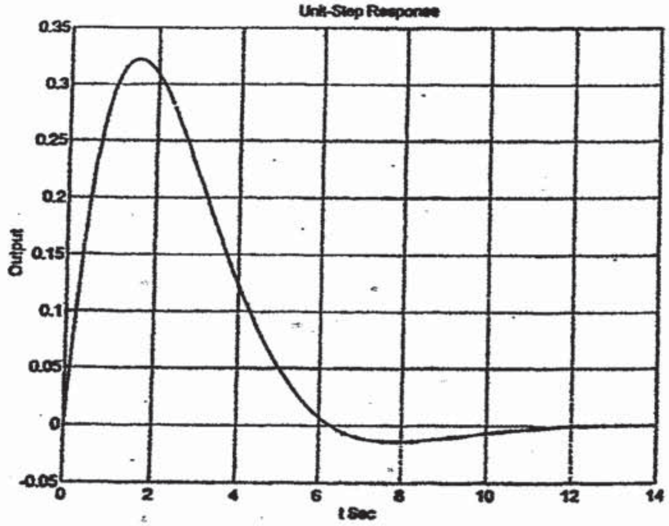






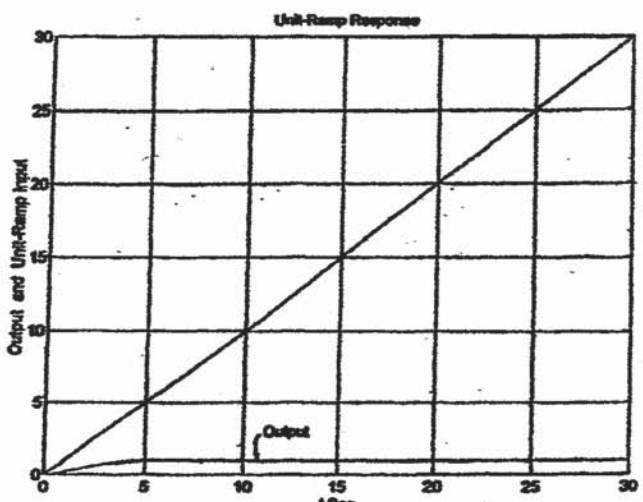
B-5-11. A MATLAB program to obtain a unit-step response of the given system is given below. The resulting unit-step response curve is shown on the next page.

```
% ***** Unit-Step Response *****

A = [-1  -0.5;1   0];
B = [0.5;0];
C = [1   0];
D = [0];
[y,x,t] = step(A,B,C,D);
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('Output')
```



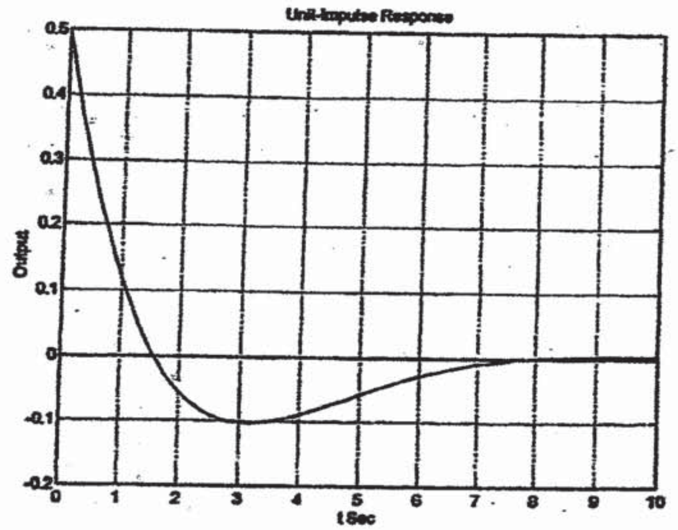
A MATLAB program to obtain a unit-ramp response of the given system is presented below, together with the unit-ramp response curve.

```
% ***** Unit-ramp response *****

A = [-1   -0.5;1   0];
B = [0.5;0];
C  = [1   0];
D = [0];

% ***** Enter matrices AA, BB, CC, and DD of the new enlarged state
% equation and output equation *****

AA = [A   zeros(2,1);C   0];
BB = [B;0];
CC = [0   0   1];
DD = [0];

% ***** Enter step-response command [z,x,t] = step(AA,BB,CC,DD) *****

[z,x,t] = step(AA,BB,CC,DD);
x3 = [0   0   1]*x'; plot(t,x3,t,t,'--')
grid
title('Unit-Ramp Response')
xlabel('t Sec')
ylabel('Output  and  Unit-Ramp Input')
text(11,3,'Output')
```



Finally, a MATLAB program to obtain a unit-impulse response of the system is given next, together with the unit-impulse response curve.

```
% ***** Unit-impulse response *****

A = [-1   -0.5;1   0];
B = [0.5;0];
C = [1   0];
D = [0];
impulse(A,B,C,D)
```



B-5-12. From the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{36}{s^2+2s+36} = \frac{36}{(s+1)^2+(\sqrt{35})^2}$$

we find that $\omega_n = 6$, $\zeta = \frac{1}{6}$, and $\omega_d = \sqrt{35}$.

Rese time:

$$t_r = \frac{\pi - \beta}{\omega_d}$$

where

$$\beta = \tan^{-1}\frac{\omega_d}{\zeta\omega_n} = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta} = \tan^{-1}\frac{\sqrt{\frac{35}{36}}}{\frac{1}{6}}$$

$$= \tan^{-1} 5.9161 = 1.4034 \text{ rad}$$

Hence

$$t_r = \frac{3.1416 - 1.4034}{\sqrt{35}} = 0.2938 \text{ sec}$$

Peak time:

$$t_p = \frac{\pi}{\omega_d} = \frac{3.1416}{\sqrt{35}} = 0.5310 \text{ sec}$$

Maximum overshoot:

$$M_p = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\pi} = e^{-\frac{\pi}{\sqrt{35}}} = e^{-0.5310}$$

