THE DESIGN OF COMPRESSION STRUCTURES FOR MINIMUM WEIGHT

by

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1. INTRODUCTION

1.1. Fundamental importance of the problem

THE primary structure of an aeroplane usually consists basically of a set of tubular beams. The main structural box of the wing or tailplane is a well-known example: a semi-monocoque fuselage is another. For any given loading condition of the aircraft the material in the tube is stressed mainly in tension, in shear, or in compression, depending on its location in the tube cross section.

The aim of the designer is to make the material fulfil these three functions in the most economical manner. In tension, he is limited only by the quality of material available. In shear, this is again substantially the case, although it is well known that very light shear webs over great depths do not develop as high an effective failing stress as do more sturdy webs. This property of dependence on the *intensity of loading* is much more marked in the case of the compression structure, which is liable to instability in various ways.

The fundamental problem of the design of such a structural element is the stabilising of the compression surface. The ingenuity of the designer in this respect has led to various types of structure being adopted: one may mention the two-spar wing box, the thick skins reinforced by stringers and supported by ribs in the case of a wing, or by frames in the case of a fuselage, and the sandwich structure. It is clear that a generalised approach cannot cover all forms of structure, and in the present case the second type only is considered.

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1.2. Type of structure considered

It is assumed that it is required to design a surface carrying a certain ultimate compressive end load. The surface consists of a skin stiffened by longitudinal stringers, and supported at intervals by ribs or frames.

As a limiting case this type of structure can include the two-spar wing, in which there are only two " stringers " which are stabilised by the spar webs. The trend of modern design has been towards the thick stiffened skin type of construction, the thick skin being needed for torsional stiffness in the case of the wings or tailplanes and for carrying pressurising loads in the case of fuselages. In addition to the reinforced skin cover there may also be spar booms which carry part of the end load.

1.3. Aims of the investigation

Of the various design conditions, the one of over-riding importance is that the structure should have adequate strength. The primary purpose of this investigation therefore is to determine the structure which has a minimum weight when its external dimensions and its strength are given.

In general the design conditions often involve other aspects besides strength. The provision of adequate stiffness may be one: ease of manufacture, and limitation of working stresses by fatigue considerations, may be others: they are not inevitable design conditions, whereas strength considerations are.

A means is therefore provided for assessing the effect of geometrical limitations which may be imposed by these other requirements on the structural weight, so that a structure can be chosen in which the weight increase, above that necessary for strength alone in the minimum weight structure, is also a minimum.

1.4. Limitations and difficulties

In a subject with so broad a scope it is inevitable that any solution involving a reasonable amount of labour is not a precise and complete one: in fact it may be assumed to be a design condition of the structure that the labour involved in design should not be prohibitive. Some reasonable limits to the range of the problem explored, and perhaps to the accuracy of the answer desired, must therefore be set.

To illustrate this point, one may note that it is necessary to include many forms of stringers so as to pick the best type for the particular structure—typical shapes being Z-section, top hat, bulb angle, and so on. A certain amount of experience is necessary in order to choose a limited range of sections which must be subjected to theoretical analysis. Even when a basic type such as the Z-section has been chosen, there are several shape parameters which can be widely varied.

When a single shape of stringer has been chosen for analysis, the situation is still far from favourable on the theoretical side. The various modes of instability are often quite heavily coupled so that analysis is difficult and depends on a large number of parameters: the failing stress may be appreciably different from the instability stress because of initial eccentricity or lateral air loading, and so on.

In view of these difficulties one may feel that it is not possible to find an optimum structure. However, the nature of an optimum is such that the weight is a mathematical minimum with respect to the design parameters: it follows therefore that if our methods of analysis provide only a reasonable approximation to the truth, the weight will still be very close to the mathematical minimum achieved by the ideal structure.

1.5. Division of the problem

Let us suppose that the compression skin-stringer combination is required to carry an end load P per unit width, and that the ribs or frames are uniformly spaced at a distance L apart along the length of the structure.

If we fix P and L, and assume that the ribs or frames possess a certain minimum stiffness, we must first determine the lightest skin-stringer combination which can carry the loading P over the bay length L. This is the first stage in the analysis: and if the spacing of the ribs or frames is completely dictated by practical considerations, one need go no farther.

In a structure of fair size it is generally possible to vary the rib or frame spacing. The second stage of the problem is then to vary the rib or frame spacing so that the total weight (ribs + skin + stringers) is a minimum.

2. CHARACTERISTICS OF THE SKIN-STRINGER COMBINATION

2.1. Conditions assumed

It is assumed that the number of stringers is sufficiently large for the instability stresses to be the same as in an infinitely wide panel (more than three or four stringers usually ensures that this is so). The ribs are assumed to supply simple support so that the combination of skin and stringers acts as a pin-ended strut of length L.

NOTATION

P = compressive end load carried per inch width of skin-stringer combination.
L = rib or frame spacing.
T = thickness of skin which has the same cross sectional area as the skin-stringer combination.
E = compression Young's modulus of skin-stringer material.
E_T = tangent modulus of skin-stringer material.
K = radius of gyration of skin-stringer combination.
f = mean stress realised by skin and stringers at failure. (Note: $f=P/T$).
d = flange width of stringer.
h = depth of stringer.
b = stringer spacing.
t_s = stringer wall thickness.
t = skin thickness.
A_s = cross sectional area of stringer.
$f_o = 3.62\,E\,(t/b)^2$.
f_b = initial local buckling stress.
f_L = secondary local buckling stress in stringer.
f_T = secondary torsional buckling stress in stringer.
D = depth of rib.
T_R = thickness of an ideal plate rib which has the same weight as the actual rib.
W = weight of skin-stringer combination per unit area.

