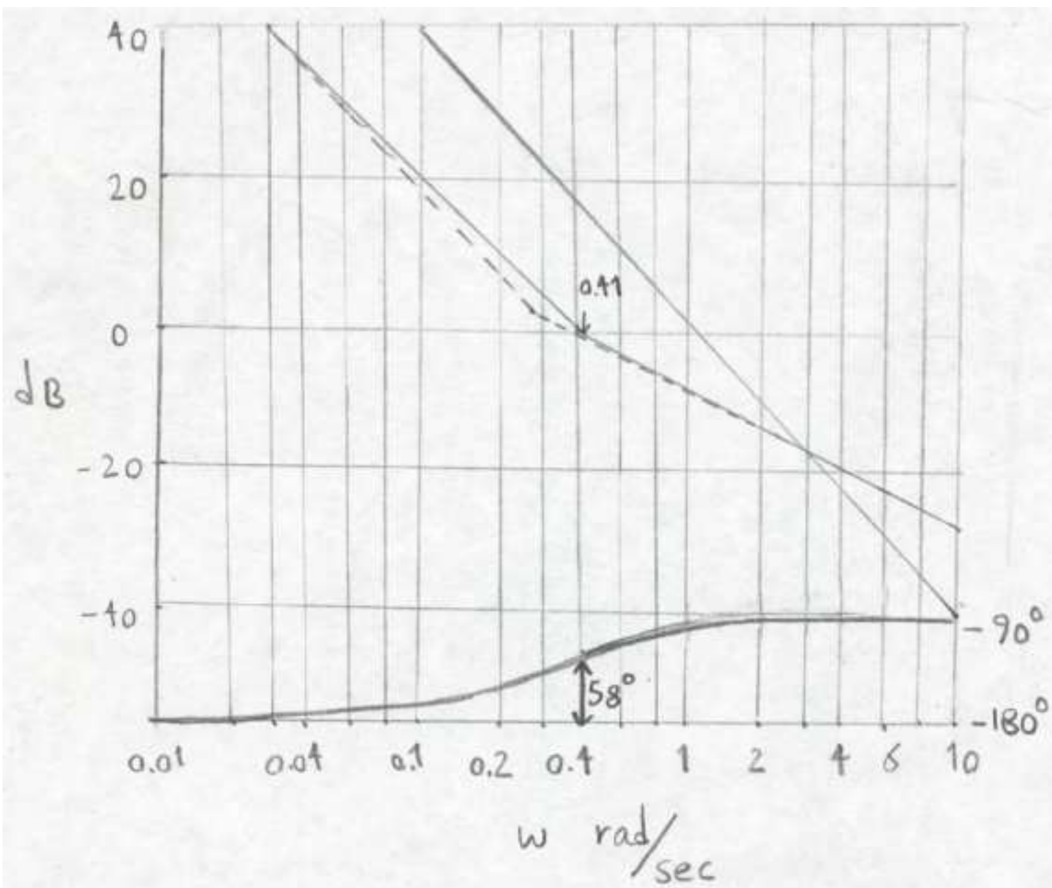
B-7-31

Choose the gain crossover frequency to be approximately 0.4 $\frac{rad}{sec}$ (as suggested in the question, so that the bandwidth of the closed-loop system is close to the chosen gain cross over frequency) and the phase margin to be approximately 60° which represents an adequate phase margin, as specified in the question.

Draw the high frequency asymptote having a slope of -20 $\frac{dB}{dec}$ (since at high frequencies you have 2 poles – 1 zero, this leads to -20 $\frac{dB}{dec}$) to cross the 0 dB line at about $\boldsymbol{\omega = 0.4}\frac{rad}{sec}$. Choose the corner frequency to be 0.25 $\frac{rad}{sec}$. Then the low-frequency asymptote with slope -40 $\frac{dB}{dec}$ (because of the 2 poles at the origin) can be drawn in the bode diagram. See the Bode diagram shown below:



The actual magnitude curve which would give a phase margin of approximately 60° crosses the 0 dB line at about $\boldsymbol{\omega = 0.4}\frac{rad}{sec}$ and the phase margin becomes approximately 58°.

