Undecidability - V

Complete Course on Theory of Computation

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DCFL's not closed under concatenation

CFL's are closed under concatenation

$\Downarrow$

$L_1 : a^n b^n \Rightarrow$ CFL

$L_2 : c^n \Rightarrow$ CFL

$L_1 \cdot L_2 \Rightarrow a^n b^n c^m$

Reversal

$1h \Rightarrow$ 7 days $\quad 1h \Rightarrow 2$ l

$\underbrace{a^n b^n}_{\text{DCFL}} \cup \underbrace{c a^n b^{2n}}_{\text{DCFL}}$

$\xrightarrow{\text{Reversal}}$

$\underbrace{b^n a^n}_{\text{DCFL}} \cup \underbrace{b^{2n} a^n c}_{\text{DCFL}}$

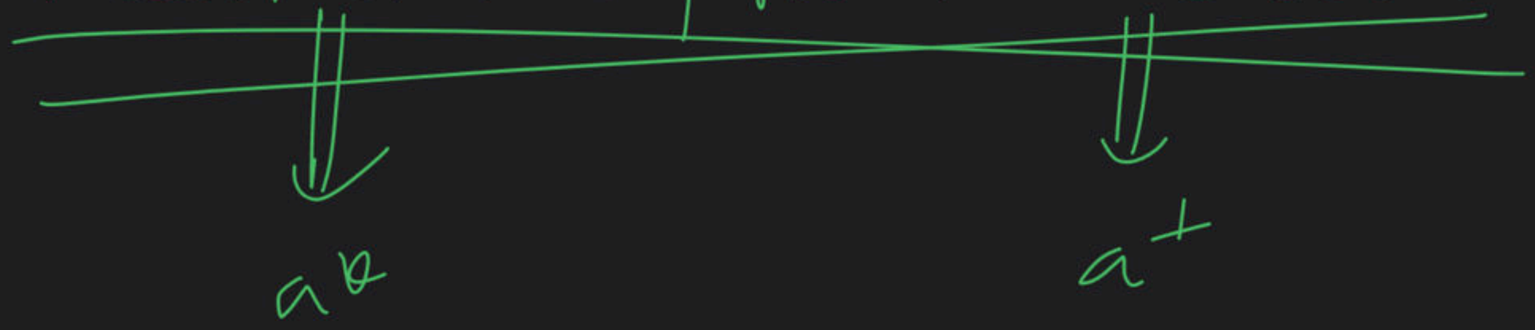
$\frac{1h}{1h}$

$\underline{\qquad\qquad\text{DCFL}\qquad\qquad}$

$\underline{\qquad\qquad\text{CFL}\qquad\qquad}$

DCFL's are not closed under Reversal.

Kleen closure | positive closure:



L

$S \rightarrow a|b$

L^*

$S \rightarrow a|b|SS|\epsilon$

L^+ $S \rightarrow a|b|SS$

CFL's are closed under K.C, P.C

$L = \{ c a^n b^n \cup a^k b^{2k} \mid n, k \geq 0 \} \Longrightarrow$ DCFL ✓

$L^* = (c a^n b^n + a^k b^{2k} \mid n, k \geq 0)^*$

$(a+b)^* = a b$

$= c a^n b^n a^k b^{2k} \mid k, n \neq 0$

$\Longleftrightarrow \Longrightarrow$ L - DCFL ✓

L^* — ~~DCFL~~

caaabbbb

caabb

Prefix

L: $a^n b^n \mid n \geq 1$

$S \rightarrow asb \mid ab$

$S \rightarrow asb \mid ab \mid as \mid a \mid \epsilon$

prefix (L)

CFL's are closed under prefix

CFL ✓

DCFL ✓

Suffix (or) Substring

L

$a^n b^n \cup a^n b^{2n}$

Suffix(L)

aaaaaaa

DCFL are not closed under Suffix

$a^n b^n \cup a^n b^{2n}$

$b^n a^n c \cup b^{2n} a^n$

~~aabb~~

aabb

abb

bb

b

ϵ

~~CFL~~

$L: a^n b^n \mid n \geq 1$

$S \rightarrow asb \mid ab$

substring

aaabb ✓

asb ✓

aas ✓

sbb ✓

suffix

aaabb ✓

asbb

sbb

bb

b

ϵ

suffix(L)

$S \rightarrow asb \mid ab \mid b \mid \epsilon \mid sb$

substring(L)

$S \rightarrow asb \mid ab \mid as \mid sb \mid a \mid b \mid \epsilon$

CFL's ~ closed under suffix

Subset

$L : (a+b+c)^*$ ⟹ RL ✓, DCFL ✓, CFL ✓

Subset(L) ⟹ $a^n b^n c^n \mid n \geq 1$ ⟹ RL ✗, DCFL ✗, CFL ✗

Subset failed by all

WWR

CFG $\Rightarrow$ CFL

$S \rightarrow a S a \mid b S b \mid \epsilon$

$\Rightarrow$ || Reversed

$S \rightarrow a S a \mid b S b \mid \epsilon$ CFG $\Rightarrow$ CFL

CFL's are closed under reversal

$a^n b^n \mid n \geq 1$

$S \rightarrow (a S b) \mid (ab)$ $\xrightarrow{\text{Rev}}$ $S \rightarrow bSa \mid ba$

$a^n b^n \mid n \geq 1$ $\quad$ $b^n a^n \mid n \geq 1$

$\left(\frac{WcW^R}{DCFL}\right)^C \Longrightarrow$?

$(WW^R) \Longrightarrow$ CFL

$\| \checkmark$

DCFL $\checkmark$

$(WW^R)^C \Longrightarrow$ CFL

Decidable problems of CFL

$\Downarrow \checkmark$

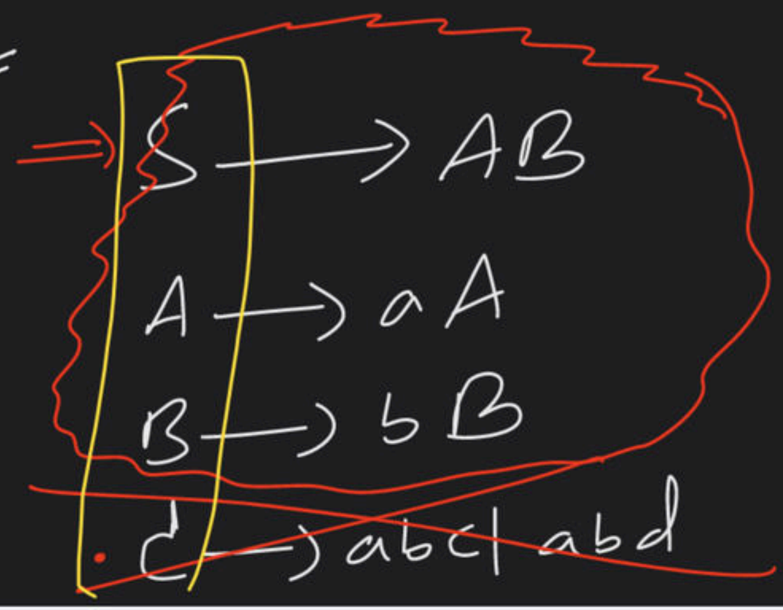
1. Emptiness of CFL
2. Finiteness " "
3. Membership " "

L

$S \rightarrow as \mid bs \mid asb$

$L = \{\}$

L



$L = \{\}$