Skewness and Kurtosis

Skewness: Skewness is the measurement of the lack of symmetry of the distribution. That is when a distribution is not symmetrical it is called a skewed distribution. In a symmetrical distribution the value of mean, median and mode are same. In a skewed these values differ.Skewness tells us about the direction of variation.

In the following distribution the value of mean=median=mode.

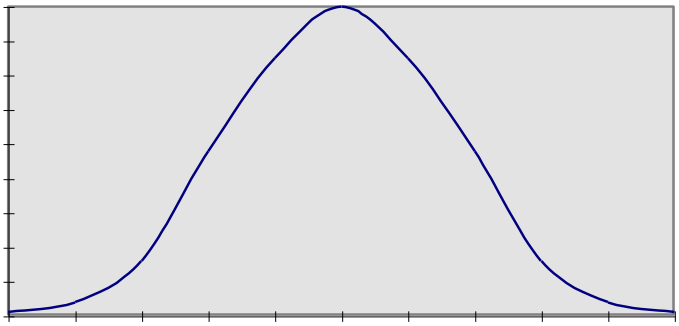

Fig-1: Symmetrical distribution

In the following two curves are not symmetric.

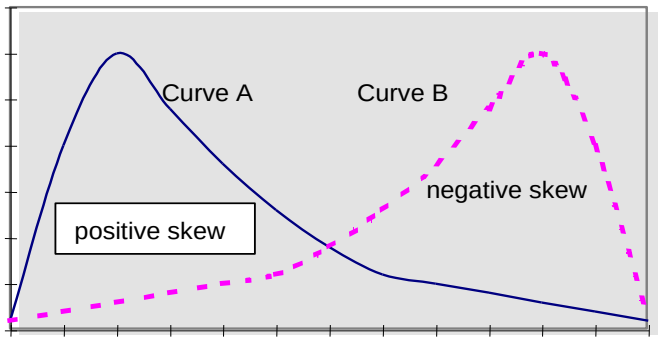

Fig-2: Skewed distribution

Two types of distributions with respect to asymmetry:

1. **Positively skewed distribution:** In the above fig-2; the curve A is positively skewed distribution. Distribution in which the tail is on the right.

In this case, Mean> Median.

2. **Negatively skewed distribution:** In the above fig-2; the curve B is negatively skewed distribution. Distribution in which the tail is on the left.

In this case, Median >Mean.

Coefficient of skewness: The method is most frequently used for measuring skewness.The formula for measuring coefficient of skewness is as follows:

$$Sk_p = \frac{3(Mean - Median)}{s}$$

Where;

Sk_p =Coefficient of skewness

Mean=sample mean

Median=sample median

s =sample standard deviation.

The range of the coefficient of skewness is lies between -3 to +3.

- If the skewness comes to less than -1 or greater than +1, the data distribution is highly skewed.
- If the skewness comes to between -1 and $-\frac{1}{2}$ or between $+\frac{1}{2}$ and +1, the data distribution is moderately skewed.
- If the skewness is between $-\frac{1}{2}$ and $+\frac{1}{2}$, the distribution is approximately symmetric.

Problem: From the following data calculate Coefficient of skewness:
15, 18,2,6,4

Sol: We know, $Sk_p = \frac{3(Mean - Median)}{s}$

$$Mean = \frac{15+18+2+6+4}{5} = 9$$

$Median$=6

$$s = \sqrt{\frac{(15-9)^2 + (18-9)^2 + (2-9)^2 + (6-9)^2 + (4-9)^2}{4}}$$

=7.07

Now, $Sk_p = \frac{3(Mean - Median)}{s}$

$$= \frac{3(9-6)}{7.07}$$
$$= 1.27$$

This indicates there is highly skewness in these data.

Kurtosis: Kurtosis refers the degree of flatness or peaked ness in the region about the mode of a frequency curve. That is, their peaked ness or flatness.
Three types of distributions with respect to kurtosis:

1. **Leptokurtic distribution:** Peaked distribution or more peaked then symmetric curve.

2. **Platykurtic distribution:** Flat distribution or less peaked then symmetric curve

3. **Mesokurtic distribution:** Normal distribution or symmetrical distribution.

Measures of Kurtosis: Kurtosis is measures by

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

Where, β_2 =coefficient of kurtosis; μ_4 =is the 4th moment; μ_2 =is the 2nd moment
If the values of β_2 is greater then 3, the curve is more peaked then symmetric curve that is leptokurtic.
If the values of β_2 is less then 3, the curve is less peaked then symmetric curve that is Platykurtic.

Moments: Moments are popularly used to describe the characteristics of a distribution.
It is denoted by μ .
For ungrouped data:
The rth moment of a variable X about the arithmetic mean $\bar{X}$ is given by:

$$\mu_r = \frac{\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^r}{N};$$

For different values of r, we shall get different moments.