CHAPTER 8

B-8-1. Referring to Equation (3-37), we have

$$K_p = \frac{R_4(R_1C_1 + R_2C_2)}{R_3R_1C_2} = 39.42$$

$$T_i = R_1C_1 + R_2C_2 = 3.077$$

$$T_d = \frac{R_1C_1R_2C_2}{R_1C_1 + R_2C_2} = 0.7692$$

First, notice that

$$(R_1C_1) + (R_2C_2) = 3.077$$

$$(R_1C_1)(R_2C_2) = 0.7692 \times 3.077 = 2.3668$$

Hence we obtain

$$R_1C_1 = 1.5385, \qquad R_2C_2 = 1.5385$$

Since we have six unknown variables and three equations, we can choose three variables arbitrarily. So we choose $C_1 = C_2 = 10\ \mu F$ and one remaining variable later. Then we get

$$R_1 = R_2 = 153.85\ k\Omega$$

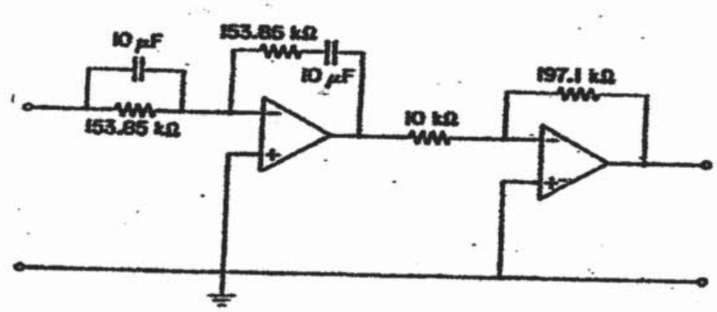
From the equation for K_p, we have

$$\frac{R_4}{R_3}\ \frac{R_1C_1 + R_2C_2}{R_1C_2} = 39.42$$

or

$$\frac{R_4}{R_3} = 39.42 \times \frac{1}{2} = 19.71$$

We now choose arbitrarily $R_3 = 10\ k\Omega$. Then, $R_4 = 197.1\ k\Omega$. The PID controller obtained is shown below.



B-8-2. For the reference input, the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{2K(as+1)(bs+1)(s+2)}{s(s+1)(s+10) + 2K(as+1)(bs+1)(s+2)}$$

Notice that the numerator is a polynomial in s of degree 3 and the denominator is also a polynomial in s of degree 3. In such a case, it is advisable to reduce the degree of numerator polynomial by one by choosing a = 0. Then the closed-loop transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{2K(bs+1)(s+2)}{s(s+1)(s+10) + 2K(bs+1)(s+2)}$$

Let us choose the value of b to be 0.5 so that the zero of the controller is located at s = -2. Then, the controller transfer function $G_c(s)$ becomes

$$G_c(s) = \frac{K(0.5s+1)}{s} = \frac{0.5K(s+2)}{s}$$

Then

$$\frac{C(s)}{R(s)} = \frac{K(s+2)^2}{s(s+1)(s+10) + K(s+2)^2}$$

The closed-loop transfer function for the disturbance input becomes as

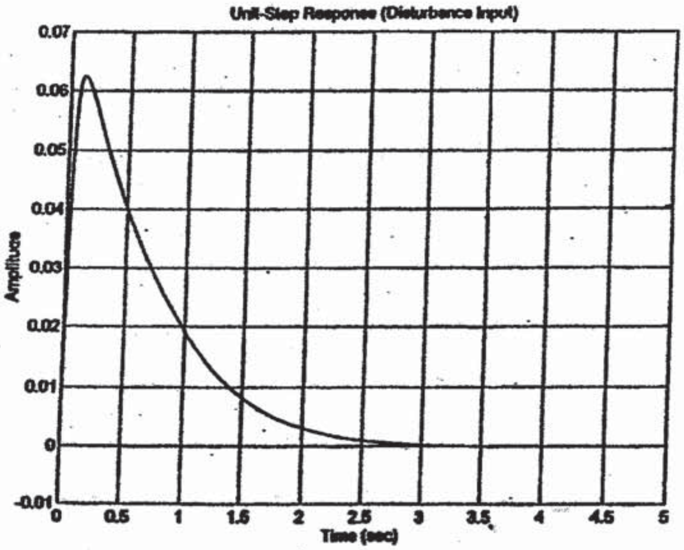
$$\frac{C_D(s)}{D(s)} = \frac{2s(s+2)}{s(s+1)(s+10) + K(s+2)^2}$$

The requirement on the response to the step disturbance input is that the response should attenuate rapidly. Let us interpret this requirement to be the settling time of 2 sec. By a simple MATLAB trial-and-error approach on the value of K, we find that K = 20 gives the settling time to be 2 sec. So we choose K = 20. With K = 20, the closed-loop transfer function $C_D(s)/D(s)$ for the disturbance input becomes

$$\frac{C_D(s)}{D(s)} = \frac{2s^2 + 4s}{s^3 + 31s^2 + 90s + 80}$$

The following MATLAB program produces the response to the unit-step disturbance input. The resulting response curve is shown on the next page.

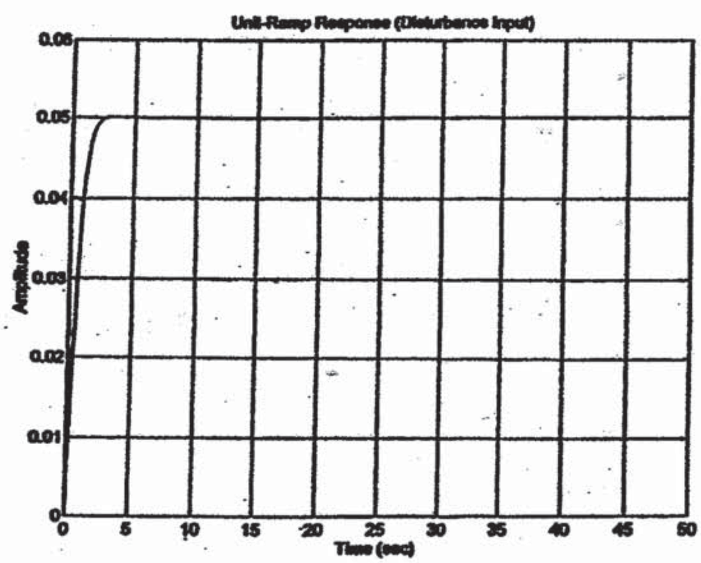
```
% ***** Unit-step response (Disturbance input) *****

numd = [0  2  4  0];
dend = [1  31  90  80];
step(numd,dend)
grid
title('Unit-Step Response (Disturbance Input)')
```



This response curve corresponds to the settling time of 2 sec. This may not be obvious. Therefore, we plot the response to the unit-ramp disturbance input. The following MATLAB is used to obtain the unit-ramp response.

```
% ***** Unit-ramp response (Disturbance input) *****

numdd = [0  0  2  4  0];
dendd = [1  31  90  80  0];
step(numdd,dendd)
grid
title('Unit-Ramp Response (Disturbance Input)')
```

The resulting response curve is shown below.



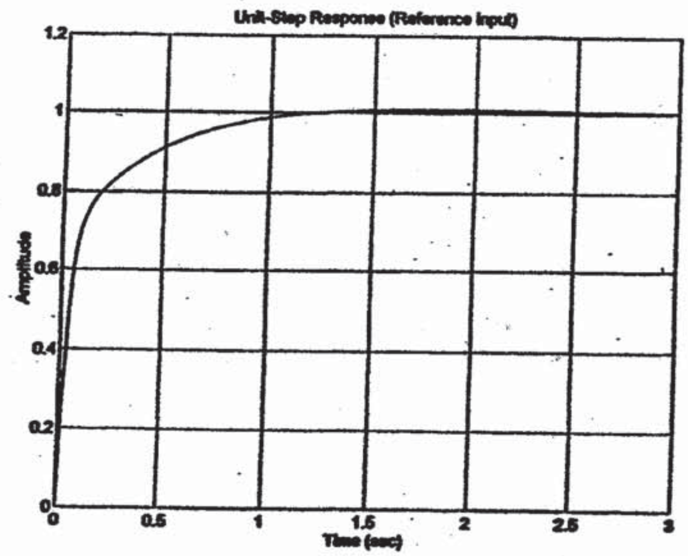
The settling time can be seen to be approximately 2 sec.

For the reference input, the closed-loop transfer function with K = 20 is

$$\frac{C(s)}{R(s)} = \frac{20s^2 + 80s + 80}{s^3 + 31s^2 + 90s + 80}$$

The MATLAB program below produces the unit-step response curve for the reference input, as shown below.

```
% ***** Unit-step response (Reference input) *****

numr = [0  20  80  80];
denr = [1  31  90  80];
step(numr,denr)
grid
title('Unit-Step Response (Reference Input)')
```



From this plot, we see that the settling time for the reference input is 2 sec. The closed-loop poles for the system are shown in the MATLAB output shown below.

```
roots(denr)

ans =

 -27.8742
  -1.5629 + 0.6537i
  -1.5629 - 0.6537i
```

The designed controller is

$$G_c(s) = \frac{20(0.5s+1)}{s} = \frac{10(s+2)}{s}$$

B-8-3. **The closed-loop transfer function of the system shown in Figure** 8-72(a) is

$$\frac{C(s)}{R(s)} = \frac{K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)}{1+K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)} \tag{1}$$

The closed-loop transfer function of the system shown in Figure 8-72(b) can **be obtained as follows: Define the input to the block $G_p(s)$ as U(s). Then,**

$$U(s) = K_p(1+T_d s)R(s) + \frac{K_p}{T_i s}\left[R(s)-C(s)\right] - K_p(1+T_d s)C(s)$$

Also, we have

$$C(s) = G_p(s)\,U(s)$$

Hence

$$\frac{C(s)}{G_p(s)} = K_p\left(1+\frac{1}{T_i s}+T_d s\right)R(s) - K_p\left(1+\frac{1}{T_i s}+T_d s\right)C(s)$$

from which we obtain

$$\frac{C(s)}{R(s)} = \frac{K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)}{1+K_p\left(1+\frac{1}{T_i s}+T_d s\right)G_p(s)}$$

This last equation is the same as Equation (1). Thus, the two systems are equivalent.

