UNIVERSITY OF VICTORIA

FINAL EXAMINATIONS – DECEMBER 2003

ELEC 360 – CONTROL THEORY AND SYSTEMS I

SECTION F 01

TO BE ANSWERED IN BOOKLETS

DURATION: 3 hours

INSTRUCTOR: Dr. P. Agathoklis

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS QUESTION PAPER HAS 6 PAGES, INCLUDING THIS COVER PAGE AND TWO ATTACHED FIGURES.

FOUR (4) PAGES OF HANDWRITTEN NOTES AND PHOTOCOPIES OF LAPLACE TRANSFORMS ARE PERMITTED.

DETACH PAGES 5 & 6 FROM THE EXAMINATION PAPER AND HAND IN WITH YOUR ANSWER BOOKLET.

Marks

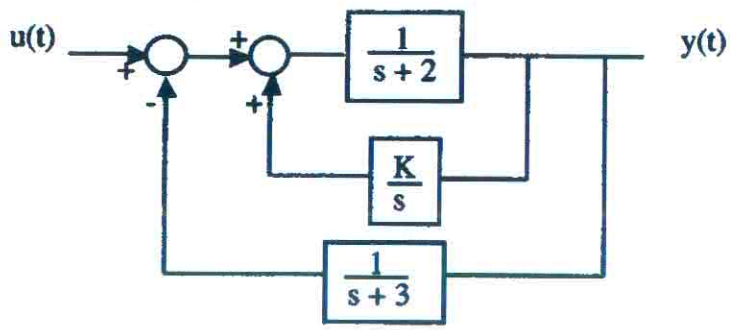
(4) 1. A transfer function has the following poles and zeros:

zeros: -3
poles: -1 + j, -1 - j

and the response to a unit ramp at steady state is y(t) = t – 1/1.5

Find the response to the unit step.

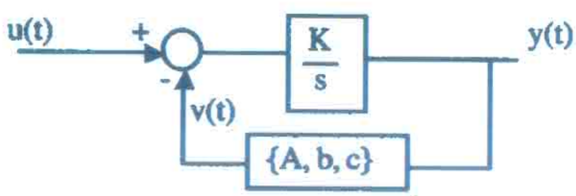
(5) 2. Consider the system



a) Find a state-space description of the system.

b) For what values of K is the system stable?

(4) 3. Consider the system given by:



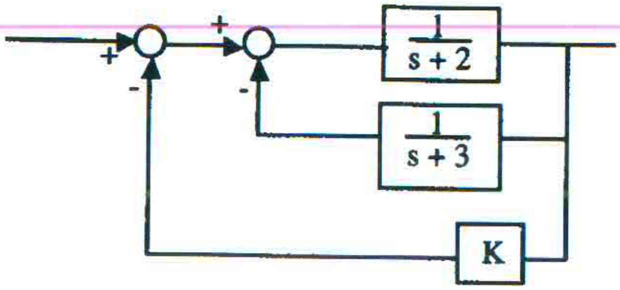
where

$$\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \underline{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} y(t)$$

$$v(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \underline{x}$$

For what values of K does the system have a steady-state error of less than 0.5?

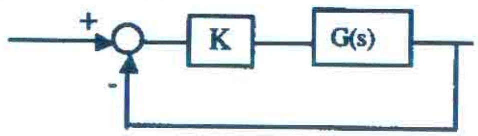
(6) 4. Consider the system



a) Sketch the root locus of the system when K goes from 0 to ∞

b) Discuss the response of the closed-loop system when K goes from 0 to ∞

(6) 5. Consider the system given by:



where $G(s) = \dfrac{(s-1)}{(s+1)(s+2)}$

a) Sketch the Bode and polar plots of G(s).

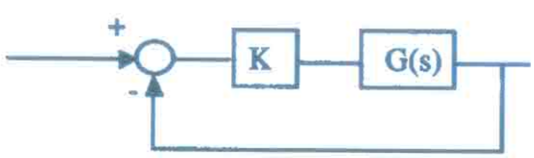
b) Discuss the stability of the closed-loop system for positive K.

(6) 6. Consider the polar plot of G(s) given on page 5:

a) What is the type of the system?

b) Find the value of the associated error constant.

c) Discuss the stability of



d) Indicate in the figure phase and gain margins.

(6) 7. The Bode plots of the open loop compensated and uncompensated system are given in page 6.

From the plot of the uncompensated system, determine:

a) The stability of the closed-loop system

b) The type of open-loop system and the value of the corresponding static error constant.

c) The phase and gain margins.

