



# CFG and PDA - V

Complete Course on Theory of Computation

# concatenation

$$\phi \cdot a = \phi$$

$$(1) a^+ \cdot b^+ \neq b^+ \cdot a^+$$

$$(3) \epsilon \cdot L_1 = L_1$$
$$L_1 \cdot \epsilon = L_1$$

$$(2) \phi \cdot L_1 = \phi$$
$$L_1 \cdot \phi = \phi$$

$$(4) \boxed{\Sigma^+ \cdot a = (a+b)^+ \cdot \underline{a}} \neq (a+b)^+$$
$$a \cdot \Sigma^+ = a(a+b)^+$$

$$(5) \boxed{(a+b)^+ \cdot \underline{a^+} = (a+b)^+ = a^+(a+b)^+}$$



$$(6) (a+b)^+ \cdot a^+ = (a+b)^+ \cdot a$$

(7) If  $L_1$  is reg &  $L_2$  also reg then  $L_1 \cdot L_2$  is always reg



⑧

$$L_1 = \text{regul} = \underline{a^n} \mid n \geq 1 = a^+$$

$$L_2 = \text{regul} = \underline{b^n} \mid n \geq 1 = b^+$$

$$L_1 \cdot L_2 = \cancel{a^n \cdot b^n} \mid m, n \geq 1 \Rightarrow \underline{a^+ b^+} \Rightarrow \text{regul}$$

9.1

$$\underline{a^n b^n \mid n \geq 1} \cdot \frac{\phi}{L_2} = \phi \text{ Regul}$$

9.2

$$\underline{a^n b^n} \cdot \frac{b}{L_2} \Rightarrow a^n b^{n+1}$$

10

$$\underline{a^n b^n} \cdot \underline{b^+} \mid m, n \geq 1 \left\{ \begin{array}{l} \text{Non} \\ \text{regul} \end{array} \right.$$



