Marks

(6) 1. (a) Find the Laplace transform of a signal *f(t)* given by:

$$f(t) = \begin{Bmatrix} 1 & for & 5 < t < 10 \\ -1 & for & 10 < t < 15 \\ 0 & else & \end{Bmatrix}$$

(b) Consider the system described by the following differential equation:

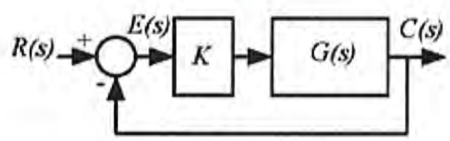
$$\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = u(t)$$

Find the response of the system when *u(t)* is a unit step, $y(0) = 1$ and $\dot{y}(0) = 0$.

(3) 2. Determine the stability of a system (using Routh Hurwitz table) given by:

$$G(s) = \frac{3s + 8}{s^3 + 6.2s^2 + 11.8s + 6.6}$$

(8) 3. Consider a system given by:



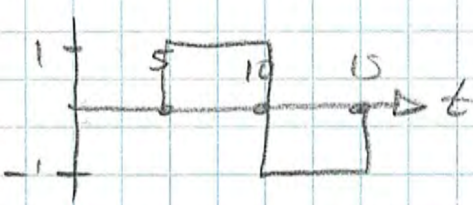
where *G(s)* is given by $G(s) = \dfrac{1}{(s+1)(s+2)}$

a.) Sketch the root-locus of the above system.
b.) Discuss how the step response of the system changes when K is changing from 0 to infinity.
c.) For the above system, find the steady-state error for both unit step and unit ramp inputs.

END

ECE 360 Midterm Feb. 2019

(6) 1. (a) $f(t) = \begin{cases} 1 & 5<t<10 \\ -1 & 10<t\,15 \\ 0 & \text{else} \end{cases}$



$f(t) = u(t-5) - 2u(t-10) + u(t-15)$

$F(s) = \dfrac{(e^{-5s} - 2e^{-10s} + e^{-15s})}{s}$

(b) $\ddot{y} + 5\dot{y} + 6y = u(t) \qquad y(0) = 1 \quad \dot{y}(0) = 0$

$s^2 Y(s) - s\,y(0) - \dot{y}(0) + 5s\,Y(s) - 5\,y(0) + 6\,Y(s) = \dfrac{1}{s}$

$Y(s)(s^2+5s+6) = \dfrac{1}{s} + s + 5 = \dfrac{s^2+5s+1}{s}$

$Y(s) = \dfrac{s^2+5s+1}{(s^2+5s+6)s} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s+3}$

$A = \left.\dfrac{s^2+5s+1}{s^2+5s+6}\right|_{s=0} = \dfrac{1}{6} = 0.17 \quad B = \left.\dfrac{s^2+5s+1}{s(s+3)}\right|_{s=-2} = \dfrac{-5}{-2} = 2.5$

$C = \left.\dfrac{s^2+5s+1}{s(s+2)}\right|_{s=-3} = \dfrac{-5}{+3} = -1.67 \qquad (1)$

$y(t) = 0.17 + 2.5e^{-2t} - 1.67e^{-3t}$ for $t \geq 0$

(3) 2.

$P(s) = s^3 + 6.2s^2 + 11.8s + 6.6$

$a_0 \qquad\qquad\qquad a_3$

s^3	1	11.8
s^2	6.2	6.6
s^1	10.74	
s^0	6.6	

(2) $b_1 = \dfrac{(6.6 - 11.8\cdot 6.2)}{-6.2} = \dfrac{-66.56}{-6.2} = 10.74$

all positive → stable.

(8) 3. $G(s) = \frac{1}{(s+1)(s+2)}$

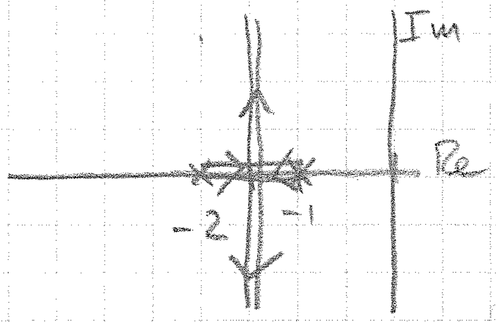
poles: $-1, -2$

asympt: $\gamma = \pm \frac{(2k+1)180^\circ}{2} = \pm 90^\circ$

$\sigma_a = \frac{-1-2}{2} = -1.5$

(a) $B(s) = 1 \quad A(s) = s^2+3s+2$

$B(s)A'(s) = 2s+3 = 0 \quad s = -\frac{3}{2} = -1.5$



(b) · overdamped

· critically damped

· underdamped

· settling time constant

· frequency increasing

· damping ratio decreasing

$\Rightarrow$ overshoot increasing

c) System type 0

$K_p = \frac{K}{2}$ ess for step

$e_{ss} = \frac{1}{1+K_p} = \frac{2}{2+K}$

ramp $K_v = \lim_{s\to 0} s\,G(s) = 0$

$e_{ss} = \frac{1}{K_v} = \infty$