2A **2.1** Let each of $\mathcal{G}$ and $\mathcal{H}$ denote a system operator that maps a function to a function; let x and y denote functions; and assume that all other variables denote numbers. Fully parenthesize each of the expressions below in order to show the implied grouping of all operations.
(a) $\mathcal{H}x(t) = t^2 + 1$;
(b) $\mathcal{G}\mathcal{H}y(t)$;
(c) $\mathcal{H}x + y$; and
(d) $x\mathcal{H}\mathcal{G}y$.

2A **Answer (a).**

$(\mathcal{H}x)(t) = [(t)^2] + 1$

2A **Answer (b).**

$[\mathcal{G}(\mathcal{H}y)](t)$

2A **Answer (c).**

$(\mathcal{H}x) + y$

2A **Answer (d).**

$(x)[\mathcal{H}(\mathcal{G}y)]$

2A **2.2** Let $\mathcal{H}$ denote a system operator that maps a function to a function; let x and y denote functions; and let all other variables denote numbers. Using strictly-correct mathematical notation, write an expression for each quantity specified below. Only use brackets for grouping when strictly required. Use $\mathcal{D}$ to denote the derivative operator.
(a) the output of the system $\mathcal{H}$ when its input is y;
(b) the output of the system $\mathcal{H}$ evaluated at $2t-1$ when the input to the system is x;
(c) the output of the system $\mathcal{H}$ evaluated at t when the input to the system is ax;
(d) the output of the system $\mathcal{H}$ evaluated at $5t$ when the input to the system is $x+y$;
(e) the derivative of the output of the system $\mathcal{H}$ when its input is ax;
(f) the output of the system $\mathcal{H}$ when its input is the derivative of ax;
(g) the sum of: 1) the output of the system $\mathcal{H}$ when its input is x; and 2) the output of the system $\mathcal{H}$ when its input is y;
(h) the output of the system $\mathcal{H}$ when its input is $x+y$; and
(i) the derivative of x evaluated at $5t-3$.

2A **Answer (a).**

$\mathcal{H}y$

2A **Answer (b).**

$\mathcal{H}x(2t-1)$

2A **Answer (c).**

$\mathcal{H}\{ax\}(t)$

2A **Answer (d).**

$\mathcal{H}\{x+y\}(5t)$

2A **Answer (e).**

$\mathcal{D}\mathcal{H}\{ax\}$, where $\mathcal{D}$ denotes the derivative operator

2A **Answer (f).**

$\mathcal{H}\mathcal{D}\{ax\}$, where $\mathcal{D}$ denotes the derivative operator

2A **Answer (g).**

$\mathcal{H}x+\mathcal{H}y$

2A **Answer (h).**

$\mathcal{H}\{x+y\}$

2A **Answer (i).**

$\mathcal{D}x(5t-3)$, where $\mathcal{D}$ denotes the derivative operator

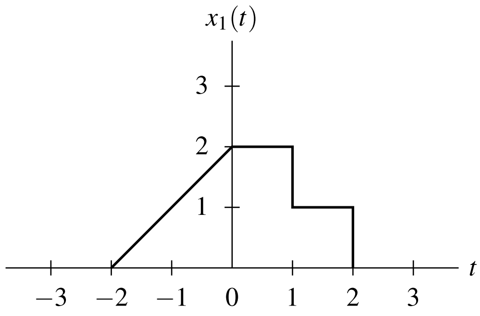
2A **3.1** Identify the independent- and dependent-variable transformations that must be applied to the function x in order to obtain each function y given below. Choose the transformations such that time shifting precedes time scaling and amplitude scaling precedes amplitude shifting. Be sure to clearly indicate the order in which the transformations are to be applied.
(a) $y(t) = x(2t-1)$;
(b) $y(t) = x(\frac{1}{2}t+1)$;
(c) $y(t) = 2x(-\frac{1}{2}t+1)+3$;
(d) $y(t) = -\frac{1}{2}x(-t+1)-1$; and
(e) $y(t) = -3x(2[t-1])-1$;
(f) $y(t) = x(7[t+3])$.

