CSC 320 - Tutorial 5

1. Context Free Grammars
2. Context Free Languages

Context Free Grammar

A context free grammar is a 4-tuple (V, Σ, R, S)

- V: is a finite set of **variables**
- Σ: is a finite set of **terminals** - disjoint from V
- R: finite set of **rules**
- $S \in V$ the **start variable**

uAv **yields** uwv, written as $uAv \Rightarrow uwv$

   Means you can get from uAv to uwv in one "step" by applying a rule on A

u **derives** v, written as $u \overset{*}{\Rightarrow} v$

   Means either $u = v$

   Or starting at u then applying a series of rules, you can get to v

   (ie. there exists a sequence $u_1, u_2, u_3, \ldots, u_k$ for $k \geq 0$ such that

   $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow u_3 \Rightarrow \ldots \Rightarrow u_k \Rightarrow v$)

Questions

1. For each of the following languages over alphabet $\Sigma = \{0, 1\}$ define the CFG (4-tuple) that recognizes the language

context free → a. $L_1 = \{w \mid w$ starts and ends with the same symbol $\}$
Grammar

consider some strings in the language:
$L(0(0\cup1)^*0 \quad$ or $\quad 1(0\cup1)^*1 \quad$ or $\quad 1 \quad$ or $\quad 0)$

Rules:
$S \rightarrow 0A0 \mid 1A1 \mid 1 \mid 0 \mid \varepsilon$

$A \rightarrow 1A \mid 0A \mid \varepsilon$

4-tuple:
$V = \{S, A\}$
$\Sigma = \{0, 1\}$ * ε is never in Σ
R
$S = S$

     b. $L_2 = \varnothing$

starting at the start variable we never derive a string

Rule:
$S \rightarrow S$ * infinite loop ∴ never derives a string

4-tuple:
$V = \{S\}$
$\Sigma = \varnothing$
R
$S = S$

c. $L_3 = \{0^n1^m \mid m, n \geq 0 \text{ and } n \leq m\}$

Consider some strings in the language:

$L_3 = \{\ \varepsilon\ ,\ 01\ ,\ 1\ ,\ 0011\ ,\ 00111\ ,\ \dots\ \}$

$\qquad\qquad \downarrow$

$\qquad\quad (n = m = 0)$

if add a zero $\Rightarrow$ need to add a 1

can add as many 1s as I want

Rules:

$S \rightarrow 0S1 \mid S1 \mid \varepsilon$

4-tuple:

$V = \{S\}$ $\qquad\qquad R$

$\Sigma = \{0, 1\}$ $\qquad\qquad S = S$

d. $L_4 = \{0^n1^m \mid 2n \leq m \leq 3n\}$

$\qquad\qquad\qquad\qquad\qquad$ * similar to 1c

$n = 0 \quad \Rightarrow \quad m = 0$

$n = 1 \quad \Rightarrow \quad 2 \leq m \leq 3$ ∴ if I add a zero $\Rightarrow$ need to add

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ two or three 1s

Rules:

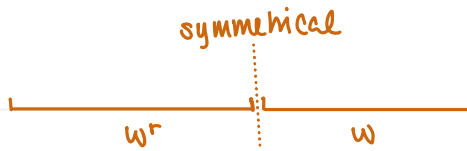$S \rightarrow 0S11 \mid 0S111 \mid \varepsilon$

4-tuple:

$V = \{S\}$

$\Sigma = \{0, 1\}$

R

$S = S$

e. $L_5 = \{w^r w\}$

w^r | w

Rules:

$S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

4-tuple:
$V = \{S\}$
$\Sigma = \{0,1\}$
R
$S = S$

2. Derive/generate the string "**aabaa**" for the following grammar:

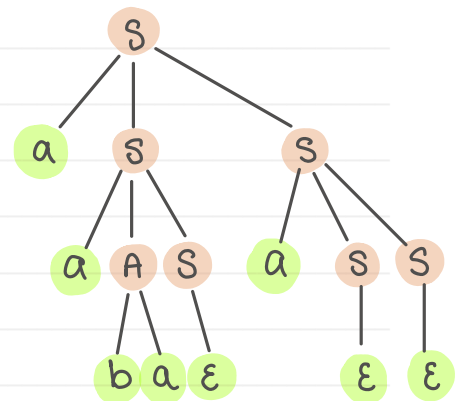$\textcircled{S} \rightarrow aAS \mid aSS \mid \varepsilon$

$A \rightarrow SbA \mid ba$

start @ the start variable
(if not explicitly told the variable on LHS of 1st rule)

$S \rightarrow a\,S\,S \rightarrow a\,a\,A\,S\,S$

$\rightarrow a\,a\,\underline{ba}\,S\,S \rightarrow a\,a\,b\,a\_S$

$\rightarrow a\,a\,b\,a\,\underline{a\,S\,S} \rightarrow a\,a\,b\,a\,a\_S$

$\rightarrow a\,a\,b\,a\,a\_$

is the string above derived ambiguously?

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