Operations on Binary Tree - Part III

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Vishvadeep Gothi • Lesson 48 • Jan 27, 2023

Data Structure
Tree 5

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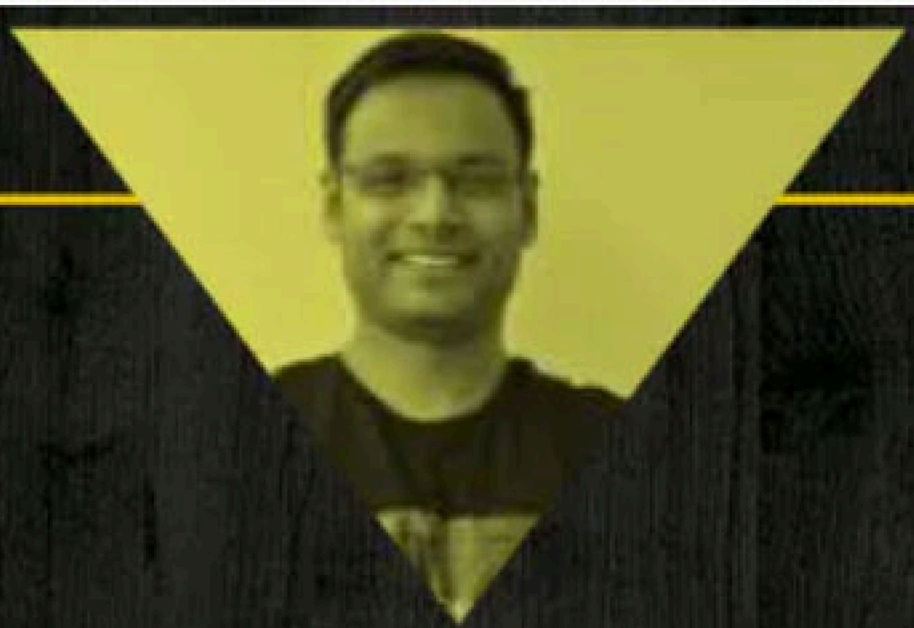


Hello!

