GATE 2024
Computer Science Engineering
(Volume - II)

TOPIC WISE GATE SOLUTIONS
2013-2023

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GATE SYLLABUS

Section 1 : Engineering Mathematics

Discrete Mathematics: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem. Computer Science and Information Technology.

Section 2 : Digital Logic

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point).

Section 3 : Computer Organization and Architecture

Machine instructions and addressing modes. ALU, data-path and control unit. Instruction pipelining, pipeline hazards. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

Section 4 : Programming and Data Structures

Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

Section 5 : Algorithms

Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide-and-conquer. Graph traversals, minimum spanning trees, shortest paths.

Section 6 : Theory of Computation

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and contex-free languages, pumping lemma. Turing machines and undecidability.

Section 7 : Compiler Design

Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation. Local optimisation, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

Section 8 : Operating System

System calls, processes, threads, inter-process communication, concurrency and synchronization. Deadlock. CPU and I/O scheduling. Memory management and virtual memory. File systems.

Section 9 : Databases

ER-model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

Section 10 : Computer Networks

Concept of layering: OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit-switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, Email.

General Aptitude (GA) :

Verbal Aptitude :

Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing.

Quantitative Aptitude :

Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability.

Analytical Aptitude :

Logic: deduction and induction Analogy, Numerical relations and reasoning.

Spatial Aptitude :

Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions.

CONTENTS

CHAPTER 1 Digital Logic

Marks Distribution of Digital Logic in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	1	4	9
2004*	4	5	14
2005*	4	4	12
2006*	1	5	11
2007*	3	5	13
2008*	4	1	6
2009	2	-	2
2010	3	2	7
2011	2	3	8
2012	2	-	2
2013	3	-	3
2014 Set-1	2	1	4
2014 Set-2	3	1	5
2014 Set-3	2	2	6

*** CS and IT combined**

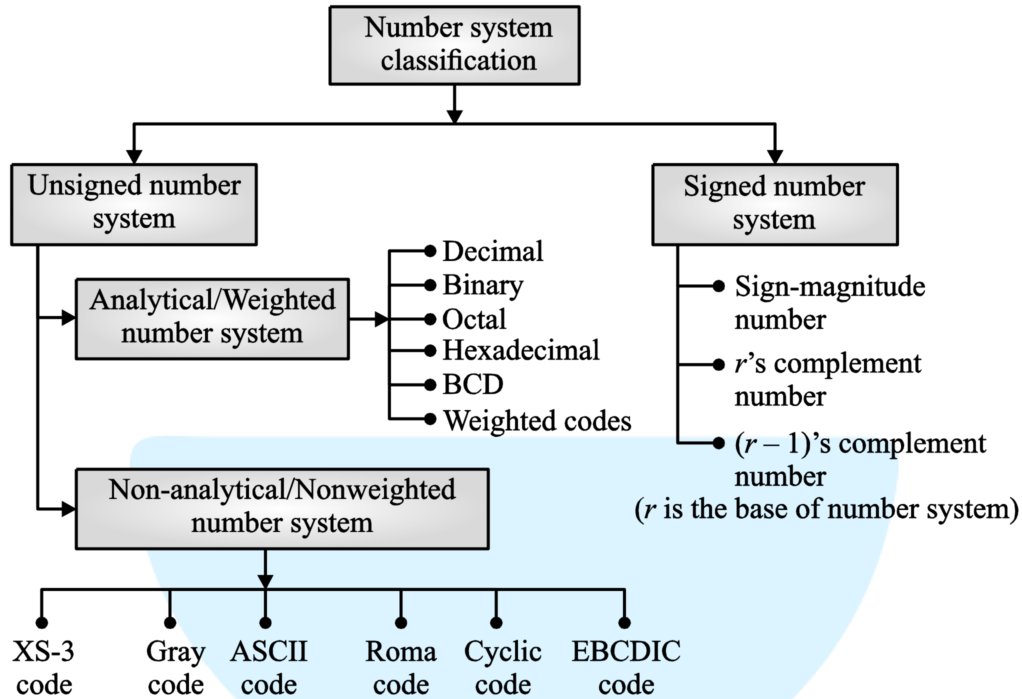
Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	1	2	5
2015 Set-2	1	2	5
2015 Set-3	-	3	6
2016 Set-1	3	2	7
2016 Set-2	3	-	3
2017 Set-1	2	1	4
2017 Set-2	1	3	7
2018	2	2	6
2019	3	2	7
2020	-	2	4
2021 Set-1	1	2	5
2021 Set-2	2	2	6

Syllabus : Digital Logic

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point).

Contents : Digital Logic

1 Number Systems



Practice Questions

2013 IIT Bombay

1.1 The smallest integer that can be represented by an 8 – bit number in 2's complement form is

(A) – 256 (B) – 128

(C) – 127 (D) 0

2014 IIT Kharagpur

1.2 The base (or radix) of the number system such that the following equation holds is ________ $\frac{312}{20} = 13.1$.

1.3 Consider the equation $(123)_5 = (x8)_y$ with x and y as unknown. The number of possible solutions is ________.

1.4 The value of a float type variable is represented using the single – precision 32-bit floating point format of IEEE – 754 standard that uses 1 bit for sign, 8 bits for biased exponent and 23 bits for mantissa. A float type variable X is assigned the decimal value of -14.25. The representation of X in hexadecimal notation is

(A) C1640000H (B) 416C0000H

(C) 41640000H (D) C16C0000H

2015 IIT Kanpur

1.5 Consider the equation $(43)_x = (y3)_8$ where x and y are unknown. The number of possible solutions is ________.

2016 IISc Bangalore

1.6 The 16-bit 2's complement representation of an integer is 1111 1111 1111 0101 its decimal representation is ________.

1.7 Let X be the number of distinct 16-bit integers in 2's complement representation. Let Y be the number of distinct 16-bit integers in sign magnitude representation. Then X – Y is ________.

2017 IIT Roorkee

1.8 The representation of the value of a 16-bit unsigned integer X in hexadecimal number system is BCA9. The representation of the value of X in octal number system is

(A) 571244 (B) 736251

(C) 571247 (D) 136251

1.9 The *n*-bit fixed-point representation of an unsigned real number X used *f* bits for the fraction part. Let $i = n - f$. The range of decimal values for X in this representation is

(A) 2^{-f} to 2^{i}

(B) 2^{-f} to $(2^{i} - 2^{-f})$

(C) 0 to 2^{i}

(D) 0 to $(2^{i} - 2^{-f})$

1.10 Given the following binary number in 32-bit (Single precision) IEEE -754 format

00111110011011010000000000000000

The decimal value closest to this floating – point number is

(A) 1.45×10^{1} (B) 1.45×10^{-1}

(C) 2.27×10^{-1} (D) 2.27×10^{1}

2018 IIT Guwahati

1.11 Consider the unsigned 8-bit fixed point binary number representation below,

$b_7\ b_6, b_5, b_4, b_3.b_2\ b_1\ b_0$

Where the position of the binary point is between b_3 and b_2. Assume b_7 is the most significant bit. Some of the decimal numbers listed cannot be represented exactly in the above representation :

(i) 31.500

(ii) 0.875

(iii) 12.100

(iv) 3.001

Which one of the following statements is true?

(A) None of (i), (ii), (iii), (iv) can be exactly represented

(B) Only (ii) cannot be exactly represented

(C) Only (iii) and (iv) cannot be exactly represented

(D) Only (i) and (ii) cannot be exactly represented

2019 IIT Madras

1.12 In 16-bit 2's complement representation, the decimal number –28 is:

(A) 1111 1111 0001 1100

(B) 0000 0000 1110 0100

(C) 1111 1111 1110 0100

(D) 1000 0000 1110 0100

1.13 Consider $Z = X - Y$, where X, Y and Z are all in sign-magnitude form X and Y are each represented in n bits. To avoid overflow, the representation of Z would require a minimum of:

(A) n bits (B) $n-1$ bits

(C) $n+1$ bits (D) $n+2$ bits

2020 IIT Delhi

1.14 Consider three registers R_1, R_2 and R_3 that store numbers in IEEE-754 single precision floating point format. Assume that R_1 and R_2 contain the values (in hexadecimal notation) 0×42200000 and $0 \times C1200000$, respectively.

If $R_3 = \dfrac{R_1}{R_2}$, what is the value stored in R_3?

(A) 0×40800000

(B) $0 \times C0800000$

(C) 0×83400000

(D) $0 \times C8500000$

2021 IIT Bombay

1.15 Let the representation of a number in base 3 be 210. What is the hexadecimal representation of the number?

(A) 15 (B) 21

(C) D2 (D) 528

1.16 Consider the following representation of a number in IEEE 754 single-precision floating point format with a bias of 127.

$S:1 \quad E:10000001$

$F:11110000000000000000000$

Here *S*, *E* and *F* denote the sign, exponent, and fraction components of the floating-point representation.

The decimal value corresponding to the above representation (rounded to 2 decimal places) is ________.

1.17 If x and y are two decimal digits and $(0.1101)_2 = (0.8xy5)_{10}$, the decimal value of $x + y$ is ________.

1.18 The format of the single-precision floating point representation of a real number as per the IEEE 754 standard is as follows :

Sign	Exponent	Mantissa

Which one of the following choices is correct with respect to the smallest normalized positive number represented using the standard?

(A) exponent $= 00000000$ and mantissa $= 00000000000000000000000$

(B) exponent $= 00000000$ and mantissa $= 00000000000000000000001$

(C) exponent $= 00000001$ and mantissa $= 00000000000000000000000$

(D) exponent $= 00000001$ and mantissa $= 00000000000000000000001$

Solutions

1.1 (B)

Given : Number of bits, $n = 8$.

Range of integer value for *n*-bit number in 2's complement signed binary number is,

$$-(2^{n-1}) \text{ to } +(2^{n-1}-1)$$

Where,

$-(2^{n-1}) \rightarrow$ Smallest integer of *n*-bit 2's complement number.

$+(2^{n-1}-1) \rightarrow$ Largest integer of *n*-bit 2's complement number.

Here, smallest integer is,

$\Rightarrow \quad -(2^{n-1})$

$\Rightarrow \quad -(2^{8-1}) \qquad (\because n = 8)$

$\Rightarrow \quad -2^7$

$\Rightarrow \quad -128$

Hence, the correct option is (B).

> **Note :** Suppose that, in the same question if examiner asked about largest integer value then answer will be $\Rightarrow +(2^{8-1}-1) = +127$.

1.2 5

Given : $\dfrac{312}{20} = 13.1$

Assume that, x is the base/radix of given number system, so above equation becomes,

$$\frac{(312)_x}{(20)_x} = (13.1)_x \qquad \ldots(i)$$

From equation (i), it is clear that $x > 3$ because highest digit in equation (i) is 3.

Now, converting equation (i) into decimal form as,

$$\frac{3x^2 + 1x + 2x^0}{2x + 0x^0} = 1x + 3x^0 + 1x^{-1}$$

$$\frac{3x^2+x+2}{2x}=1x+3+x^{-1}$$

$$3x^2+x+2=2x(1x+3+x^{-1})$$

$$3x^2+x+2=2x^2+6x+2$$

$$x^2-5x=0$$

$$x(x-5)=0$$

$$x=0 \text{ and } x=5$$

So, $x=0 \Rightarrow$ Not possible because x should be greater than 3.

$x=5 \Rightarrow$ Possible because x should be greater than 3

Thus, base/radix of given number systems is 5.

Hence, the correct answer is **5**.

1.3 **3**

Given : Equation is, $(123)_5=(x8)_y$ ...(i)

Here, it is clear that, $y>8$ and $y>x$

(here, x is a digit of a number and y is the base of that number system and base is always greater than digit/symbol of a number).

Converting equation (i) into decimal number as,

$$1\times5^2+2\times5^1+3\times5^0=x\times y^1+8\times y^0$$

$$25+10+3=xy+8$$

$$xy=30$$

So, that number of possible combination x and y under the condition $y>8$ and $y>x$ are,

x	y	xy	
1	30	30	
2	15	30	
3	10	30	
5	6	30	$\rightarrow$ Not possible because $y<8$

So, total number of possible combination for $xy=30$ is 3 only.

Hence, the correct answer is **3**.

Key Point

Generally in any number system,

- Base/radix > Maximum digit/symbol of that number system.
- Base/radix = Maximum digit/symbol of that number system + 1.

1.4 **(A)**

Given : Decimal number X is – 14.25.

Its binary equivalent is,

$$(14.25)_{10}\rightarrow(1110.010)_2$$

Now, shifting the above binary number up to three times in right to make number in IEEE format, so number becomes,

$$\Rightarrow \quad (1.110010)\times2^3$$

Since, actual number is negative, so it's sign bit (s) = 1, then we can write above number as,

$$\Rightarrow \quad X=(-1)^1\times(1.110010)\times2^3 \quad \text{...(i)}$$

According to IEEE 754, single precision floating point number representation is,

$$\Rightarrow \quad (-1)^s\times(1.m)\times2^{e-\text{Bias}} \quad \text{...(ii)}$$

Compare equation (i) and (ii),

Then $s=1,\ m=110010$

$$e-\text{Bias}=3$$

$$e=3+\text{Bias}$$

(for IEEE 754 representation, bias = 127)

$$e=3+127$$

$$e=130=10000010$$

In IEEE 754 format mantissa (m) should be 23-bit long, so m becomes,

$$m=110\ 0100\ 0000\ 0000\ 0000\ 0000$$

Now, put all values in IEEE 754 single precision floating number format as,

1	10000010	110 0100 0000 0000 0000 0000
Sign bit (s)	8-bit exponent (e)	23-bit mantissa (m)

So, its hexadecimal equivalent,

$$\underset{12=C}{1100}\ \underset{1}{0001}\ \underset{6}{0110}\ \underset{4}{0100}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}$$

$$\Rightarrow \quad (C1640000)_{16}$$

The representation of X in hexadecimal notation is, $(C1640000)_{16}$.

Hence, the correct option is (A).

Key Point

A 32-bit standard IEEE 754 single precision floating point number representation in binary is shown below,

Sign bit (s) 1-bit	Exponent (e) 8-bit	Mantissa (m) 23-bit

It is divided into 3-parts namely :

(i) Sign bit (s)

(ii) Exponent (e)

(iii) Mantissa (m)

To represent a number into above define format, a number should be in following normalized form,

$$(-1)^s \times (1.\text{mantissa}) \times 2^{e-\text{Bias}}$$

For IEEE 754 single precision floating point number representation,

$$\text{Bias} = 127 \text{ fixed}$$

1.5 **5**

Given : Equation is, $(43)_x = (3y)_8 \quad \ldots\text{(i)}$

Here, it is clear that, $x > 4$ and $y < 8$.

Where, $x \rightarrow$ Base/radix

$y \rightarrow$ Digit of base-8 number system.

Converting equation (i) into decimal form as,

$$4 \times x^1 + 3 \times x^0 = y \times 8^1 + 3 \times 8^0$$

$$4x + 3 = 8y + 3$$

$$x = 2y$$

$$y = \frac{x}{2}$$

So, that number of possible combination of x and y under the condition $y < 8$ and $x > 4$ is,

x	y	$y = \frac{x}{2}$
6	3	Satisfied
8	4	Satisfied
10	5	Satisfied
12	6	Satisfied
4	7	Satisfied
16	8	Satisfied but not consider, because $y = 8$.

Hence, the number of possible solution for x and y are **5**.

Hence, the correct answer is 5.

Note : Here, x is a base of a number $(43)_x$ and y is the digit of base-8 number system and base is always greater than digit/symbol of a number, that is why we consider $x > 4$ and $y < 8$.

1.6 **– 11**

Given : 16-bit 2's complement representation of an integer is,

$$\underbrace{1111\ 1111\ 1111\ 0101}_{\text{16-bit long}}$$

MSB ← 1 → LSB

Sign bits ←

Since, sign bit/MSB is 1, it means given number is negative.

So, taking 2's complement of this given number is,

1111 1111 1111 0101

↓ Take 2's complement

0000 0000 0000 1011

↓ Decimal value

11

Hence, decimal equivalent of given 16-bit 2's complement integer is –11.

Hence, the correct answer is **–11**.

1.7 **1**

Given : number of bits, n = 16.

In n-bit 2's complement signed binary number representation, range of integer is,

$$-(2^{n-1}) \text{ to } +(2^{n-1} - 1)$$

Total number of distinct integer in this range is,

$$\Rightarrow \quad (2^{n-1} - 1) - [-(2^{n-1})]$$

$$\Rightarrow \quad [2(2^{n-1})] - 1$$

If $n=16$, then total number of distinct integer in 2's complement signed binary number representation is,

$\Rightarrow \quad [2(2^{16-1})]-1$

$\Rightarrow \quad [2(2^{15})-1]$

$\Rightarrow \quad (2^{16}-1)=X$

In n-bit signed magnitude binary number representation, range of integer is,

$\Rightarrow \quad -(2^{n-1}-1)$ to $+(2^{n-1}-1)$

Total number of distinct integer in this range is,

$\Rightarrow \quad (2^{n-1}-1)-[-(2^{n-1}-1)]$

$\Rightarrow \quad [2(2^{n-1})]-2$

If $n=16$, the total number of distinct integer in signed magnitude binary number representation is,

$\Rightarrow \quad [2(2^{16-1})]-2$

$\Rightarrow \quad [2(2^{15})-2]$

$\Rightarrow \quad (2^{16}-2)=Y$

Thus, $\quad X-Y=(2^{16}-1)-(2^{16}-2)$

$X-Y=1$

Hence, the value of $X-Y=1$.

Key Point

(i) Range of n-bit unsigned binary numbers,

0 to 2^n-1

Total number of distinct integer in this range $=2^n$.

(ii) Range of n-bit signed binary numbers,

$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

Total number of distinct integer in this range $=2\times(2^{n-1}-1)$.

(iii) Range of n-bit 1's complement signed binary numbers,

$-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

Total number of distinct integer in this range $=2\times(2^{n-1}-1)$.

(iv) Range of n-bit 2's complement signed binary numbers,

$-(2^{n-1})$ to $+(2^{n-1}-1)$

Total number of distinct integer in this range $=(2\times2^{n-1})-1$.

1.8 (D)

Given : Hexadecimal number in 16-bit unsigned integer X is,

$X=(BCA9)_{16}$

Method 1

Now, to convert it into octal number, we will take two steps,

Step 1 : First convert each digit/symbol of X into 4-bit binary number as shown below,

$$X=(\underset{\downarrow}{B}\quad \underset{\downarrow}{C}\quad \underset{\downarrow}{A}\quad \underset{\downarrow}{9})_{16}$$

$$(1011\ 1100\ 1010\ 1001)$$

$$(BCA9)_{16} \xrightarrow{\text{Binary}} (\underset{\text{MSB}}{1}\ 01111001010100\ \underset{\text{LSB}}{1})$$

Step 2 : Now, make group of 3-bits, moving from LSB to MSB and convert each group into its decimal equivalent as shown below,

2 extra zero's are added for completing the group of 3-bit

$$\underbrace{001}_{1}\ \underbrace{011}_{3}\ \underbrace{110}_{6}\ \underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{001}_{1}$$

Thus, required octal number is $(136251)_8$.

Hence, the correct option is (D).

Tip : It is advisable to aspirant, try to convert this octal number $(136251)_8$ into hexadecimal number by yourself, this type of practice increases your thinking ability and concept clarity.

Method 2

Convert hexadecimal number $X=(BCA9)_{16}$ into decimal number as,

$\Rightarrow \quad B\times16^3+C\times16^2+A\times16^1+9\times16^0$

(In hexadecimal $B=11$, $C=12$ and $A=10$)

$\Rightarrow \quad 11\times16^3+12\times16^2+10\times16^1+9\times1$

$\Rightarrow \quad 45056+3072+160+9$

$\Rightarrow \quad (48297)_{10} \rightarrow$ Decimal value

Converting decimal number, $(48297)_{10}$ into octal number as,

		Remainder
8	48297	
8	6037	$\rightarrow$ 1
8	754	$\rightarrow$ 5
8	94	$\rightarrow$ 2
8	11	$\rightarrow$ 6
	1	$\rightarrow$ 3

So, octal number is $(136251)_8$.

Hence, the correct option is (D).

1.9 (D)

Given :

$f \rightarrow$ Fractional bit in n-bit fixed point representation.

$i \rightarrow (n-f) \rightarrow$ Integer bit in n-bit fixed point representation.

Method 1

Let us consider any decimal number X, whose 5-bit binary number will contain 2-integer bit and 3-fractional bit,

i.e. $f=3$

$i=2$

$n=5 \rightarrow$ 5-bit binary number

According to question, X is real unsigned number so its minimum value in binary form is,

$\Rightarrow \quad \underset{\text{Integer part}}{000} \;.\; \underset{\text{Fraction part}}{00}$

$\Rightarrow$ Decimal value = 0

Maximum value of X in binary form is,

$\Rightarrow \quad \underset{\text{Integer part}}{111} \;.\; \underset{\text{Fraction part}}{11}$

$\Rightarrow$ Decimal value = 7.75

So, for $n=5$ bit unsigned real number in which $i=3$ and $f=2$, then range is 0 to 7.25.

Now, apply $n=5$, $i=3$ and $f=2$ in each options and check which one satisfied to 0 to 7.75.

From option (A) :

2^{-f} to 2^i

2^{-2} to 2^3

0.25 to 8

So, option (A) is not satisfied.

From option (B) :

2^{-f} to (2^i-2^{-f})

2^{-2} to (2^3-2^{-2})

0.25 to 7.75

So, option (B) is not satisfied.

From option (C) :

0 to 2^i

0 to 2^3

0 to 8

So, option (C) is also not satisfied.

From option (D) :

0 to (2^i-2^{-f})

0 to (2^3-2^{-2})

0 to 7.75

So, option (D) is satisfied.

Hence, the correct option is (D).

Method 2

Given :

Unsigned real number X size $\rightarrow$ n-bit long

Fractional part in $X \rightarrow$ f-bit long

Integer part in $X \rightarrow$ $i=(n-f)$ bit long

Range of decimal value for only integer part of real number X is,

0 to (2^i-1)

Range of decimal value for only fractional part of real number X is,

0 to $(1-2^{-f})$

So, finally range of real number X, contain both integer and fractional part is,

0 to $(2^i-1+1-2^{-f})$

0 to (2^i-2^{-f})

Hence, the correct option is (D).

1.10 **(C)**

Given : 32-bit floating type binary number in IEEE 754 single precision format is,

$$\underset{\text{Sign bit }(s)}{0}\ \underbrace{01111100}_{\text{Exponent }(e)}\ \underbrace{1101\ 101\ 000\ \ 0000000000000}_{\text{23-bit mantissa }(m)}$$

Sign bit (s) = 0, it means number is positive.

Mantissa (m) is sufficient to calculate decimal value of number, so

$$m = 1101\,101\ \ \underbrace{0000000000000000}_{\substack{\text{Omitted these zero because it does not} \\ \text{create any effect on the number}}}$$

m becomes as,

$$m = 1101101$$

Now, converting mantissa (m) according to IEEE 754 single precision format by shifting mantissa one-time in left as,

$$\Rightarrow \quad 1.101101 \times 2^{-1}$$

Convert the above number in IEEE 754 single precision format as,

$$\Rightarrow \quad (-1)^0 \times 1.101101 \times 2^{-1}$$

So, converting fractional part ($.101101 \times 2^{-1}$) of above number into decimal as,

$$\Rightarrow \quad 1\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4} + 0\times 2^{-5} + 1\times 2^{-6} \times 2^{-1}$$

$$\Rightarrow \quad 0.227$$

$$\Rightarrow \quad 2.27\times 10^{-1}$$

The decimal value closest to given floating point number under single precision IEEE 754 format is 2.27×10^{-1}.

Hence, the correct option is (C).

1.11 **(C)**

Given : Fixed point representation of 8-bit unsigned number in binary form is,

$$\underset{\substack{\downarrow \\ \text{MSB}}}{b_7}\ b_6 b_5 b_4 b_3 \bullet b_2 b_1\ \underset{\substack{\downarrow \\ \text{LSB}}}{b_0}$$

So, 8-bit unsigned binary representation of given decimal number is,

(i) $31.500 \rightarrow 11111.100$

(ii) $0.875 \rightarrow 00000.111$

(iii) $12.100 \rightarrow 1100.000110011$

(iv) $3.001 \rightarrow 11.000000000100$

So, (iii) and (iv) can be represented in given 8-bit binary format.

Hence, the correct option is (C).

1.12 **(C)**

Given : Hexadecimal number is – 28

Number of bits $n = 16$

Now, convert decimal number $+28$ into 16-bit binary number as,

$$(+28)_{10} \xrightarrow[\text{Number}]{\text{Binary}} (0000\ 0000\ 0001\,1100)_2$$

Take 2's complement of above binary number as,

$$(0000\ 0000\ 0001\,1100)_2$$
$$\downarrow \text{2's complement}$$
$$(1111\,1111\,1110\ 0100)_2$$

So, it is the 2's complement representation of decimal number – 28.

Hence, the correct option is (C).

1.13 **(A)**

Given : $Z = X - Y$

X, Y, Z all are in sign magnitude form.

X and Y are n-bit long.

In sign magnitude representation, subtraction generally doing with 2's complement form and 1's complement form and in both forms we used the concept of addition.

So, given $Z = X - Y$ looks like subtraction but it is indirect addition.

If we add two n-bit long X and Y normally, and carry never occur, then result (Z) can be n-bit long and it requires n-bit memory to store the result (Z).

Example :

$$\begin{array}{rl} X \rightarrow & 01001........1 \rightarrow n\text{-bit long number} \\ Y \rightarrow + & 00100........0 \rightarrow n\text{-bit long number} \\ \hline \text{Result without carry} \rightarrow Z \rightarrow & 01101........1 \rightarrow n\text{-bit long number} \end{array}$$

Again, if we add two n-bit long X and Y normally, and carry occurred this time, then result

(Z) can be $(n+1)$-bit long (1 extra bit due to carry), so it requires minimum $(n+1)$ bit long memory to store the result.

Example :

$$\begin{array}{rl} & X \rightarrow \ 1001........1 \rightarrow n\text{-bit long number} \\ & Y \rightarrow +\ 1100........0 \rightarrow n\text{-bit long number} \\ \hline \text{Result with carry} \rightarrow & Z \rightarrow \underset{\uparrow \atop \text{Carry}}{1}0101........1 \rightarrow (n+1)\text{-bit long number} \end{array}$$

If we have only n-bit long memory to store $(n+1)$ bit long result (Z), then carry could not be stored and this condition is called overflow.

So, to avoid the overflow during addition of n-bit long numbers, result (Z) requires minimum of $(n+1)$ bit long memory for storage.

Hence, the correct option is (A).

1.14 **(B)**

Given : $R_1 = 0 \times 42200000$

$R_2 = 0 \times \text{C}1200000$

Method 1

R_1 in hexadecimal notation is given as,

0×42200000

Convert it into 32-bit binary number as,

$$\underset{\text{Sign bit-8}}{0}\ ,\underbrace{10000100}_{\text{8-bit exponent }(e)},\underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$$

Here, $s = 0 \Rightarrow R_1$ is positive number

$e = 10000100 \Rightarrow (132)_{10}$

Actual exponent $= e - \text{Bias}$

$= 132 - 127 = 5$

(For IEEE 754, Bias = 127 fixed)

Mantissa $(m) = \underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$

Now, represent R_1 in IEEE 754 single precision floating point format as,

$\Rightarrow$ $(-1)^s \times (1.m) \times 2^{e-\text{Bias}=\text{Actual exponent}}$

$\Rightarrow$ $(-1)^0 \times 1.01000000000000000000000 \times 2^5$

$\Rightarrow$ $1.01000000000000000000000 \times 2^5$

$\Rightarrow$ Shifted above number 5-times towards left so 2^5 reduce to 2^0.

$\Rightarrow$ 101000.000000000000000000

$\Rightarrow$ Decimal equivalent of above binary number is,

$\Rightarrow$ $+40 = R_1$

R_2 in hexadecimal notation is given as,

$0 \times C1200000$

Convert it into 32-bit binary number as,

$$\underset{\text{Sign bit-8}}{1}\ ,\underbrace{10000010}_{\text{8-bit exponent }(e)},\underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$$

Here, $s = 1 \Rightarrow R_2$ is negative number

$e = 10000010 \Rightarrow (130)_{10}$

Actual exponent $= e - \text{Bias}$

$= 130 - 127 = 3$

(For IEEE 754, Bias = 127 fixed)

Mantissa $(m) = \underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$

Now, represent R_2 in IEEE 754 single precision floating point format as,

$\Rightarrow$ $(-1)^s \times (1.m) \times 2^{e-\text{Bias}=\text{Actual exponent}}$

$\Rightarrow$ $(-1)^1 \times 1.01000000000000000000000 \times 2^3$

$\Rightarrow$ $-1.01000000000000000000000 \times 2^3$

Shifted above number 3-times towards left so 2^3 reduce to 2^0.

$\Rightarrow$ $-1010.00000000000000000000$

Decimal equivalent of above binary number is,

$\Rightarrow$ $-40 = R_2$

Thus, $R_3 = \dfrac{R_1}{R_2} = \dfrac{40}{-10} = -4$

$R_3 = -4$

Now, represent -4 into IEEE 754 single precision format as,

$s = 1 \Rightarrow$ Because R_3 is negative.

Binary value of $+4$ into 23-bit is,

$(100.00000000000000000000)_2$

Above binary number shifted 2-times towards right as,

$$(1.00000000000000000000000)\times 2^{2}$$

So, -4 can be represented according to IEEE 754 single precision format as,

$$(-1)^{1}\times(1.00000000000000000000000)\times 2^{2}$$

Here, actual exponent = $2 = e - \text{Bias}$

Exponent, $e = 2 + \text{Bias}$

(For IEEE 754 format $\text{Bias} = 127$ fixed)

$$e = 2 + 127 = 129 = (10000001)_2$$

Mantissa, $m = 00000000000000000000000$

So, $R_3 = (-4)$ in IEEE 754 binary format is,

$$\underset{\substack{\text{Sign}\\ \text{bit-8}}}{1}\ ,\ \underbrace{10000001}_{\text{8-bit exponent }(e)}\ ,\ \underbrace{00000000000000000000000}_{\text{23-bit mantissa}(m)}$$

Convert it into hexadecimal notation as,

$$\underset{12=C}{1100}\ \underset{0}{0000}\ \underset{8}{1000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}$$

So, hexadecimal notation of R_3 is,

$$(0\times C0800000)$$

Hence, the correct option is (B).

Method 2

R_1 in hexadecimal notation is given as,

$$0\times 42200000$$

Convert it into 32-bit binary number as,

$$\underset{\substack{\text{Sign}\\ \text{bit-8}}}{0}\ ,\ \underbrace{10000100}_{\text{8-bit exponent }(e)}\ ,\ \underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$$

So, R_1 can be represented according to IEEE 754 single precision format as,

$$R_1 = (-1)^{s}\times(1.m)\times 2^{e-\text{Bias}}$$

$$= (-1)^{0}\times(1.01000000000000000000000)\times 2^{132-127}$$

(For IEEE 754 format $\text{Bias} = 127$ fixed)

$$= (1.01000000000000000000000)\times 2^{5}$$

Shifting 3-times left,

$$= (1010.00000000000000000000)\times 2^{2}$$

The decimal equivalent of above number is,

$$= 10\times 2^{2}$$

$$= 10\times 4$$

$$R_1 = +40$$

Similarly, R_2 in hexadecimal notation is given as,

$$0\times C1200000$$

Convert it into 32-bit binary number as,

$$\underset{\substack{\text{Sign}\\ \text{bit-8}}}{1}\ ,\ \underbrace{10000010}_{\text{8-bit exponent }(e)}\ ,\ \underbrace{01000000000000000000000}_{\text{23-bit mantissa}(m)}$$

So, R_2 can be represented according to IEEE 754 single precision format as,

$$R_2 = (-1)^{s}\times(1.m)\times 2^{e-\text{Bias}}$$

$$= (-1)^{1}\times(1.01000000000000000000000)\times 2^{130-127}$$

(For IEEE 754 format $\text{Bias} = 127$ fixed)

$$= -(1.01000000000000000000000)\times 2^{3}$$

Shifting 3-times left,

$$= -(1010.00000000000000000000)\times 2^{0}$$

The decimal equivalent of above number is,

$$= -10\times 2^{0}$$

$$= -10\times 1$$

$$R_2 = -10$$

According to question,

$$R_3 = \frac{R_1}{R_2} = \frac{40}{-10} = -4$$

$R_3 = -4$ can be represented IEEE 754 single precision format as,

$$R_3 = (-1)^{1}\times 1.0\times 2^{2}$$

$$R_3 = (-1)^{1}\times(1.00000000000000000000000)\times 2^{2}$$

Here, sign bit $(s) = 1$

Mantissa $(m) = 00000000000000000000000$

$e - \text{Bias} = 2$

$e = 2 + \text{Bias}$

$e = 2 + 127$

$e = 129 = 10000001$

So, R_3 in IEEE 754 single precision binary format is,

$$\underset{\text{Sign bit-8}}{1}\ ,\ \underbrace{10000001}_{\text{8-bit exponent }(e)}\ ,\underbrace{00000000000000000000000}_{\text{23-bit mantissa}(m)}$$

Convert it into hexadecimal notation as,

$$\underset{12=C}{1100}\ \underset{0}{0000}\ \underset{8}{1000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}\ \underset{0}{0000}$$

So, hexadecimal notation of R_3 is,

$$(0\times C0800000)$$

Hence, the correct option is (B).

1.15 **(A)**

Given : The representation of a number in base 3 is 210.

Step 1 : Convert the given number representation to decimal representation (Base 10)

$$2\times 3^2+1\times 3^1+0\times 3^0$$

$$18+3=(21)_{10}$$

Step 2 : Now convert decimal representation i.e. $(21)_{10}$ into hexadecimal representation

$$\begin{array}{c|cl} 16 & 21 & \text{remainder} \\ \hline & 1 & \rightarrow 5 \uparrow \end{array}$$

Hexadecimal equivalent is $=(15)_{16}$

Hence, the correct option is (A).

1.16 **-7.75**

Given : $S:1 \quad E:10000001$

$$F:11110000000000000000000$$

IEEE 754 single-precision floating point format with bias of 127

Sign bit = 1, therefore actual number will be negative

Bias Exponent $=10000001$

So, its decimal equivalent is

$$=2^7+2^0$$

$$=128+1=129$$

$$e=Bias\ Exponent-127$$

$$=129-127$$

$$=2$$

Therefore, binary number is $=-1.1111\times 2^2$

Its decimal equivalent is

$$=-\left(1\times 2^2+1\times 2^1+1\times 2^0+1\times 2^{-1}1\times 2^{-2}\right)$$

$$=(-7.75)_{10}$$

Hence, the correct answer is $=(-7.75)_{10}$

1.17 **3**

Given :

x and y are two decimal digits

$$(0.1101)_2=(0.8xy5)_{10} \qquad \ldots\text{(i)}$$

Decimal equivalent of given fractional binary number $(0.1101)_2$ is

$$=1\times 2^{-1}+1\times 2^{-2}+0\times 2^{-3}+1\times 2^{-4}$$

$$=2^{-1}+2^{-2}+2^{-4}$$

$$=0.0625+0.25+0.5$$

$$=(0.8125)_{10} \qquad \ldots\text{(ii)}$$

Comparing equation (i) and (ii) we get,

$$(0.8125)_{10}=(0.8xy5)_{10}$$

Therefore, $\qquad x=1,\ y=2$

$\Rightarrow \qquad x+y=3$

Hence, the correct answer is 3.

1.18 **(C)**

In IEEE 754 representation all 1s in exponent field is reserved for special numbers.

+ (when sign bit is positive) and – (when sign bit is negative) infinities when all mantissa bits are zeroes.

SNAN (Signaling Not A Number) : when leading mantissa bit is 0 and at least one other mantissa bit is non-zero.

NAN (Quiet NAN) : when leading mantissa bit is 1.

Also, all 0s for exponent field is reserved for denormalized numbers (small numbers between 0 and ±1 which cannot be represented using normalized numbers). That is, a normalized IEEE 754 represented number (both single and double precision) must have at least one bit set in the exponent field and for the smallest exponent this will be the right most bit. Now, to make it the smallest positive normalized number in single-precision format, we can have all mantissa bits 0 which will give the numerical value as 1.

$$\underbrace{000\ldots0}_{23\text{ zeroes}} \times 2^{1-127} = 2^{-126}.$$

(Here, 1 before "." is implied in IEEE 754 representation for every normalized numbers and 127 is the exponent bias used to have negative exponents without an explicit sign bit)

Hence, the correct option is (C).

2 Boolean Algebra

Practice Questions

2013 **IIT Bombay**

2.1 Which one of the following expressions does NOT represent exclusive NOR of x and y?

(A) $xy + x'y'$ (B) $x \oplus y'$

(C) $x' \oplus y$ (D) $x' \oplus y'$

2014 **IIT Kharagpur**

2.2 Consider the following Boolean expression for F:

$$(P,Q,R,S) = PQ + \overline{P}QR + \overline{P}Q\overline{R}S .$$

The minimal sum-of-products form of F is ________.

(A) $PQ + QR + QS$

(B) $P + Q + R + S$

(C) $\overline{P} + \overline{Q} + \overline{R} + \overline{S}$

(D) $\overline{P}R + \overline{P}\,\overline{R}S + P$

2.3 The dual of a Boolean function F $\left(\mathrm{X}_1,\mathrm{X}_2,\ldots\ldots\ldots\ldots\ldots,\mathrm{X}_n,+,\cdot,'\right)$, written as F^D, is the same expression as that of F with + and · swapped. F is said to be self-dual if F = F^D. The number of self-dual function with n Boolean variables is

(A) 2^n (B) 2^{n-1}

(C) 2^{2n} (D) $2^{2^{n-1}}$

2.4 Consider the following minterm expression of

$$F\left(P,Q,R,S\right) = \sum m\left(0,2,5,7,8,10,13,15\right)$$

The minterms 2, 7, 8 and 13 are 'don't care' terms. The minimal sum-of-products form for F is ________.

(A) $Q\overline{S} + \overline{Q}S$

(B) $\overline{Q}\,\overline{S} + QS$

(C) $\overline{Q}\,\overline{R}\,\overline{S} + \overline{Q}R\overline{S} + Q\overline{R}S + QRS$

(D) $\overline{P}\,\overline{Q}\,\overline{S} + \overline{P}QS + PQS + P\overline{Q}\,\overline{S}$

2.5 Let $\oplus$ denote the exclusive OR(XOR) operation. Let '1' and '0' denote the binary constants. Consider the following Boolean expression for F over two variables P and Q:

$$F(P,Q) = (1 \oplus P) \oplus (P \oplus Q) \oplus (P \oplus Q) \oplus (Q \oplus 0)$$

The equivalent expression for F is

(A) $P + Q$ (B) $\overline{P + Q}$

(C) $P \oplus Q$ (D) $\overline{P \oplus Q}$

2015 **IIT Kanpur**

2.6 Consider the operations

$f\left(X,Y,Z\right) = X'YZ + XY' + Y'Z'$ and

$g\left(X,Y,Z\right) = X'YZ + XYZ' + XY$ Which one of the following is correct?

(A) Both $\{f\}$ and $\{g\}$ are functionally complete

(B) Only $\{f\}$ is functionally complete

(C) Only $\{g\}$ is functionally complete

(D) Neither $\{f\}$ nor $\{g\}$ is functionally complete

2.7 The number of min-term after minimizing the following Boolean expression is ________.

$$[D' + AB' + A'C + AC'D + A'C'D]'$$

2.8 The total number of prime implicants of the function $f\left(w,x,y,z\right) = \sum (0,\ 2,\ 4,\ 5,\ 6,\ 10)$ is ________.

2.9 Given the function $F = P' + QR$, where F is a function in three Boolean variables P, Q and R and $P' = !P$, consider the following statements.

(S1) $F = \sum\left(4,5,6\right)$

(S2) $F = \sum\left(0,1,2,3,7\right)$

(S3) $F = \prod\left(4,5,6\right)$

(S4) $F = \prod\left(0,1,2,3,7\right)$

Which of the following is true?

(A) (S1) – False, (S2) – True, (S3) – True, (S4) – False

(B) (S1) – True, (S2) – False, (S3) – False, (S4) – True

(C) (S1) – False, (S2) – False, (S3) – True, (S4) – True

(D) (S1) – True, (S2) – True, (S3) – False, (S4) – False

2016 IISc Bangalore

2.10 Consider the Boolean operator # with the following properties :

$x\#0 = x,\ x\#1 = \overline{x},\ x\#x = 0$, and $x\#\overline{x} = 1$.

Then $x\#y$ is equivalent to

(A) $x\overline{y} + \overline{x}y$ (B) $x\overline{y} + \overline{x}\,\overline{y}$

(C) $\overline{x}y + xy$ (D) $xy + \overline{x}\,\overline{y}$

2.11 Let, $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$ where x_1, x_2, x_3, x_4 are Boolean variables, and $\oplus$ is the XOR operator.

Which one of the following must always be TRUE?

(A) $x_1, x_2, x_3, x_4 = 0$

(B) $x_1, x_3 + x_2 = 0$

(C) $\overline{x_1} \oplus \overline{x_3} = \overline{x_2} \oplus \overline{x_4}$

(D) $x_1 + x_2 + x_3 + x_4 = 0$

2017 IIT Roorkee

2.12 Consider the Karnaugh map given below, where x represents "don't care" and blank represents 0

dc \ ba	00	01	11	10
00		×	×	
01	1			×
11	1			1
10		×	×	

Assume for all inputs (a, b, c, d), the respective complements $\left(\overline{a}, \overline{b}, \overline{c}, \overline{d}\right)$ are also available. The above logic is implemented using 2-input NOR gates only. The minimum number of gates required is _______.

2.13 Given, $f\left(w, x, y, z\right) = \Sigma_m$ (0, 1, 2, 3, 7, 8, 10) + Σ_d (5, 6, 11, 15) where d represents the don't-care condition in Karnaugh maps. Which of the following is a minimum product-of- sum (POS) form of $f\left(w, x, y, z\right)$?

(A) f = $\left(\overline{w} + \overline{z}\right)\left(\overline{x} + z\right)$

(B) f = $\left(\overline{w} + z\right)\left(x + z\right)$

(C) f = $\left(w + z\right)\left(\overline{x} + z\right)$

(D) f = $\left(w + \overline{z}\right)\left(\overline{x} + z\right)$

2.14 If $w,\ x,\ y,\ z$, are Boolean variables, then which one of the following is INCORRECT?

(A) $wx + w\left(x + y\right) + x\left(x + y\right) = x + wy$

(B) $\overline{w\overline{x}\left(y + \overline{z}\right)} + \overline{w}x = \overline{w} + x + \overline{y}z$

(C) $\left(w\overline{x}\left(y + x\overline{z}\right) + \overline{w}\,\overline{x}\right)y = x\overline{y}$

(D) $\left(w + y\right)\left(wxy + wyz\right) = wxy + wyz$

2018 IIT Guwahati

2.15 Let $\oplus$ and $\odot$ denote the Exclusive OR and Exclusive NOR operations, respectively. Which one of the following is NOT CORRECT?

(A) $\overline{P \oplus Q} = P \odot Q$

(B) $\overline{P} \oplus Q = P \odot Q$

(C) $\overline{P} \oplus \overline{Q} = P \oplus Q$

(D) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

2.16 Consider the minterm list form of a Boolean function F given below.

$$F(P, Q, R, S) = \sum m(0, 2, 5, 7, 9, 11)$$
$$+ d(3, 8, 10, 12, 14)$$

Here, m denotes a minterm and d denotes a don't care term. The number of essential prime implicants of the function F is _______.

2019 IIT Madras

2.17 Which one of the following is Not a valid identity?

(A) $x \oplus y = (xy + x'y')'$

(B) $x \oplus y = x + y$, if $xy = 0$

(C) $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

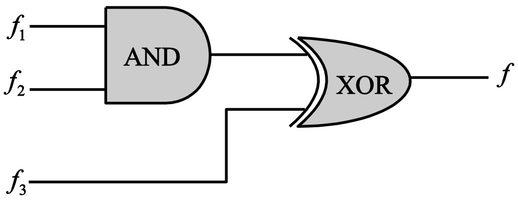
(D) $(x + y) \oplus z = x \oplus (y \oplus z)$

2.18 What is the minimum number of 2-input NOR gates required two implement a 4-variable function expressed in sum-of-minterms form as $f = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$? Assume that all the inputs and their complements are available. Answer : __________.

2.19 Consider three 4-variable function $f_1, f_2,$ and $f_3,$ which are expressed in sum-of – minterms as

$$f_1 = \Sigma(0,2,5,8,14)$$
$$f_2 = \Sigma(2,3,6,8,14,15)$$
$$f_3 = \Sigma(2,7,11,14)$$

For the following circuit with one AND gate and one XOR gate, the output function f can be expressed as:



(A) $\Sigma(7,8,11)$

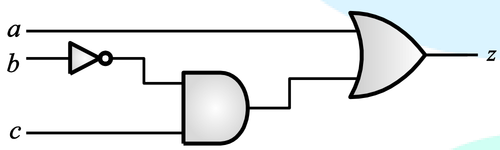
(B) $\Sigma(2,7,8,11,14)$

(C) $\Sigma(2,14)$

(D) $\Sigma(0,2,3,5,6,7,8,11,14,15)$

2020 IIT Delhi

2.20 Consider the Boolean function $z(a,b,c)$.



Which one of the following minterm lists represents the circuit given above?

(A) $z = \sum(0,1,3,7)$

(B) $z = \sum(1,4,5,6,7)$

(C) $z = \sum(2,4,5,6,7)$

(D) $z = \sum(2,3,5)$

2021 IIT Bombay

2.21 Consider the following Boolean expression $F = (X+Y+Z)(\overline{X}+Y)(\overline{Y}+Z)$.

Which of the following Boolean expressions is/are equivalent to $\overline{F}$ (complement of F)?

(A) $(\overline{X}+\overline{Y}+\overline{Z})(X+\overline{Y})(Y+\overline{Z})$

(B) $X\overline{Y}+\overline{Z}$

(C) $(X+\overline{Z})(\overline{Y}+\overline{Z})$

(D) $X\overline{Y}+Y\overline{Z}+\overline{X}\,\overline{Y}\,\overline{Z}$

2.22 Consider a Boolean function $f(w,x,y,z)$ such that

$$f(w,0,0,z) = 1$$
$$f(1,x,1,z) = x+z$$
$$f(w,1,y,z) = wz+y$$

The number of literals in the minimal sum-of-products expression of f is ______.

Solutions

2.1 (D)

Checking from options,

From option (A) :

$\Rightarrow \quad xy+\overline{x}\,\overline{y}$

$\Rightarrow \quad x \odot y$

It shows 2-input EX-NOR expression.

Hence, option (A) is incorrect.

From option (B) :

$\Rightarrow \quad x \oplus \overline{y}$

$\Rightarrow \quad \overline{x}\,\overline{y}+x\overline{\overline{y}}$

$\Rightarrow \quad \overline{x}\,\overline{y}+xy$

$\Rightarrow \quad x \odot y$

It shows 2-input EX-NOR expression.

Hence, option (B) is incorrect.

From option (C) :

$\Rightarrow \quad \overline{x} \oplus y$

$\Rightarrow \quad \overline{x}\,\overline{y}+\overline{\overline{x}}y$

$\Rightarrow \quad \overline{x}\,\overline{y}+xy$

$\Rightarrow \quad x \odot y$

It shows 2-input EX-NOR expression.

Hence, option (C) is incorrect.

From option (D) :

$\Rightarrow \quad \overline{x} \oplus \overline{y}$

$\Rightarrow \quad \overline{\overline{x}}\,\overline{y}+\overline{x}\,\overline{\overline{y}}$

$\Rightarrow \quad x\overline{y}+\overline{x}y$

$\Rightarrow \quad x \oplus y$

It does not show 2-input EX-NOR expression.

Hence, option (D) is correct.

Hence, the correct option is (D).

2.2 **(A)**

Given : 4-variable function F is,

$$F(P, Q, R, S) = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$$

Method 1

Using Boolean algebra rules,

$\bar{P}Q$ common

$$F = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$$

Q common

$$F = Q(P + \bar{P}R) + \bar{P}Q(R + \bar{R}S)$$

Apply distributive law,

$$F = Q[(P + \bar{P})(P + R)] + \bar{P}Q[(R + \bar{R})(R + S)]$$

$$(\because P + \bar{P} = 1 \text{ and } R + \bar{R} = 1)$$

$$F = Q(P + R) + \bar{P}Q(R + S)$$

$$F = PQ + \underbrace{QR + \bar{P}QR}_{QR \text{ common}} + \bar{P}QS$$

$$F = PQ + \bar{P}QS + QR(1 + \bar{P}) \quad (\because 1 + \bar{P} = 1)$$

$$F = \underbrace{PQ + \bar{P}QS}_{Q \text{ common}} + QR$$

Apply distributive law,

$$F = Q(P + \bar{P})(P + S) + QR \ (\because P + \bar{P} = 1)$$

$$F = Q(P + S) + QR$$

$$F = PQ + QS + QR$$

Hence, the correct option is (A).

Method 2

$$F(P, Q, R, S) = \underbrace{PQ + \bar{P}QR + \bar{P}Q\bar{R}S}_{\text{3-Product terms}}$$

Converting each product term into minterms as,

$$PQ \rightarrow \begin{cases} PQ\bar{R}\bar{S} \rightarrow 1100 \rightarrow 12 \rightarrow m_{12} \\ PQ\bar{R}S \rightarrow 1101 \rightarrow 13 \rightarrow m_{13} \\ PQR\bar{S} \rightarrow 1110 \rightarrow 14 \rightarrow m_{14} \\ PQRS \rightarrow 1111 \rightarrow 15 \rightarrow m_{15} \end{cases}$$

$$\bar{P}QR \rightarrow \begin{cases} \bar{P}QR\bar{S} \rightarrow 0110 \rightarrow 6 \rightarrow m_6 \\ \bar{P}QRS \rightarrow 0111 \rightarrow 7 \rightarrow m_7 \end{cases}$$

$$\bar{P}Q\bar{R}S \rightarrow 0101 \rightarrow 5 \rightarrow m_5$$

So, F can be written as,

$$F = \Sigma m(m_5, m_6, m_7, m_{12}, m_{13}, m_{14}, m_{15})$$

$$F = \Sigma m(5, 6, 7, 12, 13, 14, 15)$$

Now, K-map in SOP format as,

$PQ \backslash RS$	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$				
$\bar{P}Q$		1	1	1
PQ	1	1	1	1
$P\bar{Q}$				

QS (Quad)

QR (Quad)

PQ (Quad)

So, minimized function can be written as,

$$F = PQ + QR + QS$$

Hence, the correct option is (A).

2.3 **(D)**

Number of self-dual function for n-variation is $2^{2^{n-1}}$.

Hence, the correct option is (D).

Key Point

- Number of distinct Boolean function formed by n-variable is $\Rightarrow 2^{2^n}$.
- Number of distinct dual Boolean function formed by n-variable is $\Rightarrow 2^{2n}$
- Number of distinct combination formed by n-variable is $\Rightarrow 2^n$.

2.4 **(B)**

Given : Expression of 4-variation is,

$$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 8, 10, 13, 15) \quad \ldots(i)$$

Here, minterm 2, 7, 8, 11 are treated as don't cares term so equation (i) can be written as,

$$F(P, Q, R, S) = \Sigma m(0, 5, 10, 13, 15) + \Sigma d(2, 7, 8, 11)$$

Now, making K-map in SOP format,

$PQ \backslash RS$	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$	1			X
$\bar{P}Q$		1	X	
PQ		1	1	
$P\bar{Q}$	X		X	1

QS (Quad)

$\bar{Q}\bar{S}$ (Quad)

So, minimal sum-of-product form of F is,

$$F = \bar{Q}\,\bar{S} + QS$$

Hence, the correct option (B).

2.5 **(D)**

Given : Expression is,

$$F(P,\ Q) = [(1 \oplus P) \oplus (P \oplus Q)]$$
$$\oplus [(P \oplus Q) \oplus (Q \oplus 0)]$$

Before proceeding in solution, we may write the properties of EX-OR gate as,

$$X \oplus X = 0$$
$$X \oplus 0 = X$$
$$X \oplus \bar{X} = 1$$
$$X \oplus 1 = \bar{X}$$

So, equation (i) becomes,

$$F = [\bar{P} \oplus (P \oplus Q)] \oplus [(P \oplus Q) \oplus Q]$$
$$F = [\underbrace{\bar{P} \oplus P}_{1} \oplus Q] \oplus [P \oplus \underbrace{Q \oplus Q}_{0}]$$
$$F = [\underbrace{1 \oplus Q}_{\bar{Q}}] \oplus [\underbrace{P \oplus Q}_{P}]$$
$$F = \bar{Q} \oplus P \qquad (\because\ P \oplus \bar{Q} = P \odot Q)$$
$$F = P \odot Q = \overline{P \oplus Q}$$

Hence, the correct option is (D).

Key Point

S. No.	For X-OR gate	For X-NOR gate
1.	$A \oplus \bar{A} = 1$ $A \oplus 1 = \bar{A}$ $\bar{A} \oplus 1 = A$	$A \odot \bar{A} = 0$ $A \odot 0 = \bar{A}$ $\bar{A} \odot 0 = A$
2.	$A \oplus A = 0$ $A \oplus 0 = A$	$A \odot A = 1$ $A \odot 1 = A$
3.	$\overline{A \oplus B} = A \odot B$	$\overline{A \odot B} = A \oplus B$
4.	$\bar{A} \oplus B = A \odot B$ $A \oplus \bar{B} = A \odot B$	$\bar{A} \odot B = A \oplus B$ $A \odot \bar{B} = A \oplus B$

2.6 **(B)**

Given : Functions are,

$$f(xyz) = \bar{x}yz + x\bar{y} + \bar{y}\,\bar{z} \qquad \ldots\text{(i)}$$
$$g(xyz) = \bar{x}yz + \bar{x}y\bar{z} + xy \qquad \ldots\text{(ii)}$$

Converting equation (i) into 2-variable equation by putting $y = x$ in equation (i) as,

$$f(x,\ x,\ z) = \bar{x}xz + x\bar{x} + \bar{x}\,\bar{z} \qquad (\because\ x\bar{x} = 0)$$
$$f(x,\ z) = 0 + 0 + \bar{x}\,\bar{z}$$
$$f(x,\ z) = \overline{x + z} \rightarrow \text{NOR rule}$$

Thus, given function $f(x, y,\ z)$ is converted into universal NOR logic so, $f(x, y,\ z)$ is functionally complete.

Similarly, converting equation (ii) into 2-variable equation by putting $y = x$ in equation (ii) as,

$$g(x,\ x, z) = \bar{x}xz + \bar{x}x\bar{z} + xx$$
$$(\because\ x\bar{x} = 0 \text{ and } xx = x)$$
$$g(x,\ z) = 0 + 0 + x$$
$$g(x,\ z) = x$$

So, $g(x, y,\ z)$ can't be converted in universal logic (i.e. NAND and NOR) that's why $g(x,\ y,\ z)$ is not a functionally complete.

Hence, the correct option is (B).

2.7 **4**

Given : Function f is shown below,

$$f = \overline{[\bar{D} + A\bar{B} + \bar{A}C + A\bar{C}D + \bar{A}\bar{C}D]} \qquad \ldots\text{(i)}$$

Assume, $f = \bar{g}$

So, $\quad g = \bar{D} + A\bar{B} + \bar{A}C + A\bar{C}D + \bar{A}\bar{C}D$

Now, converting equation (ii), into canonical standard SOP form as,

$$g = \bar{D}(A + \bar{A})(B + \bar{B})(C + \bar{C})$$
$$+ AB(C + \bar{C})(D + \bar{D}) + \bar{A}C(B + \bar{B})(D + \bar{D})$$
$$+ A\bar{C}D(B + \bar{B}) + \bar{A}\bar{C}D(B + \bar{B})$$

Now expand g as,

$$g = \underbrace{\bar{A}\bar{B}\bar{C}\bar{D}}_{1} + \underbrace{\bar{A}\bar{B}C\bar{D}}_{2} + \underbrace{\bar{A}\bar{B}CD}_{3} + \underbrace{\bar{A}B\bar{C}D}_{5}$$
$$+ \underbrace{\bar{A}BC\bar{D}}_{6} + \underbrace{\bar{A}BCD}_{7} + \underbrace{A\bar{B}\bar{C}\bar{D}}_{8} + \underbrace{A\bar{B}\bar{C}D}_{9}$$
$$+ \underbrace{A\bar{B}C\bar{D}}_{10} + \underbrace{A\bar{B}CD}_{11} + \underbrace{AB\bar{C}D}_{13} + \underbrace{ABCD}_{15}$$

So, g can be written in SOP form with the help of minterms as,

$$g = \Sigma m(1,\ 2,\ 3,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 13,\ 15)$$

Now, $\bar{g} = \Sigma m(0,\ 4,\ 12,\ 14)$

So, function f becomes,

$$f = \bar{g} = \Sigma m\underbrace{(0,\ 4,\ 12,\ 14)}_{\text{4-minterms}}$$

So, f will contain 4-minterms.

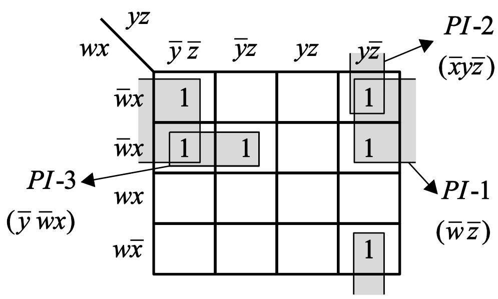
Hence, the correct answer is **4**.

2.8 **3**

Given : 4-variable function f is,

$f(w,\ x,\ y,\ z) = \Sigma m(0,\ 2,\ 4,\ 5,\ 6,\ 10)$

Now, K-map in SOP form for function f as,



So, number of Prime Implicant (PI) in function f is **3**.

2.9 **(A)**

Given : 3-variable Boolean function in SOP form is,

$$F = \bar{P} + QR \qquad \ldots\text{(i)}$$

Converting equation (i) into standard canonic SOP form as,

$$F = \bar{P}(Q+\bar{Q})(R+\bar{R}) + QR(P+\bar{P})$$

$$F = \underset{0}{\bar{P}\bar{Q}\bar{R}} + \underset{1}{\bar{P}\bar{Q}R} + \underset{2}{\bar{P}Q\bar{R}} + \underset{3}{\bar{P}QR} + \underset{7}{PQR}$$

So, F can be written with the help of minterm in SOP format as,

$$F = \Sigma m(0,\ 1,\ 2,\ 3,\ 7) \rightarrow \text{SOP form}$$

And its POS form as,

$$F = \pi M(4,\ 5,\ 6) \rightarrow \text{ POS form}$$

So, statements S_2 and S_3 are correct and S_1, S_4 are wrong.

Hence, the correct option is (A).

2.10 **(A)**

Given : Boolean operator # with its properties function is,

$$x \# 0 = x \qquad \ldots\text{(i)}$$

$$x \# 1 = \bar{x} \qquad \ldots\text{(ii)}$$

$$x \# x = 0 \qquad \ldots\text{(iii)}$$

$$x \# \bar{x} = 1 \qquad \ldots\text{(iv)}$$

From equation (iii) and (iv), we can observe that,

(i) If both input are same then this operator # makes output is zero.

(ii) If both input are different then this operator # makes output is logic-1.

Now, consider two input x, y and its truth table as,

Input		Output
x	y	$x \# y$
0	0	0
0	1	1
1	0	1
1	1	0

Both input are same

Both input are different

So, this truth table follow the function of EX-OR logic operation.

Thus, $\quad x \# y = x \oplus y = x\bar{y} + \bar{x}y$

Hence, the correct option is (A).

2.11 **(C)**

Given : $x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 0$

Let $x_1 = 1, x_2 = 1, x_3 = 1 \text{ and } x_4 = 1$

Such that

$$x_1 \oplus x_2 \oplus x_3 \oplus x_4 = 1 \oplus 1 \oplus 1 \oplus 1 = 0$$

From option (A) :

$$x_1 x_2 x_3 x_4 = 0$$

$$1 \cdot 1 \cdot 1 \cdot 1 = 0$$

$1 \neq 0$, Hence false

From option (B) :

$$x_1\, x_3 + x_2 = 0$$

$$1 \cdot 1 + 1 = 0$$

$$1 + 1 = 0$$

$1 \neq 0$; Hence false

From option (C) :

$$\bar{x}_1 \oplus \bar{x}_3 = \bar{x}_2 \oplus \bar{x}_4$$

Is always True.

From option (D) :

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$1 + 1 + 1 + 1 = 0$$

$1 \neq 0$; Hence False

So, the correct option is (C).

2.12 **1**

Given : K-map is shown below,

$dc \backslash ba$	00	01	11	10
00		X	X	
01	1			X
11	1			1
10		X	X	

$\bar{a}c$ (Quad)

So, minimize function f from above K-map in SOP form is,

$$f = \bar{a}c$$

Again, Re-write above equation as,

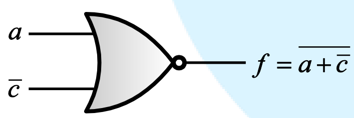
$$f = \overline{\overline{\bar{a}c}}$$

Apply De-Morgan law,

$$f = \overline{\bar{\bar{a}} + \bar{c}}$$

$$f = \overline{a + \bar{c}}$$

Now, implement $f = \overline{a + \bar{c}}$ using 2-input NOR gates as,



(Here, complements $\bar{c}$ is available)

So, minimum number of NOR gate requires is **1**.

2.13 (A)

Given : 4-variable SOP function f is,

$$f = \Sigma m(0, 1, 2, 3, 7, 8, 10) + \Sigma d(5, 6, 11, 15)$$

$\rightarrow$ SOP form

Method 1

Now, convert it into POS form as,

$$f = \pi M(4, 9, 12, 13, 14) \cdot \pi d(5, 6, 11, 15)$$

$\rightarrow$ POS form

Now, K-map in POS format is,

$wx \backslash yz$	$y+z$	$y+\bar{z}$	$\bar{y}+\bar{z}$	$\bar{y}+z$
$w+x$				
$w+\bar{x}$	0	X		X
$\bar{w}+\bar{x}$	0	0	X	0
$\bar{w}+x$		0	X	

$(\bar{x}+z)$ (Quad)

$(\bar{w}+\bar{z})$ (Quad)

So, minimized function f from above K-map in POS form is,

$$f(w, x, y, z) = (\bar{w}+\bar{z})(\bar{x}+z)$$

Hence, the correct option is (A).

Method 2

This method basically for those aspirant who are not comfortable with POS, K-map format. In this method, only SOP format of K-map used to get answer in POS format.

Here, f is given as

$$f = \Sigma m(0, 1, 2, 3, 7, 0, 10) + \Sigma d(5, 6, 11, 15)$$

$\rightarrow$ SOP form

So, $\bar{f}$ can be written as,

$$\bar{f} = \Sigma m(4, 9, 12, 13, 14) + \Sigma d(5, 6, 11, 15)$$

$\rightarrow$ SOP form

Now, making SOP K-map for $\bar{f}$ as,

$wx \backslash yz$	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$				
$\bar{w}x$	1	X		X
wx	1	1	X	1
$w\bar{x}$		1	X	

$x\bar{z}$ (Quad)

wz (Quad)

Thus, minimizing function $\bar{f}$ in SOP form from above K-map is,

$$\bar{f} = wz + x\bar{z}$$

Now, taking complement on both sides of above equation as,

$$\bar{\bar{f}} = \overline{wz + x\bar{z}}$$

Apply De-Morgan law,

$$f = (\overline{wz}) \cdot (\overline{x\bar{z}})$$

Again, apply De-Morgan law,

$$f = (\bar{w}+\bar{z})(\bar{x}+z)$$

Hence, the correct option is (A).

2.14 (C)

Checking from options,

From option (A) :

$$wx + w(x+y) + x(x+y) = x + wy$$

L.H.S. :

$\Rightarrow \quad wx + w(x+y) + x(x+y)$

$\Rightarrow \quad wx + wx + wy + xx + xy$

$(\because\ wx + wx = wx \text{ and } xx = x)$

$\Rightarrow \quad x(1 + y + w) + wy \quad (\because 1 + \text{Any literal} = 1)$

$\Rightarrow \quad x + wy = \text{R.H.S.}$

So, option (A) true.

From option (B) :

$$\overline{w\bar{x}(y+\bar{z})} + \bar{w}x = \bar{w} + x + \bar{y}z$$

L.H.S. :

$\Rightarrow \quad \overline{w\bar{x}(y+\bar{z})} + \bar{w}x$

Apply De-Morgan law,

$\Rightarrow \quad \overline{w\bar{x}} + (\overline{y+\bar{z}}) + \bar{w}x$

Again, apply De-Morgan law,

$\Rightarrow \quad (\bar{w}+x) + (\bar{y}\cdot z) + \bar{w}x$

$\Rightarrow \quad \bar{w} + (x + x\bar{w}) + \bar{y}z$

$\Rightarrow \quad \bar{w} + x(1+\bar{w}) + \bar{y}z \qquad (\because\ 1+\bar{w}=1)$

$\Rightarrow \quad \bar{w} + x + \bar{y}z = \text{R.H.S.}$

So, option (B) is true.

From option (C) :

$$[w\bar{x}(y+x\bar{z}) + \bar{w}\,\bar{x}]y = x\bar{y}$$

L.H.S. :

$\Rightarrow \quad [w\bar{x}(y+x\bar{z}) + \bar{w}\,\bar{x}]y$

$\Rightarrow \quad (w\bar{x}y + w\bar{x}x\bar{z} + \bar{w}\,\bar{x})y \qquad (\because\ x\bar{x}=0)$

$\Rightarrow \quad (w\bar{x}yy + 0 + \bar{w}\,\bar{x}y)$

$\Rightarrow \quad (w\bar{x}y + \bar{w}\,\bar{x}y)$

$\Rightarrow \quad \bar{x}y(w+\bar{w}) \qquad (\because\ w+\bar{w}=1)$

$\Rightarrow \quad \bar{x}y \neq \text{R.H.S.}$

So, option (C) is false.

From option (D) :

$$(w+y)(wxy+wyz) = (wxy+wyz)$$

L.H.S. :

$\Rightarrow \quad (w+y)(wxy+wyz)$

$\Rightarrow \quad wwxy + wwy + wxyy + wyyz$

$(\because\ ww=w,\ yy=y)$

$\Rightarrow \quad wxy + wyz + wxy + wyz$

$\Rightarrow \quad wxy + wyz = \text{R.H.S.}$

So, option (d) is true.

Hence, the correct option is (C).

2.15 (D)

Checking from options,

From option (A) :

$$\overline{P \oplus Q} = P \odot Q$$

L.H.S. :

$\Rightarrow \quad \overline{P \oplus Q}$

$\Rightarrow \quad \overline{P\bar{Q} + \bar{P}Q}$

Apply De-Morgan law,

$\Rightarrow \quad (\overline{P\bar{Q}})\cdot(\overline{\bar{P}Q})$

Again, apply De-Morgan law,

$\Rightarrow \quad (\bar{P} + \bar{\bar{Q}})\cdot(\bar{\bar{P}} + \bar{Q})$

$\Rightarrow \quad (\bar{P}+Q)(P+\bar{Q})$

$\Rightarrow \quad PQ + \bar{P}\bar{Q}$

$\Rightarrow \quad P \odot Q = \text{R.H.S.}$

So, option (A) is true.

From option (B) :

$$\bar{P} \oplus Q = P \odot Q$$

L.H.S. :

$\Rightarrow \quad \bar{P} \oplus Q$

$\Rightarrow \quad \bar{P}\bar{Q} + \bar{\bar{P}}Q$

$\Rightarrow \quad \bar{P}\bar{Q} + PQ$

$\Rightarrow \quad P \odot Q = \text{R.H.S.}$

So, option (B) is true.

From option (C) :

$\Rightarrow \quad \bar{P} \oplus \bar{Q} = P \oplus Q$

L.H.S. :

$\Rightarrow \quad \bar{P} \oplus \bar{Q}$

$\Rightarrow \quad \bar{P}\bar{\bar{Q}} + \bar{\bar{P}}\bar{Q}$

$\Rightarrow \quad \bar{P}Q + P\bar{Q}$

$\Rightarrow \quad P \oplus Q = \text{R.H.S.}$

So, option (C) is true.

From option (D) :

$$(P \oplus \bar{P})Q = (P \odot \bar{P}) \odot \bar{Q}$$

L.H.S. :

$\Rightarrow \quad (P \oplus \bar{P})Q \qquad (\because\ P \oplus \bar{P} = 1)$

$\Rightarrow \quad 1 \oplus Q \qquad (\because\ 1 \oplus Q = \bar{Q})$

$\Rightarrow \quad \bar{Q}$

R.H.S. :

$\Rightarrow \quad (P \odot \bar{P}) \odot \bar{Q} \qquad (\because\ P \odot \bar{P} = 0)$

$\Rightarrow \quad 0 \odot \bar{Q} \qquad (\because\ 0 \odot \bar{Q} = Q)$

$\Rightarrow \quad Q$

So, $\text{L.H.S.} \neq \text{R.H.S.}$

So, option (D) is false.

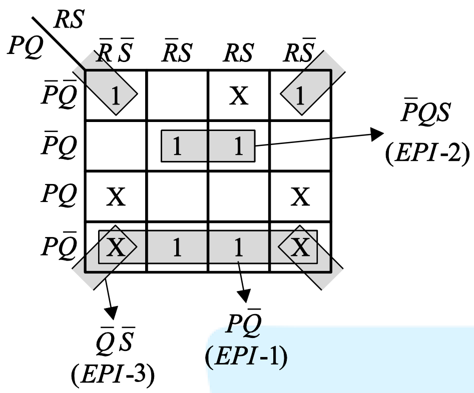
Hence, the correct option is (D).

2.16 **3**

Given : 4-variation Boolean function F is shown below,

$$F(P, Q, R, S) = \Sigma m(0, 2, 5, 7, 9, 11) + \Sigma d(3, 8, 10, 12, 14)$$

So, K-map in SOP format for function F is,



So, number of Essential Prime Implicant (EPI) = **3**

> **Note :** In the same question if examine asked about PI then answer will be 5 (i.e. $\bar{P}QS$, $P\bar{Q}$, $\bar{Q}\bar{S}$, $\bar{P}RS$, $\bar{P}\bar{Q}R$).

2.17 **(D)**

Checking from options,

From option (A) :

$x \oplus y = (\overline{xy + \bar{x}\,\bar{y}})$

R.H.S. :

$\Rightarrow \quad \overline{xy + \bar{x}\,\bar{y}}$

Apply De-Morgan law,

$\Rightarrow \quad (\overline{xy}) \cdot (\overline{\bar{x}\,\bar{y}})$

Again, apply De-Morgan law,

$\Rightarrow \quad (\bar{x} + \bar{y}) \cdot (\bar{\bar{x}} + \bar{\bar{y}})$

$\Rightarrow \quad (\bar{x} + \bar{y}) \cdot (x + y)$

$\Rightarrow \quad \bar{x}y + x\bar{y}$

$\Rightarrow \quad x \oplus y = \text{L.H.S.}$

So, option (A) shows valid identity.

From option (B) :

$x \oplus y = x + y$ if $xy = 0$

L.H.S. :

$\Rightarrow \quad x \oplus y$

$\Rightarrow \quad x\bar{y} + \bar{x}y$

$\Rightarrow \quad (x + y)(\bar{x} + \bar{y})$

$\rightarrow$ POS form of EX-OR logic gate

$\Rightarrow \quad (x + y)(\overline{xy}) \qquad (\because \overline{xy} = \bar{x} + \bar{y})$

Put $xy = 0$,

$\Rightarrow \quad (x + y) = \text{R.H.S.}$

So, option (B) is true.

From option (C) :

$(x \oplus y) \oplus z = x \oplus (y \oplus z)$

It's an associative rule so it is always valid for EX-OR logic.

So, option (C) is true.

From option (D) :

$(x + y) \oplus z = x \oplus (y + z)$

L.H.S. :

$\Rightarrow \quad (x + y) \oplus z$

$\Rightarrow \quad (\overline{x + y})z \neq (x + y)\bar{z} \qquad (\because \overline{x + y} = \bar{x}\,\bar{y})$

$\Rightarrow \quad \bar{x}\,\bar{y}z + x\bar{z} + y\bar{z}$

R.H.S. :

$\Rightarrow \quad x \oplus (y + z)$

$\Rightarrow \quad \bar{x}(y + z) + x(\overline{y + z}) \qquad (\because \overline{y + z} = \bar{y}\,\bar{z})$

$\Rightarrow \quad \bar{x}y + \bar{x}z + x(\bar{y}\,\bar{z})$

So, $\text{R.H.S.} \neq \text{L.H.S.}$

So, option (D) does not show valid identity.

Hence, the correct option is (D).

2.18 **3**

Given : 4-variable SOP form is,

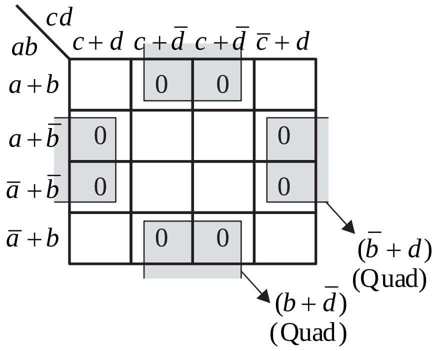
$f = \Sigma m(0, 2, 5, 7, 8, 10, 13, 15) \rightarrow$ SOP form

Here, examines asked about implementation of function using NOR gate only so it is better to solve function (f) in POS form.

So, POS form function (f) is,

$f = \pi M(1, 3, 4, 6, 9, 11, 12, 14) \rightarrow$ POS form

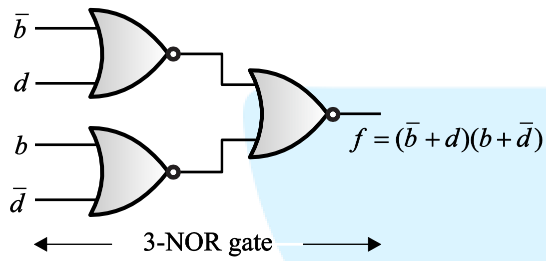
Now, making K-map in POS format for function (f) is,



Minimized form of f in POS format is,

$f=(\bar{b}+d)(b+\bar{d}) \rightarrow$ EX-NOR in POS form

So, 2-input EX-NOR gate using 2-input NOR gate is shown below,



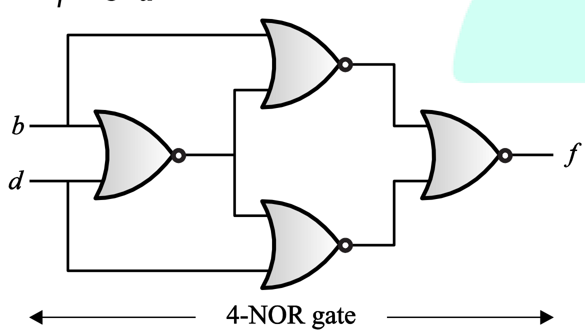
Hence, minimum number of 2-input NOR gate required is **3**.

Key Point

In this question, all the input and their complement are avilable that's why 3-NOR gate is sufficient suppose if examine says, all inputs are available but their complement is not available then it requires 4-NOR gate to implement given functions as,

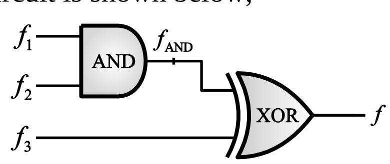
$$f=(\bar{b}+d)(b+\bar{d})$$

$$f=b \cdot d$$

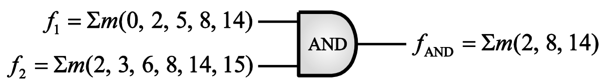


2.19 (A)

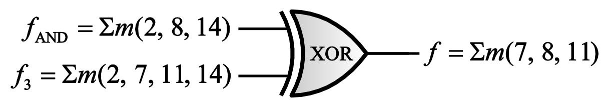
Given : Circuit is shown below,



We know that, AND gate only passes common minterms of inputs that are applied to it so,



XOR gate passes only those minterms from input which are uncommon and blocks the common minterms.



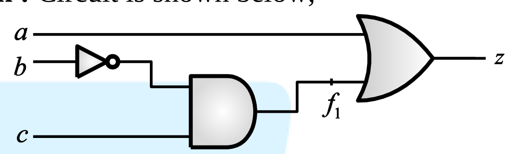
So, output function (f) can be expressed as,

$$f=\Sigma m(7,8,11)$$

Hence, the correct option is (A).

2.20 (B)

Given : Circuit is shown below,

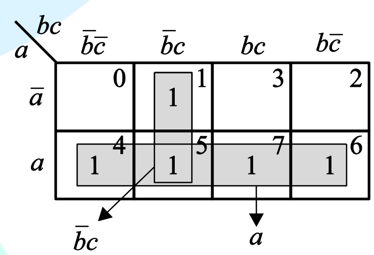


Here, $f_1=\bar{b}c$

So, $z=a+f_1$

$z=a+\bar{b}c$ ...(i)

Making K-map in SOP form according to equation (i) as,



So, minterms present in K-map showing z.

Thus, $z=\Sigma m(1,4,5,6,7)$

Hence, the correct option is (B).

2.21 (B, C, D)

Given :

$$F=(X+Y+Z)(\bar{X}+Y)(\bar{Y}+Z)$$

Method 1

The K-map for the POS function $F(X,Y,Z)$ is shown below:

X \ YZ	$Y+Z$	$Y+\bar{Z}$	$\bar{Y}+\bar{Z}$	$\bar{Y}+Z$
X	0			0
$\bar{X}$	0	0		0

So, $F=\Pi M(0,2,4,5,6)$

Complement of F in POS form

$$\bar{F} = \prod M\,(1,3,7)$$

Complement of F in SOP form

$$\bar{F} = \Sigma m(0,2,4,5,6)$$

Consider each option one by one :

From option (A) :

$$= \left(\bar{X} + \bar{Y} + \bar{Z}\right)\left(X + \bar{Y}\right)\left(Y + \bar{Z}\right)$$

$$= \prod M\,(1,3,2) \neq \bar{F}(POS)$$

Hence, option (A) is incorrect

From option (B) :

$$= X\bar{Y} + \bar{Z}$$

$$= \sum m(0,2,4,5,6) = \bar{F}\,(SOP)$$

Hence, option (B) is correct

From option (C) :

$$= \left(X + \bar{Z}\right)\left(\bar{Y} + \bar{Z}\right)$$

$$= \prod M\,(1,3,7) = \bar{F}(POS)$$

Hence, option (C) is correct

From option (D) :

$$= X\bar{Y} + Y\bar{Z} + \bar{X}\bar{Y}\bar{Z}$$

$$= \sum m(0,2,4,5,6) = \bar{F}(SOP)$$

Hence, option (D) is correct.

Hence, options B, C and D are correct.

Method 2

Given : $F = (X + Y + Z)\left(\bar{X} + Y\right)\left(\bar{Y} + Z\right)$

Taking complement of above expression;

$$\bar{F} = \overline{(X + Y + Z)(\bar{X} + Y)(\bar{Y} + Z)}$$

Applying De-Morgan's law;

$$\bar{F} = \overline{(X + Y + Z)} + \overline{(\bar{X} + Y)} + \overline{(\bar{Y} + Z)}$$

$$\bar{F} = (\bar{X}.\bar{Y}.\bar{Z}) + \bar{\bar{X}}.\bar{Y} + \bar{\bar{Y}}.\bar{Z} \quad \left[\because \bar{\bar{X}} = X\right]$$

$$\therefore \bar{F} = (\bar{X}.\bar{Y}.\bar{Z}) + (X.\bar{Y}) + (Y.\bar{Z}) \qquad \text{...Option (D)}$$

Taking $\bar{Y}$ as common we get;

$$\bar{F} = \bar{Y}\left[(\bar{X}\bar{Z}) + X\right] + Y\bar{Z}$$

$$\left[\because A + BC = (A+B)(A+C)\right]$$

$$\bar{F} = \bar{Y}\left[(X + \bar{X})(X + \bar{Z})\right] + Y\bar{Z}$$

$$\bar{F} = \bar{Y}\left[X + \bar{Z}\right] + Y\bar{Z}$$

$$\bar{F} = X\bar{Y} + \overline{YZ} + Y\bar{Z}$$

Taking $\bar{Z}$ as common

$$\bar{F} = X\bar{Y} + \bar{Z}(Y + \bar{Y}) \qquad \left[\because Y + \bar{Y} = 1\right]$$

$$\therefore \bar{F} = X\bar{Y} + \bar{Z} \qquad \text{... Option(B)}$$

Applying distributive law here we get;

$$\bar{F} = (X + \bar{Z})(\bar{Y} + \bar{Z}) \qquad \text{...Option (C)}$$

Hence, options B, C and D are correct.

2.22 **6**

Given :

Boolean function $f\left(w, x, y, z\right)$ such that

$$f\,(w, 0, 0, z) = 1$$

$$f\,(1, x, 1, z) = x + z$$

$$f\,(w, 1, y, z) = wz + y$$

Truth table for above expression is,

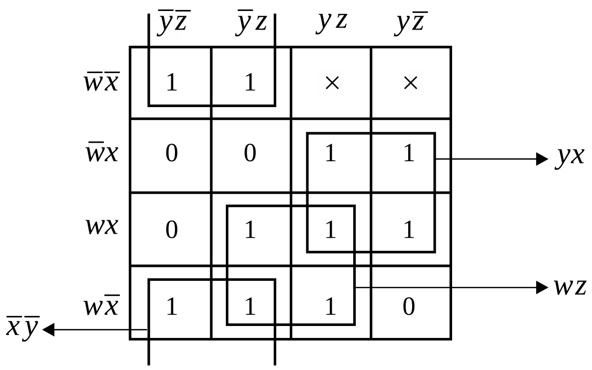
w	x	y	z	f
0	0	0	0	1
0	0	0	1	1
0	0	1	0	x
0	0	1	1	x
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

From the above table SOP function is

$$f\,(w, x, y, z)$$

$$= \sum m\,(0, 1, 6, 7, 8, 9, 11, 13, 14, 15) + d(2, 3)$$

SOP K-Map for $f\,(w, x, y, z)$:



So, minimized function in SOP format is,

$f = xy + \bar{x}\bar{y} + wz$

Therefore, the number of literals in the minimal sum of products expression of f is 6.

Hence, the correct answer is 6.

3 Combinational Circuit

Practice Questions

2013 IIT Bombay

3.1 In the following truth table, $V = 1$ if and only if the input is valid.

INPUTS				OUTPUTS		
D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	x	x	0
1	0	0	0	0	0	1
x	1	0	0	0	1	1
x	x	1	0	1	0	1
x	x	x	1	1	1	1

What function does the truth table represent?

(A) Priority encoder

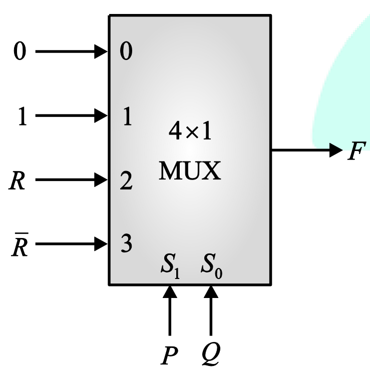
(B) Decoder

(C) Multiplexer

(D) De-multiplexer

2014 IIT Kharagpur

3.2 Consider the 4-to-1 multiplexer with two select lines S_1 and S_0 given below



The minimal sum-of-products form of the Boolean expression for the output F of the multiplexer is

(A) $\overline{P}Q + Q\overline{R} + P\overline{Q}R$

(B) $\overline{P}Q + \overline{P}Q\overline{R} + PQ\overline{R} + P\overline{Q}R$

(C) $\overline{P}QR + \overline{P}Q\overline{R} + Q\overline{R} + P\overline{Q}R$

(D) $PQ\overline{R}$

3.3 Consider the following combinational function block involving four Boolean variables x, y, a, b where x, a, b are inputs and y is the output.

```
f (x, y, a, b)
{
        if (x is 1) y = a;
        else y = b;
}
```

Which one of the following digital logic blocks is the most suitable for implementing this function?

(A) Full adder

(B) Priority encoder

(C) Multiplexer

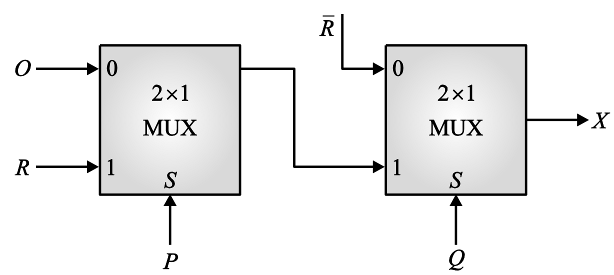
(D) Flip-flop

2015 IIT Kanpur

3.4 A half adder is implemented with XOR and AND gates. A full adder is implemented with two half adders and one OR gate. The propagation delay of an XOR gate is twice that of an AND/OR gate. The propagation delay of an AND/OR gate is 1.2 microseconds. A 4-bit ripple- carry binary adder is implemented by using four full adders. The total propagation time of this 4-bit binary adder in microseconds is ______.

2016 IISc Bangalore

3.5 Consider the two cascaded 2-to-1 multiplexers as shown in the figure.



The minimal sum of products form of the output X is

(A) $\overline{P}\,\overline{Q} + PQR$ (B) $\overline{P}Q + QR$

(C) $PQ+\overline{P}\overline{Q}R$ (D) $\overline{Q}\overline{R}+PQR$

3.6 Consider a carry look ahead adder for adding two n-bit integers, built using gates of fan-in at most 2. The time to perform addition using this adder is

(A) $\Theta(1)$ (B) $\Theta(\log(n))$

(C) $\Theta(\sqrt{n})$ (D) $\Theta(n)$

3.7 Consider an eight-bit ripple-carry adder for computing the sum of A and B, where A and B are integers represented in 2's complement form. If the decimal value of A is one, the decimal value of B that leads to the longest latency for the sum to stabilize is ______.

2017 **IIT Roorkee**

3.8 When two 8-bit numbers $A_7.........A_0$ and $B_7.........B_0$ in 2's complement representation (with A_0 and B_0 as the least significant bits) are added using a ripple-carry adder, the sum bits obtained are $S_7.........S_0$ and the carry bits are $C_7.........C_0$. An overflow is said to have occurred if

(A) The carry bit C_7 is 1

(B) All the carry bits ($C_7.........C_0$) are 1

(C) $\left(A_7.B_7.\overline{S}_7+\overline{A}_7.\overline{B}_7.S_7\right)$ is 1

(D) $\left(A_0.B_0.\overline{S}_0+\overline{A}_0.\overline{B}_0.S_0\right)$ is 1.

Solutions

3.1 **(A)**

Condition 1 :

D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	0	0	X	X	0

Here, $V=0$ it means given input (D_0, D_1, D_2, D_3) are invalid and does not perform any function.

Condition 2 :

D_0	D_1	D_2	D_3	X_0	X_1	V
1	0	0	0	0	0	1

Here, $V=1$ and output $X_0X_1=00$, when input D_0 is 1 (active),

i.e. $V=1 \Rightarrow X_0X_1=00 \Rightarrow D_0=1$

So, output $(X_0X_1)=00$ shows 2-bit binary number of decimal value 0, because D_0 is active.

Condition 3 :

D_0	D_1	D_2	D_3	X_0	X_1	V
0	1	0	0	0	1	1

Here $V=1$ and output $X_0X_1=01$, when input D_1 is 1 (active),

i.e. $V=1 \Rightarrow X_0X_1=01 \Rightarrow D_1=1$

So, output $(X_0X_1)=01$ shows 2-bit binary number of decimal 1 because D_1 is active.

Condition 4 :

D_0	D_1	D_2	D_3	X_0	X_1	V
0	0	1	0	1	0	1

Here $V=1$ and output $X_0X_1=10$, when input D_2 is 1 (active),

i.e. $V=1 \Rightarrow X_0X_1=10 \Rightarrow D_2=1$

So, output $(X_0X_1)=10$ shows 2-bit binary number of decimal 2 because D_2 is active.

Condition 5 :

D_0	D_1	D_2	D_3	X_0	X_1	V
X	X	X	1	1	1	1

Here $V=1$ and output $X_0X_1=11$, when input D_3 is 1 (active),

i.e. $V=1 \Rightarrow X_0X_1=11 \Rightarrow D_3=1$

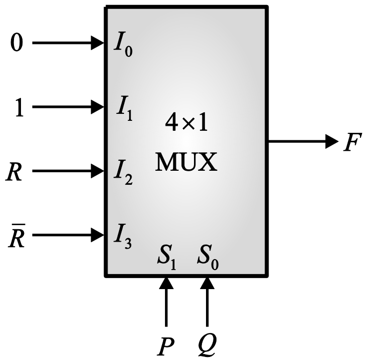
So, output $(X_0X_1)=11$ shows 2-bit binary number of decimal 3 because D_3 is active.

So, based on above 5-conditions it is clear that, given truth table is equivalent to the truth table of 4×2 priority encoder in which $V=0$, it means no-encoding and $V=1$, it means priority encoding.

Hence, the correct option is (A).

3.2 **(A)**

Given : Circuit is shown below,



Selection lines : $S_1 = P,\ S_0 = Q$

Data inputs : $I_0 = 0,\ I_1 = 1,\ I_2 = R,\ I_3 = \bar{R}$

So, output of MUX in SOP form is,

$$F = \bar{S}_1\bar{S}_0 I_0 + \bar{S}_1 S_0 I_1 + S_1\bar{S}_0 I_2 + S_1 S_0 I_3$$

$$F = \bar{P}\bar{Q}0 + \bar{P}Q1 + P\bar{Q}R + PQ\bar{R}$$

$$F = \bar{P}Q + P\bar{Q}R + PQ\bar{R}$$

$$F = (\bar{P}Q + PQ\bar{R}) + P\bar{Q}R$$

$$F = Q(\bar{P} + P\bar{R}) + P\bar{Q}R$$

Apply distributive law,

$$F = Q[(\bar{P} + P)(\bar{P} + \bar{R})] + P\bar{Q}R \qquad (\because\ P + \bar{P} = 1)$$

$$F = Q(\bar{P} + \bar{R}) + P\bar{Q}R$$

$$F = \bar{P}Q + Q\bar{R} + P\bar{Q}R$$

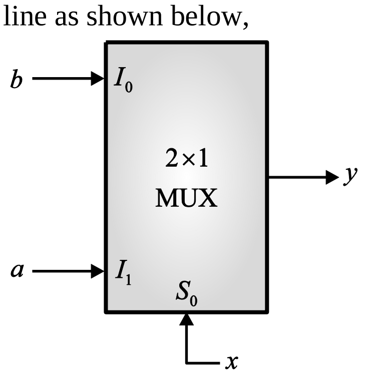
Hence, the correct option is (A).

3.3 (C)

According to question,

$$\text{Output}\,(y) \rightarrow \begin{cases} a, & \text{if } x = 1 \\ b, & \text{if } x \neq 1 \end{cases}$$

Thus, it perform the function of 2×1 MUX, when x is selection line as shown below,



When $x = 1$, it select I_1 and output $(y) = a$.

When $x = 0$, it select I_0 and output $(y) = b$.

Hence, the correct option is (C).

