Understanding Basic Control Systems

Control Systems Guide

1 Introduction to Feedback Control Systems

A basic unity feedback control system consists of the following components:

- Input signal R(s): The desired output or reference signal
- Error signal E(s): The difference between desired and actual output
- Controller gain K: Amplification factor
- Plant/Process G(s): The system being controlled
- Output signal C(s): The actual system output

2 Key Terminology

2.1 System Response Characteristics

- **Step Response**: The system's output behavior when the input changes instantaneously from zero to a constant value
- **Rise Time**: Time taken for the output to go from 10% to 90% of its final value
- **Settling Time**: Time taken for the system to reach and stay within 2% of its final value
- **Overshoot**: Maximum amount the system exceeds its final steady-state value, expressed as a percentage
- **Steady-state Error**: The difference between desired and actual output when time approaches infinity

2.2 Damping Characteristics

- **Overdamped**: System returns to steady state without oscillating
  - Slower response
  - No overshoot

- Typically occurs with small K values

- **Underdamped**: System oscillates before reaching steady state
  - Faster initial response
  - Has overshoot
  - Multiple oscillations
  - Common with larger K values

- **Critically Damped**: Fastest response without overshoot
  - Optimal balance between speed and stability
  - Occurs at specific K value

3 Effect of Gain (K) on System Response

3.1 K = 0

- System is completely unresponsive
- Output C(s) = 0 for any input
- Equivalent to open-loop system

3.2 Small K Values

- Slow, stable response
- Typically overdamped
- Large steady-state error
- No overshoot

3.3 Medium K Values

- Faster response
- May become underdamped
- Moderate overshoot
- Reduced steady-state error

3.4 Large K Values

- Very fast initial response
- Significant overshoot
- Multiple oscillations
- Small steady-state error
- Longer settling time due to oscillations

3.5 K

- Extremely oscillatory behavior
- May become unstable
- Theoretically zero steady-state error
- Not practically useful

4 Mathematical Representation

For a unity feedback system:

$$E(s) = R(s) - C(s)$$
$$C(s) = KG(s)E(s)$$
$$\text{Transfer Function } T(s) = \frac{C(s)}{R(s)} = \frac{KG(s)}{1 + KG(s)}$$

5 Universal Principles

Regardless of the specific G(s):

- Higher K generally means faster initial response
- Higher K leads to more oscillatory behavior
- There's always a trade-off between speed and stability
- Steady-state error generally decreases as K increases

The main differences between systems with different G(s) are:

- The K value at which underdamped behavior begins
- Whether there's a K value that causes instability
- The specific pattern of oscillations
- The rate at which the response characteristics change with K