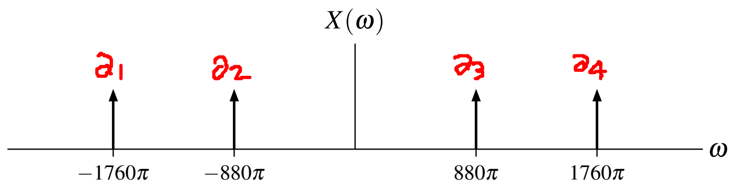
Exercise 6.123

L Answer (a).

FIRST SOLUTION. In what follows, radian measure is used for frequency, unless explicitly indicated otherwise. In the case being considered, the sampling rate is $\omega_s = 2\pi \cdot 500 = 1000\pi$. The Fourier transform X of x is plotted in the figure below. The sinusoids at 440 and 880 Hz are associated with the impulses at (angular) frequencies of $\pm 2\pi \cdot 440 = \pm 880\pi$ and $\pm 2\pi \cdot 880 = \pm 1760\pi$, respectively.



Since the Nyquist condition is clearly violated, sampling can result in aliasing. (The Nyquist rate is $2 \cdot 2\pi \cdot 880 = 3520\pi$, while the sampling rate is 1000π.) Sampling x at the frequency ω_s will yield a spectrum of the form

$$Y(\omega) = c \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s),$$

where c is a nonzero constant whose value is not important to this discussion. Clearly, Y is ω_s-periodic. Plotting Y, we obtain the graph shown below.

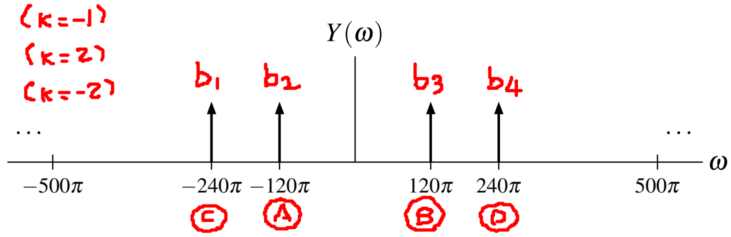
$\uparrow 1000\pi$

A $880\pi - 1000\pi = -120\pi$ $(k=1)$

B $-880\pi + 1000\pi = 120\pi$ $(k=-1)$

C $1760\pi - 2000\pi = -240\pi$ $(k=2)$

D $-1760\pi + 2000\pi = 240\pi$ $(k=-2)$



In the plot of Y, we have impulses at $\pm 120\pi$ and $\pm 240\pi$, which correspond to sinusoids with (angular) frequencies 120π and 240π, respectively. Therefore, when the signal is played back on the loudspeaker after sampling, two sinusoids will be heard, with (angular) frequencies 120π and 240π (or, equivalently, 60 Hz and 120 Hz, respectively).