Since we have chosen the corner frequency to be 0.25 $\frac{rad}{sec}$, we get

$$\boldsymbol{T_d = 4}$$

From the Bode diagram, $\boldsymbol{K_d}$ must be chosen to be -21.4 dB (by expanding the low frequency asymptote and finding the gain at $\boldsymbol{\omega = 1 \frac{rad}{sec}}$ (i.e $\boldsymbol{K_d}$ is the Static Acceleration Error Constant $\boldsymbol{K_a}$). This gives

$$\boldsymbol{K_d = -21.4\ dB}$$

$$\boldsymbol{= 0.0851}$$

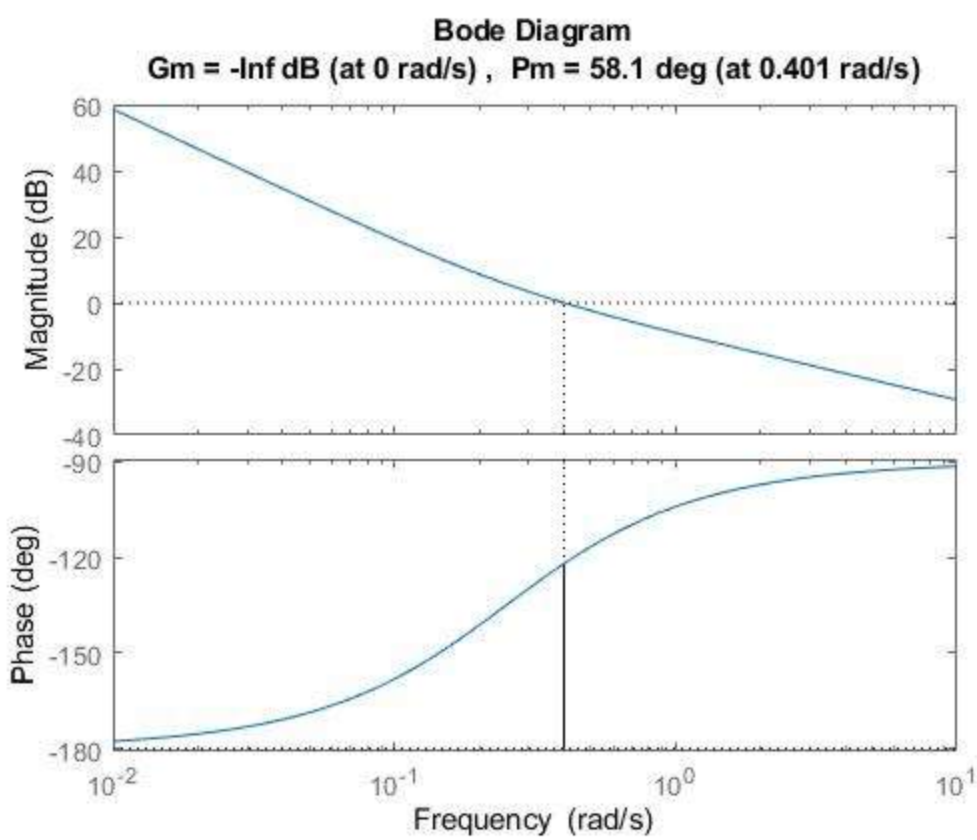
Thus

$$\boldsymbol{K_d(1 + T_d S) = 0.0851(1 + 4s)}$$

Then, the open-loop transfer function becomes

$$\boldsymbol{G(s) = \frac{0.0851(1 + 4s)}{s^2}}$$

Here are the open-loop Bode plot and the corresponding margins:



