# ELEC 260

# QUIZ 1

# SOLUTIONS

## PROBLEM 1

$$F(z) = \frac{(z-1)^2(z+1)}{z^2(z^2+2z+2)}$$

To find the poles and zeros and their orders, we factor $F(z)$.

Factor $z^2+2z+2$

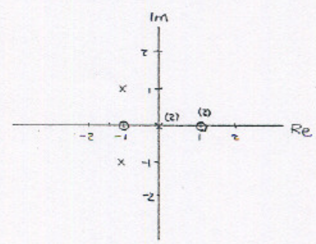
$$z = \frac{-2 \pm \sqrt{2^2-4(1)(2)}}{2} = -1 \pm j$$

So, $z^2+2z+2 = (z+1-j)(z+1+j)$.

We now can write

$$F(z) = \frac{(z-1)^2(z+1)}{z^2(z+1-j)(z+1+j)}$$

Therefore, $F(z)$ has a $1^{st}$ order zero at $-1$, a $2^{nd}$ order zero at $1$, $1^{st}$ order poles at $-1+j$ and $-1-j$, and a $2^{nd}$ order pole at $0$.




## PROBLEM 2

(a) $F_1(z) = z^2 + 3z + j\pi$

Since $F_1(z)$ is a polynomial, $F_1(z)$ is analytic everywhere (i.e., for all $z$).

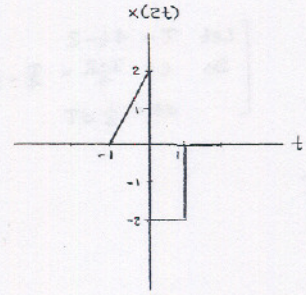
(b) $F_2(z) = \dfrac{(z-1)^2(z+1)}{(z-2)(z+2)^2}$

Since $F_2(z)$ is a rational function, it is analytic everywhere except where the denominator polynomial is zero.

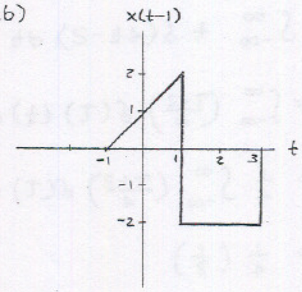
Therefore, $F_2(z)$ is analytic at all points except $2$ and $-2$.

## PROBLEM 3

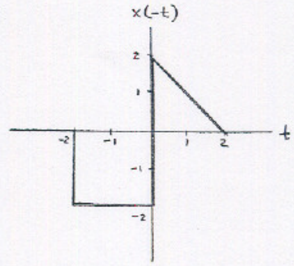
(a)



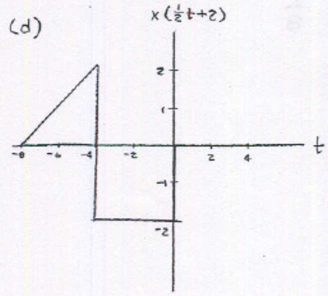

(b)

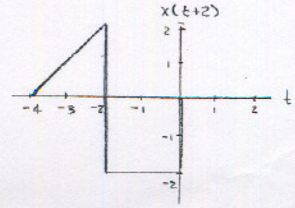



(c)




(d)



PROBLEM 4

Let $T_1$ and $T_2$ be the period of $x_1(t)$ and $x_2(t)$, respectively.

$$T_1 = \frac{2\pi}{2\pi} = 1$$

$$T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$\frac{T_1}{T_2} = \frac{1}{\left(\frac{2}{5}\right)} = \frac{5}{2}$$

Since $T_1/T_2$ is rational, $y(t)$ is periodic.

The period $T$ of $y(t)$ is given by

$$T = 2T_1 = 2.$$

PROBLEM 5

$$x_1(t) = t^3$$

Is $x_1(t)$ even?

$$x_1(-t) = (-t)^3 = -t^3$$

Since $x_1(-t) \neq x_1(t)$, $x_1(t)$ is not even.

Since $x_1(-t) = -x_1(t)$, $x_1(t)$ is odd.

PROBLEM 6

$$\int_{-\infty}^{\infty} t\,\delta(4t-2)\,dt$$

$$\left[\begin{array}{l} \text{Let } \tau = 4t-2 \\ \text{So } t = \frac{\tau+2}{4} = \frac{\tau}{4}+\frac{1}{2} \text{ and} \\ dt = \frac{1}{4}\,d\tau \end{array}\right]$$

$$= \int_{-\infty}^{\infty} \left(\frac{\tau+2}{4}\right) \delta(\tau) \left(\frac{1}{4}\right) d\tau$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} \left(\frac{\tau+2}{4}\right) \delta(\tau)\, d\tau$$

$$= \frac{1}{4}\left(\frac{1}{2}\right)$$

$$= \frac{1}{8}$$

PROBLEM 7

$$x(t) = 1\,[u(t+1) - u(t)] + (t)\,[u(t) - u(t-1)] + (2-t)\,[u(t-1) - u(t-2)]$$

$$= u(t+1) + (t-1)\,u(t) + (2-t-t)\,u(t-1) + (t-2)\,u(t-2)$$

$$= u(t+1) + (t-1)\,u(t) + (2-2t)\,u(t-1) + (t-2)\,u(t-2)$$

## PROBLEM 8

(a) Suppose that

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y_3(t)$

The system is linear if for any $x_1(t)$, and $x_2(t)$ and any complex constants $a_1$ and $a_2$, $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$.

From the system definition, we have

$$y_1(t) = x_1^2(t)$$

$$y_2(t) = x_2^2(t)$$

$$\begin{aligned} y_3(t) &= [a_1 x_1(t) + a_2 x_2(t)]^2 \\ &= a_1^2 x_1^2(t) + 2a_1 a_2 x_1(t) x_2(t) + a_2^2 x_2^2(t) \\ &= a_1^2 y_1(t) + a_2^2 y_2(t) + 2a_1 a_2 x_1(t) x_2(t) \end{aligned}$$

Since $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$, the system is not linear.

## PROBLEM 8

(b) Suppose that

$x_1(t) \longrightarrow y_1(t)$

$x_1(t-t_0) \longrightarrow y_2(t)$

The system is time invariant if for any $x_1(t)$ and any real constant $t_0$, $y_2(t) = y_1(t-t_0)$.

From the system definition, we have

$$y_1(t-t_0) = x_1^2(t-t_0)$$

$$y_2(t) = x_1^2(t-t_0)$$

Since $y_2(t) = y_1(t-t_0)$, the system is time invariant.

## PROBLEM 8

(c) Consider the inputs $x_1(t) = 1$ and $x_2(t) = -1$.

Suppose that

$x_1(t) \longrightarrow y_1(t)$

$x_2(t) \longrightarrow y_2(t)$

Then, we have

$$y_1(t) = (1)^2 = 1$$

$$y_2(t) = (-1)^2 = 1$$

Therefore, two distinct inputs yield the same output.

Consequently, the system is not invertible.