$\omega_d = \omega_n\sqrt{1-\zeta^2}$
$\sigma = \zeta\omega_n$

$\boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

Rise time $t_r = \frac{\pi - \beta}{\omega_d}$, $\beta = \tan^{-1}\frac{\omega_d}{\sigma}$

peak time $t_p = \frac{\pi}{\omega_d}$

Max. overshoot $M_p = e^{-(\sigma/\omega_d)\pi}$

5 % : $t_s = \frac{4}{\sigma}$

2 % : $t_s = \frac{3}{\sigma}$

$e_{ss} = \lim_{s\to 0} s.E(s)$

$= \frac{1}{1+G(s)}$

$e_{ss} = \lim_{s\to 0} \frac{1}{s^2 G(s)} \to p(t)$

$K_p = \lim_{s\to 0} G(s)$

$= G(0)$.

$K_p = K$

	signal error $u(t)$	$r(t)$	$p(t)$
Type 0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$
3	0	0	0

Closed loop transfer func. $= \frac{C(s)}{R(s)}$

open $= G(s)$

unit ramp $\to e_{ss} = \lim_{s\to 0}\frac{1}{sG(s)}$

Routh array

$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1}s' + a_n = 0$.

Routh array

s^n	a_0	a_2	a_4	a_6	---
s^{n-1}	a_1	a_3	a_5	a_7	---
s^{n-2}	b_1	b_2	b_3	---	
s^{n-3}	c_1	c_2	c_3	---	
⋮					
s^0	a_n				

where $b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$

$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$

$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$

$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$

* For a system to be stable, it is necessary that each term of first column of Routh array must be positive if $a_0 > 0$.

Laplace

$L(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)\,dt$

only if $f(t) = 0$ for $t < 0$

- $L(f(t-a).u(t-a)) = e^{-as}$
- $L(e^{-at} f(t)) = F(s+a)$
- $L\left(f\left(\frac{t}{a}\right)\right) = aF(s)$
- $L\left[\frac{d^n}{dt^n}f(t)\right] = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0)\dots - sf^{n-2}(0) - f^{n-1}(0)$

FVT

$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

special case

eg: $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$.

s^0 value will always be the const. value in the char. eq.

s^6	1	5	2	-8
s^5	1	3	-4	0
s^4	2	6	-8	0
s^3	$\cancel{0}$	0	0	0
s^3	8	12	0	0
s^2	3	-8	0	0
s	$\frac{12.3+8.8}{3}$	0	0	0
s^0	-8			

if u proceed further get 0's.

$A(s) = 2s^4 + 6s^2 - 8s^2$

$\frac{d}{ds}A(s) = 8s^3 + 12s = 0$.

Partial frac.

$F(s) = \frac{s^2+2s+3}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$

$b_3 = \left[\frac{s^2+2s+3}{(s+1)^3}(s+1)^3\right]_{s=-1}$

$b_2 = \left[\frac{d}{ds}\left[\frac{s^2+2s+3}{(s+1)^3}(s+1)^3\right]\right]_{s=-1}$

$b_1 = \frac{1}{2!}\left[\frac{d^2}{ds^2}\left[\frac{s^2+2s+3}{(s+1)^3}(s+1)^3\right]\right]_{s=-1}$

$F(s) = \frac{2}{(s+1)^3} \Rightarrow f(t) = \frac{2t^2 e^{-t}}{2}$

Root locus

eg: Plot the root locus for a unity feedback closed loop system whose open loop transfer function is

$G(s) = \frac{K}{s(s+4)(s^2+2s+2)}$

→ order 4

step 1: find poles & zeros.

s^2+2s+2

$\frac{-2 \pm \sqrt{4-8}}{2}$

$-1 \pm \sqrt{-1} = -1 \pm 1i$

∴ poles $= 0, -4, -1+i, -1-i$

zeroes = none.

take real part for summation

no. of branches $= p - z = 4 - 0 = 4$

step 2: Centroid

$\sigma_m = \frac{\Sigma \text{poles} - \Sigma \text{zeroes}}{p-z}$

$= \frac{-4-1-1-0}{4} = \frac{-6}{4} = -1.5$

step 3: angle of asymptotes

$\frac{(2q+1)\,180°}{p-z} =$

$q = 0, 1, 2, 3$ $(q - z - 1)$

$q=0$, $\frac{180}{4} = 60°$

$q=1$, $\frac{3.180}{4}45 = 135°$

$q=2$, $\frac{5.180}{4}45 = 225°$

$q=3$, $\frac{7.180}{4}45 = 315°$

step 4: mark these ~~po~~ angles

step 5: breakaway points.

step 5: Breakaway points.

Char. eq => $1 + G(s)H(s) = 0.$

$\therefore \; 1 + \dfrac{K}{s(s+4)(s^2+2s+2)} = 0.$

or $\dfrac{(s^2+4s)(s^2+2s+2)}{s(s+4)(s^2+2s+2)} = 0.$

or $s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s + K = 0.$ — ①.

Now, $K = -[s^4 + 2s^3 + 2s^2 + 4s^3 + 8s^2 + 8s]$

$K = -[s^4 + 6s^3 + 10s^2 + 8s]$

$\dfrac{dK}{ds} = -[4s^3 + 18s^2 + 20s + 8]$

set $\dfrac{dK}{ds} = 0$

$4s^3 + 18s^2 + 20s + 8 = 0.$

choose the root that lies w/in root locus.

approximate. breakaway pt. = $\underline{\underline{-3.09}}$,

tr → 0 – 100% underdamped
→ 10 to 90% over damped
→ 5% to 95% critically damped.

$y = 0$ → undamped system
$0 < y < 1$ → under damped
$y = 1$ → critically damped
$y > 1$ → over damped.

$$T(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2y\omega_n s + \omega_n^2}$$

ω_n = undamped natural frequency
y = damping ratio = ξ = zeta
= $\dfrac{\text{actual damping}}{\text{critical damping}}$

step 6

To find out imaginary axis, we have to form Routh's array.
Char. eq from eq ①

$s^4 + 6s^3 + 10s^2 + 8s + K = 0.$

Routh array

s^4	1	10	K
s^3	6	8	0
s^2	26/3	K	0
s^1	$\dfrac{(\frac{208}{3} - 6K)}{(26/3)}$	0	0
s	K		

=> $\dfrac{208}{3} - 6K = 0$

$\boxed{K = \dfrac{208}{18}}$

Auxillary eq.

$\dfrac{26}{3}s^2 + K = 0.$

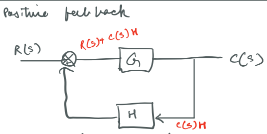
$s^2 = \dfrac{-3K}{26} = \dfrac{-4}{3}$

$s = \pm j\dfrac{2}{\sqrt{3}} = \pm 1.15i$

crossing the imaginary axis.

* Angle of dept. only necessary when imaginary poles.

Positive feedback

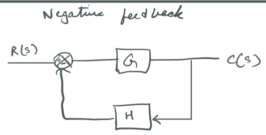


– adding feedback

$C(s) = [R(s) + C(s)H]\,G$

=> $T = \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 - G(s)H}$

Negative feedback



– subtracting feed back

Transf. $= \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1 + G(s)H}$