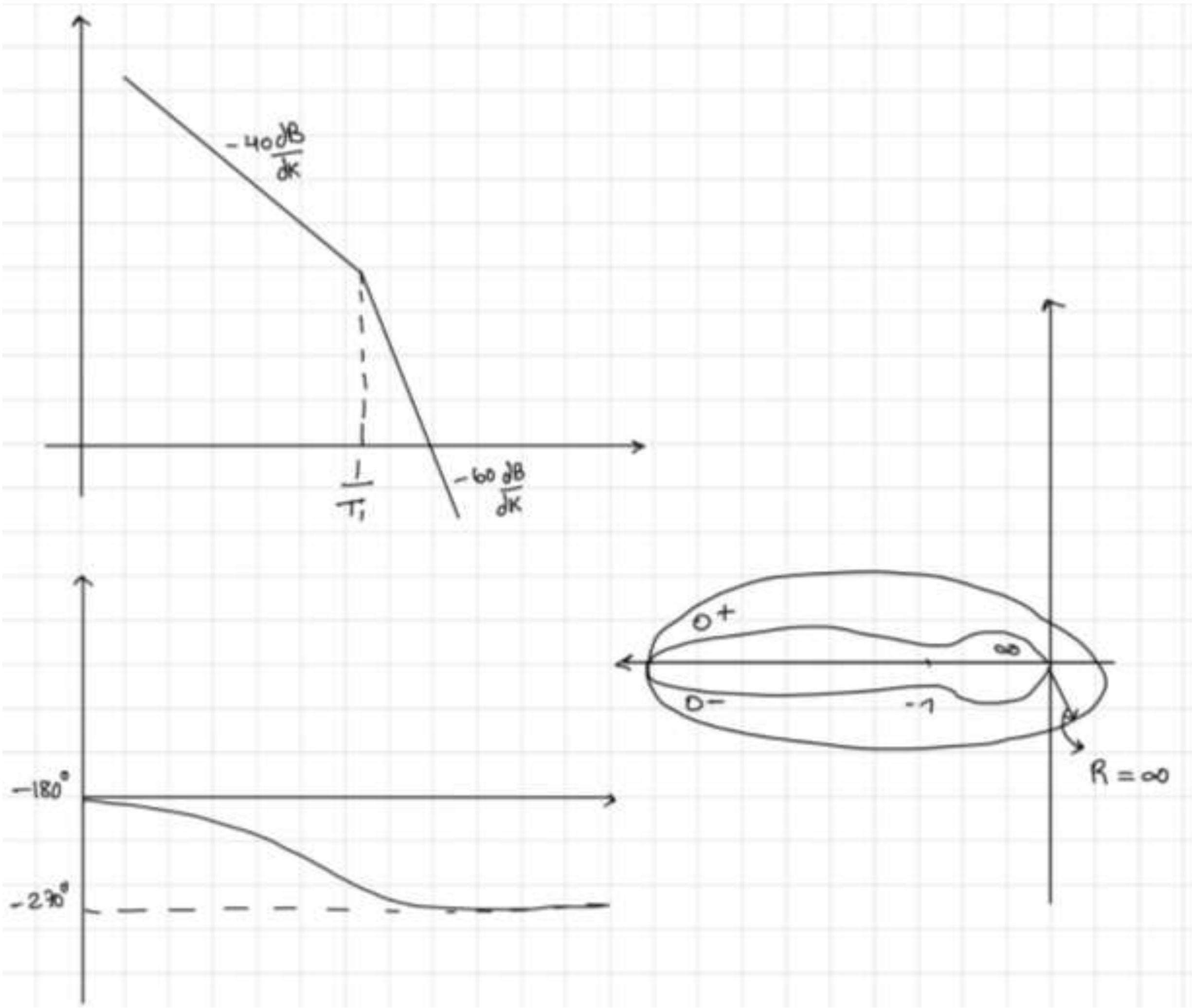
ELEC 360 Assignment #8 Solutions

B-7-9

The system with proportional control only: $\boldsymbol{G(s) = \frac{K}{s^2(T_1 s+1)}} \quad \boldsymbol{T_1 > 0}$

This gives the following Bode and Nyquist Plots:



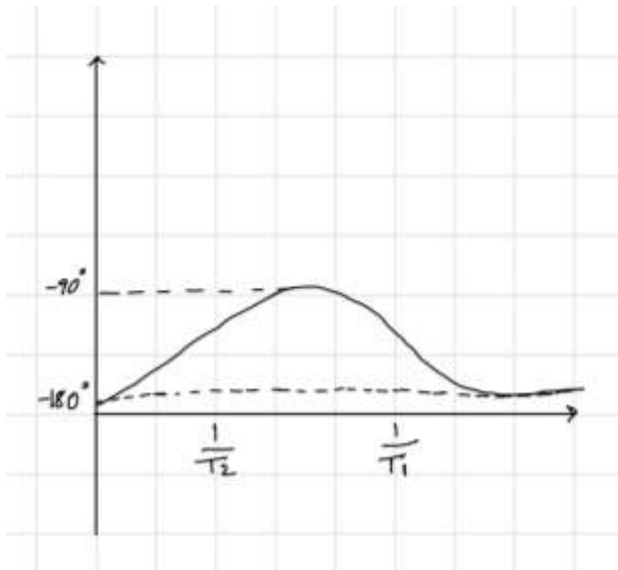
Using the Nyquist stability criterion we have: $\boldsymbol{P = 0, N = 2 \Rightarrow Z = 2}$

This implies that the closed loop is unstable for all K > 0 since it will have 2 unstable poles (*Z=2*).

