Disclaimer

This document was made with the purpose to keep record of the Python script written to design the profile of a horizontal axis wind turbine (HAWT) and the equations and procedures followed to accomplish this goal.

Nomenclature

a, a'	Induction factors
c	Chord length
C_d	Drag coefficient
C_l	Lift coefficient
C_l/C_d	Lift to drag ratio
C_n	Normal coefficient
C_t	Tangential coefficient
F	Prandtl's Loss Factor
r	Radial position on the blade length
R	Radius of the turbine rotor
Z	Number of blades
α	Attack angle
θ	Twist angle
λ	Tip speed ratio
σ	Solidity of the blade section
φ	Relative velocity angle

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Description of Blade geometry design method

Initial values [1]:

The method starts with the initial values calculations, this is, the initial chord length distribution, the calculations are performed following the equations above.

$$\lambda_{r,i} = \lambda \cdot (r_i/R) \qquad \text{Eq. (1)}$$

$$\varphi_i = 2/3 \cdot \tan^{-1}(1/\lambda_{r,i}) \qquad \text{Eq. (2)}$$

$$c_i = \frac{8 \cdot r_i}{Z \cdot C_{l,d}} \cdot (1 - \cos(\varphi_i)) \qquad \text{Eq. (3)}$$

Where $\lambda_{r,i}$ is the local tip-speed ratio, φ_i is the local relative angle, and c_i is the local chord length, the subscript represents the ith section of the blade. The chord length distribution is passed to the next section of calculations.

One-point design method [2]:

For every coordinate r_i/R, assume a and a' equal to zero. Then:

$$\tan\varphi_i = \frac{1 - a}{(1 + a') \cdot \lambda_{r,i}} \qquad \text{Eq. (4)}$$

$$F_i = \frac{2}{\pi} \cdot \cos^{-1}\left[\exp\left(-\frac{Z \cdot (1 - r_i/R)}{2 \cdot r_i/R \cdot \sin\varphi_i}\right)\right] \qquad \text{Eq. (5)}$$

$$\sigma_i = \frac{Z \cdot c_i}{2 \cdot \pi \cdot r_i} \qquad \text{Eq. (6)}$$

$$\theta_i = \varphi_i - \alpha_d \qquad \text{Eq. (7)}$$

The chord length values used in Eq. (6) are the initial values given by Eq. (3). The Lift Coefficient *Cl* and Lift to Drag ratio *Cl/Cd* are obtained from tabulated airfoil data at design attack angle α_d. Then, the normal coefficient and tangential coefficient are calculated as follows:

$$C_n = C_l \cdot \cos(\varphi_i) + C_d \cdot \sin(\varphi_i) \qquad \text{Eq. (8)}$$

$$C_t = C_l \cdot \sin(\varphi_i) - C_d \cdot \cos(\varphi_i) \qquad \text{Eq. (9)}$$

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After the previous step, an iterative process starts. The induction factors *a* and *a'* are recalculated including Prandtl's loss factor *F* as follows:

$$a = \frac{\sigma_i \cdot C_t}{4 \cdot F_i \cdot \sin^2(\varphi_i) + \sigma_i \cdot C_t} \qquad \text{Eq. (10)}$$

$$a' = \frac{\sigma_i \cdot C_n}{4 \cdot F_i \cdot \sin(\varphi_i) \cdot \cos(\varphi_i) + \sigma_i \cdot C_n} \qquad \text{Eq. (11)}$$

The relative velocity angle φ is calculated again using equation Eq. (4). Prandtl's tip factor *F* is calculated again using equation Eq. (5). The twist angle θ is calculated again using equation (7). The axial coefficient and tangential coefficient are recalculated using equations Eq. (8) and Eq. (9).

Then, using the new values of relative velocity angle, axial coefficient, and tangential coefficient, the process goes back to the equations Eq. (10) and Eq. (11), and the calculations described in the previous paragraph are repeated. The iterative method will be running until values of *a* and *a'* for that specific coordinate r_i/R converges.

The method described above must be repeated for every coordinate r_i/R defined when the blade is divided into *n* finite sections. After the blade geometry has been calculated, the chord length is recalculated using Eq. (3).

The power coefficient is then calculated using Eq. (12) [3]:

$$C_p = \frac{8}{\lambda \cdot n} \cdot \sum_{i=k}^{n} F_i \cdot \sin^2 \varphi_i \cdot (\cos \varphi_i - \lambda_{r,i} \cdot \sin \varphi_i) \cdot (\sin \varphi_i + \lambda_{r,i} \cdot \cos \varphi_i) \cdot \left[1 - \left(\frac{C_l}{C_d}\right) \cdot \cot \varphi_i\right] \cdot \lambda_{r,i}^2 \qquad \text{Eq. (12)}$$

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Validation of the proposed methodology

The equations described before were programmed in a Python script to speed up the calculations, to check the good performance of the script it was compared with an example problem in [2]; the input values are shown in the table 1.

