TECHNICAL UNIVERSITY BERLIN

GEODESY AND GEOINFORMATION SCIENCE

ADJUSTMENT CALCULATION

HOMEWORK II

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Contents

TASK I

Objective: In this task, the job is to calculate the area of the rectangle and standard deviation from the given data set.

Given:

Data Set	**Values**
Length of side 'a'	15.00 m
Length of side 'b'	24.5 m
Standard Deviation of side 'a' $\boldsymbol{\sigma_a}$	3 cm = 0.03 m
Standard Deviation of side 'b' $\boldsymbol{\sigma_b}$	4 cm = 0.04 m
Correlation coefficient between both side $\boldsymbol{\varrho_{ab}}$	0.3

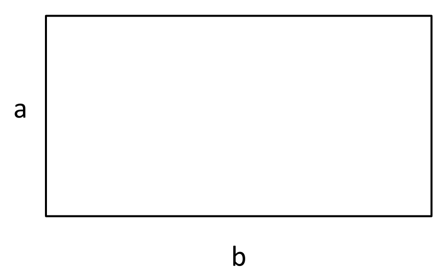


Fig. 1

Area of the rectangle A = $a \times b = 15\ m \times 24.5\ m = 367.5\ m^2$

Step I:

$$F = \begin{array}{cc} & \begin{array}{cc} a & b \end{array} \\ A & \begin{bmatrix} \frac{\partial A}{\partial a} & \frac{\partial A}{\partial b} \end{bmatrix} \end{array}$$

From the above matrix, we get the functional model. Now using the derivative, we get

$$F = [b \quad a]$$

Inserting the given values, we get

$$F = [24.5 \quad 15.00]$$

Step II: To proceed further, we need the stochastic model. So,

$$\sum_{ll} = \begin{bmatrix} \sigma_a^2 & \varrho_{ab} \\ \varrho_{ba} & \sigma_b^2 \end{bmatrix}$$

where, $\varrho_{ab} = \varrho_{ba} = \rho_{ab \times \sigma_a \times \sigma_b}$

Putting all the values from the given data set, we get

$$\sum_{ll} = \begin{bmatrix} 0.0009 & 0.0004 \\ 0.0004 & 0.0016 \end{bmatrix}$$

Step III:

Computing variance covariance matrix for the unknown with theoretical variance covariance matrix $\sum_{ll}$ is as follows

$$\sum_{xx} = F \times \sum_{ll} \times F^T$$

$$\sum_{xx} = \begin{bmatrix} 24.5 & 15.00 \end{bmatrix} \times \begin{bmatrix} 0.0009 & 0.00036 \\ 0.00036 & 0.0016 \end{bmatrix} \times \begin{bmatrix} 24.5 \\ 15.00 \end{bmatrix}$$

$$\sum_{xx} = 1.1648$$

So, the standard deviation of the area $\sigma_A = \sqrt{1.1648} = 1.0793$

Results	**Values**
Area of rectangle	$367.5\ m^2$
Standard Deviation	1.0793

TASK II

Objective: In this task, the job is to calculate the distance between electronic distance meters (EDM) and reflector as well as its standard deviation.

Given:

Data Set	**Values**
Initial time 't_0'	0.8147236863 s
Final time 't_1'	0.8147240201 s
Standard Deviation of measurement of time 'σ_t'	10^{-9} s
Velocity of light	3.00×10^{8} m/s
Remarks	c is constant and error free

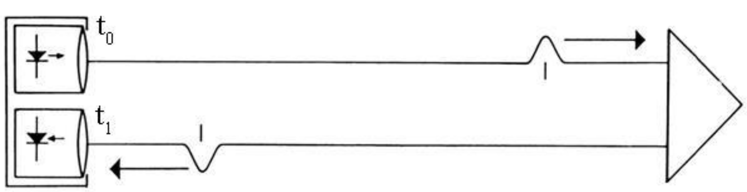


Fig. 2 EDM – impulse method

Step I:

To calculate the distance travelled by the light between the EDM and the reflector, we are using

$$\boldsymbol{s} = \boldsymbol{c}\times\boldsymbol{t} \text{ m}$$

where, s is the distance, c is the velocity of light, and t is the time

Now considering the time difference, the above equation will be

$$\boldsymbol{s} = \boldsymbol{c}\times(\boldsymbol{t_1} - \boldsymbol{t_0}) \text{ m}$$

Putting the values in the above equation,

$$\boldsymbol{s} = 3.00\times10^{8} \times(0.8147240201 - 0.8147236863) \text{ m}$$

$$= \mathbf{100.1400\ m}$$

Step II:

Now our objective is to calculate the Standard deviation of height.

$$\sigma_S = \sqrt{(\frac{\partial S}{\partial t_0})^2\sigma_{t_0}^2 + (\frac{\partial S}{\partial t_1})^2\sigma_{t_1}^2}$$

To compute the above equation, we need to know the values of $\frac{\partial S}{\partial t_0}$ and $\frac{\partial S}{\partial t_1}$.

