

# Axion misalignment as a synchronization phenomenon

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We propose a dynamical reinterpretation of axion misalignment as an emergent collective phenomenon. Drawing an explicit parallel between axion field dynamics and synchronization in coupled oscillator systems, we show that a macroscopic axion phase can arise dynamically from initially incoherent configurations through gradient-driven ordering in an expanding Universe. In this framework, the misalignment angle is not a fundamental initial condition but a collective variable that becomes well defined only once phase coherence develops. Using a simple lattice model, we illustrate how the collective phase is selected prior to the onset of axion oscillations, providing a dynamical basis for the standard misalignment picture. This perspective offers a new way of organizing axion initial-condition sensitivity, reframes anthropic small-angle arguments in terms of phase-ordering efficiency, and suggests a broader connection between fine-tuning and emergent collective dynamics in the early Universe.

## INTRODUCTION

The axion remains one of the most compelling candidates for cold dark matter, arising naturally from the Peccei–Quinn (PQ) solution to the strong  $CP$  problem [1–3]. In the standard cosmological picture, the axion relic abundance is generated by the misalignment mechanism, in which the axion field begins coherent oscillations when the Hubble rate drops below the axion mass,  $3H(T_{\text{osc}}) \simeq m_a(T_{\text{osc}})$  [4–6]. Assuming a spatially homogeneous initial field value  $\theta_i$ , the present-day abundance scales as  $\Omega_a \propto f_a^P \theta_i^2$ , up to known corrections [7, 8].

For large axion decay constants  $f_a$ , reproducing the observed dark matter density requires  $\theta_i \ll 1$ , a fact often interpreted in terms of anthropic selection [7]. This reasoning treats the initial misalignment angle as a fundamental environmental parameter, implicitly assuming that a unique, homogeneous axion phase exists prior to the onset of oscillations. However, the axion is a compact phase field whose cosmological evolution is governed by local dynamics, and the existence of a single global angle is itself a dynamical question.

In this work we point out that the axion field equations describe a system that is mathematically equivalent to a network of locally coupled, damped phase variables with time-dependent coupling set by cosmological expansion. Such systems are known to exhibit synchronization and phase-ordering phenomena, in which macroscopic collective variables emerge dynamically from initially incoherent microscopic phases. Synchronization was originally introduced by Kuramoto in the context of coupled oscillators [9, 10] and has since been recognized as a universal mechanism underlying collective phase coherence in a wide range of physical systems [11]. In spatially extended systems with local coupling, synchronization and phase ordering describe the same dynamical process viewed from microscopic and macroscopic perspectives, respectively.

Adopting this perspective, the axion misalignment angle is naturally identified with the collective phase of the

axion field, which becomes well defined only once dynamical coherence has been established. The appropriate macroscopic variables are the synchronization order parameter  $R$  and the associated collective phase  $\Psi$ ; a single “initial angle” exists only in the regime  $R \simeq 1$ . The standard misalignment formula is therefore valid only after the axion field has undergone a phase-ordering process driven by gradient interactions, Hubble damping, and the time-dependent axion potential.

This reframing does not modify the axion equations of motion, nor does it alter the standard relic abundance calculation within its usual domain of applicability. Rather, it makes explicit the dynamical assumptions implicit in the use of a homogeneous initial angle, namely that sufficient phase ordering has occurred prior to the onset of oscillations. From this perspective, anthropic small-angle arguments are reinterpreted not as statements about fundamental initial data, but as assumptions about the efficiency of collective ordering in the early Universe.

We emphasize that the synchronization perspective adopted here differs from previous uses of related language in axion physics. Synchronization has been discussed in the context of axion–Josephson or axion–superconductor analogies, where an axion phase couples to an external coherent oscillator [12], as well as in defect dynamics, where “phase locking” refers to the attachment of strings and domain walls once the axion potential turns on [13]. By contrast, we focus on synchronization among axion field degrees of freedom themselves, as a cosmological phase-ordering process that determines when a macroscopic axion angle exists and how its value is selected. Similarly, while the standard distinction between Peccei–Quinn symmetry breaking before or after inflation is usually framed in terms of horizon size and defect formation [8], we reinterpret it here in terms of the efficiency and scale of collective phase ordering.

