3.22 For each function x given below, find a single expression for x (i.e., an expression that does not involve multiple cases). Group similar unit-step function terms together in the expression for x.

(a) $x(t) = \begin{cases} -t-3 & -3 \le t < -2 \\ -1 & -2 \le t < -1 \\ t^3 & -1 \le t < 1 \\ 1 & 1 \le t < 2 \\ -t+3 & 2 \le t < 3 \\ 0 & \text{otherwise;} \end{cases}$

(b) $x(t) = \begin{cases} -1 & t < -1 \\ t & -1 \le t < 1 \\ 1 & t \ge 1; \end{cases}$ and

(c) $x(t) = \begin{cases} 4t+4 & -1 \le t < -\frac{1}{2} \\ 4t^2 & -\frac{1}{2} \le t < \frac{1}{2} \\ -4t+4 & \frac{1}{2} \le t < 1 \\ 0 & \text{otherwise.} \end{cases}$

$$x(t) = (4t+4)\left[u(t+1) - u(t+\tfrac{1}{2})\right] + 4t^2\left[u(t+\tfrac{1}{2}) - u(t-\tfrac{1}{2})\right] + (4-4t)\left[u(t-\tfrac{1}{2}) - u(t-1)\right]$$

$$= (4t+4)u(t+1) + (-4t-4+4t^2)u(t+\tfrac{1}{2}) + (-4t^2+4-4t)u(t-\tfrac{1}{2}) + (4t-4)u(t-1)$$

$$= (4t+4)u(t+1) + (4t^2-4t-4)u(t+\tfrac{1}{2}) + (-4t^2-4t+4)u(t-\tfrac{1}{2}) + (4t-4)u(t-1)$$

$$= 4\left[(t+1)u(t+1) + (t^2-t-1)u(t+\tfrac{1}{2}) + (-t^2-t+1)u(t-\tfrac{1}{2}) + u(t-1)^2\right]$$

3.24 Determine whether each system $\mathcal{H}$ given below is memoryless.

(a) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau)d\tau$;
(b) $\mathcal{H}x(t) = \text{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = x(t-1)+1$;
(d) $\mathcal{H}x(t) = \int_t^{\infty} x(\tau)d\tau$;
(e) $\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau$;
(f) $\mathcal{H}x(t) = tx(t)$; and
(g) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$.

(d) $\mathcal{H}x(t) = \int_t^{\infty} x(\tau)d\tau$

$\therefore \mathcal{H}x(t_0) = \int_{t_0}^{\infty} x(\tau)d\tau$

$\mathcal{H}x(t_0)$ depends ~~it depends~~ on $x(t)$ for $t_0 \le t \le \infty$. Therefore the system is not memoryless

(g) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

$= \int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t)\,d\tau$

$= x(t)$

$\mathcal{H}x(t_0)$ depends on $x(t)$ for $t = t_0$

$\therefore$ memoryless

3.25 Determine whether each system $\mathcal{H}$ given below is causal.

(a) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau)d\tau$;
(b) $\mathcal{H}x(t) = \text{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = x(t-1)+1$;
(d) $\mathcal{H}x(t) = \int_t^{\infty} x(\tau)d\tau$;
(e) $\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)\delta(\tau)d\tau$; and
(f) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)u(t-\tau)d\tau$.

b

$\mathcal{H}x(t) = \text{Even}\{x\}(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$

$\mathcal{H}x(t_0)$ depends on $x(t)$ for $t = \pm t_0$

$\therefore$ system not causal

f

$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)\,u(t-\tau)\,d\tau$

$= \int_{-\infty}^{t} x(\tau)\,d\tau$

$\mathcal{H}x(t_0)$ depends on $x(t)$ for $t \le t_0$

$\therefore$ causal

3.26 For each system $\mathcal{H}$ given below, determine if $\mathcal{H}$ is invertible, and if it is, specify its inverse.

