110 120 130 140 50 60 70

(50 60 70) 110 (120 130 140)

Recursion - Part II

Complete Course on Algorithm for GATE - CS & IT

70 20 60 40 10 30 50
1 2 3 4 5 6 7

(20 60 40 10 30 50) 70 ( )

20 70 60 40 ~~50~~ 30 10

(10) 20 (70 60 40 30 50)

Subbarao Lingamgunta • Lesson 34 • Jan 8, 2023

QS-Algo

$QS(a, p, r) \Longrightarrow T(n)$ $\quad r - p + 1$

$\{$

if $(p == r)$ return $(a[p]) \Longrightarrow O(1)$

else $\{$

$m = \text{Partition}(a, p, r); \Longrightarrow n$

$QS(a, p, m-1); \Longrightarrow T(m-p)$

$QS(a, m+1, r); \Longrightarrow T(r-m)$

return(a);

$\}$

$\}$

$m - \cancel{1} - p + \cancel{1}$

$r - (m+1) + 1$

$r - m - \cancel{1} + \cancel{1}$

Stack Space

BC — $\log n$

WC — n

using better program

Stack Space

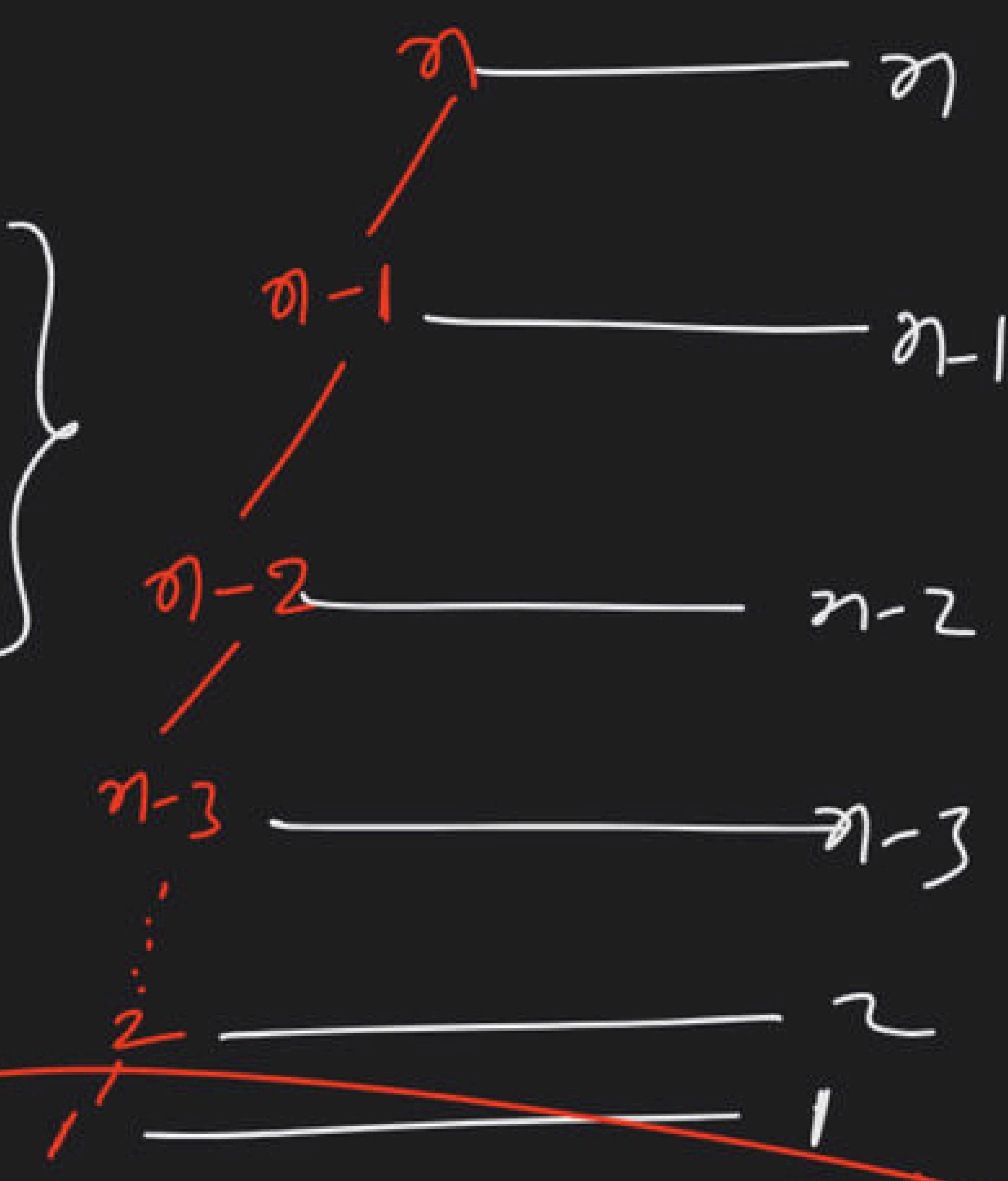
BC $\Downarrow$ 1

WC $\Downarrow$ $\log n$

Inplace.

Let $T(n)$ be the TC of above algo.

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ \boxed{n + T(m-p) + T(r-m)} & \text{if } n > 1 \end{cases}$$



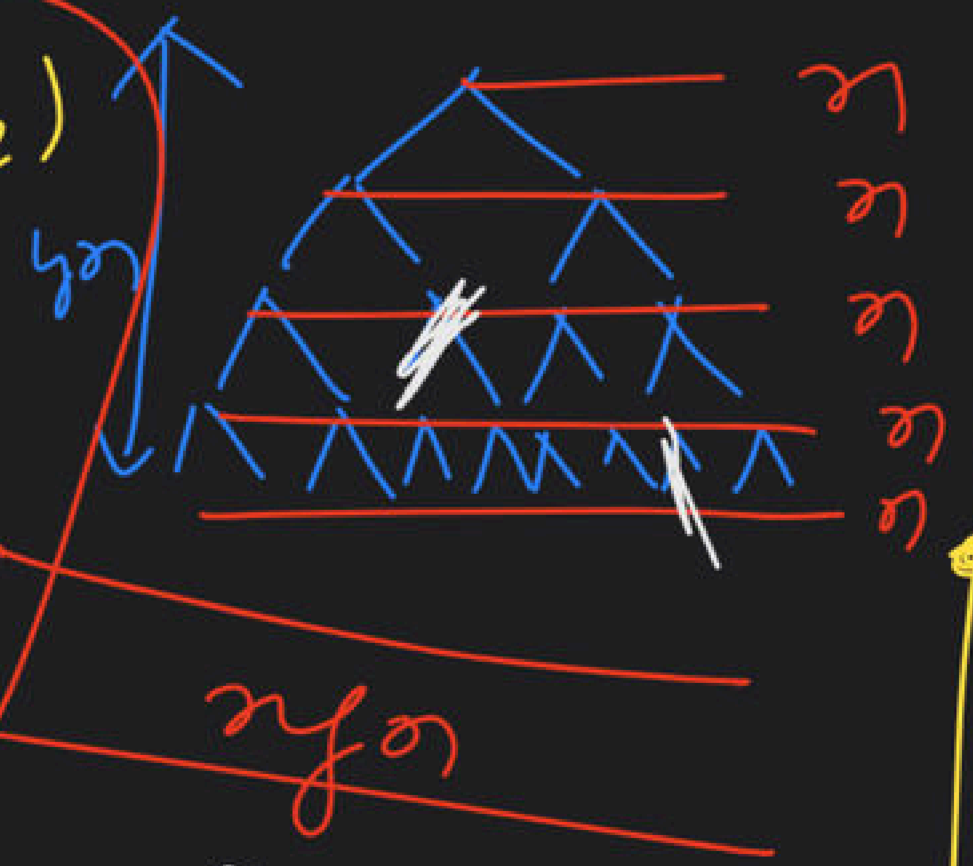
BC, AC

$$T(n) = n + T(n/2) + T(n/2)$$

$$= 2T(n/2) + n$$

$$\Big\downarrow \log n$$

$$= O(n \log n)$$



WC

$$T(n) = n + T(n-1) + T(0)$$

$$= T(n-1) + n$$

$$\Big\downarrow n$$

$$= 1 + 2 + 3 + \cdots + n$$

$$= O(n^2)$$

Consider the following sequences

(i) $1, 2, 3, 4, \ldots, n-2, n-1, n$ $\Rightarrow$ QS comparisions $\Rightarrow c_1$

(ii) $n, n-1, n-2, n-3, \ldots, 2, 1$ " " $\Rightarrow c_2$

(iii) $n, n, n, n, \ldots, n, n$ " " $\Rightarrow c_3$

(i)

