Page Replacement and Frame Allocation

Comprehensive Course on Operating System for GATE - 2024/25

Vishvadeep Gothi • Lesson 33 • May 26, 2023

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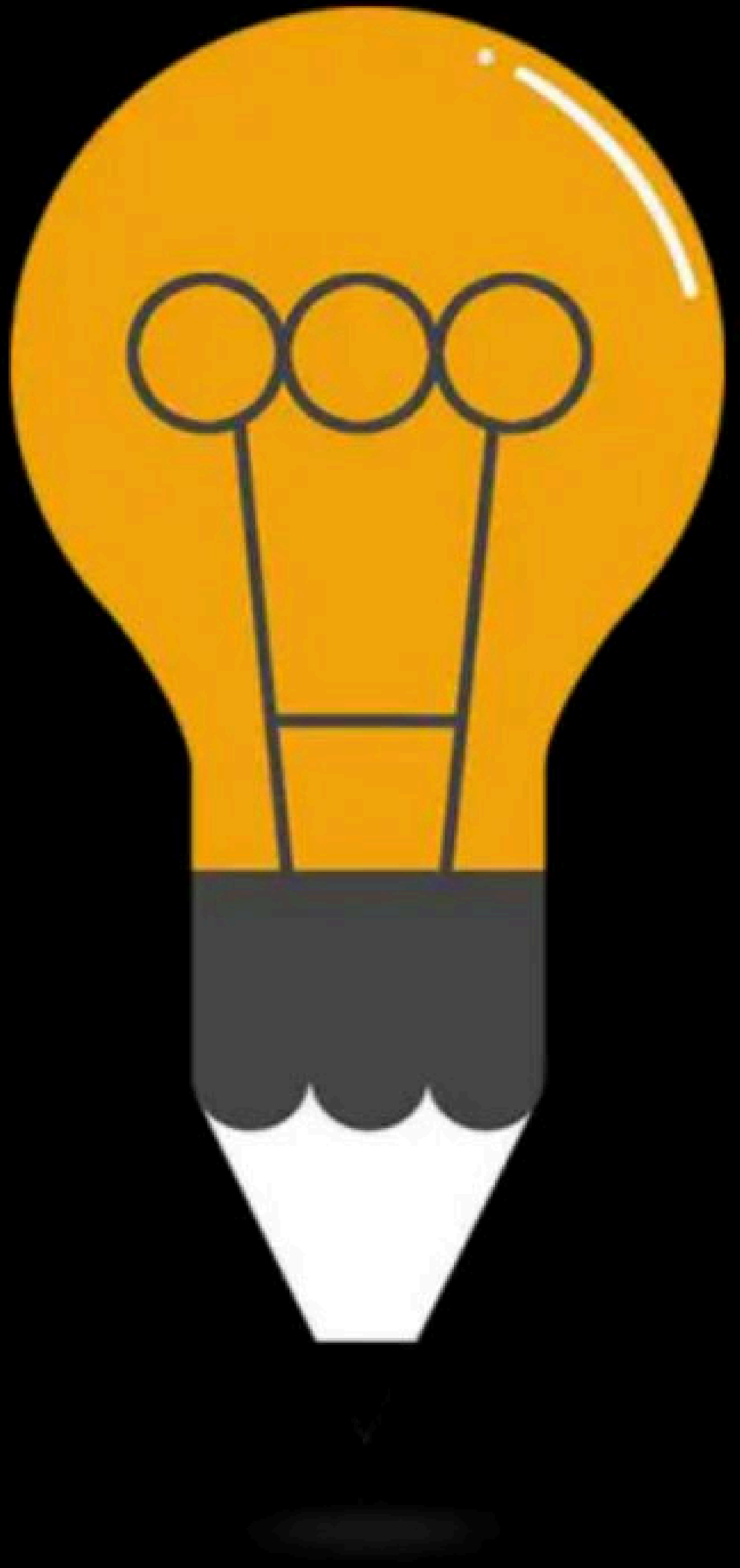

Operating System

Page Replacement

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Page Replacement Policies

1. First In First Out (FIFO)
2. Optimal Policy
3. Least Recently Used (LRU)
4. Least Frequently Used (LFU)
5. Most Frequently Used (MFU)
6. Last In First Out (LIFO)
7. Second Chance

