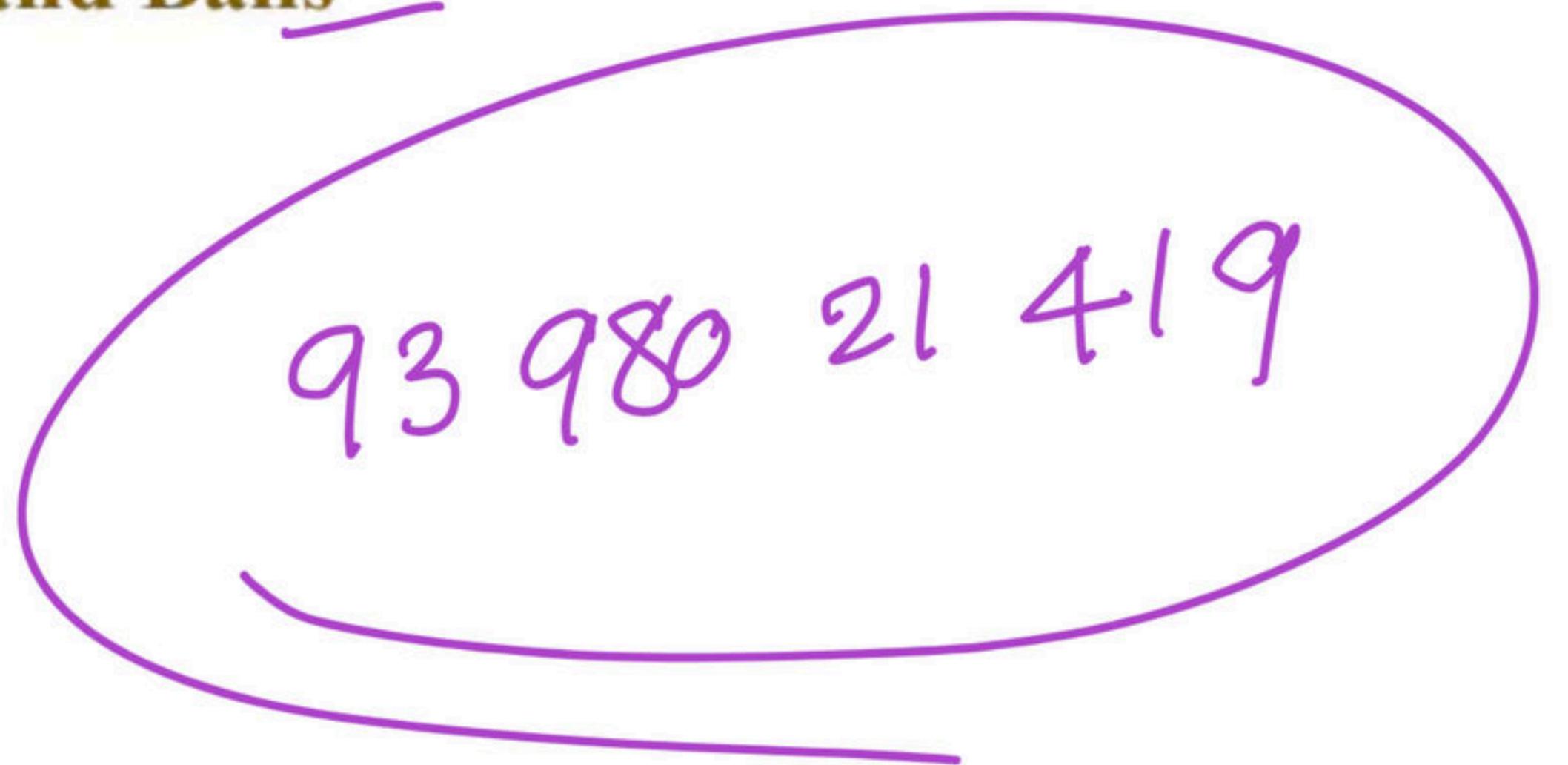


# Practice Session on Probability - Part I

Revision Course on Engineering Mathematics - GATE, CS & IT

# PROBABILITY

- Basic definitions ✓
- Problems on Coin , Dice , Cards and Balls
- Conditional Probability ✓
- Independent events ✓
- Total Probability ✓
- Bayes Theorem ✓



93 980 21 419

# PROBABILITY

- **Random Variable**  
**Mean , Variance , S.D**
- **Probability Distributions**
  - Binomial ,
  - Poisson,
  - Uniform ,
  - Exponential ,
  - Gaussian ( Normal )

# Sample space

All the possible outcomes of a random experiment .

Eg : tossing a coin

$$S = \{ \text{Head}, \text{Tail} \}$$

Rolling a dice

$$S = \{ 1, 2, 3, 4, 5, 6 \}$$

## Event

The outcomes of a random experiment are called as Event . Event is always subset of the sample space .

$$S = \{ H, T \}$$

## Favorable events

The outcomes which are favorable to my desired event .

## Mutually **exclusive** events

Two events A and B are said to be mutually exclusive ( disjoint or incompatible) if the occurrence of one event prevents the occurrence of other event , i.e the events does not occur simultaneously .

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

## Equally likely events

Occurrence of any event in a random experiment are equal then the events are said to be equally likely events .

Eg : Tossing a unbiased coin

$$S = \{ H, T \}$$

$$P(H) = \frac{1}{2}$$

$$P(T) = \frac{1}{2}$$

# Independent events

The occurrence of one event does not depend on another.

