ECE-260

Tutorial 06

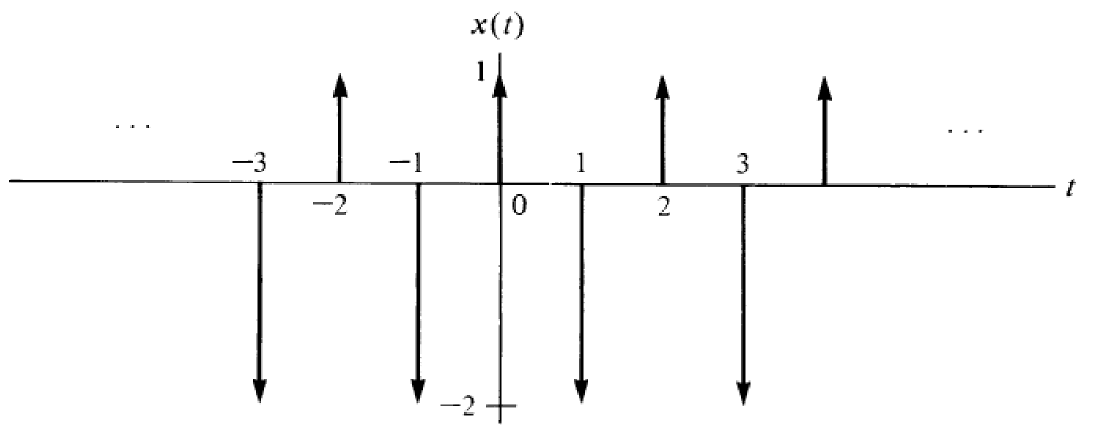
Topics covered

1) Fourier series

2) Properties of Fourier series

Question 01

Determine the Fourier series for the following signal.



The period is $T_0 = 2$, with $\omega_0 = 2\pi/2 = \pi$. The Fourier coefficients are

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Choosing the period of integration as $-\frac{1}{2}$ to $\frac{3}{2}$, we have

$$\begin{aligned} a_k &= \frac{1}{2}\int_{-1/2}^{3/2} x(t)e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2}\int_{-1/2}^{3/2} [\delta(t) - 2\delta(t-1)]e^{-jk\omega_0 t}\,dt \\ &= \frac{1}{2} - e^{-jk\omega_0} = \frac{1}{2} - (e^{-j\pi})^k \end{aligned}$$

Therefore,

$$a_0 = -\tfrac{1}{2}, \qquad a_k = \tfrac{1}{2} - (-1)^k$$

$$x(t) = \sum_{-\infty}^{+\infty} a_k e^{jk\pi t}$$

where

$$a_k = \begin{cases} -\dfrac{1}{2}, & k = 0 \\ \dfrac{1}{2} - (-1)^k, & k \neq 0 \end{cases}$$

REVIEW:

(Fourier series analysis equation).

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt,$$

$$\equiv$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Question 01

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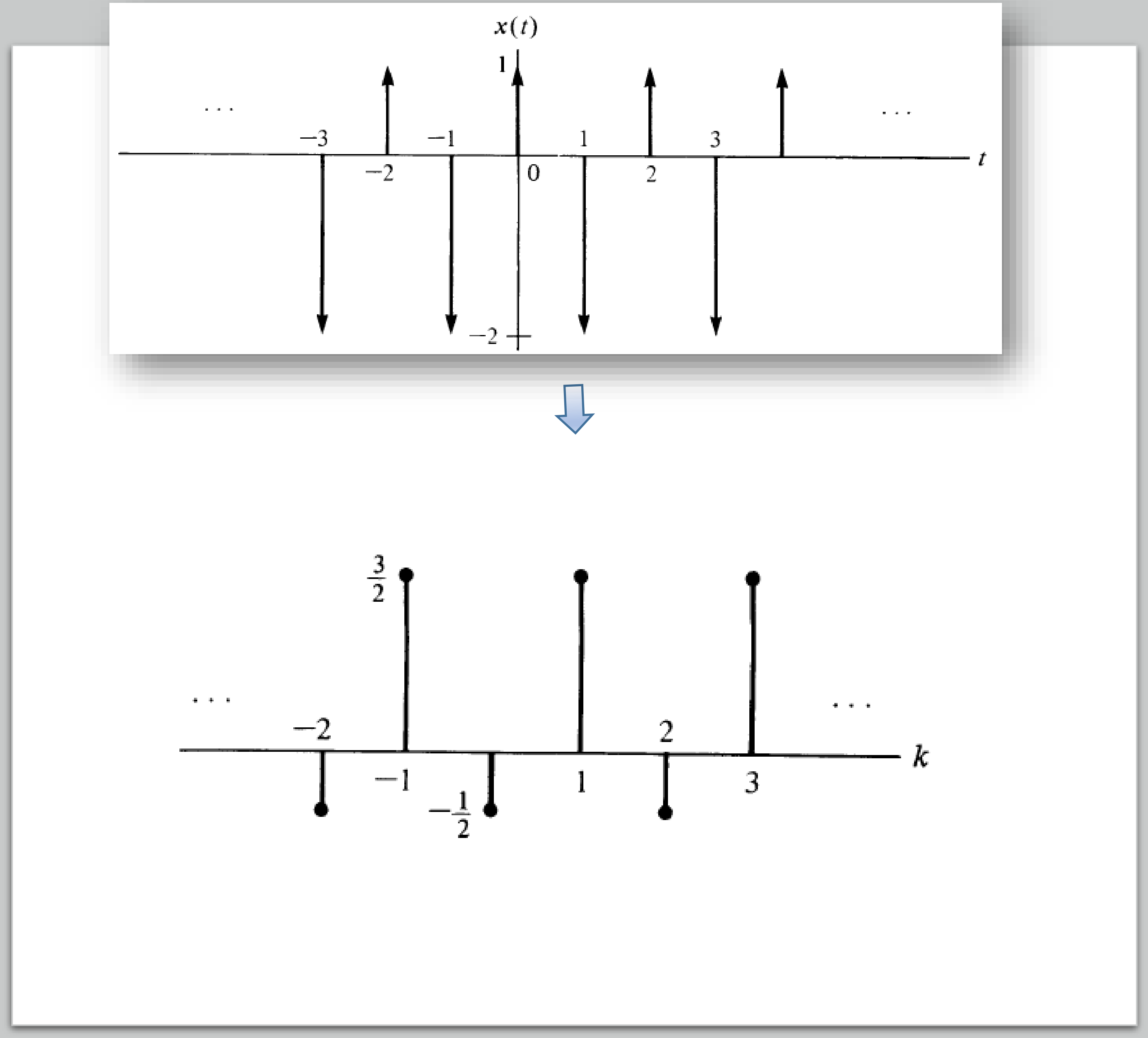


REVIEW:

(Fourier series analysis equation).

$$c_k = \frac{1}{T}\int_T x(t)e^{-jk\omega_0 t}\,dt,$$

$$\equiv$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$$

Question 02

Use symbolic MATLAB to compute the Fourier series of $y(t) = 1 + \sin(100t)$. Find and plot its magnitude and phase spectra.

REVIEW: $\omega_0 \equiv \Omega_0$

Find the Fourier series of a raised-cosine signal ($B \geq A$),

$$x(t) = B + A\cos(\Omega_0 t + \theta)$$

which is periodic of period T_0 and fundamental frequency $\Omega_0 = 2\pi/T_0$.

$$x(t) = B + \frac{A}{2}\left[e^{j(\Omega_0 t+\theta)} + e^{-j(\Omega_0 t+\theta)}\right]$$

$$= B + \frac{Ae^{j\theta}}{2}e^{j\Omega_0 t} + \frac{Ae^{-j\theta}}{2}e^{-j\Omega_0 t}$$

which gives

$$X_0 = B$$

$$X_1 = \frac{Ae^{j\theta}}{2}$$

$$X_{-1} = X_1^*$$

If we let $\theta = -\pi/2$ in $x(t)$, we get

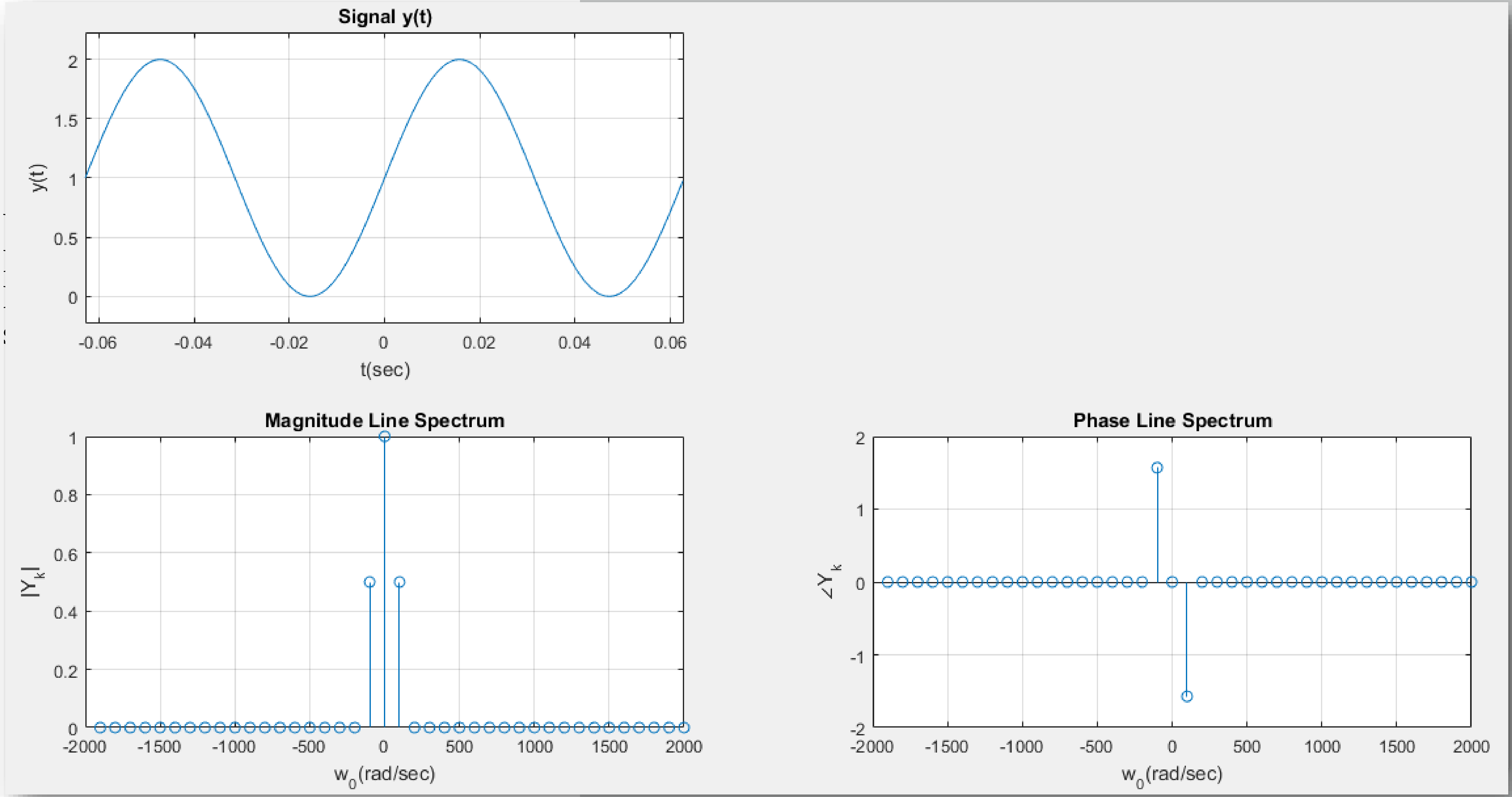
$$y(t) = B + A\sin(\Omega_0 t)$$

Question 02

Use symbolic MATLAB to compute the Fourier series of $y(t) = 1 + \sin(100t)$. Find and plot its magnitude and phase spectra.

```
% Tutorial 06_ECE 260 (Part-A)

% symbolic Fourier Series computation
% x: periodic signal
% T0: period
% N: number of harmonics
% X,w: Fourier series coefficients at harmonic frequencies

function [X, w] = fourierseries(x, T0, N)

syms t
% computation of N Fourier series coefficients
    for k = 1:N
        X1(k) = int(x*exp(-1i*2*pi*(k - 20) * t/T0), t, 0, T0)/T0;
        X(k) = subs(X1(k));
        w(k) = (k-20)*2*pi/T0; % harmonic frequencies
    end
```

```
% Tutorial 06_ECE 260 (Part-B)

% symbolic Fourier Series computation
% x: periodic signal
% T0: period
% N: number of harmonics
% X,w: Fourier series coefficients at harmonic frequencies

syms t

T0 = 2*pi/100;
x = 1 + sin(100*t);
% N = 40;

[X,w] = fourierseries(x,T0,40);

subplot(221); ezplot(x,[-T0 T0]); title('Signal y(t)'); xlabel('t(sec)'); ylabel('y(t)'); grid

subplot(223); stem(w,abs(X)); title('Magnitude Line Spectrum'); xlabel('w_{0}(rad/sec)'); ylabel('|Y_{k}|'); grid

subplot(224); stem(w,angle(X)); title('Phase Line Spectrum'); xlabel('w_{0}(rad/sec)'); ylabel('\angleY_{k}'); grid
```


Signal y(t)
y(t)
t(sec)
Magnitude Line Spectrum
$|Y_k|$
w_0(rad/sec)
Phase Line Spectrum
$\angle Y_k$
w_0(rad/sec)

Question 03

Consider the sum *z(t)* of a periodic signal *x(t)* of period $T_1 = 2$, with a periodic signal *y(t)* with period $T_2 = 0.2$. Find the Fourier coefficients Z_k of *z(t)* in terms of the Fourier coefficients X_k and Y_k of *x(t)* and *y(t)*.

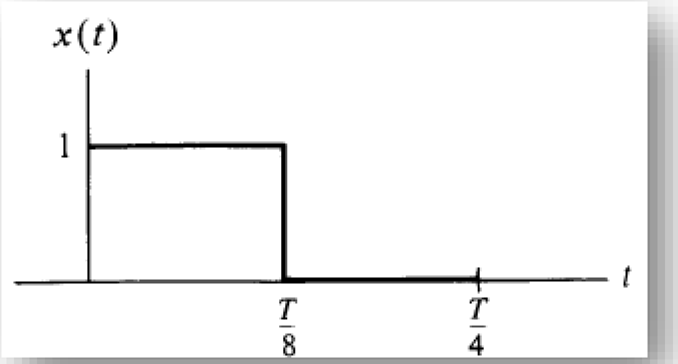
The ratio $T_2/T_1 = 1/10 = N/M$ is rational, so $z(t)$ is periodic of period $T_0 = T_1 = 10T_2 = 2$. The fundamental frequency of $z(t)$ is $\Omega_0 = \Omega_1 = \pi$, and $\Omega_2 = 10\Omega_0 = 10\pi$ is the fundamental frequency of $y(t)$. Thus, the Fourier coefficients of $z(t)$ are

$$Z_k = \begin{cases} X_k + Y_{k/10} & \text{when } \; k = 0, \pm 10, \pm 20, \ldots \\ X_k & \text{otherwise} \end{cases}$$

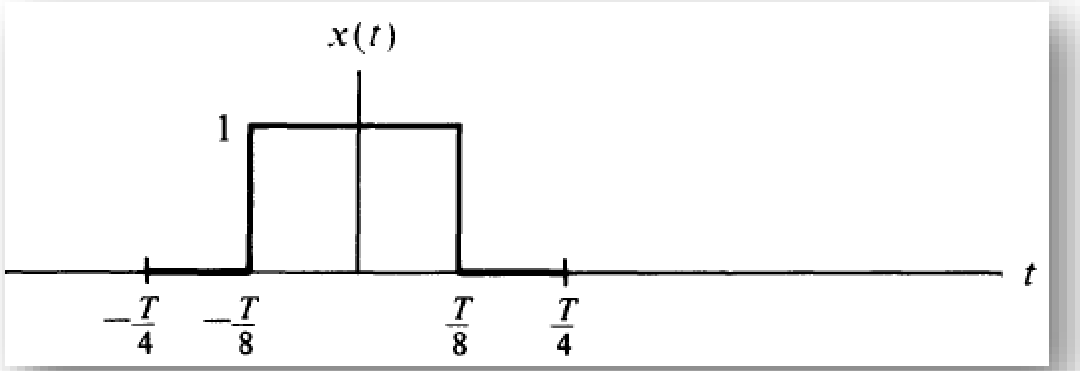
Question 04

Suppose *x(t)* is periodic with period **T** and is specified in the interval 0 < t < **T**/4 as given in figure. Sketch *x(t)* in the interval 0 < t < **T** if

- The Fourier series has only odd harmonics and *x(t)* is an even function



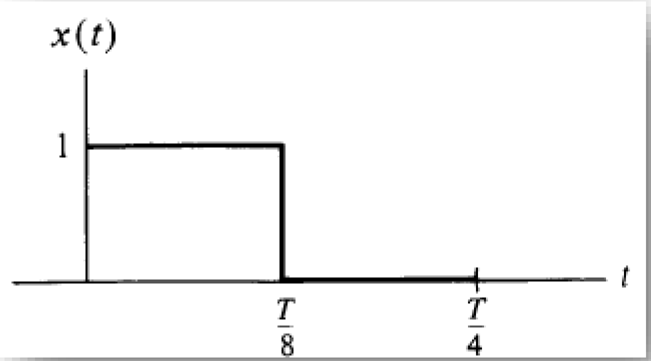
Since $x(t)$ is even, we can extend Figure



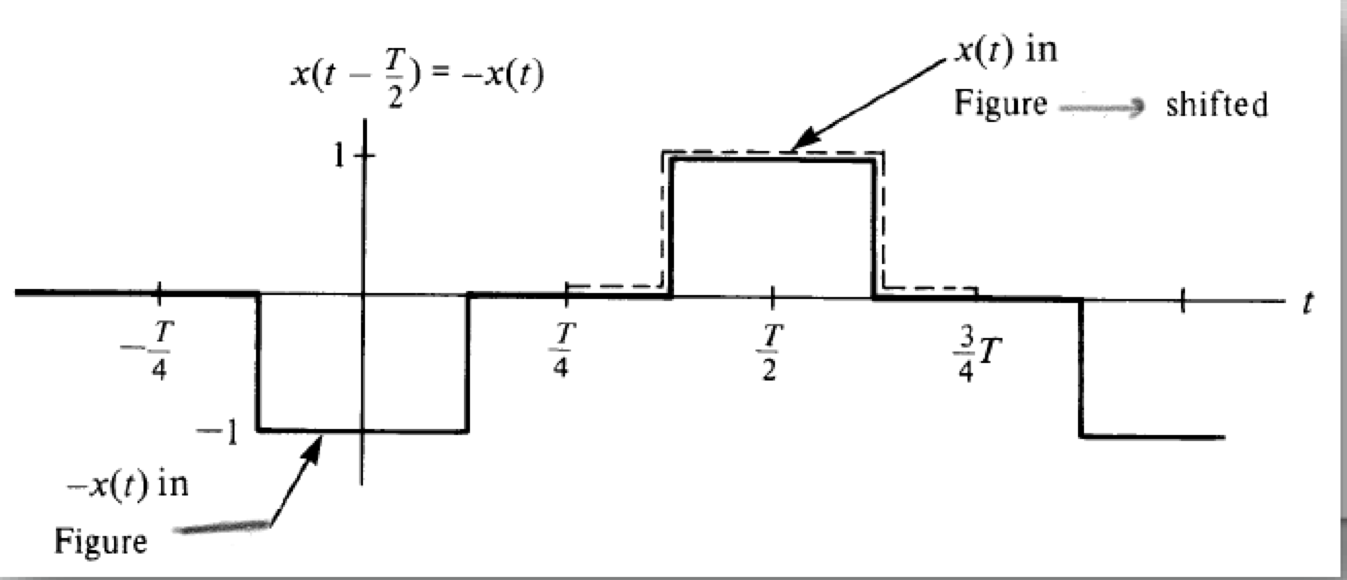
Question 04

Suppose $x(t)$ is periodic with period **T** and is specified in the interval $0 < t < \mathbf{T}/4$ as given in figure. Sketch $x(t)$ in the interval $0 < t < \mathbf{T}$ if

- The Fourier series has only odd harmonics and $x(t)$ is an even function



Since $x(t)$ has only odd harmonics, it must have the property that $x(t - T/2) = -x(t)$



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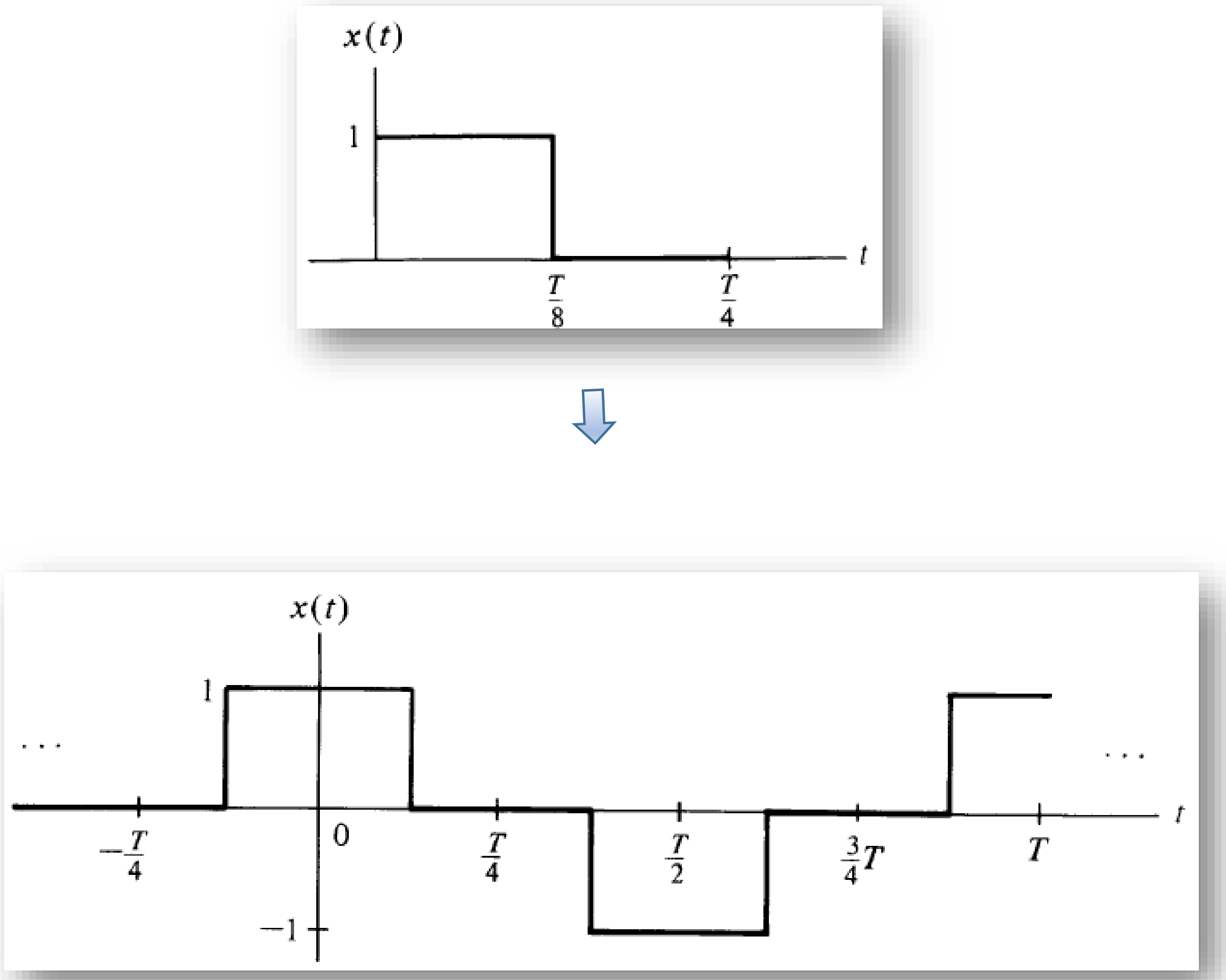