In the following sections, we make the equivalence between axion dynamics and synchronization explicit, introduce the relevant collective variables, and illustrate

their emergence with minimal numerical simulations incorporating cosmological inputs.

## AXION FIELD DYNAMICS AS A COUPLED PHASE SYSTEM

### Continuum axion equation in an expanding Universe

After Peccei–Quinn symmetry breaking, the axion field can be written as  $\phi(x, t) = f_a \theta(x, t)$ , where  $\theta$  is a compact phase variable,  $\theta \equiv \theta + 2\pi$  [2, 3]. In a spatially flat Friedmann–Robertson–Walker background, the axion phase obeys

$$\ddot{\theta}(x, t) + 3H(t)\dot{\theta}(x, t) - \frac{1}{a^2(t)}\nabla^2\theta(x, t) + m_a^2(T)\sin\theta(x, t) = 0, \quad (1)$$

where  $H(t)$  is the Hubble rate,  $a(t)$  the scale factor, and  $m_a(T)$  the temperature-dependent axion mass [4–6, 8].

Equation (1) contains three key ingredients: local phase coupling through spatial gradients, dissipation via Hubble friction, and a time-dependent restoring force that becomes effective near the QCD scale. This structure is characteristic of phase-ordering dynamics in systems with compact order parameters [14, 15].

### Discretization and emergence of a coupling network

To make the connection explicit, we discretize space on a comoving lattice with spacing  $\Delta x$  and define  $\theta_i(t) \equiv \theta(x_i, t)$ . The Laplacian is approximated by nearest-neighbor differences,

$$\nabla^2\theta(x_i) \longrightarrow \frac{1}{(\Delta x)^2} \sum_{j \in \langle i \rangle} (\theta_j - \theta_i). \quad (2)$$

Substituting into Eq. (1) yields

$$\ddot{\theta}_i + 3H\dot{\theta}_i + \sum_{j \in \langle i \rangle} J_{ij}(t)(\theta_i - \theta_j) + m_a^2(T)\sin\theta_i = 0, \quad (3)$$

with a time-dependent coupling

$$J_{ij}(t) = \frac{1}{a^2(t)(\Delta x)^2} \quad (j \in \langle i \rangle), \quad (4)$$

and  $J_{ij} = 0$  otherwise. The coupling strength therefore decreases with cosmic expansion and is fully fixed by the background geometry and the spatial resolution; it introduces no new free parameters. Discretizations of this form are standard in numerical studies of axion field dynamics and defect evolution [8, 16].

Since  $\theta$  is a compact variable, it is convenient to rewrite the linear coupling in Eq. (3) in a periodic form,

$$\sum_{j \in \langle i \rangle} J_{ij}(t)(\theta_i - \theta_j) \longrightarrow \sum_{j \in \langle i \rangle} K_{ij}(t)\sin(\theta_j - \theta_i), \quad (5)$$

which is equivalent in the regime of small phase differences and respects the underlying  $2\pi$  periodicity. The effective coupling matrix is therefore

$$K_{ij}(t) = \frac{1}{a^2(t)(\Delta x)^2}, \quad (6)$$

identical for all nearest-neighbor pairs. This representation makes the analogy with coupled phase systems manifest.

### Relation to synchronization dynamics

Equations (3) and (5) describe a network of identical phase variables with local coupling, dissipation, and a time-dependent on-site potential. This structure is mathematically equivalent to a second-order generalization of the Kuramoto model for synchronization, with vanishing intrinsic frequencies and a time-dependent coupling strength [9–11].