From the equation of S, we know that

$$\boldsymbol{s = c\times(t_1 - t_0)\ m}$$

Then,

$$\frac{\partial S}{\partial t_0} = -c \ and \ \frac{\partial S}{\partial t_1} = c$$

And, we know that $\sigma_{t_0} = \sigma_{t_1} = 10^{-9}$ s

Putting the values in the equation σ_S, we get

$$\sigma_S = \sqrt{(-c)^2(10^{-9})^2 + (c)^2(10^{-9})^2}$$

$$= 0.42\ m$$

Step III:

Now to compute the accuracy of the time measurement in order to obtain the standard deviation smaller than 1 mm.

$$\sigma_S = \sqrt{(\frac{\partial S}{\partial t_0})^2\sigma_{t_0}^2 + (\frac{\partial S}{\partial t_1})^2\sigma_{t_1}^2}$$

$$\sigma_S = \sqrt{2c^2\sigma_{t_0}^2} = \sqrt{2c^2\sigma_{t_1}^2}$$

$$\sigma_{t_0} = \sigma_{t_1} = \frac{\sigma_S}{c\sqrt{2}}$$

$$\sigma_{t_0} = \sigma_{t_1} = 4.19 \times 10^{-10}\text{s}$$

Results	**Values**
Distance between EDM and reflector	$100.1400\ m$
Standard Deviation	$0.42\ m$
Time accuracy	4.19×10^{-10}s

TASK III

Objective: The objective of the task is to calculate the height of the center of the ring of fire and the standard deviation. The second objective is to find a suitable length of the net in order to save Bobo and the cost of circus organization.

Given:

Data Set	Values
Initial speed of the canon	15 m/s
Standard deviation of the speed of canon	0.1 m/s
Releasing angle	45°
Standard deviation of the releasing angle	0.08°
Gravity	9.81 m/s^2
No. of shows per year	500

Functional relations:

$$H = \frac{v_0^2 \, sin^2\sigma}{2g}$$

and

$$L = \frac{v_0^2 \sin 2\,\sigma}{g}$$

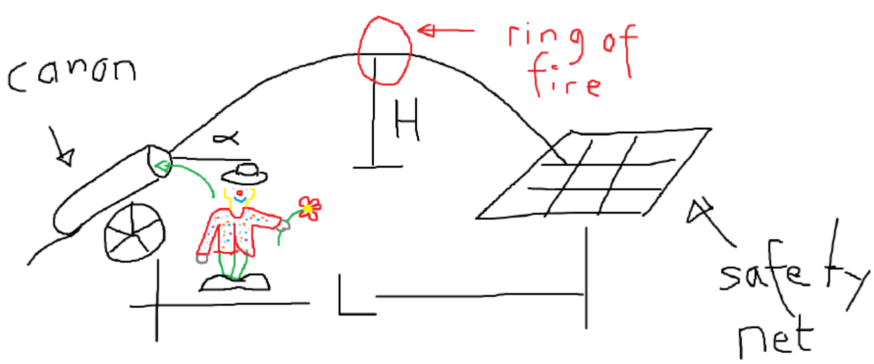


Fig. 3 Experimental Setup

Putting the values in

$$H = \frac{v_0^2 \, sin^2\sigma}{2g}$$

$$H = \frac{15^2\, sin^2 45}{2g} = 5.74\, m$$

and

$$L\ = \frac{v_0^2 \sin 2\,\sigma}{g}\ m$$

$$L\ = \frac{15^2 \sin 90}{g} = 22.9358\, m$$

Standard deviation of H =

$$\sigma_H = \sqrt{(\frac{\partial H}{\partial v_0})^2\, \sigma_{v_0}^2 + (\frac{\partial H}{\partial \alpha})^2\, \sigma_\alpha^2}$$

$$\sigma_H = \sqrt{(\frac{v_0\, sin^2\sigma}{g})^2\, \sigma_{v_0}^2 + (\frac{1}{2g} v_0^2 2\, sin\alpha\, cos\alpha)^2\, \sigma_\alpha^2}$$

