CSC 320 Midterm 2 Practice Questions

1) Select which of the following languages are context-free:

(a) $L = \{w \in \{0,1\}^* \mid w \text{ has exactly twice as many 0's as 1's}\}$

(b) $L = \{aaa, bbab, aaabbb, aa, c, ca\}$

(c) $L(R)^*$ where R is a regular expression

(d) $L = \{a^n b^{2n} c^{2n} \mid n \geq 0\}$

2) Select every true statement:

(a) If a language is context-free, then it is non-regular.

(b) If a language is context-free, then it is regular.

(c) If a language is regular, then it is context free.

(d) If a language is non-regular, then it is not context-free.

(e) If a language is not context free, then it is non-regular.

3) Let T_R denote the class of Turing-recognizable languages.

(a) For any language $L \in T_R$, there exists a nondeterministic Turing machine M with $L(M) = L$.

(b) For any language $L \in T_R$ there exists a nondeterministic finite automaton N with $L(N) = L$.

(c) Let R be a regular expression. Then $L(R) \in T_R$.

(d) Let P be a pushdown automaton. Then $L(P) \in T_R$.

(e) $\emptyset \in T_R$.

4) Consider the following CFG $G = (\{S, A, B\}, \{a, b\}, R, S)$ where the rules in R are given as follows.

$$S \to SS \mid AB$$
$$A \to Aa \mid a$$
$$B \to Bb \mid b$$

(a) Show that G is ambiguous by giving two leftmost derivations of a string in $L(G)$.

(b) Convert G to an equivalent PDA following the steps of the CFG to PDA conversion.

(c) Convert G into Chomsky Normal Form. Show all your steps.

5) Consider the language $L = \{0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } i + k = j\}$.

(a) Give a context free grammar G with $L(G) = L$.

(b) Give a state diagram for a PDA which recognizes L (without using the CFG to PDA conversion).

6) Prove that the language $L = \{0^n \mid n > 0,\ n \text{ is a prime number}\}$ is not context free using the pumping lemma for context free languages.

7) Give a high-level description of a Turing machine which recognizes the following language:

$$L = \{0^i 1^j 2^k \mid i \times j = k \text{ and } i, j, k \geq 1\}$$