2B **3.22** For each function x given below, find a single expression for x (i.e., an expression that does not involve multiple cases). Group similar unit-step function terms together in the expression for x.

(a) $x(t) = \begin{cases} -t-3 & -3 \leq t < -2 \\ -1 & -2 \leq t < -1 \\ t^3 & -1 \leq t < 1 \\ 1 & 1 \leq t < 2 \\ -t+3 & 2 \leq t < 3 \\ 0 & \text{otherwise;} \end{cases}$

(b) $x(t) = \begin{cases} -1 & t < -1 \\ t & -1 \leq t < 1 \\ 1 & t \geq 1; \quad \text{and} \end{cases}$

(c) $x(t) = \begin{cases} 4t+4 & -1 \leq t < -\frac{1}{2} \\ 4t^2 & -\frac{1}{2} \leq t < \frac{1}{2} \\ -4t+4 & \frac{1}{2} \leq t < 1 \\ 0 & \text{otherwise.} \end{cases}$

2B **Answer (c).**

We have

$$\begin{aligned} x(t) &= (4t+4)\left[u(t+1) - u\left(t+\tfrac{1}{2}\right)\right] + 4t^2\left[u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right)\right] + (4-4t)\left[u\left(t-\tfrac{1}{2}\right) - u(t-1)\right] \\ &= (4t+4)u(t+1) + (-4t-4+4t^2)u\left(t+\tfrac{1}{2}\right) + (-4t^2+4-4t)u\left(t-\tfrac{1}{2}\right) + (4t-4)u(t-1) \\ &= (4t+4)u(t+1) + (4t^2-4t-4)u\left(t+\tfrac{1}{2}\right) + (-4t^2-4t+4)u\left(t-\tfrac{1}{2}\right) + (4t-4)u(t-1) \\ &= 4\left[(t+1)u(t+1) + (t^2-t-1)u\left(t+\tfrac{1}{2}\right) + (-t^2-t+1)u\left(t-\tfrac{1}{2}\right) + (t-1)u(t-1)\right]. \end{aligned}$$

2B **3.24** Determine whether each system $\mathcal{H}$ given below is memoryless.
(a) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau) d\tau$;
(b) $\mathcal{H}x(t) = \operatorname{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = x(t-1)+1$;
(d) $\mathcal{H}x(t) = \int_{t}^{\infty} x(\tau) d\tau$;
(e) $\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau) d\tau$;
(f) $\mathcal{H}x(t) = tx(t)$; and
(g) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau) d\tau$.

2B **Answer (d).**

We have

$$\mathcal{H}x(t) = \int_{t}^{\infty} x(\tau) d\tau.$$

Consider $\mathcal{H}x(t_0)$. This quantity depends on $x(t)$ for $t_0 \leq t < \infty$. Therefore, the system is not memoryless.

2B **Answer (g).**

We have

$$\begin{aligned}\mathcal{H}x(t) &= \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau) d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)\delta(\tau-t) d\tau \\ &= x(t).\end{aligned}$$

Consider $\mathcal{H}x(t_0)$. The quantity $\mathcal{H}x(t_0)$ depends on $x(t)$ only for $t = t_0$. Therefore, the system is memoryless.

2B **3.25** Determine whether each system $\mathcal{H}$ given below is causal.
(a) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau)d\tau$;
(b) $\mathcal{H}x(t) = \operatorname{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = x(t-1)+1$;
(d) $\mathcal{H}x(t) = \int_{t}^{\infty} x(\tau)d\tau$;
(e) $\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau$; and
(f) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau$.

2B **Answer (b).**

We are given the system $\mathcal{H}$ characterized by the equation

$$\mathcal{H}x(t) = \operatorname{Even}(x)(t).$$

We can rewrite this equation as

$$\mathcal{H}x(t) = \tfrac{1}{2}[x(t)+x(-t)].$$

Now, consider $\mathcal{H}x(t_0)$ for arbitrary real t_0. This quantity depends on $x(t)$ for $t = \pm t_0$. Since $-t_0 > t_0$ for negative t_0, the system is not causal.