$a^n b^n c^n$   
 $\Downarrow$   
CFL



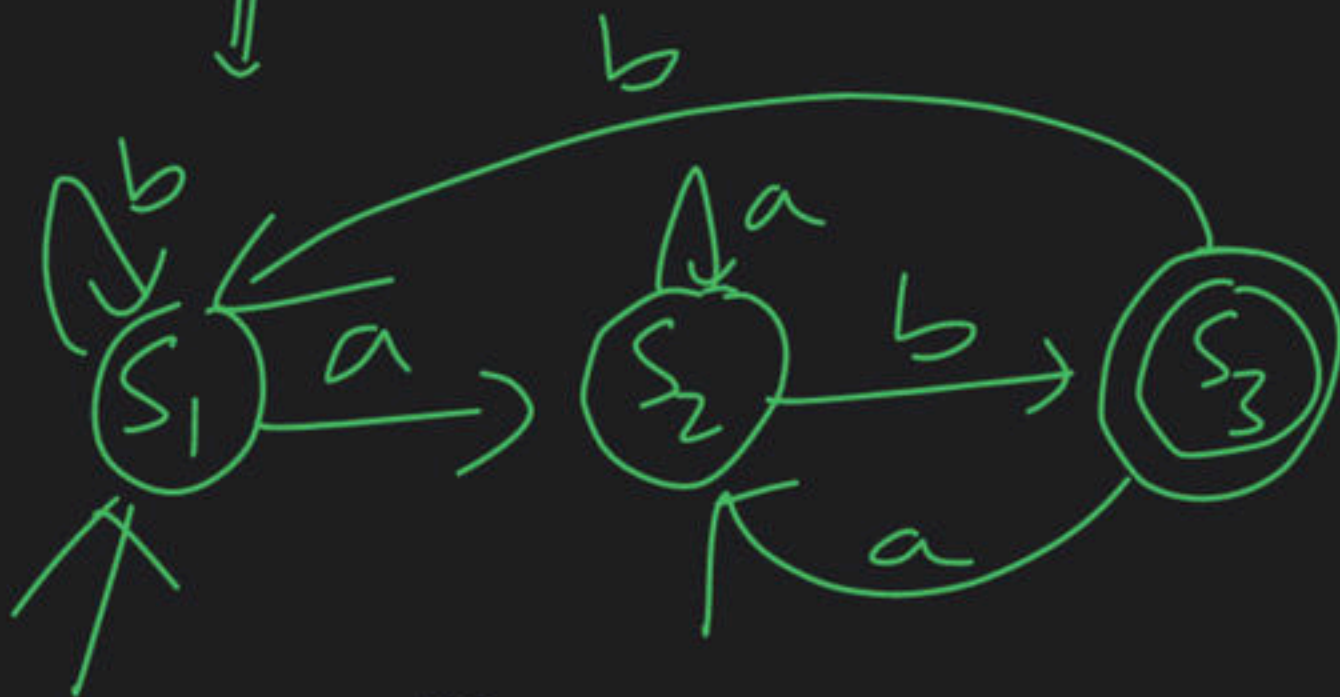
# Reversal operator

$$L = (a+b)^b ab$$

$\Downarrow$

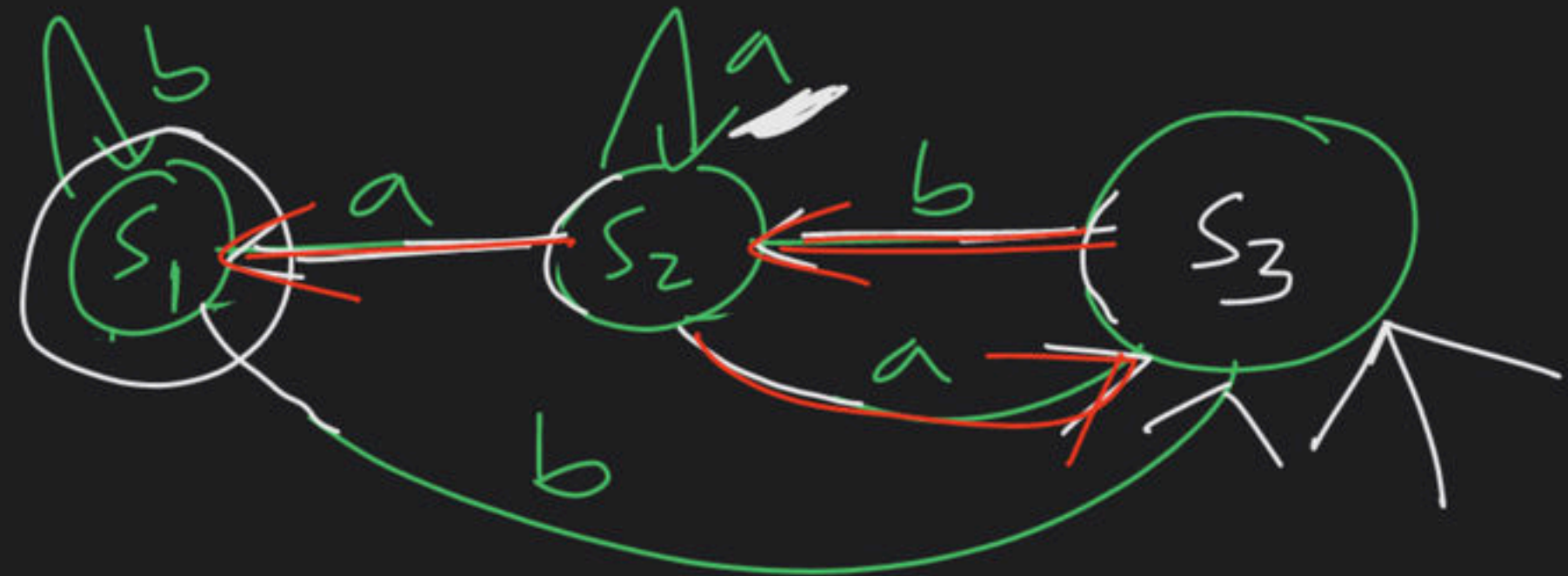
DFA

$\Downarrow$



$$L^R = ba(a+b)^b$$

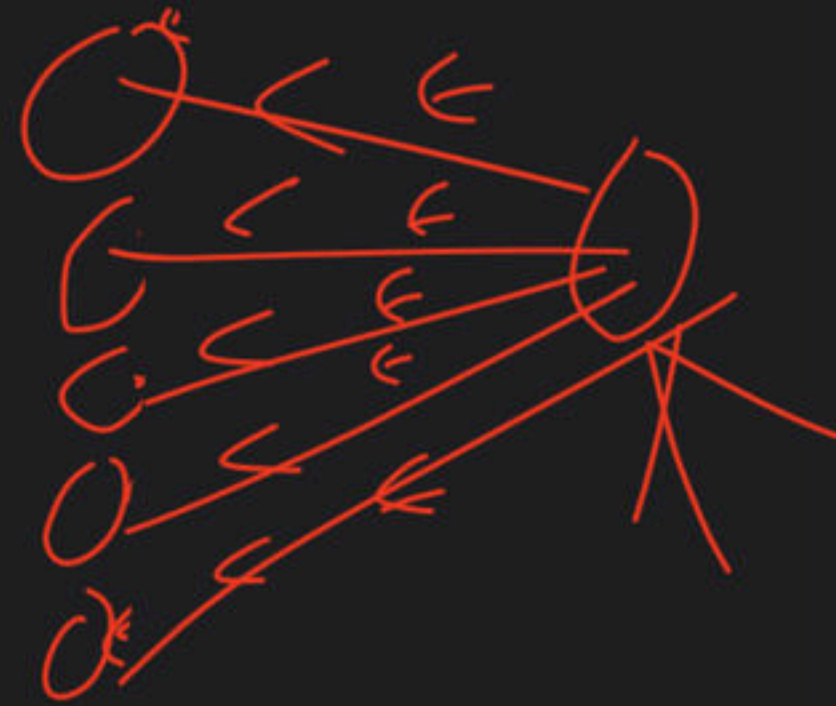
baaaaaa



① reverse edge direction

② Interchange

Initial & Final



③ DFA  $\xrightarrow{\text{Reverse}}$  DFA / NFA /  $\epsilon$ -NFA

④ After reverse it more than 1-initial  
then make it as single initial with  $\epsilon$

help of  $\epsilon$ -transition.

