Exercise 6.116

R Answer (c).

We are given that the system is characterized by the equation

$$\mathcal{D}y(t)+3y(t)+2\mathcal{I}y(t)=\mathcal{D}x(t)+5x(t).$$

To eliminate the integration operator $\mathcal{I}$ (which would cause difficulties later), we differentiate the preceding equation. (Note that $\mathcal{D}\mathcal{I}x(t)=\frac{d}{dt}\int_{-\infty}^{t}x(\tau)d\tau=x(t)$.) Taking the derivative, we obtain

$$\mathcal{D}^2y(t)+3\mathcal{D}y(t)+2y(t)=\mathcal{D}^2x(t)+5\mathcal{D}x(t).$$

differentiate

Taking the Fourier transform of the preceding equation, we obtain

take FT

$$(j\omega)^2Y(\omega)+3j\omega Y(\omega)+2Y(\omega)=(j\omega)^2X(\omega)+5j\omega X(\omega).$$

Rearranging, we have

move X to RHS;
move Y to LHS

$$\begin{aligned}
-\omega^2Y(\omega)+3j\omega Y(\omega)+2Y(\omega)&=-\omega^2X(\omega)+5j\omega X(\omega) \Rightarrow\\
(-\omega^2+3j\omega+2)Y(\omega)&=(-\omega^2+5j\omega)X(\omega) \Rightarrow\\
\frac{Y(\omega)}{X(\omega)}&=\frac{-\omega^2+5j\omega}{-\omega^2+3j\omega+2} \Rightarrow\\
\frac{Y(\omega)}{X(\omega)}&=\frac{\omega^2-5j\omega}{\omega^2-3j\omega-2}.
\end{aligned}$$

factor

divide by $X(\omega)$ and $(-\omega^2+3j\omega+2)$

multiply by $\frac{-1}{-1}$

Since the system is LTI, $H(\omega)=\frac{Y(\omega)}{X(\omega)}$. Thus, we have

$H(\omega)=\frac{Y(\omega)}{X(\omega)}$

$$H(\omega)=\frac{\omega^2-5j\omega}{\omega^2-3j\omega-2}.$$

