Basics Concepts

Comprehensive Course on Engineering Mathematics

B V Reddy • Lesson 1 • July 27, 2023

ENGINEERING MATHEMATICS

(CS-IT)

B V REDDY Sir

UNIQUE WAY OF TEACHING

- BUILIDING THE STRONG CONCEPT
- SOLVING BASIC PROBLEM TO MAKE MORE STRONG IN CONCEPTS
- SOLVING PRVIOUS GATE and ESE PROBLEMS

Preparation Strategy

1. Class notes
2. Previous paper of GATE

   ECE
   EEE
   IN
   CS
   MECH
   CIVIL
   CHE
   PI

$\boxed{2023 - 2010}$

Preparation Strategy

1. Class notes
2. Previous paper of GATE

ECE

EEE

IN

CS

MECH

CIVIL

CHE

PI

Things I will provide

1. Complete Notes
2. Short Notes
3. DPPs with all PYQs
4. My Contact No :

93980 21419 .

SOLVE ALL THE DPPs

PROBABILITY

1. $P(A) = \frac{n(A)}{n(S)}$

2. $0 \le P(A) \le 1$

Impossible event (for 0); Sure event (certain event) (for 1)

3. Sum of all probabilities =1

$$\sum P = 1$$

4. P(sample space) =1

5. $P(\overline{A}) = 1 - P(A)$

6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A & B are mutually exclusive events,

$P(A \cap B) = 0$

7. $P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$

8. $P(\overline{A \cup B}) = 1 - P(A \cup B)$

$P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B})$

9. $P(\overline{A \cap B}) = 1 - P(A \cap B)$

$P(\overline{A \cap B}) = P(\overline{A} \cup \overline{B})$

10. $P(\text{only } A) = P(A \cap \overline{B}) = P(A) - P(A \cap B)$

11. $P(\text{only } B) = P(\overline{A} \cap B) = P(B) - P(A \cap B)$

12. $P(\text{Both } A \text{ \& } B) = P(A \cap B)$

13. $P(\text{at least one}) = P(A \cup B)$

14. $P(\text{Either } A \text{ or } B) = P(A \cup B)$

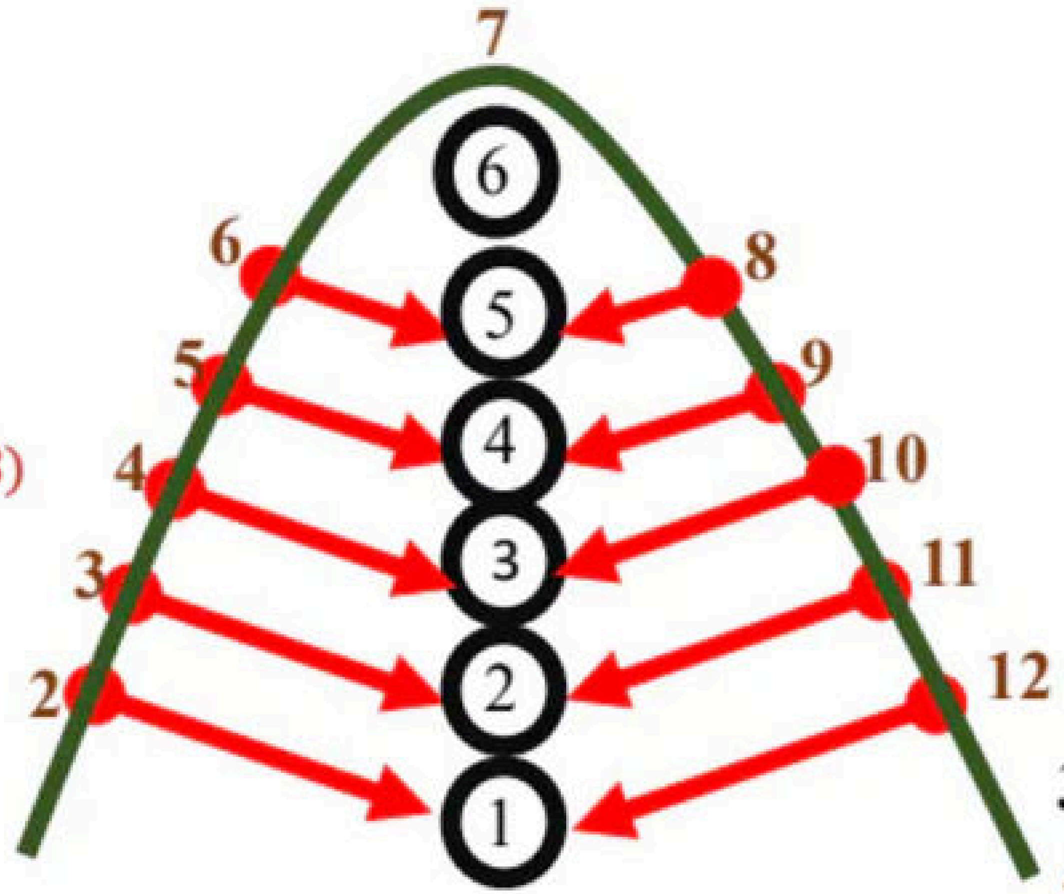
15. $P(\text{neither } A \text{ nor } B) = P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

16. $P(\text{neither } A \text{ nor } B) = P(\overline{A} \cap \overline{B}) = \overline{P(A \cup B)}$

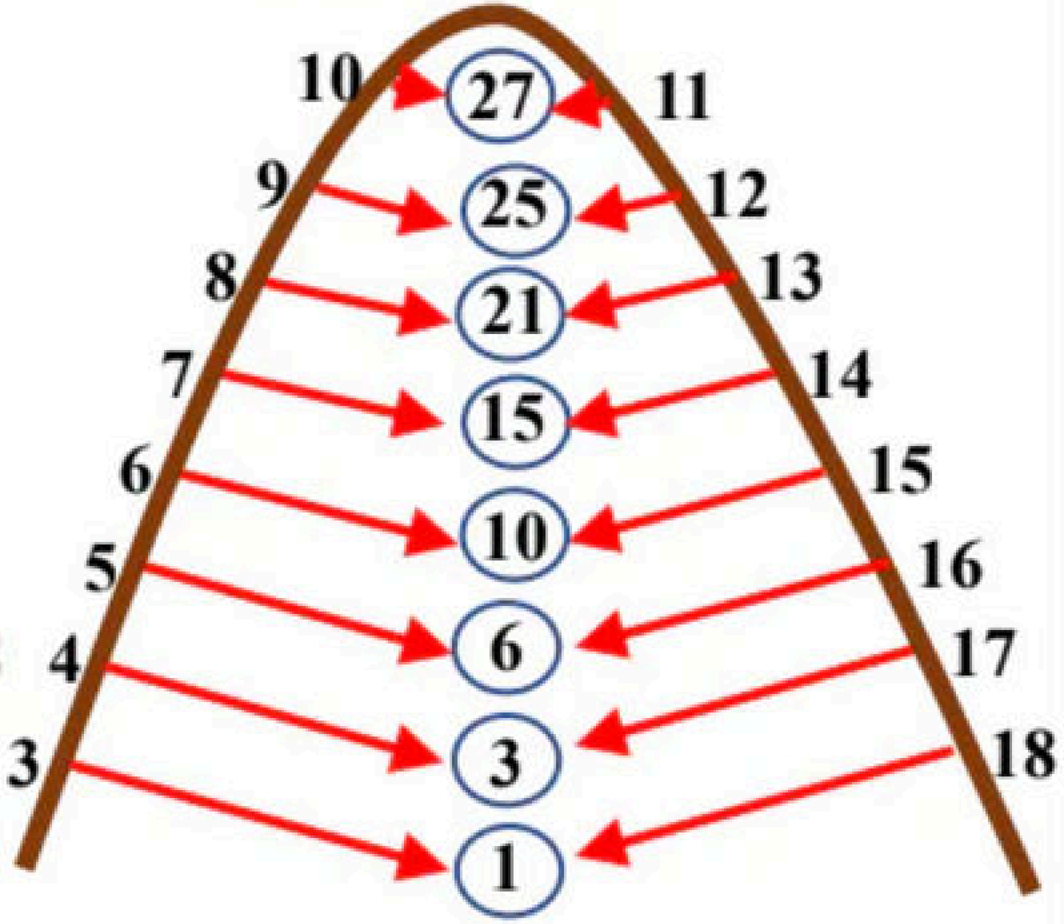
17. $P(\text{exactly one}) = P(A \Delta B) = P(A \cap \overline{B}) + P(\overline{A} \cap B)$

18. Rolling a dice

2 –Dice rolled



3 - Dice rolled



Pack of cards (52)

- Red(26)
  - Hearts(13)
  - Diamonds(13)
- Black(26)
  - Spade(13)
  - Club(13)

Each suit contains

2, 3, 4, 5, 6, 7, 8, 9, 10 → **Number cards = 9**

J, K, Q → **Face cards = 3**

J, K, Q, A → **Honour cards = 4**

20. If A, B, C are pair wise independent

$P(A \cap B) = P(A)P(B)$

$P(B \cap C) = P(B)P(C)$

$P(C \cap A) = P(C)P(A)$

21. If A, B, C are mutually independent events

$P(A \cap B) = P(A)P(B)$

$P(B \cap C) = P(B)P(C)$

$P(C \cap A) = P(C)P(A)$

$P(A \cap B \cap C) = P(A)P(B)P(C)$

22. If A, B, C, D are mutually independent events

$P(A \cap B) = P(A)P(B)$

$P(A \cap C) = P(A)P(C)$

$P(A \cap D) = P(A)P(D)$

$P(B \cap C) = P(B)P(C)$

$P(B \cap D) = P(B)P(D)$

$P(C \cap D) = P(C)P(A)$

$P(A \cap B \cap C) = P(A)P(B)P(C)$

$P(A \cap C \cap D) = P(A)P(C)P(D)$

$P(B \cap C \cap D) = P(B)P(C)P(D)$

$P(A \cap B \cap D) = P(A)P(B)P(D)$

$P(A \cap B \cap C \cap D) = P(A)P(B)P(C)\,P(D$

- The total number of conditions for mutual independence of n- events is $= 2^n - 1 - n$
- **Mutually independent events are pair wise independent but vice versa not true .**

23. $P(A/B) + P(\overline{A}/B) = 1$

$P(A/\overline{B}) + P(\overline{A}/\overline{B}) = 1$

24. If A & B independent events

- **$\overline{A}$ & B are also independent**
- **A & $\overline{B}$ are also independent**
- **$\overline{A}$ & $\overline{B}$ are also independent**

Total probability theorem

$$P(A) = \sum_{n=1}^{N} P(B_n)P\left(\frac{A}{B_n}\right)$$

Baye's Theorems

$$P\left[\frac{B_n}{A}\right] = \frac{P\left(\frac{A}{B_n}\right)p(B_n)}{\sum_{n=1}^{N} P\left(\frac{A}{B_n}\right)p(B_n)}$$

CDF

$F(x) = P(X \leq x)$

1. $F(-\infty) = 0$
2. $F(\infty) = 1$
3. $0 \leq F(x) \leq 1$
4. CDF is a non-decreasing function
5. CDF is a continuous function
6. $P(x > x_1) = 1 - F(x_1)$

PDF

$f(x) = \frac{d}{dx}(F(x))$

1. $\int_{-\infty}^{\infty} f(x)dx = 1$

Area Bounded by any pdf curve is unity

2. $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} f(x)dx$

Mean

$E[x] = \sum xp(x) \rightarrow$ for discrete R.V

$E[x] = \int xf(x)dx \rightarrow$ for continous R.V

1. $E[K] = K$.
2. $E[x+K] = E[x] + K$
3. $E[ax+b] = a\,E[x] + b$
4. $E[x+y] = E[x] + E[y]$
5. $E[ax+by] = a\,E(x) + b\,E(y)$
6. $E[x-m] = 0$ m - mean of x.
7. if x & y are independent R.V
   $E[xy] = E[x]\,E[y]$.

Variance

$Var(x) = E[(x-m)^2]$

$Vax(x) = E[x^2] - (E[x])^2$

1. $Var(K) = 0$
2. $Var(ax) = a^2\,var(x)$
3. $Var(ax+b) = a^2\,var(x)$
4. $var(x+y) = var(x) + var(y) + 2\,cov(x, y)$
5. $var(x-y) = var(x) + var(y) - 2\,cov(x, y)$

If x & y are independent random variable

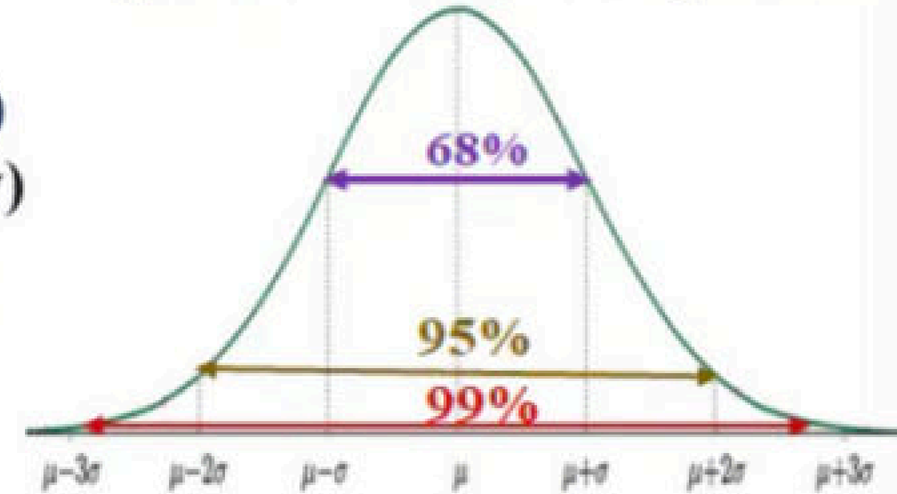
$cov(x, y) = 0$

$var(x+y) = var(x) + var(y)$

$var(x-y) = var(x) + var(y)$

Normal Density function

1. The curve is smooth, regular, bell shaped and symmetrical about mean
2. The area under the normal density function is unity .
3. The maximum value of density function occurs at $x = \mu$
   Its maximum value of density function is $= \frac{1}{\sigma\sqrt{2\pi}}$
4. If σ – increases then normal density function decreases and curve tends to be flat
5. If σ – decreases then normal density function increases and curve tends to be more peaked at mean .
6. $P(\mu - \sigma < x < \mu + \sigma) = 0.6826$
   $P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9544$
   $P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$

