

GATE 2024
Computer Science Engineering
(Volume - I)

TOPIC WISE GATE SOLUTIONS
2013-2023



GATE SYLLABUS

Section 1 : Engineering Mathematics

Discrete Mathematics: Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem. Computer Science and Information Technology.

Section 2 : Digital Logic

Boolean algebra. Combinational and sequential circuits. Minimization. Number representations and computer arithmetic (fixed and floating point).

Section 3 : Computer Organization and Architecture

Machine instructions and addressing modes. ALU, data-path and control unit. Instruction pipelining, pipeline hazards. Memory hierarchy: cache, main memory and secondary storage; I/O interface (interrupt and DMA mode).

Section 4 : Programming and Data Structures

Programming in C. Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

Section 5 : Algorithms

Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide-and-conquer. Graph traversals, minimum spanning trees, shortest paths.

Section 6 : Theory of Computation

Regular expressions and finite automata. Context-free grammars and push-down automata. Regular and context-free languages, pumping lemma. Turing machines and undecidability.

Section 7 : Compiler Design

Lexical analysis, parsing, syntax-directed translation. Runtime environments. Intermediate code generation. Local optimisation, Data flow analyses: constant propagation, liveness analysis, common subexpression elimination.

Section 8 : Operating System

System calls, processes, threads, inter-process communication, concurrency and synchronization. Deadlock. CPU and I/O scheduling. Memory management and virtual memory. File systems.

Section 9 : Databases

ER-model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

Section 10 : Computer Networks

Concept of layering: OSI and TCP/IP Protocol Stacks; Basics of packet, circuit and virtual circuit-switching; Data link layer: framing, error detection, Medium Access Control, Ethernet bridging; Routing protocols: shortest path, flooding, distance vector and link state routing; Fragmentation and IP addressing, IPv4, CIDR notation, Basics of IP support protocols (ARP, DHCP, ICMP), Network Address Translation (NAT); Transport layer: flow control and congestion control, UDP, TCP, sockets; Application layer protocols: DNS, SMTP, HTTP, FTP, Email.

General Aptitude (GA) :

Verbal Aptitude :

Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing.

Quantitative Aptitude :

Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability.

Analytical Aptitude :

Logic: deduction and induction Analogy, Numerical relations and reasoning.

Spatial Aptitude :

Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions.

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2. Divide & Conquer Method
3. Greedy Method
4. Dynamic Programming
5. Miscellaneous

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4.1 – 4.00

1. ER Model
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3. Relational Model Relational Algebra
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5. Discrete Mathematics & Graph Theory

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7. General Aptitude

- 1.** Numerical Ability
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CHAPTER 10 | C Programming

Marks Distribution of C Programming in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	3	9
2004*	3	8	19
2005*	4	4	12
2006*	-	6	12
2007*	-	5	10
2008*	1	12	25
2009	1	-	1
2010	1	2	5
2011	1	2	5
2012	1	3	7
2013	1	1	3
2014 Set-1	1	2	5
2014 Set-2	1	2	5

* CS and IT combined

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2014 Set-3	2	1	4
2015 Set-1	1	2	5
2015 Set-2	2	1	4
2015 Set-3	1	4	9
2016 Set-1	2	3	8
2016 Set-2	1	2	5
2017 Set-1	1	4	9
2017 Set-2	2	4	10
2018	2	2	6
2019	2	4	10
2020	1	2	5
2021 Set-1	-	2	4
2021 Set-2	2	2	6

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C Programming



Practice Questions

2013 IIT Bombay

1. Consider the following function :

```
int unknown(int n)
{
    int i, j, k=0;
    for (i=n/2; i<=n; i++)
        for (j=2; j<=n; j=j*2)
            k = k + n/2;
    return (k);
}
```

The return value of the function is

- (A) $O(n^2)$ (B) $O(n^2 \log n)$
(C) $O(n^3)$ (D) $O(n^3 \log n)$

2. What is the return value of $f(p, p)$ if the value of p is initialized to 5 before the call? Note that the first parameter is passed by reference, whereas the second parameter is passed by value.

```
int f(int &x, int c)
{
    c = c - 1;
    if (c == 0)
        return 1;
    x = x + 1;
    return f(x, c) * x;
}
```

- (A) 3024 (B) 6561
(C) 55440 (D) 161051

2014 IIT Kharagpur

3. Consider the following program in C language :

```
#include <stdio.h>
main()
{
    int i;
    int *pi = &i;
    scanf("%d", pi);
    printf("%d\n", i+5);
}
```

Which one of the following statements is TRUE?



- (A) Compilation fails.
(B) Execution results in a run-time error.
(C) On execution, the value printed is 5 more than the address of variable i.
(D) On execution, the value printed is 5 more than the integer value entered.
4. Consider the following C function in which size is the number of elements in the array E:
Consider the function func shown below:
- ```
int func(int num)
{
 int count = 0;
 while (num)
 {
 count++;
 num >= 1;
 }
 return (count);
}
```
- The value returned by func(435) is \_\_\_\_\_.
5. Let A be a square matrix of size n x n. Consider the following program. What is the expected output?
- ```
C = 100
for i = 1 to n do
    for j = 1 to n do
    {
        Temp = A[i][j] + C
        A[i][j] = A[j][i]
        A[j][i] = Temp - C
    }
    for i = 1 to n do
        for j = 1 to n do
            Output(A[i][j]);
```
- (A) Transpose of matrix A
(B) The matrix A itself
(C) Adding 100 to the upper diagonal elements and subtracting 100 from diagonal elements of A
(D) None of the above
6. Suppose n and p are unsigned int variables in a C program. We wish to set p to nC_3 . If n is large, which of the following statements is most likely to set p correctly?
- (A) $p = n * (n-1) * (n-2) / 6;$
(B) $p = n * (n-1) / 2 * (n-2) / 3;$
(C) $p = n * (n-1) / 3 * (n-2) / 2;$
(D) $p = n * (n-1) * (n-2) / 6.0;$
7. The value returned by the function MyX is the
`int MyX(int *E, unsigned int size)`



```
{  
    int Y = 0;  
    int Z;  
    int i, j, k;  
    for(i = 0; i < size; i++)  
        Y = Y + E[i];  
    for(i = 0; i < size; i++)  
        for(j = i; j < size; j++)  
        {  
            Z = 0;  
            for(k = i; k <= j; k++)  
                Z = Z + E[k];  
            if (Z > Y)  
                Y = Z;  
        }  
    return Y;  
}
```

- (A) Maximum possible sum of elements in any sub-array of array E.
(B) Maximum element in any sub-array of array E.
(C) Sum of the maximum elements in all possible sub-arrays of array E
(D) The sum of all the elements in the array E.

8. Consider the following pseudo code. What is the total number of multiplications to be performed?

```
D = 2  
for i = 1 to n do  
    for j = i to n do  
        for k = j + 1 to n do  
            D = D * 3
```

- (A) Half of the product of the 3 consecutive integers.
(B) One-third of the product of the 3 consecutive integers.
(C) One-sixth of the product of the 3 consecutive integers.
(D) None of the above.

9. Consider the following function

```
double f(double x)  
{  
    if (abs(x*x - 3) < 0.01)  
        return x;  
    else  
        return f(x/2 + 1.5/x);  
}
```

Give a value q (to 2 decimals) such that $f(q)$ will return q:_____.

10. Consider the C function given below.



```
int f(int j)
{
    static int i = 50;
    int k;
    if (i == j)
    {
        printf("something");
        k = f(i);
        return 0;
    }
    else
    return 0;
}
```

Which one of the following is TRUE?

- (A) The function returns 0 for all values of j.
- (B) The function prints the string something for all values of j.
- (C) The function returns 0 when $j = 50$.
- (D) The function will exhaust the runtime stack or run into an infinite loop when $j = 50$

11. Consider the C function given below. Assume that the array listA contains n (> 0) elements, sorted in ascending order.

```
int ProcessArray(int *listA, int x, int n)
{
    int i, j, k;
    i = 0;
    j = n-1;
    do
    {
        k = (i+j)/2;
        if (x <= listA[k])
            j = k-1;
        if (listA[k] <= x)
            i = k+1;
    }
    while (i <= j);
    if (listA[k] == x)
        return(k);
    else
        return -1;
}
```

Which one of the following statements about the function ProcessArray is CORRECT?

- (A) It will run into an infinite loop when x is not in listA.
- (B) It is an implementation of binary search.



- (C) It will always find the maximum element in listA.
(D) It will return -1 even when x is present in listA.

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- 12.** The output of the following C program is _____.

```
void f1 (int a, int b)
{
    int c;
c=a; a=b; b=c;
}
void f2 (int *a, int *b)
{
    int c;
c=*a; *a=*b; *b=c;
}
int main()
{
    int a=4, b=5, c=6;
    f1(a, b);
f2(&b, &c);
printf ("%d", c-a-b);
return 0;
}
```

- 13.** Consider the following C function.

```
int fun (int n)
{
    int x=1, k;
if (n==1)
return x;
for (k=1; k<n; ++k)
x = x + fun(k) * fun(n - k);
return x;
}
```

The return value of fun(5) is _____.

- 14.** Consider the following C program segment.

```
# include <stdio.h>
int main()
{
    char s1[7] = "1234", *p;
p = s1 + 2;
*p = '0';
printf("%s", s1);
}
```

What will be printed by the program?



- (A) 12 (B) 120400
(C) 1034 (D) 1204

15. Consider the following function written in the C programming language.

```
void foo (char *a)
{
    if (*a && *a != ' ')
{
    foo(a+1);
    putchar(*a);
}
}
```

The output of the above function on input “ABCD EFGH” is

- (A) ABCD EFGH (B) ABCD
(C) HGFE DCBA (D) DCBA

16. Consider the following pseudo code, where x and y are positive integers.

```
begin
q := 0
r := x
while r ≥ y do
begin
r := r - y
q := q + 1
end
end
```

The post condition that needs to be satisfied after the program terminates is

- (A) { $r=qx+y \wedge r \leq y$ }
(B) { $x=qy+r \wedge r \leq y$ }
(C) { $y=qx+r \wedge 0 < r \leq y$ }
(D) { $q+1 < r-y \wedge y > 0$ }

17. What is the output of the following C code? Assume that the address of x is 2000 (in decimal) and an integer requires four bytes of memory.

```
#include <stdio.h>
int main()
{
    unsigned int x[4][3] = {{1, 2, 3}, {4, 5, 6},
                           {7, 8, 9}, {10, 11, 12}};
    printf("%u, %u, %u", x+3, *(x+3), *(x+2)+3);
}
```

- (A) 2036, 2036, 2036



(B) 2012, 4, 2204

(C) 2036, 10, 10

(D) 2012, 4, 6

18. Consider the C program below.

```
#include < stdio.h >
int *A, stkTop;
int stkFunc(int opcode, int val)
{
    static int size=0, stkTop=0;
    switch (opcode)
    {
        case -1: size = val; break;
        case 0: if (stkTop < size) A[stkTop++] = val; break;
        default: if (stkTop) return A[--stkTop];
    }
    return -1;
}
int main()
{
    int B[20]; A = B; stkTop = -1;
    stkFunc (-1, 10);
    stkFunc ( 0, 5);
    stkFunc ( 0, 10);
    printf ("%d\n", stkFunc(1, 0) + stkFunc(1, 0));
}
```

The value printed by the above program is _____.

19. Consider the following C program.

```
#include<stdio.h>
int main( )
{
    static int a[] = {10, 20, 30, 40, 50};
    static int *p[] = {a, a+3, a+4, a+1, a+2};
    int **ptr = p;
    ptr++;
    printf("%d%d", ptr - p, **ptr);
}
```

The output of the program is _____

20. Consider the following recursive C function.

```
void get (int n)
{
    if (n < 1)
```



```
return;
get(n-1);
get(n-3);
printf("%d", n);
}
```

If get(6) function is being called in main() then how many times will the get() function be invoked before returning to the main()?

- (A) 15 (B) 25
(C) 35 (D) 45

21. Consider the following C program :

```
# include <stdio.h>
int main( )
{
    int i, j, k = 0;
j = 2 * 3 / 4 + 2.0 / 5 + 8 / 5;
k -= --j;
for (i = 0; i < 5; i++)
{
switch(i + k)
{
case 1:
case 2: printf("\n%d", i + k);
case 3: printf("\n%d", i + k);
default: printf("\n%d", i + k);
}
}
return 0;
}
```

The number of times printf statement is executed is _____.

22. Consider the following C program.

```
# include <stdio.h>
int f1(void);
int f2(void);
int f3(void);
int x = 10;
int main()
{
    int x = 1;
x += f1() + f2() + f3() + f2();
printf("%d", x);
return 0;
}
int f1()
```



```
{  
    int x = 25;  
    x++;  
    return x;  
}  
int f2( )  
{  
    static int x = 50;  
    x++;  
    return x;  
}  
int f3( )  
{  
    x *= 10;  
    return x;  
}
```

The output of the program is _____.

2016 IISc Bangalore

23. Consider the following C program.

```
void f(int, short);  
void main()  
{  
    int i = 100;  
    short s = 12;  
    short *p = &s;  
    _____; // call to f()  
}
```

Which one of the following expressions, when placed in the blank above, will NOT result in a type checking error?

- (A) f(s, *s) (B) i = f(i,s)
(C) f(i,*s) (D) f(i,*p)

24. Consider the following C program.

```
#include<stdio.h>  
void mystery(int *ptrA, int *ptrB)  
{  
    int *temp;  
    temp = ptrB;  
    ptrB = ptrA;  
    ptrA = temp;  
}  
int main()  
{  
    int a=2016, b=0, c=4, d=42;
```



```
mystery(&a, &b);  
if (a < c)  
mystery(&c, &a);  
mystery(&a, &d);  
printf("%d\n", a);  
}
```

The output of the program _____.

25. The value printed by the following program is _____.

```
void f(int* p, int m)  
{  
m = m + 5;  
*p = *p + m;  
return;  
}  
void main()  
{  
int i=5, j=10;  
f(&i, j);  
printf("%d", i+j);  
}
```

26. The following function computes the maximum value contained in an integer array p[] of size n ($n \geq 1$)

```
int max(int *p, int n)  
{  
int a=0, b=n-1;  
while (_____  
){  
if (p[a] <= p[b])  
{  
a = a+1;  
}  
else  
{  
b = b-1;  
}  
}  
return p[a];  
}
```

The missing loop condition is

- (A) $a \neq n$ (B) $b \neq 0$
(C) $b > (a + 1)$ (D) $b \neq a$

27. What will be the output of the following C program?

```
void count(int n)
```



```
{  
static int d = 1;  
printf("%d ", n);  
printf("%d ", d);  
d++;  
if(n > 1) count(n-1);  
printf("%d ", d);  
}  
int main()  
{  
count(3);  
}
```

- (A) 3 1 2 2 1 3 4 4 4 (B) 3 1 2 1 1 1 2 2 2
(C) 3 1 2 2 1 3 4 (D) 3 1 2 1 1 1 2

28. What will be the output of the following pseudo-code when parameters are passed by reference and dynamic scoping is assumed?

```
a=3;  
void n(x)  
{  
    x = x * a;  
print(x);  
}  
void m(y)  
{  
a = 1; a = y - a; n(a);  
print(a);  
}  
void main()  
{  
m(a);  
}  
(A) 6, 2      (B) 6, 6  
(C) 4, 2      (D) 4, 4
```

29. The following function computes X^Y for positive integers X and Y.

```
int exp(int X, int Y)  
{  
int res = 1, a = X, b = Y;  
while ( b != 0 )  
{
```



```
if ( b%2 == 0 )
{
    a = a*a;
    b = b/2;
}
else
{
    res = res*a;
    b = b-1;
}
}
return res;
}
```

Which one of the following conditions is TRUE before every iteration of the loop

- (A) $X^Y = a^b$ (B) $(res * a)^Y = (res * X)^b$
(C) $X^Y = res * a^b$ (D) $X^Y = (res * a)^b$

- 30 Consider the following program :

```
int f(int *p, int n)
{
    if (n <= 1)
        return 0;
    else
        return max(f(p+1,n-1),p[0]-p[1]);
}

int main()
{
    int a[] = {3,5,2,6,4};
    printf("%d", f(a,5));
}
```

Note : max(x,y) returns the maximum of x and y. The value printed by this program is _____.

2017 IIT Roorkee

31. Consider the following C code :

```
#include <stdio.h>
int *assignval (int *x, int val)
{
    *x = val;
    return x;
}
int main()
{
```



```
int *x = malloc(sizeof(int));
if (NULL == x) return;
x = assignval(x, 0);
if(x)
{
x = (int*) malloc(sizeof (int));
if (NULL == x) return;
x = assignval (x, 10);
}
printf("%d\n", *x);
free(x);
}
```

The code suffers from which one of the following problems:

- (A) Compiler error as the return of malloc is not typecast appropriately.
- (B) Compiler error because the comparison should be made as x==NULL and not as shown.
- (C) Compiles successfully but execution may result in dangling pointer.
- (D) Compiles successfully but execution may result in memory leak.

32. Consider the following function implemented in C :

```
void printxy(int x, int y)
{
    int *ptr;
x=0;
ptr=&x;
y=*ptr;
*ptr=1;
printf("%d, %d", x, y);
}
```

The output of invoking printxy(1,1) is:

- (A) 0, 0
- (B) 0, 1
- (C) 1, 0
- (D) 1, 1

33. Match the following :

- (P) static char var; (i) Sequence of memory locations to store addresses
- (Q)m = malloc(10); (ii) A variable located in data section of memory

m = NULL;

- (R) char *ptr[10]; (iii) Request to allocate a CPU register to store data
- (S) register int varl; (iv)A lost memory which cannot be freed

- (A) P-> (ii), Q-> (iv), R-> (i), S-> (iii)
- (B) P-> (ii), Q-> (i), R-> (iv), S-> (iii)
- (C) P-> (ii), Q-> (iv), R-> (iii), S-> (i)
- (D) P-> (iii), Q-> (iv), R-> (i), S-> (ii)

34. Consider the following two functions



```
void fun1(int n)
{
    if(n == 0) return;
    printf("%d", n);
    fun2(n-2);
    printf("%d", n);
}
Void fun2(int n)
{
    if(n == 0) return;
    printf("%d", n);
    fun1(++n);
    printf("%d", n);
}
```

The output printed when fun1(5) is called is

- (A) 53423122233445
- (B) 53423120112233
- (C) 53423122132435
- (D) 53423120213243

35. Consider the C functions **foo** and **bar** given below :

```
int foo(int val)
{
    int x = 0;
    while (val > 0)
    {
        x = x + foo(val--);
    }
    return val;
}
int bar(int val)
{
    int x = 0;
    while (val > 0)
    {
        x = x + bar(val-1);
    }
    return val;
}
```

Invocations of foo(3) and bar(3) will result in:



- (A) Return of 6 and 6 respectively
- (B) Infinite loop and abnormal termination respectively
- (C) Abnormal termination and infinite loop respectively
- (D) Both terminating abnormally

36. Consider the following C program.

```
#include <stdio.h>
#include <string.h>
void printlength (char *s, char *t)
{
    unsigned int c = 0;
    int len = ((strlen (s) - strlen (t)) > c) ? strlen (s) : strlen (t);
    printf ("%d\n", len);
}
void main()
{
    char *x = "abc";
    char *y = "defgh";
    printlength(x, y);
}
```

Recall that **strlen** is defined in **string.h** as returning a value of type **size_t**, which is an **unsigned int**.

The output of the program is _____.

37. The output of executing the following C program is _____.

```
# include<stdio.h>
int total(int v)
{
    static int count = 0;
while (v)
{
    count += v & 1;
    v >>= 1;
}
return count;
}
void main()
{
    static int x = 0;
int i = 5;
for (; i> 0; i--)
{
```



```
x = x + total(i);  
}  
printf ("%d\n", x) ;  
}
```

38. Consider the C program fragment below which is meant to divide x by y using repeated subtractions. The variable x, y, q and r are all unsigned int.

```
while(r >= y)  
{  
    r = r - y;  
    q = q + 1;  
}
```

Which of the following conditions on the variables x, y, q and r before the execution of the fragment will ensure that the loop terminates in a state satisfying the condition $x == (y*q + r)$?

- (A) (q == r) && (r == 0)
(B) (x > 0) && (r == x) && (y > 0)
(C) (q == 0) && (r == x) && (y > 0)
(D) (q == 0) && (y > 0)
39. Consider the following snippet of a C program. Assume that swap (&x,&y) exchanges the content of x and y :

```
int main ()  
{  
    int array[] = {3, 5, 1, 4, 6, 2};  
    int done =0;  
    int i;  
    while (done==0)  
    {  
        done =1;  
        for (i=0; i<=4; i++)  
        {  
            if (array[i] < array[i+1])  
            {  
                swap(&array[i], &array[i+1]);  
                done=0;  
            }  
        }  
        for (i=5; i>=1; i--)  
        {  
            if (array[i] > array[i-1])  
            {  
                swap(&array[i], &array[i-1]);  
                done =0;  
            }  
        }  
    }  
}
```



```
    }
}
}
printf("%d", array[3]);
}
```

The output of the program is _____

40. Consider the following C program :

```
#include <stdio.h>
int main()
{
    int m = 10;
    int n, n1;
    n = ++m;
    n1 = m++;
    n--;
    --n1;
    n -= n1;
    printf("%d", n);
    return 0;
}
```

The output of the program is _____.

41. Consider the following program

```
#include<stdio.h>
#include<string.h>
int main()
{
    char * c = "GATECSIT2017";
    char *p = c;
    printf("%d", (int)strlen(c+2[p]-6[p]-1));
    return 0;
}
```

The Output of the following program is _____.

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42. Consider the following C program.

```
#include <stdio.h>
struct Ournode
{
    char x, y, z;
};
int main()
```



```
{  
struct Ournode p = {'1', '0', 'a' + 2};  
struct Ournode *q = &p;  
printf("%c, %c", *((char *)q + 1), *((char *)q + 2));  
return 0;  
}
```

The output of this program is :

- (A) 0, c
- (B) 0, a+2
- (C) '0', 'a+2'
- (D) '0', 'c'

43. Consider the following C program:

```
#include <stdio.h>  
int counter = 0;  
int calc(int a, int b)  
{  
    int c;  
    counter++;  
    if (b == 3)  
        return (a * a * a);  
    else  
    {  
        c = calc(a, b / 3);  
        return (c * c * c);  
    }  
}  
  
int main()  
{  
    calc(4, 81);  
    printf("%d", counter);  
}
```

The output of this program is _____.

44. Consider the following C program :

```
#include <stdio.h>  
void fun1(char *s1, char *s2)  
{  
    char *temp;  
    temp = s1;  
    s1 = s2;  
    s2 = temp;  
}  
void fun2(char **s1, char **s2)  
{
```



```
char *temp;
temp = *s1;
*s1 = *s2;
*s2 = temp;
}
int main()
{
    char *str1 = "Hi", *str2 = "Bye";
fun1(str1, str2);
printf("%s %s", str1, str2);
fun2(&str1, &str2);
printf("%s %s", str1, str2);
return 0;
}
```

The output of the program above is

- (A) Hi Bye Bye Hi
- (B) Hi Bye Hi Bye
- (C) Bye Hi Hi Bye
- (D) Bye Hi Bye Hi

45. Consider the following C code. Assume that unsigned long int type length is 64 bits.

```
unsigned long int fun(unsigned long int n)
{
    unsigned long int i, j = 0, sum = 0;
for( i = n; i > 1; i = i/2) j++;
for( ; j > 1; j = j/2) sum++;
return sum;
}
```

The value returned when we call fun with the input 2^{40} is

- (A) 4
- (B) 5
- (C) 6
- (D) 40

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46. Consider the following C program :

```
#include <stdio.h>
int jumble(int x, int y)
{
    x = 2 * x + y;
return x;
}
int main()
{
```



```
int x = 2, y = 5;  
y = jumble(y, x);  
x = jumble(y, x);  
printf("%d\n", x);  
return 0;  
}
```

The value printed by program is _____.

47. Consider the following C program :

```
#include<stdio.h>  
int main()  
{  
int arr[] = {1, 2, 3, 4, 5, 6, 7, 8, 9, 0, 1, 2, 5}, *ip = arr + 4;  
printf("%d\n", ip[1]);  
return 0;  
}
```

The number that will be displayed on execution of the program is _____.

48. Consider the following C program :

```
void convert(int n)  
{  
If (n < 0)  
printf(" % d", n);  
else  
{  
convert(n / 2);  
printf(" % d", n % 2);  
}  
}
```

Which one of the following will happen when the function convert is called with any positive integer n as argument?

- (A) It will print the binary representation of n and terminate.
- (B) It will print the binary representation of n in the reverse order and terminate.
- (C) It will print the binary representation of n but will not terminate.
- (D) It will not print anything and will not terminate.

49. Consider the following C program :

```
#include<stdio.h>  
int r()  
{  
static int num=7;  
return num--;  
}  
int main()  
{
```



```
    for(r();r();r())
printf("%d ",r());
return 0;
}
```

Which one of the following values will be displayed on execution of the programs?

- (A) 41
- (B) 52
- (C) 63
- (D) 630

50. Consider the following C program :

```
#include <stdio.h>
int main()
{
float sum = 0.0, j = 1.0, i = 2.0;
while (i / j > 0.0625)
{
j = j + j;
sum = sum + i / j;
printf("%f\n", sum);
};
return 0;
}
```

The number of times variable sum will be printed When the above program is executed is _____

51. Consider the following C program :

```
#include<stdio.h>
int main()
{
    int a[] = {2, 4, 6, 8, 10};
int i, sum = 0, *b = a + 4;
for (i = 0; i < 5; i++)
sum = sum + (*b - i) - *(b - i);
printf("%d\n", sum);
return 0;
}
```

The output of above C program is _____ .

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52. Consider the following C program.

```
#include <stdio.h>
int main ()
{
int a[4][5] = {{1, 2, 3, 4, 5},
{6, 7, 8, 9, 10},
```



```
{11, 12, 13, 14, 15},  
{16, 17, 18, 19, 20}};  
printf("%d\n", *(*(a+**a+2)+3));  
return(0);  
}
```

The output of the program is _____.

53. Consider the following C functions.

```
int fun1(int n)  
{  
    static int i= 0;  
    if (n > 0)  
    {  
        ++i;  
        fun1(n-1);  
    }  
    return (i);  
}  
  
int fun2(int n)  
{  
    static int i= 0;  
    if (n>0)  
    {  
        i = i+ fun1 (n);  
        fun2(n-1) ;  
    }  
    return (i);  
}
```

The return value of fun2(5) is _____.

54. Consider the following C functions.

```
int tob (int b, int* arr)  
{  
    int i;  
    for (i = 0; b>0; i++)  
    {  
        if (b%2)  
            arr [i] = 1;  
        else arr[i] = 0;  
        b = b/2;  
    }  
    return (i);  
}
```



```
int pp(int a, int b)
{
    int arr[20];
int i, tot = 1, ex, len;
ex = a;
len = tob(b, arr);
for (i=0; i<len ; i++)
{
if (arr[i] ==1)
tot = tot * ex;
ex= ex*ex;
}
return (tot) ;
}
```

The value returned by pp(3,4) is _____.

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55. Consider the following ANSI C program.

```
#include <stdio.h>
int main()
{
    int i , j,count;
    count=0;
    i=0;
    for(j=-3;j<=3;j++)
    {
        if(j >= 0)&&(i++)
        count=count+j;
    }
    count=count+i;
    printf("%d",count);
    return 0;
}
```

Which one of the following options is correct?

- (A) The program will compile successfully and output 8 when executed.
- (B) The program will not compile successfully.
- (C) The program will compile successfully and output 10 when executed.
- (D) The program will compile successfully and output 13 when executed.

56. Consider the following ANSI C function:

```
int SimpleFunction(int Y[], int n, int x)
{
    int total = Y[0], loopIndex;
    for (loopIndex = 1; loopIndex <= n - 1; loopIndex++)
        total = x * total + Y[loopIndex];
    return total;
}
```



Let Z be an array of 10 elements with $Z[?] = 1$, for all i such that $0 < i < 9$. The value returned by `SimpleFunction(Z, 10, 2)` is _____.

57. Consider the following ANSI C function:

```
int SomeFunction (int x, int y)
{
    if (x==1) || (y==1)  return 1;
    if (x==y)  return x;
    if (x>y)  return SomeFunction(x-y,y);
    if (y>x)  return SomeFunction(x,y-x);

}
```

The value returned by `SomeFunction(15, 255)` is _____.

58. Consider the following ANSI C program.

```
#include <stdio.h>
int foo(int x, int y, int q)
{if ((x <= 0) && (y <= 0))
    return q;
if (x <= 0)
    return foo(x, y-q, q);
if (y <= 0)
    return foo(x-q, y, q);
return foo(x, y-q, q) + foo(x-q, y, q)

int main()
{
    int r = foo(15,15,10);
    printf ('%d', r);
    return 0;
}
```

The output of the program upon execution is _____.

59. Consider the following ANSI C program:

```
#include <stdio.h>
#include <stdlib.h>
struct Node{
    int value;
    struct Node *next;
};

int main()
{
    struct Node *boxE, *head, *boxN; int index = 0;
    boxE = head = (struct Node *) malloc(sizeof(struct Node));
    head->value = index;
    for (index = 1; index <= 3; index++)
    {
        boxN = (struct Node *) malloc(sizeof(struct Node));
        boxE->next = boxN;
        boxE = boxN;
    }
}
```



```
boxN->value = index;
boxE = boxN; }
for (index = 0; index <= 3; index++) {
printf ("Value at index %d is %d\n", index, head->value);
head = head->next;
printf ("Value at index %d is %d\n", index+1, head->value); }
```

Which one of the statements below is correct about the program?

- (A) Upon execution, the program goes into an infinite loop.
- (B) It has a missing return which will be reported as an error by the compiler.
- (C) Upon execution, the program creates a linked-list of five nodes.
- (D) It dereferences an uninitialized pointer that may result in a run-time error.

60. Consider the following ANSI C program:

```
int main( )
{ int a[4][5];
int i, j;
for (i = 0; i < 4; i++)
for (j = 0; j < 5; j++)
a[i][j] = 10 * i + j;
printf ("%d", *(a[1] + 9)); }
```

Find the output of the above problem

- (A) 20
- (B) 24
- (C) 30
- (D) 14

Solutions

1 (B)

Ist for loop iterates from $i = \frac{n}{2}$ to n there by incrementing i by 1 each time therefore, $\frac{n}{2}$ iteration for the Ist loop.

IInd for loop (nested) iterates from $j = 2$ to n but j gets doubled each time therefore, $\log_2 n$ iteration for the IInd loop.

Example :

j	Iteration
2	1
2^2	2
2^3	3
:	:
$2^{\log_2 n} = n$	$\log_2 n$

Inside IInd loop $\frac{n}{2}$ value gets added to k for every iteration, k having initial value 0.

Therefore total iterations will be $\frac{n}{2}$ (from Ist loop) $\times \log_2 n$ (from IInd loop) i.e. $\frac{n \log_2 n}{2}$.



Hence, final value k will be $\frac{n}{2} \times \frac{n \log_2 n}{2} = \frac{n^2 \log_2 n}{4} = O(n^2 \log_2 n)$

Hence, the correct option is (B).

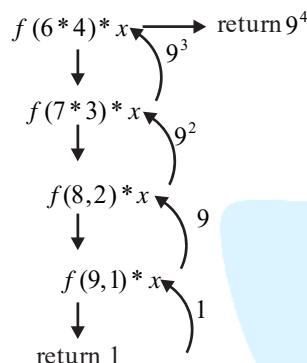
2 (B)

$$f(p, p) \Rightarrow f(5, 5)$$

$x \rightarrow$ call by reference

$c \rightarrow$ call by value

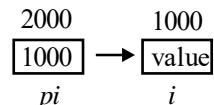
Therefore, last updated value of x will be used for each multiplication, after all the modification are done as it was call by reference.



Hence, the correct option is (B).

3 (D)

$$*pi = &i$$



pi is pointer variable which is assigned address of *i*

`scanf ("%d", pi)` `scanf` stores the input by using address of memory location, it gets through the pointer variable and hence the value is stored at the address of *i* variable (address stored in *pi* pointer)

`printf ("%d", i+5);` it will print the value that is given by user and adding (+5) into it

On execution, the value printed is 5 more than the integer value entered

Hence, the correct option is (D).

4 (A)

Binary representation 435 is 110110011

∴ MSB (most significant bit) in binary representation is 9th.

count = 0 initialized

while loop terminate when condition is false or 0,

count++; value of count incremented by 1

num >>= 1 is right shift and keeps shifting until num loses MSB which is 1 and hence at last there are no 1's available i.e. num becomes 0.

So, we keep shifting num till, num = 000000000 and increment then value of count, which result $c = 9$
Hence, the correct answer is 9.

5 (B)



Subtracting C from temp takes place in the last statement of the loop but, addition of C already took place when temp was assigned the value in the first statement of the loop so there is no effect because of C variables the result is same.

Regarding swapping the values of $A[i][j]$ with $A[j][i]$, it seems to give transpose but not

$$\therefore \begin{aligned} i &\rightarrow 1 \text{ to } n \\ j &\rightarrow 1 \text{ to } n \end{aligned}$$

Because of which there will be no effect of swapping values.

For example,

$$A[3][5] \leftrightarrow A[5][3] \quad \text{swapping takes place}$$

when $i = 3, j = 5$

but when $i = 5$ and $j = 3$, then again swapping takes place as a result it leads to no change in matrix at last.

Hence, we get same matrix itself.

Hence, the correct option is (B).

6 (B)

$n(n-1)(n-2)$ may be a big number which can go out of range. To avoid that, we break it into multiple parts.

$n(n-1)$ will be having one number even for sure. So, we won't lose precision if we divided it by 2.

Therefore first part will be

$$\frac{n(n-1)}{2}$$

and for rest of the part, we can later multiply it by $(n-2)$ and at last divided by 3.

Hence, the correct option is (B).

7 (A)

for ($i = 0; i < \text{size}; i++$)

$Y = Y + E[i];$ calculate the sum of all elements of array $E[]$, and store it to y

for ($i = 0; i < \text{size}; i++$)

 for ($j = i; j < \text{size}; j++$)

 {

$z = 0;$

 for

$(k = 1, k \leq j; k++)$

$Z = Z + E[k];$

 calculating sum of all possible subarray by starting from index i to j

 if ($z > y$)

 checking sum of subarray value with sum of all element of $E[]$

$y = z$

 largest sum of sub array will be stored in y .

 }

Hence, the correct option is (A).

8 (C)

Given : For the given pseudocode, we can write the mathematical representation for the given loops as followed :

$$\Rightarrow \sum_{i=1}^n \sum_{j=i}^n \sum_{k=j+1}^n 1$$



$$\begin{aligned}
 &\Rightarrow \sum_{i=1}^n \sum_{j=i}^n (n-j) \\
 &\Rightarrow \sum_{i=1}^n \sum_{j=i}^n n - \sum_{i=1}^n \sum_{j=i}^n j \\
 &\Rightarrow n \sum_{i=1}^n \sum_{j=i}^n 1 - \sum_{i=1}^n \sum_{j=i}^n j \\
 &\Rightarrow n \left(\sum_{i=1}^n (n-i+1) \right) - \sum_{i=1}^n \left(\frac{n(n+1)}{2} - \frac{i(i-1)}{2} \right) \\
 &\Rightarrow n^3 - \frac{n^2(n+1)}{2} + n^2 - \frac{n^2(n+1)}{2} + \frac{1}{2} \sum_{i=1}^n i^2 - \frac{1}{2} \sum_{i=1}^n i \\
 &\therefore \sum i^2 = \frac{n(n+1)(2n+1)}{6}, \sum i = \frac{n(n+1)}{2} \\
 &\Rightarrow n^3 - n^2(n+1) + n^2 + \frac{n(n+1)(2n+1)}{12} - \frac{n(n+1)}{4} \\
 &\Rightarrow \frac{n^3 - n^3 - n^2 + n^2 + n(n+1)}{4} \left(\frac{2n+1}{3} - 1 \right) \\
 &\Rightarrow \frac{n(n+1)(2n+1-3)}{12} \\
 &\Rightarrow \frac{2(n-1)n(n+1)}{12} \\
 &\Rightarrow \frac{(n-1)n(n+1)}{6}
 \end{aligned}$$

Hence, the correct option is (C).

9 (1.73)

$$\begin{aligned}
 &\text{abs}(x*x - 3) < 0.01 \\
 &\text{So, } x^2 - 3 < 0.01 \text{ and } -(x^2 - 3) < 0.01 \\
 &\Rightarrow x^2 < 3.01 \text{ and } x^2 > 2.99 \\
 &\Rightarrow x < 1.735 \text{ and } x > 1.729
 \end{aligned}$$

Corrected to 2 decimal places answer should be 1.73 or 1.74.

Solving the else part :

$$\frac{x}{2} + \frac{3}{2x} = \frac{x^2 + 3}{2x}$$

So, the new value of x will be $\frac{x^2 + 3}{2x}$ and we need it equal to x.

$$\begin{aligned}
 \frac{x^2 + 3}{2x} &= x \\
 x^2 + 3 &= 2x^2 \\
 x^2 &= 3 \\
 x &= 1.732
 \end{aligned}$$

Hence, the answer is 1.73.

**10 (D)**

if condition is true only when $i = j$ (equal) so let initial

```
j = 50
if (50 == 50)
{
    printf("something")
    k = f(i)
}
```

*here $j(50)$ is called again and again recursively, infinite time
which leads to stack overflow*

Hence, the correct option is (D).

11 (B)

Let search element be 2 (i.e. x)

List A →

0	1	2
1	2	3

```
n = 3
i = 0; j = 3 - 1 = 2
do
{
    k = (i + j) / 2           middle value of i, j
    if(x <= list A[k])
        if(2 <= list [1])   true
        j = k - 1 = 1 - 1 = 0
    if(list A[k] <= x)
        i = k + 1 = 1 + 1 = 2
}
while          (i <= j)  false
if(list A[k] == x)
    return k
```

So, we can say that this function is iterative implementation of binary search.

Hence, the correct option is (B).

12 (-5)

$f_1(a, b);$ call by value, so there is no effect on actual parameter

$f_2(&b, &c);$ call by reference, therefore value stored at address of b and c gets swapped

As a result value of variables b and c only gets swapped.

$\text{printf}(c - a - b); \quad 5 - 4 - 6 = -5$

Hence, the correct answer is -5.

13 (51)

5	1
n	x

$x = x + \text{fun}(k) * \text{fun}(n - k)$

if ($n == 1$) return x.



So let calculate value for $n=1$, to 5

```
if  $n=1$ 
fun (1)=1
if  $n=2$ ,  $x=1$ 
if ( $n==1$ )           false
for ( $k=1, k < 2; ++k$ )      so loop else 2 time
     $x = x + \text{fun}(k) \times \text{fun}(n-k);$ 
    =  $1 + \text{fun}(1) \times \text{fun}(2-1);$           fun (1) = 1
    =  $1 + \text{fun}(1) \times \text{fun}(1);$ 
    =  $1 + 1 \times 1 = 2$            .. fun (2) = 2
```

Similarly for $n=3$

$$\text{fun}(3) = 1 + \text{fun}(1) * \text{fun}(2) + \text{fun}(2) * \text{fun}(1) = 5$$

for $n=4$

$$\begin{aligned} \text{fun}(4) &= 1 + \text{fun}(1) * \text{fun}(3) + \text{fun}(2) * \text{fun}(2) + \text{fun}(3) * \text{fun}(1) \\ &= 1 + 5 + 4 + 5 = 15 \end{aligned}$$

for $n=5$

$$\begin{aligned} \text{fun}(5) &= 1 + \text{fun}(1) * \text{fun}(4) + \text{fun}(2) * \text{fun}(3) + \text{fun}(3) * \text{fun}(2) + \text{fun}(4) * \text{fun}(1) \\ &= 1 + 15 + 10 + 10 + 15 = 51 \end{aligned}$$

Hence, the correct option is (51).

14 (D)

$$s1[7] = \boxed{\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \end{array}}$$

$$p = s1 + 2;$$

p holds address of 3rd element

$$*p = '0'$$

value at the address pointed by pointer gets change to 0

$$\boxed{\begin{array}{cccc} 1 & 2 & \cancel{3} & 4 \\ & & 0 & \end{array}}$$

printf(s1)

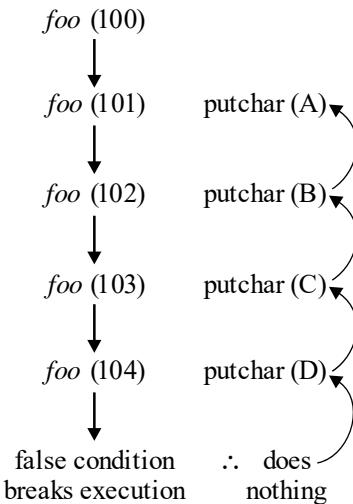
$$s1 = \boxed{\begin{array}{cccc} 1 & 2 & 0 & 4 \end{array}}$$

Hence, the correct option is (D).

15 (D)

$$\begin{aligned} *a \rightarrow & \boxed{\begin{array}{cccccccccc} A & B & C & D & & E & F & G & H \\ 100 & 101 & 102 & 103 & 104 & 105 & 106 & 107 & 108 \end{array}} \\ & (*a \& \& *a != ' ') \end{aligned}$$

As priority of != is greater than of &&, so it can be return as $((*a) \& \& (*a != ' '))$



Hence, the correct option is (D).

16 (B)

Given : In the given program x is divided by y through successive subtraction where at last r holds remainder and q holds quotient hence, remainder will be less than y i.e. $r < y$ consequently giving $x = qy + r$.

Hence, the correct option is (B).

17 (A)

1	2	3
00	04	08
4	5	6
12	16	20
7	8	9
24	28	32
10	11	12
36	40	44

Key Point

In 2D array

$x \rightarrow$ base address of 2D array \rightarrow address of first row.

$*x \rightarrow$ first row \rightarrow address of first element of first row.

$**x \rightarrow$ value of first element of first row.

$x+3$ address of Ist row +3

Will give address of IVth row & $x[3]$

$*(x+3)$ will return row $x[3]$ which indirectly represents address of first integer of IVth row i.e. 2024

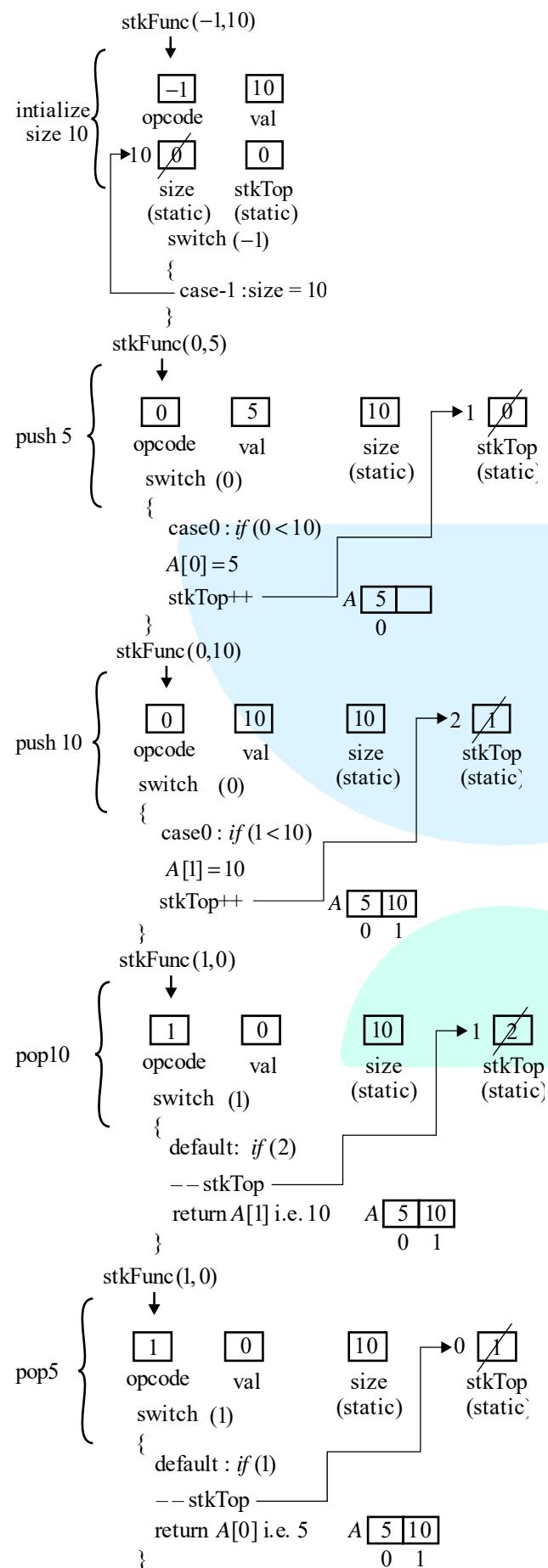
$*(x+2)$ similarly will give address of Ist integer of IIIrd row

Hence $*(x+2)+3$ {or $x[2]+3$ } will give address of 4th integer after skipping 3 integers starting from first Integer of IIIrd row $\Rightarrow 2036$

Hence, the correct option is (A).

18 (15)

Given : For the given code snippet, we draw the diagrammatical representation for the function calls, starting with initial condition, stkTop = -1





`printf(10 + 5)`

Hence, the correct answer is 15.

19 (140)

Given: `a` is an integer array `{10, 20, 30, 40, 50}`

`p` is an array of pointer to integer (Storing address of corresponding array element)

`ptr` stores the address of first element of `p` array

consider int size to be 4 byte

a

10	20	30	40	50
100	104	108	112	116

p

a	$a + 3$	$a + 4$	$a + 1$	$a + 2$
100	100+3	100+4	100+1	100+2
112	116	104	108	
200	208	216	224	232

Considering address of memory location to be 8 bytes

`ptr = p = address of first element of p = 200`, therefore `p = 200`

$\text{ptr}^{++} \Rightarrow 200 \rightarrow \begin{matrix} 208 \\ \downarrow \\ \text{next element's address} \end{matrix} \quad i.e. 208 \quad \text{ptr}$

$\text{ptr} - p$ (address - address \Rightarrow number of elements in between the address of compatible types)

$**\text{ptr} \Rightarrow *(*208)$

$\Rightarrow *(112)$

$\Rightarrow 140$

Hence, the correct answer is 140.

20 (B)

Given: $n = 6$, we need to find number of calls to `get()`

Let number of calls made by `get(n)` is given by $T(n)$

$\therefore T(n) = T_{(n-1)} + T_{(n-3)} + 1$ (1 is for the call when `get` function gets called itself)

Now

$$T(-2) \Rightarrow 1$$

$$T(-1) \Rightarrow 1$$

$$T(0) \Rightarrow 1$$

$$T(1) \Rightarrow T(0) + T(-2) + 1$$

$$\Rightarrow 1 + 1 + 1 = 3$$

$$T(2) = T(1) + T(-1) + 1$$

$$3 + 1 + 1 = 5$$

$$T(3) = T(2) + T(0) + 1$$

$$5 + 1 + 1 = 7$$

$$T(4) = T(3) + T(1) + 1$$



$$\begin{aligned}7+3+1 &= 11 \\T(5) &= T(4)+T(2)+1 \\11+5+1 &= 17 \\T(6) &= T(5)+T(3)+1 = 17+7+1 = 25\end{aligned}$$

Hence, the correct option is (B).

21 (10)

```
k = 0
j = 2 * 3 / 4 + 2.0 / 5 + 8 / 5 = 2           //type cast to int
k -= --j = k - (--j) = 0 - (1) = -1, //decreases j value to 1
for i=0
    switch (-1)                                //switch (0 + (-1))
        default:
            printf(i+k)
            printf(0 + (-1))
            printf(-1)

for i=1
    switch (0)
        default:
            printf(0)

for i=2
    Switch (1)
        case 1:
        case2:          printf(1)
        case3:          printf(1)
        default :       printf(1)

for i=3
    switch (2)
        case 2:          printf(2)
        case 3:          printf(2)
        default:         printf(2)

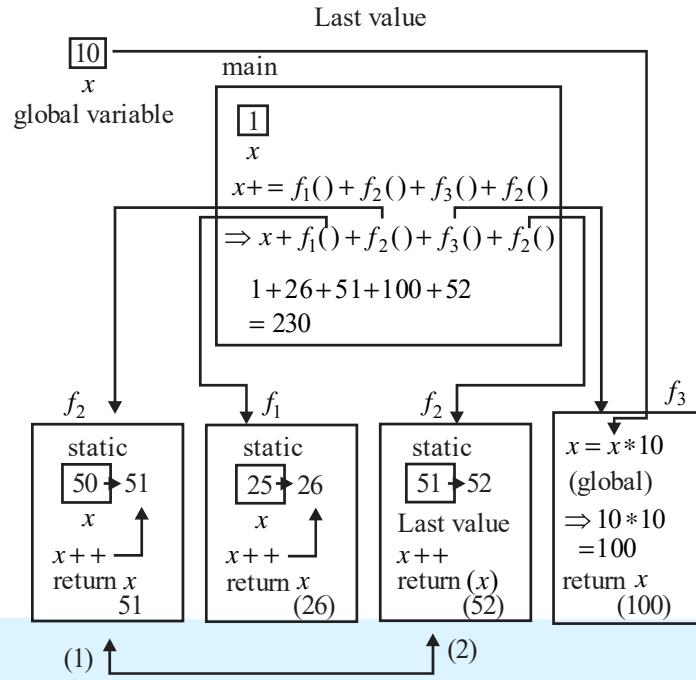
for i=4
    switch (3)
        case 3:          printf(3)
        default :        printf(3)
```

Number of time print statement is executed is 10

Hence, the correct answer is 10.

22 (230)

Given : For the given code snippet, we draw the following representation for the function calling



Hence, the correct answer is 230.

23 (D)

Option (A) is wrong, because S is not a pointer (*) variable as defined in main (), hence, it results in type checking error.

Option (B) is wrong, because no value will be returned by $f(i, s)$ since it is void type. So assignment is wrong.

Option (C) is wrong, because S is defined as “short” type variable, so passing argument as pointer will result in syntax error.

Option (D) is true because both the argument that passed from main (), match the prototype of function f.

Hence, the correct option is (D).

24 (2016)

Given :

mystery function takes two address main local variables, but inside mystery function it simply swaps the pointer values (Not changing values at corresponding address stored in pointers). Hence any call to mystery function is not making any change to main is local variable.

∴ Values in variable keep intact after all the calls to mystery ().

Hence, the correct answer is 2016.

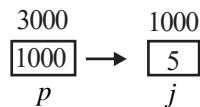
25 (30)

- 1000 2000

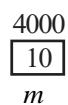


- $f(&i, j);$

- $*p = \&i$ p is pointer, holding address of i



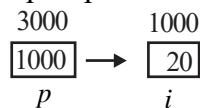
4. $m = j$



5. $m = m + 5 = 10 + 5 = 15$



6. $*p = *p + m = 5 + 15 = 20$

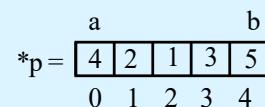


7. `printf(30)`

The value printed by the following program is 30.

Hence, the correct answer is 30.

26 (D)



Here a points to starting element index of array and b points to last element index of array and while scanning, a will keep incrementing and b will keep decrementing based on corresponding condition until both points to same element index, as a result returning highest value.

Hence, the correct option is (D).

27 (C)

```

count (3)
[3]      [1]
n          d (static)

print (3)
print (1)
d++;
[2]
if (3>1)      d    true,
count (2)
[2]      [2]
n          d (static)

print (2)
print (2)
d++;
[3]
if (2>1) true      d
count (1)

```



1	3
n	d

```

print(1)
print(3)
d ++
        4
        d

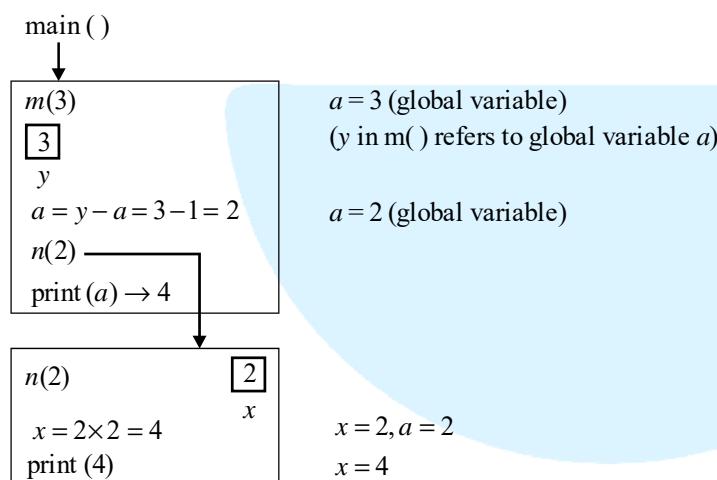
if(1>1)           false
    print(4)

```

The output of the program is 3122134

Hence, the correct option is (C).

28 (D)



Since dynamic scoping is used, also parameter are passed by reference for variable `a` which is global variable so final output is 4, 4

Hence, the correct option is (D).

29 (C)

Given: we need to find x^4 using `res`, `a` and `b` variable.

Lets take an example where $x = 5, y = 15$

Hence initial condition becomes $a = 5, b = 15 \& res = 1$

res	a	b
$1 \times 5 = 5$	5	$15 - 1 = 14$
5	$5 \times 5 = 5^2$	$14 / 2 = 7$
$5 \times 5^2 = 5^3$	5^2	$7 - 1 = 6$
5^3	$5^2 \times 5^2 = 5^4$	$6 / 2 = 3$
$5^3 \times 5^4 = 5^7$	5^4	$3 - 1 = 2$



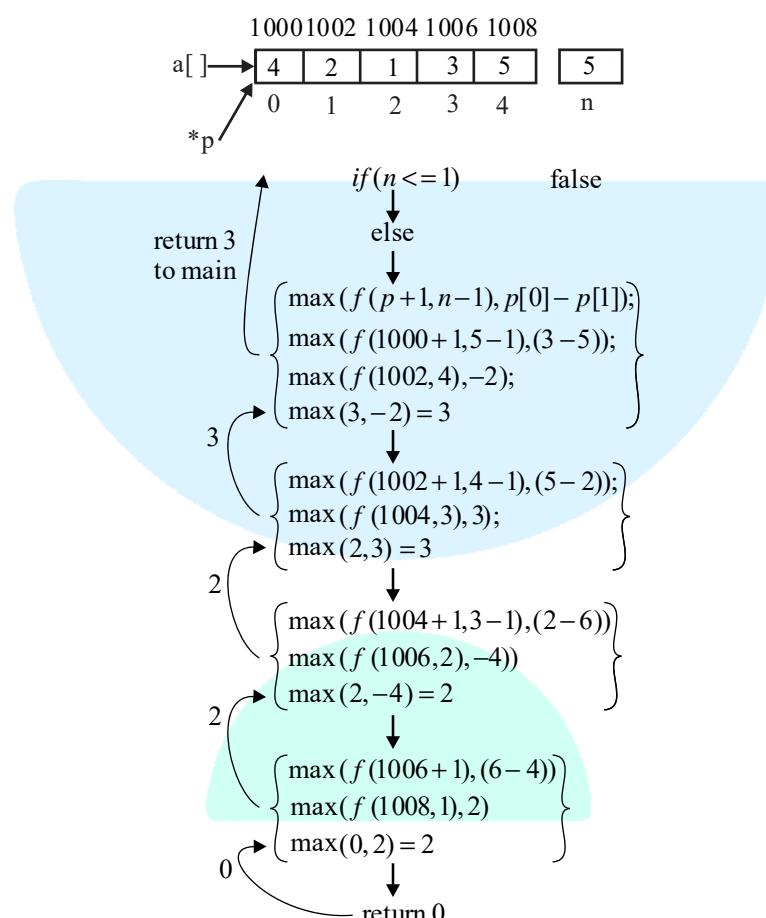
5^7	$5^4 \times 5^4 = 5^8$	1
$5^7 \times 5^8 = 5^{15}$	5^8	0

As loop proceeds, we can clearly find the pattern that if we multiple a^b to res at any point of iteration we get 5^{15} which is x^y we need.

$$\therefore x^y = \text{res} \times a^b$$

Hence, the correct option is (C).

30 (3)



Hence, the correct answer is 3.

31 (D)

Option A : In C++ we need to do typecasting. C does automatic implicit typecasting.

C compiler will work fine but C++ compiler will give error.

Option B : Null means address 0. if ($a==0$) if ($0==a$) There is no difference.

Option C : As x is always pointing to a valid memory location. Dangling pointer means if it points to a memory location which is deleted(freed). So no dangling pointer.

Option D : x will lose the previous address it was pointing to. So it will result in memory leak.
Hence, the correct option is (D).

32 (C)

Given : call to print (x, y) function is made with input of $x & y$ being 1 and 1 respectively



$\therefore x = 1$
 $y = 1$

	x	y	*ptr
input	1	1	garbage
$x = 0;$	0	1	garbage
$p \text{ tr} = \& x;$	0	1	0
$y = *p \text{ tr}$	0	0	0
$*p \text{ tr} = 1$	1	0	1 (\because \text{ptr holds the address of } x)
\therefore Final value of (x, y) getting printed is 1, 0			

Hence, the correct option is (C).

33 (A)

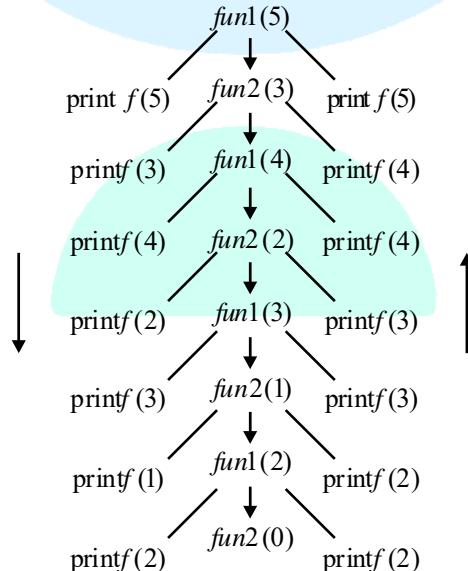
- P. static char var = A variable located in data section of memory
- Q. m = malloc (10); m = NULL; here free (m) is missing, so A lost memory is there which cannot be freed.
- R. char * ptr [10] = Sequence of memory location to store address.
- S. register int varl = Request to allocate a CPU register to store dates.

Hence, the correct option is (A).

34 (A)

Note : From fun1(), fun2() gets called but it do not modifies value of n for the statements followed afterward in fun1(). Whereas, from fun2() when fun1() gets called it first increments the value, therefore we get incremented value of n for the statements followed afterward in fun2().

Given : For the given code snippet, we draw the following representation for the function calling.



The output printed when fun1 (5) is 53423122233445

Hence, the correct option is (C).

35 (C)

```

foo (3)
  [3] [0]
  val   x
while (3 > 0)           true
  {
  
```



$x = 0 + \text{foo}(3)$ value is post decrement
}

That means foo function gets called recursively with same value & post decrement operation is performed on val, so the value 3 is passed & val is decremented later.

Hence, we got an infinite sequence of $\text{foo}(3) \rightarrow \text{foo}(3) \rightarrow \text{foo}(3) \dots$

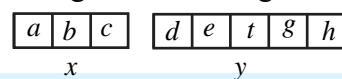
At some point, the stack would overflow hence abnormal termination

$\text{bar}(3)$ will call $\text{bar}(2)$, $\text{bar}(1)$ when called because of recursion but for any particular instance when $\text{bar}(n)$ is called the counter variable “val” for the while loop is not getting modified. Therefore, it leads infinite loop for each call to $\text{bar}(n)$ i.e. $\text{bar}(3)$, $\text{bar}(2)$, $\text{bar}(1)$ all are having infinite loop.

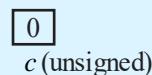
Hence, the correct option is (C).

36 (3)

`strlen` :- This function is used to find the length of the string



`printlength(x, y)`



Key Point

$(\text{ex1?ex2:ex3}) \Rightarrow \text{ex1}$ is condition, if `ex1` is true `ex2` will be executed, if false `ex3` will be executed.

$$\begin{aligned} & ((\text{strlen } (\text{s}) - \text{strlen } (\text{t})) > \text{c}) ? \text{strlen } (\text{s}) : \text{strlen } (\text{t}) \\ & = ((3 - 5 > 0)? 3 : 5) \\ & = ((-2 > 0)? 3 : 5) \end{aligned}$$

Here $(-2 > 0)$ will be returning 1 since `c` is unsigned. So 3 will assigned to len.

Hence, the correct answer is 3.

37 (23)

Note : count being static variable will retain its value throughout the function calls to function total ()

Key Point

& operator if used like `v&1` return whether least significant bit of `v` is 1 or not if 1 then returns 1 else returns 0

100100	100101
Ex: $\underline{\&000001} \rightarrow 0$ (if lsb is 0)	$\underline{\&000001} \rightarrow 1$ (if lsb is 1)
$\underline{000000}$	$\underline{000001}$

In total function

`count += v & 1` → keep adding 1 to count value if lsb of variable `v` is 1

`v >> 1` → keep shifting the bits towards RHS by 1 bit & keep iterating till `v` becomes 0

as a result it returns the number of 1+ initial value of count depending upon number of calls made to total functions

Now for loop inside mains proceeds as followed

x_i = value of `x` before addition



x_f = value of x after addition

i	x_i	count (static variable inside total function)	Total (i) (also the last value of count)	x_f
5	0	0	$0+2$ (# bits in 5)= 2	$0+2=2$
4	2	2	$2+1$ (# bit in 4)= 3	$2+3=5$
3	5	3	$3+2$ (# bits in 3)=5	$5+5=10$
2	10	5	$5+1$ (# bits in 2)= 6	$10+6=16$
1	16	6	$6+1$ (# bits in 1) = 7	$16+7=23$

Hence, the correct answer is 23.

38 (C)

Given : $x == (y * q + r)$, must hold at the end of the while loop

x = actual number/dividend

y = multiplicand/divisor

q = quotient

r = start value for the division/remainder/final remaining value

To divide a number with repeated subtraction, quotient should be initialized to 0 and should be incremented for each subtraction, and the actual number must be initialized with r .

So, if $q = 0$, then $x = r$

Hence, the correct option is (C).

39 (3)

Given : array[] = {3, 5, 1, 4, 6, 2}

Note: if we observe carefully first for loop moves minimum value towards RHS & second for loop moves maximum towards right.

while loop's Ist iteration (\because done = 0)

done = 1

Ist for loop's iterations

Counter		Array					
i	3	5	1	4	6	2	
0	5	3	1	4	6	2	done = 0
1	5	3	1	4	6	2	no change
2	5	3	4	1	6	2	done = 0
3	5	3	4	6	1	2	done = 0
4	5	3	4	6	2	1	done = 0

IInd for loop's iterations

Counter		Array					
i	5	3	4	6	2	1	
5	5	3	4	6	2	1	no change
4	5	3	4	6	2	1	no change
3	5	3	6	4	2	1	done = 0



```

2      5      6      3      4      2      1    done = 0
1      6      5      3      4      2      1    done = 0

```

while loop's IInd iteration (\therefore done = 0)

done = 1

Ist for loop's iterations

Counter	Array						
i	6	5	3	4	2	1	
0	6	5	3	4	2	1	no change
1	6	5	3	4	2	1	no change
2	6	5	4	3	2	1	done = 0
3	6	5	4	3	2	1	no change
4	6	5	4	3	2	1	no change

IInd for loop's iterations

As the array is sorted now so it will not change/swap array element.

As last value of done = 0

Therefore there will be again IIIrd iteration of while loop, but as it enters it modifies the variable done as done = 1 after which no for loop will change/swap array elements and hence last value of done remains to be 1

\therefore When it wants to check while condition for 4th iteration to begin it fails, as done = 1

\therefore final array =

6	5	4	3	2	1
---	---	---	---	---	---

Array [3] $\Rightarrow 3$

Hence, the correct answer is 3.

40 (0)

$m = 10$

$n = ++m;$



$n1 = m++;$



$n--;$



$--n1;$



$n = n - n1;$

n_1

$n = n - n1 = 10 - 10 = 0$

`printf(0);`

Hence, the correct answer is 0.

41 (2)

$C = \begin{array}{cccccccccccccc} G & A & T & E & C & S & I & T & 2 & 0 & 1 & 7 & \backslash 0 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}$



* $p = c$ p points base address of c
 $\text{strlen} (c + 2[p] - 6[p] - 1)$
 $c = \text{base address}$
 $i[p] = p[i] = *(p + i)$
 $2[p] = \text{base address} + 2 = T$
 $6[p] = \text{base address} + 6 = I$
 $c + T - I - 1$ ($T - I$, returns difference between corresponding ascii values)
 $c + 11 - 1$
 $c + 10$

It will calculate the length of string from location (base address +10) upto last (excluding Null character)

$\text{strlen}(17) = 2$

Hence, the correct answer is 2.

42 (A)

```
char x='1';
char y='0'
char z='a'+2;           \\ we can only store char in Ournode so first 'a' +2 convert into char (
'a'+2='c')
Ournode = {'1', '0', 'c'}
*q = &p ⇒ address structure variable is stored in Ournode pointer
(char *) q ⇒ structure address is being type casted into character address
```

Key Point

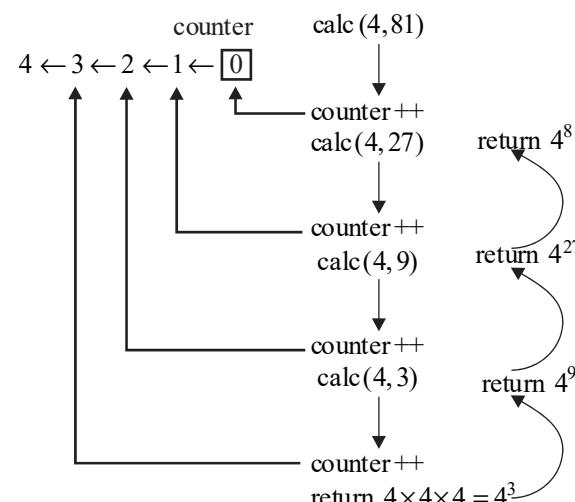
Address of any datatype + Any integer, returns address of data of similar data type from successive location by skipping some number of elements.

* $((\text{char} *) q + 1) \Rightarrow 0$
* $((\text{char} *) q + 2) \Rightarrow c$

Hence, the correct option is (A).

43 (4)

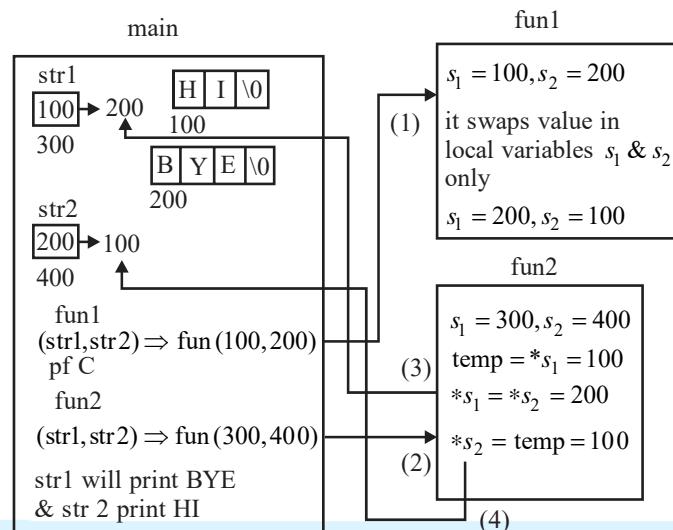
Given : counter is a global variable.





Hence, the correct answer is 4.

44 (A)



Apparently we can say fun1 is working call value & fun2 is working call by reference.
Hence, the correct option is (A).

45 (B)

Given $n = 2^{40}$

Ist for loop works as followed:

i	2^{40}	2^{39}	2^{38}	---	2^1	2^0
j	1	2	3	----	40	Exit

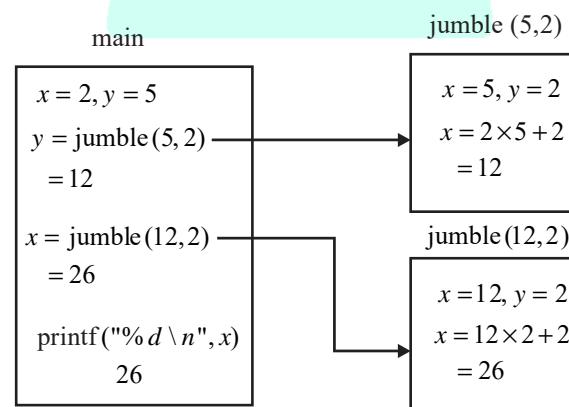
∴ Last value of $j = 40$

IInd for loop works as followed:

j	40	20	10	5	2	1
Sum	1	2	3	4	5	Exit

Hence, the correct option is (B).

46 (26)



Hence, the correct answer is 26.

47 (6)

Given : ip is an integer point.

Key Point



Pointer variable can also act like array variable, example if `char * p` is a declaration then `p[1], p [2]` will work like array variable and will return corresponding elements from contiguous addresses.

**ip = arr + 4*

ip stores the address of 5th element.

As $ip[1] \Rightarrow * (ip + 1)$

$\therefore (ip + 1)$ will refer to address of 6th element

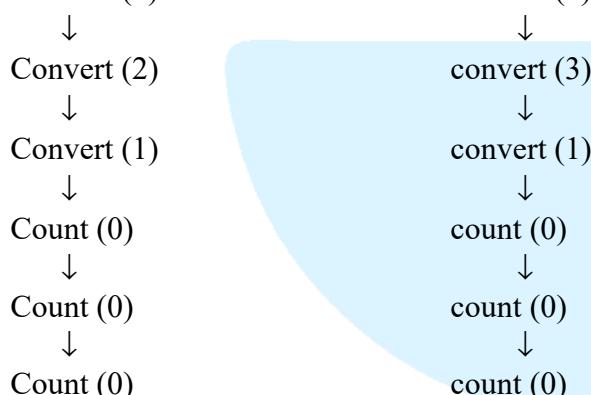
Here $*(ip + 1) \Rightarrow$ 6th element of array = 6

Hence, the correct answer is 6.

48 (D)

Given : convert (n) will keep calling itself recursively till it gets input value of n as negative number. Dividing any number by 2 respectively may approach it zero but never make it negative. Hence it will keep calling itself recursively & don't print anything.

Ex. Convert (5)



Hence, the correct option is (D).

49 (B)

Given : for $(r(); r(); r())$

General for loop (ex1; ex2; ex3)

- ex1 : for initialization
 - ex2 : for condition checking, if it is true (if non 0 value), then body of for loop will be executed, if it is false (0) terminate the for loop.
 - ex3 : defines how for loop will iterate (modification i.e., increment, decrement, etc.)

```

graph LR
    A[for (r();) initialization] --> B[↓]
    B --> C[condition]
    A --> D[↓]
    D --> E[modification]
    C --> F[↓]
    F --> G[modification]
    style C fill:none,stroke:none
    style D fill:none,stroke:none
    style F fill:none,stroke:none
    style G fill:none,stroke:none
    
```

The diagram illustrates the structure of a for loop. It consists of three main parts: initialization, condition, and modification. The initialization part is labeled "for (r()); initialization" with an arrow pointing down to "↓". The condition part is labeled "condition" with an arrow pointing down to "↓". The modification part is labeled "modification" with an arrow pointing down to "↓". Below the condition and modification labels, there is a label "checking" with an arrow pointing down to "↓".

```
{  
    print (“ % d”, r ( ));  
}
```

In each call `r()`, value of `num()` will return and decrement the value of `num` and saved to memory as `num` it is static variable.

Sequence of the call to function, `r()` is as given below

for $\begin{pmatrix} 1 & 2,5,8,11 & 4,7,10 \\ r() & r() & r() \end{pmatrix}$



`printf(%d, r())3,6,9`

#call to r()	Value returned by r()
1	7
2	6
3	5
4	4
5	3
6	2
7	1
8	0
9	
10	

Check condition

Value printed

∴ Printed value or (5, 2)

Hence, the correct option is (B).

50 (5)

$i = 2.0, j = 1.0$	
$\text{while } \left(\frac{2}{1} > 0.0625\right)$ $j = j + j = 1 + 1 = 2$ Ist print	Condition True
$\text{while } \left(\frac{2}{2} > 0.0625\right)$ $j = j + j = 2 + 2 = 4$ IInd print	Condition True
$\text{while } \left(\frac{2}{4} > 0.0625\right)$ $j = j + j = 4 + 4 = 8$ IIInd print	Condition True
$\text{while } \left(\frac{2}{8} > 0.0625\right)$ $j = j + j = 8 + 8 = 16$ IVth print	Condition True
$\text{while } \left(\frac{2}{16} > 0.0625\right)$ $j = j + j = 16 + 16 = 32$ Vth print	Condition True
$\text{while } \left(\frac{2}{32} > 0.0625\right)$	Condition False Break



Hence, the correct answer is (5).

51 (10)

Key Point

For any 1-D array variable 'a' represents address of first element, and $a+i$ refers to address of i^{th} element of an array

$*b = a + 4;$

b pointer stores the address of $a[4]$ or 5^{th} element i.e. & $a[4]$

$*b - i \Rightarrow$ means i is getting subtracted from 5^{th} element.

$*(b - i) \Rightarrow$ means i elements before the element, having b address (we have to move left to get the element starting from b address)

For loop works as follows

i	$i < 5$	Sum_i	$*b - i$	$*(b - i)$	Sum_f
0	1 (True)	0	$10 - 0 = 10$	$5^{\text{th}} \text{ el.} = 10$	$0 + 10 + 10 = 0$
1	1 (True)	0	$10 - 9 = 9$	$4^{\text{th}} \text{ el.} = 8$	$0 + 9 - 8 = 1$
2	1 (True)	1	$10 - 2 = 8$	$3^{\text{rd}} \text{ el.} = 6$	$1 + 8 - 6 = 3$
3	1 (True)	3	$10 - 3 = 7$	$2^{\text{nd}} \text{ el.} = 4$	$3 + 7 - 4 = 6$
4	1 (True)	6	$10 - 4 = 6$	$1^{\text{st}} \text{ el.} = 2$	$6 + 6 - 2 = 10$
5	0 (false)			Exit	

Hence, the correct answer is (10).

52 (19)

Given : $a[4][5] = \{ \{1, 2, 3, 4, 5\},$

$\{6, 7, 8, 9, 10\},$

$\{11, 12, 13, 14, 15\},$

$\{16, 17, 18, 19, 20\}$

}

We need to find $*(*(a + **a + 2) + 3);$

Key Point

For a 2-D array variable 'a' it always refers to address of first row $\therefore a$ and $& a[0]$ both refers to same thing, similarly $a[i]$ means i^{th} row and it also refers to first element of $a[i]$ array (1-D) $\therefore a[i]$ and $& a[i][0]$ refers to same thing.

In short

$a \rightarrow$ address of first element of 2 D array i.e., address of 1^{st} row $\Rightarrow & a[0]$

$a + i \Rightarrow$ address of 1^{st} row + $i \Rightarrow$ address of i^{th} row (i starting from 0)

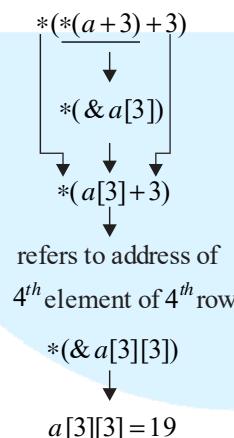
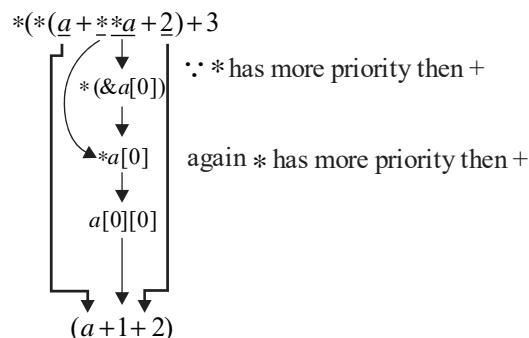
$a[i] = * (a + i) \Rightarrow$ i^{th} row or address of first element of i^{th} row



$a[i] + j \Rightarrow$ address of j^{th} element of i^{th} row

Also, $\&(*x) = *(\&x) = x$

Now



Hence, the correct answer is (19).

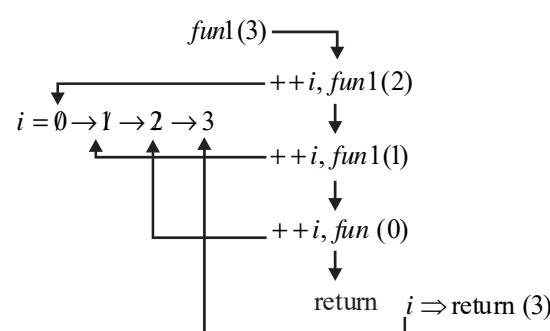
53 (55)

Given: we need to find $\text{fun2}(5)$

First we need to get what $\text{fun1}(n)$ does is it return $i+n$ where n is input given and i is the initial value during function call

\therefore it is static variable therefore its value persists during function calls.

Ex: $\text{fun1}(3) \rightarrow$



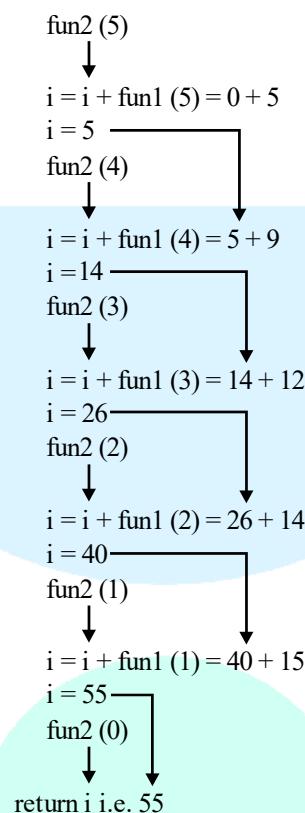
that is initial value of $i + n \Rightarrow 0 + 3 = 3$

As we know repetitive calls to $\text{fun1}()$ will be made for the n values 5, 4, 3, 2 and 1, so, lets summarize it before, through the table



-	Initial value	Final value
<i>n</i> in <i>fun1(n)</i>	<i>i</i>	<i>i</i>
5	0	5
4	5	9
3	9	12
2	12	14
1	14	15

Now, the *fun2()* function working is given below using *fun1()* return values mentioned in table given before.



So final value return *fun2(5)* is 55

Hence, the correct answer is (55).

54 (81)

Given: Need to calculate *pp(3, 4)*

$$a = 3, b = 4, \text{tot} = 1, \text{ex} = 3$$

len = *tob(b, arr)*

address of array's first element

gets passed to function *tob* here

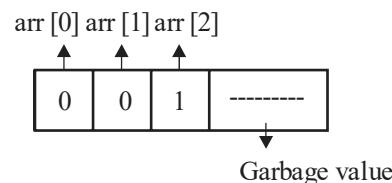
Inside *tob()* function for loop works as follow:-

i	b	loop condition	b % 2	arr [i]
0	4	$4 > 0 \Rightarrow 1$ (True)	0	$a[0] = 0$
1	2	$2 > 0 \Rightarrow 1$ (True)	0	$a[1] = 0$



2	1	$1 > 0 \Rightarrow 1$ (True)	1	$a[2]=1$
3	0	$0 > 0 \Rightarrow 0$ (False)	Exit	

So array has been initialized as



and value returned is last value of i i.e. 3

$$\therefore \text{len} = 3$$

Now inside pp(), the for loop works as follow:-

- ex_i & ex_f values refers to value of ex before & after checking if condition

i	len	loop condition	arr[i]	ex_i	If condition	Tot	ex_f
0	3	$0 < 3 \Rightarrow 1$ (True)	0	3	0	1	$3 \times 3 = 9$
1	3	$1 < 3 \Rightarrow 1$ (True)	0	9	0	1	$9 \times 9 = 81$
2	3	$2 < 3 \Rightarrow 1$ (True)	1	81	1	$1 \times 81 = \underline{\underline{81}}$	$81 \times 81 = 6561$
3	3	$3 < 3 \Rightarrow 0$ (False)		Exit			

Hence, the correct answer is (81).

55 (C)

For $j = -3$ to $j = -1$

$\text{if } ((j > 0) \& \& (i++))$ will result in false

Hence neither I nor count null change, count = 0, $i=0$

For $-j = 0$ ($j \geq 0$) result in TRUE hence $i++$ will execute

$j = 1$ ($j \geq 0 \& \& (i++) = \text{TRUE}$)

Hence, count = $0 + 1 = 1$ and $i++$

$j = 2$ ($j \geq 0 \& \& (i++) = \text{TRUE}$)

Hence count = $1 + 2 = 3$ and $i++$

$j = 3$ ($j \geq 0 \& \& (i++) = \text{TRUE}$)

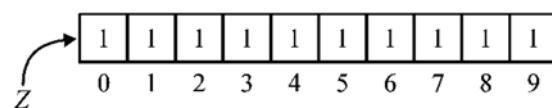
Hence, count = $3 + 3 = 6$ and $i++$

After exiting loop

$$\begin{aligned} \text{Count} &= \text{count} + i \\ &= 6 + 4 = 10 \end{aligned}$$

Hence, the correct option is (C).

56 (1023)



$$\text{total} = Y[0] = Z[0] = 1$$



$n = 10$

$x = 2$

Loop index	Total = $x * \text{total} + Y[\text{loop index}]$
1	$2 * 1 + Y[1] = 2 * 1 + 1 = 3$
2	$2 * 3 + Y[2] = 2 * 3 + 1 = 7$
3	$2 * 7 + Y[3] = 2 * 7 + 1 = 15$
4	$2 * 15 + Y[4] = 2 * 15 + 1 = 31$
5	$2 * 31 + Y[5] = 2 * 31 + 1 = 63$
6	$2 * 63 + Y[6] = 2 * 63 + 1 = 127$
7	$2 * 127 + Y[7] = 2 * 127 + 1 = 255$
8	$2 * 255 + Y[8] = 2 * 255 + 1 = 511$
9	$2 * 511 + Y[9] = 2 * 511 + 1 = 1023$

Hence, the correct answer is (1023).

57 (15)

Given :

```
int SomeFunction (int x, int y)
{
    if (x == 1) || (y == 1)) return 1;      ...(1)
    if (x == y) return x;                  ...(2)
    if (x > y) return SomeFunction(x - y, y); ...(3)
    if (y > x) return SomeFunction(x, y - x); ...(4)
}
```

Now, after calling

SomeFunction(15, 255)

{255 > 15} Therefore line (3) will execute.



SomeFunction(15, 240)

{240 > 15} Therefore line (3) will execute.



SomeFunction(15, 225)

{225 > 15} Therefore line (3) will execute.



SomeFunction(15, 210)

{210 > 15} Therefore line (3) will execute.

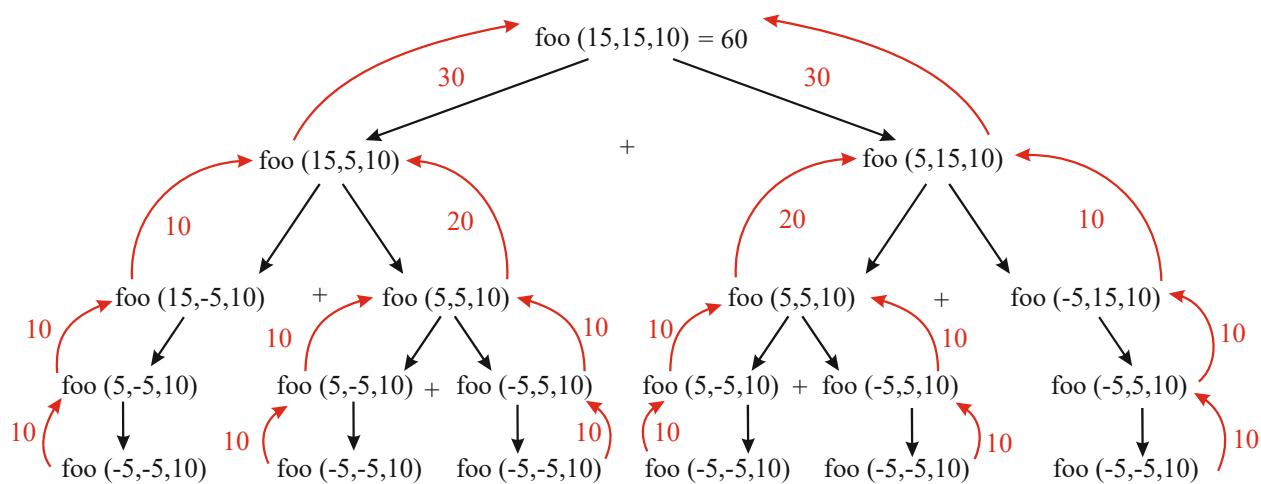
⋮

SomeFunction(15, 15)

{15 == 15} Therefore line (2) will execute.

