TECHNICAL UNIVERSITY BERLIN

GEODESY AND GEOINFORMATION SCIENCE

ADJUSTMENT CALCULATION

HOMEWORK IV

WINTER SEMESTER – 2016-17

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Contents

TASK I

Objective: In this task, our objective is to find the functional model, observation equations as well as the stochastic model. Along with that we also need to find the normal equation system and the height of the remaining points with their standard deviation. Apart from these, we have to calculate the residuals and adjusted observations. Lastly, we have to compare the adjusted unknowns, adjusted observations and residuals along with their standard deviations.

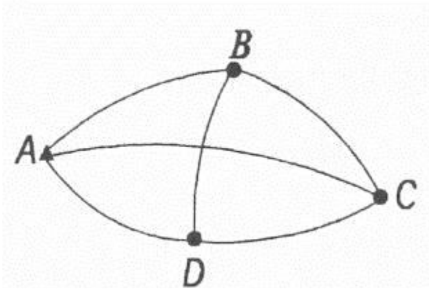

Fig. 1 Levelling Network

To proceed further, we need to define what is given and what we need to calculate for the ease of calculation.

Case A: Performing adjustments using the point A as benchmark and others as new points. So, the height of point A is

$$H_A = 378.554\,m$$

As the point is the benchmark now, so the unknows are H_B, H_C, H_D.

Functional Model: The functional model can be defined as

$$\Delta h_{AB} + H_A = H_B$$

$$\Delta h_{BC} = H_C - H_B$$

$$\Delta h_{CD} = H_D - H_C$$

$$H_A - \Delta h_{DA} = H_D$$

$$\Delta h_{BD} = H_D - H_B$$

$$\Delta h_{AC} + H_A = H_C$$

Observation Equation: The observation equation can be written as:

$$\Delta h_{AB} + H_A + v_1 = H_B$$

$$\Delta h_{BC} + v_2 = H_C - H_B$$

$$\Delta h_{CD} + v_3 = H_D - H_C$$

$$H_A - \Delta h_{DA} + v_4 = H_D$$

$$\Delta h_{BD} + v_6 = H_D - H_B$$

$$\Delta h_{AC} + H_A + v_5 = H_C$$

Residual Vector: $V = [v_1\ v_2\ v_3\ v_4\ v_5\ v_6]^T$

Variance Covariance Matrix of Observation:

$$\Sigma_{ll} = \begin{bmatrix} \sigma^2\Delta h_{AB} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2\Delta h_{BC} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2\Delta h_{CD} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2\Delta h_{DA} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2\Delta h_{BD} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2\Delta h_{AC} \end{bmatrix}$$

Co-factor matrix of the observations: $\boldsymbol{Q_{LL}} = \frac{1}{\sigma_0^2}\Sigma_{LL}$

Weight matrix of the observation: $\boldsymbol{P} = Q_{LL}^{-1}$

Vector of adjusted unknown: $\boldsymbol{\hat{X}} = [\widehat{X_1} \quad \widehat{X_2} \quad \dots \quad \widehat{X_u}]^T$

Design Matrix: $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Vectors of residuals: $\boldsymbol{v_{n,1}} = [v_1 \quad v_2 \quad \dots. \quad v_n]^T$

Observation equation: $L + v = A\hat{X}$

Normal equation: $A^T P A\hat{X} = A^T P L$

Vector of absolute values: $n = A^T P L$

Normal equations: $N\hat{X} = n$

Inversion of normal matrix: $Q_{\hat{X}\hat{X}} = N^{-1}$

Solution for the unknowns: $\hat{X} = Q_{\hat{X}\hat{X}}\, n$

Vector of residuals: $v = A\hat{X} - L$

Vector of adjusted observations: $\hat{L} = L + v$

Final Check: $\hat{L} = \varphi(\hat{X})$

Empirical reference standard dev: $s_0 = \sqrt{\frac{v^T P v}{n-u}}$

Co-factor matrix of adjusted unknown: $Q_{\hat{X}\hat{X}}$

VCM of adjusted unknown: $\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}}$

