CSC 320 Spring 2024
Assignment 5

This assignment has 5 written questions and is out of a total of 27 marks. Submit one PDF file containing your solutions on Brightspace.

Questions

1. **[4 marks]** Show that the following language L is decidable by giving a high-level description of a decider M with $L(M) = L$.

$$L = \{\langle D \rangle \mid D \text{ is a DFA over } \Sigma^* \text{ for some } \Sigma \text{ and } L(D) = \Sigma^*\}$$

   Hint: You may use a decider shown in class or tutorial as a subroutine.

2. **[4 marks]** Using a high-level TM description, give a TM M that recognizes the complement of E_{TM}, $\overline{E_{TM}} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \neq \emptyset \}$.

3. **[3 marks]** Prove that $E_{TM} = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \emptyset \}$ is not Turing-recognizable. You may use your answer from question 2 as well as any proof shown in class.

4. **[8 marks]** Consider the following language L_1:

$$L_1 = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts at least one string of form } 0^*1^*0^*\}$$

   (a) Prove that L_1 is undecidable by showing a reduction from A_{TM} to L_1.

   (b) Prove the correctness of your reduction by explaining how the decider S for A_{TM} that you create in the reduction works, illustrating that S is indeed a decider for A_{TM}, and explaining why S always halts.

5. **[8 marks]** Consider the following language L_2:

$$L_2 = \{\langle M \rangle \mid M \text{ is a TM and } M \text{ accepts exactly 2 strings}\}$$

   (a) Prove that L_2 is undecidable by showing a reduction from A_{TM} to L_2.

   (b) Prove the correctness of your reduction by explaining how the decider S for A_{TM} that you create in the reduction works, illustrating that S is indeed a decider for A_{TM}, and explaining why S always halts.