The closed-loop transfer function is

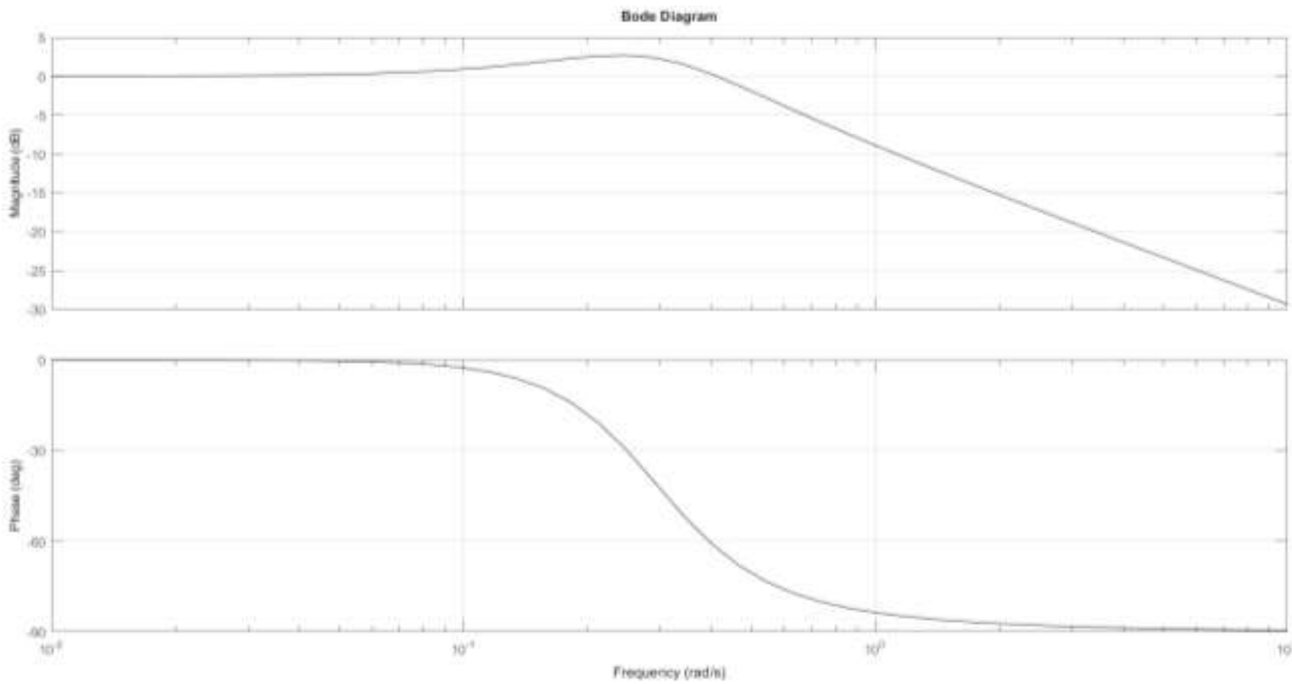
$$\frac{\boldsymbol{C(s)}}{\boldsymbol{R(S)}} = \frac{\mathbf{0.0851(1+4}\boldsymbol{s}\mathbf{)}}{\boldsymbol{s}^{\mathbf{2}} + \mathbf{0.0851(1+4}\boldsymbol{s}\mathbf{)}}$$

$$= \frac{4s+1}{11.751s^2+4s+1}$$

A Bode diagram of the closed-loop transfer function

$$\frac{C(j\omega)}{R(j\omega)} = \frac{4j\omega+1}{11.751(j\omega)^2+4j\omega+1}$$

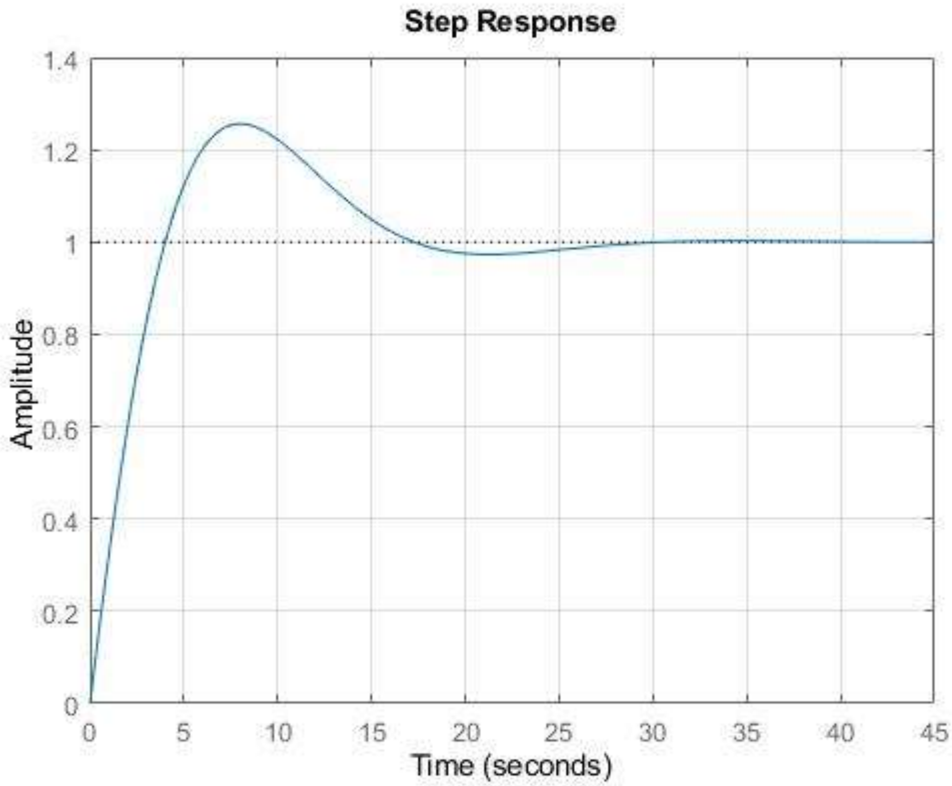
is shown below.



