Number Systems - I

Comprehensive Course on Digital Logic Design 2023/2024

B V Reddy • Lesson 17 • Nov 11, 2022

DIGITAL LOGIC CIRCUITS

(ECE /CSE/EEE/IN)

Karnaugh Map(K- Map)

K- Map

- It is a graphical technique to minimize the Boolean expressions, that may contain don't care combinations.
- K- map is a systematic method and suitable up to 5 variables .
- Gray code is used to formulate K-map, the minimization is based on the gray code property i.e. ***Logical Adjacency***.

- K- map technique is used in 2 format's
  1. If we need the answer in SOP form , then the K-map is used in minterm mode
  2. If we need the answer in POS form ,then the K- map is used in Maxterm mode

0 → 00

1 → 01

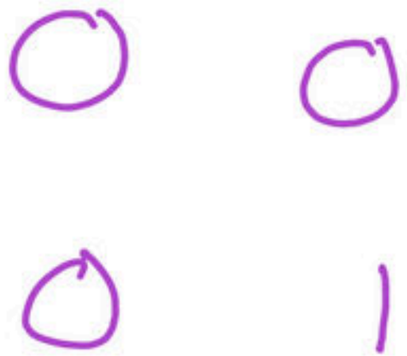
2 → 10

3 → 11

Gray Code
00
01
11
10

$$Y = \overset{00}{\bar{A}\bar{B}} + \overset{11}{AB} + \overset{10}{A\bar{B}} + \bar{A}B$$

$$Y = \underset{00}{\bar{A}\bar{B}} + \underset{10}{A\bar{B}} + AB + \bar{A}B$$



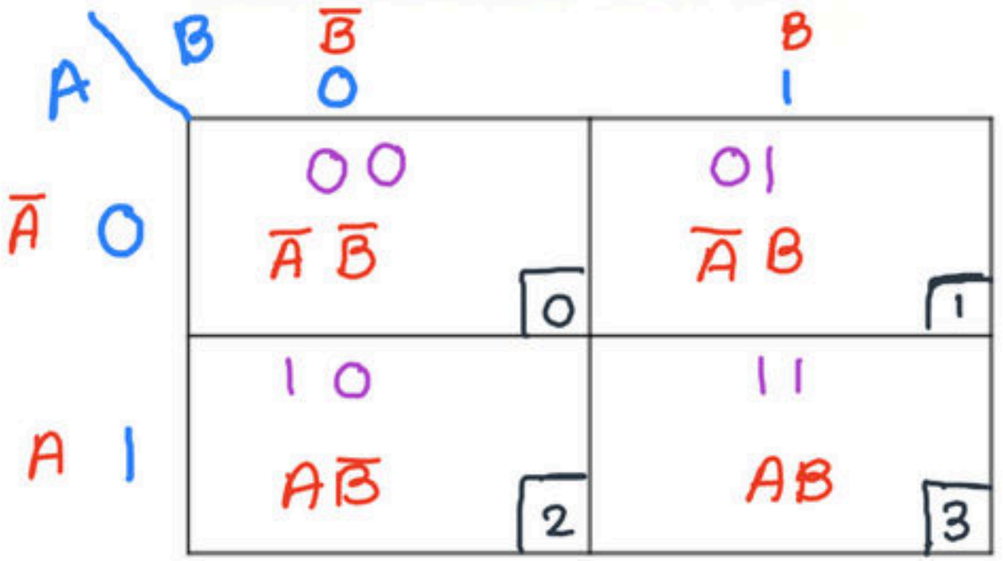
Logical Adjacency

- Two cells are said to be logical adjacent to each other , if there is only one bit change between them .
- For a n-variable K –map , for every cell there are ‘ n ‘ logical adjacent cells

