Lab 02

Tags

Experiment 2: Speed Control using a DC Motor

1. Objective

The objective of this laboratory is to develop an understanding of Proportional and Integral (PI) control as applied to a speed control application. In particular you will explore:

- Qualitative properties of proportional and integral action.
- Design of PI controllers for given specifications.
- Response of a PI controlled system to load disturbances.

2. Introduction

The following nomenclature, as described in Table 2.1, is used for the system modeling and control design.

Symbol	Description	Units
ω_m	Motor angular velocity	rad/s
u_m	Voltage from amplifier which drives the motor	V
u_e	Back-emf voltage	V
T_m	Torque generated by motor	N·m
Td	Disturbance torque externally applied to the inertial load	N·m
Vd	Disturbance voltage corresponding to Td	V
V_{sea}	Simulated disturbance voltage	V
i_m	Motor armature current	A
k_m	Motor torque constant	N·m/A
R_m	Motor armature resistance	Ω
$J_{ee(}$	Total moment of inertia of motor rotor and the load	kg·m²
K	Open-loop steady-state gain	rad/(V·s)
τ	Open-loop time constant	s
ω_n	Undamped Natural Frequency	rad
ζ	Damping Ratio	-
T_s	2% Settling Time	s
h	Sampling interval	s
k_p	Proportional gain	V·s/rad
k_i	Integral gain	V/rad
u	Control signal	V
r	Reference signal	rad/s
y	Measured process output	rad/s

Table 2.1. System speed control nomenclature.

Consider a DC motor whose angular velocity ω_m(t) is supposed to follow a certain reference signal r(t). Assuming that the speed of the motor is measured, the difference between the measured speed ω_m(t) and the desired speed r(t) can be used to control the system. The structure of such a feedback control system is given in **Fig. 2.1.** The block labeled "Motor" represents a DC motor such as the one studied in experiment 1 and has voltage u_m and disturbance torque Td as inputs and motor speed ω_m as the output. The controller is usually designed so that the output of the closed-loop system satisfies certain specifications with respect to transient and steady-state performance. Another effect of the controller is to reduce the influence of disturbances. A torque on the motor axis, such as Td, is a typical example of a load disturbance for a speed control system. In this experiment, the disturbance torque is typically a torque that you apply manually to the inertial load. The effects of such a disturbance will also be considered in this experiment.

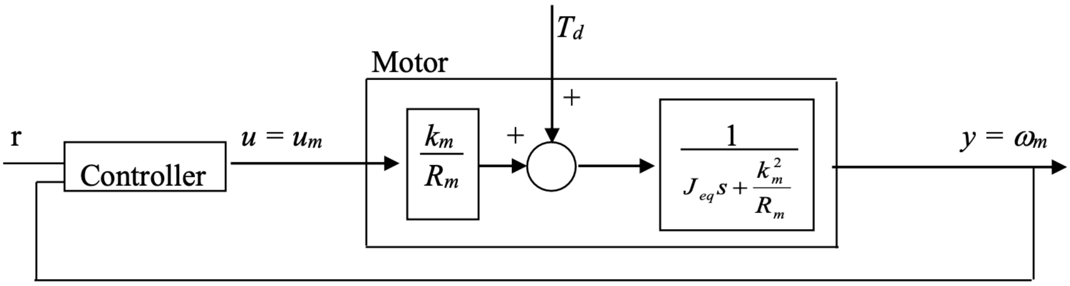

Figure 2.1. Block diagram of a speed control system

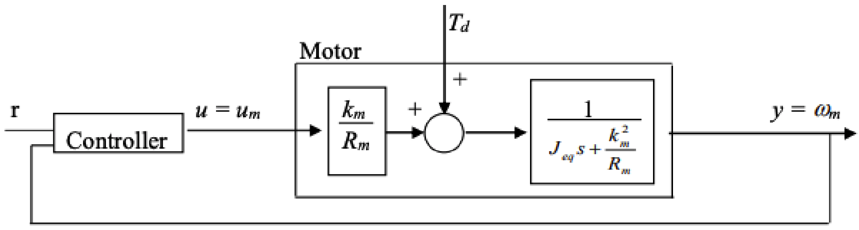

Figure 2.1. Block diagram of a speed control system

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Some of the most commonly used control algorithms are Proportional (P), Integral (I) and Proportional-Integral (PI) control. The PI controller is used for a variety of purposes and it often works very well for systems with simple dynamics with respect to transient and steady-state response. For systems with complicated dynamics, it can often give good performance provided that specifications are not too demanding. Better performance can be obtained by using more complicated controllers (PID, lead-lag compensators, etc.).

3. PI Control Law

The controller function in **Fig. 2.1** to be used in this lab can be described using the following equation:

$$u_m(t) = k_p(b_p r(t) - \omega_m(t)) + k_i \int_0^t (r(\tau) - \omega_m(\tau))d\tau \tag{1}$$

where:

- u_m(t) is the control signal,
- r(t) the reference signal (also called the set point),
- ω_m(t) the measured output.

Parameters:

- k_p = the proportional gain,
- k_i = the integral gain,
- b_{psp} = reference signal weight.

This is a linear Proportional and Integral (PI) controller and its response is governed by the above three parameters. The Proportional (P) controller is the special case of (1) with $k_i = 0$ and $b_{sp} = 1$ while Integral (I) controller is the case with $k_p = 0$.

The error signal $e(t)$ is an important signal when evaluating the performance of the system. It is given by:

$$e(t) = r(t) - \omega_m(t) \tag{2}$$

An important property of the controller with integral action is that it always leads to the correct steady-state value, provided that the closed-loop system is stable and therefore there exists an equilibrium. With this assumption, at steady-state we have:

$$r(t) = r_{ss}, \quad u(t) = u_{ss}, \quad \omega_m(t) = \omega_{mss} \tag{3}$$

Eq. (1) then implies that:

$$u_{ss} = k_p(r_{ss} - \omega_{mss}) + k_i(r_{ss} - \omega_{mss})t \tag{4}$$

Since the left-hand side is a constant, the right-hand side must also be a constant, which implies that:

$$\omega_{mss} = r_{ss}, \quad e_{ss} = 0 \tag{5}$$

where e_{ss} is the steady-state error.

3.1. Manual Tuning of PI Controller: Ziegler-Nichols

Manual tuning procedures are generally used when no mathematical model of the system is available to perform control system design. The Ziegler-Nichols method is a classical tuning rule. Typically in manual control tuning, we first set integral gain to zero and increase proportional gain until the system reaches the stability boundary. At this point, a stable output oscillation is achieved. The critical gain k_{pc}, where this occurs, and the frequency of the oscillation T_{pc} are determined. A similar test with pure integral control gives k_{ic} and T_{ic}. The values k_{pc} and k_{ic} give the ranges for the gains. Suitable values can then be determined empirically or by traditional tuning rules.

The Ziegler-Nichols closed-loop method recommends the following PI controller gain tuning:

$$k_p = 0.4\, k_{pc} \tag{6}$$

$$T_i = 0.8\, T_{pc} \tag{7}$$

$$k_i = \frac{k_p}{T_i} \quad \text{or} \quad k_i = 0.5\, \frac{k_{pc}}{T_{pc}} \tag{8}$$

J.G. Ziegler and N.B. Nichols experimentally developed in the early forties the above tuning rules based on closed-loop tests. However, the Ziegler-Nichols method suffers from one major drawback: the physical system has to tolerate to be brought into a critically stable state without catastrophic consequences. For example, sustained oscillation is generally out of the question for many industrial processes limiting the use of the Ziegler Nichols tuning rules.

4. Pre-Laboratory Assignments

You may find the short videos discussing the theory behind the preparatory questions useful:

YouTube Link

4.1. Proportional Control

4.1.1.

Using the open-loop transfer function of the DC motor with the values of K and \tau as obtained in experiment 1:

$$G(s) = \frac{\Omega_m(s)}{U_m(s)} = \frac{K}{\tau s + 1} \tag{9}$$

and proportional control,

$$u_m(t) = k_p\left(r(t) - \omega_m(t)\right) \tag{10}$$

obtain the closed-loop transfer function $G_p(s)$ between the reference signal $r(t)$ as input and the motor speed ω_m as output (Assume disturbance torque $T_d = 0$).

4.1.2.

Determine the location of the poles of the closed-loop system when the proportional gain k_p is changed. That is, derive the poles of the closed-loop system as a function of k_p. How does the unit step response of the system change when k_p is changed?

4.1.3.

Consider a step with amplitude r_0 for $r(t)$ and use the Final Value Theorem to find the value of the output signal ω_m of the closed-loop system at steady-state. How does it compare to the steady-state value of the input signal $r(t)$?

4.2. Integral Control

4.2.1.

Using the open-loop transfer function of the DC motor obtained in experiment 1, Eq. (9), and integral control:

$$u_m(t) = k_i \int_0^t \left(r(\tau) - \omega_m(\tau)\right) d\tau \tag{11}$$

4.2.2

Obtain the closed-loop transfer function G(s) between the reference signal r(t) as input and the motor speed ω_m as output (Assume disturbance torque $T_d = 0$).

4.2.2

Determine the location of the poles of the closed-loop system when the integral gain k_i is changed. That is, derive the poles of the closed-loop system as a function of k_i. How does the unit step response of the system change when k_i is changed?

4.2.3

Consider a step with amplitude r_0 for $r(t)$ and use the Final Value Theorem to find the value of the output signal ω_m of the closed-loop system at steady-state. How does it compare to the steady-state value of the input signal $r(t)$?

4.3 Proportional and Integral Control

4.3.1

Using the open-loop transfer function of the DC motor obtained in experiment 1, eq. (9), and PI control:

$$u_m(t) = k_p(b_{sp}r(t) - \omega_m(t)) + k_i \int_0^t \left(r(\tau) - \omega_m(\tau)\right) d\tau \tag{12}$$

Obtain the closed-loop transfer function G_{PI}(s) between the reference signal r(t) as input and the motor speed ω_m as output (Assume disturbance torque $T_d = 0$).

4.3.2

One possible way to design a controller is to choose controller gains which give a specified denominator polynomial for the closed-loop system (i.e., characteristic polynomial). For a second-order system, the controller gains can be chosen in a way resulting in the following transfer function:

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{13}$$

Determine the PI controller parameters k_p, b_{sp}, and k_i so that the closed-loop system, i.e. $G_{PI}(s)$, satisfies the specified transfer function. That is, derive k_p, b_{sp}, and k_i as functions of ω_n, ζ, K, and τ.

4.3.3

Determine k_p, b_{sp}, and k_i that give $\omega_n = 16\ [\mathrm{rad/sec}]$ and $\zeta = 1$. Determine also the corresponding poles of the closed-loop system and the 2% settling time T_s:

$$T_s = \frac{4}{\zeta\omega_n} \tag{14}$$

4.3.4

Using the Final Value Theorem, compute $\omega_{ss,PI}$, the steady-state value of the output signal for a unit step in the input. Roughly plot the step response of the closed-loop system.

4.4 Closed-loop System's response to disturbances

4.4.1

In Fig. 2.1, consider the case where $r(t) = 0$ and a PI controller is being used. Determine the closed-loop system transfer function $G_D(s)$:

$$G_D(s) = \frac{\Omega(s)}{T_d(s)} \tag{15}$$

where a disturbance torque T_d is applied on the inertial load as input and the motor speed ω_m is the output. Express $G_D(s)$ as a function of the following system parameters: k_p, k_i, K, τ, and J_{eq}.

4.4.2

When a proportional controller is used ($k_p \neq 0$ and $k_i = 0$), apply the Final Value Theorem to calculate the steady-state velocity, $\omega_{ss,P}$, in response to a step input disturbance torque of amplitude T_d0.

4.4.3

When an integral controller is used ($k_p = 0$ and $k_i \neq 0$), apply the Final Value Theorem to calculate the steady-state velocity, $\omega_{ss,I}$, in response to a step input disturbance torque of amplitude T_d0.

4.1 Closed-Loop Transfer Function with Proportional Control (Answers)

4.2 Integral Control (Answers)

4.3 Proportional and Integral (PI) Control (Answer)

4.4 Closed-Loop System's Response to Disturbances (Answers)

Table 2.2. Speed Control Pre-Laboratory Assignment Results

Description	Symbol	Value	Units
Open-loop steady-state gain	K	2	$rad/(Vs)$
Open-loop time constant	τ	0.5	s
PI Controller Design			
Given Damping Ratio	ζ	1	
Given Undamped Natural Frequency	ω_n	16	rad/s
Proportional Gain	k_p	15	$(Vs)/rad$

Integral Gain	k_i	256	V/rad
Closed-Loop Poles		-16	
2% Settling Time	T_s	10.25	s
Output at Steady-State using PI Control	$\omega_{ss,PI}$	1	rad
Response To Load Disturbances			
Steady-State Velocity, P control	$\omega_{ss,P}$	0.03	rad
Steady-State Velocity, I control	$\omega_{ss,I}$	0	rad