University of Victoria
Midterm Examination #2
Fall 2013

Course Name:	**ELEC 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**11197 (A01), 11198 (A02)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Name: [redacted]
Student Number: V00[redacted]

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 27

This examination is **closed book.**

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

Show all of your work!
Clearly define any new quantities (e.g., variables, functions, etc.) that you introduce in your solutions.

17.5

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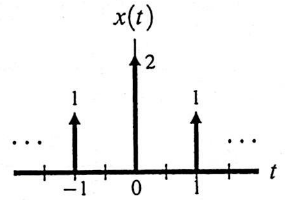
Do not write on this page unless instructed to do so.

PROBLEM 1.

Consider the periodic signal $x(t)$ with fundamental period $T = 3$ and fundamental frequency ω_0 as shown in the figure. Using the Fourier series analysis equation, find the Fourier series coefficient sequence c_k for the signal $x(t)$. Your solution must consider the single period of $x(t)$ for $-\frac{T}{2} \leq t < \frac{T}{2}$. Your answer must be simplified as much as possible and must be expressed in terms of cos and sin to whatever extent is possible. **[6 marks]**



$T = 3 \qquad w_0 = \frac{2\pi}{3}$

$$c_k = \frac{1}{3}\int_{-1}^{1} x(t)\, e^{-jkw_0 t}\, \partial t$$

$$= \frac{1}{3}\int_{-1}^{1} [u(t+1) + 2u(t) + u(t-1)]\, e^{-jkw_0 t}\, \partial t$$

$$= \frac{1}{3} e^{-jkw_0(-1)} + \frac{2}{3} e^{-jkw_0(0)} + \frac{1}{3} e^{-jkw_0(1)}$$

$$\frac{1}{3}\left(e^{jkw_0} - e^{-jkw_0}\right) + \frac{2}{3} \qquad \cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\left(\frac{2}{2}\right)\frac{1}{3}\left(e^{jk\left(\frac{2\pi}{3}\right)} - e^{-jk\left(\frac{2\pi}{3}\right)}\right) + \frac{2}{3}$$

$$\boxed{c_k = \frac{2}{3}\cos\left(\frac{2\pi}{3}k\right) + \frac{2}{3}}$$

6/6

$k = 0$

$\cancel{1 + 2 + 1 = 2}$

$$\frac{1}{3}\int_{-1}^{1} x(t)\, e^{-j(0)w_0 t}$$

$$\frac{1}{3}\int_{-1}^{1} u(t+1) + 2u(t) + u(t-1)$$

$$\frac{1}{3}(\cancel{1} + 2 + \cancel{1}) = \boxed{2/3}$$

PROBLEM 2. Let $y(t) = x_1(t) * x_2(t)$, where $x_1(t) = \operatorname{sinc} 5\pi t$ and $x_2(t) = \operatorname{sinc} 10\pi t$. Let $X_1(\omega)$, $X_2(\omega)$, and $Y(\omega)$ denote the Fourier transforms of $x_1(t)$, $x_2(t)$, and $y(t)$, respectively.

(A) Using properties of the Fourier transform and only the Fourier transform pair $\operatorname{sinc} t \overset{\mathcal{F}}{\longleftrightarrow} \pi \operatorname{rect} \frac{\omega}{2}$, find a fully simplified expression for $Y(\omega)$. You must use a systematic method and show all of your work. **[6 marks]**

$$y(t) = x_1(t) * x_2(t)$$

$$y(t) = \operatorname{Sinc} 5\pi t * \operatorname{Sinc} 10\pi t$$

$$\mathcal{F}\{x_1(t) * x_2(t)\} = X_1(\omega) X_2(\omega)$$

$$\mathcal{F}\{\operatorname{Sinc} t\} \to \pi \operatorname{rect}\left(\frac{\omega}{2}\right)$$

$$\mathcal{F}\{x(at)\} = \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

$X_1(\omega)$
$$\left[\frac{1}{5\pi}(\pi)\left(\operatorname{rect}\left(\frac{\omega}{2 \cdot 5\pi}\right)\right)\right] \text{ Mult}$$

$X_2(\omega)$
$$\left[\frac{1}{10\pi}\pi\left(\operatorname{Rect}\left(\frac{\omega}{2 \cdot 10\pi}\right)\right)\right]$$

$$\left[\frac{1}{5} \operatorname{Rect}\left(\frac{\omega}{10\pi}\right)\right]\left[\frac{1}{10} \operatorname{Rect}\left(\frac{\omega}{20\pi}\right)\right]$$

$$\underline{\frac{1}{50} \operatorname{Rect}\left(\frac{\omega}{10\pi}\right) \operatorname{Rect}\left(\frac{\omega}{20\pi}\right)} = Y(\omega) = ?$$