10						
~~10~~	20	30	40	50	60	70
1	2	3	4	5	6	7

$l = $ p, i, j

$\Downarrow$ partition(7) $\Rightarrow$ 6C, 1S

$1 = (\,)\ 10\ (20\ 30\ 40\ 50\ 60\ 70)$

$2 = $ p, i, j

$\Downarrow$ partition(6) $\Rightarrow$ 5C, 1S

$(\,)\ 20\ (30\ 40\ 50\ 60\ 70)$

$\Rightarrow$ 4C, 1S

$(\,)\ 30\ (\ldots)$ $\Rightarrow$ 3C, 1S

$(\,)\ 40\ (\ldots)$ $\Rightarrow$ 2C, 1S

$(\,)\ 50\ (\ldots)$ $\Rightarrow$ 1C, 1S

$(\,)\ 60\ (70)$

Total comp $= 6+5+4+3+2+1$

$= n-1 + n-2 + n-3 + \cdots + 1$

$= \frac{(n-1)n}{2} = O(n^2)$

$c_1 = c_2 = c_3$

Total swaps = $\underbrace{1+1+1+\dots+1}_{n-1} = n-1$

(ii)

	10	60	50	40	30	20	70
	~~70~~	~~60~~	~~50~~	~~40~~	~~30~~	~~20~~	~~10~~
i =	1	2	3	4	5	6	7
	p	j	j	j	j	j	j
	i						

$c+s$

$n^2+n \Rightarrow O(n^2)$

$\Downarrow$ QS(7) $\Longrightarrow$ 6C, 7S

$\underline{1} = \left(\begin{matrix} \cancel{70}\ 10 & 60 & 50 & 40 & 30 & 20 \\ 1 & 2 & 3 & 4 & 5 & 6 \\ p & j & j & j & j & j \end{matrix}\right) \begin{matrix} 70 \\ 7 \end{matrix} (\ )$

i

$\Downarrow$ 5C, 1S

$(\ ) \begin{matrix} 10 \\ 1 \end{matrix} \left(\begin{matrix} 60 & 50 & 40 & 30 & 20 \\ 2 & 3 & 4 & 5 & 6 \end{matrix}\right)$

$\Downarrow$ 4C, 5S

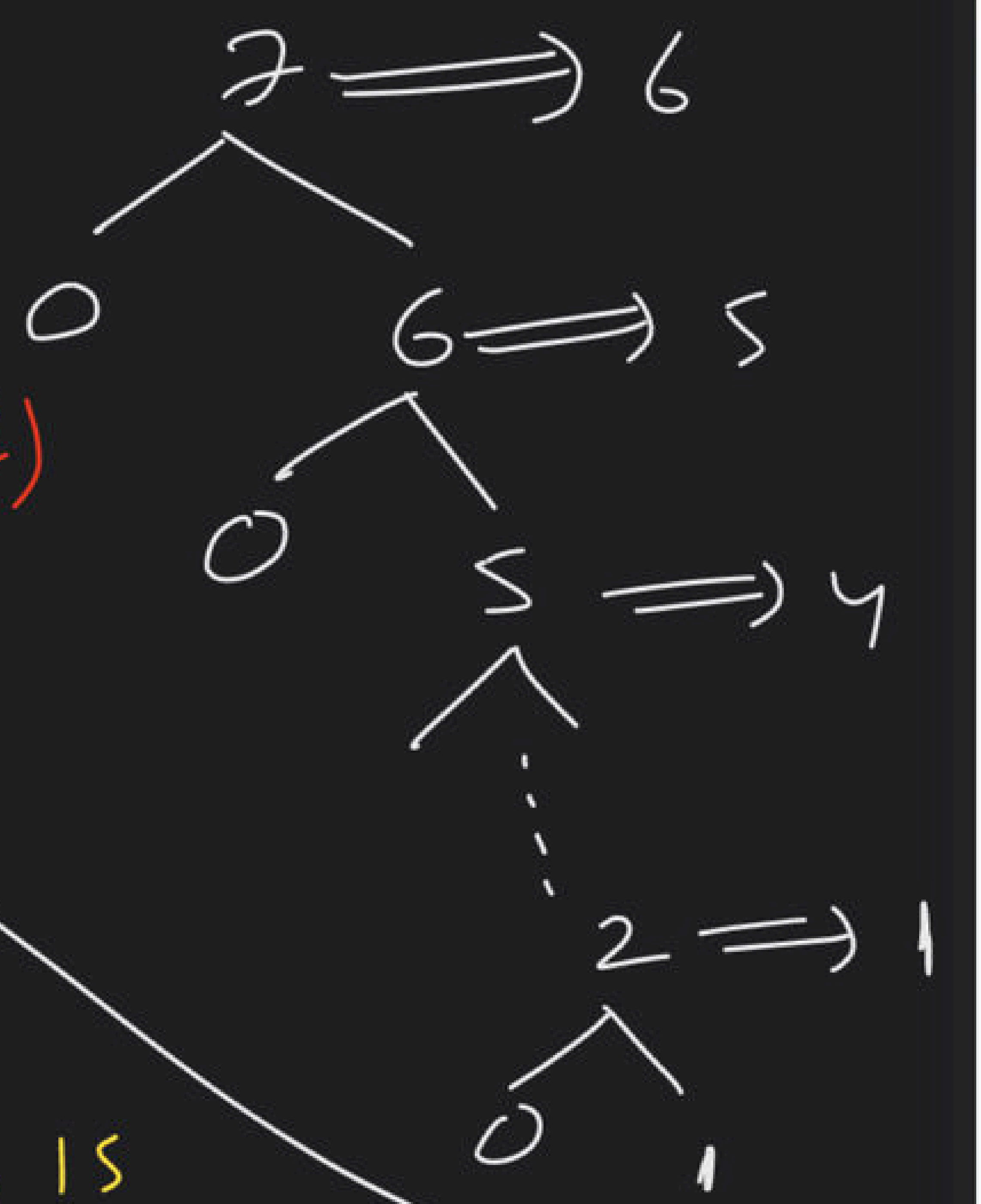
(20 - - - - - - - -) 60 ( )

