Master MLDM/DSC/CPS2 - First year
Introduction to Artificial Intelligence
Exam on Propositional and First Order Logics
October 2019

1 Truth table ($\simeq$ 2 points)

1. Using the truth table method, prove that the premises: $p \Rightarrow q$ and $r \Rightarrow p \vee q$, logically entail $r \Rightarrow q$
2. Using the truth table method, prove that $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ is valid

1.1 Solution to question 1

p	q	r	p	⇒	q	r	⇒	(p	∨	q)	r	⇒	q
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1	1	1	1	0	1	1
1	0	1	1	0	0	1	1	1	1	0	1	0	0
1	0	0	1	0	0	0	1	1	1	0	0	1	0
0	1	1	0	1	1	1	1	0	1	1	1	1	1
0	1	0	0	1	1	0	1	0	1	1	0	1	1
0	0	1	0	1	0	1	0	0	0	0	1	0	0
0	0	0	0	1	0	0	1	0	0	0	0	1	0

We can see that, for every row (rows 1, 2, 5, 6, 8) where we have a 1 in the column of $p \Rightarrow q$ and a 1 in the column of $r \Rightarrow (p \vee q)$ then we have a 1 in the column of $r \Rightarrow q$. That means **the two first formulas logically entail the third one**.

1.2 Solution to question 2

p	q	r	(p	⇒	(q	⇒	r))	⇒	((p	⇒	q)	⇒	(p	⇒	r))
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	0	1	1	1	1	0	1	0	0
1	0	1	1	1	0	1	1	1	1	0	0	1	1	1	1
1	0	0	1	1	0	1	0	1	1	0	0	1	1	0	0
0	1	1	0	1	1	1	1	1	0	1	1	1	0	1	1
0	1	0	0	1	1	0	0	1	0	1	1	1	0	1	0
0	0	1	0	1	0	1	1	1	0	1	0	1	0	1	1
0	0	0	0	1	0	1	0	1	0	1	0	1	0	1	0

We can see that, for every row of the table, the truth value of $(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$ is always true. That means **the formula is valid**.

2 Validity, unsatisfiability, contingency ($\simeq$ 3 points)

Using resolution reasoning, say whether each sentence below is valid, unsatisfiable or contingent:

1. $(\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \Rightarrow (s \vee q)$
2. $(a \Rightarrow b) \wedge (\neg a \Rightarrow (b \vee c)) \wedge (\neg c \Rightarrow \neg b) \wedge ((b \wedge c) \Rightarrow \neg a) \wedge (c \Rightarrow a)$

2.1 Solution for sentence 1

First, we try to prove the empty clause from the CNF of this sentence. That would prove the sentence is unsatisfiable. **Take care to the precedence of operators!**

The first step is to convert the sentence to CNF:

- $(\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \Rightarrow (s \vee q)$
- $\neg((\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \vee (s \vee q)$
- $\neg(\neg(\neg q \vee (\neg(p \vee \neg p) \wedge r)) \vee s) \vee (s \vee q)$
- $\neg(\neg(\neg q \vee (\neg p \wedge p \wedge r)) \vee s) \vee (s \vee q)$
- $\neg((q \wedge \neg(\neg p \wedge p \wedge r)) \vee s) \vee (s \vee q)$
- $(\neg(q \wedge \neg(\neg p \wedge p \wedge r)) \wedge \neg s) \vee (s \vee q)$
- $((\neg q \vee (\neg p \wedge p \wedge r)) \wedge \neg s) \vee (s \vee q)$
- $((\neg q \vee \neg p) \wedge (\neg q \vee p) \wedge (\neg q \vee r) \wedge \neg s) \vee (s \vee q)$
- $(\neg q \vee \neg p \vee s \vee q) \wedge (\neg q \vee p \vee s \vee q) \wedge (\neg q \vee r \vee s \vee q) \wedge (\neg s \vee s \vee q)$

So we have four clauses:

1. $\{\neg q, \neg p, s, q\}$
2. $\{\neg q, p, s, q\}$
3. $\{\neg q, r, s, q\}$
4. $\{\neg s, s, q\}$

and there is no way to prove the empty clause from this. **The sentence is not unsatisfiable**.

