1. **[7 marks]** Consider the following language, where $\Sigma = \{a, b\}$:

$$L = \{w \in \Sigma^* \mid \text{each pair of consecutive } b\text{'s in } w \text{ is separated by a substring of } a\text{'s of length } 2i, i \geq 0\}$$

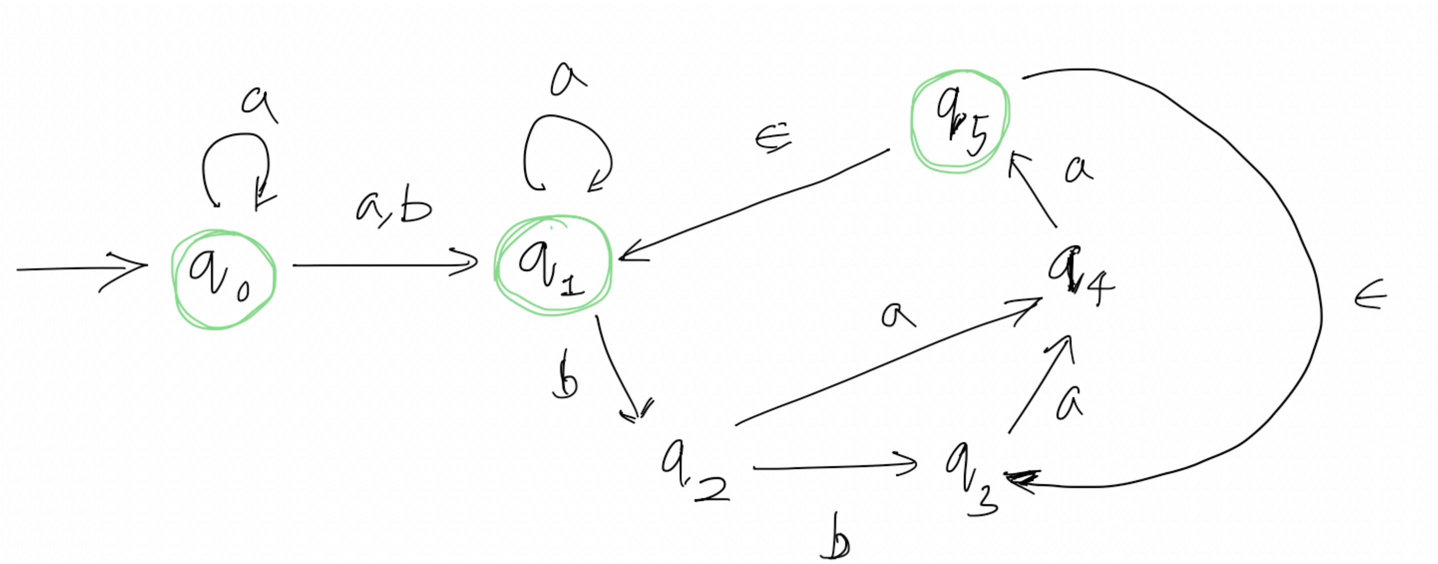
   (a) Give the state diagram and the formal 5-tuple definition of an NFA $N = (Q, \Sigma, \delta, q_0, F)$ which recognizes L, with a transition table describing δ.

   (b) Give a regular expression R where $L(R) = L$.

My Solution 1(a)

NFA N = (Q, Σ, δ, q0, F) is defined by ($\{Q_0, Q_1, Q_2, Q_3, Q_4, Q_5\}$, {a,b}, δ, Q_0, $\{Q_0, Q_1, Q_5\}$) where δ can be expressed in a transition table as

States	a	b	ε
Q_0	$\{Q_0, Q_1\}$	Q_1	∅
Q_1	Q_1	Q_2	∅
Q_2	Q_4	Q_3	∅
Q_3	Q_4	∅	∅
Q_4	Q_5	∅	∅
Q_5	∅	∅	$\{Q_1, Q_3\}$



