University of Victoria
Exam 4
Fall 2022

Course Name:	**ECE 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 11002), A02 (CRN 11003)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ______________________________

Given Name(s): ______________________________

Student Number: _______________

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

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Do not write on this page unless instructed to do so.

Question 1. Using the Fourier transform pair $e^{-|t|} \overset{\text{CTFT}}{\longleftrightarrow} \frac{2}{\omega^2+1}$ and properties of the Fourier transform, find the Fourier transform X of the function $x(t) = e^{-j2t}e^{-|3t-1|}$. You must use a **systematic method**, **show all of your work**, and you **must not skip any steps**. A correct final answer with an incorrect or incomplete justification may receive zero marks. **[5 marks]**

Question 2. The function $x(t) = -t^3 e^{-2t} u(t)$ has the Fourier transform $X(\omega) = \dfrac{-6}{(2+j\omega)^4}$.

(A) Find a **fully-simplified** formula for the magnitude spectrum of x. **[2 marks]**

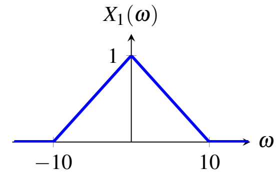
(B) Find a **fully-simplified** formula for the phase spectrum of x. **[2 marks]**

(C) Determine at what frequency/frequencies x has the most spectral information. You **must justify** your answer. **[1 mark]**

Question 3. A LTI circuit with input v_0, output v_1, and frequency response H is characterized by the equations $v_0(t) = 2i(t) + 2\int_{-\infty}^{t} i(\tau)d\tau + v_1(t)$ and $i(t) = \frac{1}{2}\int_{-\infty}^{t} v_1(\tau)d\tau$. Find a **fully-simplified** formula for H. **Show all of your work** and **do not skip any steps** in your solution. A solution with a correct final answer that cannot be **clearly understood by the marker** may receive zero marks. **[5 marks]**

Question 4.

A system with input x and output y is characterized by the equation $y(t) = [1 + \cos(15t)]x(t)$. Let X and Y denote the Fourier transforms of x and y, respectively. Let x_1 denote the function with the frequency spectrum X_1 shown in the figure.



(A) Find a **fully-simplified** expression for Y in terms of X. **[2 marks]**

(B) If $x = x_1$, find the lowest rate ω_x at which x can be sampled in order to avoid aliasing. Your answer **must be justified**. **[1 mark]**

(C) If $x = x_1$, find the lowest rate ω_y at which y can be sampled in order to avoid aliasing. Your answer **must be justified**. (A graph may be helpful in justifying/explaining your answer.) **[2 marks]**

Question 5. Consider a LTI system with input x, output y, and impulse response $h(t) = e^{-2t}u(t)$. Using the Fourier transform, find a **fully simplified** formula for y in the case that $x(t) = e^{-2t}\cos(3t)u(t)$. **[5 marks]**

END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \tfrac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \tfrac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$
$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt$$

$$\mathscr{F}x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$
$$\mathscr{F}^{-1}X(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$
$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$
$$a_k = \tfrac{1}{T}X_T(k\omega_0)$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty} \|x(t)\|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \|X(\omega)\|^2\,d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\operatorname{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\operatorname{rect}\left(\frac{t}{T}\right)$	$\|T\|\operatorname{sinc}\left(\frac{T\omega}{2}\right)$
9	$\operatorname{sinc}(Bt)$	$\frac{\pi}{\|B\|}\operatorname{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$
12	$e^{-at}\cos(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
14	$e^{at}u(-t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a-j\omega}$