So now let's try to prove the empty clause from the CNF of the negation of the sentence. That would prove the negation of the sentence is unsatisfiable, so the sentence is valid.

The first step is to convert the sentence to CNF:

- $\neg[(\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \Rightarrow (s \vee q)]$
- $\neg[\neg((\neg q \vee (\neg(p \vee \neg p) \wedge r) \Rightarrow s) \vee (s \vee q)]$
- $\neg[\neg(\neg(\neg q \vee (\neg(p \vee \neg p) \wedge r)) \vee s) \vee (s \vee q)]$
- $(\neg(\neg q \vee (\neg(p \vee \neg p) \wedge r)) \vee s) \wedge \neg s \wedge \neg q$
- $((q \wedge \neg(\neg(p \vee \neg p) \wedge r)) \vee s) \wedge \neg s \wedge \neg q$
- $((q \wedge (p \vee \neg p \vee \neg r)) \vee s) \wedge \neg s \wedge \neg q$
- $(q \vee s) \wedge (p \vee \neg p \vee \neg r \vee s) \wedge \neg s \wedge \neg q$

So we have four clauses:

1. $\{q, s\}$
2. $\{p, \neg p, \neg r, s\}$
3. $\{\neg s\}$
4. $\{\neg q\}$

We can then run the resolution principle several times:

$1 + 3 \rightarrow \{q\}(5)$

$5 + 4 \rightarrow \{\}$

We get the empty clause, so the negation of the sentence is unsatisfiable, that means **the sentence is valid**.

2.2 Solution to sentence 2

Here again, first, we try to prove the empty clause from the CNF of this sentence. That would prove the sentence is unsatisfiable. **Take care to the precedence of operators!**

- $(a \Rightarrow b) \wedge (\neg a \Rightarrow (b \vee c)) \wedge (\neg c \Rightarrow \neg b) \wedge ((b \wedge c) \Rightarrow \neg a) \wedge (c \Rightarrow a)$
- $(\neg a \vee b) \wedge (a \vee b \vee c) \wedge (c \vee \neg b) \wedge (\neg b \vee \neg c \vee \neg a) \wedge (\neg c \vee a)$

So we have five clauses:

1. $\{\neg a, b\}$
2. $\{a, b, c\}$
3. $\{c, \neg b\}$
4. $\{\neg b, \neg c, \neg a\}$
5. $\{\neg c, a\}$

We can then run the resolution principle several times:

$1 + 2 \rightarrow \{b, c\}(6)$

$6 + 3 \rightarrow \{c\}(7)$

$7 + 5 \rightarrow \{a\}(8)$

$8 + 4 \rightarrow \{\neg b, \neg c\}(9)$

$9 + 7 \rightarrow \{\neg b\}(10)$

$10 + 1 \rightarrow \{\neg a\}(11)$

$11 + 8 \rightarrow \{\}$

We get the empty clause, so **the sentence is unsatisfiable**.

Final remark: The general method to determine if a sentence S is unsatisfiable, valid or contingent, is as follows. Convert S to CNF and try to prove the empty clause. If you succeed, then **the sentence S is unsatisfiable**. If you fail, then consider the negation of the sentence S, convert it to CNF and try to prove the empty clause. If you succeed, then **the sentence S is valid**. If you fail, **the sentence S is contingent**.

3 Problem modeling and solving ($\simeq$ 5 points)

Brown, Jones, and Smith are suspected of a crime. In front of a jury they testify as follows:

Brown: "Jones is guilty and Smith is innocent."

Jones: "If Brown is guilty then so is Smith."

Smith: "I'm innocent and at least one of the others is guilty."

