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Miscellaneous - Part III

Complete Course on Algorithms - GATE

Subbarao Lingamgunta • Lesson 54 • May 22, 2023

Longest Common Subsequence (LCS)

Subsequence of a given sequence is just the given sequence only in which 0 (or) more symbols left out.

LCS(6,6) ⇒ ?

Common Subsequence:

$x = (A\ B\ B\ A\ B\ B)$ — $1\ 2\ 3\ 4\ 5\ 6$

$y = (B\ A\ A\ B\ A\ A)$ — $1\ 2\ 3\ 4\ 5\ 6$

$CS_0 = ()$

$CS_1 = (A)$

$CS_2 = (A\ B)$

$CS_3 = (A\ B\ A)$ — LCS

$CS_4 = (A\ B\ A\ B)$ ~~(B A B A)~~

Ex

$S = (A\ B\ B\ A\ B\ B)$ — $1\ 2\ 3\ 4\ 5\ 6$

$SS_1 = (A\ A)$ — $1\ 4$

$SS_2 = ()$

$SS_3 = (A\ B\ B\ A\ B\ B)$ — $1\ 2\ 3\ 4\ 5\ 6$

$SS_4 = (B\ A\ B\ A)$ — $2\ 4\ 5\ 6$

ex

$x = (A\ B\ A\ B\ A\ B)$

$y = (B\ A\ B\ A\ B\ A)$

1 2 3 4 5 6

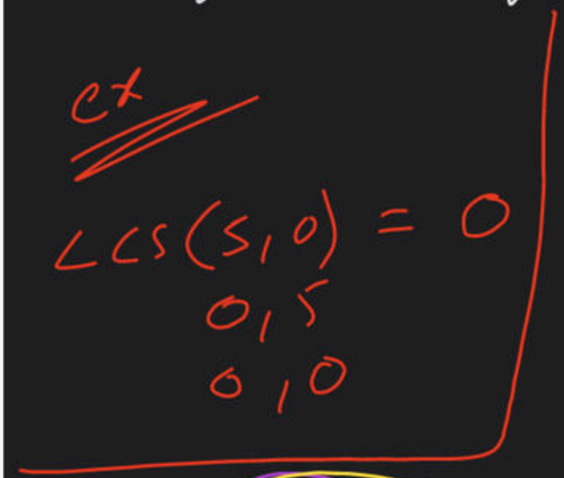
$CS_0 = ()$ $CS_1 = (A)$ $CS_2 = (A\ B)$

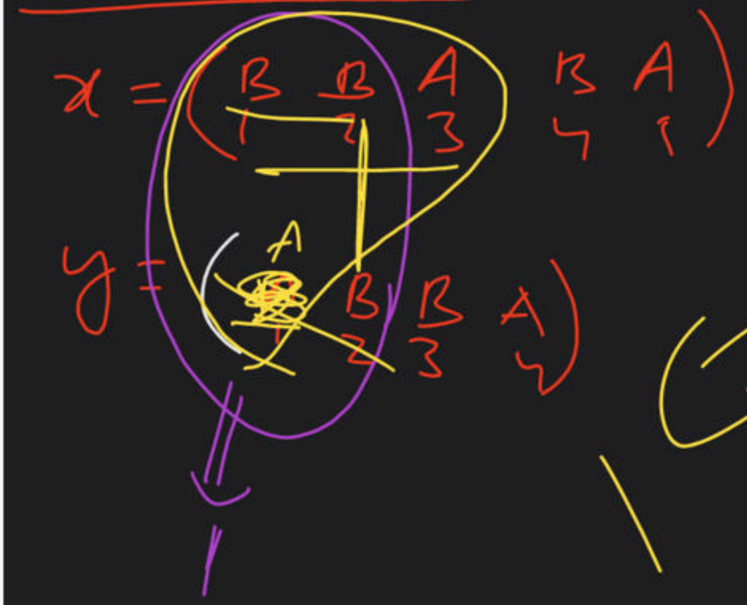
$CS_3 = (A\ B\ A)$ $CS_4 = (A\ B\ A\ B)$

