$\frac{27.5}{37} + 2 = 29.5$

ELEC 260 (SIGNAL ANALYSIS), SECTION K01
MIDTERM EXAMINATION #1

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SECTION: K01

DURATION: 50 MINUTES

ALL QUESTIONS ARE **TO BE ANSWERED ON THE EXAMINATION PAPER** IN THE SPACE PROVIDED.

STUDENTS MUST COUNT THE NUMBER OF PAGES IN THIS EXAMINATION PAPER BEFORE BEGINNING TO WRITE, AND REPORT ANY DISCREPANCY IMMEDIATELY TO THE INVIGILATOR.

THIS EXAMINATION PAPER HAS **12 PAGES** (ALL OF WHICH ARE NUMBERED).

TOTAL MARKS: 37

THIS EXAMINATION IS **CLOSED BOOK.**

THE USE OF CALCULATORS IS **NOT** PERMITTED.

SHOW ALL OF YOUR WORK!
CLEARLY DEFINE ANY NEW QUANTITIES (E.G., VARIABLES, FUNCTIONS, ETC.) THAT YOU INTRODUCE IN YOUR SOLUTIONS.

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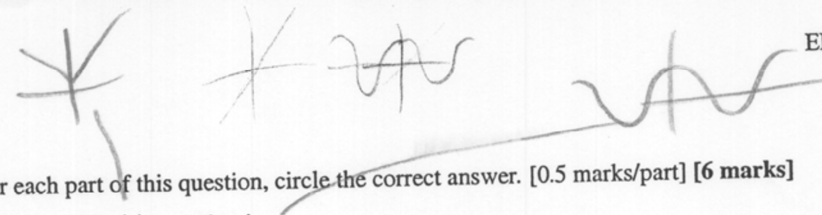
Do not write on this page unless instructed to do so.



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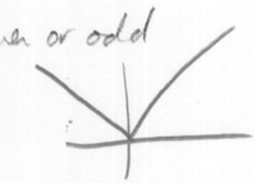
QUESTION 1. For each part of this question, circle the correct answer. [0.5 marks/part] **[6 marks]**

✗ **(A)** The function $x(t) = |t| + \cos 2\pi t$ is

i) neither even nor odd
ii) even but not odd
iii) odd but not even
iv) both even and odd

sum of even + odd is neither even or odd
odd + odd = ?
even + odd = neither



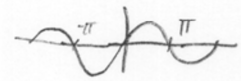
✓ **(B)** The function $y(t) = \sin 2\pi t + \cos 5\pi t$ is

$T_1 = \frac{2\pi}{2\pi} = 1$ $T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}$ $\frac{T_1}{T_2} = \frac{5}{2}$
period $= 2T_1 = 2$

i) aperiodic
ii) periodic with period 2
iii) periodic with period 5
iv) periodic with period 7
v) none of the above

✓ **(C)** The integral $\int_{-\infty}^{\infty}\left(e^{-2t^2+4t+1}\sin t\right)\delta(t-\pi)dt$ evaluates to

$e^{-2(\pi)^2+4\pi+1}\cancel{\sin(\pi)} \to 0$



i) $-\pi$
ii) -1
iii) 0
iv) 1
v) π
vi) none of the above

✗ **(D)** Consider the system with input $x(t)$ and output $y(t)$ characterized by the equation $y(t) = x(t-a)$ where a is an arbitrary real constant. Is this system memoryless?

If a=0, y(t) is memoryless
If a = other c, y(t) has memory

i) yes
ii) no
iii) insufficient information to determine

✗ **(E)** Consider the system with input $x(t)$ and output $y(t)$ characterized by the equation $y(t) = \int_{-\infty}^{t+1} x(\tau)d\tau$. Is this system causal?

i) yes
ii) no
iii) insufficient information to determine

✓ **(F)** Consider the system with input $x(t)$ and output $y(t)$ characterized by the equation $y(t) = 1/x(t)$. Is this system BIBO stable?

$y(0) = \frac{1}{0} = \infty$

i) yes
ii) no
iii) insufficient information to determine

✓ **(G)** Consider the system with input $x(t)$ and output $y(t)$ characterized by the equation $y(t) = x(-t)$. Is this system time invariant?

