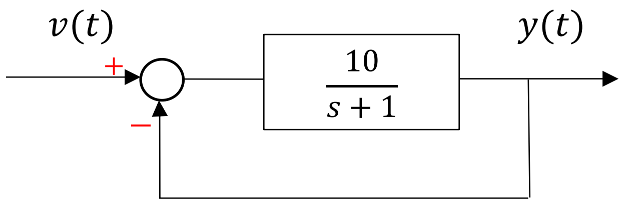
ECE 360 Assignment 7

B_7_1:

From the below diagram, we can see that:



We can get that the closed-loop transfer function is:

$$G(s) = \frac{10}{s+11}$$

$$G(j\omega) = \frac{10}{j\omega + 11} = |G(j\omega)|e^{KG(j\omega)}$$

If $v(t) = A\sin(\omega t + \theta)$, then at steady state the output will be:

$$y(t) = A|G(j\omega)|\sin(\omega t + (\theta + KG(j\omega)))$$

Using this equation, we get:

$$y_1(t) = 0.905\sin(t + 24.8°)$$

$$y_2(t) = 1.79\cos(2t - 55.3°)$$

$$y_3(t) = y_1(t) - y_2(t)$$

B_7_4

$$G(s) = \frac{10(s^2 + 0.4s + 1)}{s(s^2 + 0.8s + 9)}$$

We can get:

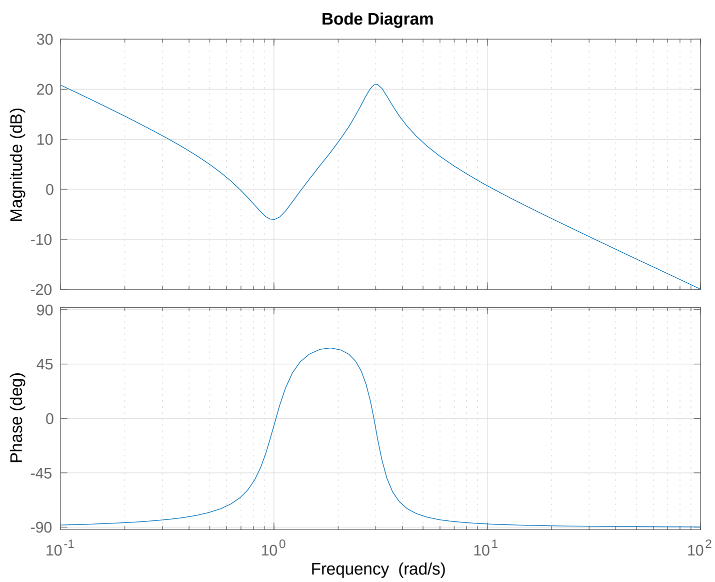
$$G(j\omega) = \frac{10((j\omega)^2 + 0.4\,j\omega + 1)}{j\omega\,((j\omega)^2 + 0.8\,j\omega + 9)} = \frac{\frac{10}{9}\left(\left(j\frac{\omega}{\omega_{n_1}}\right)^2 + 2\zeta_1\frac{j\omega}{\omega_{n_1}} + 1\right)}{j\omega\left(\left(j\frac{\omega}{\omega_{n_1}}\right)^2 + 2\zeta_2\frac{j\omega}{\omega_{n_2}} + 1\right)}$$

Where:

