University of Victoria
Exam 5
Fall 2022

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 11002), A02 (CRN 11003)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ________________________________

Given Name(s): ________________________________

Student Number: ________________

This examination paper has **10 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 24

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

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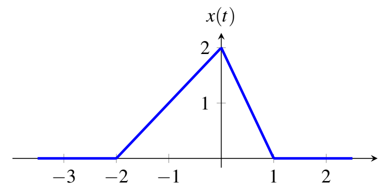
Do not write on this page unless instructed to do so.

Question 1.

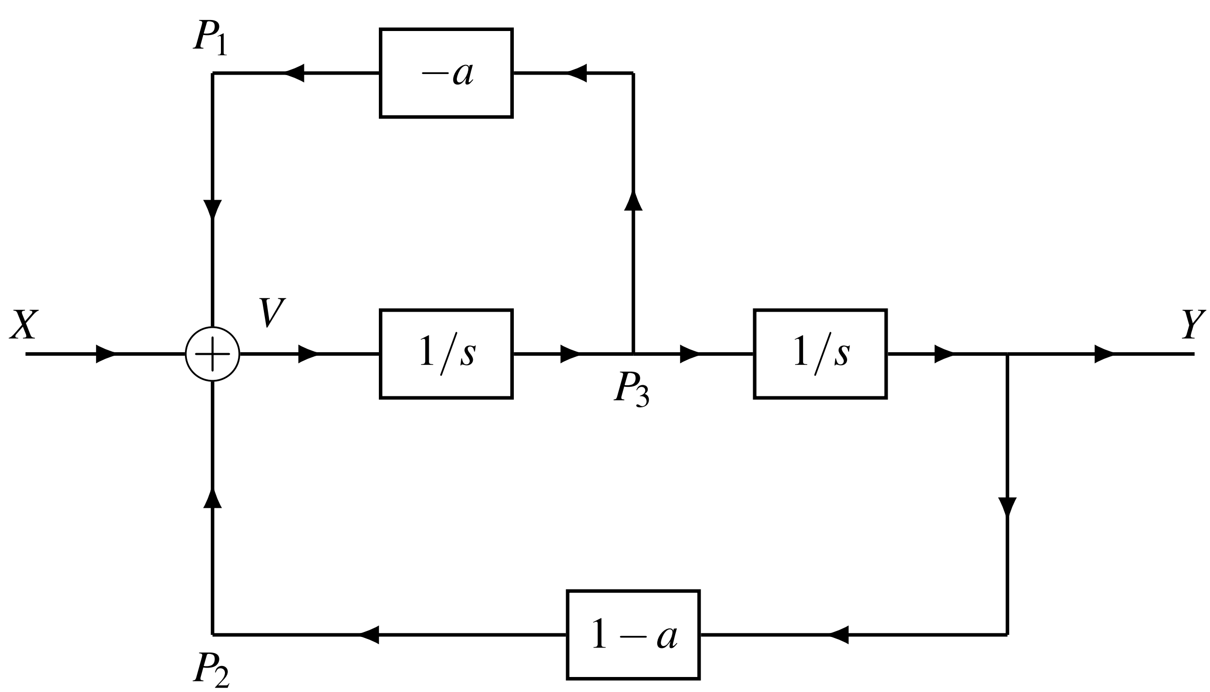
Using properties of the Laplace transform and a Laplace transform table, find the Laplace transform X of the signal x, where x is as shown in the figure to the right. Do not forget to state the **region of convergence** of X in your final answer. Your final expression for X must be **fully simplified** (e.g., it **cannot contain any derivative operators**). Your answer must be **clearly presented** and you must **show all of your work**. [Hint: Group terms before taking the Laplace transform in order to avoid the need to compute derivatives.] **[6 marks]**



Question 2.

(A) Consider an arbitrary LTI system with system function H. Clearly state the condition on H that must be satisfied for the system to be BIBO stable. **[1 mark]**

(B) Consider the system with input Laplace transform X, output Laplace transform Y, and system function H, as shown in the figure. Each subsystem in the block diagram is causal and LTI and labelled with its system function, and the Laplace transforms of several intermediate signals are labelled as V, P_1, P_2, and P_3. The symbol a appearing in the block diagram denotes a **real** constant. In order to avoid a proliferation of special cases, you can assume $a \neq 0$ and $a \neq 1$. Express each of Y, P_1, P_2, and P_3 **in terms of only the function** V **and/or constant** a. You must write each answer at the corresponding place **on the block diagram**. For example, write the answer for P_1 close to P_1 on the diagram. It is strongly advised that you **use pencil** for writing your answers in this question in order to avoid massacring the diagram if you might need to change part of an answer. **[2 marks]**



(C) Determine for what values of the real constant a the system in part (b) is BIBO stable. **Show all of your work** and **do not skip any steps**. Your solution must be **clearly presented**, otherwise marks may be deducted even if the final answer is correct. **[4 marks]**

Question 3. A (physically-realizable) LTI circuit with input v_0, output v_1, and system function H is characterized by the equations

$$v_0(t) = 2i(t) + \tfrac{1}{4}\int_{-\infty}^{t} i(\tau)d\tau + v_1(t) \quad \text{and} \quad i(t) = \tfrac{1}{4}\int_{-\infty}^{t} v_1(\tau)d\tau.$$

Find a fully simplified formula for H (including its ROC). **[6 marks]**

QUESTION 3 CONTINUED

Question 4. Find the inverse Laplace transform x of the function $X(s) = \frac{s-7}{s^2-1}$ for $-1 < \text{Re}(s) < 1$. **Show all of your work** and **do not skip any steps**. **[5 marks]**

END

Useful Formulae and Other Information

$$\mathscr{L}x(s) = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt \qquad \mathscr{L}^{-1}X(t) = x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds \qquad \mathscr{L}_{\mathrm{u}}x(s) = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}\,dt$$

$$A_k = (v-p_k)F(v)\big|_{v=p_k} \qquad A_{kl} = \frac{1}{(q_k-l)!}\left[\left[\tfrac{d}{dv}\right]^{q_k-l}\left[(v-p_k)^{q_k}F(v)\right]\right]\Big|_{v=p_k}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Bilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain	ROC
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$	$R+\mathrm{Re}\{s_0\}$
Time/Laplace-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathrm{Re}\{s\}>0\}$

Property	
Initial Value Theorem	$x(0^+) = \lim_{s\to\infty} sX(s)$
Final Value Theorem	$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

Bilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}>0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}<0$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}>0$
5	$-t^n u(-t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}<0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}>-a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}<-a$
8	$t^n e^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}>-a$
9	$-t^n e^{-at}u(-t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}<-a$
10	$\cos(\omega_0 t)u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
11	$\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
12	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$
13	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$

Unilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$
Time/Laplace-Domain Scaling	$x(at)$, $a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1 * x_2(t)$, x_1 and x_2 are causal	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)-x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_{0^-}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property	
Initial Value Theorem	$x(0^+) = \lim_{s\to\infty} sX(s)$
Final Value Theorem	$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$

Unilateral Laplace Transform Pairs

Pair	$x(t)$, $t \geq 0$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$
7	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
8	$e^{-at}\cos(\omega_0 t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
9	$e^{-at}\sin(\omega_0 t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$