21 +2 = 23

University of Victoria
Midterm Examination #1, Summer 2009
ELEC 260 (Signal Analysis)
Section(s) A01

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SECTION: [redacted]

DURATION: 50 MINUTES

ALL QUESTIONS ARE **TO BE ANSWERED ON THE EXAMINATION PAPER** IN THE SPACE PROVIDED.

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS EXAMINATION PAPER HAS **10 PAGES** (ALL OF WHICH ARE NUMBERED).

TOTAL MARKS: 25

THIS EXAMINATION IS **CLOSED BOOK.**

THE USE OF A CRIB SHEET IS **NOT** PERMITTED.

THE USE OF A CALCULATOR IS **NOT** PERMITTED.

SHOW ALL OF YOUR WORK!
CLEARLY DEFINE ANY NEW QUANTITIES (E.G., VARIABLES, FUNCTIONS, ETC.) THAT YOU INTRODUCE IN YOUR SOLUTIONS.

My question pertains to question 6.

Everything up to & including the line stating $e^{t+6}u(t+4)\left[\int_0^3 \delta(t-3)dt + \int_3^5 \delta(t-3)dt\right]$ is correct. I even evaluate $\int_0^3 \delta(t-3)dt$ to be 0 and $\int_3^5 \delta(t-3)dt$ to be 1.

The only mistake made was when I wrote in $u(t-3)$ in the final answer, which was a brain fart.

I understand ~~this~~ the final answer is incorrect, but I feel that the other work deserves some marks as it is correct, and properly evaluated.

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1 - 3   -3
1 - 2   -2
-5

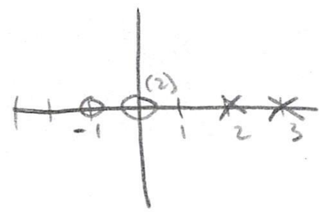
PROBLEM 1. Let $F(s) = \dfrac{s^3+s^2}{s^2-5s+6}$, where s is complex. Find the poles and zeros of $F(s)$ and determine the order of each pole and zero. **[3 marks]**

$$F(s) = \frac{s^2(s+1)}{(s-3)(s-2)}$$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5\pm 1}{2}$$

Zero Order 1: $s=-1$
Order 2: $s=0$

Poles Order 1: $s=3$, $s=2$

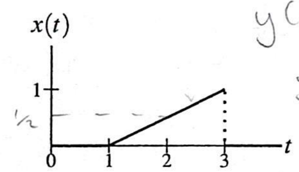


$m = \frac{1-0}{3-1} = \frac{1}{2}$

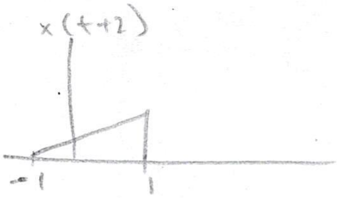
$y(t) = \frac{1}{2}x - \frac{1}{2}$

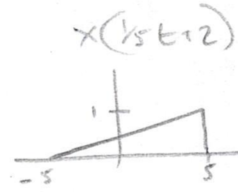
$y(2) = \frac{1}{2}$

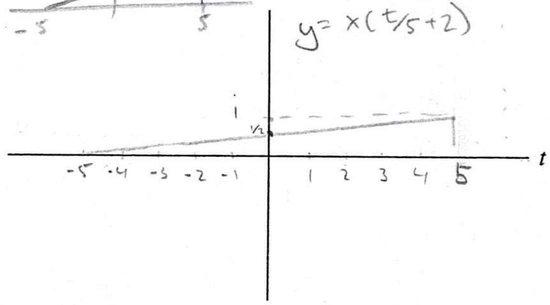
PROBLEM 2.



Suppose that we have the signal $x(t)$ shown in the graph to the right. Using the axes provided below, plot $y(t) = x(t/5+2)$. **[2 marks]**







2

PROBLEM 3. Let $x_1(t) = e^{j6t}$, $x_2(t) = \cos 15t$, and $y(t) = x_1(t) + x_2(t)$. Determine whether $y(t)$ is periodic, and if it is periodic, determine its fundamental period. **[3 marks]**

x_1: $T_1 = \frac{2\pi}{6}$ x_2: $T_2 = \frac{2\pi}{15}$ $\frac{T_1}{T_2} = \frac{2\pi/6}{\frac{2\pi}{15}} = \frac{2\pi}{6} \cdot \frac{15}{2\pi} = \frac{15}{6} = \frac{q}{p} = \frac{T_1}{T_2}$

$\boxed{\frac{15}{6} \text{ is rational, therefore } y(t) \text{ is periodic. The period of } y(t) \text{ is } T_3 = 2\pi}$

$T = pT_1 = qT_2 = 6\,\frac{2\pi}{6} = 15\,\frac{2\pi}{15} = 2\pi$

−1

PROBLEM 4. Suppose that we have a system with input $x(t)$ and output $y(t)$.

