

Heaps and Binary Heap

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Data Structure

Heap and BST PYQs

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BSTPyQs

Question GATE-1996

A binary search tree is generated by inserting in order the following integers:

50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24

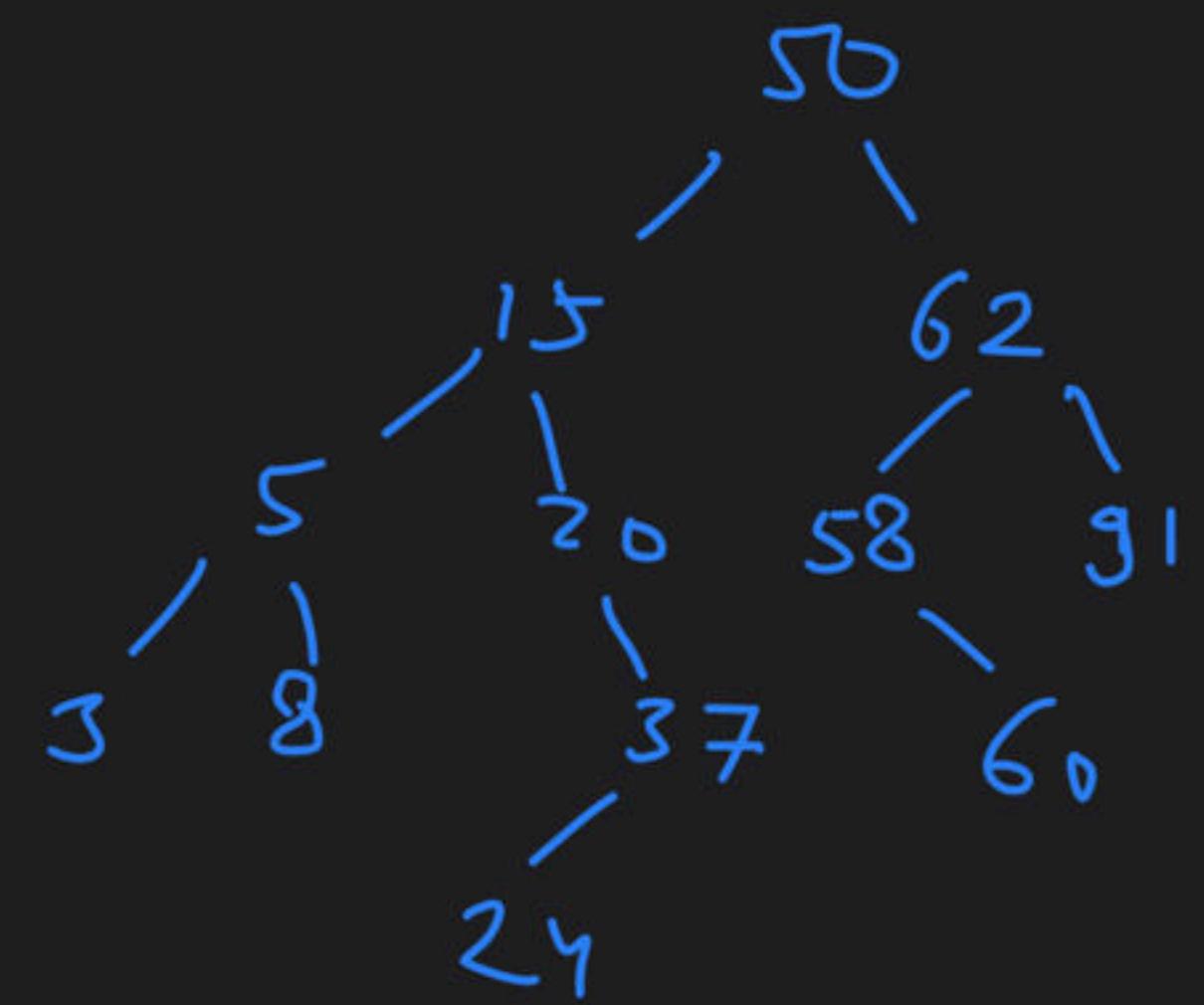
The number of nodes in the left subtree and right subtree of the root respectively is

A. (4, 7)

~~C.~~ B. (7, 4)

C. (8, 3)

D. (3, 8)



Question GATE-1996

A binary search tree is used to locate the number 43. Which of the following probe sequences are possible and which are not? Explain.

- (a) 61 52 14 17 40 43
- (b) 2 3 50 40 60 43
- (c) 10 65 31 48 37 43
- (d) 81 61 52 14 41 43
- (e) 17 77 27 66 18 43

