Recursion in Data Structure - Part II

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Vishvadeep Gothi • Lesson 41 • Jan 18, 2023

Data Structure
Recursion

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Fibonacci Series

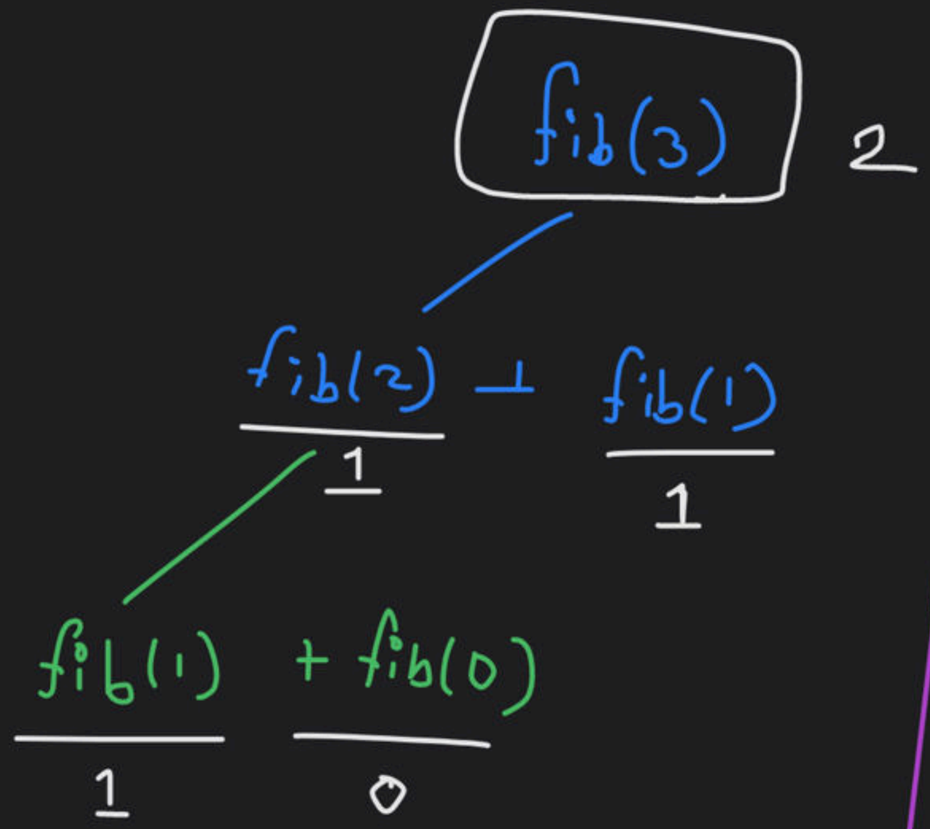
n	0	1	2	3	4	5	6	7	8	9	10
fib(n)	0	1	1	2	3	5	8	13	21	34	55

Fibonacci Series

$$fib(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ fib(n-1) + fib(n-2) & n > 1 \end{cases}$$

```
int fib (int n)
{
  if (n == 0) return 0;
  if (n == 1) return 1;
  return fib(n-1) + fib(n-2);
}
```

$n = 3$



for fib(3),

Total no. of fib() calls = 5

Total no. of additions = 2

for fib(n)

no. of functn calls (invocations) $\Rightarrow 2 * fib(n+1) - 1$

no. of additions $\Rightarrow fib(n+1) - 1$

Fibonacci Series

$$fib(n) = \begin{cases} 0 & n = 0 \\ 1 & n = 1 \\ fib(n-1) + fib(n-2) & n > 1 \end{cases}$$

Calculate the no of invocations in *fib(8)* = $2 * fib(9) - 1 = 2 * 34 - 1 = 67$

Calculate the no of additions in *fib(9)* = $fib(10) - 1 = 55 - 1 = 54$

my = (n+1)

Question

Fill in the blank:

$$pow(x,n) = \begin{cases} 1, & n = 0 \\ 0, & x = 0 \\ x, & n = 1 \\ x * pow(x, n-1), & n > 0 \\ \frac{1}{x} * pow(x, \_\_), & n < 0 \end{cases}$$

n+1

$x^{-3} = \frac{1}{x} * x^{-2}$

$\frac{1}{x} * x^{-1}$

x^3

pow(x, 3) => x * x^2

=>

x * pow(x, 2)

In prev. quest^{n}, no. of multiplicat^{n}s needed to calculate :-

① pow(x, 5) => x * x * x * x * x

↳ x * pow(x, 4)

↳ x * pow(x, 3)

↳ x * pow(x, 2)

↳ x * pow(x, 1)

Ans = 4

② pow(x, 37) => ?

