

# Problems Discussion

Comprehensive Course on Engineering Mathematics

# CALCULUS

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- 1.Limits**
- 2.Continuity**
- 3.Differentiability**
- 4.Mean value theorems**
- 5.Taylors series**
- 6.Maxima and minima**
- 7.Integration**

# Function

The relationship between input and the outputs is called as a function.

The relationship between dependent variable and the independent variable is called as a function

## Even function

A function  $f(x)$  is said to be an even function if  $f(-x) = f(x)$ .

Ex:

## Odd Function:

A function  $f(x)$  is said to be an odd function if  $f(-x) = -f(x)$ .

Ex:

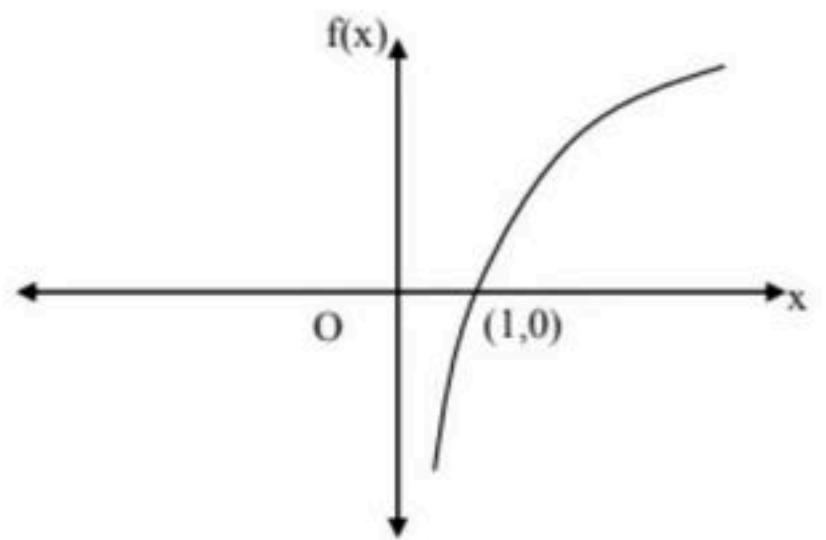
# Modulus function

Step function (Greatest integer function)(Bracket function) (Floor value function )

# Signum function

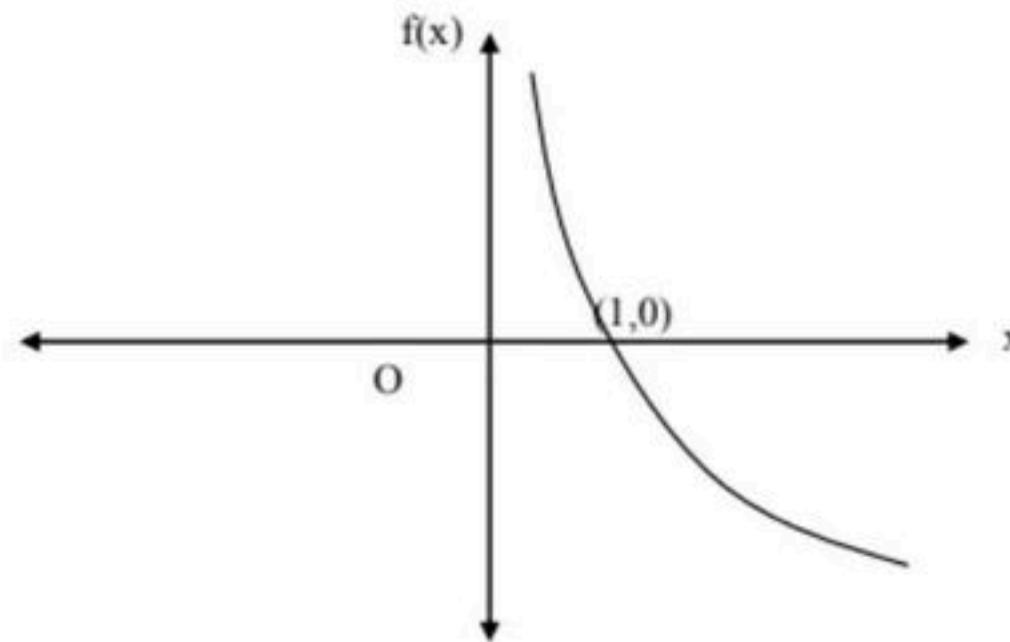
# Logarithmic function

A function of the form  $f(x) = \log_a x$



**Case – I**

For  $a > 1$



**Case – II**

For  $0 < a < 1$

14. The function  $f(x) = e^x$  is \_\_\_\_\_ **(GATE -EC-1999)**
- (a) Even      (b) Odd      (c) Neither even nor odd      (d) None

Limit of a function gives approximate value of the function in the neighbourhood of a point.

To examine the behaviour of the function  $y = f(x)$  , in the neighbourhood of a point  $x = a$  ,  
when  $f(x)$  is indeterminate at  $x= a$  .

## Limit of a function

Limit of a function  $f(x)$  is said to exist at  $x = a$  , if

## Difference between limit of a function and functional value at a given point

## Reasons for non -existence of limit

1. If LHL or RHL or both does not exits
2. If LHL and RHL both exists but they are unequal

# Indeterminate forms



# Determinate forms

# Algebra of limits

## Evaluation of Limits

1. Find the value of function at the given limit. If it is determinate , it self is the answer .
2. If the value is indeterminate of the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$  , then apply L' Hospital rule .
3. If the value is indeterminate , but not in the  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$  form, then convert to this form .
4. L' Hospital rule can be applied only in case of the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$  ,if can be applied any number of times, but check whether it is of the form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$





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$$1. \lim_{x \rightarrow 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} = \underline{\hspace{2cm}}$$

(GATE -ME- 1993)

4.  $\lim_{x \rightarrow \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} = \underline{\hspace{2cm}}$

(GATE -CS- 1995)

- (a)  $\infty$
- (b) 0
- (c) 2
- (d) Does not exist

5.  $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = \underline{\hspace{2cm}}$

(GATE -CS- 1995)

- (a)  $\infty$
- (b) 0
- (c) 1
- (d) Does not exist

8.  $\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\theta}$ , where  $m$  is an integer, is one of the following:

(GATE -CS- 1997)

- (a)  $m$
- (b)  $m\pi$
- (c)  $m\theta$
- (d) 1

11.  $\lim_{x \rightarrow 0} \frac{1}{10} \frac{1 - e^{-j5x}}{1 - e^{-jx}} = \underline{\hspace{2cm}}$

(GATE -IN-1998)

(a) 0

(b) 1.1

(c) 0.5

(d) 1

12. Limit of the function,  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{n^2 + n}}$  is \_\_\_\_\_

(GATE -EC-1999)

- (a)  $\frac{1}{2}$
- (b) 0
- (c)  $\infty$
- (d) 1

13. Value of the function  $\lim_{x \rightarrow a} (x-a)^{x-a}$  is \_\_\_\_\_ (GATE -CS-1999)

- (a) 1
- (b) 0
- (c)  $\infty$
- (d) a

16. Limit of the function  $f(x) = \frac{1-a^4}{x^4}$  as  $x \rightarrow \infty$  is given by

(GATE -CS-2000)

- (a) 1
- (b)  $e^{-a^4}$
- (c)  $\infty$
- (d) 0



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$$17. \lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} = \underline{\hspace{2cm}}$$

(GATE-IN-2001)

- (a) 0
- (b)  $\frac{1}{2}$
- (c) 1
- (d) 2

18. Limit of the following sequence as  $n \rightarrow \infty$  is \_\_\_\_\_  $x_n = n^{\frac{1}{n}}$

(GATE -CE-2002)

- (a) 0
- (b) 1
- (c)  $\infty$
- (d)  $-\infty$

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$$20. \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \underline{\hspace{2cm}}$$

(GATE-CS-2003)

- (a) 0
- (b)  $\infty$
- (c) 1
- (d) -1

21. The value of the function,  $f(x) = \lim_{x \rightarrow 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$  is \_\_\_\_\_ (GATE-CS-2004)

(a) 0

(b)  $\frac{-1}{7}$

(b)  $\frac{1}{7}$

(d)  $\infty$

$$23. \lim_{x \rightarrow 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} =$$

**(GATE-ME-2007)**

(a) 0

(b)  $\frac{1}{6}$

(c)  $\frac{1}{3}$

(d) 1

25. What is the value of  $\lim_{x \rightarrow \pi/4} \frac{\cos x - \sin x}{x - \pi/4}$  (GATE-PI-  
2007)

- (a)  $\sqrt{2}$
- (b) 0
- (c)  $-\sqrt{2}$
- (d) Limit does not exist

 unacademy  
26.  $\lim_{\theta \rightarrow 0} \frac{\sin(\theta/2)}{\theta}$  is

**(GATE-EC-2007)**

- (a) 0.5
- (b) 1
- (c) 2
- (d) not defined

27.  $\lim_{x \rightarrow \infty} \frac{x - \sin x}{x + \cos x} = \underline{\hspace{2cm}}$

(GATE-EC-2008)

(a) 1

(b) -1

(c)  $\infty$

(d)  $-\infty$

30. The value of  $\lim_{x \rightarrow 8} \frac{x^{1/3} - 2}{x - 8}$  is

(GATE-ME-2008)

(a)  $\frac{1}{16}$

(b)  $\frac{1}{12}$

(c)  $\frac{1}{8}$

(d)  $\frac{1}{4}$

31. The value of the expression  $\lim_{x \rightarrow 0} \left[ \frac{\sin(x)}{e^x - x} \right]$  is

(GATE-PI-2008)

(a) 0

(b)  $\frac{1}{2}$

(c) 1

(d)  $\frac{1}{1+e}$

34. What is the value of  $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{2n}$  ?

(GATE-CS-2010)

(a) 0

(b)  $e^{-2}$

(c)  $e^{-t/2}$

(d) 1

50. The value of  $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^2 + 2020}}{x + 7}$  is

GATE-2021 (CE)

- (a) 1
- (b) 3
- (c)  $\frac{7}{9}$
- (d) Indeterminable

53. The value of  $\lim_{x \rightarrow \infty} (1 + x^2)^{e^{-x}}$  is

**(GATE-CS-2015)**

- (a) 0
- (b)  $\frac{1}{2}$
- (c) 1
- (d)  $\infty$

55. The value of  $\lim_{x \rightarrow \infty} \frac{1 - \cos(x^2)}{2x^4}$  is

**(GATE-ME-2015)**

- (a) 0
- (b)  $\frac{1}{2}$
- (c)  $\frac{1}{4}$
- (d) undefined

57. The value of  $\lim_{x \rightarrow 0} \left( \frac{-\sin x}{2 \sin x - x \cos x} \right)$  is \_\_\_\_\_

(GATE-ME-2015)

58. The value of  $\lim_{x \rightarrow \infty} \frac{x^2 - 5x + 4}{4x^2 + 2x}$  is

GATE-2020 (CE)

(a) 0

(b) 1

(c)  $\frac{1}{4}$

(d)  $\frac{1}{2}$

Consider the limit:

GATE-2021 (CE)

$$\lim_{x \rightarrow 1} \left( \frac{1}{\ln x} - \frac{1}{x-1} \right)$$

The limit (correct up to one decimal place) is \_\_\_\_\_

60. The value of  $\lim_{x \rightarrow \infty} \frac{x \ln(x)}{1+x^2}$  is

GATE-2021(CE)

- (a) 1.0
- (b) 0.5
- (c)  $\infty$
- (d) 0



# Continuity

## Continuity

A function  $f(x)$  is said to be continuous at  $x=a$  if it satisfies the following conditions.