Cofactor matrix of adjusted obs: $Q_{\hat{L}\hat{L}} = AQ_{\hat{X}\hat{X}}A^T$

VCM of adjusted observation: $\sum_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}}$

Co-factor matrix of the residuals: $Q_{VV} = Q_{LL} - Q_{\hat{L}\hat{L}}$

VCM of the residuals: $\sum_{VV} = s_0^2\, Q_{VV}$

Residuals	Std. Dev of Residuals	Adjusted Observations	Std. Dev of Adjusted Observations	Adjusted Unknowns	Std. Dev of Adjusted Unknowns
0.0037	0.0032	10.5127	0.0023	389.0667	0.0023
-0.0002	0.0015	5.3598	0.0021	394.4265	0.0026
-0.0019	0.0023	-8.5249	0.0023	385.9016	0.0018
0.0004	0.0008	-7.3476	0.0018		
0.0019	0.0017	-3.1651	0.0020		
-0.0085	0.0074	15.8725	0.0026		

Table 1

Case B: Performing adjustments using the point A as benchmark and others as new points. So, the height of point A is

$H_B = 455.873\, m$

Functional Model: The functional model can be defined as

$$H_B - \Delta h_{AB} = H_A$$

$$\Delta h_{BC} + H = H_{C\,B}$$

$$\Delta h_{CD} = H_D - H_C$$

$$\Delta h_{DA} = H_A - H_D$$

$$\Delta h_{BD} + H_B = H_D$$

$$\Delta h_{AC} = H_C - H_A$$

Observation Equation: The observation equation can be written as:

$$H_B - \Delta h_{AB} + v_1 = H_A$$

$$\Delta h_{BC} + H_B + v_2 = H_C$$

$$\Delta h_{CD} + v_3 = H_D - H_C$$

$$\Delta h_{DA} + v_4 = H_A - H_D$$

$$\Delta h_{BD} + H_B + v_5 = H_D$$

$$\Delta h_{AC} + v_6 = H_C - H_A$$

Residual Vector: $V = [v_1\ v_2\ v_3\ v_4\ v_5\ v_6]^T$

Variance Covariance Matrix of Observation:

$$\Sigma_{ll} = \begin{bmatrix} \sigma^2\Delta h_{AB} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2\Delta h_{BC} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2\Delta h_{CD} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2\Delta h_{DA} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2\Delta h_{BD} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2\Delta h_{AC} \end{bmatrix}$$

Co-factor matrix of the observations: $\boldsymbol{Q_{LL}} = \frac{1}{\sigma_0^2}\Sigma_{LL}$

Weight matrix of the observation: $\boldsymbol{P} = Q_{LL}^{-1}$

Vector of adjusted unknown: $\boldsymbol{\hat{X}} = [\widehat{X_1} \quad \widehat{X_2} \quad \dots \quad \widehat{X_u}]^T$

Design Matrix: $$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

Vectors of residuals: $\boldsymbol{v_{n,1}} = [v_1 \quad v_2 \quad \dots. \quad v_n]^T$

Observation equation: $L + v = A\hat{X}$

Normal equation: $A^T PA\hat{X} = A^T PL$

Vector of absolute values: $n = A^T PL$

Normal equations: $N\hat{X} = n$

Inversion of normal matrix: $Q_{\hat{X}\hat{X}} = N^{-1}$

Solution for the unknowns: $\hat{X} = Q_{\hat{X}\hat{X}}\, n$

Vector of residuals: $v = A\hat{X} - L$

Vector of adjusted observations: $\hat{L} = L + v$

Final Check: $\hat{L} = \varphi(\hat{X})$

Empirical reference standard dev: $s_0 = \sqrt{\frac{v^T Pv}{n-u}}$

Co-factor matrix of adjusted unknown: $Q_{\hat{X}\hat{X}}$

VCM of adjusted unknown: $\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}}$

