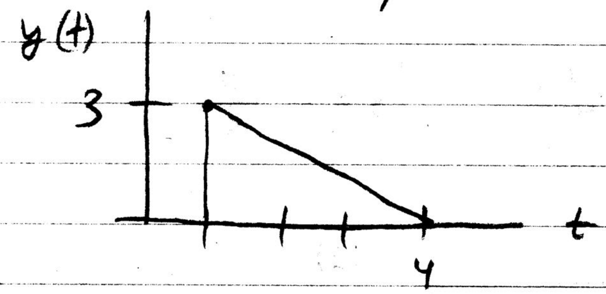
ELEC 360 Midterm, Oct 06

(4) 1. Find the Laplace transform of:



$$y(t) = 3u(t-1) - (t-1)u(t-1) + (t-4)u(t-4)$$

$$Y(s) = \frac{3e^{-s}}{s} - \frac{1}{s^2}\left(e^{-s} - e^{-4s}\right)$$

(4) 2. Find $\ddot{y}(t) + 2y(t) = u(t)$

$$u(t) = \begin{cases} 1 & \text{for } t \geq 1 \\ 0 & \text{else} \end{cases} \qquad (\dot{y}(0) = y(0) = 0)$$

$$s^2 Y(s) + 2s\,Y(s) = \frac{e^{-s}}{s} \qquad Y(s) = \frac{e^{-s}}{s^2(s+2)}$$

$$y(t) = \frac{1}{4}\left(2(t-1) - 1 + e^{-2(t-1)}\right)u(t-1) \qquad (1)\ \frac{1}{4}\left(2t - 1 + e^{-2t}\right)$$

$$y(t) = \frac{1}{4}\left(2t - 3 + e^{-2(t-1)}\right) \text{ for } t \geq 1$$

(4) 3.

$$\dot{x}_1 = -kx_2 + r(t) \qquad \dot{x}_2 = ax_1 - dx_2 + f\,r(t) - fkx_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -k \\ a & -(fk+d) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ f \end{bmatrix} r(t)$$

$$c(t) = [0, b]\,\underline{x}$$

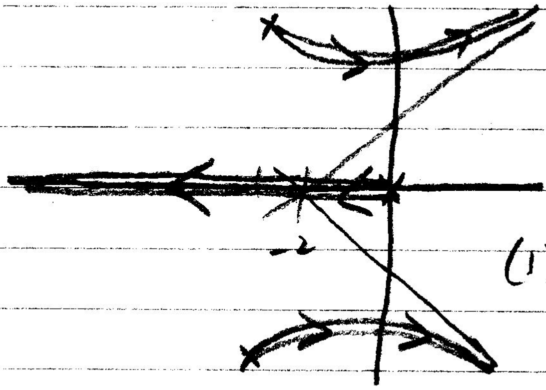
(5) 4. $$\frac{\frac{1}{s(s+6)}}{1+\frac{25}{s(s+6)}} = \frac{1}{s^2+6s+25}$$

open-loop $\dfrac{K}{s(s^2+6s+25)}$ (1) (A-6-2)

poles $s_1 = 0$ $s_{2,3} = \dfrac{-6 \pm \sqrt{36-100}}{2} = -3 \pm j4$

asymp. $\gamma = \dfrac{\pm 180}{3}(2k+1) = \pm 60°, 180°$ $\sigma_a = \dfrac{-6}{3} = -2$ (1)

$A'(s) = 3s^2 + 12s + 25 = 0 \Rightarrow s_{1,2} = -2 \pm j2.08$



(1)

b) for low K the real pole is dominant
→ first order response
With increased K, complex conjugate poles dominant and system slower
→ for high K the system is unstable (1)

5.

open-loop $\dfrac{K}{s(s^2+6s+25)} = G(s)$

(1)

$K_v = \lim_{s \to 0} s\,G(s) = \dfrac{K}{25}$ $e_{ss} = \dfrac{1}{K_v} = \dfrac{25}{K} < 0.3$

$K > 83.33$

(Note, root locus crosses imag. axis for $K=150$, wts. thus $83.3 < K < 150$)