PROBLEMS

B–6–1. Plot the root loci for the closed-loop control system with

$$G(s) = \frac{K(s+1)}{s^2}, \qquad H(s) = 1$$

B–6–2. Plot the root loci for the closed-loop control system with

$$G(s) = \frac{K}{s(s+1)(s^2+4s+5)}, \qquad H(s) = 1$$

B–6–3. Plot the root loci for the system with

$$G(s) = \frac{K}{s(s+0.5)(s^2+0.6s+10)}, \qquad H(s) = 1$$

B–6–4. Show that the root loci for a control system with

$$G(s) = \frac{K(s^2+6s+10)}{s^2+2s+10}, \qquad H(s) = 1$$

are arcs of the circle centered at the origin with radius equal to $\sqrt{10}$.

B–6–5. Plot the root loci for a closed-loop control system with

$$G(s) = \frac{K(s+0.2)}{s^2(s+3.6)}, \qquad H(s) = 1$$

B–6–6. Plot the root loci for a closed-loop control system with

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)}, \qquad H(s) = 1$$

Locate the closed-loop poles on the root loci such that the dominant closed-loop poles have a damping ratio equal to 0.5. Determine the corresponding value of gain K.

B–6–7. Plot the root loci for the system shown in Figure 6–100. Determine the range of gain K for stability.

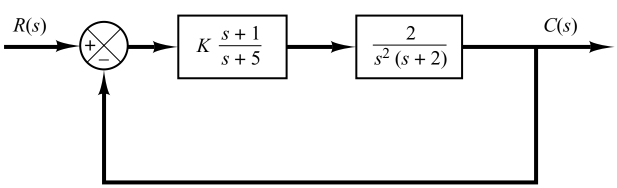

Figure 6–100
Control system.

B–6–8. Consider a unity-feedback control system with the following feedforward transfer function:

$$G(s) = \frac{K}{s(s^2+4s+8)}$$

Plot the root loci for the system. If the value of gain K is set equal to 2, where are the closed-loop poles located?

B–6–9. Consider the system whose open-loop transfer function is given by

$$G(s)H(s) = \frac{K(s-0.6667)}{s^4+3.3401s^3+7.0325s^2}$$

Show that the equation for the asymptotes is given by

$$G_a(s)H_a(s) = \frac{K}{s^3+4.0068s^2+5.3515s+2.3825}$$

Using MATLAB, plot the root loci and asymptotes for the system.

B–6–10. Consider the unity-feedback system whose feedforward transfer function is

$$G(s) = \frac{K}{s(s+1)}$$

The constant-gain locus for the system for a given value of K is defined by the following equation:

$$\left|\frac{K}{s(s+1)}\right| = 1$$

Show that the constant-gain loci for $0 \le K \le \infty$ may be given by

$$[\sigma(\sigma+1)+\omega^2]^2+\omega^2 = K^2$$

Sketch the constant-gain loci for $K = 1, 2, 5, 10$, and 20 on the s plane.

B–6–11. Consider the system shown in Figure 6–101. Plot the root loci with MATLAB. Locate the closed-loop poles when the gain K is set equal to 2.

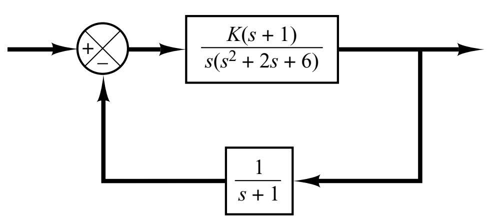

Figure 6–101
Control system.

B–6–12. Plot root-locus diagrams for the nonminimum-phase systems shown in Figures 6–102(a) and (b), respectively.

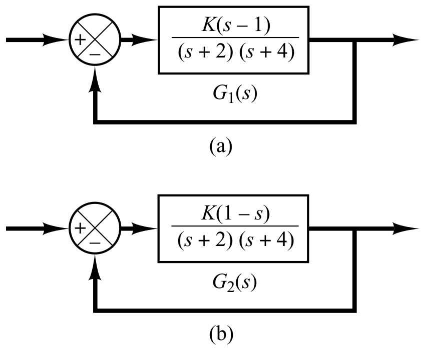

Figure 6–102 (a) and (b) Nonminimum-phase systems.

