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ELEC 260: Quiz 1

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Time: 50 minutes
This quiz is *closed book*.
The use of calculators is *not* permitted.
Answer all of the questions in the space provided.
Show all of your work!
Total Marks: 24
Total Pages: 10 (7 non-blank pages + 3 blank pages)

Useful Formulae

Quadratic formula: The roots of $az^2 + bz + c = 0$ are given by $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Euler's relation: $e^{j\theta} = \cos\theta + j\sin\theta$

1. Suppose that we have the function $F(z)$ given by

$$F(z) = \frac{(z-1)^2(z+1)}{z^2(z^2+2z+2)}$$

where z is complex.

(a) Find the poles and zeros of $F(z)$, and determine the order of each pole and zero. **(3 marks)**

zeros of $z = +1, -1$, -1 is 2nd order.

$\frac{2}{3}$

poles of $z = 0$ 2nd order

$z = (+1) \pm j$ first order

$-1 \pm j$

(-1)

$\frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm \frac{\sqrt{-4}}{2} = -1 \pm \frac{2\sqrt{-1}}{2} = -1 \pm j$

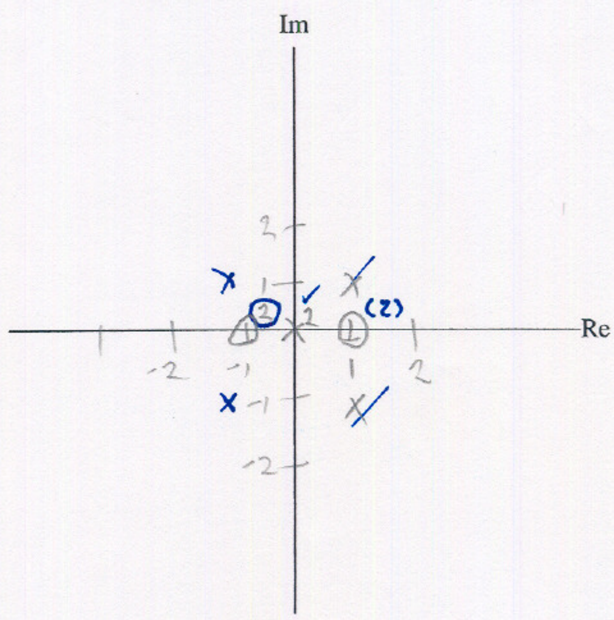
$z+1-j \quad z+1+j$

$z^2 + z + z + 2 + 1 \ldots + 1$

$z^2 + 2z + 2$

(b) Using the axes provided below, plot the poles and zeros obtained in part (a). Indicate the order of each pole and zero on the plot. (Note: Use the symbol 'x' to denote a pole and the symbol 'o' to denote a zero.) **(1 mark)**

$\frac{0}{1}$



2. Suppose that we have the functions

$$F_1(z) = z^2 + 3z + j\pi \quad \text{and}$$
$$F_2(z) = \frac{(z-1)^2(z+1)}{(z-2)(z+2)^2},$$

where z is complex.

(a) State for what values of z the function $F_1(z)$ is analytic, and give two or three sentences to justify your answer. Do not use the Cauchy-Riemann equations for this question! **(1 mark)**

$\frac{1}{1}$

a) $F_1(z)$ is Analytic everywhere because it is polynomial.

$F_2(z)$ is analytic everywhere except where it has zeros or holes ie except at $z = 1, -1$ and $z = 2, -2$ because it is a ratio of polynomials.