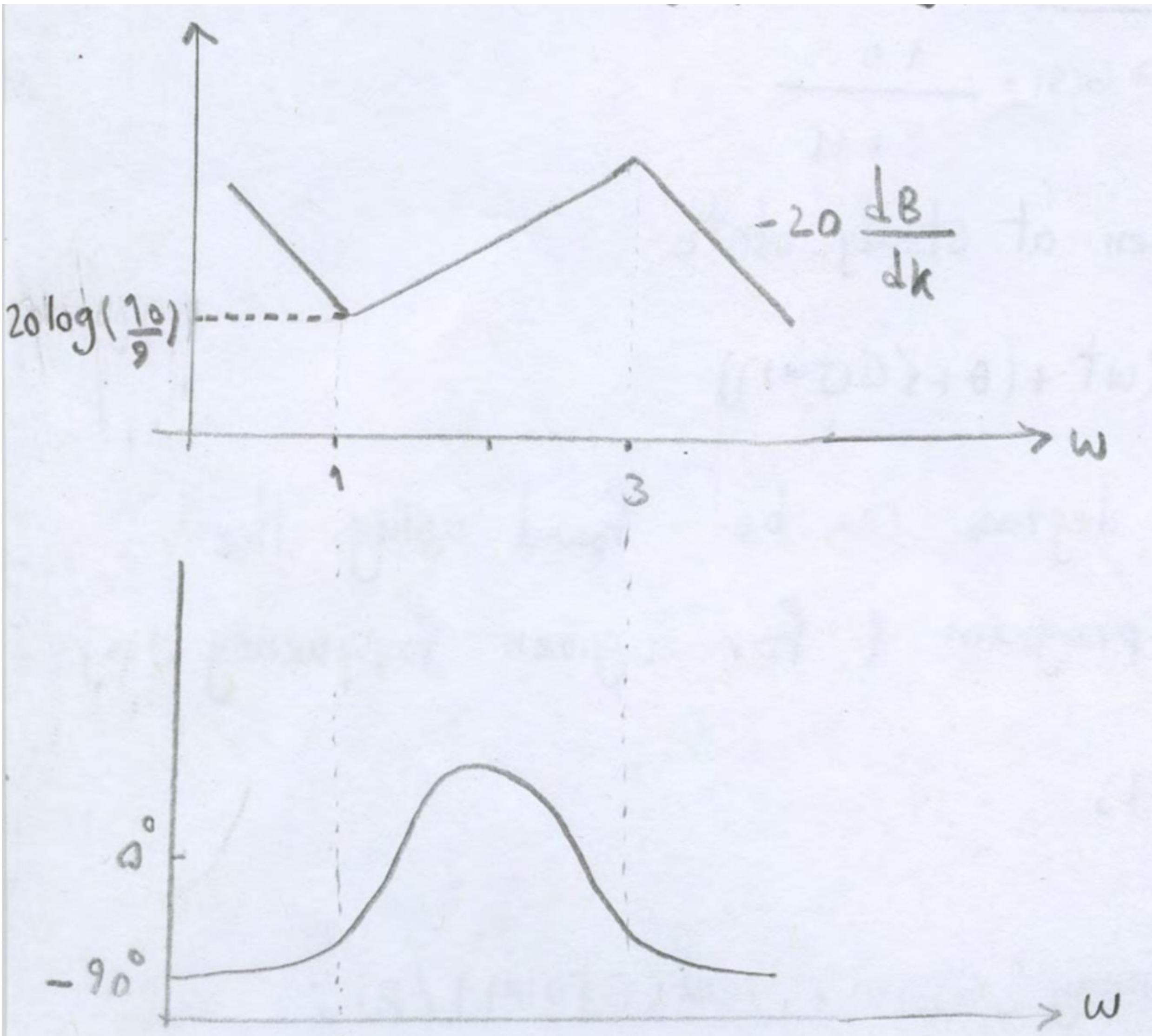
$\omega_{n_1} = 1$, $\zeta_1 = 0.2$, $\omega_{n_2} = 3$, $\zeta_2 = 0.133$

In Matlab, the exact curve depends on ζ_1 and ζ_2 can be obtained as follow:

```
num = [0 10 4 10];
den = [1 0.8 9 0];
bode(num, den)
grid on
```



A sketch of the asymptotes gives:



B_7_5

We can get the transfer function as follow:

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

Divided by ω_n^2, we can obtain the following equation,

$$G(j\omega) = \frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + 2\zeta\left(j\frac{\omega}{\omega_n}\right) + 1}$$

Therefore, we can get:

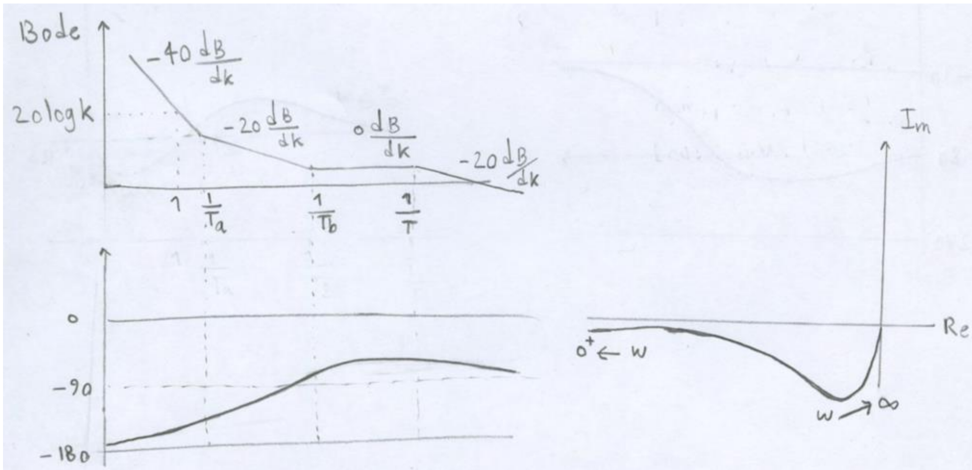
$$|G(j\omega_n)| = \left|\frac{1}{-1 + 2\zeta j + 1}\right| = \frac{1}{2\zeta}$$

B_7_7

We know about the transfer function is:

$$G(s) = \frac{K(T_a s + 1)(T_b s + 1)}{s^2(Ts + 1)}$$

Case(a), when $0 < T < T_a < T_b$, a sketch of the asymptotes gives:



Case (b): when $0 < T_a < T_b < T$, a sketch of the asymptotes gives: