Since the system is stable, $\int_0^\infty e(t)\,dt$ converges to a constant value. Noting that

$$\int_0^\infty e(t)\,dt = \lim_{s\to 0} s\,\frac{E(s)}{s} = \lim_{s\to 0} E(s)$$

we have

$$\begin{aligned}\int_0^\infty e(t)\,dt &= \lim_{s\to 0} \frac{Q(s) - P(s)}{sQ(s)} \\ &= \lim_{s\to 0} \frac{Q'(s) - P'(s)}{Q(s) + sQ'(s)} \\ &= \lim_{s\to 0} \left[Q'(s) - P'(s)\right]\end{aligned}$$

Since

$$\lim_{s\to 0} P'(s) = T_a + T_b + \cdots + T_m$$

$$\lim_{s\to 0} Q'(s) = T_1 + T_2 + \cdots + T_n$$

we have

$$\int_0^\infty e(t)\,dt = \left(T_1 + T_2 + \cdots + T_n\right) - \left(T_a + T_b + \cdots + T_m\right)$$

For a unit-step input $r(t)$, since

$$\int_0^\infty e(t)\,dt = \lim_{s\to 0} E(s) = \lim_{s\to 0} \frac{1}{1 + G(s)} R(s) = \lim_{s\to 0} \frac{1}{1 + G(s)} \frac{1}{s} = \frac{1}{\lim_{s\to 0} sG(s)} = \frac{1}{K_v}$$

we have

$$\frac{1}{K_v} = \frac{1}{\lim_{s\to 0} sG(s)} = \left(T_1 + T_2 + \cdots + T_n\right) - \left(T_a + T_b + \cdots + T_m\right)$$

Note that zeros in the left half-plane (that is, positive $T_a, T_b, \ldots, T_m$) will improve K_v. Poles close to the origin cause low velocity-error constants unless there are zeros nearby.

PROBLEMS

B–5–1. A thermometer requires 1 min to indicate 98% of the response to a step input. Assuming the thermometer to be a first-order system, find the time constant.

If the thermometer is placed in a bath, the temperature of which is changing linearly at a rate of 10°/min, how much error does the thermometer show?

B–5–2. Consider the unit-step response of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{1}{s(s + 1)}$$

Obtain the rise time, peak time, maximum overshoot, and settling time.

B–5–3. Consider the closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Determine the values of ζ and ω_n so that the system responds to a step input with approximately 5% overshoot and with a settling time of 2 sec. (Use the 2% criterion.)

B–5–4. Consider the system shown in Figure 5–72. The system is initially at rest. Suppose that the cart is set into motion by an impulsive force whose strength is unity. Can it be stopped by another such impulsive force?

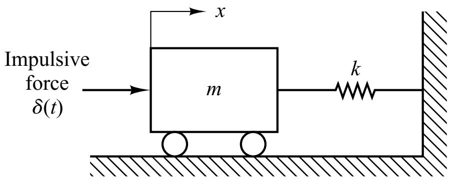

Figure 5–72
Mechanical system.

Assume that a record of a damped oscillation is available as shown in Figure 5–73. Determine the damping ratio ζ of the system from the graph.

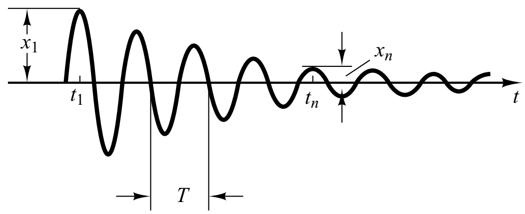

Figure 5–73
Decaying oscillation.

B–5–5. Obtain the unit-impulse response and the unit-step response of a unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{2s + 1}{s^2}$$

B–5–6. An oscillatory system is known to have a transfer function of the following form:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

B–5–7. Consider the system shown in Figure 5–74(a). The damping ratio of this system is 0.158 and the undamped natural frequency is 3.16 rad/sec. To improve the relative stability, we employ tachometer feedback. Figure 5–74(b) shows such a tachometer-feedback system.

Determine the value of K_h so that the damping ratio of the system is 0.5. Draw unit-step response curves of both the original and tachometer-feedback systems. Also draw the error-versus-time curves for the unit-ramp response of both systems.

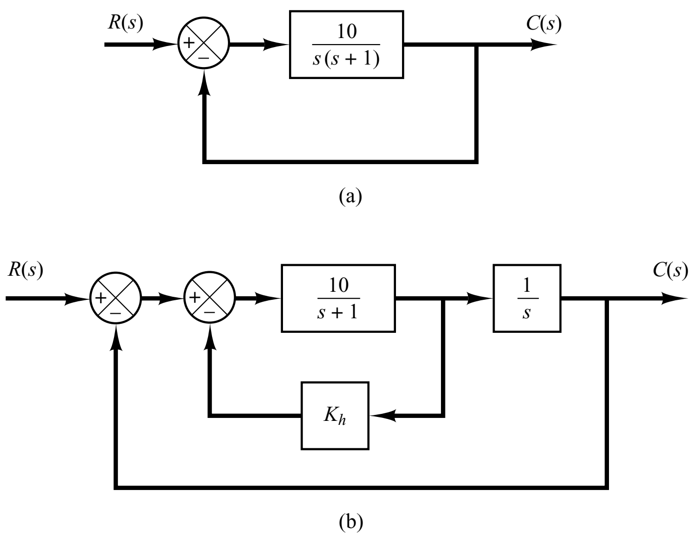

Figure 5–74
(a) Control system; (b) control system with tachometer feedback.

B–5–8. Referring to the system shown in Figure 5–75, determine the values of K and k such that the system has a damping ratio ζ of 0.7 and an undamped natural frequency ω_n of 4 rad/sec.

B–5–9. Consider the system shown in Figure 5–76. Determine the value of k such that the damping ratio ζ is 0.5. Then obtain the rise time t_r, peak time t_p, maximum overshoot M_p, and settling time t_s in the unit-step response.

B–5–10. Using MATLAB, obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system:

$$\frac{C(s)}{R(s)} = \frac{10}{s^2 + 2s + 10}$$

where $R(s)$ and $C(s)$ are Laplace transforms of the input $r(t)$ and output $c(t)$, respectively.

B–5–11. Using MATLAB, obtain the unit-step response, unit-ramp response, and unit-impulse response of the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u is the input and y is the output.

B–5–12. Obtain both analytically and computationally the rise time, peak time, maximum overshoot, and settling time in the unit-step response of a closed-loop system given by

$$\frac{C(s)}{R(s)} = \frac{36}{s^2 + 2s + 36}$$

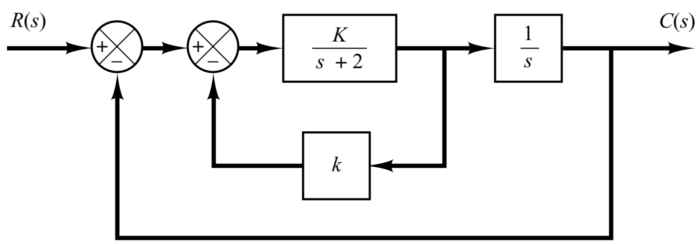

Figure 5–75
Closed-loop system.

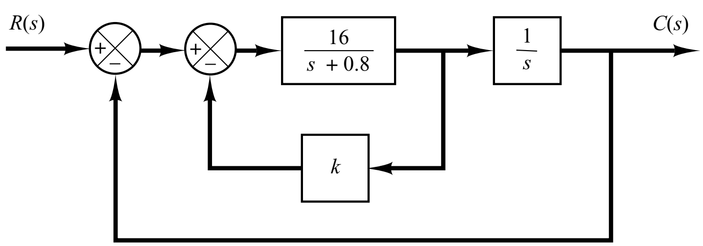

Figure 5–76
Block diagram of a system.

B–5–13. Figure 5–77 shows three systems. System I is a positional servo system. System II is a positional servo system with PD control action. System III is a positional servo system with velocity feedback. Compare the unit-step, unit-impulse, and unit-ramp responses of the three systems. Which system is best with respect to the speed of response and maximum overshoot in the step response?

B–5–14. Consider the position control system shown in Figure 5–78. Write a MATLAB program to obtain a unit-step response and a unit-ramp response of the system. Plot curves $x_1(t)$ versus t, $x_2(t)$ versus t, $x_3(t)$ versus t, and $e(t)$ versus t $[\text{where } e(t) = r(t) - x_1(t)]$ for both the unit-step response and the unit-ramp response.

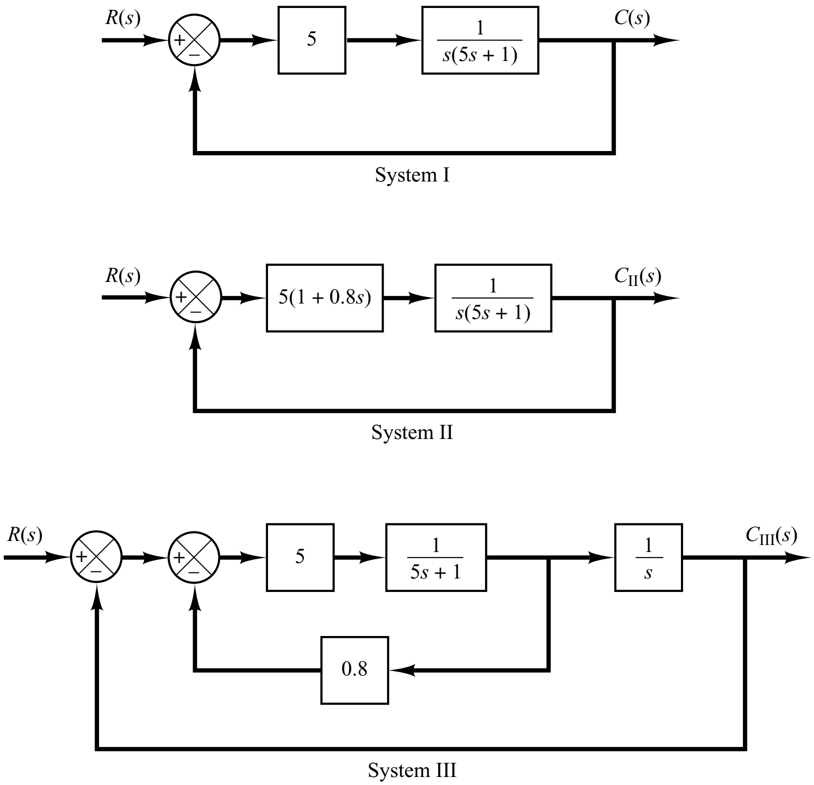

Figure 5–77
Positional servo system (System I), positional servo system with PD control action (System II), and positional servo system with velocity feedback (System III).

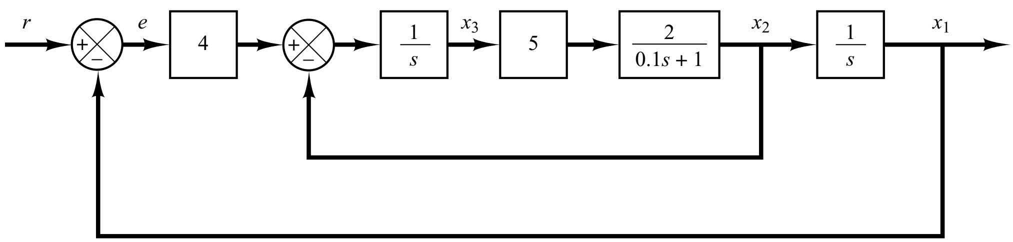

Figure 5–78
Position control system.

B–5–15. Using MATLAB, obtain the unit-step response curve for the unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{10}{s(s+2)(s+4)}$$

Using MATLAB, obtain also the rise time, peak time, maximum overshoot, and settling time in the unit-step response curve.

B–5–16. Consider the closed-loop system defined by

$$\frac{C(s)}{R(s)} = \frac{2\zeta s + 1}{s^2 + 2\zeta s + 1}$$

where $\zeta = 0.2, 0.4, 0.6, 0.8$, and 1.0. Using MATLAB, plot a two-dimensional diagram of unit-impulse response curves. Also plot a three-dimensional plot of the response curves.

B–5–17. Consider the second-order system defined by

$$\frac{C(s)}{R(s)} = \frac{s + 1}{s^2 + 2\zeta s + 1}$$

where $\zeta = 0.2, 0.4, 0.6, 0.8, 1.0$. Plot a three-dimensional diagram of the unit-step response curves.

B–5–18. Obtain the unit-ramp response of the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u is the unit-ramp input. Use the lsim command to obtain the response.

B–5–19. Consider the differential equation system given by

$$\ddot{y} + 3\dot{y} + 2y = 0, \qquad y(0) = 0.1, \qquad \dot{y}(0) = 0.05$$

Using MATLAB, obtain the response $y(t)$, subject to the given initial condition.

B–5–20. Determine the range of K for stability of a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

B–5–21. Consider the following characteristic equation:

$$s^4 + 2s^3 + (4 + K)s^2 + 9s + 25 = 0$$

Using the Routh stability criterion, determine the range of K for stability.

