Recursion - Part I

Complete Course on Algorithm for GATE - CS & IT

(a) $f(n) = n+5$

$g(n) = n$

$n+5 \leq c \cdot n, \forall n, n \geq n_0$

$c = 2$, $n_0 = 5$

$n+5 = O(n)$

(b) $f(n) = n$, $g(n) = n+5$

$n \leq c \cdot (n+5)$ $\quad n_0 = 1$

$c = 1$

$n = O(n+5)$

Subbarao Lingamgunta • Lesson 7 • Dec 1, 2022

Ex $f(n) = n^2+n+10$, $g(n) = n^2$

$f(n) = O(g(n))$?

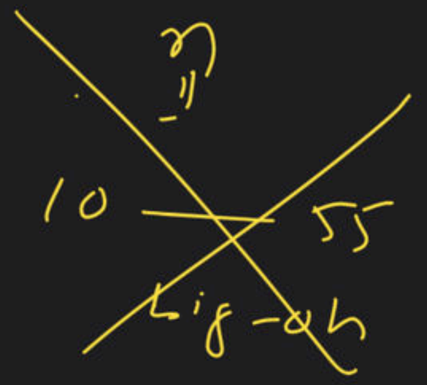
$\Updownarrow$

$n^2+n+10 \leq c \cdot n^2$, $\forall n$, $n \geq n_0$

$\Updownarrow$ $\Downarrow$

30 3

$n^2+n+10 = O(n^2)$



Ex $f(n) = n^2$, $g(n) = n$

$f(n) = O(g(n)) \iff n^2 \leq c \cdot n$, $\forall n$, $n \geq n_0 \iff n^2 \neq O(n)$

$\Downarrow$ $\Downarrow$

$\bot$

Theta - Notation

$$f(n) = \Theta(g(n))$$

iff

① $f(n) \geq c_1 \cdot g(n)$

and

② $f(n) \leq c_2 \cdot g(n)$

$\forall n, n \geq n_0$

Omega - Notation $(\Omega, \geq)$

$f(n) = \Omega(g(n))$

iff

$f(n) \geq c \cdot g(n), \forall n, n \geq n_0$

Such that $\exists$ 2 true constants $c > 0$, $n_0 \geq 1$

ex

- $f(n) = n$, $g(n) = n+5$

$f(n) = \Omega(g(n))$

$\Uparrow$

① $1 \cdot n \geq c \cdot (n+5)$, $\forall n$, $n \geq n_0$ $\Longleftrightarrow$ $n = \Omega(n+5)$

$c \downarrow \frac{1}{2}$ $\quad n_0 \downarrow 5$

$n \leq c_2 \cdot (n+5)$, $\forall n, n \geq n_0$

$\Downarrow$ $\quad\quad$ $\Downarrow$

$n = O(n+5)$

$c_1 = \frac{1}{2}$ $\quad c_2 = 1$

$n_0 = 5$

$n = \Theta(n+5)$

ex $f(n) = n^2+n+10$, $g(n) = n^2$

$f(n) = \Theta(g(n))$?

Asymptotically equal.

① $n^2+n+10 \geq c_1 \cdot n^2$, $\forall n$, $n \geq n_0$

$\Downarrow$ 1 $\qquad$ $\Downarrow$ 1

② $n^2+n+10 \leq c_2 \cdot n^2$, $\forall n$, $n \geq n_0$

$\Downarrow$ 3 $\qquad$ $\Downarrow$ 3

$n^2+n+10 = \Omega(n^2)$

$n^2+n+10 = O(n^2)$

$n^2+n+10 = \Theta(n^2)$

$c_1 = 1, c_2 = 3$

$n_0 = 3$

ex $f(n) = n$ $g(n) = n$

$f(n) = \Theta(g(n))$?

$n \geq c_1 \cdot n$, $\forall n$, $n \geq n_0$

$\Downarrow$ $\Downarrow$

1 1

$n \leq c_2 \cdot n$, $\forall n$, $n \geq n_0$

$\Downarrow$ $\Downarrow$

1 1

$c_1 = 1, c_2 = 1$

$n_0 = 1$

$n = \Theta(n)$

ex $f(n) = n$ $g(n) = n^2$

$f(n) = \Theta(g(n))$?