(i)  $f(a)$  is defined

(ii)  $\lim_{x \rightarrow a^-} f(x)$  exists i.e  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x)$

(iii)  $\lim_{x \rightarrow a} f(x) = f(a)$

•

# Reasons of discontinuity

1. If the function is not defined at a given point.
2. If the limit of the function not exists
3. If the limit of the function exists , functional value exists but both are not equal

## Left continuous (or) continuity from the left at a point

A function  $f(x)$  is said to be continuous from the left (or) left continuous at  $x=a$  if

(i)  $f(a)$  is defined

(ii)  $\lim_{x \rightarrow a^-} f(x) = f(a)$

## Right continuous (or) continuity from the right at a point

A function  $f(x)$  is said to be continuous from the right (or) right continuous at  $x=a$  if

(i)  $f(a)$  is defined

(ii)  $\lim_{x \rightarrow a^+} f(x) = f(a)$

## Continuity of a function in an open interval:

A function  $f(x)$  is said to be continuous in an open interval  $(a,b)$

if  $f(x)$  is continuous  $\forall x \in (a, b)$

## Continuity of a function on closed interval:

A function  $f(x)$  is said to be continuous on closed interval  $[a,b]$  if

(i)  $f(x)$  is continuous  $(a, b)$

(ii)  $\lim_{x \rightarrow a^+} f(x) = f(a)$

(iii)  $\lim_{x \rightarrow b^-} f(x) = f(b)$

1.  $\sin x$ ,  $\cos x$ ,  $e^x$ ,  $a^x$ ,  $|x|$ , polynomial functions are always continuous .  
2.

Function	Points of discontinuity
$\tan x$ , $\sec x$	
$\cot x$ , $\operatorname{cosec} x$	
$[x]$	
$\frac{1}{x}$	
$\operatorname{Sgn}(x)$	

3. If  $f(x)$  and  $g(x)$  are two continuous functions then  $f(x) + g(x)$ ,  $f(x) - g(x)$ ,  $f(x) \cdot g(x)$  and  $\frac{f(x)}{g(x)}$  (since  $g(x) \neq 0$ ) are also continuous.

4. Logarithmic functions are continuous in  $(0, \infty)$

# Types of discontinuity

## 1. Discontinuity of first kind (or) Removable discontinuity

- a. Missing point discontinuity
- b. Isolated point discontinuity

## 2. Discontinuity of second kind (or) Irremovable discontinuity

- a. Finite discontinuity(Jump discontinuity )
- b. Infinite discontinuity
- c. Oscillatory discontinuity

# Discontinuity of first kind (or) Removable discontinuity

## 1. Missing point discontinuity

## 2. Isolated point discontinuity

## **Discontinuity of second kind (or) Irremovable discontinuity**

### 1. Finite discontinuity

## 2. Infinite discontinuity

### 3. Oscillatory discontinuity

37. What should be the value of  $\lambda$  such that the function defined below is continuous at  $x = \frac{\pi}{2}$ ? (GATE-CE-2011)

$$f(x) = \begin{cases} \frac{\lambda \cos x}{\pi}, & \text{if } x \neq \frac{\pi}{2} \\ \frac{\pi}{2} - x \\ 1 & , \text{ if } x = \frac{\pi}{2} \end{cases}$$

42. Which one of the following functions is continuous at  $x = 3$ ?

(GATE-CS-2013)

$$(a) f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \\ \frac{x+3}{3}, & \text{if } x < 3 \end{cases}$$

$$(b) f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8 - x & \text{if } x \neq 3 \end{cases}$$

$$(c) f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 4 & \text{if } x > 3 \end{cases}$$

$$(d) f(x) = \frac{1}{x^3 - 27}, \text{ if } x \neq 3$$

# Continuity of Composite functions



# Differentiability

## Derivative of a function at a point:

If a function  $f(x)$  is defined on a neighborhood of a real number ‘ $a$ ’ and  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  exists and finite then the finite limit is called derivative or differential coefficient of  $f(x)$  at a point ‘ $a$ ’ and it is denoted by  $f'(a)$ .

$$\therefore \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a)$$



**RHD**

## Differentiability of a function in an interval

A function  $f(x)$  is said to be differentiable in an interval  $[a, b]$

**1.  $f(x)$  is continuous in  $(a, b)$**

**2.  $f^1(a^-) = \lim_{x \rightarrow a^-} \frac{f(x) - f(a)}{x - a}$  exists**

**3.  $f^1(a^+) = \lim_{x \rightarrow a^+} \frac{f(x) - f(a)}{x - a}$  exists**

**4.  $f^1(a^-) = f^1(a^+)$**

# Working Procedure to check continuity & differentiability

1. Check for continuity of the given function
2. If it is discontinuous , then the function is non differentiable ,
3. If it is continuous , find the LHD and RHD .
4. If LHD and RHD exists, and  $\text{LHD} = \text{RHD}$  , then the function is differentiable .



- If  $f(x)$  and  $g(x)$  are two differentiable functions then  $f(x)+g(x)$ ,  $f(x)-g(x)$ ,  $f(x) \cdot g(x)$ ,  $\frac{f(x)}{g(x)}$  ( $g(x) \neq 0$ ) are also differentiable.
- Polynomial functions, exponential functions, sine and cosine functions are differentiable everywhere.
- Every differentiable function is continuous but a continuous function need not be differentiable.
- If the function is discontinuous, then it is not differentiable.
- $|x|$  is continuous but not differentiable at  $x = 0$
- $|x-a|$  is continuous but not differentiable at  $x = a$
- $|ax-b|$  is continuous but not differentiable at  $x = \frac{b}{a}$
- $\text{Sgn}(x-a)$  is not differentiable at  $x = a$
- $[x]$  is not differentiable at all integers

# Reasons for non differentiability of a function

## 1. Sharp Corner

## 2. Vertical tangent

### 3. Having discontinuities at $x = a$

## 4. Function tending to infinite at $x = a$

6. The function  $f(x) = |x+1|$  on the interval  $[-2, 0]$  is \_\_\_\_\_

(GATE-EC- 1995)

- (a) continuous and differentiable
- (b) continuous on the interval but not differentiable at all points
- (c) Neither continuous nor differentiable
- (d) Differentiable but not continuous

9. If  $y = |x|$  for  $x < 0$  and  $y = x$  for  $x \geq 0$  then  
**(GATE -EC- 1997)**

- (a)  $\frac{dy}{dx}$  is discontinuous at  $x = 0$
- (b)  $y$  is discontinuous at  $x = 0$
- (c)  $y$  is not defined at  $x = 0$
- (d) Both  $y$  and  $\frac{dy}{dx}$  are discontinuous at  $x = 0$

7. If a function is continuous at a point its first derivative (GATE -EC- 1995)

- (a) may or may not exist
- (b) exists always
- (c) will not exist
- (d) has a unique value

19. Which of the following functions is not differentiable in the domain [-1, 1]?

(a)  $f(x) = x^2$

(b)  $f(x) = x - 1$

(GATE -EC-2002)

(c)  $f(x) = 2$

(d)  $f(x) = \max(1-x, x)$

22. Consider the function  $f(x) = |x|^3$ , where  $x$  is real.



Then the function  $f(x)$  at  $x = 0$  is

**(GATE -IN-2007)**

- (a) continuous but not differentiable
- (b) once differentiable but not twice
- (c) twice differentiable but not thrice
- (d) thrice differentiable

33. If  $f(x) = \sin |x|$  then the value of  $\frac{df}{dx}$  at  $x = -\frac{\pi}{4}$  is (GATE-PI-2010)

- (a) 0
- (b)  $\frac{1}{\sqrt{2}}$
- (c)  $-\frac{1}{\sqrt{2}}$
- (d) 1

36. The function  $y = |2-3x|$

(GATE-ME-2010)

- (a) is continuous  $\forall x \in \mathbb{R}$  and differentiable  $\forall x \in \mathbb{R}$
- (b) is continuous  $\forall x \in \mathbb{R}$  and differentiable  $\forall x \in \mathbb{R}$  except at  $x = \frac{3}{2}$
- (c) is continuous  $\forall x \in \mathbb{R}$  and differentiable  $\forall x \in \mathbb{R}$  except at  $x = \frac{2}{3}$
- (d) is continuous  $\forall x \in \mathbb{R}$  and differentiable  $\forall x \in \mathbb{R}$  except at  $x = 3$

46. If a function is continuous at a point, (GATE-ME-SET-3-2014)

- (a) the limit of the function may not exist at the point
- (b) the function must be derivable at the point
- (c) the limit of the function at the point tends to infinity
- (d) the limit must exist at the point and the value of limit should be same as the value of the function at the point.

# Differentiability of $Y = |f(x)|$

Q. Check the differentiability of  $y = |x|$

Q. Check the differentiability of  $y = |e^x|$

Q. Check the differentiability of  $y = |\sin x|$

Q. Check the differentiability of  $y = |x^3|$



# Differentiability of $f(x)$ , $g(x)$

Q. Check the differentiability of  $y = x|x|$

Q. Check the differentiability of  $y = \cos x|x|$

Q. Check the differentiability of  $y = (x^2 - 3x + 2)|x^2 - 5x + 6|$





# Mean Value Theorems

# Rolle's Theorem

Q. Find the value of 'C' from Rolle's theorem for the function  $f(x) = x^2 - 1$  on  $[-1, 1]$

Q. Find the value of 'C' from Rolle's theorem for the function  $f(x) = (x - a)^m(x - b)^n$  on  $[a, b]$

 Q. Find the value of 'C' from Rolle's theorem for the function  $f(x) = \log \left[ \frac{x^2+ab}{(a+b)x} \right]$  on  $[a, b]$