Cofactor matrix of adjusted obs: $Q_{\hat{L}\hat{L}} = AQ_{\hat{X}\hat{X}}A^T$

VCM of adjusted observation: $\sum_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}}$

Co-factor matrix of the residuals: $Q_{VV} = Q_{LL} - Q_{\hat{L}\hat{L}}$

VCM of the residuals: $\sum_{VV} = s_0^2\, Q_{VV}$

Residuals	**Std. Dev of Residuals**	**Adjusted Observations**	**Std. Dev of Adjusted Observations**	**Adjusted Unknowns**	**Std. Dev of Adjusted Unknowns**
0.0037	0.0032	10.5127	0.0023	445.3603	0.0023
-0.0002	0.0015	5.3598	0.0021	461.2328	0.0021
-0.0019	0.0023	-8.5249	0.0023	452.7079	0.0020
0.0004	0.0008	-7.3476	0.0018		
0.0019	0.0017	-3.1651	0.0020		
-0.0085	0.0074	15.8725	0.0026		

Table 2

Comparing Table 1 and Table 2, we find that the values of residuals, adjusted observations along with their standard deviations are same. So, we can conclude that the values doesn't depend on the benchmark.

TASK II

Objective: In this task, our objective is to find the adjusted coordinate of a point in a triangulation network. The control points and their 2D coordinates are given. To find out the adjusted coordinate we need to setup the stochastic model, functional model, calculating the value for the break off condition, residuals, adjusted observations along with their standard deviations. Finally, we need to compare the result with the result we got in Exercise 12.

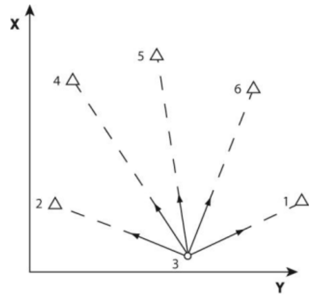

Fig. 2 Triangulation Network

Given:

Table 3: 2D coordinates of control stations

Point	Y [m]	X [m]
1	682.415	321.052
2	203.526	310.527
4	251.992	506.222
5	420.028	522.646
6	594.553	501.494

Table 4: Observed directions

Instrument station	Foresight station	Direction [gon]
3	1	206.9094
	2	46.5027
	4	84.6449
	5	115.5251
	6	155.5891

Functional Model: The functional model can be defined as

$$\alpha_{24} = r_{34} - r_{32}$$

$$\alpha_{45} = r_{35} - r_{34}$$

$$\alpha_{56} = r_{36} - r_{35}$$

$$\alpha_{61} = r_{31} - r_{36}$$

where,

$$r_{31} = \tan^{-1}\left(\frac{y_1 - y_3}{x_1 - x_3}\right) - \omega_3$$

$$r_{32} = \tan^{-1}\left(\frac{y_2 - y_3}{x_2 - x_3}\right) - \omega_3$$

$$r_{34} = \tan^{-1}\left(\frac{y_4 - y_3}{x_4 - x_3}\right) - \omega_3$$

$$r_{35} = \tan^{-1}\left(\frac{y_5 - y_3}{x_5 - x_3}\right) - \omega_3$$

$$r_{36} = \tan^{-1}\left(\frac{y_6 - y_3}{x_6 - x_3}\right) - \omega_3$$

So, the functional model can be re-write as:

$$\alpha_{24} = \tan^{-1}\left(\frac{y_4 - y_3}{x_4 - x_3}\right) - \tan^{-1}\left(\frac{y_2 - y_3}{x_2 - x_3}\right)$$

$$\alpha_{45} = \tan^{-1}\left(\frac{y_5 - y_3}{x_5 - x_3}\right) - \tan^{-1}\left(\frac{y_4 - y_3}{x_4 - x_3}\right)$$

$$\alpha_{56} = \tan^{-1}\left(\frac{y_6 - y_3}{x_6 - x_3}\right) - \tan^{-1}\left(\frac{y_5 - y_3}{x_5 - x_3}\right)$$

$$\alpha_{61} = \tan^{-1}\left(\frac{y_1 - y_3}{x_1 - x_3}\right) - \tan^{-1}\left(\frac{y_6 - y_3}{x_6 - x_3}\right)$$

