

Doubt Clearing Session

Comprehensive Course on Digital Logic Design 2023/2024

DIGITAL LOGIC CIRCUITS

(CSE)

Karnaugh Map(K- Map)

K- Map

- It is a graphical technique to minimize the Boolean expressions, that may contain don't care combinations.
- K-map is a systematic method and suitable up to 5 variables .
- Gray code is used to formulate K-map, the minimization is based on the gray code property i.e. *Logical Adjacency*.
- K-map technique is used in 2 format's
 - 1. If we need the answer in SOP form , then the K-map is used in minterm mode
 - 2. If we need the answer in POS form ,then the K- map is used in Maxterm mode

Logical Adjacency

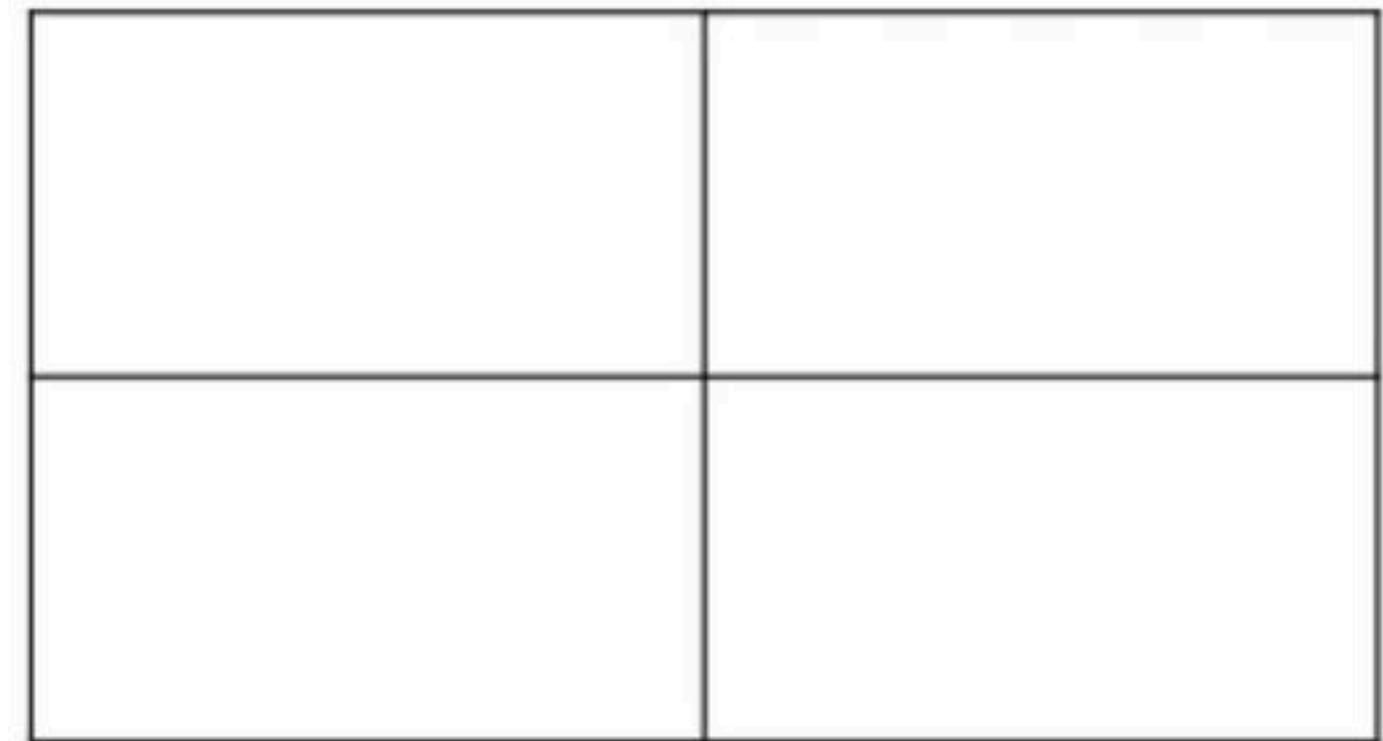
- Two cells are said to be logical adjacent to each other , if there is only one bit change between them .
- For a n-variable K –map , for every cell there are ‘ n ‘ logical adjacent cells

2- Variable K- Map

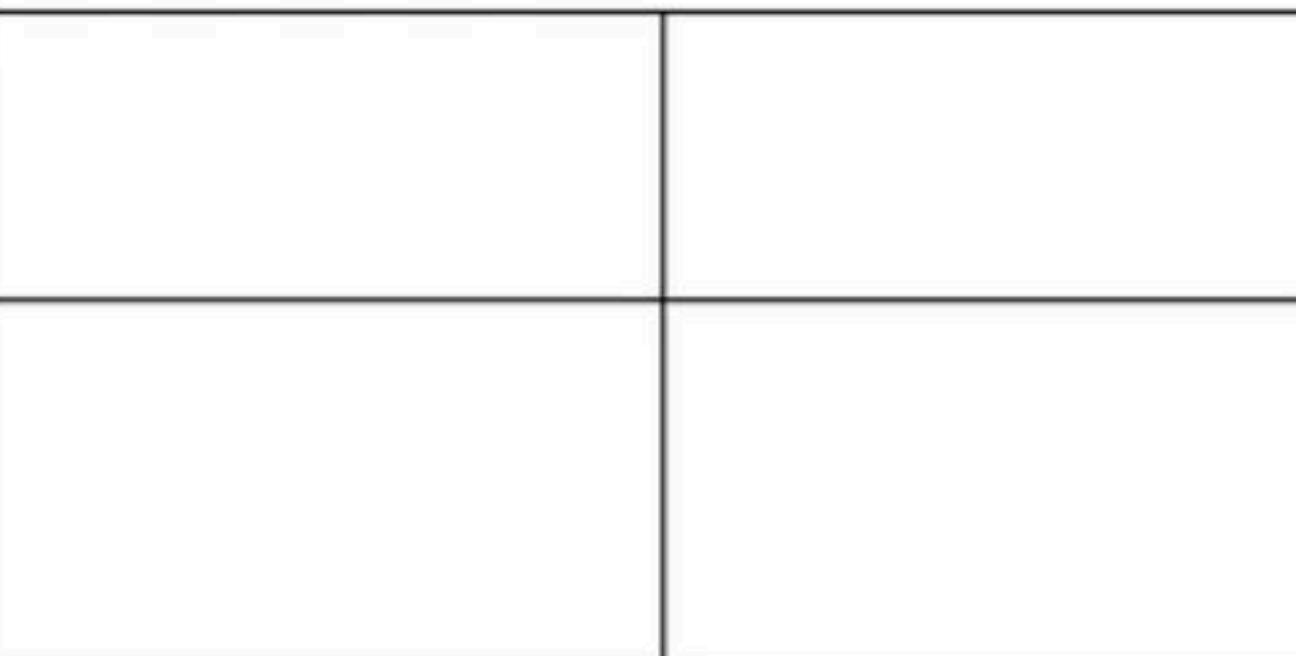
Minterm mode



Minterm mode



Maxterm mode



3-Variable K-Map

Minterm mode

3-Variable K-Map

Minterm mode

3-Variable K-Map

Maxterm mode

4- Variable K-map(minterm mode)

Adjacent cell for 8 –

Adjacent cell for 2 –

Adjacent cell for 11 –

Adjacent cell for 15 –

Adjacent cell for 0 –

Adjacent cell for 6 –

Adjacent cell for 14 –

Adjacent cell for 3 –

4- Variable K-map(minterm mode)

Adjacent cell for 8 –

Adjacent cell for 2 –

Adjacent cell for 11 –

Adjacent cell for 15 –

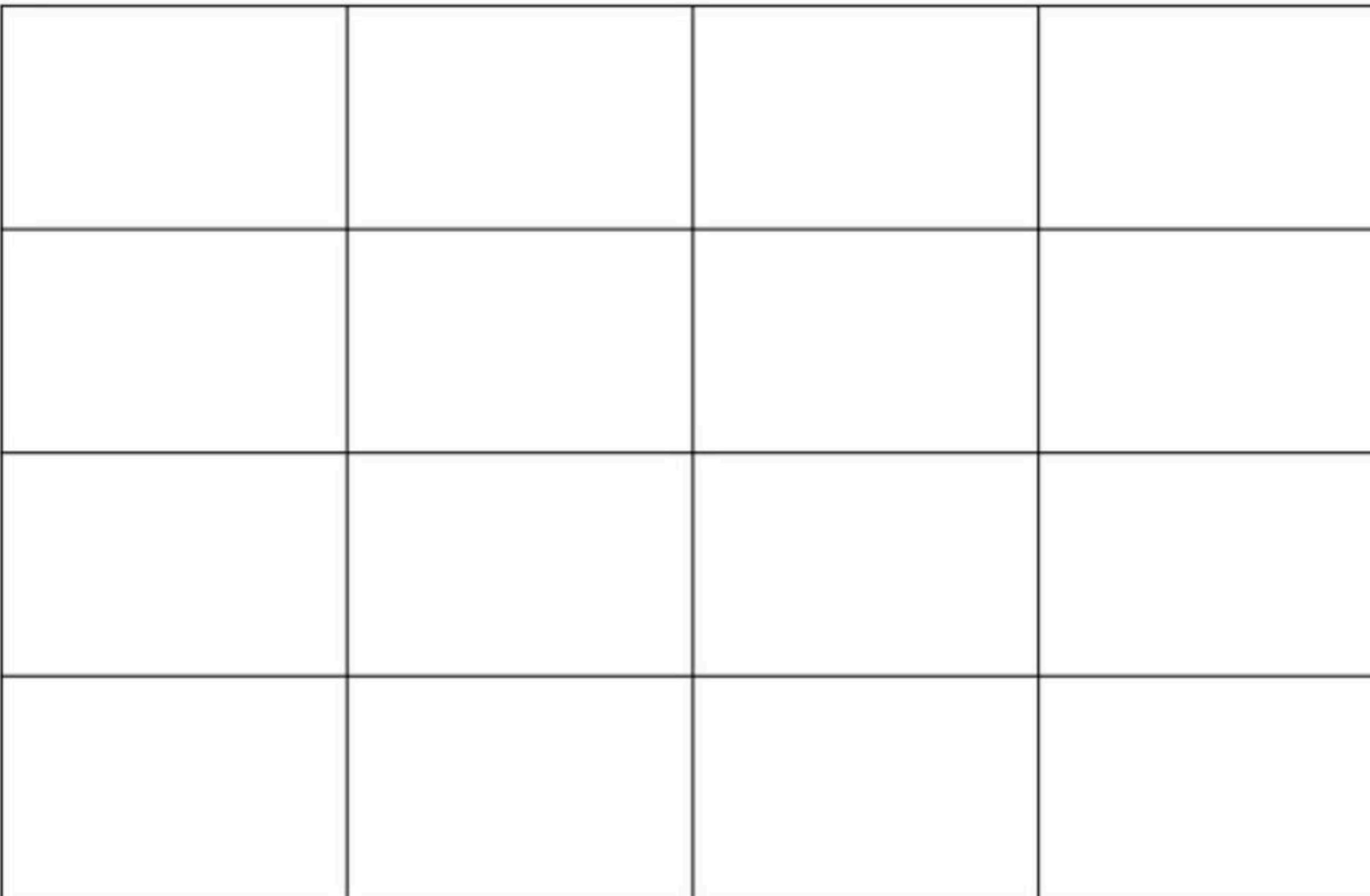
Adjacent cell for 0 –

Adjacent cell for 6 –

Adjacent cell for 14 –

Adjacent cell for 3 –

4- Variable K-map (Maxterm mode)



Minimization Steps

1. Place the corresponding minterm/maxterms in the corresponding cells.

minterm place -----> 1

Maxterm place -----> 0

1. Using the valid sub cube property go for bigger size grouping .

2. Don't care combination's may not be covered , it depends on the K-map.

3. Remove the redundant groups if any present .

Priority of grouping

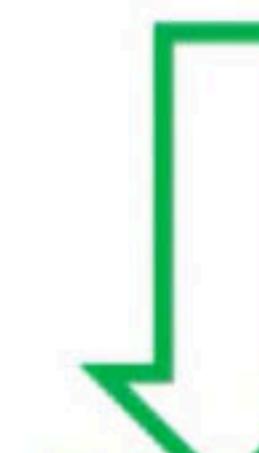
(16-cells)



Octet (8-cells)



Quad (4-cells)



Pair (2-cells)

Valid Sub cube

A collection of 2^m number of cells in a K- map is said to be forming a valid sub cube , provided inside the collection 2^m number of cells every cell is logical adjacent to m number of cells .

4- Variable K-map

Q) Minimize the following

A blue line labeled **B** connects the top-left cell to the top-right cell of the matrix.