Hence the function call will return 15

58 (60)



Hence the function call will return 60.

59 (D)

The linked list of four nodes will be created.

First node: value = 0 Second node : value = 1

Third node : value = 2 Fourth node : value = 3

The last for loop will print the index number and the values

In the last iteration when index = 3

head = head-> next // trying to access the unknown memory location because the fifth node is not there.

Hence the segmentation fault or run time error will come here.

Hence, the correct option is (D).

60 (B)

After the execution of program the content of the array will be

	[0]	[1]	[2]	[3]	[4]
[0]	0	1	2	3	4
[1]	10	11	12	13	14
[2]	20	21	22	23	24
[3]	30	31	32	33	34

$$*(a[1] + 9) = a[2][4] = 24$$

Hence, the correct option is (B).

CHAPTER 2 | Data Structures

Marks Distribution of Data Structures in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	2	7	16
2004*	6	7	20
2005*	6	10	26
2006*	4	4	12
2007*	2	11	24
2008*	1	13	27
2009	-	4	8
2010	1	2	5
2011	1	2	5
2012	-	1	2
2013	-	4	8
2014 Set-1	1	1	3
2014 Set-2	2	1	4
2014 Set-3	1	2	5

* CS and IT combined

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	2	2	6
2015 Set-2	1	3	7
2015 Set-3	3	3	9
2016 Set-1	1	2	5
2016 Set-2	2	3	8
2017 Set-1	2	1	4
2017 Set-2	1	1	3
2018	2	2	6
2019	-	1	2
2020	2	1	4
2021 Set-1	2	3	8
2021 Set-2	1	1	3

Syllabus : Data Structures

Recursion. Arrays, stacks, queues, linked lists, trees, binary search trees, binary heaps, graphs.

Contents : Data Structures

S. No. Topics

- 1.** Arrays
- 2.** Stacks and Queues
- 3.** Linked Lists
- 4.** Hashing
- 5.** Trees
- 6.** Graphs



Practice Questions

2013 IIT Bombay

**Common Data for
Questions 1.1 & 1.2**

The procedure given below is required to find and replace certain characters inside an input character string supplied in array A. The characters to be replaced are supplied in array oldc, while their respective replacement characters are supplied in array newc. Array A has a fixed length of five characters, while arrays oldc and newc contain three characters each. However, the procedure is flawed

```
void find_and_replace(char *A, char *oldc, char
                      *newc) {
    for (int i = 0; i < 5; i++)
        for (int j = 0; j < 3; j++)
            if (A[i] == oldc[j]) A[i] = newc[j];
}
```

The procedure is tested with the following four test cases

- (1) oldc = "abc", newc = "dab"
- (2) oldc = "cde", newc = "bcd"
- (3) oldc = "bca", newc = "cda"
- (4) oldc = "abc", newc = "bac"

- 1.1 The tester now tests the program on all input strings of length five consisting of characters 'a', 'b', 'c', 'd' and 'e' with duplicates allowed. If the tester carries out this testing with the four test cases given above, how many test cases will be able to capture the flaw?
 (A) Only one (B) Only two
 (C) Only three (D) All Four
- 1.2 In the above question, if array A is made to hold the string "abcde", which of the above four test cases will be successful in exposing the flaw in this procedure?
 (A) None (B) 2 only
 (C) 3 and 4 only (D) 4 only

2015 IIT Kanpur

- 1.3 A young tableau is a 2D array of integers increasing from left to right and from top to bottom, any unfilled entries are marked with ∞ , and hence there cannot be any entry to the right of, or below a ∞ . The following young tableau consists of unique entries.

1	2	5	14
3	4	6	23
10	12	18	25
31	∞	∞	∞

When an element is removed from a young tableau, other elements should be moved into its place so that the resulting table is still a young tableau (unfilled entries may be filled in with a ∞). The minimum number of entries (other than 1) to be shifted, to remove 1 from the given young tableau is _____.

- 1.4 Consider the following two C code segments. Y and X are one- and two-dimensional arrays of size n and $n \times n$ respectively, where $2 \leq n \leq 10$. Assume that in both code segments, elements of Y are initialized to 0 and each element $X[i][j]$ of array X is initialized to $i + j$. Further assume that when stored in main memory page frame.

Code segment 1:

```
//initialize elements of Y to 0
// initialize elements X[i][j] of X to i + j
for (i = 0; i < n; i++)
    Y[i] += X[0][i];
```

Code segment 2 :

```
// initialize elements of Y to 0
// initialize elements X[i][j] of X to i + j
for (i = 0; i < n; i++)
    Y[i] += X[i][0];
```

Which of the following statements is /are correct?

S1 : Final contents of array Y will be same in both code segments.



S2 : Elements of array X accessed inside the for loop shown in code segment 1 are contiguous in main memory.

S3 : Elements of array X accessed inside the for loop shown in code segment 2 are contiguous in main memory.

- (A) Only S2 is correct
- (B) Only S3 is correct
- (C) Only S1 and S2 are correct
- (D) Only S1 and S3 are correct

1.5 Suppose $c = (c[0], c[k - 1])$ is array of length k, where all the entries are from the set {0, 1}. For

any positive integer a and n, consider the following pseudocode.

```
DOSOMETHING (c, a, n)
z ← 1
for i ← 0 to k - 1
do z ←  $z^2 \bmod n$ 
if  $c[i] = 1$ 
then  $z \leftarrow (z \times a) \bmod n$ 
return z
```

if $k = 4$, $c = \{1, 0, 1, 1\}$, $a = 2$ and $n = 8$ then the output of DOSOMETHING (c, a, n) is _____.

Solutions

1.1 (B)

Given :

```
void find_and_replace(char *A, char *oldc, char
*newc) {
    for (int i = 0; i < 5; i++)
        for (int j = 0; j < 3; j++)
            if (A[i] == oldc[j]) A[i] = newc[j];
}
```

Flaw in this given procedure is that one character of array 'A' can be replaced by more than one character of newc array, which should not be so test case (3) and (4) identifies this flaw as they are containing oldc and newc array characters arranged in specific manner. Following string can reflect flaw, if tested by test case (3).

Initially, $i = j = 0$

$A = bcda$	$oldc = bca$	$newc = cda$
↑	↑	↑
$i = 0$	$j = 0$	so replaced by c

Next $i = 0$ and $j = 1$

$A = ccda$	$oldc = bca$
↑	↑
$i = 0$	$j = 1$
$newc = cda$	
↑	
$j = 1$	
$c = c$ so, replaced by d	

Likewise, character 'b' in A replaced by 'c' and then by 'd'

Same way test case (4) can also catch flaw. Hence, the correct option is (B).

1.2 (C)

Given :

```
void find_and_replace(char *A, char *oldc, char
*newc) {
    for (int i = 0; i < 5; i++)
        for (int j = 0; j < 3; j++)
            if (A[i] == oldc[j]) A[i] = newc[j];
}
```

Now for string 'abcde' in array A, both test case (3) and (4) will be successful in finding the flaw, as explained in above question.

Hence, the correct option is (C).

1.3 5

Given :

Initial Young tableau all unique entries:

1	2	5	14
3	4	6	23
10	12	18	25
31	∞	∞	∞

As per the given point in the question, if the element removed from young tableau it should be young tableau which is increasing from left to right & top to bottom.

So, when 1 is removed, smallest adjacent element should take place in such manner that there is an increasing order from left to right & top to bottom.



2	← (2)	5	14	
3	4	6	23	
10	12	18	25	
31	∞	∞	∞	

So, 2 replaced 1

Now when 2 is moved to 1, it should be replaced by smallest adjacent which is 4.

2	4	5	14	
3	(4)	6	23	
10	12	18	25	
31	∞	∞	∞	

When 4 is moved to 2, it should be replaced by its smallest adjacent which is 6.

2	4	5	14	
3	6	← (6)	23	
10	12	18	25	
31	∞	∞	∞	

When 6 is moved to 4, it should be replaced by its smallest adjacent which is 18.

2	4	5	14	
3	6	18	23	
10	12	(18)	25	
31	∞	∞	∞	

When 18 is moved to 6, it should be replaced by its smallest adjacent which is 25.

2	4	5	14	
3	6	18	23	
10	12	25	← (25)	
31	∞	∞	∞	

When 25 is moved to 18, it should be replaced by its smallest adjacent which is ∞.

2	4	5	14	
3	6	18	23	
10	12	25	∞	
31	∞	∞	∞	

So total number shifted are 2, 4, 6, 18 and 25 is (5).

Hence, the correct answer is 5.

1.4 (C)

Given : Y is one-dimensional array which is initialized with 0 for n element, $2 \leq n \leq 10$

$y[n] =$	0	0	0
	0	1		$n-1$

And $X[i][j]$ is two-dimensional array which is initialized as $i + j$, $2 \leq n \leq 10$

j	0	1	2	$n-1$
i	0	1	2	$n-1$
0	0	1	2	$n-1$
1	1	2	3	n
2	2	3	4	$n+1$
⋮	⋮	⋮	⋮	⋮
$n-1$	$n-1$	n	$n+1$	$n+n-2$

Statement S1, find value $X[0][i]$ and value of $X[i][0]$ is

Let $i = 2$

$$X[0][2] = 2$$

$$X[2][0] = 2$$

∴ S1 is true

Statement S2, $X[0][i]$

As we know in C language programming by default memory allocation is row major allocation, so X accessed the continuous memory.

Statement S3, $X[i][0]$

In this statement memory accessed is non-continuous, since it uses column order operation.

Hence, the correct option is (C).

1.5 0

Given : $k = 4$

$$c = \{1, 0, 1, 1\}, a = 2, n = 8$$

$c[4] =$	1	0	1	1
	0	1	2	3

DOSOMETHING ($c, 2, 8$)

$$z = 1$$

for $i = 0$ to 3

for $i = 0$

$$z = 1^2 \% 8$$

$$z = 1$$

if $c[0] = 1$

$$z = (1 \times 2) \% 8$$

$$z = 2$$

for $i = 1$

$$z = 2^2 \% 8$$

$$z = 4$$



```
if      c[1]=0  
for    i=2  
      z=4^2%8  
      z=0  
if      c[2]=1  
      z=(0×2)%8  
      z=0  
for    i=3
```

```
z=(0)^2 %8  
z=0  
if      c[3]=1  
      z=(0×2)%8  
      z=0  
Return 0  
Output of the program is 0.  
Hence, the correct answer is 0.
```

2

Stack and Queue



Practice Questions

2013 IIT Bombay

- 2.1 Consider the following operation along with Enqueue and Dequeue operations on queues, where k is a global parameter.

```
MultiDequeue (Q)
{
    m = k;
    while (Q is not empty) and (m > 0)
    {
        Dequeue (Q);
        m = m - 1;
    }
}
```

What is the worst-case time complexity of a sequence of n MultiDequeue operations on an initially empty queue?

- (A) $\Theta(n)$ (B) $\Theta(n + k)$
(C) $\Theta(nk)$ (D) $\Theta(n^2)$

2014 IIT Kharagpur

- 2.2 Suppose a stack implementation supports an instruction REVERSE, which reverse the order of elements on the stack, in addition to the PUSH and POP instructions. Which one of the following statements is TRUE with respect to this modified stack?

- (A) A queue cannot be implemented using this stack.
(B) A queue can be implemented where ENQUEUE takes a single instruction and DEQUEUE takes a sequence of two instructions.
(C) A queue can be implemented where ENQUEUE takes a sequence of three instructions and DEQUEUE takes a single instruction.
(D) A queue can be implemented where both ENQUEUE and DEQUEUE take a single instruction each.

2015 IIT Kanpur

- 2.3 The result evaluating the postfix expression $10\ 5 +\ 60\ 6/*8 -$ is
(A) 284 (B) 213
(C) 142 (D) 71

2016 IISc Bangalore

- 2.4 A queue is implemented using an array such that ENQUEUE and DEQUEUE operations are performed efficiently. Which one of the following statements is CORRECT (n refers to the number of items in the queue)?
(A) Both operations can be performed in $O(1)$ time
(B) At most one operation can be performed in $O(1)$ time but the worst-case time for the other operation will be $\Omega(n)$
(C) The worst-case time complexity for both operations will be $\Omega(n)$
(D) Worst case time complexity for both operations will be $\Omega(\log n)$

- 2.5 Let Q denote a queue containing sixteen numbers and S be an empty stack. Head (Q) returns the element at the head of the queue Q without removing it from Q. Similarly, Top (S) returns the element at the top of S without removing it from S. Consider the algorithm given below.

```
While Q is not empty do
    if S is Empty OR Top (S) ≤ Head (Q) then
        x := Dequeue (Q);
        Push (S, x);
    else
        x := Pop (S);
        Enqueue (Q, x);
    end
end
```

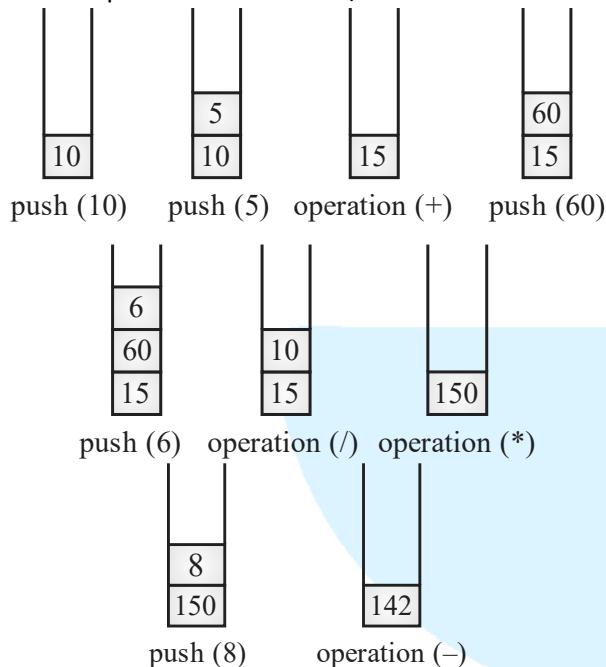
The maximum possible number of iterations of the while loop in the algorithm is _____.



Dequeue → for delete element in Queue.
So, if we perform enqueue 3 operation (1) Reverse (2) Push (3) Reverse and to dequeue an element we Pop then we can implement queue using modified stack
Hence, the correct option is (C).

2.3 (C)

Given : expression $10\ 5 + 60\ 6 * 8 -$



Hence, the correct option is (C).

2.4 (A)

Given : Implementing Queue using array such that enqueue and dequeue operation performed efficiently.

As in question, it is said that queue is implemented using array such that ENQUEUE and DEQUEUE operations are performed efficiently.

We can do both Enqueue and Dequeue operations in $O(1)$, if we fully utilize the space by circular queue implementation.

If N denote the total size of the array

FRONT as array index, where element to be deleted.

REAR as array index, after where new element to be inserted.

* For ENQUEUE

$$\text{REAR} = (\text{REAR} + 1) \bmod N$$

$\rightarrow O(1)$

* For DEQUEUE

$$\text{FRONT} = (\text{FRONT} + 1) \bmod N$$

$\rightarrow O(1)$

Condition to check whether queue is empty

$$\text{REAR} == \text{FRONT} == -1$$

Condition to check if queue is full

$$(\text{REAR} + 1) \bmod N == \text{FRONT}$$

Hence, the correct option is (A).

2.5 256

While loop will run for the maximum number of times if the Queue elements are sorted in descending order.

Suppose that initially, Queue elements are 16,15,14,13,...,2,1.

Now, 16 will be first pushed into the stack. So, now Top(S) is 16 and Head(Q) is 15, So 16 will be popped out of the stack (since, "if S is Empty OR Top(S) ≤ Head(Q)" returns false, hence else part will be executed) and enqueued into the Queue.

So, after two iterations Queue elements will be $\rightarrow 15,14,13,...,2,1,16$ and stack will be empty.

Similarly, each of the elements 15,14,13,...,2 will be pushed into the stack and immediately popped out of the stack (when popped out of the stack then also enqueue into the queue).

So, after 30 iterations stack will be empty and Queue content will be like $\Rightarrow 1,16,15,14,...,2$.

Now 1 will be Dequeued and pushed into the stack. Once 1 is pushed into the stack, it will never be popped (or we can say never be enqueued into the Queue again) because in order to Pop 1, there should be an element into the Queue which is less than 1 and that element comes at the front of the queue, since there is no element currently present in the Queue which is less than 1, there is no way to pop 1. So, after 31 iterations Queue is $\Rightarrow 16,15,14,...,2$ and stack contains 1.

Now, the problem is reduced to Queue with 15 elements.

Using the similar logic, we can say after another 29 iterations (Total = 31+29) Queue will be like $\Rightarrow 16,15,14,...,3$ and stack contains 1,2 (Top(S) is 2) and then 2 will remain there in the stack forever.

Similar way if we go on then, after $31+29+27+25+\dots+1$ iterations Queue will be empty. This is in A.P. series with $d = 2$.

$$\therefore \text{Sum} = \frac{(16 \times (1+31))}{2} = \frac{16 \times 32}{2} = 256$$



Hence, the correct answer is 256.

Key Point

$$\text{Sum of A.P.} = \frac{n}{2}(a+l)$$

Where, n is number of elements
 a is first element of series
 l is last element of series

2.6 9

Given :

Operator	Precedence	Associativity	Arity
+	High	Left	Binary
-	Medium	Right	Binary
*	Low	Left	Binary

And given expression (i) $2 - 5 + 1 - 7 * 3$

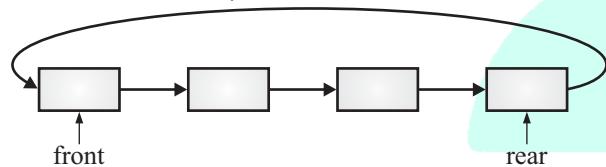
$$\begin{aligned}
 &= ((2 - ((5 + 1) - 7)) * 3 \\
 &= ((2 - (6 - 7)) * 3 \\
 &= ((2 - (-1)) * 3 \\
 &= (3 * 3) \\
 &= 9
 \end{aligned}$$

Hence, the correct answer is 9.

2.7 (B)

Given :

A circular queue is implemented using singly linked list, where front point front node of queue and rear point rear node of queue.



And pointer which pointing to next node in queue.

Statements are :

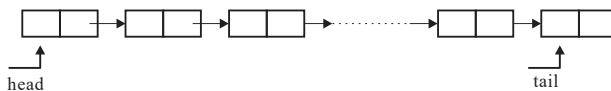
- Next pointer of front node points to the rear node.
- Next pointer of rear node points to the front node.

A pointer should point to rear to perform both insertion and deletion operation in queue in time $\Theta(1)$ if it point front deletion take $\Theta(1)$ but insertion take $\Theta(n)$, as we know in queue insertion operation is performed by rear & deletion by front.

Hence, the correct option is (B).

2.8 (B)

Given :



Insertion is performed at head it will take $\Theta(1)$ time.

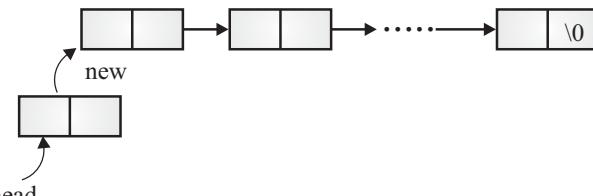
new \rightarrow [] []

Let new is a pointer to newly created node.

to insert

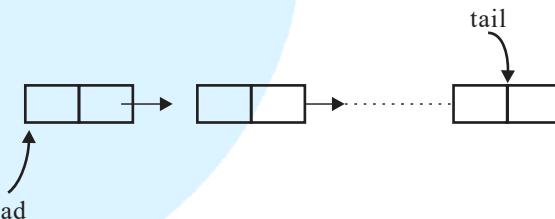
new \rightarrow next = head

head = new



Deletion is performed at tail, we have to go to one node before last node, make tail to point this.

next pointer of this node to NULL and free the last node.



While (head \neq NULL)

{

 head 2 = head 1;
 head 1 = head 1 \rightarrow next;

}

 head 2 \rightarrow next \neq NULL;
 free (head 1);
 tail = head 2;

Hence, insertion takes $\Theta(1)$ and deletion take $\Theta(n)$.

Hence, the correct option is (B).

2.9 86

Given : The following sequence of operations on an empty stack.

push(54); push(52); pop(); push(55); push(62); s = pop();

The following sequence of operations on an empty queue.

enqueue(21); enqueue(24); dequeue(); enqueue(28); enqueue(32);

q = dequeue();

s = pop() will return top of stack value, i.e.

s = 62

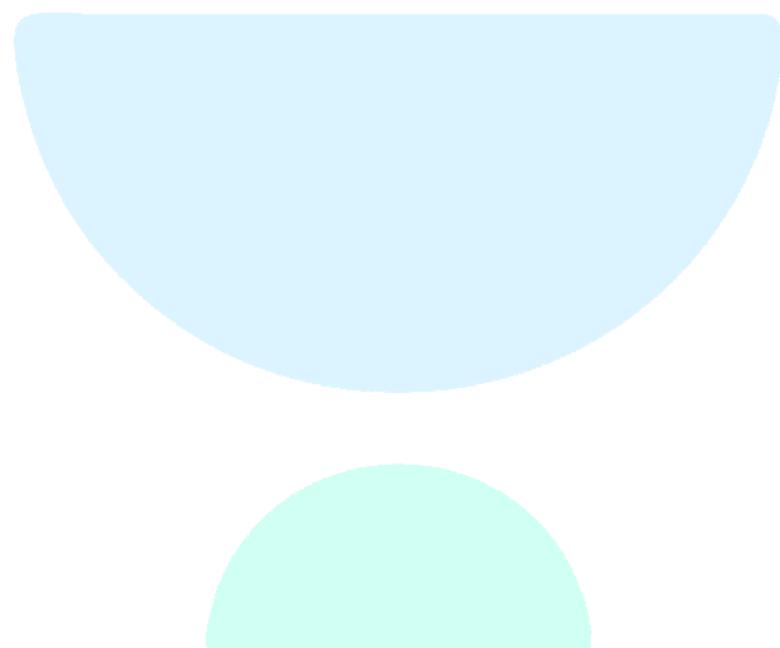
q = dequeue () will return head of queue value



$$q = 24$$

$$s + q = 62 + 24 = 86$$

Hence, the correct answer is 86.



3

Linked Lists



Practice Questions

2017 IIT Roorkee

3.1 Consider the C code fragment given below.

```
typedef struct node
{
    int data;
    node *next;
}
node;
void join (node*m, node*n)
{
    node* p = n;
    while (p->next !=
```

NULL)

{

$p = p \rightarrow next;$

}

$p \rightarrow next = m;$

}

Assuming that m and n point to valid NULL terminated linked lists, invocation of join will

(A) Append list m to the end of list n for all inputs.

(B) Either cause a null pointer dereference or append list m to the end of list n .

(C) Cause a null pointer dereference for all inputs.

(D) Append list n to the end of list m for all inputs.

Solutions

3.1 (B)

Given : C Function :

```
typedef struct node
{
    int data;
    node *next;
}
node;
void join (node*m, node*n)
{
    node* p = n;
    while (p->next != NULL)
    {
        p = p->next;
```

```
}
```

If first are not NULL : Invocation of join will append list m and to the end of list n if the list are not NULL.

Example : Before join operation.

$m = 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow null$

$n = 6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow null$

After join operation

$6 \rightarrow 7 \rightarrow 8 \rightarrow 9 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow null$

If list are Null : If the list n is empty and itself NULL, then joining and referencing would obviously create NULL point issue.

Hence, the correct option is (B).

4

Hashing



Practice Questions

2014 IIT Kharagpur

- 4.1** Consider a hash table with 9 slots. The hash function is $h(k) = k \bmod 9$. The collisions are resolved by chaining. The following 9 keys are inserted in the order: 5, 28, 19, 15, 20, 33, 12, 17, 10. The maximum, minimum, and average chain lengths in the hash table, respectively, are
 (A) 3, 0, and 1 (B) 3, 3, and 3
 (C) 4, 0, and 1 (D) 3, 0, and 2
- 4.2** Consider a hash table with 100 slots. Collisions are resolved using chaining. Assuming simple uniform hashing, what is the probability that the first 3 slots are unfilled after the first 3 insertions?
 (A) $(97 \times 97 \times 97) / 100^3$
 (B) $(99 \times 98 \times 97) / 100^3$
 (C) $(97 \times 96 \times 95) / 100^3$
 (D) $(97 \times 96 \times 95) / (3! \times 100^3)$

2015 IIT Kanpur

- 4.3** Given a hash table T with 25 slots that stores 2000 elements, the load factor α for T is _____.
- 4.4** Which one of the following hash function on integers will distribute keys most uniformly over 10 buckets numbered 0 to 9 i ranging from 0 to 2020?
 (A) $h(i) = i^2 \bmod 10$
 (B) $h(i) = i^3 \bmod 10$
 (C) $h(i) = (11 * i^2) \bmod 10$
 (D) $h(i) = (12 * i) \bmod 10$

2020 IIT Delhi

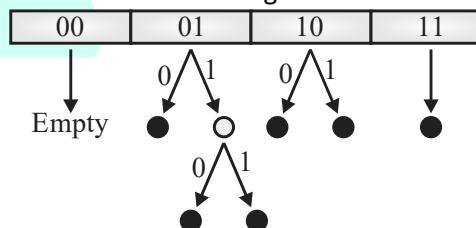
- 4.5** Consider a double hashing scheme in which the primary hash function is $h_1(k) = k \bmod 23$, and the secondary hash function is $h_2(k) = 1 + (k \bmod 19)$. Assume that the table size is 23. Then the address returned by probe 1 in the probe sequence (assume that the probe sequence

begins at probe 0) for key value $k = 90$ is _____.

2021 IIT Bombay

- 4.6** Consider a dynamic hashing approach for 4 big integer keys.
 1. The main hash table size is 4.
 2. The 2 least significant bits of a key is used to index into the main hash table
 3. Initially, the main hash table entries are empty
 4. Thereafter, when more keys are hashed into it to resolve collisions, the set of all keys corresponding to a main hash table entry is organized as a binary tree that grows on demand.
 5. First the 3rd least significant bit is used to divide the keys into left and right subtrees.
 6. To resolve more collisions, each node of the binary tree is further sub-divided into left and right subtrees based on the 4th least significant bit.
 7. A split is done only if it is needed i.e. only when there is a collision.

Consider the following state of the hash table.



Which of the following sequence of key insertions can cause the above state of the hash table (assume the key are in decimal notation)?

- (A) 9, 5, 10, 6, 7, 1
 (B) 10, 9, 6, 7, 5, 13
 (C) 9, 5, 13, 6, 10, 14
 (D) 5, 9, 4, 13, 10, 7



Solutions

4.1 (A)

Given : Hash table length = 9

Hash function $h(k) = k \bmod 9$

Key (k) = {5, 28, 19, 15, 20, 33, 12, 17, 10}

$$h(k) = k \bmod 9$$

$$h(5) = 5 \% 9 = 5$$

$$h(28) = 28 \% 9 = 1$$

$$h(19) = 19 \% 9 = 1$$

$$h(15) = 15 \% 9 = 6$$

$$h(20) = 20 \% 9 = 2$$

$$h(33) = 33 \% 9 = 6$$

$$h(12) = 12 \% 9 = 3$$

$$h(17) = 17 \% 9 = 8$$

$$h(10) = 10 \% 9 = 1$$

0		
1	28	→ 19 → 10
2	20	
3	12	
4		
5	5	
6	15	→ 33
7		
8	17	

Maximum length = $28 \rightarrow 19 \rightarrow 10 = 3$

Minimum length = 0

$$\text{Average} = \frac{0+3+1+1+0+1+2+0+1}{9} = 1$$

Hence, the correct option is (A).

4.2 (A)

Given : hash table has 100 slots.

Collision resolution method is chaining

(As in chaining, collision is resolved externally).

In uniform hashing each slot has equal probability to be picked by hash function.

First 3 slots are unfilled, we have 97 slots to be chosen, repetition is allowed because of chaining.

So, Required probability

$$= \frac{97}{100} \times \frac{97}{100} \times \frac{97}{1000}$$

Hence, the correct option is (A).

4.3 80

Key Point

$$\text{Load factor } a = \frac{\text{Number of key stored in T}}{\text{Number of slots in T}}$$

Given : Number of slots = 25

Number of keys = 2000

$$\text{Load factor } a = \frac{2000}{25} = 80$$

Hence, the correct answer is 80.

4.4 (B)

Given : Length of hash table = 10

Key = 1, 2, 3, ..., 9

If you do cube all number from 0 to 9, you get following.

Number	Cube	Last digit in cube
0	0	0
1	1	1
2	8	8
3	27	7
4	64	4
5	125	5
6	216	6
7	343	3
8	512	2
9	729	9

Therefore, all numbers from 0 to 9 are equally divided in 10 buckets. If we take for square we don't get equal distribution. In the following table 1, 4, 6 and 9 repeated so these buckets would have more entries and buckets 2, 3, 7 and 8 would be empty.

Hence, the correct option is (B).

Key Point

The last digit of cube of 0 to 9 numbers are unique.
Hence, $i^3 \bmod 10$ will result in 10 different values.

4.5 13

Given : hash function = $h_1(k) = k \bmod 23$,



$$h_2(k) = 1 + (k \bmod 19)$$

Table size = 23

Key, $k = 90$,

Probe $i = 1$

- $h_1(k) = 90 \bmod 23 = 21$
- $h_2(k) = 1 + (90 \bmod 19) = 1 + 14 = 15$

According to double hashing formula

$(h_1(k) + i \times h_2(k)) \bmod (\text{table size})$

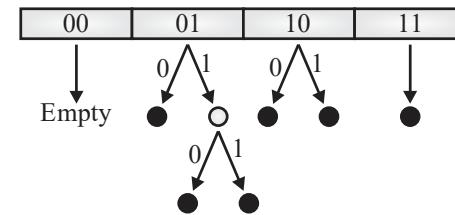
$$\begin{aligned} &= (21 + 1 \times 15) \bmod (23) \\ &= 36 \bmod 23 = 13 \end{aligned}$$

Hence, the correct answer is 13.

4.6 (B)

Given :

state of main hash table :



Following numbers are possible

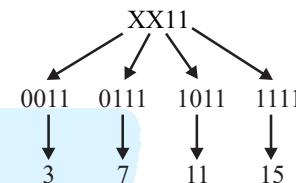
X001 → 0001-1
X001 → 1001-9

X010 → 0010-2
X010 → 1010-10

0101 → 5

1101 → 13

X110 → 0110-6
X110 → 1110-14



Among above possible values only one 4-bit combination should be present.

Hence, the correct option is (B).

5

Trees



Practice Questions

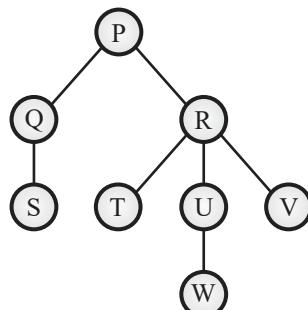
2013 IIT Bombay

- 5.1 The pre-order traversal sequence of a binary search tree is 30, 20, 10, 15, 25, 23, 39, 35, 42. Which one of the following is the post-order traversal sequence of the same tree?
- (A) 10, 20, 15, 23, 25, 35, 42, 39, 30
 - (B) 15, 10, 25, 23, 20, 42, 35, 39, 30
 - (C) 15, 20, 10, 23, 25, 42, 35, 39, 30
 - (D) 15, 10, 23, 25, 20, 35, 42, 39, 30

2014 IIT Kharagpur

- 5.2 A priority queue is implanted as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is : 10, 8, 5, 3, 2. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:
- (A) 10, 8, 7, 3, 2, 1, 5
 - (B) 10, 8, 7, 2, 3, 1, 5
 - (C) 10, 8, 7, 1, 2, 3, 5
 - (D) 10, 8, 7, 5, 3, 2, 1

- 5.3 Consider the following rooted tree with the vertex labeled P as the root:



The order in which the nodes are visited during an in-order traversal of the tree is

- (A) SQPTRWUV
- (B) SQPTURWV
- (C) SQPTWUVR
- (D) SQPTRUWV

- 5.4 Consider the pseudocode given below. The function DoSomething() takes as argument a pointer to the root of an arbitrary tree represented by the leftMostChild → rightsibling

representation. Each node of the tree is of type tree node.

```
typedef struct treeNode* treeptr;
struct treeNode
{
    treeptr leftMostChild, rightsibling;
};

int DoSomething (treeptr tree)
{
    int value = 0;
    if (tree != NULL)
    {
        if (tree->leftMostChild
            == NULL)
            value = 1;
        else
            value =
                DoSomething(tree->leftMostChild);
        value = value+DoSomething(tree->
            rightsibling);
    }
    return(value);
}
```

When the pointer to the root of a tree is passed as the argument to DoSomething, the value returned by the function corresponds to the

- (A) number of internal nodes in the tree.
- (B) height of the tree.
- (C) number of nodes without a right sibling in the tree.
- (D) number of leaf nodes in the tree.

2015 IIT Kanpur

- 5.5 Which of the following is/are correct in-order traversal sequence(s) of binary search trees (s)?

1. 3, 5, 7, 8, 15, 19, 25



2. 5, 8, 9, 12, 10, 15, 25
 3. 2, 7, 10, 8, 14, 16, 20
 4. 4, 6, 7, 9, 18, 20, 25
 (A) 1 and 4 only (B) 2 and 3 only
 (C) 2 and 4 only (D) 2 only
- 5.6** The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are
 (A) 63 and 6, respectively
 (B) 64 and 5, respectively
 (C) 32 and 6, respectively
 (D) 31 and 5, respectively
- 5.7** While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is
 (A) 65 (B) 67
 (C) 69 (D) 83
- 5.8** Consider a binary tree T has 200 leaf nodes. Then the number of nodes in T have exactly two children are _____
- 5.9** Consider the following array of elements
 $\langle 89, 19, 50, 17, 12, 15, 2, \\ 5, 7, 11, 6, 9, 100 \rangle$.
 The minimum number of interchanges needed to convert it into a max-heap is
 (A) 4 (B) 5
 (C) 2 (D) 3
- 5.10** Consider a max heap, represented by the array
 40, 30, 20, 10, 15, 16, 17, 8, 4
- | Array index | 1 | 2 | 3 | 4 | 5 |
|-------------|----|----|----|----|----|
| Value | 40 | 30 | 20 | 10 | 15 |
| | | | | | |
- | Array index | 6 | 7 | 8 | 9 |
|-------------|----|----|---|---|
| Value | 16 | 17 | 8 | 4 |
| | | | | |

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

- (A) 40, 30, 20, 10, 15, 16, 17, 8, 4, 35
 (B) 40, 35, 20, 10, 30, 16, 17, 8, 4, 15
 (C) 40, 30, 20, 10, 35, 16, 17, 8, 4, 15
 (D) 40, 35, 20, 10, 15, 16, 17, 8, 4, 30

- 5.11** An operator delete (i) for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is

implemented in an array and i-refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?
 (A) O(1)
 (B) O(d) but not O(1)
 (C) O(2^d) but not O(d)
 (D) O($d2^d$) but not O(2^d)

- 5.12** A binary tree T has 20 leaves. The number of nodes in T having two children is _____.

2016 IISc Bangalore

- 5.13** Consider the following New-order strategy for traversing a binary tree :

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order;

The new-order traversal of the expression tree corresponding to the reverse polish expression

$3\ 4\ *\ 5\ -\ 2\ ^\ 6\ 7\ * 1\ +\ -$ is given by

- (A) $+ - 1\ 6\ 7\ * 2\ ^\ 5 - 3\ 4\ *$
 (B) $- + 1\ * 6\ 7\ ^\ 2 - 5\ * 3\ 4$
 (C) $- + 1\ * 7\ 6\ ^\ 2 - 5\ * 4\ 3$
 (D) $1\ 7\ 6\ * + 2\ 5\ 4\ 3\ * - -$

- 5.14** A complete binary min-heap is made by including each integer in [1, 1023] exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _____ .

- 5.15** The number of ways in which the numbers 1, 2, 3, 4, 5, 6, 7 can be inserted in an empty binary search tree, such that the resulting tree has height 6, is _____ Note: The height of a tree with a single node is 0.

2017 IIT Roorkee

- 5.16** Let T be binary search tree with 15 nodes. The minimum and maximum possible heights of T are :

Note : The height of a tree with a single node is 0.



- (A) 4 and 15 respectively
 (B) 3 and 14 respectively
 (C) 4 and 14 respectively
 (D) 3 and 15 respectively

- 5.17** The pre-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20. Then the post-order traversal of this tree is :
 (A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20
 (B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12
 (C) 7, 2, 6, 8, 9, 10, 20, 17, 19, 15, 16, 12
 (D) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12

2018 IIT Guwahati

- 5.18** The post-order traversal of a binary tree is 8, 9, 6, 7, 4, 5, 2, 3, 1. The in-order traversal of the same tree is 8, 6, 9, 4, 7, 2, 5, 1, 3. The height of a tree is the length of the longest path from the root to any leaf. The height of the binary tree above is _____.

2019 IIT Madras

- 5.19** Let T be a full binary tree with 8 leaves. (A full binary tree has every level full.) Suppose two leaves a and b of T are chosen uniformly and independently at random. The expected value of the distance between a and b in T (i.e., the number of edges in the unique path between a and b) is (rounded off to 2 decimal places)

2020 IIT Delhi

- 5.20** The pre-order traversal of a binary search tree is 15, 10, 12, 11, 20, 18, 16, 19. Which one of the following is the post-order traversal of the tree ?
 (A) 10, 11, 12, 15, 16, 18, 19, 20
 (B) 11, 12, 10, 16, 19, 18, 20, 15
 (C) 20, 19, 18, 16, 15, 12, 11, 10
 (D) 19, 16, 18, 20, 11, 12, 10, 15
- 5.21** Consider the array representation of a binary min-heap containing 1023 elements. The

minimum number of comparisons required to find the maximum in the heap is _____.

2021 IIT Bombay

- 5.22** An articulation point in a connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components.

Let T be a DFS tree obtained by doing DFS in a connected undirected graph G. Which of the following options in/are correct?

- (A) Root of T can never be an articulation point in G.
 (B) A leaf of T can be an articulation point in G.
 (C) If u is an articulation point in G such that x is an ancestor of u and y is a descendent of v in T then all paths from x to y in G must pass through u
 (D) Root of T is an articulation point in G, if and only if it has 2 or more children.

- 5.23** Consider the following statements.

S_1 : The sequence of procedure calls corresponds to a pre-order traversal of the activation tree.

S_2 : The sequence of procedure returns corresponds to a post-order traversal of the activation tree.

Which one of the following options is correct?

- (A) S_1 is true and S_2 is true
 (B) S_1 is true and S_2 is false
 (C) S_1 is false and S_2 is true
 (D) S_1 is false and S_2 is false

- 5.24** Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing BFS starting from root. Let B denote the set of 3 elements obtained by performing DFS state starting from root. The value of $|A - B|$ is _____.

Solutions

5.1 (D)

Given :

pre-order traversal sequence of a binary search tree:

30, 20, 10, 15, 25, 23, 39, 35, 42

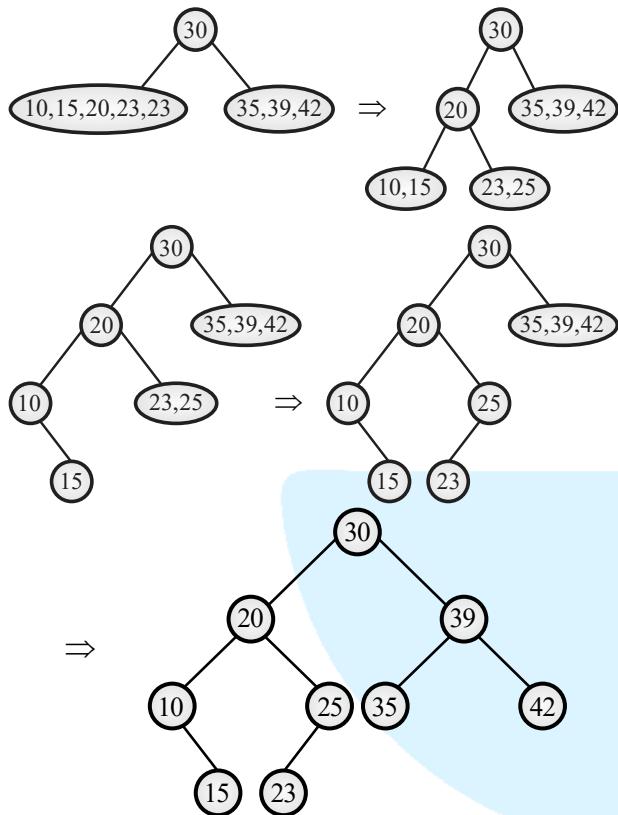
In-order traversal of binary search tree gives ascending order.

So, in-order is 10, 15, 20, 23, 25, 30, 35, 39, 42.



Key Point

Starting from pre-order and search than node in-order and make left subtree and right subtree



Key Point

In post-order traversal, we visit left subtree first then right subtree and then root.

So post-order is tree is 15, 10, 23, 25, 20, 35, 42, 39, 30.

Hence, the correct option is (D).

5.2 (A)

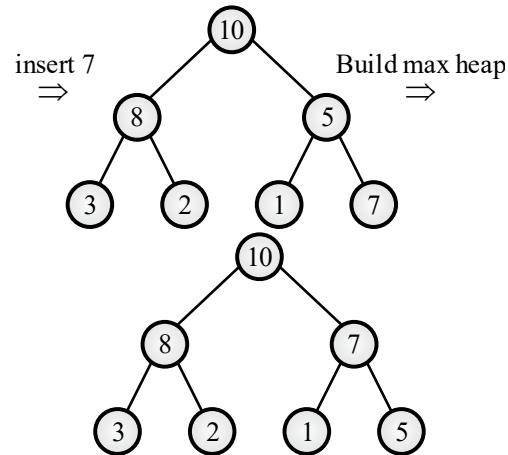
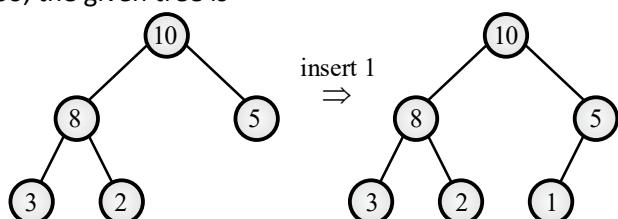
Given : The level-order traversal of the heap : 10, 8, 5, 3, 2,

Two new elements 1 and 7 are inserted into the heap in that order.

Key Point

Max heap is implemented with 5 element 10, 8, 5, 3, 2 in level order traversal.

So, the given tree is

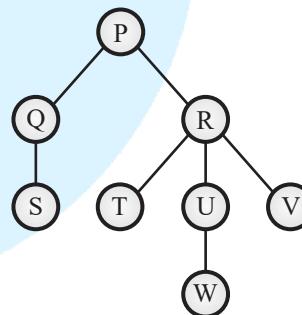


So, level order traversal of heap 10, 8, 7, 3, 2, 1, 5.

Hence, the correct option is (A).

5.3 (A)

Given :



In in-order traversal, left subtree visited first then root and then right subtree (left, root, mid, then right).

So SQPTRWUV is in-order.

Hence, the correct option is (A).

5.4 (D)

The key to solving such question is to understand or detect where/ by what condition the value (on the counter) is getting incremented each time.

Here, that condition is if (tree → left most = Null)

Which mean if there is no left most child of the tree (or the subtree or the current nodes called in recursion).

⇒ Which means there is no child to that particular node (since if there is no left most child, there is no child at all).

⇒ Which mean the node order consideration is a leaf node.



- ⇒ The function recursively counts and add to value, whenever a leaf node is encountered.
- ⇒ The function returns the number of leaf node in the tree.

Hence, the correct option is (D).

5.5 (A)

Key Point

In-order traversal of binary search tree gives ascending order.

1. Ascending order
2. $12 > 10$, not in-order
3. $10 > 10$, not in-order
4. Ascending order

Hence, the correct option is (A).

5.6 6(A)

Given :

a binary tree of height 5,

Maximum no. of node with height $h(5)$

$$= 2^{h+1} - 1$$

$$= 2^{5+1} - 1$$

$$= 63$$

Minimum no. of node with height $h(5)$

$$= h+1$$

$$= 5+1$$

$$= 6$$

Hence, the correct option is (A).

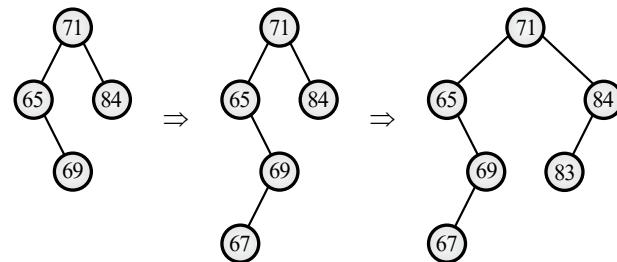
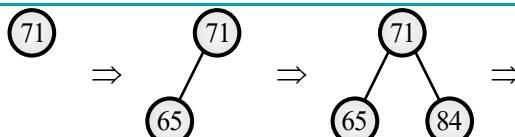
5.7 (B)

Given :

Insertion of the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST).

Key Point

In binary search tree, key (root) is smaller than right subtree and greater than left subtree.



So, the element in lowest level is 67.

Hence, the correct option is (B).

5.8 199

Given :

Number of leaf nodes in a binary tree = 200

Key Point

No. of leaf nodes = internal nodes

$$\times (\text{degree of tree} - 1) + 1$$

No. of internal nodes = x

$$200 = x \times (2-1) + 1$$

$$200 = x + 1$$

$$x = 199$$

Hence, the correct answer is 199.

5.9 (D)

Given :

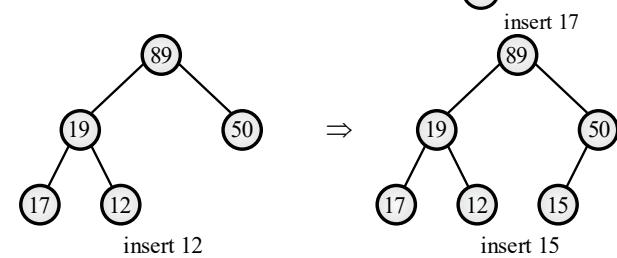
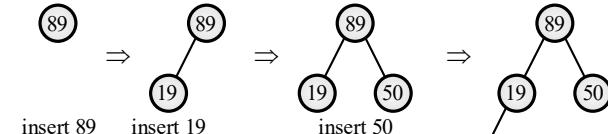
array of elements

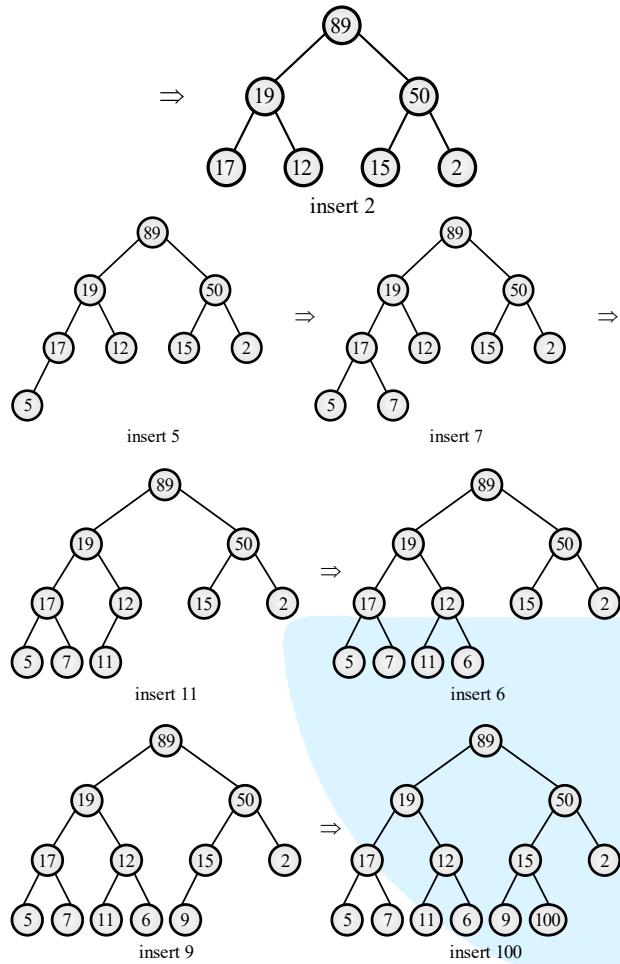
$$\langle 89, 19, 50, 17, 12, 15, 2, \\ 5, 7, 11, 6, 9, 100 \rangle$$

Key Point

In max-heap root element is maximum comparing to its children.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
89	19	50	17	12	15	12	15	2	5	7	11	6	9	100

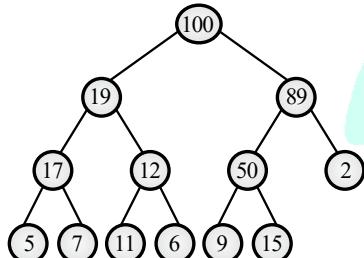




Here we need to build a max heap, since 100 is greater value than 15, 50 and 89.

So, we need to inter change them.

Resultant tree is



Therefore, we need 3 inter change value.

Hence, the correct option is (D).

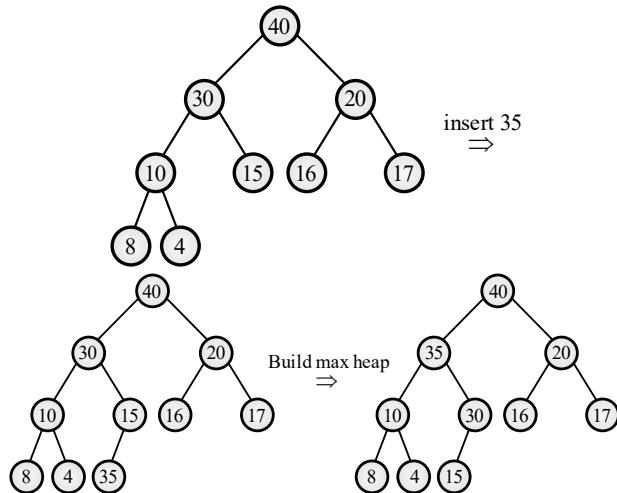
5.10 (B)

Given :

a max heap, represented by the array :

40, 30, 20, 10, 15, 16, 17, 8, 4

The max heap is



Key Point

In max heap, root value is maximum comparing to child.

So, after insertion 35, new heap is
40, 35, 20, 10, 30, 16, 17, 8, 4, 15

Hence, the correct option is (B).

5.11 (B)

Here we need to apply heapify and suppose if we are deleting root, in worst case then it would take $\Theta(n)$ time.

Hence, the correct option is (B).

5.12 19

Given:

Number of leaf nodes = 20

Number of nodes having two children is = $n-1$

Where, n = number of leaf nodes

Number of nodes having two children = $20-1=19$

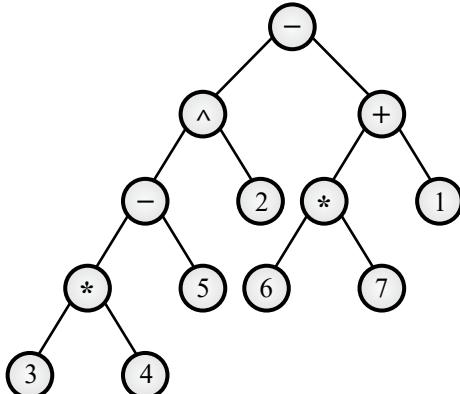
Hence, the correct answer is 19.

5.13 (C)

Given : The postfix expression,

$34*5-2^67*1+-$

The expression tree is



So, the new order of above expression tree is
 $- + 1 * 7 6 \wedge 2 - 5 * 4 3$

Hence, the correct option is (C).

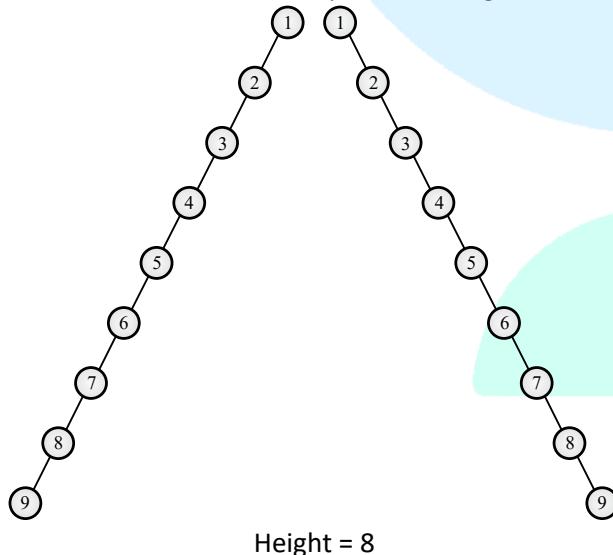
5.14 8

Given : A complete binary min-heap made by including each integer in $[1, 1023]$ exactly once.

Key Point

In min heap, root element is smaller than its children and in complete binary tree, every node will have 2 children at same level.

So, in-order to maximum depth with integer 9 is 8.



And make rest of node as child, as per given condition.

Hence, the correct answer is 8.

5.15 64

Given :

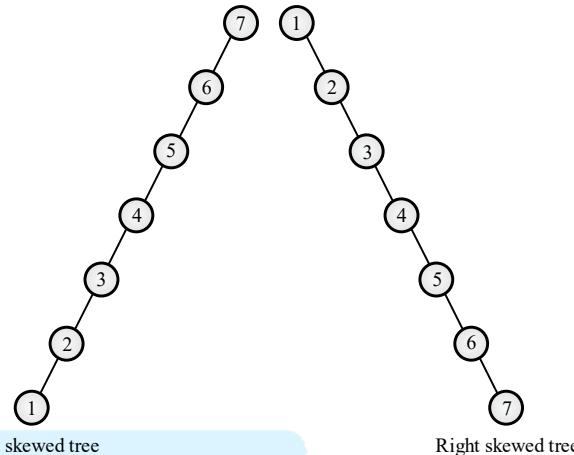
Numbers to insert : 1, 2, 3, 4, 5, 6, 7

Height of the resulting tree = 6

Key Point

In binary search tree key value should be greater than left subtree and smaller than right subtree.

So, in-order to construct BST of height 6, with given element is



So, at each level we have to choose from level 0 to level 6 and at level remaining element 1.

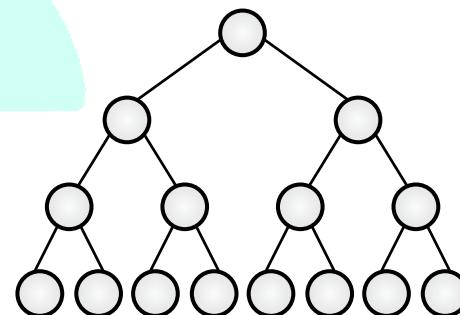
So, the number of way = $2^6 = 64$

Hence, the correct answer is 64.

5.16 (B)

Given : Binary search tree with 15 nodes.

So maximum possible height of tree with 15 node is 14, when it left or right skewed tree and height is minimum when it is complete binary search tree with 15 node height.



Hence, the correct option is (B).

5.17 (B)

Given :

Pre-order traversal :

12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20

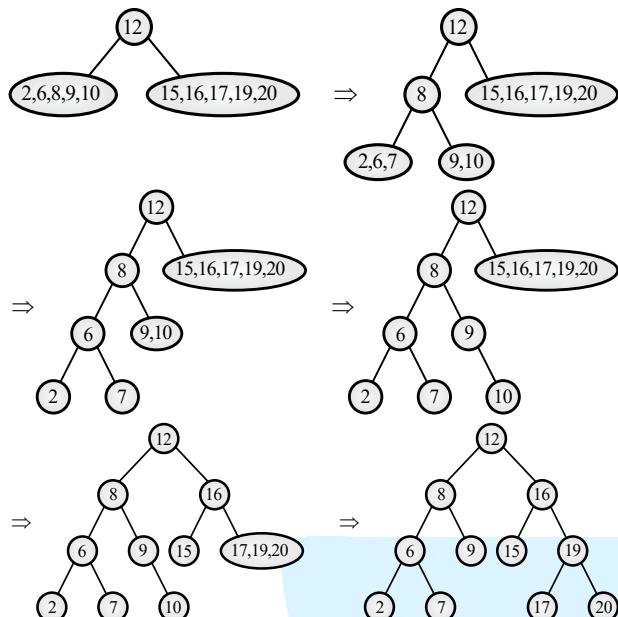
Key Point

In-order traversal of BST is ascending order

so in-order is 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20



Starting from pre-order and search that node in-order and make left subtree & right subtree.



In post-order traversal we visit left subtree then right subtree and later root.

So post-order is

2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

Hence, the correct option is (B).

5.18 4

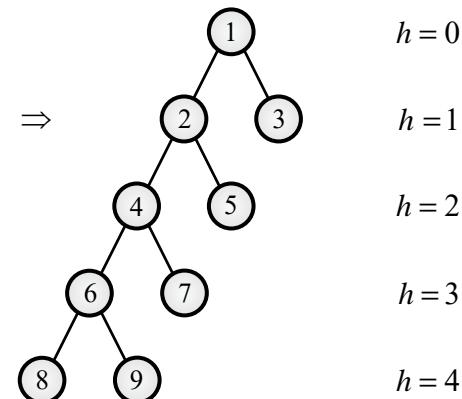
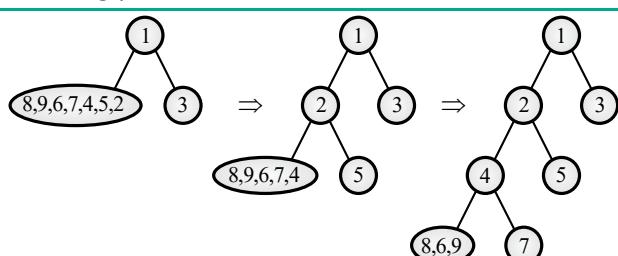
Given :

Post-order : 8, 9, 6, 7, 4, 5, 2, 3, 1

In-order : 8, 6, 9, 4, 7, 2, 5, 1, 3.

Key Point

Starting from right in post-order and search that node in-order and make left subtree & right subtree accordingly.



5.19 4.25

Given :

a full binary tree with 8 leaves.

No. of pair with path length 0 = 8

No. of pair with path length 1 = 0

No. of pair with path length 2 = 8

No. of pair with path length 3 = 0

No. of pair with path length 4 = 16

No. of pair with path length 5 = 0

No. of pair with path length 6 = 32

Total number of possible pair = $8 \times 8 = 64$

So excepted path length

$$= 0 \times \frac{8}{64} + 2 \times \frac{8}{64} + 4 \times \frac{16}{64} + \frac{6 \times 32}{64} = \frac{272}{64} = 4.25$$

Hence, the correct answer is 4.25.

5.20 (B)

Given:

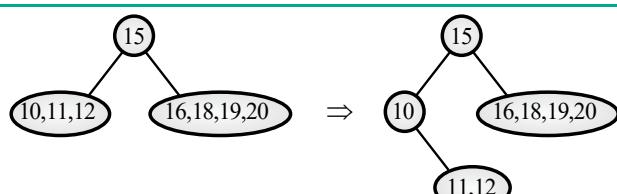
pre-order traversal :

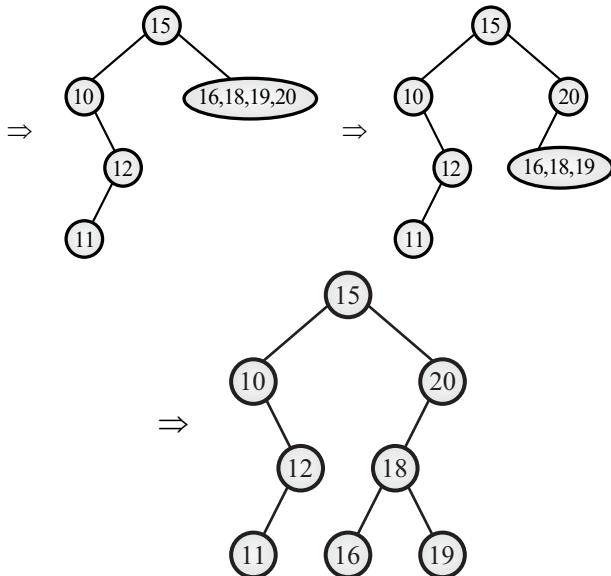
15, 10, 12, 11, 20, 18, 16, 19

In-order traversal of BST is ascending order 10, 11, 12, 15, 16, 1, 19, 20

Key Point

Starting from pre-order and search that node in in-order and make left subtree & right subtree.





Key Point

In post-order traversal we visit left subtree first, then right subtree and then root.

So post-order is 11, 12, 10, 16, 19, 18, 20, 15

Hence, the correct option is (B).

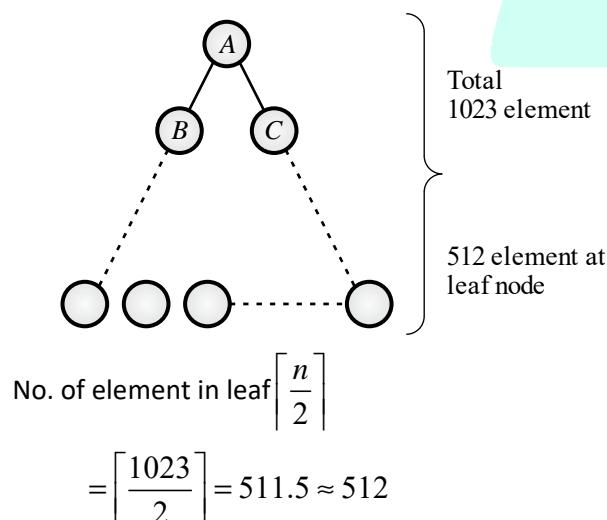
5.21 511

Given : A binary min-heap containing 1023 elements.

Key Point

Min-heap means, parents should be minimum or equal to its children so, max children could be either left or right one.

Following this logic maximum can be definitely at leaf node.

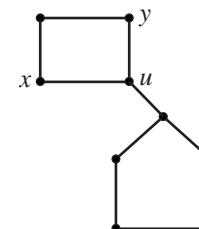


To find maximum among 512 elements, no. of comparison needed is 511.

Hence, the correct answer is 511.

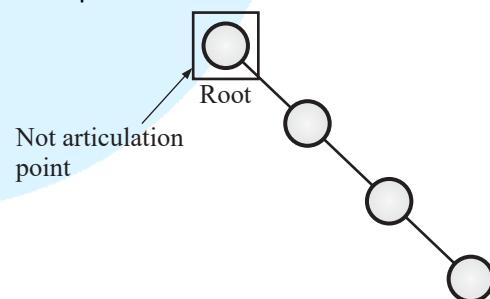
5.22 (D)

- (A) False: Check option (D) for more information.
- (B) False: This can never happen. Leaf will always have degree = 1.
- (C) False: Below is the reason to show how this is false.



If u is articulation point, then removing u generates 2 connected components, now there might be a case when x and y will belong to either one of the connected component and hence a path will exist between them without passing through u

(D) True. We need at least 2 children so that root is articulation point.



5.23 (A)

Given : Statements :

S1 : The sequence of procedure calls corresponds to a pre-order traversal of the activation tree.

S2 : The sequence of procedure returns corresponds to a post-order traversal of the activation tree.

Consider the function calls that result from a main program calling `f(5)`

//Fibonacci Series using Recursion

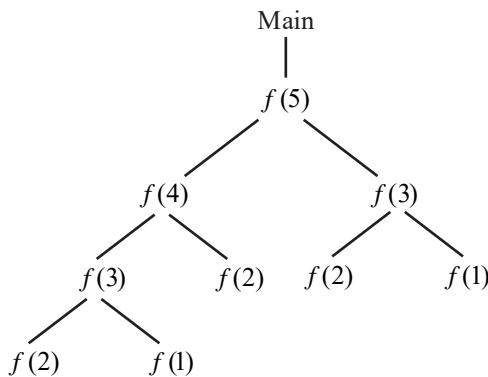
```
#include<stdio.h>
int fib(int n)
{
    if (n <= 1)
        return n;
    return fib(n-1) + fib(n-2);
}
```

```
int main ()
```



```
{
    fib(5)
    return 0;
}
```

Activation tree:



The sequence of procedure call is:

```

System starts main
    enter f(5)
        enter f(4)
            enter f(3)
                enter f(2)
                    exit f(2)
                    enter f(1)
                    exit f(1)
                    exit f(3)
                    enter f(2)
                    exit f(2)
                exit f(4)
                enter f(3)
                    enter f(2)
    
```

exit f(2)

enter f(1)

exit f(1)

exit f(3)

exit f(5)

main ends

Pre-order traversal:

f(5), f(4), f(3), f(2), f(1), f(2),
f(3), f(2), f(1)

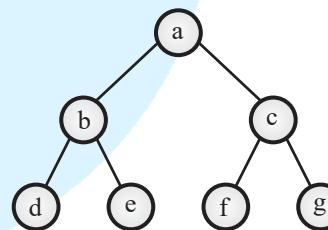
Post-order traversal:

f(2), f(1), f(3), f(2), f(4), f(2),
f(1), f(3), f(5)

Hence, the correct option is (A).

5.24 1

Given : Binary tree with 7 nodes consider the following complete binary tree



A = set of first 3 elements obtained by BFS

A = {a, b, c}

B = set of first 3 elements obtained by DFS

B = {a, b, d}

A - B = {c}

|A - B| = 1

Hence, the correct answer is 1.

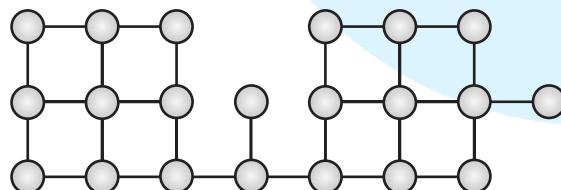
6 Graphs



Practice Questions

2014 IIT Kharagpur

- 6.1 Let G be a graph with n vertices and m edges. What is the tightest upper bound on the running time of depth first search on G , when G is represented as an adjacency matrix?
- (A) $\Theta(n)$ (B) $\Theta(n + m)$
 (C) $\Theta(n^2)$ (D) $\Theta(m^2)$
- 6.2 Suppose depth first search is executed on the graph below starting at some unknown vertex. Assume that a recursive call to visit a vertex is made only after first checking that the vertex has not been visited earlier. Then the maximum possible recursion depth (including the initial call) is.

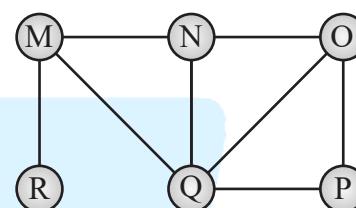


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- 6.3 Breadth first search (BFS) is started on a binary tree beginning from the root vertex. There is a vertex t at a distance four from the root. If t is the n^{th} vertex in this BFS traversal, then the maximum possible value of n is _____.
 6.4 In an adjacency list representation of an undirected simple graph $G = (V, E)$, each edge (u, v) has two adjacency list entries: $[v]$ in the adjacency list of u , and $[u]$ in the adjacency list of v . These are called twins of each other. A twin pointer is a pointer from an adjacency list entry to its twin. If $|E| = m$ and $|V| = n$, and the memory size is not a constraint, what is the time complexity of the most efficient algorithm to set the twin pointer in each entry in each adjacency list?
 (A) $\Theta(n^2)$ (B) $\Theta(n + m)$
 (C) $\Theta(m^2)$ (D) $\Theta(n^4)$

2017 IIT Roorkee

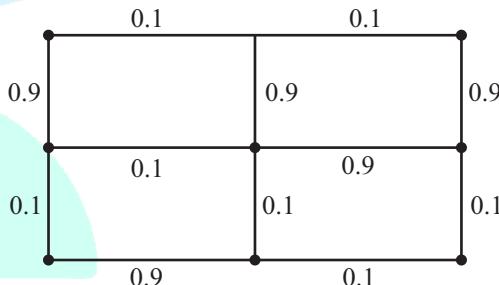
- 6.5 The Breadth First Search (BFS) algorithm has been implemented using the queue data structure. Which one of the following is a possible order of visiting the nodes in the graph below?



- (A) MNOPQR (B) NQMPOR
 (C) QMNRQP (D) POQNMR

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- 6.6 Consider the following undirected graph with edge weights as shown:



The number of minimum-weight spanning trees of the graph is _____.
 6.7 Let G be a connected undirected weighted graph consider the following two statements.

- S₁. There exists a minimum edge weight in G which is present in every MST of G .
 S₂. If every edge in G has distinct weights, then G has a unique MST.
 Which of the following is true?
 (A) S₁ is true and S₂ is false
 (B) S₁ is false and S₂ is true
 (C) Both S₁ and S₂ are true
 (D) Both S₁ and S₂ are false



Solutions

6.1 (C)

Given :

$$\text{Graph } G = (V, E)$$

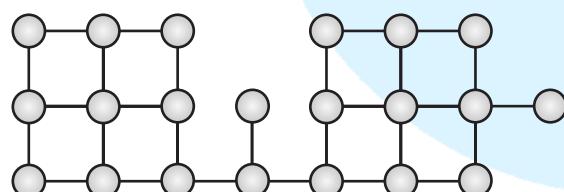
Number of vertices = n

Number of edges = m

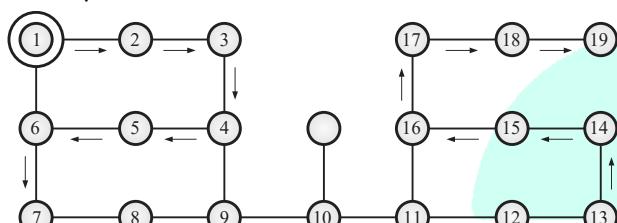
Let us take the vertices as n and the edges as m of a graph G , DFS algorithm take $\Theta(m+n)$ using adjacency matrix. So the graph is represented as $n \times n$ matrix. To do DFS for every vertex, we do transverse the corresponding row so the vertex to find all adjacent vertices. Thus the time complexity is $\Theta(n^2)$. Hence, the correct option is (C).

6.2 19

Given :



Let the path is :



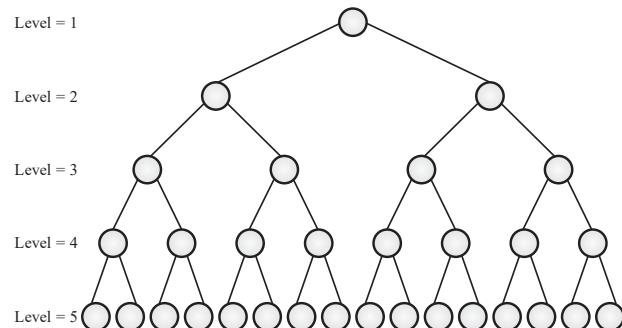
Suppose we start DFS at vertex numbered as 1 and continue calling recursive function for DFS on subsequence nodes numbered in ascending order.

The recursive calling sequence is shown as marked line in the above diagram which shows maximum possible recursion depth including the initial state 1 call is 19. Hence, the correct answer is 19.

6.3 31

Binary tree is almost 2 children, so maximum value of n , when vertex is at distance 4 is $(2^{4+1} - 1)$

$$\begin{aligned} &= 2^{4+1} - 1 = 32 - 1 \\ &= 31 \end{aligned}$$



Hence, the correct answer is 31.

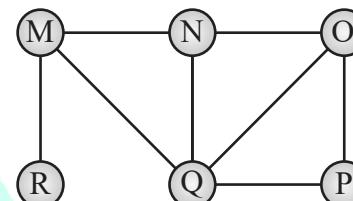
6.4 (B)

Most efficient algorithm to set the twin pointer in each entry in each adjacency list is as follows. Applying BFS on undirected graph gives you two twin pointer. Visit every vertex level wise for every vertex fill adjacent vertex in the adjacency list. BFS take $\Theta(m+n)$ time.

Hence, the correct option is (B).

6.5 (D)

Given :



Key Point

BFS : Start at root (Some arbitrary node of a graph sometime referred to as "search key") and explore the neighbor node first, before moving to the next level neighbour.

(A) MNOPQR

Adjacency list

M →	MQR	M
N →	MOQ	M N Q R
Q →		

We cannot visit in this order as O is not an adjacent to N.

(B) NQMPOR

Adjacency list

N →	QMO	N
Q →	MNPO	N Q M O
M →	NQR	N Q M O P



It is not possible to visit P before O as P is not adjacent to M.

(C)

Adjacency list	
Q → MNOP	Q
M → NQR	Q M N O P
N → MOQ	Q M N O P R

It is not possible to visit R, as O is adjacent to N.

(D)

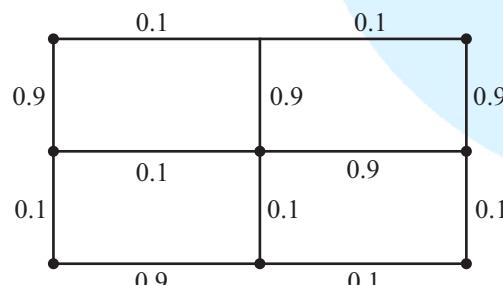
Adjacency list	
P → OQ	P
O → NQ	P O Q
Q → MNO	P O Q N
N → MOQ	P O Q N M
M → MQR	P O Q N M R
R →	

It is true

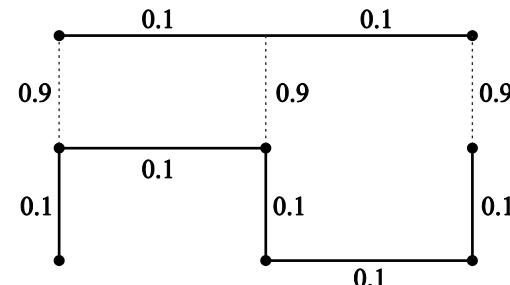
Hence, the correct option is (D).

6.6 3

Given :



MST for given graph is



Solid lines represent compulsory edges in MST and dotted lines represent optional edges in MST only one edge out of those 3 edges is possible hence answer is 3.

6.7 (B)

Given :

G is a connected undirected weighted graph,

- S₁. There exists a minimum edge weight in G which is present in every MST of G.
- S₂. If every edge in G has distinct weights, then G has a unique MST.

By using Kruskal's algorithm to find MST, we sort the edges based on their weight and start selecting edges from the smallest weight (w_small for example).

Problem with S₁: If we have multiple copies of w_small, then a specific w_small weighted edge is not guaranteed to be selected by Kruskal.

S₂ is Correct: If the sorted order of the edges contains only distinct values, the Kruskal algorithm will always select a unique set of edges resulting in a unique minimum spanning tree.

Hence, the correct option is (B).



Marks Distribution of Algorithms in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	4	6	16
2004*	4	10	24
2005*	4	8	20
2006*	10	7	24
2007*	4	10	24
2008*	4	11	26
2009	2	5	12
2010	1	3	7
2011	2	3	8
2012	3	3	9
2013	1	3	7
2014 Set-1	2	2	6
2014 Set-2	2	2	6
2014 Set-3	1	2	5

* CS and IT combined

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	2	3	8
2015 Set-2	2	3	8
2015 Set-3	2	3	8
2016 Set-1	3	3	9
2016 Set-2	2	3	8
2017 Set-1	2	2	6
2017 Set-2	1	2	5
2018	2	3	8
2019	1	3	7
2020	2	2	6
2021 Set-1	5	4	13
2021 Set-2	3	3	9

Syllabus : Algorithms

Searching, sorting, hashing. Asymptotic worst case time and space complexity. Algorithm design techniques: greedy, dynamic programming and divide-and-conquer. Graph traversals, minimum spanning trees, shortest paths.

Contents : Algorithms

S. No. Topics

- 1.** Complexity Analysis and Asymptotic Notations
- 2.** Divide and Conquer Method
- 3.** Greedy Method
- 4.** Dynamic Programming
- 5.** Miscellaneous Topics



Practice Questions

2014 IIT Kharagpur

- 1.1 Which one of the following correctly determines the solution of recurrence relation with $T(1)=1$?

$$T(n) = 2T\left(\frac{n}{2}\right) + \log n$$

- (A) $\Theta(n)$ (B) $\Theta(n \log n)$
 (C) $\Theta(n^2)$ (D) $\Theta(\log n)$

- 1.2 Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H . Suppose there are m such numbers in T . If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is _____.

2015 IIT Kanpur

- 1.3 Consider a complete binary tree where the left and the right subtrees of the root are maxheaps. The lower bound for the number of operations to convert the tree to a heap is :
- (A) $\Omega(\log n)$ (B) $\Omega(n)$
 (C) $\Omega(n \log n)$ (D) $\Omega(n^2)$

- 1.4 An unordered list contains n distinct elements. The number of comparisons to find an element in this list that is neither maximum nor minimum is :
- (A) $\Theta(n \log n)$ (B) $\Theta(n)$
 (C) $\Theta(\log n)$ (D) $\Theta(1)$

- 1.5 Consider the equality $\sum_{i=0}^n i^3 = X$ and the following choices for X
- I. $\Theta(n^4)$ II. $\Theta(n^5)$
 III. $O(n^5)$ IV. $\Omega(n^3)$

The equality above remains correct if X is replaced by

- (A) Only I
 (B) Only II
 (C) I or III or IV but not II
 (D) II or III or IV but not I

- 1.6 Consider the following C function

```
int fun1 (int n)
{
    int i, j, k, p, q = 0;
    for (i = 1; i < n; ++i)
    {
        p = 0;
        for (j = n; j > 1; j = j\2)
            ++p;
        for (k=1; k<p; k = k* 2)
            ++q;
    }
    return q;
}
```

Which one of the following most closely approximated the return value of the function `fun1`?

- (A) n^3 (B) $n(\log n)^2$
 (C) $n(\log n)$ (D) $n \log(\log n)$

- 1.7 An algorithm performs $(\log N)^{1/2}$ find operations, N insert operations, $(\log N)^{1/2}$ delete operations, and $(\log N)^{1/2}$ decrease-key operations on a set of data items with keys drawn from a linearly ordered set. For a delete operation, a pointer is provided to the record that must be deleted. For the decrease-key operation, a pointer is provided to the record that has its key decreased. Which one of the following data structures is the most suited for the algorithm to use, if the goal is to achieve the best asymptotic complexity considering all the operations?

- (A) Unsorted array
 (B) Min-heap
 (C) Sorted array



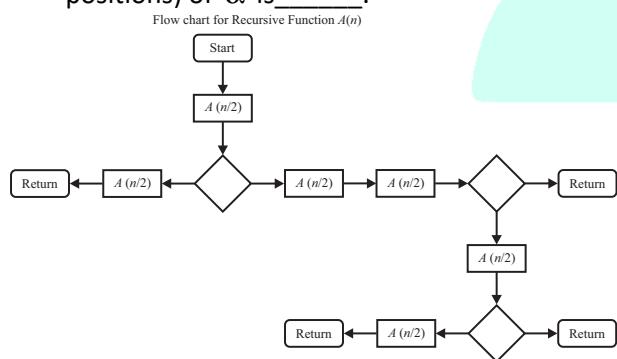
(D) Sorted doubly linked list

- 1.8** Let $f(n) = n$ and $g(n) = n^{(1+\sin n)}$, where n is a positive integer. Which of the following statements is /are correct?
 i. $f(n) = O(g(n))$ ii. $f(n) = \Omega(g(n))$
 (A) only I (B) only II
 (C) Both I and II (D) Neither I nor II

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- 1.9** N items are stored in sorted doubly linked list. For a delete operation, a pointer is provided to the record to be deleted. For a decrease-key operation, a pointer is provided to the record on which the operation is to be performed. An algorithm performs the following operations on the list in this order; $\Theta(N)$ delete $O(\log N)$ insert, $O(\log N)$ find, and $\Theta(N)$ decrease key. What is the time complexity of all these operations put together?
 (A) $\Theta(\log^2 N)$ (B) $\Theta(N)$
 (C) $\Theta(N^2)$ (D) $\Theta(N^2 \log N)$

- 1.10** The given diagram shows the flowchart for a recursive function $A(n)$. Assume that all statements, except for the recursive calls, have $O(1)$ time complexity. If the worst case time complexity of this function is $O(n^\alpha)$, then the least possible value (accurate up to two decimal positions) of α is _____.
 Flow chart for Recursive Function $A(n)$





2020 IIT Delhi

1.15 For parameters a and b , both of which are $\omega(1)$,

$T(n) = T(n^{1/a}) + 1$, and $T(b) = 1$. Then $T(n)$ is

- (A) $\Theta(\log_a \log_b n)$
- (B) $\Theta(\log_{ab} n)$
- (C) $\Theta(\log_b \log_a n)$
- (D) $\Theta(\log_2 \log_2 n)$

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1.16 Consider the following three functions :

$$f_1 = 10^n, f_2 = n^{\log n}, f_3 = n^{\sqrt{n}}$$

Which one of the following options arranges the functions in the increasing order of asymptotic growth rate?

- (A) $f_1 f_2 f_3$
- (B) $f_2 f_3 f_1$
- (C) $f_3 f_2 f_1$
- (D) $f_2 f_1 f_3$

1.17 Consider the following recurrence relation

$$T(n) = \begin{cases} T(n/2) + T(2n/5) + 7n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

Which one of the following options is correct?

- (A) $T(n) = \Theta(n)$
- (B) $T(n) = \Theta((\log n)^{5/2})$
- (C) $T(n) = \Theta(n \log n)$
- (D) $T(n) = \Theta(n^{5/2})$

1.18 For constants $a \geq 1$ and $b > 1$, consider the following recurrence defined on the non-negative integers:

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

Which of the following options is correct about the recurrence $T(n)$?

- (A) if $f(n)$ is $\Theta(n^{\log_b(a)})$, then $T(n)$ is $\Theta(n^{\log_b(a)})$
- (B) if $f(n)$ is $\Theta(n^{\log_b(a)-\epsilon})$, for some $\epsilon > 0$ then, $T(n)$ is $\Theta(n^{\log_b(a)})$
- (C) if $f(n)$ is $\frac{n}{\log_2(n)}$, then $T(n)$ is $\Theta(\log_2(n))$
- (D) if $f(n)$ is $n \log_2(n)$, then, $T(n)$ is $\Theta(n \log_2 n)$

1.19 Let P be an array containing n integers. Let t be the lowest upper bound on the number of comparisons of the array elements, required to find the minimum and maximum values in an arbitrary array of n elements. Which one of the following choices is correct?

- (A) $t > 2n - 2$
- (B) $t > 3\left\lceil \frac{n}{2} \right\rceil$ and $t \leq 2n - 2$
- (C) $t > n$ and $t \leq 3\left\lceil \frac{n}{2} \right\rceil$
- (D) $t > \lceil \log_2(n) \rceil$ and $t \leq n$

Solutions

1.1 (A)

Given : $T(n) = 2T\left(\frac{n}{2}\right) + \log n$

$$T(1) = 1$$

$$a = 2, b = 2, k = 0, p = 1$$

On comparing with extended master method

$$T(n) = aT\left(\frac{n}{b}\right) + n^k \log^p n$$

$$a > b^k$$

$$2 > 2^0$$

$$2 > 1$$

$$T(n) = \Theta(n^{\log_b a})$$

$$T(n) = \Theta(n^{\log_2 2})$$

$$T(n) = \Theta(n)$$

Hence, the correct option is (A).

1.2 110

Given : A balanced binary search tree T holding n numbers.

Finding L and H will take $O(h)$, h is $O(\log n)$, as given tree is balanced binary.



Now traverse m element from L to H in in order way $O(m)$.