(A) Clearly state, in mathematical terms, the condition that must be satisfied in order for the above system to be linear. *Be sure to* ***define all quantities*** *such as variables, functions, and constants. Otherwise, you will receive zero marks.* ***Be careful with the notation*** *that you choose to employ. If, for example, you confuse arrows and equal signs in your solution, you will probably receive zero marks.* **[2 marks]** Denote

Input 1 $x_1(\delta)$
Input 2 $x_2(\delta)$
Input 3 $x_3(\delta)$
Output 1 $y_1(\delta)$
Output 2 $y_2(\delta)$
Output 3 $y_3(\delta)$
Non zero constants a, b

$x_1(\delta) \rightarrow y_1(\delta)$
$x_2(\delta) \rightarrow y_2(\delta)$
$ax_1(\delta) + bx_2(\delta) = x_3(\delta)$
$x_3(\delta) \rightarrow y_3(\delta)$

If
$y_3(\delta) = ay_1(\delta) + by_2(\delta)$
then the system is linear.

✓ 2

(B) Suppose now that the above system is characterized by the equation $y(t) = (t^2+1)x(t)$. **Using the condition stated in part (a) of this problem,** determine whether this system is linear. **[2 marks]**

$y_1(\delta) = (t^2+1)x_1(\delta)$
$y_2(\delta) = (t^2+1)x_2(\delta)$

$ax_1(\delta) + bx_2(\delta) = x_3(\delta)$
$y_3 = (t^2+1)x_3(\delta)$
$y_3 = (t^2+1)(ax_1(\delta) + bx_2(\delta))$
$y_3 = (t^2+1)ax_1(\delta) + (t^2+1)bx_2(\delta)$
$y_3 = ay_1(\delta) + by_2(\delta)$

$\therefore$ The system is linear!

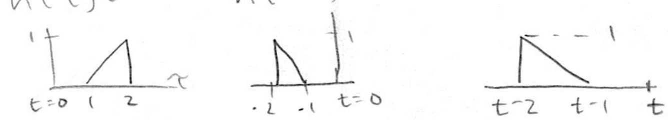
✓ 2

$y = mx + b$ $m = \frac{1-0}{2-1} = \frac{1}{1}$

$0 = 1 + b$ $b = -1$

$y = 1x - 1$
$= x - 1$

$h(\tau) =$ $h(-\tau)$ $h(t-\tau)$



$h(\tau) = \tau - 1$

$h(t-\tau) = t - \tau - 1$

$\tau = t-1$

$t-1-(t-1) = 0$

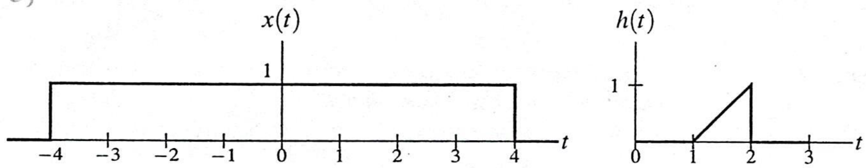
$\tau = t-2$

$t-1-(t-2)$

$t-t-1+2 = 1$

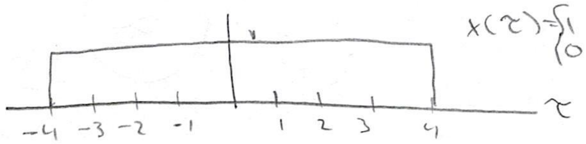
PROBLEM 5.

Using graphical methods, compute the convolution $y(t) = x(t) * h(t)$ where the signals $x(t)$ and $h(t)$ are as shown in the figure to the right. **[8 marks]**

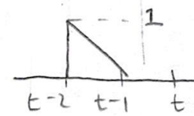


(A) Plot $x(\tau)$ and $h(t-\tau)$ versus τ. ***Be very careful** to plot these graphs correctly. If you make a mistake here, all of your subsequent work will be completely wrong, and **you will lose a very substantial number of marks** as a result.* **[2 marks]**

$x(\tau)$



$x(\tau) = \begin{cases} 1 & -4 < \tau < 4 \\ 0 & o/w \end{cases}$

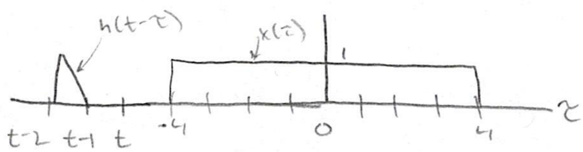


$h(t-\tau) = t - 1 - \tau$

τ

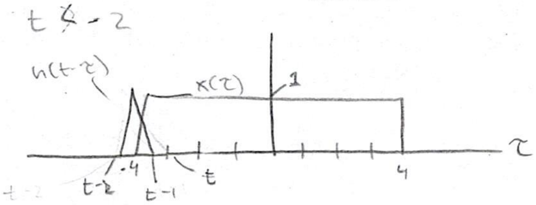
(B) For each of the cases (i.e., ranges for t) to be considered in the computation of the convolution result $y(t)$, carefully sketch and **fully label** the graph that includes both $x(\tau)$ and $h(t-\tau)$ plotted versus τ, and also indicate the corresponding range for t. **[3 marks]**

