23/27

University of Victoria
Midterm Examination #1
Summer 2014

Course Name:	**ELEC 260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**30280 (A01), 30281 (A02)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: [redacted]
Given Name(s): [redacted]
Student Number: V00 [redacted]

This examination paper has **8 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 27

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

Show all of your work!
Clearly define any new quantities (e.g., variables, functions, etc.) that you introduce in your solutions.

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Do not write on this [redacted]ss instructed to do so.

PROBLEM 1. Consider the function $f(z) = \frac{z^2-1}{z^3-4z^2+5z}$ where z is complex. Determine for what values of z the function $f(z)$ is analytic. **[3 marks]**

f(z) is not analytic ~~when~~ on its poles

where denominator = 0

$z^3 - 4z^2 + 5z$

$z(z^2 - 4z + 5)$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{4 \pm \sqrt{16-20}}{2}$$

$$\frac{4 \pm 2j}{2}$$

$2 + j$
$2 - j$

and

$z = 0$

f(z) not analytic ~~for~~

$(2 + j) = z$
$(2 - j) = z$
or
$0 = z$

PROBLEM 2. Show that the function $y(t) = \frac{1}{2}[x(t) + x(-t)]$ is even for every choice of $x(t)$. **[2 marks]**

$x(t) = x(-t)$ if even

$y(t) = \frac{1}{2}[x(t) + x(-t)]$

$y(-t) = \frac{1}{2}[x(-t) + x(t)]$

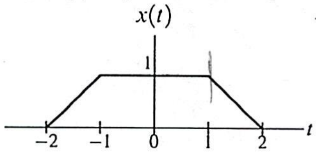
$y(-t) = \frac{1}{2}[(x(t)) + x(-t)] = y(t)$

$y(-t) = y(t)$

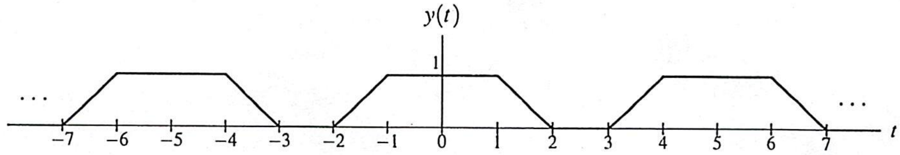
$y(t)$ is even

PROBLEM 3.

Let $x(t)$ and $y(t)$ be the functions shown in the figures below, where $x(t)$ is zero everywhere outside of the range shown in the plot and $y(t)$ is periodic with period $T = 5$.



(a)



(b)

(A) Using unit-step functions, find a single expression for $x(t)$ that is valid for all t. When stating your final answer, group together terms having the same unit-step function factor. **[4 marks]**

for $t < -2$ $\quad x(t) = 0$

$-2 \leq t < -1$ $\quad x(t) = t + 2$

$-1 \leq t < 1$ $\quad x(t) = 1$

$1 \leq t < 2$ $\quad x(t) = (-t) + 2$

$t > 2$ $\quad x(t) = 0$

$$0 + (t+2)[u(t+2) - u(t+1)] + 1[u(t+1) - u(t-1)]$$

$$+ (-t+2)[u(t-1) - u(t-2)] + 0$$

$$= t+2(u(t+2)) - \underbrace{(t+2)(u(t+1)) + u(t+1)}_{-t-2+1} - \underbrace{u(t-1) + (-t+2)u(t-1)}_{-1+-t+2} - (-t+2)(u(t-2))$$

$-u(t-1)$ $\quad -t+2u(t-1)$

$$= (t+2)(u(t+2)) - (t+1)(u(t+1)) + (-t+1)u(t-1) - (-t+2)u(t-2)$$

✓

(B) Using the result of part (a), find a single expression for $y(t)$ that is valid for all t **[1 mark]**

Replace t with $(t + 5n)$ $\quad n \in$ integer

✗

$$(t+2+5n)(u(t+2+5n)) - (t+1+5n)(u(t+1+5n)) + (-t+1+5n)u(t-1+5n) -$$

$$(-t+2+5n)\,u(t+5n-2)$$

PROBLEM 4. Suppose that we have a system $\mathscr{H}$ with input $x(t)$ and output $y(t)$.