B–6–13. Consider the mechanical system shown in Figure 6–103. It consists of a spring and two dashpots. Obtain the transfer function of the system. The displacement x_i is the input and displacement x_o is the output. Is this system a mechanical lead network or lag network?

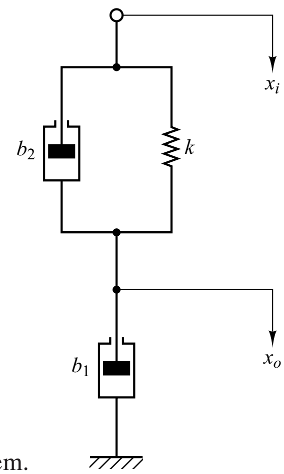

Figure 6–103 Mechanical system.

B–6–14. Consider the system shown in Figure 6–104. Plot the root loci for the system. Determine the value of K such that the damping ratio ζ of the dominant closed-loop poles is 0.5. Then determine all closed-loop poles. Plot the unit-step response curve with MATLAB.

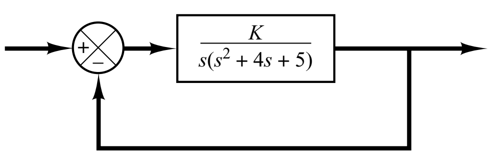

Figure 6–104 Control system.

B–6–15. Determine the values of K, T_1, and T_2 of the system shown in Figure 6–105 so that the dominant closed-loop poles have the damping ratio $\zeta = 0.5$ and the undamped natural frequency $\omega_n = 3$ rad/sec.

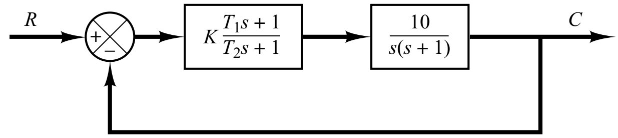

Figure 6–105 Control system.

B–6–16. Consider the control system shown in Figure 6–106. Determine the gain K and time constant T of the controller $G_c(s)$ such that the closed-loop poles are located at $s = -2 \pm j2$.

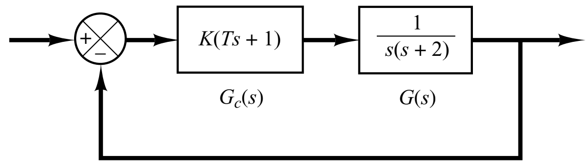

Figure 6–106 Control system.

B–6–17. Consider the system shown in Figure 6–107. Design a lead compensator such that the dominant closed-loop poles are located at $s = -2 \pm j2\sqrt{3}$. Plot the unit-step response curve of the designed system with MATLAB.

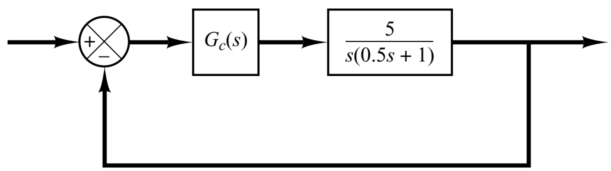

Figure 6–107 Control system.

B–6–18. Consider the system shown in Figure 6–108. Design a compensator such that the dominant closed-loop poles are located at $s = -1 \pm j1$.

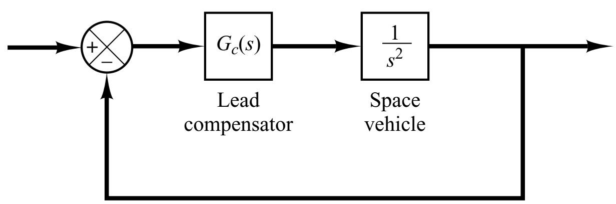

Figure 6–108 Control system.