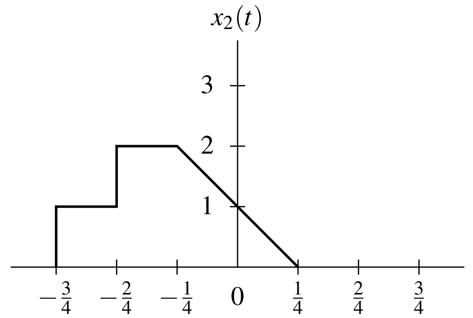
2A **Answer (f).**

We have that $y(t) = x(7[t+3])$. To obtain y from x, we apply the following transformations:

1. time shift by -21 (i.e., shift left by 21); and
2. time scale by 7 (i.e., compress horizontally by a factor of 7).

2A **3.2** Given the functions x_1 and x_2 shown in the figures below, express x_2 in terms of x_1.



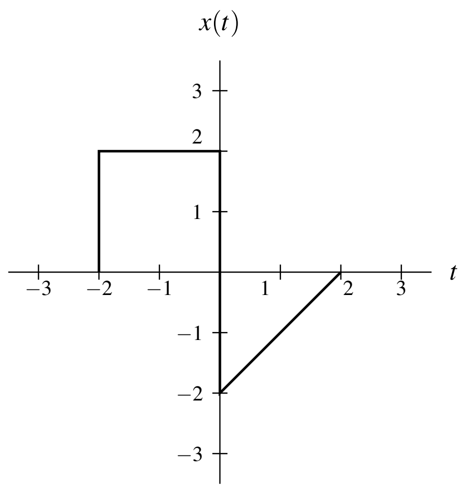


2A **Answer.**

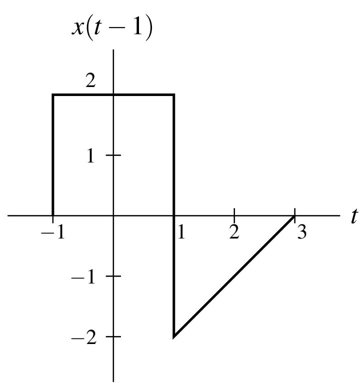
We observe that x_2 is simply a time-shifted, time-scaled, and time-reversed version of x_1. More specifically, x_2 is generated from x_1 through the following transformations (in order): 1) time shifting by 1, 2) time scaling by 4, and 3) time reversal. Thus, we have

$$x_2(t) = x_1(-4t - 1).$$

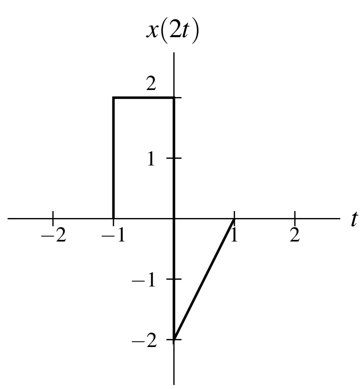
2A **3.4** Given the function x shown in the figure below, plot and label each of the following functions:
(a) $x(t-1)$;
(b) $x(2t)$;
(c) $x(-t)$;
(d) $x(2t+1)$; and
(e) $\frac{1}{4}x(-\frac{1}{2}t+1)-\frac{1}{2}$.



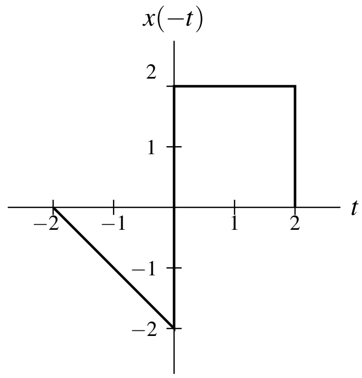
2A **Answer (a).**



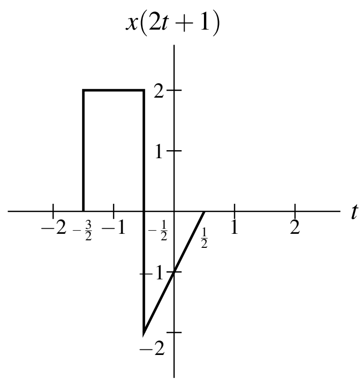
2A **Answer (b).**



2A **Answer (c).**



2A Answer (d).



