1)

$$\gamma = \prod_{j=1}^{p} \frac{1}{s_j^2} \quad (1)$$

$$\frac{\partial K^s(x_l, x_k)}{\partial \frac{1}{s_h^2}} = \exp\left\{-\frac{1}{2}\sum_{j=1}^{p} \frac{1}{s_j^2}(x_{lj} - x_{kj})^2\right\} \cdot -\frac{1}{2}(x_{lh} - x_{kh})^2$$

$$\frac{\partial L_{KCM-r-GN}}{\partial \frac{1}{s_h^2}} = \sum_{i=1}^{c} \sum_{e_k \in P_i} \left\{ -\frac{2}{|P_i|} \cdot \sum_{e_l \in P_i} K^s(x_k, x_l) \cdot -\frac{1}{2}(x_{lh} - x_{kh})^2 + \frac{1}{|P_i|^2} \sum_{e_r \in P_i} \sum_{e_s \in P_i} K^s(x_r, x_s) \cdot -\frac{1}{2}(x_{rh} - x_{sh})^2 \right\}$$

$$- w \prod_{\substack{j=1 \\ j \neq h}}^{p} \frac{1}{s_j^2}$$

$$= \pi_h - w \prod_{\substack{j=1 \\ j \neq h}}^{p} \frac{1}{s_j^2} = 0$$

$$\pi_h - \frac{w}{\frac{1}{s_h^2}} \cdot \prod_{j=1}^{p} \frac{1}{s_j^2} = 0$$

$$\pi_h = \frac{w}{\frac{1}{s_h^2}} \cdot \gamma \rightarrow \frac{1}{s_h^2} = \frac{w}{\pi_h} \cdot \gamma \quad (2)$$

(2) -> (1):

$$\gamma = \prod_{j=1}^{p} \frac{w \cdot \gamma}{\pi_j}$$

2]

$$Y = \frac{w^P \cdot Y^P}{\prod_{j=1}^{P} \pi_j} \rightarrow w^P = \frac{Y \cdot \prod_{j=1}^{P} \pi_j}{Y^P}$$

$$w = \left(\frac{Y \cdot \prod_{j=1}^{P} \pi_j}{Y^P} \right)^{1/P}$$

$$w = \frac{Y^{1/P} \cdot \left\{ \prod_{j=1}^{P} \pi_j \right\}^{1/P}}{Y}$$

3

$$\boxed{\pi_h - \frac{w}{\frac{1}{s_h^2}} \cdot \gamma = 0} \; ; \; \boxed{w = \frac{\gamma^{1/p} \cdot \left\{ \prod_{j=1}^{p} \pi_j \right\}^{1/p}}{\gamma}}$$

$$\pi_h = \frac{\gamma^{1/p} \cdot \left\{ \prod_{j=1}^{p} \pi_j \right\}^{1/p}}{\frac{1}{s_h^2}}.$$

$$\frac{1}{s_h^2} = \frac{\gamma^{1/p} \left\{ \prod_{j=1}^{p} \pi_j \right\}^{1/p}}{\pi_h}$$

onde:

$$\pi_h = \sum_{i=1}^{c} \sum_{e_k \in P_i} \left\{ \frac{1}{|P_i|} \cdot \sum_{e_l \in P_i} \cdot \kappa^s(x_k, x_l) \left(x_{kh} - x_{lh} \right)^2 - \frac{1}{2|P_i|^2} \sum_{e_r \in P_i} \sum_{e_s \in P_i} \kappa^2(x_r, x_s) \cdot \left(x_{rh} - x_{sh} \right)^2 \right\}$$