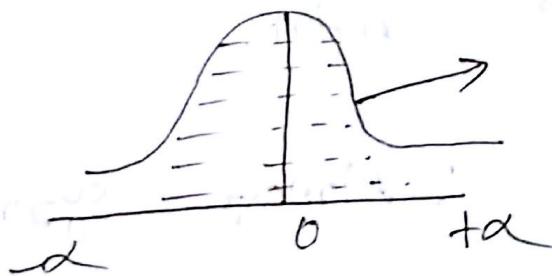


final

Skewness

①

Symmetric distribution  
Equal

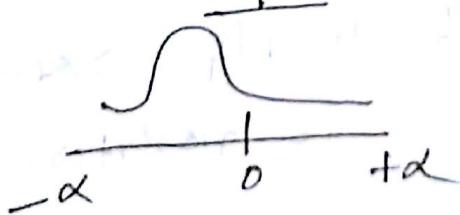


skewed

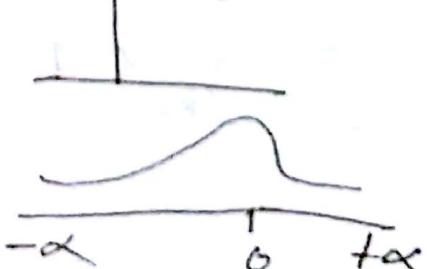
distribution

positive skewed

Postively



Negatively



## Coefficient of skewness

$$S_K = \frac{3(\text{mean} - \text{median})}{\text{S.D}}$$

$$S_K (-3 \text{ to } +3)$$

(2)

if,

①  $S_K = 0$ ; Symmetric distr.

②  $S_K = (-0.5 \text{ to } 0.5)$  ; Almost symmetric

③  $S_K > (0.5 \text{ to } 1)$  ; Moderately skewed  
which is positive.

④  $S_K (-1 \text{ to } -0.5)$  ; which is moderately skewed  
which is negative.

⑤  $S_K < -1$  ; Highly skewed which  
is negative.

⑥  $S_k > 1$ ; Highly skewed which is positive.

\* 10, 2, 11, 20, 1

Step 1: 1, 2, 10, 11, 20.

(3)

Calculated coeff of skeweness and comment interpret on your result. or Comment on the shape of the distribution.

med:

10

mean :

8.8

$$S.D = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(10-8.8)^2 + (2-8.8)^2 + (11-8.8)^2 + (20-8.8)^2}{5-1}}$$

$$\frac{1.44 + 46.24 + 4.84 + 125.44 + 60.84}{4}$$

$$2 \sqrt{59.2} = 7.73$$

$$S_K = \frac{3(\text{mean} - \text{median})}{\text{S.D}}$$

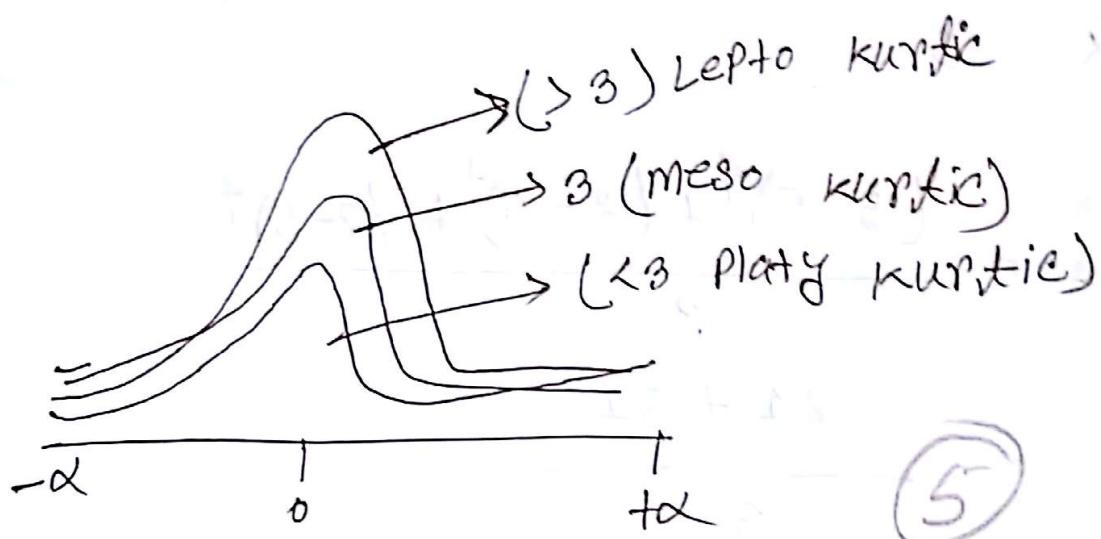
$$= \frac{3(8.8 - 10)}{7.73}$$

$$= -0.47$$

Almost symmetric

## Kurtosis

1. Lepto kurtic ;
2. Meso kurtic ;
3. Platy kurtic ;



Coefficient of kurtosis :