2.2. General case[1]

Consider a skin-stringer combination which is equivalent in area to a skin of thickness T. Let the radius of gyration of the combination be K.

Then if the mean compressive stress in the skin and stringers is f,

$$P=fT \quad . \quad . \quad . \quad (1)$$

Euler instability of the combination is given by

$$f=\frac{\pi^2 E_T K^2}{L^2} \quad . \quad . \quad (2)$$

where E_T is the tangent Young's modulus of the material. Since the skin-stringer combination can be regarded as an assembly of interconnected plates, it will also develop instability of a local plate-buckling type. The mean stress at which this local instability occurs is given by

$$f=AE_T\,(T/K)^2 \quad . \quad . \quad (3)$$

where A is a constant depending on the geometry of the cross section. There may be several modes of local buckling; we will assume that the one to which A is appropriate will involve distortions of the section sufficiently large to induce premature flexural instability and failure.

It is easily demonstrated that the lightest structure is that in which flexural instability and local instability coincide. If we impose this condition we find that

$$f=\pi^{\frac{1}{2}} A^{\frac{1}{4}}\left(\frac{PE_T}{L}\right)^{\frac{1}{2}} \quad . \quad . \quad (4)$$

Now the coefficient A depends on the type of skin-stringer combination chosen, and for the optimum structure will have a maximum value. If we call this maximum value A_0, then the highest stress which could ever be achieved is given by

$$f_0=\pi^{\frac{1}{2}} A_0^{\frac{1}{4}}\left(\frac{PE_T}{L}\right)^{\frac{1}{2}} \quad . \quad (5)$$

The implication of this result is an important one: it is that the maximum stress which could possibly be achieved is dictated entirely by the value of the structure loading coefficient P/L, which is the fundamental quantity dictating the weight of the skin-stringer combination. It is thus economic to make P as large as possible by concentrating all the compression material in the sheet-stringer cover, and almost eliminating spar booms.

In seeking to determine the best type of skin-stringer combination, we must determine the best value of A which each can achieve. At this stage the analysis becomes more detailed since a variety of types must be considered and the individual instability stresses must be explored for each type over a range of geometrical parameters. Once the value of A_0 has been determined, families of optimum structures can immediately be produced for all P and all L, so that the actual design process is extremely rapid once the general investigation is completed. As a specific example here we consider the case of Z-section stringers, which are widely used.

2.3. Instability characteristics of Z-section stringers

The Z-section stringer-skin combination can develop several separate types of instability, which may be coupled to a greater or less degree.

(*a*) *Skin buckling* (or initial buckling). This generally involves waving of the skin between stringers in a half-wavelength comparable with the stringer pitch. There will also be a certain amount of waving of the stringer web and lateral displacement of the free flange. For some proportions these latter may become larger than the skin displacements, and the mode becomes more torsional or local in nature (*see* (*b*) and (*c*)).

(*b*) *Local instability.* A secondary short-wavelength buckling may take place in which the stringer web and flange are displaced out of their own planes in a half-wavelength comparable with the stringer depth. There will be smaller associated movements of the skin and lateral displacements of the stringer free flange.

(*c*) *Torsional instability.* The stringer rotates as a solid body about a longitudinal axis in the plane of the skin, with associated smaller displacements of the skin normal to its plane and distortions of the stringer cross-section. The half-wavelength is usually of the order of three times the stringer pitch.

(*d*) *Flexural instability.* Simple strut instability of the skin-stringer combination in a direction normal to the plane of the skin. There may be small associated twisting of the stringers. The half-wavelength is generally equal to the frame spacing.

(*e*) *Inter-rivet buckling.* Buckling of the skin as a short strut between rivets: this can be avoided by using a sufficiently close rivet pitching along the stringer.

(*f*) *Wrinkling.* A mode of instability similar to inter-rivet buckling, but analogous

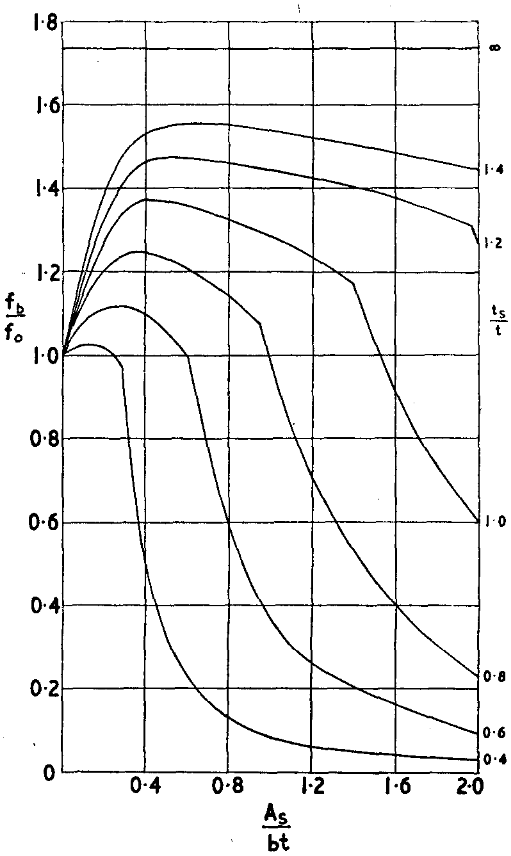

Fig. 1.
Initial buckling stress of flat panels with Z-section stringers ($d/h = 0.3$).

to wrinkling of a sandwich structure, in which the skin develops short-wavelength buckling as an elastically supported strut. For all practical skin-stringer combinations it can be avoided by keeping the line of attachment very close to the stringer web.

