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Newton Backward Difference Interpolation

If the table is too long and if the required point is close to the end of the table, we can use another formula known as Newton Backward difference formula.

Here, the reference point is x_n instead of x_0.

Backward difference Table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	y_0			
		$\nabla y_1 = y_1 - y_0$		
x_1	y_1		$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$	
		$\nabla y_2 = y_2 - y_1$		$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$
x_2	y_2		$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	
		$\nabla y_3 = y_3 - y_2$		
x_3	y_3			

where,

x. values : These are the known data points where the function $f(x)$ is given.

y-values : These are the corresponding function values $f(x)$ at each x.

∇y : First Backward difference.

$\nabla^2 y$: Second Backward difference.

$\nabla^3 y$: Third Backward difference.

$\vdots$

So on

Newton's Backward Difference Interpolation Formula

It is derived using a polynomial approximation.

The formula is :

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

where, $u = \dfrac{x - x_n}{h}$,

and h is constant difference between successive values of x.

Numerical Problem :

Q: Compute $f(0.29)$ using suitable formula from the table given below :

x :	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$:	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

Solution : Step length, $h = 0.22 - 0.20$

$= 0.02$

Last value of x, $x_n = 0.30$

Interpolation point, $x = 0.29$

$$u = \frac{x - x_n}{h} = \frac{0.29 - 0.30}{0.02}$$

$$u = -0.5$$

Backward difference table,

x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$	$\nabla^5 f(x)$
0.20	1.6596					
		0.0102				
0.22	1.6698		0.0004			
		0.0106		-0.0002		
0.24	1.6804		0.0002		0.0004	
		0.0108		0.0002		-0.0007
0.26	1.6912		0.0004		-0.0003	
		0.0112		-0.0001		
0.28	1.7024		0.0003			
		0.0115				
0.30	1.7139					

Newton Backward Interpolation formula,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

$$f(x) = f(x_n) + u\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n) + \frac{u(u+1)(u+2)}{3!}\nabla^3 f(x_n) + \frac{u(u+1)(u+2)(u+3)}{4!}\nabla^4 f(x_n) + \frac{u(u+1)(u+2)(u+3)(u+4)}{5!}\nabla^5 f(x_n)$$

$$f(x) = 1.7139 + (-0.5)\times 0.0115 + \frac{-0.5(-0.5+1)}{2!}\times 0.0003 + \frac{-0.5(-0.5+1)(-0.5+2)}{3!}\times -0.0001 + \frac{-0.5(-0.5+1)(-0.5+2)(-0.5+3)}{4!}\times -0.0003 + \frac{-0.5(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}\times -0.0007$$

$$f(x) = 1.7139 - 0.00575 - 3.75\times 10^{-5} - 6.25\times 10^{-6} + 1.171875\times 10^{-5} + 1.9140625\times 10^{-5}$$

$$= 1.708149609$$

Homework Problem

The following data represent the function $f(x) = e^x$

x	1	1.5	2	2.5
y	2.7183	4.4817	7.3891	12.1825

Estimate the value of $f(1.25)$ using

(i) Newton's forward difference interpolation

(ii) Newton's Backward difference interpolation.

Solution :

At first, we need to construct Newton difference table,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
1	2.7183			
		1.7634		
1.5	4.4817		1.144	
		2.9074		0.742
2	7.3891		1.886	
		4.7934		
2.5	12.1825			

(i) Newton Forward difference,

Step length, $h = 0.5$

first value of x, $x_0 = 1$

Interpolation point, $x = 1.25$

$$u = \frac{x - x_0}{h} = \frac{1.25 - 1}{0.5}$$

$$u = 0.5$$

Forward difference formula,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$= 2.7183 + 0.5 \times 1.7634 + \frac{0.5(0.5-1)}{2!} \times 1.144 + \frac{0.5(0.5-1)(0.5-2)}{3!} \times 0.742$$

$$= 2.7183 + 0.8817 - 0.143 + 0.046375$$

$$= 3.503375$$

$$f(1.25) = 3.503375$$

Ans.

(ii) Newton Backward difference,

Step length, $h = 0.5$

Last value of x, $x_n = 2.5$

Interpolation point, $x = 1.25$

$$u = \frac{x - x_n}{h} = \frac{1.25 - 2.5}{0.5}$$

$$u = -2.5$$

Backward difference formula,

$$y(x) = y_n + u\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$= 12.1825 + (-2.5)\times 4.7934 + \frac{-2.5(-2.5+1)}{2!}\times 1.886 + \frac{-2.5(-2.5+1)(-2.5+2)}{3!}\times 0.742$$

$$= 12.1825 - 11.9835 + 3.53625 - 0.231875$$

$$= 3.503375$$

$$f(1.25) = 3.503375$$ Ans.

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