Quotient Operation

$L = L_1 / L_2$

$L = \{ x \mid \forall xy \in L_1 \ \& \ \exists y \in L_2 \}$

① $\dfrac{abc}{b} = \phi$

② $\dfrac{a\cancel{bc}}{\cancel{bc}} = a$

③ $\dfrac{abc \cdot \cancel{\epsilon}}{\cancel{\epsilon}} = abc$

④ $\dfrac{ab\cancel{c}}{ba\cancel{c}} = \phi$

⑤ $\{a, ab\} / \{\epsilon, b, aa\}$

$\Downarrow$

$\dfrac{a}{\epsilon}, \cancel{\dfrac{a}{b}}, \cancel{\dfrac{a}{aa}}, \dfrac{ab}{\epsilon}, \dfrac{a\cancel{b}}{\cancel{b}}, \cancel{\dfrac{ab}{aa}}$

$a, ab, a \Longrightarrow \{a, ab\}$

⑥ $\dfrac{aaaaa}{\{a, aa, aaa\}} = aaaa, aaa, aa$

⑦ $\dfrac{\epsilon \cdot \cancel{ab}}{\cancel{ab}} = \epsilon$.

(8) $$\frac{a^*}{a} = \frac{a}{a}, \frac{a\cancel{a}}{\cancel{a}}, \frac{aaa}{a}, \ldots \Longrightarrow a^*$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\epsilon \quad a \quad aa$$

(9) $$\frac{a}{a^*} = \frac{a}{a}, \frac{a \cdot \epsilon}{\epsilon} \Longrightarrow \{a, \epsilon\}$$

$$\Downarrow \quad \Downarrow$$

$$\epsilon \quad a$$

$$\frac{L_1 \cdot \cancel{\epsilon}}{\cancel{L_2 \cdot \epsilon}}$$

(10) $$\frac{a^*}{b^*} = \frac{\epsilon}{\epsilon} = \frac{\epsilon}{\epsilon}, \frac{a}{\epsilon}, \frac{aa}{\epsilon}, \frac{aaa}{\epsilon}, \ldots \Longrightarrow a^*$$

(11) $$\frac{ba^*}{b} = \frac{b}{b} \Longrightarrow \epsilon$$

(12) $L_1/\phi = \phi$

(13) $\dfrac{\phi}{L_2} = \phi$

(14) $\dfrac{a^*b}{ab} = \dfrac{ab}{\cancel{ab}}, \dfrac{a\cancel{ab}}{\cancel{ab}}, \dfrac{aaab}{ab} \Rightarrow a^*$

(15) $\dfrac{\epsilon}{a} = \phi$

Note (1) If L_2 includes ϵ then L_1/L_2 = minimum L_1 will come.

(2) If L_2 is not empty then $\dfrac{\Sigma^*}{L_2} = \Sigma^*$

(3) "

then $\dfrac{L_2}{\Sigma^*} = \dfrac{TOC}{\epsilon}, \dfrac{TOC}{C}, \dfrac{TOC}{OC}$ TOC

All prefixes of L_2 $\Longleftarrow$ $\dfrac{TOC}{TOC}$

(4) If L_1 & L_2 are regular then

L_1/L_2 is also regular.

