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University of Victoria
Exam 2
Fall 2021

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 10971), A02 (CRN 10972)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: DRAKE
Given Name(s): STEPHEN
Student Number: **V00**

This examination paper has **6 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

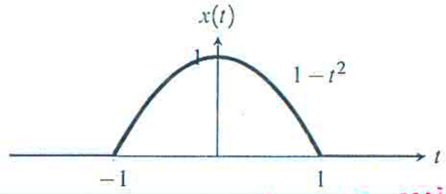
The use of a calculator is **not** permitted.

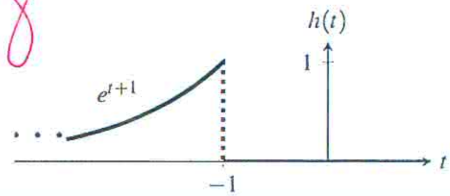
You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

Question 1. **Using the graphical method** (i.e., the method used during the lectures), compute $x * h(t)$, where x and h are as shown in the figures. (You must compute $x * h$, not $h * x$.) For each separate case in your solution, you must state the **convolution result** and the **corresponding range of** t as well as show the **fully-labelled graph** from which this result is derived. Each curve in these plots must be **labelled with its formula** (e.g., $3t+1$, e^{-t}, t^2+3, etc.). Each convolution result may be stated in the form of an integral, but the integral must be simplified as much as possible without integrating. The unit-step function must not appear anywhere in your answer. **[8 marks]**

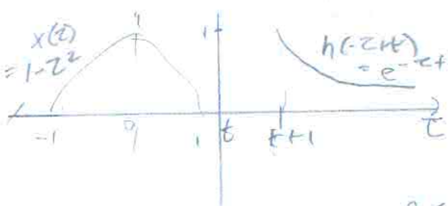
0/8





$h(-\tau)$ missed -2

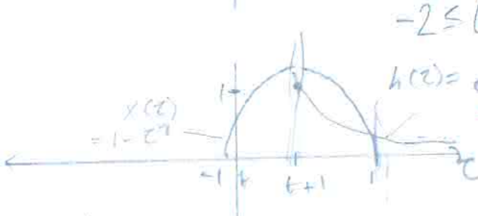
$t > 0$:



$= 0$ (no overlap)

X -2

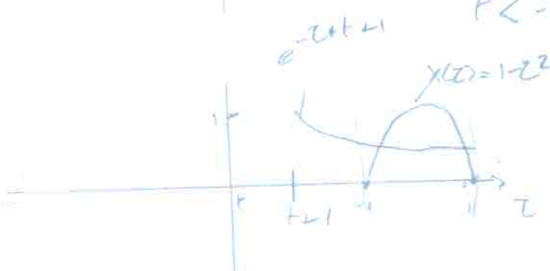
$-2 \le t < 0$



-1 X $= \int_{t+1}^{1} (1-\tau^2)\, e^{-\tau+t+1}\, d\tau$ X -1

$= \int_{t+1}^{1} e^{-\tau+t+1}\, d\tau - \int_{t+1}^{1} \tau^2 e^{-\tau+t+1}\, d\tau$

$t < -2$



$= \int_{-\infty}^{1} (1-\tau^2)\, e^{-\tau+t+1}\, d\tau$ X -2

$= \int_{-1}^{1} e^{-\tau+t+1}\, d\tau - \int_{-1}^{1} \tau^2 e^{-\tau+t+1}\, d\tau$

