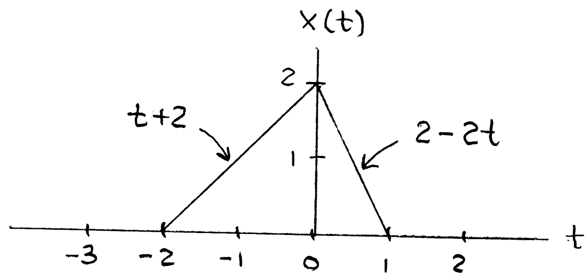
ECE 260

EXAM 5

SOLUTIONS

(FALL 2022)

QUESTION 1



$$x(t) = (t+2)[u(t+2)-u(t)] + (2-2t)[u(t)-u(t-1)]$$

$$= (t+2)u(t+2) - tu(t) - 2u(t) + 2u(t) - 2tu(t) + 2(t-1)u(t-1)$$

$$= (t+2)u(t+2) - 3tu(t) + 2(t-1)u(t-1)$$

$$X(s) = e^{2s} L\{tu(t)\}(s) - 3L\{tu(t)\}(s) + 2e^{-s} L\{tu(t)\}(s)$$

$$= e^{2s}\left(\frac{1}{s^2}\right) - 3\left(\frac{1}{s^2}\right) + 2e^{-s}\left(\frac{1}{s^2}\right)$$

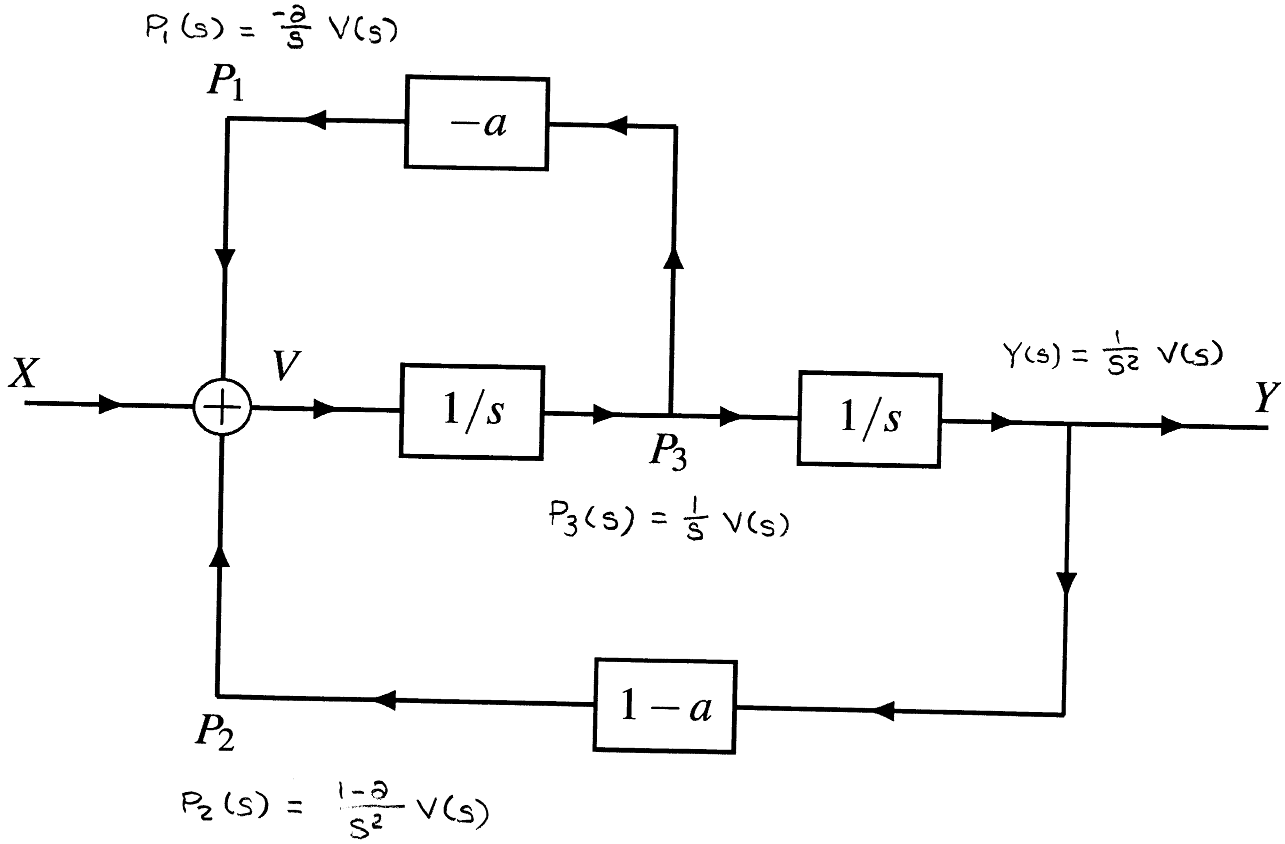
$$= \frac{e^{2s} - 3 + 2e^{-s}}{s^2} \quad \text{for all } s \in \mathbb{C}$$

