Practice Session on Calculus - Part II

Revision Course on Engineering Mathematics - GATE, CS & IT

B V Reddy • Lesson 10 • Nov 26, 2022

Linear Algebra DPP

1. The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det(A) = 100 and trace(A) = 14. The value of $|a-b|$ is ________.

(GATE-16-EC)

$tr(A) = a + 5 + 2 + b = 14$

$a + b = 7.$

$det(A) = 100 = (-1)^{2+2}\, 5 \begin{vmatrix} a & 3 & 7 \\ 0 & 2 & 4 \\ 0 & 0 & b \end{vmatrix}$

$100 = 5a \, |2b|$

$10ab = 100$

$ab = 10$

$a + b = 7.$

$a = 5 \quad b = 2$

$|a-b| = 3.$

2. The value of x for which the matrix

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ -6 & -4 & -9+x \end{bmatrix}$$

has zero as an eigen value is __________.

(GATE-16-EC)

$\lambda_1 \quad \lambda_2 \quad \lambda_3.$

$\lambda_1 \lambda_2 \lambda_3 = 0 = |A|$

$3[-63 + 7x + 52] - 2[-81 + 9x + 78] + 4[-36 + 42] = 0$

$R_3 \to R_3 + 2R_1.$

$$\begin{bmatrix} 3 & 2 & 4 \\ 9 & 7 & 13 \\ 0 & 0 & -1+x \end{bmatrix} \Rightarrow (x-1)[21-18] = 0$$

$x = 1$

3. Consider a 2×2 square matrix

$$\boldsymbol{A} = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

Where x is unknown. If the eigen values of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to

(GATE-16-EC)

(a) $+j\omega$ (b) $-j\omega$

(c) $+\omega$ (d) $-\omega$

$$\lambda_1 + \lambda_2 = 2\sigma$$

$$\lambda_1 \lambda_2 = (\sigma + j\omega)(\sigma - j\omega) = \sigma^2 - \omega x$$

$$\sigma^2 + \omega^2 = \sigma^2 - \omega x.$$

$$\omega x = -\omega^2$$

$$\boxed{x = -\omega}$$

4. Consider 3 × 3 matrix with every element being equal to 1. Its only non-zero eigenvalue is __________

(GATE-16-EE)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$\lambda_1 \; \lambda_2 \; \lambda_3$

$\lambda_1 \neq 0 \qquad \lambda_2 = \lambda_3 = 0$

$$\lambda_1 + \lambda_2 + \lambda_3 = 3.$$

$$\lambda_1 + 0 + 0 = 3$$

$$\lambda_1 = 3$$

5. A 3×3 matrix P is such that, $P^3 = P$. Then the eigen values of P are

(GATE-16-EE)

(a) 1, 1, -1
(b) 1, 0.5 + j0.866, 0.5 – j0.866
(c) 1, -0.5 + j0.866, -0.5 – j0.866
(d) 0, 1, -1

$P^3 = P$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda(\lambda+1)(\lambda-1) = 0$

$\boxed{\lambda = 0, -1, 1}$

6. Consider the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ whose eigen values are 1, -1 and 3. Then trace of $(A^3 - 3A^2)$ is ________. **(GATE-16-IN)**

$\underline{A}$

$\lambda = 1, -1, 3$

$\underline{\lambda = 1}$

$1^3 - 3 \cdot 1 = -2$

$\underline{\lambda = -1}$

$-1 - 3 = -4$

$\underline{\lambda = 3}$ $27 - 27 = 0$

trace $= -2 - 4 + 0$

$= -6.$

$A \longrightarrow \lambda$

$A^2 \longrightarrow \lambda^2$

$A^3 - 3A^2 \longrightarrow \lambda^3 - 3\lambda^2$

7. The condition for which the eigen values of matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are positive, is

(GATE-16-ME)

~~(a)~~ k > ½ (b) k > -2

(c) k > 0 (d) k < -½

$\lambda_1 + \lambda_2 = 2 + k$

$\lambda_1 \lambda_2 = 2k - 1$

$\lambda_1 + \lambda_2 > 0$

$2 + k > 0$

$\boxed{k > -2}$ ✓

$\lambda_1 \lambda_2 > 0$

$2k - 1 > 0$

$\boxed{k > \frac{1}{2}}$ ✓

8. A real square matrix A is called skew-symmetric if

(GATE-16-ME)

(a) $A^T = A$ (b) $A^T = A^{-1}$

(c) $A^T = -A$ (d) $A^T = A + A^{-1}$

$A^T = A \longrightarrow$ Symmetric

$A^T = -A \longrightarrow$ Skew Symmetric.

9. The eigen values of the matrix are $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(GATE-16-PI)

(a) i and -i ✓ (b) 1 and -1

(c) 0 and 1 (d) 0 and -1

$\lambda_1 + \lambda_2 = 0$

$i(-i) = +1.$ ✓

$\lambda_1 \lambda_2 = 0 + 1 = 1.$

10. The number of solutions of the simultaneous algebraic equations y = 3x + 3 and y = 3x + 5 is

(a) zero (b) 1

(c) 2 (d) infinite **(GATE-16-PI)**

$$\left.\begin{aligned} y &= 3x+3 \\ y &= 3x+5 \end{aligned}\right\} \quad \text{No Solution}$$

11. Two eigen values of a 3×3 real matrix P are $(2+\sqrt{-1})$ and 3. The determinant of P is ________.

(GATE-16-CSE)

$\lambda_1 = 2 - j$

$\lambda_2 = 2 + j$

$\lambda_3 = 3.$

$|P| = \lambda_1 \lambda_2 \lambda_3.$

$= (2-j)(2+j)\,3.$

$= (4+1)\,3 = 15.$

12. Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is _____.

(GATE-16-CSE)

$$\det(A^{-1})^T = |A^T|^{-1} = \frac{1}{|A|} = \frac{1}{8}$$

$$|A^T| = |A|$$

13. Let A_1, A_2, A_3 and A_4 be four matrices of dimensions 10×5, 5×20, 20×10 and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1A_2A_3A_4$ using the basic matrix multiplication method is ________.

(GATE-16-CSE)

$(A)_{\cancel{m} \times \cancel{n}} \; (B)_{n \times p}$

No. of multiplications $= mnp$.

$A_1 \; A_2 \; (A_3)_{20\times10} \; (A_4)_{10\times5} \qquad = 20\times10\times5 = 1000$

$A_1 \; (A_2)_{5\times20} \; (\quad)_{20\times5} \qquad = 5\times20\times5 = 500$

$(A_1)_{10\times5} \; (\quad)_{5\times5} \qquad = 10\times5\times5 = 250$

1750.

$(A_1)_{10\times5}$ $(A_2)_{5\times20}$ $(A_3)_{20\times10}$ $(A_4)_{10\times5}$

$= 5\times20\times10 = 100$

$(A_1)_{10\times5}$ $(\quad)_{5\times10}$ $(A_4)_{10\times5}$ $= 5\times10\times5 = 250$

$(A_1)_{10\times5}$ $(\quad)_{5\times5}$ $= 10\times5\times5 = 250$

1500

14. The eigen values of the matrix

$$A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix} \text{ are}$$

$\lambda_1 + \lambda_2 + \lambda_3 = 11.$

(GATE-17-IN)

$\lambda_1 \lambda_2 \lambda_3 = (25 + 36)$

(a) $-1, 5, 6$

(b) $1, -5 \pm j6$

(c) $1, 5 \pm j6$ ✓

(d) $1, 5, 5$ ✓

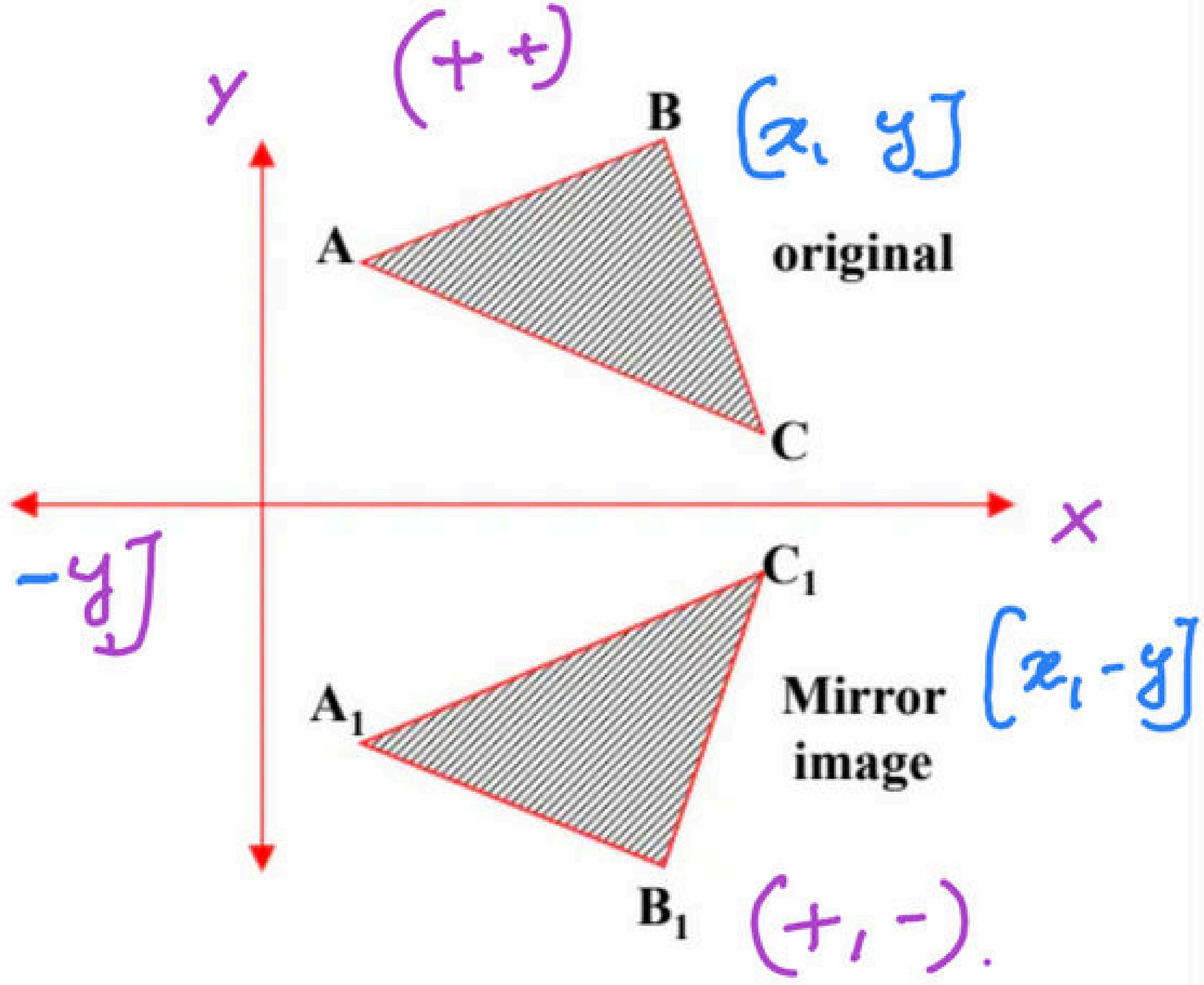
15. The figure shows a shape ABC and its mirror image $A_1B_1C_1$ across the horizontal axis (x-axis). The coordinate transformation matrix that maps ABC to $A_1B_1C_1$ is

(GATE-17-IN)

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$[x \quad y]\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = [x \quad -y]$$



16. Consider the 5×5 matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$

$|A - \lambda I| = 0$

It is given that A has only one real eigen value. Then the real eigen value of A is

(a) -2.5 (b) 0

(c) 15 (d) 25

(GATE-17-EC)

$C_1 \rightarrow C_1 + C_2 + C_3 + C_4 + C_5$

$$\begin{bmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} 15-\lambda & 2 & 3 & 4 & 5 \\ 15-\lambda & 1-\lambda & 2 & 3 & 4 \\ 15-\lambda & 5 & 1-\lambda & 2 & 3 \\ 15-\lambda & 4 & 5 & 1-\lambda & 2 \\ 15-\lambda & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 15-\lambda & 2 & 3 & 4 & 5 \\ 15-\lambda & 1-\lambda & 2 & 3 & 4 \\ 15-\lambda & 5 & 1-\lambda & 2 & 3 \\ 15-\lambda & 4 & 5 & 1-\lambda & 2 \\ 15-\lambda & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$\lambda = 15$

$\lambda = - - - -$

$$(15-\lambda)\begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1-\lambda & 2 & 3 & 4 \\ 1 & 5 & 1-\lambda & 2 & 3 \\ 1 & 4 & 5 & 1-\lambda & 2 \\ 1 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

17. The eigen values of the matrix given below

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix} \text{ are}$$

(GATE-17-EE)

(a) (0, -1, -3) (b) (0, -2, -3)

(c) (0, 2, 3) (d) (0, 1, 3)

$\lambda_1 + \lambda_2 + \lambda_3 = -4$

$\lambda_1 \lambda_2 \lambda_3 = 0$

18. The product of eigen values of the matrix P

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix} \text{ is}$$

(a) -6   (b) 2   **(GATE-17-ME)**

(c) 6   (d) -2

$$\lambda_1 \lambda_2 \lambda_3 = 2(3-6) + 0 + 1\,(8)$$

$$= -6 + 8 = +2$$

19. The determinant of a 2×2 matrix is 50. If one eigen value of the matrix is 10, the other eigen value is ________.

(GATE-17-ME)

$\lambda_1 \lambda_2 = 50$

$\lambda_1 = 10$

$\lambda_2 = 5$

20. Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigen values λ_1 and λ_2 are $x_1 = \begin{bmatrix} 70 \\ \lambda_1 - 50 \end{bmatrix}$ and $x_2 = \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}$, respectively. The value of $x_1^T x_2$ is _____

(GATE-17-ME)

$$x_1^T x_2 = \begin{bmatrix} 70 & \lambda_1 - 50 \end{bmatrix}_{1\times 2} \begin{bmatrix} \lambda_2 - 80 \\ 70 \end{bmatrix}_{2\times 1}$$

$$= 70(\lambda_2 - 80) + 70(\lambda_1 - 50)$$

$$= 70(\lambda_1 + \lambda_2) + 70(-80 - 50)$$

$$= 70(50 + 80) - 70(50 + 80) = \underline{\underline{0}}$$

21. Consider the following simultaneous equations (with c_1 and c_2 being constants):

$$3x_1 + 2x_2 = c_1$$
$$4x_1 + x_2 = c_2$$

The characteristic equation for these simultaneous equations is

(GATE-17-CE)

(a) $\lambda^2 - 4\lambda - 5 = 0$ (b) $\lambda^2 - 4\lambda + 5 = 0$
(c) $\lambda^2 + 4\lambda - 5 = 0$ (d) $\lambda^2 + 4\lambda + 5 = 0$

$$\lambda^2 - (\text{trace})\lambda + |A| = 0$$

$$A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

$3 - 8$

$$\lambda^2 - (4\lambda) - 5 = 0$$

22. If $A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}$ then AB^T is equal to

(GATE-17-CE)

(a) $\begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 40 \\ 42 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 43 & 27 \\ 34 & 50 \end{bmatrix}$ (d) $\begin{bmatrix} 38 & 32 \\ 28 & 56 \end{bmatrix}$

$$AB^T = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix} \begin{bmatrix} 3 & 8 \\ 7 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 3+35 & 8+20 \\ 18+14 & 48+8 \end{bmatrix}$$

$$= \begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$$

23. The matrix P is the inverse of a matrix Q. If I denote the identity matrix, which one of the following options is correct?

