ELEC 360 – Assignment #1 Solutions

Q.1 – a

Convolution of 2 functions:

$$f(t) = \begin{cases} 0 & , \quad t < 0 \\ \underbrace{\cos(2\omega t)}_{f_1(t)} * \underbrace{\cos(4\omega t)}_{f_2(t)} & , \quad t \geq 0 \end{cases}$$

$$L\{f_1(t) * f_2(t)\} = F_1(s) \cdot F_2(s) \Rightarrow$$

$$\Rightarrow L\{f(t)\} = F(s) = \frac{s}{s^2 + 4\omega^2} \cdot \frac{s}{s^2 + 16\omega^2} = \frac{s^2}{(s^2 + 4\omega^2)(s^2 + 16\omega^2)}$$

Q.1 – b)

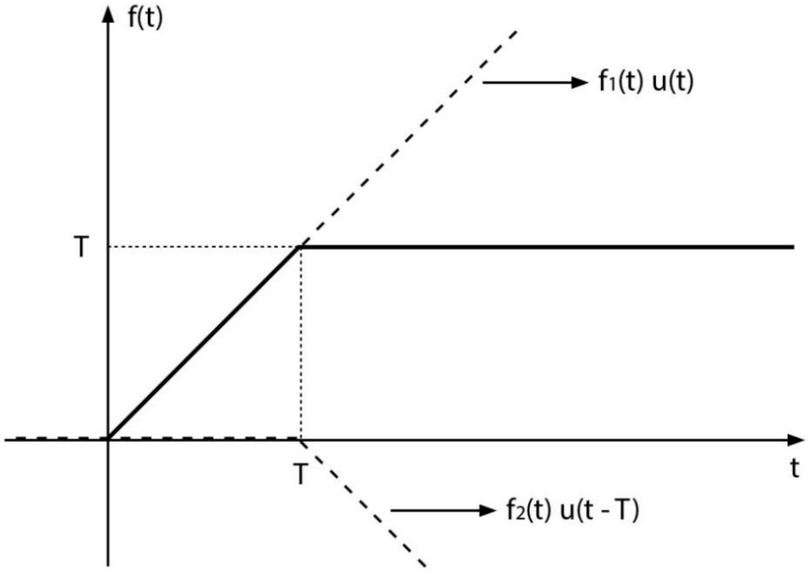
Product of 2 functions:

$$f(t) = \begin{cases} 0 & , \quad t < 0 \\ \cos(2\omega t) \cdot \cos(4\omega t) & , \quad t \geq 0 \end{cases}$$

$$f(t) = \cos(2\omega t) \cdot \cos(4\omega t) = \frac{1}{2}[\cos(2\omega t) + \cos(6\omega t)] \Rightarrow$$

$$\Rightarrow F(s) = \frac{1}{2}\left[\frac{s}{s^2 + 4\omega^2} + \frac{s}{s^2 + 36\omega^2}\right]$$

Q.2



where,

$$\begin{cases} f_1(t) = t \\ f_2(t) = -(t - T) \end{cases}$$

So, the $f(t)$ would be,

$f(t) = f_1(t)u(t) + f_2(t)u(t - T) = tu(t) - (t - T)u(t - T) \Rightarrow$

$$\Rightarrow F(s) = L\{f(t)\} = \frac{1}{s^2} - e^{-Ts}\frac{1}{s^2} = \frac{1}{s^2}(1 - e^{-Ts})$$

Q.3

$$\left.\begin{aligned} L\{u(t-r)\} &= \frac{e^{-rs}}{s} \\ L\{e^{-\alpha t}u(t)\} &= \frac{1}{s+a} \end{aligned}\right\} \Rightarrow L^{-1}\left\{\frac{5e^{-s}}{s+2}\right\} = 5e^{-2(t-1)}u(t-1)$$

Q.4

$2\ddot{x} + 7\dot{x} + 3x = u(t), \qquad x(0) = 3, \quad \dot{x}(0) = 0, \quad u(t): unit\ step$

$$\overset{L}{\Rightarrow} 2(s^2X(s) - 3s) + 7(sX(s) - 3) + 3X(s) = \frac{1}{s} \Rightarrow$$

$$\Rightarrow 2s^2X(s) - 6s + 7sX(s) - 21 + 3X(s) = \frac{1}{s} \Rightarrow$$

$$\Rightarrow X(s)(2s^2 + 7s + 3) = \frac{1}{s} + 21 + 6s \Rightarrow$$

$$\Rightarrow X(s) = \frac{6s^2 + 21s + 1}{s(2s^2 + 7s + 3)} = \frac{6s^2 + 21s + 1}{2s\left(s + \frac{1}{2}\right)(s + 3)}$$

partial fraction expansion: 3 poles $\left[0, -\frac{1}{2}, -3\right]$

$$(s = 0)\ a_0 = \cancel{s}\frac{6s^2 + 21s + 1}{2s\left(\cancel{s} + \frac{1}{2}\right)(s + 3)} = \frac{1}{3}$$

$$\left(s = -\frac{1}{2}\right)\ a_1 = \cancel{\left(s + \frac{1}{2}\right)}\frac{6s^2 + 21s + 1}{2s\cancel{\left(s + \frac{1}{2}\right)}(s + 3)} = \frac{16}{5}$$

$$(s = -3)\ a_2 = \cancel{(s + 3)}\frac{6s^2 + 21s + 1}{2s\left(s + \frac{1}{2}\right)\cancel{(s + 3)}} = -\frac{8}{15}$$

$$X(s) = \frac{\frac{1}{3}}{s} + \frac{\frac{16}{5}}{s + \frac{1}{2}} + \frac{\left(-\frac{8}{15}\right)}{s + 3} \overset{L^{-1}}{\Longrightarrow} x(t) = \frac{1}{3}u(t) + \frac{16}{5}e^{-\frac{1}{2}t} - \frac{8}{15}e^{-3t}, t \geq 0$$

Q.5

$$\ddot{x} + 3\dot{x} + 6x = 0, \qquad x(0) = 0, \quad \dot{x}(0) = 3$$

$$\overset{L}{\Rightarrow} s^2X(s) - 3 + 3sX(s) + 6X(s) = 0 \Rightarrow$$

$$\Rightarrow X(s)(s^2 + 3s + 6) = 3 \Rightarrow X(s) = \frac{3}{s^2 + 3s + 6} \Rightarrow$$

$$\Rightarrow X(s) = \frac{3}{(s+1.5)^2 + 3.75} = \frac{3}{\sqrt{3.75}} \cdot \frac{\sqrt{3.75} \longrightarrow \omega}{\underbrace{(s+1.5)^2}_{(s+a)^2} + \underbrace{3.75}_{\omega^2}}$$

$$\boxed{L\left\{\frac{\omega}{(s+a)^2 + \omega^2}\right\} = e^{-at} \sin \omega t}$$

$$\Rightarrow L^{-1}\{X(s)\} = \frac{3}{\sqrt{3.75}} e^{-1.5t} \sin\left(\sqrt{3.75} \cdot t\right) = x(t)$$