

Stack Permutation and Advance Topics

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Data Structure: Doubts & Stack

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Hello!

I am Vishvadeep Gothi

I am here because I love to teach

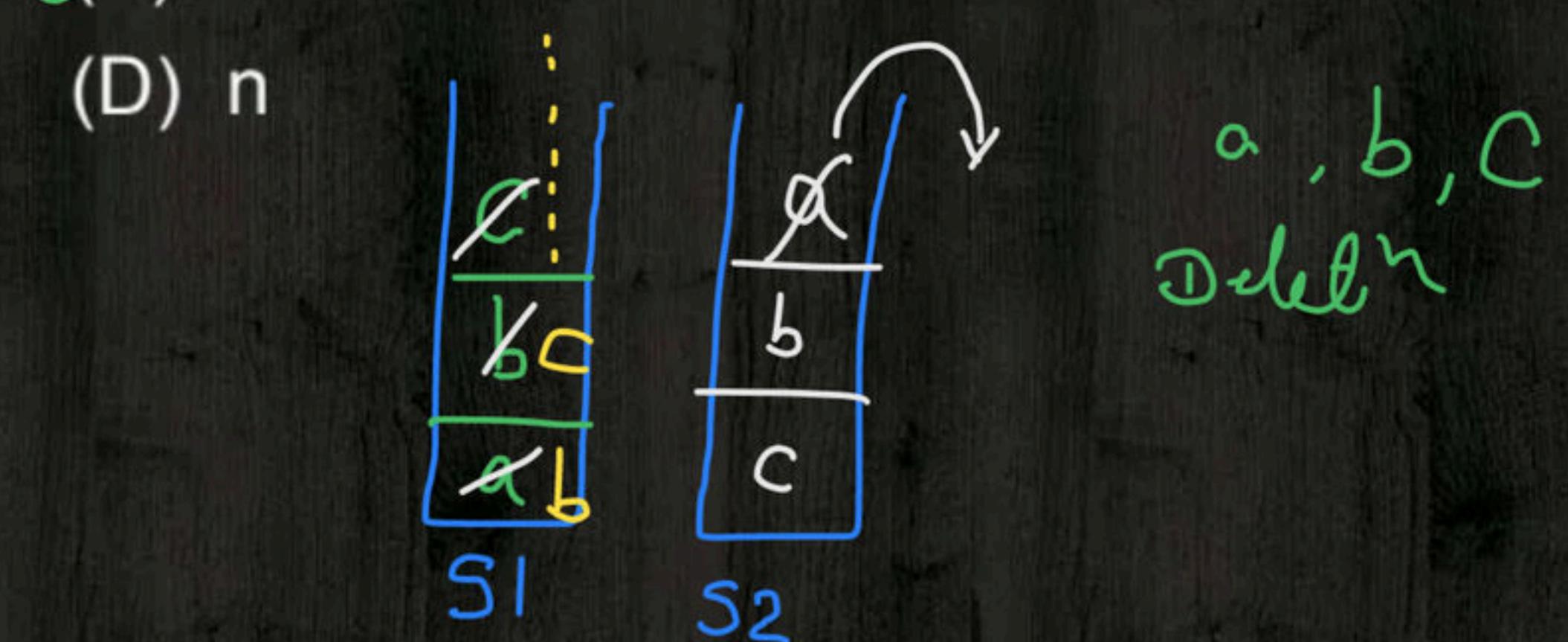
Question GATE-2001

What is the minimum number of stacks of size n required to implement a queue of size n ?

- (A) One
- (B) Two
- (C) Three
- (D) n

I have \Rightarrow Stack (LIFO)

I want \Rightarrow Queue (FIFO)



Implementation of Queue Using Stack

Best method :-

Insertⁿ (Q)

{

PUSH (S₁)

}

Deleteⁿ ()

{

1. if S₂ is not empty then
 pop from S₂

2. if S₂ is empty then
 bring all elements from S₁ to S₂
 and then pop from S₂

}

Implementation of Queue Using Stack

① Insert (a)

Insert (b)

Insert (c)

Delete ()