First In First Out (FIFO)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

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First In First Out (FIFO)

Assume:

- Number of frames = 4 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

Belady’s Anomaly

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First In First Out (FIFO)

Advantages

1. Simple and easy to implement.
2. Low overhead.

Disadvantages:

1. Poor performance.
2. Doesn’t consider the frequency of use or last used time, simply replaces the oldest page.
3. Suffers from Belady’s Anomaly

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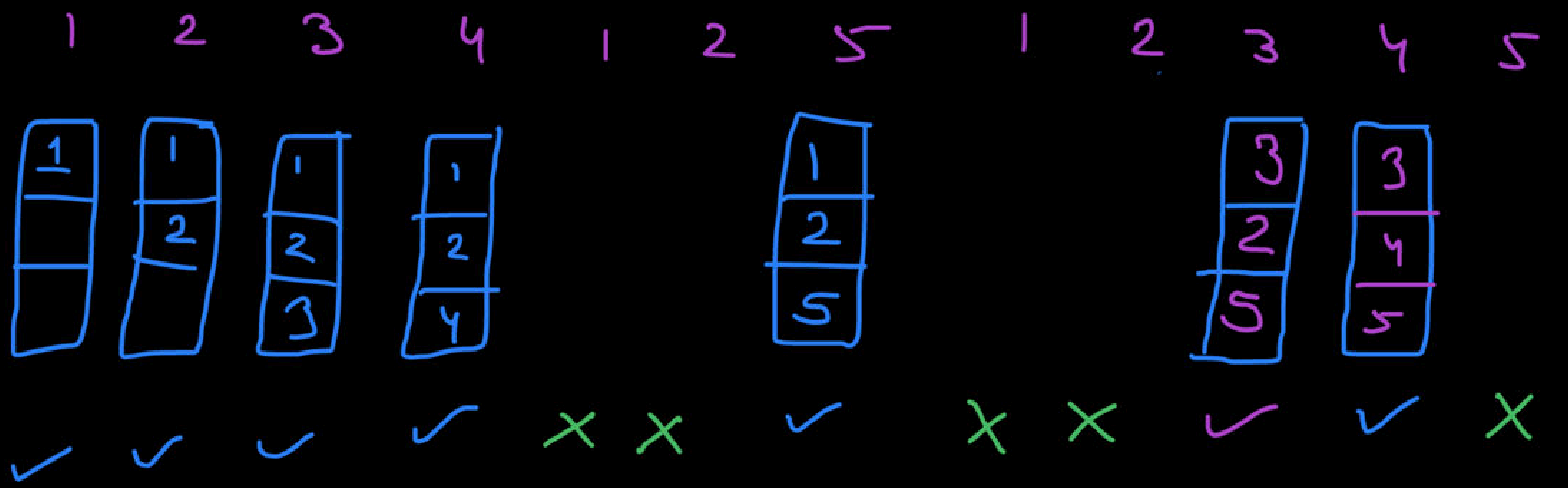
Optimal Policy

→ replace a page which will not be referred for longest period of time.

Assume:

Tie ⇒ FIFO

- ◎ Number of frames = 3 (All empty initially)
- ◎ Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5



no. of page Faults = 7

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no. of frames = 4 (all initially empty)

optimal policy

1	6	9	6	2	3	6	9	2	5	4	3	6	9	6	2	3
1	1	1		1	3				3	3					3	
	6	6		6	6				6	6					2	
		9		9	9				9	9					9	
				2	2				5	4					4	
✓	✓	✓	✗	✓	✓	✗	✗	✗	✓	✓	✗	✗	✗	✗	✓	✗

no. of p. faults = 8

p. f. rate = $\frac{8}{17}$

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Optimal Policy

Advantages

1. Easy to Implement
2. Simple data structures are used
3. Highly efficient → provides min. possible page faults

Disadvantages:

1. Requires future knowledge of the program → not practical
2. Time-consuming

→ takes a lot of time to find a page to replace.

no. of page faults in optimal policy $\leq$ no. of page faults in other policy

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Least Recently Used (LRU) → replace a page which has not been used since longest period of time.

