Exercise 6.118

L **Answer (c).** 1 2

We are given $h(t) = (\pi t)^{-1}$ and $x(t) = 1 - \frac{1}{2}\cos(2t) + \frac{1}{3}\sin(3t)$. First, we find the Fourier transform X of x. Taking the Fourier transform of x, we obtain

take FT of 2

$$\begin{aligned}
X(\omega) &= \mathcal{F}\{1\}(\omega) - \tfrac{1}{2}\mathcal{F}\{\cos(2\cdot)\}(\omega) + \tfrac{1}{3}\mathcal{F}\{\sin(3\cdot)\}(\omega) \\
&= 2\pi\delta(\omega) - \tfrac{1}{2}(\pi[\delta(\omega-2)+\delta(\omega+2)]) + \tfrac{1}{3}\left(\tfrac{\pi}{j}[\delta(\omega-3)-\delta(\omega+3)]\right) \\
&= 2\pi\delta(\omega) - \tfrac{\pi}{2}\delta(\omega-2) - \tfrac{\pi}{2}\delta(\omega+2) + \tfrac{\pi}{j3}\delta(\omega-3) - \tfrac{\pi}{j3}\delta(\omega+3).
\end{aligned}$$

from FT table

multiply

3

Next, we find the Fourier transform H of h. As our starting point, we use the Fourier transform pair

$$\operatorname{sgn}(t) \overset{\text{CTFT}}{\longleftrightarrow} \frac{2}{j\omega}.$$

duality

From this pair, we can use properties of the Fourier transform to write

$$\begin{aligned}
\frac{2}{jt} &\overset{\text{CTFT}}{\longleftrightarrow} 2\pi\operatorname{sgn}(-\omega) = -2\pi\operatorname{sgn}(\omega) \Rightarrow \\
\left(\frac{j}{2\pi}\right)\left(\frac{2}{jt}\right) &\overset{\text{CTFT}}{\longleftrightarrow} -\left(\frac{j}{2\pi}\right)(2\pi)\operatorname{sgn}(\omega) \Rightarrow \\
\frac{1}{\pi t} &\overset{\text{CTFT}}{\longleftrightarrow} -j\operatorname{sgn}(\omega).
\end{aligned}$$

linearity

simplify

4

Thus, we have

from 4

$$H(\omega) = -j\operatorname{sgn}(\omega).$$

5

Since the system is LTI, we know that $Y(\omega) = X(\omega)H(\omega)$. So, we have

substitute 3 and 5

$$\begin{aligned}
Y(\omega) &= X(\omega)H(\omega) \\
&= \left[2\pi\delta(\omega) - \tfrac{\pi}{2}\delta(\omega-2) - \tfrac{\pi}{2}\delta(\omega+2) + \tfrac{\pi}{j3}\delta(\omega-3) - \tfrac{\pi}{j3}\delta(\omega+3)\right][-j\operatorname{sgn}(\omega)] \\
&= -j2\pi\operatorname{sgn}(\omega)\delta(\omega) + \tfrac{j\pi}{2}\operatorname{sgn}(\omega)\delta(\omega-2) + \tfrac{j\pi}{2}\operatorname{sgn}(\omega)\delta(\omega+2) - \tfrac{\pi}{3}\operatorname{sgn}(\omega)\delta(\omega-3) \\
&\quad + \tfrac{\pi}{3}\operatorname{sgn}(\omega)\delta(\omega+3) \\
&= \tfrac{j\pi}{2}\operatorname{sgn}(2)\delta(\omega-2) + \tfrac{j\pi}{2}\operatorname{sgn}(-2)\delta(\omega+2) - \tfrac{\pi}{3}\operatorname{sgn}(3)\delta(\omega-3) + \tfrac{\pi}{3}\operatorname{sgn}(-3)\delta(\omega+3) \\
&= \tfrac{j\pi}{2}\delta(\omega-2) - \tfrac{j\pi}{2}\delta(\omega+2) - \tfrac{\pi}{3}\delta(\omega-3) - \tfrac{\pi}{3}\delta(\omega+3) \\
&= -\tfrac{1}{2}\left(\tfrac{\pi}{j}[\delta(\omega-2)-\delta(\omega+2)]\right) - \tfrac{1}{3}(\pi[\delta(\omega-3)+\delta(\omega+3)]).
\end{aligned}$$

multiply

equivalence property

evaluate sgn(...)

rewrite for easier FT table lookup

Taking the inverse Fourier transform of Y, we obtain

$$\begin{aligned}
y(t) &= -\tfrac{1}{2}\mathcal{F}^{-1}\left\{\tfrac{\pi}{j}[\delta(\omega-2)-\delta(\omega+2)]\right\}(t) - \tfrac{1}{3}\mathcal{F}^{-1}\{\pi[\delta(\omega-3)+\delta(\omega+3)]\}(t) \\
&= -\tfrac{1}{2}\sin(2t) - \tfrac{1}{3}\cos(3t).
\end{aligned}$$

take inverse FT

FT table for sin, cos