TECHNICAL UNIVERSITY BERLIN

GEODESY AND GEOINFORMATION SCIENCE

Prof. Dr.-Ing. Frank Neitzel

SELECTED SECTIONS OF ADJUSTMENT CALUCLATION

HOMEWORK III

[Gauss-Helmert Model]

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TASK I

(Adjustment of a Straight Line)

Objective: The main objective of this task is to perform least square adjustment in order to obtain a straight line that pass through all the points which are measured on a plane with different accuracy. To obtain this, the task has been separated in to setting up the functional model and stochastic model followed by setting up normal equation system. The calculation of adjusted unknown, residuals and adjusted observations and their standard deviation respectively. In the next part of the task, determination of the variance component for both groups of observation is required. Accordingly, performing variance component estimation for both groups of observation is also required.

1.1 *Adjustment without Variance Component Estimation Method*

1.1.1 Computational Analysis

We can obtain the a solution for this adjustment problem by applying the Gauss- Markov model or the Gauss-Helmert model. In this case we apply the Gauss-Markov model which be defined with the functional model of

$$y_i = ax_i + b \qquad \text{... (1)}$$

The adjustment model of the following problem can be defined accordingly

$$y_i + v_i = ax_i + b \qquad \text{... (2)}$$

where,

y_i = co-ordinates of the point of our observation,

x_i = co-ordinates considered as fixed parameters,

a, b = unknown parameters.

Now from the adjustment model it is quite obvious that the problem is linear so there is no requirement for the iterations and no need to introduce initial values for the unknowns and the residuals.

Vector of observation can be defined as:

$$L_{202\times1} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{202} \end{bmatrix} \qquad \text{... (3)}$$

Vector of unknown can be defined accordingly

$$X_{2\times1} = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \text{... (4)}$$

Assuming the measurement has been performed with the same standard deviation and they are not correlated.

Now the variance co-variance matrix of the observation can be defined as

$$\Sigma_{LL} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}_{202\times202} \qquad \text{... (5)}$$

Theoretical standard deviation is

$$\sigma_0 = 1 \qquad \text{... (6)}$$

Co-factor matrix of Observations

$$\boldsymbol{Q}_{\boldsymbol{LL}_{202\times 202}} = \frac{1}{\sigma_0^2}\Sigma_{LL} \qquad \text{... (7)}$$

Weight Matrix can be defined by

$$\boldsymbol{P} = Q_{LL}^{-1} \qquad \text{... (8)}$$

Design Matrix can be defined as

$$A = \begin{bmatrix} \frac{\partial y_1}{\partial a} & \frac{\partial y_1}{\partial b} \\ \frac{\partial y_2}{\partial a} & \frac{\partial y_2}{\partial b} \\ \vdots & \vdots \\ \frac{\partial y_{202}}{\partial a} & \frac{\partial y_{202}}{\partial b} \end{bmatrix} \qquad \text{... (9)}$$

Normal matrix can be defined as

$$N_{4\times 4} = A^T P A \qquad \text{... (10)}$$

Vector of absolute values can be defined as

$$n_{2\times 1} = A^T P L \qquad \text{... (11)}$$

Vector of adjusted unknown can be defined as

$$\hat{\boldsymbol{X}}_{\boldsymbol{2\times 1}} = [\hat{a} \quad \hat{b}]^T \qquad \text{... (12)}$$

Vector of residuals can be defined as

$$v = A\hat{X} - L = \begin{bmatrix} v_{y_1} \\ v_{y_2} \\ \vdots \\ v_{y_{202}} \end{bmatrix} \qquad \text{... (13)}$$

Vector of adjusted observations can be defined as

$$\hat{L} = L + v \qquad \text{... (14)}$$

Empirical reference standard deviation can be defined as

$$s_0 = \sqrt{\frac{v^T P v}{r}} \qquad \text{... (15)}$$

Co-factor matrix of adjusted unknown can be defined as

$$Q_{\hat{X}\hat{X}} = N^{-1} \qquad \text{... (16)}$$

VCM of adjusted unknown can be defined as

$$\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}} \qquad \text{... (17)}$$

Standard deviation of adjusted unknown can be defined as

$$\sigma_x = \sqrt{\Sigma_{\hat{X}\hat{X}}} \quad \text{... (18)}$$

Cofactor matrix of adjusted observation can be defined as

$$Q_{\hat{L}\hat{L}} = AQ_{\hat{X}\hat{X}}A^T \quad \text{... (19)}$$

VCM of adjusted observation can be defined as

$$\Sigma_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}} \quad \text{... (20)}$$

Standard deviation of adjusted observation can be defined as

$$\sigma_{\hat{L}} = \sqrt{\Sigma_{\hat{L}\hat{L}}} \quad \text{... (21)}$$

Co-factor matrix of the residuals can be defined as

$$Q_{VV} = Q_{LL} - Q_{\hat{L}\hat{L}} \quad \text{... (22)}$$

VCM of the residuals can be defined as

$$\Sigma_{VV} = s_0^2\, Q_{VV} \quad \text{... (23)}$$

Standard deviation of residuals can be defined as

$$\sigma_v = \sqrt{\Sigma_{VV}} \quad \text{... (24)}$$

1.1.2 Evaluation of Results

The adjusted unknown and the standard deviation has been shown in the table

Unknowns	$\hat{X}$	σ_x
A	0.98915	0.01046
B	0.02808 m	0.02610 m

Table. 1 Adjusted Unknown and Standard Deviation

The maximum value of the standard deviations of adjusted observation is 0.02610 and the minimum 0.0119. So it can be said that the difference between them is smaller. On the other hand, the difference between standard deviation of the residuals is also smaller. The maximum value of the standard deviation of the residuals is 0.1685 and the minimum value of standard deviation of the residuals is 0.1669. The graph has been plotted below to show the result.

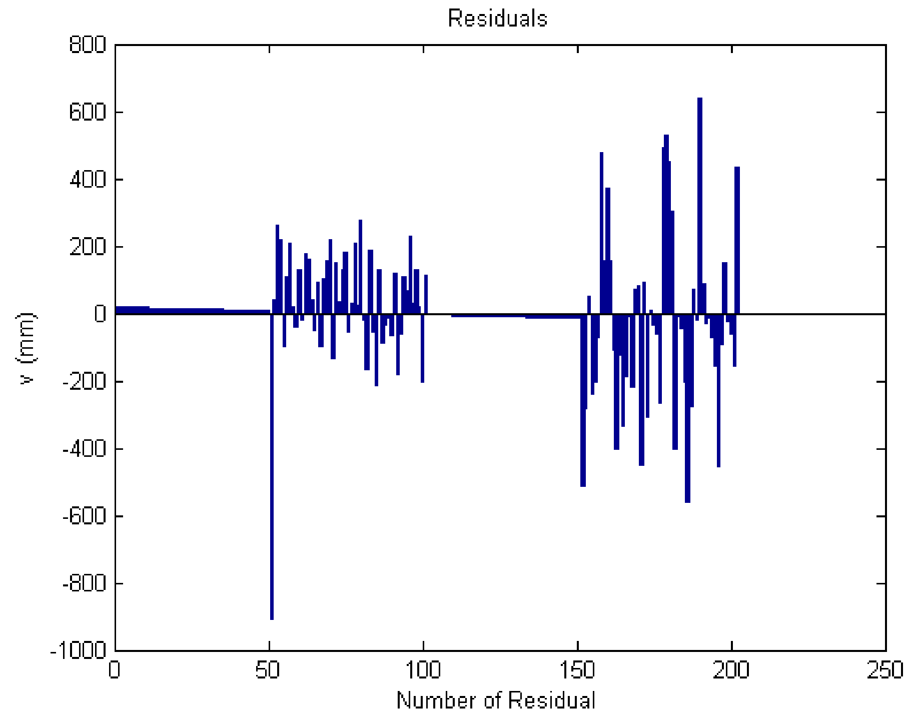

Fig. 1 Plot of residuals from the adjustment

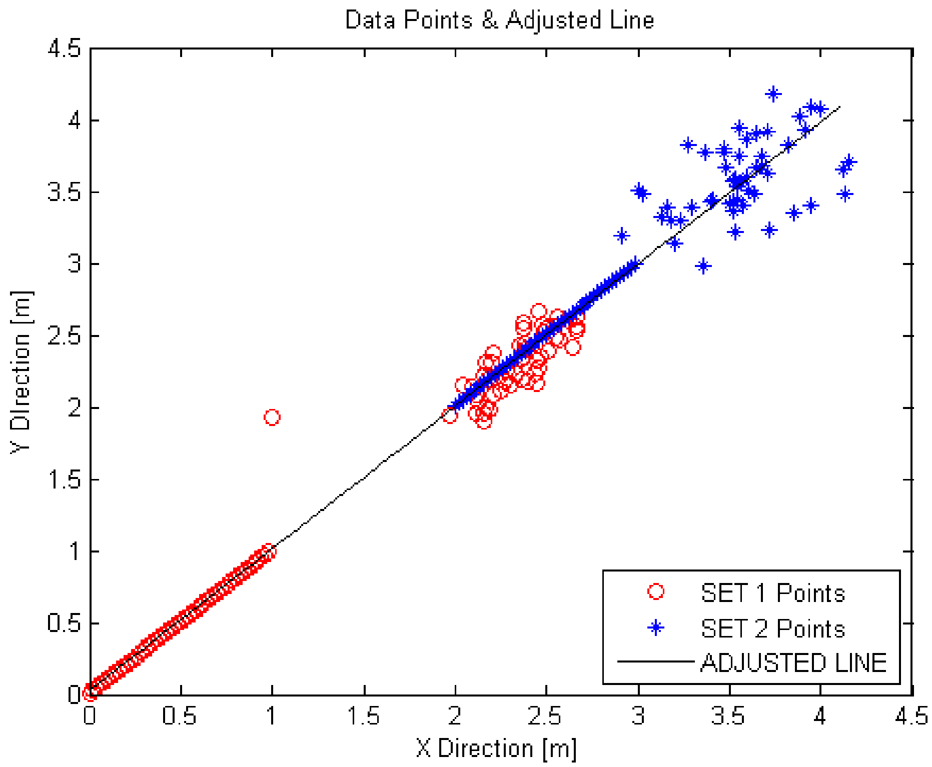

Fig. 2 Plot of data point and adjusted line

1.2 *Adjustment with Variance Component Estimation Method*

1.2.1 Computational Analysis

In this method the iteration has been introduced. The computation will end when the empirical standard deviation will be equal to 1. During the iteration the weight matrix will change accordingly.

