fem_python (1D Elements)

It is a program written in Python to solve problems with the Finite Elements Method. The program includes 1D elements, better known as “line elements” and 2D elements, both to solve Heat Transfer problems and Solid Mechanics problems.

The program is in constant development and some elements will be added, thus, this manual must be updated to include those changes.

1D Elements: Better known as line elements, these are prismatic elements, assuming isotropic behavior of the material. All of them are linear elements with 2 nodes, with the exception of Beam elements; Beam elements have cubic interpolation polynomials, but its formulation use two nodes.

- Heat Transfer:
  - Conduction Element: Based on the direct formulation of a linear element using Fourier Law:

$$[ke] = \frac{k_t \cdot A}{L} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

  - Convection Element: Based on the direct formulation of a linear element using Newton’s Cooling Law:

$$[ke] = h \cdot A \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

  - Fin Element: It is a combination of a conduction element with convection heat transfer at the outer surface:

$$[ke] = \frac{k_t \cdot A}{L} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + \frac{h \cdot p \cdot L}{6} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\{fe\} = \frac{h \cdot T_o \cdot P \cdot L}{2} \cdot \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

- Solid Mechanics:
  - Bar Element: Based on the direct formulation of a linear elastic bar using Hook's Law:

$$[ke] = \frac{E \cdot A}{L} \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

To perform calculation on frames and large structures, the element must have the ability to model inclined members; this is achieved with a transformation matrix:

$$[R] = \begin{bmatrix} lx & ly & 0 & 0 \\ 0 & 0 & lx & ly \end{bmatrix}$$

Where:

$$lx = \frac{x_1 - x_2}{L} \qquad\qquad ly = \frac{y_1 - y_2}{L}$$

The element stiffness matrix results in:

$$[Ke] = [R]^T \cdot [ke] \cdot [R]$$

  - Frame Element (Euler – Bernoulli formulation): It is an element with linear elastic bar degrees of freedom and in – plane beam degrees of freedom with cubic interpolation.

$$[ke] = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}$$

To perform calculation on frames and large structures, the element must have the ability to model inclined members; this is achieved with a transformation matrix:

$$[R] = \begin{bmatrix} lx & ly & 0 & 0 & 0 & 0 \\ -ly & lx & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & lx & ly & 0 \\ 0 & 0 & 0 & -ly & lx & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Where:

$$lx = \frac{x_1 - x_2}{L} \qquad ly = \frac{y_1 - y_2}{L}$$

The element stiffness matrix results in:

$$[Ke] = [R]^T \cdot [ke] \cdot [R]$$

- Timoshenko Beam Element

  Coming soon