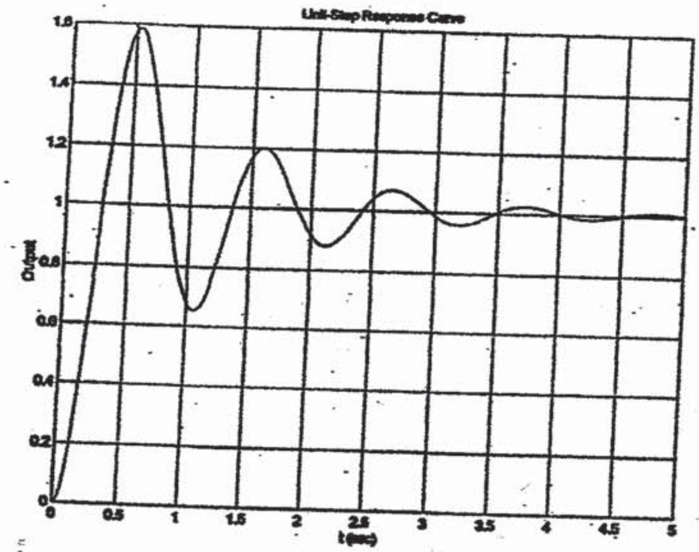
$$= 0.5880$$

Settling time (2% criterion):

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{\frac{1}{6}\times 6} = 4 \text{ sec}$$

A MATLAB program to obtain the rise time, peak time, maximum overshoot, and settling time is shown below. The unit-step response curve of this system is shown on the next page.

```
num = [0  0  36];
den = [1  2  36];
t = 0:0.001:5;
[y,x,t] = step(num,den,t);
r = 1; while y(r) <1.0001; r = r+1; end
rise_time = (r-1)*0.001

rise_time =

   0.2940

[ymax,tp] = max(y);
peak_time = (tp-1)*0.001

peak_time =

   0.5310

max_overshoot = ymax-1

max_overshoot =

   0.5880

s = 5001; while y(s)>0.98 & y(s)<1.02; s = s-1; end;
settling_time = (s-1)*0.001

settling_time =

   3.8210
```



B-5-13. The closed-loop transfer function of System I is

$$\frac{C_I(s)}{R(s)} = \frac{1}{s^2 + 0.2s + 1}$$

The closed-loop transfer function of System II is

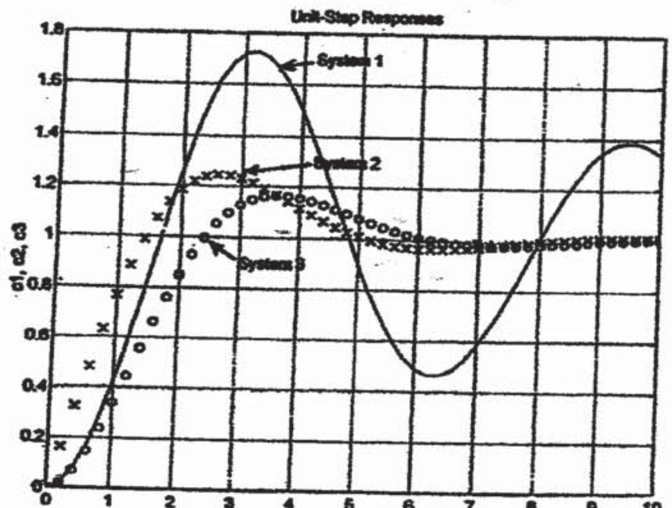
$$\frac{C_{II}(s)}{R(s)} = \frac{1 + 0.8s}{s^2 + s + 1}$$

The closed-loop transfer function of System III is

$$\frac{C_{III}(s)}{R(s)} = \frac{1}{s^2 + s + 1}$$

The unit-step response curves for the three systems are shown in Figure 1. The system utilizing proportional-plus-derivative control action exhibits the shortest rise time. The system with velocity feedback has the least maximum overshoot, or the best relative stability, of the three systems.

Figure 1



The unit-impulse response curves for the three systems are shown in Figure 2.

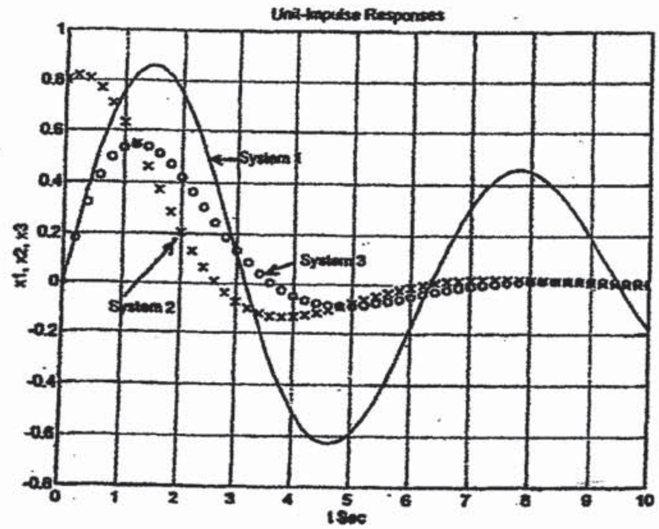


Figure 2

The unit-ramp response curves for the three systems are shown in Figure 3. System II has the advantage of quicker response and less steady error in following a ramp input.

The main reason why the System II that utilizes proportional-plus-derivative control action has superior response chracteristics is that derivative control responds to the rate of change of the error signal and can produce early corrective action before the magnitude of the error becomes large. Notice that the output of System III is the output of System II delayed by a first-order lag term 1/(1 + 0.8s).

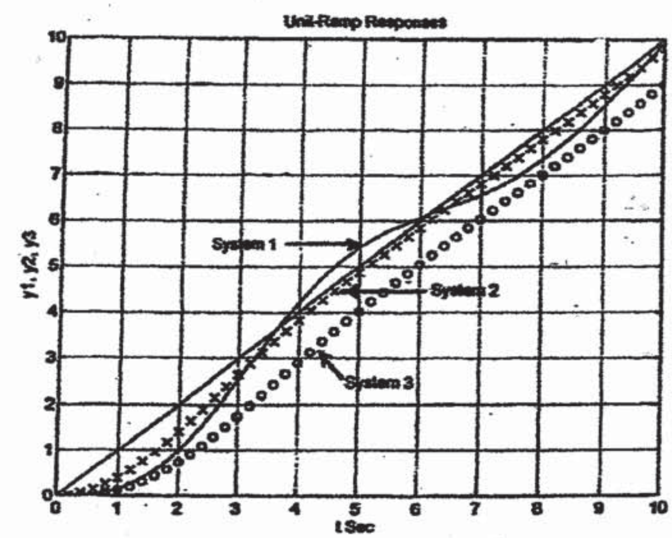


Figure 3

The MATLAB program that used to obtain Figures 1, 2, and 3 is shown below.

