

## Quantum Mechanics

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### Introduction

When talking about quantum mechanics, it is often said that matter has both wave-like and particle-like properties. Above all, that approach is an attempt to explain the non-intuitive aspect of the subject with familiar notions. In fact, a quantum-mechanical particle is an object that is a curious combination of these two aspects — an object the exact likes of which we do not see in the familiar macroscopic environment and which is therefore inherently non-intuitive. Still, quantum mechanical particles are most successfully treated like waves, and it turns out that some seemingly particle-like properties could still be explained using wave considerations. Let us dive in! The text in small print could be omitted on the first reading, the majority of IPhO problems could be solved without knowing these facts.

### Wavefunction

*In classical mechanics, the state of a particle without internal degrees of freedom<sup>1</sup> can be completely described by its **momentum and co-ordinate**; in quantum mechanics, the complete state of a particle is described by its so-called **wavefunction**  $\Psi$ , which is generally a complex number and which could be represented, for example, as a function of co-ordinates and time, like so:  $\Psi = \Psi(x, y, z, t)$ . The probability to find the particle at time  $t$  at position with co-ordinates  $x, y$  and  $z$  is then proportional<sup>2</sup> to the modulus-squared of the complex wavefunction,  $|\Psi|^2 = \Psi(x, y, z, t)\Psi^*(x, y, z, t)$ , where  $\Psi^*$  is the complex conjugate of  $\Psi$ . What is actually meant by "finding" the particle at a given point of space is actually a tricky question, we will return to it later. In short, this is a short-hand for saying that a measurement has been conducted to find the position of the particle, and the result of that measurement was  $(x, y, z)$ .*

The example of the photon could be helpful to understand the idea of the wavefunction: the real part of the photon's wavefunction is the electric field vector and the imaginary part is the magnetic field vector (this statement is true in the Gauss system of units; in SI, the wavefunction's imaginary part is the product of the magnetic induction and the speed of light). The probability to find a photon at any point is proportional to the electromagnetic energy density (light intensity) at that point.

Comparing the electron and the photon we should note that for photons one uses a vectorial wavefunction, for electrons — a scalar one. The difference arises because the angular moment of a photon about an axis through its "centre of mass" can have three different values (which correspond to linear and two circular — clockwise and counter-clockwise — polarisations). We will not discuss this nuance further.

### Energy and momentum in Quantum Mechanics

In quantum mechanics, the so-called *stationary states* or *eigenstates of energy* play an important role. In such a state, the particle has a well-defined single value of energy  $E$  (the so-called eigenvalue).

There are at least two good reasons for this. Firstly, any state of a quantum-mechanical system is representable as a superposition of stationary states (exactly in the same way as the random motion of connected oscillators is representable as a superposition of normal modes). Secondly, generally a quantum system that has been brought of the state of equilibrium will quickly find itself in a state with the lowest possible energy such that no conservation laws are violated. After all, the lowest energy is also a well-defined, unique energy.

It turns out (the reason being the Schroedinger equation, which could be considered as a postulate of quantum mechanics), that in a state with a well-defined energy  $E$  the wavefunction evolves as follows:  $\Psi(x, y, z, t) = \psi(x, y, z)e^{-iEt/\hbar}$ , so that the wavefunction oscillates with angular frequency  $\omega$ , which is related to the energy by the important formula

$$E = \hbar\omega = h\nu. \quad (1)$$

The linear frequency  $\nu = \omega/2\pi$ ; the quantity  $h$  is known as Planck's constant, while  $\hbar = h/2\pi$  is better known as the *reduced Planck's constant*.

Note that the energy in (1) is the total (kinetic plus potential) energy of a quantum-mechanical system (e.g. of a particle), while the zero-level of potential energy could be freely chosen. Note how a change of the zero level of the potential energy by a value  $U$  means that the wavefunction gets multiplied by an extra factor  $e^{iUt/\hbar}$  (this statement also holds for non-stationary states), which does not at all alter the physically tangible quantity — the probability of finding the particle — because the modulus of this factor is still just unity. There are some important practical lessons to be learned from this:

- (a) For motion and collisions of non-relativistic electrons and neutrons, one can use the expression for non-relativistic kinetic energy,  $p^2/2m$ , as the particles don't vanish and it makes sense to measure the kinetic energy with respect to the motionless state (in other words, the rest mass energy can be ignored);
- (b) For photon absorption and emission, it is natural to use the relativistic energy expression  $E = mc^2$ ;
- (c) The zero level of potential energy can be chosen arbitrarily.