2B **Answer (f).**

We are given the system $\mathcal{H}$ characterized by the equation

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau.$$

Simplifying the preceding equation, we obtain

$$\begin{aligned}\mathcal{H}x(t) &= \int_{-\infty}^{t} x(\tau)u(t-\tau)d\tau + \int_{t}^{\infty} x(\tau)u(t-\tau)d\tau \\ &= \int_{-\infty}^{t} x(\tau)d\tau.\end{aligned}$$

Now, consider $\mathcal{H}x(t_0)$ for arbitrary real t_0. We have

$$\mathcal{H}x(t_0) = \int_{-\infty}^{t_0} x(\tau)d\tau.$$

From this equation, we can see that $\mathcal{H}x(t_0)$ depends on $x(t)$ only for $t \le t_0$. Therefore, the system is causal.

2B **3.26** For each system $\mathcal{H}$ given below, determine if $\mathcal{H}$ is invertible, and if it is, specify its inverse.
(a) $\mathcal{H}x(t) = x(at-b)$ where a and b are real constants and $a \neq 0$;
(b) $\mathcal{H}x(t) = e^{x(t)}$, where x is a real function;
(c) $\mathcal{H}x(t) = \operatorname{Even}(x)(t) - \operatorname{Odd}(x)(t)$;
(d) $\mathcal{H}x(t) = \mathcal{D}x(t)$, where $\mathcal{D}$ denotes the derivative operator; and
(e) $\mathcal{H}x(t) = x^2(t)$.

2B **Answer (b).**

Let $y = \mathcal{H}x$. We have

$$y(t) = e^{x(t)}.$$

Therefore, $y(t) > 0$. Taking the natural logarithm of both sides of the above equation, we obtain

$$\ln y(t) = x(t),$$

or alternatively,

$$x(t) = \ln y(t).$$

Thus, we have found the inverse system. Therefore, the system is invertible.

2B **Answer (e).**

Consider the inputs

$$x_1(t) = -A \quad \text{and} \quad x_2(t) = A,$$

where A is a strictly positive real constant. We have

$$\mathcal{H}x_1(t) = A^2 \quad \text{and} \quad \mathcal{H}x_2(t) = (-A)^2 = A^2.$$

Therefore, the distinct inputs x_1 and x_2 both yield the same output. Since distinct inputs yield the same output, the system is not invertible.

2B **3.27** Determine whether each system $\mathcal{H}$ given below is BIBO stable.
(a) $\mathcal{H}x(t) = \int_t^{t+1} x(\tau)d\tau$ [Hint: For any function f, $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\,dx$.];
(b) $\mathcal{H}x(t) = \frac{1}{2}x^2(t) + x(t)$;
(c) $\mathcal{H}x(t) = 1/x(t)$;
(d) $\mathcal{H}x(t) = e^{-|t|}x(t)$; and
(e) $\mathcal{H}x(t) = \left(\frac{1}{t-1}\right)x(t)$.

2B **Answer (d).**

Let $y = \mathcal{H}x$. Suppose that x is bounded as

$$|x(t)| \le A < \infty.$$

We have

$$\begin{aligned} |y(t)| &= \left|e^{-|t|}x(t)\right| = \left|e^{-|t|}\right| |x(t)| \\ &= e^{-|t|} |x(t)|. \end{aligned}$$

Replacing $e^{-|t|}$ and $|x(t)|$ by their upper bounds, we have

$$|y(t)| \le (1)(A) = A < \infty.$$

Thus, $|y(t)| < \infty$. Therefore, a bounded input always yields a bounded output, and the system is BIBO stable.

2B **Answer (e).**

Consider the bounded input

$$x(t) = 1.$$

We have

$$\mathcal{H}x(t) = \frac{1}{t-1}(1) = \frac{1}{t-1}.$$

As $t \to 1$, $|\mathcal{H}x(t)| \to \infty$. Consequently, $\mathcal{H}x$ is unbounded. Since x is bounded but $\mathcal{H}x$ is unbounded, the system is not BIBO stable.

2B **3.28** Determine whether each system $\mathcal{H}$ given below is time invariant.
(a) $\mathcal{H}x(t) = \mathcal{D}x(t)$; where $\mathcal{D}$ denotes the derivative operator;
(b) $\mathcal{H}x(t) = \text{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = \int_t^{t+1} x(\tau-\alpha)d\tau$, where α is a constant;
(d) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$, where h is an arbitrary (but fixed) function;
(e) $\mathcal{H}x(t) = x(-t)$;
(f) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau)d\tau$; and
(g) $\mathcal{H}x(t) = 3x(t-1)$.