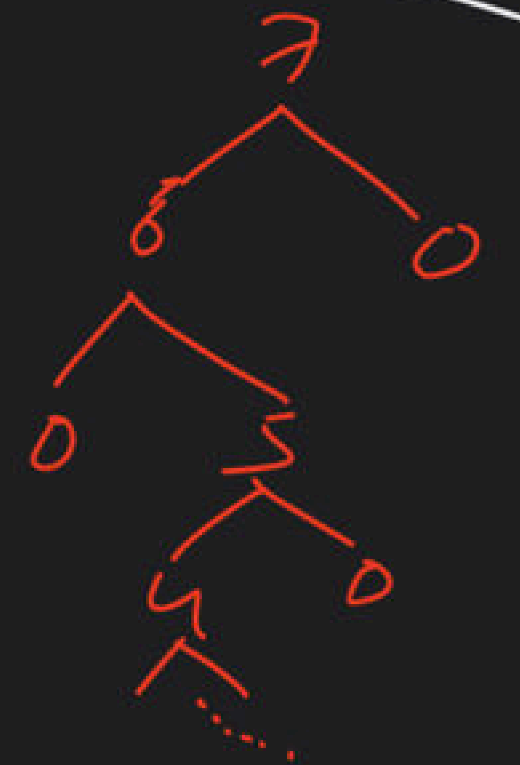
( ) 20 (30 - - - -)

3C, 1S

(30 - - - -) 50 ( )

( ) 30 (40 - - -)





$$\text{Total comparision} = 6+5+4+3+2+1$$

$$= n-1 + n-1 + n-3 + \dots + 2 + 1$$

$$= \frac{(n-1)n}{2} = O(n^2)$$

$$\text{Total swaps} = ns + 1s + (n-2)s + 1s + (n-4)s + 1s$$

$$\text{-------}$$

$$= \frac{n(n+1)}{4} \implies O(n^2)$$

$$C+S \implies O(n^2) \; [TC]$$

(iii) $7 = \cancel{10_a}\ \cancel{10_b}\ \cancel{10_c}\ \cancel{10_d}\ \cancel{10_e}\ \cancel{10_f}\ \cancel{10_g}$