(a) $\mathcal{H}x(t) = x(at - b)$ where a and b are real constants and $a \neq 0$;

(b) $\mathcal{H}x(t) = e^{x(t)}$, where x is a real function;

(c) $\mathcal{H}x(t) = \text{Even}(x)(t) - \text{Odd}(x)(t)$;

(d) $\mathcal{H}x(t) = \mathcal{D}x(t)$, where $\mathcal{D}$ denotes the derivative operator; and

(e) $\mathcal{H}x(t) = x^2(t)$.

$\boxed{b}$ let $y = \mathcal{H}x$ $\quad \therefore y(t) = e^{x(t)}$

$\therefore y(t) > 0$

Taking natural logarithm on both sides we get

$\ln y(t) = x(t)$

or, $x(t) = \ln y(t)$

$\therefore$ The system is invertible

$\boxed{e}$ $\boxed{x_1(t) = -A}$ $\boxed{x_2(t) = A}$ where A is a positive real constant.

$\therefore \mathcal{H}x_1(t) = A^2$

$\therefore \mathcal{H}x_2(t) = (-A)^2 = A^2$

both inputs have same outputs

$\therefore$ not invertible

3.27 Determine whether each system $\mathcal{H}$ given below is BIBO stable.

(a) $\mathcal{H}x(t) = \int_t^{t+1} x(\tau)d\tau$ [Hint: For any function f, $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|\,dx$.];

(b) $\mathcal{H}x(t) = \frac{1}{2}x^2(t) + x(t)$;

(c) $\mathcal{H}x(t) = 1/x(t)$;

(d) $\mathcal{H}x(t) = e^{-|t|}x(t)$; and

(e) $\mathcal{H}x(t) = \left(\frac{1}{t-1}\right)x(t)$.

(d) $\mathcal{H}x(t) = e^{-|t|}x(t)$

let $y = \mathcal{H}x$, having x bounded by $|x(t)| \leq A < \infty$

we have, $|y(t)| = |e^{-|t|}x(t)| = |e^{-|t|}|\,|x(t)|$

$= e^{-|t|}|x(t)|$

replacing $e^{-|t|}$ and $|x(t)|$: $|y(t)| \leq A = A < \infty$

$\therefore |y(t)| < \infty$

$\therefore$ bounded input $\Rightarrow$ bounded output

$\therefore$ BIBO Stable

e) considering: $x(t)=1$ we have: $\mathcal{H}x(t) = \frac{1}{t-1}[1]$

$$= \frac{1}{t-1}$$

As $t \to 1$
$|\mathcal{H}(x)| \to \infty$] x is bounded but $\mathcal{H}x$ is not bounded

∴ system not BIBO stable

3.28 Determine whether each system $\mathcal{H}$ given below is time invariant.
(a) $\mathcal{H}x(t) = \mathcal{D}x(t)$; where $\mathcal{D}$ denotes the derivative operator;
(b) $\mathcal{H}x(t) = \mathrm{Even}(x)(t)$;
(c) $\mathcal{H}x(t) = \int_t^{t+1} x(\tau - \alpha)d\tau$, where α is a constant;
(d) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$, where h is an arbitrary (but fixed) function;
(e) $\mathcal{H}x(t) = x(-t)$; and
(f) $\mathcal{H}x(t) = \int_{-\infty}^{2t} x(\tau)d\tau$.

b) $\mathcal{H}x(t-t_0) = \mathcal{H}x'(t)$ where $x'(t) = x(t-t_0)$

We have $\mathcal{H}x(t) = \mathrm{Even}\{x\}(t) = \frac{1}{2}[x(t)+x(-t)]$

$$\mathcal{H}x(t_0) = \frac{1}{2}[x(t)+x(-t)]$$

$$\therefore \mathcal{H}x(t-t_0) = \frac{1}{2}\left[x(t-t_0)+x(-[t-t_0])\right]$$

$$= \frac{1}{2}[x(t-t_0)+x(-t+t_0)]$$

$$\mathcal{H}x'(t) = \frac{1}{2}[x'(t)+x'(-t)] = \frac{1}{2}[x(t-t_0)+x(-t-t_0)]$$