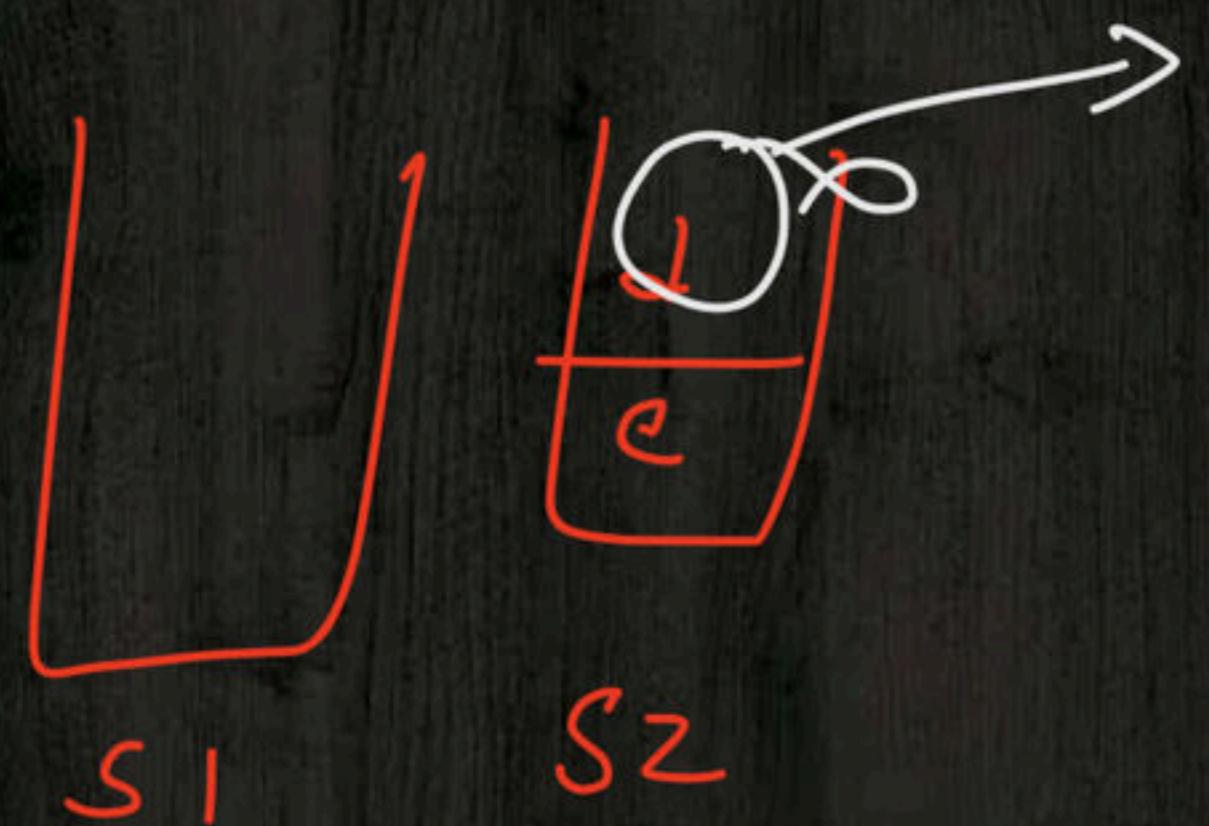
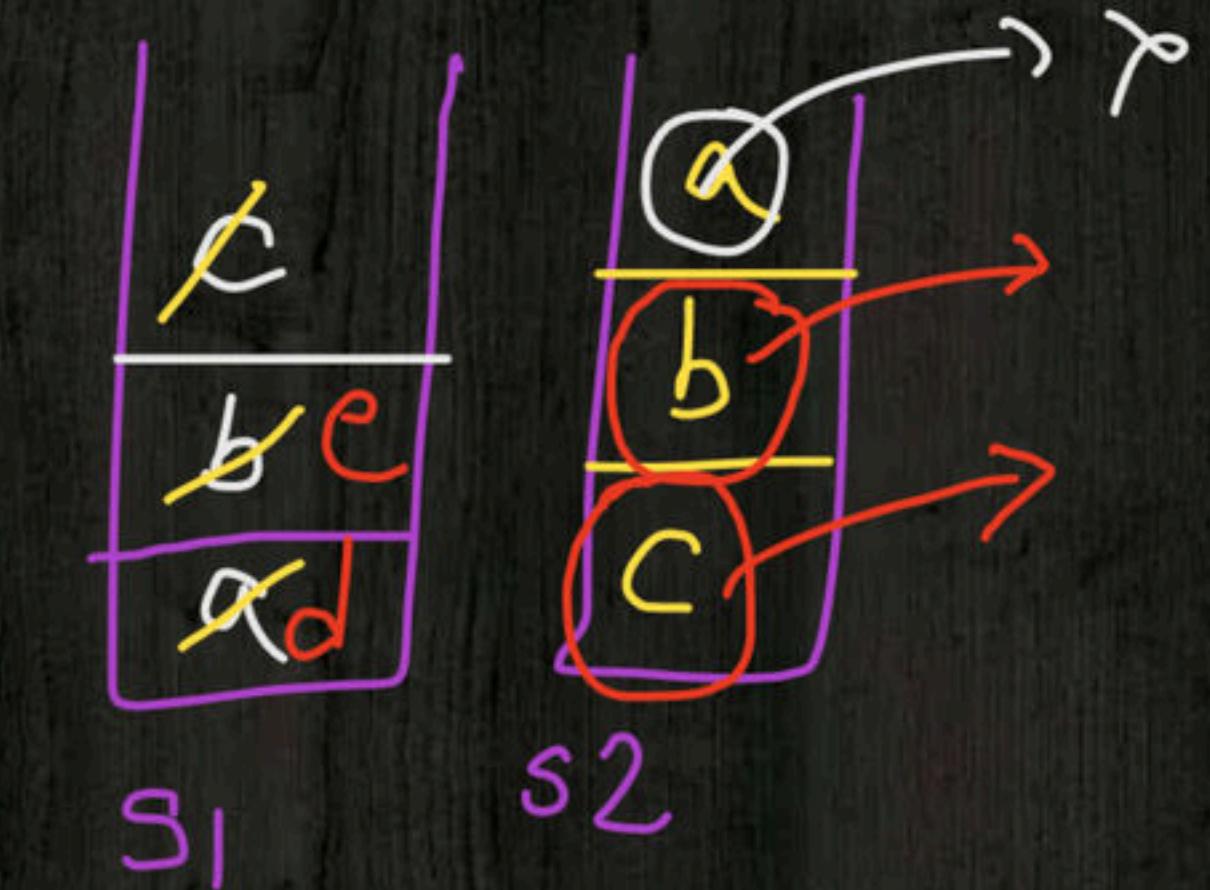
insert (d)