A, C, D

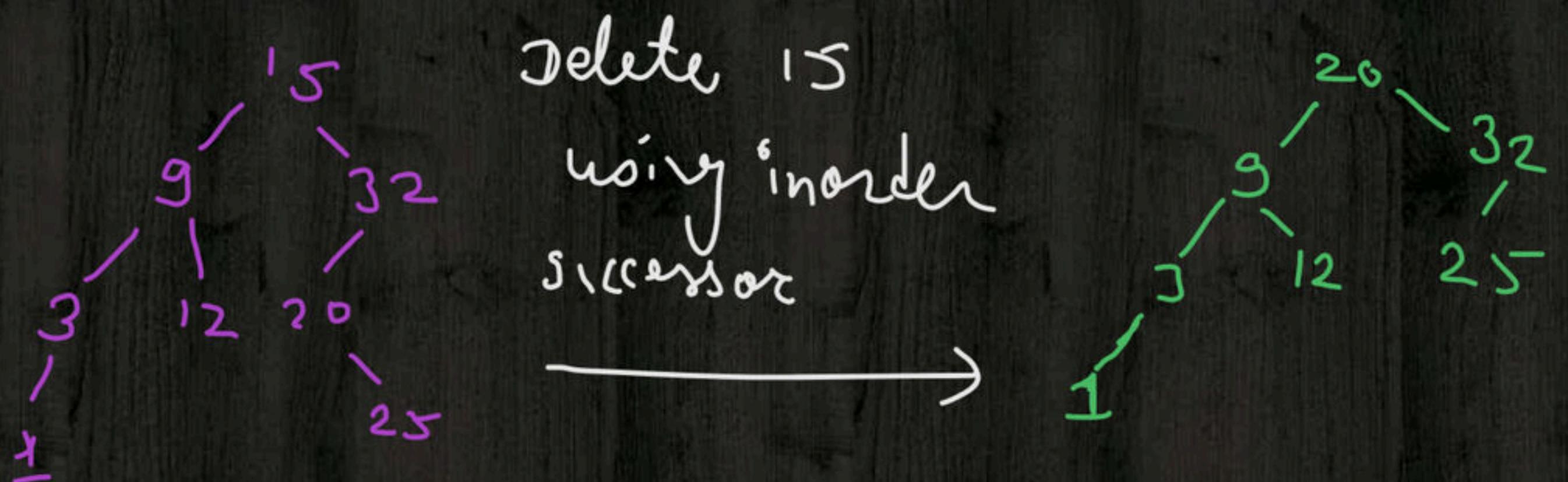
Question GATE-2001

A. Insert the following keys one by one into a binary search tree in the order specified.

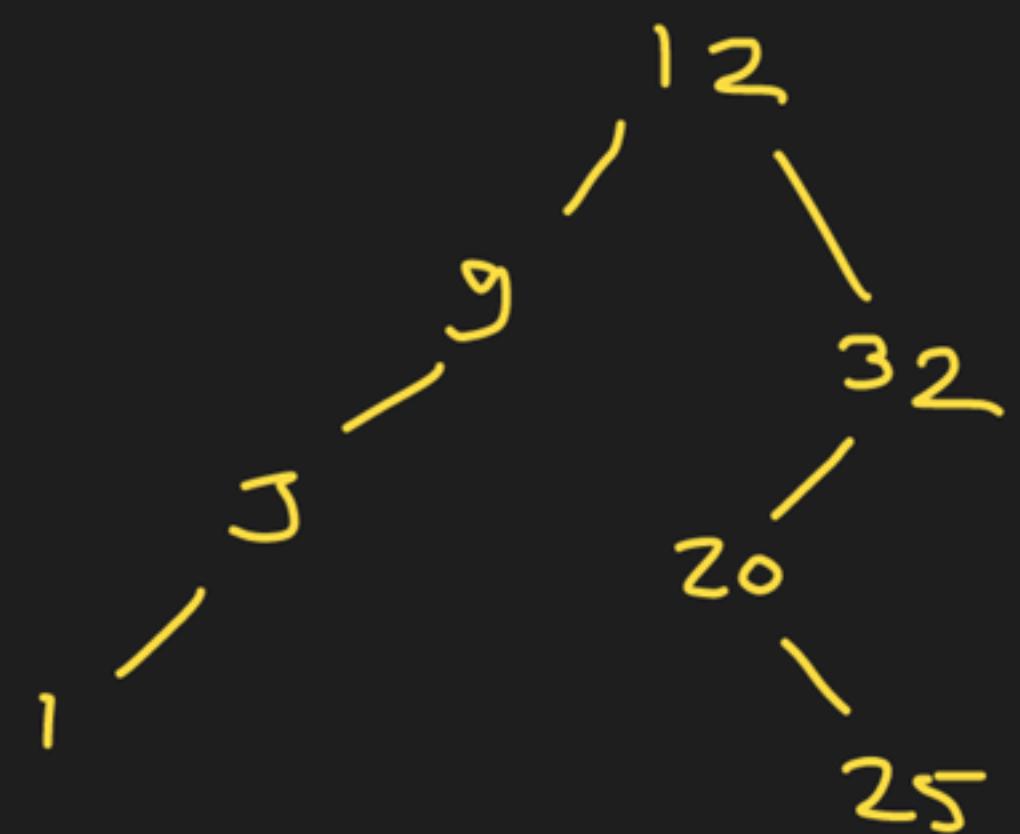
15, 32, 20, 9, 3, 25, 12, 1

Show the final binary search tree after the insertions.

B. Draw the binary search tree after deleting 15 from it.



Delete 15 using
inorder predecessor



Question GATE-2003

A data structure is required for storing a set of integers such that each of the following operations can be done in $O(\log n)$ time, where n is the number of elements in the set.

- I. Deletion of the smallest element
- II. Insertion of an element if it is not already present in the set

Which of the following data structures can be used for this purpose?

- A. A heap can be used but not a balanced binary search tree
- B. A balanced binary search tree can be used but not a heap
- C. Both balanced binary search tree and heap can be used
- D. Neither balanced search tree nor heap can be used

Question GATE-2003

Let $T(n)$ be the number of different binary search trees on n distinct elements.

