UNIVERSITY OF VICTORIA

Department of Electrical and Computer Engineering
ECE 360 – Control Systems I

Laboratory

Experiment no.: 1

Title: Modeling and Identification of a DC Motor

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Summary

Introduction

A DC motor can be analyzed using classical physical laws describing the forces, torques, speeds, and electrical characteristics of the system. On the other hand, a DC motor can be modeled in the Laplace domain by considering the parameters in its transfer function, gain K and time constant τ. By deriving expressions for system characteristics, then experimenting using the DC Motor Control Trainer (DCMCT), an understanding of control systems analysis will be gained.

Prelab Assignment

The prelab assignment answers are shown below.

Question 4.2.1

Noting motor parameters from Appendix 1, and with constant current equaling zero, we can simplify equation (2) from the manual and solve for ω_{max}:

$$\begin{aligned}
u_m(t) &= R_m i_m(t) + L_m \dot{i}_m(t) + u_e(t) \\
u_{max} &= k_m \omega_{max} \\
\omega_{max} &= \frac{u_{max}}{k_m} \\
&= \frac{15 \text{ V}}{0.0502 \text{ Nm/A}} \\
&= 298.8 \text{ s}^{-1}
\end{aligned}$$

Question 4.2.2

With zero rotation, we simplify (2):

$$\begin{aligned}
u_{max} &= R_m i_{max} \\
i_{max} &= \frac{u_{max}}{R_m} \\
&= \frac{15 \text{ V}}{10.6 \ \Omega} \\
&= 1.42 \text{ A}
\end{aligned}$$

Question 4.2.3

With input constant voltage and measured constant current, we simplify (2):

$$R_m = \frac{u_m}{i_m}$$

Question 4.2.4

With constant current, we simplify (2):

$$k_m = \frac{u_m - R_m i_m}{\omega_m}$$

Question 4.3.1

From the LT of (5), we find

$$\begin{aligned} sJ_{eq}\Omega_m(s) &= k_m I_m(s) \\ I_m(s) &= \frac{sJ_{eq}\Omega_m(s)}{k_m} \end{aligned}$$

Then, we solve for the transfer function:

$$\begin{aligned} G(s) &= \frac{\Omega_m(s)}{R_m I_m(s) + sL_m I_m(s) + k_m\Omega_m(s)} \\ &= \frac{\Omega_m(s)}{R_m\left(\frac{sJ_{eq}\Omega_m(s)}{k_m}\right) + sL_m\left(\frac{sJ_{eq}\Omega_m(s)}{k_m}\right) + k_m\Omega_m(s)} \\ &= \frac{\Omega(s)}{\Omega(s)\left(\left(\frac{L_m J_{eq}}{k_m}\right)s^2 + \left(\frac{R_m J_{eq}}{k_m}\right)s + k_m\right)} \\ &= \frac{1}{\left(\frac{L_m J_{eq}}{k_m}\right)s^2 + \left(\frac{R_m J_{eq}}{k_m}\right)s + k_m} \end{aligned}$$

Question 4.3.2

Considering L_m to be much smaller than R_m, we approximate:

$$G(s) \approx \frac{1}{\left(\frac{R_m J_{eq}}{k_m}\right)s + k_m}$$

Question 4.3.3

Rearranging the equation from the previous question, we obtain

$$\begin{aligned} G(s) &\approx \frac{\frac{k_m}{R_m J_{eq}}}{s + \frac{k_m^2}{R_m J_{eq}}} \\ &\approx \frac{214.30}{s + 10.76} \end{aligned}$$

Question 4.3.4

Rearranging the previous equation in a different way, we obtain

$$\begin{aligned} G(s) &\approx \frac{\frac{1}{k_m}}{\left(\frac{R_m J_{eq}}{k_m^2}\right)s + 1} \\ &\approx \frac{19.92}{0.093s + 1} \end{aligned}$$

Question 4.3.5

From the LT of (2), assuming $u_m(t) = 0$ and $L_m = 0$, we obtain

$$\begin{aligned} 0 &= R_m I_m(s) + k_m \Omega_m(s) \\ I_m(s) &= \frac{-k_m \Omega_m(s)}{R_m} \end{aligned}$$