Putting the values in the above equation,

$$\sigma_H = \ 0.0782\, m$$

Standard deviation of L =

$$\sigma_L = \sqrt{(\frac{\partial L}{\partial v_0})^2\, \sigma_{v_0}^2 + (\frac{\partial L}{\partial \alpha})^2\, \sigma_\alpha^2}$$

$$\sigma_L = \sqrt{(\frac{v_0\, 2sin2\alpha}{g})^2\, \sigma_{v_0}^2 + (\frac{2v_0^2}{g}\ cos2\alpha)^2\, \sigma_\alpha^2}$$

$$\sigma_L = \ 0.3061\, m$$

It is given that the number of shows performed by the circus organization is 500. So, we need to find out the standard deviation of the length of the safety net for 500 times. As the width of the net is constant, so we don't need to compute the standard deviation of the width.

$$\sigma_{L'} = \ \sqrt{2}\sigma_L$$

So, putting the value in the above equation

$$\sigma_{L'} = 6.844 \times 10^{-3} m$$

Results	**Values**
Height of the center of the ring of fire H	$5.74\ m$
Standard Deviation of H	$0.0782\ m$
Length of the center of the safety net L	$22.9358\ m$
Standard Deviation of H	$0.3061 m$

TASK IV

Objective: The objective of this task is to find the velocity of the object and its standard deviation. On the same side, we have to calculate the position of the object at a given time and its standard deviation.

Given:

Data Set	Values
Azimuth angle $\boldsymbol{\alpha_1}$	35.1550 gon
Azimuth angle $\boldsymbol{\alpha_2}$	55.1200 gon
Standard deviation of azimuth angles	0.001 gon
Time $\boldsymbol{t_1}$	9.7 s
Time $\boldsymbol{t_2}$	23.1 s
Standard deviation of time	0.1 s
Distance $\boldsymbol{S_1}$	20.005 m
Distance $\boldsymbol{S_2}$	30.001 m
Standard deviation of distance	1 mm

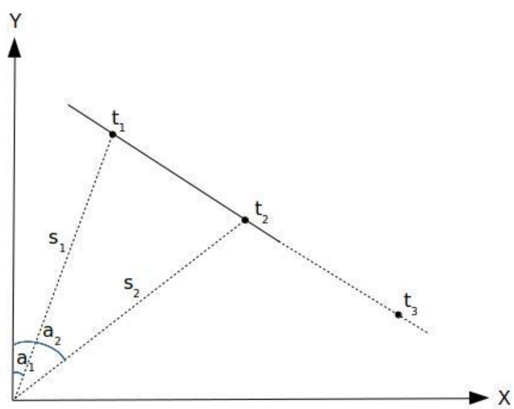


Fig. 4 Movement of a car in 2D

Part I

To compute the velocity, we need to know the distance between the two positions.

The distance between the two positions is given by

$$S = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

Now to find the value of S we need to know the coordinates (X_1, Y_1) and (X_2, Y_2)

Position of the car at $\boldsymbol{t_1}$,

$$X_1 = \boldsymbol{S_1} sin\boldsymbol{\alpha_1}$$

$$Y_1 = \boldsymbol{S_1} cos\boldsymbol{\alpha_1}$$

Similarly, position of the car at $\boldsymbol{t}_2$,

$$X_2 = \boldsymbol{S_2} sin\boldsymbol{\alpha_2}$$

$$Y_2 = \boldsymbol{S_2} cos\boldsymbol{\alpha_2}$$

Putting the values in the above equations,

(X_1, Y_1) = (10.7537, 17.4530)

(X_2, Y_2) = (22.8496, 19.4411)

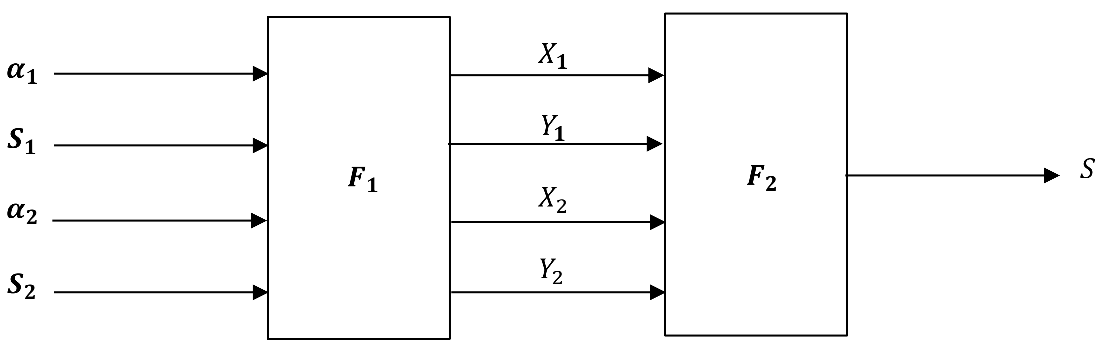
Putting the values of (X_1, Y_1) and (X_2, Y_2) in

$$S = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

$$S = \sqrt{(22.8496 - 10.7537)^2 + (19.4411 - 17.4530)^2} = 12.2582\, m$$

So, the velocity

$$v = \frac{S}{(\boldsymbol{t_2} - \boldsymbol{t_1})} = \frac{12.2582}{13.40} = 0.9148\, m/s$$