Ans = 36

Ques) write a new recursive approach to calculate pow(x, 77) with less than 15 multiplications ? Ans = 7

Sol^n

$x^8 \Rightarrow x^4 * x^4$

$\hookrightarrow x^2 * x^2$

$\hookrightarrow x * x$

$x^9 \Rightarrow x * x^8$

$\hookrightarrow x^1 * x^1$

$\hookrightarrow x^2 * x^2$

$\hookrightarrow x * x$

$x^{37} \Rightarrow x * x^{18} * x^{18}$

$\llcorner\ x^9 * x^9$

$\llcorner\ x * x^4 * x^4$

$\llcorner\ x^2 * x^2$

$\llcorner\ x * x$

Ans = 7 multiplicatⁿ

```
int pow(x, n)
{
  if (n==0) return 1;
  if (x==0) return 0;
  if (n==1) return x;
  if (n%2==0)
    {
      a = pow(x, n/2);
      return a*a;
    }
  if (n%2!=0)
    {
      a = pow(x, n/2);
      return x*a*a;
    }
```

Ques) write a recursive algo to calculate multiplication of 2 positive integers by successive additions.

Soln

$3 * 5 \Rightarrow 3 + (3 + 3 + 3 + 3)$

$3 + (3 * 4)$

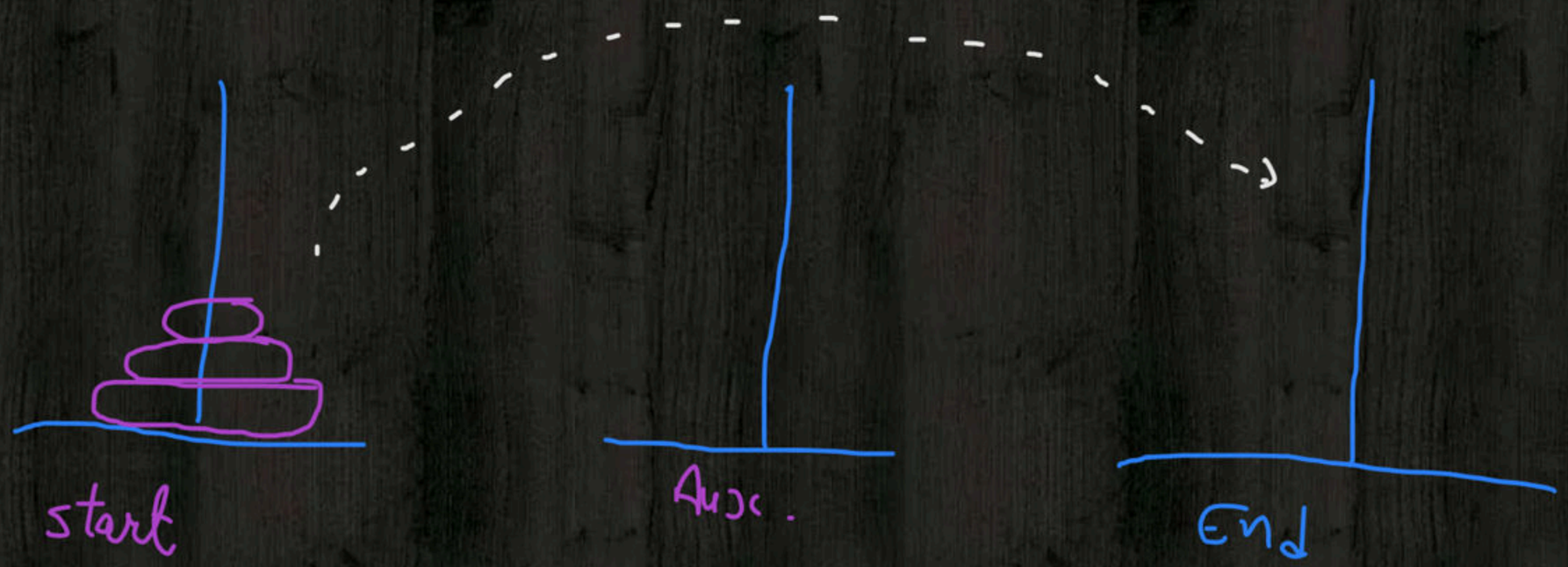
$3 + (3 * 3)$

$3 + (3 * 2)$

$3 + (3 * 1)$

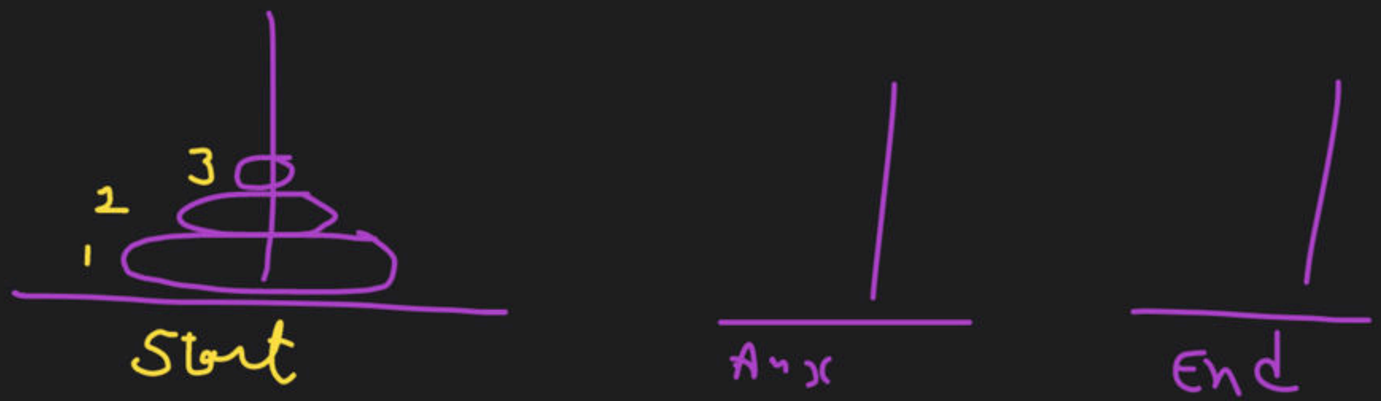
```
int mul (int a, int b)
{
   if (a == 0 || b == 0)
      return 0;
   if (a == 1)
      return b;
   if (b == 1)
      return a;
   return a + mul (a, b - 1);
}
```

Tower of Hanoi

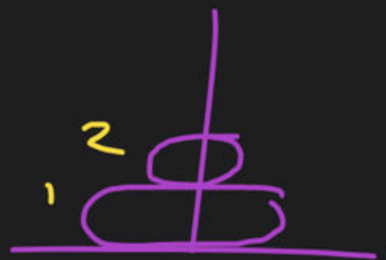


2 rules:-

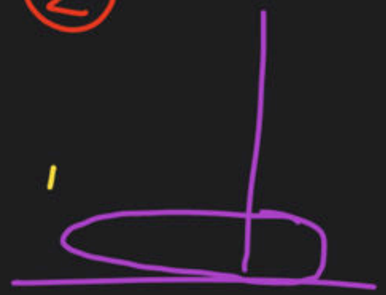
1. can move only one disk at a time
2. can place smaller disk above larger disk.

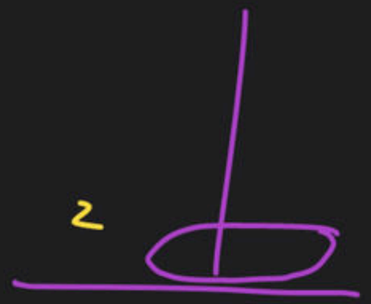


(1)



(2)





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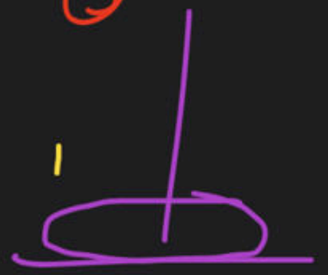
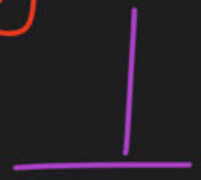
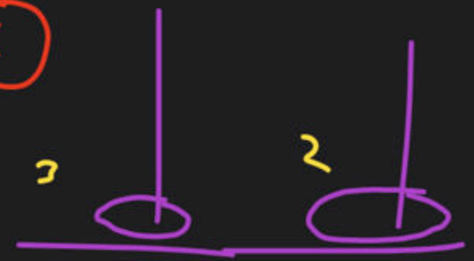


(4)