(GATE-17-CE)

(a) PQ = I but QP ≠ I

(b) QP = I but PQ ≠ I

(c) PQ = I and QP = I

(d) $PQ - QP = I$

$$A A^{-1} = I.$$

$$A^{-1} A = I.$$

24. Consider the matrix $\begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$. Which one of the following statements is TRUE for the eigenvalues and eigenvectors of this matrix?

(GATE-17-CE)

(a)eigenvalue 3 has a multiplicity of 2, and only one independent eigenvector exists.
(b) eigenvalue 3 has a multiplicity of 2, and two independent eigenvectors exists.
(c) eigenvalue 3 has a multiplicity of 2, and no independent eigenvector exists.
(d) eigenvalues are 3 and -3, and two independent eigenvectors exist

No. of independent eigen vectors = $n - \rho(A-\lambda I) = 2-1=1$

$\lambda_1+\lambda_2 = 6$

$\lambda_1 \lambda_2 = 9$

$\lambda_1 = 3 \quad \lambda_2 = 3$

$\lambda = \underline{3}.$

AM $= 2$

$A-\lambda I = \begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix}$

$\rho(A-\lambda I) = 1$

25. If the characteristic polynomial of a 3×3 matrix M over R (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30$. $a \epsilon R$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is __________.

(GATE-17-CSIT)

$\lambda^3 - 4\lambda^2 + a\lambda + 30 = 0$

$2^3 - 4(2^2) + a(2) + 30 = 0$

$a = -11$

$\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$

$\boxed{\lambda = 5}$

$$\begin{array}{c|cccc} 2 & 1 & -4 & -11 & 30 \\ & 0 & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & \boxed{0} \end{array}$$

$\lambda^2 - 2\lambda - 15 = 0$

$(\lambda - 5)(\lambda + 3) = 0$

$\lambda = 5, -3, 2$

26. Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$.

$\boxed{AA^T = I}$

$A \rightarrow$ orthogonal matrix.

Which one of the following statements about P is INCORRECT?

(GATE-17-ME)

(a) Determinant of P is equal to 1

(b) P is orthogonal

(c) Inverse of P is equal to its transpose

(d) All eigen values of P are real numbers

$A^{-1} = A^T$

$|A| = 1$

Use the code : BVREDDY, to get the maximum discount

27. For the given $\boxed{\text{orthogonal matrix Q,}}$

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is (GATE-18-CE)

(a) $Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

(b) $Q = \begin{bmatrix} -3/7 & -2/7 & 6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

(c) $Q = \begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

(d) $Q = \begin{bmatrix} -3/7 & 6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

28. Which one of the following matrices is singular?

(GATE-18-CE)

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

~~(c)~~ $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$

$|A| = 0$

$\hookrightarrow$ Singular

29. Consider matrix $A = uv^T$

Where, $u = \binom{1}{2}$, $v = \binom{1}{1}$.

Note that v^T denotes the transpose of v. The largest eigen value of A is ________.

(GATE-18-CSIT)

$$A = \underset{2\times 1}{\begin{bmatrix} 1 \\ 2 \end{bmatrix}} \underset{1\times 2}{\begin{bmatrix} 1 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 \\ 2 & 2. \end{bmatrix}$$

$$\lambda_1 + \lambda_2 = 3.$$

$$\lambda_1 \lambda_2 = 0$$

$$\lambda_1 = 3, \quad \lambda_2 = 0$$

30. Consider a non-singular 2×2 square matrix A. If trace (A) = 4 and trace (A^2) = 5, the determinant of the matrix A is __________. (Up to 1 decimal place)

(GATE-18-EC)

$$\lambda_1 + \lambda_2 = 4$$

$$\lambda_1^2 + \lambda_2^2 = 5$$

$$(\lambda_1 + \lambda_2)^2 = \lambda_1^2 + \lambda_2^2 + 2\lambda_1 \lambda_2.$$

$$16 = 5 + 2\lambda_1 \lambda_2$$

$$\lambda_1 \lambda_2 = \frac{11}{2}$$

31. Let N be a 3 by 3 matrix with real number entries. The matrix N is such that $N^2 = 0$. The eigen values of N are

(a) 0, 0, 0 (b) 0, 0, 1 **(GATE-18-IN)**

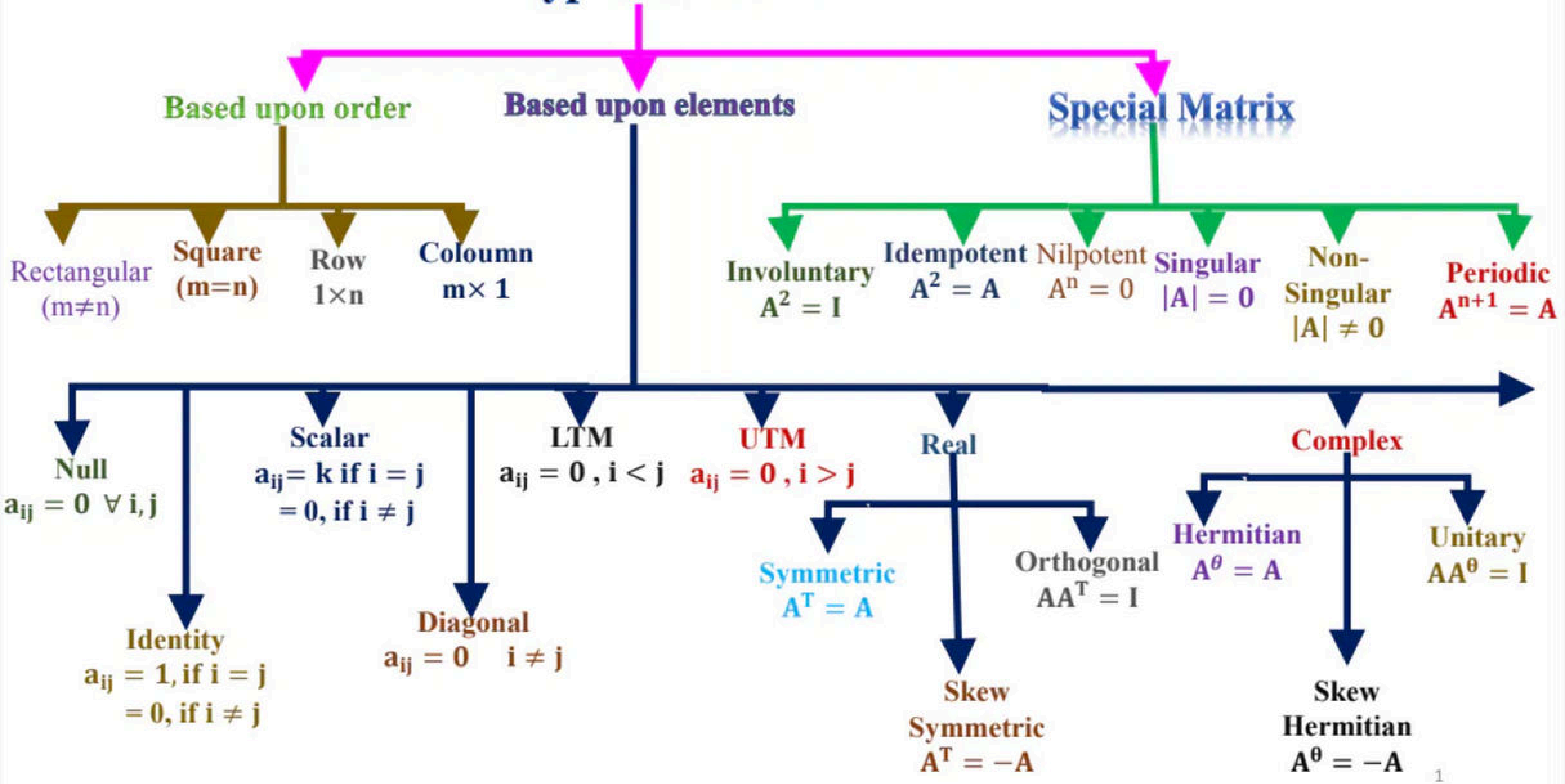
(c) 0, 1, 1 (c) 1, 1, 1

$N^2 = 0$

$\hookrightarrow$ Nipotent matrix

$\lambda = 0, 0, 0$

Types of Matrix



Trace of a matrix

Trace(A) = sum of principal diagonal elements

If A & B are square matrices of order n, then

- tr(A+B) = tr(A)+tr(B)
- tr(A-B) = tr(A)- tr(B)
- tr(AB) ≠ tr(A) tr(B)
- tr(BA) ≠tr(B) tr(A)
- tr(AB) = tr(BA)
- tr(kA) = k tr(A)
- $\mathrm{tr}(A^T) = tr(A)$
- $\mathbf{tr}\ (I_n) = n$

Transpose of a matrix

rows into columns and columns into rows.

if A= $[a_{ij}]$

then $A^T = [a_{ji}]$

- $(A^T)^T = A$
- $(KA)^T = KA^T$
- $(A - B)^T = A^T - B^T$
- $(A\,B\,C)^T = C^T B^T A^T$
- $(A^T)^n = (A^n)^T$

If A- Skew-symmetric

kA, A^3, A^5...... A^{2n+1} are Skew symmetric

A^{2n} Symmetric

If A- Symmetric

kA, $A + A^T$, AA^T, $A^T A$, A^n are symmetric

$A - A^T$, $A^T - A$ are skew-symmetric.

- ✓ $AI = A$
- ✓ $I^{-1} = I$
- ✓ $I^T = I$
- ✓ $I^n = I$
- ✓ $|I| = 1$
- ✓ $Adj\,(I) = I$

- ✓ The diagonal elements of a skew-symmetric matrix are all zero.
- ✓ Sum of all the elements of skew symmetric matrix is zero .
- ✓ Null matrix is both symmetric and skew symmetric
- ✓ if A and B are square symmetric matrices of same order then AB is symmetric if and only if AB =BA
- ✓ If A and B are skew symmetric matrices of same order then AB is skew symmetric if and only if AB = - BA.

If A and B are symmetric then

(AB + BA) -----> symmetric

(AB - BA) -----> skew symmetric

$(K_1 A + K_2 B)$ -----> symmetric

If A and B are skew symmetric

then (AB + BA) -----> symmetric

(AB - BA) -----> skew symmetric

$(K_1 A + K_2 B)$----> skew symmetric

- ❑ Every square matrix A can be uniquely expressed as a sum of symmetric & Skew- symmetric matrices.
- ❑ If $A^T = A$, then A is symmetric
- ❑ $if\ A^T = -A$. Then A is skew symmetric

Matrix Algebra

- A+B = B+A (Commutative law)
- (A+B)+C = A+ (B+C) (Associative law)
- A+(-A) = 0(null matrix)
- A+B = A+C then B = C (Left Cancellation law)
- B+A = C+A , then B= C (Right cancellation law)
- k(A+B) = kA +kB
- $(k + l)A = k A + l A$
- $(k l)A = k(lA) = l(kA)$
- $(-k)$A = - (kA)
- If AB --->exists ,
- BA -----> may or may not exist
- AB ≠ BA (not obeys commutative)
- ABC =A(BC)= B(AC) (obeys Associative)
- A(B+C) = AB+AC (obeys Distributive)
- $A^m A^n = A^{m+n}$
- $(A^m)^n = A^{mn}$
- If AB = 0 , then A and B may not be null matrix.
- If AB =0 , then BA may not be zero .
- If $A^2 = B^2$, then A and B may not be equal
- If A and B are two square matrix of same order then
  - $(A + B)^2 = A^2 + B^2 + AB + BA$
  - $(A - B)^2 = A^2 + B^2 - AB - BA$

Minor and Co-factor of an element

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

minor of a_{11} is

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$= (a_{22} a_{33} - a_{32} a_{23})$$

Co-factor of a_{ij} is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

S.No	Matrix	Determinant
1	Orthogonal matrix	$\lvert A\rvert = \pm 1$
2	Unitary matrix	$\lvert A\rvert = \pm 1$
3	Involutary	$\lvert A\rvert = \pm 1$
4	UTM LTM Diagonal Scalar Identity	$\lvert A\rvert$ = Product of principal diagonal elements
5	Idempotent	$\lvert A\rvert = 0$ or 1
6	Nilpotent	$\lvert A\rvert = 0$
7	All the elements of any row are zero	$\lvert A\rvert = 0$
8	All the elements of any column are zero	$\lvert A\rvert = 0$
9	All elements are consecutive	$\lvert A\rvert = 0$ (valid for 3rd and higher order matrix)
10	Skew symmetric of odd order	$\lvert A\rvert = 0$
11	Skew symmetric of even order	Perfect square
12	Sum of all elements of each row is zero	$\lvert A\rvert = 0$
13	Sum of all elements of each column is zero	$\lvert A\rvert = 0$

