ELEC 260

MIDTERM EXAM 1

SOLUTIONS

Question 1

(a) ii) even but not odd $x(-t) = |-t| + \cos(-2\pi t) = |t| + \cos 2\pi t = x(t)$
(b) ii) periodic with period 2 $T_1 = 1,\ T_2 = 2/5,\ \frac{T_1}{T_2} \in Q \quad T = 2T_1 = 2$
(c) iii) 0
(d) iii) insufficient information to determine
(e) ii) no
(f) ii) no
(g) ii) [redacted] no
(h) i) yes
(i) ii) no
(j) iii) delta(t)
(k) iii) h(t) is absolutely integrable
(l) i) H_1

QUESTION 2

(a) To find the poles and zeros of $F(z)$, we rewrite $F(z)$ in factored form. We have

$$F(z) = \frac{z^2 + 5z + 6}{(z-j)(z^4 - z^3)}$$

$$= \frac{(z+2)(z+3)}{(z-j)\, z^3 (z-1)}$$

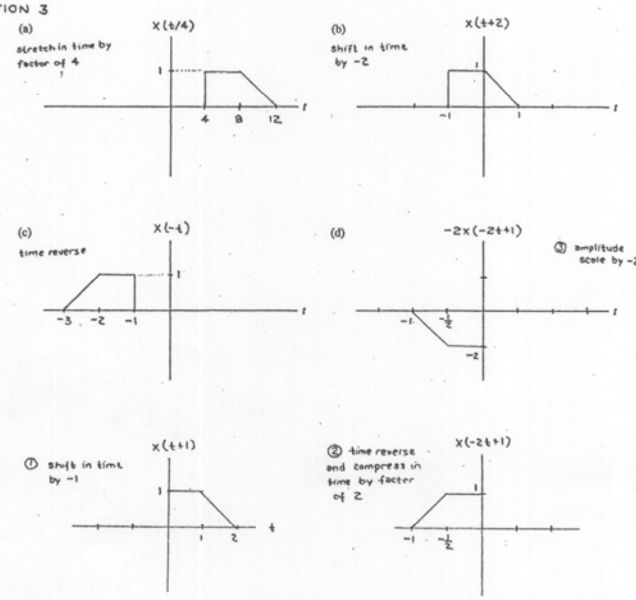
By inspection, we see that $F(z)$ has the following poles and zeros:

poles
- j (1st order)
- 0 (3rd order)
- 1 (1st order)

zeros
- -2 (1st order)
- -3 (1st order)

(b) Since $F(z)$ is a rational function, it is analytic everywhere except at its poles. Therefore, $F(z)$ is analytic everywhere except at $z \in \{j, 0, 1\}$.

QUESTION 3



QUESTION 4

We can write

$$x(t) = t\left[u(t)-u(t-1)\right] + (2-t)\left[u(t-1)-u(t-2)\right]$$

$$= t\,u(t) - t\,u(t-1) + 2\,u(t-1) - 2\,u(t-2) - t\,u(t-1) + t\,u(t-2)$$

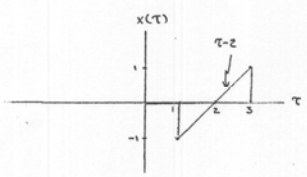
$$= (t)\,u(t) + (-2t+2)\,u(t-1) + (t-2)\,u(t-2)$$

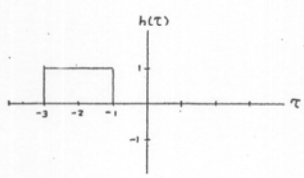
QUESTION 5

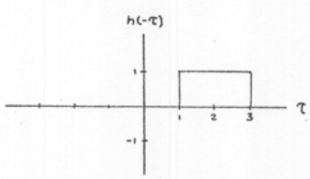
A proof of the commutative property of convolution is provided in the lecture notes.

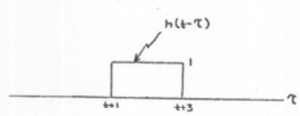
(See Section 3.3.1 on page 43.)

QUESTION 6

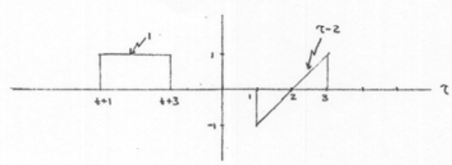




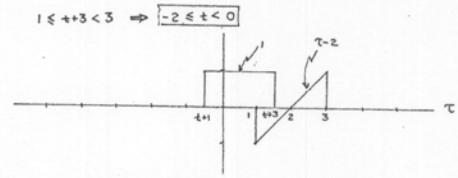




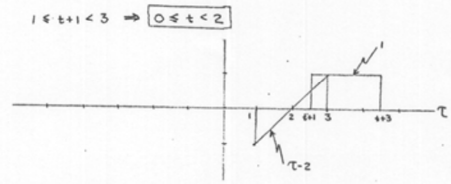
$t+3<1 \Rightarrow \boxed{t<-2}$



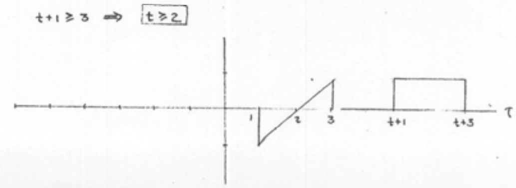
$1 \leq t+3 < 3 \Rightarrow \boxed{-2 \leq t < 0}$