# Lagrange's Mean Value Theorem

Q. Find 'C' of LMVT for  $f(x) = \sin x - \sin 2x$  in  $[0, \pi]$

 Q. If  $f'(x) = \frac{1}{3-x^2}$  and  $f(0) = 1$ , find an interval in which  $f(1)$  lies



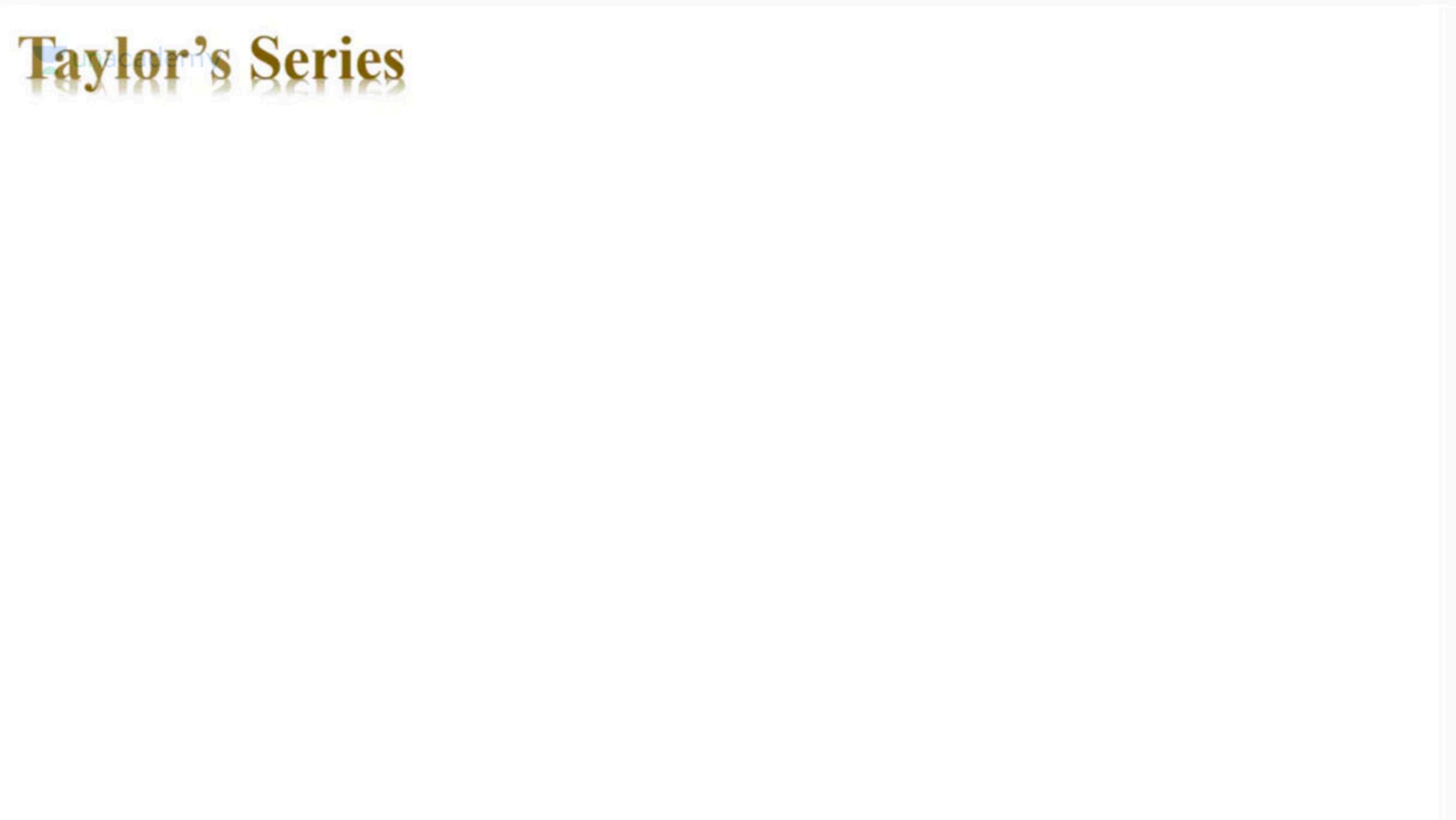
# Cauchy's Mean Value Theorem

Q. The 'C' of Cauchy's mean value theorem for  $f(x) = e^x$ ,  $g(x) = e^{-x}$  defined in  $[a, b]$  is -----

 Q. The 'C' of Cauchy's mean value theorem for  $f(x) = \frac{1}{x}$ ,  $g(x) = \frac{1}{x^2}$  defined in  $[a, b]$  is -----

Q. The 'C' of Cauchy's mean value theorem for  $f(x) = \sin x$ ,  $g(x) = \cos x$   
defined in  $[a, b]$  is -----

# Taylor's Series



# MacLaurin's Series

Q. Expand  $e^x$  by Taylor's series about  $x=0$

97. The third term in the taylor's series expansion of  $e^x$  about 'a' would be \_\_\_\_\_

- (a)  $e^a (x-a)$       (b)  $\frac{e^a}{2} (x-a)^2$       (c)  $\frac{e^a}{2}$       (d)  $\frac{e^a}{6} (x-a)^3$       GATE -1995

98. The taylor's series expansion of sin x is \_\_\_\_\_ (GATE-EC-1998)

(a)  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

(b)  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$

(c)  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

(d)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

100. unattempted The Taylor series expansion of  $\sin x$  about  $x = \frac{\pi}{6}$  is given by **(GATE-CE-2000)**

(a)  $\frac{1}{2} + \frac{\sqrt{3}}{2} \left( x - \frac{\pi}{6} \right) - \frac{1}{4} \left( x - \frac{\pi}{6} \right)^2 - \frac{\sqrt{3}}{12} \left( x - \frac{\pi}{6} \right)^3 + \dots$

(b)  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

(c)  $\frac{x - \frac{\pi}{6}}{1!} - \frac{\left(x - \frac{\pi}{6}\right)^3}{3!} + \frac{\left(x - \frac{\pi}{6}\right)^5}{5!} - \frac{\left(x - \frac{\pi}{6}\right)^7}{7!} + \dots$

(d)  $\frac{1}{2}$

101. ~~unada~~ Limit of the following series as x approaches

$$\frac{\pi}{2} \text{ is } f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

- (a)  $\frac{2\pi}{3}$       (b)  $\frac{\pi}{2}$       (c)  $\frac{\pi}{3}$       (d) 1      **(GATE-CE-2001)**

102. For the function  $e^{-x}$ , the linear approximation around  $x = 2$  is

(a)  $(3-x)e^{-2}$

(b)  $1 - x$

(c)  $\left[3 + 2\sqrt{2} - (1 + \sqrt{2})x\right]e^{-2}$

(d)  $e^{-2}$

GATE- 2007

103. For  $|x| \ll 1$ ,  $\cot h(x)$  can be approximated as

(GATE-EC-2007)

(a)  $x$

(b)  $x^2$

(c)  $\frac{1}{x}$

(d)  $\frac{1}{x^2}$

104. The expression  $e^{\ln x}$  for  $x > 0$  is equal to

- (a)  $-x$
- (b)  $x$
- (c)  $x^{-1}$

(GATE-IN-2008)

- (d)  $-x^{-1}$

105.Which of the following function would have only odd powers of x in its Taylor series expansion about the point x = 0? **(GATE-EC-2008)**

- (a)  $\sin(x^3)$
- (b)  $\sin(x^2)$
- (c)  $\cos(x^3)$
- (d)  $\cos(x^2)$

106. In the Taylor series expansion of  $e^x + \sin x$  about the point  $x = \pi$ , the coefficient of  $(x - \pi)^2$  is **(GATE-EC-2008)**

- (a)  $e^\pi$
- (b)  $0.5 e^\pi$
- (c)  $e^\pi + 1$
- (d)  $e^\pi - 1$

107. In the Taylor series expansion of  $e^x$  about  $x = 2$ , the coefficient of  $(x-2)^4$  is  
**(GATE-ME-2008)**

(a)  $\frac{1}{4!}$

(b)  $\frac{2^4}{4!}$

(c)  $\frac{e^2}{4!}$

(d)  $\frac{e^4}{4!}$

108. ~~The~~ Taylor series expansion of  $\frac{\sin x}{x - \pi}$  at  $x = \pi$  is given by (GATE-EC-2010)

(a)  $1 + \frac{(x - \pi)^2}{3!} + \dots$

(b)  $-1 - \frac{(x - \pi)^2}{3!} + \dots$

(c)  $1 - \frac{(x - \pi)^2}{3!} + \dots$

(d)  $-1 + \frac{(x - \pi)^2}{3!} + \dots$

112. The infinite series  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$  corresponds to (GATE-CE-2012)

- (a)  $\sec x$
- (b)  $e^x$
- (c)  $\cos x$
- (d)  $1 + \sin^2 x$

113. The Taylor series expansion of  $3 \sin x + 2\cos x$  is

(GATE-EC-SET-1-2014)

(a)  $2 + 3x - x^2 - \frac{x^3}{2} + \dots$

(c)  $2 + 3x + x^2 + \frac{x^3}{2} + \dots$

(b)  $2 - 3x + x^2 - \frac{x^3}{2} + \dots$

(d)  $2 - 3x - x^2 + \frac{x^3}{2} + \dots$

# Taylor's Series for functions of two variables

Q. Expand  $x^2y + 3y - 2$  in powers of  $(x-1)$  and  $(y+2)$  using Taylor's series

# Maxima & Minima

# Increasing and Decreasing functions at a point

Calculus

**Q.** Find the set values of  $\lambda$  for which the function  $f(x) = \begin{cases} x + 1 & x < 1 \\ \lambda & x = 1 \\ x^2 - x + 3 & x > 1 \end{cases}$  is strictly increasing at  $x = 1$

# Increasing and Decreasing functions on an interval

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# Monotonic Function



# Test for Monotonicity

**Q** Find the interval in which  $f(x) = x^3 - 3x^2 - 9x + 20$  is increasing and decreasing

**Q** Find the interval in which  $f^1(x) = x^2 - 5x + 6$  is increasing and decreasing

**Q** Find the interval in which  $f^1(x) = -x^2 - 5x + 6$  is increasing and decreasing

**Q** Find the interval in which  $f'(x) = x(x^2 - 4)$  is increasing or decreasing

**Q** Find the interval in which  $f'(x) = (x+2)(x-1)^2(x-5)$  is increasing or decreasing

 Q. Find the interval in which  $f^1(x) = \frac{(x-1)(x-5)}{(x-3)}$  is increasing or decreasing

 Q. Find the interval in which  $f^1(x) = \frac{(x-1)(x+5)}{(x-3)(x+4)}$  is increasing or decreasing

## Stationary points

The values of  $x$  for which  $f'(x) = 0$ , are called stationary points or turning points .

## Critical points

The values of  $x$  for which  $f'(x) = 0$ , and the points where  $f'(x)$  is not exist are called as critical points .

# Local (Relative) Maxima

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# Local (Relative) Minimum

Local  
Minimum

# Extreme Points & Extreme values

The point at which the function has a maximum or a minimum is called extreme point.

The values of the function at extreme points are called extreme values(Extrema)

# Methods to find Local Extremum

## First derivate test

# Second derivate test

62. The function  $y = x^2 + \frac{250}{x}$  at  $x = 5$  attains

(GATE-EC-1994)

- (a) Maximum
- (b) Minimum
- (c) Neither
- (d) 1

63. The function  $f(x) = x^3 - 6x^2 + 9x + 25$  has

(GATE-EE-1995)

- (a) a maxima at  $x = 1$  and a minima at  $x = 3$
- (b) a maxima at  $x = 3$  and a minima at  $x = 1$
- (c) no maxima, but a minima at  $x = 3$
- (d) a maxima at  $x = 1$ , but no minima

70. The function  $f(x) = 2x^3 - 3x^2 - 36x + 2$  has its maxima at

**(GATE-CS-2004)**

- (a)  $x = -2$  only
- (b)  $x = 0$  only
- (c)  $x = 3$  only
- (d) both  $x = -2$  and  $x = 3$

71. For the function  $f(x) = x^2 e^{-x}$ , the maximum occurs when  $x$  is equal to

- (a) 2
- (b) 1
- (c) 0
- (d) -1

72. For real  $x$ , the maximum value of  $\frac{e^{\sin x}}{e^{\cos x}}$  is

**(GATE-IN-2007)**

- (a) 1
- (b)  $e$
- (c)  $e^{\sqrt{2}}$
- (d)  $\infty$

76. Consider the function  $f(x) = (x^2 - 4)^2$  where  $x$  is a real number. Then the function has **(GATE-EE-2007)**

- (a) Only one minimum
- (b) Only two minima
- (c) Three minima
- (d) Three maxima

77. A point on the curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve  $3x^4 - 16x^3 + 24x^2 + 37$  is \_\_\_\_\_

(GATE-CS-2008)

- (a) 0
- (b) 1
- (c) 2
- (d) 3

78. Consider the function  $y = x^2 - 6x + 9$ . The maximum value of  $y$  obtained when  $x$  varies over the interval 2 to 5 is **(GATE-IN-2008)**

- (a) 1
- (b) 3
- (c) 4
- (d) 9

79. For real values of  $x$ , the minimum value of function



$$f(x) = e^x + e^{-x}$$

(a) 2

(b) 1

(c) 0.5

(GATE-EC-2008)

(d) 0

80. At  $t=0$ , the function  $f(t) = \frac{\sin t}{t}$  has

**(GATE-EE-2010)**

- (a) a minimum
- (b) a discontinuity
- (c) a point of inflection
- (d) a maximum

81. If  $e^y = x^{1/x}$  then y has a

(GATE-EC-2010)

- (a) maximum at  $x = e$
- (b) minimum at  $x = e$
- (c) maximum at  $x = e^{-1}$
- (d) minimum at  $x = e^{-1}$

83. The function  $f(x) = 2x - x^2 + 3$  has

(GATE-EE-2011)

- (a) a maxima at  $x = 1$  and a minima at  $x = 5$
- (b) a maxima at  $x = 1$  and a minima at  $x = -5$
- (c) only a maxima at  $x = 1$
- (d) only a minima at  $x = 1$

86. For  $0 \leq t < \infty$ , the maximum value of the function

$$f(t) = e^{-t} - 2e^{-2t}$$
 occurs at

- (a)  $t = \log_e 4$
- (b)  $t = \log_e 2$
- (c)  $t = 0$
- (d)  $t = \log_e 8$

87. The maximum value of the function  $f(x) = \ln(1+x) - x$  (where  $x > -1$ ) occurs at  $x = \underline{\hspace{2cm}}$ . **(GATE-EC-SET-3-2014)**

# Global (Absolute) maximum

# Global(Absolute) minimum

**Q.** Find the points of local maxima and minima if any of the following function defined in  $0 \leq x \leq 6$ ,  $f(x) = x^3 - 6x^2 + 9x + 15$ .