2.4. Failure of Z-section stringers

When the skin-stringer combination approaches its Euler instability stress, development of instabilities (*a*), (*b*), (*c*), (*e*) or (*f*) will so reduce the flexural stiffness as to cause premature collapse.

If the Euler instability stress is reasonably remote, instability (*a*) (skin buckling) will not precipitate failure, and the structure will carry increased load, with the skin buckled, until failure occurs by the onset of instability (*b*), (*c*), (*e*) or (*f*). In general an excessive margin of flexural stiffness is needed to prevent failure due to any of these latter four modes.

We will therefore consider, separately, two cases. In the first, primary local buckling causes flexural failure. In the second, skin buckling develops and flexural failure is precipitated by secondary local buckling. It will be assumed that inter-rivet buckling and wrinkling are avoided by suitable disposition of the riveting.

3. Z-SECTION STRINGERS—INITIAL LOCAL BUCKLING CAUSING FAILURE[2]

3.1. Initial local buckling

The local buckling mode which is the first to develop is a mixture of modes (*a*), (*b*) and (*c*) of Section 2.3, the predominant type of buckling being dictated by the geometry of the particular skin-stringer combination used.

The stress at which this initial buckling occurs has been determined theoretically by J. H. Argyris[3] for a wide range of skin-stringer combinations, allowance being made for all the inter-actions between the various modes of distortion. Typical results are shown in Fig. 1 in which the non-dimensional buckling stress (f_b/f_o) is plotted against A_s/bt for various values of t_s/t. (*Note*: f_o is the buckling stress of the skin if pin-edged along the stringers, and f_b the actual initial buckling stress.)

The upper portions of the curves correspond to a skin-buckling-cum-stringer local type of instability, while the lower portions of the curves correspond to a stringer torsional-cum-lateral type of instability over a longer wavelength. The change of slope in the curves takes place when these two types of initial buckling occur at the same stress.

3.2. Flexural instability

Considering a stringer associated with a pitch b of skin, the whole cross section is fully effective until initial local buckling occurs.

In general the stringer will not develop pure flexural instability: there will also be a certain amount of stringer twisting, the analysis of which is far from simple. Fortunately the type of design which this analysis will show to be most efficient is one in which flexural-torsional coupling is small, and we will therefore assume that pure flexural instability occurs. If we consider $d/h = 0.3$, the second moment of area of the skin-stringer combination is

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$$I_{NA}=\frac{0.633bt+0.37ht_s}{1.6ht_s+bt}h^3t_s \quad . \qquad (6)$$

The flexural instability stress is therefore

$$f_E=\frac{\pi^2E_T}{L^2}\times h^3t_s\times\frac{0.633bt+0.37ht_s}{(1.6ht_s+bt)^2} \qquad (7)$$

We equate this to the initial local buckling stress

$$f_b=f_b/f_o\times 3.62E_T\,(t/b)^2 \quad . \qquad (8)$$

The load per inch run is then given by

$$P=ft\left(1+\frac{1.6ht_s}{bt}\right). \quad . \qquad (9)$$

where $f=f_E=f_B$.

Combining equations (7), (8), (9) in the same manner as equations (1), (2), (3) we find

$$f=F\sqrt{\frac{PE_T}{L}} \quad . \quad . \qquad (10)$$

where F is a function of A_s/bt and t_s/t, corresponding to the quantity $\pi^{\frac{1}{2}}A^{\frac{1}{4}}$ of equation (4); it is a measure of the structural efficiency of the skin-stringer combination.

3.3. Optimum structure

The quantity $F\left(=f\sqrt{\frac{L}{PE_T}}\right)$ is plotted against A_s/bt and t_s/t in Fig. 2.

It is seen that an optimum value of A_s/bt and t_s/t exists, at which for a given P, E_T and L the stress realised will be a maximum. For this optimum design with $A_s/bt=1.5$ and $t_s/t=1.05$ we can write

$$f=0.95\sqrt{\frac{PE_T}{L}} \quad . \quad . \qquad (11)$$

It is also noticeable that a ridge of high realised stress exists for the family of designs where the two types of local buckling occur simultaneously; if for any reason the minimum-weight design cannot be used, it is economic to use designs of this family. (The general principle seems to emerge that the most efficient designs are those in which failure occurs simultaneously in all possible buckling modes.)

3.4. Design charts

If we consider only the more efficient designs, in which the two modes of local buckling occur at the same stress, the results can be presented in the form of Fig. 3.

From these curves the stress realised by a given type of structure, together with the stringer pitch and depth, and the skin and stringer thicknesses, may be found at once for any basic design conditions P, E_T and L.

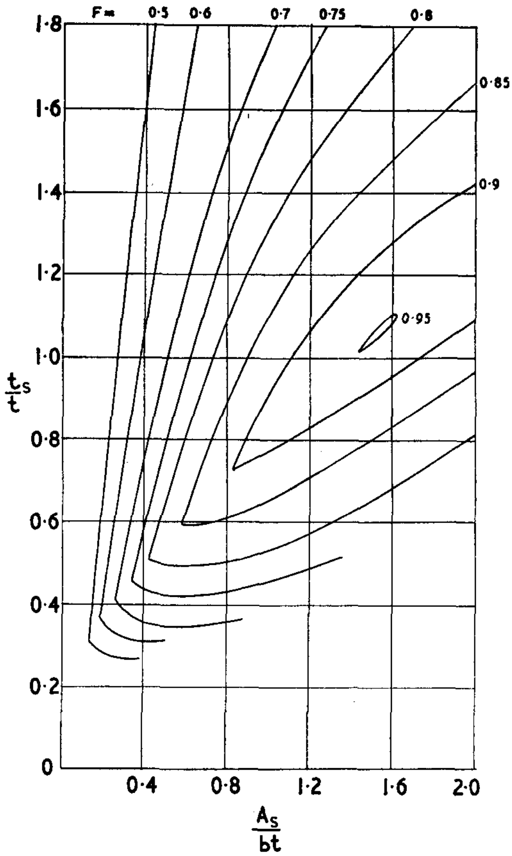

Fig.2.

