Assignment:

$$(e^{j\pi t} - e^{-j\pi t})(e^{j\pi t} - e^{-j\pi t})$$
$$= e^{2j\pi t} - e^{0} - e^{0} + e^{-2j\pi t}$$

5.1

a) $1 + \cos(\pi t) + \sin^2(\pi t)$

$$\rightarrow 1 + \frac{1}{2}(e^{j\pi t} + e^{-j\pi t}) + \left[\frac{1}{2j}(e^{j\pi t} - e^{-j\pi t})\right]^2$$

$$\rightarrow \frac{1}{2}e^{j\pi t} + \frac{1}{2}e^{-j\pi t} - \frac{1}{4}e^{2j\pi t} + \frac{3}{2} - \frac{1}{4}e^{-2j\pi t}$$

$w_0 = \pi, \; T = 2$

$$c_n = \begin{cases} 3/2 & n = 0 \\ 1/2 & n = \pm 1 \\ -1/4 & n = \pm 2 \\ 0 & \text{otherwise} \end{cases}$$

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b) $[\cos(4t)][\sin(t)]$

$$\frac{1}{2}(e^{j4t} + e^{-j4t})\frac{1}{2j}(e^{jt} - e^{-jt})$$

$$\left(\frac{1}{2}e^{4jt} + \frac{1}{2}e^{-4jt}\right)\left(\frac{1}{2j}e^{jt} - \frac{1}{2j}e^{-jt}\right)$$

$$\frac{1}{4j}e^{5jt} - \frac{1}{4j}e^{3jt} + \frac{1}{4j}e^{-3jt} - \frac{1}{4j}e^{-5jt}$$

$w_0 = 1$

$T = 2\pi$

$$c_n = \begin{cases} 1/4j & n = \pm 5, 3 \\ 0 & \text{otherwise} \end{cases}$$

c)

$$x(t) = |\sin(2\pi t)| = \left|\frac{1}{2j}\left(e^{2j\pi t} - e^{-2j\pi t}\right)\right|$$

$$= \frac{1}{2j}e^{2j\pi t} - \frac{1}{2j}e^{-2j\pi t}$$

$\omega_0 = \pi$

$T = 2$

$$c_n = \frac{1}{T}\int_T |\sin(2\pi t)| e^{-jn4\pi t}\,dt$$

$$= 2\int_0^{1/2} \sin(\underbrace{2\pi}_{b} t) e^{\overbrace{-jk4\pi}^{a} t}\,dt = 2\int_0^{1/2} e^{\overbrace{-jn4\pi}^{a} \underset{x}{t}} \sin(\underbrace{2\pi}_{b} t)\,d\underset{x}{t}$$

$$= 2\left(\frac{e^{-jn4\pi t}\left[-jn4\pi\sin(2\pi t) - 2\pi\cos(2\pi t)\right]}{(-jn4\pi)^2 + (2\pi)^2}\right)_0^{1/2}$$

$$= 2\left(\frac{e^{-jn2\pi}\left[-jn4\pi\sin(\pi) - 2\pi\cos(\pi)\right] + 2\pi}{-n^2 16\pi^2 + 4\pi^2}\right)$$

$$= 2\left(\frac{e^{-jn2\pi}(4\pi)}{4 - 16n^2(\pi^2)}\right)$$

$$c_n = \frac{1}{T}\int_T x(t)e^{-jnw_0 t}\,dt \qquad T=4 \qquad w_0 = \frac{\pi}{2}$$

$$c_n = \frac{1}{4}\int_{-2}^{2}\left(\delta(t-1) - \frac{1}{2}\delta(t+1)\right)e^{-jn\pi/2\, t}\,dt$$

$$c_n = \frac{1}{4}\int_{-2}^{2}\delta(t-1)e^{-jn\pi/2\, t}\,dt - \frac{1}{8}\int_{-2}^{2}\delta(t+1)e^{-jn\pi/2\, t}\,dt$$

$$c_n = \frac{1}{4}\left(e^{-jn\pi/2}\right) - \frac{1}{8}\left(e^{jn\pi/2}\right)$$

