7.13 For the LTI system with input x and output y and each system function H given below, find the differential equation that characterizes the system.

(a) $H(s) = \dfrac{s+1}{s^2+2s+2}$.

$$Y(s) = H(s)X(s)$$

$$\Rightarrow Y(s) = \left[\frac{s+1}{s^2+2s+2}\right] X(s)$$

$$\Rightarrow Y(s)\left[s^2+2s+2\right] = X(s)(s+1)$$

$$\Rightarrow s^2Y(s) + 2sY(s) + 2Y(s) = X(s)s + X(s)$$

Taking the inverse Laplace transform...

$$L^{-1}\{s^2Y(s)\}(t) + L^{-1}\{2sY(s)\}(t) + L^{-1}\{2Y(s)\}(t)$$
$$= L^{-1}\{X(s)s\}(t) + L^{-1}\{X(s)\}(t)$$

$$\Rightarrow D^2y(t) + 2Dy(t) + 2y(t) = Dx(t) + x(t)$$

7.14 For the causal LTI system with input x and output y that is characterized by each differential equation given below, find the system function H of the system.

(a) $\mathcal{D}^2y(t) + 4\mathcal{D}y(t) + 3y(t) = 2\mathcal{D}x(t) + x(t)$.

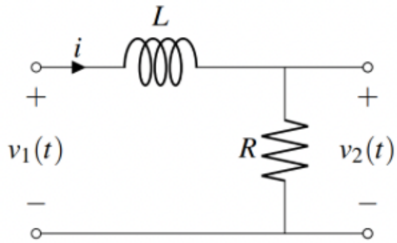
$$L\{D^2y(t)\}(s) + 4L\{Dy\}(s) + 3L\{y(t)\}(s) = 2L\{Dx\}(s) + L\{x\}(s)$$

$$\Rightarrow s^2Y(s) + 4sY(s) + 3Y(s) = 2sX(s) + X(s)$$

$$\Rightarrow \left[s^2+4s+3\right]Y(s) = \left[2s+1\right]X(s)$$

$$\Rightarrow Y(s)/X(s) = \frac{2s+1}{s^2+4s+3}$$

7.17 Consider the LTI resistor-inductor (RL) network with input v_1 and output v_2 shown in the figure below.



(a) Find the system function H of the system.
(b) Determine whether the system is BIBO stable.
(c) Determine the type of ideal frequency-selective filter that the system best approximates.
(d) Find the step response g of the system.

(a) $v_1(t) = L\,Di(t) + v_2(t)$ & $i(t) = \frac{1}{R} v_2(t)$

combining... $v_1(t) = LD\{\frac{1}{R} v_2\}(t) + v_2(t)$

$$V_1(s) = \mathcal{L}v_1(s) = \mathcal{L}\left\{\frac{L}{R} Dv_2(t) + v_2(t)\right\}(s)$$

$$= \frac{L}{R}\mathcal{L}\{Dv_2\}(s) + \mathcal{L}v_2(s)$$

$$= \frac{L}{R} sV_2(s) + V_2(s)$$

rearranging, we have

$$V_1(s) = \left[\frac{L}{R}s + 1\right] V_2(s) \Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{1}{\frac{L}{R}s+1} = \frac{R/L}{s+R/L}$$

$$H(s) = \frac{R/L}{s+R/L} \quad \text{for } \operatorname{Re}(s) > -R/L$$

(b) The rational function H has a single pole at $-R/L$. Since L and R are strictly positive quantities, we have that $-R/L < 0$. In the words, all of the poles of H are in the left-half plane. Since the system is causal, this implies that the system is stable.

(c) $$V_2(s) = H(s)V_1(s) = \frac{R/L}{s+R/L}\cdot\frac{1}{s} = \frac{R/L}{s(s+R/L)}$$

$$V_2(s) = \frac{A_1}{s+R/L} + \frac{A_2}{s} \qquad A_1 = \left(s+\frac{R}{L}\right)V_2(s)\Big|_{s=-R/L} = \frac{R/L}{s}\Big|_{s=-R/L} = -1$$

$$A_2 = sV_2(s)\Big|_{s=0} = \frac{R/L}{s+R/L}\Big|_{s=0} = 1$$

Taking the inverse Laplace transform of V_2 yield:

$$V_2(t) = \mathcal{L}^{-1}\left\{\frac{-1}{s+R/L} + \frac{1}{s}\right\}(t) = -\mathcal{L}^{-1}\left\{\frac{1}{s+R/L}\right\}(t) + \mathcal{L}^{-1}\left\{\frac{1}{s}\right\}(t)$$

$$= -\left[e^{-(R/L)t}u(t)\right]u(t) = \left[1-e^{-(R/L)t}\right]u(t)$$

$$g(t) = \left[1-e^{-(R/L)t}\right]u(t)$$

7.18 Consider a LTI system with the system function

$$H(s) = \frac{s^2+7s+12}{s^2+3s+12}.$$

Find all possible inverses of this system. For each inverse, identify its system function and the corresponding ROC. Also, indicate whether the inverse is causal and/or stable. (Note: You do not need to find the impulse responses of these inverse systems.)

$$H_{inv}(s) = \frac{1}{H(s)} = \frac{s^2+3(s)+12}{s^2+7s+12}$$

Factoring the denominator of $H_{inv}(s)$, we have

$$H_{inv}(s) = \frac{s^2+3s+12}{(s+4)(s+3)}$$

Obtaining the system function H_{inv} is rational and has poles at -4 and -3.

3 ROCs for $H_{inv}(s)$
(i) $Re(s) < -4$
(ii) $-4 < Re(s) < -3$
(iii) $Re(s) > -3$

Only the inverse system associated with this ROC is stable.

Since H_{inv} is rational and it's rational pole is -3, only the ROC of $Re(s) > -3$ is associated w/ a causal system.

7.20 In wireless communication channels, the transmitted signal is propagated simultaneously along multiple paths of varying lengths. Consequently, the signal received from the channel is the sum of numerous delayed and amplified/attenuated versions of the original transmitted signal. In this way, the channel distorts the transmitted signal. This is commonly referred to as the multipath problem. In what follows, we examine a simple instance of this problem.

Consider a LTI communication channel with input x and output y. Suppose that the transmitted signal x propagates along two paths. Along the intended direct path, the channel has a delay of T (where $T > 0$) and gain of one. Along a second (unintended indirect) path, the signal experiences a delay of $T + \tau$ and gain of a (where $\tau > 0$). Thus, the received signal y is given by $y(t) = x(t-T) + ax(t-T-\tau)$. Find the system function H of a LTI system that can be connected in series with the output of the communication channel in order to recover the (delayed) signal $x(t-T)$ without any distortion. Determine whether this system is physically realizable.

$$y(t) = x(t-T) + ax(t-T-\tau)$$

$$Y(s) = e^{-sT}X(s) + ae^{-(T+\tau)s}X(s) = \left(e^{-sT} + ae^{-(T+\tau)s}\right)X(s)$$

$$G(s) = e^{-sT} + ae^{-(T+\tau)s} = e^{-sT}(1+ae^{-s\tau})$$

We want $G(s)H(s) = e^{-sT}$

$$H(s) = \frac{e^{-sT}}{G(s)} = \frac{1}{1+ae^{-s\tau}}$$

This system (with transfer function $F(s) = ae^{-s\tau}$) simply amplifies the input signal by a and delay the signal by τ (where $\tau > 0$) and such a system is physically realizable (since we can build systems that delay and amplify signals.)

7.21 For each differential equation given below that characterizes a causal (incrementally-linear TI) system with input x and output y, solve for y subject to the given initial conditions.
(a) $\mathcal{D}^2 y(t) + 7\mathcal{D}y(t) + 12y(t) = x(t)$, where $y(0^-) = -1$, $\mathcal{D}y(0^-) = 0$, and $x(t) = u(t)$.

$$L_u\{D^2 y\}(s) + 7L_u\{Dy\}(s) + 12L_u\{y\}(s) = L_u\{x\}(s)$$

$$\Rightarrow s^2 Y(s) - sy(0^-) - y'(0^-) + 7[sY(s) - y(0^-)] + 12Y(s) = X(s)$$

$$\Rightarrow [s^2+7s+12]Y(s) = sy(0^-)+y'(0^-)+7y(0^-)+X(s)$$

$$\Rightarrow Y(s) = \frac{X(s)+sy(0^-)+y'(0^-)+7y(0^-)}{s^2+7s+12}$$

$x(t) = u(t) \ldots\ X(s) = L_u\{u\}(s) = 1/s$

$$Y(s) = \frac{1/s - s - 7}{s^2+7s+12} = \frac{-s^2-7s+1}{s(s^2+7s+12)} = \frac{-s^2-7s+1}{s(s+3)(s+4)}$$