In this language, the axion gradient term enforces local phase alignment, while Hubble friction drives relaxation toward ordered configurations. The emergence of a homogeneous axion mode is therefore not an assumption but a dynamical outcome of phase ordering. Whether and when a single global phase develops depends on the competition between gradient coupling, cosmological damping, and the turn-on of the axion potential.

## EMERGENT COLLECTIVE VARIABLES AND THE MEANING OF THE MISALIGNMENT ANGLE

### Order parameter and phase coherence

The axion field equation describes the evolution of a compact phase subject to local coupling and dissipation. In such systems, the appropriate macroscopic description is not given by individual phases but by collective variables characterizing coherence. We therefore introduce the complex order parameter

$$R(t)e^{i\Psi(t)} \equiv \frac{1}{V} \int_V d^3x e^{i\theta(x, t)}, \quad (7)$$

where  $R(t) \in [0, 1]$  measures the degree of phase coherence and  $\Psi(t)$  defines a collective phase. This definition is standard in the theory of synchronization and phase ordering [9, 11, 15].

Two limiting regimes are of particular relevance. For an incoherent field configuration with random phases, one has  $R \simeq 0$  and no global phase is well defined. Conversely, when the field has developed long-range coherence,  $R \simeq 1$  and the axion field is well approximated by a homogeneous mode,

$$\theta(x, t) \simeq \Psi(t) \quad (R \simeq 1). \quad (8)$$

Only in this regime does it become meaningful to describe the system in terms of a single axion angle.

### Reduction to the homogeneous mode

When coherence is established, the axion energy density reduces to that of the collective mode. In the small-angle limit,  $|\Psi| \ll 1$ , one finds

$$\rho_a(t) \simeq \frac{f_a^2}{2} [\dot{\Psi}^2(t) + m_a^2(T)\Psi^2(t)], \quad (9)$$

which coincides with the standard homogeneous-field expression used in misalignment calculations [4, 5, 8].

More generally, when  $R < 1$ , the energy density depends on the full field variance,

$$\rho_a(t) \propto f_a^2 m_a^2(T) \langle \theta^2 \rangle = f_a^2 m_a^2(T) [\Psi^2 + \text{Var}(\theta)], \quad (10)$$

and cannot be expressed solely in terms of a single angle. The usual misalignment formula is therefore valid only after dynamical phase ordering has rendered  $\text{Var}(\theta) \ll \Psi^2$ .

### Oscillation onset and identification of the misalignment angle

Axion oscillations begin when the axion mass becomes comparable to the Hubble rate,

$$3H(T_{\text{osc}}) \simeq m_a(T_{\text{osc}}), \quad (11)$$

after which the comoving axion number density is approximately conserved. In the standard treatment, one evaluates the energy density at  $T_{\text{osc}}$  using a homogeneous initial angle  $\theta_i$ . From the present perspective, this procedure is justified only if the axion field has already become coherent at that time,

$$R(T_{\text{osc}}) \simeq 1. \quad (12)$$

When this condition holds, the identification

$$\theta_i \equiv \Psi(T_{\text{osc}}) \quad (13)$$

is well defined, and the standard misalignment abundance follows. If instead  $R(T_{\text{osc}}) < 1$ , no unique misalignment angle exists and the axion abundance depends on the full phase distribution rather than on a single collective variable.

### Implications for small-angle assumptions

The common assumption of a small initial angle  $\theta_i \ll 1$  can thus be reinterpreted as a statement about the outcome of the axion phase-ordering dynamics. Specifically,

it presumes that a coherent collective phase  $\Psi$  exists and that its value happens to be small at the onset of oscillations. Whether this is realized depends on the competition between gradient-induced alignment, Hubble damping, and the time dependence of the axion potential, rather than on an arbitrary choice of initial conditions.

This observation does not alter the axion equations of motion nor the standard relic abundance calculation in the regime  $R \simeq 1$ . It clarifies, however, that the “initial misalignment angle” is not a fundamental input parameter but an emergent collective variable whose existence and value are set dynamically.