1	1

Q) Minimize the following

A	B
1	
1	

Q) Minimize the following

A	B
1	1
	1

Q) Minimize the following

A	B
1	1
1	

Q) Minimize the following

1	1
1	1

Q) Minimize the following

A	B
0	0

Q) Minimize the following

A	B
	0
	0

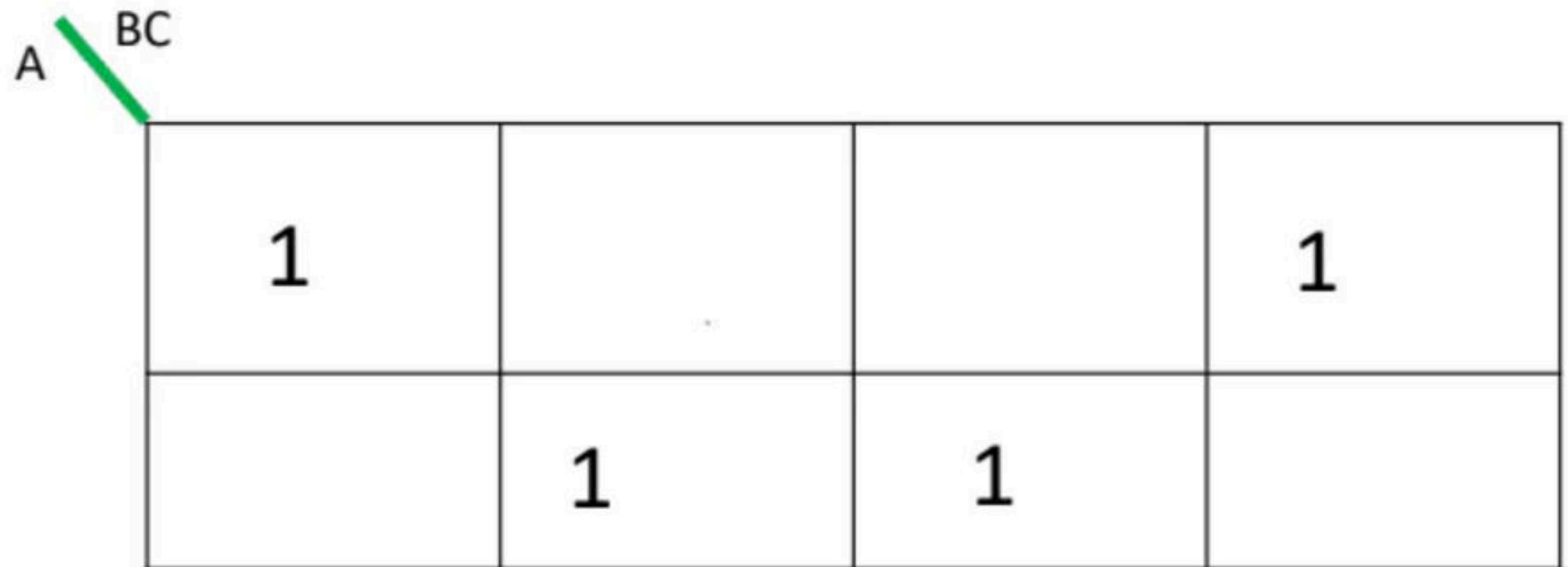
Q) Minimize the following

A	B
0	0
	0

Q) Minimize the following

0	0
0	0

Q) Minimize the following



Q) Minimize the following

A BC			1	
1			1	

$$Q) F(A,B,C) = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

$$Q) F(A, B, C) = \sum m(0, 1, 2, 3, 4, 6)$$

$$Q) F(A, B, C) = \sum m(0, 1, 3, 4)$$

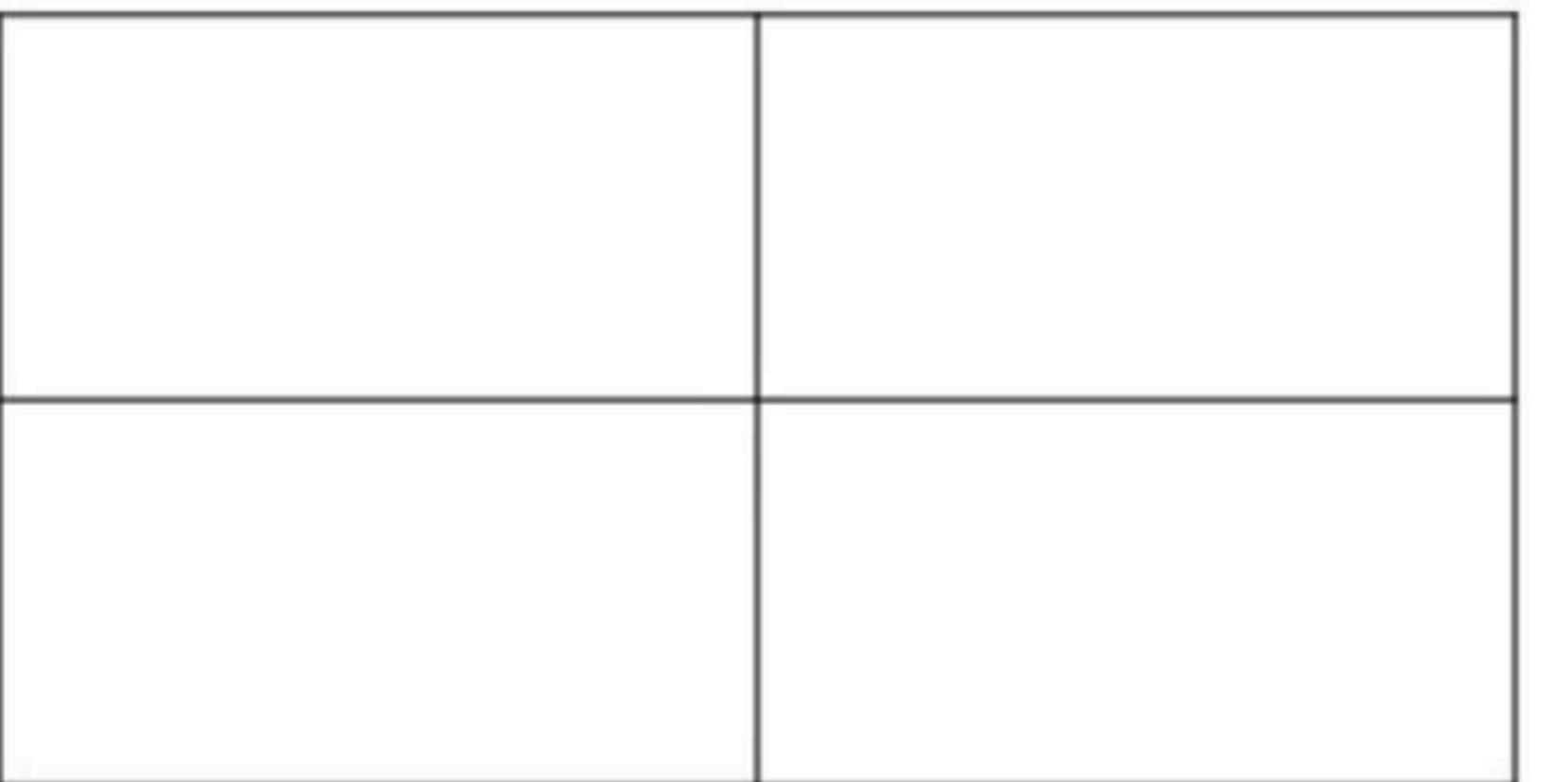
$$Q) F(A, B, C) = \sum m(0, 1, 3, 5, 6)$$

$$Q) F(A,B,C) = \prod M(2,4,7)$$

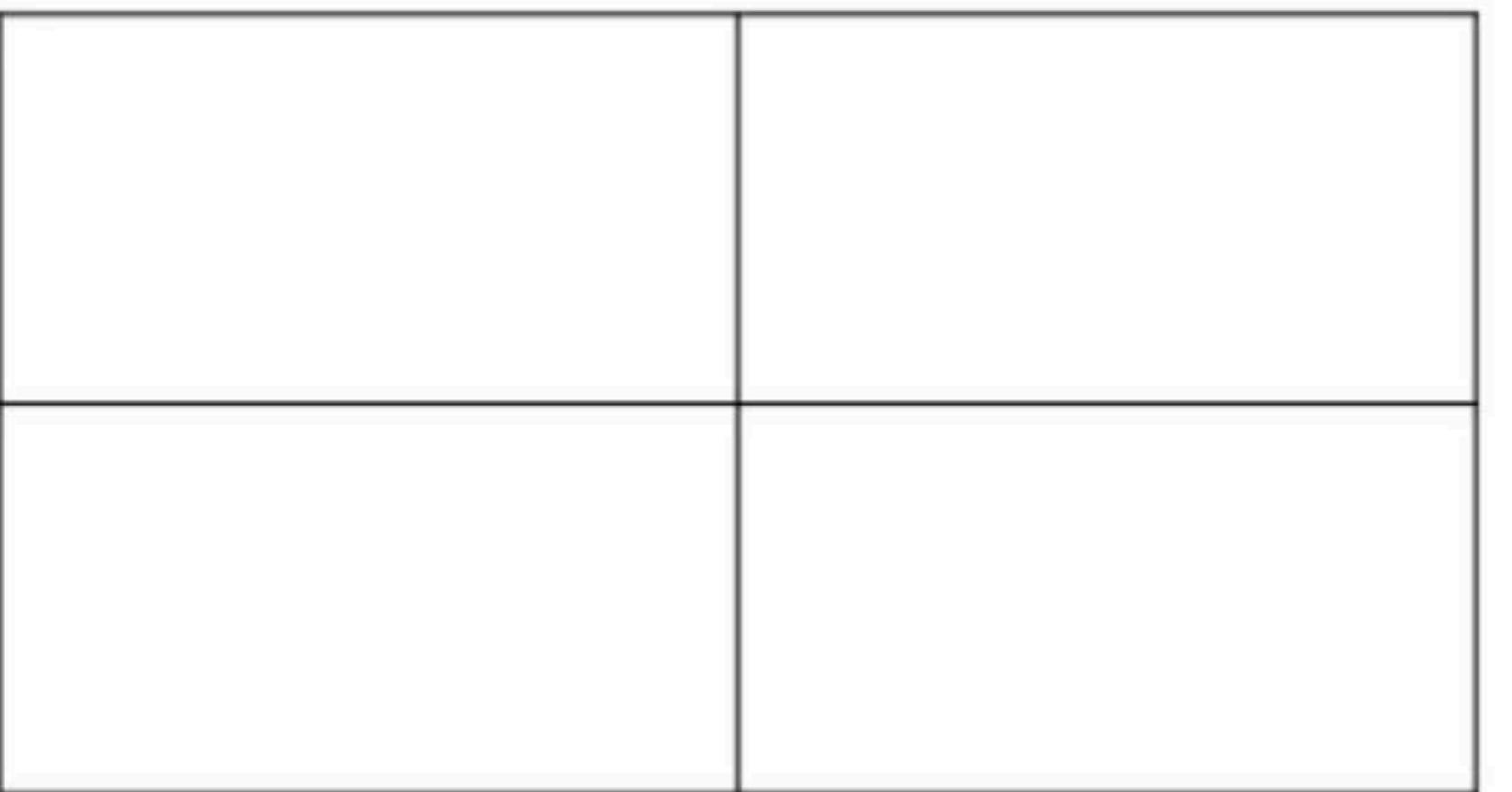
Q) Minimize the following

CD			
AB			
	1		1
	1		1
	1	1	
	1		

$$Q) F(A, B) = \sum m(0, 3) + d(2)$$



$$Q) F(A, B) = \prod M(0, 1) + d(3)$$



$$Q) F(A, B, C) = \prod M(1, 2, 6, 4, 7)$$

Q) Minimize the following

CD			
AB			
	1		1
	1	1	
	1	1	
	1		1

Q) Minimize the following

	1	1	
1			1
1			1
	1	1	

Q) Minimize the following

CD			
AB			
1	1	1	1
	1	1	1

Q) Minimize the following

CD			
AB			
1	1	1	1
1	1	1	1

Q) Minimize the following

CD			
AB			
1	1	1	1
1	1	1	1

Q) Minimize the following

CD			
AB			
1			1
1			1
1			1
1			1

$$Q) F(A, B, C, D) = \sum m(3, 4, 6, 8, 9, 11, 12, 14)$$

$$Q) F(A, B, C, D) = \prod M(3, 4, 6, 8, 9, 11, 12, 14)$$

Note :

- For a n-variable Boolean expression, the maximum number of literals a minterm contains = n
- For a n – variable k-map if grouping is done by considering 2^m number of cells , then m-number of literals are eliminated and the resulting term from that group contains (n-m) number of literals
- 8 cells – 2^3 cells → Octet -----> 3 variables eliminated
- 4 cells – 2^2 cells → Quad -----> 2 variables eliminated
- 2 cells – 2^1 cells → Pair -----> 1 variables eliminated

A

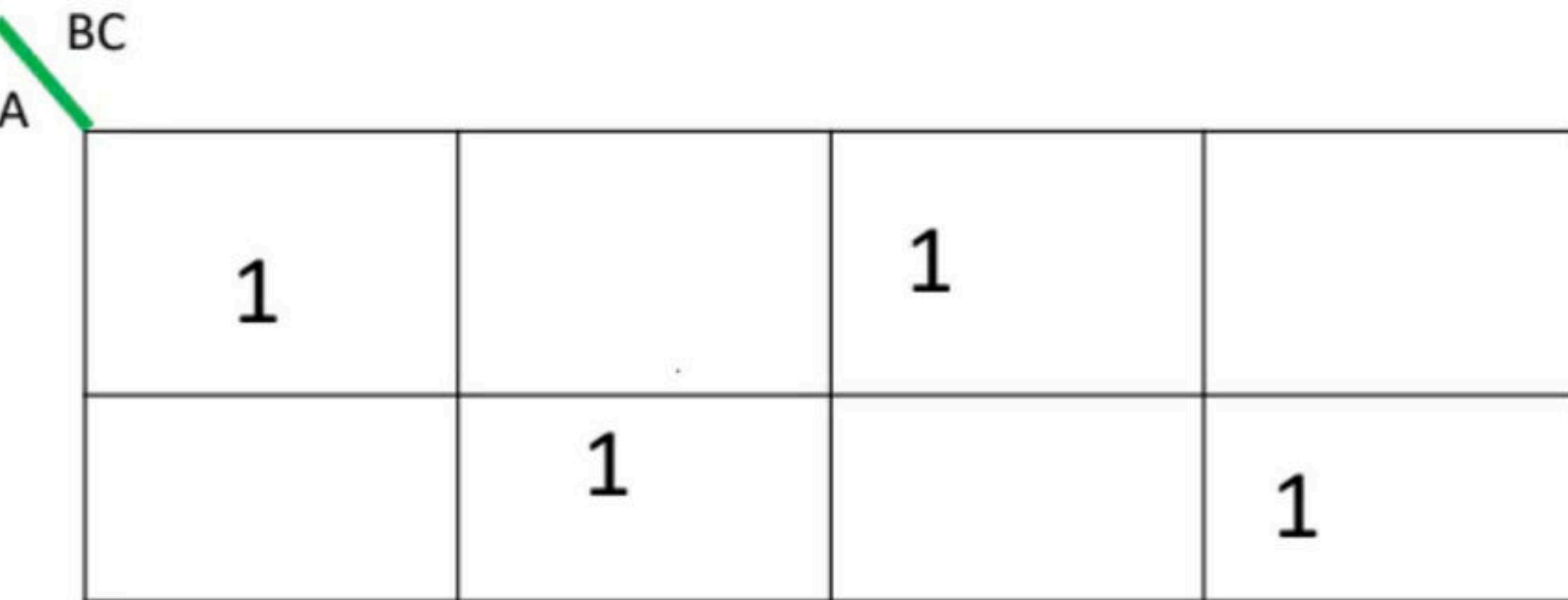
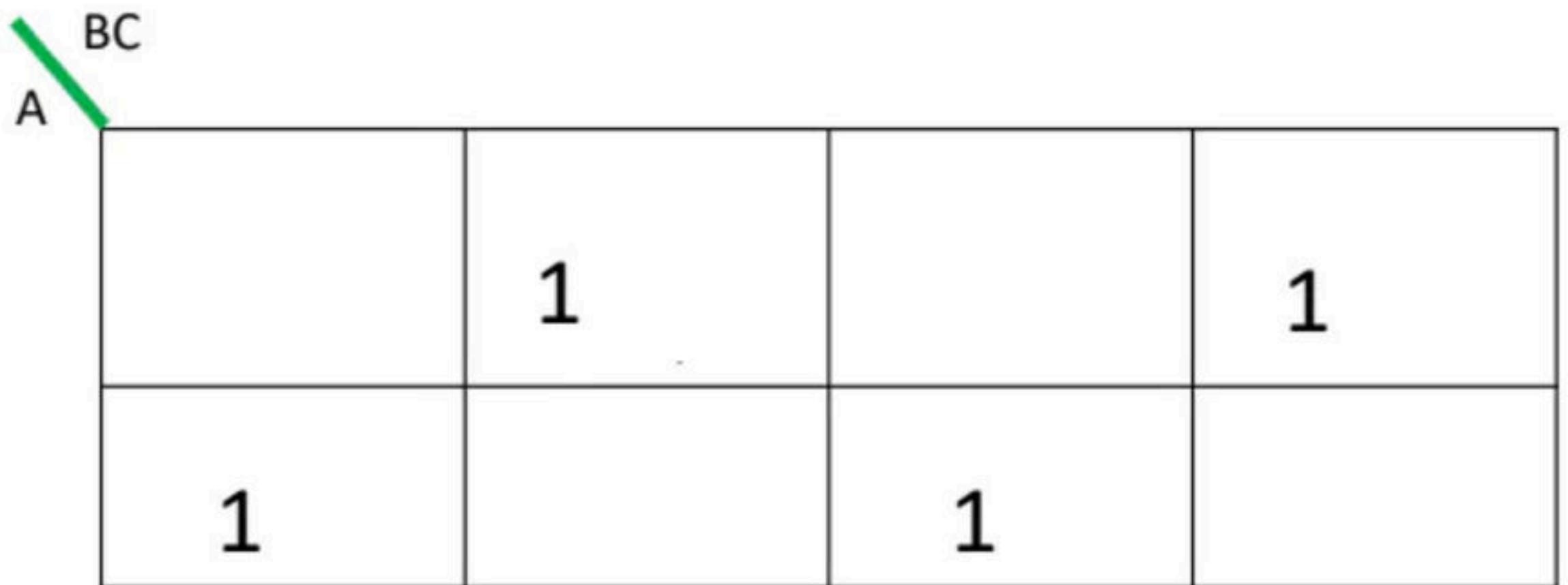
B

