CSC 320 - Tutorial 4

1. Pumping lemma for regular languages

Pumping Lemma

If L is a regular language there exists some natural number p (the pumping length) where for any string s in L with a length of at least p (ie. $s \in L$ and $|s| \geq p$) s can be divided into substrings xyz satisfying the following conditions:

1. $|xy| \leq p$
2. $|y| > 0$
3. $xy^iz \in L$ for $i \geq 0$

Questions

1. Prove that the following languages are not regular using the pumping lemma

    a. $L_1 = \{0^n 1^n 2^n \mid n \geq 0\}$

(proof by contradiction)

Assume L_1 is Regular.
then for all strings $w \in L_1$ where $|w| \geq p$ — arbitrary pumping length
the pumping lemma holds.

Pick the string $w = 0^p 1^p 2^p$, $w \in L_1$ and $|w| \geq p$

$w = \underbrace{0..\underline{y}01}_{p}\underbrace{1...1}_{p}\underbrace{2....2}_{p}$

By the pumping lemma $w = xyz$ where...

1) $|xy| \leq p$ $\therefore$ xy consists of only 0s

2) $|y| > 0$ or $y \neq \varepsilon$ $\therefore$ y consists of @ least one 0

$\therefore$ $y = 0^n$ where $0 < n \leq p$ ← combining the above conditions

3) $xy^iz \in L_1$ for all $i \geq 0$ * this is where we look for the contradiction

$i = 0$ $xz = 0^{p-n} 1^p 2^p$ $(p-n < p)$
$\therefore$ $xz \notin L_1$ * contradiction: should be in L_1 but its not

$i = 2$ $xyyz = 0^{p+n} 1^p 2^p$ $(p+n > p)$
$\therefore$ $xy^2z \notin L_1$ * contradiction

either one is sufficient to show contradiction

the pumping lemma does not hold $\therefore$ L_1 is not Regular

b. $L_2 = \{w^r w \mid w \in \{0,1\}^* \}$

Assume L_2 is Regular, then the pumping lemma holds for all $w \in L_2$ where $|w| \geq p$

pick $w = 0^p 1 1 0^p$ $\quad w \in L_2$ and $|w| = 2p+2 \geq p$

By the pumping lemma $w = xyz$ s.t.

1) $|xy| \leq p$ $\quad \therefore$ xy consists of only 0s

2) $|y| > 0$ $\quad \therefore$ y consists of @ least one 0

$\therefore$ $y = 0^n$ where $0 < n \leq p$

3) $xy^iz \in L_2$ for all $i \geq 0$

$i = 0$ $\quad xy^0z = 0^{p-n} 1 1 0^p \Rightarrow w \notin L_2$

$i = 2$ $\quad xyyz = 0^{p+n} 1 1 0^p \Rightarrow w \notin L_2$

one counter example is sufficient to show contradiction

$\therefore$ L_2 is not regular

2. Is the string $s = 0^p0^p$ a good choice to devise a contradiction to prove L_2 is not regular? Why or why not?

not all strings in a given language can yeild a contradiction.

In the 1b we saw that after pumping (up or down) the number of zeros before the 1s was not the same as the number of zeros after.

We proved that no matter how you slice w into substrings x, y, z you cannot get the pumping lemma to hold.

Consider using $w = \underbrace{0^p0^p}_{\text{even string}}$ $w \in L_2$ and $|w| \geq p$

1) $|xy| \leq p$ $\therefore$ xy consists of 0s

2) $y \neq \varepsilon$ $\therefore$ $y = 0^n$ $0 < n \leq p$

3) $xy^iz \in L_2$ for all $i \geq 0$ $0 \ldots \underbrace{00}_{y} \ldots 0$

my $i = 0$ $xz = \underset{x}{0 \ldots 0}\underset{z}{0 \ldots 0} \in L_2$ if its even

$i = 2$ $xyyz = \underset{x}{0 \ldots 0}\underset{y}{0 \ldots 0}\underset{y}{0 \ldots 0}\underset{z}{0 \ldots 0} \in L_2$ if its even

$\therefore$ the substrings x, y, z where y is even will always work $\Rightarrow$ we never get a contradiction

Practice questions:

1. Show that $L = \{0^i1^j \mid i > j\}$ is not regular
2. Show that $L = \{0^i1^j \mid i < j\}$ is not regular
3. Show that $L = \{0^i1^j \mid i \leq j\}$ is not regular
4. Argue that $L = \{0^i1^j \mid i \leq j < 121\}$ is not regular