DAC - Part IV

Complete Course on Algorithm for GATE - CS & IT

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$T(n) = T(n/2) + c$

$f(n)$	$n^{\log_b a}$
$\Downarrow$	$\Downarrow$
c	$n^{\log_2 1} \Rightarrow n^0 \Rightarrow 1$

$\Rightarrow c(\log n)^0 \Rightarrow \Theta(1 \cdot (\log n)^{0+1})$

$\Rightarrow \underline{\Theta(\log n)}$

$T(n) = T(\sqrt{n}) + \log_2 n$

$\Downarrow$

convert into $aT(n/b) + f(n)$ format

$\Downarrow$

① assume $n = 2^K \Rightarrow K = \log_2 n$

$T(2^K) = T((2^K)^{1/2}) + \log_2 2^K$

$T(2^K) = T(2^{K/2}) + K$

② assume $T(2^K) = S(K)$

$S(K) = S(K/2) + K$

$T(n) = \Theta(\log n)$

$f(K)$

$\Downarrow$

K

K^b

$\Downarrow$

$K^0 \Rightarrow K^0 = C$

$\Downarrow$

$= C \cdot K$

$S(K) = \Theta(K)$

$\Downarrow$

③ $T(2^K) = \Theta(K)$

④ $T(2^{\log n}) = \Theta(\log n)$

$T(n) = 2T(\sqrt{n}) + c$ constant

$\Downarrow$

① allume $n = 2^k \Rightarrow k = \log_2 n$

$T(2^k) = 2T(2^{k/2}) + c$

② allume $T(2^k) = S(k)$

$S(k) = 2S(k/2) + c$

$f(k)$	$k^{\log_b a}$
$\Vert$	$\Vert$
c	$k^{\log_2 2} \Rightarrow k$

$\Rightarrow$ $\frac{k}{k}$

$S(k) = \Theta(k)$ ✓

③ $S(k) = \Theta(k)$

$T(2^k) = \Theta(k)$

④ $T(2^k) = \Theta(k)$

$T(2^{\log_2 n}) = \Theta(\log_2 n)$

$T(n) = \Theta(\log_2 n)$

$T(n) = T(\sqrt{n}) + c$ (constant)

① $T(2^k) = T(2^{k/2}) + c$

② $S(k) = S(k/2) + c$

$f(k)$	k^0
$\Downarrow$	$\Downarrow$
c	$c * (\log k)^0$

$\Downarrow$

$S(k) = \theta(\log k)^1$

③ $S(k) = \theta(\log k)$

$T(2^k) = \theta(\log k)$

④ $T(2^k) = \theta(\lg k)$

$T(2^{2^n}) = \theta(\log_2(\lg 2^n))$

$$T(n) = \theta(\log_2(\log_2 n))$$

fib(n) ⟹ T(n)

```
{
if(n==0 || n==1)
  return(n)
else
return(fib(n-1)+fib(n-2))
}
```

fib(n-1) → T(n-1)

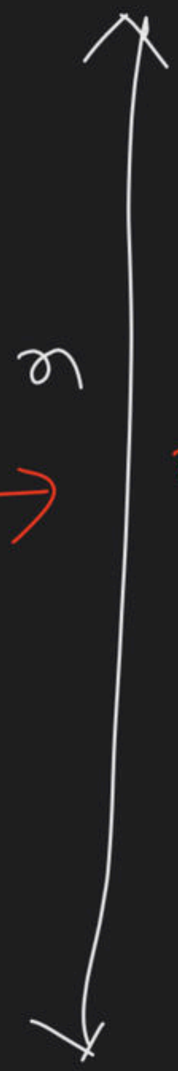
fib(n-2) → T(n-2)

