Thus

$$z - 243 = 30(x - 9) + 72(y - 3)$$

Hence a linear approximation of the given nonlinear equation near the operating point is

$$z - 30x - 72y + 243 = 0$$

PROBLEMS

B–2–1. Simplify the block diagram shown in Figure 2–29 and obtain the closed-loop transfer function $C(s)/R(s)$.

B–2–2. Simplify the block diagram shown in Figure 2–30 and obtain the closed-loop transfer function $C(s)/R(s)$.

B–2–3. Simplify the block diagram shown in Figure 2–31 and obtain the closed-loop transfer function $C(s)/R(s)$.

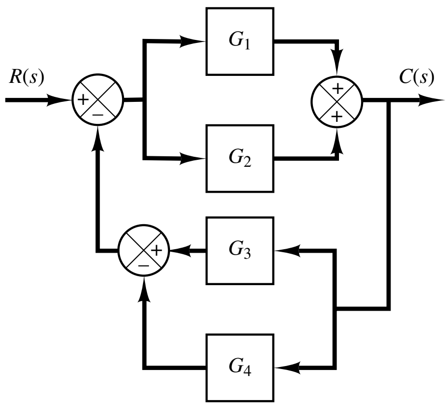

Figure 2–29
Block diagram of a system.

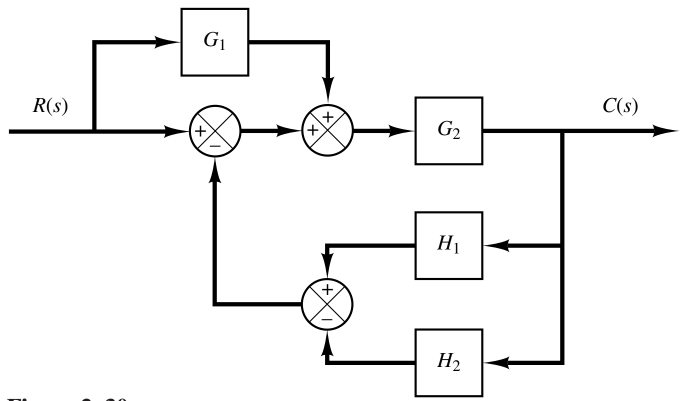

Figure 2–30
Block diagram of a system.

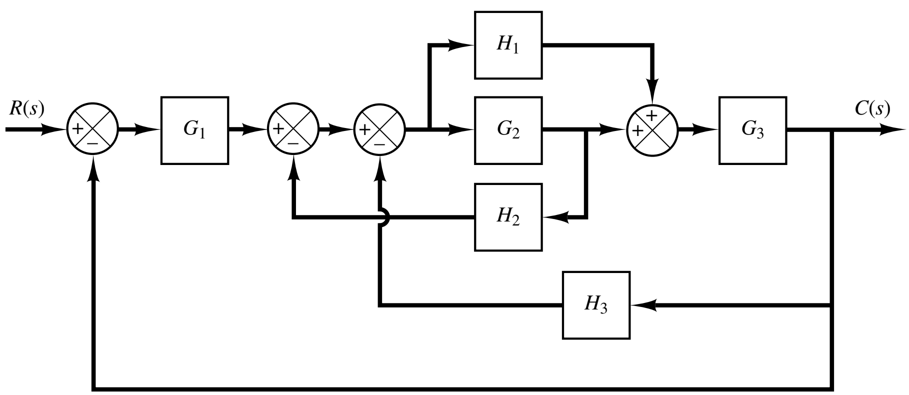

Figure 2–31
Block diagram of a system.

B–2–4. Consider industrial automatic controllers whose control actions are proportional, integral, proportional-plus-integral, proportional-plus-derivative, and proportional-plus-integral-plus-derivative. The transfer functions of these controllers can be given, respectively, by

$$\frac{U(s)}{E(s)} = K_p$$

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

$$\frac{U(s)}{E(s)} = K_p(1 + T_d s)$$

$$\frac{U(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

where $U(s)$ is the Laplace transform of $u(t)$, the controller output, and $E(s)$ the Laplace transform of $e(t)$, the actuating error signal. Sketch $u(t)$-versus-t curves for each of the five types of controllers when the actuating error signal is

(a) $e(t)$ = unit-step function

(b) $e(t)$ = unit-ramp function

In sketching curves, assume that the numerical values of K_p, K_i, T_i, and T_d are given as

$$K_p = \text{proportional gain} = 4$$
$$K_i = \text{integral gain} = 2$$
$$T_i = \text{integral time} = 2 \text{ sec}$$
$$T_d = \text{derivative time} = 0.8 \text{ sec}$$

B–2–5. Figure 2–32 shows a closed-loop system with a reference input and disturbance input. Obtain the expression for the output $C(s)$ when both the reference input and disturbance input are present.

