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ELEC 260: Quiz 2

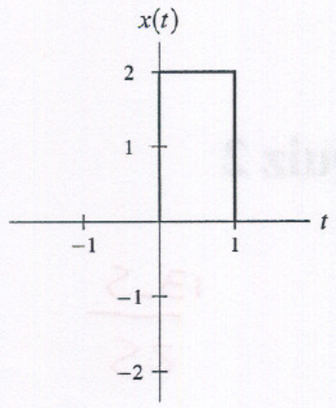
13.5/25

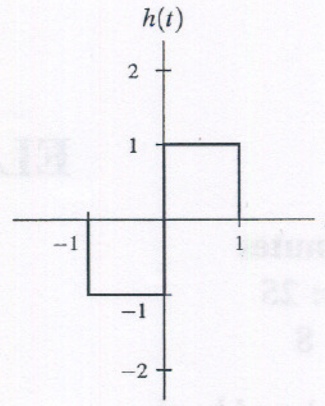
Time: 50 minutes
Total Marks: 25
Total Pages: 8

This quiz is *closed book.*
The use of calculators is *not* permitted.
Answer all of the questions in the space provided.

Show all of your work!
Clearly define any new quantities (e.g., variables, functions, etc.) that you introduce in your solutions.

1. Suppose that we have the two functions $x(t)$ and $h(t)$ shown in the graphs below.

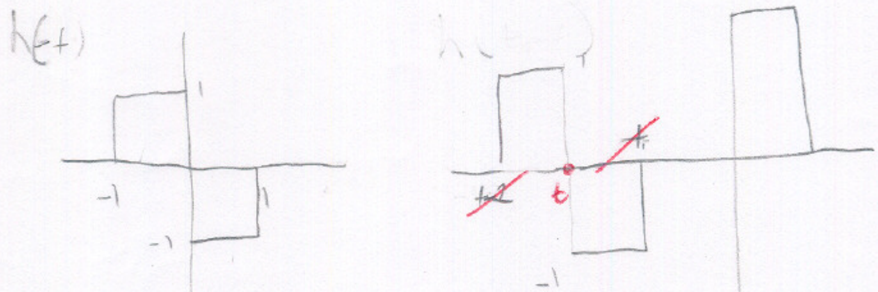




$\frac{7}{10}$

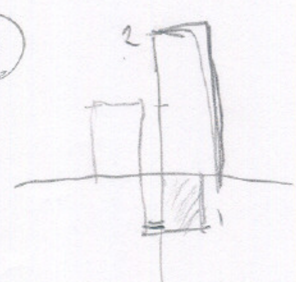
Compute the convolution $y(t) = x(t) * h(t)$. In your solution, you do not need to plot $y(t)$. **(10 marks)**

$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$



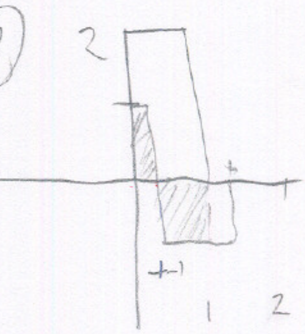
$t < 0$

1



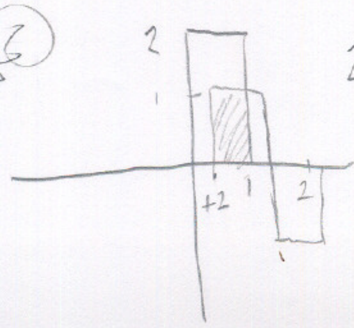
$0 < t < 1$

2



$1 < t < 2$

3



$2 < t < 3$

4

$t > 3$

(EXTRA SPACE FOR QUESTION 1 SOLUTION)

① $\int_0^t -2\,dt \quad 2t\Big|_0^t = 2t$ ×

② $\int_0^{t-1} 2\,dt + \int_{t-1}^{1} -2\,dt$ ×

$2t\Big|_0^{t-1} + 2t\Big|_{t-1}^{1}$

$2t-2 + 2-(2(t-1))$

$2t-2 + 2-(2t-2)$

$2-2t+2$

$= 2$

③ $\int_{t-2}^{1} 2t = 2t\Big|_{t-2}^{1} = 2 - 2(t-2)$

$= 2-2(t-2)$

$= 2-2t+4 = -2t+6$

$2 - 2t+4$

$-2t+6$ ✓

④ $= 0$

$$\begin{cases} 0 & t<0 \\ 2t & 0<t<1 \; × \\ 2 & 1<t<2 \; × \\ -2t+6 & 2<t<3 \; ✓ \\ 0 & t>3 \end{cases}$$

$\frac{0}{2}$

$\frac{1}{5}$

2. Show that, for any real signals $x(t)$ and $h(t)$, the following identity holds:

$$x(t) * h(t) = h(t) * x(t)$$

(i.e., convolution is commutative). **(2 marks)**

$x(\tau)$ not $x(t)$

$x(t) * h(t) = \int_{-\infty}^{\infty} x(t) h(t-\tau) d\tau$

$h(t) * x(t) = \int_{-\infty}^{\infty} h(t) x(t-\tau) d\tau$

let $T = t-\tau$, $t = T+\tau$

$= \int_{-\infty}^{\infty} x(T+\tau) h(t) d\tau$

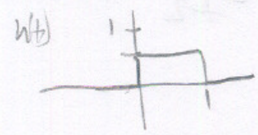
$\frac{0}{2}$

3. Suppose that we have a LTI system with input $x(t)$, output $y(t)$, and impulse response $h(t)$ where

$$h(t) = \frac{1}{2}[u(t) - u(t-1)].$$

Find an expression for the output $y(t)$ in terms of the input $x(t)$. Your final expression for $y(t)$ should be fully simplified (e.g., the final answer should not contain any unit-step functions). **(3 marks)**