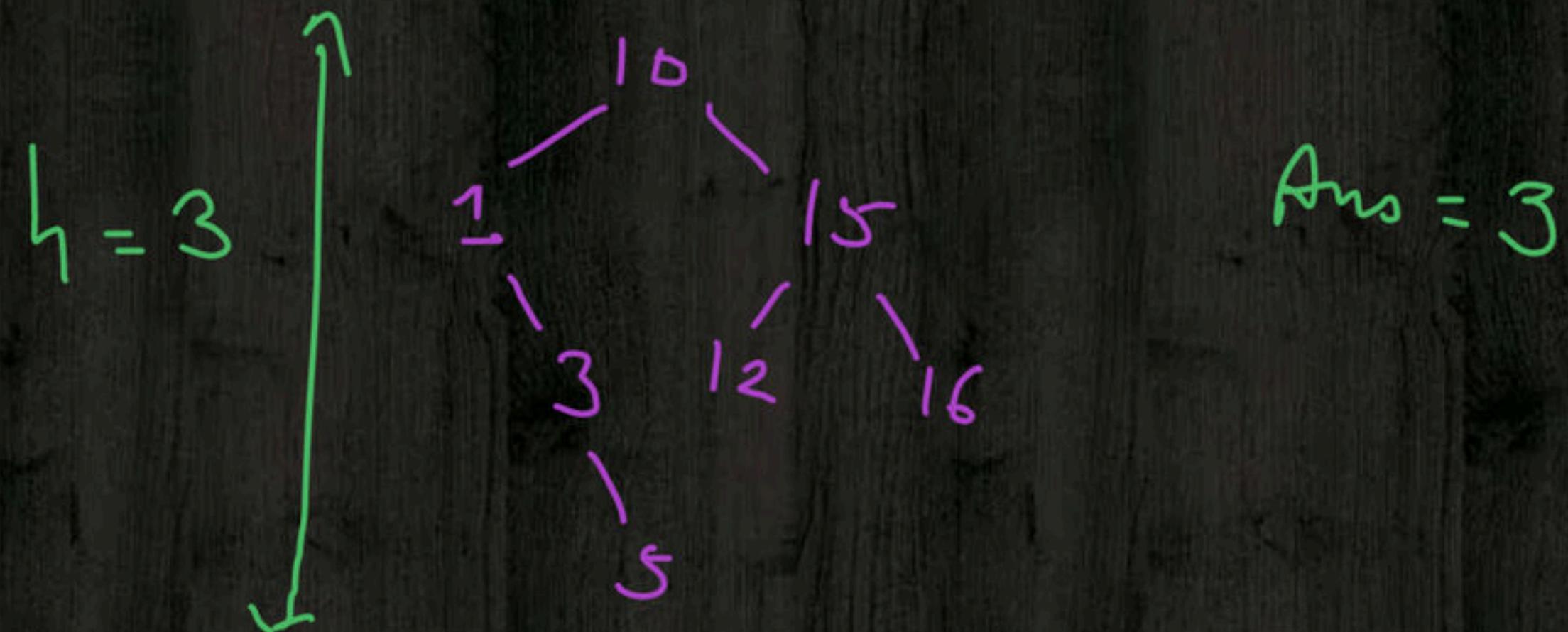
Then $T(n) = \sum_{k=1}^n T(k-1)T(x)$, where x is

- A. $n - k + 1$
- B. $n - k$
- C. $n - k - 1$
- D. $n - k - 2$

Question GATE-2004

The following numbers are inserted into an empty binary search tree in the given order: 10, 1, 3, 5, 15, 12, 16 . What is the height of the binary search tree (the height is the maximum distance of a leaf node from the root)?

- A. 2
- B. 3
- C. 4
- D. 6



Question GATE-2004

A program takes as input a balanced binary search tree with n leaf nodes and computes the value of a function $g(x)$ for each node x . If the cost of computing $g(x)$ is:

$$\min \left(\frac{\text{number of leaf-nodes}}{\text{in left-subtree of } x}, \frac{\text{number of leaf-nodes}}{\text{in right-subtree of } x} \right)$$

Then the worst-case time complexity of the program is?

- A. $\Theta(n)$
- B. $\Theta(n \log n)$
- C. $\Theta(n^2)$
- D. $\Theta(n^2 \log n)$

Question GATE-2005

The numbers $1, 2, \dots, n$ are inserted in a binary search tree in some order. In the resulting tree, the right subtree of the root contains p nodes. The first number to be inserted in the tree must be

- A. p
- B. $p + 1$
- C. $n - p$
- D. $n - p + 1$

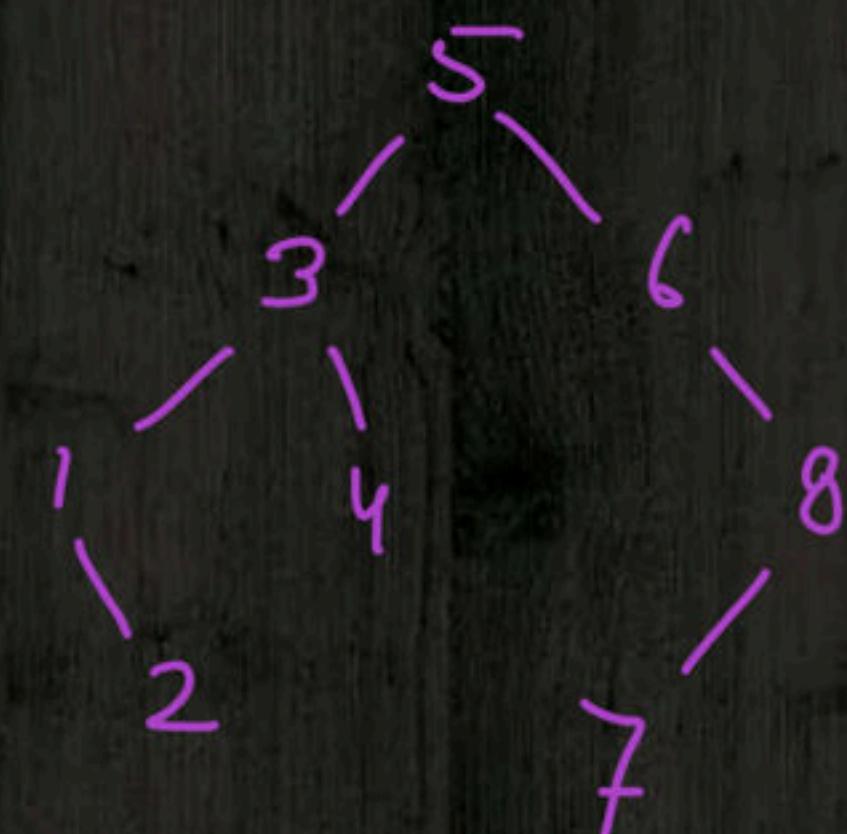
C. $n - p$

Question GATE-2005

A binary search tree contains the numbers 1,2,3,4,5,6,7,8. When the tree is traversed in pre-order and the values in each node printed out, the sequence of values obtained is 5,3,1,2,4,6,8,7. If the tree is traversed in post-order, the sequence obtained would be