$-(t-t_0) = -t+t_0$

i) yes
ii) no
iii) insufficient information to determine

✗ **(H)** A LTI system has the impulse response $h(t)$ given by $h(t) = \delta(t-1)$. Does this system have memory?

i) yes
ii) no
iii) insufficient information to determine

✓ **(I)** A LTI system has the impulse response $h(t)$ given by $h(t) = e^{at}u(-t)$, where a is an arbitrary real constant. Is this system causal?

i) yes
ii) no
iii) insufficient information to determine

✓ **(J)** Let $h(t)$ and $h^{\text{inv}}(t)$ denote the impulse responses of a LTI system and its inverse, respectively. Then, $h(t) * h^{\text{inv}}(t)$ must be equal to

i) 0
ii) 1
iii) $\delta(t)$
iv) none of the above

✗ **(K)** A LTI system with impulse response $h(t)$ is BIBO stable if and only if

i) $h(t)$ is bounded
ii) $h(t)$ is integrable
iii) $h(t)$ is absolutely integrable
iv) $h(t)$ is square integrable
v) none of the above

✗ **(L)** Suppose that we have three systems $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$. The responses of these systems to the complex exponential input e^{j2t} are given by

$$e^{j2t} \xrightarrow{\mathcal{H}_1} te^{j2t}, \quad e^{j2t} \xrightarrow{\mathcal{H}_2} 3e^{j2t}, \quad \text{and} \quad e^{j2t} \xrightarrow{\mathcal{H}_3} e^{j2t+\pi/3}.$$

Which of these systems cannot be LTI?

i) $\mathcal{H}_1$
ii) $\mathcal{H}_2$
iii) $\mathcal{H}_3$
iv) $\mathcal{H}_1$ and $\mathcal{H}_2$
v) $\mathcal{H}_1$ and $\mathcal{H}_3$
vi) $\mathcal{H}_2$ and $\mathcal{H}_3$
vii) $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$.

QUESTION 2. Suppose that we have the function

$$F(z) = \frac{z^2+5z+6}{(z-j)(z^4-z^3)}$$

where z is complex.

(a) Find the poles and zeros of $F(z)$ and determine the order of each pole and zero. **[2 marks]**

$z^2+5z+6 \rightarrow (z+3)(z+2)$ — first order zeroes $\rightarrow$ $-2, -3$ ✓

$z^3(z-1)$ — first order poles $1, +j$ ✓

third order poles 0 ✓

$\frac{2}{2}$

(b) Identify for what values of z the function $F(z)$ is analytic. [*Note: You do not need to use the Cauchy-Riemann equations. Simply state the final result and very briefly justify your answer.*] **[1 mark]**

$F(z)$ is analytic for all values except where it is divisible by zero. ?

In this example $F(z)$ is analytic everywhere except where

$z = 0, 1,$ or j.

$\frac{0.5}{1}$

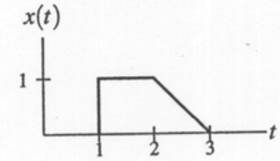
QUESTION 3.

Suppose that we have the signal $x(t)$ shown in the graph to the right. Using the axes provided below, plot each of the following signals:

$\frac{2.5+2}{6}$

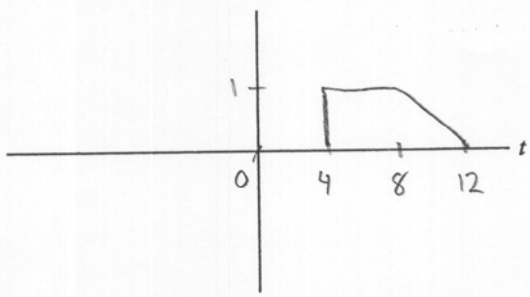
(a) $x(t/4)$, **[1 mark]**
(b) $x(t+2)$, **[1 mark]**
(c) $x(-t)$, **[1 mark]**
(d) $-2x(-2t+1)$. **[3 marks]**

It is highly recommended that you do part (d) of the question in several steps to reduce the likelihood of error (i.e., apply one transformation at a time). If you do everything in one step and make a mistake, you will probably receive zero marks.



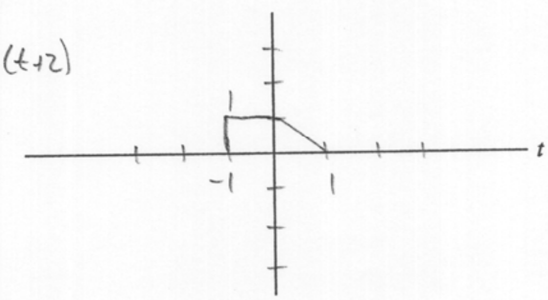
(a)

$x(\frac{1}{4}t)$



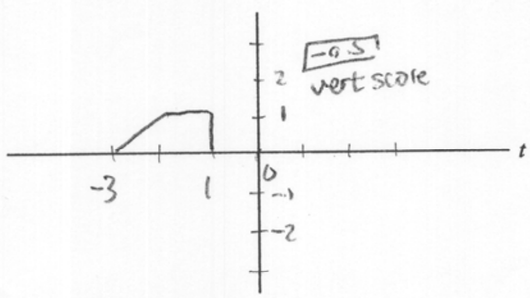
(b)

