Topics to discuss

Potential Method

Example 2 : Incrementing a binary Number

Potential Method :

We again look at incrementing a binary counter. This time, we define the potential of the counter after i^{th} INCREMENT operation, the number of 1s in the counter after the i^{th} operation.

The amortized cost $\hat{c}_i$ of the i^{th} operation with respect to potential function ϕ is defined by,

$$\hat{c}_i = c_i + \phi(D_i) - \phi(D_{i-1})$$

Incrementing a binary number

Considered the problem of implementing a k-bit binary counter that counts upward from 0.

Pseudo code :

```
INCREMENT (A)
    i = 0
    while i < A.length and A[i] == 1
        A[i] = 0
        i = i + 1
    If i < A.length
        A[i] = 1
```

Incrementing a binary number

Counter value	A[3]	A[2]	A[1]	A[0]	Actual Cost	$\emptyset(D_i)$	$\emptyset(D_{i-1})$	Potential Difference	Amortized cost
0	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	0	1	2
2	0	0	1	0	2	1	1	0	2
3	0	0	1	1	1	2	1	1	2
4	0	1	0	0	3	1	2	-1	2
5	0	1	0	1	1	2	1	1	2
6	0	1	1	0	2	2	2	0	2
7	0	1	1	1	1	3	2	1	2
8	1	0	0	0	4	1	3	-2	2

Total amortized cost of 8 operations is,

$$(0+2+2+2+2+2+2+2+2)$$

$$= 2 \times 8$$

So, for 'n' no. of increment operations,

$$= 2 \times n$$

$$\simeq O(n)$$

As, $\phi(D_i) \geq \phi(D_0)$

it implies, total amortized cost of n operations is an upper bound on the total actual cost.

Follow Now

Start Practicing

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