DP - Part VIII

Complete Course on Algorithm for GATE - CS & IT

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```
R-QS(a, P, r)
{
    r = Randomgenerated(a, P, r)
    swap(a[P], a[r])
    m = Partition(a, P, r)
        R-QS(a, P, m-1)
        R-QS(a, m+1, r)
            return(a)
}
```

BC, AC

$O(n \log n)$

WC

$O(n^2)$

10_a 10_b 10_c 10_d 10_e 10_f 10_g

$(10_a\ 10_b - - \cdots\ 10_f)\ 10_d\ ()$

$\Vert$

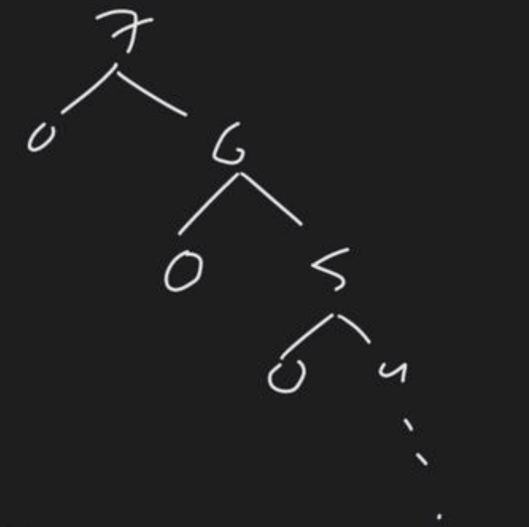
$(\quad\quad)\ 10_g\ ()$

$O(n^2)$

10 20 30 40 50 60 70

( )20( )

QS ⇒ n^2



R-QS

(10 20 30) 40 (50 60 70)

1 2 3 4 5 6 7

|| ||

( )10(20 30) (50)60(70)

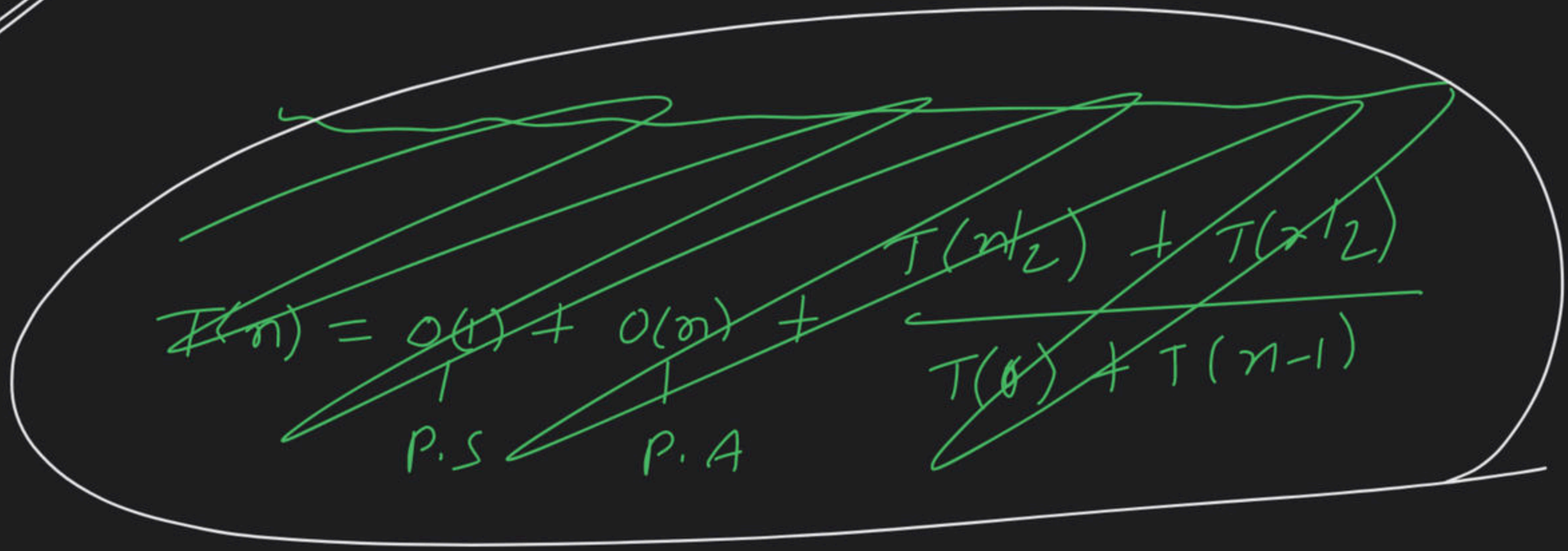
||

( )20(30)

O(nlogn)

In Quicksort the sorting of n-elements the n/5th smallest element selected as pivot using O(n) time complexities algo. Then what will be the WC-TC of QS?

