DAC - Part VII

Complete Course on Algorithm for GATE - CS & IT

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Power of an element

i/p: 2 integers $a \geq 2$, $n \geq 1$, o/p: Find a^n

2 9

$2^9 \Rightarrow 512$

$a^n = \underbrace{a \times a \times a \times a \cdots \times a}_{n \text{ times}}$

```
pow(a,n)
S=1
for(i=1; i<=n; i++)
    S = S*a
```

without DAC

$\Theta(n)$

```
Pow(a,n)
S=a
for(i=1; i<=log n; i++)
    S = S*S
```

1. $a \times a = a^2$
2. $a^2 \times a^2 = a^4$
3. $a^4 \times a^4 = a^8$

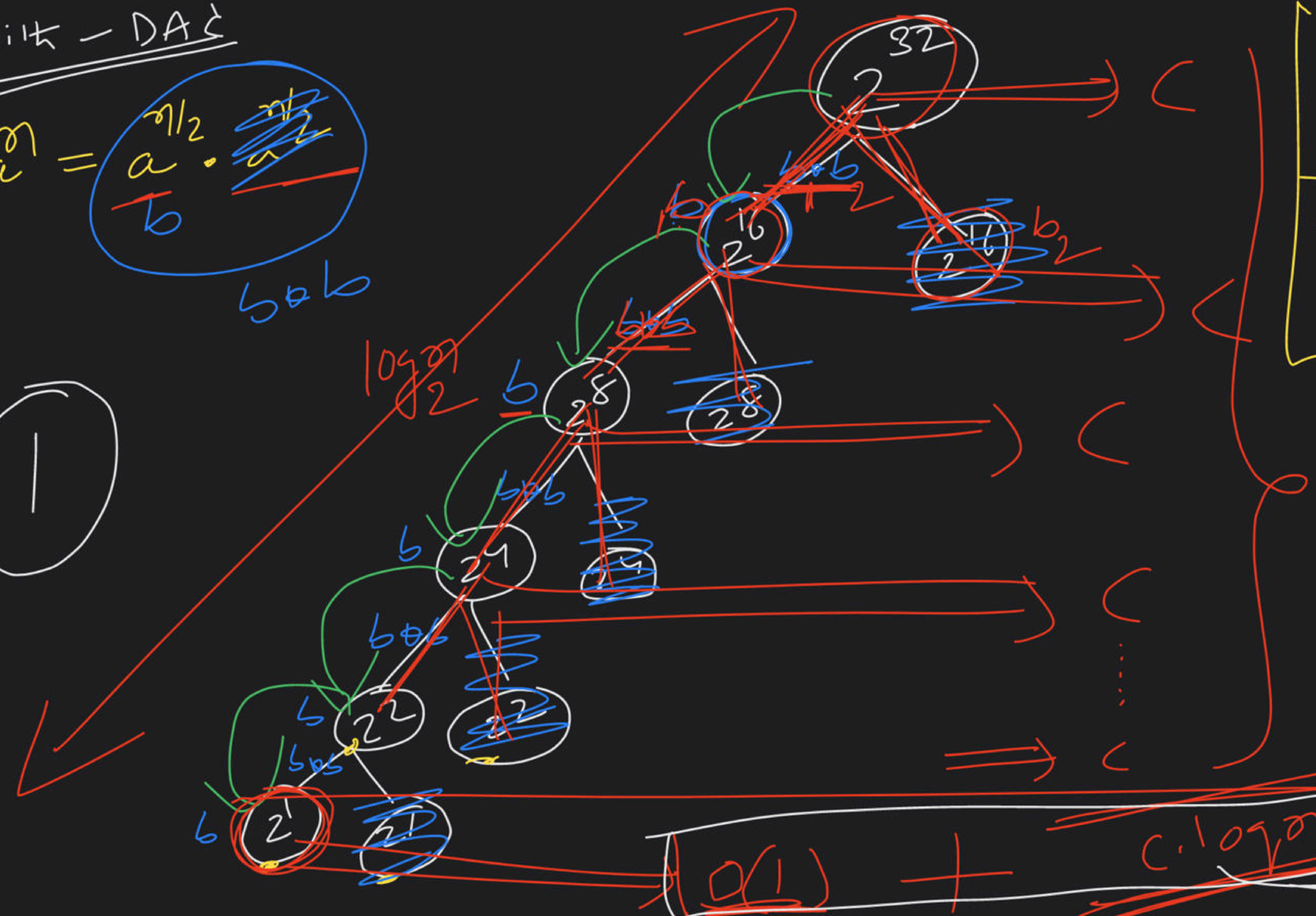
$\log n$ a^n

with - DAC

$$a^n = a^{n/2} \cdot a^{n/2}$$

b

b+b



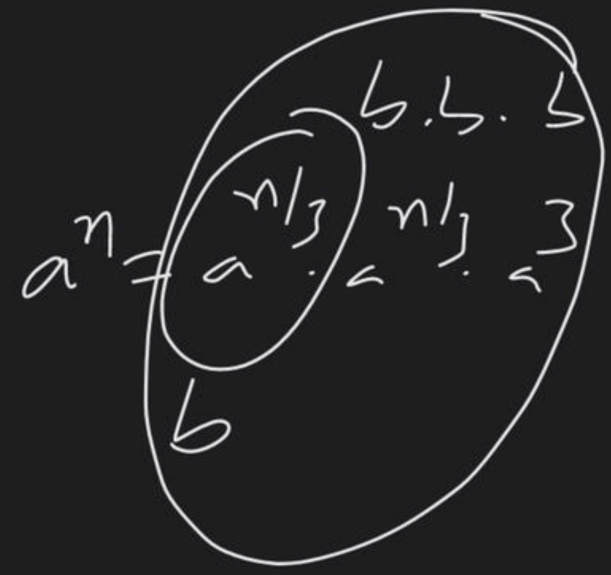
1 if n = 1
small

return(a)
solution

$$O(1) + c \cdot \log_2 n = \Theta(\log n)$$

```
DAC-Pow(a, n)
{
    if(n==1) return(a)
    else
    {