QUESTION 2

(a) A LTI system with system function H is BIBO stable if and only if the ROC of H contains the imaginary axis.

QUESTION 2

(b)



$P_1(s) = \frac{-a}{s} V(s)$
P_1
$-a$
X
V
$1/s$
$1/s$
$Y(s) = \frac{1}{s^2} V(s)$
Y
P_3
$P_3(s) = \frac{1}{s} V(s)$
$1-a$
P_2
$P_2(s) = \frac{1-a}{s^2} V(s)$

QUESTION 2 (c)

From the labelled block diagram, we have

$$Y(s) = \frac{1}{s^2} V(s)$$

$$V(s) = X(s) - \frac{a}{s} V(s) + \frac{1-a}{s^2} V(s)$$

Rearranging the second of these equations, we have

$$V(s) + \frac{a}{s} V(s) + \frac{a-1}{s^2} V(s) = X(s) \Rightarrow$$

$$\left[1 + \frac{a}{s} + \frac{a-1}{s^2}\right] V(s) = X(s) \Rightarrow$$

$$\frac{s^2 + as + a - 1}{s^2} V(s) = X(s) \Rightarrow$$

$$V(s) = \frac{s^2}{s^2 + as + a - 1} X(s)$$

Substituting the preceding formula for V into the above equation for Y, we obtain

$$Y(s) = \frac{1}{s^2}\left[\frac{s^2}{s^2 + as + a - 1} X(s)\right]$$

$$= \frac{1}{s^2 + as + a - 1} X(s)$$

Therefore, $H(s) = \dfrac{1}{s^2 + as + a - 1}$.

Now, we factor $H(s)$.

$$\frac{-a \pm \sqrt{a^2 - 4(a-1)}}{2} = \frac{-a \pm \sqrt{a^2 - 4a + 4}}{2} = \frac{-a \pm \sqrt{(a-2)^2}}{2}$$

$$= \frac{-a \pm (a-2)}{2} = \left\{\frac{-a-a+2}{2}, \frac{-a+a-2}{2}\right\} = \left\{\frac{-2a+2}{2}, \frac{-2}{2}\right\}$$