(a) $\theta(n \log n)$ (b) $\theta(n^2)$ (c) $\theta(n)$ (d) $\theta(n^3)$



$$T(n) = \underset{\text{P.S}}{n} + \underset{\text{P.A}}{n} + T(n/5 - 1) + T\left(\frac{4n}{5}\right)$$

$$= 2n + T(n/5) + T\left(\frac{4n}{5}\right)$$

$$= n + T(n/5) + T(4n/5)$$

$$= \Theta(n \log n)$$

In QS the sorting of n-numbers the n/10th ele selected as pivot. If $\Theta(n^2)$ time algo. Find what is the WC-TC of QS?

(a) $\Theta(n^2)$ (b) $\Theta(n \log n)$ (c) $\Theta(n^3)$ (d) $\Theta(n)$

$$T(n) = \underbrace{n^2 + n}_{\text{P.S}\quad\text{P.A}} + T(0) + T(n-1)$$

$$= n^2 + T(n-1)$$

$$= 1^2 + 2^2 + 3^2 + \cdots + n^2 \Longrightarrow \frac{n(n+1)(2n+1)}{6} \Longrightarrow \underline{\Theta(n^3)}$$

In QS the sorting of n numbers the n/5 th largest element selected as pivot which $\Theta(\log n)$ TC algo. Find BC-TC of QS?

(a) $\Theta(n \log n)$ (b) $\Theta(n^2)$ (c) $\Theta(n^3)$ (d) $\Theta(n)$

$$T(n) = \underbrace{\log n + n}_{\text{P.S}\quad\text{P.A}} + T(n - n/5) + T(n/5 - 1)$$

$$= n + T(4n/5) + T(n/5)$$

$$= \Theta(n \log n) \;//$$

In QS if the sorted n-numbered 300 largest elements selected as pivot using $\Theta(n^2)$ TC algo. Find BC-TC of QS?

$$T(n) = \underbrace{n^2 + n} + T(n-300) + T(299)$$

$$= n^2 + T(n-300)$$

$$= n^2 \cancel{+} \frac{n}{300} \Longrightarrow \Theta(n^3)$$

$n/2$ - element (middle element) $\Longrightarrow$ WC- n^2

$n/2$ - smallest element (median) $\Longrightarrow$ WC- $n \log n$

Selection - procedure

~~(Selection Sort)~~

SP(1, 11)

i/p: An array of n-ele, integer - K

