

CSC 320 Spring 2024 Assignment 1

This assignment has 6 written questions and is out of a total of 30 marks. Submit one PDF file containing your solutions on Brightspace.

Questions

1. [5 marks] Let $\Sigma = \{a, b\}$. Consider the following languages L_A and L_B over Σ :
- $$L_A = \{w \in \Sigma^* \mid w \text{ contains an even number of occurrences of symbol } a\}$$
- $$L_B = \{w \in \Sigma^* \mid \text{each pair of consecutive } b\text{'s in } w \text{ is separated by a substring of } a\text{'s of length } 2i, i \geq 0\}$$

Clarification: L_B is saying that between b 's there must be $2i$ a 's for some $i \geq 0$ (e.g. $baab \in L_B$ and $baaab \notin L_B$). Also, i must be an integer value ≥ 0 .

For each of the following strings, decide whether it is a member of L_A or L_B :

- (a) b
- (b) $aabbbbbaa$
- (c) $abaaabaaa$
- (d) $bbaaaabbaab$
- (e) a

Answers

1A: b is a member of L_A because there's even number of occurrences of a 's in the string (0 is an even number) and also a part of L_B because there's $2i$ number of a 's where i is equal to 0, satisfying the condition $i \geq 0$. Therefore, the string is a member of both L_A and L_B .

1B: $aabbbbbaa$ is a member of both L_A and L_B , since it has 4 a 's in the string (L_A) and the string can be rewritten as $aa-bb-0-bb-aa$ where we have pairs of b 's separated by $2i$ number of a 's (L_B) where number of a in between is 0.

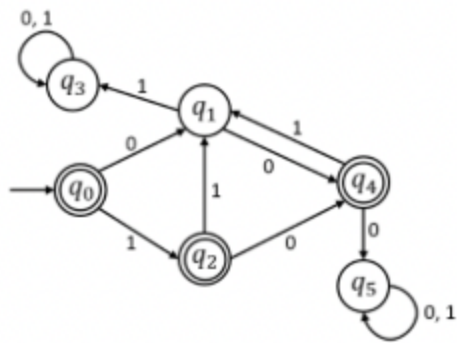
1C: $abaaabaaa$ is not a member of L_A because there's an uneven number of a 's in the string, and also not a member of L_B since it doesn't have $2i$ number of a 's in between the b pairs, where number of a in between is 3.

1D: $bbaaaabbaab$ or $bb-aa-aa-bb-aa-b$ is a member of both L_A and L_B since there's even occurrences of a in the string and there's $2i$ number of a 's in between b 's.

1E: a is not a member of L_A since there's uneven number of a 's but it's a member of L_B since there's no consecutive number of b 's to be separated by $2i$ number of a 's.

— x — 0 — x — 0 — x — 0 — x —

2. [5 marks] Consider the following DFA M :



Describe M as a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$. In particular, give

- (a) Q
- (b) Σ
- (c) δ (in the form of a transition table)
- (d) q_0
- (e) F

ANSWER 2:

The given DFA can be described as a 5-tuple $(Q, \Sigma, \delta, q_0, F)$:

Q = finite set of states $\{q_0, q_1, q_2, q_3, q_4, q_5\}$.

Σ = finite set of symbols called *alphabets* $\{0, 1\}$.

δ = Transition function $\delta: Q \times \Sigma \rightarrow Q$ has equal number of outgoing transitions for each alphabets.

q_0 = Has an *initial state* of q_0 .

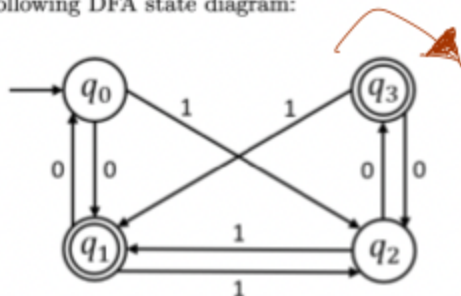
F = Has multiple finite *accept states*, $\{q_0, q_2, q_4\}$.

$(\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, \{q_0, q_2, q_4\})$

with $\delta: Q \times \Sigma \rightarrow Q$ defined by

δ	0	1
q_0	q_0	q_2
q_1	q_3	q_4
q_2	q_1	q_4
q_3	q_3	q_1
q_4	q_5	q_4
q_5	q_5	q_5

3. [5 marks] Consider the following DFA state diagram:



Transition table

δ	0	1
q_0	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_1
q_3	q_2	q_0

For each of the following strings, give the exact sequence of states that the automaton undergoes when reading the string. Furthermore, indicate whether or not the string is accepted.