(GATE-CS-1998)

**Q.** The minimum value of function  $y = x^2$  in the interval  $[1, 5]$  is

**(GATE-ME-2007)**

- (a) 0
- (b) 1
- (c) 25
- (d) undefined

1) (

**Q.** Consider the function  $f(x) = x^2 - x - 2$ . The maximum value of  $f(x)$  in the closed interval  $[-4, 4]$  is

**(GATE-EC-2007)**

- (a) 18
- (b) 10
- (c) -2.25
- (d) indeterminate

- Q.** The maximum value of  $f(x) = x^3 - 9x^2 + 24x + 5$  in the interval  $[1, 6]$  is
- (a) 21      (b) 25      (c) 41      (d) 46

**GATE- 2012**

**Q.** The maximum value of  $f(x) = 2x^3 - 9x^2 + 12x - 3$  in the interval  $0 \leq x \leq 3$  is \_\_\_\_.

**GATE-2014**

Q. Let  $f(x) = xe^{-x}$ . The maximum value of the function in the interval  $(0, \infty)$  is

(a)  $e^{-1}$       (b)  $e$       (c)  $1 - e^{-1}$       (d)  $1 + e^{-1}$

**GATE-2014**

**Q.** The minimum value of the function  $f(x) = x^3 - 3x^2 - 24x + 100$  in the interval  $[-3, 3]$  is **(GATE-EC-SET-2-2014)**

- (a) 20
- (b) 28
- (c) 16
- (d) 32

# Convex and Concave functions

# Point of Inflection

For a continuous function  $f(x)$  said to have a point of inflection at  $x = x_0$  if

1.

2.

Number of inflection points for the curve  $y = x + 2x^4$  is \_\_\_\_\_  
**(GATE-CE-1999)**

(a) 3

(b) 1

(c) 0

(d) 2

2.

 Q. At  $x=0$ , the function  $f(x)=x^3+1$  has

**(GATE-ME,PI-2012)**

- (a) a maximum value
- (b) a minimum value
- (c) a singularity
- (d) a point of inflection

Q. Find the points of inflection for  $f(x) = x^3 - 3x^2 - 7x + 8$

**Q.** Find the points of inflections of  $f(x) = e^{-x^2}$

# Maxima and minima for functions of two variables

Let  $z = f(x,y)$  be the function of two variables for which maxima or minima is to be obtained.

Step 1: find p, q, r, s and t

Step 2: equate p and q to zero for obtaining stationary points.

Step 3: find r, s and t at each stationary point.

- (a) If  $rt - s^2 > 0$  and  $r > 0$  then  $f(x, y)$  has a minimum at that stationary point.
- (b) If  $rt - s^2 > 0$  and  $r < 0$  then  $f(x, y)$  has a maximum at that stationary point.
- (c) If  $rt - s^2 < 0$  then  $f(x, y)$  has no extremum at that stationary point and such points are called saddle points.
- (d) If  $rt - s^2 = 0$  then the case is undecided.

**Q.** Given a function  $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$ , the optimal values of  $f(x, y)$  is  
**(GATE-CE-2010)**

- (a) a minimum equal to  $\frac{10}{3}$
- (b) a maximum equal to  $\frac{10}{3}$
- (c) a minimum equal to  $\frac{8}{3}$
- (d) a maximum equal to  $\frac{8}{3}$

**Q.** The function  $f(x,y) = 2x^2 + 2xy - y^3$  has

**(GATE-EC-2000)**

- (a) Only one stationary point at (0, 0)
- (b) Two stationary points at (0, 0) and (1/6, -1/3)
- (c) Two stationary points at (0, 0) and (1, -1)
- (d) No stationary point

**Q.** The function  $f(x) = 8 \log x - x^2 + 3$  attains its minimum over the interval  $[1,e]$  at  
 $x = \underline{\hspace{2cm}}$  (Here  $\log_e x$  is the natural logarithm of  $x$ .)

**(GATE-2022-ECE)**

- (a) 2
- (b) 1
- (c) e
- (d)  $\frac{1+e}{2}$

Q. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation  $y = 2x - 0.1x^2$  where  $y$  is the height of the arch in meters. The maximum possible height of the arch is

- a) 8 m
- b) 10m
- c) 12m
- d) 14m

# Constrained maxima and minima

Sometimes it is required to find the extremum of a function subject to some other conditions involving the variables. Such problems are called constrained maxima and minima problems

 Q. If the sum of the two positive numbers is 18 , then the maximum value of their product is

- a)81
- b)85
- c)72
- d)80

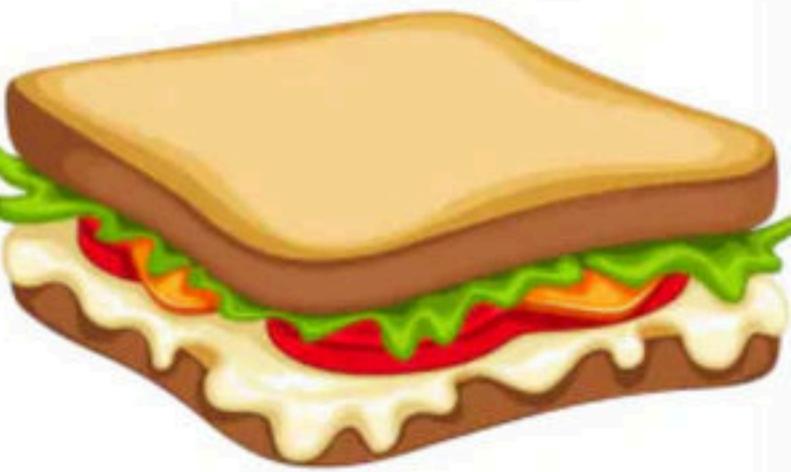
Q. If  $x^2 + y^2 = 1$  then the maximum value of  $x+y$

# Lagrange's Method

Q. Find the maximum and minimum values of  $x + y + z$  subject to  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Q. Find the point on the plane  $x + 2y + 3z = 4$ , that is closest to the origin.

# Sandwich theorem



 Q. If  $3 - \frac{x^2}{12} \leq f(x) \leq 3 + \frac{x^3}{9}$ , then the value of  $\lim_{x \rightarrow 0} f(x)$

**Q. Find  $\lim_{n \rightarrow \infty} \frac{[x] + [2x] + [3x] + \dots + [nx]}{n^2}$** , where  $[.]$  is the greatest integer function.

# Double Limit

 Q. Find  $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} (x^3 + y^3)$

Q. Find  $\lim_{y \rightarrow 2} \frac{3x^2y}{x^2+y^2+5}$

Q. Find  $\lim_{y \rightarrow 2} \frac{x^{\frac{Lt}{2}} - 1}{2x^2 + y^2}$

 unacademy  $\lim_{x \rightarrow \infty} \frac{2x-3}{x^3+4y^3}$



Q. Find  $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

# Definite Integrals



# Integration by parts.

---

$$\int u v \, dx = u \int v \, dx - \int u' \int v \, dx \, dx$$

u & v are functions of 'x'

I LATE

I — Inverse trigonometric function

L — Logarithmic function

A — Algebraic function

T — Trigonometric function

E — Exponential function.

$$\int \underset{U}{x} \underset{E}{e^x} dx$$

↓      ↓

$$\int \underset{V}{\text{sim}} \underset{A}{x^2} dx$$

↓      ↓

I L A T E .  
↑

$$\int \underset{V}{e^x} \underset{T}{\cos x} dx.$$

↓      ↓

$$\int \underset{U}{\tan^{-1}(x)} \underset{E}{e^x} dx.$$

↓      ↓

Q)  $\int \log x \, dx.$

ILATE.

$$\int (1) \log x \, dx.$$

↓      ↓  
v      u

$$\int uv = u \int v - \int u' \int v \, dx \, dx.$$

$$= \log x \int 1 \, dx - \int \frac{1}{x} x \, dx.$$

$$= \log x (x) - \int 1 \, dx$$

$$= x \log x - x = x(\log x - 1).$$

a)  $\int x e^x dx = x e^x - \int 1 e^x dx.$

$\downarrow$   
u      v

$$= x e^x - e^x.$$

$$= e^x(x-1)$$

Q)  $\int e^x \sin x dx = \sin x e^x - \int \cos x e^x dx.$

$\downarrow$   
 $v$   
 $u$

$$= \sin x e^x - [\cos x e^x + \int \sin x e^x dx]$$

$$\underline{I} = \sin x e^x - e^x \cos x - \frac{\int e^x \sin x dx}{\underline{I}}$$

$$\underline{I} + \underline{I} = e^x (\sin x - \cos x)$$

$I = \frac{e^x}{2} [\sin x - \cos x]$

$$\int x^n f(x) dx$$

Q)  $\int x \sin x dx = -x \cos x + \sin x.$

D	I.
$+x$	$\sin x$
-	$-\cos x$
$+0$	$-\sin x$

Q)  $\int x^2 \cos 2x \, dx = \frac{x^2}{2} \sin 2x + \frac{x}{2} \cos 2x - \frac{\sin 2x}{4}$ .

D	I
$+x^2$	$\cos 2x$
$-2x$	$\frac{\sin 2x}{2}$
$+2$	$-\frac{\cos 2x}{4}$
$-0$	$-\frac{\sin 2x}{8}$

$$\int x^3 e^{2x} dx = e^{2x} \left[ \frac{x^3}{2} - \frac{3x^2}{4} + \frac{6x}{8} - \frac{6}{16} \right]$$

D	I
$+x^3$	$e^{2x}$
$-3x^2$	$\frac{e^{2x}}{2}$
$+6x$	$\frac{e^{2x}}{4}$
$-6$	$\frac{e^{2x}}{8}$
$+0$	$\frac{e^{2x}}{16}$

# **Multiple Integral's**

# Double Integrals

$$\iint_R f(x,y) dx dy = \int_{y=}^{\infty} \left[ \int_{x=}^{\infty} f(x,y) dx \right] dy$$
$$= \int_{x=}^{\infty} \left[ \int_{y=}^{\infty} f(x,y) dy \right] dx.$$

$$I_1 = \int_{y=} \left[ \int_{x=} f(x,y) dx \right] dy.$$

$$I_2 = \int_x \left[ \int_{y=} f(x,y) dy \right] dx.$$

if  $I_1 = I_2$ , then  $f(x,y)$  is continuous in the region R

if  $I_1 \neq I_2$ , then  $f(x,y)$  is discontinuous.

