UNIVERSITY OF VICTORIA

FINAL EXAMINATIONS - APRIL 2014

CONTROL THEORY AND SYSTEMS I, ELEC 360, A01/A02

CRN# 21098/21099

TO BE ANSWERED IN BOOKLETS

DURATION: 3 hours
INSTRUCTOR: Dr. H.-C. Yang

STUDENT MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS FOUR (4) PAGES, INCLUDING THIS COVER, AND ONE (1) ADDITIONAL PAGE FOR TABLE.

ADDITIONAL INSTRUCTIONS

THE EXAM IS CLOSED BOOKS AND CLOSED NOTES. HOWEVER, YOU ARE ALLOWED TO BRING FOUR FORMULA SHEETS (8.5 $\times$ 11, SINGLE SIDE).

THERE ARE SEVEN PROBLEMS. THE PROBLEMS ARE FULLY INDEPENDENT. SOME OF THE QUESTIONS WITHIN THE PROBLEMS ARE ALSO INDEPENDENT.

PARTIAL CREDIT WILL BE AWARDED PROVIDED THAT YOU SHOW YOUR WORK AND THAT I CAN DECIPHER IT. IN PARTICULAR, IT IS VERY IMPORTANT TO CLEARLY SHOW ALL THE STEPS IN SOLVING A QUESTION. NOTE THAT ANSWERS TO QUESTION WITHOUT WORK SHOWN OR WITHOUT ANY EXPLANATION WILL EARN NO CREDIT.

ANSWER ALL QUESTIONS

Question 1 (8 points)

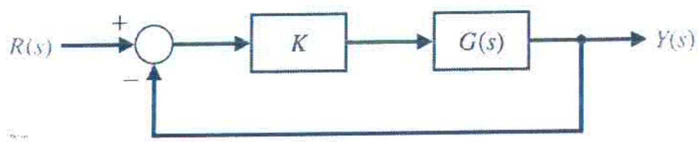
A dynamic system is described by the following differential equation

$$\ddot{y} + 3\dot{y} + 2y = \dot{u} - 2u.$$

(1) Assuming zero initial condition, determine the output of the system $y(t)$ when the input is $u(t) = e^{-t/2}$ for $t \geq 0$.
(2) Determine the steady-state output of the system when the input is $u(t) = \cos 3t$.

Question 2 (8 points)

Consider the following second order closed-loop control system with unity feedback



$$G(s) = \frac{5}{s(s+2)}.$$

(1) Express the percent overshot of the system as function of K.
(2) Express the resonant peak of the closed-loop frequency response as function of K.
(3) Express the phase margin of the system as function of K. Note that phase margin is defined based on open-loop frequency response.

Question 3 (9 points)

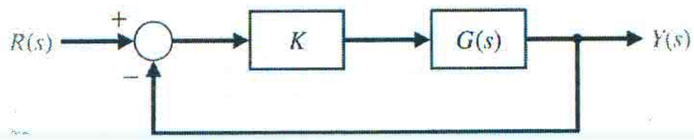
Consider a unity-feedback ($H(s) = 1$) control system with the following open-loop transfer function

$$G(s)H(s) = \frac{K(s+8)}{s(s+2)(s+4)}.$$

Determine the value range of K such that i) system is stable and ii) the steady state error for unit ramp input is less than 0.5.

Question 4 (12 points)

Consider the following closed-loop control system with proportional controller $G_c(s) = K$

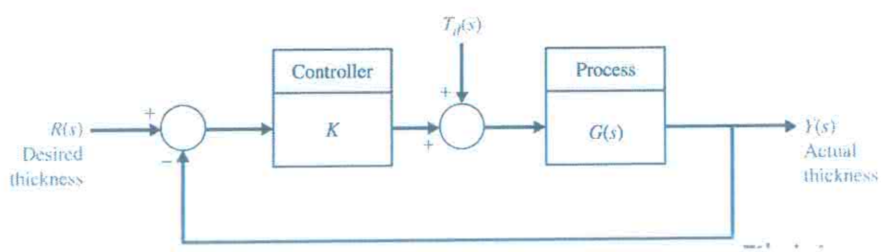


$$G(s) = \frac{1}{s(s+1)(s+9)}.$$

(1) Determine the desired area for the dominant closed-loop system poles in the s-plane such that i) the percent overshoot is less than 20%, ii) the settling time for $\delta = 2\%$ is less than 2 sec.
(2) Determine whether the system can satisfy both performance specifications in part (1) with properly selected K, by sketching the root locus of the system as K increases from 0 to ∞.
(3) Will your result in (2) change if a proportional and derivative (PD) controller with transfer function $G_c(s) = K(s+2)$ is used? Why?

