1D1P

One Day One Problem

Persiapan OSN Fisika Tingkat Nasional 2024

Solusi Day 5 – Pemfokusan Magnetik

Solusi ini diambil dari solusi resmi APhO 2005, bisa diakses melalui website berikut https://apho.olimpicos.net/.

a) In magnetic field, the particle will be deflected and follow a helical path.

Lorentz Force in a magnetic field B,

$$\frac{mv_\perp^2}{R} = ev_\perp B \tag{1}$$

Where $v_\perp$ is the transverse velocity of the electron, R is the radius of the path.

Since $v_\perp = \omega R$ ($\omega = \frac{2\pi}{T}$ is the particle angular velocity and T is the period), then,

$$m\frac{2\pi}{T} = eB \tag{2}$$

To be focused, the period of electron T must be equal to $\frac{L}{v_{//}}$, where $v_{//}$ is the parallel component of the velocity.

We also know,

$$eV = \frac{1}{2}m\left(v_\perp^2 + v_{//}^2\right) \approx \frac{1}{2}mv_{//}^2 \tag{3}$$

All the information above leads to

$$B = 2^{3/2}\pi\frac{\left(mV/e\right)^{1/2}}{L} \tag{4}$$

Numerically

$$B = 4.24 \text{ mT}$$

b) The magnetic field of the Solenoid:

$$B = \mu_0 i n \tag{5}$$

$$i = \frac{B}{n\mu_0} \tag{6}$$

Numerically

$$i = 6.75 \text{ A.}$$

The magnetic force due to the fringe field on charge q with velocity v is

$$\vec{F} = q\vec{v} \times \vec{B} \tag{1}$$

The z-component of the force obtained from the cross product is

$$F_z = q\left(v_x B_y - v_y B_x\right) = -qv_y B_x = -\frac{qv\sin\theta Bz}{b} \tag{2}$$

The vertical momentum gained by the particle after entering the fringe field

$$\Delta P_z = \int F_z dt = -\frac{qvBz\sin\theta}{b}\Delta t = -\frac{qvBz\sin\theta}{b}\frac{b}{v\cos\theta} = -qBz\tan\theta \tag{3}$$

The particle undergoes a circular motion in the constant magnetic field B region

$$m\frac{v^2}{R} = qvB \tag{4}$$

$$v = \frac{qBR}{m} = \frac{qBl}{2m\sin\theta} \tag{5}$$

Therefore,

$$\sin\theta = \frac{qBl}{2mv} \tag{6}$$

After the particle exits the fringe field at the other end, it will gain the same momentum.

The total vertical momentum gained by the particle is

$$\left(\Delta P_z\right)_{total} = 2\Delta P_z = -2qBz\tan\theta \approx -2qBz\frac{qBl}{2mv} = -\frac{q^2 B^2 zl}{mv} \tag{7}$$

Note that for small θ, we can approximate $\tan\theta \approx \sin\theta$

Meanwhile, the momentum along the horizontal plane (xy-plane) is

$$p = mv \tag{8}$$

From the geometry in figure 4, we can get the focal length by the following relation,

$$\frac{\left|\Delta P_z\right|}{p} = \frac{|Z|}{f} \tag{9}$$

$$\boxed{f = \frac{m^2 v^2}{q^2 B^2 l}} \tag{10}$$