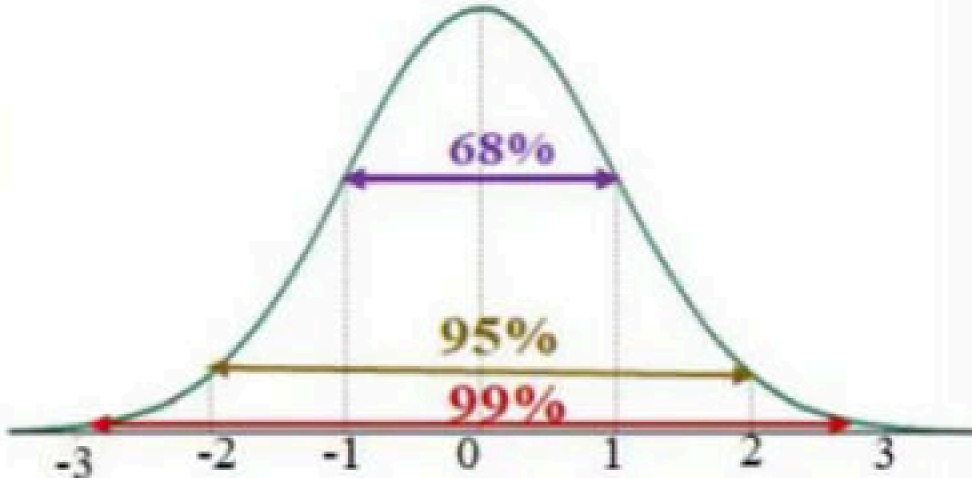


7. **Standard Normal density function**

$P(-1 \leq Z \leq 1) = 0.6826$

$P(-2 \leq Z \leq 2) = 0.9544$

$P(-3 \leq Z \leq 3) = 0.9973$



8. $P(-a \le Z \le a) = 2P(0 \le z \le a)$

9. $P(-a \le Z \le b) = P(0 \le z \le a) + P(0 \le z \le b)$

10. $P(Z \ge a) = 0 \cdot 5 - P(0 \le z \le a)$

11. $P(Z \le -a) = P(z \ge a)$

12. $P(Z \ge -a) = P(z \le a)$

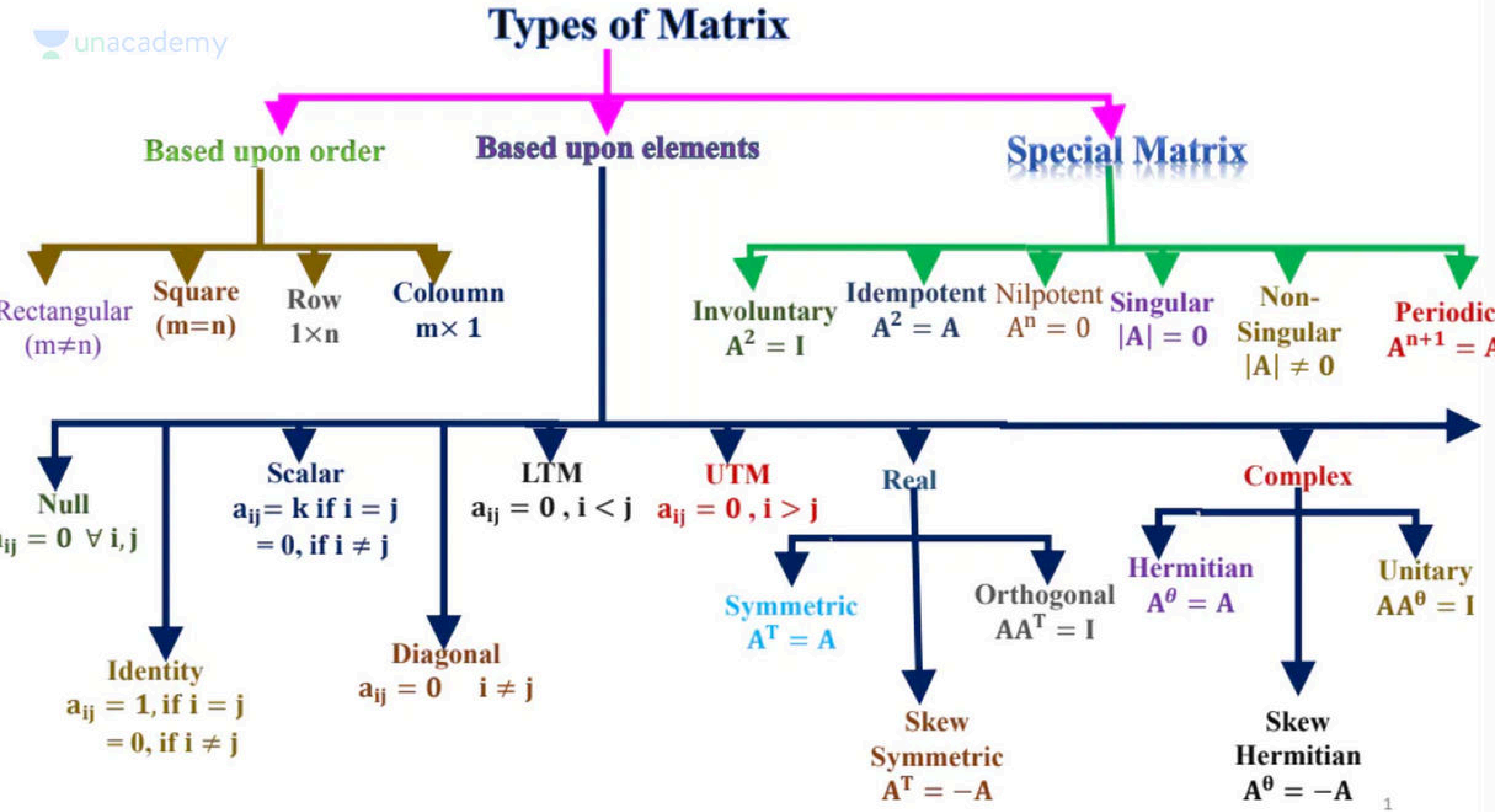
13. $P(Z \le a) = 0 \cdot 5 + P(0 \le z \le a)$

14. If $a < b$

$P(a \le Z \le b) = p(0 \le z \le b) - P(0 \le z \le a)\cdot$

Probability Distribution Functions

	Binomial	Poisson's	Uniform	Exponential	Normal
1. pdf	$B(x,n,p) = n_{C_x} p^x q^{n-x}$	$P(x,\lambda) = \dfrac{e^{-\lambda}\lambda^x}{x!}$ $\lambda = np$	$f(x) = \dfrac{1}{b-a}$ $a \le x \le b$	$f(x) = ae^{-ax}$, $x > 0$	$f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$
2. Mean	np	λ	$\dfrac{a+b}{2}$	$\dfrac{1}{a}$	Mean = μ
3. Variance	npq	λ	$\dfrac{(a-b)^2}{12}$	$\dfrac{1}{a^2}$	Variance = σ^2
3. S.D	$\sqrt{npq}$	$\sqrt{\lambda}$	$\sqrt{\dfrac{(a-b)^2}{12}}$	$\dfrac{1}{a}$	S.D = σ



Types of Matrix
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Based upon order
Based upon elements
Special Matrix
Rectangular (m≠n)
Square (m=n)
Row 1×n
Coloumn m× 1
Involuntary $A^2 = I$
Idempotent $A^2 = A$
Nilpotent $A^n = 0$
Singular $|A| = 0$
Non-Singular $|A| \neq 0$
Periodic $A^{n+1} =$
Null $a_{ij} = 0 \ \forall \, i, j$
Identity $a_{ij} = 1$, if $i = j$ $= 0$, if $i \neq j$
Scalar $a_{ij} = k$ if $i = j$ $= 0$, if $i \neq j$
Diagonal $a_{ij} = 0 \quad i \neq j$
LTM $a_{ij} = 0 \, , i < j$
UTM $a_{ij} = 0 \, , i > j$
Real
Symmetric $A^T = A$
Orthogonal $AA^T = I$
Skew Symmetric $A^T = -A$
Complex
Hermitian $A^\theta = A$
Unitary $AA^\theta = I$
Skew Hermitian $A^\theta = -A$

Trace of a matrix

Trace(A) = sum of principal diagonal elements

If A & B are square matrices of order n, then

- tr(A+B) = tr(A)+tr(B)
- tr(A-B) = tr(A)- tr(B)
- tr(AB) ≠ tr(A) tr(B)
- tr(BA) ≠tr(B) tr(A)
- tr(AB) = tr(BA)
- tr(kA) = k tr(A)
- $tr(A^T) = tr(A)$
- $\mathbf{tr}\ (I_n) = n$

Transpose of a matrix

rows into columns and columns into rows.

if A= $[a_{ij}]$ then $A^T = [a_{ji}]$

- $(A^T)^T = A$
- $(KA)^T = KA^T$
- $(A - B)^T = A^T - B^T$
- $(A\,B\,C)^T = C^T B^T A^T$
- $(A^T)^n = (A^n)^T$

If A- Skew-symmetric

kA, A^3, A^5...... A^{2n+1} are Skew symmetric

A^{2n} Symmetric

If A- Symmetric

kA, $A + A^T$, AA^T, $A^T A$, A^n are symmetric

$A - A^T$, $A^T - A$ are skew-symmetric.

- ✓ $AI = A$
- ✓ $I^{-1} = I$
- ✓ $I^T = I$
- ✓ $I^n = I$
- ✓ $|I| = 1$
- ✓ $Adj\,(I) = I$

- ✓ The diagonal elements of a skew-symmetric matrix are all zero.
- ✓ Sum of all the elements of skew symmetric matrix is zero .
- ✓ Null matrix is both symmetric and skew symmetric
- ✓ if A and B are square symmetric matrices of same order then AB is symmetric if and only if AB =BA
- ✓ If A and B are skew symmetric matrices of same order then AB is skew symmetric if and only if AB = - BA.

If A and B are symmetric then

(AB + BA) -----> symmetric

(AB - BA) -----> skew symmetric

$(K_1A + K_2B)$ -----> symmetric

If A and B are skew symmetric

then (AB + BA) -----> symmetric

(AB - BA) -----> skew symmetric

$(K_1A + K_2B)$----> skew symmetric

- ☐ Every square matrix A can be uniquely expressed as a sum of symmetric & Skew- symmetric matrices.
- ☐ If $A^T = A$, then A is symmetric
- ☐ $if\ A^T = -A$. Then A is skew symmetric

Matrix Algebra

- A+B = B+A (Commutative law)
- (A+B)+C = A+ (B+C) (Associative law)
- A+(-A) = 0(null matrix)
- A+B = A+C then B = C (Left Cancellation law)
- B+A = C+A , then B= C (Right cancellation law)
- k(A+B) = kA +kB
- $(k + l)A = k A + l A$
- $(k\ l)A = k(lA) = l(kA)$
- $(-k)$A = - (kA)
- If AB --->exists ,
- BA -----> may or may not exist
- AB ≠ BA (not obeys commutative)
- ABC =A(BC)= B(AC) (obeys Associative)
- A(B+C) = AB+AC (obeys Distributive)
- $A^m A^n = A^{m+n}$
- $(A^m)^n = A^{mn}$
- If AB = 0 , then A and B may not be null matrix.
- If AB =0 , then BA may not be zero .
- If $A^2 = B^2$, then A and B may not be equal
- If A and B are two square matrix of same order then
  - $(A + B)^2 = A^2 + B^2 + AB + BA$
  - $(A - B)^2 = A^2 + B^2 - AB - BA$

Minor and Co-factor of an element

$$\text{if } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

minor of a_{11} is

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$= (a_{22}\, a_{33} - a_{32}\, a_{23})$$

Co-factor of a_{ij} is

$$A_{ij} = (-1)^{i+j} M_{ij}$$

S.No	Matrix	Determinant
1	Orthogonal matrix	$\|A\| = \pm 1$
2	Unitary matrix	$\|A\| = \pm 1$
3	Involutary	$\|A\| = \pm 1$
4	UTM LTM Diagonal Scalar Identity	$\|A\|$= Product of principal diagonal elements
5	Idempotent	$\|A\| = 0$ or 1
6	Nilpotent	$\|A\| = 0$
7	All the elements of any row are zero	$\|A\| = 0$
8	All the elements of any column are zero	$\|A\| = 0$
9	All elements are consecutive	$\|A\| = 0$ (valid for 3rd and higher order matrix)
10	Skew symmetric of odd order	$\|A\| = 0$
11	Skew symmetric of even order	Perfect square
12	Sum of all elements of each row is zero	$\|A\| = 0$
13	Sum of all elements of each column is zero	$\|A\| = 0$

	Matrix	Determinant
14	$R_i = k\, R_j$	$\|A\| = 0$
15	$C_i = k\, C_j$	$\|A\| = 0$
16	$R_i \longleftrightarrow R_j$ $C_i \longleftrightarrow C_j$	Sign of Det changes
17	$R_i \rightarrow R_i + kR_j$ $C_i \rightarrow C_i + kC_j$	No change of Det
18	$\|AB\| = \|A\|\|B\|$.	
19	$\|A\| = \|A^T\|$	
20	$\|A^m\| = \|A\|^m$ $\|A^{-1}\| = \frac{1}{\|A\|}$ ($\|A_{n\times n}\| \neq 0$)	
21	$\|kA_{n\times n}\| = k^n \|A_{n\times n}\|$	
22	$\|A+B\| \neq \|A+B\|$	
23	$\|I\| = 1$	

Inverse of a square matrix

- If the inverse of a square matrices A exists then the matrix is called invertible matrix

$A^{-1} = \frac{1}{\lvert A\rvert} adj(A)$.
$(A^{-1})^{-1} = A$
$(AB)^{-1} = B^{-1}A^{-1}$
$(ABCD)^{-1} = D^{-1}C^{-1}B^{-1}A^{-1}$
$(kA)^{-1} = \frac{A^{-1}}{k}$
$(A^T)^{-1} = (A^{-1})^T$
$(A^\theta)^{-1} = (A^{-1})^\theta$.
$(A^n)^{-1} = (A^{-1})^n$
$(I)^{-1} = I$
$(adjA)^{-1} = \frac{A}{\lvert A\rvert}$

$A(\text{adjA}) = (adjA)A = \lvert A\rvert I$
$adj(AB) = adj(B)adj(A)$
$adj(kA) = k^{n-1}adj(A)$
$\lvert adj(A)\rvert = \lvert A\rvert^{n-1}$
$\lvert adj(kA)\rvert = k^{n(n-1)}\lvert A\rvert^{n-1}$
$\lvert adj(adj(A))\rvert = \lvert A\rvert^{(n-1)^2}$
A ---> Hermitian matrix then adj(A)---> Hermitian.
A ---> symmetric matrix then adj (A) ---> symmetric.
A--->diagonal matrix then adj(A)--->diagonal matrix
A ---->triangular matrix , then adj(A)---> triangular matrix. If A is LTM then adj(A) --->LTM If A is UTM then adj(A) --->UTM

Rank of a Matrix

A non-negative integer 'r' is said to be the rank of matrix A, if

- There exists at least one non-zero minor of order 'r'.
- all minors of order (r+1) if they exist, are zeros. Then we rite Rank of A = $\rho(A)$ = r (or)
- Rank of matrix A is the number of linearly independent rows (or columns) of A (or)
- The number of non zero rows in the Row Echelon form .

Echelon form

A matrix A of order m×n is said to be in row echelon form if

- The number of zeros before the first non-zero element in each row is less than the number of such zeros in the next non zero row.
- If there are any Zero rows , they must be below the non-zero rows.

1	Rank (null matrix) = zero
2	$\rho(A_{m\times n}) \le min\{m, n\}$.
3	If A is non singular matrix, $\rho(A_{n\times n}) = n$.
4	If A is singular matrix, then $\rho(A_{n\times n}) < n$.
5	$\rho(I_n) = n$.
6	$\rho(A^T) = \rho(A)$. $\rho(A^{-1}) = \rho(A)$. $\rho(AA^T) = \rho(A)$. $\rho(A^\theta) = \rho(A)$. $\rho(AA^\theta) = \rho(A)$
7	$\rho(AB) \le min\{\rho(A), \rho(B)\}$.
8	If A and B are square matrix of order n, then $\rho(AB) \ge \rho(A) + \rho(B) - n$
9	If $\rho(A_{n\times n}) = n$, then $\rho(adj\ A) = n$
10	$\rho(A + B) \le \{\rho(A) + \rho(B)\}$.
11	$\rho(A - B) \ge \{\rho(A) - \rho(B)\}$.

12	**If A and B are square matrix of order n, then** $\mathbf{Max}\{0, \rho(A) + \rho(B) - n\} \le \rho(AB) \le min\{\rho(A), \rho(B)\}$
13	**If A and B are square matrix of order n, then** $\lvert\rho(A) - \rho(B)\rvert \le \rho(A + B) \le min\{\rho(A) + \rho(B), m, n\}$
14	The rank of a diagonal matrix is equal to the number of non-zero diagonal elements.
15	The rank of a UTM is number of non zeros rows in the UTM. The rank of a LTM is number of non zeros rows in the LTM.
16	If $\rho(A_{n\times n}) = n - 1$, then $\rho(adj\ A) = 1$.
17	If $\rho(A_{n\times n}) < n - 1$, then $\rho(adj\ A) = 0$.