$$= \frac{1}{4}e^{-jn\pi/2} - \frac{1}{8}e^{jn\pi/2}$$

$$= \frac{1}{4}(-j)^n - \frac{1}{8}j^n$$

$x(t) = -x(t - T/2)$ $\qquad$ $\underline{c_n = 0 \text{ for all even } n}$

$\rightarrow$

$$T = \frac{T}{2}$$

$$-x(t-T/2) = -\sum_{n=-\infty}^{\infty} c_n e^{jnw_0(t-T/2)}$$

$$= -\sum_{-\infty}^{\infty} c_n e^{jnw_0 t} e^{-jnw_0 T/2} = -\sum_{n=-\infty}^{\infty} e^{-j\pi n} c_n e^{j\pi w_0 t}$$

$$= -\sum_{n=-\infty}^{\infty} (-1)^n c_n e^{j\pi w_0 t}$$

$c_n = -(-1)^n c_n$

if n is even:

if n is even

$c_n = -c_n \qquad c_n + c_n = 0 \qquad \underline{c_n = 0}$

1)

$T = 5 \quad \omega_0 = \frac{2\pi}{5} \qquad x(t) = 5e^{3t} \quad 0 \le t < 5$

$$c_n = \frac{1}{\cancel{5}} \int_0^5 \cancel{5} e^{3t} e^{-jn\omega_0 t} \, dt$$

$$c_n = \int_0^5 e^{3t - jn\omega_0 t} \, dt = \int_0^5 e^{(3 - jn\omega_0)t} \, dt$$

$$= \frac{1}{3 - jn\omega_0} e^{(3 - jn\omega_0)t} \Big|_0^5$$

$$= \frac{1}{3 - jn\omega_0} \left(e^{(3 - jn\omega_0)5} - 1 \right)$$

$$= \frac{1}{3 - jn\frac{2\pi}{5}} \left(e^{15 - \cancel{5}jn\frac{2\pi}{\cancel{5}}} - 1 \right)$$

$$= \frac{1}{\frac{3}{1} - jn\frac{2\pi}{5}} \left(e^{15} e^{-jn2\pi} - 1 \right)$$

$$= \frac{1}{\frac{15}{5} - \frac{jn2\pi}{5}} \left(e^{15} (1)^n - 1 \right) = \frac{5(e^{15} - 1)}{15 - jn2\pi}$$

2/

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function y myfunc(x)

    if (x >= 0)
        y = 0
        for n = 0:99
            y = y + exp(n * x) * cos(n * x)
        end
    else
        y = exp(x)
    end
end
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3)

$$\tilde{x}(t_a) = \frac{1}{2}[x(t_a^-) + x(t_a^+)]$$

$$y(0) = \frac{1}{2}[x(0^-) + x(0^+)]$$

$$= \frac{1}{2}[-25 + 1] = \frac{1}{\cancel{2}}(-\overset{12}{\cancel{24}}) = \boxed{-12} \quad y(0)$$

$$y(2) = \frac{1}{2}[x(2^-) + x(2^+)]$$

$$= \frac{1}{2}[e^2 - 2^2] = \boxed{\frac{1}{2}[e^2 - 4]} \quad y(2)$$

4)

$$c_h = \frac{-1}{(2+j\pi h)^2},$$

$$|c_h| = \left|\frac{-1}{(2+j\pi h)^2}\right| = \frac{|-1|}{|(2+j\pi h)^2|} = \frac{1}{\left(\sqrt{2^2+(j\pi h)^2}\right)^2}$$

$$= \boxed{\frac{1}{4-\pi^2 h^2}} \leftarrow$$

$$\arg c_h = \arg\left(\frac{-1}{(2+j\pi h)^2}\right) = \frac{\arg(-1)}{\arg(2+j\pi h)^2}$$

$$\arg(-1) - \arg((2+j\pi h)^2)$$

$$= \arg(-1) - 2\arg(2+j\pi h)$$

$$\boxed{\pi - 2\arctan\left(\frac{\pi h}{2}\right)}$$

5)

$$x(t) = 4 + 3\cos(t) + 2\cos(3t)$$

$$= 4 + 3\left[\tfrac{1}{2}\left(e^{jt} + e^{-jt}\right)\right] + 2\left[\tfrac{1}{2}\left(e^{j3t} + e^{-j3t}\right)\right]$$

$$= \underbrace{4}_{4e^{0t}} + \frac{3}{2}e^{jt} + \frac{3}{2}e^{-jt} + e^{j3t} + e^{-j3t}$$

$$c_n = \begin{cases} 4 & n = 0 \\ 3/2 & n = \pm 1 \\ 1 & n = \pm 3 \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{w_0 = 1}$$

$$y(t) = \sum_{n=0}^{3} c_n H(nw_0) e^{jnw_0 t}$$

$$y(t) = \frac{3}{2} H(1) e^{jt} + \frac{3}{2} H(-1)^{-jt} + H(3) e^{3jt} + H(-3) e^{-3jt}$$

$$y(t) = \frac{3}{2} e^{jt} - \frac{3}{2} e^{-jt} + e^{3jt} - e^{-3jt}$$

$$y(t) = \frac{3}{2}\left(e^{jt} - e^{-jt}\right) + \left(e^{3jt} - e^{-3jt}\right)$$

$$y(t) = \frac{3}{\cancel{2}}\left(\cancel{2}j\sin(t)\right) + \left(2j\sin(3t)\right)$$

$$y(t) = 3j\sin(t) + 2j\sin(3t)$$

$T=4 \quad w_0 = \frac{2\pi}{4} = \frac{\pi}{2} \qquad x(t) = \sum_{k=-\infty}^{\infty} c_n e^{jnw_0 t}$

$$c_n = \frac{1}{4}\int_{-1}^{1} 1e^{-jnw_0 t}\, dt$$

$$c_n = \frac{1}{4}\left(-\frac{1}{jnw_0} e^{-jnw_0 t}\right)\Bigg|_{-1}^{1}$$

$$= \frac{1}{4} \cdot -\frac{1}{jnw_0}\left(e^{-jnw_0} - e^{jnw_0}\right)$$

$$= -\frac{1}{2jn\pi}\left(e^{-jk\pi/2} - e^{jn\pi/2}\right)$$

$$= -\frac{1}{2jn\pi}\left(-e^{jn\pi/2} + e^{-jn\pi/2}\right)$$

$$= \frac{1}{2jn\pi}\left(e^{jn\pi/2} - e^{-jn\pi/2}\right)$$

$$= \frac{1}{\cancel{2j}n\pi}\left(\cancel{2j}\sin(n\pi/2)\right)$$

$$= \frac{1}{n\pi}\sin(n\pi/2)$$

$$x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{n\pi}\sin(n\pi/2)\, e^{jn\pi/2 t}$$

$T = 3, \; w_0 = \frac{2\pi}{3}$ interval $= -3/2 \to \frac{3}{2}$

$$c_n = \frac{1}{3}\int_{-3/2}^{3/2} \left(\delta(t-1) + -\delta(t+1)\right) e^{-jnw_0 t} \, dt$$

$$c_n = \frac{1}{3}\int_{-1}^{1} \left(\delta(t-1) - \delta(t+1)\right) e^{-jnw_0 t} \, dt$$

$$= \frac{1}{3}\int_{-\infty}^{\infty} \delta(t-1) e^{-jnw_0 t} \, dt - \frac{1}{3}\int_{-\infty}^{\infty} \delta(t+1) e^{-jnw_0 t} \, dt$$

$$= \frac{1}{3}\left(e^{-jnw_0}\right) - \frac{1}{3}\left(e^{jnw_0}\right)$$

$$= \frac{1}{3}\left(e^{-jnw_0} - e^{jnw_0}\right)$$

$$= -\frac{1}{3}\left(e^{jnw_0} - e^{-jnw_0}\right)$$

$$-\frac{1}{3}\left(2j\sin(nw_0)\right)$$

$$-\frac{1}{3}\left(2j\sin\left(\frac{2\pi n}{3}\right)\right)$$

$$x(t) \sum_{n=-\infty}^{\infty} -\frac{1}{3} 2j\sin(2\pi n/3)\, e^{jn\frac{2\pi}{3}t}$$

$$C_k = -\frac{2j}{3}\sin\left(\frac{2\pi k}{3}\right)$$

$$|C_k| = \left|-\frac{2j}{3}\sin\left(\frac{2\pi k}{3}\right)\right|$$

$$|C_k| = \left|0 - j\frac{2}{3}\sin\left(\frac{2\pi k}{3}\right)\right|$$

$$= \sqrt{0^2 + \left(\frac{2}{3}\sin\left(\frac{2\pi k}{3}\right)\right)^2}$$

$$= \sqrt{\left(\frac{2}{3}\sin\left(\frac{2\pi k}{3}\right)\right)^2} = \frac{2}{3}\left|\sin\left(\frac{2\pi k}{3}\right)\right|$$

