# Kelas Mekanika Fluida

Saturday, August 10, 2024 6:54 PM

- Fluida → Cairan, gas

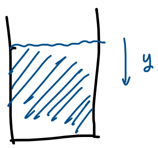
  zat apapun yang bisa mengalir

⇒ Statik → mempelajari kondisi fluida yang diam

⇒ dinamis → karakteristik gerakan fluida

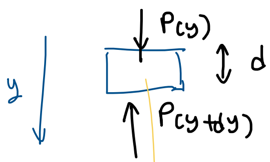
1) Statik

a) Tekanan Hidrostatis



$$P_{(y)} = \rho (g)\ y$$

⇒ Apabila massa jenis fluida fungsi posisi (y)



dw = ρg dV

dw = ρg (dA dy)

Hukum Newton (I)

$$\Rightarrow \Sigma F = 0$$

$$\left[ P_{(y+dy)} - P_{(y)} \right] dA - dw = 0$$

$$\frac{dP}{dy}(\cancel{dA\ dy}) - \rho g\ \cancel{dV} = 0$$

$$\frac{dP}{dy} = \rho g$$

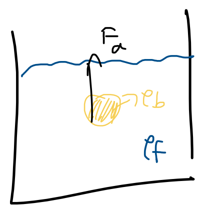
$$\Rightarrow \int dP = \int \rho g\ dy$$

$$P_{(y)} - \underbrace{P_{(0)}}_{0} = g \int_0^y \rho_{(y)}\ dy$$

$$P \ldots = g \int^y \rho_{(y)}\ dy$$

$I(y) = \sigma \int_0$

b) Gaya apung / gaya archimedes




$$F_a = \rho_f g V_{benda}$$

⇛ Untuk benda kecil dan benda dengan bentuk yang sembarang didalam fluida dgn massa jenis $\rho(y)$

$$F_a = -[\nabla P_{hidrostatis}] V_b$$

$$\boxed{F_a = -\nabla P_h \, V_b}$$

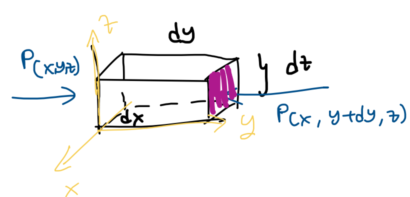
$$\nabla A = \frac{\partial A}{\partial x}\hat{x} + \frac{\partial A}{\partial y}\hat{y} + \frac{\partial A}{\partial z}\hat{z}$$

⇓ turunan parsial → mirip turunan biasa

$$A = 5xy + x^2 z$$

$$\frac{\partial A}{\partial x} = 5y\left(\frac{dx}{dx}\right) + z\frac{dx^2}{dx}$$

$$\frac{\partial A}{\partial x} = 5y + 2xz$$



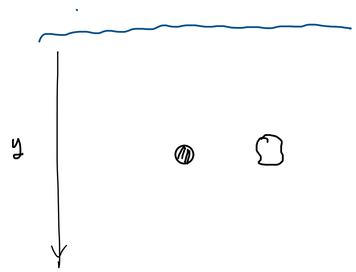

$$F_y = -[P_{(x,y+dy,z)} - P_{(x,y,z)}]\, dz\, dx\, \hat{y}$$

$$F_y = -\frac{\partial P}{\partial y}\, dy\, dz\, dx \, \hat{y}$$

$$F_y = -\frac{\partial P}{\partial y}\, V \, \hat{y}$$

⇒ Contoh Soal :

laut $B = -V\left(\frac{\partial P}{\ }\right)$ Bulk modulus



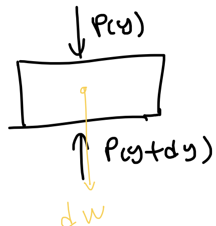

$B = - \dots \left(\frac{\partial \dots}{\partial V}\right)_T$

$\boxed{B = -V\left(\frac{dP}{dV}\right)}$ ⇒ untuk temperatur konstan

$P(y) = ?$ ⇒ (1) massa jenis fungsi y

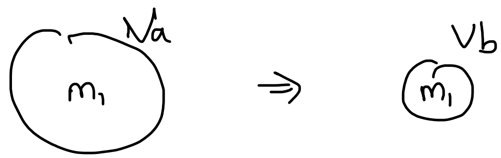
(2) Hukum newton, utk persamaan kestabilan

⇒ Hukum newton :




$$\frac{dP}{dy} = \rho_{(y)}\, g \quad \dots (1)$$

⇒ Hukum kekekalan massa :




$$\rho_a V_a = \rho_b V_b$$

$$\rho V = m = \text{constant}$$

$$d[\rho V] = 0$$

$$d\rho\, V + dV \rho = 0$$

$$dV = -\frac{d\rho}{\rho} V$$

$$\frac{dV}{d\rho} = -\frac{V}{\rho} \Rightarrow \frac{d\rho}{dV} = \frac{1}{\left(\frac{dV}{d\rho}\right)} = \frac{-\rho}{V}$$

