Unit-Ramp Response: The unit-ramp response of the compensated system may be obtained by entering MATLAB Program 7–34 into the computer. Here we converted the unit-ramp response of $G_cG/(1 + G_cG)$ into the unit-step response of $G_cG/[s(1 + G_cG)]$. The unit-ramp response curve obtained using this program is shown in Figure 7–155.

MATLAB Program 7–34

```
%*****Unit-ramp response*****

num = [40  24  3.2];
den = [1  9.02  24.18  56.48  24.32  3.2  0];
t = 0:0.05:20;
c = step(num,den,t);
plot(t,c,'-',t,t,'.')
grid
title('Unit-Ramp Response of Compensated System')
xlabel('Time (sec)')
ylabel('Unit-Ramp Input and Output c(t)')
```

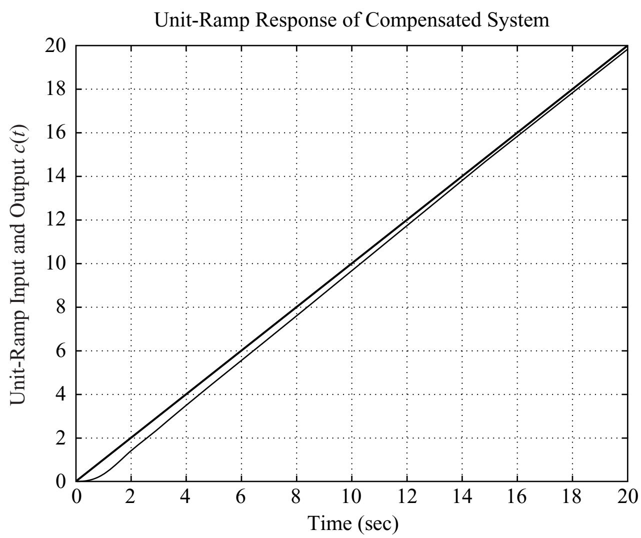

Figure 7–155 Unit-ramp response of the compensated system.

PROBLEMS

B–7–1. Consider the unity-feedback system with the open-loop transfer function:

$$G(s) = \frac{10}{s + 1}$$

Obtain the steady-state output of the system when it is subjected to each of the following inputs:

(a) $r(t) = \sin(t + 30^\circ)$

(b) $r(t) = 2\cos(2t - 45^\circ)$

(c) $r(t) = \sin(t + 30^\circ) - 2\cos(2t - 45^\circ)$

B–7–2. Consider the system whose closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{K(T_2 s + 1)}{T_1 s + 1}$$

Obtain the steady-state output of the system when it is subjected to the input $r(t) = R \sin \omega t$.

B–7–3. Using MATLAB, plot Bode diagrams of $G_1(s)$ and $G_2(s)$ given below.

$$G_1(s) = \frac{1 + s}{1 + 2s}$$

$$G_2(s) = \frac{1 - s}{1 + 2s}$$

$G_1(s)$ is a minimum-phase system and $G_2(s)$ is a nonminimum-phase system.

B–7–4. Plot the Bode diagram of

$$G(s) = \frac{10(s^2 + 0.4s + 1)}{s(s^2 + 0.8s + 9)}$$

B–7–5. Given

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

show that

$$|G(j\omega_n)| = \frac{1}{2\zeta}$$

B–7–6. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{s + 0.5}{s^3 + s^2 + 1}$$

This is a nonminimum-phase system. Two of the three open-loop poles are located in the right-half s plane as follows:

$$\text{Open-loop poles at } s = -1.4656$$

$$s = 0.2328 + j0.7926$$

$$s = 0.2328 - j0.7926$$

Plot the Bode diagram of $G(s)$ with MATLAB. Explain why the phase-angle curve starts from 0° and approaches +180°.

B–7–7. Sketch the polar plots of the open-loop transfer function

$$G(s)H(s) = \frac{K(T_a s + 1)(T_b s + 1)}{s^2(Ts + 1)}$$

for the following two cases:

(a) $T_a > T > 0, \quad T_b > T > 0$

(b) $T > T_a > 0, \quad T > T_b > 0$

B–7–8. Draw a Nyquist locus for the unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K(1 - s)}{s + 1}$$

Using the Nyquist stability criterion, determine the stability of the closed-loop system.

B–7–9. A system with the open-loop transfer function

$$G(s)H(s) = \frac{K}{s^2(T_1 s + 1)}$$

is inherently unstable. This system can be stabilized by adding derivative control. Sketch the polar plots for the open-loop transfer function with and without derivative control.

