Inverse Laplace Transform: Step-by-Step Solution

Question:

Find the inverse Laplace transform of the following function:

$$F(s) = \frac{5e^{-s}}{s+2}$$

Solution:

We are tasked with finding the inverse Laplace transform of

$$F(s) = \frac{5e^{-s}}{s+2}.$$

We will break down this problem step by step.

Step 1: Recognizing the time delay factor

The term e^{-s} represents a time shift in the time domain. According to the **time delay theorem** of Laplace transforms:

$$\mathcal{L}^{-1}\left\{e^{-s}F(s)\right\} = u(t-1)f(t-1),$$

where $u(t-1)$ is the unit step function that shifts the function by 1 unit in time. Therefore, the factor e^{-s} implies that the inverse Laplace transform will be delayed by 1 second.

Step 2: Simplifying the remaining function

Next, we focus on the remaining function $\frac{5}{s+2}$. This matches the standard form:

$$\frac{1}{s+a} \longleftrightarrow e^{-at}u(t),$$

where $a = 2$. Therefore, the inverse Laplace transform of $\frac{5}{s+2}$ is:

$$\mathcal{L}^{-1}\left\{\frac{5}{s+2}\right\} = 5e^{-2t}u(t).$$

Step 3: Applying the time shift

Using the time shift property from Step 1, we apply the shift to the result of Step 2. This gives:

$$\mathcal{L}^{-1}\left\{\frac{5e^{-s}}{s+2}\right\} = 5e^{-2(t-1)}u(t-1).$$

Here, $u(t-1)$ ensures that the function is delayed by 1 second, meaning it is zero for $t < 1$.

Final Answer:

Thus, the inverse Laplace transform of $F(s) = \frac{5e^{-s}}{s+2}$ is:

$$f(t) = 5e^{-2(t-1)}u(t-1),$$

where $u(t-1)$ is the unit step function.