

FREE-RBF-KAN: KOLMOGOROV–ARNOLD NETWORKS WITH ADAPTIVE RADIAL BASIS FUNCTIONS FOR EFFICIENT FUNCTION LEARNING

SHAO-TING CHIU* SIU WUN CHEUNG† ULISSES BRAGA-NETO*

CHAK SHING LEE† RUI PENG LI†

Abstract

Kolmogorov–Arnold Networks (KANs) have shown strong potential for efficiently approximating complex nonlinear functions. However, the original KAN formulation relies on B-spline basis functions, which incur substantial computational overhead due to De Boor’s algorithm. To address this limitation, recent work has explored alternative basis functions such as radial basis functions (RBFs) that can improve computational efficiency and flexibility. Yet, standard RBF-KANs often sacrifice accuracy relative to the original KAN design. In this work, we propose Free-RBF-KAN, a RBF-based KAN architecture that incorporates adaptive learning grids and trainable smoothness to close this performance gap. Our method employs freely learnable RBF shapes that dynamically align grid representations with activation patterns, enabling expressive and adaptive function approximation. Additionally, we treat smoothness as a kernel parameter optimized jointly with network weights, without increasing computational complexity. We provide a general universality proof for RBF-KANs, which encompasses our Free-RBF-KAN formulation. Through a broad set of experiments, including multiscale function approximation, physics-informed machine learning, and PDE solution operator learning, Free-RBF-KAN achieves accuracy comparable to the original B-spline-based KAN while delivering faster training and inference. These results highlight Free-RBF-KAN as a compelling balance between computational efficiency and adaptive resolution, particularly for high-dimensional structured modeling tasks.

1 Introduction

The Kolmogorov–Arnold Network (KAN) [Liu et al., 2024] is a neural architecture grounded in the Kolmogorov–Arnold representation theorem [Kolmogorov, 1956], which states that any multivariate continuous function can be expressed as a superposition of univariate continuous functions and addition. The original KAN leverages B-splines to model these univariate components due to their strong approximation capabilities. KAN has been shown to exhibit no spectral bias [Wang et al., 2024a], making it advantageous for learning functions with high-frequency components. However, in practice, computing B-spline bases using De Boor’s iteration and the required rescaling of the B-spline domain during training introduces significant computational overhead. To mitigate this issue, recent work has explored alternative basis functions and approaches. Li [2024] proposed FastKAN, which replaces B-splines with Gaussian radial basis functions (RBFs) and uses layer normalization to avoid

*Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX, USA.
Email: stchiu@tamu.edu, ulisses@tamu.edu.

†Center for Applied Scientific Computing, Lawrence Livermore National Laboratory, Livermore, CA, USA. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344 (LLNL-JRNL-2012535) and was supported by the LLNL-LDRD program under Project No. 25-ERD-051 and the LLNL Computing Scholar Program. Email: cheung26@llnl.gov, li50@llnl.gov, lee1029@llnl.gov.

domain rescaling, offering a simpler and faster implementation while retaining competitive accuracy. Extending this approach, FasterKAN [Delis, 2024] introduces a reflectional switch activation function (RSWAF) that approximates Gaussian RBFs without costly exponential evaluations, leading to further reductions in computational cost. Adaptive meshing via FreeKnots-KAN, as proposed in Zheng et al. [2025] and Actor et al. [2025], improved both the accuracy and stability by learning the placement of B-spline knots.

All of the aforementioned modifications to the basic KAN architecture can be viewed as extensions of radial basis function (RBF) networks [Orr et al., 1996], which replace the traditional sigmoidal nonlinearities in neural networks (NNs) with RBFs. RBF networks are universal function approximators and have been shown to offer an accurate and interpretable alternative to standard NNs [Montazer et al., 2018]. However, they do not scale well to high-dimensional problems: the RBFs themselves become unwieldy high-dimensional functions, and the number of grid centers grows exponentially with the input dimension, reflecting the classic curse of dimensionality. In contrast, RBF-KAN leverages univariate RBF basis functions within the superpositional structure prescribed by the Kolmogorov–Arnold theorem, enabling scalable function approximation even in high-dimensional settings.

In this paper, we propose Free-RBF-KAN, which adopts a hierarchical, multichannel structure rooted in the Kolmogorov–Arnold decomposition. The Free-RBF-KAN architecture integrates adaptive learning grids (i.e., *free knots*) with tunable RBF shape parameters. This formulation constrains the grid to a fixed domain while learning a mesh aligned with activation patterns. The resulting geometric refinement balances computational efficiency and expressiveness, making it well-suited for modeling structured, high-dimensional functions with varying local complexity. Each RBF kernel in Free-RBF-KAN acts as a univariate component, and their superposition constructs the multivariate function, aligning with the Kolmogorov–Arnold theorem. To enhance performance, Free-RBF-KAN uses trainable grids and a shape mechanism similar to Ramabathiran and Ramachandran [2021], Wettschereck and Dietterich [1991], Mojarrad et al. [2023]. Our method permits dynamic repositioning of both the grid points and the smoothness during training. The adaptive mesh has been integrated into KAN by Actor et al. [2025] and Zheng et al. [2025], and enhances resolution and adaptability by decoupling the mesh from fixed uniform structures. Furthermore, Actor et al. [2025] provides a theoretical justification for KAN that shows that spline-based KANs can serve as preconditioning for multichannel MLPs, which yields improved optimization landscapes and faster convergence.

As a theoretical contribution of this work, we formally extend the universal approximation theorem for RBF networks to the RBF-KAN architecture. To the best of our knowledge, this is the first universality proof established for the RBF-KAN family. In contrast to KANs that rely on B-spline bases, RBF-KAN exhibits stronger function-approximation properties: its approximation error bound is independent of the target function, and it does not require the target function to admit any specific or predesigned decomposition. This inherent universality demonstrates that RBF-KAN is not merely a strategy for reducing computational overhead, but a fundamentally powerful and flexible framework for function approximation. Furthermore, we evaluate the regression performance of Free-RBF-KAN on high-dimensional dataset and analyze its Neural Tangent Kernel (NTK). Our NTK study confirms that Free-RBF-KAN, like the original KAN, does not exhibit spectral bias.

We further explore the application of Free-RBF-KAN to physics-informed machine learning and operator learning tasks. Using a physics-informed loss in the spirit of Physics-Informed Neural Networks (PINNs) [Raissi et al., 2019], we train Free-RBF-KAN to solve a 1D heat conduction PDE and a 2D Helmholtz PDE. Notably, the PINN baseline fails to converge on the heat conduction problem, and on the Helmholtz benchmark it produces substantially larger errors than physics-informed Free-RBF-KAN, even though the PINN

uses two orders of magnitude more parameters. We also evaluate Free-RBF-KAN as the trunk network within a DeepONet [Lu et al., 2019] for learning the solution operator of a reaction–diffusion PDE. In contrast to DeepOKAN [Abueidda et al., 2025], we retain a standard MLP for the branch network to maximize performance. Free-RBF-KAN achieves lower approximation error than the standard DeepONet while requiring fewer parameters. Across all experiments, Free-RBF-KAN consistently outperforms both RBF-KAN and the original KAN.

1.1 Main Contributions

The central contribution of this work is the development of Free-RBF-KAN, a novel and highly efficient Kolmogorov–Arnold Network architecture utilizing a flexible RBF formulation. Our work provides the following specific advancements:

- **Architectural Innovation:** Free-RBF-KAN is based on a free RBF formulation that leverages adaptive meshing (centroids) and tunable sharpness factors. This innovation grants the activation functions enhanced flexibility, enabling a dynamic alignment of the mesh representation with activation patterns to improve accuracy without increasing computational complexity.
- **Theoretical Foundation:** We formally extend the RBF network universality approximation theorem to the RBF-KAN family of neural networks. Furthermore, an NTK analysis confirms that Free-RBF-KAN exhibits the desirable property of lacking spectral bias in regression, akin to the original KAN.
- **Broad Application and Efficiency:** We demonstrate the scalability of Free-RBF-KAN across diverse regimes, including general regression problems, physics-informed machine learning, and operator learning. Physics-informed Free-RBF-KAN and Free-RBF-KAN-ONet achieves comparable or superior accuracy to the original PINN and DeepONet, while using a smaller number of parameters and being clearly superior to RBF-KAN and KAN variants.

2 Related Work

A wide range of works have extended the Kolmogorov–Arnold Network (KAN) and Radial Basis Function (RBF) frameworks in both theoretical and application-driven directions. Physics-Informed RBF networks using Gaussian kernels have demonstrated superior performance on high-frequency PDEs [Bai et al., 2023], while Zeng et al. [2024] proposed RBF-PINN as an alternative to Fourier embeddings. Within the KAN framework, SS et al. [2024] introduced Chebyshev basis functions, and Wang et al. [2024b] employed third-order B-splines in KAN to encode physical laws in strong, energy, and inverse forms. Hybrid models such as BSRBF-KAN [Ta, 2024] combine B-splines and RBFs to harness the benefits of both smooth local representation and adaptive flexibility. Krisnawan et al. [2025] combines the architectures of KAN and RBF network for accurate indoor localization using RSSI-based fingerprinting. Bai et al. [2023] demonstrated that physics-informed RBF networks can outperform traditional PINNs, though these models typically use a single-layer RBF architecture. Similarly, Ta [2024] combined B-spline and RBF bases to enhance training, while Shukla et al. [2024] and Abueidda et al. [2025] introduced DeepOKAN, which uses RBF-KAN for operator learning. Li [2024] demonstrates that B-Spline can be approximated by radial basis function with Gaussian Kernel. A thorough comparison of PINN and KAN-based models is provided by Shukla et al. [2024]. On the theoretical side, classical results on

RBF networks include their universal approximation capabilities [Park and Sandberg, 1993, Ismayilova and Ismayilov, 2024], convergence and consistency properties [Xu et al., 1994], and optimal approximation results [Girosi and Poggio, 1990]. RBF networks have also been used for solving multiscale PDEs [Wang et al., 2023b]. Earlier work by Wettschereck and Dietterich [1991] focused on learning RBF centers to improve performance. Connections between RBFs and kernel machines were explored by Que and Belkin [2016], while Chen [2024] extended KAN to a Gaussian process formulation that provides error estimates. Theoretical refinements of Kolmogorov’s representation theorem include improvements by Fridman [1967], a constructive proof by Braun and Griebel [2009], and an extension by Kurkova [1991] showing that the representation can be realized through affine and sigmoidal functions—permitting the approximation of discontinuous but bounded functions. This theory underpins recent developments such as the KKAN model proposed in Toscano et al. [2025], which builds on the Kurkova–Kolmogorov–Arnold representation. A broader perspective on KAN developments is provided by Somvanshi et al. [2024], which highlights applications across scientific computing, time-series forecasting, and graph learning.