2A **3.6** Determine if each function x given below is periodic, and if it is, find its fundamental period.
(a) $x(t) = \cos(2\pi t) + \sin(5t)$;
(b) $x(t) = [\cos(4t - \frac{\pi}{3})]^2$;
(c) $x(t) = e^{j2\pi t} + e^{j3\pi t}$;
(d) $x(t) = 1 + \cos(2t) + e^{j5t}$;
(e) $x(t) = \cos(14t - 1) + \cos(77t - 3)$;
(f) $x(t) = \cos(et) + \sin(42t)$; and
(g) $x(t) = |\sin(\pi t)|$.

2A **Answer (e).**

Let T_1 and T_2 denote the periods of $\cos(14t-1)$ and $\cos(77t-3)$, respectively. We have

$$T_1 = \frac{2\pi}{14} = \frac{\pi}{7}, \quad T_2 = \frac{2\pi}{77}, \quad \text{and} \quad \frac{T_1}{T_2} = \frac{\pi/7}{2\pi/77} = \frac{77}{14} = \frac{11}{2}.$$

Since T_1/T_2 is rational, x is periodic. The period T of x is $T = 2T_1 = \frac{2\pi}{7}$.

2A **Answer (f).**

Let T_1 and T_2 denote the periods of $\cos(et)$ and $\sin(42t)$, respectively. We have

$$T_1 = \frac{2\pi}{e} \quad \text{and} \quad T_2 = \frac{2\pi}{42}.$$

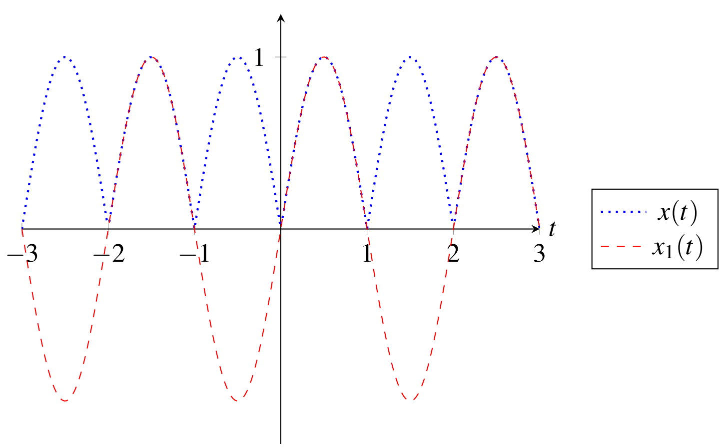
So, we have

$$\frac{T_1}{T_2} = \frac{\left(\frac{2\pi}{e}\right)}{\left(\frac{2\pi}{42}\right)} = \left(\frac{2\pi}{e}\right)\left(\frac{42}{2\pi}\right) = \frac{42}{e}.$$

Since T_1/T_2 is irrational, x is not periodic. (Note that e is irrational.)

2A **Answer (g).**

The function $x_1(t) = \sin(\pi t)$ has the fundamental period $T_1 = \frac{2\pi}{\pi} = 2$. Taking the absolute value of x_1 halves the fundamental period. This is clear from the graph of x and x_1 shown below. Therefore, the fundamental period T of x is $T = T_1/2 = 2/2 = 1$.



2A **3.9** Determine whether each function x given below is even, odd, or neither even nor odd.
(a) $x(t) = t^3$;
(b) $x(t) = t^3|t|$;
(c) $x(t) = |t^3|$;
(d) $x(t) = \cos(2\pi t)\sin(2\pi t)$;
(e) $x(t) = e^{j2\pi t}$; and
(f) $x(t) = \frac{1}{2}[e^t + e^{-t}]$.

2A **Answer (c).**

From the definition of x, we have

$$\begin{aligned} x(-t) &= \left|(-t)^3\right| \\ &= \left|-t^3\right| \\ &= \left|t^3\right| \\ &= x(t). \end{aligned}$$

Thus, we have that $x(t) = x(-t)$ for all t. Therefore, x is even.