B-8-4. **We shall first obtain the closed-loop transfer function C(s)/R(s) of the I-PD controlled system. In the absence of the disturbance D(s), the minor loop has the following transfer function:**

$$\frac{C(s)}{U(s)} = \frac{39.42}{s(s+1)(s+5)+39.42(1+0.7692s)}$$

where U(s) is the input to the minor loop. The open-loop transfer function G(s) of the system is

$$G(s) = \frac{1}{3.077s}\left[\frac{39.42}{s(s+1)(s+5)+39.42(1+0.7692s)}\right]$$

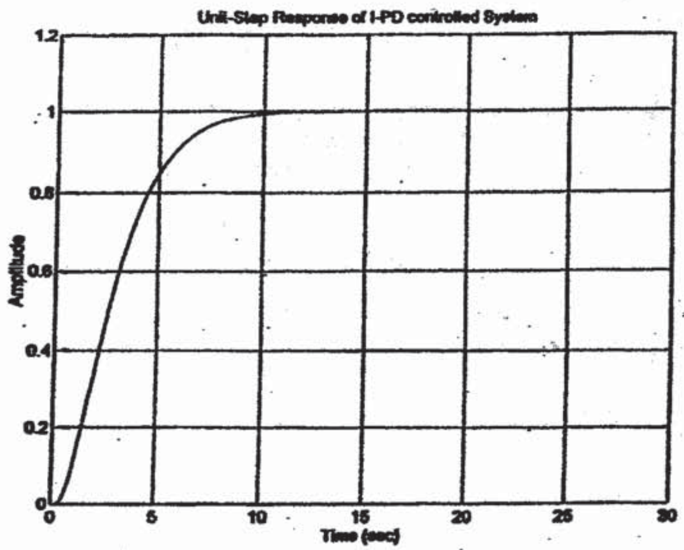
$$= \frac{12.8112}{s^4+6s^3+35.3219\,s^2+39.42s}$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{12.8112}{s^4 + 6s^3 + 35.3219s^2 + 39.42s + 12.8112}$$

The following MATLAB program produces the unit-step response. The resulting response curve is shown below.

```
% ***** Unit-step response *****

num = [0  0  0  0  12.8112];
den = [1  6  35.3219  39.42  12.8112];
t = 0:0.05:30;
step(num,den,t)
grid
title('Unit-Step Response of I-PD controlled System')
```



Notice that the response is slow but shows no overshoot. The closed-loop poles are shown in the following MATLAB output.

```
roots(den)

ans =

  -2.3514 + 4.8215i
  -2.3514 - 4.8215i
  -0.6486 + 0.1568i
  -0.6486 - 0.1568i
```

Since the dominant closed-loop poles are located very close to the $j\omega$ axis, the response speed is very slow compared with that of the closed-loop system shown in Figure 8-73(a).

B-8-5. For the PID controlled system shown in Figure 8-73(a), the closed-loop transfer function between the output and the disturbance input is

$$\frac{C(s)}{D(s)} = \frac{s}{s^2(s+1)(s+5) + 39.42(s + 0.3250 + 0.7692 s^2)}$$

$$= \frac{s}{s^4 + 6s^3 + 35.3219 s^2 + 39.42 s + 12.8112}$$

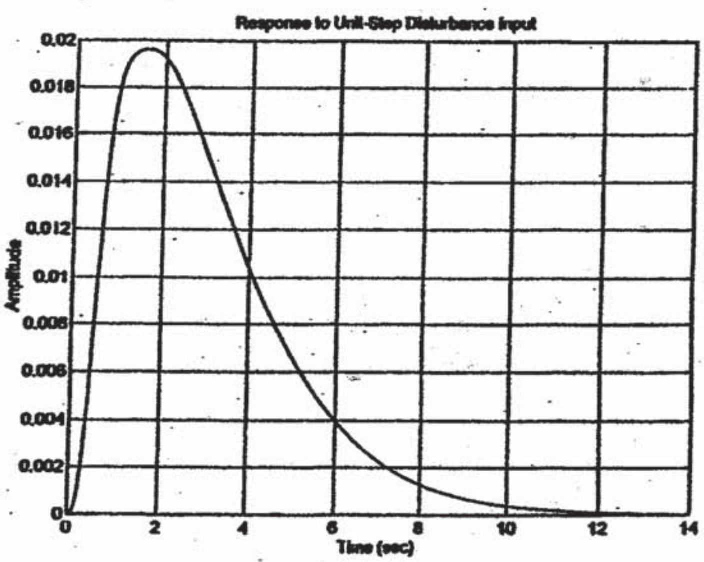
For the I-PD controlled system shown in Figure 8-73(b), the closed-loop transfer function between the output and the disturbance input can be obtained as follows:

$$\frac{C(s)}{D(s)} = \frac{s}{s^2(s+1)(s+5) + 39.42(s + 0.3250 + 0.7692 s^2)}$$

$$= \frac{s}{s^4 + 6s^3 + 35.3219 s^2 + 39.42 s + 12.8112}$$

Since the two closed-loop transfer functions are identical, we get the same unit-step response curves for the two systems. The following MATLAB program produces the response to the unit-step disturbance input. The resulting response curve is shown below.

```
% ***** Unit-step response *****

num = [0  0  0  1  0];
den = [1  6  35.3219  39.42  12.8112];
step(num,den)
grid
title('Response to Unit-Step Disturbance Input')
```



The closed-loop transfer function C(s)/R(s) of the system of Figure 8-73(a) is obtained as follows: [We assume that D(s) = 0.]

$$\frac{C(s)}{R(s)} = \frac{39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right)\frac{1}{s(s+1)(s+5)}}{1 + 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right)\frac{1}{s(s+1)(s+5)}}$$

$$= \frac{30.3215s^2 + 39.42s + 12.8112}{s^4 + 6s^3 + 35.32186s^2 + 39.42s + 12.8112}$$

The closed-loop transfer function C(s)/R(s) of the system of Figure 8-73(b) is obtained as follows: [We assume that D(s) = 0.]

$$\frac{C(s)}{R(s)} = \frac{\frac{39.42}{s(s+1)(s+5) + 39.42(1+0.7692s)}\,\frac{1}{3.077s}}{1 + \frac{39.42}{s(s+1)(s+5) + 39.42(1+0.7692s)}\,\frac{1}{3.077s}}$$

$$= \frac{12.8112}{s^4 + 6s^3 + 35.32186s^2 + 39.42s + 12.8112}$$

Note that the characteristic equation (denominator) for both systems are the same, but the numerators are different.

B-8-6. For the reference input, the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

For the disturbance input,

$$\frac{C(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

For the noise input,

$$\frac{C(s)}{N(s)} = -\frac{G_1(s)\,G_2(s)H(s)}{1 + G_1(s)G_2(s)H(s)}$$

Notice that the characteristic equations for the three closed-loop transfer functions are the same;

$$1 + G_1(s)\,G_2(s)H(s) = 0$$

That is, the characteristic equation for this system is the same regardless of which input signal is chosen as input.

B-8-7. The closed-loop transfer function C(s)/R(s) for the reference input is

$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1 G_2 G_3}{1+G_2 G_3 H_2}}{1+\dfrac{G_1 G_2 G_3 H_1}{1+G_2 G_3 H_2}} = \frac{G_1 G_2 G_3}{1+G_2 G_3 H_2 + G_1 G_2 G_3 H_1}$$

The closed-loop transfer function C(s)/D(s) for the disturbance input is obtained as follows: Noting that the feedforward transfer function is $G_3(s)$ and the feedback transfer function is $[-G_1(s)H_1(s) - H_2(s)]G_2(s)$, and that the closed-loop system is a positive-feedback system, we have

$$\frac{C(s)}{D(s)} = \frac{G_3}{1 - G_3[-G_1 H_1 - H_2]G_2}$$

$$= \frac{G_3}{1+G_1 G_2 G_3 H_1 + G_2 G_3 H_2}$$

B-8-8. For the system shown in Figure 8-76 (b), the closed-loop transfer function for the disturbance input is

$$\frac{C(s)}{D(s)} = \frac{-K\,G(s)H(s)}{1+KG(s)H(s)}$$

To minimize the effect of disturbances, the adjustable gain K should be chosen as small as possible. Thus, the answer to the question is "no".

B-8-9.

(a)

$$\frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1+G_{c1} G_{c2} G_p}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+G_{c1} G_{c2} G_p}$$

$$\frac{Y(s)}{N(s)} = \frac{-G_{c1} G_{c2} G_p}{1+G_{c1} G_{c2} G_p}$$

Hence

$$G_{yr} = G_{c1} G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

If G_{yd} is given, then G_{yn} is fixed but G_{yr} is not fixed because G_{c1} is independent of G_{yd}. Thus, two closed-loop transfer functions among three closed-loop transfer functions G_{yr}, G_{yd}, and G_{yn} are independent. Hence, the system is a two-degrees-of-freedom system.

(b)

$$\frac{Y(s)}{R(s)} = \frac{G_{c1} G_{c2} G_p}{1 + G_{c2} G_p}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c2} G_p}$$

$$\frac{Y(s)}{N(s)} = \frac{-G_{c2} G_p}{1 + G_{c2} G_p}$$

Hence

$$G_{yr} = G_{c1} G_{c2} G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

If G_{yd} is given, then G_{yn} is fixed but G_{yr} is not fixed because $G_{c1}G_{c2}$ is independent of G_{yd}. Thus, two closed-loop transfer functions among three closed-loop transfer functions G_{yr}, G_{yd}, and G_{yn} are independent. Hence, the system is a two-degrees-of-freedom system.