I am Vishvadeep Gothi

I am here because I love to teach

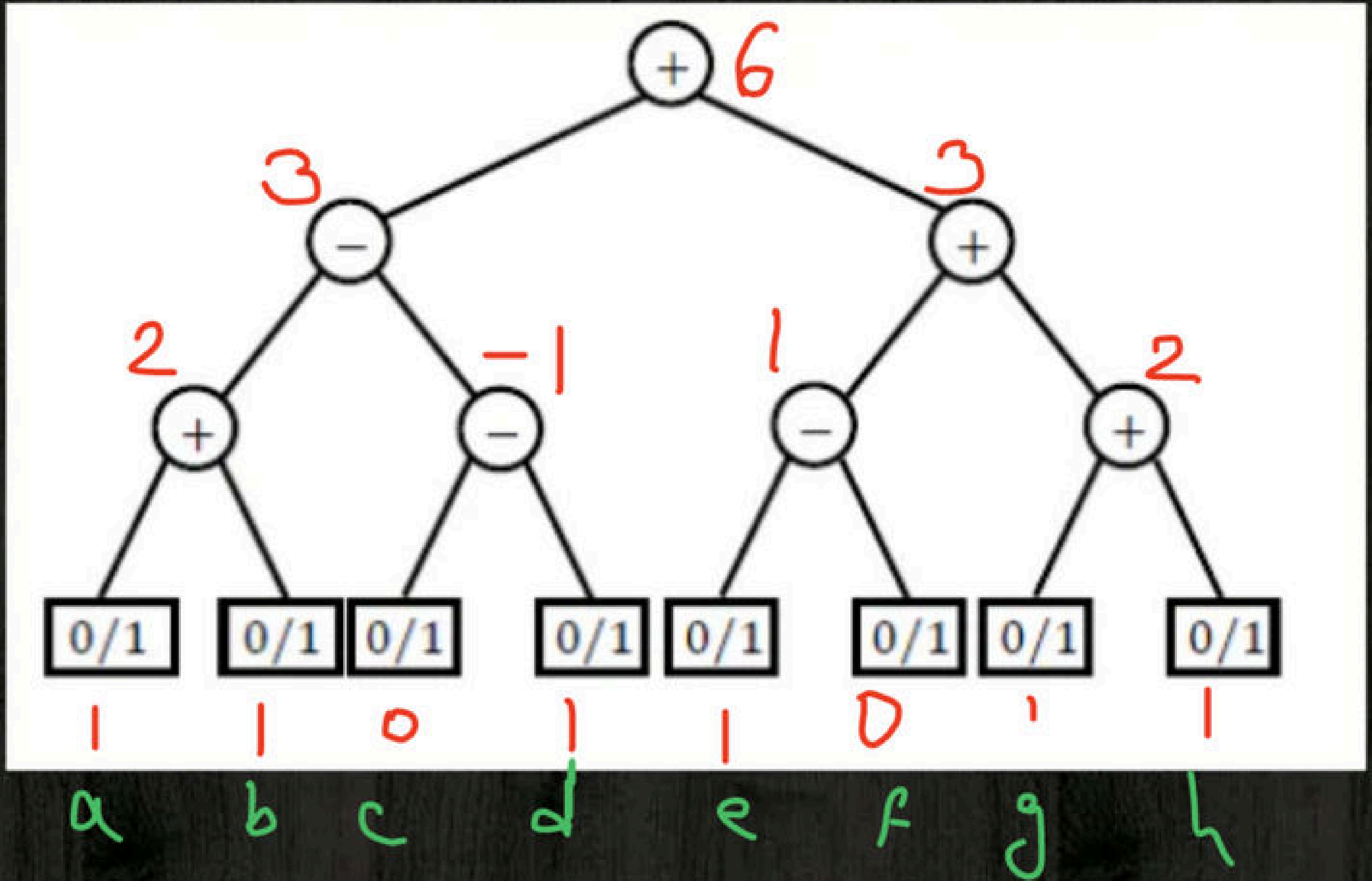
Expression Tree

Root => operator

Question GATE-2014

Ans = 6

Consider the expression tree shown. Each leaf represents a numerical value, which can either be 0 or 1. Over all possible choices of the values at the leaves, the maximum possible value of the expression represented by the tree is ____.



$$\left[(a+b)-(c-d)\right]+\left[(e-f)+(g+h)\right]$$

$$a+b-c+d+e-f+g+h$$

for max value :-

all +ve => 1
all −ve => 0

$1 + 1 - 0 + 1 + 1 - 0 + 1 + 1 \Rightarrow 6$

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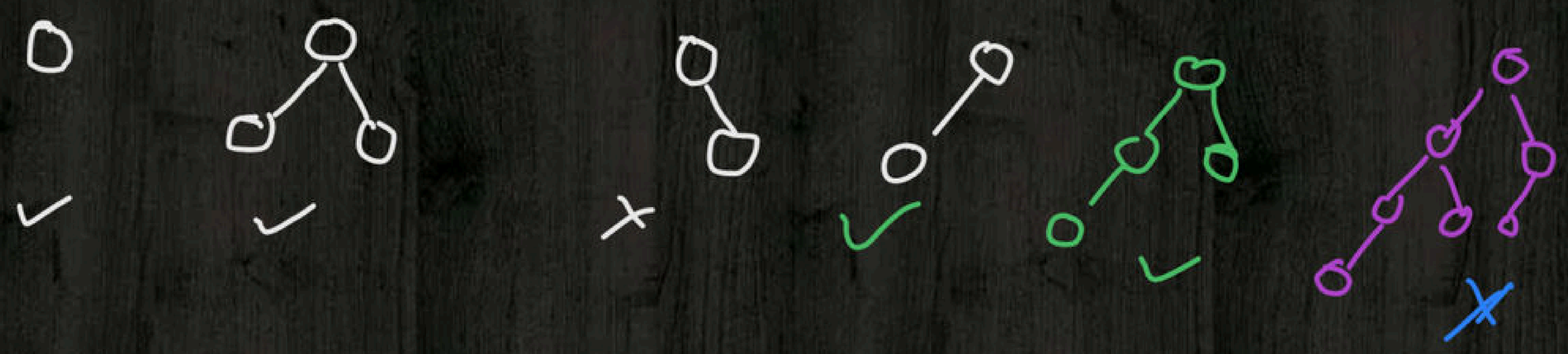
for min value :-

all +ve => 0
all −ve => 1

$0 + 0 - 1 + 0 + 0 - 1 + 0 + 0 \Rightarrow -2$

Complete Binary Tree

A BT in which all levels must have max no. of nodes. except possibly the last level. The last level nodes must be arranged from left to right



