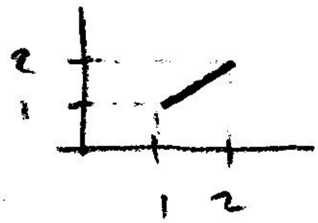
ELEC 360 Quiz #1

1. Find the Laplace transform of

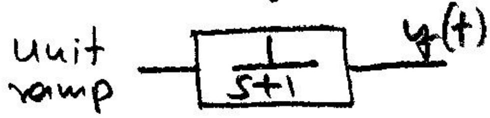


$$f(t) = t\left[u(t-1) - u(t-2)\right] \qquad u(t):\text{step}$$

$$= (t-1)u(t-1) - (t-2)u(t-2) + u(t-1) - 2u(t-2)$$

$$F(s) = \frac{e^{-s}}{s^2} - \frac{e^{-2s}}{s^2} + \frac{e^{-s}}{s} - \frac{2e^{-2s}}{s}$$

2. Find $y(t)$ for



$$Y(s) = \frac{1}{s^2}\,\frac{1}{s+1} = \frac{b_1}{s} + \frac{b_2}{s^2} + \frac{a}{1+s}$$

$$a = \left[\frac{\cancel{s+1}}{s^2\cancel{(s+1)}}\right]_{s=-1} = 1$$

$$b_2 = \left[\frac{\cancel{s^2}}{\cancel{s^2}(s+1)}\right]_{s=0} = 1$$

$$b_1 = \left[\frac{d}{ds}\left[\frac{1}{s+1}\right]\right]_{s=0} = \left[\frac{-1}{(s+1)^2}\right]_{s=0} = -1$$

$$Y(s) = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}$$

$$y(t) = t - 1 + e^{-t} \quad \text{for } t \geq 0 \text{ or}$$

$$y(t) = \left[t - 1 + e^{-t}\right]u(t)$$

3. Find the solution $y(t)$ of

$$\ddot{y} + 2\dot{y} + 2y = 0 \qquad y(0) = 2 \quad \dot{y}(0) = 0$$

$$s^2Y(s) - sy(0) - \cancel{\dot{y}(0)} + 2sY(s) - 2y(0) + 2Y(s) = 0$$

$$Y(s)(s^2 + 2s + 2) = sy(0) + 2y(0) = 2s + 4$$

s^2+2s+2 has complex conjugate roots!

$$Y(s) = \frac{2(s+2)}{(s+1)^2 + 1^2} = \frac{2(s+1)}{(s+1)^2+1^2} + \frac{2}{(s+1)^2+1^2}$$

$$y(t) = 2e^{-t}\cos t + 2e^{-t}\sin t \qquad t \geq 0$$