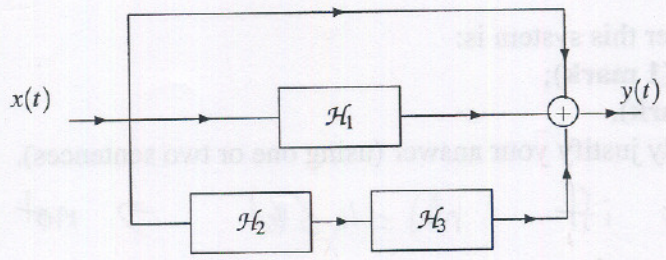
$x(t) * h(t) = \int_{-\infty}^{\infty} x \, h(t-\tau) d\tau$



$= \int_0^1 x(t) dt$

$\frac{1}{3}$

4. Suppose that we have the system shown below with the input $x(t)$ and output $y(t)$. Let $h(t)$ denote the impulse response of this system (i.e., the system with input $x(t)$ and output $y(t)$). In the diagram below, the blocks labelled $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ are LTI systems with the impulse responses $h_1(t)$, $h_2(t)$, and $h_3(t)$, respectively.



(a) Find the impulse response $h(t)$ in terms of $h_1(t)$, $h_2(t)$, and $h_3(t)$. **(2 marks)**

$$y(t) = x(t) + x(t) * h_1(t) + x(t) * h_2(t) * h_3(t)$$

$\frac{1}{2}$

(b) Determine the impulse response $h(t)$ in the specific case that

$$h_1(t) = \delta(t+1), \quad h_2(t) = \delta(t), \quad \text{and} \quad h_3(t) = \delta(t).$$

(1 mark)

if $y(t) = x(t) + x(t) * h_1(t) + x(t) * h_2(t) * h_3(t)$

then $y(t) = x(t) + x(t) * \delta(t+1) + x(t) * \delta(t) * \delta(t)$

$= x(t) + x(t+1) + x(t)$

$y(t) = 2x(t) + x(t+1)$

$\frac{0.5}{1}$

5. A LTI system has the impulse response $h(t)$ given by

$$h(t) = e^{-\alpha t}u(t)$$

where α is a real constant.

(a) Indicate whether this system is:

(i) memoryless **(1 mark)**;

(ii) causal **(1 mark)**.

In each case, briefly justify your answer (using one or two sentences).

has memory

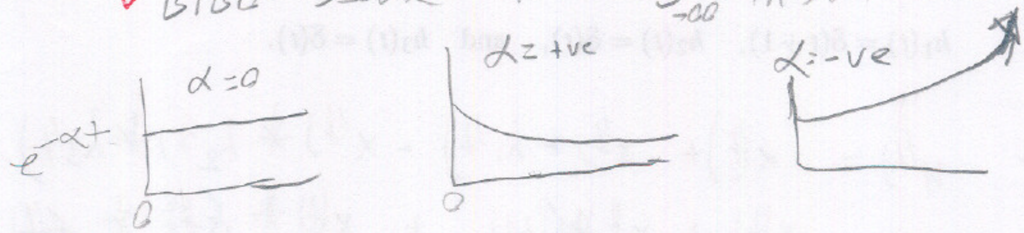
$\frac{1}{1}$ ✓ memoryless iff $h(t) = k\,\delta(t)$ $\Rightarrow$ not memoryless (memoryful?)

$\frac{1}{1}$ ✓ causal iff $h(t) = 0$ for $t < 0$ $u(t)$ means function is 0 until $t = 0$ then $e^{-\alpha t}$ after $t = 0$

so function is causal.

(b) Determine for what values of α the system is bounded-input bounded-output stable. **(3 marks)**

✓ BIBO stable iff $\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$ (Boom)



$\frac{1}{3}$

BIBO iff $\left|\frac{1}{\alpha} e^{-\alpha t}\right| < \infty$ So BIBO stable if

×

α is non negative

×

$\alpha > 0$

6. Suppose that we have two LTI systems $\mathcal{H}_1$ and $\mathcal{H}_2$ with impulse responses $h_1(t)$ and $h_2(t)$, respectively, where

$$h_1(t) = \tfrac{1}{2}\delta(t+1) \quad \text{and}$$
$$h_2(t) = 2\delta(t-1).$$

Determine whether the system $\mathcal{H}_2$ is the inverse of system $\mathcal{H}_1$. Show all of your work. Explain what you are doing! **(2 marks)**

✓ invertible if $h_1(t) * h^{inv}(t) = \delta(t)$

$\frac{1}{2}\delta(t+1) * 2\delta(t-1) = \int \delta(t+1)\,\delta(t-1)$ ×

$= \delta(t)$

$\therefore h_2 = h_1^{inv}$

$\frac{1}{2}$