(c)

$$\frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1 + G_{c2} G_p}$$

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c2} G_p}$$

$$\frac{Y(s)}{N(s)} = \frac{-G_{c2} G_p}{1 + G_{c2} G_p}$$

Hence

$$G_{yr} = G_{c1} G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

If G_{yd} is given, then G_{yn} is fixed. G_{yr} is not fixed because G_{c1} is independent of G_{yd}. Thus, the system is a two-degrees-of-freedom system.

B-8-10. Define the input signal to box G_{c3} as A(s). Then, for D(s) = 0 and N(s) = 0, we have

$$A(s) = G_{c2} R(s) + G_{c1} [R(s) - Y(s)] - G_1 G_{c3} A(s)$$

$$Y(s) = G_{c3} G_1 G_2 A(s)$$

Eliminating A(s) from the above two equations, we get

$$Y(s) = G_{c3} G_1 G_2 \frac{(G_{c1} + G_{c2}) R(s) - G_{c1} Y(s)}{1 + G_{c3} G_1}$$

or

$$(1 + G_{c3} G_1 + G_{c1} G_{c3} G_1 G_2) Y(s) = G_{c3} G_1 G_2 (G_{c1} + G_{c2}) R(s)$$

Hence,

$$\frac{Y(s)}{R(s)} = \frac{(G_{c1} + G_{c2}) G_{c3} G_1 G_2}{1 + G_{c3} G_1 + G_{c1} G_{c3} G_1 G_2} \qquad (1)$$

To find Y(s)/D(s), we may proceed as follows. For R(s) = 0 and N(s) = 0, we have

$$A(s) = G_{c1} [-Y(s)] - G_1 [D(s) + G_{c3} A(s)]$$

$$Y(s) = G_1 G_2 [D(s) + G_{c3} A(s)]$$

Hence

$$Y(s) = G_1 G_2 \left[D(s) + G_{c3} \frac{-G_{c1} Y(s) - G_1 D(s)}{1 + G_{c3} G_1} \right]$$

Simplifying, we have

$$(1 + G_{c3} G_1 + G_{c1} G_{c3} G_1 G_2) Y(s) = G_1 G_2 D(s)$$

or

$$\frac{Y(s)}{D(s)} = \frac{G_1 G_2}{1 + G_{c3} G_1 + G_{c1} G_{c2} G_1 G_2} \qquad (2)$$

Next, we shall find Y(s)/N(s). For R(s) = 0 and D(s) = 0, we have

$$A(s) = -G_{c1} [Y(s) + N(s)] - G_{c3} G_1 A(s)$$

$$Y(s) = G_{c3} G_1 G_2 A(s)$$

Therefore,

$$Y(s) = G_{c3} G_1 G_2 \frac{-G_{c1} Y(s) - G_{c1} N(s)}{1 + G_{c3} G_1}$$

or

$$\left(1+G_{c3}G_1+G_{c1}G_{c3}G_1G_2\right)Y(s) = -G_{c1}G_{c3}G_1G_2N(s)$$

Hence,

$$\frac{Y(s)}{N(s)} = \frac{-G_{c1}G_{c3}G_1G_2}{1+G_{c3}G_1+G_{c1}G_{c3}G_1G_2} \tag{3}$$

From Equations (1), (2), and (3), we get

$$G_{yn} = -G_{c1}G_{c3}G_{yd}$$

$$G_{yr} = -G_{yn} + G_{c2}G_{c3}G_{yd}$$

If G_{yd} is given, G_{yn} is independent of G_{yd} because $G_{c1}G_{c3}$ is independent of G_{yd}. G_{yr} is independent of G_{yd} and G_{yn} because $G_{c2}G_{c3}$ is independent of G_{yn} and G_{yd}. Hence, all three closed-loop transfer functions G_{yr}, G_{yd}, and G_{yn} are independent. Hence, the system is a three-degrees-of-freedom system.

B-8-11. The open-loop transfer function of the system is

$$G(s) = \frac{K(s+a)^2}{s}\,\frac{1.2}{(0.36s+1)(s+1)(1.2s+1)}$$

$$= \frac{1.2Ks^2+2.4Kas+1.2Ka^2}{0.36s^4+1.86s^3+2.5s^2+s}$$

Hence, the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{1.2Ks^2+2.4Kas+1.2Ka^2}{0.36s^4+1.86s^3+(2.5+1.2K)s^2+(1+2.4Ka)s+1.2Ka^2}$$

The requirement in this problem is that the maximum overshoot in the unit-step response is that

$$M_p < 0.1\ , \qquad M_p > 0.02$$

where M_p is the maximum overshoot. In terms of the output y to the unit-step input,

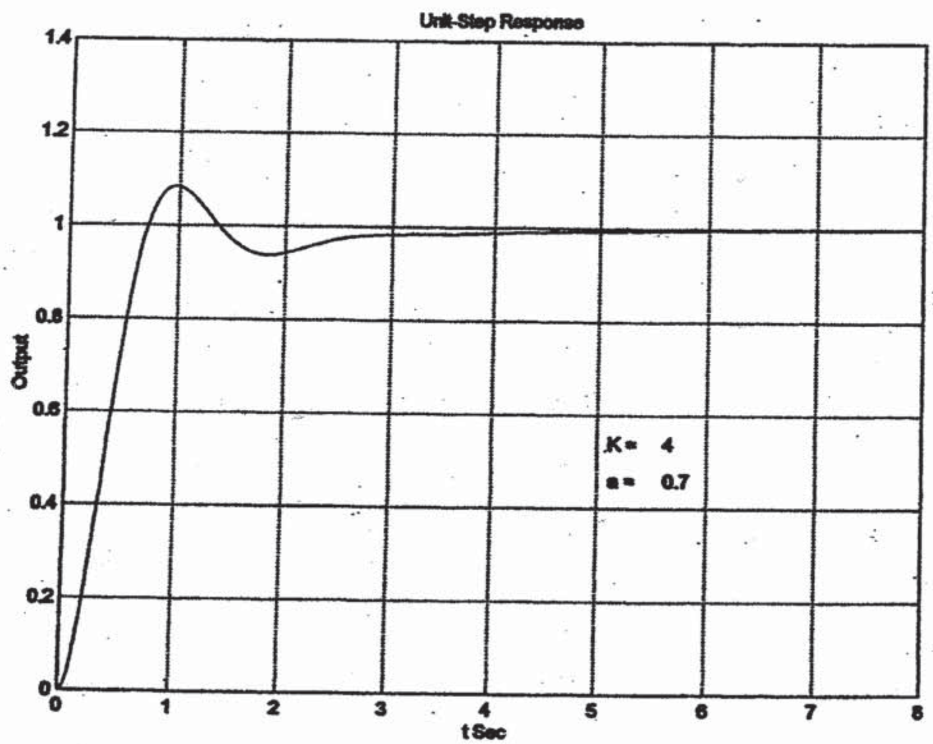
$$m < 1.1\ , \qquad m > 1.02$$

where

$$m = max(y)$$

A possible MATLAB program to obtain a set of the values of K and a that satisfies the requirement is given on the next page. The resulting unit-step response curve is shown also on the next page.

```
% ***** Search of K and a Values for 0.02<Mp<0.10 *****

t = 0:0.01:8;
for K = 4:-0.05:1;
    for a = 4:-0.05:0.4;
        num = [0  0  1.2*K  2.4*K*a  1.2*K*a^2];
        den = [0.36  1.86  2.5+1.2*K  1+2.4*K*a  1.2*K*a^2];
          y = step(num,den,t);
          m = max(y);
          if m < 1.1 & m > 1.02
          break;
        end
        end
    if m < 1.1 & m > 1.02
    break;
 end
 end
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('Output')
KK = num2str(K);
aa = num2str(a);
text(5.1,0.54,'K ='), text(5.6,0.54,KK)
text(5.1,0.46, 'a ='), text(5.6,0.46,aa)
sol = [K  a  m]

sol =

  4.0000   0.7000   1.0846
```



The selected set of K and a is

$$K = 4 \ , \quad a = 0.7$$

The maximum overshoot is 8.46 %.

B-8-12. The feedforward transfer function is

$$G(s) = \frac{1.2K(s+a)^2}{s(0.3s+1)(s+1)(1.2s+1)}$$

The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{1.2Ks^2 + 2.4Kas + 1.2Ka^2}{0.36s^4 + 1.86s^3 + (2.5+1.2K)s^2 + (1+2.4Ka)s + 1.2Ka^2}$$

The requirements in this problem are

$$1.03 < m < 1.08 \ , \quad t_s < 2 \text{ sec}$$

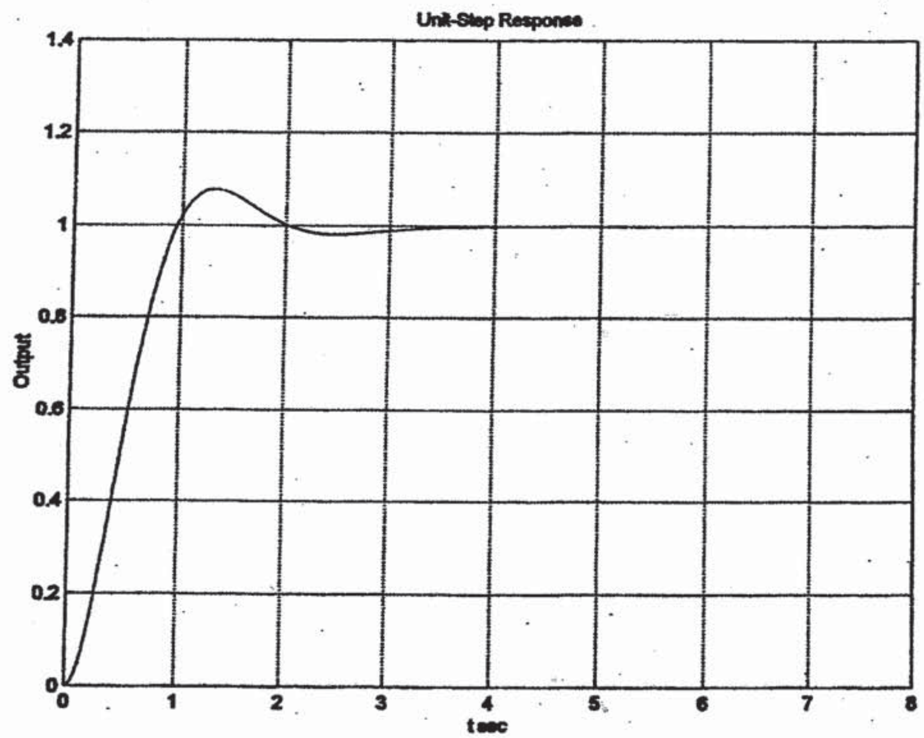
where m = maximum output. The search region is