	Matrix	Determinant
14	$R_i = k\,R_j$	$\lvert A\rvert = 0$
15	$C_i = k\,C_j$	$\lvert A\rvert = 0$
16	$R_i \longleftrightarrow R_j$ $C_i \longleftrightarrow C_j$	Sign of Det changes
17	$R_i \rightarrow R_i + kR_j$ $C_i \rightarrow C_i + kC_j$	No change of Det
18	$\lvert AB\rvert = \lvert A\rvert\lvert B\rvert$.	
19	$\lvert A\rvert = \lvert A^T\rvert$	
20	$\lvert A^m\rvert = \lvert A\rvert^m$ $\lvert A^{-1}\rvert = \frac{1}{\lvert A\rvert}$ $(\lvert A_{n\times n}\rvert \neq 0)$	
21	$\lvert kA_{n\times n}\rvert = k^n\lvert A_{n\times n}\rvert$	
22	$\lvert A+B\rvert \neq \lvert A+B\rvert$	
23	$\lvert I \rvert = 1$	

Inverse of a square matrix

- If the inverse of a square matrices A exists then the matrix is called invertible matrix

$A^{-1} = \frac{1}{\lvert A\rvert} adj(A)$.
$(A^{-1})^{-1} = A$
$(AB)^{-1} = B^{-1}A^{-1}$
$(ABCD)^{-1} = D^{-1}C^{-1}B^{-1}A^{-1}$
$(kA)^{-1} = \frac{A^{-1}}{k}$
$(A^T)^{-1} = (A^{-1})^T$
$(A^\theta)^{-1} = (A^{-1})^\theta$.
$(A^n)^{-1} = (A^{-1})^n$
$(I)^{-1} = I$
$(adJA)^{-1} = \frac{A}{\lvert A\rvert}$

$A(\text{adj}A) = (adjA)A = \lvert A\rvert I$
$adj(AB) = adj(B)adj(A)$
$adj(kA) = k^{n-1}adj(A)$
$\lvert adj(A)\rvert = \lvert A\rvert^{n-1}$
$\lvert adj(kA)\rvert = k^{n(n-1)}\lvert A\rvert^{n-1}$
$\lvert adj(adj(A))\rvert = \lvert A\rvert^{(n-1)^2}$
A ---> Hermitian matrix then adj(A)---> Hermitian.
A ---> symmetric matrix then adj (A) ---> symmetric.
A--->diagonal matrix then adj(A)--->diagonal matrix
A ---->triangular matrix , then adj(A)---> triangular matrix. If A is LTM then adj(A) --->LTM If A is UTM then adj(A) --->UTM

Rank of a Matrix

A non-negative integer 'r' is said to be the rank of matrix A, if

- There exists at least one non-zero minor of order 'r'.
- all minors of order (r+1) if they exist, are zeros. Then we rite Rank of A = $\rho(A)$ = r (or)
- Rank of matrix A is the number of linearly independent rows (or columns) of A (or)
- The number of non zero rows in the Row Echelon form .

Echelon form

A matrix A of order m×n is said to be in row echelon form if

- The number of zeros before the first non-zero element in each row is less than the number of such zeros in the next non zero row.
- If there are any Zero rows , they must be below the non-zero rows.

1	Rank (null matrix) = zero
2	$\rho(A_{m\times n}) \le min\{m,n\}$.
3	If A is non singular matrix, $\rho(A_{n\times n}) = n$.
4	If A is singular matrix , then $\rho(A_{n\times n}) < n$.
5	$\rho(I_n) = n$.
6	$\boldsymbol{\rho(A^T) = \rho(A)}$. $\boldsymbol{\rho(A^{-1}) = \rho(A)}$. $\boldsymbol{\rho(AA^T) = \rho(A)}$. $\rho(A^\theta) = \rho(A)$. $\boldsymbol{\rho(AA^\theta) = \rho(A)}$
7	$\rho(AB) \le min\{\rho(A),\rho(B)\}$.
8	If A and B are square matrix of order n , then $\rho(AB) \ge \rho(A) + \rho(B) - n$
9	If $\rho(A_{\mathrm{n}\times n}) = \mathrm{n}$, then $\rho(adj\ A) = n$
10	$\rho(A+B) \le \{\rho(A) + \rho(B)\}$.
11	$\rho(A-B) \ge \{\rho(A) - \rho(B)\}$.

12	**If A and B are square matrix of order n , then** $\mathbf{Max}\{\mathbf{0}, \boldsymbol{\rho(A) + \rho(B) - n}\} \le \boldsymbol{\rho(AB)} \le \boldsymbol{min\{\rho(A), \rho(B)\}}$
13	**If A and B are square matrix of order n , then** $\boldsymbol{|\rho(A) - \rho(B)| \le \rho(A+B) \le min\{\rho(A) + \rho(B), m, n\}}$
14	The rank of a diagonal matrix is equal to the number of non-zero diagonal elements.
15	The rank of a UTM is number of non zeros rows in the UTM . The rank of a LTM is number of non zeros rows in the LTM .
16	If $\rho(A_{\mathrm{n}\times n}) = \mathrm{n} - 1$, then $\rho(adj\ A) = 1$.
17	If $\rho(A_{\mathrm{n}\times n}) < \mathrm{n} - 1$, then $\rho(adj\ A) = 0$.

Non Homogeneous equations
(AX= B)

Find $\rho(A)$, $\rho(A \mid B)$

- $\rho(A) = \rho(AB)$
  System is consistent
  i.e solution exists
  - $\rho(A) = \rho(A|B) = n$ (no.of unknowns)
    Unique solutions
  - $\rho(A) = \rho(A|B) < n$
    Infinitely many solutions
- $\rho(A) \neq \rho(AB)$
  System is inconsistent
  i.e no solution

Homogeneous equations
(AX= 0)

Always has a solution

- $\rho(A) = n$ (no.of unknowns)
  Unique solutions (or) Zero as a solution (or) Trivial solution
- $\rho(A) < n$
  Infinitely many solutions (or) Non trivial solution

Eigen Values

- Sum of the eigen values of a matrix is equal to the trace of the matrix
- Product of the eigen values of a matrix is equal to the determinant of the matrix .
- For lower triangular matrix (upper triangular matrix or diagonal matrix or scalar matrix or identity matrix), the Eigen values are same as diagonal elements of the matrix.
- For a matrix if a+ib is an eigen value of matrix A , then a-ib is also an eigen value of matrix A.
- For a matrix if a+$\sqrt{b}$ is an eigen value of matrix A , then a- $\sqrt{b}$ is also an eigen value of matrix A.

If **λ is an Eigen value** of a matrix A , **X is the Eigen Vector** and k is a scalar then

S.No	Matrix	Eigen Values	Eigen Vectors
1	A^m	λ^m	X
2	A^{-1}	$\frac{1}{\lambda}$	X
3	A^θ	$\bar{\lambda}$	X
4	kA.	Kλ	X
5	$A \pm kI$	$A \pm k$	X
6	$(A \pm kI)^n$	$(\lambda \pm k)^n$	X
7	adjA	$\frac{\|A\|}{\lambda}$	X
8	$a_0I + a_1A + a_2A^2$	$a_0 + a_1\lambda + a_2\lambda^2$	X

Type of Matrix	Eigen values
1. Symmetric matrix and Hermitian matrix	Real values
2.Skew symmetric matrix and skew Hermitian matrix	Zero (or) purely imaginary
3.Orthogonal matrix and Unitary matrix	Magnitude of eigen value is one $\|\lambda\| = 1$
4.Idempotent matrix	0 (or) 1
5.Involuntary matrix	-1 (or) +1
6. Nilpotent matrix	0

- If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are distinct Eigen values of a square matrix A of order 'n' then the corresponding Eigen vectors $X_1, X_2, \ldots\ldots, X_n$ of matrix A are linearly independent.
- If some Eigen values of matrix A are repeated then Eigen vectors of A may or may not be linearly independent.
- The number of linearly independent eigen vectors of an eigen value 'λ' is $n - \rho(A - \lambda I)$

Cayley – Hamilton theorem

Every square matrix satisfies its own characteristic equation.

- To find higher powers of a matrix
- To find Inverse of a matrix

Diagonalization of a Matrix

$$AP = PD$$

A---Given matrix

D—Diagonal Matrix

P---Modal Matrix

- To find the matrix from Eigen vectors

$$A = PDP^{-1}$$

- To find higher powers of a matrix

$$A^n = PD^nP^{-1}$$

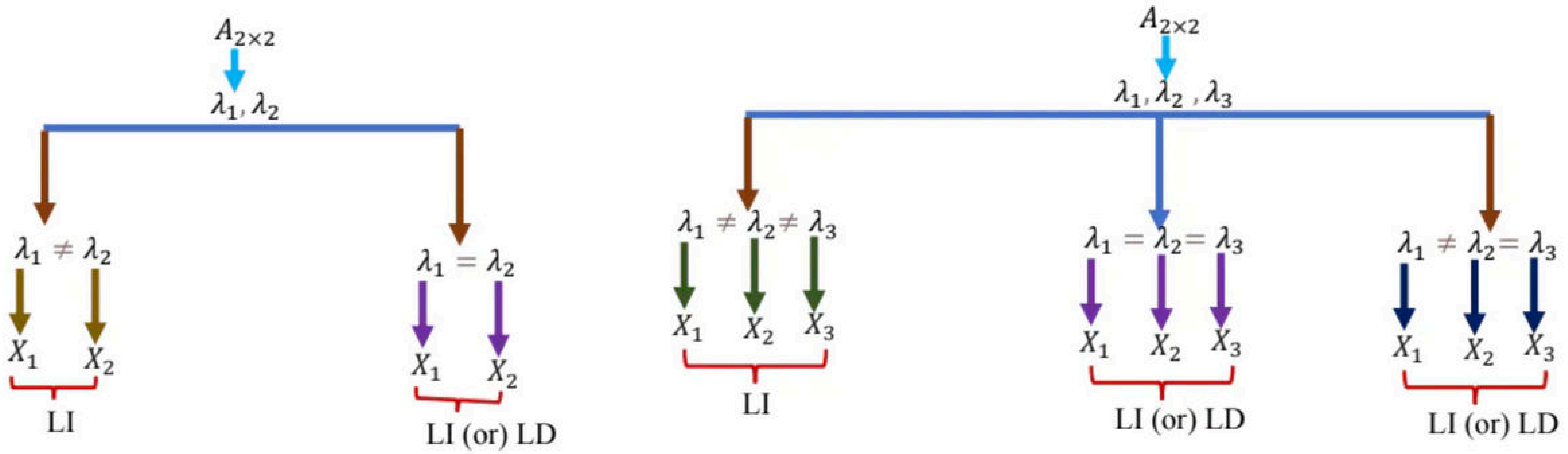
Algebraic Multiplicity of an Eigen value

- The number of times an eigen value is repeated is called as algebraic multiplicity (A.M) of that eigen value

Geometric Multiplicity of an Eigen value

- The number of linearly independent eigen vectors corresponding to an eigen value λ is called as geometric multiplicity of that eigen value λ.
- The number of linearly independent eigen vectors of an eigen value 'λ'
- $= n - \rho(A - \lambda I)$
- Algebraic multiplicity of an eigen value $\geq$ Geometric multiplicity
- Geometric multiplicity of every eigen value of a matrix is ≥ 1
- If all the eigen values of a matrix are distinct then the matrix can be diagonalizable but converse need not be true .
- A matrix is diagonalizable iff for every eigen value , geometric multiplicity is equal to algebraic multiplicity.

- Every idempotent matrix , involuntary matrix , symmetric matrix, unitary matrix can be diagonalizable .
- Nilpotent matrix can never be diagonalizable .
- If all the elements are equal then A is diagonalizable .



$$\boldsymbol{\lambda^2 - (trace\ of\ A)\lambda + |A| = 0}\cdot$$

$$\lambda^3 - (\boldsymbol{trace\ of}\ A)\lambda^2 + (\mathbf{trace(Adj)}\lambda - |A| = 0$$

32. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then det (A^{-1}) is __________ (correct to two decimal places).

(GATE-18-ME)

UTM

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{4}$$

33. The diagonal elements of a 3 by 3 matrix are -10, 5, and 0, respectively. If two of its eigenvalues are -15 each, the third eigen value is ________.

(GATE-18-PI)

$$\lambda_1 + \lambda_2 + \lambda_3 = -5$$

$$-15 - 15 + \lambda_3 = -5$$

$$\lambda_3 = 25$$

34. A 3×3 matrix has eigen values 1, 2, and 5. The determinant of the matrix is _______.

(GATE-19-INST)

$$\lambda_1 \lambda_2 \lambda_3 = (1)(2)(5) = 10$$

35. Consider the following matrix: $R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$

The absolute value of the product of Eigen values of R is ________.

(GATE-19-CSIT)

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 2 & 12 & 56 \\ 0 & 3 & 21 & 117 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 2 & 12 & 56 \\ 0 & 0 & 4 & 42 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$R_4 \to R_4 - (R_2 + R_3)$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 0 & 2 & 18 \\ 0 & 0 & 4 & 42 \end{bmatrix}$$

$$\begin{array}{r} 56 \\ 38 \\ \hline 1 \end{array}$$

$R_4 \to R_4 - 2R_3$

$$\sim \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 0 & 2 & 18 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

$|A| = 12$

36. M is a 2×2 matrix with eigen values 4 and 9. The eigen values of M^2 are

(GATE-19-EE)

(a) 2 and 3 (b) -2 and -3

(c) 4 and 9 (d) 16 and 81

37. Consider a 2×2 matrix $M = [v_1, v_2]$, where, v_1 and v_2 are the column vectors.

Suppose $M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$, where u_1^T and u_2^T are the row vectors.

Consider the following statements:

Statement 1: $\boldsymbol{u_1^T v_1 = 1}$ and $u_2^T v_2 = 1$

Statement 2: $\boldsymbol{u_1^T v_2 = 0}$ and $u_2^T v_1 = 0$

Which of the following options is correct?