3 Fundamentals

3.1 Kolmogorov–Arnold Theorem

The Kolmogorov–Arnold Network (KAN) [Liu et al., 2024] is inspired by the Kolmogorov–Arnold Theorem [Kolmogorov, 1956]. The theorem states that for any integer $n \geq 2$ there exist continuous univariate real functions $\phi^{(p,q)}(x) : [0, 1] \rightarrow \mathbb{R}$ and $\phi^{(q)} : \mathbb{R} \rightarrow \mathbb{R}$ that can represent continuous multivariate real function $f(x_1, \dots, x_d) : [0, 1]^d \rightarrow \mathbb{R}$ as

$$f(x_1, \dots, x_d) = \sum_{q=1}^{2d+1} \phi^{(q)} \left(\sum_{p=1}^d \phi^{(p,q)}(x_p) \right). \quad (1)$$

This result provides a universal representation for high-dimensional functions using only sums and compositions of univariate functions. However, the decomposition (1) may involve non-smooth inner functions $\phi^{(p,q)}$, and is hard for exact representation as shown in Girosi and Poggio [1989].

3.2 Radial Basis Functions

Radial basis functions (RBFs) are real-valued functions whose output depends solely on the distance from a central point. RBF networks are widely used in interpolation, approximation theory, and machine learning, and classical results (e.g., [Park and Sandberg, 1991]) show that they are universal approximators of continuous functions on compact domains. In the one-dimensional setting, a single-layer RBF network is takes the form

$$g(x) = \sum_{m=1}^G \omega_m K \left(\frac{x - c_m}{\sigma_m} \right), \quad (2)$$

where $G \in \mathbb{N}^+$ is the number of nodes, $x \in \mathbb{R}$ is the input, the kernel $K : \mathbb{R} \rightarrow \mathbb{R}^+$ depends only on $|x - c_m|$. Each term of (2) is parameterized by a weight ω_m , a centroid c_m , and a smoothness factor $\sigma_m > 0$. In this work, we focus on one-dimensional RBFs as fundamental building blocks for approximating multivariate functions through the superpositional structure provided by the Kolmogorov–Arnold Theorem.

4 Free RBF-KAN

The original KAN [Liu et al., 2024] is loosely inspired by the Kolmogorov–Arnold theorem. Assuming the target function is sufficiently smooth, KAN employs B-spline basis functions for the functions $\phi^{(p,q)}$ in (1). Its expressivity derives from a multilayer architecture—an extension not present in the original formulation of Kolmogorov [1956], but one that substantially improves practical performance. RBF-KAN preserves the general structure of [Liu et al., 2024], replacing the B-spline bases with one-dimensional radial basis function (RBF) kernels. Let n_l denote the number of nodes in the l -th layer and $\mathbf{x}^{(l)}$ the inputs to layer l . The multilayer RBF-KAN then satisfies the following recursive relation:

$$\mathbf{x}^{(l+1)} = \Phi^{(l)}(\mathbf{x}^{(l)}) = \begin{bmatrix} \sum_{j=1}^{n_l} \sum_{m=1}^G \omega_{1jm}^{(l)} K\left(\frac{x_j^{(l)} - c_{1jm}^{(l)}}{\sigma_{1jm}^{(l)}}\right) \\ \vdots \\ \sum_{j=1}^{n_l} \sum_{m=1}^G \omega_{n_{l+1},jm}^{(l)} K\left(\frac{x_j^{(l)} - c_{n_{l+1},jm}^{(l)}}{\sigma_{n_{l+1},jm}^{(l)}}\right) \end{bmatrix}, \quad (3)$$

where $K : \mathbb{R} \rightarrow \mathbb{R}^+$ is an RBF kernel that is assumed to be uniformly continuous. As defined previously, with centroid $c_m \in \mathbb{R}$ and smoothness parameter $\sigma_m > 0$, a common choice for K is the Gaussian kernel:

$$K\left(\frac{x - c_m}{\sigma_m}\right) = \exp\left(\frac{(x - c)^2}{\sigma^2}\right), \quad (4)$$

for which $K \in C^\infty(\mathbb{R})$. Another widely used option is the Matérn kernel with smoothness parameter, such as $\nu = 5/2$ which satisfies $K \in C^3(\mathbb{R})$. The smoothness of the chosen kernel directly determines the smoothness of the resulting RBF-KAN output. In this study, we investigate trainable centroids and smoothness parameters to provide additional flexibility and improve representation quality. Throughout the paper, *RBF-KAN* refers to networks with fixed centroids and fixed smoothness parameters, and *Free-RBF-KAN* refers to models in which both centroids and smoothness factors are learnable.

In physics-informed machine learning, residual connections through nonlinear activations and scaling have been shown to improve accuracy [Liu et al., 2024]. Following the approach of Wang et al. [2024b], we introduce an optional scaling matrix and a nonlinear activation into each layer. The output of layer l is then given by

$$\mathbf{x}^{(l+1)} = \rho_o \left(\mathbf{W}_{rbf} \odot \Phi^{(l)}(\mathbf{x}^{(l)}) + \mathbf{W} \odot \rho(\mathbf{x}^{(l)}) \right) \quad (5)$$

where ρ_o and $\rho : \mathbb{R} \rightarrow \mathbb{R}$ are componentwise activation functions. For the hidden layers, ρ is SiLU activation, following Liu et al. [2024], and ρ_o is the sigmoid nonlinearity, which generally performs better than tanh in this architecture, except for the final output layer, where ρ_o is the identity function. The parameters of the RBF-KAN network are summarized in Table 1.

4.1 Adaptive Meshing

The free-knots method for KAN has been explored in Zheng et al. [2025] and Actor et al. [2025]. In this work, we extend the underlying idea to the RBF-KAN setting. Unlike B-splines, which require maintaining a strictly ordered sequence of grid points, RBFs introduce

Trainable Param.	#Param	Description
$\omega_{ijm}^{(l)}$	$G \times n_{l+1} \times n_l$	Weight of a radial basis function
$c_{ijm}^{(l)}$	$G \times n_{l+1} \times n_l$	Centroid of a radial basis function
$\sigma_{ijm}^{(l)}$	$G \times n_{l+1} \times n_l$	Smoothness of a radial basis function
W_{rbf}	$n_{l+1} \times n_l$	Scaling factors of activation function ϕ
W	$n_{l+1} \times n_l$	Scaling factors of activation function σ

Table 1: Trainable parameters in RBF-KAN.

additional flexibility of allowing any order of centroids, as each kernel is evaluated independently and does not rely on a prescribed ordering of centroids. This allows us to develop a moving grid method in the RBF framework that can adaptively remesh the activation functions during training. It is worth noting that the free-knot adaptation for B-splines is considerably more complicated and the smoothness of B-spline bases cannot be treated as a trainable parameter. In contrast, RBFs enable both the centroid locations and the associated smoothness parameters to be learned directly and efficiently, yielding a more expressive and computationally efficient approach.

To obtain an efficient representation, we constrain the grids to lie within a prescribed domain during training. This can be achieved by reparameterization using a bounded monotonic activation function $\rho : \mathbb{R} \rightarrow [a, b]$ with $a < b$ and $\rho \in C^\infty$. Given a grid domain $\Omega_g \in (x_l, x_r)$ with $x_l < x_r$, a free parameter $\tilde{c} \in \mathbb{R}$ is mapped to a valid centroid location $c \in (x_l, x_r)$ via

$$c = x_l + \frac{x_r - x_l}{b - a} (\rho(\tilde{c}) - a) \quad (6)$$

In this work, without specification, we set $a = -1, b = 1$, and choose ρ to be the tanh function, ensuring that centroid c remains within Ω_g . This reparameterization is smooth and compatible with gradient-based optimization. For initialization, the grid points are placed uniformly within the domain.

4.2 Adaptive Smoothness

The smoothness of B-splines (determined by their polynomial orders) is fixed and must be predetermined prior to the training. In contrast, the smoothness of RBFs can be specified either by the order parameter ν in the case of the Matérn kernel, or more generally by the smoothness factor σ . To ensure that σ remains positive during gradient-based optimization, we introduce an unconstrained parameter $\tilde{\sigma} \in \mathbb{R}$ and define σ via the mapping

$$\sigma = \exp(\tilde{\sigma}). \quad (7)$$

The combination of adaptive meshing and adaptive smoothness can significantly enhance the express power of Free-RBF-KAN. As demonstrated in Zheng et al. [2025], allowing the RBF centroids to move within an extended range improves gradient smoothness and helps training stability. Although the introduction of additional trainable parameters may moderately increase the training time, Free-RBF-KAN remains substantially faster than B-spline KAN. Moreover, the inference cost of Free-RBF-KAN is identical to that of standard RBF-KAN once the centroids and smoothness parameters have been trained and fixed.