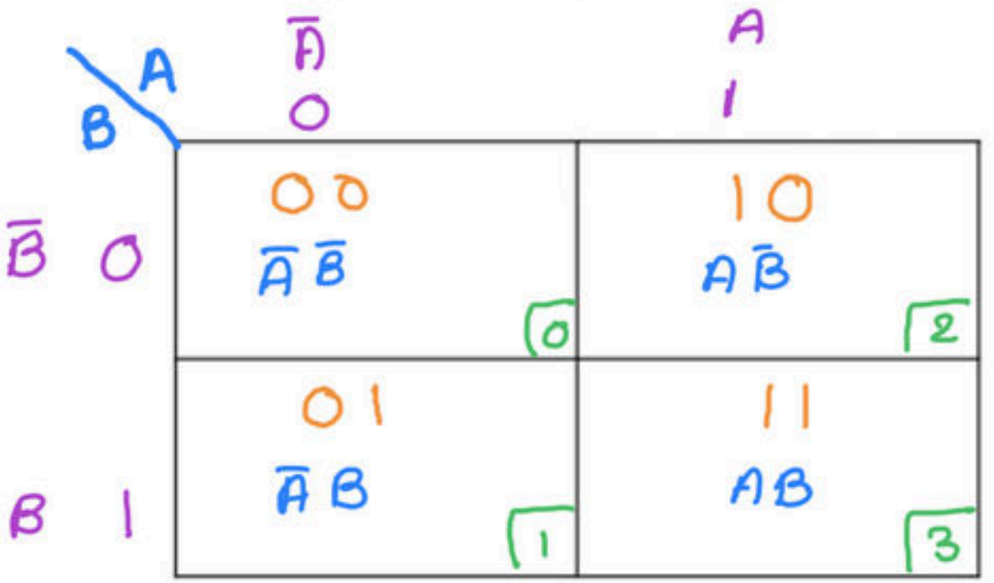
2- Variable K- Map

$f(A, B) = \_\_$

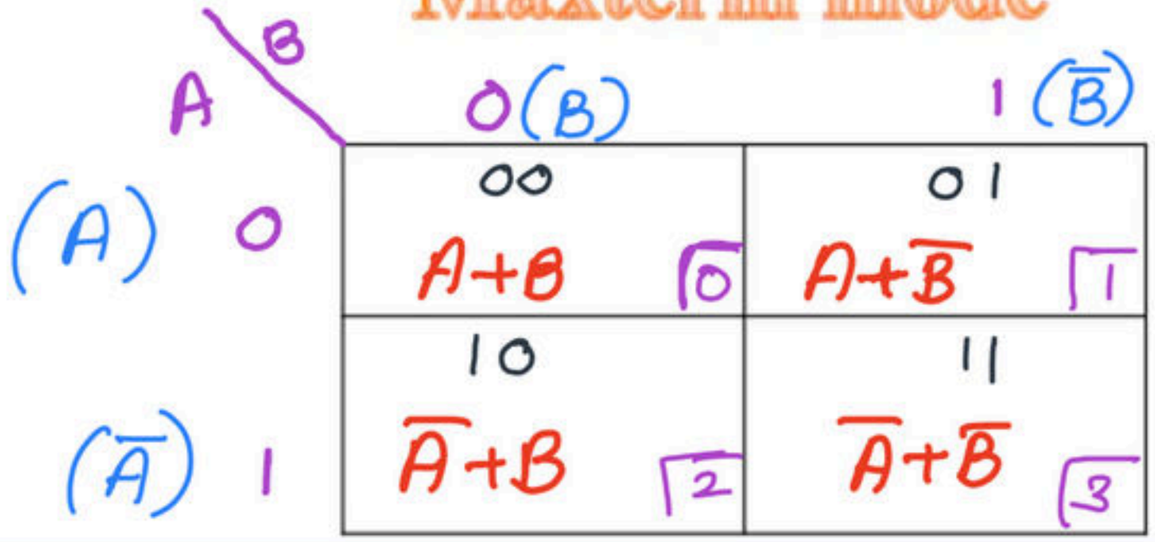
Minterm mode



Minterm mode



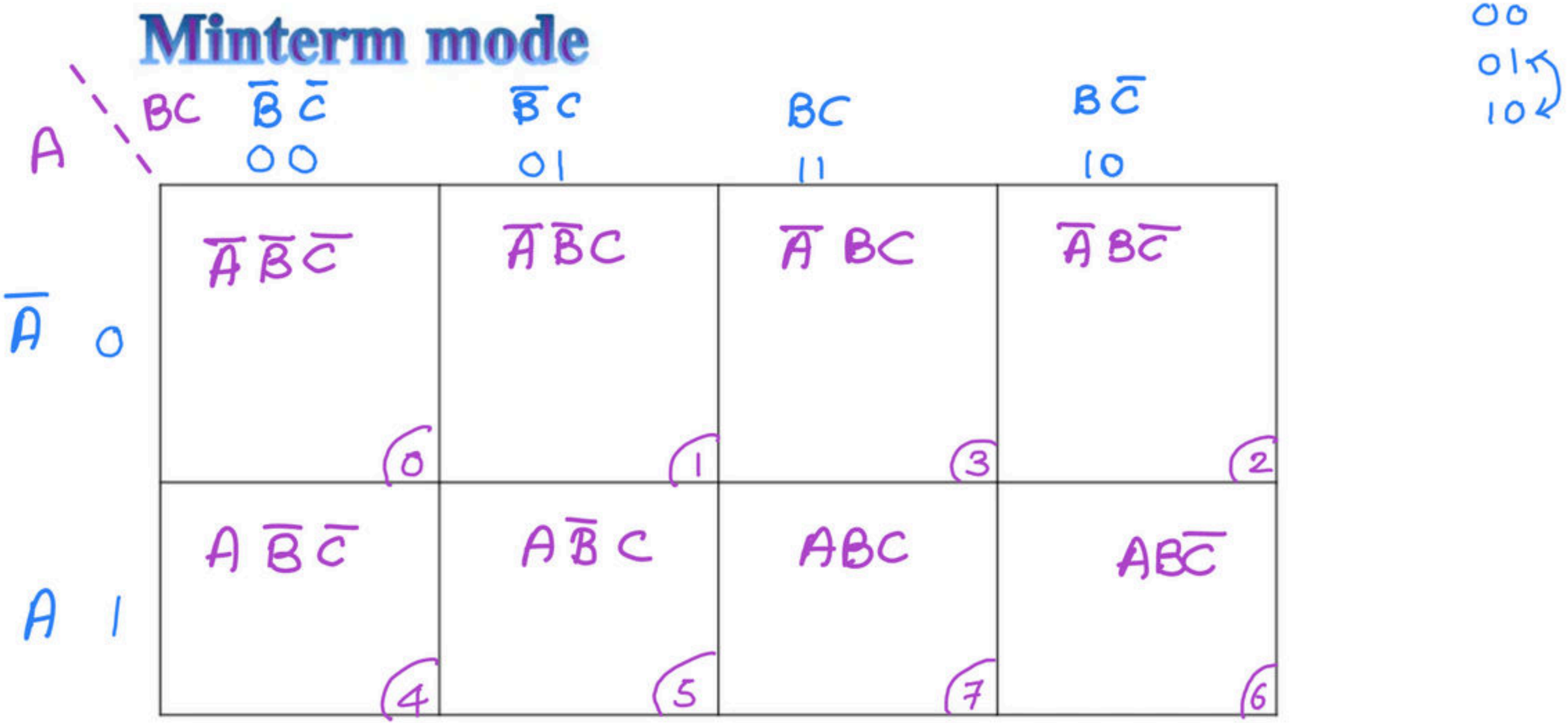
Maxterm mode



3-Variable K-Map

$f(A, B, C) =$ ———

Minterm mode



00
01
10

3-Variable K-Map

$f(A, B, C) =$ ——————.

Minterm mode

B \ AC	$\bar{A}\bar{C}$ (00)	$\bar{A}C$ (01)	AC (11)	$A\bar{C}$ (10)
$\bar{B}$ (0)	000 (0)	001 (1)	101 (5)	100 (4)
B (1)	010 (2)	011 (3)	111 (7)	110 (6)

A B C .
1 0 1

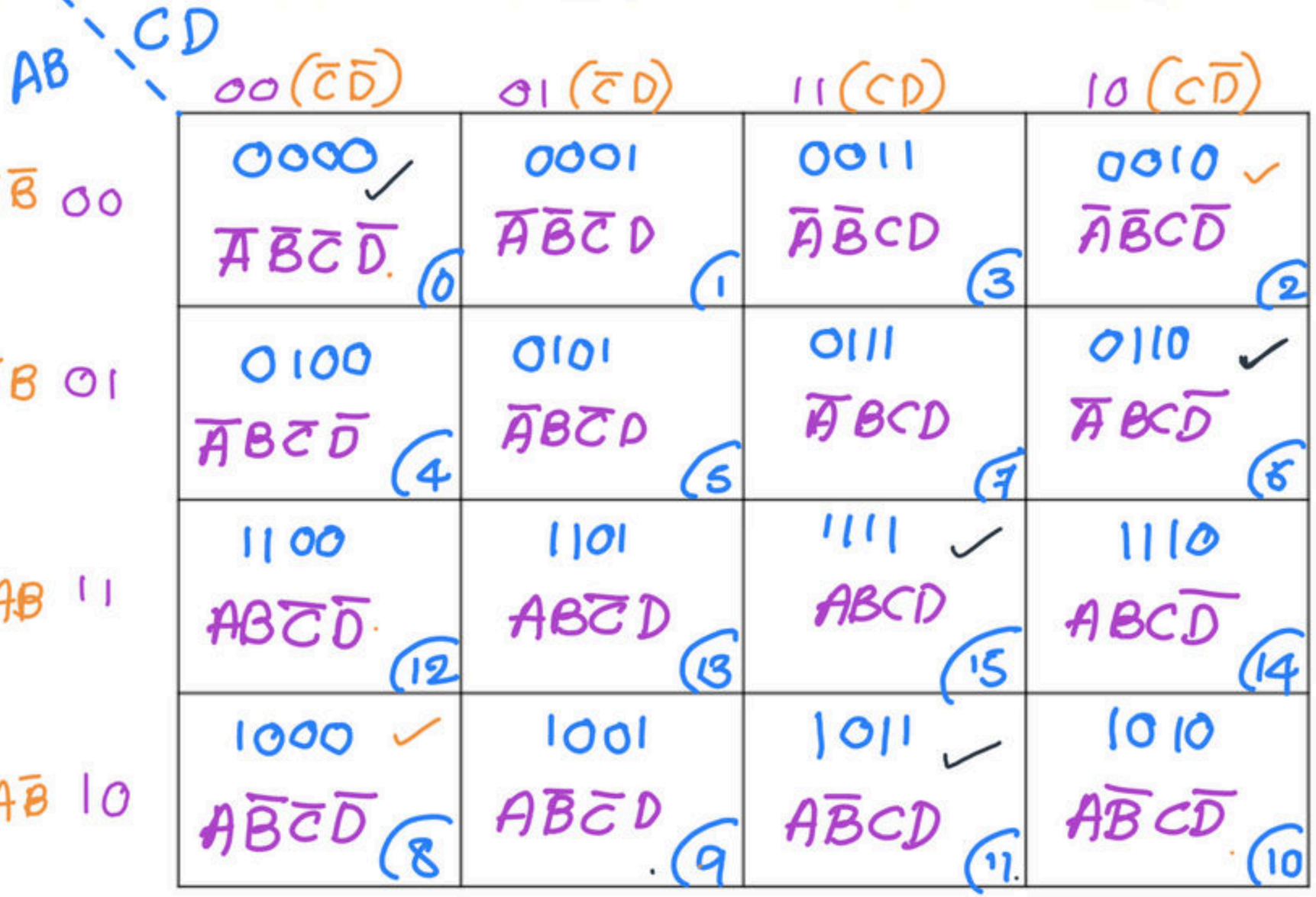
3-Variable K-Map

$f(A, B, C) = \_\_\_\_$

Maxterm mode

A \ BC	00 $(B+C)$	01 $(B+\bar{C})$	11 $(\bar{B}+\bar{C})$	10 $(\bar{B}+C)$
(A) 0	$A+B+C$ (0)	$A+B+\bar{C}$ (1)	$A+\bar{B}+\bar{C}$ (3)	$A+\bar{B}+C$ (2)
$(\bar{A})$ 1	$\bar{A}+B+C$ (4)	$\bar{A}+B+\bar{C}$ (5)	$\bar{A}+\bar{B}+\bar{C}$ (7)	$\bar{A}+\bar{B}+C$. (6)