Shashank

(EE)

yosh

(CS)

Japathi

(CSE)

- Eg : 1. when two dice are rolled , Getting ‘1’ on first die does not depend on ‘2’ on the second die .
2. when an unbiased coin is tossed two times, the event of getting a head in the first toss is independent of getting head in the second toss .

$$P(A \cap B) = P(A) P(B)$$

$A$  &  $B$  are independent

# Probability

The probability of an event A is defined as

$$P(A) = \frac{\text{number of favourable events}}{\text{total number of possible events}}$$
$$= \frac{n(A)}{n(S)}$$

# Properties of Probability

1.  $P(A) = \frac{n(A)}{n(S)}$  ✓

2.  $0 \leq P(A) \leq 1$

Impossible event

Sure event  
(certain event)

15

3. Sum of all probabilities = 1

$$\sum P = 1$$

4.  $P(\text{sample space}) = 1$

5.  $P(\overline{A}) = 1 - P(A)$

6.  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A & B are mutually exclusive events,

$$P(A \cap B) = 0$$

$$P(A \cap B) \leq P(A)$$

$$7. P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$$

$$8. P(\overline{A \cup B}) = 1 - P(A \cup B) \quad \checkmark$$

$$\underline{\underline{P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B})}}$$

$$P(A \cup B) \geq P(A)$$

$$P(A \cup B) \leq P(A) + P(B)$$

$$9. P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$P(\overline{A \cap B}) = P(\overline{A} \cup \overline{B})$$

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$$10. P(\text{only } A) = P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

