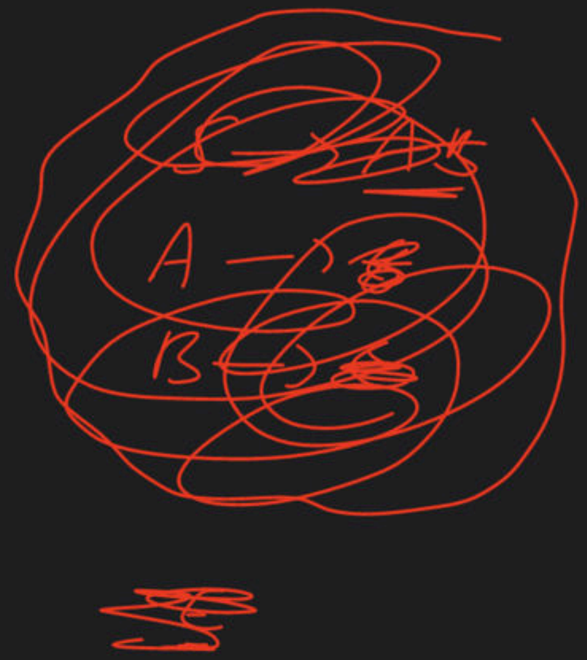
Simplification of CFG

1. Elimination of Null productions
2. " Unit productions
3. " Useless Symbols



Elimination of Null production

ex 1

$S \to aS_1b$

$S_1 \to aS_1b \mid \epsilon$

ϵ L

$S \to aS_1b \mid ab$

$S_1 \to aS_1b \mid \epsilon \mid ab$

ϵ L

S Dead

$S \to aS_1b \mid ab$

ex (2)

$S \rightarrow aS_1b \mid \epsilon$

$S_1 \rightarrow aS_1b \mid \epsilon$

L

$ab \in L$

$aabb \in L$

$\epsilon \in L$ ✓

$S \rightarrow aS_1b \mid \cancel{\epsilon} \mid ab$

$S_1 \rightarrow aS_1b \mid \cancel{\epsilon} \mid ab$

L

$ab \in L$

$aabb \in L$

$\epsilon \notin L$

If the given CFG G $\in \not\in L(G)$ then we can get equivalent CFG after eliminating Null productions.

ex (3)

$S \to ABaC$

$A \to BC$

$B \to b \mid \in$

$C \to D \mid \in$

$D \to d$

$S \to ABaC \mid ABa \mid AaC \mid BaC \mid Aa \mid a \mid Ba \mid aC$

$A \to BC \mid \in \mid b \mid d$ (1)

$B \to b \mid \in$

$C \to D \mid \in \mid d$ (2)