2B **Answer (b).**

The system is time invariant if, for every function x and every real number t_0, the following condition holds:

$$\mathcal{H}x(t-t_0) = \mathcal{H}x'(t) \quad \text{where} \quad x'(t) = x(t-t_0).$$

We have

$$\mathcal{H}x(t) = \text{Even}\{x\}(t) = \tfrac{1}{2}[x(t)+x(-t)].$$

From the definition of $\mathcal{H}$, we have

$$\begin{aligned}
\mathcal{H}x(t) &= \tfrac{1}{2}[x(t)+x(-t)], \\
\mathcal{H}x(t-t_0) &= \tfrac{1}{2}[x(t-t_0)+x(-[t-t_0])] \\
&= \tfrac{1}{2}[x(t-t_0)+x(-t+t_0)], \quad \text{and} \\
\mathcal{H}x'(t) &= \tfrac{1}{2}[x'(t)+x'(-t)] \\
&= \tfrac{1}{2}[x(t-t_0)+x(-t-t_0)].
\end{aligned}$$

Since $\mathcal{H}x(t-t_0) = \mathcal{H}x'(t)$ does not hold for all x and t_0, the system is not time invariant.

2B **Answer (d).**

The system is time invariant if, for every function x and every real number t_0, the following condition holds:

$$\mathcal{H}x(t-t_0) = \mathcal{H}x'(t) \quad \text{where} \quad x'(t) = x(t-t_0).$$

From the definition of $\mathcal{H}$, we have

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau.$$

By substitution, we have

$$\mathcal{H}x(t-t_0) = \int_{-\infty}^{\infty} x(\tau)h(t-t_0-\tau)d\tau.$$

From the definition of $\mathcal{H}$, we also have

$$\begin{aligned}
\mathcal{H}x'(t) &= \int_{-\infty}^{\infty} x'(\tau)h(t-\tau)d\tau \\
&= \int_{-\infty}^{\infty} x(\tau-t_0)h(t-\tau)d\tau.
\end{aligned}$$

Now, we use a change of variable. Let $\lambda = \tau - t_0$ so that $\tau = \lambda + t_0$ and $d\tau = d\lambda$. Applying the change of variable yields

$$\begin{aligned}
\mathcal{H}x'(t) &= \int_{-\infty}^{\infty} x(\lambda)h(t - [\lambda + t_0])d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)h(t - \lambda - t_0)d\lambda \\
&= \int_{-\infty}^{\infty} x(\lambda)h(t - t_0 - \lambda)d\lambda \\
&= \mathcal{H}x(t - t_0).
\end{aligned}$$

Since $\mathcal{H}x(t - t_0) = \mathcal{H}x'(t)$ holds for all x and t_0, the system is time invariant.

2B **3.29** Determine whether each system $\mathcal{H}$ given below is linear.
(a) $\mathcal{H}x(t) = \int_{t-1}^{t+1} x(\tau)d\tau$;
(b) $\mathcal{H}x(t) = e^{x(t)}$;
(c) $\mathcal{H}x(t) = \mathrm{Even}(x)(t)$;
(d) $\mathcal{H}x(t) = x^2(t)$; and
(e) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$, where h is an arbitrary (but fixed) function.

2B **Answer (b).**

The system is linear if, for all functions x_1 and x_2 and all complex constants a_1 and a_2, the following condition holds:

$$\mathcal{H}(a_1x_1 + a_2x_2) = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2.$$

From the definition of $\mathcal{H}$, we have that

$$\begin{aligned} a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t) &= a_1e^{x_1(t)} + a_2e^{x_2(t)} \quad \text{and} \\ \mathcal{H}\left[a_1x_1 + a_2x_2\right](t) &= e^{a_1x_1(t)+a_2x_2(t)} \\ &= e^{a_1x_1(t)}e^{a_2x_2(t)}. \end{aligned}$$

Since $\mathcal{H}(a_1x_1 + a_2x_2) = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$ does not hold for all x_1, x_2, a_1, and a_2, the system is not linear.

