Asymptotic Notation - Part II

Complete Course on Algorithm for GATE - CS & IT

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```
main()
{
    for(i=1; i<=n; i++)
    {
        for( j=1; j<=i^2; j++)
        {
            x = y+z;
        }
    }
}
```

$i=1 \quad i=2 \quad i=3 \quad \ldots \quad i=n$

$j=1^2 \quad j=2^2 \quad j=3^2 \quad \ldots \ldots \quad j=n^2$

$$\Rightarrow 1^2+2^2+3^2+4^2+\cdots+n^2$$

$$\Rightarrow \frac{n(n+1)(2n+1)}{\cancel{2}\ 6} \quad \cancel{\theta(n^2)} = \theta(n^3)$$

```
main()
{
    int i;
    for(i=1; i<=n; i++)
    {
        for(i=1; i<=n^4; i++)
        {
            for(i=1; i<=n^3; i++)
            {
                x=y+z;
            }
        }
}
```

infinite loop.

i

$\boxed{\cancel{9}}$

1600

~~1~~ ~~1~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ . . . . .

$\cancel{n^3}$

$\cancel{n^3+1}$

$\cancel{n^3+2}$

~~1~~

~~2~~ . . . .

$\cancel{n^3}$

$\cancel{n^3+1}$

$\cancel{n^3+2}$

1

Asymptotic Notation

1. Big-oh Notation $(O, \leq)$
2. Omega " $(\Omega, \geq)$
3. Theta " $(\theta, =)$

Let $f(n)$ & $g(n)$ be two +ve functions

Big-Oh-notation

- Let $f(n) = O(g(n))$

iff

- $f(n) \leq c \, g(n), \ \forall n, \ n \geq n_0$

Such that $\exists$ two true constants $c > 0, \ n_0 \geq 1$

$n+5 = O(n)$

eg

$f(n) = n+5, \quad g(n) = n$

$f(n) = O(g(n))$ iff $\boxed{n+5 \leq c \cdot n, \ \forall n, \ n \geq n_0}$

$c = 2$, $n_0 = 5$

ex ②

$f(n) = n, \quad g(n) = n+5$

$f(n) = O(g(n))$

$\Updownarrow$

$n \leq c \cdot (n+5), \quad \forall n, \; n \geq n_0$

$\Downarrow \qquad\qquad \Downarrow$

$1 \qquad\qquad 1$

$\Updownarrow$

$n = O(n+5)$

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```
main()
{
    s = 0
    for(i=1; s <= n; i++)
    {
        s = s+i
    }
}
```

$1 \leq n$

$1+2 \leq n$

$1+2+3 \leq n$

$1+2+3+4 \leq n$

$\vdots \; \underline{K}$

$1+2+3+4+\cdots+K = n$

$$\frac{K(K+1)}{2} = n$$

$$K^2 = n$$

$$K = \sqrt{n}$$

$$O(\sqrt{n})$$

```
main()
{
  for(i=1; i<=n; i++)
  {
    for(j=1; j<=n; j=j+i)
    {
      x = y+2;
    }
  }
}
```

$$\frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n}$$

$$\Longrightarrow n\left[\underbrace{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}}_{\log n}\right] \Rightarrow \log n$$

$$\Longrightarrow n \cdot \log n = O(n \log n)$$