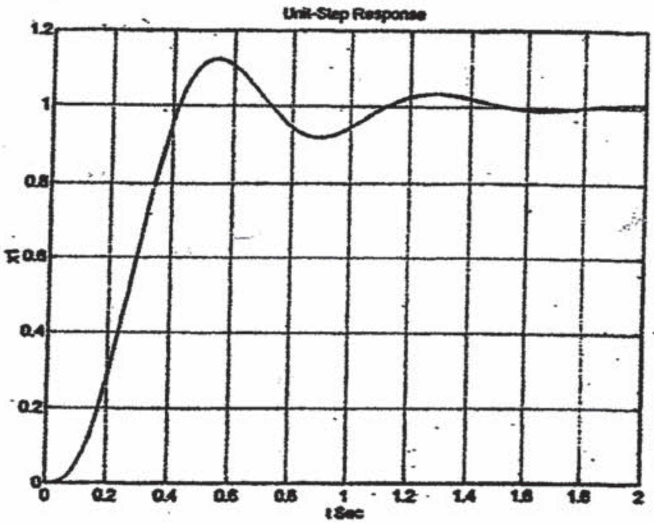
```
% ---- Obtaining unit-step, unit-impulse, and unit-ramp responses ----
% ***** Unit-step responses of three systems *****
num1 = [0  0  1];
den1 = [1  0.2  1];
num2 = [0  0.8  1];
den2 = [1  1  1];
num3 = [0  0  1];
den3 = [1  1  1];
c1 = step(num1,den1,t);
c2 = step(num2,den2,t);
c3 = step(num3,den3,t);
plot(t,c1,'-',t,c2,'x', t,c3,'o')
grid
title('Unit-Step Responses')
xlabel('t Sec')
ylabel('c1, c2, c3')
text(4.2,1.7,'System 1')
text(4.2,1.3,'System 2')
text(3,0.9,'System 3')
% ***** Unit-impulse responses of three systems *****
x1 = impulse(num1,den1,t);
x2 = impulse(num2,den2,t);
x3 = impulse(num3,den3,t);
plot(t,x1,'-',t,x2,'x', t,x3,'o')
grid
title('Unit-Impulse Responses')
xlabel('t Sec')
ylabel('x1, x2, x3')
text(3,0.5,'System 1')
text(0.8,-0.1,'System 2')
text(4.1,0.1,'System 3')
% ***** Unit-ramp responses of three systems *****
num1r = [0  0 0  1];
den1r = [1  0.2  1  0];
num2r = [0  0  0.8  1];
den2r = [1  1  1  0];
num3r = [0  0  0  1];
den3r = [1  1  1  0];
y1 = step(num1r,den1r,t);
y2 = step(num2r,den2r,t);
y3 = step(num3r,den3r,t);
plot(t,t,'--',t,y1,'-',t,y2,'x', t,y3,'o')
grid
title('Unit-Ramp Responses')
xlabel('t Sec')
ylabel('y1, y2, y3')
text(2.5,5.5,'System 1')
text(6.2,4.5,'System 2')
text(4.8,2.5,'System 3')
```

B-5-14. The closed-loop transfer function of the system is

$$\frac{X_1(s)}{R(s)} = \frac{40}{0.1s^3 + s^2 + 10s + 40}$$

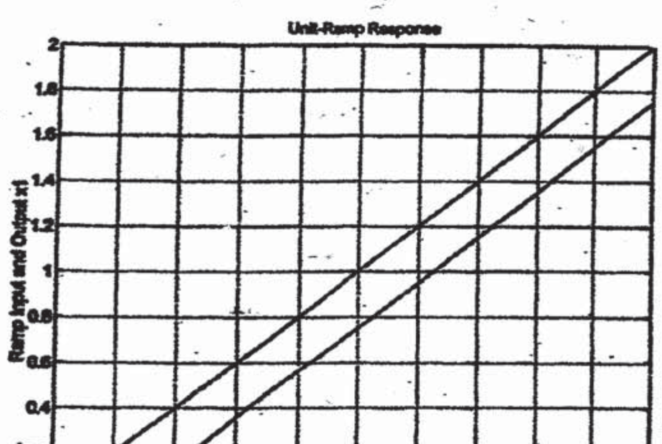
MATLAB program to obtain the unit-step response curve is given below, together with the unit-step response curve.

```
% ***** Unit-step response *****

num = [0  0  0  40];
den = [0.1  1  10  40];
t = 0:0.01:2;
x1 = step(num,den,t);
plot(t,x1,'-')
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('x1')
```



A MATLAB program to obtain the unit-ramp response curve is given below. The resulting unit-ramp response curve is also shown.

```
% ***** Unit-ramp response *****

numr = [0  0  0  0  40];
denr = [0.1  1  10  40  0];
t = 0:0.01:2;
y1 = step(numr,denr,t);
plot(t,t,'--',t,y1,'-')
grid
title('Unit-Ramp Response')
xlabel('t Sec')
ylabel('Ramp Input and Output x1')
```