## NUMERICAL ILLUSTRATION

To illustrate the dynamical emergence of a collective axion phase, we perform lattice simulations of the axion field in an expanding background. The purpose of this numerical study is not a quantitative determination of the axion relic abundance, but rather to demonstrate explicitly how the misalignment angle can arise as a collective variable from initially incoherent configurations.

We consider a one-dimensional comoving lattice with periodic boundary conditions and evolve the axion phase  $\theta_i$  according to the overdamped equation of motion, which isolates the phase-ordering dynamics

$$3H(T) \dot{\theta}_i = J(T) \Delta \theta_i - m_a^2(T) \sin \theta_i, \quad (14)$$

where  $H(T)$  is the Hubble rate in radiation domination and  $J(T) = 1/[a^2(T) \Delta x^2]$  controls gradient interactions on the lattice. The axion mass  $m_a(T)$  turns on smoothly around a reference temperature  $T_0$  and saturates at low temperature. Time evolution is implemented using logarithmic temperature steps  $\Delta \ln T$ , corresponding to uniform fractions of a Hubble time via  $dt = -d \ln T / H(T)$ .

*Emergence of coherence.* Starting from random initial phases, the system rapidly develops phase coherence as the Universe cools. This is quantified by the complex order parameter

$$R(T)e^{i\Psi(T)} \equiv \frac{1}{N} \sum_i e^{i\theta_i(T)}. \quad (15)$$

As shown in Fig. 1, the modulus  $R(T)$  grows from values close to zero at high temperature to  $\mathcal{O}(1)$  well before the onset of axion oscillations. This demonstrates explicitly that macroscopic phase coherence can arise dynamically, without imposing homogeneous initial conditions.

*Collective phase.* Once coherence is established, the collective phase  $\Psi(T)$  becomes well defined and slowly varying. This behavior is shown in Fig. 2. At early times, when  $R \simeq 0$ ,  $\Psi$  fluctuates strongly and has no physical meaning. Only after the system has ordered does  $\Psi(T)$  stabilize, providing a meaningful macroscopic axion angle.

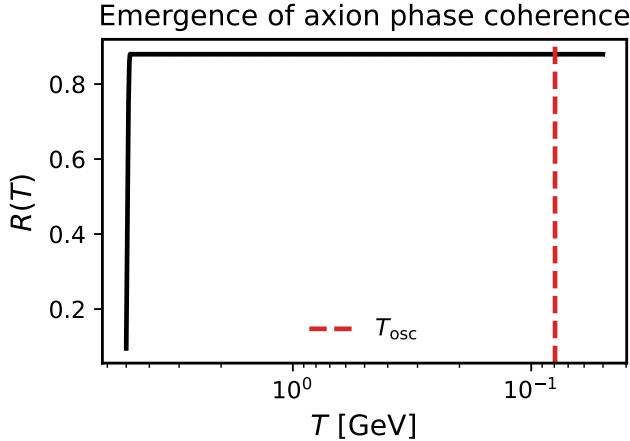


FIG. 1. Evolution of the order parameter  $R(T)$  as a function of temperature. Starting from random initial phases, the system develops macroscopic coherence ( $R = \mathcal{O}(1)$ ) well before the onset of axion oscillations.

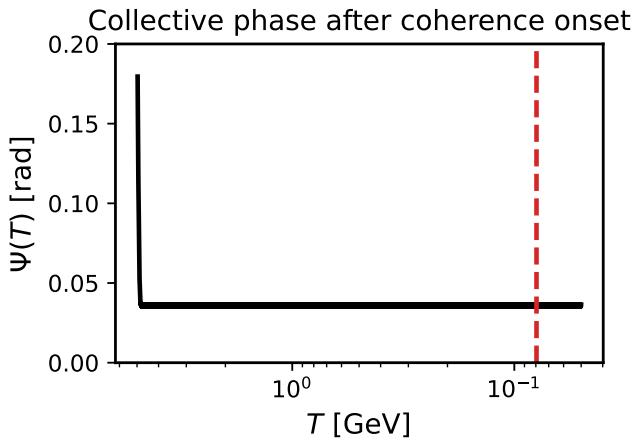


FIG. 2. Emergent collective phase  $\Psi(T)$ . At high temperature, where  $R \simeq 0$ , the phase fluctuates strongly and has no physical meaning. Once coherence is established,  $\Psi(T)$  stabilizes and becomes a well-defined macroscopic axion angle.