(GATE-19-EE)

(a) Statement 2 is true and statement 1 is false

(b) Both the statements are false

(c) Statement 1 is true and statement 2 is false

(d) Both the statements are true

$M M^{-1} = I = M^{-1} M = I$

$\underset{2\times1}{\begin{bmatrix} U_1^T \\ U_2^T \end{bmatrix}} \underset{1\times2}{\begin{bmatrix} V_1 & V_2 \end{bmatrix}} = \begin{bmatrix} U_1^T V_1 & U_1^T V_2 \\ U_2^T V_1 & U_2^T V_2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$U_1^T V_1 = 1 \quad U_1^T V_2 = 0$

$U_2^T V_1 = 0 \quad U_2^T V_2 = 1$

38. For any real, square and non-singular matrix B, the det B^{-1} is

(GATE-19-ME)

(a) zero

~~(b)~~ $(\det B)^{-1}$

(c) - (det B)

(d) det B

$$|B^{-1}| = |B|^{-1}$$

39. Consider the matrix $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

The number of distinct eigen values of P is

(a) 2 (b) 1 **(GATE-19-ME)**

(c) 3 (b) 0

UTM $\lambda = 1, 1, 1.$

40. The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

$-20+45-20 \neq 1$

(GATE-19-CE)

(a) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$ ×

(b) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$ ×

$4-9+4=-1$

$\frac{-16}{5}+\frac{12}{5}-\frac{2}{5} \neq 1$

(c) $\begin{bmatrix} -2 & -\frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

(d) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

$20-45+20 \neq 0$

$-4+9-4=1.$

41. Let A_1, A_2, A_3 and A_4 be four matrices of dimensions 10×5, 5×20, 20×10 and 10×5, respectively. The minimum number of scalar multiplications required to find the product $A_1A_2A_3A_4$ using the basic matrix multiplication method is ________.

(GATE-20-ME)

1500

42. A 4 × 4 matrix [P] is given below

(2,1)

$$[P] = \begin{bmatrix} 0 & 1 & 3 & 0 \\ -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

The eigen values of [P] are

(GATE-2020(CE))

(a) 0, 3, 6, 6 ✗ (b) 1, 2, 3, 4

(c) 1, 2, 5, 7 ✓ (d) 3, 4, 5, 7

$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 15$

$\lambda_1 \lambda_2 \lambda_3 \lambda_4 = +2 \begin{vmatrix} 1 & 3 & 0 \\ 0 & 6 & 1 \\ 0 & 1 & 6 \end{vmatrix}$

$= 2[36-1] = 70.$

$\lambda_1 \lambda_2 \lambda_3 \lambda_4 = 70.$

43. If $P = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ and $Q = \begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ then Q^TP^T is

(GATE-21-CE)

(a) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

(b) $\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix}$

(c) $\begin{bmatrix}2 & 1\\4 & 3\end{bmatrix}$

(d) $\begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}$

$$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\begin{bmatrix}1 & 3\\2 & 4\end{bmatrix} = \begin{bmatrix}0+2 & 0+4\\1+0 & 3+0\end{bmatrix} = \begin{bmatrix}2 & 4\\1 & 3\end{bmatrix}$$

44. If A is a square matrix then orthogonality property mandates

(GATE-21-CE)

(a) $AA^T = I$ (b) $AA^T = 0$

(c) $AA^T = A^{-2}$ (d) $AA^T = A^2$

$A A^T = I$

45. Let p and q be real numbers such that $p^2 + q^2 = 1$. The eigenvalues of the matrix $\begin{bmatrix} p & q \\ q & -p \end{bmatrix}$ are

(GATE-21-EE)

(a) pq and -pq
(b) 1 and 1
(c) j and -j
(d) 1 and -1 ✓

$\lambda_1 + \lambda_2 = 0$

$\lambda_1 \lambda_2 = -p^2 - q^2 = -1.$

46. Consider the following matrix :

$$\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$|A - \lambda I| = 0$

The largest eigenvalue of the above matrix is_____________.

(GATE-2021-cs)

$$\begin{bmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{bmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3 + C_4$

$$(3-\lambda) \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0.$$

$$\begin{bmatrix} 3-\lambda & 1 & 1 & 1 \\ 3-\lambda & -\lambda & 1 & 1 \\ 3-\lambda & 1 & -\lambda & 1 \\ 3-\lambda & 1 & 1 & -\lambda \end{bmatrix} = 0.$$

$$(3-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(\lambda+1)^3 = 0$$

$\lambda = 3$ ✓

$\lambda = -1, -1, -1$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$R_4 \to R_4 - R_1$

$$(3-\lambda)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & -\lambda-1 & 0 & 0 \\ 0 & 0 & -\lambda-1 & 0 \\ 0 & 0 & 0 & -\lambda-1 \end{vmatrix} = 0$$

47. A real 2 ×2 non-singular matrix A with repeated eigenvalue is given as

$$A = \begin{bmatrix} x & -3.0 \\ 3.0 & 4.0 \end{bmatrix}$$

where x is a real positive number . The value of x (rounded off to one decimal place) is________.

(GATE – 2021 – EC)

$2\lambda = 4+x \Rightarrow \lambda = \frac{4+x}{2}.$

$\lambda^2 = 4x+9.$

$\left(\frac{4+x}{2}\right)^2 = 4x+9$

$16+x^2+8x = 16x+36.$

$x^2-8x-20=0.$

$x=10 \quad x=-2$

48. The determinant of the matrix M shown below is______________.

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

(GATE – 2021 – IN)

$\frac{11am}{2pm}$

$|M| = (4-6)(4-6) = 4.$

$R_2 \to R_2 - 3R_1$

$R_4 \to R_4 - \frac{R_3}{2}$

$1 - \frac{3}{2} = -\frac{1}{2}$

$$M = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$1(-2)(4)\left(\frac{-1}{2}\right) = 4$

49. The eigen vector (s) of the matrix $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\alpha \neq 0$ is (are)

(GATE – 93)

(a) $(0, 0, \alpha)$ (b) $(\alpha, 0, 0)$

(c) $(0, 0, 1)$ (d) $(0, \alpha, 0)$

50. If A and B are real symmetric matrices of order n then which of the following is true.

(GATE – 94[CS])

(a) $A A^T = I$

(b) $A = A - 1$

(c) $AB = BA$

(d) $(AB)^T = B^T A^T$

51. The inverse of the matrix $S = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is (GATE – 95[EE])

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

52. The eigen values of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are

(GATE – 94[EE])

(a) (a + 1), 0
(b) a, 0
(c) (a – 1), 0
(d) 0, 0

53. The matrix $\begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$ is an inverse of the matrix $\begin{bmatrix} 5 & -4 \\ 1 & -1 \end{bmatrix}$ **(GATE – 94[PI])**

(a) True

(b) False

54. The value of the following determinant

$$\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$$ is (GATE – 94[PI])

(a) 8 (b) 12

(c) –12 (d) –8

55. For the following matrix $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ the number of real positive characteristic roots is

(GATE – 94[PI])

(a) one (b) two

(c) four (d) 0

56. Given matrix $L = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$ and $M = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$

then $L \times M$ is **(GATE – 95[PI])**

(a) $\begin{bmatrix} 8 & 1 \\ 13 & 2 \\ 22 & 5 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 8 \\ 2 & 13 \\ 5 & 22 \end{bmatrix}$

(d) $\begin{bmatrix} 6 & 2 \\ 9 & 4 \\ 0 & 5 \end{bmatrix}$

57. Inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is

(GATE – 97[CE])

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

58. Given the matrix $A=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}$. Its eigen values are (GATE – 95[EE])

a) 1, 2, 3 **b)-1, -2 ,-3** **c)1 ,-2 , 3** **d)-1 ,-2 ,3**

59. The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication.

(GATE – 96[CS])

(a) If $a = b$ (or) $\theta = n\pi$, n is an integer

(b) always

(c) never

(d) If $a \cos\theta \neq b \sin\theta$

60. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that $AB = I$.

Let $C = A\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $CD = I$.

Express the elements of D in terms of the elements of B.

61. The eigen values of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are

(GATE – 96[ME])

(a) 0, 0, 0

(b) 0, 0, 1

(c) 0, 0, 3

(d) 1, 1, 1

62. If the determinant of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is

(GATE – 97[CE])

(a) – 26
(b) 26
(c) 0
(d) 52

63. If A and B are two matrices if both AB and BA exists

a) Only if A has as many rows as B has columns
b) Only if the order of A and B are same
c) Only if A and B are skew symmetric
d) Only if both A and B are symmetric

64. The determinant of the matrix

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

is (GATE – 97[CS])

(a) 11 (b) – 48

(c) 0 (d) – 24

65. If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ then which of the following is a factor of Δ.

(GATE – 98[CS])

(a) $a + b$

(b) $a - b$

(c) abc

(d) $a + b + c$

66. If A is a real square matrix then AA^T is

(GATE – 98[CE])

(a) un symmetric

(b) always symmetric

(c) skew – symmetric

(d) some times symmetric

67. In matrix algebra $AS = AT$ (A, S, T, are matrices of appropriate order) implies $S = T$ only if

(GATE – 98[CE])

(a) A is symmetric

(b) A is singular

(c) A is non-singular

(d) A is skew-symmetric

68. The eigen values of the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

are

(GATE – 98[EC])

(a) 1, 1

(b) –1, –1

(c) $j, -j$

(d) 1, –1

69. $A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$. The sum of the eigen values of the matrix A is

(GATE – 98[EE])

(a) 10
(b) -10
(c) 24
(d) 22

70. If $A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ then $A^{-1} =$

(GATE – 98[EE])

(a) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1/3 & 0 \\ -2 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & -1/3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1/5 & 0 & 1/2 \\ 0 & 1/3 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1/5 & 0 & -1/2 \\ 0 & 1/3 & 0 \\ -1/2 & 0 & 1 \end{bmatrix}$

71. If $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ and

$\text{adj}(A) = \begin{bmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{bmatrix}$ then $k =$

(GATE – 99)

(a) –5 (b) 3

(c) –3 (d) 5

72. If A is any $n x n$ matrix and k is a scalar then $|kA| = \alpha |A|$ where α is

(GATE–99[CE])

(a) kn (b) n^k

(c) k^n (d) $\frac{k}{n}$

73. The number of terms in the expansion of general determinant of order n is

(GATE – 99[CE])

(a) n^2 (b) $n!$

(c) n (d) $(n+1)^2$

74. The equation $\begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & -1 \\ y & x^2 & x \end{vmatrix} = 0$ represents a parabola passing through the points.

(GATE – 99[CE])

(a) (0, 1), (0, 2), (0, –1)

(b) (0, 0), (–1, 1), (1, 2)

(c) (1, 1), (0, 0), (2, 2)

(d) (1, 2), (2, 1), (0, 0)

75. An $n \times n$ array V is defined as follows $v[i,j] = i - j$ for all i, j, $1 \le i, j \le n$ then the sum of the elements of the array V is

(GATE–2000[CS])

(a) 0
(b) $n - 1$
(c) $n^2 - 3n + 2$
(d) $n(n + 1)$

76. The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is **(GATE–2000[CS])**

(a) 4 (b) 0 (c) 15 (d) 20

77. If A, B, C are square matrices of the same order then $(ABC)^{-1}$ is equal be

(GATE–2000[CE])

(a) $C^{-1} A^{-1} B^{-1}$ (b) $C^{-1} B^{-1} A^{-1}$

(c) $A^{-1} B^{-1} C^{-1}$ (d) $A^{-1} C^{-1} B^{-1}$

78. The eigen values of the matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix} \text{are}$$

(GATE–2000[EC])

(a) 2, –2, 1, –1 (b) 2, 3, –2, 4

(c) 2, 3, 1, 4 (d) None

79. Consider the following statements

S_1: The sum of two singular matrices may be singular.

S_2: The sum of two non-singular may be non-singular.

Which of the following statements is true?

(GATE–01[CS])

(a) S_1 & S_2 are both true

(b) S_1 & S_2 are both false

(c) S_1 is true and S_2 is false

(d) S_1 is false and S_2 is true

80. The determinant of the following matrix

$$\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix}$$

(GATE– 01[CE])

(a) – 76 (b) – 28

(c) 28 (d) 72

81. The eigen values of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 9 \end{bmatrix}$ are

(GATE– 01[CE])

(a) (5.13, 9.42) (b) (3.85, 2.93)

(c) (9.00, 5.00) (d) (10.16, 3.84)

82. The product [P] $[Q]^T$ of the following two matrices [P] and [Q]

where $[P] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $[Q] = \begin{bmatrix} 4 & 8 \\ 9 & 2 \end{bmatrix}$ is

(GATE–01[CE])

(a) $\begin{bmatrix} 32 & 24 \\ 56 & 46 \end{bmatrix}$ (b) $\begin{bmatrix} 46 & 56 \\ 24 & 32 \end{bmatrix}$

(c) $\begin{bmatrix} 35 & 22 \\ 61 & 42 \end{bmatrix}$ (d) $\begin{bmatrix} 32 & 56 \\ 24 & 46 \end{bmatrix}$

83. Obtain the eigen values of the matrix

$$A = \begin{bmatrix} 1 & 2 & 34 & 49 \\ 0 & 2 & 43 & 94 \\ 0 & 0 & -2 & 104 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

(GATE – 02[CS])

(a) 1,2,–2,–1 (b) –1,–2,–1,–2

(c) 1,2,2,1 (d) None

84.. The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

(GATE–07[ME])

(a) 0
(b) 1
(c) 2
(d) infinite

85. The determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix}$ is **(GATE – 02[EE])**

(a) 100 (b) 200

(c) 1 (d) 300

86. Eigen values of the following matrix are

$$\begin{bmatrix} -1 & 4 \\ 4 & -1 \end{bmatrix}$$

(GATE – 02[CE])

(a) 3, –5
(b) –3, 5
(c) –3, –5
(d) 3, 5

87. If matrix $X = \begin{bmatrix} a & 1 \\ -a^2+a-1 & 1-a \end{bmatrix}$ and

$X^2 - X + I = O$ then the inverse of X is

(GATE – 04)

(a) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$

(b) $\begin{bmatrix} 1-a & -1 \\ a^2-a+1 & a \end{bmatrix}$

(c) $\begin{bmatrix} -a & 1 \\ -a^2+a-1 & a-1 \end{bmatrix}$

(d) $\begin{bmatrix} a^2-a+1 & a \\ 1 & 1-a \end{bmatrix}$

88. The number of different n×n symmetric matrices with each elements being either 0 or 1 is (GATE–04[CS])

(a) 2^n (b) 2^{n^2}

(c) $2^{\frac{n^2+n}{2}}$ (d) $2^{\frac{n^2-n}{2}}$

89. Let A, B, C, D be n×n matrices, each with non-zero determinant. $ABCD = I$ then $B^{-1} =$

(GATE–04[CS])

(a) $D^{-1}C^{-1}A^{-1}$ (b) CDA

(c) ABC (d) does not exist

90. The sum of the eigen values of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix} \text{ is}$$

(GATE–04[ME])

(a) 5 (b) 7

(c) 9 (d) 18

91. For what value of x will the matrix given below become singular? $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$

(GATE–04[ME])

a) -4 b) 4 c) 2 d)-2

92. Real matrices $[A]_{3\times1}$, $[B]_{3\times3}$, $[C]_{3\times5}$, $[D]_{5\times3}$, $[E]_{5\times5}$, $[F]_{5\times1}$ are given. Matrices [B] and [E] are symmetric. Following statements are made with respect to their matrices.