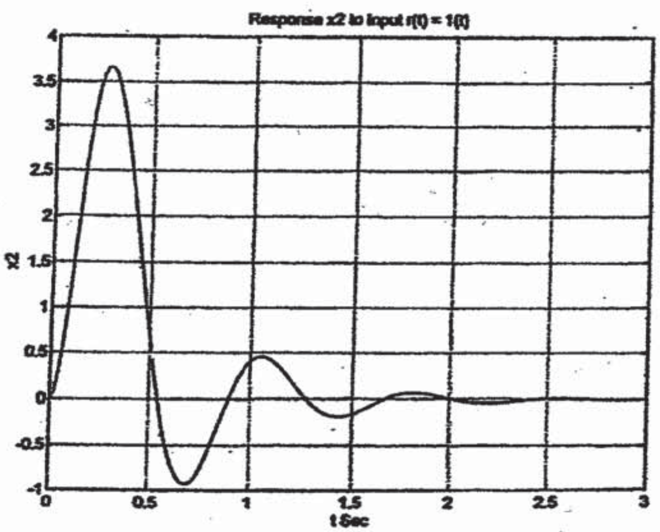


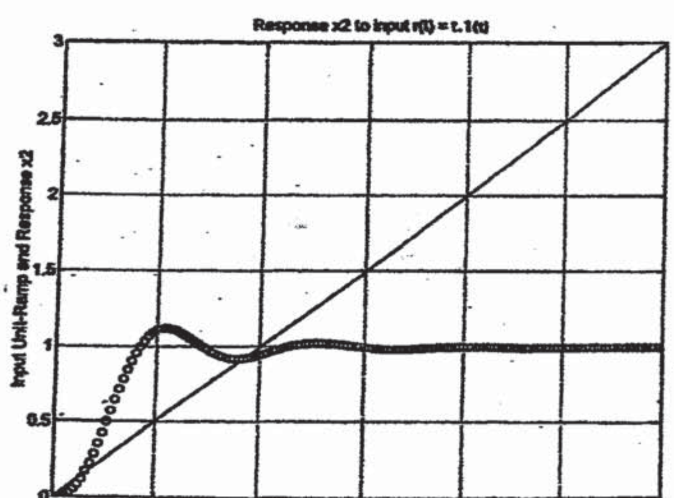
Noting that $x_2 = \frac{d}{dt} x_1$, we have

$$\frac{X_2(s)}{R(s)} = \frac{40s}{0.1s^3 + s^2 + 10s + 40}$$

The response $x_2(t)$ for the unit-step input and that for the unit-ramp input can be obtained by using the MATLAB program given on the next page. The resulting response curves [$x_2(t)$ versus t curves] are also shown on the next page.

```
% ***** MATLAB program to obtain responses x2 to inputs r(t) = 1(t) and
% r(t) = t.1(t) *****

num2 = [0  0  40  0];
den2 = [0.1  1  10  40];
t = 0:0.02:3;
x2 = step(num2,den2,t);
plot(t,x2)
grid
title('Response x2 to Input r(t) = 1(t)')
xlabel('t Sec')
ylabel('x2')

num2r = [0  0  0  40  0];
den2r = [0.1  1  10  40  0];
y2 = step(num2r,den2r,t);
plot(t,t,'--',t,y2,'o');
grid
title('Response x2 to input r(t) = t.1(t)')
xlabel('t Sec')
ylabel('Input Unit-Ramp and Response x2')
```





Next, we shall obtain $x_3(t)$ versus t curves for the unit-step input and unit-ramp input. Noting that

$$\frac{X_2(s)}{X_3(s)} = \frac{10}{0.1s+1}$$

and

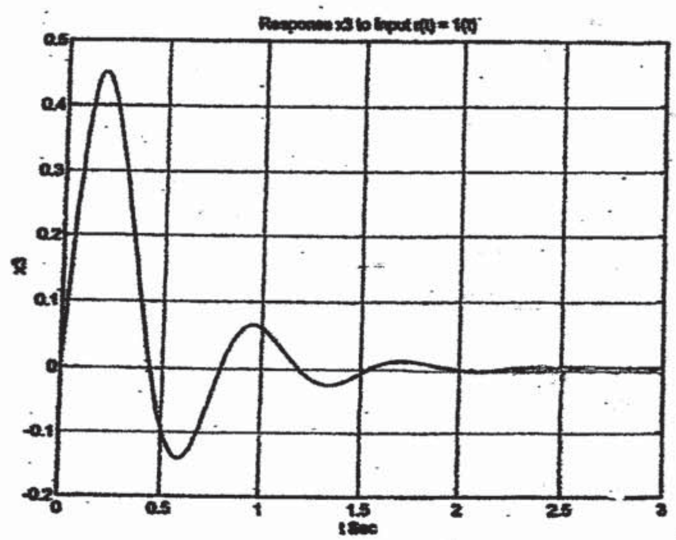
$$\frac{X_2(s)}{R(s)} = \frac{40s}{0.1s^3+s^2+10s+40}$$

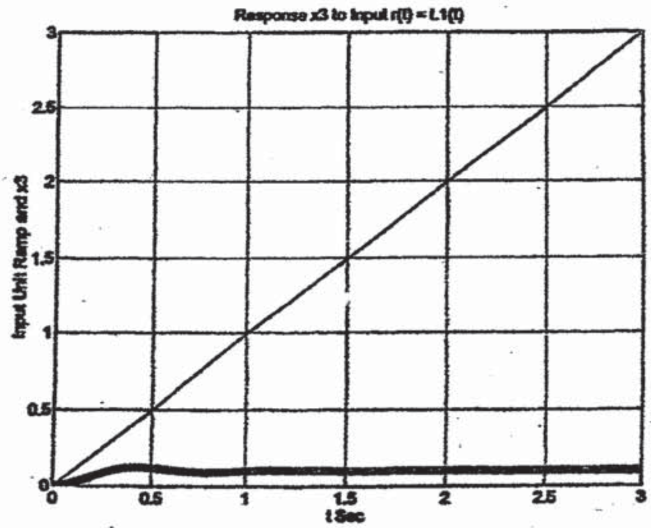
we have

$$\frac{X_3(s)}{R(s)} = \frac{X_3(s)}{X_2(s)} \cdot \frac{X_2(s)}{R(s)} = \frac{4s^2+40s}{s^3+10s^2+100s+400}$$

The following MATLAB program can be used to obtain responses $x_3(t)$ to inputs $r(t) = 1(t)$ and $r(t) = t \cdot 1(t)$. The response curves are shown below and on the next page.

```
% ***** MATLAB program to obtain response x3 to inputs
% r(t) = 1(t) and r(t) = t.1(t) *****

num3 = [0  4  40  0];
den3 = [1  10  100  400];
t = 0:0.01:3;
x3 = step(num3,den3,t);
plot(t,x3);
grid
title('Response x3 to Input r(t) = 1(t)')
xlabel('t Sec')
ylabel('x3')

num3r = [0  0  4  40  0];
den3r = [1  10  100  400  0];
y3 = step(num3r,den3r,t);
plot(t,t,'--',t,y3,'o')
grid
title('Response x3 to Input r(t) = t.1(t)')
xlabel('t Sec')
ylabel('Input Unit Ramp and x3')
```