insert (e)

Delete ()

Delete ()

Delete ()



X X | X | X | X | e

Question GATE-2006

An implementation of a queue Q , using two stacks $S1$ and $S2$, is given below:

```
void insert (Q, x) {
    push (S1, x);
}
void delete (Q) {
    if (stack-empty(S2)) then
        if (stack-empty(S1)) then (
            print("Q is empty");
            return;
        )
        else while (! (stack-empty(S1))) {
            x=pop(S1);
            push(S2,x);
        }
    x=pop(S2);
}
```

$m \Rightarrow \text{insert}^n$

$n \Rightarrow \text{delet}^m$

PUSH $\Rightarrow x$

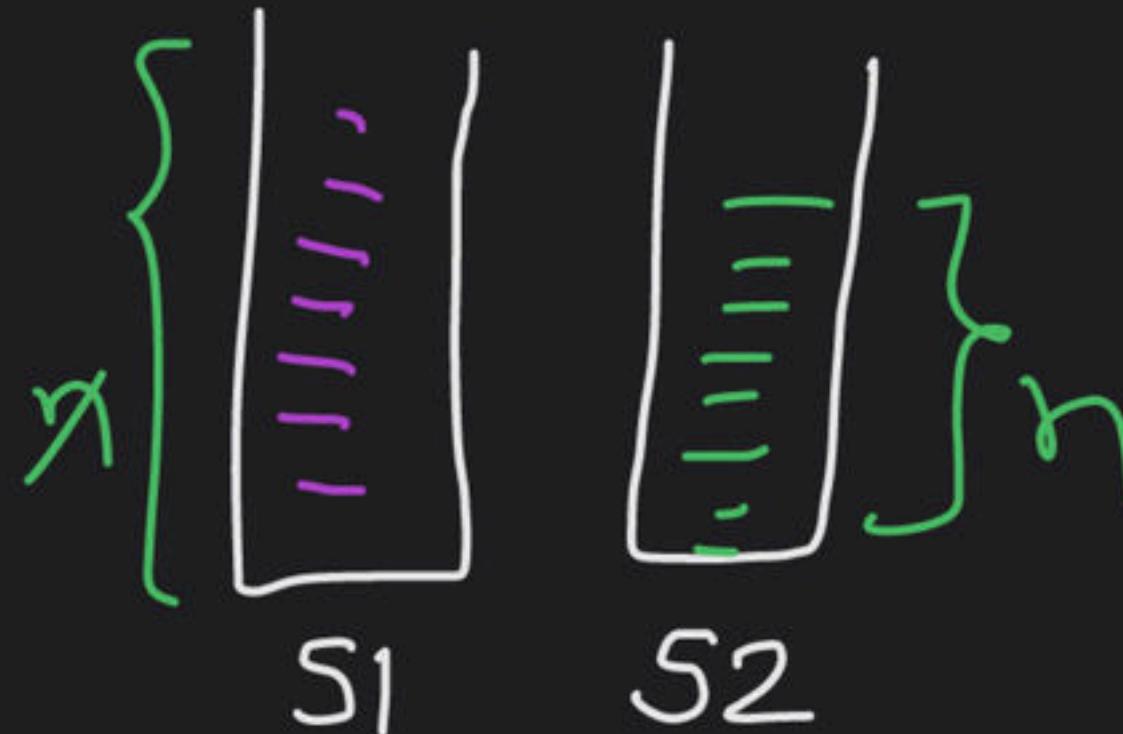
POP $\Rightarrow y$

Let n insert and $m (\leq n)$ delete operations be performed in an arbitrary order on an empty queue Q . Let x and y be the number of push and pop operations performed respectively in the process. Which one of the following is true for all m and n ?