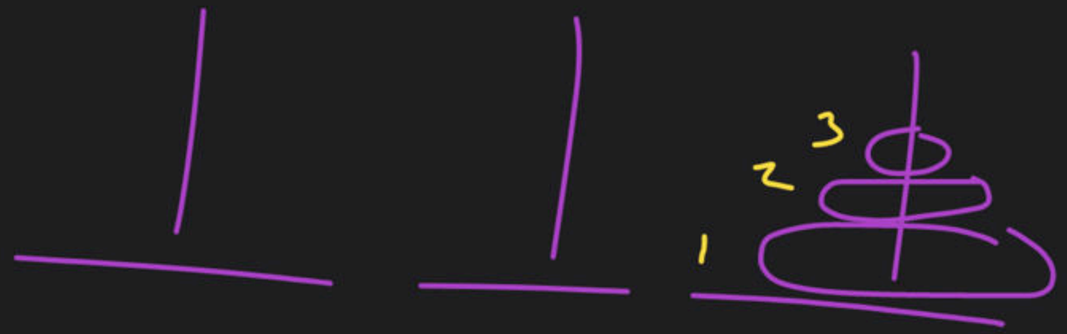


(5)



6

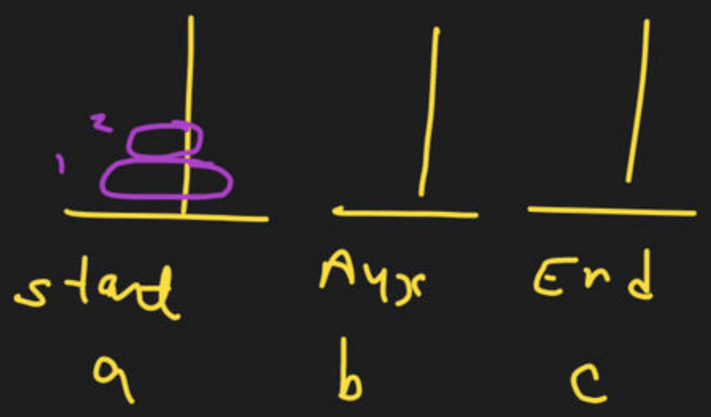
7

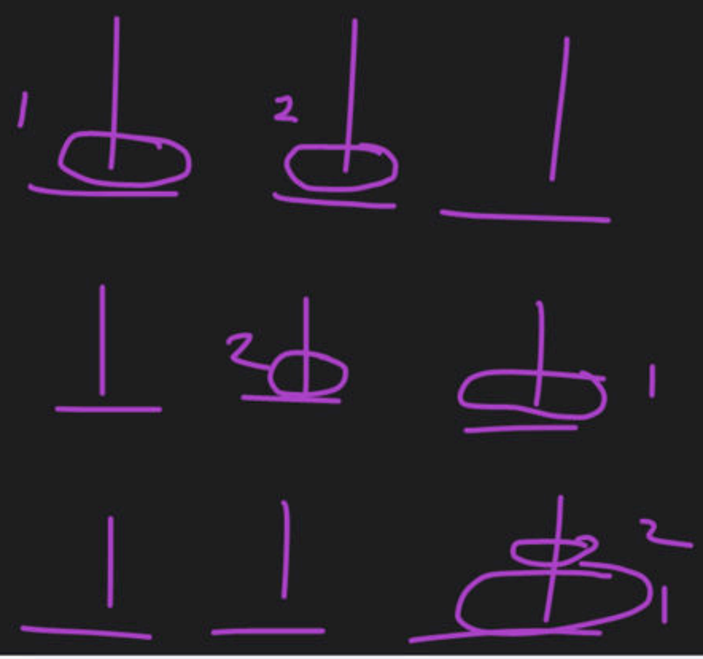


```
Tow (n, start, Aux, End)
{
   if (n > 0)
   {
      Tow (n-1, start, End , Aux)
      move a disk from start ⟶ End
      Tow(n-1, Aux, start , End)
   }
}
```