$n = \Omega(n^2)$ ✗

$n \geq c_1 \cdot n^2, \forall n, n \geq n_0$

$\Downarrow$

✗

&&

$n \leq c_2 \cdot n^2, \forall n, n \geq n_0$

$\Downarrow$ $\Downarrow$

$n = O(n^2)$ ✓

$n \neq \Theta(n^2)$

ex ②

$f(n) = n+5, \quad g(n) = n$

$f(n) = \Omega(g(n))$

$\Updownarrow$

$n+5 \geq c \cdot n, \ \forall n, n \geq n_0$

$\Downarrow$ $\quad$ $\Updownarrow$ $\quad$ $\Downarrow$

$n+5 = \Omega(n)$

ex $f(n) = n \cdot g(n) = n^2$

$f(n) = \Omega(g(n))$

$\Updownarrow$

$n \geq c \cdot n^2, \ \forall n, n \geq n_0$

$\Downarrow$

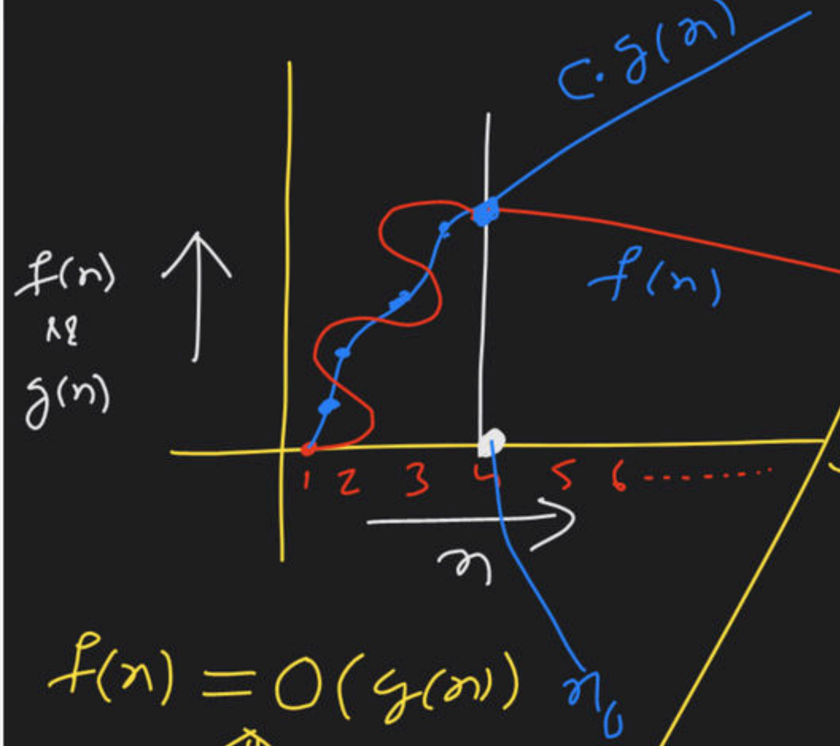