4.3 Universal Approximation

To establish the universal approximation property of the Free-RBF-KAN architecture, we rely on several classical results from approximation theory. We first recall the Kolmogorov–

Arnold representation theorem, which provides a canonical functional decomposition for multivariate continuous functions.

Lemma 4.1 (Kolmogorov-Arnold Representation Theorem [Kolmogorov, 1956, Arnol'd, 1957]). *For any continuous multi-variable function $f : [0, 1]^d \rightarrow \mathbb{R}$, there exist $2d + 1$ continuous univariate functions $\Phi^{(q)} : \mathbb{R} \rightarrow \mathbb{R}$ and $d(2d + 1)$ continuous univariate functions $\phi^{(pq)} : [0, 1] \rightarrow \mathbb{R}$ such that:*

$$f(x_1, \dots, x_d) = \sum_{q=1}^{2d+1} \Phi^{(q)} \left(\sum_{p=1}^d \phi^{(pq)}(x_p) \right). \quad (8)$$

We next recall a classical density result for ridge-function approximation.

Theorem 4.2 (Pinkus Theorem [Pinkus, 1999]). *Let $\sigma \in C(\mathbb{R})$. The set*

$$\mathcal{M}(\sigma) = \text{span}\{\sigma(\mathbf{w} \cdot \mathbf{x} - \theta) : \theta \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^n\} \quad (9)$$

is dense in $C(\mathbb{R}^n)$ on compact sets with respect to uniform convergence, if and only if σ is not a polynomial.

Restricting Theorem 4.2 to the one-dimensional setting yields a useful univariate density result.

Lemma 4.3 (Univariate Density [Leshno et al., 1993]). *Let $K : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function. The set of functions spanned by the shifts and scales of K , specifically*

$$\mathcal{S} = \text{span} \left\{ K \left(\frac{x - c}{\sigma} \right) : c, \sigma \in \mathbb{R}, \sigma \neq 0 \right\}, \quad (10)$$

is dense in the space of continuous functions $C[a, b]$ for any compact interval $[a, b] \subset \mathbb{R}$ if and only if K is not a polynomial.

Remark 4.4. Lemma 4.3 follows from Theorem 4.2 by restricting to the one-dimensional case $n = 1$ and reparameterizing the affine arguments as shifts and scales. Lemma 4.3 provides a univariate density result that will be used to approximate the univariate component functions appearing in the Kolmogorov–Arnold and KAN-type representations.

We are now ready to state the universal approximation theorem for *non-polynomial KAN* (NP-KAN).

Theorem 4.5 (Universal Approximation of NP-KAN). *Let $K : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous, non-polynomial function. For any $f \in C([0, 1]^d)$ and any $\varepsilon > 0$, there exists an NP-KAN network g of the form*

$$g(x_1, \dots, x_d) = \sum_{q=1}^{2d+1} \widehat{\Phi}^{(q)} \left(\sum_{p=1}^d \widehat{\phi}^{(pq)}(x_p) \right), \quad (11)$$

where $\widehat{\Phi}^{(q)}, \widehat{\phi}^{(pq)} \in \mathcal{S}$ defined in (10), for all $1 \leq p \leq d$ and $1 \leq q \leq 2d + 1$, such that

$$\|f - g\|_{C([0, 1]^d)} < \varepsilon. \quad (12)$$

Proof. Let $f \in C([0, 1]^d)$ and $\varepsilon > 0$ be given. By Lemma 4.1, f admits the representation (8) with continuous univariate functions $\{\Phi^{(q)}\}_{q=1}^{2d+1}$ and $\{\phi^{(pq)}\}_{1 \leq p \leq d, 1 \leq q \leq 2d+1}$.

For each $1 \leq q \leq 2d + 1$, define inner sum $S_q : [0, 1]^d \rightarrow \mathbb{R}$ by

$$S_q(x) = \sum_{p=1}^d \phi^{(pq)}(x_p) \text{ for all } x = (x_1, \dots, x_d) \in [0, 1]^d. \quad (13)$$

Since S_q is continuous on the compact set $[0, 1]^d$, its image $I_q = S_q([0, 1]^d)$ is a compact interval in \mathbb{R} . By the Heine–Cantor theorem, $\Phi^{(q)}$ is uniformly continuous on I_q . Thus, there exists a $\delta_q > 0$ such that for all $y, \hat{y} \in I_q$:

$$|y - \hat{y}| < \delta_q \implies |\Phi^{(q)}(y) - \Phi^{(q)}(\hat{y})| < \frac{\varepsilon}{2(2d + 1)}. \quad (14)$$

For each $1 \leq p \leq d$, by Lemma 4.3, there exists $\hat{\phi}^{(pq)} \in \mathcal{S}$ such that

$$\max_{x_p \in [0, 1]} |\phi^{(pq)}(x_p) - \hat{\phi}^{(pq)}(x_p)| < \frac{\delta_q}{d}. \quad (15)$$

Define $\hat{S}_q : [0, 1]^d \rightarrow \mathbb{R}$ by

$$\hat{S}_q(x_1, \dots, x_d) = \sum_{p=1}^d \hat{\phi}^{(pq)}(x_p) \text{ for all } x = (x_1, \dots, x_d) \in [0, 1]^d. \quad (16)$$

Then for all $x = (x_1, \dots, x_d) \in [0, 1]^d$, by the triangle inequality and (15), we have

$$|S_q(x) - \hat{S}_q(x)| \leq \sum_{p=1}^d |\phi^{(pq)}(x_p) - \hat{\phi}^{(pq)}(x_p)| < \sum_{p=1}^d \frac{\delta_q}{d} = \delta_q. \quad (17)$$

By (14), it follows that

$$|\Phi^{(q)}(S_q(x)) - \Phi^{(q)}(\hat{S}_q(x))| < \frac{\varepsilon}{2(2d + 1)}. \quad (18)$$

Since \hat{S}_q is continuous on $[0, 1]^d$, $\hat{I}_q = \hat{S}_q([0, 1]^d)$ is a compact interval. Applying Lemma 4.3 again, there exists $\hat{\Phi}^{(q)} \in \mathcal{S}$ such that

$$\max_{y \in \hat{I}_q} |\Phi^{(q)}(y) - \hat{\Phi}^{(q)}(y)| < \frac{\varepsilon}{2(2d + 1)}. \quad (19)$$

Using triangle inequality and the estimates (18) and (19), we arrive at

$$|\Phi^{(q)}(S_q(x)) - \hat{\Phi}^{(q)}(\hat{S}_q(x))| < \frac{\varepsilon}{2d + 1}. \quad (20)$$

Finally, summing over $q = 1, \dots, 2d + 1$ yields

$$|f(x) - g(x)| \leq \sum_{q=1}^{2d+1} |\Phi^{(q)}(S_q(x)) - \hat{\Phi}^{(q)}(\hat{S}_q(x))| < \sum_{q=1}^{2d+1} \frac{\varepsilon}{2d + 1} = \varepsilon, \quad (21)$$

for all $x \in [0, 1]^d$. This concludes the proof. \square

Corollary 4.6 (Universal Approximation of RBF-KAN). *Let $K(x)$ be the Gaussian RBF in (4), which is continuous and non-polynomial. For any continuous function $f \in C([0, 1]^d)$ and any $\varepsilon > 0$, there exists an RBF-KAN network g such that $\|f - g\|_{C([0, 1]^d)} < \varepsilon$.*

5 Numerical experiments

We now turn to numerical experiments to assess the practical performance of the proposed Free-RBF-KAN architecture. To ensure a fair and meaningful comparison, we begin by outlining the configurations of the baseline models. The original Kolmogorov–Arnold Network (KAN) used in our comparisons employs cubic B-spline basis functions (i.e., B-splines of order 3). All models across the KAN family, including KAN and the various RBF-KAN counterparts, are constructed with the same number of nodes and layers, providing a controlled setting for evaluating expressive power and computational efficiency. Unless otherwise noted, all RBF-KAN variants in this study, including Free-RBF-KAN, use the Gaussian kernel as their radial basis activation.

5.1 Functional approximation

Our first experiment examines the approximation of a nonsmooth function to illustrate the importance of adaptive meshing (see Fig. 1). Following Actor et al. [2025], we consider the function:

$$f(x, y) = \cos(4\pi x) + \sin(\pi y) + \sin(2\pi y) + |\sin(3\pi y^2)| \quad (22)$$

We evaluate a compact neural network architecture with layer sizes [2, 5, 1] across several model variants: MLP, KAN, FreeKnots-KAN, RBF-KAN, and Free-RBF-KAN. FreeKnots-KAN [Actor et al., 2025] introduces adaptive remeshing of the B-spline grid, enabling improved approximation accuracy compared to fixed-grid KANs. All models are trained using the LBFGS optimizer for 300 epochs with a learning rate of 1 on a regression dataset of 16,384 points processed in batches of 1,024.

The results show that the RBF-KAN substantially outperforms the standard KAN while using fewer parameters. Both FreeKnots-KAN and its RBF-based counterpart achieve similarly low test errors; however Free-RBF-KAN attains this accuracy fewer parameters, underscoring its efficiency. These findings demonstrate that replacing B-splines with RBFs with Gaussian kernels not only improves accuracy but also reduce model complexity. Overall, RBF-KAN exhibits strong capability in approximating nonsmooth functions, with the Gaussian kernel providing the best performance among RBF variants considered.