- A. ~~$n+m \leq x < 2n$ and $2m \leq y \leq n+m$~~
- B. $n+m \leq x < 2n$ and $2m \leq y \leq 2n$
- C. $2m \leq x < 2n$ and $2m \leq y \leq n+m$
- D. $2m \leq x < 2n$ and $2m \leq y \leq 2n$

worst case :-

Insert all n elements then delete m elements



insertn :-

$$\text{PUSH} = n \checkmark$$

deletion :-

$$\text{POP} = n$$

$$\text{PUSH} = n \checkmark$$

$$\text{POP} = m$$

Total

$$\text{PUSH} = 2n$$

$$\text{POP} = n+m$$

Best case :-

Insert m elements first, delete all of them;
then insert remaining $n-m$ elements



$$\text{Total PUSH} = n + m$$
$$\text{POP} = 2^m$$

Insertion :-

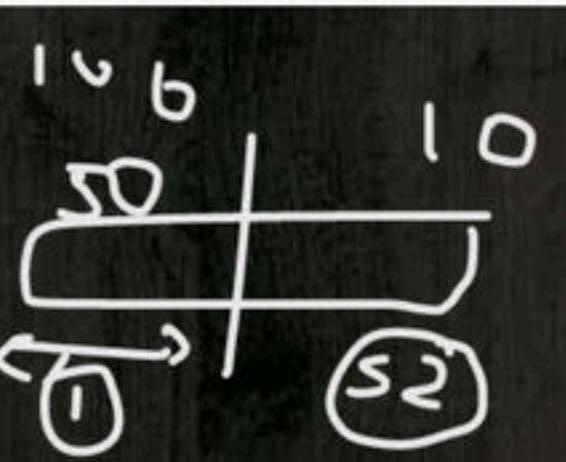
$$\text{PUSH} = m + n - m = n$$

Deletion :-

$$\text{POP} = \underline{m}$$

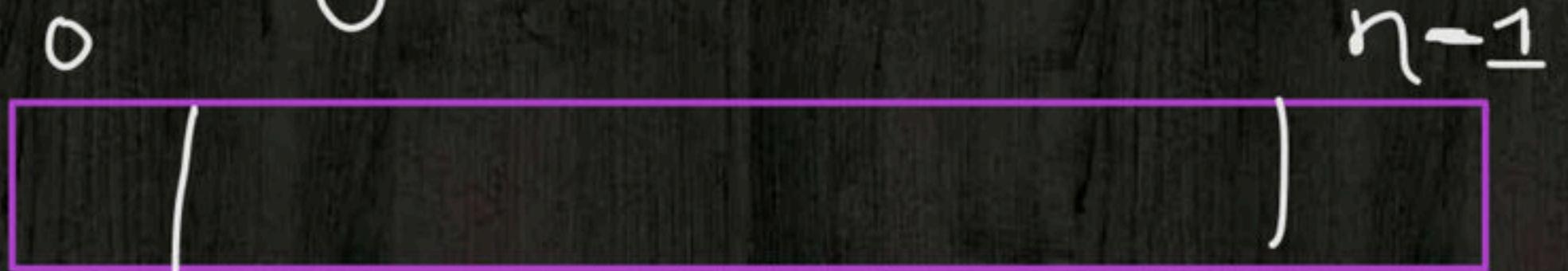
$$\text{PUSH} = m$$

$$\text{POP} = \underline{m}$$



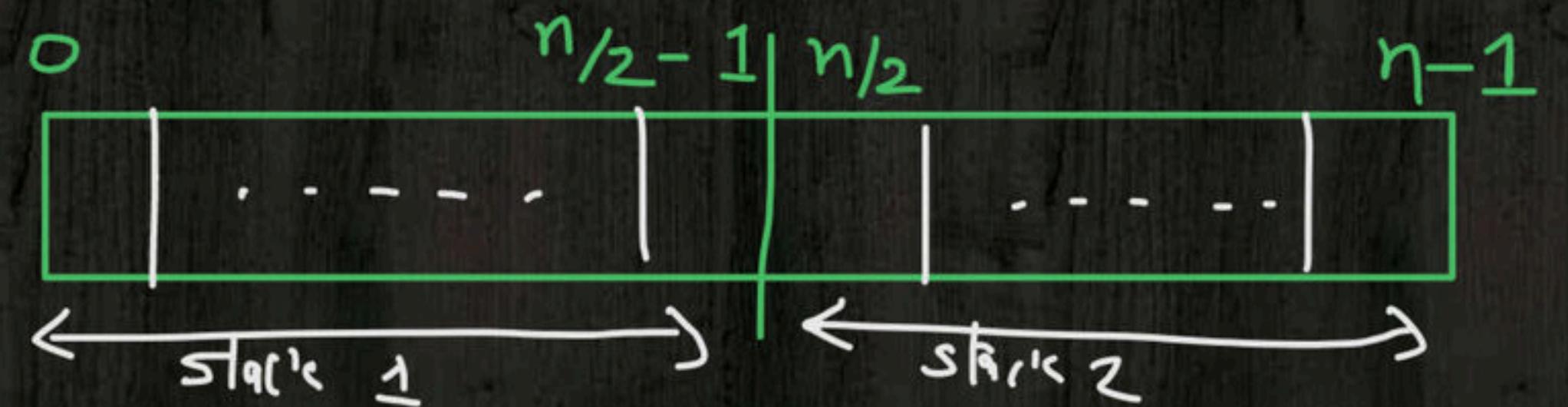
Multiple Stacks in Single Array

array



Method 1 :- \rightarrow space utilization of array is not proper

Divide array in 2 equal halves
& implement one stack in each half



Stack 1 :-

$$\text{top } 1 = -1$$

$$\text{max top } 1 = \frac{n}{2} - 1$$

Stack 2 :-

$$\text{top } 2 = \frac{n}{2} - 1$$

$$\text{max top } 2 = n - 1$$

Multiple Stacks in Single Array

$top_1 = -1$

↓

0



→ stack 1

Method 2:-

both stacks grows from 2 diff ends.