- A. 8,7,6,5,4,3,2,1
- C. 2,1,4,3,6,7,8,5

- B. 1,2,3,4,8,7,6,5
- D. 2,1,4,3,7,8,6,5



Postorder:-

2, 1, 4, 3, 7, 8, 6, 5 -

Question GATE-2006

Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number 55. Which of the following sequences CANNOT be the sequence of nodes examined?

- A. {10, 75, 64, 43, 60, 57, 55}
- B. {90, 12, 68, 34, 62, 45, 55}
- C. {9, 85, 47, 68, 43, 57, 55} ✓
- D. {79, 14, 72, 56, 16, 53, 55}

Question GATE-2007

When searching for the key value 60 in a binary search tree, nodes containing the key values 10, 20, 40, 50, 70, 80, 90 are traversed, not necessarily in the order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?

- A. 35
- B. 64
- C. 128
- D. 5040

→ 10, 20, 40, 50 ← 70, 80, 90

$$\frac{7!}{4! \times 3!} = 35$$

Question GATE-2008

A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

- I. 81, 537, 102, 439, 285, 376, 305
- II. 52, 97, 121, 195, 242, 381, 472
- III. 142, 248, 520, 386, 345, 270, 307
- IV. 550, 149, 507, 395, 463, 402, 270

Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequences list nodes in the order in which we could have encountered them in the search?

- A. II and III only
- B. I and III only
- C. III and IV only
- D.  III only

Question GATE-2008

How many distinct BSTs can be constructed with 3 distinct keys?

- A. 4
- B. 5
- C. 6
- D. 9

$$\frac{2^n C_n}{n+1}$$

$n = 3$

$$= \frac{6 C_3}{4} = \frac{6+5+4}{3+2+1+4} = 5$$

Question GATE-2008

Which of the following is TRUE?

- A. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$
- B. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a complete binary tree is $\Theta(n \log n)$
- C. The cost of searching a binary search tree is $O(\log n)$ but that of an AVL tree is $\Theta(n)$
- D. The cost of searching an AVL tree is $\Theta(n \log n)$ but that of a binary search tree is $O(n)$

Question GATE-2008

You are given the postorder traversal, P , of a binary search tree on the n elements $1, 2, \dots, n$. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

- A. $\Theta(\log n)$
- B. $\Theta(n)$
- C. $\Theta(n \log n)$
- D. None of the above, as the tree cannot be uniquely determined

Question GATE-2009

Suppose the numbers 7,5,1,8,3,6,0,9,4,2 are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?

- A. 7510324689
- B. 0243165987
- C. 0123456789
- D. 9864230157

Question GATE-2012

The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is

- A. $\Theta(n \log n)$
- C. $\Theta(n)$
- B. $\Theta(n2^n)$
- D. $\Theta(\log n)$

$$\log_2 (\text{no. of elements})$$

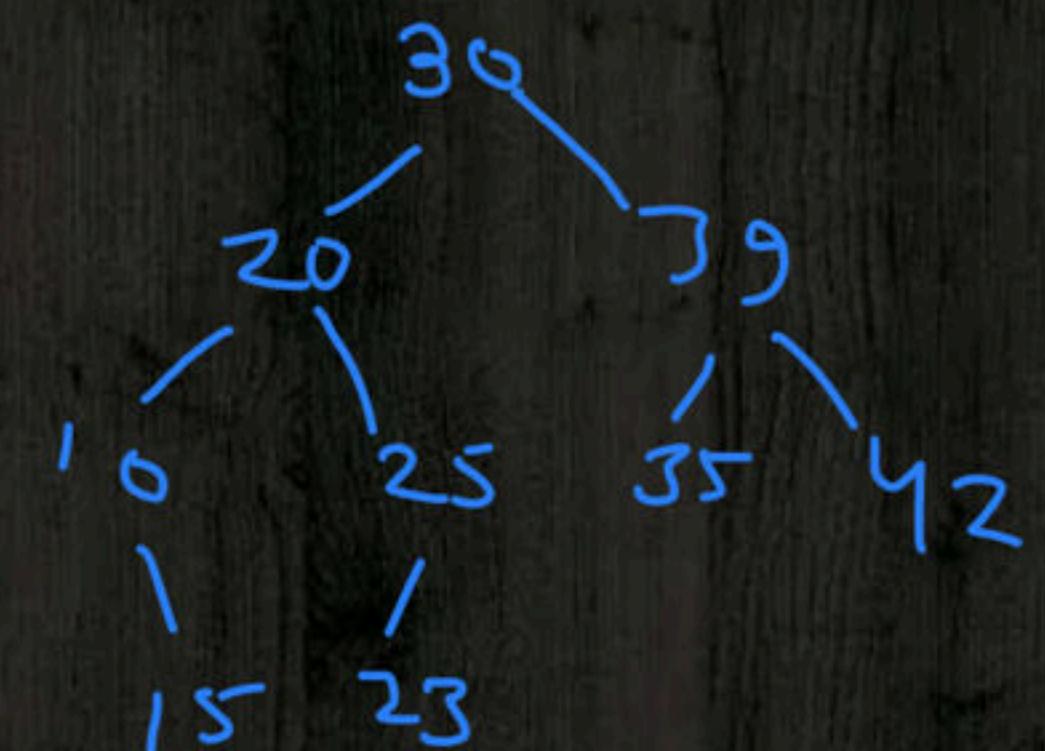
$$= \log_2 (n2^n)$$

$$= \log_2 n + \log_2 2^n = \log n + n$$

Question GATE-2013

The preorder traversal sequence of a binary search tree is 30,20,10,15,25,23,39,35,42 . Which one of the following is the postorder traversal sequence of the same tree?

