CSC 320 Spring 2024
Assignment 1

This assignment has 6 written questions and is out of a total of 30 marks. Submit one PDF file containing your solutions on Brightspace.

Questions

1. **[5 marks]** Let $\boxed{\Sigma = \{a, b\}.}$ Consider the following languages L_A and L_B over Σ:
   $L_A = \{w \in \Sigma^* \mid w \text{ contains an even number of occurrences of symbol } a\}$
   $L_B = \{w \in \Sigma^* \mid \text{each pair of consecutive } b\text{'s in } w \text{ is separated by a substring of } a\text{'s of length } 2i, i \geq 0\}$

   Clarification: L_B is saying that between b's there must be $2i$ a's for some $i \geq 0$ (e.g. $baab \in L_B$ and $baaab \notin L_B$). Also, i must be an integer value ≥ 0.

   For each of the following strings, decide whether it is a member of L_A or L_B:

   (a) b
   (b) $aabbbbaa$
   (c) $abaaabaaa$
   (d) $bbaaaabbaab$
   (e) a

<u>*Answers*</u>

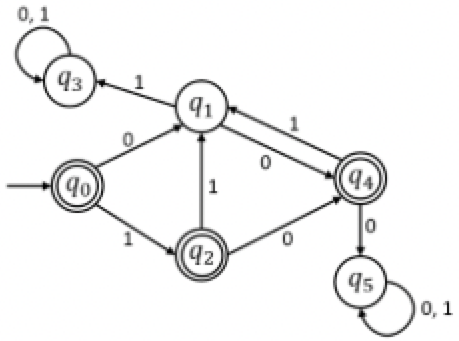
1A: ***b*** is a member of LA because there's even number of occurrences of ***a***'s in the string (0 is an even number) and also a part of LB because there's *2i number of **a**'s* where i is equal to 0, satisfying the condition ***i>=0***. Therefore, the string is a member of both LA and LB.

1B: ***aabbbbaa*** is a member of both LA and LB, since it has 4 *a*'s in the string (LA) and the string can be rewritten as ***aa-bb-0-bb-aa*** where we have pairs of *b*'s separated by <u>*2i number of **a**'s*</u> (LB) where number of ***a*** in between is 0.

1C: ***abaaabaaa*** is not a member of LA because there's an uneven number of ***a***'s in the string, and also not a member of LB since it doesn't have ***2i*** number of ***a***'s in between the ***b*** pairs, where number of ***a*** in between is 3.
1D: ***bbaaaabbaab*** or ***bb-aa-aa-bb-aa-b*** is a member of both LA and LB since there's even occurrences of a in the string and there's ***2i*** number of ***a***'s in between b's.
1E: ***a*** is not a member of LA since there's uneven number of a's but it's a member of LB since there's no consecutive number of ***b***'s to be separated by ***2i*** number of ***a***'s.

— x — o — x — o — x — o — x —

2. **[5 marks]** Consider the following DFA M:



Describe M as a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$. In particular, give

(a) Q
(b) Σ
(c) δ (in the form of a transition table)
(d) q_0
(e) F

ANSWER 2:

The given DFA can be described as a 5-tuple (Q, Σ, δ, q0, F) :

Q = finite set of states $\{q_0, q_1, q_2, q_3, q_4, q_5\}$.
Σ = finite set of symbols called *alphabets* {0, 1}.
δ = Transition function $\boldsymbol{\delta}: \boldsymbol{Q} \times \boldsymbol{\Sigma} \to \boldsymbol{Q}$ has equal number of outgoing transitions for each *alphabets*.
q0 = Has an *initial state* of q_0.
F = Has multiple finite *accept states,* $\{q_0, q_2, q_4\}$.

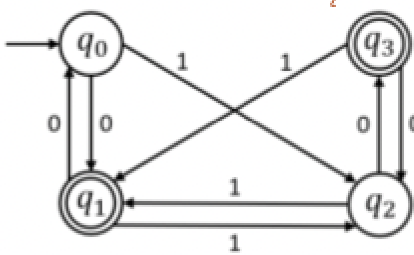
$(\{q_0, q_2, q_3, q_4, q_5\}, \{0, 1\}, \delta, q_0, \{q_0, q_2, q_4\})$

with $\delta: Q \times \Sigma \to Q$ defined by

δ	0	1
q_1	q_4	q_3
q_2	q_4	q_1
q_3	q_3	q_3
q_4	q_5	q_1
q_5	q_5	q_5
q_0	q_1	q_2

—x—o—x—o—x—o—x—

3. **[5 marks]** Consider the following DFA state diagram:



Transition table

δ	0	1
q_0	q_1	q_2
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_2	q_1

For each of the following strings, give the exact sequence of states that the automaton undergoes when reading the string. Furthermore, indicate whether or not the string is accepted.

