GT - Part VI

Complete Course on Algorithm for GATE - CS & IT

Subbarao Lingamgunta • Lesson 19 • Dec 23, 2022

$T(n) = 2T(n/2) + n\log_2 n$

$= 2\left\{2T(n/2^2) + \frac{n}{2}\log\frac{n}{2}\right] + n\log n$

$= 2^2 T(n/2^2) + n\log\frac{n}{2} + n\log n$

$= 2^2\left[2T(n/2^3) + \frac{n}{2^2}\log\frac{n}{2^2}\right] + n\log\frac{n}{2} + n\log n$

$= 2^3 T(n/2^3) + n\log\frac{n}{2^2} + n\log\frac{n}{2^1} + n\log\frac{n}{2^0}$

k 번 하면

$= 2^{\log n} T(n/2^{\log n}) + n\left[\log\left(\frac{n}{2^0}\right) + \log\left(\frac{n}{2^1}\right) + \log\left(\frac{n}{2^2}\right) + \cdots + \log\left(\frac{n}{2^{\log n-1}}\right)\right]$

$= n\,T(1) + n\left[(\log n - 0) + (\log n - 1) + (\log n - 2) + \cdots + (\log n - (\log n - 1))\right]$

$= n\cdot 1 + n[1+2+3+4+\cdots+\log n]$

$n + n\{1 + 2 + 3 + \cdots + \log n\}$

$\Rightarrow n + n\left\{\frac{\log n(\log n + 1)}{2}\right\}$

$\Rightarrow n + n(\log n)^2 \Rightarrow \Theta(n(\log n)^2)$

$T(n) = 2T(n/2) + n$

$= 2\{2T(n/2^2) + \frac{n}{2}\} + n$

$= 2^2 T(n/2^2) + n + n$

$= 2^2\{2T(n/2^3) + \frac{n}{2^2}\} + n + n$

$= 2^3 T(n/2^3) + n + n + n$

$\log n$

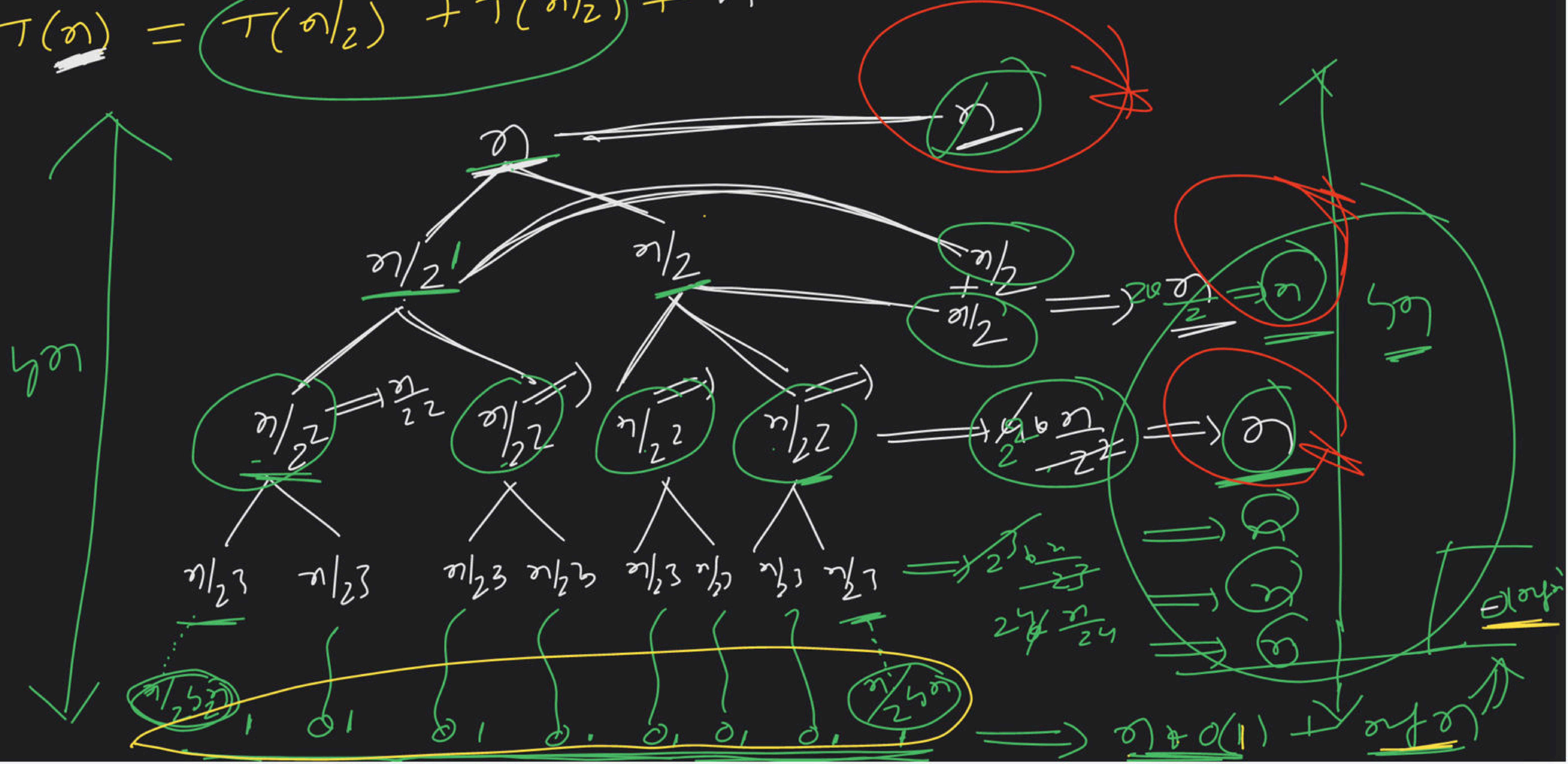
$2^{\log n} T(n/2^{\log n}) + n \cdot \log n$