Complete Binary Tree

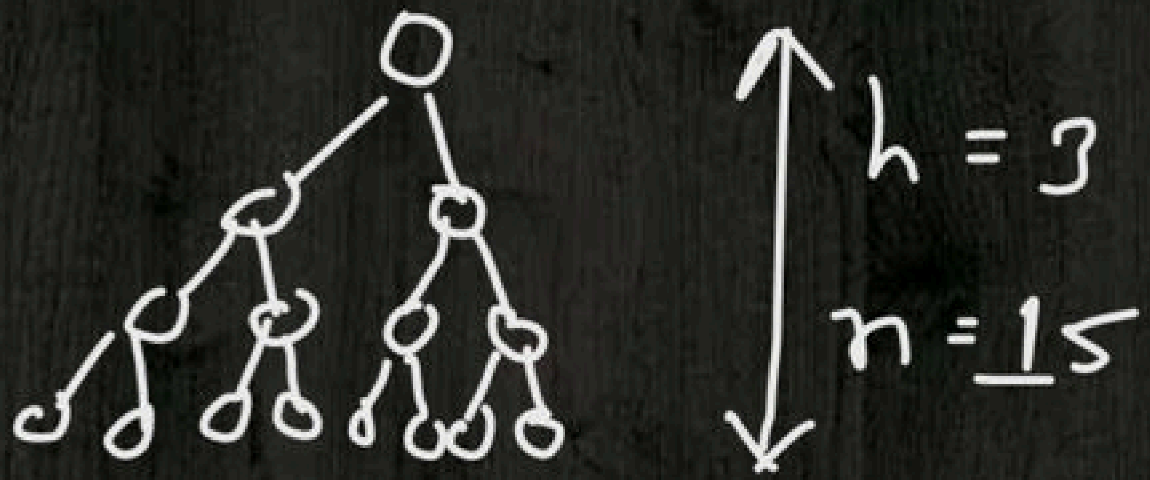
strict CBT:- all levels must have max. no. of nodes

Question

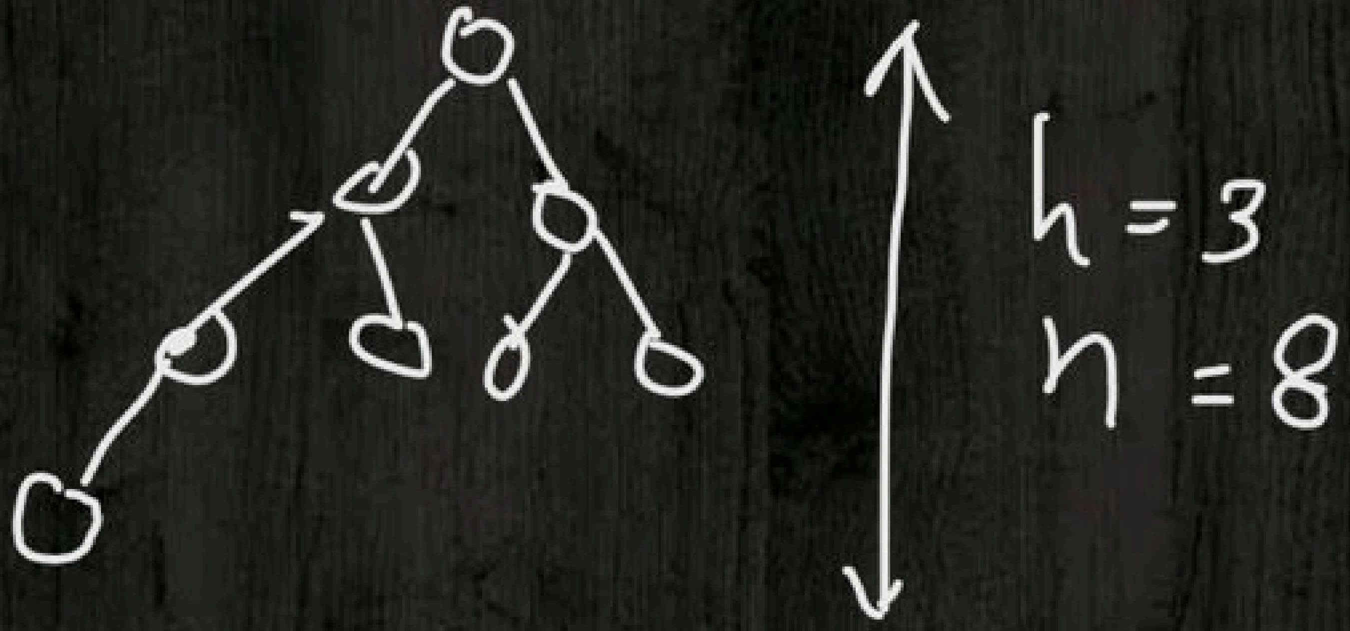
Maximum and minimum number of nodes in a complete binary tree of height h?

Note: Height of tree with single node is 0

max:- $2^{h+1} - 1$



min:- 2^h



Array Representation of CBT

Rule :- array starting from index 1.