⇒ Hubungan kompresibilitas cairan:

$$B = -V \frac{dp}{dV}$$

$$B = -V \frac{dp}{d\rho}\left(\frac{d\rho}{dV}\right)$$

$$B = +\cancel{V}\left(\frac{\rho}{\cancel{V}}\right)\frac{dp}{d\rho} = \rho\left(\frac{dp}{d\rho}\right)$$

$$\Rightarrow \frac{dp}{d\rho} = \frac{B}{\rho} \quad \ldots (I)$$

⇒ Aturan rantai di Persamaan (I)

$$\frac{dp}{dy} = \frac{dp}{d\rho}\left(\frac{d\rho}{dy}\right) = \rho(y)\, g$$

$$\frac{B}{\rho}\left(\frac{d\rho}{dy}\right) = \rho(y)\, g$$

$$\int \frac{d\rho}{\rho^2} = \int \frac{g}{B}\, dy$$

$$-\frac{1}{\rho}\Big|_{\rho_{(y=0)}} = \frac{gy}{B}\Big|_0$$

$\rho_{(y=0)} = \rho_0$ ; untuk air, $\rho_0 = 1000\ kg/m^3$

$$-\frac{1}{\rho} + \frac{1}{\rho_0} = \frac{gy}{B}$$

$$\frac{1}{\rho} = \frac{1}{\rho_0} - \frac{gy}{B}$$

$$\rho_{(y)} = \frac{B\ \rho_0}{B - gy\rho_0}$$

$\Rightarrow$ Persamaan 1

$$\frac{dP}{dy} = \rho_{(y)}\ g = \frac{B\ \rho_0\ g}{B - gy\rho_0}$$

$$\int dP = \int \frac{B\ \rho_0\ g}{B - gy\rho_0}\ dy$$

$$P_{(y)} - P_{(0)} = B\ \rho_0\ g \int \frac{dy}{B - gy\rho_0}$$

$$-g\rho_0 dy = du \Rightarrow dy = -\frac{1}{g\rho_0} du$$

$$\int \frac{dy}{B - gy} = -\frac{1}{g\rho_0}\int \frac{du}{u} = -\frac{1}{g\rho_0} \ln u$$

$$P(y) - P(0) = -\frac{B\cancel{\rho_0 g}}{\cancel{\rho_0 g}} \ln (B - \rho_0 g y)\Big|_{y=0}^{y}$$

$$P(y) = -B \ln\left(\frac{B - \rho_0 g y}{B}\right) = B$$

$$\boxed{P(y) = \underline{B} \ln\left(\frac{B}{B - \rho_0 g y}\right)}$$

$B_{air} =$

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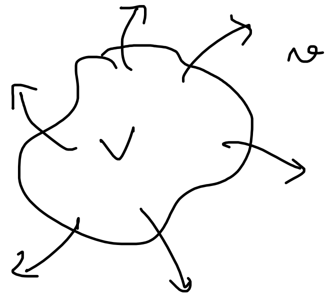
$\Rightarrow$ Fangkat suatu benda dengan volume $V$

$\Rightarrow$ $F_{angkat} = \left(\frac{dP}{dy}\right) V = \rho(y)\, g\, V = -$

2) Fluida dinamis

↳ Fluida yang inkompresibel, massa jenisnya tetap konstan, B



$$\oint \underbrace{\rho}_{\text{konstan}} (\vec{v} \cdot d\vec{A}) = 0$$

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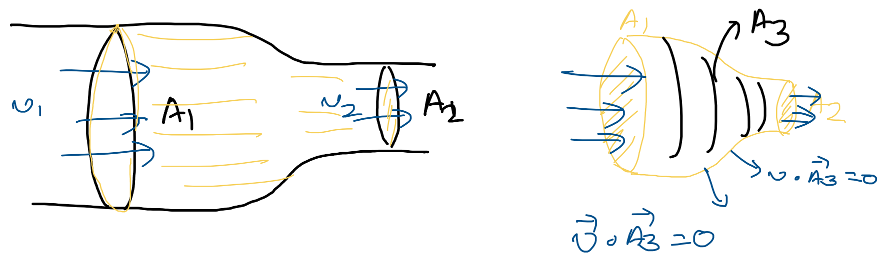
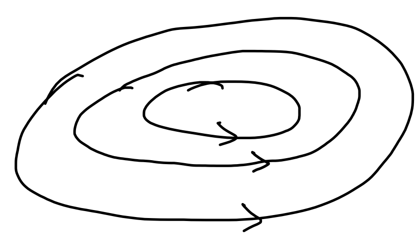



$$\boxed{v_1 A_1 = v_2 A_2}$$

↳ Fluidanya tidak ada gesekan, $\oint \vec{v} \cdot d\vec{\ell} = 0$, tidak ada pus