B–2–6. Consider the system shown in Figure 2–33. Derive the expression for the steady-state error when both the reference input $R(s)$ and disturbance input $D(s)$ are present.

B–2–7. Obtain the transfer functions $C(s)/R(s)$ and $C(s)/D(s)$ of the system shown in Figure 2–34.

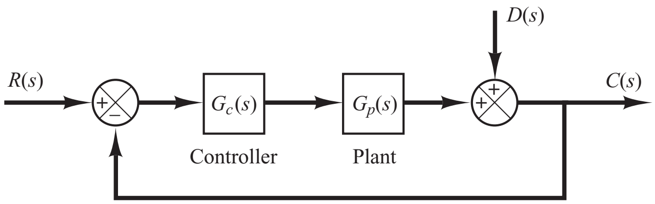

Figure 2–32
Closed-loop system.

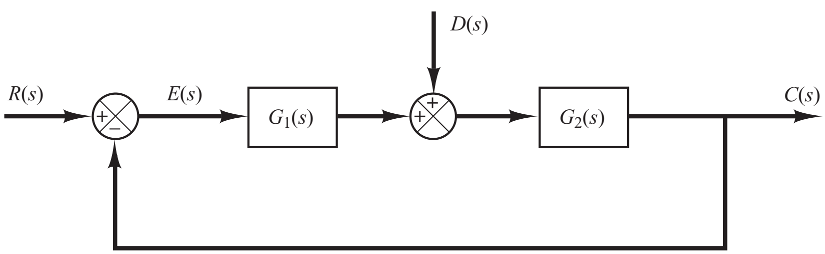

Figure 2–33
Control system.

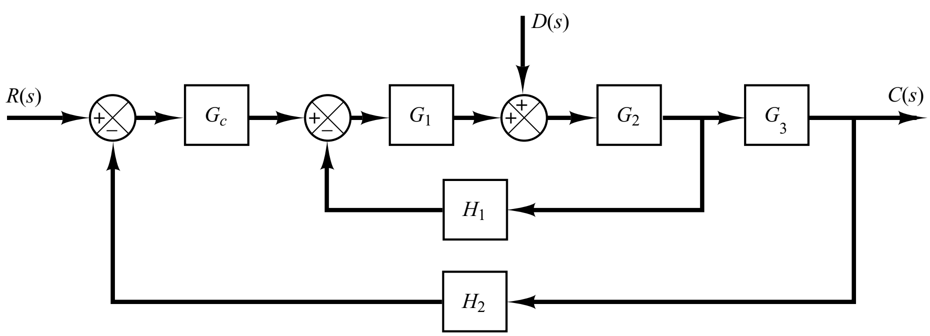

Figure 2–34
Control system.

B–2–8. Obtain a state-space representation of the system shown in Figure 2–35.

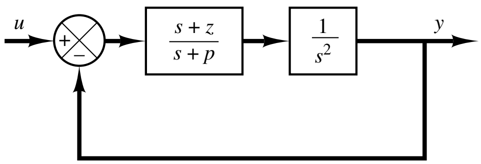

Figure 2–35
Control system.

B–2–9. Consider the system described by

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Derive a state-space representation of the system.

B–2–10. Consider the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function of the system.

B–2–11. Consider a system defined by the following state-space equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Obtain the transfer function $G(s)$ of the system.

B–2–12. Obtain the transfer matrix of the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B–2–13. Linearize the nonlinear equation

$$z = x^2 + 8xy + 3y^2$$

in the region defined by $2 \le x \le 4, 10 \le y \le 12$.

B–2–14. Find a linearized equation for

$$y = 0.2x^3$$

about a point $x = 2$.