Non Homogeneous equations
(AX= B)

Find $\rho(A)$, $\rho(A \mid B)$

- $\rho(A) = \rho(AB)$
  - System is consistent i.e solution exists
    - $\rho(A) = \rho(A|B) = n$ (no.of unknowns)
      - Unique solutions
    - $\rho(A) = \rho(A|B) < n$
      - Infinitely many solutions
- $\rho(A) \neq \rho(AB)$
  - System is inconsistent i.e no solution

Homogeneous equations
(AX= 0)

Always has a solution

- $\rho(A) = n$ (no.of unknowns)
  - Unique solutions (or) Zero as a solution (or) Trivial solution
- $\rho(A) < n$
  - Infinitely many solutions (or) Non trivial solution

Eigen Values

- Sum of the eigen values of a matrix is equal to the trace of the matrix
- Product of the eigen values of a matrix is equal to the determinant of the matrix .
- For lower triangular matrix (upper triangular matrix or diagonal matrix or scalar matrix or identity matrix), the Eigen values are same as diagonal elements of the matrix.
- For a matrix if a+ib is an eigen value of matrix A , then a-ib is also an eigen value of matrix A.
- For a matrix if a+$\sqrt{b}$ is an eigen value of matrix A , then a- $\sqrt{b}$ is also an eigen value of matrix A.

If **λ is an Eigen value** of a matrix A , **X is the Eigen Vector** and k is a scalar then

S.No	Matrix	Eigen Values	Eigen Vectors
1	A^m	λ^m	X
2	A^{-1}	$\frac{1}{\lambda}$	X
3	A^θ	$\bar{\lambda}$	X
4	kA.	Kλ	X
5	$A \pm kI$	$A \pm k$	X
6	$(A \pm kI)^n$	$(\lambda \pm k)^n$	X
7	adjA	$\frac{\lvert A \rvert}{\lambda}$	X
8	$a_0I + a_1A + a_2A^2$	$a_0 + a_1\lambda + a_2\lambda^2$	X

Type of Matrix	Eigen values
1. Symmetric matrix and Hermitian matrix	Real values
2.Skew symmetric matrix and skew Hermitian matrix	Zero (or) purely imaginary
3.Orthogonal matrix and Unitary matrix	Magnitude of eigen value is one $\lvert\lambda\rvert = 1$
4.Idempotent matrix	0 (or) 1
5.Involuntary matrix	-1 (or) +1
6. Nilpotent matrix	0

- If $\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$ are distinct Eigen values of a square matrix A of order 'n' then the corresponding Eigen vectors $X_1, X_2, \ldots\ldots, X_n$ of matrix A are linearly independent.
- If some Eigen values of matrix A are repeated then Eigen vectors of A may or may not be linearly independent.
- The number of linearly independent eigen vectors of an eigen value 'λ' is $n - \rho(A - \lambda I)$

Cayley – Hamilton theorem

Every square matrix satisfies its own characteristic equation.

- To find higher powers of a matrix
- To find Inverse of a matrix

Diagonalization of a Matrix

$$AP = PD$$

A---Given matrix

D—Diagonal Matrix

P---Modal Matrix

- To find the matrix from Eigen vectors

$$A = PDP^{-1}$$

- To find higher powers of a matrix

$$A^n = PD^nP^{-1}$$

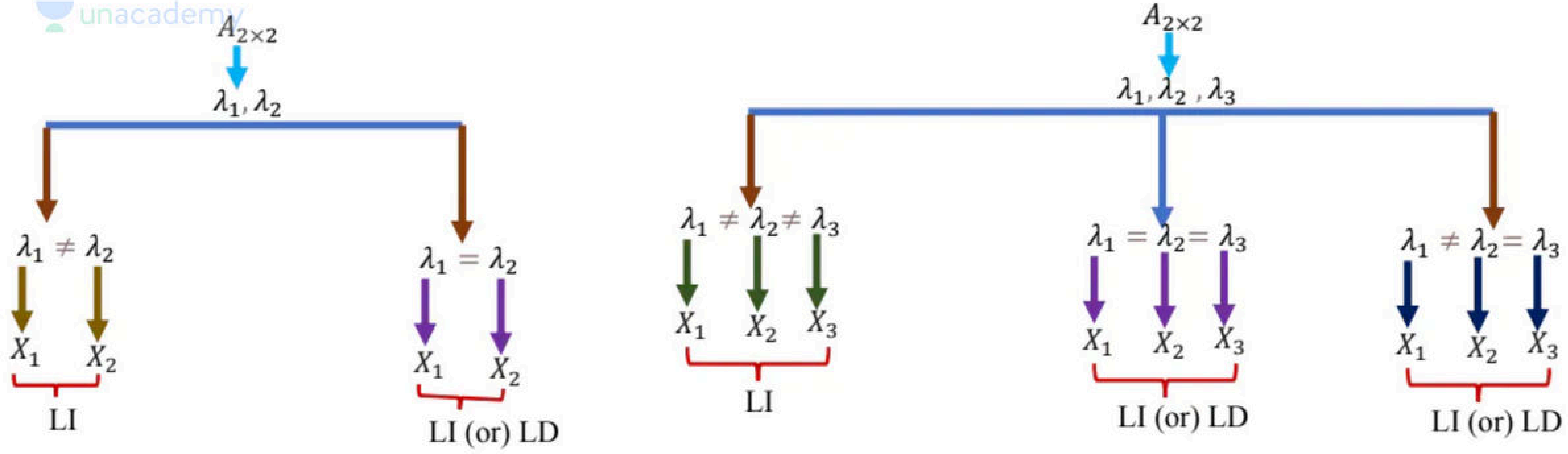
Algebraic Multiplicity of an Eigen value

- The number of times an eigen value is repeated is called as algebraic multiplicity (A.M) of that eigen value

Geometric Multiplicity of an Eigen value

- The number of linearly independent eigen vectors corresponding to an eigen value λ is called as geometric multiplicity of that eigen value λ.
- The number of linearly independent eigen vectors of an eigen value 'λ'
- $= n - \rho(A - \lambda I)$
- Algebraic multiplicity of an eigen value $\geq$ Geometric multiplicity
- Geometric multiplicity of every eigen value of a matrix is ≥ 1
- If all the eigen values of a matrix are distinct then the matrix can be diagonalizable but converse need not be true .
- A matrix is diagonalizable iff for every eigen value , geometric multiplicity is equal to algebraic multiplicity.

- Every idempotent matrix , involuntary matrix , symmetric matrix, unitary matrix can be diagonalizable .
- Nilpotent matrix can never be diagonalizable .
- If all the elements are equal then A is diagonalizable .



$$\lambda^2 - (\textit{trace of } A)\lambda + |A| = 0 \cdot$$

$$\lambda^3 - (\textit{trace of } A)\lambda^2 + (\textbf{trace}(\textbf{Adj})\lambda - |A| = 0$$

Use the code: BVREDDY, to get maximum discount

Syllabus

1. Probability

2. Linear algebra.

3. Calculus.

PROBABILITY

- Basic definitions
- Problems on Coin , Dice , Cards and Balls
- Conditional Probability
- Independent events
- Total Probability
- Bayes Theorem

PROBABILITY

- **Single Random Variable**

  Mean , Variance , S.D and Skew

- Probability Distributions

  Binominal ,

  Poisson,

  Uniform ,

  Exponential ,

  Gaussian (Normal)

- Multiple Random Variable

Experiment :

Any physical action or a process whose results are observed and noted . There are two types of experiments , there are

$H_2 + O_2 \longrightarrow H_2O$

1. Deterministic or predictable experiment
2. Non deterministic or random experiment

Deterministic experiment

An experiment whose outcomes are predictable with certainty prior to the performance of an experiment .

Eg: sum of two numbers .

$$5+3 = 8$$

Non deterministic experiment

An experiment whose outcomes cannot be predicted with certainty prior to the performance of an experiment but all possible combinations of the outcomes can be predicted .

Eg: 1. Tossing a coin ✓

2. Rolling a dice

$$\{H, T\}$$

$$\{1, 2, 3, 4, 5, 6\}.$$

Sample space

All the possible outcomes of a random experiment .

Eg : tossing a coin

S= { Head , Tail }

Rolling a dice

S = { 1,2,3,4,5,6}

There are two types of sample space

1. Continuous sample space
2. Discrete sample space

Continuous sample space

If the sample space is defined for every value of the given interval .

Eg: measuring the temperature of a room between two intervals

$T =$

10am $< t <$ 11pm.

2 3

Discrete sample space

If the sample space is defined only in discrete intervals of time .

Eg : Rolling a die

S ={ 1,2,3,4,5,6}.

Event

The outcomes of a random experiment are called as Event .Event is always subset of the sample space .

$S = \{H, T\}$

Favorable events

The outcomes which are favorable to my desired event .

$S = \{1, 2, 3, 4, 5, 6\}$

$A \rightarrow$ even no.

$A = \{2, 4, 6\}$.

$A \subset S$

Mutually exclusive events

Two events A and B are said to be mutually exclusive (disjoint or incompatible) if the occurrence of one event prevents the occurrence of other event , i.e the events does not occur simultaneously .

A & B. → Mutually exclusive events.

$A \cap B = \varphi$ (null set)

H T

$n(A \cap B) = 0$

$p(A \cap B) = 0$

Equally likely events

Occurrence of any event in a random experiment are equal then the events are said to be equally likely events .

Eg :

Tossing a fair coin.

$S = \{H, T\}$

$\boxed{\text{No biasing}}$

Unbiased coin.

$S = \{H, T\}$

Chance to get H = chance to get T = 50%

$S = \{1, 2, 3, 4, 5, 6\}$

2-H coin (Sholay coin) (Biased coin)

Chance to get H = 100 %

Chance to get T = 0 %

Chance to get H ≠ chance to get T

Independent events

The occurrence of one event does not depend on another.

$$S = \{1, 2, 3, 4, 5, 6\} \longrightarrow 1$$

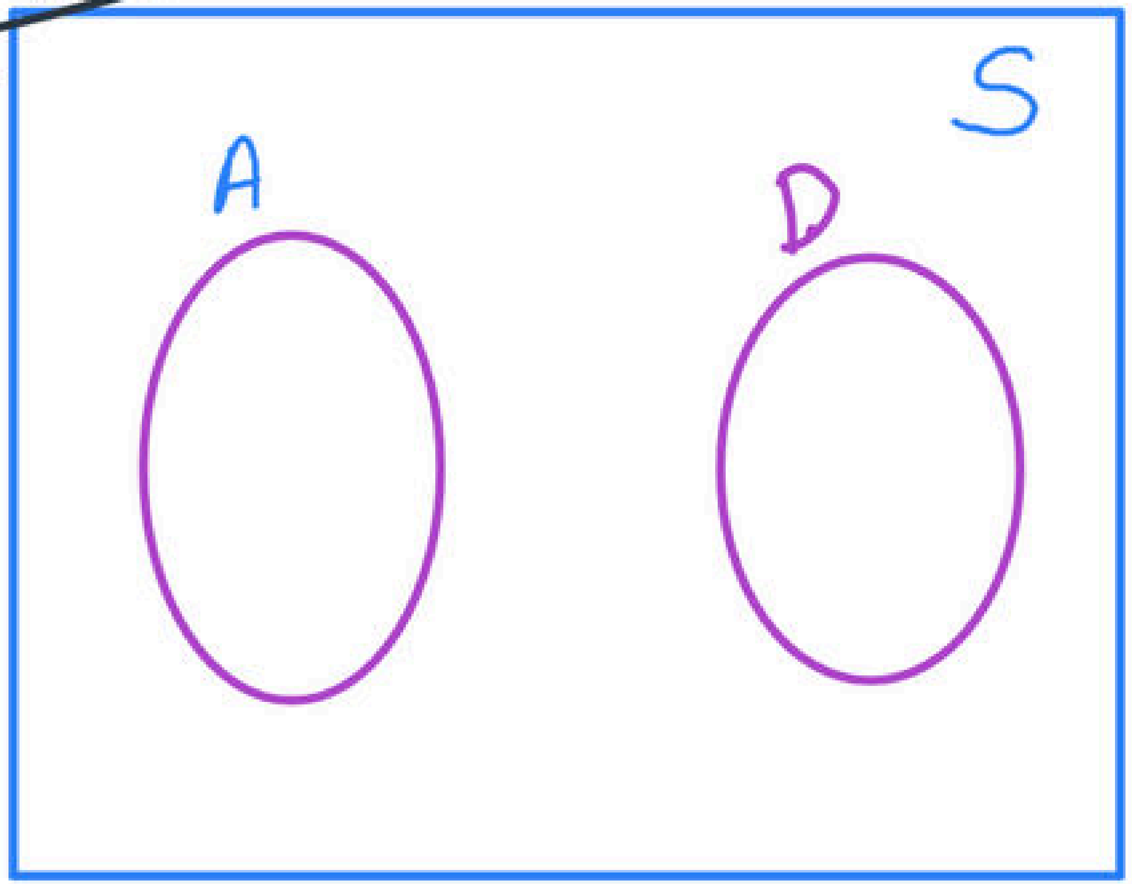
$$S = \{1, 2, 3, 4, 5, 6\} \longrightarrow 2$$

Eg : 1. when two dice are rolled , Getting '1' on first die does not depend on '2' on the second die .

2. when an unbiased coin is tossed two times, the event of getting a head in the first toss is independent of getting head in the second toss .

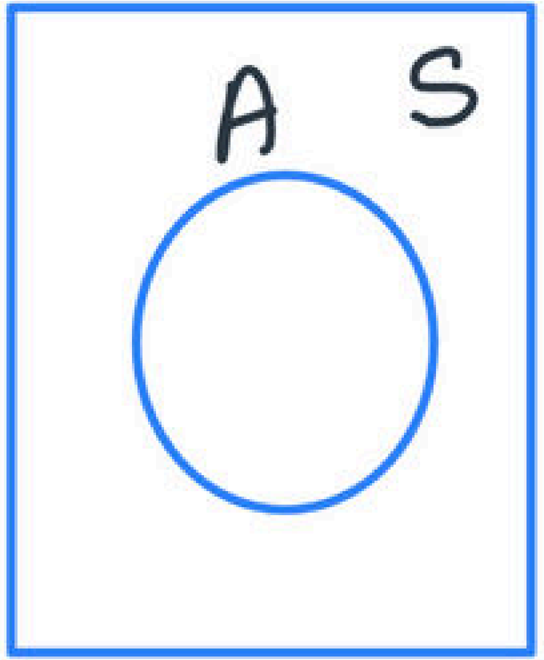
mutually exclusive events.

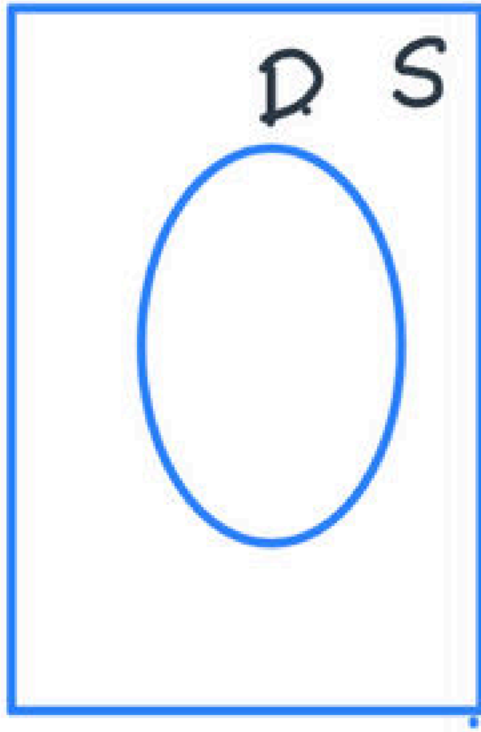
IIT Kgp.



$A \cap D = \varphi$.

IIT Kgp





BVREDDY has 3- Chidreens

First Child $\longrightarrow$ Girl $\quad S = \{B, G\}$

2^{nd} Child $\longrightarrow$ Boy. $\quad S = \{B, G\}$

3^{rd} Child $\longrightarrow$ $\quad S = \{B, G\}$

Probability

The probability of an event A is defined as

$$P(A) = \frac{number\ of\ fauourable\ events}{total\ number\ of\ possible\ events}$$

$$P(A) = \frac{n(A)}{n(S)}$$

Properties of Probability

1. $P(A) = \frac{n(A)}{n(S)}$

2. $0 \leq P(A) \leq 1$

$P(A) = 0 \longrightarrow$ impossible event

$P(A) = 1 \longrightarrow$ sure event.