$O(2^n)$

for n = 2





Tow (2, a, b, c) → a → c

Tow (1, a, c, b) → a → b

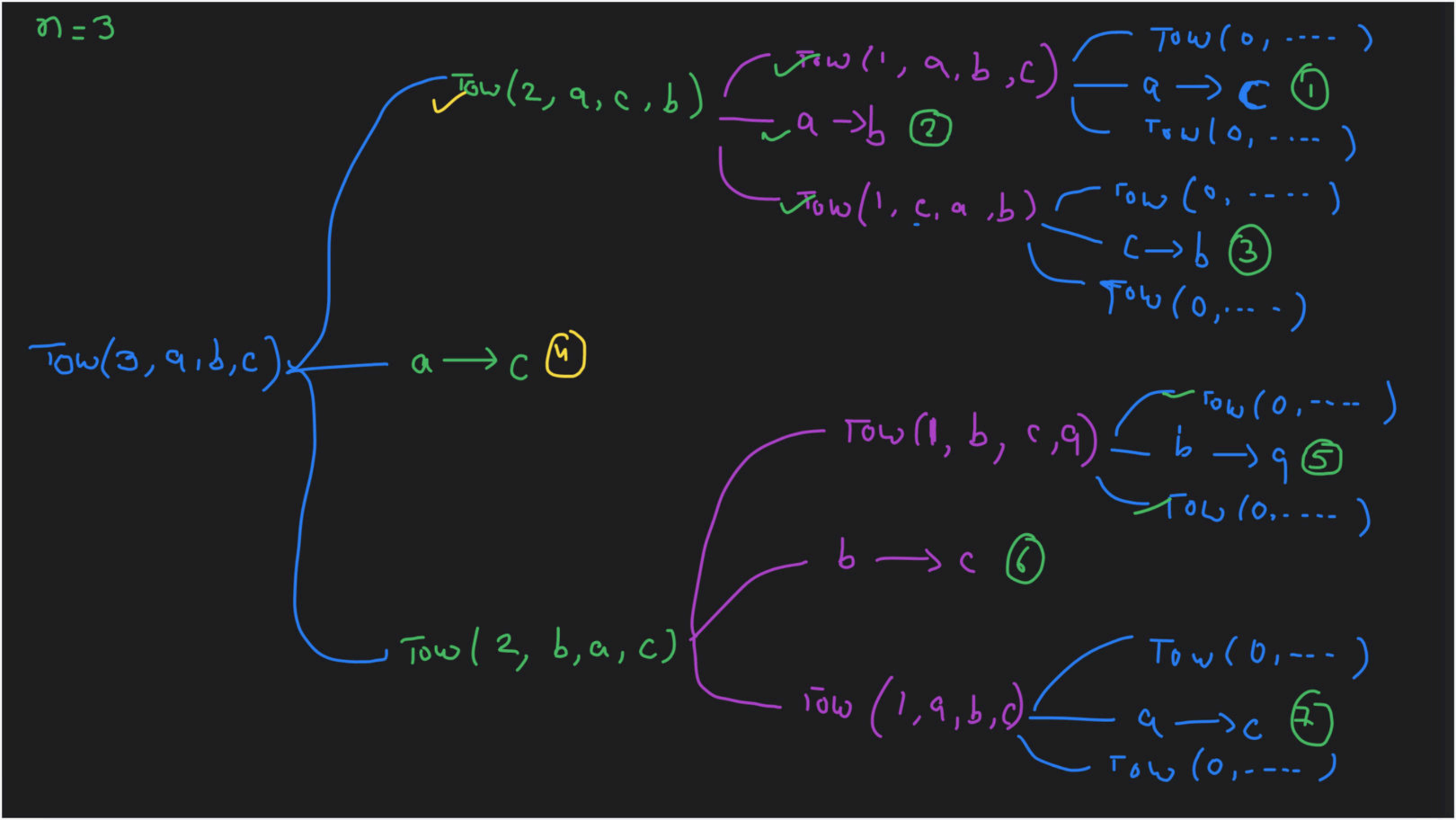
Tow (0, ...)

Tow (0, ...)

Tow (1, b, a, c) → b → c

Tow (0, ...)

Tow (0, ...


n = 3
ToH(3, a, b, c)
ToH(2, a, c, b)
ToH(1, a, b, c)
ToH(0, ----)
a → c (1)
ToH(0, ----)
a → b (2)
ToH(1, c, a, b)
ToH(0, ----)
c → b (3)
ToH(0, ----)
a → c (4)
ToH(2, b, a, c)
ToH(1, b, c, a)
ToH(0, ----)
b → a (5)
ToH(0, ----)
b → c (6)
ToH(1, a, b, c)
ToH(0, ----)
a → c (7)
ToH(0, ----)

Tower of Hanoi

Answer the following questions when there are n=3 disks:

1. How many disk moves? 7 $\rightarrow 2^n - 1$
2. How many total function invocation 15 $2^{n+1} - 1$
3. After how many invocations the first move of the disk is made? 4 $n + 1$
4. After how many invocations the last move of the disk is made? 14 $2^{n-1} - 2$

Happy Learning

Queue,
stack -
Expression,
Recursion,
Tree basics

Quiz

DPP

Ques (1) write a recursive algo to calculate division of 2 positive integers using successive subtraction ?

(2) write a recursive approach to calculate factorial of a given positive integer ?