B–6–19. Referring to the system shown in Figure 6–109, design a compensator such that the static velocity error constant K_v is 20 sec^{-1} without appreciably changing the original location $\left(s = -2 \pm j2\sqrt{3}\right)$ of a pair of the complex-conjugate closed-loop poles.

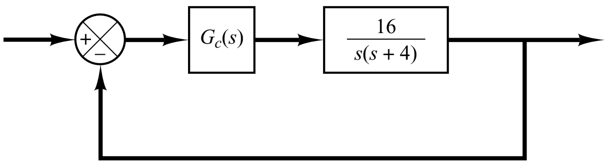

Figure 6–109
Control system.

B–6–20. Consider the angular-positional system shown in Figure 6–110. The dominant closed-loop poles are located at $s = -3.60 \pm j4.80$. The damping ratio ζ of the dominant closed-loop poles is 0.6. The static velocity error constant K_v is 4.1 sec^{-1}, which means that for a ramp input of 360°/sec the steady-state error in following the ramp input is

$$e_v = \frac{\theta_i}{K_v} = \frac{360°/\text{sec}}{4.1 \text{ sec}^{-1}} = 87.8°$$

It is desired to decrease e_v to one-tenth of the present value, or to increase the value of the static velocity error constant K_v to 41 sec^{-1}. It is also desired to keep the damping ratio ζ of the dominant closed-loop poles at 0.6. A small change in the undamped natural frequency ω_n of the dominant closed-loop poles is permissible. Design a suitable lag compensator to increase the static velocity error constant as desired.

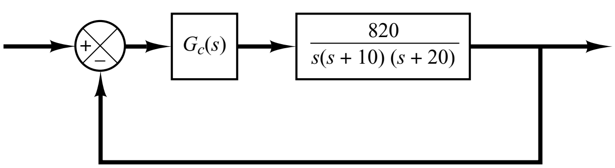

Figure 6–110
Angular-positional system.

B–6–21. Consider the control system shown in Figure 6–111. Design a compensator such that the dominant closed-loop poles are located at $s = -2 \pm j2\sqrt{3}$ and the static velocity error constant K_v is 50 sec^{-1}.

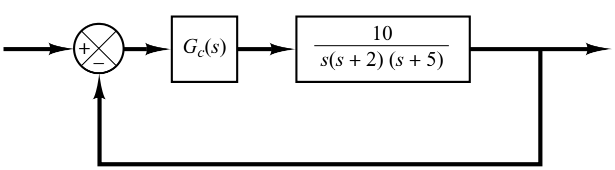

Figure 6–111
Control system.

B–6–22. Consider the control system shown in Figure 6–112. Design a compensator such that the unit-step response curve will exhibit maximum overshoot of 30% or less and settling time of 3 sec or less.

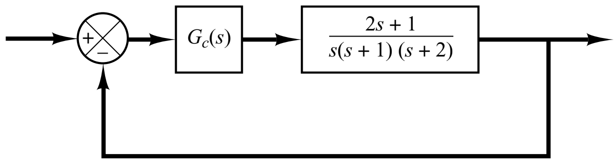

Figure 6–112
Control system.

B–6–23. Consider the control system shown in Figure 6–113. Design a compensator such that the unit-step response curve will exhibit maximum overshoot of 25% or less and settling time of 5 sec or less.

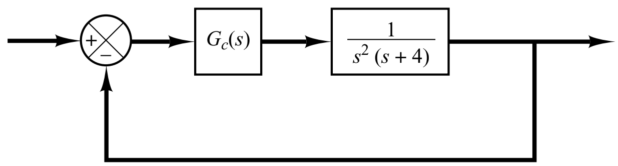

Figure 6–113
Control system.

