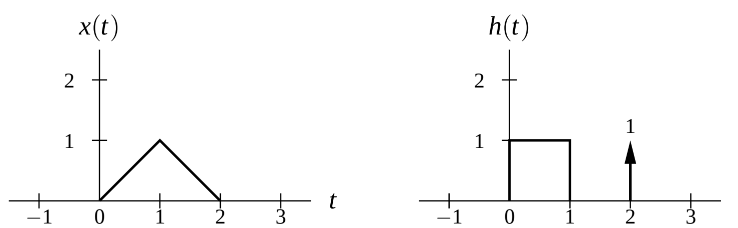
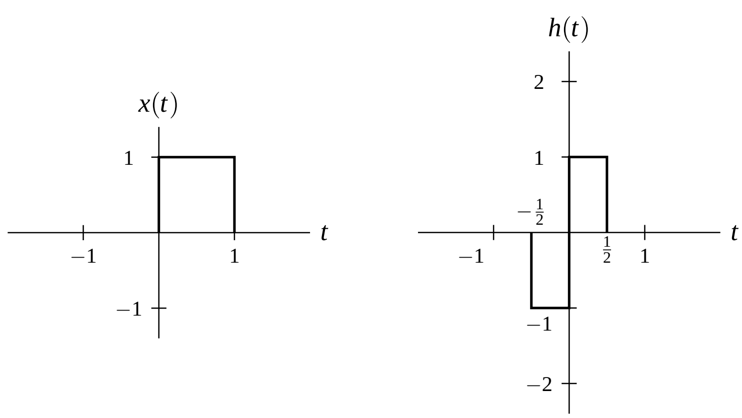
Chapter 2

Continuous-Time Linear-Time Invariant Systems (Chapter 3)

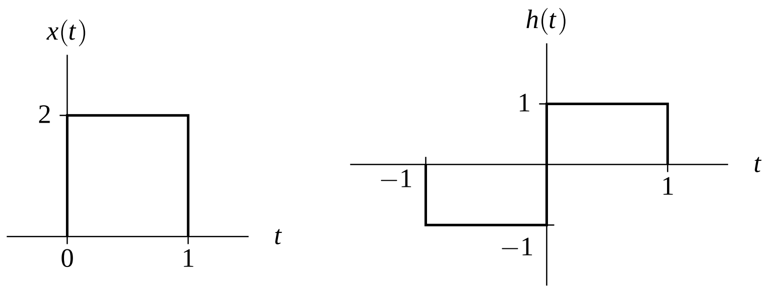
3.1 Using graphical methods, for each pair of signals $x(t)$ and $h(t)$ given in the figures below, compute the convolution $y(t) = x(t) * h(t)$.



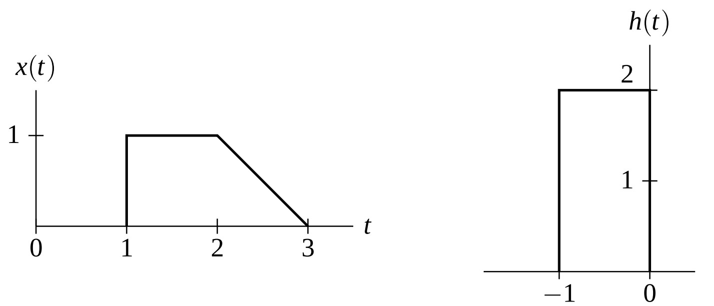
(a)



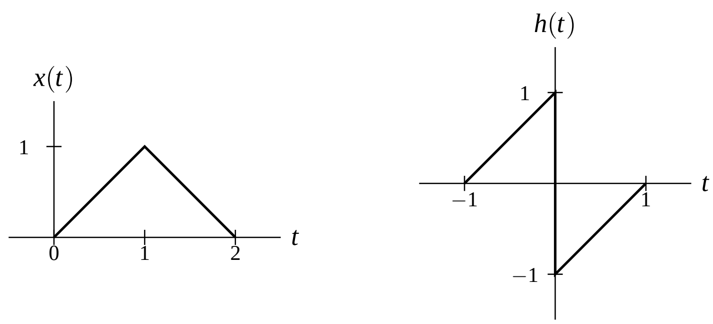
(b)



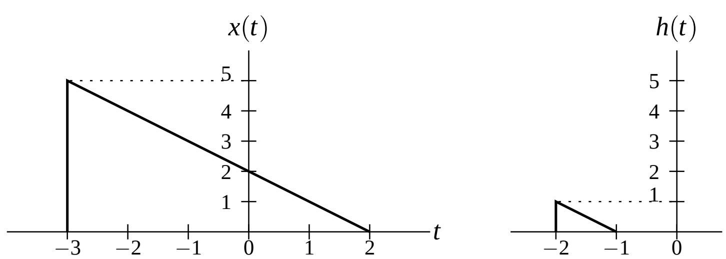
(c)



(d)