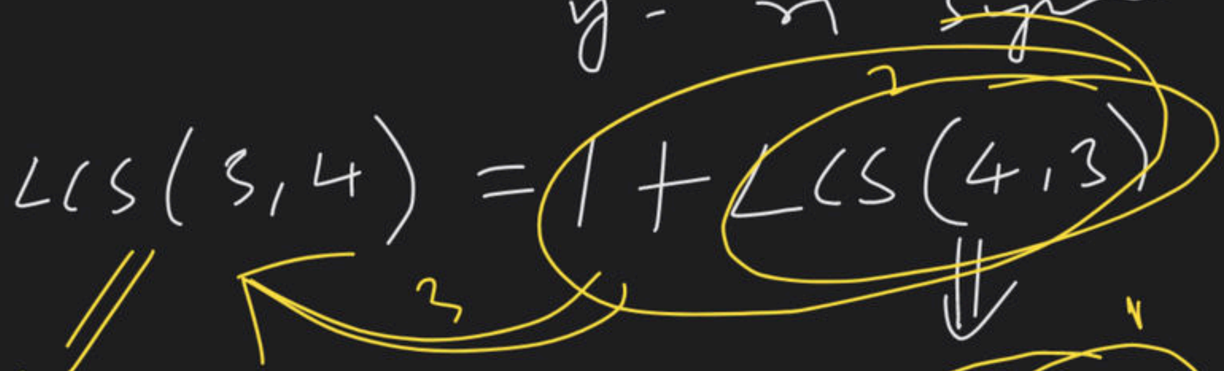
$CS_5 = (A\ B\ A\ B\ A)$ → LCS

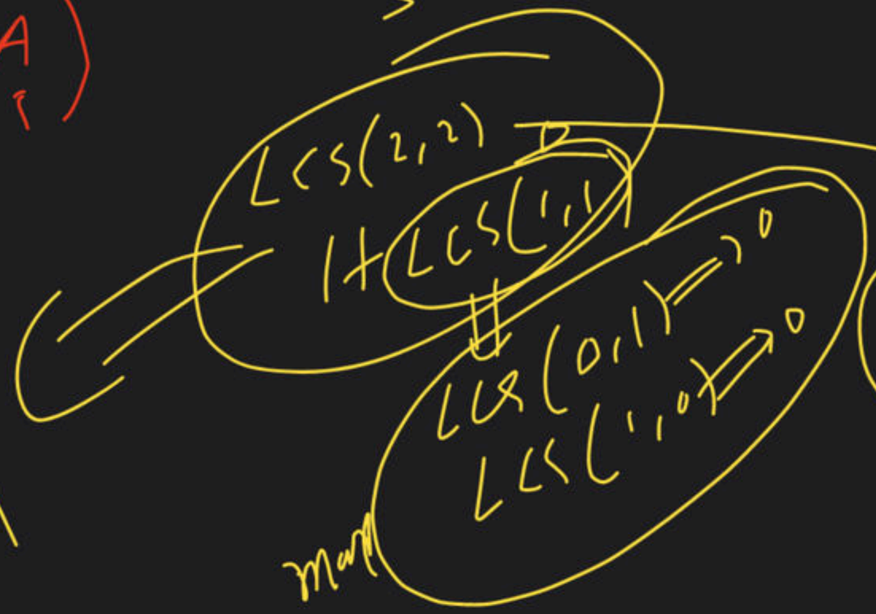
$LCS(6,6) \Rightarrow 5$

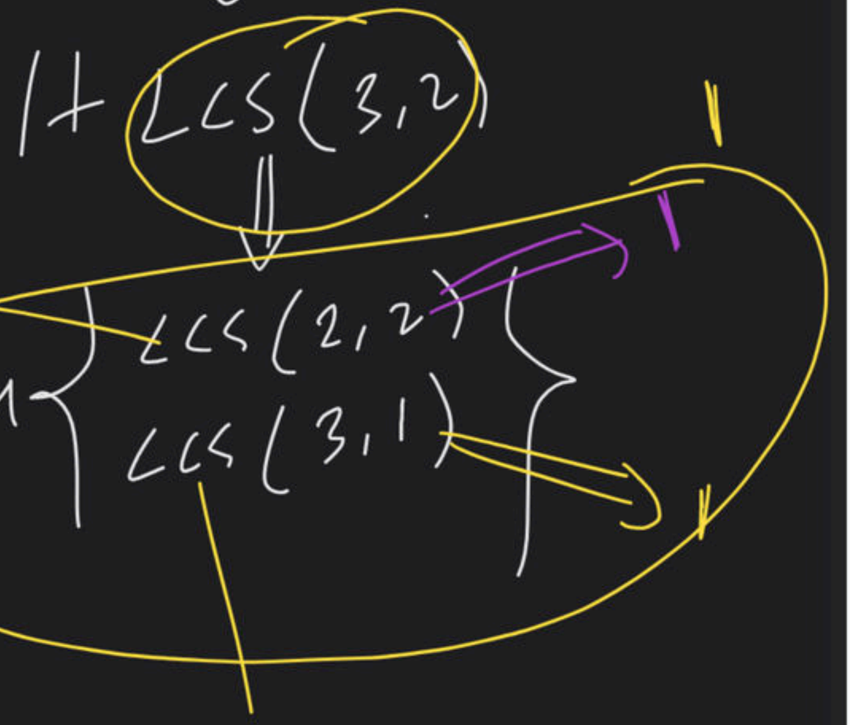
Let LCS(m,n) = The length of the largest common subsequence of 2-sequences x & y where x — m sym
y - n sym