2A **Answer (d).**

From the definition of x, we have

$$\begin{aligned} x(-t) &= [\cos(-2\pi t)][\sin(-2\pi t)] \\ &= (\cos 2\pi t)(-\sin 2\pi t) \\ &= -(\cos 2\pi t)(\sin 2\pi t) \\ &= -x(t). \end{aligned}$$

Thus, we have that $x(t) = -x(-t)$ for all t. Therefore, x is odd.

2A **3.10** Prove each of the following assertions:
(a) The sum of two even functions is even.
(b) The sum of two odd functions is odd.
(c) The sum of an even function and an odd function, where neither function is identically zero, is neither even nor odd.
(d) The product of two even functions is even.
(e) The product of two odd functions is even.
(f) The product of an even function and an odd function is odd.

2A **Answer (b).**

Let $y(t) = x_1(t) + x_2(t)$, where x_1 and x_2 are both odd. From the definition of y, we have

$$\begin{aligned} y(-t) &= x_1(-t) + x_2(-t) \\ &= [-x_1(t)] + [-x_2(t)] \\ &= -[x_1(t) + x_2(t)] \\ &= -y(t). \end{aligned}$$

Thus, $y(t) = -y(-t)$. Therefore, y is odd.

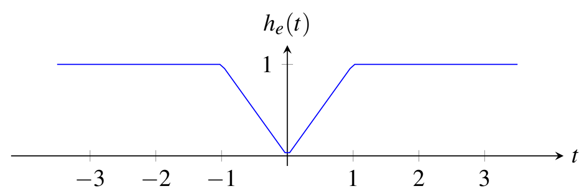
2A **3.17** Suppose that the function h is causal and has the even part h_{e} given by

$$h_{\mathrm{e}}(t) = t[u(t) - u(t-1)] + u(t-1) \quad \text{for } t \geq 0.$$

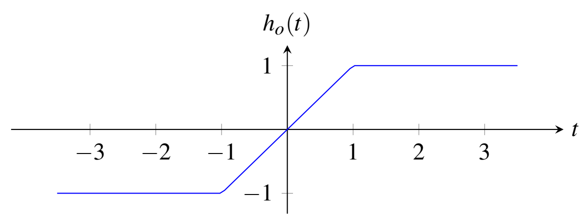
Find h.

2A **Answer.**

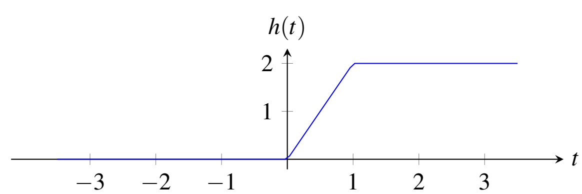
FIRST SOLUTION (USING GRAPHS AS AN AID). Since h_e is even, we can deduce $h_e(t)$ for $t < 0$ by symmetry. This yields the function in the following graph:



Since $h_o(t) = h(t) - h_e(t)$ and $h(t) = 0$ for $t < 0$, we have that for $t < 0$, $h_o(t) = -h_e(t)$. Since h_o is odd, we can infer $h_o(t)$ for $t > 0$ by symmetry. This yields the function in the following graph:



Since $h = h_e + h_o$, we can conclude that h is the function shown in the following graph:



ALTERNATIVE SOLUTION (USING ONLY EQUATIONS). We have that

$$\begin{aligned} \text{for } t \geq 0\text{:} \quad h_e(t) &= t[u(t) - u(t-1)] + u(t-1) \\ &= tu(t) + (-t+1)u(t-1). \end{aligned}$$

Since h_e is even, we can deduce that

$$\begin{aligned} \text{for } t < 0\text{:} \quad h_e(t) &= h_e(-t) \\ &= (-t)u(-t) + (t+1)u(-t-1). \end{aligned}$$