Total time complexity

$$O(\log n + m)$$

Comparing with given $O(n^a \log^b n + m^c \log^d n)$

$$a = 0, b = 1, c = 1 \text{ and } d = 0$$

$$a + 10b + 100c + 1000d$$

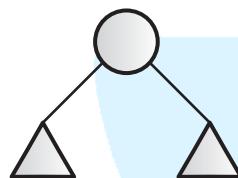
$$0 + 10(1) + 100(1) + 1000(0)$$

$$= 100$$

Hence, the correct answer is 110.

1.3 (A)

Given : A complete binary tree where the left and the right subtrees of the root are maxheaps.



To convert given complete tree into heap, do heapify.

$$\Omega(\log n)$$

Hence, the correct option is (A).

1.4 (D)

Given : An unordered list containing n distinct elements.

Take 3 element from list.

Find element which is neither maximum nor maximum among these 3 elements.

Constant time required

$$\Theta(1)$$

Hence, the correct option is (D).

1.5 (C)

$$\text{Given : } \sum_{i=0}^n i^3 = X$$

choices for X

I. $\Theta(n^4)$

II. $\Theta(n^5)$

III. $O(n^5)$

IV. $\Omega(n^3)$

Consider given sum :

$$\sum_{i=0}^n i^3 = X$$

$$0 + 1^3 + 2^3 + 3^3 + \dots + (n-1)^3 + n^3$$

$$\left(\frac{n(n+1)}{2}\right)^2 = X$$

$$X = \Theta(n^4)$$

(C) I or III or IV but not II.

Hence, the correct option is (C).

1.6 (D)

Given :

int fun1 (int n)

{

 int i, j, k, p, q = 0;

 for ($i = 1; i < n; ++i$)

 {

$p = 0$;

 for ($j = n; j > 1; j = j \setminus 2$)

$++p$;

 for ($k = 1; k < p; k = k * 2$)

$++q$;

 }

 return q;

}

Now, from above program :

for ($i = 1, i < n, ++i$) $\rightarrow n$ times

{

$p = 0$;

 for ($j = n; j > 1; j = j / 2$) $\rightarrow \log n$ times

$++p$;

 for ($k = 1; k < p; k = k * 2$) $\rightarrow \log p$ times

$++q$;

}

In the code, we can see i loop runs n times.

j loop runs $\log n$ times for each i , so value of p is $\log n$.

k loop runs $\log p$ times i.e. $\log \log n$ times for each value of i .

Total increment of q will be $n \log \log n$ times.

Hence, the correct option is (D).

1.7 (A)

Given : An algorithm performs find operations, N insert operations, delete operations, and decrease-



key operations on a set of data items with keys drawn from a linearly ordered set.

Unsorted Array

$\sqrt{\log N}$ find

Each find operation takes $\Theta(N)$ times

$$\text{So } (\log N)^{1/2} N = \Theta((\log N)^{1/2} N)$$

N insert

Each insert operation takes $\Theta(1)$

$$\text{So } N\Theta(1) = \Theta(N)$$

$\sqrt{\log N}$ delete

Each delete $O(1)$

$$\text{So } \sqrt{\log N} O(1) = \Theta(\sqrt{\log N})$$

$\sqrt{\log N}$ decrease key

Each decrease key $O(1)$

$\sqrt{\log N} O(1) = \Theta((\log N)^{1/2})$

Similarly in sorted array

$(\log N)^{1/2}$ find operation

Each find operation $O(\log N)$

$$\text{So } (\log N)^{1/2} O(\log N)$$

$O((\log N)^{3/2})$

N insert

Each insert operation takes $O(N)$

$$\text{So } N(O(N)) = O(N^2)$$

$(\log N)^{1/2}$ delete operation

Each delete $O(N)$

So $(\log N)^{1/2}$ delete operation

$(\log N)^{1/2} O(N)$

$O(N(\log N)^{1/2})$

$(\log N)^{1/2}$ decrease key

Each decrease key operation takes $O(N)$

$(\log N)^{1/2} O(N)$

$O(N(\log N)^{1/2})$

MIN Heap

$(\log N)^{1/2}$ find

Each find operation takes $O(N)$

$(\log N)^{1/2} O(N)$

$O(N(\log N)^{1/2})$

N insert

Each insert takes $\log N$ time

$O(N \log N)$

$(\log N)^{1/2}$ delete operation

Each delete takes $O(\log N)$

$$\text{So } (\log N)^{1/2} \times O(\log N)$$

$O((\log N)^{3/2})$

$(\log N)^{1/2}$ decrease key

Each decrease takes $O(\log N)$

$(\log N)^{1/2} \times O(\log N)$

$O((\log N)^{3/2})$

Sorted doubly linked list

$(\log N)^{1/2}$ find

$$\text{So } (\log N)^{1/2} O(N)$$

$= O(N(\log N)^{1/2})$

N insert

Each insert takes $O(N)$

$N \times O(N)$

$O(N^2)$

$(\log N)^{1/2}$ delete

Each delete takes $O(1)$

$$\text{So } (\log N)^{1/2} \times O(1)$$

$O((\log N)^{1/2})$

$(\log N)^{1/2}$ decrease key

Each decrease takes $O(N)$

$$\text{So } O(N(\log N)^{1/2})$$

On comparing unsorted array is best suited.

Hence, the correct option is (A).

Key Point

Summary



	Unsorted Array	Min-Heap	Sorted Array	Sorted Doubly Linked List
Find	$O(n\sqrt{\log n})$	$O(n\sqrt{\log n})$	$O(\log n)^{\frac{1}{2}}$	$O(n\sqrt{\log n})$
Insert	$O(n)$	$O(n \log n)$	$O(n^2)$	$O(n^2)$
Delete	$O(n\sqrt{\log n})$	$O(n\sqrt{\log n})$	$O(n\sqrt{\log n})$	$O(\sqrt{\log n})$
Decrease Key	$O(\sqrt{\log n})$	$O(\log n)^{\frac{1}{2}}$	$O(n\sqrt{\log n})$	$O(\sqrt{\log n})$
Overall Time Complexity	$O(n\sqrt{\log n})$	$O(n \log n)$	$O(n^2)$	$O(n^2)$

1.8 (D)**Given :** $f(n) = n$

$$g(n) = n^{(1+\sin n)}$$

Following statements :

i. $f(n) = O(g(n))$

ii. $f(n) = \Omega(g(n))$

$$-1 \leq \sin n \leq 1$$

$$n^0 \leq g(n) \leq n^2$$

We cannot compare (i) and (ii)

Neither (i) nor (ii).

Hence, the correct option is (D).

1.9 (C)**Given :** A sorted doubly linked list containing N items.

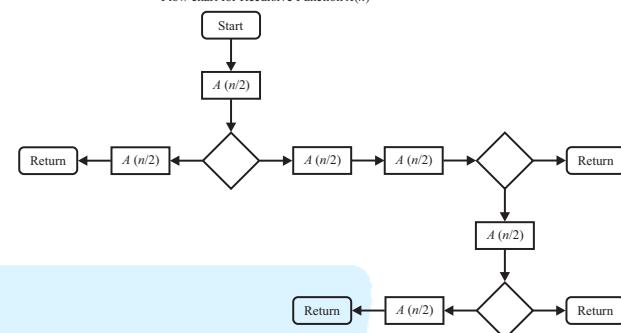
Following operations with order:

 $\Theta(N)$ delete $O(\log N)$ insert, $O(\log N)$ find, and $\Theta(N)$ decrease key. $O(N)$ delete operationsEach take $O(1)$ timeSo $O(N)$ $O(\log N)$ insert operationsEach takes $O(N)$ So $(\log N) \times O(N)$ $O(N \log N)$ $O(\log N)$ find operationsEach find takes $O(N)$ So $\log N \times O(N)$ $O(N \log N)$ $O(N)$ decrease operationsEach decrease takes $O(N)$ So $N \times O(N)$

$$O(N^2)$$

Time complexity $O(N^2)$.

Hence, the correct option is (C).

1.10 2.32**Given :**Flow chart for Recursive Function $A(n)$ 

$$T(n) = 5T\left(\frac{n}{2}\right) + C$$

Taking longest recursive path

$$a = 5, b = 2, k = 0$$

(Using Masters Theorem)

$$a > b^k$$

$$5 > 2^0$$

$$5 > 1$$

$$T(n) = \Theta(n^{\log_b a})$$

$$= \Theta(n^{\log_2 5})$$

$$= 2(n^{2.32}) \quad \alpha = 2.32$$

Hence, the correct answer is 2.32.

1.11 (B)**Given :** functions from positive integers to real numbers :

$$10, \sqrt{n}, n, \log_2 n, \frac{100}{n}$$

Arranging given functions in increasing order:

$$\frac{100}{n} < 10 < \log_2 n < \sqrt{n} < n$$

Decreasing function < Constant function < Logarithmic function < Square root < Polynomial function

Hence, the correct option is (B).



1.12 (B)

Given : $T(n) = \begin{cases} 2T(\sqrt{n})+1, & n > 2 \\ 2, & 0 < n \leq 2 \end{cases}$

$$T(n) = 2T(\sqrt{n})+1$$

Take, $n = 2^m$

$$T(2^m) = 2T(2^{m/2})+1$$

Take $T(2^m) = S(m)$

$$S(m) = 2S\left(\frac{m}{2}\right)+1$$

By applying master method

$$S(m) = \Theta(m)$$

$$T(n) = \Theta(\log n)$$

Hence, the correct option is (B).

1.13 (C)

Given :

int fun(int n)

{

```
    int i, j;
    for (i = 1; i <= n; i++)
    {
        for (j = 1; j < n; j += i)
        {
            printf("%d %d", i, j);
        }
    }
```

}

For $i = 1$

$j = 1, 2, \dots, n$

n times

$i = 2$

$j = 1, 3, 5, \dots, n$

$\frac{n}{2}$ times

$i = 3$

$j = 1, 4, 7, \dots, n$

$\frac{n}{3}$ times

$i = 4$

$j = 1, 5, 9, \dots, n$

$\frac{n}{4}$ times

Therefore we get

$$\begin{aligned} &= \frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \dots + \frac{n}{n} \\ &= n \left[1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right] \end{aligned}$$

$$= n \log n$$

$$= \Theta(n \log n)$$

Hence, the correct option is (C).

1.14 (B)

Given :

n unsorted arrays: A_1, A_2, \dots, A_n .

n is odd.

Each of A_1, A_2, \dots, A_n contains n distinct elements.

Hence, Time complexity to find median in unsorted array $O(n)$

We have to apply this for all arrays

$$n \times O(n) = O(n^2)$$

Again, apply once more on medians of all array $O(n)$ time.

$$O(n^2) + O(n)$$

$$O(n^2)$$

Hence, the correct option is (B).

1.15 (A)

Given : $T(n) = T(n^{1/a})+1$,

$$T(b)=1$$

$$a = b = \omega(1)$$

$$T(n) = T(n^{1/a})+1$$

$$T(b) = 1$$

$$T(n) = T(n^{1/a})+1$$

$$T(n^{1/a}) = T(n^{1/a^2})+1$$

$$T(n^{1/a^2}) = T(n^{1/a^3})+1$$

On applying substitution after k iterations

$$T(n) = T(n^{1/a^k})+1+1+\dots+k \text{ times}$$

$$T(n) = T(n^{1/a^k})+k$$

When $n^{1/a^k} = b$ taking \log_b on both sides

$$\frac{1}{a^k} \log_b n = \log_b b$$

$$\log_b n = a^k$$

$$a^k = \log_b n$$

$$k = \log_a \log_b n$$

$$\Theta(\log_a \log_b n)$$

Hence, the correct option is (A).



1.16 (B)

Given : $f_1 = 10^n$, $f_2 = n^{\log n}$, $f_3 = n^{\sqrt{n}}$

Taking log for all the functions

$$\begin{aligned}\log(f_1) &= n \log 10, \log(f_2) \\ &= (\log n)^2 \log(f_3) = \sqrt{n \log n}\end{aligned}$$

hence $f_2 < f_3 < f_1$

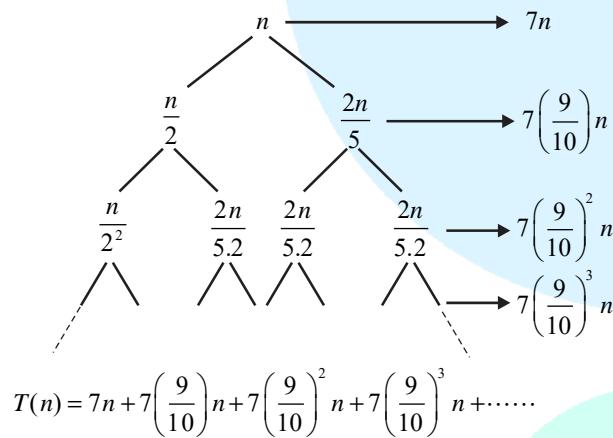
Hence, the correct option is (B).

1.17 (A)

Given :

$$T(n) = \begin{cases} T(n/2) + T(2n/5) + 7n & \text{if } n > 0 \\ 1 & \text{if } n = 0 \end{cases}$$

Recurrence tree for given recurrence relation is



Decreasing GP = $\theta(1)$

$$\begin{aligned}&= 7n * \theta(1) \\ &= O(7n) \\ &= O(n)\end{aligned}$$

Hence, the correct option is (A).

1.18 (B)

Given : $T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$ $a \geq 1$ $b > 1$

If we take $a = 2$, $b = 2$, on applying extended master theorem A, C, D are false hence option B is correct.

Hence, the correct option is (B).

1.19 (B)

Given : P is an array containing n integers.

Using min-max algorithm in worst case it will take $t \leq 2n - 2$ comparisons without divide and conquer technique.

It will take $t \leq \frac{3n}{2} - 2$ with divide and conquer technique.

Hence, the correct option is (B).

2

Divide and Conquer Method



Practice Questions

2013 IIT Bombay

- 2.1 Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?
- (A) $O(1)$ (B) $O(\log n)$
(C) $O(n)$ (D) $O(n \log n)$

2014 IIT Kharagpur

- 2.2 Let P be a Quick sort program to sort numbers in ascending order using the first element as the pivot. Let t_1 and t_2 be the number of comparisons made by P for the inputs [1 2 3 4 5] and [4 1 5 3 2] respectively. Which one of the following holds?
- (A) $t_1 = 5$ (B) $t_1 < t_2$
(C) $t_1 > t_2$ (D) $t_1 = t_2$
- 2.3 You have an array of n elements. Suppose you implement quick sort by always choosing the central element of the array as the pivot. Then the tightest upper bound for the worst-case performance is :
- (A) $O(n^2)$ (B) $O(n \log n)$
(C) $\Theta(n \log n)$ (D) $O(n^3)$

- 2.4 The minimum number of comparisons required finding the minimum and the maximum of 100 numbers is _____.

2015 IIT Kanpur

- 2.5 Which one of the following is the recurrence equation for the worst-case time complexity of the Quicksort algorithm for sorting n (≥ 2) numbers? In the recurrence equations given in the options below, c is a constant,
- (A) $T(n) = 2T(n/2) + cn$
(B) $T(n) = T(n-2) + T(1) + cn$
(C) $T(n) = 2T(n-1) + cn$
(D) $T(n) = T(n/2) + cn$

- 2.6 What are the worst-case complexities of insertion and deletion of a key in a binary search tree?

- (A) $\Theta(\log n)$ for both insertion and deletion
(B) $\Theta(n)$ for both insertion and deletion
(C) $\Theta(n)$ for insertion and $\Theta(\log n)$ for deletion
(D) $\Theta(\log n)$ for insertion and $\Theta(n)$ for deletion

- 2.7 Suppose you are provided with the following function declaration in the C programming language.

```
int partition(int a[], int n);
```

The function treats the first element of $a[]$ as a pivot, and rearranges the array so that all elements less than or equal to the pivot is in the left part of the array, and all elements greater than the pivot is in the right part. In addition, it moves the pivot so that the pivot is the last element of the left part. The return value is the number of elements in the left part. The following partially given function in the C programming language is used to find the k^{th} smallest element in an array $a[]$ of size n using the partition function. We assume $k \leq n$.

```
int kth_smallest(int a[], int n, int k)
{
    int left_end = partition(a, n);
    if (left_end+1 == k)
        return a[left_end];
    if (left_end+1 > k)
    {
        return kth_smallest(a, left_end, k);
    }
    else
    {
        return kth_smallest(a, left_end+1, k);
    }
}
```



}

The missing argument lists are respectively

(A) (a, left_end, k) and (a+left_end+1, n-left_end-l, k-left_end-l)

(B) (a, left_end, k) and (a, n-left_end-l, k-left_end-l)

(C) (a+left_end+1, n-left_end-1, k-left_end-1)
and (a, left_end, k)

(D) (a, n-left_end-l, k-left_end-l) and (a, left_end, k)

2016 IISc Bangalore

- 2.9** Assume that the algorithms considered here sort the input sequence in ascending order. If the input is already in ascending order, which of the following are TRUE?

 - I. Quick sort runs in $\Theta(n^2)$ time
 - II. Bubble sort runs in $\Theta(n^2)$ time
 - III. Merge sort runs in $\Theta(n)$ time
 - IV. Insertion sort runs in $\Theta(n)$ time

(A) I and II only (B) I and III only
(C) II and IV only (D) I and IV only

2.10 The worst-case running times of Insertion sort, Merge sort and Quick sort, respectively, are :

 - (A) $\Theta(n \log n)$, $\Theta(n \log n)$, and $\Theta(n^2)$
 - (B) $\Theta(n^2)$, $\Theta(n^2)$, and $\Theta(n \log n)$
 - (C) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n \log n)$
 - (D) $\Theta(n^2)$, $\Theta(n \log n)$, and $\Theta(n^2)$

2017 IIT Roorkee

- 2.11** Let A be an array of 31 numbers consisting of a sequence of 0's followed by a sequence of 1's. The problem is to find the smallest index i such that $A[i]$ is 1 by probing the minimum number of locations in A. The worst-case number of

probes performed by an optimal algorithm is _____.

2018 IIT Guwahati

- 2.12** The number of possible min-heaps containing each value from {1, 2, 3, 4, 5, 6, 7} exactly once is _____.

2019 IIT Madras

- ### 2.13 Consider a sequence of 14 elements

$$A = [-5, -10, 6, 3, -1, -2, 13, 4, -9, -1, 4, 12, -3, 1]$$

The Subsequence sum $S(i, j) = \sum_{k=i}^j A[k]$. Determine the maximum of $S(i, j)$ where $0 \leq i \leq j \leq 14$, (Divide and conquer approach may be used.)

- 2.14** An array of 25 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is

- 2.15** Consider the following statements :

 - I. The smallest element in a max-heap is always at a leaf node
 - II. The second largest element in a max-heap is always a child of the root node
 - III. A max-heap can be constructed from a binary search tree in $\Theta(n)$ time
 - IV. A binary search tree can be constructed from a max-heap in $\Theta(n)$ time

Which of the above statements are TRUE?

- (A) I, II and III (B) I, II and IV
 (C) I, III and IV (D) II, III and IV

2020 IIT Delhi

- 2.16** What is the worst-case time complexity of inserting n elements into an empty linked list, if the linked list needs to be maintained in sorted order?

(A) $\Theta(n)$ (B) $\Theta(n \log n)$
(C) $\Theta(n^2)$ (D) $\Theta(1)$

- 2.17** What is the worst-case time complexity of inserting n^2 elements into an AVL-tree with n elements initially?



- (A) $\Theta(n^4)$ (B) $\Theta(n^2)$
 (C) $\Theta(n^2 \log n)$ (D) $\Theta(n^3)$

- 2.18** In a balanced binary search tree with n elements, what is the worst-case time complexity of reporting all elements in range $[a, b]$? Assume that the number of reported elements is k .
 (A) $\Theta(\log n)$ (B) $\Theta(\log n + k)$
 (C) $\Theta(k \log n)$ (D) $\Theta(n \log k)$

2021 IIT Bombay

- 2.19** Let H be a binary min-heap consisting of n elements implemented as an array. What is the worst-case time complexity of an optimal algorithm to find the maximum element in H ?
 (A) $\Theta(\log n)$ (B) $\Theta(1)$
 (C) $\Theta(n \log n)$ (D) $\Theta(n)$

- 2.20** Consider the following array

23	32	45	69	72	73	89	97
----	----	----	----	----	----	----	----

Which algorithm out of the following option uses the least number of comparisons (among the array elements) to sort the above array in ascending order ?

- (A) Merge sort
 (B) Quick sort using last element as pivot
 (C) Selection sort
 (D) Insertion sort

- 2.21** What is the worst-case number of arithmetic operations performed by recursive binary search on a sorted array of size n ?

- (A) $\Theta(n^2)$ (B) $\Theta(\log_2 n)$
 (C) $\Theta(n)$ (D) $\Theta(\sqrt{n})$

- 2.22** A Binary search tree T contains n distinct elements. What is time complexity of picking an elements in T that is smaller than the maximum element in T ?

- (A) $\Theta(n \log n)$ (B) $\Theta(\log n)$
 (C) $\Theta(n)$ (D) $\Theta(1)$

Solutions

2.1 (C)

Given : A binary search tree of n nodes.

If the element to be inserted is either smaller than all the elements or larger than all the element then insertion will take $O(n)$ time.

Hence, the correct option is (C).

2.2 (C)

Given :

Input 1 : [1 2 3 4 5]

Time : t_1

Input 2 : [4 1 5 3 2]

Time : t_2

Key Point

Quick sort performs worst when input sequence is already sorted.

$$t_1 > t_2$$

Hence, the correct option is (C).

2.3 (A)

Given : An array of n elements.

We can get worst-case of $O(n^2)$ even if we choose the center element of the array as the pivot.

6 4 2 1 3 5 7
↑
pivot

Array is split into $(n-1)$ element on one side and 1 element on other side

Hence $O(n^2)$ worst-case complexity

Hence the correct option is (A).

2.4 148

Given : $n = 100$

Key Point

Using divide and conquer,
 To find minimum number of comparison require to find minimum and maximum is $T(n) = 2T\left(\frac{n}{2}\right) + 2$

After solving above recursion, we get

$$T(n) = \frac{3}{2}n - 2 = 1.5 \times n - 2$$



$$\begin{aligned} T(n) &= 1.5 \times 100 - 2 \\ &= 150 - 2 \\ &= 148 \end{aligned}$$

Hence, the correct answer is 148.

2.5 (B)

Given : Quicksort algorithm for sorting $n (\geq 2)$ numbers.

Key Point

Worst-case for quick sort happens when 1 element is on one list and $n-1$ elements on another list.

From above points we can conclude that only option (B) is correct.

Hence, the correct option is (B).

2.6 (B)

Key Point

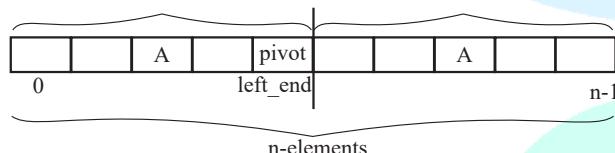
In BST both insertion and deletion takes $O(n)$ time if BST is either left skewed or right skewed.

Hence, the correct option is (B).

2.7 (A)

Given :

```
int partition(int a[], int n);
```



Left_end (is pivot position) = partition (a, n);

Note : k^{th} smallest is same as an element at index $(k-1)$ in the sorted array.

If ($\text{left_end} + 1 == k$) / k^{th} smallest found.

Return a [left_end]; // return pivot element.

if ($\text{left_end} + 1 > k$) // k^{th} smallest is before pivot.

return k^{th} _smallest (a, left_end , k); // search before pivot.

else // k^{th} smallest is after pivot (right part)

```
return kth_smallest (a+left_end+1, n-(left_end-1), k-(left_end+1));
```

Address of 1st element after pivot Number of elements in right part
(right part)

Hence, the correct option is (A).

2.8 (B)

Given : Number of elements = 64

Key Point

Merge sort algorithm take $O(n \log n)$ time for n input elements.

For some constant k ,

$$kn \log n = t$$

$$k64 \log 64 = 30 \text{ sec}$$

$$k = \frac{30}{6 \times 64} = \frac{5}{64}$$

In 6 minutes, size of input

$$= \frac{5}{64} \times n \log n = 6 \times 60$$

$$n \log n = \frac{6 \times 60}{5} \times 64$$

$$n \log n = 4608$$

$$n = 512$$

Hence, the correct option is (B).

2.9 (D)

Given : The input is already in ascending order.

Key Point

When input is already sorted then,

- (i) Quick sort takes $O(n^2)$ time
- (ii) Bubble sort takes $O(n)$ time {Best case}
- (iii) Merge sort takes $O(n \log n)$ time
- (iv) Insertion sort takes $O(n)$ time {Best case}

From above points we can conclude that only option (D) is correct.

Hence, the correct option is (D).

2.10 (D)

Key Point

Worst-case running time

- (i) Insertion sort $\rightarrow O(n^2)$
- (ii) Merge sort $\rightarrow O(n \log n)$
- (iii) Quick sort $\rightarrow O(n^2)$

From above points we can conclude that only option (D) is correct.

Hence, the correct option is (D).

2.11 (5)

Given : A is an array of 31 numbers consisting of a sequence of 0's followed by a sequence of 1's.

Here, since 0s are followed by 1 so we have a sorted sequence and we can apply binary search.



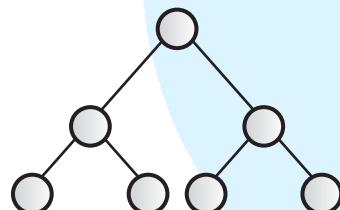
At each stage we compare with $\frac{(\text{low} + \text{high})^{\text{th}}}{2}$
 element index and if it is 1 we check left and if it is 0 we check right.
 Total worst-case no. of probes is $[\log_2 31] = 5$
 Hence, the correct answer is 5.

2.12 80**Given :** Values : {1, 2, 3, 4, 5, 6, 7}**Key Point**

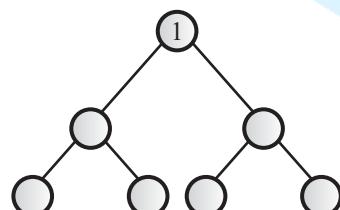
A min heap is a binary tree such that

- The data in each node is less than (or equal to) the data in that node's children.
- The binary tree is complete.

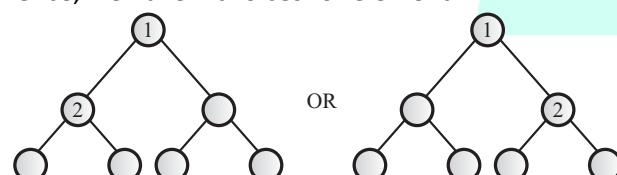
Since a min-heap is complete binary tree hence 7 nodes are distributed in 2 levels (root at level 0)



In min-heap smallest element is in root hence the structure is



In first level, second minimum element is present.
 Hence, we have 2 choices for element 2.



Now for leaf node of element 2 we have 5 elements

So total no. of ways = $5 \times 4 = 20$

Since leaf node can be arranged in 2 ways

We have $20 \times 2 = 40$ ways.

And finally left subtree of 1 i.e. root can be arranged in two ways hence total

$$40 \times 2 = 80 \text{ ways}$$

Hence, the correct answer is 80.

2.13 (29)

Given : Subsequence sum, $s(i, j) = \sum_{k=i}^j A[K]$

By observing the array, it can deduced that $s(i, j)$ will be maximum when

$$i = 2 \text{ and } j = 12.$$

$$\begin{aligned} \text{Sum} &= 6 + 3 - 1 - 2 + 13 + 4 - 9 - 1 + 9 + 1 \\ &= 29 \end{aligned}$$

Hence, the correct answer 29.

2.14 0.08**Given :** Number of elements = 25

Favorable cases = 2 (either minimum or maximum)

Key Point

In quick sort worst-case arises when pivot selected is either minimum or maximum.

$$\text{Probability} = \frac{2}{25} = 0.08$$

Hence, the correct answer is 0.08.

2.15 (A)**Given :** Statements :

- I. The smallest element in a max-heap is always at a leaf node.
- II. The second largest element in a max-heap is always a child of the root node.
- III. A max-heap can be constructed from a binary search tree in $\Theta(n)$ time.
- IV. A binary search tree can be constructed from a max-heap in $\Theta(n)$ time.

Key Point

- I. The smallest element in a max-heap is always at a leaf node.
- II. The second largest element in a max-heap is always a child of the root node
- III. A max-heap can be constructed from a binary search tree in $\Theta(n)$ time

From above points we can conclude that only statement I, II and III are true.

Hence, the correct option is (A).

2.16 (C)

Given : An empty linked list, which needs to be maintained in sorted order while inserting n elements.



Each insertion will take $\Theta(n)$ time hence n elements will take $\Theta(n^2)$ time.
Hence, the correct option is (C).

2.17 (C)

Given : An AVL-tree with n elements initially, we have to insert n^2 elements.
Each insertion will take $O(\log n)$ time hence insertion of n^2 element will take $(n^2 \log n)$ time
Hence, the correct option is (C).

2.18 (B)

Given :
a balanced binary search tree with n elements.
Time complexity to check if element 'a' and 'b' present in given balanced binary search tree = $O(\log n)$
 \therefore The BST is balanced
Time complexity to traverse all elements in range $[a, b] = \Theta(K)$
Total time complexity
 $= \Theta(\log n) + \Theta(\log n) + \Theta(K)$
 $= \Theta(\log n + K)$
Hence, the correct option is (B).

2.19 (D)

Given : H is a binary min-heap consisting of n elements implemented as an array.
In a min heap maximum element is present at leaf node we need to navigate through the leaf node i.e. $n/2$ node Hence $O(n)$.
Hence, the correct option is (D).

2.20 (D)

Given : Array is already sorted
Therefore, insertion sort will use the least number of comparisons.
Hence, the correct option is (D).

2.21 (B)

Given : Recursive binary search on a sorted array of size n .
The worst-case occurs when we are searching for a key that is smaller than the smallest element of the array or larger than the largest element of the array.
Hence, the correct option is (B).

2.22 (D)

Given : A Binary search tree T containing n distinct elements.
In BST maximal element will be in right most leaf node
If root has right sub tree then root itself is smaller than maximum value
If root does not have right subtree then root is maximum element and left child of root is the required element.
Hence, the correct option is (D).

3

Greedy Method



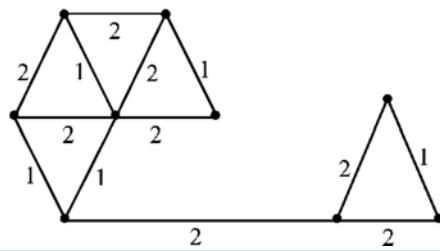
Practice Questions

2013 IIT Bombay

- 3.1 Which one of the following is the tightest upper bound that represents the number of swaps required to sort n numbers using selection sort?
- (A) $O(\log n)$ (B) $O(n)$
 (C) $O(n \log n)$ (D) $O(n^2)$

2014 IIT Kharagpur

- 3.2 Consider the tree arcs of a BFS traversal from a source node W in an unweighted, connected, undirected graph. The tree T formed by the tree arcs is a data structure for computing.
- (A) the shortest path between every pair of vertices.
 (B) the shortest path from W to every vertex in the graph.
 (C) the shortest paths from W to only those nodes that are leaves of T .
 (D) the longest path in the graph
- 3.3 Suppose P, Q, R, S, T are sorted sequences having lengths 20, 24, 30, 35, 50 respectively. They are to be merged into a single sequence by merging together two sequences at a time. The number of comparisons that will be needed in the worst case by the optimal algorithm for doing this is _____.
- 3.4 The number of distinct minimum spanning trees for the weighted graph below is _____.



2015 IIT Kanpur

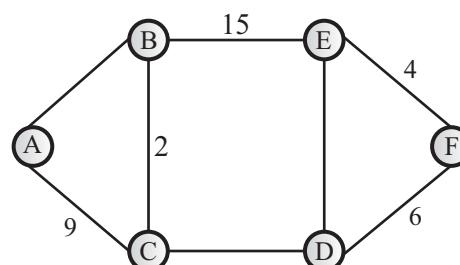
- 3.5 Let G be a connected undirected graph of 100 vertices and 300 edges. The weight of a

minimum spanning tree of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes _____.

- 3.6 Let $G = (V, E)$ be a simple undirected graph, and s be a particular vertex in it called the source. For $x \in V$, let $d(x)$ denote the shortest distance in G from s to x . A breadth first search (BFS) is performed starting at s . Let T be resultant BFS tree. If (u, v) is an edge of G that is not in T , then which one of the following CANNOT be the value of $d(u) - d(v)$?

- (A) -1 (B) 0
 (C) 1 (D) 2

- 3.7 The graph shown below has 8 edges with distinct integer edge weights. The minimum spanning tree (MST) is of weight 36 and contains the edges: $\{(A, C), (B, C), (B, E), (E, F), (D, F)\}$. The edge weights of only those edges which are in the MST are given in the figure shown below. The minimum possible sum of weights of all 8 edges of this graph is _____.



2016 IISc Bangalore

- 3.8 Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is /are TRUE?



- P : Minimum spanning tree of G does not change
Q : Shortest path between any pair of vertices
does not change

(A) P only (B) Q only
(C) Neither P nor Q (D) Both P and Q

- 3.9** Let G be a complete undirected graph on 4 vertices, having 6 edges with weights being 1, 2, 3, 4, 5 and 6. The maximum possible weight that a minimum weight spanning tree of G can have is .

- 3.10** $G = (V, E)$ is an undirected simple graph in which each edge has distinct weight, and e is a particular edge of G . Which of the following statements about the minimum spanning trees (MSTs) of G is/are TRUE?

2017 IIT Roorkee

- 3.11** Let $G = (V, E)$ be any connected undirected edge-weighted graph. The weights of the edges in E are positive and distinct.

Consider the following statements :

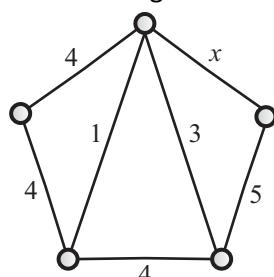
- (I) Minimum Spanning tree of G is always unique.
 - (II) Shortest path between any two vertices of G is always unique.

Which of the above statements is/are necessarily true?

- (A) (I) only
 - (B) (II) only
 - (C) Both (I) and (II)
 - (D) Neither (I) nor (II)

2018 IIT Guwahati

- 3.12** Consider the following undirected graph G :



Choose a value for x that will maximize the number of minimum weight spanning trees (MWSTs) of G . The number of MWSTs of G for this value of x is _____.

- 3.13** Consider the weights and values of items listed below. Note that there is only one unit of each item.

Item number	Weight (in Kgs)	Value (in rupees)
1	10	60
2	7	28
3	4	20
4	2	24

The task is to pick a subset of these items such that their total weight is no more than 11 Kgs and their total value is maximized. Moreover, no item may be split. The total value of items picked by an optimal algorithm is denoted by V_{opt} . A greedy algorithm sorts the items by their value-to-weight ratios in descending order and packs them greedily, starting from the first item in the ordered list. The total value of items picked by the greedy algorithm is denoted by V_{greedy} . The value of $V_{opt} - V_{greedy}$ is _____.

2020 IIT Delhi

- 3.14** Let $G = (V, E)$ be a weighted undirected graph and let T be a Minimum Spanning Tree (MST) of G maintained using adjacency lists. Suppose a new weighted edge $(u, v) \in V \times V$ is added to G . The worst case time complexity of determining if T is still an MST of the resultant graph is

(A) $\Theta(|E| + |V|)$ (B) $\Theta(|E||V|)$
(C) $\Theta(|E| \log |V|)$ (D) $\Theta(|V|)$

- 3.15** Consider a graph $G = (V, E)$, where $V = \{v_1, v_2, \dots, v_{100}\}$, $E = \{(v_i, v_j) \mid 1 \leq i < j \leq 100\}$, and weight of the edge (v_i, v_j) is $|i - j|$. The weight of minimum spanning tree of G is

- 3.16** Let $G = (V, E)$ be a directed, weighted graph with weight function $w: E \rightarrow \mathbb{R}$. For some function $f: V \rightarrow \mathbb{R}$, for each edge $(u, v) \in E$,

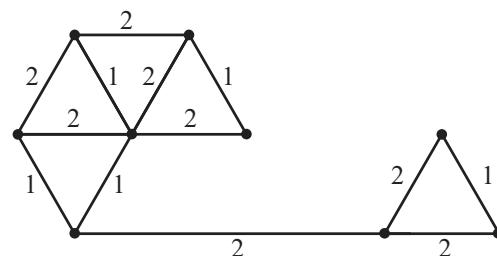


So, total no. of comparisons
 $= 43 + 64 + 93 + 158 = 358$

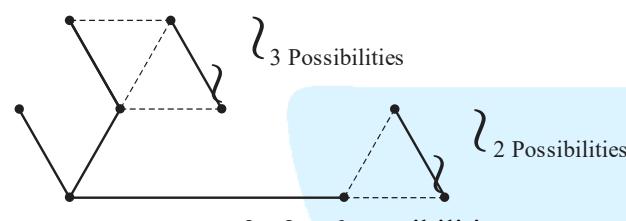
Hence, the correct answer is 358.

3.4 6

Given :



MST :



Total possibilities = $3 \times 2 = 6$ possibilities

Hence, the correct answer is 6.

3.5 995

Given : Number of vertices in $G = 100$

Number of edges in $G = 300$

The weight of MST of $G = 500$

Since number of vertices in $G = 100$

Number of edges in MST = 99 { $100 - 1 = 99$ }

Now, if each edge weight is incremented by 5 then weight in MST will be increased = $99 \times 5 = 495$

Now total weight of MST = $500 + 495 = 995$

Hence, the correct answer is 995.

3.6 (D)

Given: $G = (V, E)$ simple undirected graph, s is source, $d(x)$ shortest distance in G from s to x , T is the resultant BFS tree, (u,v) is an edge of G that is not in T .

$d(u) - d(v) = 0$ is possible when both u and v have an edge from a common node t and t is in the shortest path from s to u and v .

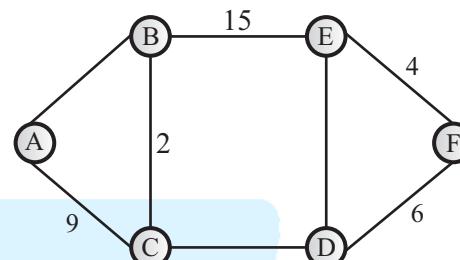
$d(u) - d(v) = 1$ is possible when v and another node t are in the shortest path from s to u and both t and v are siblings- same distance from s to both t and v causing $t-u$ edge to be in BFS tree and not $v-u$.

$d(u) - d(v) = -1$ is possible as explained above by interchanging u and v .

$d(u) - d(v) = 2$ is not possible. This is because on BFS traversal we either visit u first or v . Let's take u first. Now, we put all neighbors of u on queue. Since v is a neighbour and v is not visited before as assumed, $d(v)$ will become $d(u) + 1$. Similarly, for v being visited first. Hence, the correct option is (D).

3.7 69

Given :



Consider the cycle ABC. AC and AB are part of minimum spanning tree. So, AB should be greater than max (AC, BC) (greater and not equal as edge weights are given to be distinct) as otherwise we could add AB to the minimum spanning tree and removed the greater of AC, BC and we could have got another minimum spanning tree.

So, $AB > 9$

Similarly, for the cycle DEF, $ED > 6$

And for the cycle BCDE, $CD > 15$

So, minimum possible sum of these will be $10 + 7 + 16 = 33$. Adding the weight of spanning tree, we get the total sum of edge weights.

$$= 33 + 36 = 69$$

Hence, the correct answer is 69.

3.8 (A)

Given : G is a weighted connected undirected graph with distinct positive edge weights every edge weight is increased by the same value.

Key Point

If every edge weight is increased by the same value, then minimum spanning tree of G does not change, but shortest path may change.

Hence, the correct option is (A).

3.9 7

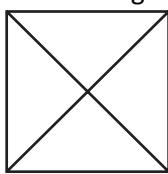
Given : Number of vertices = 4

Number of edges = 6

Edge weights = 1, 2, 3, 4, 5, 6

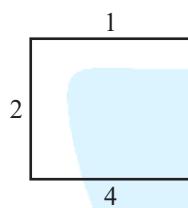


Graph with 4 vertices and 6 edges :



An edge with minimum weight cannot be included only if the MST forms a cycle by adding that edge. Here we will try to reject an edge by adding it to cycle. Since we cannot have a cycle with two edges hence edge weight 1 and 2 will be included in MST. If we reject edge weight 3 by forming a cycle, the next available edge will be 4 and weight of MST will be $1+2+4=7$.

MST :



Hence, the correct answer is 7.

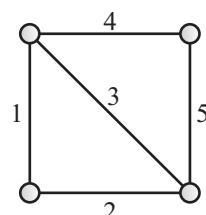
3.10 (B)

Given : $G = (V, E)$ is an undirected simple graph in which each edge has distinct weight, and e is a particular edge of G .

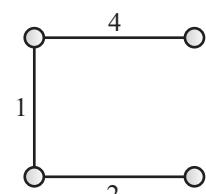
Statement 1 : If e is the lightest edge of some cycle in G , then every MST of G includes e .

This statement is false as per below counter example.

Graph :



MST :



Statement II : If e is the heaviest edge of some cycle in G , then every MST of G excludes e .

This statement is true, since for MST we will have lighter edges of the cycle.

Hence, the correct option is (B).

3.11 (A)

Given : $G = (V, E)$ is any connected undirected edge-weighted graph.

The weights of the edges in E are positive and distinct.

Key Point

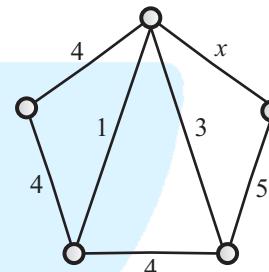
In a graph G if all edge weights are positive and distinct the MST of G is always unique but shortest path may or may not be unique.

Since, graph G has all edge weights positive and distinct the MST of G is always unique but shortest path may or may not be unique

Hence, the correct option is (A).

3.12 4

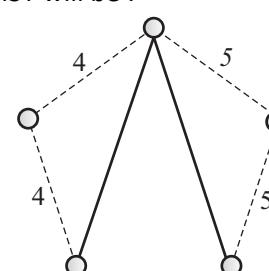
Given :



Key Point

Number of possible MST increases, when we have multiple edges with same edge weights.

If $x = 5$, then MST will be :



To maximize the number of MST, x should be 5.

So number of MST = $2 \times 2 = 4$

Hence, the correct answer is 4.

3.13 16

Given :

Item number	Weight (in Kgs)	Value (in rupees)
1	10	60
2	7	28
3	4	20
4	2	24

In case of optimal algorithm,

Adding items such that weight ≤ 11 kg



Three combination possible

Item 1 Weight = 10, Item 2,4 weight = 9

Item 2,3 Weight = 11

Highest value = 60 for item 1, weight 10

$$V_{opt} = 60$$

In case of greedy algorithm

Item number	Weight	Value	Value/weight
1	10	60	6
2	7	28	4
3	4	20	5
4	2	24	12

Item 4 and item 3 is picked, weight = $2 + 4 = 6$

Their values are 24, 20 respectively

$$\Rightarrow V_{\text{greedy}} = 24 + 20 = 44$$

$$V_{opt} - V_{\text{greedy}} = 60 - 44 = 16$$

Hence, the correct answer is 16.

3.14 (D)

Given : $G = (V, E)$ is a weighted undirected graph and T is a Minimum Spanning Tree (MST) of G maintained using adjacency lists a new weighted edge $(u, v) \in V \times V$ is added to G

- As T is a minimum spanning tree and we need to add a new edge to existing spanning tree.
- Later we need to check still T is a minimum spanning tree or not, So we need to check all vertices whether there is any cycle present after adding a new edge.
- All vertices need to traverse to confirm minimum spanning tree after adding new edge then time complexity is $O(V)$.

Hence, the correct option is (D).

3.15 99

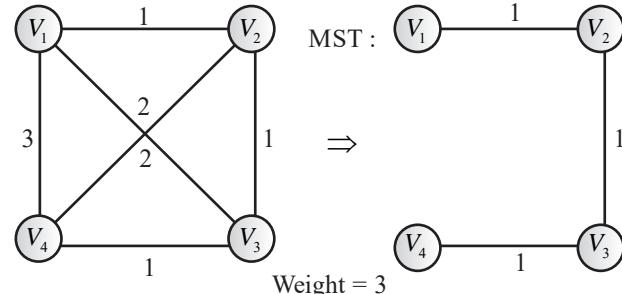
Given : Graph $G = (V, E)$, where

$$V = \{v_1, v_2, \dots, v_{100}\},$$

$$E = \{(v_i, v_j) \mid 1 \leq i < j \leq 100\},$$

weight of the edge (v_i, v_j) is $|i - j|$.

Consider the following for $n = 4$



Similarly for $n = 100$, 99 will be the cost of MST.

Hence, the correct answer is 99.

3.16 (A)

Given :

$G = (V, E)$ be a directed, weighted graph with weight function $w: E \rightarrow \mathbb{R}$. For some function $f: V \rightarrow \mathbb{R}$, for each edge $(u, v) \in E$, define $w'(u, v)$ as $w(u, v) + f(u) - f(v)$.

For any mapping of vertices to real values, the shortest paths won't change. All intermediate node values get canceled on any path you take and what you're left with is only the source and destination node values which would add up to cost on any path. Hence the shortest path would still be same.

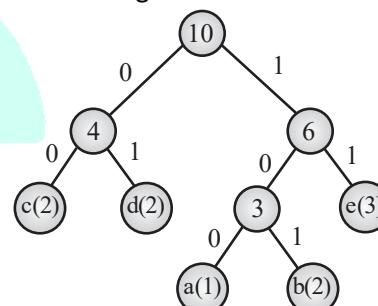
Hence, the correct option is (A).

3.17 (B)

Given :

String aabbccddeee

Using Huffman Coding-



Character	Frequency	Encoding
a	1	100
b	2	101
c	2	00
d	2	01
e	3	11

Hence length of encoded string is

$$\begin{aligned} &= 1*3 + 2*3 + 2*2 + 2*2 + 3*2 \\ &= 3 + 6 + 4 + 4 + 6 = 23 \end{aligned}$$

Hence, the correct option is (B).



3.18 (B)

Given : Statements

- S₁. There exists a minimum edge weight in G which is present in every MST of G.
- S₂. If every edge in G has distinct weights, then G has a unique MST.

By using Kruskal's algorithm to find MST, we sort the edges based on their weight and start selecting edges from the smallest weight (w_small for example).

Problem with S₁: If we have multiple copies of w_small, then a specific w_small weighted edge is not guaranteed to be selected by Kruskal.
S₂ is Correct: If the sorted order of the edges contains only distinct values, the Kruskal algorithm will always select a unique set of edges resulting in a unique minimum spanning tree.

Hence, the correct option is (B).

4

Dynamic Programming



Practice Questions

2013 IIT Bombay

- 4.1** What is the time complexity of Bellman-ford single-source shortest path algorithm on a complete graph of n vertices?
- (A) $\Theta(n^2)$ (B) $\Theta(n^2 \log n)$
 (C) $\Theta(n^3)$ (D) $\Theta(n^3 \log n)$

2014 IIT Kharagpur

- 4.2** Consider two strings $A = "qpqrr"$ and $B = "pqprqrp"$. Let x be the length of the longest common subsequence (not necessarily contiguous) between A and B and let y be the number of such longest common subsequences between A and B. Then $x+10y = \underline{\hspace{2cm}}$.

2016 IISc Bangalore

- 4.3** The Floyd-Warshall algorithm for all-pair shortest paths computation is based on :
- (A) Greedy paradigm.
 (B) Divide-and-conquer paradigm.
 (C) Dynamic programming paradigm.
 (D) neither greedy nor divide-and-conquer nor dynamic programming paradigm.
- 4.4** Consider the weighted undirected graph with 4 vertices, where the weight of edge $\{i, j\}$ is given by the entry W_{ij} in the matrix W .

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

The largest possible integer value of x , for which at least one shortest path between some pair of vertices will contain the edge with weight x is $\underline{\hspace{2cm}}$.

- 4.5** Let A_1, A_2, A_3 , and A_4 be four matrices of dimensions $10 \times 5, 5 \times 20, 20 \times 10$ and 10×5 , respectively. The minimum number of scalar multiplications required to find the product

$A_1 A_2 A_3 A_4$ using the basic matrix multiplication method is $\underline{\hspace{2cm}}$.

2018 IIT Guwahati

- 4.6** Assume that multiplying a matrix G_1 of dimension $p \times q$ with another matrix G_2 of dimension $q \times r$ requires pqr scalar multiplications. Computing the product of n matrices $G_1 G_2 G_3 \dots G_n$ can be done by parenthesizing in different ways. Define $G_i G_{i+1}$ as an explicitly computed pair for a given parenthesization if they are directly multiplied. For example, in the matrix multiplication chain $G_1 G_2 G_3 G_4 G_5 G_6$ using parenthesization $(G_1(G_2 G_3))(G_4(G_5 G_6))$, $G_2 G_3$ and $G_5 G_6$ are the only explicitly computed pairs. Consider a matrix multiplication chain $F_1 F_2 F_3 F_4 F_5$, where matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions $2 \times 25, 25 \times 3, 3 \times 16, 16 \times 1$ and 1×1000 , respectively. In the parenthesization of $F_1 F_2 F_3 F_4 F_5$ that minimizes the total number of scalar multiplications, the explicitly computed pairs is/are
- (A) $F_1 F_2$ and $F_3 F_4$ only
 (B) $F_2 F_3$ only
 (C) $F_3 F_4$ only
 (D) $F_1 F_2$ and $F_4 F_5$ only

2021 IIT Bombay

- 4.7** Define R_n to be the maximum amount earned by cutting a rod of length n meters into one or more pieces of integer length and selling them. For $i > 0$, let $p[i]$ denote the selling price of a rod whose length is i meters. Consider the array of prices:
 $p[1] = 1, p[2] = 5, p[3] = 8, p[4] = 9,$
 $p[5] = 10, p[6] = 17, p[7] = 18$



Which of the following statements is/are correct about R_7 ?

Options :

- (A) R_7 cannot be achieved by a solution consisting of three pieces.

(B) R_7 is achieved by three different solutions.

(C) $R_7 = 19$

(D) $R_7 = 18$

Solutions

4.1 (C)

Time complexity of bellman ford algorithm is relatively high $O(V.E)$ and in case $E = V^2$, $O(V^3)$.

Hence, the correct option is (C).

4.2 34

Given : $A = "qpqrr"$

$B = "pqprqrp"$

The longest common subsequence (not necessarily contiguous) between A and B is having 4 as the length. So $x = 4$ and such common subsequence are as follows

1. $pqqr$
2. $pqrr$
3. $pqrr$

So $y = 3$ (the number of longest common subsequence)

$$\text{Therefore } x + 10y = 4 + 10 \times 3 = 34$$

Hence, the correct answer is 34.

4.3 (C)

Floyd-Warshall algorithm follows dynamic programming approach because the all pair shortest paths are computed in bottom up manner.

Hence, the correct option is (C).

4.4 (C)

Given : Weighted undirected graph with 4 vertices the weight of edge $\{i, j\}$ is given by the entry W_{ij} in the matrix W .

$$W = \begin{bmatrix} 0 & 2 & 8 & 5 \\ 2 & 0 & 5 & 8 \\ 8 & 5 & 0 & x \\ 5 & 8 & x & 0 \end{bmatrix}$$

Let the node be A, B, C D.

The known shortest path between nodes (so far)

$$AB = 2(A \rightarrow B)$$

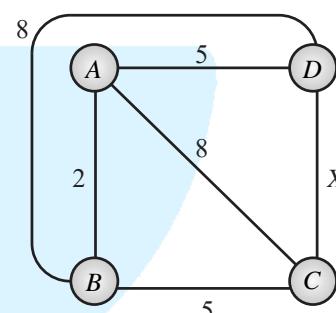
$$AC = 8(A \rightarrow C), 5 + X(A \rightarrow D \rightarrow C)$$

$$AD = 5(5 \rightarrow D)$$

$$BC = 5(B \rightarrow C)$$

$$BD = 8(B \rightarrow D), 5 + X(B \rightarrow C \rightarrow D)$$

$$CD = X(C \rightarrow D), 12(C \rightarrow B \rightarrow A \rightarrow D)$$



The largest value of ' x ' can be 12, so that it will included in the shortest path from C to D.

Hence, the correct option is (C).

4.5 1500

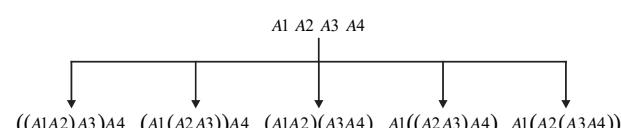
Given :

A_1, A_2, A_3 , and A_4 be four matrices of dimensions $10 \times 5, 5 \times 20, 20 \times 10$ and 10×5 , respectively.

Here A_1 is 10×5 matrix, A_2 is a 5×20 matrix and A_3 is a 20×10 matrix and A_4 is 10×5 matrix.

As we know if we multiply A and B of order $m \times n$ and $n \times p$ respectively, then number of scalar multiplication in the multiplication of A and B will be $m \times n \times p$.

Here we have four matrices A_1, A_2, A_3, A_4



The order in which we parenthesize the product affects the number of simple arithmetic operation needed to compute the product efficiency.



- * $((A_1 A_1) A_3) A_4 = (10 \times 5 \times 20) + (10 \times 20 \times 10) + (10 \times 10 \times 5)$
 $= 1000 + 2000 + 500$
 $= 3500$
- * $((A_1 (A_2 A_3)) A_4) = (10 \times 5 \times 10) + (50 \times 20 \times 10) + (10 \times 5 \times 5)$
 $= 500 + 1000 + 250$
 $= 1750$
- * $(A_1 A_2)(A_3 A_4) = (10 \times 5 \times 20) + (20 \times 10 \times 5)$
 $= 1000 + 1000$
 $= 2000$
- * $A_1 ((A_2 A_3) A_4) = (5 \times 20 \times 10) + (5 \times 10 \times 5) + (10 \times 5 \times 5)$
 $= 1000 + 250 + 250$
 $= 1500$
- * $A_1 (A_2 (A_3 A_4)) = (20 \times 10 \times 5) + (5 \times 20 \times 5) + (10 \times 5 \times 5)$
 $= 1000 + 500 + 250$
 $= 1750$

Hence, the correct answer is 1500.

4.6 (C)

Given : Matrices F_1, F_2, F_3, F_4 and F_5 are of dimensions $2 \times 25, 25 \times 3, 3 \times 16, 16 \times 1$ and 1×1000 , respectively.

Here F_1 is 2×25 matrix, F_2 is 25×3 matrix, F_3 is 3×16 matrix, F_4 is 16×1 matrix and F_5 is 1×1000 matrix.

Dimension of F_5 is 1×1000 , which cause very much multiplication cost. So evaluating F_5 at last is optimal.

Total number of scalar multiplication are

$$48 + 75 + 50 + 2000 = 2173$$

Optimal parenthesis is $((F_1 (F_2 (F_3 F_4))) F_5)$ as concluded F_3, F_4 are explicitly computed pair.

Hence, the correct option is (C).

4.7 (B, D)

Given: R_7 : Maximum amount earned by cutting rod of length 7' into 1, 2, 3, 4, 5, 6, 7 pieces the array of prices:

$$p[1] = 1, p[2] = 5, p[3] = 8, p[4] = 9$$

$$p[5] = 10, p[6] = 17, p[7] = 18$$

Now, $p[7]$ can be achieved by

$$p[6] + p[1] = 17 + 1 = 18$$

(Also, R_7 is achieved by 3 diff. solution)

$$p[5] + p[2] = 10 + 5 = 15$$

$$p[4] + p[3] = 9 + 8 = 17$$

$$p[5] + p[1] + p[1] = 10 + 1 + 1 = 12$$

$$p[4] + p[2] + p[1] = 9 + 5 + 1 = 15$$

$$p[4] + p[1] + p[1] + p[1] = 9 + 1 + 1 + 1 = 12$$

$$p[3] + p[4] = 8 + 9 = 17$$

$$p[3] + p[3] + p[1] = 8 + 8 + 1 = 17$$

$$p[3] + p[2] + p[2] = 8 + 5 + 5 = 18$$

(R_7 is achieved by 3 diff. pieces)

Hence, the correct option is (B, D).

5

Miscellaneous Topics



Practice Questions

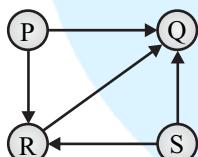
2013 IIT Bombay

- 5.1** The number of elements that can be sorted in $\Theta(\log n)$ time using heap sort is

- (A) $\Theta(1)$ (B) $\Theta(\sqrt{\log n})$
 (C) $\Theta\left(\frac{\log n}{\log \log n}\right)$ (D) $\Theta(\log n)$

2014 IIT Kharagpur

- 5.2** Consider the directed graph given below. Which one of the following is true?



- (A) The graph doesn't have any topological ordering
 (B) Both PQRS and SRPQ are topological ordering
 (C) Both PSRQ and SPRQ are topological ordering
 (D) PSRQ is the only topological ordering

- 5.3** Suppose you want to move from 0 to 100 on the number line. In each step, you either move right by a unit distance or you take a shortcut. A shortcut is simply a pre-specified pair of integers i, j with $i < j$. Given a shortcut i, j if you are at position i on the number line, you may directly move to j . Suppose $T(k)$ denotes the smallest number of steps needed to move from k to 100. Suppose further that there is at most 1 shortcut involving any number, and in particular from 9 there is a shortcut to 15. Let y and z be such that $T(9) = 1 + \min(T(y), T(z))$. Then the value of the product yz is _____.

2015 IIT Kanpur

- 5.4** Match the following :

List-I

P : Prim's algorithm for minimum spanning tree

Q : Floyd-Warshall algorithm for all pair's shortest paths

R : Merge sort

S : Hamiltonian circuit

List-II

(I) Backtracking

(II) Greedy method

(III) Dynamic programming

(IV) Divide and conquer

Codes :

- (A) P-III, Q-II, R-IV, S-I
 (B) P-I, Q-II, R-IV, S-III
 (C) P-II, Q-III, R-IV, S-I
 (D) P-II, Q-I, R-III, S-IV

- 5.5** Given below are some algorithms, and some algorithm design paradigms.

List-I

1. Dijkstra's Shortest Path
2. Floyd-Warshall algorithm to compute all pairs shortest path
3. Binary search on a sorted array
4. Backtracking search on a graph

List-II

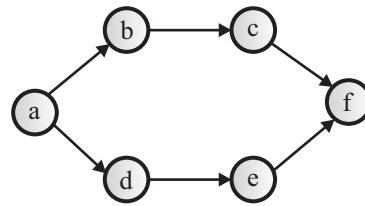
- Divide and Conquer
- Dynamic Programming
- Greedy design
- Depth-first search
- Breadth-first search

Match the above algorithms on the left to the corresponding design paradigm they follow :

- (A) 1-i, 2-iii, 3-i, 4-v,
 (B) 1 -iii, 2-ii, 3-i, 4-v
 (C) 1-iii, 2-ii, 3-i, 4-iv
 (D) 1-iii, 2-ii, 3-i, 4-v

2016 IISc Bangalore

- 5.6** Consider the following directed graph :



The number of different topological orderings of the vertices of the graph is _____.

2017 IIT Roorkee

- 5.7 Consider the following table :

Algorithms

- (P) Kruskal
- (Q) Quicksort
- (R) Floyd-Warshall

Design Paradigms

- (i) Divide and conquer

- (ii) Greedy

- (iii) Dynamic programming

Match the algorithms to the design paradigms they are based on.

- (A) (P) \leftrightarrow (ii), (Q) \leftrightarrow (iii), (R) \leftrightarrow (i)
- (B) (P) \leftrightarrow (iii), (Q) \leftrightarrow (i), (R) \leftrightarrow (ii)
- (C) (P) \leftrightarrow (ii), (Q) \leftrightarrow (i), (R) \leftrightarrow (iii)
- (D) (P) \leftrightarrow (i), (Q) \leftrightarrow (ii), (R) \leftrightarrow (iii)

- 5.8 Match the algorithms with their time complexities

Algorithm

- (P) Towers of Hanoi with n disks
- (Q) Binary search given n sorted numbers
- (R) Heap sort given n numbers at the worst case
- (S) Addition of two $n \times n$ matrices

Time complexity

- (i) $\Theta(n^2)$

- (ii) $\Theta(n \log n)$

- (iii) $\Theta(2^n)$

- (iv) $\Theta(\log n)$

- (A) (P) \rightarrow (iii), (Q) \rightarrow (iv), (R) \rightarrow (i),
 $S \rightarrow$ (ii)

- (B) (P) \rightarrow (iv), (Q) \rightarrow (iii), (R) \leftrightarrow (i),
 $(S) \rightarrow$ (ii)

- (C) (P) \rightarrow (iii), (Q) \rightarrow (iv), (R) \rightarrow (ii),
 $(S) \rightarrow$ (i)

- (D) (P) \rightarrow (iv), (Q) \rightarrow (iii), (R) \rightarrow (ii),
 $(S) \rightarrow$ (i)

2018 IIT Guwahati

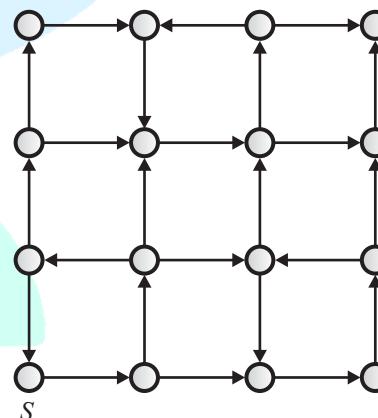
- 5.9 Consider the following program written in pseudo-code. Assume that x and y are integers.

```
Count (x, y) {
    if (y != 1) {
        if (x != 1) {
            print("*");
            Count (x/2, y);
        }
        else {
            y=y-1;
            Count (1024, y);
        }
    }
}
```

The number of times that the print statement is executed by the call Count(1024, 1024) is _____.

2021 IIT Bombay

- 5.10 Consider the following directed graph:



Which of the following is/are correct about the graph?

- (A) A depth first traversal starting at vertex S classifies three directed edges as back edges.

- (B) The graph does not have strongly connected components.

- (C) For each pair of vertices u and v, there is a directed path from u to v.

- (D) The graph does not have a topological order.



Solutions

5.1 (C)

Key Point

To sort k elements in a heap, complexity is $\Theta(k \log k)$

From the given options,

Let us assume $k = 1 \Rightarrow$ complexity = $\Theta(\log 1)$

For $k = \sqrt{\log n} \Rightarrow$ complexity

$$= \Theta(\sqrt{\log n} \cdot \log \sqrt{\log n})$$

$$= \sqrt{\log n} \times \frac{1}{2} \log(\log n)$$

For $k = \frac{\log n}{\log \log n}$, complexity

$$= \frac{\log n}{\log \log n} \times \log\left(\frac{\log n}{\log \log n}\right)$$

$$= \frac{\log n}{\log \log n} \times (\log(\log n) - \log(\log \log n))$$

$$= \log n - \frac{\log * \log \log \log n}{\log \log n}$$

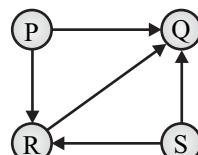
$$= \log n$$

Therefore, this can be bounded by $\log n$.

Hence, the correct option is (C).

5.2 (C)

Given :



Key Point

A topological sort of a directed graph is a linear ordering of its vertices such that for every directed edge uv from vertex u to vertex v , u comes before v in the ordering

Applying DFS by choosing P as starting vertex topological ordering is PSRQ applying DFS by choosing S as starting vertex topological ordering is SPRQ

Hence, the correct option is (C).

5.3 150

$T(k)$ is the smallest no. of steps needed to move from k to 100.

Now, it is given that

$$T(9) = 1 + \min T(y)T(z)$$

Where, $T(y)$ = steps from y to 100

$$T(z) = \text{steps from } z \text{ to 100}$$

Where y and z are two possible values that can be reached from 9.

One number that can be reached from 9 is 10. Another no. is 15, the shortcut path from 9, as given in the question.

The value of ' y ' and ' z ' are 10 and 15.

$$\text{So } y \times z = 10 \times 15 = 150$$

Hence, the correct answer is 150.

5.4 (C)

Prim's algorithm- Greedy method

Floyd warshall-dynamic programming

Merge sort – divide & conquer.

Hamiltonian circuit- Backtracking

Hence, the correct option is (C).

5.5 (C)

Dijkstra's shortest path- Greedy design

Floyd –warshall algorithm – Dynamic programming

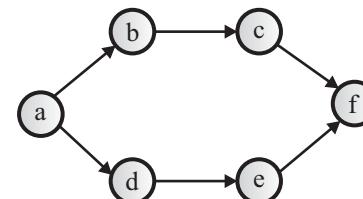
Binary search – divide and conquer.

Backtracking search – depth -first- search.

Hence, the correct option is (C).

5.6 6

Given :



Given graph starts with a and ends with f .

i.e. $a \dots f$

blank spaces can be filled with b, c, d, e such that b comes before c and d comes before e .



$$\text{so, number of ways} = \frac{4!}{2 \times 2!} = 6$$

Hence, the correct answer is 6.

5.7 (C)

Kruskal- Greedy

Quick sort – Divide & conquer

Floyd- Warshall- Dynamic programming

Hence, the correct option is (C).

5.8 (C)

Time complexity:

Tower of Hanoi- $\Theta(2^n)$

Heap sort (worst case) - $\Theta(n \log n)$

Binary search - $\Theta(\log n)$

Addition of two $n \times n$ matrices - $\Theta(n^2)$

Hence, the correct option is (C).

5.9 10230

For each y value `printf("*")` will run 10 times and once

x value reaches 1 count (1024, $y-1$) will be called

variable y can take values [2, 1024] i.e. 1023 values

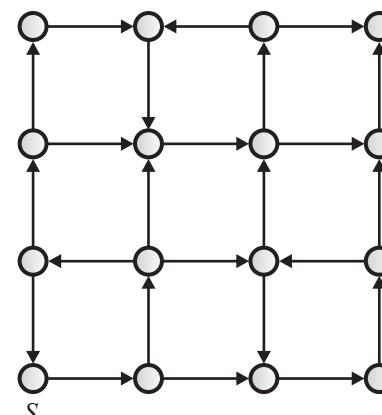
\Rightarrow no of time * will be printed = 10×1023

$$= 10230$$

Hence, the correct answer is 10230.

5.10 (A, D)

Given :



Back Edge - It is an edge (u, v) such that v is ancestor of node u but not part of DFS tree.

We can observe that,

- (A) There are only 3 back edges, if started from S.
- (B) The graph does have a strongly connected component, it has cycle.
- (C) Not all rectangular/square components form a cycle.
- (D) The graph does not have a topological order, because there's a cycle in the bottom left corner of the graph.

Hence the correct option are (A, D).

CHAPTER 4 | Databases



Marks Distribution of Databases in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	3	3	9
2004*	6	8	22
2005*	7	10	27
2006*	3	8	19
2007*	1	10	21
2008*	2	12	26
2009	-	5	10
2010	2	3	8
2011	2	2	6
2012	2	5	12
2013	1	3	7
2014 Set-1	2	3	8
2014 Set-2	2	3	8
2014 Set-3	2	3	8

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	2	2	6
2015 Set-2	2	2	6
2015 Set-3	2	2	6
2016 Set-1	3	1	5
2016 Set-2	2	2	6
2017 Set-1	2	3	8
2017 Set-2	2	3	8
2018	2	2	6
2019	2	3	8
2020	2	3	8
2021 Set-1	2	3	8
2021 Set-2	1	4	9

* CS and IT combined

Syllabus : Databases

ER-model. Relational model: relational algebra, tuple calculus, SQL. Integrity constraints, normal forms. File organization, indexing (e.g., B and B+ trees). Transactions and concurrency control.

Contents : Databases

S. No. Topics

- 1. ER Model**
- 2. Database Design : Functional Dependency & Normalization**
- 3. SQL, Relational Algebra and Calculus**
- 4. Transactions and Concurrency Control**
- 5. File Structures**



Practice Questions

2015 IIT Kanpur

- 1.1** Consider an Entity-Relationship (ER) model in which entity sets E_1 and E_2 are connected by an m: n relationship R_{12} . E_1 and E_3 are connected by a l: n (l on the side of E_1 and n on the side of E_3) relationship R_{13} .

E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute. E_2 has two single-valued attributes a_{21} and a_{22} of which a_{21} is the key attribute. E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute. The relationship do not have any attributes.

If a relational model is derived from the above ER model, then the minimum number of relations that would be generated if all the relations are in 3NF is _____.

2017 IIT Roorkee

- 1.2** An ER model of a database consists of entity types A and B. These are connected by a relationship R which does not have its own attribute. Under which one of the following conditions, can the relational table for R be merged with that of A?

- (A) Relationship R is one-to-many and the participation of A in R is total.
- (B) Relationship R is one-to-many and the participation of A in R is partial.

(C) Relationship R is many-to-one and the participation of A in R is total.

(D) Relationship R is many-to-one and the participation of A in R is partial.

2018 IIT Guwahati

- 1.3** In an Entity-Relationship (ER) model, suppose R is a many-to-one relationship from entity set E1 to entity set E2. Assume that E1 and E2 participate totally in R and that the cardinality of E1 is greater than the cardinality of E2.

Which one of the following is true about R?

- (A) Every entity in E1 is associated with exactly one entity in E2.
- (B) Some entity in E1 is associated with more than one entity in E2.
- (C) Every entity in E2 is associated with exactly one entity in E1.
- (D) Every entity in E2 is associated with at most one entity in E1.

2020 IIT Delhi

- 1.4** Which one of the following is used to represent the supporting many-one relationships of a weak entity set in an entity-relationship diagram?

- (A) Diamonds with double/bold border
- (B) Rectangles with double/bold border
- (C) Ovals with double/bold border
- (D) Ovals that contain underlined identifiers

Solutions

1.1 4

Given :

E_1 and E_2 are connected by an m: n relationship R_{12} .

E_1 and E_3 are connected by a l: n (l on the side of E_1 and n on the side of E_3) relationship R_{13}

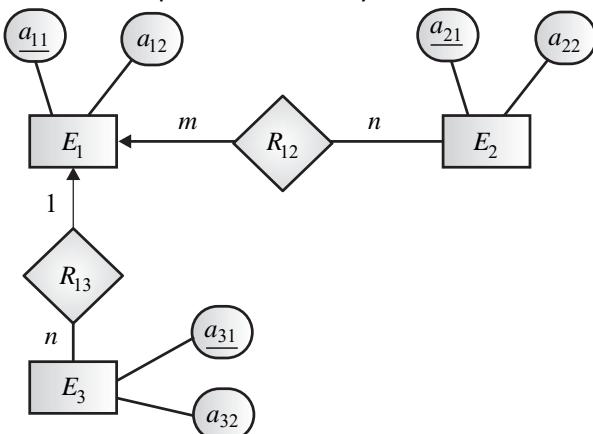
E_1 has two single-valued attributes a_{11} and a_{12} of which a_{11} is the key attribute.



E_2 has two single-valued attributes a_{21} and a_{22} of which a_{21} is the key attribute.

E_3 has two single-valued attributes a_{31} and a_{32} of which a_{31} is the key attribute.

The relationship do not have any attributes.



The relations generated are :

For $E_1 = (a_{11}, a_{12}) \{a_{11} \rightarrow a_{12}\}$

For $E_2 = (a_{21}, a_{22}) \{a_{21} \rightarrow a_{22}\}$

For E_3 and $E_1 - E_3$ relation

$E_3 R_{13} = (a_{31}, a_{32}, a_{11}) \{a_{31} \rightarrow a_{32} a_{11}\}$

For $R_{12} = (a_{11}, a_{21})$

R_{13} does not require a separate table because it is a many to one relation.

R_{12} require a separate table because of many to many relationship.

We cannot combine them further because if we combine, they will violate the condition of 3NF. So, Total minimum 4 tables required.

Hence, the correct answer is 4.

1.2 (C)

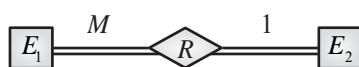


The relational table R will always be merged with entity having total participation and the type of relation should be many to one.

Hence, the correct option is (C).

1.3 (A)

Given :



Since, it is a many to one relationship, therefore-

- (1) Entity in E_1 is related to exactly one entity in E_2 .

- (2) Entity in E_2 can be related to more than one entity in E_1 .

Hence, the correct option is (A).

1.4 (A)

Weak entity set is represented by rectangle with double border.

But here it is asking for the relationship of a weak entity set that is represented by diamond with double border.

Hence, the correct option is (A).



Practice Questions

2013 | IIT Bombay

Common Data for Questions 2.1 & 2.2

Relation R has eight attributes ABCDEFGH.
Fields of R contain only atomic values.

$F = \{CH \rightarrow G, A \rightarrow BC, B \rightarrow CFH, E \rightarrow A, F \rightarrow EG\}$ is a set of functional dependencies (FDs) so that F^+ is exactly the set of FDs that hold for R.

2014 IIT Kharagpur

- 2.4** Given the following two statements:

S1 : Every table with two single-valued attributes is in 1 NF, 2 NF, 3 NF and BCNF.

S2 : $AB \rightarrow C$, $D \rightarrow E$, $E \rightarrow C$ is a minimal cover for the set of functional dependencies $AB \rightarrow C$, $D \rightarrow E$, $AB \rightarrow E$, $E \rightarrow C$

Which one of the following is **CORRECT**?

 - (A) S1 is TRUE and S2 is FALSE
 - (B) Both S1 and S2 are TRUE
 - (C) S1 is FALSE and S2 is TRUE
 - (D) Both S1 and S2 are FALSE

- 2.5** The maximum number of super keys for the relation schema R(E,F,G,H) with E as the key is _____.
2.6 Given an instance of the STUDENTS relation as shown below :

Student ID	Student Name	Student Email	Student Age	CPI
2345	Shankar	Shankar @math	X	9.4
1287	Swati	Swati @ee	19	9.5
7853	Shankar	Shankar @cse	19	9.4
9876	Swati	Swati @mech	18	9.3
8765	Ganesh	Ganesh @civil	19	8.7

For (StudentName, StudentAge) to be a key for this instance, the value X should NOT be equal to .

- 2.7** A prime attribute of a relation scheme R is an attribute that appears

 - (A) in all candidate keys of R
 - (B) in some candidate key of R
 - (C) in a foreign key of R
 - (D) only in the primary key of R

2015 IIT Kanpur

- 2.8** Consider the relation X (P, Q, R, S, T, U) with the following set of functional dependencies $F = \{\{P, R\} \rightarrow \{S, T\}, \{P, S, U\} \rightarrow \{Q, R\}\}$

Which of the following is the trivial functional dependency in F^+ , where F^+ is closure of F?

(A) $\{P, R\} \rightarrow \{S, T\}$
(B) $\{P, R\} \rightarrow \{R, T\}$
(C) $\{P, S\} \rightarrow \{S\}$
(D) $\{P, S, U\} \rightarrow \{Q\}$

2016 IISc Bangalore



- 2.9** Which of the following is **NOT** a super key in a relational schema with attributes V, W, X, Y, Z and primary key V Y?

(A) VXYZ (B) VWXZ
 (C) VWXY (D) VWXYZ

- 2.10** A database of research articles in a journal uses the following schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

The primary key is (VOLUME, NUMBER, STARTPAGE, ENDPAGE) and the following functional dependencies exist in the schema.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow TITLE

(VOLUME, NUMBER) \rightarrow YEAR

(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow PRICE

The database is redesigned to use the following schemas.

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE)

(VOLUME, NUMBER, YEAR)

Which is the weakest normal form that the new database satisfies, but the old one does not?

(A) 1 NF (B) 2 NF
 (C) 3 NF (D) BCNF

2017 IIT Roorkee

- 2.11** The following functional dependencies hold true for the relational schema R{V, W, X, Y, Z}:

V \rightarrow W

VW \rightarrow X

Y \rightarrow VX

Y \rightarrow Z

Which of the following is irreducible equivalent for this set of functional dependencies?

(A) V \rightarrow W	(B) V \rightarrow W
V \rightarrow X	W \rightarrow X
Y \rightarrow V	Y \rightarrow V
Y \rightarrow Z	Y \rightarrow Z
(C) V \rightarrow W	(D) V \rightarrow W
V \rightarrow X	W \rightarrow X
Y \rightarrow V	Y \rightarrow V
Y \rightarrow X	Y \rightarrow X

Y \rightarrow Z

Y \rightarrow Z

2019 IIT Madras

- 2.12** Let the set of functional dependencies $F = \{QR \rightarrow S, R \rightarrow P, S \rightarrow Q\}$ hold on a relation schema $X = (PQRS)$. X is not in BCNF. Suppose X is decomposed into two schemas Y and Z, where $Y = (PR)$ and $Z = (QRS)$.

Consider the two statements given below.

- I. Both Y and Z are in BCNF
 II. Decomposition of X into Y and Z is dependency preserving and lossless

Which of the above statements is/are correct.

(A) Both I and II
 (B) I only
 (C) II only
 (D) Neither I nor II

2020 IIT Delhi

- 2.13** Consider a relational table R that is in 3NF, but not in BCNF. Which one of the following statements is TRUE?

(A) R has a nontrivial functional dependency $X \rightarrow A$, where X is not a super key and A is a prime attribute.
 (B) R has a nontrivial functional dependency $X \rightarrow A$, where X is not a super key and A is a non-prime attribute and X is not a proper subset of any key.
 (C) R has a nontrivial functional dependency $X \rightarrow A$, where X is not a super key and A is a non-prime attribute and X is a proper subset of some key.
 (D) A cell in R holds a set instead of an atomic value.

2021 IIT Bombay

- 2.14** Suppose the following functional dependencies hold on a relation U with attributes P, Q, R, S, and T

$P \rightarrow QR$

$RS \rightarrow T$

Which of the following functional dependencies can be inferred from the above functional dependencies?

(A) $PS \rightarrow T$ (B) $R \rightarrow T$



- (C) $P \rightarrow R$ (D) $PS \rightarrow Q$

- 2.15** Consider the relation $R(P, Q, S, T, X, Y, Z, W)$ with the following functional dependencies.

$$PQ \rightarrow X; P \rightarrow YX; Q \rightarrow Y; Y \rightarrow ZW$$

Consider the decomposition of the relation R into the constituent relations according to the following two decomposition schemes

$$D_1 : R = \left[(P, Q, S, T); (P, T, X) \right] \\ \left[(Q, Y); (Y, Z, W) \right]$$

$$D_2 : R = \left[(P, Q, S); (T, X) \right] \\ \left[(Q, Y); (Y, Z, W) \right]$$

Which one of the following options is correct?

- (A) D_1 is a lossless decomposition, but D_2 is a lossy decomposition.
 (B) Both D_1 and D_2 are lossy decompositions.
 (C) Both D_1 and D_2 are lossless decompositions.
 (D) D_1 is a lossy decomposition, but D_2 is a lossless decomposition.

Solutions

2.1 (B)

Given : Relation R has eight attributes ABCDEFGH. Fields of R contain only atomic values.

$$\text{FD} : \{CH \rightarrow G, A \rightarrow BC, B \rightarrow CFH, E \rightarrow A, F \rightarrow EG\}$$

The attribute D does not appear on RHS of any functional dependency. Hence, D must be a part of candidate key.

$$D^+ = \{D\}$$

So, adding A, B, C, E, F, G, H to D and checking for the key of size 2

$$(AD)^+ = \{A, B, C, D, F, H, E, G\}$$

$$(BD)^+ = \{B, C, F, H, D, E, G, A\}$$

$$(CD)^+ = \{C, D\}$$

$$(ED)^+ = \{E, D, A, B, C, F, H, G\}$$

$$(FD)^+ = \{F, D, E, G, A, B, C, H\}$$

$$(GD)^+ = \{G, D\}$$

$$(HD)^+ = \{H, D\}$$

Candidate keys of size 2 are – AD, BD, ED, FD. Checking for candidate key of size 3,

- (1) We cannot add any attribute to AD, BD, ED, FD as it will just make super keys that are not minimal.

- (2) Proceeding with CD, GD and HD, here we cannot add A, B, E, F as it will just make super keys. Adding G, C, H to CD, GD, & HD.

$$(CGD)^+ = \{G, C, D\}$$

$$(CHD)^+ = \{G, H, D, G\}$$

$$(GHD)^+ = \{G, H, D\}$$

Checking for candidate key of size 4,

$$(CGHD)^+ = \{C, G, H, D\}$$

Final set of candidate keys – AD, BD, ED, FD.

Hence, the correct option is (B).

Key Point

If an attribute does not appear in RHS of any of the functional dependencies then that attribute must be part of all candidate keys.

2.2 (A)

We know the candidate keys are – AD, BD, ED, FD. $A \rightarrow BC, B \rightarrow CFH, F \rightarrow EG$ have partial dependencies like $A \rightarrow C, B \rightarrow CH, F \rightarrow G$. Hence, the relation is not in 2NF, it is in 1NF only. Hence, the correct option is (A).

Key Point

- In 1-NF, there should be no composite and multi-valued attributes.
- In 2-NF, there should be no partial dependency i.e. no non-prime attribute is dependent on any proper subset of any candidate key.



- In 3-NF, there should be no transitive dependency i.e. non-prime attribute should not depend transitively on the key.
- In BCNF, for every functional dependency $X \rightarrow Y$, X should be a super key.

2.3 (B)

Given: R (E, F, G, H, I, J, K, L, M, N)

$$EF \rightarrow G, F \rightarrow IJ$$

FD: $EH \rightarrow KL, K \rightarrow M$

$$L \rightarrow N$$

As E, F, and H are not on RHS of any functional dependency, so they must be in candidate key.

$$\{E, F, H\}^+ = \{E, F, H, G, I, J, K, L, M, N\}$$

Since, all attributes can be derived from {E, F, H} it is the key.

Hence, the correct option is (B).

Key Point

If an attribute does not appear in RHS of any of the functional dependencies then that attribute must be part of all candidate keys.

2.4 (A)

S_1 is true, as a relation with two single valued attributes is always in BCNF and hence also in 1NF, 2NF and 2NF also.

S_2 is false, because $AB \rightarrow E$ cannot be derived from the minimal cover.

In the set of FD :

$$(AB)^+ = \{A, B, C, E\}$$

But, in minimal cover-

$$(AB)^+ = \{A, B, C\}$$

Hence, the correct option is (A).

Key Point

Every table with two single-valued attributes is in 1 NF, 2 NF, 3 NF and BCNF.

2.5 8

Given : R(E,F,G,H)

Method 1

E is the key. The attributes other than E, ie. F, G, H will be added in different combination to E and result in super keys.

So, super keys are-

$$\{E, EF, EG, EH, EFG, EFH, EGH, EFGH\}$$

Hence, the answer is 8

Method 2

The possible keys (i.e. super keys) $= 2^{4-1} = 8$

Key Point

Maximum number of possible super keys for a relation with n attributes having one candidate key with one attribute only $= 2^{n-1}$

Where, n = Total number of attributes.

Hence, the correct answer is 8.

2.6 19

Given :

Student ID	Student Name	Student Email	Student Age	CPI
2345	Shankar	Shankar @math	X	9.4
1287	Swati	Swati @ee	19	9.5
7853	Shankar	Shankar @cse	19	9.4
9876	Swati	Swati @mech	18	9.3
8765	Ganesh	Ganesh @civil	19	8.7

(StudentName, StudentAge) is a key

As we know, candidate key uniquely determines all the tuples in the table.

So, if X=19, (Shankar, 19) will create ambiguity by having different values of student E mail.

Hence, X should not be equal to 19.

2.7 (B)

An attribute is called a prime attribute if it appears in atleast 1 candidate key of R. It need not to appear in all candidate keys of R.

Hence, the correct option is (B).

Key Point



A prime attribute is an attribute which is present in any of the candidate keys.

2.8 (C)

Given : $X (P, Q, R, S, T, U)$

$$PR \rightarrow ST,$$

$$PSU \rightarrow QR$$

$PS \rightarrow S$ is a trivial functional dependency because $\{S\} \subset \{P, S\}$.

Key Point

- (1) For trivial FD, if $X \rightarrow Y$ then $Y \subset X$ holds.
- For non-trivial FD, if $X \rightarrow Y$ then $X \cap Y = \emptyset$ holds.