(I) Matrix product $[F]^T \ [C]^T \ [B] \ [C] \ [F]$ is a scalar.

(II) Matrix product $[D]^T \ [F] \ [D]$ is always symmetric.

With reference to above statements which of the following applies?

(GATE–04[CE])

(a) statement (I) is true but (II) is false

(b) statement (I) is false but (II) is true

(c) both the statements are true

(d) both the statements are false

93. The eigen values of the matrix $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ are **(GATE–04[CE])**

(a) 1, 4 (b) – 1, 2

(c) 0, 5 (d) can not be determined

94. What are the eigen values of the following 2×2 matrix $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ **(GATE–05[CS])**

(a) $-1, 1$ (b) $1, 6$

(c) $2, 5$ (d) $4, -1$

95. Consider the matrices X_{4x3}, Y_{4x3} and P_{2x3}.
The order of $[P (X^T Y)^{-1} P^T]^T$ will be

(GATE–05[CE])

(a) 2×2
(b) 3×3
(c) 4×3
(d) 3×4

96. The determinant of the matrix given below

is $\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$ (GATE–05)

(a) -1 (b) 0

(c) 1 (d) 2

97. For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is –2. Which of the following is an eigen vector? **(GATE–05[EE])**

(a) $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

98. If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$ then the top row of R^{-1} is (GATE–05[EE])

(a) $[5 \quad 6 \quad 4]$

(b) $[5 \quad -3 \quad 1]$

(c) $[2 \quad 0 \quad -1]$

(d) $[2 \quad -1 \quad 0]$

99. The eigen values of the matrix M given are 15, 3 and 0.

$M=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, the value of the determinant of a matrix is **(GATE–05[PI])**

(a) 20
(b) 10
(c) 0
(d) –10

100. Identify which one of the following is an eigen vector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$

(GATE–05(IN))

(a) $[-1 \quad 1]^T$ (b) $[3 \quad -1]^T$

(c) $[1 \quad -1]^T$ (d) $[-2 \quad 1]^T$

101. If $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1/2 & a \\ 0 & b \end{bmatrix}$ then

$a + b =$

(GATE–05[EE])

(a) $\frac{7}{20}$

(b) $\frac{3}{20}$

(c) $\frac{19}{60}$

(d) $\frac{11}{20}$

102. Given an orthogonal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}. \ (AA^T)^{-1} \text{ is}$$

(GATE–05[EC])

(a) $\frac{1}{4}I_4$

(b) $\frac{1}{2}I_4$

(c) I

(d) $\frac{1}{3}I_4$

103. Given the matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$, the eigen vector is

(GATE–05[EC])

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

104. Eigen values of a matrix $S=\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1. What are the eigen values of the matrix $S^2 = SS$? **(GATE – 06[ME])**

(a) 1 and 25
(b) 6, 4
(c) 5, 1
(d) 2, 10

105. Multiplication of matrices E and F is G. Matrices E and G are

$$E=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} \text{ and } G=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

What is the matrix F?

(GATE–06[ME])

(a) $\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ (b) $\begin{bmatrix}\cos\theta & \cos\theta & 0\\ -\cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}\sin\theta & -\cos\theta & 0\\ \cos\theta & \sin\theta & 0\\ 0 & 0 & 1\end{bmatrix}$

106. For a given 2x2 matrix A, it is observed that

$$A\begin{bmatrix}1\\-1\end{bmatrix}=-1\begin{bmatrix}1\\-1\end{bmatrix} \text{ and } A\begin{bmatrix}1\\-2\end{bmatrix}=-2\begin{bmatrix}1\\-2\end{bmatrix}$$

then the matrix A is **(GATE-06[IN])**

(a) $A=\begin{bmatrix}2 & 1\\-1 & -1\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & -2\end{bmatrix}\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}$

(b) $A=\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & -1\end{bmatrix}$

(c) $A=\begin{bmatrix}1 & 1\\-1 & -2\end{bmatrix}\begin{bmatrix}-1 & 0\\0 & -2\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & -1\end{bmatrix}$

(d) $A=\begin{bmatrix}0 & -2\\1 & -3\end{bmatrix}$

107. The eigen values and the corresponding eigen vectors of a 2x2 matrix are given by

Eigen value	**Eigen vector**
$\lambda_1 = 8$	$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$
$\lambda_2 = 4$	$V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

The matrix is **(GATE–06[EC])**

(a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

108. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$. The eigen value corresponding to the eigen vector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is

(GATE–06[EC]

(a) 2 (b) 4

(c) 6 (d) 8

109. For a given matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the eigen value is 3. The other two eigen values are

(GATE–06[CE])

(a) 2, –5 (b) 3, –5

(c) 2, 5 (d) 3, 5

110. The minimum and maximum eigen values of matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are -2 and 6 respectively. What is the other eigen value?

(GATE–07[CE])

(a) 5 (b) 3

(c) 1 (d) – 1

111. The inverse of 2×2 matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$ is

(GATE – 07[CE])

(a) $\frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 7 & 2 \\ 5 & 1 \end{bmatrix}$

(c) $\frac{1}{3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} -7 & -2 \\ -5 & -1 \end{bmatrix}$

112. If a square matrix A is real and symmetric then the eigen values

(GATE – 07[ME])

(a) are always real

(b) are always real and positive

(c) are always real and non-negative

(d) occur in complex conjugate pairs

113. The determinant $\begin{vmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{vmatrix}$ equals to

(GATE–07[PI])

(a) 0

(b) $2b(b-1)$

(c) $2(1-b)(1+2b)$

(d) $3b(1+b)$

114. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A^9 equals

(GATE–07[EE])

(a) $511\,A + 510\,I$ (b) $309\,A + 104\,I$

(c) $154\,A + 155\,I$ (d) e^{9A}

115. All the four entries of 2 x 2 matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$ are non – zero and one of the eigen values is zero. Which of the following statement is true?

(GATE–08[EC])

(a) $P_{11} P_{22} - P_{12} P_{21} = 1$

(b) $P_{11} P_{22} - P_{12} P_{21} = -1$

(c) $P_{11} P_{22} - P_{21} P_{12} = 0$

(d) $P_{11} P_{22} + P_{12} P_{21} = 0$

116. The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value to 3. The sum of the other two eigen values is

(GATE–08[ME])

(a) p (b) p – 1

(c) p – 2 (d) p – 3

117. The eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ & $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a + b$?

(GATE–08[ME])

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

118. The eigen vector pair of the matrix $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ is **(GATE–08[PI])**

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

119. The inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

(GATE–08[PI])

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

120. How many of the following matrices have an eigen value 1?

$$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}, \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}, \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} \& \begin{bmatrix}-1 & 0\\0 & -1\end{bmatrix}$$

(GATE–08[CS])

(a) one
(b) two
(c) three
(d) four

121 . The product of marices $(PQ)^{-1}$ P is

(GATE–08[CE])

(a) P^{-1}

(b) Q^{-1}

(c) $P^{-1} Q^{-1} P$

(d) $P Q P^{-1}$

122. The eigen values of the matrix

$[P] = \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are (GATE–08[CE])

(a) –7 and 8
(b) –6 and 5
(c) 3 and 4
(d) 1 and 2

123. A square matrix B is symmetric if ___

(GATE–09[CE])

(a) $B^T = -B$

(b) $B^T = B$

(c) $B^{-1} = B$

(d) $B^{-1} = B^T$

124. The eigen values of the following matrix

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix} \text{ are}$$

(GATE–09[EC])

(a) 3, 3+5j, 6-j

(b) -6+5j, 3+j, 3-j

(c) 3+j, 3-j, 5+j

(d) 3, -1+3j, -1-3j

125. The characteristic equation of a 3 × 3matrix P is defined as

$$\alpha(\lambda) = |\lambda I - P| = \lambda^3 + 2\lambda + \lambda^2 + 1 = 0.$$

If I denotes identity matrix then the inverse of P will be

(GATE–08[EE])

(a) $P^2 + P + 2I$ (b) $P^2 + P + I$

(c) $-(P^2 + P + I)$ (d) $-(P^2 + P + 2I)$

126. The eigen values of a 2×2 matrix X are -2 and -3. The eigen values of matrix $(X+I)^{-1}(X+5I)$ are **(GATE–09[IN])**

(a) -3, -4 (b) -1, -2

(c) -1, -3 (d) -2, -4

127. For a matrix $[M]=\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$. The transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by (GATE–09[ME])

(a) $-\frac{4}{5}$

(b) $-\frac{3}{5}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

128. The trace and determinant of a 2×2 matrix are shown to be -2 and -35 respectively. Its eigen values are (GATE–09[EE])

(a) $-30, -5$
(b) $-37, -1$
(c) $-7, 5$
(d) $17.5, -2$

129. The value of the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$ is

(GATE–09[PI])

(a) – 28 (b) – 24

(c) 32 (d) 36

130. An eigen vector of $p = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is

(GATE–10[EE])

(a) $[-1 \;\; 1 \;\; 1]^T$

(b) $[1 \;\; 2 \;\; 1]^T$

(c) $[1 \;\; -1 \;\; 2]^T$

(d) $[2 \;\; 1 \;\; -1]^T$

131. The eigen values of a skew-symmetric matrix are (GATE–10[EC])

(a) always zero

(b) always pure imaginary

(c) either zero (or) pure imaginary

(d) always real

132. One of the eigen vector of the matrix

$A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is (GATE–10[ME])

(a) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

133. A real $n \times n$ matrix $A = [a_{ij}]$ is defined as follows $\begin{cases} a_{ij} = i, & \forall\ i = j \\ \quad = 0, & \text{otherwise} \end{cases}$.

The sum of all n eigen values of A is

(GATE 10[IN])

(a) $\dfrac{n(n+1)}{2}$

(b) $\dfrac{n(n-1)}{2}$

(c) $\dfrac{n(n+1)(2n+1)}{2}$

(d) n^2

134. X and Y are non-zero square matrices of size $n \times n$. If $XY = O_{n \times n}$ then

(GATE–10[IN])

(a) $|X| = 0$ and $|Y| \neq 0$

(b) $|X| \neq 0$ and $|Y| = 0$

(c) $|X| = 0$ and $|Y| = 0$

(d) $|X| \neq 0$ and $|Y| \neq 0$

135. Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$.

If the eigen values of A are 4 and 8 then

(GATE–10[CS])

(a) $x = 4, y = 10$ (b) $x = 5, y = 8$

(c) $x = -3, y = 9$ (d) $x = -4, y = 10$

136. The inverse of the matrix $\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$ is

(GATE–10[CE])

(a) $\frac{1}{2}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(b) $\frac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(c) $\frac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(d) $\frac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

137. If $(1,\ 0,\ -1)^T$ is an eigen vector of the following matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ then the corresponding eigen value is

(GATE–10[PI])

(a) 1
(b) 2
(c) 3
(d) 5

138. The minimum eigenvalue of the following matrix is $\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$

(GATE – 13[EC])

(a) 0 (b) 1

(c) 2 (d) 3

139. The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of lower triangular matrix [L] and an upper triangular [U]. The properly decomposed [L] and [U] matrices respectively are

(GATE–11[EE])

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

140. The matrix $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigen values -3, -3, 5. An eigen vector corresponding to the eigen value 5 is $[1\ 2\ -1]^T$. One of the eigen vector of the matrix M^3 is

(GATE–11[IN])

(a) $[1\ \ 8\ \ -1]^T$
(b) $[1\ \ 2\ \ -1]^T$
(c) $[1\ \ \sqrt[3]{2}\ \ -1]^T$
(d) $[1\ \ 1\ \ -1]^T$

141. The eigen values of the following matrix $\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$ are **(GATE–11[PI])**

(a) 4, 9 (b) 6, –8

(c) 4, 8 (d) –6, 8

142. If a matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$ the transpose of product of these two matrices i.e., $(AB)^T$ is equal to

(GATE-11[PI])

(a) $\begin{bmatrix} 28 & 19 \\ 34 & 47 \end{bmatrix}$ (b) $\begin{bmatrix} 19 & 34 \\ 47 & 28 \end{bmatrix}$

(c) $\begin{bmatrix} 48 & 33 \\ 28 & 19 \end{bmatrix}$ (d) $\begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$

143. Eigen values of a real symmetric matrix are always **(GATE–11[ME])**

(a) positive (b) negative

(c) real (d)162. [A] is a square

144. [A] is a square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and differences of these matrices are defined as

$[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$ respectively. Which of the following statements is true? **(GATE–11[CS])**

(a) Both [S] and [D] are symmetric

(b) Both [S] and [D] are skew-symmetric

(c) [S] is skew-symmetric and [D] is symmetric

(d) [S] is symmetric and [D] is skew symmetric

145. Consider the matrix as given below

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

. Which one of the following options provides the correct values of the eigen values of the matrix? (GATE–11[CS])

(a) 1, 4, 3 (b) 3, 7, 3

(c) 7, 3, 2 (d) 1, 2, 3

146. Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is

(GATE–12[EC, EE, IN])

(a) $15A + 12\,I$ (b) $19\,A + 30\,I$

(c) $17A + 15\,I$ (d) $17A + 21I$

147. For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, ONE of the normalized eigen vectors is given as

(GATE–12[ME, PI])

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$

(b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$

(d) $\begin{pmatrix} \frac{1}{5} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

148. The eigen values of matrix $\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$ are

(GATE–12[CE])

(a) –2.42 and 6.86
(b) 3.48 and 13.53
(c) 4.70 and 6.86
(d) 6.86 and 9.50

149. A matrix has eigen values –1 and –2. The corresponding eigenvectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. The matrix is

(GATE – 13[EE])

(a) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

150. The two vectors [1 1 1] and [1 a a^2]

where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$ and $j = \sqrt{-1}$ are

(GATE–11[EE])

(a) orthonormal
(b) orthogonal
(c) parallel
(d) collinear

151. Let A be an $m \times n$ matrix and B an $n \times m$ matrix. It is given that determinant (I_m+AB) = determinant (I_n+BA), where I_k is the $k \times k$ identity matrix. Using the above property, the determinant of the matrix given below is $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

(GATE – 2013[EC])

(a) 2
(b) 5
(c) 8
(d) 16

152. One pair of eigenvectors corresponding to the two eigen values of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is **(GATE – 2013[IN])**

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

153. The eigen values of a symmetric matrix are all (GATE – 2013[ME])

(a) Complex with non-zero positive imaginary part.