- A. 10,20,15,23,25,35,42,39,30
- B. 15,10,25,23,20,42,35,39,30
- C. 15,20,10,23,25,42,35,39,30
- D. 15,10,23,25,20,35,42,39,30



15, 10, 23, 25, 20, 35, 42, 39, 30

Question GATE-2013

Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?

- A. $O(1)$
- B. $O(\log n)$
- C. $O(n)$
- D. $O(n \log n)$

Question GATE-2014

Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H . Suppose there are m such numbers in T . If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is _____.

Question GATE-2015

While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

- (A) 65
- (B) 67
- (C) 69
- (D) 83

Question GATE-2015

Which of the following is/are correct in order traversal sequence(s) of binary search tree(s)?

- I. 3,5,7,8,15,19,25
 - II. 5,8,9,12,10,15,25
 - III. 2,7,10,8,14,16,20
 - IV. 4,6,7,9,18,20,25
-
- A. I and IV only
 - B. II and III only
 - C. II and IV only
 - D. II only

Question GATE-2015

What are the worst-case complexities of insertion and deletion of a key in a binary search tree?

- A. $\Theta(\log n)$ for both insertion and deletion
- B. $\Theta(n)$ for both insertion and deletion
- C. $\Theta(n)$ for insertion and $\Theta(\log n)$ for deletion
- D. $\Theta(\log n)$ for insertion and $\Theta(n)$ for deletion

Question GATE-2017

Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are:

Note: The height of a tree with a single node is 0.

- (A) 4 and 15 respectively
- (B) 3 and 14 respectively
- (C) 4 and 14 respectively
- (D) 3 and 15 respectively

Question GATE-2017

The pre-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20.

Then the post-order traversal of this tree is:

- (A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20
- (B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12
- (C) 7, 2, 6, 8, 9, 10, 20, 17, 19, 15, 16, 12
- (D) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12

Question GATE-2020

The preorder traversal of a binary search tree is 15, 10, 12, 11, 20, 18, 16, 19.
Which one of the following is the postorder traversal of the tree?

- A. 10, 11, 12, 15, 16, 18, 19, 20
- B. 11, 12, 10, 16, 19, 18, 20, 15
- C. 20, 19, 18, 16, 15, 12, 11, 10
- D. 19, 16, 18, 20, 11, 12, 10, 15

Question GATE-2020

In a balanced binary search tree with n elements, what is the worst case time complexity of reporting all elements in range $[a, b]$? Assume that the number of reported elements is k .

- A. $\Theta(\log n)$
- B. $\Theta(\log n + k)$
- C. $\Theta(k \log n)$
- D. $\Theta(n \log k)$

Question GATE-2021

A binary search tree T contains n distinct elements. What is the time complexity of picking an element in T that is smaller than the maximum element in T ?

- A. $\Theta(n \log n)$
- B. $\Theta(n)$
- C. $\Theta(\log n)$
- D. $\Theta(1)$

Question GATE-2022

Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index _____.



