Nice work

30 days DMS Nicely

8

2 to 4 marks

Graph Theory - Part I

Complete Course on Discrete Mathematics

$1k \Rightarrow 90\left(\left(\frac{1-T}{5}\right)\right)$

$\frac{29}{5} - \frac{29}{0}$

Rosen (1500 pages)

2 ⟨ 1, 1 ⟩ 2

2 ⟨ 1, 2 ⟩

2 ⟨ 2, 2 ⟩ 4

GT + Algo

GT + DS

Subbarao Lingamgunta • Lesson 1 • Sept 29, 2023

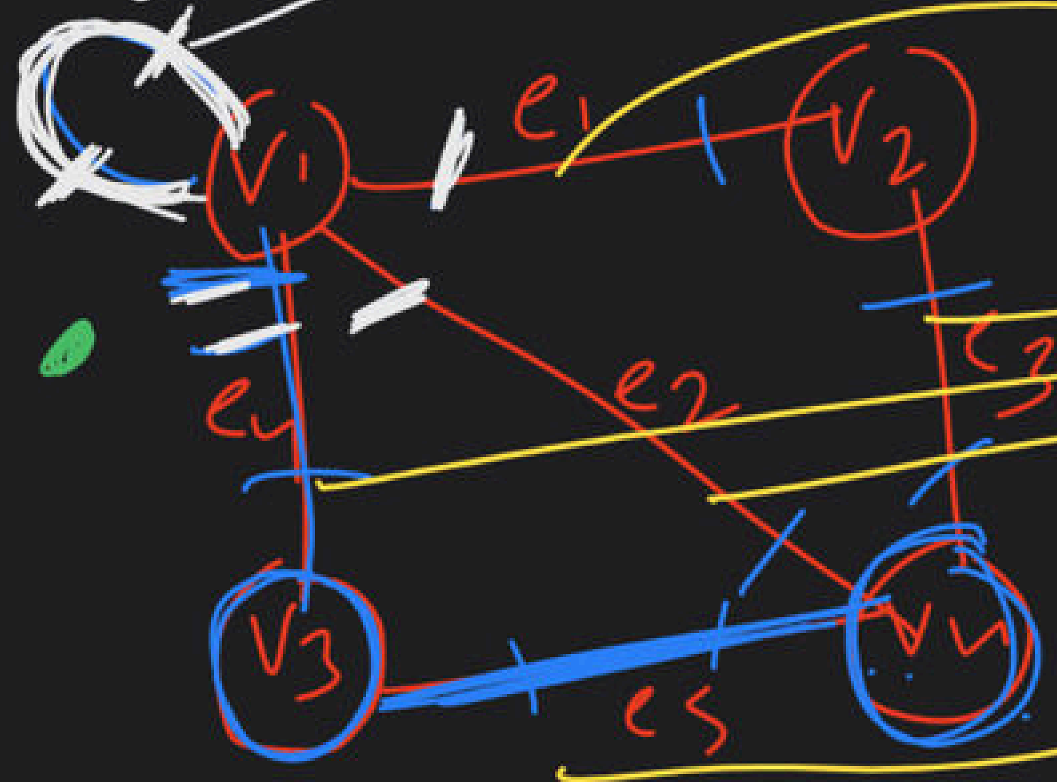
#PYQs

Graph $G = (V, E)$

E: Set of edges (or) lines

V: Set of vertices (or) Nodes

Undirected Graph



$\boxed{\text{Selfloop counts degree} \Rightarrow 2}$

$V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$

Adjacent

$\deg(V_3) = 2$

$\deg(V_4) = 3$

$\deg(V_2) = 2$

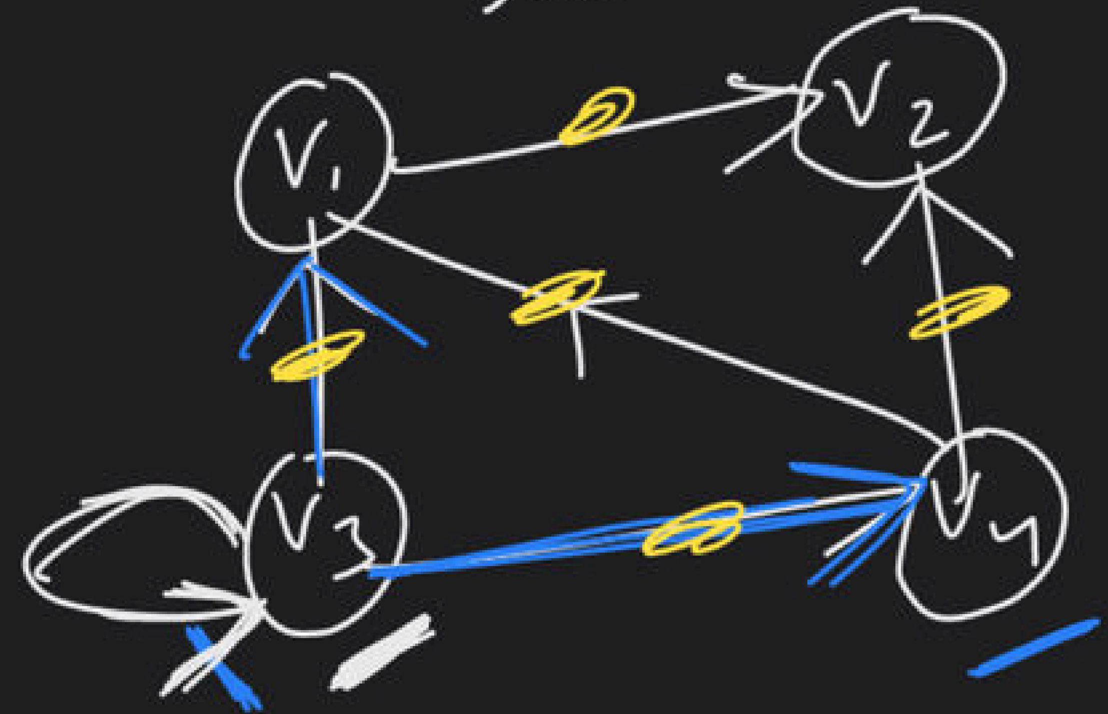
$\deg(V_1) = 3$

$2+3+2+3 \Rightarrow \frac{10}{2} = 5$

$v_1 — v_2$

$v_2 — v_1$

Directed Graph $\Rightarrow$ selfloop — I – 1, O – 1



$v_1 — v_2$ ✓

$v_2 — v_1$ ✗

$O(v_1) = 1$

$I(v_1) = 2$

$\text{outdeg}(V_3) = \cancel{2}\ 3$

$\text{Indeg}(V_3) = \cancel{0}\ 1$

$O(V_2) = 0$

$I(V_2) = 2$

$O(V_4) = 2$

$I(V_4) = 1$

$2+1+0+2 \Rightarrow 6$

$2+2+1+0 \Rightarrow 6$

no. of edges 5

Size of graph = No. of edges

Order of the graph = No. of vertices

Undirected graph

Sum of degrees $= \boxed{2 * |E|}$
even

(or)

$$\sum_{i=1}^{n} \text{degree}(V_i) = \boxed{2 * |E|}$$
even

$\frac{50}{2} = 25$ edges

$\frac{50}{2} = \frac{2E}{2}$

Sum of Indegrees = Sum of out degrees = no. of edges

(or)

Directed graph.

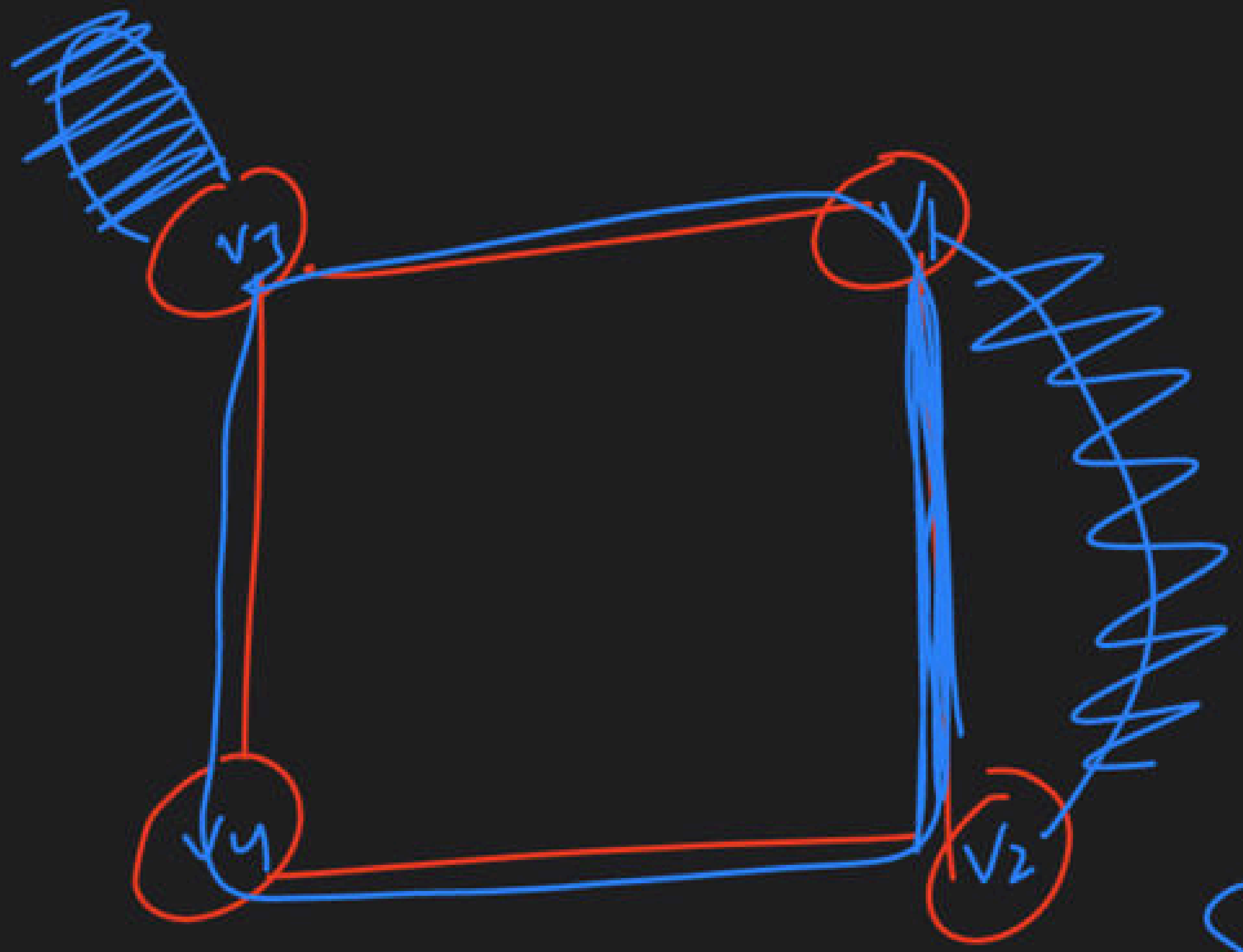
$$\sum_{i=1}^{n} \text{Indegree}(V_i) = \sum_{i=1}^{n} \text{Outdegree}(V_i) = |E|$$