To start the computation, the initial values for the variance computation for the first iteration are

$$\alpha_1 = 1 \quad \text{... (25)}$$

$$\alpha_2 = 1 \quad \text{... (26)}$$

The variance covariance matrix of the observation can be defined as

$$\Sigma_{\boldsymbol{LL}} = \begin{bmatrix} \alpha_1 & \mathbf{0} \\ \mathbf{0} & \alpha_2 \end{bmatrix} \quad \text{... (27)}$$

The theoretical standard deviation can be defined as

$$\sigma_0 = 1 \quad \text{... (28)}$$

The cofactor matrix of observation can be defined as

$$\boldsymbol{Q}_{\boldsymbol{LL}_{202\times202}} = \frac{1}{\sigma_0^2}\Sigma_{LL} \quad \text{... (29)}$$

The weight matrix can be defined as

$$\boldsymbol{P} = Q_{LL}^{-1} \quad \text{... (30)}$$

Design Matrix can be defined as

$$A = \begin{bmatrix} \frac{\partial y_1}{\partial a} & \frac{\partial y_1}{\partial b} \\ \frac{\partial y_2}{\partial a} & \frac{\partial y_2}{\partial b} \\ \vdots & \vdots \\ \frac{\partial y_{202}}{\partial a} & \frac{\partial y_{202}}{\partial b} \end{bmatrix} \quad \text{... (31)}$$

Normal matrix can be defined as

$$N_{4\times4} = A^T P A \quad \text{... (32)}$$

Vector of absolute values can be defined as

$$n_{2\times1} = A^T P L \quad \text{... (33)}$$

Vector of adjusted unknown can be defined as

$$\widehat{\boldsymbol{X}}_{\boldsymbol{2\times1}} = [\hat{a} \quad \hat{b}]^T \quad \text{... (34)}$$

Vector of residuals can be defined as

$$v = A\hat{X} - L = \begin{bmatrix} v_{y_1} \\ v_{y_2} \\ \vdots \\ v_{y_{202}} \end{bmatrix} \quad \text{... (35)}$$

Vector of adjusted observations can be defined as

$$\hat{L} = L + v \quad \ldots (36)$$

Empirical reference standard deviation can be defined as

$$s_0 = \sqrt{\frac{v^T P v}{r}} \quad \ldots (37)$$

Co-factor matrix of adjusted unknown can be defined as

$$Q_{\hat{X}\hat{X}} = N^{-1} \quad \ldots (38)$$

VCM of adjusted unknown can be defined as

$$\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}} \quad \ldots (39)$$

Standard deviation of adjusted unknown can be defined as

$$\sigma_x = \sqrt{\Sigma_{\hat{X}\hat{X}}} \quad \ldots (40)$$

Cofactor matrix of adjusted observation can be defined as

$$Q_{\hat{L}\hat{L}} = A Q_{\hat{X}\hat{X}} A^T \quad \ldots (41)$$

VCM of adjusted observation can be defined as

$$\Sigma_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}} \quad \ldots (42)$$

Standard deviation of adjusted observation can be defined as

$$\sigma_{\hat{L}} = \sqrt{\Sigma_{\hat{L}\hat{L}}} \quad \ldots (43)$$

Co-factor matrix of the residuals can be defined as

$$Q_{VV} = Q_{LL} - Q_{\hat{L}\hat{L}} \quad \ldots (44)$$

VCM of the residuals can be defined as

$$\Sigma_{VV} = s_0^2\, Q_{VV} \quad \ldots (45)$$

Standard deviation of residuals can be defined as

$$\sigma_v = \sqrt{\Sigma_{VV}} \quad \ldots (46)$$

Now to compute the redundancy for each set of measurement

$$r = \sum_{i=1}^{202} diag(Q_{VV} P)_i \quad \ldots (47)$$

where the computation for r_1 starts from 1 to 101 and for r_2 starts from 102 to 202.

The vector of residuals can be defined as

$$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad \ldots (48)$$

The weight matrix can be defined accordingly

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \quad \ldots (49)$$

Now, as there are two set of observation it is quite probable that they are not measured with same accuracy. The variance component of each group of observation has been calculated in order to obtain the closeness of the empirical standard deviation of each group of observation to the theoretical one.

Empirical reference standard deviation can be defined as

$$s_0 = \sqrt{\frac{v^T P v}{r}} \quad \text{... (50)}$$

For the first set it can be defined as

$$s_{0_1} = \sqrt{\frac{{v_1}^T P_1 v_1}{r_1}} \quad \text{... (51)}$$

where,

v_1 = sub-vector of the residuals v for the set 1,

P_1 = sub-matrix of P for the set 1,

r_1 = sub-matrix of $Q_{VV}P$.

The new parameter for the variance component can be defined as

$$\alpha_i^n = (s_0)^2 \alpha_i^{n-1} \quad \text{... (52)}$$

By applying a variance component estimation, the stochastic model can be improved under the consideration of s_{0_1} and s_{0_2} until it reached $\sigma_{0=1}$. For each iteration the weight matrix changes continuously until both the group of observations have the same standard deviation. After the end of the iteration the two set of observation has the different weight matrix. The weight of these matrices depend on the reliability of each set of observations. The stochastic model has been changed accordingly and every set of observation has the weight matrix which is proportional to its reliability.

1.2.2 Evaluation of Results

The adjusted unknown and the standard deviation has been shown in the table

Unknowns	$\hat{X}$	σ_x
A	0.98588	0.00957
B	0.02541m	0.02118 m

Table. 2 Adjusted Unknown and Standard Deviation

Now comparing the results from Table 1 and Table 2 it is evident that the values calculated in Table 1 are higher than that from Table 2. So it can be said that the results in Table 2 is more reliable than that of 1. The standard deviation of the adjusted unknown are much lower than the values in Table 1. Now, the standard deviation of the residuals having maximum value of 0.2 and minimum value of 0.12. Apart from that, standard deviation of adjusted observation having maximum value of 0.02 and minimum value of 0.01. The plot for the above computation has been shown below.

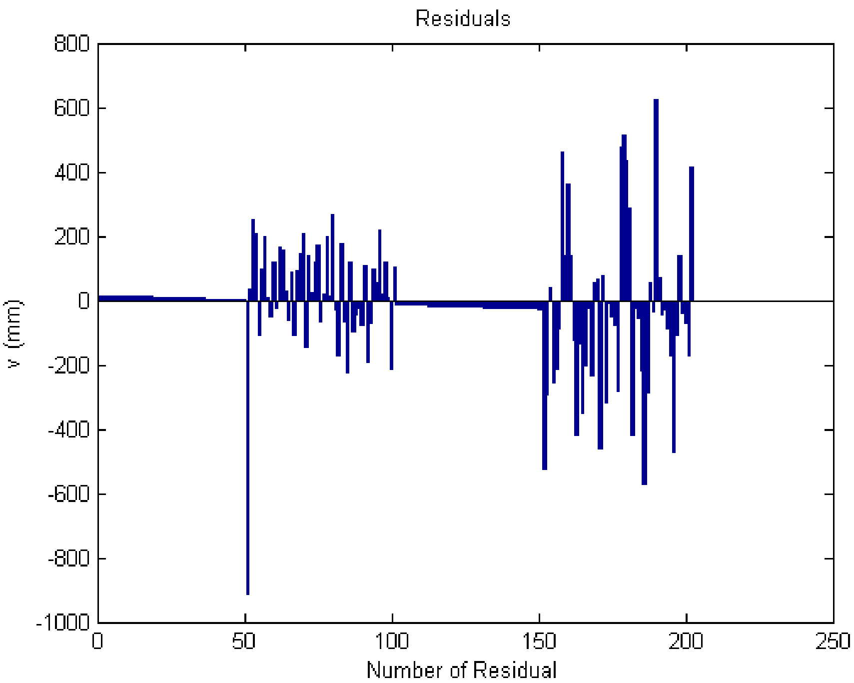

Fig. 3 Plot of the residuals from the adjustment

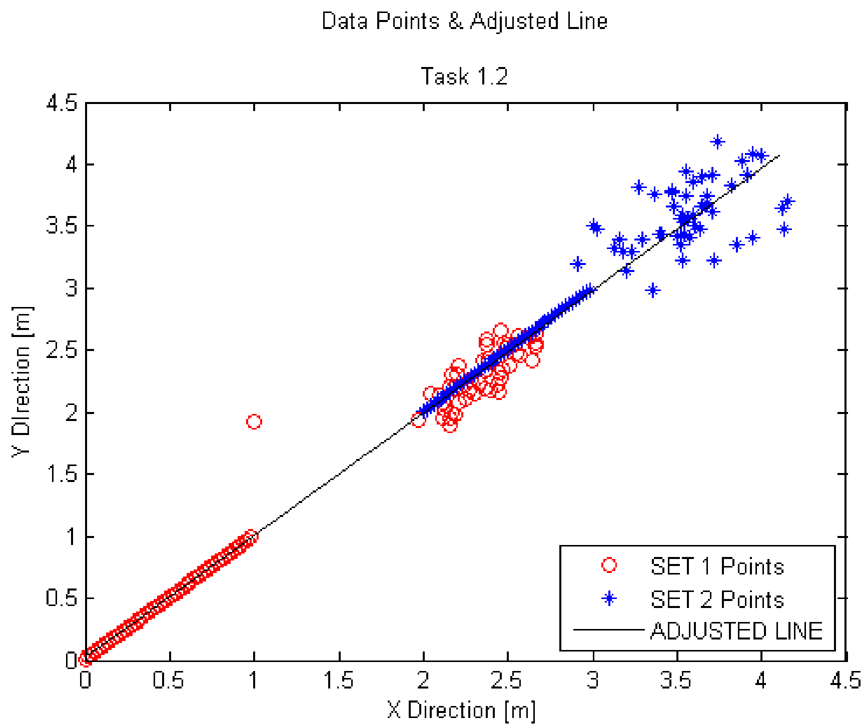

Fig. 4 Plot of the Data point and Adjusted Line

Comparing both the results, it can be said that the results in the later case is more reliable than that of the previous one. The results in the Table 1 are affected from false assumption that the observations are equally weighted and the results of the standard deviation are higher than that of Table 2.

TASK II

(Adjustment of a Plane)

Objective: The main objective of this task is to perform least square adjustment in order to fit a plane that passes through all 3D points and detect blunders using L1 adjustment with an iterative weighted L2 adjustment. Secondly, the outliers have to be removed from the observations and determination of the parameters of the plane using least square adjustment.

2.1 *Adjustment using L1-adjustment*

2.1.1 *Computational Analysis*

The 3D plane fitting problem can be solved using Gauss-Helmert Model with an additional constraint. The functional model can be defined by

$$d = x_i n_x + y_i n_y + z_i n_z \quad \text{... (53)}$$

where,

d = distance between the plane and the origin of the coordinate system that has to be estimated,

n_x, n_y, n_z = parameters of the plane that has to be estimated,

x_i, y_i, z_i = observed coordinates.

The additional constraint can be defined as

$$n_x^2 + n_y^2 + n_z^2 = 1 \quad \text{... (54)}$$

The adjustment model can be defined as

$$\Psi_i(v, X) = (x_i + v_{x_i})n_x + (y_i + v_{y_i})n_y + (z_i + v_{z_i})n_z - d \quad \text{... (55)}$$

$$\gamma(X) = n_x^2 + n_y^2 + n_z^2 - 1 \quad \text{... (54)}$$

which is the special case of Gauss-Helmert model with a condition $\Psi_i(v, X) = c_1$ and $\gamma(X) = c_2$. Iterative solution of a linearized equation system has been obtained. Initial values for the unknowns and the residuals need to be introduced. For the initial values of residuals zero has been chosen and one for the unknowns.

Now, vector of observations can be defined as

$$L_{30\times1} = \begin{bmatrix} x_1 \\ \vdots \\ x_{10} \\ y_1 \\ \vdots \\ y_{10} \\ z_1 \\ \vdots \\ z_{10} \end{bmatrix} \quad \text{... (55)}$$

Vector of unknown can be defined accordingly

$$X_{4\times1} = \begin{bmatrix} n_x \\ n_y \\ n_z \\ d \end{bmatrix} \quad \text{... (56)}$$

Vector of residuals can be defined as

$$v_{30\times1} = \begin{bmatrix} v_{x_1} \\ \vdots \\ v_{x_{10}} \\ v_{y_1} \\ \vdots \\ v_{y_{10}} \\ v_{z_1} \\ \vdots \\ v_{z_{10}} \end{bmatrix} \qquad \ldots (57)$$

As it is stated that the measurement were obtained with the same standard deviation and are uncorrelated, then weight matrix will be considered as identity matrix in the beginning. The given problem has been divided in to two section. In the first section, stochastic model will be updated and secondly adjustment will be performed in order to update the weight matrix.