and can perform insert

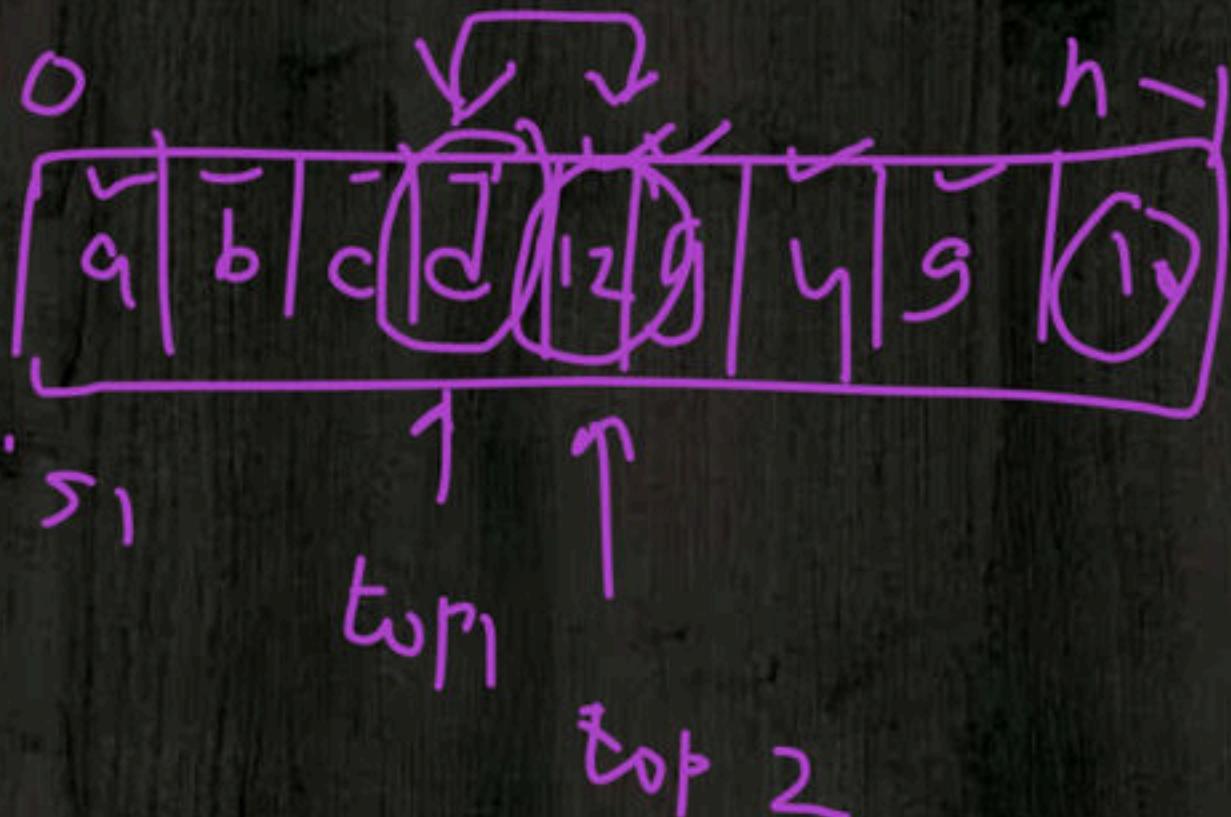
until there is a free space available

$top_2 = n$

↓

n-1

←
stack 2



overflow condtn :-

$$-\text{top } 1 = -\text{top } 2 - 1$$

or

$$\text{top } 2 = \text{top } 1 + 1$$

underflow condtn :-

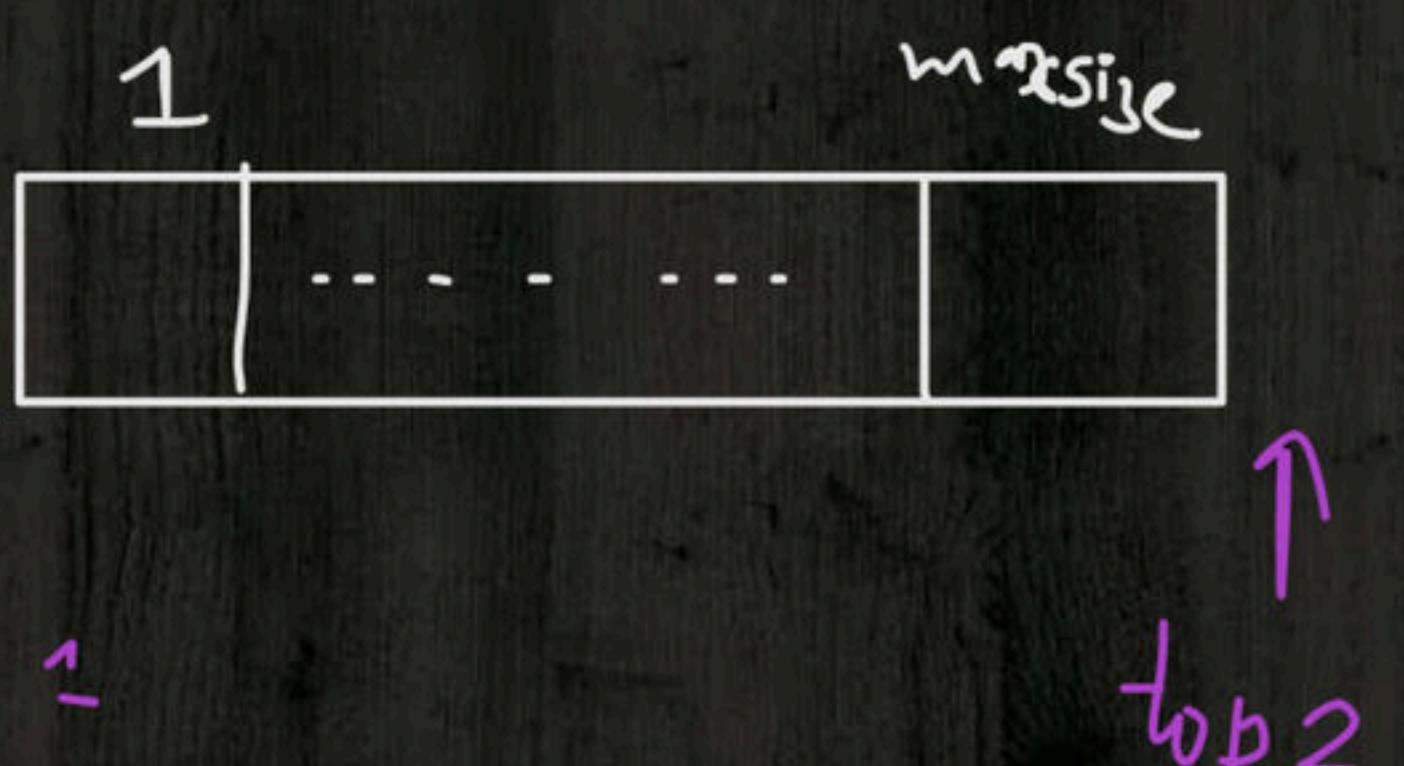
$$\text{top } 1 = -1$$

$$\text{top } 2 = n$$

