Countable & Uncountable

N = Set of natural number

$= \{1, 2, 3, 4, \ldots\ldots\}$.

$N \subset Z \subset Q$

1

Z = Set of integer

$= \{ \bar{2}, 0, \cancel{Z^+} \} \Rightarrow \{ \ldots\ldots -3\ -2\ -1\ 0\ +1\ +2\ +3\ \ldots\ldots \}$

Q = Set of rational number

$= \{ p/q$ where p & q are integer $q \neq 0 \}$

$\mathbb{R}$ = Set of real numbers

Set of rational numbers

$\Downarrow$

p/q

$p, q \in \mathbb{Z}, q \neq 0$

Set of irrational numbers

$\Downarrow$

$\sqrt{2}, \sqrt{3}, \sqrt{5} \ldots$

$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

2

CN = Set of complex number

= a + ib where a, b ∈ R

$N \subset Z \subset Q \subset R \subset CN$

3

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Set S said to be countable iff S has EP.

EP ⟹ Enumeration procedure ⟹ Every member of S can be printed in finite time.

$N = \{1, 2, 3, 4, 5, 6, 7, \ldots\ldots 10000, \ldots\ldots 1cr, \ldots\ldots\}$

$\Rightarrow$ Countably infinite (CI)

$$Z = \begin{array}{ccc} +ve & 0 & -ve \\ \Downarrow & \Downarrow & \Downarrow \\ CI & CF & CI \end{array}$$

Countably infinite (CI)

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$$\mathbb{Q} = \left.\begin{array}{llllllll}
1/1 & 1/2 & 1/3 & 1/4 & 1/5 & & \cdots & CI \\
2/1 & 2/2 & 2/3 & 2/4 & 2/5 & 2/6 & \cdots & CI \\
3/1 & 3/2 & 3/3 & 3/4 & 3/5 & & \cdots & CI \\
 & & \vdots & & & & & \\
100/1 & 100/2 & \cdots & & & & & CI \\
 & & & \vdots & & & &
\end{array}\right\} CI$$

$$\frac{50}{3}$$

$$\boxed{9 - x \neq x}$$

R = 0 1 2 3 4 - - - - - - - - - - -

- - - - 100 - - - - 1L - - - - 100cr - - - -

0.1 0.2 0.3 0.4 0.5 - - - - - -

0.15 0.16 0.17 -

6

Cantor

0.5

All we can't print

Using Cantor's Diagnolization theorem we can prove set of real number is uncountably Infinite (Uncountable)

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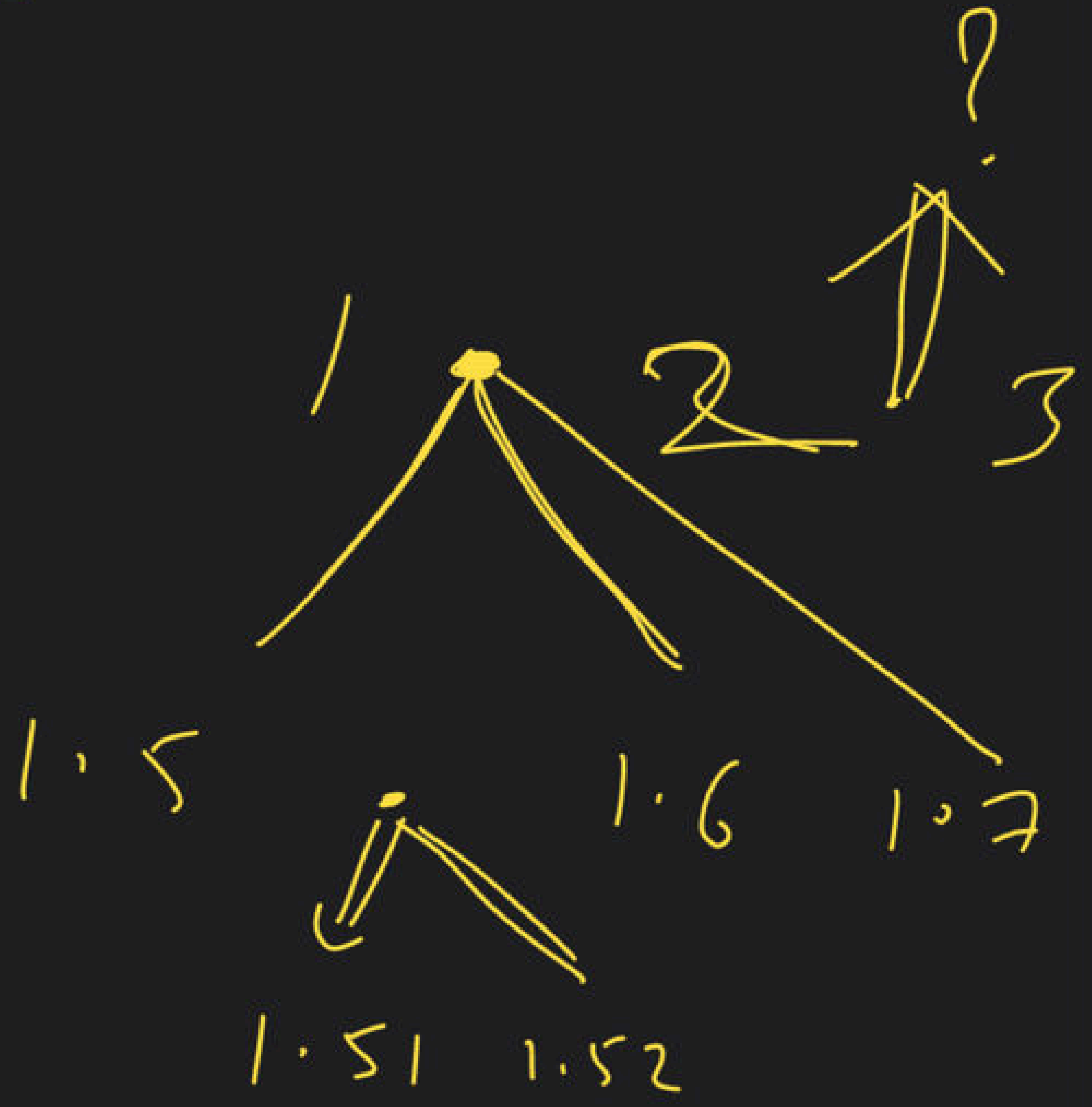

