Chapter 1

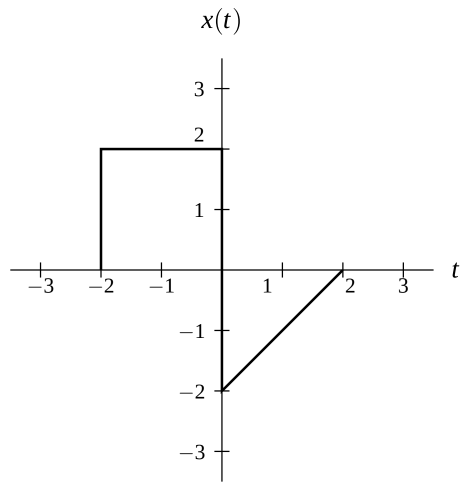
Continuous-Time Signals and Systems (Chapter 2)

2.1 Identify the time and/or amplitude transformations that must be applied to the signal $x(t)$ in order to obtain each of the signals specified below. Choose the transformations such that time shifting precedes time scaling and amplitude scaling precedes amplitude shifting. Be sure to clearly indicate the order in which the transformations are to be applied.
(a) $y(t) = x(2t-1)$;
(b) $y(t) = x(\frac{1}{2}t+1)$;
(c) $y(t) = 2x(-\frac{1}{2}t+1)+3$;
(d) $y(t) = -\frac{1}{2}x(-t+1)-1$; and
(e) $y(t) = -3x(2[t-1])-1$.

Solution.

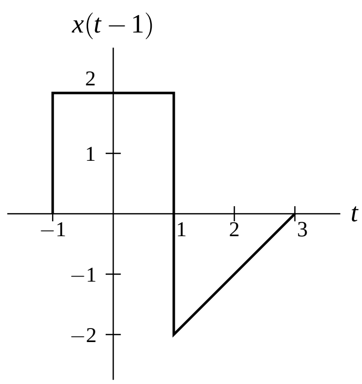
(e) To obtain $y(t)$ from $x(t)$, we apply the following transformations: 1) time shift by 2 (i.e., shift to the right by 2), 2) time scale by 2 (i.e., compress horizontally by 2), 3) amplitude scale by -3 (i.e., expand vertically by 3 and invert), and 4) amplitude shift by -1 (i.e., shift down by 1).

2.3 Given the signal $x(t)$ shown in the figure below, plot and label each of the following signals:
(a) $x(t-1)$;
(b) $x(2t)$;
(c) $x(-t)$;
(d) $x(2t+1)$; and
(e) $\frac{1}{4}x(-\frac{1}{2}t+1)-\frac{1}{2}$.



Solution.

(a)



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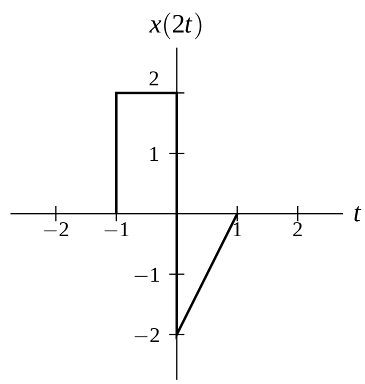

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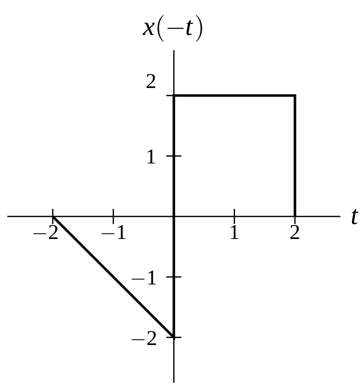

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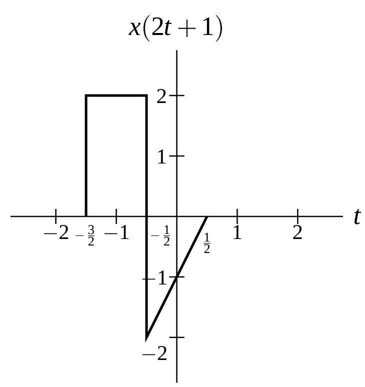
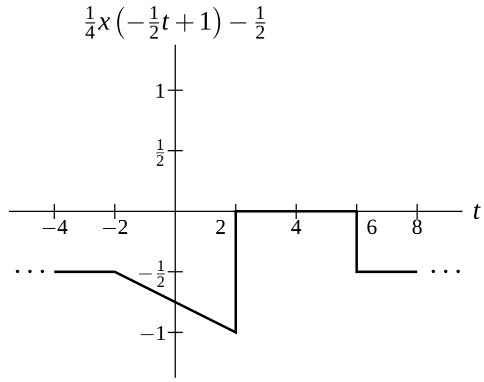


(e)



2.4 Determine whether each of the following functions is even, odd, or neither even nor odd:
(a) $x(t) = t^3$;
(b) $x(t) = t^3\,|t|$;
(c) $x(t) = \left|t^3\right|$;
(d) $x(t) = (\cos 2\pi t)(\sin 2\pi t)$;
(e) $x(t) = e^{j2\pi t}$; and
(f) $x(t) = \frac{1}{2}[e^t + e^{-t}]$.