Contours of $f\sqrt{\frac{L}{PE_T}}$ for Z-section stringers where initial buckling coincides with failure.

A useful feature now becomes apparent: if for any reason it is not possible to use the minimum-weight design, Fig. 3 shows the amount of weight penalty incurred. For example, if practical considerations demand that $b\left(\frac{E_T}{PL^3}\right)^{\frac{1}{4}}$ cannot be less than 2.0, the weight of the structure will inevitably be 21 per cent. greater than the lightest possible. If $b\left(\frac{E_T}{PL^3}\right)^{\frac{1}{4}}$ could be reduced to 1.5, the increase would only be 9 per cent.

Thus the efficiency of the structure from a weight point of view can be related quantitatively to geometrical limitations imposed by consideration of ease of manufacture, stiffness, ease of maintenance and so forth, and with such a quantitative relationship a

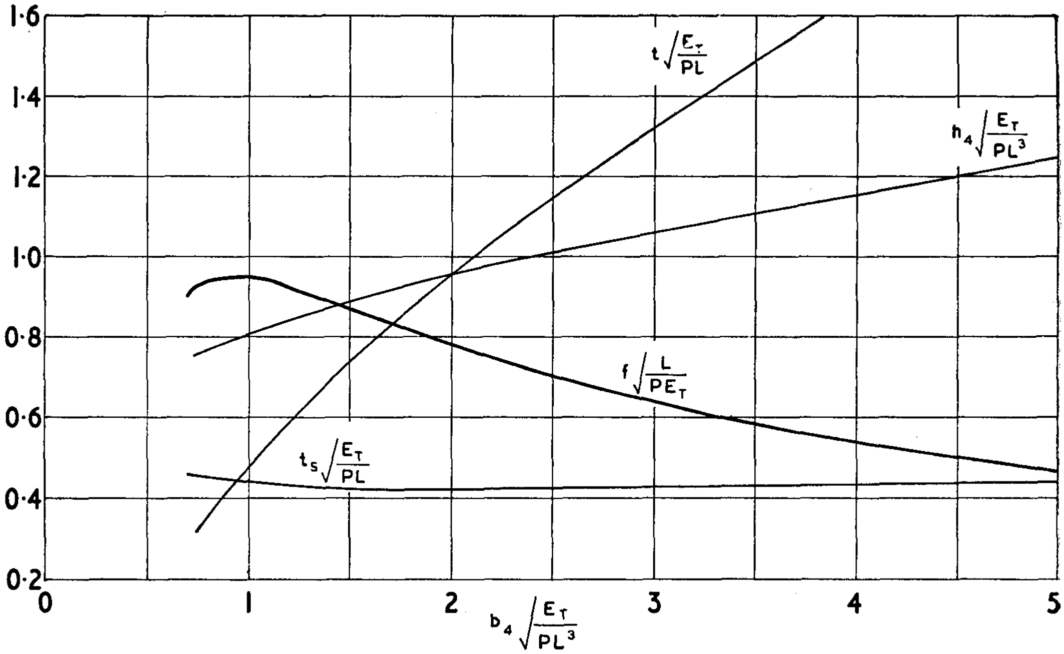

Fig. 3.

Design chart for optimum Z-section stringers where initial buckling coincides with failure.

compromise is more readily found in which allowance is made for all the various design conditions.

3.5. Effect of initial eccentricity

In practice the full theoretical value of $F\left(=f\sqrt{\frac{L}{PE_T}}\right)$ is not achieved: experimental results indicate that about 90 per cent. is an average realised value. This reduction is due to the effect of initial eccentricity of the structure, and (unlike the case of a simple strut) it occurs even at low design stresses.

Suppose that the Euler stress and skin buckling stress of a particular skin-stringer combination are both 30,000 lb./in.^{2}. When an actual compressive stress of 27,000 lb./in.^{2} is reached, it is likely that the bow due to initial eccentricity will give an additional stress (due to bending) of 3,000 lb./in.^{2} in the skin: thus the total stress in the skin will be 30,000 lb./in.^{2}, the skin will buckle and induce premature flexural failure.

A simple method of allowing for these effects is to design for a theoretical failing load per inch P (assuming no eccentricity) somewhat greater than that actually required. A more refined method, which gives slightly more efficient structures, is to provide a slightly greater margin of flexural stress than is allowed for local buckling, the actual margin depending on the standard of straightness of the stringers which can be achieved.

It should be noted that even when initial eccentricity has been allowed for, the theoretical optimum design is also the practical optimum. However, in an actual design it is convenient to eliminate errors in the estimated failing stress (due to approximations in the present theory, to the use of clad skins, or to initial eccentricity) by one or two panel tests.

Although the present analysis was intended only as a guide to determining the best skin-stringer combination, in practice it has provided a quite reliable estimate of the strength of panels, agreeing closely with test results.

4. Z-SECTION STRINGERS—SECONDARY LOCAL BUCKLING CAUSING FAILURE[4]

4.1. General

In this class of design, the skin buckles at some stress below the failing stress of the panel. Strictly speaking the mode of buckling involves movement of the stringer as well as the skin, and is in fact a complex one as mentioned in Section 3.1.

