CSC 320 Fall 2024
Assignment 2

This assignment has 7 written questions and is out of a total of 31 marks. Submit one PDF file containing your solutions on Brightspace.

Questions

1. **[7 marks]** Consider the following language, where $\Sigma = \{a, b\}$:

   $L = \{w \in \Sigma^* \mid$ each pair of consecutive b's in w is separated by a substring of a's of length $2i, i \geq 0\}$

   (a) Give the state diagram and the formal 5-tuple definition of an NFA $N = (Q, \Sigma, \delta, q_0, F)$ which recognizes L, with a transition table describing δ.

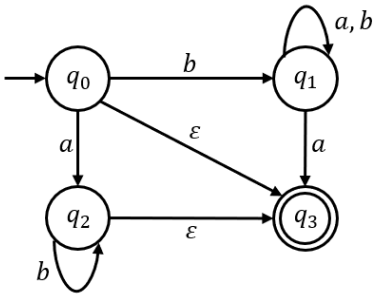
   (b) Give a regular expression R where $L(R) = L$.

2. **[3 marks]** Consider the following language, where $\Sigma = \{a, b\}$:

   $$L = \{w \in \Sigma^* \mid w \text{ starts with an } a \text{ and contains the substring } abb\}$$

   Give a regular expression R where $L(R) = L$.

3. **[4 marks]** Consider the following NFA $N = (Q_N, \Sigma, \delta_N, q_N, F_N)$:



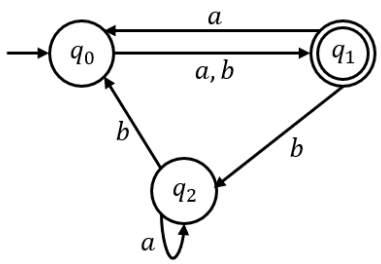
   Following the construction presented in class, give the state diagram and formal 5-tuple definition for equivalent DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$ with $L(N) = L(D)$.

4. **[4 marks]** Let $\Sigma = \{a, b, c\}$. Consider the regular expression

   $$R = (ab)^*(a \cup c)^*b$$

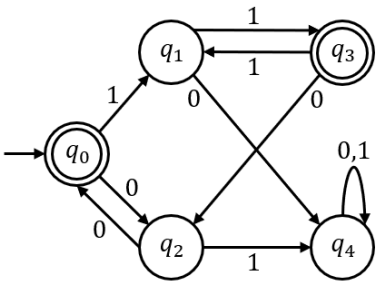
   Give a state diagram for an NFA N with $L(N) = L(R)$, following the construction in the regular expression to NFA proof shown in class.

5. **[4 marks]** Consider the following DFA M.



Following the DFA to regular expression proof shown in class, provide a regular expression R with $L(R) = L(M)$. Show all the steps in your work by drawing the state diagram for the initial GNFA as well as after each state removal. Remove states in lexicographic order (i.e. $q_{rip} = q_0, q_1, q_2$).

6. **[4 marks]** Consider the following DFA D:



Use the DFA state minimization algorithm to produce a DFA D' with a minimal number of states and $L(D) = L(D')$.

7. **[5 marks]** Consider the following language:

$$L = \{w \in \{0,1\}^* \mid w \text{ has at least twice as many 1's as 0's}\}$$

Use the pumping lemma to prove that L is non-regular and show every step in your proof.