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

1	2	3	4	1	2	5	1	2	3	4	5
1	1	1	4	4	4	5			3	3	3
	2	2	2	1	1	1			1	4	4
		3	3	3	2	2			2	2	5
✓	✓	✓	✓	✓	✓	✓	X	X	✓	✓	✓

no. of page faults = 10

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no. of frame = 4 (all initially empty)

LRU policy

6	8	7	8	9	7	5	9	2	1	9	5	7	6	2	3	1	7
6	6	6		6		5		5	5			5	5	5	3	3	3
	8	8		8		8		2	2			7	7	7	7	1	1
		7		7		7		7	1			1	6	6	6	6	7
				9		9		9	9			9	9	2	2	2	2
✓	✓	✓	×	✓	×	✓	×	✓	✓	×	×	✓	✓	✓	✓	✓	✓

no. of page faults = 13

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Least Recently Used (LRU)

Advantages

1. Efficient.
2. Doesn't suffer from Belady's Anomaly

Disadvantages:

1. Complex Implementation
2. Expensive
3. Requires hardware support

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Question

- Number of frames = 4 (All empty initially)
- Page reference sequence: 5, 7, 0, 1, 7, 6, 7, 2, 1, 6, 7, 6, 1
- Number of page faults for optimal and LRU policies?

optimal :-

5	7	0	1	7	6	7	2	1	6	7	6	1
5	5	5	5		6		6					
	7	7	7		7		7					
		0	0		0		2					
			1		1		1					
✓	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✗	✗

no. of faults = 6

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LRU :-

5	7	0	1	7	6	7	2	1	6	7	6	1
5	5	5	5		6		6					
	7	7	7		7		7					
		0	0		0		2					
			1		1		1					
✓	✓	✓	✓	✗	✓	✗	✓	✗	✗	✗	✗	✗

no. of faults = 6

p. fault rate = $\frac{6}{13}$

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Question

Consider the following page references:

2, 3, 4, 5, 6, 4, 5, 2, 7, 8, 9, 8, 9, 8, 9, 1, 6, 5, 6, 5, 3

Using optimal policy and 4 frames. Memory access time is 2ms and page fault service time is 40ms. The effective memory access time is?

2	2	2	2	7	7	9	9	9
	3	3	6	6	6	6	6	6
		4	4	4	8	8	1	1
		5	5	5	5	5	5	3

✓ ✓ ✓ ✓ ✓ × × × ✓ ✓ ✓ × × × ✓ × × × × ✓

$$\text{p.f. rate} = \frac{10}{21}$$

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$$EMAT = \frac{11}{21} * 2 * 2ms + \frac{10}{21} * (2 + 40)$$

$$= \frac{44 + 420}{21}$$

$$= \frac{464}{21}$$

$$= 22.\underline{1}$$

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MRU (Most Recently Used) :-

Replace the page which has been refered most recently.

ex:- no. of frames = 3

1	3	1	4	2	5	6	5	2	6	4	3	1
1	1		1	1	1	1	1	1	1	1		
	3		3	3	3	3	3	3	3	3		
			4	2	5	6	5	2	6	4		
✓	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✗	✗

no. of page faults = 10

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Counting Algorithms

- Counting algorithms look at the number of occurrences of a particular page and use this as the criterion for replacement.

- Such counting algorithms includes:
  - LFU (Least Frequently Used)
  - MFU (Most Frequently Used)

Tie breaker = FIFO

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Least Frequently Used (LFU)

Assume:

◎ Number of frames = 3 (All empty initially)

◎ Page reference sequence: 1 2 0 3 0 4 2 3 0 3 2

1	2	0	3	0	4	2	3	0	3	2
1	1	1	3		3	2	2			
	2	2	2		4	4	3			
		0	0		0	0	0			
✓	✓	✓	✓	✗	✓	✓	✓	✗	✗	✗

no. of faults = 7

page	freq.
1	1
2	~~1~~ ~~2~~ 3
0	~~1~~ ~~2~~ 3
3	1 ~~2~~ 3
4	1

Most Frequently Used (MFU)