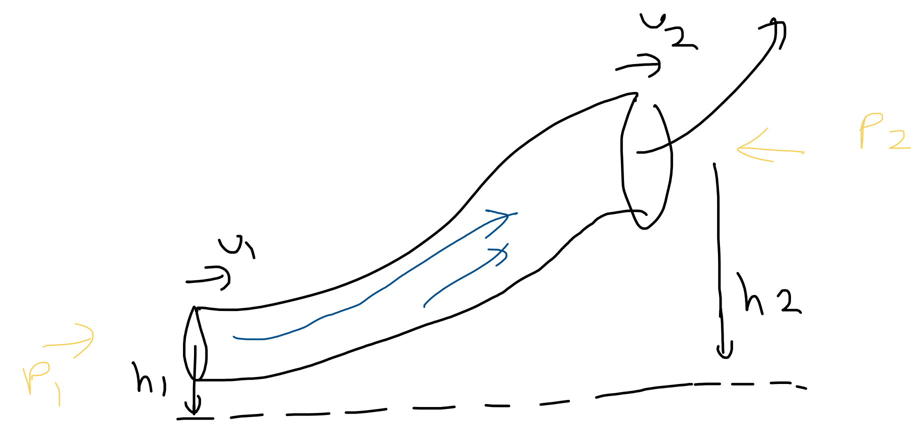


analogi h

$\oint \vec{B} \cdot d$

a) Hukum Bernoulli (dari 2 asumsi awal)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho$$




b) Viskositas , $\oint \vec{v} \cdot d\vec{\ell} \neq 0$

↳ derajat kekentalan cair

stress = gaya/s

$$f \text{ (sheer stress)} = \eta \frac{\partial v}{\partial z}$$

$\eta$ = koefisien viskositas ; air

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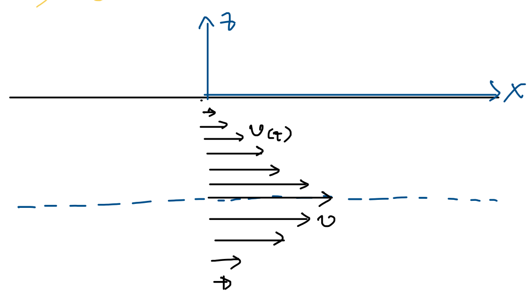
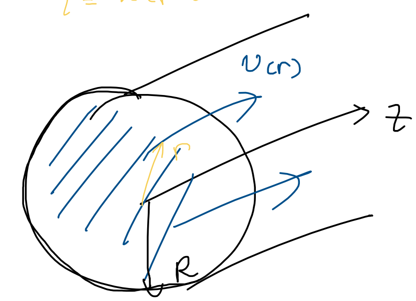


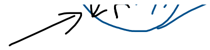



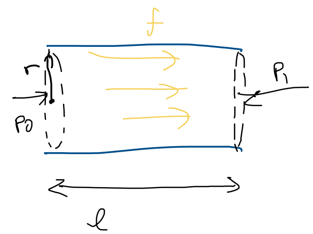


Hukum newton 2 ⇒ persamaan

$$\Sigma F = m \frac{dv}{dt}$$

steady state ⇒ $dv = 0$






$P_0 > P_1$

$P_1 - P_0 < 0$

$$\Sigma F = 0$$

$$f(2\pi r \ell) - (P_1 - P_0)\,\pi r^2 = 0$$

$$f(2\cancel{\pi} r) - \frac{(P_1 - P_0)}{\ell}\,\cancel{\pi} r^2 =$$

$$2f = \left(\frac{P_1 - P_0}{\ell}\right) r$$

$$f = \frac{P_1 - P_0}{\ell}\left(\frac{r}{2}\right)$$

$$f = \eta\left(\frac{\partial v}{\partial r}\right) = \frac{P_1 - P_0}{\ell}\,\frac{r}{2}$$

$$\eta\,\frac{\partial v}{\partial r} = -\left(\frac{P_0 - P_1}{\ell}\right)\frac{r}{2}$$

$$\eta\,\frac{dv}{dr} = -\left(\frac{\Delta P}{\Delta z}\right)\frac{r}{2}$$

$$\int_{v(0)}^{v(r)} dv = -\frac{\Delta P}{\Delta z}\,\frac{1}{\eta}\int_0^r \frac{r}{2}\,dr$$

$$v(r) - v(0) = \frac{-\Delta P}{\Delta z}\,\frac{r^2}{4\eta}$$

$\Rightarrow$ di boundary cairan dengan pipa $v($

$$v(R) = 0$$

$$v(R) - v(0) = -\frac{\partial P}{\partial z} \frac{R^2}{4\eta}$$

$$\cancel{-} v(0) = \cancel{-}\frac{\partial P}{\partial z} \frac{R^2}{4\eta}$$

$$v(r) - v(0) = -\frac{\partial P}{\partial z} \frac{r^2}{4\eta}$$

$$v(r) = \frac{\partial P}{\partial z} \frac{1}{4\eta} (R^2 -$$

$$v(r) = \frac{1}{4\eta} \frac{\partial P}{\partial z} (R^2 - r^2)$$