*Relation to oscillation onset.* The onset of axion oscillations is conventionally defined by the condition  $m_a(T_{\text{osc}}) \simeq 3H(T_{\text{osc}})$ , illustrated in Fig. 3. We find that coherence is already well established at  $T_{\text{osc}}$ , so that identifying the usual misalignment angle with the emergent collective phase,

$$\theta_i \equiv \Psi(T_{\text{osc}}), \quad (16)$$

is dynamically justified within this framework.

*Spatial structure and residual fluctuations.* Although the field becomes coherent, it does not become perfectly homogeneous. Figure 4 shows snapshots of the phase field at representative temperatures. The evolution proceeds from a highly disordered configuration at high temperature to a smooth but nonuniform profile at late

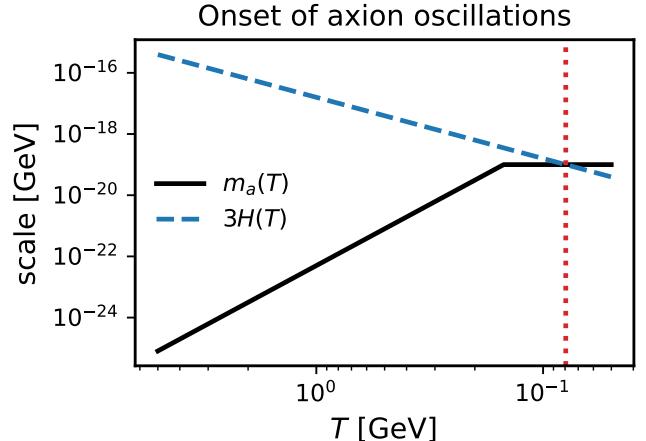


FIG. 3. Comparison between the axion mass  $m_a(T)$  and the Hubble scale  $3H(T)$ . The intersection defines the conventional oscillation temperature  $T_{\text{osc}}$ . At this temperature, the system is already in a coherent phase, justifying the identification of the misalignment angle with the emergent collective phase  $\Psi(T_{\text{osc}})$ .

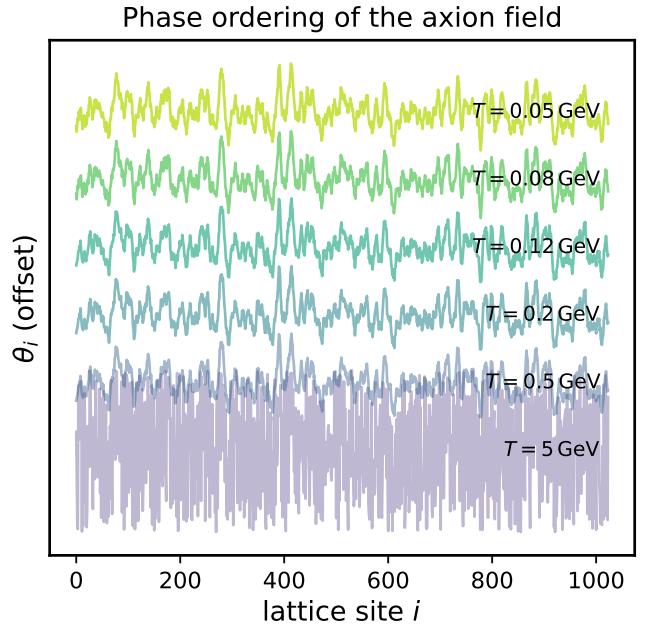


FIG. 4. Snapshots of the axion phase field  $\theta_i$  at representative temperatures (top to bottom: decreasing  $T$ ). The field evolves from a highly disordered configuration at high temperature to a smooth but nonuniform profile at late times. Residual spatial structure reflects conserved topological winding in one dimension.

times. In one spatial dimension, complete alignment is obstructed by conserved topological winding, preventing  $R$  from reaching unity.