Note: If  $L$  is regular then  $L^R$  also regular



(2)  $L = \Sigma^*$  then  $L^R = \Sigma^*$



(3)  $L = a^* \Rightarrow L^R = a^*$

(4)  $L = \emptyset \Rightarrow L^R = \emptyset$

(5)  $a(a+b)^* \xRightarrow{R} (a+b)^* a$

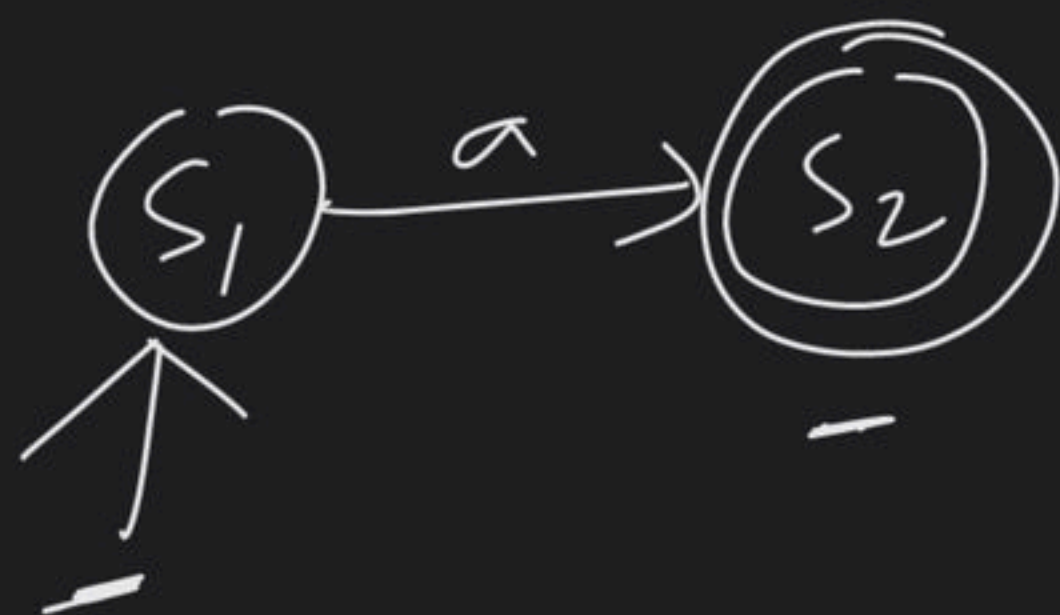
(6)  $a^* b^* \xRightarrow{R} b^* a^*$

(7)  $(a+b)^* a b (a+b)^* \xRightarrow{R} (a+b)^* b a (a+b)^*$

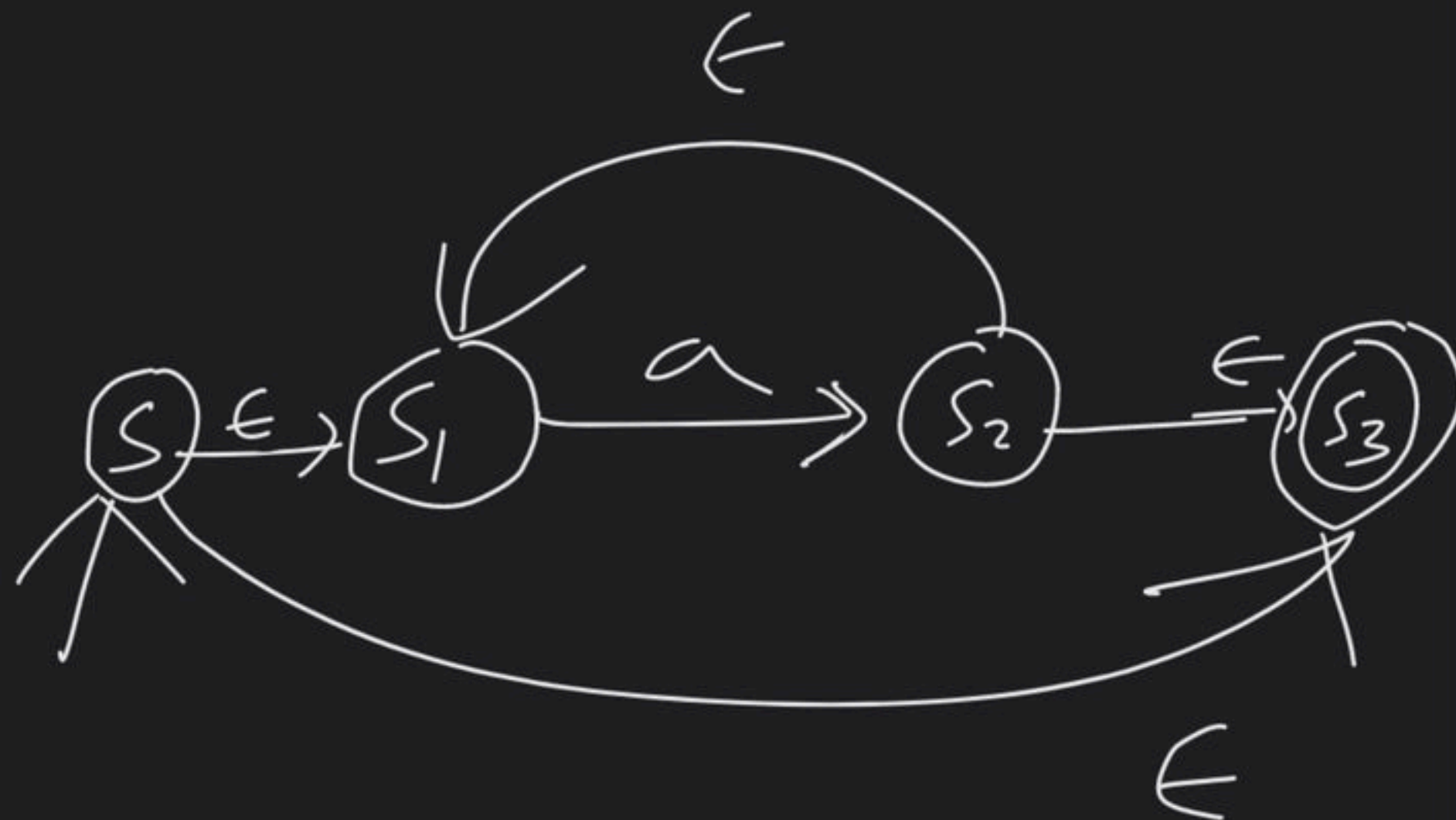
# Kleen closure

①

$$L = a$$



$\xRightarrow{L^*}$



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Note: If  $L$  is regular then  $L^*$  also regular



