ECE 260

EXAM 1

SOLUTIONS

(FALL 2024)

QUESTION 1

From ①, we have

$v(t) = 0$ for all $t<0$ $\Rightarrow$

$x(t-3)+1 = 0$ for all $t<0$ $\Rightarrow$

$x(t-3) = -1$ for all $t<0$ $\Rightarrow$

$x(t) = -1$ for all $t+3<0$ $\Rightarrow$

$x(t) = -1$ for all $t<-3$

Now, we consider ②.

$w(t) = x(t-2)+1$ is even

The function w is obtained from x by

time shifting by 2 (i.e., shift right by 2)

amplitude shifting by 1 (i.e., shift up by 1)

Therefore, x is obtained from w by

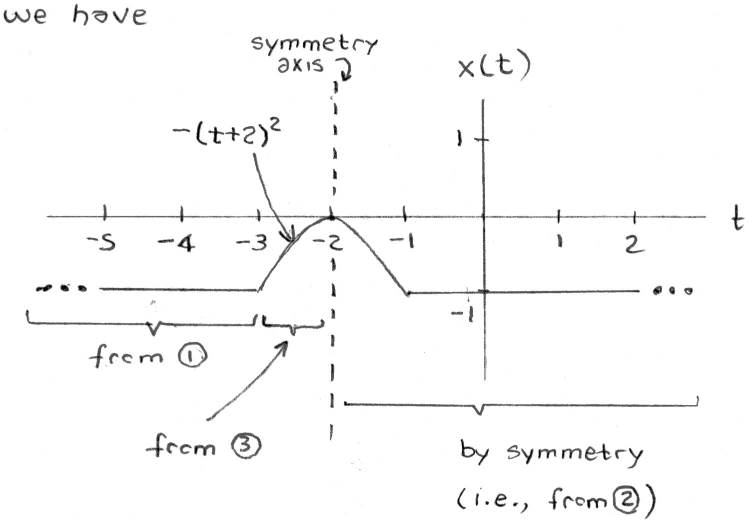
amplitude shifting by -1 (i.e., shift down by 1)

time shifting by -2 (i.e., shift left by 2)

So, w having a symmetry point at $(0,0)$ implies that x has a symmetry point at $(0,0)+(-2,-1) = (-2,-1)$.

In other words, x has even-type symmetry about the line $t=-2$.

Thus, we have



$$x(t) = \begin{cases} -(t+2)^2 & -3 \le t \le -1 \\ -1 & \text{otherwise} \end{cases}$$

QUESTION 2A

$$|H(\omega)| = \left| \frac{-5}{(-1-j\omega)^4} \right| = \frac{|-5|}{|(-1-j\omega)^4|} = \frac{5}{|-1-j\omega|^4} = \frac{5}{\left(\sqrt{\omega^2+1}\right)^4}$$

$$= \frac{5}{|\omega^2+1|^2} = \frac{5}{(\omega^2+1)^2}$$

QUESTION 2B

$$\arg H(\omega) = \arg\left[\frac{-5}{(-1-j\omega)^4}\right]$$

$$= \arg(-5) - \arg\left[(-1-j\omega)^4\right]$$

$$= \pi - 4\arg(-1-j\omega)$$

$$= \pi - 4\left[\pi + \arctan\left(\frac{-\omega}{-1}\right)\right]$$

$$= \pi - 4\pi - 4\arctan(\omega)$$

$$= -3\pi - 4\arctan(\omega)$$

or more generally,

$$\arg H(\omega) = -4\arctan(\omega) + (2k+1)\pi \quad \text{for all } k \in \mathbb{Z}$$

QUESTION 3A

$$x(t) = [t][u(t)-u(t-1)] + [2-t][u(t-1)-u(t-2)]$$

$$= tu(t) - tu(t-1) + 2u(t-1) - 2u(t-2) - tu(t-1) + tu(t-2)$$

$$= tu(t) + [-t+2-t]\,u(t-1) + [-2+t]\,u(t-2)$$

$$= tu(t) + (2-2t)\,u(t-1) + (t-2)\,u(t-2)$$

QUESTION 3B

$$y(t) = \sum_{k=-\infty}^{\infty} x(t-3k)$$

where the formula for x is as determined in part (a)

QUESTION 4A

A system $\mathcal{H}$ is said to be linear if, for all functions x_1 and x_2 and all complex constants a_1 and a_2, the following holds:

$$\mathcal{H}\{a_1 x_1 + a_2 x_2\} = a_1 \mathcal{H} x_1 + a_2 \mathcal{H} x_2$$

QUESTION 4B

$$\boxed{\mathcal{H}x(t) = 2x(t) + 3}$$

$$\mathcal{H}\{a_1 x_1 + a_2 x_2\}(t)$$

$$= 2\left[a_1 x_1(t) + a_2 x_2(t)\right] + 3$$

$$= 2a_1 x_1(t) + 2a_2 x_2(t) + 3$$

$$a_1 \mathcal{H}x_1(t) + a_2 \mathcal{H}x_2(t)$$

$$= a_1\left[2x_1(t) + 3\right] + a_2\left[2x_2(t) + 3\right]$$

$$= 2a_1 x_1(t) + 3a_1 + 2a_2 x_2(t) + 3a_2$$

$$= 2a_1 x_1(t) + 2a_2 x_2(t) + 3(a_1 + a_2)$$

since $\mathcal{H}\{a_1x_1 + a_2x_2\} = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$ does not hold for all x_1, x_2, a_1, and a_2, the system $\mathcal{H}$ is not linear (e.g., this relationship does not hold if $a_1 + a_2 \neq 1$).

QUESTION 5

```
function x = func(t)
    x = ...
      (0 <= t & t < 3) .* ...
      (t + 1) .^ 9 .* (t + 2) .^ 7 ./ (t + 3) + ...
      (3 <= t & t < 6) .* ...
      sin(pi * t) ./ (t + 1);
```

QUESTION 6

$\mathcal{H}x(t) = Ax^3(t) + B$

$x_1(t) = -1, \quad \lambda_1 = -9$

$x_2(t) = 2, \quad \lambda_2 = 9$

Since x_1 is an eigenfunction of $\mathcal{H}$ with eigenvalue λ_1

$\mathcal{H}x_1 = \lambda_1 x_1 \Rightarrow$

$A(-1)^3 + B = (-9)(-1) \Rightarrow$

$-A + B = 9$ ①

Since x_2 is an eigenfunction of $\mathcal{H}$ with eigenvalue λ_2

$\mathcal{H}x_2 = \lambda_2 x_2 \Rightarrow$

$A(2)^3 + B = (9)(2) \Rightarrow$

$8A + B = 18$ ②

solving for A and B

① → $-A + B = 9 \Rightarrow B = A + 9$ ③

② → $8A + B = 18 \underset{\text{use ③}}{\Rightarrow} 8A + A + 9 = 18 \Rightarrow 9A + 9 = 18 \Rightarrow 9A = 9 \Rightarrow$

$A = 1$

③ → $B = A + 9 = 1 + 9 = 10$

therefore,

$A = 1$

$B = 10$