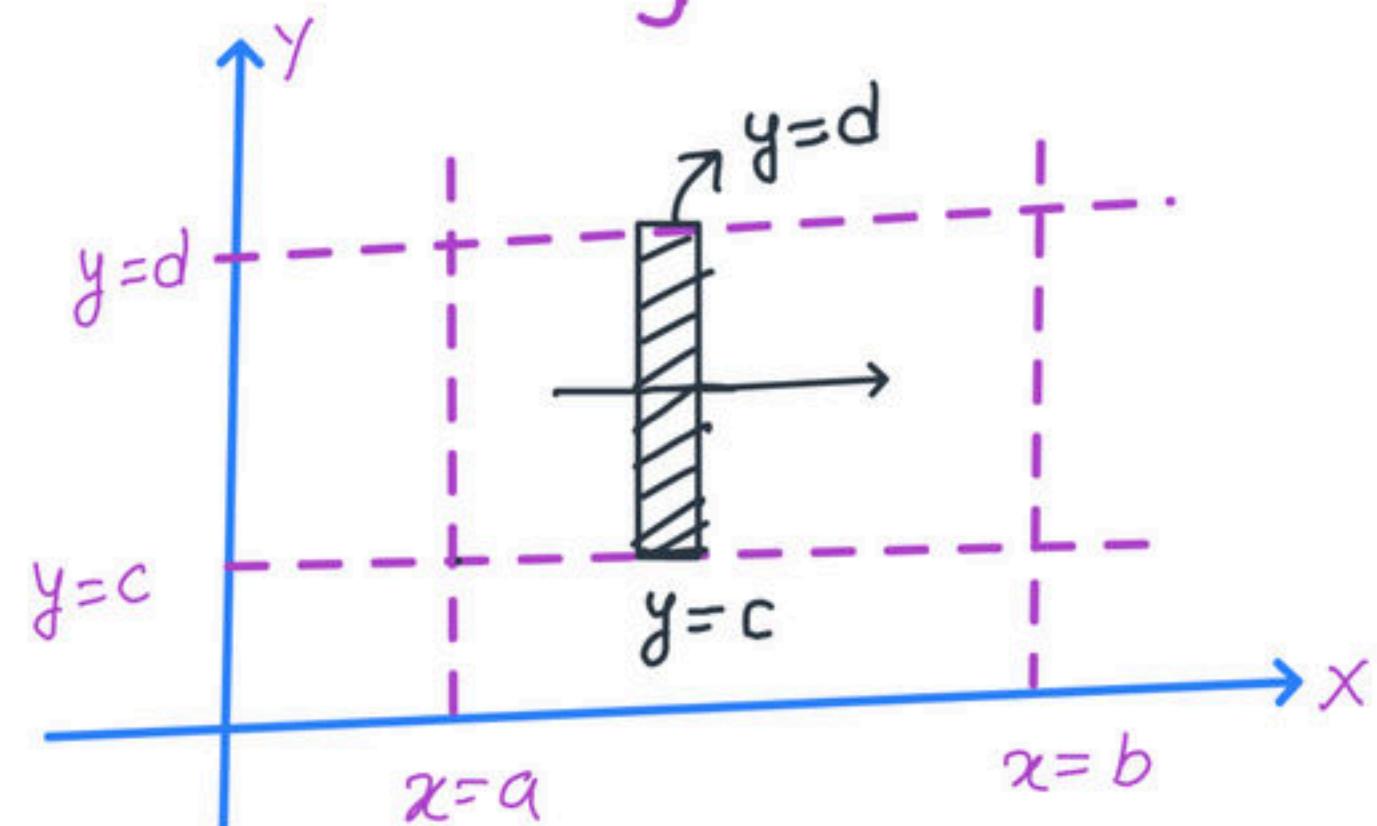
# Concept of Strip

## Vertical Strip

Case-1 : Let the region R is defined as

$$R = \left\{ (x, y) ; a \leq x \leq b, c \leq y \leq d \right\}$$

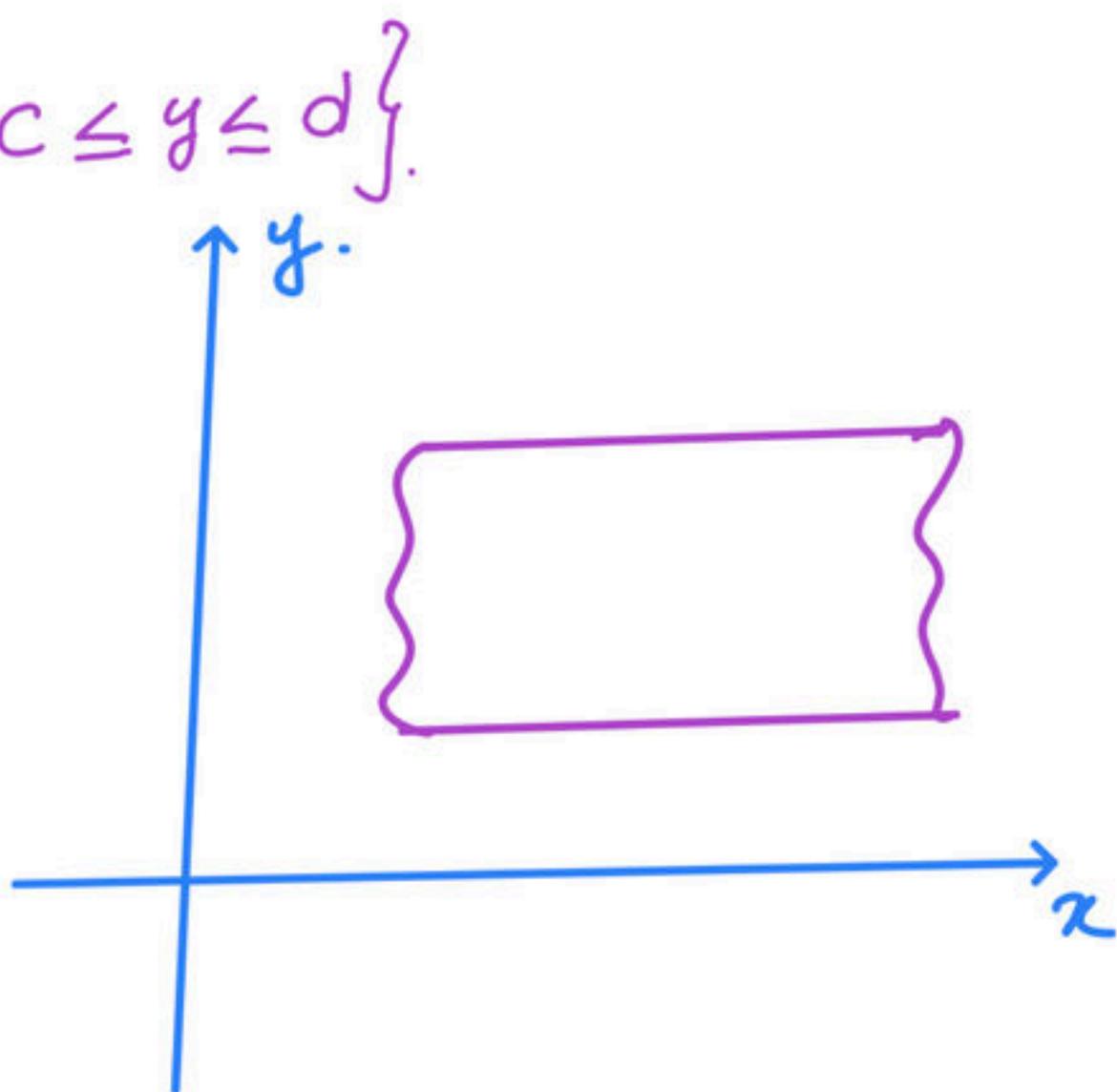
$x = a$  to  $b$   
 $y = c$  to  $d$ .



$$\iint_R f(x,y) dx dy = \int_a^b \int_c^d f(x,y) dy dx.$$

Case-2

$$R = \{(x, y) ; g(y) \leq x \leq h(y), c \leq y \leq d\}.$$



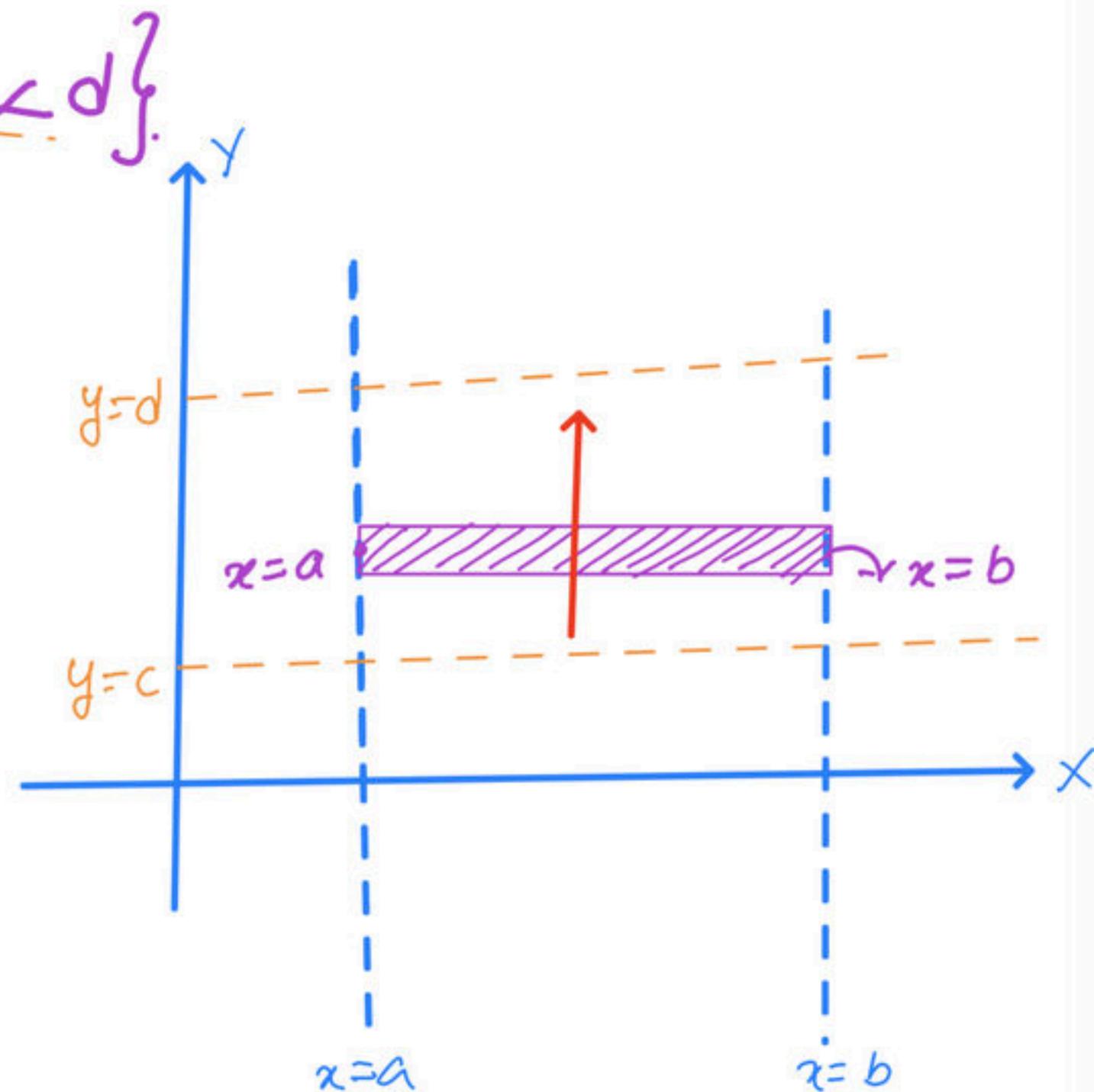
## Horizontal Strip

$$R = \{(x, y); a \leq x \leq b, c \leq y \leq d\}$$

$x = a$  to  $b$

$y = c$  to  $d$ .