Solution. (a) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= (-t)^3 \\ &= -t^3 \\ &= -x(t). \end{aligned}$$

Since $x(t) = -x(-t)$ for all t, $x(t)$ is odd.

(e) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= e^{-j2\pi t} \\ &= \cos(-2\pi t) + j\sin(-2\pi t) \\ &= \cos 2\pi t - j\sin 2\pi t. \end{aligned}$$

Thus, we can clearly see that $x(-t) \neq x(t)$ and $x(-t) \neq -x(t)$. Therefore, $x(t)$ is neither even nor odd.

(f) From the definition of $x(t)$, we have

$$\begin{aligned} x(-t) &= \tfrac{1}{2}[e^{-t} + e^{-(-t)}] \\ &= \tfrac{1}{2}[e^t + e^{-t}] \\ &= x(t). \end{aligned}$$

Thus, we can clearly see that $x(-t) = x(t)$ for all t. Therefore, $x(t)$ is even.

2.5 Prove each of the following assertions:
(a) The sum of two even signals is even.
(b) The sum of two odd signals is odd.
(c) The sum of an even signal and an odd signal is neither even nor odd.
(d) The product of two even signals is even.
(e) The product of two odd signals is even.
(f) The product of an even signal and an odd signal is odd.

Solution.

(a) Let $y(t) = x_1(t) + x_2(t)$ where $x_1(t)$ and $x_2(t)$ are even. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t) + x_2(-t).$$

Since $x_1(t)$ and $x_2(t)$ are even, we have that $x_1(-t) = x_1(t)$ and $x_2(-t) = x_2(t)$. So, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t) + x_2(t) \\ &= y(t). \end{aligned}$$

Thus, $y(t) = y(-t)$. Therefore, $y(t)$ is even.

(c) Let $y(t) = x_1(t) + x_2(t)$, where $x_1(t)$ and $x_2(t)$ are even and odd signals, respectively. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t) + x_2(-t).$$

Since $x_1(t)$ is even, $x_1(-t) = x_1(t)$. Similarly, since $x_2(t)$ is odd, $x_2(-t) = -x_2(t)$. Using these observations, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t) + [-x_2(t)] \\ &= x_1(t) - x_2(t). \end{aligned}$$

Clearly, $y(-t) \neq y(t)$ and $y(-t) \neq -y(t)$. Thus, $y(t)$ is neither even nor odd.

(f) Let $y(t) = x_1(t)x_2(t)$, where $x_1(t)$ and $x_2(t)$ are even and odd signals, respectively. From the definition of $y(t)$, we have

$$y(-t) = x_1(-t)x_2(-t).$$

Since $x_1(t)$ is even, $x_1(-t) = x_1(t)$. Similarly, since $x_2(t)$ is odd, $x_2(-t) = -x_2(t)$. Using these observations, we can simplify the above expression for $y(t)$ as

$$\begin{aligned} y(-t) &= x_1(t)[-x_2(t)] \\ &= -x_1(t)x_2(t) \\ &= -y(t). \end{aligned}$$

Thus, we have that $y(t) = -y(-t)$. Therefore, $y(t)$ is odd.

2.8 Suppose $h(t)$ is a causal signal and has the even part $h_e(t)$ given by

$$h_e(t) = t[u(t) - u(t-1)] + u(t-1) \quad \text{for } t > 0.$$

Find $h(t)$ for all t.

Solution.