4- Variable K-map(minterm mode)



$f(A, B, C, D)$

Adjacent cell for 8 – (0, 9, 10, 12)

Adjacent cell for 2 – (0, 3, 6, 10)

Adjacent cell for 11 – (9, 10, 15, 3)

Adjacent cell for 15 – (7, 11, 13, 14)

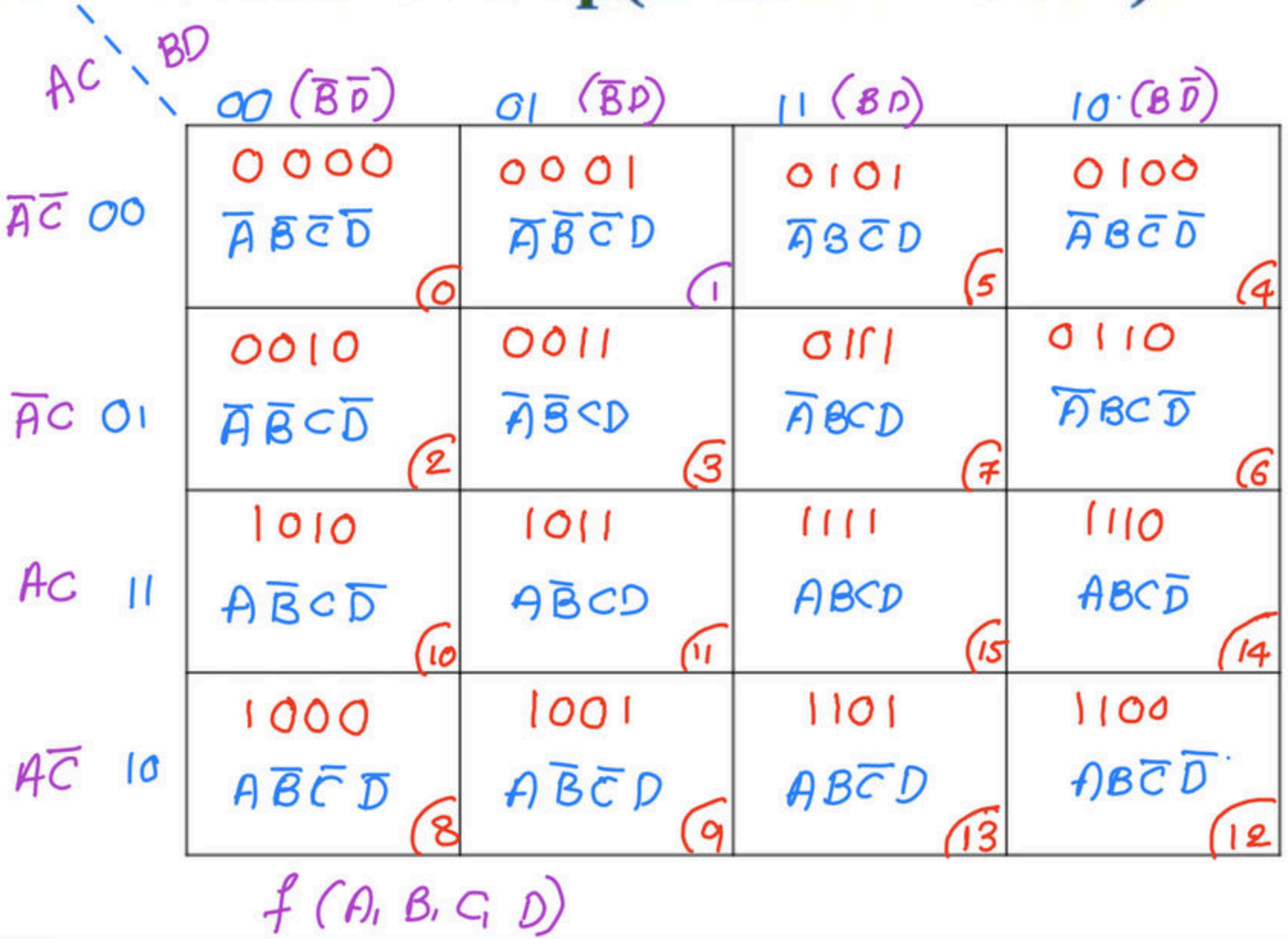
Adjacent cell for 0 – (1, 4, 2, 8)

Adjacent cell for 6 – (2, 7, 14, 4)

Adjacent cell for 14 – (6, 15, 10, 12)

Adjacent cell for 3 – (1, 2, 7, 11)

4- Variable K-map(minterm mode)



Adjacent cell for 8 – (0, 9, 10, 12)

Adjacent cell for 2 – (0, 3, 10, 6)

Adjacent cell for 11 – (3, 10, 15, 9)

Adjacent cell for 15 – (7, 11, 14, 13)

Adjacent cell for 0 – (1, 2, 8, 4)

Adjacent cell for 6 – (4, 7, 2, 14)

Adjacent cell for 14 – (6, 15, 12, 10)

Adjacent cell for 3 – (1, 2, 7, 11)

4- Variable K-map (Maxterm mode)

$f(A, B, C, D)$

AB \ CD	00 $(C+D)$	01 $(C+\bar{D})$	11 $(\bar{C}+\bar{D})$	10 $(\bar{C}+D)$
00 $(A+B)$	0000 $(A+B+C+D)$ (0)	0001 $A+B+C+\bar{D}$ (1)	0011 $A+B+\bar{C}+\bar{D}$ (3)	0010 $A+B+\bar{C}+D$ (2)
01 $(A+\bar{B})$	0100 $A+\bar{B}+C+D$ (4)	0101 $A+\bar{B}+C+\bar{D}$ (5)	0111 $A+\bar{B}+\bar{C}+\bar{D}$ (7)	0110 $A+\bar{B}+\bar{C}+D$ (6)
11 $(\bar{A}+\bar{B})$	1100 $\bar{A}+\bar{B}+C+D$ (12)	1101 $\bar{A}+\bar{B}+C+\bar{D}$ (13)	1111 $\bar{A}+\bar{B}+\bar{C}+\bar{D}$ (15)	1110 $\bar{A}+\bar{B}+\bar{C}+D$ (14)
10 $(\bar{A}+B)$	1000 $\bar{A}+B+C+D$ (8)	1001 $\bar{A}+B+C+\bar{D}$ (9)	1011 $\bar{A}+B+\bar{C}+\bar{D}$ (11)	1010 $\bar{A}+B+\bar{C}+D$ (10)

$Y = A + B$.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

$Y = \bar{A}B + A\bar{B} + AB$.

↳ SOP.

