Topics to discuss

Aggregate Analysis.

Example 3 : Dynamic Table

Pseudo code :

Initially table is empty,
T.num = T.size = 0

```
TABLE-INSERT (T, x)
    If T.size == 0
        allocate T.table with 1 slot
        T.size == 1
    If T.num == T.size
        allocate new-table with 2.T.size slots
        Insert all items in T.table into new table
        free T.table
        T.table == new-Table
        T.size == 2.T.size
    Insert x into T.table
    T.num = T.num + 1
```

In this pseudocode, we assume that T is an object representing the table.

T.table contains a pointer to the block of storage representing the table.

T.num contains the number of items in the table

T.size gives total number of the slot in the table.

Example 3 : Dynamic Table :

1

1	2

1	2	3	4

1	2	3	4	5	6	7	8

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

i	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Size	1	2	4	4	8	8	8	8	16	16	16	16	16	16	16	16
Cost	1	2	3	1	5	1	1	1	9	1	1	1	1	1	1	1

Worst case TC for single insert $\rightarrow O(n)$

[As we need to copy all previous element and then insert single element]

Worst case TC for n insert $\rightarrow O(n^2)$

Amortized Analysis :

We observe that, the i^{th} operation causes an expansion only when $i-1$ is an exact power of 2.

The amortized cost of an operation is in fact $O(1)$, as we can show using aggregate analysis.

The cost of the i^{th} operation is

$$c_i = \begin{cases} i & \text{if } i-1 \text{ is an exact power of } 2, \\ 1 & \text{otherwise.} \end{cases}$$

The total cost of n Table-Insert operations is therefore,

$$\sum_{i=1}^{n} c_i \le 1+2+3+1+5+1+1+1+9+1+1+\ldots n \text{ terms}$$

$$\le (1+1+1+\ldots n \text{ times}) + (1+2+4+8+\ldots + \log n \text{ terms})$$

$$\le n + \left(2^0+2^1+2^2+2^3+\cdots+2^{\log(n-1)}\right).$$

$$\sum_{i=1}^{n} c_i \leq n + \left(2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{\log(n-1)}\right). \qquad \left[S = \frac{a(r^n - 1)}{r-1}\right]$$

$$\leq n + \frac{2^0\left(2^{\log(n-1)+1} - 1\right)}{2-1}$$

$$\leq n + 2^{\log_2(n-1) + \log_2 2} - 1 \qquad \left[a^{\log_a b} = b\right]$$

$$\leq n + 2^{\log_2 2(n-1)} - 1$$

$$\leq n + 2(n-1) - 1$$

$$\leq n + 2n - 2 - 1$$

$$\leq 3n - 3$$

$$O(n)$$

So, $\sum_{i=1}^{n} c_i \leq 3n - 2$

$O(n)$

Cost of n insertion, $= \sum_{i=1}^{n} c_i = O(n)$

Cost of single insertion $= \frac{O(n)}{n} = O(1)$

The average cost of each insertion or amortized cost per insertion is $O(1)$.

Follow Now

Start Practicing

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