$cn = a + ib$

$\Downarrow \qquad \Downarrow$

$R \qquad R$

UCI VUI

UCI $\Rightarrow$ U

UCI

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① Subset of countable is countable.

but subset of uncountable is may be countable / uncountable.

② $C \times C = C$

③ $U \times U = U$

④ $U \times C = U$

⑤ $C \cup C \cup C = C$

⑥ $C \cup C \cup C \cup \ldots\ldots = C$ ✓

$\Downarrow$ ✓ $\Downarrow$ ✓ $\Downarrow$ ✓

⑦ $U \cup U \cup U \cup \ldots\ldots\ldots = U$

$\Downarrow$ ✗

⑨

Countable $\Longrightarrow$ N, Q, Z

Uncountable $\Longrightarrow$ R, CN, irrational number

Real num = Rational num (Q) + Irrational num

U

C U ? U

To

Cantor's theorem

If S is CI then $P(S)$ is uncountable
(or)
N, Q, Z

countable $\Longrightarrow$ N, Z, Q

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Uncountable $\Longrightarrow$ R, Irrational, CN, $P(N)$, $P(Z)$
$P(Q)$

TOC

$\Sigma = \{a, b\}$

1 $\Sigma^* = 0, 1, 2, 3, \ldots \ldots \Longrightarrow CI$

$\Downarrow$

$2^{\Sigma^*} \Longrightarrow$ Uncount

2 $L \subset \Sigma^*$

$\Downarrow$

Countable

CF CI

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$$\underbrace{\begin{array}{c} L_1 \\ \Downarrow \\ \text{Reg} \\ \Downarrow \\ C \end{array} \cup \begin{array}{c} L_2 \\ \Downarrow \\ \text{Reg} \\ \Downarrow \\ C \end{array} \cup \begin{array}{c} L_3 \\ \Downarrow \\ \text{Reg} \\ \Downarrow \\ C \end{array} \cdots\cdots = \begin{array}{c} C \\ \Downarrow \\ \text{countable} \end{array}}$$

3

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Set of all regular languages are countable.

4 Set of all CFL's also countable

5 " " CSL's " "

6 " " Recursive Language " "

7 " " REL " "

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$$\begin{matrix} \text{all} & & \text{all} & & \text{all} & & \text{all} & & \text{all} & & \text{all} & \\ \text{R.L} & \cup & \text{CFL} & \cup & \text{CSL's} & \cup & \text{R} & \cup & \text{REL} & \cup & \text{non REL} & = U \\ \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & & \Downarrow & \\ \text{CI} & & \text{CI} & & \text{CI} & & \text{CI} & & \text{CI} & & U & \end{matrix}$$

$$2^{\Sigma^*}$$

$$\Downarrow$$

$$U$$

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Set of all languages $\Rightarrow$ uncountable

TOC

Countable

$\Downarrow$

Σ^*, S.Reg, S.CFL,

S.CSL, S.K, S.REL

L

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Uncountable

$\Downarrow$

2^{Σ^*} Set of non REL

S.NReg, S.NCFL

S.NCSL, S.NRec

Dedicate Haty

CI

$A = \{1, 2, 3, \ldots\ldots\}$

3

$A = \{1, 2, 3\}$

$P(A)$ = Uncountable

$P(A) = 2^3$ - Sets

= 8 - Sets