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$$

$\mu$  = moments.

$$* \quad m_p = \frac{\sum_{i=1}^N (x_i - \bar{x})^p}{N}$$

(5, 2, 8)

$$\bar{x} = 5$$

⑥

$$m_4 = \frac{(5-5)^4 + (2-5)^4 + (8-5)^4}{3}$$

$$= \frac{81 + 81}{3}$$

$$= 54$$

$$m_2 = \frac{(5-5)^2 + (2-5)^2 + (8-5)^2}{3}$$

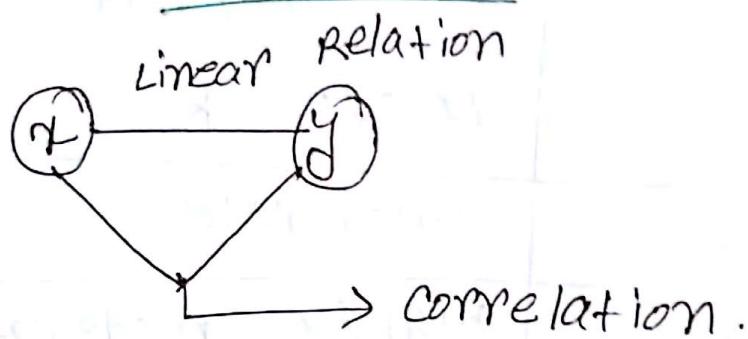
$$= \frac{9 + 9}{3}$$

$$= 6$$

$$\beta_2 = \frac{m_4}{(m_2)^2} = \frac{54}{(6)^2} = \frac{54}{36} = 1.5$$

प्राप्ति

## Correlation



Kar Pearson Correlation coefficient:

$$r^2 = \frac{\sum_{i=1}^n [(x_i - \bar{x})(y_i - \bar{y})]}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}}$$

$r=0$  No linear relation

$$r = \begin{pmatrix} -1 & +0 & +1 \end{pmatrix}$$

Nex<sup>+</sup>

$> 0 \text{ to } 0.2$	very low/very weak
$0.21 \text{ to } 0.4$	low/weak
$0.41 \text{ to } 0.6$	moderate
$0.61 \text{ to } 0.8$	highly moderate
$0.81 \text{ to } 1$	very high

Supply	Demand	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$
5	2	0.7	-1.6	0.49	2.56
6	4	1.7	0.34	2.89	0.1156
2	5	-2.3	1.34	5.29	1.79
$\bar{x} = 4.3$		$\bar{y} = 3.66$	0.1	0.08	8.67
					4.4656

$$\rho = \frac{\sum_{i=1}^n [(x_i - \bar{x})(y_i - \bar{y})]}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

8

$$= \frac{(1 \times 0.08)}{(8.67 \times 4.46)}$$

$$= \frac{8 \times 10^{-3}}{38.71}$$

$$= 2.066 \times 10^{-4}$$

coefficient of determination.