$$= \{1-a, -1\}$$

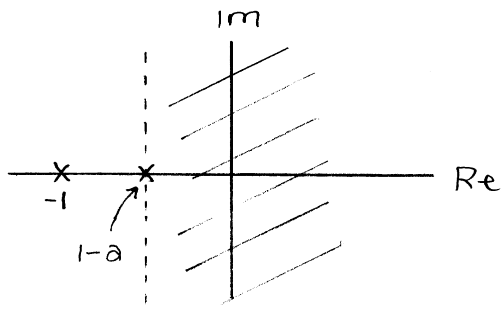
So, we have

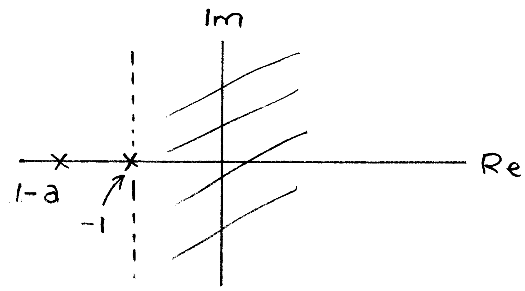
$$H(s) = \frac{1}{(s+1)(s+a-1)} \quad \text{for } \operatorname{Re}(s) > \max\{1-a, -1\}$$

QUESTION 2(c) [CONTINUED]

For the system to be BIBO stable, we require

$1 - a < 0 \Rightarrow a > 1$ (i.e., all poles of H are to the left of the imaginary axis)





QUESTION 3

$$\boxed{v_o(t) = 2i(t) + \frac{1}{4}\int_{-\infty}^{t} i(\tau)\,d\tau + v_1(t) \quad \text{and} \quad i(t) = \frac{1}{4}\int_{-\infty}^{t} v_1(\tau)\,d\tau}$$

$$V_o(s) = 2I(s) + \frac{1}{4s}I(s) + V_1(s)$$

$$I(s) = \frac{1}{4s}V_1(s)$$

$$V_o(s) = 2\left[\frac{1}{4s}V_1(s)\right] + \frac{1}{4s}\left[\frac{1}{4s}V_1(s)\right] + V_1(s)$$

$$V_o(s) = \frac{1}{2s}V_1(s) + \frac{1}{16s^2}V_1(s) + V_1(s)$$

$$V_o(s) = \left(\frac{1}{2s} + \frac{1}{16s^2} + 1\right)V_1(s)$$

$$V_o(s) = \left(\frac{8s + 1 + 16s^2}{16s^2}\right)V_1(s)$$

$$V_1(s) = \left(\frac{16s^2}{16s^2 + 8s + 1}\right)V_o(s)$$

$$\frac{V_1(s)}{V_o(s)} = \frac{16s^2}{16s^2 + 8s + 1}$$

$$H(s) = \frac{16s^2}{16s^2 + 8s + 1}$$

$$\frac{-8 \pm \sqrt{8^2 - 4(16)}}{2(16)} = \frac{-8 \pm 0}{32} = \frac{-8}{32} = \frac{-1}{4}$$

$$H(s) = \frac{16s^2}{16(s + \frac{1}{4})^2} \quad \text{for } \operatorname{Re}(s) > -\frac{1}{4}$$

QUESTION 4

$$\boxed{X(s) = \frac{s-7}{s^2-1} \quad \text{for} \quad -1 < \mathrm{Re}(s) < 1}$$

$$X(s) = \frac{s-7}{(s+1)(s-1)}$$

$$X(s) = \frac{A_1}{s+1} + \frac{A_2}{s-1}$$

$$A_1 = (s+1)\,X(s)\Big|_{s=-1} = \frac{s-7}{s-1}\Big|_{s=-1} = \frac{-8}{-2} = 4$$

$$A_2 = (s-1)\,X(s)\Big|_{s=1} = \frac{s-7}{s+1}\Big|_{s=1} = \frac{-6}{2} = -3$$

$$X(s) = \frac{4}{s+1} - \frac{3}{s-1}$$

$$\begin{aligned} x(t) &= 4\,L^{-1}\left\{\frac{1}{s+1}\right\}(t) - 3\,L^{-1}\left\{\frac{1}{s-1}\right\}(t) \\ &= 4\left[e^{-t}u(t)\right] - 3\left[-e^{t}u(-t)\right] \\ &= 4e^{-t}u(t) + 3e^{t}u(-t) \end{aligned}$$