We have that

$$\text{for } t > 0: \quad \begin{aligned} h_e(t) &= t[u(t) - u(t-1)] + u(t-1) \\ &= tu(t) + (-t+1)u(t-1). \end{aligned}$$

Since $h_e(t)$ is even, we can deduce that

$$\text{for } t < 0: \quad \begin{aligned} h_e(t) &= h_e(-t) \\ &= (-t)u(-t) + (t+1)u(-t-1). \end{aligned}$$

Let $h_o(t)$ denote the odd part of $h(t)$. Since $h(t)$ is causal and $h(t) = h_e(t) + h_o(t)$, we know that

$$\text{for } t < 0: \quad h_o(t) = -h_e(t).$$

Using this observation, we can deduce that

$$\begin{aligned}\text{for } t < 0: \quad h_o(t) &= -h_e(t) \\ &= -[(-t)u(-t) + (t+1)u(-t-1)] \\ &= (t)u(-t) + (-t-1)u(-t-1).\end{aligned}$$

Since $h_o(t)$ is odd, we have that

$$\begin{aligned}\text{for } t > 0: \quad h_o(t) &= -h_o(-t) \\ &= -[(-t)u(t) + (t-1)u(t-1)] \\ &= tu(t) + (-t+1)u(t-1).\end{aligned}$$

Therefore, we can conclude that

$$\begin{aligned}h(t) &= h_e(t) + h_o(t) \\ &= [tu(t) + (-t+1)u(t-1)] + [tu(t) + (-t+1)u(t-1)] \\ &= (2t)u(t) + (2-2t)u(t-1).\end{aligned}$$

2.9 Determine whether each of the signals given below is periodic. If the signal is periodic, find its fundamental period.
(a) $x(t) = \cos 2\pi t + \sin 5t$;
(b) $x(t) = [\cos(4t - \frac{\pi}{3})]^2$;
(c) $x(t) = e^{j2\pi t} + e^{j3\pi t}$; and
(d) $x(t) = 1 + \cos 2t + e^{j5t}$.

Solution.

(a) Let T_1 and T_2 denote the periods of $\cos 2\pi t$ and $\sin 5t$, respectively. We have

$$T_1 = \tfrac{2\pi}{2\pi} = 1 \quad \text{and} \quad T_2 = \tfrac{2\pi}{5}.$$

Since T_1/T_2 is irrational, $x(t)$ is not periodic.

(b) Since $\cos\left(4t - \frac{\pi}{3}\right)$ is periodic and the square of a periodic function is periodic, $x(t)$ is periodic. From the definition of $x(t)$, we can write

$$\begin{aligned}x(t) &= \cos^2\left(4t - \tfrac{\pi}{3}\right) \\ &= \left[\tfrac{1}{2}\left(e^{j(4t-\pi/3)} + e^{-j(4t-\pi/3)}\right)\right]^2 \\ &= \tfrac{1}{4}\left[e^{j(4t-\pi/3)(2)} + 2 + e^{-j(4t-\pi/3)(2)}\right] \\ &= \tfrac{1}{4}\left[e^{j(8t-2\pi/3)} + e^{-j(8t-2\pi/3)} + 2\right] \\ &= \tfrac{1}{4}[2\cos(8t - 2\pi/3) + 2] \\ &= \tfrac{1}{2}\cos(8t - 2\pi/3) + \tfrac{1}{2}.\end{aligned}$$

Thus, the period T of $x(t)$ is $T = \frac{2\pi}{8} = \frac{\pi}{4}$.

(c) Let T_1 and T_2 denote the periods of $e^{j2\pi t}$ and $e^{j3\pi t}$, respectively. We have

$$T_1 = \tfrac{2\pi}{2\pi} = 1, \quad T_2 = \tfrac{2\pi}{3\pi} = \tfrac{2}{3}, \quad \text{and} \quad \frac{T_1}{T_2} = \frac{1}{2/3} = \tfrac{3}{2}.$$

Since T_1/T_2 is rational, $x(t)$ is periodic. The period T of $x(t)$ is $T = 2T_1 = 2$.

2.10 Evaluate the following integrals:
(a) $\int_{-\infty}^{\infty} \sin\left(2t+\frac{\pi}{4}\right)\delta(t)dt$;
(b) $\int_{-\infty}^{t} [\cos\tau]\delta(\tau+\pi)d\tau$;
(c) $\int_{-\infty}^{\infty} x(t)\delta(at-b)dt$ where a and b are real constants and $a \neq 0$;
(d) $\int_{0}^{2} e^{j2t}\delta(t-1)dt$; and
(e) $\int_{-\infty}^{t} \delta(\tau)d\tau$.

Solution.