variation  $r^2 = 90\%$

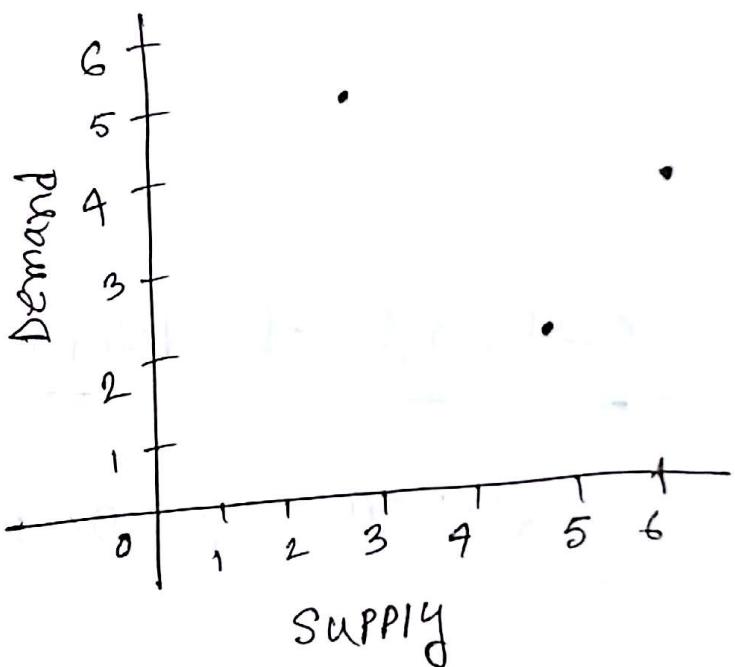
## Correlation

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

(10)

## Scatter plot

SUPPLY	Demand
5	2
6	4
2	5



Scatter plot

# Regression Analysis

(x)

(y)

(11)

- 1) Simple Regression Analysis (two variables)
  - i) independent var.
  - ii) Dependent var.

- 2) multiple Regression Analysis (more than two)
  - i) more than 1 Independent var.
  - ii) 1 Dependent var.

## Simple Regression Analysis:

Next

Estimated simple linear Regression equation / model / line:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

→ ind  
↓ de.

$\beta_0$        $\beta_1$       }  $\rightarrow$  Regression Parameter.

$\gamma_i$  = independent var.

$y_i$  = dependent var.

$$\hat{\beta}_1 = \text{slope} = \frac{\sum_{i=1}^n [(x_i - \bar{x})(y_i - \bar{y})]}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = \text{intercept} = \bar{y} - \hat{\beta}_1 \bar{x}$$

12

x	y
1	4
2	5
3	6

Income	Expenditure
20	18
17	6
15	15
12	11
$\bar{y} = 13.5$	
$\bar{x} = 12.5$	

$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$
5.5	30.25	6.5	35.25
-6.5	42.25	-6.5	42.25
2.5	6.25	1.5	3.75
-1.5	2.25	-1.5	2.25
	81		84

(13)

$$\rho_1 = \frac{\sum_{i=0}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \frac{84}{81} = 1.037$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 13.5 - (1.037 \times 12.5)$$

$$= 0.53$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$= 0.53 + 1.03x_i$$

(Ans:)

$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

→ Regression Parameter.

$\hat{\beta}_0$  = Intercept

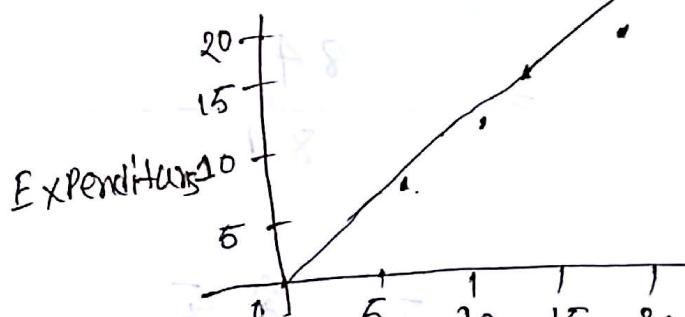
$\hat{\beta}_1$  = Slope.

(14)

Standard Error

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

Income	Expenditure
20	18
7	6
15	15
12	11



$$S_{x,y} = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}$$

$$\begin{aligned}\hat{y}_i &= -0.19 + 0.94x_i \\ &= -0.19 + 0.94 \times 20 \\ &= 18.61 \\ &\quad \text{---} \\ &6.33 \\ &\quad \text{---} \\ &13.91 \\ &\quad \text{---} \\ &11.00\end{aligned}$$

(15)

$$S_{x,y} = \sqrt{\frac{(18-18.61)^2 + (6-6.33)^2 + (15-13.91)^2 + (11-11.00)^2}{4-2}}$$

$$= \sqrt{\frac{0.37^2 + 10.89 + 1.18 + .81}{2}} = 1.11$$

~~Next~~

Q: calculating standard error?

$$\hat{y}_i = -0.19 + 0.94x_i$$

Question:

Cost, $x$	Profit
10	8
5	$x$
12	11

calculate standard error ( $S.E.$ )

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$\hat{\beta}_1 = \frac{\sum_{i=0}^n [(x_i - \bar{x})(y_i - \bar{y})]}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

(16)

