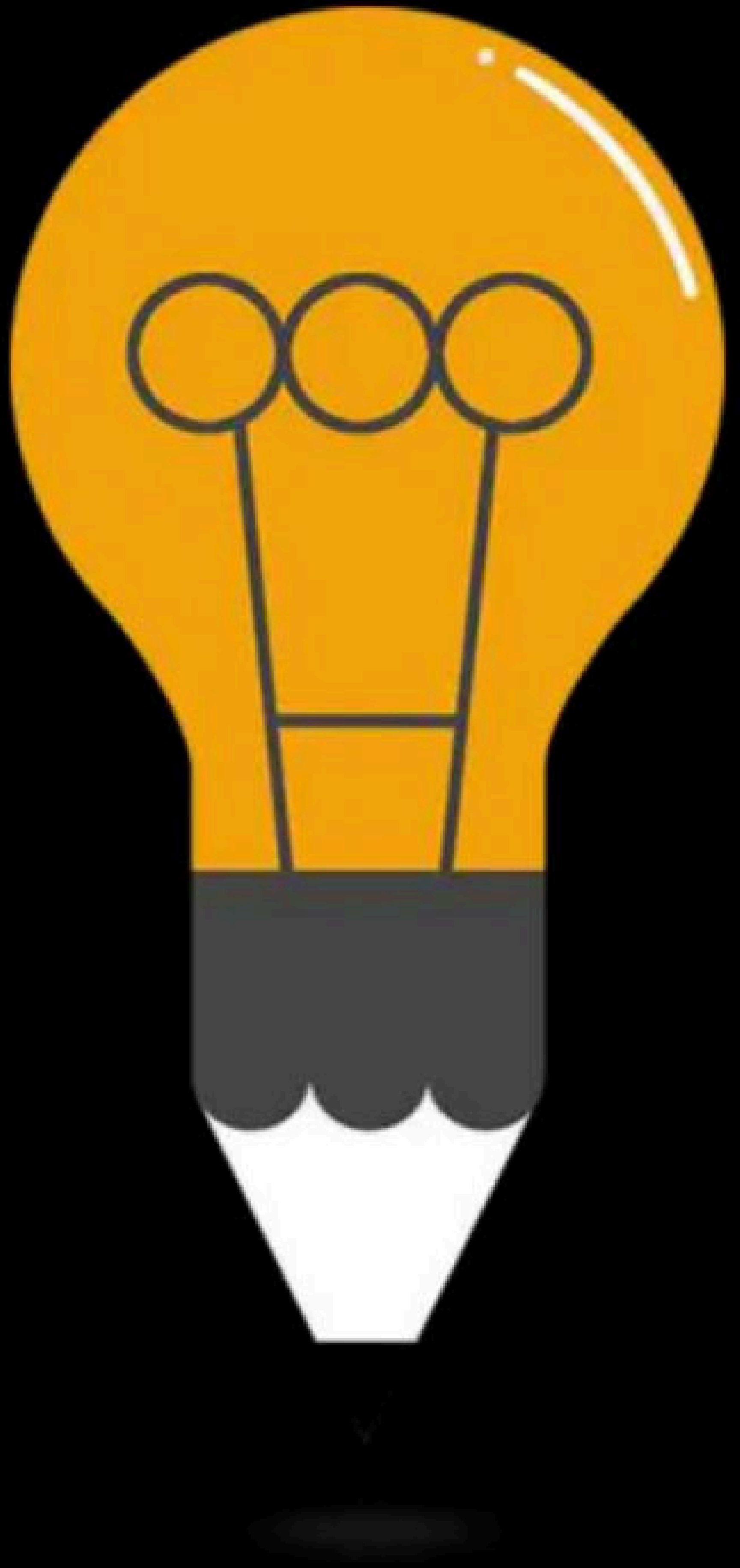
Page Replacement Algorithms

Comprehensive Course on Operating System for GATE - 2024/25

Vishvadeep Gothi • Lesson 31 • May 24, 2023



Operating System
Virtual Memory & Page Replacement

By: Vishvadeep Gothi

Virtual Memory

Demand Paging

Demand Paging:
Bring pages in memory when CPU demands

Page Fault:
When the demanded page is not available in physical memory

Effective Memory Access Time

TLB & virtual mem.:-

$$EMAT = H\,(t_{TLB} + t_{mm}) + (1-H)\Big[(1-p) * (t_{TLB} + t_{mm} + t_{mm})$$

$$+\ p * (t_{TLB} + t_{mm} + \text{p.f. service time})\Big]$$

or

$$= t_{TLB} + t_{mm} + (1-H)\big[(1-p) * t_{mm} + p * \text{service time}\big]$$

Ques) TLB hit = 90%

TLB access time = 10 nsec

mm —||—— = 200 nsec

page fault rate = 1%

p.f. service time = 100 msec

E.M.A.T. = ________ nsec

Soln :-

$$E.MAT = 10 + 200 + 0.1 [0.99 * 200 + 0.01 * 10^8 \text{ nsec}]$$

$= 100229.8$ nsec

or

$= 100.23$ μsec

P.f. S. T. = 100 ms

$= 100 * 10^{-3}$ sec

$= 100 * 10^6 * 10^{-6} * 10^{-3}$ sec $= 10^8$ ns

Dirty-bit included :- (w/o TLB)

CPU ⇒ L.A.

⇓

search in P.T. (in mm)

hit

P.A.

⇓

access mm for content

fault

P.f. service

no replacement or not dirty page

Direct bring page from s.m. to mm.

replace dirty page

write back dirty page + bring page to mm

Dirty bit with TLB :-

$$E.M.A.T. = H * (t_{TLB} + t_{mm}) + (1-H)\left[t_{TLB} + t_{mm}\right.$$