1) Model this universe using Propositional Logic, that is, provide three judicious proposition constants and convert those three testimonies to three proposition sentences.
2) Write a truth table for the three testimonies.
3) Use the above truth table to answer the following questions (explain your answer):

(a) Are the three testimonies satisfiable?

(b) The testimony of one of the suspects logically entails that of another. Say which one entails which one?

(c) Assuming that everybody is innocent, who committed perjury?

(d) Assuming that all testimonies are true, who is innocent and who is guilty?

(e) Assuming that the innocent told the truth and the guilty told lies, who is innocent and who is guilty?

3.1 Solution to question 1

The three proposition constants:

- BG: Brown is guilty
- JG: Jones is guilty
- SG: Smith is guilty

The three testimonies are translated into:

- $JG \wedge \neg SG$
- $BG \Rightarrow SG$
- $\neg SG \wedge (BG \vee JG)$

3.2 Solution to question 2

BG	JG	SG	$JG \wedge \neg SG$	$BG \Rightarrow SG$	$\neg SG \wedge (BG \vee JG)$
0	0	0	0	1	0
0	0	1	0	1	0
0	1	0	1	1	1
0	1	1	0	1	0
1	0	0	0	0	1
1	0	1	0	1	0
1	1	0	1	0	1
1	1	1	0	1	0

3.3 Solution to question 3

1. YES: row 3 has only 1s
2. $JG \wedge \neg SG \models \neg SG \wedge (BG \vee JG)$ That is: Brown testimony entails Smith testimony.
3. Everybody is innocent corresponds to raw 1, thus Smith tells the truth and Brown and Smith commited perjury.
4. Given raw 3, Jones is guilty, Brown and Smith are innocent.
5. We have to search for an assignment such that if BG (resp. JG and SG) is false (Brown is innocent) then the sentence of Brown (resp. Jones and Smith) is true and that if BG (resp. JG and SG) is true (Brown is guilty), then the sentence of Brown (resp. Jones and Smith) is false. The only assignment satisfying this restriction is truth assignment 6 (raw 6) in which Jones is innocent and Brown and Smith are guilty.

4 Problem modeling and solving ($\simeq$ 5 points)

Here are some informations about a simple world:

When Mary isn't sick, she sings, she dances with John, and Harry is jealous. When Lucy is sick and wants to run outside, John is afraid. When Mary is not happy, she cannot eat. When Mary is dancing with John or Harry, Lucy is sick. When John or Harry is jealous, Lucy is sick. Mary isn't sick. When John is afraid or Harry is jealous, Mary is not happy. When Mary sings, Lucy wants to run outside.

Model this universe using propositional logic, and then provide a resolution proof of: *Harry is jealous and Mary cannot eat.*

4.1 Solution

4.2 The proposition constants

- mis: Mary is sick
- ms: Mary sings
- mdj: Mary dances with John
- mdh: Mary dances with Harry
- hj: Harry is jealous
- jj: John is jealous
- lis: Lucy is sick
- lwr: Lucy wants to run outside
- ja: John is afraid
- mh: Mary is happy
- me: Mary can eat

4.3 Translating the sentences into propositional logic

- $\neg mis \Rightarrow ms \wedge mdj \wedge hj$
- $lis \wedge lwr \Rightarrow ja$
- $\neg mh \Rightarrow \neg me$
- $mdj \vee mdh \Rightarrow lis$
- $jj \vee hj \Rightarrow lis$
- $\neg mis$
- $ja \vee hj \Rightarrow \neg mh$
- $ms \Rightarrow lwr$
- goal (to be negated): $(hj \wedge \neg me)$

4.4 Converting to CNF

1. $mis \vee ms$
2. $mis \vee mdj$
3. $mis \vee hj$
4. $\neg lis \vee \neg lwr \vee ja$
5. $mh \vee \neg me$
6. $\neg mdj \vee lis$
7. $\neg mdh \vee lis$
8. $\neg jj \vee lis$
9. $\neg hj \vee lis$
10. $\neg mis$
11. $\neg ja \vee \neg mh$
12. $\neg hj \vee \neg mh$
13. $\neg ms \vee lwr$
14. $\neg hj \vee me$

