Topics to discuss

Aggregate Analysis.

Example 2 : Incrementing a binary number.

There are three most common techniques used in amortized analysis.

i) Aggregate Analysis / Aggregate Method
ii) Accounting Method
iii) Potential Method.

Aggregate Analysis / Aggregate Method :

In aggregate analysis, we show that for all n, a sequence of n operations takes worst-case time $T(n)$ in total.

In the worst case, the average cost, or amortized cost, per operation is therefore $T(n)/n$.

This amortized cost applies to each operation, even when there are several types of operations in the sequence.

Example 2: Incrementing a binary number

Considered the problem of implementing a k-bit binary counter that counts upward from 0.

Counter value	A[3]	A[2]	A[1]	A[0]
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1

Pseudo code :

```
INCREMENT (A)
1)    i = 0
2)    while i < A.length and A[i] == 1
3)          A[i] = 0
4)          i = i + 1
5)    If i < A.length
6)          A[i] = 1
```

$i = 0$

$0 < 4$ and $A[0] == 1$

T and $T \rightarrow T$

$A[0] = 0$

$i = 1$

$1 < 4 \rightarrow T$

$A[1] = 1$

Counter value	A[3]	A[2]	A[1]	A[0]	Cost	Total cost
0	0	0	0	0	0	0
1	0	0	0	1	1	0+1=1
2	0	0	1	0	2	1+2=3
3	0	0	1	1	1	3+1=4
4	0	1	0	0	3	4+3=7
5	0	1	0	1	1	7+1=8
6	0	1	1	0	2	8+2=10
7	0	1	1	1	1	10+1=11
8	1	0	0	0	4	11+4=15

4 bit binary counter as its value goes from 0 to 8 by a sequence of 8 increment operation.

Total cost is always less than twice the total number of increment operations.

$15 < 2 \times 8$

$\boxed{15 < 16}$

Asymptotic Analysis :

We observe some operations only flip one bit. and some operations flip more than one bit.

A single execution of INCREMENT takes time $O(k)$ in the worst case.

Thus a sequence of n INCREMENT operations on an initially zero counter takes time $O(nk)$ in the worst-case.

Aggregate Analysis or Amortized Analysis :

We can observe that not all the bits flip each time INCREMENT.

A[0] flip each time INCREMENT.

A[1] flips only every other time

i.e., A[1] to flip $\frac{n}{2}$ time.

Similarily,

A[2] flips every fourth time or $\frac{n}{4}$ time in a sequence of n INCREMENT operations.

In general, for $i = 0, 1, \ldots K-1$,

$A[1]$ flips $\frac{n}{2}$ times

$A[2]$ flips $\frac{n}{2^2}$ times

So, bit $A[i]$ flips $\frac{n}{2^i}$ times

for $i \geq K$, bit $A[i]$ does not exist, and so it cannot flip.

The total no. of flip in the sequence is thus,

$$\sum_{i=0}^{K-1} \frac{n}{2^i} < \sum_{i=0}^{\infty} \frac{n}{2^i} \qquad \left[S_\infty = \frac{a}{1-r} \right]$$

$$< \frac{n}{2^0} + \frac{n}{2^1} + \frac{n}{2^2} + \cdots \infty$$

$$< \frac{n}{1 - \frac{1}{2}}$$

$$< 2n$$

The worst case complexity for a sequence of n increment operations on an initially zero counter is therefore $O(n)$.

The average cost of each operation, or amortized cost per operation is $O(n)/n = O(1)$.

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