2B **Answer (e).**

The system is linear if, for all functions x_1 and x_2 and all complex constants a_1 and a_2, the following condition holds:

$$\mathcal{H}(a_1x_1 + a_2x_2) = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2.$$

From the definition of $\mathcal{H}$, we have that

$$a_1\mathcal{H}x_1(t) + a_2\mathcal{H}x_2(t) = a_1\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau + a_2\int_{-\infty}^{\infty} x_2(\tau)h(t-\tau)d\tau$$

and

$$\begin{aligned} \mathcal{H}\left[a_1x_1 + a_2x_2\right](t) &= \int_{-\infty}^{\infty} \left[a_1x_1(\tau) + a_2x_2(\tau)\right]h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty} a_1x_1(\tau)h(t-\tau)d\tau + \int_{-\infty}^{\infty} a_2x_2(\tau)h(t-\tau)d\tau \\ &= a_1\int_{-\infty}^{\infty} x_1(\tau)h(t-\tau)d\tau + a_2\int_{-\infty}^{\infty} x_2(\tau)h(t-\tau)d\tau. \end{aligned}$$

Comparing the preceding two equations, we conclude that $\mathcal{H}(a_1x_1 + a_2x_2) = a_1\mathcal{H}x_1 + a_2\mathcal{H}x_2$. Thus, the system is linear.

2B **3.33** For each system $\mathcal{H}$ and the functions $\{x_k\}$ given below, determine if each of the x_k is an eigenfunction of $\mathcal{H}$, and if it is, also state the corresponding eigenvalue.
(a) $\mathcal{H}x(t) = x^2(t)$, $x_1(t) = a$, $x_2(t) = e^{-at}$, and $x_3(t) = \cos t$, where a is a complex constant;
(b) $\mathcal{H}x(t) = \mathcal{D}x(t)$, $x_1(t) = e^{at}$, $x_2(t) = e^{at^2}$, and $x_3(t) = 42$, where $\mathcal{D}$ denotes the derivative operator and a is a real constant;
(c) $\mathcal{H}x(t) = \int_{t-1}^{t} x(\tau)d\tau$, $x_1(t) = e^{at}$, $x_2(t) = t$, and $x_3(t) = \sin t$, where a is a nonzero complex constant; and
(d) $\mathcal{H}x(t) = |x(t)|$, $x_1(t) = a$, $x_2(t) = t$, $x_3(t) = t^2$, where a is a strictly positive real constant.

2B **Answer (b).**

We are given

$$\mathcal{H}x(t) = \mathcal{D}x(t), \quad x_1(t) = e^{at}, \quad x_2(t) = e^{at^2}, \quad \text{and} \quad x_3(t) = 42,$$

where $\mathcal{D}$ denotes the derivative operator and a is a real constant.

Consider x_1. We have

$$\begin{aligned} \mathcal{H}x_1(t) &= \mathcal{D}x_1(t) \\ &= \mathcal{D}e^{at} \\ &= ae^{at} \\ &= ax_1(t). \end{aligned}$$

Therefore, x_1 is an eigenfunction of $\mathcal{H}$ with eigenvalue a.

Consider x_2. We have

$$\begin{aligned} \mathcal{H}x_2(t) &= \mathcal{D}x_2(t) \\ &= \mathcal{D}e^{at^2} \\ &= 2ate^{at^2}. \end{aligned}$$

Therefore, x_2 is not an eigenfunction of $\mathcal{H}$.

Consider x_3. We have

$$\begin{aligned} \mathcal{H}x_3(t) &= \mathcal{D}x_3(t) \\ &= \mathcal{D}42 \\ &= 0 \\ &= 0x_3(t). \end{aligned}$$

Therefore, x_3 is an eigenfunction of $\mathcal{H}$ with eigenvalue 0.

2B **3.201** For each mathematical function f given below, write a MATLAB function (with a name of your choosing) that takes an $m \times n$ matrix t and returns a matrix x of the same dimensions where $x_{i,j} = f(t_{i,j})$. The MATLAB function is not permitted to use any conditional statements (such as **`if`** statements) or looping constructs (such as **`for`** or **`while`** statements). (For a matrix a, the notation $a_{i,j}$ denotes the element of a in the ith row and jth column.)

(a) $f(t) = \left(\frac{t^2-1}{t^2+1}\right) e^{-|t/10|} \cos\left(\frac{t}{2\pi}\right)$;

(b) $f(t) = (t^2+1)^{-1} + te^{-|t|}\sin(2t)$;

(c) $f(t) = \begin{cases} \frac{1}{2} & 0 \leq \sin(t) < \frac{1}{\sqrt{2}} \\ 1 & \sin(t) > \frac{1}{\sqrt{2}} \\ 0 & \text{otherwise;} \end{cases}$

(d) $f(t) = \operatorname{rect}(t)$;

(e) $f(t) = \operatorname{tri}(t/2) = \begin{cases} 1-|t| & -1 \leq t \leq 1 \\ 0 & \text{otherwise;} \end{cases}$

