6.14 For each differential/integral equation below that defines a LTI system with input x and output y, find the frequency response H of the system. (Note that the prime symbol denotes differentiation.)
(a) $y''(t)+5y'(t)+y(t)+3x'(t)-x(t)=0$;
(b) $y'(t)+2y(t)+\int_{-\infty}^{t} 3y(\tau)d\tau+5x'(t)-x(t)=0$; and
(c) $y''(t)+5y'(t)+6y(t)=x'(t)+11x(t)$.

differentiate the given equation w/ respect to t.

$$\left(\frac{d}{dt}\right)^2 y(t) + 2\left(\frac{d}{dt}\right) y(t) + 3y(t) + 5\left(\frac{d}{dt}\right)^2 x(t) - \frac{d}{dt} x(t) = 0$$

Fourier Transform of both sides:

$$\mathcal{F}\left\{\left(\frac{d}{dt}\right)^2 y(t) + 2\frac{d}{dt} y(t) + 3y(t) + 5\left(\frac{d}{dt}\right)^2 x(t) - \frac{d}{dt} x(t)\right\} = 0$$

$$= (j\omega)^2 Y(\omega) + 2(j\omega) Y(\omega) + 3Y(\omega) + 5(j\omega)^2 X(\omega) - j(\omega)X(\omega) = 0$$

$$= \left[-\omega^2 + j2\omega + 3\right] Y(\omega) = \left[5\omega^2 + j\omega\right] X(\omega)$$

$$\therefore \text{Frequency Response, } H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{5\omega^2 + j\omega}{-\omega^2 + 2j\omega + 3}$$

6.15 For each frequency response H given below for a LTI system with input x and output y, find the differential equation that characterizes the system.

(a) $H(\omega) = \dfrac{j\omega}{1+j\omega}$; and

(b) $H(\omega) = \dfrac{j\omega+\frac{1}{2}}{-j\omega^3-6\omega^2+11j\omega+6}$.

<u>b</u>

$$Y(\omega) = j\omega + \frac{1}{2}$$

$$X(\omega) = -j\omega^3 - 6\omega^2 + 11j\omega + 6$$

$$\Rightarrow \left[-j\omega^2 - 6\omega^2 + 11j\omega + 6\right] Y(\omega) = \left[j\omega + \frac{1}{2}\right] X(\omega)$$

$$\Rightarrow -j\omega^3 Y(\omega) - 6\omega^2 Y(\omega) + 11j\omega Y(\omega) + 6Y(\omega) = j\omega X(\omega) + \frac{1}{2}X(\omega)$$

$$= 0$$

Taking inverse Fourier Transform

$$F^{-1}\{-j\omega^3 Y(\omega) - 6\omega^2 Y(\omega) + 11 j\omega Y(\omega) + 6Y(\omega)$$

$$-j\omega X(\omega) - \frac{1}{2}X(\omega)\} = 0$$

$$\Rightarrow F^{-1}\{j\omega^3 Y(\omega) + 6F^{-1}(j\omega^2 Y(\omega)) + 11F^{-1}(j\omega Y(\omega))$$

$$+ 6F^{-1}(Y(\omega))\}$$

$$\Rightarrow \left(\frac{d}{dt}\right)^3 y(t) + 6\left(\frac{d}{dt}\right)^2 y(t) + 11\frac{d}{dt}(y(t))$$

$$+ 6y(t) - \frac{d}{dt}x(t) - \frac{1}{2}x(t) = 0$$

6.16 For each case below, use frequency-domain methods to find the response y of the LTI system with impulse response h and frequency response H to the input x.

(a) $h(t) = \delta(t) - 300\operatorname{sinc}(300\pi t)$ and $x(t) = \frac{1}{2} + \frac{3}{4}\cos(200\pi t) + \frac{1}{2}\cos(400\pi t) - \frac{1}{4}\cos(600\pi t)$.