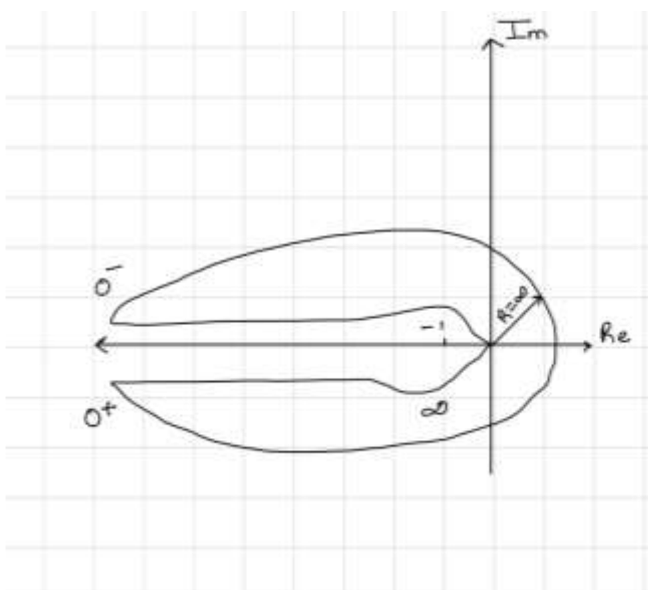
Now add derivative control, i.e $K(sT_2 + 1)$ and place the zero at a lower frequency than the pole, i.e:

$$\boldsymbol{G(s) = \frac{K(sT_2+1)}{s^2(sT_1+1)} \qquad T_2 > T_1 > 0}$$

This gives the following Phase plot for the Bode Diagram



which implies the following Nyquist plot:



Using the Nyquist stability criterion we have: $\boldsymbol{P = 0, N = 0 \implies Z = 0}$

This implies that the closed loop is stable for all K > 0 since *Z=0*.

<u>B-7-16</u>

1. In Fig. 7-158(a) we have P = 0, N = 0 => Z = 0, closed-loop system is stable.
2. In Fig. 7-158(b) we have P = 0, N = 2 => Z = 2, closed-loop system is not stable and has two unstable poles with positive real parts

B-7-23

The phase of *G(jω)* is given by:

$$G(s) = \frac{as+1}{s^2} \Rightarrow < G(j\omega) = \tan^{-1}(a\omega) - 180°$$

and for ω_0, the Gain Cross-Over Frequency we have:

$$|G(j\omega_0)| = \frac{\sqrt{a^2{\omega_0}^2+1}}{{\omega_0}^2} = 1 \quad \Longrightarrow \quad a^2{\omega_0}^2 + 1 = {\omega_0}^4$$

For ω_0 the phase of *G(jω₀)* should be -135° to give a phase margin of 45°

$$< G(j\omega_0) = \tan^{-1}(a\omega_0) - 180° = -135 \quad \rightarrow \tan^{-1}(a\omega_0) = 45°$$

and $a\omega_0 = 1$

From $a\omega_0 = 1$ and $a^2{\omega_0}^2 + 1 = {\omega_0}^4 \Longrightarrow a = \left(\frac{1}{\sqrt{2}}\right)^{\frac{1}{2}} = 0.841$

B-7-26

$$\boldsymbol{G(s) = \frac{K}{s(s^2+s+4)}}$$

For ω_0, the Gain Cross-Over Frequency, the phase of *G(jω₀)* should be -130° to give a phase margin of 50°

$$< G(j\omega_0) = -< j\omega_0 - < (1 - 0.25{\omega_0}^2 + j0.25\omega_0) = -130°$$

and

$$-90° - tan^{-1}\left(\frac{0.25\omega_0}{1-0.25{\omega_0}^2}\right) = -130°$$

and this gives: $\omega_0 = 1.491 \frac{rad}{sec}$.

This is supposed to be the Gain Cross-Over Frequency, and this implies that

$|G(j\omega_0)|=1$, which then implies for K:

$$|G(j\omega_0)| = \frac{K}{j\omega_0(-\omega_0{}^2+j\omega_0+4)}\Bigg|_{\omega_0 = 1.491} = 0.289K = 1$$

$$\text{and } K = \frac{1}{0.289} = 3.46$$

The Phase Cross-Over Frequency ω_1 is obtained form $< G(j\omega_1) = -180°$ and is

$$\omega_1 = 2\frac{rad}{sec}$$

The gain margin is given by $-20\log|G(j\omega_1)| = -20\log|G(j2)| = 1.26\ dB$