referred to the motor shaft. When J_0 and $b_0 + (K_2K_3/R_a)$ are multiplied by $1/n^2$, the inertia and viscous-friction coefficient are expressed in terms of the output shaft. Introducing new parameters defined by

$$J = J_0/n^2 = \text{moment of inertia referred to the output shaft}$$

$$B = [b_0 + (K_2K_3/R_a)]/n^2 = \text{viscous-friction coefficient referred to the output shaft}$$

$$K = K_0K_1K_2/nR_a$$

the transfer function $G(s)$ given by Equation (3–51) can be simplified, yielding

$$G(s) = \frac{K}{Js^2 + Bs}$$

or

$$G(s) = \frac{K_m}{s(T_m s + 1)}$$

where

$$K_m = \frac{K}{B}, \qquad T_m = \frac{J}{B} = \frac{R_aJ_0}{R_ab_0 + K_2K_3}$$

The block diagram of the system shown in Figure 3–29(b) can thus be simplified as shown in Figure 3–29(c).

PROBLEMS

B–3–1. Obtain the equivalent viscous-friction coefficient b_{eq} of the system shown in Figure 3–30.

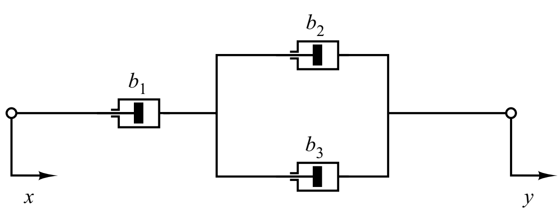

Figure 3–30
Damper system.

B–3–2. Obtain mathematical models of the mechanical systems shown in Figures 3–31(a) and (b).

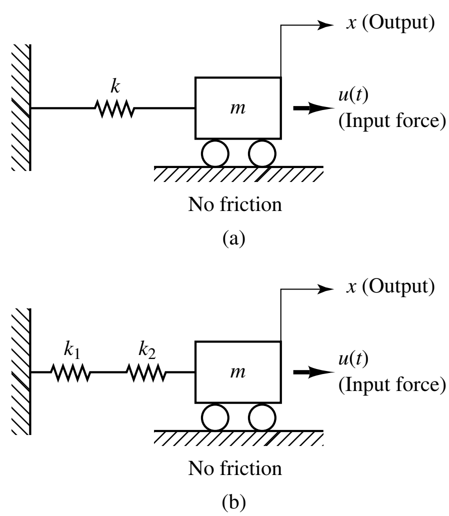

Figure 3–31
Mechanical systems.

B–3–3. Obtain a state-space representation of the mechanical system shown in Figure 3–32, where u_1 and u_2 are the inputs and y_1 and y_2 are the outputs.

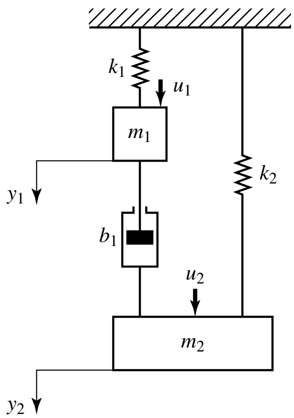

Figure 3–32 Mechanical system.

B–3–4. Consider the spring-loaded pendulum system shown in Figure 3–33. Assume that the spring force acting on the pendulum is zero when the pendulum is vertical, or $\theta = 0$. Assume also that the friction involved is negligible and the angle of oscillation θ is small. Obtain a mathematical model of the system.

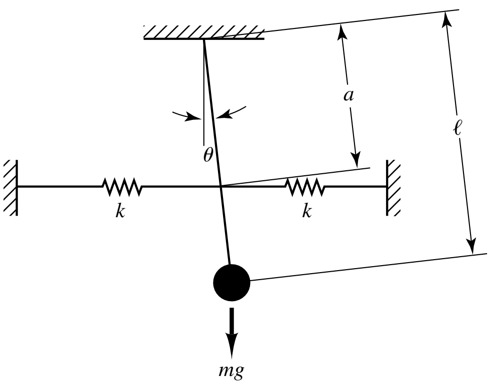

Figure 3–33 Spring-loaded pendulum system.

B–3–5. Referring to Examples 3–5 and 3–6, consider the inverted-pendulum system shown in Figure 3–34. Assume that the mass of the inverted pendulum is m and is evenly distributed along the length of the rod. (The center of gravity of the pendulum is located at the center of the rod.) Assuming that θ is small, derive mathematical models for the system in the forms of differential equations, transfer functions, and state-space equations.

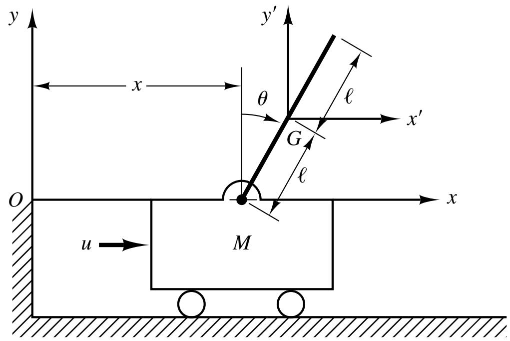

Figure 3–34 Inverted-pendulum system.

B–3–6. Obtain the transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the mechanical system shown in Figure 3–35.

