

Answer (u).

We need to compute $x * h$, where

$$x * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$x(t) = \begin{cases} 2-t & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(t) = \begin{cases} -t-2 & -3 \leq t < -2 \\ 0 & \text{otherwise} \end{cases}$$

First, we plot $x(\tau)$ and $h(t-\tau)$ versus τ in Figures (a) and (d), respectively.

Figure (e): $t < -2$

$$x * h(t) = 0$$

Figure (f): $-2 \leq t < -1$

$$x * h(t)$$

$$= \int_1^{t+3} (2-\tau)(\tau-t-2) d\tau$$

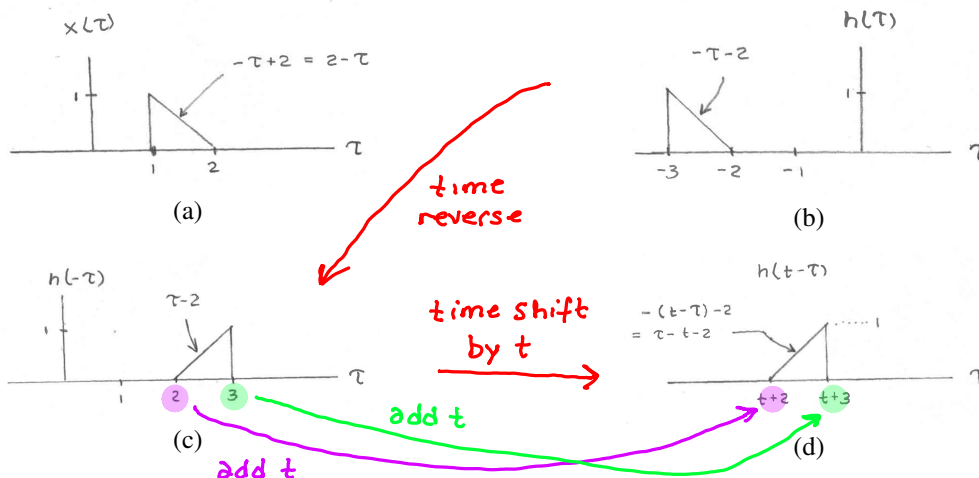
Figure (g): $-1 \leq t < 0$

$$x * h(t)$$

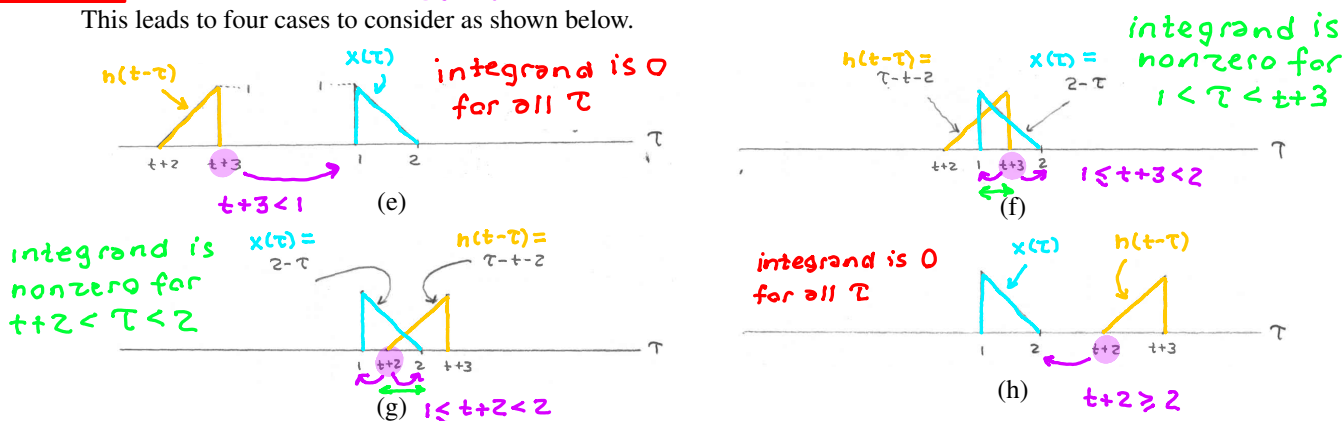
$$= \int_{t+2}^2 (2-\tau)(\tau-t-2) d\tau$$

Figure (h): $t \geq 0$

$$x * h(t) = 0$$



This leads to four cases to consider as shown below.



From Figure (e), for $t < -2$ (i.e., $t+3 < 1$), we have

$$x * h(t) = 0.$$

From Figure (f), for $-2 \leq t < -1$ (i.e., $1 \leq t+3 < 2$), we have

$$x * h(t) = \int_1^{t+3} \underbrace{(2-\tau)}_{x(\tau)} \underbrace{(\tau-t-2)}_{h(t-\tau)} d\tau.$$

From Figure (g), for $-1 \leq t < 0$ (i.e., $1 \leq t+2 < 2$), we have

$$x * h(t) = \int_{t+2}^2 \underbrace{(2-\tau)}_{x(\tau)} \underbrace{(\tau-t-2)}_{h(t-\tau)} d\tau.$$

From Figure (h), for $t \geq 0$ (i.e., $t+2 \geq 2$), we have

$$x * h(t) = 0.$$

Simplifying, we obtain

$$x * h(t) = \begin{cases} \frac{1}{6}t^3 - t - \frac{2}{3} & -2 \leq t < -1 \\ -\frac{1}{6}t^3 & -1 \leq t < 0 \\ 0 & \text{otherwise.} \end{cases}$$