ECE 260

EXAM 5 SOLUTIONS

(SUMMER 2020)

QUESTION 1

$$X(s) = \frac{5s-1}{s^2-1} \quad \text{for } -1 < Re(s) < 1$$

$$X(s) = \frac{5s-1}{(s+1)(s-1)}$$

$$X(s) = \frac{A_1}{s+1} + \frac{A_2}{s-1}$$

$$A_1 = (s+1)\left(\frac{5s-1}{(s+1)(s-1)}\right)\Bigg|_{s=-1} = \frac{5s-1}{s-1}\Bigg|_{s=-1} = \frac{-6}{-2} = 3$$

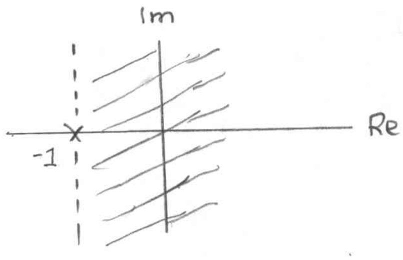
$$A_2 = (s-1)\left(\frac{5s-1}{(s+1)(s-1)}\right)\Bigg|_{s=1} = \frac{5s-1}{s+1}\Bigg|_{s=1} = \frac{4}{2} = 2$$

$$X(s) = \frac{3}{s+1} + \frac{2}{s-1}$$

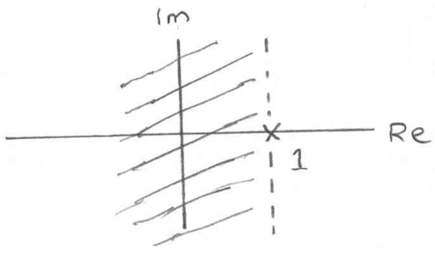
$$x(t) = 3L^{-1}\left\{\frac{1}{s+1}\right\} + 2L^{-1}\left\{\frac{1}{s-1}\right\}$$

$$= 3\left[e^{-t}u(t)\right] + 2\left[-e^{t}u(-t)\right]$$

$$= 3e^{-t}u(t) - 2e^{t}u(-t)$$



poles and ROC of $\frac{1}{s+1}$

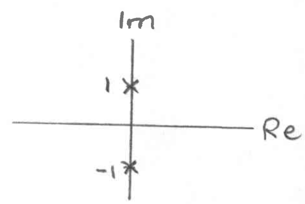


poles and ROC of $\frac{1}{s-1}$

QUESTION 2

$$H(s) = \frac{s^2+1}{s^2-1}$$

poles of G



(a) $$G(s) = \frac{1}{H(s)} = \frac{s^2-1}{s^2+1} = \frac{(s+1)(s-1)}{(s+j)(s-j)}$$

G(s) can have two possible ROCs:

① $Re(s) > 0$

② $Re(s) < 0$

Therefore, the system $\mathcal{H}$ has two inverses corresponding to:

① $G(s) = \dfrac{s^2-1}{s^2+1}$ for $Re(s) < 0$

② $G(s) = \dfrac{s^2-1}{s^2+1}$ for $Re(s) > 0$

(b) Since G is rational, the inverse system is causal if and only if the ROC of G is a RHP to the right of the rightmost pole.

Therefore, inverse system ① is not causal and inverse system ② is causal.

QUESTION 3

$$y''(t) - 5y'(t) + 6y(t) = x'(t) + 7x(t)$$

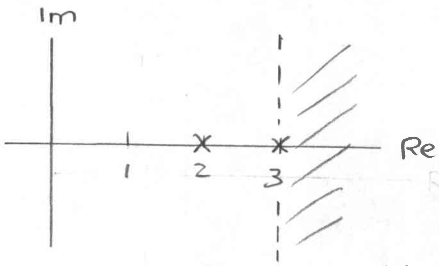
$$s^2 Y(s) - 5s\,Y(s) + 6Y(s) = s\,X(s) + 7X(s)$$

$$[s^2 - 5s + 6]\,Y(s) = [s+7]\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s+7}{s^2-5s+6} = \frac{s+7}{(s-3)(s-2)}$$

