University of Victoria
Exam 5
Fall 2024

Course Name:	**ECE260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 10960), A02 (CRN 10961)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: ________________________________

Given Name(s): ________________________________

Student Number: ________________

This examination paper has **10 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 25

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

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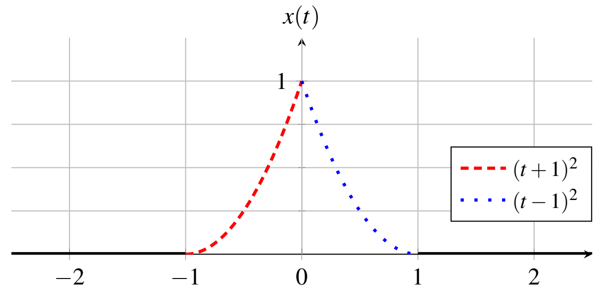
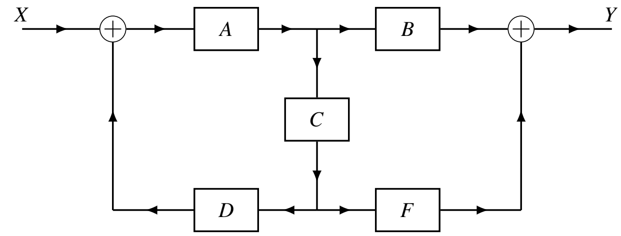
Do not write on this page unless instructed to do so.

Question 1.

Using properties of the Laplace transform and a Laplace transform table, find the Laplace transform X of the function x, where x is as shown in the figure. Except over the interval $[-1,1]$, x is zero. Do not forget to state the **region of convergence** of X in your final answer. Your final expression for X must be **fully simplified** (e.g., it **cannot contain any derivative operators**). [**Hint**: A well-chosen solution approach will avoid the need for derivatives altogether.] **[6 marks]**





Question 2.
Consider the LTI system $\mathscr{H}$ with input Laplace transform X, output Laplace transform Y, and system function H, as shown in the figure. Each subsystem in the block diagram is causal and LTI and labelled with its system function.

(A) Find a **fully-simplified** formula for H in terms of only A, B, C, D, and F. Your solution **must be clearly presented**, otherwise marks may be deducted even if the final answer is correct. If your solution introduces any new variables to refer to unlabelled signals in the block diagram, **you must label** the block diagram with those new variables (so that the marker can know to which signals these new variables refer). Failure to do this, may result in **zero marks** being given (even if the final answer is correct). **You are also encouraged to add other annotations** (e.g., formulas) to the block diagram in order to allow a clear presentation of your solution with minimal writing. **[3 marks]**

(B) Consider an arbitrary LTI system with system function G. Clearly state the condition on G that must be satisfied for the system to be BIBO stable. **[1 mark]**

QUESTION 2 CONTINUED

(C) Suppose that $A(s) = 1$, $B(s) = 0$, $C(s) = 1/s$, $D(s) = 2\alpha + 1$, and $F(s) = 1$, where α is a real constant. Determine for what values of α the system $\mathscr{H}$ is BIBO stable. **Show all of your work** and **do not skip any steps**. Your solution must be **clearly presented**, otherwise marks may be deducted even if the final answer is correct. **[3 marks]**

Question 3.

(A) A causal LTI system $\mathscr{H}$ is characterized by the differential equation $\mathscr{D}^2 y(t) = \mathscr{D}^2 x(t) + 3\mathscr{D}x(t) + 2x(t)$. Find the system function H of the system $\mathscr{H}$ (including the ROC). **[4 marks]**

(B) Determine whether $\mathscr{H}$ has an inverse that is BIBO stable. If such an inverse exists, find the system function G of this inverse system (including the ROC). Otherwise, explain why such an inverse cannot exist. **[2 marks]**

Question 4. Find the inverse Laplace transform x of the function $X(s) = \frac{s-8}{s^2-s-6}$ for $\mathrm{Re}(s) < -2$. **Show all of your work** and **do not skip any steps**. **[6 marks]**

END

Useful Formulae and Other Information

$$\mathscr{L}x(s) = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \qquad \mathscr{L}^{-1}X(t) = x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}ds \qquad \mathscr{L}_{\mathrm{u}}x(s) = X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

$$A_k = (v-p_k)F(v)\big|_{v=p_k} \qquad A_{kl} = \frac{1}{(q_k-l)!}\left[\left[\tfrac{d}{dv}\right]^{q_k-l}\left[(v-p_k)^{q_k}F(v)\right]\right]\Big|_{v=p_k}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Bilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain	ROC
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Shifting	$x(t-t_0)$	$e^{-st_0}X(s)$	R
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$	$R+\mathrm{Re}\{s_0\}$
Time/Laplace-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	aR
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathrm{Re}\{s\}>0\}$

Property	
Initial Value Theorem	$x(0^+) = \lim\limits_{s\to\infty} sX(s)$
Final Value Theorem	$\lim\limits_{t\to\infty} x(t) = \lim\limits_{s\to 0} sX(s)$

Bilateral Laplace Transform Pairs

Pair	$x(t)$	$X(s)$	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}>0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathrm{Re}\{s\}<0$
4	$t^nu(t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}>0$
5	$-t^nu(-t)$	$\frac{n!}{s^{n+1}}$	$\mathrm{Re}\{s\}<0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}>-a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\mathrm{Re}\{s\}<-a$
8	$t^ne^{-at}u(t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}>-a$
9	$-t^ne^{-at}u(-t)$	$\frac{n!}{(s+a)^{n+1}}$	$\mathrm{Re}\{s\}<-a$
10	$\cos(\omega_0t)u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
11	$\sin(\omega_0t)u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\mathrm{Re}\{s\}>0$
12	$e^{-at}\cos(\omega_0t)u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$
13	$e^{-at}\sin(\omega_0t)u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\mathrm{Re}\{s\}>-a$

Unilateral Laplace Transform Properties

Property	Time Domain	Laplace Domain
Linearity	$a_1x_1(t)+a_2x_2(t)$	$a_1X_1(s)+a_2X_2(s)$
Laplace-Domain Shifting	$e^{s_0t}x(t)$	$X(s-s_0)$
Time/Laplace-Domain Scaling	$x(at)$, $a>0$	$\frac{1}{a}X\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$X^*(s^*)$
Time-Domain Convolution	$x_1*x_2(t)$, x_1 and x_2 are causal	$X_1(s)X_2(s)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$sX(s)-x(0^-)$
Laplace-Domain Differentiation	$-tx(t)$	$\frac{d}{ds}X(s)$
Time-Domain Integration	$\int_{0^-}^{t}x(\tau)d\tau$	$\frac{1}{s}X(s)$

Property	
Initial Value Theorem	$x(0^+)=\lim\limits_{s\to\infty}sX(s)$
Final Value Theorem	$\lim\limits_{t\to\infty}x(t)=\lim\limits_{s\to 0}sX(s)$

Unilateral Laplace Transform Pairs

Pair	$x(t)$, $t\geq 0$	$X(s)$
1	$\delta(t)$	1
2	1	$\frac{1}{s}$
3	t^n	$\frac{n!}{s^{n+1}}$
4	e^{-at}	$\frac{1}{s+a}$
5	t^ne^{-at}	$\frac{n!}{(s+a)^{n+1}}$
6	$\cos(\omega_0t)$	$\frac{s}{s^2+\omega_0^2}$
7	$\sin(\omega_0t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
8	$e^{-at}\cos(\omega_0t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$
9	$e^{-at}\sin(\omega_0t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$