3. Sum of all probabilities = 1.

$\Sigma P = 1.$

4. $P(S) = 1$

5. Let A - be the event in the sample space 'S'

A — event to occur

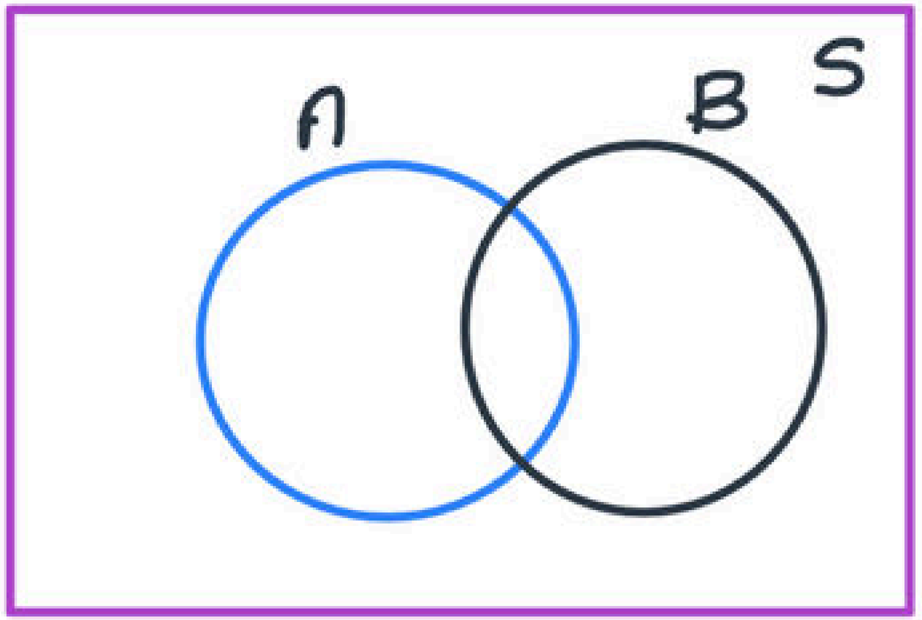
$\bar{A}$ — event not to occur.

$P(A) + \overline{P(A)} = 1.$

$$\boxed{P(A) + P(\overline{A}) = P(S).}$$

6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

7. $P(\text{only } A) = P(A) - P(A \cap B)$



8. $P(\text{only } B) = P(B) - P(A \cap B)$

9. $P(\text{at least one}) = P(\text{कम से कम एक})$

$= P(\text{only } A) + P(\text{only } B) + P(A \cap B)$.

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) + P(A \cap B)$.

$$P(\text{at least one}) = P(A) + P(B) - P(A \cap B)$$
$$= P(A \cup B)$$

10. $P[\text{Either } A \text{ or } B] = P(A) + P(B) - P(A \cap B) = P(A \cup B)$

11. $P(\text{exactly one}) = P(A \cap \bar{B}) + P(\bar{A} \cap B)$
$$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$$

$P(A \Delta B)$
$$= P(A \cup B) - P(A \cap B)$$

$$P(\text{exactly one}) = P(\text{only } A) + P(\text{only } B)$$

$$= \underline{P(A) - P(A \cap B) + P(B)} - P(A \cap B)$$

$$= P(A \cup B) - P(A \cap B)$$

$$P(A - B) \qquad P(A + B)$$

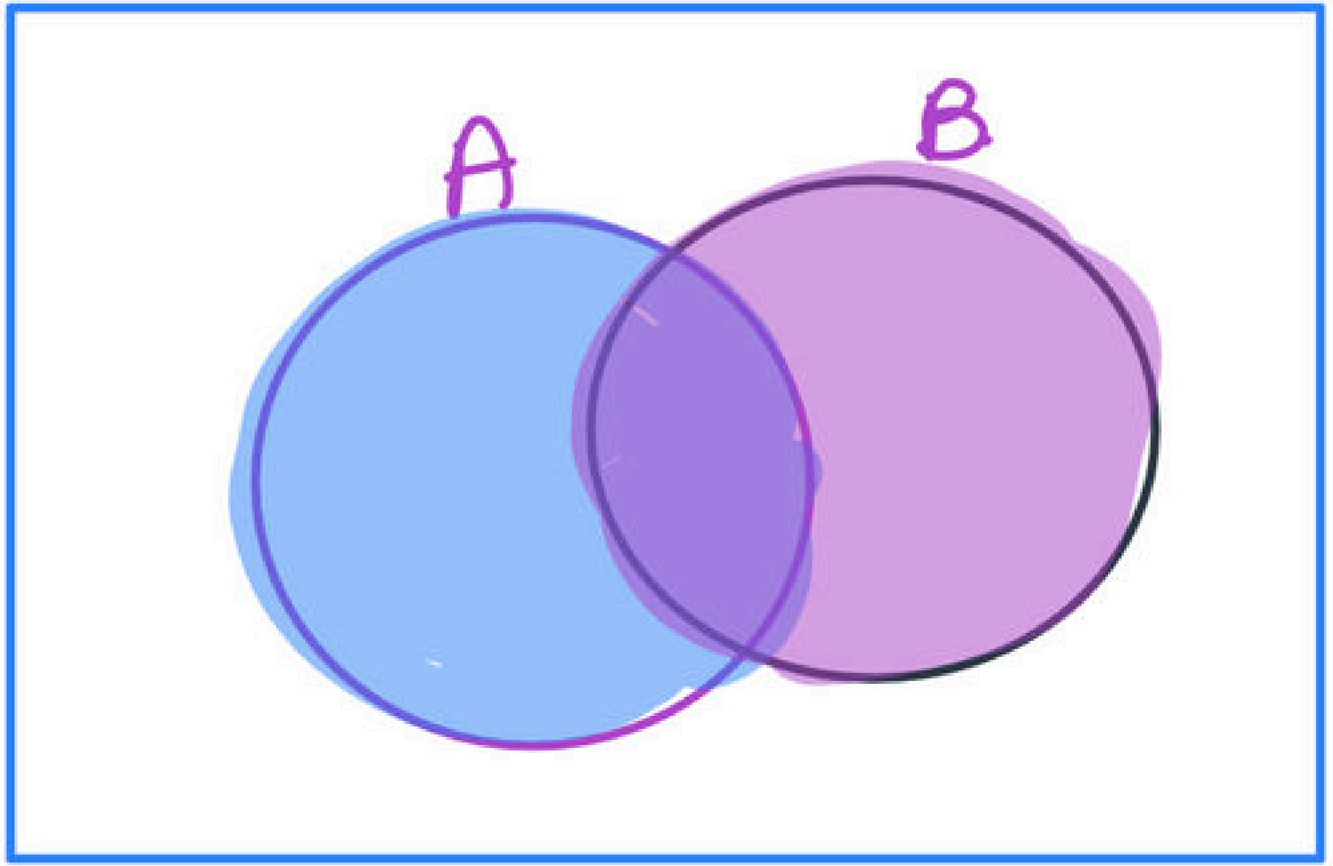
$$\downarrow$$

$$P(A \cap \overline{B})$$

$$P(A \cup B) = P(A) + P(B) - \underline{P(A \cap B)}$$

$P(A)$

$P(B)$



Rolling a dice

When two dice are rolled

$$n(S)=$$

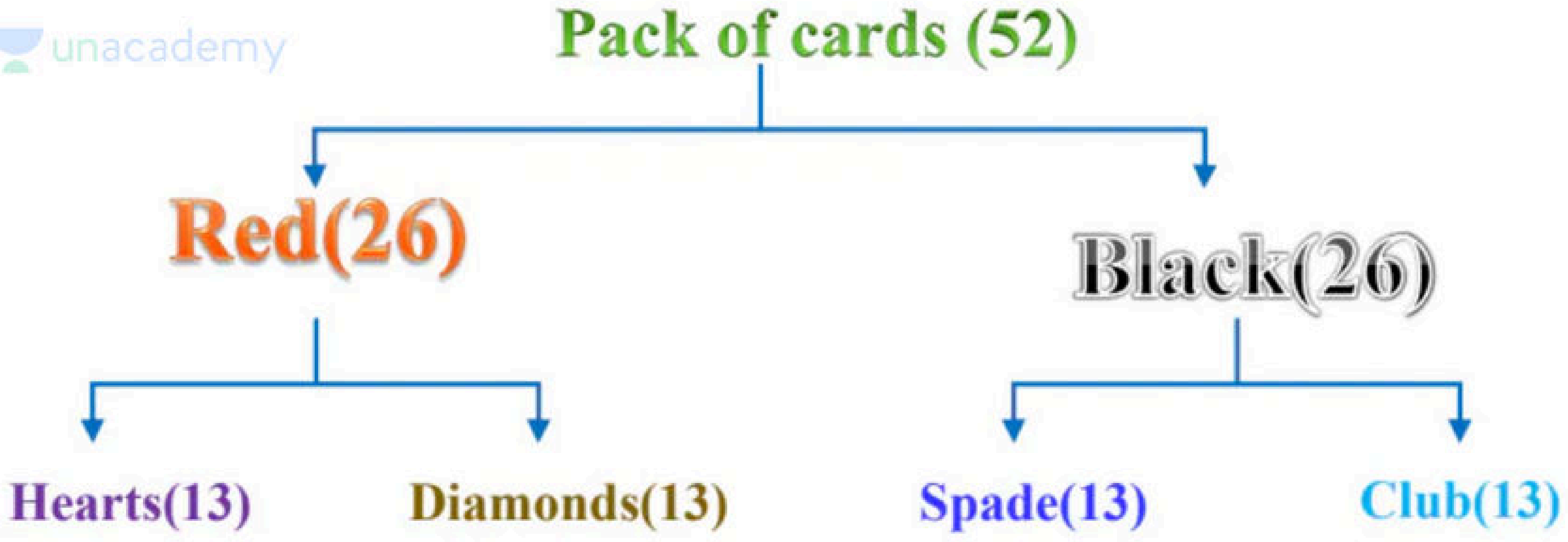
Sum of the numbers on the dice = {

Rolling a dice

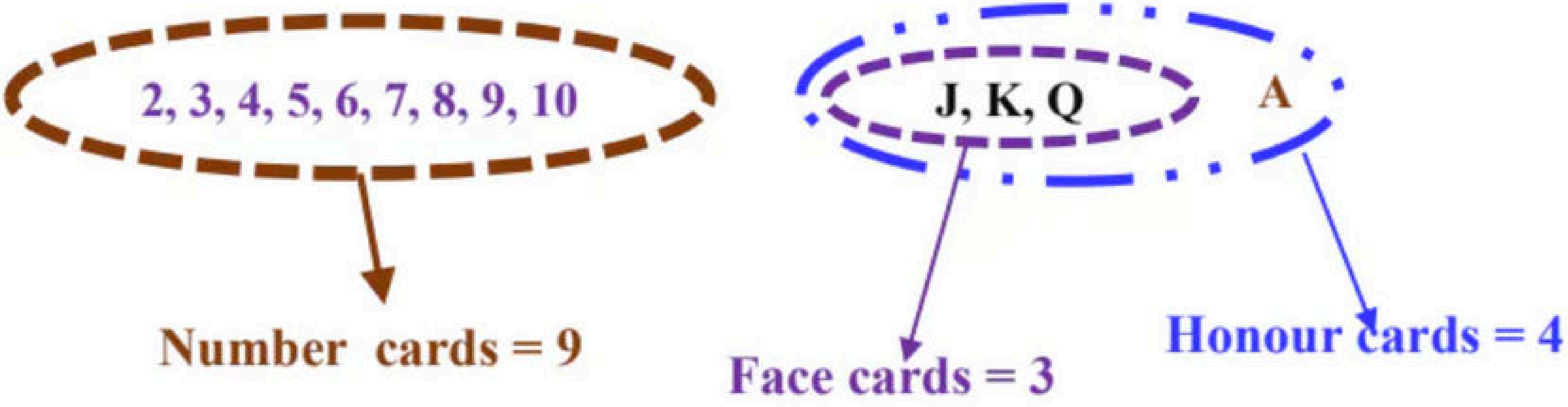
When three dice are rolled

$n(S)=$

Sum of the numbers on the dice = {



Each suit contains



1. Total number of face cards =
2. Total number of number cards =
3. Total number of honour cards =
4. Total number of Red Kings =
5. Total number of Spade Queen =
6. Total number of Black Diamonds =
7. Total number of Diamond Ace =
8. Total number of Black 2's =
9. Total number of Red face cards =
10. Total number of Spade King =
11. Total number of Club 9's =
12. Total number of Red Hearts =

1. If three coins are tossed. find the probability of getting

(i) Three heads

(ii) two tails

(iii) no heads

2. Two unbiased dice are thrown. find the probability that

(i) Both the dice show the same number

(ii) The first dice shows 6

(iii) The total number on the dice is 8

(iv) The total number on the dice is greater than 8

(v) The total number on the dice is any number from 2 to 10, both inclusive.

(vi) Total number on the dice is 13.

3. When two dice are rolled , what is the probability of getting the sum

a) sum = 4 b) sum =11 c) sum > 10

d) sum $\leq$ 10 e) $4 \leq sum \leq 11$

4. An urn contains 6 white, 4 red and 9 black balls. if three balls are drawn at random, find the probability that ,

i. Two of the balls drawn are white.

ii. One is of each color.

iii. None is red.

iv. At least one is white.

5. A committee of four people is to be appointed from three officers of production department, four officers of purchase department, Two officers of the sales department and one chartered accountant. find the probability of forming the committee in the following manner:

(i) There must be one from each category.

(ii) It should have at least one from the purchase department.

(iii) The chartered accountant must be in the committee.

6. A and B are playing a game of tossing a coin . One who gets head wins the game . If A starts the game , find the probabilities of their winning .

Joint Probability

If A and B are the two events in sample space S , which are not mutually exclusive then the joint probability of A and B can be denoted as $P(A \cap B)$.

Conditional Probability

If A and B are the two events in sample space S , then the conditional probability of A given B is defined as

$$P\left[\frac{A}{B}\right] =$$

the conditional probability of B given A is defined as

$$P\left[\frac{B}{A}\right] =$$

Properties of conditional probability

Pair wise independent events

Mutually independent events

If A, B and C are Mutually independent events then

If A, B ,C and D are Mutually independent events then

NOTE :

1. The total number of conditions for n- events are mutually independent is = ………

2. Mutually independent events are pair wise independent but vice versa not true .

If A and B are pair wise independent events

Total Probability Theorem

Total Probability Theorem

If sample space contains n – mutually exclusive events, then probability of event A defined on the sample space S can be expressed as a conditional probability .

$$P(A) = \sum_{n=1}^{N} P(B_n) P\left(\frac{A}{B_n}\right)$$

7. A box contains 5 red balls and 6 black balls , another box contains 6 red balls and 4 black balls. One box is chosen at random and one ball is drawn from it. Find the probability of getting

a)Red ball

b) Black ball

8. A coin, weighted so that $P(H) = 2/3$ and $P(T) = 1/3$ is tossed. If heads appears, then a number is selected at random from the numbers 1 through 9. If tails appears, then a number is selected at random from the numbers 1 through 5. Find the probability P that an even number is selected.

a) 67/145 b) 58/135 c) 74/157 d) 43/142

9. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is_________.

(GATE – 16 -CSE-SET2)

10.. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is

(GATE-ME- SET-3-2014)

Bayes' theorem

Bayes' theorem

If sample space S contains n- mutually exclusive events, let A is any event in the sample space , then the conditional probability of B_n given A is

$$P\left[\frac{B_n}{A}\right] = \frac{P\left(\frac{A}{B_n}\right)p(B_n)}{\sum_{n=1}^{N} P\left(\frac{A}{B_n}\right)p(B_n)}$$

11. A box contains 5 red balls and 6 white balls , another box contains 4 red ball and 6 white balls . One ball is drawn and found to be red . Find the probability that the ball is drawn from first box .

12. 25 girls out of 100 , 5 boys of 100 have color blind. One person is chosen at random and found to be colorblind . Find the probability that person is a girl .