Let h_o denote the odd part of h. Since h is causal and $h = h_e + h_o$, we know that

$$\text{for } t < 0\text{:} \quad h_o(t) = -h_e(t).$$

Using this observation, we can deduce that

$$\begin{aligned} \text{for } t < 0\text{:} \quad h_o(t) &= -h_e(t) \\ &= -[(-t)u(-t) + (t+1)u(-t-1)] \\ &= (t)u(-t) + (-t-1)u(-t-1). \end{aligned}$$

Since h_o is odd, we have that $h_o(0) = 0$ and

$$\text{for } t > 0: \quad \begin{aligned} h_o(t) &= -h_o(-t) \\ &= -[(-t)u(t) + (t-1)u(t-1)] \\ &= tu(t) + (-t+1)u(t-1). \end{aligned}$$

Therefore, we can conclude that $h(t) = 0$ for all $t < 0$ and

$$\text{for } t \geq 0: \quad \begin{aligned} h(t) &= h_e(t) + h_o(t) \\ &= [tu(t) + (-t+1)u(t-1)] + [tu(t) + (-t+1)u(t-1)] \\ &= (2t)u(t) + (2-2t)u(t-1) \\ &= (2t)[u(t) - u(t-1)] + 2u(t-1). \end{aligned}$$

2A **3.18** For each case below, for the function x (of a real variable) having the properties stated, find $x(t)$ for all t.

(a) The function x is such that:

- $x(t) = \begin{cases} t-1 & 1 \le t \le 2 \\ 3-t & 2 < t \le 3; \end{cases}$
- the function v is causal, where $v(t) = x(t-1)$; and
- the function w is odd, where $w(t) = x(t+1)$.

(b) The function x is such that:

- $x(t) = t-1$ for $0 \le t \le 1$;
- the function v is causal, where $v(t) = x(t-1)$; and
- the function w is odd, where $w(t) = x(t)+1$.

2A **Answer (b).**

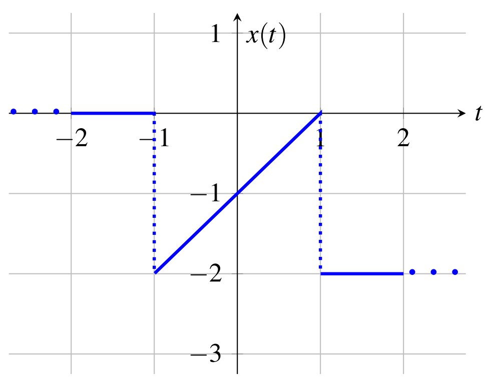
Trivially, we have that $x(t) = t-1$ for $0 \le t \le 1$. Since $v(t) = x(t-1)$ is causal, we have

$$\begin{aligned} & v(t) = 0 \text{ for all } t < 0 \\ \Rightarrow\quad & x(t-1) = 0 \text{ for all } t < 0 \\ \Rightarrow\quad & x(t) = 0 \text{ for all } t+1 < 0 \\ \Rightarrow\quad & x(t) = 0 \text{ for all } t < -1. \end{aligned}$$

Since $w(t) = x(t)+1$ is odd, x shifted upwards by 1 unit is odd (or equivalently, x would be odd if the horizontal axis were moved downwards by 1 unit). Combining the above observations, we conclude that x is the piecewise-linear function given by

$$x(t) = \begin{cases} 0 & t < -1 \\ t-1 & -1 \le t \le 1 \\ -2 & t > 1. \end{cases}$$

A plot of x is shown below.



2A **3.20** Fully simplify each of the expressions below.

(a) $\int_{-\infty}^{\infty} \sin\left(2t + \frac{\pi}{4}\right) \delta(t)dt$;
(b) $\int_{-\infty}^{t} \cos(\tau)\delta(\tau+\pi)d\tau$;
(c) $\int_{-\infty}^{\infty} x(t)\delta(at-b)dt$, where a and b are real constants and $a \neq 0$;
(d) $\int_{0}^{2} e^{j2t}\delta(t-1)dt$;
(e) $\int_{-\infty}^{t} \delta(\tau)d\tau$; and
(f) $\int_{0}^{\infty} \tau^2 \cos(\tau)\delta(\tau+42)d\tau$.