$n \neq \Omega(n^2)$

$f(n)$ $\quad$ $g(n)$

$1 \cdot n^2 \geq c(n^2+n+1)$ $\quad \forall n, n \geq n_0$

$\Downarrow$ $\quad$ $\frac{1}{2}$ $\quad$ $2 \ast$

$n^2 = \Omega(n^2+n+1)$



$f(n) = O(g(n))$

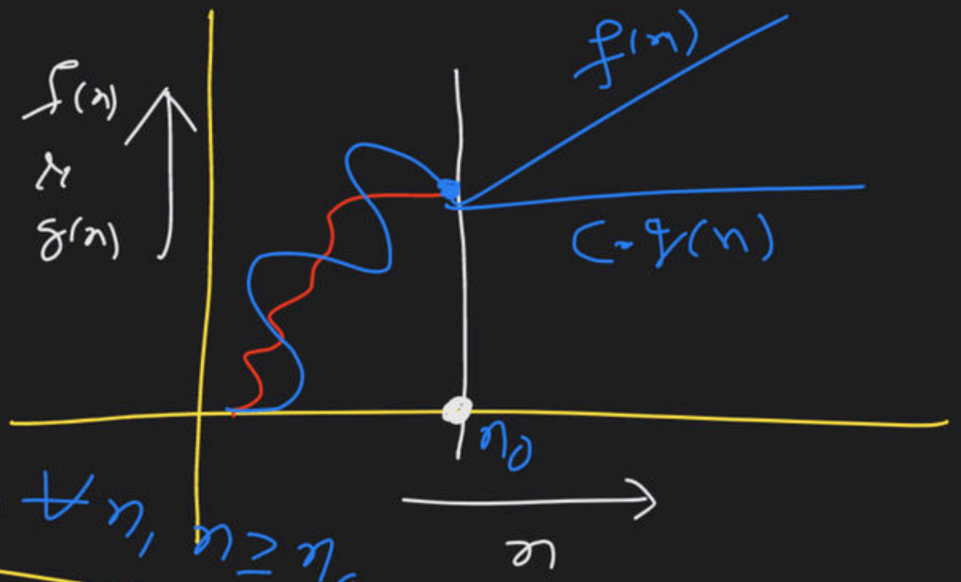
$\Updownarrow$

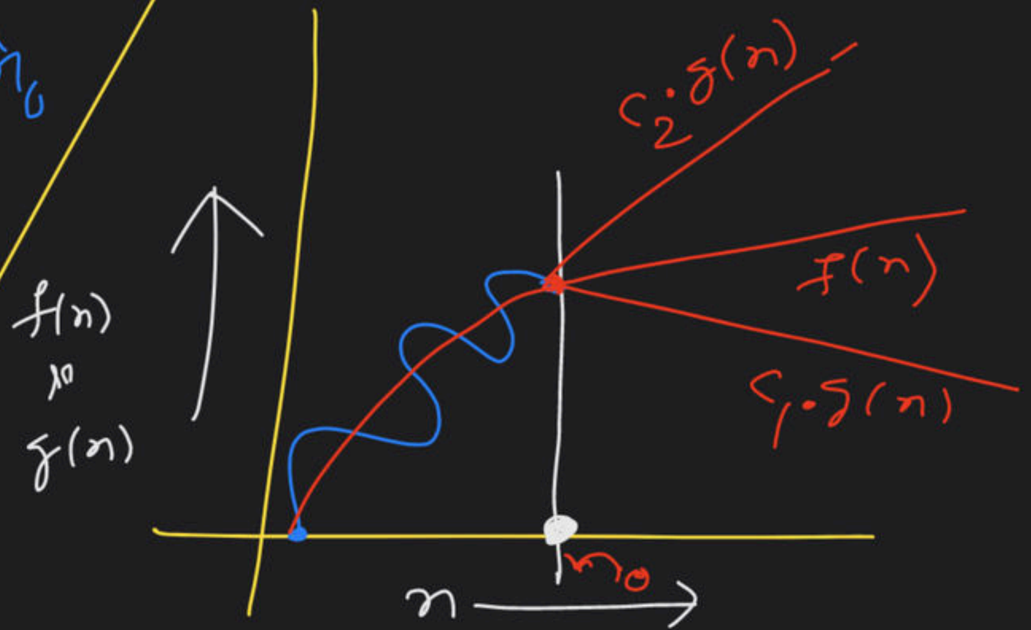
$f(n) \leq c \cdot g(n)$ $\forall n_1, n_2 \geq n_0$

$f(n) = \Omega(g(n))$

$\Updownarrow$

$f(n) \geq c \cdot g(n)$, $\forall n_1, n_2 \geq n_0$





$f(n) = \Theta(g(n))$

$\Updownarrow$

① $f(n) \geq c_1 \cdot g(n)$

&

② $f(n) \leq c_2 \cdot g(n)$