$1 \leq t+1 < 3 \Rightarrow \boxed{0 \leq t < 2}$



$t+1 \geq 3 \Rightarrow \boxed{t \geq 2}$



for $t<-2$

$$x(t) * h(t) = 0$$

for $-2 \le t < 0$

$$\begin{aligned} x(t) * h(t) &= \int_1^{t+3} (1)(\tau-2)\, d\tau \\ &= \int_1^{t+3} (\tau-2)\, d\tau \\ &= \left[\tfrac{1}{2}\tau^2 - 2\tau\right]\Big|_1^{t+3} \\ &= \tfrac{1}{2}(t+3)^2 - 2(t+3) - \left[\tfrac{1}{2} - 2\right] \\ &= \tfrac{1}{2}(t^2+6t+9) - 2t - 6 + \tfrac{3}{2} \\ &= \tfrac{1}{2}t^2 + 3t + \tfrac{9}{2} - 2t - 6 + \tfrac{3}{2} \\ &= \tfrac{1}{2}t^2 + t \end{aligned}$$

for $0 \le t < 2$

$$\begin{aligned} x(t) * h(t) &= \int_{t+1}^{3} (1)(\tau-2)\, d\tau \\ &= \int_{t+1}^{3} (\tau-2)\, d\tau \\ &= \left[\tfrac{1}{2}\tau^2 - 2\tau\right]\Big|_{t+1}^{3} \\ &= \tfrac{9}{2} - 6 - \left[\tfrac{1}{2}(t+1)^2 - 2(t+1)\right] \\ &= -\tfrac{3}{2} - \tfrac{1}{2}(t^2+2t+1) + 2(t+1) \\ &= -\tfrac{3}{2} - \tfrac{1}{2}t^2 - t - \tfrac{1}{2} + 2t + 2 \\ &= -\tfrac{1}{2}t^2 + t \end{aligned}$$

for $t \ge 2$

$$x(t) * h(t) = 0$$

$$x(t) * h(t) = \begin{cases} \tfrac{1}{2}t^2 + t & \text{for } -2 \le t < 0 \\ -\tfrac{1}{2}t^2 + t & \text{for } 0 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

QUESTION 7

(a) Suppose that

$$x_1(t) \longrightarrow y_1(t)$$
$$x_2(t) \longrightarrow y_2(t)$$
$$a_1 x_1(t) + a_2 x_2(t) \longrightarrow y_3(t)$$

If for all $a_1, a_2 \in \mathbb{C}$ and all $x_1(t), x_2(t)$ we have $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$, then the system is linear.

(b) From the system input-output equation, we can write

$$y_1(t) = x_1(t) + 1$$

$$y_2(t) = x_2(t) + 1$$

$$\begin{aligned} y_3(t) &= [a_1 x_1(t) + a_2 x_2(t)] + 1 \\ &= a_1 x_1(t) + a_2 x_2(t) + 1 \end{aligned}$$

$$\begin{aligned} a_1 y_1(t) + a_2 y_2(t) &= a_1[x_1(t) + 1] + a_2[x_2(t) + 1] \\ &= a_1 x_1(t) + a_2 x_2(t) + a_1 + a_2 \end{aligned}$$

Thus, we have

$$y_3(t) \ne a_1 y_1(t) + a_2 y_2(t).$$

Therefore, the system is not linear.

QUESTION 8

Let us denote the input to system $\mathcal{H}_2$ as $v(t)$.

From the block diagram, we have

$$\begin{aligned} v(t) &= x(t) + h_1(t) * x(t) \\ &= [\delta(t) + h_1(t)] * x(t) \end{aligned}$$

$$\begin{aligned} y(t) &= v(t) * h_2(t) + x(t) * h_3(t) \\ &= [\delta(t) + h_1(t)] * x(t) * h_2(t) + x(t) * h_3(t) \\ &= x(t) * [\delta(t) + h_1(t)] * h_2(t) + x(t) * h_3(t) \\ &= x(t) * [(\delta(t) + h_1(t)) * h_2(t) + h_3(t)] \\ &= x(t) * [h_2(t) + h_1(t) * h_2(t) + h_3(t)] \end{aligned}$$

Therefore, the impulse response $h(t)$ of the system is given by

$$h(t) = h_2(t) + h_1(t) * h_2(t) + h_3(t)$$