We now require to investigate the post-buckled behaviour, and so to arrange the geometry that the flexural instability stress is coincident with the critical stress for one or more secondary local buckling modes. These secondary local buckling modes may be of the nature of stringer local instability or stringer torsional instability, modified by restraint from the skin.

A rigorous solution to this problem is not yet in sight. Accordingly, approximate methods will be used; they are justifiable in the first place because, as has already been remarked, the weight of an optimum structure designed by such methods is very close to the true ideal minimum. A second justification appears at the end of the analysis; it is that the optimum structures determined here happen to be of such a type that the approximations used seem to be very close to the truth. We can make two general observations regarding efficient structures:—

(*a*) Coupling of modes reduces the lower instability stress and raises the higher, thus leading to an inefficient structure.

(*b*) The most efficient structure is that in which every type of instability which could cause failure occurs simultaneously.

By letting failure occur at more than about three times the skin buckling stress we achieve designs in which the stringers are relatively sturdy, and in which the coupling between skin buckling and stringer local distortion is negligible.

We now attempt to satisfy (*b*), namely that the local, torsional and flexural instability stresses shall be equal, and assuming that failure occurs at more than three times the skin buckling stress.

4.2. Instability stresses

Notation as in Section 3.1. It is assumed that $d/h = 0.3$.

(*a*) *Skin buckling stress*

$$f_b = f_b/f_o \times 3.62 E_T (t/b)^2 \quad . \quad . \quad (12)$$

(*b*) *Stringer local instability stress*

The following approximation is used for the edge stress at which stringer local instability occurs

$$f_L = 3.62 E_T (t_s/h)^2 \quad . \quad . \quad (13)$$

(*c*) *Stringer torsional instability stress*

The simple analysis developed by H. L. Cox[5] gives for the edge stress at which stringer torsional instability occurs

$$f_T = \frac{GJ - 2\sqrt{E_T H K}}{J'} \quad . \quad (14)$$

where

Stringer polar moment of inertia J' $= 0.633 h^3 t_s$

Stringer St. Venant torsion constant J $= 0.533 h t_s^3$

Bending torsional constant H $= 0.2 h^2 d^3 t_s$

Skin support stiffness $K = E t^3 / b$

Shear modulus G $= 0.385 E$

It should be noted that at edge stresses greater than about three times the skin buckling stress, the stringer rotation in the torsional instability mode is in the same direction for all stringers, owing to the effect of the skin buckles. We also note that the potential energy of the skin and lateral distortion of the stringer have been neglected; these are valid approximations in the present case.

(*d*) *Flexural instability stress*

We assume that after the skin buckles it has a constant

$$\frac{df_{\text{AVERAGE}}}{df_{\text{EDGE}}} \text{ of } 0.3.$$

For a stringer and a pitch of skin the tangent second moment of area is then

$$I_{NA} = \frac{0.19bt + 0.47ht_s}{0.3bt + 1.6ht_s} h^3 t_s. \quad . \quad (15)$$

The load per inch at flexural instability is then

$$P = \frac{\pi^2 E_T}{bL^2} \left(\frac{0.19bt + 0.47ht_s}{0.3bt + 1.6ht_s} \right) h^3 t_s \quad (16)$$

(*e*) *Actual load per inch*

The expression for the load carried is

$$bP = f_{\text{EDGE}} \left(1.6ht_s + bt \left(0.3 + 0.7 \frac{f_b}{f_{\text{EDGE}}} \right) \right) \quad (17)$$

4.3. Optimum designs

Using equations (12) - (17) we impose the condition that flexural, torsional and local buckling should occur at the same edge stress. Owing to the form of equation (14) it is not possible in this case to evolve a simple solution of the form

$$f \sqrt{\frac{L}{PE_T}} = \textit{constant},$$

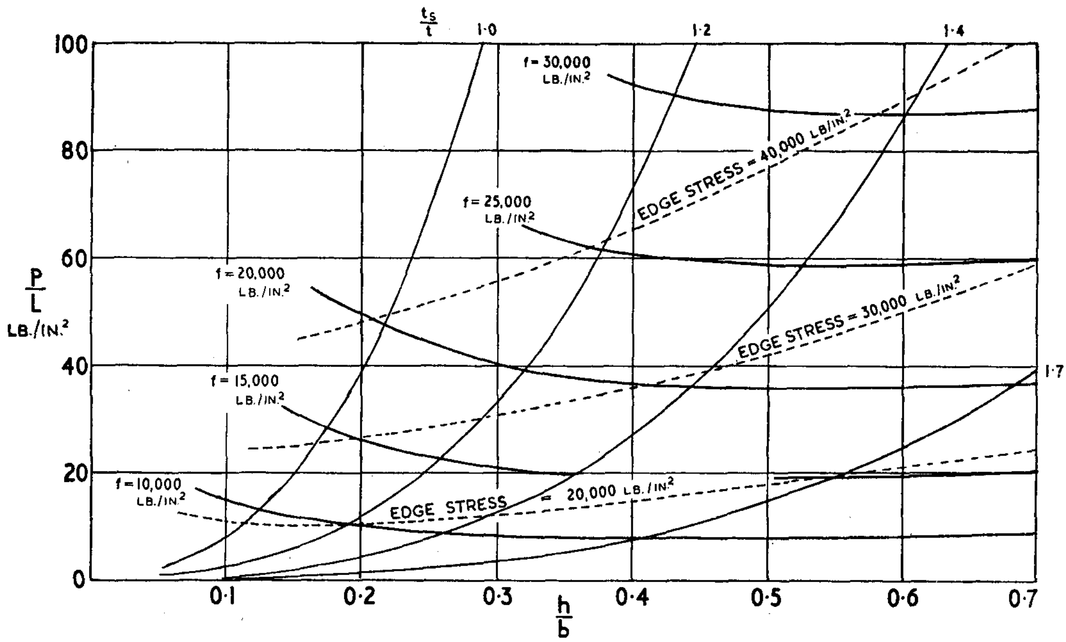

Fig. 4.
Design chart for optimum Z-section stringers where secondary buckling causes failure. (Light alloy material.)