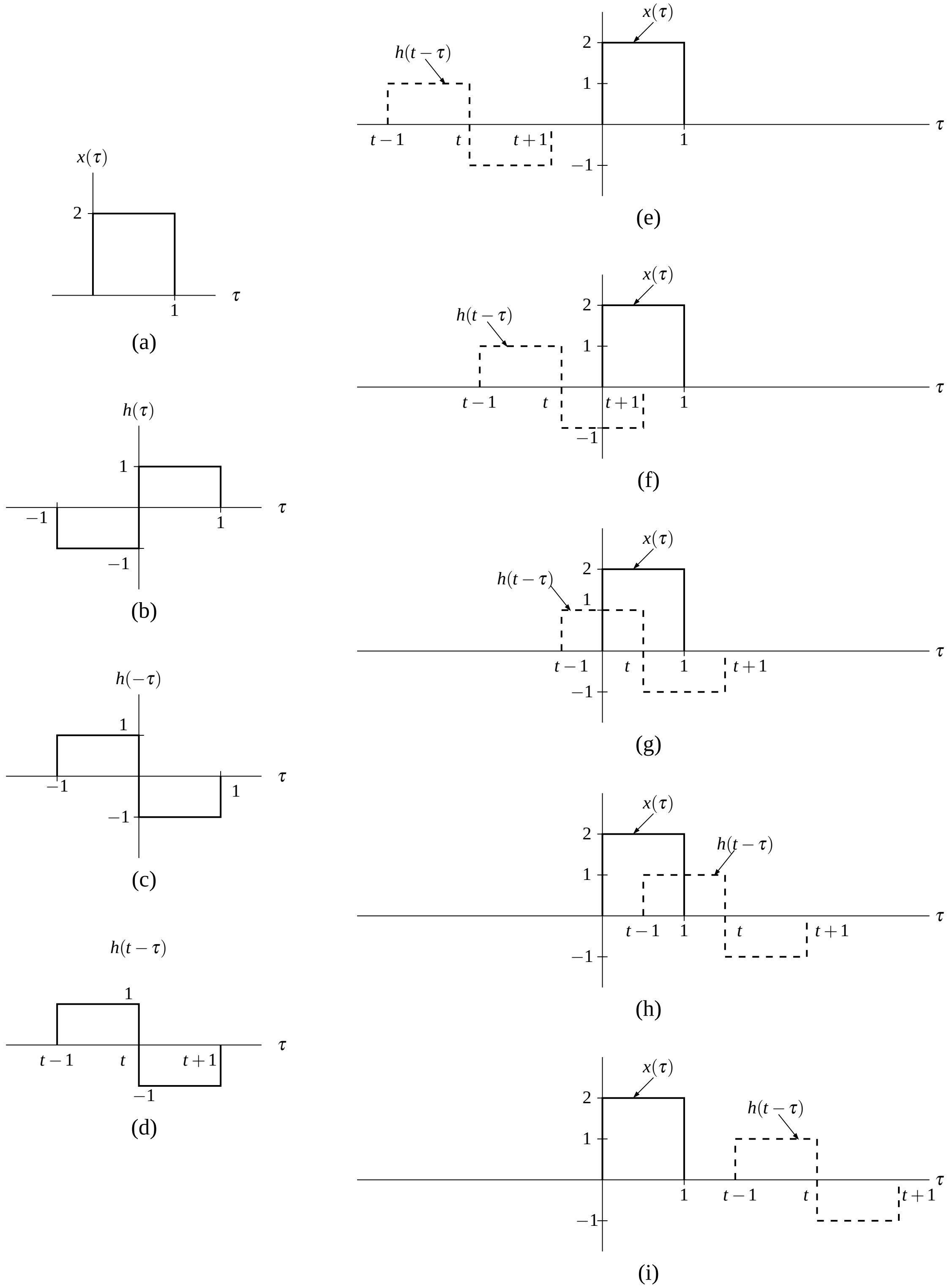
(e)



(f)

Solution.

(c)



First, we consider the case of $t < -1$. From Figure (e), we can see that

$$x(t) * h(t) = 0.$$

Second, we consider the case of $-1 \leq t < 0$. From Figure (f), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_0^{t+1} (-2) d\tau \\ &= [-2\tau]|_0^{t+1} \\ &= -2t - 2. \end{aligned}$$

Third, we consider the case of $0 \leq t < 1$. From Figure (g), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_0^t 2 d\tau + \int_t^1 (-2) d\tau \\ &= [2\tau]|_0^t + [-2\tau]|_t^1 \\ &= 2t + (-2 - [-2t]) \\ &= 4t - 2. \end{aligned}$$

Fourth, we consider the case of $1 \leq t < 2$. From Figure (h), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_{t-1}^1 2 d\tau \\ &= [2\tau]|_{t-1}^1 \\ &= 2 - [2t - 2] \\ &= 4 - 2t. \end{aligned}$$

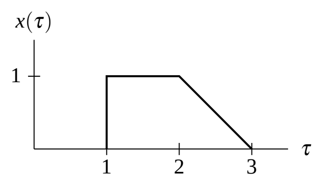
Lastly, we consider the case of $t > 2$. From Figure (i), we can see that

$$x(t) * h(t) = 0.$$

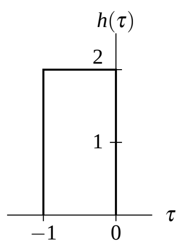
Combining the above results, we have

$$x(t) * h(t) = \begin{cases} -2t - 2 & \text{for } -1 \leq t < 0 \\ 4t - 2 & \text{for } 0 \leq t < 1 \\ 4 - 2t & \text{for } 1 \leq t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

(d)



(a)



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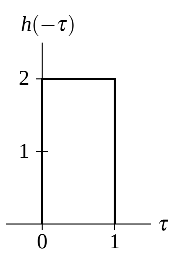

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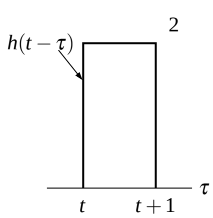

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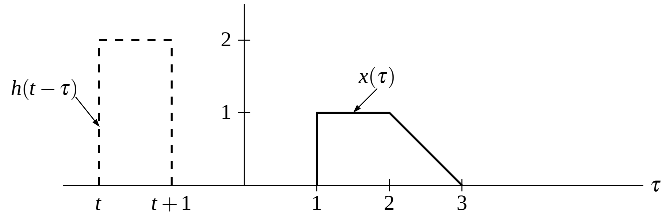
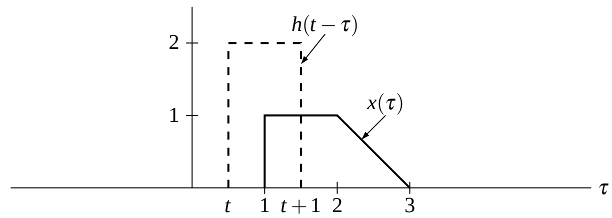
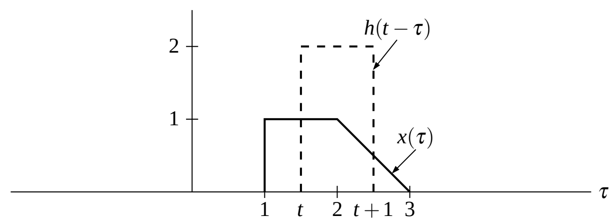