In quantum mechanics, energy conservation law could be regarded as the condition for resonance. Suppose there is a system with two energy levels  $E_1 < E_2$ . Suppose the system transfers from the low-lying energy state to the other state by absorbing a particle (a photon for concreteness) with energy  $E_3$ . Energy conservation law then reads  $E_2 = E_1 + E_3$ , which could be re-written using angular frequencies in the form  $\omega_2 - \omega_1 = \omega_3$ . On the left hand side of the equation is the angular frequency of the system's wavefunction, if we chose the zero level of energy to be at  $E = E_1$ , while on the other side of the equation is the frequency of the photon. Therefore the interaction (absorption of the photon) can only happen if the condition for resonance is fulfilled: the electromagnetic frequency is equal to the difference in frequencies of the two stationary states.

Consider a quantum-mechanical simple harmonic oscillator: a particle that moves in parabolic potential (potential energy is proportional to the square of the displacement from equilibrium). Suppose the classical angular frequency of this particle is  $\omega_0$ . This oscillator can go from one quantum-mechanical energy

<sup>1</sup>Such an internal degree of freedom could be, for example, rotation about its own axis.

<sup>2</sup>In non-relativistic quantum mechanics the particles are never created nor destroyed, and the probability to find the particle in all space is therefore unity. Hence the integral of  $|\Psi|^2$  over all space must also be equal to unity. Performing the integral fixes the constant in front of the wavefunction — we are normally referring to just that constant when we say, in the following, that the wavefunction is proportional to something.

level ( $E_i$ ) to another ( $E_j$ , corresponding to higher amplitude oscillations) by absorbing ( $n = 1, 2, 3 \dots$ ) photons. These photons are electromagnetic waves, which have to be in resonance with the oscillator, so that the electromagnetic angular frequency has to be equal to the eigenfrequency of the oscillator  $\omega_0$ . From the energy conservation law  $E_j - E_i = \hbar n \omega_0$  we deduce that the energy levels of the simple harmonic oscillator have to be of the form

$$E_n = \hbar n \omega_0 + C.$$

A rigorous solution of Schroedinger's equation shows that for the zero level of potential energy at the minimum of the potential, the constant  $C = \hbar \omega_0 / 2$ .

This result can easily be generalised unto oscillators with  $m$  degrees of freedom. From classical mechanics we know that in this case, the system has  $m$  eigenfrequencies  $\omega_j$ ,  $j = 1, 2, \dots, m$ .  $j$ -th eigenfrequency can be excited if the resonance condition is satisfied, so that the electromagnetic frequency is  $\omega_j$ . Therefore one has to employ  $m$  different integers  $n_j$  to describe the stationary energy levels of such an oscillator:

$$E = \sum_{j=1}^m \hbar \omega_j n_j + C.$$

This energy expression looks just as if we had different particles — of energies  $\omega_j$ ,  $j = 1, 2, \dots, m$  — and the integers  $n_j$  describe the numbers of these particles. In case of elastic oscillations (standing waves) of the crystal lattice these so-called *quasi-particles* (so not truly real, physical particles) are called *phonons*.

We have seen that the energy of the oscillator is *quantized*, it can only assume certain discrete values.

As we have seen, in a stationary state the time-dependence of the wavefunction is known, and therefore the interesting part is its spatial dependence  $\psi(x, y, z)$ . It turns out (from Schroedinger's Equation) that in a state with uniquely defined momentum  $\vec{p}$ , the wavefunction is a sinusoidal plane wave,  $\psi = e^{i\vec{k}\vec{r}}$ , where the wave vector  $\vec{k}$  is related to momentum through the formula

$$\vec{p} = \hbar \vec{k} \quad (2)$$

Wave vector points along the direction of the wave's propagation, and its modulus is  $k = 2\pi/\lambda$ , where  $\lambda$  is the wavelength. The function  $\psi = e^{i\vec{p}\vec{r}/\hbar}$  is called an *eigenfunction* of momentum.

Note that for a particle, the group velocity of the wavefunction considered as a wave is equal to the particle's classical velocity:  $\hbar\omega = p^2/2m = (\hbar k)^2/2m + U$ . Upon differentiating, this gives  $v_g = \frac{d\omega}{dk} = \hbar k/m = p/m = v$ . This also holds for photons:  $\hbar\omega = mc^2 = pc = \hbar kc$ , and  $v_g = \frac{d\omega}{dk} = c$ .