as for the unbuckled structure. Instead, the equations lead to families of solutions which are shown in Fig. 4 for light alloy ($E_T = 10^7$). It is notable that the realised mean stress f again depends on the structure loading coefficient P/L, and the mean stress realised by the optimum structure varies very nearly as $\sqrt{(P/L)}$ as in the case of the unbuckled designs. If we adopt this as a standard of comparison we see that the value of $F \left(= f\sqrt{\frac{L}{PE_T}}\right)$ realised

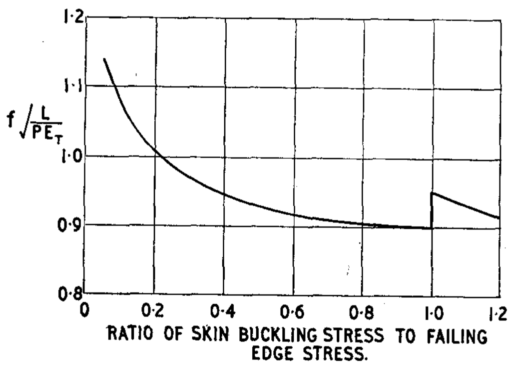

Fig. 5.
Effect of skin buckling on theoretical stress realised by optimum Z-stringer-skin combinations.

is about 1.20, which is greater than for unbuckled designs; in fact as far as failure by elastic instability is concerned the buckled structure is superior to the unbuckled one. The limitation on the buckled structure occurs when the edge stress begins to exceed that permitted by present materials, which it does for values of P/L exceeding about 100 lb./in.². For values of P/L above this, the unbuckled type of structure will tend to become more efficient.

4.4. Relative merit of buckling and non-buckling designs

In Fig. 5 the structural efficiency, as measured by $f\sqrt{\frac{L}{PE_T}}$, of the designs is plotted against the ratio of the skin buckling stress to the edge stress at failure. It is seen that the skin should either not be allowed to buckle at all, or should buckle at a comparatively low stress, if good structural efficiency is desired.

In Fig. 6 the best possible results using Z-section stringers are given for current light alloy material. It is assumed that the optimum design is used, and the mean stress thus achieved is plotted against the value of

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P/L. It is seen that the working stress, and hence the structural weight, is dictated entirely by the value of P/L which is used, and typical values of this quantity for various aircraft components are shown.

In practice, requirements of torsional stiffness may often cause the unbuckled design to be used. If a certain minimum skin thickness is required for stiffness, reference to Figs. 3 and 4 gives at once the stress realised by the best unbuckled design and the best buckled one, from which the choice of the better type of structure can be made for the particular case concerned.

5. OTHER TYPES OF STRINGER

5.1. Basis of comparison

We have now established that the structural efficiency of a skin-stringer combination can be measured by the constant F, which is $f\sqrt{\frac{L}{PE_T}}$ and which has a definite maximum value for any given type of stringer. The results of some similar calculations and tests for stringer sections other than Z are given below.

Type	Theoretical best value of $f\sqrt{\frac{L}{PE_T}}$	Realised value
Z-section, primary buckling causing failure	0.95	0.88
Z-section, secondary buckling causing failure ...	1.20	1.14
Hat section, primary buckling causing failure	0.96	0.89
Y-section, primary buckling causing failure	1.25	1.15

5.2. Structural efficiency of various types of stringer (optimum designs)

We see that of the range explored, the Y-section stringer is the most efficient, although for values of P/L less than 100 the buckled skin and Z-section is as good and in practice is more robust. Design charts similar to Fig. 3 can easily be plotted for each shape of stringer.

6. CHOICE OF MATERIALS

6.1. Extension of the theory

Consider a structure having a bay length L and sustaining a compressive load per inch width P. We have demonstrated that

$$f=F\sqrt{\frac{PE_T}{L}}$$

where F is a constant for the optimum design. The skin-stringer combination is therefore equivalent in weight to a skin of thickness T where

$$T=P/f=\frac{1}{F}\sqrt{\frac{PL}{E_T}} \quad . \quad (18)$$

If the density of the material is ρ, then the weight per unit area of skin-stringer surface is

$$W=\frac{\rho}{F}\sqrt{\frac{PL}{E_T}} \quad . \quad . \quad (19)$$

We see therefore that W is a minimum when the value of $\sqrt{E_T}/\rho$ is a maximum. (It is noteworthy that an analysis of tubular struts[6] by Pugsley also yields this result.)

On this basis Fig. 7 has been prepared to show the relative efficiency of optimum compression skin-stringer combination in various current materials. (It should be noted that for a given P and L the geometry of the structure will be different for each material, being adjusted to retain the optimum design for that particular material. It is assumed that such optimum designs realise $F=1.15$.)

6.2. Comparison of materials

The results show that present aluminium alloys are the most efficient at high values of the loading coefficient P/L; at lower values of P/L, magnesium alloys are more efficient; at lower values still, wood is efficient, and for very low values of P/L (appropriate to model aeroplanes) balsa wood is indicated as the best structural material.