$h(t) = \delta(t) - 300\operatorname{sinc} 300\pi t \qquad x(t) = \frac{1}{2} + \frac{3}{4}\cos 200\pi t - \frac{1}{4}\cos 600\pi t$

$H(\omega) = F\{\delta(t)\} - F\left\{\frac{300\pi}{\pi}\operatorname{sinc} 300\pi t\right\} = 1 - \operatorname{rect}\left(\frac{\omega}{600\pi}\right) = \begin{cases} 1 & |\omega| > 300\pi \\ 0 & \text{otherwise} \end{cases}$

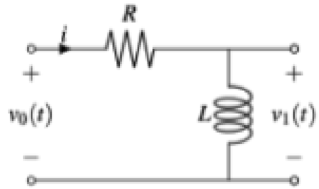
$X_1(\omega) = F\left\{\frac{1}{2}\right\} + \frac{3}{4}F\{\cos 200\pi t\} + \frac{1}{2}F\{\cos 400\pi t\} - \frac{1}{4}F\{\cos 600\pi t\}$

$Y(\omega) = X_1(\omega)H(\omega) = \frac{1}{2}\pi(\omega - 400\pi) + \delta(\omega + 400\pi) - \frac{1}{4}\pi[\delta(\omega - 600\pi) + (600\pi + \omega)]$

$$y(t) = F^{-1}\{Y(\omega)\} = \frac{1}{2} F^{-1}\{\pi[\delta(\omega - 400\pi) + \delta(\omega + 400\pi)$$

$$= \frac{1}{2}\cos 400\pi t - \frac{1}{4}\cos 600\pi t$$

6.17 Consider the LTI resistor-inductor (RL) network with input v_0 and output v_1 as shown in the figure below.



(a) Find the frequency response H of the system.
(b) Determine the magnitude and phase responses of the system.
(c) Determine the type of frequency-selective filter that this system best approximates.
(d) Find v_1 in the case that $v_0(t) = \operatorname{sgn} t$.
(e) Find the impulse response h of the system.

a

$$v_1(t) = L\frac{d}{dt}\left[\frac{1}{R}\left[v_0(t) - v_1(t)\right]\right]$$

$$= \frac{L}{R}\frac{d}{dt}v_0(t) - \frac{L}{R}\frac{d}{dt}v_1(t)$$

$$V_1(\omega) = \frac{L}{R}F\left\{\frac{d}{dt}v_0(t)\right\} - \frac{L}{R}F\left\{\frac{d}{dt}v_1(t)\right\}$$

$$= V_1(\omega) = \frac{L}{R}j\omega_0 V_0(\omega) - \frac{L}{R}j\omega V_1(\omega)$$

$$= \left[1 + \frac{L}{R}j\omega\right]V_1(\omega) = \frac{L}{R}j\omega_0 V_0(\omega)$$

$$H(\omega) = \frac{V_1(\omega)}{V_0(\omega)} = \frac{V_1(\omega)}{V_0(\omega)} = \frac{\frac{L}{R}j\omega_0}{1+\frac{L}{R}j\omega} = \frac{j\omega}{r+j\omega}$$

b

$$|H(\omega)| = \frac{|j\omega|}{|1+j\omega|} = \frac{|\omega|}{\sqrt{1+\omega^2}}$$

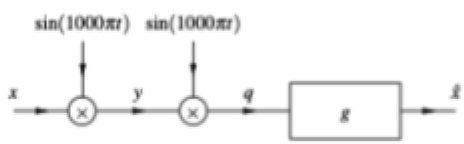
$$\arg H(\omega) = \arg j\omega - \arg(1+j\omega)$$
$$= \frac{\pi}{2}\,\mathrm{sgn}\,\omega - \tan^{-1}\omega$$

where $\arg j\omega = \begin{cases} \frac{\pi}{2} & \omega > 0 \\ -\frac{\pi}{2} & \omega < 0 \end{cases} = \frac{\pi}{2}\,\mathrm{sgn}\,\omega$

