TU BERLIN

GEODESY AND GEOINFORMATION SCIENCE

ADJUSTMENT CALCULATION

HOMEWORK I

WINTER SEMESTER – 2016-17

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Contents

TASK 1

In this task, the job is to plot the absolute frequency/ polygon graph, relative frequency/polygon graph and cumulative frequency/polygon graph for the given set of distances.

Absolute Frequency: As from the definition of the absolute frequency, we know that it is defined by the number of the times a value repeat itself. It is denoted by $\boldsymbol{f_i}$ where i represents the number of values. The sum of the absolute frequency is always equal to the number of data. It is denoted by $\boldsymbol{N}$.

Mathematically,

$f_1 + f_2 + \ldots\ldots + f_n = N$ which can by represented as $\sum_{i=1}^{n} f_i = N$

In this case, from the given set of data the computed absolute frequency for the different data are as follow:

Distances	**Absolute Frequency**
23.4562928571429	5
23.4568785714286	3
23.4574642857143	6
23.4580500000000	20
23.4586357142857	8
23.4592214285714	5
23.4598071428571	3

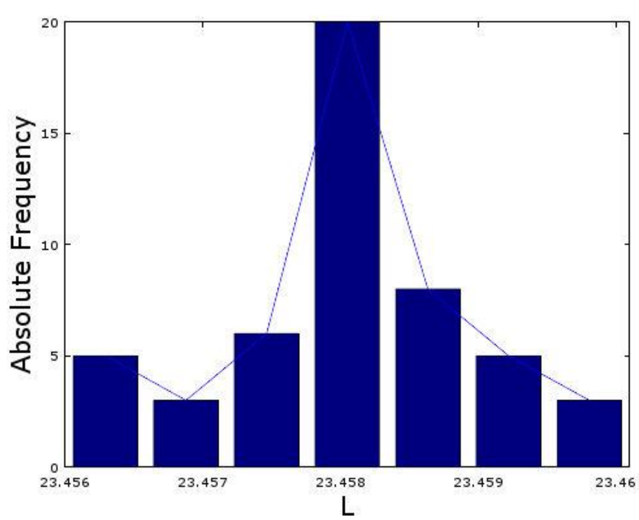

Fig.1 Absolute Frequency/Polygon Graph

Relative Frequency: From the definition of the relative frequency, it is defined as the comparison of the repeated frequency with the total number of observations. So, mathematically it can be represented as

$\boldsymbol{n_i} = \dfrac{\boldsymbol{f_i}}{\boldsymbol{N}}$ where i represents the number of values.

From the given data set, the relative frequencies are as follows:

Distances	**Absolute Frequency**	**Relative Frequency**
23.4562928571429	5	5/50=0.1
23.4568785714286	3	3/50=0.06
23.4574642857143	6	6/50=0.12
23.4580500000000	20	20/50=0.40
23.4586357142857	8	8/50=0.16
23.4592214285714	5	5/50=0.10
23.4598071428571	3	3/50=0.06

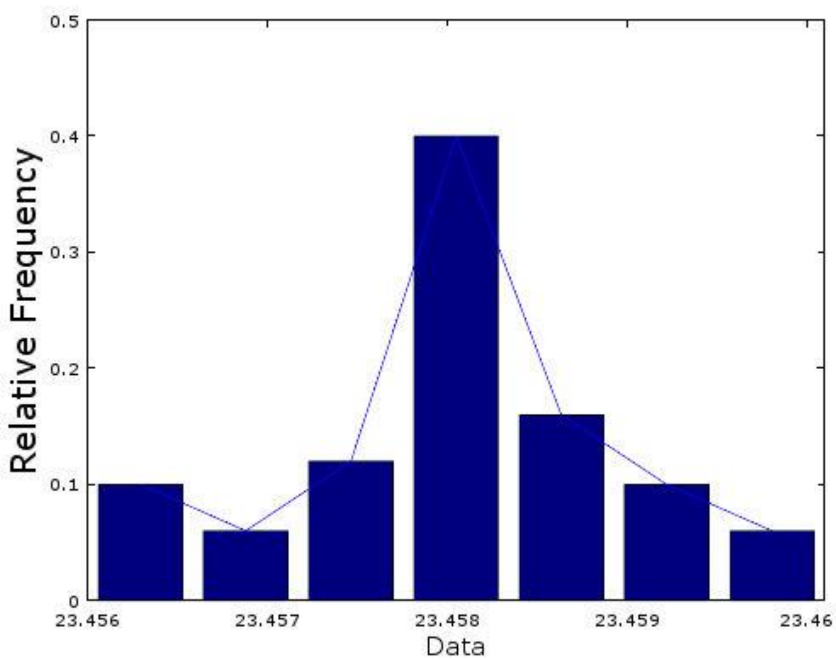

Fig.2 Relative Frequency/Polygon Graph

Cumulative Frequency: Cumulative Frequency is defined as the sum of the absolute frequencies.