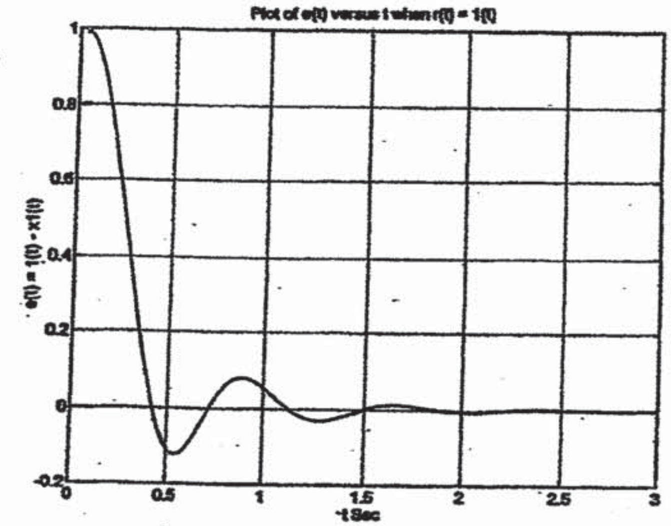


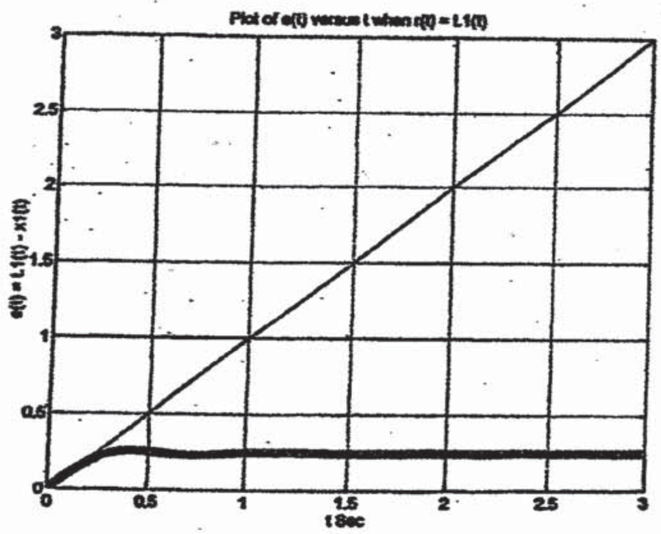


Finally, we shall obtain the error versus t curves. Plots of e(t) versus t curves when the input r(t) is a unit step or unit ramp can be obtained by use of the following MATLAB program.

```
% ----- MATLAB program to obtain e(t) versus t curves -----

% ***** Unit-step response *****

num = [0  0  0  40];
den = [0.1   1   10   40];
t = 0: 0.01:3;
x1 = step(num,den,t);
plot(t, 1 - x1);
grid
title('Plot of e(t) versus t when r(t) = 1(t)')
xlabel('t Sec')
ylabel('e(t) = 1(t) - x1(t)')

% ***** Unit-ramp response *****

numr = [0  0  0  0   40];
denr = [0.1   1   10   40   0];
y1 = step(numr,denr,t);
plot(t,t,'--',t,t' - y1,'o')
grid
title('Plot of e(t) versus t when r(t) = t.1(t)')
xlabel('t Sec')
ylabel('e(t) = t.1(t) - x1(t)')
```

The error e(t) versus t curves are shown on the next page.





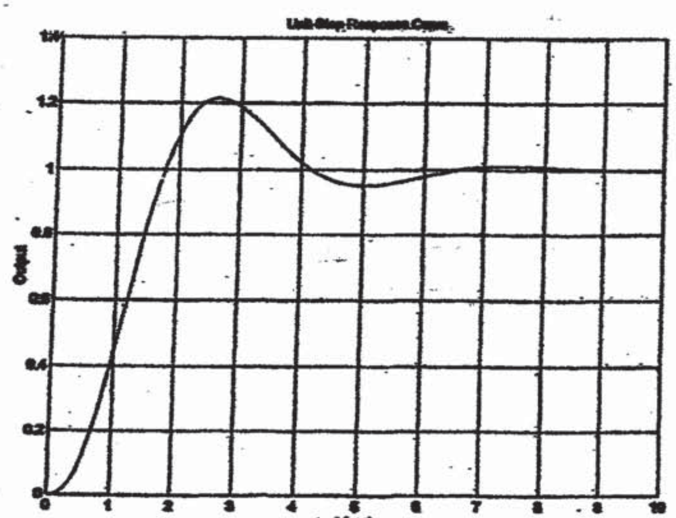
B-5-15. The closed-loop transfer function C(s)/R(s) of this system is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{\frac{10}{s(s+2)(s+4)}}{1+\frac{10}{s(s+2)(s+4)}}$$

$$= \frac{10}{s^3+6s^2+8s+10}$$

A MATLAB program to obtain the unit-step response curve as well as the rise time, peak time, maximum overshoot, and settling time is shown on the next page. The unit-step response curve is also shown on the next page.

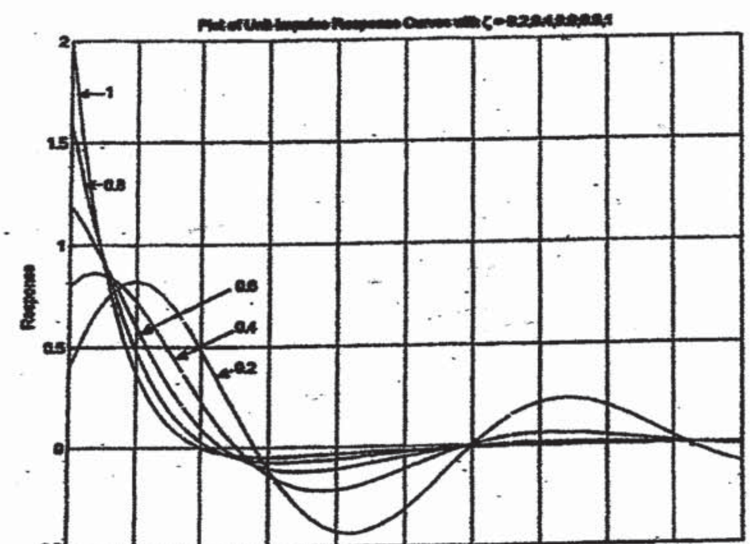
```
num = [0  0  0  10];
den = [1  6  8  10];
t = 0:0.002:10;
[y,x,t] = step(num,den,t);
plot(t,y)
grid
title('Unit-Step Response Curve')
xlabel('t (sec)')
ylabel('Output')

r = 1; while y(r) <1.0001; r = r+1; end
rise_time = (r-1)*0.002

rise_time =

   1.7720

[ymax,tp] = max(y);
peak_time = (tp-1)*0.002

peak_time =

   2.6320

max_overshoot = ymax-1

max_overshoot =

   0.2146

s = 5001; while y(s)>0.98 & y(s)<1.02; s = s-1; end;
settling_time = (s-1)*0.002

settling_time =

   5.9960
```