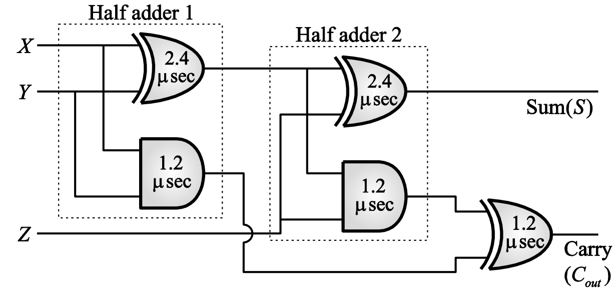
3.4 19.2

Given : $(t_{pd})_{\text{OR/AND}} = 1.2\ \mu\text{sec}$

$(t_{pd})_{\text{EX-OR}} = 2(t_{pd})_{\text{OR/AND}}$

$(t_{pd})_{\text{EX-OR}} = 2.4\ \mu\text{sec}$

Now, full adder using 2-half adder and OR gate is shown below,



So, total sum propagation delay (t_s) for sum (s) from input is,

$$t_s = (t_{pd})_{\text{EX-OR}_1} + (t_{pd})_{\text{EX-OR}_2}$$

$$t_s = (2.4\ \mu + 2.4\ \mu)\text{sec} = 4.8\ \mu\text{sec}$$

$(t_s = \text{Sum propagation delay})$

Similarly, total carry propagation delay (t_c) for carry (C_{out}) from input is,

$$t_c = (t_{pd})_{\text{EX-OR}_1} + (t_{pd})_{\text{AND}_2} + (t_{pd})_{\text{OR}}$$

$$t_c = (2.4\mu + 1.2\mu + 1.2\mu)\text{sec}$$

$$t_c = 4.8\ \mu\text{sec}$$

$(t_c = \text{Carry propagation delay})$

Thus, total propagation delay (T_{pd}) for n-bit ripple carry adder, made by n-full-adder is,

$$T_{pd} = (n-1)t_c + \max(t_s,\ t_c)$$

Where, $n =$ Number of bits used in ripple carry adder,

$t_c =$ Carry propagation delay for 1-full adder.

$t_s =$ Sum propagation delay of 1-full adder use in n-bit ripple carry adder.

Max $(t_c,\ t_s) =$ Choosing maximum between t_c and t_s.

So, $T_{pd} = [(4-1)(4.8) + \max(4.8,\ 4.8)]\ \mu\text{sec}$

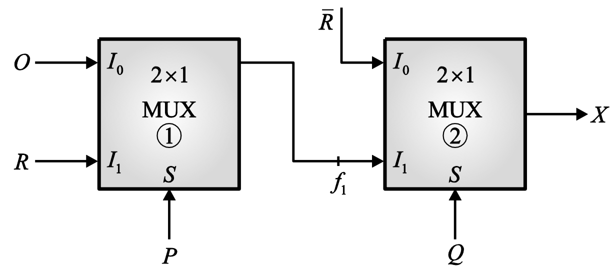
$$T_{pd} = (3\times4.8 + 4.8)\ \mu\text{sec}$$

$$T_{pd} = 19.2\ \mu\text{sec}$$

Hence, the total propagation time (t_{pd}) of 4-bit ripple carry adder is $19.2\ \mu\text{sec}$.

3.5 **(D)**

Given : Circuit is shown below,



For MUX-1 :

Selection line : $S = P$

Data input : $I_0 = 0,\ I_1 = R$

Thus, output (f_1) of MUX-1 is,

$$f_1 = I_0\bar{S} + I_1 S$$

$$f_1 = 0\bar{P} + RP = RP$$

For MUX-2 :

Selection line : $S = Q$

Data input : $I_0 = \bar{R},\ I_1 = f_1$

Thus, output (X) of MUX-2 is,

$$X = \bar{S}I_0 + SI_1$$

$$X = \bar{Q}\,\bar{R} + Qf_1$$

$$X = \bar{Q}\,\bar{R} + Q(RP)$$

$$X = \bar{Q}\,\bar{R} + QRP$$

Thus, minimal SOP form of output (X) is $\bar{Q}\,\bar{R} + PQR$.

Hence, the correct option is (D).

3.6 **(B)**

For n-bit look ahead carry adder, to generate n^{th} carry bit, it is required to perform AND logic operation between $(n+1)$ inputs.

Suppose adder contains logic gates with each logic gate having fan-in (number of input to each logic gate used in look-ahead carry adder) at most K then time taken by adder to perform addition is $\log_K(n+1)$ time and its asymptotic representation $\theta(\log_K n)$, so according to given question,

$$K = \text{Fan-in} = 2$$

So, time taken by adder to perform addition is,

$$\theta(\log_2 n) \approx \theta(\log n)$$

Hence, the correct option is (B).

Note : This question based upon time complexity analysis of look ahead carry adder.

3.7 **–1**

First you understand the term longest latency in ripple carry adder, it means, maximum delay that a ripple carry adder will take to generate last carry (C_{out}) during the addition of two n-bit numbers A and B respectively (where, A and B should be in sign magnitude format).

Longest latency is only possible when internal carry generated at each and every addition from LSB to MSB.

According to question, A and B are two 8-bit numbers in 2's complement format as,

$A = +1 = 0000\ 0001 \rightarrow$ 2's complement form

So, we choose decimal value of B in such a way that internal carry must be generated at each addition, so we take

$B = -1 = 1111\ 1111 \rightarrow$ 2's complement form

Now adding A and B as shown below,

$$\begin{array}{lcccccccccl}
 & & (1) & (1) & (1) & (1) & (1) & (1) & (1) & & \longrightarrow \text{Internal carry at each addition} \\
\text{A} = & & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \\
\text{B} = + & & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \\
\hline
\text{Last carry } (C_{out}) \longrightarrow & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 &
\end{array}$$

Hence, the decimal value of B that leads longest latency for the sum to be stabilized is **–1**.

3.8 **(C)**

First you understand what is actual mean of overflow in addition of 2's complement numbers.

It means, we don't have so much memory space to store the result of addition of 2's complement number then overflow occurred.

Example :

Suppose we have two 3-bit positive number A and B in 2's complement binary form and have only 3-bit memory to store the result of their addition as,

$A = 3 = A_2A_1A_0 = 0, 11 \rightarrow$ Positive number

$B = 2 = B_2B_1B_0 = 0, 10 \rightarrow$ Positive number

Add them,

$$\begin{array}{rcll} 3 & = A_2A_1A_0 & \rightarrow & 011 \\ +2 & = B_2B_1B_0 & \rightarrow & 010 \\ \hline 5 & S_2S_1S_0 & \underset{C_{out}}{0} & 101 \rightarrow \text{4-bit result} \end{array}$$

$C_{out} \longrightarrow$ Outer carry

Result of addition is + 5 (i.e. 0101) it require 4-bit to store in memory but we have only 3-bit memory than this condition is called overflow. So, the condition of overflow can be easily detected by MSB's A_2, B_2, S_2 as

$$A_2 = 0,\ B_2 = 0,\ S_2 = 1$$

i.e., $\bar{A}_2 \cdot \bar{B}_2 \cdot S_2 = 1$ ...(i)

Suppose we have two 3-bit negative number A and B in 2's complement binary form and have only 3-bit memory to store the result of their addition as,

$$A = -3 = A_2A_1A_0 = 1,01 \rightarrow \text{Negative number}$$

$$B = -2 = B_2B_1B_0 = 1,10 \rightarrow \text{Negative number}$$

Add them,

$$\begin{array}{rcll} -3 & = A_2A_1A_0 & \rightarrow & 101 \\ (+)-2 & = B_2B_1B_0 & \rightarrow & +110 \\ \hline -5 & S_2S_1S_0 & \underset{C_{out}}{1} & 011 \rightarrow \text{4-bit result} \end{array}$$

$C_{out} \longrightarrow$ Outer carry

Result of addition is – 5 (i.e. 1011) it require 4-bit to store in memory but we have only 3-bit memory than this condition is called overflow. So, the condition of overflow can be easily detected by MSB's A_2, B_2, S_2 as

$$A_2 = 1,\ B_2 = 1,\ S_2 = 0$$

i.e., $A_2 \cdot B_2 \cdot \bar{S}_2 = 1$ ...(ii)

So combined equation (i) and (ii), because at a time only one condition satisfied, so final condition for detecting overflow by MSB's is,

$$\bar{A}_2 \cdot \bar{B}_2 \cdot S_2 + A_2 \cdot B_2 \cdot \bar{S}_2 = 1$$

Now, extend this concept for 8-bit numbers A and B, so condition of overflow can be detected as,

$$\bar{A}_7 \cdot \bar{B}_7 \cdot S_7 + A_7 \cdot B_7 \cdot \bar{S}_7 = 1$$

Hence, the correct option is (C).

Key Point

In addition of two 2's complement numbers, overflow occurred,

- When both numbers are positive.
- When both numbers are negative.
  (i.e. both number of same sign)

Overflow can't be occurred if both numbers have different sign. (i.e. one positive number added with negative number or vice-versa).

4 Sequential Circuits

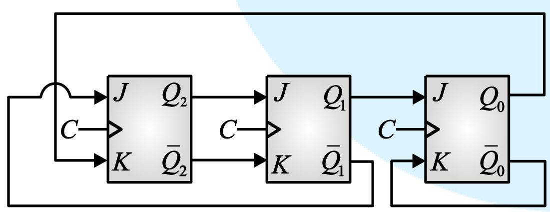
Practice Questions

2014 IIT Kharagpur

4.1 Let $k = 2^n$. A circuit is built by giving the output of an n – bit binary counter as input to an n-to-2^n bit decoder. This circuit is equivalent to a

(A) k-bit binary up counter
(B) k-bit binary down counter
(C) k-bit ring counter
(D) k-bit Johnson counter

4.2



The above synchronous sequential circuit built using JK flip-flops is initialized with $Q_2Q_1Q_0 = 000$. The state sequence for this circuit for the next 3 clock cycles (C) is _______.

(A) 001, 010, 011 (B) 111, 110, 10
(C) 100, 110, 111 (D) 100, 011, 001

2015 IIT Kanpur

4.3 Consider a 4-bit Johnson counter with an initial value of 0000. The counting sequence of this counter is

(A) 0, 1, 3, 7, 15, 14, 12, 8, 0
(B) 0, 1, 3, 5, 7, 9, 11, 13, 15, 0
(C) 0, 2, 4, 6, 8, 10, 12, 14, 0
(D) 0, 8, 12, 14, 15, 7, 3, 1, 0

4.4 A positive edge-triggered D flip-flop is connected to a positive edge-triggered JK flip-flop as follows. The Q output of the D flip-flop is connected to both the J and K inputs of the JK flip-flop, while the Q output of the JK flip-flop, is connected to the input of the D flip-flop. Initially, the output of the D flip- flop is set to logic one and the output of the JK flip-flop is cleared. Which one of the following is the bit sequence (including the initial state) generated at the Q output of the JK flip-flop when the flip-flops are connected to a free-running common clock? Assume that J = K = 1 is the toggle mode and J = K = 0 is the state-holding mode of the JK flip-flop. Both the flip-flops have non-zero propagation delays.

(A) 0110110… (B) 0100100…
(C) 011101110… (D) 011001100…

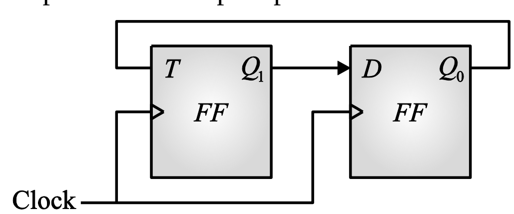
4.5 The minimum number of J-K flip-flops required to construct a synchronous counter with the count sequence (0, 0, 1, 1, 2, 2, 3, 3, 0, 0, …..) is _______.

2016 IISc Bangalore

4.6 We want to design a synchronous counter that counts the sequence 0-1-0-2-0-3 and then repeats. The minimum number of J-K flip-flops required to implement this counter is _______.

2017 IIT Roorkee

4.7 Consider a combination of T and D flip-flops connected as shown below. The output of the D flip-flop is connected to the input of the T flip-flop and the output of the T flip-flop is connected to the input of the D flip-flop.



Initially, both Q_0 and Q_1 are set to 1 (before the 1st clock cycle). The outputs

(A) $Q_1\,Q_0$ after the 3rd cycle are 11 and after the 4th cycle are 00 respectively
(B) $Q_1\,Q_0$ after the 3rd cycle are 11 and after the 4th cycle are 01 respectively

(C) $Q_1\ Q_0$ after the 3rd cycle are 00 and after the 4th cycle are 11 respectively

(D) $Q_1\ Q_0$ after the 3rd cycle are 01 and after the 4th cycle are 01 respectively

4.8 The next state table of a 2-bit saturating up-counter is given below.

Q_1	Q_0	Q_1^+	Q_0^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

The counter is built as a synchronous sequential circuit using T flip-flops. The expression for T_1 and T_0 are

(A) $T_1 = Q_1Q_0, T_0 = \overline{Q_1}\,\overline{Q_0}$

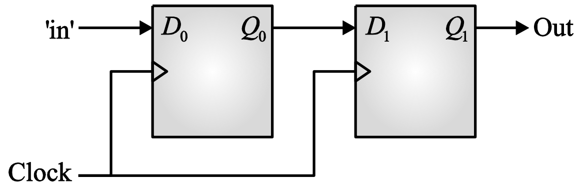
(B) $T_1 = \overline{Q_1}Q_0, T_0 = \overline{Q_1} + \overline{Q_0}$

(C) $T_1 = Q_1 + Q_0, T_0 = \overline{Q_1} + \overline{Q_0}$

(D) $T_1 = \overline{Q_1}Q_0, T_0 = Q_1 + Q_0$

2018 IIT Guwahati

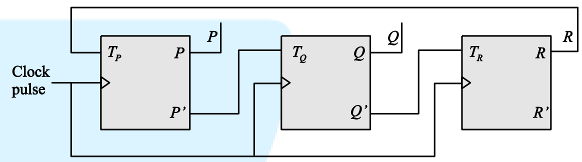
4.9 Consider the sequential circuit shown in the figure, where both flip-flops used are positive edge-triggered D flip-flops.



The number of states in the state transition diagram of this circuit that have a transition back to the same state on some value of "in" is ________.

2021 IIT Bombay

4.10 Consider a 3-bit counter, designed using T flip-flops, as shown below:



Assuming the initial state of the counter given by PQR as 000, what are the next three states?

(A) 011,101,000

(B) 001,010,111

(C) 011,101,111

(D) 001,010,000

Solutions

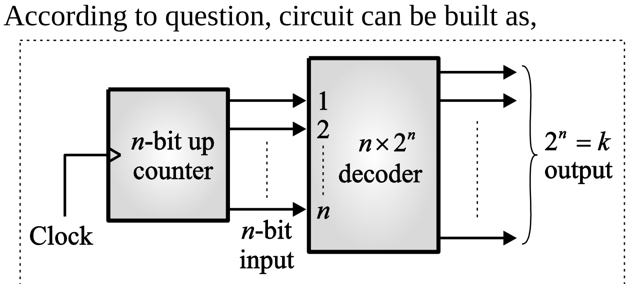
4.1 (C)

Given : $K = 2^n$

n-bit counter (Assuming up-counter)

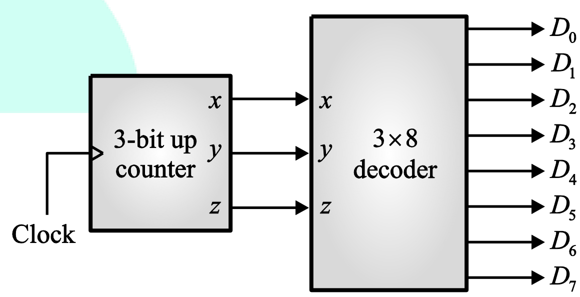
$n \times 2^n$ bit decoder.

According to question, circuit can be built as,



Circuit-1

Assume $n = 3$ bit, so above circuit can be reduced as,



So, state table of above circuit is,

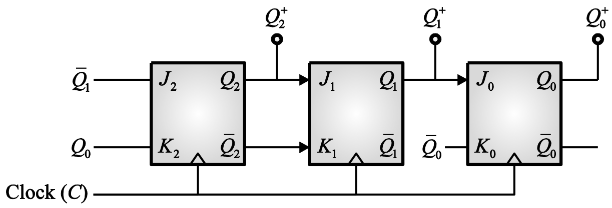
CLK	Counter output (decode input)			Decode output							
	x	y	z	D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
1	0	0	0	1	0	0	0	0	0	0	0
2	0	0	1	0	1	0	0	0	0	0	0
3	0	1	0	0	0	1	0	0	0	0	0
4	0	1	1	0	0	0	1	0	0	0	0
5	1	0	0	0	0	0	0	1	0	0	0
6	1	0	1	0	0	0	0	0	1	0	0
7	1	1	0	0	0	0	0	0	0	1	0
8	1	1	1	0	0	0	0	0	0	0	1

So, output is 8-bit (2^3 bit) long and it is equivalent to output 8-bit ring counter.

Thus, circuit-1 shown $2^n = K$ bit ring counter.
Hence, the correct option is (C).

4.2 (C)

Given : Circuit redraw here is shown below,



Method 1

It is a 3-bit synchronous sequential circuit. So, inputs to each flip-flops are,

$$J_2 = \bar{Q}_1,\ J_1 = Q_2,\ J_0 = Q_1$$

$$K_2 = Q_0,\ K_1 = \bar{Q}_2,\ K_0 = \bar{Q}_0$$

Initially all flip-flop are cleared i.e.,

$$Q_2Q_1Q_0 = 000$$

So, state table can be formed as,

Previous state			Flip-flop input						CLK	Next state		
Q_2	Q_1	Q_0	$J_2(\bar{Q}_1)$	$K_2(Q_0)$	$J_1(Q_2)$	$K_1(\bar{Q}_2)$	$J_0(Q_1)$	$K_0(\bar{Q}_0)$		Q_2^+	Q_1^+	Q_0^+
-	-	-	-	-	-	-	-	-	0	0	0	0
0	0	0	1	0	0	1	0	1	1	1	0	0
1	0	0	1	0	1	0	0	1	2	1	1	0
1	1	0	0	0	1	0	1	1	3	1	1	1

Initially

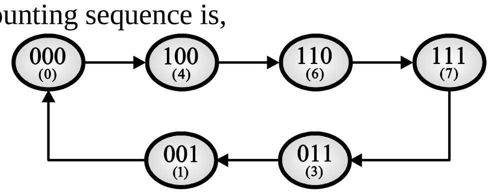
So, state sequence for next 3-cycle is 100, 110, 111.
Hence, the correct option is (C).

Method 2

Given : circuit is 3-bit Johnson counter using JK-flip-flop, so its state table is shown below,

Clock	Q_2^+	Q_1^+	Q_0^+	Decimal value
0	0	0	0	0
1	1	0	0	4
2	1	1	0	6
3	1	1	1	7
4	0	1	1	3
5	0	0	1	1
6	0	0	0	0 (Repeated)

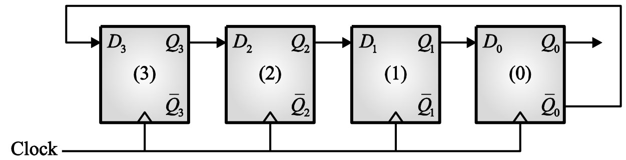
So, counting sequence is,



Here, question asked about, state sequence for next 3-cycle is $100 \rightarrow 110 \rightarrow 111$.
Hence, the correct option is (C).

4.3 (D)

4-bit Johnson counter using D-flip-flop is shown below,



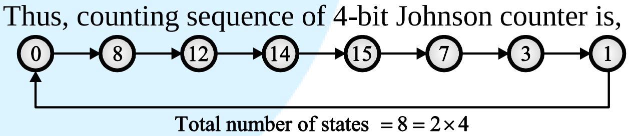
Counting sequence of 4-bit Johnson counter with initial state 0000 is shown below,

CLK	Q_3	Q_2	Q_1	Q_0	Decimal value
0	0	0	0	0	0
1	1	0	0	0	8
2	1	1	0	0	12
3	1	1	1	0	14
4	1	1	1	1	15
5	0	1	1	1	7
6	0	0	1	1	3
7	0	0	0	1	1
8	0	0	0	0	3

Initial

Repeated

Thus, counting sequence of 4-bit Johnson counter is,



Hence, the correct option is (D).

Key Point

For n-stage Johnson counter

(i) The number of used state = $2n$

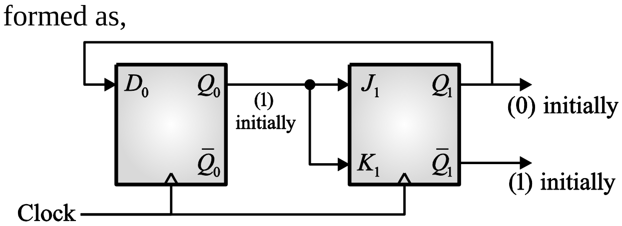
(ii) The number of unused state $= 2^n - 2n$

(iii) Maximum possible count $= 2^n - 1$

(iv) Output frequency $= \dfrac{f_{clk}}{2n}$

4.4 (A)

As per the given data in question, the circuit can be formed as,



Given :

$J = K = 1 \rightarrow$ Toggle mode.

$J = K = 0 \rightarrow$ States holding mode (previous state mode)

State table of given circuit as,

Present state		Flip-Flop inputs			CLK (C)	Next state	
Q_0	Q_1	D_0 (Q_1)	J_1 (Q_0)	K_1 (Q_0)		Q_0^+	Q_1^+
-	-	-	-	-	0	1	0
1	0	0	1	1	1	0	1
0	1	1	0	0	2	1	1
1	1	1	1	1	3	1	0
1	0	0	1	1	4	0	1
0	1	1	0	0	5	1	1
1	1	1	1	1	6	1	0

Initially

So, upto 6th, the output of JK-flip-flop (Q_1^+) is 011011……

Hence, the correct option is (A).

4.5 **2**

Given : Counting sequence is,

0, 0, 1, 1, 2, 2, 3, 3, 0, 0, …..

Maximum counting digit is count by counter

$\Rightarrow \quad 3 = (11)_2$

The decimal number-3 can be represented in 2-bit binary number as 11. So, 2-flip-flops are required to construct synchronous counter using JK-flip-flop.

Hence, minimum number of JK-flip-flop required for this counter is **2**.

4.6 **2**

Given : Counting sequence is,

$$0 \rightarrow 1 \rightarrow 0 \rightarrow 2 \rightarrow 0 \rightarrow 3$$

Maximum counting digit is count by counter

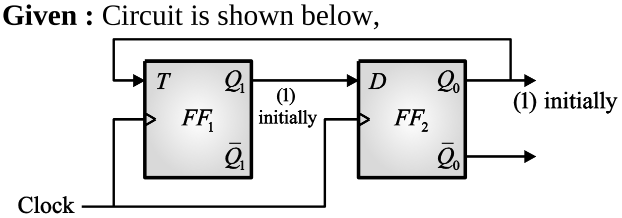
$\Rightarrow \quad 3 = 11$

The decimal number-3 can be represented in 2-bit binary number as 11. So, 2-flip-flops are required to construct synchronous counter using JK-flip-flop.

Hence, minimum number of JK-flip-flop required for implement this counter is **2**.

4.7 **(B)**

Given : Circuit is shown below,



So, input of flip-flops, used in above circuit is,

$T = Q_0$ and $D = Q_1$

So, state table can be formed as,

Present state		Flip-flop input		CLK	Next state	
Q_1	Q_0	T (Q_0)	D (Q_1)		Q_1^+	Q_0^+
-	-	-	-	0	1	1
1	1	1	1	1	0	1
0	1	1	0	2	1	0
1	0	0	1	3	1	1
1	1	1	1	4	0	1

Initially

Thus, output $Q_1^+Q_0^+$ after 3rd and 4th clock are 11, 01 respectively.

Hence, the correct option is (B).

4.8 **(B)**

According to question, two T-flip-flop can be used for the implementation of synchronous sequential circuit.

Now, next state table given as,

Present state		Next state	
Q_1	Q_2	Q_1^+	Q_2^+
0	0	0	1
0	1	1	0
1	0	1	1
1	1	1	1

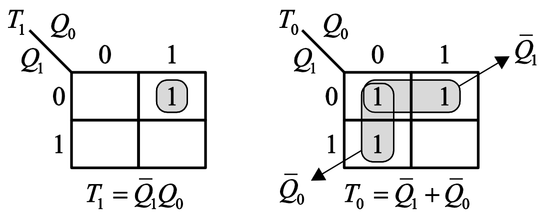
Excitation table of T-flip-flop is given as,

Q_n	Q_n^+	T
0	0	0
0	1	1
1	0	1
1	1	0

So, with the help of excitation- table of T-flip-flop, input T_1 and T_0 is decided as,

Present state		Next state		Inputs of flip-flop	
Q_1	Q_2	Q_1^+	Q_0^+	T_1	T_0
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	0	0

So, 2-variable K-map for the minimize expression of T_1 and T_0 us given as,

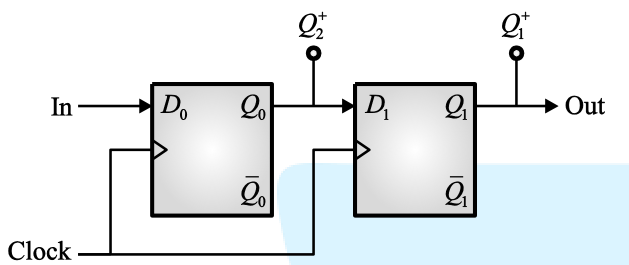


Thus, expression of $T_1 = \bar{Q}_1 Q_0$ and $T_0 = \bar{Q}_0 \bar{Q}_1$.

Hence, the correct option is (B).

4.9 **2**

Given : 2-bit sequential circuit is shown below,



Initial states of given circuit is not given in question. So, consider 4-cases

Case 1 : When initial state $Q_0 Q_1 = 00$ and input (in) is 0 or 1. So next state $(Q_0^+ Q_1^+)$ and 'out' is

Present state		Input (in)	Flip-flop inputs		CLK	Next state		Output (out)
Q_1	Q_2		$D_0 = in$	$D_1 = Q_0$		Q_0^+	Q_1^+	Q_1^+
0	0	0	0	0	1	0	0	0
0	0	1	1	0	2	1	0	0

Case 2 : When initial state $(Q_0 Q_1) = 01$ and input (in) is 0 or 1, so next state $(Q_0^+ Q_1^+)$ and 'out' is,

Present state		Input (in)	Flip-flop inputs		CLK	Next state		Output (out)
Q_0	Q_1		$D_0 = in$	$D_1 = Q_0$		Q_0^+	Q_1^+	Q_1^+
0	1	0	0	0	1	0	0	0
0	1	1	1	0	2	1	0	0

Case 3 : When initial state $(Q_0 Q_1) = 10$ and input (in) is 0 to 1, so next state $(Q_0^+ Q_1^+)$ and 'out' is,

Present state		Input (in)	Flip-flop inputs		CLK	Next state		Output (out)
Q_0	Q_1		$D_0 = in$	$D_1 = Q_0$		Q_0^+	Q_1^+	Q_1^+
1	0	0	0	1	1	0	1	1
1	0	1	1	1	2	1	1	1

Case 4 : When initial state $(Q_0 Q_1) = 11$ and input (in) is 0 to 1, so next state $(Q_0^+ Q_1^+)$ and 'out' is,

Present state		Input (in)	Flip-flop inputs		CLK	Next state		Output (out)
Q_0	Q_1		$D_0 = in$	$D_1 = Q_0$		Q_0^+	Q_1^+	Q_1^+
1	1	0	0	1	1	0	1	1
1	1	1	1	1	2	1	1	1

Combined all four cases to get compact state table as,

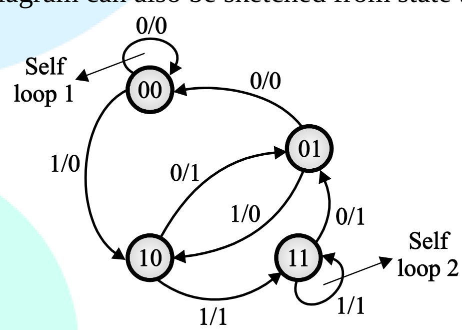
Present state		Input (in)	Next state		Out
Q_0	Q_1		Q_0^+	Q_1^+	Q_1^+
0	0	0	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1
1	1	1	1	1	1

Self repeating state Self repeating state

So, the question asking about the self-repeating state/self-loop in state-transition diagram, so from state table it is clear that, only 2-self repeating state can be found.

Thus, number of state that have transition back to same state on some value of "in" is 2.

State diagram can also be sketched from state table as,



(State transition diagram)

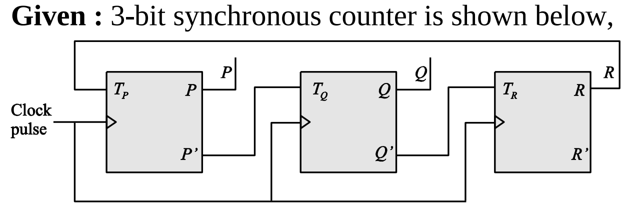
Here, $x / y \Rightarrow$ $x \rightarrow$ Input and $y \rightarrow$ Output.

Again, state diagram also showing 2-self loop.

Hence, number of state in state diagram that have a transition back to the same state on some values of 'in' is 2.

4.10 **(A)**

Given : 3-bit synchronous counter is shown below,



Next state equation for each flip-flop present in above counter,

1. $P^{+} = T_{p} \oplus P = R \oplus P$

2. $Q^{+} = T_{Q} \oplus Q = \overline{P} \oplus Q$

3. $R^{+} = T_{R} \oplus R = \overline{Q} \oplus R$

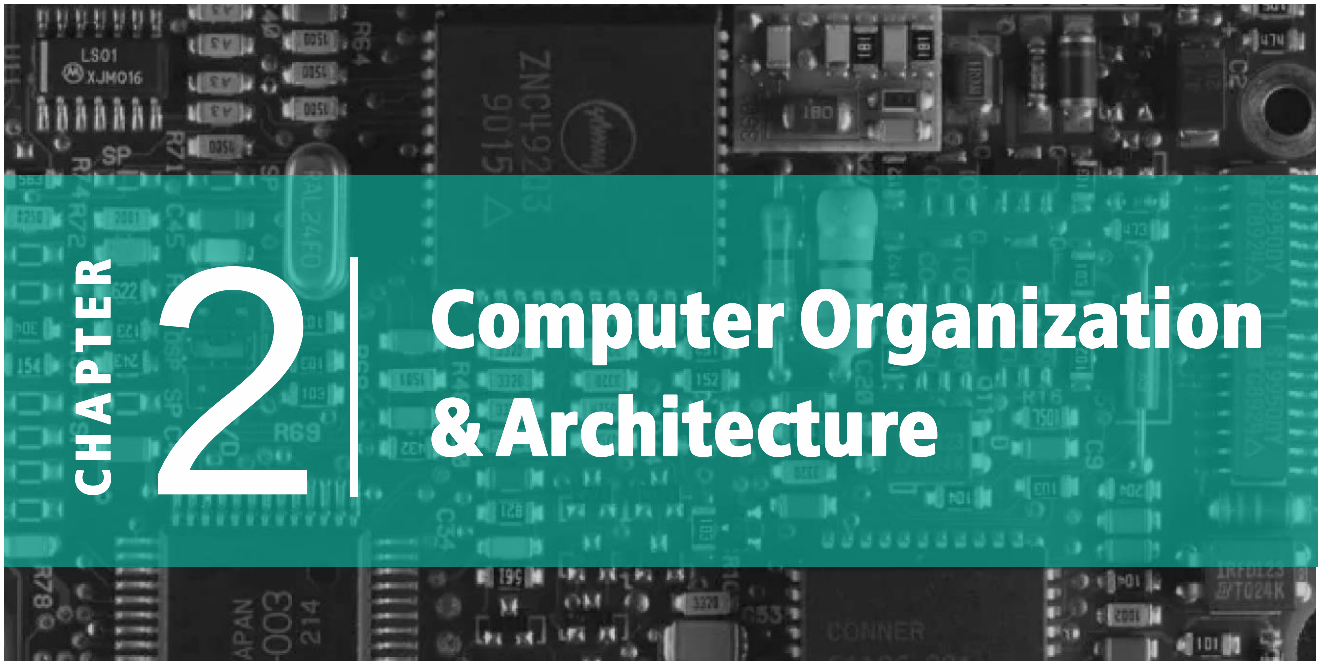
So, next state table will be

CLK	Present states			Next states		
	P	***Q***	***R***	P^{+}	Q^{+}	R^{+}
1	0	0	0	0	1	1
2	0	1	1	1	0	1
3	1	0	1	0	0	0

So, next three states are :

$$000 \rightarrow 011 \rightarrow 101 \rightarrow 000$$

Hence, the correct option is (A).]



CHAPTER 2 Computer Organization & Architecture

Marks Distribution of Computer Organization & Architecture in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	3	8
2004*	1	7	15
2005*	4	8	20
2006*	1	7	15
2007*	2	6	14
2008*	-	12	24
2009	2	4	10
2010	1	4	9
2011	2	2	6
2012	1	4	9
2013	1	3	7
2014 Set-1	1	3	7
2014 Set-2	1	3	7
2014 Set-3	1	2	5

*** CS and IT combined**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	1	1	3
2015 Set-2	1	2	5
2015 Set-3	1	2	5
2016 Set-1	1	2	5
2016 Set-2	1	5	11
2017 Set-1	3	4	11
2017 Set-2	1	3	7
2018	3	3	9
2019	2	1	4
2020	5	3	11
2021 Set-1	2	4	10
2021 Set-2	2	3	8

Syllabus : Computer Organization & Architecture

Machine instructions and addressing modes. ALU, data-path and control unit. Instruction pipelining, pipeline hazards. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

Contents : Computer Organization & Architecture

1 Machine Instruction and Addressing Format

Practice Questions

2014 IIT Kharagpur

1.1 A machine has a 32-bit architecture, with 1-word long instructions. It has 64 registers, each of which is 32 bits long. It needs to support 45 instructions, which have an immediate operand in addition to two register operands. Assuming that the immediate operand is an unsigned integer, the maximum value of the immediate operand is _______.

2015 IIT Kanpur

1.2 For computers based on three-address instruction formats, each address field can be used to specify which of the following :

S1 : A memory operand

S2 : A processor register

S3 : An implied accumulator register.

(A) Either S1 or S2

(B) Either S2 or S3

(C) Only S2 and S3

(D) All of S1, S2 and S3

1.3 Consider a processor with byte-addressable memory. Assume that all registers, including Program Counter (PC) and Program Status Word (PSW), are of size 2 bytes. A stack in the main memory is implemented from memory location $(0100)_{16}$ and it grows upward. The stack pointer (SP) points to the top element of the stack. The current value of SP is $(016E)_{16}$. The CALL instruction is of two words; the first word is the op-code and the second word is the starting address of the subroutine (one word = 2 bytes). The CALL instruction is implemented as follows :

- Store the current value of PC in the stack.
- Store the value of PSW register in the stack.
- Load the starting address of the subroutine in PC.

The content of PC just before the fetch of a CALL instruction is $(5FA0)_{16}$. After execution of the CALL instruction, the value of the stack pointer is

(A) $(016A)_{16}$ (B) $(016C)_{16}$

(C) $(0170)_{16}$ (D) $(0172)_{16}$

2016 IISc Bangalore

1.4 A processor has 40 distinct instructions and 24 general purpose registers. A 32-bit instruction word has an opcode, two register operands and an immediate operand. The number of bits available for the immediate operand field is ___.

1.5 Consider a processor with 64 registers and an instruction set of size twelve. Each instruction has five distinct fields, namely, opcode, two source register identifiers, one destination register identifier, and a twelve-bit immediate value. Each instruction must be store in memory in a byte-aligned fashion. If a program has 100 instructions, the amount of memory (in bytes) consumed by the program text is _______

2017 IIT Roorkee

1.6 Consider the C struct defined below :

```
    struct data
{   int marks [100];
    char grade;
};
struct data student;
```

The base address of student is available in register R1.

The field student grade can be accessed efficiently using

(A) Post-increment addressing mode, (R1) +

(B) Pre-decrement addressing mode, – (R1)

(C) Register direct addressing mode, R1

(D) Index addressing mode, X (R1), where X is an offset represented in 2'S complement 16-bit representation.

1.7 Consider the expression

$$(a-1)*\left(\left(\frac{(b+c)}{3}\right)+d\right).$$

Let X be the minimum number of registers required by an optimal code generation (without any register spill) algorithm for a load/store architecture, in which

(i) only load and store instructions can have memory operands and

(ii) arithmetic instructions can have only register or immediate operands

The value of X is ________.

1.8 Consider a RISC machine where each instruction is exactly 4 bytes long. Conditional and unconditional branch instructions use PC-relative addressing mode with Offset specified in bytes to the target location of the branch instruction. Further the Offset is always with respect to the address of the next instruction in the program sequence. Consider the following instruction sequence

Instr. No.	Instruction
i:	add R2, R3, R4
i+1:	sub R5, R6, R7
i+2:	cmp R1, R9, R10
i+3:	beq R1, Offset

If the target of the branch instruction is i, then the decimal value of the Offset is _____________.

2018 IIT Guwahati

1.9 The following are some events that occur after a device controller issues an interrupt while process L is under execution.

P. The processor pushes the process status of L onto the control stack.

Q. The processor finishes the execution of the current.

R. The processor executes the interrupt service routine.

S. The processor pops the process status of L from the control stack.

T. The processor loads the new PC value based on the interrupt.

Which of the following is the correct order in which the events occur?

(A) QPTRS (B) PTRSQ

(C) TRPQS (D) QTPRS

1.10 A processor has 16 integer registers $(R0, R1, \ldots, R15)$ and 64 floating-point registers $(F0, F1, \ldots, F63)$. It uses a 2-byte instruction format. There are four categories of instructions : Type-1, Type-2, Type-3, and Type-4. Type-1 category consists of four instructions, each with 3 integer register operands (3Rs). Type-2 category consists of eight instructions, each with 2 floating point register operands (2Fs). Type-3 category consists of fourteen instructions, each with one integer register operand and one floating point register operand (1R + 1F). Type-4 category consists of N instructions, each with a floating point register operand (1F).

The maximum value of N is ________.

2020 IIT Delhi

1.11 A processor has 64 registers and uses 16-bit instruction format. It has two types of instructions : I-type and R-type. Each I-type instruction contains an opcode, a register name, and a 4-bit immediate value. Each R-type instruction contains an opcode and two register names. If there are 8 distinct I-type opcodes, then the maximum number of distinct R-type opcodes is ________.

2021 IIT Bombay

1.12 Consider a computer system consisting of registers R1, R2, R3 and MEMORY[X] denotes the content at the memory location of X. Assume memory is byte addressable.

Instruction	Semantics	Size (bytes)

MOV R1 (5000)	R1← M[5000]	4
MOV R2 (R3)	R2← M[R3]	4
ADD R2,R1	R2←R1+R2	2
MOV(R3)	M[R3]← R2	4
INC R3	R3← R3+1	2
DEC R1	R1← R1–1	2
BNEZ 1004	Branch if non-zero to the absolute address	2
HALT	Stop	1

Assume that the content of the memory location 5000 is 10 and the content of the register R3 is 3000. The content of each of the memory locations from 3000 to 3010 is 50. The instruction sequence starts from the memory location 1000. All the numbers are in decimal format. Assume that the memory is byte addressable.

After the execution of the program, the content of memory location 3010 is __.

1.13 If the numerical value of a 2-byte unsigned integer on a little endian computer is 255 more than that on a big endian computer, which of the following choices represent(s) the unsigned integer on a little endian computer?

(A) 0x6665 (B) 0x0001

(C) 0x4243 (D) 0x0100

Solutions

1.1 **16383**

Given :

Instruction size = 1 word

Word size = 32 bits

Number of registers = 64

Size of registers = 32 bits

Number of instructions to be supported = 45

Number of bits required for opcode field $= \log_2 45 = 6$ bits

Number of bits required for register operands $= \log_2 64 = 6$ bits

← 32 Bits →			
OPCODE	REGISTER 1	REGISTER 2	IMMEDIATE
← 6 →	← 6 →	← 6 →	←32-18=14 Bit →

Immediate field have 14 bits, so maximum value of immediate operand is $2^{14} - 1 = 16383$

Hence, the correct answer is 16383.

1.2 **(A)**

Computers with three-address instruction formats can use each address field to specify either a processor register or a memory operand.

Hence, the correct option is (A).

Key Point

Implied accumulator register is used in one address instruction format.

1.3 **(D)**

Given :

Memory is byte-addressable

Size of registers = 2 bytes

Initial location of SP = $(0100)_{16}$

Current value of SP = $(016E)_{16}$.

Size of CALL instruction = 2 words

Word size = 2 bytes

Content of PC before the fetch of CALL instruction = $(5FA0)_{16}$.

PC and PSW both are of 2 bytes, first two operations (storing both PC and PSW) will increment the SP to 4 bytes. Hence the value of stack pointer is

$(016E)_{16} + 4 = (0172)_{16}$

Hence, the correct option is (D).

1.4 **16**

Given :

Number of instructions = 40

Number of general-purpose registers = 24

Instruction size = 32 bits

Instruction format :

1 opcode, 2 register operands, 1 immediate operand

Number of bits required for opcode field

$$= \log_2 40 = 6$$

Number of bits required for register operand

$$= \log_2 24 = 5$$

32 Bit

OPCODE	REGISTER 1	REGISTER 2	IMMEDIATE
6	5	5	32-16=16 Bits

16 Bits

Hence, the correct answer is 16.

1.5 **500**

Given :

Number of instructions = 100

Number of registers = 64

Instruction size = 12

Size of immediate value = 12 bits

Instruction format :

1 opcode, 2 source registers, 1 destination register, 1 immediate value

Number of bits required for opcode field

$$= \log_2 12 \approx 4 \text{ bits}$$

Number of bits required for register operands

$$= \log_2 64 \approx 6 \text{ bits}$$

Number of bits required for immediate value

$$= 12 \text{ bits}$$

$\Rightarrow$ Size of one instruction

$$= 4 + 2 \times 6 + 6 + 12$$

$$= 34 \text{ bits} = 4.25 \text{ Byte}$$

Since instructions are stored in byte aligned fashion

One instruction will take 5 Bytes.

$\Rightarrow$ 100 instructions will take 500 Bytes.

Hence, the correct answer is 500.

1.6 **(D)**

Given :

```
struct data
{   int marks [100];
    char grade;
};
struct data student;
```

base address of student = content of R1.

Key Point

In C, struct members are stored in the order they are declared.

Hence, Option (A) will give the next address (which is of marks).

Option (B) will give the previous address

In register direct addressing mode, the instruction contains the register in which operand is stored.

Only option (D) gives access to desired address by adding displacement X in base address available in R1.

Hence, the correct option is (D).

1.7 **2**

Given :

$$(a-1) * \left(\left(\frac{(b+c)}{3} \right) + d \right).$$

Assembly code for given expression is

Load R_1, b

Load R_2, c

ADD R_1, R_2

DIV $R_1, 3$

Load R_2, d

ADD R_1, R_2

Load R_2, a

SUB $R_2, 1$

MUL R_2, R_1

Hence minimum 2 registers required.

Hence, the correct answer is 2.

1.8 **–16**

Given :

Instruction size = 4 bytes

Instruction sequence :

Instr. No.	Instruction
i:	add R2, R3, R4
i+1:	sub R5, R6, R7
i+2:	cmp R1, R9, R10
i+3:	beq R1, Offset

Target of branch instruction = i

assume addresses start with 2000 for the first instruction.

Address	Instruction
2000	add R2, R3, R4
2004	sub R5, R6, R7
2008	cmp R1, R9, R10
2012	beq R1, Offset
2016	Next instruction

PC after instruction fetch of the instruction beq R1, Offset will be 2016, and branch target is 2000,

$PC_{target} = PC_{current} + \text{Offset} \quad 2000 = 2016 + \text{Offset}$

$\therefore \text{Offset} = -16$

Hence, the correct answer is –16.

Key Point

Meaning of the given mnemonics :

add	Addition
sub	Subtraction
cmp	Compare
beq	Branch if equal

1.9 (A)

Steps :

1. Processor finishes execution of current instruction.
2. Processor signals acknowledgement of interrupt.
3. Processor pushes process status and PC value onto control stack.
4. Processor loads new PC value based on interrupt.
5. Processor executes the interrupt service routine.
6. Processor restore the old process status and PC.

Hence, the correct option is (A).

Key Point

A device controller issues an interrupt implies it is a hardware interrupt.

1.10 32

Given :

Number of integer registers = 16

Number of floating-point registers = 64

Size of instruction = 2 bytes

Total number of bits in instruction format $= 16$ bits

Total number of possible instruction encodings $= 2^{16}$

Number of bits required for integer operand

$= \log_2 16 = 4$ bits

Number of bits required for floating point

$= \log_2 64 = 6$ bits

Number of encodings consumed by Type 1

$= 4 \times 2^{3\times4} = 2^{14}$

Number of encodings used by Type 2

$= 8 \times 2^6 \times 2^6 = 2^{15}$

Number of encodings used by Type 3

$= 14 \times 2^4 \times 2^6 = 14336$

Number of encodings left

$= 2^{16} - (2^{14} + 2^{15} + 14336) = 2048$

Number of instructions of type 4 $= \frac{2048}{64} = 32$

Hence, the correct answer is 32.

1.11 14

Given :

Number of registers = 64

Size of instruction = 16 bits

Number of distinct I-type instructions = 8

Total number of possible instruction encodings $= 2^{16}$

Number of bits required for register

$= \log_2(64) = 6$ bits

Number of bits required for immediate value $= 4$ bits

I-type instruction format

OPCODE	REGISTER	IMMEDIATE

R-type instruction format

OPCODE	REGISTER	REGISTER

Given 8 I-type instructions, let x be the number of R-type then

$$2^{16} = 8 \times 2^6 \times 2^4 + x \times 2^6 \times 2^6$$

$$\Rightarrow \quad x = 14$$

Hence, the correct answer is 14.

1.12 50

Given :

Instruction	**Semantics**	**Size (bytes)**
MOV R1 (5000)	R1← M[5000]	4
MOV R2 (R3)	R2← M[R3]	4
ADD R2,R1	R2←R1+R2	2
MOV(R3), R2	M[R3]← R2	4
INC R3	R3← R3+1	2
DEC R1	R1← R1–1	2
BNEZ 1004	Branch if non-zero to the absolute address	2
HALT	Stop	1

Content of the memory location 5000 = 10

Content of the register R3 =3000

Content of each of the memory locations from 3000 to 3010 = 50

Initially the contents of the memory are :

1000	MOV R1 (5000)
1004	MOV R2 (R3)
1008	ADD R2,R1
1010	MOV(R3), R2
1014	INC R3
1016	DEC R1
1018	BNEZ 1004
1020	HALT
⋮	⋮
3000	50
3001	50
⋮	⋮
3010	50
⋮	⋮
5000	10

The content of registers are :

R_1	
R_2	
R_3	3000

After execution of the instructions

MOV R1 (5000)

MOV R2 (R3)

contents of the register will be

R_1	10
R_2	50
R_3	3000

After execution of the instructions

ADD R2,R1

MOV(R3), R2

contents of the memory will be

3000	60
3001	50
⋮	⋮
3010	50
⋮	⋮
5000	10

After execution of the instructions

INC R3

DEC R1

contents of the register will be

R_1	9
R_2	60
R_3	3001

Since R_1 is not zero, therefore, 1004 will be stored in PC and again instruction 1 will be executed.

After executing this loop until R_1 becomes 0, i.e., 9 times, the content of memory will be

3000	60
3001	59

3002	58
3003	57
⋮	⋮
3010	**50**
⋮	⋮
5000	10

Hence, the correct answer is 50.

1.13 **(A, D)**

Consider each option one by one :

Option (A) : 0x6665

On little endian (LE) : 0x6665

On Big endian (BE) : 0x6566

Converting LE into decimal :

$$(6665)_{16} = (26213)_{10}$$

Converting BE into decimal :

$$(6566)_{16} = (25958)_{10}$$

Subtracting BE from LE,

$$26213 - 25958 = 255$$

Clearly, LE = 255 + BE

Option (B) : 0x0001

On little endian (LE) : 0x0001

On Big endian (BE) : 0x0100

Converting LE into decimal :

$$(0001)_{16} = (1)_{10}$$

Converting BE into decimal :

$$(0100)_{16} = (256)_{10}$$

Subtracting BE from LE,

$$1 - 256 = -255$$

Option (C) : 0x4243

On little endian (LE) : 0x4243

On Big endian (BE) : 0x4342

Converting LE into decimal :

$$(4243)_{16} = (16963)_{10}$$

Converting BE into decimal :

$$(4342)_{16} = (17218)_{10}$$

Subtracting BE from LE,

$$16963 - 17218 = -255$$

Option (D) : 0x0100

On little endian (LE) : 0x0100

On Big endian (BE) : 0x0001

Converting LE into decimal :

$$(0100)_{16} = (256)_{10}$$

Converting BE into decimal :

$$(0001)_{16} = (1)_{10}$$

Subtracting BE from LE,

$$256 - 1 = 255$$

Clearly, LE = 255 + BE

Hence, the correct options are (A), (D).

Key Point

Little Endian – In this scheme, low-order byte is stored on the starting address (A) and high-order byte is stored on the next address (A + 1).

Big Endian – In this scheme, high-order byte is stored on the starting address (A) and low-order byte is stored on the next address (A + 1).

Example :

Assume number 0x2568

LE representation of 0x2568 is :

$$\underset{\text{higher byte}}{25}\quad\underset{\text{lower byte}}{68}$$

BE representation of 0x2568 is :

$$\underset{\text{higher byte}}{68}\quad\underset{\text{lower byte}}{25}$$

2 Data Path and Control Unit

Practice Questions

2013 IIT Bombay

2.1 Consider the following sequence of micro-operations

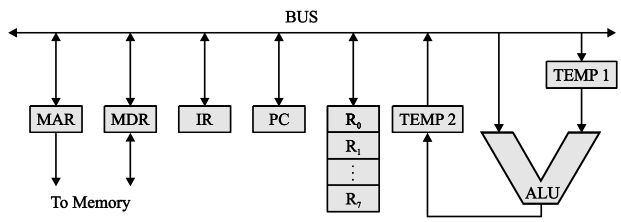
MBR ← PC
MAR ← X
PC ← Y
Memory ← MBR

Which one of the following is a possible operation performed by this sequence?

(A) Instruction fetch
(B) Operand fetch
(C) Conditional branch
(D) Initiation of interrupt service

2020 IIT Delhi

2.2 Consider the following data path diagram -



Consider an instruction: R0 ← R1 + R2. The following steps are used to execute it over the given data path. Assume that PC is incremented approximately. The subscripts *r* and *w* indicate read and write operations, respectively.

1. $R2_r, TEMP1_r, ALU_{add}, TEMP2_w$
2. $R1_r, TEMP1_w$
3. PC_r, MAR_w, MEM_r
4. $TEMP2_r, R0_w$
5. MDR_r, IR_w

Which one of the following is the correct order of execution of the above steps?

(A) 3, 5, 1, 2, 4 (B) 3, 5, 2, 1, 4
(C) 1, 2, 4, 3, 5 (D) 2, 1, 4, 5, 3

Solutions

2.1 (D)

Given :

sequence of micro-operations.

MBR ← PC
MAR ← X
PC ← Y
Memory ← MBR

MBR – Memory Buffer Register (that stores the data being transferred to and from the immediate access store)

MAR – Memory Address Register (that holds the memory location of data that needs to be accessed.)

PC – Program Counter (It contains the address of the instruction being executed at the current time)

The 1st instruction places the value of PC into MBR

The 2nd instruction places an address X into MAR.

The 3rd instruction places an address Y into PC.

The 4th instruction places the value of MBR (which was the old PC value) into Memory.

Now it can be seen from the 1st and the 4th instructions, that the control flow was not sequential and the value of PC was stored in the memory, so that the control can again come back to the address where it left the execution.

This behavior is seen in the case of interrupt handling. And here X can be the address of the location in the memory which contains the beginning address of Interrupt service routine.

And Y can be the beginning address of Interrupt service routine.

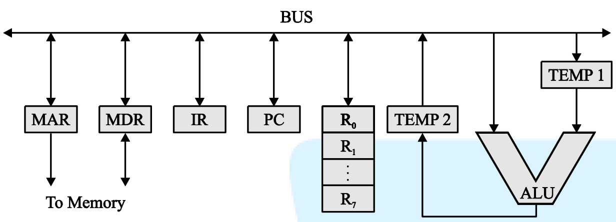
In case of conditional branch (as for option C) only PC is updated with the target address and there is no need to store the old PC value into the memory.

And in the case of Instruction fetch and operand fetch (as for option A and B), PC value is not stored anywhere else.

Hence, the correct option (D).

2.2 **(B)**

Given :



Method – 1

→ Store the content of PC into MAR by issuing memory read.

PC_r, MAR_w, MEM_r

→ Store the data from memory data register into IR.

MDR_r, IR_w

→ Store the content of R1 into temporary register TEMP1

$R1_r, TEMP1_w$

→ Add contents of R2 and TEMP1 and store result in TEMP2.

$R2_r, TEMP1_r, ALU_{add}\ TEMP2_w$

→ Store contents of TEMP2 into R0.

$TEMP2_r, R0_w$

Hence, the correct option is (B).

Method – 2

1	$R2_r, TEMP1_r, ALU_{add}, TEMP2_w$	$TEMP2 \leftarrow R2 + TEMP1$	Addition	Step 4
2	$R1_r, TEMP1_w$	$TEMP1 \leftarrow R1$	Bring to ALU	Step 3
3	PC_r, MAR_w, MEM_r	$MDR \leftarrow M[PC]$	Instructio n fetch	Step 1
4	$TEMP2_r, R0_w$	$R0 \leftarrow TEMP2$	Store result	Step 5
5	MDR_r, IR_w	$IR \leftarrow MDR$	Store the content into IR	Step 2

Hence, the correct option is (B).

3 Instruction Pipelining

Practice Questions

2013 **IIT Bombay**

3.1 Consider an instruction pipeline with five stages without any branch prediction: Fetch Instruction (FI), Decode Instruction (DI), Fetch Operand (FO), Execute Instruction (EI) and Write Operand (WO). The stage delays for FI, DI, FO, EI and WO are 5 ns, 7ns, 10 ns, 8 ns and 6 ns, respectively. There are intermediate storage buffers after each stage and the delay of each buffer is 1 ns. A program consisting of 12 instructions $I_1, I_2, I_3, \ldots, I_{12}$ is executed in this pipelined processor. Instruction I_4 is the only branch instruction and its branch target is I_9. If the branch is taken during the execution of this program, the time (in ns) needed to complete the program is

(A) 132 (B) 165

(C) 176 (D) 328

2014 **IIT Kharagpur**

3.2 Consider the following processors (ns stands for nanoseconds). Assume that the pipeline registers have zero latency.

P_1 : Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P_2 : Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.

P_3 : Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.

P_4 : Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Which processor has the highest peak clock frequency?

(A) P1 (B) P2

(C) P3 (D) P4

3.3 Consider a 6-stage instruction pipeline, where all stages are perfectly balanced. Assume that there is no cycle-time overhead of pipelining. When an application is executing on this 6-stage pipeline, the speedup achieved with respect to non-pipelined execution if 25% of the instructions incur 2 pipeline stall cycles is__________.

3.4 An instruction pipeline has five stages namely, instruction fetch (IF), instruction decode and register fetch (ID/RF), instruction execution (EX), memory access (MEM), and register write back (WB) with stage latencies 1 ns, 2.2 ns, 2 ns, 1 ns, and 0.75 ns, respectively (ns stands for nanoseconds). To gain in terms of frequency, the designers have decided to split the ID/RF stage into three stages (ID, RF1, RF2) each of latency 2.2/3 ns. Also, the EX stage is spilt into two stages (EX1, EX2) each of latency 1 ns. The new design has a total of eight pipeline stages. A program has 20% branch instructions which execute in the EX stage and produce the next instruction pointer at the end of the EX stage in the old design and at the end of the EX2 stage in the new design. The IF stage stalls after fetching a branch instruction until the next instruction pointer is computed. All instructions other than the branch instruction have an average CPI of one in both the designs. The execution times of this program on the old and the new design are P and Q nanoseconds, respectively. The value of P/Q is____________.

3.5 Consider two processors P_1 and P_2 executing the same instruction set. Assume that under identical conditions, for the same input, a program running on P_2 takes 25% less time but incurs 20% more CPI (clock cycles per instruction) as compared to the program running on P_1. If the clock frequency of P_1 is 1GHz, then the clock frequency of P_2 (in GHz) is _____.

2015 **IIT Kanpur**

3.6 Consider a non-pipelined processor with a clock rate of 2.5 gigahertz and average cycles per instruction of four. The same processor is upgraded to a pipelined processor with five stages; but due to the internal pipelined delay, the clock speed is reduced to 2 gigahertz. Assume that there are no stalls in the pipeline. The speedup achieved in this pipelined processor is ________.

3.7 Consider the sequence of machine instructions given below.

MUL	R5, R0, R1
DIV	R6, R2, R3
ADD	R7, R5, R6
SUB	R8, R7, R4

In the above sequence, R0 to R8 are general purpose registers. In the instruction shown, the first register stores the result of the operation performed on the second and the third registers. This sequence of instructions is to be executed in a pipelined instruction processor with the following 4 stages:
(1) Instruction Fetch and Decode (IF), (2) Operand Fetch (OF),
(3) Perform Operation (PO) and
(4) Write back the result (WB).
The IF, OF and WB stages take 1 clock cycle each for any instruction. The PO stage takes 1 clock cycle for ADD or SUB instruction, 3 clock cycles for MUL instruction and 5 clock cycles for DIV instruction. The pipelined processor uses operand forwarding from the PO stage to the OF stage. The number of clock cycles taken for the execution of the above sequence of instructions is ________.

3.8 Consider the following reservation table for a pipeline having three stages S_1, S_2 and S_3.

Time →

	1	2	3	4	5
S_1	X				X
S_2		X		X	
S_3			X		

The minimum average latency (MAL) is ________.

3.9 Consider the following code sequence having five instructions I1 to I5. Each of these instructions has the following format.

OP Ri, Rj, Rk

where operation OP is performed on contents of registers Rj and Rk and the result is stored in register Ri.

I1 : ADD R1, R2, R3
I2 : MUL R7, R1, R3
I3 : SUB R4, R1, R5
I4 : ADD R3, R2, R4
I5 : MUL R7, R8, R9

Consider the following three statements:
S1: There is an anti-dependence between instructions I2 and I5.
S2: There is an anti-dependence between instructions I2 and I4.
S3: Within an instruction pipeline an anti-dependence always creates one or more stalls.
Which one of above statements is/are correct?
(A) Only S1 is true
(B) Only S2 is true
(C) Only S1 and S2 are true
(D) Only S2 and S3 are true

2016 IISc Bangalore

3.10 The stage delays in a 4-stage pipeline are 800, 500, 400 and 300 picoseconds. The first stage (with delay 800 picoseconds) is replaced with a functionally equivalent design involving two stages with respective delays 600 and 350 picoseconds. The throughput increase of the pipeline is _______ percent.

3.11 Consider a 3 GHz (gigahertz) processor with a three-stage pipeline and stage latencies τ_1, τ_2, and τ_3 such that $\tau_1 = 3\tau_2/4 = 2\tau_3$. If the longest pipeline stage is split into two pipeline stages of equal latency, the new frequency is __________ GHz, ignoring delays in the pipeline registers.

3.12 Suppose the functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G, respectively. Given two instances of U_F and two instances of U_G, it is required to implement the computation $F(G(X_i))$ for $1 \le i \le 10$. Ignoring all other delays, the minimum time required to complete this computation is ____________ nanoseconds.

2017 IIT Roorkee

3.13 Instruction execution in a processor is divided into 5 stages. Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Execute (EX), and Write Back (WB). These stages take 5, 4, 20, 10, and 3 nanoseconds (ns) respectively. A pipelined implementation of the processor requires buffering between each pair of consecutive stages with a delay of 2 ns. Two pipelined implementation of the processor are contemplated :

(i) A naive pipeline implementation (NP) with 5 stages and

(ii) An efficient pipeline (EP) where the OF stage is divided into stages OF1 and OF2 with execution times of 12 ns and 8 ns respectively.

The speedup (correct to two decimal places) achieved by EP over NP in executing 20 independent instructions with no hazards is _________.

2018 IIT Guwahati

3.14 The instruction pipeline of a RISC processor has the following stages: Instruction Fetch (IF), Instruction Decode (ID), Operand Fetch (OF), Perform Operation (PO) and Writeback (WB), The IF, ID, OF and WB stages take 1 clock cycle each for every instruction. Consider a sequence of 100 instructions. In the PO stage, 40 instructions take 3 clock cycles each, 35 instructions take 2 clock cycles each, and the remaining 25 instructions take 1 clock cycle each. Assume that there are no data hazards and no control hazards.

The number of clock cycles required for completion of execution of the sequence of instruction is ______ .

2020 IIT Delhi

3.15 Consider a non-pipelined processor operating at 2.5 GHz. It takes 5 clock cycles to complete an instruction. You are going to make a 5-stage pipeline out of this processor. Overheads associated with pipelining force you to operate the pipelined processor at 2 GHz. In a given program, assume that 30% are memory instructions, 60% are ALU instructions and the rest are branch instructions. 5% of the memory instructions cause stalls of 50 clock cycles each due to cache misses and 50% of the branch instructions cause stalls of 2 cycles each. Assume that there are no stalls associated with the execution of ALU instructions. For this program, the speedup achieved by the pipelined processor over the non-pipelined processor (round off to 2 decimal places) is __________ .

2021 IIT Bombay

3.16 Consider a pipelined processor with 5 stages, Instruction Fetch (IF), Instruction Decode (ID). Execute (EX), Memory Access (MEM), and Write Back (WB). Each stage of the pipeline, except the EX-stage, takes one cycle. Assume that the ID stage merely decodes the instruction and the register read is performed in the EX-stage. The EX-stage takes one cycle for ADD instruction and two cycles for MUL instruction. Ignore pipeline register latencies. Consider the following sequence of 8 instructions:

ADD, MUL, ADD, MUL, ADD, MUL, ADD, MUL

Assume that every MUL instruction is data-dependent on the ADD instruction just before it and every ADD instruction (except the first ADD) is data-dependent on the MUL instruction just before it. The Speedup is defined as follows:

Speed up =

$$\frac{\text{ET without operand forwarding}}{\text{ET with operand forwarding}}$$

The Speedup achieved in executing the given instruction sequence 011 the pipelined processor (rounded to 2 decimal places) is_____.

3.17 A five-stage pipeline has stage delays of 150, 120. 150, 160 and 140 nanoseconds. The registers that are used between the pipeline stages have a delay of 5 nanoseconds each. The total time to execute 100 independent instructions on this pipeline, assuming there are no pipeline stalls, is ________ nanoseconds.

Solutions

3.1 (B)

Given :

Number of stages = 5

Stage delays = 5, 7, 10, 8, 6 ns

Buffer delay = 1 ns

Number of instructions = 12

Method 1

According to the question branch is taken that means after I4, I9 will be executed and further I10, I11, I12.

Space-time diagram :

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
11	FI	DI	FO	EI	WO										
12		FI	DI	FI	EI	WO									
13			FI	DI	PO	EI	WO								
14				FI	DI	FO	EI	WO							
19					Stall	Stall	Stall	FI	DI	FO	EI	WO			
110									FI	DI	FO	EI	WO		
111										FI	DI	FO	EI	WO	
112											FI	DI	FO	EI	WO

Number of cycles required $= 15$ cycles

Execution time = No. of cycles $\times$ Cycle time

Stage delay $= \max(5, 7, 10, 8, 6)$)

Total delay $= 10 + 1 = 11$ ns

Execution time $= 15 \times 11 = 165$ ns

Hence, the correct option (B).

Method 2

According to the question branch is taken that means after I4, I9 will be executed and further I10, I11, I12.

Therefore,

Number of instructions executed = 8

Number of cycles without hazards $= k + n - 1$

$= 5 + 8 - 1 = 12$ cycles

Number of stalls cycles because of branch instructions $= 4 - 1 = 3$

Total number of cycles $= 12 + 3 = 15$

Execution time $= 15 \times 11 = 165$ ns

Hence, the correct option (B).

3.2 **(C)**

Given :

Processor configurations :

P_1 : Four-stage pipeline with stage latencies 1 ns, 2 ns, 2 ns, 1 ns.

P_2 : Four-stage pipeline with stage latencies 1 ns, 1.5 ns, 1.5 ns, 1.5 ns.

P_3 : Five-stage pipeline with stage latencies 0.5 ns, 1 ns, 1 ns, 0.6 ns, 1 ns.

P_4 : Five-stage pipeline with stage latencies 0.5 ns, 0.5 ns, 1 ns, 1 ns, 1.1 ns.

Method 1

$$\text{CT for } P_1 = \frac{1}{2} = 0.5 \text{ GHz}$$

$$\text{CT for } P_2 = \frac{1}{1.5} = 0.67 \text{ GHz}$$

$$\text{CT for } P_3 = \frac{1}{1} = 1 \text{ GHz}$$

$$\text{CT for } P_4 = \frac{1}{1.1} = 0.90 \text{ GHz}$$

Hence, the correct option is (C).

Method 2

Find maximum stage delay of each process :

For P_1, maximum stage delay = 2 ns

For P_2, maximum stage delay = 1.5 ns

For P_3, maximum stage delay = 1 ns

For P_4, maximum stage delay = 1.1 ns

$$\text{Since, clock cycle time} \propto \frac{1}{\text{max (stage delays)}}$$

Processor with minimum stage delay will have highest frequency.

Therefore, P_3 has the highest clock frequency.

Hence, the correct option is (C).

📖 Key Point

$$\text{Clock cycle time} \propto \frac{1}{\text{max (stage delays)}}$$

3.3 **4**

Given :

Number of stages = 6

25% of the instructions incur 2 pipeline stall cycles.

📖 Key Point

Execution time of pipeline $= 1 +$ stall frequency $\times$ stall cycle

Execution time of non-pipeline = 6 cycles

Execution time of pipeline $= 1 +$ stall frequency $\times$ stall cycle

$= 1 + 0.25 \times 2 = 1.5$ cycles

$$\text{Speed up} = \frac{6}{1.5} = 4$$

Hence, the correct answer is 4.

3.4 **1.54**

Given :

For old design,

Number of stages = 5

Stage delays = 1, 2.2, 2, 1, 0.75 ns

For old design,

Number of stages = 8

Stage delays = 1, 0.73, 0.73, 0.73, 1, 1, 1, 0.75 ns

Key Point

Cycle time = max (stage delay)

Average execution time = (1 + frequency of branch instruction × branch penalty)×cycle time

P (Old design) :

Stage latency $= \max(1, 2.2,\ 2, 1,\ 0.75) = 2.2$ ns

Average execution time = (1 + frequency of branch instruction × branch penalty)×cycle time

$$= (1 + 0.2 \times 2) \times 2.2 = 3.08 \text{ ns}$$

Q (New design) :

Stage delay

$$= \max(1,\ 0.73,\ 0.73,\ 0.73,\ 1,\ 1,\ 1,\ 0.75)$$

$$= 1 \text{ ns}$$

Average execution time $= (1 + 0.2 \times 5) \times 1 = 2$ ns

$$\frac{P}{Q} = \frac{3.08}{2} = 1.54$$

Hence, the correct answer is 1.54.

3.5 **1.6**

Given :

Two processors P_1 and P_2

For P_1

Execution time = T_1

Number of clock cycles = C_1

Frequency = $f_1 = 1$ GHz

For P_2

Execution time = T_1

Number of clock cycles = C_1

Frequency = f_2

CPU TIME (T)

$$= \frac{\text{Instructuion Count} \times \text{Cycle Per Instruction}}{\text{Clock frequency}}$$

$$T = IC \times CPI \times f^{-1}$$

$$\frac{T \times f}{CPI} = IC$$

P_1 & P_2 executing same instruction set So,

No. of Instructions same for both

$$= I_1 = I_2 = IC$$

If P_1 takes T_1 time,

$$T_2 = 0.75 \times T_1$$

$$\frac{T_2}{T_1} = 0.75$$

If P_1 incurs C_1 clock cycles per instruction,

$$C_2 = 1.2 \times C_1$$

$$\frac{C_2}{C_1} = 1.2$$

Since IC is same for both,

$$\frac{(f_1 \times T_1)}{C_1} = \frac{(f_2 \times T_2)}{C_2} \text{ and } f_1 = 1 \text{ GHz}$$

$$f_2 = \left(\frac{C_2}{C_1}\right) \times \left(\frac{T_1}{T_2}\right) \times f_1$$

$$f_2 = \frac{1.2 \times 1 \text{ GHz}}{0.75} = 1.6 \text{ GHz}$$

Key Point

CPU time

$$(T) = \frac{I_C \times \text{CPI}}{F} = \frac{\text{instruction count} \times \text{CPI}}{\text{frequency}}$$

Hence, the correct answer is **1.6.**

3.6 **3.2**

Given :

For non-pipelined processor :

Clock rate = 2.5 GHz

Cycle per instruction (CPI) = 4

For pipelined processor :

Clock rate = 2 GHz

Number of stages (k) = 5

CPI = 1 for pipeline processor

Execution time of non-pipe lined $= \dfrac{4}{2.5} = 1.6$ ns

Execution time of pipe lined $= \dfrac{1}{2} = 0.5$ ns

Speed up $= \dfrac{1.6}{0.5} = 3.2$

Hence, the correct answer is 3.2.

3.7 **13**

Given :

sequence of machine instructions :

MUL R5, R0, R1
DIV R6, R2, R3
ADD R7, R5, R6
SUB R8, R7, R4

Number of stages (k) = 4

Number of clock cycles for IF, OF, WB stage = 1

Number of clock cycles for PO stage in case of

ADD or SUB = 1

MUL = 3

DIV = 5

Space-time diagram :

	1	2	3	4	5	6	7	8	9	10	11	12	13
11	IF	OF	PO	PO	PO	WB							
12		1F	OF	-	-	PO	PO	PO	PO	PO	W		
13			1F	-	-	-	-	-	-	OF	PO	WB	
14				-	-	-	-	-	-	IF	OF	PO	WB

Number of cycles required = 13

Hence, the correct answer is 13.

3.8 **3**

Given :

Number of stages = 3

Reservation table

Time →

	1	2	3	4	5
S_1	X				X
S_2		X		X	
S_3			X		

For given reservation table :

→ Forbidden latency = {0, 2, 4}

$\{(5-1)=4,\ 4-2,\ 2\}$

{0 is by default forbidden latency}

→ Collision vector $= C_n, C_{n-1}, ...C_0$ where $n = \max$ (forbidden latency)

⇒ Collision vector $= 10101$

→ State diagram :

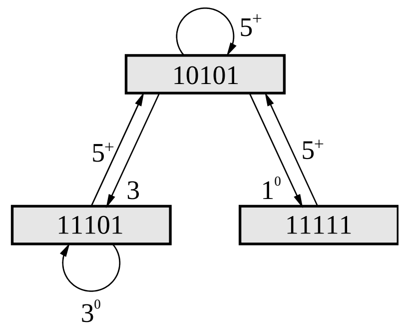
State 1 : 10101 = Reaches state 2 (11111) after 1 cycle

Reaches state 3 (11101) after 3 cycles

Reaches state 2 (10101) after 5 cycle

From greedy cycle,

$$\text{MAL} = \min\left(3, \frac{1+5}{2}\right) = 3$$



Hence, the correct answer is 3.

3.9 **(B)**

Given :

Code sequence :

I1 : ADD R1, R2, R3
I2 : MUL R7, R1, R3
I3 : SUB R4, R1, R5
I4 : ADD R3, R2, R4
I5 : MUL R7, R8, R9

statements:

S1: There is an anti-dependence between instructions I2 and I5.

S2: There is an anti-dependence between instructions I2 and I4.

S3: Within an instruction pipeline an anti-dependence always creates one or more stalls.

The given instructions can be written as below:

I1: R1 = R2 + R3
I2: R7 = R1 * R3
I3: R4 = R1 - R5
I4: R3 = R2 + R4
I5: R7 = R8 * R9

An anti-dependency, also known as write-after-read (WAR), occurs when an instruction requires a value that is later updated.

S1: There is an anti-dependence between instructions I2 and I5.

False, I2 and I5 don't form any write after read situation.

They both write R7.

S2: There is an anti-dependence between instructions I2 and I4.

True, I2 reads R3 and I4 writes it.

S3: Within an instruction pipeline an anti-dependence always creates one or more stalls.

Anti-dependency can be removed by renaming variables.

See following example.

1. B = 3
2. A = B + 1
3. B = 7

Renaming of variables could remove the dependency.

1. B = 3
N. B2 = B
2. A = B2 + 1
3. B = 7

Hence, the correct option is (B).

3.10 **33.33**

Given :

For pipeline 1,

Number of stages = 4

Stage delays = 800, 500, 400 and 300 ps

For pipeline 2,

Number of stages = 5

Stage delays = 600, 350, 500, 400 and 300 ps

Key Point

Cycle time = max (stage delay)

Pipe line 1 :

Cycle time = max (stage delay) =800 psec

$$\text{Performance} = \text{Throughput} = \frac{1}{\text{Execution time}}$$

$$\text{Throughput} = \frac{1}{800}$$

Pipe line 2 :

Cycle time = max (stage delay) = 600 psec

$$\text{Throughput} = \frac{1}{600}$$

$$\text{Throughput increase} = \frac{\frac{1}{600} - \frac{1}{800}}{\frac{1}{800}} \times 100$$

$$= 33.33\%$$

Hence, the correct answer is 33.33.

3.11 **4**

Given :

Number of stages = 3

Frequency = 3 GHz

Stage delays = τ_1, τ_2, τ_3

$$\tau_1 = \frac{3\tau_2}{4} = 2\tau_3$$

$$\tau_3 = \frac{\tau_1}{2},\ \tau_2 = \frac{4\tau_1}{3}$$

Let $\tau_1 = x$

$$\tau_2 = \frac{4x}{3} = 1.33x,\ \tau_3 = \frac{x}{2} = 0.5x$$

Cycle time = max (stage delay)

$$= \max(x, 1.33x, 0.5x)$$

$$\frac{1}{3}\text{ ns} = 1.33x$$

$$x = 0.25$$

Now longest pipeline is split into two pipeline

Stage delays $= \{x, 0.5x, 0.66x, 0.66x\}$

Cycle time = max (stage delays) $= x$ ns

$$\text{Frequency} = \frac{1}{x} = \frac{1}{0.25} = 4\text{ GHz}$$

Hence, the correct answer is 4.

Key Point

Cycle time = max (stage delay)

3.12 **28**

Given : The functions F and G can be computed in 5 and 3 nanoseconds by functional units U_F and U_G, respectively.

Method 1

there are two functional units, so each unit will get 5 instances to compute on,

F takes 5 ns and G takes 3 ns

For F to begin the computation, there should be at least one G

Hence time at which F ends computing

$$= 3 + 5 \times 5 = 28\text{ ns}$$

Hence, the correct answer is 28.

Method 2

Since there are 10 inputs and both the functional units can work in parallel, only 5 steps are required for computation.

The computation sequence is as follows :

Input	U_G (time)	U_F (time)
1, 2	3	8
3, 4	6	13
5, 6	9	18
7, 8	12	23
9, 10	15	28

Hence time at which F ends computing is 28 ns.

Hence, the correct answer is 28.

3.13 **1.50**

Given :

For naive pipeline implementation

Number of stages = 5

Stage delays = 5, 4, 20, 10, 3 ns

Buffer delay = 2 ns

For efficient pipeline implementation

Number of stages = 8

Stage delays = 5, 4, 12, 8, 10, 3 ns

Buffer delay = 2 ns

Number of instructions = 20

Key Point

Cycle time = max (stage delay) + Buffer Delay

Time taken to execute instruction in pipeline

$=(k+(n-1))\times T_p$

Case 1 (NP) :

Cycle time = max (stage delay) + buffer delay

$$=20+2=22 \text{ ns}$$

Execution time

$$=(k+n-1)\times T_p=(5+20-1)22=528 \text{ ns}$$

Case 2 (EP) :

Cycle time = max (stage delay) + buffer

$$=12+2=14 \text{ ns}$$

Execution time

$$=(k+n-1)T_P=(6+20-1)\times 14=350$$

$$\text{Speed up }=\frac{528}{350}=1.508\approx 1.50$$

Hence, the correct answer is 1.50.

3.14 **219**

Given :

Number of stages = 5

Number of clock cycles for IF, ID, OF, WB stage = 1

Number of instructions = 100

Considering each instruction takes one cycle at each stage i.e. no stall cycle

No. of cycles for 100 instruction $=(100+5-1)$

$$=104 \text{ cycles}$$

But for PO stage, 40 instruction takes 3 cycles, 35 take 2 and 25 take 1 cycle.

Therefore, 40 instruction takes 2 extra cycles, 35 take 1 extra cycles and 25 take 0 extra cycle

No. of stall cycles $=40\times 2+35\times 1=115$ cycle

Total no. of cycles $=104+115=219$

Hence, the correct answer is 219.

3.15 **2.16**

Given :

For a non-pipelined processor :

Frequency = 2.5 GHz

CPI = 5

For a pipelined processor :

Number of stages = 5

Frequency = 2 GHz

30% are memory instructions, 60% are ALU instructions and the rest are branch instructions. 5% of the memory instructions cause stalls of 50 clock cycles each due to cache misses and 50% of the branch instructions cause stalls of 2 cycles each.

Execution time of non-pipelined processor

$$=\frac{5}{2.5}=2 \text{ ns}$$

For pipelined processor,

30 % memory instruction, 60 % ALU, 10 % branch instruction

Execution time of pipelined processor

$$=0.6+0.3(0.05(1+50)+0.95)$$
$$+0.1(0.5(1+2)+0.5)$$
$$=1.85 \text{ cycle}$$
$$=\frac{1.85}{2}\text{ ns}=0.925 \text{ ns}$$

$$\text{Speed up}=\frac{2}{0.925}=2.162\approx 2.16$$

Hence, the correct answer is 2.16.

3.16 **1.875**

Given : Number of stages = 5

Number of instructions = 8

Space-time diagram :

With operand forwarding

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
ADD	IF	ID	EX	MEM	WB											
MUL		IF	ID	EX	EX	MEM	WB									
ADD			ID	ID		EX	MEM	WB								
MUL				IF		ID	EX	EX	MEM	WB						
ADD						IF	ID		EX	MEM	WB					
MUL							IF		ID	EX	EX	MEM	WB			
ADD									IF	ID		EX	MEM	WB		
MUL										IF		ID	EX	EX	MEM	WB

Execution cycle with operand forwarding = 16

Without Operand Forwarding

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
ADD	IF	ID	EX	MEM	WB										
MUL		IF	ID			EX	EX	MEM	WB						
ADD			IF			ID				EX	MEM	WB			
MUL						IF				ID			EX	EX	MEM
ADD										IF			ID		
MUL													IF		
ADD															
MUL															

	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
ADD															
MUL															
ADD															
MUL	WB														
ADD		EX	MEM	WB											
MUL		ID			EX	EX	MEM	WB							
ADD		IF			ID				EX	MEM	WB				
MUL					IF				ID			EX	EX	MEM	WB

Execution cycle without operand forwarding = 30

Speed up $= \frac{30}{16} = 1.875$

Hence, the correct answer is 1.875.

3.17 **17160**

Given :

Number of stages = 5

Stage delays = 150, 120, 150, 160, 140 ns

Buffer delay = 5 ns

Number of instructions = 100

For pipeline processor

Cycle time t_p = Max of stage delay + Buffer delay

= Max (150, 120, 150, 160, 140) + 5 ns

$= 160\,\text{ns} + 5\,\text{ns}$

$= 165\,\text{ns}$

Key Point

Cycle time = max (stage delay) + Buffer Delay

Time taken to execute instruction in pipeline

$= (k + (n-1)) \times T_p$

$\therefore \quad ET_{pipe} = (5 + 100 - 1) \times 165\,\text{ns}$

$= 17160\,\text{ns}$

Hence, the correct answer is 17160.

4 Memory Organization

Practice Questions

2013 **IIT Bombay**

4.1 In a k-way set associative cache, the cache is divided into v sets, each of which consists of k lines. The lines of a set are placed in sequence one after another. The lines in set s are sequenced before the lines in set $(s+1)$. The main memory blocks are numbered 0 onwards. The main memory block numbered j must be mapped to any one of the cache lines from

(A) $(j \bmod v)*k$ to $(j \bmod v)*k+(k-1)$

(B) $(j \bmod v)$ to $(j \bmod v)+(k-1)$

(C) $(j \bmod k)$ to $(j \bmod k)+(v-1)$

(D) $(j \bmod k)*v$ to $(j \bmod k)*v+(v-1)$

2014 **IIT Kharagpur**

4.2 An access sequence of cache block addresses is of length N and contains n unique block addresses. The number of unique block addresses between two consecutive accesses to the same block address is bounded above by k. What is the miss ratio if the access sequence is passed through a cache of associativity $A \geq k$ exercising least-recently-used replacement policy?

(A) n/N (B) $1/N$

(C) $1/A$ (D) k/n

4.3 A 4-way set-associative cache memory unit with a capacity of 16KB is built using a block size of 8 words. The word length is 32 bits. The size of the physical address space is 4 GB. The number of bits for the TAG field is___.

4.4 In designing a computer's cache system, the cache block (or cache line) size is an important parameter. Which one of the following statements is correct in this context?

(A) A smaller block size implies better spatial locality

(B) A smaller block size implies a smaller cache tag and hence lower cache tag overhead

(C) A smaller block size implies a smaller cache tag and hence lower cache hit time

(D) A smaller block size incurs a smaller cache miss penalty

4.5 If the associativity of a processor cache is doubled while keeping the capacity and block size unchanged, which one of the following is guaranteed to be NOT affected?

(A) Width of tag comparator

(B) Width of set index decoder

(C) Width of way selection multiplexor

(D) Width of processor to main memory data bus

4.6 Consider a main memory system that consists of 8 memory modules attached to the system bus, which is one word wide. When a write request is made, the bus is occupied for 100 nanoseconds (ns) by the data, address, and control signals. During the same 100 ns, and for 500 ns there after, the addressed memory module executes one cycle accepting and storing the data. The (internal) operation of different memory modules may overlap in time, but only one request can be on the bus at any time. The maximum number of stores (of one word each) that can be initiated in 1 millisecond is ________.

4.7 The memory access time is 1 nanosecond for a read operation with a hit in cache, 5 nanoseconds for a read operation with a miss in cache, 2 nanoseconds for a write operation with a hit in cache and 10 nanoseconds for a write operation with a miss in cache. Execution of a sequence of instructions involves 100 instruction fetch operations, 60 memory operand read operations and 40 memory operand write operations. The cache hit-ratio is 0.9. The average memory access time (in nanoseconds) in executing the sequence of instructions is________.

2015 IIT Kanpur

4.8 Assume that for a certain processor, a read request takes 50 nanoseconds on a cache miss and 5 nanoseconds on a cache hit. Suppose while running a program, it was observed that 80% of the processor's read requests result in a cache hit. The average read access time in nanoseconds is ________.

4.9 Consider a machine with a byte addressable main memory of 2^{20} bytes, block size of 16 bytes and a direct mapped cache having 2^{12} cache lines. Let the address of two consecutive bytes in main memory be $(E201F)_{16}$. What are the tag and cache line address (in hex) for main memory address $(E201F)_{16}$?

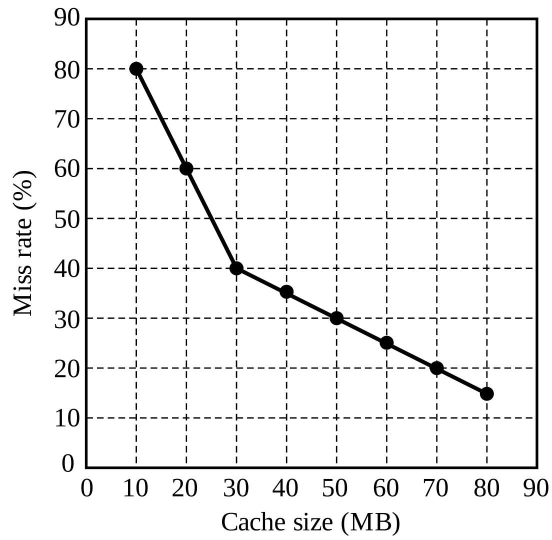
(A) E, 201 (B) F, 201

(C) E, $E20$ (D) 2, $01F$

2016 IISc Bangalore

4.10 The width of the physical address on a machine is 40 bits. The width of the tag field in a 512 KB 8-way set associative cache is ______ bits.

4.11 A file system uses an in-memory cache to cache disk blocks. The miss rate of the cache is shown in the figure. The latency to read a block from the cache is 1 ms and to read a block from the disk is 10 ms. Assume that the cost of checking whether a block exists in the cache is negligible. Available cache sizes are in multiples of 10 MB.



The smallest cache size required to ensure an average read latency of less than 6 ms is ______ MB.

4.12 A processor can support a maximum memory of 4GB, where the memory is word-addressable (a word consists of two bytes). The size of the address bus of the processor is at least ______ bits.

2017 IIT Roorkee

4.13 Consider a two-level cache hierarchy with $L1$ and $L2$ caches. An application incurs 1.4 memory accesses per instruction on average. For this application, the miss rate of $L1$ cache is 0.1; the $L2$ cache experiences, on average, 7 misses per 1000 instructions. The miss rate of $L2$ expressed correct to two decimal places is _________.

4.14 Consider a 2-way set associative cache with 256 blocks and uses LRU replacement. Initially the cache is empty. Conflict misses are those misses which occur due to contention of multiple blocks for the same cache set. Compulsory misses occur due to first time access to the block. The following sequence of accesses to memory blocks

(0, 128, 256, 128, 0, 128, 256, 128, 1, 129, 257, 129, 1, 129, 257, 129)

is repeated 10 times. The number of conflict misses experienced by the cache is__________.

4.15 A cache memory unit with capacity of N words and block size of B words is to be designed. If it is designed as direct mapped cache, the length of the TAG field is 10 bits. If the cache unit is now designed as a 16-way set-associative cache, the length of the TAG field is ________ bits.

4.16 In a two-level cache system, the access times of $L1$ and $L2$ caches are 1 and 8 clock cycles, respectively. The miss penalty from the $L2$ cache to main memory is 18 clock cycles. The miss rate of $L1$ cache is twice that of $L2$. The average memory access time (AMAT) of this cache system is 2 cycles. The miss rates of $L1$ and $L2$ respectively are

(A) 0.111 and 0.056
(B) 0.056 and 0.111
(C) 0.0892 and 0.1784
(D) 0.1784 and 0.0892

4.17 The read access times and the hit ratios for different caches in a memory hierarchy are as given below.

Cache	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

The read access time of main memory is 90 nanoseconds. Assume that the caches use the referred-words-first read policy and the write back policy. Assume that all the caches are direct mapped caches. Assume that the dirty bit is always 0 for all the blocks in the caches. In execution of a program, 60% of memory reads are for instruction fetch and 40% are for memory operand fetch. The average read access time in nanoseconds (up to 2 decimal places) is________

4.18 Consider a machine with a byte addressable main memory of 2^{32} bytes divided into blocks of size 32 bytes. Assume that a direct mapped cache having 512 cache lines is used with this machine. The size of the tag field in bits is_____________.

2018 **IIT Guwahati**

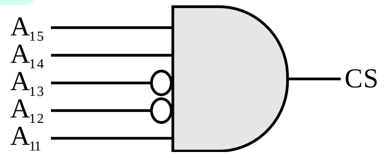
4.19 A 32-bit wide main memory unit with a capacity of 1GB is built using $256 \times M \times 4-$bit DRAM chips. The number of rows of memory cells in the DRAM chip is 2^{14}. The time taken to perform one refresh operation is 50 nanoseconds. The refresh period is 2 milliseconds. The percentage (rounded to the closest integer) of the time available for performing the memory read/write operations in the main memory unit is_____.

4.20 The size of the physical address space of a processor is 2^P bytes. The word length is 2^W bytes. The capacity of cache memory is 2^N bytes. The size of each cache block is 2^M words. For a K-way set-associative cache memory, the length (in number of bits) of the tag field is

(A) $P-N-\log_2 K$
(B) $P-N+\log_2 K$
(C) $P-N-M-W-\log_2 K$
(D) $P-N-M-W+\log_2 K$

2019 **IIT Madras**

4.21 The chip select logic for a certain DRAM chip in a memory system design is shown below. Assume that the memory system has 16 address lines denoted by A15 to A0. What is the range of address (in hexadecimal) of the memory system that can get enabled by the chip select (CS) signal?



(A) C800 to CFFF
(B) CA00 to CAFF
(C) C800 to C8FF
(D) DA00 to DFFF

4.22 A certain processor uses a fully associative cache of size 16 kB, the cache block size is 16 bytes. Assume that the main memory is byte addressable and uses a 32-bit address. How many bits are required for the Tag and the Index fields respectively in the addresses generated by the processor?

(A) 24 bits and 0 bits

(B) 28 bits and 4 bits

(C) 24 bits and 4 bits

(D) 28 bits and 0 bits

4.23 A certain processor deploys a single-level cache. The cache block size is 8 words and the word size is 4 bytes. The memory system uses a 60-MHz clock. To service a cache miss, the memory controller first takes 1 cycle to accept the starting address of the block, it then takes 3 cycles to fetch all the eight words of the block, and finally transmits the words of the requested block at the rate of 1 word per cycle. The maximum bandwidth for the memory system when the program running on the processor issues a series of read operations is___$\times 10^6$ bytes/sec

2020 IIT Delhi

4.24 A direct mapped cache memory of 1 MB has a block size of 256 bytes. The cache has an access time of 3 ns and a hit rate of 94%. During a cache miss, it takes 20 ns to bring the first word of a block from the main memory, while each subsequent word takes 5 ns. The word size is 64 bits. The average memory access time in ns (round off to 1 decimal place) is______.

4.25 A computer system with a word length of 32 bits has a 16 MB byte- addressable main memory and a 64 KB, 4-way set associative cache memory with a block size of 256 bytes. Consider the following four physical addresses represented in hexadecimal notation.

A1 = 0*x*42C8A4,

A2 = 0*x*546888,

A3 = 0*x*6A289C,

A4 = 0*x*5E4880

Which one of the following is TRUE?

(A) A1 and A4 are mapped to different cache sets.

(B) A2 and A3 are mapped to the same cache set.

(C) A3 and A4 are mapped to the same cache set.

(D) A1 and A3 are mapped to the same cache set.

2021 IIT Bombay

4.26 Assume a two-level inclusive cache hierarchy, L1 and L2, where L2 is the larger of the two. Consider the following statements.

S1: Read misses in a write through L1 cache do not result in writebacks of dirty lines to the L2

S2: Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is used with writeback caches.

Which of the following statements is correct?

(A) S1 is true and S2 is false

(B) S1 is false and S2 is true

(C) S1 is true and S2 is true

(D) S1 is false and S2 is false

4.27 Consider a set-associative cache of size $2\text{KB}\ (1\text{KB} = 2^{10}\ \text{bytes})$ with cache block size of 64 bytes. Assume that the cache is byte-addressable and a 32 -bit address is used for accessing the cache. If the width of the tag field is 22 bits, the associativity of the cache is______.

4.28 Consider a computer system with a byte-addressable primary memory of size 2^{32} bytes. Assume the computer system has a direct-mapped cache of size 32 KB $(1\text{KB} = 2^{10}\ \text{bytes})$, and each cache block is of size 64 bytes. The size of the tag field is ________ bits.

Solutions

4.1 **(A)**

Given :

Number of sets in cache $= V$

for set 0, the cache lines are numbered $0, 1, \ldots, k-1$. Now for set 1, the cache lines are numbered $k, k+1, \ldots, k+(k-1)$.

So m.m. block j will map to set (j mod V) from lines (j mod v) $\times k$ to (j mod v)$\times k + (k-1)$

Hence, the correct option is (A).

4.2 **(A)**

Given :

There are N accesses to cache. Out of these n are unique block addresses.

Therefore, minimum n misses are guaranteed due to unique block addresses.

In worst case scenario, all the k block addresses can go to the same set.

Since associativity is $\geq$ k above case is not possible hence in best case only compulsory misses occur.

So, miss ratio is $\dfrac{n}{N}$.

Hence, the correct option is (A).

4.3 **20**

Given :

Word length $= 32$ bits

Block size $= 8$ words $= 8 \times 4$ B $= 32$ Bytes

Number of bits in word offset $= 5$ bits

$$\text{Number of blocks } = \frac{16\text{ KB}}{8\text{ words}} = 512 \text{ blocks}$$

$$\text{Number of sets } = \frac{512}{4} = 128$$

$\Rightarrow$ Set off set $= 7$ bits

$\therefore$ TAG $= 32 - (7 + 5) = 20$

32

TAG	SO	WO
20	7	5

Hence, the correct answer is 20.