(a) From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{\infty} \sin\left(2t+\tfrac{\pi}{4}\right)\delta(t)dt &= \left[\sin\left(2t+\tfrac{\pi}{4}\right)\right]\Big|_{t=0} \\ &= \sin\left(\tfrac{\pi}{4}\right) \\ &= \tfrac{1}{\sqrt{2}}.\end{aligned}$$

(b) From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{t} [\cos\tau]\delta(\tau+\pi)d\tau &= \begin{cases} [\cos\tau]|_{\tau=-\pi} & \text{for } t > -\pi \\ 0 & \text{for } t < -\pi \end{cases} \\ &= \begin{cases} \cos(-\pi) & \text{for } t > -\pi \\ 0 & \text{for } t < -\pi \end{cases} \\ &= \begin{cases} -1 & \text{for } t > -\pi \\ 0 & \text{for } t < -\pi \end{cases} \\ &= -u(t+\pi).\end{aligned}$$

(c) We use a change of variable. Let $\lambda = at$ so that $t = \lambda/a$ and $d\lambda = adt$. Performing the change of variable and simplifying yields

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)\delta(at-b)dt &= \begin{cases} \int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)\left(\frac{1}{a}\right)d\lambda & \text{for } a > 0 \\ \int_{\infty}^{-\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)\left(\frac{1}{a}\right)d\lambda & \text{for } a < 0 \end{cases} \\ &= \begin{cases} \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)d\lambda & \text{for } a > 0 \\ -\frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)d\lambda & \text{for } a < 0 \end{cases} \\ &= \frac{1}{|a|}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right)\delta(\lambda-b)d\lambda \\ &= \frac{1}{|a|}\left[x\left(\frac{\lambda}{a}\right)\right]\Big|_{\lambda=b} \\ &= \frac{1}{|a|}x\left(\frac{b}{a}\right).\end{aligned}$$

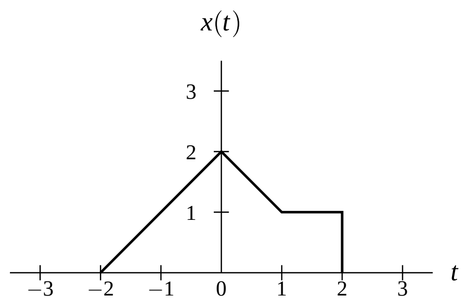
(d) Since the nonzero part of $\delta(t-1)$ is contained on the interval $[0,2]$, we can deduce from the sifting property that

$$\begin{aligned}\int_{0}^{2} e^{j2t}\delta(t-1)dt &= \left[e^{j2t}\right]\Big|_{t=1} \\ &= e^{j2}.\end{aligned}$$

(e) We have

$$\int_{-\infty}^{t} \delta(\tau)d\tau = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$$
$$= u(t).$$

2.11 Suppose that we have the signal $x(t)$ shown in the figure below. Use unit-step functions to find a single expression for $x(t)$ that is valid for all t.



Solution.

We have

$$\begin{aligned} x(t) &= [t+2][u(t+2)-u(t)] + [-t+2][u(t)-u(t-1)] + [1][u(t-1)-u(t-2)] \\ &= [t+2]u(t+2) + [-t-2-t+2]u(t) + [t-2+1]u(t-1) + [-1]u(t-2) \\ &= (t+2)u(t+2) + (-2t)u(t) + (t-1)u(t-1) + (-1)u(t-2). \end{aligned}$$

2.12 Determine whether the system with input $x(t)$ and output $y(t)$ defined by each of the following equations is linear:

(a) $y(t) = \int_{t-1}^{t+1} x(\tau)d\tau$;
(b) $y(t) = e^{x(t)}$;
(c) $y(t) = \text{Even}\{x(t)\}$; and
(d) $y(t) = x^2(t)$.

Solution.

(a) Let $y_1(t)$, $y_2(t)$, and $y_3(t)$ denote the responses of the system to the inputs $x_1(t)$, $x_2(t)$, and $a_1x_1(t)+a_2x_2(t)$, respectively, where a_1 and a_2 are complex constants. If $y_3(t) = a_1y_1(t)+a_2y_2(t)$ for all $x_1(t)$, $x_2(t)$, a_1, and a_2, then the system is linear. We have

$$\begin{aligned} y_1(t) &= \int_{t-1}^{t+1} x_1(\tau)d\tau, \\ y_2(t) &= \int_{t-1}^{t+1} x_2(\tau)d\tau, \quad \text{and} \\ y_3(t) &= \int_{t-1}^{t+1} [a_1x_1(\tau)+a_2x_2(\tau)]d\tau \\ &= \int_{t-1}^{t+1} a_1x_1(\tau)d\tau + \int_{t-1}^{t+1} a_2x_2(\tau)d\tau \\ &= a_1\int_{t-1}^{t+1} x_1(\tau)d\tau + a_2\int_{t-1}^{t+1} x_2(\tau)d\tau \\ &= a_1y_1(t)+a_2y_2(t). \end{aligned}$$

Since $y_3(t) = a_1y_1(t) + a_2y_2(t)$, the system is linear.