Hence, the correct option is (C).

2.9 (B)

Given : Relation Schema with attributes V,W,X,Y,Z and having primary key as VY.

Any superset of primary key is a super key.

Here, primary key is VY and VWXZ is not a super set as it does not contain Y.

Hence, the correct option is (B).

Key Point

SUPERKEY : it is an attribute (or set of attributes) that uniquely identify a tuple. Super key is a superset of a candidate key.

2.10 (B)

Given :

Research articles (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE)

Primary keys: (VOLUME, NUMBER, STARTPAGE, ENDPAGE)

FD's :-

$$(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow TITLE$$

$$(VOLUME, NUMBER) \rightarrow YEAR$$

$$(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow PRICE$$

After Redesign database:

(VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE) breaks into (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, PRICE) and (VOLUME, NUMBER, YEAR)

So, (VOLUME, NUMBER, STARTPAGE, ENDPAGE, TITLE, YEAR, PRICE) satisfy $(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow TITLE$ and $(VOLUME, NUMBER, STARTPAGE, ENDPAGE) \rightarrow PRICE$

Here Candidate keys are:-

VOLUME, NUMBER, STARTPAGE, ENDPAGE.

Prime attributes are:-

(VOLUME, NUMBER)

Non-prime attributes are:-

(STARTPAGE, ENDPAGE, TITLE, PRICE)

Satisfy 2NF condition i.e. No partial dependency, also satisfy 3NF and BCNF condition but weaker among them is 2NF

(VOLUME, NUMBER, YEAR) satisfy $(VOLUME, NUMBER) \rightarrow YEAR$

Here candidate keys are:-

VOLUME, NUMBER

Prime attributes are:-

(VOLUME, NUMBER)

Non prime attributes is:-

(YEAR)

Satisfy 2NF condition i.e. No partial dependency also satisfy 3NF and BCNF condition but weaker among them is 2NF

In old database.

FD:- $(VOLUME, NUMBER) \rightarrow YEAR$ do not satisfy 2NF condition i.e. No partial dependency.

Hence, the correct option is (B).

2.11 (A)

Given : R (V, W, X, Y, Z) and

Functional dependencies are-

$$V \rightarrow W$$

$$VW \rightarrow X$$

$$Y \rightarrow V$$

$$Y \rightarrow X$$

$$Y \rightarrow Z$$

$$(V)^+ = VWX$$

$$(VW)^+ = VWX$$

$$(Y)^+ = VXZHY$$

$$\text{Without } Y \rightarrow X$$

$$(Y)^+ = YVZWX$$

It is the same as before.

Hence $Y \rightarrow X$ is redundant.



Also, W can be derived from V from first functional dependency and therefore, W is redundant in second functional dependency. So, after removing it, final canonical cover is-

$$\begin{aligned}V &\rightarrow W \\V &\rightarrow X \\Y &\rightarrow V \\Y &\rightarrow Z\end{aligned}$$

So, option (A) is correct.

Hence, the correct option is (A).

2.12 (C)

Given : $X = (PQRS)$

FD : $F = \{QR \rightarrow S, R \rightarrow P, S \rightarrow Q\}$

Functional dependencies in $Y(PR)$ are-

$$R \rightarrow P$$

Hence, it is in BCNF. (Binary attribute)

Functional dependencies in $Z(QRS)$ are-

$$QR \rightarrow S$$

$$S \rightarrow Q$$

Candidate keys are QR and RS

In, $S \rightarrow Q$ S is not super key.

$\therefore Z$ is not in BCNF.

As $Y \cap Z = R \neq \emptyset$, hence it is lossless.

Also, all the dependencies are preserved

\therefore It is dependency preserving.

So, only (ii) is correct

Hence, the correct option is (C).

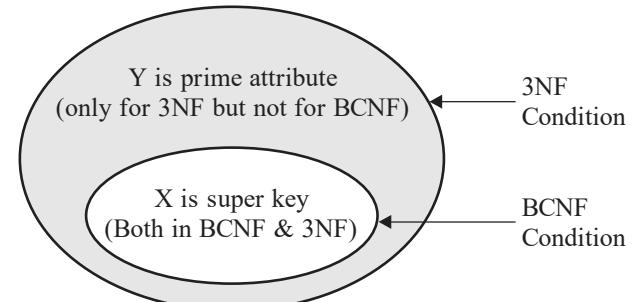
Key Point

Condition for lossless decomposition : when a table is decomposed into two tables, if the common attribute of the decomposed tables is a key in at least one of the tables then the decomposition is lossless.

2.13 (A)

Given : Relational Table R which is in 3NF but not in BCNF.

Option (A) means X is not a super key but Y is a prime attribute. Satisfies the condition of 3NF but it is not in BCNF as X is not a super key.



Option (B) means X is neither a super key nor Y is a prime attribute, hence not in 3NF.

Option (C) Means it is a partial dependency i.e. not even in 2NF.

Option (D) Means multivalued attribute is there i.e., not in 1 NF

So, option (A) is TRUE.

Hence, the correct option is (A).

Key Point

- In 1-NF, there should be no composite and multi-valued attributes.
- In 2-NF, there should be no partial dependency i.e. no non-prime attribute is dependent on any proper subset of any candidate key.
- In 3-NF, there should be no transitive dependency i.e. non-prime attribute should not depend transitively on the key.
- In BCNF, for every functional dependency $X \rightarrow Y, X$ should be a super key.

2.14 (A, C, D)

Given : FD's

$$P \rightarrow QR$$

$$RS \rightarrow T$$

Consider option (A)

$$PS \rightarrow T$$

$$(PS)^+ = P, Q, R, S, T \text{ So}$$

$$PS \rightarrow T \text{ holds}$$

Consider option (B)

$$R \rightarrow T$$

$$(R)^+ = R \text{ So}$$

$$R \rightarrow T \text{ doesn't hold}$$

Consider option (C)

$$P \rightarrow R$$

$$(P)^+ = P, Q, R \text{ So}$$

$$P \rightarrow R \text{ holds}$$



Consider option (D)

$$PS \rightarrow Q$$

$$(PS)^+ = P, Q, R, S, T \text{ So}$$

$$PS \rightarrow Q \text{ holds}$$

Hence, the correct options (A), (C) & (D) are inferred from the above FD's.

Hence, the correct options are (A), (C) and (D).

2.15 (A)

Given:

$$R(P, Q, S, T, X, Y, Z, W)$$

FD's

$$PQ \rightarrow X$$

$$P \rightarrow YX$$

$$Q \rightarrow Y$$

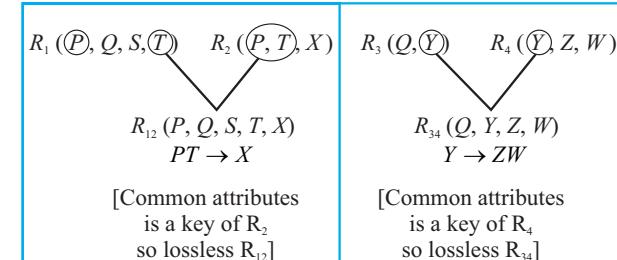
$$Y \rightarrow ZW$$

Decomposition schemes :

$$D_1: R = [(P, Q, S, T); (P, T, X); (Q, Y); (Y, Z, W)]$$

$$D_2: R = [(P, Q, S); (T, X); (Q, Y); (Y, Z, W)]$$

Decomposition D_1 :



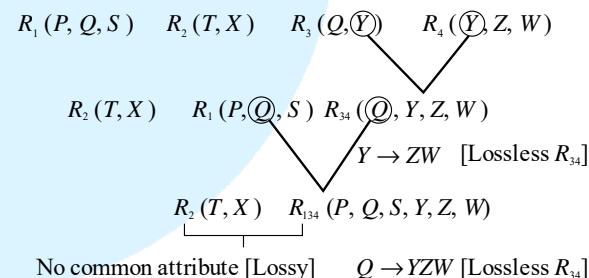
$$R_{12}(P, \underline{\underline{Q}}, S, T, X) \quad R_{34}(\underline{\underline{Q}}, Y, Z, W)$$

$$R_{1234}(P, Q, S, T, X, Y, Z, W)$$

[Common attributes is a key of R_{34} Overall lossless decomposition]

Therefore, D_1 is a lossless decomposition.

Decomposition D_2 :



Therefore, D_2 is a lossy decomposition.

Hence, the correct option is (A).



Practice Questions

2013 IIT Bombay

- 3.1** Consider the following relational schema.

Students (rollno: integer, sname: string)

Courses (courseno: integer, cname: string)

Registration (rollno: integer, courseno: integer, percent: real)

Which of the following queries are equivalent to this query in English?

"Find the distinct names of all students who score more than 90% in the course numbered 107".

I. $\text{SELECT DISTINCT } S.\text{sname} \text{ FROM Students}$

as S , Registration as R

WHERE $R.\text{rollno}=S.\text{rollno}$ AND
 $R.\text{courseno}=107$ AND $R.\text{percent}>90$

II. $\pi_{\text{sname}}(\sigma_{\text{courseno}=107 \wedge \text{percent}>90} (\text{Registration} \bowtie \text{Students}))$

III. $\{T \mid \exists S \in \text{Student}, \exists R \in \text{Registration}$

 $(S.\text{rollno} = R.\text{rollno} \wedge R.\text{courseno} = 107 \wedge R.\text{percent}>90 \wedge T.\text{sname} = S.\text{sname})\}$

IV. $\{\langle S_N \rangle \mid \exists S_R \exists R_P (\langle S_R, S_N \rangle \in \text{Students} \wedge \langle S_R, 107, R_P \rangle \in \text{Registration} \wedge R_P > 90)\}$

(A) I, II, III and IV

(B) I, II and III only

(C) I, II and IV only

(D) II, III and IV only

2014 IIT Kharagpur

- 3.2** Given the following statements:

S1: A foreign key declaration can always be replaced by an equivalent check assertion in SQL.

S2: Given the table R (a, b, c) where a and b together form the primary key, the following is a valid table definition.

```
CREATE TABLE S (
    a INTEGER,
    d INTEGER,
```

e INTEGER,

PRIMARY KEY (d),

FOREIGN KEY (a) references R

Which one of the following statements is **CORRECT**?

(A) S1 is TRUE and S2 is FALSE.

(B) Both S1 and S2 are TRUE.

(C) S1 is FALSE and S2 is TRUE.

(D) Both S1 and S2 are FALSE.

- 3.3** Given the following schema:

Employees (emp-id, first-name, last-name, hire-date, dept-id, salary)

Department (dept-id, dept-name, manager-id, location-id)

You want to display the last names and hire dates of all latest hires in their respective departments in the location ID 1700. You issue the following query:

SELECT last-name, hire-date

FROM employees

WHERE (dept-id, hire-date) IN

(SELECT dept-id, MAX(hire-date))

FROM employees JOIN

department USING (dept-id)

WHERE location-id=1700

GROUP BY dept-id);

What is the outcome?

(A) It executes but does not give the correct result.

(B) It executes and gives the correct result.

(C) It generates an error because of pairwise comparison.

(D) It generates an error because the GROUP BY clause cannot be used with table joins in a sub-query.

- 3.4** Consider a join (relation algebra) between relation $r(R)$ and $s(S)$ using the nested loop method. There are 3 buffers each of size equal to disk block size, out of which one buffer is



reserved for intermediate results. Assuming size $r(R) < \text{size } s(S)$, the join will have fewer number of disk block accesses if

- (A) Relation $r(R)$ is in the outer loop.
- (B) Relation $s(S)$ is in the outer loop.
- (C) Join selection factor between $r(R)$ and $s(S)$ is more than 0.5.
- (D) Join selection factor between $r(R)$ and $s(S)$ is less than 0.5.

- 3.5** SQL allows duplicate tuples in relations and corresponding defines the multiplicity of tuples in the result of joins. Which one of the following queries always gives the same answer as the nested query shown below:

Select * from R

where a in (select S.a from S)

- (A) Select R.* from R, S where R.a = S.a
- (B) Select distinct R.* from R, S where R.a = S.a
- (C) Select R.* from R, (select distinct a from S) as S1 where R.a = S1.a
- (D) Select R.* from R, S where R.a = S.a and is unique R

- 3.6** What is the optimized version of the relation algebra expression

$$\pi_{A_1}(\pi_{A_2}(\sigma_{F_1}(\sigma_{F_2}(r)))),$$

where A_1, A_2 are sets of attributes in r with $A_1 \subset A_2$ and F_1, F_2 are Boolean expressions based on the attributes in r ?

- (A) $\pi_{A_1}(\sigma_{(F_1 \wedge F_2)}(r))$
- (B) $\pi_{A_1}(\sigma_{(F_1 \vee F_2)}(r))$
- (C) $\pi_{A_2}(\sigma_{(F_1 \wedge F_2)}(r))$
- (D) $\pi_{A_2}(\sigma_{(F_1 \vee F_2)}(r))$

- 3.7** Consider the relational schema given below, where eld of the relation $dependent$ is a foreign key referring to $emplId$ of the relation $employee$. Assume that every employee has at least one associated dependent in the dependent relation.

Employee (emplId, empName, empAge)
dependent (depId, eld, depName, depAge)

Consider the following relational algebra query :

$$\prod_{emplId}(\text{employee}) - \prod_{emplId}(\text{employee} \bowtie_{(emplId=depId) \wedge (empAge \leq depAge)} \text{dependent})$$

The above query evaluates to the set of $emplId$ s of employees whose age is greater than that of

- (A) Some dependent.
- (B) All dependents.
- (C) Some of his/ her dependents.
- (D) All of his/her dependents.

- 3.8** Consider the following relational schema:

employee(emplId, empName, empDept)

customer(custId, custName, salesRepId, rating)
salesRepId is a foreign key referring to emplId of the employee relation. Assume that each employee makes a sale to at least one customer. What does the following query return?

SELECT empName

FROM employee E

WHERE NOT EXISTS

(SELECT custId

FROM customer C

WHERE C.salesRepId = E.emplId

AND C.rating < > 'GOOD');

- (A) Names of all the employees with at least one of their customers having a 'GOOD' rating.
- (B) Names of all the employees with at most one of their customers having a 'GOOD' rating.
- (C) Names of all the employees with none of their customers having a 'GOOD' rating.
- (D) Names of all the employees with all their customers having a 'GOOD' rating.

- 3.9** Consider the following relation

Cinema (theater, address, capacity)

Which of the following options will be needed at the end of the SQL query

SELECT P1.address

FROM Cinema P1

Such that it always finds the addresses of theaters with maximum capacity?

- (A) WHERE P1.capacity >= All (select P2.capacity from Cinema P2)
- (B) WHERE P1.capacity >= Any (select P2.capacity from Cinema P2)
- (C) WHERE P1.capacity > All (select max(P2.capacity) from Cinema P2)



(D) WHERE P1.capacity > Any (select max(P2.capacity) from Cinema P2)

2015 IIT Kanpur

- 3.10** Consider two relations $R_1(A, B)$ with the tuples $(1, 5), (3, 7)$ and $R_2(A, C) = (1, 7), (4, 9)$. Assume that $R(A, B, C)$ is the full natural outer join of R_1 and R_2 . Consider the following tuples of the form (A, B, C) : $a = (1, 5, \text{null})$, $b = (1, \text{null}, 7)$, $c = (3, \text{null}, 9)$, $d = (4, 7, \text{null})$, $e = (1, 5, 7)$, $f = (3, 7, \text{null})$, $g = (4, \text{null}, 9)$. Which one of the following statements is **CORRECT**?
- (A) R contains a, b, e, f, g but not c, d
 - (B) R contains all of a, b, c, d, e, f, g
 - (C) R contains e, f, g but not a, b
 - (D) R contains e but not f, g

- 3.11** SELECT operation in SQL is equivalent to
- (A) The selection operation in relational algebra.
 - (B) The selection operation in relational algebra, except that SELECT in SQL retains duplicates.
 - (C) The projection operation in relational algebra.
 - (D) The projection operation in relational algebra, except that SELECT in SQL retains duplicates.
- 3.12** Consider the following relations:

Student

Roll No	Student_Name
1	Raj
2	Rohit
3	Raj

Performance

Roll No	Course	Marks
1	Math	80
1	English	70
2	Math	75
3	English	80
2	Physics	65
3	Math	80

Consider the following SQL query.

SELECT S.Student_Name, sum(P.Marks)

FROM Student S, Performance P

WHERE S.Roll_No = P.Roll_No

GROUP BY S.Student_Name

The number of rows that will be returned by the SQL query is _____.

2016 IISc Bangalore

- 3.13** Consider the following databases table named water_schemes:

water_schemes		
scheme_no	district_name	capacity
1	Ajmer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

The number of tuples returned by the following SQL query is :

```
with total (name, capacity) as
Select district_name, sum (capacity)
from water_schemes
group by district_name
with total_avg (capacity) as
select avg (capacity)
from total
select name
from total, total_avg
where total.capacity ≥
total_avg.capacity
```

2017 IIT Roorkee

- 3.14** Consider the following tables T1 and T2. In table T1, P is the primary key and Q is the foreign key referencing R in table T2 with on-delete cascade and on-update cascade. In table T2, R is the primary key and S is the foreign key referencing P in table T1 with on-delete set NULL and on-update cascade. In order to delete record (3, 8) from table T1, the number of additional records that need to be deleted from table T1 is _____.

T1		T2	
P	Q	R	S
2	2	2	2



3	8	8	3
7	3	3	2
5	8	9	7
6	9	5	7
8	5	7	2
9	8		

- 3.15** Consider the following database table named top_scorer.

player	country	goals
Klose	Germany	16
Ronaldo	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Muller	Germany	10
Rahn	Germany	10

Consider the following SQL query:

```
SELECT ta.player
FROM top_scorer AS ta
WHERE ta.goals > ALL
(  SELECT tb.goals
    FROM top_scorer AS tb
    WHERE tb.country = 'Spain')
AND ta.goals > ANY
(  SELECT tc.goals
    FROM top_scorer AS tc
    WHERE tc.country = 'Germany')
```

The number of tuples returned by the above SQL query is _____.

- 3.16** Consider a database that has the relation schema EMP (EmpID, EmpName and DeptName). An instance of the schema EMP and a SQL query on it are given below.

EMP		
Emp ID	Emp Name	Dept Name
1	XYA	AA
2	XYB	AA
3	XYC	AA
4	XYD	AA

5	XYE	AB
6	XYF	AB
7	XYG	AB
8	XYH	AC
9	XYI	AC
10	XYJ	AC
11	XYK	AD
12	XYL	AD
13	XYM	AE

```
SELECT AVG (EC.Num)
FROM EC
WHERE (DeptName, Num) IN
(  SELECT DeptName,
        COUNT (Empld) AS
        EC (DeptName, Num)
FROM EMP
GROUP BY DeptName)
```

The output of executing the SQL query is _____.

- 3.17** Consider a database that has the relation schemas EMP (Empld, EmpName, DeptId), and DEPT (DeptName, DeptId). Note that the DeptId can be permitted to be NULL in the relation EMP. Consider the following queries on the database expressed in tuple relational calculus.

- $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \forall v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}]))\}$
- $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}]))\}$
- $\{t \mid \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}] \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] = v[\text{DeptId}]))\}$

Which of the above queries are safe?

- (I) and (II) only
- (I) and (III) only
- (II) and (III) only
- (I), (II), and (III)

- 3.18** Consider a database that has the relation schema CR (Student Name, Course Name). An Instance of the schema CR is as given below.

CR	
Student Name	Course Name
SA	CA



SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA
SF	CB
SF	CC

The following query is made on the database.

$$T_1 \leftarrow \pi_{\text{CourseName}}(\sigma_{\text{StudentName}=\text{SA}}(\text{CR}))$$

$$T_2 \leftarrow \text{CR} \div T_1$$

The number of rows in T_2 is _____.

2018 IIT Guwahati

- 3.19 Consider the following two tables and four queries in SQL.

Book (isbn, bname), Stock (isbn, copies)

Query 1:

```
SELECT B.isbn, S.copies
FROM Book B INNER JOIN Stock S ON B.isbn =
S.isbn;
```

Query 2:

```
SELECT B.isbn, S.copies
FROM Book B LEFT OUTER JOIN Stock S ON
B.isbn = S.isbn;
```

Query 3:

```
SELECT B.isbn, S.copies
FROM Book B RIGHT OUTER JOIN Stock S ON
B.isbn = S.isbn;
```

Query 4:

```
SELECT B.isbn, S.copies
FROM Book B FULL OUTER JOIN Stock S ON
B.isbn = S.isbn;
```

Which one of the queries above is certain to have an output that is a superset of the outputs of the other three queries?

- (A) Query 1 (B) Query 2
 (C) Query 3 (D) Query 4

- 3.20 Consider the relations $r(A, B)$ and $s(B, C)$, where $s.B$ is a primary key and $r.B$ is a foreign key referencing $s.B$. Consider the query

$$Q : r \bowtie (\sigma_{B < 5}(s))$$

Let LOJ denote the natural left outer-join operation. Assume that r and s contain no null values.

Which one of the following queries is NOT equivalent to Q ?

- (A) $\sigma_{B < 5}(r \bowtie s)$
 (B) $\sigma_{B < 5}(r LOJ s)$
 (C) $r LOJ (\sigma_{B < 5}(s))$
 (D) $\sigma_{B < 5}(r) LOJ s$

- 3.21 Consider the following four relational schemas. For each schema, all non-trivial functional dependencies are listed, The **bolded** attributes are the respective primary keys.

Schema I:

Registration(rollno, courses) Field ‘courses’ is a set-valued attribute containing the set of courses a student has registered for.

Non-trivial functional dependency:

rollno → courses

Schema II:

Registration (rollno, coursid, email)

Non-trivial functional dependencies:

rollno, courseid → email

email → rollno

Schema III:

Registration (rollno, courseid, marks, grade)

Non-trivial functional dependencies:

rollno, courseid, → marks, grade

marks → grade

Schema IV:

Registration (rollno, courseid, credit)

Non-trivial functional dependencies:

rollno, courseid → credit



courseid → credit

Which one of the relational schemas above is in 3NF but not in BCNF?

- (A) Schema I (B) Schema II
 (C) Schema III (D) Schema IV

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- 3.22** Consider the following $P(X, Y, Z)$, $Q(X, Y, T)$ and $R(Y, V)$.

P		
X	Y	Z
X1	Y1	Z1
X1	Y1	Z2
X2	Y2	Z2
X2	Y4	Z4

Q		
X	Y	T
X2	Y1	2
X1	Y2	5
X1	Y1	6
X3	Y3	1

R	
Y	V
Y1	V1
Y3	V2
Y2	V3
Y2	V4

How many tuples will be returned by the following relational algebra query?

$$\begin{aligned} & \prod_X (\sigma_{(P \cdot Y = R \cdot Y \wedge R \cdot V = V_2)} (P \times R)) \\ & - \prod_X (\sigma_{(Q \cdot Y = R \cdot Y \wedge Q \cdot T > 2)} (Q \times R)) \end{aligned}$$

- 3.23** A relational database contains two table Student and Performance as shown below :

Student	
Roll_no.	Student_name
1	Amit
2	Priya
3	Vinit

4	Rohan
5	Smita

Performance		
Roll_no.	Subject_code	Marks
1	A	86
1	B	95
1	C	90
2	A	89
2	C	92
3	C	80

The primary key of the Student table is Roll_no. For the Performance table, the columns Roll_no. and Subject_code together form the primary key. Consider the SQL query given below :

```
SELECT S.Student_name, sum (P.Marks)
FROM Student S, Performance P
WHERE P.Marks > 84
GROUP BY S.Student_name;
```

The number of rows returned by the above SQL query is _____.

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- 3.24** Consider a relational database containing the following schemas.

Catalogue		
sno	pno	cost
S1	P1	150
S1	P2	50
S1	P3	100
S2	P4	200
S2	P5	250
S3	P1	250
S3	P2	150
S3	P5	300
S3	P4	250

Suppliers		
sno	sname	location
S1	M/s Royal furniture	Delhi
S2	M/s Balaji furniture	Bangalore



S3	M/s Premium furniture	Chennai
----	-----------------------	---------

Parts		
pno	pname	Part_spec
P1	Table	Wood
P2	Chair	Wood
P3	Table	Steel
P4	Almirah	Steel
P5	Almirah	Wood

The primary key of each table is indicated by underlining the constituent fields.

```
SELECT s.sno, s.sname
FROM Suppliers s, Catalogue c
WHERE s.sno = c.sno AND
cost > (SELECT AVG (cost)
FROM Catalogue
WHERE pno = 'p4'
GROUP BY ...)
```

The number of rows returned by the above SQL query is

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- 3.25** A relation r (A, B) in relational database has 1200 tuples. The attribute A has integer values ranging 6 to 20 and the attribute B has integer values ranging from 1 to 20. Assume that the attributes A and B independently distributed. The estimated number of tuple in the output of $\sigma_{(A>10) \vee (B=18)}(r)$ is _____.

- 3.26** The following relation records the age of 500 employees of a company, where empNo (indicating the employee number) is the key :

empAge(empNo, age)

Consider the following relational algebra expression:

$$\Pi_{\text{empNo}} (\text{empAge} \bowtie_{(\text{age} > \text{age1})} \rho_{\text{empNo1}, \text{age1}} \\ (\text{empAge}))$$

What does the above expression generate?

- (A) Employee numbers of only those employees whose age is more than the age of exactly one other employee.

- (B) Employee numbers of only those employees whose age is the maximum.
 - (C) Employee numbers of all employees whose age is not the minimum.
 - (D) Employee numbers of all employees whose age is the minimum.

- 3.27** Consider the following statements S_1 and S_2 about the relational data model:

S₁: A relation cannot have more than one foreign key.

S₂: A foreign key in a relation scheme
R cannot be used to refer the tuples of
B.

Which one of the following choice is correct?

- (A) S_1 is true S_2 is false
 - (B) S_2 is true S_1 is true
 - (C) Both S_1 and S_2 are true
 - (D) Both S_1 and S_2 are false

- 3.28** The relation scheme given below is used to store information about the employees of a company, where empId is the key and deptId indicates the department to which the employee is assigned. Each employee is assigned to exactly one department.

```
emp(empld, name, gender, salary, deptId)
```

Consider the following SQL Query:

```
select dentId, count(*)
```

from emp

where gender = "female" and

where gender
(select avg(salary)

(select avg (salary) from emp)
group by deptId;

group by depth,

The above query gives, for each department in the company, the number of female employees whose salary is greater than the average salary of

- (A) Employees in the department
 - (B) Female employees in the department
 - (C) Employees in the company
 - (D) Female employees in the company



Solutions

3.1 (A)

Given :

```
Students (rollno: integer, sname: string)
Courses (courseno: integer, cname: string)
Registration (rollno: integer, courseno: integer,
percent: real)
```

All the above queries will return the same result i.e. the distinct names of all students who score more than 90 % in the course numbered 107.

Hence, the correct option is (A).

3.2 (D)

Given :

- S1:** A foreign key declaration can always be replaced by an equivalent check assertion in SQL.
- S2:** Given the table R (a, b, c) where a and b together form the primary key, the following is a valid table definition.

S_1 : By using a check constraint, we can have same effect as foreign key constraint while adding the elements but not when deleting the elements from parent table.

So, S_1 is false.

So, we can't replace it with check constraint.

S_2 : According to definition of foreign key, it must be a primary key to any other table but here a referencing R is just a prime attribute in R, not a primary key.

So, S_2 is false.

Hence, the correct option is (D).

3.3 (B)

Given : Relation schema,

Employees (emp-id, first-name, last-name, hire-date, dept-id, salary)

Department (dept-id, dept-name, manager-id, location-id)

Query :

```
SELECT last-name, hire-date
FROM employees
WHERE (dept-id, hire-date) IN
(SELECT dept-id, MAX(hire-date)
```

```
FROM employees JOIN
department USING (dept-id)
WHERE location-id=1700
GROUP BY dept-id);
```

Inner query will generate the maximum hire date for each department where location id is 1700.

The outer query will give the last name and hire date of all employees who joined on maximum hire date. Hence the query will give correct result.

Hence, the correct option is (B).

3.4 (A)

There will be fewer number of disk block accesses in a nested loop join if outer loop has smaller relation i.e. $r(R)$.

Hence, the correct option is (A).

3.5 (C)

Given : Select * from R

where a in (select S.a from S)

Since the multiplicity of duplicate tuples is maintained, i.e. multiplicity of duplicate tuples will be distributed, when there is a match between R.a and S.a, when there is a match, the value of S.a is repeated in every option except the option (C).

So, the answer of option (C) will always be same as of the given nested query.

3.6 (A)

Given : $\pi_{A_1}(\pi_{A_2}(\sigma_{F_1}(\sigma_{F_2}(r))))$,

where A_1, A_2 are sets of attributes in r with $A_1 \subset A_2$ and F_1, F_2 are Boolean expressions based on the attributes in r

As $A_1 \subset A_2$, the final result will produce only the value of A_1 hence, we can remove the projection of A_2 and the two conditional select can be combined by using one select and combining the conditions using AND.

Hence, the correct option is (A).

3.7 (D)

Given:

employee (emplId, empName, empAge)

dependent (depId, eId, depName, depAge)



Query: $\Pi_{\text{empld}}(\text{employee}) - \Pi_{\text{empld}}(\text{employee} \bowtie_{(\text{empld}=\text{eld}) \wedge (\text{empAge} \leq \text{depAge})} \text{dependent})$
where

$\Pi_{\text{empld}}(\text{employee} \bowtie_{(\text{empld}=\text{eld}) \wedge (\text{empAge} \leq \text{depAge})} \text{dependent})$

Means

All employees whose age is less than or equal to that of all of his dependents.

where as Query: $\Pi_{\text{empld}}(\text{employee}) - \Pi_{\text{empld}}(\text{employee} \bowtie_{(\text{empld}=\text{eld}) \wedge (\text{empAge} > \text{depAge})} \text{dependent})$

Means

All employees whose age is greater than that of all of his dependents.

Hence, the correct option is (D).

3.8 (D)

Given : Following relational schema:

employee(empld, empName, empDept)

customer(custId, custName, salesRepId, rating)

Query :

```
SELECT empName
FROM employee E
WHERE NOT EXISTS
(SELECT custId
FROM customer C
WHERE C.salesRepId = E.empld
AND C.rating <> 'GOOD');
```

The inner query will return customer id of those customers who did not have a 'GOOD' rating. The outer query will choose those employees where there is no customer having a not 'GOOD' rating. i.e. it will return employees with all their customers having a 'GOOD' rating.

Hence, the correct option is (D).

3.9 (A)

Given : following relation

Cinema (theater, address, capacity)

In option (A)-inner query will produce the capacity of all theatres and outer query will produce the theater address which passes the condition 'Capacity >= ALL' and only maximum capacity theatre will satisfy the condition.

In option (B)-It will return address of all theatres.

In option (C)-It will return NULL set because there is no value which is strictly greater than maximum.

In option (D)-It also return NULL.

Hence, the correct option is (A).

3.10 (C)

Given :

R_1	
A	B
1	5
3	7

R_2	
A	B
1	7
4	9

Full outer join of R_1 and R_2 -

A	B	C
1	7	e
3	Null	f
4	9	g

R contains e, f, g but not a, b.

Hence, the correct option is (C).

3.11 (D)

Select operation in SQL selects the various columns which is done by projection operation in relational calculus but SELECT in SQL retain duplicates.

Hence, the correct option is (D).

3.12 2

Given : Student

Roll No	Student_Name
1	Raj
2	Rohit
3	Raj

Performance

Roll No	Course	Marks
1	Math	80
1	English	70



2	Math	75
3	English	80
2	Physics	65
3	Math	80

Result of cross product and where condition is-

S. Roll_no	S.Studnet_name	P.Roll_no	P.course	P.marks
1	Raj	1	Maths	80
1	Raj	1	English	70
2	Rohit	2	Maths	75
2	Rohit	2	Physics	65
3	Raj	3	English	80
1	Raj	3	Maths	80

After grouping and calculating sum, we get-

S.Student_name	Sum (P.marks)
Raj	310
Rohit	140

Hence, number of rows returned = 2.

Hence, the correct answer is 2.

3.13 2

Given :

water_schemes		
scheme_no	district_name	capacity
1	Ajmer	20
1	Bikaner	10
2	Bikaner	10
3	Bikaner	20
1	Churu	10
2	Churu	20
1	Dungargarh	10

Output of first query will be-

Total (name, capacity)

Name	Capacity
Ajmer	20
Bikaner	40
Churu	30
Dungargarh	10

$$\text{Average capacity} = \frac{100}{4} = 25$$

Output of second query is

Avg (capacity)
25

Final query will return the name of those districts where total capacity is equal to or more than average capacity i.e. 25.

Name
Bikaner
Churu

Hence, the correct answer is 2.

3.14 0

Given :

T1		T2	
P	Q	R	S
2	2	2	2
3	8	8	3
7	3	3	2
5	8	9	7
6	9	5	7
8	5	7	2
9	8		

Here, Q refers to R, deleting 8 from Q will not create any issue and S refers to P but it is given that relationship is on delete set Null. So, when 3 will be deleted from T_1 , the row is T_2 with entry 3 in column S is set to Null. Hence no more deletions are preformed.

Hence, the correct answer is 0.

3.15 7

Given :

player	country	goals
Klose	Germany	16
Ronaldo	Brazil	15
G Muller	Germany	14
Fontaine	France	13
Pele	Brazil	12
Klinsmann	Germany	11
Kocsis	Hungary	11
Batistuta	Argentina	10
Cubillas	Peru	10
Lato	Poland	10
Lineker	England	10
T Muller	Germany	10
Rahn	Germany	10

Select tb.goals from top_scorer As tb WHERE tb.country = 'spain'.



This query will return nothing as there is no tuple with country = 'spain'

As empty set always return true, so first condition in WHERE clause is always satisfied.

Select tc.goals from top_scorer As tc where tc.country = 'Germany'.

This query will return the goals of German players and the second condition of WHERE clause will be satisfied if player have greater goals than only German player i.e. greater than 10.

First 7 players have goals > 10.

The answer is 7.

3.16 2.6

Given :

EMP		
Emp ID	Emp Name	Dept Name
1	XYA	AA
2	XYB	AA
3	XYC	AA
4	XYD	AA
5	XYE	AB
6	XYF	AB
7	XYG	AB
8	XYH	AC
9	XYI	AC
10	XYJ	AC
11	XYK	AD
12	XYL	AD
13	XYM	AE

Output of inner query is

Dept Name	Num
AA	4
AB	3
AC	3
AD	2
AE	1

Outer query will find average of returned count- i.e.

$$\text{Output} = \frac{(4+3+3+2+1)}{5} = \frac{13}{5} = 2.6$$

Hence, the correct answer is 2.6.

3.17 (D)

Given :

Relation schema,

EMP (EmpId, EmpName, DeptId)

DEPT (DeptName, DeptId)

TRC :

- I. $\{t | \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}]) \wedge \forall v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}])\}$
- II. $\{t | \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}]) \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] \neq v[\text{DeptId}])\}$
- III. $\{t | \exists u \in \text{EMP}(t[\text{EmpName}] = u[\text{EmpName}]) \wedge \exists v \in \text{DEPT}(t[\text{DeptId}] = v[\text{DeptId}])\}$

Let us see what is the output of above queries

- I. It gives the employees names who do not belong to any department.
- II. It gives employees names who do not belong to same department.
- III. It gives employee's names who belong to same department.

As all of them gives finite number of tuples in output, means all are safe.

Key Point

The expression is said to be safe, if it outputs a finite number of tuples, otherwise it is unsafe.

Hence, the correct option is (D).

3.18 4

Given :

CR	
Student Name	Course Name
SA	CA
SA	CB
SA	CC
SB	CB
SB	CC
SC	CA
SC	CB
SC	CC
SD	CA
SD	CB
SD	CC
SD	CD
SE	CD
SE	CA
SE	CB
SF	CA



SF	CB
SF	CC

 T_1

Course Name
CA
CB
CC

$$T_2 = CR \div T_1$$

Hence, T_2 will produce-

SA
SC
SD
SF

Hence, the correct answer is 4.

3.19 (D)

Given :

Book (isbn, bname)

Stock (isbn, copies)

Example:

Book

isbn	bname
2	A
4	B
6	C
8	D
10	E

Stock

isbn	copies
4	100
6	200
10	200
12	400

Query 1: select B. isbn, S.copies from Book B Inner join Stock S on B.isbn=S.isbn;

isbn	copies
4	100
6	200
10	200

Query 2:Select B.isbn, S.copies from Book B Left outer join Stock S on B.isbn = S.isbn;

isbn	copies
4	100

6	200
10	200
2	Null
8	Null

Query 3: Select B.isbn, S.copies from Book B Right outer join Stock S on B.isbn = S.isbn;

isbn	copies
4	100
6	200
10	200
12	400

Query 4: Select B.isbn, S.copies from Book B Full outer join Stock S on B.isbn=S. isbn;

isbn	copies
4	100
6	200
10	200
2	Null
8	Null
12	400

Query 4 is full outer join so that full order join record set superset of records compare to inner join, left outer join and right outer join.

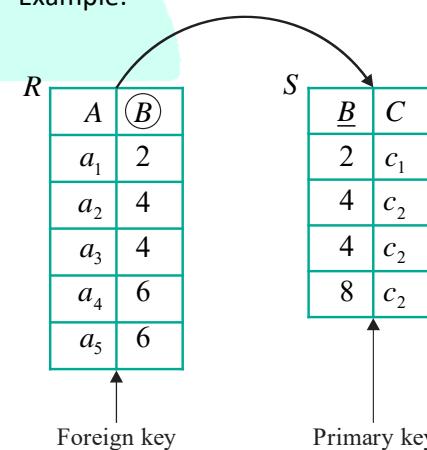
Hence, the correct option is (D).

3.20 (C)

Given : r (A, B) and s(B, C)

Query: Q: r $\bowtie (\sigma_{B < 5}(s))$

Example:



Query: Q: r $\bowtie (\sigma_{B < 5}(s))$

Result in :

A	B	C
a ₁	2	c ₁
a ₂	4	c ₂



a_3	4	c_1
-------	---	-------

Consider Query (A): $\sigma_{B<5}(r \bowtie s)$

Result in:

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

Consider Query (B): $\sigma_{B<5}(r =\bowtie s)$

Result in:

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

Consider Query (C):

$r \bowtie \sigma_{B<5}(s)$)

Result in:

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1
a_4	6	NULL
a_5	6	NULL

Consider Query (D): $\sigma_{B<5}(r) \bowtie s$

Result in:

A	B	C
a_1	2	c_1
a_2	4	c_1
a_3	4	c_1

So Query (C) not equal to given Query Q.

Hence, the correct option is (C)

3.21 (B)

Schema-I is not even in 1 NF as 'courses' is a multivalued attribute.

Schema-II is in 3NF but not in BCNF because in Email \rightarrow roll no.

Email is not a super key.

Schema-III is in 2 NF only, because it contains transitive dependency.

Schema-IV is in 1 NF, as it contains partial dependency.
Hence, the correct option is (B).

3.22 1

Given :

P		
X	Y	Z
X1	Y1	Z1
X1	Y1	Z2
X2	Y2	Z2
X2	Y4	Z4

Q		
X	Y	T
X2	Y1	2
X1	Y2	5
X1	Y1	6
X3	Y3	1

R	
Y	V
Y1	V1
Y3	V2
Y2	V3
Y2	V4

The first query returns-

X
X_2

Second query returns-

X
X_1

Difference will return-

X
X_2

Hence, the correct answer is 1.

3.23 5

Given :

Student	
Roll_no.	Student_name
1	Amit
2	Priya
3	Vinit
4	Rohan



5	Smita	
Performance		
Roll_no.	Subject_code	Marks
1	A	86
1	B	95
1	C	90
2	A	89
2	C	92
3	C	80

First, it will do cross product of student and performance where marks > 84.

After grouping the output will be-

S.Student_name	Sum (P.marks)
Amit	452
Priya	452
Vinit	452
Rohan	452
Smita	452

Hence, the correct answer is 5.

3.24 (A)

Given :

Following relational database scheme :

Catalogue		
sno	pno	cost
S1	P1	150
S1	P2	50
S1	P3	100
S2	P4	200
S2	P5	250
S3	P1	250
S3	P2	150
S3	P5	300
S3	P4	250

Suppliers		
sno	sname	location
S1	M/s Royal furniture	Delhi
S2	M/s Balaji furniture	Bangalore
S3	M/s Premium furniture	Chennai

Parts		
pno	pname	Part_spec

P1	Table	Wood
P2	Chair	Wood
P3	Table	Steel
P4	Almirah	Steel
P5	Almirah	Wood

Query :

Select AVG (cost) from Catalogue where Pno = 'P4' GROUPBY Pno.

Results in:

sno	pno	cost
S1	P1	150
S1	P2	50
S1	P3	100
S2	P4	200
S2	P5	250
S3	P1	250
S3	P2	150
S3	P5	300
S3	P4	250

First, we will apply groupby P4 and the table will become as shown below:

sno	pno	cost
S1	P1	150
S3	P1	250
S1	P2	50
S3	P2	150
S1	P3	100
S2	P4	200
S3	P4	250
S2	P5	250
S3	P5	300

Now applying groupby we will select all rows having Pno= P4 and then find the average cost.

sno	pno	cost
S2	P4	200
S3	P4	250

$$\begin{aligned} \text{Average cost} &= \frac{200+250}{2} \\ &= 225 \end{aligned}$$

Therefore, inner query will return 225.

So, Now Query will become



Select s.sno, s.sname from Supplier s, Catalogue c where s.sno = c.sno AND cost >225

So, here we need to do cross product of Supplier table s and Catalogue table c and from the cross product we will select those rows where s.sno = c.sno AND cost > 225.

Since it is given that cost > 225 so we do not need to consider rows from Catalogue table having cost > 225 while doing cross product.

Hence row number 5, 6, 8, 9 would only be taken while doing cross product.

So, after doing cross product the table will become :

s.sno	s.name	s.location	c.sno	c.pno	c.cost
S1	M/S Royal furniture	Delhi	S2	P5	250
S1	M/S Royal furniture	Delhi	S3	P1	250
S1	M/S Royal furniture	Delhi	S3	P5	300
S1	M/S Royal furniture	Delhi	S3	P4	250
S2	M/S Balaji furniture	Bangalore	S2	P5	250
S2	M/S Balaji furniture	Bangalore	S3	P1	250
S2	M/S Balaji furniture	Bangalore	S3	P5	300
S2	M/S Balaji furniture	Bangalore	S3	P4	250
S3	M/S Premium furniture	Chennai	S2	P5	250
S3	M/S Premium furniture	Chennai	S3	P1	250
S3	M/S Premium furniture	Chennai	S3	P5	300
S3	M/S Premium furniture	Chennai	S3	P4	250

Now after doing cross product only 4 would be selected from the table due to the condition s.sno = c.sno

Final Result :

s.sno	s.name	s.location	c.sno	c.pno	c.cost
S2	M/S Balaji furniture	Bangalore	S2	P5	250
S3	M/S Premium furniture	Chennai	S3	P1	250
S3	M/S Premium furniture	Chennai	S3	P5	300
S3	M/S Premium furniture	Chennai	S3	P4	250

Hence, the correct option is (A).

3.25 (820)

Given : r (A, B) has 1200 tuples

A : attribute ranging from 6 to 20.

B : attribute ranging from 1 to 20

Query: $\sigma_{(A>10) \vee (B=18)}(r)$

There are 10 distinct integer for (A>10) out of 15.

There are 1 distinct integer for (B=18) out of 20

$$\text{So, } P(A>10) = \frac{10}{15} = \frac{2}{3}$$

$$P(B=18) = \frac{1}{20}$$

$$P(A>10)P(B=18) = \frac{2}{3} \times \frac{1}{20} = \frac{1}{30}$$

Now,

$$P(A>10) \vee P(B=18) = P(A>10) + P(B=18)$$

$$P(A>10) \wedge P(B=18)$$

$$= \frac{2}{3} + \frac{1}{20} - \frac{1}{30} = \frac{40+30-2}{60} = \frac{41}{60}$$

Therefore, estimated number of tuples

$$= \frac{41}{60} \times 1200 = 820 \text{ tuples}$$

Hence, the correct answer is 820.

3.26 (C)

Given:

$\text{empAge}(\text{empNo}, \text{age})$

Relational algebra expression:-

$\Pi_{\text{empNo}}(\text{empAge} \bowtie_{(\text{age}>\text{age1})} \rho_{\text{empNo1}, \text{age1}}(\text{empAge}))$

Let's take example,

empAge	
empNo	age
1	20
2	19
3	15

empAge	
emp No	age1
1	20
2	19
3	15

The given query return:

empNo
1
2

Therefore, the query returns empNo (Employee Number) of all employees whose age is not the minimum. (Since it is greater than atleast 1 age)

Hence, the correct option is (C).



3.27 (D)

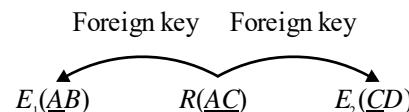
Given :

S1: A relation scheme can have at most one foreign key

This statement is false.

A relation scheme can have more than one Foreign key references to more than one table.

Example



More than 1 Foreign key can be possible

S2: A foreign key in a relation scheme R cannot be used to refer to tuples of R.

This statement is false.

A foreign key in a relation scheme R can be used to refer to tuples of R

Employee			
	Emp_id	Ename	Supplier
	E ₁	A	E ₁
	E ₂	B	E ₁
	E ₃	C	E ₂

Self-referential relationship. So, both S1 and S2 are False.

Hence, the correct option is (D).

3.28 (B)

Given :

emp (empld, name, gender, salary, deptId)

Empld	Ename	Gender	Salary	Dept Id
21 K > 20 K	E ₁	Riya	Female	21 K CS
	E ₂	Umesh	Male	19 K CS
25 K > 20 K	E ₃	Neha	Female	25 K EC
	E ₄	Sumit	Male	14 K EC
21 K > 20 K	E ₅	Pooja	Female	21 K CS

Average Salary = 20K

Query :

Select deptId, count(*) from emp where gender = "female" and salary > (select avg (salary) from emp) groupby deptId;

Here,

Inner query finds the average salary of all the employee. Therefore, the query finds, "for each department in the company, the number of female employees whose salary is greater than the average salary of employees in the company".

So, output will be

DeptId	Count(*)
CS	2
EC	1

Hence, the correct option is (B).

4

Transactions and Concurrency Control



Practice Questions

2014 IIT Kharagpur

- 4.1** Consider the following four schedules due to three transactions (indicated by the subscript) using read and write on a data item x, denoted by $r_i(x)$ and $w_i(x)$ respectively. Which one of them is conflict serializable?
- $r_1(x); r_2(x); w_1(x); r_3(x); w_2(x)$
 - $r_2(x); r_1(x); w_2(x); r_3(x); w_1(x)$
 - $r_3(x); r_2(x); r_1(x); w_2(x); w_1(x)$
 - $r_2(x); w_2(x); r_3(x); r_1(x); w_1(x)$
- 4.2** Consider the following schedule S of transactions T_1, T_2, T_3, T_4 :

T_1	T_2	T_3	T_4
Writes(X) commit	Reads(X)	Writes(X) Commit	
	Writes(Y) Reads(Z) commit		Reads(X) Reads(Y) Commit

Which one of the following statements is CORRECT?

- S is conflict-serializable but not recoverable
 - S is not conflict-serializable but is recoverable
 - S is both conflict-serializable and recoverable
 - S is neither conflict-serializable nor is it recoverable
- 4.3** Consider the transactions T_1, T_2 and T_3 are the schedules S1 and S2 given below.
- T1: $r_1(x); r_1(z); w_1(x); w_1(z)$
T2: $r_2(y); r_2(z); w_2(z)$
T3: $r_3(y); r_3(x); w_3(y)$

S1: $r_1(x); r_3(y); r_3(x); r_2(y); r_2(z); w_3(y); w_2(z); r_1(z); w_1(x); w_1(z)$

S2: $r_1(x); r_3(y); r_2(y); r_3(x); r_1(z); r_2(z); w_3(y); w_1(x); w_2(z); w_1(z)$

Which one of the following statements about the schedule is TRUE?

- Only S1 is conflict serializable
- Only S2 is conflict serializable
- Both S1 and S2 are conflict serializable
- Neither S1 nor S2 is conflict serializable.

2015 IIT Kanpur

- 4.4** consider the following partial Schedule S involving two transactions T_1 and T_2 . Only the read and the write operations have been shown. The read operation on data item P is denoted by read (P) and the write operation on data item P is denoted by write (P).

Time instance	Transaction - id	
	T1	T2
1	Read(A)	
2	Write(A)	
3		Read(C)
4		Write(C)
5		Read(B)
6		Write(B)
7		Read(A)
8		commit
9	Read(B)	

Schedule S

Suppose that the transaction T_1 fails immediately after time instance 9. Which one of the following statements is correct?

- T_2 must be aborted and then both T_1 and T_2 must be re-started to ensure transaction atomicity
- Schedule S is non-recoverable and cannot ensure transaction atomicity
- Only T_2 must be aborted and then re-started to ensure transaction atomicity



- (D) Schedule S is recoverable and can ensure atomicity and nothing else needs to be done.

4.5 Consider the following transaction involving two bank accounts x and y.

```
read(x);  
x := x - 50;  
write(x);  
read(y);  
y := y + 50;  
write(y);
```

The constraint that the sum of the accounts x and y should remain constant is that of

- (A) Atomicity
 - (B) Consistency
 - (C) Isolation
 - (D) Durability

4.6 Consider a simple checkpointing protocol and the following set of operations in the log.

```
(Start, T4);  
(Write, T4, y, 2, 3);  
(Start, T1);  
(Commit, T4);  
(Write, T1, z, 5, 7);  
(Checkpoint);  
(Start, T2);  
(Write, T2, x, 1, 9);  
(Commit, T2);  
(Start, T3);  
(Write, T3, z, 7, 2);
```

If a crash happens now and the system tries to recover using both undo and redo operations, what are the contents of the undo list and the redo list?

- (A) Undo: T3, T1; Redo: T2
 - (B) Undo: T3, T1; Redo: T2, T4
 - (C) Undo: none; Redo: T2, T4, T3, T1
 - (D) Undo: T3, T1, T4; Redo: T2

2016 IISc Bangalore

4.7 Suppose a database schedule S involves transactions T_1, \dots, T_n . Construct the precedence graph of S with vertices representing the transactions and edges representing the conflicts. If S is serializable, which one of the following orderings of the

vertices of the precedence graph is guaranteed to yield a serial schedule?

- (A) Topological order
 - (B) Depth-first order
 - (C) Breadth-first order
 - (D) Ascending order of transaction indices

4.8 Consider the following database schedule with two transactions, T_1 and T_2 .

$$S = r_2(X); r_1(X); r_2(Y); w_1(X) \\ r_1(Y); w_2(X); a_1; a_2$$

Where $r_i(Z)$ denotes a read operation by transaction T_i on a variable Z , $w_i(Z)$ denotes a write operation by T_i on a variable Z and a_i denotes an abort schedule is TRUE?

- (A) S is non-recoverable
 - (B) S is recoverable, but has a cascading abort
 - (C) S does not have a cascading abort
 - (D) S is strict

4.9 Which one of the following is NOT a part of the ACID properties of database transactions?

- (A) Atomicity
 - (B) Consistency
 - (C) Isolation
 - (D) Deadlock-free

4.10 Consider the following two phase locking protocol. Suppose a transaction T accesses (for read or write operations), a certain set of objects $\{O_1, \dots, O_k\}$. This is done in the following manner.

Step 1: T acquires exclusive locks to $\{O_1, \dots, O_k\}$ in increasing order of their addresses.

Step 2: The required operations are performed

Step 3: All locks are released

This protocol wil

- (A) Guarantee serializability and deadlock-freedom
 - (B) Guarantee neither serializability nor deadlock-freedom
 - (C) Guarantee serializability but not deadlock-freedom
 - (D) Guarantee deadlock-freedom but not serializability

2017 IIT Roorkee

4.11 Two transactions T_1 and T_2 are given as



$T_1 : r_1(X) w_1(X) r_1(Y) w_1(Y)$

$T_2 : r_2(Y) w_2(Y) r_2(Z) w_2(Z)$

Where $r_i(V)$ denotes a read operation by transaction T_i on a variable V and $w_i(V)$ denotes a write operation by transaction T_i on a variable V . The total number of conflict serializable schedules that can be formed by T_1 and T_2 is _____.

- 4.12** In a database system, unique timestamps are assigned to each transaction using Lamport's logical clock. Let TS (T_1) and TS (T_2) be the timestamps of transactions T_1 and T_2 respectively. Besides, T_1 holds a lock on the resource R, and T_2 has requested a conflicting lock on the same resource R. The following algorithm is used to prevent deadlocks in the database system assuming that a killed transaction is restarted with the same timestamp.

if $TS(T_2) < TS(T_1)$ then

T_1 is killed

else T_2 waits.

Assume any transaction that is not killed terminates eventually. Which of the following is TRUE about the database system that uses the above algorithm to prevent deadlocks?

- (A) The database system is both deadlock-free and starvation-free
- (B) The database system is deadlock-free, but not starvation-free
- (C) The database system is starvation-free, but not deadlock-free
- (D) The database system is neither deadlock-free nor starvation-free

2019 IIT Madras

- 4.13** Consider the following two statements about database transaction schedules:

- I. Strict two-phase locking protocol generates conflict serializable schedules that are also recoverable.
- II. Timestamp-ordering concurrency control protocol with Thomas' Write Rule can generate view serializable schedules that are not conflict serializable

Which of the above statements is/are TRUE?

- (A) I only

- (B) II only

- (C) Both I and II

- (D) Neither I or II

2020 IIT Delhi

- 4.14** Consider a schedule of transactions T_1 and T_2 :

T_1	RA			RC		WD		WB	Commit	
T_2		RB	WB		RD		WC			Commit

Here, RX stands for "Read (X)" and WX stands for "Write (X)". Which one of the following schedules is conflict equivalent to the above schedule?

- (A)

T_1				RA	RC	WD	WB		Commit	
T_2	RB	WB	RD				WC			Commit

- (B)

T_1	RA	RC	WD	WB				Commit		
T_2					RB	WB	RD	WC		Commit

- (C)

T_1	RA	RC	WD				WB		Commit	
T_2				RB	WB	RD		WC		Commit

- (D)

T_1				RA	RC	WD	WB	Commit	
T_2	RB	WB	RD	WC					Commit

2021 IIT Bombay

- 4.15** Suppose a database system crashes again while recovering from a previous crash. Assume check-pointing is not done by the database either during the transactions or during recovery. Which of the following statements is/are correct?

- (A) The same undo and redo list will be used while recovering again.
- (B) The system cannot recover any further.
- (C) The database will become inconsistent
- (D) All the transactions that are already undone and redone will not be recovered again.

- 4.16** Let $r_i(z)$ and $w_i(z)$ denote read and write operations respectively on a data item Z by a transaction T_i . Consider the following two schedules.

$S_1 : r_1(x); r_1(y); r_2(x); r_2(y); w_2(y); w_1(x)$

$S_2 : r_1(x); r_2(x); r_2(y); w_2(y); r_1(y); w_1(x)$



- (A) S_1 is conflict serializable, and S_2 is not conflict serializable.
 (B) S_1 is not conflict serializable and S_2 is conflict serializable.
 (C) Both S_1 and S_2 are conflict serializable.
 (D) Neither S_1 nor S_2 is conflict serializable.

4.17 Let S be the following schedule of operation of three transactions T_1 , T_2 and T_3 in a relational database system:

$R_2(Y)$, $R_1(X)$, $R_3(Z)$, $R_1(Y)$, $W_1(X)$, $R_2(Z)$, $W_2(Y)$,
 $R_3(X)$, $W_3(Z)$

P: S is conflict serializable.

Q: If T_3 commits before T_1 finishes, then S is recoverable.

Which of the following choices is correct?

- (A) Both P and Q are true
 (B) P is true and Q is false
 (C) Both P and Q are false
 (D) P is false and Q is true

Solutions

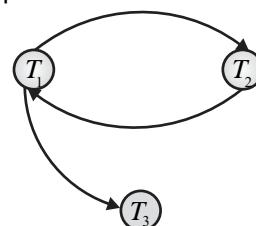
4.1 (D)

Consider option (A) :

Schedule A-

T_1	T_2	T_3
$r_1(x)$		
	$r_2(x)$	
$w_1(x)$		
		$r_3(x)$
	$w_2(x)$	

Precedence graph is



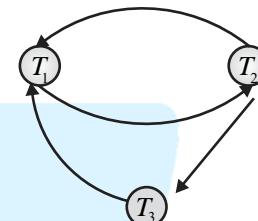
Graph have cycle, it is not conflict serializable.

Now consider option (B)

Schedule B-

T_1	T_2	T_3
	$r_2(x)$	
$r_1(x)$		
	$w_2(x)$	
		$r_3(x)$
$w_1(x)$		

Precedence graph is



→ It is also cyclic.

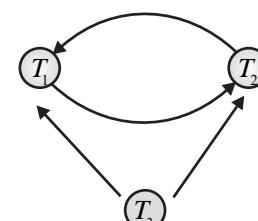
Not conflict serializable

Now consider option (D)

Schedule C-

T_1	T_2	T_3
		$r_3(x)$
	$r_2(x)$	
$r_1(x)$		
		$w_2(x)$
	$w_1(x)$	

Precedence graph is



→ It is cyclic; so not conflict serializable.

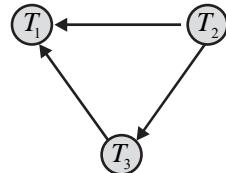
Schedule D-

T_1	T_2	T_3
	$r_2(x)$	
	$w_2(x)$	
		$r_3(x)$



$r_1(x)$		
$w_1(x)$		

Precedence graph is



→ Not cyclic, hence it is conflict serializable.

Hence, the correct option is (D).

Key Point

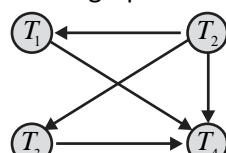
- If there is a cycle in the precedence graph of a schedule then , that schedule is not conflict-serializable.

4.2 (C)

Given :

T ₁	T ₂	T ₃	T ₄
Writes(X) commit	Reads(X)	Writes(X) Commit	
	Writes(Y) Reads(Z) commit		Reads(X) Reads(Y) Commit

Making the precedence graph-



As there is no cycle in the graph, schedule is conflict serializable. As there are no dirty read, it is recoverable also.

Hence, the correct option is (C).

Key Point

- If there is a cycle in the precedence graph of a schedule then , that schedule is not conflict-serializable.
- If there is no dirty read in a Schedule then that Schedule is recoverable.
- For a schedule to be recoverable, if some transaction T_y is reading some value updated or written by some other transaction T_x , then the transaction T_y should commit after the transaction T_x has committed.
- If there is a dirty read present in a schedule then that Schedule definitely contains Cascading rollbacks/aborts.

transaction T_y should commit after the transaction T_x has committed.

4.3 (A)

Given : T1: rl (x); rl (z); wl (x); wl (z)

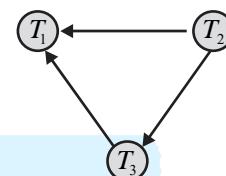
T2: r2 (y); r2 (z); w2 (z)

T3: r3 (y); r3 (x); w3 (y)

S1: rl (x); r3 (y); r3 (x); r2 (y); r2 (z); w3(y); w2 (z); rl (z); wl (x); wl (z)

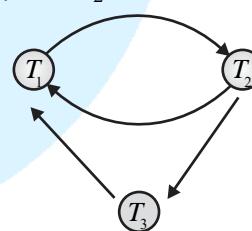
S2: rl (x); r3 (y); r2 (y); r3 (x); rl (z); r2 (z); w3 (y); wl (x); w2 (z); wl (z)

Precedence graph of S_1 –



It has no cycle, i.e. conflict serializable.

Precedence graph of S_2 –



It has a cycle, i.e. not conflict serializable.

Hence, the correct option is (A).

Key Point

- If there is a cycle in the precedence graph of a schedule then , that schedule is not conflict-serializable.
- If there is no dirty read in a Schedule then that Schedule is recoverable.
- For a schedule to be recoverable, if some transaction T_y is reading some value updated or written by some other transaction T_x , then the transaction T_y should commit after the transaction T_x has committed.
- If there is a dirty read present in a schedule then that Schedule definitely contains Cascading rollbacks/aborts.

4.4 (B)

Given :

Schedule S

	Transaction-id
--	----------------



Time instance	T1	T2
1	Read (A)	
2	Write (A)	
3		Read (A)
4		Write (A)
5		Read (B)
6		Write (B)
7		Read (A)
8		Commit
9	Read (B)	

If transaction fails, atomicity requires effect of transaction to be undone. Durability states that once transactions commits, its change cannot be undone (without running another, compensating transaction)

Recoverable schedule : A schedule exactly where, for every set of transaction T_i and T_j . If T_j reads a data items previously written by T_i , then the commit operation of T_i precedes the commit operation of T_j .

Aborting involves undoing the operation and redoing them since by the time stamp it is aborted.

Consider option (A) :

T2 must be aborted and then both T1 and T2 must be restarted to ensure transaction atomicity. It is incorrect because it says abort transaction T2 and then redo all the operations but there is no guarantee that it will succeed this time as again T1 may fail.

Now, consider option (B) :

Schedule S is non-recoverable and cannot ensure transaction atomicity.

It is correct because it is by definition an irrecoverable schedule so now even if we start to undo the action one by one (after T1 fails) in order to ensure transaction atomicity. Still we cannot undo a committed transaction i. Hence this schedule is irrecoverable by definition and also not atomic since it leaves the data base in an inconsistent state. Simply dirty read so non-recoverable.

Consider option (C) :

Only T2 must be aborted and then restarted to ensure transaction atomicity.

It is incorrect because it says abort only transaction T2 and then redo all the T2 operations. But this is dirty read problem as it is reading the data item A which is return by T1 and T1 is not committed. Again it will be dirty read problem. So incorrect.

Consider option (D) :

Schedule S is recoverable and ensure transaction atomicity and nothing else needs to be done.

Incorrect, it is clearly saying that Schedule S is recoverable but it is irrecoverable because T2 read the data item A which is written by T1 and T1 failed and rollback, at the rollback T1 start undo all operations and modified the value of A with previous value but T2 is already committed. So T2 can't change the read value of A which was earlier taken from T1.

Hence, the correct option is (B).

4.5 (B)

Given : read(x);
x: = x - 50;
write (x);
read (y);
y: = y + 50;
write(y);

Consistency ensures that whenever a transaction is executed, it must take the database from one consistent state to another.

Here sum of accounts should remain constant before and after the execution which will lead the database to consistent state.

Hence, the correct option is (B).

4.6 (A)

Given :

(Start, T4);
(Write, T4, y, 2, 3);
(Start, T1);
(Commit, T4);
(Write, T1, z, 5, 7);
(Checkpoint);
(Start, T2);
(Write, T2, x, 1, 9);
(Commit, T2);
(Start, T3);



(Write, T3, z, 7, 2);

T_1	T_2	T_3	T_4
Start			Start write (y,2,3) Commit
Write (1,5,7)			
Check point	Checkpoint start write (x, 1, 9) commit	Check point	Checkpoint
		Start write (z,7,2)	
Crash	Crash	Crash	Crash

The uncommitted write operation in T_1 and T_3 needs to be undo.

T_4 need not to be redone due to checkpoint and T_2 need to be redone as it is committed after checkpoint. So, undo T_1 & T_3 , redo T_2

Hence, the correct option is (A).

4.7 (A)

Given : A database schedule S involving transactions T_1, \dots, T_n .

If a schedule is conflict serializable, then there should be no cycle in precedence graph.

As topological sort is not possible for cyclic graph and BFS and DFS are possible for cyclic graphs.

Option (D) is also wrong as there is a possibility that transaction having more indices might come before.

Hence, the correct option is (A).

4.8 (C)

The given schedule is

T_1	T_2
$r_1(X)$	$r_2(X)$
$w_1(X)$	$r_2(Y)$
$r_1(Y)$	$w_2(X)$
a_1	a_2

(A) Since there is no dirty read. S is recoverable

Hence, option (A) is FALSE.

- (B) Since there is no dirty read, it has no cascading abort. Hence, option (B) is FALSE.
- (C) S does not have a cascading abort. Hence, this is TRUE.
- (D) The schedule is not strict because T_2 has written the value of X which was written by T_1 before T_1 has aborted. Hence, option (D) is FALSE.

Hence, the correct option is (C).

4.9 (D)

Dead lock- freedom is not one of the ACID properties.

Key Point

ACID properties are-

- A- Atomicity
- C- Consistency
- I- Isolation
- D- Durability

Hence, the correct option is (D).

4.10 (A)

Given : Two phase locking protocol 2PL over objects O_1, \dots, O_k

Step 1 : T acquires exclusive lock to O_1, \dots, O_k in increasing order of their addresses.

Step 2 : The required operations are performed.

Step 3 : All locks are released.

Two phase locking protocol is conflict serializable. So this is a modified version of the basic 2PL protocol. So serializability should be guaranteed and we can get up serializable schedule by ordering based on lock points (same as in basic 2PL)

Now in step 1 since it is mentioned as exclusive lock, only one transaction can lock the object. Due to acquiring of locks based on ordering of addresses and locks are not released until the transaction complete its operation we can prevent the circular wait condition and hence making it dead lock free.

So the correct answer should be A :

Guarantees serializability and dead lock freedom.

Hence, the correct option is (A).



4.11 54

Given : $T_1 : r_1(X) w_1(X) r_1(Y) w_1(Y)$

$T_2 : r_2(Y) w_2(Y) r_2(Z) w_2(Z)$

Number of conflict serializable on $T_1 \rightarrow T_2 : 1$

$r_1(X) w_1(X) r_1(Y) w_1(Y) r_2(Y) w_2(Y) r_2(Z) w_2(Z)$

Number of conflict serializable on $T_2 \rightarrow T_1 : 53$

$r_2(Y) w_2(Y) r_1(Y) w_1(Y)$

$r_1(X) w_1(X)$ must be before $r_1(Y)$ so that $(r_2(Y) w_2(Y))(r_1(X) w_1(X))$ can place in ${}^4C_2 = 6$ ways

1. $r_2(Y) w_2(Y) r_1(X) w_1(X) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^6C_2 = 15$ ways
 2. $r_2(Y) r_1(X) w_1(X) w_2(Y) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^4C_2 = 6$ ways
 3. $r_2(Y) r_1(X) w_2(Y) w_1(X) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^5C_2 = 10$ ways
 4. $r_1(X) w_1(X) r_2(Y) w_2(Y) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^4C_2 = 6$ ways
 5. $r_1(X) r_2(Y) w_2(Y) w_1(X) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^5C_2 = 10$ ways
 6. $r_1(X) r_2(Y) w_1(X) w_2(Y) r_1(Y) w_1(Y)$
 $r_2(Z) w_2(Z)$ can place in ${}^4C_2 = 6$ ways
- Number of ways = $15 + 6 + 10 + 6 + 10 + 6 = 53$ ways

Total conflict serializable of T_1 and T_2
= $53 + 1 = 54$ ways

Hence, the correct answer is 54.

4.12 (A)

Given :

if $TS(T_2) < TS(T_1)$ then

T_1 is killed

else T_2 waits.

As it is given that 'unique time stamps' are assigned to each transaction, so there will not be any equal timestamps.

Lamport's logical clock assign the timestamp in increasing order only.

Hence, if we use the given algorithm, the system is both deadlock free and starvation-free.

Hence, the correct option is (A).

4.13 (C)

Given : Following statements :

- I. Strict two-phase locking protocol generates conflict serializable schedules that are also recoverable.
 - II. Timestamp-ordering concurrency control protocol with Thomas' Write Rule can generate view serializable schedules that are not conflict serializable.
 1. Strict 2PL allows schedules where precedence graph do not contain cycle i.e. conflict serializable As it does not release exclusive locks until the transaction is committed, means the schedule is recoverable also.
 2. Because of blind write, the schedule generated by Thomas write rule is not conflict serializable but view serializable.
- Hence, both are TRUE.

Hence, the correct option is (C).

Key Point

- Strict 2-PL ensures conflict serializability and recoverable cascadeless schedule.
- Time-stamp ordering protocol ensures view serializability but not conflict-serializability.

4.14 (A)

Given :

T_1	RA			RC		WD		WB	Commit
T_2		RB	WB		RD		WC		Commit

Precedence graph of given schedule-



Precedence graph of option (A)



Precedence graph of option (B)



Precedence graph of option (C).



Precedence graph of option (D).



Option (B) and (D) are false because graphs is not equivalent to given graph.

Option (C) is also false because there is a swapping operation of conflicting operations $W_1(D)$ and $R_2(D)$

Hence, the correct option is (A).

4.15 (A)

Given : A database system crashes again while recovering from previous crash. In order to recover the database even on multiple crashes the same undo and redo list will be used.

Hence, the correct option is (A).

4.16 (B)

Given : Schedules

S1: $r_1(x) r_1(y) r_2(x) r_2(y) w_2(y) w_1(x)$

S2: $r_1(x) r_2(x) r_2(y) w_2(y) r_1(y) w_1(x)$

Consider S1 :

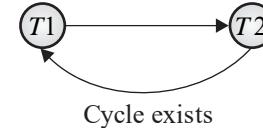
T1	T2
$r_1(x)$	
$r_1(y)$	$r_2(x)$
	$r_2(y)$
	$w_2(y)$
$w_1(x)$	

In schedule1 $r_1(y)$ is conflicting with $w_2(y)$ therefore in the precedence graph there will be edge from

$T1 \rightarrow T2$. Also $r_2(x)$ is conflicting with $w_1(x)$ therefore there will be edge from $T2 \rightarrow T1$.

Precedence graph of S1 is having cycle hence S1 is not conflict serializable.

S1 :



Cycle exists

\therefore not conflict serializable

Consider S2 :

T1	T2
$r_1(x)$	
	$r_2(x)$
	$r_2(y)$
	$w_2(y)$
$r_1(y)$	
	$w_1(x)$

In schedule2 $r_2(x)$ is conflicting with $w_1(x)$ therefore there will be edge from $T2 \rightarrow T1$ in precedence graph. $w_2(y)$ is conflicting with $r_1(y)$ therefore edge $T2 \rightarrow T1$.

Precedence graph of S2 is not having any cycle hence S2 is conflict serializable.

S2 :



No cycle

\therefore Conflict serializable

So, S1 is not conflict serializable and S2 is conflict serializable.

Hence, the correct option is (B).

4.17 (B)

Given: Transactions with operations :

T1	T2	T3
$R_1(x)$	$R_2(y)$	
$R_1(x)$		$R_3(z)$
$R_1(x)$	$R_2(z)$	
$W_1(x)$	$W_2(y)$	
		$R_3(X)$
		$W_3(z)$

Statements

P : S is conflict serializable



Q : If T3 commits before T1 finishes, then S is recoverable.

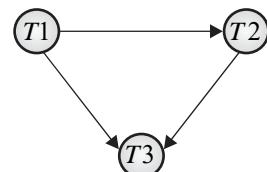
Since,

$T1 \rightarrow T2$ due to $R_1(y)$ is before $W_2(y)$

$T1 \rightarrow T3$ due to $W_1(x)$ is before $R_3(X)$

$T2 \rightarrow T3$ due to $R_2(z)$ is before $W_3(z)$ in the schedule

So precedence graph for the given schedule is:



There are no other conflicts and the discovered conflicts are not forming the cycle. Therefore, given schedule S is conflict serializable.

Therefore, statement P is True.

As we know schedule S is recoverable if T_j creating the dirty read by reading the written data by T_i and T_j commits after T_i commits. Here, transaction T3 performing dirty read of T1 and T3 should commit after T1 to make the schedule recoverable.

Therefore, statement Q is False.

Hence, the correct option is (B).

5

File Structures

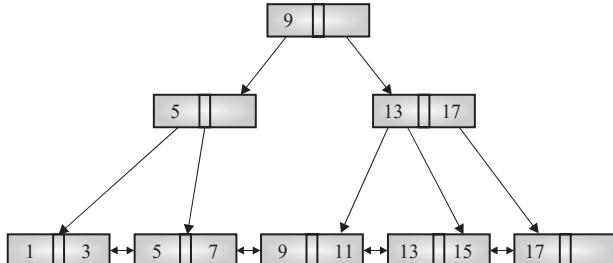


Practice Questions

2013 IIT Bombay

- 5.1 An index is clustered, if
- (A) it is on a set of fields that form a candidate key
 - (B) it is on a set of fields that include the primary key
 - (C) the data records of the file are organized in the same order as the data entries of the index
 - (D) the data records of the file are organized not in the same order as the data entries of the index

2015 IIT Kanpur

- 5.2 Consider a B+ tree in which the search key is 12 byte long, block size is 1024 byte, recorder pointer is 10 byte long and the block pointer is 8 byte long. The maximum number of keys that can be accommodated in each non-leaf node of the tree is _____.
5.3 With reference to the B+ tree index of order 1 shown below, the minimum number of nodes (including the Root node) that must be fetched in order to satisfy the following query. "Get all records with a search key greater than or equal to 7 and less than 15" is _____.


- 5.4 A file is organized so that the ordering of the data records is the same as or close to the ordering of data entries in some index. Then that index is called
- (A) Dense
 - (B) Sparse
 - (C) Clustered
 - (D) Unclustered

2016 IISc Bangalore

- 5.5 B+ Trees are considered BALANCED because.

- (A) The lengths of the paths from the root to all leaf nodes are all equal.
- (B) The lengths of the paths from the root to all leaf nodes differ from each other by at most 1.
- (C) The number of children of any two non-leaf sibling nodes differ by at most 1.
- (D) The number of records in any two leaf nodes differ by at most 1.

2017 IIT Roorkee

- 5.6 In a B+ Tree , if the search-key value is 8 bytes long , the block size is 512 bytes and the pointer size is 2 bytes, then the maximum order of the B+ Tree is _____

2019 IIT Madras

- 5.7 Which one of the following statements is NOT correct about the B+ tree data structure used for creating an index of a relational database table?
- (A) B+ Tree is a height-balanced tree
 - (B) Non-leaf nodes have pointers to data records
 - (C) Key values in each node are kept in sorted order
 - (D) Each leaf node has a pointer to the next leaf node

2020 IIT Delhi

- 5.8 Consider a database implemented using B+ tree for file indexing and installed on a disk drive with block size of 4 KB. The size of search key is 12 bytes and the size of tree/disk pointer is 8 bytes. Assume that the database has one million records. Also assume that no node of the B+ tree and no records are present initially in main memory. Consider that each record fits into one disk block. The minimum number of disk accesses required to retrieve any record in the database is _____.

2021 IIT Bombay



- 5.9 A data file consisting of 1,50,000 students records is stored on a hard disk with block size of 4096 bytes. The data file is sorted on the primary key RollNo. The size of a record pointer for this disk is 7 bytes. Each student-record has a candidate key attribute called ANum of size 12 bytes. Suppose an index file with records

consisting of two fields. ANum value and the record pointer to the corresponding student record, is built and stored on the same disk. Assume that the records of data file and index file are not split across disk blocks. The number of blocks in the index file is _____

Solutions

5.1 (C)

Clustered index is created using any column or a combination of column, which need not to be primary or candidate key. So, option A and B are not the answer. The column which is used for indexing is used to sort the rows of table. Hence the data is organized in the same order as data of the index.

Hence, the correct option is (C).

5.2 50

Given: Key size = 12 bytes

Block size = 1024 bytes

Record pointer size = 10 byte.

Block pointer size = 8 byte

For internal node-

$$(n-1)k + nb \leq \text{Block size}$$

$$(n-1)12 + n(8) \leq 1024$$

$$20n \leq 1036$$

$$n \leq 51.8$$

$$n = 51$$

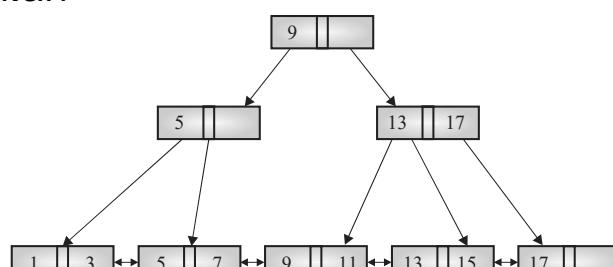
Order = 51

Number of keys = $51 - 1 = 50$

Hence, the correct answer is 50.

5.3 (*)

Given :



We need three nodes accesses to search for key 7. In B+ tree all neighboring leaf nodes are connected via block pointers and it is possible to access remaining

records after 7 using linear search for records. Hence two more nodes needed to fetch records less than 15.

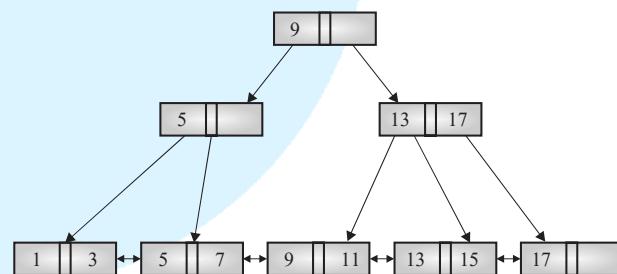
5 nodes will be fetched-

$$9 \rightarrow 5 \rightarrow (5, 7), \rightarrow (9, 11) \rightarrow (13, 15)$$

To get all records greater than or equal to 7 and less than 15.

5.4 (C)

Given:



This is the definition of clustered index.

Hence, the correct option is (C).

5.5 (A)

In both B tree and B +tree all leaf nodes are at the same level. Hence, the lengths of the paths from the root to all leaf nodes are all equal.

Hence, the correct option is (A).

5.6 52

Given : Key size = 8 bytes

Block size = 512 bytes

Block pointer size = 2 bytes

Let order be n ,

we know for B +tree,

$$(n-1) \text{ key size} + n (\text{block pointer size}) \leq \text{block size}$$

$$(n-1)8 + (n)(2) \leq 512$$

$$10n \leq 520$$

$$n \leq 52$$

$$n = 52$$

Hence, the correct answer is 52.



5.7 (B)

Properties of B+ trees are :

1. B+ tree is height balanced tree.
2. Key value is in sorted order.
3. Leaf node has pointer to next leaf node.
4. Non leaf node has pointer to a node (leaf or non leaf) and not pointer to data record.

So option (B) is not correct.

Hence the answer is option (B).

5.8 4

Given :

Search key = 12 bytes

Disk pointer = 8 bytes

Block size = 4096 bytes

Number of database records = 10^6

B+ tree index each record fits into one block.

Keys given to find levels of B+ tree : Bulk loading B+ tree design we can use

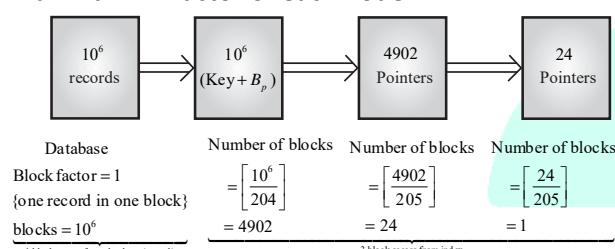
Order of B+ tree nodes =

$$8p + (p-1)12 \leq 4096$$

$$p = 205$$

Minimum levels and index :

Maximum fill factor of each node.



So, minimum 4 block access required to access record.

Hence, the correct answer is 4.

5.9 (698)

Given : Index is being built on attribute "Anum" which is candidate key, but given that file is sorted on primary key "RollNo".

This indicates that the index must be a secondary index, so "There should exist an Index record for every record of original 'student table'".

Also "Assume that Records of data file and index file are not split across disk blocks".

This data indicated unspanned strategy.

So, Size of Index record = $12+7 = 19$

No. of index record = 1, 50,000

Block factor for index,

$$(Block \ factor) \ index = \frac{4096}{19}$$

= 215 records/block

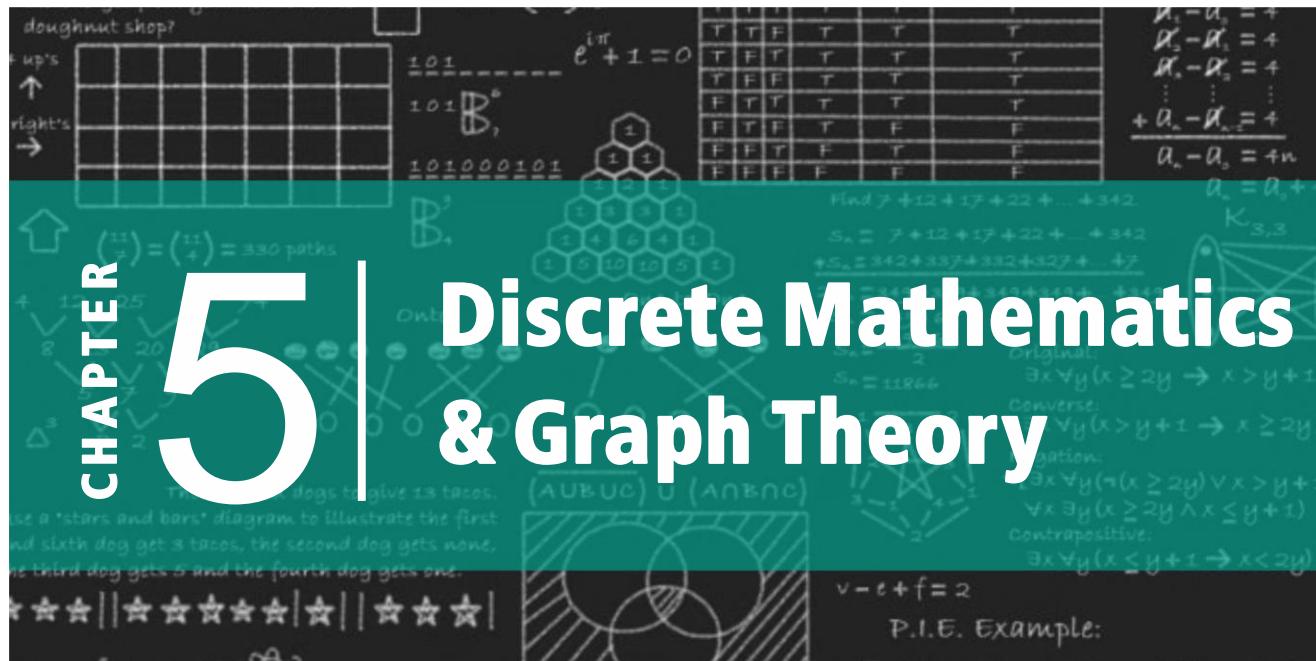
$$\text{Number of index blocks} = \frac{150000}{215} = 697.67$$

= 698 index blocks

Hence, the correct answer is 698.

CHAPTER 5

Discrete Mathematics & Graph Theory



Marks Distribution of Discrete Mathematics & Graph Theory in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	4	9	22
2004*	6	10	26
2005*	6	10	26
2006*	6	12	30
2007*	5	7	21
2008*	3	11	25
2009	4	5	14
2010	3	3	9
2011	1	2	5
2012	2	4	10
2013	2	3	8
2014 Set-1	3	5	13
2014 Set-2	2	5	12
2014 Set-3	3	5	13

* CS and IT combined

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	3	5	13
2015 Set-2	4	5	14
2015 Set-3	5	4	13
2016 Set-1	2	3	8
2016 Set-2	2	3	8
2017 Set-1	4	1	6
2017 Set-2	3	1	5
2018	3	4	11
2019	3	1	5
2020	2	4	10
2021 Set-1	2	3	8
2021 Set-2	2	3	8

Syllabus : Discrete Mathematics & Graph Theory

Propositional and first order logic. Sets, relations, functions, partial orders and lattices. Monoids, Groups. Graphs: connectivity, matching, coloring. Combinatorics: counting, recurrence relations, generating functions.

Contents : Discrete Mathematics & Graph Theory

S. No. Topics

- 1.** Mathematical Logic
- 2.** Set Theory & Algebra
- 3.** Combinatorics
- 4.** Graph Theory



Practice Questions

2013 IIT Bombay

- 1.1** What is the logical translation of the following statement?

"None of my friends are perfect".