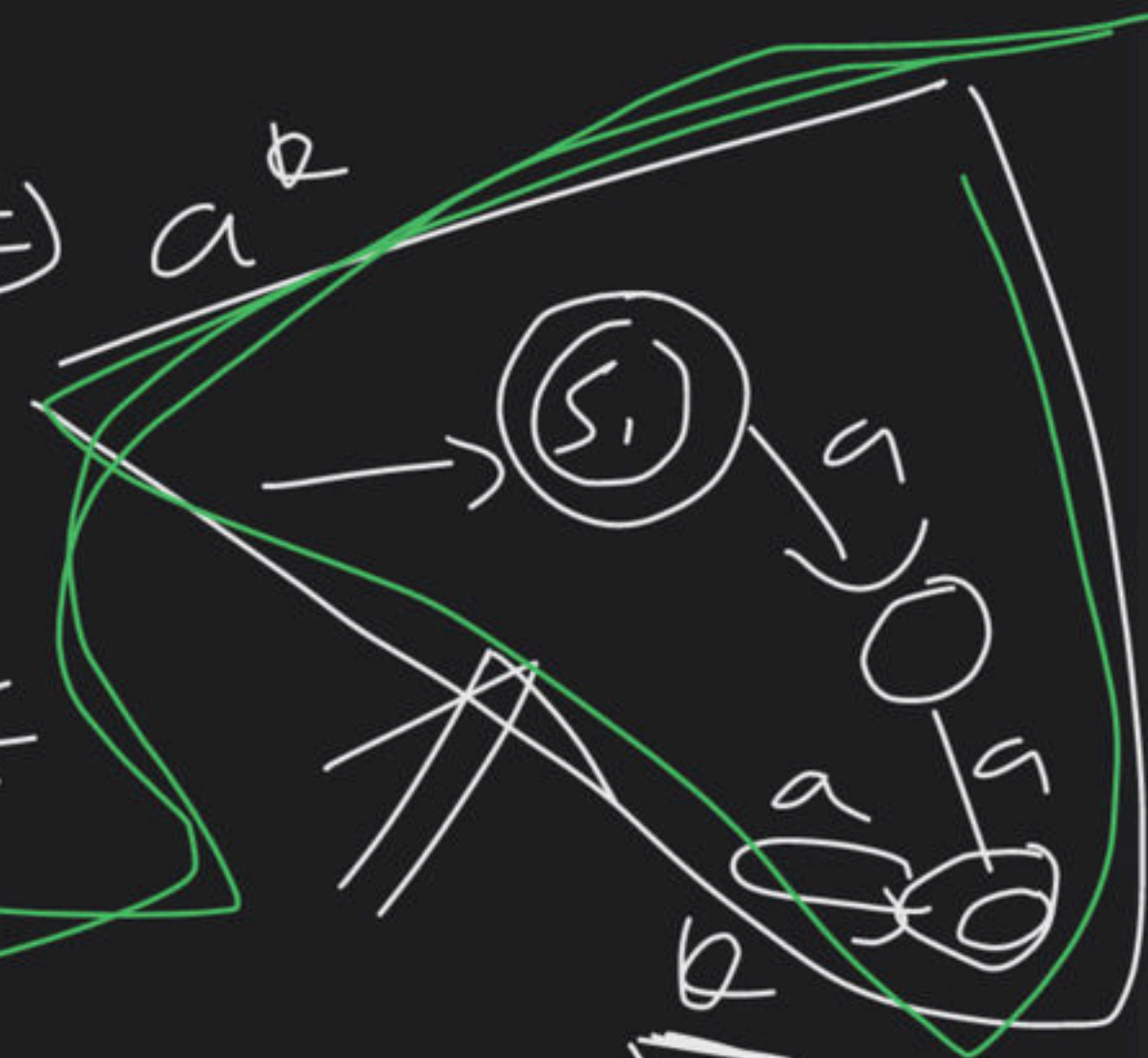
(2)  $L = \emptyset$  then  $L^0 = \epsilon$

(3)  $L = a^k b^k$  then  $L^0 \Rightarrow (a^k b^k)^0 = (a+b)^0$

(4)  $\epsilon \xrightarrow{L} \epsilon + a^+ \Rightarrow a^0 \Rightarrow L^0 = (a^0)^0 \Rightarrow a^0$

(5)  $(L^0)^0 = L^0$

(6)  $\epsilon^0 = \epsilon$



$L = \{a^p \mid p \text{ is prime}\}$

Reg ✗ CFL ✗  
CSL ✓

$L^0 = (a^p \mid p \text{ is prime})$

$= \epsilon, a^2, a^4, \underline{a^6}, a^8, a^{10}, a^3, a^9, \dots \Rightarrow$

Inf ✓

(6) of List

non  
regular

Item  $L^*$

may be reg  
(w)

may not be reg

Thank you

Dedicté Hct