1	1
1	1

A

B

	1
1	



CD

AB

1

1

1

1

1

1

1

1

CD

AB

1

1

1

1

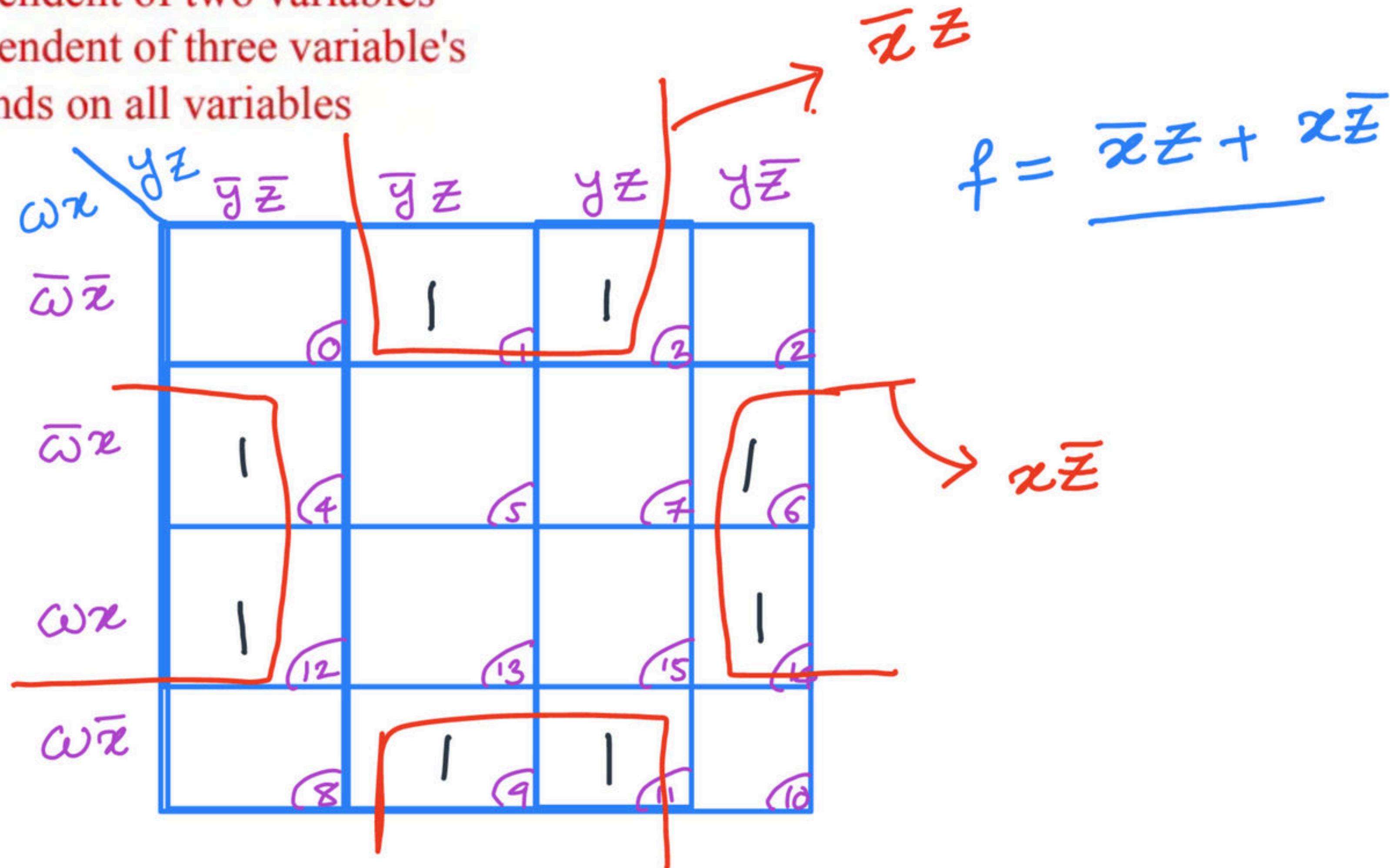
1

1

1

$$Q) F(w, x, y, z) = \sum(1, 3, 4, 6, 9, 11, 12, 14)$$

- a) Independent of one variables
- b) ~~Independent of two variables~~
- c) Independent of three variable's
- d) Depends on all variables



Q) A logic circuit implement $F = \bar{x}y + x\bar{y}\bar{z}$, it is found that $x = y = 1$ can never occur, considering this as fact , the minimized expression of F is.....

$$f = \bar{x}y + x\bar{y}\bar{z} = \sum m(2, 3, 4)$$

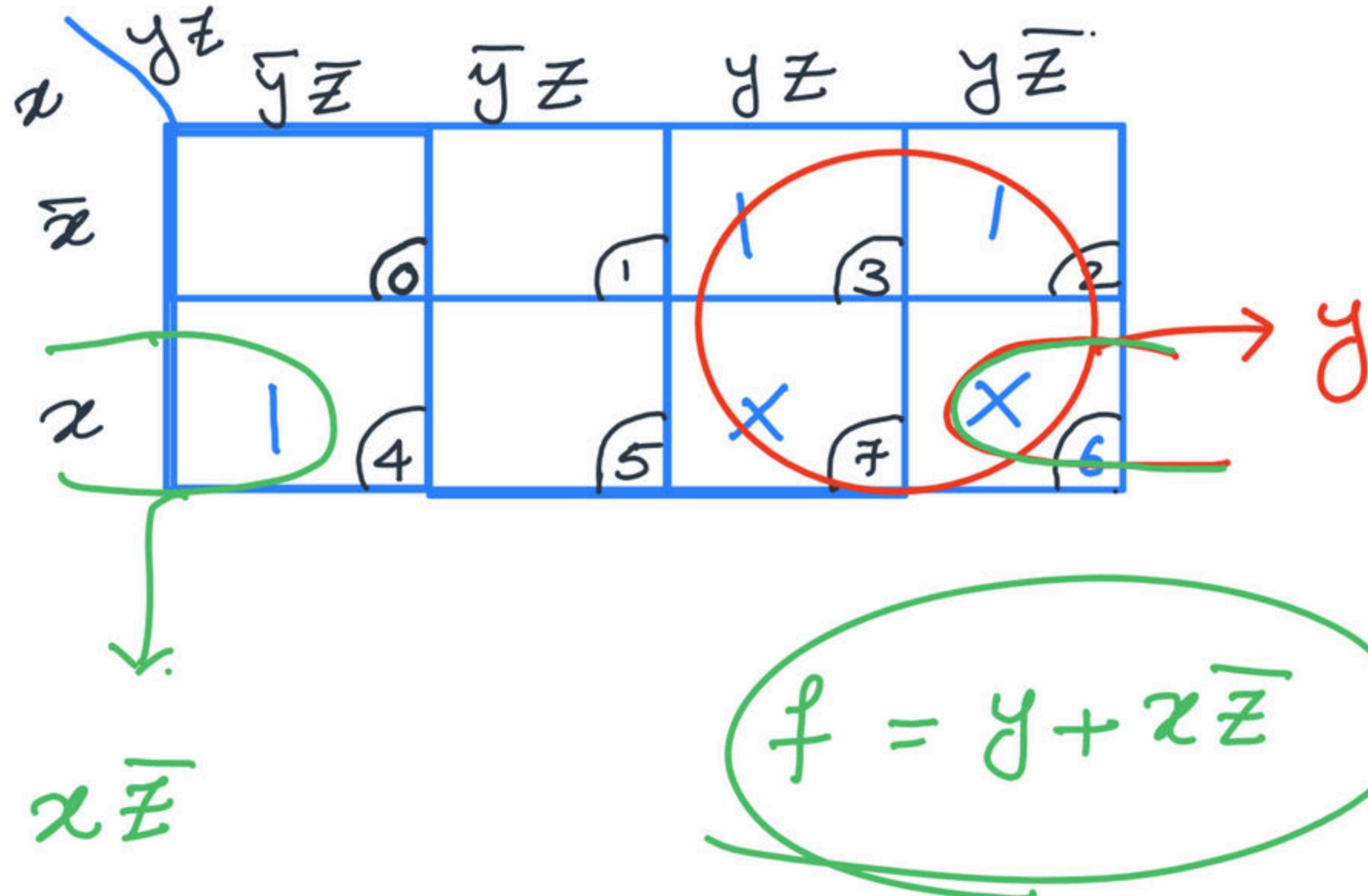
0 1 0 1 0 0

0 1 1

$$f = \sum m(2, 3, 4) + d(6, 7)$$

x	y	z	f
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$$f = \sum m(2, 3, 4) + d(6, 7)$$