- (A) $\exists x(F(x) \wedge \neg P(x))$
- (B) $\exists x(\neg F(x) \wedge P(x))$
- (C) $\exists x(\neg F(x) \wedge \neg P(x))$
- (D) $\neg \exists x(\neg F(x) \wedge P(x))$

- 1.2** Which one of the following is NOT logically equivalent to

- $$\neg \exists x(\forall y(\alpha) \wedge (\forall z(\beta)))$$
- (A) $\forall x(\exists z(\neg \beta) \rightarrow \forall y(\alpha))$
 - (B) $\forall x(\forall z(\beta) \rightarrow \exists y(\neg \alpha))$
 - (C) $\forall x(\forall y(\alpha) \rightarrow \exists z(\neg \beta))$
 - (D) $\forall x(\exists y(\neg \alpha) \rightarrow \exists z(\neg \beta))$

2014 IIT Kharagpur

- 1.3** The CORRECT formula for the sentence, "not all rainy days cold" is

- (A) $\forall d(Rainy(d) \wedge \sim Cold(d))$
- (B) $\forall d(\sim Rainy(d) \rightarrow Cold(d))$
- (C) $\exists d(\sim Rainy(d) \rightarrow Cold(d))$
- (D) $\exists d(Rainy(d) \wedge \sim Cold(d))$

- 1.4** Which one of the following Boolean expressions is NOT a tautology?

- (A) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$
- (B) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$
- (C) $(a \wedge b \wedge c) \rightarrow (c \vee a)$
- (D) $a \rightarrow (b \rightarrow a)$

- 1.5** Consider the following statements :

P : Good mobile phones are not cheap
 Q : Cheap mobile phones are not good

- L : P implies Q
 M : Q implies P
 N : P is equivalent to Q

Which of the following about L, M and N is CORRECT?

- (A) Only L is TRUE.
- (B) Only M is TRUE.
- (C) Only N is TRUE.
- (D) L, M and N are TRUE.

- 1.6** Consider the statement

"Not all that glitters is gold"

Predicate $glitters(x)$ is true if x glitters and predicate $gold(x)$ is true if x is gold.

Which one of the following logical formulae represents the above statement?

- (A) $\forall x: glitters(x) \Rightarrow \neg gold(x)$
- (B) $\forall x: gold(x) \Rightarrow glitters(x)$
- (C) $\exists x: gold(x) \wedge \neg glitters(x)$
- (D) $\exists x: glitters(x) \wedge \neg gold(x)$

- 1.7** Which one of the following propositional logic formulas is TRUE when exactly two of p , q and r are TRUE ?

- (A) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
- (B) $(\sim(p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
- (C) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$
- (D) $(\sim(p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

2015 IIT Kanpur

- 1.8** Which one of the following well-formed formulae is a tautology?

- (A) $\forall x \exists y R(x, y) \leftrightarrow \exists y \forall x R(x, y)$
- (B) $(\forall x [\exists y R(x, y) \rightarrow S(x, y)]) \rightarrow \forall x \exists y S(x, y)$
- (C) $[\forall x \exists y (P(x, y) \rightarrow R(x, y))] \leftrightarrow [\forall x \exists y (\neg P(x, y) \vee R(x, y))]$
- (D) $[\forall x \exists y (P(x, y) \rightarrow \forall x \forall y P(y, x))$

- 1.9** In a room there are only two types of people, namely Type 1 and Type 2. Type 1 people always tell the truth and Type 2 people always lie. You give a fair coin to a person in that room, without



knowing which type he is from and tell him to toss it and hide the result from you till you ask for it. Upon asking the person replies the following "The result of the toss is head if and only if I am telling the truth" Which of the following option is correct?

- (A) The result is head
- (B) The result is tail
- (C) If the person is of Type 2, then the result is tail
- (D) If the person is of Type 1, then the result is tail

1.10 Consider the following two statements.

S1 : If a candidate is known to be corrupt, then he will not be elected.

S2 : If a candidate is kind, he will be elected

Which one of the following statements follows from S1 and S2 as per sound inference rules of logic?

- (A) If a person is known to be corrupt, he is kind
- (B) If a person is not known to be corrupt, he is not kind
- (C) If a person is kind, he is not known to be corrupt
- (D) If a person is not kind, he is not known to be corrupt

1.11 Which one of the following is NOT equivalent to $p \leftrightarrow q$?

- (A) $(\neg p \vee q) \wedge (p \vee \neg q)$
- (B) $(\neg p \vee q) \wedge (q \rightarrow p)$
- (C) $(\neg p \wedge q) \vee (p \wedge \neg q)$
- (D) $(\neg p \wedge \neg q) \vee (p \wedge q)$

1.12 The binary operator \neq is defined by the following truth table.

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Which one of the following is true about the binary operator \neq ?

- (A) Both commutative and associative
- (B) Commutative but not associative

- (C) Not commutative but associative
- (D) Neither commutative nor associative

2016 IISc Bangalore

1.13 Consider the following expressions:

- (i) false
- (ii) Q
- (iii) True
- (iv) $P \vee Q$
- (v) $\neg Q \vee P$

The number of expressions given above that are logically implied by $P \wedge (P \Rightarrow Q)$ is _____.

1.14 Let p, q, r, s represent the following propositions.

- $p : x \in \{8, 9, 10, 11, 12\}$
- $q : x$ is a composite number
- $r : x$ is a perfect square
- $s : x$ is a prime number

The integer $x \geq 2$ which satisfies $\neg((p \Rightarrow q) \wedge (\neg r \vee \neg s))$ _____.

1.15 Which one of the following well-formed formulae in predicate calculus is NOT valid?

- (A) $(\forall x p(x) \Rightarrow \forall x q(x))$
 $\Rightarrow (\exists x \neg p(x) \vee \forall x q(x))$
- (B) $(\exists x p(x) \vee \exists x q(x))$
 $\Rightarrow \exists x(p(x) \vee q(x))$
- (C) $\exists x(p(x) \wedge q(x))$
 $\Rightarrow (\exists x p(x) \wedge \exists x q(x))$
- (D) $\forall x(p(x) \vee q(x))$
 $\Rightarrow (\forall x p(x) \vee \forall x q(x))$

2017 IIT Roorkee

1.16 Let p, q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction.

Then, the expression $(r \rightarrow p) \rightarrow q$ is

- (A) a tautology
- (B) a contradiction
- (C) always TRUE when p is FALSE
- (D) always TRUE when q is TRUE



1.17 The statement $(\neg p) \Rightarrow (\neg q)$ is logically equivalent to which of the statement below?

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $(\neg q) \vee p$

IV. $(\neg p) \vee q$

(A) I only

(B) I and IV only

(C) II only

(D) II and III only

1.18 Let p, q, r denote the statements "It is raining", "It is cold" and "It is pleasant", respectively. Then the statement "It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold" is represented by

(A) $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

(B) $(\neg p \wedge r) \wedge ((p \wedge q) \rightarrow \neg r)$

(C) $(\neg p \wedge r) \vee ((p \wedge q) \rightarrow \neg r)$

(D) $(\neg p \wedge r) \vee (r \rightarrow (p \wedge q))$

1.19 Consider the first-order logic sentence $F : \forall x(\exists y R(x, y))$. Assuming non-empty logical domain, which of the sentences below are implied by F ?

I. $\exists y(\exists x R(x, y))$

II. $\exists y(\forall x R(x, y))$

III. $\forall y(\exists x R(x, y))$

IV. $\neg \exists x(\forall y \neg R(x, y))$

(A) IV only

(B) I and IV only

(C) II only

(D) II and III only

2018 IIT Guwahati

1.20 Consider the first-order logic sentence

$$\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \psi(s, t, u, v, w, x, y)$$

where $\psi(s, t, u, v, w, x, y)$ is a quantifier free first order logic formula using only predicate symbols, and possibly equality, but no function symbols. Suppose φ has a model with a universe containing 7 elements.

Which one of the following statements is necessarily true?

(A) There exists at least one model of φ with universe of size less than or equal to 3.

(B) There exists no model of φ with universe of size less than or equal to 3.

(C) There exists no model of φ with universe of size greater than 7.

(D) Every model of φ has a universe of size equal to 7.

2019 IIT Madras

1.21 Consider the first order predicate formula φ :

$$\forall x[(\forall z, z|x \Rightarrow ((z = x) \vee (z = 1)))$$

$$\Rightarrow \exists w(w > x) \wedge (\forall z, z|w \Rightarrow ((w = z) \vee (z = 1)))]$$

Here ' $a|b$ ' denotes that 'a divides b', where a and b are integers. Consider the following sets :

S1. {1, 2, 3, ..., 100}

S2. Set of all positive integers

S3. Set of all integers

Which of the above sets satisfy φ :

(A) S1 and S2

(B) S1 and S3

(C) S2 and S3

(D) S1, S2 and S3

2020 IIT Delhi

1.22 Which one of the following predicate formulae is NOT logically valid?

Note that W is a predicate formula without any free occurrence of x .

(A) $\forall x(p(x) \vee W) \equiv \forall x p(x) \vee W$

(B) $\exists x(p(x) \wedge W) \equiv \exists x p(x) \wedge W$

(C) $\forall x(p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$

(D) $\exists x(p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$

2021 IIT Bombay

1.23 Choose the correct choice(s) regarding the following propositional logic assertion S :

$$((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

(A) S is a contradiction

(B) S is a tautology

(C) The antecedent of S is logically equivalent to consequent of S

(D) S is neither a tautology nor a contradiction

1.24 Let p and q be two propositions. Consider the following two formulae in propositional logic.



$$S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$$

(B) Neither S_1 nor S_2 is a tautology.

$$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$$

(C) S_1 is not a tautology but S_2 is a tautology.

(A) Both S_1 and S_2 are tautologies.

(D) S_1 is a tautology but S_2 is not a tautology.

Solutions

1.1 (D)

Given : Following statement :

"None of my friends are perfect".

Sentence form for options are:

A: Some of my friends are perfect

B: Some x who are not my friend are perfect

C: Some x who are not my friend are not perfect

D: There does not exist x such that x is my friend and is perfect

\Rightarrow None of my friends are perfect.

Hence, the correct option is (D).

1.2 (A, D)

Given : Following well-formed formulae :

$$\neg \exists x (\forall y (\alpha) \wedge (\forall z (\beta)))$$

Key Point

$$\forall x (\alpha) \equiv \neg \exists (x) (\neg \alpha)$$

Consider option (A) and (D)

Option (A)

$$\forall x (\exists z (\neg \beta) \rightarrow \forall y (\alpha)) \equiv \forall x (\neg \exists z (\neg \beta) \vee \forall y (\alpha))$$

$$\forall x (\exists z (\neg \beta) \rightarrow \forall y (\alpha)) \equiv \forall x (\forall z (\beta) \vee \forall y (\alpha))$$

$$\forall x (\exists z (\neg \beta) \rightarrow \forall y (\alpha)) \equiv \neg \exists x (\neg \forall z (\beta) \wedge \neg \forall y (\alpha))$$

Option (D)

$$\forall x (\exists y (\neg \alpha) \rightarrow \exists z (\beta)) \equiv \forall x (\neg \exists y (\neg \alpha) \vee \exists z (\beta))$$

$$\forall x (\exists y (\neg \alpha) \rightarrow \exists z (\beta)) \equiv \neg \exists x (\forall y (\alpha) \vee \exists z (\beta))$$

$$\forall x (\exists y (\neg \alpha) \rightarrow \exists z (\beta)) \equiv \neg \exists x (\neg \forall y (\alpha) \wedge \forall z (\neg \beta))$$

A and D both are not logically equivalent to given statement.

Hence, the correct option is (A, D).

{In GATE 2013, marks were given to all}

1.3 (D)

Given : Following statement :

"not all rainy days cold"

Equivalent formula is

$$\neg \forall d (\text{Rainy}(d) \rightarrow \text{Cold}(d))$$

$$\equiv \neg \forall d (\neg \text{Rainy}(d) \vee \text{Cold}(d))$$

$$\equiv \exists d (\text{Rainy}(d) \wedge \neg \text{Cold}(d))$$

Hence, the correct option is (D).

1.4 (B)

Given :

Following well-formed formulae :

$$(A) ((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$$

$$(B) (a \leftrightarrow c) \rightarrow (\neg b \rightarrow (a \wedge c))$$

$$(C) (a \wedge b \wedge c) \rightarrow (c \vee a)$$

$$(D) a \rightarrow (b \rightarrow a)$$

Method 1

Key Point

One way to check for tautology is to check for contradiction i.e. check where the formula can result in FALSE.

Also, TRUE \rightarrow FALSE evaluates to FALSE

{Only condition which contradicts the implication}

Option (A): RHS: $(a \rightarrow c)$

Assume $a \rightarrow c = \text{FALSE}$,

$\therefore a = \text{TRUE}, c = \text{FALSE}$

LHS : $((a \rightarrow b) \wedge (b \rightarrow c))$

$$\equiv (\text{TRUE} \rightarrow b) \wedge (b \rightarrow \text{FALSE})$$

Now if $b = \text{TRUE}$ then $(b \rightarrow \text{FALSE})$ results in FALSE hence option (A) will result in FALSE \rightarrow FALSE = TRUE if $b = \text{FALSE}$ then $(\text{TRUE} \rightarrow b)$ results in FALSE

Hence option (A) will result in FALSE \rightarrow FALSE = TRUE \rightarrow hence tautology.

Option (B): RHS: $(\neg b \rightarrow (a \wedge c))$

Assume $(\neg b \rightarrow (a \wedge c)) = \text{FALSE}$,

$\therefore b = \text{FALSE}, a \wedge c = \text{FALSE}$

If a and c both are FALSE then $a \rightarrow c = \text{TRUE}$

Hence the option (B) will result in TRUE \rightarrow FALSE = FALSE which contradicts the condition of Tautology hence B is not a tautology.



Option C: RHS: $(c \vee a)$

Assume $c \vee a = \text{FALSE} \therefore a \text{ or } b \text{ or both} = \text{FALSE}$ hence $a \wedge b \wedge c = \text{FALSE}$, option (c) will result in FALSE $\rightarrow \text{FALSE} = \text{TRUE}$, hence tautology

Option (D): RHS : $(b \rightarrow a)$

Assume $(b \rightarrow a) = \text{FALSE} \therefore b = \text{TRUE}, a = \text{FALSE}$
Since $a = \text{FALSE}$, option (D) will result in FALSE $\rightarrow \text{FALSE} = \text{TRUE}$

Hence tautology

Method 2

Key Point

Simplify each expression and check the value if result is true (T) then it is tautology.

$$\begin{aligned}
 \text{A. } & ((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c) \\
 & \equiv ((\sim a \vee b) \wedge (\sim b \vee c)) \rightarrow (\sim a \vee c) \\
 & \equiv \sim ((\sim a \vee b) \wedge (\sim b \vee c) \vee (\sim a \vee c)) \\
 & \equiv ((a \wedge \sim b) \vee (b \wedge \sim c)) \vee (\sim a \vee c) \\
 & \equiv ((\sim a \vee b) \vee (b \wedge \sim c) \vee (\sim a \vee c)) \\
 & \equiv \sim a \vee ((a \wedge b) \vee (b \wedge \sim c) \vee c) \\
 & \equiv (\sim a \vee a) \wedge (\sim a \wedge \sim b) \vee (\sim b \vee c) \wedge (\sim c \vee c) \\
 & \equiv (T \wedge (\sim a \vee \sim b)) \vee ((b \vee c) \wedge T) \\
 & \equiv (\sim a \vee \sim b) \vee (b \vee c) \\
 & \equiv \sim a \vee T \vee c \equiv T \\
 \text{B. } & (a \rightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c)) \\
 & \equiv \sim (\sim a \vee c) \vee (b \vee (a \wedge c)) \\
 & \equiv (a \wedge c) \vee ((b \vee a) \wedge (b \vee c)) \\
 & \equiv a \vee b \\
 \text{C. } & (a \wedge b \wedge c) \rightarrow (c \vee a) \\
 & \equiv \sim (a \wedge b \wedge c) \vee (a \vee a) \\
 & \equiv \sim a \vee b \vee \sim c \vee c \vee a \\
 & \equiv T \vee \sim b \vee T \\
 & \equiv T \\
 \text{D. } & a \rightarrow (b \rightarrow a) \\
 & \equiv \sim a \vee (\sim b \vee a) \\
 & \equiv T \vee b \equiv T
 \end{aligned}$$

Method 3: Truth Table method-

a	b	c	Option (A)	Option (B)	Option (C)	Option (D)
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T

T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	F	T	T	T
F	F	F	T	F	T	T	T

All result in tautology except option (B)

Hence, the correct option is (B).

1.5 (D)

Given :

Following statements :

P : Good mobile phones are not cheap

Q : Cheap mobile phones are not good

L : P implies Q

M : Q implies Q

N : P is equivalent to Q

Let

A: Good mobile phones

B: Cheap mobile phones

P : $(A \rightarrow \neg B) \equiv (\neg A \vee \neg B)$

Q : $(B \rightarrow \neg A) \equiv (\neg B \vee \neg A)$

Since disjunction (\vee) is commutative P is equivalent to Q i.e. $(P \rightarrow Q)$ and $(Q \rightarrow P)$

Hence, the correct option is (D).

1.6 (D)

Given : Following statement :

"Not all that glitters is gold"

glitters(x) : x glitters

gold(x) : x is gold

Not all that glitters is gold means some glitters are not gold which can be expressed as

$\exists x(\text{glitters}(x) \wedge \neg \text{gold}(x))$

Hence, the correct option is (D).

1.7 (B)

Given : exactly two of p, q and r are TRUE

p	q	r	Output(f)
T	T	T	F
T	T	F	T
T	F	T	T



T	F	F	F
F	T	T	T
F	T	F	F
F	T	F	F
F	F	T	F
F	F	F	F

$$\begin{aligned}
 f &= (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge r) + (\neg p \wedge q \wedge r) \\
 &= (p \wedge q \wedge \neg r) \vee \{(p \wedge \neg q) + (\neg p \wedge q)\} \wedge r \\
 &= (p \wedge q \wedge \neg r) \vee (\neg(p \leftrightarrow q)) \wedge r
 \end{aligned}$$

Hence, the correct option is (B).

1.8 (C)

Given :

Following well-formed formulae :

- (A) $\forall x \exists y R(x, y) \leftrightarrow \exists y \forall x R(x, y)$
- (B) $(\forall x [\exists y R(x, y) \rightarrow S(x, y)]) \rightarrow \forall x \exists y S(x, y)$
- (C) $[\forall x \exists y (P(x, y) \rightarrow R(x, y))] \leftrightarrow [\forall x \exists y (\neg P(x, y) \vee R(x, y))]$
- (D) $[\forall x \exists y (P(x, y) \rightarrow \forall x \forall y P(y, x))]$

Since in general $P \rightarrow R \equiv \neg P \vee R$

Hence $[\forall x \exists y (P(x, y) \rightarrow R(x, y))] \rightarrow [\forall x \exists y (\neg P(x, y) \vee R(x, y))]$

Hence, the correct option is (C).

1.9 (A)

Given :

Type 1 people always tell the truth

Type 2 people always lie

Statement :

"The result of the toss is head if and only if I am telling the truth"

If the person is of type 1, then

"The result of the toss is head if and only if I am telling the truth"

This sentence is true so result is head

If the person is of type 2, then

The sentence is false, either $T \leftrightarrow F$ is F or $F \leftrightarrow T$ is F since the statement "I am telling the truth" is false hence "the result is head" is true.

Hence, the correct option is (A).

1.10 (C)

Given :

Following statements :

S_1 : If a candidate is known to be corrupt, then he will not be elected.

S_2 : If a candidate is kind, he will be elected

Let P : person is corrupt

Q : person is kind

R : person is elected

$S_1 \Rightarrow P \rightarrow \neg R = \neg P \vee \neg R$

$S_2 \Rightarrow Q \rightarrow R = \neg Q \vee R$

On using resolution principle $\neg R$ and R cancel out each other hence conclusion $\neg P \vee \neg Q$

$$\equiv Q \rightarrow \neg P$$

Which means, if a person is kind then he is not known to be corrupt.

Hence, the correct option is (C).

1.11 (C)

Given : Following well-formed formula :

$$p \leftrightarrow q$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (q \rightarrow p) \quad \{\text{option B}\}$$

$$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p) \quad \{\text{option A}\}$$

$$p \leftrightarrow q \equiv (\neg p \wedge \neg q) \vee (p \wedge q) \quad \{\text{option D}\}$$

Hence, the correct option is (C).

1.12 (A)

Given :

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Given operator performs XOR operation according to the Truth Table Hence it is both commutative and Associative

Hence, the correct option is (A).

1.13 4

Given :



Following expressions:

- (i) false
- (ii) Q
- (iii) True
- (iv) $P \vee Q$
- (v) $\neg Q \vee P$

Truth table for the expression $P \wedge (P \Rightarrow Q)$ is

P	Q	$P \wedge (P \Rightarrow Q)$
T	T	T
T	F	F
F	T	F
F	F	F

For notational convenience assume
 $A = P \wedge (P \Rightarrow Q)$

So, for option (i), (ii), (iii), (iv), (v)

P	Q	A	$A \Rightarrow F$	$A \Rightarrow Q$	$A \Rightarrow T$	$A \Rightarrow (P \vee Q)$	$A \Rightarrow (\neg Q \vee P)$
T	T	T	F	T	T	T	T
T	F	F	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	F	T	T	T	T	T

If $(A \Rightarrow \text{Option } x)$ is a tautology.

Then $P \wedge (P \Rightarrow Q)$ logically implies option x.

Hence option (ii), (iii), (iv) and (v) are logically implied by $P \vee (P \Rightarrow Q)$

Hence, the correct answer is 4.

1.14 11

Given :

$p : x \in \{8, 9, 10, 11, 12\}$

$q : x$ is a composite number

$r : x$ is a perfect square

$s : x$ is a prime number

Well-formed formulae :

$$\begin{aligned} & \neg((p \rightarrow q) \wedge (\neg r \vee \neg s)) \\ & \equiv \neg((\neg p \vee q) \wedge (\neg r \vee \neg s)) \\ & \equiv (p \wedge \neg q) \vee (r \wedge s) \end{aligned}$$

It says that, $\{x \in \{8, 9, 10, 11, 12\}\}$ and x is not a composite number Or (x is a perfect square and x is a prime number)

Since x can never be a perfect square and a prime number both hence this condition is false therefore first condition must be true.

$\{x \in \{8, 9, 10, 11, 12\}\}$ and x is not a composite number

$$\Rightarrow x = 11$$

Hence, the correct answer is 11.

1.15 (D)

Given :

Following well-formed formulae :

$$(A) (\forall x p(x) \Rightarrow \forall x q(x)) \Rightarrow (\exists x \neg p(x) \vee \forall x q(x))$$

$$(B) (\exists x p(x) \vee \exists x q(x)) \Rightarrow \exists x (p(x) \vee q(x))$$

$$(C) \exists x (p(x) \wedge q(x)) \Rightarrow (\exists x p(x) \wedge \exists x q(x))$$

$$(D) \forall x (p(x) \vee q(x)) \Rightarrow (\forall x p(x) \vee \forall x q(x))$$

$\forall x$ is only one way distributive over \vee

$$\forall x (p(x) \vee q(x)) \Rightarrow (\forall x p(x) \vee \forall x q(x))$$

Let LHS be true, so we have

$$p_1 \vee q_1 \text{ (true)}$$

$$p_2 \vee q_2 \text{ (true)}$$

Now take p_1 is true and q_1 is false and p_2 is false and q_2 is true

Now LHS is true, but RHS, $\forall x p(x)$ is false ($\therefore p_2$ is false) and $\forall x q(x)$ is also false ($\therefore q_2$ is false)

So, LHS \neq RHS

Hence, the correct option is (D).

1.16 (D)

Given :

expression $(p \rightarrow q) \rightarrow r$ is a contradiction

$$\therefore (p \rightarrow q) \rightarrow r = \text{FALSE}$$

$$\text{i.e. } p \rightarrow q = \text{TRUE } r = \text{FALSE}$$

Now $(r \rightarrow p) \rightarrow q$ can be written as $\neg(r \rightarrow p) \vee q$

Which is TRUE when either $\neg(r \rightarrow p)$ or q is TRUE since $r = \text{FALSE}$, $\neg(r \rightarrow p) = \text{FALSE}$ $(r \rightarrow p) \rightarrow q$ is

TRUE when $q = \text{TRUE}$.

Hence, the correct option is (D).

1.17 (D)

Given : Following well-formed formula :

$$(\neg p) \Rightarrow (\neg q)$$

Statements :

$$\text{I. } p \Rightarrow q$$

$$\text{II. } q \Rightarrow p$$

$$\text{III. } (\neg q) \vee p$$

$$\text{IV. } (\neg p) \vee q$$



- $(\neg p) \Rightarrow (\neg q) \equiv \neg(\neg p) \vee (\neg q)$
- $(\neg p) \Rightarrow (\neg q) \equiv p \vee (\neg q)$
- $(\neg p) \Rightarrow (\neg q) \equiv (\neg q) \vee p$
- $(\neg p) \Rightarrow (\neg q) \equiv q \Rightarrow p$

Hence, the correct option is (D).

1.18 (A)

Given : p : "It is raining",

q : "It is cold"

r : "It is pleasant"

Following statement :

"It is not raining and it is pleasant, and it is not pleasant only if it is raining and it is cold"

$$\underbrace{\text{it is not raining}}_{\neg p} \text{ and } \underbrace{\text{it is pleasant}}_{r} \dots (i)$$

$$\underbrace{\text{it is not raining}}_{\neg r} \text{ only if } \underbrace{\text{it is raining and it is cold}}_{p \wedge q} \dots (ii)$$

i.e. from (i) and (ii)

$$(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$$

Hence, the correct option is (A).

1.19 (B)

Given : $F : \forall x (\exists y R(x, y))$

Following statements :

- I. $\exists y (\exists x R(x, y))$
- II. $\exists y (\forall x R(x, y))$
- III. $\forall y (\exists x R(x, y))$
- IV. $\neg \exists x (\forall y \neg R(x, y))$

Let us assume:

$R(x, y)$ is y is boyfriend of x

So, $F : \forall x (\exists y R(x, y))$ = for all girls there exist a boyfriend

- I. There exist some boys who have girlfriends (TRUE)
- II. There exists some boys for whom all the girls are girlfriend (FALSE)
- III. For all boys there exists a girlfriend (FALSE)
- IV. For all girls, there exists a boyfriend (TRUE)

Hence, I and IV are implied by F.

Hence, the correct option is (B).

1.20 (B)

Given : Following first-order logic sentence

$$\varphi \equiv \exists s \exists t \exists u \forall v \forall w \forall x \forall y \psi(s, t, u, v, w, x, y)$$

where $\psi(s, t, u, v, w, x, y)$ is a quantifier free first order logic formula using only predicate symbols, and possibly equality, but no function symbols.

φ has a model with a universe containing 7 elements.

Key Point

' \exists ' there exist quantifier decides whether a sentence belong to the model or not.

- (1) We have 7 elements in the universe, so max size of universe in a model = 7
- (2) There are 3 ' \exists ' quantifiers, so at least 3 elements in model.
- (A) Is false because of (2)
- (B) Is true
- (C) Is false because of (1)
- (D) Is false since size of models ranges from 3 to 7.

Hence, the correct option is (B).

1.21 (C)

Given : first order predicate formula φ :

$$\begin{aligned} \forall x [(\forall z, z|x \Rightarrow ((z=x) \vee (z=1))) \\ \Rightarrow \exists w (w>x) \wedge (\forall z, z|w \Rightarrow ((w=z) \vee (z=1)))] \end{aligned}$$

Here ' $a|b$ ' denotes that ' a divides b ', where a and b are integers

Following sets :

- S1. {1, 2, 3, ..., 100}
- S2. Set of all positive integers
- S3. Set of all integers

φ : If z is a prime number then there exists another prime number in the set which is larger than it.

Now, if -7 is a number which is prime (either divided by -7 or 1 only). Then there exists some number like -3 which is larger than -7 also satisfy the property

So, S_3 is correct hence S_2 is also correct.

Hence, the correct option is (C).

1.22 (C)

Given : Following well-formed formulae :

- (A) $\forall x (p(x) \vee W) \equiv \forall x p(x) \vee W$
- (B) $\exists x (p(x) \wedge W) \equiv \exists x p(x) \wedge W$
- (C) $\forall x (p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$
- (D) $\exists x (p(x) \rightarrow W) \equiv \forall x p(x) \rightarrow W$

Consider option (C)



$$\forall x(p(x) \rightarrow w) \equiv \forall x(\neg p(x) \vee w)$$

$$\forall x(p(x) \rightarrow w) \equiv (\neg \exists x p(x)) \vee w$$

$$\forall x(p(x) \rightarrow w) \equiv \exists x p(x) \rightarrow w$$

Hence, the correct option is (C).

1.23 (B, C)

Given :

Following propositional logic assertion S :

$$((P \wedge Q) \rightarrow R) \rightarrow ((P \wedge Q) \rightarrow (Q \rightarrow R))$$

Antecedent (X) :

$$(\overline{PQ}) + R \Rightarrow \overline{P} + \overline{Q} + R \quad \dots(i)$$

Consequent(Y):

$$((\overline{PQ}) + (\overline{Q} + R)) \Rightarrow (\overline{P} + \overline{Q}) + \overline{Q} + R$$

$$\Rightarrow \overline{P} + \overline{Q} + \overline{Q} + R \Rightarrow \overline{P} + \overline{Q} + R \quad \dots(ii)$$

Because Antecedent and Consequent are returning same expression, therefore $X \rightarrow Y$, will be Tautology because X and Y are coming out to be same.
For example

$$A \rightarrow A \Rightarrow \overline{A} + A \Rightarrow 1$$

Hence, the correct options are (B, C).

1.24 (C)

Given : Following well-formed formulae :

$$S_1 : (\neg p \wedge (p \vee q)) \rightarrow q$$

$$S_2 : q \rightarrow (\neg p \wedge (p \vee q))$$

Method 1

S_1 can be written as : $(p' (p+q))' + q$

$$\text{Simplifying } S_1 : (p + (p+q)') + q$$

$$= (p + (p'q')) + q$$

$$= (p + p')(p + q') + q$$

$$= p + q' + q$$

$$= 1$$

Hence S_1 is tautology.

S_2 can be written as : $q' + (p'(p+q))$

Simplifying S_2 :

$$q' + (pp' + p'q)$$

$$q' + (p'q)$$

$$q' + p'$$

Which is not always 1. Hence S_2 is not tautology.

Hence, the correct option is (C).

Method 2

$$S_1 : \sim p \wedge (p \vee q) \rightarrow q$$

If consequence is false and hypothesis is true, then we will get False in the truth table. Let assume q is false, so consequence is FALSE. can it make Hypothesis TRUE ?

Hypothesis :

$$\sim p \wedge (p \vee q) \equiv \sim p \wedge (p \vee \text{False})$$

$$\equiv \sim p \wedge (p) \equiv (\text{False})$$

Hypothesis can't be true, so we can't get False in the Truth Table.

$\therefore S_1$ is Tautology.

$S_2 : q \rightarrow \sim p \wedge (p \vee q)$: if hypothesis is true and consequence is false, then we will get False in the truth table. let assume q is True, So Hypothesis is TRUE. can it make Consequence FALSE ?

Consequence :

$$\sim p \wedge (p \vee q) \equiv \sim p \wedge (p \vee \text{TRUE})$$

$$\equiv \sim p \wedge (\text{TRUE}) \equiv \sim p$$

p can be true, so we can get False in the Truth Table.

$\therefore S_2$ is not Tautology.

Hence, the correct option is (C).



Practice Questions

2013 IIT Bombay

- 2.1** A Binary operation \oplus on a set of integers is defined as $x \oplus y = x^2 + y^2$. Which one of the following statements is TRUE about \oplus ?
- Commutative but not associative
 - Both commutative and associative
 - Associative but not Commutative
 - Neither commutative nor associative

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- 2.2** Let X and Y be finite sets and $f : X \rightarrow Y$ be a function. Which one of the following statements TRUE?
- For any subsets A and B of X , $|f(A \cup B)| = |f(A)| + |f(B)|$
 - For any subset A and B of X , $f(A \cap B) = f(A) \cap f(B)$
 - For any subset A and B of X , $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$
 - For any subsets S and T of Y , $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

- 2.3** Consider the set of all functions $f : \{0, 1, \dots, 2014\} \rightarrow \{0, 1, \dots, 2014\}$ such that $f(f(i)) = i$, for all $0 \leq i \leq 2014$. Consider the following statements :

- P : For each such function, it must be the case that for every i , $f(i) = i$.
- Q : For each such function, it must be the case that for some i , $f(i) = i$,

- R : Each such function must be onto.

Which one of the following is CORRECT?

- P, Q and R are true
- Only Q and R are true
- Only P and Q are true
- Only R is true

- 2.4** There are two elements x, y in a group $(G, *)$ such that every in the group can be written as a

product of some number of x 's and y 's in some order. It is known that

$$x * x = y * y = x * y * x * y = y * x * y * x = e$$

Where e is the identity element. The maximum number of elements in such a group is _____.

- 2.5** Consider the following relation on subset of the set S integers between 1 and 2014. For two distinct subsets U and V of S , we say $U < V$ if the minimum element in the symmetric difference of the two sets is in U . Consider the following two statements :

S1 : There is a subset of S that is larger than every other subset.

S2 : There is a subset of S that is smaller than every other subset.

Which one of the following is CORRECT?

- Both S1 and S2 are true
- S1 is true and S2 is false
- S2 is true and S1 is false
- Neither S1 nor S2 is true

- 2.6** Let S denote the set of all functions $f : \{0, 1\}^4 \rightarrow \{0, 1\}$. Denote by N the number of functions from S to the set $\{0, 1\}$. The value of $\log_2 \log_2 N$ is _____.

- 2.7** Let G be a group with 15 elements. Let L be a subgroup of G . It is known that $L \neq G$ and that the size of L is at least 4. The size of L is _____.

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- 2.8** Let $\#$ be the binary operator defined as $X \# Y = X' + Y'$ where X and Y are Boolean variables. Consider the following two statements.

$$(S1) (P \# Q) \# R = P \# (Q \# R)$$

$$(S2) Q \# R = (R \# Q)$$

Which are the following is/are true for the Boolean variables P, Q and R ?

- Only S1 is true
- Only S2 is true
- Both S1 and S2 are true
- Neither S1 nor S2 are true

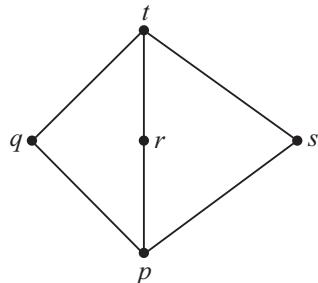


- 2.9** Suppose U is the power set of the set $S = \{1, 2, 3, 4, 5, 6\}$. For any $T \in U$, let $|T|$ denote the number of elements in T and T' denote the complement of T . For any $T, R \in U$, let T / R be the set of all elements in T which are not in R . Which one of the following is true?
- (A) $\forall X \in U (|X| = |X'|)$
- (B) $\exists X \in U \exists Y \in U \left(\begin{array}{l} |X| = 5, |Y| \\ = 5 \text{ and } X \cap Y = \emptyset \end{array} \right)$
- (C) $\forall X \in U \forall Y \in U \left(\begin{array}{l} |X| = 2, |Y| \\ = 3 \text{ and } X / Y = \emptyset \end{array} \right)$
- (D) $\forall X \in U \forall Y \in U (X / Y = Y' / X')$
- 2.10** Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R ?
- (A) Both reflexive and symmetric
- (B) Reflexive but not symmetric
- (C) Not reflexive but symmetric
- (D) Neither reflexive nor symmetric
- 2.11** The cardinality of the power set of $\{0, 1, 2, \dots, 10\}$ is ____.
- 2.12** If p, q, r, s are distinct integers such that:
- $$f(p, q, r, s) = \max(p, q, r, s)$$
- $$g(p, q, r, s) = \min(p, q, r, s)$$
- $$h(p, q, r, s) = \text{remainder of } \frac{(p \times q)}{(r \times s)} \text{ if}$$
- $$(p \times q) > (r \times s) \text{ or remainder of } \frac{(r \times s)}{(p \times q)} \text{ if}$$
- $$(r \times s) > (p \times q)$$
- Also, a function $fgh(p, q, r, s) =$
- $$f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$$
- Also, the same operations are valid with two variable functions of the form $f(p, q)$.
- What is the value of $fgh(h(2, 5, 7, 3), 4, 6, 8)$?
- 2.13** Let R be the relation on the set of positive integers such that aRb if and only if a and b distinct and have a common divisor other than 1.
- Which one of the following statements about R is true?
- (A) R is symmetric and reflexive but not transitive
- (B) R is reflexive but not symmetric and not transitive
- (C) R is symmetric but not reflexive and not symmetric
- (D) R is symmetric but not reflexive and not transitive
- 2.14** The number of onto functions (surjective) from set $X = \{1, 2, 3, 4, \}$ to set $Y = \{a, b, c\}$ is ____.
- 2.15** Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denote the set of all possible functions defined from X to Y . Let f be randomly chosen from F . The probability of f being one-to-one is ____.
- 2.16** If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$, then $\frac{g(h(x))}{h(g(x))}$ is :
- (A) $\frac{h(x)}{g(x)}$ (B) $\frac{-1}{x}$
- (C) $\frac{g(x)}{h(x)}$ (D) $\frac{x}{(1-x)^2}$
- 2.17** The binary operator \neq is defined by the following truth table.
- | p | q | $p \neq q$ |
|-----|-----|------------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
- Which one of the following is true about the binary operator \neq ?
- (A) Both commutative and associative
- (B) Commutative but not associative
- (C) Not commutative but associative
- (D) Neither commutative nor associative
- 2.18** For a set A , the power set of A is denoted by 2^A . If $A = \{5, \{6\}, \{7\}\}$, which of the following options are TRUE?
- I. $\phi \in 2^A$ II. $\phi \subseteq 2^A$
- III. $\{5, \{6\}\} \in 2^A$ IV. $\{5, \{6\}\} \subseteq 2^A$



- (A) I and III only
 - (B) II and III only
 - (C) I, II and III only
 - (D) I, II and IV only

2.19 Suppose $L = \{p, q, r, s, t\}$ is a lattice represented by the following Hasse diagram:



For any $x, y \in L$, not necessarily distinct, $x \vee y$ and $x \wedge y$ are join and meet of x, y respectively. Let

$L^3 = \{(x, y, z) : x, y, z \in L\}$ be the set of all ordered triplets of the elements of L . Let P_r be the probability that an element $(x, y, z) \in L^3$ chosen equiprobably satisfies $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Then

- (A) $P_r = 0$
 (B) $P_r = 1$
 (C) $0 < P_r \leq 1/5$
 (D) $1/5 < P_r < 1$

2.20 Let R be a relation on the set of ordered pairs of positive integers such that $((p,q),(r,s)) \in R$ if and only if $p-s = q-r$. Which one of the following is true about R ?

- (A) Both reflexive and symmetric
- (B) Reflexive but not symmetric
- (C) Not reflexive but symmetric
- (D) Neither reflexive nor symmetric

2016 IISc Bangalore

2.21 A binary relation R on $N \times N$ is defined as follows : $(a,b)R(c,d)$ if $a \leq c$ or $b \leq d$. Consider the following propositions :

$P : R$ us reflexive
 $Q : R$ is transitive

- Which one of the following statements is TRUE?

- (A) Both P and Q are true.
 - (B) P is true and Q is false.
 - (C) P is false and Q is true.
 - (D) Both P and Q are false.

2.22 Consider a set U of 23 different compounds in a Chemistry lab. There is a subset S of U of compounds, each of which reacts with exactly 3 compounds of U .

Consider the following statements :

- I. Each compound in U/S reacts with an odd number of compounds.
 - II. At least one compound in U/S reacts with an odd number of compounds.
 - III. Each compound in U/S reacts with an even number of compounds.

Which one of the above statements is ALWAYS TRUE?

2.23 A function $f : N^+ \rightarrow N^+$, defined on the set of positive integers N^+ , satisfies the following properties :

$$f(n) = f(n/2) \text{ if } n \text{ is even}$$

$$f(n) = f(n+5) \text{ if } n \text{ is odd}$$

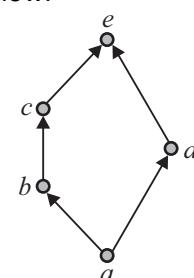
Let $R = \{i \mid \exists j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is _____.

2017 IIT Roorkee

2.24 Consider the set $X = \{a, b, c, d, e\}$ under the partial ordering

$$R = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (b,c), (b,e), (c,c), (c,e), (d,d), (d,e), (e,e)\}$$

The Hasse diagram of the partial order (X, R) is shown below.





The minimum number of ordered pairs that need to be added to R to make (X, R) a lattice is _____.

- 2.25** The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is _____.

2018 IIT Guwahati

- 2.26** Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is _____.

- 2.27** Let N be the set of natural numbers. Consider the following sets.

P : Set of Rational numbers (positive and negative)

Q : Set of functions from $\{0, 1\}$ to N

R : Set of functions from N to $\{0, 1\}$

S : Set of finite subsets of N .

Which of the sets above are countable?

(A) Q and S only

(B) P and S only

(C) P and R only

(D) P, Q and S only

2019 IIT Madras

- 2.28** Let G be an arbitrary group. Consider the following relations on G

$R_1 : \forall a, b \in G, a R_1 b$ if and only if $\exists g \in G$ such that $a = g^{-1}bg$

$R_2 : \forall a, b \in G, a R_2 b$ if and only if $a = b^{-1}$

Which of the above is/are equivalence relation/relations?

(A) R_1 and R_2

(B) R_1 only

(C) R_2 only

(D) Neither R_1 and R_2

- 2.29** Let $U = \{1, 2, \dots, n\}$. Let

$$A = \{(x, X) | x \in X, X \subseteq U\}.$$

Consider the following two statements on $|A|$.

I. $|A| = n2^{n-1}$

II. $|A| = \sum_{k=1}^n k \binom{n}{k}$

Which of the above statements is/are TRUE?

- (A) Only I
- (B) Only II
- (C) Both I and II
- (D) Neither I or II

2020 IIT Delhi

- 2.30** Let G be a group of 35 elements. Then the largest possible size of a subgroup of G other than G itself is _____.

- 2.31** Let \mathcal{R} be the set of all binary relations on the set $\{1, 2, 3\}$. Suppose a relation is chosen from \mathcal{R} at random. The probability that the chosen relation is reflexive (round off to 3 decimal places) is _____.

2021 IIT Bombay

- 2.32** Let S be a set consisting of 10 elements. The number of tuples of the form (A, B) such that A and B are subsets of S , and $A \subseteq B$ is _____.

- 2.33** A relation R is said to be circular if aRb and bRc together imply cRa . Which of the following options is/are correct?

- (A) If a relation S is transitive and circular, then S is an equivalence relation.
- (B) If a relation S is reflexive and circular, then S is an equivalence relation.
- (C) If a relation S is circular and symmetric, then S is an equivalence relation.
- (D) If a relation S is reflexive and symmetric, then S is an equivalence relation.

- 2.34** Let G be a group of order 6, and H be a subgroup of G such that $1 < |H| < 6$. Which one of the following options is correct?

- (A) Both G and H may not be cyclic.
- (B) Both G and H are always cyclic.
- (C) G is always cyclic, but H may not be cyclic.
- (D) G may not be cyclic, but H is always cyclic.

- 2.35** Consider the following sets, where $n \geq 2$

S_1 : Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$

S_2 : Set of all functions from the set $\{0, 1, 2, \dots, n^2 - 1\}$ to the set $\{0, 1, 2\}$

Which of the following is possible?

- (A) There does not exist an injection from S_1 to S_2 .



- (B) There exists a surjection from S₁ to S₂.
 (C) There does not exist a bijection from S₁ to S₂.

- (D) There exists a bijection from S₁ to S₂.

Solutions

2.1 (A)

Given : A Binary operation \oplus on a set of integers is defined as $x \oplus y = x^2 + y^2$.

$$x \oplus y = x^2 + y^2$$

$$y \oplus x = y^2 + x^2$$

$x \oplus y = y \oplus x$, hence commutative

$$(x \oplus y) \oplus (z) = (x^2 + y^2)^2 + z^2$$

$$x \oplus (y \oplus z) = x^2 + (y^2 + z^2)^2$$

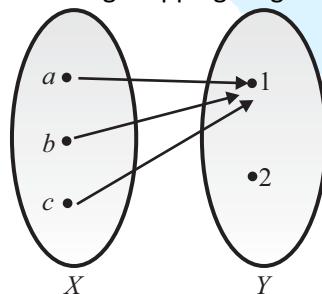
$$(x \oplus y) \oplus z \neq x \oplus (y \oplus z),$$

hence not associative

Hence, the correct option is (A).

2.2 (D)

Given : X and Y are finite sets and f is a function. Consider the following mapping diagram



Let $A = \{a\}$, $B = \{b, c\}$

$$|f(A \cup B)| = |f(a, b, c)| = |\{1\}| = 1$$

$$|f(A)| + |f(B)| = 1 + 1 = 2$$

Hence (A) is false.

$$f(A \cap B) = \{\} \text{ or } \phi$$

$$f(A) \cap f(B) = \{1\} \cap \{1\} = 1$$

Hence (B) is false.

$$|f(A \cap B)| = |\phi| = 0$$

$$\min \{|f(A)|, |f(B)|\} = \min \{1, 1\} = 1$$

Hence (C) is false.

Here we can conclude that option (D) is true

Hence, the correct option is (D).

2.3 (B)

Given : $\forall i \quad f(f(i)) = i$

$$\Rightarrow f = f^{-1}$$

For any $i \neq j$ we can map $f(i) = j$ and $f(j) = i$, but the no. of elements is odd hence at least one element will map to itself $\Rightarrow f(i) = i$, hence Q is true.

Due to the condition $f(f(i)) = i$, $f(i)$ cannot be same for two different i .

All $f(i)$ values are unique hence

co-domain = range

Function must be onto hence R is true.

Hence, the correct option is (B).

2.4 4

Given :

$$x * x = e \Rightarrow x \text{ is its own inverse}$$

$$y * y = e \Rightarrow y \text{ is its own inverse}$$

$$(x * y) * (x * y) = e \Rightarrow (x * y)^{-1} = (x * y)$$

$$(y * x) * (y * x) = e \Rightarrow (y * x)^{-1} = (y * x)$$

Now $x * y = x * e * y$

$$= x * (x * y * x * y)^* y$$

$$= (x * x)^* y * x * (y * y)$$

$$= e^* y * x * e$$

$$= y * x$$

$$x * y = y * x$$

It is commutative hence the elements are $\{x, y, x^* y, e\}$

Hence, the correct answer is 4.

2.5 (A)

Given :

$$S = \{1, 2, 3, 4, \dots, 2014\}$$

Also, for $U \subseteq S$ and $V \subseteq S$, $U < V$ if the minimum element in the symmetric difference of U and V is in U.



For $\phi \subseteq S$ and $V \subseteq S$

Symmetric difference is V only hence minimum element is in $V \Rightarrow S1$ is true.

ϕ is larger than every subset.

For $S \subseteq S$ and $V \subseteq S$

Symmetric difference will contain minimum element which is already in $S \Rightarrow S < V$

S is smaller than every subset

$S2$ is true.

Hence, the correct option is (A).

2.6 16

Given : $S = \{f : \{0, 1\}^4 \rightarrow \{0, 1\}\}$

$$|S| = 2^{2^4} = 2^{16}$$

N = number of functions from S to $\{0, 1\}$

$$N = 2^{2^{16}}$$

$$\begin{aligned} \log_2 \log_2 N &= \log_2 \log_2 2^{2^{16}} \\ &= \log_2 2^{16} \\ &= 16 \end{aligned}$$

Hence, the correct answer is 16.

2.7 5

Key Point

For any finite group G , the order of subgroup of G divides the order of group.

Given :

Order of $G = 15$

Factors of $15 = 1, 3, 5$ and 15

Since order of $1 \geq 4$ hence size of $L = 5$

Hence, the correct answer is 5.

2.8 (B)

Given : $X \# Y = X' + Y'$

$$S1 : (P \# Q) \# R = P \# (Q \# R)$$

$$(P' + Q')' + R' = P' + (Q' + R')$$

$$PQ + R' \neq P' + QR$$

Hence S1 is not true

$$S2 : Q \# R = R \# Q$$

$$Q' + R' = R' + Q'$$

Hence S2 is true.

Hence, the correct option is (B).

2.9 (D)

Given : $S = \{1, 2, 3, 4, 5, 6\}$

$$U = P(S)$$

Let $X = \{1, 2\}$

$$X' = \{3, 4, 5, 6\}$$

$$|X| = 2, |X'| = 4$$

Hence, option (A) is false.

Since there are only six elements in S therefore any subset of S with cardinality 5 will have at least 4 element common, hence $X \cap Y \neq \emptyset$.

So, option (B) is false.

Let $X = \{1, 2\}, Y = \{3, 4, 5\}$

$$\frac{X}{Y} = \{1, 2\} \neq \emptyset$$

Hence option (C) is false.

Here we can conclude that option (D) is correct.

Hence, the correct option is (D).

2.10 (C)

Given : $((p, q), (r, s)) \in R$ iff $p - s = q - r$

For reflexive $((a, b), (a, b)) \in R \quad \forall a, b \in z$

But $a - b \neq b - a$

Hence $((a, b), (a, b)) \notin R$

Hence not reflexive.

For symmetric

$$((a, b), (c, d)) \in R$$

$$\Rightarrow ((c, d), (a, b)) \in R$$

$$\Rightarrow a - d = b - c$$

$$\Rightarrow -(d - a) = -(c - b)$$

$$\Rightarrow d - a = c - b$$

$$\Rightarrow ((c, d), (a, b)) \in R$$

Hence symmetric

Hence, the correct option is (C).

2.11 2048

Key Point

If $|A| = n$ then $|P(A)| = 2^n$

Given : set is $\{0, 1, 2, \dots, 10\}$

Cardinality = 11



Cardinality of power set = $2^{11} = 2048$

Hence, the correct answer is 2048.

2.12 8

Given :

$$f(p, q, r, s) = \max(p, q, r, s)$$

$$g(p, q, r, s) = \min(p, q, r, s)$$

$$h(p, q, r, s) = \text{remainder of } \frac{(p \times q)}{(r \times s)}$$

$$\text{if } (p \times q) > (r \times s)$$

Or

$$\text{remainder of } \frac{(r \times s)}{(p \times q)}$$

$$\text{if } (r \times s) > (p \times q)$$

$$\text{Now } h(2, 5, 7, 3) = \text{remainder of } \frac{(7 \times 3)}{(2 \times 5)} = 1$$

$$fg(p, q, r, s) = f(p, q, r, s) \times g(p, q, r, s)$$

$$fg(1, 4, 6, 8) = f(1, 4, 6, 8) \times g(1, 4, 6, 8)$$

$$= 8 \times 1 = 8$$

Hence, the correct answer is 8.

2.13 (D)

Given : In relation R, aRb iff a, b are distinct and have a common divisor other than 1.

For reflexive: $(a, a) \in R \quad \forall a \in Z^+$

But a, a are same and not distinct hence $(a, a) \notin R$

Hence R is not reflexive

For symmetric: $(a, b) \in R \Rightarrow (b, a) \in R$

Let a, b are distinct

Let $a = 3, b = 6 \Rightarrow (3, 6) \in R$

Also 6, 3 are distinct and have only one common divisor other than 1 $\Rightarrow (6, 3) \in R$

Hence R is symmetric

For transitive, $(a, b) \in R$ and

$$(b, c) \in R \Rightarrow (a, c) \in R$$

Let $a = 3, b = 6, c = 2$

$$(3, 6) \in R \text{ and } (6, 2) \in R$$

But $(3, 2) \in R$ since $(3, 2)$ does not have a common divisor other than 1.

Hence R is not transitive.

Hence, the correct option is (D).

2.14 36

Given :

$$X = \{1, 2, 3, 4, \}$$

$$Y = \{a, b, c\}$$

Key Point

If $|A| = m$ and $|B| = n$ then $f : A \rightarrow B$

No. of onto function

$$= n^m - {}^n C_1 (n-1)^m + {}^n C_2 (n-2)^m - \\ \dots + {}^n C_n (n-n)^m$$

Here $m = 4$ and $n = 3$

Hence number of onto functions

$$= 3^4 - {}^3 C_1 (3-1)^4 + {}^3 C_2 (3-2)^4 - {}^3 C_3 (3-3)^4$$

$$= 81 - 3 \times 16 + 3 \times 1 = 36$$

Hence, the correct answer is 36.

2.15 0.95

Given : $|X| = 2$

$$|Y| = 20$$

F = set of all possible functions defined from X to Y.

Total functions from X to Y = 20^2

Number of one-one functions = ${}^{20} P_2$

$$\text{Probability} = \frac{\text{No. of one-one functions}}{\text{Total no. of functions}}$$

$$= \frac{20 \times 19}{20 \times 20} = 0.95$$

Hence, the correct answer is 0.95.

2.16 (A)

Given : $g(x) = 1 - x$

$$h(x) = \frac{x}{(x-1)}$$

$$g(h(x)) = 1 - \frac{x}{(x-1)} = \frac{(x-1)-x}{(x-1)}$$

$$= \frac{-1}{(x-1)}$$

$$h(g(x)) = \frac{1-x}{(1-x-1)} = \frac{1-x}{-x}$$



$$\frac{g(h(x))}{h(g(x))} = \frac{x}{(x-1)(1-x)} = \frac{h(x)}{g(x)}$$

Hence, the correct option is (A).

2.17 (A)

Given :

following truth table.

p	q	$p \neq q$
0	0	0
0	1	1
1	0	1
1	1	0

Let $p = 0, q = 1, r = 0$

From truth table,

$$p \neq q = 0 \neq 1 = 1$$

$$q \neq p = 1 \neq 0 = 1$$

Hence commutative

$$p \neq (q \neq r) = 0 \neq (1 \neq 0) = 0 \neq 1 = 1$$

$$(p \neq q) \neq r = (0 \neq 1) \neq 0 = 1 \neq 0 = 1$$

Hence associative

Hence, the correct option is (A).

2.18 (C)

Given : $A = \{5, \{6\}, \{7\}\}$

Power set i.e.

$$2^A = \{\emptyset, \{5\}, \{\{6\}\}, \{\{7\}\}, \{5, \{6\}\}, \\ \{5, \{7\}\}, \{\{6\}, \{7\}\}, \{5, \{6\}, \{7\}\}\}$$

We can conclude that

- I. $\emptyset \in 2^A$
- II. $\emptyset \subseteq 2^A$ (Since \emptyset is subset of every set)
- III. $\{5, \{6\}\} \in 2^A$

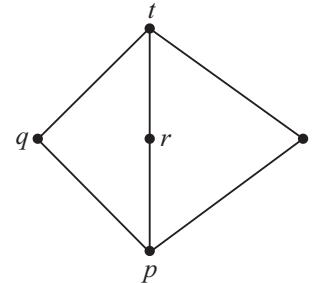
Hence, the correct option is (C).

2.19 (D)

Given :

Lattice $L = \{p, q, r, s, t\}$

Hasse diagram



$L^3 = \{(x, y, z) : x, y, z \in L\}$ be the set of all ordered triplets of the elements of L

Number of triplets in L^3 = Number of ways in which we can choose 3 elements from 5 with repetition

$$= 5 * 5 * 5 = 125.$$

Now, when we take $x = t$, then the given condition for L is

satisfied for any y and z . Here, y and z can be taken in

$$5 * 5 = 25 \text{ ways.}$$

Take $x = r, y = p, z = p$. Here also, the given condition is

satisfied. So, $P_r > 25/125 > 1/5$.

For $x = q, y = r, z = s$, the given condition is not satisfied

$$\text{as } q \vee (r \wedge s) = q \vee p = q$$

$$\text{while } (q \vee r) \wedge (q \vee s) = t \wedge t = t$$

$$\text{So, } P_r \neq 1$$

Hence, the correct option is (D).

2.20 (C)

Key Point

- (i) For reflexive $(a, a) \in R, \forall a \in S$
- (ii) For symmetric $(a, a) \in R \Rightarrow (b, a) \in R$
- (iii) For transitive $(a, a) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R$

Given : $R = \{(P, a), (r, s) | p - s = q - r\}$

For reflexive $(p, q), (p, q) \in R$

But $p - q \neq q - p$ hence it is not reflexive

For symmetric

$$(p, q), (r, s) \in R \Rightarrow ((r, s), (p, q)) \in R$$

$$(p - s) = (q - r)$$



$$-(s-p) = (r-q)$$

$$((r,s),(p,q)) \in R$$

Hence, the correct option is (C).

2.21 (B)

Given : $(a, b)R(c, d)$ if $a \leq c$ or $b \leq d$

For reflexive $(a, a) \in R$

Here $((a,b), (a, b)) \in R$

$$\Rightarrow a \leq a \text{ or } b \leq b$$

\Rightarrow True

Hence R is reflexive

For transitive

$$(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R$$

Here $((a,b), (c, d)) \in R, ((c, d), (e, f)) \in R$

$$\Rightarrow ((a, b), (e, f)) \in R$$

Let $a = 2, b = 5, c = 6, d = 3, e = 1, f = 4$

Now $((2, 5), (6, 3)) \in R$

$\because 2 \leq 6 \text{ or } 5 \leq 3 \text{ is true}$

$((6, 3), (1, 4)) \in R$

$\because 6 \leq 1 \text{ or } 3 \leq 4 \text{ is true}$

But $((2, 5), (1, 4)) \notin R$ $\because 2 \leq 1 \text{ or } 5 \leq 4 \text{ is false}$

Hence R is not transitive.

Hence, the correct option is (B).

2.22 (B)

Given : $|U| = 23$

$$|S| = 9$$

Each compound in S reacts with exactly 3 compounds of U.

Now, if a compound 'a' reacts with compound 'b' then 'b' also reacts with 'a' \Rightarrow it is a symmetric relation.

Let the compound is denoted by vertices and if the compound reacts with other compound then there is an edge.

Given : 9 out of 23 compounds reacts with exactly 3 compounds hence 9 vertices will have degree = 3 each.

Now $U \setminus S = U - S = 23 - 9 = 14$ compounds = 14 vertices

By hand shaking theorem,

$$9 \times 3 + \text{deg of 14 vertices} = 2e \text{ (even)}$$

$$27 + \text{deg of 14 vertices} = 2e \text{ (even)}$$

$$(\because \text{odd} + \text{odd} = \text{even})$$

Total degree of 14 vertices must be off. If each compound in $U \setminus S$ i.e., 14 compounds react with an odd no. of compounds then the sum will result in even. {sum of any n odd numbers such that n is even will result in even no.}

If each compound in $U \setminus S$ reacts with an even no. of compounds then again sum is even.

Hence at least one compound in $U \setminus S$ reacts with an odd number of compounds to make sum of degree odd.

Hence, the correct option is (B).

2.23 2

Given : $f : N^+ \rightarrow N^+$

$$f(n) = f\left(\frac{n}{2}\right) \text{ if } n \text{ is even}$$

$$f(n) = f(n+5) \text{ if } n \text{ is odd}$$

$$R = \{i \mid \exists j : f(j) = i\}$$

Let $f(1) = x$

$$\text{Now, } f(2) = f\left(\frac{2}{2}\right) = f(1) = x$$

$$f(3) = f(3+5) = f(8) = f\left(\frac{8}{2}\right)$$

$$= f\left(\frac{4}{2}\right) = f\left(\frac{2}{2}\right) = f(1) = x$$

$$\text{Similarly, } f(4) = f\left(\frac{4}{2}\right) = f\left(\frac{2}{2}\right) = f(1) = x$$

$$f(5) = f(5+5) = f\left(\frac{10}{2}\right) = f(5) = y$$

Hence only two values are possible, all multiple of 5 will have value = y and other will have value = x.

Hence, the correct answer is 2.

2.24 0

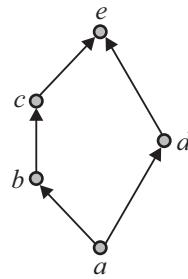
Given :

$$\text{Set } X = \{a, b, c, d, e\}$$

Partial ordering

$$R = \{(a, a), (a, b), (a, c), (a, d), (a, e), (b, b), (b, c), (b, e), (c, c), (c, e), (d, d), (d, e), (e, e)\}$$

Hasse diagram



Key Point

A Hasse diagram is called a lattice, if for every pair of elements there exists a LUB and GLB.

In the given Hasse diagram, LUB and GLB exist for every pair, hence it is already a lattice.

Hence minimum number of ordered pair that need to be added is 0.

Hence, the correct answer is 0.

2.25 271

Given :

Number of integers = 500

Let D_n denote the divisibility by n , then

$$\begin{aligned} N(D_3 \cup D_5 \cup D_7) &= N(D_3) + N(D_5) + N(D_7) \\ &\quad - N(D_{3,5}) - N(D_{5,7}) - N(D_{3,7}) + N(D_{3,5,7}) \\ &= \left[\frac{500}{3} \right] + \left[\frac{500}{5} \right] + \left[\frac{500}{7} \right] - \left[\frac{500}{3 \times 5} \right] \\ &\quad - \left[\frac{500}{5 \times 7} \right] - \left[\frac{500}{3 \times 7} \right] + \left[\frac{500}{3 \times 5 \times 7} \right] \\ &= 166 + 100 + 71 - 33 - 14 - 23 + 4 \\ &= 271 \end{aligned}$$

Hence, the correct answer is 271.

2.26 42

Given : Order of group (G) = 84

Key Point

Order of sub group always divides the order of group.

Factors of 84 = 1, 2, 4, 21, 42

Hence largest order of sub group is = 42

Hence, the correct answer is 42.

2.27 (D)

Given : For the provide sets, following inference can be drawn,

P : Set of rational numbers are countable.

Q : Set of functions from $\{0, 1\}$ to N will have N^2 elements hence countable.

R : Set of functions from N to $\{0, 1\}$ will have 2^N elements hence uncountable.

S : Set of finite subsets of N is countable.

Hence, the correct option is (D).

2.28 (B)

Given :

Following relations on group G

$R_1 : \forall a, b \in G, a R_1 b$ if and only if $\exists g \in G$ such that $a = g^{-1}bg$

$R_2 : \forall a, b \in G, a R_2 b$ if and only if $a = b^{-1}$

$R_1 : \forall a, b \in G, a R_1 b$ iff $\exists g \in G$

such that $a = g^{-1}bg$

For reflexive, $(a, a) \in R_1$

$$a = g^{-1}ag$$

Put $g = e$ (identity element)

$$a = e^{-1}ae = a$$

$(a, a) \in R_1 \Rightarrow R_1$ is reflexive

For symmetric, $(a, b) \in R_1 \Rightarrow (b, a) \in R_1$

$$a = g^{-1}bg$$

$$b = gag^{-1} = (g^{-1})^{-1}ag^{-1}$$

R_1 is symmetric

For transitive,

$$(a, b) \in R_1 \text{ and } (b, c) \in R_1 \Rightarrow (a, c) \in R_1$$

$$a = g_1^{-1}bg_1 \text{ and } b = g_2^{-1}cg_2$$

Substitute b in a

$$a = g_1^{-1}g_2^{-1}cg_2g_1$$

Since group is closed $g_1^{-1}g_2^{-1}$ and $g_1g_2 \in$ group

$$(a, c) \in R_1 \Rightarrow R_1$$
 is transitive

Hence R_1 is equivalence relation.

$R_2 : \forall a, b \in G, a R_2 b$ iff $a = b^{-1}$

For reflexive $(a, a) \in R_2$

But a need not be its own inverse in an arbitrary group, hence not reflexive.

Hence, the correct option is (B).

2.29 (C)

Given : $A = \{(x, X), x \in X \text{ and } X \subseteq U\}$



Number of k element subset of a set with n elements
 $= {}^n C_k$

No. of possible ordered pairs

$$= |A| = \sum_{k=1}^n k^n C_k = \sum_{k=1}^n k \binom{n}{k}$$

Hence II is correct.

From binomial expansion we can prove that

$$\sum_{k=1}^n k \binom{n}{k} = n2^{n-1}$$

Hence, I is correct.

Hence, the correct option is (C).

2.30 7

Key Point

The order of subgroup always divides the order of group.

Given : Order of group (G) = 35

Factors of 35 = 1, 5, 7, 35

Order of largest proper sub group = 7

Hence, the correct answer is 7.

2.31 0.125

Given : Set is {1, 2, 3}

Total no. of relations = $2^{n^2} = 2^9$

No. of reflexive relation = $2^{n(n-1)} = 2^{3 \times 2} = 2^6$

Probability that chosen relation is reflexive

$$= \frac{2^6}{2^9} = 0.125$$

Hence, the correct answer is 0.125.

2.32 59049

Given : $|S| = 10$

Let, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(A, B) such that $A \subseteq B$ $A \subseteq S$ $B \subseteq S$ $A \subseteq B$

for 1 element $S = \{1\}$

Tuples are $(\emptyset, \emptyset), (\emptyset, 1), (1, 1)$

Number of tuples $3^1 = 3$

For 2 elements $S = \{1, 2\}$

$P(S) = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

Tuples are

$(\emptyset, \emptyset), (\emptyset, 1), (\emptyset, 2), (\emptyset, \{1, 2\}), (1, 1),$
 $(1, \{1, 2\}), (2, 2), (2, \{1, 2\}) ((1, 2), (1, 2))$

Number of tuples $3^2 = 9$

Generalizing for n elements

Number of tuples = 3^n

for $n = 10$

Number of tuples = $3^{10} = 59049$

Hence, the correct answer is 59049.

2.33 (B)

Given : A relation R is said to be circular if aRb and bRc together imply cRa .

To prove:

If S is reflexive and circular then S will be equivalence relation.

Proof : S is reflexive there for element a, b, c (a, a), (b, b), (c, c) belongs to S

If (a, b) belongs to S then (b, a) must belong to S , as (a, b) and (b, b) belongs to S therefore, S is symmetric.

If (a, b) and (b, c) belongs to S then because its cyclic (c, a) must belong to S and S is symmetric too, hence (a, c) also belongs to S and S is Equivalence Relation

Hence, the correct option is (B).

2.34 (D)

Given : G is a group of order 6.

Key Point

Lagrange's Theorem: The order of every subgroup of G divides the order of G

So, Subgroup (H) of G can be the order of 1, 2, 3, 6.

Key Point

The Group of prime order is cyclic

If order of $G = 6$ which is not a prime number so **G may or may not be cyclic**.

If order of $H = 1$ or 2 or 3 , since $2, 3$ are prime numbers and Group with 1 element is always cyclic. Therefore, **H is always cyclic**.

Hence, the correct option is (D).

2.35 (B, D)

Given :

Following sets, where $n \geq 2$



S1 : Set of all $n \times n$ matrices with entries from the set $\{a, b, c\}$

S2 : Set of all functions from the set $\{0, 1, 2, \dots, n^2 - 1\}$ to the set $\{0, 1, 2\}$

S1: For $n \times n$ matrices n^2 entries will be there, for each entry we are having 3 choices, one of a, b and c. therefore total possible ways matrix can be built is 3^{n^2} , i.e. 3^{n^2} elements in Set 1

S2: Set A consists element from 0 to $n^2 - 1$, in totality n^2 elements are there, in Set B only 3 elements are there namely 0,1,2. Therefore total number of functions possible from Set A to Set B are 3^{n^2} . i.e. 3^{n^2} elements in Set 2

Therefore, both Bijection and Surjection may exist from Set1 to Set 2 as both are having same number of elements.

Hence, the correct options are (B, D).

3.

Combinatorics



Practice Questions

2014 IIT Kharagpur

- 3.1** The number of distinct positive integral factors of 2014 is _____.
3.2 There are 5 bags labeled 1 to 5. All the coins in given bag have the same weight. Some bags have coins of weight 10 gm, others have coins of weight 11 gm. I pick 1, 2, 4, 8, 16 coins respectively from bags 1 to 5. Their total weight comes out to 323 gm. Then the products of the labels of the bags having 11 gm coin is _____.
3.3 A pennant is a sequence of numbers, each number being 1 or 2. An n -pennant is a sequence of numbers with sum equal to n . For example, $(1,1,2)$ is a 4-pennant. The set of possible 1-pennant is $\{(1)\}$, the set of all possible 2-pennant is $\{(2), (1,1)\}$ and the set of all 3-pennant is $\{(2,1), (1,1,1), (1,2)\}$. Note that the pennant $(1,2)$ is not the same as the pennant $(2,1)$. The number of 10-pennant is _____.
3.4 Each of the nine words in the sentence “**The Quick brown fox jumps over the lazy dog**” is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is _____. (The answer should be rounded to one decimal place).

2015 IIT Kanpur

- 3.5** $\sum_{x=1}^{99} \frac{1}{x(x+1)} = \text{_____}.$
3.6 Let a_n represent the number of bit strings of length n containing two consecutive 1's. What is the recurrence relation for a_n ?
(A) $a_{n-2} + a_{n-1} + 2^{n-2}$
(B) $a_{n-2} + 2a_{n-1} + 2^{n-2}$

(C) $2a_{n-2} + a_{n-1} + 2^{n-2}$

(D) $2a_{n-2} + 2a_{n-1} + 2^{n-2}$

- 3.7**
- The number of divisors of 2100 is _____.
-
- 3.8**
- The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set
- $\{1, 2, 3\}$
- is _____.
-
- 2016 IISc Bangalore**

- 3.9**
- Let
- a_n
- be the number of
- n
- bit strings that do NOT contain two consecutive 1s. Which one of the following is the recurrence relation for
- a_n
- ?

(A) $a_n = a_{n-1} + 2a_{n-2}$

(B) $a_n = a_{n-1} + a_{n-2}$

(C) $a_n = 2a_{n-1} + a_{n-2}$

(D) $a_n = 2a_{n-1} + 2a_{n-2}$

- 3.10**
- The co-efficient of
- x^{12}
- in
- $(x^3 + x^4 + x^5 + x^6 + \dots)^3$
- is _____.
-
- 3.11**
- Consider the recurrence relation

$$a_1 = 8, a_n = 6n^2 + 2n + a_{n-1}. \text{ Let } a_{99} = K \times 10^4.$$

The value of K is _____.
2017 IIT Roorkee

- 3.12**
- If the ordinary generating function of a sequence
- $\{a_n\}_{n=0}^{\infty}$
- is
- $\frac{1+Z}{(1-Z)^3}$
- , then
- $a_3 - a_0$
- is equal to _____.
-
- 2018 IIT Guwahati**

- 3.13**
- Which one of the following is a closed form expression for the generating function of the sequence
- $\{a_n\}$
- , where
- $a_n = 2n+3$
- for all
- $n=0,1,2,\dots$
- ?
-
- (A)
- $\frac{3}{(1-x)^2}$
- (B)
- $\frac{3x}{(1-x)^2}$
-
- (C)
- $\frac{2-x}{(1-x)^2}$
- (D)
- $\frac{3-x}{(1-x)^2}$



2020 IIT Delhi

3.14 The number of permutations of the characters in LILAC so that no character appears in its original position, if the two L's are indistinguishable, is _____.

2021 IIT Bombay

3.15 There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing

3.1 8

Key Point

For any natural number N such that $N = a^p \times b^q \times c^r$,

Number of factors, $n = (p+1) \times (q+1) \times (r+1)$

Given : $N = 2014$

Factors of $N = 2^1 \times 19^1 \times 53^1$

$$\begin{aligned} \therefore \text{number of distinct positive factors} \\ &= (1+1) \times (1+1) \times (1+1) \\ &= 8 \end{aligned}$$

Hence, the correct answer is 8.

3.2 12

Let Number of coins of 11 gm = x

Number of coins of 10 gm = y

Given : $11x + 10y = 323$... (i)

$x + y = 31$ ($\because 1+2+4+8+16=31$) ... (ii)

Solving (i) and (ii) we get,

$$x = 13, y = 18$$

\Rightarrow number of coins of 11 gm = 13

Possible combination for 13 is $1+4+8$

i.e. 1 coin from bag 1

4 coins from bag 3

8 coins from bag 4

Hence product of label = $1 \times 3 \times 4 = 12$

Hence, the correct answer is 12.

3.3 89

Method 1

Let, number of n pennants = $f(n)$

Given:

$$f(1) = \text{number of 1 pennant} = 1$$

speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

The number of ways in which this can be done is _____.

Solutions

$$f(2) = \text{number of 2 pennant} = 2$$

$$f(3) = \text{number of 3 pennants} = 3$$

Here 3 pennants can be formed by 1 pennant and 2 pennant

$$\text{So } f(3) = f(2) + f(1)$$

$$\text{Also, similarly } f(10) = f(9) + f(8)$$

The series will become

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89$$

$$\text{Hence } f(10) = 89$$

Method 2

10-pennants can be formed in below combinations :

$$\{(1, 1, 1, 1, 1, 1, 1, 1, 1, 1), (1, 1, 1, 1, 1, 1, 2, 2), (1, 1, 1, 1, 2, 2, 2), (1, 1, 2, 2, 2, 2), (2, 2, 2, 2, 2, 2)\}$$

Number of 10 pennants = total permutations of above combinations

$$= 1 + \frac{9!}{8!} + \frac{8!}{6!2!} + \frac{7!}{4!3!} + \frac{6!}{2!4!} + 1 = 89$$

Method 3

Let there be x_1 ones and x_2 twos,

So In 10-pennant

$$x_1 + 2x_2 = 10 \quad \dots \text{(i)}$$

all possible solutions of equation (i) are :

$$(0, 5), (2, 4), (4, 3), (6, 2), (8, 1) \text{ and } (10, 0)$$

The number of ordered permutations for above solutions are

$$\frac{5!}{5!} = 1, \frac{6!}{2!4!} = 15, \frac{7!}{4!3!} = 35, \frac{8!}{6!2!} = 28, \frac{9!}{8!} = 9, \frac{10!}{10!} = 1$$

So total number of 10 pennants

$$= 1 + 15 + 35 + 28 + 9 + 1$$

$$= 89$$



Hence, the correct answer is 89.

3.4 3.9

Given : Sentence is

"The Quick Brown Fox Jumps Over The Lazy Dog"

Method 1

Each of the nine words have equal probability

So, expected length

$$\begin{aligned} &= (3+5+5+3+5+4+3+4+3) \times \frac{1}{9} \\ &= \frac{35}{9} = 3.9 \end{aligned}$$

Method 2

Let x be a random variable denoting length of word drawn.

Number of word of length 3 = 4

Probability of choosing a word of length 3 = $\frac{4}{9}$

Number of word of length 4 = 2

Probability of choosing a word of length 4 = $\frac{2}{9}$

Number of word of length 5 = 3

Probability of choosing a word of length 5 = $\frac{3}{9}$

$$E(X) = \frac{4}{9} \times 3 + \frac{2}{9} \times 4 + \frac{3}{9} \times 5 = 3.9$$

Hence, the correct answer is 3.9.

3.5 0.99

Given :

$$\begin{aligned} \text{Expression } \sum_{x=1}^{99} \frac{1}{x(x+1)} &= \sum_{x=1}^{99} \left[\frac{1}{x} - \frac{1}{x+1} \right] \\ &= \left[1 - \frac{1}{2} \right] + \left[\frac{1}{2} - \frac{1}{3} \right] + \left[\frac{1}{3} - \frac{1}{4} \right] + \dots + \left[\frac{1}{99} - \frac{1}{100} \right] \\ &= \left[1 - \frac{1}{100} \right] = \frac{99}{100} = 0.99 \end{aligned}$$

Hence, the correct answer is 0.99

3.6 (A)

Given : a_n represents the number of bit string of length n containing two consecutive 1s.

$a_1 = 0$ {no strings of length 1 contain two consecutive

1's}

$$a_2 = 1, a_3 = 3, a_4 = 8$$

Option (A) :

$$\begin{aligned} a_n &= a_{n-2} + a_{n-1} + 2^{n-2} \\ a_4 &= a_2 + a_3 + 2^2 \\ &= 1 + 3 + 4 = 8 \Rightarrow \text{True} \end{aligned}$$

Option (B) :

$$\begin{aligned} a_n &= a_{n-2} + 2a_{n-1} + 2^{n-2} \\ a_4 &= a_2 + 2a_3 + 2^2 \\ &= 1 + 6 + 4 = 11 \Rightarrow \text{False} \end{aligned}$$

Option (C) :

$$\begin{aligned} a_n &= 2a_{n-2} + a_{n-1} + 2^{n-2} \\ a_4 &= 2a_2 + a_3 + 2^2 \\ &= 2 + 2 = 9 \Rightarrow \text{False} \end{aligned}$$

Option (D) :

$$\begin{aligned} a_n &= a_{n-2} + a_{n-1} + 2^{n-2} \\ a_4 &= 2a_2 + 2a_3 + 2^2 \\ &= 2 + 6 + 4 = 12 \Rightarrow \text{False} \end{aligned}$$

Hence, the correct option is (A).

3.7 36

Key Point

If $N = p^a \times q^b \times r^c$, the number of divisor =
 $n = (p+1) \times (q+1) \times (r+1)$

Given: $N = 2100$

Divisor of $N = 2^2 \times 3^1 \times 5^2 \times 7^1$

Number of divisor

$$\begin{aligned} &= (2+1) \times (1+1) \times (2+1) \times (1+1) \\ &= 3 \times 2 \times 3 \times 2 \\ &= 36 \end{aligned}$$

Hence, the correct answer is 36.

3.8 15

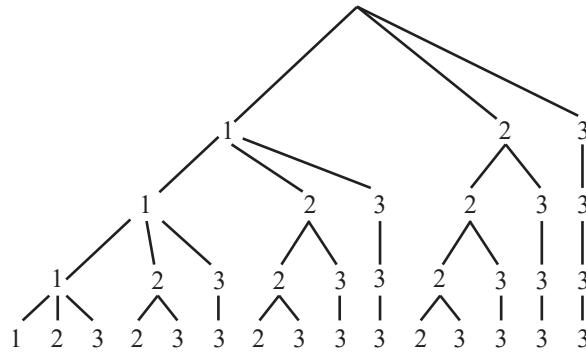
Given : set is {1, 2, 3}

Key Point

Non-decreasing means that no element is less than the element before it.

Following tree-diagram demonstrate the possible arrangements of number {1, 2, 3} in non-decreasing order.

\Rightarrow Number of 4-digit numbers = 15



Hence, the correct answer is 15.

3.9 (B)

Given : a_n = number of n -bit strings that do NOT contain two consecutive 1's.

Method 1

$$a_1 = 2 \{0,1\}$$

$$a_2 = 3 \{00, 01, 10\}$$

$$a_3 = 5 \{000, 010, 100, 001, 101\}$$

$a_4 = 8 \{0000, 0001, 0100, 0101, 1000, 1001, 0010, 1010\}$ from above observation we can conclude,

$$a_n = a_{n-1} + a_{n-2}$$

Method 2

a_n : denotes number of n -bit strings that do NOT contain two consecutive 1s (valid strings)

Last bit for a_n can be either 0 or 1,

Case 1: If it is 0, then all rest of the bits still keep it a valid string of length $n-1$ and number of such strings = a_{n-1} . Hence all the valid string of length $n-1$ also contributes to a_n by using 0 as last bit.

Case 2: If last bit is 1, we have to go on to check $(n-1)^{th}$ or simply second last bit because if its 1 too then our desired string ends with 11, which will make it invalid string, therefore the second last bit must be zero, after this we got to know that last two bit which we should append at last of a valid string is 01, hence all the valid string of length $n-2$ also contributes to a_n by using 01 as last two bits.

Hence $a_n = a_{n-1} + a_{n-2}$, is the correct recurrence relation.

Hence, the correct option is (B).

3.10 10

Given : $(x^3 + x^4 + x^5 + x^6 + \dots)^3$

Method 1

We will get x^{12} from following possibilities :

$$(x^4)^3 \rightarrow \text{Coefficient} = {}^3C_0 = 1$$

$$(x^3)^2 \cdot x^6 \rightarrow \text{Coefficient} = {}^3C_1 = 3$$

$$(x^3)(x^4)(x^5) \rightarrow \text{Coefficient} = {}^3C_2 \times {}^2C_1 = 6$$

Adding all, Coefficient of $x^{12} = 10$

Method 2

$$\begin{aligned} & (x^3 + x^4 + x^5 + x^6 + \dots)^3 \\ &= [x^3(1+x+x^2+x^3+\dots)]^3 \\ &= x^9(1+x+x^2+x^3+\dots)^3 \\ &= \frac{x^9}{(1-x^3)} = x^9 \times \sum_{k=0}^{\infty} {}^{3+k-1}C_k x^k \end{aligned}$$

To make x^{12} , put $k=3$

$$\Rightarrow \text{Coefficient of } x^{12} \text{ is } {}^{3+2}C_3 = {}^5C_3 = 10$$

Hence, the correct answer is 10.

3.11 198

Given : $a_1 = 8$

$$a_n = 6n^2 + 2n + a_{n-1}$$

$$a_n = 6n^2 + 2n + 6(n-1)^2 + 2(n-1) + a_{n-2}$$

$$\begin{aligned} a_n &= 6n^2 + 2n + 6(n-1)^2 + 2(n-1) + 6(n-2)^2 \\ &\quad + \dots + 6 \times 1^2 + 2 \times 1 \end{aligned}$$

$$\begin{aligned} a_n &= 6(n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2) \\ &\quad + 2(n + (n-1) + \dots + 2 + 1) \end{aligned}$$

$$a_n = 6 \times \frac{n(n-1)(2n+1)}{6} + 2 \times \frac{n(n+1)}{2}$$

$$a_n = n(n+1)(2n+1)$$

$$a_n = 2n(n+1)^2$$

for $n=99$

$$a_{99} = 2 \times 99 \times (99+1)^2$$

$$= 198 \times 10^4$$

Value of $K = 198$

Hence, the correct answer is 198.

3.12 15

Given :



generating function of $\{a_n\}_{n=0}^{\infty} = \frac{1+z}{(1-z)^3}$

$$\frac{1+z}{(1-z)^3} = (1+z)(1-z)^{-3}$$

$$(1-z)^{-3} = 1 + {}^3C_1 z + {}^4C_2 z^2 + {}^5C_3 z^3 + \dots \infty$$

$$(1+z)(1-z)^{-3} = (1+z) \times \\ (1 + {}^3C_1 z + {}^4C_2 z^2 + {}^5C_3 z^3 + \dots \infty)$$

$$a_0 = \text{Coefficient of } z^0 = 1$$

$$a_3 = \text{Coefficient of } z^3 = {}^5C_3 + {}^4C_2$$

$$a_3 = 10 + 6 = 16$$

$$a_3 - a_0 = 16 - 1 = 15$$

Hence, the correct answer is 15.

3.13 (D)

Given : $a_n = 2n + 3$

Key Point

Generating function for 1 is $\frac{1}{1-x}$ and for n is $\frac{x}{(1-x)^2}$

Generating function for $a_n = \frac{2x}{(1-x)^2} + \frac{3}{(1-x)}$

$$a_n = \frac{2x+3(1-x)}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

Hence, the correct option is (D).

3.14 12

Given :

L	I	L	A	C
1	2	3	4	5

- (i) two L's are indistinguishable; hence L can only be placed at position 2, 4, 5 in ${}^3C_2 = 3$ ways
- (ii) Now, one of the 2, 4, 5 is vacant and we have 3 letters, so we have only two choices for it.
- (iii) For the remaining place we have 2! ways

Total number of permutation

$$= 3 \times 2 \times 2 = 12$$

Hence, the correct answer is 12.

3.15 65

Given : There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job.
- Every computer gets at least one job.

Let C_1 be the fastest and C_3 be the slowest computers.

These two are assigned two jobs. Now out of the remaining 4 jobs we need to ensure C_2 gets at least 1. Without this constraint we can assign 4 jobs to 3 computers in $3^4 = 81$ ways. Out of these 81 ways $2^4 = 16$ will be having no jobs for C_2 .

So, number of possible ways so that C_2 gets at least one job = $81 - 16 = 65$.

Hence, the correct answer is 65.

4

Graph Theory



Practice Questions

2013 IIT Bombay

- 4.1** Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is $\frac{1}{2}$. What is the expected number of unordered cycles of length three?
- (A) $1/8$ (B) 1
 (C) 7 (D) 8

- 4.2** Which of the following statements is/are TRUE for undirected graphs?

P: Number of odd degree vertices is even.
 Q: Sum of degree of all vertices is even.