$$11. P(\text{only } B) = P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

$$P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

**12.**  $P(\text{Both A \& B}) = P(A \cap B)$

$$P(A \cap B)$$

**13.**  $P(\underline{\text{at least one}}) = P(A \cup B)$

**14.**  $P(\underline{\text{Either A or B}}) = P(A \cup B)$

**15.**  $P(\underline{\text{neither A nor B}}) = P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

**16.**  $P(\text{exactly one}) = P(A \Delta B) = P(\underline{A \cap \overline{B}}) + P(\overline{A} \cap B)$

$$18. P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

# Sample Space

1-Coin

$$S = \{ H, T \}$$

2-Coin

$$S = \{ HH, HT, TH, TT \}$$

$$00 \rightarrow TT$$

$$01 \rightarrow TH$$

$$10 \rightarrow HT$$

$$11 \rightarrow HH$$

$$TH$$

$$TT$$

$$HH$$

$$HT$$

### 3-coins

$$n(s) = 2^3$$

000  $\rightarrow$  TTT

001  $\rightarrow$  TTH

010  $\rightarrow$  THT

011  $\rightarrow$  THH

100  $\rightarrow$  HTT

101  $\rightarrow$  HTH

110  $\rightarrow$  HHT

111  $\rightarrow$  HHH

T H H
T H T
T T H
T T T
H H H
H H T
H T H
H T T

### 4-coins

0000  $\rightarrow$  TTTT

0001  $\rightarrow$  TTTH

0010  $\rightarrow$  TTHT

0011  $\rightarrow$  TTHH

0100  $\rightarrow$  THTT

0101  $\rightarrow$  THTH

0110  $\rightarrow$  THHT

0111  $\rightarrow$  THTH

1000  $\rightarrow$  HTTT

1001  $\rightarrow$  HTTH

1010  $\rightarrow$  HTHT

1011  $\rightarrow$  HTHH

1100  $\rightarrow$  HHTT

1101  $\rightarrow$  HHTH

1110  $\rightarrow$  HHHT

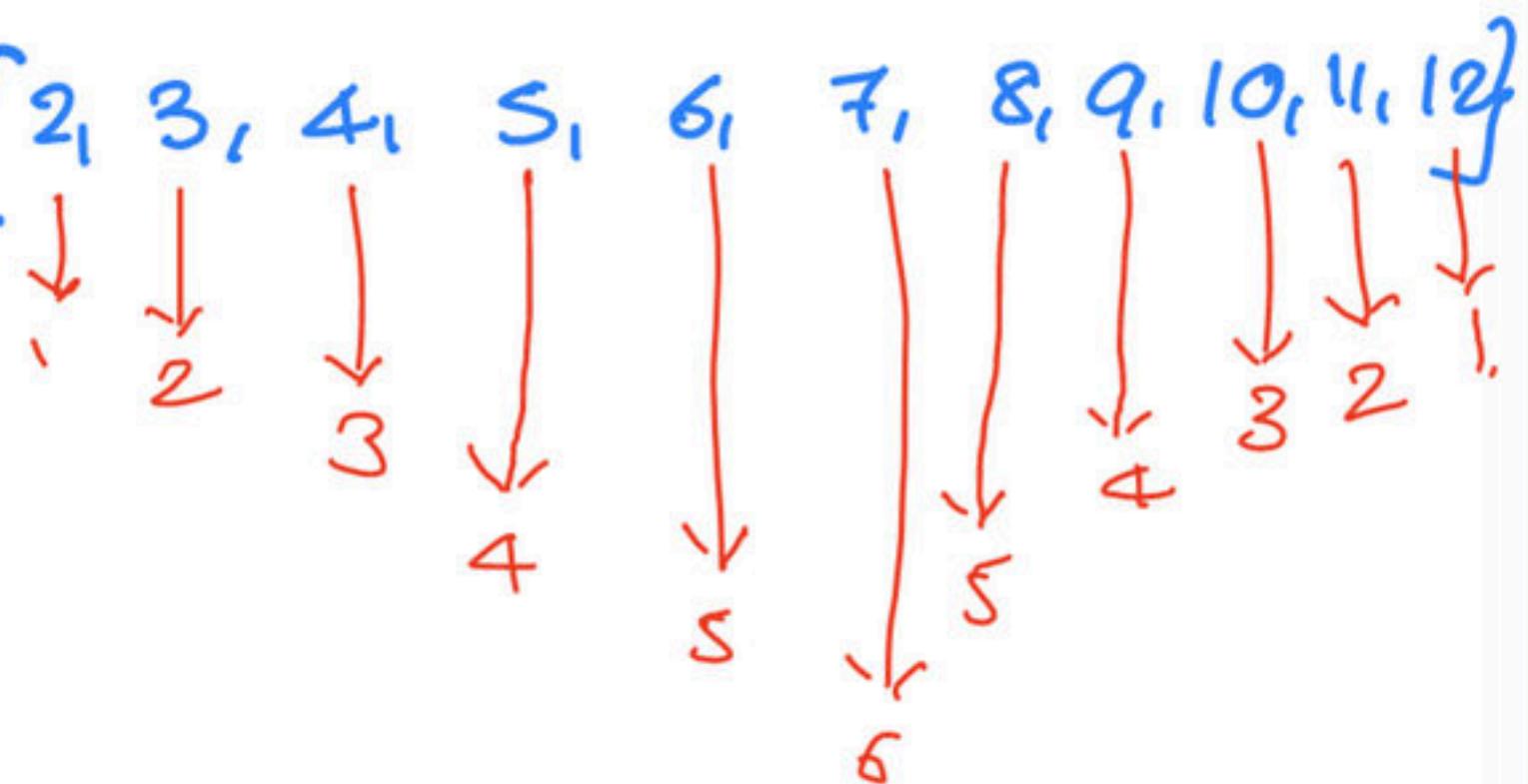
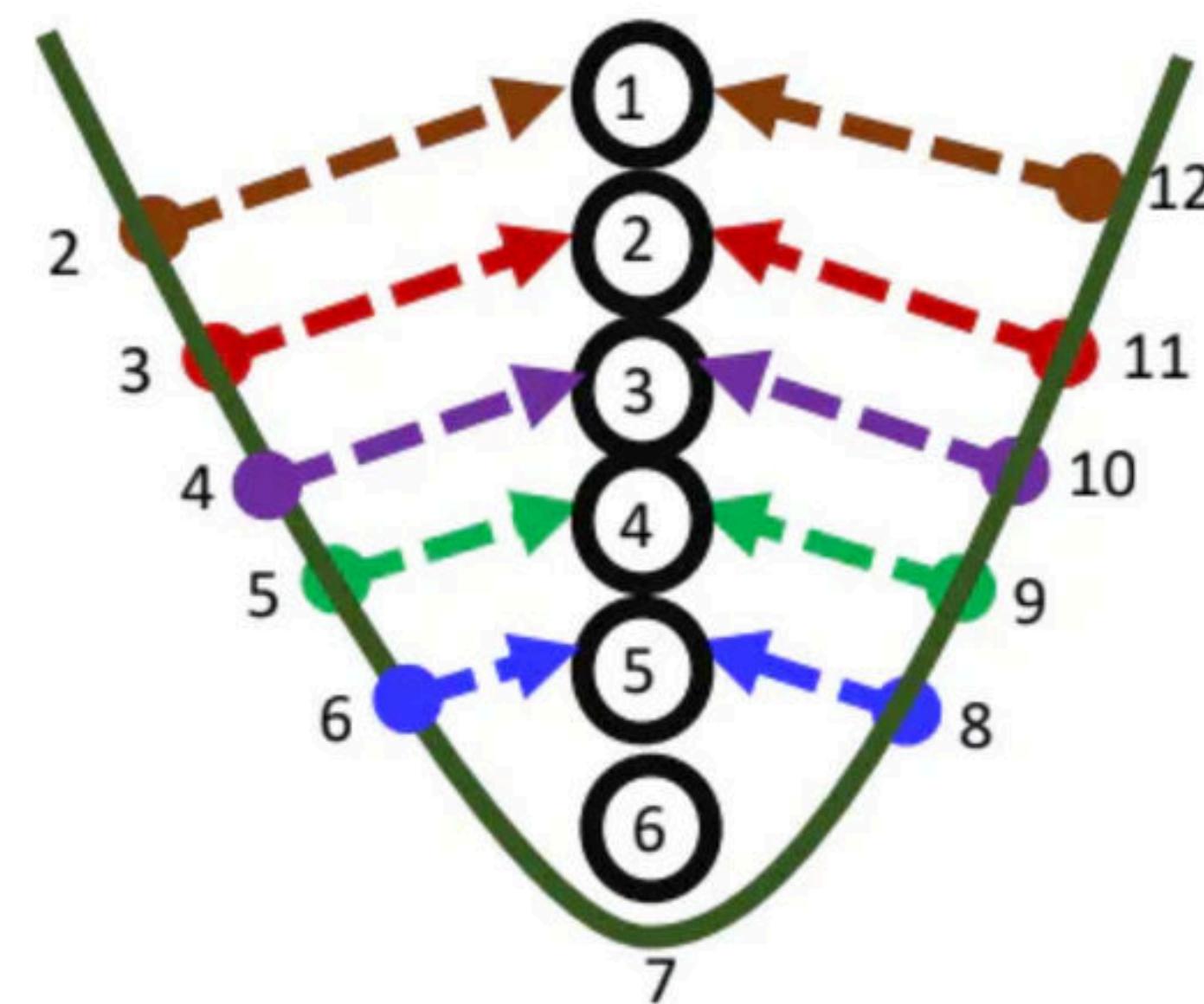
1111  $\rightarrow$  HHHH

# Rolling a dice

When two dice are rolled

$$n(S) = 6^2$$

Sum of the numbers on the dice = {  
2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}