$Y = A + B$

Minimization Steps

1. Place the corresponding minterm/maxterms in the corresponding cells.
   minterm place ------> 1
   Maxterm place ------> 0

1. Using the valid sub cube property go for bigger size grouping .

2. Don't care combination's may not be covered , it depends on the K-map.

3. Remove the redundant groups if any present .

dbn't care (X)

$\frac{100}{2} + 1 = 51$

100

0/1

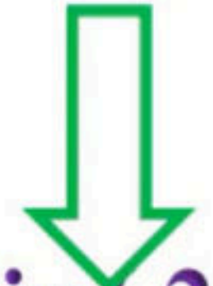
Yash	Deep raj	Praneet
38	52	10

Priority of grouping

(16-cells)

Octet (8-cells)

Quad (4-cells)



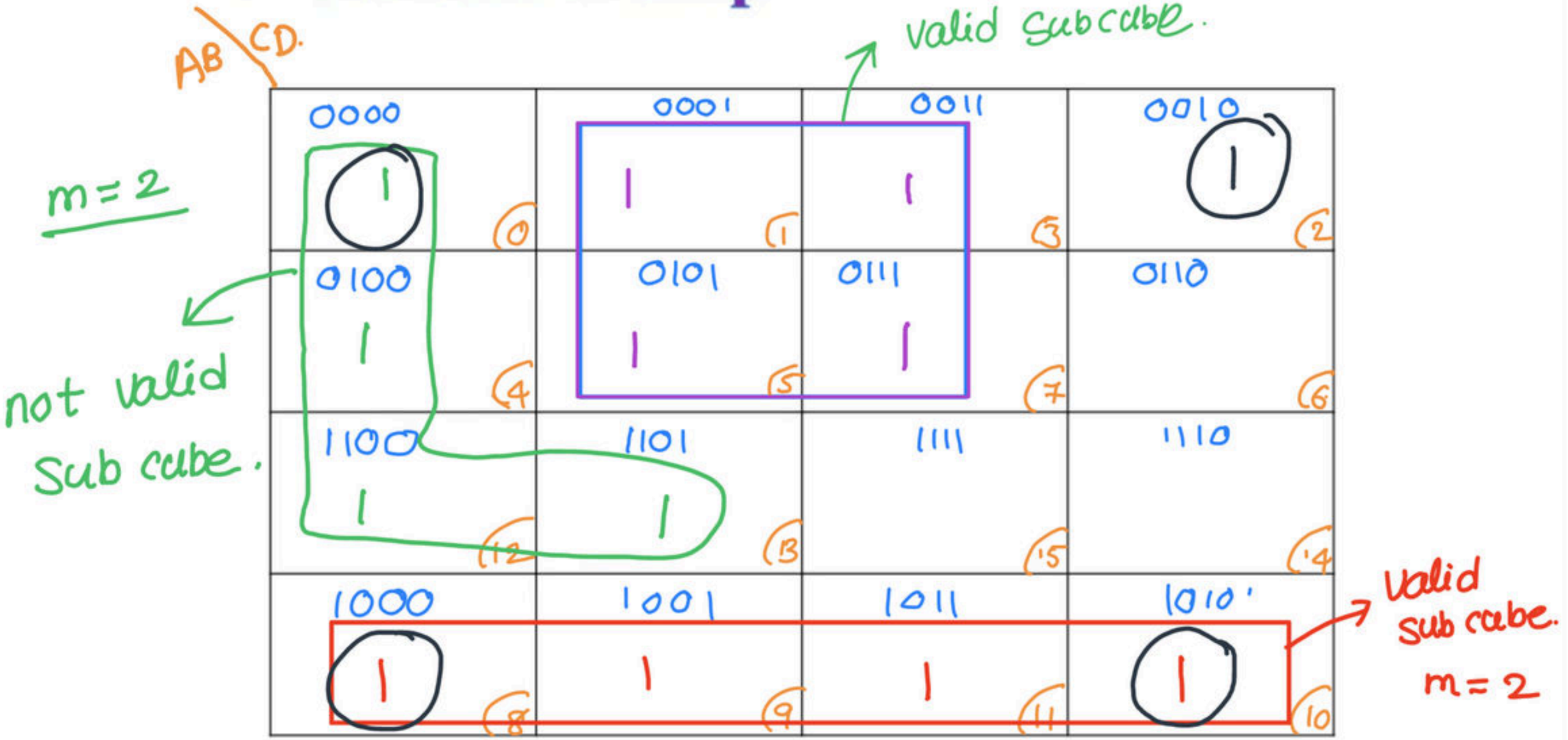
Pair (2-cells)

Valid Sub cube

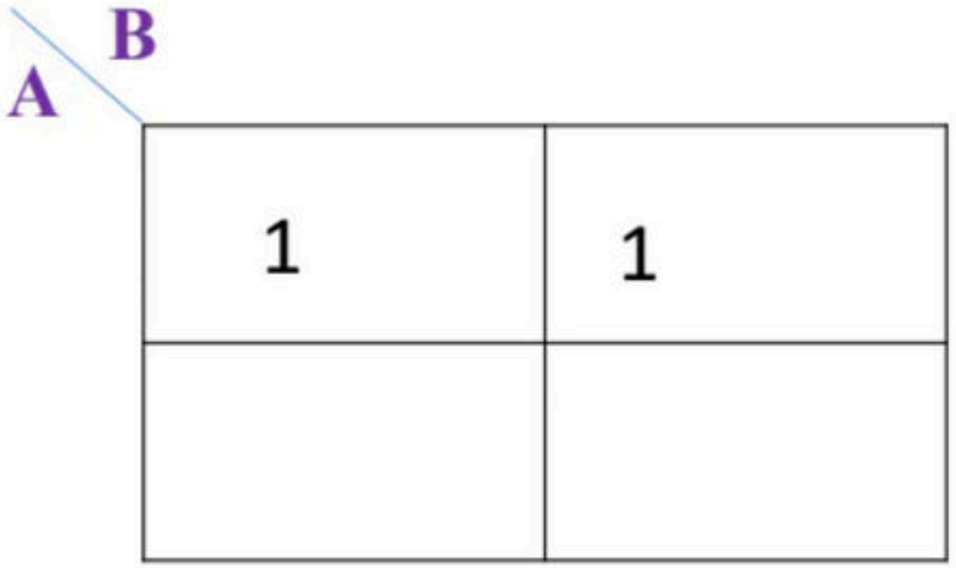
A collection of 2^m number of cells in a K- map is said to be forming a valid sub cube , provided inside the collection 2^m number of cells every cell is logical adjacent to 'm' number of cells .