(c) Let $y_1(t)$, $y_2(t)$, and $y_3(t)$ denote the responses of the system to the inputs $x_1(t)$, $x_2(t)$, and $a_1x_1(t) + a_2x_2(t)$, respectively, where a_1 and a_2 are complex constants. If $y_3(t) = a_1y_1(t) + a_2y_2(t)$ for all $x_1(t)$, $x_2(t)$, a_1, and a_2, then the system is linear. We have

$$\begin{aligned}
y_1(t) &= \tfrac{1}{2}[x_1(t) + x_1(-t)], \\
y_2(t) &= \tfrac{1}{2}[x_2(t) + x_2(-t)], \quad \text{and} \\
y_3(t) &= \tfrac{1}{2}[(a_1x_1(t) + a_2x_2(t)) + (a_1x_1(-t) + a_2x_2(-t))] \\
&= \tfrac{1}{2}[a_1x_1(t) + a_1x_1(-t) + a_2x_2(t) + a_2x_2(-t)] \\
&= \tfrac{1}{2}a_1[x_1(t) + x_1(-t)] + \tfrac{1}{2}a_2[x_2(t) + x_2(-t)] \\
&= a_1y_1(t) + a_2y_2(t).
\end{aligned}$$

Since $y_3(t) = a_1y_1(t) + a_2y_2(t)$, the system is linear.

(d) Let $y_1(t)$, $y_2(t)$, and $y_3(t)$ denote the responses of the system to the inputs $x_1(t)$, $x_2(t)$, and $a_1x_1(t) + a_2x_2(t)$, respectively, where a_1 and a_2 are complex constants. If $y_3(t) = a_1y_1(t) + a_2y_2(t)$ for all $x_1(t)$, $x_2(t)$, a_1, and a_2, then the system is linear. We have

$$\begin{aligned}
y_1(t) &= x_1^2(t) \\
y_2(t) &= x_2^2(t) \\
y_3(t) &= [a_1x_1(t) + a_2x_2(t)]^2 \\
&= a_1^2x_1^2(t) + 2a_1a_2x_1(t)x_2(t) + a_2^2x_2^2(t) \\
&\neq a_1y_1(t) + a_2y_2(t).
\end{aligned}$$

Since $y_3(t) \neq a_1y_1(t) + a_2y_2(t)$, the system is not linear.

2.13 Determine whether the system with input $x(t)$ and output $y(t)$ defined by each of the following equations is time invariant:
(a) $y(t) = \frac{d}{dt}x(t)$;
(b) $y(t) = \text{Even}\{x(t)\}$;
(c) $y(t) = \int_t^{t+1} x(\tau - \alpha)d\tau$ where α is a constant;
(d) $y(t) = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$;
(e) $y(t) = x(-t)$; and
(f) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$.

Solution.

(a) Let $y_1(t)$ and $y_2(t)$ denote the responses of the system to the inputs $x_1(t)$ and $x_2(t) = x_1(t - t_0)$, respectively, where t_0 is a constant. If $y_2(t) = y_1(t - t_0)$ for all $x_1(t)$ and t_0, then the system is time invariant. (Note that $\frac{d}{dt}x_2(t) = \frac{d}{dt}x_1(t - t_0)$.) We have

$$\begin{aligned}
y_1(t) &= \tfrac{d}{dt}x_1(t), \\
y_2(t) &= \tfrac{d}{dt}x_2(t) \\
&= \tfrac{d}{dt}x_1(t - t_0), \quad \text{and} \\
y_1(t - t_0) &= \left[\tfrac{d}{dv}x_1(v)\right]\Big|_{v=t-t_0} \\
&= \tfrac{d}{dt}x_1(t - t_0) \\
&= y_2(t).
\end{aligned}$$

Since $y_2(t) = y_1(t - t_0)$, the system is time invariant.