*Interpretation.* These simulations demonstrate that, in a simple overdamped lattice model, the axion field nat-

urally undergoes phase ordering and develops a collective phase prior to the onset of oscillations. The misalignment angle thus emerges as a macroscopic dynamical variable rather than an arbitrary initial condition. While the overdamped approximation and lattice ultraviolet modes enhance the efficiency of early-time ordering, the qualitative result—the dynamical origin of a collective axion phase—is robust and independent of the detailed modeling of the oscillatory regime.

## IMPLICATIONS AND OUTLOOK

In this work we have proposed a dynamical reinterpretation of axion misalignment in terms of collective phase ordering. By making explicit the parallel between axion field dynamics and synchronization phenomena, we have shown that a macroscopic axion phase can emerge dynamically from initially incoherent configurations through gradient-driven ordering in an expanding background.

Within this framework, the conventional misalignment angle is not a fundamental initial datum, but an emergent collective variable. A single macroscopic angle only becomes well defined once phase coherence has developed, i.e. when the order parameter  $R(T)$  becomes nonzero. The collective phase  $\Psi(T)$  is then dynamically selected, and the usual identification of the misalignment angle with  $\Psi(T_{\text{osc}})$  follows naturally. This emphasizes that both *when* a misalignment angle exists and *which* value it takes are determined by the preceding dynamics.

This perspective also offers a new interpretation of anthropic arguments based on small initial angles. In the present language, small effective misalignment corresponds to inefficient phase ordering rather than to finely tuned initial conditions. The apparent need for anthropic selection may therefore be reinterpreted as a statement about the dynamical history of phase coherence, rather than as a requirement on fundamental initial conditions.

The synchronization viewpoint provides a unifying way to understand the standard distinction between Peccei–Quinn symmetry breaking before or after inflation. Efficient phase ordering over superhorizon scales reproduces the familiar pre-inflationary scenario with a homogeneous axion field, while limited coherence or disruption by topological defects corresponds to the post-inflationary case. In this sense, the usual taxonomy can be rephrased in terms of the efficiency, scale, and topology of phase ordering dynamics. From this perspective, axion misalignment is not an arbitrary input, but the outcome of a collective dynamical process in the early Universe.

More broadly, this framework suggests a reorganization of axion initial-condition sensitivity. Isocurvature fluctuations may be reinterpreted as fluctuations of the collective variables  $\Psi$  and  $R$ , rather than as fluctuations of a fundamental microscopic angle. From this perspec-

tive, the apparent fine tuning of axion initial conditions reflects the dynamics of collective ordering rather than special microscopic choices. This raises the possibility that some other finely adjusted parameters in nature, such as the cosmological constant or the Higgs mass, could similarly arise as emergent collective quantities whose observed values are set dynamically rather than fixed at a fundamental level.

Extending the present analysis to higher spatial dimensions will allow the role of axion strings and other topological defects to be addressed more realistically, and will clarify how collective coherence emerges in their presence. Incorporating second-order (wave-like) dynamics would further elucidate the interplay between gradient-driven ordering and axion propagation at high temperature. More generally, in this framework the axion field may support partially synchronized configurations, in which coherent and incoherent regions coexist. Such “chimera” states are well known in networks of coupled oscillators and spatially extended systems [17, 18], and their possible realization in axion cosmology could provide a novel dynamical perspective on inhomogeneous coherence, defect formation, and residual fluctuations.

In this context, chimera-like configurations may provide a useful dynamical analogue of Peccei–Quinn symmetry breaking after inflation, where coherent axion domains coexist with incoherent regions and topological defects within a single cosmological patch.

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