Key Point

In direct mapping, the physical address is divided as-

Tag	Line Offset	Block/Word Offset

Physical Address

4.4 **(D)**

Consider each option one by one :

A is incorrect, smaller block size means during a memory access only a smaller part of near by addresses are brought to cache so spatial locality is reduced.

B and C is incorrect, smaller block size means more number of blocks and hence index bit go up and offset bits go down. Tag bits remain same.

D is correct, when a cache block is smaller, it could accommodate more number of blocks so hit ratio is improved, miss penalty is lowered.

Hence, the correct option is (D).

4.5 **(D)**

Given :

The associativity of a processor cache is doubled while keeping the capacity and block size unchanged.

When associativity is doubled, set offset is affected, accordingly, width of tag comparator is affected, width of set index decoder is affected.

A k-way set associative cache needs k-to-1 way selection multi plexer so doubling the associativity, width of way selection multiplexer is doubled.

Width of processor to main memory data bus is guaranteed to be NOT affected.

Hence, the correct option is (D).

4.6 **10000**

Given :

To initiate one word data to the bus takes 100 ns

So, time taken for one word $= 100$ ns

$\therefore$ Number of words that can be initiated for store in 1 ms is :

$$= \frac{1 \text{ ms}}{100 \text{ ns}} = 10000$$

Hence, the correct answer is 10000.

4.7 **1.68**

Given :

Memory access time for a read operation with hit in cache = 1 ns

Memory access time for a read operation with miss in cache = 5 ns

Memory access time for a write operation with hit in cache = 2 ns

Memory access time for a write operation with miss in cache = 10 ns

Cache hit ratio = 0.9

Number of instruction fetch operation = 100

Number of memory operand read operation = 60

Number of memory operand write operation = 40

Average read time $= 0.9 \times 1 \text{ ns} + 0.1 \times 5 \text{ ns}$

$$= 1.4 \text{ ns}$$

Number of read operation

$$= 100(\text{fetch}) + 60(\text{read})$$

{fetch is also read}

Total time required for read operation

$$= 160 \times 1.4 = 224 \text{ ns}$$

Average write time $= 0.9 \times 2 \text{ ns} + 0.1 \times 10 \text{ ns}$

$$= 2.8 \text{ ns}$$

Number of write operation $= 40$

Total time required for write operation

$$= 40 \times 2.8 = 112 \text{ ns}$$

Total time for both read and write

$$= 224 + 112 = 336$$

Average memory access time $= \frac{336}{200} = 1.68$ ns

Hence, the correct answer is 1.68.

4.8 **14**

Given :

Time taken for a read request on a cache miss $T_m = 50$ ns

Time taken for a read request on a cache hit $T_h = 5$ ns

Hit ratio $H_r = 80\% = 0.8$

Average read access time

$$= H_r \times T_h + (1 - H_r) \times T_m$$

$$= 0.8 \times 5 \text{ ns} \times 0.2 \times 50 \text{ ns} = 14 \text{ ns}$$

Hence, the correct answer is 14.

4.9 **(A)**

Given :

Block size $= 16$ Bytes $= 2^4$

$\Rightarrow$ Block offset $= 4$ bits

Number of lines $= 2^{12}$

$\Rightarrow$ Line offset $= 12$ bits

Main memory size $= 2^{20}$ Bytes

$\Rightarrow$ Physical address $= 20$ bits

$\longleftarrow$ 20 $\longrightarrow$

TAG	LINE OFFSET	BLOCK OFFSET
4	12	4

$(E\ 2\ 0\ 1\ F)_{16}$

$$(\underbrace{1100}_{\substack{E \\ \text{TAG}}}\ \underbrace{0010\ \ 0000\ \ 0001}_{\substack{2\ \ 0\ \ 1 \\ \text{Line address}}}1111)_2$$

Hence, the correct option is (A).

4.10 **24**

Given :

Physical address $= 40$ bits

$\Rightarrow$ Tag + Set + Block $= 40$ ... (i)

Cache size = number of sets $\times$ block per set $\times$ Block size

$\Rightarrow$ 512 KB = number of sets $\times 8 \times$ block size

$\Rightarrow$ Number of sets $\times$ Block size

$$= \frac{512}{8} \text{ KB} = 64 \text{ KB}$$

$\Rightarrow$ $S + B = 16$ ... (ii)

Form (i) and (ii)

Tag $= 24$ bits

Hence, the correct answer is 24.

4.11 **30**

Given :

Latency to read a block from cache $= 1$ ms

Latency to read a block from disk $= 10$ ms

Average read latency $= H_1T_1 + (1 - H_1)T_2$

$(\because H_2 = 1)$

$$= H \times 1 \text{ ms} + (1 - H) \times 10 \text{ ms}$$

average read latency ≤ 6 ms

$\Rightarrow \quad H + (1 - H) \times 10 < 6$ ms

$\Rightarrow \quad H > 0.445$

$\Rightarrow \quad$ Miss rate < 0.555

From the figure minimum cache size for above condition is 25 MB, but cache is available in multiples of 10 hence answer is 30 MB.

Hence, the correct answer is 30.

4.12 **31**

Given :

A processor can support a maximum memory of 4GB, where the memory is word-addressable.

Size of memory $=$ Number of words $\times$ Number of bits per word

$$2^{32} \text{ B} = \text{Number of words} \times 2 \text{ B}$$

$\Rightarrow \quad$ Number of words $= 2^{31}$

$\Rightarrow \quad$ Number of address lines $= 31$

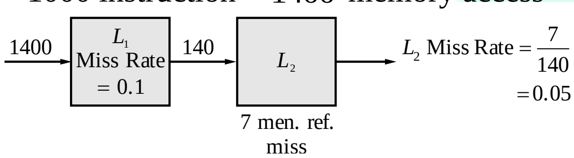
Hence, the correct answer is 31.

4.13 **0.05**

Given :

1 instruction $= 1.4$ memory access

$\Rightarrow$ 1000 instruction $= 1400$ memory access



Hence, the correct answer is 0.05.

4.14 **76**

Given :

Number of lines $= 256$

$\Rightarrow$ Number of sets $= \dfrac{256}{2} = 128$

Cache Memory

0	$\cancel{0}$ $\cancel{256}$ $\cancel{0}$ 256	128
1	$\cancel{1}$ $\cancel{257}$ $\cancel{1}$ 257	129
...		
127		

For 1st access of block 0, 256, 128, 1, 257 and 129 are compulsory miss

$\Rightarrow \quad$ 6 compulsory miss

After first access $= 4$ (conflict miss)

After second access $= 8$ (conflict miss)

$\Rightarrow \quad$ total conflict misses $= 4 + 8 \times 9 = 76$

Hence, the correct answer is 76.

4.15 **14**

Given :

Cache size $= N$ words

Number of bits to address cache size $= \log_2 N$

Block size $= B$ words

Number of bits to address block size $= \log_2 B$

Number of blocks in cache $= \dfrac{N}{B}$

Number of bits to represent blocks $= \left[\log_2 \dfrac{N}{B}\right]$

Number of sets in cache $= \dfrac{N/B}{16}$

Number of bits to represent sets

$$= \left[\log_2 \frac{N/B}{16}\right] = \left[\log_2 \frac{N}{B}\right] - \log_2 2^4$$

According to question;

$$10 + \left[\log_2 \frac{N}{B}\right] + [\log_2 B]$$

$$= X + \left[\log_2 \frac{N}{B}\right] - \log_2 2^4 + [\log_2 B]$$

$$10 = X - 4$$

$$X = 14 \text{ bits}$$

Hence, the correct answer is 14.

4.16 **(A)**

Given :

Access time of $L1$ = 1 clock cycle

Access time of $L2$ = 8 clock cycle

Miss penalty from the $L2$ cache to main memory = 18 clock cycles

Let miss rate of $L1 = x$

The miss rate of $L1$ cache is twice that of $L2$

Therefore, miss rate of $L2 = 0.5x$

The average memory access time (AMAT) of the cache system = 2 cycles

Average memory access time = $L1$ access time + $L1$ miss rate $\times$ $L2$ access time

+ $L1$ miss rate $\times$ $L2$ miss rate $\times$ M.M. access time

$$\Rightarrow \quad 2 = 1 + x \times 8 + 0.5x^2 \times 18$$

$$\Rightarrow \quad 9x^2 + 8x - 1 = 0$$

$$\Rightarrow \quad x = \frac{-8 \pm \sqrt{64+36}}{18} = \frac{2}{18} = 0.111$$

$L1$ miss rate = 0.111

Therefore, $L2$ miss rate $\dfrac{0.111}{2} = 0.056$

Hence, the correct option is (A).

4.17 **4.72**

Given :

Read access time of cache :

Cache	Read access time (in nanoseconds)	Hit ratio
I-cache	2	0.8
D-cache	2	0.9
L2-cache	8	0.9

Read access time of memory = 90 ns

60% of memory reads are for instruction fetch and 40% are for memory operand fetch.

Let, H_I = hit ratio of I-cache

H_D = hit ratio of D-cache

H_{L2} = hit ratio of L2-cache

H_M = hit ratio of memory

T_I = access time of I-cache

T_D = access time of D-cache

T_{L2} = access time of L2-cache

T_M = access time of memory

Average instruction fetch time

$$= H_I T_I + (1-H_I) H_{L2}(T_I + T_{L2})$$
$$+ (1-H_I)(1-H_{L2}) H_M (T_I + T_{L2} + T_M)$$
$$= 0.8 \times 2 + 0.2 \times 0.9 \times (8+2)$$
$$+ 0.2 \times 0.1 \times (90+8+2)$$
$$= 5.4 \text{ ns}$$

T_{avg} read = frequency $\times$ read cycle time

$$= 0.6 \times 5.4 = 3.24 \text{ ns}$$

Average data read time

$$= H_D T_D + (1-H_D) H_{L2}(T_D + T_{L2})$$
$$+ (1-H_D)(1-H_{L2}) H_M (T_D + T_{L2} + T_M)$$
$$= 0.9 \times 2 + 0.1 \times 0.9 \times (8+2)$$
$$+ 0.1 \times 0.1 \times (90+8+2)$$
$$= 3.7 \text{ ns}$$

T_{avg} read = frequency $\times$ read cycle time

$$= 0.4 \times 3.7 = 1.48 \text{ ns}$$

Total time $= 3.24 \text{ ns} + 1.48 \text{ ns} = 4.72 \text{ ns}$

Hence, the correct answer 4.72.

4.18 **18**

Given :

Main memory size $= 2^{32}$ bytes

$\Rightarrow$ 32 bit address

Block size $= 32$ Bytes

$\Rightarrow$ Block offset $= 5$ bits

Number of cache lines $= 512$

$\Rightarrow$ Line offset $= 9$ bits

32		
TAG	LO	BO
18	9	5

TAG $= 32 - (9+5) = 18$

Hence, the correct answer is 18.

4.19 **59**

Given :

One refresh operation takes 50 ns

Total number of rows $= 2^{14}$

Total time to refresh all rows $= 2^{14} \times 50$ ns

Refresh period $= 2$ ms

% time spent is refresh

$$= \frac{2^{14} \times 50 \text{ ns}}{2 \text{ ms}} \times 100 = 40.96\%$$

% time spent is read/write

$$= 100 - 40.96 = 59.04\% = 59\%$$

Hence, the correct answer is 59.

4.20 **(B)**

Given :

Physical address space $= 2^P$ Bytes

$\Rightarrow$ P bits to represent main memory.

Cache size $= 2^N$ Byte

$\Rightarrow$ N bits to represent cache memory

Tag size $= 2^X$ Bytes

$\Rightarrow$ X bits for tag

$$(\text{Size of Tag}) \times \frac{\text{Cache size}}{K} = \text{Total memory size}$$

$$\Rightarrow \quad 2^X \times \frac{2^N}{K} = 2^P$$

$$\Rightarrow \quad 2^X = 2^{P-N+\log(K)}$$

$$\Rightarrow \quad X = P - N + \log_2(K)$$

Hence, the correct option is (B).

4.21 **(A)**

Given : $A_{15} = 1, A_{14} = 1, A_{13} = 0, A_{12} = 0, A_{11} = 1$

So possible range in binary :

(1100100000000000) to (1100111111111111)

$\Rightarrow$ (C800) to (CFFF)

Hence, the correct option is (A).

4.22 **(D)**

Given :

Cache memory size $= 16$ KB

Block size $= 16$ B

$\Rightarrow$ Block offset $= 4$ bits

Main memory address $= 32$ bits

Number of lines $= \frac{16K}{16} = 2^{10}$

Fully associative means N-way

$\Rightarrow$ Number of sets $= \frac{N}{P} = \frac{2^{10}}{2^{10}} = 1$

32

TAG	WO
28	4

Hence, the correct option is (D).

Key Point

In fully associative mapping, the physical address is divided as-

Tag	Block/Word Offset

Physical Address

4.23 **160**

Given :

Time to transfer a cache block $= 1 + 3 + 8 = 12$ cycles

$\Rightarrow$ 4 Bytes $\times 8 =$ 32 Bytes in 12 cycles

So, bandwidth

$$= \frac{32}{12 \times CT} = \frac{32}{12 \times \frac{1}{60 \times 10^6}} = 160 \times 10^6$$

Hence, the correct answer is 160.

4.24 **13.5**

Given :

Block size $= 256$ Bytes

Word size $= 64$ Bits $= 8$ Bytes

$\Rightarrow$ Block size (in words)

$$= \frac{256}{8} = 32 \text{ Words}$$

Time taken to bring the block from main memory

$$= 20 + 31 \times 5 = 175 \text{ ns}$$

Average memory access time

$$= 0.94 \times 3 + (1 - 0.94) \times (175 + 3)$$

$$= 13.5 \text{ ns}$$

Hence, the correct answer is 13.5

4.25 **(B)**

Given :

Cache memory size $= 64$ KB $= 2^{16}$ B

Block size $= 256$ B $= 2^8$ B

$\Rightarrow$ Block offset $= 8$ bits

Number of lines $= \frac{2^{16}}{2^8} = 2^8$

Number of sets $= \frac{\text{Number of lines}}{\text{P} - \text{way}}$

$$= \frac{2^8}{4} = 2^6$$

$\Rightarrow$ set offset $= 6$ bits

TAG	SO	BO
10	6	8

24

$A1 = 0x42C8A4 \rightarrow C8 = 11001000$

$\left.\begin{array}{l} A2 = 0x546888 \rightarrow 68 = 01\underline{101000} \\ A3 = 0x6A289C \rightarrow 28 = 00\underline{101000} \end{array}\right\}$ LSB 6 bits

are same

$A4 = 0x5E4880 \rightarrow 48 = 01001000$

Hence, the correct option is (B).

4.26 (A)

Given :

statements.

S1: Read misses in a write through L1 cache do not result in write backs of dirty lines to the L2

S2: Write allocate policy must be used in conjunction with write through caches and no-write allocate policy is used with write back caches.

Consider each statement one by one :

S1: Write through Cache means simultaneous access for both L1 and L2 and in this case if read miss happen in L1 cache we tried to fetch it from L2 cache and if its dirty then we don't need to write back to L2 because it's Write through Cache.

S2: Write allocate policy used for write back cache not for Write through Cache.

Hence, the correct option is (A).

4.27 2

Given :

32 bit address is used for accessing the cache.

It is given that cache is Set-Associative.

The address bits get split as follows:

Tag	Set Offset	Block Offset

32 bits

Block Size = 64 B $\Rightarrow$ Block offset = 6 bits.

Given that Tag field width = 22 bits.

Therefore, width of Set Index field

$= 32 - 22 - 6 = 4$

$\Rightarrow 2^4 = 16$ sets in the cache

Cache size is 2 KB and Block size = 64 B

$\Rightarrow 2^5 = 32$ blocks present in the cache.

16 sets contain 32 blocks

$\Rightarrow$ 2 blocks per set or associativity = 2

Hence, the correct answer is 2.

Key Point

In set associative mapping, the physical address is divided as-

Tag	Set Offset	Block/Word Offset

Physical Address

4.28 17

Given :

For direct mapping technique;

Main memory size $= 2^{32}$ bytes

$\Rightarrow$ Physical Address Space (PAS)

$= \log_2 2^{32} = 32$ bit

Cache memory size $= 32\text{KB} = 2^{15}$ B

Block size/word offset, (B) = 64 bytes

we know that number of lines,

$$N = \frac{\text{Cache Memory Size}}{\text{Block Size}}$$

$$\therefore \quad N = \frac{2^{15}}{2^6} = 2^9 B$$

so, Line Offset (LO) $= \log_2 N$

$\Rightarrow \log_2 2^9 = 9$ bits

Block offset $= \log_2 B \Rightarrow \log_2 2^6 = 6$ bits

$\therefore$ tag bit = PAS – (LO + BO)

$\Rightarrow$ Tag $= 32 - (9 + 6)$ bits

$\Rightarrow$ Tag = 17 bits.

so, tag bit size in given problem is 17 bits.

Hence, the correct answer is 17.

Key Point

In direct mapping, the physical address is divided as-

Tag	Line Offset	Block/Word Offset

Physical Address

5 Input Output Organization

Practice Questions

2016 IISc Bangalore

5.1 The size of the data count register of a DMA controller is 16 bits. The processor needs to transfer a file of 29, 154 kilobytes from disk to main memory. The memory is byte addressable. The minimum number of times the DMA controller needs to get the control of the system bus from the processor to transfer the file from the disk to main memory is _____.

2020 IIT Delhi

5.2 Consider the following statements.

I. Daisy chaining is used to assign priorities in attending interrupts.

II. When a device raises a vectored interrupt, the CPU does polling to identify the source of interrupt.

III. In polling, the CPU periodically checks the status bits to know if any device needs its attention.

IV. During DMA, both the CPU and DMA controller can be bus masters at the same time.

Which of the above statements are TRUE?

(A) I and II only (B) I and II only

(C) III only (D) I and III only

2021 IIT Bombay

5.3 Consider a computer system with DMA support. The DMA module is transferring one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals. Consider a 2 MHz processor. If 0.5% processor cycles are used for DMA, the data transfer rate of the device is __________ bits per second.

Solutions

5.1 456

Given :

Data count register = 16 bits

Data count register gives the number of words the DMA can transfer in a single cycle.

$\Rightarrow$ Count value $= 2^{16} = 64$ KBytes

Data transferred is stopped at when count value becomes zero, so data count = 16 bits,

Maximum value is $2^{16} - 1 = 65535$

In one cycle DMA transfer 2^{16}B

So, minimum no. of cycles to transfer 29154 KB is $= \dfrac{29154 \times 2^{10}}{2^{16}} \approx 456$

Hence, the correct answer is 456.

📖 Key Point

Data count register gives the number of words the DMA can transfer in a single cycle.

5.2 (D)

Given :

Statements :

I. Daisy chaining is used to assign priorities in attending interrupts.

II. When a device raises a vectored interrupt, the CPU does polling to identify the source of interrupt.

III. In polling, the CPU periodically checks the status bits to know if any device needs its attention.

IV. During DMA, both the CPU and DMA controller can be bus masters at the same time.

$\rightarrow$ I is TRUE. In Daisy chaining interrupt is attended in serial manner. The device with highest priority is placed first and so on.

$\rightarrow$ II is FALSE. Vectored interrupts are achieved by assigning a unique code to each interrupting device, hence doesn't involve polling.

→ III is TRUE. Polling is a mechanism in which CPU periodically checks the status bit to know if a device needs attention.

→ IV is FALSE. During DMA only one of the CPU or DMA controller can use the BUS at a time. CPU releases the BUS after getting request from DMA.

Hence, the correct option is (D).

5.3 **80000**

Given :

Frequency of processor = 2 MHz

0.5% processor cycles are used for DMA

DMA module transfers one 8-bit character in one CPU cycle from a device to memory through cycle stealing at regular intervals.

As we know 2 MHz $\rightarrow 2\times10^{6}$ cycles per second

Now 0.5 % of these cycles are taken by DMA to transfer the data.

So total number of cycles taken to transfer the data will be $\frac{0.5}{100}\times2\times10^{6}=10,000$ and in each cycle 8 bits are transferred.

So, data transfer rate in bits per second

$=8\times10000=80,000$.

Hence, the correct answer is 80000.

CHAPTER 3 Operating System

Marks Distribution of Operating System in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	5	12
2004*	3	4	11
2005*	-	2	4
2006*	1	8	17
2007*	2	6	14
2008*	2	5	12
2009	2	5	12
2010	3	2	7
2011	4	2	8
2012	1	4	9
2013	2	4	10
2014 Set-1	2	3	8
2014 Set-2	1	3	7
2014 Set-3	1	3	7

*** CS and IT combined**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	2	4	10
2015 Set-2	2	3	8
2015 Set-3	2	2	6
2016 Set-1	1	4	9
2016 Set-2	1	3	7
2017 Set-1	2	2	6
2017 Set-2	2	2	6
2018	2	3	8
2019	2	4	10
2020	2	4	10
2021 Set-1	3	2	7
2021 Set-2	2	3	8

Syllabus : Operating System

System calls, processes, threads, inter-process communication, concurrency and synchronization. Deadlock. CPU and I/O scheduling. Memory management and virtual memory. File systems.

Contents : Operating System

1 Process Management - I

Practice Questions

2013 IIT Bombay

1.1 A scheduling algorithm assigns priority proportional to the waiting time of a process. Every process starts with priority zero (the lowest priority). The scheduler re- valuates the process priorities every *T* Time units and decides the next process to schedule. Which one of the following is TRUE if the processes have no I/O operations and all arrive at time zero?

(A) This algorithm is equivalent to the first-come-first-serve algorithm.

(B) This algorithm is equivalent to the round-robin algorithm.

(C) This algorithm is equivalent to the shortest-job- first algorithm.

(D) This algorithm is equivalent to the shortest-remaining- time- first algorithm.

2014 IIT Kharagpur

1.2 Which one of the following is FALSE?

(A) User level threads are not scheduled by the kernel.

(B) When a user level thread is blocked, all other threads of its process are blocked.

(C) Context switching between user level threads is faster than context switching between kernel level threads.

(D) Kernel level threads cannot share the code segment.

1.3 Three processes A, B and C each execute a loop of 100 iteration. In each iteration of the loop, a process performs a single computation that requires t_c CPU millisecond and then initiates a single I/O operation that lasts for t_{io} millisecond. It is assumed that the computer where the processes execute has sufficient number of I/O devices and the OS of the computer assigns different I/O devices to each process. Also the scheduling overhead of the OS is negligible. The process has the following characteristics:

Process	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The processes A, B and C are started at times 0, 5 and 10 millisecond respectively in a pure time-sharing system (Round Robin scheduling that uses a time slice of 50 milliseconds. The time in millisecond at which process C would complete its first I/O operation is ________.

1.4 An operating system uses shortest remaining time first scheduling algorithm for pre-emptive following of processes. Consider the following set of processes with their arrival time and CPU burst time (in millisecond):

Process	**Arrival time**	**Burst time**
P1	0	12
P2	2	4
P3	3	6
P4	8	5

The average waiting time (in milliseconds) of the processes is _______.

1.5 Consider the following set of processes that need to be scheduled on a single CPU. All the times are given in milliseconds.

Process Name	**Arrival time**	**Execution Time**
A	0	6
B	3	2
C	5	4
D	7	6
E	10	3

Using the shortest remaining time first scheduling algorithm, the average process turnaround time (in msec) is _______.

2015 IIT Kanpur

1.6 The maximum number of processes that can be in Ready state for a computer system with *n* CPUs is

(A) n

(B) n^2

(C) 2^2

(D) Independent of n

1.7 Consider a uniprocessor system executing three tasks T_1, T_2 and T_3 each of which is composed of an infinite sequence of jobs (or instances) which arrive periodically at intervals of 3, 7 and 20 milliseconds, respectively. The priority of each task is the inverse of its period, and the available tasks are scheduled in order of priority, with the highest priority task scheduled first. Each instance of T_1, T_2 & T_3 requires an execution time of 1, 2 and 4 milliseconds, respectively. Given that all task initially arrive at the beginning of the 1^{st} millisecond and task pre-emption are allowed, the first instance of T_3 completes its execution at the end of ________ millisecond.

1.8 For the processes listed in the following table, which of the following scheduling schemes will give the lowest turnaround time?

Process	Arrival time	Processing time
A	0	3
B	1	6
C	4	4
D	6	2

(A) First Come First Serve

(B) Non- preemptive Shortest Job First

(C) Shortest Remaining Time

(D) Round Robin with Quantum value two

1.9 Consider an arbitrary set of CPU- bound processes with unequal CPU burst lengths submitted at the same time to a computer system. Which one of the following process scheduling algorithm would minimize the average waiting time in the ready queue?

(A) Shortest remaining time first

(B) Round- robin with time quantum less than the shortest CPU burst

(C) Uniform random

(D) Highest priority first with priority proportional to CPU burst length

1.10 Consider the following processes, with the arrival time and the length of the CPU burst given in millisecond. The scheduling algorithm used is pre-emptive shortest remaining-time first.

Process	Arrival time	Burst time
P1	0	10
P2	3	6
P3	7	1
P4	8	3

The average turnaround time of these processes is _______ milliseconds.

2017 IIT Roorkee

1.11 Threads of a process share

(A) global variables but not heap

(B) head but not global variables

(C) neither global variables nor heap

(D) both heap and global variables

1.12 Consider the following CPU processes with arrival time (in milliseconds) and length of CPU bursts (in millisecond) As given below:

Process	Arrival time	Burst time
P1	0	7
P2	3	3
P3	5	5
P4	6	2

If the pre-emptive shortest remaining time first scheduling algorithm is used to schedule the process, then the average waiting time across all process is _______ milliseconds.

1.13 Which of the following is/are shared by all the threads in a process?

I. Program counter

II. Stack

III. Address space

IV. Registers

(A) I and II only

(B) III only

(C) IV only

(D) III and IV only

1.14 Consider the set of processes with arrival time (in milliseconds). CPU burst time (in milliseconds). And priority (0 is the highest priority) shown below. None of the processes have I/O burst time.

Process	Arrival Time	Burst Time	Priority
P_1	0	11	2
P_2	5	28	0
P_3	12	2	3

P_4	2	10	1
P_5	9	16	4

The average waiting time (in millisecond) of all the processes using preemptive priority scheduling algorithm is _________.

2019 **IIT Madras**

1.15 Consider the following four processes with arrival times (in milliseconds) and their length of CPU bursts (in milliseconds) as shown below:

Process	Arrival Time	Burst Time
P1	0	3
P2	1	1
P3	3	3
P4	4	Z

These processes are run on a single processor using preemptive Shortest Remaining Time First scheduling algorithm. If the average waiting time of the processes is 1 millisecond, then the value of Z is ________.

1.16 The following C program is executed on a Unix/Linux system:

```
#include <unistd.h>
int main ()
{
 int i ;
for (i=0; i<10; i++)
if (i% 2 = = 0) fork ();
return 0;
}
```

The total number of child processes created is_______.

2020 **IIT Delhi**

1.17 Consider the following set of processes, assumed to have arrived at time 0. Consider the CPU scheduling algorithms Shortest Job First (SJF) and Round Robin (RR). For RR, assume that the processes are scheduled in the order P_1, P_2, P_3, P_4.

Process	Burst Time
P1	8
P2	7
P3	2
P4	4

If the time quantum for RR is 4 ms, then the absolute value of the difference between the average turnaround times (in ms) of SJF and RR (round off to 2 decimal places) is _______.

1.18 Consider the following statements about process state transitions for a system using pre-emptive scheduling.

A running process can move to ready state.

I. A ready process can move to running state.

II. A blocked process can move to running state.

III. A blocked process can move to ready state.

Which of the above statements are TRUE?

(A) I, II, and III only

(B) II and III only

(C) I, II, and IV only

(D) I, II, III and IV only

2021 **IIT Bombay**

1.19 Three processes arrive at time zero with CPU bursts of 16, 20 and 10 milliseconds. If the scheduler has prior knowledge about the length of the CPU bursts, the minimum achievable average waiting time for these three processes in a non-pre-emptive scheduler (rounded to nearest integer) is _______ milliseconds.

1.20 Which of the following statement(s) is/are correct in the context of CPU scheduling?

(A) Turnaround time includes waiting time

(B) The goal is to only maximize CPU utilization and minimize throughput

(C) Round-robin policy can be used even when the CPU time required by each of the processes is not known apriori

(D) Implementing pre-emptive scheduling needs hardware support

Solutions

1.1 **(D)**

Given :-

The scheduler re-evaluates the process priorities every T time units and decides the next process to schedule. From above we understand that there must be a time quantum T exits, Without time quantum scheduler cannot re-evaluates the process priorities.

So,

Option (A) : This algorithm is equivalent to the first-come-first-serve algorithm.

It is false, there is no time quantum in FCFS.

Option (B) : This algorithm is equivalent to the round-robin algorithm.

It is true, In Round Robin algorithm priority is assigned based on waiting time of process. (When arrival time is not same).

Option (C) : This algorithm is equivalent to the shortest-job-first algorithm.

It is false, in shortest-job-first algorithm, priority is assigned based on Burst Time of a process.

Option (D) : This algorithm is equivalent to the shortest-remaining-time-first algorithm.

It is false, SRTF is just the pre-emptive version of SJF, those which have less Burst time after first process executed, executed before large burst time process. And in this priority is assigned based on Burst time of a process.

Hence, the correct option is (D).

1.2 **(D)**

Given :

Option (A) : User level threads are not scheduled by the Kernel.

It is true, User level threads are scheduled by thread library. Kernel does not know about it.

Option (B) : When a user level thread is blocked, all other threads of its process are blocked.

It is true

With a multithreaded Kernel, user level threads can make non-blocking system calls without getting blocked. But in this option, it is explicitly. But in this option, it is explicitly said 'a thread is blocked'.

Option (C) : Context switching between user level threads is faster than context switching between Kernel level threads

Context switching between user level threads is faster as they actually have no context is faster as they actually have no context-switch-nothing is saved and restored while for Kernel level thread, Registers, PC and SP must be saved and restored.

Option (D) : Kernel level threads cannot share the code segment

It is false

Threads can share the code segments.

They have only separate Registers and stack.

1.3 **1000**

Given :

Process	t_c	t_{io}
A	100 ms	500 ms
B	350 ms	500 ms
C	200 ms	500 ms

The gantt chart for round robin algorithm for the first iteration execution for each of the 3 processes is as follows :

A	B	C	A	B	C	B	C	B	C	B

0 50 100 150 200 250 300 350 400 450 500 550

After finishing t_c CPU at time 500 ms, C goes for I/O operation, which needs 500 ms more, so the time at which process C would complete its first I/O operations is $500+500=1000$ ms.

At 1000 ms time unit C complete its I/O.

Hence, the correct answer is 1000.

1.4 **5.5**

Given :

Process	Arrival time	Burst time
P1	0	12
P2	2	4
P3	3	6
P4	8	5

Gantt chart :

P_1	P_2	P_2	P_3	P_3	P_4	P_1

0 2 3 6 8 12 17 27

Process	AT	BT	CT	TAT	WT
P_1	0	12	27	27	15
P_2	2	4	6	4	0
P_3	3	6	12	9	3
P_4	8	5	17	9	4

Key Point

Waiting Time = Turnaround Time – Burst Time

Average waiting time = $\dfrac{15+0+3+4}{4}$ = 5.5 ms

Hence, the correct answer is 5.5.

1.5 **7.2**

Given :

Process id	AT	ET
A	0	6
B	3	2
C	5	4

D	7	6
E	10	3

Using SRTF :

A	B	A	C	E	D

0 3 5 8 12 15 21

🕮 Key Point
Turnaround Time = Completion Time – Arrival Time

$TAT(A) = 8 - 0 = 8$

$TAT(B) = 5 - 3 = 2$

$TAT(C) = 12 - 5 = 7$

$TAT(D) = 21 - 7 = 14$

$TAT(E) = 15 - 10 = 5$

Average TAT (Turnaround Time)

$$= \frac{8+2+7+14+5}{5} = 7.2$$

Hence, the correct answer is 7.2.

1.6 (D)

Maximum number of processes that can be in ready state is independent of number of processes(n).
Hence, the correct option is (D).

1.7 4

Given :

Process Name	Arrival time	Execution Time
T_1	3	1
T_2	7	2
T_3	20	4

Periodic arrival time of T1: 0, 3, 6, 9, 12, 15, 18, 21

Priority of $T_1 = 1/3$, Service time of $T_1 = 1$.

Priority of $T_2 = 1/7$, service time of $T_2 = 2$

Periodic arrival times of $T_3 : 0,\ 20,\ 40....$

Priority of $T_3 = 1/20$, service time of $T_3 : 0,\ 20,\ 40$

T_1 has highest priority and T_3 has lowest priority.

T_1	T_2	T_1	T_3	T_1	T_2	T_1	T_3

0 1 2 3 4 5 6 7 8 9 10 11 12

3: T_1 arrives gain; 6: T_1 arrives; 9: T_1 arrives

First instance of T_3 (4 units) completed at the

Hence, the correct answer is 7.

1.8 (C)

Given :

Process	Arrival time	Processing time
A	0	3
B	1	6
C	4	4
D	6	2

🕮 Key Point
Turnaround Time = Completion Time – Arrival Time

(1) FCFS :

A	B	C	D

0 3 9 13 15

$$\text{Avg TAT} = \frac{3+8+9+9}{4} = \frac{29}{4} = 7.25$$

(2) Non preemptive SJF :

A	B	D	C

0 3 9 11 15

$$\text{Avg TAT} = \frac{3+8+5+11}{4} = 6.75$$

(3) SRTF :

A	B	C	D	E

0 3 4 8 10 15

$$\text{Avg TAT} = \frac{3+14+4+4}{4} = \frac{25}{4} = 6.25$$

(4) RR :

A	B	A	C	B	D	C	B

0 2 4 5 7 9 11 13 15

$$\text{Avg TAT} = \frac{5+14+9+5}{4} = \frac{33}{4} = 8.25$$

SRTF has lowest turn around time.
Hence, the correct option is (C).

1.9 (A)

SJF (Shortest Job First) minimizes average waiting time.
Now, here as all processes arrive at the same time, SRTF (Shortest Remaining Time First) would be same as SJF.
Hence, the correct option is (A).

1.10 8.25

🕮 Key Point
Turnaround Time = Completion Time – Arrival Time
Burst time – the total time needed by a process from the CPU for its complete execution.

Gantt chart :

P_1	P_2	P_3	P_2	P_4	P_1

0 3 7 8 10 13 20

Process number	Arrival time	Burst time	Completion time	TAT
1	0	10	20	20
2	3	6	10	1
3	7	1	8	1
4	8	3	13	5

Average turn around time: $\frac{33}{4} = 8.25$

Hence, the correct answer is 8.25.

1.11 (D)

Generally, every thread of a process has their own PC and stack. Both heap and global variables are shared by every thread of a process.

Hence, the correct option is (D).

1.12 3.0

Key Point

Waiting Time = Turnaround Time – Burst Time

Using preemptive SRTF algorithm Gantt chart will be

P_1	P_2	P_4	P_1	P_3

0 3 6 8 12 17

$6 \downarrow P_2$, $8 \downarrow P_4$, $12 \downarrow P_1$, $17 \downarrow P_3$

Turn around time

$$P_1 \rightarrow 12 - 0 = 12$$

$$P_2 \rightarrow 6 - 3 = 3$$

$$P_3 \rightarrow 17 - 5 = 12$$

$$P_4 \rightarrow 8 - 6 = 2$$

Waiting time

$$P_1 \rightarrow 12 - 7 = 5$$

$$P_2 \rightarrow 3 - 3 = 0$$

$$P_3 \rightarrow 12 - 5 = 7$$

$$P_4 \rightarrow 2 - 2 = 0$$

Average waiting time $= \frac{5+0+7+0}{4} = 3.0$

Hence, the correct answer is 3.0.

1.13 (B)

All the threads share address space but other entities like, stack, PC, register are not shared and every thread will have its own.

Hence, the correct option is (B).

1.14 29

Key Point

Turnaround Time = Completion Time – Arrival Time

Process	AT	BT	Priority	CT	TAT	WT
P_1	0	11	2	49	49	38
P_2	5	28	0 (high)	33	28	0
P_3	12	2	3	51	39	37
P_4	2	10	1	40	38	28
P_5	9	16	4	67	58	42
					Total	145

P_1	P_4	P_2	P_4	P_1	P_3	P_5

0 2 5 33 40 49 51 67

Average waiting time $= \frac{145}{5} = 29$

Hence, the correct answer is 29.

1.15 2

Given :

Process	Arrival Time	Burst Time
P1	0	3
P2	1	1
P3	3	3
P4	4	Z

Gantt chart till $t = 4$:

P_1	P_2	P_1	……

0 1 2 4

Now, there are 2 processes which are waiting in ready queue P_3 and P_4

So, **case 1 :** If $z < 3$,

P_1	P_2	P_1	P_4	P_3

0 1 2 4 $4+z$ $7+z$

Hence, $z = 1$ or 2 is only possible. After solving we will get $z = 2$.

case 2 : If $z \geq 3$

P_1	P_2	P_1	P_3	P_4

0 1 2 4 7 $7+z$

Here, Average waiting time will not depend on z, hence $z > 3$ is not possible.

Key Point

Waiting Time = Turnaround Time – Burst Time

Hence, the correct answer is 2.

1.16 31

Key Point

If n times fork() is called, it creates $2^n - 1$ child process

Given :

```
#include <unistd.h>
int main ()
{
     int i ;
    for (i=0; i<10; i++)
    if (i% 2 = = 0) fork ();
    return 0;
}
```

Fork (): Statement inside the loop is executed when $i = 0, 2, 4, 6, 8$. So $n = 5$ times fork () is executed.

So number of child process is created

$$= 2^5 - 1 = 31$$

Hence, the correct answer is 31.

1.17 **5.25**

Key Point

Turnaround Time = Completion Time – Arrival Time

Given :

Process	Burst Time
P1	8
P2	7
P3	2
P4	4

SJF :

P_3	P_4	P_2	P_1

0 2 6 13 21

$$TAT = (21-0) + (13-0) + (2-0) + (6-0)$$

$$\text{Average } TAT = \frac{42}{4} = 10.5$$

RR :

P_1	P_2	P_3	P_4	P_1	P_2

0 4 8 10 14 18 27

$$TAT = (18-0) + (21-0) + (10-0) + (14-0)$$

$$= 18 + 21 + 10 + 14$$

$$\text{Average } TAT = \frac{63}{4} = 15.75$$

Hence, $|SJF(TAT) - RR(TAT)| = |10.5 - 15.75|$

$$= 5.25$$

Hence, the correct answer is 5.25.

1.18 **(C)**

Given statements :

I. A running process can move to ready state.

II. A ready process can move to running state.

III. A blocked process can move to running state.

IV. A blocked process can move to ready state.

A blocked process cannot go to running state directly.

Except (III), every option is viable.

Hence, the correct option is (C).

1.19 **12**

Given :

	Arrival Time	Burst Time
P_0	0	16
P_1	0	20
P_2	0	10

The average waiting time is minimum in case of SJF (Shortest Job First) scheduling.

Gantt chart for SJF scheduling :

P_2	P_0	P_1

0 10 26 46

Waiting time = Completion time – Burst time

Hence, waiting time for processes :

	Burst Time	Waiting Time
P_0	16	10
P_1	20	26
P_2	10	0

$$\text{Average waiting time } = \frac{(0+10+26)}{3} = \frac{36}{3} = 12$$

Hence, the correct answer is 12.

1.20 **A, C, D**

Round Robin policy can be used even when the CPU time required by each of the processes is not known in apriori. This is True because Round Robin policy depends on time quantum.

Implementing pre-emptive scheduling needs hardware support. This is True because pre-emption needs interrupt to occur.

The goal is to only maximize CPU utilization and minimize throughput, this is False

Turn around time = WT+BT

Hence, the correct options are (A), (C) and (D).

2 Process Management - II

Practice Questions

2013 **IIT Bombay**

2.1 Three concurrent processes X, Y and Z execute three different code segments that access X executes the P operation (i.e., wait) on semaphores a, b and c; process Y executes the P operation on semaphores b, c and d; process Z executes the P operation on semaphores c, d and a before entering the respective code segments. After completing the execution of its code segment, each process invokes the V operation (i.e., signal) on its three semaphores. All semaphores are binary semaphores initialized to one. Which one of the following represents a deadlock- free order of invoking the P operations by the processes?

(A) X: P(a)P(b)P(c)
Y:P(b)P(c)P(d)
Z: P(c)P(d)P(a)

(B) X: P(b)P(a)P(c)
Y:P(b)P(c)P(d)
Z:P(a)P(c)P(d)

(C) X: P(b)P(a)P(c)
Y: P(c)P(b)P(d)
Z:P(a)P(c)P(d)

(D) X: P(a)P(b)P(c)
Y:P(b)P(c)P(d)
Z:P(c)P(d)P(a)

2.2 A shared variable x, initialized to zero, is operated on by four concurrent processes W, X, Y, Z as follows. Each of the processes W and X reads X from memory, increments by one, stores it to the memory, and then terminates. Each of the processes Y and Z reads X from memory, and terminates. Each process before reading x invokes the P operation (i.e., wait) on a counting semaphore S and invokes the V operation (i.e., signal) on the semaphore S after storing x to memory. Semaphore S is initialized to two. What is the maximum possible value of X after all processes complete execution?

(A) – 2 (B) – 1
(C) 1 (D) 2

2.3 A certain computation generates two arrays a and b such that $a[i] = f(i)$ for $0 \le i < n$ and $b[i] = g(a[i])$ for $0 \le i < n$. Suppose this computation is decomposed into two concurrent processes X and Y such that X computes the array a and Y computes the array b. The processes employ two binary semaphores R and S, both initialized to zero. The array a is shared by the processes are shown below.

Process X :

```
private i;
for(i=0;i<n; i++)
{
    a[i] = f(i);
    Exit X (R,S);
}
```

Process Y :

```
private i;
for(i = 0; i<n; i++)
{   Entry Y (R,S);
    b[i] = g(a[i]);
}
```

Which one of the following represents the CORRECT implementation of Exit X and Entry Y?

(A)
```
Exit X(R, S)
{
  P (R);
  V (S);
}
Entry Y (R, S)
{
  P (S);
  V (R);
}
```

(B)
```
Exit X(R, S)
{
  V (R);
  V (S);
}
Entry Y (R, S)
{
  P (R);
  P (S);
}
```

(C)
```
Exit X (R, S)
{
  P (S);
  V (R);
}
Entry Y (R, S)
{
  V (S);
  P (R);
}
```

(D)
```
Exit X (R,S)
{
  V (R);
  P (S);
}
Entry Y (R, S)
{
  V (R);
  P (S);
}
```

2014 **IIT Kharagpur**

2.4 Consider the procedure below for the producer-Consumer problem which uses semaphores:

```
Semaphore n = 0;
Semaphore s = 1;
void producer ()
{
    while (true)
{
    produce ();
    semWait (s);
    addToBuffer ();
    semSignal (s);
    semSignal (n)
}
}
void consumer ()
{
    while (true)
{
    semWait (s);
    semWait (n)'
    remove FromBuffer ();
    semSignal (s);
    consume ();
    }
}
```

Which one of the following is **TRUE?**

(A) The producer will be able to add an item to the buffer, but the consumer can never consume it.

(B) The consumer will remove no more than one item from the buffer.

(C) Deadlock occurs if the consumer succeeds in acquiring semaphore s when the buffer is empty.

(D) The starting value for the semaphore n must be 1 and not 0 for deadlock-free operation.

2015	IIT Kanpur

2.5 The following two function P1 and P2 that share a variable B with an initial value of 2 execute concurrently.

```
P1 ()                P2 ()
{                    {
    C = B – 1;       D = 2 * B;
    B = 2 * C;       B = D – 1;
}                    }
```

The number of distinct values that B can possibly take after the execution is ________.

2.6 Two processes X and Y need to access a critical section. Consider the following synchronization construct used by both the processes

Process X :

```
/* other code for process X */
while (true)
{
    varP = true;
    while (varQ == true)
    {
        /* critical section */
        varP = false;
    }
}
/* other code for process X */
```

Process Y :

```
/* other code for process Y */
while (true)
{
    varQ = true;
    while (varP == true)
    {
        /* critical section */
        varQ = false;
    }
}
/* other code for process Y */
```

Here, varP and varQ are shared variables and both are initialized to false. Which one of the following statements is true?

(A) The proposed solution prevents deadlock but fails to guarantee mutual exclusion

(B) The proposed solution guarantees mutual exclusion but fails to prevents deadlock

(C) The proposed solution guarantees mutual exclusion and prevents deadlock

(D) The proposed solution fails to prevents deadlock and fails to guarantees mutual exclusion

2016	IISc Bangalore

2.7 Consider the following proposed solution for the critical section problem. There are n processes: $P_0......................P_{n-1}$. In the code, function pmax returns an integer not smaller than any of if arguments. For all i, t[i] is initialized to zero.

Code for P_1

do {

```
c[i]= 1;
t[i]= pmax(t[0],.......,t[n-1])+1; c[i]=0
    for every  j≠i  in {0,…..,n-1}
{
    while (c[j]);
    while (t[j]) != 0 && t[j]<=t[i]);
}
    Critical Section;
    t[i] = 0;
    Remainder Section
} while (true);
```

While one of the following is TRUE about the above solution?

(A) At most one process can be in the critical section at any time

(B) The bounded wait condition is satisfied

(C) The progress condition is satisfied

(D) It cannot cause a deadlock

2.8 Consider the following two-process synchronization solution.

Process 0

Entry: loop while (turn == 1);

(Critical section)

Exit: turn = 1;

Process 1

Entry: loop while (turn ==0);

(Critical section)

Exit: turn = 0;

The shared variable turn is initialized to zero. Which one of the following is TRUE?

(A) This is a correct two- process synchronization solution.

(B) This solution violates mutual exclusion requirement.

(C) This solution violates progress requirement.

(D) This solution violates bounded wait requirement.

2.9 Consider a non-negative counting semaphore S. The operation P(S) decrements S, and V(S) increments S. During an execution, 20 P(S) operations and 12 V(S) operation are issued in some order. The largest initial value of S for which at least one P(S) operation will remain blocked is _________.

2017 IIT Roorkee

2.10 A multithreaded program P executes with x number of threads and uses y number of locks for ensuring mutual exclusion while operating on shared memory locations. All locks in the program are non-reentrant, i.e., if a thread holds a lock l, then it cannot re-acquire lock l without releasing it. If a thread is unable to acquire a lock, it blocks until the lock becomes available. The minimum value of x and the minimum value of y together for which execution of P can result in a deadlock are:

(A) x = 1, y = 2   (B) x = 2, y = 1

(C) x = 2, y = 2   (D) x = 1, y = 1

2018 IIT Guwahati

2.11 Consider the following solution to the producer-consumer synchronization problem. The shared buffer size is *N*. Three semaphores *empty*, *full* and mutex are defined with respective initial values of 0, *N* and 1. Semaphore empty denotes the number of available slots in the buffer, for the consumer to read from. Semaphore full denotes the number of available slots in the buffer, for the producer to write to. The placeholder variables, denoted by *P*, *Q*, *R*, and *S*, in the code below can be assigned either *empty* or *full*. The valid semaphore operations are : wait ( ) and signal ( ).

Producer	Consumer

do {
wait (*P*);
wait (mutex);
// Add item to buffer
signal (mutex);
signal (*Q*);
} while (1);
	do {
	wait (*R*);
	wait (mutex);
	// Consume item from buffer
	signal (mutex);
	signal (*S*);
	} while (1);

Which one of the following assignments to *P*, *Q*, *R* and *S* will yield the correct solution?

(A) P : full, Q : full, R : empty, S : empty

(B) P : empty, Q : empty, R : full, S : full

(C) P : full, Q : empty, R : empty, S : full

(D) P : empty, Q : full, R : full, S : empty

2019 IIT Madras

2.12 Consider three concurrent processes P1, P2 and P3 as shown below, which access a shared variable D that has been initialized to 100

P1	P2	P3
:	:	:
:	:	:
D = D + 20	D = D – 50	D = D + 10
:	:	:
:	:	:

The processes are executed on a uniprocessor system running a time – shared operating system .If the minimum and maximum possible values of D after the three process have completed are X of Y respectively, then the value of Y-X is _________.

2020 IIT Delhi

2.13 Each of a set of n processes executes the following code using two semaphores a and b initialized to 1 and 0, respectively. Assume that count is a shared variable initialized to 0 and not used in CODE SECTION P.

CODE SECTION P

wait (a); count = count + 1;

if (count $==n$) signal (b);

signal (a);

wait (b); signal (b);

CODE SECTION Q

What does the code achieve?

(A) It ensures that no process executes CODE SECTION Q before every process has finished CODE SECTION P.

(B) It ensures that at most two processes are in CODE SECTION Q at any time.

(C) It ensures that all processes execute CODE SECTION P mutually exclusively.

(D) It ensures that at most $n-1$ processes are in CODE SECION P at any time.

2021 IIT Bombay

2.14 Which of the following standard C library functions will always invoke a system call when executed from a single-threaded process in a UNIX/Linux operating system?

(A) exit (B) malloc

(C) sleep (D) strlen

2.15 Consider the following pseudocode, where S is a semaphore initialized to 5 in line 2 and counter is a shared variable initialized to 0 in line 1. Assume that the increment operation in line 7 is not atomic.

```
1.  int counter = 0;
2.  Semaphore S = init(5);
3.  void parop(void)
4.  {
5.        wait(S);
6.        wait(S);
7.        counter++;
8.        signal(S);
9.        signal(S);
10. }
```

If five threads execute the function parop concurrently, which of the following program behaviour(s) is/are possible?

(A) The value of counter is 5 after all the threads successfully complete the execution of parop

(B) The value of counter is 1 after all the threads successfully complete the execution of parop

(C) The value of counter is 0 after all the threads successfully complete the execution of parop

(D) There is a deadlock involving all the threads

2.16 Consider the following multi-threaded code segment (in a mix of C and pseudo-code), invoked by two processes P1 and P2, and each of the process spawns two threads T1 and T2:

```
int x = 0;  // global
Lock L1;    // global
main () {
    create a thread to execute foo( ); // Thread T1
create a thread to execute foo( ); // Thread T2
wait for the two threads to finish execution;
  print(x);}
foo() {
  int y = 0;
```

```
Acquire L1;
x = x + 1;
y = y + 1;
Release L1;
print (y);}
```

Which of the following statement(s) is/are correct?

(A) Both P1 and P2 will print the value of x as 2.

(B) At least of P1 and P2 will print the value of x as 4.

(C) At least one of the threads will print the value of y as 2.

(D) Both T1 and T2, in both the processes, will print the value of y as 1.

2.17 Consider a computer system with multiple shared resource types, with one instance per resource type. Each instance can be owned by only one process at a time. Owning and freeing of resources are done by holding a global lock (L). The following scheme is used to own a resource instance:

```
function OWNRESOURCE(Resource R)
  Acquire lock L // a global lock
  if R is available then
    Acquire R
    Release lock L
  else
    if R is owned by another process P then
      Terminate P, after releasing all resources owned by P
      Acquire R
      Restart P
      Release lock L
    end if
  end if
end function
```

Which of the following choice(s) about the above scheme is/are correct?

(A) The scheme ensures that deadlocks will not occur

(B) The scheme may lead to live-lock

(C) The scheme may lead to starvation

(D) The scheme violates the mutual exclusion property

Solutions

2.1 **(B)**

Method 1

Trick to check deadlock in the given code proceed parallel i.e. first executed the first line of all the processes then executed the second line of all processes and so on.

	X		Y		Z
1.	$P(a)$	1	$P(b)$	1	$P(c)$
2.	$P(b)$	2	$P(c)$	2	$P(d)$
3.	$P(c)$	3	$P(d)$	3	$P(a)$

Value of a, b, c are 1 initially execute first line of all processes i.e.

$X_1 \rightarrow Y_1 \rightarrow Z_1$ after the execute of $X_1 \rightarrow Y_1 \rightarrow Z_1$ value of $\{a,b,c\}=0$. Now no process is allowed to enter into the critical section so deadlock.

Method 2

	X		Y		Z
1.	$P(a)$	1	$P(b)$	1	$P(c)$
2.	$P(b)$	2	$P(c)$	2	$P(d)$
3.	$P(c)$	3	$P(d)$	3	$P(a)$

1. Execute X_i so value of b gets 0.
2. As value of $b=0$ when we execute Y_1 it get blocked) (Y is blocked).
3. Now execute Z_1 so value of a get 0.
4. Now try to execute the second line of unblocked processes as value of $a=0$ when we try X_2 it get blocked (X is blocked)
5. Now execute Z_2, value of c gets 0.
6. As process X and Y are blocked and we can easily execute Z_3.

So, no deadlock, likewise we can check option (C) and option (D).

Hence, the correct option is (B).

2.2 **(D)**

Method 1

	X	W	Y	Z
1.	$P(S)$	$P(S)$	$P(S)$	$P(S)$
2.	$R(X)$	$R(X)$	$R(X)$	$R(X)$
3.	$X=X+1$	$X=X+1$	$X=X+2$	$X=X+2$
4.	Store to MM	Store to MM	Store to MM	Store to MM
5.	$V(S)$	$V(S)$	$V(S)$	$V(S)$

1. Start with $X.P(S) \Rightarrow S=1,$ read $X=0, X=X+1$ now before store into the memory process *X* preempt to process *Y*.
2. Y will perform $P(S) \Rightarrow S=0,$ read $X=0\; X=X-2$ then store $X=-2$ and $V(S) \Rightarrow S=1$.
3. Z will perform $P(S) \Rightarrow S=0$ and $X=-2X=X-2$, then store $X=-4$ and $V(S) \Rightarrow S=1$.
4. Now processes s will store value of variable X into M so $X=1$ and $V(S) \Rightarrow S=2$.
5. Now process W perform $P(S) \Rightarrow S=1$ read $X=1X=X+1$ store $X=2,$ $V(S) \Rightarrow S=2$.

Method 2

W

$P(S)$

$P(S)$

LOAD $X_{,}R_0$

$R_0 \to R_0+1$

Store R_0, X

$V(S)$

$V(S)$

X

LOAD $X_{,}R_0$

$R_0 \to R_0+1$

Store R_1, X

Y

$P(S)$

$P(S)$

LOAD $X_{,}R_2$

$R_2 \to R_2-2$

Store R_2, X

$V(S)$

$V(S)$

Z

LOAD $X_{,}R_3$

$R_1 \to R_1-2$

Store R_1, X

Suppose 'W' executes and makes value of semaphore $S=1$ Now it increment value of X to 1 and comes out and make $S=2$. Now 'X' executes, where the value of $X=1$ is stored in R_1 and then R_1 is incremented and value of R_1 is 2. Now 'X' gets preempted, also to be noted that value of semaphore is now '1' because the execution during process X got preempted and wasn't able to increase value S to 2. Now process Y will get completely execute and the Z will get completely execute. Now the value of 'X' will get executed and the value stored in R_1 i.e., '2' will get stored in 'X'. So the value in 'X' will be finally 2.

Hence, the correct option is (D).

2.3 (B)

Given : code -

Process X :

```
private i;
for(i=0;i<n; i++)
{
  a[i] = f(i);
  Exit X (R,S);
}
```

Process Y :

```
private i;
for(i = 0; i<n; i++)
{ Entry Y (R,S);
  b[i] = g(a[i]);
}
```

The solution is using a binary semaphore. X and Y are two different processes whatever the values inserted by the processes in the array that will be used by processes Y after wards.

Option (A) : Exit X(R, S)

```
{
   P (R);
   V (S);
}
Entry Y (R, S)
{
   P (S);
   V (R);
}
```

Suppose process X executes exit X then it will wait for R, and then process Y executes entry Y then it will wait for S. Since initially both binary semaphores are '0' no one will increment it both the processes will stuck in deadlock.

∴ Option (A) is wrong.

Option (B) : Exit X(R, S)

```
{
   V (R);
   V (S);
```

}
Entry Y (R, S)
{
P (R);
P (S);
}

It is wrong, because process X can insert multiple values in array and terminate but process Y can consume only one value.

∴ Option (B) is wrong.

Option (C) : Exit X (R, S)
{
P (S);
V (R);
}
Entry Y (R, S)
{
V (S);
P (R);
}

Here take any sequence of operation of process X and process Y. First process X will wait for S which is incremented by process X.

There is no sequence of operation in which the value of R or S overlaps.

Hence both processes execute one after another.

∴ Option (C) is wrong.

Option (D) : Exit X (R,S)
{
V (R);
P (S);
}
Entry Y (R, S)
{
V (R);
P (S);
}

Suppose first process X executes. It sets $R=1$ and then waits for S. Now after that process Y executes. It first sets $S=1$ and then decrement $R=0$. It comes again and then again sets $S=1$ (i.e., it overlaps the value of S) and then wait for R. Clearly here we lost one iteration of process X due to overlapping of value of S, and after $(n-1)$ iteration process well stock.

∴ Option (D) is wrong.

Hence, the correct option is (B).

2.4 **(C)**

Given :

Semaphore n = 0;
Semaphore s = 1;

```
void producer ()
{
   while (true)
{
   produce ();
   semWait (s);
   addToBuffer ();
   semSignal (s);
   semSignal (n)
}
}
void consumer ()
{
   while (true)
{
   semWait (s);
   semWait (n)'
   remove FromBuffer ();
   semSignal (s);
   consume ();
   }
}
```

Method 1

Consumer executes wait (S) then wait (n) and goes to sleep by decreasing n value. After consumer sleep, producer goes to the sleep by executing wail (S).

Method 2

Answer is (C), because when consumer first access the semaphore 's' it will down (S) and make $'S'=0$, but for semaphore (n), it has to wait for producer to make it 1 but as for producer it cant access the critical section because the value of $S=0$, sem wait (S) will not work, so there will be deadlock.

Hence, the correct option is (C).

2.5 **3**

Given :

$P_1()$	$P_2()$
{ {	
$C = B - 1;$	$D = 2 * B;$
$B = 2 * C;$	$B = D - 1;$
} }	

If P_1 executes after P_2 then

P_1 ( ) P_2 ( )

{

$C = B - 1;$

$D = 2 \times B$

$B = 2 \times C;$

$B = D - 1;$

}

$D = 2 \times 2$

$C = 3 - 1$

$= 4$

$= 2$

$\Rightarrow B = D - 1$

$= 4 - 1$

$= 3$

{

}

$B = 2 \times C$

$= 2 \times 2$

$B = 4$

P_2 executes after P_1 then

$P_1()$

{

$C = B - 1$

$D = 2 \times B$

$B = 2 \times C$

$B = D - 1$

}

$C = B - 1$

$D = 2 \times B$

$= 2 - 1$

$= 1$

$\Rightarrow B = 2 \times 1$

$B = 2$

$= 4 - 1$

$B = 3$

$P_2()$

{

}

$= 2 \times 2$

$= 4$

$B = D - 1$

$= 3$

If we done preemption between P_1 and P_2 then $B = 2$

$\therefore$ There are 3 different possible values

$B = 2, 3, 4$

Hence, the correct answer is 3.

2.6 **(D)**

Given :

Process X

```
/* other code for process X */
while (true)
{
  varP = true;
  while (varQ == true)
  {
      /* critical section */
      varP = false;
  }
}
/* other code for process X */
```

Process Y

```
/* other code for process Y */
while (true)
{
  varQ = true;
  while (varP == true)
  {
      /* critical section */
      varQ = false;
  }
}
/* other code for process Y */
```

As both processes can enter critical section together hence mutual exclusion is violated. After the entry of both process into critical section, one process will continuously enter the CS infinite times and the other process would never be allowed to enter the CS and the situation is like a live lock.

Hence, the correct option is (D).

2.7 **(A)**

Given : Code for P_1

```
do {
      c[i]= 1;
      t[i]= pmax(t[0],.......,t[n-1])+1; c[i]=0
      for every j≠i in {0,…..,n-1}
      {
        while (c[j]);
        while (t[j]) != 0 && t[j]<=t[i]);
      }
      Critical Section;
      t[i] = 0;
      Remainder Section
   } while (true);
```

It satisfies the mutual exclusion, so only one process can be in the critical section at any time.

We will check the four options one by one. Based on the above code option B, C, and D are not satisfied.

We can see that while $(t[j])! = 0 \,\&\&\, t[j] <= t[i];$ because of this condition deadlock is possible when $t[j] == t[i]$.

Because progress means no deadlock as no one process is able to make progress by stopping other process.

Bounded waiting is also not satisfied. In this case both deadlock and bounded waiting to be arising from the same reason as if $t[j] == t[i]$ is possible then starvation is possible means infinite waiting.

Mutual exclusion is satisfied.

All other process j started before i must have value of $t[j] < t[i]$ as function $P\max()$ return a integer smaller than any of its arguments.

Hence, the correct option is (A).

2.8 (C)

Given :

Process 0

Entry: loop while (turn == 1);

(Critical section)

Exit: turn = 1;

Process 1

Entry: loop while (turn ==0);

(Critical section)

Exit: turn = 0;

Given problem is strict alteration problem in which one process go into critical section or not is decided by non-critical section of another process.

So, progress property is not satisfied

Hence, the correct option is (C).

2.9 7

$S - 20 + 12 = -1$

$S - 8 = -1$

$S = -1 + 8 = +7$

So, the initial values of the semaphore should be '7'.

Hence, the correct answer is 7.

2.10 (D)

First you have to know multithreading, mutual exclusion and reentrant mutex. The reentrant mutex (recursive mutex, recursive lock) is particular type of mutual exclusion (mutex) device that may be locked multiple times by the same process/ thread, without causing a deadlock.

Here non re-entrant process can't own same lock multiple times, so if process tries to acquire already owned lock, will get blocked and deadlock will happen.

From the above options $X = 1$ (a single thread and $Y = 1$ (a single lock) deadlock is possible when we consider given situation in question.

Hence, the correct option is (D).

2.11 (C)

Given :

Producer	Consumer
do {	do {
wait (P);	wait (R);
wait (mutex);	wait (mutex);
// Add item to buffer	// Consume item from buffer
signal (mutex);	signal (mutex);
signal (Q);	signal (S);
} while (1);	} while (1);

Method 1

Full = N, empty = 0, mutex = 1

1. Initially buffer will be empty, so consumer should not start first, so option b, D are eliminated

2. With option A consumer will never consume its, so it is wrong.

3. Option 'C' is correct answer which proper functionality of produce and consumer

Method 2

Producer: " producer first checks whether any free rolots available in the buffer or not, here, wait $(P) \Rightarrow$ wait (full)

After producing an item, the item is added to buffer, hence at the end producer increases number of items.

Hence, signal $(Q) \Rightarrow$ signal (empty)

Consumer: before the consumer should take an item from buffer, it should check how many items are present on the buffer for consumption and it should be greater than 0.

Hence, wait $(R) \Rightarrow$ wait (empty)

After consuming an item, on free slot is increased on buffer.

Hence, signal $(S) \Rightarrow$ signal (full)

$\therefore$ C is correct option

Hence, the correct option is (C).

2.12 80

Given :

P1	P2	P3
:	:	:
:	:	:

D = D + 20	D = D – 50	D = D + 10
:	:	:
:	:	:

Initial value of $D = 100$

Minimum value occurs when P_1, P_2, P_3 read and P_1, P_3 update the value

P_1 Update $D = 110$

P_3 Update $D = 120$

And last P_2 will update the $d = 100 - 50 = 50 = X$ maximum value will occur when P_2 and P_3 read the initial value of D and update, first own P_2 and update $D = 50$, P_3 will run and update

$D = 100 + 10 = 110$

Now P_1 will read the value of $D = 110$ and update:

$D = D + 20 = 110 + 20$

$D = 130 = Y$

$Y - X = 130 - 50 = 80$

Hence, the correct option is 80.

2.13 (C)

Given :

CODE SECTION P

wait (a); count = count + 1;

if (count == n) signal (b);

signal (a);

wait (b); signal (b);

CODE SECTION Q

All the processes running the given code will remain block due to wait (b) until the value the value of count becomes n.

When the value of count will become equals ton, value of b changes to 1 which will subsequently allow a process to go into process section θ.

Hence, no process can go to section θ until all the process executes code sati on P.

Hence, the correct option is (C).

2.14 (A), (C)

Sleep : It is a blocking system call the OS would consider the process as blocked.

Exit : the exit call in Linux is also a system call.

Strlen: it is a library function.

Malloc: malloc is a library call and not always a system call

Hence, the correct options are (A) and (C).

2.15 (A), (B), (D)

Let T_1, T_2, T_3, T_4, T_5 be the five threads.

Option A, is possible if all threads T_1 to T_5 will execute one by one in any sequence.

Option B, is also possible. Let T_1 execute line 5 and 6 and suppose the value of counter, is which is zero is store in T_1 is register and then context switch takes place and control switches to T_2 (assume) then T_2 will also execute line 5 and 6 (S will become 1 now, hence no other process can enter) and then line 7 (counter = 1) then

Now, S will be 3 hence 1 more process can enter into critical section, Let T_3 come and completes, similarly T_4 and T_5 also come and complete.

Now, finally T_1 get chance control since it has already saved counter value 0, hence counter will be 1 when T_1 resumes.

Hence option (B) is possible.

Option (C), not possible, refer above explanation.

Option (D), is possible, when T_1 executes lines and then context switch takes place and control does to T_2, similarly T_2 executes line 5 and control goes to T_3 because of context switch and so on.

In this above scenario all thread T_1 to T_5 will to continuously waiting to enter into its critical section but since $S = 0$, hence no process can execute line 6 success fully. Hence deadlock.

Hence, the correct options are (A), (B) and (D).

2.16 (A), (D)

Given :

```
int x = 0;  // global
Lock L1;    // global
main () {
    create a thread to execute foo( ); // Thread T1
create a thread to execute foo( ); // Thread T2
wait for the two threads to finish execution;
```

```
    print(x);}
  foo() {
    int y = 0;
    Acquire L1;
    x = x + 1;
    y = y + 1;
    Release L1;
    print (y);}
```

Each process has its own address space.

P1:

Two threads T11,T12 are created in main.

Both execute foo function and threads don't wait for each other. Due to explicit locking mechanism here, mutual exclusion is there and hence no race condition inside foo().

y being thread local, both the threads will print the value of y as 1.

Due to the wait in main, the print(x) will happen only after both the threads finish. So, x will have become 2.

PS: Even if x was not assigned 0 explicitly in C all global and static variables are initialized to 0 value.

P2:

Same thing happens here as P1 as this is a different process. For sharing data among different processes mechanisms like shared memory, files, sockets etc. must be used.

Hence, the correct options are (A) and (D).

2.17 (A), (B), (C)

Given :

```
  function OWNRESOURCE(Resource R)
    Acquire lock L // a global lock
    if R is available then
      Acquire R
      Release lock L
    else
        if R is owned by another process P then
      Terminate P, after releasing all resources
        owned by P
      Acquire R
      Restart P
      Release lock L
      end if
    end if
  end function
```

Mutual exclusion is not violated.

Also, there will be no deadlock because of forceful pre-emption of resources.

This may lead to starvation if the process is keeps on coming and pre-empting each other like P1 is pre-empted by P2 and P2 is pre-empted by P3.

Live-lock is also possible due to continuous pre-emption of resources.

For option (c) consider two processes P1 and P2 now P1 enter the code acquires lock and resource.

Now P2 enters the else part kills P1 and acquire R and restart P1 Now P1 again acquire lock and kills the process P2 this continues creating a live lock scenario but there is ambiguity in the code since "Release R" is not written anywhere so ambiguity is regarding how the process will release Resource R. According to the code, the only way to release the resource is by getting killed.

Hence, the correct options are (A), (B) and (C).

3 Deadlock

Practice Questions

2014 **IIT Kharagpur**

3.1 An operating system uses the banker' algorithm for deadlock avoidance when managing the allocation of three resource types X, Y, and Z to three processes P0, P1, and P2. The table given below presents the current system state. Here, the Allocation matrix shows the current number of resources of each type allocated to each process and the Max matrix shows the maximum number of resources of each type required by each process during its execution.

	Allocation			Max		
	X	Y	Z	X	Y	Z
P0	0	0	1	8	4	3
P1	3	2	1	6	2	0
P2	2	1	1	3	3	3

There are 3 units of type X, 2 units of type Y and 2 units of type Z still available. The system is currently in a safe state. Consider the following independent requests for additional resources in the current state:

REQ1 : P0 requests 0 units of X, 0 units of Y and 2 units of Z

REQ2 : P1 requests 2 units of X, 0 units of Y and 0 units of Z

Which one of the following is **TRUE**?

(A) Only REQ1 can be permitted.

(B) Only REQ2 can be permitted.

(C) Both REQ1 and REQ2 can be permitted.

(D) Neither REQ1 nor REQ2 can be permitted.

3.2 A system contains three programs and each requires three tape units for its operation. The minimum number of tape units which the system must have such that deadlocks never arise is ________.

2015 **IIT Kanpur**

3.3 A system has 6 identical resources and N processes competing for them. Each process can request at most 2 resources. Which one of the following values of N could lead to a deadlock?

(A) 1 (B) 2

(C) 3 (D) 6

3.4 Consider the following policies for preventing deadlock in a system with mutually exclusive resources.

I. Processes should acquire all their resources at the beginning of execution. If any resource is not available, all resources acquired so far are released.

II. The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.

III. The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.

IV. The resources are numbered uniquely; a process is allowed to request only for a resource with resource number larger than its currently held resources.

Which of the above policies can be used for preventing deadlock?

(A) Any one of I and III but not II or IV

(B) Any one of I, III, and IV but not II

(C) Any one of II and III but not I or IV

(D) Any one of I, II, III, and IV

2017 **IIT Roorkee**

3.5 A system shares 9 tape drives. The current allocation and maximum requirement of tape drives for that processes are shown below:

Process	Current Allocation	Maximum Requirement
P1	3	7
P2	1	6
P3	3	5

Which of the following best describes current state of the system?

(A) Safe, Deadlocked

(B) Safe, Not Deadlocked

(C) Not Safe, Deadlocked

(D) Not Safe, Not Deadlocked

2018 **IIT Guwahati**

3.6 Consider a system with 3 processes that share 4 instances of the same resource type. Each process can request a maximum of K instances. Resource instances can be requested and released only one at a time. The largest value of K that will always avoid deadlock is _____.

3.7 In a system, there are three types of resources: E, F and G. Four processes P_0, P_1, P_2 and P_3 execute concurrently. At the outset, the processes have declared their maximum resource requirements using a matrix named Max as given below. For example, Max $[P_2, F]$ is the maximum number of instances of F that P_2 would require. The number of instances of the resources allocated to the various processes at any given state is given by a matrix named Allocation.

Consider a state of the system with the Allocation matrix as shown below, and in which 3 instances of E and 3 instances of F are the only resources available.

Allocation			
	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max			
	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

From the perspective of deadlock avoidance, which one of the following is true?

(A) The system is in *safe* state.

(B) The system is not in *safe* state, but would be *safe* if one more instance of E were available

(C) The system is not in *safe* state, but would be *safe* if one more instance of F were available

(D) The system is not in *safe* state, but would be *safe* if one more instance of G were available.

Solutions

3.1 **(B)**

Given :

	Allocation	**Max**	**Need**	**Need with additional requests**
	X Y Z	X Y Z	X Y Z	X Y Z
P_0	0 0 1	8 4 3	8 4 2	8 4 4 (REQ_1)
P_1	3 2 0	6 2 0	3 0 0	5 0 0 (REQ_2)
P_2	2 1 1	3 3 3	1 2 2	1 2 2

	Available X Y Z		
	3 2 2	$\Rightarrow$	P_2 can satisfy its need
	+ 2 1 1		P_2 releases its allocation
New Available	5 3 3	$\Rightarrow$	P_1 satisfies its need
	+ 3 2 0		P_1 releases its allocation
New Available	8 5 3	$\Rightarrow$	P_0 cannot satisfy with additional requests

Therefore, only REQ_2 can be permitted.

Hence, the correct option is (B).

3.2 **7**

Given :

Number of process = 3

Need of each process = 3

For a system to be deadlock free,

Sum of maximum need of processes < Number of processes + Number of resources

$(3 \times 2 \text{ tape units}) + 1$ tape unit = 7

Hence, the correct answer is 7.

3.3 **(D)**

Let's assume that each process requests 2 resources each. Now there are total 6 identical resources available. Give 1 resources to every process then there will be deadlock because now each process will wait for another resource which is not available. Since there are total 6 resources so for deadlock to be possible there should be 6 process available. Hence, the value of N is 6.

Hence, the correct option is (D).

3.4 **(D)**

Given policies:

I. Processes should acquire all their resources at the beginning of execution. If any resource is not available, all resources acquired so far are released.

II. The resources are numbered uniquely, and processes are allowed to request for resources only in increasing resource numbers.

III. The resources are numbered uniquely, and processes are allowed to request for resources only in decreasing resource numbers.

IV. The resources are numbered uniquely; a process is allowed to request only for a resource with resource number larger than its currently held resources.

All of the given policies can be used for preventing deadlock.

Hence, the correct option is (D).

3.5 (B)

Given :

Process	Current Allocation	Maximum Requirement	Need
P_1	3	7	4
P_2	1	6	5
P_3	3	5	2

Number of resources = 9

Therefore, Available = 9 – 7 = 2

From current available need of process P_3 can be satisfied, releasing 3 additional resources. After execution of P_3 number of available resources = 5.

Now, form current available need of any of the two processes P_1 or P_2 can be satisfied.

Safe sequence $\Rightarrow P_3 \rightarrow P_1 \rightarrow P_2$

Safe and not deadlock.

Hence, the correct option is (B).

3.6 2

Given :

Number of process = 3

Number of resources = 4

Need of each process = 2

Total we have 4 instances, if each process requires max of 2 instances then this case any of P_1, P_2 or P_3 can request the remaining instance and can fulfil the requirement. After completion of that process, remaining process can continue with its released instance.

$\therefore$ Hence 2 is correct answer.

Hence, the correct answer is 2

3.7 (A)

Given :

Allocation	E	F	G
P_0	1	0	1
P_1	1	1	2
P_2	1	0	3
P_3	2	0	0

Max	E	F	G
P_0	4	3	1
P_1	2	1	4
P_2	1	3	3
P_3	5	4	1

Method 1

Current available = $(E, F, G) = (3, 3, 0)$

After P_0, allocation of P_0 added to available

So, now available becomes $\Rightarrow \dfrac{E\ F\ G}{4\ 3\ 1}$

With this available P_0 added to available. So now after P_2, available is $\Rightarrow \dfrac{E\ F\ G}{5\ 3\ 4}$

With this available P_1 can complete. Hence after P_1, available is $\Rightarrow \dfrac{E\ F\ G}{6\ 4\ 6}$

At the end P_3 also can complete.

Hence the system is in safe state.

$\therefore$ Hence A is correct option.

Method 2

	Max	Allocation	Need
	E F G	E F G	E F G
P_0	4 3 1	1 0 1	3 3 0
P_1	2 1 4	1 1 2	1 0 2
P_2	1 3 3	1 0 3	0 3 0
P_3	5 4 1	2 0 0	3 4 1

Available = $(E, F, G) = (3, 3, 0)$

Safe sequence:

(P_0, P_2, P_1, P_3)

$P_0 : P_0$ can be allotted (3, 3, 0)

After completion available = (4, 3, 1)

$P_2 : P_2$ can be allotted (0, 3, 0)

After completion available = (5, 3, 4)

$P_1 : P_1$ can be allotted (1, 0, 2)

After completion available = (6, 4, 6)

$P_3 : P_3$ can be allotted (3, 4, 1)

After completion available = (8, 4, 6)

Hence, the correct option is (A).

4 Memory Management and Virtual Memory

Practice Questions

2013 IIT Bombay

Common Data for Questions 4.1 & 4.2

A computer uses 46-bit virtual address, 32-bit physical address, and a three- level paged page table organization. The page table base register stores the base address of the first- level table (T_1), which occupies exactly one page. Each entry of T_1 store the base address of a page of the second- level table (T_2). Each entry of T_2 store the base address of a page of the third-level table (T_3). Each entry of T_3 store a page table entry (PTE). The PTE is 32 bits in size. The processor used in the computer has a 1 MB 16-way set associative virtually indexed physically tagged cache. The cache block size is 64 bytes.

4.1 What is the size of a page in KB in this computer?

(A) 2 (B) 4

(C) 8 (D) 16

4.2 Consider data given in the above question. What is the minimum number of page colours needed to guarantee that no two synonyms map to different sets in the processor cache of this computer?

(A) 2 (B) 4

(C) 8 (D) 16

2014 IIT Kharagpur

4.3 A system use 3 page frames for storing process pages in main memory. It uses the Least Recently Used (LRU) page replacement policy. Assume that all the page frames are initially empty. What occur total number of page faults that will occur while processing the page reference string given below?

4, 7, 6, 1, 7, 6, 1, 2, 7, 2

(A) 4 (B) 5

(C) 6 (D) 7

4.4 Assume that there are 3 page frames which are initially empty. If the page reference string is 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6, the number of page faults using the optimal replacement policy is ____.

4.5 A computer has twenty physical page frames which contain page numbered 101 through 120. Now a program accesses the pages numbered 1, 2…………, 100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(A) Least - recently - used

(B) First - in - first - out

(C) Last - in - first - out

(D) Most - recently - used

4.6 Consider a paging hardware with a TLB. Assume that the entire page table and all the pages are in the physical memory. It takes 10 millisecond to search the TLB and 80 millisecond to access the physical memory. If the TLB hit ratio is 0.6, the effective memory access time (in milliseconds) is _____.

2015 IIT Kanpur

4.7 Consider a system with byte- addressable memory, 32-bit logical addresses, 4 kilobyte page size and page table entries of 4 bytes each. The size of the page table in the system in megabytes is ________.

4.8 Consider a main memory with five page frames and the following sequence of page references: 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3. which one of the following is true with respect to page replacement policies First in First out (FIFO) and Least Recently Used (LRU)?

(A) Both incur the same number of page fault

(B) FIFO incurs 2 more page fault than LRU

(C) LRU incurs 2 more page faults than FIFO

(D) FIFO incurs 1 more page faults than LRU

4.9 A computer system implements 8 kilobyte pages and a 32- bit physical address space. Each page table entry contains a valid bit, a dirty bit, three permission bits, and the translation. If the maximum size of the page table of a process is 24 megabytes, the length of the virtual address supported by the system is __________ bits.

4.10 A computer system implements a 40 bit virtual address, page size of 8 kilobytes, and a 128-entry translation look-aside buffer (TLB) organized into 32 sets each having four ways. Assume that the TLB tag does not store any process id. The minimum length of the TLB tag in bits is ________.

(A) 20 (B) 10

(C) 11 (D) 22

4.11 Consider six memory partitions of sizes 200 KB, 400 KB, 600 KB 500 KB, 300 KB and 250 KB, where KB refers to kilobyte. These partitions need to be allotted to four processes of size 357 KB, 210 KB, 468 KB and 491 KB in that order. If the best fit algorithm is used, which partitions are NOT allotted to any process?

(A) 200 KB and 300 KB

(B) 200 KB and 250 KB

(C) 250 KB and 300 KB

(D) 300 KB and 400 KB

2016 IISc Bangalore

4.12 Consider a computer system with 40-bit virtual addressing and page size of sixteen kilobytes. If the computer system has a one- level page table per process and each page table entry requires 48 bits, then the size of the per- process page table is______ megabytes.

4.13 Consider a computer system with ten physical page frames. The system is provided with an access sequence $(a_1, a_2, ..., a_{20}, a_1, a_2, ...a_{20})$, where each a_i is a distinct virtual page number. The difference in the number of page faults between the last – in- first- out page replacement policy and the optimal page replacement policy is ________.

4.14 In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

(A) LRU (Least Recently Used)

(B) OPT (Optimal Page Replacement)

(C) MRU (Most Recently Used)

(D) FIFO (First In First Out)

2017 IIT Roorkee

4.15 Recall that Belady's anomaly is that the page-fault rate may increase as the number of allocated frames increases. Now, consider the following statements:

S1 : Random page replacement algorithm (where a page chosen at random is replaced) Suffers from Belady's anomaly

S2 : LRU page replacement algorithm suffers from Belady's anomaly

Which of the following is CORRECT?

(A) S1 is true, S2 is true

(B) S1 is true, S2 is false

(C) S1 is false, S2 is true

(D) S1 is false, S2 is false

2018 IIT Guwahati

4.16 Consider a process executing on an operating system that uses demand paging. The average time for a memory access in the system is M units if the corresponding memory page is available in memory, and D units if the memory access causes a page fault. It has been experimental measured that the average time taken for a memory access in the process is X units.

Which one of the following is the correct expression for the page fault rate experienced by the process?

(A) (D – M) / (X – M)

(B) (X – M) / (D – M)

(C) (D – X) / (D – M)

(D) (X – M) / (D – X)

2019 IIT Madras

4.17 Assume that in a certain computer, the virtual addresses are 64 bits long and the physical addresses are 48 bits long. The memory is word addressable. The page size is 8k Band the word size is 4 bytes. The Translation Look-aside Buffer (TLB) in the address translation path has 128 valid entries. At most how many distinct virtual addresses can be translated without any TLB miss?

(A) 16 x 210 (B) 8 x 220

(C) 4 x 220 (D) 256 x 210

2020 IIT Delhi

4.18 Consider allocation of memory to a new process. Assume that none of the existing holes in the memory will exactly fit the process's memory requirement. Hence, a new hole of smaller size will be created if allocation is made in any of the existing holes. Which one of the following statement is TRUE?

(A) The hole created by first fit is always larger than the hole created by next fit.

(B) The hole created by worst fit is always larger than the hole created by first fit.

(C) The hole created by best fit is never larger than the hole created by first fit.

(D) The hole created by next fit is never larger than the hole created by best fit.

4.19 Consider a paging system that uses 1-level page table residing in main memory and a TLB for address translation. Each main memory access takes 100 ns and TLB lookup takes 20 ns. Each page transfer to/from the disk takes 5000 ns. Assume that the TLB hit ratio is 95%, page fault rate is 10%. Assume that for 20% of the total page faults, a dirty page has to be written back to disk before the required page is read from disk. TLB update time is negligible. The average memory access time in ns (round off to 1 decimal places) is ________.

2021 IIT Bombay

4.20 In the context of operating systems, which of the following statements is/are correct with respect to paging?

(A) Paging helps solve the issue of external fragmentation

(B) Page size has no impact on internal fragmentation

(C) Paging incurs memory overheads

(D) Multi-level paging is necessary to support pages of different sizes

4.21 Consider a three-level page table to translate a 39 bits virtual address to a physical address As shown below:

Level 1 offset	Level 2 offset	Level 3 offset	Page Offset
9 bits	9 bits	9 bits	12 bits

The page size is 4KB (1 KB = 2^{10} bytes) and page table entry size at every level is 8 bytes. A Process P is currently using 2GB (1 GB = 2^{30} bytes) virtual memory which is mapped to 2GB Of physical memory. The minimum amount of memory required for the page table of P across All level is _____ KB.

Solutions

4.1 (C)

Given :

Let the page size of x bits

Size of $T_1 = 2^x$ bytes.

Now numbers of entries in $T_1 = 2^x / 4$

Number of entries in $T_1 =$ Number of second level page tables

Total size of second level page tables $= 2^x / 4 \times 2^x$

Similarly, number of entries in II - level page table =

Number of III level page table $= (2^x/4) \times (2^x/4)$

Total size of III level page table

$$=(2^x/4)\times(2^x/4)\times(2^x)$$

Similarly, in III level all entries :

$$2^x/4\times 2^x/4\times 2^x/4$$

$$2^{3x}-6$$

Number of pages virtual memory :

$$2^{46}/2^x=2^{(46-x)}$$

Total number of pages in the III-level page tables = Number of page in virtual memory

$$2^{(3x-6)}=2^{(46-x)}$$

$$4x=52$$

$$x=13$$

That means page size of 13 bits or page size

$=2^{13}$ bytes = 8 KB

Hence, the correct option is (C).

4.2 (C)

Given :

Number of block $=\dfrac{2^{20}}{2^6}=2^{14}$

Number of sets $=\dfrac{2^{14}}{2^4}=2^{10}$

$VA\rightarrow 46$ bits

30	10	6
Tag	Set	Block offset

In VIPT if the no. of bits of page offset = set + block offset then only one page colour is coefficient but we need 8 colours because the no. of bits where the cache set index and physical page number overlap is 3. So 2^3 page colour is required.

Hence, the correct option is (C).

Key Point

page table size = Number of entries in page table × page table entry size

4.3 (C)

Given :

Number of page frame = 3

Reference string 4, 7, 6, 1, 7, 6, 1, 2, 7, 2

		6	6	6	6	6	6	7	7
	7	7	7	7	7	7	2	2	2
4	4	4	1	1	1	1	1	1	1
4	7	6	1	7	6	1	2	7	2

∴ 6 page faults occurs in LRU.

Hence, the correct option is (C).

4.4 7

Given :

Number of page frame = 3

Reference string 1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6

Optimal page replacement policy.

		3	4	4	4
	2	2	2	2	2
1	1	1	1	1	1
1	2	3	4	2	1

4	4	4	6	6
2	2	2	2	2
5	3	3	3	3
5	3	2	4	6

∴ 7 page faults.

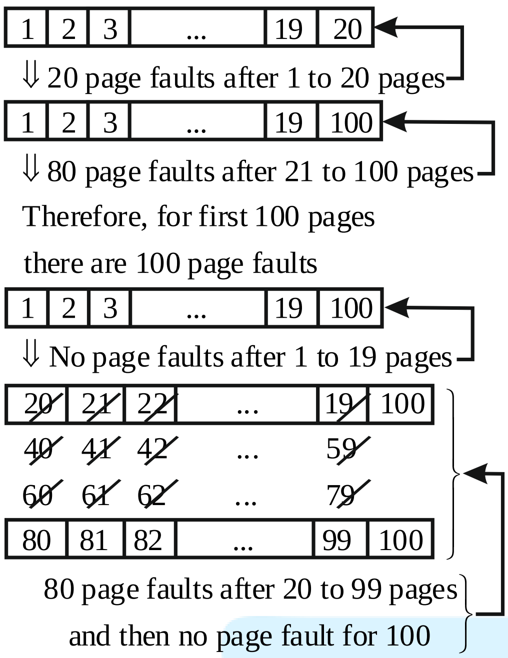
Hence, the correct answer is 7.

4.5 (D)

Given :

Pages :

1, 2, ... 20, ..., 100



Hence, the correct option is (D).

4.6 **122**

Given :

Hit ratio of TLB = $H_{TLB} = 0.6$

Memory access time = $T_M = 80$

TLB access time = $T_{TLB} = 10$

Effective memory access time

$$= H_{TLB}(T_L + T_N) + (1 - H_{TLB})(T_{TLB} + T_M + T_M)$$
$$= 0.6(1.0 + 80) + (1 - 0.6)(10 + 80 + 80)$$
$$= 0.6(90) + 0.4(170) = 122$$

Note : In case of TLB miss 1st T_M is page access time while 2nd T_M is main memory access time.

Hence, the correct answer is 122.

Key Point

Effective memory access time

$$= H_{TLB}(T_L + T_N) + (1 - H_{TLB})(T_{TLB} + T_M + T_M)$$

4.7 **4**

Given :

Page table entry size = 4 bytes

Page table size = Number of page table entries × Page table entry size

= Number of pages × 4 bytes

$$= \frac{2^{32}}{2^{12}} \times 4 \text{ bytes} = 4\,\text{MB}$$

Hence, the correct answer is 4.

Key Point

Page table size = Number of page table entries × Entry size

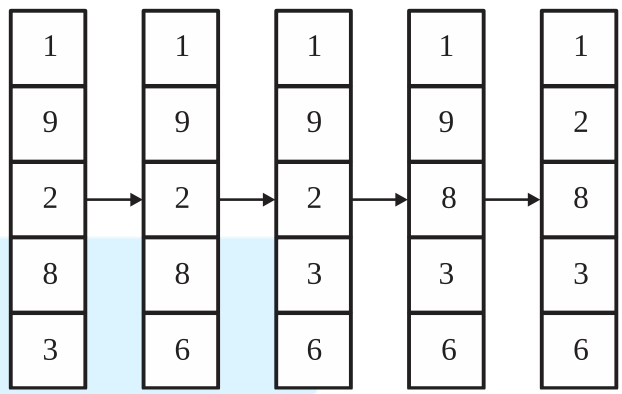
4.8 **(A)**

Given :

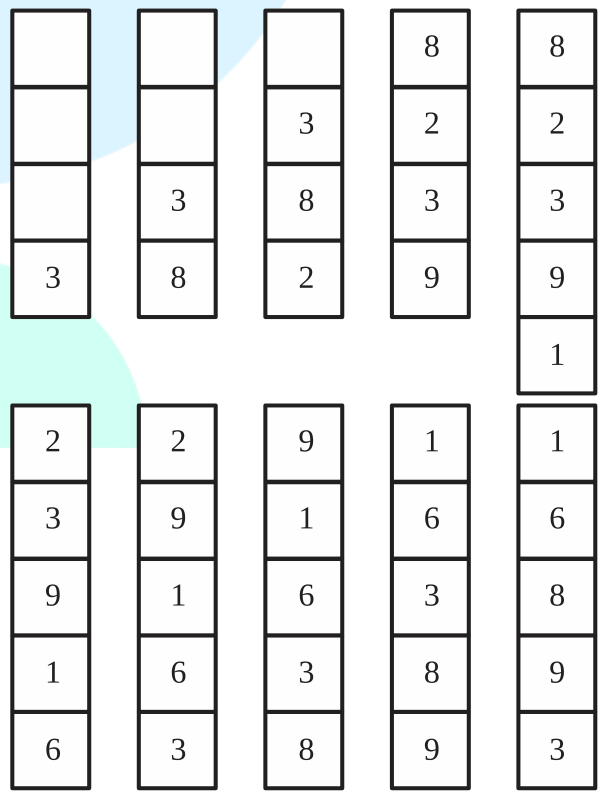
Number of page frame = 3

Reference string 3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3

FIFO :



FIFO 9 page faults.



1	8	9	3
8	9	3	9
9	3	6	6
3	6	2	2
6	2	1	1

Here LRU also increase 9 page faults. Both FCFS and LRU incur the same number of page faults.

Hence, the correct option is (A).

4.9 36

Given :

Page size = 8 kB = 13 bit offset

Number of frame bits $= 32 - 13 = 19\,\text{bits}$

Page table entry = valid + dirty + permission bits + translation (frame bits)

$= 1 + 1 + 3 + \text{Frame bits}$

$= 5 + 19 = 24$ bits.

Page table size $= 24$ Mbytes

Number of pages = Number of page table entries

$$= \frac{24\,\text{M bytes}}{24\,\text{bits}} = 8\,\text{M} = 2^{23}\ \text{pages}$$

$\therefore$ 23 bits needed for page and 13 bits offset length of virtual address $= 23 + 13 = 36$ bits.

Hence, the correct answer is 36.

Key Point

Page table entry = valid + dirty + permission bits + translation (frame bits)

4.10 (D)

Given :

Page size = 8 KB $= 2^{13}$ bytes

13 bits required for offset.

Number of sets in cache $= 32$

5 bits required for index (set offset)

← 40 bits →

	5	13

$\therefore$ 22 tag bits are required.

Hence, the correct option is (D).

4.11 (A)

The best fit algorithm find the smallest sufficient partition.

	357	491	468		210
200 kB	400 kB	600 kB	500 kB	300 kB	250 kB

$\therefore$ The summary partitions 200 KB and 200 KB are not allowed.

Hence, the correct option is (A).

4.12 384

Given :

page table entry size = 48 bits

Page table size = Number of entries in page table $\times$ page table entry size

$$= \left(\frac{2^{40}}{2^{14}}\right) \times 48\ \text{bits} = 2^{26} \times 6\ \text{bytes}$$

$$= 64\,\text{M} \times 6\,\text{B} = 384\,\text{MB}$$

Hence, the correct answer is 384.