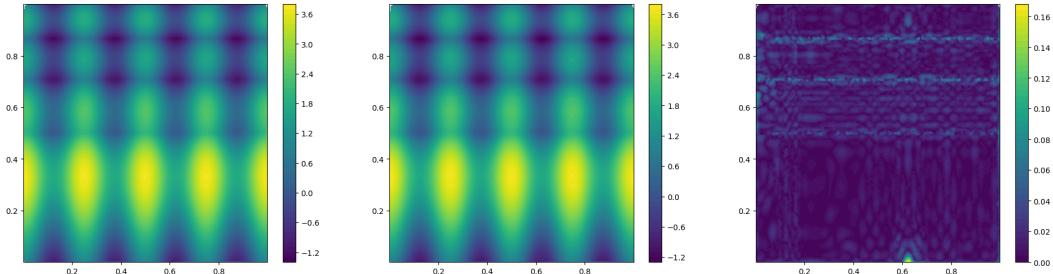


Figure 1: (Left) Analytical solution; (Middle) Prediction by Free-RBF-KAN; (Right) Error residual.

5.2 Spectral bias

Our next experiment investigates the spectral bias of Free-RBF-KAN. Unlike multilayer perceptrons (MLPs), which are known to exhibit spectral bias, KAN has been shown to avoid this limitation [Wang et al., 2024b]. We assess whether this property extends to

Model	Layers	Basis	Free Grids	#Param.	Test MSE
MLP	[2,10,10,10,1]	Tanh		261	5.26e-1
KAN	[2,5,1]	Spline		195	3.96e-3
FreeKnots-KAN	[2,5,1]	Spline	✓	307	2.57e-4
RBF-KAN	[2,5,1]	RBF-Gaussian		120	6.05e-4
Free-RBF-KAN	[2,5,1]	RBF-Gaussian	✓	290	2.39e-4

Table 2: Performance on nonsmooth 2D function.

RBF-KAN by conducting a spectral bias analysis using the Neural Tangent Kernel (NTK) framework, following the methodology in Wang et al. [2024b]. Detailed NTK derivation can be found in Wang et al. [2024b, Sec. 4.1] and Wang et al. [2021, Sec. 3.1]. The NTK analysis is performed on a multiscale regression task:

$$f(x) = 0.1 \sin(50\pi x) + \sin(2\pi x), \quad x \in [0, 1], \quad (23)$$

using MLP, KAN, and RBF-KAN variants. The input domain is discretized into 100 uniform grid points. The MLP architecture consists of 4 hidden layers with 100 neurons each, while all KAN variants use 3 hidden layers with 5 neurons. KAN employs cubic B-spline activations with 20 grid points, followed by a tanh normalization layer. RBF-KAN and Free-RBF-KAN use the same grid specifications but replace the B-splines with Gaussian RBFs. All models are trained using the mean square error (MSE) loss:

$$\mathcal{L}(\theta) = \frac{1}{n} \sum_{i=1}^n (f(x_i; \theta) - y_i)^2. \quad (24)$$

All KAN variants successfully learn the multiscale regression problem and outperform the MLP baseline, as shown in Fig. 2 for the approximated solutions and in Fig. 3 for the training loss. Moreover, the NTK eigenvalue spectra, shown in Fig. 4, reveal that RBF-KAN maintains a broader and less rapidly decaying NTK eigenvalue spectrum comparable to that of KAN and MLP, indicating an absence of spectral bias (Section 5.2). Free-RBF-KAN exhibits an even wider eigenvalue spectrum after 9000 training steps, suggesting that the additional flexibility provided by adaptive centroids and shape parameters promotes faster convergence.

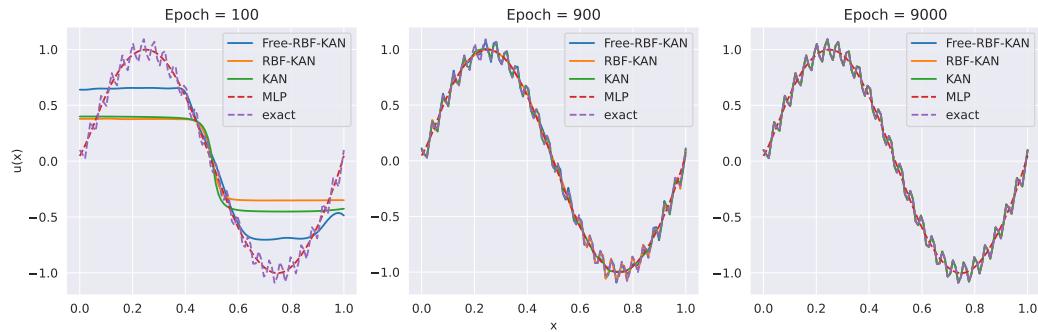


Figure 2: The approximation of f in (23) using MLP, KAN, RBFKAN, and Free-RBKFAN.

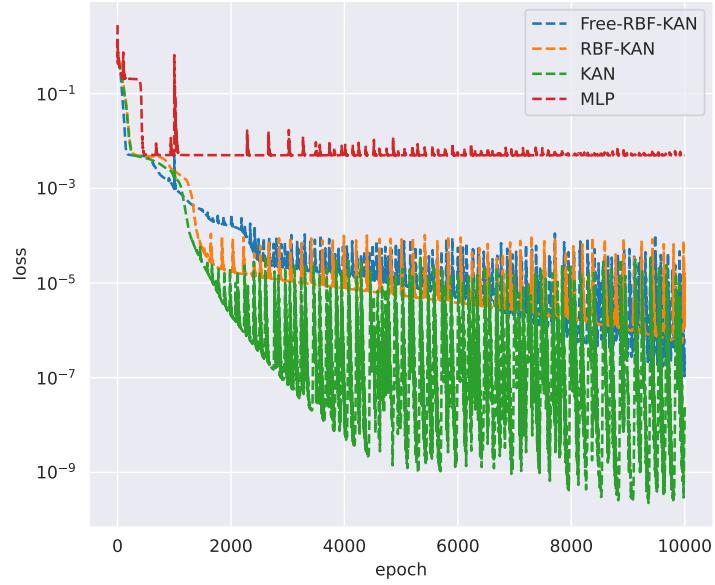


Figure 3: The training loss of approximating f in Eq. (23) using MLP, KAN, RBF-KAN, and Free-RBF-KAN.

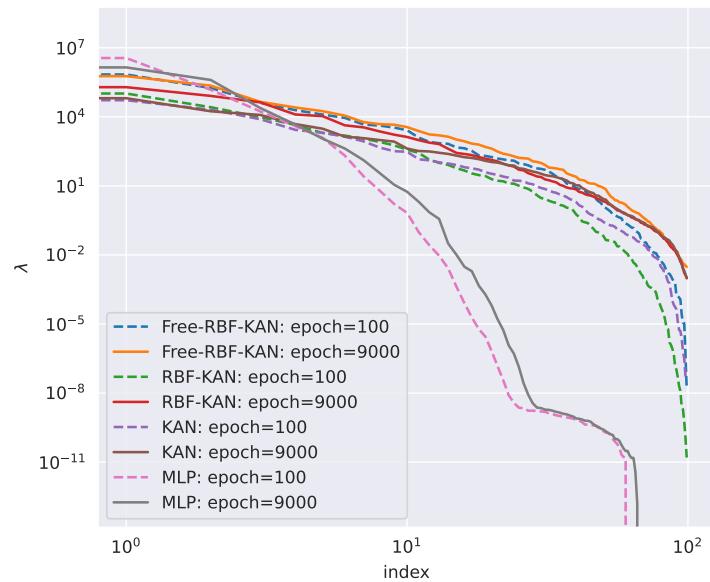


Figure 4: The NTK analysis on the spectral bias of MLP, KAN, RBF-KAN, Free-RBF-KAN in approximating f in Eq. (23)

Overall, the NTK analysis demonstrates that RBF-KAN not only trains more efficiently than the B-spline-based KAN but also retains the ability to represent multiscale features more effectively than MLPs. Furthermore, Free-RBF-KAN offers additional improvements in convergence behavior due to its adaptive smoothness and mesh refinement capabilities.

5.3 High-dimensional regression problem

In the next set of experiments, we evaluate model performance on a high-dimensional regression task using the MNIST dataset¹. This setting is nontrivial for traditional RBF networks, as placing and optimizing RBF centroids directly in high-dimensional spaces notoriously difficult and computationally expensive. On the other hand, RBF-KAN leverages a hierarchical architecture composed of univariate RBF functions, making it naturally suited with deep structures and far more effective at mitigating the curse of dimensionality.

For both RBF-KAN and KAN, we use 10 grid points for each activation function. The implementation of KAN follows the setup in Liu et al. [2024]. To enhance performance, we remove the residual activation and apply a sigmoid function for normalization. The MNIST data are normalized, and we use a batch size of 64. All networks share the same architecture with layer sizes $[28 \times 28, 64, 10]$. Training is performed for 20 epochs using the Adam optimizer with a learning rate of 1e-3. Test loss is evaluated on a test dataset as shown in Table 3.

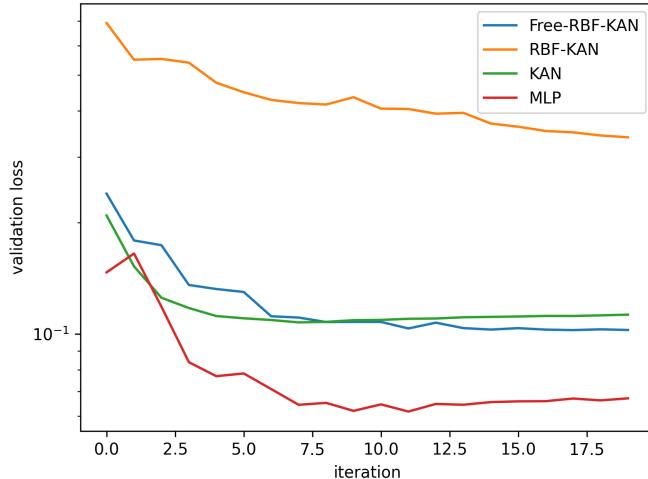


Figure 5: Validation loss during training on MNIST dataset.