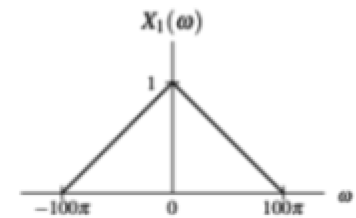
6.24 Consider the system shown below in Figure A with input x and output $\hat{x}$, where this system contains a LTI subsystem with impulse response g. The Fourier transform G of g is given by

$$G(\omega) = \begin{cases} 2 & |\omega| \le 100\pi \\ 0 & \text{otherwise.} \end{cases}$$

Let X, $\hat{X}$, Y, and Q denote the Fourier transforms of x, $\hat{x}$, y, and q, respectively.



(a)



(b)

(a) Suppose that $X(\omega) = 0$ for $|\omega| > 100\pi$. Find expressions for Y, Q, and $\hat{X}$ in terms of X.
(b) If $X = X_1$ where X_1 is as shown in Figure B, sketch Y, Q, and $\hat{X}$.

a)

$$Y(\omega) = F\{x(t)\sin 1000\pi t\}$$

$$= F\left\{\frac{1}{2j}\left[e^{j1000\pi t} - e^{-j1000\pi t}\right]x(t)\right\}$$

$$= \frac{1}{2j}F\left\{e^{j1000\pi t}x(t)\right\} - \frac{1}{2j}F\left\{e^{-j1000\pi t}x(t)\right\}$$

$$= \frac{1}{2j}X(\omega - 1000\pi) - \frac{1}{2j}X(\omega + 1000\pi)$$

$$Q(\omega) = F\{y(t)\sin 1000\pi t\}$$

$$= F\left\{\frac{1}{2j}\left[e^{j1000\pi t} - e^{-j1000\pi t}\right]y(t)\right\}$$

$$= \frac{1}{2j}Y(\omega-1000\pi) - \frac{1}{2j}Y(\omega+1000\pi)$$

$$\hat{X}(\omega) = G(\omega)Q(\omega)$$

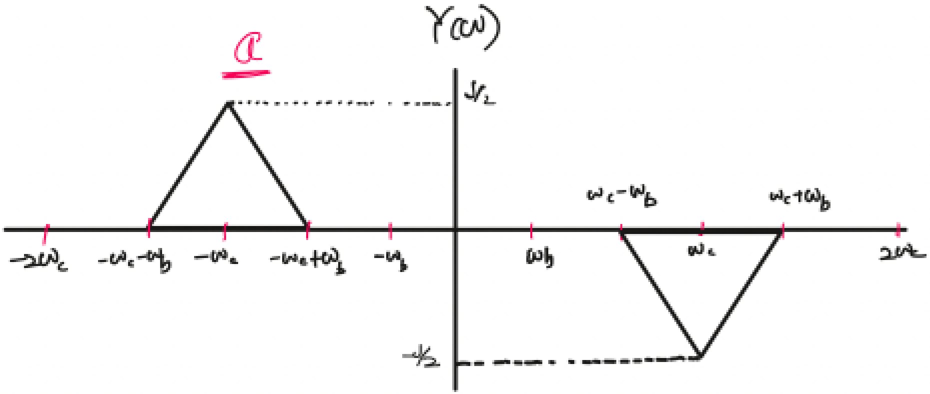
$$Q(\omega) = \frac{1}{2j}Y(\omega-1000\pi) - \frac{1}{2j}Y(\omega+1000\pi)$$

$$= -\frac{1}{4}X(\omega-2000\pi) + \frac{1}{4}X(\omega) + \frac{1}{4}(\omega) + \frac{1}{4}(\omega) - \frac{1}{4}X(\omega+2000\pi)$$

$$= \frac{1}{2}X(\omega) - \frac{1}{4}X(\omega-2000\pi) - \frac{1}{4}X(\omega+2000\pi)$$