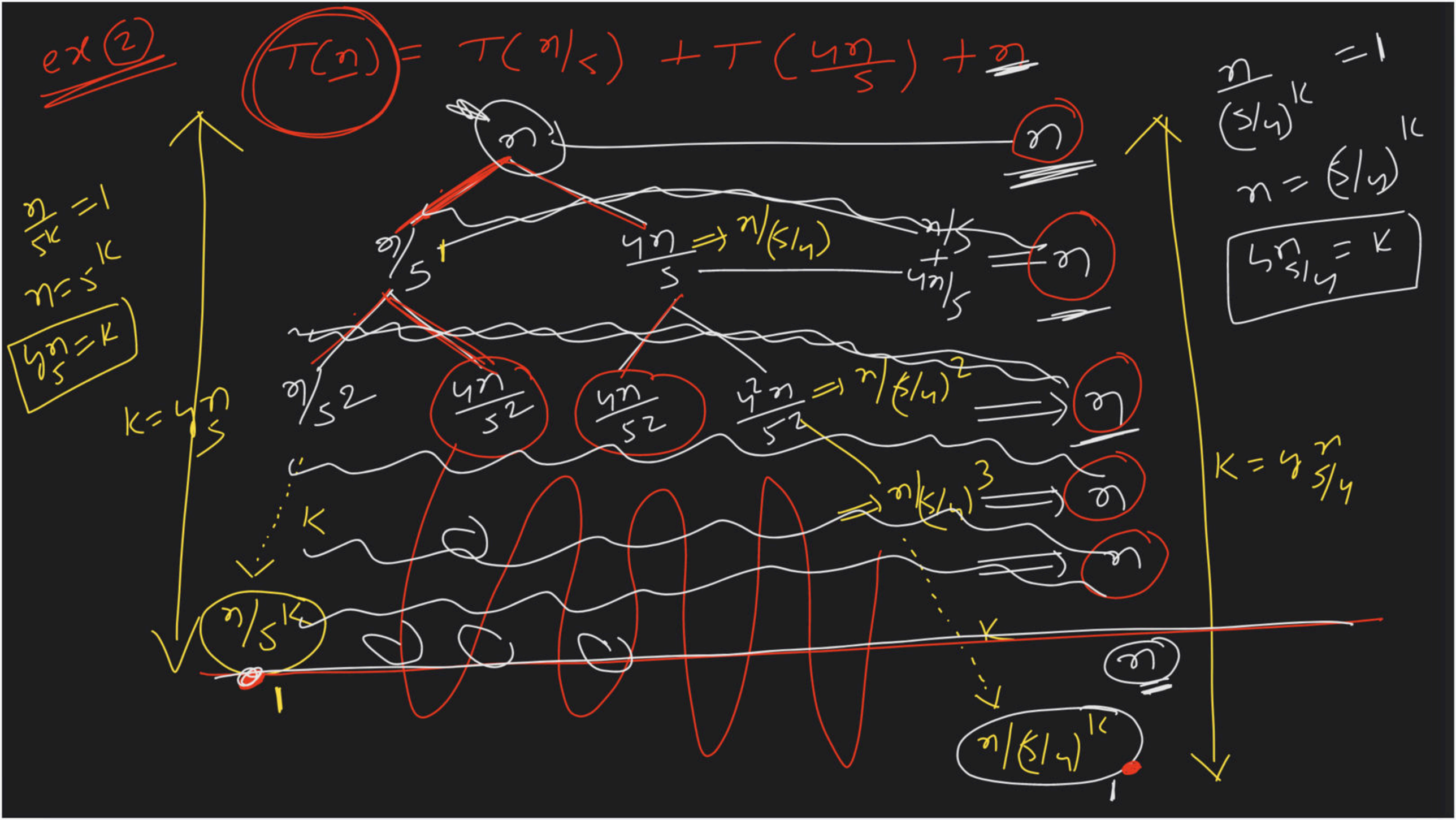
$n \cdot T(1) + n \log n$

$n \cdot O(1) + n \log n \Rightarrow \Theta(n \log n)$

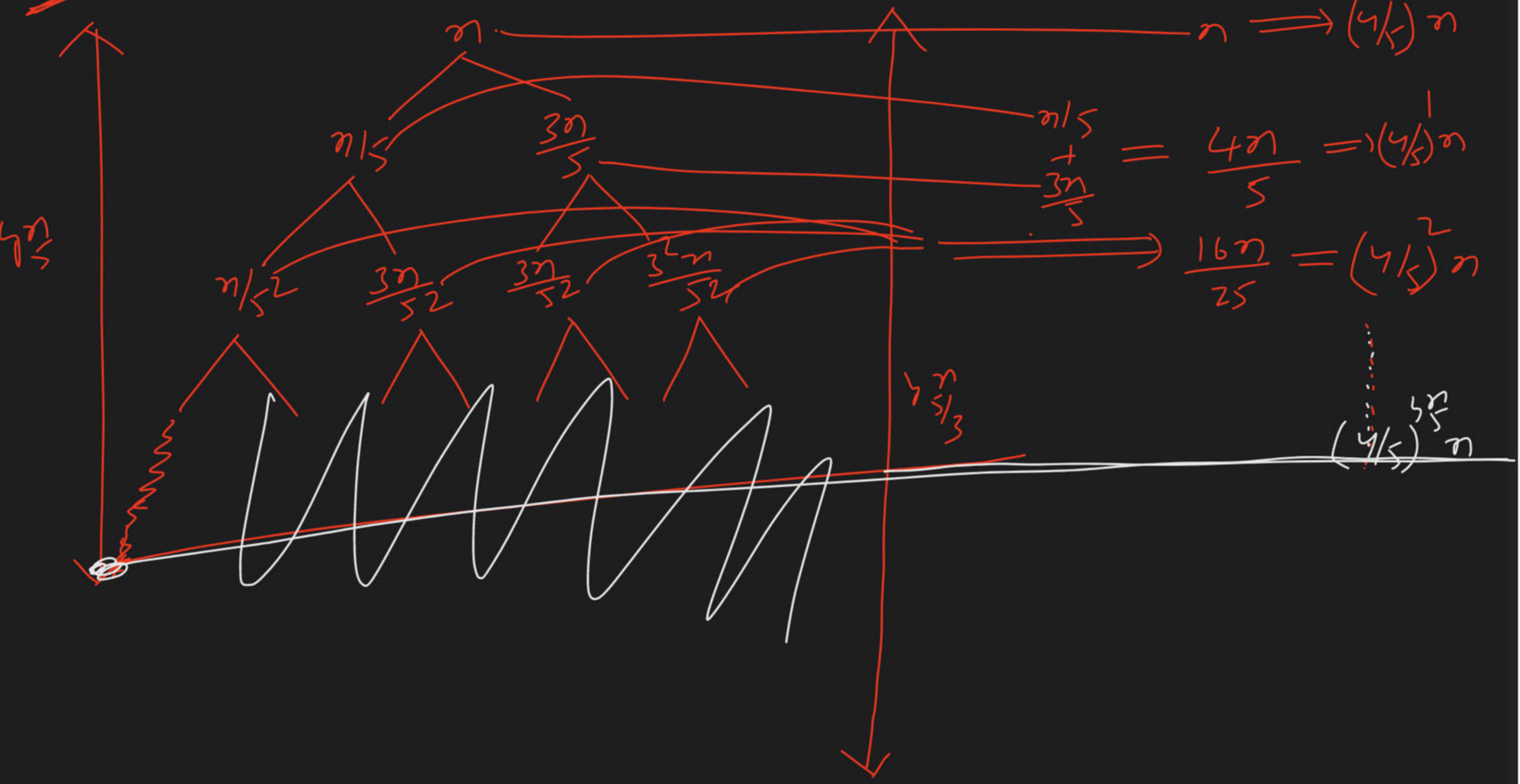
Recursive Tree method

$$T(n) = T(n/2) + T(n/2) + n \implies 2T(n/2) + n$$




ex (2)
T(n) = T(n/5) + T(4n/5) + n
n/5^k = 1
n = 5^k
4n/5 = k
k = 4n/5
n/5
4n/5
n/5^2
4n/5^2
4n/5^2
4^2 n/5^2
n/(5/4)^2
n/(5/4)^3
k
n/5^k
n/(5/4)^k
n/(5/4)^k = 1
n = (5/4)^k
4n/5/4 = k
k = 4n/5/4

$T(n) = T(n/5) + T(\frac{3n}{5}) + n$



$$T(n) \leq n\left[\underbrace{(4/5)^0 + (4/5)^1 + (4/5)^2 + \cdots + (4/5)^{\log_{5/3} n}}_{O(1)} \quad \underbrace{50}\right]$$

$$\leq n \cdot O(1)$$

$$\leq n$$

$$T(n) = O(n)$$

$$T(n) \geq n\left[\underbrace{(4/5)^0 + (4/5)^1 + (4/5)^2 + \cdots + (4/5)^{\log_5 n}}_{O(1)} \quad \underbrace{30}\right]$$

