Question 1

Assume for a contradiction that L is context-free. Let p be the pumping length.

Choose $s = a^p b^{2p} c^{3p}$

Since $s \in L$ and $|s| \geq p$, we can write $s = uvxyz$ such that the propertries of the PL hold.

By property 2, $|vxy| \leq p$, we have the following cases:

Case 1) vxy consists of one type of symbol (a's, b's or c's)

Case 2) vxy consists of two types of symbols (a...ab...b or b...bc...c)

Case 1) By property 1, v and/or y is a non-empty substring of the one type of symbol (a, b, or c)

The string uv^2xy^2z will result in increasing the number of the one type of symbol without changing the number of the other two types of symbols. So, uv^2xy^2z is not of form $a^n b^{2n} c^{3n}$

Hence, $uv^2xy^2z \notin L$

Case 2) By property 1, v and/or y is a nonempty substring of two symbols (eg. $v = a...ab..b$) or v and/or y is one type of symbol (eg. $v = a..a$, $y = b...b$)

In the two symbol case, the string uv^2xy^2z will result in symbols out of order, ie not of form a...ab...bc...c

In the one symbol case, uv^2xy^2z will increase the number of one or two types of symbols, but not the third type of symbol. So, the string will not be of form $a^n b^{2n} c^{3n}$.

Hence, $uv^2xy^2z \notin L$

We have shown that we cannot write $s = uvxyz$ satisfying all conditions of the PL. Therefore, L is not context-free.

Question 2

Assume for a contradiction that L is context-free. Let p be the pumping length.

Choose $s = 0^p 1^p 0^{p+1} 1^{p+1}$

Since $s \in L$ and $|s| \geq p$, s can be rewritten as $s = uvxyz$ such that the conditions of the PL hold.

By property 2, $|vxy| \leq p$, we have the following cases: $\overbrace{\underbrace{0...0}_{p}\underbrace{1...1}_{p}}^{\text{section 1}}\overbrace{\underbrace{0...0}_{p+1}\underbrace{1...1}_{p+1}}^{\text{section 2}}$

Case 1) vxy consists of one type of symbol

Case 2) vxy consists of 0's followed by 1's (0...01...1)

Case 3) vxy consists of 1's followed by 0's (1...10...0)

Case 1) vxy is in the 0's of section 1, 1's of section 1, 0's of section 2, or 1's of section 2. By property 1, v and/or y is a non-empty substring of one type of symbol in the section.

The string uv^2xy^2z will increase the number of one type of symbol in the section without increasing the number of the other type of symbol in the same section. So, uv^2xy^2z will not be of form $0^n1^n0^m1^m$.

Hence $uv^2xy^2z \notin L$.

Case 2) In this case, we need to consider the sections differently

a) vxy is of form $0...01...1$ and is in section 1
By property 1, v and/or y is non empty.

If v <u>or</u> y consists of two symbols (eg. $v = 0...01...1$), uv^2xy^2z will not be of form $0..01..10..01..1$, so $uv^2xy^2z \notin L$

If v and/or y consists of one symbol (eg. $v = 0...0$, $y = 1...1$), <u>uv^2xy^2z</u> will result in the section 1 having more (or equal) number of 0's or 1's than section 2, so $uv^2xy^2z \notin L$.

b) vxy is of form $0..01..1$ and is in section 2.

Similar problem as a) but we take <u>uv^0xy^0z</u>, which will result in less (or equal) number of 0's or 1's in section 2 than section 1.

So, $uv^0xy^0z \notin L$

Case 3) vxy consists of $1...10...0$ somewhere in the middle of s (between section 1 and 2)
By property 1, v and/or y is non empty.

If v <u>or</u> y consists of two symbols, uv^2xy^2z will not be of form $0...01...10...01...1$, so $uv^2xy^2z \notin L$.

If v and/or y consists of one symbol, then uv^2xy^2z will increase the one type of symbol in a section without increasing the other type of symbol in the same section, which will not be of form $0^n1^n0^m1^m$.

So, $uv^2xy^2z \notin L$.

We have shown that there is no way to rewrite $s = uvxyz$ satisfying all properties of the PL.

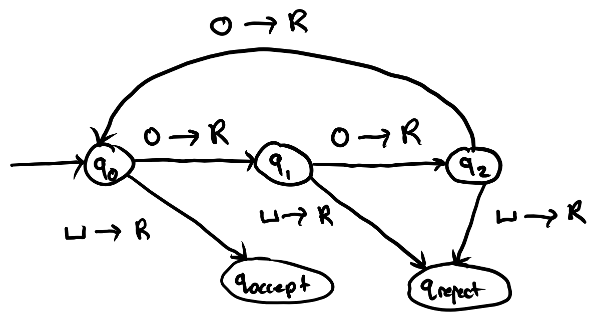
Therefore, L is not context-free.

<u>Question 3</u>

a) 1. If the tape is empty, accept
2. Scan the 0's on the tape from left to right in groups of 3
If there are ever less than 3 0's in a group, reject
3. If we reach the end of the input string, accept

b)

M



$M = (\{q_0, q_1, q_2, q_{accept}, q_{reject}\}, \{0\}, \{0, \sqcup\}, \delta, q_{accept}, q_{reject})$

where δ is described by the state diagram.