ECE 260

EXAM 1

SOLUTIONS

(SUMMER 2024)

QUESTION 1

Let T_1 and T_2 denote the fundamentol periods of x_1 and x_2, respectively.

$$T_1 = \frac{2\pi}{21}$$

$$T_2 = \frac{2\pi}{15}$$

$$\frac{T_1}{T_2} = \frac{\left(\frac{2\pi}{21}\right)}{\left(\frac{2\pi}{15}\right)} = \left(\frac{2\pi}{21}\right)\left(\frac{15}{2\pi}\right) = \frac{15}{21} = \frac{5}{7}$$

since $\frac{T_1}{T_2} \in \mathbb{Q}$, x is periodic

$$T = 7T_1 = 7\left(\frac{2\pi}{21}\right) = \frac{2\pi}{3}$$

QUESTION 2

Let $x(t) = x_1(t) + x_2(t)$ where

$$x_1(t) = \int_{-\infty}^{t-1} \delta\left(\tfrac{1}{2}\tau - \tfrac{3}{2}\right) d\tau \quad \text{and} \quad x_2(t) = \int_{-\pi}^{\pi} \tau^2 \sin(\tau)\, \delta(\tau - 10)\, d\tau$$

$$\begin{aligned}
x_1(t) &= \int_{-\infty}^{t-1} \delta\left(\tfrac{1}{2}\tau - \tfrac{3}{2}\right) d\tau \\
&= \int_{-\infty}^{\frac{1}{2}(t-1)-\frac{3}{2}} \delta(\lambda)(2)\, d\lambda \\
&= 2 \int_{-\infty}^{\frac{1}{2}t-2} \delta(\lambda)\, d\lambda \\
&= \begin{cases} 2 & \frac{1}{2}t - 2 \geq 0 \\ 0 & \text{otherwise} \end{cases} \\
&= \begin{cases} 2 & \frac{1}{2}t \geq 2 \\ 0 & \text{otherwise} \end{cases} \\
&= \begin{cases} 2 & t \geq 4 \\ 0 & \text{otherwise} \end{cases} \\
&= 2u(t-4)
\end{aligned}$$

$$\begin{aligned}
x_2(t) &= \int_{-\pi}^{\pi} \tau^2 \sin(\tau)\, d(\tau - 10)\, d\tau \\
&= \int_{-\pi}^{\pi} 0\, d\tau \\
&= 0
\end{aligned}$$

$$\begin{aligned}
x(t) &= x_1(t) + x_2(t) \\
&= 2u(t-4)
\end{aligned}$$

QUESTION 3(A)

A system $\mathcal{H}$ is said to be time invariant if, for every function x and every real constant t_0, the following condition holds:

$$S_{t_0} \mathcal{H} x = \mathcal{H} S_{t_0} x,$$

where $S_{t_0} x(t) = x(t - t_0)$ (i.e., S_{t_0} is an operator that time shifts a function by t_0).

QUESTION 3(B)

$$\boxed{\mathcal{H}x(t) = x(-2t)}$$

$$S_{t_0}\underbrace{\mathcal{H}x}_{}(t) = S_{t_0} v_1(t) = v_1(t-t_0) = x(-2[t-t_0]) = x(-2t+2t_0)$$

$v_1 = \mathcal{H}x \Rightarrow v_1(t) = x(-2t)$

$$\mathcal{H}\underbrace{S_{t_0}x}_{}(t) = \mathcal{H}v_2(t) = v_2(-2t) = x(-2t-t_0)$$

$v_2 = S_{t_0}x \Rightarrow v_2(t) = x(t-t_0)$

Since $S_{t_0}\mathcal{H}x = \mathcal{H}S_{t_0}x$ for all x and all t_0 does <u>not</u> hold, $\mathcal{H}$ is not time invariant.