(a) $w_1 = \varepsilon$
(b) $w_2 = 0111$
(c) $w_3 = 1000000100$
(d) $w_4 = 1010110$
(e) $w_5 = 11111111$

ANSWER 3:

a) Since q_0 is not a final (accept) state, the string is not accepted.

ε

b) Since q_2 is not a final (accept) state, the string is not accepted.

0 1 1

c) Since q_1 is a final (accept) state, the string is accepted.

1 0 0 0 0 0 0 1 0 0

d) Since q_0 is not a final (accept) state, the string is not accepted.

1 0 1 0 1 1 0

2 2 1 0 2 1 0

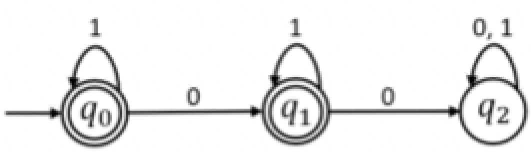
e) Since q_1 is a final (accept) state, the string is accepted.

1 1 1 1 1 1 1 1

2 1 2 1 2 1 2 1

— x — 0 — x — 0 — x — 0 — x —

4. **[5 marks]** Consider the state diagram of the following DFA F. Describe the language $L(F)$ that the automaton recognizes using set notation.



① 1
② 0 1

ANSWER 4: The state diagram of the above DFA ***F*** recognizes the language L(F) such that:

L(F) = { w ∈ Σ∗ | |w| ≥ 1, if w begins with 1 } or
{ w ∈ Σ∗ | |w| ≥ 2, if w starts with 0 and ends with 1}

— x — 0 — x — 0 — x — 0 — x —

5. **[5 marks]** Let $\Sigma = \{a, b\}$. For the following language L_A over Σ, construct a deterministic finite automaton A with $L(A) = L_A$. For the automaton, give the state diagram and the formal 5-tuple definition of the DFA $A = (Q, \Sigma, \delta, q_0, F)$ with a transition table describing δ.

$$L_A = \{w \in \Sigma^* \mid w \text{ starts with an } a \text{ and contains the substring } abb\}$$

ANSWER 5:

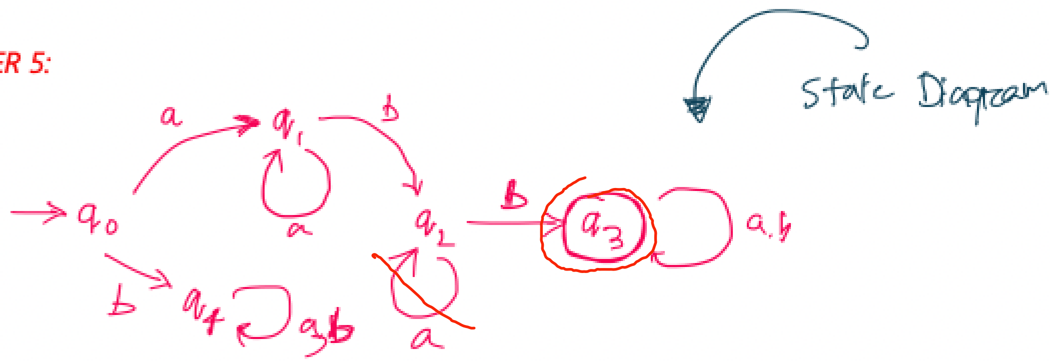


Figure: Constructed DFA **A**

The given DFA **A** can be described as a 5-tuple (Q, Σ, δ, q0, F) :

Q = Finite set of states $\{q_0, q_1, q_2, q_3\}$.
Σ = Finite set of symbols called *alphabets* {a, b}.
δ = Transition function $\boldsymbol{\delta}$: $\boldsymbol{Q \times \Sigma \rightarrow Q}$ has equal number of outgoing transitions for each *alphabets*.
q_0 = Has an *initial state* of q_0.
F = Has one finite *accept states,* $\{q_3\}$.

So $(\{q_0, q_1, q_2, q_3\}, \{a, b\}, \boldsymbol{\delta}, q_0, \{q_3\})$ defined by:

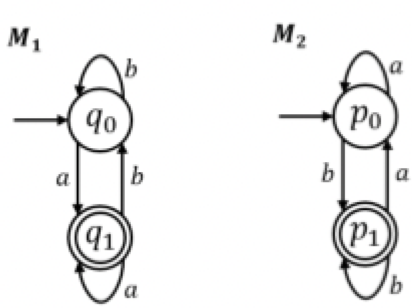
Formal Definition

δ	a	b
q_0	q_1	q_4
q_1	q_1	q_2
q_2	q_2	q_3
q_3	q_3	q_3
q_4	q_4	q_4

Transition Table

— x — o — x — o — x — o — x —

6. **[5 marks]** Consider the state diagrams of the following automata M_1 and M_2:



Give a state diagram and transition table for a DFA M that recognizes the language $L(M) = L(M_1) \cap L(M_2)$. For this, you must use the construction from the proof that builds a DFA that recognizes the intersection of two regular languages.