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Cost	Profit	$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
10	8	1	-1.66	1	-1.66
5	$x$	-4	-1.66	16	6.64
12	11	3	2.34	9	7.02

26      13

$$\bar{x} = 9$$

$$\bar{y} = 8.66$$

$$\hat{\beta}_1 = \frac{13}{26}$$

$$= \frac{1}{2} = 0.5$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 8.66 - 0.5 \times 9$$

$$= 4.16$$

$$\hat{y}_1 = \hat{\beta}_0 + \hat{\beta}_1 x_1$$

$$= 4.16 + 0.5 \times 10$$

$$= 9.16$$

$$\hat{y}_2 = \hat{\beta}_0 + \hat{\beta}_1 x_2$$

$$= 4.16 + 0.5 \times 5$$

$$= 6.66$$

$$\hat{y}_3 = \hat{\beta}_0 + \hat{\beta}_1 x_3$$

$$= 4.16 + 0.5 \times 12$$

$$= 10.16$$

$$S_{(x,y)} = \sqrt{\frac{\sum (y_i - \bar{y}_i)^2}{n-2}}$$

$$= \sqrt{\frac{(10-9.16)^2 + (5-6.66)^2 + (12-10.16)^2}{3-2}}$$

$$= \sqrt{0.7056 + 2.75 + 3.34}$$

$$= 2.60$$

(Ans.)

(7)

## Correlation

### Correlation:

A correlation is a linear relationship between two variables. Correlation measures the linear association between two variables.

### Example:

- \* Is there any relationship between height and shoe size.
- \* Number of cigarettes smoked per day and lung cancer.
- \* Supply and demand.

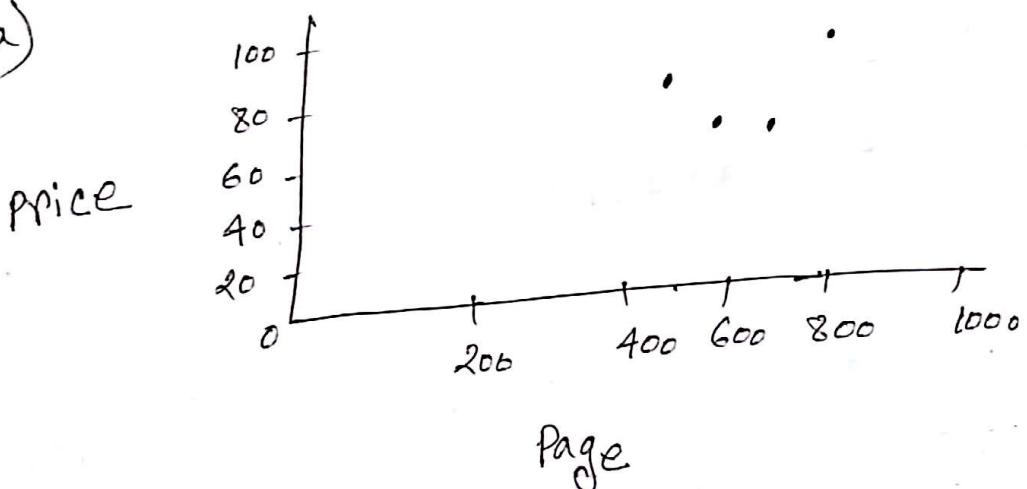
### Math:

Book	Page (X)	Price
History	500	84
Algebra	700	75
Management	800	99
Sociology	600	72

- i) draw scatter diagram
- ii) Determine the coefficient of correlation.
- iii) Interpret the result.

Solution:

a)



b) Here:

we know:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{2600}{4} = 650.$$

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{330}{4} = 82.5.$$

$x$	$y$	$(x_i - \bar{x})$	$y_i - \bar{y}$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
500	84	-150	1.5	22500	2.25	-225
700	75	50	-7.5	2500	56.25	-375
800	99	150	16.5	22500	272.25	2475
600	72	-50	-10.5	2500	110.25	525

$$\begin{array}{|c|c|c|c|} 
 \hline 
 & \sum_{i=1}^n (x_i - \bar{x})^2 & \sum_{i=1}^n (y_i - \bar{y})^2 & \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) \\ 
 \hline 
 & = 50000 & = 441 & = 2400 \\ 
 \hline 
 \end{array}$$

Now,

From the correlation coefficient.

$$\begin{aligned}
 r &= \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}} \\
 &= \frac{2400}{\sqrt{50000 \times 441}} \\
 &= 0.511.
 \end{aligned}$$

c. Interpretation: The correlation between the number of pages and the selling price of

the book is 0.511. This indicates a moderate association between the variable.