B–7–10. Consider the closed-loop system with the following open-loop transfer function:

$$G(s)H(s) = \frac{10K(s + 0.5)}{s^2(s + 2)(s + 10)}$$

Plot both the direct and inverse polar plots of $G(s)H(s)$ with $K = 1$ and $K = 10$. Apply the Nyquist stability criterion to the plots, and determine the stability of the system with these values of K.

B–7–11. Consider the closed-loop system whose open-loop transfer function is

$$G(s)H(s) = \frac{Ke^{-2s}}{s}$$

Find the maximum value of K for which the system is stable.

B–7–12. Draw a Nyquist plot of the following $G(s)$:

$$G(s) = \frac{1}{s(s^2 + 0.8s + 1)}$$

B–7–13. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the system.

B–7–14. Consider a unity-feedback control system with the following open-loop transfer function:

$$G(s) = \frac{s^2 + 2s + 1}{s^3 + 0.2s^2 + s + 1}$$

Draw a Nyquist plot of $G(s)$ and examine the stability of the closed-loop system.

B–7–15. Consider the unity-feedback system with the following $G(s)$:

$$G(s) = \frac{1}{s(s-1)}$$

Suppose that we choose the Nyquist path as shown in Figure 7–156. Draw the corresponding $G(j\omega)$ locus in the $G(s)$ plane. Using the Nyquist stability criterion, determine the stability of the system.

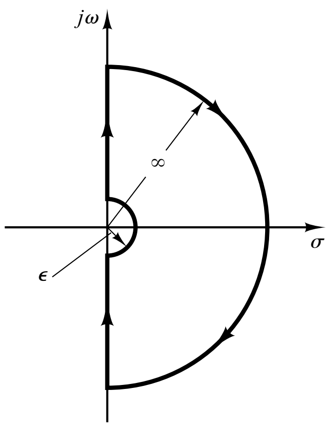

Figure 7–156
Nyquist path.

B–7–16. Consider the closed-loop system shown in Figure 7–157. $G(s)$ has no poles in the right-half s plane.

If the Nyquist plot of $G(s)$ is as shown in Figure 7–158(a), is this system stable?

If the Nyquist plot is as shown in Figure 7–158(b), is this system stable?

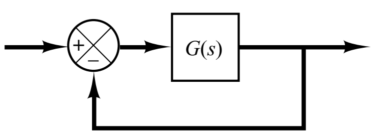

Figure 7–157
Closed-loop system.

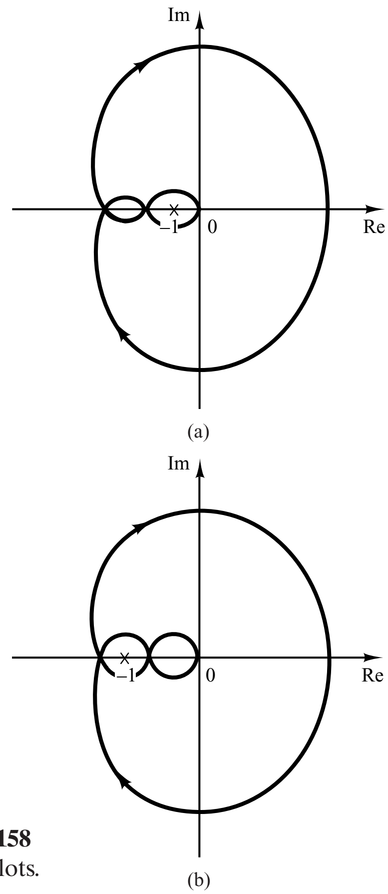

Figure 7–158
Nyquist plots.

B–7–17. A Nyquist plot of a unity-feedback system with the feedforward transfer function $G(s)$ is shown in Figure 7–159.

If $G(s)$ has one pole in the right-half s plane, is the system stable?

If $G(s)$ has no pole in the right-half s plane, but has one zero in the right-half s plane, is the system stable?

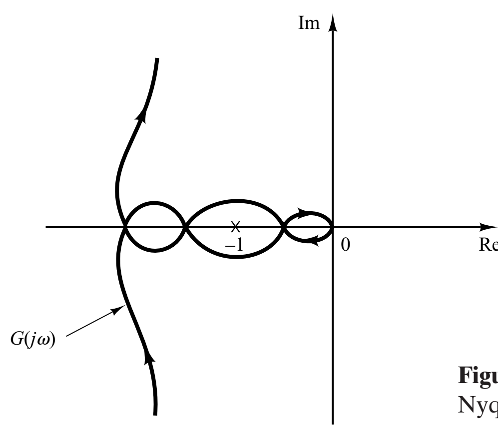

Figure 7–159
Nyquist plot.

B–7–18. Consider the unity-feedback control system with the following open-loop transfer function $G(s)$:

$$G(s) = \frac{K(s+2)}{s(s+1)(s+10)}$$

Plot Nyquist diagrams of $G(s)$ for $K = 1, 10$, and 100.