From the plot of the compensated system, determine:

a) The compensator used

b) The new phase and gain margins

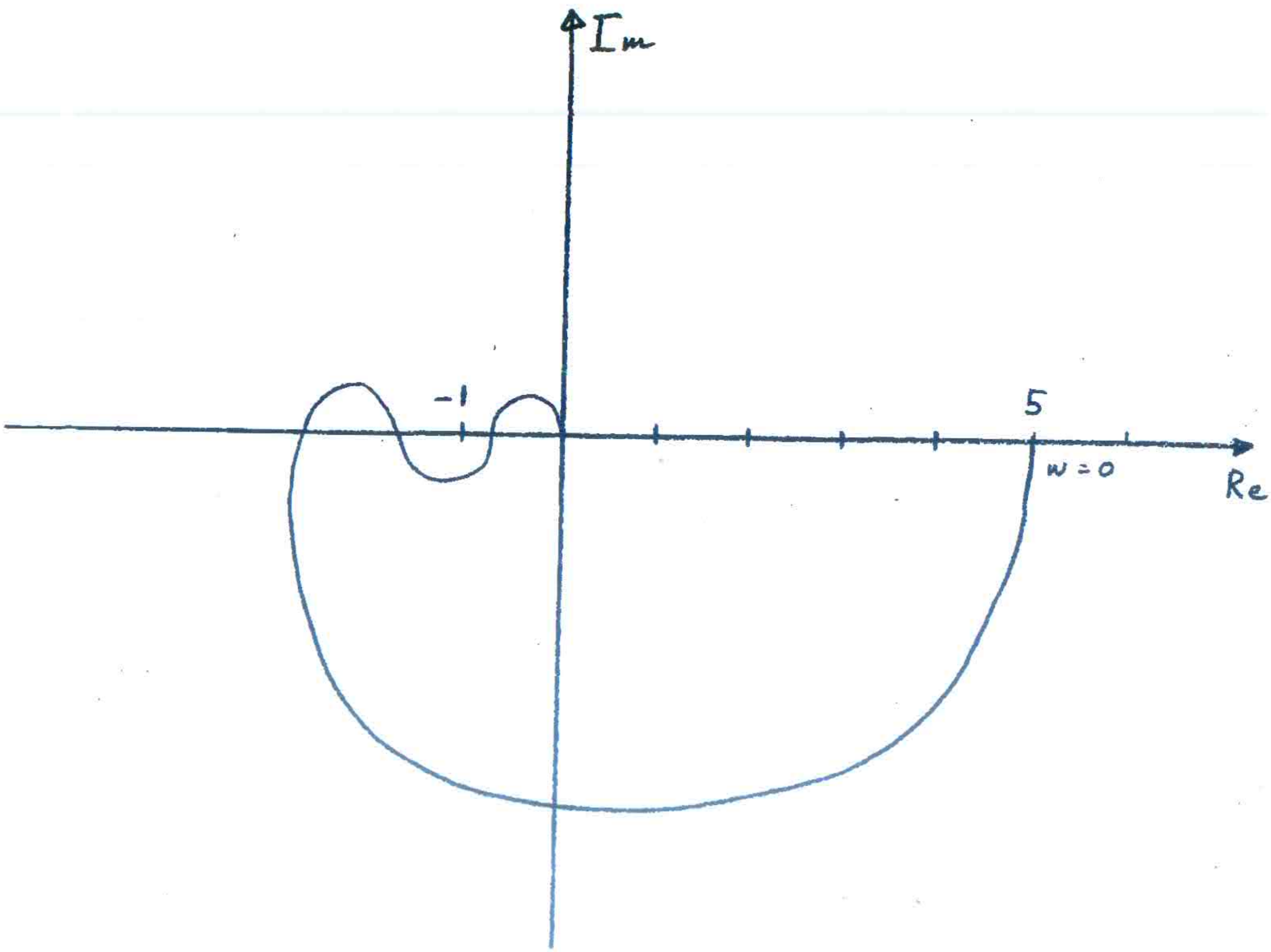
c) Discuss the effects of using a compensator – what has been improved and how?

Justify your answers and indicate in the attached figure (page 6) the corresponding quantities.