13. Three boxes numbered I II and III contain 1-white , 2 black and 3 red balls,: 2 white 1 black and 1 red ball ; 4 white 5 black and 3 red balls respectively. One box is randomly selected and a balls drawn from it . If the ball is red then find the probability that it is from box II

14. In a certain college, 4% of the men and 1% of the women are taller than 1.8m. Further more, 60% of the students are women. Now if a student is selected at random and is taller than 1.8m, what is the probability that the student is a woman ?

a)3/11 b) 4/11 c) 5/11 d) 6/11

15. We are given three urns as follows. Urn A contains 3 red and 5 white marbles, Urn B contains 2 red and 1 white marble, Urn C contains 2 red and 3 white marbles. An urn is selected at random and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A?

a) 45/173 b) 37/165 c) 27/109 d) 39/185

16. A box contains three coins, two of them fair and one two headed. A coin is selected at random and tossed twice. If heads appears both times, what is the probability that the coin is two headed?

a) 2/3 b)1/3 c)3/4 d)1/2

17. An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the opposite colour is put in to the urn. A second marble is drawn from the urn. If both marbles where of the same colour . What is the probability that they were both white?

a) 5/6 b)7/8 c)8/9 d)9/10

18. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

a) $\frac{3}{23}$ b) $\frac{6}{23}$ c) $\frac{3}{10}$ d) $\frac{3}{5}$

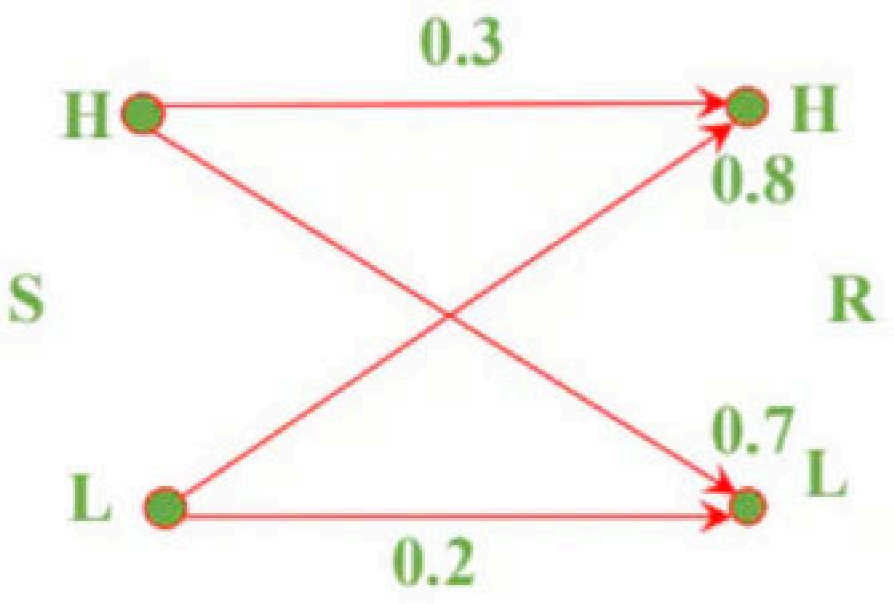
2

19. Consider two identical bags B_1, and B_2, each containing 10 balls of identical shapes and sizes. Bag B_1, contains 7 Red and 3 Green balls, while bag B_2, contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B_1 (rounded off to one decimal place) is____________.

(GATE- 2020(IN))

20. A sender (S) transmits a signal , which can be one of the two kinds : H and L with probabilities 0.1 and 0.9 respectively , to a receiver (R) .

In the graph below , the weight of edge (u , v) is the probability of receiving *v* when *u* is transmitted , where *u* , v ∈(H , L) . For example , the probability that the received signal is L given the transmitted signal was H , is 0.7 .



If the received signal is H , the probability that the transmitted signal was H (rounded to 2 decimal places) is________________.

(GATE-2021-cs)

21. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting positive report is 0.01, the probability that a person tested gets an incorrect report is **(GATE-IN-2009)**

(a) 0.0027 (b) 0.0173 (c) 0.1497 (d) 0.2100

22. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty ? **(GATE-CS-2010)**

(a) $pq + (1 - p)(1 - q)$ (b) $(1 - q)p$ (c) $(1 - p)q$ (d) pq

22. An automobile plant contacted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to reliable, is made by Y is **(GATE-CE-2012)**

(a) 0.288 (b) 0.334 (c) 0.667 (d) 0.720

23. The probability that student knows the correct answer to a multiple choice question is 2/3 . If the student does not known the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4 . Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is **GATE- 2013 ME**

(a) 2/3 (b) 3/4 (c) 5/6 (d) 8/9

24. In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day ?

(GATE-PI- SET-1-2014)

(a) 3/10 (b) 9/11 (c) 14/17 (d) 27/41

25. A box contains the following three coins ,

I. A fair coin with head on one face and tail on the other face .

II . A coin with heads on both the faces .

III . A coin with tails on both the faces.

A coin is picked randomly from the box and tossed . Out of the two remaining coins in the box , one coin is then picked randomly and tossed . If the first toss results in a head , the probability of getting a head in the second toss is

(GATE – 2021 – EC)

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

26. A and B are playing a game of tossing a coin . One who gets head wins the game . If A starts the game , find the probabilities of their winning .

27. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that sum of the numbers is

a) An even number

b) Odd number

28. A bag has r red balls and b black balls . All balls are identical except for their colours . In a trial , a ball is randomly drawn from the bag , its colour is noted and the ball is placed back into the bag along with another ball of the same colour . Note that the number of balls in the bag will increase by one , after the trial . A sequence of four such trials is conducted . Which one of the following choices gives the probability of drawing a red ball in the fourth trial ?

(GATE-2021-cs)

(a) $\frac{r}{r+b}$

(b) $\frac{r}{r+b+3}$

(c) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(d) $\frac{r+3}{r+b+3}$

29. A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8?

a) 1/3 b) 1/4 c) 1/5 d) 2/3

30. A point is selected at random inside a circle. Find the probability p that the point is closer to the Centre of the circle than to its circumference?

a) 1/3 b) 1/4 c) 1/5 d) 2/3

31. In a class of 100 students, 40 failed in mathematics, 30 failed in physics, 25 failed in Chemistry, 20 failed in math's and physics, 15 failed in physics and chemistry, 10 failed in Chemistry and math's, 5 failed in math's, physics and chemistry. If a student is selected at Random then the probability that he passed in all three subjects is

a) 0.4 b) 0.45 c) 0.55 d) 0.65

32. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students Have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses ?

(a) 15 (b) 20 (c) 25 (d) 35

33. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(GATE-CS-1995)

(a) 18/25 (b) 2/5 (c) 5/12 (d) 19/25

34. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is

(GATE-CS -2005)

a) $\frac{1}{36}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$

35. The probability that there are 53 Sundays in a randomly chosen leap year is

(GATE-IN-2005)

(a) 1/7 (b) 1/14 (c) 1/28 (d) 2/7

36. The box 1 contains chips numbers 3, 6, 9, 12 and 15. The box 2 contains chips numbers 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is

(GATE-IN-2011)

(a) 6/25 (b) 2/5 (c) 5/12 (d) 19/25

37. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _______ **(GATE-EC-SET-1-2014)**

38. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is (n-3) is **(GATE-EE- SET-1-2014)**

(a) 2^{-n} (b) 0 (c) ${}^{n}C_{n-3}2^{-n}$ (d) 2^{-n+3}

39. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _____ **(GATE-CS- SET-2-2014)**

40. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If P(.) denotes the probability of the event, the maximum value of P(A) P(B) is ______

(GATE-CS- SET-3-2014)

41. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate1 has failed, the probability that both gates 2 and 3 will fail is

GATE-04

(a) 0.240 (b) 0.200 (c) 0.040 (d) 0.008

42. A fair dice is rolled twice. The probability that an odd number will follow an even number is

(GATE-EC-2005)

(a) 6/25 (b) 2/5 (c) 1/4 (d) 19/25

43. An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. Probability of getting at least one head is _______ **(GATE-ME-2011)**

(a) 6/25 (b) 2/5 (c) 1/4 (d) 31/32

44. Consider a die with the property that the probability of a face with 'n' dots showing up is proportional to 'n'. The probability of the face with three dots showing up is _______

(GATE-EE- SET-2-2014)

45. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

46. The probability of an event B is P1 . The probability of events A and B occur together is P2 , while the probability that A or B occur together is P3 . The probability of event A in terms of P1, P2 and P3 is

a) P1+P2

b) P3+P2

c) P3+P1

d) P3- P1+ P2

47. Assume for simplicity that N people, all born in April (a month of 30 days) are collected in a room, consider the event of at least two people in the room being born on the same date of the month (even if in different years e.g. 1980 and 1985). What is the smallest N so that the probability of this exceeds 0.5 is ?

(a) 20 (b) 7 (c) 15 (d) 16 **GATE- 2009 EE**

48. There are five bags each containing identical sets of ten distinct chocolates . One chocolate is picked from each bag . The probability that at least two chocolates are identical is____________.

(GATE-2021-cs)

(a) 0.6976 (b) 0.3024

(c) 0.4235 (d) 0.8125

49. Two coins R and S are tossed. The 4 joint events H_RH_S, T_RT_S, H_RT_S, T_RH_S have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE ? **(GATE-EE-2015)**

(a) The coin tosses are independent (b) R is fair, R is not

(c) S is fair, R is not (d) The coin tosses are dependent

RANDOM VARIABLE

RANDOM VARIABLE

Random variable can be considered to be a function , that assigns real values to every element in the sample space .

Discrete Random variable

If the random variable takes only finite values , then it is called as discrete random variable.

Continuous Random variable

If the random variable takes infinite values , then it is called as continuous random variable.

Cumulative Distribution Function (CDF)

Properties of CDF

Probability Density Function (PDF)

Properties of PDF

Q. If the probability density function of a random variable x is given by

$$f(x) = \begin{cases} \dfrac{kx^2}{0} & -1 \leq x \leq 1 \\ 0, & elesewhere \end{cases}$$

the value of k is________________.

(GATE- 2020(PI

Q. The constant C such that the function

$$f(x) = Cx^2, \quad 0 < x < 3.$$
$$= 0. \quad \text{Otherwise}$$

is a probability density function.

a) 1

b) 1/9

c) 1/8

d) 1/3

Q. A random variable x has density function $f(x) = C/(x^2 + 1)$ where $-\infty < x < \infty$. then the value of the constant C

a) $\frac{1}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{1}{2\pi}$ d) $\frac{-1}{\pi}$

Q. $P_x(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relation M and N is **(GATE-IN-2008)**

a) $M + \frac{2}{3}N = 1$

b) $2M + N = 3$

(c) $M + N = 1$

(d) $M + N = 3$

Q. A continuous random variable X has a probability density function

$f(x) = e^{-x}, 0 < x < \infty$. Then $P\{X > 1\}$ is **(GATE-EE-2013)**

(a) 0.368 (b) 0.5 (c) 0.632 (d) 1.0

Q. Find the value of λ such that the function f(x) is a valid probability density function

$f(x) = \lambda(x-1)(2-x)$ for $1 < x < 2$ ____ **(GATE-CE-2013)**

a) 5
b) 6
c) 2
d) 1

Q. For the function $f(x) = a + bx$, $0 \le x \le 1$, to be a valid probability density function, which one of the following statements is correct?

(GATE-17-CE)

(a) $a = 1, b = 4$

(b) $a = 0.5, b = 1$

(c) $a = 0, b = 1$

(d) $a = 1, b = -1$

Q. The graph of a function f(x) is shown in the figure. For f(x) to be valid probability density function, the value of h is

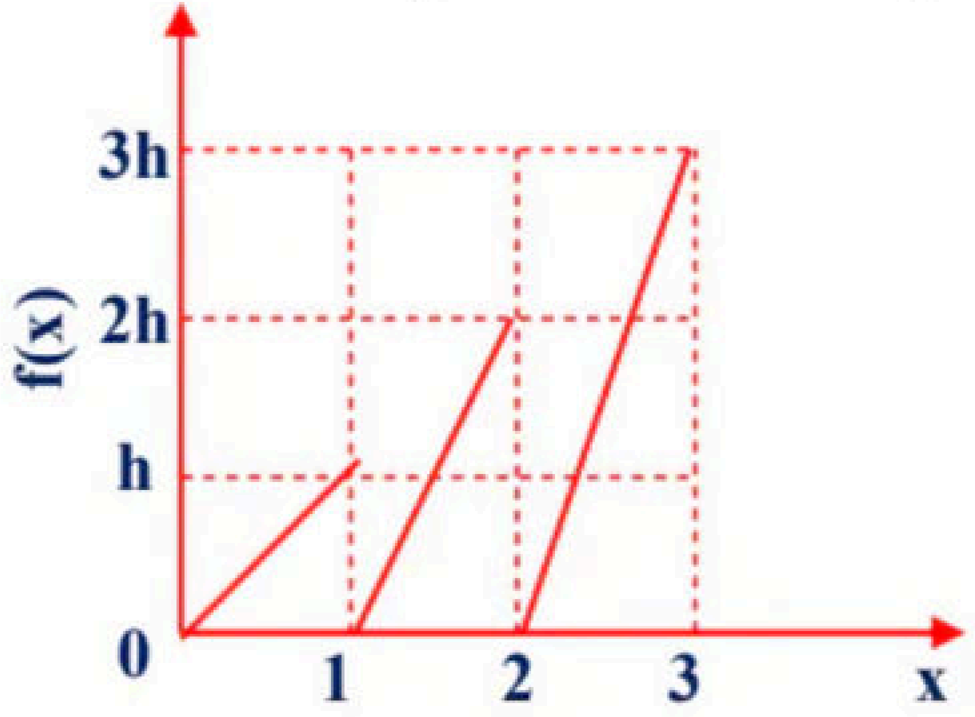
(GATE-18-CE)

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 1

(d) 3



Q. The probability density function of a random variable x be given as $f(x) = ae^{-2|x|}$; the value of a is

GATE- 2022 (EE)

Mean (Average Value) (Expectation) (1^{st} Moment)

Q. The random variable X takes on the values 1, 2 (or) 3 with probabilities $\frac{2+5P}{5}, \frac{1+3P}{5}, and\ \frac{1.5+2P}{5}$ respectively the values of P and E(X) are respectively

(a) 0.05, 1.87 (b) 1.90, 5.87 (c) 0.05, 1.10 (d) 0.25, 1.40

GATE- 2007(PI)

Q. X is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \frac{1}{8}(x+1), for\ 2 < x < 4 \\ 0, \quad otherwise \end{cases}$$

. Then E(x) = _______

Q. let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, E[X], is _**(GATE-EC- SET-2-2014)**

Properties of Mean

Q. If independent random variables X_1, X_2, X_3 have mean 1, then the mean of the variable is $\frac{x_1+3x_2+4x_3}{2}$

(A) 1 (B) 2

(C) 3 (D) 4

Q. If a random variable X has a mean 5, then the expectation

i. E(3X)

ii. E(3X-1) (GATE-17-CSIT)

Variance

Q. in the following table, x is a discrete random variable and p(x) is the probability density. The standard deviation of x is

x	1	2	3
p(x)	0.3	0.6	0.1

(a) 0.18 (b) 0.36 (c) 0.54 (d) 0.6

Q. discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true?

GATE- 2009 (EC)

x	1	2	3	4	5
P(X=x)	0.1	0.2	0.4	0.2	0.1

a) Both the student and the teacher are right
b) Both the student and the teacher are wrong
c) The student is wrong but the teacher is right
d) The student is right but the teacher is wrong

Properties of Variance

Q. If the difference between the expectation of the square of a random variable $[E(X^2)]$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then,

(GATE-CS-2011)

(a) $R = 0$ (b) $R < 0$ (c) $R \geq 0$ (d) $R > 0$

Q. Let X be a real-valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds true is **(GATE-EC-2014)**

(a) $(E[X])^2 > E[X^2]$ (b) $E[X^2] \geq (E[X])^2$ (c) $E[X^2] = (E[X])^2$ d) $E[X^2] > (E[X])^2$

Q. If f(x) and g(x) are two probability density functions,

$$f(x) = \begin{cases} \frac{X}{a} + 1 & : \ -a \le x < 0 \\ -\frac{X}{a} + 1 & : \ \ 0 \le x \le a \\ 0 & : \ \ 0 \le x \le a \end{cases} \qquad g(x) = \begin{cases} -\frac{X}{a} & : -a \le x < 0 \\ \frac{X}{a} & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

Which of the following statements is true?

(GATE-16-CE-SET2)

(a) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are same.

(b) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different.

(c) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same.

(d) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are different.