So, initially the weight matrix can be defined as

$$P_0 = I_{30\times30} \qquad \ldots (58)$$

Now, the weight matrix given in the equation 58 is used for first iteration. Now for the next iterations the weight matrix can be defined as

$$P = \begin{bmatrix} \frac{1}{|v_1| + c} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \frac{1}{|v_{30}| + c} \end{bmatrix} \qquad \ldots (59)$$

where,

v_i = residual of the observation from the previous iteration

$c = 10^{-5}$

The cofactor matrix of the observation can be defined as

Co-factor matrix of Observations

$$\boldsymbol{Q_{LL}} = \boldsymbol{P^{-1}} \qquad \ldots (60)$$

Vector of condition equation can be defined as

$$\Psi_{10\times1} = \begin{bmatrix} (x_1 + v_{x_1})n_x + (y_1 + v_{y_1})n_y + (z_1 + v_{z_1})n_z - d \\ (x_2 + v_{x_2})n_x + (y_2 + v_{y_2})n_y + (z_2 + v_{z_2})n_z - d \\ \vdots \\ (x_{10} + v_{x_{10}})n_x + (y_{10} + v_{y_{10}})n_y + (z_{10} + v_{z_{10}})n_z - d \end{bmatrix} \qquad \ldots (61)$$

The design matrix has been split in to 3 design matrices which can be defined as

$$A = \begin{bmatrix} \frac{\partial \Psi_1}{\partial n_x} & \frac{\partial \Psi_1}{\partial n_y} & \frac{\partial \Psi_1}{\partial n_z} & \frac{\partial \Psi_1}{\partial d} \\ \frac{\partial \Psi_2}{\partial n_x} & \frac{\partial \Psi_2}{\partial n_y} & \frac{\partial \Psi_2}{\partial n_z} & \frac{\partial \Psi_2}{\partial d} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \Psi_{10}}{\partial n_x} & \frac{\partial \Psi_{10}}{\partial n_y} & \frac{\partial \Psi_{10}}{\partial n_z} & \frac{\partial \Psi_{10}}{\partial d} \end{bmatrix} \quad \dots (62)$$

$$B = \begin{bmatrix} \frac{\partial \Psi_1}{\partial v_{x_1}} & \frac{\partial \Psi_1}{\partial v_{x_2}} & \cdots & \frac{\partial \Psi_1}{\partial v_{z_{10}}} \\ \frac{\partial \Psi_2}{\partial v_{x_1}} & \frac{\partial \Psi_2}{\partial v_{x_2}} & \cdots & \frac{\partial \Psi_2}{\partial v_{z_{10}}} \\ \vdots & \vdots & \cdots & \vdots \\ \frac{\partial \Psi_{10}}{\partial v_{x_1}} & \frac{\partial \Psi_{10}}{\partial v_{x_2}} & \cdots & \frac{\partial \Psi_{10}}{\partial v_{z_{10}}} \end{bmatrix} \quad \dots (63)$$

$$C = \begin{bmatrix} \frac{d\gamma}{\partial n_x} & \frac{d\gamma}{\partial n_y} & \frac{d\gamma}{\partial n_z} & \frac{d\gamma}{\partial d} \end{bmatrix} \quad \dots (64)$$

Vector of constraint can be defined as

$$c_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{10\times 1} \quad \dots (65)$$

$$c_2 = [1] \quad \dots (66)$$

Vector of misclosures can be defined as

$$w_1 = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{10} \end{bmatrix}_{10\times 1} \quad \dots (67)$$

$$w_2 = [0] \quad \dots (68)$$

Normal Matrix can be defined as

$$N = \begin{bmatrix} -A^T(BQ_{LL}B^T)A & C^T \\ C & 0 \end{bmatrix} \quad \dots (69)$$

The vector n can be defined as

$$n = \begin{bmatrix} A^T(BQ_{LL}B^T)^{-1} \\ w_1 \\ w_2 \end{bmatrix} \quad \dots (70)$$

Solution of the normal equation can be defined as

$$\hat{x} = N^{-1}n = \begin{bmatrix} d\hat{n}_x \\ d\hat{n}_y \\ d\hat{n}_z \\ d\hat{d} \\ k_c \end{bmatrix} \quad \dots (71)$$

where,

k_c = Lagrange multiplier of the constraint

Lagrange multiplier of the observations can be defined as

$$k = (BQ_{LL}B^T)^{-1}\left(A\begin{bmatrix} d\hat{n}_x \\ d\hat{n}_y \\ d\hat{n}_z \\ d\hat{d} \end{bmatrix} + w_1\right) \quad \text{... (72)}$$

The estimated residuals can be defined as

$$v = Q_{LL}B^T k \quad \text{... (73)}$$

The estimation of the unknown parameters is given by the equation

$$\hat{X} = X^0 + \begin{bmatrix} d\hat{n}_x \\ d\hat{n}_y \\ d\hat{n}_z \\ d\hat{d} \end{bmatrix} \quad \text{... (74)}$$

After two different iterations the following matrices can be calculated

$$\hat{L} = \begin{bmatrix} x_1 + v_{x_1} \\ x_2 + v_{x_2} \\ \vdots \\ z_{10} + v_{z_{10}} \end{bmatrix}_{30\times 1} \quad \text{... (75)}$$

Empirical Reference Standard deviation can be defined as

$$s_0 = \sqrt{\frac{v^T P v}{r}} \quad \text{... (76)}$$

Cofactor matrix of adjusted unknown can be defined as

$$Q_{\hat{X}\hat{X}} = -N^{-1} \quad \text{... (77)}$$

VCM of adjusted unknown can be defined as

$$\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}} \quad \text{... (78)}$$

Standard deviation of adjusted unknown can be defined as

$$\sigma_x = \sqrt{\Sigma_{\hat{X}\hat{X}}} \quad \text{... (79)}$$

Co-factor matrix of the residuals can be defined as

$$Q_{VV} = Q_{LL}(BQ_{LL}B^T)^{-1}BQ_{LL} \quad \text{... (80)}$$

VCM of the residuals can be defined as

$$\Sigma_{VV} = s_0^2\, Q_{VV} \quad \text{... (81)}$$

Standard deviation of residuals can be defined as

$$\sigma_v = \sqrt{\Sigma_{VV}} \quad \text{... (82)}$$

Cofactor matrix of adjusted observation can be defined as

$$Q_{\hat{L}\hat{L}} = Q_{LL} - Q_{VV} \quad \dots (83)$$

VCM of adjusted observation can be defined as

$$\Sigma_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}} \quad \dots (84)$$

Standard deviation of adjusted observation can be defined as

$$\sigma_{\hat{L}} = \sqrt{\Sigma_{\hat{L}\hat{L}}} \quad \dots (85)$$

2.1.2 *Results*

The values for the adjusted unknowns are shown in the table.

Unknowns	X	$\hat{X}$	σ_x
n_x	1	0.2669	0.000828
$\hat{n}_y$	1	0.5354	0.000249
$\hat{n}_z$	1	0.8013	0.000309
d	1	25.0442 [m]	0.021019 [m]

Table. 3 Adjusted Unknown and Standard Deviation

Now, it is clear from the above result that the standard deviation for the unknown d is much higher than the one of the parameters of the normal vector.

The results for the adjusted observation, residuals and their standard deviation has been shown below.

	L (m)	$\hat{L}$ (m)	$\sigma_{\hat{L}}$ (mm)	v (m)	σ_v (mm)
x_1	17.995	17.995	0.00241643	0.00000498	0.00004840
x_2	14.995	14.995	0.001980576	0.00000073	0.00051299
x_3	13.991	13.991	0.002417903	0.00000499	0.00002390
x_4	17.01	17.01	0.00201407	-0.00000105	0.00050471
x_5	15.994	15.994	0.002416907	-0.00000498	0.00004207
x_6	17.996	17.996	0.002417219	0.00000498	0.00003735
x_7	9.976	9.976	0.002355174	-0.00000439	0.00025978
x_8	19.004	19.004	0.002415025	-0.00000496	0.00006342
x_9	14.022	14.022	0.002418373	-0.00000499	0.00000190
x_{10}	9.983	9.983	0.002415379	0.00000497	0.00005999
y_1	56.994	56.994	0.003414496	0.00001999	0.00019441
y_2	58.988	58.988	0.001812218	0.00000158	0.00111082
y_3	50.993	50.993	0.003424977	0.00002010	0.00009626
y_4	50.018	50.018	0.001881305	-0.00000236	0.00113242
y_5	57.003	57.003	0.003417885	-0.00002002	0.00016913
y_6	52.996	52.996	0.003420102	0.00002004	0.00015025

y_7	53.011	53.011	0.003031355	-0.00001579	0.00093397
y_8	50.008	50.008	0.003404562	-0.00001988	0.00025407
y_9	57.997	57.997	0.003428336	-0.00002013	0.00000766
y_{10}	54.979	54.979	0.003407062	0.00001991	0.00024051
z_1	-12.823	-12.823	0.002418219	0.00411718	0.04005082
z_2	-13.152	-13.152	0.001283543	0.00000257	0.00180445
z_3	-7.493	-7.493	0.002425641	0.01699503	0.08140875
z_4	-7.83	-7.83	0.001332469	-0.00000400	0.00192009
z_5	-12.153	-12.153	0.002420619	-0.00546107	0.04613328
z_6	-10.155	-10.155	0.002422189	0.00693727	0.05200097
z_7	-7.487	-7.487	0.002146893	-0.00010970	0.00648700
z_8	-8.485	-8.485	0.002411184	-0.00238333	0.03045929
z_9	-9.474	-9.474	0.00242802	-2.69182785	1.02470377
z_{10}	-8.807	-8.807	0.002412954	0.00266729	0.03222620

Table. 4 Adjusted Unknown, Observation, Residuals and Standard Deviation

Now it is evident from the table that the value of the standard deviation of the residuals of the z coordinate is much higher than that of x and y coordinates. To detect the blunder, a computational graph has been shown below.

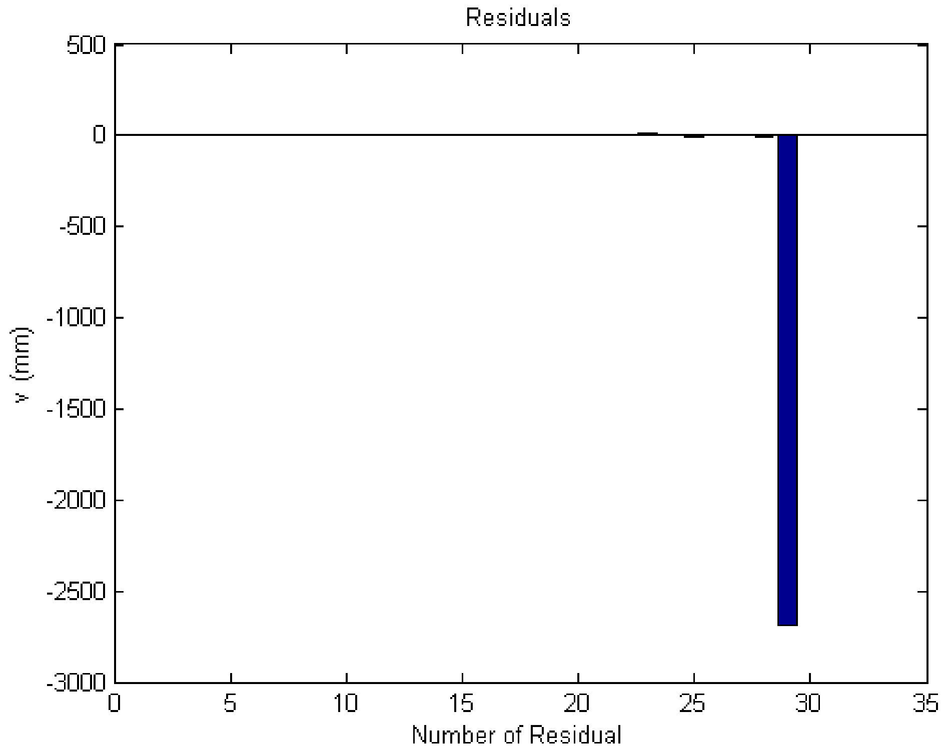

Fig. 5 Plot of the residuals

So, it is evident from the graph and Table that the blunder is z_9.