$$f = B + AB.$$

$$f = B.$$

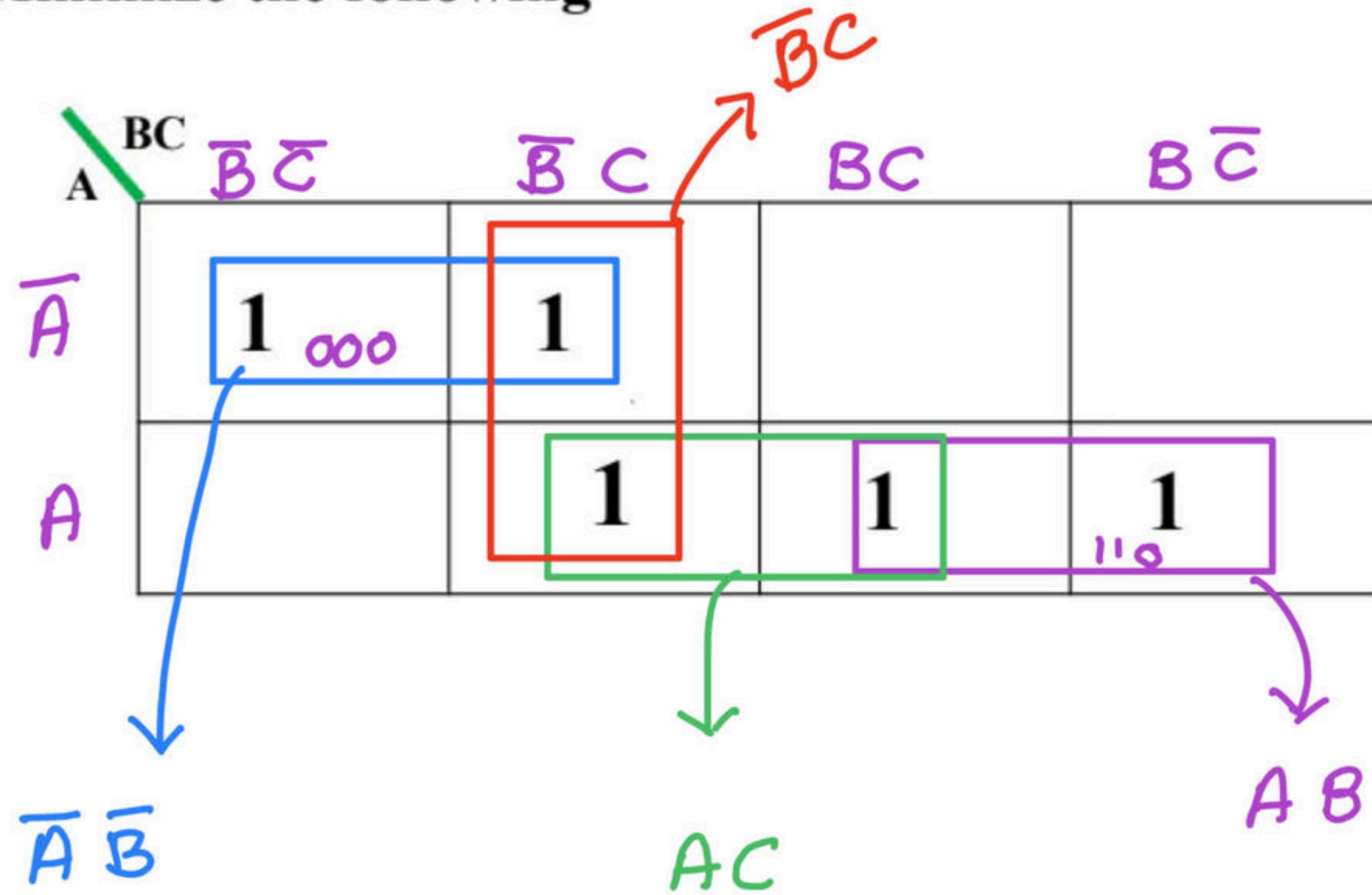
$$2^2 = 2^m$$

$$m = 2$$

$$n = 3$$

$$f = y + x\bar{z}$$

Q) Minimize the following



$$f = \bar{A}\bar{B} + AB + \bar{B}C \text{ (or)}$$

$$f = \bar{A}\bar{B} + AB + AC.$$

$$f = \bar{A}\bar{B} + AB + \bar{B}C$$

No. of literals = 6

$$f = \bar{A}\bar{B} + AB + AC.$$

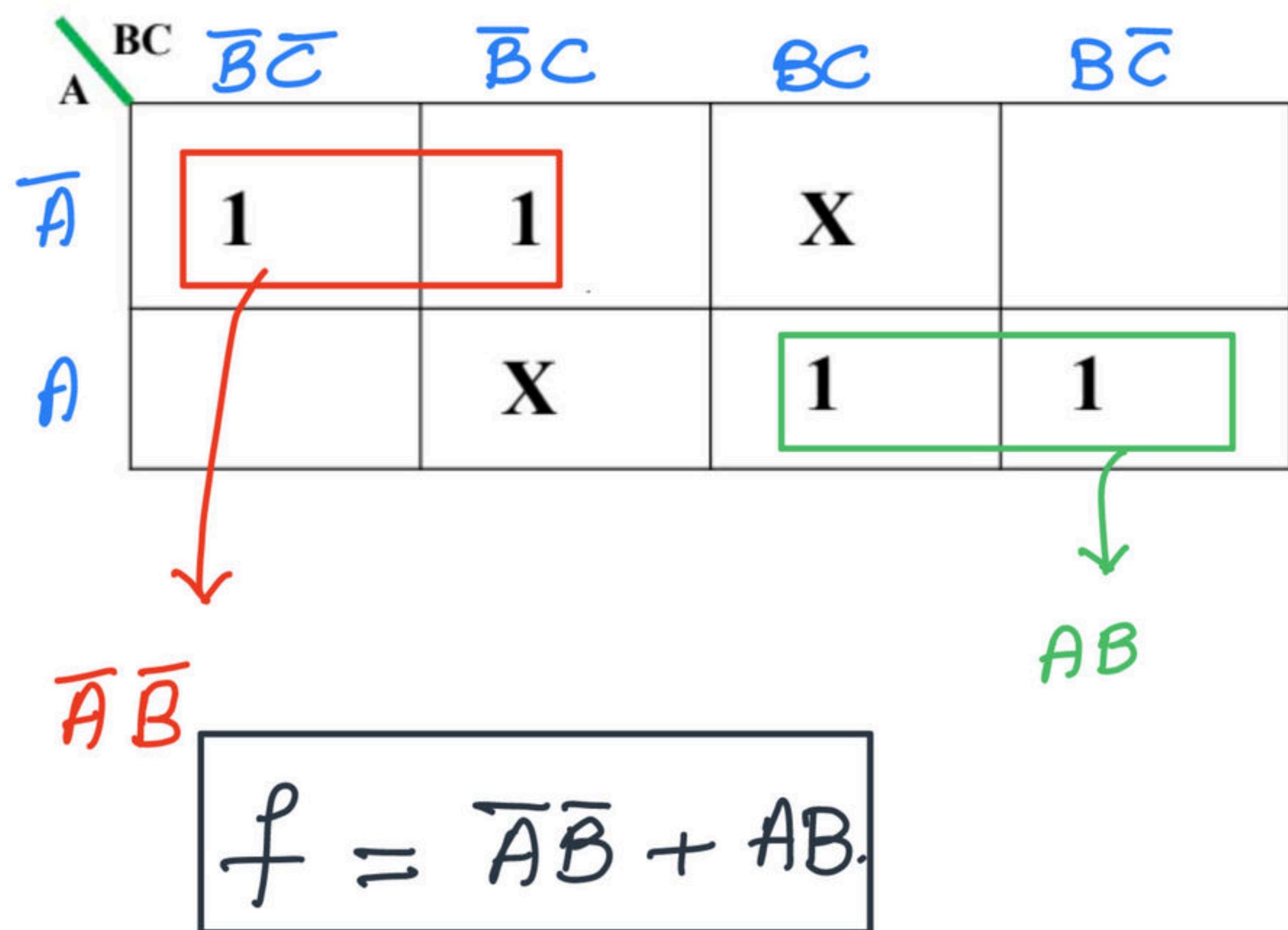
No. of literals = 6.

both

NOTE:

Minimal expression may not be Unique

Q) Minimize the following



if $x = 1$

$$f = \overline{A}\overline{B} + AB + C$$

No. of literals = 5

if $x = 0$

$$f = \overline{A}\overline{B} + AB$$

No. of literals = 4.

Minterm :

Each product term in the canonical SOP expression is called a minterm

Maxterm :

Each sum term in the canonical POS expression is called a maxterm

Implicant , Prime Implicant , Essential Prime Implicant

Implicant : Each minterm in canonical SOP expression is known as Implicant .

Prime Implicant (PI):

Prime Implicant is a product term , obtained by combining maximum possible cells in the K- Map. While doing so make sure that a smaller group is not completely inside a bigger group .

Essential Prime Implicant (EPI) :

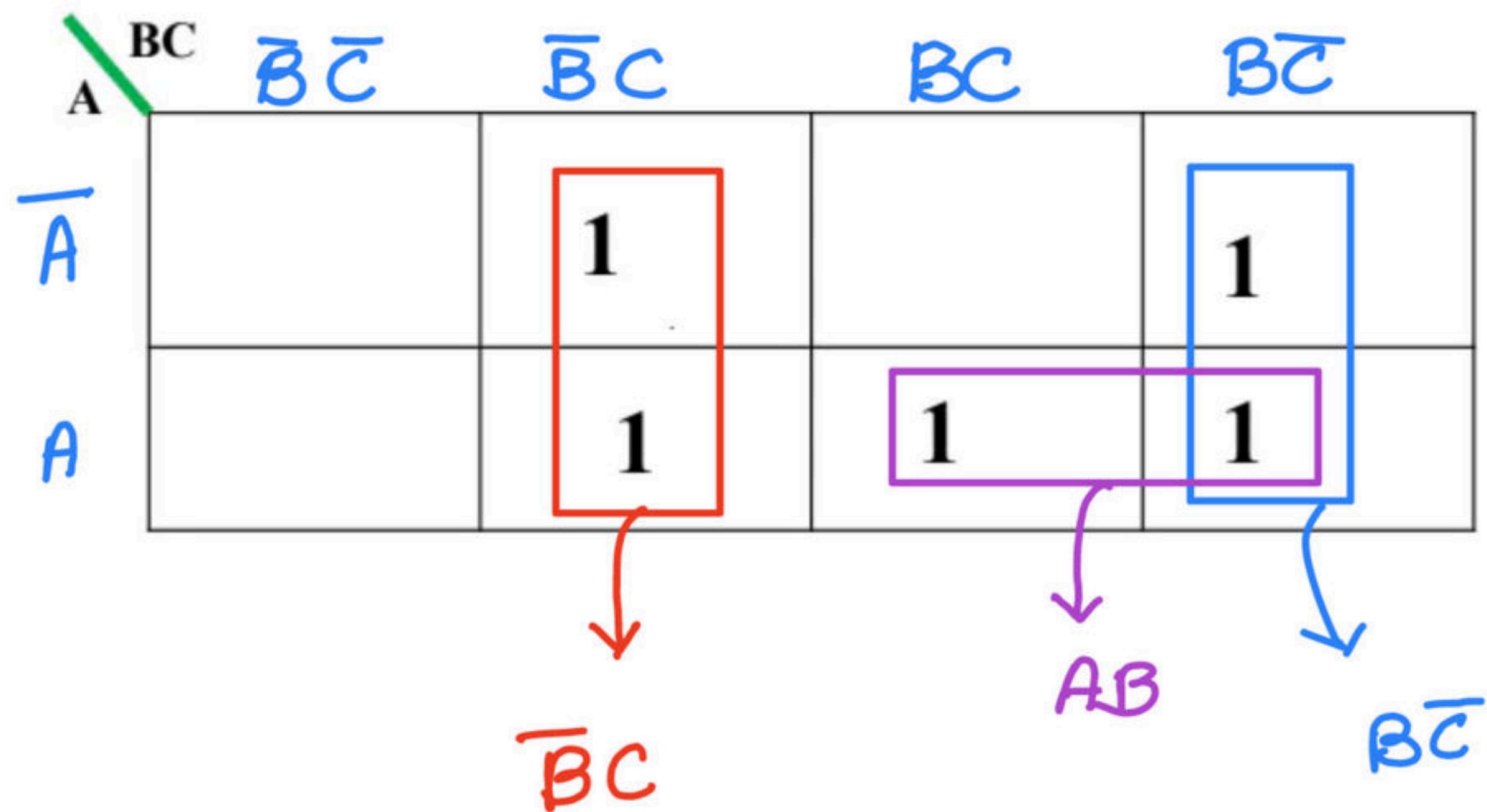
A prime Implicant is an EPI , if and only if it contains at least one minterm which is not covered by multiple groups .

All EPI's are PI's , but vice versa not true

$$\text{EPI} \leq \text{PI}$$

The minimal expression = All EPI's + Optional PI's

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$f = \bar{B}C + B\bar{C} + AC \quad (\text{Gy})$$

$$f = \bar{B}C + B\bar{C} + AB$$

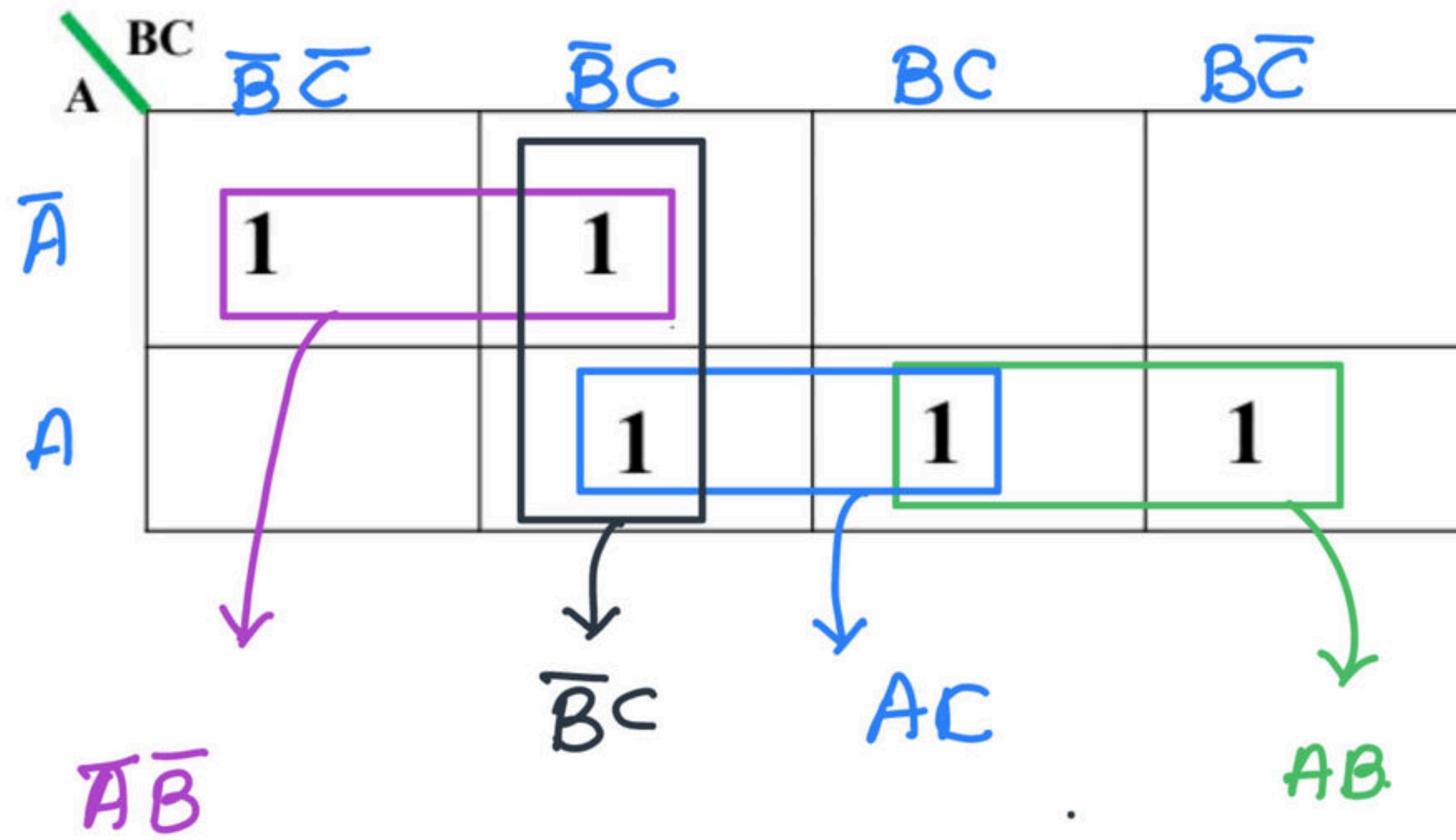
PI = 4

$\bar{B}C$, AC, \cancel{AB} , $\cancel{B\bar{C}}$.