(b) Complex with non-zero negative imaginary part.

(c) real

(d) Pure imaginary

154. What is the minimum number of multiplications involved in computing the matrix product PQR? Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns and matrix R has 4 rows and 1 column ________.

(GATE – 2013[CE])

155. Which one of the following does NOT equal $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$?

(GATE – 2013[CS])

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

156. For matrices of same dimension M, N and scalar c, which one of these properties DOES NOT ALWAYS hold?

(GATE–14–EC–SET1)

(a) $\left(M^{T}\right)^{T} = M$

(b) $(cM)^{T} = c(M)^{T}$

(c) $(M + N)^{T} = M^{T} + N^{T}$

(d) $MN = NM$

157. A real (4×4) matrix A satisfies the equation $A^2 = I$, where I is the (4×4) identity matrix. The positive eigen value of A is ________.

(GATE–14–EC–SET1)

158. Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which is obtained by reversing the order of the columns of the identity matrix I_6. Let $P = I_6 + \alpha J_6$, where α is a non – negative real number. The value of α for which det (P) = 0 is ______.

(GATE–14–EC–SET1)

159. The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is _______.

(GATE–14–EC–SET2)

160. The maximum value of the determinant among all 2×2 real symmetric matrices with trace 14 is ________.

(GATE–14–EC–SET2)

161. Which one of the following statements is NOT true for a square matrix A?

(GATE–14–EC–SET3)

(a) If A is upper triangular, the eigen values of A are the diagonal elements of it

(b) If A is real symmetric, the eigen values of A are always real and positive

(c) If A is real, the eigen values of A and A^T are always the same

(d) If all the principal minors of A are positive, all the eigen values of A are also positive

162. A system matrix is given as follows

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -6 & -11 & 6 \\ -6 & -11 & 5 \end{bmatrix}.$$

The absolute value of the ratio of the maximum eigen value to the minimum eigen value is ______.

(GATE–14–EE–SET1)

163. Which one of the following statements is true for all real symmetric matrices?

(GATE–14–EE–SET2)

(a) All the eigen values are real

(b) All the eigen values are positive

(c) All the eigen values are distinct

(d) Sum of all the eigen values is zero

164. A scalar valued function is defined as $f(x) = x^T Ax + b^T x + c$, where A is a symmetric positive definite matrix with dimension n×n; b and x are vectors of dimension n×1. The minimum value of f(x) will occur when x equals.

(GATE-14–IN–SET1)

(a) $\left(A^T A\right)^{-1} B$

(b) $-\left(A^T A\right)^{-1} B$

(c) $-\left(\frac{A^{-1}B}{2}\right)$

(d) $\frac{A^{-1}B}{2}$

165. For the matrix A satisfying the equation given below, the eigen values are

$$[A]\begin{bmatrix}1 & 2 & 3\\7 & 8 & 9\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$$

(GATE–14–IN–SET1)

(a) (1, -j, j)
(b) (1, 1, 0)
(c) (1, 1, -1)
(d) (1, 0, 0)

166. Given that the determinant of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is -12, the determinant of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is **(GATE-14-ME-SET1)**

(a) -96

(b) -24

(c) 24

(d) 96

167. One of the eigen vectors of the matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is

(GATE–14–ME–SET2)

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$
(b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$
(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$
(d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

168. Consider a 3×3 real symmetric matrix S such that two of its eigen values are $a \neq 0$, $b \neq 0$ with respective eigen vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$. If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals

(GATE–14–ME–SET3)

(a) a (b) b

(c) ab (d) 0

169. Which one of the following equations is a correct identity for arbitrary 3×3 real matrices P,Q and R?

(GATE–14–ME–SET4)

(a) $P(Q+R) = PQ + RP$

(b) $(P-Q)^2 = P^2 - 2PQ + Q^2$

(c) $\det(P+Q) = \det P + \det Q$

(d) $(P+Q)^2 = P^2 + PQ + QP + Q^2$

170. Given the matrices $J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and

$K = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, the product $K^T JK$ is ______.

(GATE–14–CE–SET1)

171. The sum of Eigen values of the matrix, [M]

is where $[M] = \begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}$

(GATE–14–CE–SET1)

(a) 915
(b) 1355
(c) 1640
(d) 2180

172. The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is

(GATE–14–CE–SET2)

173. The value of the dot product of the eigenvectors corresponding to any pair of different eigen values of a 4×4 symmetric positive definite matrix is ______.

(GATE–14–CS– SET1)

174. The product of the non-zero eigen values of

the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$ is ______.

(GATE–14–CS–SET2)

175. Which one of the following statements is TRUE about every n×n matrix with only real eigen values? **(GATE-14-CS-SET3)**

(a) If the trace of the matrix is positive and the determinant is negative, at least one of its eigen values is negative.

(b) If the trace of the matrix is positive, all its eigen values are positive.

(c) If the determinant of the matrix is positive, all its eigen values are positive.

(d) If the product of the trace and determinant of the matrix is positive, all its eigen values are positive.

176. The value of 'P' such that the vector $\begin{bmatrix}1\\2\\3\end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix}4 & 1 & 2\\P & 2 & 1\\14 & -4 & 10\end{bmatrix}$ is ——. **(GATE–15–EC–SET1)**

177. The value of 'x' for which all the eigen-values of the matrix given below are real is

$$\begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

(GATE–15–EC– SET2)

(a) $5 + j$ (b) $5 - j$

(c) $1 - 5j$ (d) $1 + 5j$

178. For $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

(GATE–15–EC–SET3)

(a) $\sec^2 x$ (b) $\cos 4x$

(c) 1 (d) 0

179. If the sum of the diagonal elements of a 2×2 matrix is -6, then the maximum possible value of determinant of the matrix is ________.

(GATE–15–EE– SET1)

180. The necessary condition to diagonalize a matrix is that (GATE – 01[IN])

(a) all its eigen values should be distinct

(b) its eigen vectors should be independent

(c) its eigen values should be real

(d) the matrix is non-singular

181. The smallest and largest Eigen values of the following matrix are: $\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}$

(GATE – 15 – CE – Set 1)

(a) 1.5 and 2.5 (b) 0.5 and 2.5

(c) 1.0 and 3.0 (d) 1.0 and 2.0

182. The two Eigen Values of the matrix $\begin{bmatrix} 2 & 1 \\ 1 & p \end{bmatrix}$ have a ratio of 3:1 for p = 2. What is another value of 'p' for which the Eigen values have the same ratio of 3:1?

(GATE – 15 – CE – Set 2)

(a) –2 (b) 1

(c) 7/3 (d) 14/3

183. If any two columns of a determinant

$$P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$$ are interchanged, which one of the following statements regarding the value of the determinant is CORRECT?

(GATE – 15 – ME – Set 1)

(a) Absolute value remains unchanged but sign will change.

(b) Both absolute value and sign will change.

(c) Absolute value will change but sign will not change.

(d) Both absolute value and sign will remain unchanged.

184. At least one eigenvalue of a singular matrix is (GATE – 15 – ME – Set 2)

(a) positive (b) zero

(c) negative (d) imaginary

185. The lowest eigen value of the 2×2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is ____. **(GATE – 15 – ME – Set 3)**

186. For a given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$,

where $i = \sqrt{-1}$, the inverse of matrix P is

(GATE – 15 – ME – Set 3)

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

187. Consider the following 2×2 matrix A where two elements are unknown and are marked by a and b. The eigenvalues of this matrix are −1 and 7. What are the values of a and b?

$$A = \begin{pmatrix} 1 & 4 \\ b & a \end{pmatrix}$$ **(GATE – 15 – CS – Set 1)**

(a) $a = 6, b = 4$ (b) $a = 4, b = 6$

(c) $a = 3, b = 5$ (d) $a = 5, b = 3$

188. The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is ______.

(GATE – 15 – CS – Set 2)

189. Perform the following operations on the maxtrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

(i) Add the third row to the second row

(ii) Subtract the third column from the first column.

The determinant of the resultant matrix is _______. (GATE – 15 – CS – Set 2)

190. In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigen value 1 are

(GATE – 15 – CS – Set 3)

(a) $\{\alpha(4, 2, 1) \mid \alpha \neq 0, \alpha \in R\}$

(b) $\{\alpha(-4, 2, 1) \mid \alpha \neq 0, \alpha \in R\}$

(c) $\{\alpha(\sqrt{2}, 0, 1) \mid \alpha \neq 0, \alpha \in R\}$

(d) $\{\alpha(-\sqrt{2}, 0, 1) \mid \alpha \neq 0, \alpha \in R\}$

191. In matrix equation $[A]\{X\} = \{R\}$.

$$[A] = \begin{bmatrix} 4 & 8 & 4 \\ 8 & 16 & -4 \\ 4 & -4 & 15 \end{bmatrix}, \{X\} = \begin{Bmatrix} 2 \\ 1 \\ 4 \end{Bmatrix} \text{ and } \{R\} = \begin{Bmatrix} 32 \\ 16 \\ 64 \end{Bmatrix}$$

One of the eigen values of matrix [A] is

(a) 16 (b) 15 **(GATE-19-ME)**

(c) 4 (d) 8

192. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix.

The determinant of B is _________ (up to 1 decimal place)

(GATE-18-EC)

193. Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix}1\\4\end{bmatrix}$

Consider the following statements.

(I) P does not have an inverse

(II) P has a repeated eigenvalue

(III) P cannot be diagonalized

Which one of the following options is correct?

(a) Only I and III are necessarily true

(b) Only II is necessarily true

(c) Only I and II are necessarily true

(d) Only II and III are necessarily true

(GATE-18-CSIT)

194. Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^{n} \sum_{j=1}^{n} {A_{ij}}^2 = 50$.

Consider the following statements.

(I) One eigenvalue must be in [-5, 5]

(II) The eigenvalue with the largest magnitude must be strictly greater than 5

Which of the above statements about eigenvalues of A is/are necessarily ***correct***?

(GATE-17-CSIT)

(a) Both (I) and (II) (b) (I) only

(c) (II) only (d) Neither (I) nor (II)

195. The matrix $A = \begin{bmatrix} \frac{3}{2} & 0 & \frac{1}{2} \\ 0 & -1 & 0 \\ \frac{1}{2} & 0 & \frac{3}{2} \end{bmatrix}$ has three distinct eigen values and one of its eigen vectors is $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Which one of the following can be another eigen vector of A?

(GATE-17-EE)

(a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

196. If the entries in each column of a square matrix M add up to 1, then an eigen value of M is

(a) 4 (b) 3

(c) 2 (d) 1

(GATE-16-CE)

197. Among the following, the pair of vectors orthogonal to each other is

(GATE – 95[ME])

(a) [3, 4, 7], [3, 4, 7]

(b) [1, 0, 0], [1, 1, 0]

(c) [1, 0, 2], [0, 5, 0]

(d) [1, 1, 1], [−1, −1, −1]

198. Let $M^4 = I$ (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(GATE – 16 – EC – Set 1)

(a) M^{4k+1} (b) M^{4k+2}

(c) M^{4k+3} (d) M^{4k}

199. Let $A_{n \times n}$ be matrix of order n and I_{12} be the matrix obtained by interchanging the first and second rows of I_n. Then A I_{12} is such that its first. (GATE – 97[CS])

(a) row is the same as its second row

(b) row is the same as second row of A

(c) column is the same as the second column of A

(d) row is a zero row.

200. If the vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then one of the eigen value of A is **(GATE – 98[EE])**

(a) 1
(b) 2
(c) 5
(d) –1

201. If $A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{bmatrix}$ the matrix A^4, calculated by the use of Cayley – Hamilton theorem **(GATE – 93)**

202. A 5×7 matrix has all its entries equal to 1.

Then the rank of a matrix is

(GATE – 94[EE])

(a) 7 (b) 5

(c) 1 (d) Zero

203. The rank of $(m \times n)$ matrix $(m < n)$ cannot be more than

(GATE – 94[EC])

(a) m (b) n

(c) mn (d) None

204. The rank of the matrix $\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is

(GATE – 94[CS])

(a) 0
(b) 1
(c) 2
(d) 3

205. If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called

(GATE – 94[PI])

(a) Non-singular (b) singular

(c) transpose (d) minor

206. Rank of the matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is 3

(GATE – 94[ME])

(a) True (b) False

207. The rank of the following (n+1) x (n+1) matrix, where 'a' is a real number is

$$\begin{bmatrix} 1 & a & a^2 & . & . & . & a^n \\ 1 & a & a^2 & . & . & . & a^n \\ . & & & & & & \\ . & & & & & & \\ 1 & a & a^2 & . & . & . & a^n \end{bmatrix}$$

(GATE – 95[EE])

(a) 1 (b) 2

(c) n (d) depends on the value of a

208. The rank of the matrix $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$ is

(GATE – 98[CS])

(a) 3 (b) 1

(c) 2 (d) 4

209. Consider the following two statements.

(GATE–2000[CE])

(I) The maximum number of linearly independent column vectors of a matrix A is called the rank of A.

(II) If A is $n x n$ square matrix then it will be non-singular if rank of A = n

(a) Both the statements are false

(b) Both the statements are true

(c) (I) is true but (II) is false

(d) (I) is false but (II) is true

210. The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is

(GATE–2000[IN])

(a) 0
(b) 1
(c) 2
(d) 3

211. The rank of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ is

(GATE – 02[CS])

(a) 4 (b) 2 (c) 1 (d) 0

212. Given matrix $[A] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$, the rank of the matrix is **(GATE – 03[CE])**

(a) 4
(b) 3
(c) 2
(d) 1

213. Let $A = [a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i.j$. Then the rank of A is

(GATE–07[IN])

(a) 0 (b) 1

(c) $n - 1$ (d) n

214. $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$; $B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$

If the rank of matrix A is N, then the rank of matrix B is

(GATE–14–EE–SET3)

(a) N/2 (b) N – 1

(c) N (d) 2 N

215. The following system of equations

$$x_1 + x_2 + x_3 = 3$$

$$x_1 - x_3 = 0$$

$$x_1 - x_2 + x_3 = 1 \text{ has}$$

(GATE – 94[EC]

(a) Unique solution

(b) No solution

(c) Infinite number of solutions

(d) Only one solution

216. Let $AX = B$ be a system of linear equations where A is an $m \times n$ matrix B is an $n \times 1$ column matrix which of the following is false?