As shown in Fig. 5, RBF-KAN exhibits progressively improved performance when adaptive grids and adaptive smoothness are incorporated. Prior work [Yu et al., 2024] has reported that KAN performs worse than MLP on the MNIST dataset. While RBF-KAN still lags behind MLP in accuracy and requires longer training time, Free-RBF-KAN achieves better accuracy than the standard KAN, and substantially narrows the performance gap to MLP. In terms of the training time, the additional flexibility of Free-RBF-KAN does not introduce noticeable overhead compared to KAN, yet it consistently yields improved accuracy. These results demonstrate that the adaptivity in both the grid points and the

¹<https://docs.pytorch.org/vision/main/generated/torchvision.datasets.MNIST.html>

kernel smoothness can enhance the model performance without significantly compromising computational efficiency. Among all the KAN-based methods, RBF-KAN achieves the fastest training speed but also the lowest accuracy. As observed in prior work, all KAN variants remain less competitive than MLP in terms of parameter efficiency, accuracy, and training time. Nonetheless, Free-RBF-KAN provides clear improvements in both accuracy and efficiency compared to RBF-KAN and standard KAN.

Type	Layers	Basis	# Param.	Test loss	Training time (sec.)
MLP	[28 × 28, 64, 10]	Tanh	509410	6.702e-02	81.58
KAN	[28 × 28, 64, 10]	B-Spline	762240	1.166e-01	97.95
RBF-KAN	[28 × 28, 64, 10]	RBF	508160	2.020e-01	82.27
Free-RBF-KAN	[28 × 28, 64, 10]	RBF	525120	8.789e-02	85.81

Table 3: Model performance on MNIST dataset. The Free-RBF-KAN, which is RBF-KAN with trainable centroids and smoothness parameters of RBFs, yields the best loss.

5.4 Physics-Informed Machine Learning

Our next set of experiments focuses on physics-informed machine learning. We begin by briefly reviewing the formulation of Physics-Informed Neural Networks (PINNs), which incorporate physical laws, typically expressed as partial differential equations (PDEs), into the learning process. For further details on the PINN framework and training methodology, we refer the reader to Raissi et al. [2019]. Consider a PDE defined on a domain Ω , subject to governing dynamics and boundary constraints. Such a system can be written in the general form

$$\begin{aligned} \mathcal{N}_{\mathbf{x}}[u(\mathbf{x})] &= f(\mathbf{x}), \quad \mathbf{x} \in \Omega \\ \mathcal{B}_{\mathbf{x}}[u(\mathbf{x})] &= g(\mathbf{x}), \quad \mathbf{x} \in \partial\Omega \end{aligned} \tag{25}$$

where \mathcal{N} is a differential operator that governs the behavior of the unknown solution u , and \mathcal{B} is the boundary operator. The functions f and g represent the source term and boundary condition, respectively. Here, Ω denotes the domain of interest, and $\partial\Omega$ its boundary. PINNs solve PDEs by approximating the solution u with a neural network $\hat{u}(\mathbf{x}; \omega)$, where ω denotes the trainable parameters. Physical laws are incorporated into the training objective through a loss function that penalizes violations of the PDE and boundary conditions. Automatic differentiation (AD) is used to efficiently compute all required derivatives at training points. The total loss combines contributions from the PDE residual evaluated at interior collocation points and the boundary residual, and is given by

$$\begin{aligned} \mathcal{L}(\omega) &= \frac{\lambda_c}{N_c} \sum_{i=1}^{N_c} |\mathcal{N}_{\mathbf{x}}[\hat{u}(\mathbf{x}_i; \omega)] - f(\mathbf{x}_i)|^2 \\ &\quad + \frac{\lambda_b}{N_b} \sum_{i=1}^{N_b} |\hat{u}(\mathbf{x}_i; \omega) - g(\mathbf{x}_i)|^2 \end{aligned} \tag{26}$$

where N_c and N_b denote the number of collocation points sampled in the interior of the domain and the boundary, respectively. The coefficients λ_c and λ_b weight the interior and boundary losses and, unless otherwise specified, are set to $\lambda_c = \lambda_b = 1$. The sampling points are typically drawn uniformly from Ω and $\partial\Omega$. Training proceeds by minimizing $\mathcal{L}(\omega)$ via gradient-based optimization to adjust the network parameters, so that the output solution satisfies both the governing PDE and the boundary conditions.

In the remainder of this section, we demonstrate the ability of RBF-KAN on physics-informed machine learning settings. Specifically, we investigate two representative PDE problems: a 2D heat conduction with a high frequency force term, and a 2D Helmholtz equation with smooth sinusoidal source. In both cases, the models are trained using the strong-form PINN loss defined in Eq. (26) to learn the corresponding solutions.

5.4.1 Heat Conduction in 2D

The heat conduction problem from Wang et al. [2021] has been shown to be more effectively learned by KAN than by MLP [Wang et al., 2024b]. To evaluate whether the proposed RBF-KAN can also handle multiscale physics-informed problems, we compare RBF-KAN and Free-RBF-KAN against MLP and the original KAN. The governing heat conduction equation can be written as

$$\begin{aligned} u_t &= \frac{1}{(K\pi)^2} u_{xx}, \quad x \in [0, 1], t \in [0, 1] \\ u(x, 0) &= \sin(K\pi x), \quad x \in [0, 1] \\ u(0, t) &= u(1, t) = 0, t \in [0, 1] \end{aligned} \tag{27}$$

and its analytical solution is given by

$$u(x, t) = e^{-t} \sin(K\pi x). \tag{28}$$

In this experiment, we set $K = 50$. We randomly sample 4000 interior collocation points and 200 boundary points for each boundary segment. The MLP baseline has 4 hidden layers with width 40, while all KAN variants use an architecture with layer sizes [2,5,5,1] and 30 grid points per activation. The KAN model uses cubic B-spline activations, and the grid range of KAN variations is set as $(x_l, x_r) = (0, 1)$, same as Wang et al. [2024b]. All models are trained using the Adam optimizer with a learning rate 10^{-3} and an exponential learning rate scheduler with $\gamma = 0.999$ applied each epoch. Training is performed for 15,000 epochs.

The results presented in Fig. 6 show that both RBF-KAN and Free-RBF-KAN can successfully learn this high frequency problem. While RBF-KAN attains slightly lower accuracy than KAN, it trains in roughly half the time required by KAN. Incorporating adaptive grids and smoothness further improves the performance: Free-RBF-KAN achieves higher accuracy than both KAN and RBF-KAN. Although the additional flexibilities in Free-RBF-KAN increase the training time slightly, Free-RBF-KAN remains substantially faster than KAN, as shown in (Table 4).

Model	Layers	Basis	# Param.	L^∞ -loss	Training time
MLP	[2,40,40,40,1]	Tanh	5081	1	60
Free-RBF-KAN	[2,5,5,1]	RBF-Gaussian	2000	2.41e-3	138
RBF-KAN	[2,5,5,1]	RBF-Gaussian	1280	2.78e-3	124
KAN	[2,5,5,1]	Spline	1400	6.52e-3	267

Table 4: Performance on 2D heat conduction problem. Grid size is set 30 for all KAN approaches. The training times are measured in seconds.

5.4.2 Helmholtz Equation in 2D

To further demonstrate the benefits of adaptive meshing, we examine physics-informed learning for a 2D Helmholtz equation with Dirichlet boundary conditions, following the