$$T(n) \geq n \cdot O(1)$$

$$T(n) = \Omega(n)$$

$$\boxed{T(n) = \theta(n)}$$

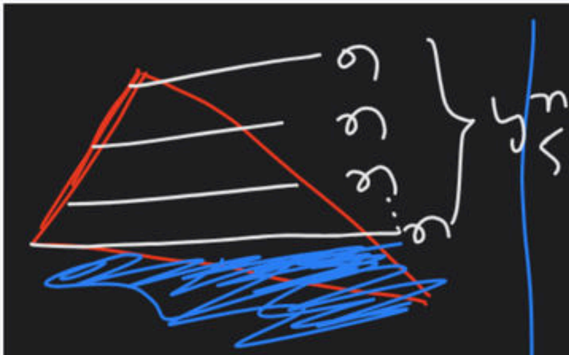
$$n \leq T(n) \leq n$$

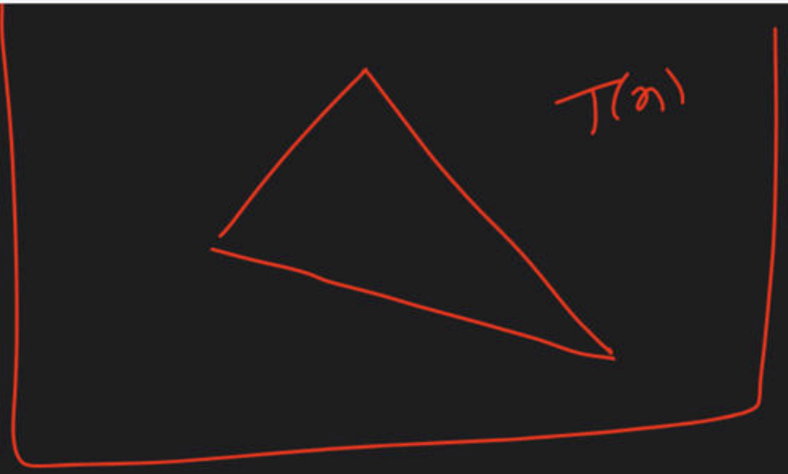
$$T(n) = T(n/5) + T(3n/5) + c_{\text{constant}}$$

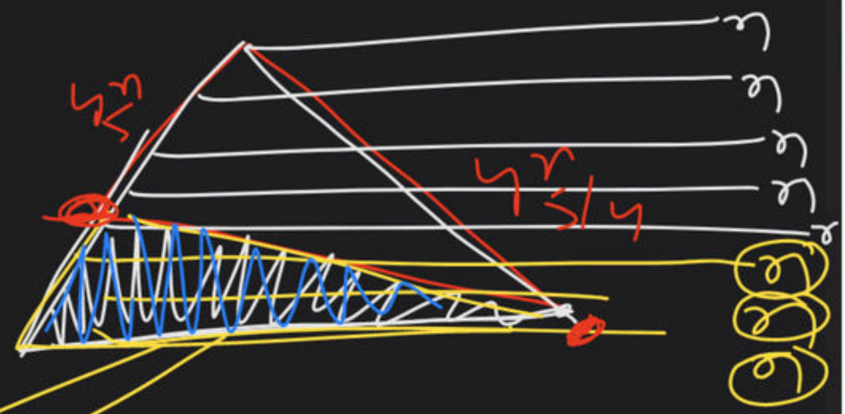
$$T(n) = T(n/5) + T(3n/5) + n^2$$

$$T(n) = T(n/2) + T(n/5) + T(n/4) + n$$

$$T(n) = T(n-1) + T(n/5) + T(n-2) + c$$







$T(n) = O(n \log_{5/4} n)$

$T(n) = \Omega(n \log_{5/4} n)$

$T(n) \leq n + n + n + \cdots + n$ ($\log_{5/4} n$ terms)

