Inverse Laplace Transforms

Problem 1

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right)$$

Step-by-Step Solution:

1. **Decompose the expression**:
We can rewrite $s^2 - 1$ as $(s-1)(s+1)$, which suggests a partial fraction decomposition:

$$\frac{1}{s^2-1} = \frac{A}{s-1} + \frac{B}{s+1}$$

Solving for A and B, we get:

$$1 = A(s+1) + B(s-1)$$

Setting $s = 1$, we find $A = \frac{1}{2}$, and setting $s = -1$, we find $B = -\frac{1}{2}$.
Therefore:

$$\frac{1}{s^2-1} = \frac{1/2}{s-1} - \frac{1/2}{s+1}$$

2. **Inverse Laplace Transform**:
Using the standard inverse Laplace transform formula:

$$\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at},$$

we can now invert both terms:

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right) = \frac{1}{2}e^{t} - \frac{1}{2}e^{-t}.$$

This is also the hyperbolic sine function:

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right) = \sinh(t).$$

Problem 2

$$\mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right)$$

Step-by-Step Solution:

1. **Recognize the standard form**:
The expression $\frac{s}{s^2+a^2}$ is a standard Laplace transform pair. We know that:

$$\mathcal{L}^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos(at).$$

2. **Apply the formula**:
Here, $a^2 = 9$, so $a = 3$. Therefore:

$$\mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right) = \cos(3t).$$