$\boxed{\phi \cdot \epsilon = \phi}$

$abcd \cdot \epsilon = abcd$

$\epsilon \cdot abcd = abcd$

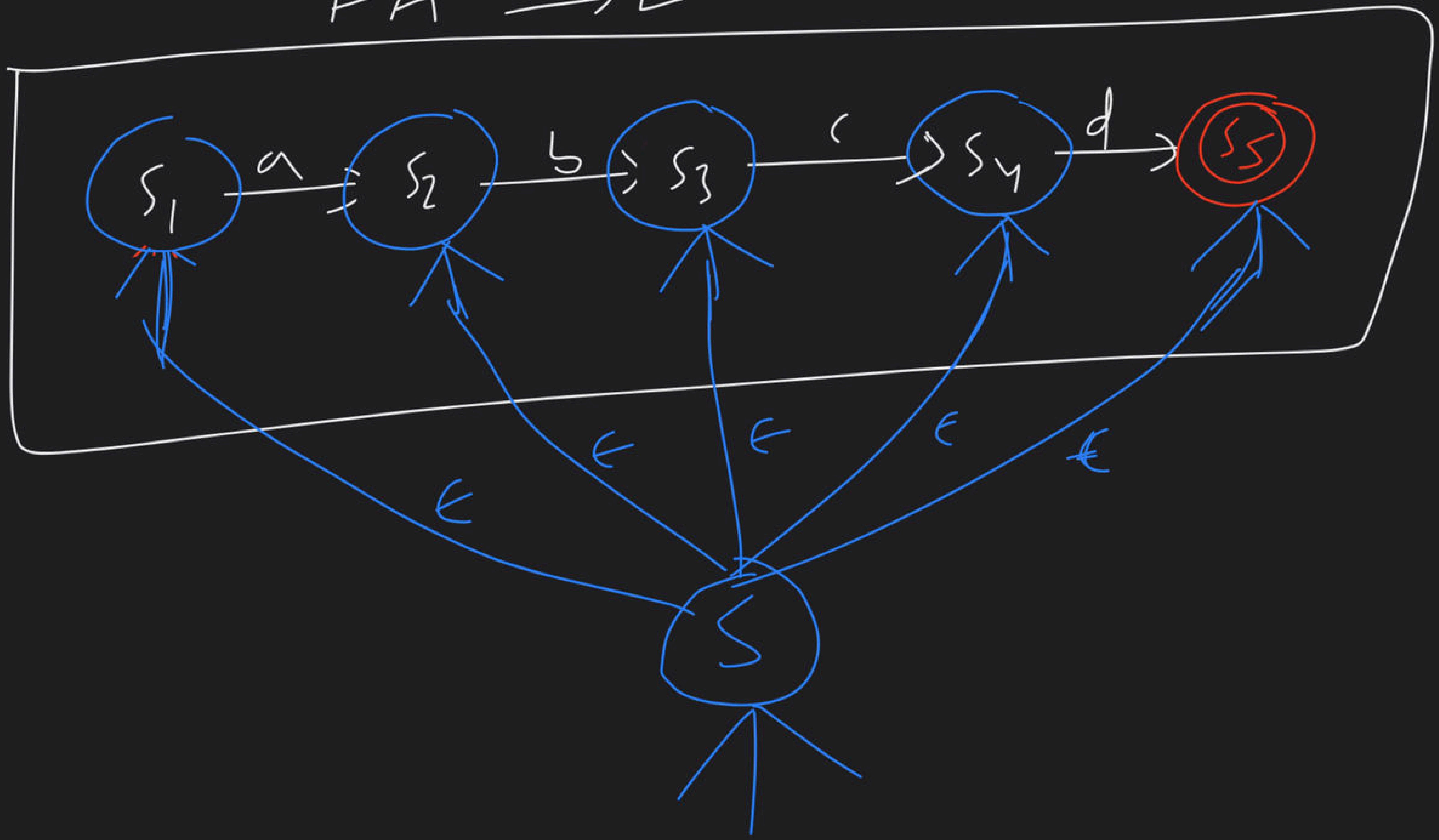
$L = \{abcd\}$

$prefix(L) = \{\epsilon, a, ab, abc, abcd\}$

$Suffix(L) = \{\epsilon, d, cd, bcd, abcd\}$

$Substring(L) = \{\epsilon, a, b, c, d, ab, bc, cd, bcd, abc, abcd$

FA ⟹ L



If L is regular then

make-all-final $\Longleftarrow$ prefix(L)

make-all-start $\Longleftarrow$ Suffix(L) are also regular

make-all-F & S $\Longleftarrow$ Substring(L)

Substitution

It is a mapping from

Σ to $P(\Delta^*)$

(or)

It is a mapping from each Symbol of Σ to one of the Regular Language over Δ.

$\Sigma \longrightarrow \Delta^*$

$\Sigma \longrightarrow P(\Delta^*)$

R.L

ex $\Sigma = \{a, b\}$ $\Delta = \{0, 1\}$

$S(a) = 0^k$ $S(b) = 0^k 1$

1. $L_1 = \{ab\}$

$S(L_1) = ?$

$\Downarrow$

$S(ab)$

$\Downarrow$

$S(a) \cdot S(b)$

$0^k \cdot 0^k 1 \Longrightarrow 0^k 1$

2. $L_2 = \{a^k\}$

$\Downarrow$

$S(L_2)$

$\Downarrow$

$S(a^k) \Longrightarrow [S(a)]^k \Longrightarrow (0^k)^k \Longrightarrow 0^k$

1. $S(abcd)$

$\Downarrow$

$S(a) \cdot S(b) \cdot S(c) \cdot S(d)$

2. $S(a^k) = [S(a)]^k$

3. $S(a+b)$

$\Downarrow$

$S(a) + S(b)$