$D \to d$

L

Unit productions

$A \rightarrow B$

S.V S.V

} Unit production

$A \rightarrow a$

S.V S.T

~~U.P~~

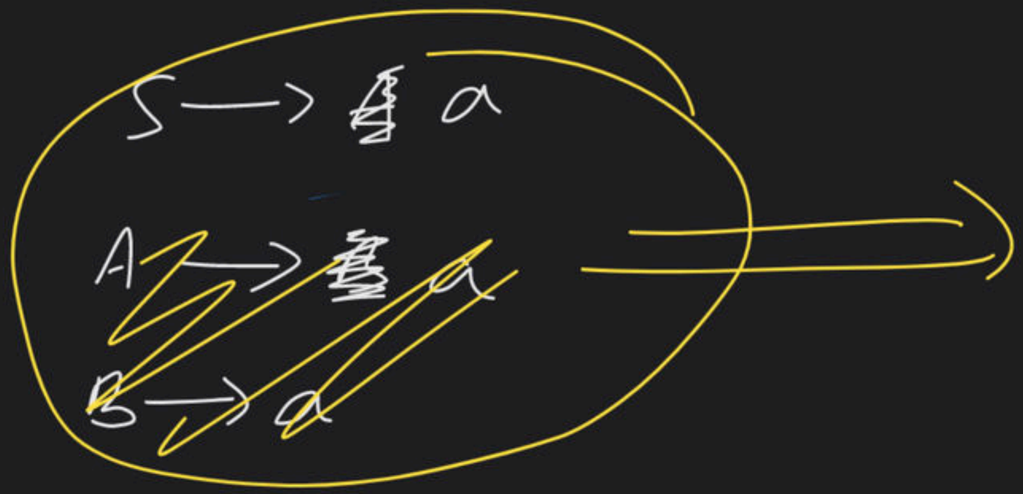
Ex ①

$S \rightarrow A$

$A \rightarrow B$

$B \rightarrow a$

$\Longrightarrow$



$\Longrightarrow S \rightarrow a$

ex ②

$S \rightarrow Aa \mid B$

$B \rightarrow A \mid bb$

$A \rightarrow a \mid bc \mid B$

E · U · P

$S \rightarrow Aa \mid B \mid bb \mid a \mid bc$

$B \rightarrow A \mid bb \mid a \mid bc$

$A \rightarrow a \mid bc \mid B \mid A \mid bb$

Useless Symbol Elimination

$S \rightarrow ABC \mid A$

$B \rightarrow bB$

$C \rightarrow a$

$D \rightarrow b$

$A \rightarrow a$

$\Longrightarrow$

(1) Eliminate those Variables not reachable from start

$S \rightarrow ABC \mid A$

$A \rightarrow a$, $B \rightarrow bB$, $C \rightarrow a$

(2) Eliminate those productions not producing any string

$S \rightarrow A$ $\quad$ $A \rightarrow a$

ex

$S \rightarrow as \mid AB \mid \epsilon$

$A \rightarrow bA$

$B \rightarrow AA$

(1)

(2) ~~$A \rightarrow bA$~~

~~$B \rightarrow AA$~~

$S \rightarrow as \mid \epsilon \mid$ ~~AB~~

$S \rightarrow as \mid \epsilon$

ex (2)