4.5 Resolution proof

Here is one possible resolution

$$14 + 3 \rightarrow mis \vee me \ (15)$$

$$15 + 10 \rightarrow me \ (16)$$

$$16 + 5 \rightarrow mh \ (17)$$

$$17 + 12 \rightarrow \neg hj \ (18)$$

$$18 + 3 \rightarrow mis \ (19)$$

$$19 + 10 \rightarrow \{\}$$

5 Unification ($\simeq$ 2 points)

For each pair of logical sentences below, say whether they are unifiable or not. In case they are unifiable give their most general unifier, in case they are not unifiable, explain why.

1. p(A,a) and p(b,B)
2. r(f(Y),Y,b) and r(f(X), f(X), Z)
3. s(a,X,c) and s(Y,d,Z)
4. t(a,f(X,Y),b) and t(Z,f(a),W)
5. u(X,X,b) and v(a,a,Y)
6. w(a,b,c,c,b,a) and w(X,Y,Z,Z,Y,X)
7. x(a,f(X,g(a,Y),Z,W),b,T) and x(X,f(a,T,c,b),Y,g(X,W))
8. y(a,f(X,g(a,Y),Z,W),b,T) and y(X,f(a,T,c,b),Y,g(Y,W))

5.1 Solution

1. **Unifiable.** $\{A = b, B = a\}$
2. Depends if the occurs check is activated or not.
   (a) **Not unifiable** if occurs check is activated
   (b) **Unifiable** if occurs check is not activated. $\{Y = X, X = f(X), Z = b\}$
3. **Unifiable.** $\{X = d, Y = a, Z = c\}$
4. **Not unifiable**. f has not the same arity in the two sentences
5. **Not unifiable.** the functor of the first term is u/3 and the functor of the second term is v/3.
6. **Unifiable.** $\{X = a, Y = b, Z = c\}$
7. **Unifiable.** $\{X = a, Y = W, W = b, Z = c, T = g(a, b)\}$
8. **Not unifiable**. X unifies with a, then T unifies with g(a,Y), then Z unifies with c, W unifies with b, Y unifies with b and finally T, with is now g(a,b) should unify with g(b,b) which is not possible.

6 Validity, unsatisfiability, contingency ($\simeq$ 4 points)

Using resolution reasoning, say whether each sentence below is valid, unsatisfiable or contingent

1. $\forall X.\forall Y.((p(X) \land p(Y)) \Rightarrow q(X,Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X,Y))$
2. $(\exists X.q(X) \land (\forall X.(p(X) \Rightarrow \neg q(X)))) \Rightarrow \exists X.\neg p(X)$

6.1 Solution to question 1

First step, we try to prove the sentence is unsatisfiable. To do so, we have to transform it to a CNF and then try to prove the empty clause.

- $\forall X.\forall Y.((p(X) \land p(Y)) \Rightarrow q(X,Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X,Y))$
- $\neg\forall X.\forall Y.(\neg(p(X) \land p(Y)) \lor q(X,Y)) \lor \forall X.(\neg p(X) \lor \exists Y.q(X,Y))$
- $\exists X.\exists Y.((p(X) \land p(Y)) \land \neg q(X,Y)) \lor \forall X.(\neg p(X) \lor \exists Y.q(X,Y))$
- $\exists X.\exists Y.((p(X) \land p(Y)) \land \neg q(X,Y)) \lor \forall Z.(\neg p(Z) \lor \exists W.q(Z,W))$
- $((p(a) \land p(b)) \land \neg q(a,b)) \lor \forall Z.(\neg p(Z) \lor q(Z,f(Z)))$
- $((p(a) \land p(b)) \land \neg q(a,b)) \lor (\neg p(Z) \lor q(Z,f(Z)))$
- $(p(a) \lor \neg p(Z) \lor q(Z,f(Z))) \land (p(b) \lor \neg p(Z) \lor q(Z,f(Z))) \land (\neg q(a,b) \lor \neg p(Z) \lor q(Z,f(Z)))$

So we have three clauses:

1. $\{p(a), \neg p(Z), q(Z,f(Z))\}$
2. $\{p(b), \neg p(Z), q(Z,f(Z))\}$
3. $\{\neg q(a,b) \lor \neg p(Z) \lor q(Z,f(Z))\}$

But we cannot do anything with this. This means **the sentence is not unsatisfiable**.

