University of Victoria
Exam 3
Fall 2024

Course Name:	**ECE260**
Course Title:	**Continuous-Time Signals and Systems**
Section(s):	**A01, A02**
CRN(s):	**A01 (CRN 10960), A02 (CRN 10961)**
Instructor:	**Michael Adams**
Duration:	**50 minutes**

Family Name: _____

Given Name(s): _____

Student Number: _____

This examination paper has **9 pages**, all of which are numbered.

Students must count the number of pages in this examination paper before beginning to write, and report any discrepancy immediately to the invigilator.

All questions are **to be answered on the examination paper** in the space provided.

Total Marks: 24

This examination is **closed book**.

The use of a crib sheet is **not** permitted.

The use of a calculator is **not** permitted.

You must **show all of your work!**

You must **clearly define any new quantities** introduced in your answers (such as variables, functions, operators, and so on).

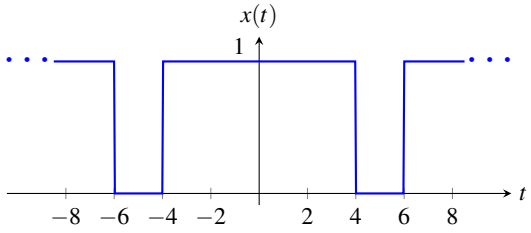
Question 1.

(A) A LTI system $\mathscr{H}$ has impulse response $h(t) = \frac{6}{\pi}\cos(9t)\,\mathrm{sinc}(3t)$. Find a fully-simplified expression for the frequency response H of this system. Note that $\int_{-\infty}^{\infty} \mathrm{sinc}(at)e^{-j\omega t}dt = \frac{\pi}{|a|}\,\mathrm{rect}\left(\frac{\omega}{2a}\right)$ for nonzero real a. **[5 marks]**

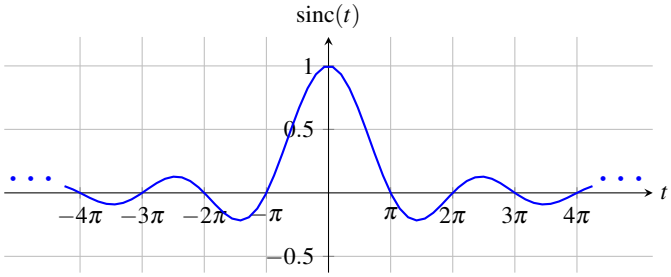
(B) Plot H. Identify the type of frequency-selective filter that $\mathscr{H}$ best approximates and specify the range of frequencies corresponding to the passband of the filter. **[1 mark]**

Question 2.

(A) Consider the periodic function x with fundamental period $T = 10$ and fundamental frequency ω_0 shown in the figure to the right. Using the Fourier series analysis equation, find the Fourier series coefficient sequence c for the function x. Your solution **must consider the single period of** $x(t)$ **for** $-\frac{T}{2} \leq t < \frac{T}{2}$. Your final answer must be **fully simplified** and expressed **in terms of the** sinc **function** to whatever extent is possible. **[7 marks]**



(B) Determine the frequency or frequencies at which x (from part (a) of this question) has the most spectral information. You must **clearly express the rationale/logic** by which you arrived at your final answer. No marks will be given for a correct final answer with a missing or incorrect justification. An accurate plot of the sinc function is shown in the figure to the right in case this might be helpful. **[1 mark]**



Question 3.

(A) Consider the function $v(t) = 4 + 2\cos(3t)$. Let T and ω_0 denote the fundamental period and fundamental frequency of v, respectively. Find the Fourier-series coefficient sequence c for v. Be sure to explicitly state the values for T and ω_0 in your answer. **[3 marks]**

(B) Consider a LTI system with input x, output y, and frequency response H, where

$$H(\omega) = \begin{cases} 1 & \text{for } \omega \geq 0 \\ -1 & \text{for } \omega < 0. \end{cases}$$

Suppose that $x = v$, where v is the function from part (a) of this question. Using the result of part (a), find a fully simplified expression for $y(t)$. **[3 marks]**

Question 4. Consider a T-periodic function x with Fourier series coefficient sequence c. If x is real valued, its Fourier series can be expressed in what is known as the combined trigonometric form. The sum x_n of the first n terms of the Fourier series (of x) expressed in combined trigonometric form is given by

$$x_n(t) = \begin{cases} c_0 + 2\sum_{k=1}^{n-1} |c_k| \cos\left[(2\pi/T)kt + \arg(c_k)\right] & n \geq 2 \\ c_0 & n = 1. \end{cases}$$

Write a MATLAB function called `evaluate_fs` that takes an n-element vector $c = \begin{bmatrix} c_0 & c_1 & \cdots & c_{n-1} \end{bmatrix}$ (where $n \geq 1$), a positive real number T, and an arbitrary real number t, and returns $x_n(t)$. The only functions that `evaluate_fs` is permitted to call are the functions **`cos`**, **`abs`**, **`angle`** (i.e., argument), and **`length`** (i.e., get the number of rows/columns in a vector). Your code must **use proper indentation** and **must not be excessively long**. Be sure to **use correct syntax** in your answer, since syntax clearly matters here. **[4 marks]**

Line #	Line of Code
1	
2	
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END

USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$
$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$
$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

θ				
Degrees	Radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	0	1	0
30	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90	$\frac{\pi}{2}$	1	0	undefined
135	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
180	π	0	-1	0
225	$\frac{5\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	1
270	$\frac{3\pi}{2}$	-1	0	undefined
315	$\frac{7\pi}{4}$	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	-1

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$$
$$c_k = \frac{1}{T}\int_T x(t)e^{-j(2\pi/T)kt}dt$$

Fourier Series Properties

Property	Time Domain	Fourier Domain
Linearity	$\alpha x(t) + \beta y(t)$	$\alpha a_k + \beta b_k$

Property	
Even Symmetry	x is even $\Leftrightarrow$ a is even
Odd Symmetry	x is odd $\Leftrightarrow$ a is odd
Real / Conjugate Symmetry	x is real $\Leftrightarrow$ a is conjugate symmetric

$$H(\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$$