$$E.M.A.T. = (1-p) * (2\,t_{mm}) + p\left[t_{mm} + (1-m) * \text{time needed to serve when replaced page not dirty} + m * \text{time needed to serve when replaced page is dirty}\right]$$

for P.T.

m = fraction of pages dirty & replaced

Ques)

$p = 2\%$

$M = 5\%$

$t_{mm} = 100$ nsec

P.f.S.T. when page dirty = 200 μsec

— " ———— " not dirty = 100 μsec

E.M.A.T. = ______ nsec

Sol^n

E.M.A.T. = $0.98 * 2 * 100 \text{ nsec} +$

$0.02 [100 + 0.95 * 100000$

$+ 0.05 * 200000]$

$= 2298$ nsec

Dirty bit with TLB:-

$$E.M.A.T. = H * (t_{TLB} + t_{mm}) +$$

$$(1-H)\left[t_{TLB} + \underset{\substack{\uparrow \\ \text{for P.T.}}}{t_{mm}} + (1-p) * t_{mm} + p * \left((1-m) * \underset{\text{for non-dirty page}}{\text{P.F.S.T.}} + m * \text{P.F.S.T. for dirty pag}\right)\right]$$

Ques) H = 85 %

$t_{TLB} = 20\ nsec$

$t_{mm} = 200\ nsec$

$p = 1\ \%$

$m = 2\ \%$

P.F.S.T. with dirty page = 5000 usec

— " — without — " — = 2000 usec

Sol^n

$$E.M.A.T. = 0.85 * (20 + 200) + 0.15 \Big[20 + 200 + 0.99 * 200 + 0.01 * \big(0.98 * 2000000 + 0.02 * 5000000 \big) \Big]$$

$$= \boxed{3339.7\ nsec}$$

Ans

p.f.s.T. for non-dirty page = Page replacement algo time + time to bring a page from sm to mm + time to update P.T.

$\Downarrow$

t_{mm}

---

p.f.s.T. for dirty page = Page replacement algo time + write back time for dirty page from mm to s.m. + time to bring faulted page from sm to mm + time to update P.T.

Ques) TLB Hit = 90%

t_{TLB} = 15 nsec

t_{mm} = 120 nsec

p = 2%

m = 10%

time taken to transfer a page b/w mm & sm = 1500 μsec

E.M.A.T. = ______ ?