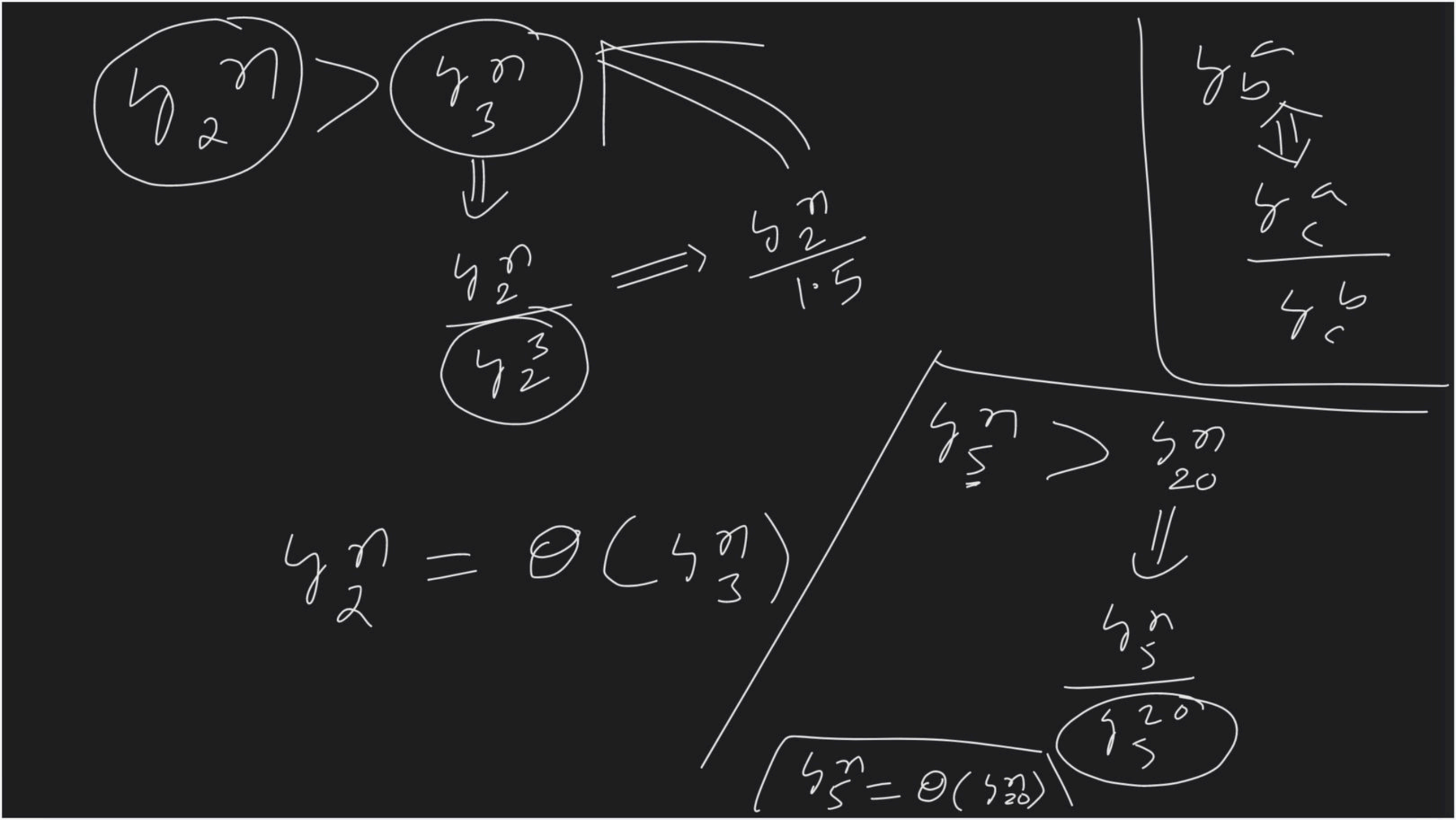
$T(n) \leq n \log_{5/4} n \implies T(n) = O(n \log_{5/4} n)$

$T(n) \geq n + n + n + \cdots + n$ ($\log_5 n$ terms) $\implies T(n) \geq n \log_5 n$

$n \log_5 n \leq T(n) \leq n \log_{5/4} n$

$T(n) = \Omega(n \log_5 n)$

$T(n) = \Theta(n \log n)$


5_2^n
5_3^n
$\frac{5_2^n}{5_2^3} \Rightarrow \frac{5_2^n}{1.5}$
$5_2^n = \Theta(5_3^n)$
$5_5^n > 5_{20}^n$
$\frac{5_5^n}{5_5^{20}}$
$5_5^n = \Theta(5_{20}^n)$
$\frac{5_c^c}{5_c^b}$