-0.5

$\frac{5.5}{6}$

(B) Let ω_{x_1} and ω_y denote the Nyquist rates (expressed in radian measure) for $x_1(t)$ and $y(t)$. Find ω_{x_1} and ω_y. [Recall that the Nyquist rate is the lowest rate at which a signal can be sampled without aliasing occurring.] **[2 marks]**

$\omega_s \geq 2\omega_n$

$\boxed{\cancel{X_1(\omega)}} = 10\pi$

ω_n

$\boxed{\cancel{X_2(\omega)}} = 20\pi$

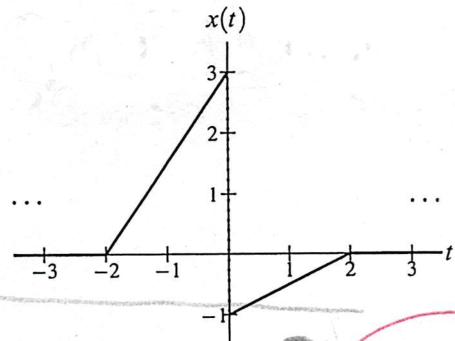
$2(10\pi) = \boxed{\cancel{20\pi}}$

$2(20\pi) = \boxed{\cancel{40\pi}}$

zero

PROBLEM 3.

Consider the (piecewise-linear) periodic signal $x(t)$, as shown in the figure, with fundamental period $T = 6$ and fundamental frequency ω_0. Let $\hat{x}(t)$ denote the Fourier series representation of $x(t)$. That is, $\hat{x}(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$, where the c_k are the Fourier series coefficients. Find $\hat{x}(0)$ and $\hat{x}(1)$. **[2 marks]**



~~c_n $x(t)$~~

$c_n = \frac{1}{6}$ $\qquad \omega_0 = \frac{2\pi}{6} = \pi/3$

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} 1/6\, e^{jk(\pi/3)t}$$

$$\hat{x}(0) = \sum_{k=-\infty}^{\infty} 1/6\, e^{0} = \cancel{1/6}$$

$$\hat{x}(1) = \sum_{k=-\infty}^{\infty} 1/6\, e^{jk\pi/3}$$

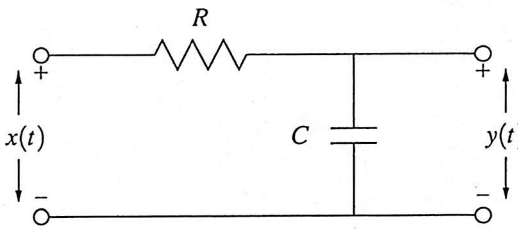
0

Wrong approach

plug & val

PROBLEM 4.

Consider the LTI circuit shown in the figure to the right with input $x(t)$ and output $y(t)$, where $R = 15$ and $C = \frac{1}{5}$. This circuit can be shown to be characterized by the differential equation $x(t) = RC\frac{d}{dt}y(t) + y(t)$. Let $h(t)$ denote the impulse response of the system. Let $X(\omega)$, $Y(\omega)$, and $H(\omega)$ denote the Fourier transforms of $x(t)$, $y(t)$, and $h(t)$, respectively.



(A) Find a fully simplified expression for $H(\omega)$. **[3 marks]**

$$x(t) = RC\frac{\partial}{\partial t}y(t) + y(t)$$

$$\mathcal{F}\{x(t)\} = 3\mathcal{F}\{\tfrac{\partial}{\partial t}y(t)\} + \mathcal{F}\{y(t)\}$$

$$X(\omega) = 3j\omega Y(\omega) + Y(\omega)$$

$$X(\omega) = Y(\omega)(3j\omega + 1)$$

$$[3j\omega + 1][X(\omega)]$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \boxed{\frac{1}{3j\omega + 1}}$$

$$Y(\omega) = H(\omega)X(\omega)$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$

$$\mathcal{F}\{\tfrac{\partial}{\partial t}x(t)\} = j\omega X(\omega)$$

3

(B) Find a fully simplified expression for $h(t)$. **[2 marks]**

$$\frac{1}{3j\omega + 1} = \frac{1}{1 + 3j\omega}\left(\frac{3}{3}\right)$$

$$\frac{3}{3 + 9j\omega} = \frac{3}{9(\frac{1}{3} + j\omega)} = \tfrac{1}{3}\mathcal{F}^{-1}\left\{\frac{1}{(\frac{1}{3} + j\omega)}\right\}$$

$$\boxed{= \tfrac{1}{3}e^{-\frac{1}{3}t}u(t)}$$

$$\frac{1}{a + j\omega} \xrightarrow{\mathcal{F}} e^{-at}u(t)$$

2

PROBLEM 5.