B–7–19. Consider a negative-feedback system with the following open-loop transfer function:

$$G(s) = \frac{2}{s(s+1)(s+2)}$$

Plot the Nyquist diagram of $G(s)$. If the system were a positive-feedback one with the same open-loop transfer function $G(s)$, what would the Nyquist diagram look like?

B–7–20. Consider the control system shown in Figure 7–160. Plot Nyquist diagrams of $G(s)$, where

$$G(s) = \frac{10}{s[(s+1)(s+5) + 10k]} = \frac{10}{s^3 + 6s^2 + (5 + 10k)s}$$

for $k = 0.3, 0.5$, and 0.7.

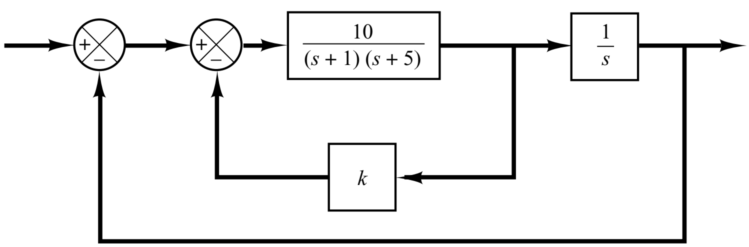

Figure 7–160
Control system.

B–7–21. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

There are four individual Nyquist plots involved in this system. Draw two Nyquist plots for the input u_1 in one diagram and two Nyquist plots for the input u_2 in another diagram. Write a MATLAB program to obtain these two diagrams.

B–7–22. Referring to Problem **B–7–21**, it is desired to plot only $Y_1(j\omega)/U_1(j\omega)$ for $\omega > 0$. Write a MATLAB program to produce such a plot.

If it is desired to plot $Y_1(j\omega)/U_1(j\omega)$ for $-\infty < \omega < \infty$, what changes must be made in the MATLAB program?

B–7–23. Consider the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{as + 1}{s^2}$$

Determine the value of a so that the phase margin is 45°.

B–7–24. Consider the system shown in Figure 7–161. Draw a Bode diagram of the open-loop transfer function $G(s)$. Determine the phase margin and gain margin.

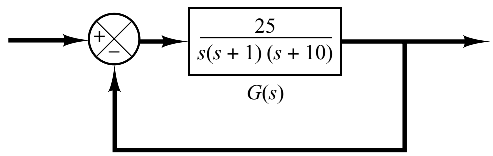

Figure 7–161
Control system.

B–7–25. Consider the system shown in Figure 7–162. Draw a Bode diagram of the open-loop transfer function $G(s)$. Determine the phase margin and gain margin with MATLAB.

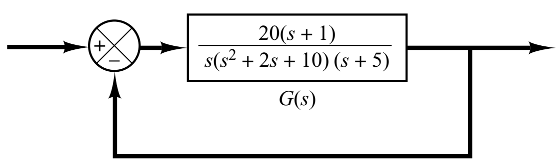

Figure 7–162
Control system.

B–7–26. Consider a unity-feedback control system with the open-loop transfer function

$$G(s) = \frac{K}{s(s^2 + s + 4)}$$

Determine the value of the gain K such that the phase margin is 50°. What is the gain margin with this gain K?

B–7–27. Consider the system shown in Figure 7–163. Draw a Bode diagram of the open-loop transfer function, and determine the value of the gain K such that the phase margin is 50°. What is the gain margin of this system with this gain K?

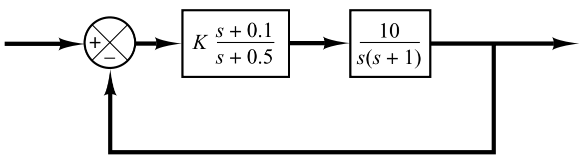

Figure 7–163
Control system.

B–7–28. Consider a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s^2 + s + 0.5)}$$

Determine the value of the gain K such that the resonant peak magnitude in the frequency response is 2 dB, or $M_r = 2$ dB.

B–7–29. A Bode diagram of the open-loop transfer function $G(s)$ of a unity-feedback control system is shown in Figure 7–164. It is known that the open-loop transfer function is minimum phase. From the diagram, it can be seen that there is a pair of complex-conjugate poles at $\omega = 2$ rad/sec. Determine the damping ratio of the quadratic term involving these complex-conjugate poles. Also, determine the transfer function $G(s)$.