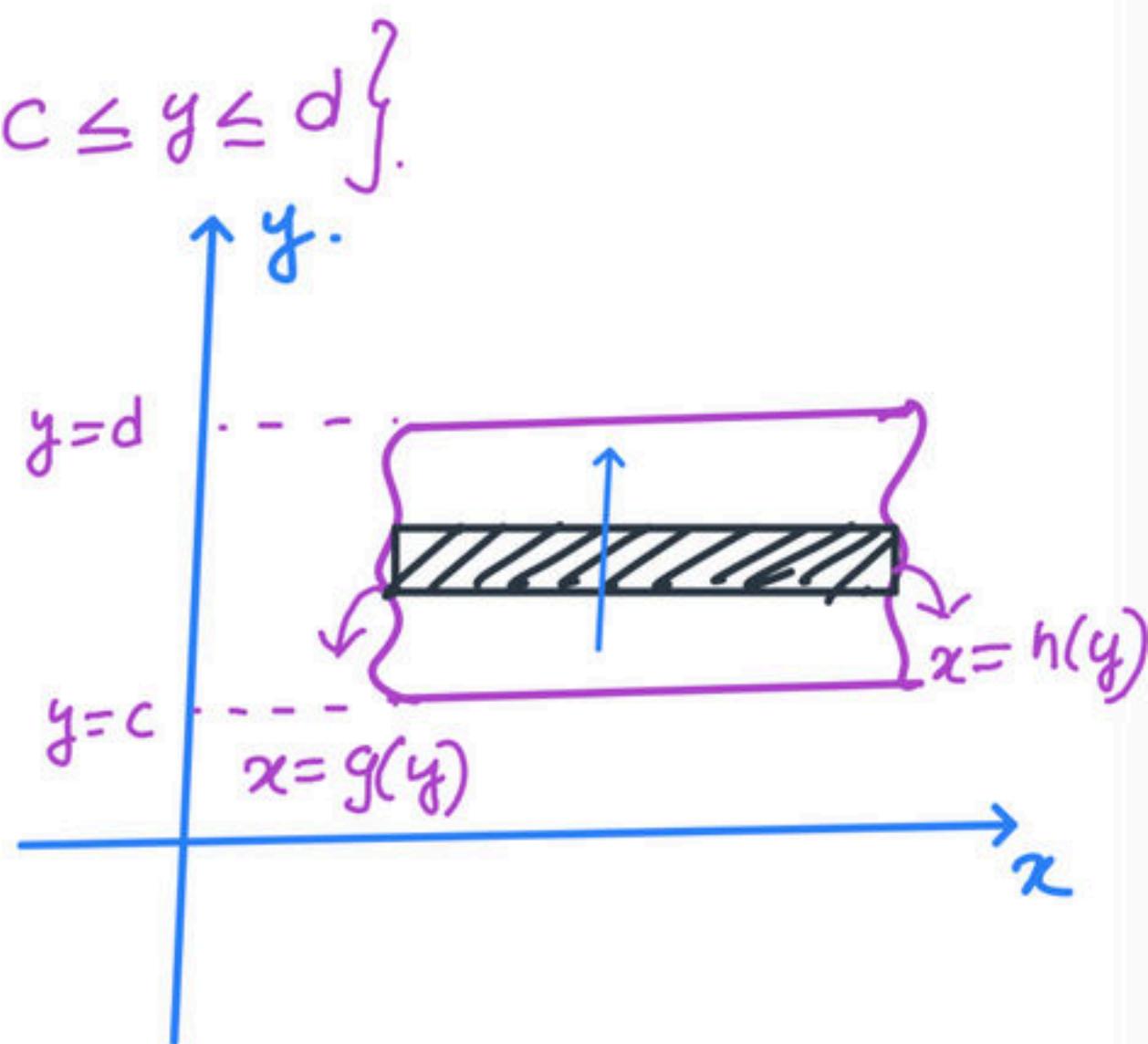
$$\iint_R f(x, y) dx dy = \int_a^b \int_c^d f(x, y) dx dy.$$



Case-2

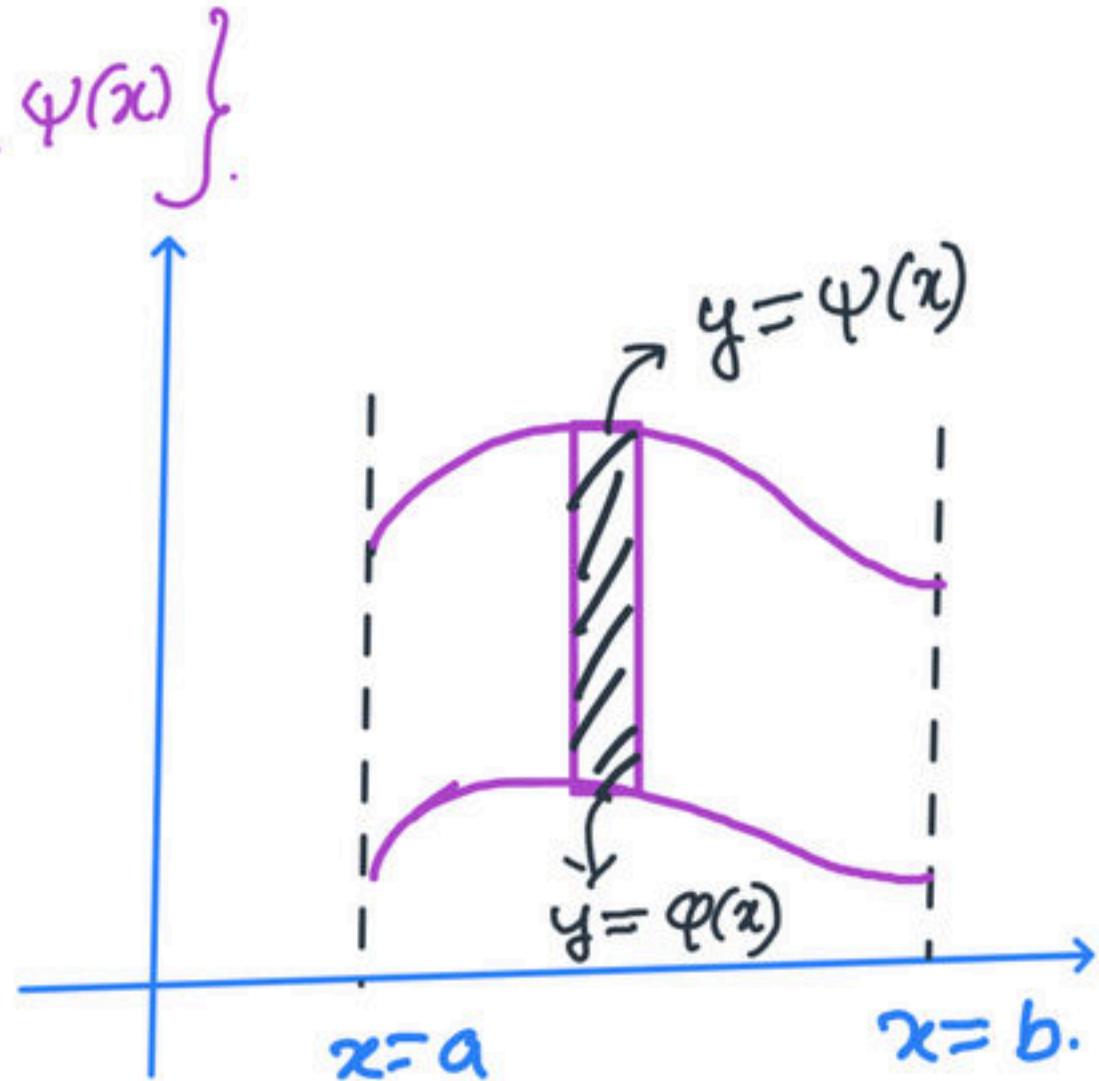
$$R = \{(x, y) ; g(y) \leq x \leq h(y), c \leq y \leq d\}.$$

$$\iint_R f(x, y) dx dy = \int_{y=c}^{y=d} \int_{x=g(y)}^{x=h(y)} f(x, y) dx dy.$$



$$R = \{(x, y) ; a \leq x \leq b, \varphi(x) \leq y \leq \psi(x)\}$$

$$\iint_R f(x, y) dx dy = \int_{x=a}^b \int_{y=\varphi(x)}^{\psi(x)} f(x, y) dy dx.$$



**Q:**  $\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$

**(GATE-EC-2000)**

- (a) 0
- (b)  $\pi$
- (c)  $\pi/2$
- (d) 2

Q. Evaluate  $\int_1^2 \int_3^4 (xy + e^y) dy dx$

$$\int_{x=1}^2 \int_{y=3}^4 (xy + e^y) dy dx = \int \left[ \frac{xy^2}{2} + e^y \right]_3^4 dx.$$

$$x=1$$

$$= \int_{x=1}^2 \left[ \frac{x}{2} (16 - 9) + e^4 - e^3 \right] dx.$$

$$x=1$$

$$= \int_{x=1}^2 \left[ \frac{7x}{2} + e^4 - e^3 \right] dx.$$

$$x=1$$

$$= \int_{x=1}^2 \left[ \frac{7x}{2} + e^4 - e^3 \right] dx.$$

$$= \frac{\frac{7x^2}{2}}{4} + e^4 x - e^3 x \Big|_1^2.$$

$$= \frac{7}{4}(4-1) + e^4 - e^3.$$

$$= \frac{21}{4} + e^4 - e^3.$$

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249.  $\int_0^2 \int_0^3 xy \, dx \, dy$

- (A) 0      (B) 9      (C) 8      (D) 1

250.

$$\int_0^{\frac{\pi}{2}} \int_{-1}^1 x^2 y^2 dx dy$$

- (A)  $\frac{\pi^3}{36}$       (B)  $\frac{\pi}{0}$       (C) -1      (D) 0

251. Evaluate  $\int_{-1}^2 (1 + |x|) dx$

- (A) 3.5
- (C) 4

- (B) 5.5
- (D) None of these

252.  $\int_0^{\pi} \sin^5 x \cos^9 x dx = \underline{\hspace{2cm}}$

**253.** Let  $f(x)$  be any bounded real valued

function in the interval  $[a, b]$ .

Consider the following statements:

A:  $\int_{-a}^a f(x) dx = \int_0^a \{f(x) + f(-x)\} dx$

B:  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$

Then which of the following is appropriate?

- (A) A and B both are true and they are interdependent
- (B) A and B are true independently
- (C) A is true and B is false always
- (D) A is true and B is true in special case

254. For which value of  $n$ ,

$\int_0^{\frac{\pi}{2}} \frac{dx}{16\cos^2 x + 25\sin^2 x}$  becomes equal to  $n\pi$ .