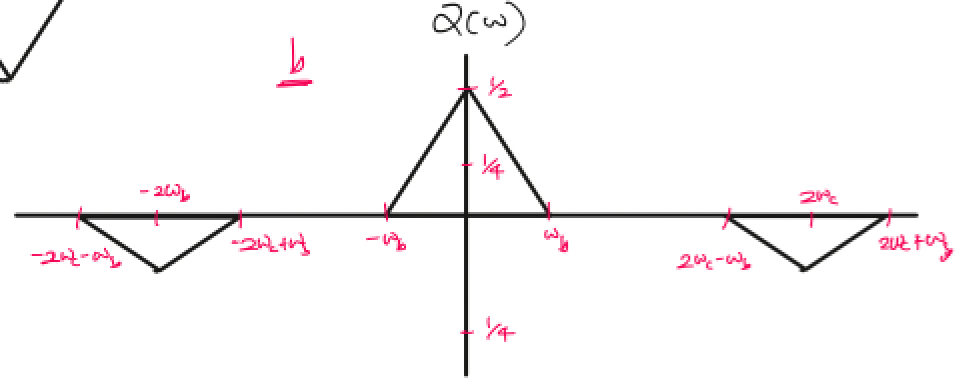
combining... $\hat{X}(\omega) = G(\omega)Q(\omega) = 2\cdot\frac{1}{2}\cdot X(\omega) = X(\omega)$

a



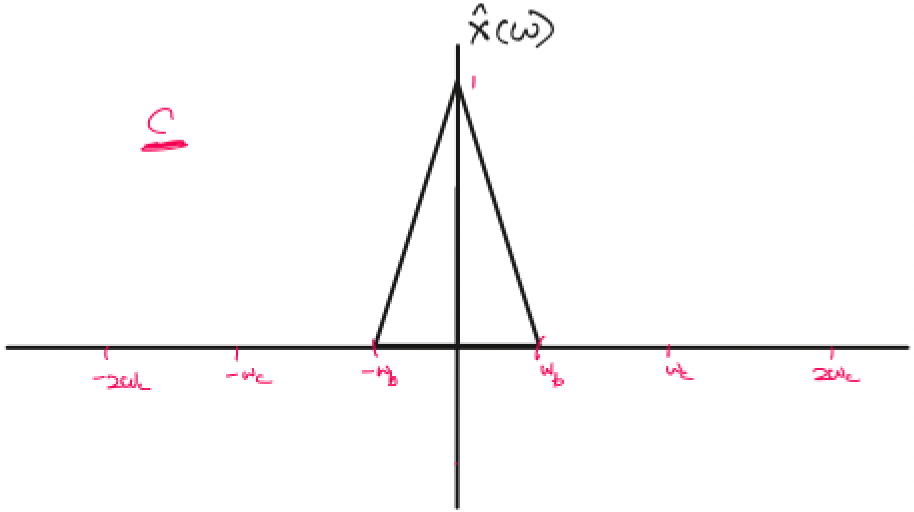
Y(ω)
1/2
-1/2
-2ωc
-ωc-ωb
-ωc
-ωc+ωb
-ωb
ωb
ωc-ωb
ωc
ωc+ωb
2ωc

b



Q(ω)
1/2
1/4
1/4
-2ωc
-2ωc-ωb
-2ωc+ωb
-ωb
ωb
2ωc
2ωc-ωb
2ωc+ωb

c



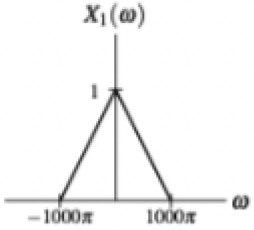
X̂(ω)
1
-2ωc
-ωc
-ωb
ωb
ωc
2ωc

6.26 Consider the system shown below in Figure A with input x and output y. Let X, P, S, H, and Y denote the Fourier transforms of x, p, s, h, and y, respectively. Suppose that

