Exam 3 - exam3

continuous signal (University of Victoria)

University of Victoria
Exam 3
Summer 2022

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 30305), A02 (CRN 30306)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: _____

Given Name(s): _____

Student Number: **V00**_____

This examination paper has **7 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

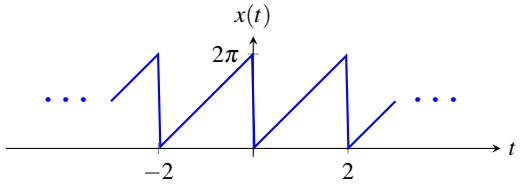
The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

Question 1.

Consider the periodic function x with fundamental period $T = 2$ and fundamental frequency ω_0 shown in the figure. Using the Fourier series analysis equation, find the Fourier series coefficient sequence c for the function x. Your solution must consider the single period of $x(t)$ for $0 \leq t < T$. Your final answer must be **fully simplified**. You may use the fact that $\int xe^{ax}dx = \frac{1}{a^2}e^{ax}(ax-1)+C$ for $a \neq 0$. **[8 marks]**



Question 2. Using the MATLAB programming language, write a function called `count_nonzero` that takes a matrix `m` as input and returns the number of nonzero elements in `m`. Your code **must not call any other functions** other than `width` and `height`. (The `width` function takes a matrix as a parameter and returns the number of columns in the matrix. The `height` function is similar except it returns the number of rows.) Your code must **use proper indentation** and **must not exceed 15 lines** in length. Be sure to use correct syntax in your answer, since syntax clearly matters here. **[2 marks]**

Line #	Line of Code
1	
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Question 3. A LTI system has the impulse response h given by $h(t) = e^{-3t}u(t)$. Using Fourier series techniques, find the response y of the system to the input $x(t) = 10 + 2\cos(t) + 2j\sin(2t)$. (Do not use convolution.) Your final answer for y must be **fully simplified**, but may contain complex sinusoids. **Do not skip any steps** in your solution and **show all of your work**. You must **make clear** the approach being used in your answer. You may use the fact that $\int_0^\infty e^{-at}e^{-jbt}dt = \frac{1}{a+jb}$ for all (strictly) positive real a and all real b. **[8 marks]**

Question 4. Consider the periodic function x with fundamental period $T = 2$, where

$$x(t) = \begin{cases} 1 + e^t & -1 < t < 0 \\ e^{-2t} & 0 \leq t \leq 1. \end{cases}$$

Suppose that x has the Fourier series representation $y(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$. Find $y(0)$ and $y(1)$. **Show all of your work**, and **fully justify** your answer. **[3 marks]**

Question 5. Let x denote a periodic function with fundamental period $T = 2$, fundamental frequency ω_0, and the Fourier series coefficient sequence c, where $c_k = \dfrac{e^{j3k}(j2k+1)}{(j2k-1)^3}$.

(A) Find a **fully-simplified** expression for the (Fourier series) magnitude spectrum of x. **[3 marks]**

(B) Determine the frequency (or frequencies) at which x has the most spectral information. You **must make clear** how you arrived at your answer. **[1 mark]**

END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

θ				
Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
135	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
180	π	0	-1	0
225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
270	$\frac{3\pi}{2}$	-1	0	undefined
315	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$
$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$

Property	
Even Symmetry	x is even $\Leftrightarrow$ a is even
Odd Symmetry	x is odd $\Leftrightarrow$ a is odd
Real / Conjugate Symmetry	x is real $\Leftrightarrow$ a is conjugate symmetric

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$