QUESTION 4

```
function x = func(t)
    x = (t >= -4 & t < 0) .* ...
      ((t .* sin(2 * pi * t)) ./ ((abs(t) + 1) .^ 2)) ...
      + (t >= 0 & t < 4) .* ...
      ((t .^ 2 .* cos(4 * pi * t)) ./ (t .^ 2 + 1));
end
```

QUESTION 5

② w is causal ; $w(t) = x(t+1) - 2$

$w(t) = 0$ for all $t<0$

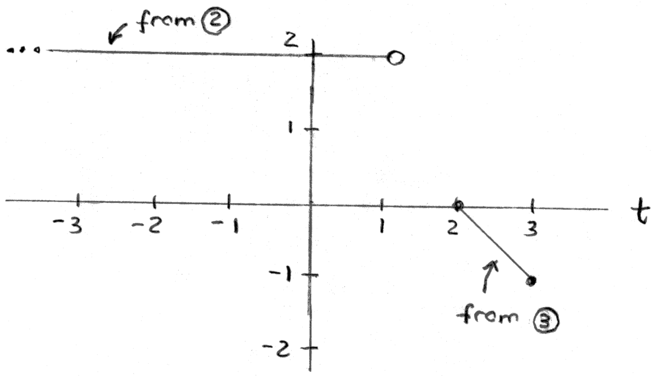
$x(t+1) - 2 = 0$ for all $t<0$

$x(t+1) = 2$ for all $t<0$

$x([t-1]+1) = 2$ for all $t-1<0$

$x(t) = 2$ for all $t<1$

③ $x(t) = 2-t$ for $2 \leq t \leq 3$



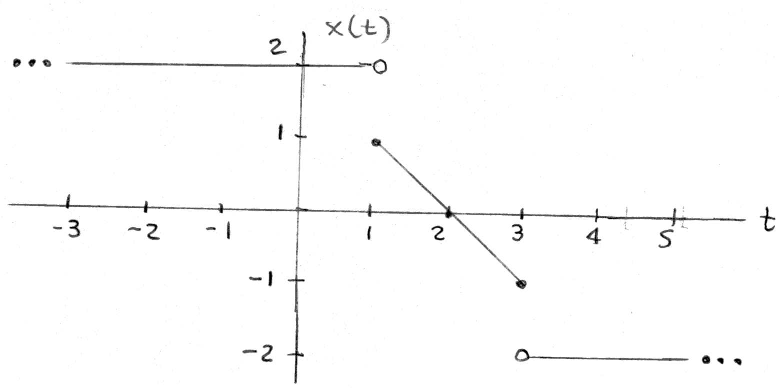
① v is odd ; $v(t) = x(t+2)$

v is obtained from x by time shifting x by -2

so x is obtained from v by time shifting v by 2

therefore, v having odd symmetry about 0 implies

x has odd symmetry about $0+2=2$



$$\therefore \quad x(t) = \begin{cases} 2 & t<1 \\ 2-t & 1 \leq t \leq 3 \\ -2 & t>3 \end{cases}$$

QUESTION 6

$$\boxed{\mathcal{H}x(t) = \int_t^{t+1} x(\tau)\, d\tau \;; \qquad x_1(t) = e^{jt}}$$

$$\begin{aligned}
\mathcal{H}x_1(t) &= \int_t^{t+1} x_1(\tau)\, d\tau \\
&= \int_t^{t+1} e^{j\tau}\, d\tau \\
&= \left[\tfrac{1}{j} e^{j\tau}\right]\Big|_t^{t+1} \\
&= -j\left[e^{j(t+1)} - e^{jt}\right] \\
&= -j\left[e^{j} e^{jt} - e^{jt}\right] \\
&= -j\left[e^{j} - 1\right] e^{jt} = j\left[1 - e^{j}\right] e^{jt} \\
&= e^{-j\pi/2}\left[e^{j} - 1\right] e^{jt} = e^{j\pi/2}\left[1 - e^{j}\right] e^{jt}
\end{aligned}$$

$\therefore$ x_1 is an eigenfunction of $\mathcal{H}$ with eigenvalue λ_1, where

$$\lambda_1 = j(1 - e^{j}) = e^{j\pi/2}(1 - e^{j})$$