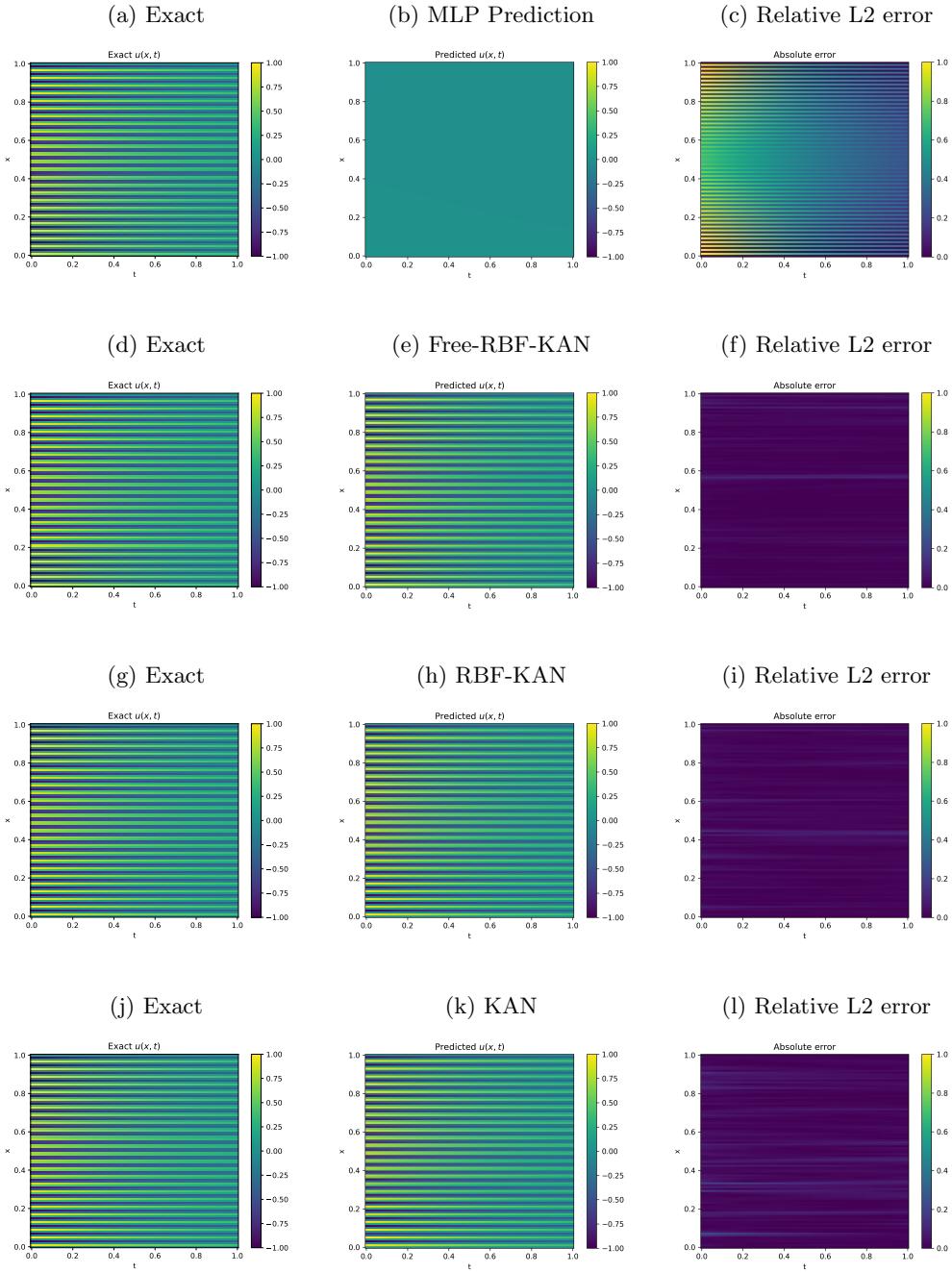


Figure 6: Predicted solutions from MLP, Free-RBF-KAN, RBF-KAN, and KAN on the heat conduction problem.

setup in Wang et al. [2020]. The problem is defined as

$$\begin{aligned} u_{xx}(x, y) + u_{yy}(x, y) + k^2 u(x, y) &= q(x, y), \quad x, y \in \Omega \\ u(x, y) &= 0 \quad x, y \in \partial\Omega \end{aligned} \quad (29)$$

where the forcing term is given by

$$q(x, y) = -(a_1\pi)^2 \sin(a_1\pi x) \sin(a_2\pi y) - (a_2\pi)^2 \sin(a_1\pi x) \sin(a_2\pi y) \quad (30)$$

$$+ k^2 \sin(a_1\pi x) \sin(a_2\pi y), \quad (31)$$

and the analytical solution is

$$u(x, y) = \sin(a_1\pi x) \sin(a_2\pi y), \quad (32)$$

We set $\Omega \in [-3, 3] \times [-3, 3]$ and choose $a_1 = a_2 = k = 1$. For each training epoch, we randomly sampled 4,000 collocation points in the interior and 100 boundary points on each edge of the domain. The MLP baseline has 5 hidden layers with 128 nodes each, whereas all the KAN variants have 2 hidden layers of 5 nodes per layer and 10 grids for each activation function. Training is performed using the Adam optimizer with a learning rate of 10^{-3} .

As shown in Fig. 7, both the MLP and Free-RBF-KAN architectures successfully approximate the PDE solution. Although minor inaccuracies remain near the boundaries, these errors could be further reduced by explicitly enforcing boundary conditions within the network architecture [Wang et al., 2023a]. In contrast, the standard RBF-KAN produces inferior approximations, highlighting the performance gains achieved through adaptive centroids and kernel shapes in the Free-RBF approach. Notably, the original KAN fails to capture the solution over the entire domain, a limitation that is likely due to the high smoothness demands of the target function. While increasing the B-spline order could potentially improve KAN’s expressive capacity, doing so would incur substantial computational overhead.

Model	Layers	Basis	# Param.	L^2 -loss	Training time
MLP	[2,128,128,128,1]	Tanh	50049	4.15e-2	39
Free-RBF-KAN	[2,5,5,1]	RBF-Gaussian	640	3.35e-2	62
RBF-KAN	[2,5,5,1]	RBF-Gaussian	400	3.67e-1	49
KAN	[2,5,5,1]	Spline	600	1.58	153

Table 5: Performance on 2D Helmholtz problem. Grid size is set 10 for all KAN approaches. The training times are measured in seconds.

The timing results show that KAN is significantly slower than the other approaches, indicating that the B-spline-based formulation is computationally expensive when combined with AD, as shown by the training timings in Table 5. In contrast, RBF-KAN achieves a substantial speed-up. The adaptive variant, namely Free-RBF-KAN, further improves the mean squared error (MSE), outperforming both MLP and standard RBF-KAN. These results demonstrate that RBF-KAN retains the advantages of both RBF networks and KAN-like architectures while avoiding the curse of dimensionality.

5.5 DeepONet with KAN Variants

Our final set of experiments investigates the performance of different architectures within the DeepONet framework. DeepONet [Lu et al., 2019] consists of a branch network and a trunk network; in our study, we evaluate how various choices for the trunk network,

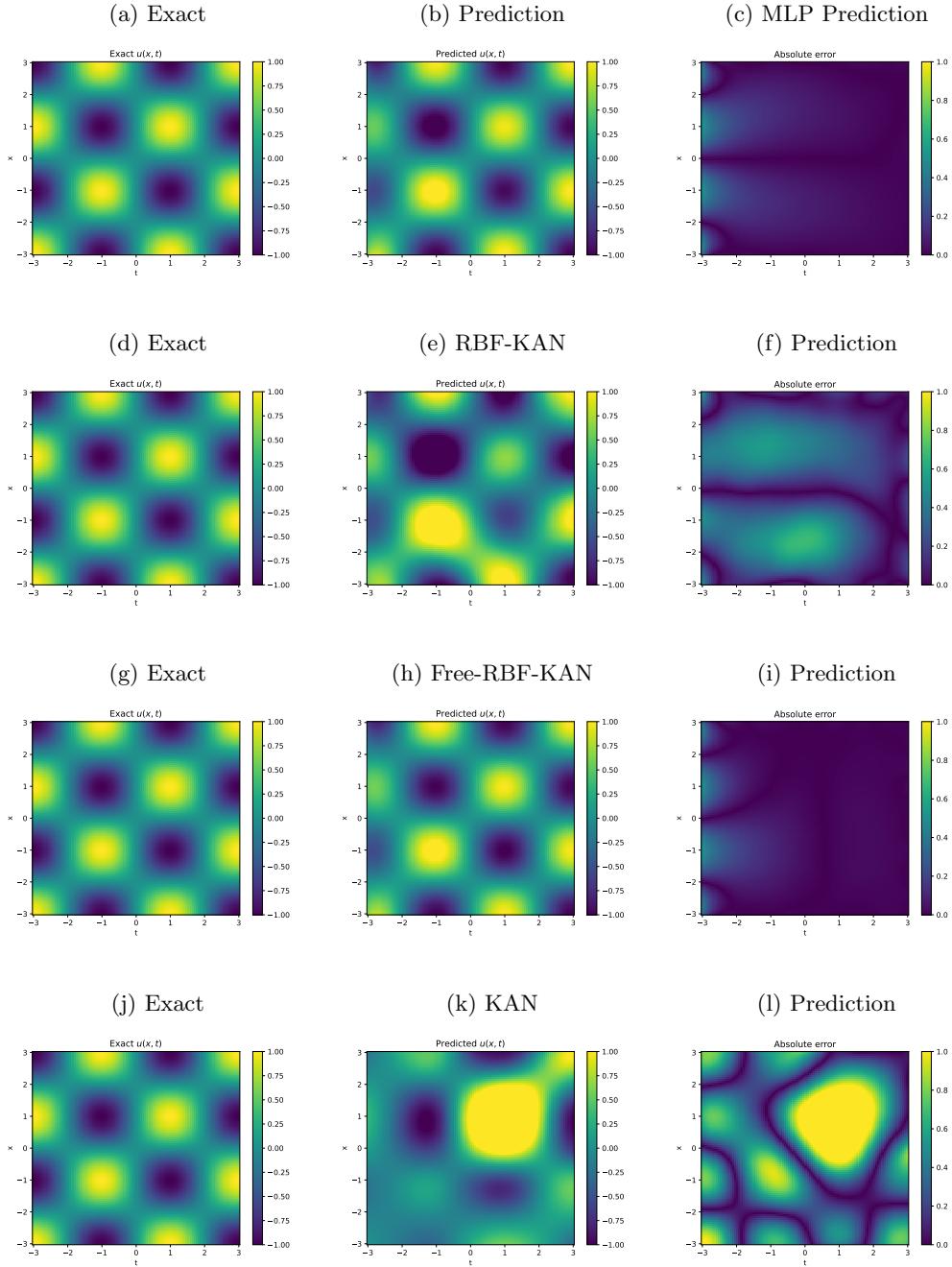


Figure 7: The exact solutions (left column), the predicted solutions from MLP, RBF-KAN, Free-RBF-KAN and KAN (middle column), and the errors (right column) for the 2D Helmholtz equation.

including MLP, KAN, RBF-KAN, and Free-RBF-KAN, affect operator-learning accuracy, while the branch network is kept fixed as an MLP. The goal is to learn the solution operator $\mathcal{G} : f \mapsto u(x, t)$ for the 1D reaction diffusion equation.

$$\frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} + ku^2 + f(x), \quad (x, t) \in (0, 1] \times (0, 1], \quad (33)$$

All DeepONet variations share the same branch network structure with 100 inputs, 100 outputs, 4 hidden layers of width 40. The trunk net is replaced by MLP, KAN, RBF-KAN, or Free-RBF-KAN, following the configurations summarized in Table 6. All models were trained with the Adam optimizer using a learning rate of 10^{-3} , and an exponential learning rate scheduler ($\gamma = 0.95$) for 10,000 steps. Training data consist of 50 forcing functions sampled from a Gaussian Random Field (GRF) with the length scale 0.2, where in each sample 10 locations of sensor observation $u(x, t)$ are randomly selected for training. Performance was evaluated by the relative mean squared error (RMSE) on the full 100×100 spatiotemporal grid, and averaged over 30 random test seeds.