$$2 \le K \le 4 \ , \quad 0.5 \le a \le 3$$

The step size is 0.05 for both K and a. A MATLAB program to obtain the first set of K and a that satisfies the requirements is shown below.

```
% ***** Search of K and a Values for 0.03 < Mp < 0.08 and ts < 2 sec *****

t = 0:0.01:8;
for K = 4:-0.05:2;
   for a = 3:-0.05:0.5;
      num = [0  0  1.2*K  2.4*K*a  1.2*K*a^2];
      den = [0.36  1.86  2.5+1.2*K  1+2.4*K*a  1.2*K*a^2];
        y = step(num,den,t);
        m = max(y);
        s = 801; while y(s) > 0.98 & y(s) < 1.02;
        s = s-1; end;
        ts = (s-1)*0.01;
       if m < 1.08 & m > 1.03 & ts < 2.0
       break;
       end
       end
   if m < 1.08 & m > 1.03 & ts < 2.0
   break
   end
   end
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t sec')
ylabel('Output')
solution = [K  a  m  ts]

solution =

   2.6000   0.8500   1.0774   1.8400
```

The first set of K and a that satisfies the requirements is K = 2.6 and a = 0.85. The maximum overshoot M_p and settling time t_s are 7.74 % and 1.84 sec, respectively. The response curve with K = 2.6 and a = 0.85 is shown below.



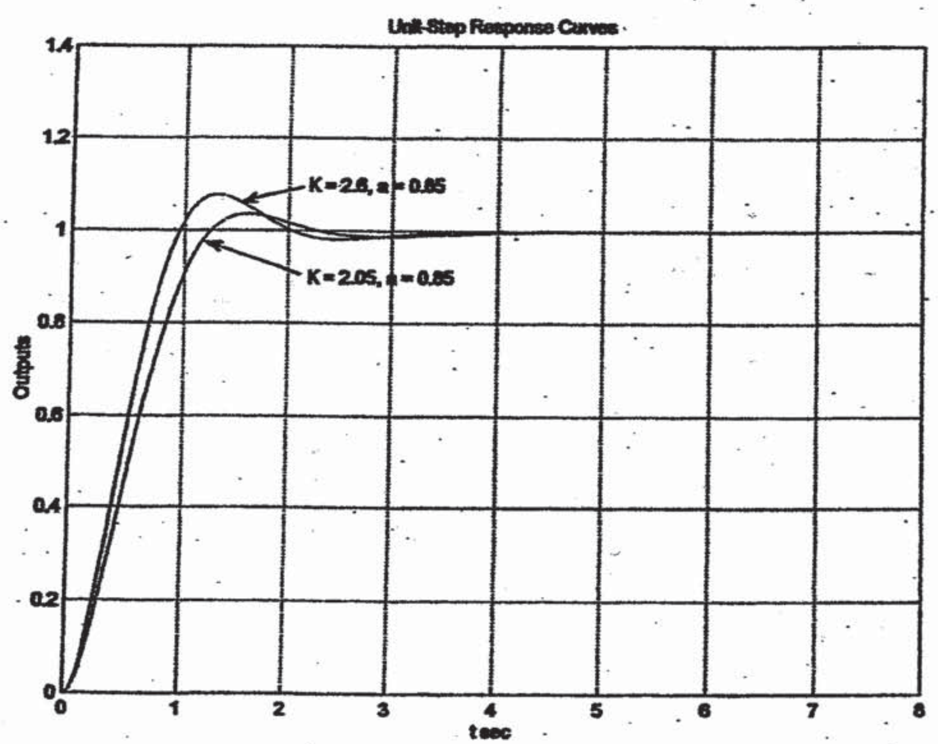
Next, we shall obtain all possible sets of K and a that satisfy the requirements. The following MATLAB program produces the desired result.

```
% ***** Search of all possible Sets of K and a Values for 0.03 < Mp < 0.08
% and ts < 2 sec *****

t = 0:0.01:8;
k = 0;
for i = 1:41;
    K(i) = 4.05 - i*0.05;
       for j = 1:51;
       a(j) = 3.05 - j*0.05;
       num = [0  0  1.2*K(i)  2.4*K(i)*a(j)  1.2*K(i)*a(j)*a(j)];
       den = [0.36  1.86  2.5+1.2*K(i)  1+2.4*K(i)*a(j)  1.2*K(i)*a(j)*a(j)];
         y = step(num,den,t);
        m = max(y);
         s = 801; while y(s) > 0.98 & y(s) < 1.02;
         s = s-1; end;
        ts = (s-1)*0.01;
       if m < 1.08 & m > 1.03 & ts < 2.0
       k = k+1;
       table(k,:) = [K(i)  a(j)  m  ts];
       end
    end
 end
table(k,:) = [K(i)  a(j)  m  ts]
```

```
table =

    2.6000    0.8500    1.0774    1.8400
    2.5500    0.8500    1.0737    1.8500
    2.5000    0.8500    1.0700    1.8600
    2.4500    0.8500    1.0662    1.8800
    2.4000    0.8500    1.0624    1.8900
    2.3500    0.8500    1.0585    1.9000
    2.3000    0.8500    1.0546    1.9100
    2.2500    0.8500    1.0507    1.9200
    2.2000    0.8500    1.0468    1.9300
    2.1500    0.8500    1.0428    1.9400
    2.1000    0.8500    1.0388    1.9400
    2.0500    0.8500    1.0348    1.9400
    2.0000    0.5000    0.9552    8.0000

sorttable = sortrows(table,3)

sorttable =

    2.0000    0.5000    0.9552    8.0000
    2.0500    0.8500    1.0348    1.9400
    2.1000    0.8500    1.0388    1.9400
    2.1500    0.8500    1.0428    1.9400
    2.2000    0.8500    1.0468    1.9300
    2.2500    0.8500    1.0507    1.9200
    2.3000    0.8500    1.0546    1.9100
    2.3500    0.8500    1.0585    1.9000
    2.4000    0.8500    1.0624    1.8900
    2.4500    0.8500    1.0662    1.8800
    2.5000    0.8500    1.0700    1.8600
    2.5500    0.8500    1.0737    1.8500
    2.6000    0.8500    1.0774    1.8400

K = sorttable(13,1)

K =

    2.6000

a = sorttable(13,2)

a =

    0.8500

num = [0  0  1.2*K  2.4*K*a  1.2*K*a^2];
den = [0.36  1.86  2.5+1.2*K  1+2.4*K*a  1.2*K*a^2];
y = step(num,den,t);
plot(t,y)
grid
hold
Current plot held
```

```
K = sorttable(2,1)

K =

   2.0500

a = sorttable(2,2)

a =

   0.8500

num = [0  0  1.2*K  2.4*K*a  1.2*K*a^2];
den = [0.36  1.86  2.5+1.2*K  1+2.4*K*a  1.2*K*a^2];
y = step(num,den,t);
plot(t,y)
title('Unit-Step Response Curves')
xlabel('t sec')
ylabel('Outputs')
text(2.2,1.1,'K = 2.6, a = 0.85')
text(2.2,0.9,'K = 2.05, a = 0.85')
```

There are 12 sets of the values of K and a that satisfy the requirements. All sets produces similar response curves. The best choice of the set depends on the system objective. If a small maximum overshoot is desired, then K = 2.05 and a = 0.85 will be the best choice. If the shorter settling time is more important than a small maximum overshoot, then K = 2.6 and a = 0.85 will be the best choice. The unit-step response curves for the two cases are shown below.



B-8-13.

$$G_p(s) = \frac{3(s+5)}{s(s+1)(s^2+4s+13)}$$

The closed-loop transfer functions Y(s)/D(s) and Y(s)/R(s) are given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1+G_{c1}(s)G_p(s)} = \frac{G_p}{1+G_{c1}G_p}$$

and

$$\frac{Y(s)}{R(s)} = \frac{[G_{c1}(s)+G_{c2}(s)]G_p(s)}{1+G_{c1}(s)G_p(s)} = \frac{(G_{c1}+G_{c2})G_p}{1+G_{c1}G_p}$$