- (A)  $\frac{1}{40}$     (B)  $\frac{1}{50}$     (C)  $\frac{1}{20}$     (D)  $\frac{1}{30}$

255. Evaluate  $\int_0^1 \int_1^2 (x^2 + y^2) dx dy$

- (A)  $-\frac{8}{3}$     (B)  $\frac{8}{3}$     (C) 0    (D) 1

256. The value of  $\int_{-4}^7 |x| dx$  is

- (a) 30.5
- (c) 32.5

- (b) 30
- (d) 32

257. The value of  $\int_0^{1.5} x[x^2] dx$ , where  $[x]$  is a step function, is

(a)  $\frac{4}{3}$

(b)  $\frac{1}{2}$

(c)  $\frac{2}{3}$

(d)  $\frac{3}{4}$

258. The value of  $\int_0^\pi x \sin^8(x) \cos^6(x) dx$  is

(a)  $\frac{\pi^2}{512}$

(b)  $\frac{105\pi^2}{512}$

(c)  $\frac{105\pi}{86016}$

(d)  $\frac{5\pi^2}{4096}$

259. The value of  $\int_{x=1}^a \int_{y=1}^b \frac{1}{xy} dx dy$  is \_\_\_\_\_.

- (a)  $(\log a)(\log b)$
- (b)  $\log(ab)$
- (c)  $\log a - \log b$
- (d)  $\log(a + b)$

260.  $\int_1^3 \int_1^2 xy^2 \, dx \, dy =$

(a) 10

(b) 11

(c) 13

(d) 12

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261.  $\int_0^1 \int_0^2 \int_0^2 x^2 yz \, dx \, dy \, dz =$

(a)  $-\frac{7}{3}$

(b)  $\frac{7}{3}$

(c)  $\frac{7}{2}$

(d)  $-\frac{7}{2}$

262. The value of  $\int_{x=0}^1 \int_{y=0}^2 xy \, dx \, dy$  is \_\_\_\_\_.

263.  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right) dx = \underline{\hspace{2cm}}$

- (a) 0
- (b) 1
- (c)  $\frac{\pi}{2}$
- (d) none of these

264.  $\int_{-\pi}^{\pi} \sin^4 x \, dx =$

(a)  $\pi$

(b)  $\frac{\pi}{2}$

(c)  $\frac{3\pi}{4}$

(d) 0

265.

$$\int_{-1}^2 \frac{|x|}{x} dx = \dots$$

266.  $\int_0^{\pi} |\cos x| dx =$

267.  $\int_0^n [x] dx = \underline{\hspace{2cm}}$ , where  $[x]$  is a step function  
and 'n' is an integer.

(a)  $\frac{n(n+1)}{2}$

(b)  $\frac{n(n-1)}{2}$

(c)  $\frac{n}{2}$

(d)  $\frac{n+1}{2}$

268.  $\int_0^{\frac{\pi}{2}} \log(\tan x) dx =$

(a) 0

(c)  $\frac{\pi}{4}$

(b)  $\frac{\pi}{2}$

(d)  $\pi$

269. Let  $\frac{d}{dx}[F(x)] = \frac{e^{\sin x}}{x}$ ,  $x > 0$ .

If  $\int_1^4 \left( \frac{2e^{\sin x^2}}{x} \right) dx = F(k) - F(1)$

then  $k = \underline{\hspace{2cm}}$ .

270.  $\int_0^{\pi/4} \log(1 + \tan x) dx =$

(a) 0

(b)  $(\pi/2) \log 2$

(c)  $(\pi/8) \log 2$

(d)  $(-\pi/4) \log 2$

271.  $\int_0^{\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0

(b)  $3\pi/256$

(c)  $3\pi/128$

(d)  $5\pi/128$

272.  $\int_0^{2\pi} \sin^4 x \cos^6 x \, dx =$

(a)  $3\pi/128$

(b)  $3\pi/256$

(c)  $3\pi/64$

(d) 0

273.  $\int_0^{2\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0

(b)  $3\pi/128$

(c)  $5\pi/128$

(d)  $3\pi/256$

274. 
$$\int_0^{\pi/2} \int_0^{\pi/2} \sin(x+y) dx dy$$

(GATE-EC-2000)

- (a) 0
- (b)  $\pi$
- (c)  $\pi/2$
- (d) 2

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275. The value of the integral is  $I = \int_0^{\pi/4} \cos^2 x \, dx$  (GATE-CE-2001)

(a)  $\frac{\pi}{8} + \frac{1}{4}$

(b)  $\frac{\pi}{8} - \frac{1}{4}$

(c)  $\frac{-\pi}{8} - \frac{1}{4}$

(d)  $\frac{-\pi}{8} + \frac{1}{4}$

276.

The value of the integral  $\int_{-\pi/2}^{\pi/2} (x \cos x) dx$  is

(GATE-PI-2008)

- (a) 0
- (b)  $\pi - 2$
- (c)  $\pi$
- (d)  $\pi + 2$

277. The value of the following definite integral in  $\int_{-\pi/2}^{\pi/2} \frac{\sin 2x}{1 + \cos x} dx = (\text{GATE-ME-2002})$

- (a) -2log 2
- (b) 2
- (c) 0
- (d) None

- 278.** The value of the following improper integral is  $\int_0^1 x \log x \, dx =$  **(GATE-ME-2002)**
- (a) 1/4
  - (b) 0
  - (c) -1/4
  - (d) 1

279.  $\int_{-a}^a [\sin^6 x + \sin^7 x] dx$  is equal to

(GATE-ME-2004)

(a)  $2 \int_0^a \sin^6 x dx$

(b)  $2 \int_0^a \sin^7 x dx$

(c)  $2 \int_0^a (\sin^6 x + \sin^7 x) dx$

(d) zero

280. The value of  $\iint\limits_{0 \ 0}^{3 \ x} (6 - x - y) \ dx \ dy$  is \_\_\_\_\_

(GATE-CS-2008)

(a) 13.5

(b) 27.0

(c) 40.5

(d) 54.0

281.

$$\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx dy dz$$

(A)  $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \frac{3}{8}$

(B)  $\frac{e^{4a}}{4} - \frac{3e^{2a}}{4}$

(C)  $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} - \frac{3}{8}$

(D) None

282.  $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} (x^2 + y^2) dx dy =$

(a)  $\frac{2}{35}$

(b)  $-\frac{3}{35}$

(c)  $\frac{3}{35}$

(d)  $-\frac{2}{35}$

283.  $\int_0^1 \int_{4y}^4 e^{x^2} dx dy =$

(a)  $\frac{(e^{16} - 1)}{8}$

(b)  $-\frac{(e^{16} + 1)}{8}$

(c) 0

(d)  $-\frac{(e^{16} - 1)}{8}$

284.

$$\int_0^4 \int_0^{x^2} e^{y/x} dy dx =$$

(a)  $3e^4$

(c)  $-3e^4$

(b)  $3e^4 + 7$

(d)  $3e^4 - 7$

286.  $\iiint_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz =$

(a) 1

(b) 2

(c) 3

(d) 0

287. The integral  $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$  is equal to \_\_\_\_\_.

(GATE-16-EC)

288.  $\int_{1/\pi}^{\pi} \frac{\cos(1/x)}{x^2} dx = \underline{\hspace{2cm}}$

(GATE-CS-2015)

**289.** Given the following statements about a function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , select the right option:

- P: If  $f(x)$  is continuous at  $x = x_0$ , then it is also differentiable at  $x = x_0$
- Q: If  $f(x)$  is continuous at  $x = x_0$ , then it may not be differentiable at  $x = x_0$
- R: If  $f(x)$  is differentiable at  $x = x_0$ , then it is also continuous at  $x = x_0$

**(GATE-16-EC)**

- (a) P is true, Q is false, R is false
- (b) P is false, Q is true, R is true
- (c) P is false, Q is true, R is false
- (d) P is true, Q is false, R is true

290. The value of

$$\int_0^{\frac{\pi}{2}} \int_0^{2a \cos \theta} r \sin \theta \, dr \, d\theta$$

is \_\_\_\_\_.

(a)  $\frac{a^2}{2}$

(b)  $2a^2$

(c)  $\frac{2a^2}{3}$

(d)  $4a^2$

292.  $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dx dy =$

(a)  $-\frac{\pi}{16}$

(b)  $\frac{\pi}{16}$

(c)  $\frac{\pi}{8}$

(d)  $-\frac{\pi}{8}$

301. The value of  $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz dz dy dx$  is \_\_\_\_\_

(A)  $\frac{13}{9} - \frac{\ln 3}{6}$

(B)  $\frac{7}{6} - \frac{\ln 3}{6}$

(C)  $\frac{1}{6} - \ln 3$

(D)  $\frac{3}{2} - \ln 3$

**302.** The value of  $\int_0^1 \int_0^2 \int_1^2 x^2 y z dz dy dx$  is \_\_\_\_\_

- (A) 0
- (B) 1
- (C) 2
- (D) 3

304. The value of  $\int_{-1}^2 \int_{x^2}^{x+2} dy dx = \underline{\hspace{2cm}}$

(A)  $\frac{7}{2}$

(B)  $\frac{9}{2}$

(C)  $\frac{11}{2}$

(D)  $\frac{5}{2}$

 305. The value of  $\int_0^1 \int_0^1 \frac{dydx}{\sqrt{1-x^2} \sqrt{1-y^2}} = \underline{\hspace{2cm}}$

(A)  $\frac{\pi^2}{4}$

(B)  $\frac{\pi^2}{2}$

(C)  $\frac{\pi^2}{8}$

(D)  $\frac{\pi^2}{16}$

**306.** The value of  $\int_0^{100\pi} |\sin x| dx$  is \_\_\_\_\_

- (A) 100
- (B)  $100\pi$
- (C)  $200\pi$
- (D) 200

 307. The value of integral  $\int_{-1}^1 \ln \left( \frac{2-x\cos x}{2+x\cos x} \right) dx$  is \_\_\_\_\_

- (A)  $x\ln(2 + x\cos x)$
- (B)  $x\ln(2 - x\cos x)$
- (C)  $x\cos x$
- (D) 0

**308.** If  $f(x) = \int_x^0 \sin t^2 dt$  then  $f'(x)$  is \_\_\_\_\_

- (A)  $2x \sin x^2$
- (B)  $-\sin x^2$
- (C)  $2x \cos x^2$
- (D)  $\cos x^2$

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Q. Evaluate  $\int_0^5 \int_0^{x^2} x(x^2 + y^2) dx dy$

Q. The value of  $\iint\limits_{0,0}^{3,x} (6-x-y) dx dy$  is \_\_\_\_\_

(GATE-CS-2008)

~~(a) 13.5~~

(b) 27.0

(c) 40.5

(d) 54.0

$$\begin{aligned}
 & \iint\limits_{x=0}^{3} \int\limits_{y=0}^x (6-x-y) dy dx = \int\limits_{x=0}^3 \left[ 6y - xy - \frac{y^2}{2} \right]_0^x dx \\
 & = \int\limits_{x=0}^3 \left[ 6x - x^2 - \frac{x^2}{2} \right] dx \\
 & = \int\limits_{x=0}^3 \left[ 6x - \frac{3}{2}x^2 \right] dx
 \end{aligned}$$

$$= \int_{x=0}^3 \left[ 6x - \frac{3}{2}x^2 \right] dx.$$

$$= \left. \frac{6x^2}{2} - \frac{3}{2} \cdot \frac{x^3}{3} \right|_0^3.$$

$$= \left. 3x^2 - \frac{x^3}{2} \right|_0^3.$$

$$= 3(9) - \frac{27}{2}$$

$$= \frac{27}{2} = \underline{\underline{13.5}}$$

**Q.** The value of the double integral  $\int_0^{1/x} \int_x^{1/x} \frac{x}{1+y^2} dx dy = \underline{\hspace{2cm}}$  **(GATE-EC-1993)**