        mid = ⌊n/3⌋
        b = DAC-pow(a, mid)
        c = b*b*b
        return(c)
    }
}
```

(2)



Let $T(n)$ be the TC of above algo to find a^n

RR-TC (3)

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ O(1) + T(n/2) + O(1) & \text{if } n > 1 \end{cases}$$

$T(n) = T(n/2^1) + C$ (const)

$= T(n/2^2) + C + C$

$= T(n/2^3) + C + C + C$

$\vdots$ $k = \log n$

$= T(n/2^{\log n}) + \underbrace{C + C + C \cdots + C}_{\log n}$

Stack space

$= T(1) + C \cdot \log n$

$= O(1) + C \cdot \log n$ else

$= C \cdot \log n = \Theta(\log n)$ Time

if

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log_2 n = k$

$T(n)$ ನಲ್ಲಿ

$\Downarrow$

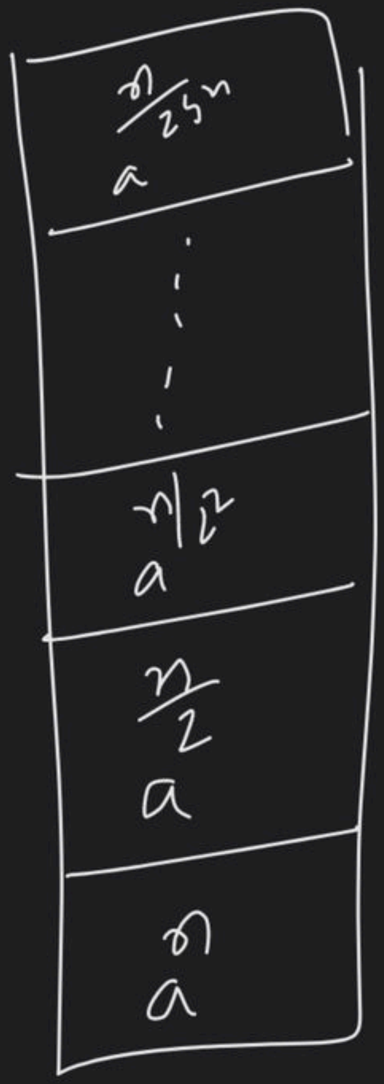
C

$n^0 \Rightarrow C \cdot (\log n)^0$

$\Downarrow$

$\Theta(C \cdot (\log n)^0)$

$\Theta(\log n)$


n/2^n
a
n/2^2
a
n/2
a
n
a