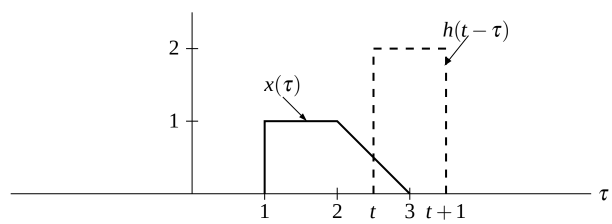
(e)



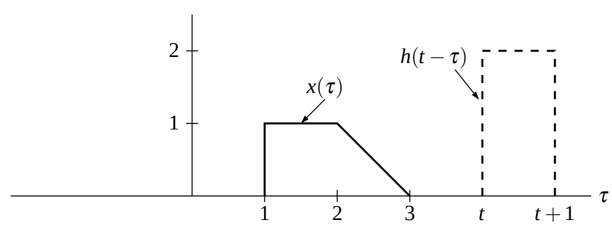
(f)



(g)



(h)



(i)

First, we consider the case of $t < 0$. From Figure (e), we can see that

$$x(t) * h(t) = 0.$$

Second, we consider the case of $0 \le t < 1$. From Figure (f), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_1^{t+1} 2 d\tau \\ &= [2\tau]|_1^{t+1} \\ &= 2t + 2 - [2] \\ &= 2t. \end{aligned}$$

Third, we consider the case of $1 \le t < 2$. From Figure (g), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_t^2 2 d\tau + \int_2^{t+1} 2(-\tau+3) d\tau \\ &= [2\tau]|_t^2 + [-\tau^2 + 6\tau]|_2^{t+1} \\ &= 4 - 2t + [-(t+1)^2 + 6t + 6] - [-4 + 12] \\ &= 4 - 2t - (t^2 + 2t + 1) + 6t + 6 - 8 \\ &= -t^2 + 2t + 1. \end{aligned}$$

Fourth, we consider the case of $2 \le t < 3$. From Figure (h), we can see that

$$\begin{aligned} x(t) * h(t) &= \int_t^3 2(-\tau+3) d\tau \\ &= [-\tau^2 + 6\tau]|_t^3 \\ &= -9 + 18 - (-t^2 + 6t) \\ &= 9 + t^2 - 6t \\ &= t^2 - 6t + 9. \end{aligned}$$

Last, we consider the case of $t > 3$. From Figure (i), we can see that

$$x(t) * h(t) = 0.$$

Combining the above results, we have

$$x(t) * h(t) = \begin{cases} 2t & \text{for } 0 \le t < 1 \\ -t^2 + 2t + 1 & \text{for } 1 \le t < 2 \\ t^2 - 6t + 9 & \text{for } 2 \le t < 3 \\ 0 & \text{otherwise.} \end{cases}$$

3.2 For each pair of signals $x(t)$ and $h(t)$ given below, compute the convolution $y(t) = x(t) * h(t)$.
(a) $x(t) = e^{at}u(t)$ and $h(t) = e^{-at}u(t)$ where a is a nonzero real constant;
(b) $x(t) = e^{-j\omega_0 t}u(t)$ and $h(t) = e^{j\omega_0 t}u(t)$ where ω_0 is a strictly positive real constant;
(c) $x(t) = u(t-2)$ and $h(t) = u(t+3)$;
(d) $x(t) = u(t)$ and $h(t) = e^{-2t}u(t-1)$;
(e) $x(t) = u(t-1) - u(t-2)$ and $h(t) = e^t u(-t)$.

Solution.

(a) We have

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{a\tau}u(\tau)e^{-a(t-\tau)}u(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{a\tau}u(\tau)e^{-at}e^{a\tau}u(t-\tau)d\tau \\
&= e^{-at}\int_{-\infty}^{\infty} e^{2a\tau}u(\tau)u(t-\tau)d\tau \\
&= e^{-at}\int_{0}^{t} e^{2a\tau}d\tau \quad \text{for } t \geq 0 \\
&= e^{-at}[\tfrac{1}{2a}e^{2a\tau}]\big|_0^t \quad \text{for } a \neq 0 \\
&= \tfrac{1}{2a}e^{-at}[e^{2a\tau}]\big|_0^t \\
&= \tfrac{1}{2a}e^{-at}[e^{2at}-1] \\
&= \tfrac{1}{2a}[e^{at}-e^{-at}].
\end{aligned}$$

Thus, we have that

$$\begin{aligned}
y(t) &= \begin{cases} \frac{1}{2a}[e^{at}-e^{-at}] & \text{for } t \geq 0 \\ 0 & \text{otherwise} \end{cases} \\
&= \tfrac{1}{2a}[e^{at}-e^{-at}]u(t).
\end{aligned}$$

(c) We have

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} u(\tau-2)u(t-\tau+3)d\tau \\
&= \int_{2}^{t+3} d\tau \quad \text{for } t \geq -1 \\
&= [\tau]\big|_2^{t+3} \\
&= t+3-2 \\
&= t+1.
\end{aligned}$$

Thus, we have that

$$\begin{aligned}
y(t) &= \begin{cases} t+1 & \text{for } t \geq -1 \\ 0 & \text{otherwise} \end{cases} \\
&= [t+1]u(t+1).
\end{aligned}$$

