22/25

UNIVERSITY OF VICTORIA
MIDTERM EXAMINATION #1
SUMMER 2012

CRN:	30127 (A01), 31086 (A02)
COURSE NUMBER:	ELEC 260
COURSE NAME:	CONTINUOUS-TIME SIGNALS AND SYSTEMS
SECTION(S):	A01, A02
INSTRUCTOR:	MICHAEL ADAMS
DURATION:	50 MINUTES

NAME: [redacted]

STUDENT NUMBER: V0[redacted]

ALL QUESTIONS ARE **TO BE ANSWERED ON THE EXAMINATION PAPER** IN THE SPACE PROVIDED.

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS EXAMINATION PAPER HAS **10 PAGES** (ALL OF WHICH ARE NUMBERED).

TOTAL MARKS: 25

THIS EXAMINATION IS **CLOSED BOOK**.

THE USE OF A CRIB SHEET IS **NOT** PERMITTED.

THE USE OF A CALCULATOR IS **NOT** PERMITTED.

SHOW ALL OF YOUR WORK!
CLEARLY DEFINE ANY NEW QUANTITIES (E.G., VARIABLES, FUNCTIONS, ETC.) THAT YOU INTRODUCE IN YOUR SOLUTIONS.

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PROBLEM 1.

3

(A) Let $F(s) = \frac{(s+4)(s-2)^7}{s^3(s+5)^{10}}$, where s is complex. Find the poles and zeros of the function $F(s)$ and determine the order of each pole and zero. Determine for what values of s the function $F(s)$ is analytic. **[3 marks]**

zeros: $(s+4)(s-2)^7 = 0$

at $s = -4$ (1^{st} order), 2 (7^{th} order)

poles: $s^3(s+5)^{10} = 0$

at $s = 0$ (3^{rd} order), $s = -5$ (10^{th} order)

This function is a rational polynomial ∴ analytic for all reals except when denominator = 0.
This occurs at $s = -5$ and $s = 0$.

1

(B) Let $H(\omega) = \frac{3+4j}{(1-j\omega)^5}$, where ω is real. Find a fully simplified expression for $|H(\omega)|$. **[2 marks]**

① Convert to polar

$r = \sqrt{3^2+4^2}$
$= 5$

$\arg = \tan^{-1}\left(\frac{4}{3}\right)$

$r = \sqrt{1^2+\omega^2}$

$\arg = \tan^{-1}(-\omega)$

$$\frac{5e^{\tan^{-1}\left(\frac{4}{3}\right)}}{\sqrt{1^2+\omega^2}\cdot e^{5\tan^{-1}(-\omega)}} \rightarrow \frac{5}{\sqrt{1^2+\omega^2}}e^{\tan^{-1}\left(\frac{4}{3}\right)-5\tan^{-1}(-\omega)}$$

$$|H(\omega)| = \left|\sqrt{\frac{5^2}{\sqrt{1^2+\omega^2}^2}}\right|$$

$$= \sqrt{\frac{25}{1^2+\omega^2}}$$

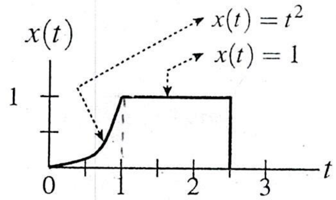
$$= \sqrt{\frac{25}{1+\omega^2}}$$

-1

2

PROBLEM 2.

Suppose that we have the signal $x(t)$ shown in the figure. Use unit-step functions to find a single expression for $x(t)$ that is valid for all t. **When stating your final answer,** *group together terms having the same unit-step function factor.* **[3 marks]**



$$= (t^2)\left[u(t) - u(t-1)\right] + \left[u(t-1) - u(t-2.5)\right]$$

$$= t^2 u(t) - t^2 u(t-1) + u(t-1) - u(t-2.5)$$

$$\boxed{= t^2 u(t) + (1-t^2)u(t-1) - u(t-2.5)}$$

3

PROBLEM 3. Evaluate the integral $\int_{-\infty}^{t+1} (\tau^3 + \tau^2 + \tau + 1)\delta(\tau - 1)d\tau$. Your answer must be stated in **fully simplified** form. **[3 marks]**

$$\int_{-\infty}^{t+1} (\tau^3 + \tau^2 + \tau + 1)\,\delta(\tau - 1)\,d\tau$$

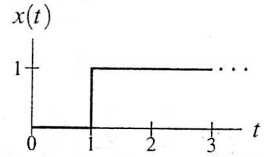
Delta function within bounds of integration ∴ use SIFTING property