EPI = 2

$\bar{B}C$, $B\bar{C}$

Q) Find the number of Prime Implicants and Essential Prime Implicants



PI = 4
EPI = 2

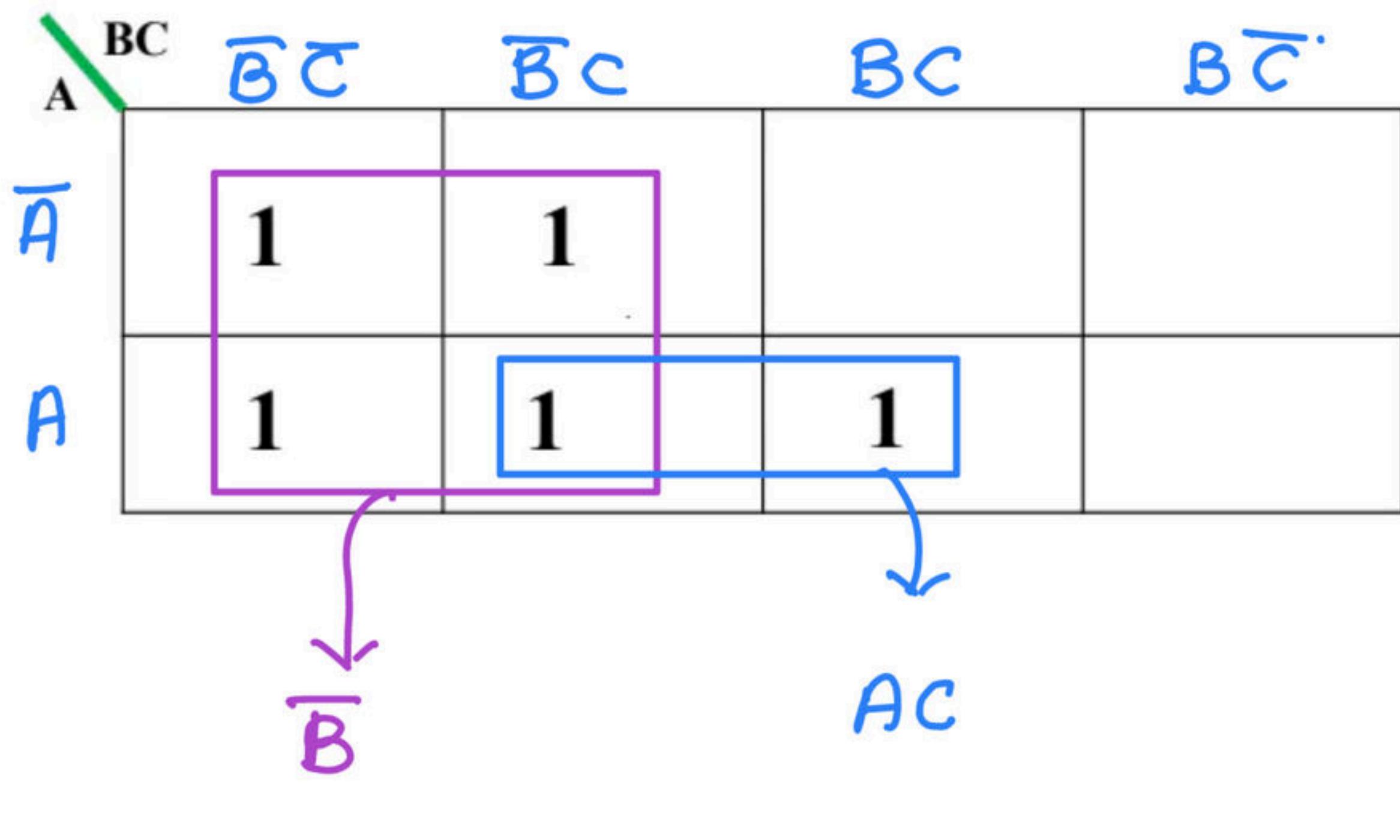
\overline{AB} \overline{BC} AC AB

\overline{AB}, AB

$\checkmark f = \overline{AB} + AB + AC \text{ } (\alpha)$

$\checkmark f = \overline{AB} + AB + \overline{BC}$.

Q) Find the number of Prime Implicants and Essential Prime Implicants



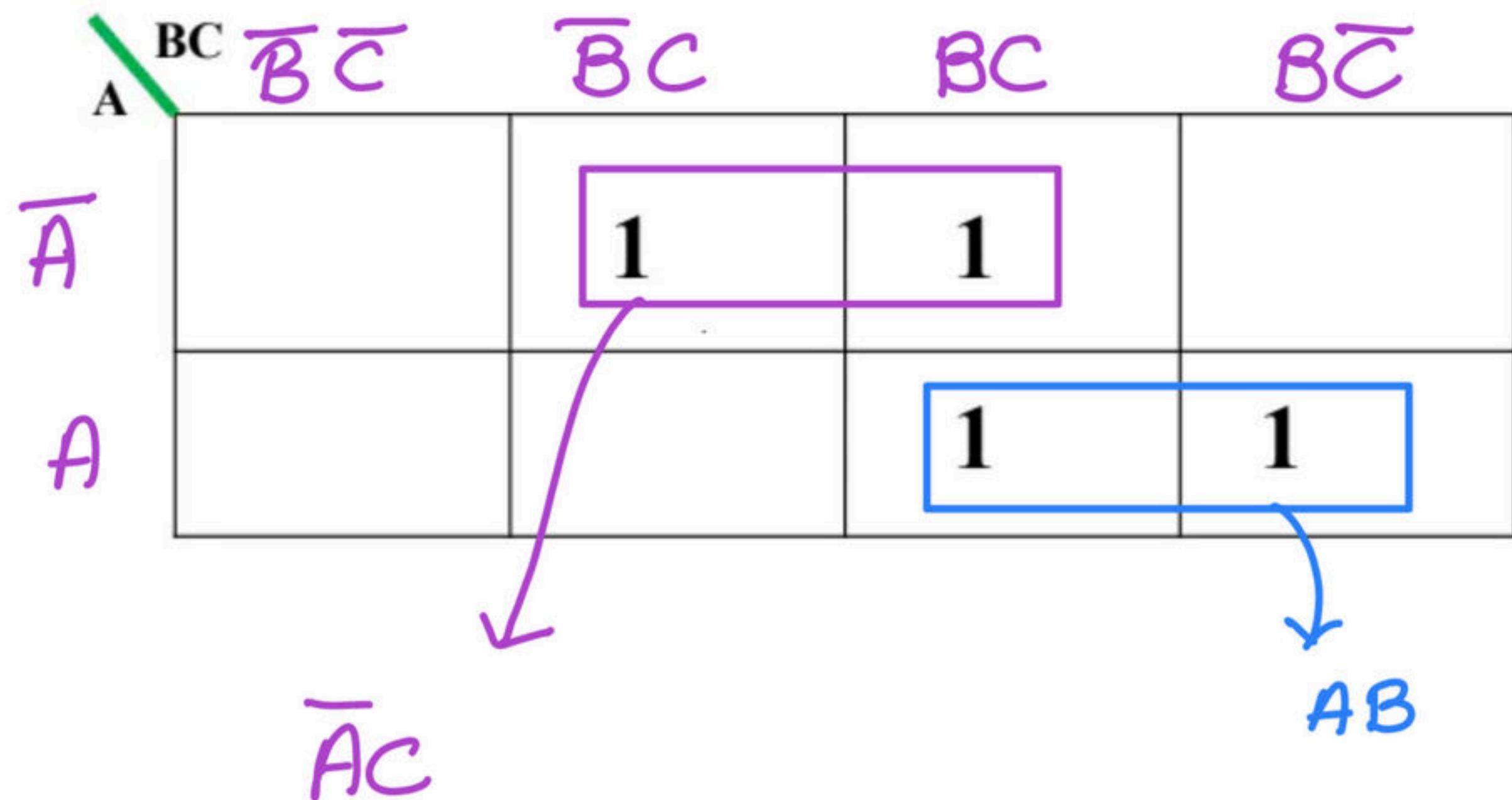
PI = 2

EPI = 2

\bar{B} , AC

$$f = \underline{\bar{B}} + AC.$$

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$f = \bar{A}C + AB + 0$$

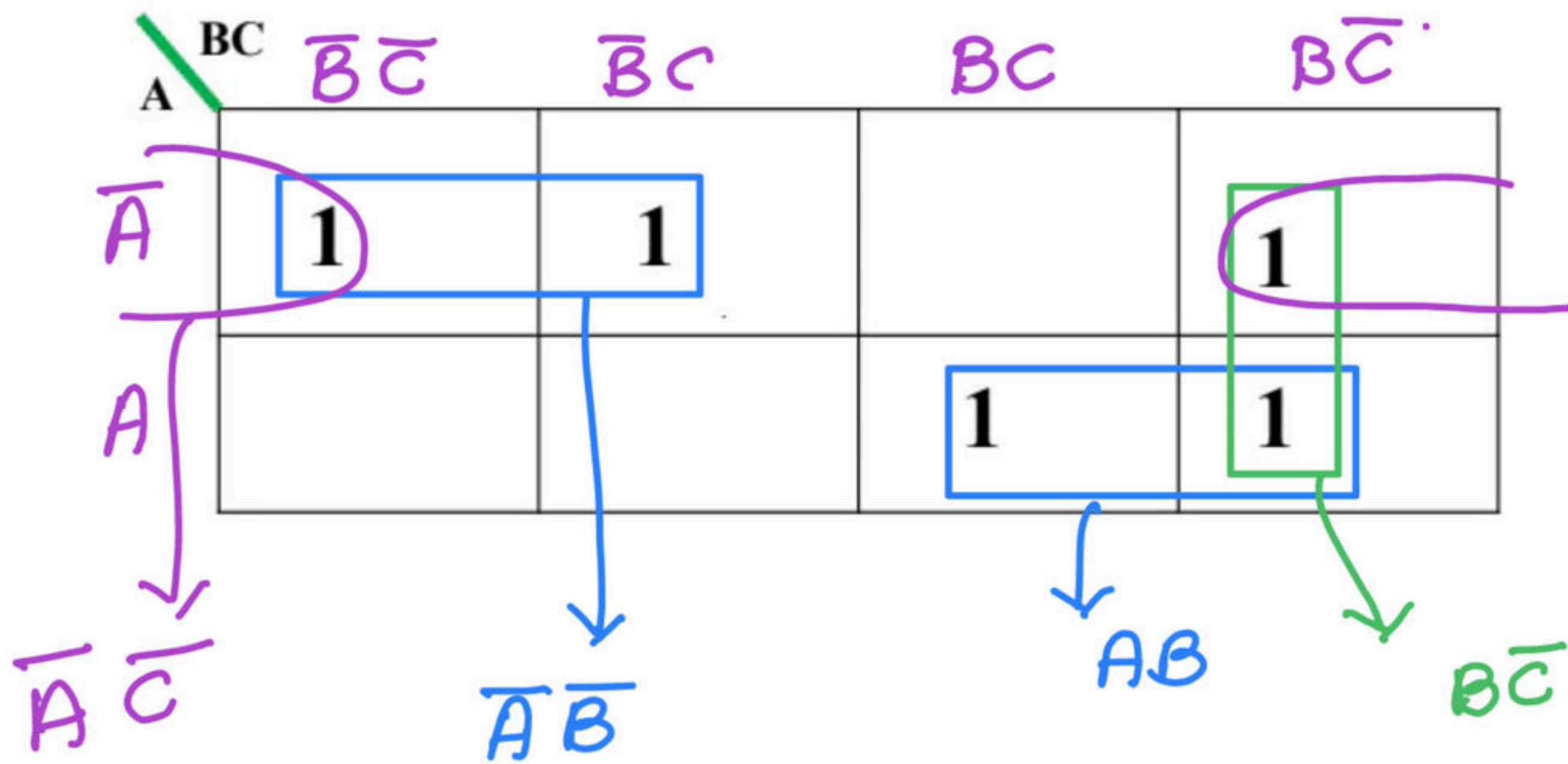
PI = 3

$\bar{A}C$, $\frac{BC}{\downarrow}$, AB

EPI = 2

$\bar{A}C$, AB

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$\underline{\text{PI}} = 4$$

\overline{AC} $\overline{A}\overline{B}$ AB $B\overline{C}$

$$\underline{\text{EPI}} = 2$$

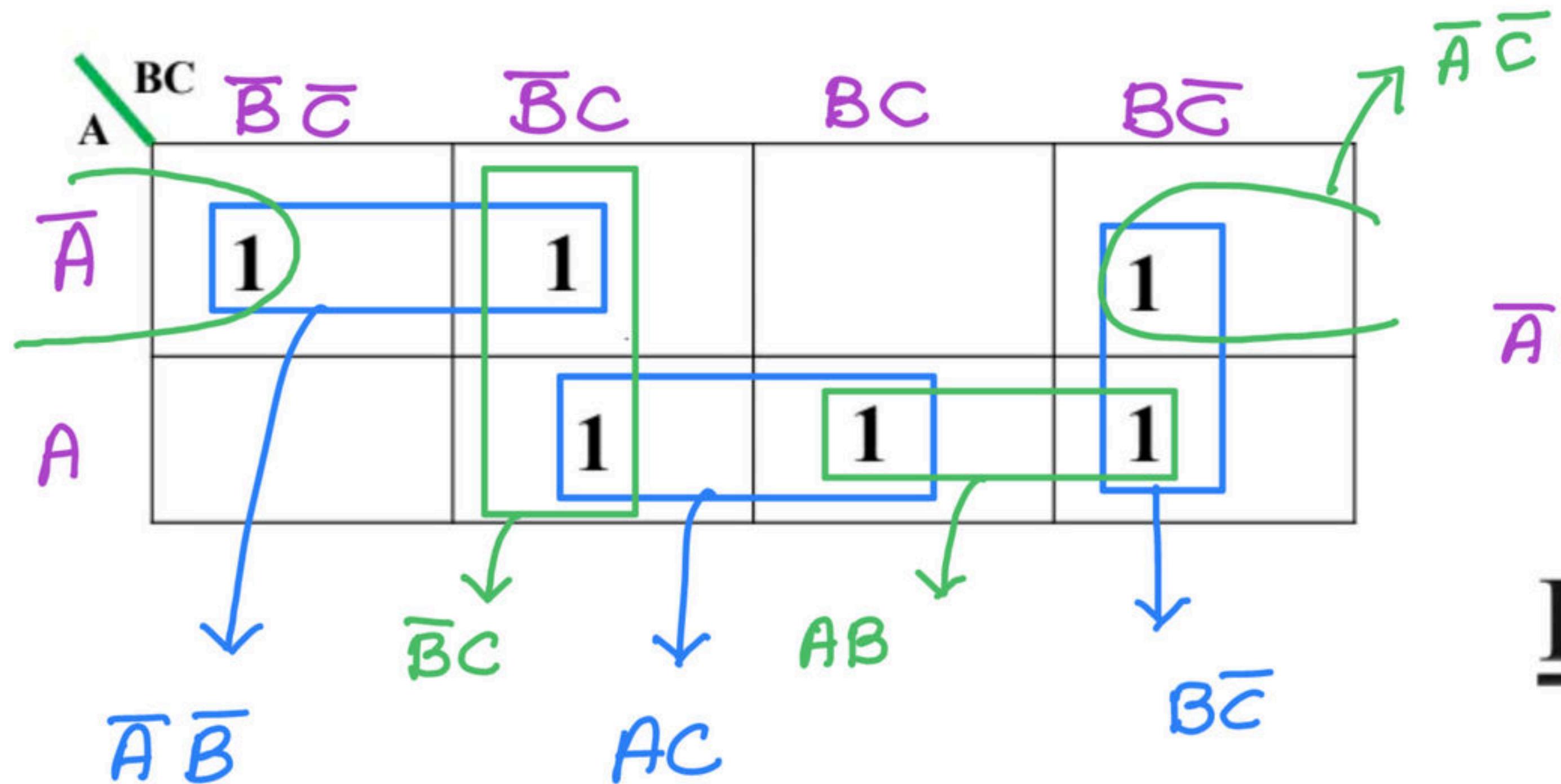
$\overline{A}\overline{B}$, AB

$$f = \overline{A}\overline{B} + AB + B\overline{C}$$

(or)

$$f = \overline{A}\overline{B} + AB + \overline{A}\overline{C}$$

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$\underline{\text{PI}} = 6$$