(d) We have

$$\begin{aligned}
x(t) * h(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} u(\tau)e^{-2(t-\tau)}u(t-\tau-1)d\tau \\
&= \begin{cases} e^{-2t}\int_0^{t-1} e^{2\tau}d\tau & \text{for } t > 1 \\ 0 & \text{otherwise} \end{cases} \\
&= [u(t-1)]e^{-2t}\int_0^{t-1} e^{2\tau}d\tau \\
&= [u(t-1)]e^{-2t}\left[\tfrac{1}{2}e^{2\tau}\right]\Big|_0^{t-1} \\
&= \tfrac{1}{2}[u(t-1)]e^{-2t}\left[e^{2\tau}\right]\Big|_0^{t-1} \\
&= \tfrac{1}{2}[u(t-1)]e^{-2t}[e^{2t-2}-1] \\
&= \tfrac{1}{2}[u(t-1)][e^{-2}-e^{-2t}] \\
&= [-\tfrac{1}{2}e^{-2t}+\tfrac{1}{2}e^{-2}]u(t-1).
\end{aligned}$$

(e) We have

$$\begin{aligned}
x(t) * h(t) &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} [u(\tau-1)-u(\tau-2)]e^{t-\tau}u(-[t-\tau])d\tau.
\end{aligned}$$

In order to evaluate the above integral, we can see that there are three cases to consider: i) $t < 1$, ii) $1 \le t < 2$, and iii) $t \ge 2$. First, we consider the case of $t < 1$. We have

$$\begin{aligned}
x(t) * h(t) &= \int_1^2 e^{t-\tau}u(\tau-t)d\tau \\
&= \int_1^2 e^{t-\tau}(1)d\tau \\
&= e^t\int_1^2 e^{-\tau}d\tau \\
&= e^t\left[-e^{-\tau}\right]\Big|_1^2 \\
&= e^t[-e^{-2}+e^{-1}] \\
&= e^t[e^{-1}-e^{-2}] \\
&= e^{t-1}-e^{t-2}.
\end{aligned}$$

Second, we consider the case of $1 \le t < 2$. We have

$$\begin{aligned}
x(t) * h(t) &= \int_1^2 e^{t-\tau}u(\tau-t)d\tau \\
&= \int_t^2 e^{t-\tau}(1)d\tau \\
&= e^t\int_t^2 e^{-\tau}d\tau \\
&= e^t\left[-e^{-\tau}\right]\Big|_t^2 \\
&= e^t[-e^{-2}+e^{-t}] \\
&= 1-e^{t-2}.
\end{aligned}$$

Third, we consider the case of $t \geq 2$. We have

$$\begin{aligned}
x(t) * h(t) &= \int_1^2 e^{t-\tau} u(\tau - t) d\tau \\
&= \int_1^2 e^{t-\tau}(0) d\tau \\
&= 0.
\end{aligned}$$

Combining these results, we have

$$x(t) * h(t) = \begin{cases} e^{t-1} - e^{t-2} & \text{for } t < 1 \\ 1 - e^{t-2} & \text{for } 1 \leq t < 2 \\ 0 & \text{for } t \geq 2. \end{cases}$$

3.3 Let $y(t) = x(t) * h(t)$. Given that

$$v(t) = \int_{-\infty}^{\infty} x(-\tau - b) h(\tau + at) d\tau,$$

where a and b are constants, express $v(t)$ in terms of $y(t)$.

Solution.

From the definition of $v(t)$, we have

$$v(t) = \int_{-\infty}^{\infty} x(-\tau - b) h(\tau + at) d\tau.$$

Now, we employ a change of variable. Let $\lambda = -\tau - b$ so that $\tau = -\lambda - b$ and $d\tau = -d\lambda$. Applying this change of variable and simplifying, we obtain

$$\begin{aligned}
v(t) &= \int_{\infty}^{-\infty} x(\lambda) h([-\lambda - b] + at)(-1) d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda) h(at - b - \lambda) d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda) h([at - b] - \lambda) d\lambda \\
&= [x(\rho) * h(\rho)]|_{\rho = at - b} \\
&= y(at - b).
\end{aligned}$$

Therefore, we have that $v(t) = y(at - b)$.

3.4 Consider the convolution $y(t) = x(t) * h(t)$. Assuming that the convolution $y(t)$ exists, prove that each of the following assertions is true:
(a) If $x(t)$ is periodic then $y(t)$ is periodic.
(b) If $x(t)$ is even and $h(t)$ is odd, then $y(t)$ is odd.

Solution.

(a) From the definition of convolution, we have

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= \int_{-\infty}^{\infty} x(\tau) h(t - \tau) d\tau.
\end{aligned}$$

Suppose that $x(t)$ is periodic with period T. Then, we have $x(t) = x(t+T)$ and we can rewrite the above integral as

$$y(t) = \int_{-\infty}^{\infty} x(\tau+T)h(t-\tau)d\tau.$$

Now, we employ a change of variable. Let $\lambda = \tau + T$ so that $\tau = \lambda - T$ and $d\lambda = d\tau$. This yields

$$\begin{aligned}
y(t) &= \int_{-\infty}^{\infty} x(\lambda)h(t-[\lambda-T])d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)h(t+T-\lambda)d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)h([t+T]-\lambda)d\lambda \\
&= [x(v)*h(v)]|_{v=t+T} \\
&= y(v)|_{v=t+T} \\
&= y(t+T).
\end{aligned}$$

Therefore, $y(t)$ is periodic with period T.

3.7 Find the impulse response of the LTI system characterized by each of the equations below. In each case, the input and output of the system are denoted as $x(t)$ and $y(t)$, respectively.

(a) $y(t) = \int_{-\infty}^{t+1} x(\tau)d\tau$;

(b) $y(t) = \int_{-\infty}^{\infty} x(\tau+5)e^{\tau-t+1}u(t-\tau-2)d\tau$;

(c) $y(t) = \int_{-\infty}^{t} x(\tau)v(t-\tau)d\tau$ and

(d) $y(t) = \int_{t-1}^{t} x(\tau)d\tau$.

Solution.