$x(t+2)$



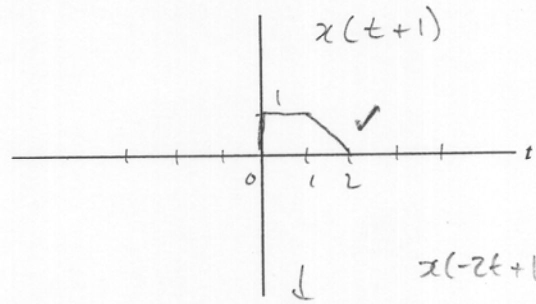
(c)

$x(-t)$

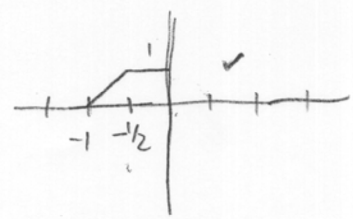


(d)

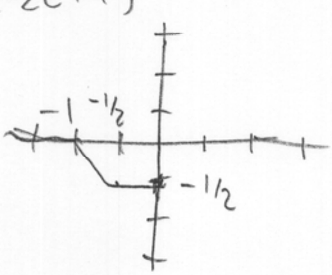
$\frac{2}{3}$



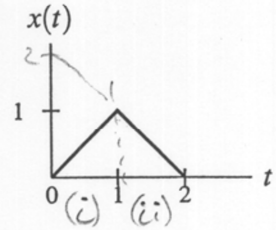
$x(-2t+1)$



$-2x(-2t+1)$



QUESTION 4.



Suppose that we have the signal $x(t)$ shown in the figure. Use unit-step functions to find a single expression for $x(t)$ that is valid for all t. **When stating your final answer,** *group together terms having the same unit-step function factor.* **[5 marks]**

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(i) $v_1(t) = t[u(t) - u(t-1)] = tu(t) - tu(t-1)$

(ii) $v_2(t) = (2-t)[u(t-1) - u(t-2)] = 2u(t-1) - tu(t-1) - 2u(t-2) + tu(t-2)$

$x(t) = v_1(t) + v_2(t) = tu(t) + (2-2t)u(t-1) + (t-2)u(t-2)$ ✓

QUESTION 5. Show that, for any real signals $x(t)$ and $h(t)$, the following identity holds:

$$x(t) * h(t) = h(t) * x(t)$$

(i.e., convolution is commutative). *Do not skip any steps in the proof.* **[2 marks]**

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$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$ let $\lambda = t-\tau$ $\tau = t-\lambda$ $d\tau = -d\lambda$

$= \int_{\infty}^{-\infty} x(t-\lambda)h(\lambda)(-d\lambda)$

$= \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$ ← defⁿ of convolution

$x(t) * h(t) = h(t) * x(t)$ ✓

QUESTION 6.

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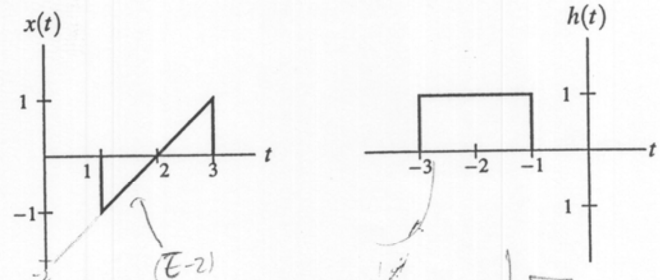
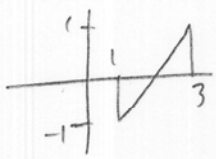
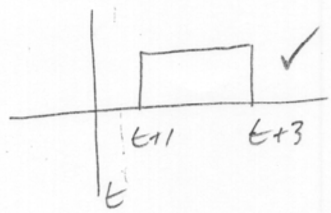
Using graphical methods, compute the convolution $y(t) = x(t) * h(t)$ where the signals $x(t)$ and $h(t)$ are as shown in the figure to the right. **[8 marks]**



$x(\tau)$ $\quad$ $h(t-\tau)$

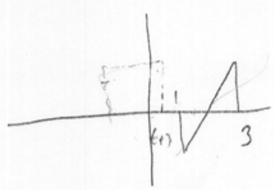




$t+3 < 1$

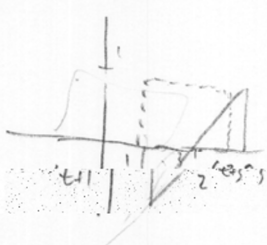
$t < -2$

i) $t < -2$



$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = 0$$ ✓

ii) $-2 \leq t < 0$



$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{1}^{t+3} (1)(\tau-2)d\tau$$ ✓

$$= \frac{\tau^2}{2} - 2\tau \Big|_{1}^{t+3} = \left[\frac{(t+3)^2}{2} - 2(t+3)\right] - \left[\frac{1^2}{2} - 2\right]$$