$\bar{A}\bar{B}$ $\bar{B}C$ AC AB $B\bar{C}$ $\bar{A}\bar{C}$

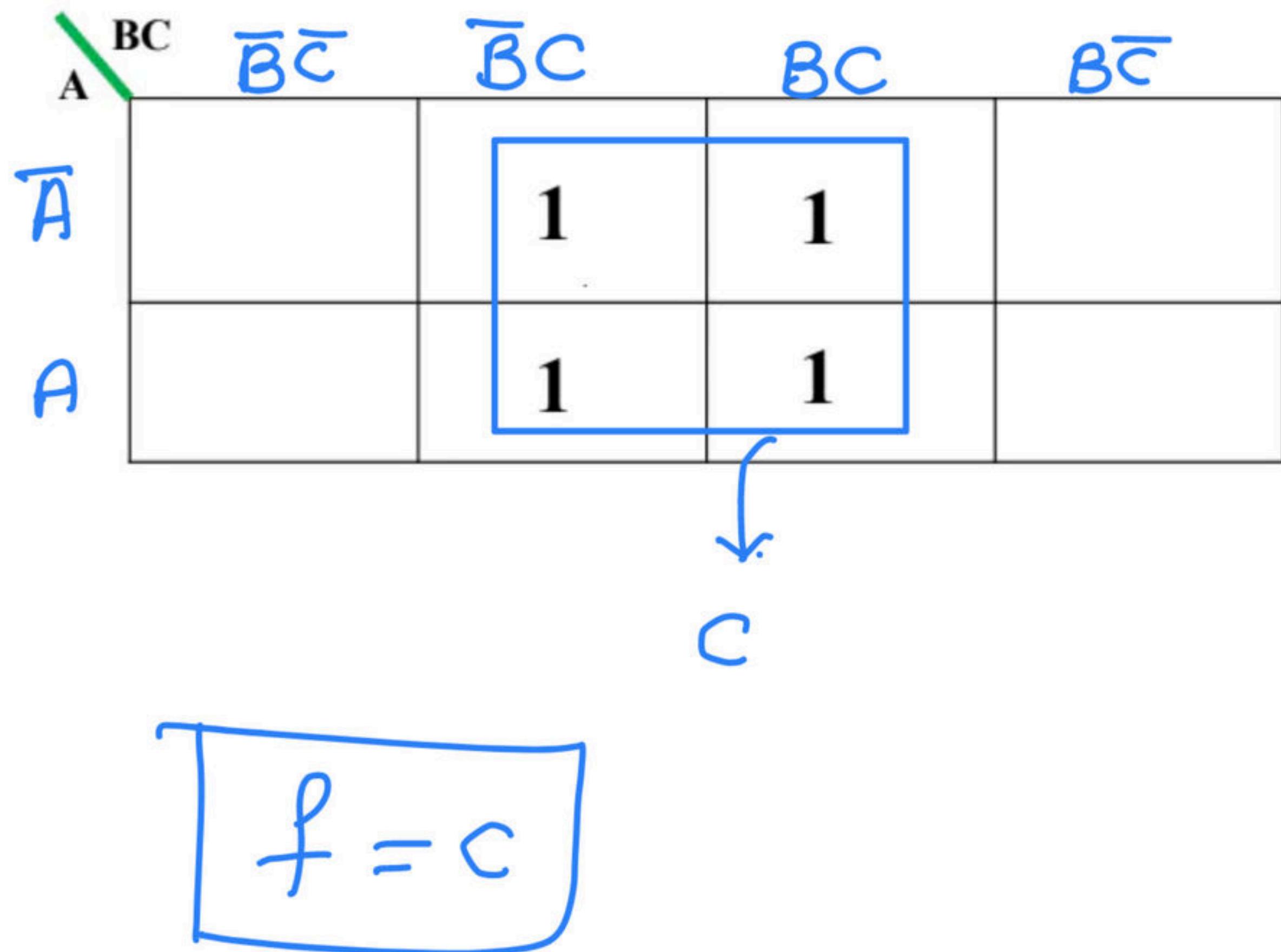
$$\underline{\text{EPI}} = 0$$

$$f = 0 + \bar{A}\bar{B} + AC + B\bar{C}$$

(or)

$$f = 0 + \bar{B}C + AB + \bar{A}\bar{C}$$

Q) Find the number of Prime Implicants and Essential Prime Implicants



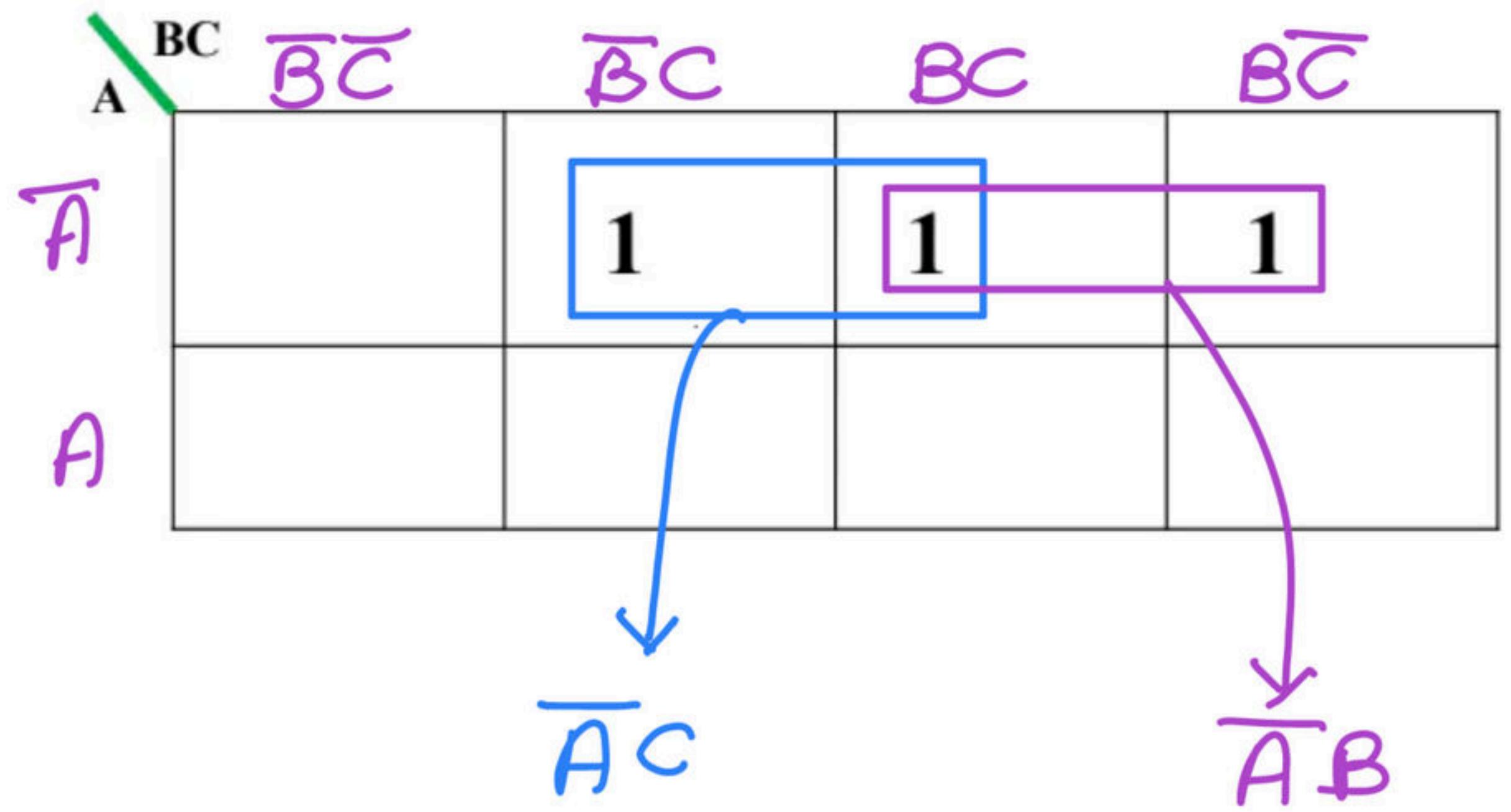
$$\underline{\text{PI}} = 1$$

C

$$\underline{\text{EPI}} = 1$$

C .

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$f = \overline{AC} + \overline{AB}$$

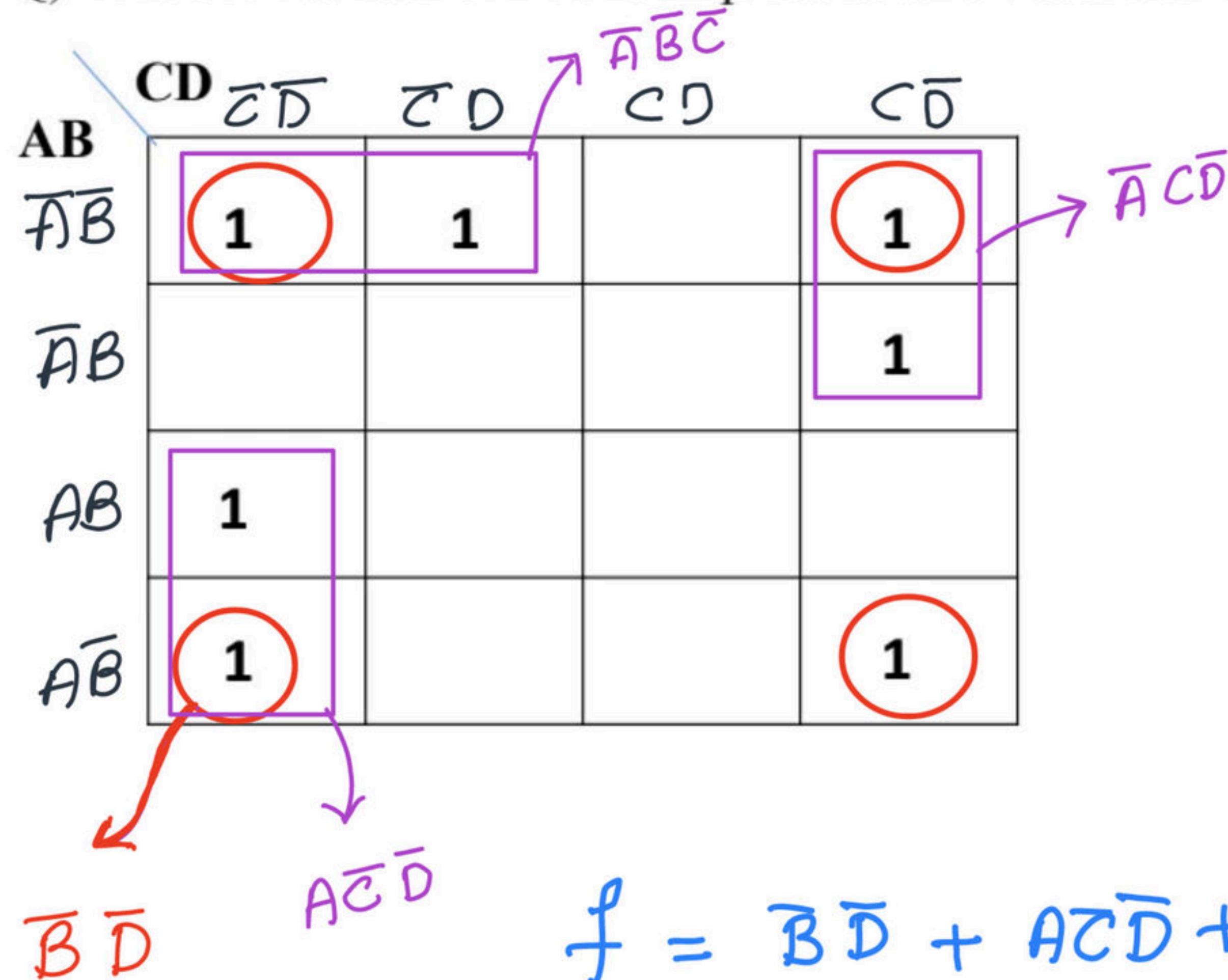
PI = 2

\overline{AC} \overline{AB}

EPI = 2

$\overline{AC} . \overline{AB}$

Q) Find the number of Prime Implicants and Essential Prime Implicants



$$\underline{\text{PI}} = 4$$

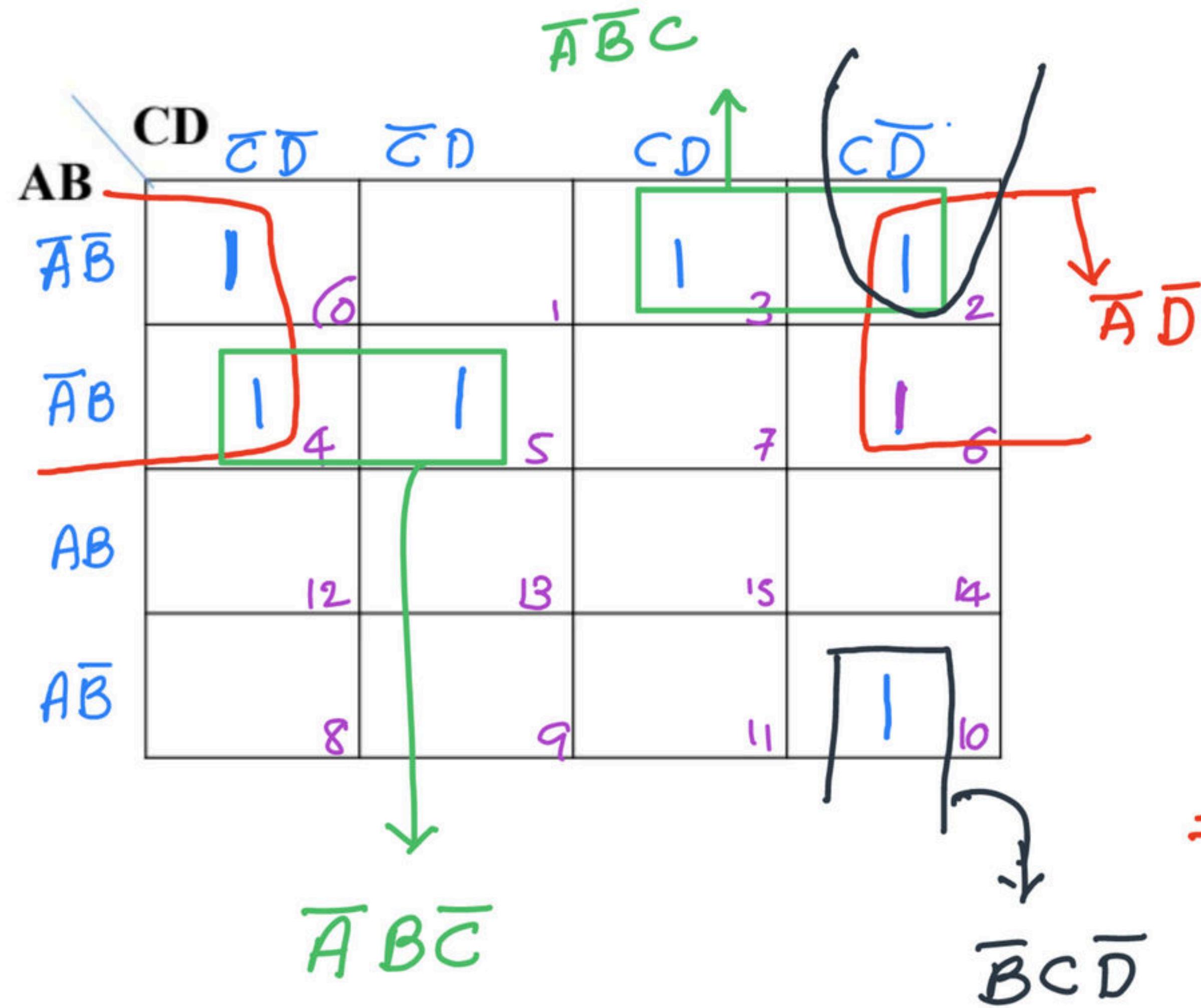
$\bar{B}\bar{D}, A\bar{C}\bar{D}, \bar{A}\bar{B}\bar{C}, \bar{A}C\bar{D}$