### Wavefunction as a function of momentum

As seen before, the wavefunction of a particle with well-defined momentum is a sinusoidal plane wave that fills the whole space, so that the position of the particle is entirely unknown.

The square of the modulus of the wavefunction  $|\psi(\vec{r})|^2 \equiv |\psi(x, y, z)|^2$  gives the probability to find the particle at a given point. This statement should be considered as a postulate given on empirical (experimental) basis. The question of what is actually meant by "finding the particle" will be addressed later. For the moment, it is important to note that the wavefunction can not only be given as a function of the co-ordinate  $\vec{r}$ , but also as that of the momentum  $\vec{p} \equiv (p_x, p_y, p_z)$ , or  $\psi = \psi(\vec{p}) \equiv \psi_p$ . Here, we are dealing with a mathematical trick (called "Fourier analysis"), where an arbitrary wavefunction can be represented as the sum of various states of well-defined momentum,  $\psi_p e^{i\vec{p}\vec{r}/\hbar}$ . In this representation, the square of the absolute value of each component's amplitude  $|\psi_p|^2$  gives the probability to find the particle with the corresponding momentum  $\vec{p}$  (generally, the

amplitude  $\psi_p$  is complex number whose argument gives the phase-shift of the corresponding component wave).

Let us approach the issue mathematically. Fourier analysis tells us, that any function  $f(x)$  could be represented as the sum of sinusoidal functions,  $f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f_k e^{ikx} dk$ , where the quantity  $f_k = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$ , which depends on the wave vector  $k$ , is called a Fourier component of the function  $f(x)$ . Changing the variables  $k \rightarrow p/\hbar$ , we can write the given integral for the wavefunction in the form

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi_p e^{ipx/\hbar} dp, \quad (3)$$

where the factor

$$\psi_p = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-ipx/\hbar} dx. \quad (4)$$

Formulae (3,4) are not for learning by heart at this point, but to supply evidence to this statement: each wavefunction can be represented as a superposition of states (wavefunctions  $\psi = e^{ipx/\hbar}$ ) such that the momentum has a well-defined unique value  $p$ ; the amplitudes  $\psi_p$  of those states can be regarded as providing the dependence of the given wavefunction on momentum.

To make a further comparison, consider the state where the particle has a well-defined unique co-ordinate  $x_0$ ; then its wavefunction is  $\psi = \delta(x - x_0)$ . The function  $\delta(x)$  is called the Dirac delta-function (the area under the graph of this function is 1 and it only has a non-zero value at point  $x = 0$ ).

Now we can express an arbitrary wavefunction  $\psi(x)$  as a superposition where every component has a unique co-ordinate:

$$\psi(x) = \int \psi(x_0) \delta(x - x_0) dx_0.$$

Note that on the right hand side of the equality sign, the prefactor in front of the state of well-defined position  $\delta(x - x_0)$  is the initial function  $\psi(x)$  at position  $x = x_0$ , whose modulus-squared is the probability to find the particle at  $x_0$ . Therefore it should not be surprising that in (3), the modulus-squared of the prefactor in front of  $e^{ipx/\hbar}$  (which is  $|\psi_p|^2$ ) gives the probability to find the particle with momentum  $p$ .

The transfer from the function  $\psi(x)$  to another  $\psi_p$  could be regarded as a spatial rotation in the functional (*Hilbert*) space (where the function  $\psi$  is a vector), because just as in the ordinary spatial rotation formulae, the new co-ordinates of the vector  $\psi_p$  are represented through the linear combination of the old co-ordinates  $\psi(x)$ . The only difference is that as the number of co-ordinates in ordinary spaces is finite, the linear combination is written with a sum; in the Hilbert space, however, the set of co-ordinates is uncountable (is a continuum), and therefore the sum is replaced by the integral. While in the ordinary space a co-ordinate-independent vector is signified with a small arrow upon the symbol, in the Hilbert space the notation  $|\psi\rangle$  is normally used. Therefore  $|\psi\rangle$  represents the wavefunction for which it is undecided or unimportant whether we should investigate its dependence based on the co-ordinate  $x$ , momentum  $p$  or a different physical quantity altogether.