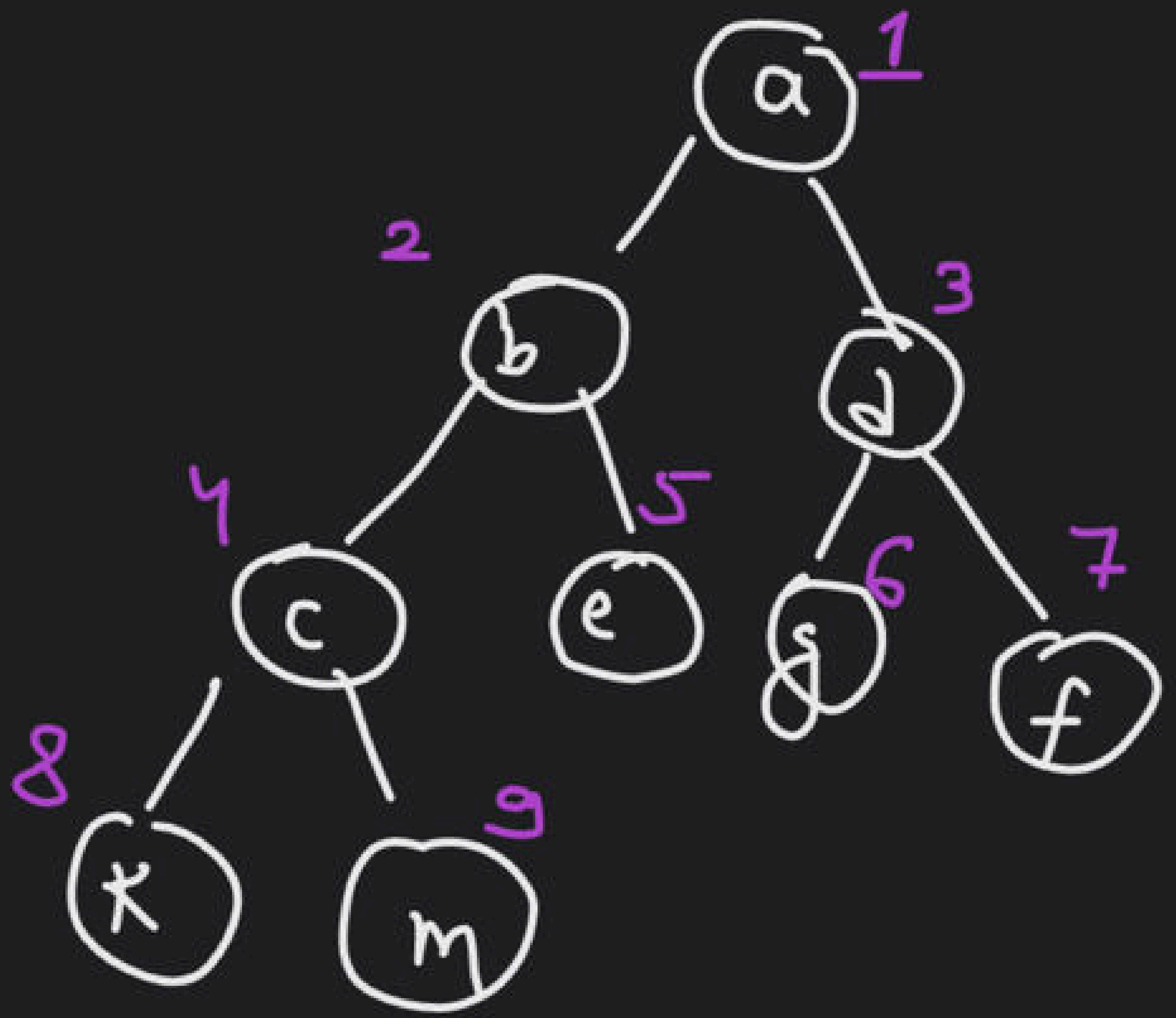
Root at index => 1

for any node at index i,

Left child => index $2i$

Right child => index $2i+1$

parent => index $\left\lfloor \frac{i}{2} \right\rfloor$



array

1	2	3	4	5	6	7	8	9	
a	b	d	c	e	g	f	k	m	

If array indexing starts from zero.