$$\underline{\text{EPI}} = 4$$

$\bar{B}\bar{D}, A\bar{C}\bar{D}, \bar{A}\bar{B}\bar{C}, \bar{A}C\bar{D}$

$$f = \bar{B}\bar{D} + A\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}C\bar{D}$$

$$Q) F(A, B, C, D) = \sum m(0, 2, 3, 4, 5, 6, 10)$$



$$PI = 4$$

$$\overline{AB}\overline{C}, \overline{BC}\overline{D}, \overline{A}\overline{B}C, \overline{A}\overline{D}$$

$$EPI = 4$$

$$\overline{AB}\overline{C}, \overline{BC}\overline{D}, \overline{A}\overline{B}C, \overline{A}\overline{D}$$

$$f = \overline{AB}\overline{C} + \overline{BC}\overline{D} + \overline{A}\overline{B}C + \overline{A}\overline{D}$$

$$\overline{BC}\overline{D}$$

NOTE:

- The minimal expression = (All EPI's) + (Optional PI's)
- If all PI's are EPI's , then the minimal expression is unique
- The sufficient condition for a K-map to have unique solution is

~~The number of EPI's = Number of PI's~~

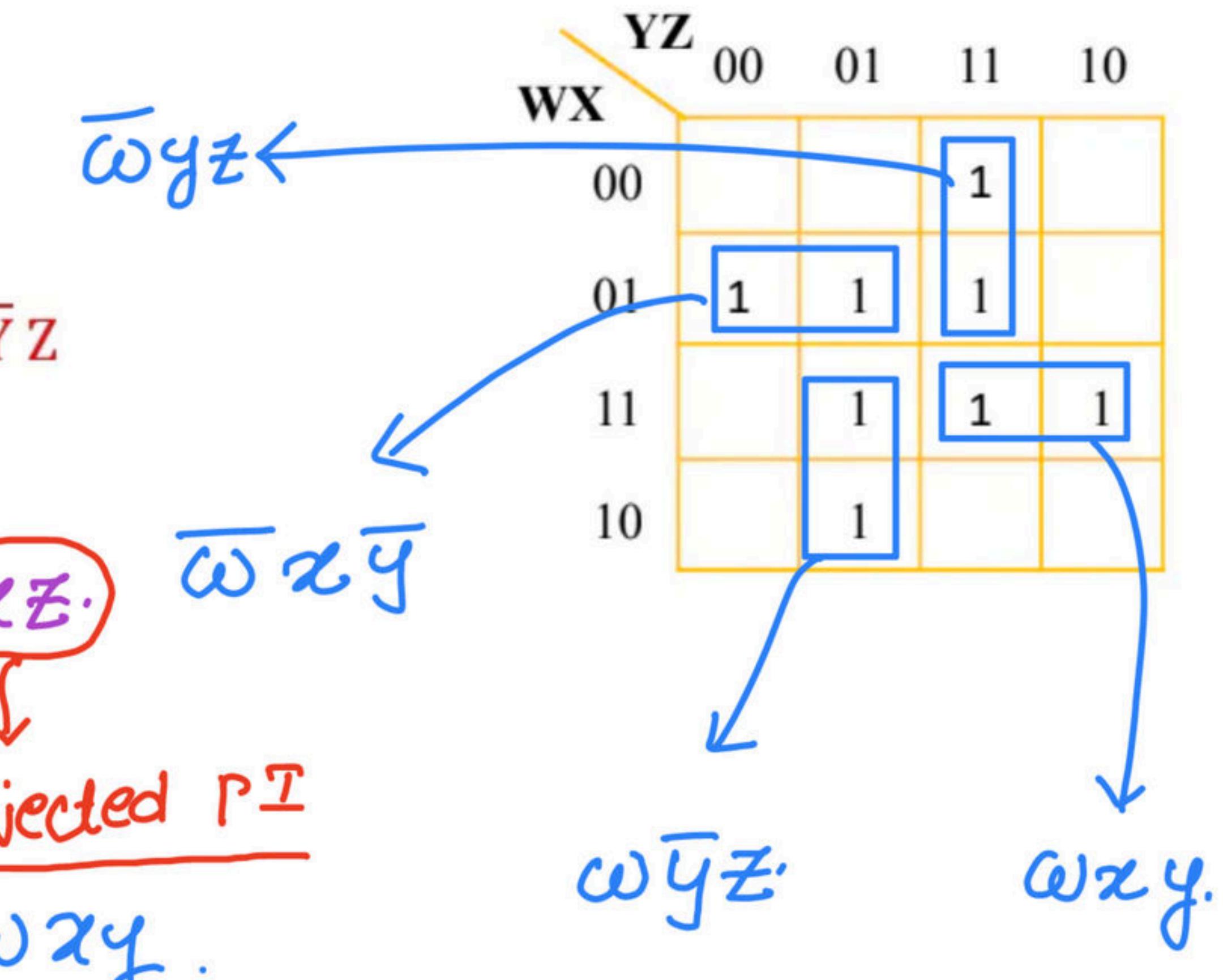
Number of EPI's = Number of PI's

Q. What is the minimized logic expression corresponding to the given Karnaugh-map?

- (a) XZ
- ~~(b) $\bar{W}X\bar{Y} + \bar{W}YZ + W\bar{Y}Z + WX\bar{Y}$~~
- (c) $\bar{W}X\bar{Y} + \bar{W}YZ + W\bar{Y}\bar{Z} + WX\bar{Y}$
- (d) $XZ + \bar{W}YZ + \bar{W}X\bar{Y} + WX\bar{Y} + W\bar{Y}Z$

$$\underline{PT = 5}$$

$\bar{W}YZ, \bar{W}X\bar{Y}, W\bar{Y}Z, WXY, XZ$



$$\underline{EPI = 4.}$$

$\bar{W}YZ, \bar{W}X\bar{Y}, W\bar{Y}Z, WXY$

$$f = \bar{W}YZ + \bar{W}X\bar{Y} + W\bar{Y}Z + WXY + 0$$

Rejected PT

$$Q) F(A, B, C, D, E) = \sum m(0, 2, 4, 7, 10, 12, 13, 18, 23, 26, 28, 29)$$

Q. How many min terms (excluding redundant terms) does the minimal switching function

$f(v, w, x, y, z) = x + \bar{y}z$ originally have

- (a) 16
- (b) 20
- (c) 24
- (d) 32

Q. Consider the Karnaugh map given below:

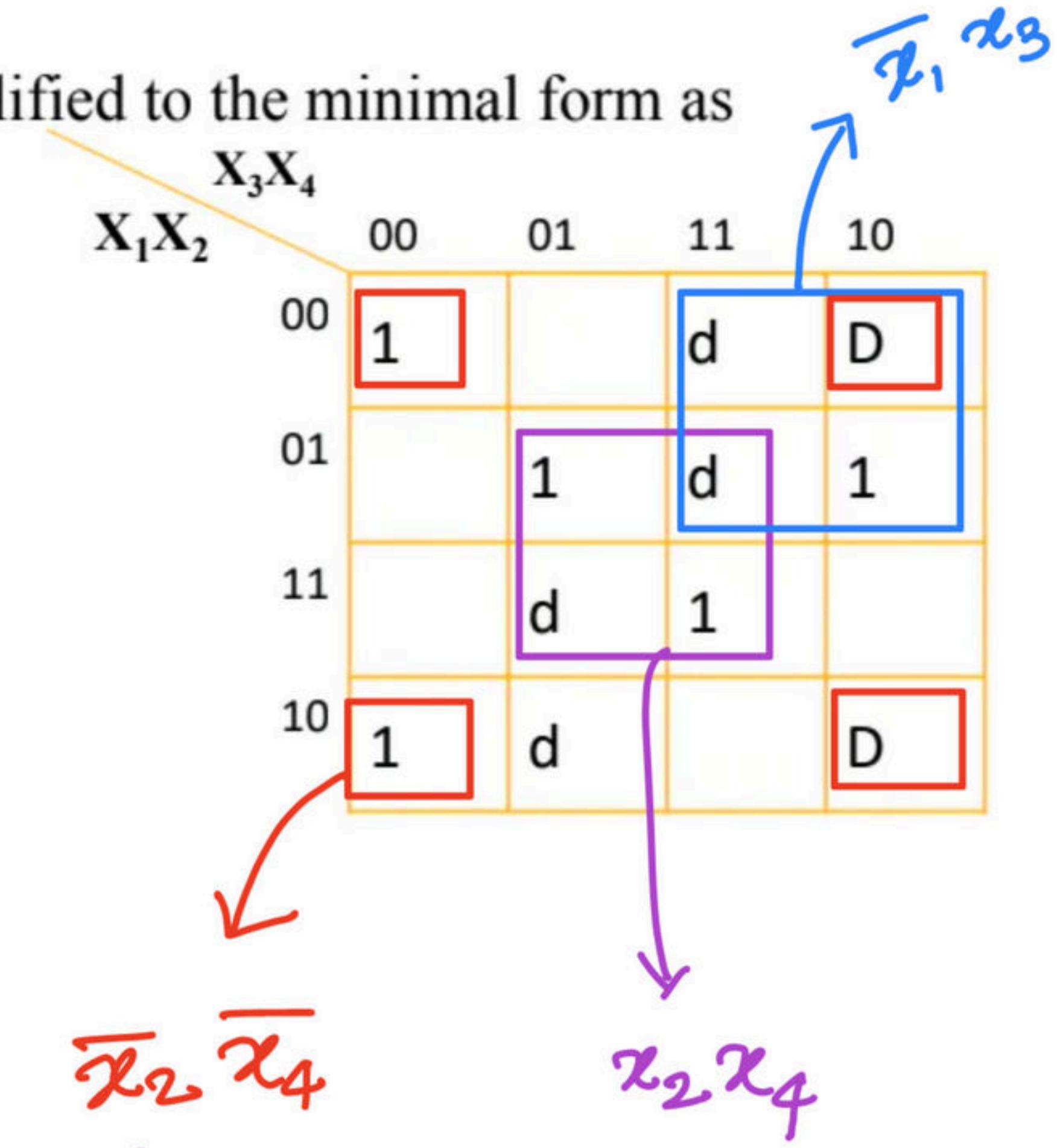
The function represented by this map can be simplified to the minimal form as

- (a) $X_1 \bar{X}_2 \bar{X}_4 + X_2 X_4 + X_1 \bar{X}_3$
- (b) $X_1 X_2 X_4 + X_2 X_4 + X_1 \bar{X}_2 \bar{X}_3 \bar{X}_4$
- (c) ~~$\cancel{X}_2 X_4 + \bar{X}_2 \bar{X}_4 + X_3 \bar{X}_1$~~
- (d) $X_1 \bar{X}_2 \bar{X}_4 + \bar{X}_1 X_2 \bar{X}_3 X_4 + X_1 X_2$

$$\underline{PI = 3} \cdot \underline{EPIT = 3}$$

$\bar{x}_2 \bar{x}_4$, $x_2 x_4$ $\bar{x}_1 \bar{x}_3$

$$f = \bar{x}_2 \bar{x}_4 + x_2 x_4 + \bar{x}_1 \bar{x}_3$$



Q. Find the number of EPI's in the following function F

$$F(W,X,Y,Z) = \sum m(0, 2, 5, 7, 8, 10, 12, 14, 15)$$

- (a) 5
 - (b) 4
 - (c) 3
 - (d) 2

Q. For the following function $F(A, B, C, D) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$ the number of essential prime implicants and non-essential prime implicants respectively are?

- (a) 1,3
- (b) 2,2
- (c) 3, 1
- (d) 2,3

Q. Consider the Boolean function, $F(w, x, y, z) = wy + xy + \overline{wxyz} + \overline{wxy} + xz + \overline{x}yz$. Which one of the following is the complete set of essential prime implicants?

- (a) $w, \overline{y}, xz, \overline{xz}$
- (b) w, y, xz
- (c) y, \overline{xyz}
- (d) y, xz, \overline{xz}

Q. The number of essential prime Implicants of the given minterms $F(A,B,C.,D) = \sum m(0,1,4,5,8,11,12,13,15)$ is _____.

