UNIVERSITY OF VICTORIA
Midterm Exam
(March 06, 2024)

Course Name & Number	Control Theory and Systems I ECE 360
Instructor:	Dr. Sana Shuja
Duration:	50 minutes

- This exam has a total of 8 pages including this cover page, and there are 3 questions on the exam worth a total of 50 points.
- Students must count the number of pages and report any discrepancy immediately to the Invigilator.
- This exam is to be answered on this question paper and to be returned.
- This is a closed book, closed notes exam, and only a scientific calculator is allowed. Formula required for this exam are at the end of the exam.
- All Questions are to be solved.
- Use of mobile phones, and/or any communication device is strictly prohibited.

Question 1: (10 points)

Find the transfer function, $G(s) = \theta_2(s)/T(s)$, for the rotational mechanical system shown in Figure 1 below.

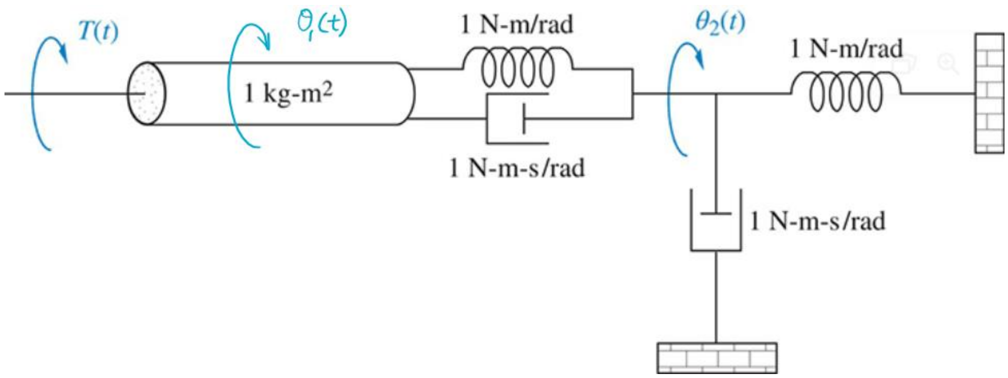

Figure 1

Solution

Writing the equations of motion,

$$(s^2+s+1)\theta_1(s) - (s+1)\theta_2(s) = T(s)$$
$$-(s+1)\theta_1(s) + (2s+2)\theta_2(s) = 0$$

where $\theta_1(s)$ is the angular displacement of the inertia.

Solving for $\theta_2(s)$,

$$\theta_2(s) = \frac{\begin{vmatrix} (s^2+s+1) & T(s) \\ -(s+1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2+s+1) & -(s+1) \\ -(s+1) & (2s+2) \end{vmatrix}} = \frac{(s+1)T(s)}{2s^3+3s^2+2s+1}$$

From which, after simplification,

$$G(s) = \frac{\theta_2(s)}{T(s)} = \frac{1}{2s^2+s+1}$$

Question 2: (20 points)

Find the value of K_1 and K_2 in the system of Fig. 2 that will result in a step response with a peak value of 1.5 sec. and a settling time of 3 sec.

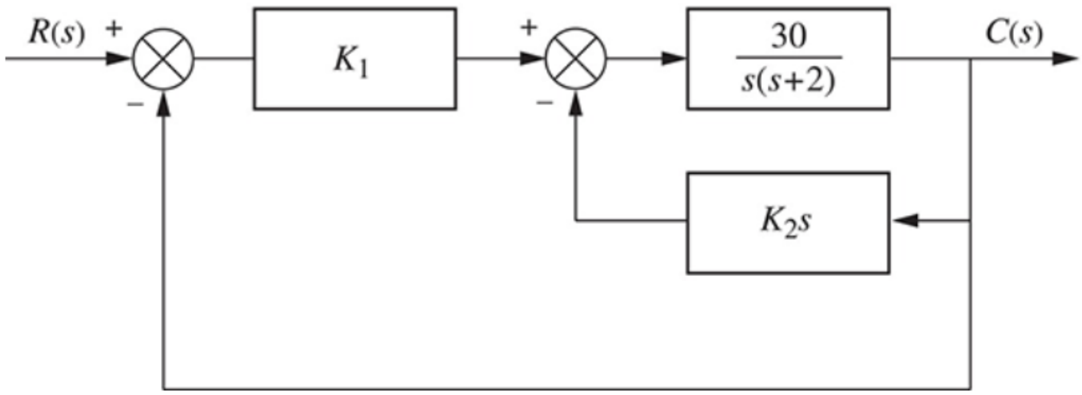

Figure 2

Solution

We first find ξ, ω_n necessary for the specifications.

We have $T_s = \frac{4}{\xi\omega_n} = 3$ and

$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 1.5.$

Eliminating ω_n from both equations we get $\frac{3\pi\xi}{4\sqrt{1-\xi^2}} = 1.5.$

Cross-multiplying, squaring both sides and solving,

we get $\xi = \sqrt{\frac{4}{4+\pi^2}} = 0.537.$ $\omega_n = 2.4829.$

The closed loop transfer function of the system is:

$$T(s) = \frac{\frac{30K_1}{s(s+2)}}{1 + \frac{30K_1}{s(s+2)} + \frac{30K_2 s}{s(s+2)}} = \frac{30K_1}{s^2 + (30K_2+2)s + 30K_1}$$

From which we get that

$30K_1 = \omega_n^2$

or $K_1 = 0.2055$

and $30K_2 + 2 = 2\xi\omega_n = 2.667$

or $K_2 = 0.0222$.

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Question 3: (20 points)

For the closed loop transfer function T(s) given below, use Ruth-Herwitz stability criteria to find the range of K for which there will be only two closed-loop, right-half-plane poles.

$$T(s) = \frac{K(s+2)}{s^4 + 3s^3 - 3s^2 + (K+3)s + (2K-4)}$$

Solution

s4	1	- 3	2K-4
s3	3	K+3	0
s2	$-\frac{K+12}{3}$	2K-4	0
s1	$\frac{K(K+33)}{K+12}$	0	0
s0	2K-4	0	0

For K < -33: 1 sign change;
For –33 < K < -12: 1 sign change;
For –12 < K < 0: 1 sign change;
For 0 < K < 2: 3 sign changes;
For K > 2: 2 sign changes.
Therefore, K > 2 yields two right-half-plane poles.