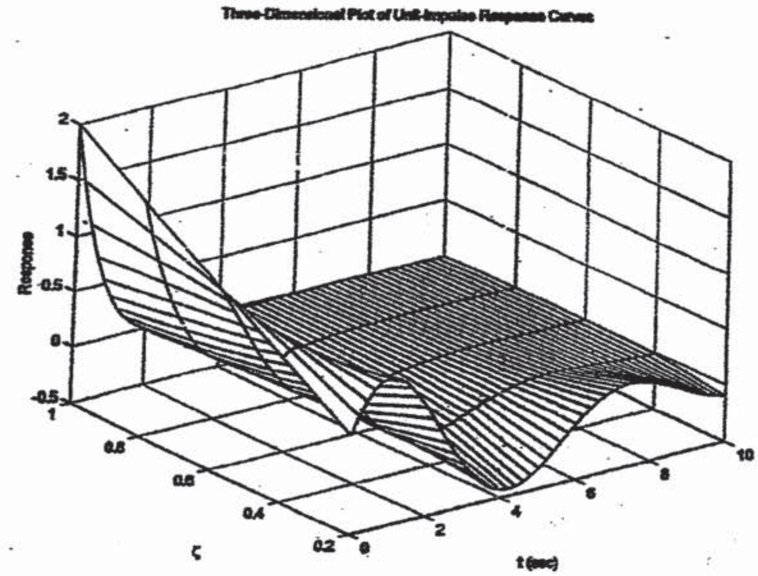


B-5-16. A MATLAB program that produces a two-dimensional diagram of unit-impulse response curves and a three-dimensional plot of the response curves is given below.

```
% To plot a Two-Dimensional Diagram.

t = 0:0.2:10;
zeta = [0.2  0.4  0.6  0.8  1];
    for n = 1:5;
    num = [0  2*zeta(n)  1];
    den = [1  2*zeta(n)  1];
    [y(1:51,n),x,t] = impulse(num,den,t);
    end
plot(t,y)
grid
title('Plot of Unit-Impulse Response Curves with \zeta = 0.2,0.4,0.6,0.8,1')
xlabel('t (sec)')
ylabel('Response')
text(2.5,0.4,'0.2')
text(2.5,0.6,'0.4')
text(2.5,0.8,'0.6')
text(0.5,1.3,'0.8')
text(0.5,1.75,'1')

% To plot a Three-Dimensional Diagram.

mesh(t,zeta,y')
title('Three-Dimensional Plot of Unit-Impulse Response Curves')
xlabel('t (sec)')
ylabel('\zeta')
zlabel('Response')
```

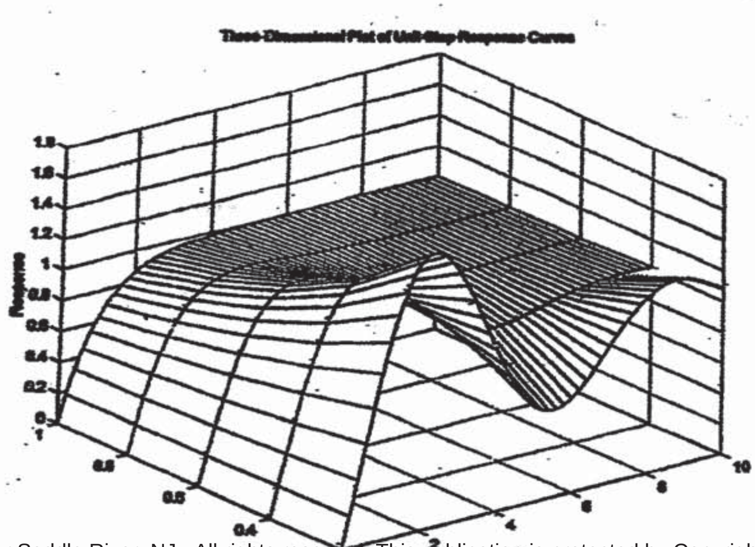
The two-dimensional diagram and three-dimensional diagram produced by this MATLAB program are shown below and on the next page, respectively.





B-5-17. A MATLAB program to produce a three-dimensional diagram of the unit-step response curves is shown below. The resulting three-dimensional plot is also shown below.

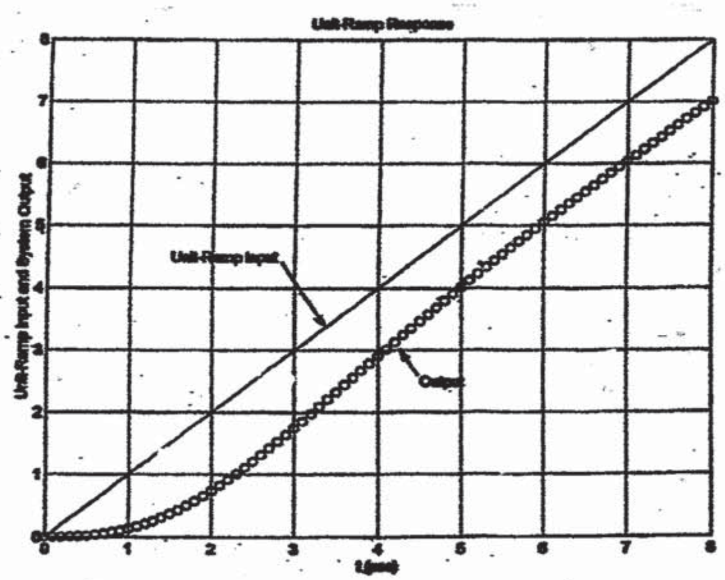
```
t = 0:0.2:10;
zeta = [0.2  0.4  0.6  0.8  1];
   for n = 1:5;
   num = [0  1  1];
   den = [1  2*zeta(n)  1];
   [y(1:51,n),x,t] = step(num,den,t);
   end
mesh(t,zeta,y')
title('Three-Dimensional Plot of Unit-Step Response Curves')
xlabel('t (sec)')
ylabel('\zeta')
zlabel('Response')
```



B-5-18. A MATLAB program to obtain the unit-ramp response curve of the given system is given below.

```
% MATLAB Program for Problem B-5-18

A = [0  1;-1  -1];
B = [0;1];
C = [1  0];
D = 0;
t = 0:0.1:8;
u = t;
y = lsim(A,B,C,D,u,t);
plot(t,u,'-',t,y,'o')
grid
title('Unit-Ramp Response')
xlabel('t (sec)')
ylabel('Unit-Ramp Input and System Output')
text(1.5,4.5,'Unit-Ramp Input')
text(4.5,2.5,'Output')
```

The resulting response curve is shown below, together with the unit-ramp input.



