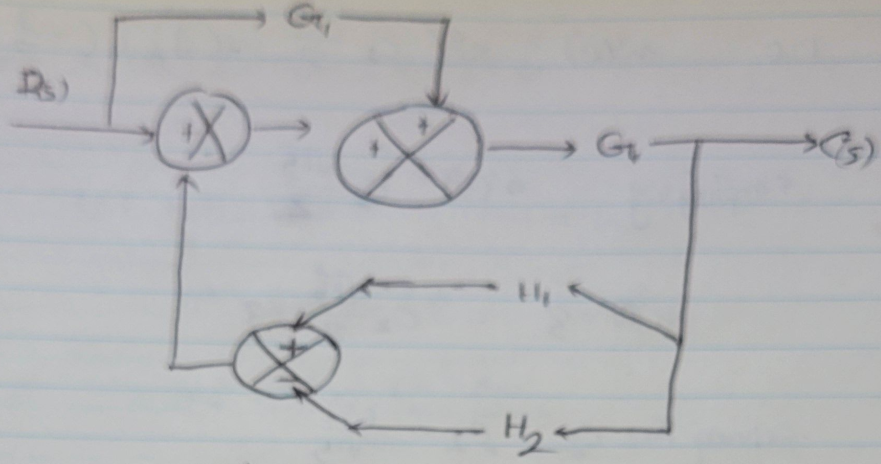
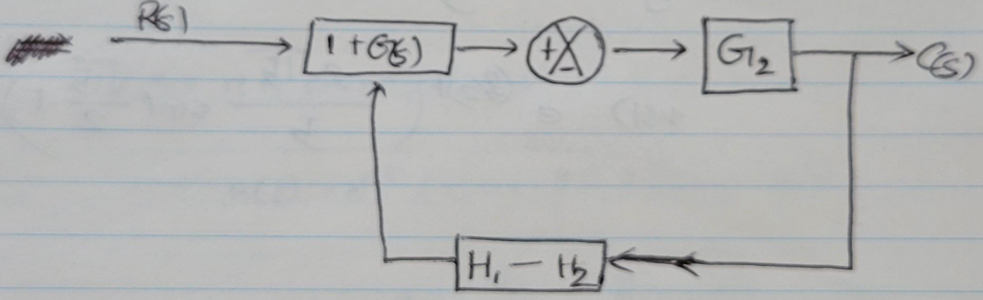
B-2-3



$\Downarrow\Uparrow$



$\Downarrow\Uparrow$

$$\left[\cancel{R}\,[R(s)][1+G_1(s)] - [C(s)][H_1-H_2]\right][G_2] = C(s)$$

$$\Rightarrow \frac{C(s)}{G(s)} = \frac{G_2(1+G_1)}{1+G_2(H_1-H_2)}$$

① Input $R(s)$ $\qquad R(s) - \cancel{}) H_3(s)C(s) = E_1(s)$

② Forwarding block $G(s)$) $\quad X_1(s) = G_1(s)\, E_1(s) = G_1(s)\big(R(s) - H_3(s)(C(s))\big)$

③ Summing junction 2:

$$E_2(s) = X_1(s) - H_2(s)\, C(s) = \big(G_1(s)\big(R(s) - H_3(s)\, C(s)\big) - H_2(s)(C(s))\big)$$

④ $G_2(s)$

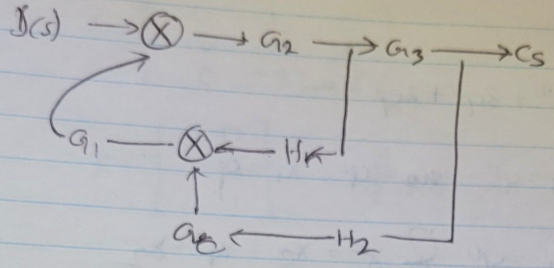
$$X_2(s) = G_2(s)\, E_2(s) = G_2(s)\left[G_1(s)\, (R(s) - H_3(s)C(s)) - H_2(s)C(s)\right]$$

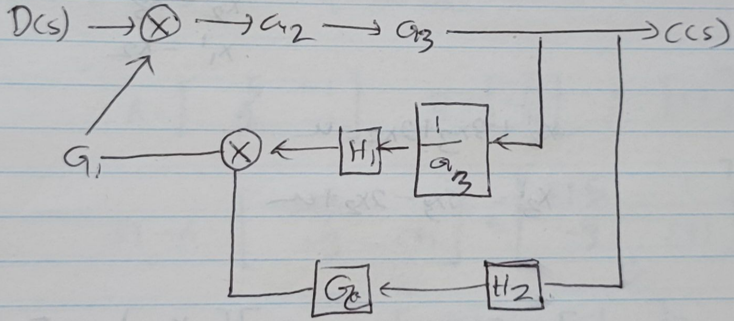
⑤ $E_3(s) = X_2(s) + H_1(s)\, C(s)$

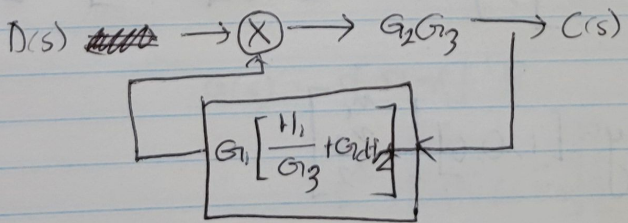
⑥ $C(s) = G_3(s)\, E_3(s) = G_3(s)\, \big(G_2(s)\left[G_1(s)\, (R(s) - H_3(s)C(s)\right]$

$+ H_1(s)\, C(s)\big)$

⑦ $$\boxed{\frac{C(s)}{R(s)} = \frac{G_3(s)G_2(s)G_1(s)}{1 + G_3(s)\big(G_2(s)\, G_1(s)\, H_3(s) + G_2(s)H_2(s) - H_1(s)\big)}}$$







$$\frac{C(s)}{D(s)} = \frac{G_2G_3}{1+\left[G_1\left(\frac{H_1}{G_3}+G_5H_2\right)\right](G_2G_3)} = \frac{G_2G_3}{1+G_1G_2(H_1G_5G_3H_2)}$$

3-2-(1)

$y''' + 3y'' + 2y' = u$

$x_3 = y''$ $x_2 = y'$ $x_1 = y$

$x_3' = y'''$ & $x_2' = x_3$ $x_1' = x_2$

From differential equation:

$x_3' + 3x_2' + 2x_1' = u$ where

$x_2' = x_3$

$x_1' = x_2$

$x_3' + 3x_3 + 2x_2 = u$

$x_3' = -3x_3 - 2x_2 + u$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$\therefore y = [1, 0, 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\therefore$ The state representation is $x' = Ax + Bu$

$y = Cx + Du$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & -3 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = [1, 0, 0] \quad D = 0$$

B-2 (1)

$x' = Ax + Bu \quad y = Cx + Du$

$$\therefore A = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad C = [1\ 2], \quad D = 0$$

$$sI - A = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix} = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix} = s^2 + 6s + 8$$

$$\therefore \frac{1}{|sI - A|} = \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

$$\therefore G(s) = C[sI - A]^{-1} B$$

$$= [1 2] \left[\frac{1}{s^2+6s+8}\right] \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \left[\frac{1}{s^2+6s+8}\right] (2s + 10 + 62 - 3)$$

$$= \frac{12s + 59}{s^2 + 6s + 8}$$