Turing Machine

In the following, B is the blank symbol. Initially, all the tape's cells are filled with blank symbols, except those used by the input string. Tape(s) is(are) bi-infinite. Symbols R and L respectively encodes a movement of the head to the right and to the left, and if needed, symbol V will encode no movement: the head remains on the same cell.

Exercise 1 *Deterministic Turing Machine*

Let M a Turing machine be defined by:

$$(Q = \{q_0, q_1, q_{Accept}\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, B\}, \delta, q_0, F = \{q_A\}, B)$$

δ is defined as follows:

State	0	1	B
q_0	$q_1, 0, R$	$q_0, 1, R$	q_{Accept}, B, V
q_1	$q_0, 0, R$	$q_1, 1, R$	—

Question 1 *Give some details about the elements defining the machine. Draw the transition diagram corresponding to the definition of M. Simulate the computation of M with the following initial configurations: $C = q_0011$, $C' = q_00011B$, $C'' = q_011$? In each case, provide the values of time and space complexity of the computation. What is the complexity of $t_M(w), w \in \Sigma^*$? What does this machine compute ?*

Exercise 2 *Two-Tapes Deterministic Turing Machine*

Let M a two-tapes Turing machine be defined by:

$$(Q = \{q_C, q_R, q_T, q_A\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, B\}, \delta, q_C, F = \{q_A\}, B)$$

δ is defined as follows:

State	0,0	1,1	0,B	1,B	B,B	B,0	B,1
q_C	—	—	$q_C, 0, 0, R, R$	$q_C, 1, 1, R, R$	q_R, B, B, V, L	—	—
q_R	—	—	—	—	q_T, B, B, L, R	$q_R, B, 0, V, L$	$q_R, B, 1, V, L$
q_T	$q_T, 0, 0, L, R$	$q_T, 1, 1, L, R$	—	—	q_A, B, B, V, V	—	—

Convention: The cell of this table indexed by (row) q_C and (column) "$0, B$" encodes the following transition: in state q_C reading a 0 on the first cell and a B on the second, machine changes to (remains in) state q_C, writes a 0 on the first tape, a 0 on the second tape, moves the first tape's head to the right, and the second tape's head to the right.

Question 2 *Draw the transition diagram corresponding to the definition of M. Simulate the computation of M with the following initial configuration: q_C010 for the first tape, second tape containing only blank symbols? Same question with q_C011B as content of the first tape? Explain with words what are the states q_C, q_R and q_T made for? What does this machine compute?*

Exercise 3 *Non-Deterministic Turing Machine*

Let M a Non-deterministic Turing machine be defined by:

$$(Q = \{q_0, q_1, q_2, q_f\}, \Sigma = \{0, 1\}, \Gamma = \{0, 1, B\}, \delta, q_0, F = \{q_f\}, B)$$

Transition function δ is defined by the following set of transitions:

$$\{(q_0, 0, q_0, 1, R), (q_0, 0, q_1, 1, R), (q_1, 1, q_2, 0, L), (q_2, 1, q_0, 1, R), (q_1, B, q_f, B, R)\}$$

Question 3 *Draw the corresponding transition diagram. Draw the tree of computation with initial configuration $C = q_0 00011$. What is the time complexity for this computation? What is the value of $r = \max_{q \in Q, a \in \Gamma} |\{(q, a, q', z, Z) \in \delta\}|$? What would be the number of steps of the simulation of this non-deterministic computation by a deterministic Turing machine when the initial configuration is $C = q_0 00011$?*

Q1: Details: see the slides, Definition 3.

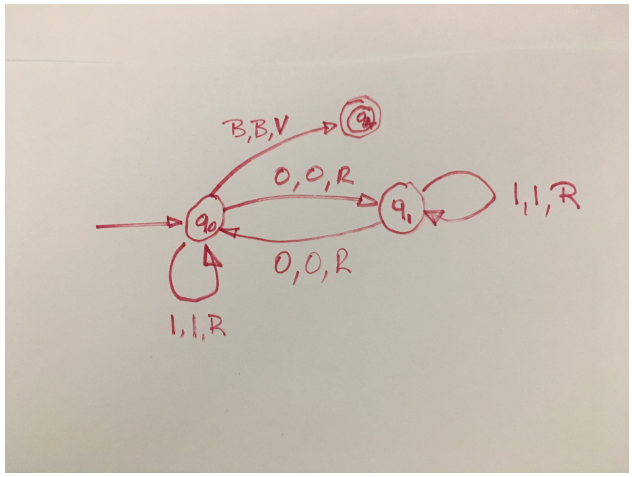

FIGURE 1 – Transition diagram Question 1

For the computations, see `http://turingmachinesimulator`, Examples/Even amount of zeros. $t_M(011) = 3, s_M(011) = 3$, $t_M(0011) = 5, s_M(0011) = 5$, $t_M(11) = 3, s_M(11) = 3$. Notice that after the transition from q_0 to q_{Accept} the head remains on the same cell, but the content of this cell is used (must be a blank symbol for this transition).