Table 1. Parameters used to calculate the dimensionless chord and the twist distribution.

Parameter	**Value**
Tip-speed ratio λ	6
Number of blades Z	3
Design attack angle α_d	4
Lift coefficient *Cl*	0.8
Drag coefficient *Cd*	0.012

The dimensionless chord *c/R* distribution is shown in figure 1, it is observed that both curves are very similar, but they disagree at the tip of the blade. The same behavior is observed in the figure 2, where the twist distribution is shown. However, the results obtained have the same behavior, around almost all the blade span, which is a good performance of the presented methodology.

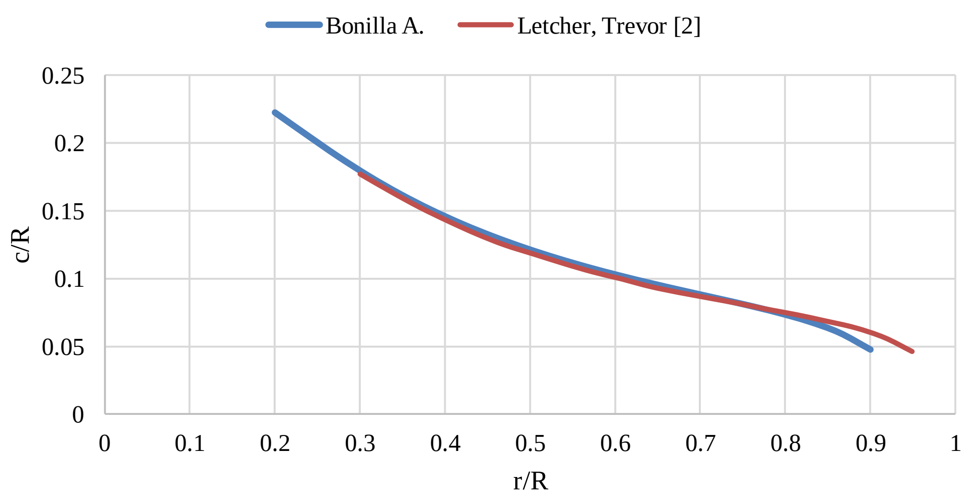

Figure 1. Dimensionless chord, *c/R*, for one-point design for the parameters described in table 1

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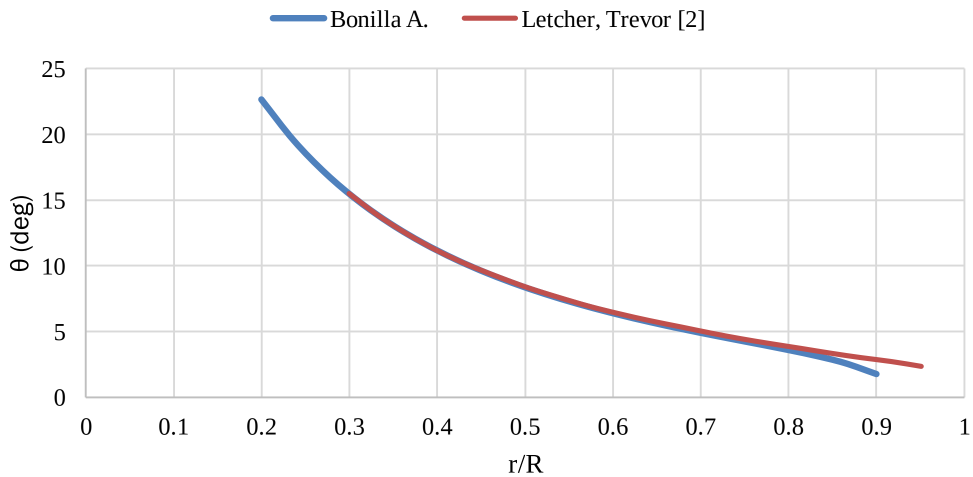

Figure 2. Twist distribution for one-point design for the parameters described in table 1.

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References

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2. Letcher, Trevor M. (2017). **Wind Energy Engineering. A Handbook for Onshore and Offshore Wind Turbines**. Chapter 9, Design of Horizontal-Axis Wind Turbines, pp. 178-181. An imprint of Elsevier.
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