2.2 Adjustment using L2 adjustment after removing blunders

2.2.1 Computation Analysis

The first part of this task is to remove the blunder and perform L2 adjustment. The functional model in this task will remain same. The weight matrix reduced from 30 to 27.

2.2.2 Results

The values for the adjusted unknowns are shown in the table.

Unknowns	X	$\hat{X}$	σ_x
n_x	1	0.2667	0.000563
$\hat{n}_y$	1	0.5351	0.000442
$\hat{n}_z$	1	0.8016	0.000313
d	1	25.0215 [m]	0.026509 [m]

Table. 5 Adjusted Unknown and Standard Deviation

The results for the adjusted observation, residuals and their standard deviation has been shown below.

	L (m)	$\hat{L}$ (m)	$\sigma_{\hat{L}}$ (mm)	v (m)	σ_v (mm)
x_1	17.995	17.995	0.005536094	0.00088329	0.00153224
x_2	14.995	14.995	0.005536094	0.00008031	0.00153224
x_3	13.991	13.991	0.005536094	0.00266811	0.00153224
x_4	17.01	17.01	0.005536094	-0.00092240	0.00153224
x_5	15.994	15.994	0.005536094	-0.00128190	0.00153224
x_6	17.996	17.996	0.005536094	0.00099013	0.00153224
x_7	9.976	9.976	0.005536094	-0.00096903	0.00153224
x_8	19.004	19.004	0.005536094	-0.00132366	0.00153224
x_{10}	9.983	9.983	0.005536094	-0.00012485	0.00153224
y_1	56.994	56.994	0.004852701	0.00177187	0.00307366
y_2	58.988	58.988	0.004852701	0.00016110	0.00307366
y_3	50.993	50.993	0.004852701	0.00535219	0.00307366
y_4	50.018	50.018	0.004852701	-0.00185033	0.00307366
y_5	57.003	57.003	0.004852701	-0.00257148	0.00307366
y_6	52.996	52.996	0.004852701	0.00198620	0.00307366
y_7	53.011	53.011	0.004852701	-0.00194386	0.00307366
y_8	50.008	50.008	0.004852701	-0.00265526	0.00307366
y_{10}	54.979	54.979	0.004852701	-0.00025044	0.00307366
z_1	-12.823	-12.823	0.003434409	0.00265432	0.00460445
z_2	-13.152	-13.152	0.003434409	0.00024134	0.00460445
z_3	-7.493	-7.493	0.003434409	0.00801776	0.00460445
z_4	-7.83	-7.83	0.003434409	-0.00277186	0.00460445
z_5	-12.153	-12.153	0.003434409	-0.00385217	0.00460445

z_6	-10.155	-10.155	0.003434409	0.00297539	0.00460445
z_7	-7.487	-7.487	0.003434409	-0.00291196	0.00460445
z_8	-8.485	-8.485	0.003434409	-0.00397766	0.00460445
z_{10}	-8.807	-8.807	0.003434409	-0.00037516	0.00460445

Table. 6 Adjusted Unknown, Observation, Residuals and Standard Deviation

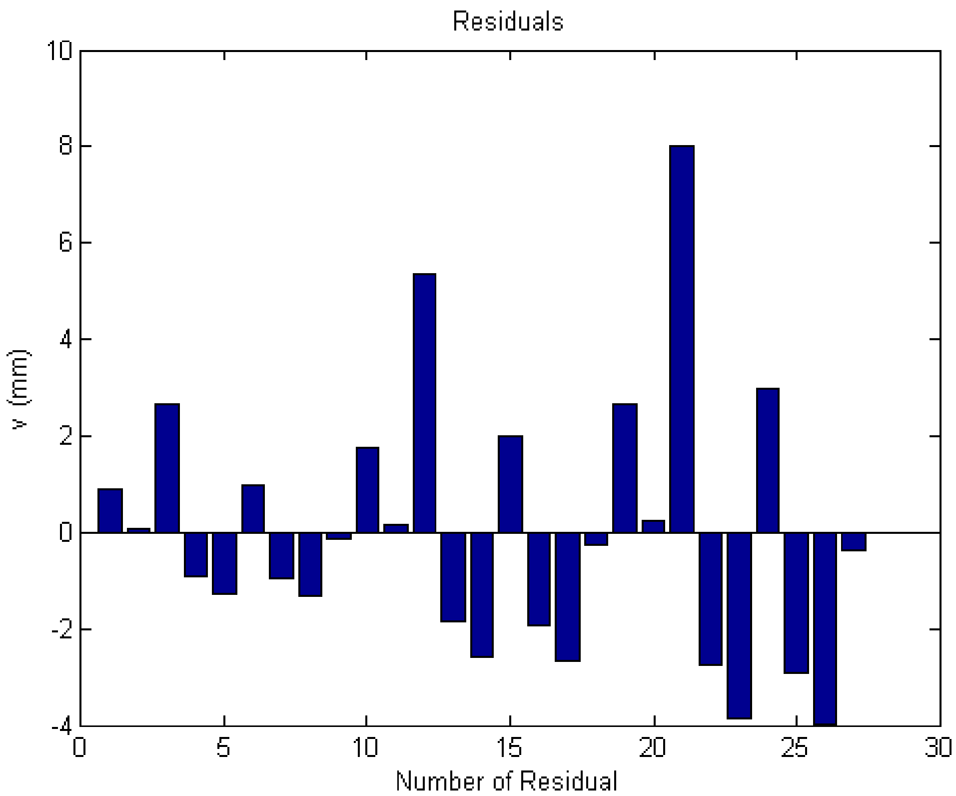

Fig. 6 Plot of the Residuals

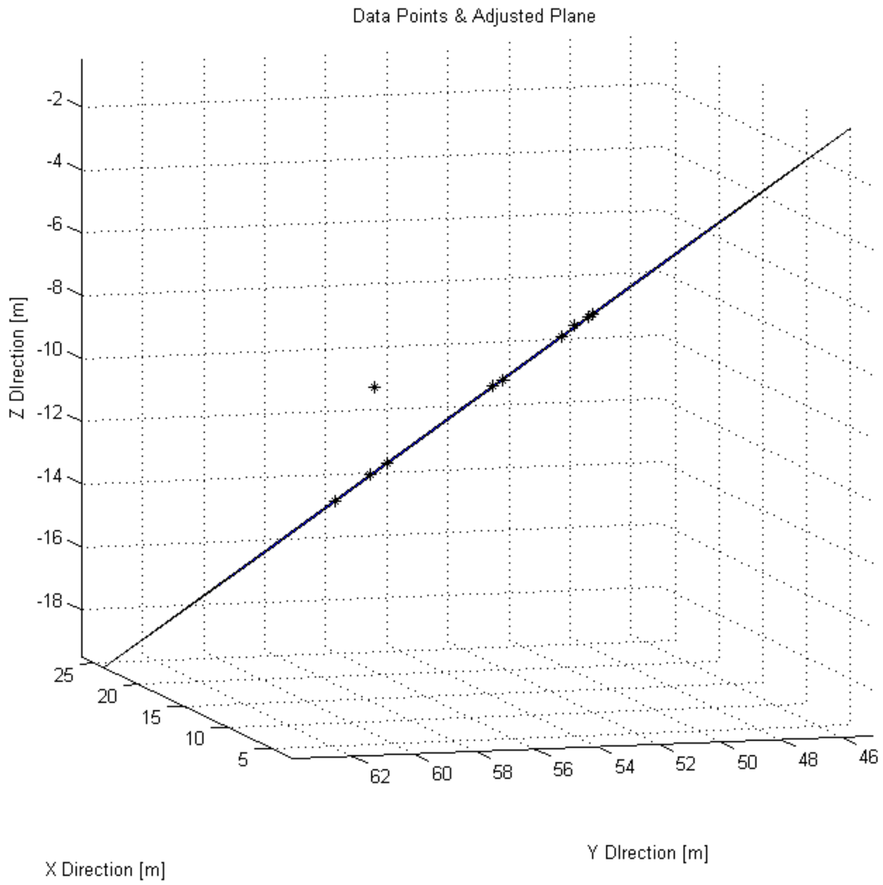

Fig. 7 Observed Point and Adjusted Plane

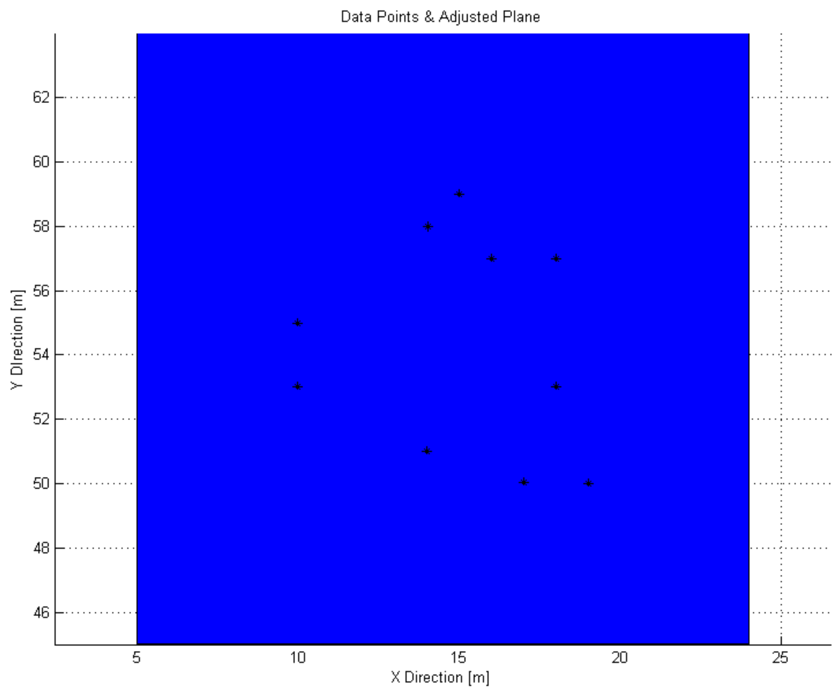

Fig. 8 Observed Point and Adjusted Plane

TASK III

(Adjustment of a Circle)

Objective: The main objective of this task is to perform adjustment in order to determine the parameters of a circle applying least square adjustment within the Gauss-Helmert Model. First of all, the given data viz. Azimuth and distance has to be converted to co-ordinate points. Secondly, from the co-ordinate points it is required to calculate the adjusted unknowns, residuals and adjusted observations along with their standard deviation.