So, mathematically it can be represented as

Distances	**Absolute Frequency**	**Relative Frequency**	**Cumulative Frequency**
23.4562928571429	5	5/50=0.1	5
23.4568785714286	3	3/50=	5+3=8
23.4574642857143	6	6/50=	8+6=14
23.4580500000000	20	20/50=	14+20=34
23.4586357142857	8	8/50=	34+8=42
23.4592214285714	5	5/50=	42+5=47
23.4598071428571	3	3/50=	47+3=50

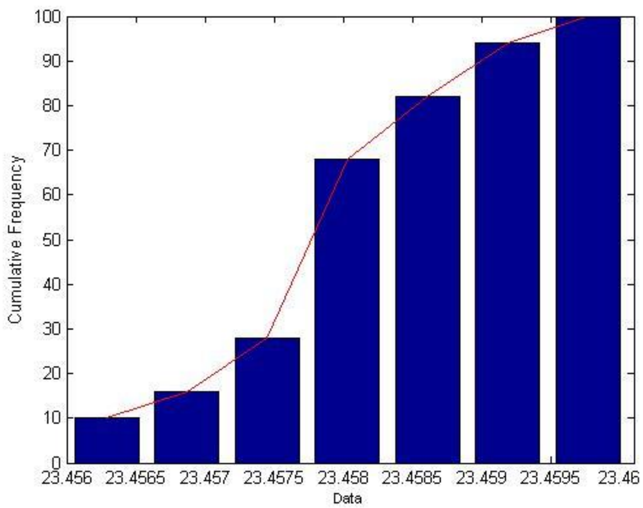

Fig.3 Cumulative Frequency/Polygon Graph

Mean Value	**23.458**
Variance	**8.9112e-007**
Standard Deviation for Single Observation	**9.4399e-004**
Standard Deviation for Arithmetic Mean	**1.3350e-004**

Question: Number of times the measurement of distance for arithmetic means s_l <=0.1mm

We need to compute it as $\frac{s_l^2}{s_l} = 89.1122$. Taking the round figure, it will be 90 times.

TASK 2

In this task, the data of measured height of the levelling network has been provided. The main objective is to find the difference between the forward levelling and the backward levelling. Simultaneously, the standard deviation for the single measurement as well as for the arithmetic mean is also required,

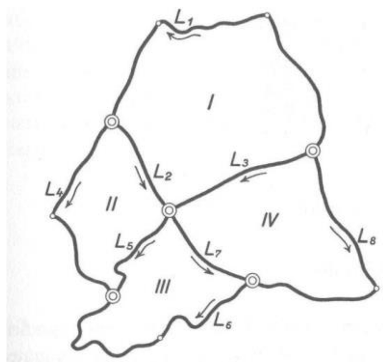

Fig 2. Sketch of a levelling network

From the given data set, the difference between the forward levelling and the backward levelling can be evaluated from the given formula:

$$\boldsymbol{d_j = l_j - l_j'} \qquad \text{for } j = 1,2,3\ldots$$

Forward Leveling (I)	**Backward Levelling(II)**	**Difference (I-II)**
-45.7352	-45.7377	0.0025
43.6127	43.6132	-0.0005
12.4046	12.4067	-0.0021
40.8439	40.8438	0.0001
3.2470	3.2491	-0.0021
-2.8467	-2.8463	-0.0004
15.1068	15.1075	0.0007
10.8990	10.8986	0.0004

Standard Deviation: Standard deviation of a single observation can be defined mathematically as

$$S_l = \sqrt{\frac{1}{2n}\overrightarrow{d_{1,n}}^{T}\ \overrightarrow{d_{n,1}}}$$

$S_l = \sqrt{\frac{d_1^2+d_2^2+\cdots d_n^2}{2n}} = \sqrt{\frac{\sum d^2}{2n}}$ Empherical Standard deviation of a single observation.