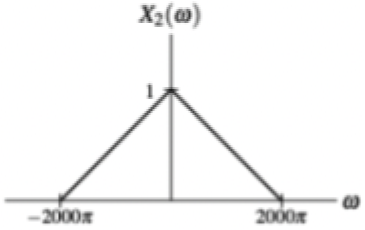
$$p(t) = \sum_{n=-\infty}^{\infty} \delta\left(t - \tfrac{n}{1000}\right) \quad \text{and} \quad H(\omega) = \tfrac{1}{1000}\operatorname{rect}\left(\tfrac{\omega}{2000\pi}\right).$$

(a) Derive an expression for S in terms of X. Derive an expression for Y in terms of S and H.
(b) Suppose that $X = X_1$, where X_1 is as shown in Figure B. Using the results of part (a), plot S and Y. Indicate the relationship (if any) between the input x and output y of the system.
(c) Suppose that $X = X_2$, where X_2 is as shown in Figure C. Using the results of part (a), plot S and Y. Indicate the relationship (if any) between the input x and output y of the system.

(a)



(b)



(c)

a

$p(t)$ is periodic $\quad T = \frac{1}{1000} \quad \omega_0 = 2000\pi$

$$p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2000\pi kt}$$

$$c_k = \left(\frac{1}{1000}\right)^{-1} \int_{-\frac{1}{2000}}^{\frac{1}{2000}} \delta(t) e^{-j2000\pi kt}\, dt$$

$$s(t) = x(t)p(t) = x(t)\left(1000 \sum_{-\infty}^{\infty} e^{j2000\pi kt}\right)$$

$$= 1000 \sum_{k=-\infty}^{\infty} x(t) e^{j2000\pi kt}$$

$$y(t) = s(t) * h(t)$$

$$Y(\omega) = H(\omega)\, S(\omega) = \begin{cases} 1/1000\, S(\omega) & |\omega| < 1000\pi \\ 0 & \text{otherwise} \end{cases}$$

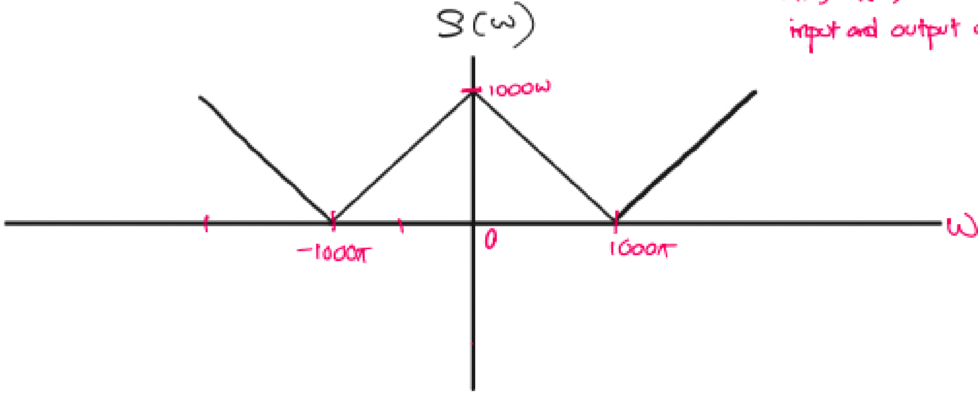
$$= \begin{cases} \sum_{k=-\infty}^{\infty} X(\omega - 2000\pi k) & |\omega| < 1000\pi \\ 0 & \text{otherwise} \end{cases}$$