(A) Clearly state, in mathematical terms, the condition that must be satisfied in order for the system $\mathscr{H}$ to be time invariant. Be sure to define all quantities such as variables, functions, and constants. Otherwise, you will receive zero marks. Be careful with the notation that you choose to employ. If, for example, you confuse arrows and equal signs in your solution, you will probably receive zero marks. **[2 marks]**

input

$X_1(t) \rightarrow y_1(t)$

for

$X_2(t) = X_1(t - t_0)$

$X_2(t) \rightarrow Y_2(t)$

$Y_2(t) = Y_3(t - t_0)$

This equation must be true for the system to be time invariant

(B) Suppose now that the system $\mathscr{H}$ is characterized by the equation $y(t) = 1 + x(-t)$. Using the condition stated in part (a), determine whether this system is time invariant. **[2 marks]**

$Y(t) = 1 + x(-t)$

$Y_1(t) = 1 + X_1(-t)$ let $X_2(t) = X_1(t - t_0)$

Shift

$Y_2(t) = 1 + X_1(-t - t_0)$

$Y_3(t - t_0) = 1 + X_3(-(t - t_0))$

$= 1 + X_3(-t + t_0)$

$Y_3(t - t_0) \neq Y_2(t)$

NOT T.I

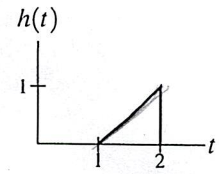
$\frac{1}{e^{-\infty}}$

$e^{t} = 0$ $\quad e^{0} = 1$

$\ln(0) = 1$

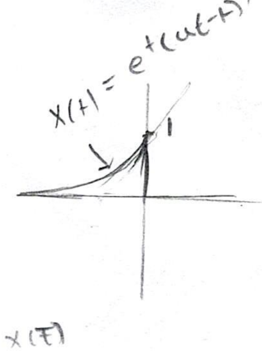
PROBLEM 5.

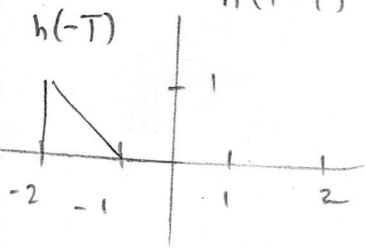
Using graphical methods (i.e., the method used during the lectures), compute the convolution $x(t) * h(t)$, where $x(t) = e^t u(-t)$ and $h(t)$ is as shown in the figure. For each separate case in your solution, you must state the the convolution result and the corresponding range of t. Each convolution result may be stated in the form of an integral, but the integral must be simplified as much as possible without integrating. In addition, for each separate case, you must show the fully-labelled graph from which your answer is derived. In the preceding sentence, the words "fully labelled" imply (amongst other things) that the equation of each curve on a graph must be labelled with its equation. You cannot use the commutative property of convolution in your solution. That is, you must compute directly compute $x(t) * h(t)$. You cannot instead compute $h(t) * x(t)$. **[7 marks]**



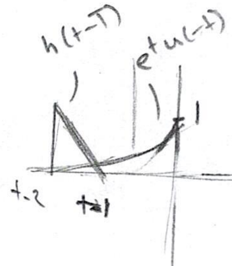
$h(t) = t(u(t-1) - u(t-2))$ $\qquad X(t) * h(t)$

$h(t-T) = (t-T)(u(t-T-1) - u(t-T-2))$ $\quad \int_{-\infty}^{\infty} X(T)\, h(t-T)\, dT$