END

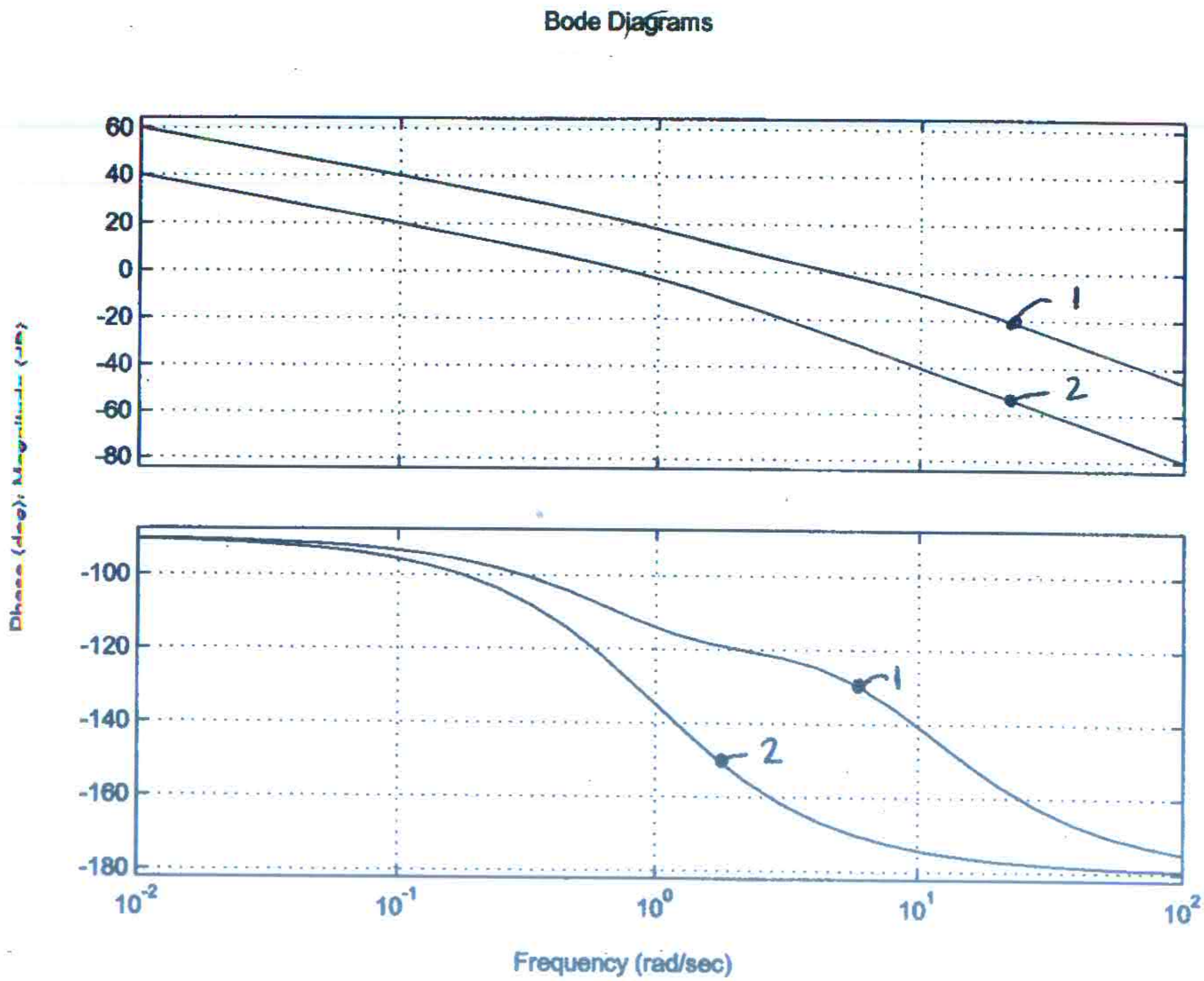
Name: ______________________ **Student No.:** ______________

Figure for Question 6



Name: ______________________ **Student No.:** ______________

Figure for Question 7



1. Uncompensated system
2. Compensated system

2003-Q1

$P_1 = -1+j$, $P_2 = -1-j$

$Z_1 = -3$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{(s+3)\cdot K}{(s+1-j)(s+1+j)} = \frac{K(s+3)}{s^2+2s+2}$$

unit-ramp input. $R(s) = \frac{1}{s^2}$

$$C(s) = \frac{C(s)}{R(s)} R(s) = \frac{K(s+3)}{s^2(s^2+2s+2)} = K\left[\frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+2s+2}\right]$$

$$= K\left[\frac{-1}{s} + \frac{1.5}{s^2} + \frac{s+0.5}{s^2+2s+2}\right]$$

$$\Rightarrow C(s) = \underbrace{\frac{-k}{s} + \frac{1.5k}{s^2}}_{\Downarrow} + \cancel{\frac{(s+0.5)k}{s^2+2s+2}}$$

since at steady state, $e^{-at}\cos + e^{-at}\sin = 0$.