(a) Let $h(t)$ denote the impulse response of the system.

$$\begin{aligned}
h(t) &= \int_{-\infty}^{t+1} \delta(\tau)d\tau \\
&= \begin{cases} 1 & \text{for } t > -1 \\ 0 & \text{for } t < -1 \end{cases} \\
&= u(t+1).
\end{aligned}$$

(b) Let $h(t)$ denote the impulse response of the system.

$$\begin{aligned}
h(t) &= \int_{-\infty}^{\infty} \delta(\tau+5)e^{\tau-t+1}u(t-\tau-2)d\tau \\
&= e^{-5-t+1}u(t-[-5]-2) \\
&= e^{-t-4}u(t+3).
\end{aligned}$$

(c) Let $h(t)$ denote the impulse response of the system. We have

$$h(t) = \int_{-\infty}^{t} \delta(\tau)v(t-\tau)d\tau.$$

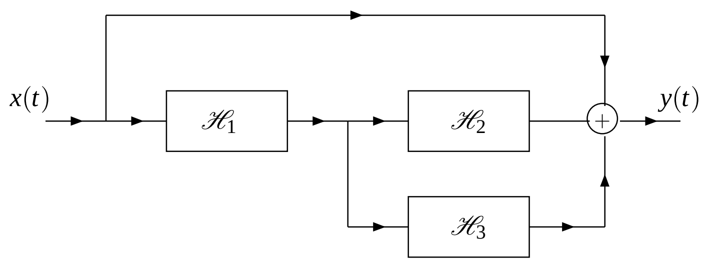
Now, we employ a change of variable. Let $\lambda = \tau - t$ so that $\tau = \lambda + t$ and $d\lambda = d\tau$. Applying this change of variable, we obtain

$$\begin{aligned} h(t) &= \int_{-\infty}^{0} \delta(\lambda + t) v(t - [\lambda + t]) d\lambda \\ &= \int_{-\infty}^{0} \delta(\lambda + t) v(-\lambda) d\lambda. \end{aligned}$$

From the equivalence property of the unit-impulse function, we can write

$$\begin{aligned} h(t) &= \int_{-\infty}^{0} \delta(\lambda + t) v(t) d\lambda \\ &= v(t) \int_{-\infty}^{0} \delta(\lambda + t) d\lambda \\ &= v(t) u(t). \end{aligned}$$

3.8 Consider the system with input $x(t)$ and output $y(t)$ as shown in the figure below. Suppose that the systems $\mathscr{H}_1$, $\mathscr{H}_2$, and $\mathscr{H}_3$ are LTI systems with impulse responses $h_1(t)$, $h_2(t)$, and $h_3(t)$, respectively.



(a) Find the impulse response $h(t)$ of the overall system in terms of $h_1(t)$, $h_2(t)$, and $h_3(t)$.
(b) Determine the impulse response $h(t)$ in the specific case that

$$h_1(t) = \delta(t+1), \quad h_2(t) = \delta(t), \quad \text{and} \quad h_3(t) = \delta(t).$$

Solution.

(a) Let $v(t)$ denote the output of the system $\mathscr{H}_1$. From the block diagram, we have

$$\begin{aligned} v(t) &= x(t) * h_1(t) \\ y(t) &= v(t) * [h_2(t) + h_3(t)] + x(t). \end{aligned}$$

Combining these equations yields

$$\begin{aligned} y(t) &= v(t) * [h_2(t) + h_3(t)] + x(t) \\ &= [x(t) * h_1(t)] * [h_2(t) + h_3(t)] + x(t) \\ &= x(t) * [h_1(t) * [h_2(t) + h_3(t)]] + x(t) \\ &= x(t) * [h_1(t) * h_2(t) + h_1(t) * h_3(t)] + x(t) * \delta(t) \\ &= x(t) * [h_1(t) * h_2(t) + h_1(t) * h_3(t) + \delta(t)]. \end{aligned}$$

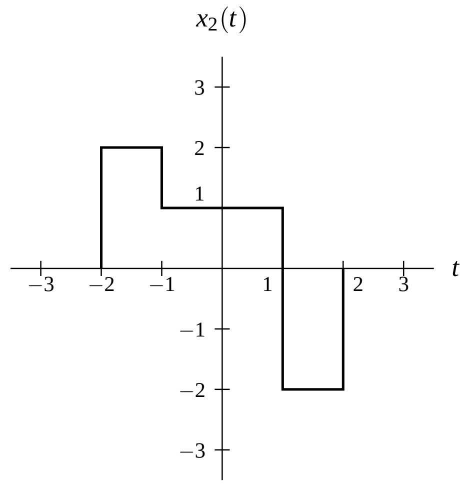
Therefore, we have

$$h(t) = h_1(t) * h_2(t) + h_1(t) * h_3(t) + \delta(t).$$

(b) Substituting the given expression for $h_1(t)$, $h_2(t)$, and $h_3(t)$ into the expression for $h(t)$, we obtain

$$\begin{aligned} h(t) &= \delta(t+1) * \delta(t) + \delta(t+1) * \delta(t) + \delta(t) \\ &= \delta(t+1) + \delta(t+1) + \delta(t) \\ &= 2\delta(t+1) + \delta(t). \end{aligned}$$

3.9 Consider a LTI system whose response to the signal $x_1(t) = u(t) - u(t-1)$ is the signal $y_1(t)$. Determine the response $y_2(t)$ of the system to the input $x_2(t)$ shown in the figure below in terms of $y_1(t)$.



Solution.
First, we express $x_2(t)$ in terms of $x_1(t)$. This yields

$$x_2(t) = 2x_1(t+2) + x_1(t+1) + x_1(t) - 2x_1(t-1).$$

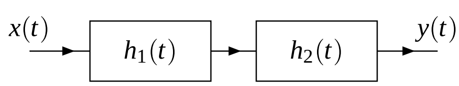
Then, we observe that the system is LTI. This implies that

$$2x_1(t+2) + x_1(t+1) + x_1(t) - 2x_1(t-1) \to 2y_1(t+2) + y_1(t+1) + y_1(t) - 2y_1(t-1).$$

Therefore, we have

$$y_2(t) = 2y_1(t+2) + y_1(t+1) + y_1(t) - 2y_1(t-1).$$

3.10 Suppose that we have the system shown in the figure below with input $x(t)$ and output $y(t)$. This system is formed by the interconnection of two LTI systems with the impulse responses $h_1(t)$ and $h_2(t)$.