Assume that $G_{c1}(s)$ is a PID controller with a filter and has the following form:

$$G_{c1}(s) = \frac{K(s+a)^2}{s}\,\frac{s^2+4s+13}{(s+5)(s+27)}$$

The characteristic equation for the system is

$$1+G_{c1}G_p = 1 + \frac{K(s+a)^2}{s}\,\frac{s^2+4s+13}{(s+5)(s+27)}\,\frac{3(s+5)}{s(s+1)(s^2+4s+13)}$$

$$= 1 + \frac{3K(s+a)^2}{s^2(s+1)(s+27)}$$

With a trial-and-error search of K and a with MATLAB, we find a possible set of K and a as follows:

$$K = 58, \quad a = 1.4$$

With this chosen set of K and a, the controller $G_{c1}(s)$ becomes as follows:

$$G_{c1}(s) = \frac{58(s+1.4)^2}{s}\,\frac{s^2+4s+13}{(s+5)(s+27)}$$

$$= \frac{58s^2+162.4s+113.68}{s}\,\frac{s^2+4s+13}{(s+5)(s+27)}$$

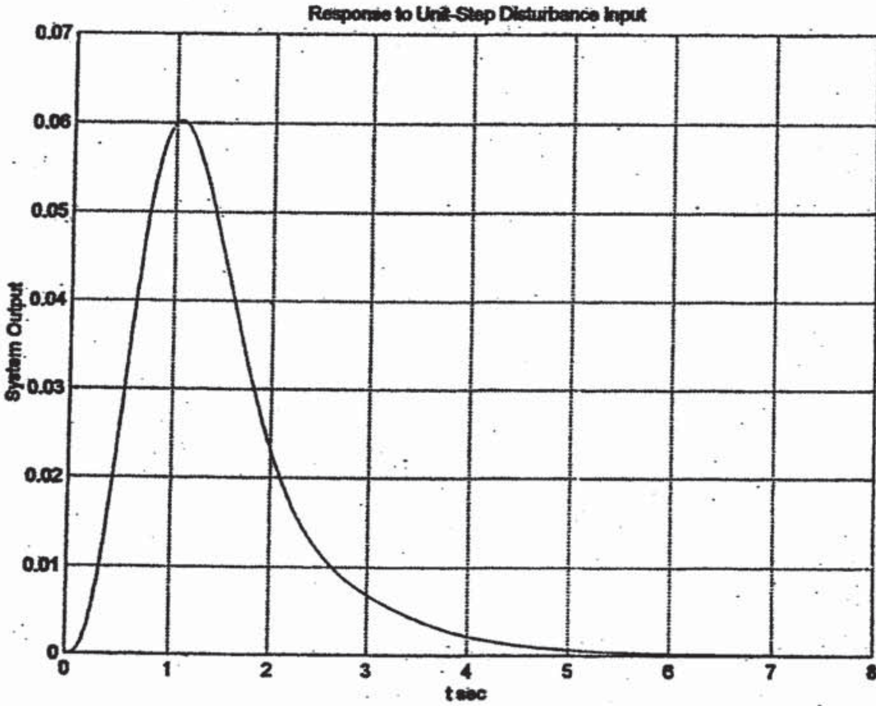
The closed-loop transfer function Y(s)/D(s) is obtained as

$$\frac{Y(s)}{D(s)} = \frac{\dfrac{3(s+5)}{s(s+1)(s^2+4s+13)}}{1+\dfrac{3\times 58(s+1.4)^2}{s^2(s+1)(s+27)}}$$

$$= \frac{3s^3+96s^2+405s}{(s^2+4s+13)(s^4+28s^3+201s^2+487.2s+341.04)}$$

The response curve when D(s) is a unit-step disturbance is shown in the next page.

Next, we consider the response to reference inputs. The closed-loop transfer function Y(s)/R(s) is



$$\frac{Y(s)}{R(s)} = \frac{(G_{c1} + G_{c2})G_p}{1 + G_{c1}G_p} = (G_{c1} + G_{c2})\frac{Y(s)}{D(s)}$$

Define $G_{c1}(s) + G_{c2}(s) = G_c(s)$. Then

$$\frac{Y(s)}{R(s)} = G_c\frac{Y(s)}{D(s)} = G_c\frac{G_p}{1+G_{c1}G_p}$$

$$= \frac{G_c\dfrac{3s^3+96s^2+405s}{s^2+4s+13}}{s^4+28s^3+201s^2+487.2s+341.04}$$

To satisfy the requirements on the responses to the ramp reference input and acceleration reference input, we use the zero-placement approach. That is, we choose the numerator of Y(s)/R(s) to be the sum of the last three terms of the denominator, or

$$G_c\frac{3s^3+96s^2+405s}{s^2+4s+13} = 201s^2+487.2s+341.04$$

from which we get

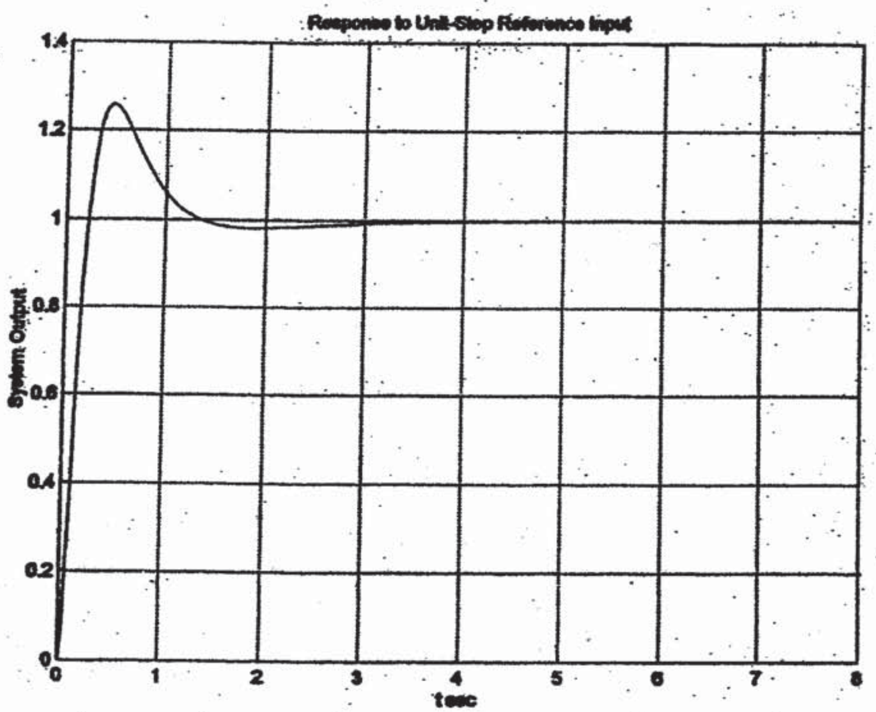
$$G_c(s) = \frac{(201s^2+487.2s+341.04)(s^2+4s+13)}{3(s^3+32s^2+135s)}$$

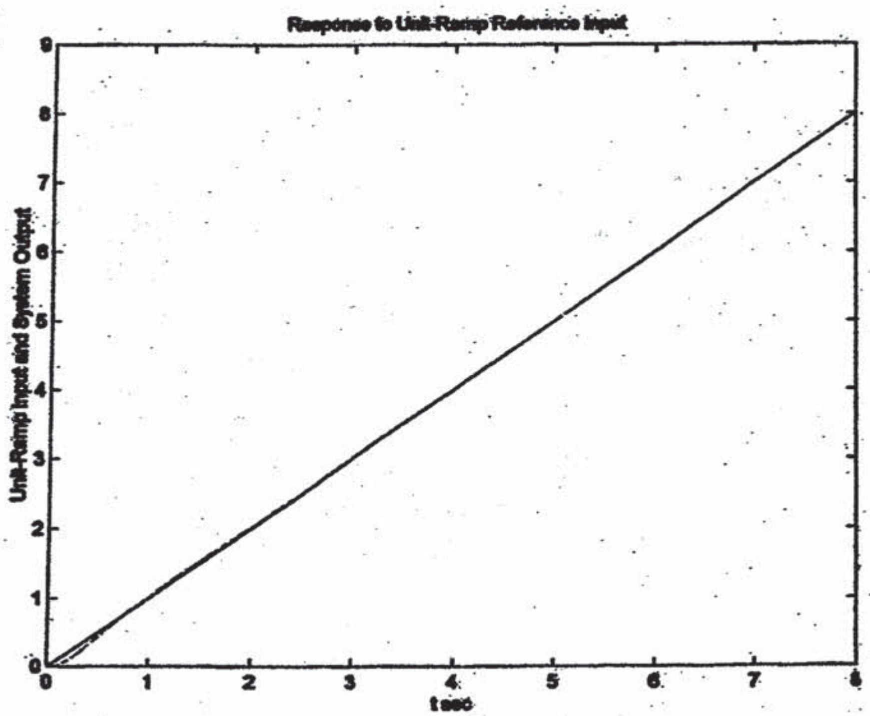
$$= \frac{67s^2+162.4s+113.68}{s}\,\frac{s^2+4s+13}{(s+5)(s+27)}$$

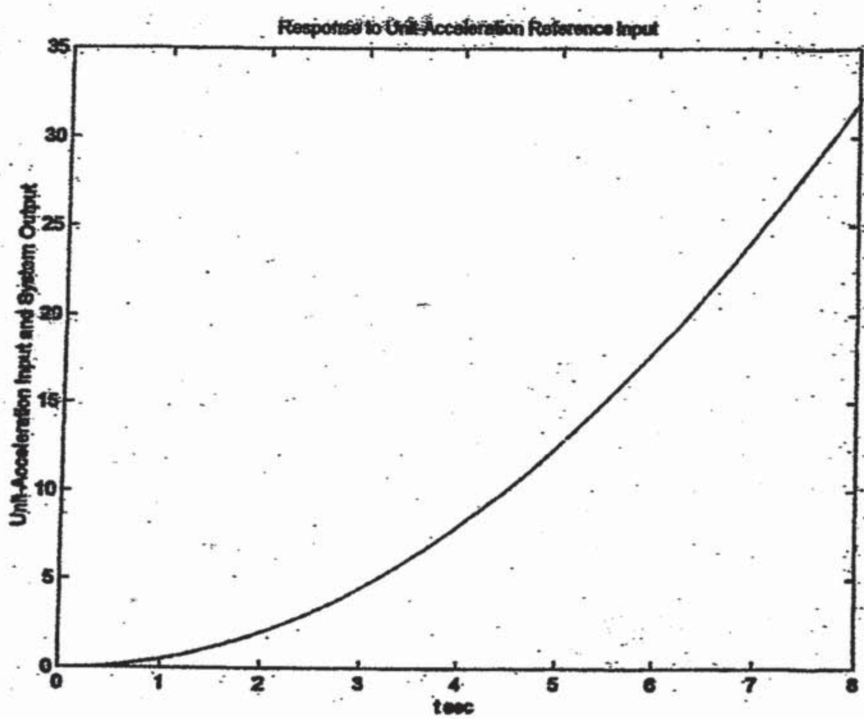
The closed-loop transfer function Y(s)/R(s) now becomes

$$\frac{Y(s)}{R(s)} = \frac{201 s^2 + 487.2 s + 341.04}{s^4 + 28 s^3 + 201 s^2 + 487.2 s + 341.04}$$

The response curves to the unit-step reference input, unit-ramp reference input, and unit-acceleration reference input are shown below and on the next page.