3.1 *Computational Analysis*

To convert the given data set in to cartesian co-ordinates, the following functional model has been used.

$$y = D \sin t \qquad \text{... (86)}$$

$$x = D \cos t \qquad \text{... (87)}$$

It is stated that the standard deviation of the azimuth is $\sigma_t = 1mgon$ and the standard deviation of the distance is $\sigma_D = 1mm$

After setting up the functional model, it is required to set up the stochastic model for further approach.

$$\Sigma_{ll} = \begin{bmatrix} \sigma_{D_1}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{t_1}^2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \sigma_{D_{12}}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{t_{12}}^2 \end{bmatrix}_{24\times 24} \qquad \text{... (88)}$$

Now to compute the propagation of variance covariance matrix, it is necessary to compute the design matrix. As the functional model is nonlinear, the design matrix has to be computed from the Jacobian Matrix. The design matrix has been shown in the code.

Now putting all the values in the given equation,

$$\Sigma_{xx} = F \times \Sigma_{ll} \times F^T \qquad \text{... (89)}$$

Now, the data set is ready to compute the parameters of a circle applying least square adjustment within the Gauss-Helmert Model.

The functional model for the circle can be defined as

$$(x_i - x_m)^2 + (y_i - y_m)^2 = r^2 \qquad \text{... (90)}$$

where,

x_i, y_i = co-ordinate points of the circle

x_m, y_m = centre of the circle

r = radius of the circle

The equation 90 can be written as

$$[(x_i + v_{x_i} - x_m]^2 + [(y_i + v_{y_i}) - y_m]^2 - r^2 = 0 \qquad \text{... (91)}$$

which is the form of non-linear Gauss – Helmert Model. The approach is to choose appropriate initial value for the unknowns and the residuals.

Now, vector of observations can be defined as

$$L_{30\times1}=\begin{bmatrix}x_1\\ \vdots\\ x_{12}\\ y_1\\ \vdots\\ y_{12}\end{bmatrix} \quad \text{... (92)}$$

Vector of unknown can be defined accordingly

$$X_{4\times1}=\begin{bmatrix}x_m\\ y_m\\ r\end{bmatrix} \quad \text{... (93)}$$

Vector of residuals can be defined as

$$v_{30\times1}=\begin{bmatrix}v_{x_1}\\ \vdots\\ v_{x_{12}}\\ v_{y_1}\\ \vdots\\ v_{y_{12}}\end{bmatrix} \quad \text{... (94)}$$

Variance Covariance matrix of observation

$$\Sigma_{\boldsymbol{LL}}=\Sigma_{xx} \quad \text{... (95)}$$

Co-factor matrix of observation can be defined as

$$\boldsymbol{Q}_{\boldsymbol{LL}_{24\times24}}=\frac{1}{\sigma_0^2}\Sigma_{LL} \quad \text{... (96)}$$

The weight matrix can be defined as

$$\boldsymbol{P}=Q_{LL}^{-1} \quad \text{... (97)}$$

Vector of condition equation can be defined as

$$\Psi_{10\times1}=\begin{bmatrix}[(x_1+v_{x_1}-x_m]^2+[(y_1+v_{y_1})-y_m]^2-r^2\\ [(x_2+v_{x_2}-x_m]^2+[(y_2+v_{y_2})-y_m]^2-r^2\\ \vdots\\ [(x_{12}+v_{x_{12}}-x_m]^2+[(y_{12}+v_{y_{12}})-y_m]^2-r^2\end{bmatrix} \quad \text{... (98)}$$

Design Matrix A can be defined as

$$A=\begin{bmatrix}\frac{\partial\Psi_1}{\partial x_m} & \frac{\partial\Psi_1}{\partial y_m} & \frac{\partial\Psi_1}{\partial r}\\ \frac{\partial\Psi_2}{\partial x_m} & \frac{\partial\Psi_2}{\partial y_m} & \frac{\partial\Psi_2}{\partial r}\\ \vdots & \vdots & \vdots\\ \frac{\partial\Psi_{12}}{\partial x_m} & \frac{\partial\Psi_{12}}{\partial y_m} & \frac{\partial\Psi_{12}}{\partial r}\end{bmatrix} \quad \text{... (99)}$$

Design Matrix B can be defined as

$$B = \begin{bmatrix} \frac{\partial \Psi_1}{\partial v_{x_1}} & \cdots & \frac{\partial \Psi_1}{\partial v_{y_{12}}} \\ \frac{\partial \Psi_2}{\partial v_{x_1}} & \cdots & \frac{\partial \Psi_2}{\partial v_{y_{12}}} \\ \vdots & \cdots & \vdots \\ \frac{\partial \Psi_{12}}{\partial v_{x_1}} & \cdots & \frac{\partial \Psi_{12}}{\partial v_{y_{12}}} \end{bmatrix} \quad \ldots (100)$$

The vector of misclosures can be defined as

$$w = -Bv^0 + \Psi(v^0, x^0) - c \quad \ldots (101)$$

which can be written as

$$w_{12\times 1} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{12} \end{bmatrix}_{12\times 1} \quad \ldots (102)$$

Normal Matrix can be defined as

$$N = \begin{bmatrix} (BQ_{LL}B^T) & A \\ A^T & 0 \end{bmatrix} \quad \ldots (103)$$

Solution of the normal equation can be defined as

$$\hat{x} = N^{-1}n \quad \ldots (104)$$

The estimated residuals can be defined as

$$v = Q_{LL}B^T k \quad \ldots (105)$$

Vector of adjusted observation can be defined as

$$\hat{L} = \begin{bmatrix} x_1 + v_{x_1} \\ x_2 + v_{x_2} \\ \vdots \\ y_{10} + v_{y_{10}} \end{bmatrix}_{24\times 1} \quad \ldots (106)$$

Cofactor matrix of adjusted unknown can be defined as

$$N^{-1} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \quad \ldots (107)$$

where

$$Q_{11} = -(A^T(BQ_{LL}B^T)^{-1}.A)^{-1} \quad \ldots (108)$$

$$Q_{12} = (-BQ_{LL}B^T)^{-1}.(AQ_{22}) \quad \ldots (109)$$

$$Q_{21} = Q_{12}{}^T \quad \ldots (110)$$

$$Q_{22} = (-BQ_{LL}B^T)^{-1}(I_{12} - AQ_{21}) \quad \ldots (111)$$

Empirical Reference Standard deviation can be defined as

$$s_0 = \sqrt{\frac{v^T P v}{r}} \quad \ldots (112)$$

VCM of adjusted unknown can be defined as

$$\Sigma_{\hat{X}\hat{X}} = s_0^2 Q_{\hat{X}\hat{X}} \quad \dots (113)$$

Standard deviation of adjusted unknown can be defined as

$$\sigma_x = \sqrt{\Sigma_{\hat{X}\hat{X}}} \quad \dots (114)$$

Co-factor matrix of the residuals can be defined as

$$Q_{VV} = Q_{LL} B^T Q_{LL} B Q_{LL} \quad \dots (115)$$

VCM of the residuals can be defined as

$$\Sigma_{VV} = s_0^2\, Q_{VV} \quad \dots (116)$$

Standard deviation of residuals can be defined as

$$\sigma_v = \sqrt{\Sigma_{VV}} \quad \dots (117)$$

Cofactor matrix of adjusted observation can be defined as

$$Q_{\hat{L}\hat{L}} = Q_{LL} - Q_{VV} \quad \dots (118)$$

VCM of adjusted observation can be defined as

$$\Sigma_{\hat{L}\hat{L}} = s_0^2 Q_{\hat{L}\hat{L}} \quad \dots (119)$$

Standard deviation of adjusted observation can be defined as

$$\sigma_{\hat{L}} = \sqrt{\Sigma_{\hat{L}\hat{L}}} \quad \dots (120)$$

3.2 *Results*

The results for the adjusted unknowns has been shown below

Unknowns	$\hat{X}$	$\sigma_{\hat{X}}$
x_m	100.0001	0.0010
y_m	70.0000	0.00090
r	39.9994	0.00063

Table. 7 Adjusted Unknown and Standard Deviation

	L	$\hat{L}$	$\sigma_{\hat{L}}$	v	σ_v
x_1	69.99981419	69.99732956	0.003001749	-0.0024846	0.0019901
x_2	139.9970089	139.9994822	0.001118023	0.0024733	0.0019811
x_3	90.00024527	89.99999205	0.003500871	-0.0002532	0.0002462
x_4	134.6417697	134.6404321	0.002288304	-0.0013376	0.0013007
x_5	104.6401258	104.6408665	0.002200712	0.0007406	0.0023283
x_6	119.9994179	119.9992045	0.002823469	-0.0002134	0.0006708
x_7	109.9982715	109.999434	0.001200511	0.0011625	0.0025148
x_8	99.99790614	99.99705152	0.002331754	-0.0008546	0.0018488
x_9	104.6390709	104.6393198	0.001240947	0.0002489	0.0020379
x_{10}	79.99855931	79.99829199	0.00176551	-0.0002673	0.0021890
x_{11}	89.99921633	89.9982841	0.001552461	-0.0009322	0.0014110
x_{12}	65.35737593	65.35875706	0.001291129	0.0013811	0.0020905
y_1	69.99951542	69.99873594	0.001854137	-0.0007795	0.0005787
y_2	59.99834561	60.00069296	0.001080921	0.0023473	0.0017426
y_3	50.00313911	50.0017622	0.001835605	-0.0013769	0.0005446
y_4	65.36170977	65.35877581	0.001366445	-0.0029340	0.0011605
y_5	35.35847136	35.36049708	0.001437345	0.0020257	0.0015564
y_6	79.99831123	79.99874558	0.001684632	0.0004343	0.0003337
y_7	30.00008672	30.00064471	0.001174358	0.0005580	0.0022779
y_8	100.0009501	100.0008483	0.001662073	-0.0001018	0.0004154
y_9	35.35885757	35.35955291	0.001455382	0.0006953	0.0026906
y_{10}	120.0000475	119.9997944	0.001485169	-0.0002531	0.0009792
y_{11}	50.00007692	50.00036373	0.002099621	0.0002868	0.0027249
y_{12}	134.6407551	134.6405921	0.001205684	-0.0001630	0.0015488

Table. 8 Adjusted Unknown, Observation, Residuals and Standard Deviation

From the table it can be stated that, adjusted observation and unknowns do not differ from the intial values. It can be stated that the application of least square adjustment within the Gauss-Helmert model seem to provide the expected result. The choice for the initial values for the unknown and the residuals seems to be appropriate. To check whether the observed points fit the adjusted circle or not, a plot for that one has been given below.

