Topics to discuss

Potential Method

Example 1 : Stack Operation

Potential Method :

Instead of representing prepaid work as credit stored with specific objects in the data structure, the potential method of amortized analysis represents the prepaid work as "Potential Energy". or just "Potential", which can be released to pay for future operations.

We will perform n operations, starting with an initial data structure D_0. For each $i = 1, 2, \ldots, n$, we let c_i be the actual cost of the i^{th} operation and D_i be the data structure that results after applying the i^{th} operation to data structure D_{i-1}.

A potential function ϕ maps each data structure D_i to a real number $\phi(D_i)$, which is a potential associated with data structure D_i.

The amortized cost $\hat{c}_i$ of the i^{th} operation with respect to potential function ϕ is defined by,

$$\hat{c}_i = c_i + \phi(D_i) - \phi(D_{i-1})$$

Total amortized cost,

$$\sum_{i=1}^{n} \hat{c}_i = \sum_{i=1}^{n} \left(c_i + \phi(D_i) - \phi(D_{i-1})\right)$$

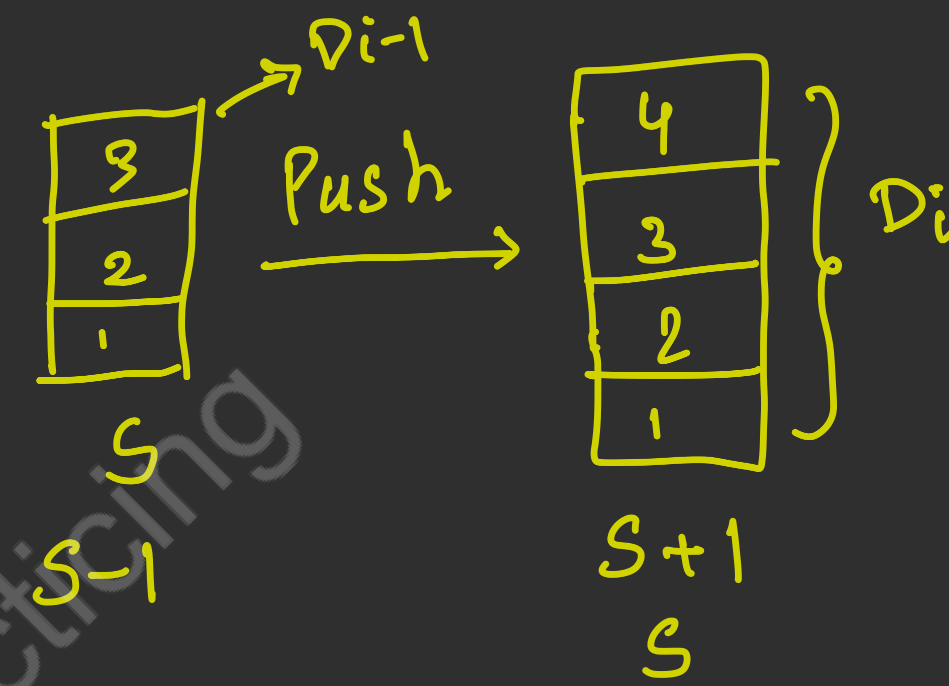
$$= \sum_{i=1}^{n} c_i + \phi(D_n) - \phi(D_0)$$

For Push Operation :

