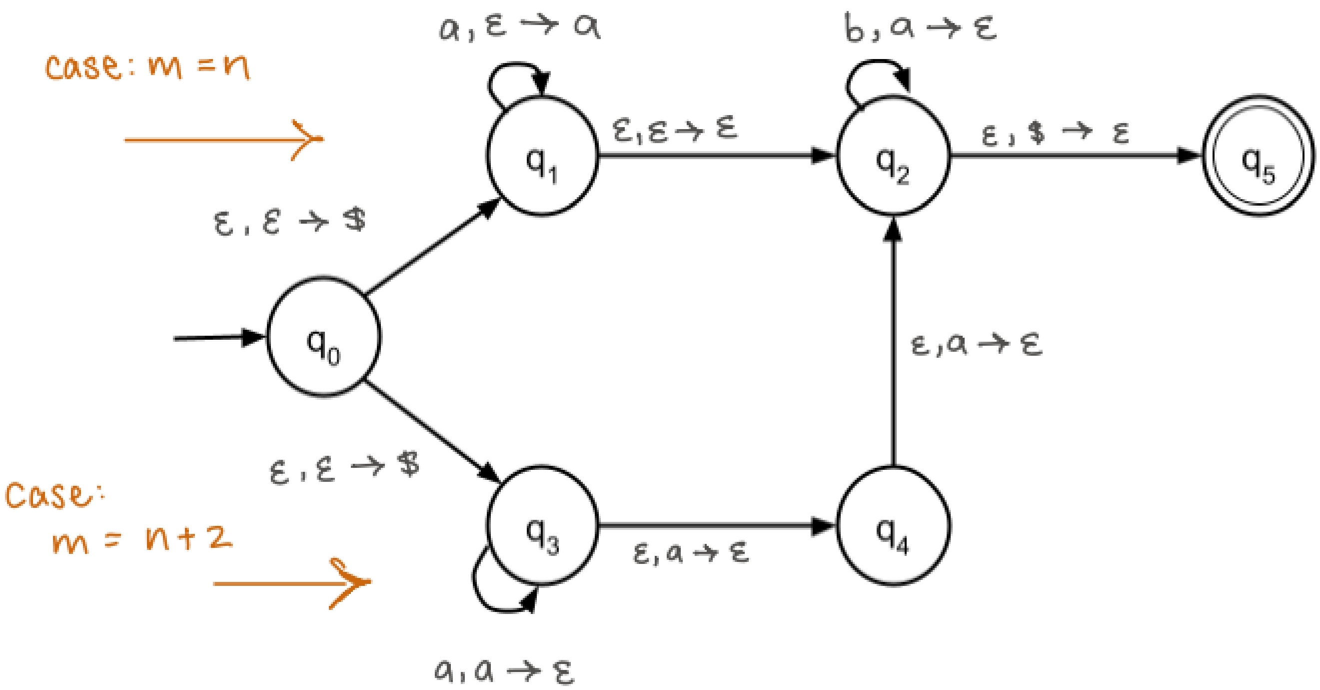
CSC320 - Tutorial 6

1. Complete the state diagram by adding transitions so that the constructed PDA recognizes the language L (read , pop → push)

   L = $\{a^m b^n \mid m, n \geq 0$ **and** (**either** $m = n$ **or** $m = n + 2$)

   branches



Chomsky Normal Form

A context-free grammar G = (V, Σ, R S) is in chomsky normal form (CNF) if every rule is in the form:

$A \rightarrow BC$ where A, B, $C \in V$ (B and C cannot be he start variable)

or

$A \rightarrow a$ where $A \in V$ and $a \in \Sigma$

and

$S \rightarrow \varepsilon$ is only permitted where S is the start variable

CNF Steps

1. Add a new start variable
2. Eliminate all ε-rules ($A \rightarrow \varepsilon$)
3. Eliminate all unit rules ($A \rightarrow B$)
4. Convert remaining rules to be in the form $A \rightarrow BC$ or $A \rightarrow a$

Push-down Automaton

A push-down automaton is defined as a 6-tuple (Q, Σ, Γ, δ, q_0, F)

Q: finite set of states

Σ: finite set of input alphabet

Γ: finite stack alphabet ($\Sigma \subseteq \Gamma$)

δ: $Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \rightarrow P(Q \times \Sigma_\varepsilon)$

$q_0 \in Q$: start state

$F \subseteq Q$: set of accept states

2. Convert the following CFG into CNF

$S \to AAA \mid \varepsilon$

$A \to aa \mid Aa \mid \varepsilon$

1. add new start state

$S \to AAA \mid \varepsilon$
$A \to aa \mid Aa \mid \varepsilon$

$\Rightarrow$

$S_0 \to S$
$S \to AAA \mid \varepsilon$
$A \to aa \mid Aa \mid \varepsilon$

2. remove rule $A \to \varepsilon$

$S_0 \to S$
$S \to AAA \mid \varepsilon$
$A \to aa \mid Aa \mid \cancel{\varepsilon}$