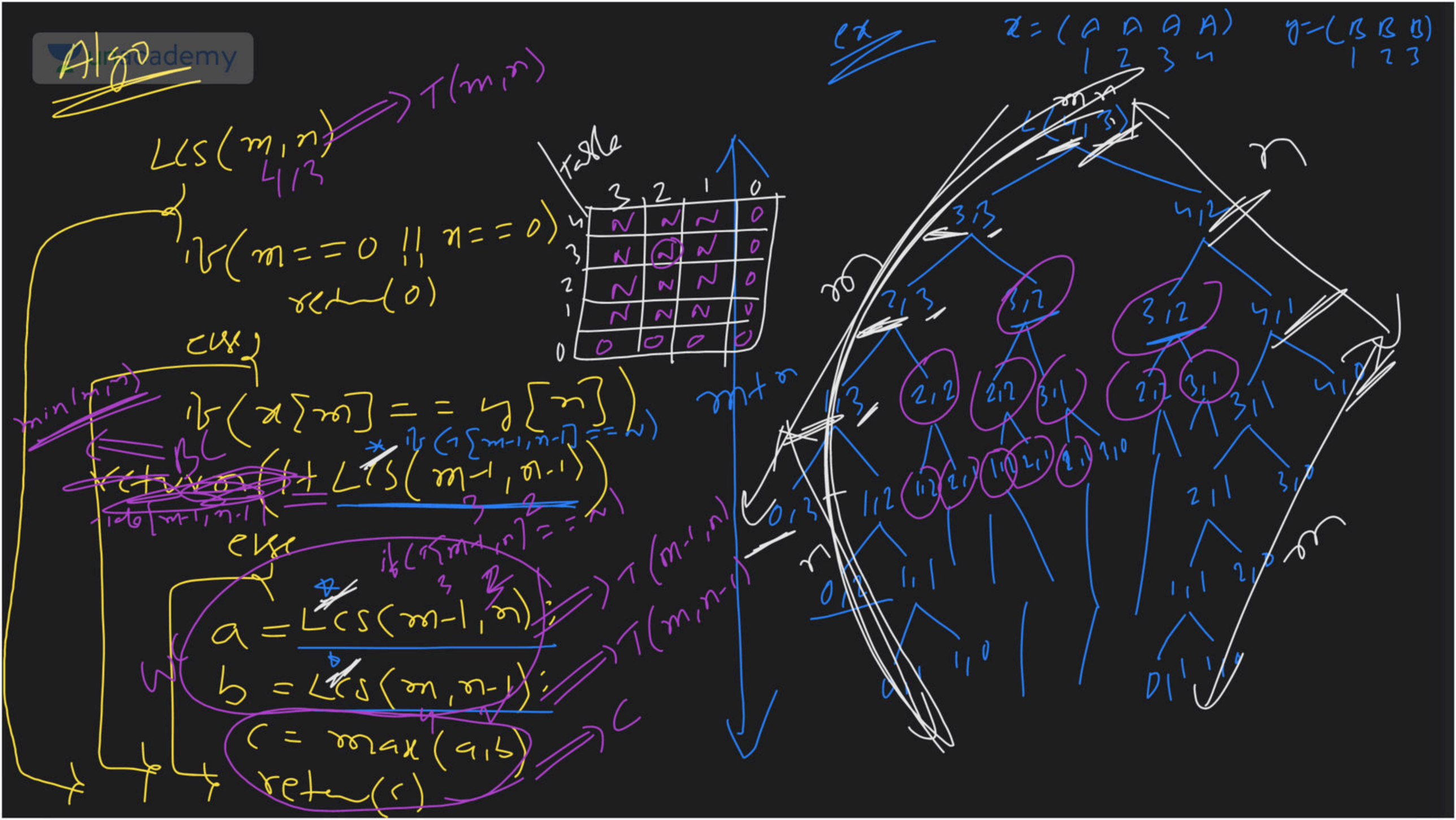











Algo
LCS(m,n)
if(m==0 || n==0)
return(0)
else
if(x[m]==y[n])
return(1+LCS(m-1,n-1))
else
a=LCS(m-1,n);
b=LCS(m,n-1);
c=max(a,b)
return(c)
T(m,n)
table
m+n
T(m-1,n)
T(m,n-1)
c
ex
x=(A A A A)
y=(B B B)
m
n

Wt $T(m,n)$ = TC of LCS(m,n)

WC-RR

$$T(m,n) = T(m-1,n) + T(m,n-1) + c$$

$$= 2^0 c + 2^1 c + 2^2 c + \cdots + 2^{m+n} c$$

$$= c\left\{2^0 + 2^1 + 2^2 + \cdots + 2^{m+n}\right\}$$

$$= c \cdot 2^{m+n}$$

$$= O(2^{m+n})$$

$$= O\left[2^m \cdot 2^n\right]$$

TC WC | TC $\min(m,n)$

Without DP

Space-complex

↓

extra space

↓

stack

↓

$O(m+n)$

With DP

In LCS(m,n) how many DFC?

Ans: LCS$(4,3)$

3 2

3 1

2 1

1 1

0 1

0 3+1

4+1

LCS$(4,3)$ = $(4-1)(3+1)$ − DFC

LCS(m,n) = $(m+1)\cdot(n+1)$ − DFC

= mn − DFC

= $mn + O(1)$

= $O(mn)$ TC WC

Space

↓

extra space

stack

↓

$m+n$

DFC

↓

mn

$O(mn)$