B-5-6

From the graph, we have value for the following variables:

$$x_1,\ x_n, T, t_n (= nT)$$

ζ can be obtained from the above using

$$\frac{x_1}{x_2} = \frac{e^{-\zeta\omega_n t_1}}{e^{-\zeta\omega_n (t_1+T)}} = \frac{1}{e^{-\zeta\omega_n T}} = e^{\zeta\omega_n T}$$

$$\frac{x_1}{x_n} = \frac{1}{e^{-\zeta\omega_n (n-1)T}} = e^{(n-1)\zeta\omega_n T}$$

Logarithmic decrement $\Rightarrow \ln\frac{x_1}{x_2} = \frac{1}{n-1}\ln\frac{x_1}{x_n} = \zeta\omega_n T$

$$= \zeta\omega_n \frac{2\pi}{\omega_d} = \frac{2\pi\,\zeta}{\sqrt{1-\zeta^2}}$$

Define,

$$\frac{1}{n-1}\ln\frac{x_1}{x_n} = \frac{2\pi\ \zeta}{\sqrt{1-\zeta^2}} = \Delta$$

Then,

$$4\pi^2\zeta^2 = \Delta^2(1-\ \zeta^2) \Rightarrow \zeta^2 = \frac{\Delta^2}{4\pi^2+\Delta^2}$$

$$\Rightarrow \quad \zeta = \frac{\Delta}{\sqrt{4\pi^2+\Delta^2}} = \frac{\left(\frac{1}{n-1}\right)(\ln\frac{x_1}{x_n})}{\sqrt{4\pi^2 + \left(\frac{1}{n-1}\right)^2 (\ln\frac{x_1}{x_n})^2}}$$

An alternative solution is

$$\frac{x_1}{x_n} = \frac{e^{-\sigma t_1}}{e^{-\sigma t_n}} = e^{-\sigma(t_1-t_n)}$$

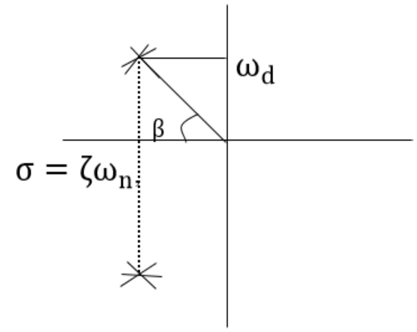
$$\Rightarrow \ln\left(\frac{x_1}{x_n}\right) = -\ \sigma(t - t_n) \Rightarrow \quad \sigma = \frac{\ln\ (\frac{x_1}{x_n})}{(t_n-t_1)}$$

Using the above equation for sigma and

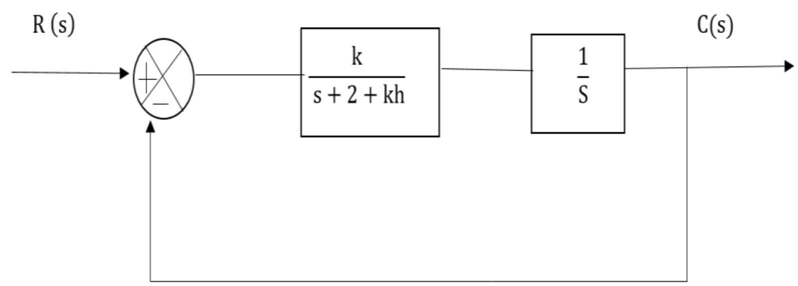
$$\omega_d = \frac{2\pi}{T}$$

$$\zeta = \cos\beta = \frac{\sigma}{\omega_n} = \frac{\sigma}{\sqrt{\sigma^2 + \omega_d^2}} = \frac{\frac{\ln(\frac{X_1}{X_n})}{(t_n - t_1)}}{\sqrt{\frac{(\ln(\frac{X_1}{X_n}))^2}{(t_n - t_1)^2} + \frac{4\pi^2}{T^2}}}$$

$$\Rightarrow \zeta = \frac{T\ln((\frac{X_1}{X_n}))}{\sqrt{(T\ln(\frac{X_1}{X_n}))^2 + 4\pi^2 (t_n - t_1)^2}}$$



B-5-8



From Fig. 5-75 we obtain

and $$\frac{C(s)}{R(s)} = \frac{k}{s^2 + 2s + khs + k}$$

Note that, $k = \omega_n^2 = 4^2 = 16$

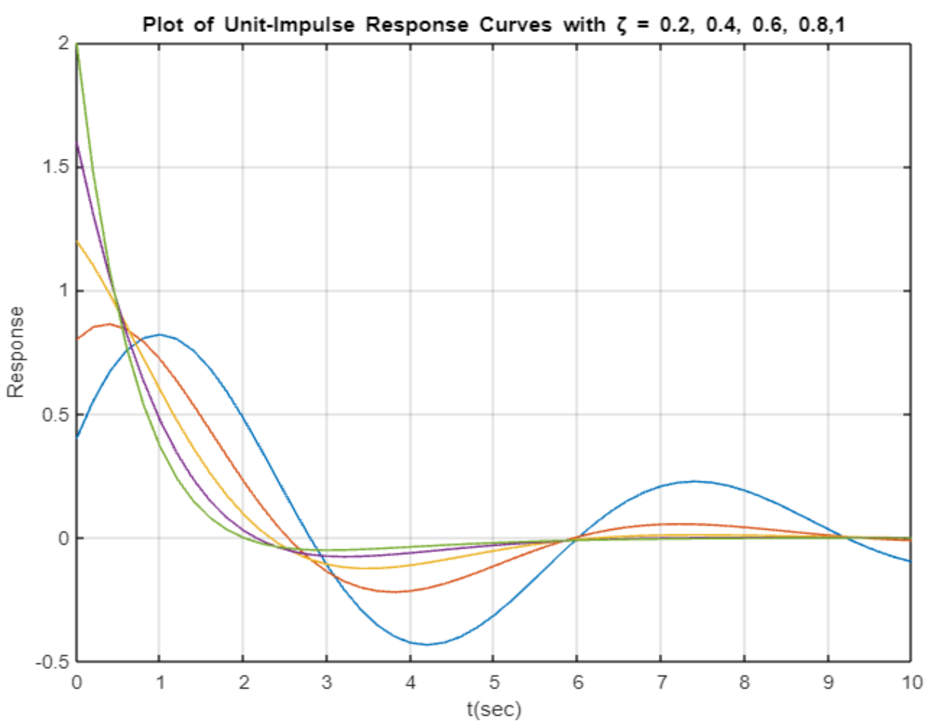
Since, $2\zeta\omega_n = 2 + kh \Rightarrow 2 * 0.7 * 4 = 2 + 16h$

