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1D1P

One Day One Problem

Persiapan OSN Fisika Tingkat Nasional 2024

Solusi Day 6 – Balon Udara

Solusi ini diambil dari solusi resmi IPhO 2004, bisa diakses melalui link berikut https://ipho.olimpicos.net/.

[Part A]

(a) [1.5 points]

Using the ideal gas equation of state, the volume of the helium gas of n moles at pressure $P+\Delta P$ and temperature T is

$$V = nRT/(P+\Delta P) \qquad \text{(a1)}$$

while the volume of n' moles of air gas at pressure P and temperature T is

$$V = n'RT/P. \qquad \text{(a2)}$$

Thus the balloon displaces $n' = n\dfrac{P}{P+\Delta P}$ moles of air whose weight is $M_A n' g$.

This displaced air weight is the buoyant force, i.e.,

$$F_B = M_A n g \frac{P}{P+\Delta P}. \qquad \text{(a3)}$$

(Partial credits for subtracting the gas weight.)

(b) [2 points]

The pressure difference arising from a height difference of z is $-\rho g z$ when the air density ρ is a constant. When it varies as a function of the height, we have

$$\frac{dP}{dz} = -\rho g = -\frac{\rho_0 T_0}{P_0}\frac{P}{T}g \qquad \text{(b1)}$$

where the ideal gas law $\rho T/P =$ constant is used. Inserting Eq. (2.1) and $T/T_0 = 1 - z/z_0$ on both sides of Eq. (b1), and comparing the two, one gets

$$\gamma = \frac{\rho_0 z_0 g}{P_0} = \frac{1.16\times 4.9\times 10^4 \times 9.8}{1.01\times 10^5} = 5.52. \qquad \text{(b2)}$$

The required numerical value is 5.5.

[Part B]

(c) [2 points]

The work needed to increase the radius from r to $r+dr$ under the pressure difference ΔP is

$$dW = 4\pi r^2 \Delta P dr\,, \tag{c1}$$

while the increase of the elastic energy for the same change of r is

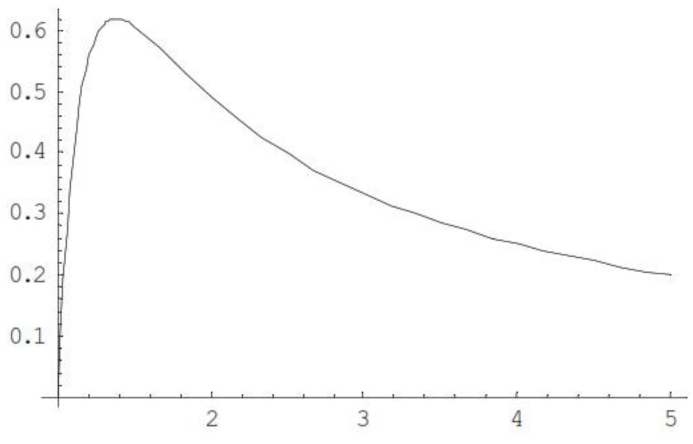
$$dW = \left(\frac{dU}{dr}\right)dr = 4\pi\kappa RT(4r - 4\frac{r_0^6}{r^5})dr\,. \tag{c2}$$

Equating the two expressions of dW, one gets

$$\Delta P = 4\kappa\, RT(\frac{1}{r} - \frac{r_0^6}{r^7}) \;=\; \frac{4\kappa RT}{r_0}\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right). \tag{c3}$$

This is the required answer.

The graph as a function of λ (>1) increases sharply initially, has a maximum at $\lambda = 7^{1/6}$ =1.38, and decreases as λ^{-1} for large λ. The plot of $\Delta P/(4\kappa RT/r_0)$ is given below.



(d) [1.5 points]

From the ideal gas law,

$$P_0V_0 = n_0RT_0 \tag{d1}$$

where V_0 is the unstretched volume.

At volume $V = \lambda^3 V_0$ containing n moles, the ideal gas law applied to the gas inside at $T = T_0$ gives the inside pressure P_{in} as

$$P_{\text{in}} = nRT_0 / V = \frac{n}{n_0\lambda^3}P_0 \quad . \tag{d2}$$

On the other hand, the result of (c) at $T = T_0$ gives

$$P_{\text{in}} = P_0 + \Delta P = P_0 + \frac{4\kappa RT_0}{r_0}\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right) = \left(1 + a\left(\frac{1}{\lambda} - \frac{1}{\lambda^7}\right)\right)P_0 . \tag{d3}$$

Equating (d2) and (d3) to solve for a,

$$a = \frac{n/(n_0\lambda^3) - 1}{\lambda^{-1} - \lambda^{-7}} . \tag{d5}$$

Inserting n/n_0=3.6 and λ=1.5 here, a=0.110.

[Part C]

(e) [3 points]

The buoyant force derived in problem (a) should balance the total mass of M_T=1.12 kg.

Thus, from Eq. (a3), at the weight balance,

$$\frac{P}{P+\Delta P}=\frac{M_\mathrm{T}}{M_A n}. \tag{e1}$$

On the other hand, applying again the ideal gas law to the helium gas inside of volume $V=\frac{4}{3}\pi r^3=\lambda^3\frac{4}{3}\pi r_0{}^3=\lambda^3 V_0$, for arbitrary ambient P and T, one has

$$(P+\Delta P)\lambda^3=\frac{nRT}{V_0}=P_0\frac{T}{T_0}\frac{n}{n_0} \tag{e2}$$

for n moles of helium. Eqs. (c3), (e1), and (e2) determine the three unknowns P, ΔP, and λ as a function of T and other parameters. Using Eq. (e2) in Eq. (e1), one has an alternative condition for the weight balance as

$$\frac{P}{P_0}\frac{T_0}{T}\lambda^3=\frac{M_\mathrm{T}}{M_A n_0}\quad . \tag{e3}$$

Next using (c3) for ΔP in (e2), one has

$$P\lambda^3+\frac{4\kappa RT}{r_0}\lambda^2(1-\lambda^{-6})=P_0\frac{T}{T_0}\frac{n}{n_0}$$

or, rearranging it,

$$\frac{P}{P_0}\frac{T_0}{T}\lambda^3=\frac{n}{n_0}-a\lambda^2(1-\lambda^{-6}), \tag{e4}$$

where the definition of a has been used again.

Equating the right hand sides of Eqs. (e3) and (e4), one has the equation for λ as

$$\lambda^2(1-\lambda^{-6})=\frac{1}{an_0}(n-\frac{M_\mathrm{T}}{M_A})=4.54. \tag{e5}$$

The solution for λ can be obtained by

$$\lambda^2\approx 4.54/(1-4.54^{-3})\approx 4.54:\ \lambda_f\cong 2.13. \tag{e6}$$

To find the height, replace $(P/P_0)/(T/T_0)$ on the left hand side of Eq. (e3) as a function of the height given in (b) as

$$\frac{P}{P_0}\frac{T_0}{T}\lambda^3 = (1 - z_f / z_0)^{\gamma-1}\lambda_f^3 = \frac{M_{\text{T}}}{M_A n_0} = 3.10 \,. \qquad \text{(e7)}$$

Solution of Eq. (e7) for z_f with λ_f=2.13 and $\gamma-1$=4.5 is

$$z_f = 49 \times \left(1 - (3.10/2.13^3)^{1/4.5}\right) = 10.9 \text{ (km)}. \qquad \text{(e8)}$$

The required answers are $\lambda_f = 2.1$, and $z_f = 11$ km.

Referensi: International Physics Olympiad (IPhO) 2004 Korea, Question No. 2