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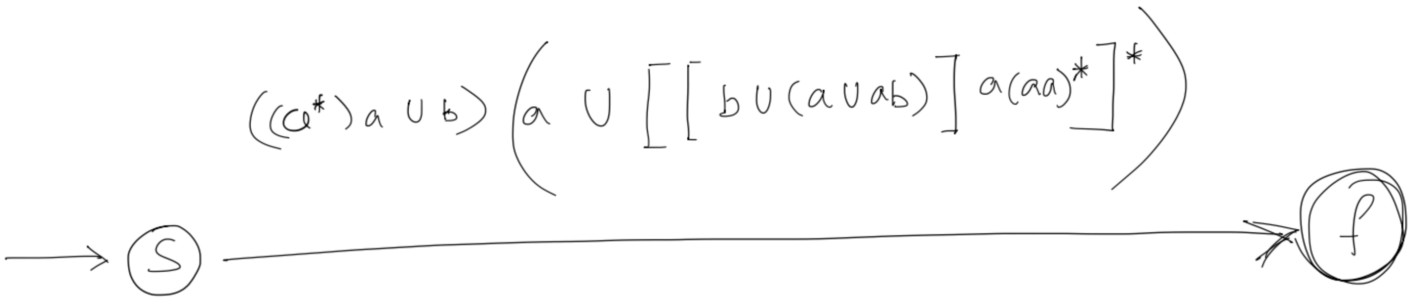
My Solution 1(b)



2. **[3 marks]** Consider the following language, where $\Sigma = \{a, b\}$:

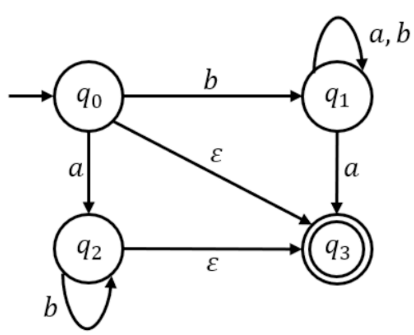
$$L = \{w \in \Sigma^* \mid w \text{ starts with an } a \text{ and contains the substring } abb\}$$

Give a regular expression R where $L(R) = L$.

My Solution 2: [(a ∪ b)* (abb) (a ∪ b)*]

Example(s): a(abb)a, aab(abb)ba, aaaabbbb(abb)bbbbaaaa

3. **[4 marks]** Consider the following NFA $N = (Q_N, \Sigma, \delta_N, q_N, F_N)$:



Following the construction presented in class, give the state diagram and formal 5-tuple definition for equivalent DFA $D = (Q_D, \Sigma, \delta_D, q_D, F_D)$ with $L(N) = L(D)$.

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My Solution 3:

