B-5-26

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{Ks+b}{s^2+as+b}$$

Hence, $$(s^2+as+b)\ G(s) = (Ks+b)(1+G(s))$$

$$=> G(s) = \frac{Ks+b}{s\,(s+a-K)}$$

The steady state error in the unit-ramp response is

$$e_{ss} = \frac{1}{K_v} = \lim_{s\to 0}\frac{1}{s\,G(s)} = \lim_{s\to 0}\frac{s(\,s+a-K)}{s(\,Ks+b)} = \frac{a-K}{b}$$

Since this is a type (I) system (one integrator in the open-loop), the steady state error for step is 0 and ∞ for a parabola respectively.

B-5-27

The close-loop transfer function is $\frac{K}{js^2+Bs+K}$

For a unit-ramp input, $R(s)=\frac{1}{s^2}$

Thus, $$\frac{E(s)}{R(s)} = \frac{R(s)-C(s)}{R(s)} = \frac{R(s)(1-G(s))}{R(s)} = \frac{Js^2+Bs}{Js^2+Bs+K}$$

$$=> E(s) = \frac{Js^2+Bs}{js^2+Bs+K}\ \frac{1}{s^2}$$

The steady state error is

$$e_{ss} = e(\infty) = \lim_{s\to 0} s\,E(s) = \frac{B}{K}$$

Further we have that

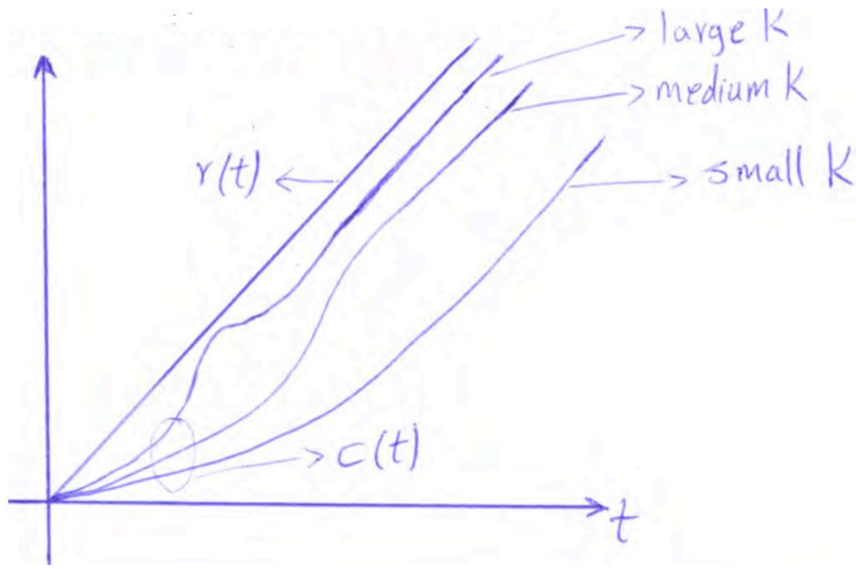
$$\frac{K}{J} = \omega_n^2 \quad 2\zeta\omega_n = \frac{B}{J} \quad => \quad \zeta = \frac{B}{2\sqrt{JK}}$$

We see that we can reduce the steady state e_{ss} by increasing the gain K or decreasing the viscous-friction coefficient B. Increasing the gain or decreasing the viscous-friction coefficient, however, causes the damping ratio to decrease, with the result that the transient response of the system will become more oscillatory. Doubling K decrease e_{ss} to half of its original value, while ζ is decreased to 0.707 of its original value since ζ is inversely proportional to the square root of K.

On the other hand, decreasing B to half of its original value decreases both e_{ss} and ζ to the halves of their original values, respectively. Therefore, it is advisable to increase the value of K rather than to decrease the value of B.

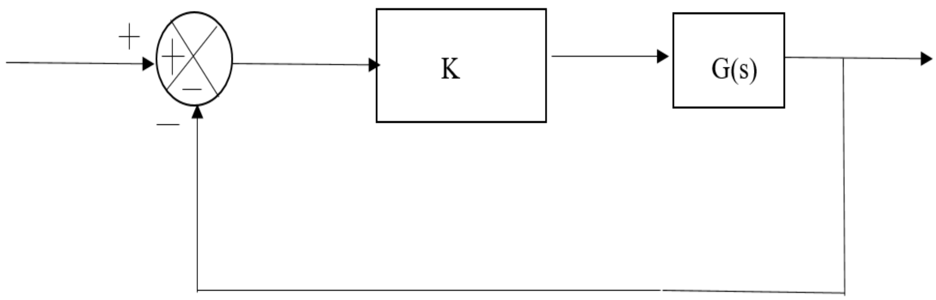
Consider now a ramp input signal. After the transient response has died out, at the steady state the output signal will follow the input signal with a steady state error, i.e there will be a constant positional error between the input and the output.

