

### Exercise 4.108

**L** Answer (d).

We are given a **LTI** system with impulse response

$$h(t) = e^{-t} \sin(t) u(t). \quad (1)$$

We have

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)| dt &= \int_{-\infty}^{\infty} |e^{-t} \sin(t) u(t)| dt && \text{substitute (1)} \\ &= \int_0^{\infty} |e^{-t} \sin(t)| dt && u(t) = 0 \text{ for all } t < 0 \\ &= \int_0^{\infty} |e^{-t}| |\sin(t)| dt && |ab| = |a| |b| \\ &= \int_0^{\infty} e^{-t} |\sin(t)| dt. && e^{-t} > 0 \text{ for all } t \in \mathbb{R} \end{aligned}$$

Now, we use the fact that  $|\sin(t)| \leq 1$  in order to replace  $|\sin(t)|$  by its upper bound of 1, yielding an inequality. In particular, we have

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)| dt &\leq \int_0^{\infty} e^{-t} |1| dt && \text{replace } |\sin(t)| \text{ by its upper bound of 1} \\ &= \int_0^{\infty} e^{-t} dt && \text{drop 1} \\ &= [-e^{-t}]_0^{\infty} && \text{integrate} \\ &= [0 - (-1)] && \text{simplify} \\ &= 1. \end{aligned}$$

Thus, we have that

$$\int_{-\infty}^{\infty} |h(t)| dt \leq 1. \quad \text{conclusion from above}$$

Since  $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ , the system is **BIBO stable**.