(c) Let $y_1(t)$ and $y_2(t)$ denote the responses of the system to the inputs $x_1(t)$ and $x_2(t) = x_1(t-t_0)$, respectively, where t_0 is a constant. If $y_2(t) = y_1(t-t_0)$ for all $x_1(t)$ and t_0, then the system is time invariant. We have

$$\begin{aligned} y_1(t) &= \int_t^{t+1} x_1(\tau-\alpha)d\tau \quad \text{and} \\ y_2(t) &= \int_t^{t+1} x_2(\tau-\alpha)d\tau \\ &= \int_t^{t+1} x_1(\tau-\alpha-t_0)d\tau. \end{aligned}$$

Now, we apply a change of variable. Let $\lambda = \tau - t_0$ so that $\tau = \lambda + t_0$ and $d\lambda = d\tau$. Applying this change of variable yields

$$\begin{aligned} y_2(t) &= \int_{t-t_0}^{t+1-t_0} x_1(\lambda-\alpha)d\lambda \\ &= y_1(t-t_0). \end{aligned}$$

Since $y_2(t) = y_1(t-t_0)$, the system is time invariant.

(e) Let $y_1(t)$ and $y_2(t)$ denote the responses of the system to the inputs $x_1(t)$ and $x_2(t) = x_1(t-t_0)$, respectively, where t_0 is a constant. If $y_2(t) = y_1(t-t_0)$ for all $x_1(t)$ and t_0, then the system is time invariant. We have

$$\begin{aligned} y_1(t) &= x_1(-t), \\ y_2(t) &= x_2(-t) \\ &= x_1(-t-t_0), \quad \text{and} \\ y_1(t-t_0) &= x_1(-[t-t_0]) \\ &= x_1(-t+t_0) \\ &\neq y_2(t). \end{aligned}$$

Since $y_2(t) \neq y_1(t-t_0)$, the system is not time invariant.

2.14 Determine whether the system with input $x(t)$ and output $y(t)$ defined by each of the following equations is causal and/or memoryless:
(a) $y(t) = \int_{-\infty}^{2t} x(\tau)d\tau$;
(b) $y(t) = \operatorname{Odd}\{x(t)\}$;
(c) $y(t) = x(t-1)+1$;
(d) $y(t) = \int_t^{\infty} x(\tau)d\tau$; and
(e) $y(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau$.

Solution. If the output $y(t)$ at $t = t_0$ for any arbitrary t_0 depends only on the input $x(t)$ for $t \leq t_0$, then the system is causal. If the output $y(t)$ at $t = t_0$ for any arbitrary t_0 depends only on the input $x(t)$ at $t = t_0$, then the system is memoryless.

(a) From the equation

$$y(t) = \int_{-\infty}^{2t} x(\tau)d\tau,$$

we can see that $y(t)|_{t=t_0}$ depends on $x(t)$ for $-\infty < t \leq 2t_0$. Therefore, the system is not causal (since $2t_0 > t_0$ for any positive t_0) and the system is not memoryless.

(c) We have

$$y(t) = x(t-1)+1.$$

Consider $y(t)|_{t=t_0}$. This quantity depends on $x(t)$ for $t = t_0 - 1$. Therefore, the system is causal (since $t_0 - 1 < t_0$) and the system is not memoryless (since $t_0 - 1 \neq t_0$).

(e) We have

$$\begin{aligned} y(t) &= \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau \\ &= \int_{-\infty}^{t} x(0)\delta(\tau)d\tau \\ &= x(0)\int_{-\infty}^{t} \delta(\tau)d\tau \\ &= x(0)u(t). \end{aligned}$$

Consider $y(t)|_{t=t_0}$. If $t_0 > 0$, this quantity depends on $x(t)$ for $t = 0$. Otherwise, this quantity does not depend on $x(t)$ at all. Therefore, the system is causal, since $y(t_0)$ only depends on $x(t)$ for $t \leq t_0$). Also, the system is not memoryless (since for $t_0 > 0$, $y(t_0)$ depends on $x(t)$ for $t \neq t_0$).

2.15 Determine whether the system with input $x(t)$ and output $y(t)$ defined by each of the equations given below is invertible. If the system is invertible, specify its inverse.
(a) $y(t) = x(at - b)$ where a and b are real constants and $a \neq 0$;
(b) $y(t) = e^{x(t)}$;
(c) $y(t) = \text{Even}\{x(t)\} - \text{Odd}\{x(t)\}$; and
(d) $y(t) = \frac{d}{dt}x(t)$.

Solution. A system is invertible if any two distinct inputs always produce distinct outputs.

(a) We have

$$y(t) = x(at - b).$$

Now, we employ a change of variable. Let $\lambda = at - b$ so that $t = \frac{1}{a}[\lambda + b]$. Since we are told that $a \neq 0$, we do not need to worry about division by zero. Applying the change of variable yields

$$y(\tfrac{1}{a}[\lambda + b]) = x(\lambda),$$

or alternatively,

$$x(t) = y(\tfrac{1}{a}[t + b]).$$

Thus, we have found the inverse system. Therefore, the system is invertible (since we have just found its inverse).