Assume:

- Number of frames = 3 (All empty initially)
- Page reference sequence: 1 2 0 3 0 4 2 3 0 3 2

Page	freq.
1	~~1~~ 2
2	~~1~~ 2
0	~~1~~ ~~2~~ 3
3	~~1~~ ~~2~~ 3
4	1

1	2	0	3	0	4	2	3	0	3	2
1	1	1	3		3			3		2
	2	2	2		2			0		0
		0	0		4			4		4
✓	✓	✓	✓	✗	✓	✗	✗	✓	✗	✓

no. of page faults = 7

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Last In First Out (LIFO)

↳ replace a page which has been brought to mm last.

Assume:

- ◎ Number of frames = 3 (All empty initially)
- ◎ Page reference sequence: 1, 2, 3, 4, 1, 2, 5, 1, 2, 3, 4, 5

no. of faults = 8

1	1	1	1			1			1	1	1
	2	2	2			2			2	2	2
		3	4			5			3	4	5
✓	✓	✓	✓	✗	✗	✓	✗	✗	✓	✓	✓

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frame = 3 | MRU algo

1	2	3	4	1	2	5	1	2	3	4	5
1	1	1	1			1		2	3		
	2	2	2			5		5	5		
		3	4			4		4	4		
✓	✓	✓	✓	✗	✗	✓	✗	✓	✓	✗	✗

no. of faults = 7

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Question GATE-2016

Consider a computer system with ten physical page frames. The system is provided with an access sequence $a_1, a_2, \ldots, a_{20}, a_1, a_2, \ldots, a_{20}$ where each a_i page number. The difference in the number of page faults between the last-in-first-out page replacement policy and the optimal page replacement policy is ________ $31 - 30 = 1$

optimal :-

no. of faults = 30

$a_1 \quad a_2 \cdots a_{10} \; a_{11} \quad a_{19} \quad a_{20} \quad a_1 \quad a_2 \cdots a_9 \quad a_{10} \quad a_{11} \cdots a_{19} \quad a_{20}$

a_1	a_1	a_1
a_2	a_2	a_2
⋮	⋮	⋮
		a_9
a_{10}	a_{11}	a_{20}

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LIFO :-

$a_1 \quad a_2 \quad \cdots\cdots \quad a_{10} \quad a_{11} \quad \cdots\cdots \quad a_{20} \quad \overbrace{a_1 \cdots\cdots a_9}^{\text{hit}} \quad a_{10} \quad \cdots \quad a_{20}$

a_1	a_1	a_1	a_1
a_2	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	a_9	a_9
a_{10}	a_{11}	a_{20}	a_{10}

no. of faults = 31

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Question GATE-2014

Hits = 19 + 19 + 2 => 40
Faults = 260

A computer has twenty physical page frames which contain pages numbered 101 through 120. Now a program accesses the pages numbered 1, 2, …, 100 in that order, and repeats the access sequence THRICE. Which one of the following page replacement policies experiences the same number of page faults as the optimal page replacement policy for this program?

(A) Least-recently-used 300
(B) First-in-first-out 300
(C) Last-in-first-out 300
(D) Most-recently-used 300

1 2 3

101
⋮
~~120~~ ~~1~~ 2 3

1 ---- 19 20 ...... 99 > —— 19

1	1	1
2	2	2
⋮	⋮	⋮
19		18
	20	99
100	100	100

101 1 1 1 21 21

: 2 2 22

: 3

120 : :

: :

20 :

Assume:- frames are initially empty, then solve Questn.

H/w.