Consider the system with input $x(t)$ and output $y(t)$ as shown in Figure A. Let $X(\omega)$ and $Y(\omega)$ denote the Fourier transforms of $x(t)$ and $y(t)$, respectively. Suppose that the Fourier transform $X_1(\omega)$ of $x_1(t)$ is as shown in Figure B.

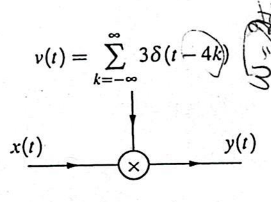


Figure A

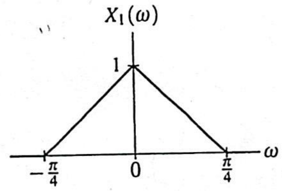


Figure B

(A) Find a fully simplified expression for $Y(\omega)$ in terms of $X(\omega)$. [Hint: Use the method discussed in the lectures in order to avoid dragging convolution into your solution.] **[5 marks]**

$Y(\omega) = X(\omega)\, V(\omega)$

$\sum_{k=-\infty}^{\infty} 3\delta(t-4k)$

Periodic $t = 4$

$\omega_0 = \frac{2\pi}{4} = \pi/2$

$c_n = \frac{3}{4}\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt$

$c_n = \frac{3}{4}(1) = 3/4$

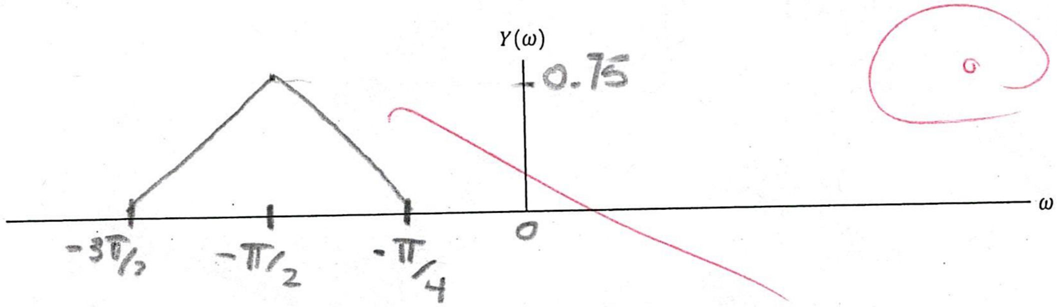
$x(t) = c_n e^{-j\omega t}$

$\sum_{-\infty}^{\infty} 3/4\, e^{-j(\pi/2)t}$

$y(\omega) = \mathcal{F}\left\{3/4 \sum_{-\infty}^{\infty} x(t) e^{-j\pi/2 t}\right\}$

$y(\omega) = 3/4\, X(\omega + \pi/2)$

(B) In the case that $x(t) = x_1(t)$, plot $Y(\omega)$ using the axes provided. **[1 mark]**



EXTRA CREDIT SECTION

The problems in this section are strictly for extra credit (i.e., bonus marks).

PROBLEM 6. Name the artist and title for the song that plays throughout most of the duration of the AMC trailer for the series finale of Breaking Bad. In order to receive extra credit, both the artist and title must be correct (including spelling). **[0.5 marks]**

Artist: ______________________________

Title: ______________________________

zero

I am the one who knocks

- Walter White

END

USEFUL FORMULAE AND OTHER INFORMATION

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

$$\mathscr{F}\{x(t)\} = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$\mathscr{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$

$$c_k = \tfrac{1}{T}X_T(k\omega_0)$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time-Domain Shifting	$x(t - t_0)$	$e^{-jk\omega_0 t_0}a_k$
Time Reversal	$x(-t)$	a_{-k}

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2 d\omega$	

Fourier Transform Pairs

$x(t)$	$X(\omega)$
$\delta(t)$	1
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
1	$2\pi\delta(\omega)$
$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$e^{-at}u(t)$, $\text{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
$t^{n-1}e^{-at}u(t)$, $\text{Re}\{a\} > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$