$25 = 25 = E$

Sum of degrees is always even

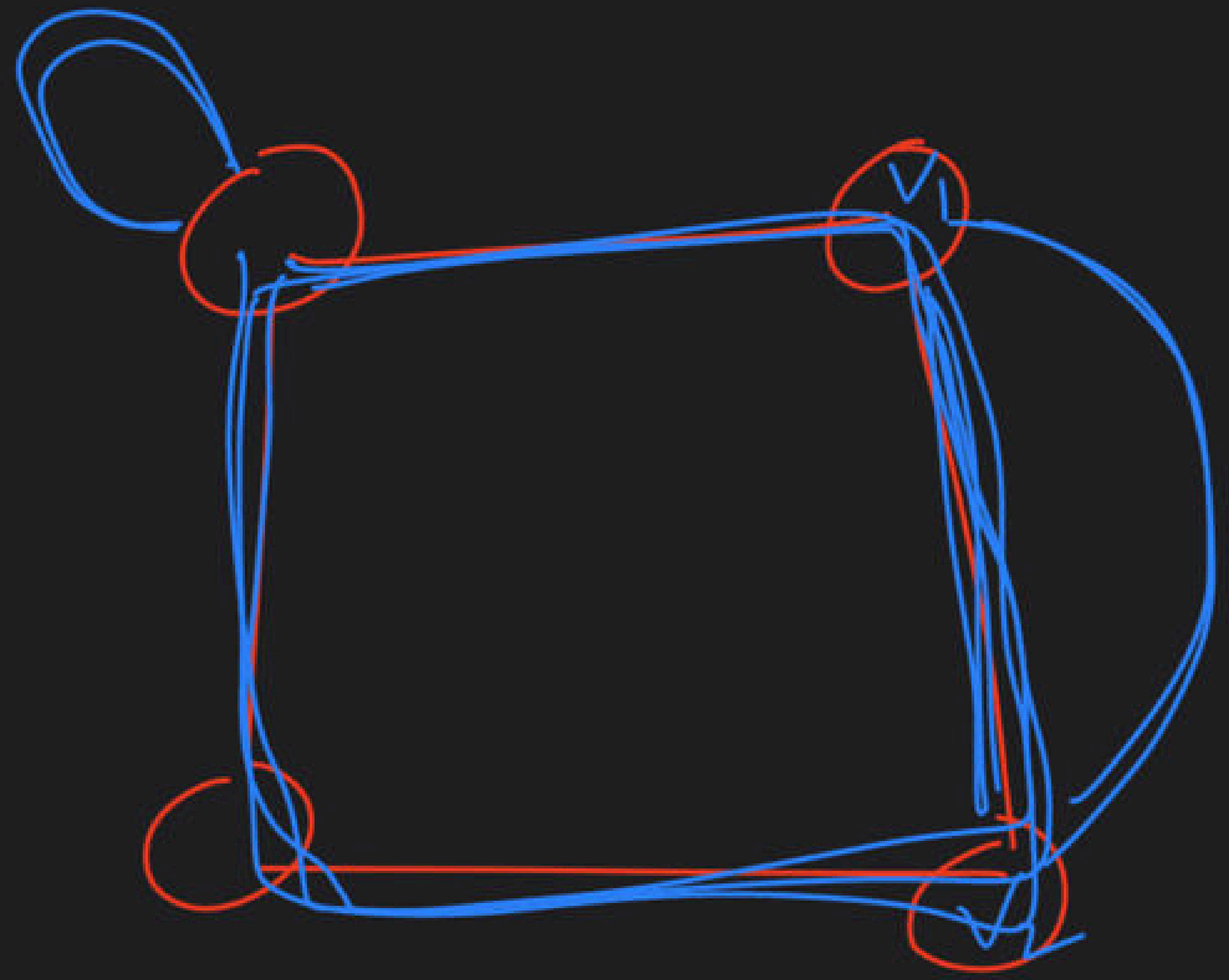
Graphs

Simple Graph



Self loops
&
parallel edges ✗

Multi Graph



Self loop allowed ✓
Parallel (or) edges ✓

If simple graph contain n-vertices

then what is max-degree possible to any vertex $= n-1$

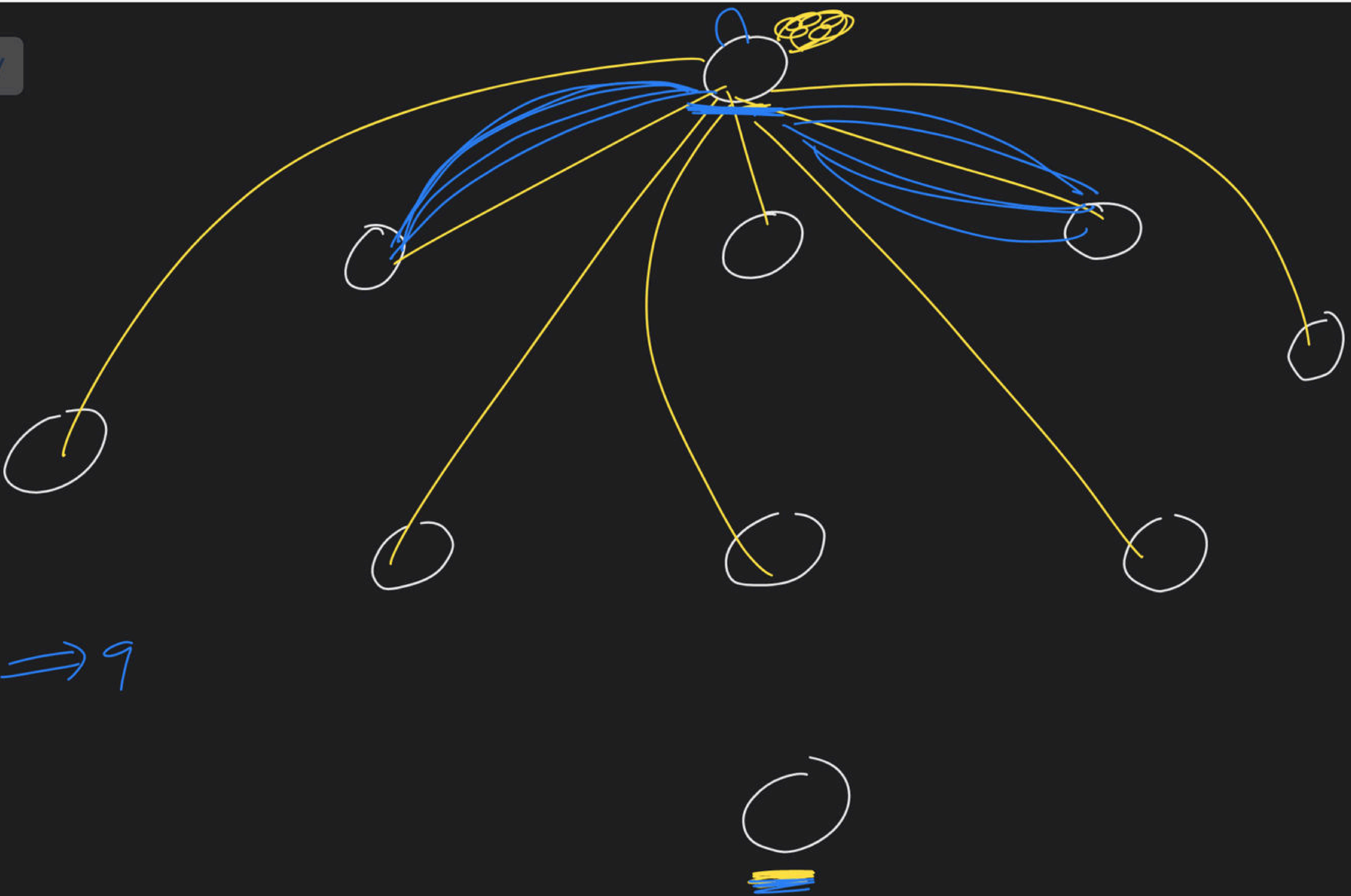
min- " " " " " $= 0$

Multi Graph

$\Downarrow$

$\boxed{\text{max-degree} = \infty}$

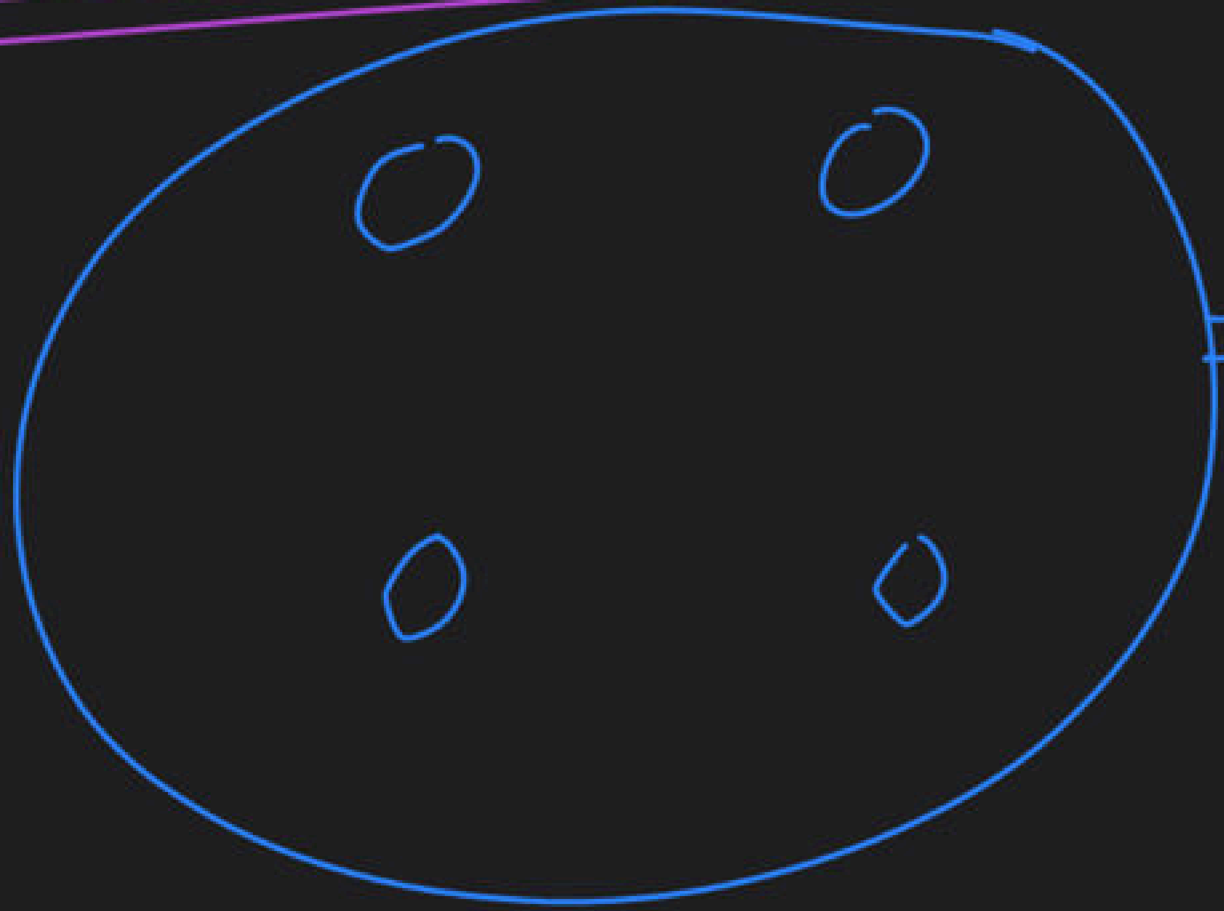
min-degree $= 0$



$10 \Rightarrow 9$

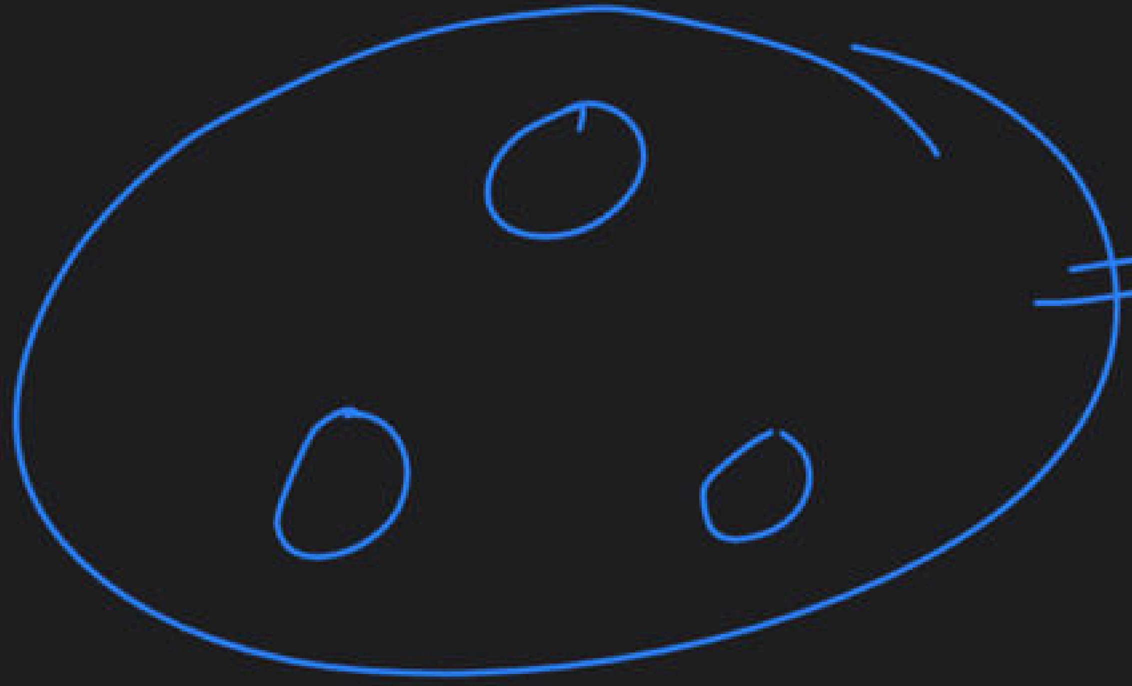
Simple Graphs

25V ⇒ Null Graph

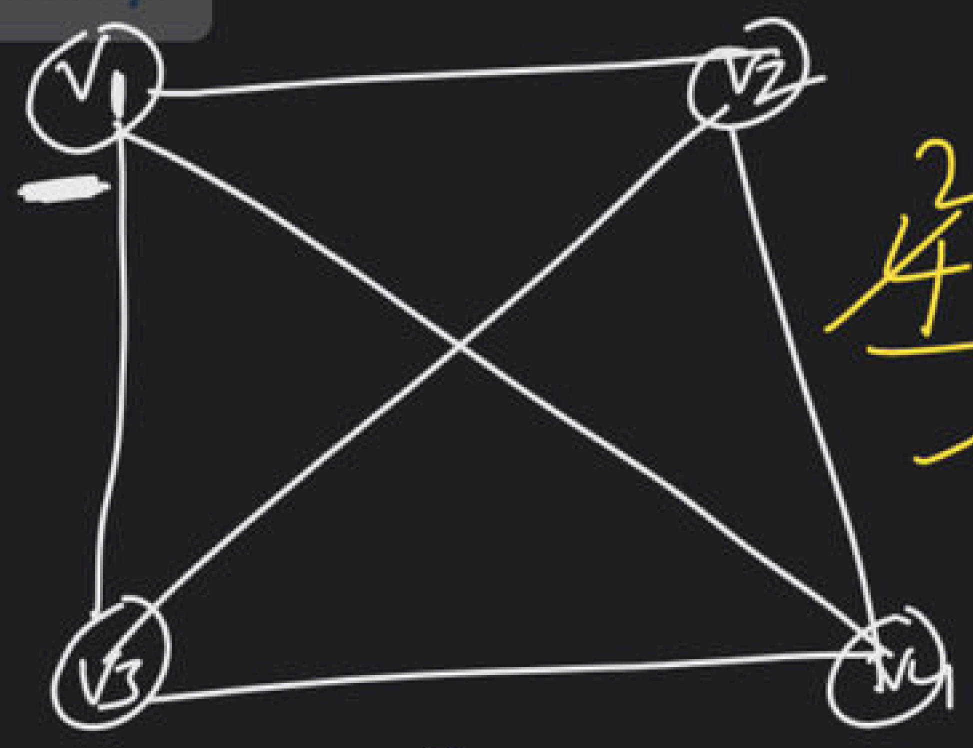