From this diagram we see that the bandwidth is approximately 0.5 $\frac{rad}{sec}$. The step response of the closed-loop system is given below. It can be seen that it has an acceptable overshoot since the phase margin is 58°

Note: To make the close-loop system faster, you would need to increase the closed-loop bandwidth by increasing the gain crossover frequency which can be increased by increasing the gain $\boldsymbol{K_d}$. Try in Matlab $\boldsymbol{K_d} = 0.2$.

This is the closed loop step response for $K_d = 0.0851$



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Note: This problem can be solved using the Matlab program lead.m which can be downloaded from the course web page.

Let us use the following lead compensator:

$$G_c(s) = K_c\alpha\frac{T_s + 1}{\alpha T_s + 1} = K_c\frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

Since K_v is specified as 4.0 sec^{-1}, we have

$$K_v = \lim_{s\to 0} sK_c\alpha\frac{T_s + 1}{\alpha T_s + 1}\frac{K}{s(0.1s + 1)(s + 1)} = K_c\alpha K = 4$$

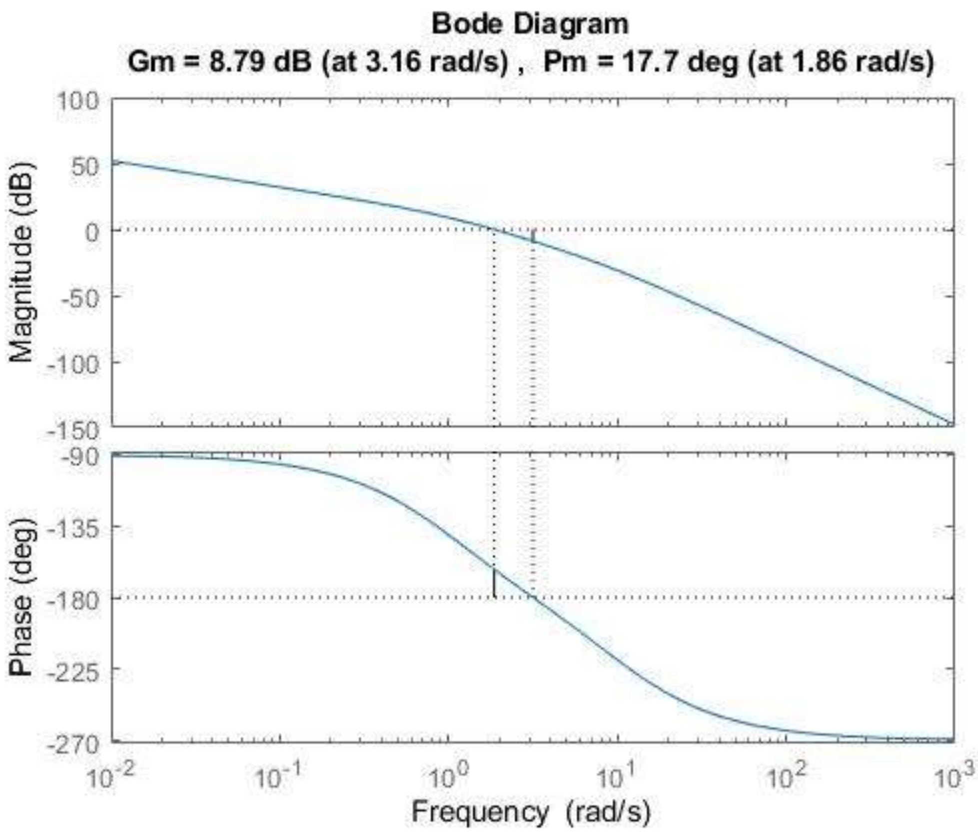
Let us set K=1 and define $K_c\alpha = \widehat{K}$. Then