The formulae above assume a one-dimensional motion, which can be described with one co-ordinate  $x$  and momentum  $p$ . In three dimensions, a single integral is replaced by triple integration (over  $x$ ,  $y$  and  $z$  or over  $p_x$ ,  $p_y$  and  $p_z$ ), the product  $px$  by the scalar product  $\vec{p} \cdot \vec{r}$  and the factor  $\frac{1}{\sqrt{2\pi\hbar}}$  by a factor  $(2\pi\hbar)^{-3/2}$ .

### The uncertainty principle

The uncertainty principle is a purely mathematical result, which comes from the connection between the two representations of the wavefunction: in the co-ordinate and momentum space (formulae 3,4). It is possible to prove that if one defines the co-ordinate and momentum uncertainties  $\delta x$  and  $\delta p$  as root-mean-square deviations from the corresponding average values  $\bar{x}$  and  $\bar{p}$ , then the following inequality holds:

$$\delta x \cdot \delta p \geq \hbar/2. \quad (5)$$

Mathematically, one can write these definitions as

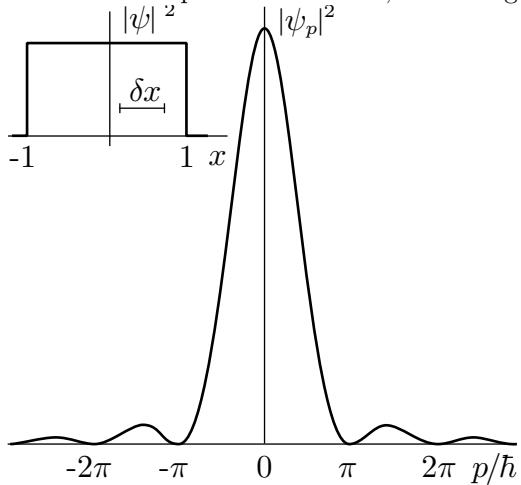
$$\begin{aligned} \bar{x} &\equiv \int_{-\infty}^{\infty} |\psi(x)|^2 x dx, & \bar{p} &\equiv \int_{-\infty}^{\infty} |\psi_p|^2 p dp, \\ \delta x &\equiv \sqrt{\int_{-\infty}^{\infty} |\psi(x)|^2 (x - \bar{x})^2 dx}, & & \\ \delta p &\equiv \sqrt{\int_{-\infty}^{\infty} |\psi_p|^2 (p - \bar{p})^2 dp}. \end{aligned}$$

In inequality (5) equality is observed if the wavefunction is a Gaussian,  $\psi(x) = \pi^{1/4}(\delta x)^{-1/2} e^{-x^2/4(\delta x)^2}$ ; in all other cases, it is a strict inequality.

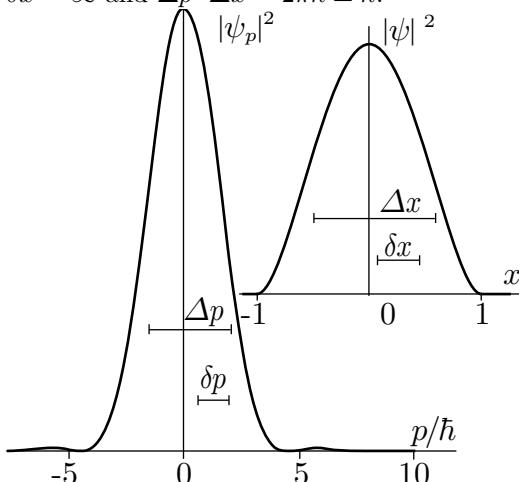
The uncertainty principle is often used in quantum-mechanical estimations, where one replaces the inequality sign with an approximate equality and the root-mean-square deviations with characteristic widths  $\Delta x$  and  $\Delta p$ . In which case would we get a more precise estimate, is it with the expression

$$\Delta x \cdot \Delta p \sim h, \quad (6)$$

$\Delta x \cdot \Delta p \sim \hbar$  or  $\Delta x \cdot \Delta p \sim \hbar/2$ ? The answers could differ by up to  $4\pi \approx 13$  times and an error this big should be avoided even in estimates (even though when estimating, one cannot discard as wrong even those answers that differ from the true answer by a factor of, say,  $(2\pi)^3 \approx 248$ , by carefully thinking through one's actions it is normally possible to achieve the difference by less than a factor of two). Let us investigate this question on two concrete example wavefunctions, see the figures.