Now creating the design matrix $\boldsymbol{F_1}$

$$\begin{array}{cc} & \begin{array}{cccc} \alpha_1 & s_1 & \alpha_2 & s_2 \end{array} \\ \begin{array}{c} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{array} & \begin{bmatrix} \frac{\partial X_1}{\partial \alpha_1} & \frac{\partial X_1}{\partial s_1} & \frac{\partial X_1}{\partial \alpha_2} & \frac{\partial X_1}{\partial s_2} \\ \frac{\partial Y_1}{\partial \alpha_1} & \frac{\partial Y_1}{\partial s_1} & \frac{\partial Y_1}{\partial \alpha_2} & \frac{\partial Y_1}{\partial s_2} \\ \frac{\partial X_2}{\partial \alpha_1} & \frac{\partial X_2}{\partial s_1} & \frac{\partial X_2}{\partial \alpha_2} & \frac{\partial X_2}{\partial s_2} \\ \frac{\partial Y_2}{\partial \alpha_1} & \frac{\partial Y_2}{\partial s_1} & \frac{\partial Y_2}{\partial \alpha_2} & \frac{\partial Y_2}{\partial s_2} \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 17.0316 & 0.5246 & 0 & 0 \\ -10.4941 & 0.8514 & 0 & 0 \\ 0 & 0 & 19.4411 & 0.7616 \\ 0 & 0 & -22.8496 & 0.6480 \end{bmatrix}$$

Now creating the design matrix $\boldsymbol{F_2}$

$$\begin{array}{cc} & \begin{array}{cccc} X_1 & Y_1 & X_2 & Y_2 \end{array} \\ S & \begin{bmatrix} \frac{\partial S}{\partial X_1} & \frac{\partial S}{\partial Y_1} & \frac{\partial S}{\partial X_2} & \frac{\partial S}{\partial Y_2} \end{bmatrix} \end{array}$$

$$[0.9868 \quad -0.1622 \quad 0.9868 \quad 0.1622]$$

So,

$$\boldsymbol{F} = \boldsymbol{F_1} \times \boldsymbol{F}_2$$

Now, determining the stochastic model $\sum_{ll}$

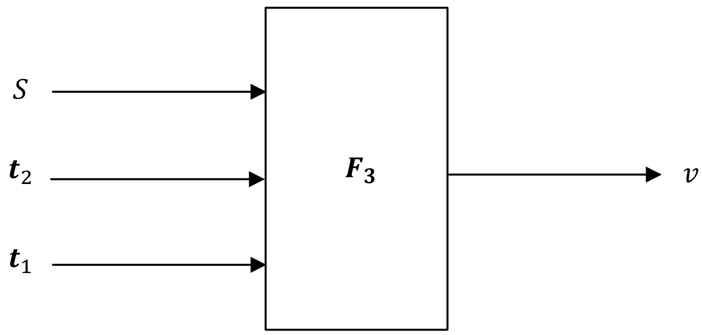
$$\begin{array}{cc} & \begin{array}{cccc} \alpha_1 & s_1 & \alpha_2 & s_2 \end{array} \\ \begin{array}{c} \alpha_1 \\ S_1 \\ \alpha_2 \\ S_2 \end{array} & \begin{bmatrix} \alpha_{\alpha_1}^2 & 0 & 0 & 0 \\ 0 & \alpha_{s_1}^2 & 0 & 0 \\ 0 & 0 & \alpha_{\alpha_2}^2 & 0 \\ 0 & 0 & 0 & \alpha_{s_2}^2 \end{bmatrix} \end{array}$$

Therefore, the variance covariance propagation matrix is

$$\sum_{xx} = F \times \sum_{ll} \times F^T$$

Standard deviation of distance = $\sqrt{0.0012} = 0.03646\, m$

From the previous section, the equation for velocity is $v = \frac{S}{(\boldsymbol{t_2} - \boldsymbol{t_1})} = \frac{12.2582}{(23.1-9.7)} = 0.9148\ m/s$