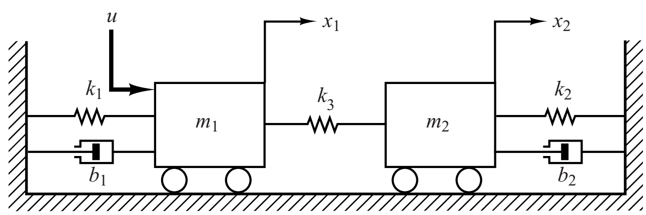

Figure 3–35 Mechanical system.

B–3–7. Obtain the transfer function $E_o(s)/E_i(s)$ of the electrical circuit shown in Figure 3–36.

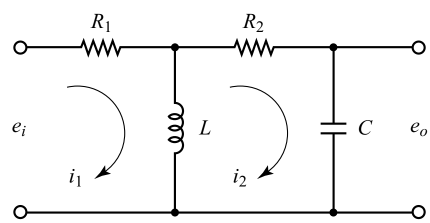

Figure 3–36 Electrical circuit.

B–3–8. Consider the electrical circuit shown in Figure 3–37. Obtain the transfer function $E_o(s)/E_i(s)$ by use of the block diagram approach.

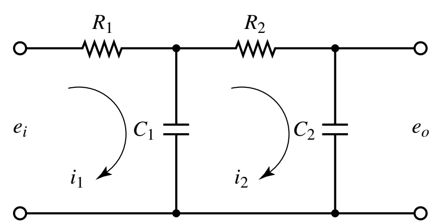

Figure 3–37 Electrical circuit.

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B–3–9. Derive the transfer function of the electrical circuit shown in Figure 3–38. Draw a schematic diagram of an analogous mechanical system.

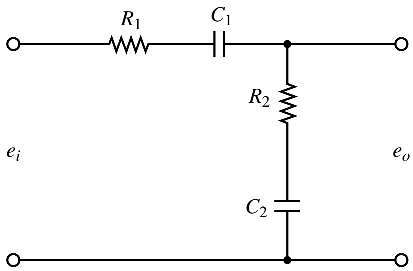

Figure 3–38 Electrical circuit.

B–3–10. Obtain the transfer function $E_o(s)/E_i(s)$ of the op-amp circuit shown in Figure 3–39.

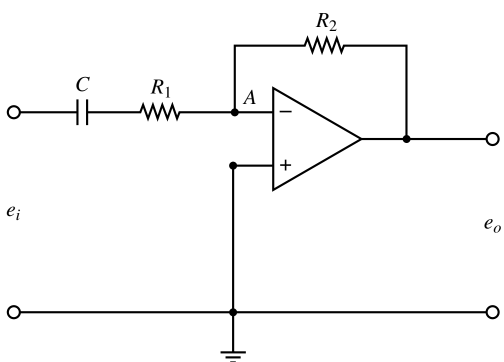

Figure 3–39 Operational-amplifier circuit.

B–3–11. Obtain the transfer function $E_o(s)/E_i(s)$ of the op-amp circuit shown in Figure 3–40.

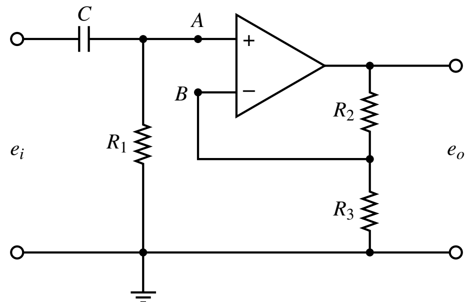

Figure 3–40 Operational-amplifier circuit.

B–3–12. Using the impedance approach, obtain the transfer function $E_o(s)/E_i(s)$ of the op-amp circuit shown in Figure 3–41.

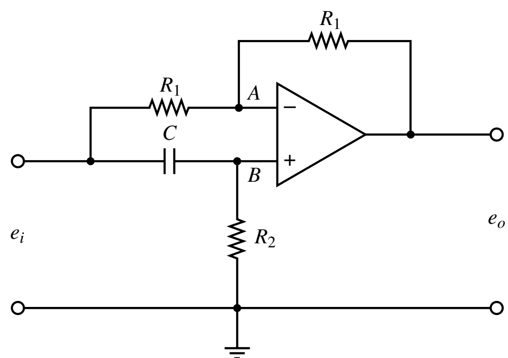

Figure 3–41 Operational-amplifier circuit.

B–3–13. Consider the system shown in Figure 3–42. An armature-controlled dc servomotor drives a load consisting of the moment of inertia J_L. The torque developed by the motor is T. The moment of inertia of the motor rotor is J_m. The angular displacements of the motor rotor and the load element are θ_m and θ, respectively. The gear ratio is $n = \theta/\theta_m$. Obtain the transfer function $\Theta(s)/E_i(s)$.

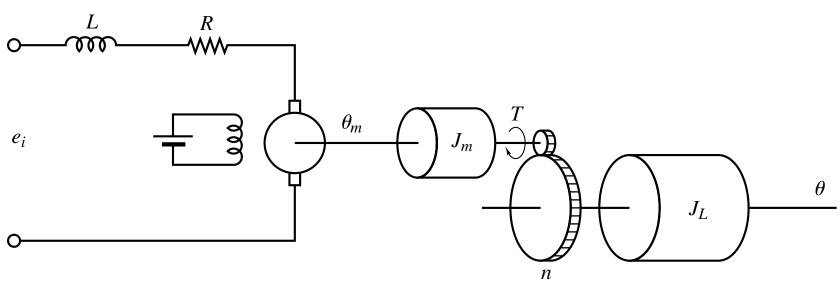

Figure 3–42 Armature-controlled dc servomotor system.