ELEC 360 – Assignment #6 Solutions

B – 6 – 6)

The open-loop transfer function is.

$$G(s) = \frac{K(s+9)}{s(s^2+4s+11)} = \frac{KB(s)}{A(s)}$$

with poles: $s = 0$, $s = -2 \pm j\sqrt{7}$ and a zero at $s = -9$.
The asymptotes have angles $\gamma = \pm(2K+1)\frac{180}{3-1} = \pm 90$
and meet the read axis at $\sigma_a = 2.5$.

For break-away point we have

$$A'(s)B(s) - A(s)B'(s) = 0$$

$$(3s^2 + 8s + 11)(s+9) - (s^3 + 4s^2 + 11s) = 0$$
$$2s^3 + 31s^2 + 72s + 99 = 0$$
$$s_1 = -13.02, \qquad s_{2,3} = -1.24 \pm j1.51$$

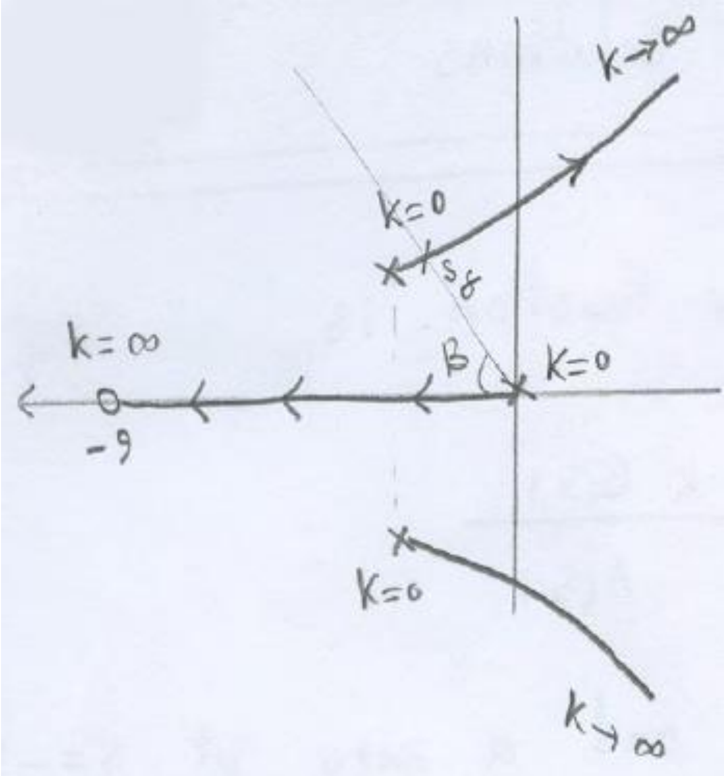
All three are rejected, since they cannot be on the root locus.

The poles with the damping ratio $\zeta = 0.5$ can be obtained graphically using the Matlab program *rlocus_demo.m* from the course web page (www.ece.uvic.ca/~panagath/ELEC360/trans/rlocus_demo.m)

For $\cos\beta = \zeta = 0.5$, the dominant closed-loop poles having the damping ratio $\zeta = 0.5$ can be located as the intersection of the root loci and lines from the origin having angles $\pm 60°$ (or use the grid generated by the program).

This leads to the complex conjugate poles $s_\zeta = -1.5 \pm j2.6$ (approx.)
The third pole other than s_ζ is at about $s = -1$.

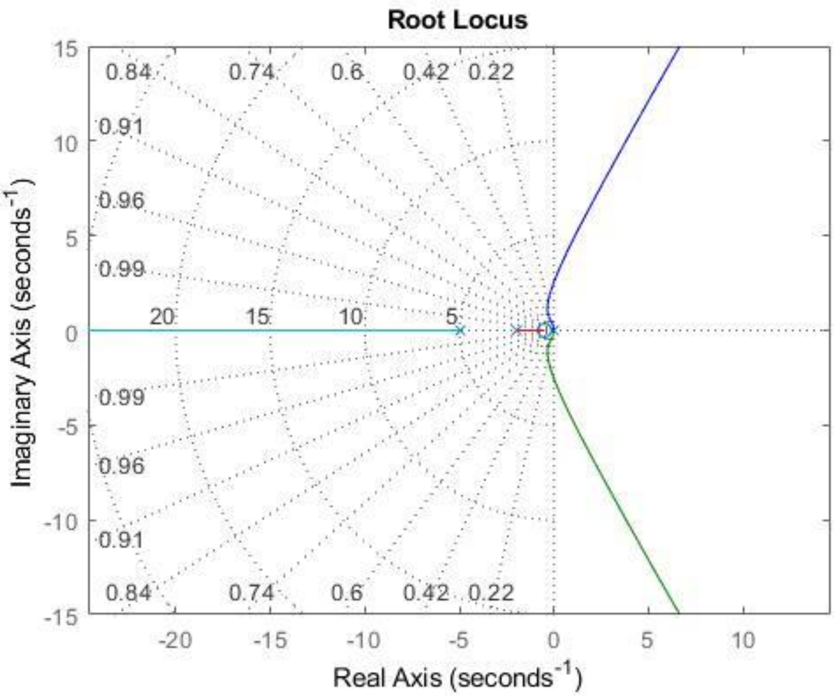
The gain value correspond to these dominant closed-loop poles is $K = 1$.