$$C(t) = -k\,u(t) + 1.5kt = 1.5kt - k = t - \frac{1}{1.5} \Rightarrow \boxed{k = \frac{1}{1.5}}$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{\frac{1}{1.5}(s+3)}{s^2+2s+2}}$$

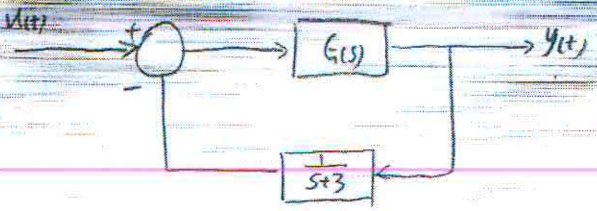
unit-step input $R(s) = \frac{1}{s}$

$$C(s) = \frac{C(s)}{R(s)} R(s) = \frac{\frac{1}{1.5}(s+3)}{s(s^2+2s+2)} = \frac{1}{1.5}\left[\frac{A}{s} + \frac{Bs+C}{s^2+2s+2}\right] \quad \frac{1}{1.5}\left[\frac{1.5}{s} + \frac{-1.5s-2}{s^2+2s+2}\right]$$

$$\Rightarrow C(s) = \frac{1}{s} + \frac{-s-\frac{2}{1.5}}{s^2+2s+2} \Rightarrow \begin{cases} A_1 = -1 \\ A_2 = -\frac{2}{1.5} \\ a = 2 \\ b = 2 \end{cases} \Rightarrow \begin{cases} \alpha = \frac{a}{2} = 1 \\ \beta = \sqrt{b - \alpha^2} = 1 \\ B_1 = \frac{A_2 - A_1\alpha}{\beta} = -\frac{2}{1.5} + 1 = \frac{-0.5}{1.5} \end{cases} = -e^{-t}\cos t - \frac{0.5}{1.5}e^{-t}\sin t$$

$$\Rightarrow C(t) = \left(1 - e^{-t}\cos t - \frac{0.5}{1.5}e^{-t}\sin t\right) \quad t \geq 0$$

(20[illegible] Q2)



$$G(s) = \frac{\frac{1}{s+2}}{1-\frac{1}{s+2}\cdot\frac{K}{s}} = \frac{s}{s(s+2)-K} = \frac{s}{s^2+2s-K}$$

$$\frac{Y(s)}{U(s)} = \frac{G(s)}{1+G(s)\cdot\frac{1}{s+3}} = \frac{\frac{s}{s^2+2s-K}}{1+\frac{s}{s^2+2s-K}\cdot\frac{1}{s+3}} = \frac{s(s+3)}{(s^2+2s-K)(s+3)+s}$$

$$\Rightarrow \quad \frac{Y(s)}{U(s)} = \frac{s^2+3s}{s^3+5s^2+(7-K)s-3K}$$

$n=3$: $\quad b_0=0,\ b_1=1,\ b_2=3,\ b_3=0$

$\quad a_0=1,\ a_1=5,\ a_2=7-K,\ a_3=-3K.$

$$\Rightarrow \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & 3K\\ 1 & 0 & K-7\\ 0 & 1 & -5\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 3\\ 1\end{bmatrix}}_{b} u$$

$$y = \underbrace{\begin{bmatrix}0 & 0 & 1\end{bmatrix}}_{c}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \qquad d=0$$

//

see back.

Closed-loop: $\frac{Y(s)}{U(s)} \Rightarrow U(s) = s^3 + 5s^2 + (7-k)s - 3k$

$$\begin{array}{c|ccc} s^3 & 1 & 7-k & 0 \\ s^2 & 5 & -3k & 0 \\ s^1 & b_1 & 0 & \\ s^0 & -3k & & \end{array}$$

$$b_1 = -\frac{1}{5}\begin{vmatrix} 1 & 7-k \\ 5 & -3k \end{vmatrix} = \frac{5(7-k)+3k}{5}$$

$$= \frac{35-5k+3k}{5} = \frac{35-2k}{5}$$

$\Downarrow$

$1 > 0$

$5 > 0$

$\frac{35-2k}{5} > 0 \Rightarrow 35-2k > 0 \Rightarrow k < 17.5$

$-3k > 0 \Rightarrow k < 0$

$\Rightarrow \boxed{k<0}$ for stable closed-loop system.

