ECE 260

EXAM 3

SOLUTIONS

(SUMMER 2022)

QUESTION 1

$$C_K = \frac{1}{T}\int_0^T x(t)\, e^{-j(2\pi/T)Kt}\, dt$$

$$= \frac{1}{2}\int_0^2 \pi t\, e^{-j\pi K t}\, dt$$

$$= \frac{\pi}{2}\int_0^2 t\, e^{-j\pi K t}\, dt \qquad K \neq 0$$

$$= \frac{\pi}{2}\left[\frac{1}{(-j\pi K)^2} e^{-j\pi K t}(-j\pi K t - 1)\right]\Bigg|_0^2$$

$$= \frac{\pi}{2}\left(\frac{1}{-\pi^2 K^2}\right)\left[e^{-j\pi K t}(-j\pi K t - 1)\right]\Bigg|_0^2$$

$$= \frac{1}{-2\pi K^2}\left[-j2\pi K - 1 - [-1]\right]$$

$$= \frac{-j2\pi K}{-2\pi K^2}$$

$$= \frac{j}{K}$$

We still must consider $K=0$. If $K=0$, we have

$$C_K = \frac{\pi}{2}\int_0^2 t\, dt$$

$$= \frac{\pi}{2}\left[\frac{1}{2}t^2\right]\Bigg|_0^2$$

$$= \frac{\pi}{2}\left(\frac{1}{2}\right)\left[t^2\right]\Bigg|_0^2$$

$$= \frac{\pi}{4}\left[4 - 0\right]$$

$$= \pi$$

Therefore, we conclude

$$C_K = \begin{cases} \pi & K = 0 \\ \frac{j}{K} & \text{otherwise.} \end{cases}$$

QUESTION 2

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function count = count_nonzero(m)
    count = 0;
    for r = 1 : height(m)
        for c = 1 : width(m)
            if m(r, c) ~= 0
                count = count + 1;
            end
        end
    end
end
```

QUESTION 3:

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt$$
$$= \int_{-\infty}^{\infty} e^{-3t} u(t)\, e^{-j\omega t}\, dt$$
$$= \int_{0}^{\infty} e^{-3t} e^{-j\omega t}\, dt$$
$$= \frac{1}{3+j\omega}$$

$$x(t) = 10 + 2\cos(t) + 2j\sin(2t)$$
$$= 10 + 2\left[\tfrac{1}{2}(e^{jt} + e^{-jt})\right] + 2j\left[\tfrac{1}{2j}(e^{j2t} - e^{-j2t})\right]$$
$$= 10 + e^{jt} + e^{-jt} + e^{j2t} - e^{-j2t}$$
$$= -e^{-j2t} + e^{-jt} + 10 + e^{jt} + e^{j2t}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_0 t} \quad \text{where } \omega_0 = 1 \quad (\text{and } T = 2\pi)$$

$$c_k = \begin{cases} -1 & k=-2 \\ 1 & k=-1 \\ 10 & k=0 \\ 1 & k=1 \\ 1 & k=2 \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \sum_{k=-\infty}^{\infty} c_k H(k\omega_0)\, e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} c_k H(k)\, e^{jkt}$$

$$= (-1)H(-2)e^{-j2t} + (1)H(-1)e^{-jt} + (10)H(0) + (1)H(1)e^{jt} + (1)H(2)e^{j2t}$$

$$= \frac{-1}{3-j2}e^{-j2t} + \frac{1}{3-j}e^{-jt} + \frac{10}{3} + \frac{1}{3+j}e^{jt} + \frac{1}{3+j2}e^{j2t}$$

$$= \frac{(-1)(3+j2)}{9+4}e^{-j2t} + \frac{3+j}{9+1}e^{-jt} + \frac{10}{3} + \frac{3-j}{9+1}e^{jt} + \frac{3-j2}{9+4}e^{j2t}$$

$$= \frac{-3-j2}{13}e^{-j2t} + \frac{3+j}{10}e^{-jt} + \frac{10}{3} + \frac{3-j}{10}e^{jt} + \frac{3-j2}{13}e^{j2t}$$

$$= \frac{-3}{13}e^{-j2t} - \frac{j2}{13}e^{-j2t} + \frac{3}{10}e^{-jt} + \frac{j}{10}e^{-jt} + \frac{10}{3} + \frac{3}{10}e^{jt} - \frac{j}{10}e^{jt} + \frac{3}{13}e^{j2t} - \frac{j2}{13}e^{j2t}$$

$$= \frac{3}{13}(e^{j2t} - e^{-j2t}) - \frac{j2}{13}(e^{j2t} + e^{-j2t}) + \frac{3}{10}(e^{jt} + e^{-jt}) - \frac{j}{10}(e^{jt} - e^{-jt}) + \frac{10}{3}$$

$$= \frac{j6}{13}\sin(2t) - \frac{j4}{13}\cos(2t) + \frac{3}{5}\cos(t) + \frac{1}{5}\sin(t) + \frac{10}{3}$$

QUESTION 4

The function x satisfies the Dirichlet conditions.

Consequently, we have

$$y(0) = \frac{1}{2}\left[x(0^-) + x(0^+)\right]$$

$$= \frac{1}{2}\left[2 + 1\right]$$

$$= \frac{3}{2}$$

$$y(1) = \frac{1}{2}\left[x(1^-) + x(1^+)\right]$$

$$= \frac{1}{2}\left[e^{-2} + 1 + e^{-1}\right]$$

$$= \frac{1 + e^{-1} + e^{-2}}{2} = \frac{e^2 + e + 1}{2e^2}$$

QUESTION 5

$$c_k = \frac{e^{j3k}(j2k+1)}{(j2k-1)^3}$$

(a) $$|c_k| = \left|\frac{e^{j3k}(j2k+1)}{(j2k-1)^3}\right| = \frac{|e^{j3k}||j2k+1|}{|(j2k-1)^3|}$$

$$= \frac{|j2k+1|}{|j2k-1|^3} = \frac{\sqrt{4k^2+1}}{\left(\sqrt{4k^2+1}\right)^3} = \frac{1}{4k^2+1}$$

(b) In the formula for c_k,
$4k^2+1$ has a minimum of 1 at $k=0$.

So c_k has a maximum of $\frac{1}{1}=1$ at $k=0$.

Therefore, x has the most spectral information at the frequency $k\omega_0 = 0\omega_0 = 0$.