Exercise 6.121

L Answer (f).

We are given the function

$$x(t) = x_1 * x_2(t), \quad 1$$

where

$$2 \quad x_1(t) = e^{-t}u(t) \quad \text{and} \quad x_2(t) = \operatorname{sinc}(10t). \quad 3$$

Let X, X_1, and X_2 denote the Fourier transforms of x, x_1, and x_2, respectively. To begin, we find X. From the convolution property of the Fourier transform, we have

$$X(\omega) = X_1(\omega)X_2(\omega).$$

FT of 1

4

From a table of Fourier transform pairs, we have

FT of 2

FT of 3

$$X_1(\omega) = \frac{1}{1+j\omega} \quad \text{and} \quad X_2(\omega) = \tfrac{\pi}{10}\operatorname{rect}(\omega/20).$$

5 6

Thus, we have

$$X(\omega) = \tfrac{\pi}{10}\operatorname{rect}(\omega/20)\left(\frac{1}{1+j\omega}\right).$$

substitute 5 and 6 into 4

Since $\frac{1}{1+j\omega}$ is nonzero for all ω and $\operatorname{rect}(\omega/20)$ is nonzero only if $\omega \in [-10, 10]$, $X(\omega)$ is only nonzero if $\omega \in [-10, 10]$. Therefore, by the sampling theorem, we have that

highest magnitude frequency is 10

$$\omega_s > 2(10) = 20.$$

sampling theorem: $\omega_s > 2\omega_b$

So, $\omega_s > 20$