Q. Two continuous random variables X and Y are related as

$$Y = 2X + 3$$

Let σ_X^2 and σ_Y^2 of denote the variance of X and Y , respectively . The variances are related as

(GATE – 2021 – EC)

(a) $\sigma_Y^2 = 25\sigma^2 X$

(b) $\sigma_Y^2 = 5\sigma_X^2$

(c) $\sigma_Y^2 = 2\sigma_X^2$

(d) $\sigma_Y^2 = 4\sigma_X^2$

Q. X and Y are two independent random variables with variances one and two respectively. Let $Z = X - Y$. The variance of Z is

(GATE-18-IN)

(a) 0 (b) 1

(c) 2 (d) 3

Probability Distributions

Binominal ,

Poisson,

Uniform ,

Exponential ,

Gaussian (Normal)

Binominal Distribution

Only two possible outcomes

- **Success or failure**
- **True or false**
- **Yes or no**
- **Good or Defective**

Q. Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is ______. **(GATE-ME- SET-2-2014)**

Q. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?

(GATE-ME-2008)

(a) 1/4 (b) 3/8 (c) 1/2 (d) ¾

Q. The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up, 3 are defective is

(GATE-IN-2015)

(a) 0.001 (b) 0.057 (c) 0.0107 (d) 0.3

Q. The probability of getting a “head” in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a “head” is obtained. If the tosses are independent, then the probability of getting “head” for the first time in the fifth toss is ____________.

(GATE- 16 – EC – SET3)

Q. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 and gives guarantee of a replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to replaced is________________. (GATE-16-ME-SET2)

Q. N- coins are tossed at a time find the probability of getting head

a) Odd number of times

b) Even number of times

Mean of Binominal distribution =

variance of Binominal distribution =

Standard deviation of Binominal distribution =

Q. For a given biased coin , the probability that the outcome of a toss is a head is 0.4 . This coin is tossed 1,000 times . Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses . The standard deviation of X (rounded to 2 decimal places) is__________________.

(GATE-2021-cs)

Q. In an examination, student can choose the order in which two questions (QuesA and QuesB) must be attempted.

- If the first question is answered wrong , the student gets zero marks .
- If the first questions is answered correctly and the second question is not answered correctly , the student gets the marks only for the first question .
- If both the questions are answered correctly , the student gets the sum of the marks of the two questions .

The following table shows the probability of correctly answering a question and the marks of the question respectively.

Question	Probability of answering correctly	Marks
Question A	0.8	10
Question B	0.5	20

Assuming that the student always wants to maximize her expected marks in the examination , in which order should student attempt the questions and what is the expected marks for that order (assume that the questions are independent) ?

(GATE-2021-CS)

(a) First QuesA and then QuesB . Expected marks 14 .
(b) First QuesB and then QuesA . Expected marks 22 .
(c) First QuesB and then QuesA . Expected marks 14 .
(d) First QuesA and then QuesB . Expected marks 16 .

Q. Each of the seven words in the sentence "Hema Sai and Owasi are good enemies" is written on a separate piece of paper. These seven pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is ________. (The answer should be rounded to one decimal place.)

(GATE-CS- SET-2-2014)

Q. Let the random variable X represent the number of times a fair coin needs to be tossed till first time heads appear for the first time. The expectation of X is _______

Q. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is ___

(GATE-EC-2015)

Q. Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is _______

(GATE-EC-2015)

Q. On a rainy day an umbrella salesman can earn Rs. 300, and on a fair day (no rain) he loses Rs. 60. What is his expected income per day, if the probability for a rainy day is 0.3

a) Rs 24 b) Rs 36 c) Rs 48 d) Rs 64

Poisson's Distribution

Poisson's distribution is a limiting case of binomial distribution as n---->∞ and p ----> 0.

Q. The probability that an individual suffers a bad reaction from injection of a serum is 0.001. Determine the probability that out of 2000 individuals, exactly 3 individuals suffer a bad reaction.

a) 0.12 b) 0.08 c) 0.18 d) 0.003

Q. If X follows poisson distribution such that $P(X = 1) = P(X = 2)$ then $P(X = 0) =$

a) e^{-1} b) e^{-2} c) e^{-3} d) e^{-4}

Q. In a certain factory of turning razor blades, there is a small chance (1/500) for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing at least one defective blade in a consignment of 10,000 packets.

a) 9802 b) 198 c) 2 d)196

Q. a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

(GATE-EE-2000)

(a) 90 and 9 (b) 9 and 90 (c) 81 and 9 (d) 9 and 81

Q. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval ?

(GATE-CS-2013)

(a) $8/(2e^3)$ (b) $9/(2e^3)$ (c) $17/(2e^3)$ (d) $26/(2e^3)$

Q. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is **(ME- SET-4-2014)**

(a) 0.029 (b) 0.034 (c) 0.039 (d) 0.044

Q. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days in independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ________

(GATE-CE- SET-2-2014)

Q. An observer counts 240veh/h at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is ____ **(CE- SET-2-2014)**

Q. If a random variable X satisfies the Poission's distribution with a mean value of 2, then the probability that $X > 2$ is **(GATE-PI-2010)**

(a) $2e^{-2}$ (b) $1-2e^{-2}$ (c) $3e^{-2}$ (d) $1-3e^{-2}$

Uniform Distribution

The probability density function of the uniform distributed random variable X is

Mean of uniform distribution =

Variance of uniform distribution =

Standard deviation of uniform distribution =

Q. The standard deviation of a uniformly distributed random variable b/w 0 and 1 is

(GATE-ME-2009)

Exponential Distribution

The probability density function of the exponential distributed random variable X is

Mean of exponential distribution =

Variance of exponential distribution =

Standard deviation of exponential distribution =

Q. The life of a bulb (in hours) is a random variable with an exponential distribution

$$f(t) = \alpha e^{-\alpha t} \quad 0 \leq t \leq \infty$$

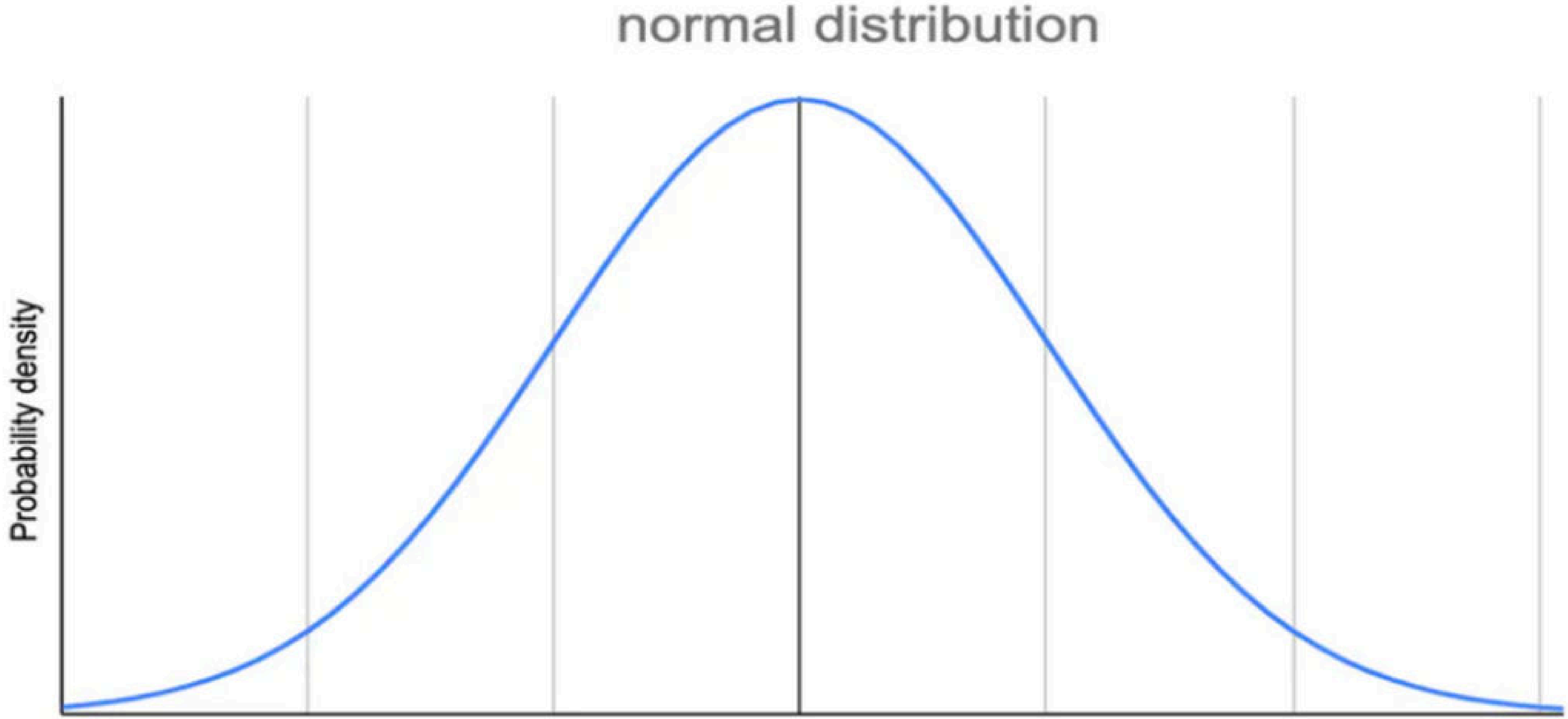
The probability that its value lies b/w 100 and 200 hours is **(GATE-PI-2005)**

Q. assume that the duration in minutes of a telephone conversion follows the exponential distribution $f(x) = \frac{1}{5} e^{\frac{-x}{5}} \qquad x > 0$. The probability that the conversation will exceed five minutes is **(GATE-IN-2007)**

Normal Distribution (Gaussian Distribution)

Properties of Normal Density function

1. The curve is smooth, regular, bell shaped and symmetrical about mean .



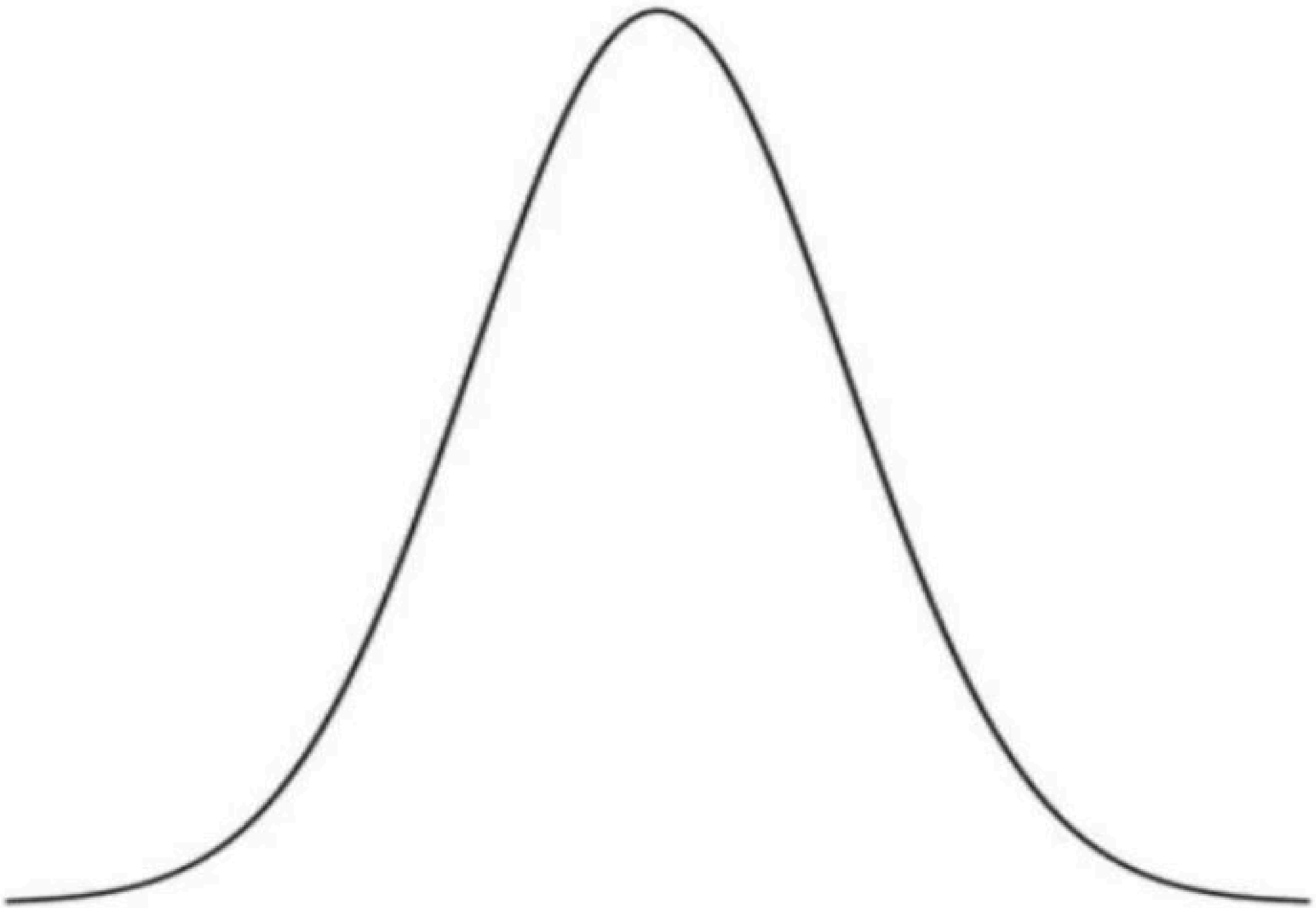
2. The area under the normal density function is unity .

3. The maximum value of density function occurs at x =

its maximum value of density function is =

4. If σ – increases then normal density function decreases and curve tends to be flat

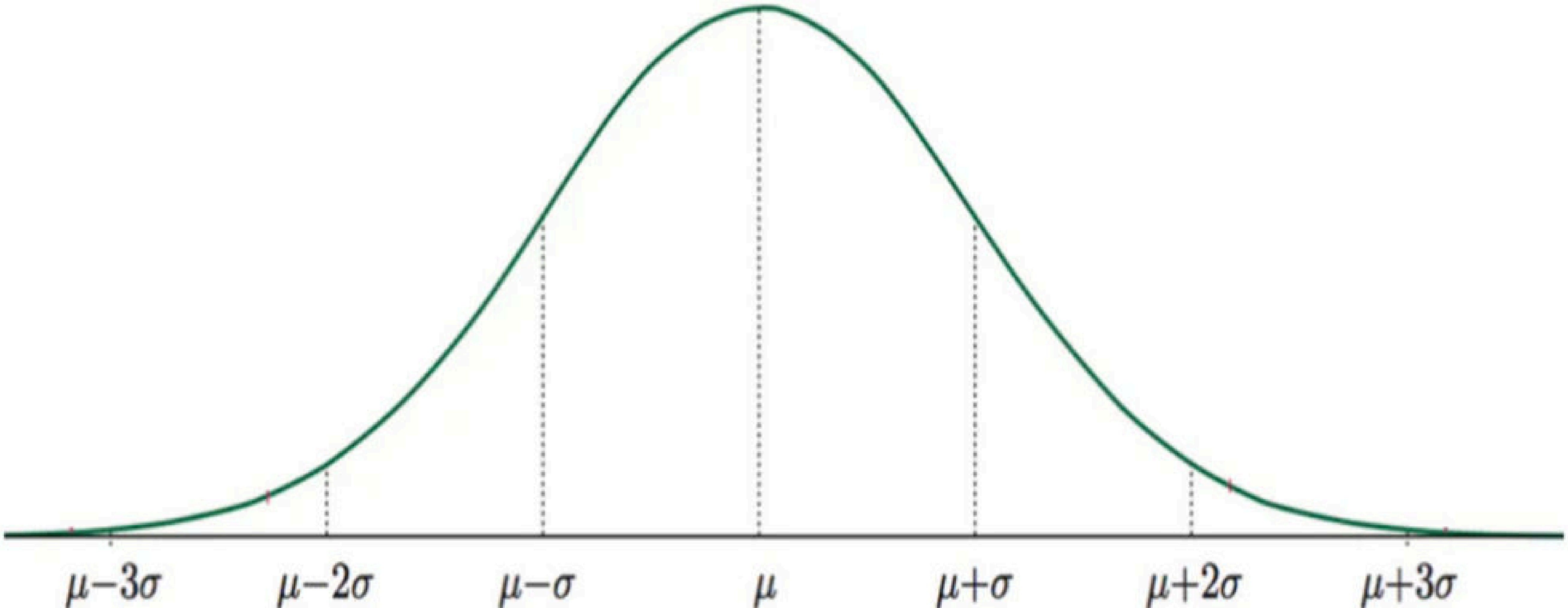
5. If σ – decreases then normal density function increases and curve tends to be more peaked at mean .