Since $\mathcal{H}x(t-t_0) \neq \mathcal{H}x'(t)$ for all x and t_0

∴ system is not time invariant

d) $\mathcal{H}x(t-t_0) = \mathcal{H}x'(t)$ ... $x'(t) = x(t-t_0)$

$$\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$\mathcal{H}x(t-t_0) = \int_{-\infty}^{\infty} x(\tau)h(t-t_0-\tau)d\tau$$

From the definition of $\mathcal{H}$.

$$\mathcal{H}x'(t) = \int_{-\infty}^{\infty} x'(\tau')\, h(t-\tau')\, d\tau'$$

$$= \int_{-\infty}^{\infty} x(\tau' - t_0)\, h(t-\tau')\, d\tau'$$

Letting $\lambda = \tau' - t_0$ or, $\tau' = \lambda + t_0$ or, $\boxed{d\tau' = d\lambda}$

$$\mathcal{H}x'(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t - [\lambda + t_0])\, d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda) h(t - \lambda - t_0)\, d\lambda$$

$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t - t_0 - \lambda)\, d\lambda$$

$$= \mathcal{H}x(t - t_0)$$

Since

$\mathcal{H}x(t - t_0) = \mathcal{H}x'(t)$

for all x and t_0,

System is time invariant.

3.29 Determine whether each system $\mathcal{H}$ given below is linear.

(a) $\mathcal{H}x(t) = \int_{t-1}^{t+1} x(\tau)d\tau$;

(b) $\mathcal{H}x(t) = e^{x(t)}$;

(c) $\mathcal{H}x(t) = \mathrm{Even}(x)(t)$;

(d) $\mathcal{H}x(t) = x^2(t)$; and

(e) $\mathcal{H}x(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$, where h is an arbitrary (but fixed) function.

(b) $a_1 \mathcal{H}x_1(t) + a_2 \mathcal{H}x_2(t) = a_1 e^{x_1(t)} + a_2 e^{x_2(t)}$

$\Rightarrow \mathcal{H}[a_1 x_1 + a_2 x_2](t) = e^{a_1 x_1(t) + a_2 x_2(t)}$

$= e^{a_1 x_1(t)} e^{a_2 x_2(t)}$

$\mathcal{H}(a_1 x_1 + a_2 x_2) = a_1 \mathcal{H}x_1 + a_2 \mathcal{H}x_2$

doesn't hold for all x_1, x_2, a_1 and a_2

$\therefore$ System not linear.

e) $a_1 \mathcal{H}x_1(t) + a_2 \mathcal{H}x_2(t) = a_1 \int_{-\infty}^{\infty} x_1(\tau) h(t-\tau)\, d\tau + a_2 \int_{-\infty}^{\infty} x_2(\tau) h(t-\tau)\, d\tau$ —— (I)

and,

$$\mathcal{H}[a_1 x_1 + a_2 x_2](t) = \int_{-\infty}^{\infty} [a_1 x_1(\tau) + a_2 x_2(\tau)] h(t-\tau)\, d\tau$$

$$= a_1 \int_{-\infty}^{\infty} x_1(\tau) h(t-\tau)\, d\tau + a_2 \int_{-\infty}^{\infty} x_2(\tau) h(t-\tau)\, d\tau$$ —— (II)

Since (I) = (II), system is linear.

$$\mathcal{H}(a_1 x_1 + a_2 x_2) = a_1 \mathcal{H}x_1 + a_2 \mathcal{H}x_2$$

3.33 For each system $\mathcal{H}$ and the functions $\{x_k\}$ given below, determine if each of the x_k is an eigenfunction of $\mathcal{H}$, and if it is, also state the corresponding eigenvalue.