$\Rightarrow$

$S_0 \to S$
$S \to AAA \mid AA \mid A \mid \varepsilon$
$A \to aa \mid Aa \mid a$

3. remove rule $S \to \varepsilon$

$S_0 \to S$
$S \to AAA \mid AA \mid A \mid \cancel{\varepsilon}$
$A \to aa \mid Aa \mid a$

$\Rightarrow$

this is ok because S_0 is the start variable

$S_0 \to S \mid \varepsilon$
$S \to AAA \mid AA \mid A$
$A \to aa \mid Aa \mid a$

4. remove unit rules

$S_0 \to S \mid \varepsilon$
$S \to AAA \mid AA \mid A$
$A \to aa \mid Aa \mid a$

$\Rightarrow$

$S_0 \to \overbrace{AAA \mid AA \mid aa \mid Aa \mid a}^{S} \mid \varepsilon$
$S \to AAA \mid AA \mid \underbrace{aa \mid Aa \mid a}_{A}$
$A \to aa \mid Aa \mid a$

Best to work bottom-up!

5. fix-up remaining rules

$S_0 \to AAA \mid AA \mid aa \mid Aa \mid a \mid \varepsilon$
$S \to AAA \mid AA \mid aa \mid Aa \mid a$
$A \to aa \mid Aa \mid a$

$\Rightarrow$

$S_0 \to XA \mid AA \mid YY \mid AY \mid a \mid \varepsilon$
$S \to XA \mid AA \mid YY \mid AY \mid a$
$A \to YY \mid AY \mid a$
$X \to AA$
$Y \to a$

new rules

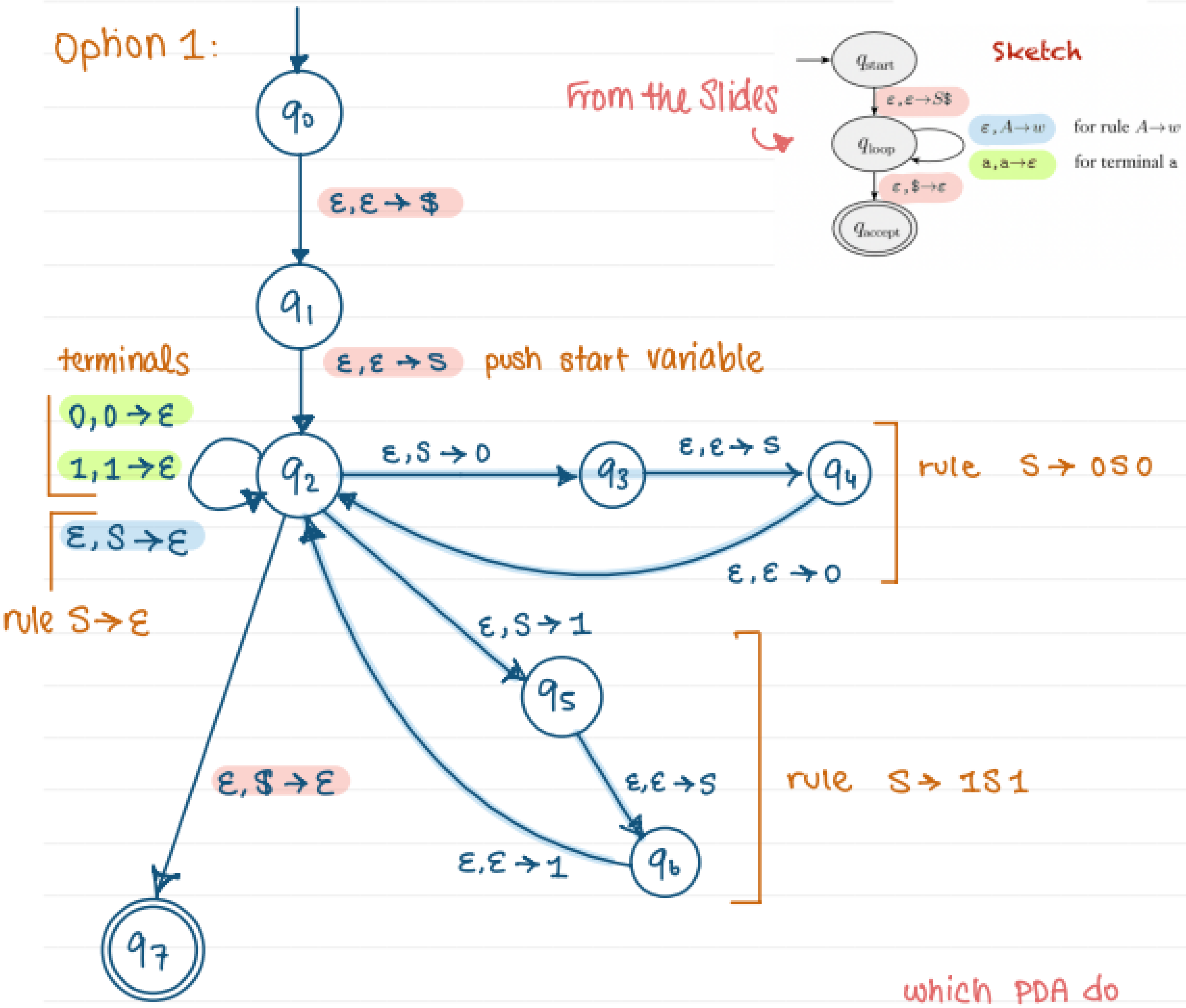
3. Construct a PDA that recognizes the same language as the following context-free grammars