Key Point

Page table size = Number of entries in page table $\times$ page table entry size

4.13 1

Example : 1, 2, 3, 4, 1, 2, 3, 4 with '2' frames

LIFO : 1, 2, 3, 4, 1, 2, 3, 4

	2	3	2	2	2	3	4
1	1	1	1	1	1	1	1
F	*F*	*F*	*F*	*F*	*F*	*F*	*F*

Total page faults = 7

Optimal : 1, 2, 3, 4, 1, 2, 3, 4

	2	3	4	4	4	4	4
1	1	1	1	2	2	3	3

Total page faults = 4

$7 - 6 = 1$

Hence, the correct answer is 1.

4.14 (D)

Because of Belady's anomaly, it will happen in FIFO phase replacement algorithm.

Hence, the correct option is (D).

4.15 (B)

Given :

Following statements:

S1 : Random page replacement algorithm (where a page chosen at random is replaced)

Suffers from Belady's anomaly

S2 : LRU page replacement algorithm suffers from Belady's anomaly

Considering each statement

Random page replacement algorithm can behave like only algorithm probably FCFS too, hence it can suffer from Belay's anomaly.

LRU page replacement algorithm doesn't suffer from Belady's anomaly.

Hence, the correct option is (B).

4.16 (B)

Given :

Let P be the page fault rate.

Key Point

Average memory access time = (1 – page fault rate) × memory access time when no page fault + Page fault rate × Memory access time when page fault.

X = (1-P) M + P \ D

X = M + P(D - M)

P = (X - M) / (D - M)

Hence, the correct option is (B).

4.17 (D)

Given :

1 word $= 4$ bytes

Page size $= 8\text{KB} = 2^{13\text{B}}$

Number of words in 1 page $= \dfrac{2^{13}}{2^{2}} = 2^{11}$

TLB can hold only 128 valid entries so, at most 128×2^{11} memory address can be addressed without TLB miss.

$128 \times 2^{11} = 256 \times 2^{10}$

Hence, the correct option is (D).

4.18 (C)

The hole created by best fit is never larger the hole is created by first fit.

Hence, the correct option is (C).

4.19 154.5

Given :

Main memory access time = 100 ns

Translation look aside buffer = 20 ns

TLB hit ratio = 95%

Page fault ratio = 10%

EMAT $= 0.95 \times (20 + 100) + 0.05 \times (0.9) \times$
$(20 + 100 + 100) + 0.1 \times [0.2 \times$
$(20 + 100 + 5000 + 5000) + 0.8 \times$
$(20 + 100 + 5000) + 0.8$
$\times (20 + 100 + 5000) = 154.5\,\text{ns}$

Hence, the correct answer is 154.5.

4.20 (A), (C)

Paging is the technique of non-contiguous allocation of physical memory which solves external fragmentation.

Paging may cause internal fragmentation depends on the page size and process size in last page, if page size is larger and process size is smaller.

Implementation of Paging requires management of Page tables which resides in pages and hence consume additional memory.

Multilevel paging is required when size of page table exceeds the size of page. Since page table reside in page, it is further divided to pages of same size.

Hence, the correct options are (A) and (C).

4.21 4108

Given :

Virtual address = 39 bits

Page size = 4 kB

Physical address = 2 GB

Page table entry size = 8 B

Level 1 offset	Level 2 offset	Level 3 offset	Page Offset
9 bits	9 bits	9 bits	12 bits

Three level pages table with address division (9, 9, 9, 12)

9 bits for a level means 2^9 entries in one-page table of that level

For process P

Process size = 2 GB

Number of pages of process

$$= \frac{\text{Process size}}{\text{1 page size}} = \frac{\text{2 GB}}{\text{4 kB}}$$

$$= 2^{19} \text{ pages}$$

At level 3 page table has only 2^9 entries.

So, one-page table of level 3 can point to 2^9 pages of virtual memory only.

So we need 2^{10} level-3 page tables of process P.

So we need 2^{10} entries in level-2

But level 2 page table has only 2^9 entries

So one page table of level 2 can only point to 2^9 page tables of level-3, so we need 2 level-2 page tables.

So, we need 1 level-1 page table to point to level-2 page tables

So, number of 1-level PT = 1

Number of 2-level PT = 2

Number of 3-level PT $= 2^{10}$

Total $PT = (1+2+2^{10}) = 1027$

Note that all the page tables, at every level have same size which is $2^9 \times 8B = 2^{12} B = 4 \text{ kB}$

So, total page tables size

$$= 1027 \times 4 \text{ kB} = 4108 \text{ kB}$$

Hence, the correct answer is 4108.

5 File System and Device Management

Practice Questions

2013 IIT Bombay

5.1 Consider a hard disk with 16 recording surfaces (0-15) having 16384 cylinders (0-16383) and each cylinder contains 64 sectors (0-63). Data storage capacity in each sector is 512 bytes. Data are organized cylinder-wise and the addressing format is $\langle \text{cylinder no., surface no., sector no.} \rangle$. A file of size 42797 KB is stored in the disk and the starting disk location of the file is $\langle 1200, 9, 40 \rangle$. What is the cylinder number of the last sector of the file, if it is stored in a contiguous manner?

(A) 1281 (B) 1282

(C) 1283 (D) 1284

2014 IIT Kharagpur

5.2 Suppose a disk has 201 cylinders, numbered from 0 to 200. At some time, the disk arm is at cylinder 100, and there is a queue of disk access requests for cylinders 30, 85, 90, 100, 105, 110, 135 and 145. If Shortest- Seek Time First (SSTF) is being used for scheduling the disk access, the request for cylinder 90 is serviced after servicing ______ number of requests.

5.3 A FAT (File allocation table) based file system is being used and the total overhead of each entry in the FAT is 4 bytes in size. Given a 100×10^6 bytes disk on which the file system is stored and data block size is 10^3 bytes, the maximum size of a file that can be stored on this disk in units of 10^6 bytes is _________.

2015 IIT Kanpur

5.4 Suppose the following disk request sequence (track number) for a disk with 100 tracks is given: 45, 20, 90, 10, 50, 60, 80, 25, 70. Assume that the initial position of the R/W head is on track 50. The additional distance that will be traversed by the R/W head when the Shortest Seek Time First (SSTF) algorithm is used compared to the SCAN (Elevator) algorithm (assuming that SCAN algorithm moves towards 100 when it starts execution) is _________ tracks.

5.5 Consider a disk pack with a seek time of 4 milliseconds and rotational speed of 10000 rotations per minute (RPM). It has 600 sectors per track and each sector can store 512 bytes of data. Consider a file stored in the disk. The file contains 2000 sectors. Assume that every sector access necessitates a seek, and the average rotational latency for accessing each sector is half of the time for one complete rotation. The total time (in milliseconds) needed to read the entire file is____.

5.6 Consider a typical disk that rotates at 15000 rotations per minute (RPM) and has a transfer rate of 50×10^6 bytes/sec. If the average seek time of the disk is twice the average rotational delay and the controller's transfer time is 10 times the disk transfer time, the average time (in milliseconds) to read or write a 512-byte sector of the disk is _____.

2016 IISc Bangalore

5.7 Consider a disk queue with requests for I/O to blocks on cylinders 47, 38, 121, 191, 87, 11, 92, 10. The C-LOOK scheduling algorithm is used. The head is initially at cylinder numbers on its servicing pass. The cylinders are numbered from 0 to 199. The total head movement (in number of cylinders) incurred while servicing these requests is _________.

2017 IIT Roorkee

5.8 In a file allocation system, which of the following allocation scheme(s) can be used if no external fragmentation is allowed?

I. Contiguous

II. Linked

III. Indexed

(A) I and III only

(B) II only

(C) III only

(D) II and III only

2018 IIT Guwahati

5.9 Consider a storage disk with 4 platters (numbered as 0, 1, 2 and 3), 200 cylinders (numbered as 0, 1, ... , 199), and 256 sectors per track (numbered as 0, 1, ... 255). The following 6 disk requests of the form [sector number, cylinder number, platter number] are received by the disk controller at the same time:
[120, 72, 2], [180, 134, 1], [60, 20, 0], [212, 86, 3], [56, 116, 2], [118, 16, 1]
Currently head is positioned at sector number 100 of cylinder 80, and is moving towards higher cylinder numbers. The average power dissipation in moving the head over 100 cylinders is 20 milliwatts and for reversing the direction of the head movement once is 15 milliwatts. Power dissipation associated with rotational latency and switching of head between different platters is negligible.
The total power consumption in milliwatts to satisfy all of the above disk requests using the Shortest Seek Time First disk scheduling algorithm is __________.

2019 IIT Madras

5.10 The index node (i-node) of a Unix-like file system has 12 direct, one single-indirect and one double-indirect pointer. The disk block size is 4 KB and the disk block addresses 32-bits long. The maximum possible file size is (rounded off to 1 decimal place) __ GB.

2021 IIT Bombay

5.11 Consider a linear list-based directory implementation in a file system. Each directory is a list of nodes, where each node contains the file name along with the file metadata, such as the list of pointers to the data blocks. Consider a given directory foo.
Which of the following operations will necessarily require a full scan of foo for successful completion?

(A) Creation of a new file in foo

(B) Deletion of an existing file from foo

(C) Renaming of an existing file in foo

(D) Opening of an existing file in foo

Solutions

5.1 (D)

Given :

Number of surfaces = 16

Number of cylinders = 16384

Number of sectors = 64

Data storage capacity = 512 bytes

File size = 42797 KB

First convert $\langle 1200, 9, 40\rangle$ into sector address.

$$(1200\times 16\times 64)+(9\times 64)+40=1229416$$

Number of sectors to store file

$$=\frac{(42797\text{KB})}{512}=85594$$

Last sector to store file $=1229416+85594$

$$=1315010$$

Now,

cylinder number $=\dfrac{1315010}{(16\times 64)}=1284.189$

remaining sectors = 194

surface number $=\dfrac{194}{64}=3.031$

remaining sectors = 2

$\therefore \langle 1284, 3, 1\rangle$ is last sector address.

Hence, the correct option is (D).

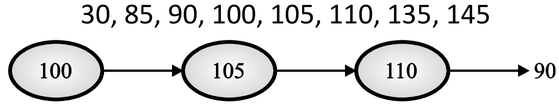
5.2 3

Given :

Number of cylinders = 201

Disk access requests :

30, 85, 90, 100, 105, 110, 135, 145



90 is serviced after servicing the 3 requests

Hence, the correct answer is 3.

5.3 **99.6**

Given :

Total overhead of each entry = 4 bytes

Disk size = 100×10^6 bytes

Data block size = 10^3 bytes

Total size for each entry $= 1004$ bytes

Number of entries in FAT

$$= \frac{100 \times 10^6}{1004} = 0.099601$$

Maximum size of a file

$$= 0.099601 \times 10^3 \text{ bytes}$$

$$= 99.601 \times 10^6 \text{ bytes}.$$

Hence, the correct answer is 99.6.

5.4 **10**

Given :

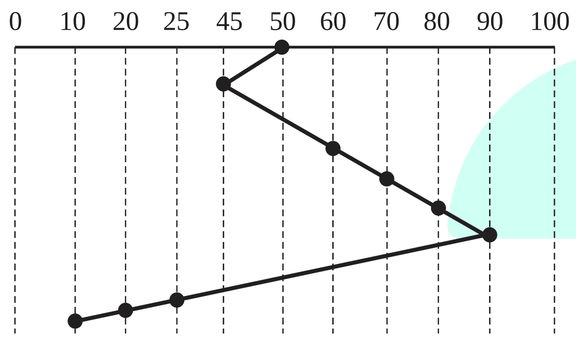
Number of tracks = 100

Disk access request :

45, 20, 90, 10, 50, 60, 80, 25, 70

Initial position of head = 50

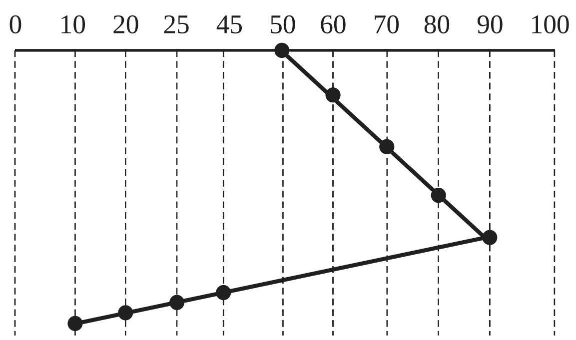
SSTF :



SSTF distance $= (50-45)+(90-45)+(90-10)$

$$= 5+45+80 = 130$$

SCAN :



SCAN distance $= (90-50)+(90-10)$

$$= 40+80 = 120$$

$\therefore$ SSTF distance – SCAN distance

$$= 130-120 = 10$$

Hence, the correct answer is 10.

5.5 **14020**

Given :

Seek time = 4 ms

Rotational speed = 10000 RPM

Number of sectors/track = 600

Size of each sector = 512 bytes

Number of sectors = 2000

Since each sector requires a seek,

Total time $= 2000 \times$ (seek time + Average rotational latency + data transfer time)

Since data transfer rate is not given, we can take that in 1 rotation, all data in a track is read. i.e., in $\frac{60}{10000} = 6\ ms$, 600×512 bytes are read. So, time to read 512 bytes $= \frac{6}{600} ms = 0.01\ ms$

Key Point

Average read/write time = Average seek time + Average rotational delay + Data transfer time

$$= 2000 \times (4 + \frac{60 \times 1000}{2 \times 10000} + 0.01)$$

$$= 2000 \times (7.01\ ms) = 14020\ ms.$$

Hence, the correct answer is 14020.

5.6 **6.1**

Given :

Rotational speed = 15000 RPM

Disk transfer rate = 50×10^6 bytes/sec

average seek time of the disk = $2 \times$ average rotational delay

controller's transfer time = $10 \times$ disk transfer time

Average time to read/write = Average seek time + Average rotational delay + Effective transfer time

Rotational delay $= \frac{60}{15} = 4\,\text{ms}$

Average rotational delay $= \frac{1}{2} \times 4 = 2\,\text{ms}$

Average seek time $= 2 \times 2 = 4\,\text{ms}$

Disk transfer time $= \dfrac{512 \text{ Bytes}}{50 \times 10^6 \text{ Bytes/sec}}$

$= 0.0102\,\text{ms}$

Effective transfer time $= 10 \times \text{disk transfer time}$

$= 0.102\,\text{ms}$

Key Point

Average read/write time = Average seek time + Average rotational delay + Data transfer time

So, Average time to read/write

$= 4 + 2 + 0.01 + 0.10$

$= 6.11\text{ms} \approx 6.1\text{ms}$

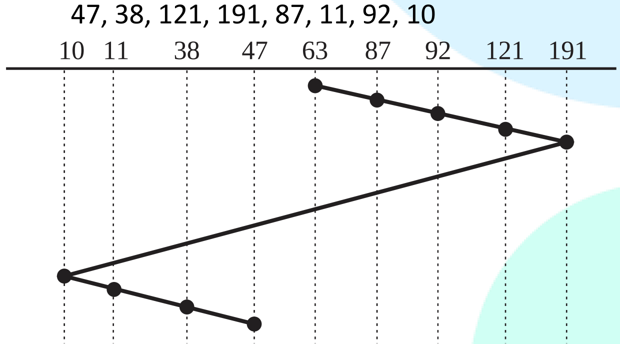
Hence, the correct answer is 6.1.

5.7 **346**

Given :

Disk access request :

47, 38, 121, 191, 87, 11, 92, 10



$(84-23)+(92-87)+(127-92)+$

$(191-121)+(191-10)+(11-10)+$

$(38-11)+47-38$

$= 24-5+29+70+181+1+27+9 = 346$

Hence, the correct answer is 346.

5.8 **(D)**

Linked and indexed allocation are non-contagious so, they will not suffer from external fragmentation.

Hence, the correct option is (D).

5.9 **85**

Given :

Number of platters = 4

Number of cylinders = 200

Number of sectors/track = 256

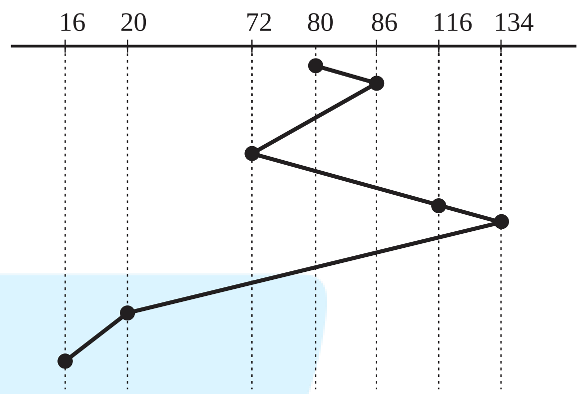
Disk access request :

[120, 72, 2], [180, 134, 1], [60, 20, 0],

[212, 86, 3], [56, 116, 2], [118, 16, 1]

average power dissipation in moving the head over 100 cylinders = 20 milliwatts

average power dissipation for reversing the direction of the head movement once = 15 milliwatts



$(86-80)+(86-72)+(134-72)$

$+(134-16)+62+118+14 = 200$

$100 \rightarrow 20$

$200 \rightarrow ?$

$\dfrac{200}{100} \times 20 = 40$

3 direction changes $3 \times 15 = 45$

$\therefore \quad 40 + 45 = 85$

Hence, the correct answer is 85.

5.10 **4**

Given :

Number of direct pointers = 12

Number of single indirect pointers = 1

Number of double indirect pointers = 1

Size of disk block $= 4\,\text{KB}$

Disk block address $= 32\,\text{bit} = 4\,\text{B}$

Number of address in one disk block

$= \dfrac{4\,\text{kB}}{4\,\text{B}} = 2^{10}$

12 direct, 1 single indirect, 1 double indirect pointer.

Maximum possible file size

$= (12 + 2^{10} + 2^{10} \times 2^{10}) \times 4\,\text{kB}$

$= 4.00395\,\text{GB} = 4\,\text{GB}$

Hence, the correct answer is 4.

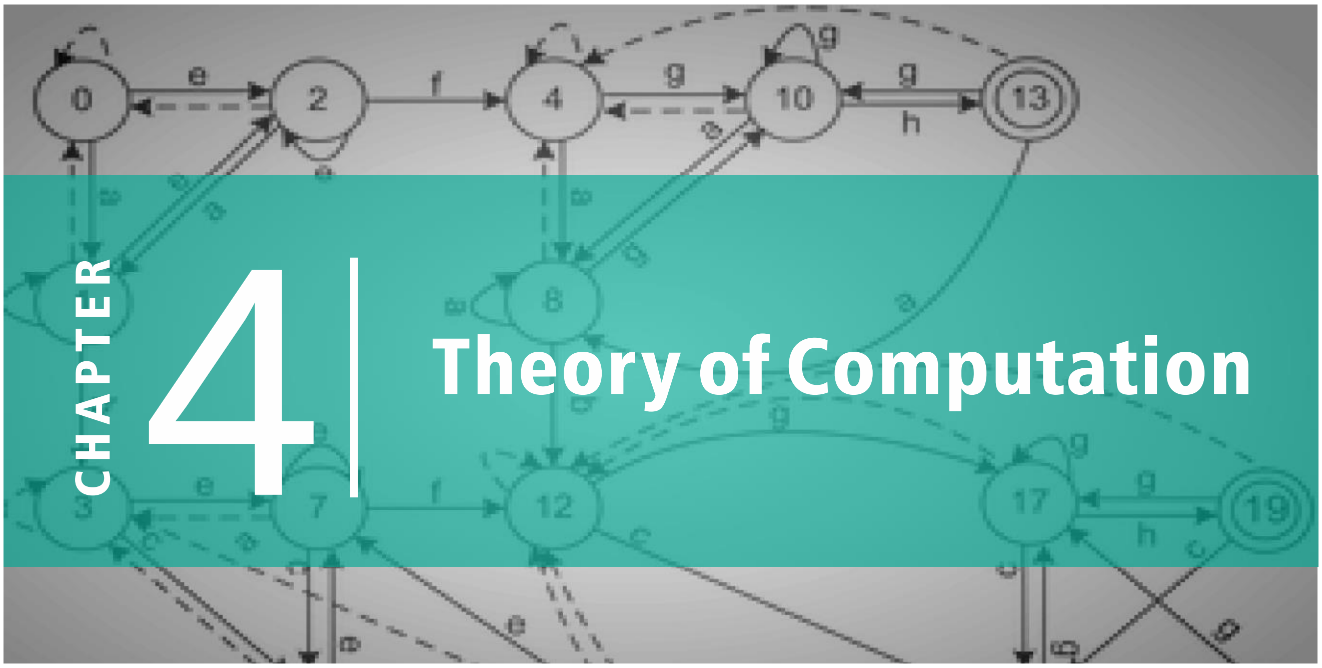
5.11 **(A, C)**

while creating new file we have to check whether same name file exist or not , so we **always** need full scan for this.

for deleting file , file can be present at any node (best case if present as first node, worst if present at last). So, we **don't always** need to full scan list.

while creating new file we have to check whether same name file exist or not , so we **always** need full scan for this

Hence, the correct options are (A) and (C).



CHAPTER 4 Theory of Computation

Marks Distribution of Theory of Computation in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	6	15
2004*	1	4	9
2005*	-	7	14
2006*	2	5	12
2007*	2	5	12
2008*	3	5	13
2009	4	3	10
2010	1	3	7
2011	3	3	9
2012	4	1	6
2013	2	3	8
2014 Set-1	2	2	6
2014 Set-2	2	2	6
2014 Set-3	2	2	6

*** CS and IT combined**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	1	2	5
2015 Set-2	1	2	5
2015 Set-3	1	1	3
2016 Set-1	3	3	9
2016 Set-2	3	4	11
2017 Set-1	2	4	10
2017 Set-2	3	3	9
2018	2	3	8
2019	2	3	8
2020	3	3	9
2021 Set-1	2	3	8
2021 Set-2	3	3	9

Syllabus : Theory of Computation

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and contex-free languages, pumping lemma. Turing machines and undecidability.

Contents : Theory of Computation

1 Finite Automata

Practice Questions

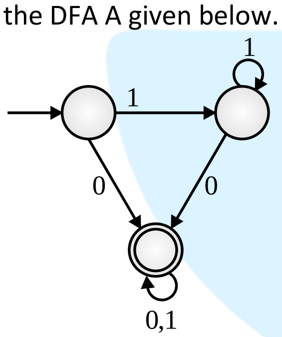
2013 IIT Bombay

1.1 Consider the languages $L_1 = \phi$ and $L_2 = \{a\}$. Which one of the following represents $L_1L_2^* \cup L_1^*$?

(A) $\{\varepsilon\}$ (B) ϕ

(C) a^* (D) $\{\varepsilon, a\}$

1.2 Consider the DFA A given below.



Which of the following are FALSE?

1. Complement of $L(A)$ is context – free
2. $L(A) = L((11*0+0)(0+1)*0*1*)$
3. For the language accepted by A, A is the minimal DFA.
4. A accepts all strings over {0, 1} of length at least 2.

(A) 1 and 3 only (B) 2 and 4 only

(C) 2 and 3 only (D) 3 and 4 only

2014 IIT Kharagpur

1.3 Which one of the following is TRUE?

(A) The language $L = \{a^n b^n \mid n \geq 0\}$ is regular.

(B) The language $L = \{a^n \mid n \text{ is prime}\}$ is regular.

(C) The language

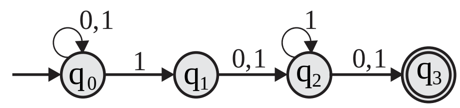
$$L = \left\{ \begin{array}{l} w \mid w \text{ has } 3k+1\ b\text{'s for} \\ \text{some } k \in N \text{ with } \Sigma = \{a,b\} \end{array} \right\}$$

is regular.

(D) The language

$L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0,1\}\}$ is regular.

1.4 Consider the finite automaton in the following figure.



What is the set of reachable states for the input string 0011?

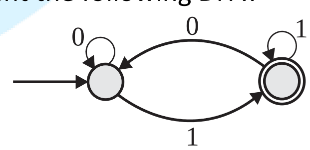
(A) $\{q_0, q_1, q_2\}$ (B) $\{q_0, q_1\}$

(C) $\{q_0, q_1, q_2, q_3\}$ (D) $\{q_3\}$

1.5 The length of the shortest string NOT in the language (over $\Sigma = \{a,b\}$) of the following regular expression is ______.

$$a^*b^*(ba)^*a^*$$

1.6 Which of the regular expression given below represent the following DFA?



1. $0^*1(1+00^*1)^*$
2. $0^*1^*1+11^*0^*1$
3. $(0+1)^*1$

(A) 1 and 2 only (B) 1 and 3 only

(C) 2 and 3 only (D) 1, 2, and 3

1.7 Let $L_1 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (110)'s as (011)'s}.

Let $L_2 = \{w \in \{0,1\}^* \mid w$ has at least as many occurrences of (000)'s as (111)'s}. Which one of the following is TRUE?

(A) L_1 is regular but not L_2

(B) L_2 is regular but not L_1

(C) Both L_1 and L_2 are regular.

(D) Neither L_1 nor L_2 are regular

2015 IIT Kanpur

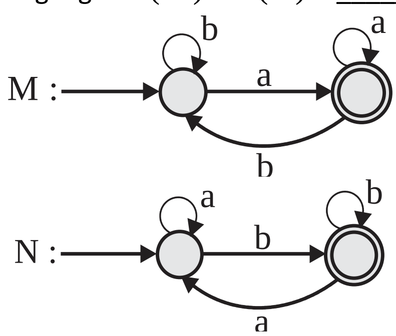
1.8 Let *L* be the language represented by the regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0,1\}$.

What is the minimum number of states in a DFA that recognizes $\overline{L}$ (complement of L)?

(A) 4 (B) 5

(C) 6 (D) 8

1.9 Consider the DFAs M and N given below. The number of states in a minimal DFA that accepts the languages $L(M) \cap L(N)$ is ______.



1.10 The number of states in the minimal deterministic finite automaton corresponding to the regular expression (0 +1)*(10) is _____.

1.11 Consider the alphabet $\Sigma = \{0,1\}$, the null/empty string λ and the set of strings X_0, X_1 and X_2 generated by the corresponding non-terminals of a regular grammar X_0, X_1, and X_2 are related as follows.

$$X_0 = 1X_1$$

$$X_1 = 0X_1 + 1X_2$$

$$X_2 = 0X_1 + \{\lambda\}$$

Which one of the following choices precisely represents the strings in X_0?

(A) $10(0^* + (10)^*)1$

(B) $10(0^* + (10)^*)^*1$

(C) $1(0+10)^*1$

(D) $10(0+10)^*1 + 110(0+10)^*1$

2016 IISc Bangalore

1.12 Which of the following languages is generated by the given grammar?

$$S \rightarrow aS \mid bS \mid \varepsilon$$

(A) $\{a^n b^m \mid n, m \geq 0\}$

(B) $\{w \in \{a,b\}^* \mid w$ has equal number of a's and b's$\}$

(C) $\{a^n \mid n \geq 0\} \cup \{b^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$

(D) $\{a,b\}^*$

1.13 Which one of the following regular expressions represents the language : the set of all binary strings having two consecutive 0's and two consecutive 1's?

(A) $(0+1)^*0011(0+1)^* + (0+1)^*1100(0+1)^*$

(B) $(0+1)^*(00(0+1)^*11 + 11(0+1)^*00)(1+1)^*$

(C) $(0+1)^*00(0+1)^* + (0+1)^*11(0+1)^*$

(D) $00(0+1)^*11 + 11(0+1)^*00$

1.14 The number of states in the minimum sized DFA that accepts the language defined by the regular expression $(0+1)^*(0+1)(0+1)^*$ is _________.

1.15 Language L_1 is defined by the grammar:

$$S_1 \rightarrow aS_1b \mid \varepsilon$$

Language L_2 is defined by the grammar:

$$S_2 \rightarrow abS_2 \mid \varepsilon$$

Consider the following statements :

$\mathrm{P}: L_1$ is regular

$\mathrm{Q}: L_2$ is regular

Which one of the following is TRUE?

(A) Both P and Q are true

(B) P is true and Q is false

(C) P is false and Q is true

(D) Both P and Q are false

1.16 Consider the following two statements :

I. If all states of an NFA are accepting states then the language accepted by the NFA is Σ^*.

II. There exists a regular language *A* such that for all languages *B*, $A \cap B$ is regular.

Which one of the following is CORRECT?

(A) Only I is true

(B) Only II is true

(C) Both I and II are true

(D) Both I and II are false

2017 IIT Roorkee

1.17 Consider the language L given by the regular expression $(a+b)*b(a+b)$ over the alphabet $\{a, b\}$. The smallest number of states needed in a deterministic finite-state automaton (DFA) accepting L is ______.

1.18 The minimum possible number of states of a deterministic finite automaton that accepts the regular language $L = \{w_1 a w_2 \mid w_1, w_2 \in \{a,b\}^*, |w_1| = 2, |w_2| \geq 3\}$ is __________.

1.19 Let δ denote the transition function and $\hat{\delta}$ denote the extended transition function of the $\varepsilon - NFA$ whose transition table is give below :

δ	ε	a	b
$\rightarrow q_0$	$\{q_2\}$	$\{q_1\}$	$\{q_0\}$
q_1	$\{q_2\}$	$\{q_2\}$	$\{q_3\}$
q_2	$\{q_0\}$	ϕ	ϕ
q_3	ϕ	ϕ	$\{q_2\}$

Them $\hat{\delta}(q_2, aba)$ is

(A) ϕ (B) $\{q_0, q_1, q_3\}$

(C) $\{q_0, q_1, q_2\}$ (D) $\{q_0, q_2, q_3\}$

2018 IIT Guwahati

1.20 Let N be an NFA with n states. Let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?

(A) $k \geq 2^n$ (B) $k \geq n$

(C) $k \leq n^2$ (D) $k \leq 2^n$

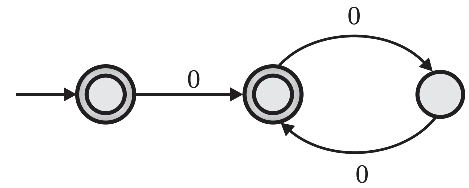
1.21 Given a language L, define L^i as follows :

$$L^0 = \{\varepsilon\}$$

$$L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

The order of a language L is defined as the smallest k such that $L^k = L^{k+1}$.

Consider the language L_1 (over alphabet 0) accepted by the following automaton.



The order of L_1 is _______.

2019 IIT Madras

1.22 If L is a regular language over $\Sigma = \{a, b\}$, which one of the following languages is NOT regular?

(A) $L.L^R = \{xy \mid x \in L, y^R \in L\}$

(B) $\{ww^R \mid w \in L\}$

(C) $\text{Prefix}(L) = \{x \in \Sigma^* \mid \exists y \in \Sigma^* \text{such that } xy \in L\}$

(D) $\text{Suffix}(L) = \{y \in \Sigma^* \mid \exists y \in \Sigma^* \text{such that } xy \in L\}$

1.23 Let Σ be the set of all bijections from $\{1, ..., 5\}$ to $\{1, ..., 5\}$, where id denotes the identity function, i.e. $id(j) = j, \forall j$. Let $\circ$ denote composition on functions. For a string $x = x_1 \ x_2 ... x_n \in \Sigma^n$, $n \geq 0$, let $\pi(x) = x_1 \circ x_2 \circ ..... \circ x_n$.

Consider the language $L = \{x \in \Sigma^* \mid \pi(x) = id\}$.

The minimum number of states in any DFA accepting L is ________.

1.24 For $\Sigma = \{a, b\}$, let us consider the regular language $L = \{x \mid x = a^{2+3k} \text{ or } x = b^{10+12k}, k \geq 0\}$. Which one of the following can be a pumping length (the constant guaranteed by the pumping lemma) for L?

(A) 3 (B) 5

(C) 9 (D) 24

2020 IIT Delhi

1.25 Consider the following statements.

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.

II. The class of regular languages is closed under infinite union.

Which of the above statements is/are TRUE?

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

1.26 Which one of the following regular expressions represents the set of all binary strings with an odd number of 1's?

(A) $((0+1)*1(0+1)*1)*10*$

(B) $(0*10*10*)*0*1$

(C) $10*(0*10*10*)*$

(D) $(0*10*10*)*10*$

1.27 Consider the following language.

$$L = \left\{ \begin{array}{l} x \in \{a,b\}^* \mid \text{number of } a\text{'s in } x \text{ is} \\ \text{divisible by 2 but not divisible by 3} \end{array} \right\}$$

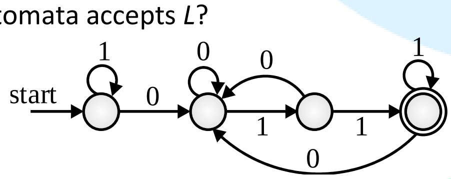
The minimum number of states in a DFA that accepts L is ________.

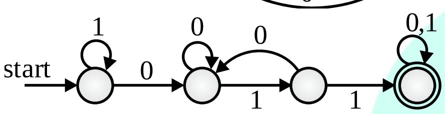
2021 **IIT Bombay**

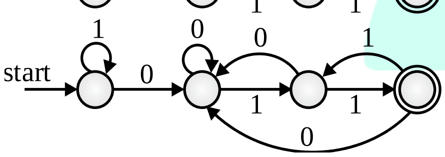
1.28 Consider the following language.

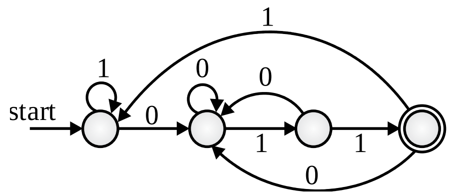
$$L = \{w \in \{0,1\}^* \mid w \text{ ends with the substring } 011\}$$

Which one of the following deterministic finite automata accepts L?

(A) 

(B) 

(C) 

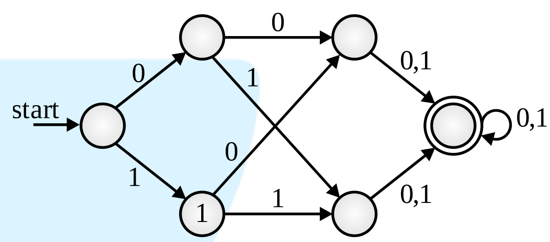
(D) 

1.29 Let $L \subseteq \{0,1\}^*$ be an arbitrary regular language accepted by a minimal DFA with k states. Which one of the following languages must necessarily be accepted by a minimal DFA with k states?

(A) $L-\{01\}$ (B) $L \cup \{01\}$

(C) $\{0,1\}^* - L$ (D) $L \cdot L$

1.30 Consider the following deterministic finite automaton (DFA).



The number of strings of length 8 accepted by the above automaton is ________.

1.31 Which of the following regular expressions represent (s) the set of all binary numbers that are divisible by three? Assume that the string ε is divisible by three.

(A) $(0+1(01^*0)^*1)^*$

(B) $(0+11+10(1+00)^*01)^*$

(C) $(0^*(1(01^*0)^*1)^*)^*$

(D) $(0+11+11(1+00)^*00)^*$

Solutions

1.1 **(A)**

Given :

$$L_1 = \phi,\ L_2 = \{a\}$$

$$L_1^* = \phi^* = \varepsilon$$

And, $L_1 \cdot L_2^* = \phi \cdot (L_2)^* = \phi$

[Since concatenation of empty language with any language will give the empty language]

Therefore,

$$L_1 \cdot L_2^* \cup L_1^* = \phi \cdot (L_2)^* \cup L_1^*$$

$$L_1 \cdot L_2^* \cup L_1^* = \phi \cup \phi^*$$

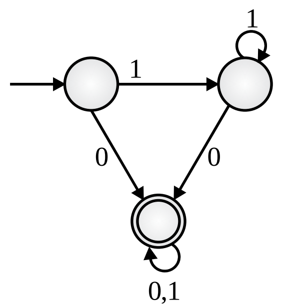
$$L_1 \cdot L_2^* \cup L_1^* = \phi \cup \{\varepsilon\}$$

$$L_1 \cdot L_2^* \cup L_1^* = \{\varepsilon\}$$

Hence, the correct option is (A).

1.2 **(D)**

Given :



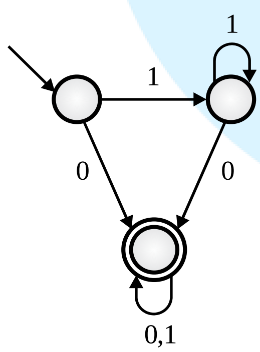
Consider the statements:

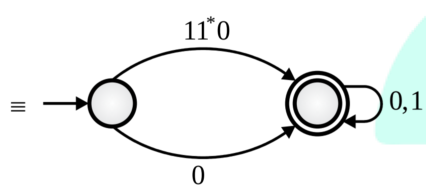
1. Complement of $L(A)$ is context free.

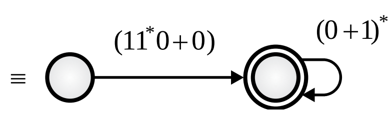
   This statement is true, since A is regular and regular languages are closed under complementation. Complement of $L(A)$ is regular. Regular languages are subset of context-free, hence complement of $L(A)$ is context-free.

2. $L(A) = L((11^*0+0)(0+1)^*0^*1^*)$

   Converting DFA into regular expression using state elimination method







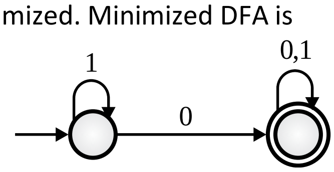
   Regular expression is : $(11^*0+0)(0+1)^*$

   $= (11^*0+0)(0+1)^*0^*1^*$

   Therefore statement 2 is true.

3. For the language accepted by A, A is the minimal DFA.

   Using equivalence theorem, given DFA can be minimized. Minimized DFA is



   Therefore, statement 3 is false

4. A accepts all strings over {0, 1} of length at least 2.

   Set of strings accepted by given DFA is {0, 00, 01, 10, 001, 010, ...}

   Since the given DFA fails to accept 11 hence, statement 4 is false.

Hence the correct option is (D).

1.3 (C)

Consider each option one by one :

Option (A) : $L = \{a^n b^n \mid n \geq 0\}$

Since comparison is required therefore,

$L = \{a^n b^n \mid n \geq 0\}$ is CFL.

Hence (A) is false.

Option (B) : $L = \{a^n \mid n \text{ is prime}\}$

$L = \{a^n \mid n$ is prime} is CSL.

Hence (B) is false.

Option (C) : $L = \left\{ \begin{array}{l} w \mid w \text{ has } 3k+1\ b\text{'s for} \\ \text{some } k \in N \text{ with } \Sigma = \{a,b\} \end{array} \right\}$

Regular expression for language *L* is :

$$a^*b(a^*ba^*ba^*b)^+a^*$$

Hence, (C) is regular.

Option (D) : $L = \{ww \mid w \in \Sigma^* \text{ with } \Sigma = \{0,1\}\}$

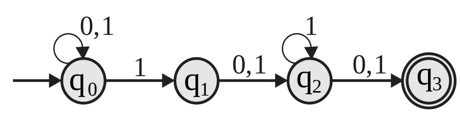
It is CSL.

Hence, (D) is false.

Hence, the correct option is (C).

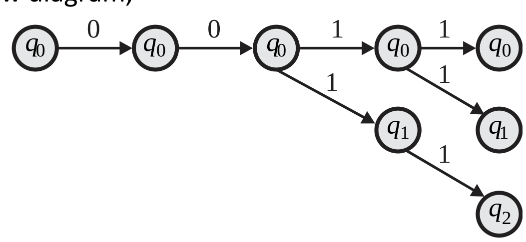
1.4 (A)

Given :



input string = 0011

Possible reachable states can be concluded from below diagram,



Hence, q_0, q_1 and q_2 are reachable from q_0 on input 0011

Hence, the correct option is (A).

1.5 **3**

Given :

Regular expression is $a^*b^*(ba)^*a^*$

we will start checking for shortest string from length 0,

$\{\varepsilon\}$ is in L

$\{a, b\}$ is in L

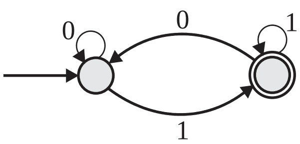
$\{aa, ab, ba, bb\}$ is in L

In length 3, *bab* does not belong to L.

Hence, the correct answer is 3.

1.6 **(B)**

Given : DFA



Given regular expressions are ;

1. $0^*1(1+00^*1)^*$
2. $0^*1^*1+11^*0^*1$
3. $(0+1)^*1$

Method 1

Using Arden's theorem

$$q_0 = \varepsilon + q_0 \cdot 0 + q_1 \cdot 0$$

$$q_0 = (\varepsilon + q_1 \cdot 0) \cdot 0^*$$

$$q_0 = q_0 \cdot 1 + q_1 \cdot 1$$

$$R.E. = (\varepsilon + q_1 0) \cdot 0^*1 + q_1 \cdot 1$$

$$R.E. = \varepsilon \cdot 0^*1 + q_1 \cdot 00^*1 + q_1 \cdot 1$$

$$= 0^*1 + q_1 \cdot (00^*1 + 1)$$

$$= 0^*1 + (00^*1 + 1)^*$$

Reducing above regular expression

$$= 0^*1((00^* + \varepsilon)1)^*$$

It produces the strings like

$1, 01, 11, 001, 111, 101, \ldots$

$\therefore$ Strings that are ending with 1

$$= (0+1)^*1 \cong 0^*1(00^*1+1)^*$$

$\therefore$ I and III are correct.

Checking II : $0^*1^*1+11^*0^*1$

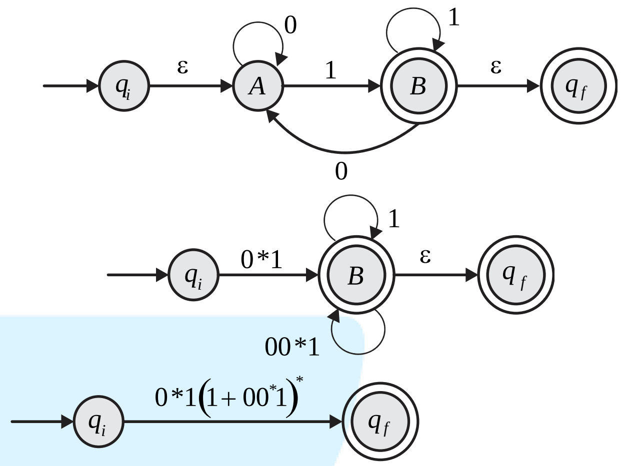
We can't generate strings like $0101, 11011, \ldots$

$\therefore$ Only I and III are correct option.

Hence, the correct option is (B).

Method 2

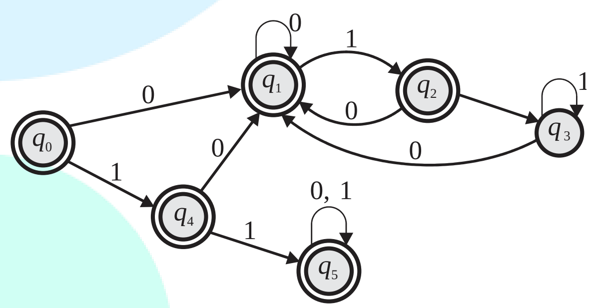
State Elimination method



Hence, the correct option is (B).

1.7 **(A)**

DFA for L_1 can be constructed as follows:



Therefore, L_1 is regular.

L_2 requires comparisons, hence DFA cannot be constructed.

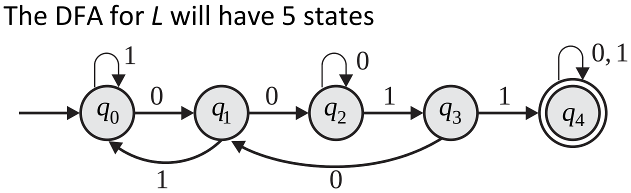
Hence, the correct option is (A).

1.8 **(B)**

Given :

L generated by regular expression $\Sigma^*0011\Sigma^*$ where $\Sigma = \{0,1\}$

The DFA for *L* will have 5 states

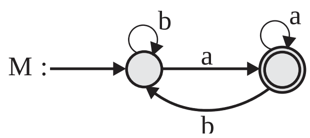


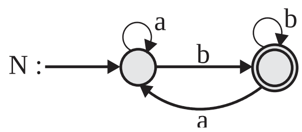
For complement of *L* we will convert all final states to non- final states and non-final final. Hence, total states are 5.

Hence, the correct option is (B).

1.9 **1**

Given :





$L(M) = (a+b)^* a = \{a, aa, ba, aaa, aba, bba, \ldots\}$

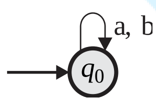
$L(M)$ is string ending with *a.*

$L(N) = (a+b)^* b = \{a, aa, bb, aab, abb, bbb, \ldots\}$

$L(N)$ is string ending with *b.*

So, $L(M) \cap L(N) = \{\ \}$

$\Rightarrow$ Only 1 state is needed



Hence, the correct answer is 1.

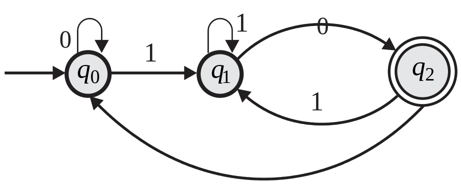
1.10 **3**

Given :

Regular expression is $(0+1)^*(10)$

$\Rightarrow$ All strings ending with 10

Minimal DFA will be as follows :



Hence, the correct answer is 3.

1.11 **(C)**

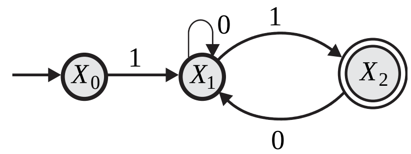
Given :

$$X_0 = 1X_1$$

$$X_1 = 0X_1 + 1X_2$$

$$X_2 = 0X_1 + \{\lambda\}$$

Converting the given transitions to a state diagram :



From the given diagram, we can easily derive X_0,

$$X_0 = 1(0+10)^*1$$

Hence, the correct option is (C).

1.12 **(D)**

Given :

$$S \to aS | bS | \varepsilon$$

The language generated by above grammar is

$$L = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, \ldots.\}$$

i.e. the above grammar will accept all strings of form: $(a+b)^*$

Hence, the correct option is (D).

1.13 **(B)**

The regular expression for set of all binary strings having two consecutive 0s and two consecutive 1s is :

$$(0+1)^* 00 (0+1)^* 11 (0+1)^* + (0+1)^* 11 (0+1)^* 00 (0+1)^*$$

After taking common, the above regular expression becomes,

$$(0+1)^* \left(00(0+1)^* 11 + 11(0+1)^* 00\right)(0+1)^*$$

Hence, the correct option is (B).

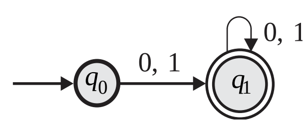
1.14 **2**

Given :

Regular expression is $(0+1)^*(0+1)(0+1)^*$

Which can also be written as $= (0+1)^+$

DFA will be as follows :



Number of states in minimal DFA = 2

Hence, the correct answer is 2.

1.15 **(C)**

Given :

Language L_1 is defined by the grammar:

$$S_1 \to aS_1b|\varepsilon$$

Language L_2 is defined by the grammar:

$$S_2 \to abS_2|\varepsilon$$

$$L_1 = \{a^n b^n \mid n \geq 0\}$$

$\Rightarrow$ L_1 is CFL.

$$L_2 = \{(ab)^n \mid n \geq 0\}$$

$\Rightarrow$ L_2 is regular.

Hence, the correct option is (C).

1.16 (B)

Given :

Statements

I. If all states of an NFA are accepting states then the language accepted by the NFA is $\sum *$.

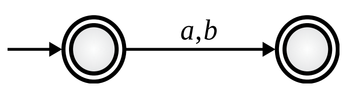
II. There exists a regular language A such that for all languages B, $A \cap B$ is regular.

Consider statement I

Statement I is False. Even if all states of an NFA are accepting states, still it may not accept $\sum *$.

Example :

Let $\sum = \{a, b\}$



$L = \{\varepsilon, a, b\}$ and not $\sum *$

Consider statement II

II is correct.

Example :

Let, $A = \{\ \}$

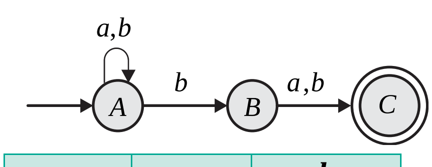
Then for all languages *B*,

$A \cap B = \{\ \}$ which is Regular (Always).

Hence, the correct option is (B).

1.17 4

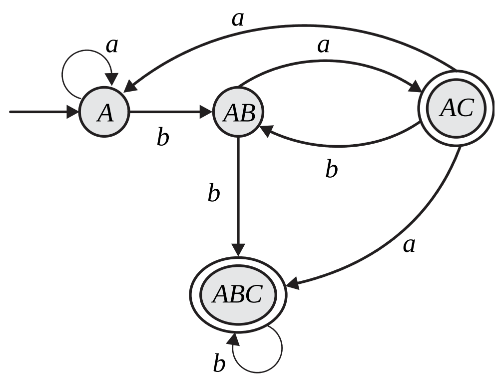
NFA for given regular expression is :



	a	b
$\to A$	A	$\{A, B\}$
$\{A, B\}$	$\{A, C\}$	$\{A, B, C\}$
$\{ABC\}$	$\{AC\}$	$\{A, B, C\}$
$\{AC\}$	$\{A\}$	$\{AB\}$

Converting NFA to DFA :

Thus from above table :



Final states will be states containing *C*.

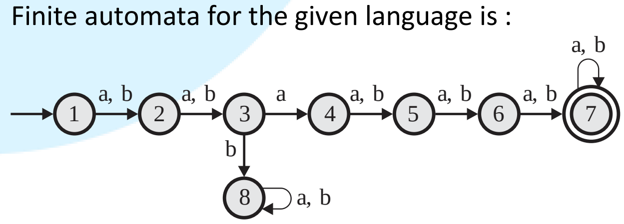
Hence, the correct answer is 4.

1.18 8

Given :

$$L = \{w_1 a w_2 \,|\, w_1, w_2 \in \{a, b\}^*, |w_1| = 2, |w_2| \geq 3\}$$

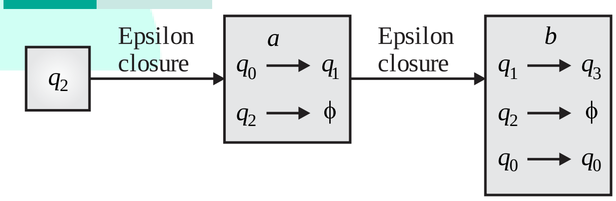
Finite automata for the given language is :

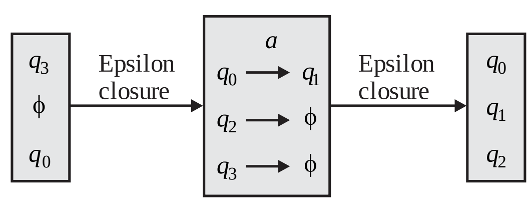


Hence, minimum no. of states needed are 8.

Hence, the correct answer is 8.

1.19 (C)





Starting state : q_2 and input string is "*aba*"

Step 1 : Find Epsilon closure of $q_2 = \{q_2, q_0\}$

Step 2 : Find transitions on *a* :

$$q_0 \to q_1$$

$$q_2 \to \phi$$

Step 3 : Find epsilon closure of

$$q_1 = \{q_1, q_2, q_0\}$$

Step 4 : Find transitions on *b* :

$$q_1 \to q_3$$
$$q_0 \to q_0$$
$$q_2 \to \phi$$

Step5 : Find epsilon closure of $q_0 = \{q_0, q_2\}$ UNION epsilon closure of $q_3 = \{q_3\}$

$$\{q_0, q_2\} \cup \{q_3\} = \{q_0, q_2, q_3\}$$

Step 6 : Find transitions on *a* :

$$q_0 \to q_1$$
$$q_2 \to \phi$$
$$q_3 \to \phi$$

Step 7 : Find epsilon closure of $q_1 : \{q_1, q_0, q_2\}$

Hence, the correct option is (C).

1.20 **(D)**

Key Point

If NFA is having n states then equivalent DFA can have at most 2^n states.

Hence, number of states in minimal DFA must be less than 2^n

$\Rightarrow$ $k \le 2^n$

Hence, the correct option is (D).

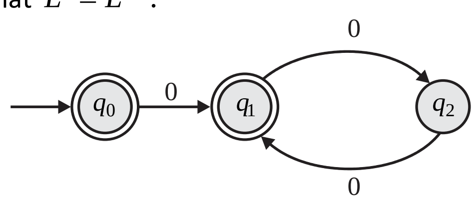
1.21 **2**

Given :

A language *L*, define L^i as follows :

$$L^0 = \{\varepsilon\}$$
$$L^i = L^{i-1} \cdot L \text{ for all } i > 0$$

The order of a language *L* is defined as the smallest *k* such that $L^k = L^{k+1}$.



$$L_1 = \varepsilon + 0(00)^*$$
$$L_1^0 = \{\varepsilon\}$$

Now, $L_1^1 = L_1^0 \cdot L_1$

$$L_1^1 = \{\varepsilon\} \cdot \{\varepsilon + 0(00)^*\}$$
$$L_1^1 = \{\varepsilon + 0(00)^*\}$$
$$L_1^1 = L_1$$

Since, $L_1^0 \ne L_1^1$

$\Rightarrow$ Order of $L_1 \ne 0$

$$L_1^2 = L_1^1.L_1$$
$$L_1^2 = \{\varepsilon + 0(00)^*\} \cdot \{\varepsilon + 0(00)^*\}$$
$$L_1^2 = \{\varepsilon + 0(00)^* + 0(00)^* 0(00)^*\}$$
$$L_1^2 = \{0^*\}$$

Clearly, $L_1^1 \ne L_1^2$

$\Rightarrow$ Order of $L_1 \ne 1$

$$L_1^3 = L_1^2.L_1$$
$$L_1^3 = \{0^*\} \cdot \{\varepsilon + 0(00)^*\}$$
$$L_1^3 = \{0^* + 0^*\, 0(00)^*\}$$
$$L_1^3 = \{0^*\}$$

Clearly, $L_1^2 = L_1^3$

$\Rightarrow$ Order of $L_1 = 2$

Hence, the correct answer is 2.

1.22 **(B)**

Consider each option one by one :

Option (A) : $L \cdot L^R = \{xy \mid x \in L, y^R \in L\}$

Option A is Concatenations of two regular languages *L* and L^R. Note that L^R is also regular because of closure property.

Option (B) : $\{ww^R \mid w \in L\}$

This is clearly a CFL.

Option (C) : $\text{Prefix}(L) = \left\{ \begin{matrix} x \in \Sigma^* \mid \exists y \in \Sigma^* \\ \text{such that } xy \in L \end{matrix} \right\}$

Regular language are closed under prefix.

Option (D) : $\text{Suffix}(L) = \left\{ \begin{matrix} y \in \Sigma^* \mid \exists y \in \Sigma^* \\ \text{such that } xy \in L \end{matrix} \right\}$

Regular language are closed under suffix.

Hence. The correct option is (B).

1.23 **120**

Number of bijective functions from {1,...,5} to $\{1,\ldots,5\} = 5! = 120$.

Since, the alphabet set Σ is having 120 elements - say for example first 120 ASCII characters representing each of these 120 distinct functions. Out of these 120 bijective functions there is exactly one identity function - say it is denoted by the ASCII character I in our example alphabet set. Now, say we make an NFA with 120 states such that from the initial state we move to state represented by function f_i for the symbol corresponding to f_i . i.e., in our ASCII set, for symbol 'A' we move to state A, for symbol 'B' we move to state B etc. and for symbol I we say in same state.

- For first symbol of input string, we stay in start state if the symbol is I
- For any other 119 symbols possible we move to the corresponding state for that symbol
- Now say for symbol A we moved to state A and the second symbol is K where the function represented by K is the inverse of the function represented by A. In this case we move back to the start state
- We can assume each of the state represents a permutation of 1,2,...5
- From any state, represented by a permutation of 1,2,...5 say s, for the next symbol b we move to the state given by f_b applied on s.
- When the string ends, if we happens to be in start state, or equivalently we simulated an Identity function, then we accept. Else reject.

  If we see the above NFA is actually a DFA and we cannot minimize it. So, we will need minimum 120 states to recognize L.

Hence, the correct answer is 120.

1.24 (D)

According to PUMPING LEMMA, there must be repetition (DFA then it repeats some states, and regular grammar repeats its nonterminal in derivation) for all acceptable strings.

Therefore, minimum Pumping Length should be 11, because string with length 10 (i.e., w = b10) does not repeat anything, but string with length 11 (i.e., w = b11) will repeat states.

Therefore, pumping length for given language should greater than 10, which is 24.

Hence, the correct option is (D).

1.25 (D)

Given :

I. If $L_1 \cup L_2$ is regular, then both L_1 and L_2 must be regular.

II. The class of regular languages is closed under infinite union.

To understand the meaning of given statements let's take example,

Consider I.

Let $L_1 = \{a^n b^n \mid n \geq 0\}$

$L_2 = \{(a+b)^*\} = \Sigma^*$

$\Rightarrow \quad L_1 \cup L_2 = \Sigma^*$ (which is regular)

But L_1 is CFL

Therefore, I is False.

Consider II.

Let $L = \{a^n b^n \mid n \geq 0\}$

L can be written as

$$L = \{ab\} \cup \{aabb\} \cup \{aaabbb\} \cup \ldots.$$

Since, *L* is infinite union of Regular languages and is CFL and not Regular.

Therefore, II is False.

Hence, the correct option is (D).

1.26 (*)

Consider each option one by one :

Option (A) : $((0+1)^*1(0+1)^*1)^*10^*$

The language generated by regular expression is

$$L(A) = \{1, 10, 100, 111, 11110, \ldots.\}$$

Since 001 $\notin L(A)$, hence this option is false

Option (B) : $(0^*10^*10^*)^*0^*1$

The language generated by regular expression is

$$L(B) = \{1, 01, 001, 111, 1101, \ldots.\}$$

Since 10, 100 $\notin L(B)$, hence this option is false.

Option (C) : $10^*(0^*10^*10^*)^*$

The language generated by regular expression is

$$L(C) = \{1, 10, 100, 111, 1000, 1011 \ldots.\}$$

Since 01, 001 $\notin L(C)$, hence this option is false.

Option (D) : $(0^*10^*10^*)^*10^*$

The language generated by regular expressions is

$$L(D)=\{1,10,100,111,1000,1011,1101,....\}$$

Since 01, 001 $\notin L(D)$, hence this option is false.

None of the given option is correct.

1.27 **6**

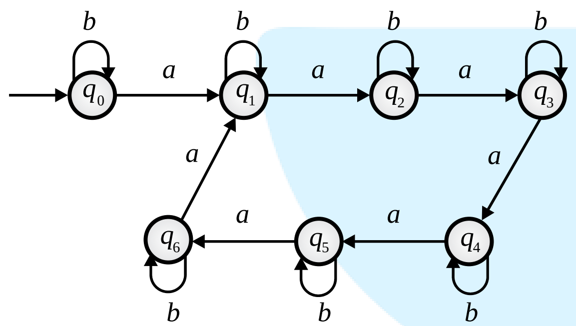
Given :

$$L=\left\{\begin{matrix} x\in\{a,b\}^* \mid \text{number of } a\text{'s in } x \text{ is} \\ \text{divisible by 2 but not divisible by 3} \end{matrix}\right\}$$

$\Rightarrow$ $L=\{x\in\{a,b\}^*$ | number of a's can be 2, 4, 8, 10, 14, 16, ...}

$\Rightarrow$ $L=\{a^2,a^4a^8,a^{10},a^{14},....\}$

DFA for above language can be constructed as-



State transition table for above DFA is

Current state	Next state	
	Input = a	Input = b
$\rightarrow q_0$	q_1	q_0
q_1	q_1	q_1
$*q_2$	q_3	q_2
q_3	q_4	q_3
$*q_4$	q_5	q_4
q_5	q_6	q_5
q_6	q_1	q_6

Now, to minimize the DFA, we use equivalence theorem, partitioning the set of states, we have

$$P_0=\underbrace{\{q_0,q_1,q_3,q_5,q_6\}}_{G_1},\underbrace{\{q_2,q_4\}}_{G_2}$$

New state transition table is

Current State	Next State	
	Input = a	Input = b
$\rightarrow q_0$	G_1	G_1
q_1	G_2	G_1
$*q_2$	G_1	G_2
q_3	G_2	G_1
$*q_4$	G_1	G_2
q_5	G_1	G_1
q_6	G_1	G_1

By observing above state transition table, new partition will be,

$$P_1=\underbrace{\{q_0,q_5,q_6\}}_{G_1},\underbrace{\{q_1,q_3\}}_{G_2},\underbrace{\{q_2,q_4\}}_{G_3}$$

Now, new state transition table is

Current state	Next state	
	Input = a	Input = b
$\rightarrow q_0$	G_2	G_1
q_1	G_3	G_2
q_2	G_2	G_3
q_3	G_3	G_2
q_4	G_1	G_3
q_5	G_1	G_1
q_6	G_2	G_1

By observing above state transition table, new partition will be,

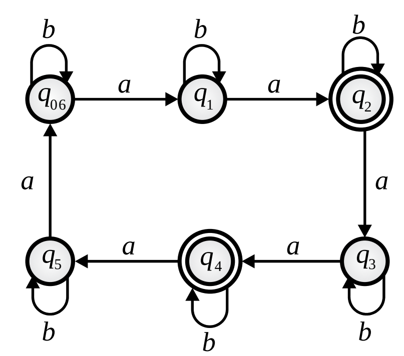
$$P_2=\{q_0,q_6\},\{q_1,q_3\},\{q_2\},\{q_4\},\{q_5\}$$

Similarly in the next iteration, new partition will be

$$P_3=\{q_0,q_6\},\{q_1\},\{q_2\},\{q_3\},\{q_4\},\{q_5\}$$

This is the set of states in minimized DFA.

So, the minimized DFA is



Number of states = 6

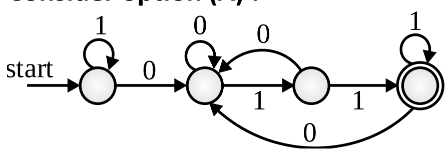
Hence, the correct answer is 6.

1.28 (D)

Given :

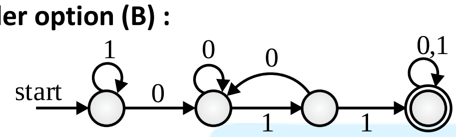
$$L=\{w\in\{0,1\}^* \mid w \text{ ends with the substring } 011\}$$

Consider option (A) :



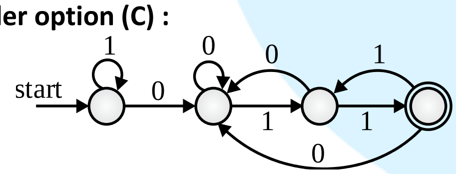
Due to self loop of alphabet 1 on state q_3, the DFA can accept strings ending with 111 therefore, this option is false

Consider option (B) :



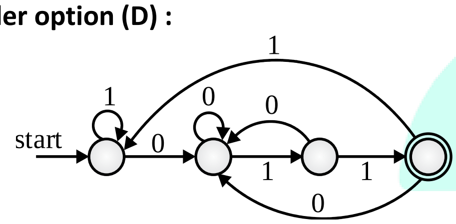
Due to self loop of alphabet 0,1on state q_3 this DFA can accept strings ending with $(0+1)^*$ therefore this option is false.

Consider option (C) :



Due to the indirect loop of alphabet 1 on state q_2, q_3 this DFA can accept string ending with 111,
Therefore this option is FALSE.

Consider option (D) :



This DFA accepts only those strings which ends with substring 011, Therefore this is the required answer.

Hence, the correct option is (D).

1.29 (C)

Given :

$$L\subseteq\{0,1\}^*$$

Since given language L has k states in its minimal DFA and we can convert the given DFA to a complemented DFA just by changing the non-final states to a final state and the final state to non-final state, keeping number of states constant.

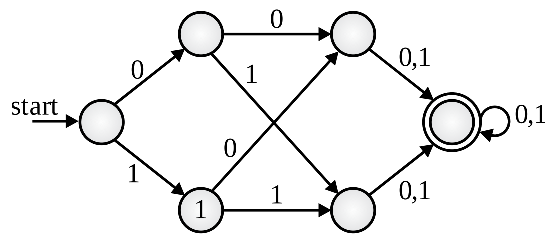
Therefore $\overline{L}$ must be accepted by a minimal DFA with K states.

$$\overline{L}=\{0,1\}^*-L$$

Hence, the correct option is (C).

1.30 256

Given :



The language accepted by this DFA is

$$L=\{w \mid w\in\{0,1\}^*, |w|\geq 3\}$$

Regular expression for DFA is: $(0+1)^3(0+1)^*$

Therefore, number of 8 length string that can be accepted is = Number of 8 length string that can be formed using $\{0,1\}=2^8=256$

Hence, the correct answer is 256.

1.31 (A, B, C)

Given :

$L=$ {all binary numbers that are divisible by 3}

$$L=\{\varepsilon,0,11,00,000,011,110,....\}$$

Consider each option :

Option (A) : $(0+1(01^*0)^*1)^*$

The language generated by this regular expression is

$$L(A)=\{\varepsilon,0,11,00,000,011,110,1100,...\}$$

It generates all strings that are divisible by 3.

Option (B) : $(0+11+10(1+00)^*01)^*$

The language generated by this regular expression is

$$L(B)=\{\varepsilon,0,00,11,011,000,110,...\}$$

It generates all the numbers that are divisible by 3

Option (C) : $(0^*(1(01^*0)^*1)^*)^*$

The language generated by this regular expression is

$$L(C)=\{\varepsilon,0,00,11,000,110,011,...\}$$

It generates all the numbers that are divisible by 3.

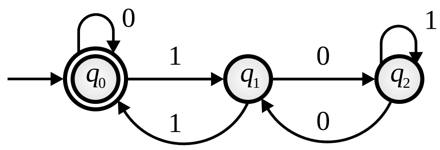
Option (D) : $(0+11+11(1+00)^*00)^*$

The language generated by this regular expression is

$$L(D)=\{\varepsilon,0,00,11,000,011,...,1111100,...\}$$

It generates $1111100=124$ which is not divisible by 3, Hence, the correct options are (A), (B), (C).

Note : DFA for the given language is



2 Pushdown Automata

Practice Questions

2014 IIT Kharagpur

2.1 Consider the following languages over the alphabet $\sum = \{0,1,c\}$:

$$L_1 = \{0^n 1^n \mid n \geq 0\}$$

$$L_2 = \{wcw^r \mid w \in \{0,1\}^*\}$$

$$L_3 = \{ww^r \mid w \in \{0,1\}^*\}$$

Here, w^r is the reverse of the string *w*.
Which of these language are deterministic context-free languages?

(A) None of the languages

(B) Only L_1

(C) Only L_1 and L_2

(D) All the three languages.

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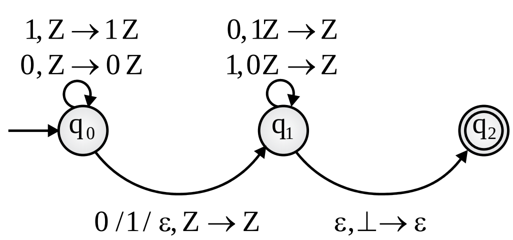
2.2 Consider the NPDA

$$\left(\begin{array}{l} Q = \{q_0, q_1, q_2\},\ \sum = \{0,1\}, \\ \Gamma = \{0,1,\perp\},\ \delta,\ q_0,\ \perp,\ F = \{q_2\}, \end{array}\right)$$

where (as per usual convention) *Q* is the set of states, $\sum$ is the input alphabet, Γ is the stack alphabet, δ is the state transition function, q_0 is the initial state, $\perp$ is the initial stack symbol, and *F* is the set of accepting states. The state transition is as follows:



Which one of the following sequences must follow the string 101100 so that the overall string is accepted by the automaton?

(A) 10110 (B) 10010

(C) 01010 (D) 01001

2.3 Which of the following languages are context-free?

$$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$$

$$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$$

$$L_3 = \{a^m b^n \mid m = 2n+1\}$$

(A) L_1 and L_2 only

(B) L_1 and L_3 only

(C) L_2 and L_3 only

(D) L_3 only

2016 IISc Bangalore

2.4 Consider the following context-free grammars :

$G_1 : S \to aS \mid B, B \to b \mid bB$

$G_2 : S \to aA \mid bB, A \to aA \mid B \mid \varepsilon, B \to bB \mid \varepsilon$

Which one of the following pairs of languages is generated by G_1 and G_2, respectively?

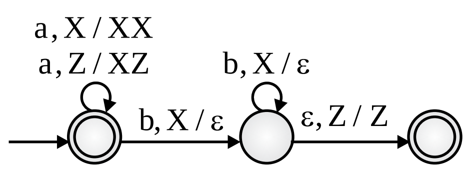
(A) $\{a^m b^n \mid m > 0\, or\, n > 0\}$ and $\{a^m b^n \mid m > 0\, and\, n > 0\}$

(B) $\{a^m b^n \mid m > 0$ and $n > 0\}$ and $\{a^m b^n \mid m > 0$ or $n \geq 0\}$

(C) $\{a^m b^n \mid m \geq 0$ or $n > 0\}$ and $\{a^m b^n \mid m > 0$ and $n > 0\}$

(D) $\{a^m b^n \mid m \geq 0$ and $n > 0\}$ and $\{a^m b^n \mid m > 0$ or $n > 0\}$

2.5 Consider the transition diagram of a PDA given below with input alphabet $\sum = \{a, b\}$ and stack alphabet $\Gamma = \{X, Z\}$. Z is the initial stack symbol. Let L denote the language accepted by the PDA.



Which one of the following is TRUE?

(A) $L = \{a^n b^n \mid n \geq 0\}$ and is not accepted by any finite automata

(B) $L=\{a^n|n\geq 0\}\cup\{a^nb^n|n\geq 0\}$ and is not accepted by any deterministic PDA

(C) L is not accepted by any Turing machine that halts on every input

(D) $L=\{a^n|n\geq 0\}\cup\{a^nb^n|n\geq 0\}$ and is deterministic context-free

2.6 Consider the following languages :

$$L_1=\{a^nb^mc^{n+m}|m,n\geq 1\}$$

$$L_2=\{a^nb^nc^{2n}|n\geq 1\}$$

Which one of the following is TRUE?

(A) Both L_1 and L_2 are context free.

(B) L_1 is context free while L_2 is not context free.

(C) L_2 is context free while L_1 is not context free.

(D) Neither L_1 nor L_2 is context free.

2017 IIT Roorkee

2.7 Consider the following context-free grammar over the alphabet $\sum=\{a,b,c\}$ with S as the start symbol :

$$S\rightarrow abScT|abcT$$

$$T\rightarrow bT|b$$

Which one of the following represents the language generated by the above grammar?

(A) $\{(ab)^n(cb)^n|n\geq 1\}$

(B) $\left\{\begin{matrix}(ab)^n cb^{m_1}cb^{m_2}\ldots cb^{m_n} \mid \\ n,m_1,m_2,\ldots,m_n\geq 1\end{matrix}\right\}$

(C) $\{(ab)^n(cb^m)^n|m,n\geq 1\}$

(D) $\{(ab)^n(cb^n)^m|m,n\geq 1\}$

2.8 Identify the language generated by the following grammar, where S is the start variable.

$$S\rightarrow XY$$

$$X\rightarrow aX|a$$

$$Y\rightarrow aYb|\varepsilon$$

(A) $\{a^mb^n\mid m\geq n,n>0\}$

(B) $\{a^mb^n\mid m\geq n,n\geq 0\}$

(C) $\{a^mb^n\mid m>n,n\geq 0\}$

(D) $\{a^mb^n\mid m>n,n>0\}$

2.9 If G is a grammar with productions

$$S\rightarrow SaS|aSb|bSa|SS|\varepsilon$$

Where S is the start variable, then which one of the following string is not generated by G?

(A) *abab* (B) *aaab*

(C) *abbaa* (D) *babba*

2.10 Consider the following languages.

$$L_1=\{a^p|p \text{ is a prime number}\}$$

$$L_2=\{a^nb^mc^{2m}|n\geq 0,m\geq 0\}$$

$$L_3=\{a^nb^nc^{2n}|n\geq 0\}$$

$$L_4=\{a^nb^n|n\geq 1\}$$

Which of the following are CORRECT?

I. L_1 is context free but not regular

II. L_2 is not context –free.

III. L_3 is not context-free but recursive

IV. L_4 is deterministic context-free

(A) I, II and IV only

(B) II and III only

(C) I and IV only

(D) III and IV only

2018 IIT Guwahati

2.11 Consider the following languages :

I. $\left\{\begin{matrix}a^mb^nc^pd^q \mid m+p=n+q, \\ \text{where } m,n,p,q\geq 0\end{matrix}\right\}$

II. $\left\{\begin{matrix}a^mb^nc^pd^q \mid m=n \text{ and } p=q, \\ \text{where } m,n,p,q\geq 0\end{matrix}\right\}$

III. $\left\{\begin{matrix}a^mb^nc^pd^q \mid m=n=p \text{ and } p\neq q, \\ \text{where } m,n,p,q\geq 0\end{matrix}\right\}$

IV. $\left\{\begin{matrix}a^mb^nc^pd^q \mid mn=p+q, \\ \text{where } m,n,p,q\geq 0\end{matrix}\right\}$

Which of the languages above are context-free?

(A) I and IV only

(B) I and II only

(C) II and III only

(D) II and IV only

2019 IIT Madras

2.12 Which one of the following languages over $\Sigma=\{a,b\}$ is NOT context-free?

(A) $\{ww^R \mid w \in \{a,b\}^*\}$

(B) $\{wa^nb^nw^R \mid w \in \{a,b\}^*, n \geq 0\}$

(C) $\{wa^nw^Rb^n \mid w \in \{a,b\}^*, n \geq 0\}$

(D) $\{a^nb^i \mid i \in \{n, 3n, 5n\}^*, n \geq 0\}$

2020 IIT Delhi

2.13 Consider the following languages.

$$L_1 = \{wxyx \mid w, x, y \in (0+1)^+\}$$

$$L_2 = \left\{xy \mid x, y \in (a+b)^*, |x| = |y|, x \neq y\right\}$$

Which one of the following is TRUE?

(A) L_1 is regular and L_2 is context-free.

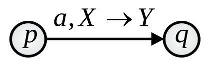
(B) L_1 is context-free but not regular and L_2 is context-free.

(C) Neither L_1 nor L_2 is context-free.

(D) L_1 is context-free but L_2 is not context-free.

2021 IIT Bombay

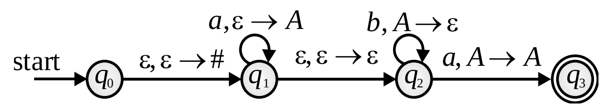
2.14 In a pushdown automaton $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, a transition of the form,



where $p, q \in Q, a \in \Sigma \cup \{\varepsilon\}$ and $X, Y \in \Gamma \cup \{\varepsilon\}$, represents

$$(q, Y) \in \delta(p, a, X).$$

Consider the following pushdown automaton over the input alphabet $\Sigma = \{a, b\}$ and stack alphabet $\Gamma = \{\#, A\}$.



The number of strings of length 100 accepted by the above pushdown automaton is ________.

2.15 For a string w, we define w^R to be the reverse of w. For example, if w = 01101 then w^R = 10110.

Which of the following languages is/are context-free?

(A) $\{wxw^Rx^R \mid w, x \in \{0,1\}^*\}$

(B) $\{ww^Rxx^R \mid w, x \in \{0,1\}^*\}$

(C) $\{wxw^R \mid w, x \in \{0,1\}^*\}$

(D) $\{wxx^Rw^R \mid w, x \in \{0,1\}^*\}$

Solutions

2.1 (C)

Given :

$$L_1 = \{0^n1^n \mid n \geq 0\}$$

$$L_2 = \{wcw^r \mid w \in \{0,1\}^*\}$$

$$L_3 = \{ww^r \mid w \in \{0,1\}^*\}$$

Since, L_1 requires only one comparison to perform and DPDA can be designed for L_1. Therefore, L_1 is DCFL.

Since, L_2 requires only one comparison to perform and DPDA can be designed for L_2. Therefore, L_1 is DCFL.

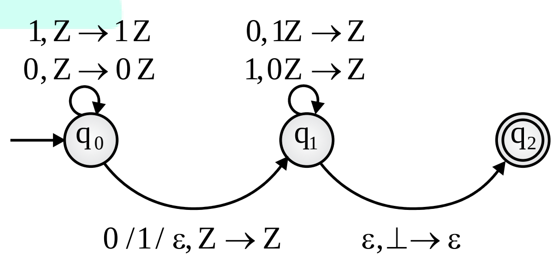
L_3 is not DCFL, as we cannot determine the middle point of the input string.

Hence, the correct option is (C).

2.2 (B)

Given :

$$\left(\begin{array}{l} Q = \{q_0, q_1, q_2\},\ \Sigma = \{0,1\}, \\ \Gamma = \{0, 1, \perp\},\ \delta,\ q_0,\ \perp,\ F = \{q_2\}, \end{array}\right)$$



In q_0 state for a '1', a '1' is pushed and for a '0' a '0' is pushed.

In q_1 state, for a '0' a '1' is popped and for a '1' a '0' is popped.

So, the given PDA is accepting all strings of the form $x0x_r'$ or $x1x_r'$ or xx_r', where x_r' is the reverse of the 1's complement of x.

The given string 101100 has 6 letters and we are given 5 letter strings.

So we have $\underbrace{10110}_{x}\,0$

1's complement of $x = 01001$

So, $x_r' = (01001)_r = 10010$

Hence, the correct option is (B).

2.3 (B)

Given :

$$L_1 = \{a^m b^n a^n b^m \mid m, n \geq 1\}$$

$$L_2 = \{a^m b^n a^m b^n \mid m, n \geq 1\}$$

$$L_3 = \{a^m b^n \mid m = 2n+1\}$$

Consider L_1 :

Here (a^m, b^m) and (b^n, a^n) can be compared using only one stack as follows :

Directly push all *m a*'s and then all *n b*'s.

Now for each *a* out of *n a's*, pop one *b*.

Now, only *m a's* are left in stack.

Now upon arrival of *m b's*, pop each *a* of *m a*'s.

In this manner, we are able to compare using only one stack. Hence, L_1 is a CFL.

Consider L_2 :

L_2 is not a CFL. Since one stack is not enough for comparison.

Consider L_3 :

Here (a^m, b^n) can be compared using only one stack as follows :

Firstly push all *a's* into stack and for every *b* pop 2 *a's*.

At the end will be left with only one *a*.

Therefore, L_3 is a CFL.

Hence, the correct option is (B).

2.4 (D)

Given :

$$G_1 : S \to aS \mid B, B \to b \mid bB$$

$$G_2 : S \to aA \mid bB, A \to aA \mid B \mid \varepsilon, B \to bB \mid \varepsilon$$

Language generated by G_1 is :

$$L(G_1) = \{b, ab, bb, aab, abb, bbb, \ldots\}$$

$$\Rightarrow \quad \{a^m b^n \mid m \geq 0 \text{ and } n > 0\}$$

Language generated by G_2 is :

$$L(G_2) = \{a, b, aa, ab, aaa, aab, abb, bbb, \ldots\}$$

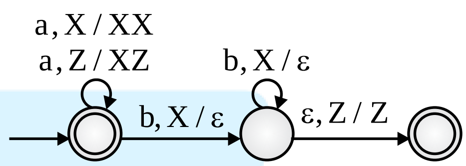
$$\Rightarrow \quad \{a^m b^n \mid m \geq 0 \text{ or } n > 0\}$$

Hence, the correct option is (D).

2.5 (D)

Given :

PDA with $\sum = \{a, b\}$ and $\Gamma = \{X, Z\}$



Since PDA has the final states.

I[st] final state (q_0) accept strings of form $\{a^n \mid n \geq 0\}$

II[nd] final state (q_2) accept strings of form $\{a^n b^n \mid n \geq 0\}$

Therefore,

$$L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$$

Language is DCFL accepted by DPDA.

Since, *L* is not regular, it cannot be accepted by Finite Automata.

Hence, the correct option is (D).

2.6 (B)

Given :

$$L_1 = \{a^n b^m c^{n+m} \mid m, n \geq 1\}$$

$$L_2 = \{a^n b^n c^{2n} \mid n \geq 1\}$$

Consider $L_1 = \{a^n b^m c^{n+m} \mid m, n \geq 1\}$

L_1 is CFL, PDA can be designed as follows :

First we will push *n a's* into stack, then *m b's* into stack, then with each *c* of $(n+m)$ *c's* pop one *b*, when no *b's* left on stack, with each *c* pop one *a*. When all *c's* are read, if the stack gets empty, then, string is accepted.

Consider $L_2 = \{a^n b^n c^{2n} \mid n \geq 1\}$

L_2 requires more than one comparison, therefore, L_2 is CSL and not CFL : L_2 cannot be implemented using a PDA.

Hence, the correct option is (B).

2.7 (B)

Given :

$$S \rightarrow abScT \mid abcT$$
$$T \rightarrow bT \mid b$$

Consider the production $S \rightarrow abScT$

This production generates equal number of (*ab*)'s and *c*'s but after each *c* there is *T* which goes to $T \rightarrow bT$

So, with each *c* there can be one or more *b*'s (one because of production $T \rightarrow b$ and more because of $T \rightarrow bT$) and these *b*'s are independent.

$$= (ab)^n (cT)^n$$
$$= \{(ab)^n cb^{m_1} cb^{m_2} \ldots cb^{m_n} \mid n, m_1, m_2 \ldots m_n \geq 1\}$$

Example :

ababcbbcbb is the part of the language and *ababcbbbbbbcbb* is also the part of the language.

So option (A) and (C) are incorrect as both say equal number of *b*'s after each *c*.

In option (D) equal number of (*ab*)'s and *c*'s is not satisfied.

Only option (B) satisfies these two conditions.

Hence, the correct option is (B).

2.8 (C)

Given :

$$S \rightarrow XY$$
$$X \rightarrow aX \mid a$$
$$Y \rightarrow aYb \mid \varepsilon$$

The language generated by the given grammar is :

$$L = \{a, aa, aaa, aab, aaaa, aaab, \ldots\}$$
$$L = \{a^m b^n \mid m > n, n \geq 0\}$$

X generates at least one '*a*', while *Y* generates equal number of *a's* and *b's* (including ε).

Hence, the correct option is (C).

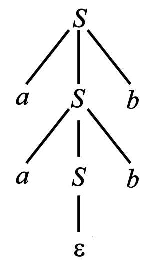
2.9 (D)

To check if a string is generated by the grammar, we need to check if a parse tree can be constructed or not.

Consider each option one by one :

Option (A) : *abab*

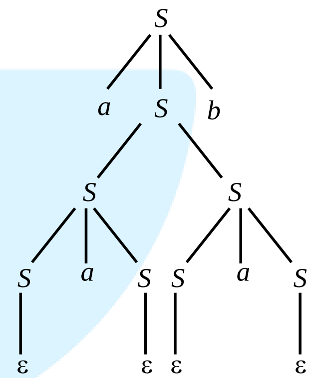
Parse tree to generate *abab* -



Therefore, we can conclude that *abab* can be generated by G.

Option (B) : *aaab*

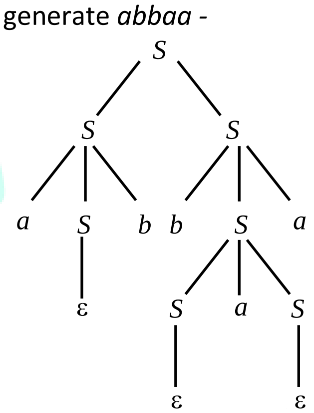
Parse tree to generate *aaab* -



Therefore, we can conclude that *aaab* can be generated by G.

Option (C) : *abbaa*

Parse tree to generate *abbaa* -



Therefore, we can conclude that *abbaa* can be generated by G.

Option (D) : *babba*

Since, we cannot construct parse tree for this string,

Hence, we can conclude that *babba* cannot be generated by G.

Hence the correct option is (D).

2.10 (D)

Consider $L_1 = \{a^p \mid p \text{ is a prime number}\}$

Since there is no pattern for prime numbers they can't be identified by both FSA and PDA.
Also since there is no bound on the length so we need infinite amount of space which can be provided by Turing machine. There exist a Turing machine which halt on every input hence, by definition of NDTM halts on every input, the language will be CSL.

Consider $L_2 = \{a^n b^m c^{2m} \mid n \geq 0, m \geq 0\}$

Since, there is only one comparison, which is among *b* and *c* therefore L_2 is CFL.

Consider $L_3 = \{a^n b^n c^{2n} \mid n \geq 0\}$

Since, there are more than one comparison, which is among *a*, *b* and *c*, therefore L_3 is CSL.

Consider $L_4 = \{a^n b^n \mid n \geq 1\}$

Since, there is only one comparison, which is among *a* and *b* therefore, L_4 is CFL. DPDA can be designed hence DCFL.

Hence the correct option is (D).

2.11 (B)

Given :

I. $\left\{\begin{matrix} a^m b^n c^p d^q \mid m+p=n+q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

II. $\left\{\begin{matrix} a^m b^n c^p d^q \mid m=n \text{ and } p=q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

III. $\left\{\begin{matrix} a^m b^n c^p d^q \mid m=n=p \text{ and } p \neq q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

IV. $\left\{\begin{matrix} a^m b^n c^p d^q \mid mn=p+q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

Consider $\left\{\begin{matrix} a^m b^n c^p d^q \mid m+p=n+q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

Grammar for above language is –

$$S \to aSd \mid ABC \mid \varepsilon$$
$$A \to aAb \mid ab \mid \varepsilon$$
$$B \to bBc \mid bc \mid \varepsilon$$
$$C \to cCd \mid cd \mid \varepsilon$$

Above Grammar is CFG so it will generate CFL.

Consider $\left\{\begin{matrix} a^m b^n c^p d^q \mid m=n \text{ and } p=q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

Grammar for above language is –

$$S \to AB \mid \varepsilon$$
$$A \to aAb \mid ab$$
$$B \to cBd \mid cd$$

Above Grammar is CFG so it will generate CFL.

Consider $\left\{\begin{matrix} a^m b^n c^p d^q \mid m=n=p \text{ and } p \neq q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

Here more than one comparison is performed so it is not CFL.

Consider $\left\{\begin{matrix} a^m b^n c^p d^q \mid mn=p+q, \\ \text{where } m,n,p,q \geq 0 \end{matrix}\right\}$

Here more than one comparison is performed so it is CSL but NOT CFL.

Hence, the correct option is (B).

2.12 (C)

Consider each option one by one :

Option (A) : $\{ww^R \mid w \in \{a,b\}^*\}$

Only one comparison is performed therefore, it is CFL.

Option (B) : $\{wa^n b^n w^R \mid w \in \{a,b\}^*, n \geq 0\}$

Since the comparisons between (w, w^R) and (a^n, b^n) can be done using only one stack, therefore, it is CFL.

Option (C) : $\{wa^n w^R b^n \mid w \in \{a,b\}^*, n \geq 0\}$

Since when comparison has to be made between w and w^R, a^n will be on top of stack and comparison will not be possible.

Therefore, it is not CFL.

Option (D) : $\{a^n b^i \mid i \in \{n, 3n, 5n\}^*, n \geq 0\}$

Given language can be re-written as

$\{a^n\, b^n \mid n \geq 0\} \cup \{a^n\, b^{3n} \mid n \geq 0\} \cup \{a^n\, b^{5n} \mid n \geq 0\}$

Since it is union of three CFLs and CFLs are closed under union.

Therefore, it is CFL.

Hence, the correct option is (C).

2.13 (A)

Given :

$$L_1 = \{wxyx \mid w, x, y \in (0+1)^+\}$$
$$L_2 = \{xy \mid x, y \in (a+b)^*, |x| = |y|, x \neq y\}$$

Consider L_1 : L_1 is a regular language, it can be represented by a regular expression.

$(a+b)(a+b)^+0(a+b)^+0+(a+b)^+1(a+b)^+1$

Consider L_2 : since comparison is performed in L_2, therefore, L_2 is a CFL.

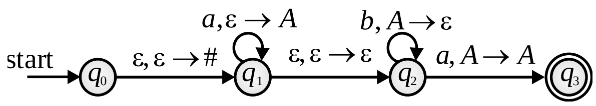
Hence, the correct option is (A).

2.14 **50**

Given :

PDA over the input alphabet $\Sigma = \{a, b\}$

And stack alphabet $\Gamma = \{\#, A\}$



PDAs can accept strings in two ways

1. By emptying the stack.
2. By reaching the final state.

In the given DFA, we can observe that in the transition from initial state $q_0 \to q_1; \#$ is pushed into the stack and is not popped out on any subsequent state.

So, we can conclude that the PDA does not accept by emptying the stack.

Now, in state q_1, only alphabet a is accepted and in state q_2, only alphabet b is accepted. This implies that the language is of the form $a^p b^q$. Also every time when a is accepted A is pushed into stack $(\varepsilon \to A)$ and when b is accepted A is popped out of stack $(A \to \varepsilon)$

To reach final state, we need at least one A to make the transition, which implies that the number of A's pushed is atleast one more than the number of A's popped.

Therefore, language accepted by the PDA is

$$L = \{a^p b^q \mid p \geq q + 1\}$$

In the question, it is given that

Length of string = 100

$\Rightarrow \quad p + q = 100$

So the possible values, in the form (p, q) are

$$\{(100, 0), (99, 1), ... (51, 49)\}$$

$\Rightarrow$ Number of such strings = 50

Hence the correct answer is 50.

2.15 **(B, C, D)**

If the language is accepted by PDA, then it is CFL.

Consider each option:

Option (A) : $\{w x w^R x^R \mid w, x \in \{0,1\}^*\}$

Let w is pushed into stack to match with w^R, and then x is pushed into stack to match with x^R.

Now to match w with w^R, *w* should be at the top of stack but the top of the stack contains x.

So PDA cannot be designed, it is not CFL.

Option (B) : $\{w w^R x x^R \mid w, x \in \{0,1\}^*\}$

This language can be split as

$$L_1 = \{w w^R \mid w \in \{0,1\}^*\}$$

$$L_2 = \{x x^R \mid x \in \{0,1\}^*\} \text{ such that } L = L_1 \cdot L_2$$

Since L_1 is CFL, L_2 is CFL and CFLs are closed under concatenation therefore, given language is CFL.

Option (C) : $\{w x w^R \mid w, x \in \{0,1\}^*\}$

This language can be written as $0(0+1)^* 0 + 1(0+1)^* 1$ which is regular, so it is CFL too.

Option (D) : $\{w x x^R w^R \mid w, x \in \{0,1\}^*\}$

PDA can be constructed for this language, first push *w* and x into the stack then to match with x^R, pop x; later to match with w^R pop *w*. Hence this language is CFL too,

Hence the correct option are (B), (C) and (D).

3 Turing Machine

Practice Questions

2016 IISc Bangalore

3.1 Consider the following languages.

$L_1 = \{\langle M \rangle \mid M$ takes at least 2016 steps on some input$\}$,

$L_2 = \{\langle M \rangle \mid M$ takes at least 2016 steps on all inputs$\}$ and

$L_3 = \{\langle M \rangle \mid M$ accepts $\varepsilon\}$,

Where for each Turing machine *M*, $\langle M \rangle$ denotes a specific encoding of *M*.

Which one of the following is TRUE?

(A) L_1 is recursive and L_2, L_3 are not recursive

(B) L_2 is recursive and L_1, L_3 are not recursive

(C) L_1, L_2 are recursive and L_3 is not recursive

(D) L_1, L_2, L_3 are recursive

2017 IIT Roorkee

3.2 Let *A* and *B* be finite alphabets and let # be a symbol outside both *A* and *B*. Let *f* be a total function from *A** to *B**. We say *f* is computable if there exists a Turing machine *M* which given an input $x \in A^*$, always halts with *f*(*x*) on its tape. Let L_f denote the languages $\{x \# f(x) \mid x \in A^*\}$.

Which of the following statement is TRUE :

(A) *f* is computable if and only if L_f is recursive

(B) *f* is computable if and only if L_f is recursively enumerable

(C) If *f* is computable then L_f is recursive, but not conversely

(D) If *f* is computable then L_f is recursive, but not conversely

2019 IIT Madras

3.3 Consider the following sets:

S_1 : Set of all recursively enumerable languages over the alphabet {0, 1}

S_2 : Set of all synthetically valid C programs

S_3 : Set of all languages over the alphabet {0, 1}

S_4 : Set of all non-regular languages over the alphabet {0, 1}

Which of the above sets are uncountable?