- (a) $w_1 = \varepsilon$
 (b) $w_2 = 0111$
 (c) $w_3 = 1000000100$
 (d) $w_4 = 1010110$
 (e) $w_5 = 11111111$

ANSWER 3:

a) Since q_0 is not a final (accept) state, the string is not accepted.

ε

b) Since q_2 is not a final (accept) state, the string is not accepted.

0 1 1 1

c) Since q_1 is a final (accept) state, the string is accepted.

1 0 0 0 0 0 0 0 1 0 0

d) Since q_0 is not a final (accept) state, the string is not accepted.

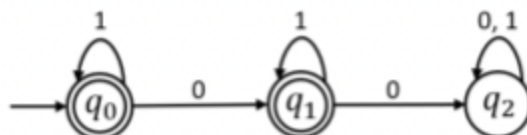
1 0 1 0 1 1 0
 2 2 1 0 2 1 0

e) Since q_1 is a final (accept) state, the string is accepted.

1 1 1 1 1 1 1 1
 2 1 2 1 2 1 1 1

— x — 0 — x — 0 — x — 0 — x —

4. [5 marks] Consider the state diagram of the following DFA F . Describe the language $L(F)$ that the automaton recognizes using set notation.



① 1
 ② 0, 1

ANSWER 4: The state diagram of the above DFA F recognizes the language $L(F)$ such that:

$$L(F) = \{ w \in \Sigma^* \mid |w| \geq 1, \text{ if } w \text{ begins with } 1 \} \text{ or } \\ \{ w \in \Sigma^* \mid |w| \geq 2, \text{ if } w \text{ starts with } 0 \text{ and ends with } 1 \}$$

— x — 0 — x — 0 — x — 0 — x —

5. [5 marks] Let $\Sigma = \{a, b\}$. For the following language L_A over Σ , construct a deterministic finite automaton A with $L(A) = L_A$. For the automaton, give the state diagram and the formal 5-tuple definition of the DFA $A = (Q, \Sigma, \delta, q_0, F)$ with a transition table describing δ .

$$L_A = \{w \in \Sigma^* \mid w \text{ starts with an } a \text{ and contains the substring } abb\}$$

ANSWER 5:

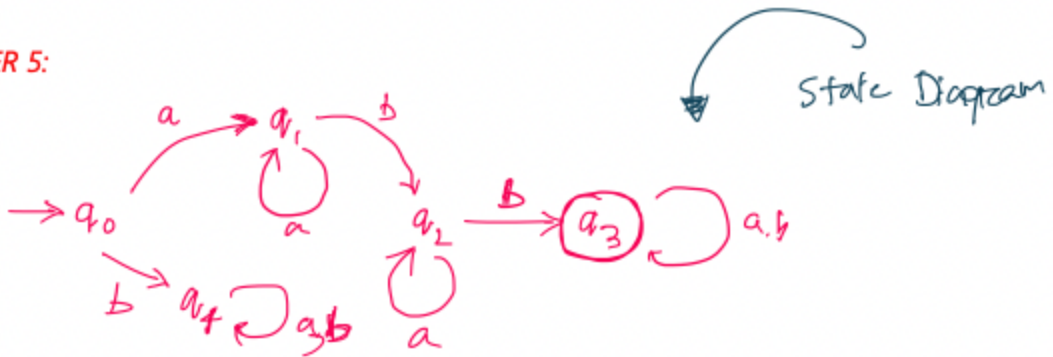


Figure: Constructed DFA **A**

The given DFA **A** can be described as a 5-tuple $(Q, \Sigma, \delta, q_0, F)$:

Q = Finite set of states $\{q_0, q_1, q_2, q_3\}$.

Σ = Finite set of symbols called *alphabets* $\{a, b\}$.

δ = Transition function $\delta: Q \times \Sigma \rightarrow Q$ has equal number of outgoing transitions for each *alphabets*.

q_0 = Has an *initial state* of q_0 .

F = Has one finite *accept states*, $\{q_3\}$.

So $(\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_3\})$ defined by:

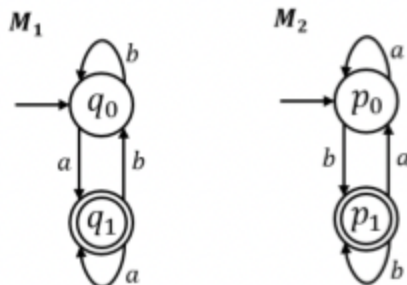
δ	a	b
q_0	q_1	q_4
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_3
q_4	q_4	q_4

Transition Table

Formal Definition



6. [5 marks] Consider the state diagrams of the following automata M_1 and M_2 :



Give a state diagram and transition table for a DFA M that recognizes the language $L(M) = L(M_1) \cap L(M_2)$. For this, you must use the construction from the proof that builds a DFA that recognizes the intersection of two regular languages.