$$\arg c_k = \arg\left(-j\frac{2}{3}\sin\left(\frac{2\pi k}{3}\right)\right)$$

$$\arctan(-\infty) = -\pi/2$$

$$\arctan(\infty) = \pi/2$$

$$x(t) = \frac{1}{2}\cos(4t) + \sin(6t)$$

$$= \frac{1}{2}\left[\frac{1}{2}\left(e^{j4t} + e^{-j4t}\right)\right] + \frac{1}{2j}\left(e^{j6t} - e^{-j6t}\right)$$

$$= \frac{1}{4}\left(e^{j2\cdot 2t} + e^{-j2\cdot 2t}\right) + \frac{1}{2j}\left(e^{j2\cdot 3t} - e^{-j2\cdot 3t}\right)$$

$w_0 = 2$

$$= \frac{1}{4}e^{j2\cdot 2t} + \frac{1}{4}e^{-j2\cdot 2t} + \frac{1}{2j}e^{j3\cdot 2t} - \frac{1}{2j}e^{-j3\cdot 2t}$$

$$C_n = \begin{cases} 1/4 & n = \pm 2 \\ 1/2j & n = 3 \\ -1/2j & n = -3 \end{cases}$$

$x(t) = \sin^2 t$

$$x(t) = \left[\frac{1}{2j}\left(e^{jt} - e^{-jt}\right)\right]^2$$

$$= \left[-\frac{1}{4}\left(e^{jt} - e^{-jt}\right)^2\right]$$

$$= -\frac{1}{4}\left(e^{jt} - e^{-jt}\right)\left(e^{jt} - e^{-jt}\right)$$

$$= -\frac{1}{4}\left(e^{2jt} - e^{0} - e^{0} + e^{-2jt}\right)$$

$$= -\frac{1}{4}e^{2jt} + \frac{1}{2} - \frac{1}{4}e^{-2jt}$$

$w_0 = 2$

$$c_k = \begin{cases} -1/4 & k = \pm 1 \\ 1/2 & k = 0 \end{cases}$$

$$H(jw) = \begin{cases} 1 & |w| \geq 5 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = 1 + 2\cos(2t) + \frac{1}{2}\cos(4t) + 1/2\cos(6t)$$

$$y(t) = ?$$

$$x(t) = 1 + 2\frac{1}{2}(e^{j2t} + e^{-j2t}) + \frac{1}{4}(e^{j4t} + e^{-j4t}) + \frac{1}{4}(e^{j6t} + e^{-j6t})$$

$$w_0 = 2 \quad = 1 + e^{j2t} + e^{-j2t} + \frac{1}{4}e^{j4t} + \frac{1}{4}e^{-j4t} + \frac{1}{4}e^{j6t} + \frac{1}{4}e^{-j6t}$$

$$c_k = \begin{cases} 1 & k = 0, \pm 1 \\ 1/4 & k = \pm 2, \pm 3 \\ 0 & \text{otherwise} \end{cases}$$

$w_0 = 2$

$$c_0 \overset{\to 0}{H(\cancel{0}\ 2)} e^0 + c_1 \overset{\to 0}{H(2)} e^{j2t} + c_{-1} \overset{\to 0}{\cancel{H(-2)}} e^{-j2t} + c_2 \overset{\to 0}{\cancel{H(4)}} e^{j4t} + c_{-2} \overset{\to 0}{\cancel{H(-4)}} e^{-j4t}$$
$$+ c_3 H(6) e^{j6t} + c_{-3} H(-6)^{-j6t}$$

$$1/4\, e^{j6t} + \frac{1}{4} e^{-j6t}$$

$$\frac{1}{4}(e^{j6t} + e^{-j6t})$$

$$\frac{1}{\cancel{4}_{\,2}}(\cancel{2}\cos(6t))$$

$$\boxed{\frac{1}{2}\cos(6t)}$$

a) $x(t) = e^{-t} \quad -1 < t < 1 \quad T = 2 \quad \omega_0 = \frac{2\pi}{2} = \pi$