2A **Answer (a).**

From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{\infty} \sin\left(2t + \tfrac{\pi}{4}\right) \delta(t)dt &= \left[\sin\left(2t + \tfrac{\pi}{4}\right)\right]\Big|_{t=0} \\ &= \sin\left(\tfrac{\pi}{4}\right) \\ &= \tfrac{1}{\sqrt{2}}.\end{aligned}$$

2A **Answer (b).**

From the sifting property of the unit-impulse function, we have

$$\begin{aligned}\int_{-\infty}^{t} [\cos\tau]\delta(\tau+\pi)d\tau &= \begin{cases} [\cos\tau]|_{\tau=-\pi} & t \geq -\pi \\ 0 & t < -\pi \end{cases} \\ &= \begin{cases} \cos(-\pi) & t \geq -\pi \\ 0 & t < -\pi \end{cases} \\ &= \begin{cases} -1 & t \geq -\pi \\ 0 & t < -\pi \end{cases} \\ &= -u(t+\pi).\end{aligned}$$

2A **Answer (c).**

We use a change of variable. Let $\lambda = at$ so that $t = \lambda/a$ and $d\lambda = adt$. Performing the change of variable and simplifying yields

$$\begin{aligned}\int_{-\infty}^{\infty} x(t)\delta(at-b)dt &= \begin{cases} \int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \delta(\lambda-b)\left(\frac{1}{a}\right) d\lambda & a > 0 \\ \int_{\infty}^{-\infty} x\left(\frac{\lambda}{a}\right) \delta(\lambda-b)\left(\frac{1}{a}\right) d\lambda & a < 0 \end{cases} \\ &= \begin{cases} \frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \delta(\lambda-b) d\lambda & a > 0 \\ -\frac{1}{a}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \delta(\lambda-b) d\lambda & a < 0 \end{cases} \\ &= \frac{1}{|a|}\int_{-\infty}^{\infty} x\left(\frac{\lambda}{a}\right) \delta(\lambda-b) d\lambda \\ &= \frac{1}{|a|}\left[x\left(\frac{\lambda}{a}\right)\right]\Big|_{\lambda=b} \\ &= \frac{1}{|a|} x\left(\frac{b}{a}\right).\end{aligned}$$

2A **Answer (f).**

We have

$$\begin{aligned}\int_{0}^{\infty} (\tau^2\cos\tau)\delta(\tau+42)d\tau &= \int_{0}^{\infty} 0 d\tau \\ &= 0.\end{aligned}$$

(Note that the integrand is zero over the entire integration interval, since $\delta(\tau+42) = 0$ for all $\tau \neq -42$.)

2A **D.101** Indicate whether each of the following is a valid MATLAB identifier (i.e., variable/function name):
(a) `4ever`
(b) `$rich$`
(c) `foobar`
(d) `foo_bar`
(e) `_foobar`

2A **Answer (a).**

A MATLAB identifier cannot begin with a numeric character. Thus, `4ever` is not a valid identifier.

2A **Answer (b).**

A MATLAB identifier cannot contain the `$` character. Thus, `$rich$` is not a valid identifier.

2A **Answer (c).**

The name `foobar` is a valid identifier.

2A **Answer (d).**

The name `foo_bar` is a valid identifier.

2A **Answer (e).**

A MATLAB identifier cannot begin with an underscore character. Thus, `_foobar` is not a valid identifier.

2A **D.106** Consider the vector v defined by the following line of code:

```
v = [0 1 2 3 4 5]
```

Write an expression in terms of v that yields a new vector of the same dimensions as v, where each element t of the original vector v has been replaced by the given quantity below. In each case, the expression should be as short as possible.
(a) $2t-3$;
(b) $1/(t+1)$;
(c) t^5-3; and
(d) $|t|+t^4$.

2A **Answer (a).**

```
2 * v - 3
```

2A **Answer (b).**

```
1 ./ (v + 1)
```

2A **Answer (c).**

```
v .^ 5 - 3
```

2A **Answer (d).**

```
abs(v) + v .^ 4
```