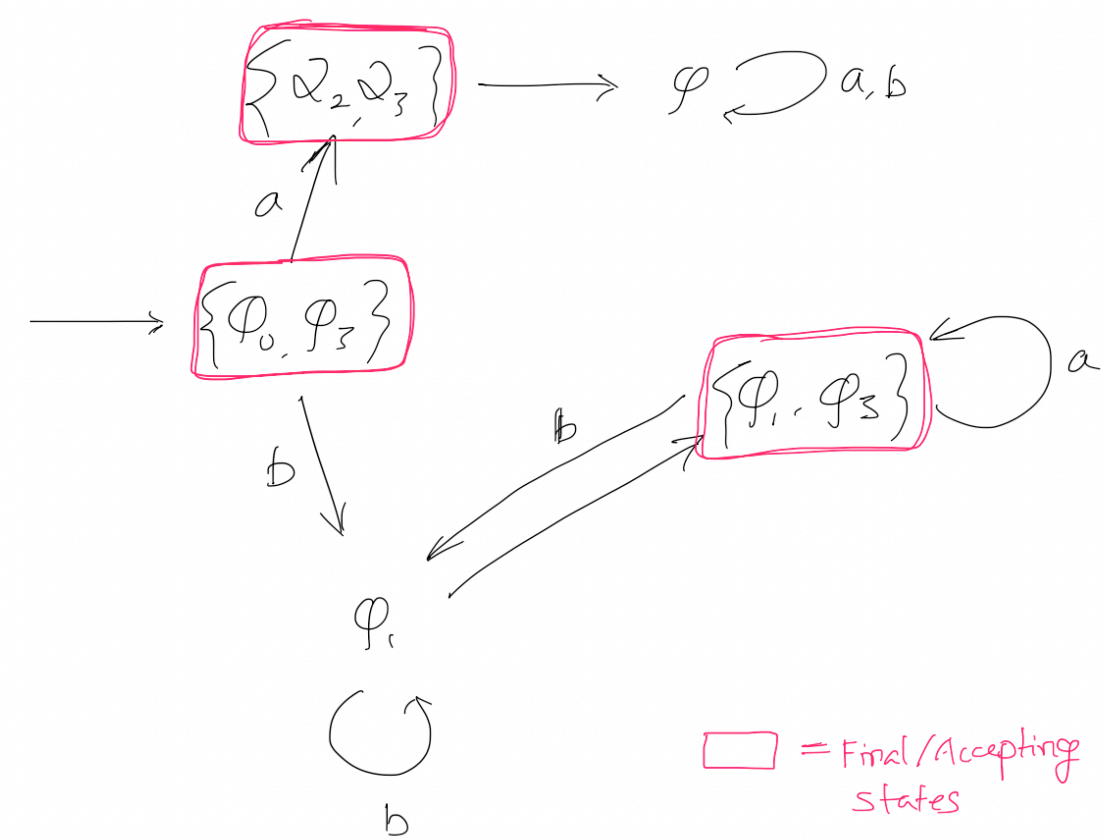
NFA N = (Q_N, Σ, δ_N, q_N, F_N) is defined by ({Q_0,Q_1,Q_2,Q_3}{a,b},δ, Q_0, {Q_3}) where δ can be expressed as in a transition table as

States	a	b	ε
Q_0	Q_2	Q_1	Q_3
Q_1	{Q_1,Q_3}	Q_1	∅
Q_2	∅	Q_2	Q_3
Q_3	∅	∅	∅

Now NFA **N** can be expressed as DFA **D** and can be defined by ({Q_1, {Q_0,Q_3},{Q_1,Q_3},{Q_2,Q_3},∅},{a,b},δ, Q_0,{Q_0,Q_3},{Q_1,Q_3},{Q_2,Q_3}) where δ can be expressed as in a transition table as:

States	a	b
Q_1	{Q_1,Q_3}	Q_1
{Q_0,Q_3}	{Q_2,Q_3}	{Q_1,Q_3}
{Q_1,Q_3}	{Q_1,Q_3}	Q_1
{Q_2,Q_3}	∅	{Q_2,Q_3}
∅	∅	∅

The state diagram along with the final states in red as:



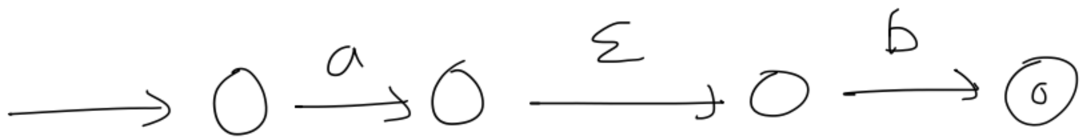
4. **[4 marks]** Let $\Sigma = \{a, b, c\}$. Consider the regular expression

