Homework 2: Propagation of observation errors

- Propagation of variances and covariances -

Surname, Given Name:	Matriculation number:	Deadline:
		05.01.2017

Test Certificate

1. Received on: ____________ ____________
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Objective

This exercise deals with the propagation of variances of correlated and uncorrelated observations. Moreover, it focuses on the development of a functional relationship and the calculation of one or several unknown parameters.

Task 1:

The sides $a = 15.00\,m$ and $b = 24.5\,m$ of a rectangle are determined with a standard deviation $\sigma_a = 3\,cm$ and $\sigma_b = 4\,cm$. The correlation coefficient between both sides is $\varrho_{ab} = 0.3$.

- Calculate the area of the rectangle and its standard deviation

Task 2:

Figure 1 illustrates the principle of the impulse method for electronic distance meters (EDM). The transmitter sends an impulse which propagates to the reflector and returns. The receiver receives a small fraction of the emitted impulse at time $t_0 = 0.8147236863$ s immediately and also the impulse which has travelled along the distance s and back at time $t_1 = 0.8147240201$ s. The time stamps have been measured with a standard deviation of $\sigma_t = 10^{-9}$ s. The speed of light is a constant parameter and can be assumed to be error free.

- Calculate the distance s between EDM and reflector as well as its standard deviation.
- How accurate do you have to measure the time in order to obtain a distance with a standard deviation smaller than 1 mm.

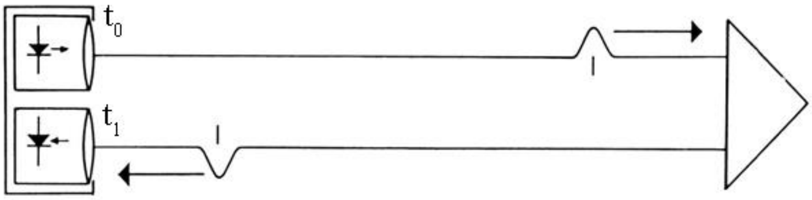

Figure 1: EDM – impulse method (www.kern-aarau.ch)

Task 3: Save Bobo's health

A circus purchased a very precise but expensive canon in order to fire Bobo the clown through a burning ring.

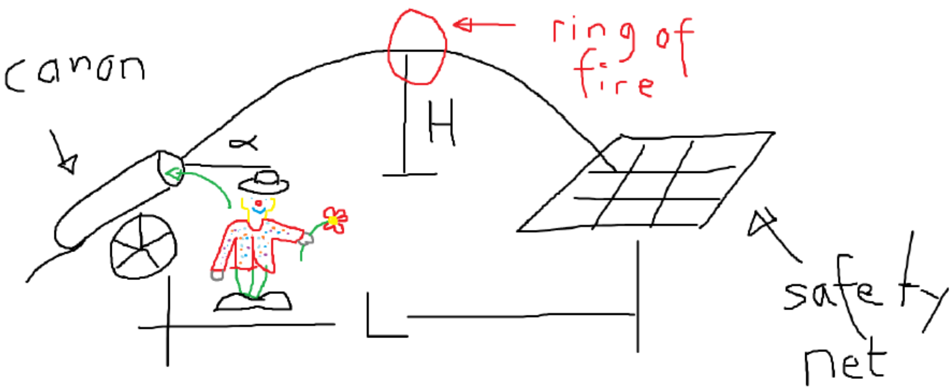

Figure 2: Experimental setup

- Calculate the height H of the center of the ring of fire and its standard deviation.
- Calculate the length L of the center of the safety net and the accompanied standard deviation.

For the propagation the following assumptions can be made:

- The initial speed of the canon is very precise and can be assumed to be 15 m/s with a standard deviation of 0.1 m/s.
- The release angle α measures 45° and its angular precision sums up to 0.08°.
- Finally the gravity shall be considered as a constant value: 9.81 m/s²

The functional relations for your tasks follow:

$$H = \frac{v_0^2 \sin^2 \alpha}{2g}$$

as well as

$$L = \frac{v_0^2 \sin 2\alpha}{g}$$

As the liquidity for safety equipment is somewhat limited, savings have to be made. The safety net is a product sold by centimetre with a constant width and each centimetre costs 100€ and needs to be replaced each year. The circus has 500 shows per year and in case Bobo will miss the safety net, he needs to go to the doctor. In such a case the average treatment expenses will amount to 150€.

While the circus director is grasping person your task is as follows:

- What is the optimal required size for the safety net that will save Bobo's health and the Circus' money?

Task 4:

A car is moving on a straight line in two dimensions (2D) with a constant velocity. The following quantities are observed in two individual positions, as depicted in Figure 3 with the accompanied standard deviations

- azimuth angles $a_1 = 35.1550$ gon and $\alpha_2 = 55.1200$ gon, with $\sigma_\alpha = 0.001$ gon
- distances $s_1 = 20.005$ m and $s_2 = 30.001$ m , with $\sigma_s = 1$ mm
- time $t_1 = 9.7$ s and $t_2 = 23.1$ s, with $\sigma_t = 0.1$ s

You tasks are:

- Estimate the velocity of the object v, as well as the standard deviation σ_v. Explain clearly all the steps needed for the results.
- Estimate the position of the object (coordinates y_3 and x_3 in 2D) at the time $t_3 = 30$ s as well as the standard deviations σ_{y_3} and σ_{x_3} .

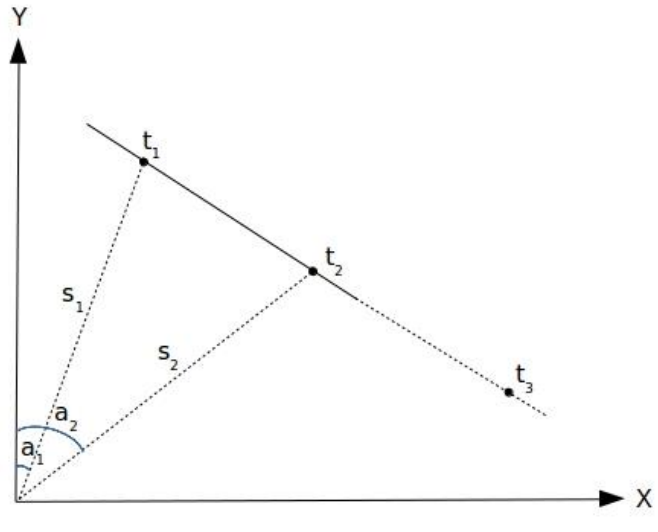

Figure 3: Movement of a car in 2D