$$H(s) = \frac{s+7}{(s-2)(s-3)} \quad \text{for } Re(s) > 3$$

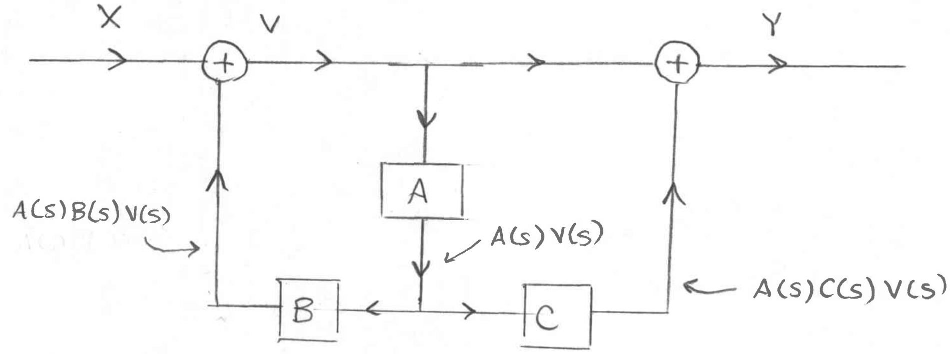
poles and ROC of H



(causal so RHP)

QUESTION 4

(a)



$$V(s) = X(s) + A(s)B(s)V(s) \Rightarrow X(s) = [1 - A(s)B(s)]\, V(s)$$

$$Y(s) = V(s) + A(s)C(s)V(s) \Rightarrow Y(s) = [1 + A(s)C(s)]\, V(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{[1 + A(s)C(s)]\, V(s)}{[1 - A(s)B(s)]\, V(s)} = \frac{1 + A(s)C(s)}{1 - A(s)B(s)}$$

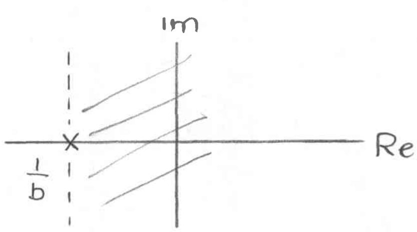
(b)

$$H(s) = \frac{1 + A(s)C(s)}{1 - A(s)B(s)} = \frac{1 + (1)(1)}{1 - (1)(bs)} = \frac{2}{1 - bs} = \frac{-2}{bs - 1}$$

$$= \frac{-2}{b(s - 1/b)}$$

[Note: $A(s) = 1$, $B(s) = bs$, and $C(s) = 1$]

poles and ROC of H



RHP ROC since causal

for BIBO stability, ROC of H contains imaginary axis

System is BIBO stable if and only if $\frac{1}{b} < 0$

or equivalently $b < 0$.

QUESTION 5

$$y''(t) + 5y'(t) + 6y(t) = \delta(t)$$

$$y(0^-) = 1 \, , \quad y'(0^-) = -1$$

$$s^2 Y(s) - sy(0^-) - y'(0^-) + 5[sY(s) - y(0^-)] + 6Y(s) = 1$$

$$s^2 Y(s) - sy(0^-) - y'(0^-) + 5sY(s) - 5y(0^-) + 6Y(s) = 1$$

$$[s^2 + 5s + 6]\,Y(s) = 1 + sy(0^-) + y'(0^-) + 5y(0^-)$$

$$Y(s) = \frac{1 + sy(0^-) + y'(0^-) + 5y(0^-)}{s^2 + 5s + 6}$$

$$= \frac{1 + s - 1 + 5}{s^2 + 5s + 6} = \frac{s+5}{(s+2)(s+3)}$$

$$Y(s) = \frac{A_1}{s+2} + \frac{A_2}{s+3}$$

$$A_1 = (s+2)\left(\frac{s+5}{(s+2)(s+3)}\right)\Bigg|_{s=-2} = \frac{s+5}{s+3}\Bigg|_{s=-2} = \frac{3}{1} = 3$$

$$A_2 = (s+3)\left(\frac{s+5}{(s+2)(s+3)}\right)\Bigg|_{s=-3} = \frac{s+5}{s+2}\Bigg|_{s=-3} = \frac{2}{-1} = -2$$

$$Y(s) = \frac{3}{s+2} - \frac{2}{s+3}$$

$$y(t) = 3\,\mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$$

$$= 3e^{-2t} - 2e^{-3t} \qquad t \geq 0$$