$$\widehat{K} = 4$$

Next, plot a bode diagram of $\dfrac{4}{s(0.1s+1)(s+1)} = \dfrac{4}{0.1s^3+1.1s^2+s}$

The following Matlab program produces the Bode diagram shown below.

```
num = [0 0 4];
den = [0.1 1.1 1 0];
margin(num,den);
```



From this plot, the phase and gain margins are 17.7º and 8.8 dB, respectively. Since the specifications call for a phase margin of 45º, let us choose

$$\phi_m = 45° - 17.7° + 11.3° = 40°$$

(This means that 11.3º has been added to compensate for the shift in the gain crossover frequency.) The maximum added phase lead is 40º. Since

$$sin\phi_m = \frac{1-\alpha}{1+\alpha} \ (\phi_m = 40°)$$

α is determined as 0.2174. Let us choose, instead of 0.2174, α to be 0.21, or

$$\alpha = 0.21$$

Next step is to determine the corner frequencies $\omega = \frac{1}{T}$ and $\omega = \frac{1}{\alpha T}$ of the lead compensator. Note that the maximum phase-lead angle ϕ_m occurs at the geometric mean of the two corner frequencies, or $\omega = \frac{1}{\sqrt{\alpha}T}$. The amount of the modification in the magnitude curve at $\omega = \frac{1}{\sqrt{\alpha}T}$ due to the inclusion of the term $\frac{(T_s+1)}{(\alpha T_s+1)}$ is

$$\left|\frac{1+j\omega T}{1+j\omega\alpha T}\right|_{\omega=\frac{1}{\sqrt{\alpha}\,T}} = \frac{1}{\sqrt{\alpha}}$$

Note that

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.21}} = 2.1822 = 6.7778\ dB$$

We need to find the frequency point where, when the lead compensator is added, the total magnitude becomes 0dB. The magnitude where $G(j\omega)$ is -6.7778 dB corresponds to $\omega = 2.81\frac{rad}{sec}$. We select this frequency to be the new gain crossover frequency ω_c. Then we obtain

$$\frac{1}{T} = \sqrt{\alpha}\omega_c = \sqrt{0.21} \times 2.81 = 1.2877$$

$$\frac{1}{\alpha T} = \frac{\omega_c}{\sqrt{\alpha}} = \frac{2.81}{\sqrt{0.21}} = 6.139$$