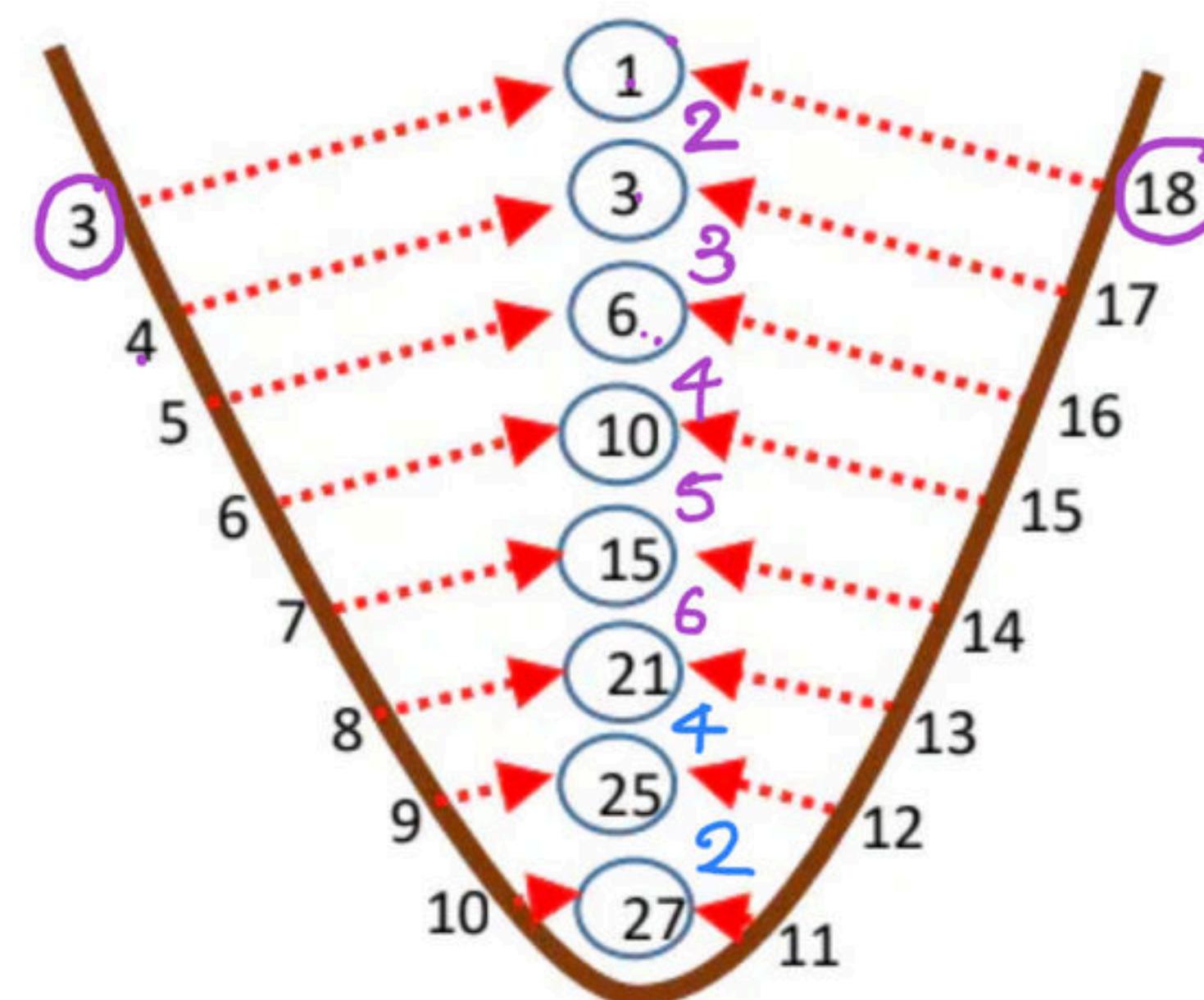
# Rolling a dice

When three dice are rolled

$$n(S) = 6^3$$

1 1 1  
1 2 1

Sum of the numbers on the dice = { 3, 4, 5, 6, 7, 8, 9, 10 }



6 6 6

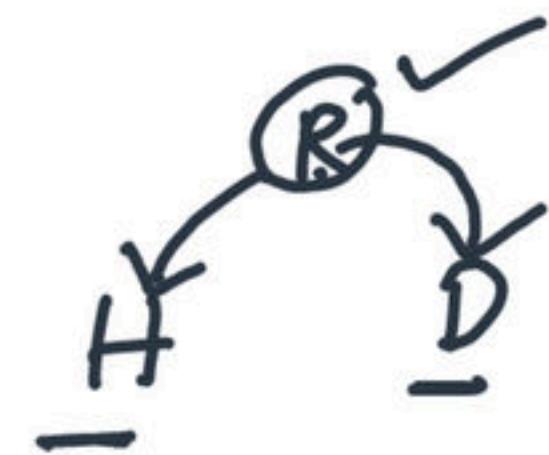
→ 18

## Pack of cards (52)



MHRD

12,400/-



Each suit contains

2, 3, 4, 5, 6, 7, 8, 9, 10

Number cards = 9

J, K, Q

Face cards = 3

Honour cards = 4

1. Total number of face cards =  $3 \times 4 = 12$

2. Total number of number cards =  $9 \times 4 = 36$

3. Total number of honour cards =  $4(4) = 16$

4. Total number of Red Kings = 2

5. Total number of Spade Queen = 1

6. Total number of Black Diamonds = 0

7. Total number of Diamond Ace = 1

8. Total number of Black 2's = 2

9. Total number of Red face cards =  $2(3) = 6$

10. Total number of Spade King = 1

11. Total number of Club 9's = 1

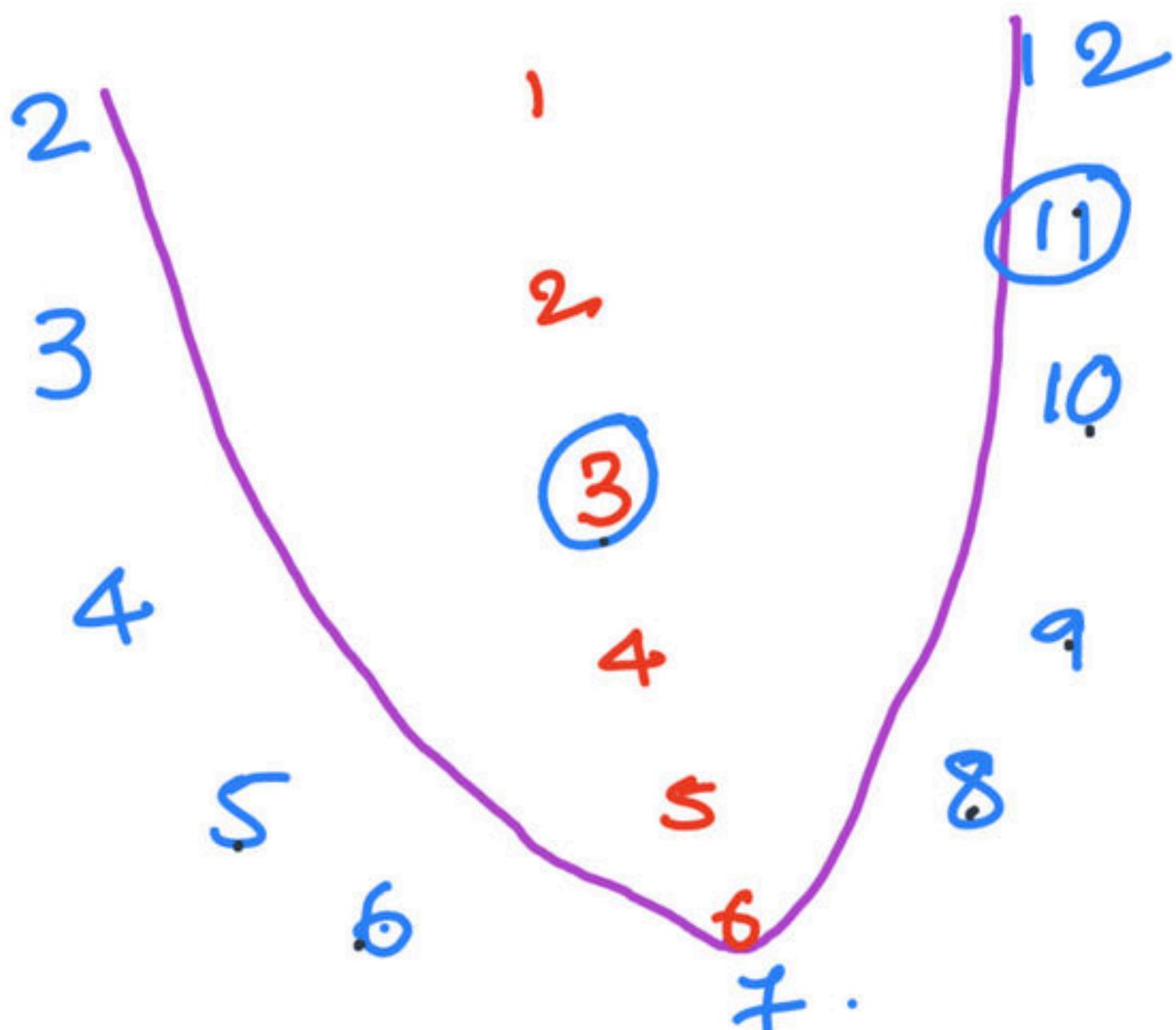
12. Total number of Red Hearts = 13

MHRD

1. When two dice are rolled, what is the probability of getting the sum

- a) sum = 4
- b) sum = 11
- c) sum > 10
- d) sum  $\leq$  10

- e)  $4 \leq \text{sum} \leq 11$



$$P(S=4) = \frac{3}{36}$$

$$P(S=11) = \frac{2}{36}$$

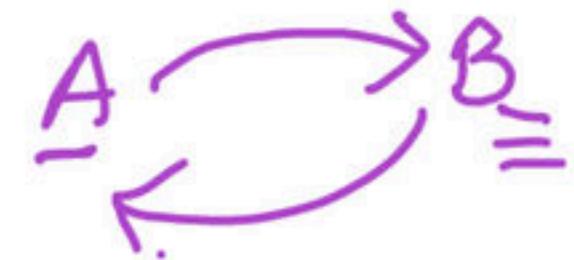
$$P(S > 10) = \frac{3}{36}$$

