Exercise 5.102

L **Answer (e).**

From the Fourier series analysis equation, we have

$$
\begin{aligned}
c_k &= \tfrac{1}{T}\int_0^T x(t)e^{-jk(2\pi/T)t}\,dt && \text{Fourier series analysis equation} \\
&= \tfrac{1}{5}\int_0^5 5e^{3t}e^{-jk(2\pi/5)t}\,dt && \text{substitute given } x \\
&= \int_0^5 e^{3t-jk2\pi t/5}\,dt && \text{combine exponentials} \\
&= \int_0^5 e^{[3-jk2\pi/5]t}\,dt. \quad 1 && \text{factor exponent}
\end{aligned}
$$

Since $3 - jk(2\pi/5) \neq 0$ for all (integer) k, no degenerate cases can arise during integration. Therefore, there is only one case to consider. For all k, we have

$$
\begin{aligned}
c_k &= \frac{1}{3-jk2\pi/5}\left[e^{t(3-jk2\pi/5)}\right]\Big|_0^5 && \text{integrate } 1 \\
&= \frac{5\left[e^{15-j2\pi k}-1\right]}{15-j2\pi k} && \text{evaluate at 0 and 5} \\
&= \frac{5\left[e^{15}(1)^k-1\right]}{15-j2\pi k} && e^{j2\pi}=1 \\
&= \frac{5(e^{15}-1)}{15-j2\pi k}. && 1^k = 1
\end{aligned}
$$