Question 05

Suppose *x(t)* is periodic with period **T** and is specified in the interval 0 < t < **T**/4 as given in figure. Sketch *x(t)* in the interval 0 < t < **T** if

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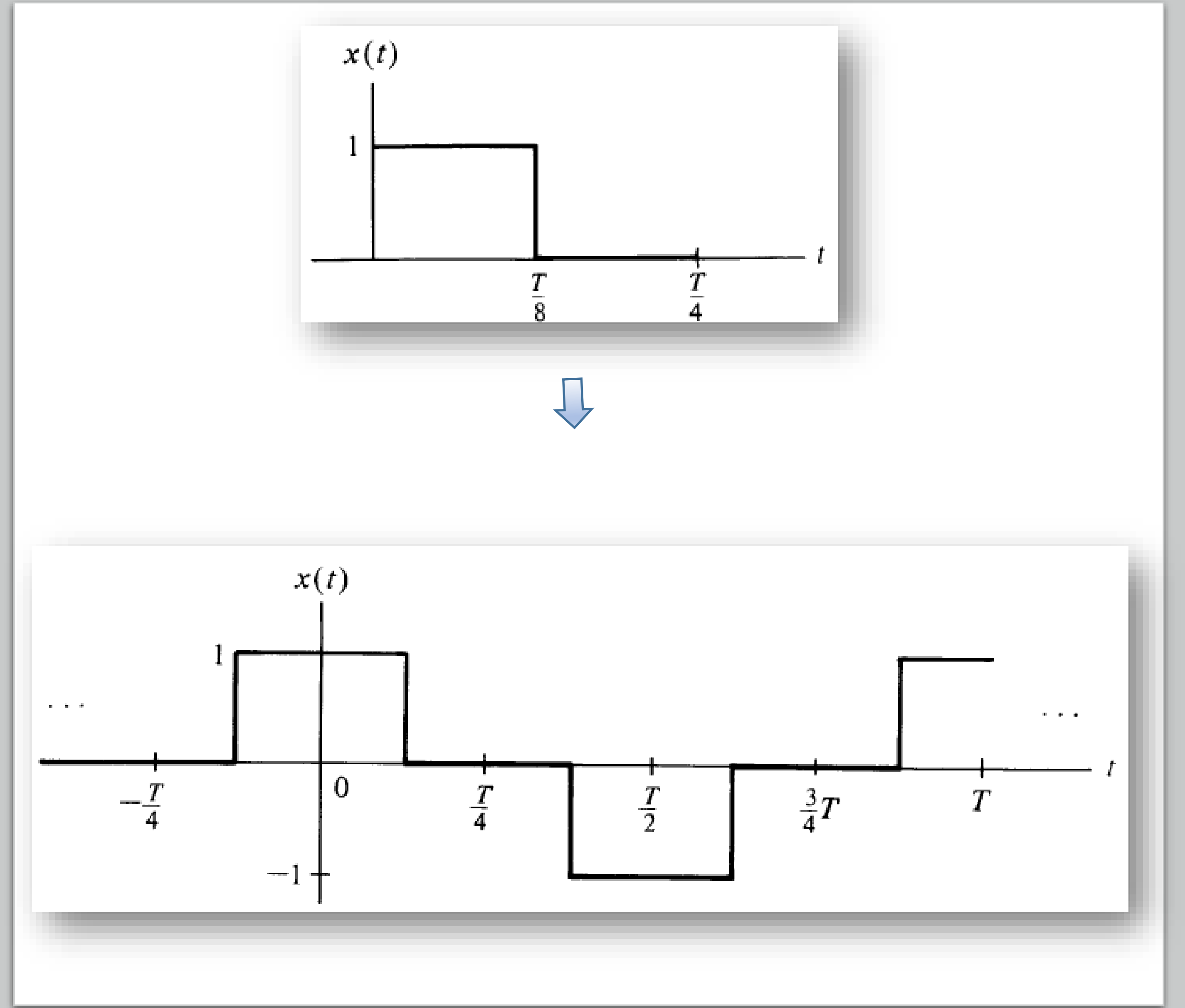

End of Tutorial 06

Concept Review

- Linearity of Fourier Series

- *Same fundamental frequency*: If $x(t)$ and $y(t)$ are periodic signals with the same fundamental frequency Ω_0, then the Fourier series coefficients of $z(t) = \alpha x(t) + \beta y(t)$ for constants α and β are

$$Z_k = \alpha X_k + \beta Y_k$$

where X_k and Y_k are the Fourier coefficients of $x(t)$ and $y(t)$.

- *Different fundamental frequencies*: If $x(t)$ is periodic of period T_1, and $y(t)$ is periodic of period T_2 such that $T_2/T_1 = N/M$, for nondivisible integers N and M, then $z(t) = \alpha x(t) + \beta y(t)$ is periodic of period

$T_0 = MT_2 = NT_1$, and its Fourier coefficients are

$$Z_k = \alpha X_{k/N} + \beta Y_{k/M} \qquad \text{for integers } k \text{ such that } k/N, \text{ and } k/M \text{ are integers}$$

where X_k and Y_k are the Fourier coefficients of $x(t)$ and $y(t)$.

Concept Review

The most important property of LTI systems is the eigenfunction property.

> *Eigenfunction property*: In steady state, the response to a complex exponential (or a sinusoid) of a certain frequency is the same complex exponential (or sinusoid), but its amplitude and phase are affected by the frequency response of the system at that frequency.

Suppose that the impulse response of an LTI system is $h(t)$ and that $H(s) = \mathcal{L}[h(t)]$ is the corresponding transfer function. If the input to this system is a periodic signal $x(t)$, of period T_0, with Fourier series

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t} \qquad \Omega_0 = \frac{2\pi}{T_0}$$

then according to the eigenfunction property the output in the steady state is

$$y_{ss}(t) = \sum_{k=-\infty}^{\infty} [X_k H(jk\Omega_0)]\, e^{jk\Omega_0 t}$$