(c) We have

$$\begin{aligned} y(t) &= \text{Even}\{x(t)\} - \text{Odd}\{x(t)\} \\ &= \tfrac{1}{2}[x(t) + x(-t)] - \tfrac{1}{2}[x(t) - x(-t)] \\ &= \tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t) - \tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t) \\ &= x(-t). \end{aligned}$$

So, $y(t) = x(-t)$. Thus, we have $x(t) = y(-t)$. Therefore, the system is invertible (since we have just found the inverse, namely $x(t) = y(-t)$).

(d) Consider an input $x_1(t)$ of the form

$$x_1(t) = A$$

where A is a constant. Such an input will always yield the output $y_1(t)$ given by

$$\begin{aligned} y_1(t) &= \tfrac{d}{dt}x_1(t) \\ &= \tfrac{d}{dt}A \\ &= 0. \end{aligned}$$

Therefore, any constant input will produce the same output (namely, an output of zero). Since distinct inputs yield the same output, the system is not invertible.

2.16 Determine whether the system with input $x(t)$ and output $y(t)$ defined by each of the equations given below is BIBO stable.
(a) $y(t) = \int_t^{t+1} x(\tau)d\tau$;
(b) $y(t) = \frac{1}{2}x^2(t) + x(t)$; and
(c) $y(t) = 1/x(t)$.
[Hint for part (a): For any function $f(x)$, $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\,dx$.]

Solution. A system is BIBO stable if its response $y(t)$ to any bounded input $x(t)$ is always bounded. That is,

$$|x(t)| \leq A \Rightarrow |y(t)| \leq B,$$

where A and B are finite constants.

(a) Suppose that $|x(t)| \leq A < \infty$ (i.e., $x(t)$ is bounded by A). Taking the absolute value of both sides of the input-output equation for the system, we obtain

$$|y(t)| = \left|\int_t^{t+1} x(\tau)d\tau\right|.$$

Using the fact that $\left|\int_\alpha^\beta f(x)dx\right| \leq \int_\alpha^\beta |f(x)|\,dx$, we can write

$$\begin{aligned} |y(t)| &\leq \int_t^{t+1} |x(\tau)|\,d\tau \\ &\leq \int_t^{t+1} A d\tau \\ &= [A\tau]\big|_t^{t+1} \\ &= A(t+1) - At \\ &= A \\ &< \infty. \end{aligned}$$

Thus, we have that

$$|x(t)| \leq A < \infty \Rightarrow |y(t)| \leq A < \infty.$$

Therefore, the system is BIBO stable.

(b) Suppose that $x(t)$ is bounded as

$$|x(t)| \leq A.$$

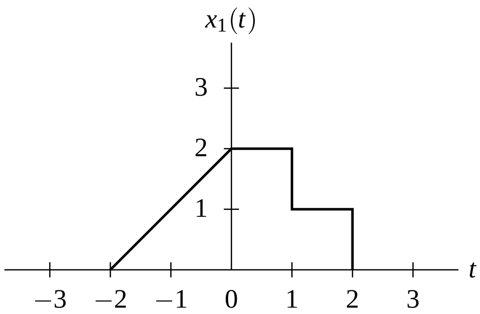
Then, we have

$$\begin{aligned} y(t) &\leq \tfrac{1}{2}A^2 + A \\ &< \infty. \end{aligned}$$

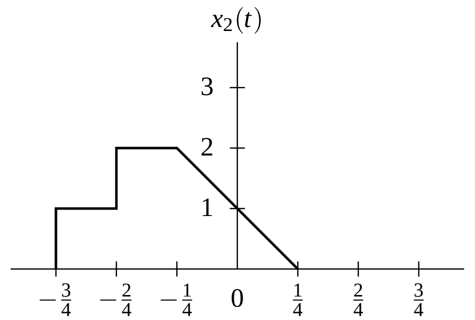
Therefore, a bounded input always yields a bounded output, and the system is BIBO stable.

(c) Consider the input $x(t) = 0$. This will produce the output $y(t) = \frac{1}{0} = \infty$. Since a bounded input produces an unbounded output, the system is not BIBO stable.

2.19 Given the signals $x_1(t)$ and $x_2(t)$ shown in the figures below, express $x_2(t)$ in terms of $x_1(t)$.



(a)



(b)

Solution.