$t_M(w) = |w| + 1$ if w contains an even number of symbol 0 and $t_M(w) = |w|$ if not. We would write $t_M(w) = O(|w|)$. M accepts strings containing an even number of symbol 0 (notice that 0 is a even number...).

Q2: For the computations, see website `http://turingmachinesimulator`, Examples/2 Tapes/Fast binary palindrome.

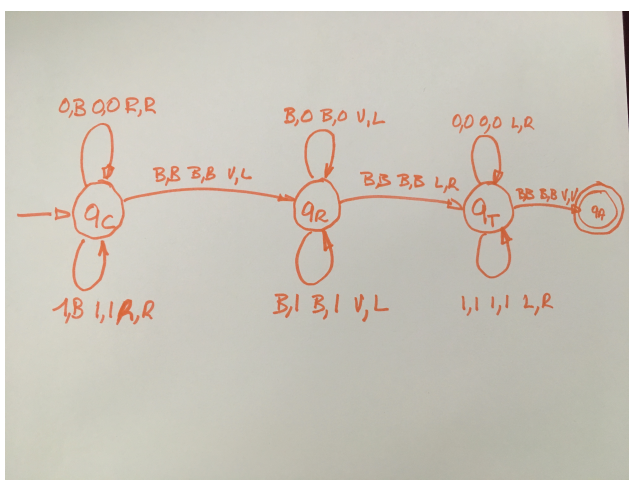

FIGURE 2 – Transition diagram Question 2

In q_C, M copies the input string onto the second tape, C stands for Copy. In q_R, the head of the second tape goes to the left without changing the content, until it reaches the first symbol B on the left of this content. The head of the first tape remains on the first B after the input word. R stands for Return. In q_T, M compares the contents of both tapes, reading the first tape form right to left, and the second from left to right. q_A is reached if symbols on both tapes are equals, that is, the input word can be read both ways and thus is a palindrome.

Q3:

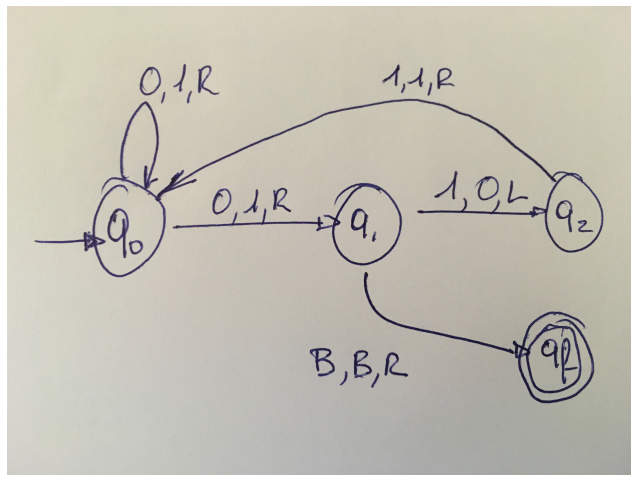

FIGURE 3 – Transition diagram Question 3

$t_M(00011) = 10$, $r = 2$. At worst, the simulation of this non-deterministic computation by a deterministic machine would require the simulation of all paths of length at most 10 in a binary tree, that is $\sum_{i=1}^{10} i.2^i$ (using the proof of the equivalence between NDTM and DTM, this would mean generating all strings y of lengths 1 to 10).

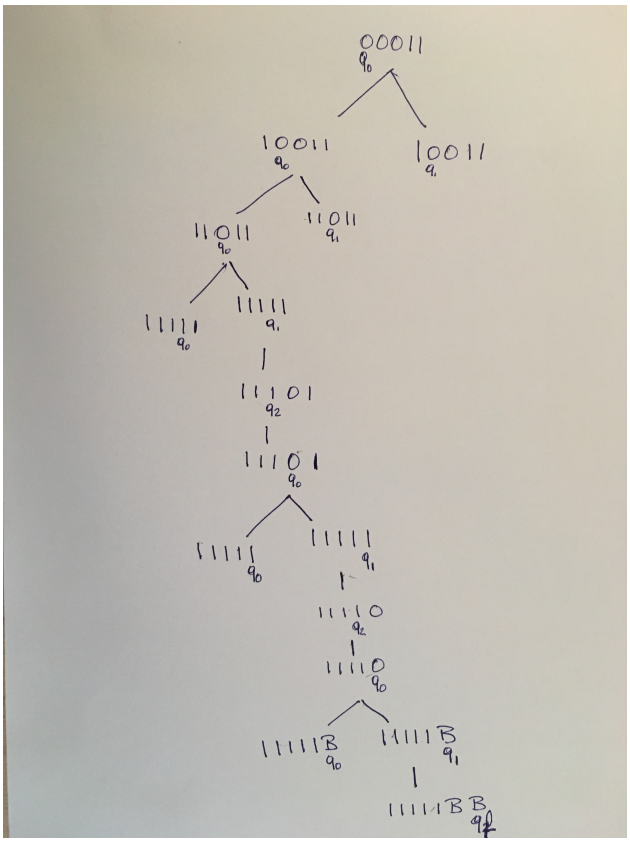

FIGURE 4 – Tree of computation with $w = 00011$