$$R = (ab)^*(a \cup c)^*b$$

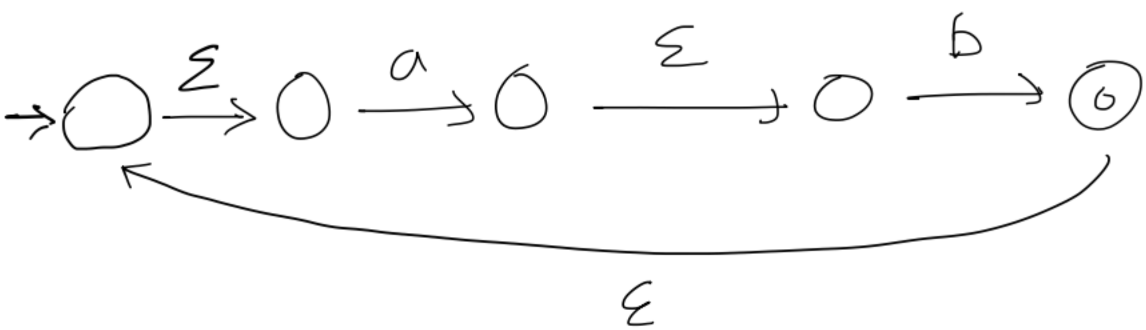
Give a state diagram for an NFA N with $L(N) = L(R)$, following the construction in the regular expression to NFA proof shown in class.

My Solution 4:

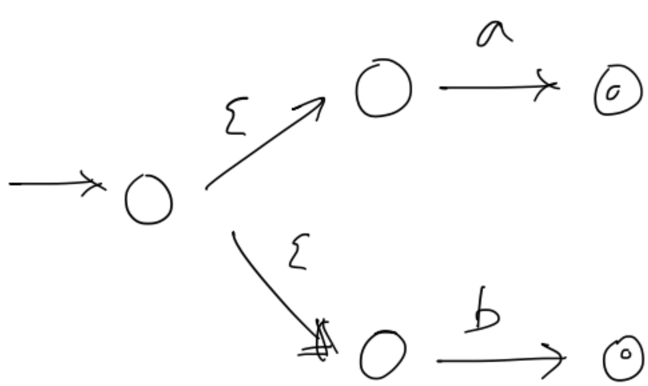
ab :



$(ab)^*$:



$(a \cup c)$:



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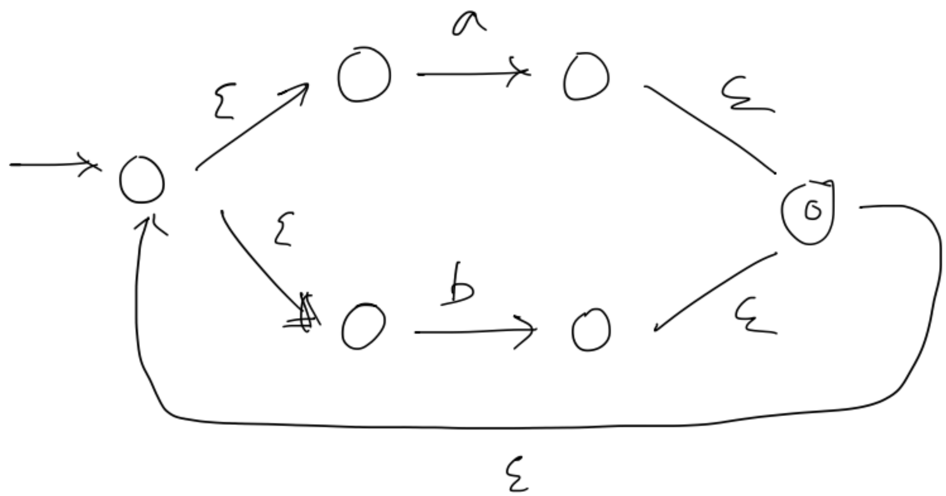
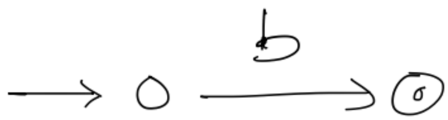
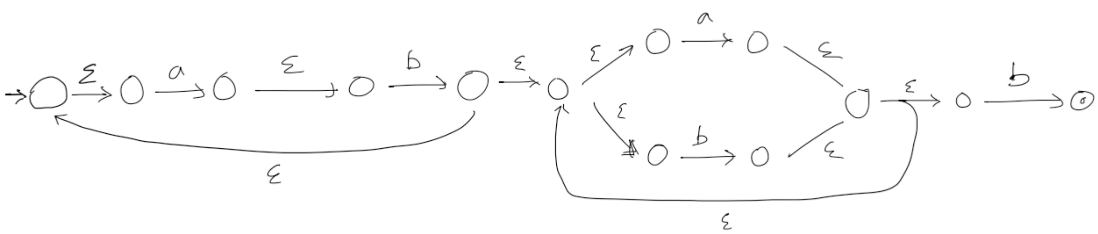
$(a \cup c)^* :$