4V ✓
0-edges ⇒ 4V
Null Graph (or) Null Graph

Null Graph ⇒ 0-edges



3V
Null Graph ⇒ Null Graph

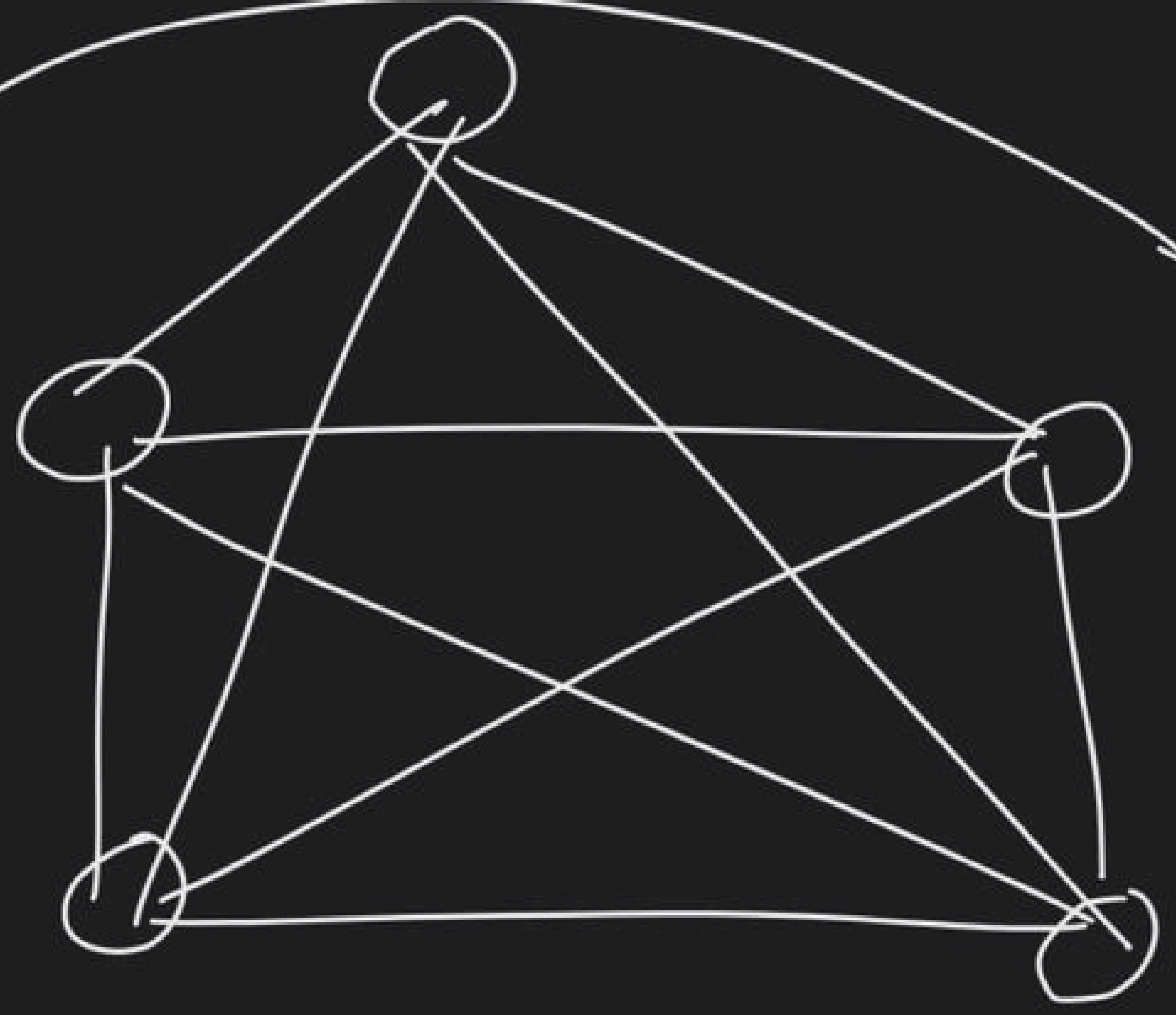


$$\frac{\cancel{4}^{2} \times 3}{\cancel{2}} \Rightarrow 6$$

4V – Complete Graph

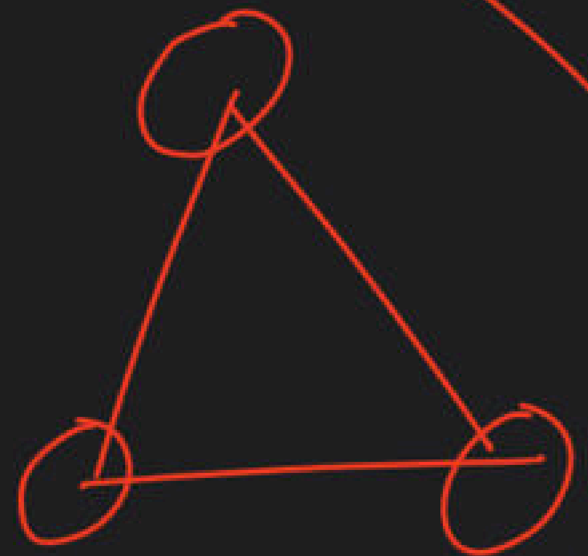
$\Downarrow$

K_4



$K_5 \Rightarrow \frac{5 \times 4}{2} \Rightarrow 10$

K_3



$\Rightarrow \frac{3 \times 2}{2} \Rightarrow 3$

$$\boxed{E(K_n) = \frac{n(n-1)}{2}}$$

$K_{25} \Longrightarrow \text{Edges} \Longrightarrow \frac{25 \times 24}{2} \Longrightarrow 25 \times 12 \Longrightarrow \underline{\underline{300}}$