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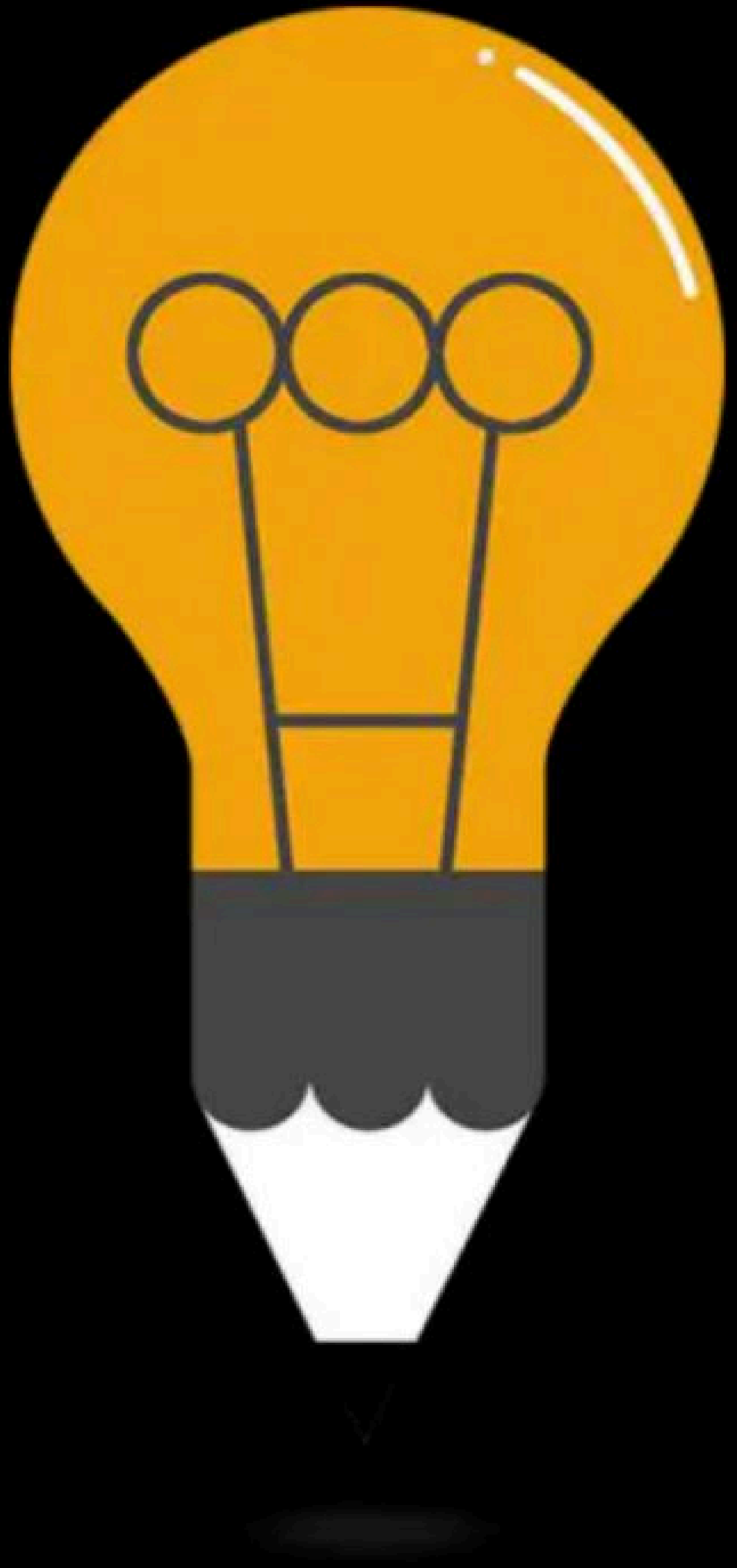

DPP

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Question 1

A main memory can hold 3 page frames and initially all of them are vacant. Consider the following stream of page requests :

2, 3, 2, 4, 6, 2, 5, 6, 1, 4, 6

If the stream uses FIFO replacement policy, the hit ratio h will be?

(a) 11/3
(b) 1/11
(c) 3/11
(d) 2/11

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Question 2

Consider the following page references:

2, 3, 4, 5, 6, 4, 5, 2, 7, 8, 9, 8, 9, 8, 9, 1, 6, 5, 6, 5, 3

Using optimal policy and 4 frames. Memory access time is 2ms without page fault and page fault service time is 40ms. The effective memory access time is?

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Question 3

A virtual memory system has only 2-page frames which are empty initially. Using demand paging the following sequence of page reference is passed through this system.

9, 8, 7, 8, 7, 9, 7, 9, 8, 9

Minimum possible number of page faults?

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Happy Learning.!