(f) $f(t) = \begin{cases} e^t & t < 0 \\ 1 & 0 \leq t < 1 \\ e^{1-t} & t \geq 1; \end{cases}$

(g) $f(t) = \begin{cases} |\sin(\pi t)| & |t| \leq 1 \\ |t|-1 & 1 < |t| \leq 2 \\ 1 & \text{otherwise;} \end{cases}$

(h) $f(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & t = 0 \\ 1 & t > 0; \quad \text{and} \end{cases}$

(i) $f(t) = \begin{cases} t^{-2} & t \geq 1 \\ 0 & \text{otherwise.} \end{cases}$

[Hint: Be careful to avoid division by zero.]

2B **Answer (a).**

```
function x = func_4(t)
    x = (t .^ 2 - 1) ./ (t .^ 2 + 1) .* exp(-abs(t / 10)) .* ...
      cos(t / (2 * pi));
end
```

2B **Answer (f).**

```
function x = func_1(t)
    x = (t < 0) .* exp(t) + (t >= 0 & t < 1) + (t >= 1) .* exp(1 - t);
end
```


2B **D.3** Let T_C, T_F, and T_K denote the temperature measured in units of Celsius, Fahrenheit, and Kelvin, respectively. Then, these quantities are related by

$$T_F = \tfrac{9}{5}T_C + 32 \quad \text{and}$$
$$T_K = T_C + 273.15.$$

Write a program that generates a temperature conversion table. The first column of the table should contain the temperature in Celsius. The second and third columns should contain the corresponding temperatures in units of Fahrenheit and Kelvin, respectively. The table should have entries for temperatures in Celsius from -50 to 50 in steps of 10.

2B **Answer.**

The temperature conversion table can be produced with the following code:

Listing D.1: temperature_conversion_table.m

```
display(sprintf('%-8s %-8s %-8s', 'Celsius', 'Fahrenheit', 'Kelvin'));
for celsius = -50 : 10 : 50
    fahrenheit = 9 / 5 * celsius + 32;
    kelvin = celsius + 273.15;
    display(sprintf('%8.2f %8.2f %8.2f', celsius, fahrenheit, kelvin));
end
```

The code produces the following output:

Listing D.2: Output of temperature conversion program

```
Celsius  Fahrenheit Kelvin
  -50.00   -58.00   223.15
  -40.00   -40.00   233.15
  -30.00   -22.00   243.15
  -20.00    -4.00   253.15
  -10.00    14.00   263.15
    0.00    32.00   273.15
   10.00    50.00   283.15
   20.00    68.00   293.15
   30.00    86.00   303.15
   40.00   104.00   313.15
   50.00   122.00   323.15
```

2B **D.4** (a) Write a function called `unitstep` that takes a single real argument t and returns $u(t)$, where

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Modify the function from part (a) so that it takes a single vector argument $t = \begin{bmatrix} t_1 & t_2 & \cdots & t_n \end{bmatrix}^T$ (where $n \geq 1$ and $t_1, t_2, \ldots, t_n$ are real) and returns the vector $\begin{bmatrix} u(t_1) & u(t_2) & \cdots & u(t_n) \end{bmatrix}^T$. Your solution must employ a looping construct (e.g., a for loop).

(c) With some ingenuity, part (b) of this exercise can be solved using only two lines of code, without the need for any looping construct. Find such a solution. [Hint: In MATLAB, to what value does an expression like "`[-2 -1 0 1 2] >= 0`" evaluate?]

2B **Answer (a).**

This exercise can be solved with code such as that shown below.

Listing D.3: `unitstep1.m`

```
function x = unitstep(t)
    % unitstep - Compute the unit-step function.

    if t >= 0
        x = 1;
    else
        x = 0;
    end
end
```

2B **Answer (b).**

This exercise can be solved with code such as that shown below.

Listing D.4: `unitstep2.m`

```
function x = unitstep(t)
    % unitstep - Compute the unit-step function.

    % Create a vector of zeros with the same size as the input vector.
    x = zeros(size(t));

    % Correctly set the elements in the result vector that should be one.
    m = length(x);
    for i = 1 : m
        if t(i) >= 0
            x(i) = 1;
        end
    end
end
```

2B **Answer (c).**

This exercise can be solved with code such as that shown below.

Listing D.5: `unitstep3.m`

```
function x = unitstep(t)
```



```
    % unitstep - Compute the unit-step function.

    x = (t >= 0);
end
```