$$\begin{aligned} P(S \leq 10) &= 1 - P(S > 10) \\ &= 1 - \frac{3}{36} \end{aligned}$$

$$P(4 \leq S \leq 11) = \frac{3 + 4 + 5 + 6 + 5 + 4 + 3 + 2}{36} = \frac{32}{36}.$$

2. A and B are playing a game of tossing a coin . One who gets head wins the game . If A starts the game , find the probabilities of their winning .

$$P(H) = \frac{1}{2}$$



$$P(\bar{H}) = \frac{1}{2}.$$

$$\begin{aligned}
 \text{A's winning} &= P(H) + \overbrace{P(\bar{H}) P(H)}^{\text{Probability of winning on first turn}} + \overbrace{P(\bar{H}) P(\bar{H}) P(H)}^{\text{Probability of winning on second turn}} + \dots \\
 &= \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} + \dots \\
 &= \frac{1}{2} \left[ 1 + \frac{1}{4} + \frac{1}{8} + \dots \right] = \frac{1}{2} \left[ \frac{1}{1 - \frac{1}{4}} \right] \\
 &= \frac{2}{3}.
 \end{aligned}$$

$$\begin{aligned}
 B's \text{ winning} &= \overline{P(H)} P(H) + \overline{P(H)} \overline{P(H)} \overline{P(H)} p(H) + \overline{P(H)} \overline{P(H)} \overline{P(H)} \overline{P(H)} P(H) \\
 &= \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{24} + \frac{1}{26} + \dots \\
 &= \frac{1}{4} \left[ 1 + \frac{1}{4} + \frac{1}{4^2} + \dots \right] \\
 &= \frac{1}{4} \left[ \frac{1}{1 - \frac{1}{4}} \right] = \frac{1}{3}.
 \end{aligned}$$

