Exercise 5.3

L Answer (b).

We are given the T-periodic function x, where

$$1 \quad x(t) = \delta(t) + 6\delta(t-1) + 6\delta(t-2) \quad \text{and} \quad T = 3.$$

From the Fourier series analysis equation, we have

$$\begin{aligned}
c_k &= \tfrac{1}{T}\int_0^T x(t)e^{-j(2\pi/T)kt}\,dt && \text{Fourier series analysis equation}\\
&= \tfrac{1}{3}\int_0^3 [\delta(t)+6\delta(t-1)+6\delta(t-2)]\,e^{-j(2\pi/3)kt}\,dt && \text{substitute given } x \text{ in } 1\\
&= \int_0^3 \left[\tfrac{1}{3}\delta(t)+2\delta(t-1)+2\delta(t-2)\right]e^{-j(2\pi/3)kt}\,dt && \text{move } \tfrac{1}{3} \text{ inside integral}\\
&= \int_0^3 \tfrac{1}{3}\delta(t)e^{-j(2\pi/3)kt}\,dt + \int_0^3 2\delta(t-1)e^{-j(2\pi/3)kt}\,dt + \int_0^3 2\delta(t-2)e^{-j(2\pi/3)kt}\,dt && \text{integrate each term separately}\\
&= \tfrac{1}{3}e^{-j(2\pi/3)k(0)} + 2e^{-j(2\pi/3)k(1)} + 2e^{-j(2\pi/3)k(2)} && \text{change limits and use sifting property}\\
&= \tfrac{1}{3} + 2e^{-j(2\pi/3)k} + 2e^{-j(4\pi/3)k} && \text{multiply constants}\\
&= \tfrac{1}{3} + e^{-j\pi k}(2)\left[e^{-j(\pi/3)k}+e^{j(\pi/3)k}\right] && \text{factor out average exponent}\\
&= \tfrac{1}{3} + (-1)^k\cos(\pi k/3). && \text{Euler and } e^{-j\pi} = -1
\end{aligned}$$