The potentially most efficient material is that with the largest value of $\sqrt{E}/\rho$ and

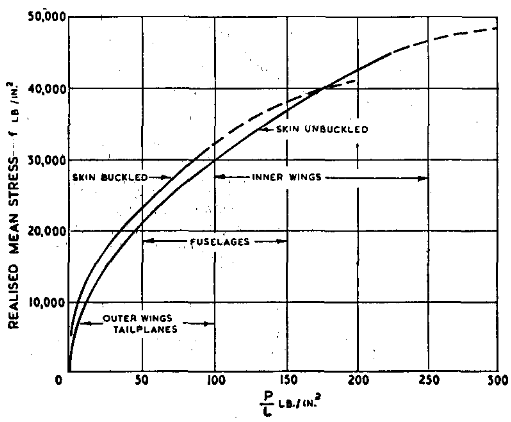

Fig. 6.
Theoretical stress realised by optimum Z-stringers and skin in aluminium alloy.

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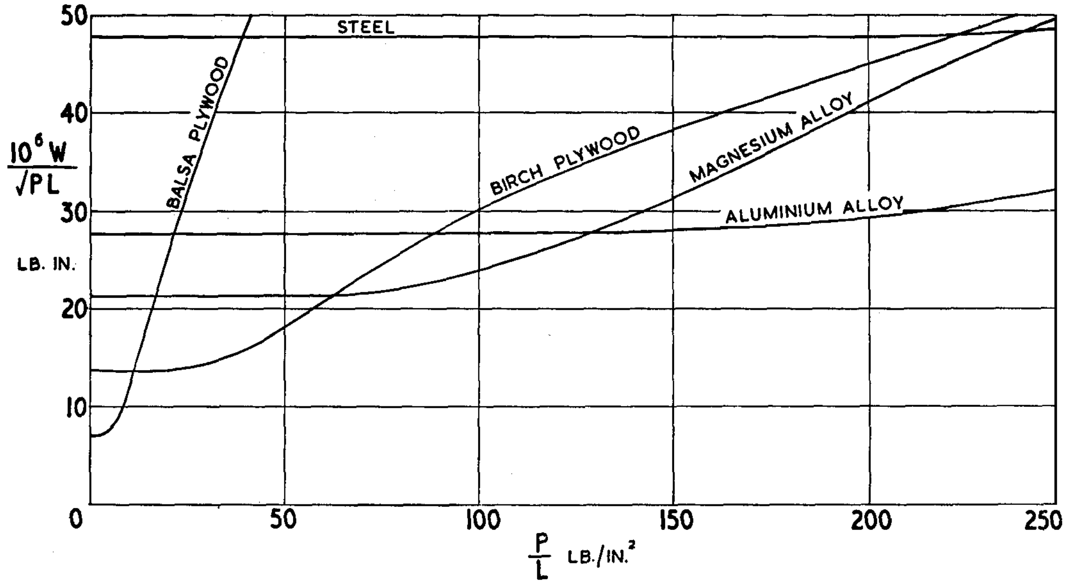

Fig. 7.
Relative weight of optimum skin-stringer combination in current materials.

magnesium alloys promise very well in this respect, since a 100 per cent. increase in proof stress would make them more efficient in general application than any increased strength aluminium alloy. A further promising development is an aluminium alloy with its Young's modulus increased by 30 per cent.: comparison with best present results is made in Fig. 8, where the effect of increased alloy strength is also shown.

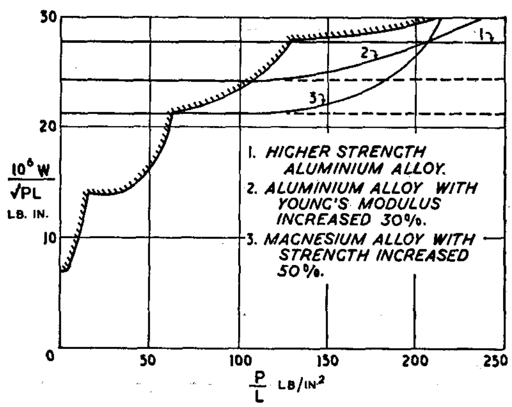

Fig. 8.
Relative weight of optimum skin-stringer combination in possible future materials.

7. CONSIDERATIONS AFFECTING RIB SPACING[7]

7.1. Object of analysis

Since the stress achieved by the skin-stringer combination varies as $\sqrt{(P/L)}$, a light combination for a wing is achieved by placing the ribs close together. When this is done, however, the weight of ribs may be considerable, and the structure of minimum weight in fact may be associated with a wider rib spacing, the increase of weight of the skin and stringers being more than offset by the saving on ribs.

Theoretical investigation of this problem is made difficult by the complex nature of the loads on a rib, and consequent ignorance of rib weights.

7.2. Determination of rib weight

The design of rib is a detail matter and the weight of a rib cannot be predicted in practice from pure theoretical considerations. Accordingly a more or less empirical approach must be adopted.

It can be demonstrated that in order to carry air load the strength of the rib must be directly proportional to the rib spacing, whereas to prevent general instability the stiffness of the rib must be inversely

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proportional to the rib spacing. Since some parts of the rib will be designed by one consideration, some by the other, and some by minimum practical sizes, it appears likely that the weight per rib will not vary very greatly with the rib spacing.

Figure 9 has been prepared on this basis for some weighed bracing ribs in wings, tailplanes and fins on various aircraft. The low scatter of the values indicates that the ignoring of the effect of rib spacing does not introduce any serious error. In default of better information we will then assume that the weight of a rib does not depend on the rib spacing within practical limits.

