ERROR ANALYSIS

Q. What is Numerical Methods?

Ans: A **numerical method** is a complete and unambiguous set of procedures for the solution of a problem, together with computable error estimates. The study and implementation of such methods is the province of numerical analysis.

Q. What are uses of numerical methods in computer science?

Ans: There are so many uses for numerical methods, it is impossible to list them all. But essentially, we can cover first the basic math problems they can be used for, which are often:

- Computing integrals and derivatives
- Solving differential equations
- Building models based on data, be it through interpolation, Least Square, or other methods
- Root finding and numerical optimization
- Estimating the solution to a set of linear and nonlinear equations
- Computational geometry

There's other areas I haven't listed, but that's some of the common fundamental uses. With respect to real world problems, here are some examples where numerical methods are used:

- Development and computation of optimal control algorithms
- Development of high fidelity simulations to model viscous flow around a race car to see if the wing designs generate sufficient down force
- Machine learning algorithms, like estimating optimal weights of parametric models using only subsets of the full dataset (like stochastic gradient descent)
- Photorealistic renderer
- Design optimization based on simulation and multi-objective optimization formulations
- Game Engines

There are many more uses for numerical methods out there, but this will hopefully show a range of areas to prove its uses are broad.

Numerical Error:

There are two kinds of numbers such as exact and approximate numbers. The exact numbers are $1, 2, 5, \ldots, \frac{2}{5}, \frac{3}{2}, \ldots \ldots, \pi, e, \ldots, etc.$ The approximate numbers are representations of exact numbers to a certain degree of accuracy. Thus, 3.1416 is an approximate number of π and 3.14159265 is another approximate number of π.

Error: The error of a quantity is the difference between it's true value and approximate. It is denoted by E. If the true value is X and approximate value is x then the error of the quantity is given by,

$$E = X - x$$

Absolute Error: The absolute error of a quantity is the absolute value of the difference between the true value X and the approximate value x. It is denoted by E_A.

$$i.e,\ E_A = |X - x|$$

Relative Error: The relative error of a quantity is the ratio of it's absolute error to it's true value. It is denoted by E_R.

$$i.e,\ E_R = \frac{E_A}{X}$$

Percentage Error: The percentage error of a quantity is 100 times of it's relative error. It is denoted by E_p.

$$i.e,\ E_p = 100E_R$$

Problem-01: An approximate value of π is 3.1428571 and true value is 3.1415926. Find the absolute, relative and percentage errors.

Solution: We have, true value $X = 3.1415926$ and approximate value

The absolute error is,

$$\begin{aligned} E_A &= |X - x| \\ &= |3.1415926 - 3.1428571| \\ &= |-0.0012645| \\ &= 0.0012645 \end{aligned}$$

The relative error is,

$$\begin{aligned} E_R &= \frac{E_A}{X} \\ &= \frac{0.0012645}{3.1415926} \\ &= 0.000402 \end{aligned}$$

The percentage error is,

A **round-off error**, also called rounding **error**, is the difference between the calculated approximation of a number and its exact mathematical value due to rounding.

$$\begin{aligned} E_p &= 100E_R \\ &= 100 \times 0.000402 \\ &= 0.0402 \end{aligned}$$

Note: If the number X is rounded to N decimal places, then $E_A = \frac{1}{2}\left(10^{-N}\right)$

Problem-02: Find the absolute, relative and percentage errors of the number 8.6 if both of its digits are correct.

Solution: The given number is $X = 8.6$

Since both digits are correct so $N = 1$

The absolute error is,

$$\begin{aligned} E_A &= \frac{1}{2}\left(10^{-1}\right) \\ &= 0.05 \end{aligned}$$

The relative error is,

$$\begin{aligned} E_R &= \frac{E_A}{X} \\ &= \frac{0.05}{8.6} \\ &= 0.0058 \end{aligned}$$

The percentage error is,

$$\begin{aligned} E_p &= 100E_R \\ &= 100 \times 0.0058 \\ &= 0.58 \end{aligned}$$

Problem-03: Evaluate the sum $S = \sqrt{2} + \sqrt{3} + \sqrt{5}$ to 4 significant digits and find its absolute, relative and percentage errors.

Solution: we have, $\sqrt{2} = 1.414, \quad \sqrt{3} = 1.732, \quad \sqrt{5} = 2.236$

$$\begin{aligned} \therefore \ S &= 1.414 + 1.732 + 2.236 \\ &= 5.382 \end{aligned}$$

Since the values of $\sqrt{2}, \sqrt{3}\ and\ \sqrt{5}$ are rounded of three decimal places, so $N = 3$ .e

The absolute error is,

$$E_A = \frac{1}{2}\left(10^{-3}\right) + \frac{1}{2}\left(10^{-3}\right) + \frac{1}{2}\left(10^{-3}\right)$$
$$= 0.0005 + 0.0005 + 0.0005$$
$$= 0.0015$$

The absolute error shows that the sum is correct to 3 significant digits only.

Hence we take $S = 5.38$

The relative error is,

$$E_R = \frac{E_A}{X}$$
$$= \frac{0.0015}{5.38}$$
$$= 0.00028$$

The percentage error is,

$$E_p = 100E_R$$
$$= 100 \times 0.00028$$
$$= 0.028$$

For Practices

1) Define percentage error, absolute error with example.

2) Evaluate the sum **S =** $\sqrt{\mathbf{3}} + \sqrt{\mathbf{5}} + \sqrt{\mathbf{7}} + \sqrt{\mathbf{11}}$ to **4** significant digits and find its percentage error, absolute error, relative error?

3) Find the absolute, relative and percentage errors of the number 5.2356 if **4** significant digits are correct.

4) Evaluate the sum **S =** $\sqrt{\mathbf{11}} + \sqrt{\mathbf{21}} + \sqrt{\mathbf{31}}$ to **5** significant digits and find its E_A, E_R, E_P ?

5) Find the absolute, relative and percentage errors of the number 0.3576 if **2** significant digits are correct.