B-5-19. By taking the Laplace transform of the differential equation:

$$\ddot{y} + 3\dot{y} + 2y = 0, \qquad y(0) = 0.1, \qquad \dot{y}(0) = 0.05$$

we obtain

$$s^2Y(s) - sy(0) - \dot{y}(0) + 3[sY(s) - y(0)] + 2Y(s) = 0$$

By substituting the given initial condition, we get

$$(s^2 + 3s + 2)Y(s) = 0.1s + 0.35$$

Solving this last equation for Y(s), we obtain

$$Y(s) = \frac{0.1s + 0.35}{s^2 + 3s + 2} = \frac{0.1s + 0.35}{(s+1)(s+2)} = \frac{0.25}{s+1} - \frac{0.15}{s+2}$$

The inverse Laplace transform of Y(s) gives

$$y(t) = 0.25e^{-t} - 0.15e^{-2t}$$

This is the solution of the given differential equation.

MATLAB solution:

Let us obtain a state space equation for the system. Define

$$x_1 = y$$

$$x_2 = \dot{y}$$

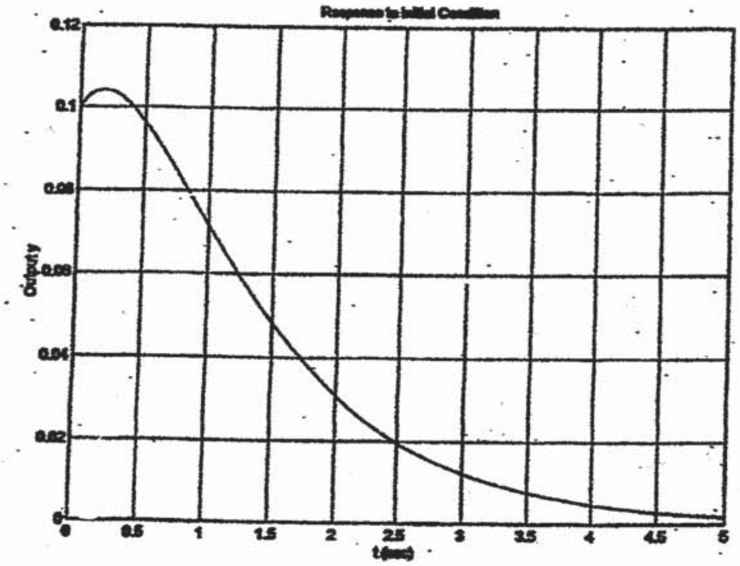
Then, the state space equation and the output equation become as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix}$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

A possible MATLAB program to obtain the response y(t) is given in the following. The resulting response curve is shown below.

```
A=[0  1;-2  -3];
B=[0; 0];
C=[1  0];
D=0;
t=0:0.01:5;
y=initial(A,B,C,D,[0.1;0.05],t);
plot(t,y)
grid
title('Response to Initial Condition')
xlabel('t (sec)')
ylabel('Output y')
```



B-5-20.

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+1)(s+2)+K}$$

The characteristic equation is

$$s^3 + 3s^2 + 2s + K = 0$$

The Routh array becomes

$$\begin{array}{lll} s^3 & 1 & 2 \\ s^2 & 3 & K \\ s^1 & \frac{6-K}{3} & \\ s^0 & K & \end{array}$$

For stability we require $6 > K$ and $K > 0$, or

$$6 > K > 0$$

B-5-21. For the characteristic equation

$$s^4 + 2s^3 + (4+K)s^2 + 9s + 25 = 0$$

the Routh array of coefficients is

$$\begin{array}{llll}
s^4 & 1 & 4+K & 25 \\
s^3 & 2 & 9 & 0 \\
s^2 & \frac{2K-1}{2} & 25 & \\
s^1 & \frac{18K-109}{2K-1} & 0 & \\
s^0 & 25 & &
\end{array}$$

For stability, we require

$$\frac{2K-1}{2} > 0 \quad , \quad \frac{18K-109}{2K-1} > 0$$

or

$$K > 0.5 \quad , \quad 18K > 109$$

Hence

$$K > \frac{109}{18} = 6.056$$

For stability, K must be greater than 109/18.

B-5-22. The closed-loop transfer function C(s)/R(s) is

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+1)(s^2+6s+25)+K(s-2)}$$

$$= \frac{K(s-2)}{s^3+7s^2+(31+K)s+25-2K}$$

For stability, the denominator of this last equation must be a stable polynomial. For the characteristic equation

$$s^3+7s^2+(31+K)s+25-2K=0$$

the Routh array becomes as follows:

$$\begin{array}{lll}
s^3 & 1 & 31+K \\
s^2 & 7 & 25-2K \\
s^1 & \frac{192+9K}{7} & 0 \\
s^0 & 25-2K &
\end{array}$$

Since K is assumed to be positive, for stability, we require

$$12.5 > K > 0$$

B-5-23. From Figure 5-80(b) we have

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + KK_h s + K} = \frac{\frac{K}{J}}{s^2 + \frac{KK_h}{J} s + \frac{K}{J}}$$