The design matrix $\boldsymbol{F_3}$ is

$$\begin{array}{cc} & \begin{array}{ccc} S & t_2 & t_1 \end{array} \\ S & \begin{bmatrix} \frac{\partial v}{\partial S} & \frac{\partial v}{\partial t_2} & \frac{\partial v}{\partial t_1} \end{bmatrix} \end{array}$$

The stochastic model is given as

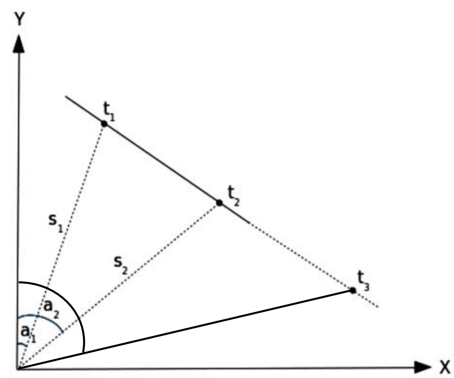
$$\begin{array}{cc} & \begin{array}{ccc} S & t_2 & t_1 \end{array} \\ \begin{array}{c} S \\ t_2 \\ t_1 \end{array} & \begin{bmatrix} 0.0346 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix} \end{array}$$

The variance covariance propagation is

$$\sum_{xx} = \boldsymbol{F_3} \times \sum_{ll} \times F_3^T$$

Therefore, the standard deviation of velocity is $\sqrt{0.001} = 0.0332\ m/s$

Part II:



From the previous result, the distance travelled by the car from $\boldsymbol{t_1}$ to $\boldsymbol{t}_2$ is 12.2582 m.

Now, the distance travelled by the car from $\boldsymbol{t_2}$ to $\boldsymbol{t}_3$ is $velocity \times time\ difference$

So, from the previous result, the velocity is 0.9148 m/s

Time difference is $30.00 - 23.1 = 6.9\ s$

Therefore, distance travelled = $6.9 \times 0.9148\ m = 6.31\ m$

To compute the coordinate, we need to know the angle between them.

So, slope of distance from $\boldsymbol{t_1}$ to $\boldsymbol{t}_2$ is

$$tan^{-1}(\frac{Y_2 - Y_1}{X_2 - X_1})$$

$$= tan^{-1}\left(\frac{19.4411 - 17.4530}{22.8496 - 10.7537}\right)$$

$$= 10.3705\ gon$$

Now, $Y_3 = 19.4411 - 6.31 \times \sin\left(10.3705 \times \frac{10}{9}\right) = 18.4177\ m$

$$X_2 = 22.8496 + 6.31 \times \cos\left(10.3705 \times \frac{10}{9}\right) = 29.0761\ m$$

To compute the standard deviation, we need to compute the functional model.

$$F_1 = \begin{bmatrix} sin\alpha_1 & 0 & s_1 cos\alpha_1 & 0 & 0 & 0 \\ cos\alpha_1 & 0 & -s_1 sin\alpha_1 & 0 & 0 & 0 \\ 0 & sin\alpha_2 & 0 & s_2 cos\alpha_2 & 0 & 0 \\ 0 & cos\alpha_2 & 0 & -s_2 sin\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} \frac{x_1 - x_2}{s} & \frac{y_1 - y_2}{s} & -\frac{x_1 - x_2}{s} & -\frac{y_1 - y_2}{s} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} \frac{-1}{t_1 - t_2} & \frac{s}{(t_1 - t_2)^2} & \frac{-s}{(t_1 - t_2)^2} \end{bmatrix}$$

Now calculating,

$$F = F_3 . F_2 . F_1$$

Putting all the values in the above matrix, we get

$$F = [-0.0506 \quad -1.0976 \quad 0.0650 \quad 1.0987 \quad 0.0701 \quad -0.0701]$$

Now the variance covariance matrix,

$$\sum_{ll} = \begin{bmatrix} \sigma_{s_1}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{s_2}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\alpha_1}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\alpha_2}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{t_1}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{t_2}^2 \end{bmatrix}$$

Now putting all the values in the given equation,

$$\sum_{xx} = F \times \sum_{ll} \times F^T$$

Putting all the values in the above equation

$$\sum_{xx} = 0.0001007$$

So, the standard deviation will be $\sqrt{0.001007} = 0.01\ m$

Results	**Values**
Velocity of the object	$0.9148\ m/s$
Standard Deviation of velocity	$0.0332\ m/s$
Position of object at time = 30 s	$29.0761m, 18.4177\ m$
Standard Deviation of current co-ordinates	$0.01m$

Reference

Lecture notes, Prof. Dr. Frank Neitzel, 2016-17

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