Date: 19-Oct-2018 Time: 9:30 to 10:20 AM

Student Name: ____

Student Number: _______________________

Instructions:

1) Please answer the questions in the space provided on the paper.
2) Please verify if the exam has 10 pages including this page and the Laplace Table.
3) Please answer all the questions.
4) Please show all the steps (formula, substitution, and simplification) to secure maximum marks.
5) Please write down the appropriate units to secure maximum marks.
6) Please write down all assumptions you make to secure maximum marks.
7) Please write legibly. In case your answer is unclear, marker's decision is final.
8) For Section A, please encircle the option of your choice

Materials Allowed:

1) You can use any type of calculator. In case you use a programmable calculator, you will have to clear the calculator's memory before starting the Quiz.
2) You are allowed to use up to 1 single-sided A4 aid sheet. The aid sheet can be handwritten or typed/printed. There is no restriction on the content of the aid sheet.
3) You don't have to hand-in the aid sheet at the end of the exam. You can re-use it for the final.

Section A 5 x 1 = 5 marks

1) Unit step response of a standard first order system is shown in Figure 1. Then the steady state error when the system is subjected to unit-ramp input will be

(1 mark)

(a) 0; (b) -4

(c) 4; (d) None of the above

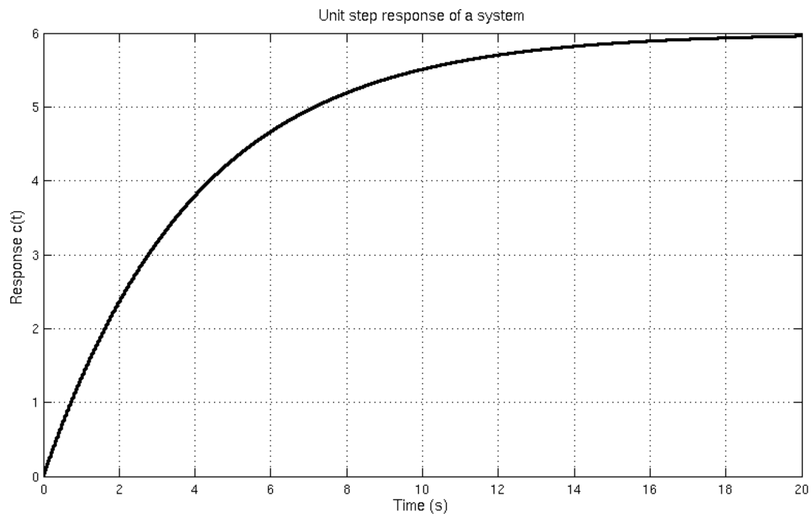


Figure 1. Unit step response of a system

Solution:

Approach 1:

The response of a standard first order system for unit step is given by the expression $c(t) = K[1 - e^{-\frac{t}{T}}]$ where K is the gain and T is the time constant of the system. The response of the same standard system for unit ramp input can be obtained by integrating the response of the step input (assuming the system is linear & time invariant). The unit ramp response is then found to be $c(t) = K[t - T + Te^{-\frac{t}{T}}]$.

Steady state error can be obtained by

$$e_{ss} = \lim_{t\to\infty}[r(t) - c(t)] = \lim_{t\to\infty}\left[t - K(t - T + Te^{-\frac{t}{T}})\right]$$

The error will tend towards infinity as time tends to infinity.

Approach 2:

The standard first order transfer function is to be assumed as the closed loop transfer function of a system with unity negative feedback. From the response graph given, the transfer function of the assumed closed loop system can be synthesized as $\frac{C(s)}{R(s)} = \frac{6}{4s+1}$. The open loop transfer function G(s) can be obtained by using the relation

$\frac{C(s)}{R(s)} = \frac{6}{4s+1} = \frac{G(s)}{1+G(s)}$. Re-arranging, $G(s) = \frac{6}{4s-5}$. As the open-loop transfer function is a Type 0 system, the steady state error for unit ramp input must be infinity.