Question 5 (8 points)

Consider the closed-loop control system with proportional controller given below



$$G(s) = \frac{1}{s(s+10)}.$$

Can we select proper values for gain K such that the following specifications are satisfied simultaneously? If yes, specify the value range for K.
i) the steady state response of the system to a unit step disturbance is less than 0.01.
ii) the phase margin of the system is greater than 45°.

Question 6 (6 points)

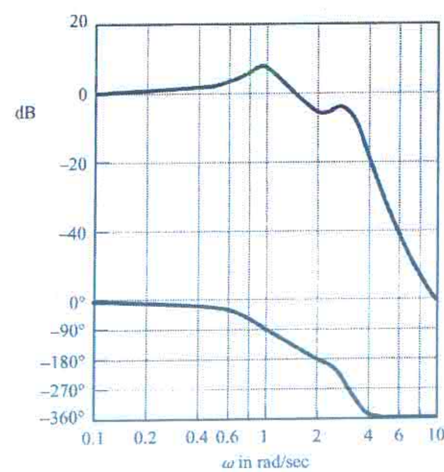
Apply the Nyquist stability criterion to determine the stability of a closed-loop control system with the following open-loop transfer function

$$G(s)H(s) = \frac{K(s+2)}{s(s-3)},$$

i.e. find the value range of K for which the system is stable.

Question 7 (4 points)

The Bode diagrams of the open-loop frequency response of a close-loop control system is given below.



(1) What is the approximate value of phase margin based on the given Bode diagrams?
(2) (**Bonus question 5 points**) Sketch the polar plot of the open-loop frequency response and determine the stability of the closed-loop system if the open loop system is known to be stable.

END

Transform Pairs

$f(t)$	$F(s)$
Unit impulse $\delta(t)$	1
Unit step $1(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{s^n}$
$t^n \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{1}{(s+a)^n}$
$t^n e^{-at} \quad (n = 1, 2, 3, \ldots)$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

(continues on next page)

Table 2–1 *(continued)*

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2}\, t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1,\ \ 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1,\ \ 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2}(\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega}(\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

Properties of Laplace Transforms

$$\mathscr{L}[Af(t)] = AF(s)$$

$$\mathscr{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$$

$$\mathscr{L}_\pm\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$$

$$\mathscr{L}_\pm\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$$

$$\mathscr{L}_\pm\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} \overset{(k-1)}{f}(0\pm)$$
$$\text{where } \overset{(k-1)}{f}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$$

$$\mathscr{L}_\pm\left[\int f(t)\,dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t)\,dt\right]_{t=0\pm}$$

$$\mathscr{L}_\pm\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^{n} \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$$

$$\mathscr{L}\left[\int_0^t f(t)\,dt\right] = \frac{F(s)}{s}$$

$$\int_0^\infty f(t)\,dt = \lim_{s\to 0} F(s) \quad \text{if } \int_0^\infty f(t)\,dt \text{ exists}$$

$$\mathscr{L}[e^{-at}f(t)] = F(s+a)$$

$$\mathscr{L}[f(t-\alpha)1(t-\alpha)] = e^{-\alpha s}F(s) \qquad \alpha \geq 0$$

$$\mathscr{L}[tf(t)] = -\frac{dF(s)}{ds}$$

$$\mathscr{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$$

$$\mathscr{L}[t^nf(t)] = (-1)^n \frac{d^n}{ds^n}F(s) \qquad (n = 1, 2, 3, \ldots)$$

$$\mathscr{L}\left[\frac{1}{t}f(t)\right] = \int_s^\infty F(s)\,ds \qquad \text{if } \lim_{t\to 0} \frac{1}{t}f(t) \text{ exists}$$

$$\mathscr{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$$

$$\mathscr{L}\left[\int_0^t f_1(t-\tau)f_2(\tau)\,d\tau\right] = F_1(s)F_2(s)$$

$$\mathscr{L}[f(t)g(t)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(p)G(s-p)\,dp$$

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