So we negate the sentence and transform it to a CNF.

- $\neg(\forall X.\forall Y.((p(X) \land p(Y)) \Rightarrow q(X,Y)) \Rightarrow \forall X.(p(X) \Rightarrow \exists Y.q(X,Y)))$
- $\neg(\neg\forall X.\forall Y.(\neg(p(X) \land p(Y)) \lor q(X,Y)) \lor \forall X.(\neg p(X) \lor \exists Y.q(X,Y)))$
- $\forall X.\forall Y.(\neg(p(X) \land p(Y)) \lor q(X,Y)) \land \neg\forall X.(\neg p(X) \lor \exists Y.q(X,Y))$
- $\forall X.\forall Y.(\neg p(X) \lor \neg p(Y) \lor q(X,Y)) \land \exists X.(p(X) \land \forall Y.\neg q(X,Y))$
- $\forall X.\forall Y.(\neg p(X) \lor \neg p(Y) \lor q(X,Y)) \land \exists Z.(p(Z) \land \forall W.\neg q(Z,W))$
- $\forall X.\forall Y.(\neg p(X) \lor \neg p(Y) \lor q(X,Y)) \land (p(a) \land \forall W.\neg q(a,W))$
- $(\neg p(X) \lor \neg p(Y) \lor q(X,Y)) \land (p(a) \land \neg q(a,W))$

So we have three clauses:

1. $\{\neg p(X), \neg p(Y), q(X,Y)\}$
2. $\{p(a)\}$
3. $\{q(a,W\}$

We can then run the resolution principle several times:

$1 + 2 \rightarrow \{\neg p(Y), q(a,Y)\}(4)$

$4 + 2 \rightarrow \{q(a,a)\}(5)$

$5 + 3 \rightarrow \{\}$

We succeed to prove the empty clause, that means the negation of the sentence is unsatisfiable and then **the sentence is valid**.

6.2 Solution to question 2

First step, we try to prove the sentence is unsatisfiable. To do so, we have to transform it to a CNF and then try to prove the empty clause.

- $(\exists X.q(X) \wedge (\forall X.(p(X) \Rightarrow \neg q(X)))) \Rightarrow \exists X.\neg p(X)$
- $\neg(\exists X.q(X) \wedge (\forall X.(\neg p(X) \vee \neg q(X)))) \vee \exists X.\neg p(X)$
- $(\forall X.\neg q(X) \vee \exists X.(p(X) \wedge q(X))) \vee \exists X.\neg p(X)$
- $(\forall X.\neg q(X) \vee \exists Y.(p(Y) \wedge q(Y))) \vee \exists Z.\neg p(Z)$
- $(\forall X.\neg q(X) \vee (p(a) \wedge q(a))) \vee \neg p(b)$
- $(\neg q(X) \vee (p(a) \wedge q(a))) \vee \neg p(b)$

So we have two clauses:

1. $\{\neg q(X), p(a), \neg p(b)\}$
2. $\{\neg q(X), q(a), \neg p(b)\}$

But we cannot do anything with this. This means **the sentence is not unsatisfiable**.

So we negate the sentence and transform it to a CNF.