b $X(\omega)=0$ for $|\omega|<1000\pi$

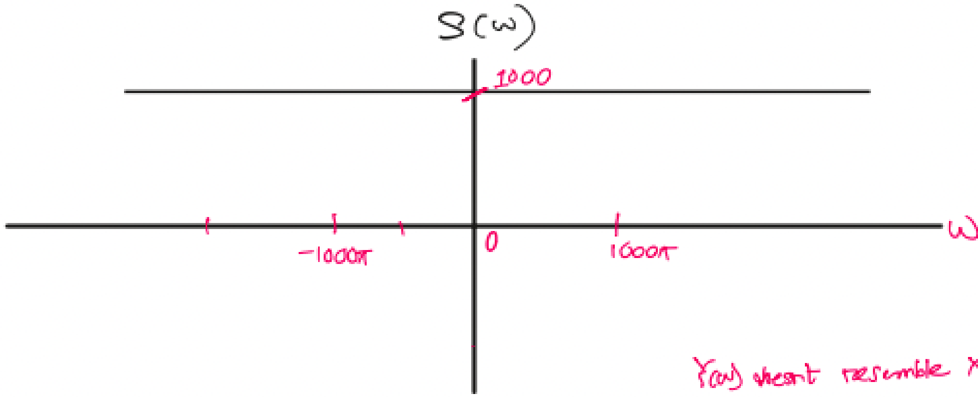
plot of $S(\omega)$

$X(\omega)=Y(\omega)$

input and output of system are identical



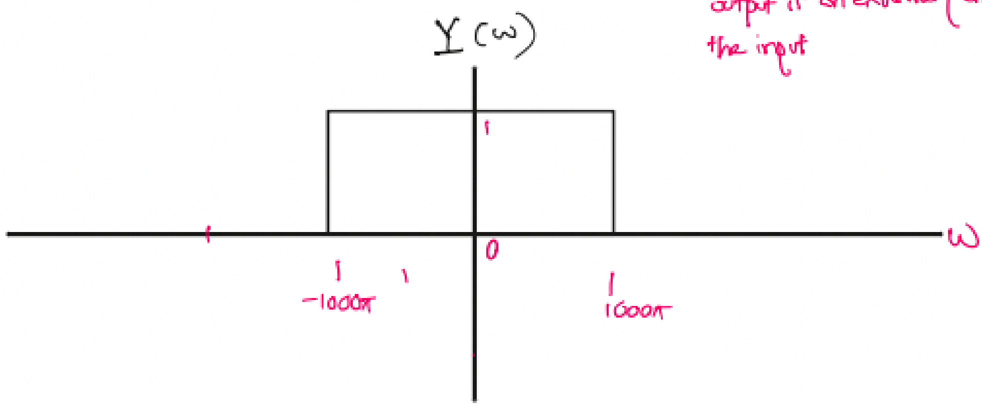
c



$Y(\omega)$ doesn't resemble $X(\omega)$

$x(t)$ input and $y(t)$ output are not related

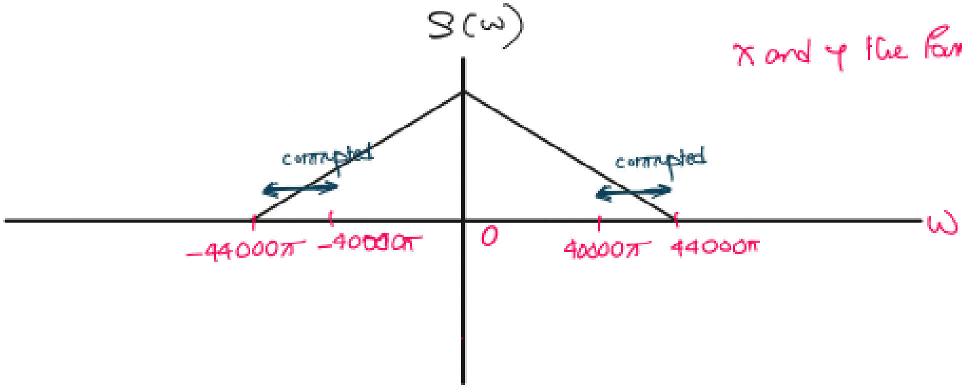
output is an extremely distorted version of the input