(A) S_1 and S_2 (B) S_3 and S_4

(C) S_2 and S_3 (D) S_1 and S_4

2020 IIT Delhi

3.4 Consider the language $L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$ and the following statements.

I. *L* is deterministic context-free.

II. *L* is context-free but not deterministic context-free.

III. *L* is not LL(*k*) for any *k*.

Which of the above statements is/are TRUE?

(A) I only (B) II only

(C) I and III only (D) III only

Solutions

3.1 (C)

Given :

$L_1 = \{\langle M \rangle \mid M$ takes atleast 2016 steps on some input$\}$

$L_2 = \{\langle M \rangle \mid M$ takes atleast 2016 steps on all inputs$\}$

$L_3 = \{\langle M \rangle \mid M$ accepts ε }

L_1 is recursive. As we have to check if there exist any string in Σ^* for which *M* takes more than or equal to 2016 steps. Since input string set is finite and we have to run *M* for finite number of steps. L_1 is recursive. We can stop giving input once we find a string taking atleast 2016 steps. Therefore, L_1 is recursive.

L_2 is recursive , because for any string, if *M* halts in less than 2016 steps then the string is not present in L_2 and if any string reaches upto 2016 steps then that string is in L_2, no matter how many more steps it takes. Therefore, L_2 is recursive.

L_3 is not recursive. As whether ε belongs to *L*(*M*) have been asked. Which according to rice theorem, is a non- trivial property. Hence, L_3 is undecidable therefore L_3 is not recursive.

Hence, the correct option is (C).

3.2 (A)

Given :

$$L_f = \{x \# f(x) | x \in A^*\}$$

Since in this question the way computable is defined based on halting Turing machine, it means computable is same as Recursive. So, clearly f is computable if L_f is recursive.

Hence, the correct option is (A).

3.3 (B)

Given :

S_1 : Set of all recursively enumerable languages over the alphabet {0, 1}

S_2 : Set of all synthetically valid C programs

S_3 : Set of all languages over the alphabet {0, 1}

S_4 : Set of all non-regular languages over the alphabet {0, 1}

Consider each set

S_1 : Every recursively enumerable language have a Turing machine. Since set of all Turing machines is countable.

Hence, set of all recursively enumerable language is countable.

S_2 : Since a one to one equivalence is possible for all valid C programs and all valid TM encodings. And since set of all valid TM encodings is countable.

Therefore set of all syntactically valid C programs is also countable.

S_3 : set of all languages over the alphabet {0, 1} $= 2^{\Sigma^*}$ = uncountable, since power set of an infinite set is uncountable.

S_4 : Since, set of all languages is uncountable. And set of all regular languages is countable. Now, Let,

$L_1 =$ Set of all regular languages

$L_2 =$ Set of all non –regular language

$L =$ Set of all languages

Clearly, $L = L_1 \cup L_2$

For *L* to be uncountable, L_2 have to be uncountable.

Hence, the correct option is (B).

3.4 (C)

Given:

$$L = \{a^n \mid n \geq 0\} \cup \{a^n b^n \mid n \geq 0\}$$

$$L = L_1 \cup L_2$$

Where, $L_1 = \{a^n / n \geq 0\}$ is regular

$L_2 = \{a^n b^n / n \geq 0\}$ is DCFL

$\Rightarrow$ *L* is DCFL since DCFL are closed under union with regular language.

$\Rightarrow$ I is True and II is False.

In III, for checking whether it is LL(*k*) for any *k*.

suppose taking any strings :

$$a^{k+1} \text{ and } a^{k+1} b^{k+1}$$

Since, upon looking at first *k* symbols, we can not tell whether it is first string or second string. Hence, it is not LL(*k*).

Hence, III is True

Hence, the correct option is (C).

4 Properties of Languages

Closure Property of Languages :

Operation	RL	DCFL	CFL	CSL	Rec	RE
Union	Yes	No	Yes	Yes	Yes	Yes
Intersection	Yes	No	No	Yes	Yes	Yes
Set difference	Yes	No	No	Yes	Yes	No
complementation	Yes	Yes	No	Yes	Yes	No
Intersection with a regular language	Yes	Yes	Yes	Yes	Yes	Yes
Union with a regular language	Yes	Yes	Yes	Yes	Yes	Yes
Concatenation	Yes	No	Yes	Yes	Yes	Yes
Kleene star	Yes	No	Yes	Yes	Yes	Yes
Kleene plus	Yes	No	Yes	Yes	Yes	Yes
Reversal	Yes	Yes	Yes	Yes	Yes	Yes
Epsilon-free homomorphism	Yes	No	Yes	Yes	Yes	Yes
Homomorphism	Yes	No	Yes	No	No	Yes
Inverse homomorphism	Yes	Yes	Yes	Yes	Yes	Yes
Epsilon-free substitution	Yes	No	Yes	Yes	Yes	Yes
Substitution	Yes	No	Yes	No	No	Yes
Subset	No	No	No	No	No	No

Decidability Property of Languages :

Problem	RL	DCFL	CFL	CSL	Rec	RE
Membership Problem	D	D	D	D	D	UD
Emptiness Problem	D	D	D	UD	UD	UD
Completeness Problem	D	UD	UD	UD	UD	UD
Equality Problem	D	D	UD	UD	UD	UD
Subset Problem	D	UD	UD	UD	UD	UD
$L_1 \cap L_2 = \phi$	D	UD	UD	UD	UD	UD
Finiteness	D	D	D	UD	UD	UD
Complement is of same type	D	D	UD	D	D	UD
Intersection is of same type	D	UD	UD	UD	UD	UD
Is L regular?	D	D	UD	UD	UD	UD

D : Decidable, UD : Undecidable

Mapping Reducibility of Languages :

Basic Theorems about $\leq_m$:

Theorem 1 : If $A \leq_m B$ and *B* is Turing-decidable then *A* is Turing-decidable.

Theorem 2 : If $A \le_m B$ and *A* is undecidable then *B* is undecidable.

Theorem 3 : If $A \le_m B$ and *B* is Turing-recognizable then *A* is Turing-recognizable.

Theorem 4 : If $A \le_m B$ and *A* is not Turing-recognizable then *B* is not Turing-recognizable.

Practice Questions

2013 IIT Bombay

4.1 Which of the following is/are undecidable?

1. G is a CFG. Is $L(G) = \phi$?
2. G is a CFG. Is $L(G) = \Sigma^*$?
3. M is a Turing machines. Is $L(M)$ regular?
4. A is a DFA and N is an NFA. Is $L(A) = L(N)$?

(A) 3 only (B) 3 and 4 only

(C) 1, 2 and 3 only (D) 2 and 3 only

4.2 Consider the following languages

$$L_1 = \{0^p 1^q 0^r \mid p, q, r \ge 0\}$$

$$L_2 = \{0^p 1^q 0^r \mid p, q, r \ge 0, p \ne r\}$$

Which one of the following statements is **FALSE**?

(A) L_2 is context-free

(B) $L_1 \cap L_2$ is context-free

(C) Complement of L_2 is recursive

(D) Complement of L_1 is context-free but not regular

4.3 Which of the following statement is/are FALSE?

1. For every non-deterministic Turing machine, there exists an equivalent deterministic Turing machine
2. Turing recognizable languages are closed under union and complementation
3. Turing decidable languages are closed under intersection and complementation
4. Turing recognizable languages are closed under union and intersection

(A) 1 and 4 only (B) 1 and 3 only

(C) 2 only (D) 3 only

2014 IIT Kharagpur

4.4 Let Σ be a finite non-empty alphabet and let 2^{Σ^*} be the power set of Σ^*. Which one of the following is TRUE?

(A) Both 2^{Σ^*} and Σ^* are countable

(B) 2^{Σ^*} is countable and Σ^* is uncountable

(C) 2^{Σ^*} in uncountable and Σ^* is countable

(D) Both 2^{Σ^*} and Σ^* are uncountable

4.5 Which one of the following problems is undecidable?

(A) Deciding if a given context-free grammar is ambiguous.

(B) Deciding if a given string is generated by a given context-free grammar.

(C) Deciding if the language generated by a given context-free grammar is empty.

(D) Deciding if the language generated by a given context-free grammar is finite.

4.6 If $L_1 = \{a^n \mid n \ge 0\}$ and $L_2 = \{b^n \mid n \ge 0\}$, consider

(i) $L_1 \cdot L_2$ is a regular language

(ii) $L_1 \cdot L_2 = \{a^n b^n \mid n \ge 0\}$

Which one of the following is CORRECT?

(A) Only (i)

(B) Only (ii)

(C) Both (i) and (ii)

(D) Neither (i) nor (ii)

4.7 Let $A \le_m B$ denotes that language A is mapping reducible (also known as many-to-one reducible) to language B. Which one of the following is FALSE?

(A) If $A \le_m B$ and *B* is recursive then *A* is recursive.

(B) If $A \le_m B$ and *A* is undecidable then *B* is undecidable.

(C) If $A \leq_m B$ and B is recursively enumerable then A is recursively enumerable.

(D) If $A \leq_m B$ and B is not recursively enumerable then A is not recursively enumerable.

4.8 Let L be a language and $\bar{L}$ be its complement. Which one of the following is NOT a viable possibility?

(A) Neither L nor $\bar{L}$ is recursively enumerable (r.e.).

(B) One of L and $\bar{L}$ is r.e. but not recursive; the other is not r.e.

(C) Both L and $\bar{L}$ are r.e. but not recursive.

(D) Both L and $\bar{L}$ are recursive.

4.9 Let $\langle M \rangle$ be the encoding of a Turing machines as a string over $\sum = \{0,1\}$.

Let $L = \{\langle M \rangle | M$ is a Turing machine that accepts a string of length 2014}.

Then, L is

(A) Decidable and recursively enumerable

(B) Undecidable but recursively enumerable

(C) Undecidable and not recursively enumerable

(D) Decidable but not recursively enumerable

2015 IIT Kanpur

4.10 Which of the following languages is /are regular?

$L_1 : \{wxw^R \,|w, x \in \{a,b\}^* |w|, |x| > 0, w^R$ is the reverse of string w}

$L_2 : \{a^n b^m \,|m \neq n \, and \, m, n \geq 0\}$

$L_3 : \{a^p b^q c^r \,|p, q, r \geq 0\}$

(A) L_1 and L_3 only (B) L_2 only

(B) L_2 and L_3 only (D) L_3 only

4.11 For any two languages L_1 and L_2 such that L_1 is context-free and L_2 is recursively enumerable but not recursive, which of the following is/are necessarily true?

I. $\bar{L}_1$ (complement of L_1) is recursive

II. $\bar{L}_2$ (complement of L_2) is recursive

III. $\bar{L}_1$ is context-free

IV. $L_1 \cup L_2$ is recursively enumerable

(A) I only (B) III only

(C) III and IV only (D) I and IV only

4.12 Consider the following statements.

I. The complements of every Turing decidable language is Turing decidable

II. There exists some language which is in NP but is not Turing decidable

III. If L is a language in NP, L is Turing decidable

Which of the above statements is /are true?

(A) Only II (B) Only III

(C) Only I and II (D) Only I and III

2016 IISc Bangalore

4.13 Which of the following decision problems are undecidable?

I. Given NFAs N_1 and N_2, is $L(N_1) \cap L(N_2) = \phi$?

II. Given a CFG $G = (N, \Sigma, P, S)$ and a string $x \in \Sigma^*$, does $x \in L(G)$?

III. Given CFGs G_1 and G_2 is $L(G_1) = L(G_2)$?

IV. Given a TM M, is $L(M) = \phi$?

(A) I and IV only

(B) II and III only

(C) III and IV only

(D) II and IV only

4.14 Consider the following types of languages :

L_1 : Regular,

L_2 : Context-free,

L_3 : Recursive,

L_4 : Recursively enumerable.

Which of the following is/are TRUE?

I. $\bar{L}_3 \cup L_4$ is recursively enumerable

II. $\bar{L}_2 \cup L_3$ is recursive

III. $L_1^* \cap L_2$ is context-free

IV. $L_1 \cup \bar{L}_2$ is context-free

(A) I only

(B) I and III only

(C) I and IV only
(D) I, II and III only

4.15 Let X be a recursive language and Y be a recursively enumerable but not recursive language. Let W and Z be two languages such that $\overline{Y}$ reduce to W and Z reduces to $\overline{X}$ (reduction means the standard many-one reduction). Which one of the following statements is TRUE?

(A) W can be recursively enumerable and Z is recursive.
(B) W can be recursive and Z is recursively enumerable.
(C) W is not recursively enumerable and Z is recursive.
(D) W is not recursively enumerable and Z is not recursive.

2017 IIT Roorkee

4.16 Let L_1, L_2 be any two context-free languages and R be any regular language. Then which of the following is/are CORRECT?

(i) $L_1 \cup L_2$ is context free
(ii) $\overline{L}_1$ is context free
(iii) $L_1 - R$ is context free
(iv) $L_1 \cap L_2$ is context free

(A) (i), (ii) and (iv) only
(B) (i) and (iii) only
(C) (ii) and (iv) only
(D) (i) only

4.17 Consider the context-free grammar over the alphabet $\{a, b, c\}$ given below. S and T are non-terminals.

$$G_1 : S \rightarrow aSb \,|\, T,\ T \rightarrow cT \,|\, \varepsilon$$
$$G_2 : S \rightarrow bSa \,|\, T,\ T \rightarrow cT \,|\, \varepsilon$$

The language $L(G_1) \cap L(G_2)$ is

(A) Finite
(B) Not finite but regular
(C) Context free but not regular
(D) Recursive but not context free

4.18 Consider the following language over the alphabet $\sum = \{a, b, c\}$.

Let $L_1 = \{a^n b^n c^m \,|\, m, n \geq 0\}$ and
$L_2 = \{a^m b^n c^n \,|\, m, n \geq 0\}$.

Which of the following are context-free languages?

I. $L_1 \cup L_2$ II. $L_1 \cap L_2$

(A) I only
(B) II only
(C) I and II
(D) Neither I nor II

4.19 Let $L(R)$ be the language represented by regular expression R. Let $L(G)$ be the language generated by a context free grammar G. Let $L(M)$ be the language accepted by a Turing machine M.

Which of the following decision problems are undecidable?

I. Given a regular expression R and a string w, is $w \in L(R)$?
II. Given a context free grammar G, is $L(G) = \phi$?
III. Given a context free grammar G, is $L(G) = \sum^*$ for some alphabet $\sum$?
IV. Given a Turing machine M and a string w, is $w \in L(M)$?

(A) I and IV only
(B) II and III only
(C) II, III and IV only
(D) III and IV only

2018 IIT Guwahati

4.20 The set of all recursively enumerable languages is

(A) closed under complementation.
(B) closed under intersection.
(C) a subset of the set of all recursive languages.
(D) an uncountable set.

4.21 Consider the following problems. $L(G)$ denotes the language generated by a grammar G. $L(M)$ denotes the language accepted by a machine M.

(I) For an unrestricted grammar G and a string w, whether $w \in L(G)$
(II) Given a Turning machine M, whether $L(M)$ is regular
(III) Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$

(IV) Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language.

Which one of the following statements is correct?

(A) Only I and II are undecidable

(B) Only III is undecidable

(C) Only II and IV are undecidable

(D) Only I, II and III are undecidable

2021 IIT Bombay

4.22 Suppose that L_1 is a regular language and L_2 is a context-free language. Which one of the following languages is NOT necessarily context-free?

(A) $L_1 \cap L_2$ (B) $L_1 \cdot L_2$

(C) $L_1 - L_2$ (D) $L_1 \cup L_2$

4.23 Let $\langle M \rangle$ denote an encoding of an automaton M. Suppose that $\Sigma = \{0,1\}$. Which of the following languages is/are NOT recursive?

(A) $L = \{\langle M \rangle \mid M$ is a DFA such that $L(M) = \phi\}$

(B) $L = \{\langle M \rangle \mid M$ is a DFA such that $L(M) = \Sigma^*\}$

(C) $L = \{\langle M \rangle \mid M$ is a PDA such that $L(M) = \phi\}$

(D) $L = \{\langle M \rangle \mid M$ is a PDA such that $L(M) = \Sigma^*\}$

4.24 For a Turing machine $M, \langle M \rangle$ denotes an encoding of M. Consider the following two languages.

$L_1 = \{\langle M \rangle \mid M$ takes more than 2021 steps on all inputs$\}$

$L_2 = \{\langle M \rangle \mid M$ takes more than 2021 steps on some input$\}$

Which one of the following options is correct?

(A) Both L_1 and L_2 are decidable.

(B) L_1 is decidable and L_2 is undecidable.

(C) L_1 is undecidable and L_2 is decidable.

(D) Both L_1 and L_2 are undecidable.

4.25 Let L_1 be a regular language and L_2 be a context-free language. Which of the following languages is/are context-free?

(A) $L_1 \cap \overline{L_2}$

(B) $\overline{\overline{L_1} \cup \overline{L_2}}$

(C) $L_1 \cup (L_2 \cup \overline{L_2})$

(D) $(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$

4.26 Consider the following two statements about regular languages:

S1: Every infinite regular language contains an undecidable language as a subset.

S2: Every finite language is regular.

Which one of the following choices is correct?

(A) Only S1 is true

(B) Only S2 is true

(C) Both S1 and S2 are true

(D) Neither S1 nor S2 is true

Solutions

4.1 (D)

Consider each statement :

Statement 1 :

It is emptiness for CFG. This problem is decidable.

Statement 2 :

It is everything for CFG. Whether CFG will generate all possible string is undecidable.

Statement 3 :

This problem is regularity for Recursive / Recursive Enumerable. It is undecidable.

Statement 4 :

This problem is equivalence for regular languages. It is decidable.

Hence, the correct option is (D)

4.2 (D)

Given :

$$L_1 = \{0^p 1^q 0^r \mid p, q, r \geq 0\}$$

$$L_2 = \{0^p 1^q 0^r \mid p, q, r \geq 0,\ p \neq r\}$$

Consider each option one by one :

Option (A) : L_2 is context-free.

L_2 is context-free . So A is correct.

Option (B) : $L_1 \cap L_2$ is context-free.

Since, intersection of a CFL with regular is CFL. So, B is correct.

Option (C) : Complement of L_2 is recursive.

Since L_2 is CFL, so it is recursive too and recursive language are closed under complement. Hence, complement of L_2 is recursive.

Option (D) : Complement of L_1 is context-free but not regular.

Since L_1 is regular. So, complement of L_1 is regular.

So D is false.

Hence, the correct option is (D).

4.3 (C)

Consider each statement :

Statement 1 :

For every non-deterministic Turing machine, there exits an equivalent deterministic Turing machine.

This statement is true. Since non- deterministic Turing machine are equivalent to deterministic Turing machine.

Statement 2 :

Turing recognizable languages are closed under union and complementation.

This statement is false. Since Turing recognizable language (recursive enumerable language) is not closed under complementation.

Statement 3 :

Turing decidable languages are closed under intersection and complementation.

This statement is true. Since Turing decidable language (recursive language) is closed under complementation and intersection.

Statement 4 :

Turing recognizable languages are closed under union and intersection

This statement is false since turning recognizable language (Recursive enumerable language) are closed under union and intersection.

Hence, the correct option is (C).

4.4 (C)

Σ - finite non- empty alphabet

2^{Σ^*} - power set of Σ^*

Since $\Sigma \rightarrow$ finite non-empty alphabet

Σ^* will be countably infinite.

Example :

Let $\Sigma = \{a, b\}$

$\Sigma^* = \{\varepsilon, a, b, aa, ab, ba, bb, aaa, ...\}$

Clearly, Σ^* is countable and infinite

Now 2^{Σ^*} will be uncountable set because the power set of any countably infinite set is uncountable.

Hence, the correct option is (C)

4.5 (A)

Consider each option :

Option (A) :

Context free grammar is not closed under ambiguity. A set is closed under an operation means when we operate an element of that set with that operator we get an element from that set. Here, context free grammar generates a context free language and set of all context free languages is also a set. But, ambiguity is not an operation and hence we can never say that CFG is closed under ambiguity.

Option (B) :

Deciding if a given string is generated by a given context-free grammar. It is membership problem for CFG which is decidable.

Option (C) :

Deciding if the language generated by a given context-free grammar is empty. It is emptiness problem for CFG which is decidable.

Option (D) :

Deciding if the language generated by a given context-free grammar is finite. It is finiteness problem for CFG which is decidable.

Hence, the correct option is (A).

4.6 (A)

Given :

$$L_1 = \{a^n \mid n \geq 0\}$$

$$L_2 = \{b^n \mid n \geq 0\}$$

L_1 and L_2 both are regular.

Since, regular languages are closed under concatenation, thus $L_1 \cdot L_2$ is regular.

Hence, I is correct and II is Incorrect.

Hence, the correct option is (A).

4.7 **(D)**

Given : $A \le_m B$ denotes language A is mapping reducible to language B.

$A \le_m B$ also means A cannot be harder than B. So, clearly options A, B, C are correct.

But in option (D), if B is not recursively enumerable then A can be recursively enumerable.

Hence, the correct option is (D).

4.8 **(C)**

Given :

A language L and its complement $\bar{L}$ out of all options, option (C) does not seems to be viable possibility.

Since, if *L* is Recursive Enumerable but not recursive then $\bar{L}$ will be surely not Recursive Enumerable.

If *L* and $\bar{L}$ are Recursive Enumerable then both have to be recursive.

Hence, the correct option is (C).

4.9 **(B)**

Given :

$\Sigma = \{0,1\}$

$L = \{\langle M \rangle \mid$ *M* is Turing machine that accepts a string of length 2014}

Since, there are finite number of strings of length '2014'. So, a Turing machine will take the input string of length '2014'. If, input string is present in the language then Turing machine will halt in final state. But if input string is not present in the language it may reject it by halting in non-final state or may also go in an infinite loop and never halt.

Hence, '*L*' is undecidable and recursively enumerable.

Hence, the correct option is (B).

4.10 **(A)**

Given :

$L_1 : \{wxw^R \mid w, x \in \{a,b\}^* |w|, |x| > 0, w^R$ is the reverse of string w}

$L_2 : \{a^n b^m \mid m \ne n \, and \, m, n \ge 0\}$

$L_3 : \{a^p b^q c^r \mid p, q, r \ge 0\}$

$L_1 : L_1$ is regular with regular expression:

$1(0+1)^* 1 + 0(0+1)^* 0$

$L_2 : L_2$ is CFL. Since we need to compare number of a's and b's

$L_3 : L_3$ is regular

Hence, the correct option is (A).

4.11 **(D)**

Given :

L_1 is context-free

L_2 is recursively enumerable but not recursive.

Consider each statement :

Statement I : $\bar{L}_1$ is recursive.

Since L_1 is CFL and recursive too. And, recursive languages are closed under complementation. Hence. $\bar{L}_1$ will be recursive. So, I is True.

Statement II : $\bar{L}_2$ is recursive

Since L_2 is Recursive Enumerable but not recursive. Hence, $\bar{L}_2$ will not be recursively enumerable. So, II is false.

Statement III : $\bar{L}_1$ is context-free

Since, CFL are not closed under complementation. $\bar{L}_1$ may or may not be CFL. So, III is false.

Statement IV : $L_1 \cup L_2$ is recursively enumerable

Since, L, is CFL and hence recursive too. $\bar{L}_1$ will be recursive and hence REE too. Hence, $\bar{L}_1 \cup L_2$ is Recursive Enumerable. Since Recursive Enumerable are closed under union. Hence, IV is True.

Hence, the correct option is (D).

4.12 **(D)**

Consider each statement :

Statement I : The complements of every Turing decidable language is Turing decidable.

The solution to a decision problem is either "yes" or "no", and hence if we can decide a problem, we have also decided its complement-just reverse "yes" and "no". (This is applicable for decidability and not for acceptance). So, I is true.

Statement II : There exists some language which is in NP but is not Turing decidable.

It is false. Because NP class is defined as the class of languages that can be solved in polynomial time by a non-deterministic Turing machine. So, none of the NP class problems is undecidable.

Statement III : If L is a language in NP, *L* is Turing decidable.

It is true for same reason as II.

Hence, the correct option is (D).

4.13 (C)

A problem is undecidable, when there is no algorithm to find the solution for it.

Consider each statement :

Statement I :

I is disjointness problem of the regular languages. It is decidable.

Statement II :

II is membership problem of CFG which is decidable.

Statement III :

III is equivalence problem of the CFG which is undecidable.

Statement IV :

IV is emptiness problem for a Turing machine. It is undecidable.

Hence, the correct option is (C).

4.14 (D)

Given :

L_1 is Regular

L_2 is CFL

L_3 is Recursive

L_4 is Recursively enumerable

Consider each statement :

Statement I : $\bar{L}_3 \cup L_4$ is recursively enumerable

Since L_3 is Rec Implies $\bar{L}_3$ will also be recursive and hence Recursive Enumerable too. Hence $\bar{L}_3 \cup L_4$ will be Recursive Enumerable. Since Recursive Enumerable are closed under union. Hence, I is True.

Statement II : $\bar{L}_2 \cup L_3$ is recursive

Since L_2 is CFL implies $\bar{L}_2$ may or may not be CFL. Since CFL are not closed under complementation. But L_2 is recursive too implies $\bar{L}_2$ will be recursive. Hence $\bar{L}_2 \cup L_3$ will be recursive. Since, recursive languages are closed under union. Hence, II is True.

Statement III : $L^*_1 \cap L_2$ is context-free

Since, L_1 is regular implies L_1^* is also regular (closed under kleene closure). Since L_2 is CFL.

So, $L_1^* \cap L_2$ is also CFL (CFL closed under intersection with regular). Hence, III is True.

Statement IV : $L_1 \cup \bar{L}_2$ is context-free

Here, L_1 is regular. $\bar{L}_2$ may or may not be CFL (CFL not closed under complement).

Hence, $L_1 \cup \bar{L}_2$ may or may not be CFL. Hence, IV is false.

Hence, the correct option is (D).

4.15 (C)

Given :

X is Recursive language

Y is Recursive Enumerable but not recursive language

$\bar{Y}$ is reducible to *W* $(\bar{Y} \le W)$

Z is reducible to $\bar{X}$ $(Z \le \bar{X})$

Y is Recursive Enumerable but not recursive implies $\bar{Y}$ will not be Recursive Enumerable. *X* is recursive implies $\bar{X}$ will be recursive.

$\bar{Y} \le W$ means that $\bar{Y}$ cannot be harder than *W*.

Since $\bar{Y}$ is not Recursive Enumerable implies *W* will not be Recursive Enumerable either.

$Z \le \bar{X}$ means that *Z* cannot be harder than $\bar{X}$. $\bar{X}$ is recursive implies Z is recursive.

Hence, the correct option is (C).

4.16 (B)

Given :

L_1, L_2 are two context-free languages and *R* is any regular language.

Consider each statement :

(i) $L_1 \cup L_2$ is context free

Since CFLs are closed under union, $L_1 \cup L_2$ is context free.

(ii) $\bar{L}_1$ is context free

Since CFLs are not closed under complement, $\bar{L}_1$ may or may not context free.

(iii) $L_1 - R$ is context free

$$L_1 - R = L_1 \cap \overline{R}$$

Since, CFL are closed under intersection with a CFL, $L_1 - R$ is context free

(iv) $L_1 \cap L_2$ is context free

Since CFLs are not closed under intersection, $L_1 \cap L_2$ may or may not context free.

Hence, the correct option is (B).

4.17 (B)

Given :

$$G_1 : S \to aSb \mid T,\ T \to cT \mid \varepsilon$$

$$G_2 : S \to bSa \mid T,\ T \to cT \mid \varepsilon$$

The language generated by G_1 is

$$L(G_1) = \{a^n c^m b^n \mid n, m \geq 0\} \cup \{c^m \mid m \geq 0\}$$

The language generated by G_2 is

$$L(G_2) = \{b^n c^m a^n \mid n, m \geq 0\} \cup \{c^m \mid m \geq 0\}$$

$$L(G_1) \cap L(G_2) = \{c^m \mid m \geq 0\}$$

So, the resultant language is not finite but it is regular.

Hence, the correct option is (B).

4.18 (A)

Given :

$$L_1 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$L_2 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Consider language I. $L_1 \cup L_2$

Since L_1 and L_2 both are CFLs and CFLs are closed under union.

So $L_1 \cup L_2$ will also be CFL.

Consider language II. $L_1 \cap L_2$

$$L_1 \cap L_2 = \{a^n b^n c^n \mid n >= 0\}$$

Here more than one comparison is performed therefore, $L_1 \cap L_2$ is a CSL and not a CFL.

Hence, the correct option is (A).

4.19 (D)

Consider each statement :

Statement I : Given a regular expression *R* and a string *w*, is $w \in L(R)$?

This statement is membership problem of regular language which is decidable.

Statement II : Given a context free grammar *G*, is $L(G) = \phi$?

This statement is emptiness problem of CFL. Which is decidable.

Statement III : Given a context free grammar *G*, is $L(G) = \sum^*$ for some alphabet $\sum$?

This statement is accept everything problem of CFL. Which is undecidable.

Statement IV : Given a Turing machine *M* and a string *w*, is $w \in L(M)$?

This statement is membership problem of recursively enumerable language. Which is undecidable.

Hence, the correct option is (D).

4.20 (B)

Recursively enumerable languages are not closed under complementation.

Recursively enumerable language are closed under intersection.

Set of all recursively enumerable languages are proper super set of all recursive languages.

Set of all recursively enumerable languages are infinite but countable.

Hence, the correct option is (B).

4.21 (D)

Consider each statement :

Statement I : For an unrestricted grammar *G* and a string *w*, whether $w \in L(G)$.

Membership of Unrestricted grammar is undecidable. So, I is undecidable.

Statement II : Given a Turning machine *M*, whether $L(M)$ is regular

Regularity problem for Turing machine sis undecidable. So, II is undecidable.

Statement III : Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$

Given two grammars G_1 and G_2, whether $L(G_1) = L(G_2)$ is undecidable. Because when nothing is mentioned about the type of the Grammar, It, by default, should be taken as A Valid Grammar i.e. Type 0 Grammar which itself covers All the Grammars. So, now the given problem is nothing but "Equivalence of two Recursive Enumerable Grammars i.e. Equality of Two Recursive Enumerable languages" Problem. Which is Undecidable. So, III is undecidable.

Statement IV : Given an NFA *N*, whether there is a deterministic PDA *P* such that *N* and *P* accept the same language.

Given an NFA N, whether there is a deterministic PDA P such that N and P accept the same language is Decidable because. We can always say that there will definitely be a DPDA (and for that matter PDA too) which will accept the same language that NFA N is accepting. So, IV is decidable.

Hence, the correct option is (D).

4.22 (C)

Given :

L_1 is a regular language

L_2 is a context free language.

Consider each option :

Option (A) : $L_1 \cap L_2$

Since CFL is closed under intersection with regular, therefore $L_1 \cap L_2$ is CFL.

Option (B) : $L_1.L_2$

Every regular language is a CFL and CFLs are closed under concatenation, therefore, $L_1.L_2$ is a CFL.

Option (C) : $L_1 - L_2 = L_1 \cap \overline{L}_2$

Since CFLs are not closed under complementation, therefore, $\overline{L}_2$ need not be CFL, which means $L_1 - L_2$ need not be CFL

Option (D) : $L_1 \cup L_2$

Since CFLs are closed under union operation therefore, $L_1 \cup L_2$ is CFL.

Hence the correct option is (C).

4.23 (D)

Given:

$\langle M \rangle$ is an automaton, $\Sigma = \{0,1\}$

Consider each option one by one-

Option (A) :

$$L = \{\langle M \rangle \mid M \text{ is a DFA such that } L(M) = \phi\}$$

Since for every regular language there exists a unique minimal DFA, and emptiness problem of regular languages are decidable hence L is recursive.

Option (B) :

$$L = \{\langle M \rangle \mid M \text{ is a DFA such that } L(M) = \Sigma^*\}$$

Similar to option (A), completeness problem of regular languages is decidable, therefore L is recursive.

Option (C) :

$$L = \{\langle M \rangle \mid M \text{ is a PDA such that } L(M) = \phi\}$$

Since every PDA can be converted to CFL and vice-versa and emptiness problem of CFL is decidable, therefore L is recursive.

Option (D) :

$$L = \{\langle M \rangle \mid M \text{ is a PDA such that } L(M) = \Sigma^*\}$$

Since completeness problem of CFL is undecidable, therefore we can conclude that L is not recursive.

Hence the correct option is (D).

4.24 (A)

Given :

$L_1 = \{\langle M \rangle \mid M$ takes more than 2021 steps on all inputs}

$L_2 = \{\langle M \rangle \mid M$ takes more than 2021 steps on some input}

Consider L_1, if for all inputs, M takes more than 2021 steps i.e., if we monitor 2021+1 steps for all possible inputs then L_1 can be decidable.

Consider L_2, if for none of inputs M takes more than 2021 steps that means we can answer "no" for L_2, else "yes"

Hence L_2 is also decidable.

Hence the correct option is (A).

4.25 (B, C, D)

Given :

L_1 is a regular language

L_2 is a context- free language.

Consider each option one by one :

Option (A) : $L_1 \cap \overline{L}_2$

Since CFLs are not closed under complementation, $\overline{L}_2$ need not be CFL. Therefore, $L_1 \cap \overline{L}_2$ may or may not be CFL.

Option (B) : $\overline{\overline{L_1} \cup \overline{L_2}}$

$$\overline{\overline{L_1} \cup \overline{L_2}} = \overline{\overline{L_1}} \cap \overline{\overline{L_2}} = L_1 \cap L_2$$

Since CFLs are closed under intersection with regular language. Therefore, $L_1 \cap L_2$ is CFL.

Option (C) :

$$L_1 \cup (L_2 \cup \overline{L_2})$$

$$L_1 \cup (L_2 \cup \overline{L_2}) = L_1 \cup (\Sigma^*)$$

$$L_1 \cup (L_2 \cup \overline{L_2}) = \Sigma^*$$

Since Σ^* is regular, hence CFL too,

$L_1 \cup (L_2 \cup \overline{L_2})$ is CFL.

Option (D) :

$$(L_1 \cap L_2) \cup (\overline{L_2} \cap L_2)$$

$$(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2) = (L_1 \cup \overline{L_1}) \cap L_2$$

$$(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2) = L_2$$

Since L_2 is CFL, therefore,

$(L_1 \cap L_2) \cup (\overline{L_1} \cap L_2)$ is CFL.

Hence the correct option are (B), (C) and (D).

4.26 (c)

Consider each statement

S1: Every infinite regular language contains an undecidable language as a subset.

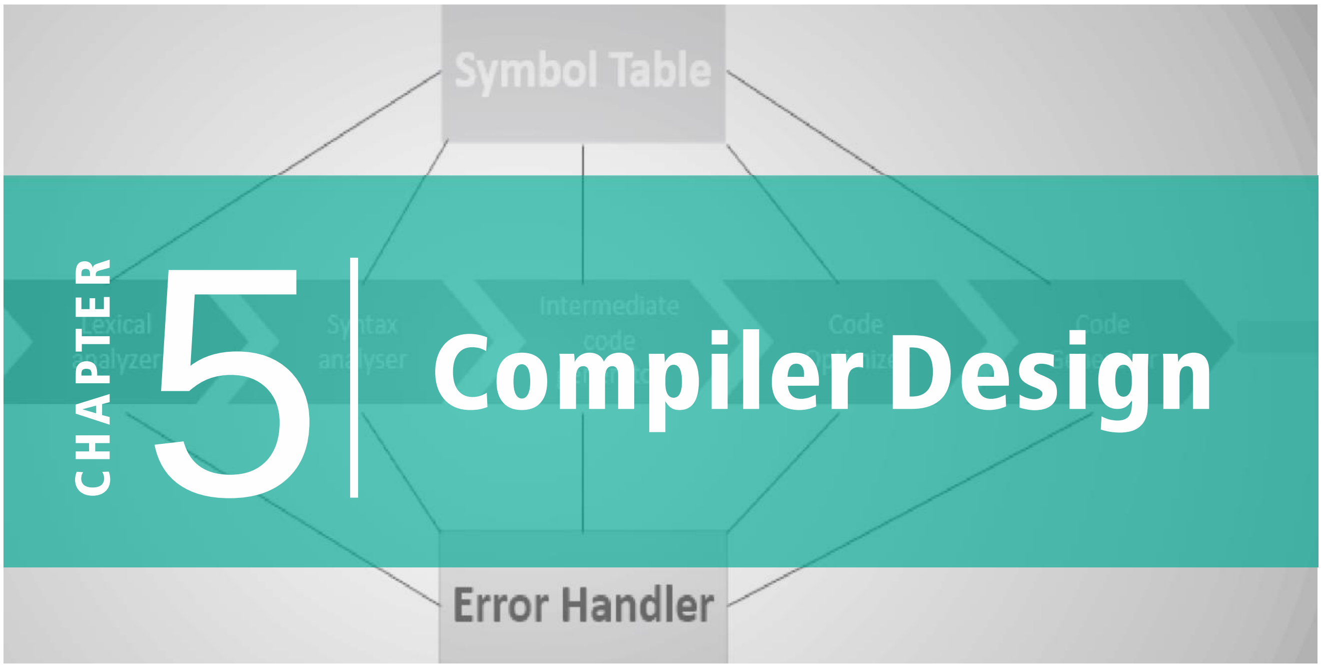
True. We can construct a subset N of A that we will prove non-regular by using pumping lemma.

S2: Every finite language is regular.

True. Every finite language is Regular. Because we can draw DFA for it.

Hence the correct option is (C).

CHAPTER 5 Compiler Design



Marks Distribution of Compiler Design in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	5	13
2004*	3	1	5
2005*	1	4	9
2006*	1	5	11
2007*	1	5	11
2008*	2	1	4
2009	1	-	1
2010	2	1	4
2011	1	1	3
2012	-	2	4
2013	1	3	7
2014 Set-1	1	1	3
2014 Set-2	2	2	6
2014 Set-3	2	1	4

*** CS and IT combined**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	1	2	5
2015 Set-2	2	1	4
2015 Set-3	1	1	3
2016 Set-1	1	2	5
2016 Set-2	1	1	3
2017 Set-1	2	1	4
2017 Set-2	2	1	4
2018	1	2	5
2019	2	2	6
2020	2	1	4
2021 Set-1	2	3	8
2021 Set-2	2	2	6

Syllabus : Compiler Design

Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation. Local optimisation, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

Contents : Compiler Design

1 Lexical Analysis

Practice Questions

2016 IISc Bangalore

1.1 Match the following :

List-I

(P) Lexical analysis

(Q) Top down parsing

(R) Semantic analysis

(S) Runtime environment

List-II

(i) Leftmost derivation

(ii) Type checking

(iii) Regular expressions

(iv) Activation records

(A) $\mathrm{P} \leftrightarrow \mathrm{i}, \mathrm{Q} \leftrightarrow \mathrm{ii}, \mathrm{R} \leftrightarrow \mathrm{iv}, \mathrm{S} \leftrightarrow \mathrm{iii}$

(B) $\mathrm{P} \leftrightarrow \mathrm{iii}, \mathrm{Q} \leftrightarrow \mathrm{i}, \mathrm{R} \leftrightarrow \mathrm{ii}, \mathrm{S} \leftrightarrow \mathrm{iv}$

(C) $\mathrm{P} \leftrightarrow \mathrm{ii}, \mathrm{Q} \leftrightarrow \mathrm{iii}, \mathrm{R} \leftrightarrow \mathrm{i}, \mathrm{S} \leftrightarrow \mathrm{iv}$

(D) $\mathrm{P} \leftrightarrow \mathrm{iv}, \mathrm{Q} \leftrightarrow \mathrm{i}, \mathrm{R} \leftrightarrow \mathrm{ii}, \mathrm{S} \leftrightarrow \mathrm{iii}$

2017 IIT Roorkee

1.2 Match the following according to input (from the left column) to the compiler phase (in the right column) that process is :

List-I

(P) Syntax tree

(Q) Character stream

(R) Intermediate representation

(S) Token stream

List-II

(i) Code generator

(ii) Syntax analyzer

(iii) Semantic analyzer

(iv) Lexical analyzer

(A) $\mathrm{P} \to (\mathrm{ii}), \mathrm{Q} \to (\mathrm{iii}), \mathrm{R} \to (\mathrm{iv}), \mathrm{S} \to (\mathrm{i})$

(B) $\mathrm{P} \to (\mathrm{ii}), \mathrm{Q} \to (\mathrm{i}), \mathrm{R} \to (\mathrm{iii}), \mathrm{S} \to (\mathrm{iv})$

(C) $\mathrm{P} \to (\mathrm{iii}), \mathrm{Q} \to (\mathrm{iv}), \mathrm{R} \to (\mathrm{i}), \mathrm{S} \to (\mathrm{ii})$

(D) $\mathrm{P} \to (\mathrm{i}), \mathrm{Q} \to (\mathrm{iv}), \mathrm{R} \to (\mathrm{ii}), \mathrm{S} \to (\mathrm{iii})$

2018 IIT Guwahati

1.3 A lexical analyzer uses the following patterns to recognize three tokens T_1, T_2, and T_3 over the alphabet {*a*, *b*, *c*}.

$$T_1 : a?(b|\,c)^*a$$

$$T_2 : b?(a|\,c)^*b$$

$$T_3 : c?(b|\,a)^*c$$

Note that '*x*?' means 0 or 1 occurrence of the symbol *x*. Note also that the analyzer outputs the token that matches the longest possible prefix.

If the string bbaacabc is processes by the analyzer, which one of the following is the sequence of tokens it outputs?

(A) $T_1T_2T_3$ (B) $T_1T_1T_3$

(C) $T_2T_1T_3$ (D) T_3T_3

1.4 Which one of the following statements is FALSE?

(A) Context-free grammar can be used to specify both lexical and syntax rules.

(B) Type checking is done before parsing.

(C) High-level language programs can be translated to different Intermediate Representations.

(D) Arguments to a function can be passed using the program stack.

Solutions

1.1 (B)

Lexical analysis uses regular expression (Finite automata) to recognize identifiers.

Top down parsing uses leftmost Derivation, to generate the string of the language.

Type checking is done at semantic analysis phase.

Activation records are related to Run Time Environment

Hence, the correct option is (B).

1.2 (C)

Syntax tree is input to semantic analyzer. Character stream is input to lexical analyzer. Intermediate code is input to code generator. Token stream is input to syntax analyzer.

Key Point

In the question, we asked to match according to input, so read the question properly before matching.

Hence, the correct option is (C).

1.3 (D)

Given :

Three tokens T_1, T_2, and T_3 over the alphabet {*a*, *b*, *c*}.

$$T_1 : a?(b|\ c)^*a$$

$$T_2 : b?(a|\ c)^*b$$

$$T_3 : c?(b|\ a)^*c$$

String bbaacabc can be broken as bbaac and abc.

T_3 can generate bbaac and again

T_3 can also generate abc, so $T_3\ T_3$ is the sequence of tokens it outputs.

Key Point

While tokenization, we check for longest matching prefix in the given string.

Hence, the correct option is (D).

1.4 (B)

Consider each options one by one :

Option (A) : Context free grammar can be used to specify both lexical and syntax rules.

Since lexical rules are nothing but regular expressions we can use CFGs to represent such rules (Every Type-3 grammar is Type-2 grammar). Additionally, syntax rules can be represented by CFGs.

Option (B) : Type checking is done before parsing.

Type checking is done during semantic analysis phase which comes after parsing.

Option (C) : High level language programs can be translated to different intermediate representations.

We have various types of intermediate code representations, ex. 3-address code, postfix notations, syntax trees.

Option (D) : Arguments to a function can be passed using the program stack.

Program stack holds the activation record of the function called, which stores function parameters, return value, return address etc.

Hence, the correct option is (B).

2 Parsing Techniques

Practice Questions

2013 IIT Bombay

2.1 What is the maximum number of reduce moves that can be taken by a bottom up parser for a grammar with no epsilon and unit production (i.e. of type $A \to \varepsilon$ and $A \to a$) to parse a string with n tokens?

(A) $n/2$ (B) $n-1$

(C) $2n-1$ (D) 2^n

2.2 Consider the following two sets of LR(1) items of an LR (1) grammar

$X \to c.X, c/d \quad X \to c.X, \$$

$X \to .cX, c/d \quad X \to .cX, \$$

$X \to .d, c/d \quad X \to .d, \$$

Which of the following statements related to merging of the two sets in the corresponding LALR parser is/are FALSE?

1. Cannot be merged since look aheads are different.
2. Can be merged but will result in S-R conflict.
3. Can be merged but will result in R-R conflict.
4. Cannot be merged since goto on c will lead to two different sets.

(A) 1 only (B) 2 only

(C) 1 and 4 only (D) 1, 2, 3 and 4

2014 IIT Kharagpur

2.3 A canonical set of items is given below

$S \to L. > R$

$Q \to R.$

On input symbol <the set has

(A) A shift reduce conflict and a reduce-reduce conflict.

(B) A shift-reduce conflict but not a reduce-reduce conflict.

(C) A reduce –reduce conflict but not a shift-reduce conflict.

(D) Neither a shift-reduce nor a reduce conflict.

2.4 Consider the grammar defined by the following production rules, with two operators* and +

$S \to T * P$

$T \to U | T * U$

$P \to Q + P | Q$

$Q \to id$

$U \to id$

Which one of the following is TRUE?

(A) + is left associative, while * is right associative

(B) + is right associative, while * is left associative

(C) Both + and * are right associative

(D) Both + and * are left associative

2015 IIT Kanpur

2.5 Which one of the following is TRUE at any valid state in shift-reduce parsing?

(A) Viable prefixes appear only at the bottom of the stack and not inside

(B) Viable prefixes appear only at the top of the stack and not inside

(C) The stack contains only a set of viable prefixes

(D) The stack never contains viable prefixes

2.6 Match the following :

List-I

P. Lexical analysis

Q. Parsing

R. Register allocation

S. Expression evaluation

List-II

1. Graph coloring
2. DFA minimization
3. Post-order traversal
4. Production tree

Codes :

(A) $P-2, Q-3, R-1, S-4$

(B) $P-2, Q-1, R-4, S-3$

(C) $P-2, Q-4, R-1, S-3$

(D) $P-2, Q-3, R-4, S-1$

2.7 Among simple LR (SLR), canonical LR, and look – ahead LR (LALR), which of the following pairs identify the method that is very easy to implement and the method that is the most powerful, in that order

(A) SLR, LALR

(B) Canonical LR, LALR

(C) SLR, canonical LR

(D) LALR, canonical LR

2.8 Consider the following grammar G

$S \to F|H$

$F \to p|c$

$H \to d|c$

Where S, F, and H are non-terminal symbol, p, d, and c are terminal symbol. Which of the following statements (s) is/are correct?

S1 : LL(1) can parse all string that are generated using grammar G

S2 : LR(1) can parse all strings that are generated using grammar G

(A) Only S1

(B) Only S2

(C) Both S1 and S2

(D) Neither S1 nor S2

2016 IISc Bangalore

2.9 Which one of the following grammars is free from left recursion?

(A) S→AB
A→Aa|b
B→c

(B) S→Ab|Bb|c
A→Bd|ε
B→e

(C) S→Aa|B
A→Bb|Sc|ε
B→d

(D) S→Aa|Bb|c
A→Bd|ε
B→Ae|ε

2017 IIT Roorkee

2.10 Consider the following grammar :

$P \to xQRS$

$Q \to yz|z$

$R \to w|\varepsilon$

$S \to y$

What is FOLLOW(Q)?

(A) {R} (B) {w}

(C) {w, y} (D) {w, ∉}

2.11 Which of the following statements about parser is/are CORRECT?

(i) Canonical LR is more powerful than SLR.

(ii) SLR is more powerful than LALR.

(iii) SLR is more powerful than Canonical LR.

(A) (i) only

(B) (ii) only

(C) (iii) only

(D) (ii) and (iii) only

2.12 Consider the following expression grammar G:

$E \to E-T|\, T$

$T \to T+F|\, F$

$F \to (E)|\, id$

Which of the following grammars is not left recursive, but is equivalent to G?

(A)
$E \to E-T|\, T$
$T \to T+F|\, F$
$F \to (E)|\, id$

(B)
$E \to TE'$
$E' \to -TE'|\, \varepsilon$
$T \to T+F|\, F$
$F \to (E)|\, id$

(C)
$E \to TX$
$X \to -TX|\, \varepsilon$
$T \to FY$
$Y \to +FY|\, \varepsilon$
$F \to (E)|\, id$

$E \to TX \mid (TX)$

(D) $X \to -TX \mid +TX \mid \varepsilon$

$T \to id$

2019 IIT Madras

2.13 Consider the grammar given below:

$S \to Aa$

$A \to BD$

$B \to b \mid \varepsilon$

$D \to d \mid \varepsilon$

Let a,b,d, and $ be indexed as follows:

a	b	d	$
3	2	1	0

Compute the FOLLOW set of the non-terminal B and write the index values for the symbols in the FOLLOW set in the descending order. (For example, if the FOLLOW set is {a, b, d,$}, then the answer should be 3210)

2.14 Which one of following kinds of derivation is used by LR parsers?

(A) Leftmost

(B) Leftmost in reverse

(C) Rightmost

(D) Rightmost in reverse

2.15 Consider the augmented grammar given below:

$S' \to S$

$S \to \langle L \rangle \mid id$

$L \to L,S \mid S$

Let I_0 = CLOSURE ({[S' → ·S]}). The number of items in the set GOTO (I_0, <) is______________.

2020 IIT Delhi

2.16 Consider the productions A → PQ and A → XY. Each of the five non-terminals A, P,Q,X, and Y has two attributes: s is a synthesized attribute, and i is an inherited attribute. Consider the following rules.

Rule 1: P.i=A.i+2, Q.i=P.i+A.i, and A.s=P.s+Q.s

Rule 2: X.i=A.i+Y.s and Y.i=X.s+A.i

Which one of the following is TRUE?

(A) Both Rule 1 and Rule 2 are L-attributed

(B) Only Rule 1 is L-attributed

(C) only Rule 2 is L-attributed

(D) Neither Rule 1 nor Rule 2 is L-attributed

2.17 Consider the following grammar.

S → aSB | d

B → b

The number of reduction steps taken by a bottom-up parser while accepting the string aaadbbb is _________.

2021 IIT Bombay

2.18 Consider the following context-free grammar where the set of terminals is a, b, c, d, f.

$S \to d\,a\,T \mid R\,f$

$T \to a\,S \mid b\,a\,T \mid \varepsilon$

$R \to c\,a\,T\,R \mid \varepsilon$

The following is a partially-filled LL(1) parsing table.

	a	b	c	d	f	$
S			(1)	$S \to daT$	(2)	
T	$T \to aS$	$T \to baT$	(3)		$T \to \varepsilon$	(4)
R			$R \to caTR$		$R \to \varepsilon$	

Which one of the following choices represents the correct combination for the numbered cells in the parsing table ("blank" denotes that the corresponding cell is empty)?

(A) (1) $S \to Rf$ (2) $S \to Rf$ (3) $T \to \varepsilon$ (4) $T \to \varepsilon$

(B) (1) blank (2) $S \to Rf$ (3) blank (4) blank

(C) (1) $S \to Rf$ (2) blank (3) blank (4) $T \to \varepsilon$

(D) (1) blank (2) $S \to Rf$ (3) $T \to \varepsilon$ (4) $T \to \varepsilon$

2.19 Consider the following statements.

S_1: Every SLR(1) grammar is unambiguous but there are certain unambiguous grammars that are not SLR(1).

S_2 : For any context-free grammar, there is a parser that takes at most $O(n^3)$ time to parse a string of length n.

Which one of the following option is correct?

(A) S_1 is true and S_2 is false

(B) S_1 is false and S_2 is true

(C) S_1 is true and S_2 is true

(D) S_1 is false and S_2 is false

2.20 Consider the following augmented grammar with {#, @, <, >, a, b, c} as the set of terminals.

$S' \to S$

$S \to S \# cS$

$S \to SS$

$S \to S @$

$S \to < S >$

$S \to a$

$S \to b$

$S \to c$

Let $I_0 = \text{CLOSURE}(\{S' \to \bullet S\})$. The number of items in the set $\text{GOTO}(\text{GOTO}(I_0 <), <)$ is_______.

Solutions

2.1 (B)

Consider the grammar

$A \to BC$

$B \to aa$

$C \to bb$

Now suppose the string is aabb

$A \to BC \to aaC \to aabb$

$n = 4$ and number of reduction is 3,

So, $n-1$

Hence, the correct option is (B).

2.2 (D)

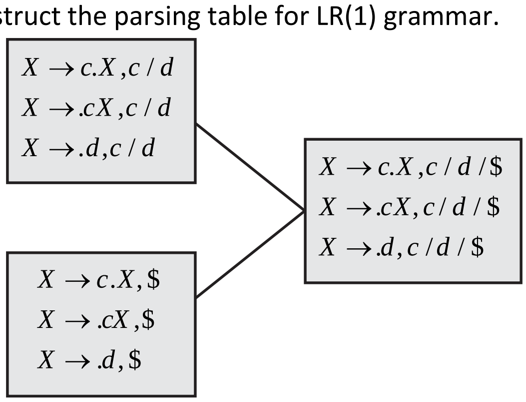
Given :

$X \to c.X, c/d \qquad X \to c.X, \$$

$X \to .cX, c/d \qquad X \to .cX, \$$

$X \to .d, c/d \qquad X \to .d, \$$

Construct the parsing table for LR(1) grammar.



Statement 1 : Cannot be merged since look aheads are different.

Because, merging is done even look aheads are different.

False

Statement 2 : Can be merged but will result in S-R conflict.

After merging, there is no SR conflict in the above table.

False

Statement 3 : Can be merged but will result in R-R conflict.

After merging, there is R-R conflict in the above table.

False

Statement 4 : Cannot be merged since goto on c will lead to two different sets.

because goto is carried on Non terminal symbols, not on terminal symbols and C is a terminal sysmbol.

False

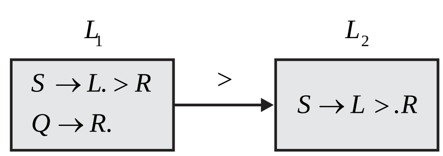
Hence, the correct option is (D).

2.3 (D)

Given :

$S \to L. > R$

$Q \to R.$



Since, there is only one reduced item, there is no conflict.

Key Point

Conflict may occurs if two or more items are present in the particular state, and one of them is reduce. There is no conflict, if one item is present in the state.

Hence, the correct option is (D).

2.4 **(B)**

Given :

$S \rightarrow T*P$

$T \rightarrow U|T*U$

$P \rightarrow Q+P|Q$

$Q \rightarrow id$

$U \rightarrow id$

Method 1

$T \rightarrow T*U$

T is generating $T*U$ recursively, so * is left associative.

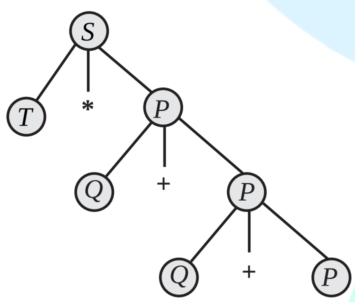
Similarly,

$P \rightarrow Q+P$

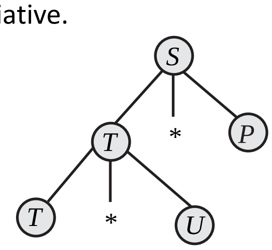
Right recursion is there, so + is right associative.

Method 2

Draw parse tree and check.



+ is right associative.



* is left associative

Hence, the correct option is (B).

2.5 **(C)**

The prefixes of right sentential forms that can appear on the stack of a shift reduce parser are called viable prefixes.

Key Point

Stack contains only a set of viable prefixes.

Hence, the correct option is (C).

2.6 **(C)**

Lexical analysis uses DFA.

Parsing makes production tree.

Register allocation can be modelled by Graph coloring.

Expression evaluation is done using tree traversal.

Hence, the correct option is (C).

2.7 **(C)**

SLR parser is simple and easy to implement as it has small parse tables. CLR parser is most powerful power among all LR parsers (SLR, LALR, CLR, LR (0)).

Key Point

LALR power is a simplified version of CLR parser.

CLR parser is also called LR (1) parser.

Hence, the correct option is (C).

2.8 **(D)**

Given :

Grammar G :

S→ F | H

F→ p |c

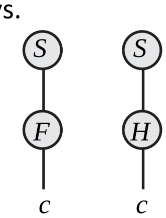
H→ d | c

Statements :

S1 : LL(1) can parse all string that are generated using grammar G

S2 : LR(1) can parse all strings that are generated using grammar G

Given grammar is ambiguous, since string "c" can be generated in 2 ways.



(I) (II)

Ambiguous grammar can be neither LL nor LR since every LL and every LR grammar is unambiguous.

Therefore, neither S1 nor S2 is correct.

Hence, the correct option is (D).

2.9 **(B)**

Considering each Option one by one:

Option (A) :

S→AB

A→Aa|b

B→c

Option (A) has immediate left recursion "A→Aa"

Option(B) :

S→Ab|Bb|c

A→Bd|ε

B→e

Option (B) is free from left recursion. There is no direct left recursion and indirect left recursion too .

Option(C) :

S→Aa|B

A→Bb|Sc|ε

B→d

Option (C) contains indirect left recursion S→Aa→Sca.

Option (D) :

S→Aa|Bb|c

A→Bd|ε

B→Ae|ε

Option (D) also has indirect left recursion A→Bd→Aed

Hence, the correct option is (B).

2.10 (C)

Given :

$P \to xQRS$ .... (i)

$Q \to yz / z$

$R \to w / \varepsilon$

$S \to y$

Follow (Q) = First of (RS)

= First of R

= w, $\underset{First(S)=y}{\overset{\varepsilon}{\downarrow}}$

= {w, y}

Key Point

Follow never contain epsilon.

Hence, the correct option is (C).

2.11 (A)

In terms of power

LR (0) < SLR < LALR < CLR

Hence, the correct option is (A).

2.12 (C)

Given :

Grammar , G

$E \to E - T \mid T$

$T \to T + F \mid F$

$F \to (E) \mid id$

Since, the grammar given above is left recursive, we need to remove left recursion ,If Grammar is of form $A \to A\alpha \mid \beta$ then after removal of left recursion it should be written as

$A \to \beta A'$

$A' \to \alpha A' \mid \varepsilon$

Rewriting after removing left recursion :

$E \to TE'$

$E' \to -TE' \mid \varepsilon$

$T \to FT'$

$T' \to +FT' \mid \varepsilon$

$F \to (E) \mid id$

Now replace E′ with X and T′ with Y

$E \to TX$

$X \to -TX \mid \varepsilon$

$T \to FY$

$Y \to +FY \mid \varepsilon$

$F \to (E) \mid id$

Hence, the correct option is (C).

2.13 31

Given :

$S \to Aa$

$A \to BD$

$B \to b / \varepsilon$

$D \to d / \varepsilon$

a, b, d and $ indexed as follows:

a	b	d	$
3	2	1	0

Follow (B) = {d, a}

Hence, their index in descending order is 31.

Hence, the correct answer is 31.

2.14 (D)

LR parser is a bottom up parser.

Hence it uses right most derivation in Reverse order.

Hence, the correct option is (D).

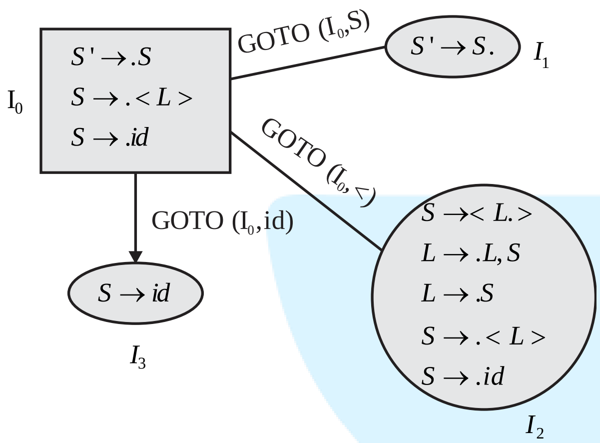
2.15 5

Given :

$S' \rightarrow S$

$S \rightarrow \langle L \rangle \mid id$

$L \rightarrow L,S \mid S$



Total number of items in set GOTO ($I_0, <$) is 5.

Hence, the correct answer is 5.

2.16 (B)

Given:

Productions:- A $\rightarrow$ PQ and A $\rightarrow$ XY

Rules:-

Rule 1: P. i = A. i+2, Q. i= P.i + A.i

And A.s = P.s + Q.s

Rule 1 is L- attributed by definition.

Rule 2: X. i = A. I +Y. s and Y. i = X.s +A. i

Rule 2 is Not L- attributed, since X take value from right sibling ($A \rightarrow XY$)

Key Point

In L-attributed definitions:

1. A parent can take its attribute values from any child (which is S–attributed and Every S–attributed is also L–Attributed).
2. A child can take its attribute values from the parent as well as from any left sibling but not from any right sibling.

Hence, the correct option is (B).

2.17 7

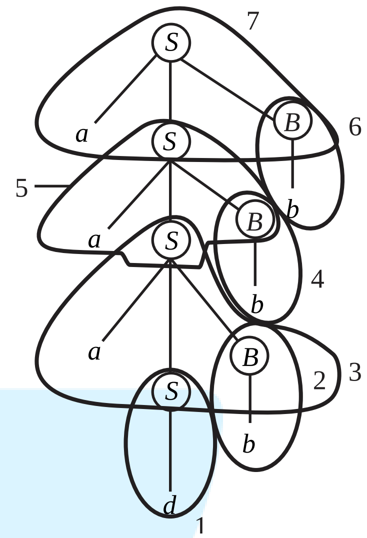
Given :

$$S \rightarrow aSB \mid d$$

$$B \rightarrow b$$

The string is aaadbbb

Parse tree :



In parse tree, all the non-terminals are reductions.

So, 7= reduction steps are required.

Hence, the correct answer is 7.

2.18 (A)

Given :

Context free grammar is

$$S \rightarrow daT \mid Rf$$

$$T \rightarrow aS \mid baT \mid \varepsilon$$

$$R \rightarrow caTR \mid \varepsilon$$

Where $\{a, b, c, d, f\}$ are set of terminals. Now, find first and follow of R,S and T.

	First	Follow
R	$\{c, \varepsilon\}$	$\{f\}$
S	$\{c, d, f\}$	$\{\$\}$
T	$\{a, b, \varepsilon\}$	$\{\$\}$

Fill these entries in LL(1) parsing table

	a	b	c	d	f	$\$$
S			$S \rightarrow Rf$	$S \rightarrow daT$	$S \rightarrow Rf$	
T	$T \rightarrow aS$	$T \rightarrow baT$	$T \rightarrow \varepsilon$		$T \rightarrow \varepsilon$	$T \rightarrow \varepsilon$
R			$R \rightarrow caTR$		$R \rightarrow \varepsilon$	

Hence, the correct option is (A).

2.19 (C)

Given : Statements

S_1 : Every SLR (1) parser parses unambiguous grammars but some unambiguous grammars were not parsed by SLR (1).

S_2 : For any context free grammar, there is a parser that takes at most $O(n^3)$ time to parse a string of length *n*.

Key Point

All unambiguous grammars might not be parsed by SLR (1) parser.

CYK algorithm will take $O(n^3)$ time to parse the given string.

Hence, the correct option is (C).

2.20 8

Given :

Grammar is

$$S' \rightarrow S$$
$$S \rightarrow S\#cS$$
$$S \rightarrow SS$$
$$S \rightarrow S@$$
$$S \rightarrow <S>$$
$$S \rightarrow a$$
$$S \rightarrow b$$
$$S \rightarrow c$$

Where $\{\#, @, <, >, a, b, c\}$ are set of terminals.

Let I_0 is the closure $(\{S' \rightarrow .S\})$.

$$\boxed{\begin{array}{c} S' \rightarrow .S \\ S' \rightarrow .S\#cS \\ S' \rightarrow .SS \\ S' \rightarrow .S@ \\ S' \rightarrow .<S> \\ S \rightarrow .a \\ S \rightarrow .b \\ S \rightarrow .c \end{array}} \xrightarrow{<} \boxed{\begin{array}{c} S \rightarrow <.S> \\ S \rightarrow .S\#cS \\ S \rightarrow .SS \\ S \rightarrow .S@ \\ S \rightarrow .<S> \\ S \rightarrow .a \\ S \rightarrow .b \\ S \rightarrow .c \end{array}} \xrightarrow{<} \boxed{\begin{array}{c} S \rightarrow <.S> \\ S \rightarrow .S\#cS \\ S \rightarrow .SS \\ S \rightarrow .S@ \\ S \rightarrow .<S> \\ S \rightarrow .a \\ S \rightarrow .b \\ S \rightarrow .c \end{array}}$$

I_0 GOTO(GOTO $(I_0, <), <$) contains 8 items.

Hence, the correct answer is 8.

3 Syntax Directed Translation

Practice Questions

2014 IIT Kharagpur

3.1 One of the purpose of using intermediate code in compilers is to

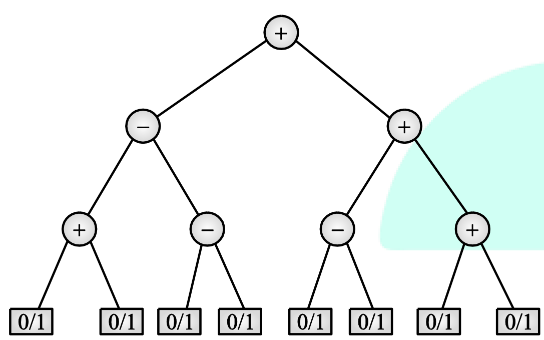
(A) Make parsing and semantic analysis simpler.

(B) Improve error recovery and error reporting.

(C) Increase the chances of reusing the machine-independent code optimizer in other compilers.

(D) Improve the register allocation.

3.2 Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ___.



2016 IISc Bangalore

3.3 Consider the following Syntax Directed Translation Scheme (SDTS), with non-terminals {S,A} and terminals {a,b}.

$$S \to aA \qquad \{\text{print } 1\}$$

$$S \to a \qquad \{\text{print } 2\}$$

$$A \to Sb \qquad \{\text{print } 3\}$$

Using the above SDTS, the output printed by a bottom up parser, for the input aab is :

(A) 1 3 2 (B) 2 2 3

(C) 2 3 1 (D) Syntax error

3.4 The attributes of three arithmetic operators in some programming language are given below.

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
−	Medium	Right	Binary
*	Low	Left	Binary

The value of the expression 2 – 5 + 1 – 7 * 3 in this language is ___________ ?

2018 IIT Guwahati

3.5 Which one of the following statements is FALSE?

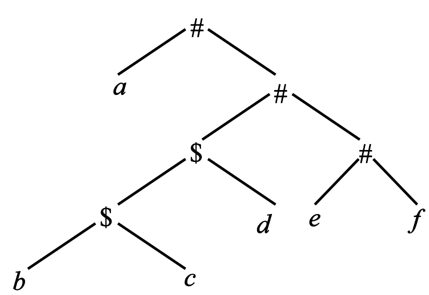
(A) Context-free grammar can be used to specify both lexical and syntax rules.

(B) Type checking is done before parsing.

(C) High-level language programs can be translated to different Intermediate Representations.

(D) Arguments to a function can be passed using the program stack.

3.6 Consider the following parse tree for the expression a # b $ c $ d # e # f, involving two binary operators $ and #.



Which one of the following is correct for the given parse tree?

(A) $ has higher precedence and is left associative; # is right associative

(B) # has higher precedence and is left associative; $ is right associative

(C) \$ has higher precedence and is left associative; # is left associative

(D) # has higher precedence and is right associative; \$ is left associative

2019 **IIT Madras**

3.7 Consider the following grammar and the semantic actions to support the inherited type declaration attributes. Let X_1, X_2, X_3, X_4, X_5, and X_6 be the placeholders for the non-terminals D, T, L or L_1 in the following table :

Production rule	Semantic action
$\mathrm{D} \to \mathrm{T\ L}$	$X_1 \cdot \text{type} = X_2 \cdot \text{type}$
$\mathrm{T} \to \text{int}$	$\mathrm{T} \cdot \text{type} = \text{int}$
$\mathrm{T} \to \text{float}$	$\mathrm{T} \cdot \text{type} = \text{float}$
$\mathrm{L} \to \mathrm{L_1}, \text{id}$	$X_3 \cdot \text{type} = X_4 \cdot \text{type}$ $\text{addType(id.entry, } X_5 \cdot \text{type)}$
$\mathrm{L} \to \text{id}$	$\text{addType(id.entry, } X_6 \cdot \text{type)}$

Which one of the following are the appropriate choices for X_1, X_2, X_3 and X_4 ?

(A) $X_1 = L,\ X_2 = T,\ X_3 = L_1,\ X_4 = L$

(B) $X_1 = T,\ X_2 = L,\ X_3 = L_1,\ X_4 = T$

(C) $X_1 = L,\ X_2 = L,\ X_3 = L_1,\ X_4 = T$

(D) $X_1 = T,\ X_2 = L,\ X_3 = T,\ X_4 = L_1$

2021 **IIT Bombay**

3.8 Consider the following grammar (that admits a series of declarations, followed by expressions) and the associated syntax directed translation (SDT) actions, given as pseudo-code

$P \to D * E *$

$D \to$ int ID {record that ID. lexeme is of type int}

$D \to$ bool ID {record that ID. Lexeme is of type bool}

$E \to E_1 + E_2$ {check that E_1.type $= E_2$.type $=$ int; set E.type := int}

$E \to !E_1$ {check that E_1.type = bool; set E.type := bool};

$E \to$ ID {set E.type := int}

With respect to the above grammar, which one of the following choices is correct?

(A) The actions can be used to correctly type-check any syntactically correct program.

(B) The actions can be used to type-check syntactically correct integer variable declarations and integer expressions.

(C) The actions can be used to type-check syntactically correct Boolean variable declarations and Boolean expressions.

(D) The actions will lead to an infinite loop.

3.9 Consider the following ANSI C program

```
int main () {
        Integer x;
        return 0;
}
```

Which one of the following phases in a seven-phase C compiler will throw an error?

(A) Lexical analyzer

(B) Syntax analyzer

(C) Semantic analyzer

(D) Machine dependent optimizer

Solutions

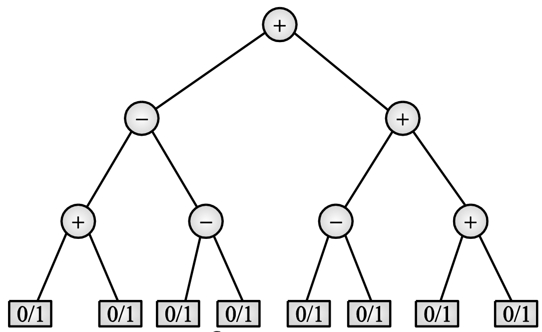
3.1 **(C)**

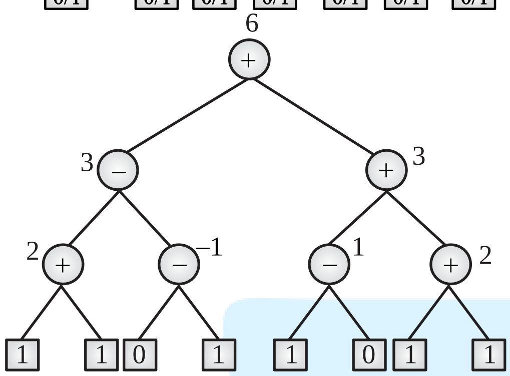
The code is converted into intermediate code, which is platform independent. It can be optimized using code optimizers. It increase the changes of reusing the machine independent code optimizer in other compilers.

Hence, the correct option is (C).

3.2 **6**

Given :





Hence, the correct answer is 6.

3.3 **(C)**

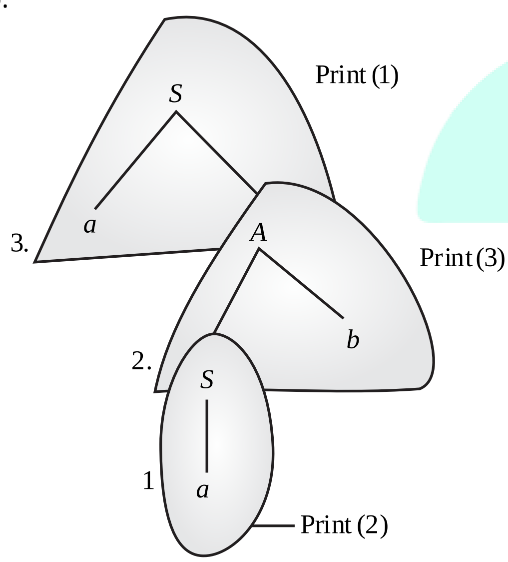
Given :

$S \to aA$ {print 1}

$S \to a$ {print 2}

$A \to Sb$ {print 3}

Given Input string is 'aab' and starting symbol is S. Bottom up parser builds the parse tree from bottom to up.



The output will be 231

Hence, the correct option is (C).

3.4 **9**

Given :

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
–	Medium	Right	Binary
*	Low	Left	Binary

+ Operator has higher precedence so

+ Operation will execute first

$= 2-5+1-7*3$

$= 2-6-7*3$

$= 2-(-1)*3$ (– operator is right associative and higher precedence than k)

$= 3*3$

$= 9$ (* operator has lowest precedence).

Hence, the correct answer is 9.

3.5 **(B)**

In compiler type checking is done after parsing.

Key Point

Parsing is done at syntax analysis phase. Type checking is done at semantic analysis phase.

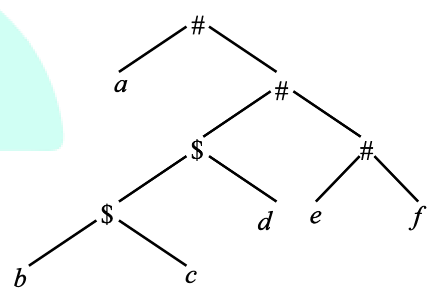
Syntax analysis phase is done before semantic analysis phase.

Hence, the correct option is (B).

3.6 **(A)**

Given :

Pares tree for the expression a # b $ c $ d # e # f given below :



$ has higher precedence than # and is left associative.

has lower precedence than $ and is right associative.

Hence, the correct option is (A).

3.7 **(A)**

Given :

Production rule	Semantic action
$D \to T\ L$	$X_1 \cdot \text{type} = X_2 \cdot \text{type}$
$T \to \text{int}$	$T \cdot \text{type} = \text{int}$

$T \rightarrow float$	$T \cdot type = float$
$L \rightarrow L_1, id$	$X_3 \cdot type = X_4 \cdot type$ $addType(id.entry, X_5 \cdot type)$
$L \rightarrow id$	$addType(id.entry, X_6 \cdot type)$

A node in a parse tree can inherit an attribute either from its parent or siblings. This means for a production.

$$S \rightarrow AB$$

A can inherit values from either S or B and similarly B can inherit values from either S or A.

In the given productions for

$L \rightarrow L_1$, id

L_1 can inherit from *L* or, or id with only *L* being a non-terminal. So this means X_3 must be L_1 and X_4 must be *L* as X_i is a placeholder for non-terminals.

Hence, the correct option is (A).

3.8 **(B)**

Given :

Following grammar and the associated Syntax Directed Translation (SDT) actions :-

$P \rightarrow D * E *$

$D \rightarrow$ int ID {record that ID. lexeme is of type int}

$D \rightarrow$ bool ID {record that ID. Lexeme is of type bool}

$E \rightarrow E_1 + E_2$ {check that $E_1.type = E_2.type = int;$ set $E.type := int$}

$E \rightarrow !E_1$ {check that $E_1.type = bool;$ set $E.type := bool$};

$E \rightarrow$ ID {set $E.type := int$}

The actions can used to type-check syntactically correct integer variable declarations and integer expressions.

Hence, the correct option is (B).

3.9 **(C)**

Given :

Following ANSI C program :

```
int main ( ) {
    Integer X;
    return 0;
}
```

In the above code, Integer will be treated as identifier and leading to undeclared variable, so it will throw an error by semantic analyzer.

Hence, the correct option is (C).

4 Code Generation and Optimization

Practice Questions

2014 IIT Kharagpur

4.1 Which one of the following is FALSE?

(A) A basic block is a sequence of instructions where control enters the sequence at the beginning and exists at the end.

(B) Available expression analysis can be used for common sub-expression elimination.

(C) Live variable analysis can be used for dead code elimination.

(D) $x = 4*5 \Rightarrow x = 20$ is an example of common sub-expression elimination.

4.2 Which one of the following is NOT performed during compilation?

(A) Dynamic memory allocation

(B) Type checking

(C) Symbol table management

(D) Inline expansion

4.3 For a C program accessing $X[i][j][k]$, the following intermediate code is generated by a complier. Assume that the size of an integer is 32 bits and the size of character is 8 bits.

$$t0 = i*1024$$
$$t1 = j*32$$
$$t2 = k*4$$
$$t3 = t1 + t0$$
$$t4 = t3 + t2$$
$$t5 = X[t4]$$

Which one of the following statements about the source code for the C program is CORRECT?

(A) X is declared as "int $X[32][32][8]$"

(B) X is declared as "int $X[4][1024][32]$"

(C) X is declared as "$char\ X[4][32][8]$" (D) X is declared as "$char\ X[32][16][2]$"

4.4 Which of the following statements are CORRECT?

1. Static allocation of all data areas by a compiler makes it impossible to implement recursion.
2. Automatic garbage collection is essential to implement recursion.
3. Dynamic allocation of activation records is essential to implement recursion.
4. Both heap and stack are essential to implement recursion.

(A) 1 and 2 only (B) 2 and 3 only

(C) 3 and 4 only (D) 1 and 3 only

4.5 Consider the basic block given below.

$$a = b + c$$
$$c = a + d$$
$$d = b + c$$
$$e = d - b$$
$$a = e + b$$

The minimum number of nodes and edges present in the DAG representation of the above basic block respectively are

(A) 6 and 6 (B) 8 and 10

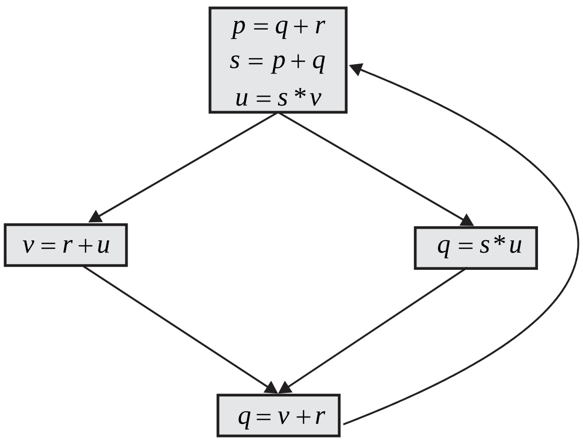
(C) 9 and 12 (D) 4 and 4

4.6 The minimum number of arithmetic operations required to evaluate the polynomial $P(x) = x^5 + 4x^3 + 6x + 5$ for a given value of *x*, using only one temporary variable is ______.

2015 IIT Kanpur

4.7 A variable *x* is said to be live at a statement S_i in a program if the following three conditions hold simultaneously:

1. There exists a statement S_j that uses x
2. There is a path from S_i to S_j in the flow graph corresponding to the program
3. The path has no intervening assignment to x including at S_i and S_j



The variables which are live both at the statement in basic block 2 and at the statement in basic block 3 of the above control flow graph are

(A) p, s, u (B) r, s, u

(C) r, u (D) q, v

4.8 The least number of temporary variables required to create a three-address code in static single assignment form for the expression q + r/3 + s – t * 5 + u * v/w is ______.

4.9 In the context of abstract-syntax-tree (AST) and control-flow-graph (CFG), which one of the following is TRUE?

(A) In both AST and CFG, let node N_2 be the successor of node N_1. In the input program, the code corresponding to N_2 is present after the code corresponding to N_1

(B) For any input program, neither AST nor CFG will contain a cycle

(C) The maximum number of successors of a node in an AST and a CFG depends on the input program

(D) Each node in AST and CFG corresponds to at most one statement in the input program

4.10 Consider the intermediate code given below.

(1) $i = 1$

(2) $j = 1$

(3) $t_1 = 5 * i$

(4) $t_2 = t_1 + j$

(5) $t_3 = 4 * t_2$

(6) $t_4 = t_3$

(7) $a[t_4] = -1$

(8) $j = j + 1$

(9) if $j <= 5$ goto (3)

(10) $i = i + 1$

(11) If $i < 5$ goto (2)

The number of nodes and edges in the control-flow-graph constructed for the above code, respectively, are

(A) 5 and 7 (B) 6 and 7

(C) 5 and 5 (D) 7 and 8

2016 IISc Bangalore

4.11 A student wrote two context-free grammars G1 and G2 for generating a single C-Like array declaration. The dimension of the array is at least one. For example,

$\text{int } a[10][3];$

The grammars use D as the start symbol, and use six terminal symbols int; id[ ] num.

Grammar G1	Grammar G2
$D \rightarrow \text{int } L;$	$D \rightarrow \text{int } L;$
$L \rightarrow id[E$	$L \rightarrow id\ E$
$E \rightarrow num]$	$E \rightarrow E[num]$
$E \rightarrow num][E$	$E \rightarrow [num]$

Which of the grammar correctly generate the declaration mentioned above?

(A) Both G1 and G2

(B) Only G1

(C) Only G2

(D) Neither G1 nor G2

4.12 Consider the following code segment.

x = u - t;

y = x * v;

x = y + w;

y = t - z;

y = x * y;

The minimum number of total variables required to convert the above code segment to static single assignment form is ___________.

2017 IIT Roorkee

4.13 Consider the following intermediate program in three address code

$p = a - b$

$q = p * c$

$p = u * v$
$q = p + q$

Which one of the following corresponds to a static single assignment form of the above code?

(A) $p_1 = a - b$
$q_1 = p_1 * c$
$p_1 = u * v$
$q_1 = p_1 + q_1$

(B) $p_3 = a - b$
$q_4 = p_3 * c$
$p_4 = u * v$
$q_5 = p_4 + q_4$

(C) $p_1 = a - b$
$q_1 = p_2 * c$
$p_3 = u * v$
$q_2 = p_4 + q_3$

(D) $p_1 = a - b$
$q_1 = p * c$
$p_2 = u * v$
$q_2 = p + q$

4.14 Consider the following grammar :

$stmt \rightarrow if$ expr then expr else expr;
stmt| 0
expr→ term relop term |term
term →id |number
id → $a|b|c$
Number →[0-9]

Where relop is relational operator (e.g., <,>,..), 0 refers to the empty statement, and if, then, else are terminals. Consider a program P following the above grammar containing then if terminals. The number of control flow paths in P is ______.

For example the program

If e_1 then e_2 else e_3 has 2 control flow paths, $e_1 \rightarrow e_2$ and $e_1 \rightarrow e_3$

2021 IIT Bombay

4.15 Consider the following C code segment:

$$a = b + c;$$
$$e = a + 1;$$
$$d = b + c;$$
$$f = d + 1;$$
$$g = e + f;$$

In a compiler, this code segment is represented internally as a directed acyclic graph (DAG). The number of nodes in the DAG is ______.

4.16 In the context of compilers, which of the following is/are NOT an intermediate representation of the source program?

(A) Three address code
(B) Abstract Syntax Tree (AST)
(C) Control Flow Graph (CFG)
(D) Symbol table

4.17 Consider the following ANSI C code segment:

```
z = x + 3 + y → f1 + y → f2;
for (i = 0; i < 200; i = i + 2)
{
    if (z > i)
    {
            p = p + x + 3;
            q = q + y → f1;
    }
    else
    {
            p = p + y → f2;
            q = q + x + 3;
    }
}
```

Assume that the variable y points to a struct (allocated on the heap) containing two fields f1 and f2, and the local variables x, y, z, p, q, and i are allotted registers. Common sub-expression elimination (CSE) optimization is applied on the code. The number of addition and the dereference operations (of the form y → f1 or y → f2) in the optimized code, respectively, are:

(A) 403 and 102
(B) 203 and 2
(C) 303 and 102
(D) 303 and 2

4.18 For a statement S in a program, in the context of liveness analysis, the following sets are defined:

USE(S) : the set of variables used in S

IN(S) : the set of variables that are live at the entry of S

OUT(S) : the set of variables that are live at the exit of S

Consider a basic block that consists of two statements, S_1 followed by S_2. Which one of the following statements is correct?

(A) $\text{OUT}(S_1) = \text{IN}(S_2)$

(B) $\text{OUT}(S_1) = \text{IN}(S_1) \cup \text{USE}(S_1)$

(C) $\text{OUT}(S_1) = \text{IN}(S_2) \cup \text{OUT}(S_2)$ (D) $\text{OUT}(S_1) = \text{USE}(S_1) \cup \text{IN}(S_2)$

Solutions

4.1 (D)

Consider each option one by one :

Option (A) : A basic block is a sequence of instructions where control enters the sequence at the beginning and exits at the end is True.

Option (B) : Available expression analysis can be used for common subexpression elimination is true.

Option (C) : Live variable analysis can be used for dead code elimination is True.

Option (D) : $x = 4*5 \Rightarrow x = 20$ is an example of constant folding but not for common subexpression elimination.

Hence, the correct option is (D).

4.2 (A)

Dynamic memory allocation is performed during run time not during compile time.

Hence, the correct option is (A).

4.3 (A)

Given :

$$t0 = i*1024$$
$$t1 = j*32$$
$$t2 = k*4$$
$$t3 = t1 + t0$$
$$t4 = t3 + t2$$
$$t5 = X[t4]$$

$$X[t_4] = X[t_3 + t_2]$$
$$X[t_4] = X[t_1 + t_0 + t_2]$$
$$X[t_4] = X[i \times 1024 + j \times 32 + k \times 4]$$
$$X[t_4] = X[i \times 1024 + j \times 32 + k \times 4] \quad \text{... (1)}$$

To access $X[i][j][k]$:

$$= X[(i \times 32 \times 8 + j \times 8 + k) \times 4] \quad \text{... (2)}$$

$\therefore$ Comparing (1) & (2) are equivalent.

Hence, the correct option is (A).

4.4 (D)

(1) Recursion cannot be implemented using static allocation.

(2) Automatic garbage collection is not essential to implement recursion.

(3) Dynamic allocation of activation records is essential to implement recursion.

(4) Heap is not needed for function calls. It is generally used for dynamic memory allocation by user (or programmer).

Hence, the correct option is (D).

4.5 (A)

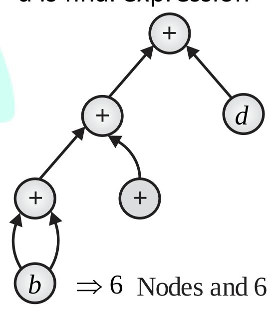
Given :

Basic block

$$a = b + c$$
$$c = a + d$$
$$d = b + c$$
$$e = d - b$$
$$a = e + b$$

$$\left.\begin{aligned} a &= b + c \\ c &= a + d \\ d &= b + c \\ e &= d - b \\ a &= e + b \end{aligned}\right\} \begin{aligned} &\Rightarrow a = e + b \\ &\Rightarrow a = d - b - b = d \\ &\Rightarrow a = b + c \\ &\Rightarrow a = b + a + d \\ &\Rightarrow a = b + b + c + d \end{aligned}$$

$\therefore$ b + b + c + d is final expression



$\Rightarrow 6$ Nodes and 6 edges

Hence, the correct option is (A).

4.6 7

Given :

$$P(x) = x^5 + 4x^3 + 6x + 5$$
$$P(x) = x(x^4 + 4x^2 + 6) + 5$$
$$P(x) = x(x(x^3 + 4x) + 6) + 5$$
$$P(x) = x(x(x(x^2 + 4)) + 6) + 5$$
$$P(x) = x\Big(x\big(x\big(x(x) + 4\big)\big) + 6\Big) + 5$$

Let T be a temporary variable to store intermediate results.

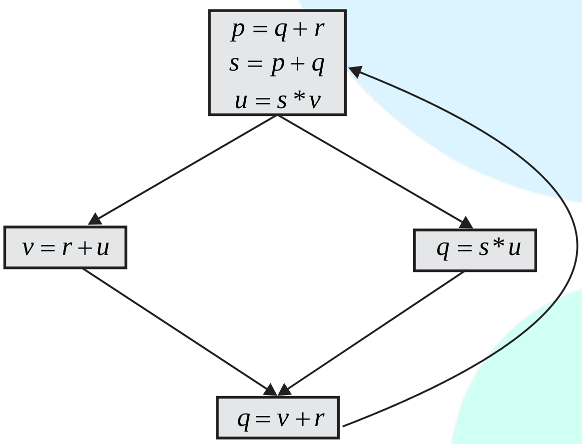
1. $T = x * x$
2. $T = T + 4$
3. $T = x * T$
4. $T = x * T$
5. $T = T + 6$
6. $T = x * T$
7. $T = T + 5$

Thus, we need 7 operations if we are to use only one temporary variable.

4.7 (C)

Given :

1. There exists a statement S_j that uses x
2. There is a path from S_i to S_j in the flow graph corresponding to the program
3. The path has no intervening assignment to x including at S_i and S_j



Live variable analysis is useful in compiler to find variables in each program that may be needed in future.

p and *s* are assigned to in 1 and there is no intermediate use of them before that. Hence, *p* and *s* are not live in both 2 and 3.

q is assigned to in 4 and hence is not live in both 2 and 3.

v is live at 3 but not at 2.

u is live at 3 and also at 2 if we consider a path of length 0 from 2–2.

So, *r* and *u* is live.

Key Point

A variable is live if it holds a value that may be needed in future. In other words, it is used in future before any new assignment.

Hence, the correct option is (C).

4.8 8

Given :

Expression : q + r/3 + s – t * 5 + u * v/w

$$t_1 = r/3$$
$$t_2 = q + t_1$$
$$t_3 = t_2 + s$$
$$t_4 = t \times 5$$
$$t_5 = t_3 - t_4$$
$$t_6 = u \times v$$
$$t_7 = t_6 / w$$
$$t_8 = t_5 + t_7$$

$\therefore$ 8 temporary variables are required.

Hence, the correct answer is 8.

4.9 (C)

(A) In CFG, code of N_2N_2 may be present before N_1N_1 when there is a loop or goto.

(B) CFG contains cycle when input program has loop.

(C) Successors in AST and CFG depend on Input program.

(D) In CFG a single rode may belong to a block of statements.

Hence, the correct option is (C).

4.10 (B)

Given :

(1) $i = 1$

(2) $j = 1$

(3) $t_1 = 5 * i$

(4) $t_2 = t_1 + j$

(5) $t_3 = 4 * t_2$

(6) $t_4 = t_3$

(7) $a[t_4] = -1$

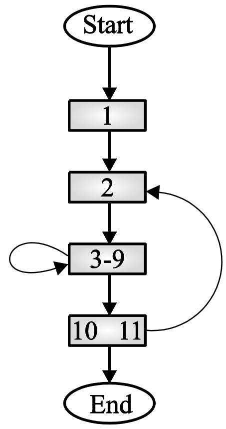
(8) $j = j + 1$

(9) if $j <= 5 \text{ goto } (3)$

(10) $i = i + 1$

(11) If $i < 5 \text{ goto } (2)$

Basic block corresponding to above code is



Number of nodes = 6

Number of edges = 7

Hence, the correct option is (B).

4.11 **(A)**

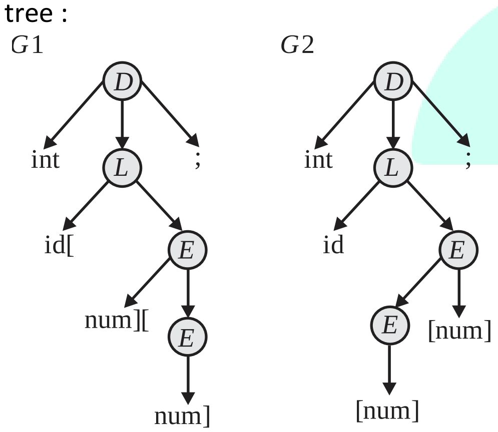
Given :

Grammar

G1	**G2**
$D \to \text{int } L;$	$D \to \text{int } L;$
$L \to id[E$	$L \to id\ E$
$E \to num]$	$E \to E[num]$
$E \to num][E$	$E \to [num]$

Both G1 and G2 can generate int a[10][3]

Parse tree :



Hence, the correct option is (A).