4- Variable K-map

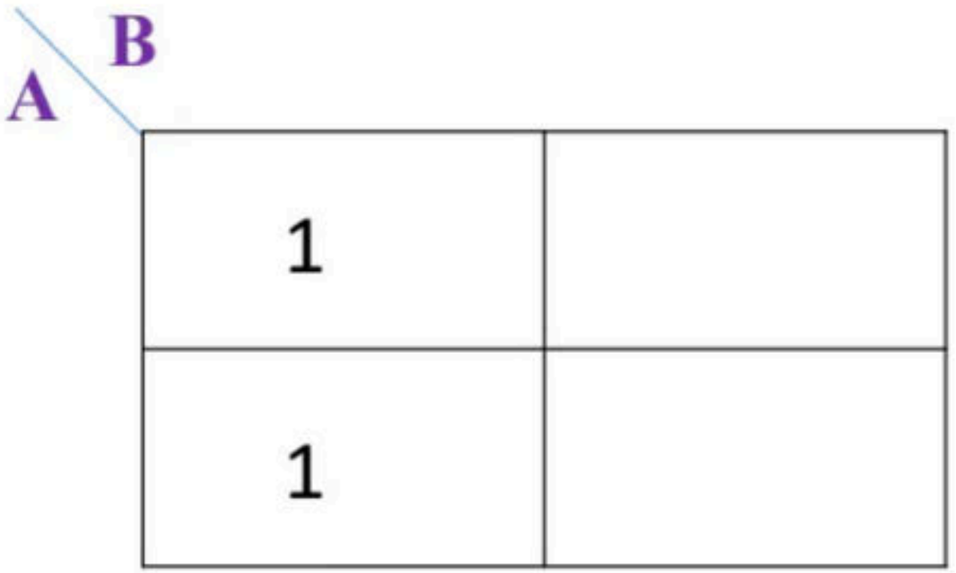
$4 = 2^2$ $\quad m = 2$



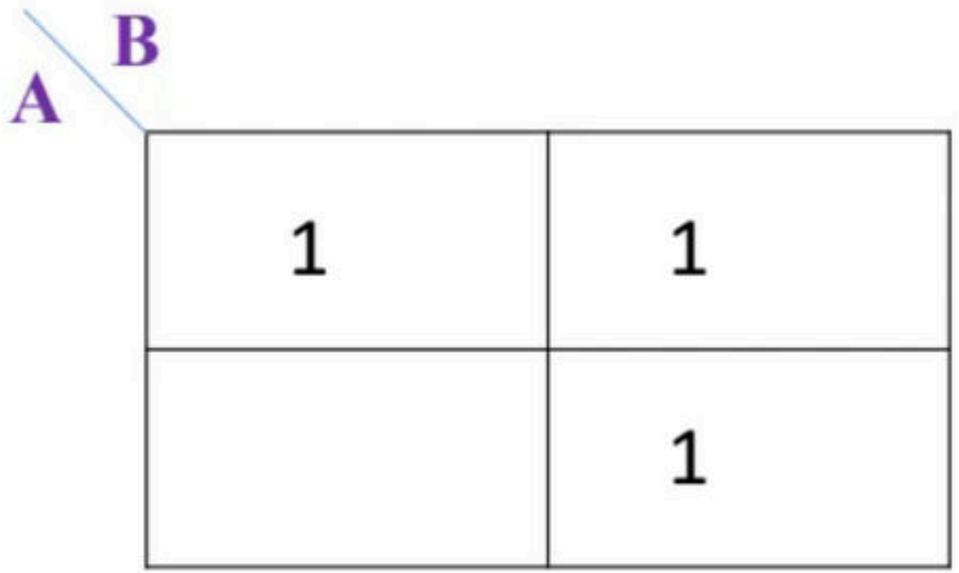
Q) Minimize the following



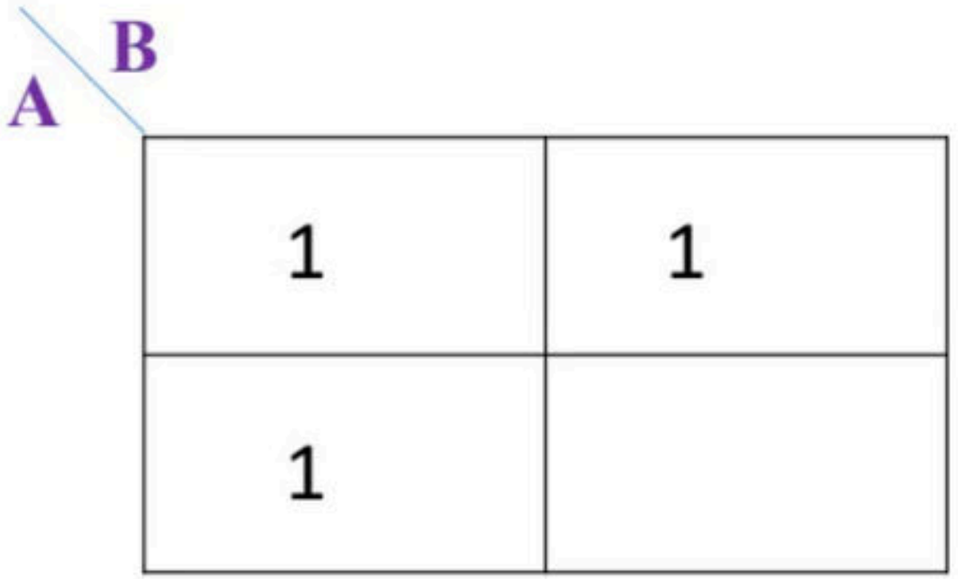
Q) Minimize the following



Q) Minimize the following



Q) Minimize the following



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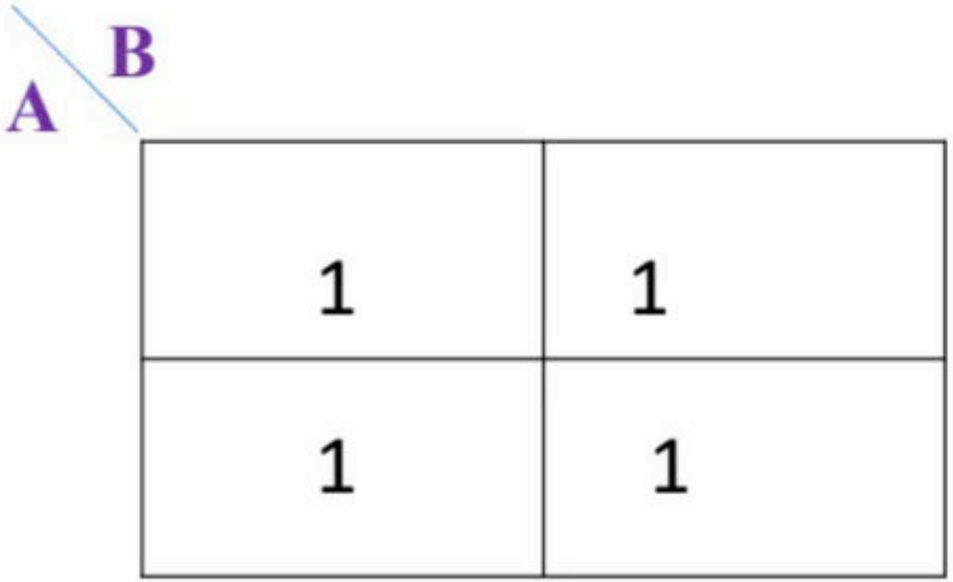

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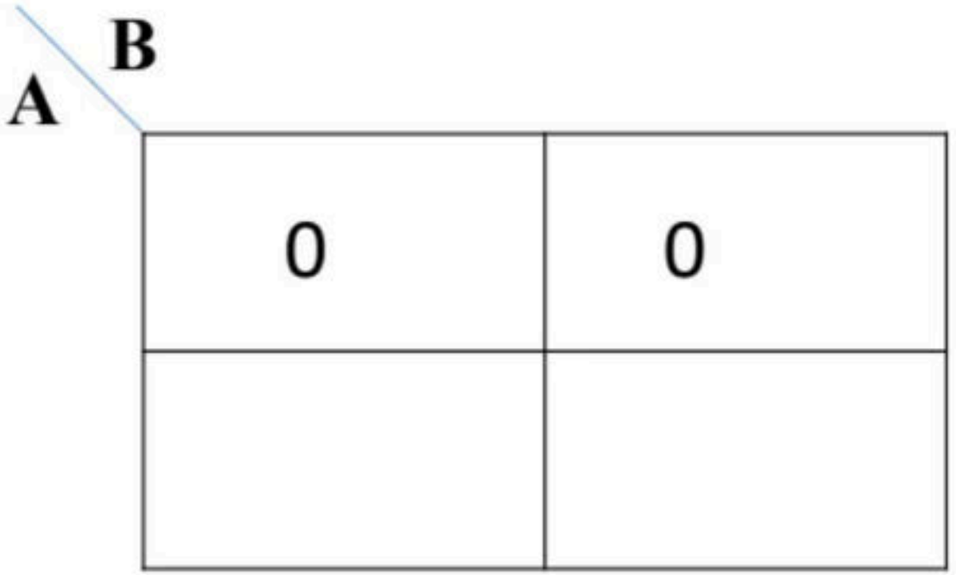

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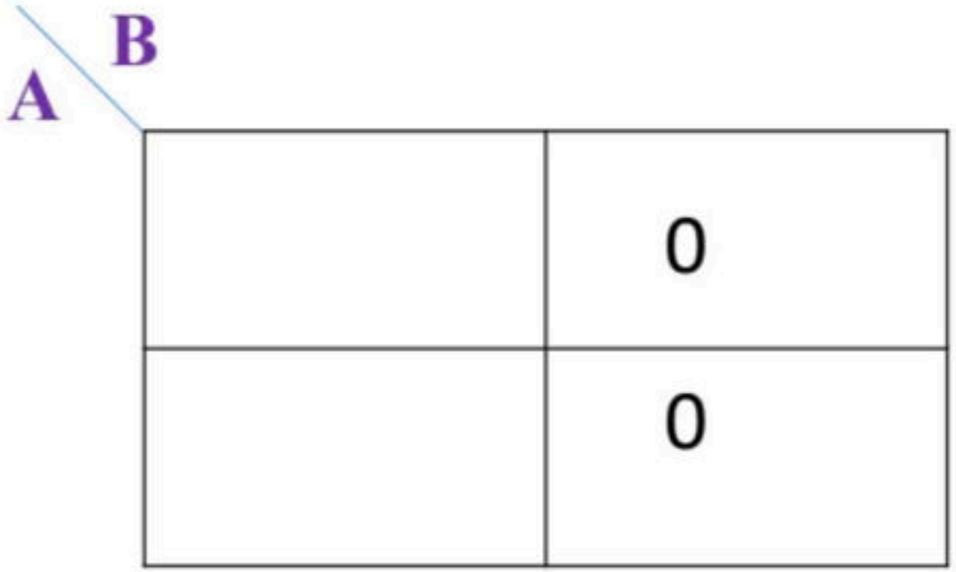