Then, from the LT of (5), we obtain $T_d(s)$:

$$\begin{aligned} sJ_{eq}\Omega_m(s) &= k_m I_m(s) + T_d(s) \\ T_d(s) &= sJ_{eq}\Omega_m(s) - k_m I_m(s) \end{aligned}$$

Combining, we obtain the disturbance transfer function:

$$\begin{aligned} G_d(s) &= \frac{\Omega_m(s)}{sJ_{eq}\Omega_m(s) + \frac{k_m^2 \Omega_m(s)}{R_m}} \\ &= \frac{1}{sJ_{eq} + \frac{k_m^2}{R_m}} \end{aligned}$$

Question 4.3.6

Noting that $G(s) = \frac{k_m}{R_m} G_d(s)$, the block diagram is shown in figure 1.

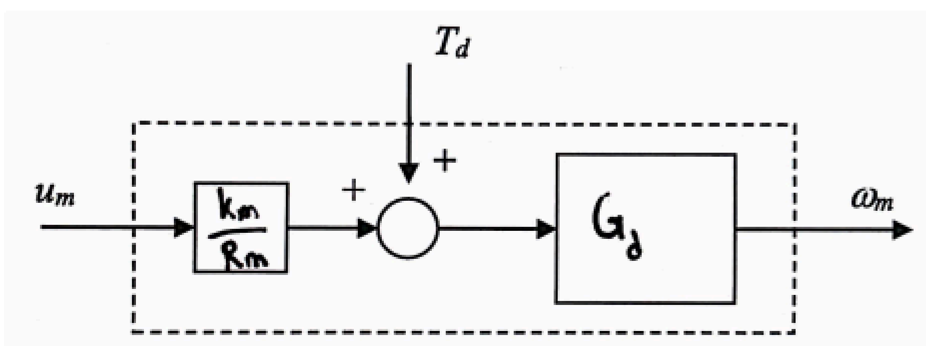


Figure 1: Block diagram

Experimental Results

Static relationship

Using the DCMCT, it was determined that the offset control set the applied DC voltage to the motor, while the amplitude control set the amplitude of the waveform added to the offset. Setting the amplitude to 0 V and the offset to the maximum value of 5 V, it was determined that the maximum speed of the system is 97 rad/s.

In the lab, data to analyze the static behavior of the system was recorded. First, by holding the motor shaft in place while adjusting a DC input, the current was obtained over several samples to experimentally obtain the resistance of the motor, shown in table 1.

u_m (V)	Measured current (A)	Bias-corrected current (A)	R_m (Ω)
-5	-0.420	-0.417	11.990
-2	-0.140	-0.137	14.599
1	0.070	0.073	13.699
2	0.130	0.133	15.038
5	0.375	0.378	13.228

Table 1: Static motor readings.

Next, the motor torque constant k_m was experimentally obtained by allowing the motor to rotate freely and recording the rotational speed over several samples, shown in table 2.

u_m (V)	Measured speed (rad/s)	k_m (Vs/rad)
5	97	0.052
2	38	0.053
1	18	0.056
-2	-38	0.053
-5	-98	0.051

Table 2: Free motor readings.

Dynamic model

First, a bump-test was performed which serves to analyze the step response of the system. A pulse train was generated and fed to the motor, with the resulting waveforms shown in figure 1.