(GATE – 96[CS])

(a) The system has a solution, if $\rho(A) = \rho(A/B)$

(b) If $m = n$ and B is a non – zero vector then the system has a unique solution.

(c) If $m < n$ and B is a zero vector then the system has infinitely many solutions.

(d) The system will have a trivial solution when $m = n$, B is the zero vector and rank of A is n.

217. In the Gauss – elimination for a solving system of linear algebraic equations, triangularization leads to

(GATE – 96[ME])

(a) diagonal matrix

(b) lower triangular matrix

(c) upper triangular matrix

(d) singular matrix

218. For the following set of simultaneous equations (GATE – 97[ME])

$$1.5x - 0.5y + z = 2$$

$$4x + 2y + 3z = 9$$

$$7x + y + 5z = 10$$

(a) the solution is unique

(b) infinitely many solutions exist

(c) the equations are incompatible

(d) finite many solutions exist

219. Consider the following set of equations

$x + 2y = 5,$

$4x + 8y = 12,$

$3x + 6y + 3z = 15.$ This set

(GATE - 98[CS])

(a) has unique solution

(b) has no solution

(c) has infinite number of solutions

(d) has 3 solutions

220. Consider the following system of linear equations (GATE – 03[CS])

$$\begin{bmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$

Notice that the 2^{nd} and 3^{rd} columns of the coefficient matrix are linearly dependent.

For how many value of α, does systems of equations have infinitely many solutions.

(a) 0
(b) 1
(c) 2
(d) infinitely many

221. A system of equations represented by $AX = 0$ where X is a column vector of unknown and A is a square matrix containing coefficients has a non-trivial solution when A is. **(GATE – 03)**

(a) non-singular (b) singular

(c) symmetric (d) Hermitian

222. What values of x, y, z satisfy the following system of linear equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$$

(GATE – 04)

(a) $x = 6, y = 3, z = 2$

(b) $x = 12, y = 3, z = -4$

(c) $x = 6, y = 6, z = -4$

(d) $x = 12, y = -3, z = 4$

223. How many solutions does the following system of linear equations have

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$

(GATE–04[CS])

(a) infinitely many

(b) two distinct solutions

(c) unique

(d) none

224. Consider the following system of equations in three real variable x_1, x_2 and x_3:

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + 5x_3 = 2$$
$$-x_1 + 4x_2 + x_3 = 3$$

This system of equations has

(GATE–05[CE])

(a) no solution

(b) a unique solution

(c) more than one but a finite number of solutions.

(d) an infinite number of solutions.

225. Consider the system of equations,

$A_{nxn} X_{nx1} = \lambda X_{nx1}$ where λ is a scalar.

Let (λ_i, X_i) be an eigen value and its corresponding eigen vector for real matrix A. Let I_{nxn} be unit matrix. Which one of the following statement is not correct?

(GATE–05[CE])

(a) For a homogeneous nxn system of linear equations $(A-\lambda I)X = 0$, having a non trivial solution, the rank of $(A-\lambda I)$ is less than n.

(b) For matrix A^m, m being a positive integer, (λ_i^m, X_i^m) will be eigen pair for all i.

(c) If $A^T = A^{-1}$ then $|\lambda_i| = 1$ for all i.

(d) If $A^T = A$ then λ_i are real for all i.

226. The number of linearly independent solutions of the system of equations

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \text{ is equal to}$$

(GATE – 94[EE])

(a) 1 (b) 2

(c) 3 (d) 0

227. In the matrix equation PX= Q which of the following is a necessary condition for the existence of atleast one solution for the unknown vector X.

(GATE–05[EE])

(a) Augmented matrix [P|Q] must have the same rank as matrix P.

(b) vector Q must have only non-zero elements.

(c) matrix P must be singular

(d) matrix P must be square

228. A is a 3×4 matrix and $AX = B$ is an inconsistent system of equations. The highest possible rank of A is

(GATE–05[ME])

(a) 1 (b) 2 (c) 3 (d) 4

229. Let A be 3×3 matrix with rank 2.

Then $AX = O$ has

(GATE – 05[IN])

(a) only the trivial solution $X = O$

(b) one independent solution

(c) two independent solutions

(d) three independent solutions

230. A system of linear simultaneous equations is given as Ax = b

where $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ & $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Then the rank of matrix A is

(GATE–06[IN])

(a) 1 (b) 2 (c) 3 (d) 4

231. A system of linear simultaneous equations is given as $Ax = b$

Where $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Which of the following statement is true?

(GATE–06[IN])

(a) x is a null vector

(b) x is unique

(c) x does not exist

(d) x has infinitely many values

232. Solution for the system defined by the set of equations $4y + 3z = 8$, $2x - z = 2$ & $3x + 2y = 5$ is (GATE–06[CE])

(a) $x = 0,\ y = 1,\ z = 4/5$

(b) $x = 0,\ y = 1/2,\ z = 2$

(c) $x = 1,\ y = 1/2,\ z = 2$

(d) non existent

233. Let A be an $n \times n$ real matrix such that $A^2 = I$ and Y be an n-dimensional vector. Then the linear system of equations $Ax = Y$ has

(GATE–07[IN])

(a) no solution

(b) unique solution

(c) more than one but infinitely many dependent solutions.

(d) infinitely many dependent solutions

234. For what values of α and β the following simultaneous equations have an infinite number of solutions

$$x + y + z = 5,$$

$$x + 3y + 3z = 9,$$

$$x + 2y + \alpha z = \beta$$

(GATE–07[CE])

(a) 2, 7

(b) 3, 8

(c) 8, 3

(d) 7, 2

235. The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

(GATE–07[ME])

(a) 0
(b) 1
(c) 2
(d) infinite

236. If A is square symmetric real valued matrix of dimension 2n, then the eigen values of A are

(GATE – 07[PI])

(a) 2n distinct real values

(b) 2n real values not necessarily distinct

(c) n distinct pairs of complex conjugate numbers

(d) n pairs of complex conjugate numbers, not necessarily distinct

237. $q_1, q_2, q_3, \ldots\ldots q_m$ are n-dimensional vectors with $m < n$. This set of vectors is linearly dependent. Q is the matrix with $q_1, q_2, q_3, \ldots\ldots q_m$ as the columns. The rank of Q is

(GATE–07[EE])

(a) less than m (b) m

(c) between m and n (d) n

238. $X = [\, x_1 \;\; x_2 \; \ldots\ldots. \; x_n]^T$ is an n-tuple non-zero vector. The $n \times n$ matrix $V = XX^T$

(GATE–07[CE])

(a) has rank zero

(b) has rank 1

(c) is orthogonal

(d) has rank n

239. Let x and y be two vectors in a 3–dimensional space and $<x, y>$ denote their dot product. Then the determinant det $\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix}$ = ______.

(GATE–07[EE])

(a) is zero when x and y are linearly independent

(b) is positive when x and y are linearly independent

(c) is non-zero for all non-zero x and y

(d) is zero only when either x (or) y is zero

240. If the rank of a 5x6 matrix Q is 4 then which one of the following statements is correct?

(GATE–08[EE])

(a) Q will have four linearly independent rows and four linearly independent columns

(b) Q will have four linearly independent rows and five linearly independent columns

(c) QQ^T will be invertible.

(d) Q^TQ will be invertible.

241. A is m x n full rank matrix with m > n and I is an identity matrix.

Let matrix $A^{+} = (A^{T}A)^{-1}A^{T}$. Then which one of the following statement is false?

(GATE–08[EE])

(a) $AA^{+}A = A$ (b) $(AA^{+})^{2} = AA^{+}$

(c) $A^{+}A = I$ (d) $AA^{+}A = A^{+}$

242. The system of linear equations

$$\left.\begin{array}{l}4x + 2y = 7\\ 2x + y = 6\end{array}\right\}\text{has}$$

(GATE–08[EC])

(a) a unique solution

(b) no solution

(c) an infinite no. of solutions

(d) exactly two distinct solution.

243. For what values of 'a' if any will the following system of equations in x, y and z have a solution?

$$2x + 3y = 4,$$
$$x + y + z = 4,$$
$$x + 2y - z = a$$

(GATE–08[ME])

(a) any real number

(b) 0

(c) 1

(d) there is no such value

244. The following system of equations

$x_1 + x_2 + 2x_3 = 1,\ x_1 + 2x_2 + 3x_3 = 2,$

$x_1 + 4x_2 + \alpha x_3 = 4$ has a unique solution.

The only possible value(s) for α is/are

(GATE–08[CS])

(a) 0 (b) either 0 (or) 1

(c) one of 0, 1 (or) –1 (d) any real value expect 5

245. The following system of equations

$$x + y + z = 3,$$
$$x + 2y + 3z = 4,$$
$$x + 4y + kz = 6$$

will not have a unique solution for k equal to (GATE–08[CE])

(a) 0 (b) 5

(c) 6 (d) 7

246. The value of x_3 obtained by solving the following system of linear equations is

$$x_1 + 2x_2 - 2x_3 = 4$$

$$2x_1 + x_2 + x_3 = -2$$

$$-x_1 + x_2 - x_3 = 2$$

(GATE–09[PI])

(a) –12 (b) –2 (c) 0 (d) 12

247. For the set of equations

$$x_1 + 2x_2 + x_3 + 4x_4 = 2,$$

$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6.$$

The following statement is true

(GATE–10[EE])

(a) only the trivial solution

$x_1 = x_2 = x_3 = x_4 = 0$ exist

(b) there are no solutions

(c) a unique non-trivial solution exist

(d) multiple non-trivial solution exist

248. The value of q for which the following set of linear equations $2x + 3y = 0$, $6x + qy = 0$ can have non-trivial solution is

(GATE–10[PI])

(a) 2 (b) 7

(c) 9 (d) 11

249. The system of equations $x + y + z = 6$, $x + 4y + 6z = 20$, $x + 4y + \lambda z = \mu$ has no solution for values of λ and μ given by

(GATE–11[EC])

(a) $\lambda = 6, \mu = 20$

(b) $\lambda = 6, \mu \neq 20$

(c) $\lambda \neq 6, \mu = 20$

(d) $\lambda \neq 6, \mu \neq 20$

250. Consider the following system of equations

$2x_1 + x_2 + x_3 = 0$, $x_2 - x_3 = 0$ and $x_1 + x_2 = 0$.

This system has **(GATE–11[ME])**

(a) a unique solution

(b) no solution

(c) infinite number of solutions

(d) five solutions

251. $x + 2y + z = 4, 2x + y + 2z = 5, x - y + z = 1$

The system of algebraic equations given above has

(GATE–12[ME, PI])

(a) a unique solution of $x=1$, $y=1$ and $z=1$

(b) only the two solutions of $x=1$, $y=1$, $z=1$ and $x=2$, $y=1$, $z=0$

(c) infinite number of solutions.

(d) no feasible solution.

252. The equation

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ has}$$

(GATE – 13[EE])

(a) no solution

(b) only one solution

(c) non-zero unique solution

(d) multiple solutions

253. Choose the CORRECT set of functions, which are linearly dependent.

(GATE – 2013[ME])

(a) $\sin x$, $\sin^2 x$ and $\cos^2 x$

(b) $\cos x$, $\sin x$ and $\tan x$

(c) $\cos 2x$, $\sin^2 x$ and $\cos^2 x$

(d) $\cos 2x$, $\sin x$ and $\cos x$

254. The system of linear equations

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix}$$ has

(GATE–14–EC–SET2)

(a) a unique solution

(b) infinitely many solutions

(c) no solution

(d) exactly two solutions

255. Which one of the following statements is NOT true for a square matrix A?

(GATE–14–EC–SET3)

(a) If A is upper triangular, the eigen values of A are the diagonal elements of it

(b) If A is real symmetric, the eigen values of A are always real and positive

(c) If A is real, the eigen values of A and A^T are always the same

(d) If all the principal minors of A are positive, all the eigen values of A are also positive

256. Given a system of equations

$$x + 2y + 2z = b_1$$

$$5x + y + 3z = b_2$$

Which of the following is true its solutions

(GATE–14–EE–SET1)

(a) The system has a unique solution for any given b_1 and b_2

(b) The system will have infinitely many solutions for any given b_1 and b_2

(c) Whether or not a solution exists depends on the given b_1 and b_2

(d) The system would have no solution for any values of b_1 and b_2

257. Which one of the following describes the relationship among the three vectors, $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+4\hat{k}$?

(GATE–14–ME–SET1)

(a) The vectors are mutually perpendicular

(b) The vectors are linearly dependent

(c) The vectors are linearly independent

(d) The vectors are unit vectors

258. The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is

(GATE–14–CE–SET2)

259. The system of equations, given below, has

$$x + 2y + 4z = 2$$
$$4x + 3y + z = 5$$
$$3x + 2y + 3z = 1$$

(GATE–14–PI–SET1)

(a) a unique solution

(b) Two solution

(c) no solution

(d) more than two solutions

260. Consider the following system of equations:

$$3x + 2y = 1$$

$$4x + 7z = 1$$

$$x + y + z = 3$$

$$x - 2y + 7z = 0$$

The number of solutions for this system is

(GATE–14–CS–SET1)

261. Consider the system of linear equations:

$$x - 2y + 3z = -1$$

$$x - 3y + 4z = 1 \text{ and}$$

$$-2x + 4y - 6z = k,$$

The value of 'k' for which the system has infinitely many solutions is _______.

(GATE–15–EC–SET1)

262. The maximum value of 'a' such that the matrix $\begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$ has three linearly independent real eigenvectors is

(GATE 15–EE–SET1)

(a) $\frac{2}{3\sqrt{3}}$ (b) $\frac{1}{3\sqrt{3}}$

(c) $\frac{1+2\sqrt{3}}{3\sqrt{3}}$ (d) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

263. We have a set of 3 linear equations in 3 unknowns. 'X ≡ Y' means X and Y are equivalent statements and 'X ≢ Y' means X and Y are not equivalent statements.

P: There is a unique solution.

Q: The equations are linearly independent.

R: All eigen values of the coefficient matrix are non zero.

S: The determinant of the coefficient matrix is nonzero.

Which one of the following is TRUE?