4.12 **10**

Given :

x = u – t;

y = x * v;

x = y + w;

y = t – z;

y = x * y;

In Static Single Assignment when we assign the values, the variables to which the value is being assigned should be unique.

T1 = u – t

T2 = T1∗v

T3 = T2+w

T4 = t–z

T5 = t3∗t4

So, the variables are T1, T2, T3, T4, T5, u, t, v, w, z

Hence, we need total 10 variables.

4.13 **(B)**

Given :

$p = a - b$

$q = p * c$

$p = u * v$

$q = p + q$

The correct code is

$p_3 = a - b$

$q_4 = p_3 \times c$

$q_4 = u \times v$

$q_5 = p_4 + q_4$

Key Point

According to static single assignment

1. A variable cannot be used more than once in the LHS.
2. A variable should be initialized almost once.

Hence, the correct option is (B).

4.14 **1024**

Given :

$stmt \to if$ expr then expr else expr; stmt| 0

expr $\to$ term relop term |term

term $\to$ id |number

id $\to a|b|c$

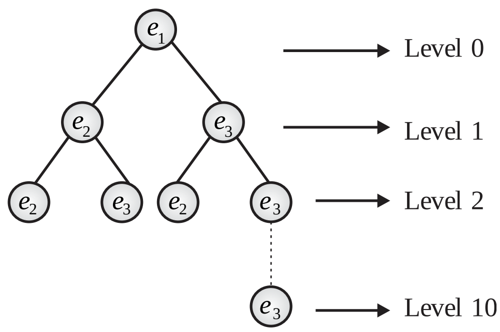
Number $\to$ [0-9]

If e_1 then e_2 else e_3

So, it has 2 paths

$e_1 \to e_2$ and $e_1 \to e_3$

For every statement, there will be 2 paths.



Using multiplication law of counting, we get

$$= 2 \times 2 \times 2 \times 2 - 10 \text{ times} = 1024$$

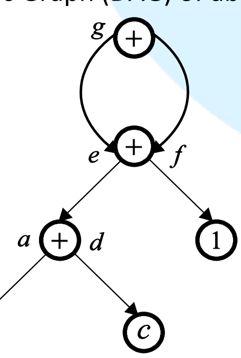
Hence, the correct answer is 1024.

4.15 **6**

Given : C code segment is :

$$a = b + c;$$
$$e = a + 1;$$
$$d = b + c;$$
$$f = d + 1;$$
$$g = e + f;$$

The Directed Acyclic Graph (DAG) of above code is



Key Point

In DAG, Root node and internal nodes are operator and leaf nodes are operand.

Operands are taken only one time, and we can use them multiple times. But for each operation we have to take operator node every time.

Hence, the correct answer is 6.

4.16 **(D)**

Three address code, Abstract syntax tree (AST), Control Flow Graph (CFG), post fix evaluation are the intermediate representation of the source program.

Symbol table is not an intermediate representation of program.

Key Point

All the phase of compiler uses symbol table for various purposes.

Hence, the correct option is (D).

4.17 **(D)**

Given :

```
z = x + 3 + y → f1 + y → f2;
for (i = 0; i < 200; i = i + 2)
{
  if (z > i)
  {
    p = p + x + 3;
    q = q + y → f1;
  }
  else
  {
    p = p + y → f2;
    q = q + x + 3;
  }
}
```

Let, $a = y \rightarrow f1$

$b = y \rightarrow f2$

$c = x + 3$

$z = a + b + c$

for $(i = 0; i < 200; i = i + 2)$

{ if $(z > i)$

{ $p = p + c;$

$q = q + a;$

}

else

{ $p = p + b;$

$q = q + c;$

}

}

Total 300 additions (for each iteration 3 addition operations $(p + c, q + a)$ or $(p + b, q + c), i + z)$

$c = x + 3$ ... (i)

$z = c + a + b$ ... (ii)

303 addition operations and 2 dereference operations (for $y \to f1, y \to f2$).

Hence, the correct option is (D).

4.18 (A)

Given :

Following sets are defined :

USE (*S*) : the set of variables used in *S*

IN (*S*) : the set of variables that are live at the entry of *S*

OUT (*S*) : the set of variables that are live at the exit of *S*

IN [EXIT] $= \phi;$

For (each basic block *B* other than EXIT)

IN [*B*] $= \phi;$

While (changes to any IN occur)

for (each basic block *B* other than EXIT)

{ OUT [*B*] $= U_{s_{asuccesorofB}}$ IN [*S*];

IN [*B*] = $\text{use}_B \cup$ (OUT [*B*] $- def_B$);

Hence, the correct option is (A).

CHAPTER 6 | Computer Network

Marks Distribution of Computer Network in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	3	8
2004*	3	4	11
2005*	5	2	9
2006*	1	5	11
2007*	2	6	14
2008*	1	4	9
2009	-	5	10
2010	2	3	8
2011	2	2	6
2012	3	3	9
2013	3	2	7
2014 Set-1	2	3	8
2014 Set-2	3	2	7
2014 Set-3	3	3	9

*** CS and IT combined**

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	4	2	8
2015 Set-2	2	3	8
2015 Set-3	2	3	8
2016 Set-1	2	4	10
2016 Set-2	3	4	11
2017 Set-1	2	3	8
2017 Set-2	3	2	7
2018	3	2	7
2019	2	4	10
2020	2	2	6
2021 Set-1	2	3	8
2021 Set-2	2	3	8

Syllabus : Computer Network

Concept of layering: OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit-switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, Email.

Contents : Computer Network

1 Concepts of Layering and LAN Technologies

Practice Questions

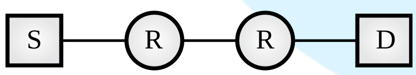
2013 IIT Bombay

1.1 Determine the maximum length of the cable (in km) for transmitting data at a rate of 500 Mbps in an Ethernet LAN with frames of size 10,000 bits. Assume the signal speed in the cable to be 2,00,000 km/s.

(A) 1 (B) 2

(C) 2.5 (D) 5

1.2 Assume that source S and destination D are connected through two intermediate routers labeled R. Determine how many times each packet has to visit the network layer and the data link layer during a transmission from S to D.



(A) Network layer – 4 times and Data link layer – 4 times

(B) Network layer – 4 times and Data link layer – 3 times

(C) Network layer – 4 times and Data link layer – 6 times

(D) Network layer – 2 times and Data link layer – 6 times

2014 IIT Kharagpur

1.3 In the following pairs of OSI protocol layer/sub-layer and its functionality, the INCORRECT pair is

(A) Network layer and Routing

(B) Data Link Layer and Bit synchronization

(C) Transport layer and End-to-end process communication

(D) Medium Access Control sub-layer and Channel sharing

2016 IISc Bangalore

1.4 In an Ethernet local area network, which one of the following statements is TRUE?

(A) A station stops to sense the channel once it starts transmitting a frame.

(B) The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.

(C) A station continues to transmit the packet even after the collision is detected.

(D) The exponential back off mechanism reduces the probability of collision on retransmissions.

2019 IIT Madras

1.5 Consider that 15 machines need to be connected in a LAN using 8-port Ethernet switches. Assume that these switches do not have any separate uplink ports. The minimum number of switches needed is _______.

Solutions

1.1 (B)

Given :

The rate of data transmission = 500 Mbps

Frame size = 10,000 bits

Signal speed in cable = 2,00,000 km/s

Transmission delay $\geq 2\times$Propagation delay

$$\frac{10,000\,\text{bits}}{500\,\text{Mbps}} \geq 2\times\frac{d}{2\times10^5\,\text{km per sec}}$$

$\Rightarrow \quad d \leq 2\,\text{km}$

$\Rightarrow \quad d = 2\,\text{km}$

Hence, the correct option is (B).

Key Point

In Ethernet LAN, following relation holds :

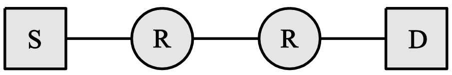
Transmission time $\geq 2\times$ propagation time

$\Rightarrow \quad T_t \geq 2\times T_p$

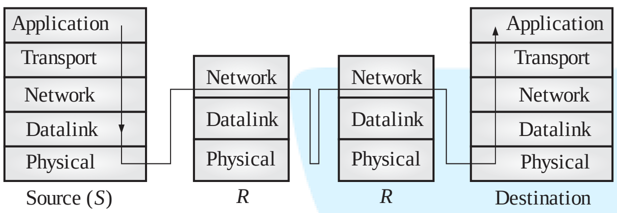
1.2 **(C)**

Given :

Source S and destination D are connected through two intermediate routers labeled R.



From above source and destination the representation of layers are as follows.



It is clear from the above figure that network layer is visited four times and data link layer is visited six times .

Hence, the correct option is (C).

1.3 **(B)**

Consider each option one by one :

Option (A) : Network layer and Routing

It is true. since, Network layer does Routing.

Option (B) : Data Link Layer and Bit synchronization.

It is false. Because bit synchronization is not provided by Data link layer.

Option (C) : Transport layer and End-to-end process communication.

It is true. since, Transport layer provides end-to-end process communication.

Option (D) : Medium Access Control sub-layer and Channel sharing.

It is true. Medium access control sub-layer of data link layer provides channel sharing.

Key Point

Bit synchronization is provided by physical layer.

Hence, the correct option is (B).

1.4 **(D)**

Consider each option one by one :

Option (A) : A station stops to sense the channel once it starts transmitting a frame. False, because if it stops sensing channel then how it sense if collision occur.

Option (B) : The purpose of the jamming signal is to pad the frames that are smaller than the minimum frame size.

False, because the purpose of jamming signal is to tell all other station that collision has occurred.

Option (C) : A station continues to transmit the packet even after the collision is detected.

False, station stops transmission as soon as collision has detected.

Option (D) : The exponential back off mechanism reduces the probability of collision on retransmissions.

True, probability of collision on retransmission reduces exponentially.

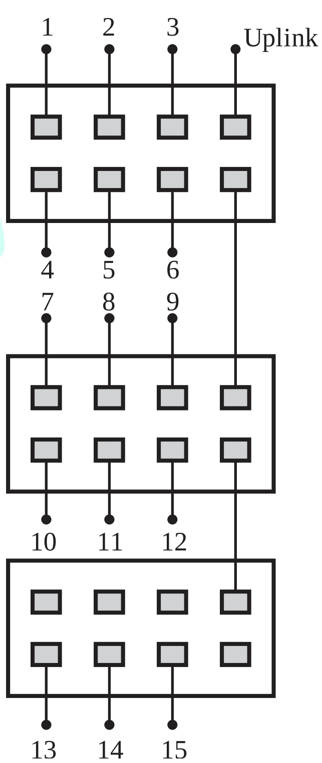
Hence, the correct option is (D).

1.5 **3**

Given :

There are 15 machines connected in a LAN by using 8 port Ethernet switches,

Therefore,



So, from above figure the minimum number of switches needed is 3.

Hence, the correct answer is 3.

2 Data Link Layer

Practice Questions

2014 IIT Kharagpur

2.1 A bit-stuffing based framing protocol uses an 8-bit delimiter pattern of 01111110. If the output bit-string after stuffing is 01111100101, then the input bit-string is :

(A) 0111110100 (B) 0111110101

(C) 0111111101 (D) 0111111111

2.2 Consider a selective repeat sliding window protocol that uses a frame size of 1 KB to send data on a 1.5 Mbps link with a one-way latency of 50 msec. To achieve a link utilization of 60%, the minimum number of bits required to represent the sequence number field is ________.

2015 IIT Kanpur

2.3 A link has transmission speed of 10^6 bits/sec. It uses data packets of size 1000 bytes each. Assume that the acknowledgment has negligible transmission delay and that its propagation delay is the same as the data propagation delay. Also, assume that the processing delays at nodes are negligible. The efficiency of the stop-and-wait protocol in this setup is exactly 25%. The value of the one way propagation delay (in milliseconds) is ________.

2.4 Consider a network connecting two systems located 8000 Km apart. The bandwidth of the network is 500×10^6 bits per second. The propagation speed of the media is 4×10^6 meters per second. It needs to design a Go-Back-N sliding window protocol for this network. The average packet size is 10^7 bits. The network is to be used to its full capacity. Assume that processing delays at nodes are negligible. Then, the minimum size in bits of the sequence number field has to be ______.

2.5 Suppose that the stop-and-wait protocol is used on a link with a bit rate of 64 kilobits per second and 20 milliseconds propagation delay. Assume that the transmission time for the acknowledgment and the processing time at nodes are negligible. Then the minimum frame size in bytes to achieve a link utilization of at least 50 % is ______.

2.6 Consider a LAN with four nodes S_1, S_2, S_3 and S_4. Time is divided into fixed-size slots, and a node can begin its transmission only at the beginning of a slot. A collision is said to have occurred if more than one node transmit in the same slot. The probability of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are $0.1, 0.2, 0.3$ and 0.4 respectively. The probability of sending a frame in the first slot without any collision by any of these four stations is _______.

2.7 Consider a CSMA/CD network that transmits data at a rate of 100 Mbps (10^8 bits per second) over a 1 km (kilometer) cable with no repeaters. If the minimum frame size required for this network is 1250 bytes, what is the signal speed (km/sec) in the cable?

(A) 8000 (B) 10000

(C) 16000 (D) 20000

2016 IISc Bangalore

2.8 Consider a 128×10^3 bits/second satellite communication link with one way propagation delay of 150 milliseconds. Selective retransmission (repeat) protocol is used on this link to send data with a frame size of 1 kilobyte. Neglect the transmission time of acknowledgement. The minimum number of bits required for the sequence number field to achieve 100% utilization is ______.

2.9 Sender uses the Stop-and-Wait ARQ protocol for reliable transmission of frames. Frames are of size 1000 bytes and the transmission rate at the sender is 80 Kbps (1 Kbps = 1000 bits/second). Size of an acknowledgment is 100 bytes and the transmission rate at the receiver is 8 Kbps. The one-way propagation delay is 100 milliseconds.

Assuming no frame is lost, the sender throughput is ______ bytes/second.

2.10 A network has a data transmission bandwidth of 20×10^6 bits per second. It uses CSMA/CD in the MAC layer. The maximum signal propagation time from one node to another node is 40 microseconds. The minimum size of a frame in the network is _______ bytes.

2017 IIT Roorkee

2.11 A computer network uses polynomials over GF(2) for error checking with 8 bits as information bits and uses $x^3 + x + 1$ as the generator polynomial to generate the check bits. In this network, the message 01011011 is transmitted as :

(A) 01011011010 (B) 01011011011

(C) 01011011101 (D) 01011011100

2.12 Consider two hosts X and Y, connected by a single direct link of rate 10^6 bits/sec. The distance between the two hosts is 10,000 km and the propagation speed along the link is 2 $\times 10^8$ m/sec. Host X sends a file of 50,000 bytes as one large message to host Y continuously. Let the transmission and propagation delays be p milliseconds and q milliseconds respectively. Then the value of p and q are

(A) p = 50 and q = 100

(B) p = 50 and q = 400

(C) p = 100 and q = 50

(D) p = 400 and q = 50

2.13 The values of parameters for the Stop-and-Wait ARQ protocol are as given below :

- Bit rate of the transmission channel = 1 Mbps.
- Propagation delay from sender to receiver = 0.75 ms.
- Time to process a frame = 0.25 ms.
- Number of bytes in the information frame = 1980.
- Number of bytes in the acknowledge frame = 20.
- Number of overhead bytes in the information frame = 20.

Assume there are no transmission errors. Then, the transmission efficiency (expressed in percentage) of the Stop-and-Wait ARQ protocol for the above parameters is ______ (correct to 2 decimal places).

2018 IIT Guwahati

2.14 Consider a simple communication system where multiple nodes are connected by a shared broadcast medium (like Ethernet or wireless). The nodes in the system use the following carrier-sense based medium access protocol. A node that receives a packet to transmit will carrier-sense the medium for 5 units of time. If the node does not detect any other transmission, it starts transmitting its packet in the next time unit. If the node detects another transmission, it waits until this other transmission finishes, and then begins to carrier-sense for 5 time units again. Once they start to transmit, nodes do not perform any collision detection and continue transmission even if a collision occurs. All transmissions last for 20 units of time. Assume that the transmission signal travels at the speed of 10 meters per unit time in the medium.

Assume that the system has two nodes P and Q, located at a distance d meters from each other. P start transmitting a packet at time t = 0 after successfully completing its carrier-sense phase. Node Q has a packet to transmit at time t = 0 and begins to carrier-sense the medium.

The maximum distance d (in meters, rounded to the closest integer) that allows Q to successfully avoid a collision between its proposed transmission and P's ongoing transmission is _______.

2021 IIT Bombay

2.15 Assume that a 12-bit Hamming code word consisting of 8-bit data and 4 check bits is

$$d_8 d_7 d_6 d_5 c_8 d_4 d_4 d_3 d_2 c_4 d_1 c_2 c_1$$

where the data bits and the check bits are given in the following tables :

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_4	c_3	c_2	c_1
y	0	1	0

Which one of the following choices gives the correct values of *x* and *y*?

(A) *x* is 0 and *y* is 0

(B) *x* is 0 and *y* is 1

(C) *x* is 1 and *y* is 0

(D) *x* is 1 and *y* is 1

2.16 Consider the sliding window flow-control protocol operating between a sender and a receiver over a full-duplex error-free link. Assume the following:

- The time taken for processing the data frame by the receiver is negligible.
- The time taken for processing the acknowledgement frame by the sender is negligible.
- The sender has infinite number of frames available for transmission.
- The size of the data frame is 2,000 bits and the size of the acknowledgement frame is 10 bits.
- The link data rate in each direction is 1 Mbps ($=10^6$ bits per second).
- One way propagation delay of the link is 100 milliseconds.

The minimum value of the sender's window size in terms of the number of frames, (rounded to the nearest integer) needed to achieve a link utilization of 50% is ________.

2.17 Consider the cyclic redundancy check (CRC) based error detecting scheme having the generator polynomial X^3+X+1. Suppose the message $m_4m_3m_2m_1m_0=11000$ is to be transmitted. Check bits $c_2c_1c_0$ are appended at end of the message by the transmitter using the above CRC scheme. The transmitted bit string is denoted by $m_4m_3m_2m_1m_0c_2c_1c_0$. The value of the check bit sequence $c_2c_1c_0$ is

(A) 101 (B) 110

(C) 100 (D) 111

2.18 Consider a network using the pure ALOHA medium access control protocol, where each frame is of length 1,000 bits. The channel transmission rate is 1 Mbps (=10^6 bits per second). The aggregate number of transmissions across all the nodes (including new frame transmissions and retransmitted frames due to collisions) is modelled as a Poisson process with a rate of 1,000 frames per second. Throughput is defined as the average number of frames successfully transmitted per second. The throughput of the network (rounded to the nearest integer) is _____.

Solutions

2.1 **(B)**

Given :

Bit stuffing is used in framing.

8-bit delimeter pattern is 0111 1110

The output bit-string after stuffing is 011111100101.

The stuffed bit is $011111\underline{0}0101$

So input bit string must be 0111110101.

Hence, the correct option is (B).

2.2 **5**

Given :

SR protocol is used ,

Where,

Frame size = 1 KB

Bandwidth, $BW=1.5\,\text{Mbps}$

Propagation delay, $T_p=50$ msec

Efficiency = 60 %

Transmission delay

$$T_t=\frac{L}{B}=\frac{(1\times8\times10^3)}{(1.5\times10^6)}=5.33\,\text{msec}$$

Propagation delay, $T_p=50$ msec

$$\text{Efficiency} = \frac{\text{Window size}}{(1+2a)} = 0.6$$

$$\text{Efficiency} = \frac{w_s}{\left(1+\frac{2\times 50}{5.33}\right)} = 0.6$$

So, window size = 11.58

Since, SR protocol is used,

$\Rightarrow$ Available sequence numbers $\geq W_s + W_r$

$\Rightarrow$ Available sequence numbers $\geq 2\times W_s$

$\Rightarrow$ Available sequence numbers ≥ 23.172

So, Available sequence numbers = 24

Sequence Bits required $= [\log_2 24] = 5$

Hence, the correct answer is 5.

2.3 **12**

Given :

Bandwidth, $B = 10^6$ bits/sec

Packet size, L = 1000 bytes

Efficiency = 25 %

$$\text{Transmission time } = T_t = \frac{L}{B}$$

$$= 1000 \text{ bytes}/10^6 \text{ bits/sec} = 8 \sec$$

And, propagation delay $= T_p$

In stop and wait protocol,

$$\text{Efficiency} = \frac{1}{1+2a} \qquad \left[\text{where, } a = \frac{T_p}{T_t}\right]$$

$$\Rightarrow \quad \frac{1}{1+2a} = 25\ \%$$

$$\Rightarrow \quad \frac{1}{1+2a} = \frac{1}{4}$$

$$\Rightarrow \quad a = \frac{3}{2}$$

$$\Rightarrow \quad \frac{T_p}{T_t} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{T_p}{8\,\text{msec}} = \frac{3}{2}$$

$\Rightarrow$ T_p = 12 msec

Hence, the correct answer is 12.

2.4 **8**

Given :

Distance, $d = 8000$ km

Bandwidth, $B = 500\times 10^6$ bps

Speed, $b = 4\times 10^6$ m/sec

Average packet size $= 10^7$ bits

$$\text{Propagation delay, } T_p = \frac{8000\times 10^3}{(4\times 10^6)} = 2\sec$$

Time taken to transmit one frame,

$$T_t = \frac{10^7}{(500\times 10^6)} = 0.02\sec$$

Since, it is given that the network is to be used to its full capacity,

$\therefore \quad \eta = 100\%$

$$\eta = \frac{w_s}{1+2a} = 1$$

$$w_s = 1+2a$$

$$w_s = 1 + \frac{2\times 2}{0.02} = 201$$

$\Rightarrow$ Sender window size, w_s = 201

And, receiver window size, w_r = 1

Since, GBN protocol is used,

$\Rightarrow$ Available sequence numbers $\geq w_s + w_r$

$\Rightarrow$ Available sequence numbers $\geq 201+1$

$\Rightarrow$ Available sequence numbers ≥ 202

So, total sequence numbers required = 202

Hence, minimum number of bits required for sequence numbers is $\lceil \log_2 202 \rceil$ =8

Hence, the correct answer is 8.

2.5 **320**

Given :

Bandwidth, B = 1 Mbps

Propagation delay, T_p = 20 m sec

Efficiency = 50%

Let, frame size = L

And transmission delay $= T_t$

Since, for stop and wait protocol,

$$\text{Efficiency} = \frac{T_t}{T_t + 2\times T_p}$$

$$= \frac{T_t}{T_t + 2\times T_p} = 50\%$$

$$2T_t = T_t + 2T_p$$

$$\Rightarrow \quad \frac{T_p}{T_t} = \frac{1}{2}$$

$$\Rightarrow \quad T_t = 2\times T_p$$

$$\Rightarrow \quad \frac{L}{B} = 2\times 20 \text{ ms}$$

$\Rightarrow \quad L = 40\times 10^{-3} \times 64\times 10^{3}$ bits

$\Rightarrow \quad L =$ 320 bytes

Hence, the correct answer is 320.

2.6 **0.4404**

Given :

A LAN with four nodes S_1, S_2, S_3 and S_4.

Time is divided into fixed-size slots

The probability of generation of a frame in a time slot by S_1, S_2, S_3 and S_4 are 0.1, 0.2, 0.3 and 0.4 respectively.

Let *P* is the probability of sending a frame in the first slot without any collision by any of these four stations.

$$P = P(S_1)P(\neg S_2)P(\neg S_3)P(\neg S_4)$$
$$+P(\neg S_1)P(S_2)P(\neg S_3)P(\neg S_4)$$
$$+P(\neg S_1)P(\neg S_2)P(S_3)P(\neg S_4)$$
$$+P(\neg S_1)P(\neg S_2)P(\neg S_3)P(S_4)$$
$$= 0.1\times 0.8\times 0.7\times 0.6 + 0.9\times 0.2\times 0.7\times 0.6$$
$$+0.9\times 0.8\times 0.3\times 0.6 + 0.9\times 0.8\times 0.7\times 0.4$$
$$= 0.4404$$

Hence, the correct answer is 0.4404.

2.7 **(D)**

Given :

A CSMA/CD network that transmits data at a rate of 100 Mbps over a 1 km (kilometer) cable with no repeaters.

Minimum frame size is 1250 bytes, for a collision to be detected, the frame size should be such that the transmission time of the frame should be greater than twice the propagation delay

So, $\dfrac{1250\times 8}{10^8} \geq \dfrac{2\times 1}{x}$

$\Rightarrow$ x = 20000 km/sec

Hence, the correct option is (D).

2.8 **4**

Given :

A 128×10^3 bits/second bandwidth satellite communication link, where

Propagation delay $= 150$ milliseconds

Frame size for retransmitting data $= 1$ kb

For SR protocol $\eta = \dfrac{w_s}{1+2a}$

$$\eta = 100\%$$

Therefore, $\dfrac{w_s}{1+2a} = 1$

$$w_s = 1 + 2a$$

$$w_s = \text{Window size}$$

$$a = \frac{\text{Propagation time}}{\text{transmission time}}$$

$$= \frac{150}{1024\times \frac{8}{128}} = \frac{150}{64} = 2.34$$

$$\Rightarrow \quad w_s = 1 + 2a = 5.6875 \approx 6$$

Available seq. numbers $\geq w_s + w_r$

In selective repeat,

$w_s = w_r = N$ (say)

$$2\times N = 2\times 6 = 12$$

Avail seq. numbers ≥ 12

So, minimum seq. number is 12

No. of bits for seq. number = $\lceil \log_2 12 \rceil = 4$

Hence, the correct answer is 4.

2.9 **2500**

Given :

Frame size $= 1000$ bytes

Transmission rate at the sender $= 80$ kbps

Size of acknowledgement $= 100$ bytes

Transmission rate at the receiver $= 8$ kbps

One way propagation delay $= 100$ milliseconds

Total time = frame_Transmission_Time + $2\times$ Propagation_delay +ACK_transmission_Time.

Frame_transmission_time $= \dfrac{(1000\times 8)}{80\times 1000} = 0.1\text{sec}$

$2\times$ Propagation_Delay $= 2\times 100 \text{ ms} = 0.2\text{sec}$

$$\text{ACK\_transmission\_time} = \frac{100\times 8}{8\times 1000} = 0.1\text{sec}$$

Total time = 0.1 + 0.2 + 0.1 = 0.4 sec

Since, Throughput = no. of bytes sent per second

And, in 0.4 second we can send 1 frame

$\Rightarrow$ in 0.4 second we can send 1000 bytes

$\Rightarrow$ in 1 second we can send 1000/0.4 = 2500 bytes

So, throughput = 2500 bytes

Hence, the correct answer is 2500.

2.10 **200**

Given :

A CSMA/CD network.

Bandwidth of the network is 20×10^6 bps.

Signal propagation time given is $40\ \mu$s.

Since, CSMA/CD

Transmission Delay = RTT (Round trip time)

Hence, $\dfrac{\text{Data size}}{\text{Bandwidth}} = \text{RTT}$

$\Rightarrow$ Data size = Bandwidth $\times$ 2 $\times$ T_p

$$\text{Data size} = (20\times 10^6)\times 2\times 40\times 10^{-6}$$
$$= 20\times 2\times 40$$
$$= 1600 \text{ bits}$$
$$= 200 \text{ bytes}$$

Hence, the correct answer is 200.

2.11 **(C)**

Given :

Message 01011011

The generator polynomial has degree 3. So, we append 3 zeroes to the original message.

```
1011 ) 01011011000
       0000
       ----
        1011
        1011
        ----
         00001100
              1011
              ----
               1110
               1011
               ----
                101
```

CRC generated = 101

$\therefore$ M = 01011011101

Hence, the correct option is (C).

2.12 **(D)**

Given :

The rate of the link by which hosts are connected $= 10^6$ bits/sec

Distance between hosts $= 10,000$ km

Propagation speed $= 2\times 10^8$ m/s

Size of file $= 50,000$ bytes

$$T_t = \frac{\text{Length}}{\text{Bandwidth}}$$
$$= \frac{50000\times 8}{10^6}$$
$$= \frac{40}{100} = 0.4\text{s} = 400\,\text{ms}$$
$$T_p = \frac{\text{Distance}}{\text{Velocity}}$$
$$= \frac{10000\times 10^3}{2\times 10^8}$$
$$= \frac{1}{20} = 0.05\text{s} = 50\ \text{ms}$$
$$p = 400, q = 50$$

Hence, the correct option is (D).

2.13 **89.34**

Given :

Bit rate of the transmission channel

= 1 Mbps.

- Propagation delay from sender to receiver = 0.75 ms.
- Time to process a frame = 0.25 ms.
- Number of bytes in the information frame = 1980.
- Number of bytes in the acknowledge frame = 20.
- Number of overhead bytes in the information frame = 20.

$$\text{Efficiency} = \frac{(T_t)_{info}}{(T_t)_{info} + (T_{proc.})_{info} + 2\times T_p + (T_t)_{ack}} \quad \text{...(i)}$$

where,

$(T_t)_{info}$ = information frame transmission time

$(T_{proc.})_{info}$ = processing time of information frame = 0.25 ms

T_p = propagation delay = 0.75 ms

$(T_t)_{ask}$ = transmission time of acknowledge frame

$$(T_t)_{info} = \frac{\text{Information frame size}}{\text{Bandwidth}}$$

$$= \frac{2000 \times 8}{1 \times 10^6} = 16 \text{ ms}$$

$$(T_t)_{ask} = \frac{\text{Acknowledgement frame size}}{\text{Bandwidth}}$$

$$= \frac{20 \times 8}{1 \times 10^6} = 0.16 \text{ ms}$$

Now, putting all these values in equation (i),

$$\text{Efficiency} = \frac{16}{16 + 2 \times 0.75 \times 0.25 + 0.16}$$

$= 89.34\ \%$

Hence, the correct answer is 89.34.

2.14 50

Given :

A simple communication system, where multiple nodes are connected by a shared broadcast medium. The system has two nodes *P* and *Q*, located at a distance *d* meters from each other.

P starts transmission at $t = 0$. If *P*'s first bit reaches *Q* within *Q*'s sensing window, then *Q* won't transmit and there shall be no collision.

Q senses carrier till $t = 5$ at $t = 6$, it starts its transmission.

If the first bit of *P* reaches *Q* by $t = 5$, collision can be averted.

Since, signal speed is 10 m/time,

So, we can conclude that max distance between *P* and *Q* can be 50 meters.

Hence, the correct answer is 50.

2.15 (B)

Given :

A 12-bit Hamming code word consisting of 8-bit data and 4 check bits is

$$d_8 d_7 d_6 d_5 c_8 d_4 d_4 d_3 d_2 c_4 d_1 c_2 c_1$$

where the data bits and the check bits are given in the following tables :

Data bits							
d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
1	1	0	x	0	1	0	1

Check bits			
c_4	c_3	c_2	c_1
y	0	1	0

Checking for $c_1 : 1 \quad x \quad 0 \quad 0 \quad 1 \quad 0$

$x = 0$ (For even parity)

Checking for $c_8 : y \quad 0 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0$

$y = 1$ (For even parity)

Hence, the correct option is (B).

2.16 51

Given :

A sliding window protocol, which is full duplex error free

The size of data frame is 2000 bits

The size of ACK frame is 10 bits

The link rate in each direction is 1 Mbps.

One way propagation delay is 100 millisecond

$$(T_t)_{data} = \frac{\text{Data frame size}}{\text{Link rate}} = \frac{2000\text{ bits}}{1\text{Mbps}}$$

$$= 2 \text{ milliseconds}$$

$$(T_t)_{ack} = \frac{\text{Ack frame size}}{\text{link rate}}$$

$$= \frac{10\text{ bits}}{1\text{Mbps}} = 10 \times 10^{-3}$$

$$= 10^{-2} \text{ milliseconds}$$

$$= 0.01 \text{ milliseconds}$$

$$\text{Efficiency, } \eta = \frac{N \times (T_t)_{data}}{(T_t)_{data} + 2 \times T_p + (T_t)_{ack}}$$

$$N = \left\lceil \frac{\eta.\left((T_t)_{data} + 2T_p + (T_t)_{ack}\right)}{(T_t)_{data}} \right\rceil$$

$$N = \left\lceil \frac{0.5(0.002 + 0.2 + 0.00001)}{0.002} \right\rceil$$

$$N = \lceil 50.5025 \rceil = 51$$

Hence, the correct answer is 51.

2.17 (C)

Given :

A CRC based error detecting scheme having generator polynomial

$$x^3 + x + 1$$

and the message $m_4 m_3 m_2 m_1 m_0 = 11000$. Check bits $c_2 c_1 c_0$ appended at the end of the message.

$$x^3 + x + 1 = 1011$$

Append 3 zeroes at the end of message 11000 $= 110000000$

$$\begin{array}{l} 1011\overline{)11000000} \\ \quad\ \ \underline{1011} \\ \quad\ \ \ 1110 \\ \quad\ \ \ \underline{1011} \\ \quad\ \ \ \ 1010 \\ \quad\ \ \ \ \underline{1011} \\ \quad\ \ \ \ 0010 \\ \quad\ \ \ \ \underline{0000} \\ \quad\ \ \ \ \ 0100 \\ \quad\ \ \ \ \ \underline{0000} \\ \quad\ \ \ \ \ \ 100 \end{array}$$

Value of $c_2 c_1 c_0 = 100$

Hence, the correct option is (C).

Key Point

If divisor has x terms pad $x-1$ zeroes at the end is dividend

2.18 **135.3**

Given :

A pure ALOHA medium access control protocol,

Frame size = 1,000 bits

Bandwidth = 1 Mbps

The aggregate number of transmissions across all the nodes is modelled as a Poisson process with a rate of 1,000 frame per second.

Now,

Transmission time of 1 frame $= \frac{1000}{10^6} = 1 \text{ ms}$

Transmission time of 1000 frame

$$= 1000 \times 1 \text{ ms} = 1 \text{ second}$$

In 1 second, we are using the full load $G = 1$.

$$\eta = G \times e^{-2G}$$

$$\eta = 0.1353$$

According to question throughput is defined as average number of frames successfully transmitted per second

Throughput $= 0.1353 \times 1000 = 135.3$

The given answer range was 130 to 140.

Hence, the correct answer is 135.3.

3 Network Layer

Practice Questions

2013 IIT Bombay

3.1 In an IPv4 datagram, the M bit is 0, the value of HLEN is 10, the value of total length is 400 and the fragment offset value is 300. The position of the datagram, the sequence numbers of the first and the last bytes of the payload, respectively are :

(A) Last fragment, 2400 and 2789
(B) First fragment, 2400 and 2759
(C) Last fragment, 2400 and 2759
(D) Middle fragment, 300 and 689

2014 IIT Kharagpur

3.2 Consider the following three statements about link state and distance vector routing protocols, for a large network with 500 network nodes and 4000 links.

[S1] : The computational overhead in link state protocols is higher than in distance vector protocols.

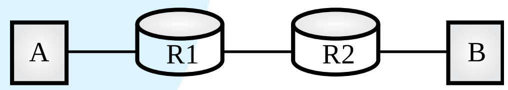
[S2] : A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

[S3] : After a topology change, a link state protocol will converge faster than a distance vector protocol.

Which one of the following is correct about S1, S2, and S3?

(A) S1, S2, and S3 are all true.
(B) S1, S2, and S3 are all false.
(C) S1 and S2 are true, but S3 is false.
(D) S1 and S3 are true, but S2 is false.

3.3 Consider the store and forward packet switched network given below. Assume that the bandwidth of each link is 10^6 bytes/sec. A user on host *A* sends a file of size 10^3 bytes to host *B* through routers R_1 and R_2 in three different ways. In the first case a single packet containing the complete file is transmitted from *A* to *B*. In the second case, the file is split into 10 equal parts, and these packets are transmitted from *A* to *B*. In the third case, the file is split into 20 equal parts and these packets are sent from *A* to *B*. Each packet contains 100 bytes of header information along with the user data. Consider only transmission time and ignore processing, queuing and propagation delays. Also assume that there are no errors during transmission. Let T_1, T_2 and T_3 be the times taken to transmit the file in the first, second and third case respectively. Which one of the following is CORRECT?



(A) $T_1 < T_2 < T_3$
(B) $T_1 > T_2 > T_3$
(C) $T_2 = T_3, T_3 < T_1$
(D) $T_1 = T_3, T_3 > T_2$

3.4 Which of the following is TRUE about the interior gateway routing protocols – Routing Information Protocol (RIP) and Open Shortest Path First (OSPF)

(A) RIP uses distance vector routing and OSPF uses link state routing
(B) OSPF uses distance vector routing and RIP uses link state routing
(C) Both RIP and OSPF use link state routing
(D) Both RIP and OSPF use distance vector routing

3.5 An IP router implementing Classless Inter-domain Routing (CIDR) receives a packet with address 131.23.151.76. The router's routing table has the following entries :

Prefix	Outer Interface Identifier
131.16.0.0/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

The identifier of the output interface on which this packet will be forwarded is ______.

3.6 Host A (on TCP/IPv4 network A) sends an IP datagram D to host B (also on TCP/IPv4 network B). Assume that no error occurred during the transmission of D. When D reaches B, which of the following IP header field (s) may be different from that of the original datagram D?

i. TTL

ii. Checksum

iii. Fragment Offset

(A) i only (B) i and ii only

(C) ii and iii only (D) i, ii and iii

3.7 An IP router with a Maximum Transmission Unit (MTU) of 1500 bytes has received an IP packet of size 4404 bytes with an IP header of length 20 bytes. The values of the relevant fields in the header of the third IP fragment generated by the router for this packet are :

(A) MF bit: 0, Datagram Length:1444; Offset:370

(B) MF bit: 1, Datagram Length:1424; Offset:185

(C) MF bit: 1, Datagram Length:1500; Offset:370

(D) MF bit: 0, Datagram Length:1424; Offset:2960

3.8 Every host in an IPv4 network has a 1 second resolution real-time clock with battery backup. Each host needs to generate up to 1000 unique identifiers per second. Assume that each host has a globally unique IPv4 address. Design a 50–bit globally unique ID for this purpose. After what period (in seconds) will the identifiers generated by a host wrap around?

2015 IIT Kanpur

3.9 Two hosts are connected via a packet switch with 10^7 bits per second links. Each link has a propagation delay of 20 microseconds. The switch begins forwarding a packet 35 microseconds after it receives the same. If 10000 bits of data are to be transmitted between the two hosts using a packet size of 5000 bits, the time elapsed between the transmission of the first bit of data and the reception of the last bit of the data in microseconds is ______.

3.10 Which of the following fields of an IP header is NOT modified by a typical IP router?

(A) Check sum

(B) Source address

(C) Time to Live (TTL)

(D) Length

3.11 Host A sends a UDP datagram containing 8880 bytes of user data to host B over an Ethernet LAN. Ethernet frames may carry data up to 1500 bytes (i.e. MTU = 1500 bytes). Size of UDP header is 8 bytes and size of IP header is 20 bytes. There is no option field in IP header. How many total number of IP fragments will be transmitted and what will be the contents of offset field in the last fragment?

(A) 6 and 925 (B) 6 and 7400

(C) 7 and 1110 (D) 7 and 8880

3.12 Consider the following routing table at an IP router :

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

For each IP address in Group I Identify the correct choice of the next hop from Group II using the entries from the routing table above.

Group I	**Group II**
(i) 128.96.171.92	(a) Interface 0
(ii) 128.96.167.151	(b) Interface 1
(iii) 128.96.164.151	(c) R2
(iv) 128.96.165.121	(d) R3
	(e) R4

(A) i-a, ii-c, iii-e, iv-d

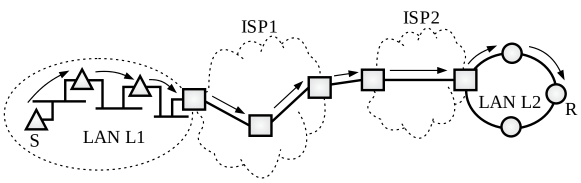
(B) i-a, ii-d, iii-b, iv-e

(C) i-b, ii-c, iii-d, iv-e

(D) i-b, ii-c, iii-e, iv-d

3.13 In the network 200.10.11.144/27, the fourth octet (in decimal) of the last IP address of the network which can be assigned to a host is ______.

3.14 In the diagram shown below, L1 is an Ethernet LAN and L2 is a Token-Ring LAN. An IP packet originates from sender S and traverses to R, as shown. The links within each ISP and across the two ISPs, are all point-to-point optical links. The initial value of the TTL field is 32. The maximum possible value of the TTL field when R receives the datagram is _______.



2016 IISc Bangalore

3.15 An IP datagram of size 1000 bytes arrives at a router. The router has to forward this packet on a link whose MTU (maximum transmission unit) is 100 bytes. Assume that the size of the IP header is 20 bytes.

The number of fragments that the IP datagram will be divided into for transmission is ______.

2017 IIT Roorkee

3.16 Consider the following statements about the routing protocols. Routing Information Protocol (RIP) and Open Shortest Path First (OSPF) in an IPv4 network.

I. RIP uses distance vector routing

II. RIP packets are sent using UDP

III. OSPF packets are sent using TCP

IV. OSPF operation is based on link-state routing

Which of the above statements are CORRECT?

(A) I and IV only
(B) I, II and III only
(C) I, II and IV only
(D) II, III and IV only

3.17 The maximum number of IPv4 router addresses that can be listed in the record route (RR) option field of an IPv4 header is ______.

2018 IIT Guwahati

3.18 Consider an IP packet with a length of 4,500 bytes that includes a 20–byte IPv4 header and 40–byte TCP header. The packet is forwarded to an IPv4 router that supports a Maximum Transmission Unit (MTU) of 600 bytes. Assume that the length of the IP header in all the outgoing fragments of this packet is 20 bytes. Assume that the fragmentation offset value stored in the first fragment is 0.

The fragmentation offset value stored in the third fragment is ______.

2019 IIT Madras

3.19 Suppose that in an IP-over-Ethernet network, a machine X wishes to find the MAC address of another machine Y in its subnet. Which one of the following techniques can be used for this?

(A) X sends an ARP request packet to the local gateway's IP address which then finds the MAC address of Y and sends to X
(B) X sends an ARP request packet to the local gateway's MAC address which then finds the MAC address of Y and sends to X
(C) X sends an ARP request packet with broadcast MAC address in its local subnet
(D) X sends an ARP request packet with broadcast IP address in its local subnet

3.20 Consider three machines M, N, and P with IP addresses 100.10.5.2, 100.10.5.5 and 100.10.5.6 respectively. The subnet mask is set to 255.255.255.252 for all the three machines. Which one of the following is true?

(A) M, N, and P all belong to the same subnet
(B) Only M and N belong to the same subnet
(C) Only N and P belong to the same subnet
(D) M, N, and P belong to three different subnets

2020 IIT Delhi

3.21 Consider the following statements about the functionality of an IP based router.

I. A router does not modify the IP packets during forwarding.

II. It is not necessary for a router to implement any routing protocol.

III. A router should reassemble IP fragments if the MTU of the outgoing link is larger than the size of the incoming IP packet.

Which of the above statements is/are TRUE?

(A) I and II only (B) I only
(C) II and III only (D) II only

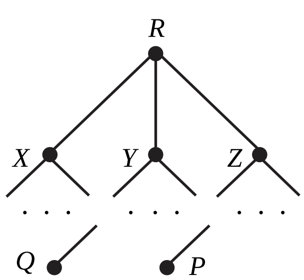
3.22 An organization requires a range of IP address to assign one to each of its 1500 computers. The organization has approached an Internet Service Provider (ISP) for this task. The ISP uses CIDR and serves the requests from the available IP address space 202.61.0.0/17. The ISP wants to assign an address space to the organization which will minimize the number of routing entries in the ISP's router using route aggregation. Which of the following address spaces are potential candidates from which the ISP can allot any one of the organization?

I. 202.61.84.0/21
II. 202.61.104.0/21
III. 202.61.64.0/21
IV. 202.61.144.0/21

(A) I and II only
(B) II and III only
(C) III and IV only
(D) I and IV only

2021	IIT Bombay

3.23 Consider a computer network using the distance vector routing algorithm in its network layer. The partial topology of the network is shown below.



The objective is to find the shortest-cost path from the router R to routers P and Q. Assume that R does not initially know the shortest routes to P and Q. Assume that R has three neighboring routers denoted as X, Y and Z. During one iteration, R measures its distance to its neighbors X, Y, and Z as 3, 2 and 5, respectively. Router R gets routing vectors from its neighbours that indicate that the distance to router P from routers X, Y and Z are 7, 6 and 5, respectively. The routing vector also indicates that the distance to router Q from routers X, Y and Z are 4, 6 and 8 respectively. Which of the following statement(s) is/are correct with respect to the new routing table o R, after updation during this iteration?

(A) The distance from R to P will be stored as 10
(B) The distance from R to Q will be stored as 7
(C) The next hop router for a packet from R to P is Y
(D) The next hop router for a packet from R to Q is Z

3.24 Consider the following two statements.

S_1 : Destination MAC address of an ARP reply is a broadcast address.

S_2 : Destination MAC address of an ARP request is a broadcast address.

Which one of the following choices is correct?

(A) Both S_1 and S_2 are true
(B) S_1 is true and S_2 is false
(C) S_1 is false and S_2 is true
(D) Both S_1 and S_2 are false

3.25 Consider two hosts P and Q connected through a router R. The maximum transfer unit (MTU) value of the link between P and R is 1500 bytes, and between R and Q is 820 bytes.

A TCP segment of size 1400 bytes was transferred from P to Q through R, with IP identification value as 0x1234. Assume that the IP header size is 20 bytes. Further, the packet is allowed to be fragmented, i.e., Don't Fragment (DF) flag in the IP header is not set by P.

Which of the following statements is/are correct?

(A) Two fragments are created at R and the IP datagram size carrying the second fragment is 620 bytes.
(B) If the second fragment is lost, R will resend the fragment with the IP identification value 0x1234.
(C) If the second fragment is lost, P is required to resend the whole TCP segment.
(D) TCP destination port can be determined by analysing only the second fragment.

Solutions

3.1 **(C)**

Given :

Value of M bit = 0

Value of HLEN = 10

Value of total length = 400

Fragment offset value = 300

HLEN is given so header length will be $=10\times4=40$

Payload = Total length – Header length

Payload $400-40=360$ bytes (0 to 359)

$=300\times8=2400$

Position of datagram = Fragment offset × 8

Last fragment $=2400+359=2759$

> **Key Point**
>
> Check carefully, whether HLEN is given or header length is given.
>
> HLEN value ranges from 5 to 15. Header length value ranges from 20 to 60.

From the above key point we can conclude that the answer is option (C).

Hence, the correct option is (C).

3.2 **(D)**

[S1] : The computational overhead in link state protocols is higher than in distance vector protocols.

True, because LSR is based upon global knowledge whereas DVR is based upon local info.

[S2] : A distance vector protocol (with split horizon) avoids persistent routing loops, but not a link state protocol.

False, persistent looping can be avoided with the help of split horizon in DVR. But there is no concept of persistent looping in LSR, in LSR only temporary loop exit and can automatically solved by system or router

[S3] : After a topology change, a link state protocol will converge faster than a distance vector protocol.

True, a topology change, a link state protocol will converge faster than a distance vector protocol.

Hence, the correct option is (D).

3.3 **(D)**

> **Key Point**
>
> Transmission time = Packet size /Bandwidth

Given :

File size $=1000$ bytes

Header size $=100$ bytes

Bandwidth $=10^6$ bytes/sec

Case I : Calculation of T_1 :

Transmission time for one link

$$=\frac{(1000+100)}{10^6}$$

$=1100$ microsecond

Total time $=3\times1100$

$=3300$ microsecond

Case II : Calculation of T_2 :

Transmission time for one link for I[st] packet

$$=\frac{(100+100)}{10^6}$$

$=200$ microsecond

Total time $=3\times200+9\times200$

$=2400$ micro second

Case III : Calculation of T_3 :

Transmission time for one link and one packet

$$=\frac{(50+100)}{10^6}$$

$=150$ microsecond

Total time $=3\times150+19\times150$

$=3300$ microsecond

Hence, the correct option is (D).

3.4 **(A)**

> **Key Point**
>
> 1. Both routing information Protocol (RIP) and Open Shortest Path First (OSPF) are Interior Gateway protocol, i.e., they both are used within an autonomous system.
> 2. RIP is based on distance vector routing.
> 3. OSPF is based on link state Routing.

From above key point we can conclude that most suitable option is (A).

Hence, the correct option is (A).

3.5	1

Given :

Prefix	Outer Interface Identifier
131.16.0.0/12	3
131.28.0.0/14	5
131.19.0.0/16	2
131.22.0.0/15	1

Address - 33.23.151.76

Coming to the first field of given routing table.

We can see that the given address matches with first and fourth entries.

Whenever we have multiple matches, the packet is forwarded to the one with longest subnet mask.

Checking with first entry : 131.16.0.0/12

131.0001 0111.151.76

<u>131.0001 0000.0.0</u>

$\Rightarrow$ 131.16.0.0 Matched

Checking with second entry : 131.28.0.0/14

131.000101 11.151.76

<u>131.000101 11.0.0</u>

$\Rightarrow$ 131.20.0.0 Not Matched.

Checking with third entry : 131.19.0.0/16

131.00010111. 151.76

<u>131.00010111. 0.0</u>

$\Rightarrow$ 131.23.0.0 Not Matched.

Checking with fourth entry : 131.22.0.0/15

131.0001011 1.151.76

<u>131.0001011 0.0.0</u>

$\Rightarrow$ 131.22.0.0 Matched.

Since 1st and 4th entry are matched, among them 4th entry has longer mask bit than 1st bit therefore, it will be chosen for forwarding.

Key Point

- Find out subnet mask for each entry and BITWISE AND with given packet address. Longest Prefix matching is used to decide among two. When one destination address matches more than one forwarding table entry. The most specific of the matching table entries is used as the interface.

Hence, the correct answer is 1.

3.6	(D)

TTL (time-to-live) value decremented on every single hop. Hence TTL is changed on every hop. Since TTL changes, check sum of the packet will also change.

If packet has a size greater the MTU (Maximum Transmission Unit) of network, then fragmentation offset can also be changed.

Hence, the correct option is (D).

3.7	(A)

Given :

IP packet length = 4404

IP header of length = 20.

So data is 4384.

MTU (Maximum Transmission Unit) is 1500, which include header size so $1500-20=1480$ bytes data will be transfer.

Now, router divide total data in 3 parts

1480 1480 1424

Since it is a last packet

Therefore, $MF=0$ and offset is

$$=\frac{1480+1480}{8}=\frac{2960}{8}=370$$

Hence, the correct option is (A).

3.8	256

Given :

Each host needs to generate 1000 unique identifiers per second which requires $[\log 1000]=10$ bits

Now these 10 bits along with 32 bit globally unique IP address will give a globally unique 42 bit IDs which stays constant.

Since, we are allowed 50 bits, we can use the next 8 bits using the clock which changes every second.

Thus our IDs will wrap around once in $2^8=256$ seconds

Hence, the correct answer is 256.

3.9	1575

Given :

No. of packets sent $= \dfrac{10000}{5000} = 2$

Bandwidth $= 10^7$ bps

Propagation delay, $T_p = 20\ \mu\text{sec}$

Time for the first packet to reach destination

= t1 + t2 + t3

= 520 µs + 35 µs + 520 µs

= 1075 µs

where, t1 = Time for the first packet to reach switch

= Transmission delay + Propagation delay

= $(5000/10^7)\times10^6$ µs + 20µs

= 520µs.

t2 = Switch delay = 35µs

t3 = Time for the first packet to reach from switch to destination

= Transmission time + Propagation delay

= 520µs.

So, first packet is received at destination at $2\times520+35=1075\,\mu\text{s}$

After first 500µs, we can start sending the second packet into the link.

Time for the second packet to reach destination

= 500 + t4 + t5 + t6

= 500 µs +520 µs +35 µs +520 µs

= 1575 µs

where, t4 = Time for the second packet to reach switch

= Transmission time + Propagation delay

= $(5000/10^7)\times10^6$ µs+20µs

= 520µs.

t5 = Switch delay = 35µs

t6 = Time for the first packet to reach from switch to destination

= Transmission time + Propagation delay

= $(5000/10^7)\times10^6$ µs+20µs

= 520µs.

So, the time elapsed between the transmission of the first bit of data and the reception of the last bit of the data = 1575 µs

Hence, the correct answer is 1575.

3.10 (B)

Key Point

Length and check sum can be modified when IP fragmentation happens.

TTL is decremented by every router on the route to destination.

So only source address cannot change.

From the above key point we can conclude that the answer is option (B).

Hence, the correct option is (B).

3.11 (C)

Given :

Size of datagram = 8880 bytes

MTU = 1500 bytes

Size of UDP header = 8 bytes

Size of IP header = 20 bytes

Total packet size of UDP

$= 8880 + 8 = 8888$ bytes

Since MTU = 150 bytes and header size of $IP = 20$ bytes

Size of IP fragments $= 1500 - 20 = 1480$ bytes

Total data (UDP datagram) is transmitted as IP fragments

Number of fragments $= \dfrac{8888}{1480} \cong 7$

Fragment offset field is used to indicate the starting position of the data in the fragment

Offset of last fragment $= \dfrac{1480\times6}{8} = 1110$

Hence, the correct option is (C).

3.12 (A)

Given :

Network No.	Net Mask	Next Hop
128.96.170.0	255.255.254.0	Interface 0
128.96.168.0	255.255.254.0	Interface 1
128.96.166.0	255.255.254.0	R2
128.96.164.0	255.255.252.0	R3
0.0.0.0	Default	R4

1.
$$\begin{array}{c} 128.96.171.92 \\ \underline{255.255.254.0} \\ 128.96.170.0 \end{array}$$

Will forward to interface 0.

2.
$$\begin{array}{c} 128.96.167.151 \\ \underline{255.255.254.0} \\ 128.96.166.0 \end{array}$$

Will forward to interface R2.

3.
$$\begin{array}{c} 128.96.163.151 \\ \underline{255.255.254.0} \\ 128.96.162.0 \end{array}$$

Does not match with any given interface

4.
$$\begin{array}{c} 128.96.164.121 \\ \underline{255.255.254.0} \\ 128.96.164.0 \end{array}$$

Will forward to interface R3.

Hence, the correct option is (A).

3.13 **158**

Given :

Network 200.10.11.144/27

144 in binary = 10010000

Out of this 3 bits in left are subnet bits. (27 bits are used for subnet, which means first 3 bytes and leftmost 3 bits from the last byte).

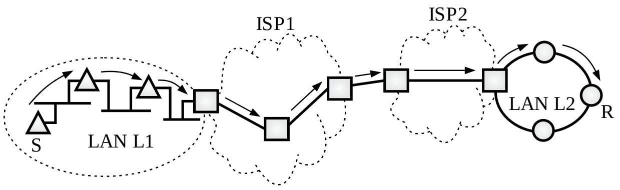
So in 4^{th} octet in the last IP address of the network which can be assigned to a host is 10011110.

So, 10011110 is 158 in decimal.

Hence, the correct answer is 158.

3.14 **26**

Given :



L1 is an Ethernet LAN and L2 is a Token-Ring LAN.

An IP Packet originates from Sender *S* and transverse to *R*.

There are 5 routers, so Network layer will be visited 5 times and 1 time on destination. So TTL value will decrement 6 times, hence TTL $= 26$.

A receiver decrements TTL value and then checks, whether it is 0 (or) not.

So, 26 is correct answer.

Hence, the correct answer is 26.

3.15 **13**

Given :

Size of datagram = 1000 bytes

MTU = 100 bytes

Header size = 20 bytes

Size of the IP header = 20 bytes

So, the size of data that can be transmitted in one fragment $= 100 - 20 = 80$ bytes

(Size of data to be transmitted = Size of datagram – size of header)

So, size of data $= 1000 - 20 = 980$ bytes

We have a datagram of size 1000 bytes

So, the cell $= \dfrac{980}{80} \cong 13$ fragments

Hence, the correct answer is 13.

3.16 **(C)**

Key Point

1. RIP uses distance vector routing (DVR) protocol.
2. RIP uses the UDP as its transport protocol with port no 520.
3. OSPF uses link state routing (LSR) protocol.
4. OSPF encapsulates its data directly into Packets and does not use either TCP or UDP.

From above key point we can conclude that statement (III) is false and only statements I, II and IV are correct.

Hence, the correct option is (C).

3.17 **9**

In IPV4, 40 bytes are reserved for options and padding.

For record route to store, 1 byte is used to store type of option

1 byte for length and 1 byte for pointer.

We can store at most $\left\lfloor \frac{37}{4} \right\rfloor = 9 \text{ router addresses.}$

Hence, the correct answer is 9.

3.18 (C)

Given :

MTU = 600 bytes and IP header = 20 bytes. So, payload will be $600-20=580$ bytes.

Nearest number which is multiple of 8 is 576, so fragment size will be 576.

Offset value of 3rd fragment

$$= \frac{576 \times (3-1)}{8} = 144$$

Hence, the correct option is (C).

3.19 (C)

If a machine X wants to find the MAC address of 4 then it will send ARP packet. Since it is used for discovering physical address associated with the given network address.

Hence, the correct option is (C).

3.20 (B)

Given :

Three machines *M*, *N* and *P* whose IP address are 100.10.5.2, 100.10.5.5 and 100.10.5.6 respectively.

Subnet mask for all 3 machines

$$= 255.255.255.252$$

Perform bitwise AND to check which subnet it belongs

For M :

$$\begin{array}{c} 100.10.5.2 \\ 255.255.255.252 \\ \hline 100.10.5.0 \end{array}$$

For N :

$$\begin{array}{c} 100.10.5.5 \\ 255.255.255.252 \\ \hline 100.10.5.4 \end{array}$$

For P :

$$\begin{array}{c} 100.10.5.6 \\ 255.255.255.252 \\ \hline 100.10.5.4 \end{array}$$

N and P belongs to same network, while M belongs to same or different network.

Hence, the correct option is (B).

3.21 (D)

Consider each statement one by one :

I. A router does not modify the IP packets during forwarding.

False. A router modifies the IP packets during forwarding because some fields like TTL (Time to live) change.

II. It is not necessary for a router to implement any routing protocol.

True. A router need not implement any routing protocol. It can just forward packets in all the directions without doing any routing.

III. A router should reassemble IP fragments if the MTU of the outgoing link is larger than the size of the incoming IP packet.

False. Router does not assemble the packets. Assembling is done at destination system.

Hence, the correct option is (D).

3.22 (B)

Given :

Number of hosts = 1500

For 1500 hosts, 11 bits are required

$$\underbrace{202.61.0}_{\text{NID}}\underbrace{0000}_{\text{SID}}\underbrace{000.00000000}_{\text{HID}}$$

Consider each configuration :

I. $202.61.84.0/21 = \underbrace{202.61.0}_{\text{NID}}\underbrace{1010}_{\text{SID}}\underbrace{100.0}_{\text{HID}}$

All HID bits should be 0, therefore above configuration is wrong.

II. $202.61.104.0/21 = \underbrace{202.61.0}_{NID}\underbrace{1101}_{SID}\underbrace{000.0}_{HID}$

III. $202.61.64.0/21 = \underbrace{202.61.0}_{\text{NID}}\underbrace{1000}_{\text{SID}}\underbrace{000.0}_{\text{HID}}$

IV. $202.61.144.0/21 = \underbrace{202.61.1}_{\text{NID}}\underbrace{0010}_{\text{SID}}\underbrace{000.0}_{\text{HID}}$

We cannot change bits of NID, in above configuration last NID bit is 1 which should be 0.

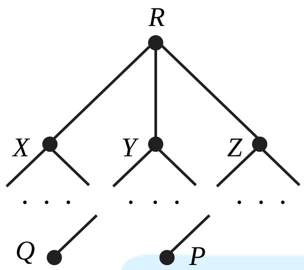
Hence, the correct option is (B).

3.23 **(B, C)**

Given :

A computer network using the distance vector routing algorithm in its network layer.

The partial topology is given below



Routing vector from R :

X	Y	Z
3	2	5

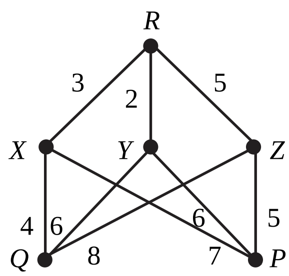
Routing vector from P :

X	Y	Z
7	6	5

Routing vector from Q :

X	Y	Z
4	6	8

Now, if we draw complete topology of the network.



As per Distance Vector Routing Algorithm,

R to P $= \min\{R-X-P, R-Y-P, R-Z-P\}$

$= \min\{10, 8, 11\}$

$= 8$ through Y.

R to Q $= \min\{R-X-Q, R-Y-Q, R-Z-Q\}$

$= \min 7, 8, 13\}$

$= 7$ through X.

Hence, the correct option are (B), (C).

3.24 **(C)**

Consider each statement one by one :

S_1 : Destination MAC address of an ARP reply is a broadcast address.

S_1 is false, as ARP reply is unicast.

S_2 : Destination MAC address of an ARP request is a broadcast address.

S_2 is true.

Hence, the correct option is (C).

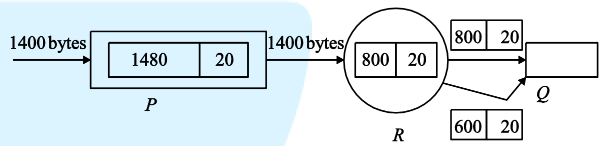
3.25 **(A), (C)**

Given :

Two hosts P and Q and they are connected through a Router.

MTU between P and R = 1500 bytes.

MTU between R and Q = 820 bytes.



For first link, segment size is less than MTU, so no fragmentation will be there.

For second link (R and Q), MTU is 820 bytes.

So we need to fragment

$\rightarrow$ first fragment will be $800 + 20$ (IP header + Data) $= 820\,B$.

$\rightarrow$ second fragment will be $1400 - 800 + 20 = 620\,B$.

So, option (A) is correct.

Now, option (C) is if the second fragment is lost, P is required to send the whole TCP segment, it is true.

Hence, the correct option is (A), (C).

4 Transport Layer

Practice Questions

2013 IIT Bombay

4.1 The transport layer protocols used for real time multimedia, file transfer, DNS and email, respectively are

(A) TCP, UDP, UDP and TCP
(B) UDP, TCP, TCP and UDP
(C) UDP, TCP, UDP and TCP
(D) TCP, UDP, TCP and UDP

2014 IIT Kharagpur

4.2 Which of the following socket API functions converts an unconnected active TCP socket into a passive socket?

(A) connect (B) bind
(C) listen (D) accept

4.3 Let the size of congestion window of a TCP connection be 32 KB when a timeout occurs. The round trip time of the connection is 100 msec and the maximum segment size used is 2 KB. The time taken (in msec) by the TCP connection to get back to 32 KB congestion window is ________.

2015 IIT Kanpur

4.4 Identify the correct order in which a server process must invoke the function calls accept, bind, listen, and recv according to UNIX socket API.

(A) listen, accept, bind, recv
(B) bind, listen, accept, recv
(C) bind, accept, listen, recv
(D) accept, listen, bind, recv

4.5 Suppose two hosts use a TCP connection to transfer a large file. Which of the following statements is/are FALSE with respect to the TCP connection?

I. If the sequence number of a segment is m, then the sequence number of the subsequent segment is always m+1.

II. If the estimated round trip time at any given point of time is t sec, the value of the retransmission timeout is always set to greater than or equal to t sec.

III. The size of the advertised window never changes during the course of the TCP connection.

IV. The number of unacknowledged bytes at the sender is always less than or equal to the advertised window.

(A) III only
(B) I and III only
(C) I and IV only
(D) II and IV only

4.6 Assume that the bandwidth for a TCP connection is 1048560 bits/sec. Let α be the value of RTT in milliseconds (rounded off to the nearest integer) after which the TCP window scale option is needed. Let β be the maximum possible window size with window scale option. Then the values of α and β are

(A) 63 milliseconds, 65535×2^{14}
(B) 63 milliseconds, 65535×2^{16}
(C) 500 milliseconds, 65535×2^{14}
(D) 500 milliseconds, 65535×2^{16}

4.7 Consider the following statements.

I. TCP connections are full duplex
II. TCP has no option for selective acknowledgement
III. TCP connections are message streams

(A) Only I is correct
(B) Only I and III are correct
(C) Only II and III are correct
(D) All of I, II and III are correct

2016 IISc Bangalore

4.8 For a host machine that uses the token bucket algorithm for congestion control, the token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second. Tokens arrive at a rate to sustain output at a rate of 10 megabytes per second. The token bucket is currently full and the machine needs to send 12 megabytes of data. The minimum time required to transmit the data is _______ seconds.

2017 IIT Roorkee

4.9 Consider a TCP client and a TCP server running on two different machines. After completing data transfer, the TCP client calls close to terminate the connection and a FIN segment is sent to the TCP server. Server-side TCP responds by sending an ACK, which is received by the client-side TCP. As per the TCP connection state diagram (RFC 793), in which state does the client-side TCP connection wait for the FIN from the server-side TCP?

(A) LAST-ACK (B) TIME-WAIT

(C) FIN-WAIT-1 (D) FIN-WAIT-2

4.10 Consider socket API on a Linux machine that supports connected UDP sockets. A connected UDP socket is a UDP socket on which connect function has already been called. Which of the following statements is/are CORRECT?

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.

II. A process can successfully call connect function again for an already connected UDP socket.

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

2018 IIT Guwahati

4.11 Consider a long-lived TCP session with an end-to-end bandwidth of 1 Gbps ($=10^9$ bits-per-second). The session starts with a sequence number of 1234. The minimum time (in seconds, rounded to the closet integer) before this sequence number can be used again is ______.

4.12 Consider the following statements regarding the slow start phase of the TCP congestion control algorithm. Note that cwnd stands for the TCP congestion window and MSS window denotes the Maximum Segments Size:

(i) The cwnd increases by 2 MSS on every successful acknowledgment

(ii) The cwnd approximately doubles on every successful acknowledgment

(iii) The cwnd increases by 1 MSS every round trip time

(iv) The cwnd approximately doubles every round trip time

Which one of the following is correct?

(A) Only (ii) and (iii) are true

(B) Only (i) and (iii) are true

(C) Only (iv) is true

(D) Only (i) and (iv) are true

2020 IIT Delhi

4.13 Consider a TCP connection between a client and a server with the following specifications; the round trip time is 6 ms, the size of the receiver advertised window is 50 KB, slow-start threshold at the client is 32 KB, and the maximum segment size is 2 KB. The connection is established at time t = 0. Assume that there are no timeouts and errors during transmission. Then the size of the congestion window (in KB) at time $t+60$ ms after all acknowledgements are processed is _______.

2021 IIT Bombay

4.14 A TCP server application is programmed to listen on port number P on host S. A TCP client is connected to the TCP server over the network.

Consider that while the TCP connection was active, the server machine S crashed and rebooted. Assume that the client does not use the TCP keepalive timer.

Which of the following behaviors is/are possible?

(A) If the client was waiting to receive a packet, it may wait indefinitely.

(B) The TCP server application on S can listen on P after reboot.

(C) If the client sends a packet after the server reboot, it will receive a RST segment.

(D) If the client sends a packet after the server reboot, it will receive a FIN segment.

4.15 Consider the three-way handshake mechanism followed during TCP connection establishment between hosts P and Q. Let X and Y be two random 32-bit starting sequence numbers chosen by P and Q respectively. Suppose P sends a TCP connection request message to Q with a TCP segment having SYN bit = 1, SEQ number = X, and ACK bit = 0. Suppose Q accepts the connection request.

Which one of the following choices represents the information present in the TCP segment header that is sent by Q to P?

(A) SYN bit = 1, SEQ NO = X+1, ACK bit = 0, ACK No = Y, FIN bit = 0

(B) SYN bit = 0, SEQ NO = X+ 1, ACK bit = 0, ACK No = Y, FIN bit = 1

(C) SYN bit = 1, SEQ NO = Y, ACK bit = 1, ACK No = X+1, FIN bit = 0

(D) SYN bit = 1, SEQ NO = Y, ACK bit = 1, ACK No = X, FIN bit = 0

Solutions

4.1 (C)

TCP (Transmission control protocol) and UDP (User datagram protocol) are two main transport layer protocol.

For real time multimedia, timely delivery is more important than corrections. Here, UDP is used.

For file transfer, correctness, reliability is necessary .here TCP is used.

DNS uses both TCP and UDP. But when efficiency is required, UDP is preferred.

Email uses SMTP protocol which uses TCP protocol.

Hence, the correct option is (C).

4.2 (C)

Listen() function converts an unconnected active TCP socket into a passive socket.

Hence, the correct option is (C).

4.3 1100

Given :

Size of congestion window = 32 kB, .

RTT (Round trip time) = 100 msec.

The maximum segment size = 2 kB.

Now,

$$\text{Threshold} = \frac{\text{Size of congestion window}}{2}$$

$$\text{Threshold, } Th = \frac{32}{2} = 16$$

RTT	kB
1	2
2	4
3	8
4	16
5	18
6	20
7	22
8	24
9	26
10	28
11	30
	32

To reach the congestion window size of 32 kB, 11 RTT required,

So time taken (in msec) = 11×100 ms

$= 1100$ msec

Key Point

$$\text{Threshold} = \frac{\text{Size of congestion window}}{2}$$

Hence, the correct answer is 1100.

4.4 (B)

The correct order is :

bind : Binds the socket to an address.

listen : Waits for connection to the socket.

accept: Accept a connection to the socket.

recv: Receive data from connection.

Hence, the correct option is (B).

4.5 (B)

I is false. TCP sequence number of a segment is the byte number of the first byte in the segment.

III is false. Receiver window changes when TCP data is process by application layer of receiver side.

Hence, the correct option is (B).

4.6 (C)

Given : bandwidth = 1048560 bits/sec

In TCP, when the bandwidth delay product increase beyond 64 KB or 65535 bytes receiver window scaling is needed.

Calculation of α :

$$\alpha \times 1048560 \times 1000 = 65535 \times 8$$

$$\Rightarrow \quad \alpha = 0.5 \text{ sec}$$

$$\Rightarrow \quad \alpha = 500 \text{ msec}$$

Calculation of β :

Since window size is to be increased from 2^{16} B to 2^{30} B.

Or, from 65535 to 65535×2^{14}

So, $\beta = 65535 \times 2^{14}$

Hence, the correct option is (C).

4.7 (A)

Consider each statement one by one

I. TCP connections are full duplex

True. In TCP, as sender and receiver can send segments at the same time, it is full duplex.

II. TCP has no option for selective acknowledgement

False. TCP has options for selective acks.

III. TCP connections are message streams

False. As each byte is counted in TCP segment, and sequence number of first byte is kept into header, TCP is byte stream protocol.

Hence, the correct option is (A).

4.8 1.1

Given :

A host machine, which was token bucket algorithm.

The token bucket has a capacity of 1 megabyte and the maximum output rate is 20 megabytes per second.

Token arrive at a rate to sustain output at a rate of 10 megabyte per second.

Rate at which, it is emptying is (20-10) Mbps.

Time taken to empty token bucket of 1 MB is $\frac{1}{10}$. i.e. 0.1 sec.

Data send in this time is $0.1 \times 20 = 2$ MB

Create at which bucket is emptying is different from rate at which data is send).

Data left to send is $12 - 2 = 10$ MB.

Now bucket is empty and rate of token arriving is less than that of going out so effective data speed will be 10 MBps.

Tine to send remaining 10 MB will be 1 sec.

So, total time is $1 + 0.1 = 1.1$ sec

Hence, the correct answer is 1.1.

4.9 (D)

Key Point

Client has sent FIN segment to the server and moves to FIN-WAIT-1, i.e, waiting for the ACK for own FIN segment. There are two possibilities here.

1. If client receives ACK for its FIN then client will move to FIN-WAIT-2 and will wait for matching FIN form server side.
2. Client has sent FIN segment but didn't get ACK till the time. Instead of ACK, client received FIN from server side.

Here we encounter first case.

So, option D is correct answer.

Hence, the correct option is (D).

4.10 (B)

Consider each statement one by one

I. A connected UDP socket can be used to communicate with multiple peers simultaneously.

I is false. On a particular UDP socket, a server can accept only one request at a time. For connecting with multiple peers simultaneously, multiple connections need to be made in UDP server which is not possible as UDP server handles each client requests iteratively.

II. A process can successfully call connect function again for an already connected UDP socket.

II is true. A process can successfully call connect function again for a already connected UDP socket.

Hence, the correct option is (B).

4.11 34.35

Given :

Bandwidth $=$ 1 Gbps $= 10^9$ bits/sec

We are required to find the wrap around time.

Since, no. of sequence numbers available $= 2^{32}$

And, 1 sequence no. corresponds to 1 byte

$$T_{wrap\ around} = \frac{2^{32} \times 8}{10^9} = 34.35 \text{ s}$$

Key Point

Maximum transmission unit is defined as the Wrap around Time is the time taken to start reusing the same sequence number

Wrap Around Time $= \frac{2^{32}}{\text{BW}}$ sec

Where BW $=$ Bandwidth also it can be said that the time taken to repeat the sequence number as per the requirement.

Hence, the correct answer is 34.35.

4.12 (C)

Every time an ACK is received by the sender, the congestion window is increased by 1 segment

$\therefore$ cwnd = cwnd + 1

cwnd increases exponentially on every RTT.

Hence, the correct option is (C).

4.13 44

Given :

A TCP connection between a client and a server with the specifications;

The RTT is 6 ms.

The size of the receiver advertised window is 50 kB

Slow-start threshold at the client is 32 kB, and MSS is 2 kB.

Here, $t + 60$ is $\frac{60}{6} = 10$ RTT

Now, 1st transmission : 2 kB

2nd transmission : 4 kB

3rd transmission : 8 kB

4th transmission : 16 kB

5th transmission : 32 kB

6th transmission : 34 kB

7th transmission : 36 kB

8th transmission : 38 kB

9th transmission : 40 kB

10th transmission : 42 kB

after 10th transmission,

Congestion window size will be 44 kB.

Hence, the correct answer is 44.

4.14 (A), (B), (C)

Consider each option one by one :

Option (A) : If the client was waiting to receive a packet, it may wait indefinitely.

It is correct, because client doesn't have a keep-alive timer, and the server after a reboot, forgets that any connection with the client existed.

Option (B) : The TCP server application on S can listen on P after reboot.

It is correct as once the current session will be terminated (due to reboot) and new connection can take place on the same port.

Option (C) : If the client sends a packet after the server reboot, it will receive a RST segment.

It is also correct as explained in option (A).

Option (D) : If the client sends a packet after the server reboot, it will receive a FIN segment.

It is false as the FIN is used to close connection.

Hence, the correct option is (A), (B) and (C).

4.15 (C)

Given :

TCP connection establishment between hosts P and Q.

X and Y are two random 32-bit starting sequence numbers chosen by P and Q respectively.

P sends a TCP connection request message to Q with a TCP segment having SYN bit = 1, SEQ number = X, and ACK bit = 0.

Host P sends the first SYN packet with SEQ number = X, SYN flag =1 and ACK flag = 0 as it's a connection request.

Host Q will reply back with a SYN packet and acknowledging the arrival of P's SYN packet.

Host Q will send a packet with SYN flag = 1, SEQ number = Y, to synchronize and establish the connection, and ACK flag = 1 to acknowledge the P's SYN packet, with ACK number = X+1 because ACK number denotes the sequence number of next expecting Byte.

Then P will reply back with an ACK packet to complete the three-way handshake.

FIN flag is used to terminate the connection, and will not be used here, FIN flag = 0.

Hence, the correct option is (C).

5 Application Layer

Practice Questions

2015 **IIT Kanpur**

5.1 In one of the pairs of protocols given below, both the protocols can use multiple TCP connections between the same client and the server. Which one is that?

(A) HTTP, FTP
(B) HTTP, TELNET
(C) FTP, SMTP
(D) HTTP, SMTP

2016 **IISc Bangalore**

5.2 Which of the following is/are example(s) of stateful application layer protocol?

(i) HTTP (ii) FTP
(iii) TCP (iv) POP3

(A) (i) and (ii) only
(B) (ii) and (iii) only
(C) (ii) and (iv) only
(D) (iv) only

5.3 Which one of the following protocols is NOT used to resolve one form of address to another one?

(A) DNS (B) ARP
(C) DHCP (D) RARP

5.4 Identify the correct sequence in which the following packets are transmitted on the network by a host when a browser requests a webpage from a remote server, assuming that the host has just been restarted.

(A) HTTP GET request, DNS query, TCP SYN
(B) DNS query, HTTP GET request, TCP SYN
(C) DNS query, TCP SYN, HTTP GET request
(D) TCP SYN, DNS query, HTTP GET request

2018 **IIT Guwahati**

5.5 Match the following :

	Field		Length in bits
P.	UDP Header's Port Number	I.	48
Q.	Ethernet MAC Address	II.	8
R.	IPv6 Next Header	III.	32
S.	TCP Header's Sequence Number	IV.	16

(A) P-III, Q-IV, R-II, S-I
(B) P-II, Q-I, R-IV, S-III
(C) P-IV, Q-I, R-II, S-III
(D) P-IV, Q-I, R-III, S-II

2019 **IIT Madras**

5.6 Which of the following protocol pairs can be used to send and retrieve e-mails (in that order)?

(A) IMAP POP3
(B) SMTP, POP3
(C) SMTP MIME
(D) IMAP, SMTP

2020 **IIT Delhi**

5.7 Assume that you have made a request for a web page through your web browser to a web server. Initially the browser cache is empty. Further, the browser is configured to send HTTP requests in non-persistent mode. The web page contains text and five very small images. The minimum number of TCP connections required to display the web page completely in your browser is _________.

Solutions

5.1 **(A)**

HTTP, FTP is used for multiple TCP

Connections between the same client and the server.

Key Point

HTTP works in Multiple Connections
1. Persistent HTTP Connection.
2. Non-persistent HTTP Connection.

FTP also uses Multiple Connections.
1. Control Connection.
2. Data Connection.

Hence, the correct option is (A).

5.2 **(C)**

Stateless protocol is a communication protocol in which no information is retained by either sender or receiver.

HTTP is stateless protocol.

FTP and POP3 are statefull application layer protocol.

TCP is statefull protocol, but it is not an application layer protocol.

It is transport layer protocol.

Hence, the correct option is (C).

5.3 **(C)**

DNS (Domain Name Server) is used to convert host name to IP address or vice versa.

ARP (Address Resolution Protocol) is used to convert IP to MAC

RARP (Reverse Address Resolution Protocol) MAC to IP

DHCP is used to assign IP dynamically which means it does not resolve addresses.

Hence, the correct option is (C).

5.4 **(C)**

Firstly we will send DNS request to get IP address. Ex- dns request can be made in the form like 'www.gateacademy.com'.

Then we will establish connection with IP we got using TCP.

After we have established the connection using tcp, http protocol will come into picture. A request is made to load the web page using 'http get request'.

Hence, the correct option is (C).

5.5 **(C)**

UDP Header's Port number	16
Ethernet MAC address	48
IPv6 Next Header	8
TCP Header's Sequence number	32

Hence, the correct option is (C).

5.6 **(B)**

SMTP is a push protocol, used to send emails from client to mail servers.

IMAP and POP3 are pull protocols used for retrieving mails from servers.

Hence, SMTP and POP3 pair can be used to send and retrieve e mails.

Hence, the correct option is (B).

5.7 **6**

Given :

Number of images = 5

Since, given HTTP connection works in non-persistent mode,for every individual object a separate connection should be established.

So, we have given a web page contains text and 5 images.

So, 1 TCP connection required for text and 5 TCP connections required for images.

So, total 6 TCP connections required.

Hence, the correct answer is 6.

CHAPTER 7

GATE-2022 & 2023 Question

1 GATE-2022 [IIT Kharagpur]

General Aptitude Part

Q.1 to Q.5 Carry One Mark Each

Question 1

The __________ is too high for it to be considered ________ .

(A) Fare/Fair (B) Fair/ Fare

(C) Faer/ Fair (D) Fare/fare

Ans. A

Sol. Fare : The amount of money you pay to travel by Bus, Train, Taxi, etc.

Fair : appropriate and acceptable in a particular situation.

The **fare** is too high for it to be considered **fair**.

Hence, the correct option is (A).

Question 2

A palindrome is a word that reads the same forward and backward. In a game of words, a player has the following two plates painted with letters

$\boxed{\text{A}}$ $\boxed{\text{D}}$

From the additional plates gives in the options, which one of the combinations of additional plates would allow the players to construct in five letter palindrome. The players should use all five plates exactly once. The plates can be rotated in their plane.

(A) $\boxed{\text{R}}$ $\boxed{\text{∀}}$ $\boxed{\text{ꓤ}}$ (B) $\boxed{\text{Z}}$ $\boxed{\text{Ǝ}}$ $\boxed{\text{D}}$

(C) $\boxed{\text{I}}$ $\boxed{\text{⅂}}$ $\boxed{\text{Y}}$ (D) $\boxed{\text{D}}$ $\boxed{\text{ᗡ}}$ $\boxed{\text{⅂}}$

Ans. A

Sol. Given :

A palindrome is a word that reads the same forward and backward.

In a game of words, a player has the two plates painted with letters $\boxed{\text{A}}$ $\boxed{\text{D}}$

The player need to construct a five letter palindrome by using those given two letter plates.

The player should use all five plates exactly one and the plates can be rotated in their plane.

According to option A,

$\boxed{\text{R}}$ $\boxed{\text{∀}}$ $\boxed{\text{ꓤ}}$

Can be used by rotating and combining with given letter plates.

The palindrome in a word will look live

$\boxed{\text{R}}$ $\boxed{\text{A}}$ $\boxed{\text{D}}$ $\boxed{\text{A}}$ $\boxed{\text{R}}$

That word will give same meaning either reads forward or backwards.

Hence, the correct option is (A).

Question 3

Let r be a root of the expression $x^2 + 2x + 6 = 0$

Then the value of the expression $(r+2)(r+3)(r+4)(r+5)$ is

(A) -51 (B) 126

(C) -126 (D) 51

Ans. **C**

Sol. **Given :** r is a root of the expression $x^2+2x+6=0$

$$r^2+2r+6=0\,,\; r^2+2r=-6$$
$$=(r+2)(r+3)(r+4)(r+5)$$
$$=(r^2+5r+6)(r^2+9r+20)$$
$$=(r^2+2r+3r+6)(r^2+2r+7r+20)$$
$$r^2+2r=-6,$$
$$=(-6+3r+6)\,(-6+7r+20)$$
$$=3r(7r+14)$$
$$=21r(r+2)$$
$$=21(r^2+2r)=21\times(-6)=-126$$

Hence, the correct option is (C).

Question 4

A function $y(x)$ is defined in the interval $[0,1]$ on the x-axis as

$$y(x)=\begin{cases}2 \text{ if } 0\le x\le \frac{1}{3}\\ 3 \text{ if } \frac{1}{3}\le x\le \frac{3}{4}\\ 1 \text{ if } \frac{3}{4}\le x\le 1\end{cases}$$

Which one of the following is the area under the curve for the interval [0, 1] on the x–axis?

(A) $\frac{13}{6}$ (B) $\frac{6}{5}$

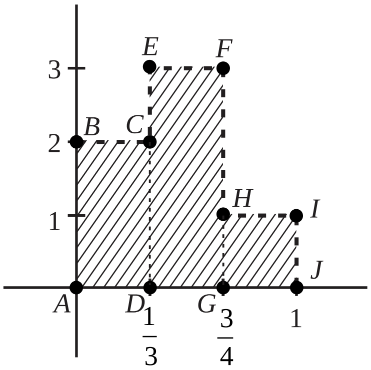
(C) $\frac{6}{13}$ (D) $\frac{5}{6}$

Ans. **A**

Sol. **Given :** A function $y(x)$ is defined in the interval [0, 1] on the x-axis as

$$y(x)=\begin{cases}2 \text{ if } 0\le x\le \frac{1}{3}\\ 3 \text{ if } \frac{1}{3}\le x\le \frac{3}{4}\\ 1 \text{ if } \frac{3}{4}\le x\le 1\end{cases}$$

Graph of $y(x)$ vs x, is drawn as shown below figure,



The Area under the curve for the interval [0, 1] on the x-axis will be define as

= Area of $ABCD$ + Area of $DEFG$ + Area of $GHIJ$

Area of the rectangle = Length $\times$ Breadth

$$= 2\times\frac{1}{3}+3\times\left(\frac{3}{4}-\frac{1}{3}\right)+1\left(1-\frac{3}{4}\right)$$

$$=\frac{2}{3}+\frac{15}{12}+\frac{1}{4}$$

$$=\frac{2}{3}+\frac{3}{2}$$

$$=\frac{13}{6}$$

Hence, the correct option is (A).

Question 5

Given below are four statements

Statement 1 : All students are inquisitive

Statement 2 : Some students are inquisitive

Statement 3 : No students are inquisitive

Statement 4 : Some students are not inquisitive

For the given four statements, find the statements what cannot be true, simultaneously, assuming that there is at least one student in the class

(A) Statement 1 and Statement 3 (B) Statement 3 and Statement 4

(C) Statement 2 and Statement 4 (D) Statement 1 and Statement 2

Ans. **A**

Sol. **Given :**

Statement 1 : All student are inquisitive.

Statement 2 : some student are inquisitive

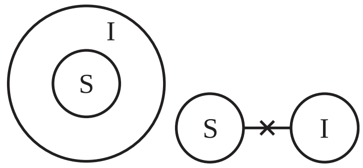
Statement 3 : No student are inquisitive.

Statement 4 : Some student are not inquisitive

For finding the statement which cannot be true, simultaneously, we will go through with all options,

According to option (A).

Statement 1 and statement 3 can not be true simultaneously by the venn diagram of the statement



We can say that statement 1 and statement 3 cannot be true simultaneously.

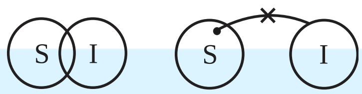
According to option (B)

Statement 3 and Statement 4 can not be true simultaneously. By the venn diagram of the statements

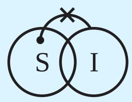
We can say that both the statement can be true simultaneously. As when no student is inquisitive some students are also not inquisitive.

According to option (C).

Statement 2 and statement 4 can not be true simultaneously by the venn diagram of the statements



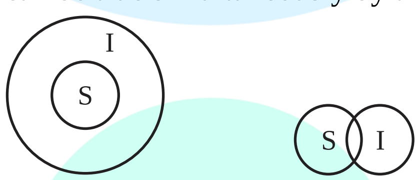
We can say that both the statement can be true simultaneously by that diagram.



By this diagram it is possible of the same time that some students are inquisitive and some students are not inquisitive.

According to option (D).

Statement 1 and statement 2 can be true simultaneously by the venn diagram of the statement.



By this diagram we can say that both the statement can be true simultaneously.

As, when all students are inquisitive then some students are already inquisitive.

Hence, the correct option is (A).

Q.6 to Q.10 Carry Two Marks Each

Question 6

A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn it is not replaced. If an orange ball is drawn it is replaced with another orange ball. First ball is drawn. What is the probability of getting an orange ball in the next drawn?

(A) $\frac{23}{50}$ (B) $\frac{19}{50}$

(C) $\frac{8}{25}$ (D) $\frac{1}{2}$

Ans. A

Sol. **Given :**

A box contains five balls of same size and shape.

Three of them are green coloured balls and two of them are orange coloured balls.

Balls are drawn from the box one at a time.

If a green ball is drawn it is not replaced.

If an orange ball is drawn it is replaced with another orange ball.

For the probability of getting an orange ball in the next drawn we have two different cases.

Case I : One Green and one Orange

$$P(E) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$$

As we can't replace ball after getting green ball so when we are drawing orange ball as that time only 4 balls are remaining in box.

Case II : One Orange and one orange

$$P(E) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

As with orange we can replace ball so when we are drawing record orange ball at time 5 balls are present in box.

$$P(E) = \text{ case 1 + case 2}$$

$$P(E) = \frac{6}{20} + \frac{4}{25} = \frac{23}{50}$$

Hence, the correct option is (A).

Question 7

Some people believe that "what gets measured, improves". Some other believe that "what gets measured, gets gamed". One possible reason for the difference in the beliefs is the work culture in organization. In organizations with good work culture, metrics help improve outcomes. However the same metrics are counterproductive in organizations with poor work culture.

Which one of the following is the correct logical inference based on the information in the above passage?

(A) Metrics are useful in organization with poor work culture

(B) Metrics are always counterproductive in organizations with good work culture

(C) Metrics are never useful in organization with good work culture

(D) Metrics are useful in organizations with good work culture

Ans. **D**

Sol. According to the given information,

Option (A); can not be inferred as metrics are counterproductive in organization with poor work culture.

Option (B); can not be inferred as metrics are counterproductive in organization with poor work culture not good work culture.

Option (C); can not be inferred as metrics help improve outcomes, in organization with good work culture.

Option (D); can be inferred as metrics are useful in organization with good work culture, is mentioned.

Hence, the correct option is (D).

Question 8

The corners and the mid point of a triangle are name using distinct letters P, Q, R, S, T and U but not necessarily in the same order. Consider the following statements

- The line joining P and R is parallel to the line joining Q and S
- P is placed on the side opposite to the corner T
- S and U cannot be placed in the same side

Which one of the following statements is correct based on the above information?

(A) P cannot be placed at a corner (B) U cannot be placed at a mid point

(C) S cannot be placed at a corner (D) R cannot be placed at a corner

Ans. **C**

Sol. **Given :**

The corners and midpoint of a triangle are name using distinct P, Q, R, S, T and U.

The line joining P and R is parallel to the line joining Q and S.

P is placed on the side opposite to the care T.

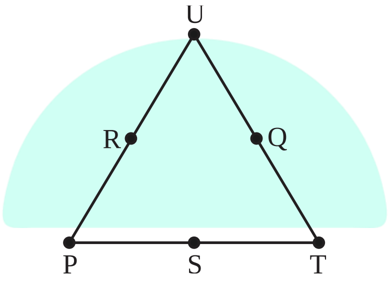
S and U cannot be placed in the same side.

with this given information we can draw multiple diagram which satisfy all the given conditions.

To prove given statement in the options which one is correct, we will draw different diagrams for each option.

According to option A,

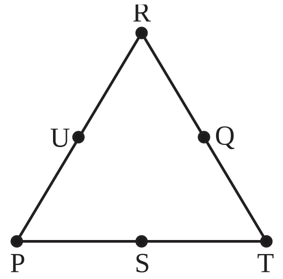
P cannot be placed at a corner but by the figure shown below,



P can be placed at a corner.

According to option B,

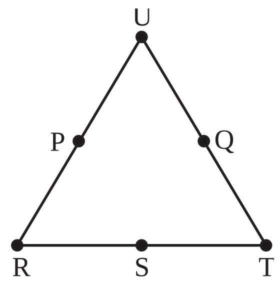
U cannot be placed at a mid point but by the figure shown below,



U can be placed at a midpoint.

According to option D,

R cannot be placed at a corner by but by the figure shown below,



R can be placed at a corner.

According to option C,

S cannot be placed at a corner

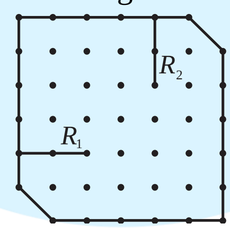
By all the possible diagram only that condition is not possible.

Hence, the correct option is (C).

Question 9

A plot of land must be divided between four families. They instant their individual plots to be similar in shape, not necessarily in area. The land has equally spaced placed marked as dots on the below figure. Two ropes R_1 and R_2 are already present and cannot be moved.

What is the least number of additional straight ropes needed to needs to divided plot? A straight rope can pass through three poles that are aligned in a straight line



(A) 2 (B) 3

(C) 4 (D) 5

Ans. **B**

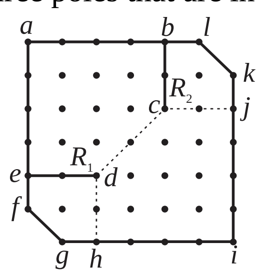
Sol. **Given :**

A plot of land must be divided between four families.