(b) State for what values of z the function $F_2(z)$ is analytic, and give two or three sentences to justify your answer. Do not use the Cauchy-Riemann equations for this question! **(2 marks)**

$F_2(z)$ is Analytic everywhere except $z = 1, -1, 2, -2$

The points are removable singularities because they can be encapsulated within a disc, ie they are point discontinuities,

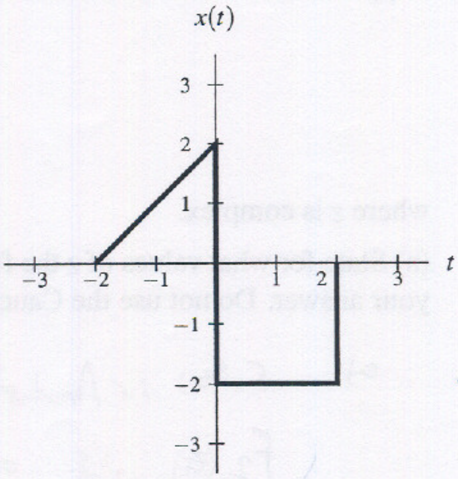
$\frac{0}{2}$

$F_2(z)$ is a rational function.

A rational function is analytic everywhere except where the denominator polynomial becomes 0.

In this case $F_2(z)$ not analytic for $z = \pm 2$.

3.



Suppose that we have the function $x(t)$ shown in the graph to the right. Using the axes provided below, plot the signals:

(a) $x(2t)$, **(1 mark)**
(b) $x(t-1)$, **(1 mark)**
(c) $x(-t)$, **(1 mark)**
(d) $x\left(\frac{1}{2}t+2\right)$. **(2 marks)**

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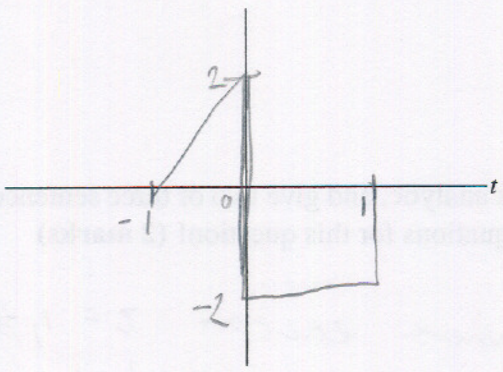
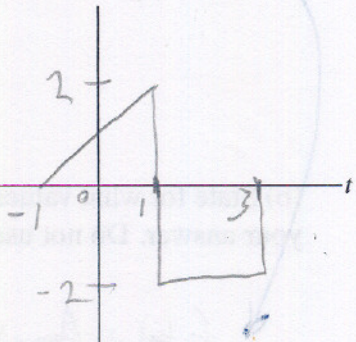
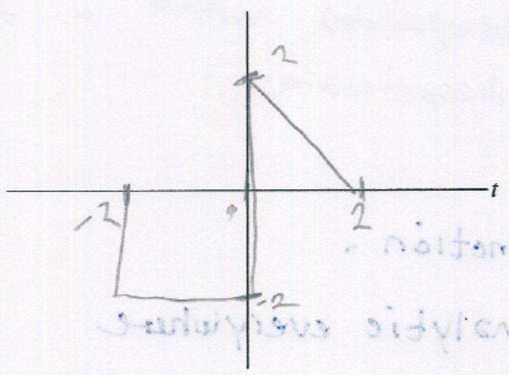
(a)



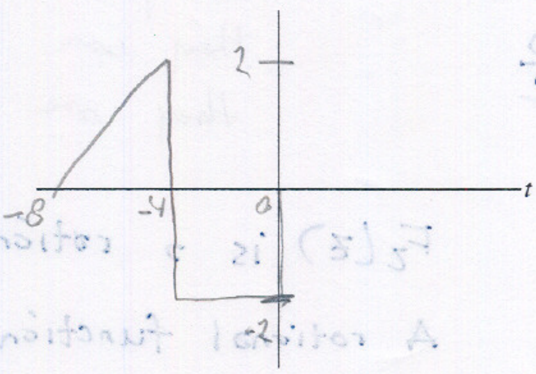
(b)



(c)



(d)



4. Let $x_1(t) = \sin 2\pi t$ and $x_2(t) = \cos 5\pi t$. Let $y(t) = x_1(t) + x_2(t)$. Determine whether $y(t)$ is periodic. If $y(t)$ is periodic, find its period. **(2 marks)**