Fig. 8 visualizes the model predictions for a representative test function. Among the evaluated architectures, Free-RBF-KAN provides the most accurate approximation, slightly outperforming the MLP in capturing fine-scale features. In contrast, both the standard RBF-KAN and the original KAN exhibit inferior performance. These results underscore the importance of adaptive RBFs and suggest that the fixed B-spline bases used in KAN may lack sufficient smoothness to effectively represent the target solution.

The MLP trunk employs fully connected layers of width 40, yielding the largest model size. In contrast, the KAN-based trunks (KAN, RBF-KAN, Free-RBF-KAN) use a much smaller hidden dimension of 4 and 20 grid points for their basis functions. KAN uses cubic B-splines (order 3), whereas the RBF-KAN variants employ Gaussian kernels. The results presented in Table 6 show that Free-RBF-KAN achieves the highest accuracy among all tested architectures while also requiring fewer parameters than the standard KAN.

Type	Layers (Trunk)	Basis	# Param	Rel. L2 error	Training time
MLP	[2, [40]*4, 100]	Tanh	18921	2.08e-2	78
Free-RBF-KAN	[2, 4, 4, 4, 100]	Gaussian	11185	1.94e-2	88
RBF-KAN	[2, 4, 4, 4, 100]	Gaussian	10625	3.7e-2	84
KAN	[2, 4, 4, 4, 100]	B-Spline	11945	6.15e-2	96

Table 6: Benchmark on nonlinear diffusion reaction problems. KAN has B-Spline with order 3. The training times are measured in seconds.

6 Conclusion

This work introduces Free-RBF-KAN, a Kolmogorov–Arnold Network architecture enhanced with radial basis functions and adaptive meshing and shape, providing a compelling alternative to the standard B-spline-based KAN. We establish that Free-RBF-KAN serves as a universal approximators of any uniform continuous function, and our experiments across synthetic, nonsmooth, high-frequency, multiscale, and physics-informed learning tasks consistently demonstrate the advantages of this formulation.

One of the key contributions of this work lies in demonstrating that Free-RBF-KAN achieves higher accuracy than the B-Spline-based KAN while requiring substantially less training time. The use of a Gaussian kernel, which is in C^∞ , makes RBF-KAN naturally suitable for physic-informed learning tasks that involve derivatives of any order. In contrast, KAN relies on De Boor’s recursion to evaluate high order B-splines, a process that is

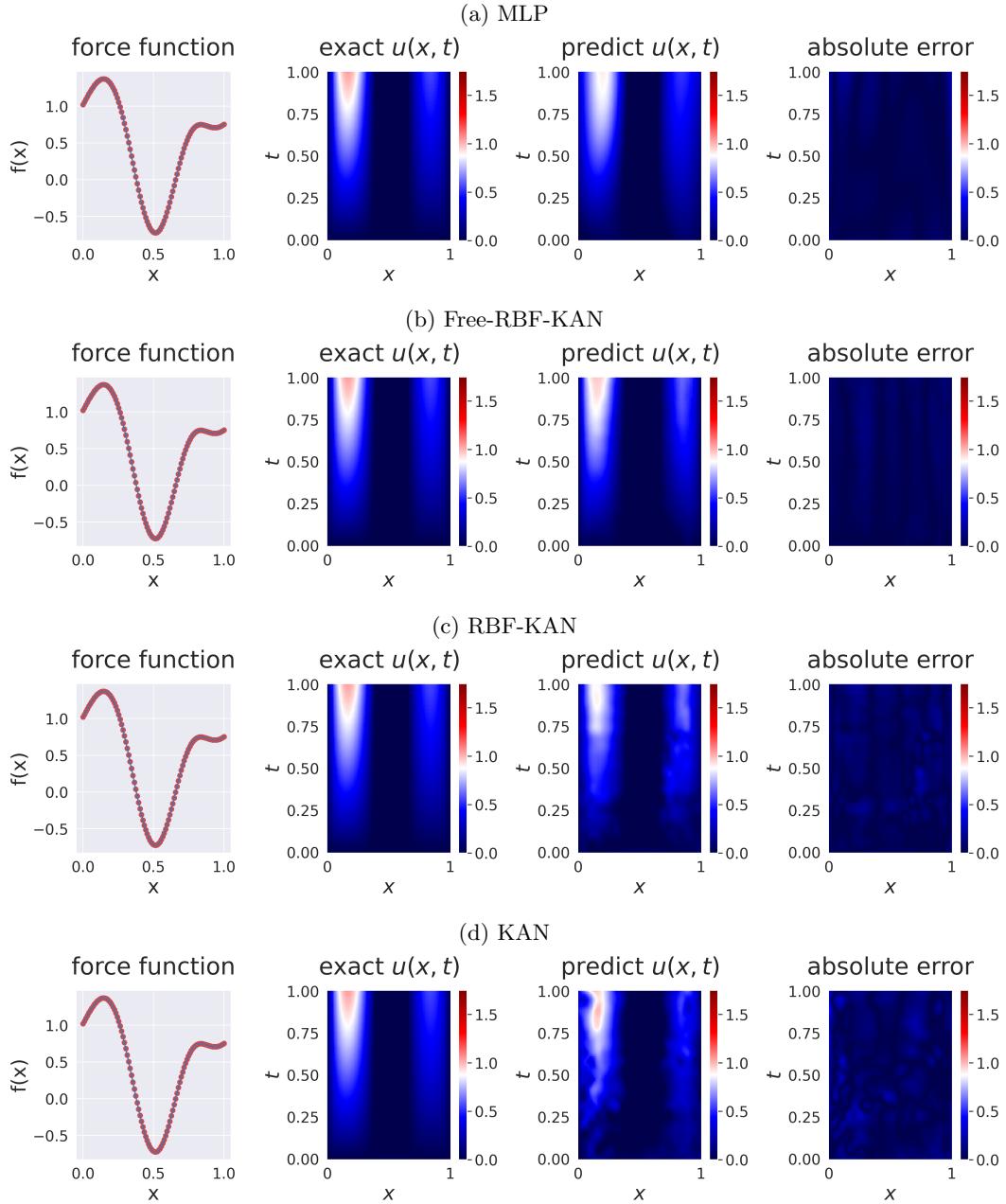


Figure 8: The prediction of DeepONet variations on reaction-diffusion PDE. All models has same branch structure, but with different trunk architecture. A sampled force function is used to generate the solution.

cumbersome and computationally expensive when combined with automatic differentiation. For example, in the 2D Helmholtz problem (Section 5.4.2), Free-RBF-KAN attains the lowest test error among all KAN variants while reducing training time to roughly one-third of that required by standard KAN. These results show that adaptive radial basis functions not only enhance representational power but also yield significant improvements in computational efficiency. Moreover, the spectral bias analysis using Neural Tangent Kernel (NTK) theory reinforces that RBF-KAN preserves the favorable learning properties of KAN. The eigenvalue spectra indicate that RBF-KAN maintains a more balanced representation across frequencies with low spectral bias throughout training. This behavior supports the claim that RBF-KAN generalizes more effectively in high-frequency regimes, where MLPs typically struggle, while also training faster than standard KAN.

Importantly, the adaptive mesh and shape mechanisms further strengthen model flexibility. In nonsmooth function regression (Section 5.1), Free-RBF-KAN matches the accuracy of FreeKnots-KAN while using fewer parameters, highlighting the effectiveness of RBFs in capturing localized features without relying on computationally expensive B-spline evaluations. The parameterization tricks for smoothness and centroid constraints ensure gradient-friendly learning without constraint optimization or gradient clipping. Despite these advantages, RBF-KAN is not a universal replacement for MLP or standard KAN in all settings. For the high dimensional regression problem with the MNIST dataset (Section 5.3), MLPs still outperform all KAN variants in both accuracy and training speed. This suggests that for unstructured image data, the hierarchical function decomposition of KAN may be less suitable than the dense representation of MLPs as also noted in Yu et al. [2024]. However, in multiscale regression and physics-informed learning, where interpretability, sparsity, and efficiency are prioritized, Free-RBF-KAN offers clear and compelling advantages. For operator learning, we introduce a new DeepONet variant by integrating Free-RBF-KAN into the trunk network to provide adaptive basis functions. This modification yields substantial improvements in operator learning, which we attribute to the inherent adaptivity of Free-RBF-KAN. By dynamically adjusting both the mesh and the kernel shape, the model achieves a more expressive functional representation, resulting in increased accuracy without a corresponding increase in computational cost. The global smoothness of the learned representation can be controlled by the choice of the Matérn kernel, although such smoothness is not required for universal approximation. This flexibility can be advantageous when modeling nonsmooth functions. Additionally, Free-RBF-KAN offers tunable stiffness through its adaptive shape parameters, providing an expressive and computationally efficient alternative to fixed-kernel approaches.

Lastly, this work bridges the gap between theoretical foundations and practical architectural design. By unifying universal approximation results for RBFs with Kolmogorov’s representation theorem (Section 4.3), we establish that RBF-KAN retains strong expressive capabilities. The empirical results supported by NTK analysis and PDE-constrained learning experiments indicate that RBF-KAN is a compelling candidate for scientific machine learning applications, particularly in multiscale and high-dimensional settings.