Simple Graph $\Longrightarrow$ 25 V

min edges — 0 $\Longrightarrow$ Null Graph

max edges — 300 $\Longrightarrow$ Complete Gr

1

25 V

complete Graph

min-edge — 300

max edge — 300

2

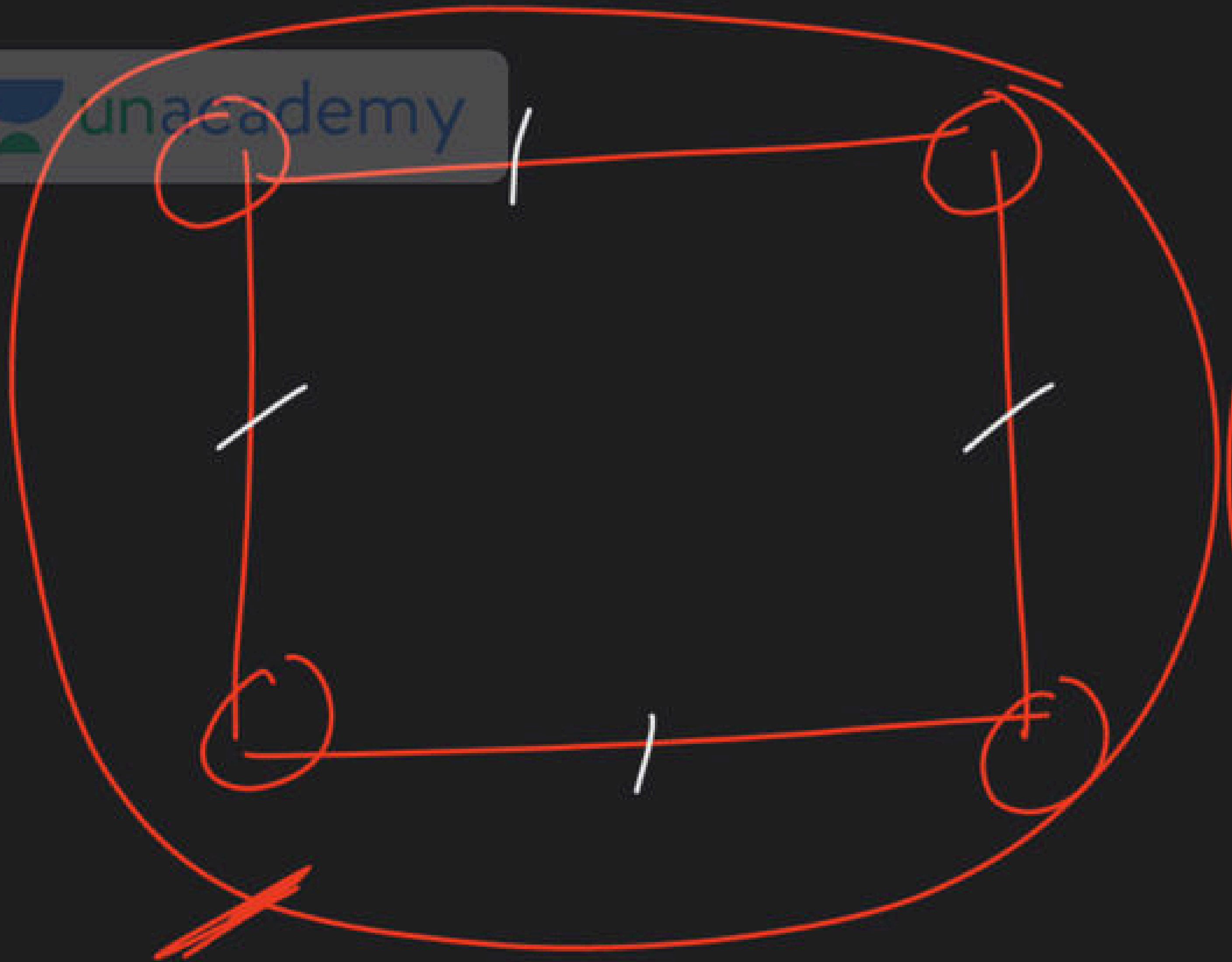
25 V

Null Graph

min-edge — 0

max edge — 0

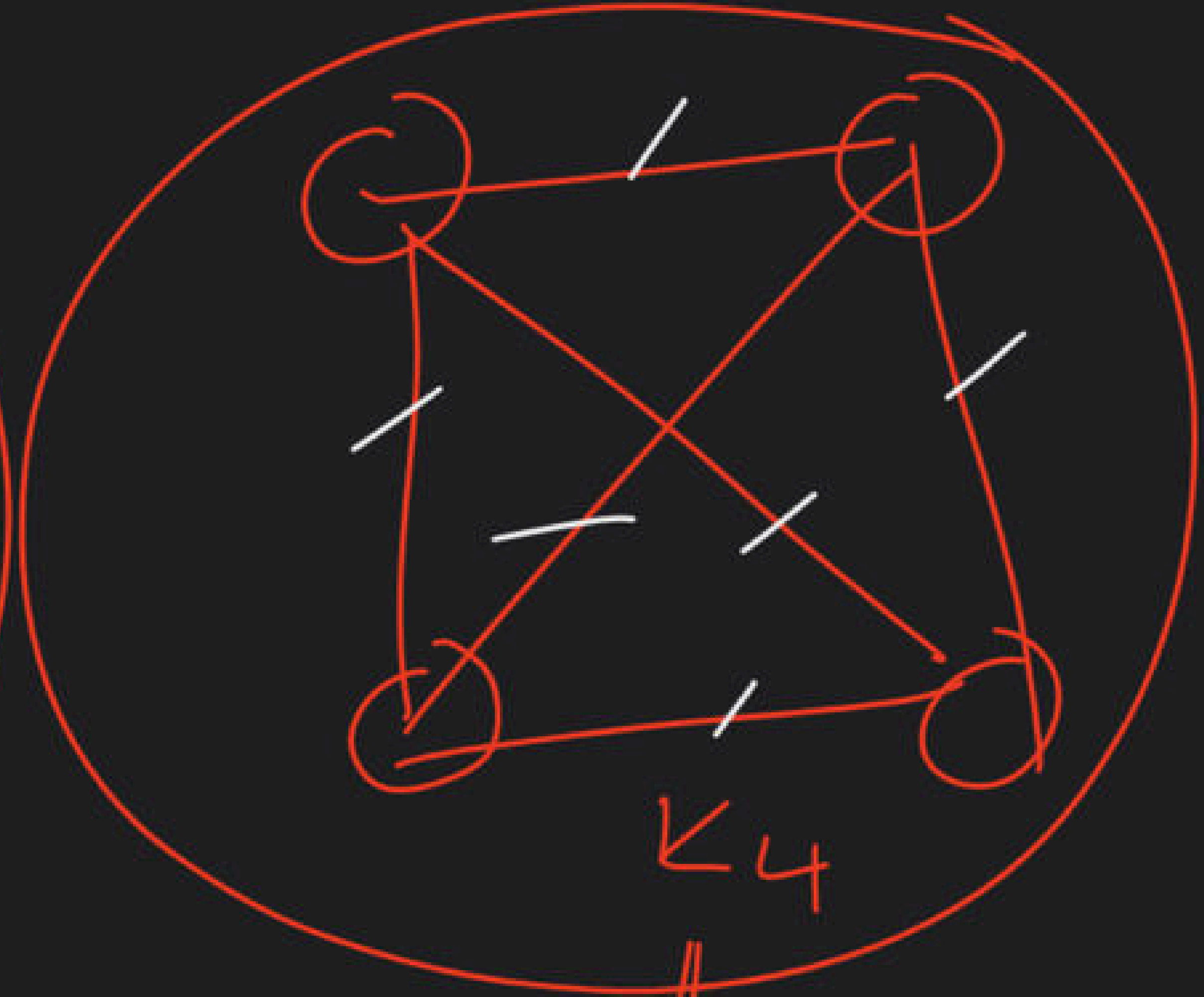
3



2-Regular Graph

$\Downarrow$

$$\frac{4*2}{2}=4$$



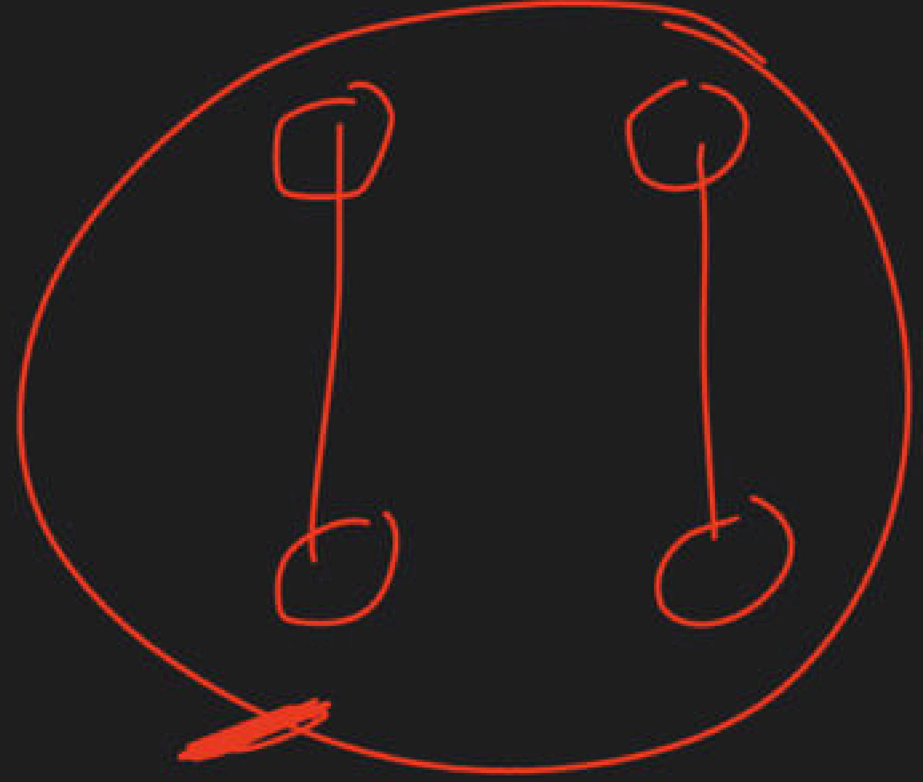
K_4

$\Downarrow$

3-regular graph

$\Downarrow$

