Exercise 5.102

L Answer (m).

We are given

$$x(t) = \sum_{k=-\infty}^{\infty} 3\delta(t-4k).$$

Clearly, x is periodic with period $T = 4$. From the Fourier series analysis equation, we have

$$\begin{aligned}
c_k &= \frac{1}{T}\int_0^T x(t)e^{-j(2\pi/T)kt}\,dt \\
&= \tfrac{1}{4}\int_0^{4^-} \sum_{k=-\infty}^{\infty} 3\delta(t-4k)e^{-j(2\pi/4)kt}\,dt \\
&= \tfrac{3}{4}\int_0^{4^-} \sum_{k=-\infty}^{\infty} \delta(t-4k)e^{-j(\pi/2)kt}\,dt \\
&= \tfrac{3}{4}\int_0^{4^-} \delta(t)e^{-j(\pi/2)kt}\,dt \\
&= \tfrac{3}{4}\int_{-\infty}^{\infty} \delta(t)e^{-j(\pi/2)kt}\,dt \\
&= \tfrac{3}{4}\left[e^{-j(\pi/2)kt}\right]\Big|_{t=0} \\
&= \tfrac{3}{4}.
\end{aligned}$$

Fourier series analysis equation

substitute given function for x

move 3 outside integral and simplify exponent

only k=0 term is nonzero

change limits since δ zero except at origin

sifting property

simplify

Thus, we have

$$c_k = \tfrac{3}{4}.$$