Standard deviation of an arithmetic mean from both direction is given by:

$$S_{\bar{l}} = \frac{S_l}{\sqrt{2}} = \sqrt{\frac{\sum d^2}{2n}}$$

From the given data set and the using the codes in Octave the values are:

Standard deviation of a single observation:

Standard deviation of a single observation:	**0.0011945**
Standard deviation of an arithmetic mean	**0.00071**

TASK 3

Correlation Coefficient: From the definition of the correlation coefficient we know that,

$$\sum_{ll} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

Now the provided data is given as

$$\sum_{ll} = \begin{bmatrix} 0.01 & 0.0135 \\ 0.0135 & 0.09 \end{bmatrix}$$

Comparing with the given values and the mathematical expression,

$\sigma_1^2 = 0.01, \sigma_{12} = \sigma_{21} = 0.0135, \sigma_2^2 = 0.09$

So, the correlation will become

$$\frac{\sigma_{12}}{\sigma_1 \sigma_2} = \frac{0.0135}{\sqrt{0.01}\sqrt{0.09}} = 0.45$$

where, σ_{12} = covariance (1,2) and σ_1 = standard deviation of data 1

Variance Covariance Matrix Set up for different Standard Deviation and Correlation Coefficient:

$\boldsymbol{\sigma_1 = 0.01m, \sigma_2 = 0.025m, \sigma_{12} = 0.85}$

Now in this case we need to determine the Variance Covariance matrix from the correlation coefficients. Simply the reverse of the previous method. So, from the definition

$$\sum_{ll} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

Putting the values, we get

$$\sum_{ll} = \begin{bmatrix} 0.0001 & 0.0002125 \\ 0.0002125 & 0.000625 \end{bmatrix}$$

$\boldsymbol{\sigma_1 = 1.0mgon, \sigma_2 = 0.025m, \sigma_{12} = -0.50}$

In this case, one value is given in mgon. So, converting the value to the metric system. The value becomes 0.0000157 degree. Now putting the values,

$$\Sigma_{ll} = \begin{bmatrix} 2.4\text{x}10^{-10} & -1.962 \times 10^{-7} \\ -1.962 \times 10^{-7} & 0.000625 \end{bmatrix}$$

TASK 4

The main objective of this task is to calculate the determination of distribution and probability density function as well as standard deviation from a graphical function.

The provided probability density function is as follow

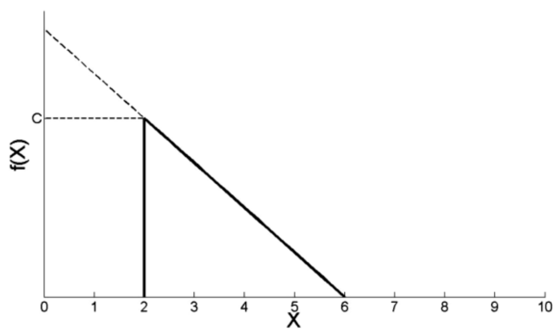

Fig 3. Probability density function

Co-efficient 'c' : As we know that the area under a probability density function is always 1. So, using that property, if we calculate the area of the triangle then,

Δ PDF = 1

or, $\frac{1}{2}\,.(6-2).c = 1$

This implies that the value of c = 0.5

Graphical Representation of the PDF: The given function can be represented as

F(x) = ax+b where a and b are constants.

So, for the value of 2 the equation becomes *F(2) = 2a+b = 0.5*

And for the value of 6 the equation becomes *F(6) = 6a+b = 0*

Computing both the equations, the values of constant are as

a = $-\frac{1}{8}$ and b = $\frac{3}{4}$

F(x) = $\int_2^6 f(x)dx = ax + b = -\frac{1}{8}x + \frac{3}{4}$

Distribution Function: From the graph is can be realized that the values less than 2 will be zero and for the values greater that 6 will be zero similarly. So that limit of the function will be

$$F(x) = \begin{cases} 0 & x < 2 \\ \dfrac{6-x}{8} & 2 \leq x \leq 6 \\ 0 & x > 6 \end{cases}$$

Expectation E(x): From the definition of expectation we know that

$E(x) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{2}^{6} \frac{6x-x^2}{8} dx = \frac{10}{3}$

Standard Deviation: From the definition of the standard deviation, we can write it as,

S = $\sqrt{var(x)}$ where var(x) = $E(x^2) - E(x)^2$

To compute the value of var(x), the value of $E(x^2)$ is also required. It can be computed similarly

$$E(x^2) = \int_{-\infty}^{+\infty} xf(x)dx = \int_{2}^{6} \frac{6x^3 - x^5}{8} dx = 12$$

So, var(x) = $12 - \frac{100}{9} = \frac{8}{9}$

From the value we get, $\sqrt{var(x)} = \sqrt{\frac{8}{9}}$ = 0.9428

Probability p(x<0) : The probability of the random variable X will be zero for the realization x<0.

Probability p(x=3) : The probability of the random variable X will be zero for the realization x=3 as the limit for the integration will be [3,3] which is mathematically equals to zero.

Probability [4,5] : To calculate this

$$\int_{4}^{5} f(x)dx = \int_{4}^{5} \frac{6-x}{8} dx = 0.1875$$

Reference

Lecture notes, Prof. Dr. Frank Neitzel, 2016-17

http://www.math.uah.edu/

mathsisfun.com

Wikipedia.com

http://www.statisticshowto.com/