- (A) P only
 (B) Q only
 (C) Both P and Q
 (D) Neither P nor Q

2014 IIT Kharagpur

- 4.3** Let $G = (V, E)$ be a directed graph, where V is the set of vertices and E the set of edges. Then which one of the following graphs has the same strongly connected components as G ?

- (A) $G_1 = (V, E_1)$ where $E_1 = \{(u, v) \notin E\}$
 (B) $G_2 = (V, E_2)$ where $E_2 = \{(u, v) \in E\}$
 (C) $G_3 = (V, E_3)$ where

$$E_3 = \left\{ (u, v) \text{ there is a path of } \begin{cases} \text{length } \leq 2 \text{ from } u \text{ to } v \text{ in } E \end{cases} \right\}$$

 (D) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated.

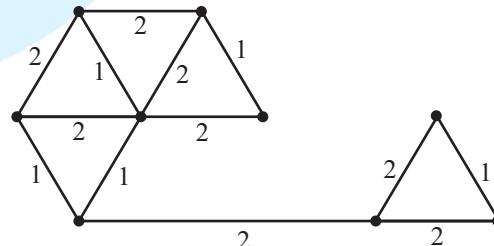
- 4.4** Consider a unidirectional graph G where self-loops are not allowed. The vertex set of G is $\{(i, j) : 1 \leq i \leq 12, 1 \leq j \leq 12\}$. There is an edge between (a, b) and (c, d) if $|a - c| \leq 1$ and $|b - d| \leq 1$. The number of edges in this graph is _____.

- 4.5** An ordered n -tuple (d_1, d_2, \dots, d_n) with $d_1 \geq d_2 \geq \dots \geq d_n$ is called graphic if there exists a simple undirected graph with n vertices having degrees d_1, d_2, \dots, d_n respectively. Which of the following 6-tuples is NOT graphic?
- (A) $(1, 1, 1, 1, 1, 1)$
 (B) $(2, 2, 2, 2, 2, 2)$
 (C) $(3, 3, 3, 1, 0, 0)$
 (D) $(3, 2, 1, 1, 1, 0)$

- 4.6** The maximum number of edges in a bipartite graph on 12 vertices is _____.

- 4.7** A cycle on n vertices is isomorphic to its complement. The value of n is _____.

- 4.8** The number of distinct minimum spanning trees for the weighted graph below is _____.



- 4.9** If G is a forest with n vertices and k connected components, how many edges does G have?

- (A) $\lfloor n/k \rfloor$ (B) $\lceil n/k \rceil$
 (C) $n-k$ (D) $n-k+1$

- 4.10** Let δ denote the minimum degree of a vertex in a graph. For all planar graphs on n vertices with $\delta \geq 3$, which one of the following is TRUE?

- (A) In a planar embedding, the number of faces is at least $\frac{n}{2} + 2$
 (B) In any planar embedding, the number of faces is less than $\frac{n}{2} + 2$
 (C) There is a planar embedding in which the number of faces is less than $\frac{n}{2} + 2$



- (D) There is a planar embedding in which the number of faces is at most $\frac{n}{\delta+1}$

2015 IIT Kanpur

- 4.11** Let G be a connected planar graph with 10 vertices. If the number of edges on each face is three, then the number of edges in G is _____.

4.12 A graph is self-complementary if it is isomorphic to its complement. For all self-complementary graphs on n vertices, n is

 - (A) A multiple of 4
 - (B) Even
 - (C) Odd
 - (D) Congruent to 0 mod 4, 1 mod 4.

4.13 In a connected graph, a bridge is an edge whose removal disconnects a graph. Which one of the following statements is true?

 - (A) A tree has no bridge
 - (B) A bridge cannot be part of a simple cycle
 - (C) Every edge of a clique with size ≥ 3 is a bridge (A clique is any complete subgraph of a graph)
 - (D) A graph with bridge cannot have a cycle

4.14 Let G be connected undirected graph of 100 vertices and 300 edges. The weight of a minimum spanning tree of G is 500. When the weight of each edge of G is 500. When the weight of each edge of G is increased by five, the weight of a minimum spanning tree becomes _____.

2016 IISc Bangalore

- 4.15** The minimum number of colors that is sufficient to vertex-color any planar graph is ____.

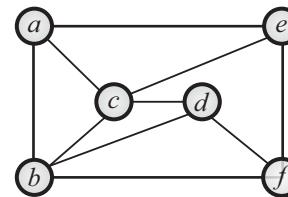
2017 IIT Roorkee

- 4.16** Let T be a tree with 10 vertices. The sum of the degree of all the vertices in T is _____.

4.17 G is undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is _____.

2018 IIT Guwahati

- 4.18** The chromatic number of the following graph is



- 4.19** Let G be a simple undirected graph. Let T_D be a depth first search tree of G . Let T_B be a breadth first search tree of G . Consider the following statements.

- (I) No edge of G is a cross edge with respect to T_D . (A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D .)

(II) For every edge (u, v) of G , if u is at depth i and v is at depth j in T_B , then $|i - j| = 1$.

(A) I only
(B) II only
(C) Both I and II
(D) Neither I nor II

- 4.20** Let G be a graph with $100!$ vertices, with each vertex labelled by a distinct permutation of the numbers $1, 2, \dots, 100$. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v . Let y denote the degree of a vertex in G , and z denote the number of connected components in G . Then, $y + 10z = \underline{\hspace{2cm}}$.

2019 IIT Madras

- 4.21** Let G be an undirected complete graph on n vertices $n > 2$. Then, the number of different Hamiltonian cycles in G is equal to

(A) $n!$ (B) $(n-1)!$
(C) 1 (D) $\frac{(n-1)!}{2}$

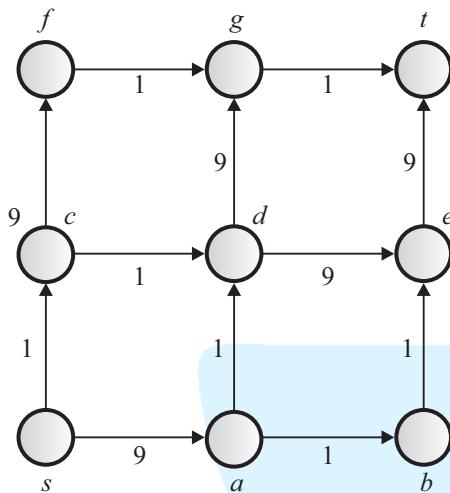
2020 IIT Delhi

- 4.22** Graph G is obtained by adding vertex s to $K_{3,4}$ and making s adjacent to every vertex of $K_{3,4}$. The minimum number of colors required to edge-color G is _____.

2021 IIT Bombay



- 4.23** In a directed acyclic graph with a source vertex s , the quality-score of a directed path is defined to be the product of the weights of the edges on the path. Further, for a vertex v other than s the quality-score of v is defined to be the maximum among the quality-scores of all the paths from s to v . The quality-score of s is assumed to be 1.



The sum of the quality-score of all the vertices in the graph shown above is _____.

- 4.24** In an undirected connected planar graph G , there are eight vertices and five faces. The number of edges in G is _____.

- 4.25** Let $G = (V, E)$ be an undirected unweighted connected graph. The diameter of G is defined as :

$$\text{diam}(G) = \max_{\forall u, v \in G} \{\text{the length of shortest path between } u \text{ and } v\}$$

Let M be the Adjacency matrix of G . Define a graph G_2 on the same set of vertices with adjacency matrix N , where

$$N_{ij} = \begin{cases} 1 & \text{if } M_{ij} > 0 \text{ or } P_{ij} > 0 \text{ where } P = M^2 \\ 0 & \text{otherwise} \end{cases}$$

Which one of the following statements is true?

- (A) $\text{diam}(G) < \text{diam}(G_2) \leq 2 \text{ diam}(G)$
- (B) $\text{diam}(G_2) = \text{diam}(G)$
- (C) $\text{diam}(G_2) \leq \lceil \text{diam}(G)/2 \rceil$
- (D) $\lceil \text{diam}(G)/2 \rceil < \text{diam}(G_2) < \text{diam}(G)$

Solutions

4.1 (C)

Given : Number of vertices = 8

$$\text{Probability of edge between a pair of vertices} = \frac{1}{2}$$

Now, a cycle of length 3 requires 3 vertices

Number of ways to choose 3 vertices

$$= {}^8 C_3 = 56$$

Probability of cycle formation = probability of edge between each pair.

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$\text{Expected number of cycles} = 56 \times \frac{1}{8} = 7$$

Hence, the correct option is (C).

4.2 (C)

Given : Following statements for undirected graphs :

P: Number of odd degree vertices is even.

Q: Sum of degree of all vertices is even.

- (i) Handshaking theorem.

Sum of degree of all vertices = $2 \times$ number of edges

- (ii) Number of odd degree vertices is even.

Hence both are correct

Hence, the correct option is (C).

4.3 (B)

Given : Following graphs :

- (A) $G_1 = (V, E_1)$ where $E_1 = \{(u, v) \notin E\}$

- (B) $G_2 = (V, E_2)$ where $E_2 = \{(u, v) \in E\}$

- (C) $G_3 = (V, E_3)$ where

$$E_3 = \left\{ (u, v) \mid \begin{array}{l} \text{there is a path of} \\ \text{length} \leq 2 \text{ from } u \text{ to } v \text{ in } E \end{array} \right\}$$

- (D) $G_4 = (V_4, E)$ where V_4 is the set of vertices in G which are not isolated.

Key Point

In a directed graph a strongly connected components will have a path from each vertex to every other vertex.

Key Point



Hence, changing the direction of all the edges, won't change the strongly connected components.
Hence, the correct option is (B).

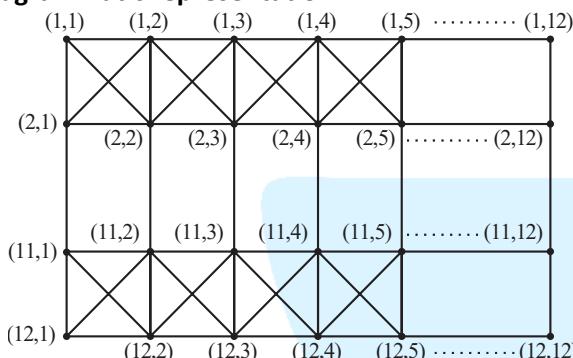
4.4 506

Given : $G = \{(i, j) | 1 \leq i \leq 12, 1 \leq j \leq 12\}$

There is an edge between (a, b) and (c, d) if $|a - c| \leq 1$ and $|b - d| \leq 1$

So, edges will exist between vertices if the distance is at most 1.

Diagrammatic representation :



From above diagram :

Horizontal edges on each row = 11

\Rightarrow Total horizontal edges = $11 \times 12 = 132$

Similarly,

Total vertical edges = $11 \times 12 = 132$

There are 11×11 such square boxes and each box contains 2 diagonal edges.

\Rightarrow Total diagonal edges = $11 \times 11 \times 2 = 242$

Total edges = $132 + 132 + 242$
= 506

Hence, the correct answer is 506.

4.5 (C)

Given :

Following 6-tuples

- (A) (1,1,1,1,1,1)
- (B) (2,2,2,2,2,2)
- (C) (3,3,3,1,0,0)
- (D) (3,2,1,1,1,0)

Key Point

After applying Havels Hakimi Theorem.

- (i) If at the end, even no. of 1's \Rightarrow sequence is valid.
- (ii) If at any step, negative number comes \Rightarrow sequence is invalid

- (iii) If at the end, odd no. of 1's \Rightarrow sequence is invalid.

Apply Havels Hakimi Theorem on option C

3, 3, 3, 1, 0, 0

2, 2, 0, -1, -1

So, sequence is invalid (case (ii))

Hence this tuple is not graphic

Hence, the correct option is (C).

4.6 36

Key Point

Number of edges in a complete bipartite graph is mn , where m and n are no. of vertices on each side It is maximum when $m = n$.

Given :

Total no. of vertices = 12

To get maximum number of edges distribute 6 edges on each side.

Hence $m = n = 6$

\Rightarrow Maximum number of edges = $6 \times 6 = 36$

Hence, the correct answer is 36.

4.7 5

Key Point

- (i) A cycle graph of n -vertices has n edges.

- (ii) If a graph is isomorphic to its complement G^c then

$$e = \frac{n(n-1)}{4}$$

Where n : number of vertices

e : number of edges

Given : a cycle graph of n vertices which is isomorphic to its complement

$$n = \frac{n(n-1)}{4}$$

$$n(n-1) = 4n$$

$$n^2 - n = 4n$$

$$n^2 - 5n = 0$$

$$n = 0, 5$$

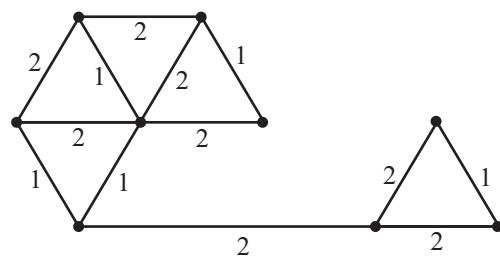
Since it is a cycle graph $n = 0$ is not possible

Hence $n = 5$

Hence, the correct answer is 5.



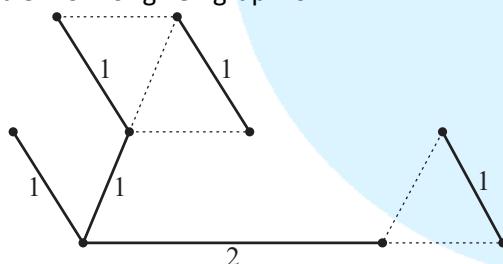
4.8 6

Given :**Key Point**

Use Kruskal's algorithm to find MST. Kruskal's algorithm.

1. Sort all the edges in non-decreasing order
2. Pick the smallest edge, check if it forms cycle, if not add it.
3. Repeat this step until all the vertices are connected.

Possible MST for given graph is:



In above diagram, solid line represents compulsory edges while dotted line represent choice.

So, we have $3 \times 2 = 6$ Choices

Number of distinct minimum spanning trees = 6

Hence, the correct answer is 6.

4.9 (C)

Key Point

(i) A forest is a collection of trees.

(ii) A tree with n vertices has $(n-1)$ edges.

Given : G is the forest with n vertices and k components. If there are n vertices then $\Rightarrow (n-1)$ edges.

If we remove one edge it will become 2 components hence to make k component, $k-1$ edges are removed

\Rightarrow Number of remaining edges

$$\begin{aligned} &= (n-1)-(k-1) \\ &= (n-k) \end{aligned}$$

Hence, the correct option is (C).

4.10 (A)

Given : δ denote the minimum degree

$\delta \leq$ average degree

$$\delta \leq \frac{2e}{n}$$

Put $\delta = 3$

$$3n \leq 2e$$

(i)

From Euler's formula, $R = e - n + 2$ (ii)

From (i) and (ii)

$$n + R - 2 \geq \frac{3n}{2}$$

$$R \geq \frac{n}{2} + 2$$

Number of faces / regions is at least $\left(\frac{n}{2} + 2\right)$

Hence, the correct option is (A).

4.11 24

Key Point

Euler formula: $n - e + R = 2$

Given : $n = 10$

$$R = e - 8 \quad \dots(i)$$

Since every edge is bounded by exactly 3 edges, and every edge is double counted,

$$\text{Hence, } e = \frac{3R}{2} \Rightarrow R = \frac{2e}{3} \quad \dots(ii)$$

From equation (i) and (ii)

$$\begin{aligned} \frac{2e}{3} &= e - 8 \\ e &= 24 \end{aligned}$$

Hence, the correct answer is 24.

4.12 (D)

Key Point

If a graph G is isomorphic to its complement G^c then

$$e = \frac{n(n-1)}{4}$$

Since, $n(n-1)$ should be divisible by 4

So, n must be congruent to 0 mod 4 or 1 mod 4

Hence, the correct option is (D).

4.13 (B)

Key Point



- (i) Bridge / Cut-edge: A single edge whose removal disconnects the graph.
- (ii) Every edge in a tree is a bridge
- (iii) In a cycle every vertex is connected to two other vertices hence it does not have a bridge.

Hence, the correct option is (B).

4.14 995

Key Point

If a graph has n vertices, then minimum spanning tree has $n-1$ edges.

Given : Number of vertices = 100

So, Number of edges in MST = 99

Initial weight of MST = 500

Now, weight of each edge is increased by 5

So, Total increase in weight = 99×5
= 495

So, Final weight of MST = $500 + 495$
= 995

Hence, the correct answer is 995.

4.15 4

Given : A planar graph

Key Point

Four Colour Theorem: The vertices of every planar graph can be colored with at most four colours.

Hence, the correct answer is 4.

4.16 18

Given : A tree T with 10 vertices

Key Point

- (i) A tree with n vertices has $(n-1)$ edges.
- (ii) Sum of all degree = $2 \times$ number of edges.

$$n=10$$

$$\Rightarrow \text{Number of edges} = n-1 = 10-1 = 9$$

$$\therefore \text{Sum of all degree} = 2 \times 9 = 18$$

Hence, the correct answer is 18.

4.17 16

Given : Number of vertices = n

$$\text{Number of edges} = 25$$

Each vertex has at least degree 3.

From handshaking theorem

$$3n = 2 \times 25$$

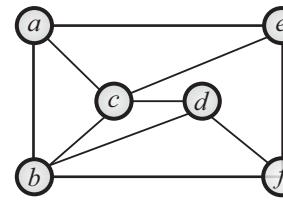
$$n = \frac{2 \times 25}{3} = 16.66$$

$$n = 16$$

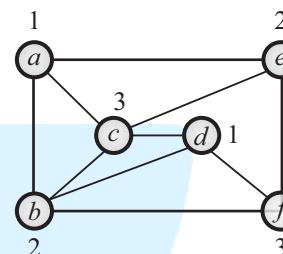
Hence, the correct answer is 16.

4.18 3

Given :



Given graph can be colored as follows:



So, number of colors required = 3

Chromatic number = 3

Hence, the correct answer is 3.

4.19 (A)

Given : Following statements :

- (I) No edge of G is a cross edge with respect to T_D .
(A cross edge in G is between two nodes neither of which is an ancestor of the other in T_D .)
- (II) For every edge (u, v) of G , if u is at depth i and v is at depth j in T_B , then $|i-j|=1$.

Key Point

Undirected graph can't have cross edges in DFS forest.

Hence, the correct option is (A).

4.20 109

Given : Number of vertices = 100!

Since an edge will exist if a vertex label can be obtained by swapping two adjacent numbers in another vertex.

Therefore, each vertex is connected to 99 other vertex

Hence, it is a regular graph.

Hence degree of each vertex = 99

$$\Rightarrow d = 99$$

Since it is a connected graph, number of connected

$$\text{Components} = z = 1$$



$$y + 10z = 99 + 10 \times 1 = 109$$

Hence, the correct answer is 109.

4.21 (C, D)

Given : An undirected complete graph G

Number of vertices = n $n > 2$

Key Point

The number of different Hamiltonian cycles in a complete undirected labeled graph on n vertices is $\frac{(n-1)!}{2}$

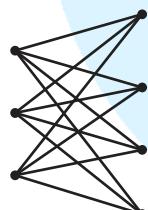
If the graph is unlabeled number of Hamiltonian cycles become 1.

Both (C) and (D) are correct as per the official answer key.

Hence, the correct option is (C, D).

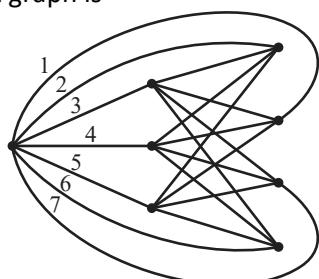
4.22 7

Given:



Now another vertex S is added to $K_{3,4}$, such that S is connected to all vertices.

The resultant graph is



Key Point

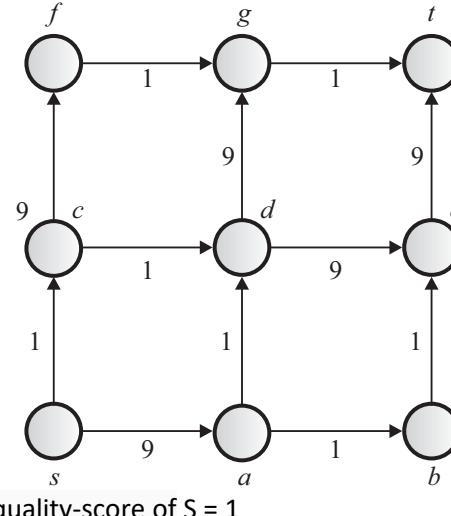
Edge coloring for graph is assignment of colors to the edges such that no two edges incident on same vertex have the same color.

So, the minimum number of colors required = 7

Hence, the correct answer is 7.

4.23 929

Given :



Therefore,

$$\text{quality-score of C} = 1 \quad \because (S \rightarrow C)$$

$$\text{quality-score of F} = 1 \times 9 \quad \because (S \rightarrow C \rightarrow F)$$

$$\text{quality-score of A} = 9 \quad \because (S \rightarrow A)$$

$$\text{quality-score of D} = 9 \times 1 \quad \because (S \rightarrow A \rightarrow D)$$

$$\text{quality-score of G} = 9 \times 1 \times 9$$

$$\therefore (S \rightarrow A \rightarrow D \rightarrow G)$$

$$\text{quality-score of B} = 9 \times 1$$

$$\therefore (S \rightarrow A \rightarrow B)$$

$$\text{quality-score of E} = 9 \times 1 \times 9$$

$$\therefore (S \rightarrow A \rightarrow D \rightarrow E)$$

$$\text{quality-score of T} = 9 \times 1 \times 9 \times 9$$

$$\therefore (S \rightarrow A \rightarrow D \rightarrow E \rightarrow T)$$

Sum of quality-score of all vertices

$$= 1 + 1 + 9 + 9 + 9 + 81 + 9 + 81 + 729 = 929$$

Hence, the correct answer is 929.

4.24 11

Given : Number of vertices = $V = 8$

Number of faces = $F = 5$

Key Point

Euler's formula for a planar graph:

$$V - E + F = 2$$

Where, V = number of vertices

E = number of edges in graph

F = number of faces in graph.

$$\text{So, } 8 - E + 5 = 2$$

This implies $E = 11$

Hence, the correct answer is 11.



4.25 (C)

Given : $G = (V, E)$ is an undirected unweighted connected graph.

$\text{Diam } (G) = \max_{\forall u, v \in G} \{\text{the length of shortest path between } u \text{ and } v\}$

G_2 is a graph on the same set of vertices with adjacency matrix N , where

$$N_{ij} = \begin{cases} 1 & \text{if } M_{ij} > 0 \text{ or } P_{ij} > 0, \text{ where } P = M^2 \\ 0 & \text{otherwise} \end{cases}$$

From the definition of adjacency matrix of G_2 , it is clear that any two vertices that were adjacent in G are also adjacent in G_2 . Also, for any two vertices m, n of G which were not adjacent in G will be adjacent in G_2 iff they are adjacent to some other common vertex(just try to find the value of $N(m, n)$ for some m, n . You will see why this double implication is true).

Also, take any path between any two vertex, say x and y in G . Let the vertices along this path be x, x_1, x_2, \dots, y . Now there will be a new path corresponding to this in G_2 which will be x, x_2, x_4, \dots, y because of the same reason as discussed above (for e.g., in G_2 , x and x_2 will become adjacent as both were adjacent to x_1 in G).

So, the length of all paths between any 2 vertex will become almost half.

Hence, the correct option is (C).

CHAPTER 6 | Engineering Mathematics

Marks Distribution of Engineering Mathematics in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2003	1	4	9
2004	5	6	17
2005	3	4	11
2006	2	4	10
2007	3	5	13
2008	3	8	19
2009	-	3	6
2010	1	3	7
2011	2	4	10
2012	3	2	7
2013	4	-	4
2014 Set-1	4	3	10
2014 Set-2	3	3	9
2014 Set-3	3	3	9

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2015 Set-1	3	2	7
2015 Set-2	1	2	5
2015 Set-3	2	2	6
2016 Set-1	3	2	7
2016 Set-2	4	-	4
2017 Set-1	2	3	8
2017 Set-2	3	4	11
2018	3	2	7
2019	3	2	7
2020	1	2	5
2021 Set-1	2	2	6
2021 Set-2	3	2	7

Syllabus : Engineering Mathematics

Linear Algebra : Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus : Limits, continuity and differentiability. Maxima and minima. Mean value theorem. Integration.

Probability and Statistics : Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem. Computer Science and Information Technology.

Contents : Engineering Mathematics

S. No. Topics

1. Linear Algebra
2. Calculus
3. Probability



Practice Questions

2013 IIT Bombay

- 1.1 Which one of the following does NOT equal to

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} ?$$

(A) $\begin{vmatrix} 1 & x(x+1) & x \\ 1 & y(y+1) & y \\ 1 & z(z+1) & z \end{vmatrix}$

(B) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(C) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(D) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

2014 IIT Kharagpur

- 1.2 The value of the dot product of the Eigen vectors corresponding to any pair of different Eigen values of a 4×4 symmetric positive definite matrix is _____.

- 1.3 Consider the following system of equation

$$3x + 2y = 1$$

$$4x + 7z = 1$$

$$x + y + z = 3$$

$$x - 2y + 7z = 0$$

The number of solutions for this system is _____.

- 1.4 If the matrix A is such that

$$A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$$

then the determinant of A is equal to _____.

- 1.5 The product of the non-zero Eigen values of the

$$\text{matrix } \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ is } _____.$$

- 1.6 Which one of the one following statement is True about every $n \times n$ matrix with only real eigenvalues?

(A) If the trace of the matrix is positive and the determinant of the matrix is negative at least one of its eigenvalues is negative.

(B) If the trace of the matrix is positive, all its eigenvalues are positive.

(C) If the determinant of the matrix is positive all its eigenvalues are positive.

(D) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

- 1.7 If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vectors space V , then the smallest possible dimension of \mathbb{R} is _____.

2015 IIT Kanpur

- 1.8 In the LU decomposition of the matrix $\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$, if the diagonal elements of U are both 1, then the lower diagonal entry l_{22} of L is _____.

- 1.9 Consider the following 2×2 matrix A where two elements are unknown and are marked by a and b . The eigenvalues of this matrix are -1 and 7 . What are the values of a and b ?

$$A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}$$

(A) $a=6, b=4$ (B) $a=4, b=6$

(C) $a=3, b=5$ (D) $a=5, b=3$



- 1.10** The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is _____.
- 1.11** Perform the following operations on the matrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$
- Add the third row to the second row
 - Subtract the third column from the first column.
- The determinant of the resultant matrix is _____.
- 1.12** In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the Eigen values is 1. The Eigen vectors corresponding to the Eigen value 1 are
- $\{\alpha(4,2,1) | \alpha \neq 0, \alpha \in R\}$
 - $\{\alpha(-4,2,1) | \alpha \neq 0, \alpha \in R\}$
 - $\{\alpha(\sqrt{2},0,1) | \alpha \neq 0, \alpha \in R\}$
 - $\{\alpha(-\sqrt{2},0,1) | \alpha \neq 0, \alpha \in R\}$
- 1.13** If the following system has non-trivial solution,
- $$px + qy + rz = 0$$
- $$qx + ry + pz = 0$$
- $$rx + py + qz = 0$$
- Then which one of the following options is TRUE ?
- $p - q + r = 0$ or $p = q = -r$
 - $p + q - r = 0$ or $p = -q = r$
 - $p + q + r = 0$ or $p = q = r$
 - $p - q + r = 0$ or $p = -q = -r$
- 2016 IISc Bangalore**
- 1.14** Two Eigen values of a 3×3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is _____.
- 1.15** Consider the systems, each consisting of m linear equations in n variables.
- If $m < n$, than all such system have a solution
 - If $m > n$, then none of these systems has a solution
 - If $m = n$, then there exists a system which has a solution
- Which one of the following is CORRECT?
- I, II and III are true
 - Only II and III are true
 - Only III is true
 - None of them is true
- 1.16** Suppose that the Eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is _____.
- 2017 IIT Roorkee**
- 1.17** Let c_1, \dots, c_n can be scalars, not all zero, such that $\sum_{i=1}^n c_i a_i = 0$ where a_i are column vectors in R^n . Consider the set of linear equations $Ax = b$ where, $A = [a_1, \dots, a_n]$ and $b = \sum_{i=1}^n a_i$. The set of equations has
- a unique solution at $x = J_n$ where J_n denotes a n -dimensional vector of all 1
 - no solution
 - infinitely many solutions
 - finitely many solutions
- 1.18** Let A be $n \times n$ real valued square symmetric matrix of rank 2 with $\sum_{i=1}^n \sum_{j=1}^n A_{ij}^2 = 50$. Consider the following statements.
- One eigenvalue must be in $[-5, 5]$
 - The eigenvalue with the largest magnitude must be strictly greater than 5. Which of the above statements about eigenvalues of A is/are necessarily CORRECT?
- Both (I) and (II)
 - (I) only
 - (II) only
 - Neither (I) nor (II)
- 1.19** Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ and



$Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices.

Then the rank of $P+Q$ is _____

- 1.20** If the characteristic polynomial of a 3×3 matrix M over \mathbb{R} (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30$, $a \in \mathbb{R}$, and one eigenvalue of M is 2, then the largest among the absolute values of the eigenvalues of M is _____

2018 IIT Guwahati

- 1.21** Consider a matrix $A = uv^T$ where $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Note that v^T denotes the transpose of v . The largest eigenvalue of A is _____.

- 1.22** Consider a matrix P whose only eigenvectors are the multiples of $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

Consider the following statements.

- (I) P does not have an inverse
- (II) P has a repeated eigenvalue
- (III) P cannot be diagonalized

Which one of the following options is correct?

- (A) Only I and III are necessarily true
- (B) Only II is necessarily true
- (C) Only I and II are necessarily true
- (D) Only II and III are necessarily true

2019 IIT Madras

- 1.23** Let X be a square matrix. Consider the following two statements on X

- I. X is invertible
- II. Determinant of X is non-zero

Which one of the following is TRUE?

- (A) I implies II; II does not imply I.
- (B) II implies I; I does not imply II.
- (C) I does not imply II; II does not imply I.

(D) I and II are equivalent statements.

- 1.24** Consider the following matrix :

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

The absolute value of the product of Eigen values of R is _____.

2020 IIT Delhi

- 1.25** Let A and B be two $n \times n$ matrices over real numbers. Let $\text{rank}(M)$ and $\det(M)$ denote the rank and determinant of a matrix M , respectively. Consider the following statements.

- I. $\text{rank}(AB) = \text{rank}(A) \text{rank}(B)$
- II. $\det(AB) = \det(A) \det(B)$
- III. $\text{rank}(A+B) \leq \text{rank}(A) + \text{rank}(B)$
- IV. $\det(A+B) \leq \det(A) + \det(B)$

Which of the above statements are TRUE?

- (A) I and II only
- (B) I and IV only
- (C) II and III only
- (D) III and IV only

2021 IIT Bombay

- 1.26** Consider the following matrix

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The largest eigenvalue of the matrix is _____.

- 1.27** Suppose that P is a 4×5 matrix such that every solution of the equation $Px = 0$ is a scalar multiple of $[2 \ 5 \ 4 \ 3 \ 1]^T$. The rank of P is _____

Solutions

1.1 (A)

Given : Matrix $A = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$



Then option (B) can be obtained by $C_2 \rightarrow (C_2 + C_1)$

and $C_3 \rightarrow (C_3 + C_1)$

Option (C) can be obtained by $R_1 \rightarrow (R_1 - R_2)$ and

$R_2 \rightarrow (R_2 - R_3)$

Option (D) can be obtained by $R_1 \rightarrow (R_1 + R_2)$ and

$R_2 \rightarrow (R_2 + R_3)$

Option (A) cannot be obtained by any row/column operation.

Hence, the correct option is (A).

1.2 0

The Eigen vectors corresponding to distinct Eigen values of real symmetric matrix are orthogonal. Dot product of orthogonal vectors is always 0.

Hence, the correct answer is 0.

1.3 1

Given : System of equations are

$$3x + 2y = 1$$

$$4x + 7z = 1$$

$$x + y + z = 3$$

$$x - 2y + 7z = 0$$

It is in form of non homogeneous equation

$$AX = B$$

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 4 & 0 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 7 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 3 \\ 0 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Augmented matrix is given by,

$$[A : B] = \left[\begin{array}{ccc|c} 3 & 2 & 0 & 1 \\ 4 & 0 & 7 & 1 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{array} \right]$$

$$R_2 \rightarrow R_2 - R_1$$

$$[A : B] = \left[\begin{array}{ccc|c} 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \\ 1 & 1 & 1 & 3 \\ 1 & -2 & 7 & 0 \end{array} \right]$$

$$R_4 \rightarrow R_4 - R_2$$

$$R_3 \rightarrow R_3 - R_2$$

$$[A : B] = \left[\begin{array}{ccc|c} 3 & 2 & 0 & 1 \\ 1 & -2 & 7 & 0 \\ 0 & 3 & -6 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\rho[A : B] = \rho(A) = 3 = \text{no. of variables}$$

So, we have unique solution.

Hence, the correct answer is 1.

1.4 0

$$\text{Given : } A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} [1 \ 9 \ 5]$$

$$A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

Here we can see that, R_2 is dependent on R_1 as $R_2 = -2R_1$ by the properties of determinant if two rows are dependent than $|A| = 0$.

Hence, the correct answer is 0.

1.5 6

$$\text{Given : } A = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Characteristic equation is given by,



$$|A - \lambda I| = \begin{vmatrix} 1-\lambda & 0 & 0 & 0 & 1 \\ 0 & 1-\lambda & 1 & 1 & 0 \\ 0 & 1 & 1-\lambda & 1 & 0 \\ 0 & 1 & 1 & 1-\lambda & 0 \\ 1 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

$$|A - \lambda I| = [1-\lambda] \begin{vmatrix} 1-\lambda & 1 & 1 & 0 \\ 1 & 1-\lambda & 1 & 0 \\ 1 & 1 & 1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

$$+ 1 \begin{vmatrix} 0 & 1-\lambda & 1 & 1 \\ 0 & 1 & 1-\lambda & 1 \\ 0 & 1 & 1 & 1-\lambda \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

$$= -[1-\lambda]^2 \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}$$

$$+ 1 \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} \quad \dots(i)$$

Determinant of $\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}$

$$= (1-\lambda)[(1-\lambda)^2 - 1] - 1[1-\lambda-1] + 1[1-(1-\lambda)]$$

$$= (1-\lambda)[\lambda^2 + 1 - 2\lambda - 1] - 1[-\lambda] + 1[1 - 1 + \lambda]$$

$$= 1 - \lambda[\lambda^2 - 2\lambda] + 2\lambda$$

$$= \lambda^2 - 2\lambda - \lambda^3 + 2\lambda^2 + 2\lambda$$

$$= -\lambda^3 + 3\lambda^2$$

... (ii)

From equation (i) and (ii)

$$= -[1-\lambda]^2[-\lambda^3 + 3\lambda^2] + 1[-\lambda^3 + 3\lambda^2] = 0$$

$$= [-\lambda^3 + 3\lambda^2][1 - (1-\lambda)^2] = 0$$

$$= [-\lambda^3 + 3\lambda^2][1 - (1 + \lambda^2 - 2\lambda)] = 0$$

$$= \lambda^2(3-\lambda)(2\lambda - \lambda^2) = 0$$

$$\therefore \lambda = 0, \lambda = 2, \lambda = 3$$

The product of non zero Eigen value is $2 \times 3 = 6$.

Hence, the correct answer is 6.

1.6 (A)

Given : A $n \times n$ matrix

From properties of eigen value, we know that

Sum of eigen values = Trace of matrix

Product of eigen value = Determinant of matrix

From option (A) if determinant of the matrix is negative then one of its eigen value must be negative also the sum may result in positive value.

Hence, the correct option is (A).

1.7 2

Given : V_1 and V_2 are two 4 dimensional subspace of 6 dimension vector V .

$$\dim(V_1 + V_2) = \dim(V_1) + \dim(V_2) - \dim(V_1 \cap V_2)$$

Minimum dimension of $(V_1 \cap V_2)$ will be

$$\dim(V_1 \cap V_2) = \dim(V_1) + \dim(V_2) - \max \dim(V_1 + V_2)$$

$$\dim(V_1 \cap V_2) = 4 + 4 - 6$$

$$\dim(V_1 \cap V_2) = 2$$

Hence, the correct answer is 2.

1.8 5

Given : $A = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$

We know that, $A = LU$

$$\text{where } L = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \text{ and } U = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

$$\text{given } u_{11} = u_{22} = 1$$

$$\begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} l_{11} & l_{11}u_{12} \\ l_{21} & l_{21}u_{12} + l_{22} \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 4 & 9 \end{bmatrix}$$

By comparing we get,

$$l_{11} = 2, l_{21} = 4$$

$$u_{12} = 1$$

$$l_{21}u_{12} + l_{22} = 9$$

$$4 \times 1 + l_{22} = 9$$

$$l_{22} = 9 - 4 = 5$$

Hence, the correct answer is 5.



1.9 (D)

Given : $A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}_{2 \times 2}$

and Eigen values are -1 and 7

$$\lambda_1 = -1, \lambda_2 = 7$$

From properties of eigen values of matrix

Sum of Eigen values = Trace of matrix

$$\lambda_1 + \lambda_2 = 1 + a$$

$$-1 + 7 = 1 + a$$

$$a = 6 - 1 = 5$$

And product of Eigen values = Determinant of matrix

$$\lambda_1 \cdot \lambda_2 = |A|$$

$$\lambda_1 \cdot \lambda_2 = \begin{vmatrix} 1 & 4 \\ b & a \end{vmatrix} = a - 4b = 5 - 4b$$

$$-1 \times 7 = 5 - 4b$$

$$4b = 5 + 7$$

$$b = \frac{12}{4} = 3$$

Hence, $a = 5$ and $b = 3$

Hence, the correct option is (D).

1.10 6

Given : $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

Characteristic equation is,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda) - 10 = 0$$

$$\lambda^2 - 5\lambda + 4 - 10 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0$$

$$(\lambda - 6)(\lambda + 1) = 0$$

Eigen values are 6 and -1

Larger Eigen value = 6

Hence, the correct answer is 6.

1.11 0

Given : $A = \begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

(i) Perform $R_2 \rightarrow R_2 + R_3$

$$A = \begin{bmatrix} 3 & 4 & 45 \\ 20 & 11 & 300 \\ 13 & 2 & 195 \end{bmatrix}$$

(ii) Perform $C_3 \rightarrow C_3 - C_1$

$$A = \begin{bmatrix} 3 & 4 & 42 \\ 20 & 11 & 280 \\ 13 & 2 & 182 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 4 & 42 \\ 20 & 11 & 280 \\ 13 & 2 & 182 \end{vmatrix}$$

$$|A| = 3(182 \times 11 - 2 \times 280) - 4(182 \times 20 - 13 \times 280) + 42(2 \times 20 - 13 \times 11)$$

$$|A| = 4326 + 0 - 4326 = 0$$

Hence, the correct answer is 0.

1.12 (B)

Given : Matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

One of the Eigen values is 1.

\therefore Eigen vector corresponding to $\lambda = 1$ is $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

We know that,

$$[A - \lambda I][X] = 0$$

$$\lambda = 1$$

Let us consider $[A - I][X] = 0$

$$\left[\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right] \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-X_2 + 2X_3 = 0$$



$$X_1 + 2X_2 = 0$$

Let $X_2 = K$

$$\therefore X_1 = -2K$$

$$X_3 = \frac{K}{2}$$

$$X_2 = K$$

\therefore Most suitable option will be option (B),

$$X_1 = -4, X_2 = 2, X_3 = \frac{1}{2}$$

Hence, the correct option is (B).

1.13 (C)

Given : $px + qy + rz = 0$

$$qx + ry + pz = 0$$

$$rx + py + qz = 0$$

It is in form of homogeneous equation

$$AX = 0$$

$$A = \begin{bmatrix} p & q & r \\ q & r & p \\ r & p & q \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

For non trivial solution $|A| = 0$

$$|A| = \begin{vmatrix} p & q & r \\ q & r & p \\ r & p & q \end{vmatrix}$$

$$|A| = p(qr - p^2) - q(q^2 - pr) + r(pq - r^2) = 0$$

$$|A| = p^3 + q^3 + r^3 - 3pqr = 0$$

So, $p + q + r = 0$ or $p = q = r$

Hence, the correct option is (C).

Key point

If $\rho(A) = \rho(A : B)$ and $|A| \neq 0$

Then system has non-trivial solution

1.14 15

Given : $P_{3 \times 3}$ has Eigen values $2 + \sqrt{-1}$ and 3. If

$\lambda = 2 + \sqrt{-1} = (2 + i)$ is one Eigen value then $2 - i$ is also an Eigen value because complex roots always occur in pairs.

Determinant of matrix = Product of eigen value

$$|P| = (2+i)(2-i)3 = (4+1)3$$

$$|P| = 15$$

Hence, the correct answer is 15.

1.15 (C)

I. Consider 2 equations in 3 variables.

$$x - y + z = 1$$

$$-x + y - z = 2$$

This system has no solution (inconsistent)

$$x = 1 \text{ and } y = 1$$

\therefore I is false

II. Consider 3 equations in two variables.

$$x + y = 2, x - y = 0,$$

$$3x + y = 4$$

This system has a unique solution

\therefore II is false

III. Consider a system with 2 equations and 2 variables $x + y = 2$ and $x - y = 0$.

The system has a solution $x = 1$ and $y = 1$

\therefore III is true

Hence, the correct option is (C).

1.16 0.125

Given : Matrix A, with eigen values

$$\lambda = 1, 2, 4$$

We know that, from property of eigen values

Determinant of a matrix = Product of Eigen values of that matrix

$$\therefore |A| = 1 \times 2 \times 4 = 8$$

$$|A^{-1}| = \frac{1}{|A|} = \frac{1}{8}$$

$$\therefore |(A^{-1})^T| = |A^{-1}| = \frac{1}{8} = 0.125$$

Hence, correct answer is 0.125.

Key point

$$(1) |A^T| = |A|$$

$$(2) |(A^{-1})^T| = |A^{-1}|$$

1.17 (D)

$\sum_i c_i a_i = 0$ with $\exists i: c_i \neq 0$ indicates that column vectors of A are linearly dependent. Determinant of matrix A would be zero. Therefore either $Ax = b$ has no solution or infinitely many solutions.

From $\sum_i a_i = b$, it is clear that a n -dimensional vector of all 1 is a solution of equation $Ax = b$.



Hence, $Ax = b$ will have infinitely many solutions.

Hence, the correct option is (C).

1.18 (D)

$$\text{Let } A = \begin{bmatrix} 0 & 5 \\ 5 & 0 \end{bmatrix}$$

Characteristic equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 5 \\ 5 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 25 = 0$$

$$\lambda = \pm 5$$

Therefore second statement is false.

Since, the rank of matrix A is 2, therefore at least one Eigen value would be zero for $n \geq 3$.

$$\text{For } n = 2, \lambda_1^2 + \lambda_2^2 \leq \sum_{i=1}^n \sum_{j=1}^n A_{ij}^2$$

$$\lambda_1^2 + \lambda_2^2 \leq 50$$

Both λ_1 and λ_2 would be real because A is a real symmetric matrix. Which implies that atleast one eigen value would be in $[-5, 5]$.
Hence, correct answer is (B).

1.19 2

Given :

$$P = \begin{bmatrix} 1 & 1 & - \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} \text{ and } Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

$$P+Q = \begin{bmatrix} 1 & 1 & - \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} + \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$$

$$P+Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix}$$

Performing elementary transformation

$$R_2 \rightarrow R_2 + R_1$$

$$P+Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 8 & 8 \\ 8 & 8 & 8 \end{bmatrix}$$

We, can see that there are two linearly independent rows so $\rho(P+Q) = 2$

Hence, the correct answer is 2.

1.20 5

Given: characteristic equation is

$$\lambda^3 - 4\lambda^2 + a\lambda + 30 = 0 \quad \dots (\text{i})$$

one eigen value is 2

Putting $\lambda = 2$ in equation (i), as eigen value satisfy characteristic equation

$$\text{So, } (2)^3 - 4(2)^2 + a(2) + 30 = 0$$

$$2a = -30 - 8 + 16$$

$$2a = -22$$

$$a = -11$$

So, the equation become,

$$\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0 \quad \dots (\text{ii})$$

On solving equation (ii), we get

$$\lambda = 2, 5, -3$$

Maximum eigen value is 5.

Hence, the correct answer is 5.

1.21 3

$$\text{Given : } u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A = uv^T$$

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix} [1 \ 1]$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Characteristic equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) - 2 = 0$$

$$\lambda^2 - 3\lambda = 0$$

$$\lambda = 0, 3$$



So, maximum value is 3.

Hence, the correct answer is 3.

1.22 (D)

Given : Eigenvectors of p are the multiples of $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

Therefore, we can conclude that eigenvectors are not independent.

So, Eigen values are repeated

Statement II is true.

As matrix does not have linearly independent vector so it can not be diagonalized

Statement III is true.

Hence, the correct option is (D).

Key Point

- If the Eigen values of a matrix are distinct, then the Eigen vectors are independent.
- A necessary and sufficient condition for a matrix $A_{n \times n}$ to be diagonalizable is that the matrix must have n linearly independent Eigen vectors.

1.23 (D)

Given : X is square matrix which is invertible and whose determinant $|X| \neq 0$.

We know that, inverse of matrix A exist only when determinant $|A| \neq 0$

So, statement II implies I and I implies II

So, both statement are equivalent statement

Hence, the correct option is (D).

1.24 12

Given : $R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & 64 \\ 1 & 5 & 25 & 125 \end{bmatrix}$$

Performing elementary transformation

$$R_4 \rightarrow R_4 - R_3$$

$$R_3 \rightarrow R_3 - R_2$$

$$R_2 \rightarrow R_2 - R_1$$

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 1 & 7 & 37 \\ 0 & 1 & 9 & 61 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$R_3 \rightarrow R_3 - R_2$$

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 0 & 2 & 18 \\ 0 & 0 & 2 & 24 \end{bmatrix}$$

$$R_4 \rightarrow R_4 - R_3$$

$$R = \begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 1 & 5 & 19 \\ 0 & 0 & 2 & 18 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

Matrix R is an upper triangular matrix so eigen values are principal diagonal elements.

$$\lambda = 1, 1, 2, 6$$

We know that from property of eigen values

Product of Eigen values = Product of leading diagonal elements

$$|R| = 1 \times 1 \times 2 \times 6$$

$$|R| = 12$$

Hence, the correct answer is 12.

Key Point

- Product of Eigen values = $\text{Det}(A) = |A|$
- In any triangular matrix Eigen values are leading diagonal elements itself.

1.25 (C)

Given : A and B are two $n \times n$ matrix

From property of rank of matrices

$$\text{Rank}(AB) = \min(\rho(A), \rho(B))$$

Statement I is false

$$\det(AB) = |A| \cdot |B|$$

Statement II is false



$$\text{Rank}(A+B) \leq \rho(A) + \rho(B)$$

Statement III is true

Let, $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

$$A+B = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, |A|=2, |B|=2$$

$$\det(A+B)=8$$

$$|A|+|B|=4$$

$$\det(A+B) \neq |A|+|B|$$

Statement IV is false

Hence, the correct option is (C).

1.26 3

Given : $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

Characteristic equation is given by

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -\lambda & 1 & 1 & 1 \\ 1 & -\lambda & 1 & 1 \\ 1 & 1 & -\lambda & 1 \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

Performing elementary transformation

$$R_1 \rightarrow R_1 - R_2$$

$$R_2 \rightarrow R_2 - R_3$$

$$R_3 \rightarrow R_3 - R_4$$

$$\begin{vmatrix} -\lambda-1 & 1+\lambda & 0 & 0 \\ 0 & -\lambda-1 & 1+\lambda & 0 \\ 0 & 0 & -\lambda-1 & 1+\lambda \\ 1 & 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(-\lambda-1) \begin{vmatrix} -\lambda-1 & 1+\lambda & 0 \\ 0 & -\lambda-1 & 1+\lambda \\ 1 & 1 & -\lambda \end{vmatrix}$$

$$-(1+\lambda) \begin{vmatrix} 0 & 1+\lambda & 0 \\ 0 & -\lambda-1 & 1+\lambda \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(-\lambda-1) \begin{vmatrix} -\lambda-1 & 1+\lambda & 0 \\ 0 & -\lambda-1 & 1+\lambda \\ 1 & 1 & -\lambda \end{vmatrix}$$

$$+ \begin{vmatrix} 0 & 1+\lambda & 0 \\ 0 & -\lambda-1 & 1+\lambda \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$(-\lambda-1) \begin{vmatrix} -\lambda-1 & 2(1+\lambda) & 0 \\ 0 & 2(-\lambda-1) & 2(1+\lambda) \\ 2 & 2 & -2\lambda \end{vmatrix} = 0$$

$$(1+\lambda) \begin{bmatrix} (-\lambda-1)(4(\lambda+\lambda^2)-4(1+\lambda)) \\ + 2(4(1+\lambda)^2) \end{bmatrix} = 0$$

$$(1+\lambda)[(-\lambda-1)4(\lambda^2-1)+8(1+\lambda)^2] = 0$$

$$(1+\lambda)^3[-4(\lambda-1)+8] = 0$$

$$(1+\lambda)^3(12-4\lambda) = 0$$

$$(1+\lambda)^3 = 0 \Rightarrow \lambda = 0, 0, 0$$

$$\text{And, } 12-4\lambda = 0$$

$$\lambda = \frac{12}{4} = 3$$

Largest eigenvalue of matrix is 3

Hence, the correct answer is 3.

1.27 4

Given : P is 4×5 matrix

Number of rows $R=4$

Number of columns $C=5$

So, number of variable = number of columns = 5

Since, Px is a homogeneous.

Rank = $\min(R, C)$

As number of variables is greater than number of rows

\therefore Rank = number of rows = 4

Hence, the correct answer is 4.

2

Calculus



Practice Questions

2013 IIT Bombay

- 2.1 Which of the following functions is continuous at $x = 3$?

$$(A) f(x) = \begin{cases} 2 & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \\ \frac{x+3}{3} & \text{if } x < 3 \end{cases}$$

$$(B) f(x) = \begin{cases} 4 & \text{if } x = 3 \\ 8-x & \text{if } x \neq 3 \end{cases}$$

$$(C) f(x) = \begin{cases} x+3 & \text{if } x \leq 3 \\ x-4 & \text{if } x > 3 \end{cases}$$

$$(D) f(x) = \frac{1}{x^3 - 27} \quad \text{if } x \neq 3$$

2014 IIT Kharagpur

- 2.2 If $\int_0^{2\pi} |x \sin x| dx = K\pi$, then the value of K is equal to _____.

- 2.3 Let the functions

$$f(\theta) = \begin{vmatrix} \sin \theta & \cos \theta & \tan \theta \\ \sin(\pi/6) & \cos(\pi/6) & \tan(\pi/6) \\ \sin(\pi/3) & \cos(\pi/3) & \tan(\pi/3) \end{vmatrix}$$

Where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ . Which of the following statement is / are TRUE?

I: There exists $\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) = 0$

II: There exists $\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) \neq 0$

- (A) I only
(B) II only
(C) Both I and II

- (D) Neither I nor II

- 2.4 The functions $f(x) = x \sin x$ satisfies the following equations:

$f''(x) + f'(x) + t \cos x = 0$. The value of t is _____.

- 2.5 A functions $f(x)$ is continuous in the interval $[0,2]$. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. which one of the following statements must be true ?

(A) There exists a y in the interval $(0,1)$ such that $f(y) = f(y+1)$

(B) For every y in the interval $(0,1)$, $f(y) = f(2-y)$

(C) The maximum value of the functions in the interval $(0,2)$ is 1

(D) There exists a y in the interval $(0,1)$ such that $f(y) = -f(2-y)$

- 2.6 A non-zero polynomial $f(x)$ of degree 3 has roots at $x=1, x=2$ and $x=3$, Which one of the following must be TRUE?

(A) $f(0)f(4) < 0$

(B) $f(0)f(4) > 0$

(C) $f(0) + f(4) > 0$

(D) $f(0) + f(4) < 0$

- 2.7 The value of the integral given below is

$$\int_0^\pi x^2 \cos x dx$$

(A) -2π (B) π

(C) $-\pi$ (D) 2π

2015 IIT Kanpur

- 2.8 The value of $\lim_{x \rightarrow \infty} (1+x^2)^{e^{-x}}$ is

(A) 0 (B) $\frac{1}{2}$

(C) 1 (D) ∞



- 2.9** $\int_{1/\pi}^{2/\pi} \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx = \underline{\hspace{2cm}}$.

2.10 $\lim_{x \rightarrow \infty} x^{1/x}$ is
 (A) ∞ (B) 0
 (C) 1 (D) Not defined

2.11 Let $f(x) = x^{-(1/3)}$ and A denote the area of the region bounded by $f(x)$ and the X-axis, when x varies from -1 to 1. Which of the following statements is/are True ?
 1. f is continuous [-1,1]
 2. f is not bounded in [-1,1]
 3. A is nonzero and finite
 (A) 2 only (B) 3 only
 (C) 2 and 3 only (D) 1,2 and 3

2.12 If for non-zero x

$$af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 25 \text{ where } a \neq b \text{ then}$$

 $\int_1^2 f(x)dx$ is.
 (A) $\frac{1}{a^2 - b^2} \left[a(\ln 2 - 25) + \frac{47b}{2} \right]$
 (B) $\frac{1}{a^2 - b^2} \left[a(2 \ln 2 - 25) - \frac{47b}{2} \right]$
 (C) $\frac{1}{a^2 - b^2} \left[a(2 \ln 2 - 25) + \frac{47b}{2} \right]$
 (D) $\frac{1}{a^2 - b^2} \left[a(\ln 2 - 25) - \frac{47b}{2} \right]$

2.15 If $f(x) = R \sin\left(\frac{\pi x}{2}\right) + S$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)dx = \frac{2R}{\pi}$, then the constants R and S are, respectively
 (A) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (B) $\frac{2}{\pi}$ and 0
 (C) $\frac{4}{\pi}$ and 0 (D) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

2.16 The value of $\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$
 (A) is 0
 (B) is -1
 (C) is 1
 (D) does not exist

2.17 Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base 'b', The solutions of this equations in the same base 'b' are $x = 5$ and $x = 6$. Then $b = \underline{\hspace{2cm}}$.

2018 IIT Guwahati

2.18 The value of $\int_0^{\pi/4} x \cos(x^2) dx$ correct to three decimal places (assuming that $\pi = 3.14$) is $\underline{\hspace{2cm}}$.

2019 IIT Madras

2.19 Compute $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$
 (A) 1
 (B) 53/12

2016 IISc Bangalore

- 2.13** Let $f(x)$ be a polynomial and $g(x) = f'(x)$ be its derivative. If the degree of $(f(x) + f(-x))$ is 10, then the degrees of $(g(x) - g(-x))$ is

2.14 $\lim_{x \rightarrow 4} \frac{\sin(x-4)}{x-4} =$ _____

2017 IIT Roorkee

- 2.15** If $f(x) = R \sin\left(\frac{\pi x}{2}\right) + S$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x) dx = \frac{2R}{\pi}$, then the constants R and S are, respectively

(A) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (B) $\frac{2}{\pi}$ and 0
 (C) $\frac{4}{\pi}$ and 0 (D) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

2.16 The value of $\lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$

(A) is 0
 (B) is -1
 (C) is 1
 (D) does not exist

2.17 Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficients in a base 'b'. The solutions of this equations in the same base 'b' are $x = 5$ and $x = 6$. Then $b = \underline{\hspace{2cm}}$.

2018 IIT Guwahati

2.18 The value of $\int_0^{\pi/4} x \cos(x^2) dx$ correct to three decimal places (assuming that $\pi = 3.14$) is $\underline{\hspace{2cm}}$.

2019 IIT Madras

2.19 Compute $\lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

(A) 1
 (B) $53/12$
 (C) $108/7$
 (D) Limit does not exist

2020 IIT Delhi



2021 IIT Bombay

2.21 Consider the following expression

$$\lim_{x \rightarrow -3} \frac{\sqrt{2x+22}-4}{x+3}$$

The value of above expression (rounded to 2 decimal places) is _____.

2.22 Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function on the interval $[-3, 3]$ and a differentiable function in the interval $(-3, 3)$ such that for every x in the interval, $f'(x) \leq 2$. If $f(-3) = 7$ then $f(3)$ is at most _____.

Solutions

2.1 (A)

From the given option (A) we have,

$$\text{Left hand limit at } x=3 \text{ is } f(x) = \frac{x+3}{3} = 2 \quad x > 3$$

$$\text{Right hand limit at } x=3 \text{ is } f(x) = x-1 = 2 \quad x < 3$$

Which is equal to $f(x)$ at $x=3$

So $f(x)$ is continuous at $x=3$

Hence, the correct option is (A).

2.2 4

$$\text{Given : } I = \int_0^{2\pi} |x \sin x| dx = K\pi$$

Breaking the limits of integration

$$\int_0^\pi x \sin x dx - \int_\pi^{2\pi} x \sin x dx = K\pi$$

Integrating by parts

$$[x(-\cos x)]_0^\pi - \int_0^\pi 1 \cdot (-\cos x) \cdot dx$$

$$-\left[[x(-\cos x)]_\pi^{2\pi} - \int_\pi^{2\pi} 1 \cdot (-\cos x) \cdot dx \right] = K\pi$$

$$[-x \cos x + \sin x]_0^\pi - [-x \cos x + \sin x]_\pi^{2\pi} = K\pi$$

$$[-\pi \cos \pi + \sin \pi - 0]$$

$$-[-2\pi \cos 2\pi + \sin 2\pi]$$

$$-(-\pi \cos \pi + \sin \pi) = K\pi$$

$$[\pi + 2\pi + \pi + 0] = K\pi$$

$$4\pi = K\pi$$

$$K = 4$$

Hence, the correct answer is 4.

2.3 (C)

$$\text{Given : } f(\theta) = \begin{vmatrix} \sin \theta & \cos \theta & \tan \theta \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) & \tan\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & \tan\left(\frac{\pi}{3}\right) \end{vmatrix}$$

Function $f(\theta)$ is a trigonometric function, so it is continuous and differentiable

When $\theta = \frac{\pi}{6}$, the value of $f(\theta) = 0$, since row 1 and row 2 become identical

When $\theta = \frac{\pi}{3}$, the value of $f(\theta) = 0$, since row 1 and row 3 become identical

$$\text{Therefore } f\left(\frac{\pi}{6}\right) = f\left(\frac{\pi}{3}\right) = 0$$

Thus, the given function is continuous and differentiable in given range and

$$f\left(\frac{\pi}{6}\right) = f\left(\frac{\pi}{3}\right) = 0$$

So, According to Rolle's mean value theorem,
There exist atleast one θ that gives

$$f'(\theta) = 0$$

Also there may exist a θ such that

$$f'(\theta) \neq 0$$

Hence statement I and statement II both are true.

Hence, the correct option is (C).

2.4 - 2

$$\text{Given : } f(x) = x \sin x$$

Differentiating w.r.t. x

$$f'(x) = \sin x + x \cos x$$

Again differentiating w.r.t. x

$$f''(x) = \cos x + \cos x - x \sin x$$



$f''(x) = 2 \cos x - x \sin x$
Given, $f''(x) + f(x) + t \cos x = 0$
So, $2 \cos x - x \sin x + x \sin x + t \cos x = 0$
 $2 \cos x + t \cos x = 0$
 $t = -2$

Hence, the correct answer is -2.

2.5 (A)

Given : A function $f(x)$ is continuous in the interval $[0, 2]$. $f(0) = f(2) = -1$ and $f(1) = 1$.

From Option (A) :

Let us define a new function g :

$$g(y) = f(y) - f(y+1)$$

Since function $f(y)$ is continuous in $[0, 2]$, $g(y)$ would also be continuous in $[0, 1]$

$$g(0) = f(0) - f(1)$$

$$g(0) = -1 - 1 = -2$$

$$g(1) = f(1) - f(2) = 1 + 1 = 2$$

Since $g(y)$ is continuous and goes from negative to positive value in $[0, 1]$, at some point $g(y)$ would be 0 in $(0, 1)$.

So, when, $g(y) = 0 \Rightarrow f(y) = f(y+1)$ for some $y \in (0, 1)$.

Hence, the correct option is (A).

2.6 (A)

Given : $x=1, x=2, x=3$ are roots

Let $f(x) = \alpha(x-1)(x-2)(x-3)$

[α is a constant]

$$f(0) = \alpha(0-1)(0-2)(0-3) = -6\alpha$$

$$f(4) = \alpha(4-1)(4-2)(4-3) = 6\alpha$$

$$f(0) \cdot f(4) = -36\alpha^2 < 0$$

So, $f(0) \cdot f(4) < 0$

Hence, the correct option is (A).

2.7 (A)

Given : $I = \int_0^\pi x^2 \cos x dx$

Integration by parts

$$I = \left[x^2 \int \cos x dx - \int \frac{d}{dx} x^2 \int \cos x dx \right]_0^\pi$$

$$I = \left[x^2 \sin x - \int 2x \sin x dx \right]_0^\pi$$

$$I = \left[x^2 \sin x - 2x \int \sin x dx + \int \frac{d}{dx} 2x \int \sin x dx \right]_0^\pi$$

$$I = \left[x^2 \sin x + 2x \cos x + \int 2(-\cos x) dx \right]_0^\pi$$

$$I = \left[x^2 \sin x + 2x \cos x - 2 \sin x \right]_0^\pi$$

$$I = \left[\pi^2 \sin \pi + 2\pi \cos \pi - 2 \sin \pi - (0 + 2(0) \cos 0 - 2 \sin 0) \right]_0^\pi$$

$$I = [0 - 2\pi - 0 - 0]$$

$$I = -2\pi$$

Hence, the correct option is (A).

2.8 (C)

Given : $f(x) = \lim_{x \rightarrow \infty} (1+x^2)^{e^{-x}}$

Taking log both side

$$\log f(x) = \lim_{x \rightarrow \infty} \log (1+x^2)^{e^{-x}}$$

$$\log f(x) = \lim_{x \rightarrow \infty} e^{-x} \log (1+x^2)$$

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{\log (1+x^2)}{e^x}$$

$$\log f(x) = \frac{\infty}{\infty}$$

For $\frac{\infty}{\infty}$ form, Applying L' Hospital's Rule

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} \log (1+x^2)}{\frac{d}{dx} e^x}$$

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{\frac{1}{1+x^2} (2x)}{e^x}$$

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{2x}{e^x (1+x^2)} = \frac{\infty}{\infty}$$

For $\frac{\infty}{\infty}$ form, Again applying L' Hospital's rule,

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} 2x}{\frac{d}{dx} [e^x (1+x^2)]}$$

$$\log f(x) = \lim_{x \rightarrow \infty} \frac{2}{e^x (2x) + (1+x^2)e^x}$$



$$\log f(x) = \lim_{x \rightarrow \infty} \frac{2}{2xe^x + (1+x^2)e^x}$$

$$\log f(x) = \frac{2}{\infty} = 0$$

$$f(x) = e^0 = 1$$

Hence, the correct option is (C).

2.9 -1

Given : $I = \int_{1/\pi}^{2/\pi} \frac{\cos\left(\frac{1}{x}\right)}{x^2} dx$

$$\text{Let } \frac{1}{x} = t$$

Differentiating with respect to x

$$-\frac{1}{x^2} dx = dt$$

$$\text{When } x = \frac{1}{\pi} \text{ then } t = \pi$$

$$\text{and when } x = \frac{2}{\pi} \text{ then } t = \frac{\pi}{2}$$

$$\therefore I = \int_{\pi}^{\pi/2} -\cos t dt = -[\sin t]_{\pi}^{\pi/2}$$

$$I = -\left[\sin \frac{\pi}{2} - \sin \pi \right]$$

$$I = -[1 - 0] = -1$$

Hence, the correct answer is -1.

2.10 (C)

Given : $y = \lim_{x \rightarrow \infty} x^{1/x}$

Taking log both side

$$\log y = \lim_{x \rightarrow \infty} \log x^{1/x}$$

$$\log y = \lim_{x \rightarrow \infty} \frac{1}{x} \log x \quad \dots (i)$$

$$\log y = \frac{\log \infty}{\infty} = \frac{\infty}{\infty} \text{ indeterminate form}$$

Applying L' Hospital rule on RHS in equation (i)

$$\log y = \lim_{x \rightarrow \infty} \frac{\frac{d}{dx} \log x}{\frac{d}{dx} x}$$

$$\log y = \lim_{x \rightarrow \infty} \frac{1/x}{1} = \lim_{x \rightarrow \infty} \frac{1}{x}$$

$$\log y = \frac{1}{\infty} = 0$$

$$y = e^0$$

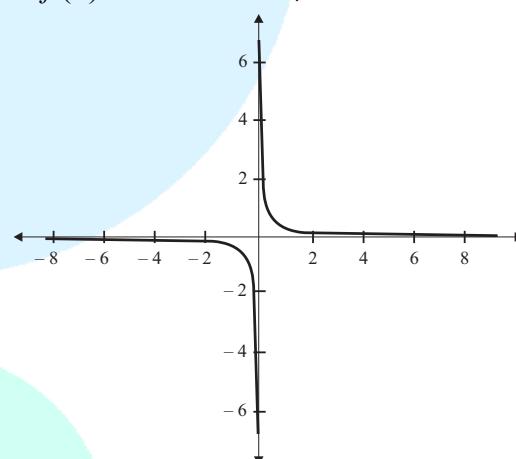
$$\text{So, } y = 1$$

Hence, the correct option is (C).

2.11 (C)

Given : $f(x) = x^{-\frac{1}{3}}$

Plot of $f(x)$ is shown below,



Statement I : f is continuous in $[-1, 1]$

We need to check continuity at $x = 0$

$$\begin{aligned} \text{Left hand limit} &= \lim_{x \rightarrow 0^-} \frac{1}{\sqrt[3]{x}} = \lim_{h \rightarrow 0} \frac{1}{\sqrt[3]{0-h}} \\ &= \lim_{h \rightarrow 0} \frac{-1}{\sqrt[3]{h}} = -\infty \end{aligned}$$

$$\text{Right hand limit} = \lim_{x \rightarrow 0^+} \frac{1}{\sqrt[3]{x}} = \lim_{h \rightarrow 0} \frac{1}{\sqrt[3]{0+h}} = \infty$$

Left hand limit \neq Right hand limit

\therefore Statement 1 is False

Statement II : f is not bounded in $[-1, 1]$. Since at $x = 0$ it goes to $-\infty$ and $+\infty$ so, the function is not bounded

\therefore Statement 1 is True

Statement III : Area $A = \int_{-1}^1 f(x) dx$



$$A = \int_{-1}^1 x^{-1/3} dx = \left[\frac{x^{1-\frac{1}{3}}}{1-\frac{1}{3}} \right]_{-1}^1$$

$$A = \left[\frac{3}{2} x^{2/3} \right]_{-1}^1$$

$$A = \frac{3}{2} + \frac{3}{2} = 3$$

Area is non zero and finite

So, statement III is true

Hence, the correct option is (C).

2.12 (A)

$$\text{Given : } af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 25 \quad \dots (\text{i})$$

Integrating both sides, within limit 1 to 2

$$\begin{aligned} a \int_1^2 f(x) dx + b \int_1^2 f\left(\frac{1}{x}\right) dx &= \int_1^2 \frac{1}{x} - 25 dx \\ a \int_1^2 f(x) dx + b \int_1^2 f\left(\frac{1}{x}\right) dx &= [\log(x) - 25x]_1^2 = \log 2 - 25 \quad \dots (\text{ii}) \end{aligned}$$

Replacing x by $\frac{1}{x}$ in (i), we get

$$af\left(\frac{1}{x}\right) + bf(x) = x - 25$$

Integration both sides, we get

$$\begin{aligned} a \int_1^2 f\left(\frac{1}{x}\right) dx + b \int_1^2 f(x) dx &= \left[\frac{x^2}{2} - 25x \right]_1^2 = -\frac{47}{2} \quad \dots (\text{iii}) \end{aligned}$$

Eliminate $\int_1^2 f\left(\frac{1}{x}\right) dx$ between (ii) and (iii) by multiplying

equation (ii) by a and equation (iii) by b and subtracting

$$(a^2 - b^2) \int_1^2 f(x) dx = a(\log 2 - 25) + b \times \frac{47}{2}$$

$$\int_1^2 f(x) dx = \frac{1}{(a^2 - b^2)} \left[a(\log 2 - 25) + \frac{47b}{2} \right]$$

$$\int_1^2 f(x) dx = \frac{1}{(a^2 - b^2)} \left[a(\log 2 - 25) + \frac{47b}{2} \right]$$

Hence, the correct option is (A).

2.13 9

Given : If $f(x)$ is a polynomial of degree 'n'

Then $f'(x)$ is a polynomial of degree $(n-1)$

$f(x) + f(-x)$ is a polynomial of degree 10

$\therefore g(x) - g(-x)$ is a polynomial of degree 9

Hence, the correct answer is 9.

Key Point

If degree of $\{f(x) + f(-x)\} = n$, then the largest even exponent of x in $f(x) = n$.

If $g(x) = f'(x)$

Then, the largest odd exponent of x in $g(x) = n-1$

\therefore Degree of $\{g(x) - g(-x)\} = n-1$

2.14 1

$$\text{Given : } f(x) = \lim_{x \rightarrow 4} \frac{\sin(x-4)}{x-4}$$

Method 1

Let, $x-4 = t$

$$f(x) = \lim_{x \rightarrow 4} \frac{\sin(x-4)}{x-4} = \lim_{t \rightarrow 0} \frac{\sin t}{t}$$

$\therefore f(x) = 1$

Hence, the correct answer is 1.

Method 2

Also, at $x = 4$ the function

$$f(x) = \frac{\sin 0}{0} = \frac{0}{0}$$

For $\frac{0}{0}$ form, applying L' Hospital's rule, we get

$$f(x) = \lim_{x \rightarrow 4} \left[\frac{\frac{d}{dx} \sin(x-4)}{\frac{d}{dx}(x-4)} \right] = \lim_{x \rightarrow 4} \frac{\cos(x-4)}{1}$$

$\therefore f(x) = \cos 0 = 1$

Hence, the correct answer is 1.

2.15 (C)

$$\text{Given : } f(x) = R \sin\left(\frac{\pi x}{2}\right) + S, f'\left(\frac{1}{2}\right) = \sqrt{2}$$

and $\int_0^1 f(x) dx = \frac{2R}{\pi}$,

differentiating $f(x)$ with respect to x



$$f'(x) = R \cos\left(\frac{\pi x}{2}\right) \times \frac{\pi}{2}$$

$$f'\left(\frac{1}{2}\right) = R \cos\left(\frac{\pi}{4}\right) \times \frac{\pi}{2}$$

$$\sqrt{2} = R \cos\left(\frac{\pi}{4}\right) \times \frac{\pi}{2}$$

$$R = \frac{\sqrt{2} \times \sqrt{2} \times 2}{\pi} = \frac{\pi}{4}$$

$$f(x) = \frac{4}{\pi} \sin\left(\frac{\pi x}{2}\right) + S$$

$$\int_0^1 f(x) dx = \int_0^1 \left[\frac{4}{\pi} \sin\left(\frac{\pi x}{2}\right) + S \right] dx$$

$$\frac{2R}{\pi} = \frac{4}{\pi} \left[-\cos\left(\frac{\pi x}{2}\right) \times \left(\frac{2}{\pi}\right) \right]_0^1 + S [x]_0^1$$

$$\frac{8}{\pi^2} = \frac{8}{\pi^2} (-0+1) + S$$

$$S = 0$$

Hence the correct option is (C).

2.16 (C)

$$\text{Given: } A = \lim_{x \rightarrow 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} \dots (i)$$

$$\text{Put } x=1$$

$$A = \frac{1-2+1}{1-3+2} = \frac{0}{0}$$

For $\frac{0}{0}$ form, applying L' Hospital's rule in equation (i)

$$A = \lim_{x \rightarrow 1} \frac{\frac{d}{dx}(7x^6 - 10x^4)}{\frac{d}{dx}(3x^2 - 6x)}$$

$$A = \lim_{x \rightarrow 1} \frac{7x^6 - 10x^4}{3x^2 - 6x}$$

$$A = \lim_{x \rightarrow 1} \frac{x(7x^5 - 10x^3)}{x(3x-6)}$$

$$A = \frac{7(1)^5 - 10(1)^3}{3(1)-6} = \frac{7-10}{3-6} = \frac{-3}{-3} = 1$$

Hence, the correct option is (C)

2.17 8

$$\text{Given: } x^2 - 13x + 36 = 0 \text{ in base } b$$

Changing to base 10

$$(13)_b = 1 \times b + 3 \times b^0 = b + 3$$

$$(36)_b = 3 \times b + 6 \times b^0 = 3b + 6$$

So, quadratic equation becomes

$$x^2 - (b+3)x + (3b+6) = 0 \dots (i)$$

Since $x=5$ and $x=6$ are solution of this equation

Put $x=5$ in equation (i),

$$5^2 - (b+3)5 + (3b+6) = 0$$

$$25 - 5b - 15 + 3b + 6 = 0$$

$$-2b = -16$$

$$b = 8$$

Hence, the correct answer is 8

2.18 0.289

$$\text{Given: } I = \int_0^{\pi/4} x \cos(x^2) dx$$

$$\text{Let } x^2 = t$$

Differentiating with respect to x

$$2xdx = dt$$

$$xdx = \frac{dt}{2}$$

$$\text{When } x=0 \rightarrow t=0$$

$$\text{and when } x=\frac{\pi}{4} \rightarrow t=\left(\frac{\pi}{4}\right)^2$$

$$I = \int_0^{(\pi/4)^2} \cos t dt$$

$$I = \int_0^{\pi^2/16} \cos t dt = [\sin t]_0^{\pi^2/16}$$

$$I = \sin\left(\frac{\pi^2}{16}\right) - \sin(0)$$

$$I = 0.28898 - 0 \cong 0.289$$

Hence, the correct answer is 0.289

2.19 (C)

$$\text{Given: } y = \lim_{x \rightarrow 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$$

$$y = \frac{(3)^4 - 81}{2(3)^2 - 5(3) - 3} = \frac{0}{0}$$

For $\frac{0}{0}$ form, applying L' Hospital's rule

$$y = \lim_{x \rightarrow 3} \frac{\frac{d}{dx}(x^4 - 81)}{\frac{d}{dx}(2x^2 - 5x - 3)}$$



$$y = \lim_{x \rightarrow 3} \frac{4x^3}{4x-5}$$

$$y = \frac{4 \times (3)^3}{4 \times 3 - 5} = \frac{108}{7}$$

Hence, the correct option is (C)

2.20 (A)

- I. Let, $f(x) = e^{-x}$

$$f'(x) = -e^{-x}$$

$$f'(0) = -1$$

$f'(x) < 0$ on the interval $[0, 1]$, so this is not an increasing function hence, statement I is false

- II. $f(x) = x^2 - \sin x$

$$f'(x) = 2x - \cos x$$

$$\text{at } x=0 \quad f'(0) = 2(0) - 1 = -1 < 0$$

Since in the question, it is asked for increasing everywhere hence II is false.

- III. $\sqrt{x^3 + 1}$

$$f'(x) = \frac{1}{2} \left(\frac{3x^2}{\sqrt{x^3 + 1}} \right) > 0$$

$\Rightarrow f(x)$ is increasing over $[0, 1]$

So, statement III is true

Hence, the correct option is (A).

Key Point

A function $f(x)$ is said to be increasing if $f'(x) > 0$ at each point in an interval.

2.21 0.25

$$\text{Given : } y = \lim_{x \rightarrow -3} \frac{\sqrt{2x+22} - 4}{x+3}$$

$$y = \frac{\sqrt{-6+22} - 4}{-3+3}$$

$$y = \frac{4-4}{-3+3}$$

$$y = \frac{0}{0}$$

For $\frac{0}{0}$ form, applying L'Hospital's rule:

$$y = \lim_{x \rightarrow -3} \frac{\frac{d}{dx}(\sqrt{2x+22} - 4)}{\frac{d}{dx}(x+3)}$$

$$y = \lim_{x \rightarrow -3} \frac{2}{2\sqrt{2x+22}}$$

$$y = \frac{1}{\sqrt{-6+22}} = \frac{1}{\sqrt{16}} = \frac{1}{4} = 0.25$$

Hence, the correct answer is 0.25

2.22 19

Given : The function f is continuous on interval $[-3, 3]$ and differentiable in interval $(-3, 3)$ and $f'(x) \leq 2$.

By using Lagrange's mean value theorem,

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$

Here, $a = -3$ and $b = 3$

$$\text{So, } f'(x) = \frac{f(3) - f(-3)}{3 - (-3)}$$

As $f'(x) \leq 2$ is given

$$2 \geq \frac{f(3) - 7}{3 + 3}$$

$$2 \times 6 \geq f(3) - 7$$

$$f(3) \leq 12 + 7$$

$$f(3) \leq 19$$

Hence, the correct answer is 19.

3

Probability



Practice Questions

2013 IIT Bombay

- 3.1 Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a poison's distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this cars during any given minute in this interval?

(A) $\frac{8}{(2e^3)}$

(B) $\frac{9}{(2e^3)}$

(C) $\frac{17}{(2e^3)}$

(D) $\frac{26}{(2e^3)}$

- 3.2 Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7?

(A) $\frac{13}{90}$

(B) $\frac{12}{90}$

(C) $\frac{78}{90}$

(D) $\frac{77}{90}$

2014 IIT Kharagpur

- 3.3 Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is _____.

- 3.4 Four fair six sided dice are rolled. The probability that the sum of the results being 22 is $\frac{X}{1296}$.
The value of X is _____.

- 3.5 The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by p . Then $100p =$ _____.

- 3.5 Each of the nine words in the sentence "The quick brown fox jumps over the lazy dog" is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is _____.

(The answer should be rounded to one decimal place.)

- 3.6 The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _____.

- 3.7 Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A) P(B)$ is _____

- 3.8 In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

(A) $\frac{3}{23}$

(B) $\frac{6}{23}$

(C) $\frac{3}{10}$

(D) $\frac{3}{5}$

2015

- 3.9 Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $\Pr[X_i = 0] = \Pr[X_i = 1] = 1/2$ for $i = 1, 2, 3$. Define another random variable $Y = X_1 X_2 \oplus X_3$ where \oplus denotes XOR.
Then $\Pr[Y = 0 / X_3 = 0] =$ _____.

2016 IISc Bangalore



- 3.10** A probability density function on the interval $[a, b]$ is given by $\frac{1}{x^2}$ and outside this interval the value of the function is zero. The value of a is _____.

3.11 Consider the following experiment.
Step 1 : Flip a fair coin twice.
Step 2 : If the outcomes are (TAILS, HEADS) then output Y and stop.
Step 3 : If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.
Step 4 : If the outcomes are (TAILS, TAILS), then go to Step 1.
The probability that the output of the experiment is Y is (up to two decimal places) _____.

3.12 Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is _____.

2017 IIT Roorkee

- 3.13** P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(A) $\frac{4}{5}$ (B) $\frac{5}{6}$
(C) $\frac{7}{8}$ (D) $\frac{11}{12}$

- 3.14** If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X + 2)^2]$ equals ____.

3.15 Let X be a Gaussian random variable with mean 0 and variance σ^2 . Let $Y = \max(X, 0)$, where $\max(a, b) = a$ if $a > b$ and $\max(a, b) = b$ if $b \geq a$. Then $E[Y]$ equals ____.

(a, b) is the maximum of a and b. The median of Y is _____.

- 3.16** For any discrete random variable X , with probability mass function

$$P(X = j) = p_j, p_j \geq 0, \quad j \in \{0, \dots, N\}, \text{ and}$$

$\sum_{j=0}^N p_j = 1$ define the polynomial

function $g_x(z) = \sum_{j=0}^N p_j z^j$. For a certain

discrete random variable Y , there exists a scalar $\beta \in [0,1]$ such that

$$g_Y(z) = (1 - \beta + \beta z)^N$$

The expectation of Y is

- (A) $N\beta(1-\beta)$
 - (B) $N\beta$
 - (C) $N(1-\beta)$
 - (D) Not expressible in terms of N and β alone

2018 IIT Guwahati

- 3.17** Consider Guwahati (G) and Delhi (D) whose temperatures can be classified as high (H), medium (M) and low (L). Let $P(H_G)$ denote the probability that Guwahati has high temperature. Similarly, $P(M_G)$ and $P(L_G)$ denotes the probability of Guwahati having medium and low temperatures respectively. Similarly, we used $P(H_D)$, $P(M_D)$ and $P(L_D)$ for Delhi.

The following table gives the conditional probabilities for Delhi's temperature given Guwahati's temperature.

	H_D	M_D	L_D
H_G	0.40	0.48	0.12
M_G	0.10	0.65	0.25
L_G	0.01	0.50	0.49

Consider the first row in the table above. The first entry denotes that if Guwahati has high temperature (H_G) then the probability of Delhi also having a high temperature (H_D) is 0.40; i.e., $P(H_D/H_G) = 0.40$. Similarly, the next two



entries are $P(M_D/H_G) = 0.48$ and $P(L_D/H_G) = 0.12$. Similarly for the other rows. If it is known that $P(H_G) = 0.2, P(M_G) = 0.5$, and $P(L_G) = 0.3$, then the probability (correct to two decimal places) that Guwahati has high temperature given that Delhi has high temperature is _____.

- 3.18** Two people, P and Q , decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q . Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is _____.

2019 IIT Madras

- 3.19** Suppose Y is distributed uniformly in the open interval $(1, 6)$. The probability that the polynomial $3x^2 + 6xY + 3Y + 6$ has only real roots is (rounded off to 1 decimal place) _____.

- 3.20** Two numbers are chosen independently and uniformly at random from the set $\{1, 2, \dots, 13\}$. The probability (rounded off to 3 decimal places) that their 4-bit (unsigned) binary representations have the same most significant bit is _____.

2020 IIT Delhi

- 3.21** For $n > 2$, let $a \in \{0, 1\}^n$ be a non-zero vector. Suppose that x is chosen uniformly at random from $\{0, 1\}^n$. Then the probability that $\sum_{i=1}^n a_i x_i$ is an odd number is _____.

2021 IIT Bombay

- 3.22** In an examination, a student can choose the order in which two questions (QuesA and QuesB) must be attempted.

- If the first question is answered wrong, the student gets zero marks.
- If the first question is answered correctly and the second question is not answered

correctly, the student gets the marks only for the first question.

- If both the questions are answered correctly, the student gets the sum of the marks of the two questions.

The following table shows the probability of correctly answering a question and the marks of the question respectively.

Question	Probability of answering correctly	Marks
QuesA	0.8	10
QuesA	0.5	20

Assuming that the student always wants to maximize her expected marks in the examination, in which order should she attempt the questions and what is the expected marks for that order (assume that the questions are independent)?

- (A) First QuesA and then QuesB. Expected marks 14.
- (B) First QuesB and then QuesA. Expected marks 22.
- (C) First QuesB and then QuesA. Expected marks 14.
- (D) First QuesA and then QuesB. Expected marks 16.

- 3.23** The lifetime of a component of a certain type is a random variable whose probability density function is exponentially distributed with parameter 2. For a randomly picked component of this type, the probability that its lifetime exceeds the expected lifetime (rounded to 2 decimal places) is _____.

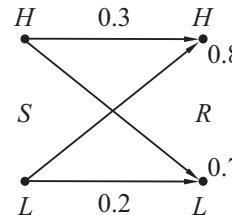
- 3.24** A bag has r red balls and b black balls. All balls are identical except for their colours. In a trial, a ball is randomly drawn from the bag, its colour is noted and the ball is placed back into the bag along with another ball of the same colour. Note that the number of balls in the bag will increase by one, after the trial. A sequence of four such trials is conducted. Which one of the following choices gives the probability of drawing a red ball in the fourth trial?



- (A) $\frac{r}{r+b}$
 (B) $\frac{r+3}{r+b+3}$
 (C) $\frac{r}{r+b+3}$
 (D) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

3.25 For a given biased coin, the probability that the outcome of a toss is head is 0.4. This coin is tossed 1000 times. Let X denotes the random variable whose value is the number of times that head appeared in those 1000 tosses. The standard deviation of X (rounded to 2 decimal points) is _____.