We observe that $x_2(t)$ is simply a time-shifted, time-scaled, and time-reversed version of $x_1(t)$. More specifically, $x_2(t)$ is generated from $x_1(t)$ through the following transformations (in order): 1) time shifting by 1, 2) time scaling by 4, and 3) time reversal. Thus, we have

$$x_2(t) = x_1(-4t-1).$$

Chapter 9

MATLAB (Appendix E)

E.101 Indicate whether each of the following is a valid MATLAB identifier (i.e., variable/function name):
(a) `4ever`
(b) `$rich$`
(c) `foobar`
(d) `foo_bar`
(e) `_foobar`

Solution.

(a) A MATLAB identifier cannot begin with a numeric character. Thus, `4ever` is not a valid identifier.

(b) A MATLAB identifier cannot contain the `$` character. Thus, `$rich$` is not a valid identifier.

(c) The name `foobar` is a valid identifier.

(d) The name `foo_bar` is a valid identifier.

(e) A MATLAB identifier cannot begin with an underscore character. Thus, `_foobar` is not a valid identifier.

E.102 Let T_C, T_F, and T_K denote the temperature measured in units of Celsius, Fahrenheit, and Kelvin, respectively. Then, these quantities are related by

$$T_F = \tfrac{9}{5}T_C + 32 \quad \text{and}$$
$$T_K = T_C + 273.15.$$

Write a program that generates a temperature conversion table. The first column of the table should contain the temperature in Celsius. The second and third columns should contain the corresponding temperatures in units of Fahrenheit and Kelvin, respectively. The table should have entries for temperatures in Celsius from -50 to 50 in steps of 10.

Solution.

The temperature conversion table can be produced with the following code:

Listing 9.1: `temperature_conversion_table.m`

```
display(sprintf('%-8s %-8s %-8s', 'Celsius', 'Fahrenheit', 'Kelvin'));
for celsius = -50 : 10 : 50
    fahrenheit = 9 / 5 * celsius + 32;
    kelvin = celsius + 273.15;
    display(sprintf('%8.2f %8.2f %8.2f', celsius, fahrenheit, kelvin));
end
```

The code produces the following output:

Listing 9.2: Output of temperature conversion program

```
Celsius  Fahrenheit Kelvin
  -50.00   -58.00   223.15
  -40.00   -40.00   233.15
  -30.00   -22.00   243.15
  -20.00    -4.00   253.15
  -10.00    14.00   263.15
    0.00    32.00   273.15
   10.00    50.00   283.15
   20.00    68.00   293.15
   30.00    86.00   303.15
   40.00   104.00   313.15
   50.00   122.00   323.15
```

E.106 Suppose that the vector `v` is defined by the following line of code:

```
v = [0 1 2 3 4 5]
```

Write an expression in terms of `v` that yields a new vector of the same dimensions as `v`, where each element t of the original vector `v` has been replaced by the given quantity below. In each case, the expression should be as short as possible.
(a) $2t-3$;
(b) $1/(t+1)$;
(c) t^5-3; and
(d) $|t|+t^4$.

Solution.

(a) `2 * v - 3`
(b) `1 ./ (v + 1)`
(c) `v .^ 5 - 3`
(d) `abs(v) + v .^ 4`

E.107 (a) Write a function called `unitstep` that takes a single real argument t and returns $u(t)$, where

$$u(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Modify the function from part (a) so that it takes a single vector argument $t = \begin{bmatrix} t_1 & t_2 & \cdots & t_n \end{bmatrix}^T$ (where $n \geq 1$ and $t_1, t_2, \ldots, t_n$ are real) and returns the vector $\begin{bmatrix} u(t_1) & u(t_2) & \cdots & u(t_n) \end{bmatrix}^T$. Your solution must employ a looping construct (e.g., a for loop).
(c) With some ingenuity, part (b) of this problem can be solved using only two lines of code, without the need for any looping construct. Find such a solution. [Hint: In MATLAB, to what value does an expression like "`[-2 -1 0 1 2] >= 0`" evaluate?]

Solution.

(a) This problem can be solved with code such as that shown below.

Listing 9.3: `unitstep1.m`

```
function x = unitstep(t)

if t >= 0
    x = 1;
else
    x = 0;
end
```

(b) This problem can be solved with code such as that shown below.

Listing 9.4: `unitstep2.m`

```
function x = unitstep(t)

% Create a vector of zeros with the same size as the input vector.
x = zeros(size(t));

% Correctly set the elements in the result vector that should be one.
m = length(x);
for i = 1 : m

    if t(i) >= 0
        x(i) = 1;
    end

end
```

(c) This problem can be solved with code such as that shown below.

Listing 9.5: `unitstep3.m`

```
function x = unitstep(t)

x = (t >= 0);
```