Exercise 5.109

$\boxed{\text{L}}$ **Answer (c).**

We are given a LTI system $\mathcal{H}$ with the impulse response h and frequency response H, where

$$h(t) = \tfrac{4}{\pi}\cos(20t)\operatorname{sinc}(2t).$$

We are also given the integral table entry

$$\int_{-\infty}^{\infty} \operatorname{sinc}(at)e^{-j\omega t}dt = \frac{\pi}{|a|}\operatorname{rect}\left(\frac{\omega}{2a}\right).$$

From the relationship between h and H, we have

$$\begin{aligned}
H(\omega) &= \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt && \text{formula for H} \\
&= \int_{-\infty}^{\infty} \tfrac{4}{\pi}\cos(20t)\operatorname{sinc}(2t)e^{-j\omega t}dt && \text{substitute given h} \\
&= \int_{-\infty}^{\infty} \tfrac{4}{\pi}\left(\tfrac{1}{2}\right)\left(e^{j20t}+e^{-j20t}\right)\operatorname{sinc}(2t)e^{-j\omega t}dt && \text{Euler} \\
&= \tfrac{2}{\pi}\int_{-\infty}^{\infty} \left(e^{j20t}+e^{-j20t}\right)\operatorname{sinc}(2t)e^{-j\omega t}dt && \text{pull out constant} \\
&= \tfrac{2}{\pi}\int_{-\infty}^{\infty} e^{j20t}\operatorname{sinc}(2t)e^{-j\omega t}dt + \tfrac{2}{\pi}\int_{-\infty}^{\infty} e^{-j20t}\operatorname{sinc}(2t)e^{-j\omega t}dt && \text{split into 2 integrals} \\
&= \tfrac{2}{\pi}\int_{-\infty}^{\infty} \operatorname{sinc}(2t)e^{j(20-\omega)t}dt + \tfrac{2}{\pi}\int_{-\infty}^{\infty} \operatorname{sinc}(2t)e^{j(-20-\omega)t}dt. && \text{combine exponentials}
\end{aligned}$$

Performing integration using the given integral table entry, we have

$$\begin{aligned}
H(\omega) &= \tfrac{2}{\pi}\left[\tfrac{\pi}{2}\operatorname{rect}\left(\tfrac{1}{4}(20-\omega)\right)\right] + \tfrac{2}{\pi}\left[\tfrac{\pi}{2}\operatorname{rect}\left(\tfrac{1}{4}(-20-\omega)\right)\right] && \text{integrate using given formula} \\
&= \tfrac{2}{\pi}\left[\tfrac{\pi}{2}\operatorname{rect}\left(\tfrac{1}{4}(\omega-20)\right)\right] + \tfrac{2}{\pi}\left[\tfrac{\pi}{2}\operatorname{rect}\left(\tfrac{1}{4}(\omega+20)\right)\right] && \text{rect is even} \\
&= \operatorname{rect}\left[\tfrac{1}{4}(\omega-20)\right] + \operatorname{rect}\left[\tfrac{1}{4}(\omega+20)\right] && \text{cancel factors} \\
&= \begin{cases} 1 & |\omega| \in [18,22] \\ 0 & \text{otherwise.} \end{cases} && \text{write as multi-case formula}
\end{aligned}$$

Since H is real and nonnegative, we trivially have that

$$|H(\omega)| = H(\omega). \qquad \text{since } H(\omega) \in \mathbb{R} \text{ and } H(\omega) \geq 0$$

From the form of $|H(\omega)|$, we conclude that $\mathcal{H}$ is an ideal bandpass filter with the passband $|\omega| \in [18,22]$.