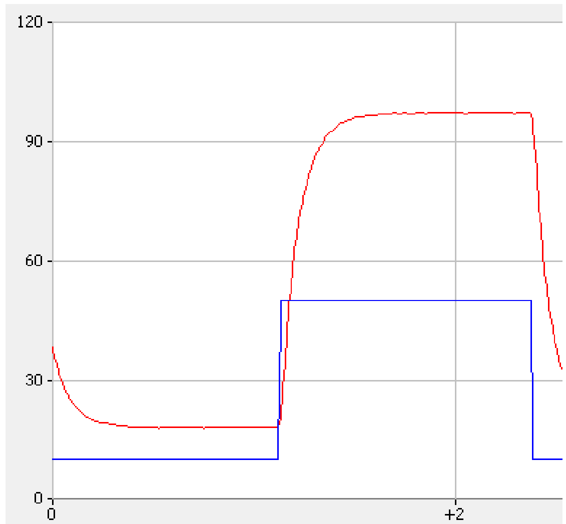
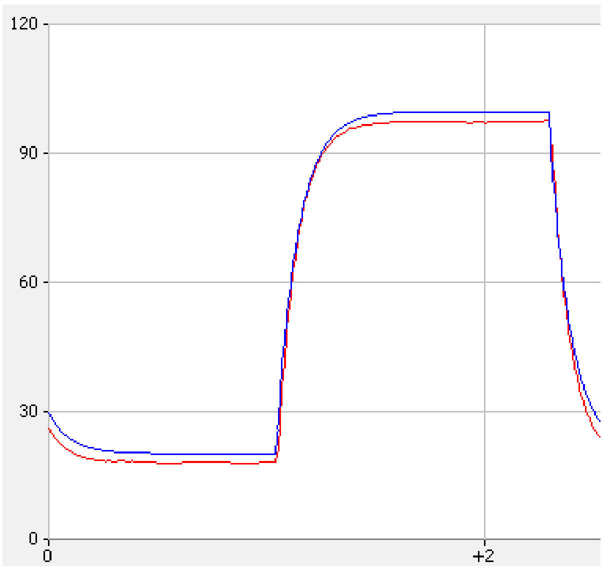


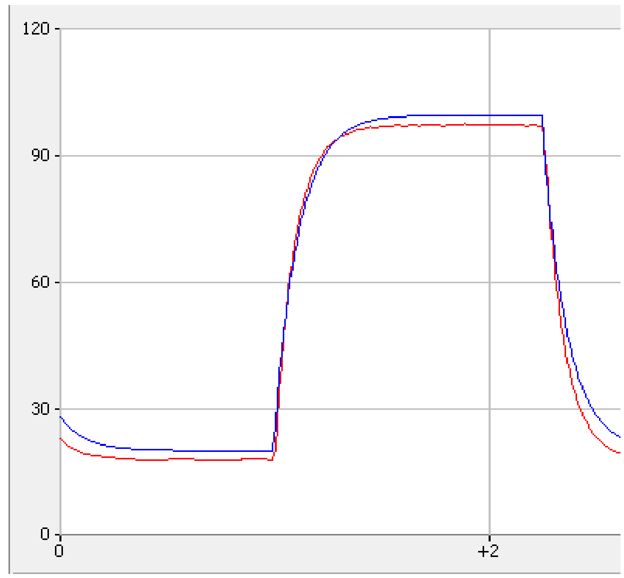
Figure 1: Bump-test waveform. Input voltage in blue, rpm in red.

Model validation

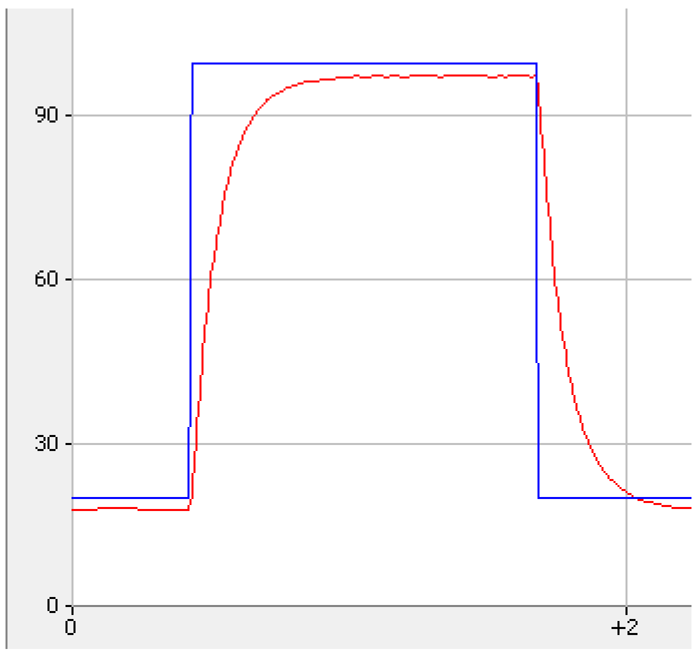
To validate the pre-lab, static relationship, and bump-test models, the estimated transfer function parameters for each (described in the Discussion section) were applied to the model, with the resulting waveforms shown in figure 2.



(a)



(b)



(c)

Figure 2: DC motor modeled using transfer function parameters obtained from the (a) prelab (b) static relationship (c) bump-test.