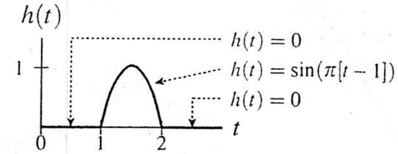
$$= (\tau^3 + \tau^2 + \tau + 1)\Big]_{\tau=1}$$

$$= \begin{cases} (1+1+1+1) = \underline{\underline{4}} & \text{If } t > 0 \\ 0 & \text{otherwise} \end{cases}$$

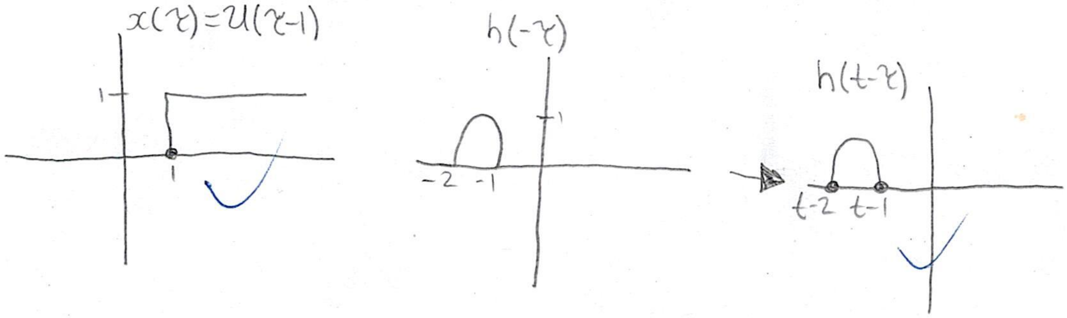
PROBLEM 4.

Using graphical methods, compute the convolution $y(t) = x(t) * h(t)$ where the signals $x(t)$ and $h(t)$ are as shown in the figure to the right. **[8 marks]**





(A) Plot $x(\tau)$ and $h(t-\tau)$ versus τ. ***Be very careful*** *to plot these graphs correctly. If you make a mistake here, all of your subsequent work will be completely wrong, and* ***you will lose a very substantial number of marks*** *as a result.* **[2 marks]**



3

(B) For each of the cases (i.e., ranges for t) to be considered in the computation of the convolution result $y(t)$, carefully sketch and **fully label** the graph that includes both $x(\tau)$ and $h(t-\tau)$ plotted versus τ, and also indicate the corresponding range for t. **[3 marks]**

