

## Exercise 6.25

**L Answer.**

A LTI system with input  $x$ , output  $y$ , and impulse response  $h$  is characterized by the equation

$$y(t) = x * h(t).$$

In the Fourier domain, the preceding equation is equivalent to

$$Y(\omega) = X(\omega)H(\omega).$$

Since the system bandwidth is  $B$  (i.e.,  $H(\omega) = 0$  for all  $|\omega| > B$ ), we have

$$Y(\omega) = \begin{cases} X(\omega)H(\omega) & |\omega| \leq B \\ 0 & \text{otherwise.} \end{cases}$$

In other words, the output spectrum  $Y$  is guaranteed to satisfy

$$Y(\omega) = 0 \quad \text{for } |\omega| > B.$$

Thus, if the input spectrum  $X$  contains information at frequencies in the range  $[-B, B]$ , this information will be completely eliminated in the output spectrum  $Y$  and, therefore, irrecoverably lost. For this reason, it is not possible to (reliably) transmit, over a communication channel with bandwidth  $B$ , a signal  $x$  with bandwidth greater than  $B$ .

take FT

$H(\omega) = 0$  for all  $|\omega| > B$

consider case  $|\omega| > B$