Doubt Clearing Session

Complete Course on Theory of Computation

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Closure Properties of Regular Languages

L5

Union

① $L_1 = \Sigma^*$ then $L_1 \cup L_2 = L_1$

② $L_1 = \phi$ " $L_1 \cup L_2 = L_2$

③ $L_1 = a^* b^*$ & $L_2 = a^n b^n$ then $L_1 \cup L_2 = a^* b^* = L_1$

④ $L_1 = (a+b)^* b (a+b)^*$ $L_2 = (a+b)^* a (a+b)^*$

then $L_1 \cup L_2 = (a+b)^+$

⑤ $L_1 = a^* b^*$ $L_2 = (a+b)^*$ then $L_1 \cup L_2 = (a+b)^* = L_2$

⑥ If L_1 is regular & L_2 is non-regular over same alphabet Σ then $L_1 \cup L_2 = ?$

$\Downarrow$

$$\underset{R}{\phi} \cup \underset{NR}{a^n b^n} = \underset{NR}{a^n b^n}$$

$\Downarrow$

may be regular

may not be "

$$\boxed{\underset{R}{(a+b)^*} \cup \underset{NR}{a^n b^n} = \underset{R}{(a+b)^*}}$$

⑦ If $L_1 \cup L_2$ is regular then L_1 ?

L_2 ?

$$a^n b^n \cup \overline{a^n b^n} = \Sigma^*$$

$$\Sigma^* \cup a^n b^n = \Sigma^*$$

$$a^* b^* \cup \Sigma^* = \Sigma^*$$

L_1 – may be regular | may not be re

L_2 – may be regular | ''

