Topic to discuss

→ Proof / Derive Newton Forward Interpolation formula.

Proof Newton Forward Interpolation Formula

Let $y = f(x)$ be a function.

and $x_0, x_1, x_2 \ldots$ are the equidistant data point.

Let h is the step length or constant difference between two successive value of x.

$$\Rightarrow h = x_1 - x_0$$

x	$y = f(x) = y(x)$
x_0	y_0
$x_1 = x_0 + h$	y_1
$x_2 = x_0 + 2h$	y_2
$x_3 = x_0 + 3h$	y_3
$\vdots \quad \vdots$	$\vdots$
$x_n = x_0 + nh$	y_n

we know, $y = f(x)$

$$y_0 = f(x_0)$$

$$y_1 = f(x_1) = f(x_0 + h)$$

$$y_2 = f(x_2) = f(x_0 + 2h)$$

$\vdots$

and so on.

Suppose x is interpolation point and we want to find function value of x which is $f(x)$. and close to top of the given data point.

We Know, $\quad y = f(x) = y(x)$

and, $x = x_0 + uh$

$$\Rightarrow u = \frac{x - x_0}{h}$$

Step-1:

We Know, Forward Shift Operator E such that

$$Ef(x) = f(x+h)$$

$$E^u f(x) = f(x+uh) \quad [\text{Replace } x \to x_0]$$

$$\Rightarrow E^u f(x_0) = f(x_0 + uh)$$

$$\Rightarrow E^u y_0 = f(x)$$

$$\Rightarrow f(x) = E^u y_0$$

$$f(x) = E^u y_0 \quad [E = 1+\Delta \;;\; \Delta \text{ is forward difference operator}]$$

$$f(x) = (1+\Delta)^u y_0 \quad \text{—— (1)}$$

Step-2

We know Binomial Expansion Theorem such that,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

Applying this in equation —(1)

$$f(x) = (1+\Delta)^u y_0$$

$$f(x) = \left[1 + u\Delta + \frac{u(u-1)}{2!}\Delta^2 + \frac{u(u-1)(u-2)}{3!}\Delta^3 + \cdots\right] y_0$$

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

(Proved)

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