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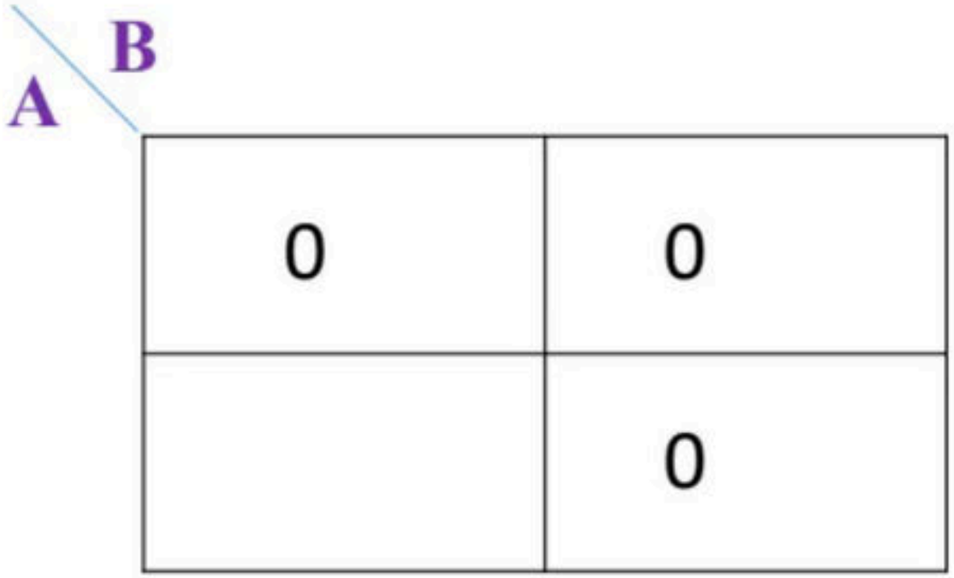

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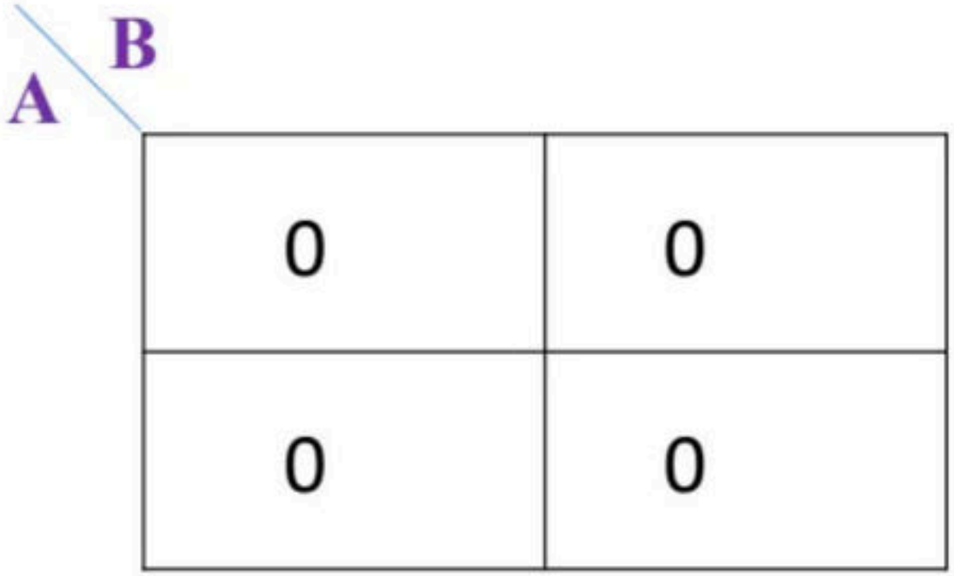
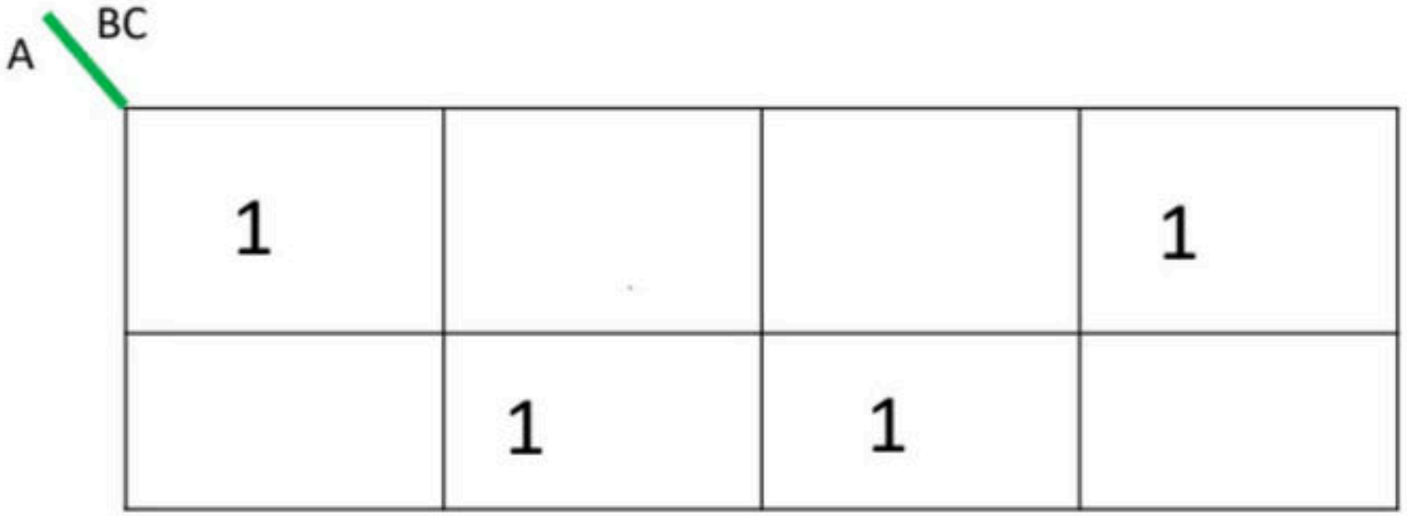