P.F.S.T. (non-dirty) = 1500 μsec + 120 nsec

= 1500000 + 120 nsec

= 1500120 nsec

P.F.S.T. (dirty) = 1500 μsec + 1500 μsec + 120 ns

= 3000120 nsec

$$E.M.A.T. = 0.9 * (15 + 120) + 0.1\left[15 + 120 + 0.98 * 120 + 0.02\left(0.9 * 1500120 + 0.1 * 3000120\right)\right]$$

$$= 3447 \text{ nsec}$$

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GATE – 2020

Consider a paging system that uses 1-level page table residing in main memory and a TLB for address translation. Each main memory access takes 100 ns and TLB lookup takes 20 ns. Each page transfer to/from the disk takes 5000 ns. Assume that the TLB hit ratio is 95%, page fault rate is 10%. Assume that for 20% of the total page faults, a dirty page has to be written back to disk before the required page is read from disk. TLB update time is negligible. The average memory access time in ns (round off to 1 decimal places) is ________

$\boxed{154.5 \text{ to } 155.5}$

$$E.M.A.T. = 0.95 * (20 + 100) + 0.05 * [20 + 100 + 0.9 * 100 + 0.1 * [0.8 * (5000 + 100) + 0.2 (5000 + 5000 + 100)]]$$

$$= 155 \text{ nsec}$$

Page Replacement

Page Replacement Policies

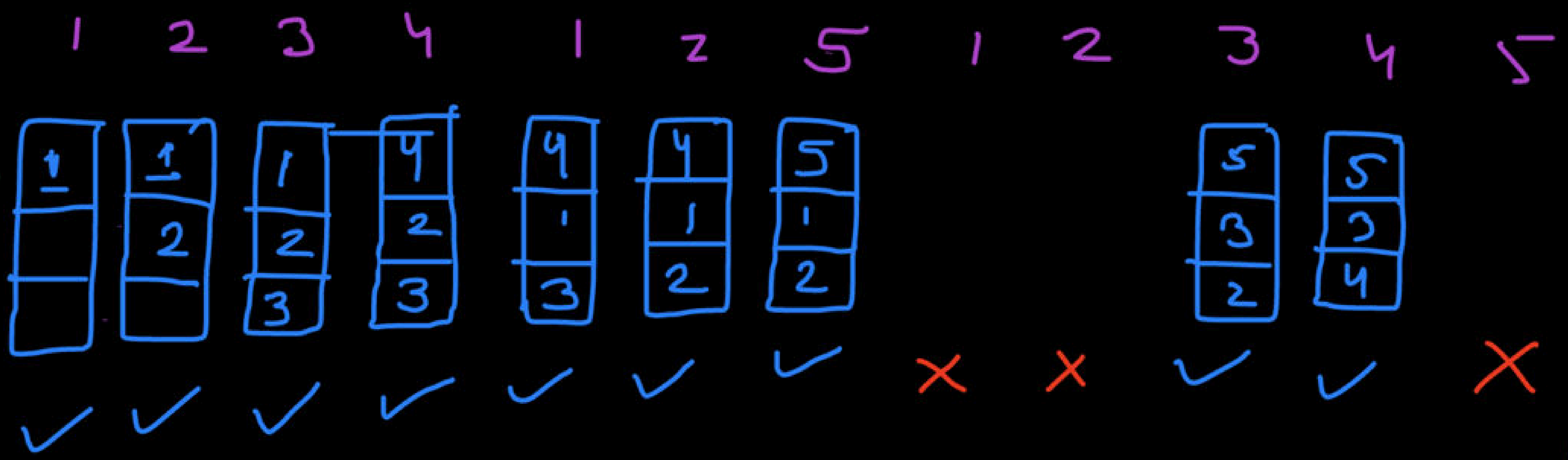
1. First In First Out (FIFO)
2. Optimal Policy
3. Least Recently Used (LRU)
4. Least Frequently Used (LFU)
5. Most Frequently Used (MFU)
6. Last In First Out (LIFO)
7. Second Chance

First In First Out (FIFO)

→ Replace the page which comes in mm first.