For each pair of $h_1(t)$ and $h_2(t)$ given below, find the output $y(t)$ if the input $x(t) = u(t)$.
(a) $h_1(t) = \delta(t)$ and $h_2(t) = \delta(t)$;
(b) $h_1(t) = \delta(t+1)$ and $h_2(t) = \delta(t+1)$;
(c) $h_1(t) = e^{-3t}u(t)$ and $h_2(t) = \delta(t)$.

Solution. Let $h(t)$ denote the impulse response of the overall system.

(b) We have

$$\begin{aligned}
h(t) &= h_1(t) * h_2(t) \\
&= \delta(t+1) * \delta(t+1) \\
&= \int_{-\infty}^{\infty} \delta(\tau+1)\delta(t-\tau+1)d\tau \\
&= \delta(t+2).
\end{aligned}$$

The output $y(t)$ is given by

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= u(t) * \delta(t+2) \\
&= u(t+2).
\end{aligned}$$

(c) First, we calculate the impulse response $h(t)$ of the system. We have that

$$\begin{aligned}
h(t) &= h_1(t) * h_2(t) \\
&= [e^{-3t}u(t)] * \delta(t) \\
&= e^{-3t}u(t).
\end{aligned}$$

Now, we compute the convolution $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$. By drawing graphs of $x(\tau)$ and $h(t-\tau)$ and applying the appropriate logic, we conclude that there are two cases to consider for the computation of $y(t)$: $t < 0$ and $t \geq 0$. In the case that $t < 0$, we have that $y(t)$ is trivially zero. So, we now consider the case that $t \geq 0$. For $t \geq 0$, we have that $y(t)$ is given by

$$\begin{aligned}
y(t) &= x(t) * h(t) \\
&= h(t) * x(t) \\
&= \int_{-\infty}^{\infty} e^{-3\tau}u(\tau)u(t-\tau)d\tau \\
&= \int_{0}^{t} e^{-3\tau}d\tau \quad \text{for } t > 0 \\
&= [-\tfrac{1}{3}e^{-3\tau}]|_0^t \\
&= -\tfrac{1}{3}[e^{-3t} - 1] \\
&= -\tfrac{1}{3}e^{-3t} + \tfrac{1}{3}.
\end{aligned}$$

Thus, we have that

$$\begin{aligned}
y(t) &= \begin{cases} -\frac{1}{3}e^{-3t} + \frac{1}{3} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases} \\
&= [-\tfrac{1}{3}e^{-3t} + \tfrac{1}{3}]u(t).
\end{aligned}$$

3.12 Consider the LTI systems with the impulse responses given below. Determine whether each of these systems is causal and/or memoryless.

(a) $h(t) = (t+1)u(t-1)$;

(b) $h(t) = 2\delta(t+1)$;

(c) $h(t) = \frac{\omega_c}{\pi}\operatorname{sinc}\omega_c t$;

(d) $h(t) = e^{-4t}u(t-1)$;
(e) $h(t) = e^{t}u(-1-t)$;
(f) $h(t) = e^{-3|t|}$; and
(g) $h(t) = 3\delta(t)$.

Solution.

A LTI system with impulse response $h(t)$ is memoryless if $h(t) = 0$ for all $t \neq 0$. Therefore, the systems in (a), (b), (c), (d), (e), and (f) all have memory, while the system in (g) is memoryless.

A LTI system with impulse response $h(t)$ is causal if $h(t) = 0$ for all $t < 0$. Therefore, the systems in (a), (d), and (g) are causal, while the systems in (b), (c), (e), and (f) are not causal.

3.13 Consider the LTI systems with the impulse responses given below. Determine whether each of these systems is BIBO stable.
(a) $h(t) = e^{at}u(-t)$ where a is a strictly positive real constant;
(b) $h(t) = (1/t)u(t-1)$;
(c) $h(t) = e^{t}u(t)$;
(d) $h(t) = \delta(t-10)$;
(e) $h(t) = \operatorname{rect} t$; and
(f) $h(t) = e^{-|t|}$.

Solution.

(a) A LTI system with impulse response $h(t)$ is BIBO stable if and only if $h(t)$ is absolutely integrable. From the given $h(t)$, we have

$$\begin{aligned}
\int_{-\infty}^{\infty} |h(t)|\,dt &= \int_{-\infty}^{\infty} \left|e^{at}u(-t)\right| dt \\
&= \int_{-\infty}^{\infty} e^{at}u(-t)dt \\
&= \int_{-\infty}^{0} e^{at}dt \\
&= \left[\tfrac{1}{a}e^{at}\right]\Big|_{-\infty}^{0} \quad \text{for } a \neq 0 \\
&= \tfrac{1}{a}\left[e^{at}\right]\Big|_{-\infty}^{0} \\
&= \tfrac{1}{a}[1-0] \\
&= \tfrac{1}{a} \\
&< \infty.
\end{aligned}$$

Therefore, the system is BIBO stable.

(b) A LTI system with impulse response $h(t)$ is BIBO stable if and only if $h(t)$ is absolutely integrable. From the given $h(t)$, can write

$$\begin{aligned}
\int_{-\infty}^{\infty} |h(t)|\,dt &= \int_{-\infty}^{\infty} \left|\tfrac{1}{t}u(t-1)\right| dt \\
&= \int_{-\infty}^{\infty} \tfrac{1}{t}u(t-1)dt \\
&= \int_{1}^{\infty} \tfrac{1}{t}dt \\
&= [\ln t]|_{1}^{\infty} \\
&= \ln\infty - \ln 1 \\
&= \infty.
\end{aligned}$$

Therefore, the system is not BIBO stable.