7.3. Determination of best rib spacing

Consider unit width of compression structure carrying a load per inch P at a mean stress f. The ribs are a distance L apart, while each rib has a depth D and is equivalent in weight to a plate of thickness T_R. We will assume for the present that all stresses are within the elastic range.

Then

$$f=F\sqrt{\frac{PE}{L}}$$

and the equivalent thickness T of the skin-stringer combination is

$$T=P/f=\frac{1}{F}\sqrt{\frac{E}{PL}}$$

When we include the weight of ribs, the total equivalent thickness becomes

$$\left(T+\frac{DT_R}{L}\right)$$

Now

$$T+\frac{DT_R}{L}=\frac{1}{F}\sqrt{\frac{PL}{E}}+\frac{DT_R}{L}$$

For the structure of minimum weight L must be chosen so as to make this quantity a minimum. Differentiating with respect to L and equating to zero we get (assuming F and E constant)

$$\frac{1}{2F}\sqrt{\frac{P}{EL}}-\frac{DT_R}{L^2}=0 \quad . \quad (20)$$

and

$$L=\left(\frac{4F^2D^2T_R{}^2E}{P}\right)^{\frac{1}{3}} \quad . \quad . \quad (21)$$

This is the optimum rib spacing, and we find that

Weight of skin and stringers per unit surface area $=\frac{2}{3}\rho\left(\frac{DT_RP}{E}\right)^{\frac{1}{3}}\left[\left(\frac{1}{4F^2}\right)^{\frac{1}{3}}+\left(\frac{2}{F^2}\right)^{\frac{1}{3}}\right]$

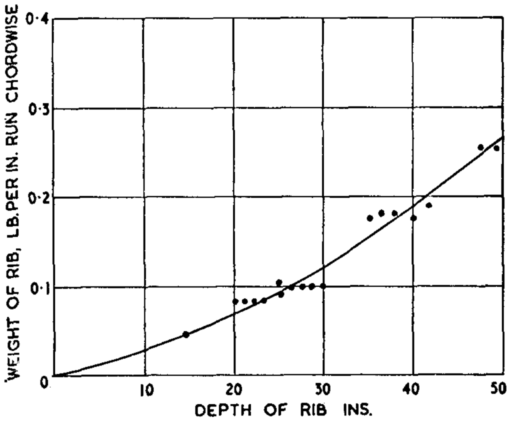

Fig. 9.
Typical weights of bracing ribs.

Weight of ribs per unit skin surface area

$$=\frac{1}{3}\rho\left(\frac{DT_RP}{E}\right)^{\frac{1}{3}}\left[\left(\frac{1}{4F^2}\right)^{\frac{1}{3}}+\left(\frac{2}{F^2}\right)^{\frac{1}{3}}\right]$$

7.4. Practical application

The foregoing analysis shows that the weight of ribs should be equal to about half the weight of the surfaces designed by compression. The analysis assumes that

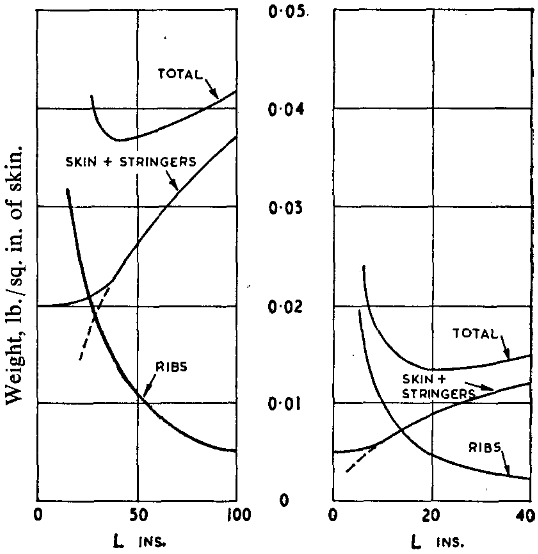

Fig. 10.
Examples of the variation of total weight with rib spacing.

P=10,000 lb./in.	P=2,500 lb./in.
Z-stringers.	Z-stringers.
Alum. Alloy.	Alum. Alloy.
Ribs 60 in. deep.	Ribs 25 in. deep.

secondary stringer stresses due to lateral loading are unimportant: this must be checked in each case but is generally an acceptable approximation.

It is also noteworthy that the depth and equivalent thickness of the ribs have as much influence on the configuration and weight of the structure as the load per inch itself.

In many cases, owing to practical considerations, the quantities F and E will vary somewhat with rib spacing; in such cases the nature of their variation always causes the optimum weight of ribs to be slightly less than half that of the skin and stringers. A satisfactory and rapid approach is to consider a range of values of L and find the total weight for each value; this has been done in Fig. 10 for a typical wing structure and for a large tailplane as examples.

By the nature of an optimum it is permissible to use a frame spacing slightly different from the optimum one without much weight penalty. It is advisable in practice to use a spacing slightly greater than that for pure minimum weight, since a simpler and more robust structure results.

8. CONCLUSIONS

No analysis of this subject can claim to be comprehensive, and the present one is no exception. Simplifications of the problem have been made, with the object of producing a fairly quick approximation to the optimum skin-stringer-rib combination; this simplification is essential because of the large number of variables involved. One must expect more detailed methods and a certain amount of testing to be used when checking the overall strength and stiffness once a structure has been fixed. It is of interest to note that the divergences in strengths from those found by the present simplified theory have been found, so far, to be quite small.

The importance of the structure loading coefficient P/L has been demonstrated, and design charts of the type derived here (based on P/L) permit a good approximation to the final optimum structure to be quite rapidly obtained, while continuous allowance is made for other practical design factors.

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Part of the analysis has been developed and published by workers in America, at the same time as it was developed independently in this country.

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