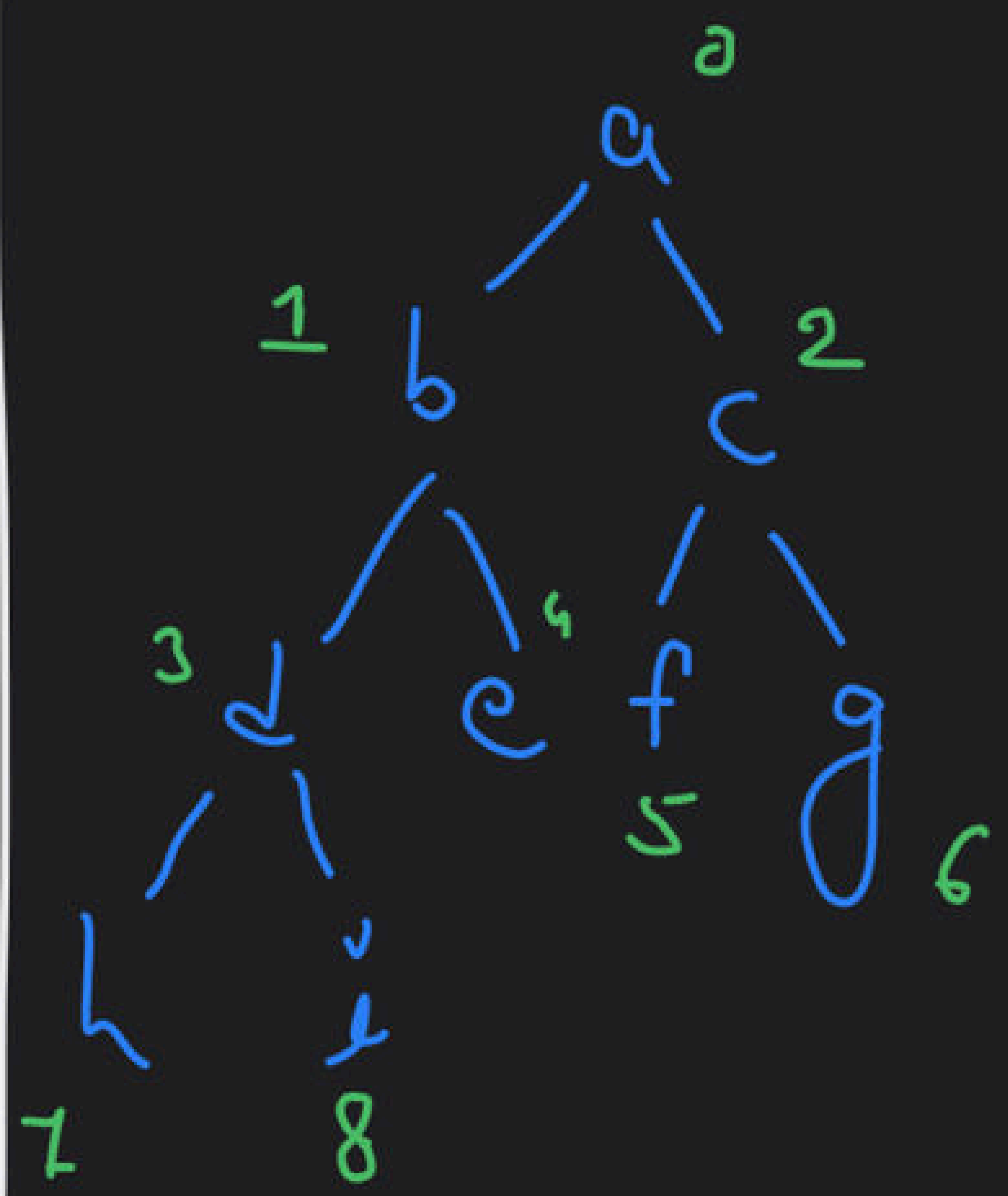
root at index $\Rightarrow 0$

for any node at index i,

L.C at index $\Rightarrow 2i+1$

R.C. at index $\Rightarrow 2i+2$

Parent at index $\Rightarrow \left\lfloor \frac{i-1}{2} \right\rfloor$



Question GATE-2006

A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. The root is stored at X[1]. For any node stored at X[i], The left child, if any, is stored in X[2i] and right child, if any, is stored in X[2i+1]. To be able to store any binary tree on n vertices the minimum size of X should be?