$$\frac{4*3}{2}=6$$



1-regular graph

$\Downarrow$

$$\frac{4*1}{2}=2$$

Null Graph

$\Downarrow$

0-regular graph

$\Downarrow$

$$\frac{4*0}{2} = 0$$

$4V,\ 2R \Longrightarrow \text{Edges} \Longrightarrow \frac{4*2}{2}=4$

$24V,\ 3\text{-}R \implies \text{edges} \implies \frac{24 * 3}{2} = 12 * 3 = 36$

$V,\ K\text{-Reg} \implies \text{edges} \implies \frac{V * K}{2}$

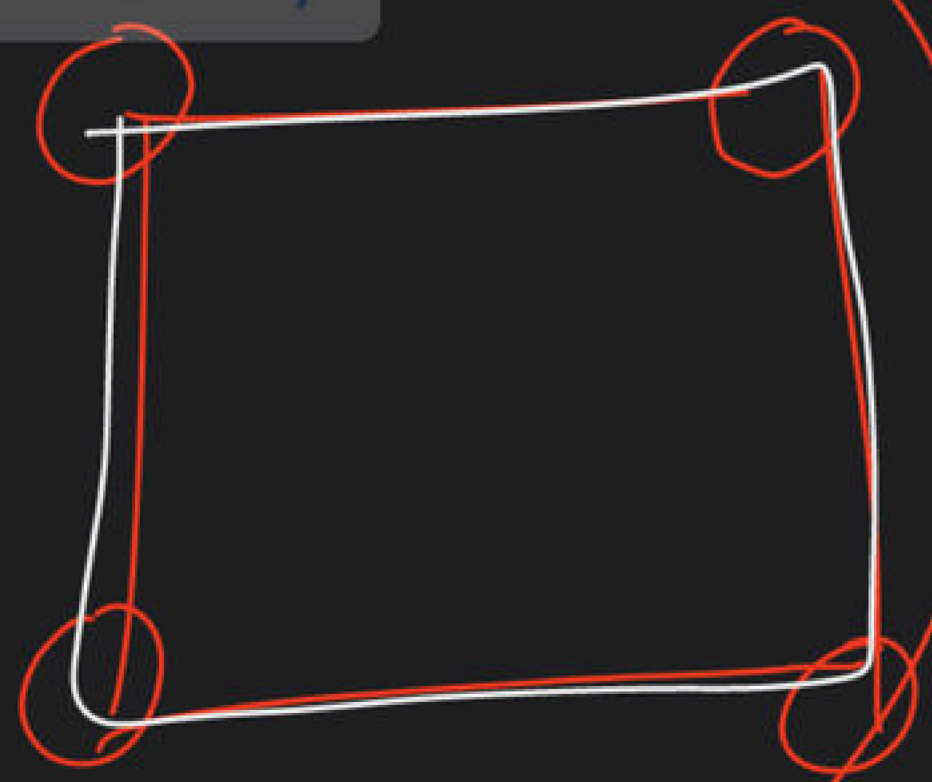
$\cancel{2+3+2+4}$

$2+2+2+2$

$4 * 2$