Examples of the unit-ramp response of the system for three different values of K are illustrated below.



B-6-1

From,



where

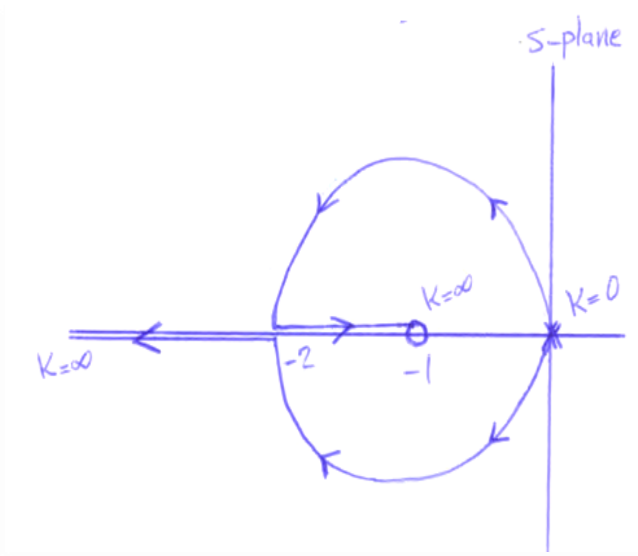
$$G(s) = \frac{s+1}{s^2} = \frac{B(s)}{A(s)}$$

We have,
open-loop poles : 0, 0
open loop zeroes : -1
asymptotes : $\gamma = \frac{\pm 180^0\,(2k+1)}{2-1} = \pm\,180^0$ i.e, real axis
(σ_α for $\pm\,180^0$is not required)

Break-in break-away points:

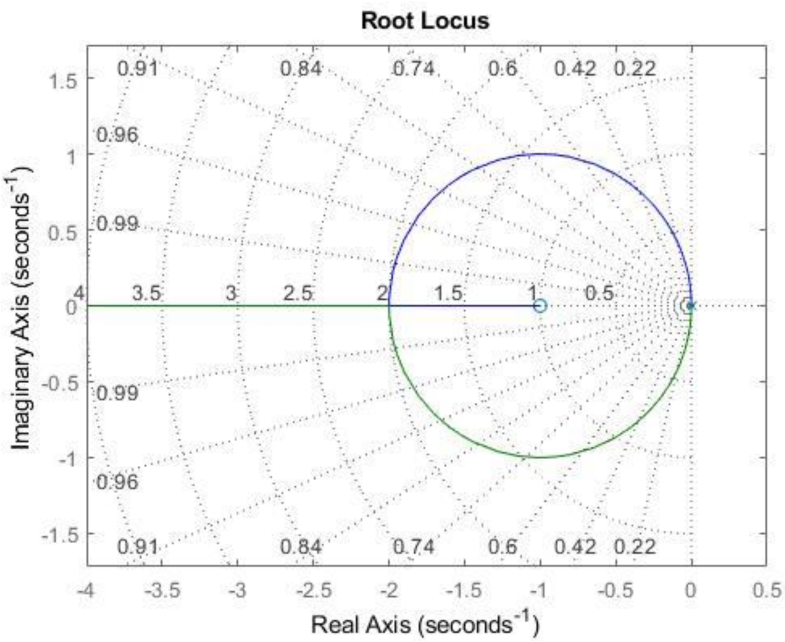
$$2s(s+1) - s^2 = 0 \Rightarrow \begin{cases} s_1 = 0 \\ s_2 = -2 \end{cases}$$

implying the following root locus sketch



The root locus can be plotted using the following matlab code:

```
>> num = [0 1 1];
>> den = [1 0 0];
>> rlocus(num, den);
>> axis('square'); grid;
```