$10_g\ 10_b\ 10_c\ 10_d\ 10_e\ 10_f\ 10_a$

$1\ 2\ 3\ 4\ 5\ 6\ 7$

$j \implies 6C, 7S$

$(10_g\ 10_b\ 10_c\ 10_d\ 10_e\ 10_f)\ (10_a)\ (\ )$

$1\ 2\ 3\ 4\ 5\ 6$

$\Longrightarrow$ 5C, 6S

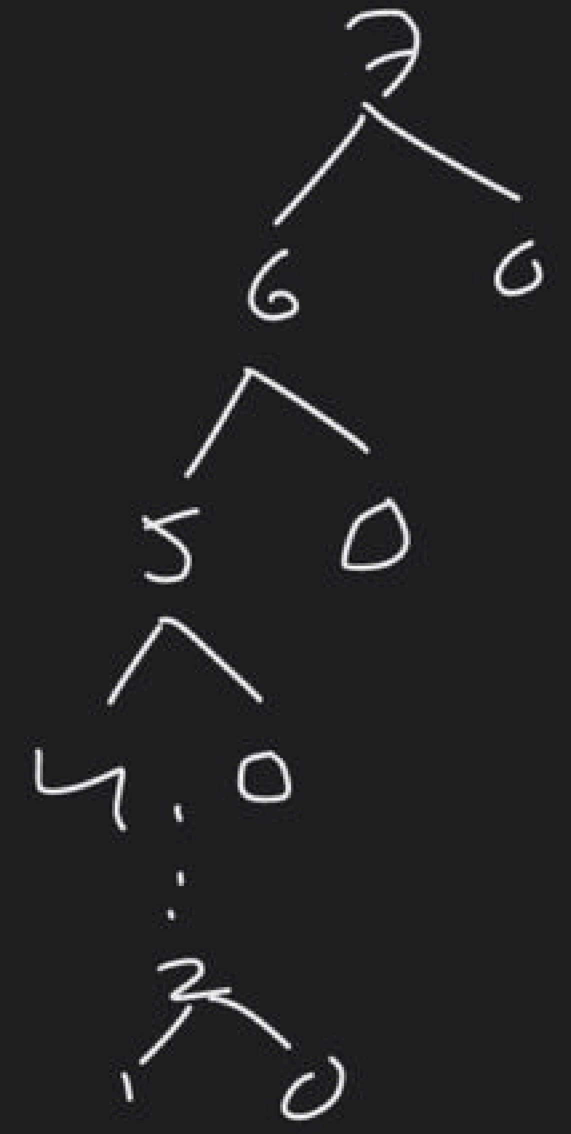
(————————) 10C

$\Longrightarrow$ 4C, 5S

(————————) 10C

$\Longrightarrow$ 1C, 2S

(————————) 10C



Total comps $= 6+5+4+3+2+1$

$= n-1 + n-2 + \cdots + 1 \Longrightarrow \frac{(n-1)(n)}{2} = O(n^2)$

Total swaps $= n + n-1 + n-2 + \cdots + 1 = \frac{n(n+1)}{2} = O(n^2)$

$C+S = 2n^2 \Longrightarrow O(n^2)$

BC, AC

$$T(n) = T(n/2) + T(n/2) + n \Longrightarrow \Theta(n \log n)$$

$$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + n \Longrightarrow \Theta(n \log_{3/2} n)$$

$$T(n) = T(n/5) + T\left(\frac{4n}{5}\right) + n \Longrightarrow \Theta(n \log_{5/4} n)$$

$$T(n) = T(n/10) + T\left(\frac{9n}{10}\right) + n \Longrightarrow \Theta(n \log_{10/9} n)$$

$$\vdots$$

$$T(n) = T(\alpha n) + T((1-\alpha) n) + n \Longrightarrow \Theta(n \log n)$$

$$0 < \alpha < 1$$

$$\max\left\{\log_{\frac{1}{\alpha}} n, \log_{\frac{1}{1-\alpha}} n\right\}$$

Independent for BC && AC

Partition
$\Downarrow$
fraction of n one side
remaining other side

Worst case

$$\boxed{T(n) = n + T(0) + T(n-1)} \Rightarrow \frac{n}{1} * n \Rightarrow n^2$$

$$T(n) = n + T(1) + T(n-2) \Rightarrow \frac{n}{2} * n \Rightarrow O(n^2)$$

$$T(n) = n + T(5) + T(n-6) \Rightarrow \frac{n}{6} * n \Rightarrow O(n^2)$$

$\vdots$

$$T(n) = n + T(c) + T(n-(c+1)) \Rightarrow \frac{n}{c+1} * n \Rightarrow O(n^2)$$

where c is constant & $c \geq 0$

Partition ⇒ oneside constant ele & another side remaining ele.