- $\neg((\exists X.q(X) \wedge (\forall X.(p(X) \Rightarrow \neg q(X)))) \Rightarrow \exists X.\neg p(X))$
- $\neg(\neg(\exists X.q(X) \wedge (\forall X.(\neg p(X) \vee \neg q(X)))) \vee \exists X.\neg p(X))$
- $(\exists X.q(X) \wedge (\forall X.(\neg p(X) \vee \neg q(X)))) \wedge \forall X.p(X))$
- $(\exists X.q(X) \wedge (\forall Y.(\neg p(Y) \vee \neg q(Y)))) \wedge \forall Z.p(Z))$
- $(q(a) \wedge (\forall Y.(\neg p(Y) \vee \neg q(Y)))) \wedge \forall Z.p(Z))$
- $(q(a) \wedge (\neg p(Y) \vee \neg q(Y))) \wedge p(Z))$

So we have three clauses:

1. $\{q(a)\}$
2. $\{\neg p(Y), \neg q(Y)\}$
3. $\{p(Z)\}$

We can then run the resolution principle several times:

$$1 + 2 \rightarrow \{\neg p(a)\}(4)$$

$$4 + 3 \rightarrow \{\}$$

We succeed to prove the empty clause, that means the negation of the sentence is unsatisfiable and then **the sentence is valid**.

7 Resolution reasoning ($\simeq$ 3 points)

Given the following first-order sentences:
a) $\forall X.(p(X) \Rightarrow \exists Y.q(Y))$
b) $\neg\exists X.(q(X) \wedge \exists Y.\neg w(Y))$
c) $\forall X.((p(X) \wedge w(X)) \Rightarrow s(X))$
d) $p(mary)$
Using resolution reasoning, show that: s(mary)

7.1 Solution

We convert each sentence to a CNF and also the negation of the conclusion.
For the first sentence we have:

- $\forall X.(p(X) \Rightarrow \exists Y.q(Y))$
- $\forall X.(\neg p(X) \vee \exists Y.q(Y))$
- $\forall X.(\neg p(X) \vee q(f(X)))$
- $(\neg p(X) \vee q(f(X)))$
- $\{\neg p(X), q(f(X))\}$

For the second sentence we have:

- $\neg\exists X.(q(X) \wedge \exists Y.\neg w(Y))$
- $\forall X.(\neg q(X) \vee \neg\exists Y.\neg w(Y))$
- $\forall X.(\neg q(X) \vee \forall Y.w(Y))$
- $(\neg q(X) \vee w(Y))$
- $\{\neg q(X), w(Y)\}$

For the third sentence we have:

- $\forall X.((p(X) \wedge w(X)) \Rightarrow s(X))$
- $\forall X.(\neg((p(X) \wedge w(X)) \vee s(X))$
- $\forall X.((\neg p(X) \vee \neg w(X)) \vee s(X))$
- $(\neg p(X) \vee \neg w(X)) \vee s(X))$
- $\{\neg p(X), \neg w(X), s(X)\}$

For the fourth sentence we have:

- $\{p(mary)\}$

For the negation of the conclusion we have:

- $\{\neg s(mary)\}$

So we have five clauses:

1. $\{\neg p(X), q(f(X))\}$
2. $\{\neg q(X), w(Y)\}$
3. $\{\neg p(X), \neg w(X), s(X)\}$
4. $\{p(mary)\}$
5. $\{\neg s(mary)\}$

We can then run the resolution principle several times:

$$5 + 3 \rightarrow \{\neg p(mary), \neg w(mary)\}(6)$$

$$6 + 4 \rightarrow \{\neg w(mary\}(7)$$

$$7 + 2 \rightarrow \{\neg q(X\}(8)$$

$$4 + 1 \rightarrow \{q(f(mary))\}(9)$$

$$9 + 8 \rightarrow \{\}$$

We succeed to prove the empty clause, that means this set of clauses is unsatisfiable so that proves sentences a), b), c) and d) logically entail s(mary).

