# CENG460 Assignment 4

1. Let $01111110$ be the FLAG. To transmit the flowing bit string
   $011110000111110001111110$ at the data link layer, what is the string actually transmitted after bit stuffing?

   Solution: $011110000111110000111110010$

2. The following character encoding is used in a data link protocol: A: $01111000$; B:$01111100$; FLAG: $01111110$; ESC: $11100000$
   Show the bit sequence transmitted (in binary) for the four-character frame: "A B ESC FLAG" when FLAG bytes with byte stuffing is used.

   Solution: $011110001111110011100000111000001110000001111110$
   (A B ESC ESC ESC FLAG)

3. An 8-bit byte with binary value $10101100$ is to be encoded using Hamming code. What is the binary value after encoding? [Hint: use $4$ check bits]

   Solution:

   | bit position: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
   |---|---|---|---|---|---|---|---|---|---|---|---|---|
   | (binary) | 1 | 10 | 11 | 100 | 101 | 110 | 111 | 1000 | 1001 | 1010 | 1011 | 1100 |
   | data bits: | | | 1 | | 0 | 1 | 0 | | 1 | 1 | 0 | 0 |
   | check bits: | 0 | 1 | | 1 | | | | 0 | | | | |
   | codeword: | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |

4. A bit stream $10011100$ is transmitted using the standard CRC method. The generator polynomial is $x^3 + 1$.

   a) Show the actual bit string transmitted.

   Solution: Divide (modulo 2) $10011100000$ by $1001$, the remainder is $101$. The actual bit string is $10011100101$.

   b) Suppose one bit is inverted during transmission. Show that this error is detected at the receiver's end.

   Solution: Let $T(x)$ be polynomial corresponding to the original bit string. For a single bit error at $(i+1)$-th bit (from the right), the received bit string corresponds to $T(x) + E(x)$ (modulo 2), where $E(x) = x^i$. Since $x^i$ is not divisable by $G(x) = x^3 + 1$ (modulo 2), any single error bit can be detected with the CRC method.

5. With the 2-D parity method, a block of bits with $n$ rows and $k$ columns uses horizontal and vertical parity bits for error correction or detection. During the class, we show that 2-D parity method can correct single bit errors.

(a) Whether the 2-D method can detect double errors? Triple errors?

Solution: For any two blocks data with 2-D parity bits, the minimal Hamming distance is $4$. Therefore, the 2-D parity method can detect double errors, and triple errors.

(b) Find an example of a pattern of six errors that cannot be detected by the use of 2-D parity method.

$$
\text{Solution:} \quad
\begin{array}{cccccc}
- & 1 & 1 & 0 & - \\
- & 1 & 0 & 1 & - \\
- & 0 & 1 & 1 & - \\
\end{array}
$$

For any three rows $i_1$, $i_2$, and $i_3$, and any three columns $j_1$, $j_2$, and $j_3$, a pattern of six errors in positions $(i_1, j_1)$, $(i_1, j_2)$, $(i_2, j_2)$, $(i_2, j_3)$, $(i_3, j_1)$, and $(i_3, j_3)$ will fail to be detected.