a. $S \rightarrow 0S0 \mid 1S1 \mid \varepsilon$

Language from tutorial 5:

e. $L_5 = \{w^r w\}$

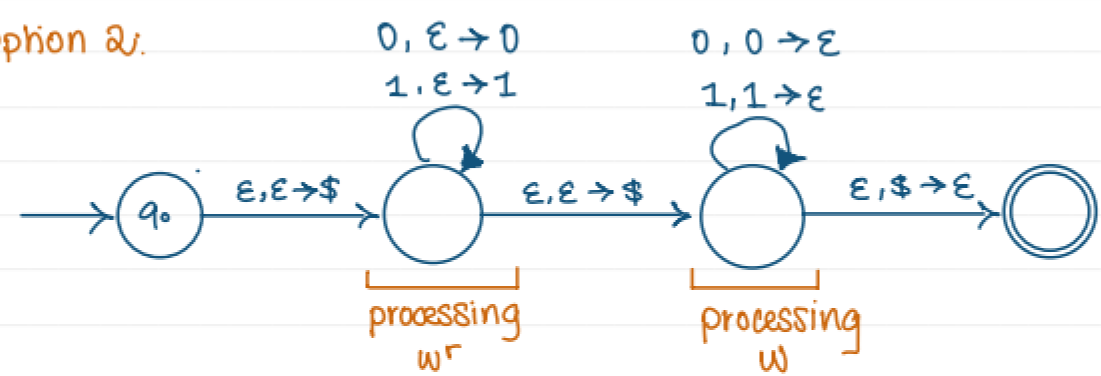
Option 1:



From the Slides

which PDA do prefer ?

Option 2.

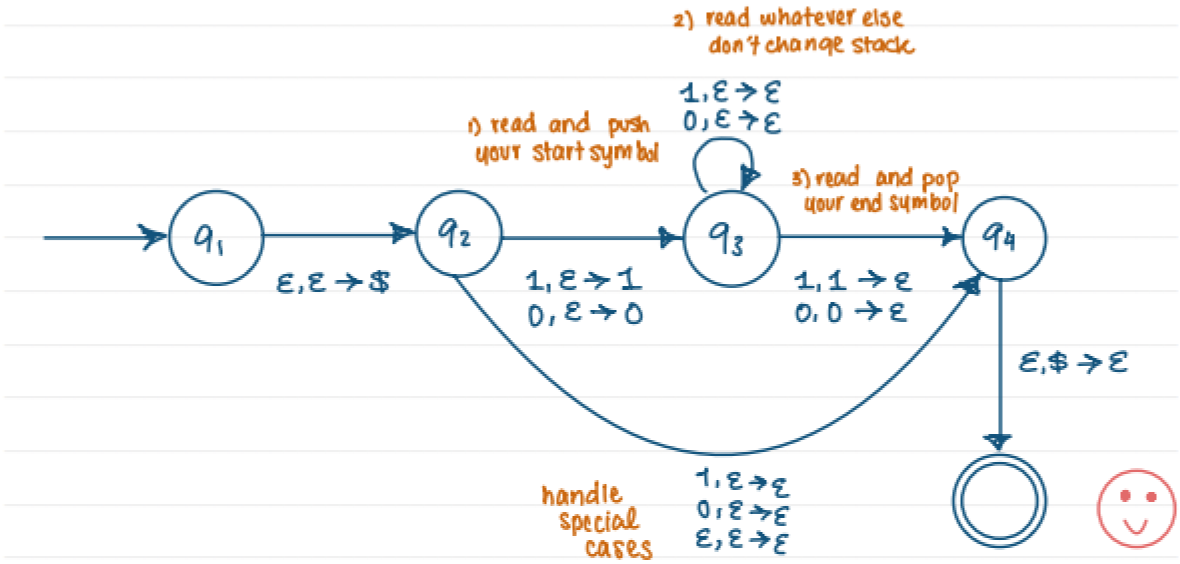


b. $S \rightarrow 0A0 \mid 1A1 \mid 1 \mid 0 \mid \varepsilon$

$A \rightarrow 1A \mid 0A \mid \varepsilon$

also from tutorial 5

a. $L_1 = \{w \mid w$ starts and ends with the same symbol $\}$



notice this Grammar has more rules ∴ following the standard conversion can get crowded.