The first figure describes an equal probability for the particle to be found along the  $x$  axis if its co-ordinate is in the range  $-1 \leq x \leq 1$ , while outside of this range the particle cannot possibly be found. This is exactly the type of distribution that arises when an electron, photon etc. passes through a single slit; in that case the probability distribution as a function of momentum,  $|\psi_p|^2$ , is exactly the same as the intensity distribution in the diffraction pattern (on the screen). We will normally consider the typical width of the diffraction maximum to be of the order of  $\Delta p = \pi\hbar$  (it is approximately the width of the peak at half-maximum). The probability  $|\psi_p|^2$  oscillates and the oscillation amplitude is inversely proportional to the square of momentum (see fig.); therefore the root-mean-square deviation is not finite,  $\delta p = \infty$ . The uncertainty in the position is, according to the integral above,  $\delta x = 1/\sqrt{3}$ , which is about 3,5 times smaller than the width of the slit  $\Delta = 2$ . Thus, we find  $\delta p \cdot \delta x = \infty$  and  $\Delta p \cdot \Delta x = 2\pi\hbar \equiv h$ .



The second figure represents the probability distribution  $|\psi(x)|^2 = (1 - x^2)^2$  for  $|x| \leq 1$  and  $|\psi(x)|^2 = 0$  for  $|x| \geq 1$  of finding the particle. In this case the wavefunction  $\psi(x)$  is continuous and therefore the probability distribution in momentum space is localised more strongly than in the case of the slit, and the root-mean-square deviation of momentum is, in fact, finite:  $\delta p \approx 1,34$ . By computing the integral it is not hard to find  $\delta x = 1/\sqrt{7}$ , so that  $\delta x \cdot \delta p \approx 0,507\hbar$  (and the solution is close to the absolute minimum  $\hbar/2$ ). Calculations show that the widths of the two peaks at half-maximum are  $\Delta p \approx 3,6\hbar$  and  $\Delta x \approx 1,1$ . Thus  $\Delta x \cdot \Delta p \approx 4\hbar \approx 0,63h$ .

The two given examples show that the estimate (6) is fitting when one takes the characteristic widths of the interval as the uncertainties  $\Delta p$  and  $\Delta x$ ; the estimate (5) is valid for the root-mean-square deviations (unless the localisation is weak and the root-mean-square deviation is actually infinite).

The uncertainty principle for the pair  $p_x$  and  $x$  holds because the wavefunction for a state of definite  $p_x$  is a sinusoidal function of  $x$  (and any state can be represented as a superposition of sinusoids for different  $p_x$ ). Obviously the same thing holds for  $p_y$  and  $y$  as well as for  $p_z$  and  $z$ , but if we remember formula (1), then it also holds for  $E$  and  $t$  (and, in fact, also for angular momentum and the rotation angle) — the variables forming such pairs are called *conjugate* variables. The connection between  $E$  and  $t$  is typically useful for describing for the excited state of an atom or a molecule: if theoretically the value of the energy in a stationary state is  $E_n$ , then due to instabilities, collisions etc. the atom or molecule cannot reside in that state for longer than some characteristic time  $\tau$ . Thus, its energy in the excited state will not be exactly  $E_n$ , but can be in the range of width  $\Gamma$  around the central value  $E_n$ , where

$$\Gamma \cdot \tau \sim h. \quad (7)$$

The quantity  $\Gamma$  is called the width of the energy level and  $\tau$  is known as the excited level's lifetime.

#### Quasi-classical approximation

If one wishes to obtain the precise forms of the stationary state wavefunctions and corresponding energy levels, one will generally have to solve Schrödinger's equation. However, one can advance quite far (and sometimes even get the correct result) by employing the so-called quasi-classical approximation (also known as the WKB(J) approximation in the English-language literature). In this approach, one first considers a particle moving in a known potential as a classical particle and finds the dependence of its momentum  $p$  on co-ordinate  $x$ . Then one says that the particle's wavefunction is almost a sinusoid with a variable wave vector:  $\psi(x) \approx \exp[\frac{i}{\hbar} \int p(x)dx]$ . This is, of course, an approximate relationship — the equality only holds if the particle is free (the momentum is constant). Still, the equality almost holds if the relative change of momentum in one wavelength is small [so that  $|p(x) - p(x + \hbar/p)| \ll p(x)$ ]. Suppose this variable-wavelength wave propagates along the  $x$ -axis or along a circular trajectory (e.g. in the case of an electron orbiting around an atom). If the particle moves in a potential well so deep that its walls keep reflecting it back, then the wave is also reflected and starts bouncing (oscillating) back and forth. The wave pattern is stable if there is an integer number of wavelengths in one oscillation period (then a stationary wave is formed). For example, if we are dealing with a potential well