6. $P(\mu - \sigma < x < \mu + \sigma) =$

$P(\mu - 2\sigma < x < \mu + 2\sigma) =$

$P(\mu - 3\sigma < x < \mu + 3\sigma) =$

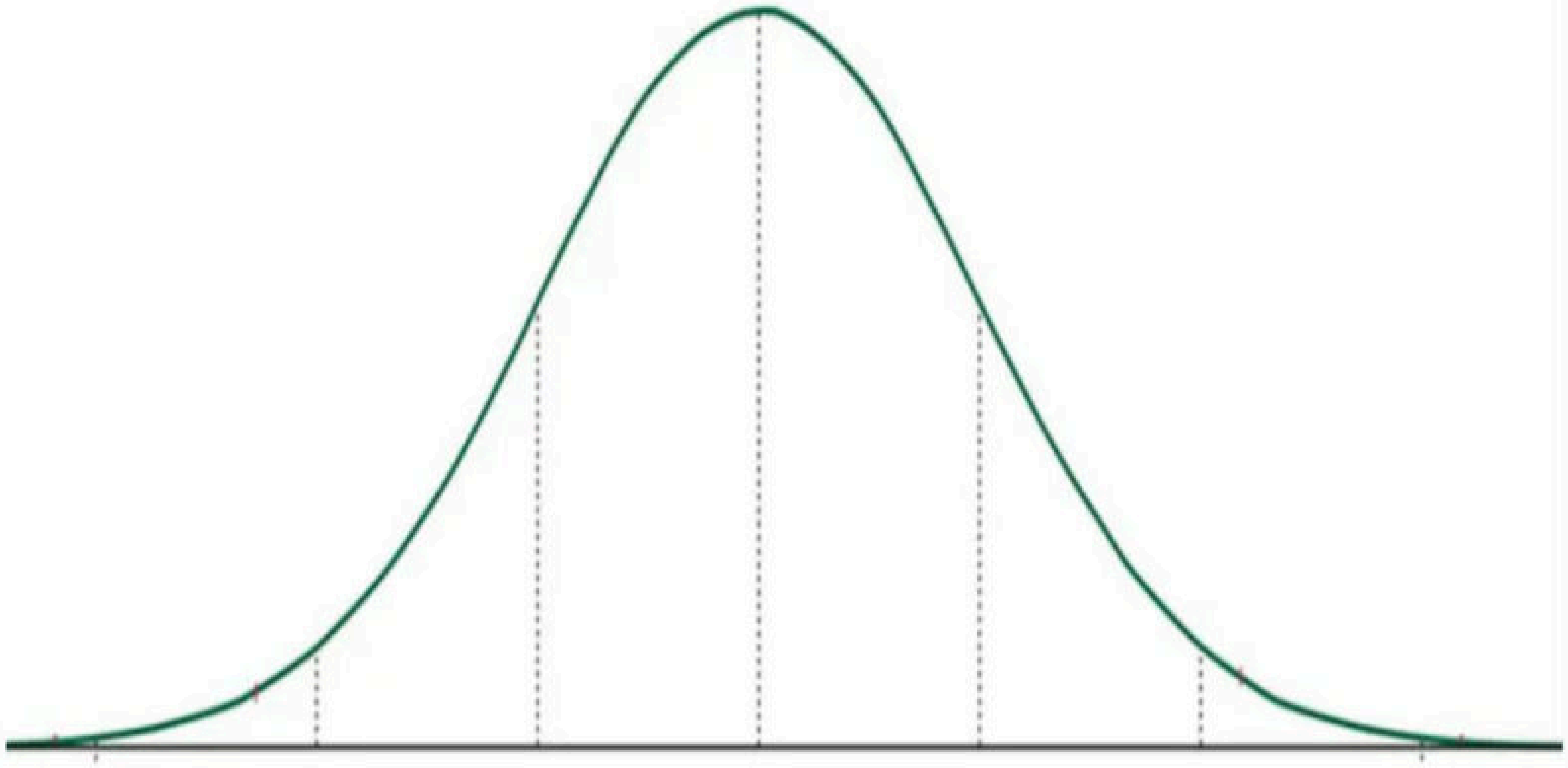


7. The standard normal distribution curve

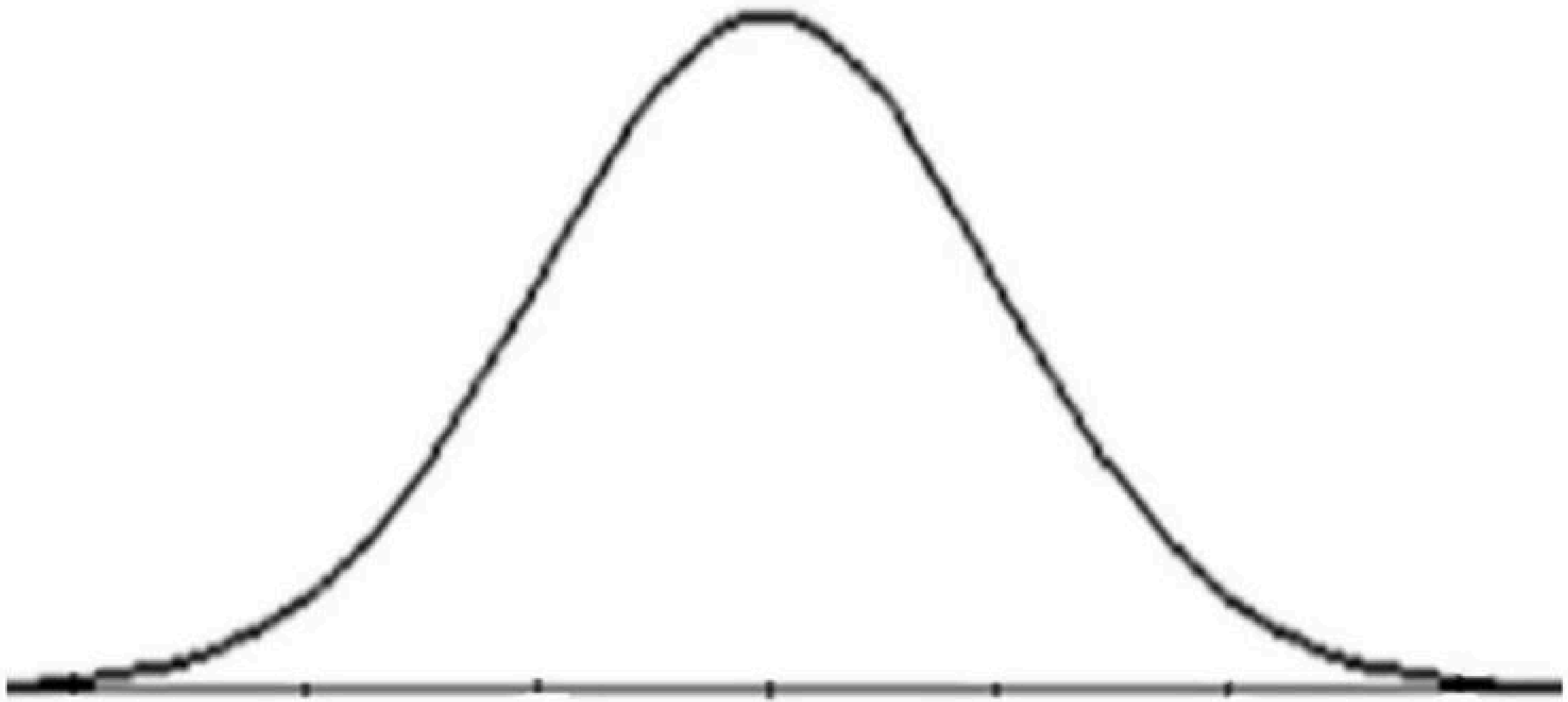
8. $P(-1 \leq Z \leq 1) =$

$P(-2 \leq Z \leq 2) =$

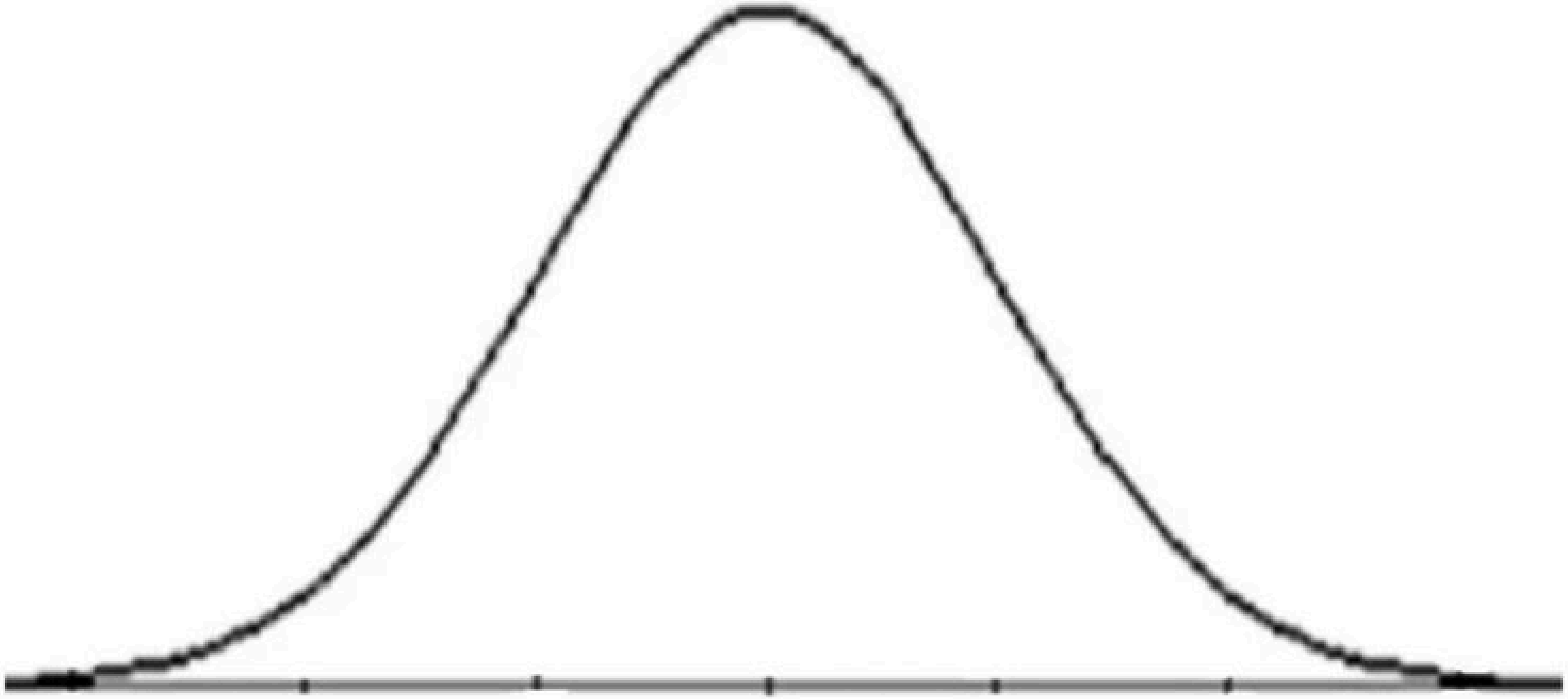
$P(-3 \leq Z \leq 3) =$



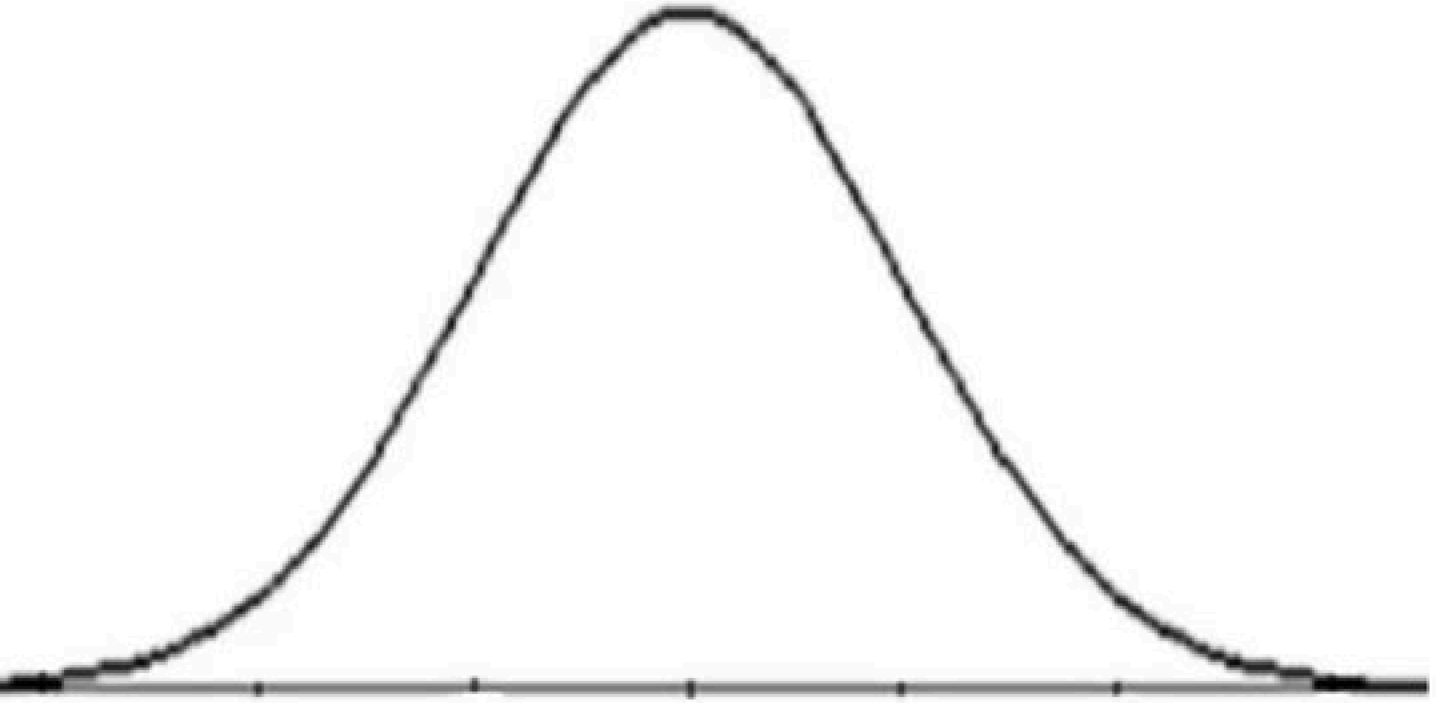
9. $P(-a \leq Z \leq a) =$



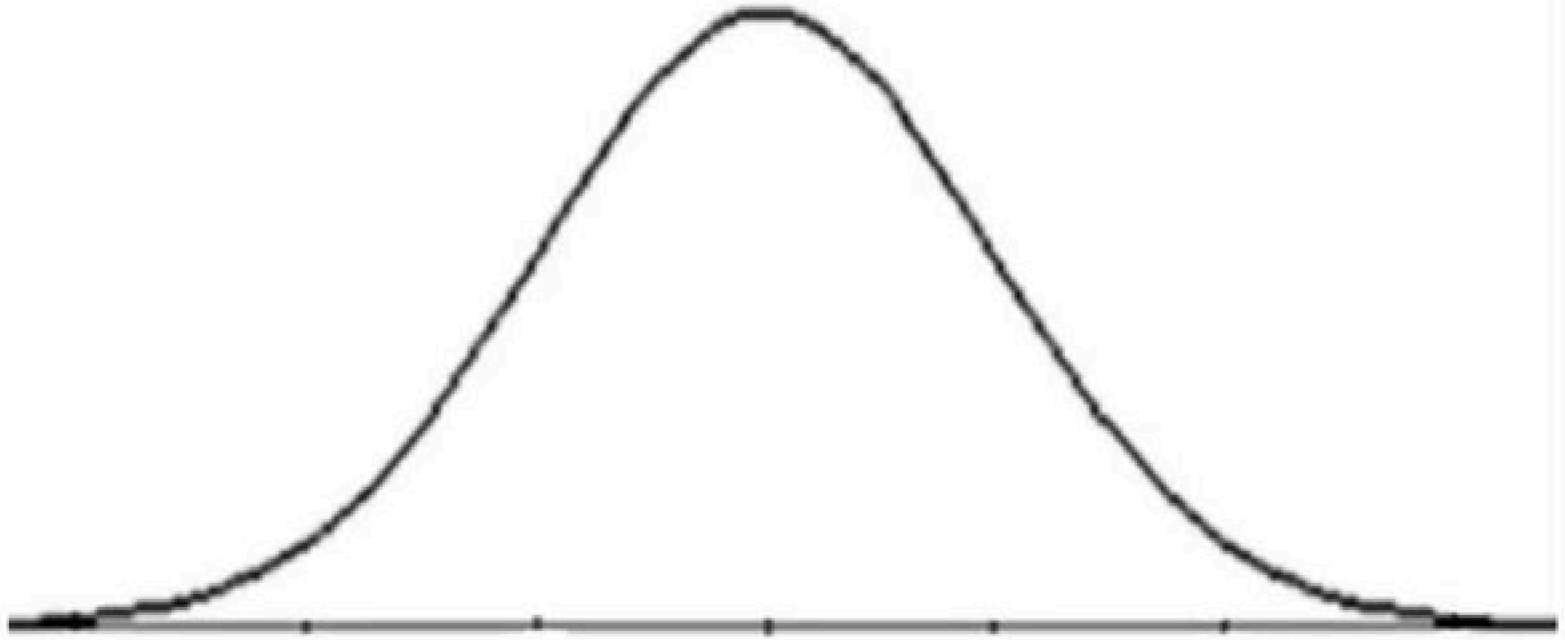
10. $P(-a \le Z \le b) =$



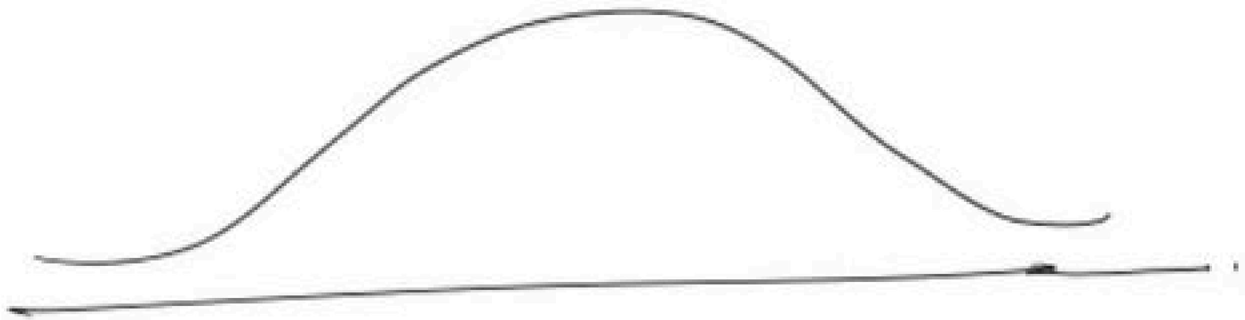
11. $P(Z \geq a) =$



12. $P(Z \leq -a) =$



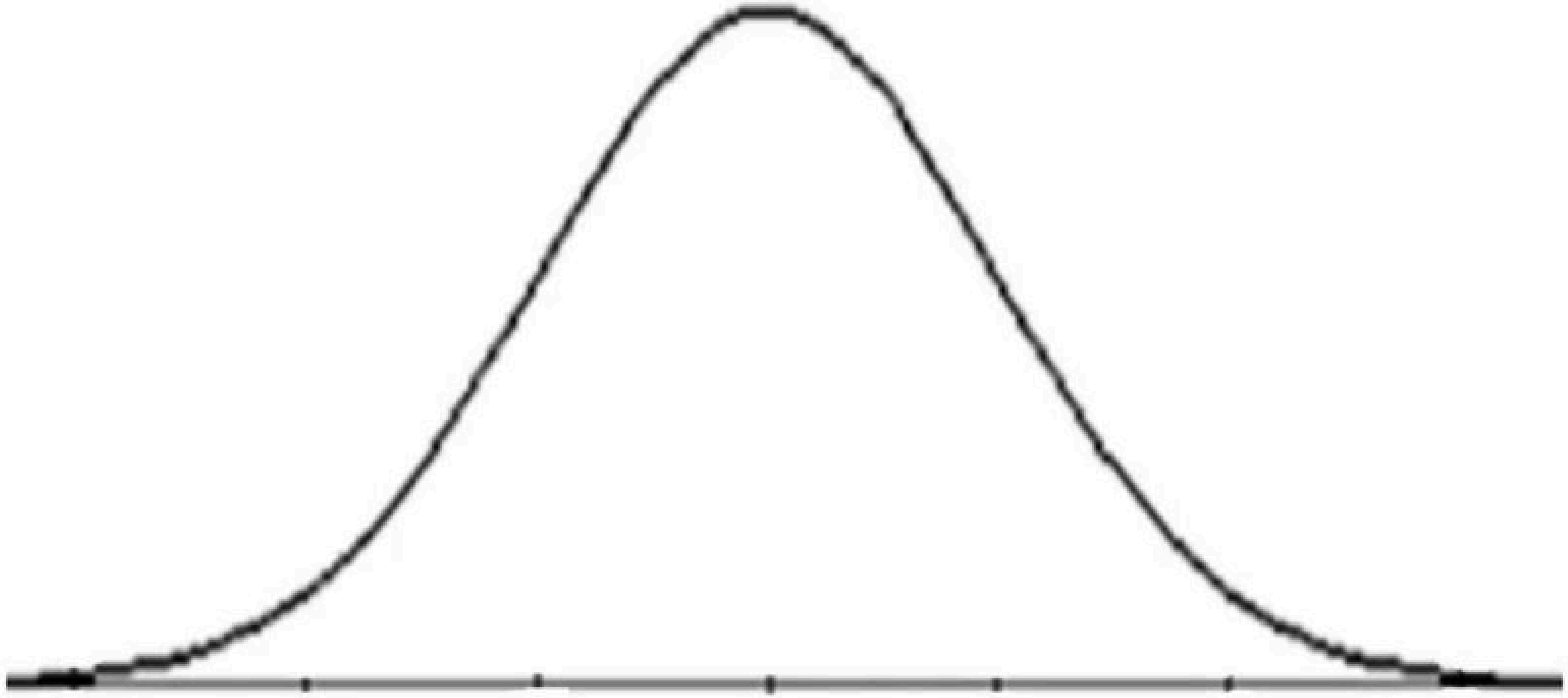
13. $P(Z \geq -a) =$



14. $P(Z \leq a) =$

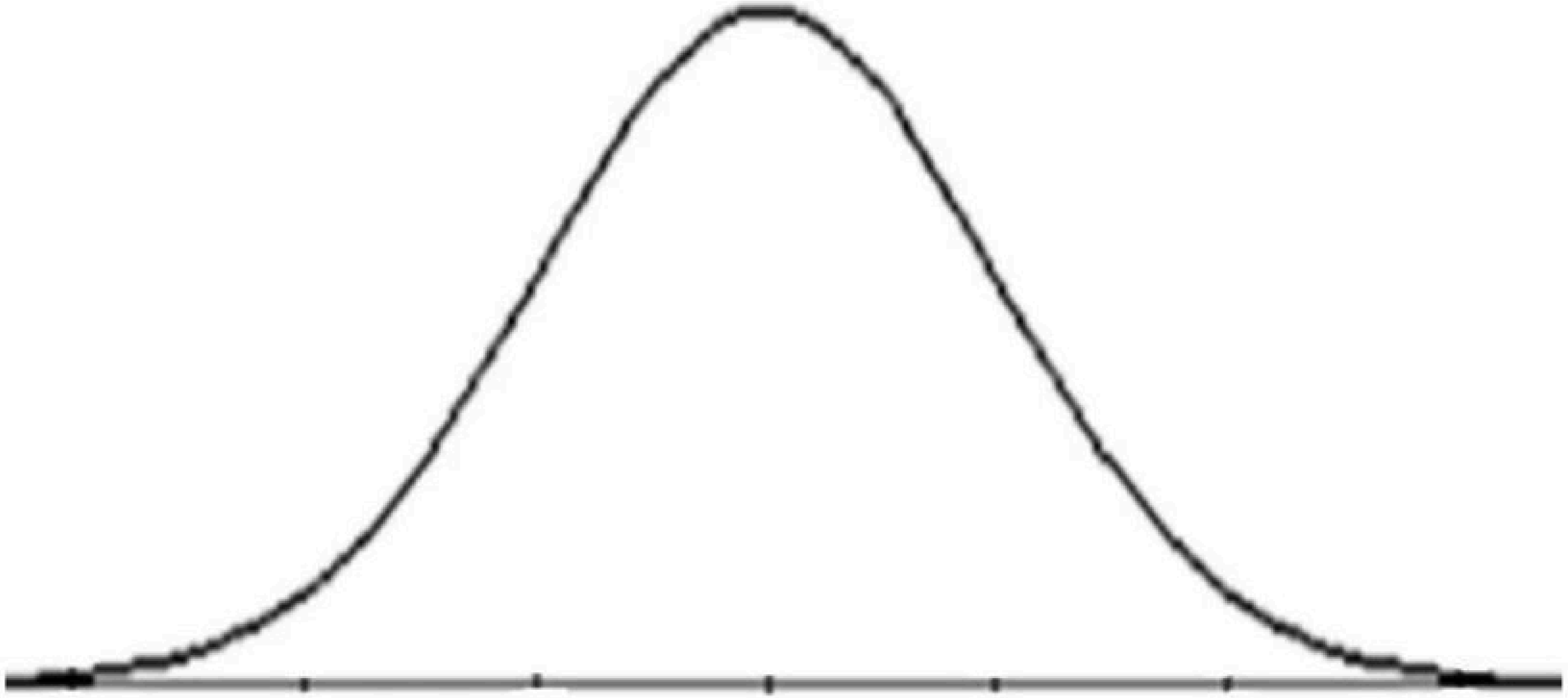
15. If $a < b$

$P(a \le Z \le b) =$



16. If $a < b$

$$P(-a \leq Z \leq -b) =$$



Q. Area under normal curve between Z = 0and Z = 1.2 is 0. 3849. Which of the following statements is false.

a) $P(Z > 1.2) = 0.1151$

b) $P(Z < 1.2) = 0.8849$

c) $P(-1.2 < Z < 1.2) = 0.7698$

d) $P(Z > -1.2) = 0.1151$

Q. Suppose that the temperature during june is normally distributed with mean 20^0 c and standard deviation 3.33^0. Find the probability P that the temperature is between 21.11^0c and 26^0c (Area under the normal curve between Z = 0 and Z = 1.80 is 0.4772 and between Z = 0 and Z = 0.33 is 0. 1293)

a) 0.3479 b) 0.6065 c) 0.8479 d) 0.1065

Q. Suppose the waist measurements of 500 boys are normally distributed with mean 66cm and standard deviation 5cm. Find the number of boys with waists ≤ 70cm
(Area under the normal curve between z = 0 and Z = 0.8 is 0.2881)

a) 394 b) 288 c) 788 d) 112

Q. Among 10,000random digits, find the probability P that the digit 3 appears at most 950 times. (Area under normal between Z = 0 and Z = 1.67 is 0.4525)

a) 0.4525 b) 0.9525 c) 0.91 d) 0.0475

Q. The mean inside diameter of a sample of 200 washers produced by a machine is 12mm and the standard deviation is 0.02mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 11.97 to 12.03mm. Other wise the washers are considered to be defective. Determine the percentage of non defective washers produced by the machine, assuming the diameters are normally distributed.(Area under the normal curve between $Z = 0$ and $Z = 1.5$ is 0.4332)

a) 43.32% b) 86.64% c) 93.32% d) 54.68%

Q. A normal random variable X has the following probability density function

$$f_x(x) =. \frac{1}{\sqrt{8\pi}} e^{-\left\{\frac{(X-1)^2}{8}\right\}}, -\infty < x < \infty$$

Then $\int_1^\infty f_X(x)dx =$

(GATE-16-PI-SET1)

(a) 0 (b) $\frac{1}{2}$ (c) $1 - \frac{1}{e}$ (d) 1

Q. Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will

(a) 0 (b) 0.5 (c) 1 (d) 10σ **(IN-2008)**

Q. For a random variable x($-\infty < x < \infty$) following normal distribution, the mean is $\mu = 100$ if the probability is $P = \alpha$ for $\alpha \geq 100$. Then the probability of x laying b/w 90 and 110 i.e., $P(90 \leq x \leq 110)$ and equal to **(GATE-PI-2008)**