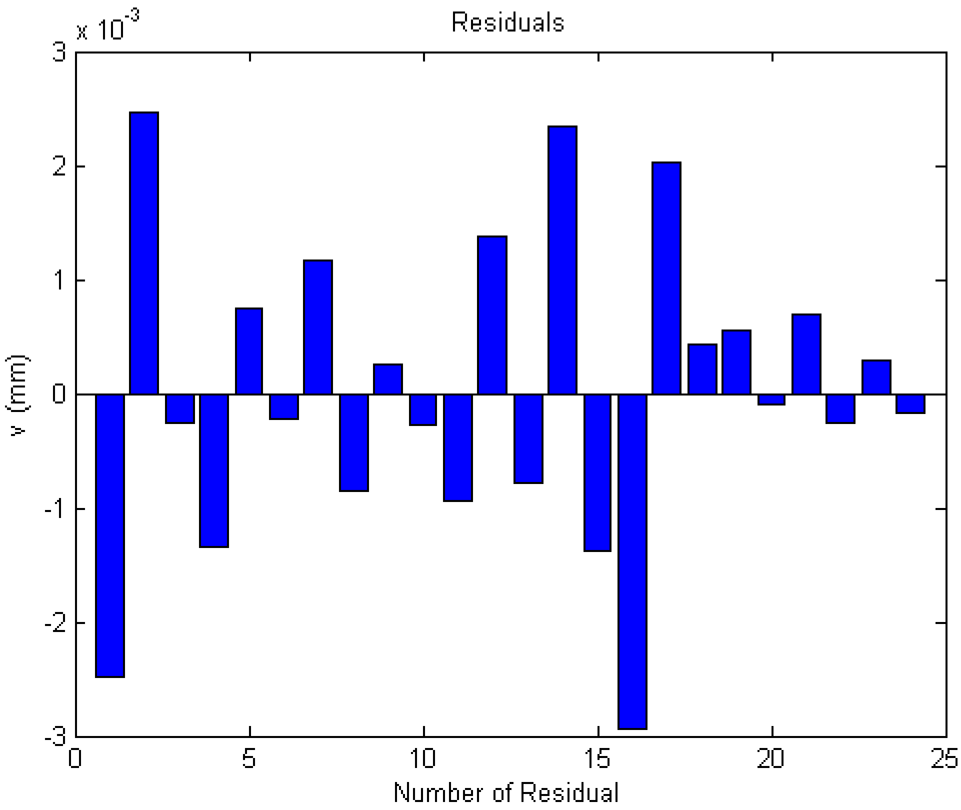

Fig. 9 Plot of the Residuals

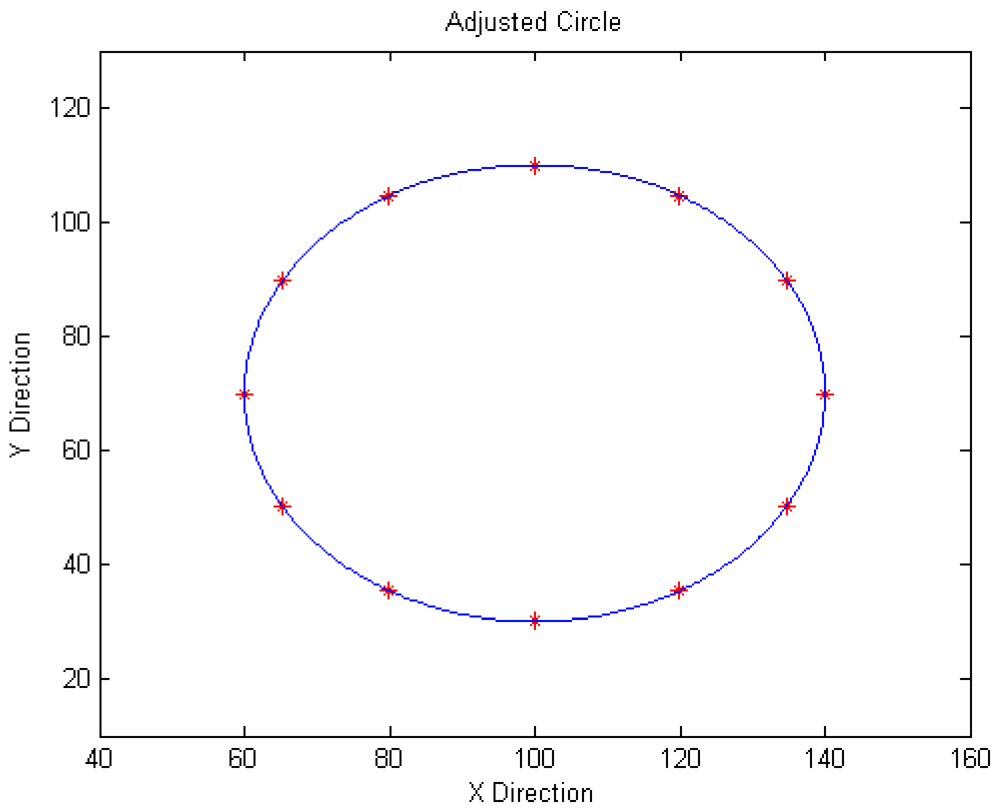

Fig. 10 Plot of the Observed Points and Adjusted Circle

References

[1] Selected Sections of Adjustment Calculation Lecture Notes, Prof. Dr.-Ing. Frank Neitzel, 2017

[2] Adjustment Calculation I Lecture Notes, Prof. Dr.-Ing. Frank Neitzel, 2016

Codes

Task 1 Part 1

```
clc;
clear all;
close all;
format long g

% Measured 2D coordinates
set1=load('Straightline_set1.txt');
set2=load('Straightline_set2.txt');
y=[set1(:,2);set2(:,2)];
x=[set1(:,1);set2(:,1)];
L = [y];

%Number of condition equations
no_n=length(L);

%Number of unknowns
no_u=2;

%Redundancy
r=no_n-no_u;

%--------------------------------------------------------------------------
%  stochastic model
%--------------------------------------------------------------------------
%Weight matrix
S_LL= eye(length(L));
sigma_0 = 1;
Q_LL= 1/sigma_0^2 *S_LL;
P = inv(Q_LL);

%--------------------------------------------------------------------------
%  Adjustment
%--------------------------------------------------------------------------
%Designmatrix A
A = [x,ones(no_n,1)];

%Normal matrix
N= A'*P*A;

%Vector of absolute values
n= A'*P*L;

% Inversion of normal matrix / Cofactor matrix of the unknowns
Q_xx=inv(N);

%Adjusted unknowns
X_hat=Q_xx*n;

% Vector of residuals
```

```
v=A*X_hat-L;

% Vector of adjusted observations
L_hat=L+v;

% Final Check
if (L_hat-(X_hat(1)*x+X_hat(2)))<10^-15
    disp('Everything fine')
else
    disp('There is a problem')
end

% Empirical reference standard deviation
s_0=sqrt(v'*P*v/r);

% VC matrix of adjusted unknowns
S_XX_hat=(s_0^2)*Q_xx;

% Standard deviation of the adjusted unknowns
s_X=sqrt(diag(S_XX_hat));

% Cofactor matrix of adjusted observations
Q_LL_hat=A*Q_xx*A';

% VC matrix of adjusted observations
S_LL_hat=(s_0^2)*Q_LL_hat;

% Standard deviation of the adjusted observations
s_L_hat=sqrt(diag(S_LL_hat));

% Cofactor matrix of the residuals
Q_vv=Q_LL-Q_LL_hat;

% VC matrix of residuals
S_vv=(s_0^2)*Q_vv;

% Standard deviation of the residuals
s_v=sqrt(diag(S_vv));

figure
bar(v*1000)
xlabel('Number of Residual')
ylabel('v (mm)')
title('Residuals')

xseries=min(x):0.1:max(x);
figure
plot(set1(:,1),set1(:,2),'ro'), hold on
plot(set2(:,1),set2(:,2),'b*'), hold on
plot(xseries,xseries*X_hat(1)+X_hat(2),'k'), hold on
xlabel('X Direction [m]')
ylabel('Y DIrection [m]')
title('Data Points & Adjusted Line')
legend('SET 1 Points','SET 2 Points','ADJUSTED
LINE','location','southeast')
```

Task 1 Part 2

```
clc;
clear all;
close all;
format long g

% Measured 2D coordinates
set1=load('Straightline_set1.txt');
set2=load('Straightline_set2.txt');
y=[set1(:,2);set2(:,2)];
x=[set1(:,1);set2(:,1)];
L = [y];

%Initial values for VCE
alpha1=1;
alpha2=1;

%-------------------------------------------------------------------------
%  Adjustment
%-------------------------------------------------------------------------
s_01=inf;
s_02=inf;
delta=10^-12;
iteration=0;

while abs(1-s_01)>delta || abs(1-s_01)>delta
%-------------------------------------------------------------------------
%  stochastic model
%-------------------------------------------------------------------------
%Weight matrix
S_LL= diag([alpha1*ones(length(L)/2,1);alpha2*ones(length(L)/2,1)]);
sigma_0 = 1;
Q_LL= 1/sigma_0^2 *S_LL;
P = inv(Q_LL);

%Number of condition equations
no_n=length(L);

%Number of unknowns
no_u=2;

%Redundancy
r=no_n-no_u;
%Designmatrix A
A = [x,ones(no_n,1)];

%Normal matrix
N= A'*P*A;

%Vector of absolute values
n= A'*P*L;

% Inversion of normal matrix / Cofactor matrix of the unknowns
Q_xx=inv(N);

%Adjusted unknowns
X_hat=Q_xx*n;
```

```
% Vector of residuals
v=A*X_hat-L;

% Vector of adjusted observations
L_hat=L+v;

% Final Check
if (L_hat-(X_hat(1)*x+X_hat(2)))<10^-15
    disp('Everything fine')
else
    disp('There is a problem')
end

% Empirical reference standard deviation
s_0=sqrt(v'*P*v/r);

% VC matrix of adjusted unknowns
S_XX_hat=(s_0^2)*Q_xx;

% Standard deviation of the adjusted unknowns
s_X=sqrt(diag(S_XX_hat));

% Cofactor matrix of adjusted observations
Q_LL_hat=A*Q_xx*A';

% VC matrix of adjusted observations
S_LL_hat=(s_0^2)*Q_LL_hat;

% Standard deviation of the adjusted observations
s_L_hat=sqrt(diag(S_LL_hat));

% Cofactor matrix of the residuals
Q_vv=Q_LL-Q_LL_hat;

% VC matrix of residuals
S_vv=(s_0^2)*Q_vv;

% Standard deviation of the residuals
s_v=sqrt(diag(S_vv));

% Redundancy Matrix
QP=Q_vv*P;

%set 1
v1=v(1:length(L)/2,1);
r1=sum(diag(QP(1:length(L)/2,1:length(L)/2)));
P1=P(1:length(L)/2,1:length(L)/2);
s_01=sqrt((v1'*P1*v1)/r1);
alpha1=(s_01^2)*alpha1;

%set 2
v2=v(length(L)/2+1:end,1);
r2=sum(diag(QP(length(L)/2+1:end,length(L)/2+1:end)));
P2=P(length(L)/2+1:end,length(L)/2+1:end);
s_02=sqrt((v2'*P2*v2)/r2);
alpha2=(s_02^2)*alpha2;

iteration=iteration+1;
```

```
end

figure
bar(v*1000)
xlabel('Number of Residual')
ylabel('v (mm)')
title('Residuals')

xseries=min(x):0.1:max(x);
figure
plot(set1(:,1),set1(:,2),'ro'), hold on
plot(set2(:,1),set2(:,2),'b*'), hold on
plot(xseries,xseries*X_hat(1)+X_hat(2),'k'), hold on
xlabel('X Direction [m]')
ylabel('Y DIrection [m]')
title({'Data Points & Adjusted Line','  ','Task 1.2'})
legend('SET 1 Points','SET 2 Points','ADJUSTED
LINE','location','southeast')
```