Case 1 $t - 1 < -4 \Rightarrow \underline{t < -3}$



$t - 1 > -4$ $t - 2 < -4$ $\Rightarrow$ $\underline{-3 < t < -2}$

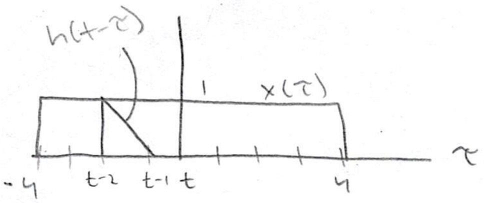
$t > -3$ $t < -2$



$t - 1 < 4$ $t - 2 > -4$

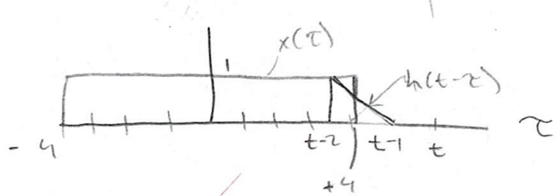
$t < 5$ $t > -2$

$\underline{-2 < t < 5}$



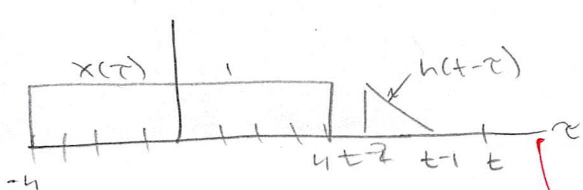
$t - 1 > 4$ $t - 2 < 4$

$t > 5$ $t < 6$ $5 < t < 6$



$t - 2 > 4$

$\underline{t > 6}$



$t < -3$

$x(t) * h(t) = 0$

$-3 < t < -2$

$x(t) * h(t) = \int_{-4}^{t-1} (1)(t-\tau-1)\,d\tau$

$-2 < t < 5$

$x(t) * h(t) = \int_{t-2}^{t-1} (1)(t-\tau-1)\,d\tau$

$5 < t < 6$

$x(t) * h(t) = \int_{t-2}^{4} 1\,(t-\tau-1)\,d\tau$

$t > 6$

$x(t) * h(t) = 0$

(C) **Use the graphs from part (b)** to determine the convolution result $y(t)$. **You may state your final answer in terms of integrals. DO NOT INTEGRATE!!!** There should not be any unit-step functions appearing in your answer. **[3 marks]**

$$x(t) * h(t) = \begin{cases} 0 & t < -3 \\ \int_{-4}^{t-1} (t-1-\tau)\,d\tau & -3 < t < -2 \\ \int_{t-2}^{t-1} (t-1-\tau)\,d\tau & -2 < t < 5 \\ \int_{t-2}^{4} (t-1-\tau)\,d\tau & 5 < t < 6 \\ 0 & t > 6 \end{cases}$$

PROBLEM 6. Consider the system with input $x(t)$ and output $y(t)$ as defined by the equation

$$y(t) = \int_0^5 e^{t+\tau+3} u(t+\tau+1)x(\tau-3)d\tau.$$

Find the impulse response $h(t)$ of this system. **[3 marks]**

$$h(t) = \int_0^5 e^{t+\tau+3} u(t+\tau+1)\,\delta(\tau-3)\,d\tau \qquad x(t)\delta(t-t_0) = x(t_0)$$

$$= \int_0^5 e^{t+3+3} u(t+3+1)\,\delta(\tau-3)\,d\tau$$

$$= e^{t+6} u(t+4) \int_0^5 \delta(\tau-3)\,d\tau$$

$$= e^{t+6} u(t+4) \Big[\underbrace{\int_0^{3^-} \delta(\tau-3)\,d\tau}_{0} + \underbrace{\int_3^5 \delta(\tau-3)\,d\tau}_{1} \Big] \qquad u(t-3) = \begin{cases} 0 & t<3 \\ 1 & t>3 \end{cases}$$

$$= e^{t+6} u(t+4)\, u(t-3)$$

$$\boxed{h(t) = e^{t+6} u(t+4) u(t-3)}$$

$\frac{2}{3}$

-1

PROBLEM 7. Using the MATLAB programming language, write a function called `myfunc` that takes a single input argument x and returns a single value y, where x and y are real numbers and

$$y = \sum_{k=1}^{25} \frac{1}{x^k + 1}.$$

Be sure to use correct syntax in your answer, since syntax clearly matters here. **[2 marks]**

```
function y = myfunc(x) {
  y=0;
  for k=[1:1:25] {
    den = x^k+1;
    frac= 1/den;
    y += frac;
  }
  return y;
}
```

END

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