HeapPyQs

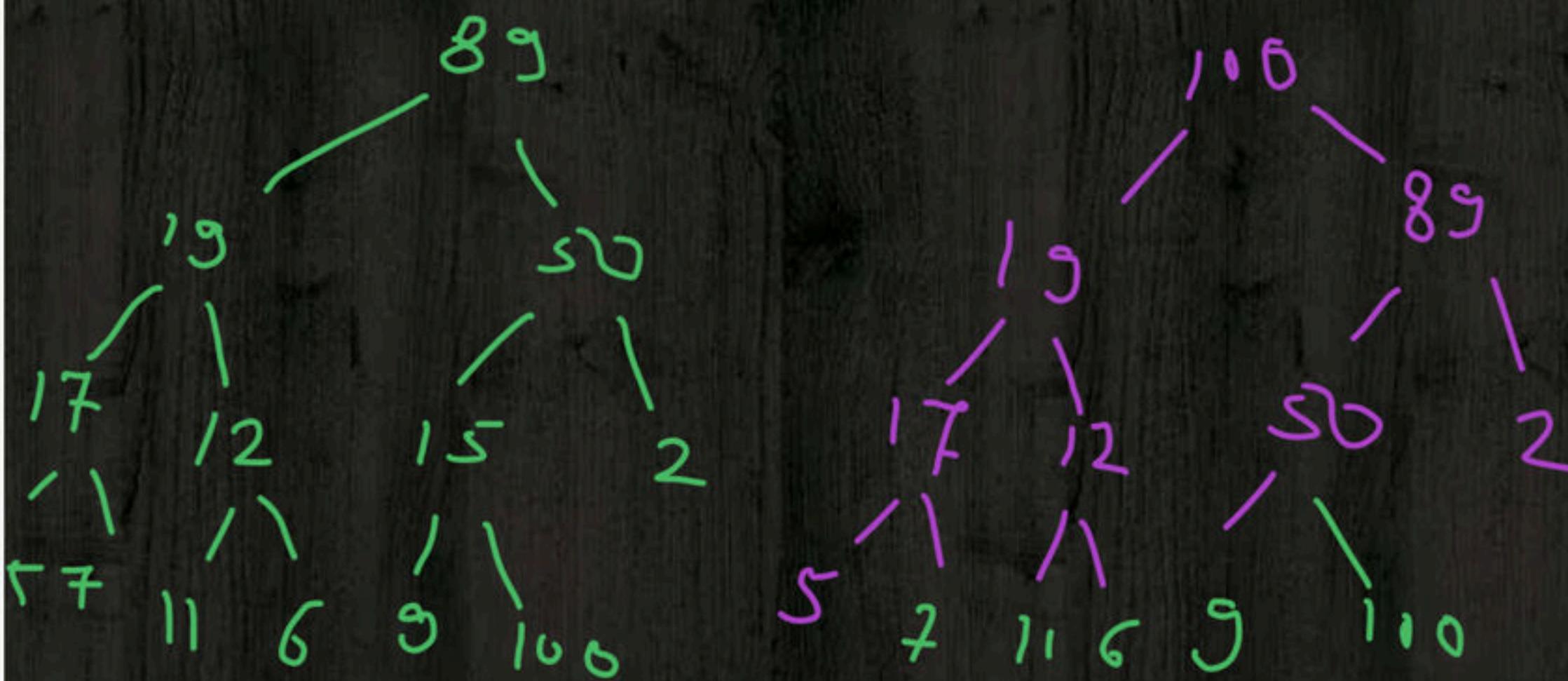
Question GATE-2015

Consider the following array of elements.

$\langle 89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100 \rangle$

The minimum number of interchanges needed to convert it into a max-heap is

- A. 4
- B. 5
- C. 2
- D. 3

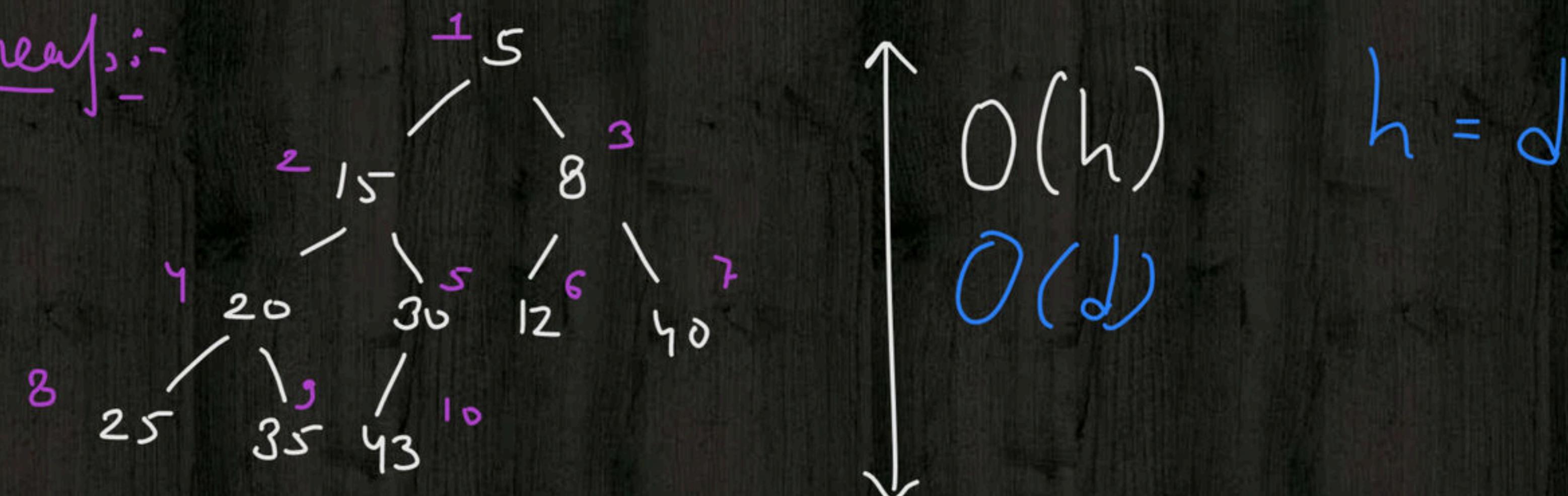


Question GATE-2016

An operator $\text{delete}(i)$ for a binary heap data structure is to be designed to delete the item in the i -th node. Assume that the heap is implemented in an array and i refers to the i -th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

- A. $O(1)$
- B. $O(d)$ but not $O(1)$
- C. $O(2^d)$ but not $O(d)$
- D. $O(d \cdot 2^d)$ but not $O(2^d)$