B–5–22. Consider the closed-loop system shown in Figure 5–79. Determine the range of K for stability. Assume that $K > 0$.

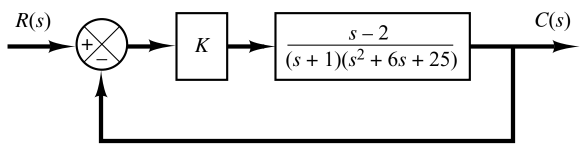

Figure 5–79 Closed-loop system.

B–5–23. Consider the satellite attitude control system shown in Figure 5–80(a). The output of this system exhibits continued oscillations and is not desirable. This system can be stabilized by use of tachometer feedback, as shown in Figure 5–80(b). If $K/J = 4$, what value of K_h will yield the damping ratio to be 0.6?

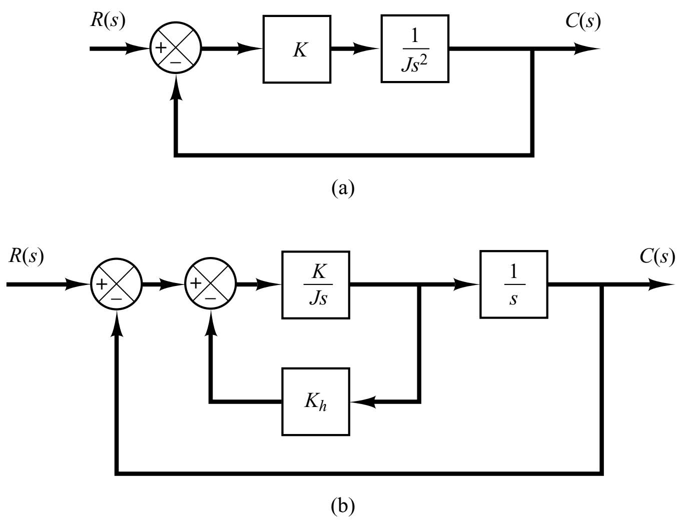

Figure 5–80
(a) Unstable satellite attitude control system;
(b) stabilized system.

B–5–24. Consider the servo system with tachometer feedback shown in Figure 5–81. Determine the ranges of stability for K and K_h. (Note that K_h must be positive.)

B–5–25. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -b_3 & 0 & 1 \\ 0 & -b_2 & -b_1 \end{bmatrix}$$

($\mathbf{A}$ is called Schwarz matrix.) Show that the first column of the Routh's array of the characteristic equation $|s\mathbf{I} - \mathbf{A}| = 0$ consists of 1, b_1, b_2, and $b_1 b_3$.

B–5–26. Consider a unity-feedback control system with the closed-loop transfer function

$$\frac{C(s)}{R(s)} = \frac{Ks + b}{s^2 + as + b}$$

Determine the open-loop transfer function $G(s)$.

Show that the steady-state error in the unit-ramp response is given by

$$e_{ss} = \frac{1}{K_v} = \frac{a - K}{b}$$

B–5–27. Consider a unity-feedback control system whose open-loop transfer function is

$$G(s) = \frac{K}{s(Js + B)}$$

Discuss the effects that varying the values of K and B has on the steady-state error in unit-ramp response. Sketch typical unit-ramp response curves for a small value, medium value, and large value of K, assuming that B is constant.

B–5–28. If the feedforward path of a control system contains at least one integrating element, then the output continues to change as long as an error is present. The output stops when the error is precisely zero. If an external disturbance enters the system, it is desirable to have an integrating element between the error-measuring element and the point where the disturbance enters, so that the effect of the external disturbance may be made zero at steady state.

Show that, if the disturbance is a ramp function, then the steady-state error due to this ramp disturbance may be eliminated only if two integrators precede the point where the disturbance enters.

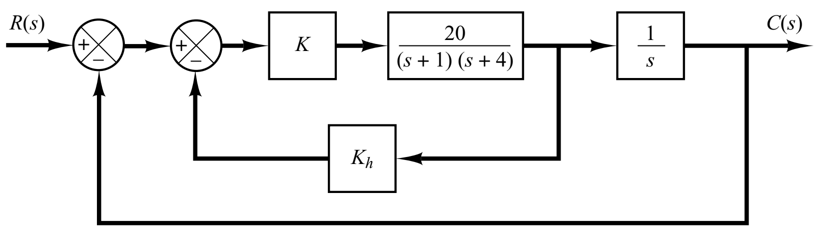

Figure 5–81
Servo system with tachometer feedback.