Topic to discuss

→ Proof / Derive Newton Backward Interpolation formula.

Proof Newton Backward Interpolation Formula

Let $y = f(x)$ be a function.
and $x_0, x_1, x_2 \ldots$ are the equidistant data point.
Let h is the step length or constant difference between two successive value of x.

$$\Rightarrow h = x_1 - x_0$$

x	$y = f(x) = y(x)$
x_0	y_0
$x_1 = x_0 + h$	y_1
$x_2 = x_0 + 2h$	y_2
$x_3 = x_0 + 3h$	y_3
$\vdots$	$\vdots$
$x_n = x_0 + nh$	y_n

So, we have, $y = f(x)$

$$y_0 = f(x_0)$$

$$y_1 = f(x_1) \Rightarrow y_1 = f(x_0 + h)$$

$$y_2 = f(x_2) \Rightarrow y_2 = f(x_0 + 2h)$$

$\vdots$

and so on.

Suppose x is interpolation point and we want to find function value of x which is $f(x)$. and close to last entries of the given data table.

We know, $y = f(x) = y(x)$

and $x = x_n + uh$

$h = \frac{x - x_n}{h}$ [since x is close to end of table]

Step-1:

We know, Forward Shift Operator E such that

$$Ef(x) = f(x+h)$$

$$\Rightarrow E^u f(x) = f(x + uh) \quad [\text{Replacing } x \rightarrow x_n]$$

$$\Rightarrow E^u f(x_n) = f(x_n + uh)$$

$$\Rightarrow E^u y_n = f(x)$$

$$\therefore f(x) = E^u y_n$$

$$f(x) = E^u y_n \quad [E^{-1} = 1-\nabla \;;\; \nabla \text{ is backward difference operator}]$$

$$\Rightarrow f(x) = (E^{-1})^{-u} y_n$$

$$\Rightarrow f(x) = (1-\nabla)^{-u} y_n \quad \text{——— (1)}$$

<u>Step-2</u>

We Know Binomial Expansion Theorem such that,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\cdots$$

Applying this in eqn. —(1) $\quad [x \to -\nabla \;;\; n \to -u]$

$$f(x) = (1-\nabla)^{-u} y_n$$

$$f(x) = \left[1 + (-\nabla)(-u) + \frac{-u(-u-1)}{2!}(-\nabla)^2 + \frac{-u(-u-1)(-u-2)}{3!}(-\nabla)^3 + \cdots\cdots\right] y_n$$

$$f(x) = \left[1 + u\nabla + \frac{u(u+1)}{2!}\nabla^2 + \frac{u(u+1)(u+2)}{3!}\nabla^3 + \cdots\right] y_n$$

$$f(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots\cdots$$

(Proved)

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