Finally, the transfer function parameters were manually adjusted to attempt to find the best-fitting model. The values K = 0.087 rad/Vs and τ = 19.3s were found to fit best, with the results shown in figure 3.

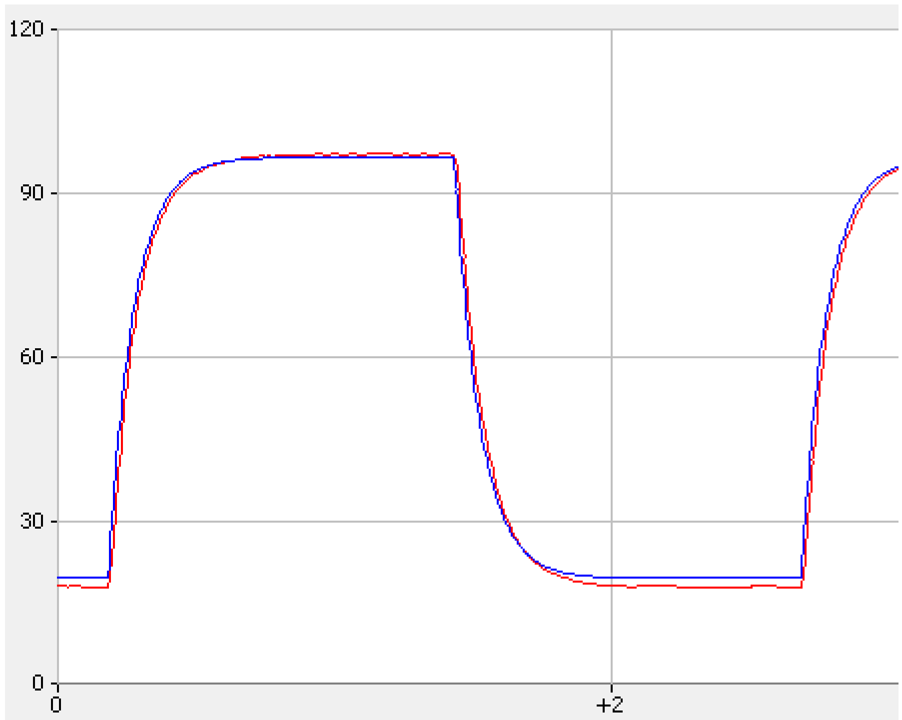


Figure 3: Best-fit model waveforms.

Discussion

Static relationship

From the readings in table 1, an average motor resistance of R_m = 13.710 Ω was obtained. The resistance is rated at 10.6 Ω, resulting in an absolute error of 3.11 Ω. Since this value was obtained experimentally assuming a linear system, small error is to be expected due to differences between the physical values and the control parameters.

From the readings in table 2, an average motor torque constant k_m = *0.053* Vs/rad was obtained. The torque constant is rated at 0.0502 Nm/A, the units of which are equivalent, and the value is very close, within 0.0003 Nm/A.

Using the obtained values above, estimates for the transfer function parameters were obtained using derivations obtained in the pre-lab and compared with the pre-lab values in table 3.

	Gain (rad/Vs)	Time constant (s)
Pre-lab	0.093	19.92
Static experiments	0.108	18.87

Table 3: Open-loop transfer function parameters compared with static observations.

The values obtained theoretically are similar to the values obtained experimentally, indicating the validity of the experiment.

Dynamic model

Using the procedure described in the lab manual section 5.3.1 along with the bump-test data shown in figure 2(c), estimates for the transfer function parameters were obtained and compared with the pre-lab values in table 4.

	Gain (rad/Vs)	Time constant (s)
Pre-lab	0.093	19.92
Bump test	0.086	19.92

Table 4: Open-loop transfer function parameters compared with bump-test observations.

Conclusion

The open-loop transfer function parameters for a DC motor were obtained theoretically and experimentally through various methods. All parameter values obtained are collected in table 5.

	Gain (rad/Vs)	Time constant (s)
Pre-lab	0.093	19.92
Static relationship	0.108	18.87
Bump test	0.086	19.92
Best fit	0.087	19.3

Table 5: Transfer function parameters summary.

All obtained values are similar, though the error on the static relationship test is notable with the largest K and the smallest τ. However, as shown by figures 2 and 3, all models result in a decent model of the DC motor.