Observation Equation: The observation equation can be written as

$$\alpha_{24} = \tan^{-1}\left(\frac{y_4 - y_3}{x_4 - x_3}\right) - \tan^{-1}\left(\frac{y_2 - y_3}{x_2 - x_3}\right) + v_{\alpha 24}$$

$$\alpha_{45} = \tan^{-1}\left(\frac{y_5 - y_3}{x_5 - x_3}\right) - \tan^{-1}\left(\frac{y_4 - y_3}{x_4 - x_3}\right) + v_{\alpha 45}$$

$$\alpha_{56} = \tan^{-1}\left(\frac{y_6 - y_3}{x_6 - x_3}\right) - \tan^{-1}\left(\frac{y_5 - y_3}{x_5 - x_3}\right) + v_{\alpha 56}$$

$$\alpha_{61} = \tan^{-1}\left(\frac{y_1 - y_3}{x_1 - x_3}\right) - \tan^{-1}\left(\frac{y_6 - y_3}{x_6 - x_3}\right) + v_{\alpha 61}$$

Stochastic Model:

$$\Sigma_{ll} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Co-factor matrix of the observations:	$\boldsymbol{Q_{LL}} = \frac{1}{\sigma_0^2}\Sigma_{LL}$
Weight matrix of the observation:	$\boldsymbol{P} = Q_{LL}^{-1}$
Normal equation:	$A^T PA\hat{X} = A^T PL$
Vector of absolute values:	$n = A^T PL$
Normal equations:	$N\hat{X} = n$
Inversion of normal matrix:	$Q_{\hat{X}\hat{X}} = N^{-1}$
Solution for the unknowns:	$\hat{X} = Q_{\hat{X}\hat{X}}\, n$
Vector of residuals:	$v = A\hat{X} - L$
Vector of adjusted observations:	$\hat{L} = L + v$
Final Check:	$\hat{L} = \varphi(\hat{X})$

Empirical reference standard dev: $s_0 = \sqrt{\frac{v^T P v}{n-u}}$

Co-factor matrix of adjusted unknown: $Q_{\hat{X}\hat{X}}$

VCM of adjusted unknown: $\sum_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}}$

Cofactor matrix of adjusted obs: $Q_{\hat{L}\hat{L}} = A Q_{\hat{X}\hat{X}} A^T$

VCM of adjusted observation: $\sum_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}}$

Co-factor matrix of the residuals: $Q_{VV} = Q_{LL} - Q_{\hat{L}\hat{L}}$

VCM of the residuals: $\sum_{VV} = s_0^2\, Q_{VV}$

Co-ordinates of Point 3	**Standard Deviation**
x_3 = 242.8585 m	0.0044
y_3 = 493.6969 m	0.0121

Residuals	**Standard Deviation of Residuals**	**Adjusted Observations**	**Standard Deviation of Adjusted Observations**
0.1057×10^{-4}	$0.3254 \times \mathbf{10^{-4}}$	0.5991 rad	0.1508×10^{-4} rad
0.1743×10^{-4}	$0.3266 \times \mathbf{10^{-4}}$	0.4851 rad	0.1483×10^{-4} rad
-0.4775×10^{-4}	$0.3459 \times \mathbf{10^{-4}}$	0.6293 rad	0.0949×10^{-4} rad
0.3502×10^{-4}	$0.2481 \times \mathbf{10^{-4}}$	0.8062 rad	0.2590×10^{-4} rad

In spite of having different approach and different functional model, the result in both the cases are same. So we can infer that the results will be same if we use angle or direction.

Reference

Lecture notes, Prof. Dr. Frank Neitzel, 2016-17

http://www.math.uah.edu/

mathsisfun.com

Wikipedia.com

http://www.statisticshowto.com/