Notice that the maximum overshoot in the unit-step response is approximately 25 % and the settling time is approximately 1.25 sec. The steady-state errors in the ramp response and acceleration response are zero. Therefore, the designed controller $G_c(s)$ is satisfactory.

Finally, we determine $G_{c2}(s)$. Noting that

$$G_{c2}(s) = G_c(s) - G_{c1}(s)$$

where

$$G_c(s) = \frac{67s^2 + 162.4s + 113.68}{s} \frac{s^2+4s+13}{(s+5)(s+27)}$$

and

$$G_{c1}(s) = \frac{58s^2 + 162.4s + 113.68}{s} \frac{s^2+4s+13}{(s+5)(s+27)}$$

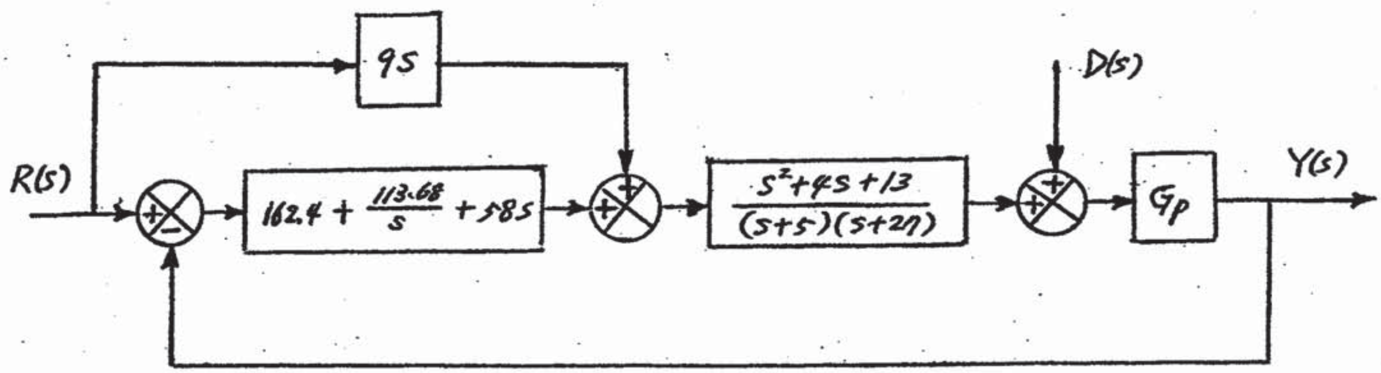
we have

$$G_{c2}(s) = 9s \frac{s^2+4s+13}{(s+5)(s+27)}$$

The block diagram of the designed system is shown on the next page. Note that

$$\frac{s^2+4s+13}{(s+5)(s+27)}$$

is a filter and is a part of the controller.



B-8-14.

$$G_p(s) = \frac{2(s+1)}{s(s+3)(s+5)}$$

The closed-loop transfer functions Y(s)/D(s) and Y(s)/R(s) are given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1+G_p(s)\left[G_{c1}(s)+G_{c2}(s)\right]} = \frac{G_p}{1+(G_{c1}+G_{c2})G_p}$$

and

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}(s)\,G_p(s)}{1+G_p(s)\left[G_{c1}(s)+G_{c2}(s)\right]} = \frac{G_{c1}G_p}{1+(G_{c1}+G_{c2})G_p}$$

Let us define $G_{c1} + G_{c2} = G_c$. Then

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+G_c G_p}$$

Let us assume that $G_c(s)$ is a PID controller and has the following form:

$$G_c(s) = \frac{K(s+a)^2}{s}$$

The characteristic equation for the system is

$$1+G_c G_p = 1 + \frac{K(s+a)^2}{s}\,\frac{2(s+1)}{s(s+3)(s+5)}$$

In what follows, we shall use the root-locus approach to determine the values of K and a. After trial-and-error analysis with MATLAB, we choose the dominant closed-loop poles to be at $s = -4 \pm j0.2$.

The angle deficiency at the desired closed-loop pole at $s = -4 + j0.2$ is obtined as follows:

$$-177.1376^\circ - 177.1376^\circ - 168.6901^\circ - 11.3099^\circ + 176.1859^\circ + 180^\circ$$
$$= -178.0893^\circ$$

(Note that the poles are at s = 0, s = 0, s = −3, s = −5 and the zero is at s = −1.) The double zero at s = −a must contribute 178.0893°. (Each zero must contribute 89.04465°.) By a simple calculation, we find

$$a = 4.0033$$

The controller $G_C(s)$ is then determined as

$$G_c(s) = \frac{K(s+4.0033)^2}{s}$$

The constant K must be determined by use of the magnitude condition. This condition is

$$\left| G_c(s) G_p(s) \right|_{s=-4+j4.2} = 1$$

Since

$$\left| \frac{K(s+4.0033)^2}{s} \cdot \frac{2(s+1)}{s(s+3)(s+5)} \right|_{s=-4+j4.2} = 1$$

we obtain

$$K = \left| \frac{s^2(s+3)(s+5)}{(s+4.0033)^2\, 2(s+1)} \right|_{s=-4+j4.2} = 69.3333$$

Hence

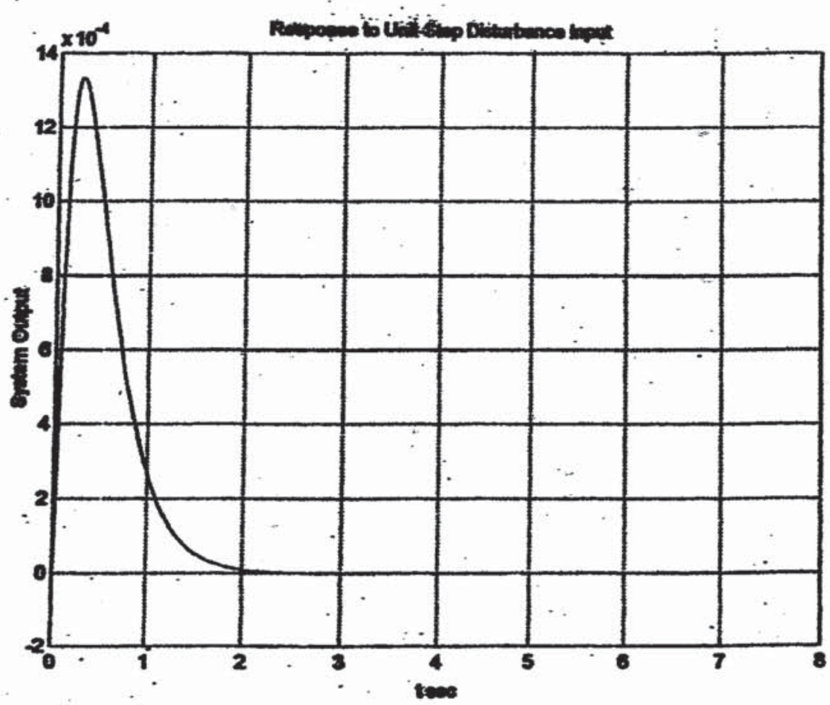
$$G_c(s) = \frac{69.3333\,(s+4.0033)^2}{s}$$

Then the closed-loop transfer function Y(s)/D(s) can be obtained as

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+G_c G_p} = \frac{\dfrac{2(s+1)}{s(s+3)(s+5)}}{1+\dfrac{69.3333\,(s+4.0033)^2}{s}\,\dfrac{2(s+1)}{s(s+3)(s+5)}}$$

$$= \frac{2s^2+2s}{s^4+146.6666\,s^3+1263.9146\,s^2+3332.5759\,s+2222.3279}$$

The response curve when D(s) is a unit-step disturbance is shown below.



Next, we consider the responses to reference inputs. The closed-loop transfer function Y(s)/R(s) is

$$\frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1+(G_{c1}+G_{c2})G_p} = G_{c1}\frac{Y(s)}{D(s)}$$

$$= \frac{(2s^2+2s)G_{c1}}{s^4+146.6666s^3+1263.9146s^2+3332.5759s+2222.3279}$$

To satisfy the requirements on the responses to the ramp reference input and acceleration reference input, we use the zero-placement approach. That is, we choose the numerator of Y(s)/R(s) to be the sum of the last three terms of the denominator, or