2) The commonly used definition of rise time for a second-order overdamped system subjected to unit step input is

(1 mark)

(a) The time required for the response to rise from 10% to 90% of its final value

(b) The time required for the response to rise from 0% to 100% of its final value

(c) The time required for the response to rise from 5% to 95% of its final value

(d) The time required for the response to rise from 0% to 50% of its final value

Solution:

As per the definition given in the textbook page 170, for an overdamped system, the 10% to 90% rise time is commonly used.

3) The characteristic polynomial of a closed loop system is found to be $s^4 + 2s^3 + 3s^2 + 4s + K$. Find the range of K for which the system is stable

(1 mark)

(a) K > 0

(b) 2< K < 0

(c) 2 > K > 0

(d) None of the above

Solution:

Using Routh's stability criterion, the value of K for which the system is stable is 2 > K > 0

Routh's table can be formed as shown below. For stability, the first column entries must be positive. Hence, *K* must be greater than 0 and less than 2.

s^4	1	3	K
s^3	2	4	0
s^2	1	K	0
s^1	4-2K	0	0
s^0	K	0	0

4) Poles of a fourth order closed-loop control system are found to be at
$s_1 = -10$; $s_2 = -10$; $s_3 = -0.5 + 2j$; $s_4 = -0.5 - 2j$
The transient response of the system for a unit step input can then be approximated to (1 mark)

(a) A curve with a damping ratio of 0.25

(b) A curve with a damping ratio of 0.50

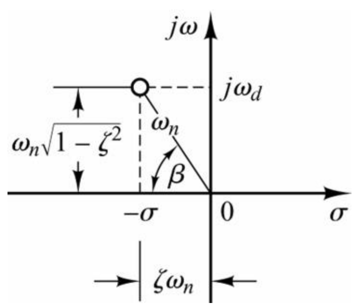
(c) A curve with a damping ratio of 0.75

(d) A curve with a damping ratio of 1.00

Solution:

The 4th order system can be approximated to a second order system with only the complex conjugate pair of roots. This is possible because (i) the complex conjugate poles have a real term that is close to the imaginary axis. (ii) the real roots are far away (at least 10 times further away) from the real part of the complex roots. Hence, the transient response characteristics is largely determined by the dominant complex poles.

If the system is approximated to a second order system whose roots are at $s_3 = -0.5 + 2j$; $s_4 = -0.5 - 2j$ then, the natural frequency ω_n and the damping ratio ζ can be obtained from the real and imaginary parts of the roots. From the figure given below, one can equate $\zeta\omega_n = 0.5$ and $\omega_n\sqrt{(1-\zeta^2)} = 2$. By solving these two equations, the value of ζ is found.



5) For unity negative feedback closed loop control system with the transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b},$$

the steady state error for unit ramp input is found to be $\frac{a-K}{b}$. The open-loop transfer function would be

(1 mark)

(a) Type 0 system (b) Type 1 system

(c) Type 2 system (d) None of the above

Solution:

Assuming $a > K$ and $a, b, K > 0$, as the steady state error for unit ramp input is a finite value, the open-loop transfer function should be a Type 1 system.

The result can also be confirmed by solving for the open loop transfer function. As the system is said to be a unity negative feedback system, the closed loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks+b}{s^2+as+b} = \frac{G(s)}{1+G(s)}$$

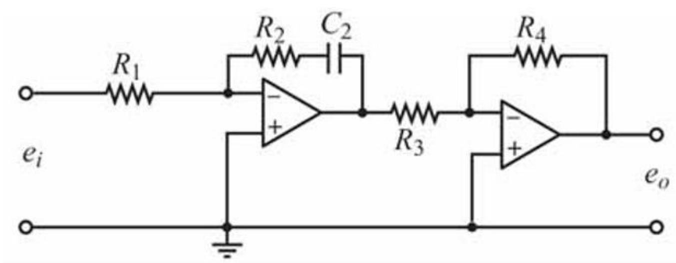
Re-arranging, and solving for G(s),

$$G(s) = \frac{Ks+b}{s(s+a-K)}$$

The open loop transfer function is a Type 1 system.

