ECE 260

EXAM 4

SOLUTIONS

(SUMMER 2020)

QUESTION 1

$x(t) = v(t)\cos(10t)$ and $y(t) = x(t)\cos(10t - \frac{\pi}{3})$

(a) $$x(t) = v(t)\cos(10t)$$
$$= \frac{1}{2}(e^{j10t} + e^{-j10t})\, v(t)$$
$$= \frac{1}{2}e^{j10t} v(t) + \frac{1}{2}e^{-j10t} v(t)$$
$$X(\omega) = \frac{1}{2}V(\omega-10) + \frac{1}{2}V(\omega+10)$$

(b) $$y(t) = x(t)\cos(10t - \frac{\pi}{3})$$
$$= \frac{1}{2}(e^{j(10t-\pi/3)} + e^{-j(10t-\pi/3)})\, x(t)$$
$$= \frac{1}{2}e^{-j\pi/3}e^{j10t}x(t) + \frac{1}{2}e^{j\pi/3}e^{-j10t}x(t)$$
$$Y(\omega) = \frac{1}{2}e^{-j\pi/3}X(\omega-10) + \frac{1}{2}e^{j\pi/3}X(\omega+10)$$

(c) $$Y(\omega) = \frac{1}{2}e^{-j\pi/3}X(\omega-10) + \frac{1}{2}e^{j\pi/3}X(\omega+10)$$
$$= \frac{1}{2}e^{-j\pi/3}\left[\frac{1}{2}V(\omega-20) + \frac{1}{2}V(\omega)\right]$$
$$+ \frac{1}{2}e^{j\pi/3}\left[\frac{1}{2}V(\omega) + \frac{1}{2}V(\omega+20)\right]$$
$$= \frac{1}{4}e^{-j\pi/3}V(\omega-20) + \frac{1}{4}e^{-j\pi/3}V(\omega) + \frac{1}{4}e^{j\pi/3}V(\omega)$$
$$+ \frac{1}{4}e^{j\pi/3}V(\omega+20)$$
$$= \frac{1}{4}e^{-j\pi/3}V(\omega-20) + \frac{1}{4}(e^{j\pi/3} + e^{-j\pi/3})V(\omega)$$
$$+ \frac{1}{4}e^{j\pi/3}V(\omega+20)$$
$$= \frac{1}{4}e^{-j\pi/3}V(\omega-20) + \frac{1}{4}\left[2\cos\left(\frac{\pi}{3}\right)\right]V(\omega) + \frac{1}{4}e^{j\pi/3}V(\omega+20)$$
$$= \frac{1}{4}e^{-j\pi/3}V(\omega-20) + \frac{1}{2}V(\omega) + \frac{1}{4}e^{j\pi/3}V(\omega+20)$$

QUESTION 2

$$x(t) = \begin{cases} 8t^2+1 & 0 \le t < \frac{1}{2} \\ t - \frac{3}{2} & \frac{1}{2} \le t < \frac{3}{2} \\ \pi & \frac{3}{2} \le t < 2 \end{cases}$$

The function x satisfies the Dirichlet conditions.

So, we have

$$\tilde{x}(\tfrac{1}{2}) = \tfrac{1}{2}\left[x(\tfrac{1}{2}^-) + x(\tfrac{1}{2}^+)\right]$$

$$= \tfrac{1}{2}\left[3 + (-1)\right]$$

$$= 1$$

$$\tilde{x}(\tfrac{3}{2}) = \tfrac{1}{2}\left[x(\tfrac{3}{2}^-) + x(\tfrac{3}{2}^+)\right]$$

$$= \tfrac{1}{2}\left[0 + \pi\right]$$

$$= \frac{\pi}{2}$$

QUESTION 3

$$H(\omega) = \frac{5j\omega + 3}{7j\omega^3 - 2j\omega^2 + 11}$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{5j\omega + 3}{7j\omega^3 - 2j\omega^2 + 11}$$

$$[7j\omega^3 - 2j\omega^2 + 11]\, Y(\omega) = [5j\omega + 3]\, X(\omega)$$

$$7j\omega^3\, Y(\omega) - 2j\omega^2\, Y(\omega) + 11\, Y(\omega) = 5j\omega\, X(\omega) + 3\, X(\omega)$$

$$-7\,(j\omega)^3\, Y(\omega) + 2j\,(j\omega)^2\, Y(\omega) + 11\, Y(\omega) = 5\,(j\omega)\, X(\omega) + 3\, X(\omega)$$

$$-7\, y'''(t) + 2j\, y''(t) + 11\, y(t) = 5\, x'(t) + 3\, x(t)$$

[Note: The prime symbol denotes a derivative.]

QUESTION 4

$y(t) = t\,x(-t)$

$v_1(t) = x(-t) \iff V_1(\omega) = X(-\omega)$

$y(t) = t\,v_1(t) \iff Y(\omega) = j\,V_1'(\omega)$

$$\begin{aligned} Y(\omega) &= j\,V_1'(\omega) \\ &= j\left[-X'(\omega)\right] \\ &= -j\,X'(\omega) \end{aligned}$$

[Note: The prime symbol denotes a derivative.]

QUESTION 5

x is bandlimited to frequencies in $[-a, a]$

$y(t) = 5x(5t)$

(a) By the sampling theorem, we have

$$\omega_x > 2(a)$$
$$= 2a$$

Therefore, the sampling rate must exceed $2a$.

(b) $Y(\omega) = \frac{1}{5}\left[5X(\omega/5)\right]$

$$= X(\omega/5)$$

Therefore, y is bandlimited to frequencies in $[-5a, 5a]$.

By the sampling theorem, we have

$$\omega_y > 2(5a)$$
$$= 10a$$

Therefore, the sampling rate must exceed $10a$.