$$Y(s) = A_1/s + A_2/s+3 + A_3/s+4$$

$$A_1 = sY(s)\Big|_{s=0} = \frac{-s^2-7s+1}{(s+3)(s+4)}\Big|_{s=0} = \frac{1}{12}$$

$$A_2 = (s+3)Y(s)\Big|_{s=-3} = \frac{-s^2-7s+1}{s(s+4)}\Big|_{s=-3} = -13/3$$

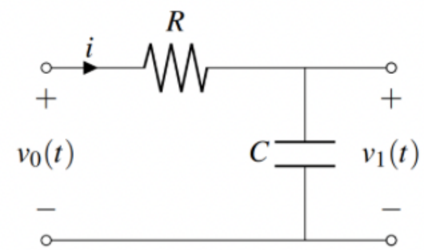
$$A_3 = (s+4)Y(s)\Big|_{s=-4} = \frac{-s^2-7s+1}{s(s+3)}\Big|_{s=-4} = 13/4$$

$$Y(s) = \frac{1}{12}\left(\frac{1}{s}\right) - \frac{13}{3}\left(\frac{1}{s+3}\right) + \frac{13}{4}\left(\frac{1}{s+4}\right)$$

$$y(t) = \frac{1}{12} L_u^{-1}\left\{\frac{1}{s}\right\}(t) - \frac{13}{3} L_u^{-1}\left\{\frac{1}{s+3}\right\}(t) + \frac{13}{4} L_u^{-1}\left\{\frac{1}{s+4}\right\}(t)$$

$$= \frac{1}{12} - \frac{13}{3}e^{-3t} + \frac{13}{4}e^{-4t} \quad \ldots \text{ for } t > 0^-$$

7.22 Consider the resistor-capacitor (RC) network shown in the figure below, where $R = 1000$ and $C = \frac{1}{1000}$.



(a) Find the differential equation that characterizes the relationship between the input v_0 and output v_1.
(b) If $v_1(0^-) = 2$, and $v_0(t) = 2e^{-3t}$, find v_1.

(a)

$$V_1(t) = \frac{1}{C}\int_{-\infty}^{t} \frac{1}{R}\left[V_0(\tau) - V_1(\tau)\right] d\tau$$

$$\Rightarrow V_1(t) - \frac{1}{C}\int_{-\infty}^{t} \frac{1}{R}\left[V_0(\tau) - V_1(\tau)\right] d\tau = 0$$

doievcthe...

$$Dv_1(t) - \frac{1}{C}\left(\frac{1}{R}\left[V_0(t) - V_1(t)\right]\right) = 0$$

$$\Rightarrow Dv_1(t) - \frac{1}{RC}V_0(t) + \frac{1}{RC}v_1(t) = 0$$

(b)