with vertical walls and flat horizontal bottom of length  $L$ , then the momentum inside the well is constant and the stationary wave condition gives  $2Lp = nh$ , which means that  $p = nh/2L$  and the  $n$ -th energy level is  $E_n = p^2/2m = n^2\hbar^2/8L^2m$ .

In a hydrogen atom, an electron of mass  $m$  orbits around the nucleus in a symmetric potential well  $U = -kZe^2/r$  (where  $k = \frac{1}{4\pi\varepsilon_0}$ ). The kinetic energy of an electron moving in a circular orbit is  $p^2/2m = -U/2$ , therefore  $p^2r = kmZe^2$ . We write the stationary wave condition as  $2\pi rp = 2\pi\hbar n$  (where  $n$  is the number of stationary waves), and then  $rp = n\hbar$ .

This last relation shows that the angular momentum  $rp$  is quantized. It turns out that this conclusion is more general and is not only characteristic of circular orbits: the angular momentum with respect to a fixed axis can only be an integer number of  $\hbar$  (an exception to this rule are the so-called *Fermi particles* or *fermions* (for example, the electron): the internal angular momentum — so-called *spin* — of fermions can only be a half-integer number of  $\hbar$ . In the case of electrons, it is always  $\pm\hbar/2$ ).

Combining these two last results, we find  $p = kmZe^2/n\hbar$  and therefore the total energy

$$E_n = U/2 = -p^2/2m = -\frac{m}{2}(kZe^2/n\hbar)^2.$$

## Quantum tunnelling

Quantum tunnelling is a striking non-classical effect, where a particle of energy  $E$  is capable of passing through a potential energy barrier  $U > E$  in a way, where beneath the barrier it acquires a negative kinetic energy  $p^2/2m = E - U$ . A negative kinetic energy means an imaginary momentum and wave vector  $k = \frac{i}{\hbar}\sqrt{2m(U-E)}$ , and the wavefunction is no longer a sinusoid, but a decaying exponential function:  $\psi(x) = e^{-x\sqrt{2m(U-E)/\hbar}}$ . Thus, if the width of the barrier  $L$  is large [ $L\sqrt{2m(U-E)} \gg \hbar$ ], then the particle's wavefunction drops exponentially below the barrier, and the particle cannot pass through (is reflected). On the other hand, if  $L\sqrt{2m(U-E)} \sim \hbar$ , then the wavefunction still decays (the particle is reflected back with a certain probability), but not extremely so: there is a non-negligible probability of *tunnelling*.

A neat approach relates quantum tunnelling to the uncertainty principle for energy: the condition for tunnelling could be rewritten in the form  $2(U-E) \cdot \frac{L}{\sqrt{2(U-E)/m}} \sim \hbar$  and could be interpreted as follows: the particle “borrows”, for a short time  $\tau = L/v$  [where  $v = \sqrt{2(U-E)/m}$ ] the energy  $2(U-E)$ , out of which  $U-E$  is spent to overcome the barrier and a further  $U-E$  is left as kinetic energy to pass the barrier; the energy could not be borrowed for any longer than the uncertainty principle allows.

## Measurement in quantum mechanics

The classical or so-called Copenhagen interpretation of quantum mechanics states that as a result of a quantum-mechanical measurement, the wavefunction *collapses*. Suppose an experiment to measure the particle's momentum is performed. Before the measurement, the wavefunction of the particle was a superposition of states of well-defined momentum,  $\psi = \int \psi_p e^{ipx/\hbar} dp$ .

Upon the measurement, a certain result  $p_m$  is obtained: thus, from the state  $\int \psi_p e^{ipx/\hbar} dp$  a state  $\psi_{p_m} e^{ip_m x/\hbar}$  has been obtained. A repeated measurement of the particle's momentum from now on will always find the same value  $p_m$  of the particle's momentum, unless the particle is affected by something else. This change of state from a superposition of *eigenfunctions* of the measured quantity — which also included the eventual result among many other contributions — into the state corresponding to the eigenfunction of *only* the measured result

is exactly what is known as the *collapse* of the wavefunction. Recall that an eigenfunction of a physical quantity is defined as a wavefunction that the particle must have in order to have this physical quantity well-defined. The probability that any of these possible results is found is proportional to the modulus-squared of the corresponding amplitude in the initial superposition. An important lesson to be learnt is that every measurement changes the state of the system.