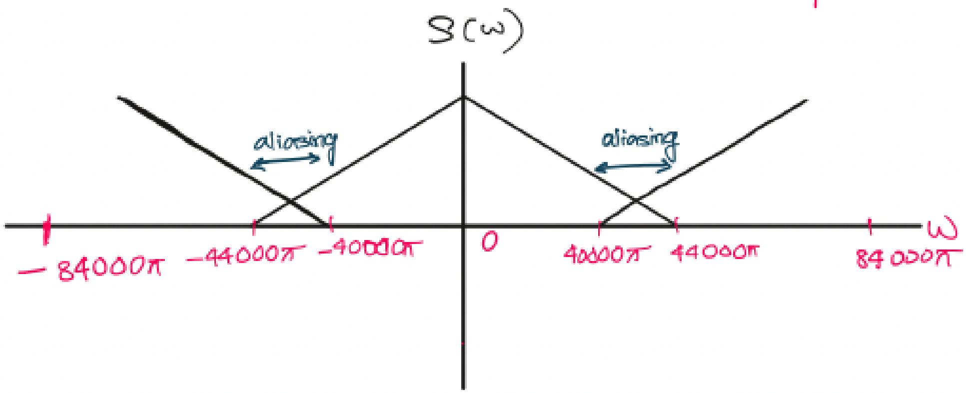
6.27 A function x is bandlimited to 22 kHz (i.e., only has spectral content for frequencies f in the range $[-22000, 22000]$). Due to excessive noise, the portion of the spectrum that corresponds to frequencies f satisfying $|f| > 20000$ has been badly corrupted and rendered useless. (a) Determine the minimum sampling rate for x that would allow the uncorrupted part of the spectrum to be recovered. (b) Suppose now that the corrupted part of the spectrum were eliminated by filtering prior to sampling. In this case, determine the minimum sampling rate for x.

y denoting signal from x after sampling and reconstruction.

X and Y the Fourier transform of x and y



aliasing only occurs in the corrupted part of the spectrum.



c

$\omega_s = 2(40000\pi) = 80000\pi$

∴ A sampling rate of 80000π rad/s

a

freqw.m

/Users/arfaz/Library/CloudStorage/OneDrive-UniversityofVictoria/0 UVIC/2 ENGR Y3/4 ECE 260 A01 T03/1 Assignments/0 Solutions/2 ...

```
function [freqresp, omega] = freqw(ncoefs, dcoefs, omega)
    % Evaluate the frequency response of H(ω) at given ω points
    freqresp = polyval(ncoefs, omega) ./ polyval(dcoefs, omega);

    % If no output arguments were specified, plot the frequency response
    if nargout == 0
        % Compute the magnitude and phase responses
        magresp = abs(freqresp);
        phaseresp = angle(freqresp);

        % Plot the magnitude response
        subplot(2, 1, 1);
        plot(omega, magresp);
        title('Magnitude Response');
        xlabel('Frequency (rad/s)');
        ylabel('Magnitude (unitless)');

        % Plot the phase response
        subplot(2, 1, 2);
        plot(omega, phaseresp);
        title('Phase Response');
        xlabel('Frequency (rad/s)');
        ylabel('Angle (rad)');
    end
end
```

```
>> % Define the coefficients for the numerator and denominator
ncoefs = [16];
dcoefs = [1, -1i * 5.2263, -13.6569, 1i * 20.9050, 16.0000];

% Define the range of ω values
omega = linspace(-5, 5, 500);

% Call the freqw function without output arguments to plot the responses
freqw(ncoefs, dcoefs, omega);
>>
```