By substituting K/J = 4 into this last equation, we obtain

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4K_h s + 4}$$

Since $\omega_n = 2$, $\zeta = 0.6$, and $2\zeta\omega_n = 4K_h$, we have

$$K_h = \frac{2\zeta\omega_n}{4} = 0.6$$

B-5-24. From the block diagram of Figure 5-81, we have

$$\frac{C(s)}{R(s)} = \frac{20K}{s^3 + 5s^2 + (4 + 20KK_h)s + 20K}$$

The stability of this system is determined by the denominator polynomial (characteristic polynomial). The Routh array of the characteristic equation

$$s^3 + 5s^2 + (4 + 20KK_h)s + 20K = 0$$

is

$$\begin{array}{lll} s^3 & 1 & 4 + 20KK_h \\ s^2 & 5 & 20K \\ s^1 & 4 + 20KK_h - 4K & 0 \\ s^0 & 20K & \end{array}$$

For stability, we require

$$4 + 20KK_h - 4K > 0, \qquad 20K > 0$$

or

$$5KK_h > K - 1, \qquad K > 0$$

The stable region in the K-K_h plane is the region that satisfies these two inequalities. Figure 1 shows the stable region in the K-K_h plane. If a point in the K-K_h plane (that is, a combination of K and K_h values) lies in the shaded region, then the system is stable. Conversely, if a point in the K-K_h plane lies in the nonshaded region, the system is unstable. The dividing curve is defined by $5KK_h \doteq K - 1$. (Any point above this dividing curve corresponds to a stable combination of K and K_h.)

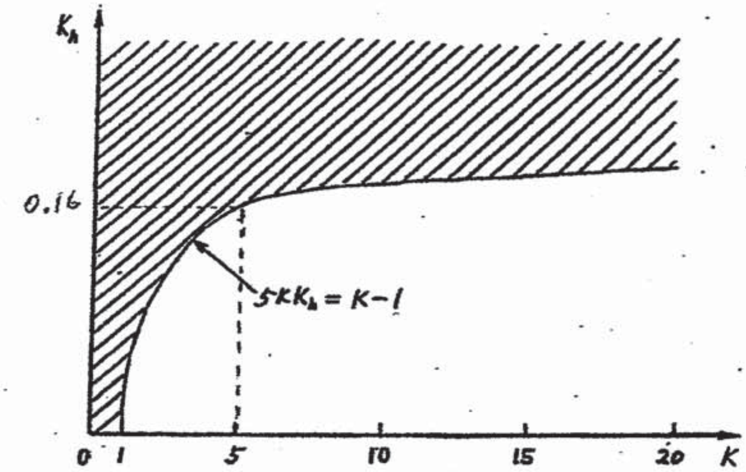


Figure 1

B-5-25.

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & -1 & 0 \\ b_3 & s & -1 \\ 0 & b_2 & s+b_1 \end{vmatrix}$$

$$= s^3 + b_1 s^2 + (b_2 + b_3) s + b_1 b_3$$

The Routh array is

$$\begin{array}{lll} s^3 & 1 & b_2 + b_3 \\ s^2 & b_1 & b_1 b_3 \\ s^1 & b_2 & 0 \\ s^0 & b_1 b_3 & \end{array}$$

Thus, the first column of the Routh array of the characteristic equation consists of 1, b_1, b_2, and $b_1 b_3$.

B-5-26.

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$

Hence

$$(s^2+as+b)\, G(s) = (Ks+b)[1+G(s)]$$

or

$$G(s) = \frac{Ks+b}{s(s+a-K)}$$

The steady-state error in the unit-ramp response is

$$e_{ss} = \frac{1}{K_v} = \lim_{s \to 0} \frac{1}{sG(s)} = \lim_{s \to 0} \frac{s(s+a-K)}{s(Ks+b)} = \frac{a-K}{b}$$

B-5-27. The closed-loop transfer function is

$$= \frac{K}{Js^2+Bs+K}$$

For a unit-ramp input, $R(s) = 1/s^2$. Thus,

$$\frac{E(s)}{R(s)} = \frac{R(s)-C(s)}{R(s)} = \frac{Js^2+Bs}{Js^2+Bs+K}$$

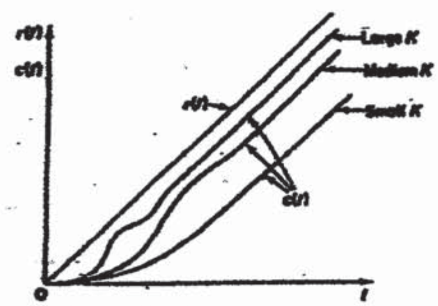
or

$$E(s) = \frac{Js^2+Bs}{Js^2+Bs+K}\,\frac{1}{s^2}$$

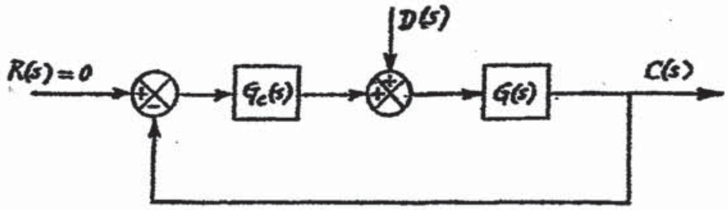
The steady-state error is

$$e_{ss} = e(\infty) = \lim_{s \to 0} sE(s) = \frac{B}{K}$$

We see that we can reduce the steady-state error e_{ss} by increasing the gain K or decreasing the viscous-friction coefficient B. Increasing the gain or decreasing the viscous-friction coefficient, however, causes the damping ratio to decrease, with the result that the transient response of the system will become more oscillatory. Doubling K decreases e_{ss} to half of its original value, while ζ is decreased to 0.707 of its original value since ζ is inversely proportional to the square root of K. On the other hand, decreasing B to half of its original value decreases both e_{ss} and ζ to the halves of their original values, respectively. Therefore, it is advisable to increase the value of K rather than to decrease the value of B. After the transient response has died out and a steady state is reached, the output velocity becomes the same as the input velocity. However, there is a steady-state positional error between the input and the output. Examples of the unit-ramp response of the system for three different values of K are illustrated to the right.



B-5-28 . Consider the system shown below.



From the diagram we obtain

$$\frac{C(s)}{D(s)} = \frac{G(s)}{1 + G(s)G_c(s)}$$

For a ramp disturbance $d(t) = at$, we have $D(s) = a/s^2$. Hence,

$$c(\infty) = \lim_{s\to 0} sC(s) = \lim_{s\to 0} \frac{sG(s)}{1+G(s)G_c(s)} \frac{a}{s^2} = \lim_{s\to 0} \frac{a}{sG_c(s)}$$

$c(\infty)$ becomes zero if $G_c(s)$ contains double integrators.