$n = 4$

A. log n
B. n
C. 2n+1
D. $\mathbf{2^n - 1}$

array size = 15

Full Binary Tree (strict BT, 2-Tree, 2-T)

A BT in which each node has either 0 child or 2 children

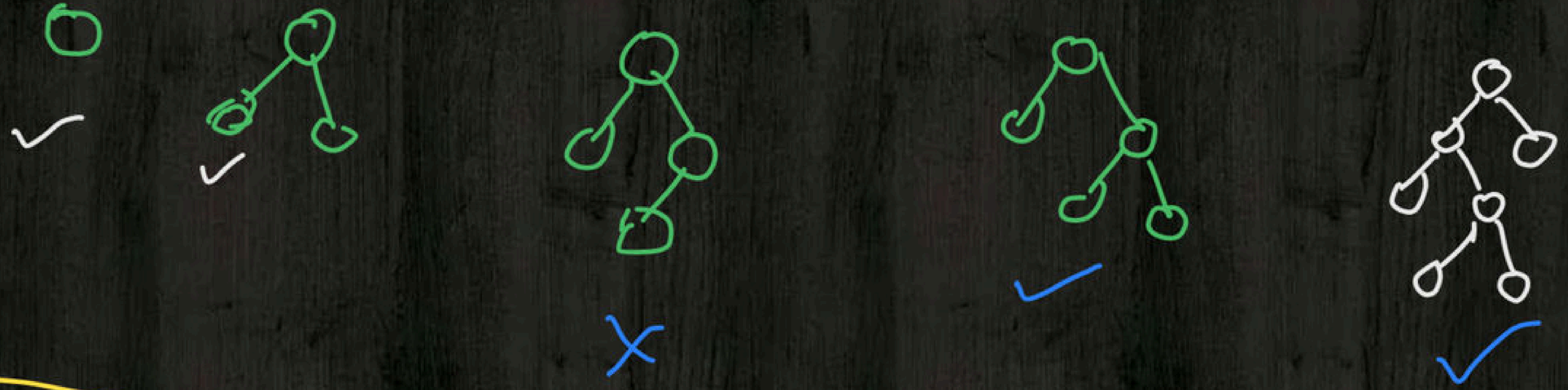
or

A BT in which every internal node has exactly 2 children.

or

A BT in which no any node has 1 child.

Full Binary Tree



for BT,

total no. of leaf nodes (L) = Total no of internal nodes with 2 children (I_2) + 1

$$L = I_2 + 1$$

for full BT,

$$\boxed{L = I + 1}$$

as we know,

$$N = L + I$$

$$\boxed{N = 2I + 1}$$

$$\boxed{N = 2L - 1}$$

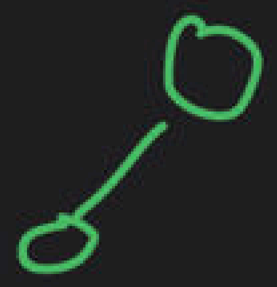
$$\boxed{I = \frac{N-1}{2}}$$

$$\boxed{L = \frac{N+1}{2}}$$

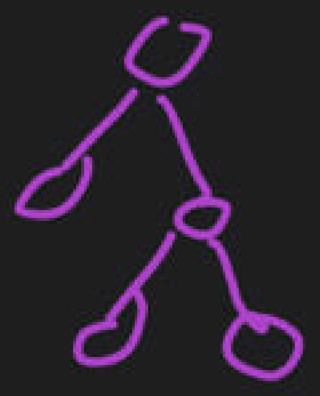
In full BT, total no. of nodes is always odd.

1. Every C B.T. is full BT also ⇒ false
2. Every full B.T. is Complete BT also ⇒ false

1. counter example

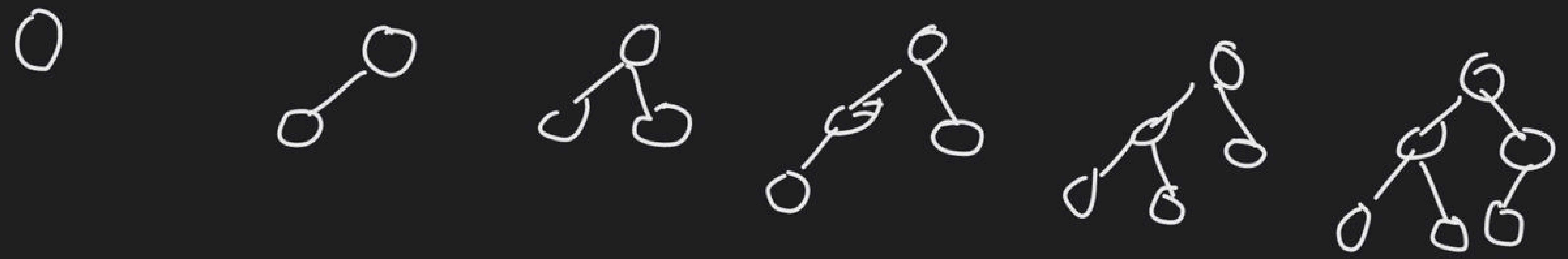


2. counter example



Ques) no. of leaf nodes in a complete BT of n nodes?