b

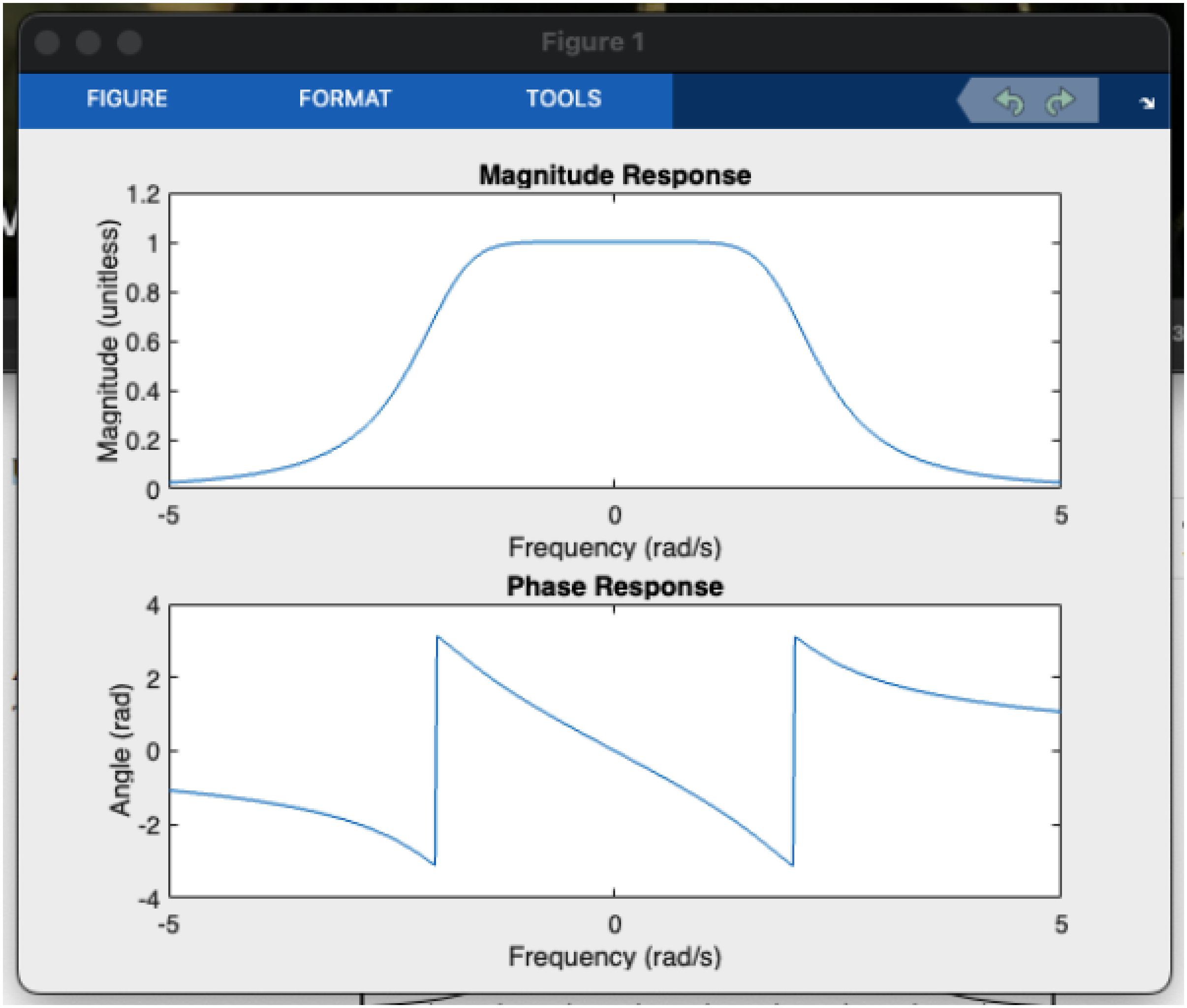


Figure 1
FIGURE
FORMAT
TOOLS
Magnitude Response
Magnitude (unitless)
1.2
1
0.8
0.6
0.4
0.2
0
-5
0
5
Frequency (rad/s)
Phase Response
Angle (rad)
4
2
0
-2
-4
-5
0
5
Frequency (rad/s)