$K_{25} = 24\text{-Regular graph}$ | $K_n = n-1 \text{ regular graph}$

cycle-graph

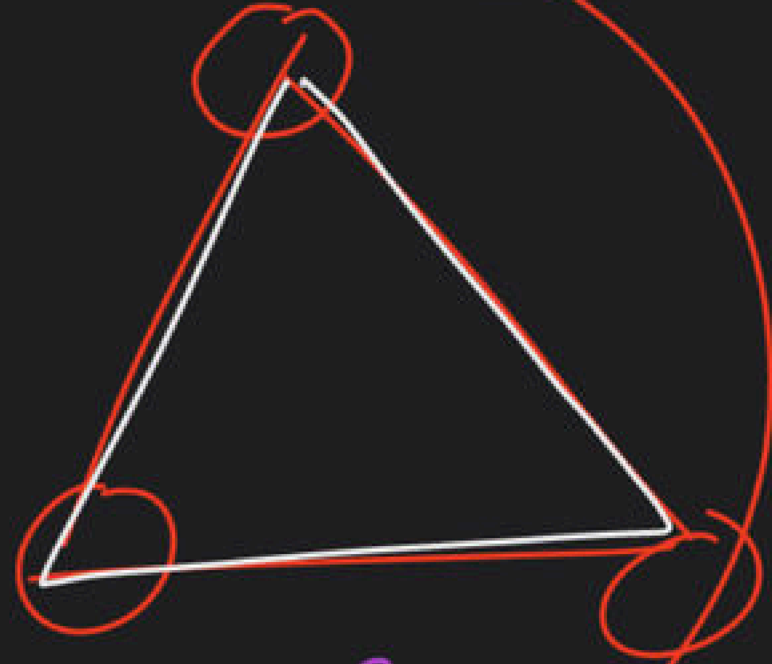


2-Reg

4V, 4e

$\Downarrow$

C_4

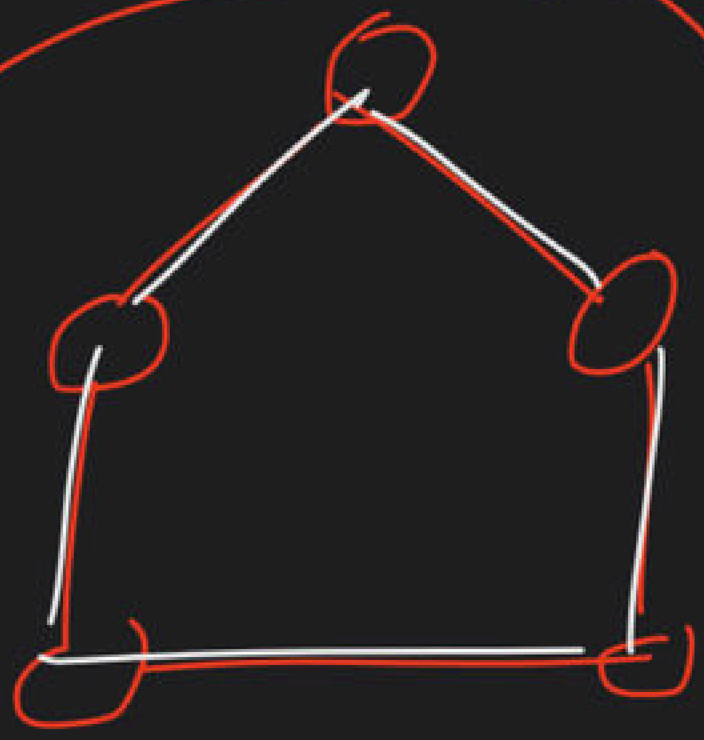


2-Reg

3V, 3e

$\Downarrow$

C_3



2-reg

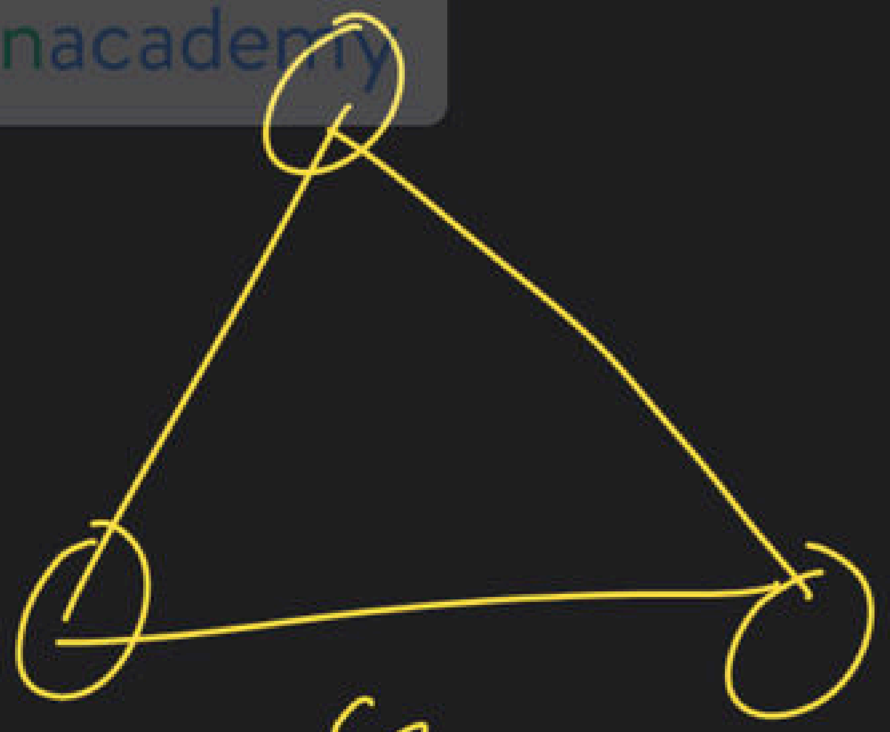
5V, 5e

$\Downarrow$

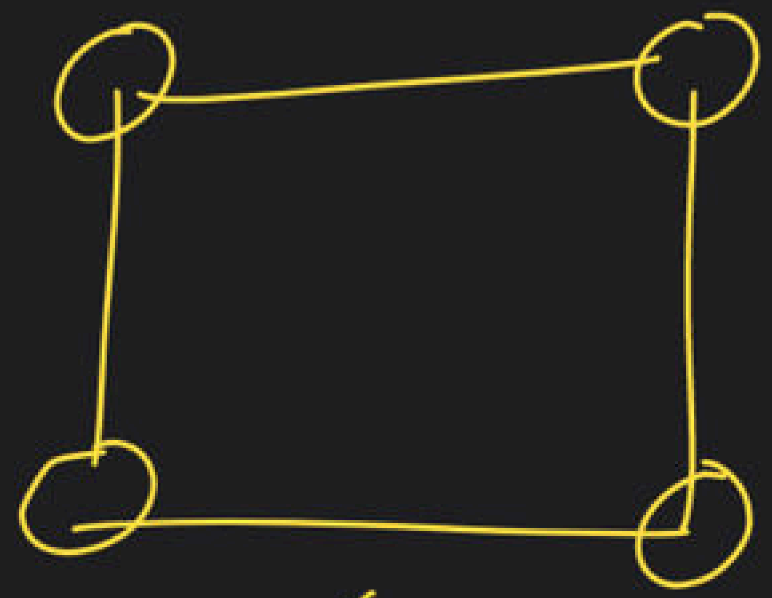
C_5

$C_n \Longrightarrow$ n-V, n-e

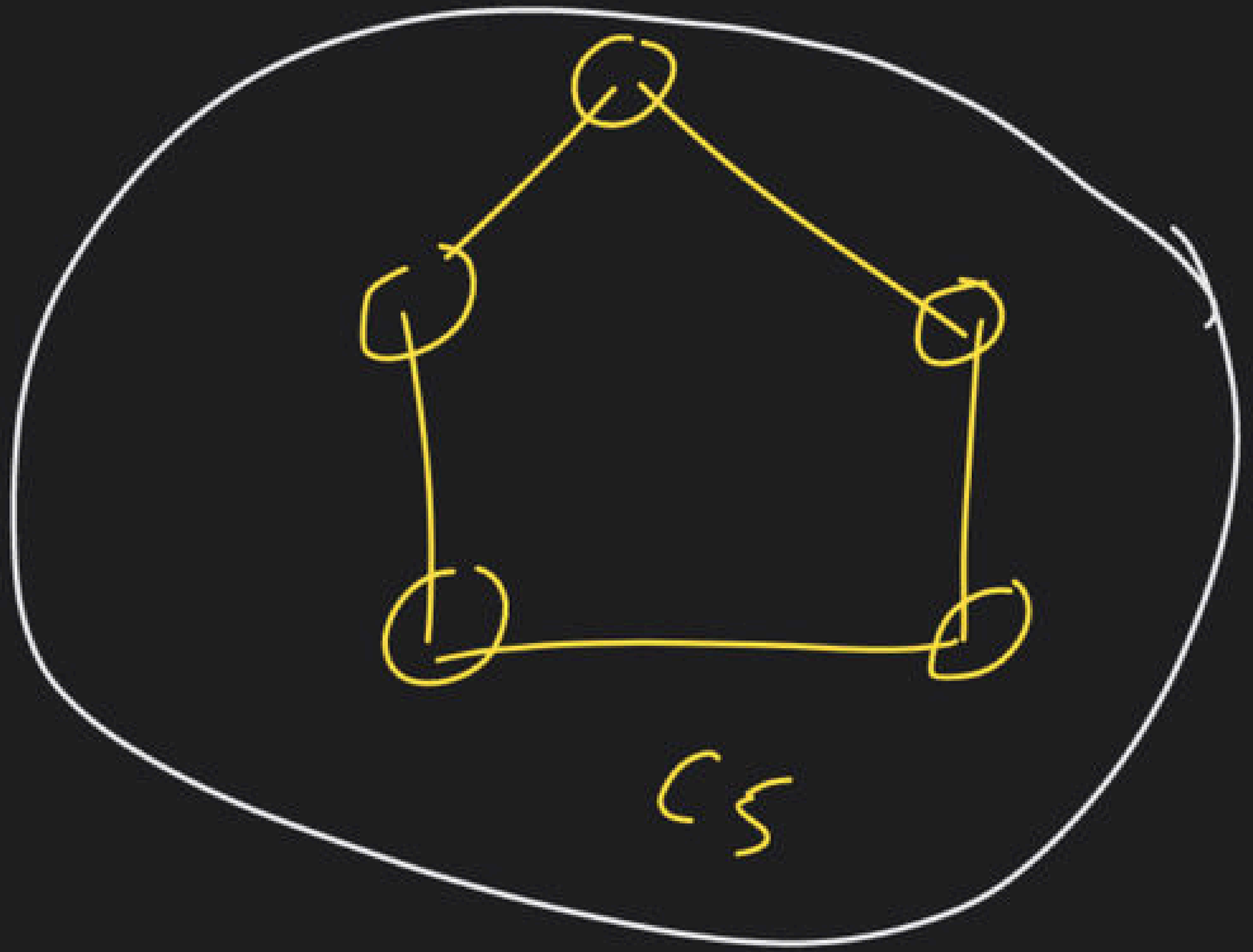
every C_n, $n \geq 3$ is 2-Reg graph