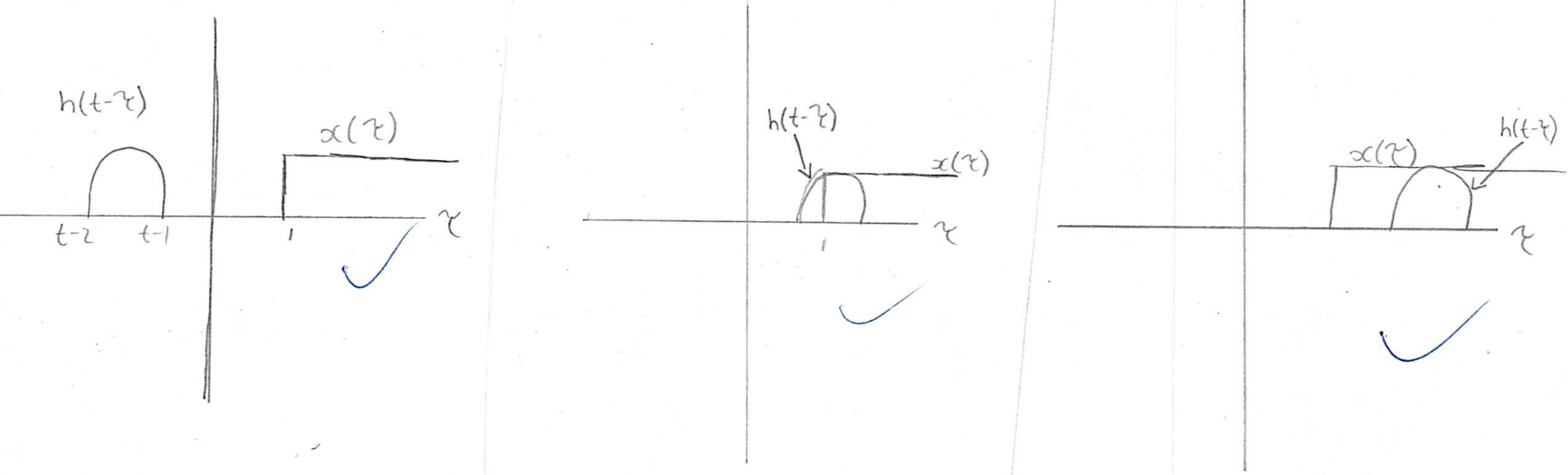
critical points at 2, 3 → 3 ranges

$t-2<1$
$t-1>1$

$t-1<1$
$t<2$

$2<t<3$

$t>3$



$t < 2$

No overlap $\therefore = 0$

$2 < t < 3$

$$\int_{t-1}^{3} h(\tau - t)\, d\tau$$

$t > 3$

$$\int_{t-2}^{t-1} h(\tau - t)\, d\tau$$

1.5

(C) Use the graphs from part (b) to determine the convolution result $y(t)$. **You may state your final answer in terms of integrals. DO NOT INTEGRATE!!! [3 marks]**

$$y(t) = \begin{cases} 0 & \text{when } t < 2 \\ \displaystyle\int_{t-1}^{3} \frac{\sin(\pi[\tau - 1])}{\tau}\, d\tau & \text{when } 2 < t < 3 \\ \displaystyle\int_{t-1}^{t-2} \frac{\sin(\pi[\tau - 1])}{\tau}\, d\tau & \end{cases}$$

-1.5

PROBLEM 5. Suppose that we have a system $\mathscr{H}$ with input $x(t)$ and output $y(t)$.

1.5

(A) Clearly state, in mathematical terms, the condition that must be satisfied in order for the system $\mathscr{H}$ to be linear. Be sure to **define all quantities** such as variables, functions, and constants. Otherwise, you will receive **zero marks**. Be careful with the notation that you choose to employ. If, for example, you confuse arrows and equal signs in your solution, you will probably receive zero marks. **[2 marks]**

Let $\qquad a, b$ are nonzero real constants

$x_1(t) \to y_1(t)$

$x_2(t) \to y_2(t)$

$x_3(t) \to y_3(t)$

$x_3(t) = ax_1(t) + bx_2(t)$

-0.5

If

$y_3(t) = ay_1(t) + by_2(t)$

System is LINEAR.

2

(B) Suppose now that the system $\mathscr{H}$ is characterized by the equation $y(t) = x(t) + 1$. **Using the condition stated in part (a)**, determine whether this system is linear. **[2 marks]**

Let

$y_1(t) = x_1(t) + 1 \qquad ay_1(t) = ax_1(t) + a$

$y_2(t) = x_2(t) + 1 \qquad by_2(t) = bx_2(t) + b$

$x_3(t) = ax_1(t) + 1 + bx_2(t) + 1$

$ax_1(t) + bx_2(t) + 2 \neq ax_1(t) + a + bx_2(t) + b$

System is NOT linear.

2

PROBLEM 6. Using the MATLAB programming language, write a function called `maximum` that takes a row-vector or column-vector as input and returns the maximum element in that vector. (You may safely assume that the input vector does not have zero length.) Your code must not call any other functions, except possibly **`length`** or **`size`**. Be sure to use correct syntax in your answer, since syntax clearly matters here. **[2 marks]**

```
function y=maximum(x)
    temp = 0 ;
    for i=1 : size(x)
        If x[i] >temp
            temp = x[i]
        end
    end
    y= temp ;
```

END

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