# Change of order of integration

Q. Evaluate  $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx$

Q. Evaluate  $\int_0^a \int_y^a \frac{x}{x^2+y^2} dy dx$

Q. By reversing the order of integration  $\int_0^{2x} \int_y^{2x} f(x, y) dy dx$  may be represented as

(a)  $\int_0^2 \int_{x^2}^{2x} f(x, y) dy dx$

(b)  $\int_0^2 \int_y^{\sqrt{y}} f(x, y) dx dy$

**(GATE-EC-1995)**

(c)  $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y) dx dy$

(d)  $\int_{x^2}^{2x} \int_0^2 f(x, y) dy dx$

Q. Changing the order of integration in double integral  $I = \int_0^8 \int_{x/4}^2 f(x, y) dy dx$  leads to

$$I = \int_r^s \int_p^q f(x, y) dx dy . \text{ What is } q?$$

**(GATE-EC-2005)**

(a)  $4y$

(b)  $16y^2$

(c)  $x$

(d)  $8$

## Triple integrals

$$\iiint_R f(x, y, z) dx dy dz = \int_x \int_y \int_z f(x, y, z) dz dy dx.$$

**Q. Evaluate**  $\iiint_R (x + y + z) dx dy dz$  where  $R: 0 \leq x \leq 1, 1 \leq y \leq 2, 2 \leq z \leq 3$

$$\begin{aligned} & \int_{z=2}^3 \int_{y=1}^2 \int_{x=0}^1 (x+y+z) dx dy dz = \int_{z=2}^3 \int_{y=1}^2 \left[ \frac{x^2}{2} + xy + xz \right]_{x=0}^1 dy dz \\ & = \int_{z=2}^3 \int_{y=1}^2 \left[ \frac{1}{2} + y + z \right] dy dz \\ & = \int_{z=2}^3 \left[ \frac{y}{2} + \frac{y^2}{2} + yz \right]_{y=1}^2 dz \end{aligned}$$

$$= \int_{z=2}^3 \left[ \frac{y}{2} + \frac{y^2}{2} + yz \right]^2 dz$$

$$= \int_{z=2}^3 \left[ \frac{1}{2}(1) + \frac{1}{2}(3) + z \right] dz$$

$$= \int_{z=2}^3 (2+z) dz$$

$$= 2z + \frac{z^2}{2} \Big|_2^3$$

$$= 2(1) + \frac{1}{2}(9-4)$$

$$= 2 + \frac{5}{2} = 4 \cdot \underline{\underline{5}}$$

Q. Evaluate  $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x + y + z) dx dy dz$

 $x - z$  $x + z$ 

$$y = x - z \text{ to } x + z$$

$$x = 0 \text{ to } z$$

$$z = -1 \text{ to } 1.$$

$$\int_{z=-1}^1 \int_{x=0}^z \int_{y=x-z}^{x+z} (x + y + z) dy dx dz = \int_{z=-1}^1 \int_{x=0}^z \left( xy + y^2 + xz \right) \Big|_{x-z}^{x+z} dx dz$$

$$= \int_{z=-1}^1 \int_{x=0}^z \left( xy + \frac{y^2}{2} + xz \right) dx dz$$

$$= \int_{z=-1}^1 \int_{x=0}^z \left[ x(z+x-z-x) + \frac{(x+z)^2 - (x-z)^2}{2} + x(x+z) - x(x-z) \right] dx dz$$

$$= \int_z^1 \int_x^z [2xz + 2xz + 2xz] dx dz$$

$$= \int_{z=-1}^1 \int_{x=0}^3 6xz dx dz$$

$$= \frac{3z^4}{4} \Big|_{-1}^1$$

$$= \int_{z=-1}^1 \left. \frac{6x^2 z}{2} \right|_0^3 dz$$

$$= \frac{3}{4} [1 - 1] = \underline{\underline{0}}$$

$$= \int_{-1}^1 3z^3 dz$$

# Change of variables

**Q.** To evaluate the double integral  $\int_0^8 \left( \int_{y/2}^{(y/2)+1} \left( \frac{2x-y}{2} \right) dx \right) dy$ , we make the substitution

$u = \left( \frac{2x-y}{2} \right)$  and  $v = \frac{y}{2}$ . The integral will reduce to

**(GATE-EE-SET-2-2014)**

- (a)  $\int_0^4 \left( \int_0^2 2udu \right) dv$
- (b)  $\int_0^4 \left( \int_0^1 2udu \right) dv$
- (c)  $\int_0^4 \left( \int_0^1 udu \right) dv$
- (d)  $\int_0^4 \left( \int_0^{21} 2udu \right) dv$

**Q.** By a change of variables  $x = uv$ ,  $y = v/u$  in a double integral, the integral  $f(x,y)$  changes to  $\int_{uv} \int_{v/u} \phi(u,v) \, du \, dv$ . Then  $\phi(u,v)$  is \_\_\_\_\_ (GATE-EE-2005)

(a)  $\frac{2v}{u}$

(b)  $2uv$

(c)  $v^2$

(d) 1

# Area bounded by the curves

**326.** The area of the region enclosed by the curve  $y = x^2$  and the straight-line  $x + y = 2$  is

- (A) 3
- (B)  $27/2$
- (C)  $9/2$
- (D) 9

**327.** The area of the region bounded by the curve  $x^2 = 2y$  and  $y^2 = 2x$  is

- (A)  $1/3$
- (B)  $2/3$
- (C)  $4/3$
- (D)  $4$

**328.** Area enclosed by the curves  $y^2 = x$  and  $y^2 = 2x - 1$  lying in the first quadrant is

- (A)  $1/6$
- (B)  $1/4$
- (C)  $1/2$
- (D)  $1/3$

**329.** The value of  $\int \int xy(x + y)dx dy$  over the area between  $y = x^2$  and  $y = x$

- (A)  $1/90$
- (B)  $1/45$
- (C)  $3/56$
- (D)  $1/15$

330. The area bounded by the parabola  $2y = x^2$  and the lines  $x = y - 4$  is equal to  
(a) 6      (b) 18      (c)  $\infty$       (d) None      (GATE-ME-1995)

331. ~~un~~Area bounded by the curve  $y = x^2$  and the lines  $x = 4$  and  $y = 0$  is given by

(a) 64

(b)  $\frac{64}{3}$

(c)  $\frac{128}{3}$

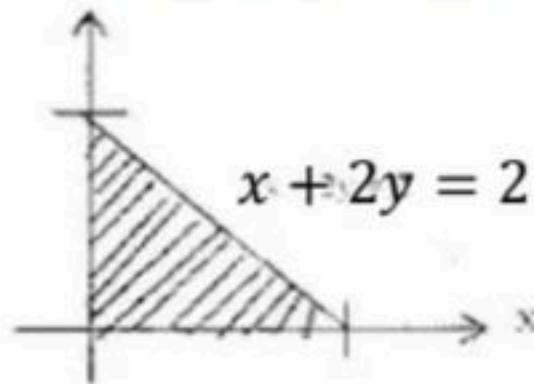
(d)  $\frac{128}{4}$

**(GATE-EE-1997)**

332. The area enclosed between the parabola  $y = x^2$  and the straight line  $y = x$  is

- (a)  $1/8$
- (b)  $1/6$
- (c)  $1/3$
- (d)  $1/2$

333. Consider the shaded triangular region P shown in the figure. What is  $\iint_P xy \, dx \, dy$ ?



(GATE-ME-2008)

- (a)  $\frac{1}{6}$
- (b)  $\frac{2}{9}$
- (c)  $\frac{7}{16}$
- (d) 1

336. The volume under the surface  $z(x, y) = x + y$  and above the triangle in the  $xy$  plane defined by  $\{0 \leq y \leq x \text{ and } 0 \leq x \leq 12\}$  is \_\_\_\_\_. (GATE-EC-SET-1-2014)

337. The volume generated by revolving the area bounded by the parabola  $y^2 = 8x$  and the line  $x = 2$  about y-axis is **(GATE-EE-1994)**

- (a)  $\frac{128\pi}{5}$
- (b)  $\frac{5}{128\pi}$
- (c)  $\frac{127}{5\pi}$
- (d) None of the above

338. The volume of an object expressed in spherical co-ordinates is given by

$$V = \int_0^{2\pi} \int_0^{\pi/3} \int_0^1 r^2 \sin\phi \, dr \, d\phi \, d\theta. \text{ The value of the integral } \quad (\text{GATE-EE-2004})$$

(a)  $\frac{\pi}{3}$

(b)  $\frac{\pi}{6}$

(c)  $\frac{2\pi}{3}$

(d)  $\frac{\pi}{4}$

339. The volume enclosed by the surface  $f(x, y) = e^x$  over the triangle bounded by the line  $x = y$ ,  $x = 0$ ,  $y = 1$  in the  $xy$  plane is \_\_\_\_\_ **(GATE-EE-2015)**

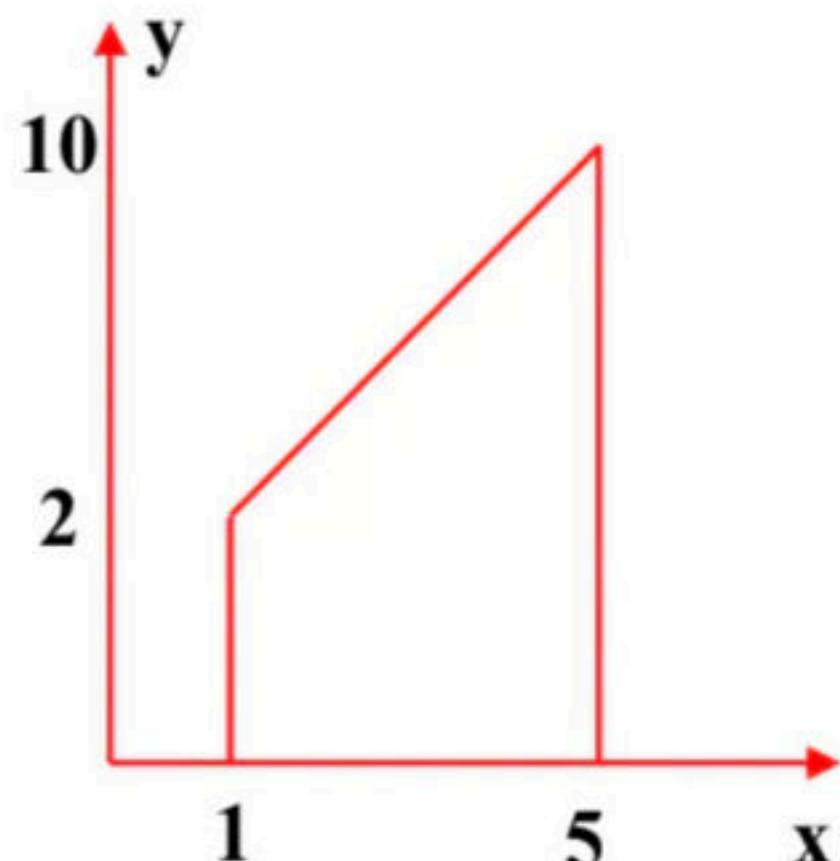
**342.** A triangle in the  $x$ - $y$  plane is bounded by the straight lines  $2x = 3y$ ,  $y = 0$  and  $x = 3$ . The volume above the triangle and under the plane  $x + y + z = 6$  is \_\_\_\_\_.

**(GATE-16-EC)**

**343.** Let  $I = c \iint_R xy^2 dx dy$ , where  $R$  is the region shown in the figure and  $C = 6 \times 10^{-4}$ . The value of  $I$  equals \_\_\_\_\_.

(Give the answer up to two decimal places)

**(GATE-17-EE)**



349. The value of integral  $\iint_D 3(x^2 + y^2) dx dy$   
where D is the shaded triangular region shown in the diagram is \_\_\_\_\_ (rounded off nearest integer).

(GATE-2022-ECE)