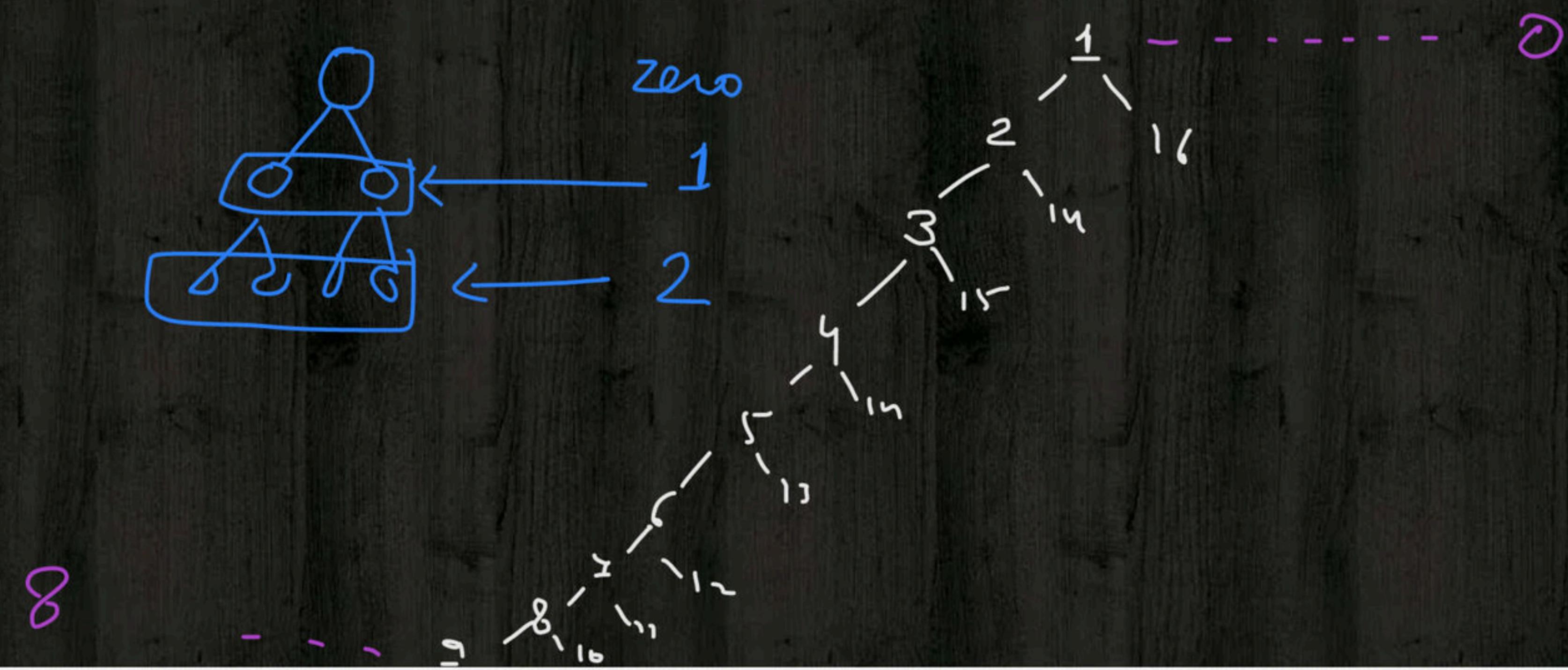
Ex:- min-heap :-



$$Ans = 8$$

Question GATE-2016

A complete binary min-heap is made by including each integer in $[1, 1023]$ exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _____.



Ans = 80

Question GATE-2018

The number of possible min-heaps containing each value from $\{1, 2, 3, 4, 5, 6, 7\}$ exactly once is _____

$$T(n) = \sum_{k=0}^{n-1} C_k * T(k) * T(n-k-1) = {}^6C_3 * T(3) *$$

$$\begin{aligned} k &= \text{no. of nodes in left subtree of root} \\ n &= \text{Total no. of nodes} \end{aligned} \quad \begin{aligned} &= 20 * 2 * 2 \\ &= 80 \end{aligned}$$

Question GATE-2019

Consider the following statements:

- I. The smallest element in a max-heap is always at a leaf node
- II. The second largest element in a max-heap is always a child of a root node
- III. A max-heap can be constructed from a binary search tree in $\theta(n)$ time
- IV. A binary search tree can be constructed from a max-heap in $\theta(n)$ time

Which of the above statements are TRUE?

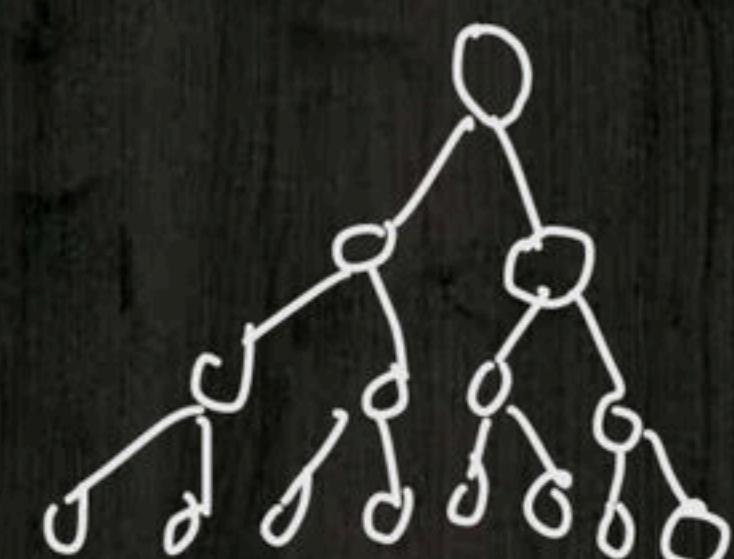
- A. I, II and III
- B. I, II and IV
- C. I, III and IV
- D. II, III and IV

Question GATE-2020

Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is _____.

In min-heap, max element should be present on leaf node.