## Probability

1. Random Experiment
2. Event.
3. Mutually Exclusive event
4. Sample Space .

$$P(A) = \frac{m}{n}$$

$$= \frac{\text{Favorable no. of outcome}}{\text{Total no. of outcome}}$$

total 20 .

$$SAK = 10$$

$$MIP = 8$$

$$POL = 2$$

$$P(POL) = \frac{2}{20}$$

A Imposiab event:

$$P(A) = 0$$

"Digital Electronics"

Addition Rule (two events)

→ Specific addition rule:

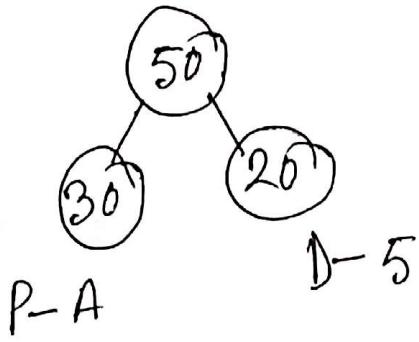
$$P(A \text{ or } B) = P(A) + P(B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B).$$

General addition rule:

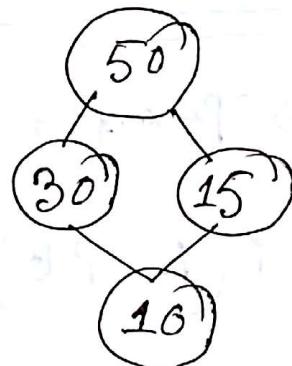
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$$\begin{aligned}
 P(P \text{ or } D) &= P(P) + P(D) \\
 &= 30/50 + 20/50
 \end{aligned}$$

$$\begin{aligned}
 P(P \text{ or } D) &= P(P) + P(D) - P(P \cap D) \\
 &= 30/50 + 15/50 - P(P \cap D) \\
 &= 30/50 + 15/50 - 10/50
 \end{aligned}$$



Laws / Properties of Probability

1.  $0 \leq P(A) \leq 1$
  2.  $\sum_{i=1}^n P(A_i) = 1$
- $\Rightarrow P(A_1) + P(A_2) + \dots + P(A_n) = 1$

$$SUK = 5$$

$$P(SUK) = 5/20$$

$$min = 10$$

$$P(min) = 10/20$$

$$UH = 5$$

$$P(UH) = 5/20$$

## Regression Analysis

Regression Analysis: is concerned with the study of the dependence of one variable (the dependent variable) on one or more other variable (the independent variables). In regression analysis we use independent variables to estimating or predicting the average change of the dependent variables.

### Examples:

The marking director of a company may want to know how the demand for

the company's product is related, to say advertising expenditure.

\* Suppose the sales manager of a company say X, wants to determine how the number of credit cards sell is related to the number of calls.

Some terms related to regression analysis:

\* Scatter plot: is a chart that portrays the relationship between two variables.

\* Dependent variable: the dependent variable is the variable being predicted or estimated. It is denoted by y.

\* Independent variable: The independent variable provides the basis for estimation. It is the predictor variable, if is denoted by x.

Linear Regression model: The equation the describes how  $y$  is related to  $x$  and an error terms is called the regression model.

the linear regression equation is:

$$y_i = \beta_0 + \beta_1 x_i + e_i$$

the estimated linear regression equation is given as:

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

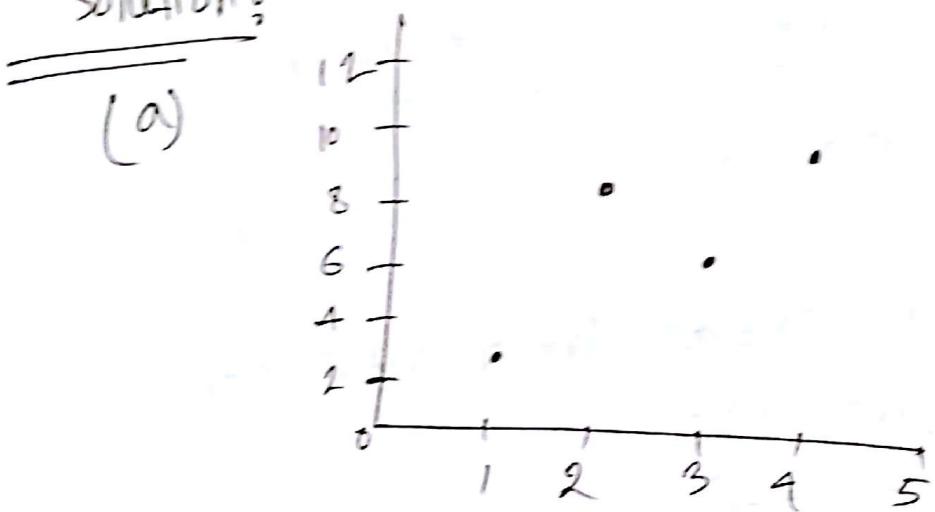
Month	Advertising	Sales revenue
July	2	7
August	1	3
September	3	8
October	4	10

- Draw Scatter Plot
- Determine the estimated regression equation.

c. Interpret the value  $\beta_0$  and  $\beta_1$

d. Estimate Sales when \$3 million is spent on advertising.

Solution:



(b) we know.  $\hat{y}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{10}{4} = 2.5 \text{ and}$$

$$\bar{y} = \frac{\sum_{i=1}^n y_i}{n} = \frac{28}{4} = 7$$

Advertising expense(x)	Sales revenue(y)	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
2	7	-0.5	0	0.25	0
4	3	-1.5	-4	2.25	6
3	8	0.5	4	0.25	0.5
4	10	1.5	3	2.25	4.5

$$\left[ \begin{array}{l} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^n (x_i - \bar{x}) = 5 \end{array} \right] \quad 11$$

$$\beta_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{11}{5}$$

$$\beta_1 = 2.2$$

$$\beta_0 = \bar{y} - \bar{\beta}_1 \bar{x} = 7 - 2.2 * 2.5$$

$$\beta_0 = 1.5$$

Now if we put the values of estimated regression equation is

$$\hat{y}_i = 1.5 + 2.2x_i$$

### C. Interpretation:

Here ~~slope~~, Slope is  $\hat{\beta}_1 = 2.2$  This means that an increase of \$ 1 million in advertising cost, the sales revenue will increase \$ 2.2 million.

and  $\hat{\beta}_0 = 1.5$  means that, is if there is no advertisement cost, then sales revenue would be \$ 1.5 million.

④ Now if  $x=3$  then

$$\hat{y} = 1.5 + 2.2 \times 3 = 8.1$$

So when advertisement cost is \$ 3 million, the sales revenue would be 8.1 million.

## Standard error

The Standard Error of Estimate measures the scatter, or dispersion, of the observed values around the line of regression. The formula that is used to compute the standard error.

$$S_{(y|x)} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}}$$

Example: From the previous example we calculate the value of  $\hat{y}_i$  as we know.

Standard Error:

$$S_{(y|x)} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}}$$

$y_i$	$\hat{y}_i$	$(y_i - \hat{y}_i)$	$(y_i - \hat{y}_i)^2$
2	5.9	1.1	1.21
3	3.7	-0.7	0.49
8	8.1	-0.1	0.01
10	10.3	-0.3	0.09

$$\sum_{i=1}^n (y_i - \hat{y}_i) = 1.8$$

Here  $n = 4$

Now by putting value we get:

$$S(y, \nu) = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n-2}}$$

$$= \sqrt{\frac{1.8}{2}}$$

$$= 0.95$$

So the Standard error is 0.95.

Coefficient of Determination  $r^2$ :

The coefficient of determination tells the percent of the variation in the dependent.

variable that is explained (determined) by the model and the explanatory variable.

Interpretation of  $r^2$ : Suppose  $r^2 = 92.7\%$

Interpretation: Almost 93% of the variability of the dependent variable explained by the independent variables.

Relationship between regression and correlation:

$$\hat{r} = \hat{\beta}_1 \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Regression versus Correlation:

In correlation the primary objective is to measure the strength or degree of linear association between two variables.

- \* In regression analysis, we are not primarily interested in such a measure. Instead, we try to estimate or predict the average value of one variable on the basis of the fixed values of other variables.
- \* In correlation analysis there is no distinction between the dependent and explanatory variables.

In regression analysis there is an asymmetry in the treatment of dependent and explanatory variables. The dependent variable is assumed to be statistical, random, or stochastic. The explanatory variables, on the other hand, are assumed to have fixed values.