Divided plots should be similar in shape not necessarily in area.

Two ropes R_1 and R_2 are already present in the plot and cannot be moved.

A straight rope can pass through three poles that are in a straight line.



In the figure shown above we can clearly see the least number of additional straight ropes shown by dotted line needed to divided plot in four parts.

The plots are abcde, bcjkl, cdhij and efghd.

Which are same in shape but not in size.

Hence, the correct option is (B).

Question 10

In a currently conducted National Entrance Test, Boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates. Which one of the following is the correct logical inference based on the information provided in the above passage?

(A) Equal number of boys and girls appeared for the test

(B) The number of boys who qualified the test is less than the number of girls who qualified.

(C) The number of boys who appeared for the test is less than the number of girls who appeared.

(D) Equal number of boys and girls qualified.

Ans. B

Sol. **Given :** In a currently conducted National Entrance Test,

Percentage of boys who appeared for the test is 65%

Percentage of girls who appeared for the test is 100% – 65% = 35%

Percentage of qualified girls is 60% of appeared girls

Accounted percentage of total qualified boys and girls is equals to appeared percentage of girls.

Let, the number of students is x,

Number of boys appeared for the test is $0.65x$

Number of girls appeared for the test is $0.35x$

Number of qualified girls $= 0.35x \times \frac{3}{5} = 0.21x$

Number of qualified boys $= 0.35x - 0.21x = 0.14x$

So, the number of boys who qualified the test is less than the number of girls who qualified.

Hence, the correct option is (B).

Technical Part

Q.11 to Q.35 Carry One Mark Each

Question 11 **698808206**

Which one of the following statement is TRUE for all positive function $f(n)$?

(A) $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial

(B) $f(n^2) = o(f(n)^2)$

(C) $f(n^2) = O(f(n^2))$, when $f(n)$ is an exponential function

(D) $f(n^2) = \Omega(f(n)^2)$

Ans. A

Sol. A : It need not be true for a function which is decreasing.

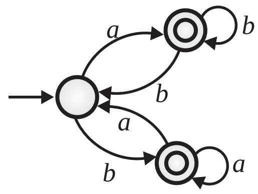
B : An exponential function may be increasing or decreasing, so this condition may not always be true.

C : It always holds because if we square the input variable, then the highest order in the polynomial will also get squared.

D : It is not true in cases when $f(n)$ is a polynomial function.

Question 12 **698808207**

Which one of the following regular expression correctly represents the language of the finite automaton given below?

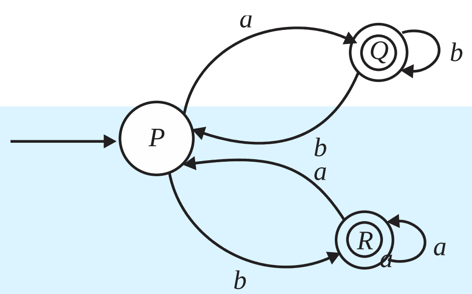


(A) $ab*bab*+ba*aba*$ (B) $(ab*b)*ab*+(ba*a)*ba*$

(C) $(ab*b+ba*a)*(a*+b*)$ (D) $(ba*a+ab*b)*(ab*+ba*)$

Ans. **D**

Sol. **Given : From**



Finite automata

We have two final state Q, R

First, find the regular expression for set of all strings that end up at the initial state P when running the given NFA.

Lets call P.

Note that, P is the initial state so NFA execution starts at P. Now to end up on state P at the end of the string we can do any of the following, in any order, any number of times:

1. Read "a" go to state Q then read any number of b's, then read b to come back to state P. so, 1 ab*b

2. Read "b" go to state R, them read any number of a's, then read a to come back

So, $P = (1+2)*$

$$= [(ab*b)+(bc*a)]*$$

Because we can do 1 or 2 in any order, any number of times:

Now, we want to find language of the given NFA if, Whose final states are Q,R

So, $R\ \varepsilon\ (iV) = Q + R$

Where Q is the regular expression for set of all strings that end up at the state Q R is the regular expression for set of all strings.

$$Q = P_{ab}*$$

$$R = Pba$$

So, $R\ \varepsilon\ (iv) = Q + R$

$$= Pab*+Pba*$$

$$= P\ (ab*+ba*)$$

$$R\ \varepsilon\ (iv) = [(ab^*b) + (ba^*a)]^*$$

$$(ab^* + ba^*)$$

$$\Rightarrow \quad (ab^*b + ba^*a)(ab^* + ba^*)$$

Hence, the correct option is (D).

Question 13 **698808208**

Which of the following statements is TRUE?

(A) The LALR (1) parser for a grammar G cannot have reduce – reduce conflict if the LR (1) parser for G does not have reduce – reduce conflict

(B) Symbol table is accessed only during the lexical analysis phase.

(C) Data flow analysis is necessary for run time memory management.

(D) LR (1) parsing is sufficient for Deterministic Context Free Language.

Ans. **D**

Sol. **Given statements are as follows :**

Statement A: The LALR (1) parser for a grammar G cannot have reduce – reduce conflict if the LR (1) parser for G does not have reduce – reduce conflict.

False, because the LALR (1) parser for a grammar G can have reduce-reduce conflict, even through LR (1) parser for G does not have reduce-reduce conflict.

Statement B: Symbol table is accessed only during the lexical analysis phase.

False, symbol table can be accessed by most of the phases of a compiler, like beginning with lexical analysis, and continuing through optimization.

Statement C: Data flow analysis is necessary for run time memory management.

False, data flow analysis is used for the optimization of codes in control flow graph.

Statement D: LR (1) parsing is sufficient for Deterministic Context Free Language.

True, LR (1) parsing is sufficient for Deterministic Context Free Language.

Hence, the correct option is (D).

Question 14 **698808209**

In relational data model which is true.

(A) A relation with only two attribute is always in BCNF

(B) BCNF decompositions preserve functional dependencies

(C) Every relation has at least one non prime attribute

(D) If all attributes of a relation are prime attributes, then relation is in BCNF.

Ans. **A**

Sol. **Given :**

In relational data model,

Following statements are as follows:

Statement A : Relation with only two attribute is always in BCNF

True,

$$R(\mathrm{XY})$$

$$\{X \to Y\} \Rightarrow BCNF$$

$$\{Y \to X\} \Rightarrow BCNF$$

$$\{X \to Y, Y \to X\} \Rightarrow BCNF$$

$$\{\text{no non trival functional dependency}\} \Rightarrow BCNF$$

$\therefore$ A relation with two attributes is always in BCNF.

Statement B: BCNF decompositions preserve functional dependencies

False, BCNF can be used to obtain a lossless join decomposition into 3NF but does not ensure dependency preservation.

Statement C: Every relation has at least one non prime attribute

False, Let relation R with attributes (A, B, C) with functional dependency set

$$\{AB \to C, C \to A\}$$

In this relation all attributes are prime.

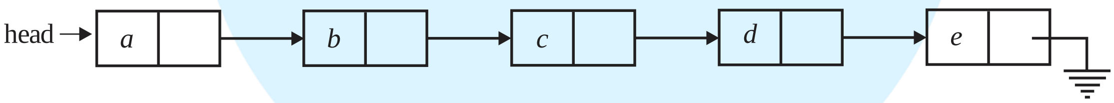
Statement D: If all attributes of a relation are prime attributes, then relation is in BCNF.

False, If all attributes of a relation are prime attributes, then relation is in 3NF, not in BCNF.

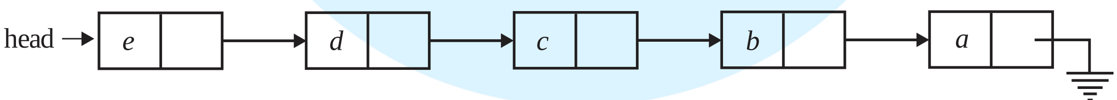
Hence, the correct option is (A).

Question 15 **698808210**

Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversed linked list should look like



Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in O (1) space?

(A) The best algorithm for the problem takes $\theta(n)$ time in the worst case.

(B) The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.

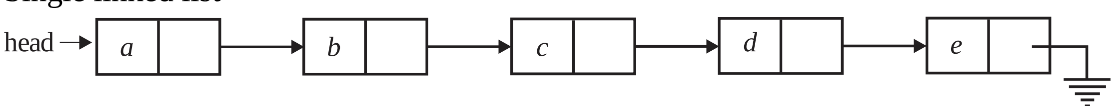
(C) The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.

(D) It is not possible to reverse a singly linked list in O (1) space.

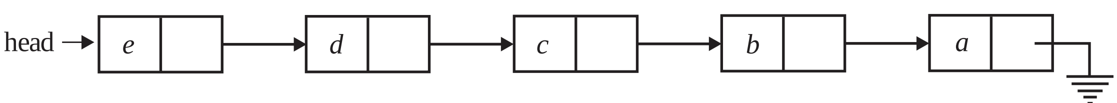
Ans. **A**

Sol. **Given :**

Single linked list



And the reversed linked list should look like



So, From given statements, statement A is suitable for the time complexity of algorithms that solve the given problem in O (1) space.

So, Best algorithm for the problem takes $\theta(n)$ time in worst case.

Hence, the correct option is (A).

Question 16 **698808211**

Suppose we are given n keys, m hash table slots and two simple uniform hash functions h_1 and h_2. Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected no. of keys in a slot?

(A) $\frac{n}{m}$ (B) $\frac{m}{n}$

(C) $\frac{n}{2m}$ (D) $\frac{2n}{m}$

Ans. **A**

Sol. **Given :** *n* keys, *m* hash table slots, and two simple uniform hash functions h_1 and h_2.

Hashing scheme uses h_1 for the odd keys and h_2 for the even keys.

Uniform hash function definition: $Pr[h(x)=i]=1/m$

which means for every x, we have an equal probability of mapping to any of slot i.

The hash function given in the question is Uniform:

Take any slot i and calculate the probability of mapping some arbitrary x to i

i.e. $Pr[h(x)=i]=?$

Let the probability of choosing h_1 is p and choosing h_2 is 1−p then

$$Pr[h(x)=i]=pPr[h_1(x)=i]+(1-p)Pr[h_2(x)=i]$$

$\Rightarrow$ $Pr[h(x)=i]=p/m+1-p/m$ (since h_1 and h_2 both are uniform hash functions)

$=1/m$

As you see, value of p does not matter but we can calculate p as = number of even keys / Total keys

Since *h* is uniform, we can say it will distribute all keys uniformly. This means if there are 50 keys and 10 slots then each slot will get 5 keys. i.e., n/m is answer.

But we can do this using probability.

Let X = the Number of items in slot 1. (Note that I am. only talking about some random slot, say slot 1)

X_i

= {1 if i^{th} item maps to slot 1; otherwise, 0}

So, we can say $X = X_1 + X_2 + \ldots X_n$

They are asking E[X]

$$E[X]=E[X_1+X_2+\ldots X_n]$$

$$E[X]=E[X_1]+E[X_2]+\ldots E[X_n]$$

$P(X_i=1)=1/m$ because whatever happens within the black box (h_1 or h_2), the overall hash function will be uniform.

$$E[X_i]=1P(X_i=1)+0P(X_i=0)=1/m$$

So, $E[X]=n/m.$

Hence, the correct option is (A).

Question 17 **698808212**

Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?

(A) DMA based I/O transfer (B) Interrupt driven I/O transfer

(C) Polling based I/O transfer (D) Programmed I/O transfer

Ans. **A**

Sol. from the given options, option (A) DMA based input output transfer facilitates transfer of bulk data from secondary memory or hard disk to main memory without the use of CPU.

Hence, the correct option is (A).

Question 18 **698808213**

Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation R1+R2, which one of the following values of R1 and R2 gives an arithmetic overflow?

(A) R1 = 1011 and R2 = 1110 (B) R1 = 1100 and R2 = 1010

(C) R1 = 0011 and R2 = 0100 (D) R1 = 1001 and R2 = 1111

Ans. **B**

Sol. **Given :**

R1 and R2 are two 4 – bit registers and they stored numbers in 2's complement form.

We know that the range of 2's complement numbers representable with 4 bits are –8 to +7.

Then, if the result of the operation R1 + R2 is out of the range –8 to 7, then it is over flow

So, from given options,

Option (A) :

$R1$ = 1011 and R2 = 1110

$$R1 = -1\times 2^3 + 0\times 2^2 + 0\times 2^1 + 1\times 2^0 = -7$$

$$R2 = -1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 1\times 2^0 = -1$$

$$R1 + R2 \ = -8(\text{no overflow})$$

Option (B) :

$R1$ = 1100 and R2 = 1010

$$R1 = -1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 = -4$$

$$R2 = -1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 0\times 2^0 = -6$$

$$R1 + R2 = -10(\text{over flow})$$

Option (C) :

$R1$ = 0011 and R2 = 0100

$$R1 = -1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 = -5$$

$$R2 = -1\times 2^3 + 1\times 2^2 + 1\times 2A1 + 0\times 2^0 = -2$$

$$R1 + R2 = -7(\text{ no overflow})$$

Option (D) :

$R1$ = 1001 and R2 = 1111

$$R1 = 0\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 = 3$$

$$R2 = 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 = 4$$
$$R1 + R2 = 7(\text{no overflow})$$

Hence, the correct option is (B).

Question 19 **698808214**

Consider the following threads, $T_1, T_2,$ and T3 executing on a single processor, synchronized using three binary semaphore variables, S_1, S_2 and S_3, operated upon using standard wait() and signal(). The threads can be context switched in any order and at any time.

T_1	T_2	T_3
while (true)	while (true)	while (true)
{	{	{
wait (S_3);	wait (S_1);	wait (S_2);
print ("C");	print ("B");	print ("A");
signal (S_2);	signal (S_3);	signal (S_1);
}	}	}

Which initialization of the semaphores would print the sequence BCABCABCA….?

(A) $S_1 = 1;\ S_2 = 1; S_3 = 1$ (B) $S_1 = 1; S_2 = 1; S_3 = 0$

(C) $S_1 = 1; S_2 = 0; S_3 = 0$ (D) $S_1 = 0; S_2 = 1; S_3 = 1$

Ans. **C**

Sol. **Given :**

A single processer in which three thread T_1, T_2 and T_3 are executing.

And they are synchronized by using three binary semaphore variables such that S_1, S_2 and S_3

They are operated by using standard wait ( ) & signal ( ) operation.

So, for checking that which initialization of semaphores would print the sequence of BCABCABCA…

Then, initially if $S_1 = 1, S_2 = 0, S_3 = 0,$

Process P_2 can successfully execute wait (S_1); while P_1 and P_3 remain stuck at wait (S_3); and wait (S_2); respectively.

After process P_2 prints B it executes signal (S_3); and gets stuck at wait (S_1);

Here B gets printed in this process.

After this Process P_1 can successfully execute wait (S_3); and then it executes print("C");, after which it executes signal (S_2); and then gets stuck at wait (S_3);

Here C gets printed in this process.

After this Process P_3 can successfully execute wait (S_2): and then it executes print ("A");, after which it executes signal (S_1): and then gets stuck at wait (S_2);

Here A gets printed in this process.

After this Process P_2 can execute wait (S_1); successfully.

The process thus keeps repeating and the pattern printed is BCABCABCA…

Hence, the correct option is (C).

Question 20 **698808215**

Consider the following two statements with respect to matrices

$$A_{m\times n}, B_{n\times m}, C_{n\times n} \text{ and } D_{n\times n}$$

Statement 1 : $Tr(AB) = Tr(BA)$

Statement 2 : $Tr(CD) = Tr(DC)$

Tr represents the trace of matrix, which one of the following holds _______.

(A) S1 is wrong and S2 is correct

(B) S1 is correct and S2 is wrong

(C) Both statements are correct

(D) Neither of the statements correct

Ans. **C**

Sol. **Given :** Two statements with respect to matrices $A_{m\times n}, B_{n\times m}, C_{n\times n}$ and $D_{n\times n}$

Statement 1 : $Tr(AB) = Tr(BA)$

Statement 2 : $Tr(CD) = Tr(DC)$

Then, from question.

The eigen values (counting multiplicity) of AB are the same as those of BA.

This is a corollary of theorem in second edition of matrix analysis by Horn and Johnson.

Paraphrasing from the cited theorem. If A is an m by n matrix and B is an n by m matrix with $n > m$ then the characteristic polynomial P_{BA} of BA is related to the characteristic polynomial P_{AB} of AB.

Since, eigen values are the same, thus the sum of eigen values (i.e. the trace) is also the same.

Hence, the correct option is (C).

Question 21 **698808216**

What is printed by ANSI C program?

```
#include <stdio.h>
int main (int argc, char *argv [])
{
    int x=1, z[2]={10,11};
    int *p = NULL;
     p= &x;
    *p = 10;

     p = &z[1];
    *(& z [0]+1) +=3;
    printf("%d, %d, %d\n", x , z [0], z [1]);
    return 0;
}
```

(A) 10, 10, 14

(B) 1, 10, 11

(C) 1, 10, 14

(D) 10, 14, 11

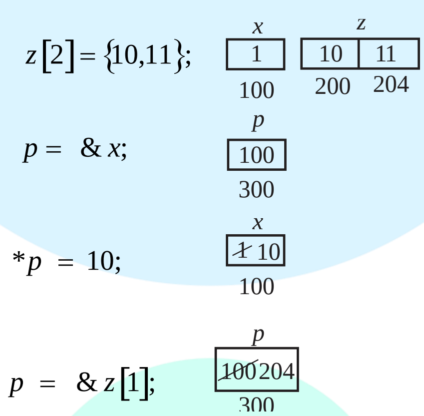
Ans. A

Sol. **Given :**

ANSI C program :

```
    #include <stdio.h>
    int main (int argc, char *argv [])
{
    int x=1, z[2]={10,11};
    int *p = NULL;
     p= &x;
    *p = 10;

     p = &z[1];
    *(& z [0]+1) +=3;
    printf("%d, %d, %d\n", x , z [0], z [1]);
    return 0;
}
```



*   (& z [0]+1) +=3;

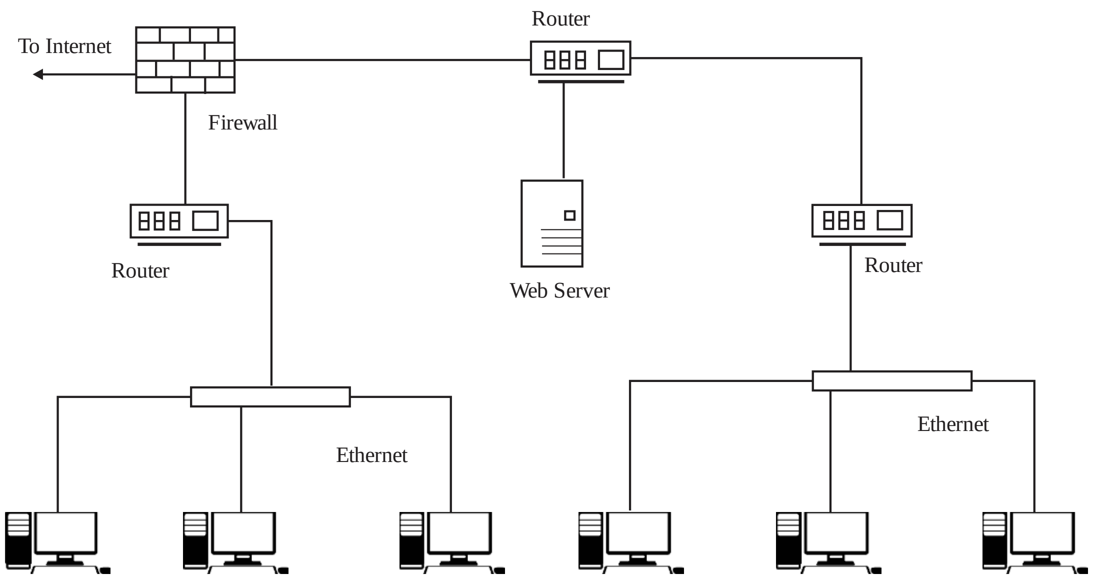
$\Rightarrow$ *(address of IInd element of array) + = 3;

$\Rightarrow$ i.e. add 3 to IInd element of array, therefore second elements becomes 14.

Hence, the correct option is (A).

Question 22 **698808217**

Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers shown below :



What is the number of subnets in the enterprise network?

(A) 3 (B) 12

(C) 6 (D) 8

Ans. **C**

Sol. **Given :** A network which has Two Ethernet segments, a web server and a firewall.

And they are connected through three routers.

So, from given figure,

This is just like non equal sub netting where router 2 has the half of the addresses, Router 1 has other half, which is further divided into two subnets which is Web server and router 3, So total of 3 subnets possible.

Hence, the correct option is (C).

Question 23 **698808218**

Which of the following statements is/are true?

(A) Every subset of recursively enumerable Language is recursive

(B) If Language L & its complement $\bar{L}$ are both recursively enumerable then L must be recursive

(C) Complement of context free language must be recursive.

(D) If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be Deterministic Context Free Language.

Ans. **B, C, D**

Sol. **Given :**

Statement A : Every subset of recursively enumerable Language is recursive

False, Since, $\sum^*$ is a recursively enumerable Language but there are many languages which are subset of $\sum^*$ which are not Recursive.

Statement B : If Language L & its complement $\bar{L}$ are both recursively enumerable then L must be recursive

True, If L is recursively enumerable Language, then for all member strings of L, the TM of L will halt within finite time. If $\bar{L}$ is recursively enumerable Language, then for all nonmember strings of L. The TM of $\bar{L}$ will halt within finite time. Thus for both member and nonmember strings of L and $\bar{L}$ we have a TM which halts within finite time. Thus L is a Recursive Language.

Statement C : Complement of context free language must be recursive.

True, Complement of CFL is a CSL in the worst case which is Recursive.

Statement D : If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be Deterministic Context Free Language.

True, If L_1 and L_2 are regular then $L_1 \cap L_2$ is Regular thus also deterministic context free Language.

Hence, the correct options are (B), (C), (D).

Question 24 **698808219**

Let WB and WT be 2 set associative cache organizations that use LRU algorithm for cache block replacement. WB is Write Back cache and WT is Write through cache. Which of the following statements are false?

(A) Each cache block in WB and WT has a dirty bit

(B) Every write hit in WB leads to a data transfer from cache to main memory

(C) Eviction of a block from WT will not lead to data transfer from cache to main memory

(D) A read miss in WB will never lead to eviction of a dirty block from WB

Ans. **A, B, D**

Sol. **Given :**

WB and WT is two set associative cache organization,

Where WB belongs to write back cache and WT belongs to write through cache.

LRU algorithm used for cache block replacement.

So, from given statements:

Statement A : Each cache block in WB and WT has a dirty bit

Statement B : Every write hit in WB leads to a data transfer from cache to main memory. False, for the hit operation no need to fetch the data from the main memory.

Statement C : Eviction of a block from WT will not lead to data transfer from cache to main memory

Statement D : A read miss in WB will never lead to eviction of a dirty block from WB

Hence, the correct options are (A), (B), (D).

Question 25 **698808220**

Consider the following three relations in a relational database.

Employee (eId, Name)

Brand (bId, bName)

Own (eId, bId)

Which of the following relational algebra expressions return the set of eIds who own all the brands?

(A) $\pi_{eId}(\pi_{eId,bId}(\text{Own}) / \pi_{bId}(\text{Brand}))$

(B) $\pi_{eId}(\pi_{eId,bId}(\text{Own}) / \pi_{bId}(\text{Own}))$

(C) $\pi_{eId}(\text{Own}) - \pi_{eId} / \pi_{eId}(\text{Own}) \times \pi_{bId}(\text{Brand}) - \pi_{eId,bId}(\text{Own}))$

(D) $\pi_{eId}(\pi_{eId}(\text{Own}) \times \pi_{bId}(\text{Own})) / \pi_{bId}(\text{Brand})$

Ans. A, C

Sol. Given :

Three relations in relational database are as follows:

Employee (eId, ename)

Brand (bId, bname)

Own (eId, bId)

To find set of eIds who own all the brand we need to divide the table own by table brand like.

(A) $\pi_{eId}(\pi_{eId,bId}(\text{Own}) / \pi_{bId}(\text{Brand}))$ and the division operation can be represented using $(\times, -)$ operations like

(C) $\pi_{eId}(\text{Own}) - \pi_{eId} / \pi_{eId}(\text{Own}) \times \pi_{bId}(\text{Brand}) - \pi_{eId,bId}(\text{Own}))$

Hence, the correct options are (A), (C)

Question 26 **698808221**

Which of the following statements is/are TRUE with respect to deadlocks?

(A) In a system where each resource has more than one instance, a cycle in its wait-for-graph indicates the presence of a deadlock.

(B) In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

(C) Circular wait is a necessary condition for the formation of deadlock

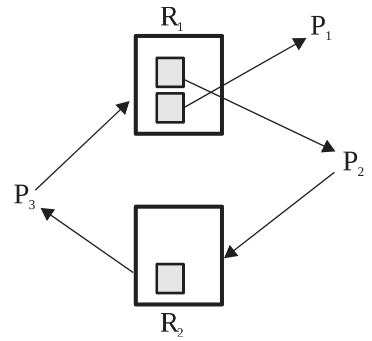
(D) If the current allocation of resources to processes leads the system to unsafe state then deadlock will necessarily occur.

Ans. B, C

Sol. Given :

Statement A : In a system where each resource has more than one instance, a cycle in its wait-for-graph indicates the presence of a deadlock.

False, A cycle exist in the system but since P_1 does not need any more resources it will finish after some time and P_3 will get resource R_1 this even in the presence of cycle the deadlock does not exist here



Statement B : In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

True, When every edge is the assignment edge, that is, no process needs any more resources than the ones that are already allocated, thus deadlock doesn't exist here.

Statement C: Circular wait is a necessary condition for the formation of deadlock

True, Circular wait is a necessary condition for Deadlock but it is not sufficient. There are following 4 necessary conditions for the occurrence of deadlock

1. Mutual exclusion
2. Hold and wait
3. No preemption
4. Circular wait

Statement D : If the current allocation of resources to processes leads the system to unsafe state then deadlock will necessarily occur.

False, Even if the allocation results in an unsafe state, some processes still may release the resources that are allocated to them for a while which may lead to elimination of deadlock from the system.

Hence, the correct options are (B), (C).

Question 27 **698808222**

Which of the following is /are true for group G?

(A) If for all $x, y \in G$, $(xy)^2 = x^2y^2$, then G is commutative.

(B) If for all $x \in G, x^2 = 1$, then G is commutative, here 1 is identity element of G

(C) If the order of G is 2, then G is commutative

(D) If G is commutative then a subgroup of G need not to be commutative

Ans. A, B, C

Sol. **A. True**

Since $(ab)^2 = a^2b^2 \rightarrow (ab)(ab)=(aa)(bb)$

We know that in a group, left and right cancellation is allowed, so

ba = ab, Hence Commutative

B. True

Key Point: $x^2 = 1$ means each element is inverse of itself.

Let a,b be two elements in group G. Consider the element ab.

Now, $(ab)^2=1 \rightarrow ab.ab=1$

Now, multiply both sides on the right with b,

ababb = 1b $\rightarrow$ aba = b

Now, multiply both sides on the right with a,

abaa = ba $\rightarrow$ ab = ba

Hence, G is abelian.

C. True

Theorem : All groups with less than 6 elements are abelian.

D. False

Theorem : Every subgroup of an abelian group has to be abelian.

Question 28 **698808223**

Suppose binary search tree with 1000 distinct elements is also complete binary tree. The tree is sorted using array representation of Binary Heap Tree. Assuming that the array indices start with 0, the 3rd largest element of tree is stoled at index _____?

Ans. 509

Sol. **Given :**

Binary search tree which has 1000 distinct elements & it also work as complete binary tree.

Array representation of binary heap tree is used for sorting of tree,

So, the largest element in the BST is the right most element with index $510(2^9 - 2$ as the indexing of the array is starting with 0) its parent is the second-largest element and its left child is the third-largest element which is of the index $510 - 1 = 509$.

Hence, the correct Answer is 509.

Question 29 **698808224**

Consider the augmented grammar with $\{+, *, (,), id\}$ as the set of terminals.

$$S' \to S$$
$$S \to S + R \mid R$$
$$R \to R * P \mid P$$
$$P \to (S) \mid id$$

If I_0 is the set of two $LR(0)$ items $\{[S' \to S.], [S \to S.+R]\}$, then go to $(\text{Closure}(I_0), +)$ contains exactly ______ items.

Ans. **5**

Sol. **Given :**

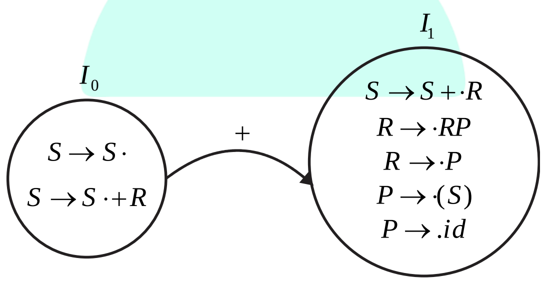
Augmented grammar

$$S' \to S$$
$$S \to S + R \mid R$$
$$R \to R * P \mid P$$
$$P \to (S) \mid id$$

And set of terminals = $\{+, *, (,), id\}$

And there is a condition if I_0 is the set of two LR (0) Items such that $\{[S' \to S.], [S \to S.+R]\}$,

Then go to $(\text{Closure}(I_0), +)$ contains :



$\therefore$ 5 item

Hence, the correct answer is 5.

Question 30 **698808225**

Consider a simple undirected graph of 10 vertices if the graph is disconnected then the maximum number of edges it can have is__________.

Ans. **36**

Sol. **Given :**

Undirected graph which has 10 vertices and we have to find the maximum number of edges when the graph is disconnected.

So, suppose we have 1 vertex on one side and other n-1 vertices on another side. To make it connected maximum possible edges (if consider it as complete graph) is

$$C_2^{n-1} \text{ which is } \frac{(n-1)(n-2)}{2}$$

Thus to make it a disconnected graph we have 1 separate vertex on another side which is not connected. Thus the maximum possible edges is

$$C_2^{n-1} = {}^9C_2 = \frac{9\times 8}{2} = 36$$

Hence, the correct answer is 36.

Question 31 **698808226**

Consider a Relation R (A, B, C, D, E) with the following three functional dependencies.

$AB \to C,\ BC \to D,\ C \to E$

The number of super keys in the relation R is _______.

Ans. 8

Sol. Given :

Relation R with attributes (A, B, C, D, E) and

There are three functional dependencies such that

$AB \to C$

$BC \to D$

$C \to E$

So, from these functional dependencies,

Candidate key = AB

$\therefore$ Number of super keys $2^{5-2} = 2^3 = 8$ super keys

Hence, the correct answer is 8.

Question 32 **698808227**

The number of arrangement of 6 identical ball in 3 identical bins__________.

Ans. 7

Sol. Given :

6 identical balls

For which we have to find out the number of arrangement for these balls into three identical bins

Now, it is the case of distribution with identical objects and identical boxes.

We simply need to partition number 6 in to maximum 3 parts and count the partition.

(6,0,0), (5,1,0), (4,2,0), (4,1,1), (3,2,0), (3,1,1), (2,2,2).

Hence, the correct answer is 7.

Question 33 **698808228**

A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (rounded off to decimal two places) needed after the optimization such that it should not increase the average memory access time is ______.

Ans. 0.85

Sol. **Given :**

a cache

which has,

Hit rate = 0.8

$\therefore$ miss rate = 0.2

Access latency = 10 ns

Miss penality = 100 ns

$$\therefore \quad \text{AMAT}_{old} = 0.8(10) + 0.2(100) = 8 + 20 = 28$$

Where, AMAT is average memory access time.

For Optimized cache.

where access latency = 15 ns = hit time

Let x be the cache hit rate after optimization.

$$\therefore \quad \text{AMAT}_{\text{new}} = x(15) + (1-x)100$$

$$= 15x + 100 - 100x = 100 - 85x$$

$$\text{AMAT}_{\text{Old}} \geq \text{AMAT}_{\text{new}}$$

$$\rightarrow \quad 100 - 85x \leq 28$$

$$\rightarrow \quad 72 = 85x$$

$$\rightarrow \quad x = \frac{72}{85} = 0.85$$

The required hit rate = 0.85

Hence, the correct answer is 0.85.

Question 34 **698808229**

The value of following limit is ________.

$$\lim_{x \to 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

Ans. **– 0.5**

Sol. **Given :**

$$\lim_{x \to 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}} \text{ is in the form of } \frac{0}{0} \text{ form at}$$

If we apply L'Hopitals rule then

$$= \lim_{x \to 0^+} \left[\frac{\frac{d}{dx}(\sqrt{x})}{\frac{d}{dx}(1 - e^{2\sqrt{x}})} \right] = \lim_{x \to 0^+} \left[\frac{\frac{1}{2\sqrt{x}}}{0 - e^{2\sqrt{x}} \cdot \frac{2}{2\sqrt{x}}} \right]$$

$$= \lim_{x \to 0^+} \left[\frac{1}{-e^{2\sqrt{x}}(2)} \right] = \lim_{x \to 0^+} \left[\frac{1}{-e^{2\sqrt{0}}(2)} \right] = \frac{-1}{2} = -0.5$$

Question 35 **698808230**

Consider the resolution of domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that query resolution mechanism is used in the resolution. The number of DNS query response pairs involved in completely resolving the domain name is _______.

Ans. **4**

Sol. **Given :** Domain name www.gate.Org.in

There is no resource records are cached any where across the DNS servers,

In the iterative query the DNS resolver go to the these three servers which is root server, TLD DNS server, authoritative server. So there will be three pairs of request and response here.

Hence, the correct answer is 4.

Q.36 to Q.65 Carry Two Marks Each

Question 36 **698808231**

Which one of the following is closed form for the generating function of the sequence $\{a_n\}_{n\geq 0}$ defined below?

$$a_n = \begin{cases} n+1, & n = \text{odd} \\ 1, & \text{Otherwise} \end{cases}$$

(A) $\dfrac{x(1+x^2)}{(1-x^2)^2} + \dfrac{1}{1-x}$

(B) $\dfrac{x(3-x^2)}{(1-x^2)^2} + \dfrac{1}{1-x}$

(C) $\dfrac{2x}{(1-x^2)^2} + \dfrac{1}{1-x}$

(D) $\dfrac{x}{(1-x^2)^2} + \dfrac{1}{1-x}$

Ans. **A**

Sol. **Given :**

Generating function

$$a_n = \begin{cases} n+1, & n = \text{odd} \\ 1, & \text{otherwise} \end{cases}$$

Of the sequence $\{a_n\}_{n\geq 0}$

So, $$G(x) = \sum_{r=0}^{\infty} a_r x^r$$

$$G(x) = \sum_{r=0\,(\text{even})}^{\infty} a_r x^r + \sum_{r=1(\text{odd})}^{\infty} a_r x^r$$

$$G(x) = \sum_{r=0 \text{ even}}^{\infty} x^r + \sum_{r=1 \text{ odd}}^{\infty} (r+1)x^r$$

As we know,

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots\ldots$$

$$\frac{1}{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 + x^{10} + \ldots\ldots \quad \ldots(i)$$

$$\frac{2x}{\left(1-x^2\right)^2} = 2x + 4x^3 + 6x^5 + 8x^7 + 10x^4\ldots\ldots \quad \ldots(ii)$$

Add equation (i) and (ii)

$$\frac{1}{1-x^2}+\frac{2x}{\left(1-x^2\right)^2}=1+2x+x^2+4x^3+x^4+6x^5+x^6+......$$

$$\frac{1+x-x}{1-x^2}+\frac{2x}{\left(1-x^2\right)^2}=\sum_{r=0(\text{even})}^{\infty}x^r+\sum_{r=1\,(\text{odd})}^{\infty}(r+1)x^r$$

$$\Rightarrow \quad \frac{1+x}{1-x^2}-\frac{x}{1-x^2}+\frac{2x}{\left(1-x^2\right)^2}=G(x)$$

$$\Rightarrow \quad G(x)=\frac{1}{1-x}+\frac{x}{1-x^2}\left(-1+\frac{2}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{1}{1-x}+\frac{x}{1-x^2}\left(\frac{-1+x^2+2}{1-x^2}\right)$$

$$\Rightarrow \quad \frac{1}{1-x}+\frac{x\left(1+x^2\right)}{\left(1-x^2\right)^2}$$

Hence, the correct option is (A).

Question 37 **698808232**

Consider simple undirected unweighted graph with at least 3 vertices. If A is adjacency matrix of graph then the number of 3-cycle in the graph is given by trace of:

(A) A^3 (B) $\frac{A^3}{2}$

(C) $\frac{A^3}{3}$ (D) $\frac{A^3}{6}$

Ans. **D**

Sol. **Given :**

An undirected unweighted graph which has atleast 3 vertices

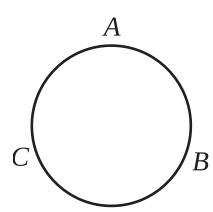
And A is adjacency matrix of graph.

So, as we know,

Diagonal element of A^3 gives number of paths of length 3,

From any vertex to itself (cycle of 3).

For each participating vertex, each cycle will be counted thrice. As given below (ABCA, BCAB and CABC).



Furthermore, since the graph is undirected, every cycle will be counted twice. So overall every cycle of length 3 in A^3 will be counted 6 times, so we divide by 6 also. Therefore, the number of Cycle is trace of $\frac{A^3}{6}$.

Hence, the correct option is (D).

Question 38 **698808233**

Which one of the following statements is False?

(A) The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address

(B) If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in cache miss, then the word will always be present in the main memory.

(C) In a system that uses hashed page tables, if two distinct virtual address VI and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same

(D) The memory access time using a given inverted page table is always same for all incoming virtual addresses.

Ans. **D**

Sol. **Given :**

Statement A : The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.

True, TLB Lookups:

1. Sequential search of the TLB
2. Direct mapping: assigns each virtual page to a specific slot in the TLB e.g., use upper bits of VPA to index TLB (Translation lookaside buffer)
3. Set associativity: use N TLB banks to perform lookups in parallel
4. Fully associative cache: allows looking up all TLB entries in parallel
5. Typically :
   a. TLBs are small and fully associative
   b. Hardware caches use direct mapped or set-associative cache

Statement B : If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in cache miss, then the word will always be present in the main memory.

True, a cache stores a copy of data from memory in a fast storage near the CPU. In case of TLB hit, we got the physical address (in main memory). We look into cache before accessing main memory. In case of a cache miss, we will definitely find the word in main memory as there was a TLB hit.

Statement C : In a system that uses hashed page tables, if two distinct virtual address VI and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same

True, a hashed page table lookup may require many memory references to search the desired virtual address and its corresponding frame number because there is no guarantee on the number of entries in the linked list.

Statement D : The memory access time using a given inverted page table is always same for all incoming virtual addresses.

False, when a memory reference takes place, this virtual address is matched by the memory-mapping unit and the Inverted Page table is searched for a match and the corresponding frame number is obtained.

If the match is found at the it entry then the physical address of the process is sent as the real address otherwise if no match is found then Segmentation Fault is generated. Finding a match requires searching the entire table. Depending on the match, the memory access time will vary.

Hence, the correct option is (D).

Question 39 **698808234**

Let $R_i(z)$ and $W_i(z)$ denote read and write operations on data element z by a transaction T_i, consider schedule S with four transactions

$S: R_4(x) R_2(x) R_3(x) R_1(y) W_1(y) W_2(x) W_3(y) R_4(y)$

Which one of the following serial schedules is conflict equivalent to S?

(A) $T_4 \to T_1 \to T_3 \to T_2$ (B) $T_1 \to T_4 \to T_3 \to T_2$

(C) $T_3 \to T_1 \to T_4 \to T_2$ (D) $T_1 \to T_3 \to T_4 \to T_2$

Ans. **D**

Sol. **Given :** A data element z.

In which transactions T_i occur by the operations read and write

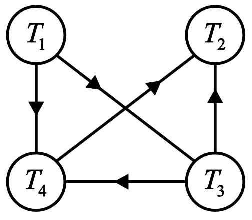
Where Read is denoted by $R_i(z)$ & write is denoted by $W_i(z)$

& there is schedule S for 4 transections i.e.,

$S: R_4(x) R_2(x) R_3(x) R_1(y) W_1(y) W_2(x) W_3(y) R_4(y)$

So, from given schedule :

T_1	T_2	T_3	T_4
			$R(x)$
	$R(x)$		
		$R(x)$	
$R(x)$			
$W(y)$			
	$W(x)$		
		$W(y)$	
			$R(y)$



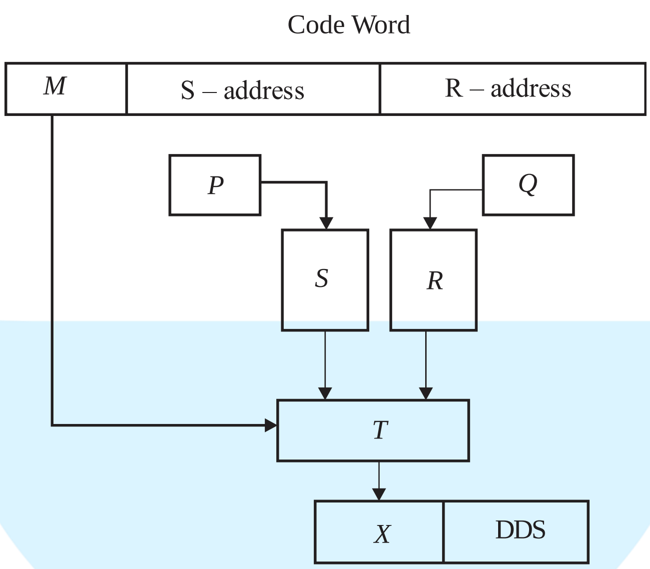
Now, Apply topological sort then the sequence is

$T_1 T_3 T_4 T_2$

Hence, the correct option is (D).

Question 40 **698808235**

Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T?



(A) P is 10: 2^{10} decoder, Q is 5: 2^{5} decoder, T is 2:1 Multiplexer

(B) P is 10: 2^{10} decoder, Q is 5: 2^{5} decoder, T is 2:1 encoder

(C) P is 10: 1 Multiplexer, Q is 5: 1 Multiplexer, T is 2:1 Multiplexer

(D) P is 1:10 de - multiplexer Q is 1:5 de - multiplexer T is 2:1 Multiplexer

Ans. **A**

Sol. **Given :** Digital display system (DDS).

Where, it show the contents of register X and length of data in register X = 16 bit

Where, S is 1024 word memory segment and R is 32 word register file.

Based on the value of M, T selects an input for loading in register X.

P & Q work as interface for choosing time addressed word.

So, based on the Address provided to P, one of the words of the memory unit S has to be selected. this function is performed by a Decoder r To be able to address 1024 words, we need 10 bits , and to select one out of 2^{10} addresses , we need 1G: 2^{10} Decoder which is unit P.

Similarly to decoder Address R we need $5:2^{5}$ decoder.

Now one of the two inputs has to be selected to be loaded into X, selecting one input among many to place in the output. This function is performed by a Multiplexer here we need a 2x1 MUX.

Hence, the correct answer is (A).

Question 41 **698808236**

Consider three floating point numbers A, B and C stored in registers R_A, R_B and R_C, respectively as per IEEE-754 single precision floating point format. The 32-bit content stored in these registers (in hexadecimal form) are as follows.

$R_A = 0xC1400000$	$R_B = 0x42100000$	$R_C = 0x41400000$

Which one of the following is false?

(A) $A+C=0$ (B) $B=3C$

(C) $C=A+B$ (D) $(B-C)>0$

Ans. **C**

Sol. **Given :**

Three registers which store three floating point numbers A, B & C in IEEE-754 single precision floating point format.

Size of register = 32 bit

$R_A = 0xC1400000$	$R_B = 0x42100000$	$R_C = 0x41400000$

$$A = 1100 \quad 0001 \quad 0100 \quad 0000$$

$\text{Biased Exponent} = 130, \text{ Exponent} = 130 - 127 = 3$

$\text{Decimal} = -1.1 \times 2^3 = -1100 = -12$

$$B = 0100 \quad 0010 \quad 0001 \quad 0000$$

$\text{Biased Exponent} = 132, \text{ Exponent} = 132 - 127 = 5$

$\text{Decimal} = +1.001 \times 2^5 = 100100 = 36$

$$C = 0100 \quad 0001 \quad 0100 \quad 0000$$

$\text{Biased Exponent} = 128 + 2 = 130, \text{ Exponent} = 130 - 122 = 3$

$\text{Decimal} = +1.1 \times 2^3 = 1100 = +12$

$$A = -12$$
$$B = 36$$
$$C = +12$$

Hence, the correct option is (C).

Question 42 **698808237**

Consider four processes P, Q, R and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S all at time $t = 0$. There is exactly one context switch from S to Q, exactly one context switch from R to Q. Exactly two context switches from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible a CPU burst time (in time units) of these process?

(A) $P=3, Q=7, R=7, S=3$ (B) $P=4, Q=10, R=6, S=2$

(C) $P=4, Q=12, R=5, S=4$ (D) $P=2, Q=9, R=5, S=1$

Ans. **A**

Sol. **Given :**

Four process P,Q,R,S which are schedule on a CPU by round robin algorithm

Where, time quantum = 4 units,

Process arrived in the order P, Q, R, S at time $t = 0$.

So, from given conditions we can draw gantt chart according to given options.

Valid Required Contexts switches are 1 – S to Q, 1 – R to Q, 2 – Q to R and no S to P.

Option (A) :

A :

P	Q	R	S	Q	R

0 3 7 11 14 17 20

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S and 1 – S to Q

Option (B) :

B :

P	Q	R	S	Q	R	Q

0 4 8 12 14 18 20 22

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

Option (C) :

C :

P	Q	R	S	Q	R	Q

0 4 8 12 16 20 21 25

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

Option (D) :

D :

P	Q	R	S	Q	R	Q

0 2 6 10 11 15 16 17

Contexts switches are 1 – P to Q, 2 – Q to R, 1 – R to S, 1 S to Q and 1 – R to Q

So, from option B, C, D it satisfy all the condition given in the question,

So, the answer is option (A).

Hence, the correct option is (A).

Question 43 **698808238**

What is printed by ANSI C programme

```
    #include<stdio.h>
    int main (int argc, char * argv [])
{
    int a [3][3][3]= {{1, 2, 3, 4, 5, 6, 7, 8, 9}, {10, 11, 12, 13, 14, 15, 16, 17, 18}, {19, 20, 21,
    22, 23, 24, 25, 26, 27}};
    int i= 0, j= 0, k=0;
    for (i = 0; i< 3; i++) {
    for(k = 0; k<3; k++)
    printf("%d", a [i][j][k]);
    printf("\n");
}
    return 0;
}
```

(A) 1 2 3
10 11 12
19 20 21

(B) 1 4 7
10 13 16
19 22 25

(C) 1 2 3
4 5 6
7 8 9

(D) 1 2 3
13 14 15
25 26 27

Ans. **A**

Sol. **Given :**

Code, in which

int a [3][3][3] = {{1, 2, 3, 4, 5, 6, 7, 8, 9}, {10, 11, 12, 13, 14, 15, 16, 17, 18},
{19, 20, 21, 22, 23, 24, 25, 26, 27}};

```
int i= 0, j= 0, k=0;
for (i = 0; i< 3; i++) {
for(k = 0; k<3; k++)
```

So, from given code

$$j = k = i = 0$$

$$\begin{aligned} a[3][3][3] = \{ & \{1, 2, 3, ...9\}, \rightarrow a[0] \\ & \{10, 11, ...18\}, \rightarrow a[1] \\ & \{19, ...27\}\} \rightarrow a[2] \end{aligned}$$

In short, there are total three 2–D arrays, and in the code we need to find first row of each 2D array i.e.

1, 2, 3 First row of first 2-D array
10, 11, 12 First row of second 2-D array
19, 20, 21 First row of third 2-D array

So, the output of the code is
$$\begin{matrix} 1 & 2 & 3 \\ 10 & 11 & 12 \\ 19 & 20 & 21 \end{matrix}$$

Hence, the correct option is (A).

Question 44 **698808239**

What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[])
{
    char a = 'P';
    char b = 'x';
    char c = (a & b) + '*';
    char d = (a | b) - '-';
    char e = (a ^ b) + '+';
    printf("%c%c%c\n", c, d, e);
```

return 0;

}

ASCII encoding for relevant characters are given below :

A	B	C	...	Z
65	66	67	...	90
a	b	c	...	z
97	98	99	...	122

*	+	–
42	43	45

(A) z K S

(B) 122 75 83

(C) * – +

(D) P × +

Ans. **A**

Sol. **Given :**

ANSI C program :

```
#include<stdio.h>
int main(int argc, char *argv[])
{
    char a = 'P';
    char b = 'x';
    char c = (a & b) + '*';
    char d = (a | b) - '-';
    char e = (a ^ b) + '+';
    printf("%c%c%c\n", c, d, e);
    return 0;
}
```

ASCII value of P = 80 and $x = 120$

$$c = \underbrace{a \& b}_{\substack{\downarrow \\ 80}} + \underbrace{'*'}_{\substack{\downarrow \\ 42}} \rightarrow 122 \Rightarrow z$$

$$d = \underbrace{a/b}_{\substack{\downarrow \\ 120}} - \underbrace{'-'}_{\substack{\downarrow \\ 45}} \rightarrow 75 \Rightarrow K$$

$$e = \underbrace{a\hat{}b}_{\substack{\downarrow \\ 40}} + \underbrace{'+'}_{\substack{\downarrow \\ 43}} \rightarrow 83 \Rightarrow S$$

Hence, the correct option is (A).

Question 45 **698808240**

Consider solving the following system of simultaneous equations using LU decomposition.

$x_1 + x_2 - 2x_3 = 4$, $x_1 + 3x_2 - x_3 = 7$ and $2x_1 + x_2 - 5x_3 = 7$

where L and U are denoted as $L=\begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$, $U=\begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

(A) $L_{32}=2, U_{33}=-\frac{1}{2}, x_1=-1$ (B) $L_{32}=2, U_{33}=2, x_1=-1$

(C) $L_{32}=-\frac{1}{2}, U_{33}=2, x_1=0$ (D) $L_{32}=-\frac{1}{2}, U_{33}=-\frac{1}{2}, x_1=0$

Ans. **D**

Sol. **Given :**

Simultaneous equations :

$$x_1+x_2-2x_3=4$$
$$x_1+3x_2-x_3=7$$
$$2x_1+x_2-5x_3=7$$

and L and U Which is denoted by $L=\begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}$ and $U=\begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$.

So, $LU=A$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}=\begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ L_{21}U_{11} & L_{21}U_{12}+U_{22} & L_{21}U_{13}+U_{23} \\ L_{31}U_{11} & L_{31}U_{12}+L_{32}U_{22} & L_{31}U_{13}+L_{32}U_{23}+U_{33} \end{bmatrix}=\begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

Comparing $A_{11,} A_{12}$ and A_{13}

$$U_{11}=1$$
$$U_{12}=1$$
$$U_{13}=-2$$

Comparing A_{21},

$$L_2.U_{11}=1$$

Putting value of U_{11}

$$L_{21}\times 1=1$$
$$L_{21}=1$$

Comparing A_{22}

$$L_{21}U_{12}+U_{22}=3$$

Putting value of L_{21} and U_{12}

$$1 \times 1 + U_{22} = 3$$

$$U_{22} = 2$$

Comparing A_{23}

$$L_{21}U_{13} + U_{23} = -1$$

Putting value of L_{21} and U_{13}

$$1 \times -2 + U_{23} = -1$$

$$U_{23} = 1$$

Comparing A_{31}

$$L_{31}U_{11} = 2$$

Putting value of U_{11}

$$L_{31} \times 1 = 2$$

$$L_{31} = 2$$

Comparing A_{32}

$$L_{31}U_{12} + L_{32}U_{22} = 1$$

Putting value of L_{31}, U_{12} and U_{22}

$$2 \times 1 + L_{32} \times 2 = 1$$

$$L_{32} \times 2 = -1$$

$$L_{32} = \frac{-1}{2}$$

Comparing A_{33}

$$L_{31}U_{13} + L_{32}U_{23} + U_{33} = -5$$

$$2 \times -2 + \frac{-1}{2} \times 1 + U_{33} = -5$$

$$-4 - \frac{1}{2} + U_{33} = -5$$

$$\boxed{U_{33} = -\frac{1}{2}}$$

Hence, the correct option is (D).

Question 46 **698808241**

Which of the following statements is / are undecidable?

(A) Given two Turing machines M_1 and M_2 decide of $L(M_1) = L(M_2)$.

(B) Given a Turing machine M, decide if $L(M)$ is regular.

(C) Given a Turing machine M, decide if M accepts all strings.

(D) Given a Turing machine, M decide if M takes more than 1073 steps on every string.

Ans. **A, C, B**

Sol. For a given Turing machine, M decide that m takes more than 1073 steps on every string is decidabe only.

Remaining are undecidable

(A), (C) and (B) are undecidable.

Hence, the correct options are (A), (C) & (B).

Question 47 **698808242**

Consider language

$$L_1 = \left\{a^n \omega a^n \mid \omega \in \{a,b\}^*\right\}$$

$$L_2 = \left\{\omega x \omega^R \mid \omega, x \in \{a,b\}^*, |\omega|, |x| > 0\right\}$$

Note ω^R is reversal of string ω which one of the following is true?

(A) L_1, L_2 are regular

(B) L_1 and L_2 are context free language

(C) L_1 is regular and L_2 is context free language

(D) L_1 and L_2 are context free language but not regular

Ans. **A, B, C**

Sol. **Given :**

Languages $L_1 \,\&\, L_2$

and w^R is reversal of string w

$$L_1 = \{a^n w a^n \,|\, w \in (a,b)^*\} \text{ is regular}$$

$$L_2 = \{w x w^R \,|\, w, x \in (a,b)^*, |w|, |x| > 0\} \text{ is also regular}$$

So, now from language L_1

L_1 is regular, because when we put the value of n is 0, then it will create a subset $\{w \,|\, w \in \{ab\}^*\}$

Which contains all possible strings.

So, if the subset of L_1 is $(a+b)^*$ then, $L_1 = (a+b)^*$.

Now, from language L_2, L_2 is also regular, because when we put "a"and "b" in place of w, then we get the expression which is regular such that $a(a+b)^+ a + b(a+b)^* b$, which covers all the other string which can be obtained by putting w as "aa", "ab","bb","ba".

$\therefore$ Both L_1 and L_2 are regular

Any language which is regular is context free language as well.

Hence, the correct options are (A), (B) & (C).

Question 48 **698808243**

Consider the following languages

$$L_1 = \left\{ww \,|\, w \in (a, b)^*\right\},\ L_2 = \{a^n b^n c^m \,|\, m, n \ge 0\} \text{ and } L_3 = \{a^m b^n c^n \,|\, m, n \ge 0\}$$

Which of the following statements are false?

(A) L_1 is not Context Free Language but L_2 and L_3 are Deterministic Context Free Language.

(B) Neither L_1 nor L_2 is Context Free Language.

(C) L_2, L_3 and $L_2 \cap L_3$ all are Context Free Language.

(D) Neither L_1 nor its complement is Context Free Language.

Ans. B, C, D

Sol. Given :

Languages

$$L_1 = \{ww \mid w \in (a, b)^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Now, from given languages:

$L_1 = \{ww \mid w \in \{a, b\}^*\}$ this language is not CFL, because it has the strings which has straight order.

but complement of $\bar{L}_1$ is CFL.

$$L_2 = \{a^n b^n c^m \mid nm \geq 0\} \text{ is CFL}$$

$$L_3 = \{a^n b^m c^m \mid n, m \geq 0\} \text{ is CFL}$$

L_2 & L_3 they both are CFL because they both have only 1 comparison.

$L_2 \cap L_3$ is not CFL.

$\therefore$ from these conditions,

(B), (C) and (D) are false

Hence, the correct options are (B), (C), (D)

Question 49 **698808244**

Consider a simple undirected weighted graph G, all of whose edge weights are distinct. Which of the following statements about the Minimum Spanning Tree of G is/are true?

(A) Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with minimum weight such that one of its vertices is in S and the other in V\S. Such an edge will always be part of any Minimum Spanning Tree of G.

(B) G can have multiple Minimum Spanning Tree.

(C) One or both of the edges with the third smallest and the fourth-smallest weight are part of any MST of G.

(D) The edge with the second-smallest weight is always part of any Minimum Spanning Tree of G.

Ans. A, C, D

Sol. Given :

Undirected weighted graph G

Where, edge weights are distinct.

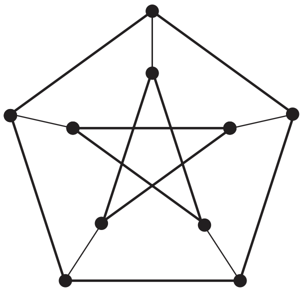
So, from given statements,

The smallest edge is always part of the MST. The graph does not have multiple spanning trees, as all the edge weights are unique. The second and third-smallest edge will be part of the MST if the number of vertices are greater than $n > 3$ and 4 respectively.

Hence the correct options are (A, C, D).

Question 50 **698808245**

The following simple undirected graph is referred to as the Peterson graph.

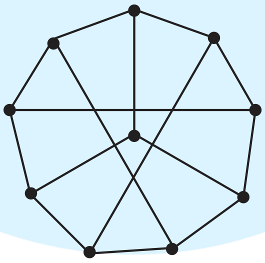


Which of the following statements is/are TRUE?

(A) The chromatic number of the graph is 3.

(B) The graph has a Hamiltonian path.

(C) The following graph is isomorphic to the Peterson graph.



(D) The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

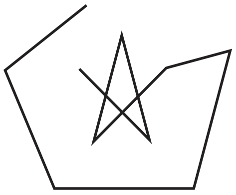
Ans. **A, B, C**

Sol. **Given :**

Undirected graph which is referred as the Peterson graph, then from

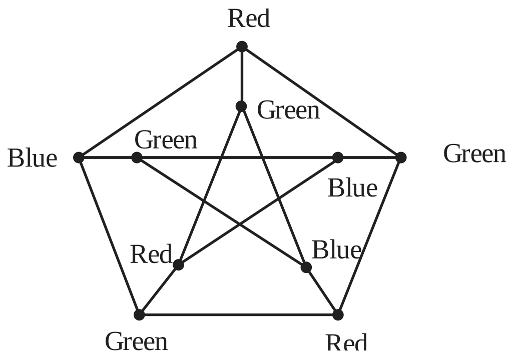
Statement A : The chromatic number of the graph is 3.

True,



Statement B : The graph has a Hamiltonian path.

True,



Statement C : The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

False,

A set of vertices I is called an independent set if no two vertices in set I are adjacent to each other or in other words the set of non-adjacent vertices is called an independent set.

It is 4. From option B by simply keeping all the green vertices together we can simply observe that largest independent set contains 4 vertices.

Statement D : The following graph is isomorphic to the Peterson graph.

True,

We can say given graphs are isomorphic if they have :

1. Equal number of vertices
2. Equal number of edges
3. Same degree sequence
4. Same number of circuit of particular length

Note : In most graphs checking the first three conditions is enough.

Hence, the correct options are (A), (B) & (D).

Question 51 **698808246**

Consider the following recurrence :

$$f(1) = 1$$
$$f(2n) = 2f(n) - 1, \text{for } n \geq 1;$$
$$f(2n+1) = 2f(n) + 1, \text{for } n \geq 1;$$

Then, which of the following statement is/are TRUE?

(A) $f(2^n + 1) = 2^n + 1$ (B) $f(5.2^n) = 2^{n+1} + 1$

(C) $f(2^n - 1) = 2^n - 1$ (D) $f(2^n) = 1$

Ans. **B, C, D**

Sol. **Given :**

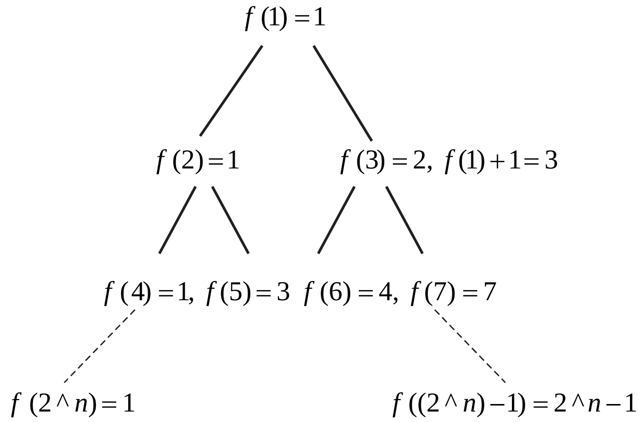
Recurrence function :

$f(1) = 1$

$f(2n) = 2f(n) - 1, \text{for } n \geq 1;$ {i.e. for even input to function f}

$f(2n+1) = 2f(n) + 1, \text{for } n \geq 1;$ {i.e. for odd input to function f}

Options (C) and (D) are true as given in the following diagram.



For option (A) :

$2^n + 1 =$ definitly odd

$\therefore \quad f(2^n+1) = 2f(2^{n-1}) + 1 = 2(1) + 1 \quad \{\text{As we know } f(2^{n-1}) = 1\}$

$= 3$

Therefore, option (A) is false.

For option (B) :

5.2^n = Definitely even number for all $n \geq 1$

$$f(5\cdot 2^n) = 2f(5\cdot 2^{n-1}) - 1$$
$$= 2\left(2f(5\cdot 2^{n-2}) - 1\right) - 1$$
$$= 2^2 f(5\cdot 2^{n-2}) - 2 - 1$$
$$= 2^2\left(2f(5\cdot 2^{n-3}) - 1\right) - 2 - 1$$
$$= 2^3 f(5\cdot 2^{n-3}) - 2^2 - 2 - 1$$
$$\vdots$$
$$= 2^n f(5\cdot 2^0) - 2^{n-1} - 2^{n-2} - 2^{n-3} \ldots - 2 - 1$$
$$= 2^n\left(f(5)\right) - \left(1 + 2 + 2^2 + \ldots 2^{n-2} + 2^{n-1}\right)$$
$$= 2^n(3) - (2^n - 1)$$
$$= 2^n(2+1) - (2^n - 1)$$
$$= 2^{n+1} + 2^n - 2^n + 1$$
$$= 2^{n+1} + 1$$

Therefore, option (B) is false.

Hence, the correct options are (B), (C) & (D).

Question 52 **698808247**

Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices?

(A) The diagonal entries of A^2 are the degrees of the vertices of the graph

(B) If the graph is connected then none of the entries of $A^{n-1} + I_n$ can be zero

(C) If the sum of all the elements of A is at most $2(n-1)$ then the graph must be acyclic

(D) If there is at least a 1 in each of A's rows and columns, then the graph must be connected

Ans. **A**

Sol. **Option (A) : True.**

Let's think about what $(A^2)_{i,i}$, the $i-th$ term on the diagonal is. We have

$$(A^2)_{ii} = (A \times A)_{ii} = \sum jA_{i,\ j}A_{j,i}$$

But $A_{ij} = A_{j,i,}$ assuming that the graph is undirected, and $A_{ji} = 1(i \sim j), i.e.$ is 1 is i and j are adjacent and 0 otherwise. Thus $A_{ij}A_{j,i} = A^2_{i,j} = A_{i,j} = 1(i \sim j)$. So the sum is just the number of j such that $i \sim j$, which precisely the degree is.

This work if vertices in your graph may have a single self-loop, provided you count that as 1 (not 2) for the degree. Indeed, the term in the sum when $j = i$ is just $A^2_{i,i,}$ but you need this to be equal to $A_{i,i}$. Thus it needs to be 0 or 1.

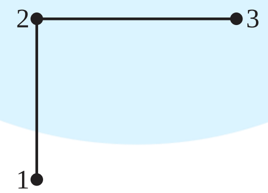
It does not work more generally, however, as $A^2_{ii} \neq A_{i,j}$ if $A_{i,j} > 1$.

$$(A^T A)_{i,j} = \sum{}_i A^2_{i,j} = \deg_{out}(i),$$

even for an undirected graph, provided the graph is simple.

Option (B) : False.

Take following connected graph with $n = 3$,



Adjacency matrix, $(A) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$$A^{n-1} = A^{3-1} = A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{n-1} + I_n = A^2 + I_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

We can see that $A^{n-1} + I_n$ have some entries as zero. So option (B) false.

Option (C) : False.

A cyclic graph is a graph containing at least one graph cycle. Consider a graph with 10 vertices where only three vertices form a cycle while reset are isolated vertices (that is a disconnected graph)

In such a case, sum of all the elements of A is $(1+1+1) = 3$ and 3 is less than $2(10-1) = 2*9 = 18$.

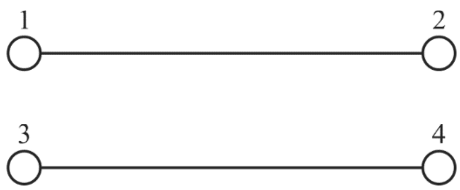
But the graph is still cyclic.

Option (D) : False.

Considering the following adjacency matrix :

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

The corresponding graph is :



The graph is not connected.

Hence, the correct option is (A).

Question 53 **698808248**

Which is /are eigen vector for given matrix $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$

(A) $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

(C) $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$ (D) $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

Ans. A, C, D

Sol. Given :

Matrix $\begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$

Then from given options :

Option (A) : $\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 6 \\ 6 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$

$$\text{Option (B)}: \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -9 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$$

$$\text{Option (C)}: \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{Option (D)}: \begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} -7 \\ -5 \\ 12 \\ 25 \end{bmatrix}$$

Hence the correct options are (A, C, D).

Question 54 **698808249**

Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB and a word length of 16 bits. During the execution of a program, four data words P, Q, R, and S are accessed in that order 10 times (i.e., PQRSPQRS…). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R, and S are 0xA248, 0xC28A, 0xCA8A, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache?

(A) Every access to S is a hit.

(B) Once P is brought to the cache it is never evicted.

(C) At the end of the execution only R and S reside in the cache.

(D) Every access to R evicts Q from the cache.

Ans. **A, B, D**

Sol. **Given :**

Direct Cache memory

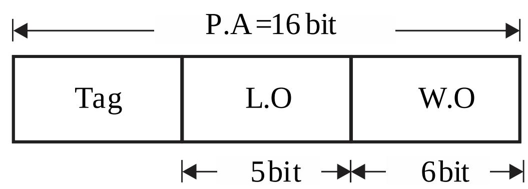
Where, cache memory size $= 2\text{ kB}$, block size $= 64\text{ Bytes}$, physical address space $= 64\text{ kB}$, word length $= 16\text{ bits}$.

$\therefore$ Cache line $\dfrac{2\text{kB}}{64\text{B}} = 32$

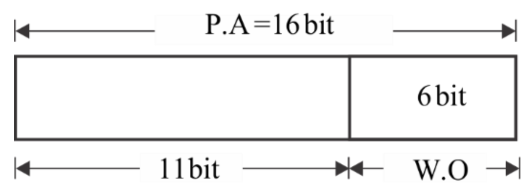
$\therefore$ Line offset $= 5\text{bit}$

Bits in Physical address = $\log_2$ (Main memory size = $\log_2$ (64 kB) = 16 bit)

Physical address format for direct mapped cache



Physical address format for main memory



Tag in main memory

		Tag in cache	L.O	W.O
P	$= A\ 2\ 4\ 8 =$	1 0 1 0, 0	0 1 0, 0 1	0 0, 1 0 0 0
Q	$= C\ A\ 8\ A =$	1 1 0 0, 1	0 1 0, 1 0	0 0, 1 0 1 0
R	$= C\ 2\ 8\ A =$	1 1 0 0, 0	0 1 0, 1 0	0 0, 1 0 1 0
S	$= A\ 2\ 6\ 2 =$	1 0 1 0, 0	0 1 0, 0 1	1 0, 0 0 1 0

- P and S are in same block of memory physical address
- Q and S are different block of main memory but mapped to same cache line.

Hence, the correct options are (A) (B) & (D).

Question 55 **698808250**

Consider routing table of an organization's router shown below :

Subnet number	Subnet mask	Next hop
12.20.164.0	255.255.252.0	R_1
12.20.170.0	255.255.254.0	R_2
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
(default)		R_3

Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table?

(A) 12.20.164.0/20 (B) 12.20.164.0/22

(C) 12.20.164.0/21 (D) 12.20.168.0/22

Ans. **A**

Sol. **Given :** Routing table of an organization's router shown below :

Subnet number	Subnet mask	Next hop
12.20.164.0	255.255.252.0	R_1
12.20.170.0	255.255.254.0	R_2
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1

(default)		R_3

(A) 12.20.1010 0100.00000000, here the network address is 12.20.1010 0100.00000000 and this network covers all the required IP addresses and hence the required answer.

(B) 12.20.10100 100.00000000, here the network address is 12.20.10100 100.00000000, but this network does not have IP address 12.20.170.0, hence this cannot be the answer.

(C) 12.20.101010 00.00000000, here in this network address we do not have IP address 12.20.164.0, hence this cannot be the answer.

(D) 12.20.101001 00.00000000, here in this network we do not have IP address 12.20.170.0, hence this cannot be the answer.

So, 12.20.164.0/20 will be the network – ID of aggregated route.

Hence the correct option is (A).

Question 56 **698808251**

Consider relational database with the four schemas and their respective instances :

Student ($\underline{sNo}$, sName, dNo)

Department ($\underline{dNo}$, dName)

Course ($\underline{cNo}$, cName, dNo)

Register ($\underline{sNo}$, dNo)

Student

sNo	sName	dNo
S1	James	D01
S2	Rocky	D01
S3	Jackson	D02
S4	Jane	D01
S5	Milli	D03

Department

dNo	dName
D01	CSE
D02	EEE

Course

cNo	cName	dNo
C_{11}	DS	D01
C_{12}	DS	D01
C_{21}	DE	D02
C_{22}	PT	D02
C_{23}	CV	D03

Register

sNo	cNo
S01	C_{17}

S01	C_{12}
S02	C_{11}
S03	C_{21}
S03	C_{22}
S03	C_{23}
S04	C_{11}
S04	C_{12}
S05	C_{11}
S05	C_{21}

SELECT * FROM Student AS S WHERE NOT EXIST
(SELECT cNo FROM Course WHERE dNo = “D01”
EXCEPT
SELECT cNo FROM Register WHERE sNo = S.sNo)
The number of rows returned by the above SQL query is _____.

Ans. **2**

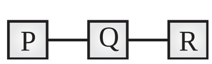
Sol. **Given :**
Relational database
Where, four schemas and their respective instances are as follows :
Student ($\underline{sNo}$, sName, dNo)
Department ($\underline{dNo}$, dName)
Course ($\underline{cNo}$, cName, dNo)
Register ($\underline{sNo}$, dNo)
So from given query,
Select * from student S where not exist
((Select cNo from course where dNo = D01 except))
(Select cNo from register where sNo = S.sno)
The above query is co-related sub query so we need to execute inner query for every row of outer table.
Hence it gives 2 rows in output

S1	J	D01
S4	M	D03

Hence the correct answer is 2.

Question 57 **698808252**

Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q-R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is _________.

Ans. 0.5

Sol. **Given :** Network with three routers P, Q, R as shown in the figure.

Once Q-R fails then Q will immediately update its distance to R to ∞. But P will still be having same finite valve (which is 2).

Now it depends on P and Q who is sending distance vector first.

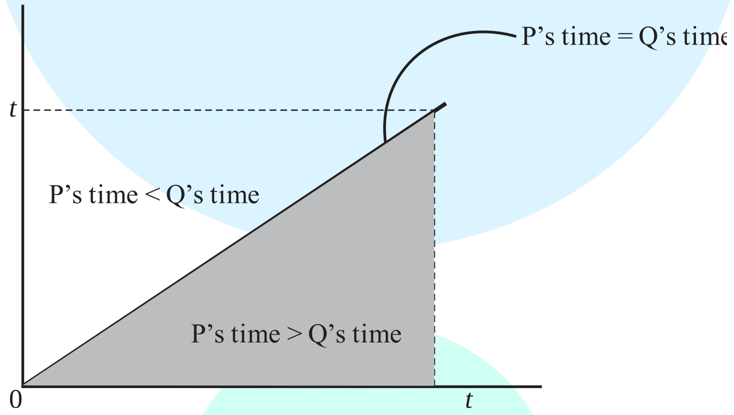
If Q sends then system becomes stable immediately but if P sends first then it will be count to infinity.

Since it is given in question that both have same average rate hence probability is also $\frac{1}{2} \rightarrow$ OR 0.5 that P sends first than Q.

Hence the correct answer is 0.5.

OR

We can also calculate the answer more mathematically considering that time is continuous variable.



Considering the probability represented by shaded area :

$$P = \frac{\text{Area of Triangle}}{\text{Area of Square}}$$

$$= \frac{\frac{1}{2}t^2}{t^2} = \frac{1}{2} = 0.5$$

Hence, the correct answer is 0.5.

Question 58 **698808253**

Let G (V,E) be a directed graph $V = \{1,2,3,4,5\}$ is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A :

$$A[i][j] = \begin{cases} 1, & 1 \le j \le i \le 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j] = 1$ indicates a directed edge from node i to node j. A directed spanning tree of G, rooted at $r \in V$, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V. The number of such directed spanning trees rooted at vertex 5 is __________.

Ans. 24

Sol. Given :

A directed graph G with V vertices and E edges

And it is defined by adjacency matrix A :

$$A[i][j] = \begin{cases} 1, & 1 \le j \le i \le 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j] = 1$ indicates a directed edge from node i to node j

So, when we consider a graph of two elements, we get only 1 possible MST $(2->1)$ When we consider a graph of three elements, we get 2 possible MSTs $(3->1, 3->2 \text{ or } 2->1, 3->2)$. Similarly, When we consider a graph of four elements, we get only $3\times2\times1$ possible MSTs. Similarly. When we consider a graph of five elements, we get only $4\times3\times2\times1 = 24$ possible MSTs.

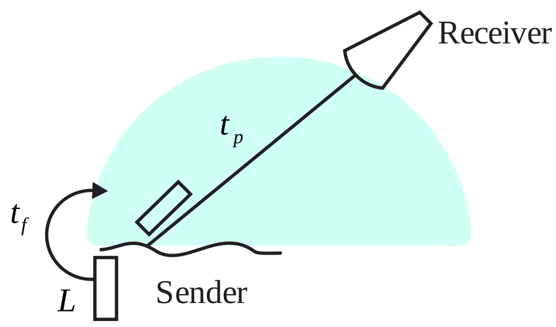
Hence the correct answer is 24.

Question 59 **698808254**

Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a speed of 3×10^8 m/s. The time taken (in milliseconds runoff to 2 decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is ______.

Ans. 7.08

Sol. Given :



A sender (earth station)

& a receiver (satellite)

Where, Distance = 2100 km , Speed = 3×10^8 m/s

Bandwidth = 10^8 m/s , Data size = 1000 Bytes.

Now, $$t_p = \frac{d}{s} = \frac{2100\,\text{km}}{3\times10^8\,\text{m/s}} = \frac{2100\times10^3\,\text{m}}{3\times10^8\,\text{m/s}} = 7\times10^{-3}\,\text{sec} = 7\text{ms}$$

$$t_t = \frac{L}{R} = \frac{1000\,\text{bytes}}{100\ \text{Mbps}} = \frac{1000\times8\ \text{bits}}{100\times10^6\,\text{bits/sec}} = \frac{8}{100}\times10^{-3}\,\text{sec} = 0.08\,\text{ms}$$

$$T = t_t + t_p = 0.08 + 7ms = 7.08\ \text{ms}$$

Hence the correct answer is 7.08 ms.

Question 60 **698808255**

Consider the data transfer using TCP over a one Gbps link. Assuming that the maximum segment life time (MSL) is set to 60 second, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is______.

Ans. **33**

Sol. **Given :** TCP connection

Where, Maximum segment lifetime = 60 second, Bandwidth = 1 Gbps.

$4\times3\times2\times1=24$, $n=$ Sequence Number bits

$\Rightarrow \quad 60\,\text{sec}=\dfrac{2^n\times8\,bits}{10^9\,bits/\text{sec}}$

$\Rightarrow \quad 60\times10^9=2^{n+3}$

$\Rightarrow \quad \log_2(60\times10^9)=\text{n}+3$

$\Rightarrow \quad \log 60+9\times\log_2 10=\text{n}+3$

$\Rightarrow \quad 5.9+29.87=n+3$

$\Rightarrow \quad 35.79=n+3$

$\Rightarrow \quad n+3=36$

$\Rightarrow \quad n=33$

Hence the correct answer is 33.

Question 61 **698808256**

A processor X_1 operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X_2 operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X_1 and X_2. If the BPU has a prediction accuracy of 80%, the speed up (rounded off to two decimal places) obtained by X_2 over X_1 in executing P is____________.

Ans. **1.42**

Sol. **Given :**

2 processors X_1 and X_2

Where, Cycle time = 0.5 ns and Prediction accuracy of BPU = 80%.

$$\text{Speed Up } x_2 \text{ over } x_1=\frac{\text{Execution time using } x_1}{\text{Execution time using } x_2}=\frac{(\text{Avg CPI})_{x_1}\times\text{Cycle time}}{(\text{Avg CPI})_{x_2}\times\text{Cycle time}}$$

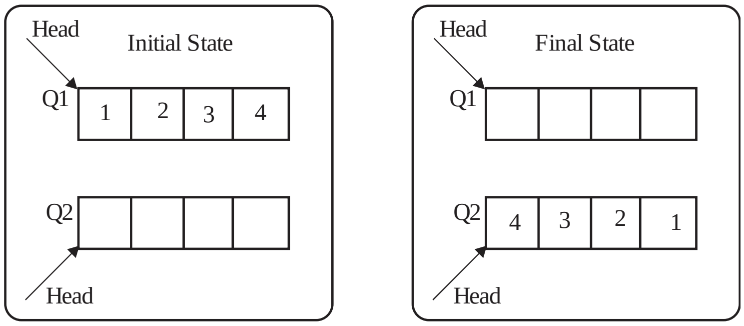
$$=\frac{(1+0.3\times2)\times t_p}{\big(1+0.3\times(0.8\times0+0.2\times2)\big)\times t_p}$$

$$=\frac{1.6}{1.12}=1.42$$

Hence the correct answer is 1.42.

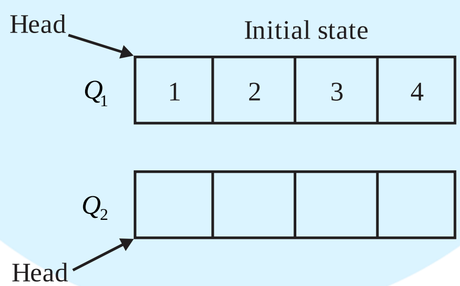
Question 62 **698808257**

Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are Enqueue (Q,element) and Dequeue(Q). The minimum number of Enqueue operations on Q1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the Final State in the figure) without using any additional storage is ________.

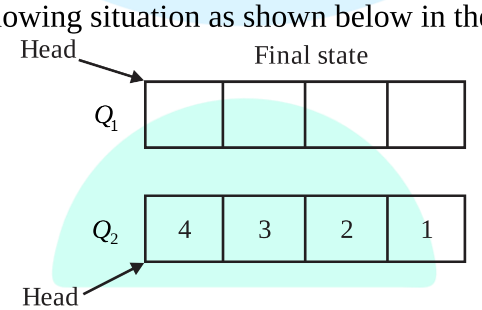


Ans. **0**

Sol. **Given :** Queues Q_1 containing four elements and Q_2 containing none as shown below in the figure.



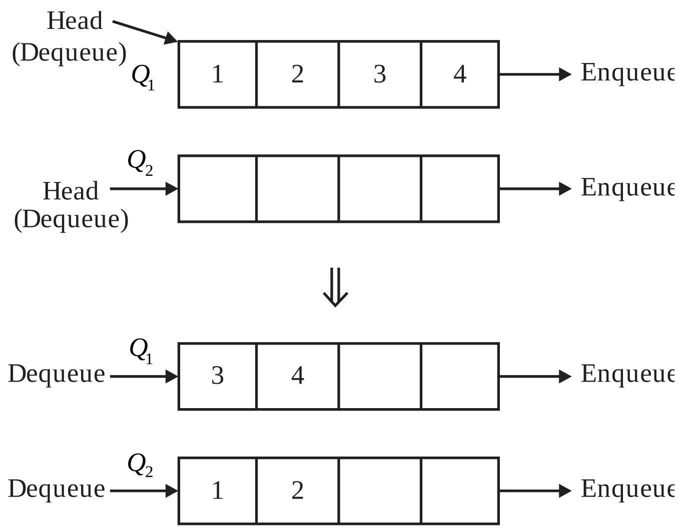
We have to obtain the following situation as shown below in the figure.



As we know that;

If queue is just having 2 elements then we can reverse a queue in place by just using simple enqueue and dequeue and if more than 2 elements we cannot reverse so, first divide 4 elements into 2-2 element each.

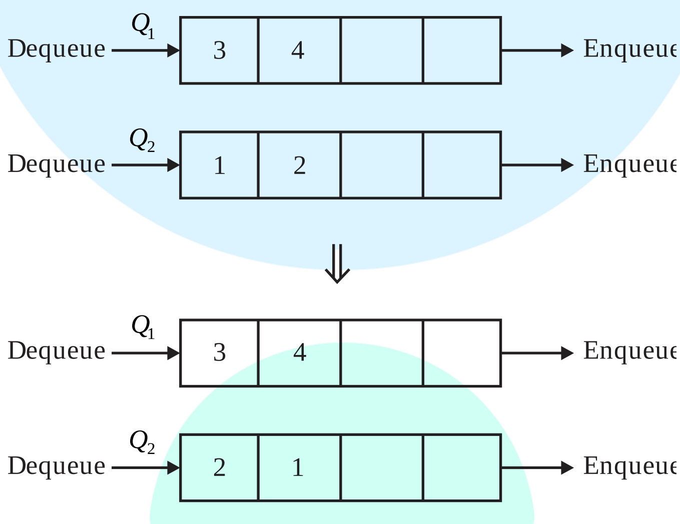
Step 1 : Enqueue 1 in Q_2, it is already sorted. Also Enqueue 2 in Q_2 i.e 1, 2.



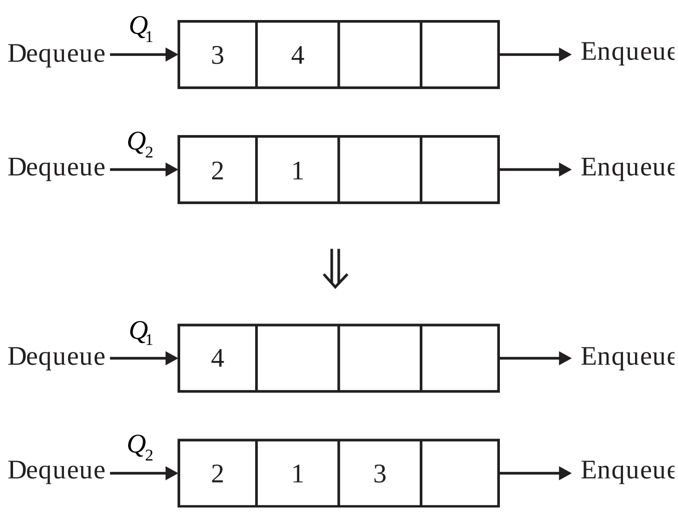
Till now, No enqueue to Q_1.

Now reverse Q_2. This doesn't cost any Enqueue to Q_1.

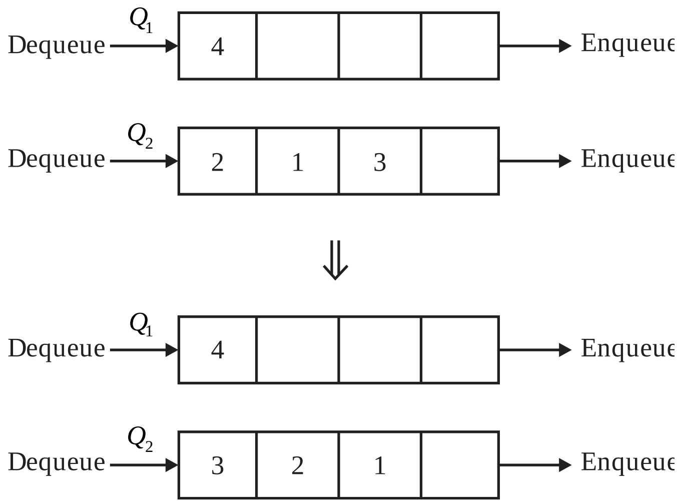
Step 2 : Perform 1 Dequeue and 1 Enqueue simultaneously for 1 time 2 in Q_2 i.e. 2, 1 (head pointing at 2 now)



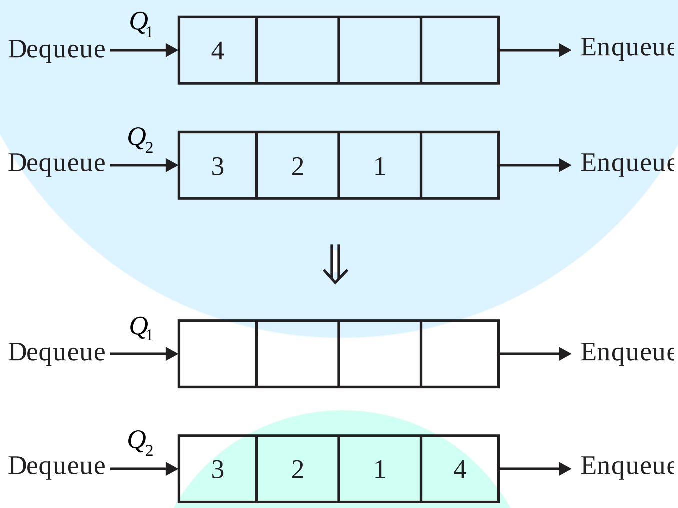
Step 3 : Enqueue 3 in Q_2 i.e 2, 1, 3



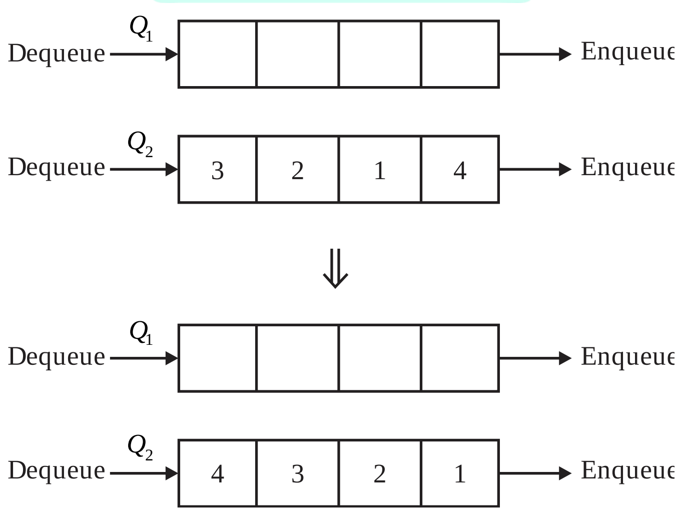
Step 4 : Perform 1 Dequeue and 1 Enqueue simultaneously for 2 times in Q_2 i.e 2, 1, 3 $\rightarrow$ 1, 3, 2 $\rightarrow$ 3, 2, 1 (head pointing at 3 now)



Step 5 : Enqueue 4 in Q_2 i.e 3, 2, 1, 4

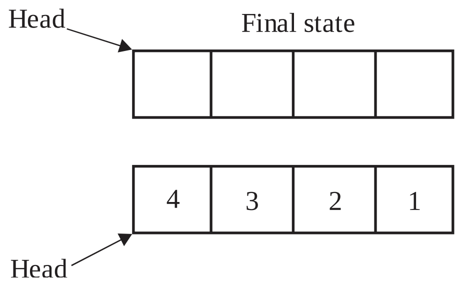


Step 6 : Perform 1 Dequeue and 1 Enqueue simultaneously for 3 times in Q_2 i.e 3, 2, 1, 4 $\rightarrow$ 2, 1, 4, 3 $\rightarrow$ 1, 4, 3, 2 $\rightarrow$ 4, 3, 2, 1



So the number of enqueue operations in Q_1 is 0.

Thus, we have obtain the final state as shown below



Hence, the correct answer is 0.

Question 63 **698808258**

Consider two file-systems A and B. that use contiguous allocation and linked allocation respectively A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50th and 51st block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are n_A and n_B respectively, then the value of $n_A + n_B$ is _________.

Ans. **153**

Sol. **Given :**

Two file systems A and B,

Which uses contiguous allocation and linked allocation respectively,

Data blocks stored in A and B = 100

So, for contiguous allocation, we have to store all the blocks in a sequence, if we have to store a block in the middle, we have to first shift all the blocks after that one place right, for which we need one disk read and one disk write for each block (2 disk accesses). Now since we are storing one block in contiguous allocation at 51^{st} position, the blocks (51^{st} to 100^{th}) have to be moved one step right, for which 2*50 disk accesses are required, then one access to store the new block, thus $n_A = 100 + 1 = 101$.

In Linked allocation, we must read 50 blocks to find the middle. Then we must write the new block somewhere with the next block pointing to the block after the 50^{th} block. Then we must write the 50^{th} block to point to this new block, thus

$n_B = 50 + 1 + 1 = 52$

Hence our required answer $n_A + n_B$ is 153.

Question 64 **698808259**

Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1

the page fault rate, defined as the ratio of number of page faults to the number of memory accesses. (2 decimal) is _______.

Ans. **0.6**

Sol. **Given :**

Demand paging system with four page frames

Where initial frame is empty and LRU page replacement policy used in this.

Page reference string are as follows :

7,2,7,3,2,5,3,4,6,7,7,1,5,6,1

~~7~~	4	5
~~2~~	6	
~~3~~	1	
~~5~~	7	

Total page faults $= 9$

Total Access $= 15$

Thus page fault rate $= 9/15 = 0.6$

Hence the correct answer is 0.6.

Question 65 **698808260**

Consider the following grammar along with translation rules:

$S \to S_1 \# T \qquad \{S_{.val} = S_{1.val} * T_{.val}\}$

$S \to T \qquad \{S_{.val} = T_{.val}\}$

$T \to T_1 \% R \qquad (T_{.val} = T_{1.val} \div R_{.val})$

$T \to R \qquad \{T_{.val} = R_{.val}\}$

$R \to id \qquad \{R_{.val} = id_{.val}\}$

Here # and % are operators and id is a token that represents an integer and $id_{.val}$ represents the corresponding integer value. The set of non-terminals is $\{S, T, R, P,\}$ and a subscripted non-terminal indicates an instance of the non-terminal.

Using this translation scheme, the computed value of $S_{.val}$ for root of the parse tree for the expression 20#10%5#8%2%2 is ______________.

Ans. **80**

Sol. **Given :**

Grammar with translation rules :

$S \to S_1 \# T \qquad \{S_{.val} = S_{1.val} * T_{.val}\}$

$S \to T \qquad \{S_{.val} = T_{.val}\}$

$T \to T_1 \% R \qquad (T_{.val} = T_{1.val} \div R_{.val})$

$T \to R \qquad \{T_{.val} = R_{.val}\}$

$R \to id \qquad \{R_{.val} = id_{.val}\}$

Where, # and % are operators &

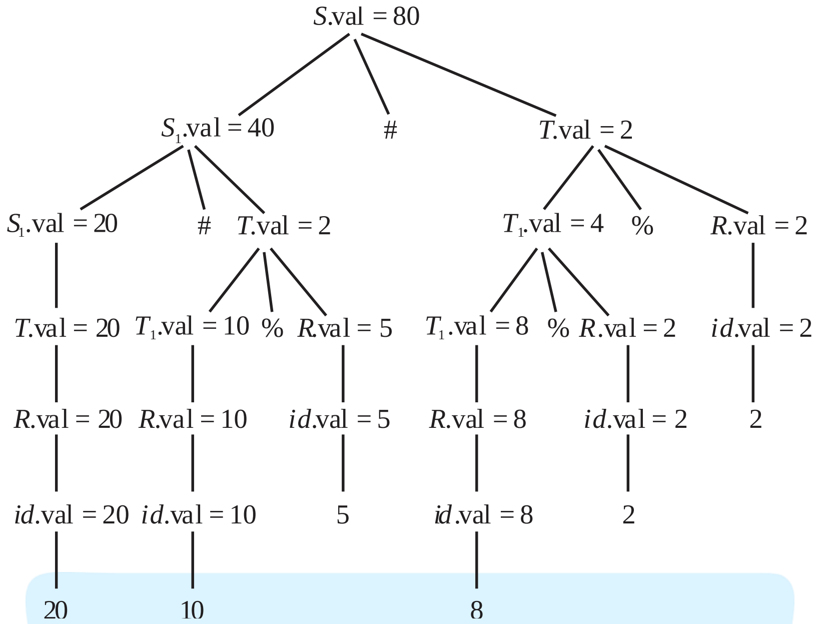
and *id* is a token which represents an integer

& id_{val} represents corresponding integer value.

Set of non terminals $= \{S, T, R, P,\}$

So, from given conditions :

Annotated parse tree :



$\therefore \quad S.\text{val} = 80$

Hence, the correct answer is 80.

❑❑❑

2 GATE-2023 [IIT Kanpur]

General Aptitude Part

Q.1 to Q.5 Carry One Mark Each

Question 1

We reached the station late, and _____ missed the train.

(A) mostly (B) near

(C) utterly (D) nearly

Ans. (D)

Sol. **Given :** We reached the station late, and **nearly** missed the train.

Hence, the correct option is (D).

Question 2

Kind : _______ : : Often : Frequently

(By word meaning)

(A) Type (B) Cruel

(C) Mean (D) Kindly

Ans. (A, D)

Sol. **Given :** Often is related with frequency. As often means, may times and frequently means continuously they both are related in same meaning with each other.

In the same way of relationship kind is related to type, as meaning of kind and type are same.

Option (B) cruel and option (C) mean cannot be related with kind according to the given relationship.

Option (D) kindly, cannot be related with kind as it does not satisfied the characteristics of given relationship.

Hence, the correct option is (A, D).

Question 3

A series of natural numbers $F_1, F_2, F_3, F_4, F_5, F_6, F_7, \ldots\ldots.$ obeys $F_{n+1} = F_n + F_{n-1}$ for all integers $n \geq 2$. If $F_6 = 37$, and $F_7 = 60$, then what is F_1?

(A) 4 (B) 8

(C) 5 (D) 9

Ans. (A)

Sol. **Given :**

A series of natural numbers $F_1, F_2, F_3, F_4, F_5, F_6, F_7, \ldots\ldots.$ obeys $F_{n+1} = F_n + F_{n-1}$ for all integers $n \geq 2$. If $F_6 = 37$, and $F_7 = 60$.

Let $n = 2$,

$F_3 = F_2 + F_1$ …(i)

$F_4 = F_3 + F_2$ …(ii)

$F_5 = F_4 + F_3$ ...(iii)

$F_6 = F_5 + F_4$ ...(iv)

$F_7 = F_6 + F_5$ ...(v)

For put the value of F_7 in equation (v), we get

$60 = 37 + F_s$

$F_s = 23$

Now, put the value of F_5 in equation (iv), we get

$F_6 = F_5 + F_4$

$37 = 23 + F_4$

$F_4 = 14$

Now, put value at F_4 is equation (iii), we get

$F_5 = F_4 + F_3$

$23 = 14 + F_3$

$F_3 = 9$

Now, put the value of F_3 in equation (ii), we get

$14 = 9 + F_2$

$F_2 = 5$

Now, put the value of F_2 in equation (i), we get

$F_3 = F_2 + F_1$

$9 = 5 + F_1$

$F_1 = 4$

Hence, the correct option is (A).

Question 4

A survey of certain year found that 90% of pregnant women received medical care at least once before giving birth. Of these women, 60% received care from doctors, while 40% received from other healthcare providers.

Given this information, which one of the following statements can be inferred with certainty?

(A) Less than half of pregnant women received medical care at least once from a doctor.

(B) More than half of pregnant women received medical care at least once from a doctor.

(C) Less than half of pregnant women received medical care at most once from a doctor.

(D) More than half of pregnant women received medical care at most once from a doctor.

Ans. **(B)**

Sol. **Given :** A survey for a certain year found that 90% of pregnant women received medical care at least once before giving birth of these women, 60% received medical from doctors.

With this given data option (A) can be inferred with certainty, as half of 90% will be 45% and from these 90%, 60% pregnant women received medical care from doctor, and 60% of 90% is 54% which is more then the percentage of half of pregnant women.

Hence, the correct option is (B).

Question 5

Looking at the surface of a smooth 3-dimensional object from the outside, which one of the following options is TRUE?

(A) The surface of the object may be concave in some places and convex in other places.

(B) The surface of object must be concave everywhere.

(C) The surface of object must be convex everywhere.

(D) The object can have edges, but no corners.

Ans. **(A)**

Sol. **Given :** We can combine the convex lens and the concave lens and the combined lens is called a convexo-concave or concavo-convex lens for which one side is convex and other side is concave. Since, convex and concave lenses are 3-dimensional object because each one is formed from two spheres (a three-dimensional object) and so the combined object is also a 3-dimensional object.

Hence, (B) and (C) are eliminated and (A) is correct.

Now, option (D), if you consider the edge as an straight line then for a finite three-dimensional object, option (D) is wrong because where at least two lines or straight edges meet, it creates a corner and according to the definition of smoothness, it should not have a sudden rise or fall and so it will not be a smooth object and so if edge means straight edges.

Hence, the correct option is (A).

Q.6 to Q.10 Carry Two Marks Each

Question 6

The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues. Studies conducted by competent health experts concluded that a complete lack of physical exercise among its working population was one of the leading causes of their health issues. As one of the measures to address the problem, the Government of Zombieland has decided to provide monetary incentives to those who ride bicycles to work.

Based only on the information provided above, which one of the following statements can be logically inferred with certainty?

(A) All the working population of Zombieland will henceforth ride bicycles to work.

(B) Riding bicycles will ensure that all of the working population of Zombieland is free of health issues.

(C) The health experts suggested to the Government of Zombieland to declare riding bicycles as mandatory.

(D) The Government of Zombieland believes that riding bicycles is a form of physical exercise.

Ans. **(D)**

Sol. **Given :** The country of Zombieland is in distress since more than 75% of its working population is suffering from serious health issues.

Studies conducted by competent health experts conducted that a complete lack of physical exercise among its working population was one of the leading cause of their heath issues.

According to this information the government of Zombieland has to take action for the physical fitness of its working population.

So, they decided to provide monetary incentives to those who ride bicycles to work.

Therefore, we can say the government of Zombieland believes that riding bicycles is a form of physical exercise.

Hence, the correct option is (D).

Question 7

Consider two functions of time (t),

$f(t) = 0.01t^2$

$g(t) = 4t$

Where $0 < t < \infty$.

Now consider the following two statements :

(i) For some $t > 0$, $g(t) > f(t)$

(ii) There exists a T, such that $f(t) > g(t)$ for all $t > T$

Which one of the following options are TRUE?

(A) only (ii) is correct

(B) both (i) and (ii) are correct

(C) neither (i) nor (ii) is correct

(D) only (i) is correct

Ans. (B)

Sol. **Given :** Two functions of time (t),

$f(t) = 0.01t^2$

$g(t) = 4t$

Where $0 < t < \infty$.

Statements (i), for some $t > 0$, $g(t) > f(t)$ is true.

For example, if $t = 1$, $g(t) = 4$, $f(t) = 0.01$

Hence, $g(t) > f(t)$ for some $t = 1$.

Statement (ii), there exists a T, such that $f(t) > g(t)$ for all $t > T$. There exist $T = 400$ such that $f(t) > g(t) \forall t > 400$, it is true.

Here, both statement (i) and (ii) are correct.

Hence, the correct option is (B).

Question 8

Which one of the following sentence sequences creates a coherent narrative?

(i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.

(ii) She begins to pant by the time she has climbed all the stairs.

(iii) Mina has bought vegetables and rice at the market, so her bags are heavy.

(iv) He was leaning against the parapet, watching the traffic below.

(A) (iv), (ii), (i), (iii)

(B) (iii), (ii), (i), (iv)

(C) (ii), (iii), (i), (iv)

(D) (i), (ii), (iv), (iii)

Ans. (B)

Sol. **Given :**

(i) Once on the terrace, on her way to her small room in the corner, she notices the man right away.

(ii) She begins to pant by the time she has climbed all the stairs.

(iii) Mina has bought vegetables and rice at the market, so her bags are heavy.

(iv) He was leaning against the parapet, watching the traffic below.

We will make pair of two sentence for the sequences which creates a coherent narrative.

The pairs will be (iii) and (ii) which gives a meaningful narrative in the same way (i) and (iv).

According to the options, sequence in option (B) will give best coherent meaning.

Hence, the correct option is (B).

Question 9

$f(x)$ and $g(y)$ are functions of x and y, respectively, and $f(x) = g(y)$ for all real values of x and y. Which one of the following options is necessarily TRUE for all x and y?

(A) $f(x) = 0$ and $g(y) = 0$

(B) $f(x) = g(y) =$ constant

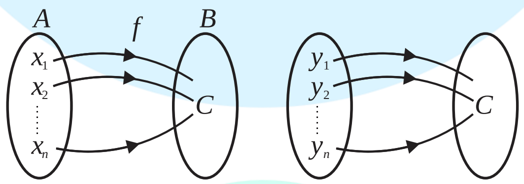
(C) $f(x) \neq$ constant and $g(y) \neq$ constant

(D) $f(x) + g(y) = f(x) - g(y)$

Ans. **(B)**

Sol. **Given :** $f(x)$ and $g(x)$ are functions of x and y and $f(x) = g(y)$ for all real values of x and y.

Here, for all values of x and y it is necessary that image of 'x' under f is same as image at 'y' is same as image of 'y' using 'g' for all real values of x and y i.e. $f(x) = g(y) = \text{Constant}$.

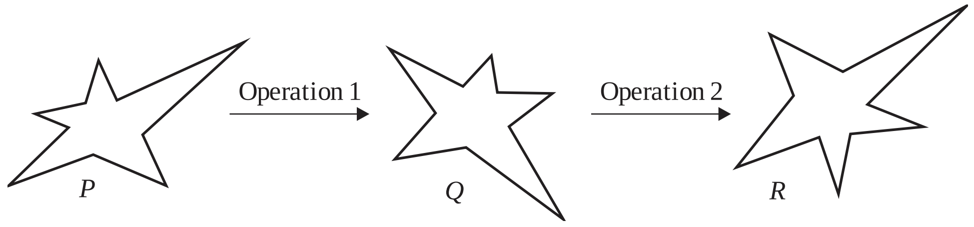


$f(x) = g(y) \forall y \in R$

Hence, the correct option is (B).

Question 10

Which one of the options best describes the transformation of the 2-dimensional figure P to Q, and then to R as shown?



(A) **Operation 1 :** A clockwise rotation by 90^0 about an axis perpendicular to the plane of the figure.

Operation 2 : A reflection along a vertical line.

(B) **Operation 1 :** A clockwise rotation by 90^0 about an axis perpendicular to the plane of the figure

Operation 2 : A reflection along a horizontal line

(C) **Operation 1 :** A counter clockwise rotation by 90^0 about an axis perpendicular to the plane of the figure.

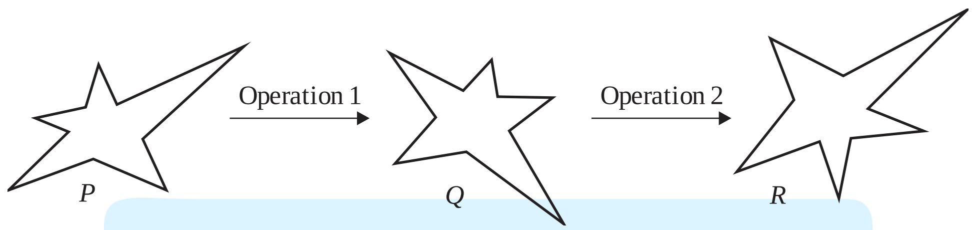
Operation 2 : A reflection along a horizontal line

(D) **Operation 1 :** A counter clockwise rotation by 180^0 about an axis perpendicular to the plane of the figure.

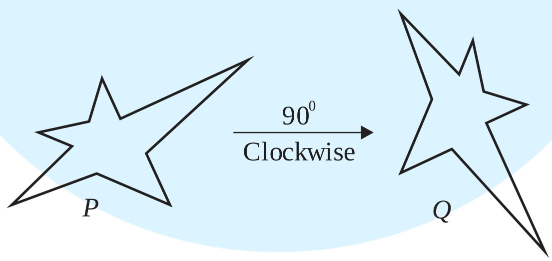
Operation 2 : A reflection along a vertical line.

Ans. (B)

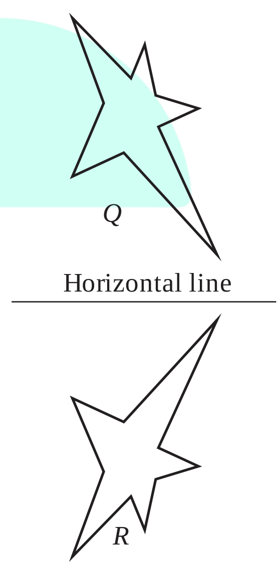
Sol. Given : The transformation of the 2-dimensional figure P to Q, and then to R as shown in below figure,



We can clearly see from P to Q, operation 1 is a clockwise rotation by 90^0 about an axis perpendicular to the plane of the figure.



Form Q to R, operation 2 is a reflection along a horizontal line.



Hence, the correct option is (B).

Technical Part

Q.1 to Q.25 Carry One Mark Each

Question 1

Consider the following statements regarding the front-end and back-end of a compiler. S1: The front-end includes phases that are independent of the target hardware.

S2: The back-end includes phases that are specific to the target hardware.

S3: The back-end includes phases that are specific to the programming language used in the source code.

Identify the CORRECT option

(A) Only S1 is TRUE.

(B) Only S1 and S2 are TRUE

(C) S1, S2, and S3 are all TRUE

(D) Only S1 and S3 are TRUE

Ans. **(B)**

Sol. S1: The front end or analysis phase consists of lexical, syntax and semantic analysis.

It takes source language and produces intermediate code representation. It is independent of target hardware. So, S1 is true.

S2: The bock-end or synthesis phase consists of code optimization and target code generation phases which takes intermediate code and generates target code as output. It is dependent on target hardware. S2 is true.

S3: Back-end phase is independent of source program as its task is to convert the intermediate code to target code. S3 is false.

Hence, the correct option is (B).

Question 2

Which one of the following sequences when stored in an array at locations A[1],...,A[10] forms a max-heap ?

(A) 23, 17, 10, 6, 13, 14, 1, 5, 7, 12

(B) 23, 17, 14, 7, 13, 10, 1, 5, 6, 12

(C) 23, 17, 14, 6, 13, 10, 1, 5, 7, 15

(D) 23, 14, 17, 1, 10, 13, 16, 12, 7, 5

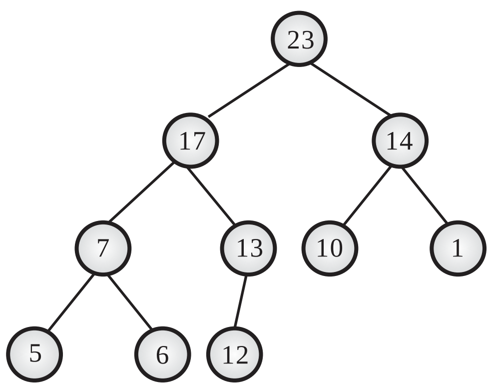
Ans. **(B)**

Sol. Here, wave to check all options for finding which one satisfies property of max-heap. i.e.

(i) Heap is a complete binary Tree

(ii) Parent element is always greater than child element value.

Upon checking only



Hence, the correct option is (B).

Question 3

Let SLLdel be a function that deletes a node in a singly-linked list given a pointer to the node and a pointer to the head of the list. Similarly, let DLLdel be another function that deletes a node in a doubly-linked list given a pointer to the node and a pointer to the head of the list.

Let n denote the number of nodes in each of the linked lists. Which one of the following choices is TRUE about the worst-case time complexity of SLLdel and DLLdel ?

(A) SLLdel is $O(1)$ and DLLdel is $O(n)$

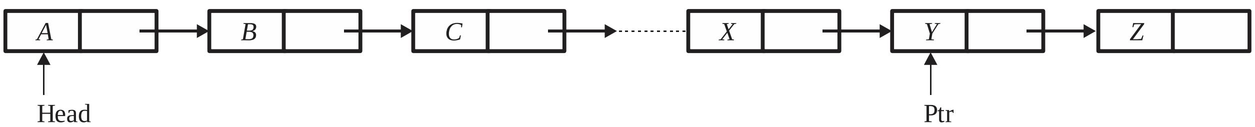
(B) Both SLLdel and DLLdel are $O(\log(n))$

(C) Both SLLdel and DLLdel are $O(1)$

(D) SLLdel is $O(n)$ and DLLdel is $O(1)$

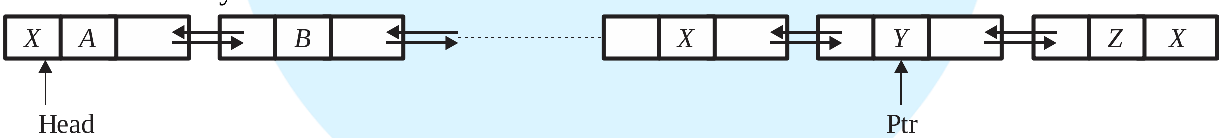
Ans. **(D)**

Sol. Given a single linked list SLL:



Here, were given head pointer and ptr pointer of the node to be deleted (Here y) We have to traverse from head node till node before the one pointed by ptr (here X) which take $O(n)$ time in worst case.

Given a doubly linked list DLL:



Simply we can do it as:

pt $\rightarrow$prev $\rightarrow$next = ptr $\rightarrow$ next

ptr $\rightarrow$next $\rightarrow$prev= ptr $\rightarrow$prev

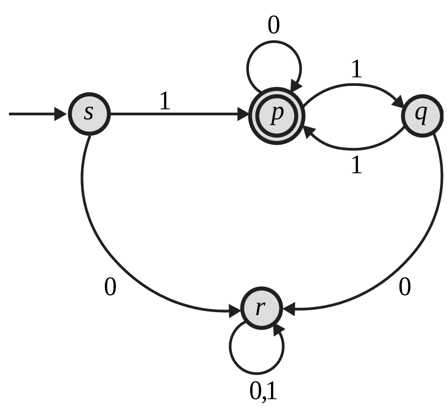
delete (ptr)

since it can be performed in $O(1)$ time:

Hence, the correct option is (D).

Question 4

Consider the Deterministic Finite-state Automaton (DFA) A shown below. The DFA runs on the alphabet {0, 1}, and has the set of states {s, p, q, r}, with s being the start state and p being the only final state



Which one of the following regular expressions correctly describes the language accepted by A?

(A) $1(0{*}11){*}$

(B) $0(0 + 1){*}$

(C) $1(0 + 11)*$ (D) $1(110*)*$

Ans. (C)

Sol. The given F.A. generate strings like:

$\Sigma = \{1,10,110,100,1011,1000....\}$

So, start with 1 to reach final state from where wave two choices os $(0+11)^*$

Hence R.E. is $1(0+11)^*$.

Checking options:

(a) Dosen't generate 10

(b) Dosen't generate any os it starts with 0.

(d) Dosen't generate 10.

Hence, the correct option is (C).

Question 5

The Lucas sequence L_n is defined by the recurrence relation :

$L_n = L_n{-}1 + L_n{-}2$, for $n \geq 3$,

with $L_1 = 1$ and $L_2 = 3$.

Which one of the options given is TRUE?

(A) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$ (B) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{3}\right)^n$

(C) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{3}\right)^n$ (D) $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n$

Ans. (A)

Sol. Lunar sequence

Put $n = 1$ in option we will get

$$l_1 = \frac{1+\sqrt{5}}{2} + \frac{1-\sqrt{5}}{3} = \frac{2}{2} = 1$$

given that $l_1 = 1$

Put $n = 2$ in ophus we will get

$$l_2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 + \left(\frac{1-\sqrt{5}}{2}\right)^2 = 2\left(\frac{1}{4} + \frac{5}{4}\right) = 2\left(\frac{6}{4}\right) = 3$$

Given that $l_2 = 3$

$$l_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Hence, the correct option is (A).

Question 6

Which one of the options given below refers to the degree (or arity) of a relation in relational database systems?

(A) Number of attributes of its relation schema.

(B) Number of tuples stored in the relation.

(C) Number of entries in the relation.

(D) Number of distinct domains of its relation schema.

Ans. **(A)**

Sol. By definition: " The degree of relation is the number of attributes it contains".

Hence, the correct option is (A).

Question 7

Suppose two hosts are connected by a point-to-point link and they are configured to use Stop-and-Wait protocol for reliable data transfer. Identify in which one of the following scenarios, the utilization of the link is the lowest.

(A) Longer link length and lower transmission rate

(B) Longer link length and higher transmission rate

(C) Shorter link length and lower transmission rate

(D) Shorter link length and higher transmission rate

Ans. **(B)**

Sol. Transmission Time $(T_t) = \dfrac{\text{Length of Packet (L)}}{\text{Band width (B.W)}}$

Propagation Time $(T_p) = \dfrac{\text{Link Length (O)}}{\text{Speed (V)}}$

Link Utilisation = Efficiency $= \dfrac{T_t}{T_t + 2T_p} = \dfrac{L/Bw}{\dfrac{L}{Bw} + 2\left(\dfrac{D}{V}\right)} = \dfrac{L}{Bw\left(T_t + 2\dfrac{D}{V}\right)}$

So, Efficiency $\propto \dfrac{1}{D(\text{Link Length})}$

Also, Transmission rate = Band width and Efficiency $\propto \dfrac{1}{Bw}$

For low utilization, we need longer link length and higher transmission rate.

Hence, the correct option is (B).

Question 8

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

Let det(A) and det(B) denote the determinants of the matrices A and B, respectively.

Which one of the options given below is TRUE?

(A) det(A) = det(B)

(B) det(B) = −det(A)

(C) det(A)=0

(D) det(AB) = det(A) + det(B)

Ans. **(B)**

Sol. $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix} B = \begin{bmatrix} 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{bmatrix}$

$R_1 \leftrightarrow R_3$

det B = – det A

Hence, the correct option is (B).

Question 9

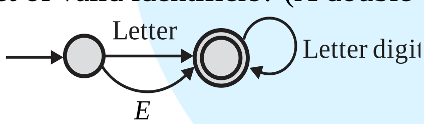
Consider the following definition of a lexical token id for an identifier in a programming language, using extended regular expressions :

Letter $\rightarrow [A-Za-z]$

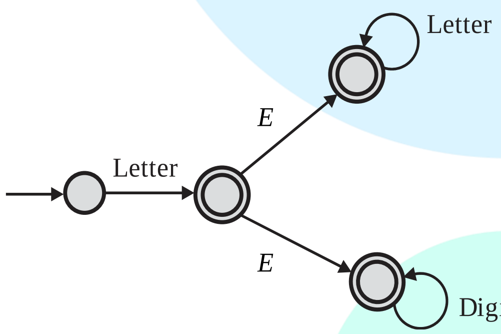
digit $\rightarrow [0-9]$

id $\rightarrow$ letter (letter/digit)*

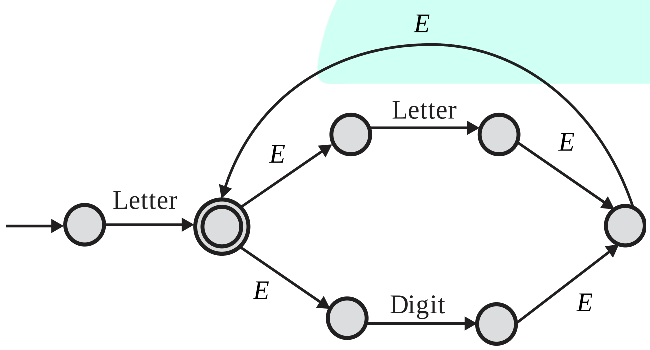
Which one of the following Non - deterministic. Finite - state Automate with $\in$ - transitions accepts the set of valid identifiers? (A double-circle denotes a final state)



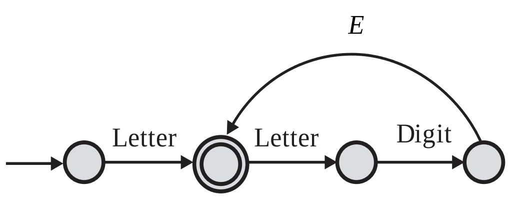
(A)



(B)



(C)



(D)

Ans. **(C)**

Sol. The regular expression is:

Letter (letter+digit)*.

The R.E. accepts strings starting with “letter” and followed by any number of “letter” “digits”.

Checking all options:

(a) is false as it also accepts empty string $\in$.

(b) is false as it dosen’t accept letter. Digit.letter.

(d) is false as letter digit not accepted.

Only option “c” is true.

Hence, the correct option is (C).

Question 10

An algorithm has to store several keys generated by an adversary in a hash table. The adversary is malicious who tries to maximize the number of collisions. Let k be the number of keys, m be the number of slots in the hash table, and $k > m$. Which one of the following is the best hashing strategy to counteract the adversary?

(A) Division method, i.e., use the hash function $h(k) = k \bmod m$.

(B) Multiplication method, i.e., use the hash function $h(k) = [m(kA - [kA])]$, where A is a carefully chosen constant.

(B) Universal hashing method

(C) If k is a prime number, use Division method. Otherwise, use Multiplication method.

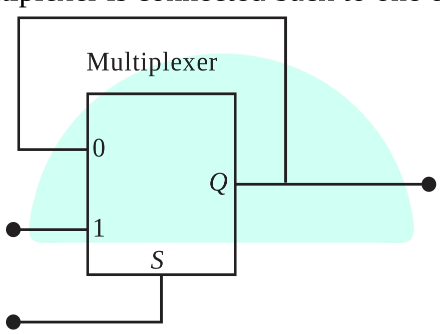
Ans. **(C)**

Sol. Here, the attacker is trying to maximize collision and to minimize it we have to use a method that randomly assigns keys to the slots. So option “C”- Universal hasting is best.

Hence, the correct option is (C).

Question 11

The output of a 2-input multiplexer is connected back to one of its inputs as shown in the figure.



Match the functional equivalence of this circuit to one of the following options

(A) D Flip-flop (B) D Latch

(C) Half-adder (D) Demultiplexer

Ans. **(B)**

Sol. The output equation of above 2×1 mux is:

$V = \overline{S}\, I_0 + S.I_1$

So, when S = 0

$Y = I_0 = Q$, the previous state value

When S = 1

$Y = I_1$, (Output = Input)

Thus, it is D, Latch

Hence, the correct option is (B).

Question 12

Which one or more of the following need to be saved on a context switch from one thread (T1) of a process to another thread (T2) of the same process?

(A) Page table base register
(B) Stack pointer
(C) Program counter
(D) General purpose registers

Ans. **(B), (C), (D)**

Sol. (A) Page Table Base Register holds base address of page table for currently executing thread since, thread switch between same process so there's no need of updation:

(B) Each thread has its own stack. So stack painter needs to be saved.

(C) PC register contains address of next instruction to be executed by current thread. So it needs to be saved when switch occurs.

(D) These registers are used to store temporary data during thread execution and needs to be saved before thread switches.

Hence, the correct options are (B), (C) & (D).

Question 13

Which one or more of the following options guarantee that a computer system will transition from user mode to kernel mode?

(A) Function Call
(B) malloc Call
(C) Page Fault
(D) System Call

Ans. **(C), (D)**

Sol.

- Function calls and malloc calls do not necessarily result in transition to kernel mode.
- System call guarantees that computer system will transition from user mode to kernel mode as using system calls a user requests services from 0S and transition from user mode to kernel mode.
- Page fault occures when program requests access of page not currently in memory (Physical memory) so, OS needs to handle page fault and may need to allocate physical memory.

Hence, the correct options are (C) & (D).

Question 14

Which of the following statements is/are CORRECT?

(A) The intersection of two regular languages is regular.
(B) The intersection of two context-free languages is context-free.
(C) The intersection of two recursive languages is recursive.
(D) The intersection of two recursively enumerable languages is recursively enumerable.

Ans. **(A), (C), (D)**

Sol. Intersection options are closed under Regular Language, Recursive Language and Recursively Enumerable Language.

For Context Enumerable Language

Let $L_1 = a^n b^n c^m / m, n \geq 0$ and $L_2 = a^m b^n c^n / m, n \geq 0$

Both L_1 and L_2 are CFL languages.

But $L_1 \cap L_2 = a^n b^n c^n / n \geq 0$ is a non-CFL language

So (B) is False

Hence, the correct options are (A), (C) & (D).

Question 15

Which of the following statements is/are INCORRECT about the OSPF (Open Shortest Path First) routing protocol used in the Internet?

(A) OSPF implements Bellman-Ford algorithm to find shortest paths.

(B) OSPF uses Dijkstra's shortest path algorithm to implement least-cost path routing.

(C) OSPF is used as an inter-domain routing protocol.

(D) OSPF implements hierarchical routing.

Ans. **(A), (C)**

Sol. OSPF uses Dijkstra's algorithm to compute the shortest path tree for each route, the cost of a route is calculated by gathering link state information from available routers.

Also, OSPF is hierarchical routing protocol, using are 0 (autonomous system) at top of hierarchy.

So, A and C are False.

Hence, the correct options are (A) & (C).

Question 16

Geetha has a conjecture about integers, which is of the form

$$\forall x\left(P(x) \Rightarrow 3yQ(x,y)\right),$$

where P is a statement about integers, and Q is a statement about pairs of integers. Which of the following (one or more) option(s) would imply Geetha's conjecture?

(A) $\exists x(P(x)) \wedge \forall y Q(x,y)$

(B) $\forall_x \forall y Q(x,y)$

(C) $\exists y \forall x\left(P(x) \Rightarrow Q(x,y)\right)$

(D) $\exists x\left(P(x) \wedge \exists y Q(x,y)\right)$

Ans. **(B), (C)**

Sol. Here, domain is set of integers, So Elements

$x, y\, E\{.....-3,-2,-1,0,1,2,......\}$

Expression $E = \forall x\{P(x) \Rightarrow \exists_y\, Q(x,y)\}$

Which says if x is P then ' always exists a y such such that Q (x,y).

Now, checking options.

For option (A) :

$$\text{Is} \underbrace{\exists[P(x)] \wedge \forall_y \theta(x,y)}_{\text{LHS}} \rightarrow \underbrace{E}_{\text{RHS}} \text{True?}$$

Here, for LHS to be true. Say there exists an $x = (6)$ for which $p(x) = p(c) =$ True and for all $y\theta(6,y)$ is True.

Now, RHS : $E = \forall x < \underbrace{p(x)}_{A} \Rightarrow \underbrace{\exists y 2(x,y)}_{B}$

$A =$ True for say $x = 7$, so $p(7) =$ True

For B, say there doesn't exist any y such that $Q(7,y) =$ True. Hence $A \Rightarrow B$

$T \Rightarrow F$ is false.

So, its case of True $\Rightarrow$ False as LHS is True and A
RHS is False
Therefore it becomes case of True $\rightarrow$ False and eventually its False:

For option (B) :

$$\underbrace{\forall x \forall y Q(x,y)}_{LHS} \rightarrow \underbrace{E}_{RHS} \text{ True?}$$

Here, LHS is True for all values of x and y.

Now Ans: $E = \forall x \underbrace{[P(x)]}_{A} \Rightarrow \underbrace{\exists y Q(x,y)]}_{B}$

Now, since,
$\forall x \forall y Q(x,y) =$ True.
$\exists y Q(x,y) =$ True too.
So, For $A \Rightarrow B$
A $\Rightarrow$ True is always True.
As RHS is True. Its case of
True $\rightarrow$ True which is true.

For option (C) :

$$\rightarrow \underbrace{\exists y \forall x [P(x) \Rightarrow Q(x,y)]}_{\text{LHS}} \rightarrow \underbrace{E}_{RHS} \text{ True?}$$

For LHS to be true, there exists some y say y = 2 for which for all x which are satisfying property p implies property Q (x,y)

Now, RHS: = $E = \forall x \left[\underbrace{P(x)}_{A} \Rightarrow \underbrace{\exists y Q(x,y)}_{B} \right]$

$B = \exists y Q(x,y)$ is always, true as there exists at least one y (we assumed y = 2) such that $\forall x Q(x,2)$ is True. So B is true
The case becomes $A \Rightarrow B$
$A \Rightarrow$ True which is true
So, L.H.S $\rightarrow$ R.H.S. is true.

For Option (D) :

$$\underbrace{\exists \times [P(x) \wedge \exists y 2(x,y)]}_{\text{LHS}} \rightarrow \underbrace{E}_{RHS} \text{True?}$$

For LHS to be true, assume $x = 6$ for which properly $P(6)$ is true and there exists a y assume $y = 2$ such that $Q(6,2)$ is true.

Now, RHS $= E = Vx \left[\underbrace{P(x)}_{A} \Rightarrow \underbrace{\exists y Q(x,y)}_{B} \right]$

Say $x = 3$ and $P(x) =$ True
But there exists no y for $Q(3,y)$ to be true.
Hence, it becomes $A \Rightarrow B$
True $\Rightarrow$ False so RHS is False

Also, it becomes case of True $\rightarrow$ False which is false.

Hence, the correct options are (B) & (C).

Question 17

Which one or more of the following CPU scheduling algorithms can potentially cause starvation?

(A) First-in First-Out

(B) Round Robin

(C) Priority Scheduling

(D) Shortest Job First

Ans. (A), (C), (D) or (C), (D)

Sol.

- SJF and priority scheduling are prone to starvation as for SJF the shorter jobs might keep coming and longer burst time jobs have to keep waiting.
- Also, for priority scheduling the higher priority job might keep coming causing lower priority jobs to starve.
- Round Robin never cause starvation as every job gets a fixed time quantum to execute, which is finite and evry job get time for execution.
- For FCFs, in case of infinite loop like
  while (1);
  then it cause starvation
  so until there's special case of a task running forever there'll be no starvation.

Hence, the correct options are (A), (C) & (D).

Question 18

Let $f(x) = x^3 + 15x^2 - 33x - 36$ be a real-valued function. Which of the following statements is/are TRUE?

(A) $f(x)$ does not have a local maximum.

(B) $f(x)$ has a local maximum.

(C) $f(x)$ does not have a local minimum.

(D) $f(x)$ has a local minimum

Sol. $f(x) = x^3 + 15x^2 - 33x - 36$

$f'(x) = 3x^2 + 30x - 33$

$f''(x) = 6x + 30$

$f'(x) = 0$

$3x^2 + 30x - 33 = 0$

$x^2 + 10x - 11 = 0$

$(x+11)(x-1) = 0$

$x = -11,\ x = 1$

$f''(x) = 6x + 30$

at $x = -11$

$f''(-11) = -66 + 30$

$= -36 < 0$

Local maxima

at $x = 1$

$f''(1) = 6 + 30$

$= 36 > 0$

Local minima

$\therefore$ $f(x)$ has a local maximum.

$\therefore$ $f(x)$ has a local minimum.

Question 19

Let f and g be functions of natural numbers given by $f(n) = n$ and $g(\text{n}) = n^2$. Which of the following statements is/are TRUE?

(A) $f \in O(g)$ (B) $f \in \Omega(g)$

(C) $f \in o(g)$ (D) $f \in \theta(g)$

Ans. **(A), (C)**

Sol. Given, $f(n), n$ and $g(n) = n^2$

(A) $f \in 0(g) \Rightarrow f(n) = 0(g(n))$

$f(n) \in 0(n^2)$, is True

(B) $f(n) = \Omega(g(n))$

$f(n) = \Omega(n^2)$, is False

(C) $f \in 0(g) \Rightarrow f(n) = (g(n))$

$f(n) = 0(n^2)$ is True

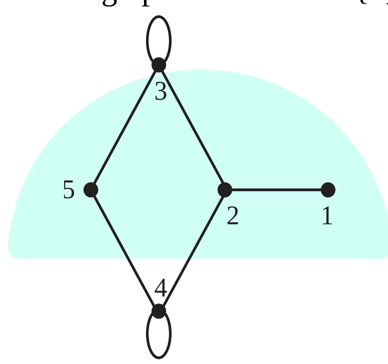
(D) $f \in 0(g) \Rightarrow f(n) = \theta(g(n))$

$f(n) = \theta(n^2)$ is False

Hence, the correct options are (A) & (C).

Question 20

Let A be the adjacency matrix of the graph with vertices {1, 2, 3, 4, 5}.



Let λ1, λ2, λ3, λ4, and λ5 be the five eigenvalues of A. Note that these eigenvalues need not be distinct. The value of λ1 + λ2 + λ3 + λ4 + λ5 = _______.

Sol.

$$A = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 0 \\ 3 & 0 & 1 & 1 & 0 & 1 \\ 4 & 0 & 1 & 0 & 1 & 1 \\ 5 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Sum of Eigen values,

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 = T_r(A)$

= 0 + 0 + 1 + 1 +0

= 2

Question 21

The value of the definite integral $\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)\,dz\,dy\,dx$ is________. (Rounded off to the nearest integer)

Sol. $\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2y - z^3)dz\,dy\,dx$

$$\int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(4x^2ydzdydx) - \int_{-3}^{3}\int_{-2}^{2}\int_{-1}^{1}(z^3dz)dydx$$

$$\int_{-3}^{3}\int_{-2}^{2}4x^2y(z)_{-1}^{1}dydx - 0$$

$$\int_{-3}^{3}\int_{-2}^{2}8x^2ydydx$$

$$\int_{-3}^{3}8x^2\left(\int_{-2}^{2}dy\right)dx$$

$$\int_{-3}^{3}8x^2(0)dx = 0$$

Question 22

A particular number is written as 132 in radix-4 representation. The same number in radix-5 representation is_________.

Ans. **110**

Sol. **Given :** $(132)_4 = (1\times 4^2 + 3\times 4 + 2)_{10}$

$= (30)_{10} = (25+5+0)_{10}$ (Power of 5)

$= (110)_s$

Hence, the correct answer is 110.

Question 23

Consider a 3-stage pipelined processor having a delay of 10 ns (nanoseconds), 20 ns, and 14 ns, for the first, second, and the third stages, respectively. Assume that there is no other delay and the processor does not suffer from any pipeline hazards. Also assume that one instruction is fetched every cycle. The total execution time for executing 100 instructions on this processor is __________ns.

Ans. **2040**

Sol. Given delays 10ns, 20 ns, 14 ns

There's no buffer delay or hazard and one instruction is fetched every cycles.

Total instruction $(n) = 100$

Pipeline delay $(T_p) = m_a(10, 20, 14) = 20\,\text{ns}$

Number of stages $(k) = 3$

So, Total execution time, $T = [k+(n-1)] \times T_p$

$= (3+100-1) \times 20$

$= 2040$ ns

Hence, the correct answer is 2040.

Question 24

A keyboard connected to a computer is used at a rate of 1 keystroke per second. The computer system polls the keyboard every 10 ms (milli seconds) to check for a keystroke and consumes $100\,\mu s$ (micro seconds) for each poll. If it is determined after polling that a key has been pressed, the system consumes an additional 200 µs to process the keystroke. Let T_1 denote the fraction of a second spent in polling and processing a keystroke.

In an alternative implementation, the system uses interrupts instead of polling. An interrupt is raised for every keystroke. It takes a total of 1 ms for servicing an interrupt and processing a keystroke. Let T_2 denote the fraction of a second spent in servicing the interrupt and processing a keystroke.

The ratio $\frac{T_1}{T_2}$ is__________. (Rounded off to one decimal place)

Ans. **10.2**

Sol. Computer system polls keyboard every 10 ms.

In one second, it polls $\frac{1\text{s}}{10\,\text{ms}} = \frac{1000\,\text{ms}}{10\,\text{ms}} = 100$ times

Each poll take $100\,\mu s$

So, Total polling time = $100 \times 100\,\mu s = 10 \times 10^3\,\mu s = 10\,\text{ms}$

Also, it takes $200\,\mu s$ for processing keystroke i.e. 0.2 ms

Total time spent in polling $(T_1) = (10+0.2)\,\text{ms} = 10.2\,\text{ms}$

In interrupt system, when there's keystroke CPU executes corresponding interrupt service routine i.e. ISR taking 1 ms. So, $T_2 = 1\text{ms}$

Now, $\frac{T_1}{T_2} = \frac{10.2\,\text{ms}}{1\text{ms}} = 10.2$

Hence, the correct answer is 10.2.

Question 25

The integer value printed by the ANSI-C program given below is__________.