$S \rightarrow AB$

$A \rightarrow a$

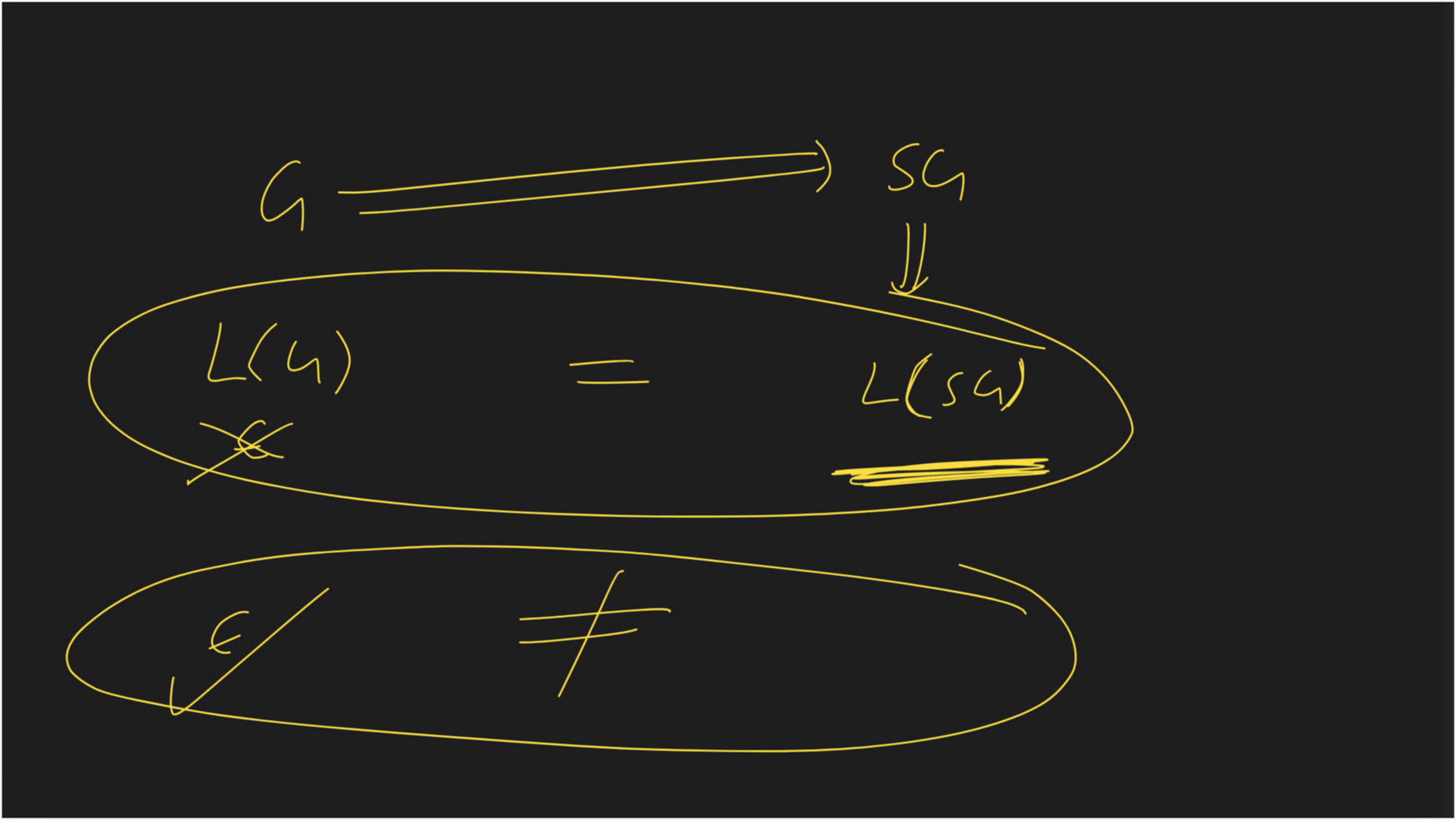
$B \rightarrow b \mid \epsilon$

Null

$S \rightarrow AB \mid A$

$A \rightarrow a$

$B \rightarrow b$


G
SG
L(G)
=
L(SG)
ϵ
≠

For every ε-free CFG we can write equivalent Simplified CFG

Normal Forms

1. CNF (Chomsky Normal Form)
2. GNF (Greibach ")

$V \rightarrow VV \mid T$

(or)

$A \rightarrow BC$

$B \rightarrow b$

$C \rightarrow c$

$V \rightarrow aV^{*}$

(or)

$A \rightarrow bCDE$

$C \rightarrow a$ // $D \rightarrow e$ // $E \rightarrow g$ //

CNF ($V \rightarrow VV \mid T$)

ex ①

$S \rightarrow AA \mid a$

$A \rightarrow SA \mid b$

CNF ✓

ex ②

$S \rightarrow AS \mid \underbrace{AAS}_{C} \mid a$

$A \rightarrow SA \mid aa$

~~CNF~~

① Simplification
no-change

② convert CNF

$S \rightarrow AS \mid CS \mid a$

$C \rightarrow AA$

$A \rightarrow SA \mid DD$

$D \rightarrow a$

$S \rightarrow ABa$ (D, E)

$A \rightarrow aab$ (E, F)

$B \rightarrow Ac$

$\Longrightarrow$ (1) Simply no-change

(2) convert

$S \rightarrow ABa$

$S \rightarrow DE$

$E \rightarrow a$, $D \rightarrow AB$

$B \rightarrow Ac$

$A \rightarrow EF$

$F \rightarrow EG$

$G \rightarrow b$

$A \rightarrow aab$

$B \rightarrow AH$

$H \rightarrow c$

Note

(1) From the given CNF grammar to generate n-length string we need $2n-1$ productions

(2) For every ϵ-free CFG we can construct equivalent CNF.

GNF $(V \rightarrow TV^*)$

ex

~~CNF~~

~~GNF~~

(1) Simplify
no-simplification

convert into GNF

$S \rightarrow aAB \mid bBB \mid bB$

$A \rightarrow aA \mid bB \mid b$

$B \rightarrow b$

ex

$S \rightarrow aSb \mid ab$

~~GNF~~

① Simplify
non-sim

② Conversion

$S \rightarrow aSB \mid aB$

$B \rightarrow b$

Note

To generate n-length string from GNF
we need n-productions.