A      B

$q^0$      $\textcircled{q^1}$

$$P = \frac{1}{2}$$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{A's winning} = \frac{q^0}{q^0 + q^1} = \frac{1}{1+q} = \frac{2}{3}.$$

$$\text{B's winning} = \frac{q^1}{q^0 + q^1} = \frac{q}{1+q}.$$

$$\begin{array}{cccc}
 A & B & C & D \\
 q^0 & q^1 & q^2 & q^3
 \end{array}$$

$$A's \text{ winning} = \frac{1}{1+q+q^2+q^3}.$$

$$D's \text{ winning} = \frac{q^3}{1+q+q^2+q^3}.$$

$$B's \text{ winning} = \frac{q}{1+q+q^2+q^3}$$

$$C's \text{ winning} = \frac{q^2}{1+q+q^2+q^3}$$

## Joint Probability

If A and B are the two events in sample space S , which are not mutually exclusive then the joint probability of A and B can be denoted as  $P(A \cap B)$  .

# Conditional Probability

If A and B are the two events in sample space S , then the conditional probability of A given B is defined as

$$P\left[\frac{A}{B}\right] = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(B \cap A)$$

the conditional probability of B given A is defined as

$$P\left[\frac{B}{A}\right] = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

# Properties of conditional probability

## Pair wise independent events

If A and B are pair wise independent events

## Mutually independent events

If A, B and C are Mutually independent events  
then

# Total Probability Theorem

If sample space contains  $n$  – mutually exclusive events, then probability of event A defined on the sample space S can be expressed as a conditional probability .

$$P(A) = \sum_{n=1}^N P(B_n)P\left(\frac{A}{B_n}\right)$$

3. A box contains 5 red balls and 6 black balls , another box contains 6 red balls and 4 black balls. One box is chosen at random and one ball is drawn from it. Find the probability of getting
- a) Red ball
  - b) Black ball

I

5-R	6-B
-----	-----

II

<u>6-R</u>	4-B
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$$P(R) = \frac{1}{2} \cdot \frac{5}{11} + \frac{1}{2} \cdot \frac{6}{10}$$

$$P(B) = \frac{1}{2} \cdot \frac{6}{11} + \frac{1}{2} \cdot \frac{4}{10}$$

## Bayes' theorem

If sample space S contains n- mutually exclusive events, let A is any event in the sample space , then the conditional probability of  $B_n$  given A is

$$P\left[\frac{B_n}{A}\right] = \frac{P\left(\frac{A}{B_n}\right)p(B_n)}{\sum_{n=1}^N P\left(\frac{A}{B_n}\right)p(B_n)}$$

4. A box contains 5 red balls and 6 white balls , another box contains 4 red ball and 6 white balls . One ball is drawn and found to be red. Find the probability that the ball is drawn from first box .

I

5-R	6-W
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II.

