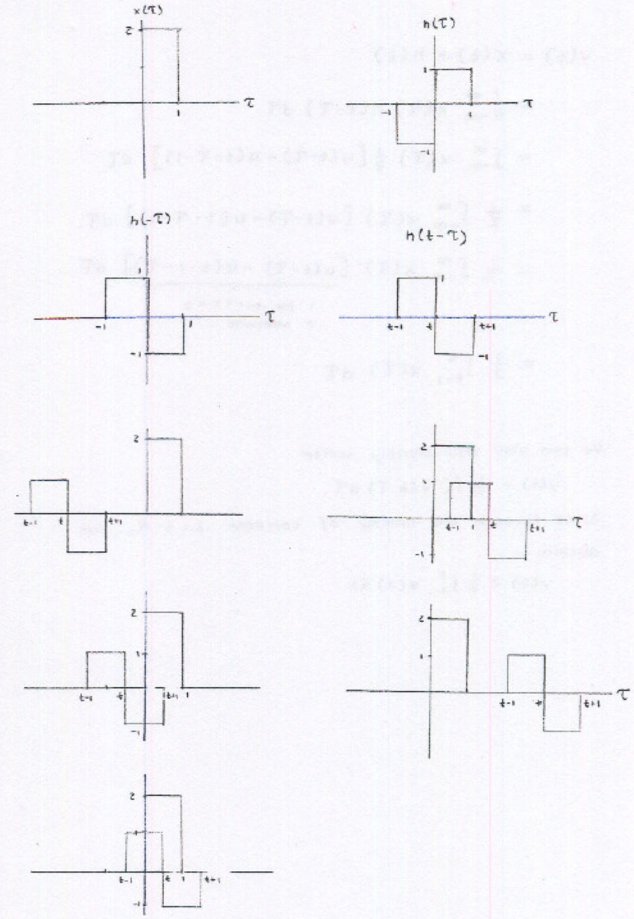
ELEC 260

QUIZ 2

SOLUTIONS

PROBLEM 1



<u>for $t < -1$</u>

$x(t) * h(t) = 0$

<u>for $-1 \le t < 0$</u>

$$x(t) * h(t) = \int_0^{t+1} (-2)\, d\tau = [-2\tau]_0^{t+1} = -2t-2$$

<u>for $0 \le t < 1$</u>

$$x(t) * h(t) = \int_0^t 2\, d\tau + \int_t^1 (-2)\, d\tau$$

$$= [2\tau]_0^t + [-2\tau]_t^1$$

$$= 2t + (-2 - [-2t])$$

$$= 4t - 2$$

<u>for $1 \le t < 2$</u>

$$x(t) * h(t) = \int_{t-1}^1 2\, d\tau$$

$$= [2\tau]_{t-1}^1$$

$$= 2 - [2t-2]$$

$$= 4 - 2t$$

<u>for $t > 2$</u>

$x(t) * h(t) = 0$

<u>final result</u>

$$x(t) * h(t) = \begin{cases} -2t-2 & -1 \le t < 0 \\ 4t-2 & 0 \le t < 1 \\ 4-2t & 1 \le t < 2 \\ 0 & \text{otherwise} \end{cases}$$

PROBLEM 2

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

Let $\lambda = t - \tau$ so that $\tau = t - \lambda$ and $d\lambda = -d\tau$.
Using this change of variable, we have

$$x(t) * h(t) = \int_{\infty}^{-\infty} x(t-\lambda)\, h(\lambda)\, (-d\lambda)$$

$$= \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$= h(t) * x(t)$$

PROBLEM 3

$$y(t) = x(t) * h(t)$$
$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} x(\tau)\, \tfrac{1}{2}\left[u(t-\tau) - u(t-\tau-1)\right] d\tau$$
$$= \tfrac{1}{2}\int_{-\infty}^{\infty} x(\tau)\left[u(t-\tau) - u(t-\tau-1)\right] d\tau$$
$$= \tfrac{1}{2}\int_{-\infty}^{\infty} x(\tau)\,\underbrace{\left[u(t-\tau) - u(t-1-\tau)\right]}_{\substack{1 \text{ for } t-1<\tau<t \\ 0 \text{ otherwise}}} d\tau$$
$$= \tfrac{1}{2}\int_{t-1}^{t} x(\tau)\, d\tau$$

We can also equivalently write

$$y(t) = \tfrac{1}{2}\int_0^1 x(t-\tau)\, d\tau.$$

Since through the change of variable $\lambda = t-\tau$, we obtain

$$y(t) = \tfrac{1}{2}\int_{t-1}^{t} x(\lambda)\, d\lambda$$

PROBLEM 4

(a) $$y(t) = x(t) + x(t) * h_1(t) + x(t) * h_2(t) * h_3(t)$$
$$= x(t) * \delta(t) + x(t) * h_1(t) + x(t) * h_2(t) * h_3(t)$$
$$= x(t) * \left[\delta(t) + h_1(t) + h_2(t) * h_3(t)\right]$$

$$h(t) = \delta(t) + h_1(t) + h_2(t) * h_3(t)$$

(b) $$h(t) = \delta(t) + \delta(t+1) + \delta(t) * \delta(t)$$
$$= \delta(t) + \delta(t+1) + \delta(t)$$
$$= 2\,\delta(t) + \delta(t+1)$$

PROBLEM 5

PART A

(i) A LTI system is memoryless if its impulse response $h(t)$ satisfies $h(t) = 0$ for all $t \neq 0$.

In this case, $h(t) \neq 0$ for $t > 0$ (e.g., at $t = 1$)

So the system has memory.

(ii) A LTI system is causal if its impulse response $h(t)$ satisfies $h(t) = 0$ for all $t < 0$.

In this case, $h(t) = 0$ for all $t < 0$.

Therefore, the system is causal

PART B

A LTI system is BIBO stable if and only if its impulse response $h(t)$ satisfies $\int_{-\infty}^{\infty} |h(t)|\, dt < \infty$.

For $\alpha = 0$, we have

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |e^0 u(t)|\, dt = \int_{-\infty}^{\infty} u(t)\, dt = \int_0^{\infty} dt = [t]\Big|_0^{\infty} = \infty$$

For $\alpha \neq 0$, we have

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-\infty}^{\infty} |e^{-\alpha t} u(t)|\, dt$$
$$= \int_{-\infty}^{\infty} |e^{-\alpha t}|\, u(t)\, dt$$
$$\left.\begin{aligned} &= \int_0^{\infty} e^{-\alpha t}\, dt \\ &= \left[-\tfrac{1}{\alpha} e^{-\alpha t}\right]\Big|_0^{\infty} \end{aligned}\right\} \alpha \neq 0$$

The above integral is only finite if $\alpha > 0$, which yields

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \left[0 - \left(-\tfrac{1}{\alpha}(1)\right)\right] = \tfrac{1}{\alpha}.$$

Therefore, the system is BIBO stable if $\alpha > 0$.

PROBLEM 6

The systems H_1 and H_2 are inverses if

$$h_1(t) * h_2(t) = \delta(t).$$

$$h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\tau)\, h_2(t-\tau)\, d\tau$$
$$= \int_{-\infty}^{\infty} \tfrac{1}{2}\delta(\tau+1)\, 2\,\delta(t-\tau-1)\, d\tau$$
$$= \int_{-\infty}^{\infty} \delta(\tau+1)\, \delta(t-\tau-1)\, d\tau$$
$$= \delta(t - [-1] - 1)$$
$$= \delta(t)$$

Therefore, the system are inverses.