o/p: Find K^{th} smallest element in array.

ex

A	80	50	30	100	150	180	130	90	75	55	60
	1	2	3	4	5	6	7	8	9	10	11

K, 8

⇓ partition(n) ⟹ m

m = 6

m = (50 30 75 55 60) 80 (100 150 180 130 90)

1 2 3 4 5 6 7 8 9 10 11

⇓ SP(7, 11)

Part

m = 8

m = (90) 100 (150 180 130)

7 8 9 10 11

11

SP(9, 11)

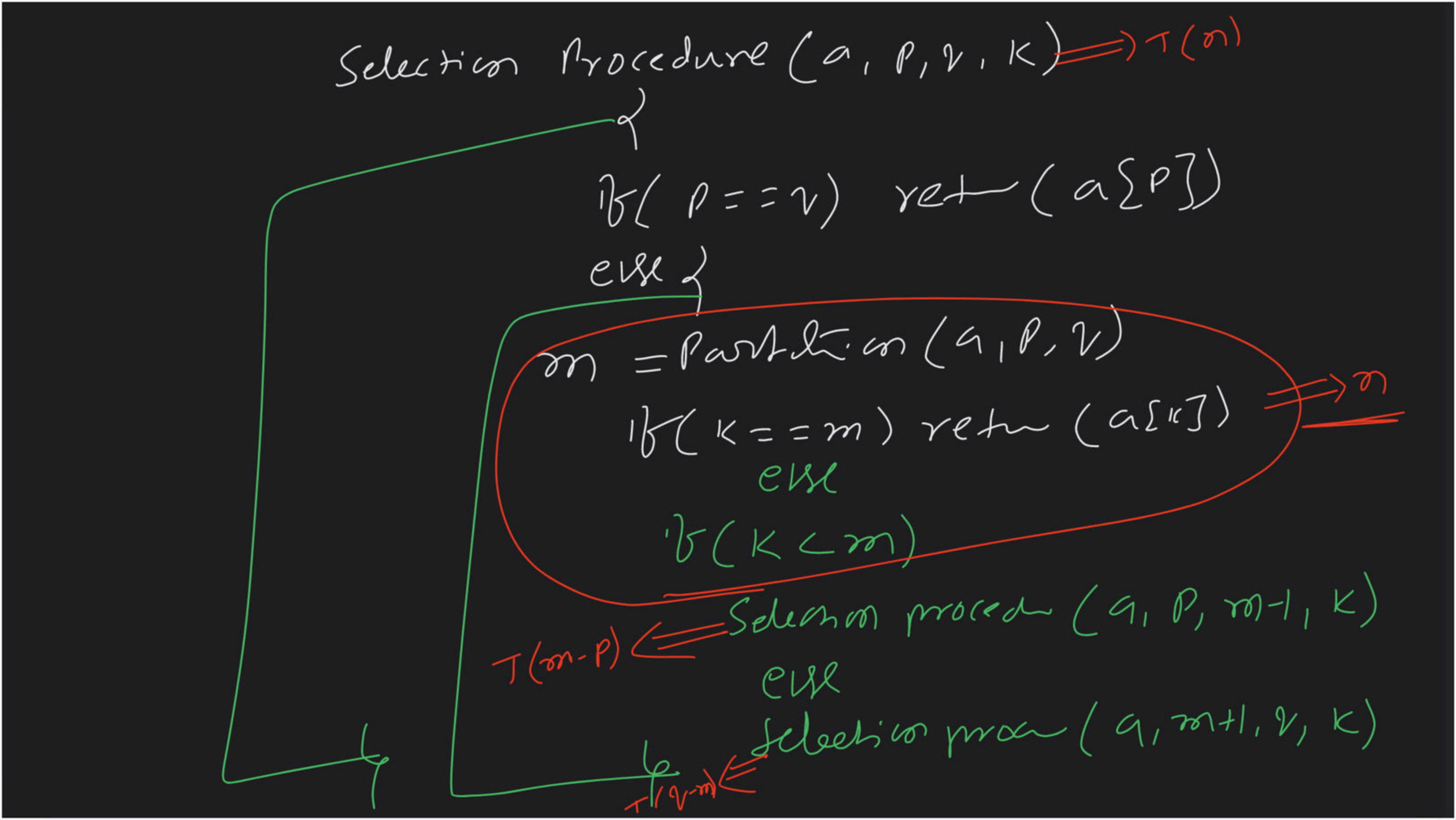
partition(9, 10)

$$m = \overset{10}{(}\ 130\ \overset{9}{)}\ 150\ \overset{10}{(}\ 180\ \overset{11}{)}$$

⇓

1-ele

return


Selection Procedure (a, p, r, k)
T(n)
{
if(p == r) ret (a[p])
else {
m = Partition (a, p, r)
if(k == m) ret (a[k])
else
if(k < m)
Selection proced (a, p, m-1, k)
T(m-p)
else
Selection proc (a, m+1, r, k)
T(r-m)
n

Let $T(n)$ be TC of above algo

RR

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ n + T(n-p) & \text{if } n > 1 \end{cases}$$

(or)

$T(q-n)$

BC, AC

$T(n) = n + T(n/2)$

$\downarrow$ $\log n$

$= n + n/2 + n/2^2 + \cdots + \frac{n}{2^{\log n}}$

$= n[1/2 + 1/2^2 + 1/2^3 + \cdots] = O(n)$

WC

$T(n) = n + T(n-1)$

$\downarrow$ n

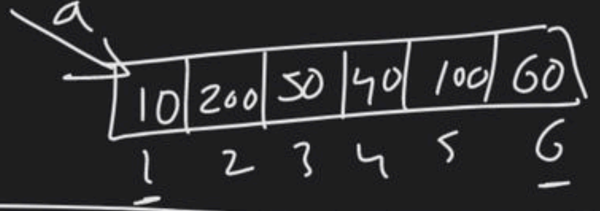
$= 1 + 2 + 3 + \cdots + n$

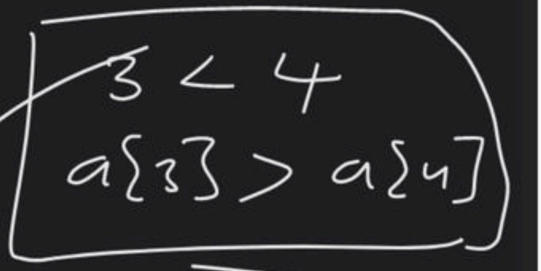
$= O(n^2)$

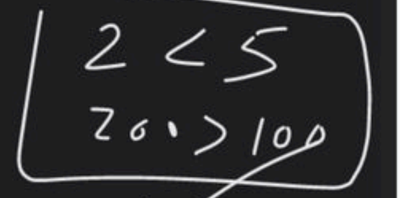
Counting no. of inversions

i/p: Array of n- distinct elements

o/p: count inversions







ex

50	10	60	20	11	21	31	80	90	3
1	2	3	4	5	6	7	8	9	10

50 ⇒ 10, 11, 21, 31, 3

10 ⇒ 3 60 ⇒ 11, 21, 31, 3 20 ⇒ 11, 21, 31, 3

11 ⇒ 3 21 ⇒ 3 31 ⇒ 3 80 ⇒ 3, 90 ⇒ 3

⇒ 19

18—DS

remaining class
complete from previous course