(GATE–15–EE–SET2)

(a) $P \equiv Q \equiv R \equiv S$ (b) $P \equiv R \not\equiv Q \equiv S$

(c) $P \equiv Q \not\equiv R \equiv S$ (d) $P \not\equiv Q \not\equiv R \not\equiv S$

264. Let A be an $n \times n$ matrix with rank $r (0 < r < n)$. Then $AX = 0$ has p independent solutions, where p is

(GATE – 15 – IN)

(a) r
(b) n
(c) $n - r$
(d) $n + r$

265. For what value of 'p' the following set of equations will have no solution?

$2x + 3y = 5$

$3x + py = 10$ (GATE – 15 – CE – Set 1)

266. Let $A = [a_{ij}]$, $1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i.j.$ The rank of A is :

(GATE – 15 – CE – Set 2)

(a) 0 (b) 1

(c) n – 1 (d) n

267. If the following system has non – trivial solution

$$px + qy + rz = 0$$
$$qx + ry + pz = 0$$
$$rx + py + qz = 0$$

Then which one of the following Options is TRUE? (GATE – 15 – CS – Set 3)

(a) $p - q + r = 0$ or $p = q = -r$

(b) $p + q - r = 0$ or $p = -q = r$

(c) $p + q + r = 0$ or $p = q = r$

(d) $p - q + r = 0$ or $p = -q = -r$

268. Let the eigen values of a 2×2 matrix A be 1, -2 with eigen vectors x_1 and x_2 respectively. Then the eigen values and eigen vectors of the matrix $A^2 - 3A + 4I$ would respectively, be

(GATE-16-EE)

(a) 2, 14; x_1, x_2

(b) 2, 14; x_1, x_2: $x_1 - x_2$

(c) 2, 0; x_1, x_2

(d) 2, 0; $x_1 + x_2$, $x_1 - x_2$

269. Let A be a 4×3 real matrix which rank 2. Which one of the following statement is **TRUE**?

(a) Rank of A^T is less than 2 **(GATE-16-EE)**

(b) Rank of A^TA is equal to 2

(c) Rank of A^TA is greater than 2

(d) Rank of A^TA can be any number between 1 and 3

270. The solution to the system of equations is

$$\begin{bmatrix} 2 & 5 \\ -4 & 3 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 2 \\ -30 \end{Bmatrix}$$

(GATE-16-ME)

(a) 6, 2
(b) -6, 2
(c) -6, -2
(d) 6, -2

271. The number of linear independent eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

____________.

(GATE-16-ME)

272. The number of solutions of the simultaneous algebraic equations $y = 3x + 3$ and $y = 3x + 5$ is

(a) zero (b) 1

(c) 2 (d) infinite **(GATE-16-PI)**

273. Consider the following linear system

$$\boldsymbol{x + 2y - 3z} = \mathrm{a}$$
$$2x + 3y + 3z = b$$
$$5x + 9y - 6z = c$$

This system is consistent if a, b, c satisfies the equation

(GATE-16-CE)

(a) $7\mathrm{a} - \mathrm{b} - \mathrm{c} = 0$ (b) $3\mathrm{a} + \mathrm{b} - \mathrm{c} = 0$

(c) $3\mathrm{a} - \mathrm{b} + \mathrm{c} = 0$ (d) $7\mathrm{a} - \mathrm{b} + \mathrm{c} = 0$

274. Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution

II. If $m > n$, then none of these systems has a solution

III. If $m = n$, then there exists a system which has a solution

Which one of the following is **CORRECT**?

(a) **I**, **II** and **III** are true

(b) Only **II** and **III** are true

(c) Only **III** is true

(d) None of them is true

(GATE-16-CSE)

275. If V is a non-zero vector of dimension 3×1, then the matrix $A = VV^T$ has a rank = ________

(GATE-17-IN)

276. The rank of the matrix $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$ is __________

(GATE-17-EC)

277. The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is

(GATE-17-EC)

(a) 0 (b) 1

(c) 2 (d) 3

278. Consider the following simultaneous equations (with c_1 and c_2 being constants):

$$3x_1 + 2x_2 = c_1$$
$$4x_1 + x_2 = c_2$$

The characteristic equation for these simultaneous equations is

(GATE-17-CE)

(a) $\lambda^2 - 4\lambda - 5 = 0$ (b) $\lambda^2 - 4\lambda + 5 = 0$

(c) $\lambda^2 + 4\lambda - 5 = 0$ (d) $\lambda^2 + 4\lambda + 5 = 0$

279. The rank of the following matrix is $\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}$

(a) 1

(b) 2

(c) 3

(d) 4

280. Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. The number of distinct real values of k for which the equation $Ax = 0$ has infinitely many solutions is __________.

(GATE-18-EC)

281. Let M be a real 4×4 matrix. Consider the following statements:

S_1: M has 4 linearly independent eigenvectors

S_2: M has 4 distinct eigen values.

S_3: M is non-singular (invertible).

Which one among the following is TRUE?

(a) S_1 implies S_2 (b) S_1 implies S_3 **(GATE-18-EC)**

(c) S_2 implies S_1 (d) S_3 implies S_2

282. Consider the following system of linear equation

$$3x + 2ky = -2$$
$$kx + 6y = 2$$

Here x and y are the unknowns and k is a real constant. The value of k for which there are infinite number of solutions is

(a) 3 (b) 1

(b) -3 (d) -6 **(GATE-18-IN)**

283. The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

(GATE-18-ME)

(a) 1 (b) 2

(c) 3 (d) 4

284. The rank of the matrix, $M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, is ________.

(GATE-19-EE)

285. The set of equations $x + y + z = 1$, $ax - ay + 3z = 5$, $5x - 3y + az = 6$ has infinite solutions, if a =

(GATE-19-ME)

(a) 4 (b) 3

(c) –4 (d) -3

286. Euclidean norm (length) of the vector $[4 \quad -2 \quad -6]^T$ is

(GATE-19-CE)

(a) $\sqrt{56}$ (b) $\sqrt{24}$

(c) $\sqrt{48}$ (d) $\sqrt{12}$

287. Consider the system of equations

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 2 & -3 \\ 4 & 4 & -6 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

The value of x_3 (round off to the nearest integer), is______________.

(GATE-2020(CE)

288. Let A and B be two $n \times n$ matrices over real numbers. Let rank(M) and det(M) denote the rank and determinant of a matrix M, respectively. Consider the following statements:

1. $\text{rank}(AB) = \text{rank}(A)\,\text{rank}(B)$

II. $\det(AB) = \det(A)\det(B)$

III. $\text{rank}(A + B) \le \text{rank}(A) + \text{rank}(B)$

IV. $\det(A + B) \le \det(A) + \det(B)$

Which of the above statements are TRUE?

(GATE-2020 (CS)

(a) III and IV only (b) II and III only

(c) I and IV only (d) I and II only

289. The rank of the matrix $\begin{bmatrix} 1 & 2 & 2 & 3 \\ 3 & 4 & 2 & 5 \\ 5 & 6 & 2 & 7 \\ 7 & 8 & 2 & 9 \end{bmatrix}$ is

(GATE-21-CE)

(a) 4 (b) 2 (c) 1 (d) 3

290. The rank of the matrix $\begin{bmatrix} 5 & 0 & -5 & 0 \\ 0 & 2 & 0 & 1 \\ -5 & 0 & 5 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ is

(GATE-21-CE)

(a) 1 (b) 2 (c) 3 (d) 4

291. Consider an $n \times n$ matrix A and a non-zero $n \times 1$ vector p. Their product $Ap = \alpha^2 p$, where $\alpha \in \Re$ and $\alpha \notin \{-1, 0, 1\}$. Based on the given information, the eigen value of A^2 is:

(GATE-21-ME)

(a) α (b) α^2 (c) $\sqrt{\alpha}$ (d) α^4

292. Suppose that P is a 4×5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[\ 2\ 5\ 4\ 3\ 1\ ]^T$. The rank of P is

(GATE-2021-cs)

293. Consider the rows vectors $v = (1, 0)$ and $w = (2, 0)$. The rank of the matrix $M = 2v^T v + 3w^T w$, where the superscript T denotes the transpose, is

(GATE – 2021 – IN)

(a) 3 (b) 2

(c) 4 (d) 1

294. Let $c_1, \ldots, c_n$ be scalars, not all zero, such that $\sum_{i=1}^{n} c_i a_i = 0$ where a_i are column vectors in R^n. Consider the set of linear equations $Ax = b$

Where $A = [a_1, \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

(GATE-17-CSIT)

(a) A unique solution at $x = j_n$ where j_n denotes a n-dimensional vector of all 1

(b) no solution

(c) infinitely many solutions

(d) finitely many solutions

295. P and Q are two square matrices of the same order. Which of the following statement(s) is/are correct?

(GATE-2022-CE)

(a) If P and Q are invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$
(b) If P and Q are invertible, then $[QP]^{-1} = P^{-1}Q^{-1}$
(c) If P and Q are invertible, then $[PQ]^{-1} = P^{-1}Q^{-1}$
(d) If P and Q are not invertible, then $[PQ]^{-1} = Q^{-1}P^{-1}$

296. The matrix M is defined as

$$M = \left[\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}\right]$$

and has eigenvalues 5 and 2. The matrix Q is formed as

$Q = M^3 - 4M^2 - 2M$

Which of the following is/are the eigenvalue(s) of matrix Q

(GATE-2022-CE)

(a) 15 (b) 25

(c) -20 (d) -30

297. Consider the following two statements with respect to the matrices $A_{m\times n}, B_{n\times m}, C_{n\times n}, D_{n\times n}$

Statement 1: $tr(AB) = tr(BA)$

Statement 2: $tr(CD) = tr(DC)$

where tr() represents the trace of a matrix. Which one of the following holds?

(GATE-2022-CSE)

(a) Statement 1 is correct and Statement 2 is wrong.

(b) Statement 1 is wrong and Statement 2 is correct.

(c) Both Statement 1 and Statement 2 are correct.

(d) Both Statement 1 and Statement 2 are wrong.

298. Consider solving the following system of simultaneous equations using LU decomposition.

$x_1 + x_2 - 2x_2 = 4$

$x_2 + 3x_2 - x_2 = 7$

$2x_1 + x_2 - 5x_2 = 7$

Where L and U are denoted as

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}, \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & L_{33} \end{bmatrix}$$

Which one of the following is the correct combination of values for L_{32}, U_{33}, and x_1?

(GATE-2022-CSE)

(a) $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$

(b) $L_{32} = 2, U_{33} = 2, x_1 = -1$

(c) $L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$

(d) $L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$

299. Which of the following is/are the eigenvector(s) for the matrix given below?

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix}$$

(GATE-2022-CSE)

(a) $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$

300. Consider a system of linear equations Ax=b, where

$$A = \begin{bmatrix} 1-\sqrt{2} & 3 \\ -1 & \sqrt{2}-3 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

This system of equations admits__________

(GATE-2022-ECE)

(a) a unique solution for x
(b) infinitely many solutions for x
(c) no solutions for x
(d) exactly two solutions for x

301. Consider a matrix 3×3 A whose $(i, j)^{th}$ element $= (i - j)^3$, then the matrix A will be

(GATE-2022-EEE)

(a) Symmetric (b) Skew symmetric

(c) Unitary (d) Null

302. Consider matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & 1 & 1 \end{bmatrix}$, the matrix A is satisfy the equation $6A^{-1} = A^2 + cA + dI$ where c and d are scalars and I is the identity matrix, the (c + d) is equal to

(GATE-2022-EEE)

(a) 5
(b) 17
(c) −6
(d) 11

303. e^A denotes the exponential of a square matrix A. Suppose λ is an eigen value and v is the corresponding eigen vector of matrix A.

Consider the following two statement

Statement 1: e^ is an eigen value of e^a

Statement 2: v is an eigen vector of e^a.

Which one of the following option is correct.?

(GATE-2022-EEE)

(a) Statement 1 is true and Statement 2 is false
(b) Statement 1 is false and Statement 2 is true
(c) Both the statements are correct
(d) Both statements are false

304. Given $M = \begin{bmatrix} 2 & 3 & 7 \\ 6 & 4 & 7 \\ 4 & 6 & 14 \end{bmatrix}$, which of the following statement(s) is/are correct?

(GATE-2022-IN)

(a) The rank of M is 2
(b)The rank of M is 3
(c) The rows of M are linearly independent
(d) The determinant of M is 0

305. The matrix $A = \begin{bmatrix} 4 & 3 \\ 9 & -2 \end{bmatrix}$ has eigen values - 5 and 7.

The eigenvector(s) is/are____________

(GATE-2022-IN)

(a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 13 \\ 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$

306. If $A = \begin{bmatrix} 10 & 2k+5 \\ 3k-5 & k+5 \end{bmatrix}$ is a symmetric matrix, the value of k is________________

(GATE-2022-ME)

(a) 8

(b) 10

(c) −0.4

(d) $\frac{1+\sqrt{1561}}{12}$

307. The system of linear equations in real (x, y) given by

$$(x\ y)\begin{bmatrix} 2 & 5-2\alpha \\ \alpha & 1 \end{bmatrix} = (0\ 0)$$

involves a real parameter α and has infinitely many non-trivial solutions for special value(s) of α. Which one or more among the following options is/are non-trivial solution(s) of (x, y) for such special value(s) of α?

(GATE-2022-ME)

(a) $x = 2, y = -2$ (b) $x = -1, y = 4$

(c) $x = 1, y = 1$ (d) $x = 4, y = -2$

308. A is a 3×5 real matrix of rank 2. For the set of homogeneous equations $Ax = 0$, where 0 is a zero vector and x is a vector of unknown variables, which of the following is/are true?

(GATE-2022-ME)

(a) The given set of equations will have a unique solution.

(b) The given set of equations will be satisfied by a zero vector of appropriate size.

(c) The given set of equations will have infinitely many solutions.

(d) The given set of equations will have many but a finite number of solutions.

309. If the sum and product of eigen values of a 2×2 matrix $\begin{bmatrix} 3 & p \\ p & q \end{bmatrix}$ are 4 and -1 respectively, then $|p|$ is__________ (in integer).

(GATE-2022-ME)

310. Matrix A as product of two other matrices is given by

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \end{bmatrix}$$

The value of det(A) is______________ [round off to nearest integer]

(GATE-2022-PI)

311. If a matrix is squared, then

(GATE-2022-PI)

(a) both eigenvalues and eigenvectors are retained.
(b) eigenvalues get squared but eigenvectors are retained.
(c) both eigenvalues and eigenvectors must change.
(d) eigenvalues are retained but eigenvectors change.