B-6-2

The open-loop transfer function

$$G(s)H(s) = \frac{K}{s(s+1)(s^2+4s+5)} = \frac{KB(s)}{A(s)}$$

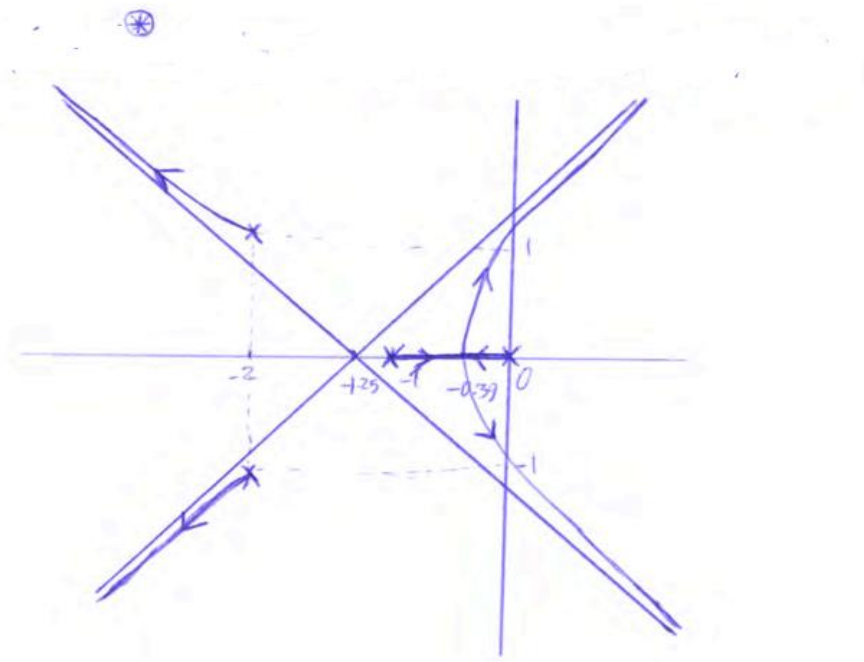
has poles : 0, -1, -2 $\pm$ j

asymptotes: $\gamma = \pm(2k+1)\,45^0 \Rightarrow \begin{cases}\gamma = \pm 45^0 \\ \gamma = \pm 135^0\end{cases}$, $\sigma_\alpha = -1.25$

The equation for the break –in points becomes,:

$$A'(s)B(s) - \overset{0}{\cancel{B'(s)}}A(s) = 4s^3 + 15s^2 + 18s + 5 = 0 \Rightarrow \begin{cases} s_1 = -0.39 \\ \cancel{s_{2,3} = -1.68 \pm j\,0.6}\end{cases}$$

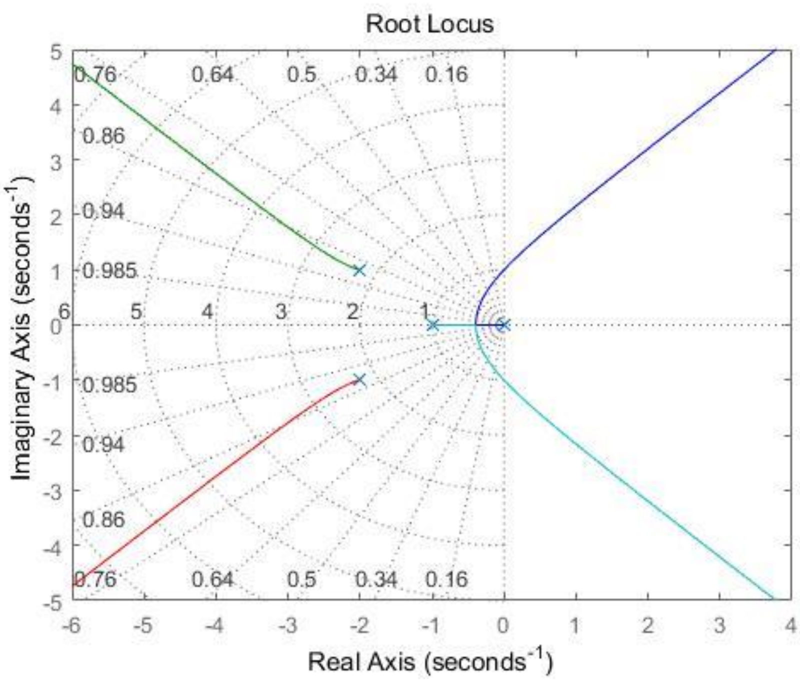
$s_{2,3}$ are rejected (Using $K = -\frac{A(s)}{B(s)}$ it can be found that K corresponding to s_1 is positive, while K is not positive for s_2 and s_3).



The root-locus can be sketched

The root locus can be plotted using the following matlab code:

```
>> num = [0 0 0 0 1];
>> den = [1 5 9 5 0];
>> rlocus(num,den);
>> axis('square');
```



<u>B-6-5</u>

The open-loop transfer function

$$G(s)H(s) = \frac{k(s+0.2)}{s^2(s+3.6)}$$

zeroes	: -0.2
poles	: 0, 0, -3.6
asymtotes	: $\pm 90^0$, , $\sigma_\alpha = -1.7$

Break-in, Break-away points $s = \begin{cases} 0 \\ -0.43 \\ -1.67 \end{cases}$

Since all 3 solutions are located in parts of the real axis which is part of the root locus, then 0 and -1.67 are break-away and -.43 is a break-in point.

A matlab code to obtain the root-locus plot is shown below as well as the root locus plot.

```
num = [0 0 1 0.2];

den = [1 3.6 0 0];
rlocus(num, den)
v = [-6 2 -4 4]; axis(v); axis('equal')
grid
title ('Root-Locus Plot   (Problem B-6-5)')
```