$$(2s^2+2s)G_{c1} = 1263.9146\,s^2+3332.5759\,s+2222.3279$$

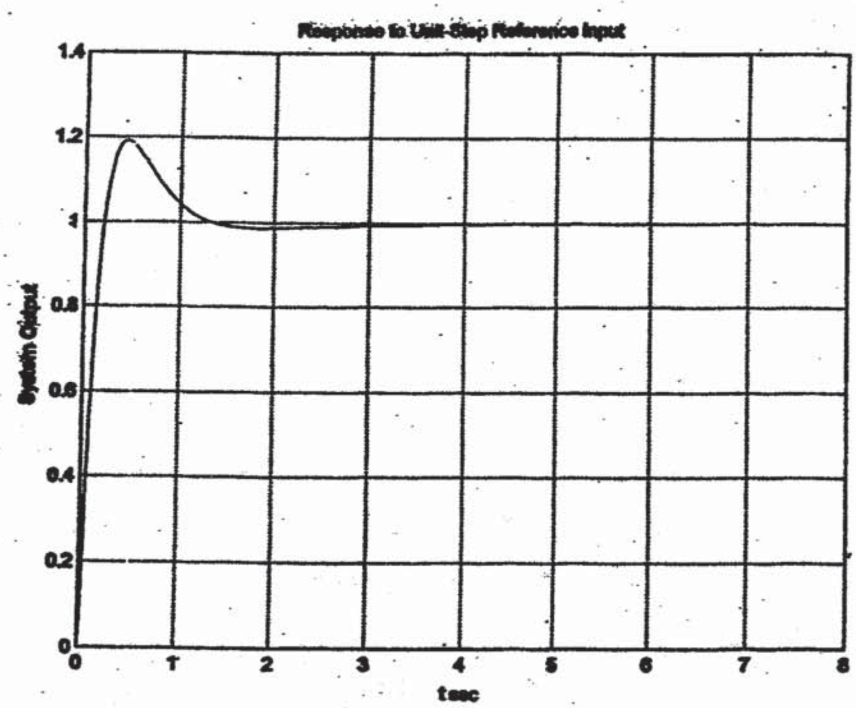
from which we get

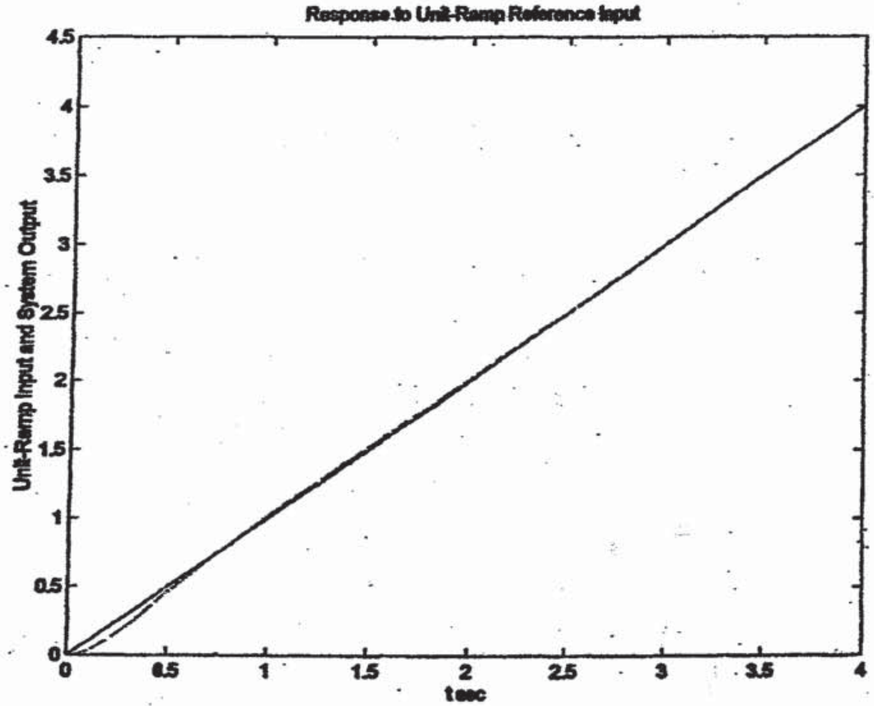
$$G_{c1} = \frac{631.9573s^2+1666.2880s+1111.1640}{s(s+1)}$$

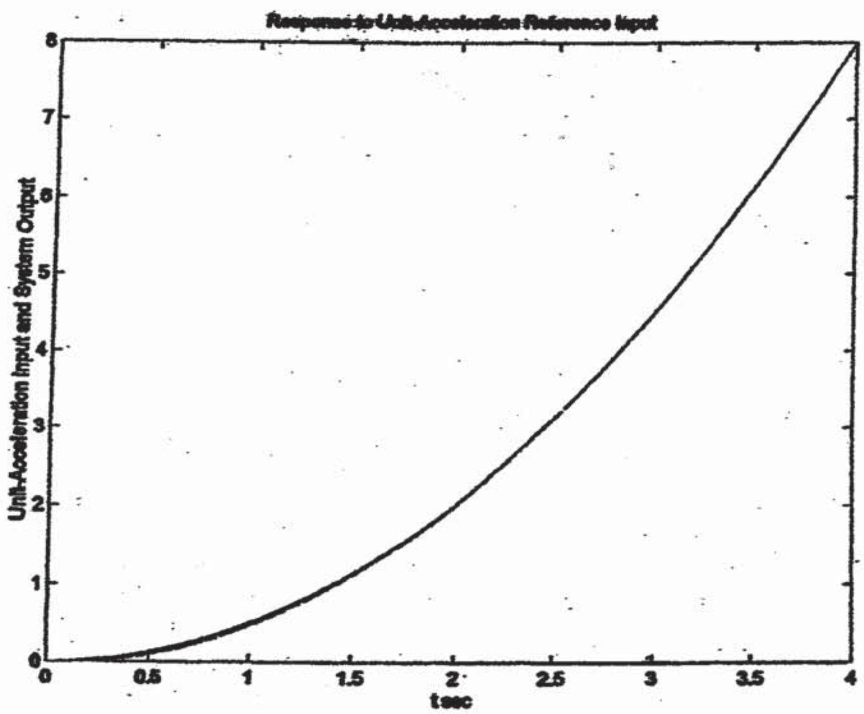
Hence, the closed-loop transfer function Y(s)/R(s) becomes as

$$\frac{Y(s)}{R(s)} = \frac{1263.9146\,s^2+3332.5759\,s+2222.3279}{s^4+146.6666s^3+1263.9146\,s^2+3332.5759\,s+2222.3279}$$

The response curves to the unit-step reference input, unit-ramp reference input, and unit-acceleration reference input are shown below and on the next page.







The maximum overshoot in the unit-step response is approximately 19 % and the settling time is approximately 1.3 sec (2% criterion) or 1.0 sec (5% criterion). The steady-state errors in the ramp response and acceleration response are zero. Therefore, the designed controller $G_c(s)$ is satisfactory.

Finally, we determine $G_{c2}(s)$. Noting that

where
$$G_{c2}(s) = G_c(s) - G_{c1}(s)$$

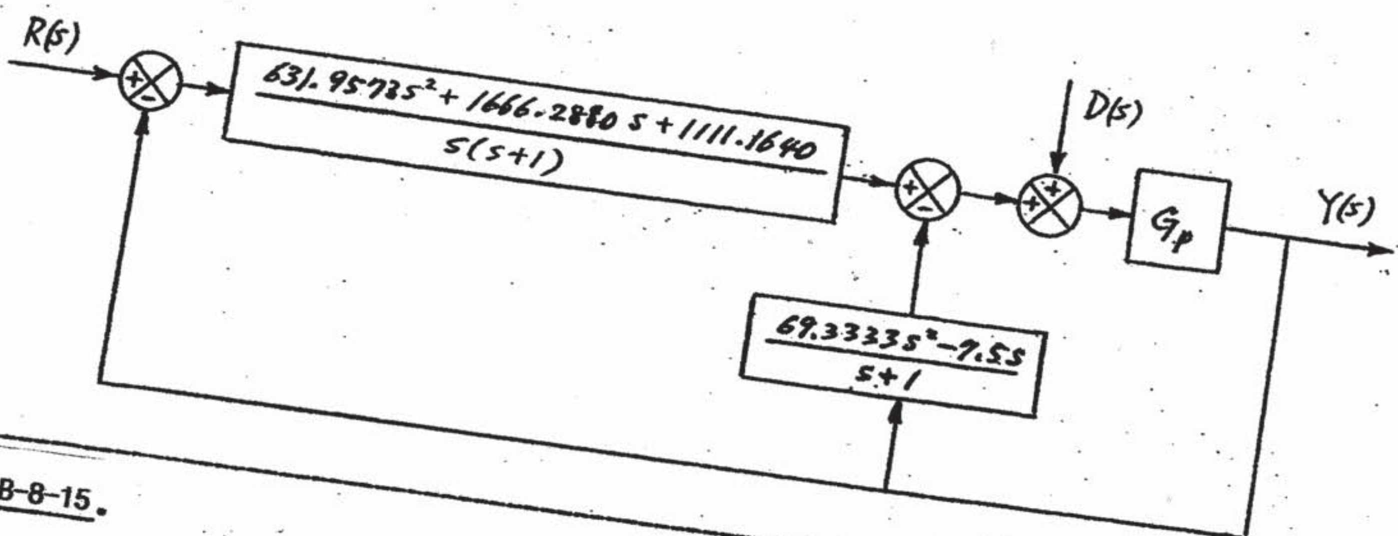
and
$$G_c(s) = \frac{69.3333\,(s+4.0033)^2}{s}$$

$$G_{c1}(s) = \frac{631.9573\,s^2 + 1666.2880\,s + 1111.1640}{s(s+1)}$$

we have

$$G_{c2}(s) = \frac{69.3333\,s^2 - 7.5\,s}{s+1}$$

A block diagram of the designed system is shown below.