X

Question 2. A LTI system $\mathscr{H}$ has the system function H, where $H(s) = \dfrac{1}{e^s(s+2)}$. Find the output y of the system in response to the input $x(t) = 4 + 5e^{-3t}$. **[4 marks]**

$$y(t) = \{\mathcal{H}x\}(t) = \frac{1}{e^{(4+5e^{-3t})}(4+5e^{-3t}+2)}$$

$$\frac{0}{4}$$

$Hx = \lambda x$ $\quad H^{-1}(s) = e^{s}(s+2)$

$\Rightarrow x = 3t$

$\lambda =$

$Hx = 16 + 20e^{-3t}$

Question 3. Show that, for any two arbitrary functions x and h, $x * h(t) = h * x(t)$ (i.e., convolution is commutative). **Do not skip any steps** in your proof. **[3 marks]**

$$x * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(-\tau + t)\, d\tau$$

Let $u = -\tau + t$
$du = -d\tau$

✓

$$= -\int_{\infty}^{-\infty} x(-u + t)\, h(u)\, du$$

✓

$$= \int_{-\infty}^{\infty} h(u)\, x(-u + t)\, du$$

3/3

$$= h * x(t)$$

Question 4.

(A) A LTI system $\mathscr{H}_1$ with impulse response h_1 is characterized by the equation $\mathscr{H}_1 x(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$. Find h_1. **[2 marks]**

$x(t) = \delta(t)$

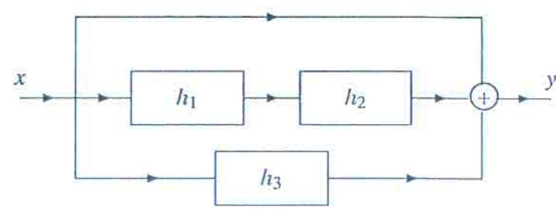
$\Rightarrow h_1(t) = \int_{-\infty}^{t-1} \delta(\tau)\, d\tau$

$h_1(t) = \begin{cases} 1, & t \geq 1 \\ 0, & t < 1 \end{cases}$

1/2

$= \delta(t-1)$ X

(B) Consider the system with input x and output y as shown in the figure. The three subsystems in the block diagram are LTI and are labelled with their impulse responses h_1, h_2, and h_3. Suppose that $h_2(t) = 2\delta(t)$, $h_3(t) = \delta(t-3)$, and h_1 is the function found in part (a). Find the impulse response h of the system (with input x and output y). **Show all of your work** and **do not skip any steps** in your answer. **[3 marks]**



$y = (x * \delta) + (x * h_1) * h_2 + (x * h_3)$

$= x * (\delta + h_1 * h_2 + h_3)$

$= x * h$ ✓

$\Rightarrow h(t) = \{\delta + h_1 * h_2 + h_3\}(t)$

$\delta(t) + 2\int_{-\infty}^{\infty} \delta(\tau-1)\delta(-\tau+t)\, d\tau + \delta(t-3)$

let $u = \tau - 1$ $du = d\tau$ $\tau = u+1$

$= \delta(t) + 2\int_{-\infty}^{\infty} \delta(u)\delta(-u+(t-1))\, d\tau + \delta(t-3)$

$= \delta(t) + 2(\delta(t) * \delta(t-1)) + \delta(t-3)$

$= \delta(t) + 2\underline{\delta(t-1)} + \delta(t-3)$

2/3

$h(t) = \begin{cases} 1, & t = 0 \\ 2, & t = 1 \\ 1, & t = 3 \\ 0, & \text{otherwise} \end{cases}$

Question 5.

(a) For an arbitrary LTI system with impulse response h, state the condition on h that must be satisfied in order for the system to be BIBO stable. **[1 mark]**

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

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(b) Consider the LTI system $\mathscr{H}$ with impulse response $h(t) = tu(t)$. **Using the condition stated in your answer to part (a)** of this question, determine whether the system $\mathscr{H}$ is BIBO stable. **[4 marks]**

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{\infty} |t\, u(t)|\, dt$$

$$= \int_{0}^{\infty} |t|\, dt \qquad (u(t)=0,\ t<0)$$

$$= \int_{0}^{\infty} t\, dt$$

$$= \frac{1}{2}\left[t^2\right]_0^{\infty}$$

$$= \frac{1}{2}(\infty) \not< \infty$$

$\Rightarrow$ Not BIBO stable

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END