Dimensi Sains
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1D1P

One Day One Problem

Persiapan OSN Fisika Tingkat Nasional 2024

Solusi Day 3 – Piston Masif dengan Pegas di Dalam Kontainer

Solusi ini diambil langsung dari pembahasan resmi APhO 2005, bisa diakses melalui website berikut https://apho.olimpicos.net/.

a) Gas Volume

At the initial condition, the system is in equilibrium and the spring is unstreched; therefore

$$P_0 A = mg \qquad \text{or} \qquad P_0 = \frac{mg}{A} \tag{1}$$

The initial volume of gas

$$V_0 = \frac{nRT_0}{P_0} = \frac{nRT_0 A}{mg} \tag{2}$$

The work done by the gas from ½ V_0 to V

$$W_{gas} = \int_{V_0/2}^{V} P dV = \int_{V_0/2}^{V} \frac{P_0 V_0^\gamma}{V^\gamma} dV = \frac{P_0 V_0^\gamma}{1-\gamma}\left(V^{1-\gamma} - \left(\frac{V_0}{2}\right)^{1-\gamma} \right) \tag{3}$$

Equation (3) can also be obtained by calculating the internal energy change (without integration)

$$W_{gas} = -\Delta E = -nC_V\left(T - T_0'\right) \tag{4}$$

where T_0' is the temperature when the gas volume is $V_0/2$.

The change of the gravitational potential energy

$$\Delta_{PE} = mg\Delta h = mg\frac{V - \frac{1}{2}V_0}{A} \tag{5}$$

The change of the potential energy of the spring

$$\begin{aligned}\Delta_{spring} &= \frac{1}{2}kx^2 - \frac{1}{2}kx_0^2 \\ &= \frac{1}{2}\left(\frac{mgA}{V_0}\right)\left(\frac{V_0 - V}{A}\right)^2 - \frac{1}{2}\left(\frac{mgA}{V_0}\right)\left(\frac{V_0 - V_0/2}{A}\right)^2 \\ &= \frac{1}{2}\frac{mgV_0}{A}\left(1 - \frac{V}{V_0}\right)^2 - \frac{1}{8}\left(\frac{mgV_0}{A}\right)\end{aligned} \tag{6}$$

The kinetic energy

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{4gV_0}{5A} = \frac{2mgV_0}{5A} \tag{7}$$

By conservation of energy, we have

$$W_{gas} = \Delta_{PE} + \Delta_{spring} + KE \tag{8}$$

$$\frac{P_0V_0^{\gamma}}{1-\gamma}\left(V^{1-\gamma} - \left(\frac{V_0}{2}\right)^{1-\gamma}\right) = mg\frac{V - \frac{V_0}{2}}{A} + \frac{1}{2}\frac{mgV_0}{A}\left(1 - \frac{V}{V_0}\right)^2 - \frac{1}{8}\frac{mgV_0}{A} + \frac{2}{5}\frac{mgV_0}{A} \tag{9}$$

$$\frac{mgV_0}{A(1-\gamma)}\left(\frac{V^{1-\gamma}}{V_0^{1-\gamma}} - \left(\frac{1}{2}\right)^{1-\gamma}\right) = mg\frac{V - \frac{V_0}{2}}{A} + \frac{mgV_0}{2A}\left(1 - \frac{V}{V_0}\right)^2 + \frac{11}{40}\frac{mgV_0}{A} \tag{10}$$

Let $s = V/V_0$, so the above equation becomes

$$\frac{1}{(1-\gamma)}\left(s^{1-\gamma} - \left(\frac{1}{2}\right)^{1-\gamma}\right) = \left(s - \frac{1}{2}\right) + \frac{1}{2}(1-s)^2 + \frac{11}{40} \tag{11}$$

With γ= 5/3 we get

$$0=\frac{1}{2}s^2+\frac{11}{40}+\frac{3}{2}\left(s^{-2/3}-\left(\frac{1}{2}\right)^{-2/3}\right) \tag{12}$$

Solving equation (12) numerically, we get

$$s_1 = 0.74 \text{ and } s_2 = 1.30$$

Therefore $V_1 = 0.74V_0 = 0.74\frac{nRT_0A}{mg} = 1.88\text{m}^3$ *or* $V_2 = 1.30V_0$=3.31 m^3.

b) Small Oscillation (2 points)

The equation of motion when the piston is displaced by x from the equilibrium position is

$$m\ddot{x} = -kx - PA + mg \tag{13}$$

P is the gas pressure

$$P=\frac{P_0V_0^\gamma}{V^\gamma}=\frac{P_0V_0^\gamma}{(V_0-Ax)^\gamma}=\frac{P_0}{\left(1-\frac{Ax}{V_0}\right)^\gamma} \tag{14}$$

Since $Ax \ll V_0$ then we have $P \approx P_0\left(1+\gamma\frac{Ax}{V_0}\right)$, therefore

$$\begin{aligned} m\ddot{x} &\approx -kx - P_0A\left(1+\gamma\frac{Ax}{V_0}\right)+mg \\ m\ddot{x} &= -\left(k+P_0A\left(\gamma\frac{A}{V_0}\right)\right)x \\ m\ddot{x} &= -\left(\frac{mgA}{V_0}+\frac{mg}{A}A\left(\gamma\frac{A}{V_0}\right)\right)x \\ m\ddot{x} &+ (1+\gamma)\frac{mgA}{V_0}x = 0 \end{aligned} \tag{15}$$

The frequency of the small oscillation is

$$f = \frac{1}{2\pi}\sqrt{(1+\gamma)\frac{gA}{V_0}} = \frac{1}{2\pi}\sqrt{(1+\gamma)\frac{mg^2}{nRT_0}} \tag{16}$$

Numerically $f = 0.114$ Hz.