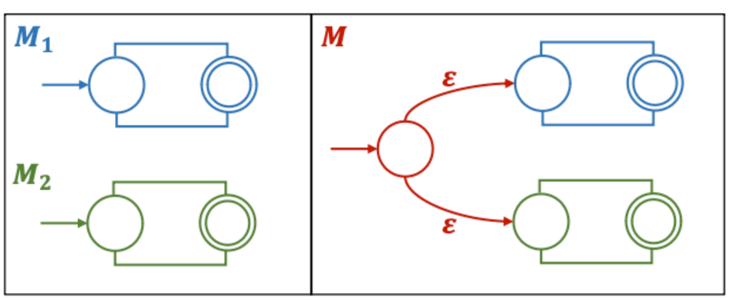
$b :$



$\therefore\ (ab)^* (a \cup c)^* b =$



I followed the state diagrams figures and replicated them in my solution.



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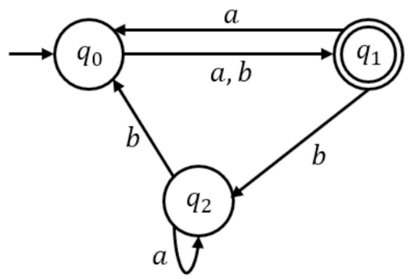
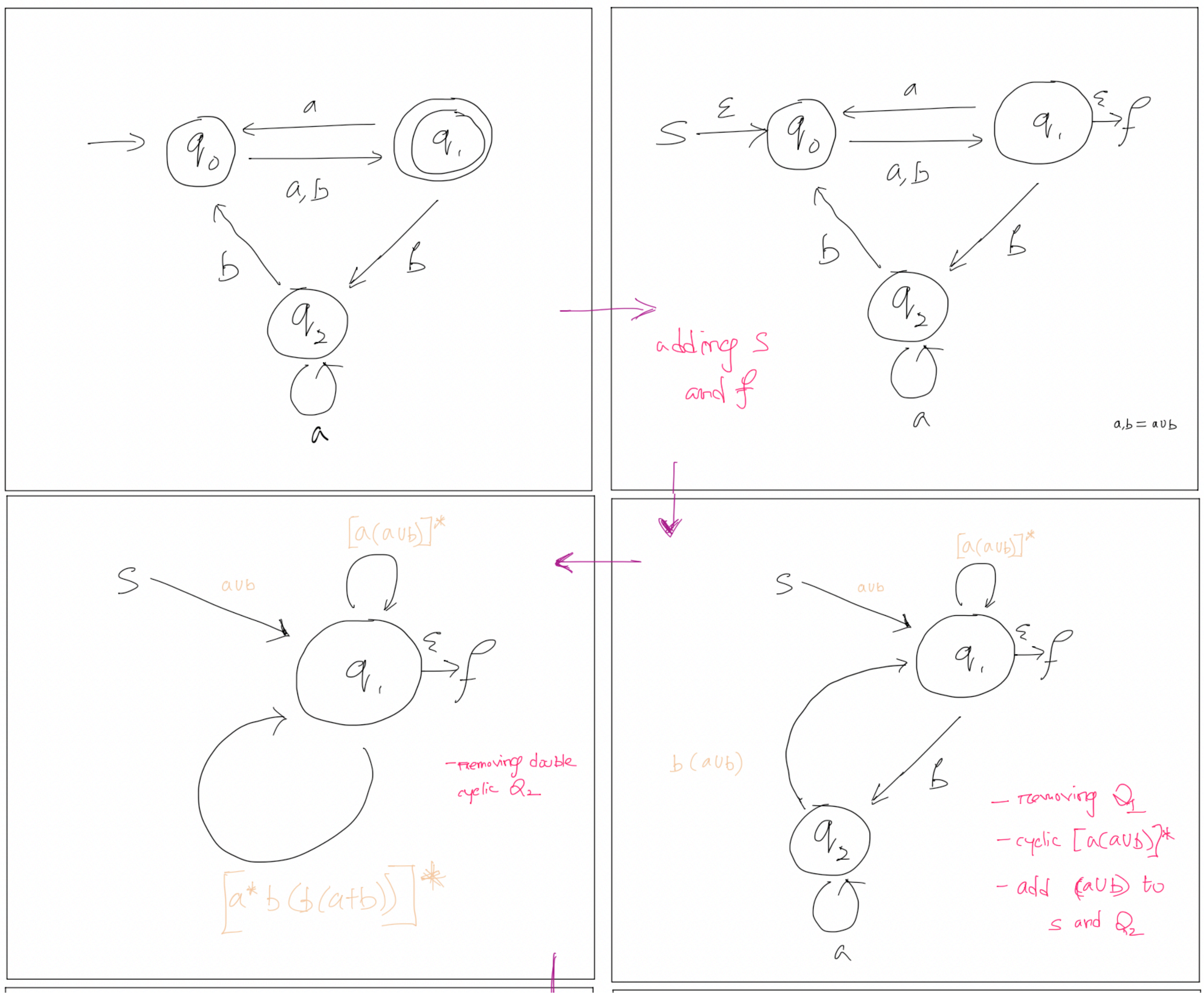
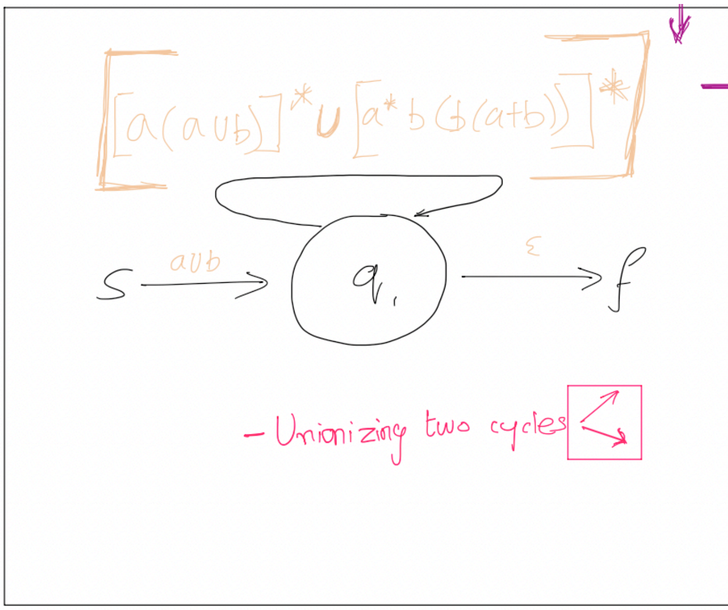
5. **[4 marks]** Consider the following DFA M.