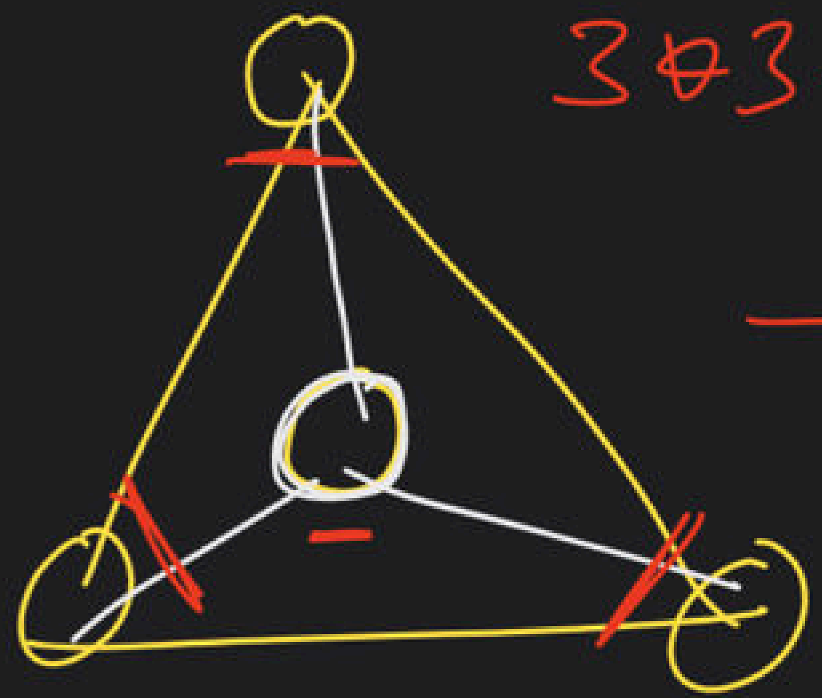
C_3



C_4



C_5

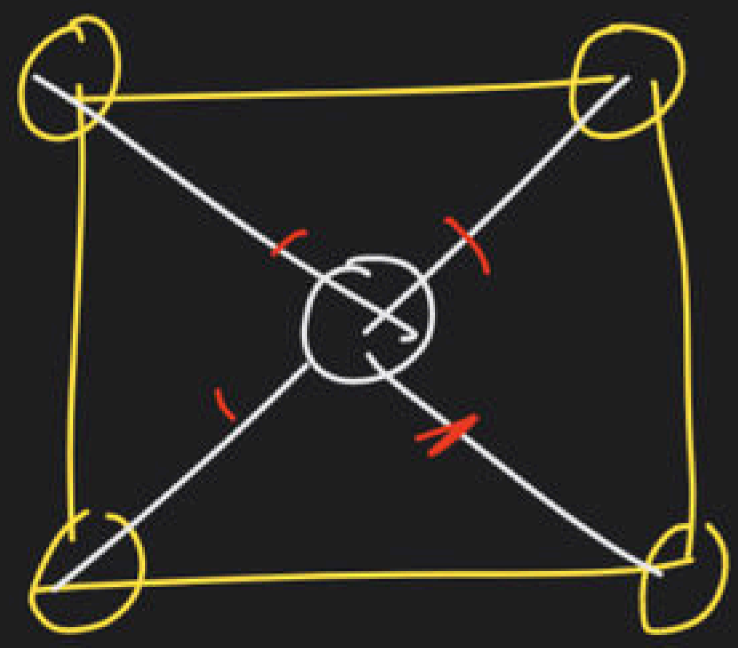


$3*3+1*3$

$\frac{12}{2} = 6$

W_4 (wheel graph)
4V

$2(n-1)$

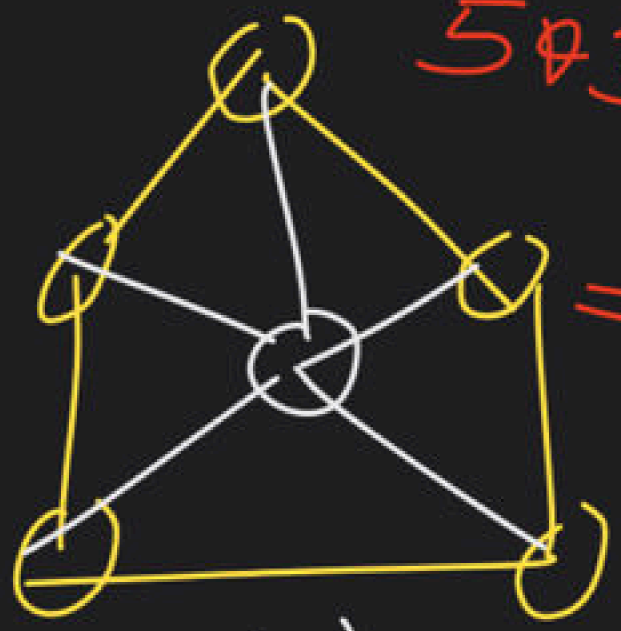


W_5

$4*3+1*4$

$\frac{16}{2} = 8$

$2(n-1)$



$5*3+1*5$

$\Rightarrow \frac{20}{2}$

$\Rightarrow 10$

W_6

$2(n-1)$

$W_n \Longleftarrow \underline{C_{n-1}}$

$n \geq 4$

$\cancel{\le 2} \quad \cancel{\le 4}$

$$E(W_n) = (n-1) * 3 + 1 * (n-1)$$