## Multiplication Rule

or  $\rightarrow U \rightarrow (+)$

and  $\rightarrow \cap \rightarrow (x)$

1. Specific multiplication rule:

$$P(A \cap B) = P(A) \times P(B)$$

$$\Rightarrow P(A \text{ and } B) = P(A) \times P(B)$$

[Independent events] 

2. General multiplication rule:

(Dependent events.)

$$P(A \cap B) = P(A) \cdot P(B/A)$$

  $\rightarrow$  conditional prob.

$$\Rightarrow P(A \text{ and } B) = P(A) \cdot P(B/A)$$

Example: A company has two large computers. The prob. That the newer one will breakdown on any particular month is 0.05. The prob. That

The older one will breakdown on any particular month is 0.1

What is the ~~prob~~ prob. That they will both breakdown in a particular month?

$$P(A) = 0.05 \quad P(A \cap B) = P(A) \times P(B)$$

$$P(B) = 0.1 \quad \text{and} \quad P(A|B) = 0.05 \times 0.1$$

$$= 6.005 \quad (\text{Ans:}).$$

## Complement Rule

$$P(\bar{A}) = 1 - P(A)$$

Example: Rahim can hit a target 5 out of 7

Karim n n n n Guvning charge

find the chances/probability that the

target is hit ones they both they?

$$P(R) = \frac{5}{7}$$

$$P(K) = \frac{6}{11}$$

~~\*\*~~  $P(R \text{ or } K) = P(R) + P(K) - P(R \text{ and } K)$

$$= \frac{5}{7} + \frac{6}{11} - [P(R), P(K)]$$

$$= \frac{5}{7} + \frac{6}{11} - \left( \frac{5}{7} * \frac{6}{11} \right)$$

$$\boxed{P(R \text{ and } K)} \\ = P(R) * P(K)$$

Complementary

$$P(R) = \frac{5}{7}$$

$$P(K) = \frac{6}{11}$$

$$P(\bar{R}) = 1 - \frac{5}{7} = \frac{2}{7}$$

$$P(\bar{K}) = 1 - \frac{6}{11} = \frac{5}{11}$$

$$P(R \cap K) = P(R) \cdot P(K)$$

$$= \frac{2}{7} \times \frac{5}{12}$$

Karim and Rahmin  
করিম এবং রহমিন

$$1 - P(\bar{R} \cap \bar{K}) = P(R \cup K)$$

$$\Rightarrow P(R \cup K) = 1 - \frac{16}{27}$$

$$= \frac{27 - 16}{27}$$

প্রক্রিয়া  
পদ্ধতি

1. marginal probability

2. joint

3. conditional

Condition Probability:

$$P(A/B) = \frac{P(A \text{ and } B)}{P(B)} \quad | \quad P(B) \neq 0$$

$$\text{or } P(B/A) = \frac{P(B \text{ and } A)}{P(A)} ; P(A) \neq 0$$

Marginal Probability:

$$N = 130$$

$$P(\text{Yes}) = \frac{90}{130}$$

$$P(\text{male}) = \frac{70}{130}$$

$$P(\text{female}) = \frac{60}{130}$$

	Yes	No	total
Femal	50	10	60
male	40	30	70
total	90	40	130

Joint Probability

$$P(\text{male and } N) = \frac{30}{130}$$

$$P(\text{female and yes}) = \frac{50}{130}$$

Conditional Probability

$$P(\text{Male}/\text{yes}) = \frac{P(\text{Male and Yes})}{P(\text{Yes})}$$

$$= \frac{40/130}{90/130} = \frac{40}{90}$$

$$P(F/N) = \frac{10/130}{40/130} = \frac{10}{40}$$

Exam A humburger chain found that 75% of all customers use salad, 80%, use ketchup and 65% use both what are the probabilities that.

- i) a ketchup used salad
- ii) a salad n ketchup.

$$P(S) = 0.75 = \frac{75}{100}$$

$$P(K) = 0.80$$

$$P(S \text{ and } K) = 0.65$$

$$\text{i)} P(S/K) = \frac{P(S \text{ and } K)}{P(K)} = \frac{0.65}{0.80} \quad (\text{Ans:})$$

$$\text{ii)} P(K/S) = \frac{P(K \text{ and } S)}{P(S)} = \frac{0.80}{0.65} \quad (\text{Ans:})$$