3.26 A sender (S) transmits a signal, which can be one of the two kinds: H and L with probabilities 0.1 and 0.9 respectively, to a receiver (R). In the graph below, the weight of edge (u, v) is the probability of receiving v when u is transmitted, where $u, v \in \{H, L\}$. For example, the probability that the received signal is L given the transmitted signal was H , is 0.7.



If the received signal is H , the probability that the transmitted signal was H (rounded to 2 decimal places) is _____.

3.27 Consider the two statements.

S_1 : There exist random variables X and Y such that

$$E[(X - E(X))(Y - E(Y))]^2 > \text{Var}[X]\text{Var}[Y]$$

S_2 : For all random variables X and Y ,

$$\text{Cov}[X, Y] = E[|X - E[X]| | Y - E[Y]|]$$

Which one of the following choices is correct?

- (A) Both S_1 and S_2 are true
 (B) Both S_1 and S_2 are false
 (C) S_1 is false, but S_2 is true
 (D) S_1 is true, but S_2 is false

Solutions

3.1 (C)

Given that mean = 3, hence by poison distribution,

$$P(r) = \frac{m^r e^{-m}}{r!}$$

Probability of observing fewer than three cars

$$= P(x < 3)$$

$$P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$P(x < 3) = \frac{e^{-m} m^0}{0!} + \frac{e^{-m} m^1}{1!} + \frac{e^{-m} m^2}{2!}$$

$$P(x < 3) = e^{-3} + e^{-3} 3 + \frac{e^{-3} 9}{2}$$

$$P(x < 3) = \frac{e^{-3} 2 + e^{-3} 6 + e^{-3} 9}{2} = \frac{17e^{-3}}{2}$$

Hence, the correct option is (C).

3.2 (D)

Total 2-digit numbers between 1 and 100 is 90 (10 to 99).

$$\text{i.e. } n(S) = 90$$

The numbers between 1 to 100 which are not divisible by 7 are 77.

$$\text{i.e. } n(E) = 77$$

The required probability is,

$$P(E) = \frac{n(E)}{n(S)} = \frac{77}{90}$$

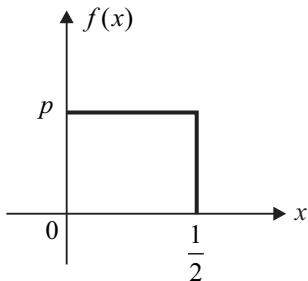
Hence, the correct option is (D).

3.3 0.25

Given: A stick of unit length.

Let ' x ' be the length of the shorter stick.

Now ' x ' is uniformly distribution between 0 to $\frac{1}{2}$



By the property of probability density function,
Area under pdf = 1

$$p \times \frac{1}{2} = 1$$

$$p = 2$$

Hence required expected length

$$E(x) = \int_0^{1/2} x f(x) dx = \int_0^{1/2} 2x dx = \frac{1}{4} = 0.25$$

Hence, the correct answer is 0.25.

3.4 10

Given : Four six face dice is rolled

For the sum to be 22 we have two possible cases

Case (i) : Three 6's and one 4

Number of ways we can obtain this is ${}^4C_3 = 4$ ways

Case (ii) : Two 6's and two 5's

This can be obtained in ${}^4C_2 = 6$ ways

So, required value of X = 6 + 4 = 10

Hence, the correct answer is 10.

3.5 11.9

Given : Total number of computers = 10

Total working computer = 4

P is the probability of picking at least 3 working computer out of four randomly chosen computer

Then,

$$P = P(3 \text{ working computer}) \\ + P(4 \text{ working computer})$$

$$P = \frac{{}^4C_3 {}^6C_1}{{}^{10}C_4} + \frac{{}^4C_4}{{}^{10}C_4}$$

$$P = \frac{25}{210}$$

$$100P = 100 \times \frac{25}{210} = 11.9$$

Hence, the correct answer is 11.9.

3.6 3.88

Given : A nine word sentence is

"The quick brown fox jumps over the lazy dog"

- (3) (5) (5) (3) (5) (4) (3) (4) (3)

Let X be the length of word drawn. So, X has following probability distributions.

X	3	4	5
P(X)	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

Expected length will be $E(X) = \sum X P(X)$

$$E(X) = 3 \times \frac{4}{9} + 4 \times \frac{2}{9} + 5 \times \frac{3}{9}$$

$$E(X) = \frac{12+8+15}{9} = \frac{35}{9} = 3.88$$

Hence, the correct answer is 3.88

3.7 0.26

Number of integers in the set which are divisible by 2 or 3 or 5

$$n(2 \cup 3 \cup 5) = n(2) + n(3) + n(5) - n(2 \cap 3) -$$

$$n(3 \cap 5) - n(5 \cap 2) + n(2 \cap 3 \cap 5)$$

$$n(2 \cup 3 \cup 5) = 50 + 33 + 20 - 16 - 10 - 6 + 3 = 74$$

The number of integers between 1 and 100, which are not divisible by 2 or 3 or 5

$$= 100 - 74 = 26$$

$$\text{So, required probability} = \frac{26}{100} = 0.26$$

Hence, the correct answer is 0.26.

3.8 0.25

Given : S is a sample space and A and B are two mutually exclusive events.

Also, $P(A \cup B) = S$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B)$$

[∵ As A and B are mutually exclusive event $P(A \cap B) = 0$]

$$P(A) + P(B) = 1$$

Let, $P(A) = x$ then $P(B) = 1 - x$



So, $P(A) \cdot P(B) = x(1-x)$

Assume, $f(x) = x(1-x)$

For maxima minima $f'(x) = 0$

$$f'(x) = x - 2x = 0$$

$x = \frac{1}{2}$ is a stationary point.

$$f''(x) = -2 < 0 \text{ at } x = \frac{1}{2}$$

$f(x)$ is maximum at $x = \frac{1}{2}$

$$\text{Maximum value of } f(x) = \frac{1}{2} \left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

$$\text{Maximum value of } P(A) \cdot P(B) = \frac{1}{4} = 0.25$$

Hence, the correct answer is 0.25

3.9 (C)

Given : Number of families = N

Number of family having three children

$$= 50\% \text{ of } N = \frac{N}{2}$$

Number of family having two children

$$= 30\% \text{ of } N = \frac{3N}{10}$$

Remaining families with 1 children

$$= N - \left(\frac{N}{2} + \frac{3N}{10} \right) = \frac{N}{5}$$

The probability that randomly child belongs to family with two children is given by

$$P = \frac{\text{Number of families with 2 children}}{\text{Total number of families}} = \frac{3N}{N} = \frac{3}{10}$$

Hence, the correct option is (C).

3.10 0.75

Given : Random variable X_i for $i = 1, 2, 3$ are independent and identically distributed.

$$P(X_i = 0) = P(X_i = 1) = \frac{1}{2} \text{ for } i = 1, 2, 3 \dots$$

$$Y = X_1 X_2 \oplus X_3$$

$$P\left(\frac{Y=0}{X_3=0}\right) = \text{Probability of } Y=0 \text{ when } X_3=0$$

$$X_3=0$$

$$Y = X_1 X_2 \oplus 0$$

For $Y = 0, X_1 X_2$ Should be zero

Sample space for $X_1 X_2 = \{00, 01, 10, 11\}$

Favorable event when $X_1 X_2 = 0$ are $\{00, 01, 10\}$

$$\text{So, } P\left(\frac{Y=0}{X_3=0}\right) = \frac{\text{Number of favourable event}}{\text{Sample space}}$$

$$P\left(\frac{Y=0}{X_3=0}\right) = \frac{3}{4} = 0.75$$

Hence, the correct answer is 0.75.

3.11 0.5

$$\text{Given : } f(x) = \begin{cases} \frac{1}{x^2}, & \text{for } a \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Area of probability density function = 1

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int_a^1 \frac{1}{x^2} dx = 1$$

$$\left[\frac{-1}{x} \right]_a^1 = 1$$

$$\frac{1}{a} - 1 = 1$$

$$a = \frac{1}{2} = 0.5$$

Hence, the correct answer is 0.5

3.12 0.33

Given : A fair coin is tossed twice. Sample space will be

$$\left\{ \underbrace{HH, HT, TH}_{N} \underbrace{TT}_{Y} \text{ Flip again twice} \right\}$$

Probability of TH when coin is tossed twice

$$= \frac{1}{4}$$

$$\text{So, } P(Y) = \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} + \dots$$

$$P(Y) = \frac{1}{4} \left[1 + \left(\frac{1}{4} \right) + \left(\frac{1}{4} \right)^2 + \dots \right]$$



$$P(Y) = \frac{\frac{1}{4}}{1 - \frac{1}{4}} \quad \left[\because 1 + r + r^2 + \dots = \frac{1}{1-r} \right]$$

$$P(Y) = \frac{1}{3} = 0.33$$

Hence, the correct answer is 0.33.

3.13 0.55

Given : The shop has an equal number of LED bulbs of two different types

Therefore,

Probability of taking type 1 bulb = 0.5

Probability of taking type 2 bulb = 0.5

The probability of an LED bulb lasting more than 100 hrs (100+) given that it is of type 1

$$P(100+ / T_1) = 0.7$$

The probability of an LED bulb taking more than 100 hr, given that it is of type 2 $P(100+ / T_2) = 0.4$

$$\begin{aligned} P(100+) &= P(100+ / T_1) \times P(T_1) \\ &\quad + P(100+ / T_2) P(T_2) \end{aligned}$$

$$P(100+) = 0.5 \times 0.7 + 0.5 \times 0.4$$

$$P(100+) = 0.35 + 0.20$$

$$P(100+) = 0.55$$

Hence, the correct answer is 0.55.

3.14 (A)

Given : Probability that P applies for job $P(P) = \frac{1}{4}$

Probability that P applies for job given that Q applies

$$\text{for job } P\left(\frac{P}{Q}\right) = \frac{1}{2}$$

Probability that Q applies for job given that P applies

$$\text{for job } P\left(\frac{Q}{P}\right) = \frac{1}{3}$$

From conditional probability we know that,

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\text{So } P\left(\frac{P}{Q}\right) = \frac{P(P \cap Q)}{P(Q)} = \frac{1}{2} \dots (\text{i})$$

$$P\left(\frac{Q}{P}\right) = \frac{P(P \cap Q)}{P(P)} = \frac{1}{3}$$

$$P(P \cap Q) = \frac{1}{3} \times P(P) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

So equation (i) becomes

$$P(Q) = \frac{P(P \cap Q)}{1/2} = \frac{1/12}{1/2} = \frac{1}{6}$$

We need to find $P\left(\frac{\bar{P}}{\bar{Q}}\right)$

$$P\left(\frac{\bar{P}}{\bar{Q}}\right) = \frac{P(\bar{P} \cap \bar{Q})}{P(\bar{Q})} = \frac{1 - P(P \cup Q)}{1 - P(Q)}$$

$$= \frac{1 - [P(P) + P(Q) - P(P \cap Q)]}{1 - P(Q)}$$

$$= \frac{1 - \left[\frac{1}{4} + \frac{1}{6} - \frac{1}{12}\right]}{1 - \frac{1}{6}} = \frac{4}{5}$$

$$P\left(\frac{\bar{P}}{\bar{Q}}\right) = \frac{4}{5}$$

Hence, the correct option is (A).

3.15 54

Given : A random variable X has a Poisson distribution with mean $\lambda = 5$

We know that for Poisson distribution

$$E(X) = \text{Var}(X) = \lambda$$

$$\text{So } E(X) = \text{Var}(X) = 5$$

$$E[(X+2)^2] = E[X^2 + 4X + 4]$$

$$E[(X+2)^2] = E[X^2] + 4E[X] + E[4]$$

Since $E[X^2]$ = mean square value

$$\text{And } \text{Var}(X) = E[X^2] - [E[X]]^2$$

$$5 = E[X^2] - 5^2$$

$$E[X^2] = 5 + 25 = 30$$

$$\text{So, } E[(X+2)^2] = 30 + 4 \times 5 + 4$$

$$E[(X+2)^2] = 30 + 24$$

$$E[(X+2)^2] = 54$$

Hence, the correct answer is 54.

3.16 0

Given : X is Gaussian random variable with mean 0 and variance σ^2 .



$$Y = \max(X, 0)$$

The median of Y defined as the least value of m such that

$$P(Y \leq m) > \frac{1}{2}$$

Given, $Y = \max(X, 0)$

$$\text{then : } P(Y \leq 0) = P(Y = 0) = P(X \leq 0) = \frac{1}{2}$$

because X is symmetrically distributed about mean

Therefore, median of Y is 0.

Hence, the correct answer is 0.

3.17 (B)

Given : X is a discrete random variable with probability mass function

$$P(X = j) = P_j, P_j \geq 0 \quad j \in \{0, 1, \dots, N\}$$

$$\sum_{i=0}^N P_i = 1$$

$$\text{Also, } g_x(z) = \sum_{j=0}^N P_j z^j$$

For discrete random variable Y

$$g_y(z) = (1 - \beta + \beta z)^N$$

let $N = 2$

$$g_y(z) = (1 - \beta + \beta z)^2$$

Put $1 - \beta = K$

$$g_y(z) = (K + \beta z)^2 = K^2 + 2\beta K z + \beta^2 z^2$$

Y	0	1	2
$P(Y)$	K^2	$2\beta K$	β^2

$$E(Y) = \sum Y p(Y)$$

$$E(Y) = 0 \cdot K^2 + 1 \cdot 2\beta K + 2\beta^2 = 2\beta(K + \beta)$$

$$E(Y) = 2\beta(1 - \beta + \beta)$$

$$E(Y) = 2\beta$$

$$\text{So, } E[Y] = N\beta$$

Hence, the correct option is (B).

3.18 0.60

$$\text{Given : } P(H_G) = 0.2$$

$$P(M_G) = 0.5$$

$$P(L_G) = 0.3$$

$$P(H_D | H_G) = 0.40$$

$$P(M_D | H_G) = 0.48$$

$$P(L_D | H_G) = 0.12$$

From Baye's theorem

$$P(A / B) = \frac{P(A \cap B)}{P(B)}$$

$$\text{So, } P(H_G | H_D) = \frac{P(H_G \cap H_D)}{P(H_D)}$$

$$P(H_G | H_D) = P(H_D | H_G) \times \frac{P(H_G)}{P(H_D)}$$

$$P(H_G | H_D) = \frac{0.40 \times 0.2}{P(H_D)}$$

Now,

$$P(H_D) = P(H_D | H_G) \times P(H_G)$$

$$+ P(H_D | M_G) \times P(M_G)$$

$$+ P(H_D | L_D) \times P(L_D)$$

$$P(H_D) = 0.4 \times 0.2 + 0.1 \times 0.5 + 0.01 \times 0.3$$

$$P(H_D) = 0.133$$

$$\text{So, } P(H_G | H_D) = \frac{0.4 \times 0.2}{0.133} = 0.6015 \approx 0.60$$

Hence, the correct answer is 0.60.

3.19 0.023

When two dice are thrown independently then sample space will be

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

Given that, the person with lower number wins.

If both get same number, then it will be a tie.

$$P(\text{Tie}) = \frac{6}{36} = \frac{1}{6}$$

[∴ Out of 36 outcomes 6 pairs have equal number]

$$P(\text{No Tie}) = 1 - \frac{1}{6} = \frac{5}{6}$$



To make sure that one of them wins on the third trial, first two trials should be a tie followed by no tie

So, $P(\text{Win}) = P(\text{Tie}) \times P(\text{Tie}) \times P(\text{No tie})$

$$P(\text{Win}) = \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{216} = 0.023$$

Hence, the correct answer is 0.023.

3.20 0.8

Given : Y is uniformly distributed in $(1, 6)$

$$\therefore f(Y) = \begin{cases} \frac{1}{b-a} & 1 < Y < 6 \\ 0 & \text{otherwise} \end{cases}$$

$$f(Y) = \frac{1}{6-1} = \frac{1}{5}$$

Polynomial is $3x^2 + 6xY + 3Y + 6$

For real roots, $b^2 - 4ac \geq 0$

$$(6Y)^2 - 4 \times 3 \times (3Y + 6) \geq 0$$

$$Y^2 - 2Y + Y - 2 \geq 0$$

$$(Y-2)(Y+1) \geq 0$$

$$Y \geq 2$$

$$Y \in [2, 6]$$

$$\text{Now, } p(2 \leq Y < 6) = \int_2^6 f(Y) dy$$

$$p(2 \leq Y < 6) = \int_2^6 \frac{1}{5} dy$$

$$p(2 \leq Y < 6) = \frac{1}{5} [y]_2^6 = \frac{6-2}{5} = 0.8$$

Hence, the correct answer is 0.8

3.20 0.503

Given: Two numbers are chosen randomly from set {1, 2, 3, ..., 13}

4 bit binary representation of this set will be,

Decimal	Binary			
	MSB			LSB
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1

8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1

From above table we can conclude that

[1 to 7] have same MSB bit as 0.

Number of elements = 7

[8-13] have same MSB bit as 1.

Number of elements = 6

From question, it is given that

"Two numbers are chosen independently and uniformly at random".

Now, Probability of picking the number with same

$$MSB(0) = \frac{{}^7C_1 \times {}^7C_1}{13 \times 13} = \frac{49}{169}$$

Probability of picking the number with same

$$MSB(1) = \frac{{}^6C_1 \times {}^6C_1}{13 \times 13} = \frac{36}{169}$$

$$\text{Total probability} = \frac{49}{169} + \frac{36}{169} = \frac{85}{169} = 0.503$$

Hence, the correct answer is 0.503.

3.21 0.5

Given : $a \in \{0, 1\}^n$ $n > 2$

Let $n = 3$

$$a \in \{001, 010, 011, 100, 101, 110, 111\}$$

$$x \in \{000, 001, 010, 011, 100, 101, 110, 111\}$$

Now choose $a = (0, 0, 1)$

Out of 8 combinations of x , only 4 combinations of $a_i x_i$ will produce odd number i.e. (0, 0, 1) (0, 1, 1) (1, 0, 1) (1, 1, 1)

So, probability that $\sum_{i=1}^n a_i x_i$ is an odd number

$$= \frac{4}{8} = 0.5$$

Hence, the correct answer is 0.5

3.22 (D)

Let X be random variable which represents total marks record.

$P(x)$ be probability of getting those marks



$P(\text{answering Ques A correctly}) = 0.8$

$P(\text{answering Ques B correctly}) = 0.5$

X	0	10	20	30
$P(x)$	$0.2 \times 0.5 = 0.1$	$0.8 \times 0.5 = 0.4$	$0.5 \times 0.2 = 0.1$	$0.8 \times 0.5 = 0.4$
$\sum P(x) = 1$				

Case I, if Question A is attempted first and it is correct.

$$E(x) = \Sigma(x)P(x)$$

$$E(x) = 0.4 \times 10 + 0.4 \times 30$$

$$E(x) = 4 + 12 = 16$$

Case II, If Question B is attempted first and is correct.

$$E(x) = \Sigma(x)P(x)$$

$$E(x) = 0.1(20) + 0.4(30)$$

$$E(x) = 2 + 12 = 14$$

So, Case I is giving maximum expected marks.

Hence, the correct option is (D).

3.23 0.367

Given : A component with random variable whose probability distribution function is exponentially distributed with parameter μ .

Let, t = lifetime of component

For exponentially distributed random variable

$$f(t) = \begin{cases} \mu e^{-\mu t}, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{Expected value} = \frac{1}{\text{parameter}} = \frac{1}{\mu}$$

The probability that lifetime of randomly picked component will exceed its expected life time

$$P\left(t > \frac{1}{\mu}\right) = \int_{1/\mu}^{\infty} f(t) dt$$

$$P\left(t > \frac{1}{\mu}\right) = \int_{1/\mu}^{\infty} \mu e^{-\mu t} dt = -\left[e^{-\mu t}\right]_{1/\mu}^{\infty} = -\left[0 - e^{-1}\right]$$

$$P\left(t > \frac{1}{\mu}\right) = \frac{1}{e} = \frac{1}{2.713} = 0.367$$

Hence, the correct answer is 0.367

3.24 (A)

There are 10 favorable ways to calculate the probability of red ball in 4th trial

$$(RBR)R = R(BRR)R = 1 \text{ way}$$

Or $(RRR) = 3 \text{ ways or } (BBB)R = 3 \text{ ways}$

$$P(RRRR) = \frac{r}{r+b} \times \frac{r+1}{r+1+b} \times \frac{r+2}{r+2+b} \times \frac{r+3}{r+3+b} \dots (\text{i})$$

$$P(BBBR) = \frac{b}{r+b} \times \frac{b+1}{r+b+1} \times \frac{b+2}{r+b+2} \times \frac{r}{r+b+3} \dots (\text{ii})$$

$$P(RRBR) = \frac{3!}{2!} \times \frac{r}{r+b} \times \frac{r+1}{r+b+1} \times \frac{b}{r+b+2} \times \frac{r+2}{r+b+3} \dots (\text{iii})$$

$$P(BBRR) = \frac{3!}{2!} \times \frac{b}{r+b} \times \frac{b+1}{r+b+1} \times \frac{r}{r+b+2} \times \frac{r+1}{r+b+3} \dots (\text{iv})$$

$$\text{Required probability} = (\text{i}) + (\text{ii}) + (\text{iii}) + (\text{iv})$$

$$= r(r+1)(r+2)(r+3) + b(b+1)(b+2) \\ = \frac{r+3r(r+1)b(r+2)+3b(b+1)r(r+1)}{(r+b)(r+b+1)(r+b+2)(r+b+3)}$$

On solving it we get,

$$= \frac{r(r+1+b)}{(r+b)(r+b+1)} = \frac{r}{r+b}$$

Hence, the correct option is (A).

3.25 15.49

Given : Probability of head, $p = 0.4$

Probability of tail, $q = 1 - p = 0.6$

Coin is tossed 1000 times, $n = 1000$

Let X is a random variable whose value is number of times head appeared in those 1000 tosses

We know that, for binomial distribution

$$\text{mean} = np$$

$$\text{variance} = npq$$

$$\text{Where } q = 1 - p$$

$$\text{So, standard deviation, } \sigma = \sqrt{npq}$$

$$\sigma = \sqrt{1000 \times 0.4 \times 0.6} = \sqrt{240} = 15.49$$

Hence, the correct answer is 15.49.

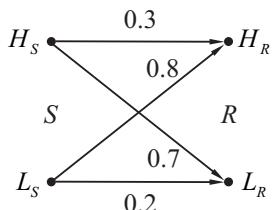


3.26 **0.04**

Given : On sender side

$$P(H_S) = 0.1 \text{ and } P(L_S) = 0.9$$

Probability of received signal is L given transmitted signal is H is $P\left(\frac{L}{H}\right) = 0.7$



If received signal is H, then probability that transmitted was H is

$$P\left(\frac{H_S}{H_R}\right) = \frac{P(H_R / H_S)P(H_S)}{P(H_R)}$$

$$P\left(\frac{H_S}{H_R}\right) = \frac{0.3 \times 0.1}{P(H_R / H_S)P(H_S) + P(H_R / L_S)P(L_S)}$$

$$P\left(\frac{H_S}{H_R}\right) = \frac{0.3 \times 0.1}{0.3 \times 0.1 + 0.8 \times 0.9}$$

$$P\left(\frac{H_S}{H_R}\right) = \frac{0.03}{0.75} = 0.04$$

Hence, the correct answer is 0.04.

3.27 **(B)**

Given :

$$S_2 : Cov[X, Y] = E[|X - E[X]| | Y - E[Y]|]$$

Case-I: If $x > \bar{x}$ and $y > \bar{y}$ then above is true.

Case-II: If $x < \bar{x}$ and $y < \bar{y}$ then above is true.

Case-III: If $x > \bar{x}$ but $y < \bar{y}$ then above is false.

Case-IV: If $y < \bar{y}$ but $y > \bar{y}$ then above is false.

Since given expression is not always true. So we can conclude that it is false.

S_1 : Since true statement is

$$(E[X - E(X)])(E[Y - E(Y)])^2 < Var[X]Var[Y]$$

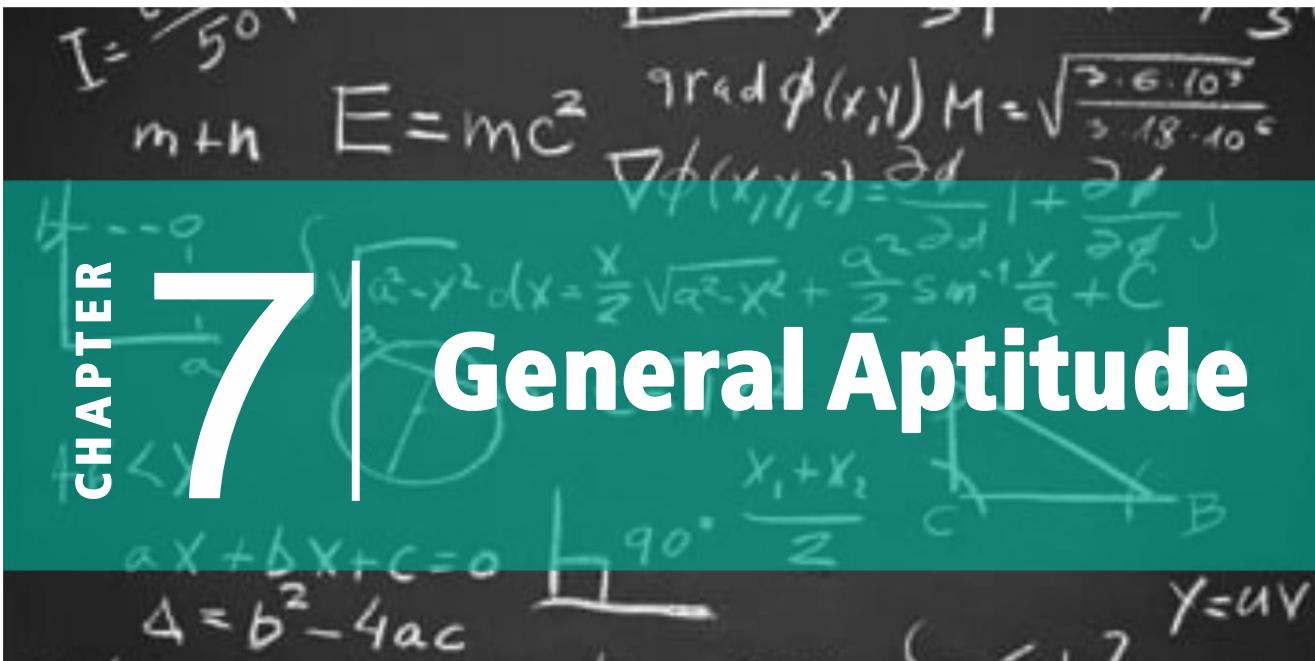
So, both S_1 and S_2 are false

Hence, the correct option is (B).

CHAPTER

7

General Aptitude



Marks Distribution of General Aptitude in Previous Year GATE Papers.

Exam Year	1 Mark Ques.	2 Marks Ques.	Total Marks
2010	5	5	15
2011	5	5	15
2012	5	5	15
2013	5	5	15
2014 Set-1	5	5	15
2014 Set-2	5	5	15
2014 Set-3	5	5	15
2015 Set-1	5	5	15
2015 Set-2	5	5	15
2015 Set-3	5	5	15

Exam Year	1 Mark Ques.	2 Mark Ques.	Total Marks
2016 Set-1	5	5	15
2016 Set-2	5	5	15
2017 Set-1	5	5	15
2017 Set-2	5	5	15
2018	5	5	15
2019	5	5	15
2020	5	5	15
2021 Set-1	5	5	15
2021 Set-2	5	5	15

Syllabus : General Aptitude

Verbal Aptitude :

Basic English grammar: tenses, articles, adjectives, prepositions, conjunctions, verb-noun agreement, and other parts of speech Basic vocabulary: words, idioms, and phrases in context Reading and comprehension Narrative sequencing.

Quantitative Aptitude :

Data interpretation: data graphs (bar graphs, pie charts, and other graphs representing data), 2- and 3-dimensional plots, maps, and tables Numerical computation and estimation: ratios, percentages, powers, exponents and logarithms, permutations and combinations, and series Mensuration and geometry Elementary statistics and probability.

Analytical Aptitude :

Logic: deduction and induction Analogy, Numerical relations and reasoning.

Spatial Aptitude :

Transformation of shapes: translation, rotation, scaling, mirroring, assembling, and grouping Paper folding, cutting, and patterns in 2 and 3 dimensions.

Contents : General Aptitude

S. No. Topics

- 1. Numerical Ability**
- 2. Logical Reasoning**
- 3. Verbal Ability**

1

Numerical Ability



Practice Questions

2013 IIT Bombay

- 1.1 What will be the maximum sum of 44, 42, 40,.....?
(A) 502 (B) 504
(C) 506 (D) 500
- 1.2 Find the sum of the expression
$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

(A) 7 (B) 8
(C) 9 (D) 10
- 1.3 Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is probability that the selected number is not divisible by 7?
(A) $\frac{13}{90}$ (B) $\frac{12}{90}$
(C) $\frac{78}{90}$ (D) $\frac{77}{90}$
- 1.4 A tourist covers half of his journey by train at 60 km/h , half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h . The average speed of the tourist in km/h during his entire journey is
(A) 36 (B) 30
(C) 24 (D) 18
- 1.5 The current erection cost of a structure is Rs.13,200 . If the labour wages per day increase by $\frac{1}{5}$ of the current wages and the working hours decrease by $\frac{1}{24}$ of the current period, then the new cost of erection in Rs. is
(A) 16,500 (B) 15,180
(C) 11,000 (D) 10,120

2014 IIT Kharagpur

- 1.6 At what time between 6 am and 7 am will the minute hand and hour hand of a clock make an angle closest to 60° ?
(A) 6:22 am (B) 6:27 am
(C) 6:38 am (D) 6:45 am
- 1.7 If $\left(z + \frac{1}{z}\right)^2 = 98$, compute $\left(z^2 + \frac{1}{z^2}\right)$.
- 1.8 What is the average of all multiples of 10 from 2 to 198?
(A) 90 (B) 100
(C) 110 (D) 120
- 1.9 The value of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}$ is
(A) 3.464 (B) 3.932
(C) 4.000 (D) 4.444
- 1.10 Which number does not belong in the series below?
2, 5, 10, 17, 26, 37, 50, 64
(A) 17 (B) 37
(C) 64 (D) 26
- 1.11 Consider the equation,
$$(7526)_8 - (Y)_8 = (4364)_8$$
Where $(X)_N$ stands for X to the base N. Find Y.
(A) 1634 (B) 1737
(C) 3142 (D) 3162
- 1.12 Round-trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is Rs 100, a group of 5 tourists purchasing round trip tickets will be charged Rs. _____.
- 1.13 In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether



they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

- 1.14** The table below has question-wise data on the performance of students in an examination. The marks for each question are also listed. There is no negative or partial marking in the examination.

Q. No.	Mar	Answered Cor rect ly	Answered Wr ong ly	Not Atte mpte d
1.	2	21	17	6
2	3	15	27	2
3	2	23	18	3

What is the average of the marks obtained by the class in the examination?

2015 IIT Kanpur

- 1.16** If the list of letters, P, R, S, T, U is an arithmetic sequence, which of the following are also in arithmetic sequence?

 - $2P, 2R, 2S, 2T, 2U$
 - $P - 3, R - 3, S - 3, T - 3, U - 3$

I. $2P, 2R, 2S, 2T, 2U$
 II. $P-3, R-3, S-3, T-3, U-3$

- ### III. P^2, R^2, S^2, T^2, U^2

- 1.17** The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, are listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

Q. No.	Mar	Answered Cor rect ly	Answered Wr ong ly	Not atte mpt ed
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

What is the average of the marks obtained by the class in the examination?

- (A) 0.20 (B) 0.25
(C) 0.30 (D) 0.33

1.19 The probabilities that a student passes in Mathematics, Physics and Chemistry are m , p and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m , p , c

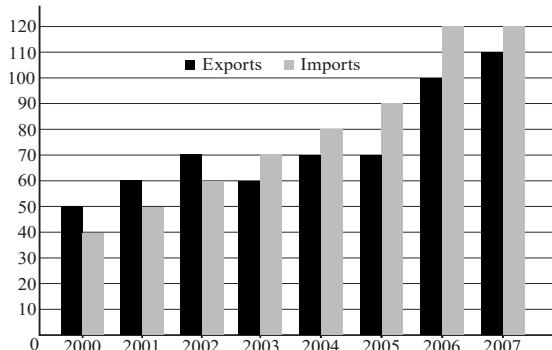
I. $p + m + c = 27/20$
II. $p + m + c = 13/20$
III. $(p) \times (m) \times (c) = 1/10$

(A) Only relation I is true
(B) Only relation II is true
(C) Relations II and III are true
(D) Relations I and III are true

- 1.20** The exports and imports (in crores of Rs.) of a country from the year 2000 to 2007 are given in the following bar chart. In which



year is the combined percentage increase in imports and exports the highest?



2017 IIT Roorkee

right-handed. The number of women at the table is

2018 | IIT Guwahati

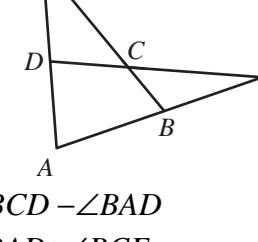


- (C) 720 (D) 0

1.32 In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift Rs 750 to each male senior citizen in the town and Rs 1000 to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only $\frac{8}{9}$ th of the eligible men and $\frac{2}{3}$ rd of the eligible women claimed the gift. How much money (in Rupees) did the philanthropist give away in total?
 (A) 1,50,000 (B) 2,00,000
 (C) 1,75,000 (D) 1,51,000

1.33 If $pqr \neq 0$ and $p^{-x} = \frac{1}{q}$, $q^{-y} = \frac{1}{r}$, $r^{-z} = \frac{1}{p}$, what is the value of the product xyz ?
 (A) -1 (B) $\frac{1}{pqr}$
 (C) 1 (D) pqr

1.34 In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?
 (A) 2 : 3 (B) 1 : 1
 (C) 3 : 2 (D) 2 : 1

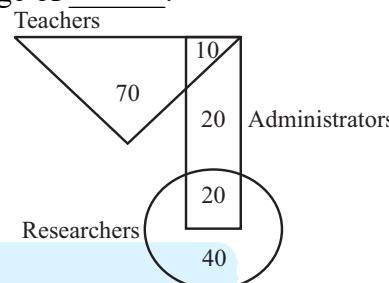
1.35 In the figure below, $\angle DEC + \angle BFC$ is equal to _____.

 (A) $\angle BCD - \angle BAD$
 (B) $\angle BAD + \angle BCF$
 (C) $\angle BAD + \angle BCD$
 (D) $\angle CBA + \angle ADC$

1.36 A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?
 (A) Three green faces and four red faces.
 (B) Four green faces and three red faces.

(C) Five green faces and two red faces.
(D) Six green faces and one red face.

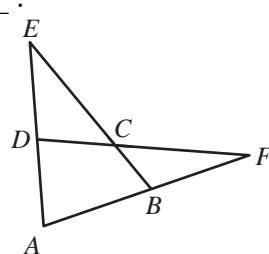
2019 IIT Madras

- 1.37 In the given diagram, teachers are represented in the triangle, researchers in the circle and administrators in the rectangle. Out of the total number of the people, the percentage of administrators shall be in the range of _____. 



- (A) 0 to 15 (B) 16 to 30
(C) 31 to 45 (D) 46 to 60

1.35 In the figure below, $\angle DEC + \angle BFC$ is equal to _____.



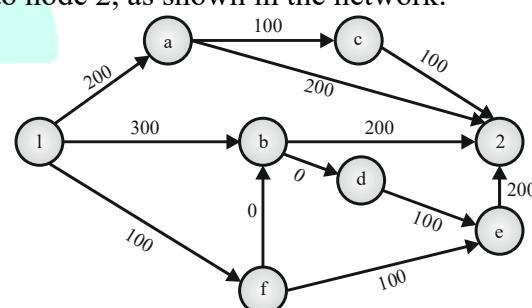
- (A) $\angle BCD - \angle BAD$
 (B) $\angle BAD + \angle BCF$
 (C) $\angle BAD + \angle BCD$
 (D) $\angle CBA + \angle ADC$

- 1.36** A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(A) Three green faces and four red faces.
(B) Four green faces and three red faces.

2020 IIT Delhi

- 1.39** There are multiple routes to reach from node 1 to node 2, as shown in the network.



The cost of travel on an edge between two nodes is given in rupees. Node ‘a’, ‘b’, ‘c’, ‘d’, ‘e’ and ‘f’ are toll booths. The toll price at toll booths marked ‘a’ and ‘e’ is Rs. 200, and is Rs. 100 for the other toll booths. Which is the cheapest route from node 1 to node 2?



(C) 1-a-c-2 (D) 1-f-b-2

2021 IIT Bombay

- 1.40** If $\left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$, then the value of x is

(A) 6 (B) 4
(C) 8 (D) 2

- 1.41** The ratio of boys to girls in class is 7 to 3. Among the options below, an acceptable value for the total number of students in the class is

(A) 21 (B) 73
(C) 37 (D) 50

- 1.42** Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3 : 4. Discount is calculated as the difference between marked price and selling price. The profit percentage is calculated as the ratio of difference between selling price and cost price, to the cost price.

Items	Cost (~)	Profit (%)	Marked Price (~)
P	5400	-	5860
Q	-	25	10000

$$\text{Profit \%} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100$$

The discount on item Q , as a % of its marked price, is _____.

(A) 25 (B) 12.5
(C) 10 (D) 5

- 1.43** The number of students in three classes is in the ratio 3 : 13 : 6. If 18 students are added to each class, the ratio changes to 15 : 35 : 21.

The total number of students in all the three classes in the beginning was

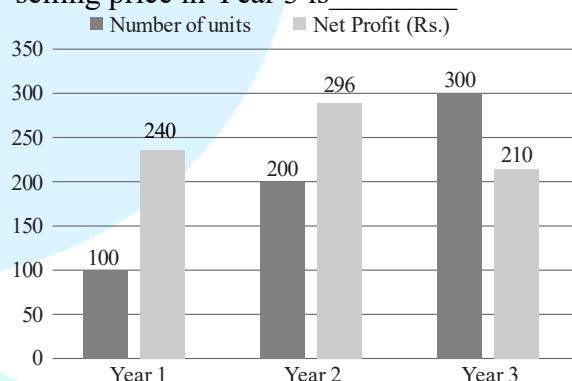
(A) 110 (B) 66
(C) 22 (D) 88

- 1.44** There are five bags each containing identical sets of ten distinct chocolates. One chocolate

is picked from each bag. The probability that at least two chocolates are identical is _____.

(A) 0.8125 (B) 0.6976
(C) 0.3024 (D) 0.4235

- 1.45** The number of units of a product sold in three different years and the respective net profits are presented in the figure above. The cost/unit in Year 3 was Rs.1, which was half the cost/unit in Year 2. The cost/unit in Year 3 was one-third of the cost/unit in Year 1. Taxes were paid on the selling price at 10%, 13% and 15% respectively for the three years. Net profit is calculated as the difference between the selling price and the sum of cost and taxes paid in that year. The ratio of the selling price in Year 2 to the selling price in Year 3 is _____.



(A) 3 : 4 (B) 1 : 1
(C) 1 : 2 (D) 4 : 3

- 1.46** We have 2 rectangular sheets of paper, M and N , of dimensions 6 cm \times 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is _____.

(A) $\frac{9}{\pi}$ (B) 3π
(C) $\frac{\pi}{2}$ (D) $\frac{3}{\pi}$



- 1.47** If θ is the angle, in degrees, between the longest diagonal of the cube and any one of the edges of the cube, then, $\cos\theta =$

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{\sqrt{2}}$
 (C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

Solutions

1.1 (C)

Given : 44, 42, 40 ...

The given series is decreasing by 2. For the sum to be maximum, the series should not include negative numbers. So, the series should be 44, 42, 40, 38 ... 2, 0

$$\text{Sum} = 0 + 2 + 4 \dots 44$$

$$\text{Sum} = 2(1+2+3 \dots 22)$$

Given series is an A.P. series.

Therefore, sum of the series is given by,

$$\text{Sum} = \frac{n}{2}(a+l)$$

where, n = number of terms

a = first term, l = last term

$$\text{Sum} = 2 \times \left[\frac{22}{2}(1+22) \right] = 506$$

Hence, the correct option is (C).

1.2 (B)

$$\text{Let } S = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \dots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

$$S = \left(\frac{\sqrt{2}-\sqrt{1}}{(\sqrt{2})^2 - (\sqrt{1})^2} \right) + \left(\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \right) + \dots + \left(\frac{\sqrt{81}-\sqrt{80}}{(\sqrt{81})^2 - (\sqrt{80})^2} \right)$$

$$S = (\sqrt{2}-\sqrt{1}) + (\sqrt{3}-\sqrt{2}) + \dots + (\sqrt{81}-\sqrt{80})$$

$$S = \sqrt{81} - 1 = 9 - 1 = 8$$

Hence, the correct option is (B).

1.3 (D)

Two digit numbers divisible by 7 are, 14, 21,98.

This is a A.P. series. So,

$$T_n = a + (n-1)d$$

$$98 = 14 + (n-1)7$$

$$\frac{84}{7} = (n-1)$$

$$n = 13$$

There are 13 two digit numbers which are divisible by 7 and between 1 to 100, there are 90 two digit numbers.

$$\text{So, } 90 - 13 = 77$$

Therefore, probability that selected number is not divisible by 7 is,

$$P(E) = \frac{77}{90}$$

Hence, the correct option is (D).

1.4 (C)

Let ' l ' be the total distance.

Average speed,

$$S_{\text{avg}} = \frac{\text{Total distance covered}}{\text{Total time}}$$

$$S_{\text{avg}} = \frac{l}{\frac{l}{2} + \frac{l}{4} + \frac{l}{4}} = \frac{1}{\frac{1}{120} + \frac{1}{120} + \frac{1}{40}}$$

$$S_{\text{avg}} = \frac{120}{5}$$

$$S_{\text{avg}} = 24 \text{ km/h}$$

Hence, the correct option is (C).

1.5 (B)

Let ' X ' be the labour wages and ' R ' be the working hours.

Now, total cost = $X \times R = 13200$



Increase in wages = $\frac{1}{5}$ i.e. 20%

So, revised wages = $X + \frac{X}{5} = \frac{6X}{5}$

$$X' = \frac{6X}{5}$$

Decrease in labour time = $\left(\frac{1}{24}\right)\%$

So, revised time = $\left(R - \frac{R}{24}\right)$

$$R' = \frac{23}{24}R$$

Therefore, revised total cost

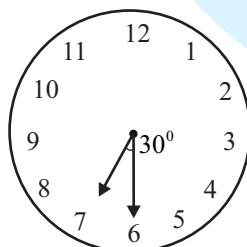
$$X'R' = 1.2X \times \frac{23}{24}R = 1.15XR$$

$$X'R' = 1.15 \times 13200 = 15180$$

Hence, the correct option is (B).

1.6 (A)

Method 1



For any clock the angle between any two consecutive numbers (out of 1 to 12) = 30° .

As hour needle moves in one hour or 60 mins from 6 to 7 i.e. 30° .

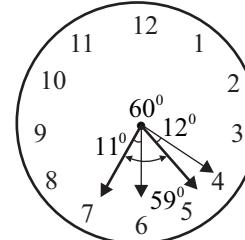
At 6 : 22 am, The hour needle moves by angle

$$\frac{22}{60} \times 30^\circ = 11^\circ$$

The minute needle moves from 4 to 5 i.e. in 5 minutes by 30° .

Thus, within 2 min (from 6 : 20 to 6 : 22), the angle by which it moves

$$= \frac{2}{5} \times 30^\circ = 12^\circ$$



Thus, the angle between two needles at 6:22 am is 59° .

Hence, the correct option is (A).

Method 2

We can also measure the angle by this formula.

$$\text{Angle} = 30H - \frac{11}{2}M$$

From option (A) :

Here, Hour (H) = 6

and minute (M) = 22

$$\text{Hence, Angle} = 30 \times 6 - \frac{11}{2} \times 22 = 59^\circ$$

From option (B) :

$$\text{Angle} = 30 \times 6 - \frac{11}{2} \times 27 = 31.5^\circ$$

From option (C) :

$$\text{Angle} = 30 \times 6 - \frac{11}{2} \times 38 = 29^\circ$$

From option (D) :

$$\text{Angle} = 30 \times 6 - \frac{11}{2} \times 45 = 67.5^\circ$$

Hence, the correct option is (A).

1.7 96

Given : $\left(z + \frac{1}{z}\right)^2 = 98$

$$z^2 + \frac{1}{z^2} + \left(2 \times z \times \frac{1}{z}\right) = 98$$

$$z^2 + \frac{1}{z^2} = 96$$

Hence, the correct answer is 96.

1.8 (B)

The all multiple of 10 from 2 to 198 are

10, 20, 30...190

It is an A.P. series :

First term (a) = 10



Common difference (d) = 10

Number of terms (n) = 19

The sum of A.P. series for n terms is,

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$S_n = \frac{19}{2} [20 + 18 \times 10] = 1900$$

$$\text{Average} = \frac{\text{Sum of terms}}{\text{Number of terms}} = \frac{1900}{19} = 100$$

Hence, the correct option is (B).

1.9 (C)

$$\text{Let } y = \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots \infty}}}$$

$$y = \sqrt{12 + y}$$

$$y^2 = 12 + y$$

$$y^2 - y - 12 = 0$$

$$y = \frac{1 \pm \sqrt{1+48}}{2} = \frac{1 \pm 7}{2} = 4, -3$$

Since, square root of a positive number is always a positive number. Therefore, y should also be a positive number.

Thus, the value of given sum must be,

$$y = 4 \quad [y \neq -ve]$$

Hence, the correct option is (C).

1.10 (C)

Given :

The series 2, 5, 10, 17, 26, 37, 50, 64.

The difference in consecutive numbers is

$$5 - 2 = 3, \quad 10 - 5 = 5, \quad 17 - 10 = 7,$$

$$26 - 17 = 9,$$

$$37 - 26 = 11, \quad 50 - 37 = 13,$$

$$64 - 50 = 14$$

The difference is 3, 5, 7, 9, 11, 13 and thus next difference should be 15 (not 14).

So, the last number should be $50 + 15 = 65$

Which means that 64 doesn't belong to the given series.

Hence, the correct option is (C).

1.11 (C)

Method 1

$$\text{Given : } (7526)_8 - (Y)_8 = (4364)_8$$

$$(Y)_8 = (7526)_8 - (4364)_8$$

$$(7526)_8 \xrightarrow{\text{convert to decimal}}$$

$$\left[7 \times 8^3 + 5 \times 8^2 + 2 \times 8^1 + 6 \times 8^0 \right] = (3926)_{10}$$

$$(4364)_8 \xrightarrow{\text{convert to decimal}}$$

$$\left[4 \times 8^3 + 3 \times 8^2 + 6 \times 8^1 + 4 \times 8^0 \right] = (2292)_{10}$$

Now,

$$\begin{array}{r} (3926)_{10} \\ -(2292)_{10} \\ \hline (1634)_{10} \end{array}$$

$$(1634)_{10} \xrightarrow{\text{convert to octal}}$$

8	1634	
8	204	2
8	25	4
	3	1

$$(y_8) = (3142)_8$$

Hence, the correct option is (C).

Method 2

$$(Y)_8 = (7526)_8 - (4364)_8$$

$$(Y)_8 = \frac{7526}{-4364} = (3142)_8$$

Hence, the correct option is (C).

1.12 850

One way single person fare = Rs. 100

Total two way trip fare for 5 persons

$$= (2 \times 100) \times 5 = \text{Rs.} 1000$$

Round trip (two way trip) discount,

= 10% of the total fare

$$= 10\% \text{ of Rs. } 1000 = \text{Rs. } 100$$

Group discount = 5% of total fare

$$= 5\% \text{ of Rs. } 1000 = \text{Rs. } 50$$

Thus, total discount

$$= \text{Rs. } 100 + \text{Rs. } 50 = \text{Rs. } 150$$

So, the net charge of 5 persons in two way trip

$$= \text{Rs. } 1000 - \text{Rs. } 150 = \text{Rs. } 850$$

Hence, the correct answer is 850.

1.13 48

Total number of persons = 300

Vehicles	Persons (Men + Women)
Car	$40 + 34 = 74$



Scooter	$30 + 20 = 50$
Both car and scooter	$60 + 46 = 106$
Do not own vehicle	$20 + 50 = 70$

Total number of person who do not own a scooter

$$= 74 + 70 = 144$$

% Respondents who do not own a scooter

$$= \frac{\text{Number of person who do not own scooter}}{\text{Total number of person}}$$

$$= \frac{144}{300} \times 100 = 48\%$$

Hence, the correct answer is 48.

1.14 (C)

Given :

Q. No.	Marks	Answered Correctly	Answered Wrongly	Not Attempted
1.	2	21	17	6
2	3	15	27	2
3	2	23	18	3

From the above table :

- (i) For Question No. (1),
Total number of students

$$= 21 + 17 + 6 = 44$$

- (ii) For Question No. (2),
Total number of students

$$= 15 + 27 + 2 = 44$$

- (iii) For Question No. (3),
Total number of students

$$= 23 + 18 + 3 = 44$$

So, there are a total of 44 students only.

Marks obtained by all the 44 students :

- (i) For Question No. (1),

$$= 21 \times 2 = 42$$

- (ii) For Question No. (2),

$$= 15 \times 3 = 45$$

- (iii) For Question No. (3),

$$= 23 \times 2 = 46$$

Average marks

$$= \frac{\text{Total marks}}{\text{Total number of students}} \\ = \frac{42 + 45 + 46}{44} = \frac{133}{44} = 3.02$$

Hence, the correct option is (C).

1.15 (D)

Let gross domestic product during 2012-2013 increases from USD 'x' to USD 'y'.

The exchange rate Rs. 50 /USD changes to Rs. 60 /USD.

So, the GDP increases from $50x$ to $60y$ and increment rate of this is 7%,

$$\text{Thus, } \frac{60y - 50x}{50x} = 0.07$$

$$60y - 50x = 3.5x$$

$$60y = 53.5x$$

$$y = 0.8916x$$

Therefore, the GDP grows at

$$= \frac{y - x}{x} \times 100 = \frac{0.8916x - x}{x} \times 100 \\ = -10.83\%$$

[– ve sign indicates decrease in GDP]

It has decreased by $10.83\% \approx 11\%$

Hence, the correct option is (D).

1.16 (B)

P, R, S, T, U is an arithmetic sequence.

They must have a common difference as,

$$R - P = S - R = T - S = U - T \dots (i)$$

- (i) Multiplying equation (i) by 2,

$$2R - 2P = 2S - 2R = 2T - 2S = 2U - 2T$$

This shows that sequence (I) $\{2P, 2R, 2S, 2T, 2U\}$ is an arithmetic sequence.

- (ii) From sequence (II),

$$(R - 3) - (P - 3) = R - P$$

$$(S - 3) - (R - 3) = S - R$$

$$(T - 3) - (S - 3) = T - S$$

$$(U - 3) - (T - 3) = U - T$$

So, from equation (i),

$$(R - 3) - (P - 3) = (S - 3) - (R - 3)$$

$$= (T - 3) - (S - 3) = (U - 3) - (T - 3)$$

This shows that sequence (II) $\{P - 3, R - 3, S - 3, T - 3, U - 3\}$ is also an arithmetic sequence.

$$\text{As } R - P = S - R$$

- (iii) From sequence (III),

$$R^2 - P^2 = (R - P)(R + P)$$

$$R^2 - P^2 = (S - R)(R + P)$$

$$\text{So, } R^2 - P^2 \neq S^2 - R^2$$



Therefore, sequence (III) is not an arithmetic sequence.

Hence, the correct option is (B).

1.17 (C)

Given :

There are total five questions and there is no negative marking for wrong answer.

Q. No.	Marks	Answered correctly	Total students (Answered correctly + Answered wrongly + Not attempted)	Total marks
1	2	21	$21+17+6 = 44$	42
2	3	15	$15+27+2 = 44$	45
3	1	11	$11+29+4 = 44$	11
4	2	23	$23+18+3 = 44$	46
5	5	31	$31+12+1 = 44$	155

So, there are a total of 44 students only.

Marks obtained by all the 44 students :

- (i) For Question No. (1),
 $= 21 \times 2 = 42$
- (ii) For Question No. (2),
 $= 15 \times 3 = 45$
- (iii) For Question No. (3),
 $= 11 \times 1 = 11$
- (iv) For Question No. (4),
 $= 23 \times 2 = 46$
- (v) For Question No. (5),
 $= 31 \times 5 = 155$

Average marks

$$= \frac{\text{Total marks}}{\text{Total number of students}} \\ = \frac{42+45+11+46+155}{44} = \frac{299}{44} = 6.795$$

Hence, the correct option is (C).

1.18 (A)

Given : Set $A = \{2, 3, 4, 5\}$ ($n_1 = 4$)

Set $B = \{11, 12, 13, 14, 15\}$ ($n_2 = 5$)

Total number of ways to select two numbers

$$= {}^4C_1 \times {}^5C_1 = 4 \times 5 = 20$$

The below table shows the sum of all 20 combination of elements in sets A and B .

Set - B				
Set - A	11	12	13	14
2	13	14	15	16
3	14	15	16	17
4	15	16	17	18
5	16	17	18	19
				20

Thus, the total number of ways (sum = 16) is 4.

So, the required probability $= \frac{4}{20} = \frac{1}{5} = 0.2$

Hence, the correct option is (A).

1.19 (D)

Given :

Probabilities that a student passes in :

Mathematics = m

Physics = p

Chemistry = c

Probability that student passes in **at least one** is 75% = 0.75

$$1 - [\text{Fails in all}] = 0.75$$

$$1 - [\text{Fails in } m \text{ AND fails in } p \text{ AND fails in } c] = 0.75$$

$$1 - [(1-m)(1-p)(1-c)] = 0.75$$

$$(1-m)(1-p)(1-c) = 0.25$$

$$1 - c - m + mc - p + pc + pm - pmc = 0.25$$

$$1 - (p + m + c) + (pc + pm + mc) - pmc = 0.25$$

$$(p + m + c) - (pc + pm + mc) + pmc = 1 - 0.25$$

$$(p + m + c) - (pc + pm + mc) + pmc = 0.75$$

... (i)

Probability that student passing in **at least two** is 50% = 0.5

[(Either passes in Physics and Math)

or (Passes in Physics and Chemistry)

or (Passes in Math and Chemistry)]

or (Passes in all) = 0.5

$$[pm(1-c) + pc(1-m) + mc(1-p)] + pmc = 0.5$$

$$pm + pc + mc - 3pmc + pmc = 0.5$$

$$pm + pc + mc = 0.5 + 2pmc \quad \dots \text{(ii)}$$

Probability that student passing in **exactly two** is 40% = 0.4

$$pm(1-c) + pc(1-m) + mc(1-p) = 0.4$$

$$pm + pc + mc - 3pmc = 0.4$$

$$pm + pc + mc = 0.4 + 3pmc \quad \dots \text{(iii)}$$



From equations (ii) and (iii),

$$0.4 + 3pmc = 0.5 + 2pmc$$

$$pmc = 0.1 = \frac{1}{10} \quad \dots(iv)$$

Thus, relation (III) is true.

And also,

$$pc + pm + mc = 0.4 + (0.1 \times 3) = 0.7 \quad \dots(v)$$

From equations (i), (iv) and (v),

$$p + m + c - 0.7 + 0.1 = 0.75$$

$$p + m + c = 1.35 = \frac{27}{20}$$

Thus, relation (I) is also true.

Hence, the correct option is (D).

1.20 2006

According to the given graph below shown table can be drawn,

In the year 2001,

% Increase in imports,

$$\frac{50 - 40}{40} \times 100\% = 25\%$$

% Increase in exports,

$$\frac{60 - 50}{50} \times 100\% = 20\%$$

In the year 2002,

% Increase in imports,

$$\frac{60 - 50}{50} \times 100\% = 20\%$$

% Increase in exports,

$$\frac{70 - 60}{60} \times 100\% = 16.67\%$$

In the year 2003,

% Increase in imports,

$$\frac{70 - 60}{60} \times 100\% = 16.67\%$$

% Increase in exports,

$$\frac{60 - 70}{70} \times 100\% = -14.2\%$$

In the year 2004,

% Increase in imports,

$$\frac{80 - 70}{70} \times 100\% = 14.2\%$$

% Increase in exports,

$$\frac{70 - 60}{60} \times 100\% = 16.67\%$$

In the year 2005,

% Increase in imports,

$$\frac{90 - 80}{80} \times 100\% = 12.5\%$$

% Increase in exports = 0%

In the year 2006,

% Increase in imports,

$$\frac{120 - 90}{90} \times 100\% = 33.34\%$$

% Increase in exports,

$$\frac{100 - 70}{70} \times 100\% = 42.8\%$$

In the year 2007,

% Increase in imports = 0%

% Increase in exports,

$$\frac{110 - 100}{100} \times 100\% = 10\%$$

Year %	% Increase in imports	% Increase in exports	Total % increase
2001	25%	20%	45%
2002	20%	16.67%	36.67%
2003	16.67%	-14.2%	2.47%
2004	14.2%	16.67%	30.87%
2005	12.5%	0%	12.5%
2006	33.34%	42.8%	76.14%
2007	0%	10%	10%

From above table, it is clear that years **2006** is the year with highest combined % increase in import and export.

Hence, the correct answer is 2006.

1.21 (D)

Given : $X = y \times 162$

Method 1

Checking from the options,

Option (A) :

$$X = 24 \times 162 = 3888$$

$$(X)^{1/3} = 15.7$$

Incorrect option.



Option (B) :

$$X = 27 \times 162 = 4374$$

$$(X)^{1/3} = 16.3$$

Incorrect option.

Option (C) :

$$X = 32 \times 162 = 5184$$

$$(X)^{1/3} = 17.3$$

Incorrect option.

Option (D) :

$$X = 36 \times 162 = 5832$$

$$(X)^{1/3} = 18$$

Hence, the correct option is (D).

Method 2

Take the prime factor of $162 = 2 \times 81 = 2 \times 9 \times 9$

$$= 2 \times 3 \times 3 \times 3 \times 3$$

Hence, to make it perfect cube, the value of y should be $y = 2 \times 2 \times 3 \times 3 = 36$

Hence, the correct option is (D).

1.22 (C)

To find a k -digit number that does NOT contain the digits 0, 5 or 9.

The total number of ways to fill any place

$$= 10 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)$$

Now, if this number does not contain (0, 5, 9) then there are 7 other ways to fill each of the k digits.

These remaining digits are following :

$$1, 2, 3, 4, 6, 7, 8$$

Probability of filling these 7 digits will be,

$$= \left(\frac{7}{10} \right)^k = (0.7)^k$$

Hence, the correct option is (C).

1.23 (A)

Let the number X is 47×10^{28} .

$$\text{Then, } X^3 = 47^3 \times 10^{84} = 103823 \times 10^{84}$$

$$X^3 = 1.03823 \times 10^{89}$$

So, there are 90 i.e. $(1 + 89)$ digits in X^3 .

Hence, the correct option is (A).

1.24 (A)

Given : There are 5 buildings V, W, X, Y and Z .

V is to the West of W : $(V) - W$

Z is to the East of X : $X - (Z)$

Z is also to the West of V : $(X - Z) - V - W$

W is to the West of Y : $(X - Z - V - W) - Y$

The building in the middle is V .

Hence, the correct option is (A).

1.25 (D)

Given : Red socks = 3, Green socks = 4,

Blue socks = 3

The probability of choosing two socks of same

$$\text{colour} = \frac{{}^3C_2 + {}^4C_2 + {}^3C_2}{{}^{10}C_2}$$

$$= \frac{\frac{3!}{2!1!} + \frac{4!}{2!2!} + \frac{3!}{2!1!}}{10!} = \frac{3+6+3}{45}$$

$$= \frac{12}{45} = \frac{4}{15}$$

Hence, the correct option is (D).

1.26 (A)

Given :

(i) Women are not right handed.

So, Women \Rightarrow Left handed.

(ii) Person in immediate right of a woman

\Rightarrow Left handed.

(iii) At least 2 Men, 2 Women and 3 right handed person are seated.

Assuming the least given conditions,

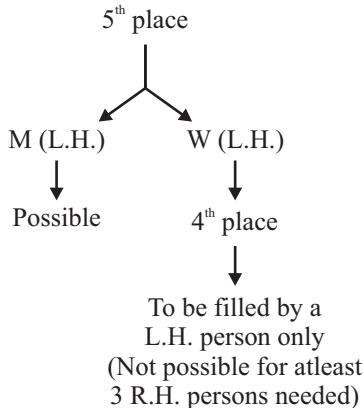
Let 1st place be occupied by a woman, then, 6th place must be occupied by a left handed (L.H.) person.

Again, let this 6th place also be occupied by a woman (W). $[W \rightarrow L.H.]$

Now, minimum 2 women are seated.

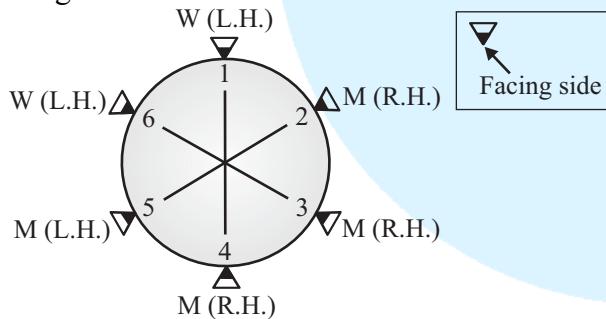
Since, a woman is in 6th place, a left handed person must sit in 5th place.

Following conditions possible for the fifth place,



So, the 5th place be occupied by a man (L.H. only). Since, there should be at least 3 R.H. persons, they can be seated in 2nd, 3rd and 4th places. As a woman cannot be a R.H. person, only R.H. men are seated in 2nd, 3rd and 4th place respectively.

Based on the above discussion, a possible seating arrangement can be as follows :



Thus, only two women can sit at the table.
Hence, the correct option is (A).

1.27 (D)

Given : These are four people (Arun, Gulab, Neel and Sweta) and four colour (Red, Pink, Blue and White).

Total ways of selection without any restriction

$$= 4 + 4 + 4 + 4 = 16$$

[Because every person can choose shirt by 4 ways]

Since, Arun dislikes red.

And Sweta dislikes white.

Therefore, the number of ways to choose the shirts, so that no one has a shirt with colour he or she dislikes

$$= 16 - 2 = 14$$

Hence, the correct option is (D).

1.28 (B)

Given : A test has 20 Questions for 100 marks. There are multiple choice and essay type Questions.

Let, Number of multiple choice questions = X

Then, according to question,

Number of essay questions = $20 - X$

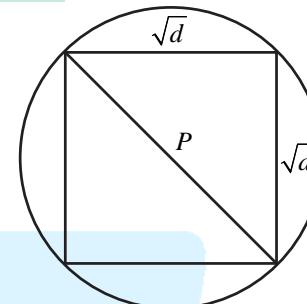
Then, $3X + 11(20 - X) = 100$

$$220 - 8X = 100$$

$$X = \frac{120}{8} = 15$$

Hence, the correct option is (B).

1.29 (D)



Area of square = d

Therefore, side one side of square = \sqrt{d}

Diagonal of square = $\sqrt{d+d} = \sqrt{2d}$

Ara of circle = πr^2

$$= \pi \left(\frac{\sqrt{2d}}{2} \right)^2 = \pi \frac{d}{2} = \frac{1}{2} \pi d$$

Hence, the correct option is (D).

1.30 (C)

Number is divided either by 20 or by $76 K \times \text{LCM}(20, 42, 76) + \text{constant difference}$

$$= 7980 K + 7 \quad (K \text{ is natural number})$$

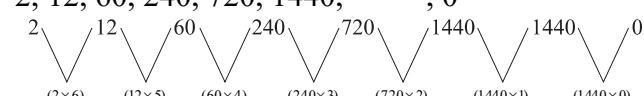
Least number will be $7980 + 7 = 7987$

Hence, the correct option is (C).

1.31 (B)

Given : Sequence :

2, 12, 60, 240, 720, 1440, -----, 0



So, 1440 must be answer.

Hence, the correct option is (B).

1.32 (B)

$$\text{Male} + \text{Female} = 300 \quad \dots (i)$$

$$\text{Total money} = \frac{8}{9} M \times 750 + \frac{2}{3} F \times 1000$$



$$= \frac{6000}{9} \times M + \frac{6000}{9} \times F$$

$$= \frac{6000}{9} \times (M + F)$$

From equation (i),

$$\text{Total money} = \frac{6000}{9} \times (300) = 200000$$

Hence, the correct option is (B).

1.33 (C)

Method 1

Given : $p^{-x} = \frac{1}{q}$, $q^{-y} = \frac{1}{r}$, $r^{-z} = \frac{1}{p}$

Put $x=1$, $y=1$, $z=1$

$$p^{-1} = \frac{1}{q}, \quad q^{-1} = \frac{1}{r}, \quad r^{-1} = \frac{1}{p}$$

i.e. $\frac{1}{p} = \frac{1}{q}$, $\frac{1}{q} = \frac{1}{r}$, $\frac{1}{r} = \frac{1}{p}$

which is true i.e. $\frac{1}{p} = \frac{1}{q} = \frac{1}{r}$

So, $xyz = 1$

Hence, the correct option is (C).

Method 2

$$p^x = q, \quad q^y = r, \quad r^z = p$$

$$x \log p = \log q$$

$$y \log q = \log r$$

$$z \log r = \log p$$

$$x = \frac{\log q}{\log p}$$

$$y = \frac{\log r}{\log q}$$

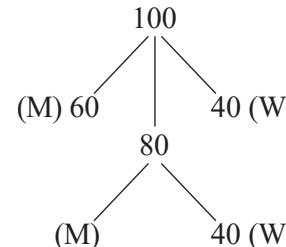
$$z = \frac{\log p}{\log r}$$

$$x \times y \times z = 1$$

Hence, the correct option is (C).

1.34 (B)

Let total number of people are 100.



So, M must be $60 - 40 = 20$

Ratio of male to female

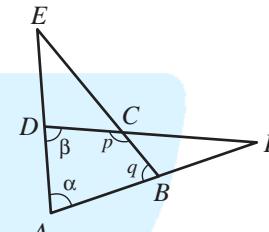
$$40 : 40$$

$$1 : 1$$

Hence, the correct option is (B).

1.35 (A)

Given :



$$\angle E + \angle F = ?$$

$$\alpha + q + E = 180^\circ \quad \dots \text{(i)}$$

$$\alpha + \beta + F = 180^\circ \quad \dots \text{(ii)}$$

$$\alpha + \beta + p + q = 360^\circ \quad \dots \text{(iii)}$$

$$\text{Equation (i)} + \text{(ii)} = \text{(iii)}$$

$$\alpha + q + E + \alpha + \beta + F = \alpha + \beta + p + q$$

$$E + F = p - \alpha$$

Hence, the correct option is (A).

1.36 (C)

For green, two red face

$$P(G) = \frac{4}{6} = \frac{2}{3}$$

$$P(R) = \frac{1}{3}$$

$$n = 7$$

Option A :

$$P(G=3) = {}^7C_3 \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^4$$

$$P(G=3) = \frac{35 \times 2^3}{(3)^7} = \frac{35 \times 2^3}{(3)^7}$$

Option B :

$$P(G=4) = {}^7C_4 \times \left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^3$$



$$P(G=4) = \frac{35 \times 2^4}{(3)^7} = \frac{35 \times 2^4}{(3)^7}$$

Option C :

$$P(G=5) = {}^7C_5 \times \left(\frac{2}{3}\right)^5 \times \left(\frac{1}{3}\right)^2$$

$$P(G=5) = \frac{21 \times 2^5}{(3)^7} = \frac{42 \times 2^4}{(3)^7}$$

Option D :

$$P(G=6) = {}^7C_6 \times \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)$$

$$P(G=6) = \frac{7 \times 2^6}{(3)^7} = \frac{28 \times 2^4}{(3)^7}$$

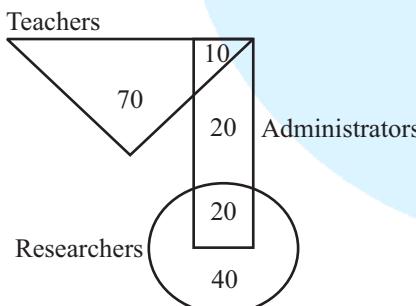
Option C is maximum value.

So, five green faces and two red faces.

Hence, the correct option is (C).

1.37 (C)

Given :



$$\begin{aligned} \text{Total number of people} &= 70 + 10 + 20 + 20 + 40 \\ &= 160 \end{aligned}$$

Out of which number of people who are administrators = $10 + 20 + 20 = 50$

$$\% \text{ of administrators} = \frac{50}{160} \times 100 = 31.25\%$$

Hence, the correct option is (C).

1.38 (B)

Given :

- (i) Speed of first car is 50 km/h.
- (ii) Speed of second car is 60 km/h.

Let us assume first car A is stationary than speed of second car B will be $(60 - 50)\text{ km/h}$
= 10 km/h

So, the number hours required for the distance between A and B to be 20 km

$$= \frac{20 \text{ km}}{10 \text{ km/h}} = 2 \text{ hours}$$

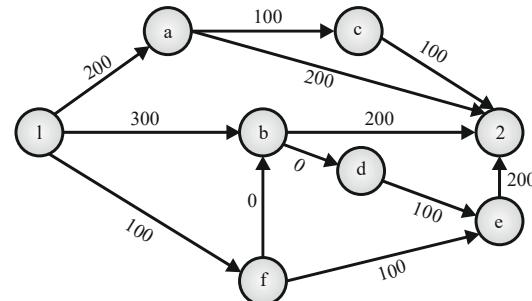
A \rightarrow 20 km \rightarrow B

So, the distance between the two cars will be 20 km in two hours.

Hence, the correct option is (B).

1.39 (D)

Given :



Checking from the options

Option (A) :

$$\begin{aligned} 1 - f - c - 2 &= 100 + 100 \\ + 100 + 200 + 200 &= \text{Rs } 700 \end{aligned}$$

Option (B) :

$$1 - b - 2 = 300 + 100 + 200 = \text{Rs } 600$$

Option (C) :

$$\begin{aligned} 1 - a - c - 2 &= 200 + 200 \\ + 100 + 100 + 100 &= \text{Rs } 700 \end{aligned}$$

Option (D) :

$$\begin{aligned} 1 - f - b - 2 &= 100 + 100 \\ + 0 + 100 + 200 &= \text{Rs } 500 \end{aligned}$$

Clearly, route $1 - f - b - 2$ is the cheapest route from node 1 to node 2.

Hence, the correct option is (D).

1.40 (B)

$$\text{Given : } \left(x - \frac{1}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2 = x + 2$$

$$\begin{aligned} \left[x^2 + \frac{1}{4} - x\right] - \left[x^2 + \frac{9}{4} - 3x\right] &= x + 2 \\ 2x - \frac{8}{4} &= x + 2 \end{aligned}$$

$$\therefore x = 4$$

Hence, the correct option is (B).

Method 2



$$\left(x - \frac{1}{2} - x + \frac{3}{2}\right) \left(x - \frac{1}{2} + x - \frac{3}{2}\right) = x + 2$$

$$2x - 2 = x + 2$$

$$x = 4$$

Hence, the correct option is (B).

1.41 (D)

Given :

Ratio of boys to girls = 7 : 3

i.e. Boys : Girls = 7 : 3

Let Number of boys = $7x$

Let Number of girls = $3x$

\therefore Total number of strength

$$= 7x + 3x = 10x$$

Only 50 satisfies the constraint

Hence, the correct option is (D).

1.42 (C)

Given :

Ratio of the cost price of P to Q = 3 : 4

Cost of P = $3x = 5400$

$$x = 1800$$

Cost of Q = $4x = 7200$

Profit % of Q = 25%

Selling price of

$$Q = 7200(\text{C.P.}) + 7200 \times 25\% (\text{Profit}) \\ = 9000$$

$$\text{Discount \%} = \frac{MP - SP}{MP} \times 100\%$$

$$\text{Discount \% for } Q = \frac{1000}{10000} \times 100\% \\ = 10\%$$

Hence, the correct option is (C).

1.43 (D)

Given :

The ratio of number of students of three classes is = 3 : 13 : 6

Let the number of students in each class is $3x, 13x, 6x$

Therefore the ratio will become $3x : 13x : 6x$

After implementing the given condition,

$$3x + 18 = 15y \quad \dots(\text{i})$$

$$13x + 18 = 35y$$

$$x + 18 = 21y \quad \dots(\text{ii})$$

$$2 \times 3x + 18 \times 2 = 15y \times 2$$

$$6x + 18 = 21y$$

$$6x + 36 = 30y$$

$$6x + 18 = 21y$$

$$x = 4 \qquad \qquad \qquad y = 2$$

\therefore The total number of students in all the three classes in the beginning

$$= 22x = 22 \times 4 = 88$$

Hence, the correct option is (D).

1.44 (B)

Given : Five bags each containing identical sets of ten distinct chocolates

Let, $P(\bar{x})$ denotes No two chocolates are identical

$$P(\bar{x}) = \frac{10 \times 9 \times 8 \times 7 \times 6}{10^5} = \frac{30240}{10^5} = 0.3024$$

Now for at least two chocolates are identical

$$= P(x)$$

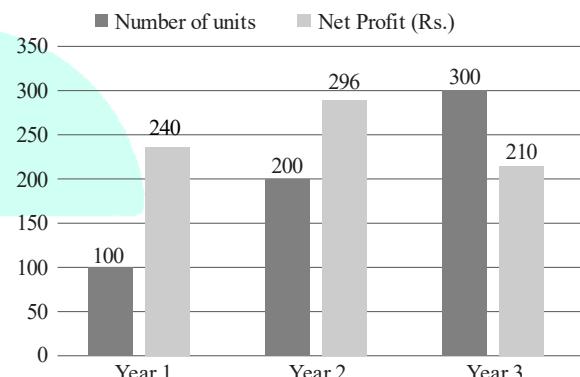
$$P(x) = 1 - P(\bar{x})$$

$$P(x) = 1 - 0.3024 = 0.6976$$

Hence, the correct option is (B).

1.45 (D)

Given :



Cost per unit of year 3 = Rs.1

and Cost per unit of year 3

$$= \frac{\text{cost per unit of year 2}}{2}$$

So, cost per unit of year 2

$$= 2 \times \text{cost per unit of year 3}$$

$$= 2 \times 1 = 2.$$

Let, Selling price of year 2 = SP2 and

Selling price of year 3 = SP3.



We have taxes in year 2 and year 3 as 13% and 15% of selling price respectively.

$$\text{So, Tax in year 2} = 13 \times \frac{\text{SP2}}{100} = 0.13 \times \text{SP2}$$

$$\text{Tax in year 3} = 15 \times \frac{\text{SP3}}{100} = 0.15 \times \text{SP3}$$

Profit in year 2 = Selling price in year 2 – (cost of all units + tax in year 2)

$$296 = \text{SP2} - (200 \times 2 + 0.13 \times \text{SP2})$$

$$296 = \text{SP2} - 400 - 0.13 \times \text{SP2}$$

$$296 + 400 = 0.87 \times \text{SP2}$$

$$696 \times \frac{100}{87} = \text{SP2}$$

$$\text{SP2} = 800.$$

Profit in year 3 = Selling price in year 3 – (cost of all units + tax in year 3)

$$210 = \text{SP3} - (300 \times 1 + 0.15 \times \text{SP3})$$

$$210 = \text{SP3} - 300 - 0.15 \times \text{SP3}$$

$$210 + 300 = 0.85 \times \text{SP3}$$

$$510 \times \frac{100}{85} = \text{SP3}$$

$$\text{SP3} = 600$$

So, ratio of Selling Price in year 2 to Selling Price in year 3

$$= \frac{800}{600} = \frac{4}{3}$$

Hence, the correct option is (D).

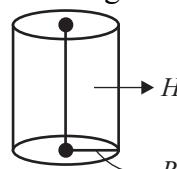
1.46 (A)

Given :

The dimension of rectangular sheet M and N = $6 \text{ cm} \times 1 \text{ cm}$

According to the question,

M is folded along shortest side to form a cylinder,



$$2\pi R = 6 \text{ cm}$$

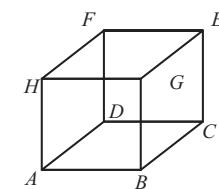
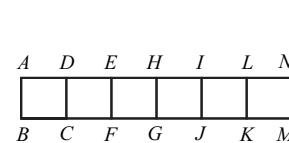
$$R = \frac{3}{\pi} \text{ cm}$$

Volume of a right circular cylinder = $\pi R^2 H$

$$\text{Volume of folded figure} = \pi \times \frac{3}{\pi} \times \frac{3}{\pi} \times 1$$

$$= \frac{9}{\pi} \text{ cm}^3$$

N is cutted as square to form a cube i.e.



Since, Volume of cube = $(\text{Side})^3$

Thus, Volume of formed cube from N = 1 cm^3

The ratio of volume of cylinder to cube

$$= \frac{9/\pi}{1} = 9 : \pi$$

Thus, ratio of the volume of the cylinder to that of the cube is $9 : \pi$.

Hence, the correct option is (A).

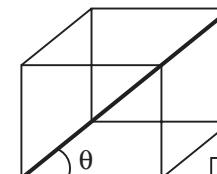
Key Point

Volume of a right circular cylinder = $\pi R^2 H$

Volume of cube = $(\text{Side})^3$

1.47 (A)

Given :



$$\cos \theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Diagonal of a cube = $\sqrt{3}a$

Adjacent side = Each side = a

$$\cos \theta = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Hence, the correct option is (A).

Key Point

Diagonal of a cube = $\sqrt{3}a$

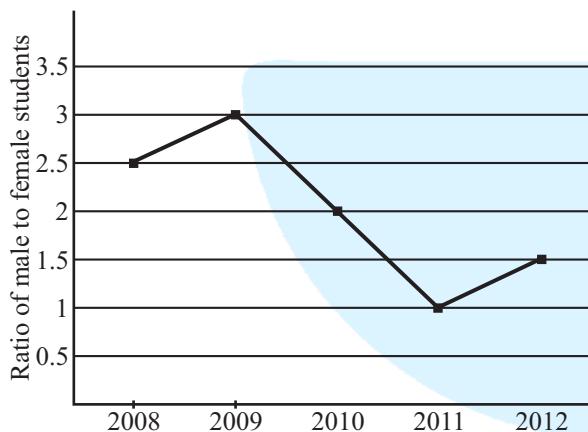
$$\cos \theta = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$



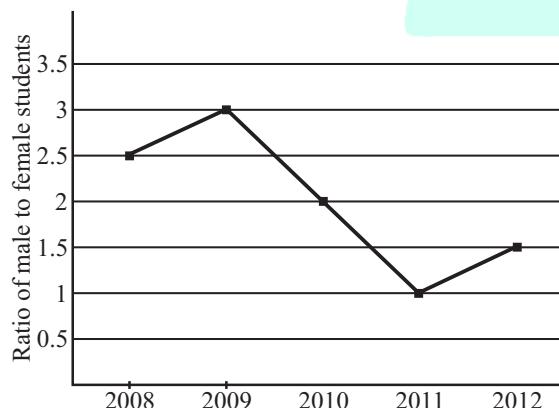
Practice Questions

2014 IIT Kharagpur

- 2.1 The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what percent did the number of male students increase in 2009?



- 2.2 The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students in 2011 and 2012 is equal, what is the ratio of male students in 2012 to male students in 2011?



- (A) 1 : 1 (B) 2 : 1
 (C) 1.5 : 1 (D) 2.5 : 1
- 2.3 The roots of $ax^2 + bx + c = 0$ are real and positive. a, b and c are real. Then $ax^2 + b|x| + c = 0$ has
- (A) No roots (B) 2 real roots

(C) 3 real roots (D) 4 real roots

- 2.4 When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? _____.
- 2.5 If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include
- (A) 2, 4 (B) 2, 14
 (C) 4, 52 (D) 14, 52

2015 IIT Kanpur

- 2.6 Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements :

- Each step is $\frac{3}{4}$ foot high.
 - Each step is 1 foot wide.
- (A) Statement I alone is sufficient, but statement II alone is not sufficient.
 (B) Statement II alone is sufficient, but statement I alone is not sufficient.
 (C) Both statements together are sufficient, but neither statement alone is sufficient.
 (D) Statement I and II together are not sufficient.

- 2.7 The given statement is followed by some courses of action. Assuming the statement to be true decide the correct option.

Statement :

There has been a significant drop in the water level in the lakes supplying water to the city.

- The water supply authority should impose a partial cut in supply to tackle the situation.



- II. The government should appeal to all the residents through mass media for minimal use of water.

III. The government should ban the water supply in lower areas.

(A) Statements I and II follow.
(B) Statements I and III follow.
(C) Statements II and III follow.
(D) All statements follow.

2.8 The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5 : 4. There are 40 males in Electrical Engineering. What is the difference between the number of female students in the Civil department and the female students in the Mechanical department?

Department	Percentage
Computer Science	40%
Civil	30%
Electrical	20%
Mechanical	10%

2.9 Based on the given statements, select the most appropriate option to solve the given question.

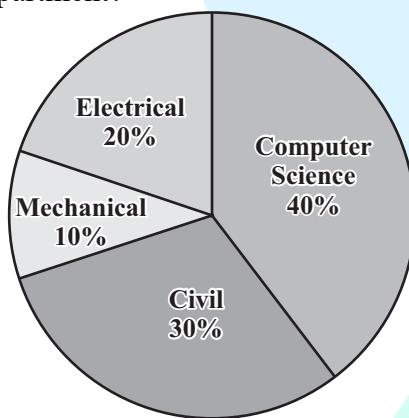
What will be the total weight of 10 poles each of same weight?

Statements :

I. One fourth of the weight of a pole is 5 kg.
II. The total weight of these poles is 160 kg more than the total weight of two poles.

(A) Statement I alone is not sufficient.
(B) Statement II alone is not sufficient.
(C) Either I or II alone is sufficient.
(D) Both statement I and II together are not sufficient.

2.10 Four branches of a company are located at *M, N, O* and *P*. *M* is north of *N* at a distance



- of 4 km; P is south of O at a distance of 2 km; N is south-east of O by 1 km. What is the distance between M and P in km?

- 2.11** If ROAD is written as URDG, then SWAN should be written as

- 2.12** The head of a newly formed government desires to appoint five of the six selected members P, Q, R, S, T and U to portfolios of Home, Power, Defense, Telecom and Finance. U does not want any portfolio if S gets one of the five. R wants either Home or Finance or no portfolio. Q says that if S gets either Power or Telecom, then she must get the other one. T insists on a portfolio if P gets one.

Which is the valid distribution of portfolio?

(A) P-Home, Q-Power, R-Defense,
S-Telecom, T-Finance

(B) R- Home, S-Power, P-Defense
O-Telecom, T-Finance

(C) P- Home, Q-Power, T-Defense,
S-Telecom, U-Finance

(D) Q- Home, U-Power, T-Defense
R-Telecom, P-Finance

- 2.13** Consider a function $f(x) = 1 - |x|$ in $-1 \leq x \leq 1$. The value of x at which the function attains a maximum, and the maximum value of the function are :

- ? 14 If p, q, r, s are distinct integers such that

$$f(p, q, r, s) \equiv \max(p, q, r, s)$$

$$g(p, q, r, s) = \min(p, q, r, s)$$

$k(p, q, r, s)$ = remainder of

$n(p, q, r, s)$ – remainder of

$$(p \times q) / (r \times s) \approx$$

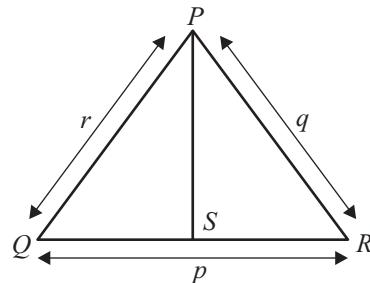
remainder of $(r \times s) / (p \times q)$
 if $(r \times s) > (p \times q)$
 Also a function $fg h(p, q, r, s) = f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$

Also the same operations are valid with two variable function of the form $f(p, q)$.