$$L\{Dv_1\}(s) - \frac{1}{RC}L\,v_0(s) + \frac{1}{RC}L\,v_1(s) = 0$$

$$\Rightarrow sV_1(s) - V_1(0^-) - \frac{1}{RC}V_0(s) + \frac{1}{RC}V_1(s) = 0$$

$$\Rightarrow \left[s + 1/RC\right] V_1(s) = V_1(0^-) + \frac{1}{RC}V_0(s)$$

$$\Rightarrow V_1(s) = \frac{(1/RC)V_0(s) + V_1(0^-)}{s + 1/RC}$$

$$V_0(s) = L\{2e^{-3t}\}(s) = \frac{2}{s+3}$$

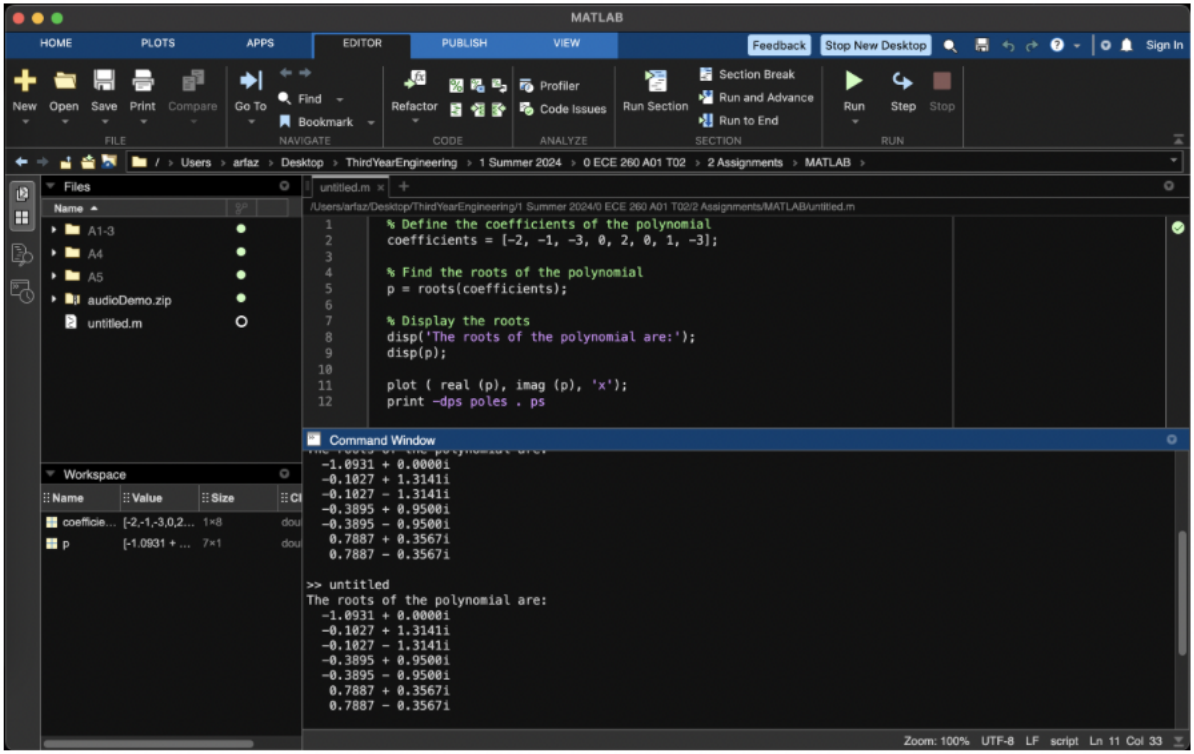
$$V_1(s) = \frac{(7/s+3) + 2}{s+1} = \frac{2s+8}{(s+1)(s+3)} = \frac{2(s+4)}{(s+1)(s+3)}$$

$$V_1(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3} \quad \ldots \quad A_1 = (s+1)V_1(s)\Big|_{s=-1} = \frac{2s+8}{s+3}\Big|_{s=-1} = 3$$

$$A_2 = (s+3)V_1(s)\Big|_{s=-3} = \frac{2s+8}{s+1}\Big|_{s=-3} = -1$$

$$\therefore V_1(s) = \frac{3}{s+1} - \frac{1}{s+3}$$

Laplace transform $= 3L^{-1}\{1/s+1\}(t) - L^{-1}\{1/s+3\}(t) = 3e^{-t} - e^{-3t}$


MATLAB
HOME PLOTS APPS EDITOR PUBLISH VIEW
Feedback Stop New Desktop Sign In
/Users/arfaz/Desktop/ThirdYearEngineering/1 Summer 2024/0 ECE 260 A01 T02/2 Assignments/MATLAB/untitled.m
% Define the coefficients of the polynomial
coefficients = [-2, -1, -3, 0, 2, 0, 1, -3];
% Find the roots of the polynomial
p = roots(coefficients);
% Display the roots
disp('The roots of the polynomial are:');
disp(p);
plot (real (p), imag (p), 'x');
print -dps poles . ps
Command Window
-1.0931 + 0.0000i
-0.1027 + 1.3141i
-0.1027 - 1.3141i
-0.3895 + 0.9500i
-0.3895 - 0.9500i
0.7887 + 0.3567i
0.7887 - 0.3567i
>> untitled
The roots of the polynomial are:
-1.0931 + 0.0000i
-0.1027 + 1.3141i
-0.1027 - 1.3141i
-0.3895 + 0.9500i
-0.3895 - 0.9500i
0.7887 + 0.3567i
0.7887 - 0.3567i