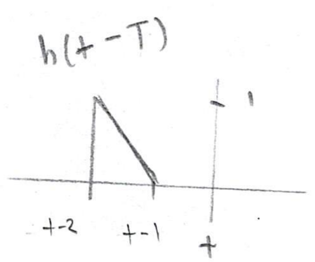


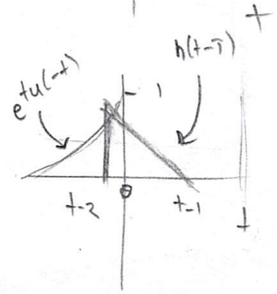
$\int_{-\infty}^{\infty} e^{t} u(-t)\, h(t-T)\, dT$



$$\int_{t-2}^{t-1} \overbrace{((t-T)(u(t-T-1) - u(t-T-2))}^{h(t-T)}\ \overbrace{e^{T} u(-T)}^{x(T)}\, dT$$

− 1.5 when $t < 1$

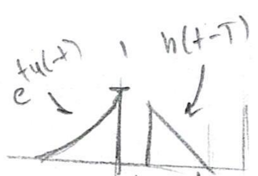




$$\int_{t-2}^{0} \overbrace{((t-T)(u(t-T-1) - u(t-T-2))}^{h(t-T)}\ \overbrace{e^{T} u(-T)}^{X(T)}\, dT$$

when $1 \leq t < 2$

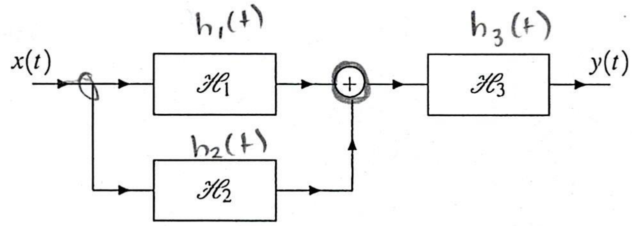
− 1.5 simplify



when
$t-2 > 0$
or when $t \geq 2$

Convolution = 0

PROBLEM 6. Consider the system shown in the figure below with input $x(t)$ and output $y(t)$. Let $h(t)$ denote the impulse response of this system. In the figure, the blocks labelled $\mathscr{H}_1$, $\mathscr{H}_2$, and $\mathscr{H}_3$ are LTI systems with the impulse responses $h_1(t)$, $h_2(t)$, and $h_3(t)$, respectively.



(A) Express the impulse response $h(t)$ of the overall system in terms of $h_1(t)$, $h_2(t)$, and $h_3(t)$. **[1 mark]**

$$\left[(x(t) * h_1(t)) + (x(t) * h_2(t)) \right] * h_3(t)$$

$$(x(t) * h_1(t) * h_3(t)) + (x(t) * h_2(t) * h_3(t))$$

$$x(t) * \underbrace{((h_1(t) * h_3(t)) + (h_2(t) * h_3(t)))}_{h(t)}$$

(B) Determine the impulse response $h(t)$ in the specific case that $h_1(t) = \delta(t)$, $h_2(t) = e^{-2t}u(t-1)$, and $h_3(t) = \delta(t-3)$. **[3 marks]**

$$(h_1(t) * h_3(t)) + (h_2(t) * h_3(t))$$

$$\delta(t) * \delta(t-3) \quad + \quad e^{-2t}u(t-1) * \delta(t-3)$$

identity

$$\int_{-\infty}^{\infty} e^{-2\tau} u(\tau-1)\, \delta(t-(\tau-3))\, d\tau$$

$$\delta(t-3) \quad + \quad e$$

$$\delta(t - \tau + 3)$$

$$\delta(-\tau + t + 3)$$

$$\delta(\tau - (t-3)) \qquad \delta(t) = \delta(-t)$$

Sifting with $t-3$

$$= e^{-2(t-3)} u((t-3)-1)$$

$$= \delta(t-3) + e^{-2(t-3)} u(t-4)$$

PROBLEM 7. Using the MATLAB programming language, write a function called `myfunc` that takes a single input argument n and returns a single value s, where n is a positive integer and s is the real number computed according to the formula

$$s = \begin{cases} \sum_{k=1}^{n} \frac{1}{k^2} & n \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Be sure to use correct syntax in your answer, since syntax clearly matters here. **[2 marks]**

```
function s = myfunc (n);
    count = 1;
    s = 0;

    while count <= (n)
        s= s + (1 / (count ^ 2));
        count = count + 1;
    end;

return;
```

~~0.5~~