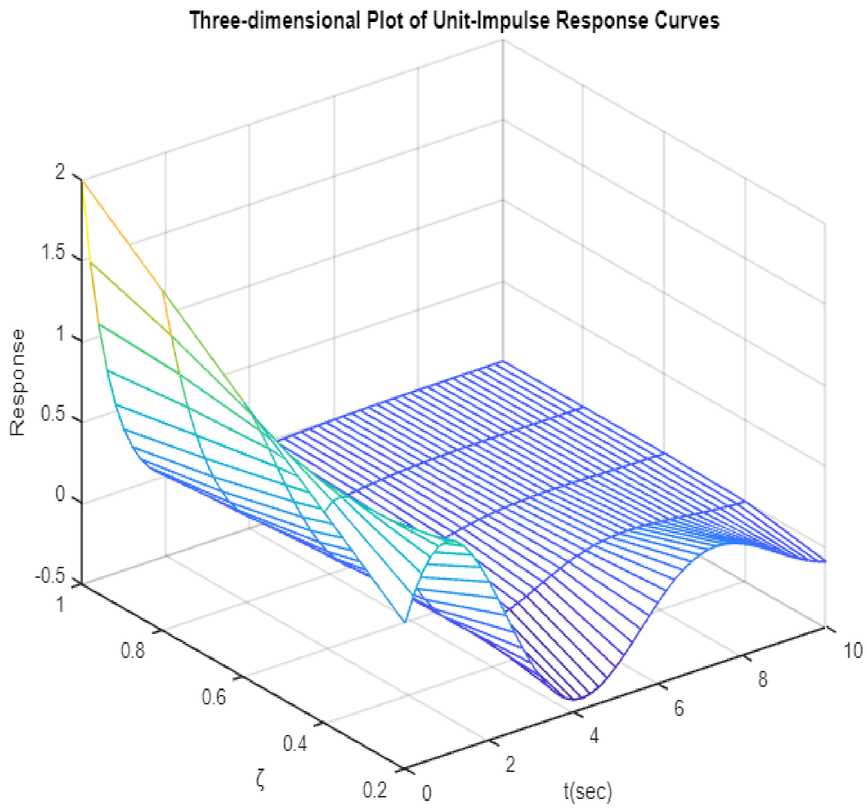
$\Rightarrow h = 0.225$

B-5-16

The matlab code and plots are given below

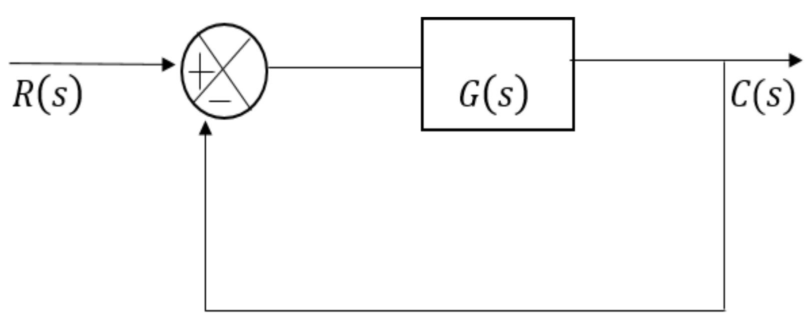
```
clear all
close all
clc
% To plot a Two-Dimentional Diagram
t= 0: .2: 10;
zeta =[.2 .4 .6 .8 1];
for n= 1:5
   num =[0 2*zeta(n) 1];
   den =[1 2*zeta(n) 1];
   [y(1:51,n),x,t] = impulse(num, den,t);
end
figure(1)
plot(t,y)
grid
title('Plot of Unit-Impulse Response Curves with \zeta = 0.2, 0.4, 0.6, 0.8,1')
xlabel('t(sec)')
ylabel('Response')


% To plot a Three-Dimentional Diagram
figure(2)
mesh(t, zeta,y')
title('Three-dimensional Plot of Unit-Impulse Response Curves')
xlabel('t(sec)')
ylabel('\zeta')
zlabel('Response')
```





B-5-20

From



We obtain,

$$\frac{C(s)}{R(s)} = \frac{k}{s(s+1)(s+2)+k}$$

The characteristic equation is

$$s^3 + 3s^2 + 2s + k = 0$$

The Routh array becomes

$$\begin{array}{ccc}
\mathbf{s^3} & 1 & 2 \\
\mathbf{s^2} & 3 & k \\
\mathbf{s^1} & \frac{6-k}{3} & \\
\mathbf{s^0} & K &
\end{array}$$

For stability we require $0 < k$ and $k < 6$, or

$$0 < K < 6$$