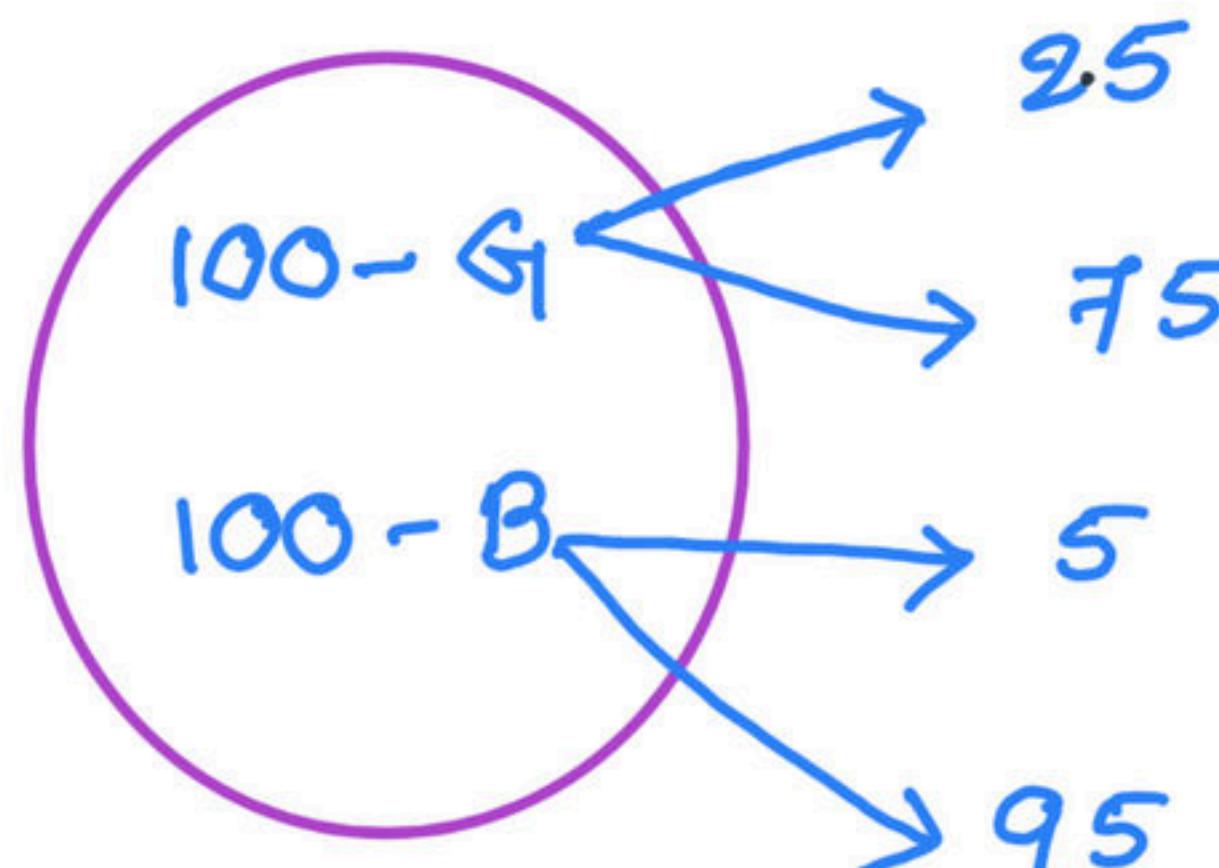
4-R	6-W
-----	-----

$$P(R) = \frac{1}{2} \cdot \frac{5}{11} + \frac{1}{2} \cdot \frac{4}{10}$$

↓      ↓  
(I)      (II)

$$P = \frac{\frac{1}{2} \left( \frac{5}{11} \right)}{\frac{1}{2} \left( \frac{5}{11} \right) + \frac{1}{2} \left( \frac{4}{10} \right)} =$$

5. 25 girls out of 100 , 5 boys of 100 have color blind. One person is chosen at random and found to be colorblind . Find the probability that person is a girl .



$$P(G) = \frac{100G_1}{200C_1} = \frac{100}{200} = \frac{1}{2}$$

$$P(B) = \frac{100B_1}{200C_1} = \frac{1}{2}.$$

$$P(C|G) = \frac{25G_1}{100C_1} = \frac{25}{100} = \frac{1}{4}$$

$$P(C|B) = \frac{5B_1}{100C_1} = \frac{1}{20}$$

$$P(C) = \left( \frac{1}{2}, \frac{1}{4} \right) + \left( \frac{1}{2}, \frac{1}{20} \right); \quad B.$$

$G_1$

$$P = \frac{\frac{1}{2} \cdot \frac{1}{4}}{\frac{1}{2} \cdot \frac{1}{4} + \frac{1}{2} \cdot \frac{1}{20}} = \frac{5}{6}.$$

6. A box contains 2 white and 3 black balls , a sample of size 4 is made ,what is the probability that the sample is in the order {white , black , white , black }

7. A box contains 6 red balls, 4 white balls, and 5 blue balls . Three balls are drawn successively from the box . Find the probability that they are drawn in the order red , white and blue if each ball is

- a) Replaced
- b) Not replaced

8. A box contains 52 badges numbered 1 to 52 . Suppose that the numbers 1 through 13 are considered lucky. A sample of size 2 is drawn from the box with replacement . What is the probability that

- a) Both badges drawn will be lucky
- b) Neither badges will be lucky
- c) Exactly one of the badges drawn will be lucky
- d) At least one of the badges will be lucky

9. In a experiment of drawing a card from a pack the event of getting a spade is denoted by  $A$ , and getting a face card is denoted as  $B$ , find the probabilities of  $A \cup B$  and  $A \cap B$

10. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that sum of the numbers is

- a) An even number
- b) Odd number

11. Find the ratio of probabilities getting sum 6 with 4 , 3 and 2 dice are rolled respectively

12. A box contains 4 point contact diodes and 6 alloy junction diodes, what is the probabilities that 3 diodes picked at random contain at least 2 point contact diodes

3.

13. A box contains 5 black , 4 white and 6 red balls . Two balls are drawn without replacement , what is the probability that the first will be white and second will be black

14. One card is drawn from a regular deck of 52 cards, what is the probability of the card being either red or king

15. One card is selected from an ordinary 52 card deck with an event A as select a king , B as select a jack or queen C as select a heart , find  $P(A \cap B)$  ,  $P(B \cap C)$  and  $P(C \cap A)$

16. There are 3 black and 4 white balls in one bag, 4 black and 3 white balls in the second bag . A die is rolled and the first bag is selected if it is 1 or 3 and second bag for the remaining , find the probability of drawing a black ball from the selected bag

17. A box contains 3 coins, one is fair ,one is two headed and one coin is weighted so that the probability of heads appearing is  $1/3$  . A coin is selected at random and tossed , find the probability that head appears.

18. Three boxes numbered I II and III contain 1-white , 2 black and 3 red balls,: 2 white 1 black and 1 red ball ; 4 white 5 black and 3 red balls respectively. One box is randomly selected and a ball drawn from it . If the ball is red then find the probability that it is from box II

19. Three coins are tossed at a time find the probability of getting

- a) At most one tail
- b) At least one tail
- c) At least one head and at most one tail

20. Four coins are tossed at a time find the probability of getting at most 2 head and at most 2 tail

21. Four coins are tossed at a time find the probability of getting at most 2 head and at least 1 tail

22. Six coins are tossed at a time find the probability of getting at least 2 head and at least 2 tails

23. N- coins are tossed at a time find the probability of getting head

- a) Odd number of times
- b) Even number of times

24. Three horses A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. What is the probability that B or C wins?

- a) 2/7
- b) 3/7
- c) 4/7
- d) 6/7

25. A card is selected at random from an ordinary pack of 52 cards. Probability of selecting a Spade card or a face card is

a)  $3/32$

b)  $23/52$

c)  $22/52$

d)  $25/52$

26. Let two items be chosen from a lot containing 12 items of which 4 are defective. What is the probability that at least one item is defective?

- a)  $19/33$
- b)  $14/33$
- c)  $1/11$
- d)  $13/33$

27. A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8?

- a)  $1/3$
- b)  $1/4$
- c)  $1/5$
- d)  $2/3$

28. A point is selected at random inside a circle. Find the probability  $p$  that the point is closer to the Centre of the circle than to its circumference?

