UNIVERSITÉ JEAN MONNET, SAINT-ETIENNE

Optimization and Operational Research TP Report

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1 UNCONSTRAINED OPTIMIZATION

1.1 PROBLEM 1

We formulated the solution using CVXPY. The equation was then solved, to compare and check. By doing it with CVXPY, we got the result being Optimal Value as 40.73, Value of x_1 was 4 and of x_2 was 2.55. We solved the question by hand with Newton's Method and the solution was the same.

1.2 PROBLEM 2

Yes. The problem is Convex. We solved it using CVXPY and found the values of the three variables upto 6 decimal points. These values were, for all x_1, x_2 and x_3 were 0.000826. We couldn't solve this one by hand by using Newton's Method and thus could not conclude.

1.3 PROBLEM 3

Yes, the problem was reformulated using CVXPY. After solving it with the regular procedure, we found the answer being x_1 equal to 3.5 and x_2 being equal to 0.5. We then did the question using the log barrier function and found the values of x_1 and x_2 to be the same. We tried to change the value of the parameter by hand ourselves with trial and error method by randomly putting in the value and doing it. By this we found that the optimal value of first variable x1 remained the same but the value of x2 was changed in the direction of increasing to the lowest value of 0.50 at mu = $10^(-4)$. At $10^(-1)$, it was 0.53 and for $10^(0)$ it was 0.83 and likewise increasing with increasing value.

Regarding the cvxpy.Parameter() command, we checked it's documentation stating,

Parameters are constant expressions whose value may be specified after problem creation. The only way to modify a problem after its creation is through parameters. For example, you might choose to declare the hyper-parameters of a machine learning model to be Parameter objects; more generally, Parameters are useful for computing trade-off curves, which we think has been implemented with our mu variable in the code.

2 Modeling Constrained Problems

2.1 Water Resources Problem

- We had to minimize the cost, making our objective function
- We had 4 constraints, one stating the maximum amount of water from stream should be <= 100,000 Liters.
- Water from stream must be used.
- The quantity of pollutants in the stream should be <= 100 $*$ the sum of both.
- The town needs 500,000 liters of water to function.

  Using these constraints and objective function, we formulated this in CVXPY. The problem was found to be optimal, with optimal cost being 45000.00 €. The value of water to be taken from reservoir and stream was 400,000 Liters and 100,000 Liters respectively.

2.2 Good Smelling Perfume Design

- All the four blends are used to make one perfume.
- We had 11 constraints in total.
- Optimal Price was found to be 63 €
- The percentage values of each blend in this optimal mixture was found to be
  - Blend 1 : 14%
  - Blend 2 : 14%
  - Blend 3 : 30%
  - Blend 4 : 42%
- Problem status is Optimal

2.3 Roadway Expenses

- The benefit function was given from urban and rural expenditure on roadways.
- Two constraints were formed by these B_u and B_r values.
- The maximum expenditure in both rural and urban can be 200M€ total, making our third constraint.
- The other constraints were made by putting all the costs and benefits >= 0, as cost can never be negative.
- Values for Optimal Benefit and Optimal Value/Benefit from each rural and urban was found.
- Optimal Benefit's value is 55348.89M€

2.4 Designing our own optimization problem

In our optimization problem, we tried to minimize the cost for the week (5 days), and on weekends we eat outside. The table looks like,
The other values here are the values of calorie intake, done each day and then the cost of the

	Day 1	Day 2	Day 3	Day 4	Day 5
Pasta	0	131	43	0	43
Apple	95	190	43	0	43
Milk	87	0	174	87	87
Oats	389	0	194	389	194
Rice	206	0	0	184	200
Cost	6	2	3	4	7

meal of that day (in €)

- We had to minimize the cost.
- Total calories of the whole week behind the food should not exceed 1000
- Total rice consumption of the week should not exceed 600 gm i.e 780 calories i.e., 0.78 of the total permeable calories consumption through the week.
- Calorie content of day 1 cannot go less 30.
- Calorie content of day 4 can not be less than 70 and others at 50.

We solved for the optimal values, and gave the results.

3 DATA ANALYSIS

3.1 SUPPORT VECTOR MACHINES

We tried to train a support vector machine with l_1 norm. We generated the data-set, with *make-blobs* of the sklearn library. We wanted to make a classifier by minimizing the convex function.

$$f(\beta, v) = (1/m) \sum_i \left(1 - y_i(\beta^T x_i - v)\right)_+ + \lambda \|\beta\|_1$$

We found the optimal values of the both norms and plotted the scatter plot.

3.2 LINEAR REGRESSION

We calculate it with using eucledian distance and the expression and model with CVXPY,

$$\sum_{i=1}^{m} (a_i^T x - b_i)^2,$$