Question GATE-2004

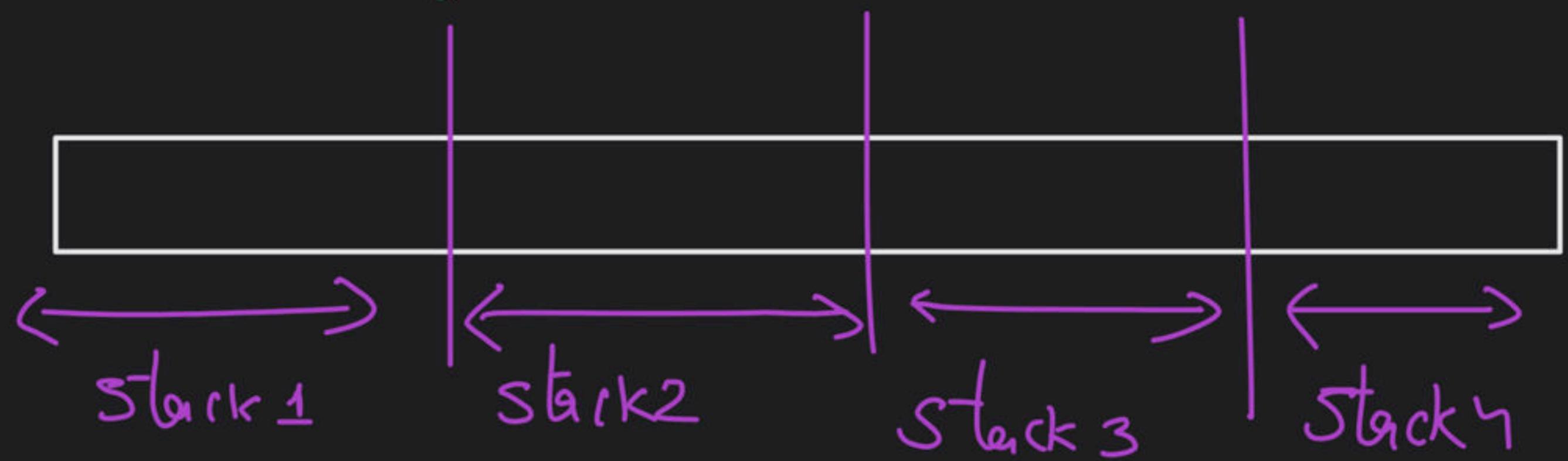
A single array $A[1..MAXSIZE]$ is used to implement two stacks. The two stacks grow from opposite ends of the array. Variables $top1$ and $top2$ ($top1 < top2$) point to the location of the topmost element in each of the stacks. If the space is to be used efficiently, the condition for “stack full” is