(2003-Q3) $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $c = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$$\frac{V(s)}{Y(s)} = c(sI - A)^{-1} b \Rightarrow \frac{V(s)}{Y(s)} = \frac{3s+4}{(s+2)(s+1)}$$

Open-loop: $\frac{Y(s)}{U(s)} = \frac{K(3s+4)}{s(s+2)(s+1)}$

type1 system:

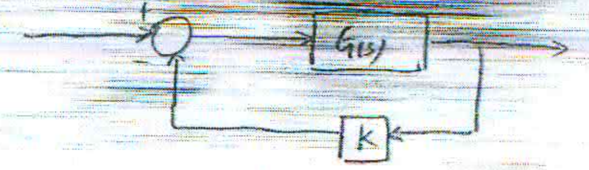
unit step $e_{ss} = 0$ — free to choose any k.

unit ramp $e_{ss} = \frac{1}{k_v}$ — Only this one is possible to have a $e_{ss} < 0.5$.

unit parabolic $e_{ss} = \infty$

$$K_v = \lim_{s \to 0} s \cdot \left(\frac{Y(s)}{U(s)}\right) = \lim_{s \to 0} \cancel{s} \cdot \frac{K(3s+4)}{\cancel{s}(s+2)(s+1)} = \frac{4K}{2}$$

$$e_{ss} = \frac{1}{K_v} = \frac{2}{4K} < 0.5 \Rightarrow \boxed{K > 1}$$



$$G(s) = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}\cdot\frac{1}{s+3}} = \frac{s+3}{(s+2)(s+3)+1}$$

$\Rightarrow$ open-loop: $\dfrac{C(s)}{R(s)} = \dfrac{K\cdot(s+3)}{(s+2)(s+3)+1}$ $\Rightarrow$ $m=1,\ n=2$

$z_1=-3$

$p_{1,2} = -2.5 \pm 0.87j$

$(s+2)(s+3)+1=0$

$\boxed{\text{rule1}}$ # of branch = 2

$\boxed{\text{rule4}}$ asym slope: $r = \dfrac{\pm 180^\circ(2K+1)}{n-m} = \pm 180^\circ \Rightarrow \cancel{\text{rule5}}$

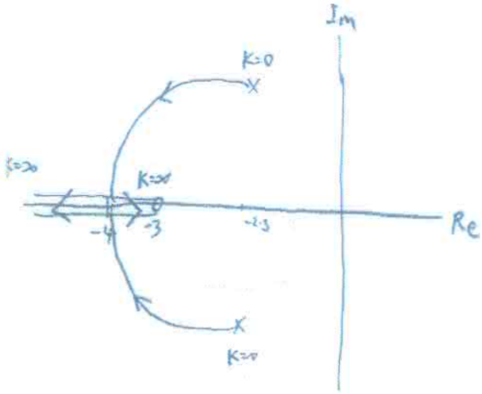
$\boxed{\text{rule6}}$ break point:

$$\left.\begin{aligned} B(s) &= s+3 \\ A(s) &= (s+2)(s+3)+1 = s^2+5s+7 \end{aligned}\right\} \Rightarrow A'(s)B(s) - A(s)B'(s) = 0$$

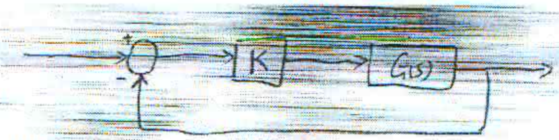
$$\Rightarrow (2s+5)(s+3) - (s^2+5s+7) = 0 \Rightarrow s=-4,\ s=-2.\ (\checkmark, \times).$$

$\boxed{\text{check}}$ $A(s) + K\,B(s) = 0 \Rightarrow K = \dfrac{-A(s)}{B(s)} = \dfrac{-(s^2+5s+7)}{s+3}$

$$\left.\begin{aligned} s_1=-4 &\Rightarrow K_1 = 3 \\ s_2=-2 &\Rightarrow K_2=-1 \end{aligned}\right\}$$



(5)