- a)  $1/3$
- b)  $1/4$
- c)  $1/5$
- d)  $2/3$

29. Let A and B be events with  $P(A) = 3/8$ ,  $P(B) = 1/2$  and  $P(A \wedge B) = 1/4$  then which of the Following is false.

- a)  $P(A^C \vee B^C) = 3/4$
- b)  $P(A^C \wedge B^C) = 3/8$
- c)  $P(A \wedge B^C) = 1/8$
- d)  $P(B \wedge A^C) = 5/8$

30. Of 120 students, 60 are studying French, 50 are studying Spanish and 20 are studying French and Spanish. If a student is selected at random then which of the following is not correct.

- a) Probability that the student is studying French or Spanish is 0.75.
- b) Probability that the student is studying neither French nor Spanish is 0.25.
- c) Probability that the student is studying Spanish but not French is 0.25.
- d) Probability that the student is studying French but not Spanish is 0.3

31. In a class of 100 students, 40 failed in mathematics, 30 failed in physics, 25 failed in Chemistry, 20 failed in math's and physics, 15 failed in physics and chemistry, 10 failed in Chemistry and math's, 5 failed in math's, physics and chemistry. If a student is selected at Random then the probability that he passed in all three subjects is

- a) 0.4
- b) 0.45
- c) 0.55
- d) 0.65

32. Let a pair of dice be tossed. If the sum is 6, find the probability that one of the dice is a 2.

- a)  $1/5$
- b)  $2/5$
- c)  $3/5$
- d)  $4/5$

33. A man visits a couple who have two children. One of the children, a boy, comes in to the room . Find the probability that the other is also a boy

a)  $1/3$

b)  $2/3$

c)  $1/2$

d)  $3/4$

34. Let A and B be events with  $P(A) = 3/8$ ,  $P(B) = 5/8$  and  $P(A \cup B) = 3/4$ . Find the conditional probability  $P(A|B)$

a)  $1/3$

b)  $2/5$

c)  $3/4$

d)  $1/2$

35. In certain college, 25% of the students failed mathematics, 15% of the students failed in Chemistry, and 10% of the students failed in both math's and chemistry. A student is Selected at random. If he failed chemistry, what is the probability that he failed in math's?

- a)  $\frac{2}{3}$
- b)  $\frac{2}{5}$
- c)  $\frac{3}{5}$
- d)  $\frac{1}{5}$

36. A die is rolled. If the number appeared is odd, what is the probability that it is prime?

a)  $\frac{1}{3}$

b)  $\frac{2}{3}$

c)  $\frac{3}{4}$

d) 1

37. In a certain college, 4% of the men and 1% of the women are taller than 1.8m. Further more, 60% of the students are women. Now if a student is selected at random and is taller than 1.8m, what is the probability that the student is a woman ?

- a)  $3/11$
- b)  $4/11$
- c)  $5/11$
- d)  $6/11$

38. We are given three urns as follows. Urn A contains 3 red and 5 white marbles, Urn B contains 2 red and 1 white marble, Urn C contains 2 red and 3 white marbles. An urn is selected at random and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A?

- a)  $45/173$
- b)  $37/165$
- c)  $27/109$
- d)  $39/185$

39. A coin, weighted so that  $P(H) = 2/3$  and  $P(T) = 1/3$  is tossed. If heads appears, then a number is selected at random from the numbers 1 through 9. If tails appears, then a number is selected at random from the numbers 1 through 5. Find the probability  $P$  that an even number is selected.

- a)  $67/145$
- b)  $58/135$
- c)  $74/157$
- d)  $43/142$

40. A box contains three coins, two of them fair and one two headed. A coin is selected at random and tossed twice. If heads appears both times, what is the probability that the coin is two headed?

a)  $\frac{2}{3}$

b)  $\frac{1}{3}$

c)  $\frac{3}{4}$

d)  $\frac{1}{2}$

41. An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the opposite colour is put in to the urn. A second marble is drawn from the urn. If both marbles were of the same colour . What is the probability that they were both white?

- a)  $5/6$
- b)  $7/8$
- c)  $8/9$
- d)  $9/10$

42. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

**(GATE-EC-2003)**

- (a) 100%
- (b) 50%
- (c) 49%
- (d) none

43. In a population of  $N$  families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

a)  $\frac{3}{23}$

b)  $\frac{6}{23}$

c)  $\frac{3}{10}$

d)  $\frac{3}{5}$

44. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses ?

- (a) 15      (b) 20      (c) 25      (d) 35

45. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

**(GATE-CS-1995)**

(a)  $18/25$

(b)  $2/5$

(c)  $5/12$

(d)  $19/25$

46. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced ?

- (a)  $1/26$
- (b)  $1/52$
- (c)  $1/169$
- (d)  $1/221$

47. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is

(GATE-CS -2005)

- a)  $\frac{1}{36}$
- b)  $\frac{1}{6}$
- c)  $\frac{1}{4}$
- d)  $\frac{1}{3}$

48. Two dice are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is **(GATE-PI-2005)**

(a)  $4/36$

(b)  $7/36$

(c)  $9/36$

(d)  $10/36$

49. The probability that there are 53 Sundays in a randomly chosen leap year is

**(GATE-IN-2005)**

- (a)  $1/7$       (b)  $1/14$

- (c)  $1/28$       (d)  $2/7$

50. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads ?  
**(GATE-EC-2009)**

(a)  $\left(\frac{1}{2}\right)^2$

(b)  $10c_2 \left(\frac{1}{2}\right)^2$

(c)  $\left(\frac{1}{2}\right)^{10}$

(d)  $10c_2 \left(\frac{1}{2}\right)^{10}$