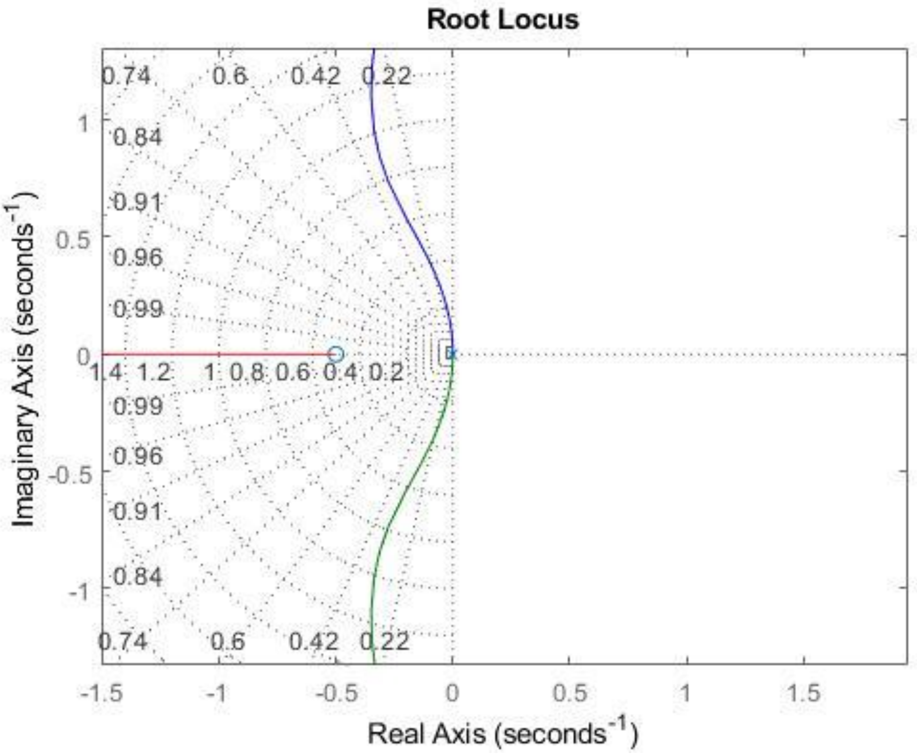
Root-locus sketch for B-6-6.

B – 6 – 7: A sketch of the root locus can be obtained using the usual rules. Here we will use a short matlab program to generate the root locus:

```
num=[2 1];
den=poly([0 0 -2 -5]);
system=tf(num,den);
rlocus(system)
axis('equal')
grid
```



By zooming closer to the origin we have:



The range of K for stability can be determined by use of Routh stability criterion. Since the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{2K(s+1)}{s^4 + 7s^3 + 10s^2 + 2Ks + 2K}$$

the characteristic equation for the system is $s^4 + 7s^3 + 10s^2 + 2Ks + 2K = 0$

The Routh array of coefficient becomes:

$$\begin{array}{llll}
s^4 & 7 & 10 & 2K \\
s^3 & 7 & & 2K \\
s^2 & \dfrac{70-2K}{7} & & 2K \\
s^1 & \dfrac{\dfrac{(70-2K)2K}{7} - 14K}{\dfrac{70-2K}{7}} & & 0 \\
s^0 & 2K & &
\end{array}$$

For stability, we require:

$$70 > 2K$$
$$42 - 4K > 0$$
$$K > 0$$

Thus, the range of k for stability is $10.5 > K > 0$.

B – 6 – 11)

This question can be solved using the Matlab by generating a similar short program as in question B-6-7. Here the solution is found using the closed loop denominator.
The term $(s+1)$ in the feedforward transfer function and the term $(s+1)$ in the feedback transfer function cancel each other. We have a *pole-zero cancellation*, which is acceptable in the stable region. This leads to the following closed-loop denominator polynomial

$s(s^2+2s+6)+K=0$

And for $K=2$

$s_1=-0.37,\qquad s_2=-0.81+j2.18,\qquad s_3=-0.81-j2.18$

B – 6 – 16)

The closed-loop transfer function $\frac{C(s)}{R(s)}$ is given by

$$\frac{C(s)}{R(s)}=\frac{K(Ts+1)}{s(s+2)+K(Ts+1)}$$

Since the closed-loop pales are specified to be

$$s=-2\pm j2$$

We obtain

$$s(s+2)+K(Ts+1)=(s+2+j2)(s+2-j2)$$

or

$$s^2+(2+KT)s+K=s^2+4s+8$$

Hence, we require

$$2+KT=4,\qquad K=8$$

Which results in

$$T=0.25,\qquad K=8$$