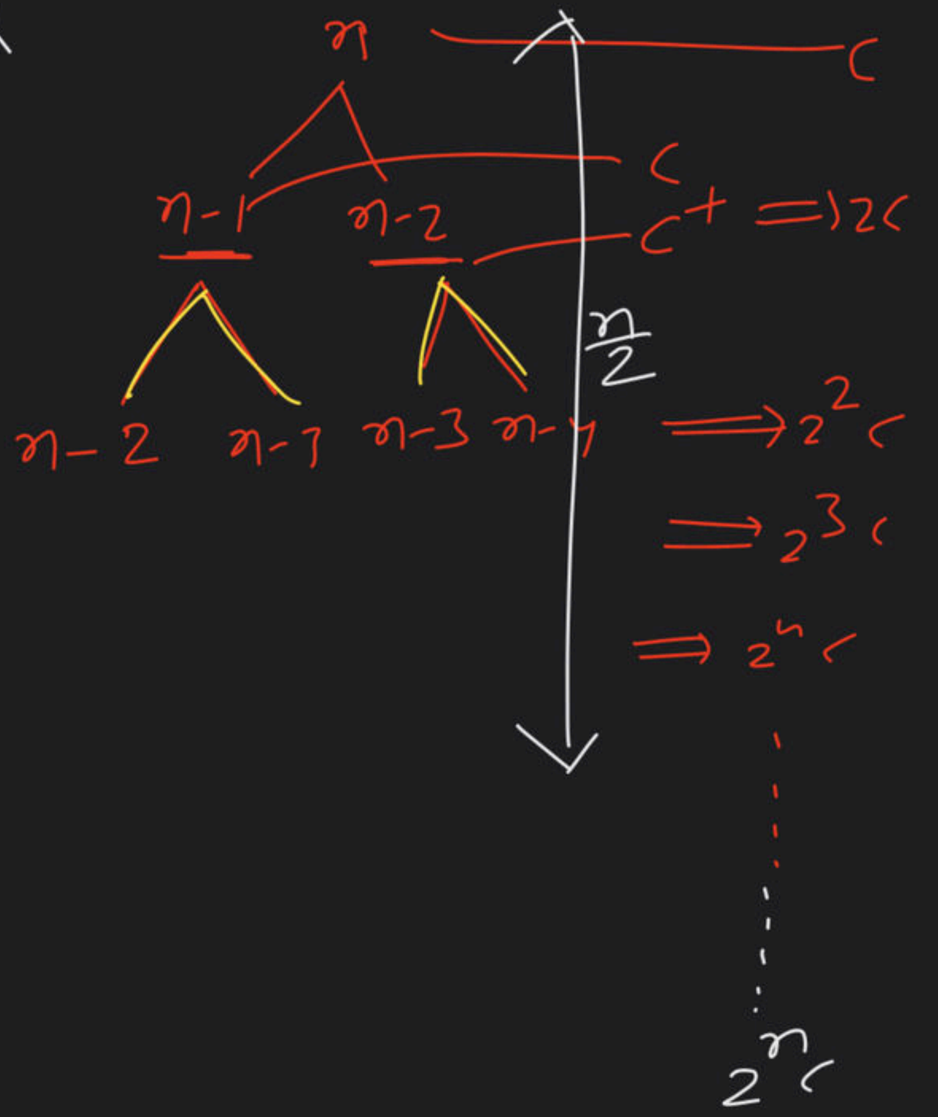
C

Time

$T(n) = T(n-1) + T(n-2) + C$

min min





$$T(n) \leq c\left[2^0 + 2^1 + 2^2 + \cdots + 2^n\right]$$

$$\leq c \cdot 2^n$$

$$= O(2^n)$$

$$T(n) \geq c\left[2^0 + 2^1 + 2^2 + \cdots + 2^{n/2}\right]$$

$$\geq c \cdot 2^{n/2}$$

$$= \Omega(2^{n/2})$$

$$2^{n/2} \leq T(n) \leq 2^n$$

a. $T(n) = O(2^n)$

b. $T(n) = \Omega(2^{n/2})$

c. both are true

Divide & Conquer