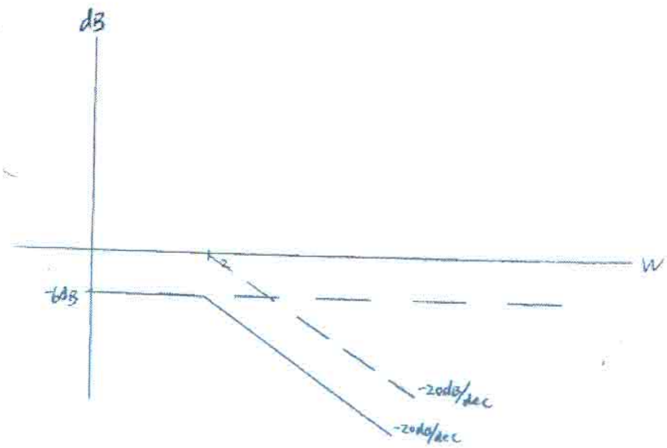
Open-loop: $G(s)H(s) = \dfrac{K(s-1)}{(s+1)(s+2)}$ non-miniphase system.

① Bode: $G(j\omega)H(j\omega) = \dfrac{K(j\omega-1)}{2(1+j\omega)(1+\frac{j\omega}{2})}$

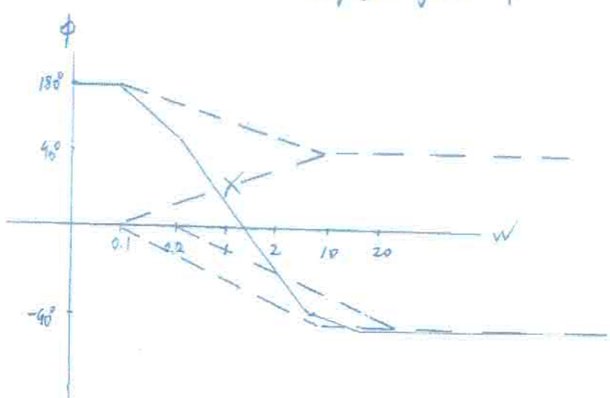
(a) mag: Let $G_1(j\omega)H_1(j\omega) = \dfrac{K\cdot\frac{1}{2}(j\omega+1)}{\cancel{(1+j\omega)}(1+\frac{j\omega}{2})}$

$\Rightarrow |G_1(j\omega)H_1(j\omega)| = |G(j\omega)H(j\omega)|$.

$20\log\frac{1}{2} = -6dB$.

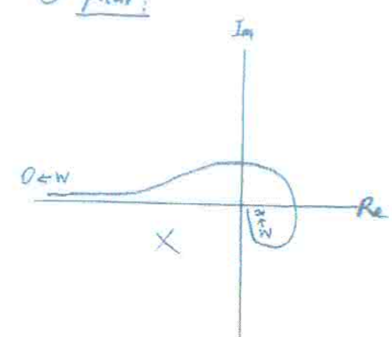


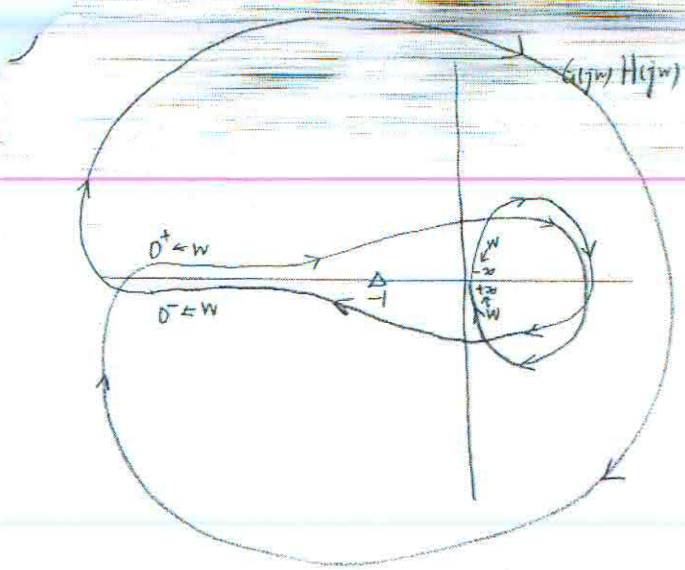
(b) phase: Let $\left.\begin{aligned} A_1(j\omega) &= j\omega+1 \\ A(j\omega) &= j\omega-1 \end{aligned}\right\} \Rightarrow \underline{/A(j\omega)} = 180^\circ - \underline{/A_1(j\omega)}$



$180^\circ \sim -90^\circ$.

② polar:





$$\left.\begin{array}{l} N = 2 \\ P = 0 \end{array}\right\} \Rightarrow Z = N + P = 2 \Rightarrow \text{closed-loop sys has 2 } \underline{\text{unstable}} \text{ poles}$$

for all $K > 0$