(c) A LTI system with impulse response $h(t)$ is BIBO stable if and only if $h(t)$ is absolutely integrable. From the given $h(t)$, we can write

$$\begin{aligned}\int_{-\infty}^{\infty}|h(t)|\,dt &= \int_{-\infty}^{\infty}\left|e^t u(t)\right|dt \\ &= \int_{-\infty}^{\infty} e^t u(t) dt \\ &= \int_0^{\infty} e^t dt \\ &= [e^t]\big|_0^{\infty} \\ &= \infty.\end{aligned}$$

Therefore, the system is not BIBO stable.

(f) A LTI system with impulse response $h(t)$ is BIBO stable if and only if $h(t)$ is absolutely integrable. From the given $h(t)$, we can write

$$\begin{aligned}\int_{-\infty}^{\infty}|h(t)|\,dt &= \int_{-\infty}^{\infty}\left|e^{-|t|}\right|dt \\ &= \int_{-\infty}^{\infty} e^{-|t|} dt \\ &= \int_{-\infty}^{0} e^{-|t|} dt + \int_0^{\infty} e^{-|t|} dt \\ &= \int_{-\infty}^{0} e^{t} dt + \int_0^{\infty} e^{-t} dt \\ &= [e^t]\big|_{-\infty}^{0} + [-e^{-t}]\big|_0^{\infty} \\ &= 1+1 \\ &= 2 \\ &< \infty.\end{aligned}$$

Therefore, the system is BIBO stable.

3.14 Suppose that we have two LTI systems with impulse responses

$$h_1(t) = \tfrac{1}{2}\delta(t-1) \quad \text{and} \quad h_2(t) = 2\delta(t+1).$$

Determine whether these systems are inverses of one another.

Solution.

These systems are inverses if $h_1(t) * h_2(t) = \delta(t)$. We have

$$\begin{aligned}h_1(t) * h_2(t) &= \int_{-\infty}^{\infty} h_1(\tau)h_2(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} \tfrac{1}{2}\delta(\tau-1)2\delta(t-\tau+1)d\tau \\ &= \int_{-\infty}^{\infty} \delta(\tau-1)\delta(t-\tau+1)d\tau \\ &= \delta(t).\end{aligned}$$

Therefore, the systems are inverses of one another.

Chapter 9

MATLAB (Appendix E)

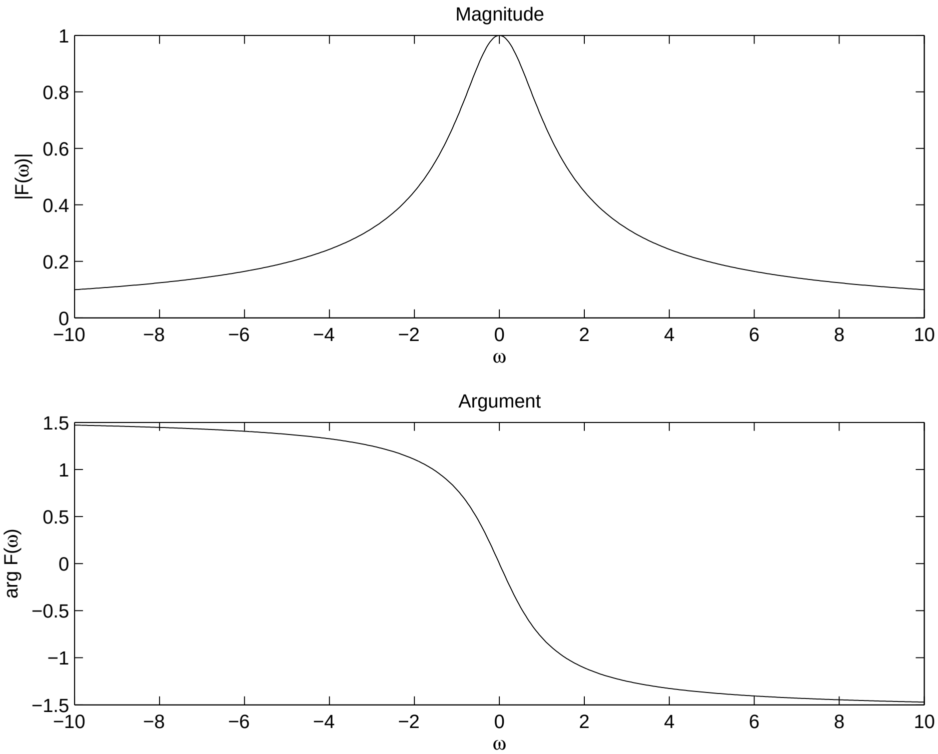
E.103 Let $F(\omega)$ denote the complex-valued function of the real variable ω given by

$$F(\omega) = \frac{1}{j\omega + 1}.$$

Write a program to plot $|F(\omega)|$ and $\arg F(\omega)$ for ω in the interval $[-10, 10]$. Use `subplot` to place both plots on the same figure.

Solution.

```
w = linspace(-10, 10, 500);
f = (j * w + 1) .^ (-1);
subplot(2, 1, 1);
plot(w, abs(f));
title('Magnitude');
xlabel('\omega');
ylabel('|F(\omega)|');
subplot(2, 1, 2);
plot(w, unwrap(angle(f)));
title('Argument');
xlabel('\omega');
ylabel('arg F(\omega)');
```



E.108 In this problem, we consider an algorithm for generating an ordered list of n points in the plane $\{p_0, p_1, \ldots, p_{n-1}\}$. The first point p_0 is chosen as the origin (i.e., $p_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$), with the remaining points being given by the formula

$$p_i = p_{i-1} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^{i-1} \begin{bmatrix} i \\ 0 \end{bmatrix}.$$

(a) Using MATLAB, write a function called `drawpattern` that takes n and θ as input arguments (in that order) with θ being specified in degrees, and then computes and plots the points $\{p_0, p_1, \ldots, p_{n-1}\}$ connected by straight lines (i.e., draw a line from p_0 to p_1, p_1 to p_2, p_2 to p_3, and so on). When performing the plotting, be sure to use `axis('equal')` in order to maintain the correct aspect ratio for the plot. For illustrative purposes, the plots produced for two sets of θ and n values are shown in Figure 9.1.