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5



✓ ⇒ page fault

no. of page faults = 9

p.f. rate = $\frac{9}{12}$

Page hit rate = $\frac{3}{12}$

First In First Out (FIFO)

no. of page faults = 10

Assume:

- Number of frames = 4 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

1	2	3	4	1	2	5	1	2	3	4	5
1	1	1	1			5	5	5	5	4	4
	2	2	2			2	1	1	1	1	5
		3	3			3	3	2	2	2	2
			4			4	4	4	3	3	3
✓	✓	✓	✓	×	×	✓	✓	✓	✓	✓	✓

Belady's Anomaly

for some page reference sequence, increasing no. of frames may increase no. of page faults.

=> only "FIFO" suffers from Belady's anomaly

First In First Out (FIFO)

Advantages

1. Simple and easy to implement.
2. Low overhead. → Time to obtain a page to replacement

Disadvantages:

1. Poor performance. → more page faults
2. Doesn't consider the frequency of use or last used time, simply replaces the oldest page.
3. Suffers from Belady's Anomaly

Optimal Policy

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Optimal Policy

Advantages

1. Easy to Implement
2. Simple data structures are used
3. Highly efficient

Disadvantages:

1. Requires future knowledge of the program
2. Time-consuming

Least Recently Used (LRU)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Least Recently Used (LRU)

Advantages

1. Efficient.
2. Doesn't suffer from Belady’s Anomaly

Disadvantages:

1. Complex Implementation
2. Expensive
3. Requires hardware support

Question

- Number of frames = 4 (All empty initially)
- Page reference sequence: 5, 7, 0, 1, 7, 6, 7, 2, 1, 6, 7, 6, 1
- Number of page faults for optimal and LRU policies?

Question

Consider the following page references:

2, 3, 4, 5, 6, 4, 5, 2, 7, 8, 9, 8, 9, 8, 9, 1, 6, 5, 6, 5, 3

Using optimal policy and 4 frames. Memory access time is 2ms and page fault service time is 40ms. The effective memory access time is?

Counting Algorithms

- Counting algorithms look at the number of occurrences of a particular page and use this as the criterion for replacement.

- Such counting algorithms includes:
  - LFU (Least Frequently Used)
  - MFU (Most Frequently Used)

Least Frequently Used (LFU)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1 2 0 3 0 4 2 3 0 3 2

Most Frequently Used (MFU)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1 2 0 3 0 4 2 3 0 3 2