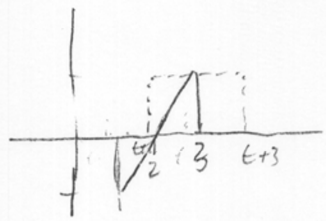
$$= \frac{t^2+6t+9}{2} - \frac{4t-12}{2}) - \frac{1}{2} + \frac{4}{2}$$

$$= \frac{t^2}{2} + t$$

EXTRA SPACE FOR QUESTION 6 SOLUTION

iii)

$0 \le t < 2$



$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$= \int_{t+1}^{3} (1)(\tau-2)\,d\tau \checkmark$$

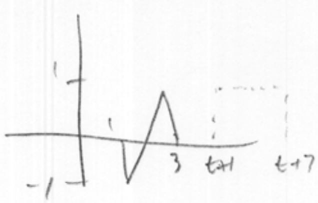
$$\frac{\tau^2}{2} - 2\tau \Big|_{t+1}^{3}$$

$$= \left[\frac{(3)^2}{2} - 2(3)\right] - \left[\frac{(t+1)^2}{2} - 2(t+1)\right]$$

$$= \left[\frac{9}{2} - \frac{12}{2}\right] - \left[\frac{t^2+2t+1}{2} - \frac{4t+4}{2}\right]$$

$$= -\frac{t^2}{2} + t$$

iv)



$$\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = 0 \checkmark$$

$t > 2$

$$y(t) = \begin{cases} 0 & t < -2 \\ \frac{t^2}{2} - t & -2 \le t < 0 \\ -\frac{t^2}{2} + t & 0 \le t < 2 \\ 0 & t > 2 \end{cases}$$

QUESTION 7. Suppose that we have a system with input $x(t)$ and output $y(t)$.

(a) Clearly state, in mathematical terms, the condition that must be satisfied in order for the above system to be linear. *Be sure to define all quantities such as variables, functions, and constants. Otherwise, you will receive zero marks.* **[1 mark]**

input $x(t)$ produces output $y(t)$ variable $\rightarrow t$

$\frac{0}{1}$

$$x_1(t) = y_1(t)$$
$$x_2(t) = y_2(t)$$
$$x_3(t) = ay_1(t) + by_2(t) \quad \text{where } a \text{ and } b \text{ are nonzero constants}$$
$$x_3(t) = ax_1(t) + bx_2(t)$$

(b) Suppose now that the above system is characterized by the equation

$$y(t) = x(t) + 1.$$

Using the condition stated in part (a) of this question, determine whether this system is linear. **[3 marks]**

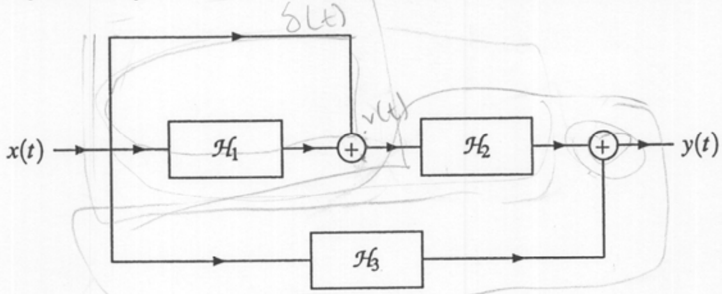
$$y_1(t) = x_1(t) + 1 \qquad y_2(t) = x_2(t) + 1$$

$\frac{3}{3}$

$$y_3(t) = [ax_1(t) + bx_2(t)] + 1$$
$$= ax_1(t) + bx_2(t) + 1$$
$$y_3(t) \neq ay_1(t) + by_1(t)$$

$\therefore$ System is not linear.

QUESTION 8. Suppose that we have the system shown below with the input $x(t)$ and output $y(t)$. Let $h(t)$ denote the impulse response of this system (i.e., the system with input $x(t)$ and output $y(t)$). In the diagram below, the blocks labelled $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$ are LTI systems with the impulse responses $h_1(t)$, $h_2(t)$, and $h_3(t)$, respectively.



$\frac{2.5}{3}$

Find the impulse response $h(t)$ of the overall system in terms of $h_1(t)$, $h_2(t)$, and $h_3(t)$. **[3 marks]**

$$h(t) = \left[\left[h_1(t) + \delta(t)\right] * h_2(t)\right] + h_3(t)$$

$$h(t) = h_1(t) * h_2(t) + \underbrace{\delta(t) * h_2(t)}_{= ?} + h_3(t)$$

$\boxed{-0.5}$

$$V(t): \left[H_1(t) + \delta(t)\right] * H_2 + H_3(t)$$

$$H_2 * H_1 + \cancel{H_2 * \delta(t)}\, H_2 + H_3(t)$$

$$H_2 * H_1 + H_2 + H_3(t)$$

EXTRA SPACE FOR SOLUTIONS

END