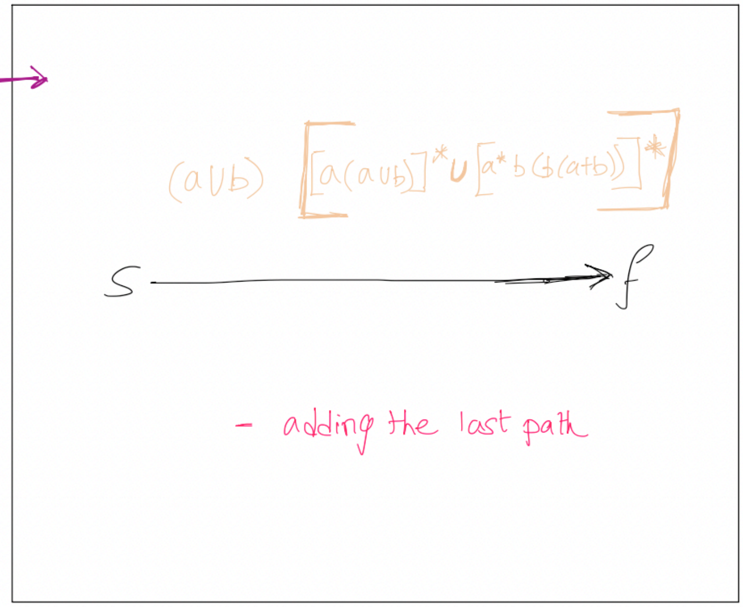


Following the DFA to regular expression proof shown in class, provide a regular expression R with $L(R) = L(M)$. Show all the steps in your work by drawing the state diagram for the initial GNFA as well as after each state removal. Remove states in lexicographic order (i.e. $q_{rip} = q_0, q_1, q_2$).

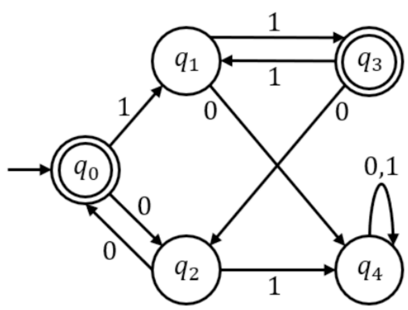
My Solution 5:







6. [**4 marks**] Consider the following DFA D:



Use the DFA state minimization algorithm to produce a DFA D' with a minimal number of states and $L(D) = L(D')$.

My Solution 6:

We can write the transition state for the DFA ***D*** in the following way:

States	1	0
Q_0	Q_1	Q_2
Q_1	Q_3	Q_4
Q_2	Q_4	Q_0
Q_3	Q_1	Q_2
Q_4	Q_4	Q_4

0 Equivalence through separating the normal and the final states = $\{Q_1, Q_2, Q_4\}$ $\{Q_0, Q_3\}$

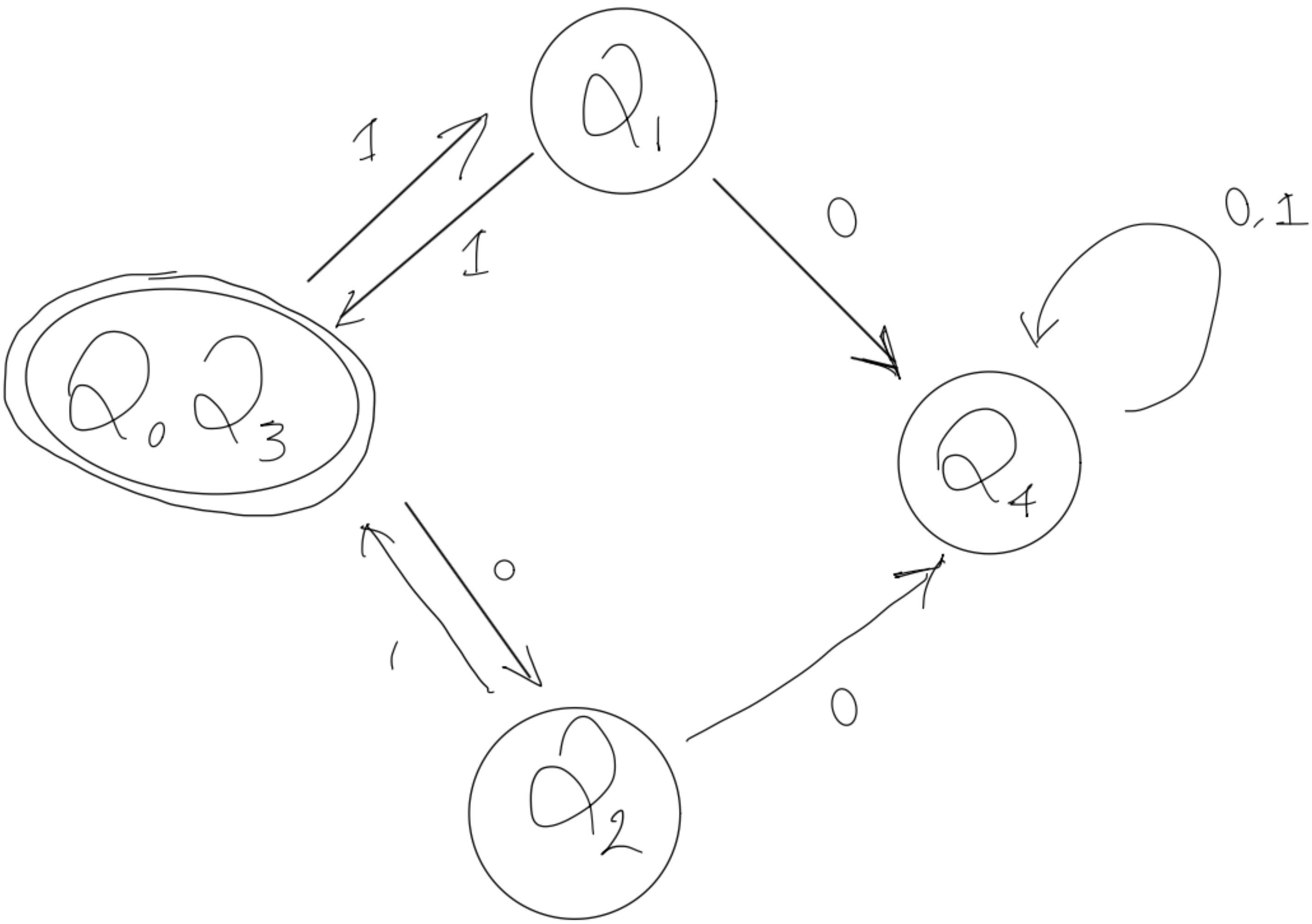
1 Equivalence through separating the normal states associated = $\{Q_1\}$ $\{Q_2\}$ $\{Q_0, Q_3\}$ $\{Q_4\}$

2 Equivalence through separating the normal states associated = $\{Q_1\}$ $\{Q_2\}$ $\{Q_0, Q_3\}$ $\{Q_4\}$

… **2 Equivalence** through separating the normal states associated = $\{Q_1\}$ $\{Q_2\}$ $\{Q_0, Q_3\}$ $\{Q_4\}$

States	1	0
Q_0Q_3	Q_1	Q_2
Q_1	Q_0Q_3	Q_4
Q_2	Q_4	Q_0Q_3
Q_4	Q_4	Q_4

And here's the minimized DFA for the given **D**:



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7. **[5 marks]** Consider the following language:

$$L = \{w \in \{0,1\}^* \mid w \text{ has at least twice as many 1's as 0's}\}$$

Use the pumping lemma to prove that L is non-regular and show every step in your proof.

My Solution 7:

Assuming L is regular and letting p be the pumping length by the pumping lemma we choose a string a^p(abb). Since |s| >= p, the pumping lemma says that s can be split into 3 parts xyz where

|xy| <= p
|y| >0
xy^{iz} is in L for all *i >=0*

The analysis considers all possible substrings 'y' within the language 'L' defined by the condition of containing the substring 'abb'. If 'y' contains only 'a's, pumping it will violate the condition by introducing more 'a's than 'b's. Similarly, if 'y' consists solely of 'b's, pumping it will disrupt the 'abb' substring. Even if 'y' contains 'ab', pumping will alter the balance of 'a's and 'b's, violating the condition. Thus, for every case, pumping 'y' results in strings outside of 'L', indicating that 'L' is non-regular.