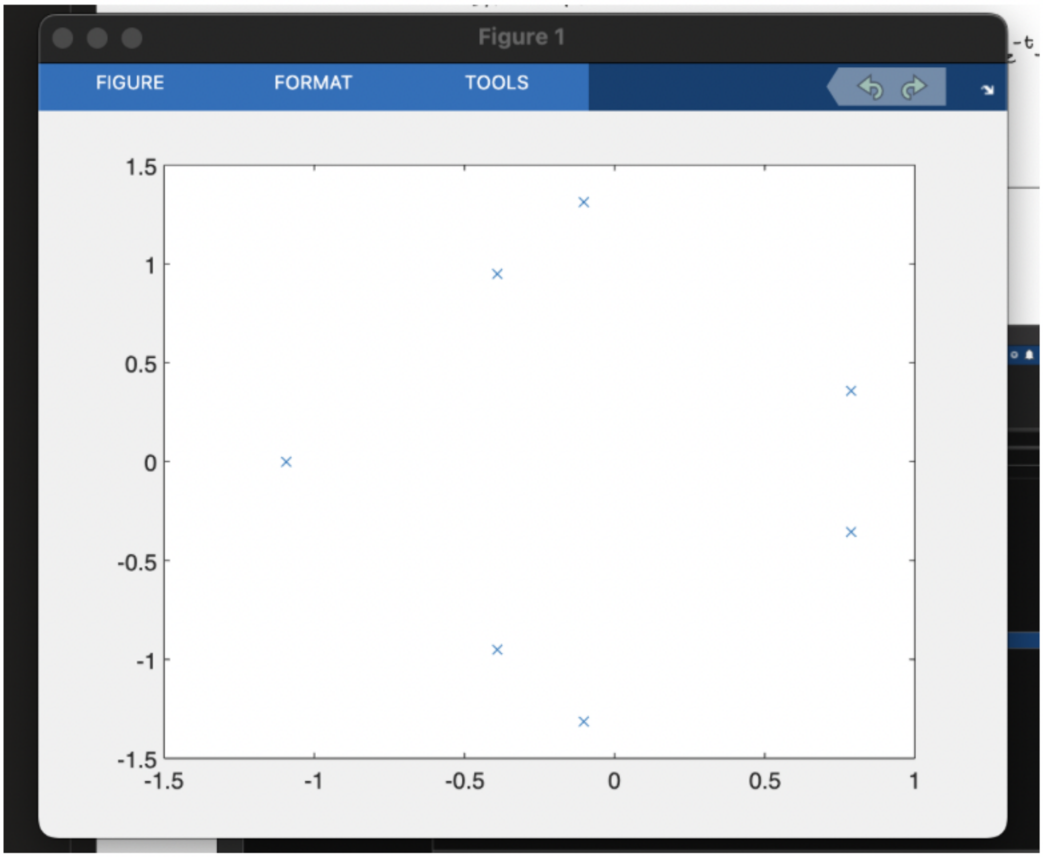


Figure 1
FIGURE FORMAT TOOLS
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5 -1 -0.5 0 0.5 1

untitled.m

/Users/arfaz/Desktop/ThirdYearEngineering/1 Summer 2024/0 ECE 260 A01 T02/2 Assignments/MATLAB/untitled.m

```matlab
% Define the numerator and denominator coefficients of the transfer function
tfnum = [0 0 0 0 1];
tfdenom = [1.0000 2.6131 3.4142 2.6131 1.0000];

% Define the final simulation time
finaltime = 20;

% Create the transfer function system
sys = tf(tfnum, tfdenom);

% Plot the step response
subplot(2, 1, 1); % Create a subplot with 2 rows, 1 column, at position 1
step(sys, finaltime); % Plot the step response
title('Step Response'); % Add a title to the subplot
xlabel('Time (seconds)'); % Label the x-axis
ylabel('Amplitude'); % Label the y-axis
grid on; % Turn on the grid for better readability

% Plot the impulse response
subplot(2, 1, 2); % Create a subplot at position 2
impulse(sys, finaltime); % Plot the impulse response
title('Impulse Response'); % Add a title to the subplot
xlabel('Time (seconds)'); % Label the x-axis
ylabel('Amplitude'); % Label the y-axis
grid on; % Turn on the grid for better readability
```