Task 2 Part 1

```
clc;
clear all;
close all;
format long g

% Break-off condition and initial settings
ep = 1e-12;
c = 1e-5;
max_delta_v = inf;
epsilon=10^-12;
delta= 10^-12;

%Number of iterations
it = 0;
iteration=0;

%-----------------------------------------------------------------------
%   Observations and initial values for unknowns
%-----------------------------------------------------------------------

data=load('plane.txt');

x=data(:,2);
y=data(:,3);
z=data(:,4);
L=[x;y;z];

%Initial values for unknowns
nx=1;
ny=1;
nz=1;
d=1;

n_v=[nx, ny, nz]';

X_0=[n_v;d];

%Initial values for the residuals
```

```
vx= zeros(length(x),1);
vy= zeros(length(y),1);
vz= zeros(length(z),1);

v=[vx;vy;vz];

%Number of condition equations
no_n=length(data);

%Number of unknowns
no_u=length(X_0);

%Redundancy
r=no_n-no_u+1;

%-----------------------------------------------------------------------------
%  Initial stochastic model
%-----------------------------------------------------------------------------
%Weight matrix
P= eye(3*no_n,3*no_n);
Q_ll=inv(P);

%-----------------------------------------------------------------------------
%  Adjustment
%-----------------------------------------------------------------------------

while max_delta_v>ep
 v_appr=v;
 max_PsiU=inf;
 max_X_hat=inf;
 while or(max_X_hat>delta,max_PsiU>epsilon)

    %Condition equations Psi_i
    Psi= nx.*(x+vx)+ny.*(y+vy)+nz.*(z+vz)-d;

    %Designmatrix A
    A=[x+vx y+vy z+vz -ones(no_n,1)];

    %Designmatrix B
    B1=nx*eye(no_n,no_n);
    B2=ny*eye(no_n,no_n);
    B3=nz*eye(no_n,no_n);
    B=[B1, B2, B3];
    %Designmatrix C
    C= [2*nx 2*ny 2*nz 0];

    %Vector of misclosures
    w_1= -B*v+Psi;
    w_2= nx^2+ny^2+nz^2-1;

    %Normal matrix
    N= [-A'*(B*Q_ll*B')^-1*A C'; C 0];

    %Vector of the right hand side
    n= [A'*(B*Q_ll*B')^-1*w_1; -w_2];

    %Inversion of normal matrix
    N_inv=inv(N);
```

```
    %Solution of normal equation
    x_hat=N_inv*n;

    %Adjusted unknowns
    X_hat= X_0+x_hat(1:end-1);

    %Update of the unknowns
    X_0=X_hat;

    nx=X_0(1);
    ny=X_0(2);
    nz=X_0(3);
    d=X_0(4);

    %Lagrangian Multipliers
    k_1= -(B*Q_ll*B')^-1*(A*x_hat(1:end-1)+w_1);

    %Residuals
    v= Q_ll*B'*k_1;

    %Update of the residuals
    vx= v(1:no_n);
    vy= v(no_n+1:2*no_n);
    vz= v(2*no_n+1:end);

    %Check 1
    max_X_hat = max(abs(x_hat));
    %Check 2
    PsiU = nx.*(x+vx)+ny.*(y+vy)+nz.*(z+vz)-d;
    max_PsiU = max(abs(PsiU));

    %Update number of iterations
    iteration=iteration+1;

end

    %Update of the stochastic model
    P=  diag(1./(abs(v)+c));
    Q_ll=inv(P);

    %Check
    max_delta_v= max(abs(v-v_appr));

    %Update number of iterations
    it=it+1;

end

%Vector of adjusted observations
L_hat=L+v;

%Empirical reference standard deviation
s_0=sqrt(v'*inv(Q_ll)*v/r);

%VC matrix of adjusted unknowns
Q_xx= -N_inv(1:no_u,1:no_u);
S_XX_hat=s_0^2*Q_xx;
```

```
%Standard deviation of the adjusted unknows
s_X=sqrt(diag(S_XX_hat));

%Cofactor matrix of the residuals
Q_vv= Q_ll*B'*(B*Q_ll*B')^-1*B*Q_ll;

%VC matrix of residuals
S_vv=s_0^2*Q_vv;

%Standard deviation of the residuals
s_v=sqrt(diag(S_vv));

%Cofactor matrix of adjusted observations
Q_LL_hat=Q_ll-Q_vv;

%VC matrix of adjusted observations
S_LL_hat=s_0^2*Q_LL_hat;

%Standard deviation of the adjusted observations
s_L_hat=sqrt(diag(S_LL_hat));

figure
bar(v*1000)
xlabel('Number of Residual')
ylabel('v  (mm)')
title('Residuals')
result.L=[L(1:30,1)];
% result.x=x;
result.s_L_hat=[s_L_hat(1:30,1)];
result.v=[v(1:30,1)];
result.s_v=[s_v(1:30,1)];
result= struct2table(result);
writetable(result, 'task2_1.xls');
```

Task 2 Part 2

```
clc;
clear all;
close all;
format long g

%--------------------------------------------------------------------------
%   Observations and initial values for unknowns
%--------------------------------------------------------------------------

data=load('plane.txt');

x=[data(1:8,2);data(10,2)];
y=[data(1:8,3);data(10,3)];
z=[data(1:8,4);data(10,4)];
L=[x;y;z];

%Initial values for unknowns
nx=1;
ny=1;
nz=1;
d=1;
```

```
n_v=[nx, ny, nz]';

X_0=[n_v;d];

%Initial values for the residuals
vx= zeros(length(x),1);
vy= zeros(length(y),1);
vz= zeros(length(z),1);

v=[vx;vy;vz];

%Number of condition equations
no_n=length(x);

%Number of unknowns
no_u=length(X_0);

%Redundancy
r=no_n-no_u+1;

%---------------------------------------------------------------------------
%  stochastic model
%---------------------------------------------------------------------------
%Weight matrix
Q_ll= eye(3*no_n, 3*no_n);

%---------------------------------------------------------------------------
%  Adjustment
%---------------------------------------------------------------------------
%break-off condition
epsilon=10^-12;
delta= 10^-12;
max_PsiU=inf;
max_X_hat=inf;

%Number of iterations
iteration=0;

%Iteration
while or(max_X_hat>delta,max_PsiU>epsilon)

    %Condition equations Psi_i
    Psi= nx.*(x+vx)+ny.*(y+vy)+nz.*(z+vz)-d;

    %Designmatrix A
    A=[x+vx y+vy z+vz -ones(no_n,1)];

    %Designmatrix B
    B1=nx*eye(no_n,no_n);
    B2=ny*eye(no_n,no_n);
    B3=nz*eye(no_n,no_n);
    B=[B1, B2, B3];
    %Designmatrix C
    C= [2*nx 2*ny 2*nz 0];

    %Vector of misclosures
    c1 = zeros(no_n,1);
```

```
    c2 = 1;
    w_1= -B*v+Psi-c1;
    w_2= nx^2+ny^2+nz^2-1;

    %Normal matrix
    N= [-A'*(B*Q_ll*B')^-1*A C'; C 0];

    %Vector of the right hand side
    n= [A'*(B*Q_ll*B')^-1*w_1; -w_2];

    %Inversion of normal matrix
    N_inv=inv(N);

    %Solution of normal equation
    x_hat=N_inv*n;

    %Adjusted unknowns
    X_hat= X_0+x_hat(1:end-1);

    %Update of the unknowns
    X_0=X_hat;

    nx=X_0(1);
    ny=X_0(2);
    nz=X_0(3);
    d=X_0(4);

    %Lagrangian Multipliers
    k_1= -(B*Q_ll*B')^-1*(A*x_hat(1:end-1)+w_1);

    %Residuals
    v= Q_ll*B'*k_1;

    %Update of the residuals
    vx= v(1:no_n);
    vy= v(no_n+1:2*no_n);
    vz= v(2*no_n+1:end);

    %Check 1
    max_X_hat = max(abs(x_hat));
    %Check 2
    PsiU = nx.*(x+vx)+ny.*(y+vy)+nz.*(z+vz)-d;
    max_PsiU = max(abs(PsiU));

    %Update number of iterations
    iteration=iteration+1;

end

Q_xx= -N_inv(1:no_u,1:no_u);

%Vector of adjusted observations
L_hat= [x;y;z]+v;

%Empirical reference standard deviation
s_0=sqrt(v'*inv(Q_ll)*v/r);

%VC matrix of adjusted unknowns
```

```
S_XX_hat=s_0^2*Q_xx;

%Standard deviation of the adjusted unknows
s_X=sqrt(diag(S_XX_hat));

%Cofactor matrix of the residuals
Q_vv= Q_ll*B'*(B*Q_ll*B')^-1*B*Q_ll;

%VC matrix of residuals
S_vv=s_0^2*Q_vv;

%Standard deviation of the residuals
s_v=sqrt(diag(S_vv));

%Cofactor matrix of adjusted observations
Q_LL_hat=Q_ll-Q_vv;

%VC matrix of adjusted observations
S_LL_hat=s_0^2*Q_LL_hat;

%Standard deviation of the adjusted observations
s_L_hat=sqrt(diag(S_LL_hat));

figure
bar(v*1000)
xlabel('Number of Residual')
ylabel('v (mm)')
title('Residuals')

xyframe=[min(x)-5,min(y)-5;min(x)-5,max(y)+5;
         max(x)+5,max(y)+5;max(x)+5,min(y)-5];
zframe=(X_hat(4)-X_hat(1)*xyframe(:,1)-X_hat(2)*xyframe(:,2))/X_hat(3);
figure
patch(xyframe(:,1),xyframe(:,2),zframe, 'b');hold on
plot3(data(:,2),data(:,3),data(:,4),'k*')
xlabel('X Direction [m]')
ylabel('Y DIrection [m]')
zlabel('Z DIrection [m]')
title('Data Points & Adjusted Plane')
grid on
axis equal
```