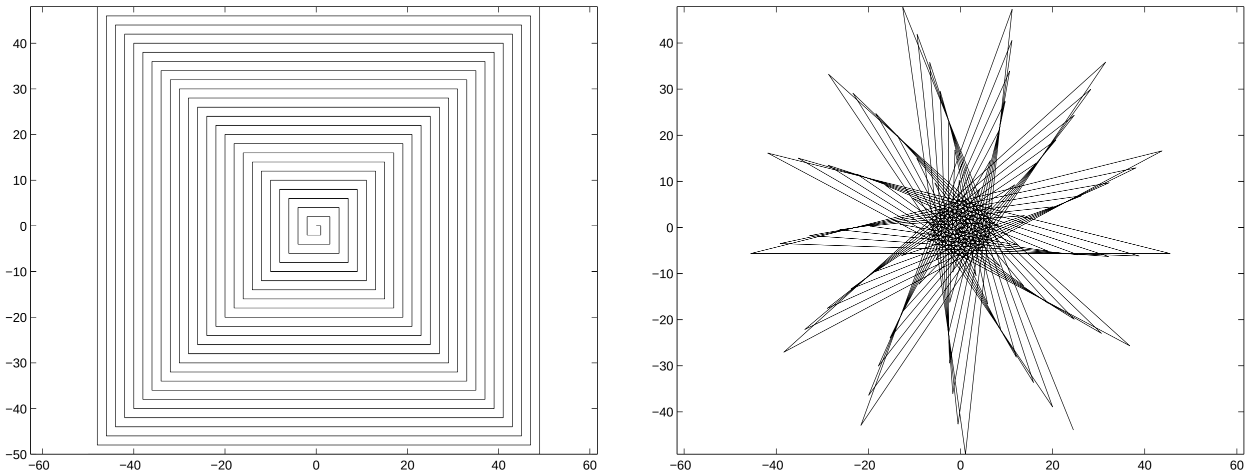
(b) Generate the plots obtained by invoking `drawpattern` with $n = 100$ and θ set to each of the following values: 89°, 144°, and 154°. [Note: In MATLAB, the `sin` and `cos` functions take values in radians, not degrees.]

Solution.

(a) The `drawpattern` function can be implemented using the code below.

Listing 9.1: `drawpattern.m`

```
function drawpattern(n, theta)

% Convert from degrees to radians.
t = theta * pi / 180;

% Generate the list of points.
p = [0 0]';
x = p';
for i = 1 : (n - 1)
```

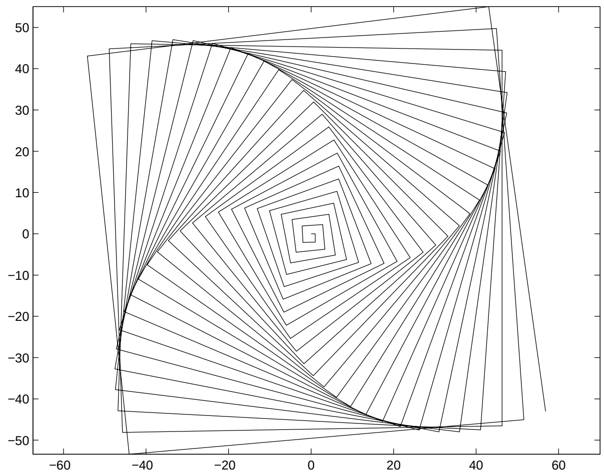


(a) (b)

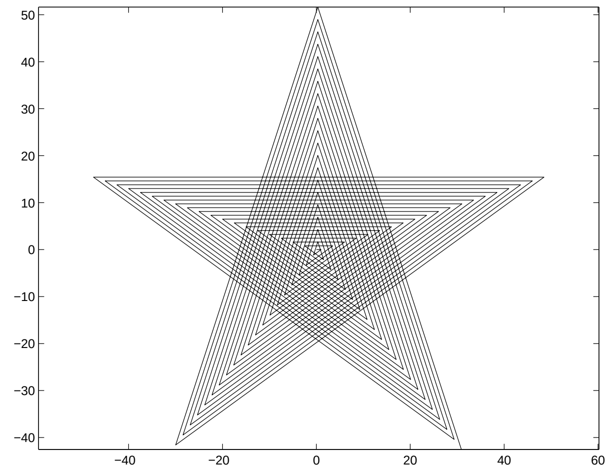
Figure 9.1: Sample plots obtained from `drawpattern`. (a) $\theta = 90^\circ$ and $n = 100$; (b) $\theta = 166^\circ$ and $n = 100$.

```
    p = p + [cos(t) sin(t); -sin(t) cos(t)] ^ (i - 1) * [i 0]';
    x = [x; p'];
end

% Plot the list of points.
plot(x(:, 1), x(:, 2));
axis('equal');

% Print the plot to a file.
% eval(sprintf('print -dps data/drawpattern_%d_%d.ps', theta, n))
```

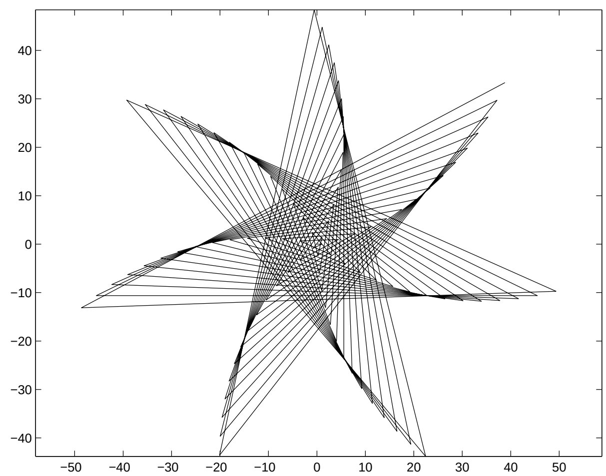
(b) The plots obtained with the `drawpattern` function are shown below.



$\theta = 89^\circ, n = 100$



$\theta = 144^\circ, n = 100$



$\theta = 154^\circ, n = 100$