Q) Minimize the following



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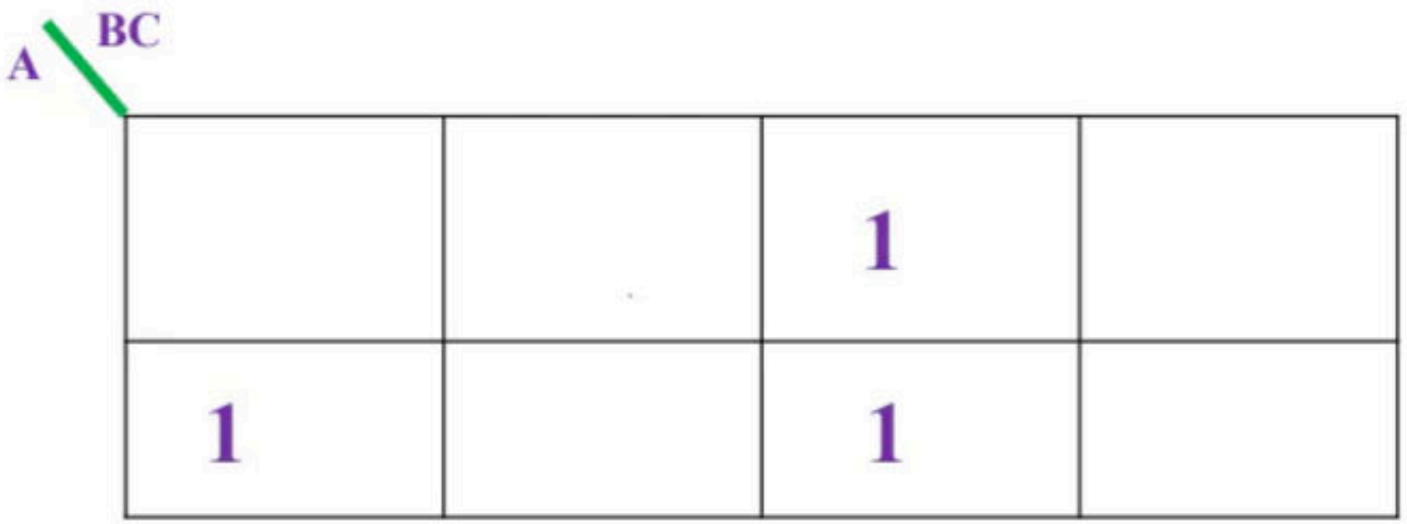
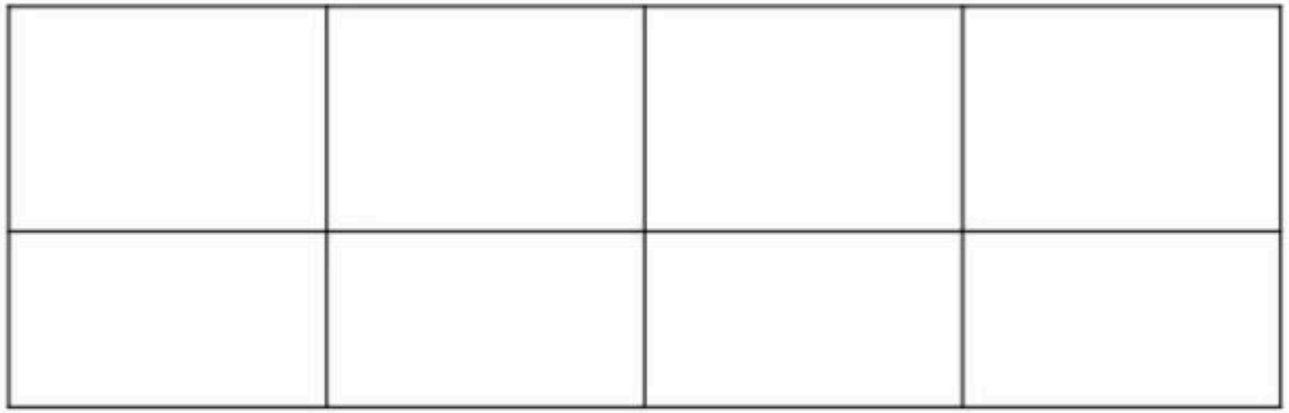