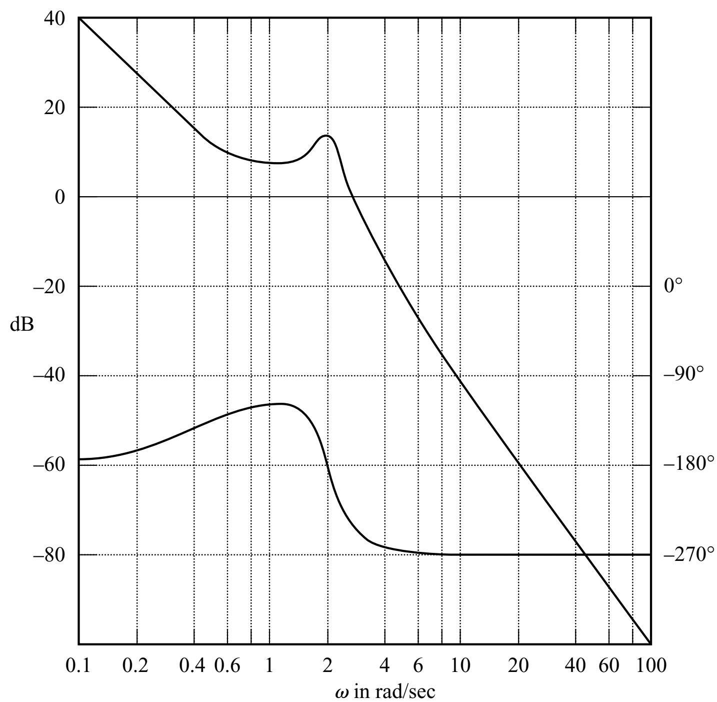

Figure 7–164
Bode diagram of the open-loop transfer function of a unity-feedback control system.

B–7–30. Draw Bode diagrams of the PI controller given by

$$G_c(s) = 5\left(1 + \frac{1}{2s}\right)$$

and the PD controller given by

$$G_c(s) = 5(1 + 0.5s)$$

B–7–31. Figure 7–165 shows a block diagram of a space-vehicle attitude-control system. Determine the proportional gain constant K_p and derivative time T_d such that the bandwidth of the closed-loop system is 0.4 to 0.5 rad/sec. (Note that the closed-loop bandwidth is close to the gain crossover frequency.) The system must have an adequate phase margin. Plot both the open-loop and closed-loop frequency response curves on Bode diagrams.

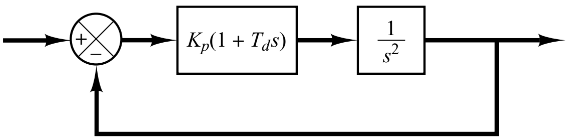

Figure 7–165
Block diagram of space-vehicle attitude-control system.

B–7–32. Referring to the closed-loop system shown in Figure 7–166, design a lead compensator $G_c(s)$ such that the phase margin is 45°, gain margin is not less than 8 dB, and the static velocity error constant K_v is 4.0 sec^{-1}. Plot unit-step and unit-ramp response curves of the compensated system with MATLAB.

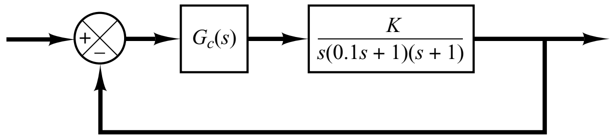

Figure 7–166
Closed-loop system.

B–7–33. Consider the system shown in Figure 7–167. It is desired to design a compensator such that the static velocity error constant is 4 sec^{-1}, phase margin is 50°, and gain margin is 8 dB or more. Plot the unit-step and unit-ramp response curves of the compensated system with MATLAB.

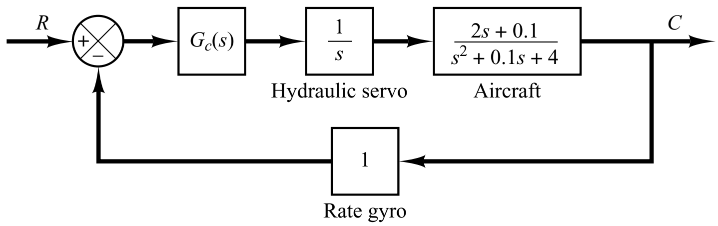

Figure 7–167
Control system.

B–7–34. Consider the system shown in Figure 7–168. Design a lag–lead compensator such that the static velocity error constant K_v is 20 sec^{-1}, phase margin is 60°, and gain margin is not less than 8 dB. Plot the unit-step and unit-ramp response curves of the compensated system with MATLAB.

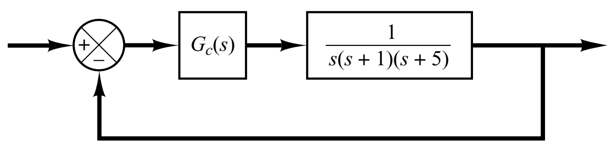

Figure 7–168
Control system.