Task 3

```
clc;
clear all;
close all;
format long g;

%Observations

x1=[29.5172;37.5116;45.6540;53.0294;58.4460;60.0143;54.8881;41.5742;26.4944
;18.5546;18.2422;22.6366];

y1=[156.522;161.952;159.215;148.658;131.716;111.227;92.194;82.295;87.464;10
4.404;125.101;143.625];
 L=[y1;x1];

 D = y1 ;
```

```
 t = x1;
 t_rad = t*pi/200;

%Vector of observations
 obs = [D t_rad];

%Functional model to convert into radian
 y = D.*sin(t_rad);
 x = D.*cos(t_rad);

%Stochastic model
 s_D = 0.001;
 s_t = 0.001;
 s_t_rad = s_t*pi/200;

 for i = 1:12
   S_LL_D(i,:) = [s_D^2 s_t_rad^2];
 end

%Rearrange for stochastic model
 S_LL_D_new(1,1) = S_LL_D(1,1);
 S_LL_D_new(2,1) = S_LL_D(1,2);
 S_LL_D_new(3,1) = S_LL_D(2,1);
 S_LL_D_new(4,1) = S_LL_D(2,2);
 S_LL_D_new(5,1) = S_LL_D(3,1);
 S_LL_D_new(6,1) = S_LL_D(3,2);
 S_LL_D_new(7,1) = S_LL_D(4,1);
 S_LL_D_new(8,1) = S_LL_D(4,2);
 S_LL_D_new(9,1) = S_LL_D(5,1);
 S_LL_D_new(10,1) = S_LL_D(5,2);
 S_LL_D_new(11,1) = S_LL_D(6,1);
 S_LL_D_new(12,1) = S_LL_D(6,2);
 S_LL_D_new(13,1) = S_LL_D(7,1);
 S_LL_D_new(14,1) = S_LL_D(7,2);
 S_LL_D_new(15,1) = S_LL_D(8,1);
 S_LL_D_new(16,1) = S_LL_D(8,2);
 S_LL_D_new(17,1) = S_LL_D(9,1);
 S_LL_D_new(18,1) = S_LL_D(9,2);
 S_LL_D_new(19,1) = S_LL_D(10,1);
 S_LL_D_new(20,1) = S_LL_D(10,2);
 S_LL_D_new(21,1) = S_LL_D(11,1);
 S_LL_D_new(22,1) = S_LL_D(11,2);
 S_LL_D_new(23,1) = S_LL_D(12,1);
 S_LL_D_new(24,1) = S_LL_D(12,2);

 S_LL_new = diag(S_LL_D_new);

%Design Matrix
 F = zeros(length(24), length(24));
 F(1,1) = sin(t_rad(1));
 F(1,2) = D(1)*cos(t_rad(1));
 F(2,1) = cos(t_rad(1));
 F(2,2) = -1*D(1)*sin(t_rad(1));
 F(3,3) = sin(t_rad(2));
 F(3,4) = D(2)*cos(t_rad(2));
 F(4,3) = cos(t_rad(2));
 F(4,4) = -1*D(2)*sin(t_rad(2));
 F(5,5) = sin(t_rad(3));
 F(5,6) = D(3)*cos(t_rad(3));
 F(6,5) = cos(t_rad(3));
```

```
F(6,6) = -1*D(3)*sin(t_rad(3));

F(7,7) = sin(t_rad(4));
F(7,8) = D(4)*cos(t_rad(4));
F(8,7) = cos(t_rad(4));
F(8,8) = -1*D(4)*sin(t_rad(4));
F(9,9) = sin(t_rad(5));
F(9,10) = D(5)*cos(t_rad(5));
F(10,9) = cos(t_rad(5));
F(10,10) = -1*D(5)*sin(t_rad(5));
F(11,11) = sin(t_rad(6));
F(11,12) = D(6)*cos(t_rad(6));
F(12,11) = cos(t_rad(6));
F(12,12) = -1*D(6)*sin(t_rad(6));
F(13,13) = sin(t_rad(7));
F(13,14) = D(7)*cos(t_rad(7));
F(14,13) = cos(t_rad(7));
F(14,14) = -1*D(7)*sin(t_rad(7));
F(15,15) = sin(t_rad(8));
F(15,16) = D(8)*cos(t_rad(8));
F(16,15) = cos(t_rad(8));
F(16,16) = -1*D(8)*sin(t_rad(8));
F(17,17) = sin(t_rad(9));
F(17,18) = D(9)*cos(t_rad(9));
F(18,17) = cos(t_rad(9));
F(18,18) = -1*D(9)*sin(t_rad(9));
F(19,19) = sin(t_rad(10));
F(19,20) = D(10)*cos(t_rad(10));
F(20,19) = cos(t_rad(10));
F(20,20) = -1*D(10)*sin(t_rad(10));
F(21,21) = sin(t_rad(11));
F(21,22) = D(11)*cos(t_rad(11));
F(22,21) = cos(t_rad(11));
F(22,22) = -1*D(11)*sin(t_rad(11));
F(23,23) = sin(t_rad(12));
F(23,24) = D(12)*cos(t_rad(12));
F(24,23) = cos(t_rad(12));
F(24,24) = -1*D(12)*sin(t_rad(12));

%VCM of the unknowns
S_xx = F*S_LL_new*F';




%---------------------------------------------------------------------------
%   Observations and initial values for the unknowns
%---------------------------------------------------------------------------
%Observations
obs_new = [y x];

%Vector of observations
L_new(1,1) = obs_new(1,1);
L_new(2,1) = obs_new(1,2);
L_new(3,1) = obs_new(2,1);
L_new(4,1) = obs_new(2,2);
L_new(5,1) = obs_new(3,1);
L_new(6,1) = obs_new(3,2);
L_new(7,1) = obs_new(4,1);
L_new(8,1) = obs_new(4,2);
L_new(9,1) = obs_new(5,1);
```

```
 L_new(10,1) = obs_new(5,2);
 L_new(11,1) = obs_new(6,1);
 L_new(12,1) = obs_new(6,2);
 L_new(13,1) = obs_new(7,1);
 L_new(14,1) = obs_new(7,2);
 L_new(15,1) = obs_new(8,1);
 L_new(16,1) = obs_new(8,2);
 L_new(17,1) = obs_new(9,1);
 L_new(18,1) = obs_new(9,2);
 L_new(19,1) = obs_new(10,1);
 L_new(20,1) = obs_new(10,2);
 L_new(21,1) = obs_new(11,1);
 L_new(22,1) = obs_new(11,2);
 L_new(23,1) = obs_new(12,1);
 L_new(24,1) = obs_new(12,2);

%Initial values for the unknowns
 ym = 70;
 xm = 100;
 rm = 40;

 X_0 = [ym xm rm]';

%Number of observations
 no_n = 12

%Number of unknowns
 no_u = length(X_0);

%Redundancy
 r = no_n-no_u;

%--------------------------------------------------------------------------
%  Stochastic model
%--------------------------------------------------------------------------
%VC Matrix of the observations
 S_lL_new = S_xx;

%Theoretical reference standard deviation
 sigma_0 = 1;      %a priori

%Cofactor matrix of the observations
 Q_ll = 1/sigma_0^2*S_lL_new;

%Weight matrix
 P = inv(Q_ll);

%--------------------------------------------------------------------------
%  Adjustment
%--------------------------------------------------------------------------
%break-off conditions
 epsilon = 10^-12;
 delta = 10^-12;
 max_x_hat = 10^Inf;

%Number of iterations
 iteration = 0;

%Initialization A and B
```

```
 A = zeros(no_n,no_u);
 B = zeros(no_n,no_n*2);

%Initial values for the residuals
 vy = zeros(no_n,1);
 vx = zeros(no_n,1);


while max_x_hat>epsilon

    %Psi function
     psi = ((x+vx)-xm).^2 + ((y+vy)-ym).^2 - rm^2;

   %Matrices with the elements from the Jacobian matrices J1, J2
     A = [-2.*((y+vy)-ym) -2*((x+vx)-xm) -2*ones(no_n,1)*rm];

     B(1,1)  = 2*((y(1)+vy(1))-ym);
     B(1,2)  = 2*((x(1)+vx(1))-xm);
     B(2,3)  = 2*((y(2)+vy(2))-ym);
     B(2,4)  = 2*((x(2)+vx(2))-xm);
     B(3,5)  = 2*((y(3)+vy(3))-ym);
     B(3,6)  = 2*((x(3)+vx(3))-xm);
     B(4,7)  = 2*((y(4)+vy(4))-ym);
     B(4,8)  = 2*((x(4)+vx(4))-xm);
     B(5,9)  = 2*((y(5)+vy(5))-ym);
     B(5,10)  = 2*((x(5)+vx(5))-xm);
     B(6,11)  = 2*((y(6)+vy(6))-ym);
     B(6,12)  = 2*((x(6)+vx(6))-xm);
     B(7,13)  = 2*((y(7)+vy(7))-ym);
     B(7,14)  = 2*((x(7)+vx(7))-xm);
     B(8,15)  = 2*((y(8)+vy(8))-ym);
     B(8,16)  = 2*((x(8)+vx(8))-xm);
     B(9,17)  = 2*((y(9)+vy(9))-ym);
     B(9,18)  = 2*((x(9)+vx(9))-xm);
     B(10,19)  = 2*((y(10)+vy(10))-ym);
     B(10,20)  = 2*((x(10)+vx(10))-xm);
     B(11,21)  = 2*((y(11)+vy(11))-ym);
     B(11,22)  = 2*((x(11)+vx(11))-xm);
     B(12,23)  = 2*((y(12)+vy(12))-ym);
     B(12,24)  = 2*((x(12)+vx(12))-xm);

    %Rearrange vector of residuals
     v_obs = [vy vx];

     v_xy(1,1)  = v_obs(1,1);
     v_xy(2,1)  = v_obs(1,2);
     v_xy(3,1)  = v_obs(2,1);
     v_xy(4,1)  = v_obs(2,2);
     v_xy(5,1)  = v_obs(3,1);
     v_xy(6,1)  = v_obs(3,2);
     v_xy(7,1)  = v_obs(4,1);
     v_xy(8,1)  = v_obs(4,2);
     v_xy(9,1)  = v_obs(5,1);
     v_xy(10,1)  = v_obs(5,2);
     v_xy(11,1)  = v_obs(6,1);
     v_xy(12,1)  = v_obs(6,2);
     v_xy(13,1)  = v_obs(7,1);
     v_xy(14,1)  = v_obs(7,2);
     v_xy(15,1)  = v_obs(8,1);
     v_xy(16,1)  = v_obs(8,2);
```

```
    v_xy(17,1) = v_obs(9,1);
    v_xy(18,1) = v_obs(9,2);
    v_xy(19,1) = v_obs(10,1);
    v_xy(20,1) = v_obs(10,2);
    v_xy(21,1) = v_obs(11,1);
    v_xy(22,1) = v_obs(11,2);
    v_xy(23,1) = v_obs(12,1);
    v_xy(24,1) = v_obs(12,2);

   %Vector of misclosures
    omega = -B*v_xy + psi;

   %Normal matrix
    N_ext = [B*Q_ll*B' A; A' zeros(3,3)];

   %Vector of right hand side of normal equations
    n_ext = [-omega; 0; 0; 0];

   %Inversion of normal matrix / Cofactor matrix of the unknowns
    Q_xx_ext = inv(N_ext);

   %Solution of the normal equations
    x_hat = Q_xx_ext*n_ext;

   %Vector of residuals
    v_new = Q_ll*B'*x_hat(1:no_n);  %no need the Lagrange multipliers

   %Update
    v = v_new;

    vy = v(1:2:end);
    vx = v(2:2:end);

    X_0 = X_0 + x_hat(end-no_u+1:end);

   %Update number of iterations
   iteration = iteration+1;

   ym = X_0(1);
   xm = X_0(2);
   rm = X_0(3);

   %Check 1
   max_x_hat = max(abs(x_hat(end-no_u+1:end)));

end

%Objective function
 vTPv = v_new'*P*v_new;

%Vector of adjusted observations
 L_hat = L_new + v_new;

%Rearrange
 y_new = L_hat(1:2:end);
 x_new = L_hat(2:2:end);

%Final check for the linearization
```

```
 %Finalcheck = max(abs(L_hat(5:8,1)-a*L_hat(1:4,1)-b));
 Finalcheck = max(abs((y_new-ym).^2 + (x_new-xm).^2 -
(ones(no_n,1)*rm).^2));

 if Finalcheck<=delta
    disp('Everything is fine!')
else
    disp('Something is wrong')
end

%Empirical reference standard deviation
 s_0 = sqrt(vTPv/r);     %a posteriori

%VC matrix of adjusted unknowns
 C_xx = -s_0^2*Q_xx_ext(end-no_u+1:end, end-no_u+1:end);

%Standard deviation of the adjusted unknonws
 s_X = sqrt(diag(C_xx));

%Cofactor matrix of the residuals
 Q_vv = Q_ll*B'*Q_xx_ext(1:no_n, 1:no_n)*B*Q_ll;

%VC matrix of residuals
 C_vv = s_0^2*Q_ll*B'*Q_xx_ext(1:no_n, 1:no_n)*B*Q_ll;

%Standard deviation of the residuals
 s_v = sqrt(diag(C_vv));

%Cofactor matrix of adjusted observations
 Q_ll_hat = Q_ll-Q_vv;

%VC matrix of adjusted observations
 S_ll_hat = s_0^2*Q_ll_hat;

%Standard deviation of the adjusted observations
 s_l_hat = sqrt(diag(S_ll_hat));

%Results for the unknowns
 ym = X_0(1);
 xm = X_0(2);
 rm = X_0(3);



%--------------------------------------------------------------------------
% Plot
%--------------------------------------------------------------------------
%Plot residuals
figure (1)
bar(v, 'b')
xlabel('Number of Residual')
ylabel('v (mm)')
title('Residuals')

%Plot adjusted measurements
ang=0:0.01:2*pi;
xp=rm*cos(ang);
yp=rm*sin(ang);
figure (2)
```

```
plot(x_new, y_new, 'r*'),hold on
plot(xm+xp,ym+yp,'b'), hold on
xlim([40 160]);
ylim([10 130])
xlabel('X Direction')
ylabel('Y Direction')
title('Adjusted Circle')
```