Q. The minimized SOP expression of the given minterms
 $F(A, B, C, D) = \sum m(0, 1, 4, 5, 7, 8, 9, 12, 13, 14, 15)$ is

- (a) $F = \bar{C} + AB + BD$
- (b) $F = \bar{C} + AB$
- (c) $F = (A + D) \cdot B$
- (d) $F = \bar{C} + BD$

Q. Given the Boolean function F in the three variables A,B and C as $F(A,B,C) = \sum m(2,5,7)$ then the minimized POS expression is

(a) $F = (\bar{A} + C)(A + B)(A + \bar{C})$

(c) Both (a) and (b)

(b) $F = (\bar{A} + C)(B + C)(A + \bar{C})$

(d) None

Q. K-map of a Boolean function is given below, after simplification choose the correct option.

		CD	00	01	11	10
		AB	00	01	11	10
AB	00	1	1	1	1	
	01	1	1	1	1	
	11	1	1	0	1	
	10	1	1	1	1	

(a) $\bar{A} + \bar{B}$

(c) $\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{A}\bar{C}$

(b) $\bar{A} + \bar{B} + \bar{C}$

(d) $\bar{A} + \bar{B} + \bar{C} + \bar{D}$

Q. Simplify $f(A,B,C,D) = \sum m(0,1,4,5,9,11,14,15) + \sum d(10,13)$

(a) $\overline{AC} + AC$

(c) $\overline{AC} + \overline{BC} + \overline{CD}$

(b) $\overline{AC} + \overline{D}$

(d) $\overline{AC} + AC + \overline{CD}$

Q. For the k -map shown below, the minimized logical expression in SOP form is

		CD	
		00	01
AB	00	1	1
	01		
11		1	1
10	1	1	1

- a. $\bar{A}\bar{B}\bar{C} + AC + \bar{B}CD + \bar{A}\bar{B}\bar{D}$
- b. $\bar{A}\bar{B}\bar{C} + \bar{B}\bar{D} + AC$
- c. $\bar{A}\bar{B}\bar{C} + AC + \bar{A}\bar{B}CD + A\bar{B}\bar{C}D$
- d. $\bar{A}\bar{B}\bar{C} + AC + \bar{A}\bar{B}\bar{D}$

Q. Consider a function $F = A \odot B \odot C \odot D$. The total number of essential prime implicants of the given function will be _____.

Q. A logical function is given as $F(A, B, C, D) = \Sigma m (0, 4, 5, 10, 11, 13, 15)$.

The number of Essential Prime Implicants in the minimized expression will be

_____.

Which of the following logic circuit is realization of the function F whose Karnaugh map is as shown below

		AB	00	01	11	10
		C	0	1	1	
		1	1			1
0	1					

- (A)
-
- ```
graph LR; A((A)) -->|not| AB[AB]; AB -->|and| ABC[ABC]; ABC -->|and| AF[AF]; AF --> F((F))
```
- (B)
- 
- ```
graph LR; B((B)) -->|not| BC[BC]; BC -->|and| CAB[CAB]; CAB -->|and| CF[CF]; CF --> F((F))
```
- (C)
-
- ```
graph LR; A((A)) -->|and| AC[AC]; AC -->|and| ABC[ABC]; ABC -->|and| AF[AF]; AF --> F((F))
```
- (D)
- 
- ```
graph LR; A((A)) -->|and| AB[AB]; AB -->|and| ABC[ABC]; ABC -->|and| AF[AF]; AF --> F((F))
```

The total number of possible minimal expressions for the four variable Boolean function:

$$F(A,B,C,D) = \sum m(0,2,3,5,7,8,9,10,11,13,15) \text{ is } \underline{\hspace{2cm}}$$

Simplified form of the Boolean function

$F(A, B, C, D, E) = \Sigma m(0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31)$ is

(A) $F = \bar{A}\bar{B}\bar{E} + \bar{B}\bar{D}\bar{E} + ADE$

(B) $F = \bar{A}\bar{B}\bar{C} + B\bar{D}E + ACE$

(C) $F = \bar{A}\bar{B}\bar{E} + BCE + ACE$

(D) $F = \bar{A}\bar{B}\bar{E} + B\bar{D}E + ACE$

A logic circuit implements the following Boolean function

$F(W, X, Y, Z) = \Sigma m(2, 3, 6, 7, 8, 12)$ It is found that the input combination for which $W = Y$ can never occur. Then minimal expression for F is

(A) $WY + \bar{Y}Z$

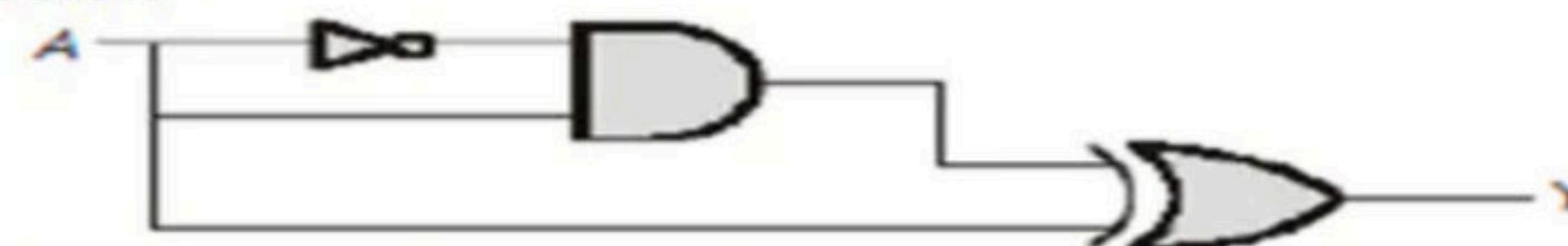
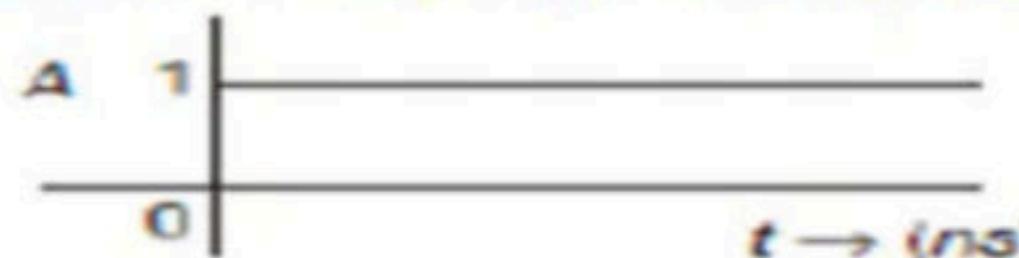
(B) $\bar{Y} + Z$

(C) $W\bar{Y} + Z$

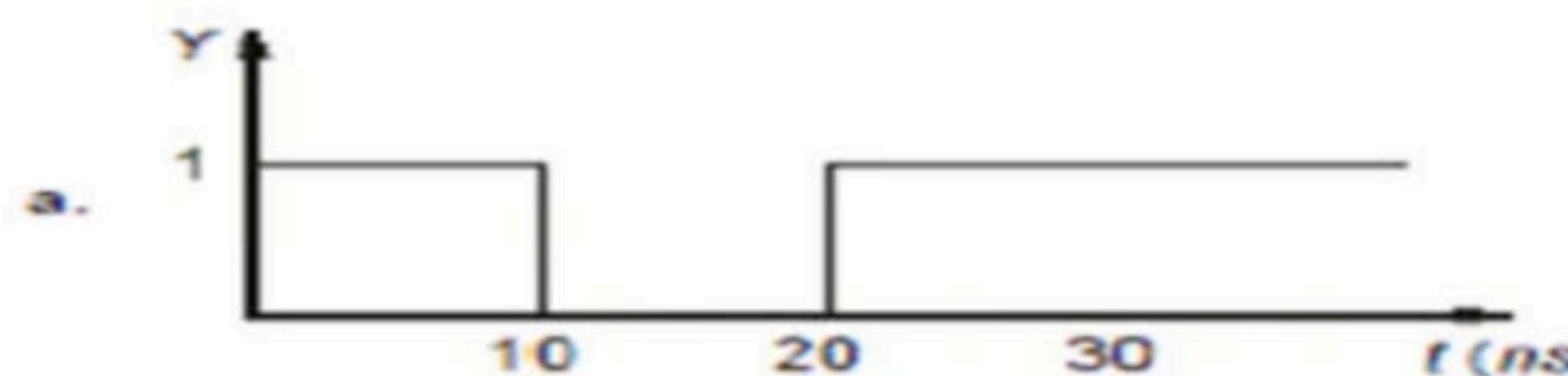
(D) $Y + \bar{Z}$

Answer : D

Consider the circuit shown in figure below



If propagation delay of NOT gate is 10 nsec, AND gate is 20 nsec and X -OR gate is 10 nsec. If A is connected to VCC at $t = 0$, then waveform for output Y is



Five soldiers **A, B, C, D** and **E** volunteer to perform an important military task if their following conditions are satisfied

- (i) either **A** or **B** or both must go
- (ii) either **C** or **E** but both must not go
- (iii) either both **A** and **C** go or neither goes
- (iv) If '**D**' goes, then '**E**' must also go
- (v) If '**B**' goes, then **A** and **C** must also go

The minimal combination of soldiers who can get the arrangement will be

Ans : $AC\bar{D}\bar{E}$

How many Boolean functions of the type
 $f(x, y, z) = f(\bar{x}, \bar{y}, \bar{z})$ are available with
three variables ?

- (a) 4
- (b) 8
- (c) 32
- (d) 16

The logic expression $Y = \Sigma m (0, 3, 6, 7, 10, 12, 15)$ is equivalent to

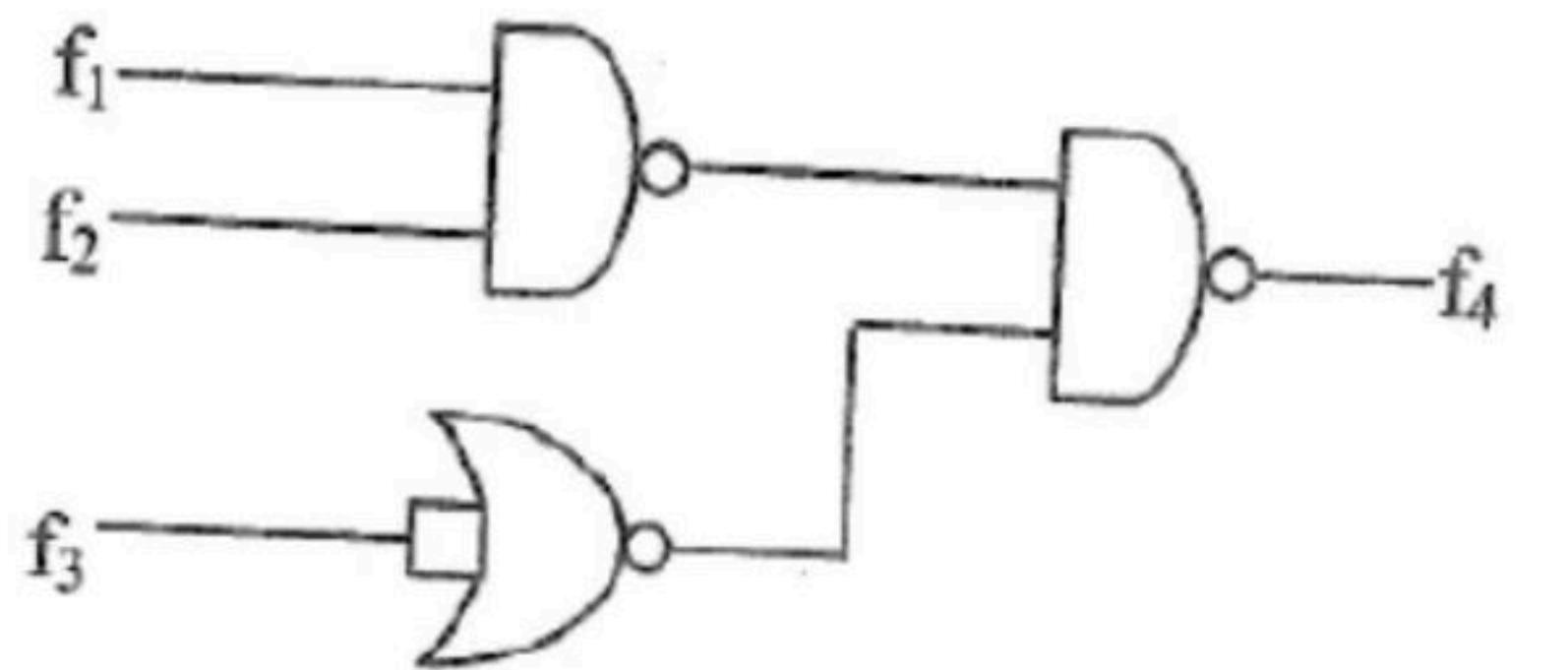
- (a) $Y = \prod M (0, 3, 6, 7, 10, 12, 15)$
- (b) $Y = \prod M (1, 2, 4, 5, 8, 9, 11, 13, 14)$
- (c) $Y = \Sigma m (1, 2, 4, 5, 8, 9, 11, 13, 14)$
- (d) $Y = \Sigma m (3, 0, 10, 12)$

Pick up correct statements from the following:

1. In an n - variable K - map combining 16 adjacent cells containing 1's as a single group will generate a prime implicant which is independent of four variables.
2. D - latch cannot be implemented with multiplexer circuit.
3. Lockout problem occurs only in ripple counters.
4. The minimum size of ROM required to implement 3 - bit binary to gray / gray to binary code converter is $2^4 \times 3$.
5. Minimum number of NOR gates (2 - input) required to implement half subtractor circuit are five.

- (a) 1, 2, 3 (b) 2, 3, 4
(c) 1, 4, 5 (d) 3, 4, 5

Consider the logic circuit shown in figure.



The functions f_1 , f_2 and f_4 are

$$f_1(w, x, y, z) = \sum m(8, 9, 10)$$

$$f_2(w, x, y, z) = \sum m(7, 8, 12, 13, 14, 15)$$

$$f_4(w, x, y, z) = \sum m(8, 9)$$

Then $f_3(w, x, y, z)$ is

- | | |
|----------------------|-------------------------|
| a) $\sum m(9, 10)$ | (b) $\sum m(9)$ |
| c) $\sum m(1, 8, 9)$ | (d) $\sum m(8, 10, 15)$ |

How many minimum number of NAND gates are required to implement the following Boolean equation, by taking inputs as A, B, C, D, E, F

$$F = \overline{A}C + \overline{D}F + \overline{B}C + \overline{E}F$$

- (a) 4
- (b) 5
- (c) 6
- (d) 7

Which of the following statements are correct?

1. The Dual of i^{th} minterms is maxterm $(2^n - i - 1)$.
 2. Sum of all minterms is always 0.
 3. Minterms and its corresponding Maxterms form a complement pair.
 4. Minterms are standard product terms.
- | | |
|-------------|-------------|
| (a) 1, 3, 4 | (b) 3, 4 |
| (c) 2, 3, 4 | (d) 1, 2, 4 |

Match List I with List II and select the correct answer using the codes given below the lists:

List - I

- A. Boolean Algebra
- B. K-Map
- C. Clock
- D. Parity

List - II

- 1. Minimization
- 2. Synchronous circuits
- 3. Error correction
- 4. DeMorgan's theorem

Codes:

A	B	C	D
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- (a) 4 1 2 3
- (b) 4 3 2 1
- (c) 3 4 2 1
- (d) 4 2 1 3