Thanks All

D · H

$S \rightarrow aBCDE$

$B \rightarrow b$

$C \rightarrow c$

$D \rightarrow d$

$E \rightarrow e$

$S \rightarrow AB$

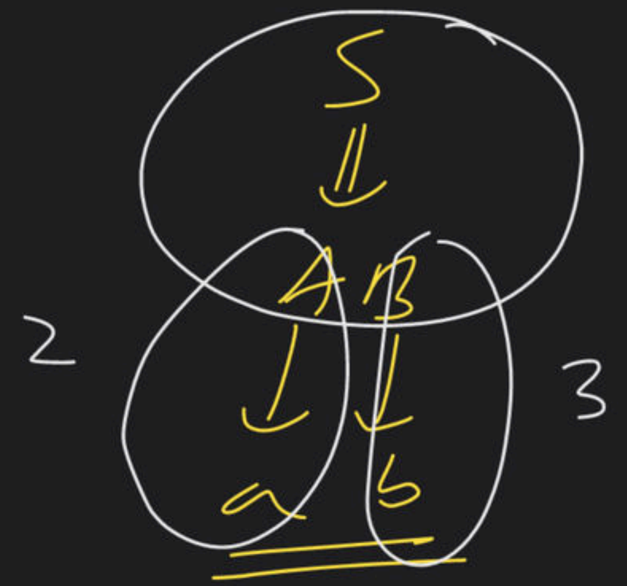
$A \rightarrow a$

$B \rightarrow b$

CNF

i/p: ab

2



1

2

3

$2n-1$

$2*2-1 \Rightarrow 3$

$S \rightarrow ab$

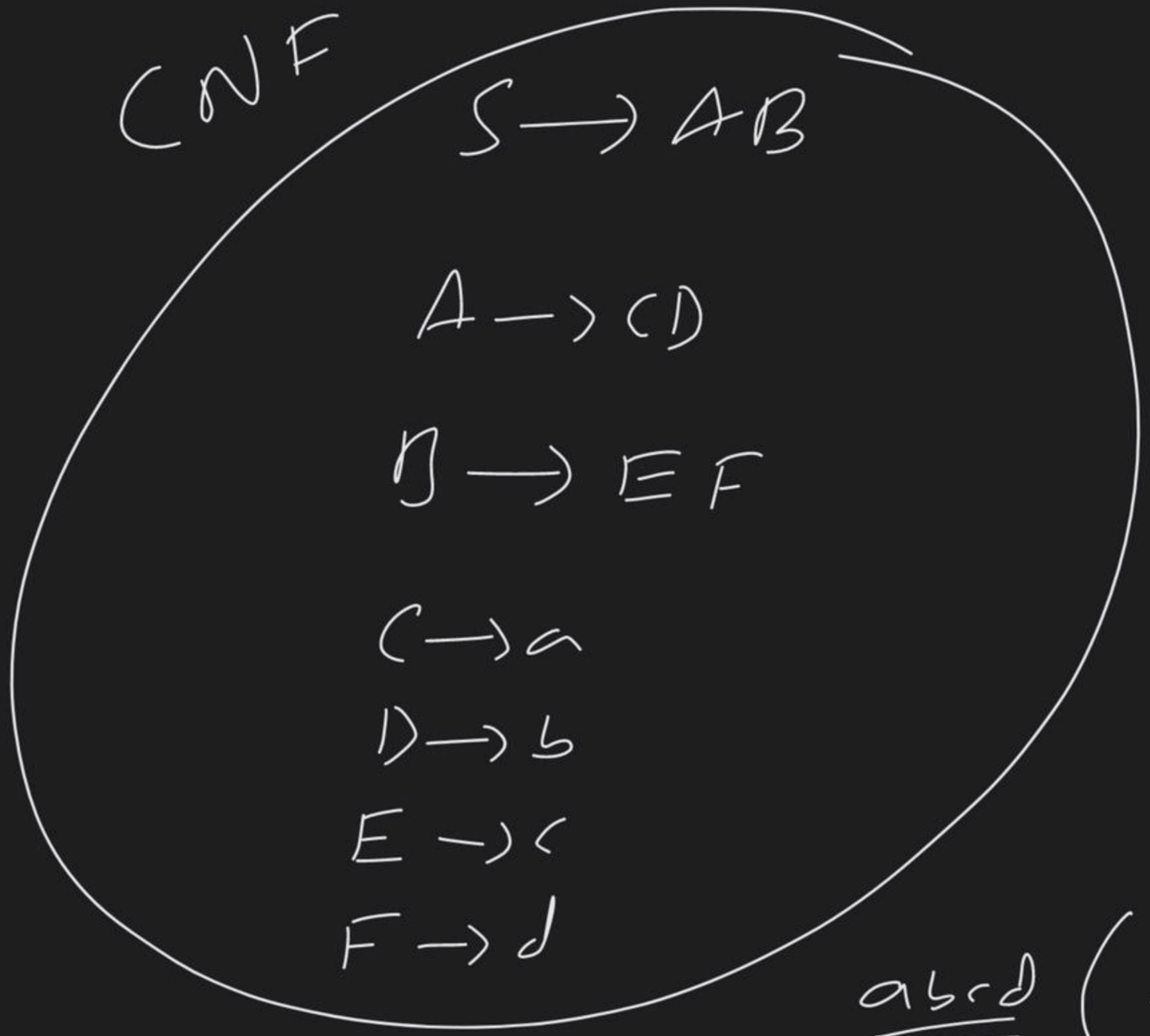
$S \rightarrow A$

$A \rightarrow B$

$B \rightarrow C$

$C \rightarrow D$

$D \rightarrow ab$



abcd (2 by 4 - 1)