Exercise 6.1

L Answer (b).

We are asked to find the Fourier transform of the function

$$x(t) = e^{-4t}u(t-1).$$

Let X denote the Fourier transform of x. From the Fourier transform analysis equation, we have

$$\begin{aligned}
X(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt && \text{Fourier transform analysis equation} \\
&= \int_{-\infty}^{\infty} e^{-4t}u(t-1)e^{-j\omega t}dt && \text{substitute given } x \\
&= \int_{-\infty}^{\infty} e^{-(4+j\omega)t}u(t-1)dt && \text{combine exponentials} \\
&= \int_{1}^{\infty} e^{-(4+j\omega)t}dt && \text{drop part of integration interval where integrand is 0} \\
&= \left[\frac{-1}{4+j\omega}e^{-(4+j\omega)t}\right]\Bigg|_{1}^{\infty} && \text{integrate} \\
&= \frac{-1}{4+j\omega}\left[e^{-(4+j\omega)t}\right]\Bigg|_{1}^{\infty} && \text{pull out factor} \\
&= \frac{-1}{4+j\omega}\left[0-e^{-(4+j\omega)}\right] && \text{evaluate at } \infty \text{ and } 1 \\
&= \frac{1}{4+j\omega}\left[e^{-(4+j\omega)}\right] && \text{cancel minus signs} \\
&= \frac{e^{-4}e^{-j\omega}}{4+j\omega}. && \text{write as single fraction}
\end{aligned}$$

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