Last In First Out (LIFO)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

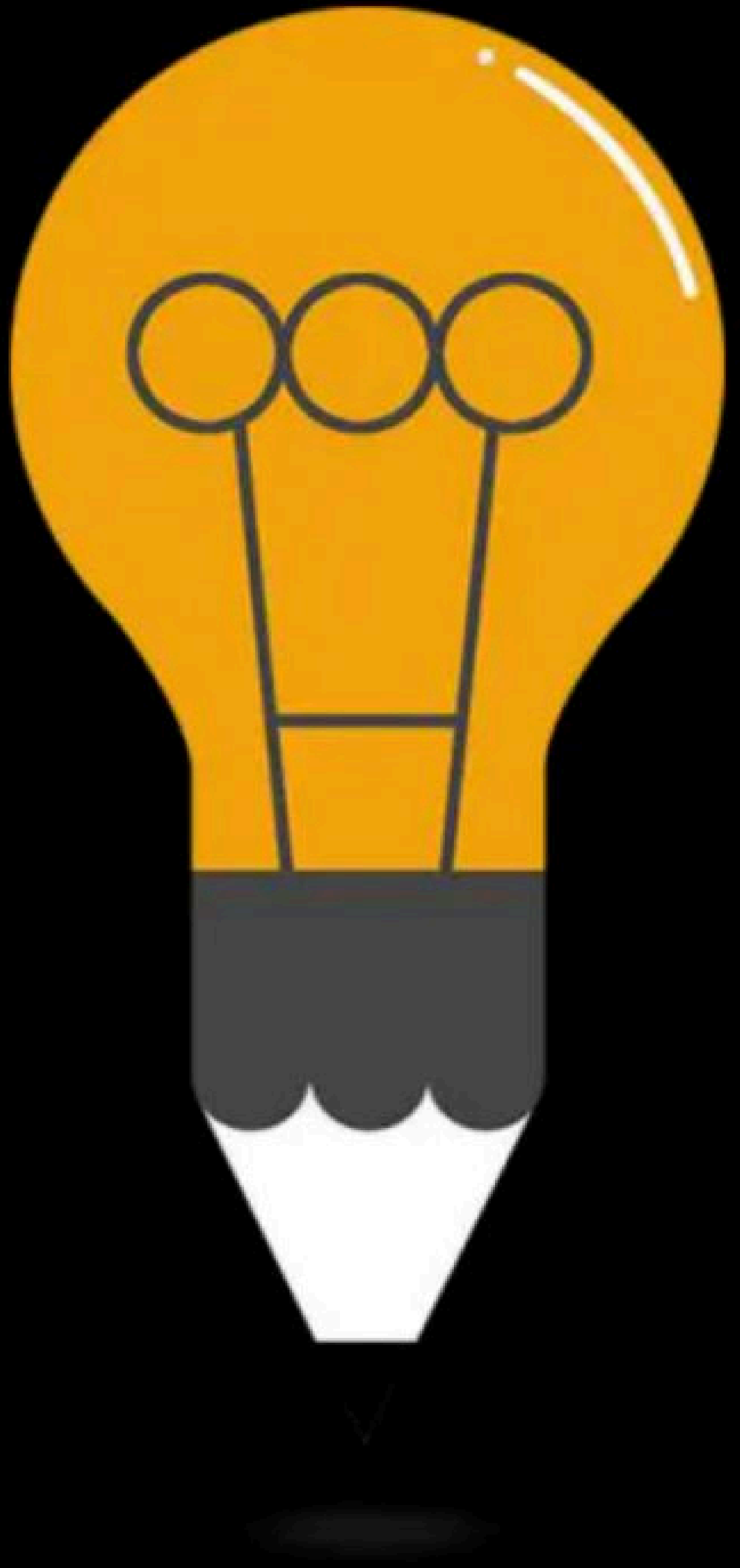
Question GATE-2016

Consider a computer system with ten physical page frames. The system is provided with an access sequence a_1, a_2, …, a_{20}, a_1, a_2, …, a_{20} where each a_i number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is __________

Question GATE-2014

A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, …, 100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(A) Least-recently-used
(B) First-in-first-out
(C) Last-in-first-out
(D) Most-recently-used



DPP

By: Vishvadeep Gothi

GATE-1993

The following page addresses, in the given sequence, were generated by a program:

1 2 3 4 1 3 5 2 1 5 4 3 2 3

This program is run on a demand paged virtual memory system, with main memory size equal to 4 pages. Indicate the page references for which page faults occur for the following page replacement algorithms.

A. LRU
B. FIFO

Assume that the main memory is initially empty

GATE-1994

A memory page containing a heavily used variable that was initialized very early and is in constant use is removed then

A. LRU page replacement algorithm is used

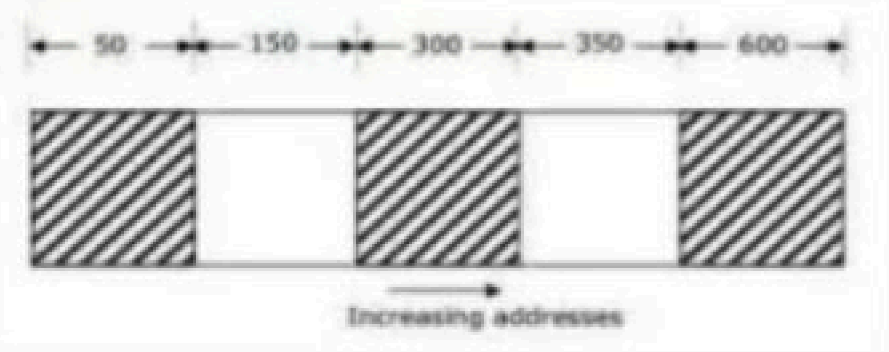
B. FIFO page replacement algorithm is used

C. LFU page replacement algorithm is used

D. None of the above

GATE-1994

Consider the following heap (figure) in which blank regions are not in use and hatched region are in use.