Ans.)



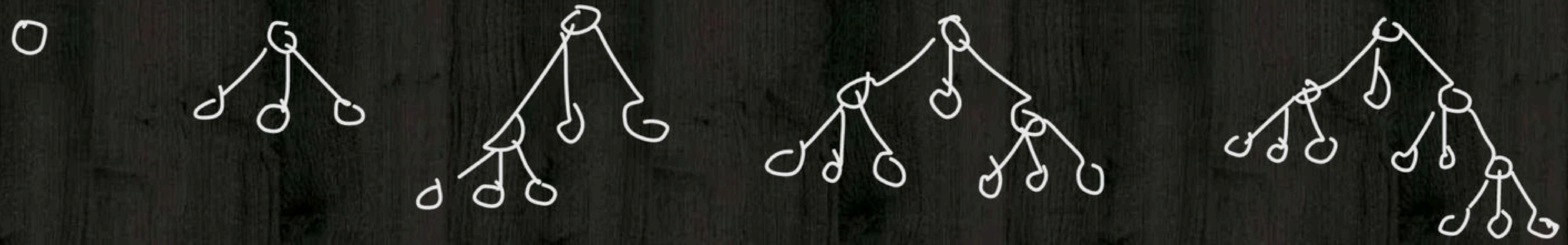
n	1	2	3	4	5	6
L	1	1	2	2	3	3

$$\left\lfloor \frac{n+1}{2} \right\rfloor \text{ or } \left\lceil \frac{n}{2} \right\rceil$$

3-Tree

complete 3-ary Tree or 3-T

every node has either 0 child or 3 children



I	0	1	2	3	4	5
L	1	3	5	7	9	11

$$\boxed{L = 2I + 1}$$

$$N = I + L$$

$$\boxed{N = 3I + 1}$$

$$\boxed{N = \frac{3L - 1}{2}}$$

keys $I = \frac{L-1}{2}$

$$N = \frac{L-1}{2} + L$$

$$N = \frac{3N-1}{2}$$

3-Tree

k-Tree

k-ary complete tree

each node has either 0 child or k-children.

