# Introduction to AI

François Jacquenet

Laboratoire Hubert Curien
Université de Saint-Etienne
Francois.Jacquenet@univ-st-etienne.fr

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# General idea

**Problem: given a sentence S, say whether S is valid, unsatisfiable or contingent**

Convert S to a clause $\Phi$ in CNF

**if** $\Phi \vdash \{\}$ **then** $\Phi$ is unsatisfiable

**else if** $\neg\Phi \vdash \{\}$ **then** $\Phi$ is valid

**otherwise** $\Phi$ is contigent

# Example 1

Given the sentence $p \wedge (p \Rightarrow \neg q) \wedge q$
The clausal form is: $\{p\}, \{\neg p, \neg q\}, \{q\}$

| | | |
|---|---|---|
| 1. | $\{p\}$ | Premise |
| 2. | $\{\neg p, \neg q\}$ | Premise |
| 3. | $\{q\}$ | Premise |
| 4. | $\{\neg q\}$ | 1, 2 |
| 8. | $\{\}$ | 3, 4 |

Conclusion: the sentence $p \wedge (p \Rightarrow \neg q) \wedge q$ is unsatisfiable

# Example 2

Given the sentence $(p \Rightarrow q) \vee (q \Rightarrow p)$

The clausal form is: $\{\neg p, q, \neg q, q\}$

We can not prove anything from that, so the sentence $(p \Rightarrow q) \vee (q \Rightarrow p)$ is not unsatisfiable

Consider the negation of this sentence: $\neg((p \Rightarrow q) \vee (q \Rightarrow p))$

The clausal form is: $\{p\}$, $\{\neg q\}$, $\{q\}$, $\{\neg p\}$

| | | |
|---|---|---|
| 1. | $\{p\}$ | Premise |
| 2. | $\{\neg q\}$ | Premise |
| 3. | $\{q\}$ | Premise |
| 4. | $\{\neg p\}$ | Premise |
| 8. | $\{\}$ | 1, 4 |

Conclusion: the sentence $\neg((p \Rightarrow q) \vee (q \Rightarrow p))$ is unsatisfiable so the sentence $(p \Rightarrow q) \vee (q \Rightarrow p)$ is valid

# Example 3

Given the sentence $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$

The clausal form is: $\{\neg p, q\}$, $\{\neg p, \neg q\}$

We can not prove the empty clause from that, so the sentence $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$ is not unsatisfiable

Consider the negation of this sentence: $\neg((p \Rightarrow q) \vee (p \Rightarrow \neg q))$

The clausal form is: $\{p\}$, $\{p, q\}$, $\{\neg q, p\}$, $\{\neg q, q\}$

and here again we can not prove the empty clause from that, so the sentence $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$ is not valid

Conclusion: the sentence $(p \Rightarrow q) \vee (p \Rightarrow \neg q)$ is contingent