(a) $\mathcal{H}x(t) = x^2(t)$, $x_1(t) = a$, $x_2(t) = e^{-at}$, and $x_3(t) = \cos t$, where a is a complex constant;

(b) $\mathcal{H}x(t) = \mathcal{D}x(t)$, $x_1(t) = e^{at}$, $x_2(t) = e^{at^2}$, and $x_3(t) = 42$, where $\mathcal{D}$ denotes the derivative operator and a is a real constant;

(c) $\mathcal{H}x(t) = \int_{t-1}^{t} x(\tau)d\tau$, $x_1(t) = e^{at}$, $x_2(t) = t$, and $x_3(t) = \sin t$, where a is a nonzero complex constant; and

(d) $\mathcal{H}x(t) = |x(t)|$, $x_1(t) = a$, $x_2(t) = t$, $x_3(t) = t^2$, where a is a strictly positive real constant.

x_1 eigenfunction of eigenvalue a

$$\mathcal{H}x_1(t) = \mathcal{D}x_1(t) = \mathcal{D}e^{at} = ae^{at} \rightleftharpoons ax(t) = ax_1(t)$$

$$\mathcal{H}x_2(t) = \mathcal{D}x_2(t) = \mathcal{D}e^{at} = \mathcal{D}e^{at^2} = 2ate^{at^2}$$

x_2 not an eigenfunction

$$\mathcal{H}x_3(t) = \mathcal{D}x_3(t) = \mathcal{D}x_3(t) = \mathcal{D}\cdot 42 = 0 = 0x_3(t)$$

x_3 eigenfunction w/ eigenvalue 0

D3A

test.m

/Users/arfaz/Desktop/test.m

```
fprintf('%-8s %-8s %-8s\n', 'Celsius', 'Fahrenheit', 'Kelvin');
for celsius = -50 : 10 : 50
    fahrenheit = 9 / 5 * celsius + 32;
    kelvin = celsius + 273.15;
    fprintf('%8.2f %8.2f %8.2f\n', celsius, fahrenheit, kelvin);
end
```

Command Window

```
>> unitStepFunction1
Not enough input arguments.

Error in unitStepFunction1 (line 3)
    if t >= 0

>> D3
Celsius  Fahrenheit Kelvin
  -50.00   -58.00   223.15
  -40.00   -40.00   233.15
  -30.00   -22.00   243.15
  -20.00    -4.00   253.15
  -10.00    14.00   263.15
    0.00    32.00   273.15
   10.00    50.00   283.15
   20.00    68.00   293.15
   30.00    86.00   303.15
   40.00   104.00   313.15
   50.00   122.00   323.15
>>
```

D4A

unitStepFunction1.m

/Users/arfaz/Library/CloudStorage/OneDrive-UniversityofVictoria/0 UVIC/2 ENGR Y3/4 ECE 260 A01 T03/1 Assignments/0 Solutions/unitStepFunction1.m

```
function x = unitStepFunction1(t)
    % unitstep - Compute the unit-step function.
    if t >= 0
        x = 1;
    else
        x = 0;
    end
end
```

Command Window

D4B

unitStepFunction2.m

/Users/arfaz/Library/CloudStorage/OneDrive-UniversityofVictoria/0 UVIC/2 ENGR Y3/4 ECE 260 A01 T03/1 Assignments/0 Solutions/uni...

```
function x = unitStepFunction2(t)
    x = zeros(size(t));
    m = length(x);
    for i = 1 : m
        if t(i) >= 0
            x(i) = 1;
        end
    end
end
```

Command Window

D4C

unitStepFunction3.m

/Users/arfaz/Library/CloudStorage/OneDrive-UniversityofVictoria/0 UVIC/2 ENGR Y3/4 ECE 260 A01 T03/1 Assignments/0 Solutions/uni...

```
function x = unitStepFunction3(t)
    x = (t >= 0);
end
```

Command Window