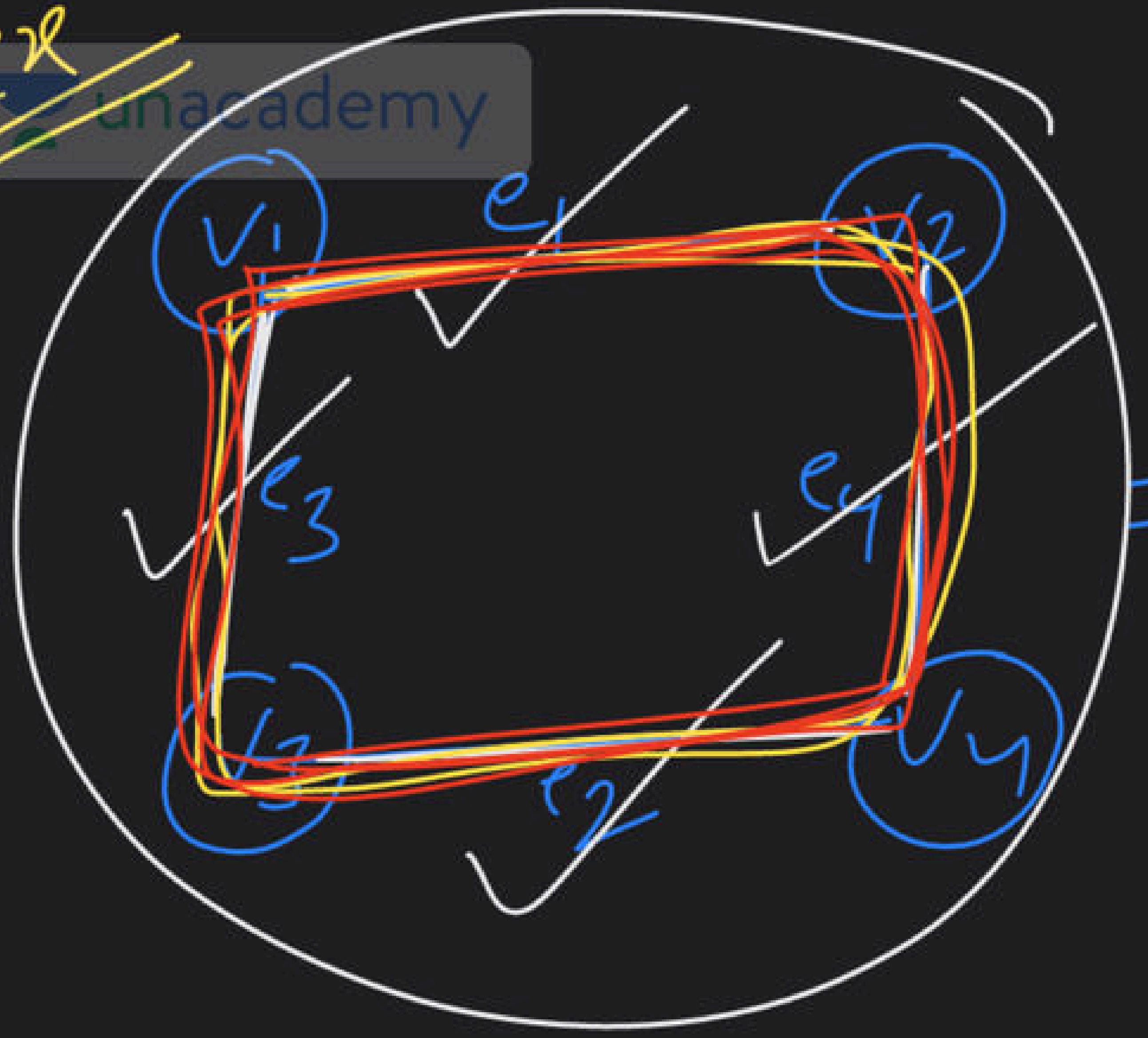
$$= (n-1)[3+1] \Rightarrow 4(n-1)$$

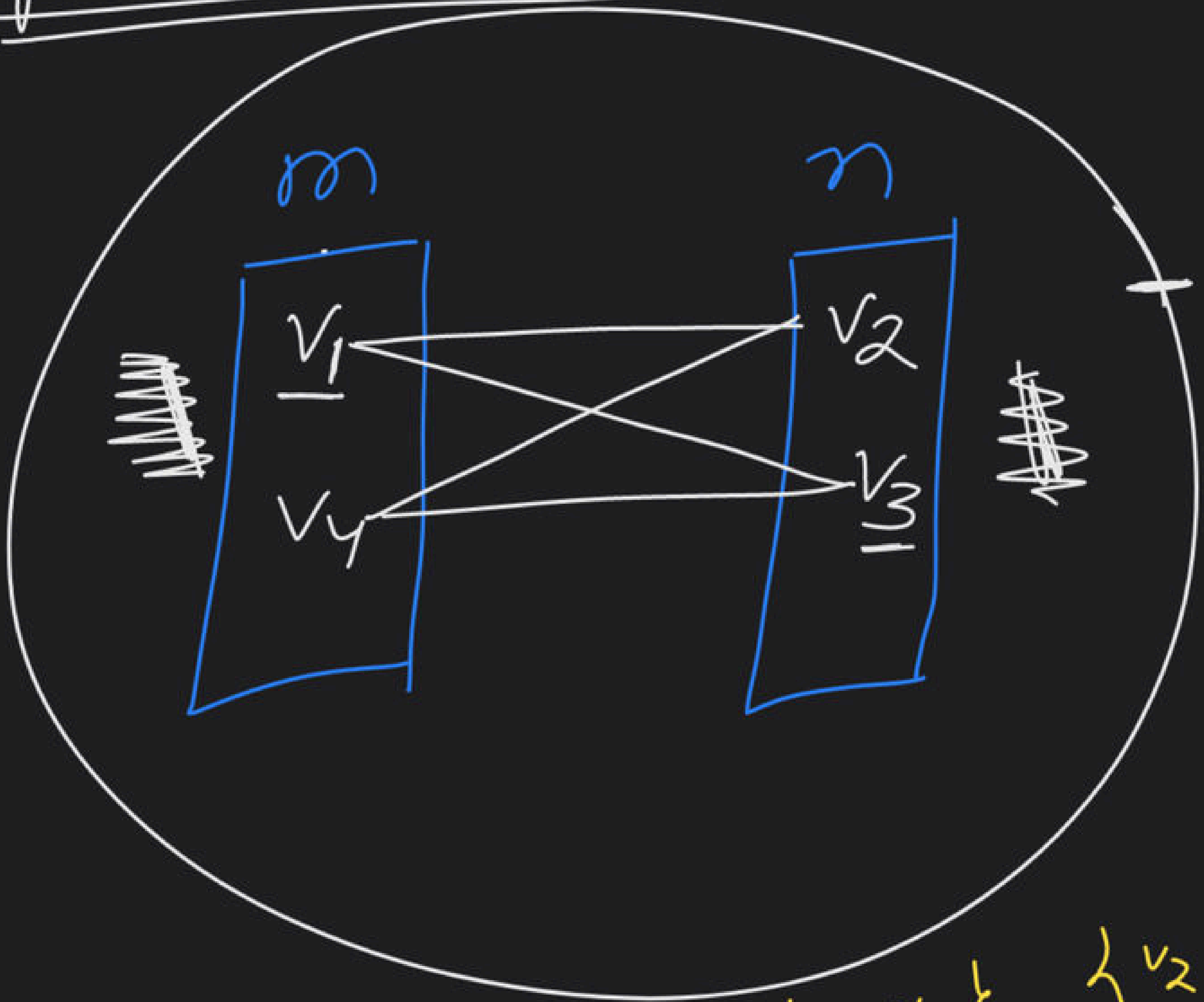
$$= \frac{4(n-1)}{2}$$

$$= \boxed{2(n-1)}$$

Bipartite Graph

ex





① with in m no edge

② with in n no edge

③ edge always from m to n

④ {v1 v4} {v2 v3} {v1 v2 v3 v4}

$m \cup n = V$

$m \cap n = \phi$

⑤ {v1 v4} {v2 v3}

⑥ all edges should cover

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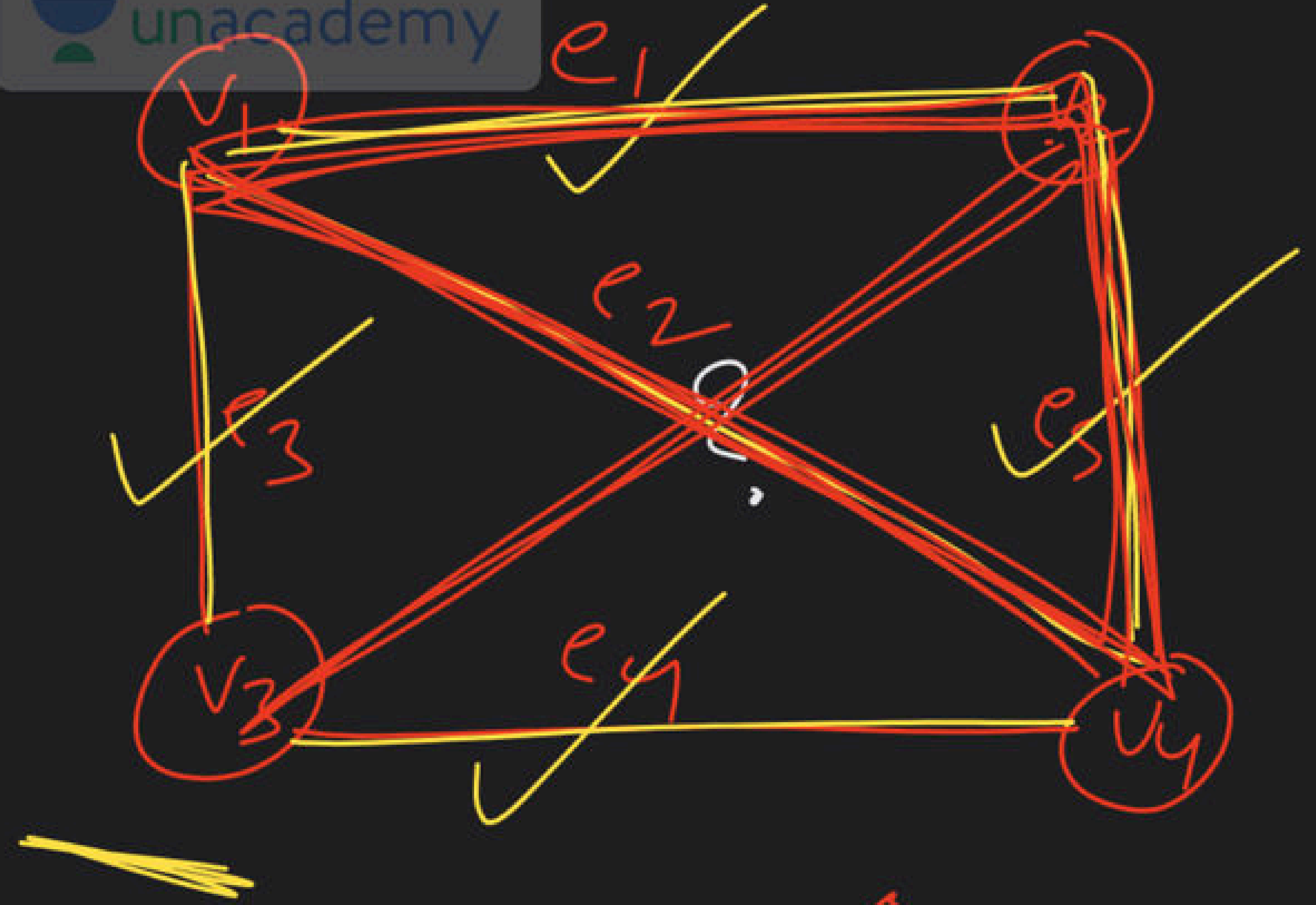

not Bipartite Graph