$$\boxed{L = (k-1)I + 1}$$

$$\boxed{N = kI + 1}$$

$$\boxed{N = \frac{kL - 1}{k-1}}$$

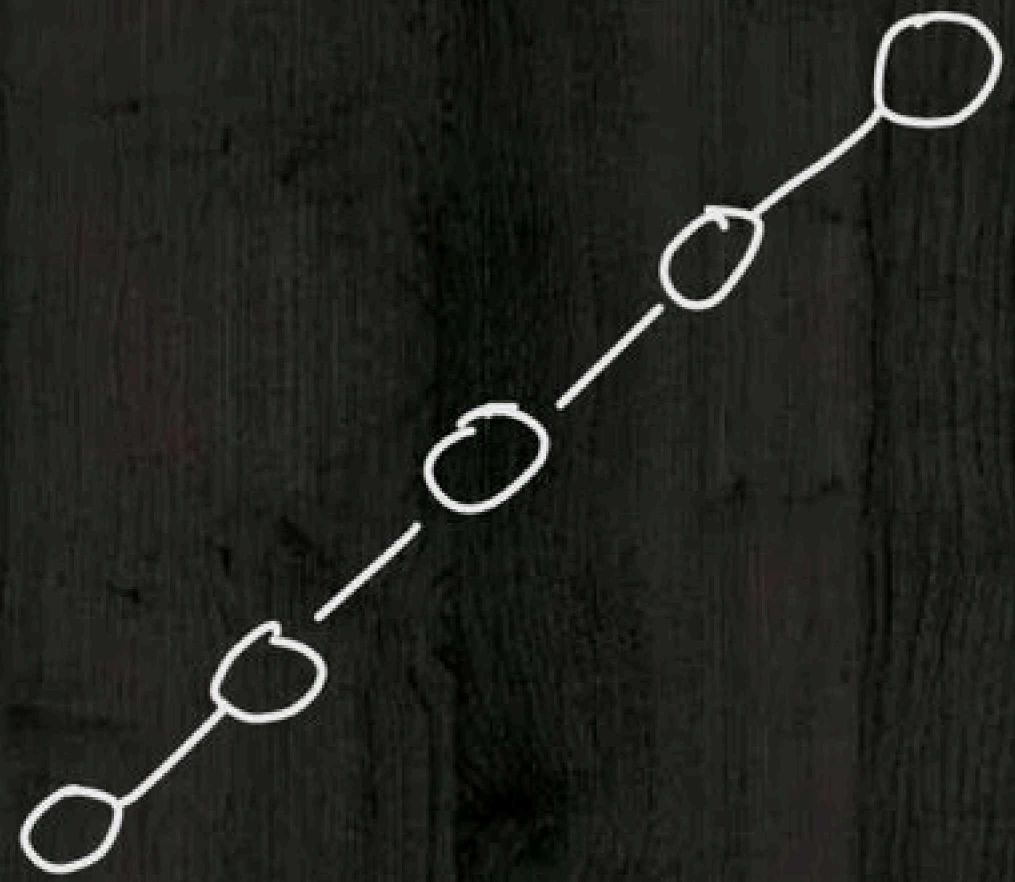
$$N = I + L$$

Put $I = \frac{L-1}{k-1}$

$$N = \frac{L-1}{k-1} + L \Rightarrow N = \frac{\cancel{L} - 1 + Lk - \cancel{L}}{k-1}$$

Left Skewed Binary Tree

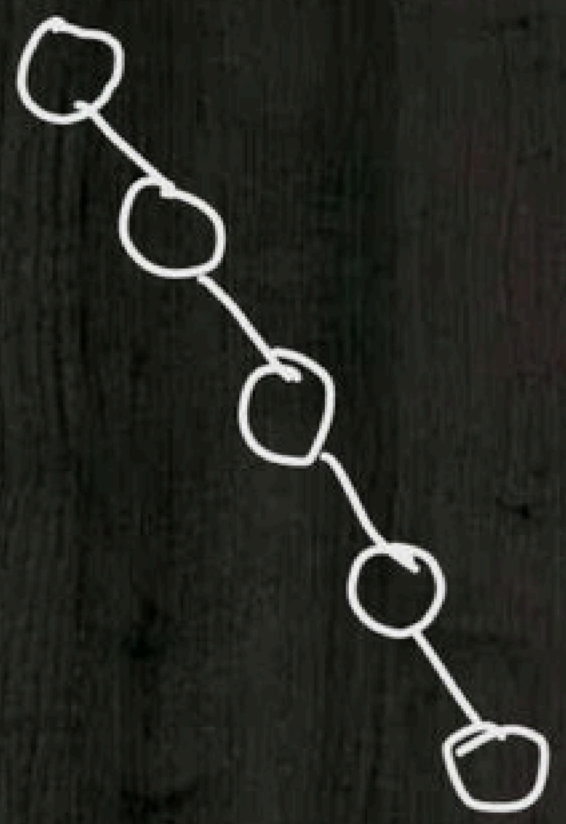
every internal node has only left-child



=> It has only single leaf node

Right Skewed Binary Tree

every internal node has only right child.



$\Rightarrow$ no. of leaf node $= \underline{1}$

DPP

Question

Consider a left skewed binary tree of height h.
How many minimum nodes to be inserted into this tree to convert this tree into a full binary tree of height h.

Note: Height of tree with single node is 0

Question

Consider a right skewed binary tree of height h.
How many minimum nodes to be inserted into this tree to convert this tree into a full binary tree of height h.

Note: Height of tree with single node is 0

Question

Consider a left skewed binary tree of height h.
How many minimum nodes to be inserted into this tree to convert this tree into a complete binary tree of height h.

Note: Height of tree with single node is 0

Question

Consider a right skewed binary tree of height h.
How many minimum nodes to be inserted into this tree to convert this tree into a complete binary tree of height h.

Note: Height of tree with single node is 0

Question

Consider a complete binary tree of height h.
How many minimum nodes to be inserted into this tree to convert this tree into a complete binary tree of height h+1, in the best case and worst case both.

Note: Height of tree with single node is 0

Question

Consider a full binary tree with 31 nodes. Number of leaf nodes and internal nodes are?

Question

Consider a full binary tree with 50 internal nodes. Number of leaf nodes and total number of nodes are?

Question

Is it possible to have a full binary tree with 178 nodes?

Question GATE-2002

The number of leaf nodes in a rooted tree of n nodes, with each node having 0 or 3 children is:

(A) $n/2$
(B) $(n - 1)/3$
(C) $(n - 1)/2$
(D) $(2n + 1)/3$

Question GATE-2005

In a complete k-ary tree, every internal node has exactly k children. The number of leaves in such a tree with n internal node is:

A. nk
B. $(n-1)k+1$
C. $n(k-1)+1$
D. $n(k-1)$

Question GATE-2010

In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

(A) 0

(B) 1

(C) (n-1)/2

(D) n-1

Happy Learning