Potential change, $\phi(D_i) - \phi(D_{i-1})$

$= s+1 - s$

$= 1$



Amortized cost,

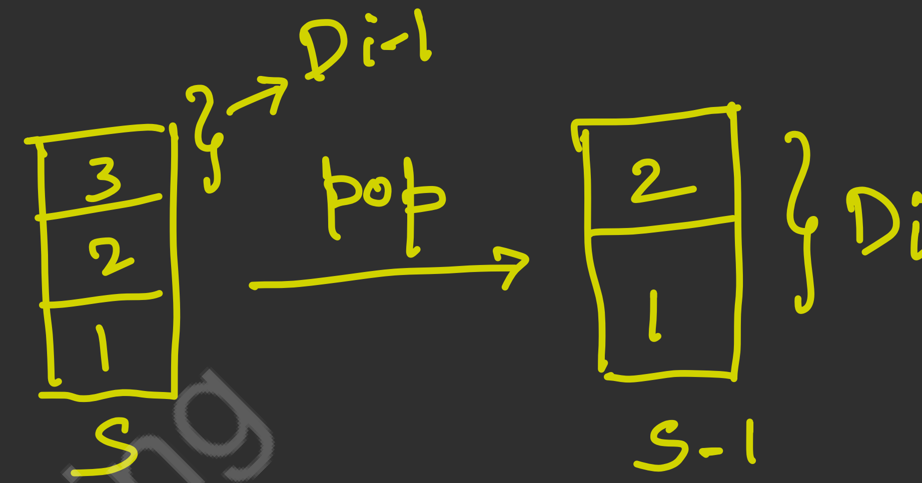
$\hat{c}_i = c_i + \phi(D_i) - \phi(D_{i-1})$

$= 1 + 1$

$= 2$

$\approx O(1)$

For Pop operation,



Potential change, $\phi(D_i) - \phi(D_{i-1})$

$= S-1 - S$

$= -1$

Amortized cost,

$\hat{C_i} = C_i + \phi(D_i) - \phi(D_{i-1})$

$= 1 - 1$

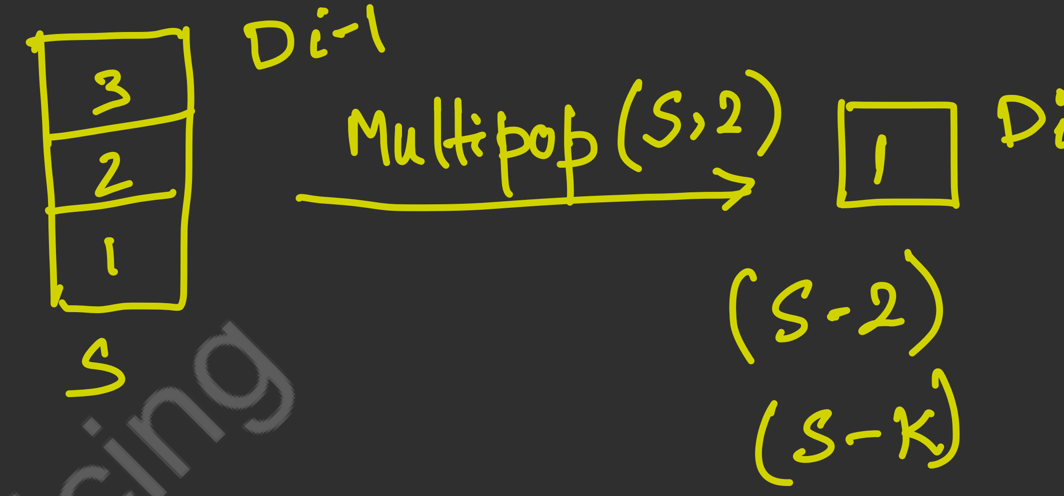
$= 0$

$\approx O(1)$

For Multipop operation,

Potential Change, $\phi(D_i) - \phi(D_{i-1})$

$$= S-K-S$$
$$= -K$$



Amortized cost,

$$\hat{C}_i = C_i + \phi(D_i) - \phi(D_{i-1})$$
$$= K - K$$
$$= 0$$
$$\simeq O(1)$$

The amortized cost of each of the three operations is $O(1)$. and thus the total amortized cost of a sequence of n operations is $O(n)$.

As, $\phi(D_i) \geq \phi(D_0)$

it implies, total amortized cost of n operations is an upper bound on the total actual cost.

Follow Now

Start Practicing

i._am._arfin

Arfin Parween