Section B 1 x 5 = 5 marks

6) For the op-amp circuit given below, obtain the transfer function Eo(s)/Ei(s) in its simplified form using complex impedance approach. Write down all the assumptions used in the derivation. (5 marks)



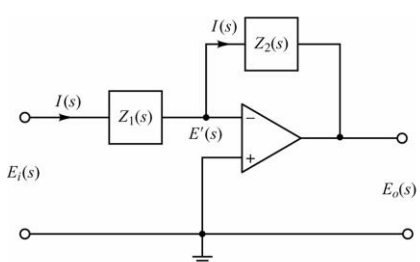
Solution:

Assumptions:

1) The op-amps are ideal.
    a. Input impedance of the op-amps are infinite.
    b. No current can flow through the op-amp's input terminals.
    c. The op-amp in the right hand side of the circuit does not load the circuit to its left.
    d. The voltage difference between the input terminals of the op-amp is zero.

Because of assumption (c), the two circuits can be analyzed independently.

In either circuit, the signal is connected to the inverting terminal of the op-amp. If complex impedance approach is used, the two op-amp circuits can be generalized in the standard form shown below.



The transfer function of such a general op-amp circuit is obtained using circuit equations. At the node near the negative input terminal, $\frac{E_i - E'}{Z_1} = -\frac{E_o - E'}{Z_2}$. This equation is valid because of the assumption (b).

Using assumption (d), as the non-inverting input terminal is grounded, the inverting input terminal must also be at zero potential. Hence, $E' = 0\ V$.

Thus, $\frac{E_o}{E_i} = -\frac{Z_2}{Z_1}$

For the first circuit from the left, $Z_1 = R_1$; $Z_2 = (R_2 + 1/sC_2) = (1+sR_2C_2)/(sC_2)$;

For the second circuit from the left, $Z_1 = R_3$; $Z_2 = R_4$

Therefore, the transfer function of the complete circuit can be derived as:

$$\frac{E_o(s)}{E_i(s)} = \frac{E_{o1}(s)}{E_i(s)} \cdot \frac{E_o(s)}{E_{o1}(s)} = -\frac{1+sR_2C_2}{sR_1C_2} \times -\frac{R_4}{R_3} = \frac{R_4 + sR_4R_2C_2}{sR_1R_3C_2}$$

Table below is from the Appendix of the book *Modern Control Engineering* authored by K. Ogata

Table A–1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab}\left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt})\right]$	$\frac{1}{s(s+a)(s+b)}$

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
21	$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t - \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{s}{s^2+2\zeta\omega_n s+\omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t}\sin(\omega_n\sqrt{1-\zeta^2}\,t + \phi)$ $\phi = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)}$
25	$1 - \cos\omega t$	$\frac{\omega^2}{s(s^2+\omega^2)}$
26	$\omega t - \sin\omega t$	$\frac{\omega^3}{s^2(s^2+\omega^2)}$
27	$\sin\omega t - \omega t\cos\omega t$	$\frac{2\omega^3}{(s^2+\omega^2)^2}$
28	$\frac{1}{2\omega}t\sin\omega t$	$\frac{s}{(s^2+\omega^2)^2}$
29	$t\cos\omega t$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
30	$\frac{1}{\omega_2^2-\omega_1^2}(\cos\omega_1 t - \cos\omega_2 t) \qquad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2+\omega_1^2)(s^2+\omega_2^2)}$
31	$\frac{1}{2\omega}(\sin\omega t + \omega t\cos\omega t)$	$\frac{s^2}{(s^2+\omega^2)^2}$

The End