B-8-15.

$$G_p(s) = \frac{1}{s^2}$$

The closed-loop transfer functions $Y(s)/D(s)$ and $Y(s)/R(s)$ are given by

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+(G_{c1}+G_{c2})\,G_p}$$

and

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}\,G_p}{1+(G_{c1}+G_{c2})\,G_p}$$

Let us define $G_{c1} + G_{c2} = G_c$. Then

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+G_c\,G_p}$$

Assume that G_c is a PID controller and has the following form:

$$G_c(s) = \frac{K(s+a)^2}{s}$$

The characteristic equation for the system is

$$1 + G_c G_p = 1 + \frac{K(s+a)^2}{s} \frac{1}{s^2}$$

In what follows, we shall use the root-locus approach to determine the values of K and a. Let us choose the dominant closed-loop poles at

$$s = -7 \pm j1$$

Then, the angle deficiency at the desired closed-loop pole at s = -7 + j1 is obtained as follows:

$$-171.8699^\circ \times 3 + 180^\circ = -335.6097^\circ$$

The double zero at s = -a must contribute 335.6097°. (Each zero must contribute 167.80485°.) By a simple calculation, we find a = 2.3729. The controller $G_c(s)$ is then determined as

$$G_c(s) = \frac{K(s+2.3729)^2}{s}$$

where K is determined as

$$K = \left| \frac{s^3}{(s+2.3729)^2} \right|_{s=-7+j1} = 15.7767$$

Hence

$$G_c(s) = \frac{15.7767(s+2.3729)^2}{s}$$

Then the closed-loop transfer function Y(s)/D(s) can be obtained as

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1+G_c G_p} = \frac{\frac{1}{s^2}}{1 + \frac{15.7767(s+2.3729)^2}{s} \frac{1}{s^2}}$$

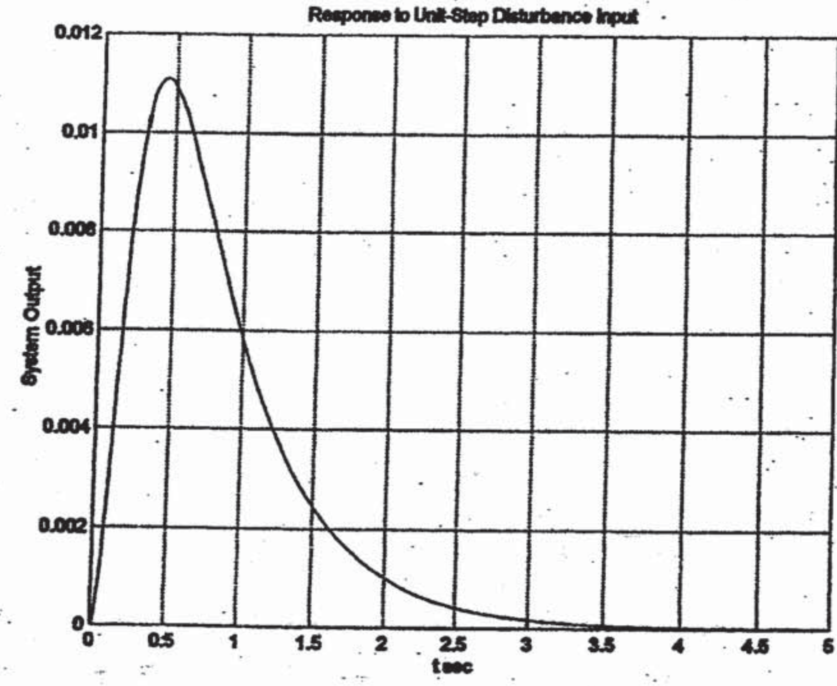
$$= \frac{s}{s^3 + 15.7767s^2 + 74.8731s + 88.8331}$$

The response curve when D(s) is a unit-step disturbance is shown in the next page.

Next, we consider the response to reference inputs. The closed-loop transfer function Y(s)/R(s) is

$$\frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1+(G_{c1}+G_{c2})G_p} = G_{c1} \frac{Y(s)}{D(s)}$$

$$= \frac{s G_{c1}}{s^3 + 15.7767s^2 + 74.8731s + 88.8331}$$



To satisfy the requirements on the responses to the ramp reference input and acceleration reference input, we use the zero-placement approach. That is, we choose the numerator of Y(s)/R(s) to be the sum of the last three terms of the denominator, or

$$sG_{c1} = 15.7767s^2 + 74.8731s + 88.8331$$

from which we get

$$G_{c1}(s) = \frac{15.7767s^2 + 74.8731s + 88.8331}{s}$$

Hence, the closed-loop transfer function Y(s)/R(s) becomes as

$$\frac{Y(s)}{R(s)} = \frac{15.7767s^2 + 74.8731s + 88.8331}{s^3 + 15.7767s^2 + 74.8731s + 88.8331}$$

The response curves to the unit-step reference input, unit-ramp reference input, and unit-acceleration reference input are shown on the following two pages. Notice that the maximum overshoot in the response to the unit-step reference input is 18 % and the settling time is approximately 0.7 sec. The steady-state errors in the ramp response and acceleration response are zero. Therefore, the designed controller $G_c(s)$ is satisfactory.

Finally, we determine G_{c2}. Noting that

$$G_c(s) = G_{c1}(s) + G_{c2}(s)$$

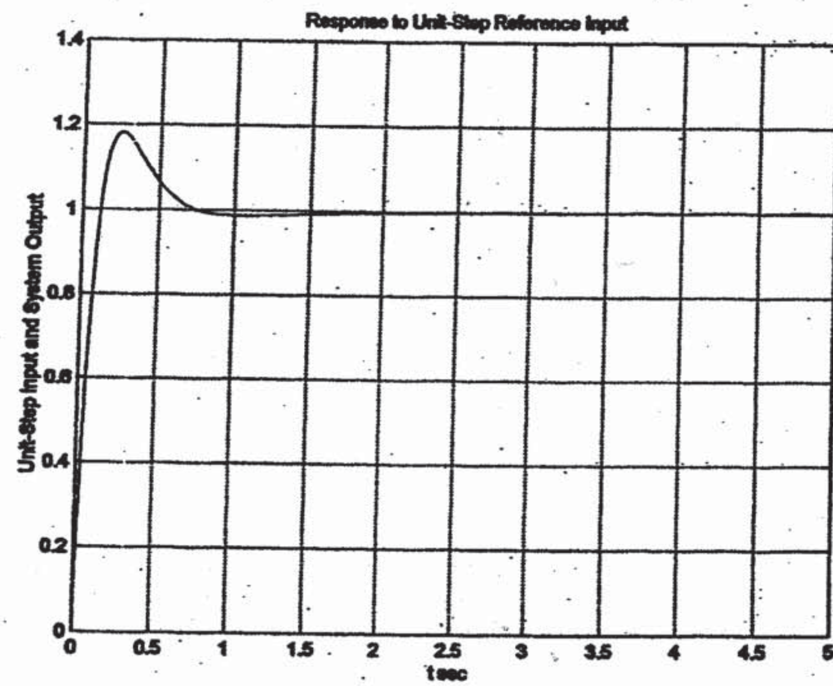
where

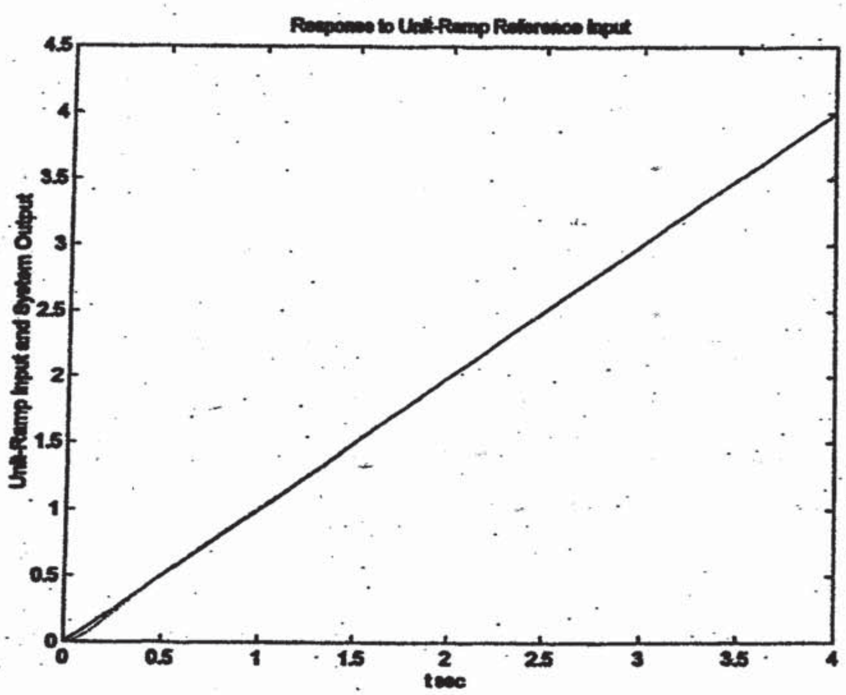
$$G_c(s) = \frac{15.7767s^2 + 74.8731s + 88.8331}{s}$$

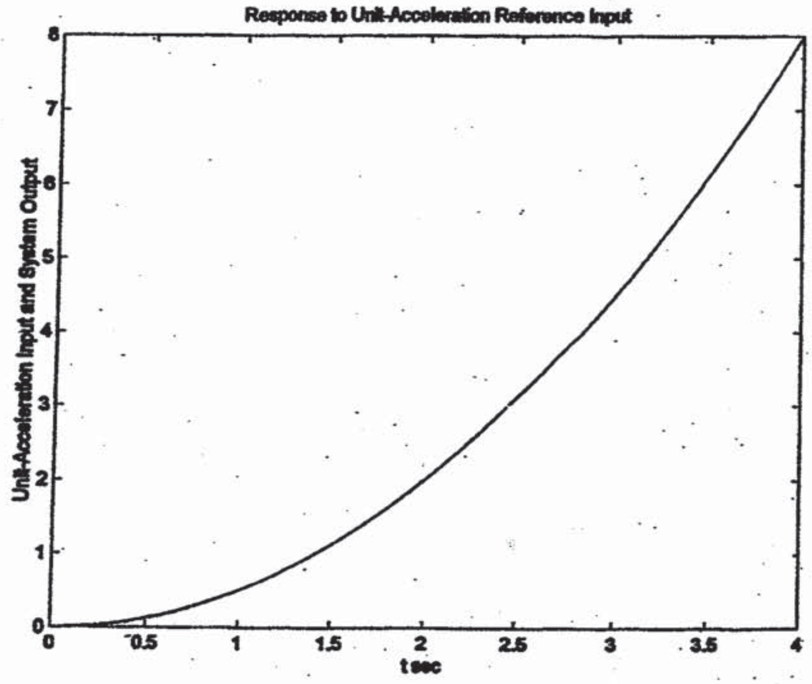
and

$$G_{c1}(s) = \frac{15.7767s^2 + 74.8731s + 88.8331}{s}$$

we obtain $G_{c2}(s) = 0$. This means that we do not need $G_{c2}(s)$ to get the desired result.







A block diagram of the designed system is shown below.