(a) $1 - 2\alpha$ (b) $1 - \alpha$ (c) $1 - \alpha/2$ (d) 2α

Q. The standard normal cumulative probability distribution function can be approximated as $F(X_N) = \frac{1}{1+\exp(-1.7255 X_N |X_N|^{0.12})}$ where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be b/w 90 cm and 102 cm is

(GATE-CE-2009)

(a) 66.7% (b) 50.0% (c) 33.3% (d) 16.7%

Q. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000mm and 200mm, respectively. The probability that the annual precipitation will be more than 1200mm is **(GATE-CE-2012)**

(a) <50% (b) 50% (c) 75% (d) 100%

Q. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geq 2U)$ is **(EC-2013)**

(a) 4/9 (b) 1/2 (c) 2/3 (d) 5/9

Q. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X<0\}$ is

(GATE-ME-2013)

(a) 0.5 (b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0 (d) 1.0

Q. The area (in percentage) under standard normal distribution curve of variable within limits from -3 to + 3 is

(GATE-16-ME- SET3)

Q. Let X be a zero mean unit variance Gaussian random variable. $E[|X|]$ is equal to ______

(GATE-EC- SET-4-2014)

Q. A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs.500 and a standard deviation of Rs.50. The percentage of savings account holders, who maintain an average daily balance more than Rs.500 is______ **(GATE-ME- SET-4-2014)**

Q. f (x) is a continuous, real valued random variable defined over the interval (-∞ , ∞)and its occurrence is defined by the density function given as :

$f(x) = \frac{1}{\sqrt{2\pi b^2}} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where 'a' and 'b' are the statistical attributes of the random {x}.

The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi b^2}} e^{\frac{1}{2}\left(\frac{x-a}{b}\right)^2} dx$ is **(GATE-CE- SET-2-2014)**

(a) 1 (b) 0.5 (c) π (d) $\pi/2$

Q. A cab was involved in hit and run accident at night. You are given following data about cabs in the city and the accident

(i) 85% of the cabs in the city are green and remaining cabs are blue.

(ii) A witness identified the cab involved in the accident as blue

(iii) it is known than the witness can correctly identify the cab colour only 80% of the time

Which of the following options is closest to the probability that the accident was caused by the blue cab?

(GATE-18-EC)

(a) 12% (b) 15%

(c) 41% (b) 80%

Q. Let X1, X2 be two independent normal random variables with means μ_1, μ_2 and standard deviations σ_1 and σ_2 respectively. Consider $Y = X_1 - X_2$, $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1, \sigma_2 = 2$, then, **(GATE-18-ME)**

(a) Y is normally distributed with mean 0 and variance 1
(b) Y is normally distributed with mean 0 and variance 5
(c) Y has mean 0 and variance 5, but not normally distributed
(d) Y has mean 0 and variance 1, but not normally distributed

Q. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x **(GATE-EC-2013)**

(a) $F(x) - G(x) \leq 0$ (b) $F(x) - G(x) \geq 0$

(c) $(F(x) - G(x)).x \leq 0$ (d) $(F(x) - G(x)).x \geq 0$