```
#include<stdio.h>
int funcp(){
        static int x = 1;
        x++;
        return x;
}
int main(){
        int x,y;
```

```
        x = funcp();
        y = funcp()+x;
        printf("%d\n", (x+y));
        return 0;
}
```

Ans. 7

Sol. When $x = \text{fun}\, cp()$ is called,

Static int $x = 1$ and $x++$ changes

Static x to 2

In main $x = 2$ is assigned.

Again $y = \text{fun cp}() + x = \text{fun cp}() + 2$

In $\text{fun cp}()$ the static $x = 2$ and $x++$ makes static $x = 3$

In main, $y = 3 + 5 = 5$

As, $x = 2$ and $y = 5$

So, $x + y = 7$ which is printed out

Hence, the correct answer is 7.

Q.26 to Q.55 Carry Two Marks Each

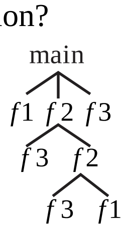
Question 26

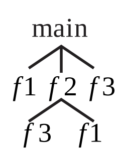
Consider the following program :

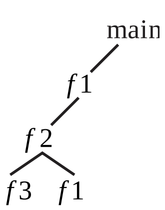
int main()	int f1()	int f2(int X)	int f3()

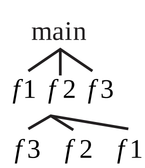
{
    f1();
    f2(2);
    f3();
    return(0);
}
	{
	    return(1);
	}
		{
		    f3();
		    if (X==1)
		        return f1();
		    else
		        return (X*f2(X-1));
		}
			{
			    return(5);
			}

Which one of the following options represents the activation tree corresponding to the main function?

(A)


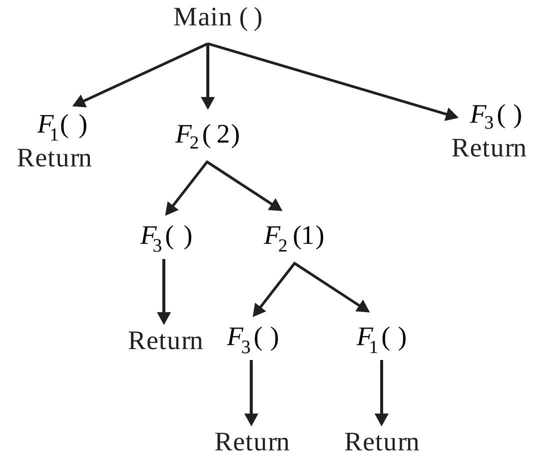
(B)


(C)


(D)


Ans. (A)

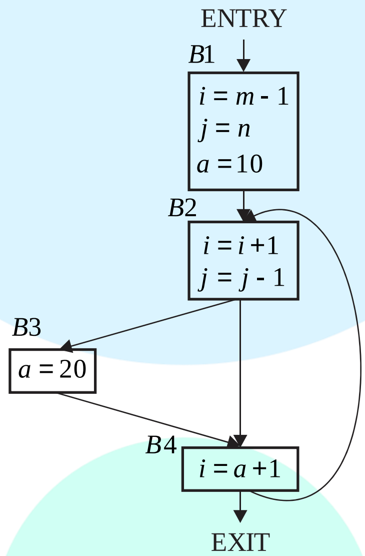
Sol. Following the execution sequence



Hence, the correct option is (A).

Question 27

Consider the control flow graph shown.



Which one of the following choices correctly lists the set of live variables at the exit point of each basic block?

(A) B1: {}, B2: {a}, B3: {a}, B4: {a}

(B) B1: {i, j}, B2: {a}, B3: {a}, B4: {i}

(C) B1: {a, i, j}, B2: {a, i, j}, B3: {a, i}, B4: {a}

(D) B1: {a, i, j}, B2: {a, j}, B3: {a, j}, B4: {a, i, j}

Ans. **(D)**

Sol. A variable $'V'$ is live (for statement b) if there exist a path from this statement to another statement is $'a'$ in CFG such that for each $b <= k < a$ and V is defined in any statement k in CFG.

For B_1 : There's path to B_2, B_3 and B_4.

In this path $'i'$ and $'J'$ are live as they're both used before modifying.

$'a'$ is not live as it's used in B_3 before being used in B_4.

Another path is $B_2 \rightarrow B_4$

Where all 3 $'a'$, $'i'$ and $'J'$ are live so live variable at exit of $B_1 = \{a, i, J\}$.

For B_2 : Similarly for path B_3, B_4 $'J'$ is live for path B_4 $'a'$ and $'J'$ are live.

For B_3 : Same as B_1 as path for exit is same.

So, live variable at exits of $B_4 = \{a, i, J\}$.

Hence, the correct option is (D).

Question 28

Consider the two functions incr and decr shown below.

```
incr(){                      decr(){
    wait(s);                     wait(s);
    X = X+1;                     X = X-1;
    signal(s);                   signal(s);
}                            }
```

There are 5 threads each invoking incr once, and 3 threads each invoking decr once, on the same shared variable X. The initial value of X is 10.

Suppose there are two implementations of the semaphore s, as follows:

I-1: s is a binary semaphore initialized to 1.

I-2: s is a counting semaphore initialized to 2.

Let V1, V2 be the values of X at the end of execution of all the threads with implementations I-1, I-2, respectively.

Which one of the following choices corresponds to the minimum possible values of V1, V2, respectively?

(A) 15, 7

(B) 7, 7

(C) 12, 7

(D) 12, 8

Ans. **(C)**

Sol. For implementation I_1 :

Binary semaphore $s = 1$ and $incr()$ called 5 times and $decr()$ called 3 times by threads

So, it alternate as sequence.

$incr()$, $decr()$, $incr()$, $decr()$, $incr()$, $decr()$ which all make X as same value 10 nd then two $incre()$ making $V_1 = 12$

For implementation I_2 :

Counting semaphore $s = 2$.

So, one possible sequence is

$decr()$

{

wait(s); //s becomes '1'

red X; //X becomes/reads and stores value 10.

Run incr ( ) 5 times

$X = 9$ //write X.

Two more decr ( ) so, $X = 7$.

Value $V_2 = 7$.

Hence, the correct option is (C).

Question 29

Consider the context-free grammar G below

$S \rightarrow aSb | X$

$X \rightarrow aX | Xb | a | b$

where S and X are non-terminals, and a and b are terminal symbols. The starting non-terminal is S.

Which one of the following statements is CORRECT?

(A) The language generated by G is $(a+b)^*$

(B) The language generated by G is $a^*(a+b)b^*$

(C) The language generated by G is $a^*b^*(a+b)$

(D) The language generated by G is not a regular language

Ans. **(B)**

Sol. Option A : Since E can't be generated by G so, option (A) is false, which accept E.

Option C : Since $'ba'$ G but is accepted by $a^*b^*(a+b)$ $10(C)$ s false.

Option D : It is false since the language generated has satisfied all conditions of being regular language.

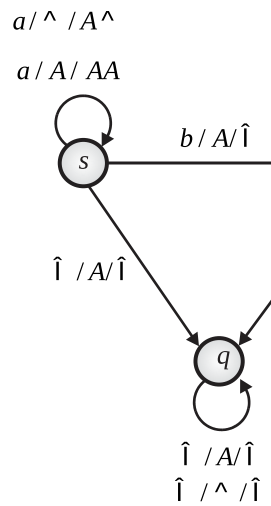
Option B : It is true as it accepts all strings generated by $a^*(a+b)b^*$.

Hence, the correct option is (B).

Question 30

Consider the pushdown automaton (PDA) P below, which runs on the input alpha-bet $\{a, b\}$, has stack alphabet $(\perp, A)$, and has three states $\{s, p, q\}$, with s being the start state. A transition from state u to state v ¸labelled $c / X / \gamma$, where c is an input symbol or $\in$, X is a stack symbol, and γ is a string of stack symbols, represents the fact that in state u, the PDA can read c from the input, with X on the top of its stack, pop X from the stack, push in the string γ on the stack, and go to state v. In the initial configuration, the stack has only the symbol $\perp$ in it.

The PDA accepts by empty stack.



Which one of the following options correctly describes the language accepted by P?

(A) $\{a^m b^n \mid 1 \le m \text{ and } n < m\}$ (B) $\{a^m b^n \mid 0 \le n \le m\}$

(C) $\{a^m b^n \mid 0 \le m \text{ and } 0 \le n\}$ (D) $\{a^m \mid 0 \le m\} \cup \{b^n \mid 0 \le n\}$

Ans. **(A)**

Sol. Option B : This option is not true since E is not accepted by P.

Option C : This option is false since E is not accepted by P.

Option D : This option is false since E is not accepted by P.

Option A : It accept all strings generated by P.

Hence, the correct option is (A).

Question 31

Consider the given C-code and its corresponding assembly code, with a few operands U1–U4 being unknown. Some useful information as well as the semantics of each unique assembly instruction is annotated as inline comments in the code. The memory is byte-addressable.

```
//C-code                          ;assembly-code (; indicates comments)
                                  ;r1-r5 are 32-bit integer registers
                                  ;initialize r1=0, r2=10
                                  ;initialize r3, r4 with base address of a, b

int a[10], b[10], i;              L01: jeq r1, r2, end    ;if(r1==r2) goto end
// int is 32-bit                  L02: lw r5, 0(r4)       ;r5 <- Memory[r4+0]
for (i=0; i<10;i++)               L03: shl r5, r5, U1     ;r5 <- r5 << U1
a[i] = b[i] * 8;                  L04: sw r5, 0(r3)       ;Memory[r3+0] <- r5
                                  L05: add r3, r3, U2     ;r3 <- r3+U2
                                  L06: add r4, r4, U3
                                  L07: add r1, r1, 1
                                  L08: jmp U4             ;goto U4
                                  L09: end
```

Which one of the following options is a CORRECT replacement for operands in the position (U1, U2, U3, U4) in the above assembly code?

(A) (8, 4, 1, L02) (B) (3, 4, 4, L01)
(C) (8, 1, 1, L02) (D) (3, 1, 1, L01)

Ans. (B)

Sol. Here an analyzing code we can observe that. We are to shift value of r_s left by u places. In code were multiplying element of b by 8. So, $u_1 = 3$, which is same as multiplying value by 8 or 2^3.

Also, r_3 and r_4 stores storing address of next element of arrays a and v. Since t's 32 bit system and size of int is 4B so well increment by 4 so, value of u_2 and u_3 s 4.

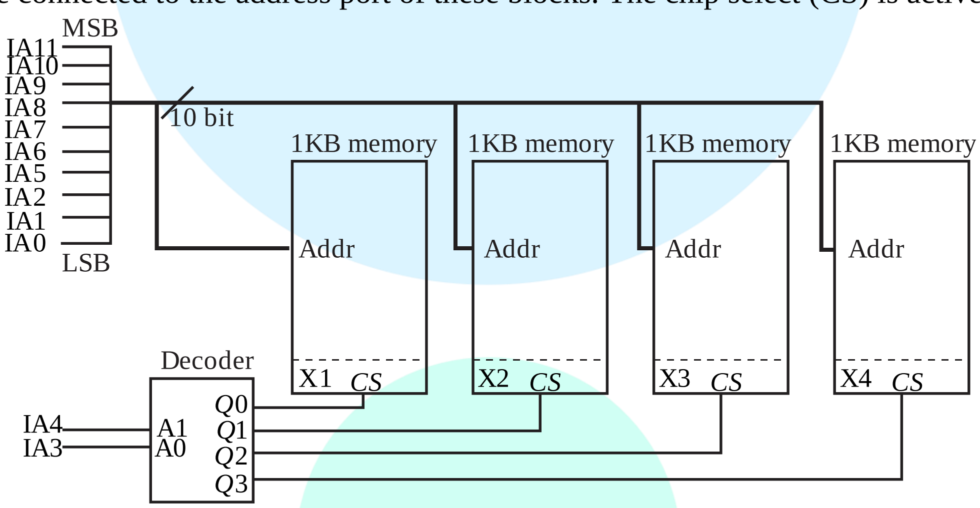
We have to jump to short of code i.e. $\angle 01$ so that for-loop can be run.

So, u_4 is $\angle 01$.

Hence, the correct option is (B).

Question 32

A 4 kilobyte (KB) byte-addressable memory is realized using four 1 KB memory blocks. Two input address lines (IA4 and IA3) are connected to the chip select (CS) port of these memory blocks through a decoder as shown in the figure. The remaining ten input address lines from IA11–IA0 are connected to the address port of these blocks. The chip select (CS) is active high.



The input memory addresses (IA11–IA0), in decimal, for the starting locations (Addr=0) of each block (indicated as X1, X2, X3, X4 in the figure) are among the options given below. Which one of the following options is CORRECT?

(A) (0, 1, 2, 3) (B) (0, 1024, 2048, 3072)
(C) (0, 8, 16, 24) (D) (0, 0, 0, 0)

Ans. (C)

Sol. The addresses are of length 12 bits.

The 2:4 decoder with input IA_3 and IA_4 decides which chip is selected.

Possible values of $IA_4 IA_3$ ate 4.

For starting address the valued $I_{11}.....I_5$ remains '0' and we're focused on value from $I_4......I_0$.

So,

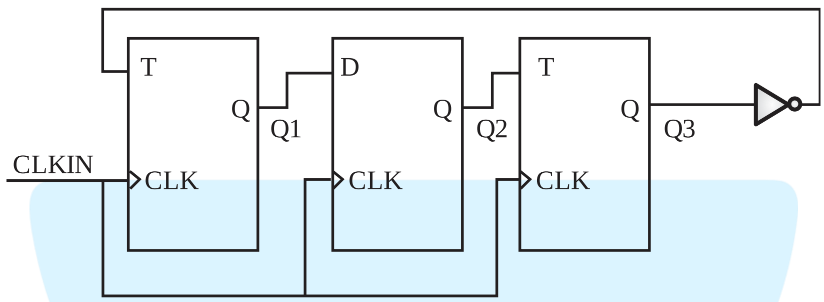
I_4	I_2	Resulting value/Starting address
0	0 (X_0)	0(00000)

0	1 (X_1)	$8(01000)$
1	0 (X_2)	$16(10000)$
1	1 (X_3)	$24(11000)$

Hence, the correct option is (C).

Question 33

Consider a sequential digital circuit consisting of T flip-flops and D flip-flops as shown in the figure. CLKIN is the clock input to the circuit. At the beginning, Q1, Q2 and Q3 have values 0, 1 and 1, respectively.



Which one of the given values of (Q1, Q2, Q3) can NEVER be obtained with this digital circuit?

(A) (0, 0, 1) (B) (1, 0, 0)

(C) (1, 0, 1) (D) (1, 1, 1)

Ans. **(A)**

Sol. Given circuit is made from two T flip flops and one D flp-flop.

Here, $T_1 = \bar{Q}_3$

So, $Q_1^+ = T_1 \oplus Q_1 = \bar{Q}_3 \oplus Q_1 = Q_3 \odot Q_1$

Also, $D_2 = Q_1$

So, $Q_2^+ = Q_1$

Now, $T_3 = Q_2$

$Q_3^+ = T_3 \oplus Q_2 = Q_3 \oplus Q_2$

The state table looks like :

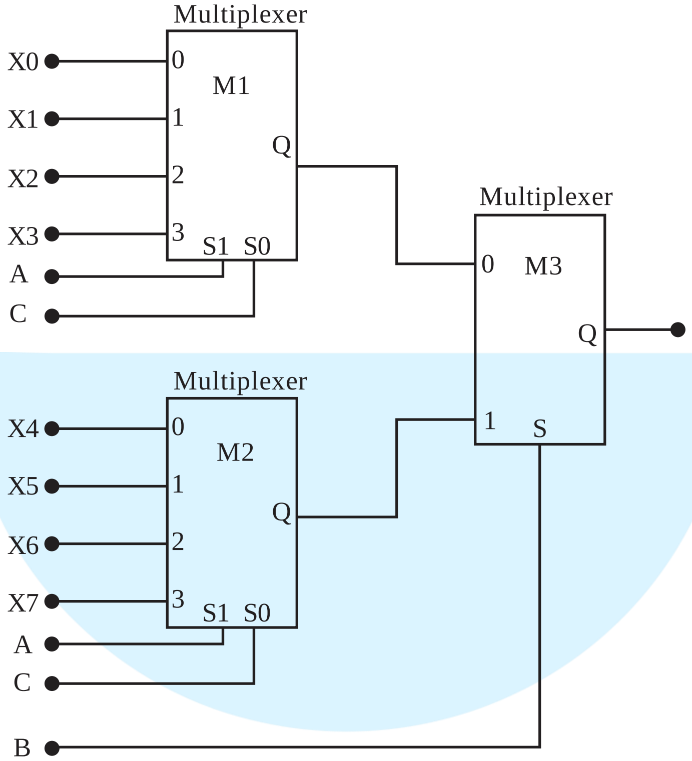
Q_1	Q_2	Q_3	Q_1^+	Q_2^+	Q_3^+
0	1	1	0	0	0
0	0	0	1	0	0
1	0	0	0	1	0
0	1	0	1	0	1
1	0	1	1	1	1
1	1	1	1	1	0
1	1	0	0	1	1

So, the missing state is 001.

Hence, the correct option is (A).

Question 34

A Boolean digital circuit is composed using two 4-input multiplexers (M1 and M2) and one 2-input multiplexer (M3) as shown in the figure. X0–X7 are the inputs of the multiplexers M1 and M2 and could be connected to either 0 or 1. The select lines of the multiplexers are connected to Boolean variables A, B and C as shown.



Which one of the following set of values of (X0, X1, X2, X3, X4, X5, X6, X7) will realise the Boolean function $\bar{A} + \bar{A}.\bar{C} + A.\bar{B}.C$?

(A) (1, 1, 0, 0, 1, 1, 1, 0) (B) (1, 1, 0, 0, 1, 1, 0, 1)

(C) (1, 1, 0, 1, 1, 1, 0, 0) (D) (0, 0, 1, 1, 0, 1, 1, 1)

Ans. **(C)**

Sol. **Given :**

$F = \bar{A} + \bar{A}\cdot\bar{C} + A\bar{B}C$.

Since final output is given by 2×1 mux m_3

So, MSB (most significant bit) is selection line of m_3 i.e. B .

Also we can observe from M_1 and M_2 , that the selection lines $(S_1, S_0) = (A, C)$ so the function has C as LSB

The function $F(B, A, C) = \bar{A} + \bar{A}\bar{C} + A\bar{B}C$ is implemented using inputs as :

	B	A	C	
X_0	0	0	0	$1 \rightarrow$ Due to A'
X_1	0	0	1	$1 \rightarrow$ Due to A'
X_2	0	1	0	0

X_3	0	1	1	$1 \to$ Due to $AB'C$
X_4	1	0	0	$1 \to$ Due to A'
X_5	1	0	1	$1 \to$ Due to A'
X_6	1	1	0	0
X_7	1	1	1	0

So, $(X_0, X_1.........X_7) = (11011100)$

Hence, the correct option is (C).

Question 35

Consider the IEEE-754 single precision floating point numbers P=0xC1800000 and Q=0x3F5C2EF4. Which one of the following corresponds to the product of these numbers (i.e., P × Q), represented in the IEEE-754 single precision format?

(A) 0x404C2EF4 (B) 0x405C2EF4

(C) 0xC15C2EF4 (D) 0xC14C2EF4

Ans. (C)

Sol. Here, $P = 0 \times 1800000 = 1100\ \ 0001\ \ 1000\ 0000\ \ 0000\ \ 0000\ 0000\ 0000$

In $1EEE$ 754 single precision format.

$(S) \to$ sign bit $= 1$

Biased exponent $= 131$

Actual exponent $= 131 - 127 = 4$

$(m) \to$ Mantissa = 0000 0000 0000 0000 0000 000

The number $= (-1)^s \times (1.m) \times 2^e = (-1)^1 \times 1.0 \times 2^4 = -16$

Similarity, $Q = 0 \times 3F5C2EE4 =$ 0011 1111 0101 1100 0010 1110 1111 0100

sign $= 0$

Biased exponent $= 126$

Actual exponent $= 126 - 127 = -1$

So, $Q = 1.10111000010111011110100 \times 2^{-1}$

$P*Q = -1.10111000010111011110100 \times 2^{-1} \times 2^4$

$P*Q = -1.10111000010111011110100 \times 2^3$

sign $= 1$

Biased exponent $= 127 + 3 = 130 = 10000010$.

The number in $1EEE$ 754 format is :

1	1000 0010	1011 1000 0101 1101 1110 100
Sign bit	Exponent	Mostissa

$0 \times C15C2EF4$

Hence, the correct option is (C).

Question 36

Let A be a priority queue for maintaining a set of elements. Suppose A is implemented using a max-heap data structure. The operation Extract-Max(A) extracts and deletes the maximum element from A. The operation Insert(A, key) inserts a new element key in A. The properties of a max-heap are preserved at the end of each of these operations.

When A contains n elements, which one of the following statements about the worst case running time of these two operations is TRUE?

(A) Both Extract-Max(A) and Insert(A, key) run in O(l).

(B) Both Extract-Max(A) and Insert(A, key) run in $O(\log(n))$.

(C) Extract-Max(A) runs in $O(l)$ whereas Insert(A, key) runs in $O(n)$.

(D) Extract-Max(A) runs in $O(l)$ whereas Insert(A, key) runs in $O(\log(n))$.

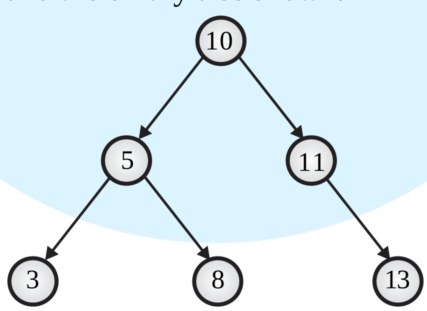
Ans. **(B)**

Sol. Since both extract-max (A) and Insert (A) needs to perform heapify ( ) operation, both take $0(\log(n))$ time.

Hence, the correct option is (B).

Question 37

Consider the C function foo and the binary tree shown.




```
typedef struct node {
        int val;
        struct node *left, *right;
} node;

int foo(node * ) {
        int retval;
        if (p == NULL)
             return 0;
        else {
             retval = p->val + foo(p->left) + foo(p->right);
             printf("%d ", retval);
             return retval;
        }
}
```

When foo is called with a pointer to the root node of the given binary tree, what will it print?

(A) 3 8 5 13 11 10

(B) 3 5 8 10 11 13

(C) 3 8 16 13 24 50 (D) 3 16 8 50 24 13

Ans. (C)

Sol. **Given :** $\text{Retval} = P \to val + f00(P \to \text{left}) + f00(P \to \text{right})$

Now, considering given tree with root 10.

$\text{Retval} = 10 + f00(P \to \text{left}) + foo(P \to \text{right})$

So, until we execute leaf node we won't get to return to root node.

Also child-Nodes 3, 8 and 13 will return value of leaf nodes, i.e. 3,8 and 13 only.

So, 5 node will return $5+3+8=16$.

11 node will return $11+13=24$

10 node will return $10+16+24=50$

Since there's no rule about evaluation order of parameters of '+' but considering/assuming left right rule by default we get output as : $3, 8, 16, 13, 24, 50$.

Hence, the correct option is (C).

Question 38

Let U = {1, 2,...,n}, where n is a large positive integer U with $|A| = |B| = k$ and $A \cap B = \phi$. We say that a permutation of U separates A from B if one of the following is true.

- All members of A appear in the permutation before any of the members of B.
- All members of B appear in the permutation before any of the members of A.

How many permutations of U separate A from B?

(A) $n!$ (B) $\binom{n}{2k}(n-2k)!$

(C) $\binom{n}{2k}(n-2k)!(k!)^2$ (D) $2\binom{n}{2k}(n-2k)!(k!)^2$

Ans. (D)

Sol. $U = \{1,2,.....n\}$

$k < n < 1000$

$|A| = k$

$|B| = k$

$A \cap B = \phi$

Permutation of $\cup$ separates of A from B

$2\binom{n}{2k}(n-2k)!(k!)^2$

Hence, the correct option is (D).

Question 39

Let $f: A \to B$ be an onto (or surjective) function, where A and B are nonempty sets. Define an equivalence relation $\sim$ on the set A as

$a \sim a_2$ if $f(a_1) = f(a_2)$,

Where $a_1, a_2 \in A$. Let $\varepsilon = \{[x] : x \in A\}$ be the set of all the equivalence classes under ~. Define a new mapping $F : \varepsilon \to B$ as

$F(|x|) = f(x)$, for all the equivalence classes $[x]$ in ε.

Which of the following statements is/are TRUE?

(A) F is NOT well-defined.

(B) F is an onto (or surjective) function.

(C) F is a one-to-one (or injective) function.

(D) F is a bijective function.

Ans. **(B), (C), (D)**

Sol. The equivalence relation on set A is defined as :

$a_1 \sim a_2$ if $f(a_1) = f(a_2)$

Where $a_1, a_2\ EA$.

Consider $a_i, b_i c_i ....EA$ and $\alpha, \beta, r..........EB$ and the mapping a_3 :

$a_1 \to \alpha,\ a_2 \to \alpha,\ a_3 \to \alpha.......a_n \to \alpha$

Similarity,

$b_1 \to \beta,\ b_2 \to \beta,\ b_3 \to ....b_n \to \beta$ and so on.........

According to equivalent, equivalent loss is :

$[a_1] = [a_2] = [a_3] = ... = [a_m]$

$[b_1] = [b_2] = [b_3] = ... = [b_n]$.........and so on.

So, set of equivalence classes under relation is :

$\varepsilon = \{[a_1], [b_1], [c_1] .....\}$

Now, given new mapping $F = \varepsilon \to B$ as :

$F([x]) = F(x)$ for all $[x]\ E\varepsilon$

It means mapping will be :

$a_1 \to \alpha, b_1 \to \beta, c_1 \to r$ ....and so on.

Since, all distinct a_1, b_1, c_1...............maps to different element of set B. SO F is injective.

We've considered $\{a_1, b_1, c_1......\}$ as leaders of their equivalent class.

We can also consider $\{a_2, b_2, c_2......\}$

Also, its cleared from mapping of F that all the elements of set B are

So, F is subjective.

Since F is both injective and subjective so F is bijective.

Also, we con observe that is well-defined function since its bijective.

Hence, the correct options are (B), (C), (D).

Question 40

Suppose you are asked to design a new reliable byte-stream transport protocol like TCP. This protocol, named myTCP, runs over a 100 Mbps network with Round Trip Time of 150 milliseconds and the maximum segment lifetime of 2 minutes.

Which of the following is/are valid lengths of the Sequence Number field in the myTCP header?

(A) 30 bits

(B) 32 bits

(C) 34 bits

(D) 36 bits

Ans. **(B), (C) (D)**

Sol. Given bandwidth (BW) = 100 Mbps

So, in 1 second we can send 100×10^{6} bits

$=125\times10^{3}\,B$

In 120 seconds $=120\times125\times10^{3}\,B$

$=15000\times10^{5}\,B$

Since n lifetime of 120 seconds 15×10^{8} bytes are generated, so

Sequence number bits $=\log_2(15\times10^{8})$

≥ 30.48

≥ 31 bits

Hence, the correct options are (B), (C) & (D).

Question 41

Let X be a set 2^{X} and denote the powerset of X.

Define a binary operation Δ on 2^{X} as follows :

$$A\Delta B=(A-B)\cup(B-A).$$

Let $H=(2^{X},\Delta)$. Which of the following statements about H is/are correct?

(A) H is a group.

(B) Every element in H has an inverse, but H is NOT a group.

(C) For every $A\in 2^{X}$, the inverse of A is the complement of A.

(D) For every $A\in 2^{X}$, the inverse of A is A.

Ans. **(A), (D)**

Sol. The symmetric difference is similar to EXOR operation in digital logic.

Now left check it for following properties :

1. Colored : Operator Δ is defined as

   $A\Delta B=(A-B)\cup(B-A)$

   $=(A\cup B)-(A\cap B)$

   $\because$ 2^{X} is power set of X, so it contains all subset of X.

   So, $A\Delta BE2^{X}\,\forall A,B,E2^{X}$.

2. Associativity : IT's similar to XOR operation which is associative always.

3. Identity : s_{ay} $C\in 2^{X}$ is identity element.

   So, $A\Delta C=C\Delta A=A$ for $C\in 2^{X}$

   $A\Delta C=(A\cup C)-(A\cap C)=A$ which is possible when $C=\Phi$.

   So, identity exists.

4. Inverse : $A\Delta B=B\Delta A=\Phi$ then $A\Delta B$ are inverse

   01 each option given $A,B\in 2^{X}$.

   So, $(A\cup B)-(A\cap B)=\Phi$

   It's possible $(A\cup B)=(A\cap B)$

If $A = B$ then its possible.

Hence, every element of 2^X is it's own inverse and H is a group.

Hence, the correct options are (A) & (D).

Question 42

Suppose in a web browser, you click on the www.gate-2023.in URL. The browser cache is empty. The IP address for this URL is not cached in your local host, so a DNS lookup is triggered (by the local DNS server deployed on your local host) over the 3-tier DNS hierarchy in an iterative mode. No resource records are cached anywhere across all DNS servers.

Let RTT denote the round trip time between your local host and DNS servers in the DNS hierarchy. The round trip time between the local host and the web server hosting www.gate-2023.in is also equal to RTT. The HTML file associated with the URL is small enough to have negligible transmission time and negligible rendering time by your web browser, which references 10 equally small objects on the same web server.

Which of the following statements is/are CORRECT about the minimum elapsed time between clicking on the URL and your browser fully rendering it?

(A) 7 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(B) 5 RTTs, in case of persistent HTTP with pipelining.

(C) 9 RTTs, in case of non-persistent HTTP with 5 parallel TCP connections.

(D) 6 RTTs, in case of persistent HTTP with pipelining.

Ans. (C), (D)

Sol. **Case I :**

Persistent HTTP : TCP connection is established once and multiple files are transmitted in single connection.

Pipelined : New HTTP request can be sent to server without receiving acknowledgement of previous.

S_{ay} a client Alfiya request a page from server, following steps are followed in order before Alfiya gets all data needed :

(i) There's 3-tier DNS hierarchy, 100 kup is done in iterative mode taking 3 RTT.

(ii) 1 RTT is used for TCP connection establishment.

(iii) 1 RTT is used to fetch HTML base file.

(iv) 1 RTT is for all other 10 objects.

So, n total 6 RTT's are used.

Case-II : Non persistent HTTP with 5 parallel connections :

Non persistent : TCP connection is made for each HTTP request and closed.

5-prallel connections : 5 objects could be sent parallel at same time.

So, in this case the client Alfiya have to wait for request to be fulfilled as following steps need to be completed.

(i) 3 RTT for DNS resolution.

(ii) 1 RTT for TCP connection establishment.

(iii) 1 RTT for fetch base HTML page.

(iv) 1 RTT for TCP connection establishment.

(v) 1 RTT to get 5 objects parallely (5 still test)

(vi) 1 RTT for TCP connection establishment

(viii) 1 RTT for getting remaining 5 objects.

In total it takes RTT's.

Hence, the correct options are (C) & (D).

Question 43

Consider a random experiment where two fair coins are tossed. Let A be the event that denotes HEAD on both the throws, B be the event that denotes HEAD on the first throw, and C be the event that denotes HEAD on the second throw. Which of the following statements is/are TRUE?

(A) A and B are independent.

(B) A and C are independent.

(C) B and C are independent.

(D) $\text{Prob}(B|C) = \text{Prob}(B)$

Ans. **(C, D)**

Sol. A : Head on both $H\ H$

B : Head on 1st

$H\ T$

$H\ H$

C : Head on second

$H\ H$ $\quad$ $T\ H$

$$P(A) = \frac{1}{4}$$

$$P(B \cap C) = \frac{1}{4}$$

$$P(B).P(C) = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$$

$P(B \cap C) = P(B).P(C)$ $\quad$ ($\therefore$ B and C are independent)

$$P(B) = \frac{1}{2}$$

$$P(A \cap B) = \frac{1}{4}$$

$$P(A).P(B) = \frac{1}{4}.\frac{1}{2} = \frac{1}{8}$$

$P(A \cap B) \neq P(A).P(B)$ $\quad$ ($\therefore$ A and B are not independent)

$$P(C) = \frac{1}{2}$$

$$P(A \cap C) = \frac{1}{4}$$

$$P(A).\text{P(C)} = \frac{1}{4}.\frac{1}{2} = \frac{1}{8}$$

$P(A \cap C) \neq P(A).P(C)$ $\quad$ ($\therefore$ A and C are not independent)

$$P\left(\frac{B}{C}\right)=\frac{P(B\cap C)}{P(C)}=\frac{1/4}{1/2}=\frac{1}{2}=P(B)$$

Hence, the correct option is (C, D).

Question 44

Consider functions Function 1 and Function 2 expressed in pseudocode as follows :

Function_1	Function_2

while $n>1$ do
for $i=1$ to n do
$x=x+1$;
end for
$n=\lfloor n/2\rfloor$;
end while
	for $i=1$ to $100*n$ do
	$x=x+1$;
	end for

Let $f_1(n)$ and $f_2(n)$ denote the number of times the statement "$x=x+1$" is executed in **Function_1** and **Function_2**, respectively.

Which of the following statements is/are TRUE?

(A) $f_1(n)\in\Theta(f_2(n))$ (B) $f_1(n)\in o(f_2(n))$

(C) $f_1(n)\in\omega(f_2(n))$ (D) $f_1(n)\in O(n)$

Ans. **(A), (D)**

Sol. Analysing function 1 first, we observe the number of times inner loop rum is halved every iteration

So, Number of times $n+\frac{n}{2}+\frac{n}{4}+\frac{n}{3}+......+1$

Inner loop runs $A(n)=n\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{3}+......\frac{1}{n}\right)$

Say, it takes p timer, so

$$A(n)=n\left\{\frac{1-\left(\frac{1}{2}\right)^p}{1-\frac{1}{2}}\right\}$$

Since, for large p, we can say $1-\left(\frac{1}{2}\right)^p\approx 1$

So, $A(n)=\frac{n}{\frac{1}{2}}=2n$

So, $f_1(n)=0(n)$

Now, Function 2 runs for 100 n times.

So, $f_2(n)=0(100n)$

$=0(n)$

So, $f_1(n)\in\theta(f_2(n))$

Also, $f_1(n) \in 0(f_2(n))$ is true.

Hence, the correct options are (A) & (D).

Question 45

Let G be a simple, finite, undirected graph with vertex set $\{v_1, \ldots, v_n\}$. Let $\Delta(G)$ denote the maximum degree of G and let $\mathbb{N} = \{1, 2, \ldots\}$ denote the set of all possible colors. Color the vertices of G using the following greedy strategy :

for $i = 1, \ldots, n$

color $(v_i) \leftarrow \min\{j \in \mathbb{N} : \text{no neighbour of } v_i \text{ is colored } j\}$

Which of the following statements is/are TRUE?

(A) This procedure results in a proper vertex coloring of G.

(B) The number of colors used is at most $\Delta(G) + 1$.

(C) The number of colors used is at most $\Delta(G)$.

(D) The number of colors used is equal to the chromatic number of G.

Ans. **(A), (B)**

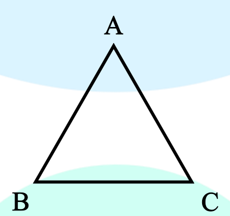
Sol. **For Option (A) :** Its, true as

"Color (vi) $\leftarrow$ min <JEN = No neighbor of vi is colored J>"

So, it ensures proper coloring.

For Option (B) : We can take example of a cycle of length 3.

Here, $\Delta G = 2$



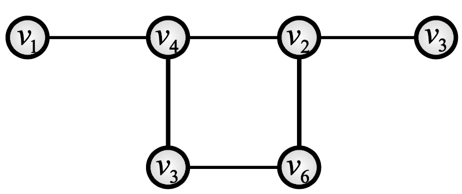
But we need 3 colours to color it Also, number of neighbor's can't be more than the degree, i.e . ΔG .

So, at most $\Delta G + 1$ colours needed.

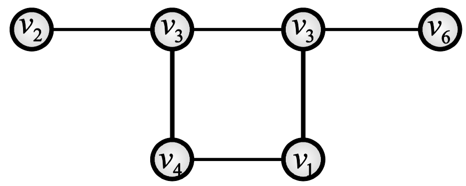
This option is true.

For Option (C) It's False as wave explained above.

For Option (D) It is not always the case as sometimes we might see that greedy coloring might not be giving optimal result. Consider example.



Uses 3 color and



Uses 3 color

When coloured greedily in order $(v_1, v_2......v_6)$. 90, number of color used $\neq$ Chromatic number of graph

Hence, the correct options are (A) & (B).

Question 46

Let $U = \{1,2,3\}$. Let 2^U denote the powerset of U. Consider an undirected graph G whose vertex set is 2^U. For any $A, B \in 2^U, (A, B)$ is an edge in G if and only if (i) $A \neq B$, and (ii) either $A \subseteq B$ or $B \subseteq A$. For any vertex A in G, the set of all possible orderings in which the vertices of G can be visited in a Breadth First Search (BFS) starting from A is denoted by $B(A)$.

If $\emptyset$ denotes the empty set, then the cardinality of $B(\emptyset)$ is _______.

Ans. **5040**

Sol. Here, given that u = <1,2,3>

Vertex set = Power set of U = 2^u = { Φ , <1>, <2>, <3>, <1,2>, <1,3> <1,2,3>}

So, humber of vertices = 8

Now, there's edge between A and B iff either of them is proper subset of another.

Since, Φ is proper subset of all other vertices except itself, so its' connected to all 7 vertices.

Since it can be visited in any order.

So, cardinality of B(Φ)=7! = 5040

Hence, the correct answer is 5040.

Question 47

Consider the following two-dimensional array D in the C programming language, which is stored in row-major order :

int D[128][128];

Demand paging is used for allocating memory and each physical page frame holds 512 elements of the array D. The Least Recently Used (LRU) page-replacement policy is used by the operating system. A total of 30 physical page frames are allocated to a process which executes the following code snippet :

```
for (int i = 0; i < 128; i++)
   for (int j = 0; j < 128; j++)
         D[j][i] *= 10;
```

The number of page faults generated during the execution of this code snippet is _______.

Ans. **4096**

Sol. Given array D[128] [123] is stored in Row – major Order.

Number of physical frames available = 30

Number of elements in 1 frame = 512

So, number of pages to accommodate all element of array $D = \frac{123 \times 128}{512} = 32$.

Since we need 32 frames and were given only 30 so, collision will occur.

Also number of rows per frame $= \frac{512}{128} = 4$

So, in 30 frames we can per store 120 rows

Thus in 1^{st} iteration, It' cause 32 page faults.

For 128 iterations it' cause $128 \times 32 = 4096$ faults.

Hence, the correct answer is 4096.

Question 48

Consider a computer system with 57-bit virtual addressing using multi-level tree-structured page tables with L levels for virtual to physical address translation. The page size is 4 KB (1 KB = 1024 B) and a page table entry at any of the levels occupies 8 bytes.

The value of L is _______.

Ans. **5**

Sol. Virtual address is 57 bits, page size is 4kB $= 2^{12}$B

Number of page $= \frac{2^{57}}{2^{12}} = 2^{45}$

Page Table Entry = 8kB

So, Each page can contain $\frac{4kB}{8B} = 2^9$ page entries

So, We need 9 bits to index page table.

So, number of levels $= \left\lceil \frac{45}{9} \right\rceil = 5$

Hence, the correct answer is 5.

Question 49

Consider a sequence a of elements $a_0 = 1, a_1 = 5, a_2 = 7, a_3 = 8, a_4 = 9,$ and $a_5 = 2.$ The following operations are performed on a stack S and a queue Q, both of which are initially empty.

I : push the elements of a from a_0 to a_5 in that order into S.

II : enqueue the elements of a from a_0 to a_5 in that order into Q.

III : pop an element from S.

IV : dequeue an element from Q.

V : pop an element from S.

VI : dequeue an element from Q.

VII : dequeue an element from Q and push the same element into S.

VIII : Repeat operation VII three times.

IX : pop an element from S.

X : pop an element from S.

The top element of S after executing the above operations is _______.

Ans. **8**

Sol. Given Elements $(a_0, a_1, ...a_4) = (i1,5,7,8,9,2)$

Step – 1 :

2	← TOS
9	
8	
7	
5	
1	

Step -2 :

1	3	7	8	9	2
↑ Head					↑ Tail

Step-3 :

9	← TOS
8	
7	
5	
1	

Step – 4 :

5	7	8	9	2
↑ Head				↑ Tail

Step – 5 :

8	← TOS
7	
5	
1	

Step – 6 :

7	8	9	2
↑ Head			↑ Tail

Step – 7 :

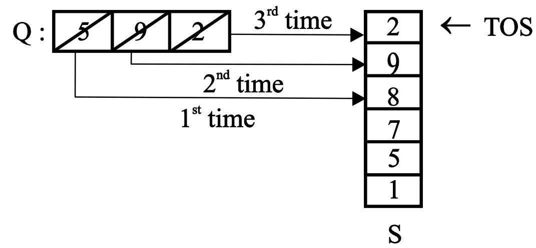
Q :

8	9	2
↑ Head		↑ Tail

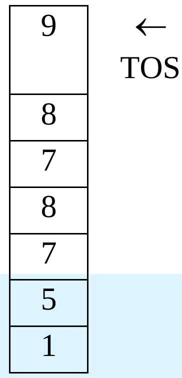
7	← TOS
8	
7	
5	
1	

S

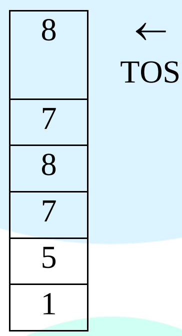
Step-8 :



Step – 9 :



Step – 10 :



∴ '8' in Top of stack So answer is 8.

Hence, the correct option is 8.

Question 50

Consider the syntax directed translation given by the following grammar and semantic rules. Here *N*, *I*, *F* and B are non-terminals. *N* is the starting non-terminal, and #, **0** and **1** are lexical tokens corresponding to input letters "#", "0" and "1", respectively. *X.val* denotes the synthesized attribute (a numeric value) associated with a non-terminal *X*. I_1 and F_1 denote occurrences of *I* and *F* on the right hand side of a production, respectively. For the tokens **0** and **1**, **0**.val = 0 and **1**.val = 1.

$N \to I\#F$	$N.val \to I.val + F.val$
$I \to I_1B$	$I.val \to (2I_1.val) + B.val$
$I \to B$	$I.val = B.val$
$F \to BF_1$	$F.val = \frac{1}{2}(B.val + F_1.val)$
$F \to B$	$F.val = \frac{1}{2}\text{B}.val$
$B \to 0$	$B.val = \mathbf{0}.val$

$B \rightarrow \mathbf{1}$ $\qquad\qquad\qquad\qquad$ $B.val = \mathbf{1}.val$

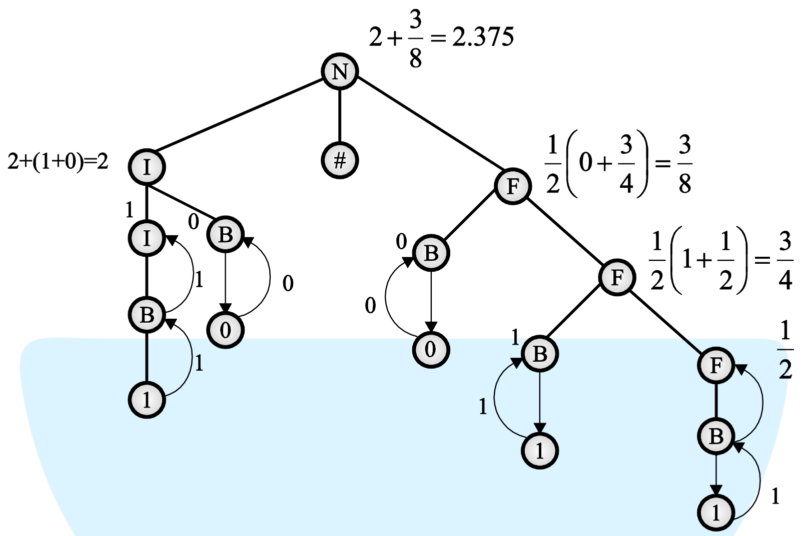
The value computed by the translation scheme for the input string

10 # 011

is ________. (Rounded off to three decimal places)

Ans. **2.375**

Sol.



Hence, the correct answer is 2.375.

Question 51

Consider the following table named Student in a relational database. The primary key of this table is rollNum.

Student :

rollNum	Name	gender	marks
1	Naman	M	62
2	Aliya	F	70
3	Aliya	F	80
4	James	M	82
5	Swati	F	65

The SQL query below is executed on this database.

```
SELECT *
FROM Student
WHERE gender = 'F' AND
      marks > 65;
```

The number of rows returned by the query is ________.

Ans. **2**

Sol. Were to return female students with marks greater than 65.

Output is :

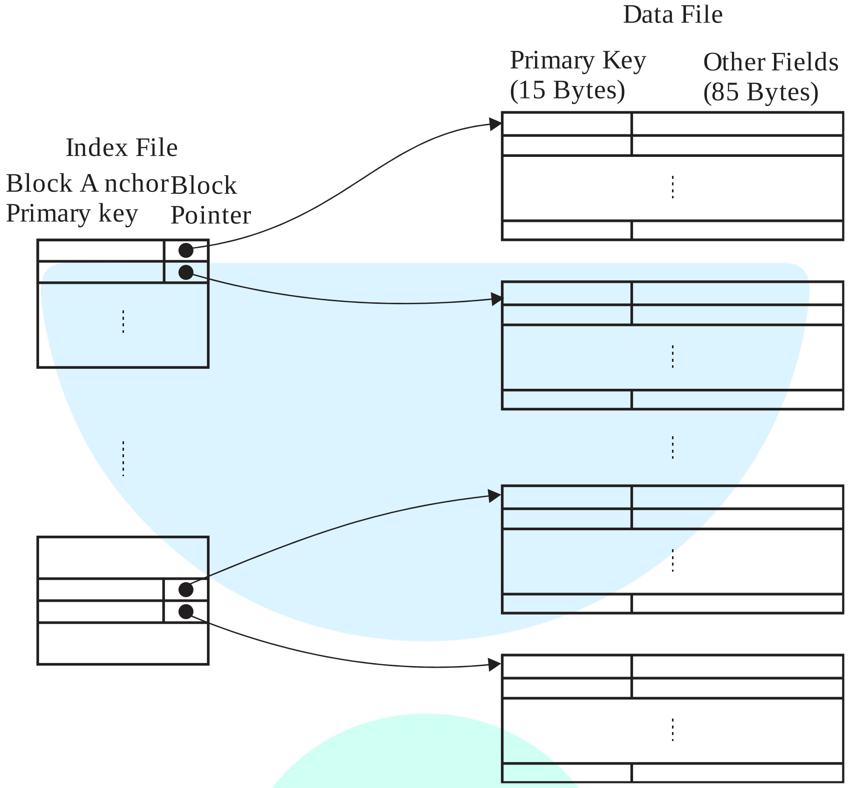
Roll	Name	Gender	Marks
2	Aliya	F	70

3	Aliya	F	80

Hence, the correct answer is 2.

Question 52

Consider a database of fixed-length records, stored as an ordered file. The database has 25,000 records, with each record being 100 bytes, of which the primary key occupies 15 bytes. The data file is block-aligned in that each data record is fully contained within a block. The database is indexed by a primary index file, which is also stored as a block-aligned ordered file. The figure below depicts this indexing scheme.



Suppose the block size of the file system is 1024 bytes, and a pointer to a block occupies 5 bytes. The system uses binary search on the index file to search for a record with a given key. You may assume that a binary search on an index file of b blocks takes $\lceil \log_2 b \rceil$ block accesses in the worst case.

Given a key, the number of block accesses required to identify the block in the data file that may contain a record with the key, in the worst case, is ________.

Ans. **6**

Sol. Given database is stored as ordered file and indexed by primary index file

There're 25,000	records
100 B	records size
15 B	Primary key size
5 B	Pointer size
1024 B	Block size

It's stored in unspanned organization.

So, Number of records per block $= \left\lfloor \dfrac{\text{Block size}}{\text{Record size}} \right\rfloor = \left\lfloor \dfrac{1024 \text{ B}}{100 \text{ B}} \right\rfloor = 10$

Number of data blocks needed $= \left\lceil \dfrac{\text{Number of records}}{\text{Number of records per block}} \right\rceil = \dfrac{25000}{10} = 2500$

No. of Index records per block $= \left\lfloor \dfrac{\text{Block size}}{\text{primany key size} + \text{Pointer size}} \right\rfloor = \left\lfloor \dfrac{1024}{15+5} \right\rfloor = \left\lfloor \dfrac{1024}{20} \right\rfloor = 51$

Number of index block needed $= \left\lceil \dfrac{2500}{51} \right\rceil = 50$

Applying binary search, $\lceil \log_2(50) \rceil = 6$

Hence, the correct answer is 6.

Question 53

Consider the language L over the alphabet {0, 1}, given below :

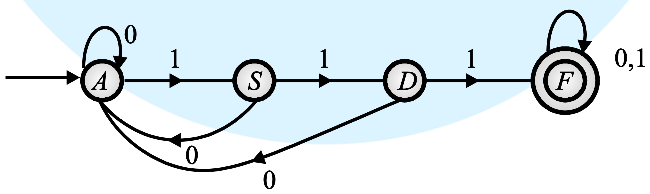
$L = \{\omega \in \{0,1\}^* \mid \omega$ does not contain three or more consecutive 1's$\}$.

The minimum number of states in a Deterministic Finite-State Automaton (DFA) for L is ________.

Ans. **4**

Sol. Say L : set of strings containing 3 consecutive is.

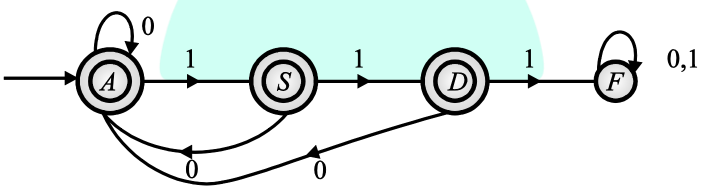
The MDFA for L $\rightarrow$



I: set of strings not containing 3 consecutive

So, no. of state remains same i.e. 4

MDFA For $I \rightarrow$



Hence, the correct answer is 4.

Question 54

An 8-way set associative cache of size 64 KB (1 KB = 1024 bytes) is used in a system with 32-bit address. The address is sub-divided into TAG, INDEX, and BLOCK OFFSET.

The number of bits in the TAG is ________.

Ans. **19**

Sol. **Given :** Cache Size $CS = 64 \times B = 2^{16}$B

System use 32 bit address (A)

So, TAG = A :- $\log_2(CS) + \log_2(P)$

$= 32 - \log_2(2^{16}) + \log_2(8)$

$= 32 - 16 + 8 = 19$

Hence, the correct option is 19.

Question 55

The forwarding table of a router is shown below.

Subnet Number	Subnet Mask	Interface ID
200.150.0.0	255.255.0.0	1
200.150.64.0	255.255.224.0	2
200.150.68.0	255.255.255.0	3
200.150.68.64	255.255.255.224	4
Default		0

A packet addressed to a destination address 200.150.68.118 arrives at the router. It will be forwarded to the interface with ID _______.

Ans. **3**

Sol. We will perform AND operation between IP and Subnet mask and see it we get same subnet –ID or not and well do longest prefix match.

So, checking subnet – 4.

IP $\wedge$ subnet = (200.150.68.118) $\wedge$(255.255.255.224) = (200.150.68.96)

Subnet ID didn't match.

Checking subnet 3

(200.150.68.118) $\wedge$ (255.255.255.0).(200.150.68.0)

Subnet ID matches.

If ll be forwarded to 3.

Hence, the correct answer is 3.

❑❑❑