The sequence of requests for blocks of sizes $300, 25, 125, 50$ can be satisfied if we use

A. either first fit or best fit policy (any one)

B. first fit but not best fit policy

C. best fit but not first fit policy

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GATE-1995

Which of the following page replacement algorithms suffers from Belady's anamoly?

A. Optimal replacement

B. LRU

C. FIFO

D. Both (A) and (C)

GATE-1995

The address sequence generated by tracing a particular program executing in a pure demand based paging system with 100 records per page with 1 free main memory frame is recorded as follows. What is the number of page faults?

0100, 0200, 0430, 0499, 0510, 0530, 0560, 0120, 0220, 0240, 0260, 0320, 0370

A. 13
B. 8
C. 7
D. 10

GATE-1997

Dirty bit for a page in a page table

A. helps avoid unnecessary writes on a paging device
B. helps maintain LRU information
C. allows only read on a page
D. None of the above

GATE-1997

Locality of reference implies that the page reference being made by a process

A. will always be to the page used in the previous page reference

B. is likely to be to one of the pages used in the last few page references

C. will always be to one of the pages existing in memory

D. will always lead to a page fault

GATE-1997

Thrashing

A. reduces page I/O
B. decreases the degree of multiprogramming
C. implies excessive page I/O
D. improve the system performance

GATE-2001

Consider a virtual memory system with FIFO page replacement policy. For an arbitrary page access pattern, increasing the number of page frames in main memory will

A. always decrease the number of page faults
B. always increase the number of page faults
C. sometimes increase the number of page faults
D. never affect the number of page faults

GATE-2002

The optimal page replacement algorithm will select the page that

A. Has not been used for the longest time in the past
B. Will not be used for the longest time in the future
C. Has been used least number of times
D. Has been used most number of times

GATE-2004

The minimum number of page frames that must be allocated to a running process in a virtual memory environment is determined by

A. the instruction set architecture
B. page size
C. number of processes in memory
D. physical memory size

GATE-2005

Increasing the RAM of a computer typically improves performance because:

A. Virtual Memory increases

B. Larger RAMs are faster

C. Fewer page faults occur

D. Fewer segmentation faults occur

GATE-2007

A virtual memory system uses First In First Out (FIFO) page replacement policy and allocates a fixed number of frames to a process. Consider the following statements:

P: Increasing the number of page frames allocated to a process sometimes increases the page fault rate.

Q: Some programs do not exhibit locality of reference.

Which one of the following is TRUE?

A. Both P and Q are true, and Q is the reason for P

B. Both P and Q are true, but Q is not the reason for P.

C. P is false but Q is true

D. Both P and Q are false.

GATE-2007

A process has been allocated 3 page frames. Assume that none of the pages of the process are available in the memory initially. The process makes the following sequence of page references (reference string): $\mathbf{1, 2, 1, 3, 7, 4, 5, 6, 3, 1}$

If optimal page replacement policy is used, how many page faults occur for the above reference string?

A. 7
B. 8
C. 9
D. 10

GATE-2007

A process, has been allocated 3 page frames. Assume that none of the pages of the process are available in the memory initially. The process makes the following sequence of page references (reference string): $1, 2, 1, 3, 7, 4, 5, 6, 3, 1$

Least Recently Used (LRU) page replacement policy is a practical approximation to optimal page replacement. For the above reference string, how many more page faults occur with LRU than with the optimal page replacement policy?

A. 0
B. 1
C. 2
D. 3

GATE-2007

The address sequence generated by tracing a particular program executing in a pure demand paging system with 100 bytes per page is

$$0100, 0200, 0430, 0499, 0510, 0530, 0560, 0120, 0220, 0240, 0260, 0320, 0410.$$

Suppose that the memory can store only one page and if x is the address which causes a page fault then the bytes from addresses x to $\mathrm{x}+99$ are loaded on to the memory.

How many page faults will occur?

