UNIVERSITY OF VICTORIA

MIDTERM –October 2024

ELEC 360 – CONTROL THEORY AND SYSTEMS I

SECTION A01

INSTRUCTOR: Dr. P. Agathoklis

DATE: October 25, 2024

DURATION: 50 minutes

TWO (2) PAGES OF NOTES AND PHOTOCOPIES OF LAPLACE TRANSFORMS ARE PERMITTED.

TO BE ANSEWERED IN BOOKLETS

ANSWER All QUESTIONS

Marks

(5) 1. Consider the system described by the following differential equation:

$$\ddot{y}(t) + 4\dot{y}(t) + 3y(t) = w(t)$$

where $w(t) = \begin{cases} 1 & for \quad 0 \leq t \leq 2 \\ 0 & else \end{cases}$

Find the response of the system for $y(0)=\dot{y}(0)=0$.

Consider a system given by:

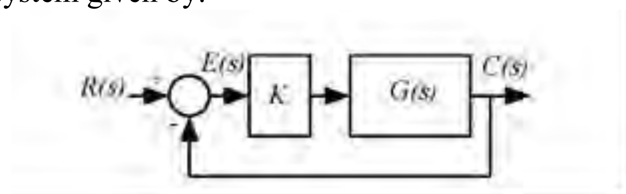


Figure 1. System for Questions 2, 3 and 4.

where *G(s)* is given by $G(s) = \frac{s+4}{s^2+4s+3}$

(6) 2. For the system in figure 1:

- Sketch the root-locus of the above system.
- Discuss the transient part of the unit step response of the closed-loop system when K goes from 0 to ∞. Justify your answers.

(3) 3. For the system in figure 1, find for what values of K (positive or negative) is the closed-loop system stable.

(2) 4. a. For the system in figure 1, find for what values of K is the steady state error (E(s)) less than 0.5 for a unit step input?

b. For the system of figure 1 find for what values of K is the steady state error less than 0.5 for a unit ramp input.

END

ELEC 360 Midterm 2024

(5) 1. $\ddot{y} + 4\dot{y} + 3y = w(t)$ $\quad y(0) = \dot{y}(0) = 0$

$Y(s)(s^2+4s+3) = \frac{1}{s}(1-e^{-2s}) \quad w(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & \text{else} \end{cases}$

$Y(s) = \frac{(1-e^{-2s})(1)}{s(s^2+4s+3)} = \left(\frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+3}\right)(1-e^{-2s})$

$A = \frac{1}{3} = 0.33 \qquad B = \frac{1}{(-1)(2)} = -0.5$

$C = \frac{1}{(-3)(-2)} = \frac{1}{6} = 0.167$

$y(t) = \left(\frac{1}{3} - \frac{e^{-t}}{2} + \frac{e^{-3t}}{6}\right)u(t) - \left(\frac{1}{3} - \frac{e^{-(t-2)}}{2} + \frac{e^{-3(t-2)}}{6}\right)u(t-2)$

(6) 2. $G(s) = \frac{K(s+4)}{s^2+4s+3} \qquad A(s) = s^2+4s+3$

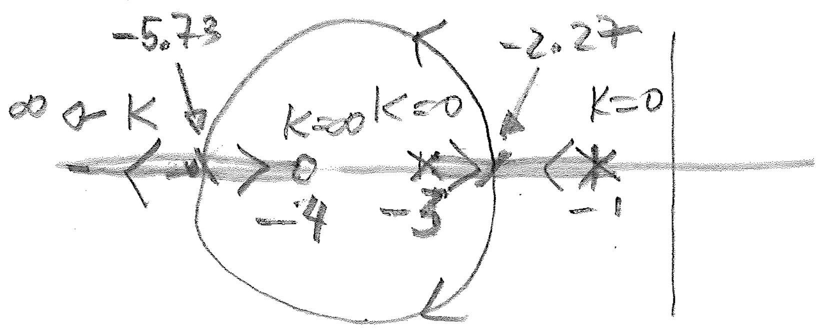
$B(s) = s+4$

poles: $-1, -3$ zeros: -4 $\quad \gamma = \pm 180^\circ$

$A'(s) \cdot B(s) - B'(s)A(s) = (2s+4)(s+4) - (s^2+4s+3)$

$2s^2 + 8s + \cancel{4s} + 16 - \cancel{s^2} - \cancel{4s} - 3 =$

$s^2 + 8s + 13 = 0 \qquad s_1 = -5.73 \quad s_2 = -2.27$



②

K ↑ - overdamped (faster)
- critically damped
- underdamped
  • faster, ζ ↓ M_p ↑
  • faster, ζ ↑ M_p ↓

→ critically damped
→ overdamped
- stable for all $K>0$.

(3) 3. Closed-loop: $G_c(s) = \dfrac{K(s+4)}{s^2+4s+3+Ks+4K}$

$s^2 + s(4+K) + (3+4K)$

second order ⇒ $4+K>0 \quad K>-4$,
$3+4K>0 \quad K>-\frac{3}{4}$

$\Rightarrow K > -\frac{3}{4}$

(2) 4. type 0, no integrators in open-loop

a) $K_p = G(0) = \dfrac{4K}{3} \quad e_{ss} = \dfrac{1}{1+K_p} = \dfrac{3}{3+4K}$

$\dfrac{3}{3+4K} < 0.5 \rightarrow 3 < 1.5 + 2K$

$K > \dfrac{1.5}{2} = 0.75$

b) $e_{ss} = \infty$ for all stable K.