8 Problem modeling and solving ($\simeq$ 6 points)

Here are some informations about a simple world:

1. If someone is young and plays the guitar then that person is happy.
2. If someone is old and plays the violin then that person is happy.
3. If someone plays the drums then he's happy.
4. Mary is young, owns a guitar, has learned the guitar and loves Paul.
5. John is old and he knows how to dance.
6. If a person owns an instrument and has learned to play it then that person plays that instrument.
7. If a person is a genius and a musical instrument is a string instrument, then that person plays that instrument.
8. If a person has built a musical instrument then that person plays that instrument.
9. Lindsey has built a violin.
10. The guitar and violin are string instruments.
11. George is a genius.
12. For any person p1 and p2,
    a) If p1 is happy and loves p2, then p1 is a happy lover of p2.
    b) If p1 knows how to dance and p2 is happy then p1 is a happy dancer with p2.
    c) If p1 is a happy dancer with p2 then p1 dances with p2.
    d) If p1 is a happy lover of p2 then p1 dances with p2.
    e) If p1 has built and instrument, then p1 dances with this instrument.

Using resolution reasoning in first order logic, answer the question: *"Who dances with whom/what?"*

8.1 Solution

8.1.1 Model in First Order Logic

First we have to translate each english sentence to a first order logic sentence.

1. If someone is young and plays the guitar then that person is happy.
   $young(P) \wedge plays(P, guitar) \Rightarrow happy(P)$
2. If someone is old and plays the violin then that person is happy.
   $old(P) \wedge plays(P, violin) \Rightarrow happy(P)$
3. If someone plays the drums then he's happy.
   $plays(P, drums) \Rightarrow happy(P)$
4. Mary is young
   $young(mary)$
5. Mary owns a guitar
   $owns(mary, guitar)$
6. Mary has learned the guitar
   $hasLearned(mary, guitar)$
7. Mary loves Paul.
   $loves(mary, paul)$

8. John is old.
   $old(john)$
9. John knows how to dance.
   $knowsHowToDance(john)$
10. If a person owns an instrument and has learned to play it then that person plays that instrument.
    $owns(P, I) \wedge hasLearned(P, I) \Rightarrow plays(P, I)$
11. If a person is a genius and a musical instrument is a string instrument, then that person plays that instrument.
    $genius(P) \wedge isAString(I) \Rightarrow plays(P, I)$
12. If a person has built a musical instrument then that person plays that instrument.
    $hasBuilt(P, I) \Rightarrow plays(P, I)$
13. Lindsey has built a violin.
    $hasBuilt(lindsey, violin)$
14. The guitar is a string instrument.
    $isAString(guitar)$
15. The violin is a string instrument.
    $isAString(violin)$
16. George is a genius.
    $genius(george)$
17. If p1 is happy and loves p2, then p1 is a happy lover of p2.
    $happy(P1) \wedge loves(P1, P2) \Rightarrow happyLover(P1, P2)$
18. If p1 knows how to dance and p2 is happy then p1 is a happy dancer with p2.
    $knowsHowToDance(P1) \wedge happy(P2) \Rightarrow happyDancer(P1, P2)$
19. If p1 is a happy dancer with p2 then p1 dances with p2.
    $happyDancer(P1, P2) \Rightarrow dancesWith(P1, P2)$
20. If p1 is a happy lover of p2 then p1 dances with p2.
    $happyLover(P1, P2) \Rightarrow dancesWith(P1, P2)$
21. If p1 has built and instrument, then p1 dances with this instrument.
    $hasBuilt(P1, I) \Rightarrow dancesWith(P1, I)$
22. Finally, the conclusion to prove is:
    $\exists Person.\exists PersonOrObject.dancesWith(Person, PersonOrObject)$

8.1.2 Convert First Order Logic sentences in CNF

The transformations are quite obvious (don't forget to negate the conclusion), we have this set of clauses:

1. $\{\neg young(P), \neg plays(P, guitar), happy(P)\}$
2. $\{\neg old(P), \neg plays(P, violin), happy(P)\}$
3. $\{\neg plays(P, drums), happy(P)\}$
4. $\{young(mary)\}$
5. $\{owns(mary, guitar)\}$
6. $\{hasLearned(mary, guitar)\}$
7. $\{loves(mary, paul)\}$
8. $\{old(john)\}$
9. $\{knowsHowToDance(john)\}$
10. $\{\neg owns(P, I), \neg hasLearned(P, I), plays(P, I)\}$
11. $\{\neg genius(P), \neg isAString(I), plays(P, I)\}$
12. $\{\neg hasBuilt(P, I), plays(P, I)\}$
13. $\{hasBuilt(lindsey, violin)\}$
14. $\{isAString(guitar)\}$
15. $\{isAString(violin)\}$
16. $\{genius(george)\}$
17. $\{\neg happy(P1), \neg loves(P1, P2), happyLover(P1, P2)\}$
18. $\{\neg knowsHowToDance(P1) \wedge \neg happy(P2), happyDancer(P1, P2)\}$
19. $\{\neg happyDancer(P1, P2), dancesWith(P1, P2)\}$
20. $\{\neg happyLover(P1, P2), dancesWith(P1, P2)\}$
21. $\{\neg hasBuilt(P1, I), dancesWith(P1, I)\}$
22. $\{\neg dancesWith(Person, PersonOrObject)\}$

8.1.3 Find some resolution proofs

Now we have to apply the resolution principle to derive the empty clause and observe the unifications of $Person$ and $PersonOrObject$ that will allow to derive the empty clause. These unifications will be the answers of the question of the problem (I only show below the unification of $Person$ and $PersonOrObject$ with constants, there are also various unifications with other variables).

First way to prove the empty clause:

$22 + 19 \rightarrow \{\neg happyDancer(Person, PersonOrObject)\}(23)$

$23 + 18 \rightarrow \{\neg knowHowToDance(Person), \neg happy(PersonOrObject)\}(24)$

$24 + 9 \rightarrow \{\neg happy(PersonOrObject)\}(25)$ with **{Person=john}**

$25 + 1 \rightarrow \{\neg young(PersonOrObject), \neg plays(PersonOrObject, guitar)\}(26)$

$26 + 4 \rightarrow \{\neg plays(mary, guitar)\}(27)$ with **{PersonOrObject=mary}**

$27 + 10 \rightarrow \{\neg owns(mary, guitar), \neg hasLearned(mary, guitar)\}(28)$

$28 + 5 \rightarrow \{\neg hasLearned(mary, guitar)\}(29)$

$29 + 6 \rightarrow \{\}$

So we have a first solution that is: **John dances with Mary**.

Second way to prove the empty clause:

$22 + 20 \rightarrow \{\neg happyLover(Person, PersonOrObject)\}(30)$

$30 + 17 \rightarrow \{\neg happy(Person), \neg loves(Person, PersonOrObject)\}(31)$

$31 + 1 \rightarrow \{\neg young(Person), \neg plays(Person, guitar), \neg loves(Person, PersonOrObject)\}(32)$

$32 + 4 \rightarrow \{\neg plays(mary, guitar), \neg loves(mary, PersonOrObject)\}(33)$ with **{Person=mary}**

$33 + 10 \rightarrow \{\neg owns(mary, guitar), \neg hasLearned(mary, guitar), \neg loves(mary, PersonOrObject)\}(34)$

$34 + 5 \rightarrow \{\neg hasLearned(mary, guitar), \neg loves(mary, PersonOrObject)\}(35)$

$35 + 6 \rightarrow \{\neg loves(mary, PersonOrObject)\}(36)$

$36 + 7 \rightarrow \{\}$ with **{PersonOrObject=paul}**

So we have a second solution that is: **Mary dances with Paul**.

Third way to prove the empty clause:

$22 + 21 \rightarrow \{\neg hasBuilt(Person, PersonOrObject\}(37)$

$37 + 13 \rightarrow \{\}$ with **{Person=lindsey, PersonOrObject=violin}**

So we have a third solution that is: **Lindsey dances her violin**.

Note that it is not necessary to use all clauses to produce the empty clause. This means that there is unnecessary information in the text provided that is not useful to prove the conclusion that is being attempted to be proven.