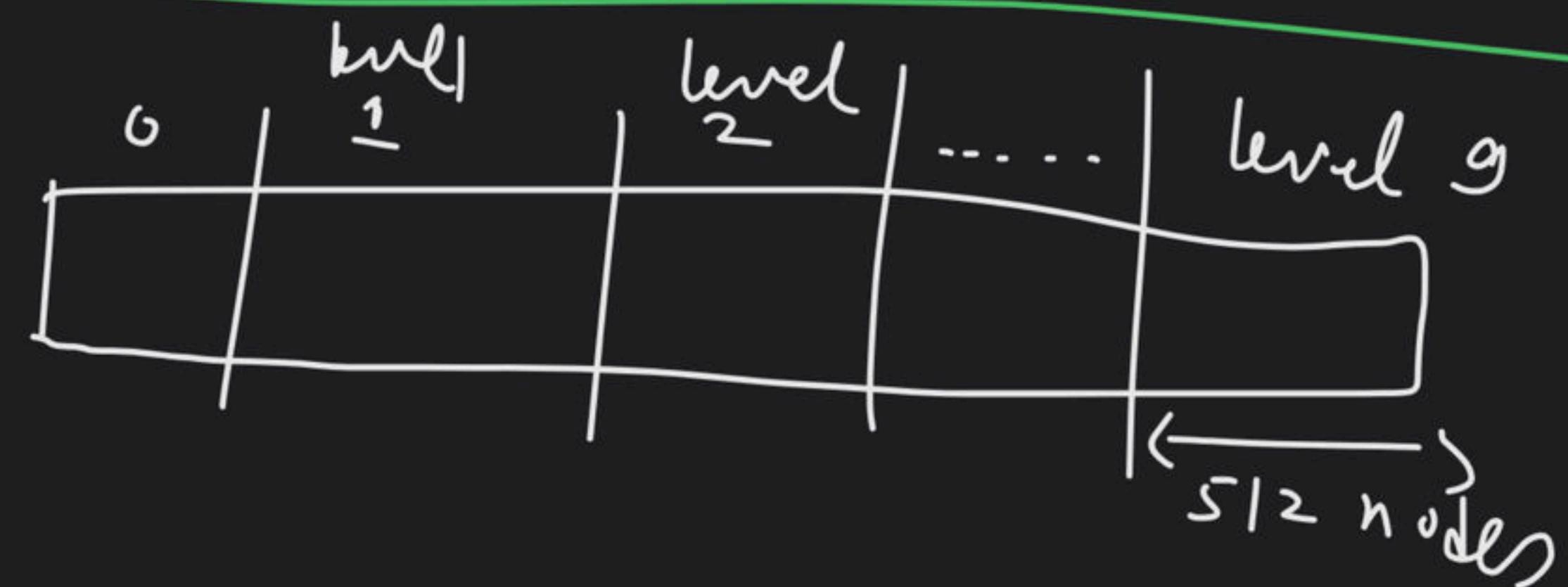
↪ get max of only leaf nodes \Rightarrow max of heap.



$$\text{leaf nodes} = \frac{2^{10} - 1}{2}$$

to find max of 512 elements,

No. of Comparisons = $512 - 1 = 511$ Ans.



Ques) Consider an array which represent min-heap of n distinct integers. The smallest index of array which contains a leaf node is _____ ?

Sol

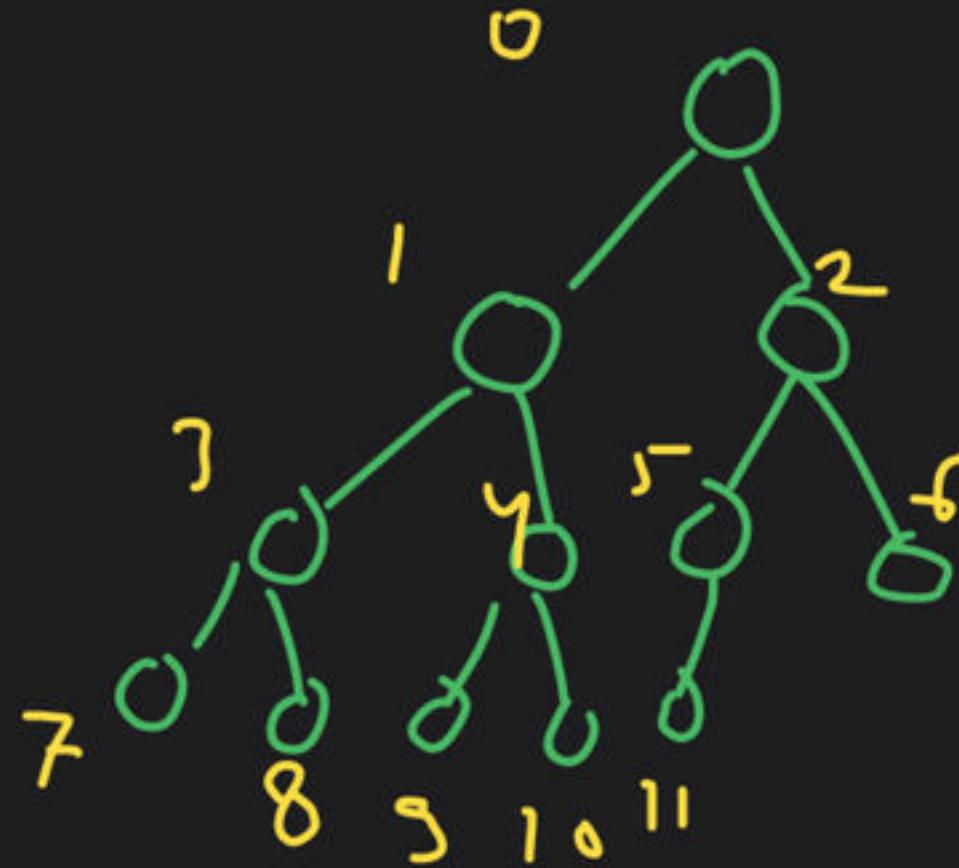
Index starts from 0

$$\left\lfloor \frac{n}{2} \right\rfloor$$

Index starts from 1

$$\left\lfloor \frac{n}{2} \right\rfloor + 1 \text{ or}$$

$$\left\lceil \frac{n+1}{2} \right\rceil$$



$$\left\lfloor \frac{n}{2} \right\rfloor$$



$$\left\lceil \frac{n+1}{2} \right\rceil \text{ or } \left\lfloor \frac{n}{2} \right\rfloor + 1$$

$$n = 15 \\ L = 7$$

$$n = 13 \\ L = 6$$

$$n = 14 \\ L = 7$$

$$n = 12 \\ L = 6$$

$$h = 10 \quad n = 12 \\ L = 6 \quad L = 7 \\ h = 11 \quad n = 13 \\ L = 6 \quad L = 7$$

Happy Learning