$$\frac{1}{2}\int_{-1}^{1} e^{-t} \cdot e^{-jk\pi t}\, dt \qquad -t - jk\pi t$$

$$-t(1+jk\pi)$$

$$\frac{1}{2}\int_{-1}^{1} e^{(-1-jk\pi)t}\, dt$$

$$\frac{1}{2}\left(\frac{1}{-(1+jk\pi)} e^{-t(1+jk\pi)}\right)\Bigg|_{-1}^{1}$$

$$\frac{1}{-2(1+jk\pi)}\left(e^{-1-jk\pi} - e^{1+jk\pi}\right)$$

$$\frac{1}{2(1+jk\pi)}\left(e^{1+jk\pi} - e^{-(1+jk\pi)}\right)$$

$$\frac{1}{2(1+jk\pi)}\left(e^{1}\cdot(e^{j\pi})^{k}\right) - \left(e^{-1}\cdot(e^{-j\pi})^{k}\right)$$

$$\frac{1}{2(1+jk\pi)}\left(e^{1}\cdot(-1)^{k}\right) - \left(e^{-1}\cdot(-1)^{k}\right)$$

$$\frac{(-1)^{k}\left(e^{1} - e^{-1}\right)}{2(1+jk\pi)}$$

1) $T = 5 \quad \omega_0 = \frac{2\pi}{5}$

$$c_n = \frac{1}{\cancel{5}} \int_0^5 \cancel{5} e^{3t} \cdot e^{-jn\omega_0 t} \, dt$$

$$= \int_0^5 e^{3t} \cdot e^{-jn\frac{2\pi}{5}t} \, dt$$

$$= \int_0^5 e^{(3 - jn\omega_0)t} \, dt$$

$$= \frac{1}{3 - jn\omega_0} e^{(3 - jn\omega_0)t} \Big|_0^5$$

$$= \frac{1}{3 - jn\omega_0} \left(e^{(3 - jn\omega_0)(5)} - 1 \right)$$

$$= \frac{1}{3 - jn\frac{2\pi}{5}} \left(e^{15} - 1 \right)$$

4)

4) $T = 2 \quad w_0 = \pi$

$$|C_n| = \left|\frac{-1}{(2+j\pi n)^2}\right| = \frac{1}{|2+(j\pi n)|^2} = \frac{1}{\left(\sqrt{2^2+(j\pi n)^2}\right)^2} = \frac{1}{4+\pi^2 n^2}$$

$$\arg C_n = \arg(-1) - \arg(2+j\pi n)^2$$

$$= \pi - 2\arg(2+j\pi n)$$

$$= \boxed{\pi - 2\arctan\left(\frac{\pi n}{2}\right)}$$

5) $4 + 3\frac{1}{2}\left(e^{jt} + e^{-jt}\right) + e^{j3t} + e^{-j3t}$

$k=0$, $\omega_0=1$: $\underset{0}{4} + \frac{3}{2}\underset{1}{e^{jt}} + \frac{3}{2}\underset{-1}{e^{-jt}} + \underset{3}{e^{j3t}} + \underset{-3}{e^{-j3t}}$ $\quad \omega_0 = 1$

$$c_k = \begin{cases} 3/2 & k = \pm 1 \\ 1 & k = \pm 3 \\ 4 & k = 0 \end{cases}$$

$$y(t) = H(0) + \frac{3}{2}e^{jt}H(1) + \frac{3}{2}e^{-jt}H(-1) + e^{j3t}H(3) + e^{-j3t}H(-3)$$

$$0 + \frac{3}{2}e^{jt} - \frac{3}{2}e^{-jt} + e^{j3t} - e^{-j3t}$$

$$= \frac{3}{2}\left(e^{jt} - e^{-jt}\right) + \left(e^{j3t} - e^{-j3t}\right)$$

$$\frac{3}{\cancel{2}}\left(\cancel{2}j\sin(t)\right) + 2j\sin(3t)$$

$$3j\sin(t) + 2j\sin(3t)$$

3)

$$y(0) = \frac{1}{2}\left[x(0^-) + x(0^+)\right]$$

$$= \frac{1}{2}\left[-(5)^2 + 1\right] = \frac{1}{2}\left[-25 + 1\right]$$

$$= \frac{1}{2}\left[-24\right] = \boxed{-12}$$

$$y(2) = \frac{1}{2}\left[x(2^-) + x(2^+)\right]$$

$$= \frac{1}{2}\left[e^2 + -4\right] = \boxed{\frac{1}{2}(e^2 - 4)}$$

2) function y myfunc (x)

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if (x >= 0)
   y = 0
   for n = 0 : 49

      y = y + exp(n * x) * cos(n * x)

   end
else

   y = exp(x)

end
end
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