This interpretation works perfectly well in practice, yet can be unsatisfying from the philosophical point of view. The issue is that the interaction of a particle with a macroscopic measurement device is treated differently from inter-particle interactions. In this last case, no collapse happens. For concreteness, let us look at the interaction of two particles, e.g. electrons. If one particle is described by the wavefunction that is (in the case of spatial representation) a function of three spatial co-ordinates and time,  $\psi(\vec{r}, t)$ , then the two-particle wavefunction is already a function of six co-ordinates:  $\Psi = \Psi(\vec{r}_1, \vec{r}_2, t)$ , where  $\vec{r}_1$  and  $\vec{r}_2$  are the two particle's respective position vectors. If the particles do not interact, this seven-variable function becomes a product of two four-variable functions,  $\Psi(\vec{r}_1, \vec{r}_2, t) = \psi_1(\vec{r}_1, t)\psi_2(\vec{r}_2, t)$  (much in the same way as the combined probability of two independent events is the product of two individual probabilities); in case of interactions such separation of variables is no longer possible: one has to solve the seven-dimensional Schroedinger equation. Interactions of more than two particles are treated analogously, and for  $n$  particles one deals with a wavefunction that depends on  $3n+1$  variables. And there is not even a mention of the collapse of the wavefunction! So how does this collapse come into play? Classical interpretation leaves this question unanswered. The tumultuous interplay between the macroscopic and microscopic worlds gives birth to various paradoxes, such as the Schrödinger's cat and the teleportation (Einstein-Podolsky-Rosen) paradox.

An alternative interpretation of quantum mechanics, the so-called *many-worlds interpretation* due to Hugh Everett and others, is capable of justifying both the collapse of the wavefunction and the mentioned paradoxes. The practical implications of this interpretation are mostly the same as in the classical interpretation. An additional advantage, however, is that the modulus-squared of the wavefunction is identified with the probability not by the means of a postulate, but via a mathematical proof. In this interpretation, the physical reality is postulated to correspond to the Universe's wavefunction  $\Psi$ , which is a function of co-ordinates of all the particles in the Universe (including those comprising the living beings); it evolves all the time according to the Schrödinger equation and experiences no collapses whatsoever. Thus,  $\Psi$  includes all the possible measurement results for any conceivable measurement. For simplicity, let us consider the wavefunction of the combined system “observer (experimenter)+particle” and let us suppose that there are only two possible outcomes to this measurement, ‘1’ and ‘2’. Before and after the measurement, the observer and the particle do not interact, and so one can separate the variables and represent the system's initial state as the product  $|M_0\rangle|O_0\rangle$ , where  $|M_0\rangle$  is the initial state of the observer, while the initial state of the particle is the superposition of two possible states,  $|O_0\rangle = \alpha|O_1\rangle + \beta|O_2\rangle$ ; here  $|O_1\rangle$  and  $|O_2\rangle$  are the particles' states in which the measured quantity is well-defined. After the measurement, the state of system assumes the form  $\alpha|M_1\rangle|O_1\rangle + \beta|M_2\rangle|O_2\rangle$ , where  $|M_j\rangle$  is the state of the observer according to whom the result of the measurement is ‘ $j$ ’. But to ensure that a concrete observer have one and only one opinion of the result of the measurement, we say that the worlds have split, or that the Universe has branched into two: in one branch, there is an observer with the state  $|M_1\rangle$ , in the other — with the state  $|M_2\rangle$ . In which Universe each concrete individual (including yourself) will end up is purely arbitrary; in either Universe it seems that a wavefunction collapse has occurred (so that  $|M_1\rangle$  sees the particle in the pure state  $|O_1\rangle$ ).

The branching of the Universe does not only happen during a measurement, but rather at any time when an energy dissipation (irreversible process) occurs. Due to irreversibility, the disconnected worlds cannot reconnect again. One can say that all the quantum mechanical probabilities come to life in different worlds, and us here are only witnesses to one possible realisation. There may be a world where the dinosaurs never perished.