What is the value of $fg(h(2, 5, 7, 3), 4, 6, 8)$?

- 2.15** In a triangle PQR , PS is the angle bisector of $\angle QPR$ and $\angle QPS = 60^\circ$. What is the length of PS ?

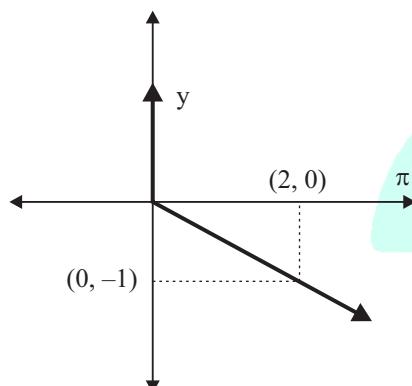


- (A) $\frac{(q+r)}{qr}$ (B) $\frac{qr}{(q+r)}$
 (C) $\sqrt{(q^2+r^2)}$ (D) $\frac{(q+r)^2}{qr}$

- 2.16** A function $f(x)$ is linear and has a value of 29 at $x = -2$ and 39 at $x = 3$. Find its value at $x = 5$.

- (A) 59 (B) 45
 (C) 43 (D) 35

- 2.17** Choose the most appropriate equation for the function drawn as a thick line, in the plot below.



- (A) $x = y - |y|$ (B) $x = -(y - |y|)$
 (C) $x = y + |y|$ (D) $x = -(y + |y|)$

2016 IISc Bangalore

- 2.18** Among 150 faculty members in an institute, 55 are connected with each other through Facebook® and 85 are connected through WhatsApp®. 30 faculty members do not have Facebook® or WhatsApp® accounts.

The number of faculty members connected only through Facebook® accounts is _____.

- (A) 35 (B) 45
 (C) 65 (D) 90

- 2.19** Pick the odd one from the following options.

- (A) CADBE (B) JHKIL
 (C) XVYWZ (D) ONPMQ

- 2.20** In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?



- (A) 21 (B) 27
 (C) 30 (D) 36

- 2.21** If 'relftaga' means carefree, 'otaga' means careful and 'fertaga' means careless, which of the following could mean 'aftercare'?

- (A) Zentaga (B) Tagafer
 (C) Tagazen (D) Relffer

- 2.22** A shaving set company sells 4 different types of razors, Elegance, Smooth, Soft and Executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78 and Executive at Rs. 173 per piece. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter/ Product	Eleganc e	Smoot h	Soft	Executiv e
Q1	27300	20009	1760	9999
Q2	25222	19392	1844	8942
Q3	28976	22429	1954	10234
Q4	21012	18229	1659	10109

Which product contributes the greatest fraction to the revenue of the company in that year?

- (A) Elegance (B) Executive
 (C) Smooth (D) Soft



- 2.23** Consider the following statements relating of the level of poker play of four players P , Q , R and S .

- I. P always beats Q
 - II. R always beats S
 - III. S loses to P only sometimes
 - IV. R always loses to Q

Which of the following can be logically inferred from the above statements?

- (i) P is likely to beat all the three other players
 - (ii) S is the absolute worst player in the set

(A) (i) only

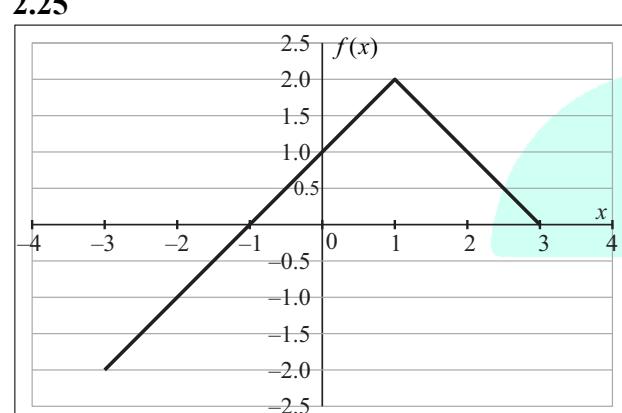
(B) (ii) only

(C) (i) and (ii)

(D) neither (i) nor (ii)

- 2.24** In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of

(A) n^4 (B) 4^n
 (C) 2^{2n-1} (D) 4^{n-1}



Choose the correct expression for $f(x)$ given in the graph.

- (A) $f(x) = 1 - |x - 1|$
 (B) $f(x) = 1 + |x - 1|$
 (C) $f(x) = 2 - |x - 1|$
 (D) $f(x) = 2 + |x - 1|$

- 2.26** A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The

resulting surface area of the body (in square units) after the removal is _____.

- 2.27** If $f(x) = 2x^7 + 3x - 5$, which of the following is a factor of $f(x)$?

- (A) $(x^3 + 8)$ (B) $(x - 1)$
 (C) $(2x - 5)$ (D) $(x + 1)$

- 2.28** In a process, the number of cycles of failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles of failure. When the load is halved, it takes 10000 cycles for failure. The load for which the failure will happen in 5000 cycles is

2017 IIT Roorkee

- 2.29** Rahul, Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali, Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?

- (A) Rahul and Murali
 - (B) Srinivas and Arul
 - (C) Srinivas and Murali
 - (D) Srinivas and Rahul

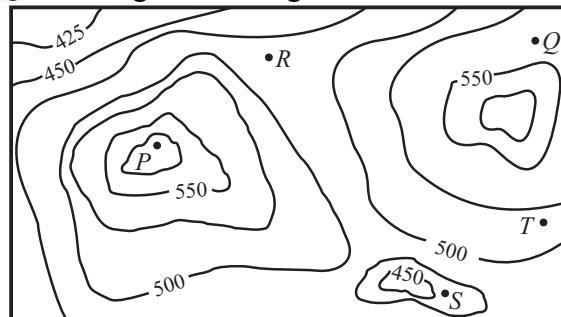
- 2.30** There are three boxes. One contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?

- (A) The box labelled ‘Apples’
 - (B) The box labelled ‘Apples and Oranges’
 - (C) The box labelled ‘Oranges’
 - (D) Cannot be determined

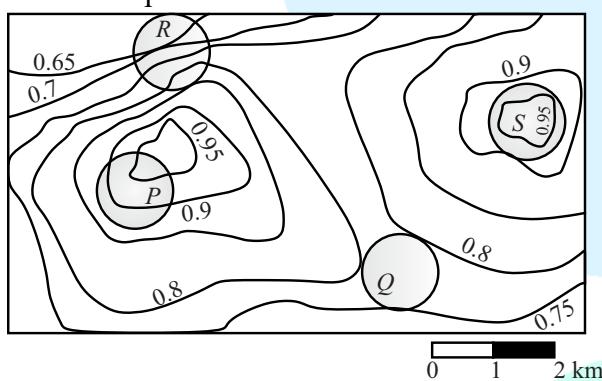
- 2.31** A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood the water



level rises to 525 m, which of the villages P , Q , R , S , T gets submerged?



- (A) P, Q (B) P, Q, T
 (C) R, S, T (D) Q, R, S
- 2.32** An air pressure contour line joins locations in a region having the same atmospheric pressure. The following is an air pressure contour plot of a geographical region. Contour lines are shown at 0.05 bar intervals in this plot.



If the possibility of a thunderstorm is given by how fast air pressure rises or drops over a region, which of the following regions is most likely to have a thunderstorm?

- (A) P (B) Q
 (C) R (D) S

- 2.33** The expression $\frac{(x+y)-|x-y|}{2}$ is equal to

- (A) the maximum of x and y
 (B) the minimum of x and y
 (C) 1
 (D) None of the above

- 2.34** The number of roots of $e^x + 0.5x^2 - 2 = 0$ in the range $[-5, 5]$ is

- (A) 0 (B) 1
 (C) 2 (D) 3

2.35 The police arrested four criminals – P , Q , R and S . The criminals knew each other. They made the following statements :

- P says “ Q committed the crime”
 Q says “ S committed the crime”
 R says “I did not do it”
 S says “What Q said about me is false”

Assume only one of the arrested four committed the crime and only one of the statements made above is true. Who committed the crime?

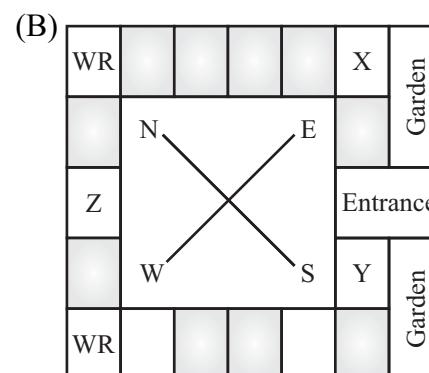
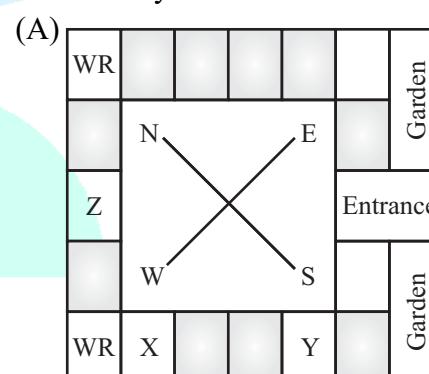
- (A) P (B) R
 (C) S (D) Q

- 2.36** Three of the five students allocated to a hostel put in special requests to the warden. Given the floor plan of the vacant rooms, select the allocation plan that will accommodate all their requests.

Request by X: Due to pollen allergy, I want to avoid a wing next to the garden.

Request by Y : I want to live as far from the washrooms as possible, since I am very sensitive to smell.

Request by Z: I believe in Vaastu and so want to stay in the South – west wing.





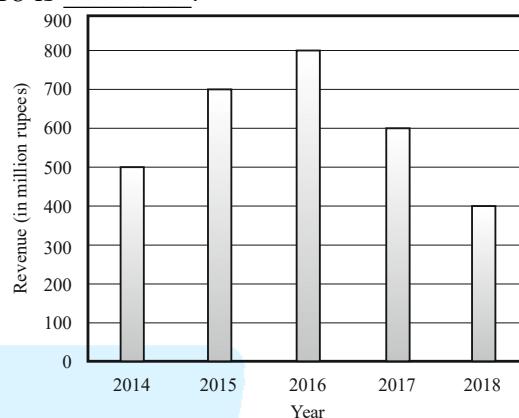
(C)

(D)

WR	X	Y	Z	Garden
	N	E	Entrance	Garden
	W	S		Garden
WR	X			Garden

2020 IIT Delhi

- 2.40** The total revenue of a company during 2014–2018 is shown in the bar graph. If the total expenditure of the company in each year is 500 million rupees, then the aggregated profit or loss (in percentage) on the total expenditure of the company during 2014–2018 is .



- (A) 20% profit (B) 20% loss
(C) 16.67 % loss (D) 16.67% profit

2.41 The dawn of the 21st century witnessed the melting glaciers oscillating between giving too much and too little to billions of people who depend on them for fresh water. The UN climate report estimates that without deep cuts to man-made emissions, at least 30% of the northern hemisphere's surface permafrost could melt by the end of the century. Given this situation of imminent global exodus of billions of people displaced by rising seas, nation-states need to rethink their carbon footprint for political concerns, if not for environmental ones.

Which one of the following statements can be inferred from the given passage?

- (A) Nation-states do not have environmental concerns.
 - (B) Billions of people are responsible for man-made emissions.
 - (C) Billions of people are affected by melting glaciers.
 - (D) Nation-states are responsible for providing fresh water to billions of people.

- 2.42** Goods and Services Tax (GST) is an indirect tax introduced in India in 2017 that is imposed on the supply of goods and services,

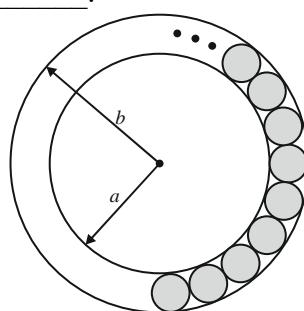


and it subsumes all indirect taxes except few. It is a destination-based tax imposed on goods and services used, and it is not imposed at the point of origin from where goods come. GST also has a few components specific to state governments, central government and Union Territories (UTs).

Which one of the following statements can be inferred from the given passage?

- (A) GST is imposed at the point of usage of goods and services.
 - (B) GST does not have a component specific to UT.
 - (C) GST includes all indirect taxes.
 - (D) GST is imposed on the production of goods and services.

- 2.43** The figure below shows an annular right with outer and inner radii as b and a , respectively. The annular space has been painted in the form of blue colour circles touching the outer and inner periphery of annular space. If maximum n number of circles can be painted, then the unpainted area available in annular space is



- (A) $\pi \left[(b^2 - a^2) - n(b-a)^2 \right]$
 (B) $\pi \left[(b^2 - a^2) + n(b-a)^2 \right]$
 (C) $\pi \left[(b^2 - a^2) - \frac{n}{4}(b-a)^2 \right]$
 (D) $\pi \left[(b^2 - a^2) + \frac{n}{4}(b-a)^2 \right]$

- 2.44** Two straight lines are drawn perpendicular to each other in X-Y plane. If α and β are the acute angles the straight lines make with the X-axis, then $\alpha + \beta$ is _____.
 (A) 120° (B) 60°

2021 IIT Bombay

- 2.45** Given below are two statements 1 and 2, and two conclusions I and II.

Statement 1 : All bacteria are microorganisms.

Statement 2 : All pathogens are microorganisms.

Conclusion I : Some pathogens are bacteria.

Conclusion II : All pathogens are not bacteria.

Based on the above statements and conclusions, which one of the following options is logically CORRECT?

Options

- (A) Only conclusion I correct.
 - (B) Only conclusion II is correct.
 - (C) Either conclusion I or II is correct.
 - (D) Neither conclusion I nor II is correct.

- 2.46** A polygon is convex if, for every pair of points P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.

Which one of the following is NOT a convex polygon?

- (A) 
(B) 
(C) 
(D) 

- 2.47** Six students P, Q, R, S, T and U , with distinct heights, compare their heights and make the following observations.

Observation I : S is taller than R

Observation II : Q is the shortest of all.

Observation III : U is taller than only one student

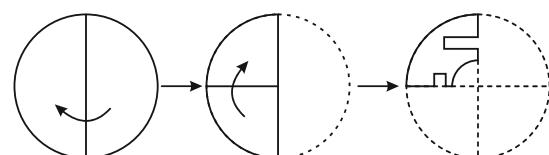
Observation IV : T is taller than S but is not tallest

The number of students that are taller than R is the same as number of student shorter than R .

- (A) T (B) R
 (C) S (D) P



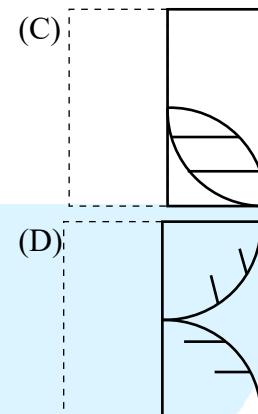
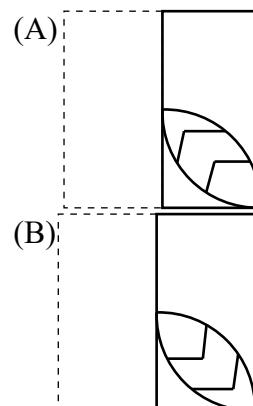
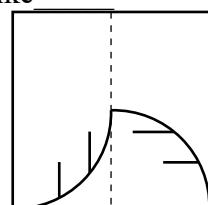
2.48



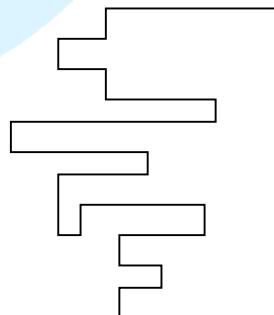
A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _____.

- (A)
- (B)
- (C)
- (D)

2.49 A transparent square sheet shown below is folded along the dotted line. The folded sheet will look like

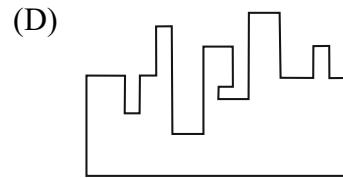
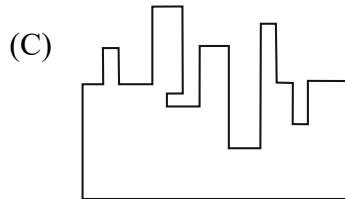


2.50



A jigsaw puzzle has 2 pieces. One of the pieces is shown above. Which one of the given options for the missing piece when assembled will form a rectangle? The piece can be moved, rotated or flipped to assemble with the above piece.

- (A)
- (B)



Solutions

2.1 140

Ratio of male to female students in year 2008,

$$\frac{M_{2008}}{F_{2008}} = 2.5$$

[‘F’ denotes female and ‘M’ denotes male]

In year 2009,

$$\frac{M_{2009}}{F_{2009}} = 3$$

$$\frac{M_{2009}}{F_{2009}} = \frac{3}{M_{2008}} = \frac{3}{2.5}$$

$$\frac{M_{2009}}{M_{2008}} \times \frac{F_{2008}}{F_{2009}} = 1.2 \quad \dots (i)$$

Number of female students get doubled in 2009

$$\text{i.e. } F_{2009} = 2F_{2008}$$

From equation (i),

$$\frac{M_{2009}}{M_{2008}} \times \frac{1}{2} = 1.2$$

$$\frac{M_{2009}}{M_{2008}} = 2.4$$

% increase of students in year 2009

$$\begin{aligned} &= \frac{M_{2009} - M_{2008}}{M_{2008}} \times 100 \\ &= \left(\frac{M_{2009}}{M_{2008}} - 1 \right) \times 100 \\ &= (2.4 - 1) \times 100 = 140\% \end{aligned}$$

Hence, the correct answer is **140**.

2.2 (C)

Let, x_1 = Number of male students in 2011

x_2 = Number of male students in 2012

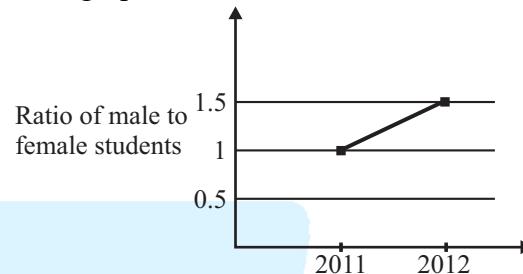
y_1 = Number of female students in 2011

y_2 = Number of female students in 2012

According to the question,

$$y_1 = y_2 = y$$

From the graph,



The ratio of male to female student in 2011,

$$\frac{x_1}{y_1} = \frac{x_1}{y} = 1$$

$$x_1 = y \quad \dots (i)$$

and the ratio of male to female students in 2012,

$$\frac{x_2}{y_2} = \frac{x_2}{y} = 1.5$$

$$x_2 = 1.5y \quad \dots (ii)$$

So the required ratio,

$$\frac{x_2}{x_1} = \frac{1.5y}{y} = 1.5 : 1$$

Hence, the correct option is (C).

2.3 (D)

Given : $ax^2 + bx + c = 0$

The quadratic equation has real and positive roots where a, b and c are real.

So, the roots are :

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For these roots to be real,

$$\sqrt{b^2 - 4ac} > 0 \quad \dots (i)$$

$$\text{Now, } ax^2 + b|x| + c = 0$$

Taking positive sign for x ($x > 0$),

$$ax^2 + bx + c = 0$$



Roots are,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Taking negative sign for x ($x < 0$),

$$ax^2 - bx + c = 0$$

Roots are,

$$x = \frac{+b + \sqrt{b^2 - 4ac}}{2a}, \quad \frac{+b - \sqrt{b^2 - 4ac}}{2a}$$

From equation (i), it is proved that all of these four roots are real.

Hence, the correct option is (D).

Key Point

For roots of a quadratic equation to be positive,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} > 0$$

$$-b \pm \sqrt{b^2 - 4ac} > 0$$

$$\text{So, } -b + \sqrt{b^2 - 4ac} > 0 \Rightarrow b < \sqrt{b^2 - 4ac}$$

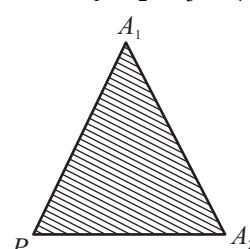
$$-b - \sqrt{b^2 - 4ac} > 0 \Rightarrow \sqrt{b^2 - 4ac} < -b$$

Thus,

$$b < \sqrt{b^2 - 4ac} < -b$$

2.4 6

A tetrahedron is a triangular pyramid which has four corners. The internal planes will be made if the point P inside is connected with any two corners out of four (A_1, A_2, A_3, A_4).



Any two corners out of 4 can be selected in 4C_2 ways.

So, new triangular planes,

$${}^4C_2 = \frac{4!}{2!(4-2)!} = \frac{4 \times 3}{2 \times 1} = 6$$

Hence, the correct answer is 6.

2.5 (D)

Given : If x is a real number and

$$|x^2 - 2x + 3| = 11$$

$$\text{Then, } x^2 - 2x + 3 = -11$$

$$x = 4, -2$$

$$\text{and } x^2 - 2x + 3 = 11$$

$$x = \frac{2 \pm \sqrt{52}}{2}$$

Since, x is real, the possible values of x are,

$$x = 4, -2$$

$$\text{Now, } y = |-x^3 + x^2 - x| \quad (\text{let})$$

$$\text{For } x = -2,$$

$$y = |-(-2)^3 + (-2)^2 - (-2)|$$

$$y = |8 + 4 + 2| = |14| = 14$$

$$\text{For } x = 4$$

$$y = |-(4)^3 + 4^2 - 4|$$

$$y = |-52| = 52$$

Hence, the correct option is (D).

2.6 (A)

Given :

Two floors in a certain buildings are 9 feet apart.

Thus, the height of successive floors = 9 feet

And, the height of each step = $\frac{3}{4}$ feet

So, number of steps = $\frac{9}{(3/4)} = 12$ steps

To find the number of steps, width of step is not required. Hence, only statement I is needed.

Hence, the correct option is (A).

2.7 (A)

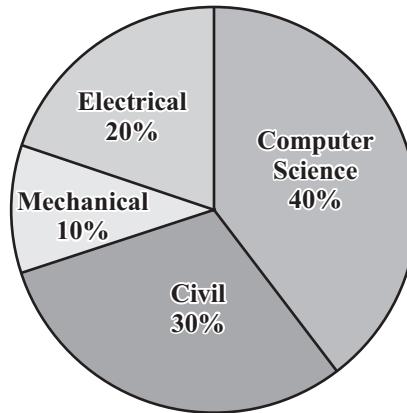
The first two statements are appropriate to tackle the situation.

The third statement implies to put a ban on water supply in lower areas which is morally wrong and should not be done.

Hence, the correct option is (A).

2.8 32

Given : Pie chart :-



Let total number of students = x

Then according to the given pie chart, the number of students are :

Branch	Number of students (In % of x)
Electrical	20% of $x = 0.2x$
Mechanical	10% of $x = 0.1x$
Civil	30% of $x = 0.3x$
Computer Science	40% of $x = 0.4x$

According to question,

The ratio of male to female students in each department = 5 : 4.

and the number of male students in Electrical is,

$$= \frac{5}{5+4} \times 0.2x = 40$$

$$\text{So, } \frac{x}{9} = 40$$

$$x = 360$$

So, the number of female students in Civil

$$= \frac{4}{5+4} \times 0.3x = \frac{4}{9} \times 0.3 \times 360 = 48$$

and the number of female students in Mechanical

$$= \frac{4}{5+4} \times 0.1x = \frac{4}{9} \times 0.1 \times 360 = 16$$

Thus, the required difference is given by,

$$\begin{aligned} &= \text{Female}_{(\text{Civil})} - \text{Female}_{(\text{Mechanical})} \\ &= 48 - 16 = 32 \end{aligned}$$

Hence, the correct answer is 32.

2.9 (C)

Let the weight of each pole = x

Statement (I) : One fourth of the weight of a pole is 5 kg.

$$\frac{x}{4} = 5 \text{ kg} \Rightarrow x = 20 \text{ kg}$$

Total weight of 10 poles = $10x = 200 \text{ kg}$

Which means that statement (I) is alone sufficient.

Statement (II) :

Total weight of 10 poles = $10x$

Total weight of 2 poles = $2x$

Given $10x - 2x = 160 \text{ kg}$

$$\Rightarrow x = 20 \text{ kg}$$

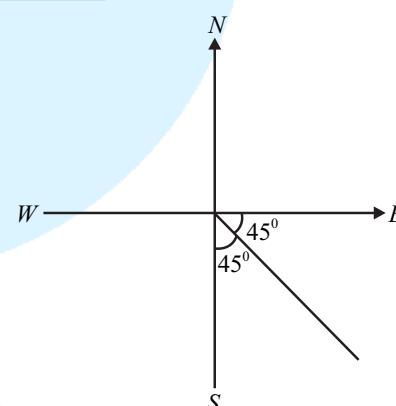
Total weight = $10x = 200 \text{ kg}$

Which means that statement (II) is alone sufficient.

So, either (I) or (II) alone is sufficient.

Hence, the correct option is (C).

2.10 (A)

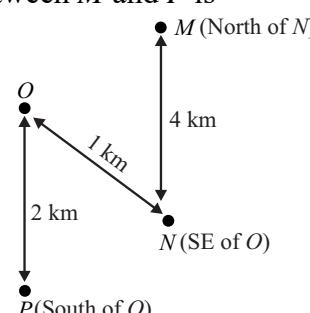


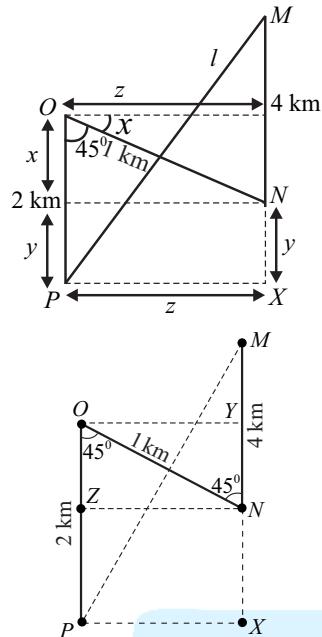
M is north of N , at a distance of 4 km.

P is south of O , at a distance of 2 km

N is south-east of O by 1 km

Distance between M and P is





From ONY triangle,

$$NY = NO \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$PX = OY = NO \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$NY = OZ = \frac{1}{\sqrt{2}}$$

$$\text{Thus, } ZP = NX = 2 - \frac{1}{\sqrt{2}} = 1.3$$

$$MX = 4 + 1.3 = 5.3$$

By Pythagoras theorem in $\triangle MXP$,

$$(MP)^2 = (MX)^2 + (PX)^2$$

$$(MP)^2 = 5.3^2 + 0.707^2 \quad [PX = OY]$$

$$MP = 5.34 \text{ km}$$

Hence, the correct option is (A).

2.11 (B)

Given : ROAD is written as URDG.

By following A – Z alphabetic chart :

R – U : In between two alphabets

O – R : In between two alphabets

A – D : In between two alphabets

D – G : In between two alphabets

So, S T U V

W X Y Z

A B C D

N O P Q

Hence, the correct option is (B).

2.12 (B)

Given :

- (i) There are 5 portfolios, home, power, defense, telecom and finance.
- (ii) 6 selected members P, Q, R, S, T and U are to be appointed in 5 portfolios.
- (iii) U does not want any portfolio if S gets one of the five.
- (iv) R wants either Home or Finance or no portfolio.
- (v) Q says that if S gets either Power or Telecom, then she must get the other one. T insists on a portfolio if P gets one.

R wants Home or Finance or no portfolio. In option (A) and (D), R has Defense and Telecom. So, both options are incorrect.

U does not want a portfolio if S gets one of the five. In option (C), S and U both have portfolios. So, option (C) is also incorrect.

Hence, the correct option is (B).

2.13 (C)

Given : $f(x) = 1 - |x|$, $-1 \leq x \leq +1$

Method 1

$$\text{As } |x| = \begin{cases} -x, & x < 0 \\ 0, & x = 0 \\ +x, & x > 0 \end{cases}$$

$$f(x) = 1 - |x| = \begin{cases} 1+x, & x < 0 \\ 1, & x = 0 \\ 1-x, & x > 0 \end{cases}$$

For $-1 \leq x \leq +1$,

$$f(x) = \begin{cases} 1+x, & -1 \leq x < 0 \\ 1, & x = 0 \\ 1-x, & x > 0 \end{cases}$$

$$f'(x) = \begin{cases} 1, & -1 \leq x < 0 \\ 0, & x = 0 \\ -1, & 0 < x \leq 1 \end{cases}$$

Since, $f'(x) = 0$ at $x = 0$, so $x = 0$ is a stationary point

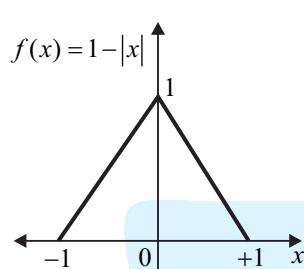
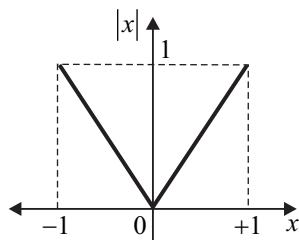
Therefore,



$$f(x) = 1 - |x| = 1$$

Hence, the correct option is (C).

Method 2



So for maximum value, $f(x) = 1$ at $x = 0$.

Hence, the correct option is (C).

2.14 8

If p, q, r and s are distinct integers then,

$$f(p, q, r, s) = \max(p, q, r, s)$$

$$g(p, q, r, s) = \min(p, q, r, s)$$

$h(p, q, r, s) = \text{Remainder of}$

$$\begin{cases} \frac{p \times q}{r \times s}, & \text{if } (p \times q) > (r \times s) \\ \frac{r \times s}{p \times q}, & \text{if } (r \times s) > (p \times q) \end{cases}$$

$$fgh(p, q, r, s) = f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$$

To determine,

$$fg(h(2, 5, 7, 3), 4, 6, 8) = ?$$

Firstly calculating $h(2, 5, 7, 3)$

Here, $p = 2, q = 5, r = 7, s = 3$

$$p \times q = 2 \times 5 = 10$$

$$r \times s = 7 \times 3 = 21$$

As $r \times s > p \times q$

$$h(2, 5, 7, 3) = \text{Remainder of } \frac{r \times s}{p \times q}$$

$$h(2, 5, 7, 3) = \text{Remainder}\left(\frac{21}{10}\right)$$

$$h(2, 5, 7, 3) = 1$$

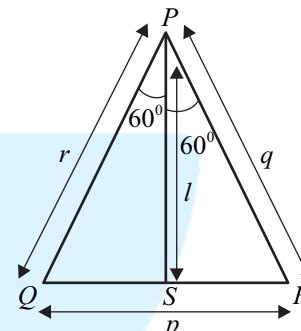
$$\begin{aligned} \text{Thus, } fg(h(2, 5, 7, 3), 4, 6, 8) &= fg(1, 4, 6, 8) \\ &= f(1, 4, 6, 8) \times g(1, 4, 6, 8) \\ &= \max(1, 4, 6, 8) \times \min(1, 4, 6, 8) \\ &= 8 \times 1 = 8 \end{aligned}$$

Hence, the correct answer is 8.

2.15 (B)

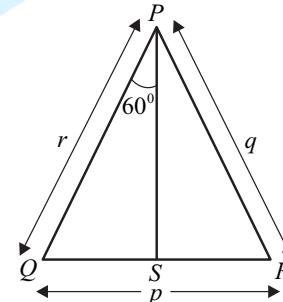
Method 1

$$\text{Using sine rule, } l = \frac{qr}{q+r}$$



Hence, the correct option is (B).

Method 2



Since, $\angle QPR = 120^\circ$. So, it may be an isosceles triangle.

Assuming ΔPQR to be isosceles.

$$\text{Thus, } r = q, \angle QSP = 90^\circ$$

From triangle,

$$\cos 60^\circ = \frac{PS}{PQ} = \frac{PS}{r}$$

$$PS = r \cos 60^\circ = r/2$$

From option (B),

$$\frac{qr}{q+r} = \frac{r^2}{2r} = \frac{r}{2}$$

Hence, the correct option is (B).

2.16 (C)



Given :

- (i) $f(x)$ is linear.
- (ii) At $x = -2$, $f(x) = 29$
At $x = 3$, $f(x) = 39$

Let $f(x) = mx + C$,

Then, at $x = -2$,

$$\begin{aligned} f(x) &= m \times -2 + C \\ 29 &= -2m + C \quad \dots \text{(i)} \end{aligned}$$

and at $x = 3$,

$$\begin{aligned} f(x) &= m \times 3 + C \\ 39 &= 3m + C \quad \dots \text{(ii)} \end{aligned}$$

From equation (i) and (ii),

$$m = 2, C = 33$$

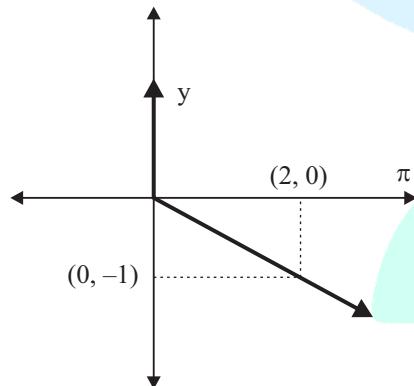
So, at $x = 5$,

$$\begin{aligned} f(x) &= mx + C = 2 \times 5 + 33 \\ f(x) &= 43 \end{aligned}$$

Hence, the correct option is (C).

2.17 (B)

Given : Plot is shown below,



From above figure,

when $y = -1 \Rightarrow x = 2$

From option (A) :

$$\begin{aligned} x &= y - |y| \\ x &= -1 - |-1| = -2 \neq 2 \quad \text{[Incorrect]} \end{aligned}$$

From option (B) :

$$\begin{aligned} x &= -y[y - |y|] \\ x &= -1[-1 - |-1|] = 2 \quad \text{[Correct]} \end{aligned}$$

From option (C) :

$$x = y + |y|$$

$$x = -1 + |-1| = 0 \neq 2 \quad \text{[Incorrect]}$$

From option (D) :

$$\begin{aligned} x &= -[y + |y|] = -[-1 + |-1|] \\ x &= -(-1 + 1) = 0 \neq 2 \quad \text{[Incorrect]} \end{aligned}$$

Hence, the correct option is (B).

2.18 (A)

Given : Total faculty members = 150

The faculty members having Facebook® account = FB = 55

The faculty members having WhatsApp® = W = 85

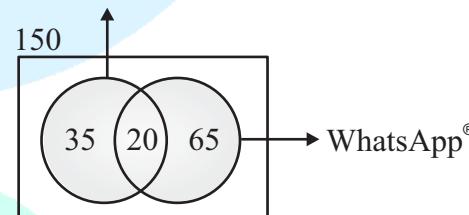
The faculty members do not have Facebook® (or) WhatsApp® accounts = 30.

The faculty members having any account = $150 - 30 = 120$.

The faculty members having both the accounts = $(FB + W) - 120 = (55 + 85) - 120 = 20$

Therefore, the number of faculty members connected only through Facebook® accounts = $55 - 20 = 35$

Facebook®



Hence, the correct option is (A).

2.19 (D)

(A)	1	2
	3	4

(B)

8	9
10	11
J	H
22	23
X	V
24	25
Y	W
26	Z

(C)

14	13
15	16
O	N
P	M
17	Q

(D)



For options (A), (B) and (C) a common pattern follows.

The 2nd, 4th, 1st, 3rd, 5th letters are consecutive alphabets.

For example, in option (B) : JHKIL

H - 2nd letter

I - 4th letter

J - 1st letter

K - 3rd letter

L - 5th letter

Option (D) does not follow this pattern.

Hence, the correct option is (D).

2.20 (C)

Method 1

In given 2×4 rectangle grid, the following type of rectangles are present :

One figured rectangles = 8

Two figured rectangles = 10

Three figured rectangles = 4

Four figured rectangles = 5

Six figured rectangles = 2

Eight figured rectangles = 1

So, total number of rectangles = 30

Hence, the correct option is (C).

Method 2

No. of rows in given cell = 2

No. of column in given cell = 4

Formula :

$$\begin{aligned}\text{Total number of cells} &= \frac{R(R+1)}{2} \times \frac{C(C+1)}{2} \\ &= \frac{2(2+1)}{2} \times \frac{4(4+1)}{2} = 30\end{aligned}$$

Hence, the correct option is (C).

Method 3

Formula : Total number of cells =

Addition of no. of rows × Addition of no. of columns

$$= (1+2) \times (1+2+3+4) = 3 \times 10 = 30$$

Hence, the correct option is (C).

2.21 (C)

Given :

(i) ‘relftaga’ means carefree.

(ii) ‘otaga’ means careful.

(iii) ‘fertaga’ means careless.

Here, it is clear that ‘taga’ means care and if taga comes at the end in code language then meaning has ‘care’ at the front.

So, option (A) and (D) are incorrect.

In ‘fertaga’, ‘fer’ means less.

So, ‘Tagafer’ means less care.

Therefore, ‘Tagazen’ means aftercare.

Hence, the correct option is (C).

2.22 (B)

Elegance	Smooth	Soft	Executive
27300	20009	17602	9999
25222	19392	18445	8942
28976	22429	19544	10234
21012	18229	16595	10109
102510	80059	72186	39284
Rs. 48	Rs. 63	Rs. 78	Rs. 173
102510 ×48	80059 ×63	72186 ×78	39284 ×173
=4920480	=5043717	=5630508	=6796132

From the above table it is clear that, more revenue is obtained from executive.

Hence, the correct option is (B).

2.23 (A)

Given :

	Winner		Loses
P v/s Q	P	Always	Q
R v/s S	R	Always	S
S v/s P	P/S	Sometimes	P/S
R v/s Q	Q	Always	R

From the given data, it can be clearly inferred that P always beats Q. Since, Q always beats R, P can also beat R. P only sometimes beats S. Hence, P is likely to beat all the other three player. So, statement (i) can be logically inferred.

S only sometimes loses to P i.e. sometimes S wins as well, meaning S beats the best player sometimes. Hence, S can't be absolute worst player in the set.



So, statement (ii) can not be logically inferred.
Hence, the correct option is (A).

2.24 (B)

Given : For a quadratic equation, the product of roots, $\alpha\beta=4$

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} = \frac{\alpha^n + \beta^n}{\frac{1}{\alpha^n} + \frac{1}{\beta^n}} = \frac{\alpha^n + \beta^n}{\frac{\beta^n + \alpha^n}{\alpha^n \beta^n}}$$

$$\alpha^n \times \beta^n = (\alpha\beta)^n = (4)^n$$

Hence, the correct option is (B).

2.25 (C)

At $x=3$,

From given graph, $f(x)$ must be equals to zero.

From option (A) :

$$f(x) = 1 - |x - 1|$$

At $x=3$,

$$f(3) = 1 - |3 - 1| = 1 - 2 = -1 \neq 0$$

So, it is an incorrect option.

From option (B) :

$$f(x) = 1 + |x - 1|$$

At $x=3$,

$$f(3) = 1 + |3 - 1| = 1 + 2 = 3$$

So, it is an incorrect option.

From option (C) :

$$f(x) = 2 - |x - 1|$$

$$\text{At } x=3, f(3) = 2 - |3 - 1| = 2 - 2 = 0$$

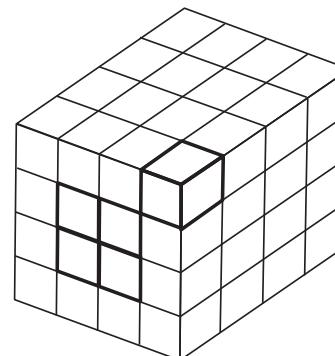
It is the correct option.

Hence, the correct option is (C).

2.26 (D)

Given :

- (i) Total number of cubes = 64
- (ii) Total number of cubes that are removed from each corner = 8



Method 1

Area of first face of cube after removal,

$$= 12 \times 1 = 12 \text{ m}^2 \quad [\text{Existing surface}]$$

For six faces, area = $12 \times 6 = 72 \text{ m}^2$

After removing 8 cubes, visible surface area at each corner = $03 \times 8 \times 1 = 24 \text{ m}^2$

$$\text{Total surface area} = 72 + 24 = 96 \text{ m}^2$$

Hence, the correct option is (D).

Method 2

Removed cube will leave same area as they covered before so still area will remain same

Area of one surface = 16 unit^2

and total number of surfaces = 6

$$\text{Total surface area} = 16 \times 6 = 96 \text{ unit}^2$$

Hence, the correct option is (D).

2.27 (B)

Given : $f(x) = 2x^7 + 3x - 5$

From option (A) :

$$x^3 + 8 = 0$$

$$x^3 = -8$$

$$x = -2$$

For $x = -2$,

$$f(x) = 2x^7 + 3x - 5$$

$$f(-2) = 2(-2)^7 + 3(-2) - 5$$

$$f(-2) = -256 - 6 - 5 = -267$$

So, this is not a factor of $f(x)$

From option (B) :

$$x - 1 = 0$$

$$x = 1$$

For $x = 1$,



$$f(x) = 2x^7 + 3x - 5$$

$$f(1) = 2(1)^7 + 3(1) - 5 = 5 - 5 = 0$$

So, $(x-1)$ is a factor of $f(x)$.

Hence, the correct option is (B).

2.28 (B)

Since, the number of cycles to failure decreases exponentially with an increase in load. The general equation is given by,

$$y = ne^{-mx}$$

where, y = number of cycle failure and x is load

Given : $y = 100$ and $x = 80$

$$\text{Therefore, } 100 = ne^{-80m} \quad \dots(\text{i})$$

When load is halved, it takes 10000 cycle for failure.

$$10000 = ne^{-40m} \quad \dots(\text{ii})$$

From equation (i) and (ii),

$$100 = e^{40m}$$

$$m = \frac{\ln 100}{40} \quad \dots(\text{iii})$$

For 5000 cycles to failure,

$$5000 = ne^{-xm} \quad \dots(\text{iv})$$

From equation (ii) and (iv),

$$2 = e^{m(x-40)}$$

$$m(x-40) = \ln 2$$

From equation (iii),

$$x-40 = \frac{\ln 2}{m}$$

$$x-40 = \frac{\ln 2}{\ln 100} \times 40$$

$$x-40 = \frac{0.693}{4.605} \times 40$$

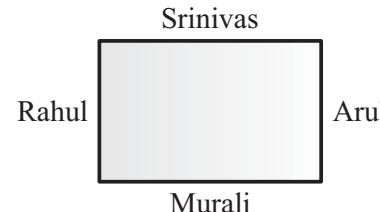
$$x = 46.02$$

Hence, the correct option is (B).

2.29 (C)

Given :

- (i) Rahul, Murali, Srinivas and Arul are sitting around a square table.
- (ii) Rahul is just left to Murali.
- (iii) Srinivas is just right to Arul.



It can be concluded that Rahul is opposite to Arul and Srinivas is opposite to Murali.

Hence, the correct option is (C).

2.30 (B)

Given : All boxes have been labelled incorrectly.

So,

Statement 1 : Box labelled "Apples" is either "Oranges" or "Apples and Oranges".

Statement 2 : Box labelled "Oranges" is either "Apples" or "Apples and Oranges".

Statement 3 : Box labelled "Apples and Oranges" is either "Apples" or "Oranges".

After opening the box labelled "Apples and Oranges", if we get Apple in it then it is sure that the box labelled "Apples and Oranges" is actually Apple.

So, Box labelled "Oranges" can have either only 'oranges' or 'apples and oranges'. Since, all boxes are incorrectly labelled, box labelled as oranges can not have only oranges. Therefore, it must be having oranges and apple and box labelled "Apples" has to be "Oranges".

Hence, the correct option is (B).

2.31 (C)

It is given that contour lines are at 25 m interval in the given figure.

On calculating distance,

$$P = 575 \text{ m}$$

$$Q = 525 \text{ m}$$

$$R = 475 \text{ m}$$

$$S = 475 \text{ m}$$

$$T = 500 \text{ m}$$

Also, given that flood level rises to 525 m i.e. a village with a height less than 525 m will get submerged.

Thus, the villages which will be submerged if water level rises to 525 m are R, S and T.

Hence, the correct option is (C).

2.32 (C)



Most number of contours is passing through the region R . So, this region is most likely to have a thunderstorm.

Hence, the correct option is (C).

2.33 (B)

$$\text{Let, } f(x) = \frac{(x+y) - |x-y|}{2} \quad \dots$$

(i)

$|x-y|$ can be in one of the two following forms

$$\begin{array}{c} |x-y| \\ \swarrow \quad \searrow \\ +(x-y) = x-y \quad -(x-y) = -x+y \end{array}$$

Taking $|x-y| = +(x-y)$

From equation (i),

$$f(x) = \frac{(x+y) - |x-y|}{2}$$

$$f(x) = \frac{x+y-x+y}{2} = \frac{2y}{2} = y$$

$(x > y) y$ is minimum

Taking $|x-y| = -(x-y)$

From equation (i),

$$f(x) = \frac{(x+y) - [-(x-y)]}{2}$$

$$f(x) = \frac{x+y+x-y}{2} = x$$

$(x < y) x$ is minimum

Thus, from both cases, function $f(x)$ is equal to minimum of x and y .

Hence, the correct option is (B).

2.34 (C)

$$f(x) = e^x + 0.5x^2 - 2$$

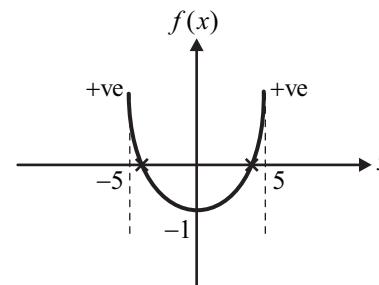
$$\text{At } x=0, f(0) = e^0 + 0 - 2 = 1 - 2$$

$$f(0) = -1$$

$$\text{At } x=-5, f(-5) = 10.5 \quad (+\text{ve})$$

$$\text{At } x=5, f(5) = 158.91 \quad (+\text{ve})$$

Based on these assertions, the approximate graph of $f(x)$ can be drawn as below :



From the above graph, it is clear that $f(x) = 0$ is having two roots in the range of $[-5, 5]$.

Hence, the correct option is (C).

2.35 (B)

Given :

(i) Out of four criminals P, Q, R and S , only one committed the crime

(ii) Only one criminal is telling the truth.

If P is telling the truth then Q and R both committed the crime which is not possible.

If Q is telling the truth then S and R both committed the crime which is again not possible.

If R is telling the truth then Q and S will contradict each other.

If S is telling the truth only R committed the crime.

Hence, the correct option is (B).

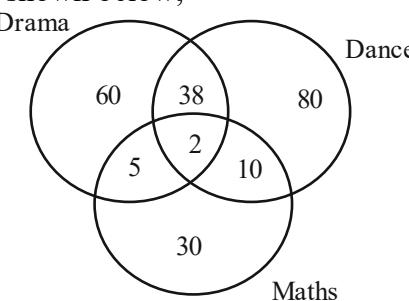
2.36 (D)

According to given data, Z believes in vastu and wants to stay in the south-west wing. In all the given options, Only option (D) satisfies this request.

Hence, the correct option is (D).

2.37 (C)

According to the given data, Venn diagram can be drawn as shown below,



Let total number of students is x .

75% of the students are not in any club. So,

$$x = 60 + 80 + 30 + 38 + 10 + 5 + 2 + 0.75x$$

$$0.25x = 225$$



$$x = 900$$

Hence, the correct option is (C).

2.38 (C)

Let the cost of the gift be x .

Since, two of them are not contributing, each friend will pay 150 more. So,

$$x = \frac{8x}{10} + 8 \times 150$$

$$\frac{2x}{10} = 1200$$

$$x = 6000$$

Hence, the correct option is (C).

2.39 (C)

Given : $P = 3$, $R = 27$, $T = 243$

The series of consecutive letters are

P, Q, R, S and T

Now, $P = 3^1$, $Q = 3^2$, $R = 3^3$, $S = 3^4$, $T = 3^5$

i.e., $Q + S = 3^2 + 3^4 = 9 + 81 = 90$

Hence, the correct option is (C).

2.40 (A)

The total expenditure of the company during 2014 to 2018 $= 500 \times 5 =$ Rs. 2000 millions

Total revenue of the company during 2014 to 2018 $= 500 + 700 + 800 + 600 + 400 =$ Rs. 3000 millions

$$\text{So, profit \%} = \frac{3000 - 2500}{2500} \times 100 = 20\%$$

Hence, the correct option is (A).

2.41 (C)

From the given passage : ‘Global exodus of billions of people displaced by rising seas’ clearly implies that billions of people are affected by melting glaciers.

Hence, the correct option is (C).

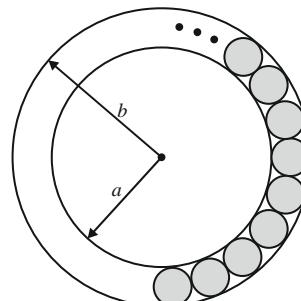
2.42 (A)

The given passage mentions that GST is ‘a destination based tax imposed on goods and services used’, therefore it can be inferred that GST is imposed at the point of usage of goods and services.

Hence, the correct option is (A)

2.43 (C)

Given :



$$\text{Surface area of the ring} = \pi b^2 - \pi a^2$$

$$\text{Radius of painted circle on the ring} = \frac{b-a}{2}$$

$$\text{Area of each painted circle} = \frac{\pi}{4} (b-a)^2$$

\therefore Area of n circles that can be painted

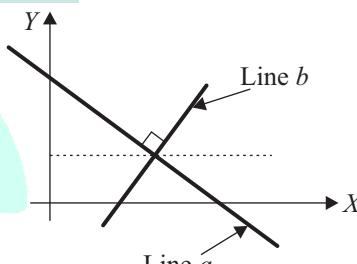
$$= \frac{n\pi}{4} (b-a)^2$$

\therefore The unpainted area available in annular space is given by

$$\text{Area} = \pi \left[(b^2 - a^2) - \frac{n}{4} (b-a)^2 \right]$$

Hence, the correct option is (C).

2.44 (C)



Line a and b are perpendicular to each other in $X-Y$ plane. Let α be the acute angle made by line a with X axis. Let β be the acute angle made by line b with X axis.

The angle made by a line with the dotted line (shown in figure) is same as the angle made by it with the X -axis.

$$\therefore \alpha + \beta = 90^\circ$$

Hence, the correct option is (C).

2.45 (D)

Given :

S1 : All bacteria are microorganisms.



S2 : All pathogens are microorganisms.
According to the statement given in the question we can form Venn diagram as shown below,
There are two possibilities,

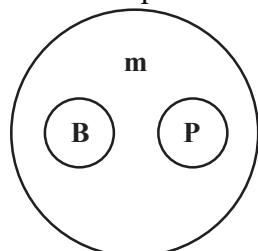


Fig. (a)

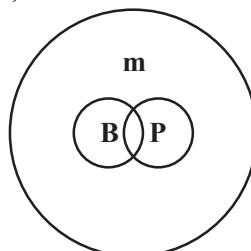


Fig. (b)

Conclusion 1 : Some pathogens are bacteria.

From fig. (a) conclusion 1 does not follow.

Conclusion 2 : All pathogens are not bacteria.

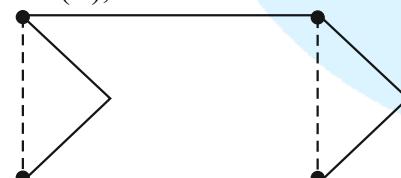
From possibility of case present in fig. (b) conclusion 2 does not follow.

Hence, the correct option is (D).

2.46 (C)

Method 1

From option (C),

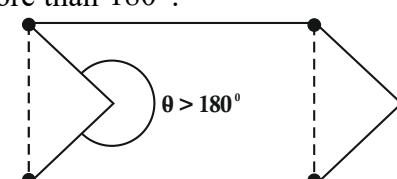


Clearly, we can see that some part is outside the segment line.

Hence, the correct option is (C).

Method 2

Concave polygon is one who have one interior angle more than 180° .



Hence, the correct option is (C).

2.47 (C)

Given:

S is taller than R i.e. $S > R$

Q is shorter of all i.e. $>> Q$

U is taller than only 1 student i.e. $U > \square$

T is taller than S but is not tallest i.e.

$\square > T > S$

Combining all drafted information & make possible case.

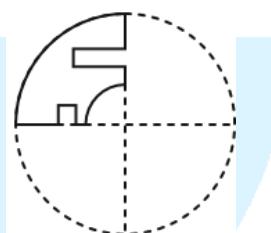
- 1- P
- 2- T
- 3- S
- 4- R
- 5- U
- 6- Q

Now, it is clear that the numbers of students taller than R is the same as the numbers of students shorter than S.

Hence, the correct option is (C).

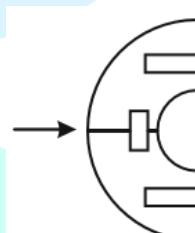
2.48 (A)

Given : Initial figure is given below

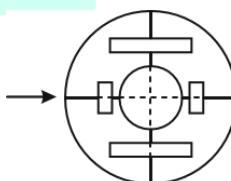


To get the answer, we can simply retrace the steps followed in reverse direction.

Step 1 : Create the water image



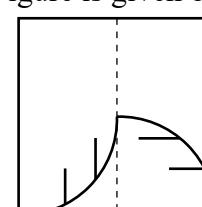
Step 2 : Create the mirror image



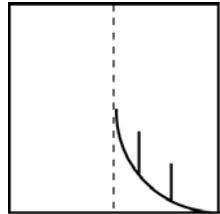
Hence, the correct option is (A).

2.49 (A)

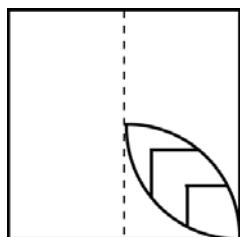
Given : Initial Figure is given below,



Mirror image of the left part of given image is



After combining the above and right part of given image we get,



Hence, the correct option is (A).

2.50 (B)

For assembling the 2 pieces to form a rectangle,
First; flip the figure to left side and rotate it to 90^0 clock wise direction and assume to put it on question figure.

Hence, the correct option is (B).

3

Verbal Ability



Practice Questions

2013 IIT Bombay

- 3.1 Which one of the following option is the closest in meaning to the word given below?

Nadir

- (A) Highest (B) Lowest
 (C) Medium (D) Integration

- 3.2 Complete the sentence :

Universalism is to particularism as diffuseness is to _____.

- (A) Specificity (B) Neutrality
 (C) Generality (D) Adaptation

- 3.3 Were you a bird, you _____ in the sky.

- (A) Would fly
 (B) Shall fly
 (C) Should fly
 (D) Shall have flown

- 3.4 Choose the grammatically incorrect sentence :

- (A) He is of Asian origin.
 (B) They belonged to Africa.
 (C) She is an European.
 (D) They migrated from India to Australia.

- 3.5 After several defeats in wars, Robert Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.

Which of the following assertions is best supported by the above information?

- (A) Failure is the pillar of success
 (B) Honesty is the best policy
 (C) Life begins and ends with adventures
 (D) No adversity justifies giving up hope

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- 3.6 Match the columns.

Column 1

1. Eradicate
 2. Distort
 3. Saturate
 4. Utilize

Column 2

- P. Misrepresent
 Q. Soak
 Completely
 R. Use
 S. Destroy
 utterly

- (A) 1:S, 2:P, 3:Q, 4:R
 (B) 1:P, 2:Q, 3:R, 4:S
 (C) 1:Q, 2:R, 3:S, 4:P
 (D) 1:S, 2:P, 3:R, 4:Q

- 3.7 Choose the word that is opposite in meaning to the word "coherent".

- (A) sticky
 (B) well-connected
 (C) rambling
 (D) friendly

- 3.8 Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite _____.

- (A) superb (B) medium
 (C) mediocre (D) exhilarating

- 3.9 Which of the following options is the closest in meaning to the phrase underlined in the sentence below?

It is fascinating to see life forms cope with varied environmental conditions.





statements below can be inferred from this passage?

- (A) Kaliningrad was historically Russian in its ethnic make up.
- (B) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia.
- (C) Koenigsberg was renamed Kaliningrad, as that was its original Russian name.
- (D) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia.

3.18 The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region.

Which one of the following statements, if true, does not contradict this conclusion?

- (A) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent.
- (B) More cases of dengue are now reported because of an increase in the municipal office's administrative efficiency.
- (C) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test.
- (D) The number of people with malarial fever (also contracted from mosquito bites) has increased this year.

3.19 A dance programme is scheduled for 10:00 am, some students are participating in the programme and they need to come an hour earlier than the start of the event. These students should be accompanied by a parent. Other students and parents should come in time for the programme. The instruction you think that is appropriate for this is

- (A) Students should come at 9:00 am and parents should come at 10:00 am.
- (B) Participating students should come at 9:00 am accompanied by a parent, and

other parents and students should come by 10:00 am.

- (C) Students who are not participating should come by 10:00 am and they should not bring their parents. Participating students should come at 9:00 am.
- (D) Participating students should come before 9:00 am parents who accompany them should come at 9:00 am. All others should come at 10:00.

3.20 By the beginning of the 20th century, several hypotheses were being proposed, suggesting a paradigm shift in our understanding of the universe. However, the clinching evidence was provided by experimental measurements of the position of a star which was directly behind our sun.

Which of the following inference (s) may be drawn from the above passage?

- (i) Our understanding of the universe changes based on the positions of stars.
 - (ii) Paradigm shifts usually occur at the beginning of centuries.
 - (iii) Stars are important objects in the universe.
 - (iv) Experimental evidence was important in confirming this paradigm shift.
- (A) (i), (ii) and (iv) (B) (iii) only
(C) (i) and (iv) (D) (iv) only

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3.21 Which one of the following combinations is incorrect?

- (A) Acquiescence - Submission
- (B) Wheedle - Roundabout
- (C) Flippancy - Lightness
- (D) Profligate - Extravagant

3.22 Select the pair that best expresses a relationship similar to that expressed in the pair :

Children : Pediatrician

- (A) Adult: Orthopaedist
- (B) Females: Gynaecologist
- (C) Kidney: Nephrologist
- (D) Skin: Dermatologist



- 3.23 Didn't you buy _____ when you went shopping?
(A) any paper (B) much paper
(C) no paper (D) a few paper
- 3.24 We _____ our friend's birthday and we _____ how to make it up to him.
(A) completely forgot---don't just know
(B) forgot completely---don't just know
(C) completely forgot---just don't know
(D) forgot completely---just don't know
- 3.25 Extreme focus on syllabus and studying for tests has become such a dominant concern of Indian students that they close their minds to anything _____ to the requirements of the exam.
(A) related (B) extraneous
(C) outside (D) useful
- 3.26 The Tamil version of _____ John Abraham-starrer *Madras Cafe* _____ cleared by the Censor Board with no cuts last week, but the film's distributors _____ no takers among the exhibitors for a release in Tamil Nadu _____ this Friday.
(A) Mr., was, found, on
(B) a, was, found, at
(C) the, was, found, on
(D) a, being, find at
- 3.27 Which of the following options is the closest in meaning to the sentence below?
She enjoyed herself immensely at the party.
(A) She had a terrible time at the party.
(B) She had a horrible time at the party.
(C) She had a terrific time at the party.
(D) She had a terrifying time at the party.
- 3.28 Select the alternative meaning of the underlined part of the sentence.
The chain snatchers took to their heels when the police party arrived.
(A) took shelter in a thick jungle.
(B) open indiscriminate fire.
(C) took to flight.
(D) unconditionally surrendered.
- 3.29 Choose the statement where underlined word is used correctly.
(A) The industrialist had a personnel jet.
(B) I write my experience in my personnel diary.
(C) All personnel are being given the day off.
(D) Being religious is a personnel aspect.
- 3.30 A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is
(A) fabric (B) textile
(C) fibre (D) apparel
- 3.31 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage :
(A) Since the report lacked needed information, it was of no use to them.
(B) The report was useless to them because there were no needed information in it.
(C) Since the report did not contain the needed information, it was not real useful to them.
(D) Since the report lacked needed information, it would not had been useful to them.
- 3.32 Alexander turned his attention towards India, since he had conquered Persia.
Which one of the statements below is logically valid and can be inferred from the above sentence
(A) Alexander would not have turned his attention towards India had he not conquered Persia.
(B) Alexander was not ready to rest on his laurels, and wanted to march to India.
(C) Alexander was completely in control of his army and could command towards India.
(D) Since Alexander's kingdom extended to Indian borders after the conquest of Persia, he was keen to move further.
- 3.33 Most experts feel that in spite of possessing all the technical skills required to be a batsman of the highest order, he is unlikely to be so due to lack of requisite temperament.



He was guilty of throwing away his wicket several times after working hard to lay a strong foundation. His critics pointed out that until he addresses to this problem, success at the highest level will continue to elude him.

Which of the statement(s) below is/are logically valid and can be inferred from the above passage?

- (i) He was already a successful batsman at the highest level
 - (ii) He has to improve his temperament in order to become a great batsman
 - (iii) He failed to make many of his good starts count
 - (iv) Improving his technical skills will success
- (A) (iii) and (iv) (B) (ii) and (iii)
(C) (i), (ii) and (iii) (D) (ii) only

2016 IISc Bangalore

3.34 Find the odd one in the following group of words.

mock, deride, praise, jeer

- (A) mock (B) deride
(C) praise (D) jeer

3.35 The man who is now Municipal Commissioner worked as _____.
_____.

- (A) the security guard at a university.
(B) a security guard at the university.
(C) a security guard at university.
(D) the security guard at the university.

3.36 A rewording of something written or spoken is a _____.
_____.

- (A) Paraphrase (B) Paradox
(C) Paradigm (D) Paraffin

3.37 Archimedes said, "Give me a lever long enough and a fulcrum on which to place it and I will move the world." The sentence above is an example of a _____ statement.
_____.

- (A) Figurative (B) Collateral
(C) Literal (D) Figurine

3.38 Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phrase in the above sentence.

- (A) put up with (B) put in with
(C) put down to (D) put up against

3.39 All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

- (i) Ooty is not a hill-station.
 - (ii) No hill-station can have more than one lake.
- (A) (i) only
(B) (ii) only
(C) both (i) and (ii)
(D) neither (i) nor (ii)

3.40 Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

- (A) I will not leave the place until minister does not meet me.
(B) I will leave the place until the minister does not meet me.
(C) I will not leave the place until the minister meet me.
(D) I will not leave the place until the minister meets me.

3.41 Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

- (i) The author believes that computers are not good for us.



(ii) Mobile computers and the internet are both intended inventions.

- (A) (i) only
- (B) (ii) only
- (C) both (i) and (ii)
- (D) neither (i) nor (ii)

3.42 Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

- (A) India is a country of exactly seventeen languages.
- (B) Linguistic pluralism is the only indicator of a nation's diversity.
- (C) Indian currency notes have sufficient space for all the Indian languages.
- (D) Linguistic pluralism is strong evidence of India's diversity.

2017 IIT Roorkee

3.43 Choose the option with words that are not synonyms.

- (A) Aversion, dislike
- (B) Luminous, radiant
- (C) Plunder, loot
- (D) Yielding, resistant

3.44 Research in the workplace reveals that people work for many reasons ____.

- (A) money beside.
- (B) beside money.
- (C) money besides.
- (D) besides money.

3.45 After Rajendra Chola returned from his voyage to Indonesia, he ____ to visit the temple in Thanjavur.

- (A) was wishing (B) is wishing
- (C) wished (D) had wished

3.46 Saturn is _____ to be seen on a clear night with the naked eye.

- (A) Enough bright
- (B) Bright enough

(C) As enough bright
(D) Bright as enough

3.47 "The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history".

Which of the following statements best reflects the author's opinion?

- (A) Nationalist are highly imaginative.
- (B) History is viewed through the filter of nationalism.
- (C) Our colonial past never happened.
- (D) Nationalism has to be adequately and properly imagined.

3.48 "We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to something seemingly more urgent—namely ideology. This was a country where all gestures, even the most private were interpreted in political terms".

The author's belief that ideology is not as important as literature is revealed by the word :

- (A) 'Culture'
- (B) 'Seemingly'
- (C) 'Urgent'
- (D) 'Political'

2018 IIT Guwahati

3.49 "From where are they bringing their books? _____ bringing _____ books from _____."

The words that best fill the blanks in the above sentence are

- (A) Their, they're, there
- (B) They're, their, there
- (C) There, their, they're
- (D) They're, there, there

3.50 "A _____ investigation can sometimes yield new facts, but typically organized ones are more successful."

The word that best fills the blank in the above sentence is

- (A) meandering
- (B) timely
- (C) consistent
- (D) systematic

2019 IIT Madras



3.51 A court is to a judge as _____ is to a teacher.

- (A) A student (B) A punishment
(C) A syllabus (D) A school

3.52 The expenditure on the project _____ as follows: equipment Rs. 20 lakhs, salaries Rs. 12 lakhs, and contingency Rs. 3 lakhs.

- (A) Break down (B) Break
(C) Breaks down (D) Breaks

3.53 The search engine's business model _____ around the fulcrum of trust.

- (A) Revolves (B) Plays
(C) Sinks (D) Bursts

3.54 "A recent High Court Judgement has sought to dispel the idea of begging as a disease – which leads to its stigmatization and criminalization – and to regard it as a symptom. The underlying disease is the failure of the state to protect citizens who fall through the social security net."

Which one of the following statements can be inferred from the given passage?

- (A) Beggars are lazy people who beg because they are unwilling to work
(B) Beggars are created because of the lack of social welfare schemes
(C) Begging is an offence that has to be dealt with firmly
(D) Begging has to be banned because it adversely affects the welfare of the state

2020 IIT Delhi

3.55 Select the word that fits the analogy:

Cook : Cook :: Fly : _____.

- (A) Flyer (B) Flying
(C) Flew (D) Flighter

3.56 His knowledge of the subject was excellent but his classroom performance was _____.

- (A) desirable
(B) extremely poor
(C) praiseworthy
(D) good

3.57 Raman is confident of speaking English _____ six months as he has been practising regularly _____ the last three weeks.

- (A) during, for (B) within, for
(C) for, in (D) for, since

2021 IIT Bombay

3.58 _____ is to surgery as writer is to _____. Which one of the following options maintains a similar logical relation in the above sentence?

- (A) Doctor, book
(B) Hospital, library
(C) Medicine, grammar
(D) Plan, outline

3.59 Pen: Write :: Knife : _____

Which one of the following options maintains a similar logical relation in the above?

- (A) Sharp (B) Cut
(C) Blunt (D) Vegetables.

3.60 Gauri said that she can play the keyboard _____ her sister.

- (A) as worse as
(B) as nicest as
(C) as better as
(D) as well as

3.61 Consider the following sentences:

- (i) Everybody in the class is prepared for the exam.
(ii) Babu invited Danish to his home because he enjoys playing chess.

Which of the following is the CORRECT observation about the above two sentences?

(A) (i) is grammatically incorrect and (ii) is unambiguous

(B) (i) is grammatically correct and (ii) is unambiguous

(C) (i) is grammatically correct and (ii) is ambiguous

(D) (i) is grammatically incorrect and (ii) is ambiguous

3.62 Some people suggest Anti-Obesity Measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core



problems that cause obesity, poverty and income inequality.

Which one of the following statements summarizes the passage?

- (A) AOM are addressing the core problems and are likely to succeed.
- (B) The proposed AOM addresses the core problems that cause obesity.
- (C) AOM are addressing the problem superficially.
- (D) If obesity reduces, poverty will naturally reduce, since obesity causes poverty.

3.63 Listening to music during exercise improves exercise performance and reduces discomfort. Scientists researched whether listening to music while studying can help students learn better and the results were inconclusive. Students who needed external stimulation for studying fared worse while

students who did not need any external stimulation benefited from music.

Which one of the following statements is the CORRECT inference of the above passage?

- (A) Listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.
- (B) Listening to music has a clear positive effect both on physical exercise and on learning.
- (C) Listening to music has a clear positive effect on learning in all students. Music has a positive effect only in some students who exercise.
- (D) Listening to music has no effect on learning and a positive effect on physical exercise.

Solutions

3.1 (B)

Nadir : It means the lowest point which is in opposite to the word Zenith (highest point).

Hence, the correct option is (B).

3.2 (A)

Universalism : Complete devotion in the interest of one and all.

Particularism : Special devotion in a particular interest.

Universalism and Particularism are opposite words. Similarly, we require a word which is opposite to diffuseness.

Specificity : Means the quality of being particular or specific.

Neutrality : Means not taking any side in a conflict.

Generality : Covering range of things rather than being specific. Similar to diffuseness.

Adaptation : Act of changing something or yourself to make it suitable for a situation.

Diffuseness refers to spreading or causing to spread in all directions. Specificity is an appropriate opposite.

Hence, the correct option is (A).

3.3 (A)

Refer to the concept of Solution 3.2

The first clause in the given question is in past tense (unrealistic situation). So, second conditional sentence concept will be applied. Therefore the correct assertion is,

Were you a bird, you would fly in the sky.

Hence, the correct option is (A).

3.4 (C)

From option (C) : She is an European

Usage of article ‘an’ depends more on sound of the first alphabet of the word than on it being a vowel. If the sound is vowel, ‘an’ is used and if the sound is of consonant then ‘a’ is used.

In option (C), ‘European’ has a sound of ‘yoo’ which is a consonant. Therefore, appropriate article will be ‘a’ and not ‘an’.

Hence, the correct option is (C).

3.5 (D)



According to the given passage, it is clear that option (B) and (C) are wrong because there is no context about honesty or adventure.

Between option (A) and (D), option (D) is more suitable because the passage explains how bruce was motivated by the fact that despite having a near-impossible goal, the spider never gave up and continued its attempt to create its net without giving up hope.

Hence, the correct option is (D).

3.6 (A)

Eradicate : Destroy utterly, root out, uproot, abolish.

Distort : Misrepresent, subvert, warp.

Saturate : Soak completely, fill something so thoroughly that no more can be absorbed.

Utilize : Use, consume, tap, make a good thing.

This indicates that the correct sequence must be S, P, Q and R.

Hence, the correct option is (A).

3.7 (C)

Coherent : clear and easy to understand.

Sticky : Something that attaches itself or stick to things.

Well-connected : Properly joint together.

Friendly : Behaving in pleasant and kind way.

Rambling : Lengthy and confusing.

Rambling is clearly opposite in meaning to coherent.

Hence, the correct option is (C).

3.8 (C)

There is a contrast in the sentence, ‘he could not understand why the judges were awarding her the first prize because he thought that her performance was’ not up to the mark OR not very good.

Superb : Excellent.

Medium : A way of doing something.

Mediocre : Not very good.

Exhilarating : Feeling happy or thrilling.

As her performance was not so good, the word ‘Mediocre’ suffices the requirement.

Hence, the correct option is (C).

3.9 (B)

‘Cope with’ means to ‘adapt to’ which means adjust to something.

Adopt to : Take legally.

Adept in : Skillful or Skilful.

Accept with : Agree.

Hence, the correct option is (B).

3.10 (C)

‘The buck stops here’ means that responsibility for something should not be passed to someone else i.e. assume the final responsibility.

Hence, the correct option is (C).

3.11 (A)

Post-colonial : Existing after the end of colonial rule.

Since, British ruled India, it can be said that India is a post-colonial country because it was a former British colony.

Hence, the correct option is (A).

3.12 (D)

The statement clearly says that ‘while trying to collect an envelope from under the table, Mr. X fell down’ which is in past tense. The statement continues with ‘and’ so the rest of the part of the sentence should also be in past tense (and lost consciousness). Therefore, part (IV) of the sentence is incorrect.

Hence, the correct option is (D).

3.13 (B)

When a simple sentence is converted to question form and if the sentence is in past tense then in the question form, verb is changed to present tense with a ‘did’ added to the sentence.

Hence, the correct option is (B).

3.14 (C)

From option (A) :

knows (present tense),
will have (future tense)

From option (B) :



knew (past tense)
had (past tense)

From option (C) :

had known (Past perfect tense)
could have (perfect conditional)

From option (D) :

should have known (Present perfect tense)
would have (future tense)

In a type three conditional sentence, if the ‘if clause’ is in past perfect tense then the main clause will include should have / could have / would have + verb (III). Therefore in the given question, the correct assertion is,

If she had known how to calibrate the instrument, she could have done the experiment.

Hence, the correct option is (C).

3.15 (C)

Option (A) can not be inferred from the passage because the palghat gap is causing the high rainfall and high temperatures, not the other way around. Option (B) and (D) are nowhere nearly mentioned in the passage.

Therefore, it can be inferred from the passage that the low terrain of the palghat gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala.

Hence, the correct option is (C).

3.16 (B)

As it has been indicated that doctors will be able to eradicate psychiatric diseases such as depression and schizophrenia through gene therapy. So, it can be inferred that these psychiatric diseases have a genetic basis.

Hence, the correct option is (B).

3.17 (B)

The author states in the passage that after World War-II Kaliningrad became Russian territory but it is not contiguous with Russia because it used to have a German majority population.

Hence, the correct option is (B).

3.18 (D)

Municipal authorities admitted that it has been unable to control the mosquito population which is why the number of people with malarial fever has increased this year.

Hence, the correct option is (D).

3.19 (B)

The passage states that students participating in the programme, should come with a parent one hour earlier (9 am) and other students and parents should come in time for the programme (10 am) which clearly directs to option (B).

Hence, the correct option is (B).

3.20 (D)

The experimental measurements of the position of a star directly behind our sun gives proof for the hypothesis.

Hence, the correct option is (D).

3.21 (B)

Acquiescence : The reluctant acceptance of something without protest.

Submission : The action of accepting or yielding to a superior force.

Wheedle : Get to do something by gentle urging special attention or flattery.

Roundabout : Not straight forward or direct.

Flippancy : Lack of respect or seriousness.

Lightness : Lack of depth or seriousness.

Profligate : Recklessly extravagant or wasteful in the use of resources.

Extravagant : Lack of restraint or exceeding what is reasonable.

Only the combination in option (B) have different meanings.

Hence, the correct option is (B).

3.22 (B)

Children : Pediatrician

Pediatricians are the doctors who are specialist in treatment and care of children only.

Option (A) : Adult : Orthopedist

Orthopedists are the doctors who are specialist in treatment and care of bones.

[Includes men, women and kids]



Option (B) : Females : Gynecologist

Gynecologists are the doctors who are specialists in treatment of the female reproductive systems [Specific for women]

Option (C) : Kidney : Nephrologist

Nephrologists are the doctors who are specialists in treatment of kidney related problems. [Includes men, women and kids]

Option (D) : Skin : Dermatologist

Dermatologists are the doctors who are specialists in treatment of skin.

[Includes men, women and kids]

Only option (B) expresses relationship similar to the given pair.

Hence, the correct option is (B).

3.23 (A)

'Much' is used for uncountable quantities, therefore it is inappropriate.

'No paper' is also inappropriate.

'A few' refers to a count of not more than 'two or three'. Therefore, a few **paper** is also inappropriate.

So, for the given sentence, "**any paper**" is the best and only suitable option.

Hence, the correct option is (A).

3.24 (C)

'Completely' is an adverb which should be used before the verb '**forgot**'. And '**just**' should be used before "**don't know**".

The words in option (C) are arranged in right sequence.

Hence, the correct option is (C).

3.25 (B)

The given sentence states that due to extreme focus on syllabus and tests, students have closed their minds.

Now, option (A) is incorrect because students are focused on studies so they can't close their minds to anything related to the requirements of the exam.

Extraneous means irrelevant or unrelated to the subject being dealt with. It is perfectly suitable and correct because students have closed their minds to anything that is unrelated to the requirements of the exam.

Hence, the correct option is (B).

3.26 (C)

The Tamil version of the John Abraham starrer Madras Cafe was cleared by the Censor board with no cuts last week, but the film's distributors found no takers among the exhibitors for a release in Tamil Nadu on this Friday.

Hence, the correct option is (C).

3.27 (C)

The phrase "enjoyed herself immensely" means she had a fantastic or terrific time.

Hence, the correct option is (C).

3.28 (C)

"Took to their heels" is an idiom which means running away or "took to flight".

Hence, the correct option is (C).

3.29 (C)

Personnel : A person or a group of people employed in an organization or place of work.

Personal : Relating to a particular person or individual.

Only in the statement in option (C) word personnel is used appropriately.

Hence, the correct option is (C).

3.30 (D)

Fabric : A cloth made by weaving or knitting fibres.

Textile : Any cloth or goods produced by weaving or knitting.

Fibre : A fine, threadlike piece, as of cotton, jute or asbestos.

Apparel : A garment, clothing especially outerwear or attire.

Hence, the correct option is (D).

3.31 (A)

Correct versions :

Option (B) : The report was useless to them because there **was** no needed information in it.

Option (C) : Since the report did not contain the needed information, it was not **really** useful to them.



Option (D) : Since the report lacked needed information, it would not **have** been useful to them.

Therefore only option (A) is correct with respect to grammar and usage.

Hence, the correct option is (A).

3.32 (A)

Given statement has no information about Alexander's army or Indian borders. Hence, option (C) and (D) are incorrect.

Option (B) states that Alexander was not ready to rest on his laurels. Laurel means a type of ancient crown which is an indication of victory. The given statement does not comment on the state of mind of Alexander i.e. whether he wanted to conquer different countries or not.

Therefore option (B) is in correct.

Hence, the correct option is (A).

3.33 (B)

Option (A) clearly contradicts the essence of the passage. Hence, it is incorrect.

Passage states that due to lack of requisite temperament, it is highly unlikely that he will become a batsman of higher order. So, he has to improve his temperament to become a great batsman.

The passage states that after laying a strong foundation i.e. after a good head start, he throws away his wicket i.e. He failed to make many of his good starts count.

Therefore, option (B) can be inferred from the passage.

Hence, the correct option is (B).

3.34 (C)

Mock, deride and jeer are synonyms which means mockery or making fun. Therefore, the odd one is '**praise**'. Praise means admiration, compliment, magnification.

Hence, the correct option is (C).

3.35 (B)

Options (A) and (D) are incorrect because the post of security guard is a general post. So 'the security' is not correct. The word 'university' is

always prefixed with 'the' being important. So, option (C) is also incorrect.

Hence, the correct option is (B).

3.36 (A)

Paraphrase : A restatement of a text, passage or a rewording of something written or spoken.

Paradox : A self-contradictory statement.

Paradigm : Pattern or mode.

Paraffin : Soft solid derived from petroleum.

Hence, the correct option is (A).

3.37 (A)

Figurative : Representing by a figure or resemblance or expressing one thing in terms, normally denoting another with which it may be regarded as analogous.

Collateral : It is an asset which is offered to secure a loan.

Literal : Taking words in their basic or usual meaning.

Figurine : A small model of human.

Hence, the correct option is (A).

3.38 (A)

"cope with" means to put up with or tolerate.

Hence, the correct option is (A).

3.39 (D)

Given statements do not clearly indicate whether Ooty is a hill-station or not because having lakes does not necessarily mean Ooty is a hill-station. Hence, statement (i) can not be logically inferred. Statement (ii) is also not true, because in given sentences, for a place to be a hill-station, at least one lake is compulsory but nothing is mentioned about how many number of lakes.

Hence, the correct option is (D).

3.40 (D)

Word 'until' itself negates the upcoming sentence, hence 'does not' is not required. According to rules of present simple tense, verb 'meet' should have 's', since the object is noun.

Hence, the correct option is (D).

3.41 (D)



Ubiquitous : Very common or existing everywhere.

The author wonders if all these developments are good or required (the author is not sure) therefore statement (i) can not be inferred.

Statement (i) in the question is nowhere nearly mentioned in the passage.

It is clearly said in the passage that internet is an unintended consequence which makes statement (ii) wrong.

Hence, the correct option is (D).

3.42 (D)

The word ‘at least’ gives away the fact that there are more than 17 languages. So, option (A) is incorrect.

Option (C) is nowhere related to the passage.

The last statement of the passage shows that linguistic pluralism is a strong evidence of India’s diversity.

Hence, the correct option is (D).

3.43 (D)

Yielding : Tending to give up under pressure.

Resistant : Offer resistance or opposing.

Luminous : Bright or shining.

Radiant : Glowing brightly

Aversion : Strong dislike

Plunder : Rob or steal goods

Loot : Rob, sack

Yielding and resistant are not synonyms.

Hence, the correct option is (D).

3.44 (D)

‘Besides’ means in addition to or apart from. The statement says that ‘people work for many reasons’ which will continue with apart from money. So, the correct assertion is,

“Research in the workplace reveals that people work for many reasons **besides money**”.

Hence, the correct option is (D).

3.45 (C)

When the main clause is in the past tense or past perfect tense, the subordinate clause must be in the past or past perfect tense. So, the correct assertion is,

“After Rajendra Chola returned from his voyage to Indonesia, he **wished** to visit the temple in Thanjavur”.

Hence, the correct option is (C).

3.46 (B)

With adjectives and adverb, **enough** comes after adjectives and adverbs.

With nouns, **enough** comes before noun.

In the given question, enough is used with bright which is an adjective, so enough will come after the adjective.

So **bright enough** is the **correct option**.

Hence, the correct option is (B).

3.47 (B)

The author said in the passage that everything related to colonial past has a hold of nationalist imagination. Nationalism that is improperly represented is not history i.e. History is viewed through the filter of nationalism.

This statement best reflects the author’s opinion based on the given paragraph.

Hence, the correct option is (B).

3.48 (B)

Seemingly means external appearance as disguised from true character.

‘Considering them important only when they were handmaidens (means helping) to something seemingly more urgent’ but in reality it is not so urgent.

So, ideology is not as important as literature is revealed by the word “seemingly” only.

Hence, the correct option is (B).

3.49 (B)

They’re used for pointing group.

Their is pointing people.

There is used for people.

Hence, the correct option is (B).

3.50 (A)

Meandering : wandering aimlessly / indirect.

Hence, the correct option is (A).

3.51 (D)



Court is a place where judge rules, so according to given options, a school is a place where teacher rules.

Hence, the correct option is (D).

3.52 (C)

'Break down' means separating something into smaller parts. In the given sentence the expenditure is divided into different parts and the sentence is in present simple tense. Therefore, the correct usage will be "breaks down".

Hence, the correct option is (C).

3.53 (A)

Fulcrum : Something that plays a central or essential role.

Here the correct word in the sentence will be 'revolves' because the given sentence emphasizes on the fact that search engine's model is based on trust.

Hence, the correct option is (A).

3.54 (B)

As it is stated in the passage that state fails in protecting citizens who fall through the security net i.e. state is not providing enough welfare schemes for its people, which is the underlying or hidden reason that beggars are created.

Hence, the correct option is (B).

3.55 (A)

From the given analogy, first pair "Cook - cook" is a verb - noun pair, thus "fly - flyer" is the correct pair.

Hence, the correct option is (A).

3.56 (B)

Since the sentence mentions 'but', therefore the blank will be the opposite in meaning of word 'excellent', i.e., extremely poor

Hence, the correct option is (B).

3.57 (B)

Given statements implies a time frame, thus within six months would be grammatically correct. For the second blank, it indicates a time

period of three week, therefore for would be appropriate to use.

Hence, the correct option is (B).

3.58 (A)

Given logical relation is Profession and Work
Therefore Doctor is to Surgery as Writer is to Book

Hence, the correct option is (A).

3.59 (B)

Given : Relation shown is question is of object and its purpose

Pen is used to Write,

Similarly, Knife is used to Cut.

Hence, the correct option is (B).

3.60 (D)

The structure as...as is used to compare things that are of similar proportion. In this case the first as acts as an adverb modifying the adjective or adverb that goes after it. The second as can act as a preposition or conjunction. If it is used as a preposition, it will be followed by a noun or pronoun.

"As X as" is a comparison of equals.

"Better than" is not.

Therefore, better, worse, nicest cannot be used in equality comparison.

Hence, the correct option is (D).

3.61 (C)

It is an ambiguous statement because it has more than one possible meaning as who enjoys playing chess is not clear from the given sentence. It may be Babu or Dinesh.

Hence, the correct option is (C).

3.62 (C)

Given : AOM sidestep addressing the core problems and "superficially" means apparently/seemingly. So, AOM addressing the problem superficially.

Hence, the correct option is (C).

3.63 (A)



Given :

1. Listening to music during exercise improves exercise performance and reduces discomfort.
2. Effect of music on students depends on the type of students.

Therefore, listening to music has a clear positive effect on physical exercise. Music has a positive effect on learning only in some students.

Hence, the correct option is (A).