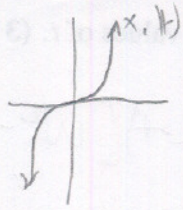
$T_0 = \frac{2\pi}{2\pi} = 1$ $\quad T_1 = \frac{2\pi}{5\pi} = \frac{2}{5}$ $\quad \frac{T_0}{T_1} = \frac{1}{\frac{2}{5}} = \frac{5}{2}$ = ratio of integers

∴ periodic

$\frac{2}{2}$

LCM of $\frac{5}{2} = 2$ $\quad 2 \cdot 1 = 2$ $\quad$ period $= 2$

5. Let $x_1(t) = t^3$. Determine whether $x_1(t)$ is even, odd, or neither of these. **(1 mark)**



is odd $\quad x_1(t) = -x_1(-t)$ = defn of odd.

$\frac{1}{1}$

6. Use the sifting property of the impulse function, in order to evaluate the following integral: $\int_{-\infty}^{\infty} t\delta(4t-2)dt$ **(2 marks)**

let $\tau = 4t$ $\quad t = \frac{\tau}{4}$ $\quad dt = \frac{d\tau}{4}$ $\quad \tau = 4t - 2$

$4t = \tau + 2$

$t = \frac{\tau + 2}{4}$

$t = \frac{1}{4}\tau + \frac{1}{2}$

$dt = \frac{1}{4}d\tau$

$= \int_{-\infty}^{\infty} \underbrace{\frac{1}{16}\tau}_{t} (\tau - 2)\, \overset{dt}{d\tau}$

$\frac{1.5}{2}$

$= \frac{1}{16}\cancel{\tau}(2)$ $\quad$ should be $\frac{1}{16}(2)$

$= \frac{1}{16} \cdot 8$

$= \frac{1}{2}$

$\frac{1}{4}\int_{-\infty}^{\infty} \left(\frac{1}{4}\tau + \frac{1}{2}\right)\delta(\tau)d\tau$

$\frac{1}{4} \cdot \frac{1}{2}$

7. Suppose that we have the signal $x(t)$ given by

$$x(t) = \begin{cases} 1 & \text{for } -1 \le t < 0 \\ t & \text{for } 0 \le t < 1 \\ 2-t & \text{for } 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

A plot of $x(t)$ is shown below.

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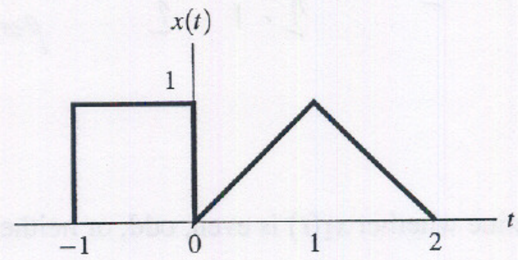

Use unit-step functions to find a single expression for $x(t)$ that is valid for all values of t. **(3 marks)**

✓ $x(t) = 1\,u(t+1) - u(t) + t[u(t) - u(t-1)] + (2-t)[u(t-1) - u(t-2)]$

$= u(t+1) - u(t) + tu(t) - tu(t-1) + 2u(t-1) - 2u(t-2) - tu(t-1) + tu(t-2)$

8. Suppose that we have the system with input $x(t)$ and output $y(t)$ as defined by

$$y(t) = [x(t)]^2.$$

$x(t) = \pm\sqrt{y(t)}$

Determine whether the above system is:
(a) linear, **(2 marks)**
(b) time invariant, **(1 mark)**
(c) invertible. **(1 mark)**
(Hint for part (c): Consider the response of the system to the inputs $x_1(t) = 1$ and $x_2(t) = -1$.)

$\frac{0}{2}$ X (a) not linear because $y(t)$ $a\,x(t_1) + b\,x(t_2) \neq a\sqrt{y(t)} + b\sqrt{y(t)}$
since it could also be $= a\sqrt{y(t)} - b\sqrt{y(t)}$

$\frac{0}{1}$ X (b) the system is not time invariant because $x(t+5)^2 \neq y(t+5)$

$\frac{0.5}{1}$ (c) not invertible because two inputs can have the same output

give example $-\frac{1}{2}$