B–6–24. Consider the system shown in Figure 6–114, which involves velocity feedback. Determine the values of the amplifier gain K and the velocity feedback gain K_h so that the following specifications are satisfied:

1. Damping ratio of the closed-loop poles is 0.5
2. Settling time $\leq$ 2 sec
3. Static velocity error constant $K_v \geq 50$ sec^{-1}
4. $0 < K_h < 1$

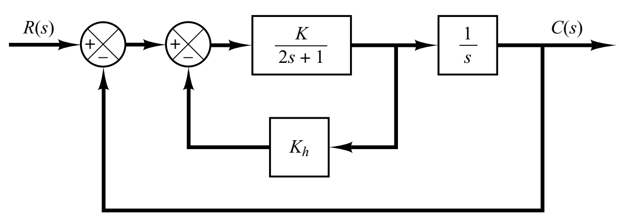

Figure 6–114
Control system.

B–6–25. Consider the system shown in Figure 6–115. The system involves velocity feedback. Determine the value of gain K such that the dominant closed-loop poles have a damping ratio of 0.5. Using the gain K thus determined, obtain the unit-step response of the system.

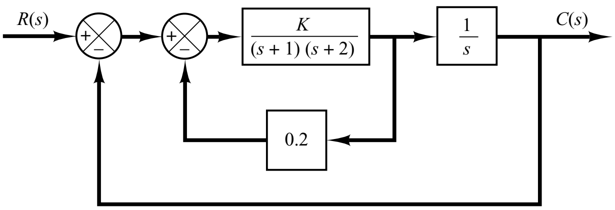

Figure 6–115
Control system.

B–6–26. Consider the system shown in Figure 6–116. Plot the root loci as a varies from 0 to ∞. Determine the value of a such that the damping ratio of the dominant closed-loop poles is 0.5.

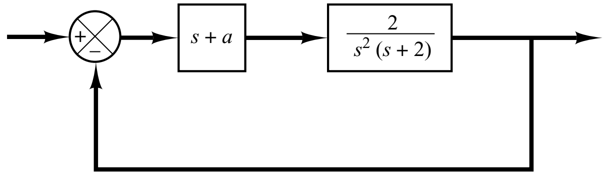

Figure 6–116
Control system.

B–6–27. Consider the system shown in Figure 6–117. Plot the root loci as the value of k varies from 0 to ∞. What value of k will give a damping ratio of the dominant closed-loop poles equal to 0.5? Find the static velocity error constant of the system with this value of k.

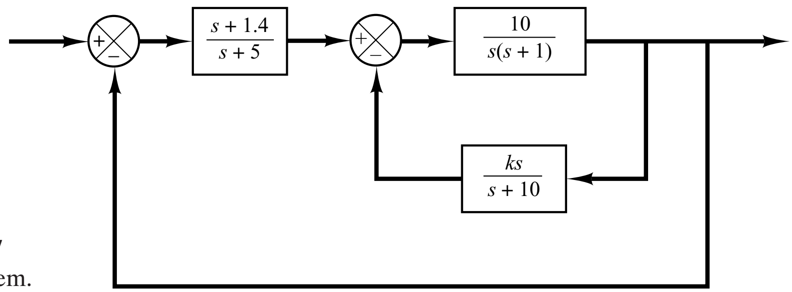

Figure 6–117
Control system.

B–6–28. Consider the system shown in Figure 6–118. Assuming that the value of gain K varies from 0 to ∞, plot the root loci when $K_h = 0.1, 0.3$, and 0.5.

Compare unit-step responses of the system for the following three cases:

(1) $K = 10, \quad K_h = 0.1$

(2) $K = 10, \quad K_h = 0.3$

(3) $K = 10, \quad K_h = 0.5$

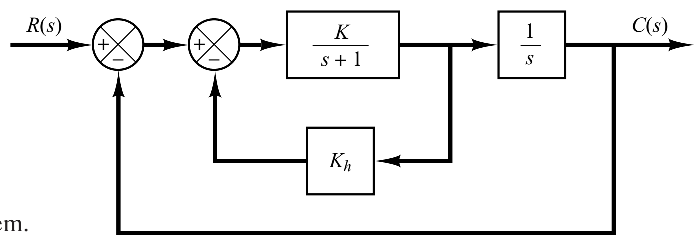

Figure 6–118
Control system.