University of Victoria
Exam 4
Summer 2020

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 30295), A02 (CRN 30296)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

This examination paper has **4 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

The answer to each question is to be uploaded as a **separate PDF document** to the **answer-submission area** for the exam on CourseSpaces **prior to the end of the exam period**.

Total Marks: 24

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must show all of your work and explain all nontrivial steps!
Clearly define any new quantities (e.g., variables, functions, etc.) that you introduce in your solutions.

1 Academic-Integrity Pledge

If you did not sign and submit the academic-integrity pledge (shown below) to the academic-integrity pledge submission area on CourseSpaces prior to the exam (as you were strongly recommended to do), you are required to include this signed pledge as part of your exam submission, as **not submitting such a pledge would constitute refusal to abide by the rules of the exam, which will result in an automatic grade of zero**.

Academic Honesty and Integrity Pledge for Online Exam in ECE 260

Department of Electrical and Computer Engineering
Faculty of Engineering
University of Victoria

Academic honesty and integrity are essential principles of the University of Victoria (UVic) and engineering as a profession. All UVic students are expected to behave as honest and responsible members of an academic community. Engineering students have an even greater responsibility to maintain the highest level of academic honesty and integrity as they prepare to enter a profession with those principles as a cornerstone. Cheating on exams or projects, plagiarizing or any other form of academic dishonesty are clear violations of these principles.

As a student of the Faculty of Engineering at UVic, I solemnly pledge to follow the policies, principles, rules, and guidelines of the University with respect to academic honesty. By signing this pledge, I promise to adhere to exam requirements and maintain the highest level of ethical principles during the exam period. Furthermore, by signing this pledge, I also acknowledge that I have read, in full, the document titled "Online Exams for ECE 260" (on the course web site) so that I am aware of all of the procedures and rules that apply to writing online exams in this course.

Printed Name	Student ID
Signature	Date (yyyy-mm-dd)

2 General Comments

In order to not lose (possibly many or all) marks for your answer to a question, it is required that you:

1. show **all** of your work;
2. **do not skip** any steps in your answer; and
3. for each nontrivial step in your answer, **include a brief comment** to explain what you are doing; for example, identify any special properties or identities/relationships being used; in many cases, a few words in point form will suffice (e.g., "used XXX property", "used XXX identity", "from definition of XXX");

The answer to each question must uploaded as it is completed. Each answer should be placed in a separate file (in PDF format). In the case of a question with multiple parts (e.g., parts (a), (b), and so on), all parts of the question can be answered in the same file.

3 Questions

Question 1. The functions v, x, and y are related by the equations

$$x(t) = v(t)\cos(10t) \quad \text{and} \quad y(t) = x(t)\cos(10t - \tfrac{\pi}{3}).$$

Let V, X, and Y denote the Fourier transforms of v, x, and y, respectively.
(a) Find a fully simplified expression for X in terms of V. **[1 mark]**
(b) Find a fully simplified expression for Y in terms of X. **[3 marks]**
(c) Find a fully simplified expression for Y in terms of V. **[2 marks]**

In each of parts (a), (b), and (c), you **must show all of your work** and you **must not skip any steps** in your solution. A correct final answer that skips steps may **receive a mark of zero**.

Question 2. The function

$$x(t) = \begin{cases} 8t^2 + 1 & 0 \le t < \frac{1}{2} \\ t - \frac{3}{2} & \frac{1}{2} \le t < \frac{3}{2} \\ \pi & \frac{3}{2} \le t < 2 \end{cases}$$

has the Fourier transform representation $\tilde{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$, where X denotes the Fourier transform of x. Find $\tilde{x}\left(\frac{1}{2}\right)$ and $\tilde{x}\left(\frac{3}{2}\right)$. You must **show all of your work** and **fully justify** your answer. **[2 marks]**

Question 3. A LTI system with the input x and output y has the frequency response $H(\omega) = \frac{5j\omega+3}{7j\omega^3-2j\omega^2+11}$. Find the differential equation that characterizes this system. **[6 marks]**

Question 4. Using properties of the Fourier transform, find the Fourier transform Y of the function y in terms of the Fourier transform X of the function x, where $y(t) = tx(-t)$. You must use a systematic method. You must **show all of your work** and you **must not skip any steps**. A correct final answer with an incorrect or incomplete justification may receive zero marks. **[6 marks]**

Question 5. A function x is bandlimited to frequencies in the range $[-a, a]$. The function y is related to x as given by $y(t) = 5x(5t)$.
(a) Find the lowest frequency ω_x at which x can be sampled to avoid aliasing. **[1 mark]**
(b) Find the lowest frequency ω_y at which y can be sampled to avoid aliasing. **[3 marks]**

In each of parts (a) and (b), you must **show all of your work** and **fully justify** your answer.

END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$\mathscr{F}x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$
$$\mathscr{F}^{-1}X(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$
$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$
$$a_k = \tfrac{1}{T}X_T(k\omega_0)$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\,d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\operatorname{sgn} t$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
6	$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\operatorname{rect}\frac{t}{T}$	$\lvert T\rvert \operatorname{sinc}\frac{T\omega}{2}$
9	$\frac{\lvert B\rvert}{\pi}\operatorname{sinc} Bt$	$\operatorname{rect}\frac{\omega}{2B}$
10	$e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$