Topic to disscuss

→ Newton Forward difference Interpolation

→ Forward difference table

→ Forward difference Formula

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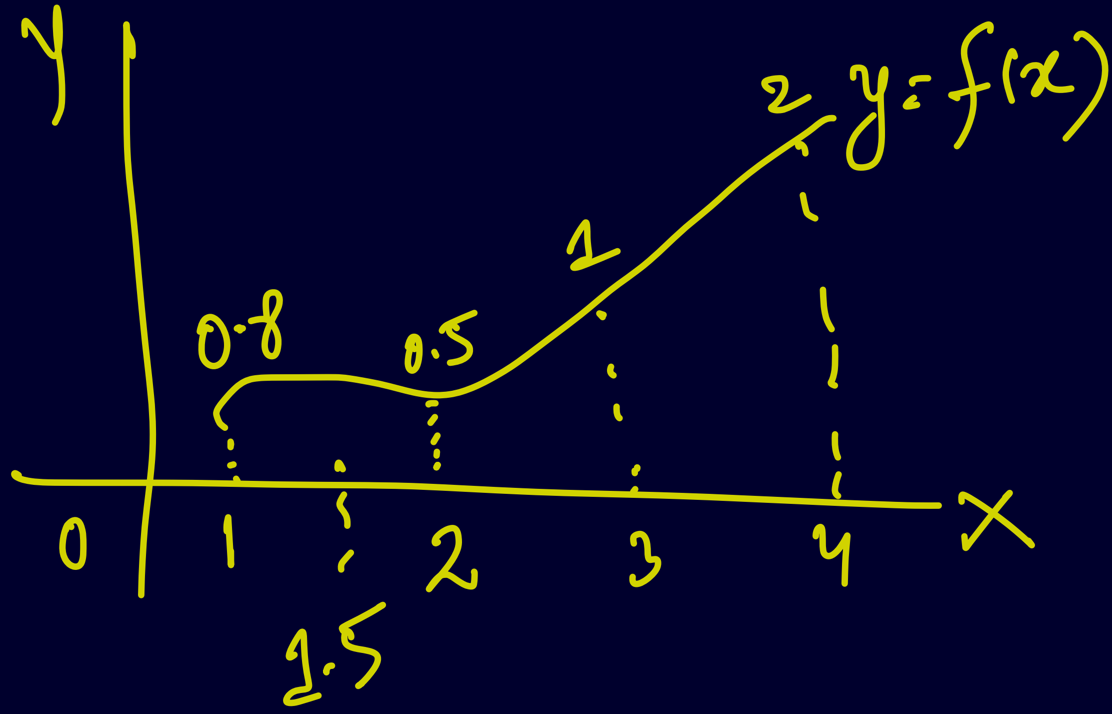
→ Homework Problem with solution.

Newton Forward Difference Interpolation

It is a method used to estimate the value of a function y at any point x, given a set of tabulated values for the function at equally spaced points. This technique is particularly useful when the values of x are equally spaced, and we are looking to estimate a value of y for some x that lies close to the beginning of the given data.



Forward difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
x_1	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
x_2	y_2	$\Delta y_2 = y_3 - y_2$		
x_3	y_3			

where,

x. values : These are the known data points where the function $f(x)$ is given.

y-values : These are the corresponding function values $f(x)$ at each x.

Δy : First forward difference.

$\Delta^2 y$: Second forward difference.

$\Delta^3 y$: Third forward difference.

$\vdots$

So on

Number of differences :

For n points, the maximum difference to compute is $\Delta^{n-1} y$.

If we have 6 data points, then maximum forward difference is indeed $\Delta^{5} y$

Newton's Forward Difference Interpolation Formula

It is derived using a polynomial approximation.

The formula is :

$$y(x) = y_0 + u\,\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots$$

where, $u = \dfrac{x - x_0}{h}$,

and h is constant difference between successive values of x.

u is reduced variable.

Numerical Problem

Q: Use Newton's Forward interpolation formula to find the value of $y = f(x)$ for $x = 12$

x	10	15	20	25	30	35
y	35.3	32.4	29.2	26.1	23.2	20.5

Solution : Given,

Step length or interval, $\boxed{h = 5}$

first value of x, $\boxed{x_0 = 10}$

Interpolation point, $\boxed{x = 12}$

Reduced variable, $u = \dfrac{x - x_0}{h}$

$$u = \frac{12 - 10}{5}$$

$$\boxed{u = 0.4}$$

Forward difference Table,

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
10	35.3					
		-2.9				
			-0.3			
15	32.4			0.4		
		-3.2			-0.3	
			0.1			0.2
20	29.2			0.1		
		-3.1			-0.1	
			0.2			
25	26.1			0.0		
		-2.9				
30	23.2		0.2			
		-2.7				
35	20.5					

Newton Forward Interpolation formula,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$+ \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0$$

$$= 35.3 + 0.4\times(-2.9) + \frac{0.4(0.4-1)}{2!}\times -0.3 + \frac{0.4(0.4-1)(0.4-2)}{3!}\times 0.4$$

$$+ \frac{0.4(0.4-1)(0.4-2)(0.4-3)}{4!}\times -0.3 + \frac{0.4(0.4-1)(0.4-2)(0.4-3)(0.4-4)}{5!}\times 0.2$$

$$= 35.3 + (-1.16) + 0.036 + 0.0256 + 0.01248 + 0.00599$$

$$y(12) = 34.2200704$$ Ans.

Homework Problem

The following data give the melting point of an alloy of zinc and lead. θ is the temperature and x is the percentage of lead. Using a suitable interpolation formula, find θ when $x = 48$.

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Solution : Given,

Step length or interval : $\boxed{h = 50 - 40 = 10}$

first value of x : $\boxed{x_0 = 40}$

Interpolation point : $\boxed{x = 48}$

Reduced Variable : $u = \dfrac{x - x_0}{h}$

$$= \frac{48 - 40}{10}$$

$$\boxed{u = 0.8}$$

Forward difference table,

x	θ	$\Delta\theta$	$\Delta^2\theta$	$\Delta^3\theta$	$\Delta^4\theta$	$\Delta^5\theta$
40	184					
		20				
			2			
50	204			0		
		22			0	
			2			0
60	226			0		
		24				
			2		0	
70	250					
		26		0		
			2			
80	276					
		28				
90	304					

Newton forward Interpolation formula,

$$\theta(x) = \theta_0 + u\Delta\theta_0 + \frac{u(u-1)}{2!}\Delta^2\theta_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3\theta_0$$

$$\theta(48) = 184 + (0.8 \times 20) + \frac{0.8(0.8-1)}{2!} \times 2 + 0$$

$$= 184 + 16 - 0.16$$

$$= 199.84$$

So, θ at $x = 48$ is 199.84

Ans.

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