A. 0
B. 4
C. 7
D. 8

GATE-2007

A demand paging system takes 100 time units to service a page fault and 300 time units to replace a dirty page. Memory access time is 1 time unit. The probability of a page fault is p. In case of a page fault, the probability of page being dirty is also p. It is observed that the average access time is 3 time units. Then the value of p is

A. 0.194
B. 0.233
C. 0.514
D. 0.981

GATE-2008

Assume that a main memory with only 4 pages, each of 16 bytes, is initially empty. The CPU generates the following sequence of virtual addresses and uses the Least Recently Used (LRU) page replacement policy.

$$0, 4, 8, 20, 24, 36, 44, 12, 68, 72, 80, 84, 28, 32, 88, 92$$

How many page faults does this sequence cause? What are the page numbers of the pages present in the main memory at the end of the sequence?

A. 6 and $1, 2, 3, 4$
B. 7 and $1, 2, 4, 5$
C. 8 and $1, 2, 4, 5$
D. 9 and $1, 2, 3, 5$

GATE-2009

In which one of the following page replacement policies, Belady's anomaly may occur?

A. FIFO
B. Optimal
C. LRU
D. MRU

GATE-2010

A system uses FIFO policy for system replacement. It has 4 page frames with no pages loaded to begin with. The system first accesses 100 distinct pages in some order and then accesses the same 100 pages but now in the reverse order. How many page faults will occur?

A. 196
B. 192
C. 197
D. 195

GATE-2012

Consider the virtual page reference string

$1, 2, 3, 2, 4, 1, 3, 2, 4, 1$

on a demand paged virtual memory system running on a computer system that has main memory size of 3 page frames which are initially empty. Let LRU, FIFO and $\mathrm{OPTIMAL}$ denote the number of page faults under the corresponding page replacement policy. Then

A. $\mathrm{OPTIMAL} < \mathrm{LRU} < \mathrm{FIFO}$
B. $\mathrm{OPTIMAL} < \mathrm{FIFO} < \mathrm{LRU}$
C. $\mathrm{OPTIMAL} = \mathrm{LRU}$
D. $\mathrm{OPTIMAL} = \mathrm{FIFO}$

GATE-2014

Assume that there are 3 page frames which are initially empty. If the page reference string is $1, 2, 3, 4, 2, 1, 5, 3, 2, 4, 6$ the number of page faults using the optimal replacement policy is__________.

GATE-2014

A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered $1, 2, ..., 100$ in that order, and repeats the access sequence **THRICE**. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

A. Least-recently-used
B. First-in-first-out
C. Last-in-first-out
D. Most-recently-used

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GATE-2014

A system uses 3 page frames for storing process pages in main memory. It uses the Least Recently Used (**LRU**) page replacement policy. Assume that all the page frames are initially empty. What is the total number of page faults that will occur while processing the page reference string given below?

$$4, 7, 6, 1, 7, 6, 1, 2, 7, 2$$

GATE-2015

Consider a main memory with five-page frames and the following sequence of page references: $3, 8, 2, 3, 9, 1, 6, 3, 8, 9, 3, 6, 2, 1, 3$. Which one of the following is true with respect to page replacement policies First In First Out (FIFO) and Least Recently Used (LRU)?

A. Both incur the same number of page faults
B. FIFO incurs 2 more page faults than LRU
C. LRU incurs 2 more page faults than FIFO
D. FIFO incurs 1 more page faults than LRU

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GATE-2016

Consider a computer system with ten physical page frames. The system is provided with an access sequence $(a_1, a_2, \ldots, a_{20}, a_1, a_2, \ldots a_{20})$, where each a_i is a distinct virtual page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is__________.

GATE-2016

In which one of the following page replacement algorithms it is possible for the page fault rate to increase even when the number of allocated frames increases?

A. LRU (Least Recently Used)
B. OPT (Optimal Page Replacement)
C. MRU (Most Recently Used)
D. FIFO (First In First Out)

GATE-2017

Recall that Belady's anomaly is that the page-fault rate may *increase* as the number of allocated frames increases. Now, consider the following statement:

$S1$: *Random page replacement* algorithm (where a page chosen at random is replaced) suffers from Belady's anomaly.

$S2$: *LRU page replacement* algorithm suffers from Belady's anomaly.

Which of the following is CORRECT?

A. $S1$ is true, $S2$ is true
B. $S1$ is true, $S2$is false
C. $S1$ is false, $S2$is true
D. $S1$ is false, $S2$is false

Happy Learning.!