(8) If L_1 - Regular

L_2 - Regular

then

$L_1 \cup L_2$ = Regular always

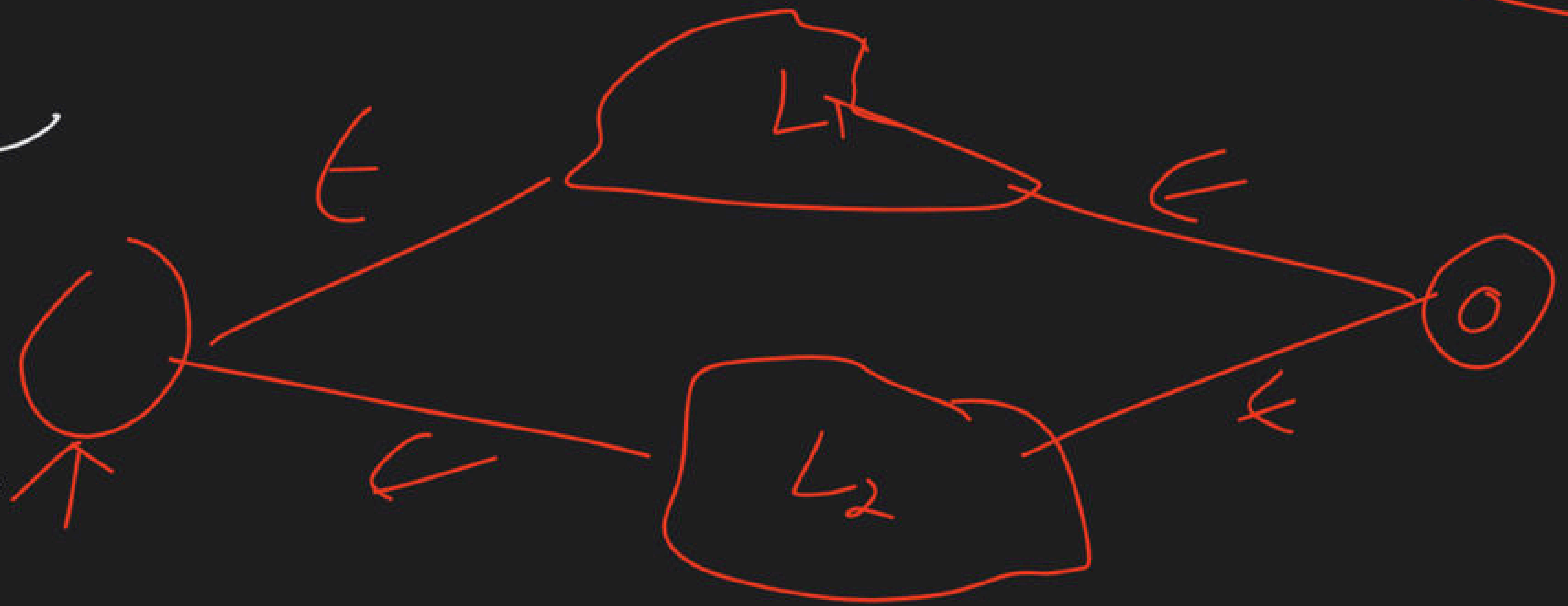
$A = \{1,2,3,4\}$ = F

$A^c = \Sigma^* - A$

= Infinite set

Not closed

So Regular Language closed under Union operation



Intersection

1. $\phi \cap L = \phi$

2. $(a+b)^* \cap L = L$

3. $a^* \cap (aa)^* = (aa)^*$

4. $a^* b \cap ab^* = \{ab\}$

5. $a^* b^* \cap b^* a^* = \{\epsilon, a^*, b^*\} \Longrightarrow a^* + b^*$

6. $a^+ b^+ \cap b^+ a^+ = \phi$

7. $ab^+ \cap ba^+ = \phi$

8. If L_1 is regl & L_2 also regular
then
$L_1 \cap L_2$ is always regular.

(9) If $L_1 \cap L_2$ is regu then L_1 — ?

L_2 — ?

$$\underbrace{a^n b^n}_{} \cap \overline{a^n b^n} = \phi$$

NR NR R

may be regu

$$\underset{NR}{a^n b^n} \cap \underset{R}{\phi} = \underset{R}{\phi}$$

may not be r—

$$\underset{R}{a^* b^*} \cap \underset{R}{b^* a^*} = \underset{R}{a^* + b^*}$$

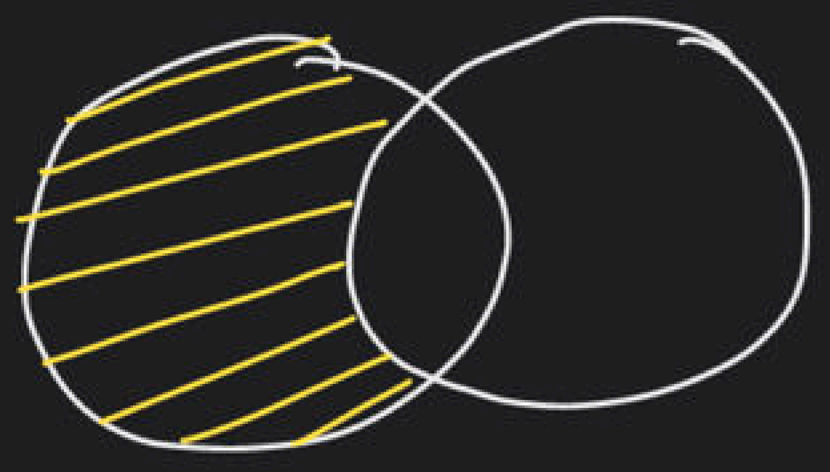
Complementation

1. $L = \phi$ then $L^c = \Sigma^* - \phi = \Sigma^*$
2. $L = \Sigma^*$ " $L^c = \Sigma^* - \Sigma^* = \phi$
3. $L = a(a+b)^*$ then $L^c = b(a+b)^* + \epsilon$
4. $L = (a+b)^* b (a+b)^*$ then $L^c = a^*$
5. If L is regular then L^c also regular
6. If L is not regular then L^c also non-regular

Difference

1. $\phi - L_1 = \phi$
2. $L_1 - \phi = L_1$
3. $\Sigma^* - L_1 = (L_1)^c$
4. $L_1 - \Sigma^* = \phi$
5. $a^* - b^* = a^+$
6. $a^*b^* - (a^* + b^*) = a^+b^+$
7. $(a^* + b^*) - a^*b^* = \phi$
8. If L_1 is regular & L_2 is regular then $L_1 - L_2$ is also regular

$$L_1 - L_2 \iff L_1 \cap L_2^c$$



Subset

$L_1 \Rightarrow$ Regular $\Rightarrow ((a+b)^*)$

$L_2 \subset L_1$

$\Downarrow$

$a^n b^n$

$\Downarrow$

non-regular

$\Longrightarrow$ If L is regular then its subset may not be regular, so R.L are not closed under subset

Note

RL, CFL, CSL, REL

$\Downarrow$ all are not closed under subset operation

Thanks All

DH, FM, S