A promising direction for future work is to explore reformulations of RBF-KAN that allow it to be expressed as an MLP with Gaussian activation functions, similar to the approach in Actor et al. [2025]. Under this perspective, the centroids and sharpness parameters can be reinterpreted as biases and weights in the linear layers of an MLP. Such a transformation could enable RBF-KAN to benefit from the computational efficiencies and mature optimization techniques developed for standard MLP architectures, potentially further improving scalability and performance.

References

- Diab W Abueidda, Panos Pantidis, and Mostafa E Mobasher. Deepokan: Deep operator network based on kolmogorov arnold networks for mechanics problems. *Computer Methods in Applied Mechanics and Engineering*, 436:117699, 2025.
- Jonas A. Actor, Graham Harper, Ben Southworth, and Eric C. Cyr. Leveraging kans for expedient training of multichannel mlps via preconditioning and geometric refinement. *arXiv preprint arXiv:2505.18131*, 2025. URL <https://arxiv.org/abs/2505.18131>.
- Vladimir Igorevich Arnol'd. On functions of three variables. In *Doklady Akademii Nauk*, volume 114, pages 679–681. Russian Academy of Sciences, 1957.
- Jinshuai Bai, Gui-Rong Liu, Ashish Gupta, Laith Alzubaidi, Xi-Qiao Feng, and YuanTong Gu. Physics-informed radial basis network (pirbn): A local approximating neural network for solving nonlinear partial differential equations. *Computer Methods in Applied Mechanics and Engineering*, 415:116290, 2023.
- Jürgen Braun and Michael Griebel. On a constructive proof of kolmogorov's superposition theorem. *Constructive approximation*, 30(3):653–675, 2009.
- Andrew Siyuan Chen. Gaussian process kolmogorov-arnold networks. *arXiv preprint arXiv:2407.18397*, 2024.
- Athanasis Delis. Fasterkan. <https://github.com/AthanasisDelis/faster-kan/>, 2024.
- Buma L Fridman. An improvement in the smoothness of the functions in an kolmogorov's theorem on superpositions. In *Doklady Akademii Nauk*, volume 177, pages 1019–1022. Russian Academy of Sciences, 1967.
- Federico Girosi and Tomaso Poggio. Representation properties of networks: Kolmogorov's theorem is irrelevant. *Neural Computation*, 1(4):465–469, 1989.
- Federico Girosi and Tomaso Poggio. Networks and the best approximation property. *Biological cybernetics*, 63(3):169–176, 1990.
- Aysu Ismayilova and Muhammad Ismayilov. On the universal approximation property of radial basis function neural networks. *Annals of Mathematics and Artificial Intelligence*, 92(3):691–701, 2024.
- A.N. Kolmogorov. On the representation of continuous functions of several variables as superpositions of continuous functions of a smaller number of variables. *Dokl. Akad. Nauk*, 108(2), 1956.
- Aditya Bagus Krisnawan, Prasetiyono Hari Mukti, and Mauridhi Hery Purnomo. Rbf-kan: Integrated approach for accurate indoor localization in dense grid rssi fingerprint. In *2025 17th International Conference on Knowledge and Smart Technology (KST)*, pages 46–51. IEEE, 2025.
- Vera Kurkova. Kolmogorov's theorem is relevant. *Neural computation*, 3(4):617–622, 1991.
- Moshe Leshno, Vladimir Ya Lin, Allan Pinkus, and Shimon Schocken. Multilayer feedforward networks with a nonpolynomial activation function can approximate any function. *Neural networks*, 6(6):861–867, 1993.

- Ziyao Li. Kolmogorov-arnold networks are radial basis function networks. *arXiv preprint arXiv:2405.06721*, 2024. URL <https://arxiv.org/abs/2405.06721>.
- Ziming Liu, Yixuan Wang, Sachin Vaidya, Fabian Ruehle, James Halverson, Marin Soljacić, Thomas Y Hou, and Max Tegmark. Kan: Kolmogorov-arnold networks. *arXiv preprint arXiv:2404.19756*, 2024.
- Lu Lu, Pengzhan Jin, and George Em Karniadakis. Deeponet: Learning nonlinear operators for identifying differential equations based on the universal approximation theorem of operators. *arXiv preprint arXiv:1910.03193*, 2019.
- Fatemeh Nassajian Mojarrad, Maria Han Veiga, Jan S Hesthaven, and Philipp Öffner. A new variable shape parameter strategy for rbf approximation using neural networks. *Computers & Mathematics with Applications*, 143:151–168, 2023.
- Gholam Ali Montazer, Davar Giveki, Maryam Karami, and Homayon Rastegar. Radial basis function neural networks: A review. *Comput. Rev. J*, 1(1):52–74, 2018.
- Mark JL Orr et al. Introduction to radial basis function networks, 1996.
- Jooyoung Park and Irwin W Sandberg. Universal approximation using radial-basis-function networks. *Neural computation*, 3(2):246–257, 1991.
- Jooyoung Park and Irwin W Sandberg. Approximation and radial-basis-function networks. *Neural computation*, 5(2):305–316, 1993.
- Allan Pinkus. Approximation theory of the mlp model in neural networks. *Acta Numerica*, 8:143–195, 1999. doi: 10.1017/S0962492900002919.
- Qichao Que and Mikhail Belkin. Back to the future: Radial basis function networks revisited. In *Artificial intelligence and statistics*, pages 1375–1383. PMLR, 2016.
- M. Raissi, P. Perdikaris, and G. E. Karniadakis. Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378:686–707, 2019. ISSN 0021-9991. doi: 10.1016/j.jcp.2018.10.045. URL <https://www.sciencedirect.com/science/article/pii/S0021999118307125>.
- Amuthan A Ramabathiran and Prabhu Ramachandran. Spinn: sparse, physics-based, and partially interpretable neural networks for pdes. *Journal of Computational Physics*, 445: 110600, 2021.
- Khemraj Shukla, Juan Diego Toscano, Zhicheng Wang, Zongren Zou, and George Em Karniadakis. A comprehensive and fair comparison between mlp and kan representations for differential equations and operator networks. *Computer Methods in Applied Mechanics and Engineering*, 431:117290, 2024.
- Shriyank Somvanshi, Syed Aaqib Javed, Md Monzurul Islam, Diwas Pandit, and Subasish Das. A survey on kolmogorov-arnold network. *arXiv preprint arXiv:2411.06078*, 2024. URL <https://arxiv.org/abs/2411.06078>.
- Sidharth SS, Keerthana AR, Anas KP, et al. Chebyshev polynomial-based kolmogorov-arnold networks: An efficient architecture for nonlinear function approximation. *arXiv preprint arXiv:2405.07200*, 2024.

- Hoang-Thang Ta. Bsrbf-kan: A combination of b-splines and radial basis functions in kolmogorov-arnold networks. *arXiv preprint arXiv:2406.11173*, 2024. URL <https://arxiv.org/abs/2406.11173>.
- Juan Diego Toscano, Li-Lian Wang, and George Em Karniadakis. Kkans: Kurkova-kolmogorov-arnold networks and their learning dynamics. *Neural Networks*, page 107831, 2025.
- Sifan Wang, Yujun Teng, and Paris Perdikaris. Understanding and mitigating gradient pathologies in physics-informed neural networks. arxiv e-prints. *arXiv preprint arXiv:2001.04536*, 2020.
- Sifan Wang, Hanwen Wang, and Paris Perdikaris. On the eigenvector bias of fourier feature networks: From regression to solving multi-scale pdes with physics-informed neural networks. *Computer Methods in Applied Mechanics and Engineering*, 384:113938, 2021.
- Sifan Wang, Shyam Sankaran, Hanwen Wang, and Paris Perdikaris. An expert's guide to training physics-informed neural networks. *arXiv preprint arXiv:2308.08468*, 2023a.
- Yixuan Wang, Jonathan W Siegel, Ziming Liu, and Thomas Y Hou. On the expressiveness and spectral bias of kans. *arXiv preprint arXiv:2410.01803*, 2024a.
- Yizheng Wang, Jia Sun, Jinshuai Bai, Cosmin Anitescu, Mohammad Sadegh Eshaghi, Xiaoying Zhuang, Timon Rabczuk, and Yinghua Liu. Kolmogorov arnold informed neural network: A physics-informed deep learning framework for solving forward and inverse problems based on kolmogorov arnold networks. *arXiv preprint arXiv:2406.11045*, 2024b.
- Zhiwen Wang, Minxin Chen, and Jingrun Chen. Solving multiscale elliptic problems by sparse radial basis function neural networks. *Journal of Computational Physics*, 492: 112452, 2023b.
- Dietrich Wettschereck and Thomas Dietterich. Improving the performance of radial basis function networks by learning center locations. *Advances in neural information processing systems*, 4, 1991.
- Lei Xu, Adam Krzyżak, and Alan Yuille. On radial basis function nets and kernel regression: Statistical consistency, convergence rates, and receptive field size. *Neural Networks*, 7(4): 609–628, 1994.
- Runpeng Yu, Weihao Yu, and Xinchao Wang. Kan or mlp: A fairer comparison. *arXiv preprint arXiv:2407.16674*, 2024.
- Chengxi Zeng, Tilo Burghardt, and Alberto M Gambaruto. Rbf-pinn: Non-fourier positional embedding in physics-informed neural networks. *arXiv preprint arXiv:2402.08367*, 2024.
- Liangzewi Nathan Zheng, Wei Emma Zhang, Lin Yue, Miao Xu, Olaf Maennel, and Weitong Chen. Free-knots kolmogorov-arnold network: On the analysis of spline knots and advancing stability. *arXiv preprint arXiv:2501.09283*, 2025.