- (A) $(top1 = MAXSIZE/2)$ and $(top2 = MAXSIZE/2 + 1)$
- (B) $top1 + top2 = MAXSIZE$
- (C) $(top1 = MAXSIZE/2)$ or $(top2 = MAXSIZE)$
- (D) ~~$top1 = top2 - 1$~~



if more than 2 stacks implemented in single array

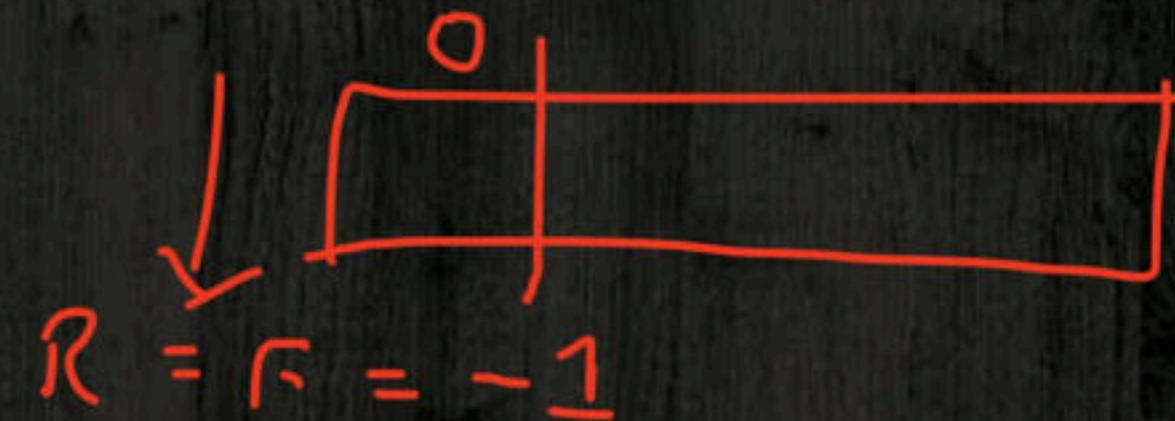
so \Rightarrow give equal space to each stack.



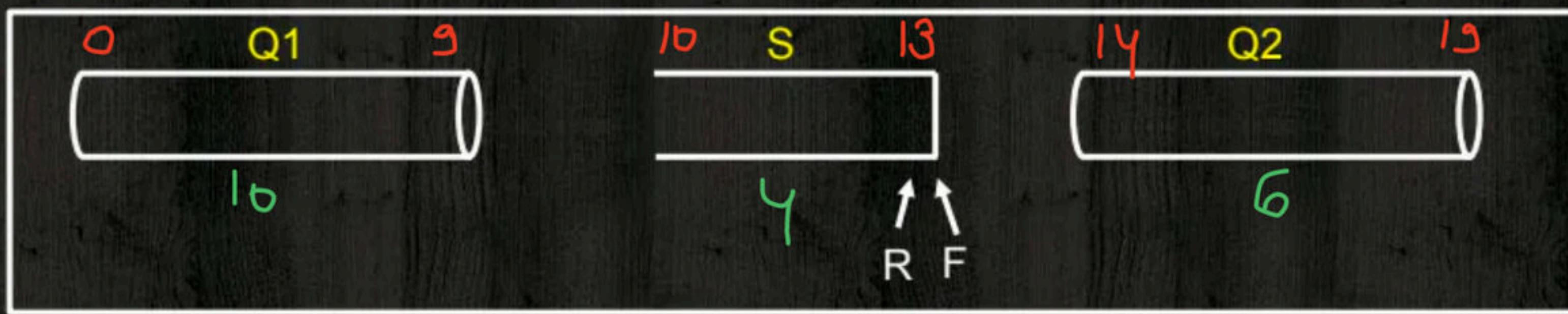


DPP

Question 1



Two queues Q1, Q2 and a stack S are implemented using a single array of size 20. Array has indexes 0 to 19 and has a base address $(504)_{10}$. The stack S and Queue Q2 are initially empty. The Q1, Q2 and S can be visualized as follows:



$$\text{Size of } Q1 = 10, Q2 = 6$$

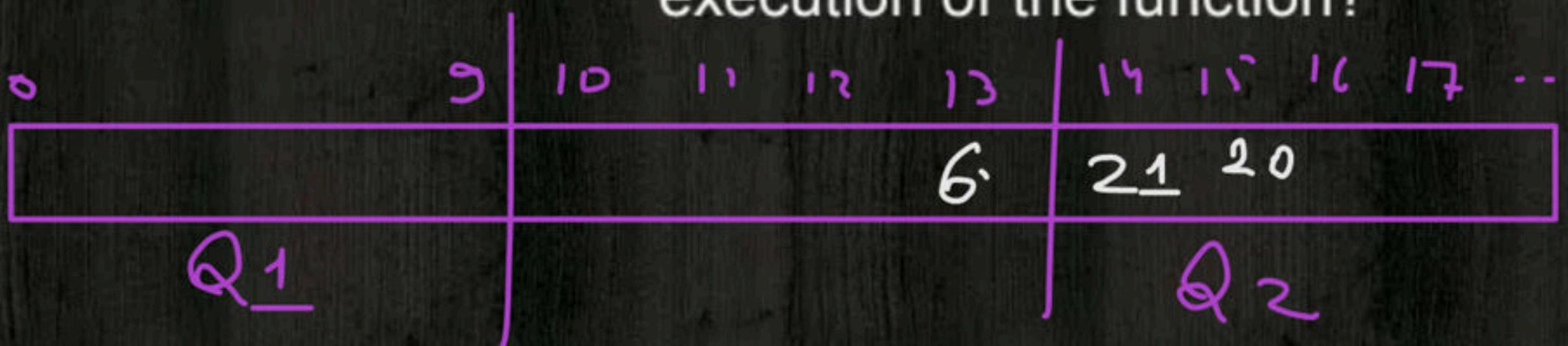
The queue Q contains values: 5,9,3,4,0,7,9,4,0,6 from front to rear in that order. The array is of integers where each element takes 2 locations in memory. Consider the following program fragment executed on this array (on Q1, Q2 and S).

Question 1 cont..

$$Q_1 \Rightarrow \cancel{5}, \cancel{9}, \cancel{3}, \cancel{1}, \cancel{0}, \cancel{7}, \cancel{2}, \cancel{4}, \cancel{1}, \cancel{6}$$

Void doSomething()

What are the addresses of elements 21 and 6 after execution of the function?



$$\begin{aligned}x &= 49446 \\y &= 497 \\z &= 441320\end{aligned}$$

$$y = 137$$

3 = ~~14~~ 20

$$\text{loc}(A[14]) = 504 + 2 \times 14 = 532$$

for 21

$$\text{loc}(A[13]) = 504 + 2 \times 13 = 530$$

for 6

Question 2

Let S be a stack of size $n \geq 1$. Starting with the empty stack, suppose we push the first n natural numbers in sequence, and then perform n pop operations. Assume that Push and Pop operations take X seconds each, and Y seconds elapse between the end of one such stack operation and the start of the next operation. For $m \geq 1$, define the stack-life of m as the time elapsed from the end of $\text{Push}(m)$ to the start of the pop operation that removes m from S . The average stack-life of an element of this stack is

- A. $n(X + Y)$
- B. $3Y + 2X$
- C. $n(X + Y) - X$
- D. $Y + 2X$

Question 3

Let Q denote a queue containing sixteen numbers and S be an empty stack. $\text{Head}(Q)$ returns the element at the head of the queue Q without removing it from Q . Similarly $\text{Top}(S)$ returns the element at the top of S without removing it from S . Consider the algorithm given below.

```
while Q is not Empty do
    if S is Empty OR Top(S) ≤ Head (Q) then
        x:= Dequeue (Q);
        Push (S, x);
    else
        x:= Pop(S);
        Enqueue (Q, x);
    end
end
```

The maximum possible number of iterations of the **while** loop in the algorithm is _____.

Question 4

Consider an array of size n to implement m number of stacks (numbered 0 to $m-1$).
The empty stack has following specifications:

for $0 \leq i < m$

$$\text{top}[i] = \text{bottom}[i] = i * \left\lfloor \frac{n}{m} \right\rfloor$$

for $i=m$

$$\text{bottom}[i] = n-1$$

$\frac{n}{m} \Rightarrow$ integer

Where bottom is constant index and top is variable (moving) index.

Question 4 continue

Fill in the blank:

1.

```
void push (int i, char item)
{
    if(____)
    {
        printf("Overflow");
    }
    else
    {
        stack[++top [i]] = item;
    }
}
```

Question 4 continue

Fill in the blank:

2.

```
void pop(int i)
{
    if(____)
    {
        printf("Underflow");
    }
    else
    {
        stack[top[i]]--;
    }
}
```

Happy Learning