Hence

$$G_c(s) = K_c \frac{s + 1.2877}{s + 6.1319}$$

and

$$K_c = \frac{\widehat{K}}{\alpha} = \frac{4}{0.21}$$

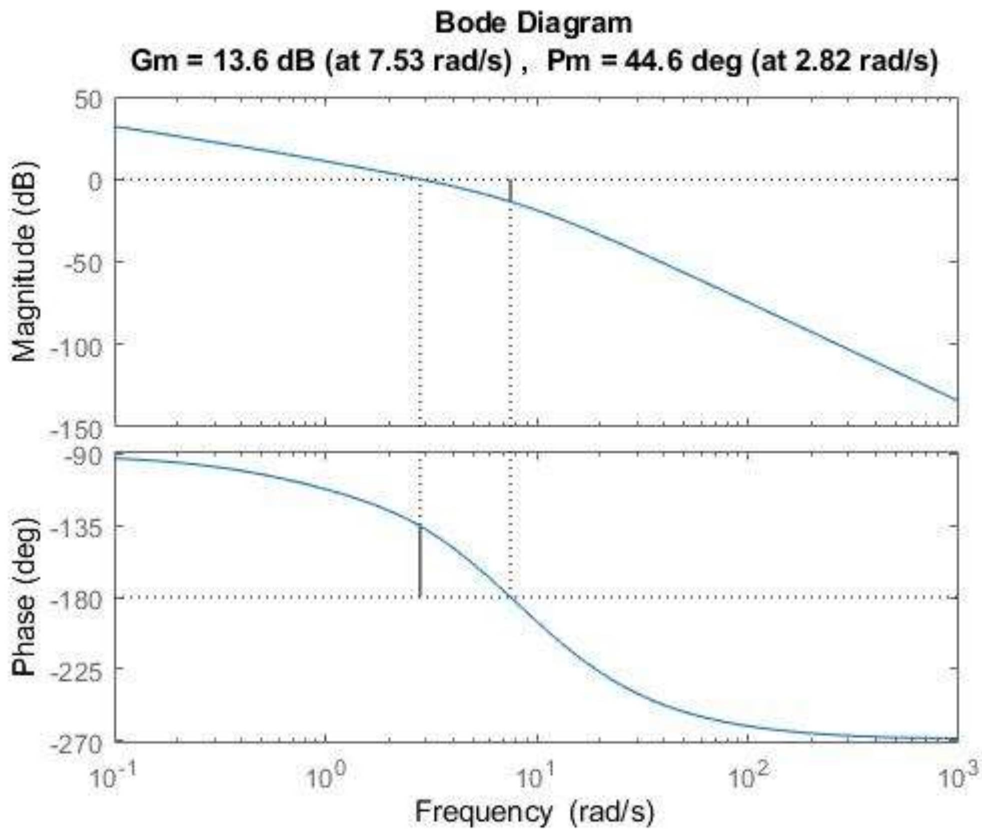
Thus

$$G_c(s) = \frac{4}{0.21}\,\frac{s + 1.2877}{s + 6.1319} = 4\,\frac{0.7766s + 1}{0.16308s + 1}$$

The open-loop transfer function becomes:

$$G_c(s)G(s) = 4\frac{0.7766s+1}{0.16308s+1}\frac{1}{s(0.1s+1)(s+1)}$$
$$= \frac{3.1064s+4}{0.01631s^4+0.2744s^3+1.2631s^2+s}$$

The bode diagram and the margins can be obtained using Matlab:

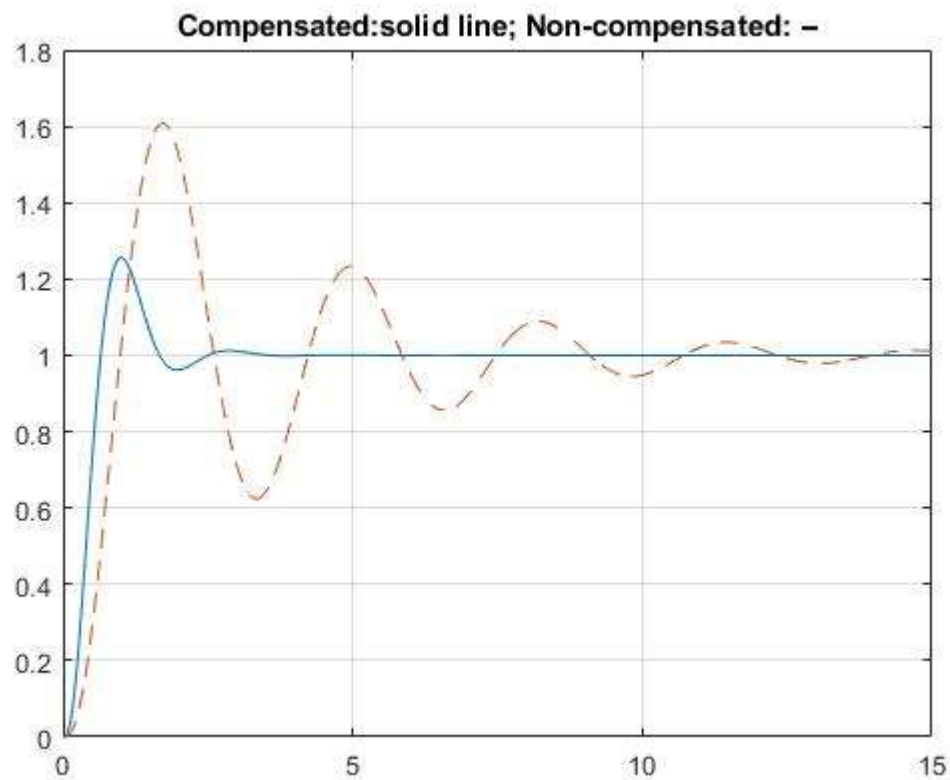


This open-loop transfer function yields phase margin of 44.6º and gain margin of 13.6 dB. So, the requirements on the phase margin and gain margin are satisfied. The closed-loop transfer function of the designed system is

$$\frac{C(s)}{R(S)} = \frac{3.1064s+4}{0.01631s^4+0.2794s^3+1.2631s^2+4.1064s+4}$$

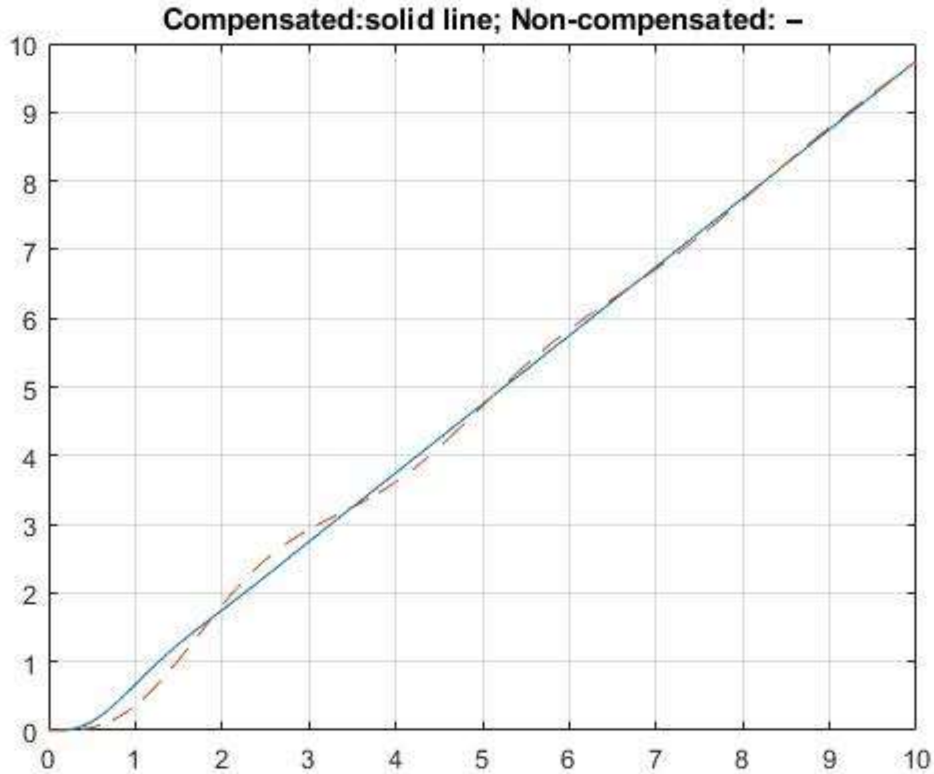
The following Matlab program produces the closed-loop unit-step response curves for the Compensated and the Non-compensated system shown on the next page.

```
tt=[0:0.02:15];
num = [0 0 4];
den1=[0.1 1.1 1 4];
yyo=step(num,den1,tt);
numc = [0 0 0 3.1064 4];
denc = [0.01631 0.2794 1.2631 4.1064 4];
yy1=step(numc,denc,tt);
figure
plot(tt,yy1,tt,yyo,'--')
title ('Compensated:solid line; Non-compensated: --');
grid
```



Similarly the following Matlab program produces the closed-loop unit-ramp responses for the Compensated and Non-Compensated systems: below,

```
tt1=[0:0.02:10];
yyo1=step(num,[den1 0],tt1);
yy2=step(numc,[denc 0],tt1);
figure
plot(tt1,yy2,tt1,yyo1,'--')
title ('Compensated:solid line; Non-compensated: --');
grid
```


Compensated:solid line; Non-compensated: –
10
9
8
7
6
5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10