ECE 260

EXAM 5

SOLUTIONS

(FALL 2020)

QUESTION 1

$$X(s) = \frac{4s+1}{s(s+1)} \quad \text{for } -1 < Re(s) < 0$$

$$X(s) = \frac{A_1}{s} + \frac{A_2}{s+1}$$

$$A_1 = sX(s)\Big|_{s=0} = \frac{4s+1}{s+1}\Big|_{s=0} = 1$$

$$A_2 = (s+1)X(s)\Big|_{s=-1} = \frac{4s+1}{s}\Big|_{s=-1} = \frac{-3}{-1} = 3$$

$$X(s) = \frac{1}{s} + \frac{3}{s+1}$$

$$x(t) = \underbrace{L^{-1}\left\{\frac{1}{s}\right\}(t)}_{Re(s)<0} + 3\underbrace{L^{-1}\left\{\frac{1}{s+1}\right\}(t)}_{Re(s)>-1}$$

$$= -u(-t) + 3\left[e^{-t}u(t)\right]$$

$$= -u(-t) + 3e^{-t}u(t)$$

QUESTION 2

$$y''(t) + 4y'(t) + 3y(t) = u(t)$$

$$y(0^-) = 0, \quad y'(0^-) = 1$$

$$s\left[sY(s) - y(0^-)\right] - y'(0^-) + 4\left[sY(s) - y(0^-)\right] + 3Y(s) = \frac{1}{s}$$

$$s^2 Y(s) - s y(0^-) - y'(0^-) + 4s Y(s) - 4y(0^-) + 3Y(s) = \frac{1}{s}$$

$$\left[s^2 + 4s + 3\right] Y(s) - s y(0^-) - y'(0^-) - 4y(0^-) = \frac{1}{s}$$

$$\left[s^2 + 4s + 3\right] Y(s) = \frac{1}{s} + s y(0^-) + y'(0^-) + 4y(0^-)$$

$$\left[s^2 + 4s + 3\right] Y(s) = \frac{1}{s} + 1$$

$$\left[s^2 + 4s + 3\right] Y(s) = \frac{s+1}{s}$$

$$Y(s) = \frac{s+1}{s(s^2+4s+3)} = \frac{s+1}{s(s+1)(s+3)} = \frac{1}{s(s+3)}$$

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+3}$$

$$A_1 = sY(s)\Big|_{s=0} = \frac{1}{s+3}\Big|_{s=0} = \frac{1}{3}$$

$$A_2 = (s+3)Y(s)\Big|_{s=-3} = \frac{1}{s}\Big|_{s=-3} = -\frac{1}{3}$$

$$Y(s) = \frac{1}{3}\left(\frac{1}{s}\right) - \frac{1}{3}\left(\frac{1}{s+3}\right)$$

$$y(t) = \frac{1}{3} L^{-1}\left\{\frac{1}{s}\right\}(t) - \frac{1}{3} L^{-1}\left\{\frac{1}{s+3}\right\}(t)$$

$$= \frac{1}{3} - \frac{1}{3} e^{-3t} \qquad t \geq 0$$

QUESTION 3

$$H(s) = \frac{s^2+1}{s^3+6s^2+11s+6} \quad \text{for } Re(s) > -1$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+1}{s^3+6s^2+11s+6}$$

$$(s^3+6s^2+11s+6)\,Y(s) = (s^2+1)\,X(s)$$

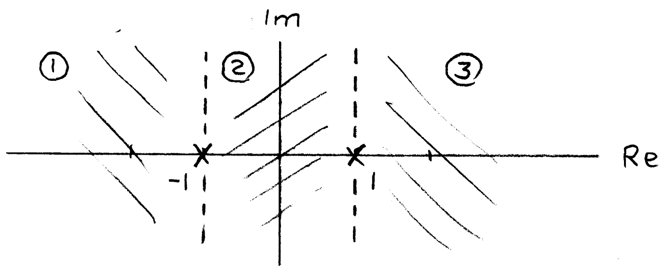
$$s^3\,Y(s) + 6s^2\,Y(s) + 11s\,Y(s) + 6Y(s) = s^2\,X(s) + X(s)$$

$$y'''(t) + 6y''(t) + 11y'(t) + 6y(t) = x''(t) + x(t)$$

QUESTION 4

$$H(s) = \frac{(s+1)(s-1)}{(s+2)(s-2)} \quad \text{for } \mathrm{Re}(s) > 2$$

(a) $$H_{inv}(s) = \frac{1}{H(s)} = \frac{(s+2)(s-2)}{(s+1)(s-1)}$$



The algebraic expression for H_{inv} is the same for all inverse systems, and is as given above. Each inverse system has a different ROC for H.

The ROCs are

① $\mathrm{Re}(s) < -1$

② $-1 < \mathrm{Re}(s) < 1$

③ $\mathrm{Re}(s) > 1$

(b) System ② is BIBO stable since the ROC contains the imaginary axis

(c) system ③ is causal since the ROC is a RHP.

QUESTION 5

(a) $V(s) = X(s) + \frac{6\beta s}{s-1} V(s)$ ①

$Y(s) = \frac{2\beta s}{s-1} V(s)$ ②

from ①, we have

$\left(1 - \frac{6\beta s}{s-1}\right) V(s) = X(s) \Rightarrow \frac{s-1-6\beta s}{s-1} V(s) = X(s) \Rightarrow$

$V(s) = \frac{s-1}{s-6\beta s-1} X(s)$ ③

substituting ③ into ②, we have

$Y(s) = \left(\frac{2\beta s}{s-1}\right)\left(\frac{s-1}{s-6\beta s-1}\right) X(s) = \frac{2\beta s}{s-6\beta s-1} X(s)$

$H(s) = \frac{2\beta s}{s-6\beta s-1} = \frac{2\beta s}{(1-6\beta)s-1} = \frac{2\beta s}{(1-6\beta)(s-\frac{1}{1-6\beta})}$ for $Re(s) > \frac{1}{1-6\beta}$

(b) for BIBO stability, the ROC of H must contain the imaginary axis.

therefore, we have

$\frac{1}{1-6\beta} < 0 \Rightarrow 1-6\beta < 0 \Rightarrow 1 < 6\beta \Rightarrow \beta > \frac{1}{6}$