Q) F(A,B,C) = $\bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$

Q) $F(A, B, C) = \sum m(0,1,2,3,4,6)$



Q) $F(A, B, C) = \sum m(0,1,3,4)$

Q) $F(A, B, C) = \sum m(0,1,3,5,6)$

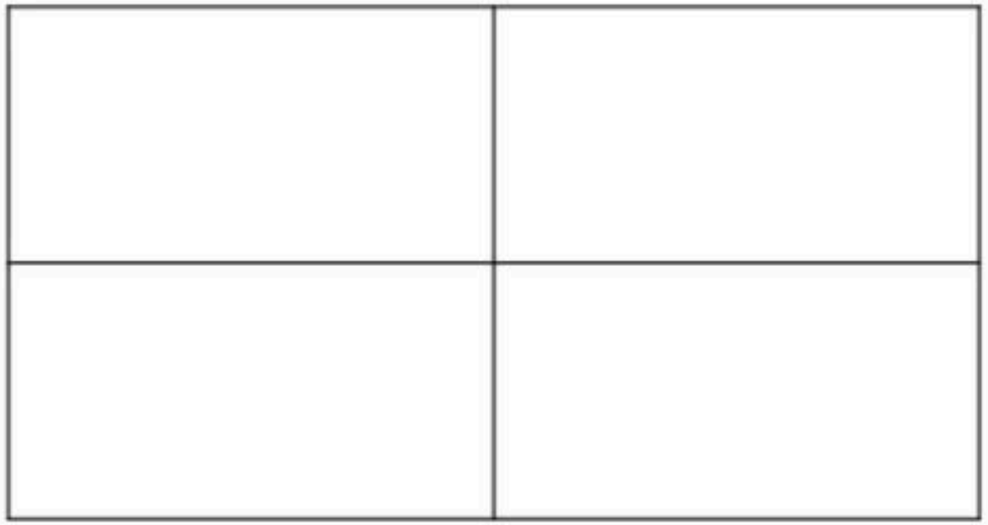
Q) F (A,B,C) = $\prod M(2,4,7)$

Q) Minimize the following

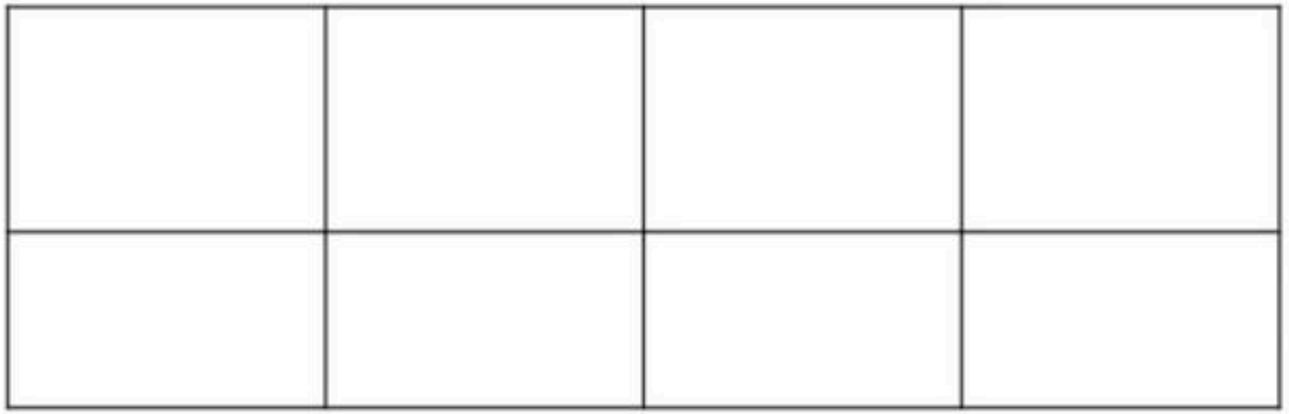
AB \ CD				
	1			1
		1		1
		1	1	
		1		

Q) $F(A, B) = \sum m(0,3) + d(2)$

Q) F (A, B) = $\prod M\ (0,1) + d(3)$



Q) F (A, B, C) = $\prod M\ (1, 2, 6, 4, 7)$



Q) Minimize the following

AB \ CD				
	1			1
		1	1	
		1	1	
	1			1

Q) Minimize the following

AB \ CD				
		1	1	
	1			1
	1			1
		1	1	

Q) Minimize the following

AB \ CD				
	1	1	1	1
		1	1	1

Q) Minimize the following

AB \ CD				
	1	1	1	1
	1	1	1	1

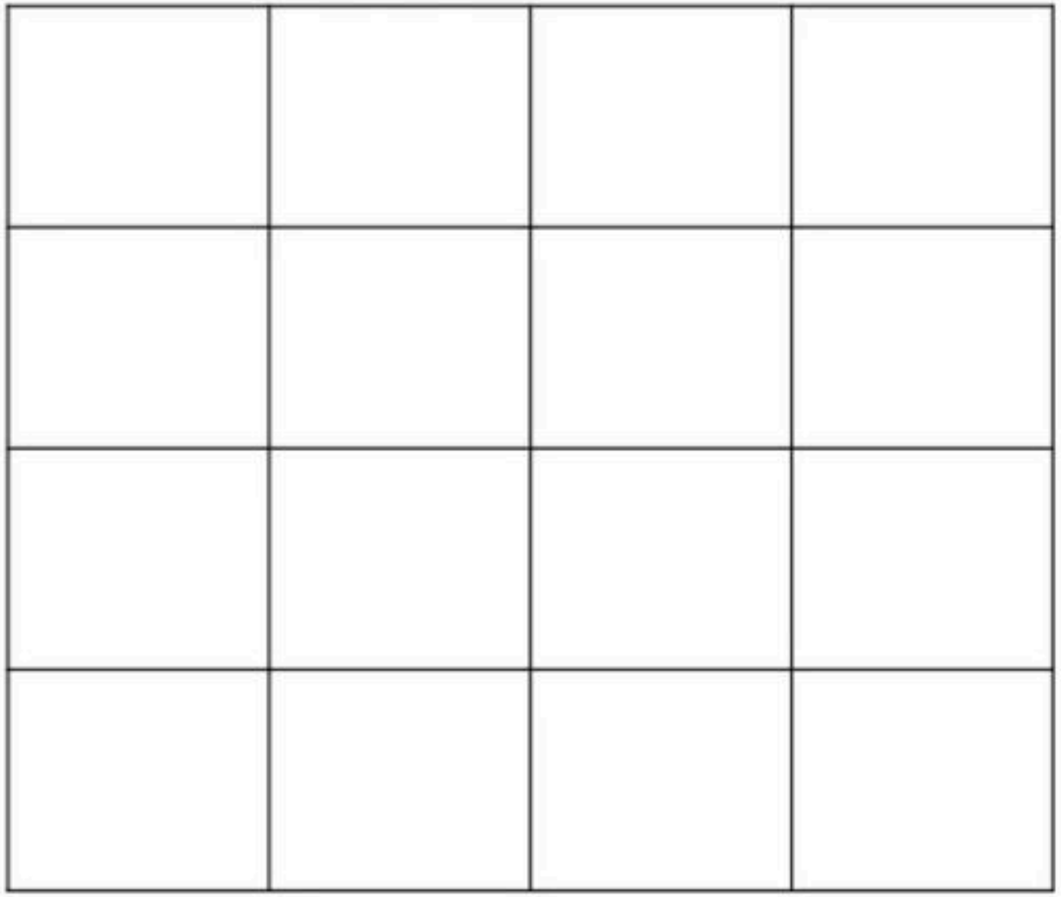
Q) Minimize the following

AB \ CD				
	1	1	1	1
	1	1	1	1

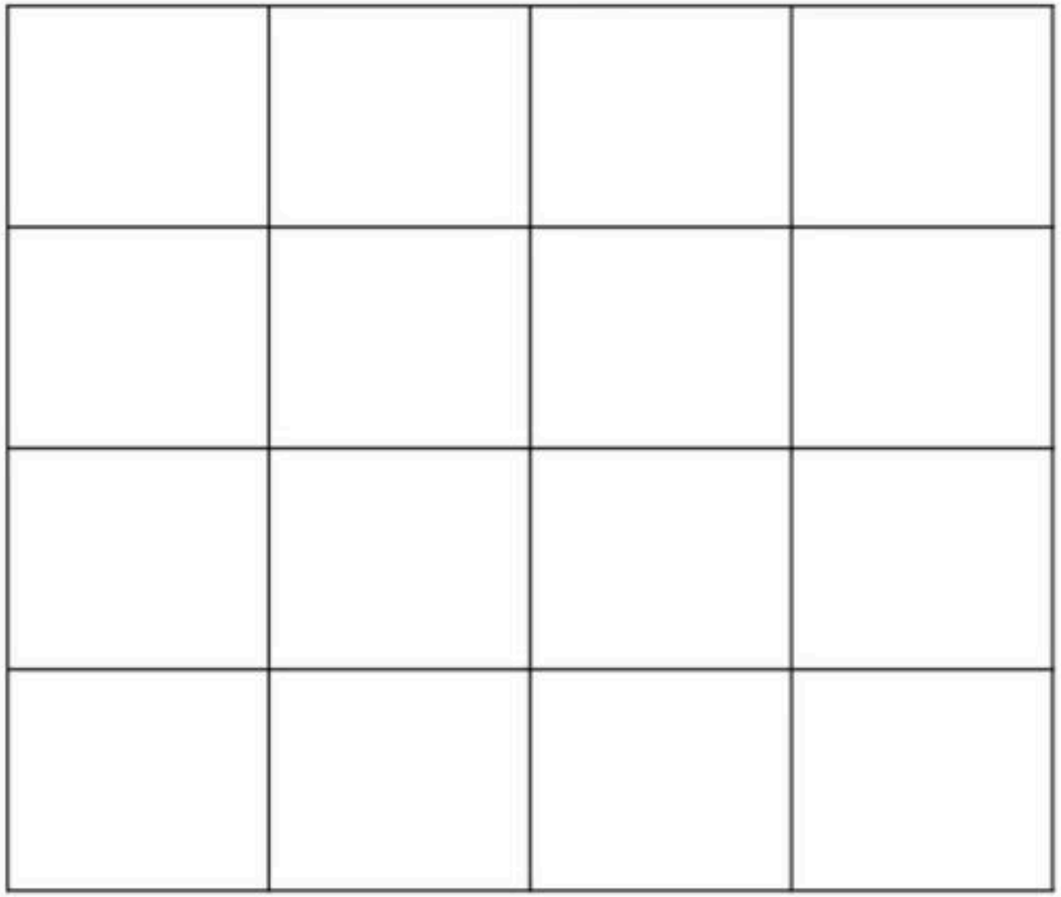
Q) Minimize the following

AB \ CD				
	1			1
	1			1
	1			1
	1			1

Q) F (A, B, C ,D)= $\sum m\,(3, 4, 6, 8, 9, 11, 12, 14\ )$



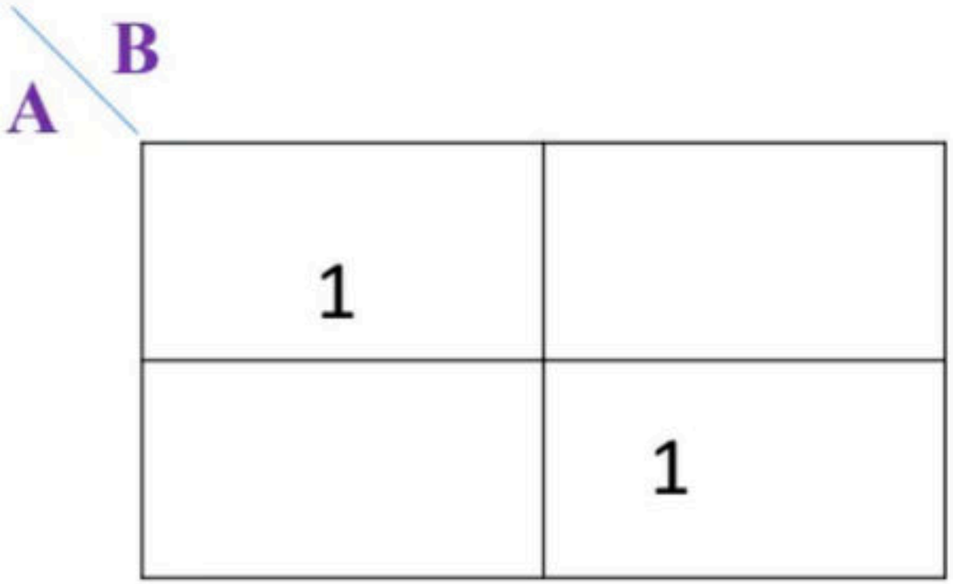
Q) F (A, B, C, D)= $\prod M$ (3 , 4, 6, 8, 9, 11,12,14)

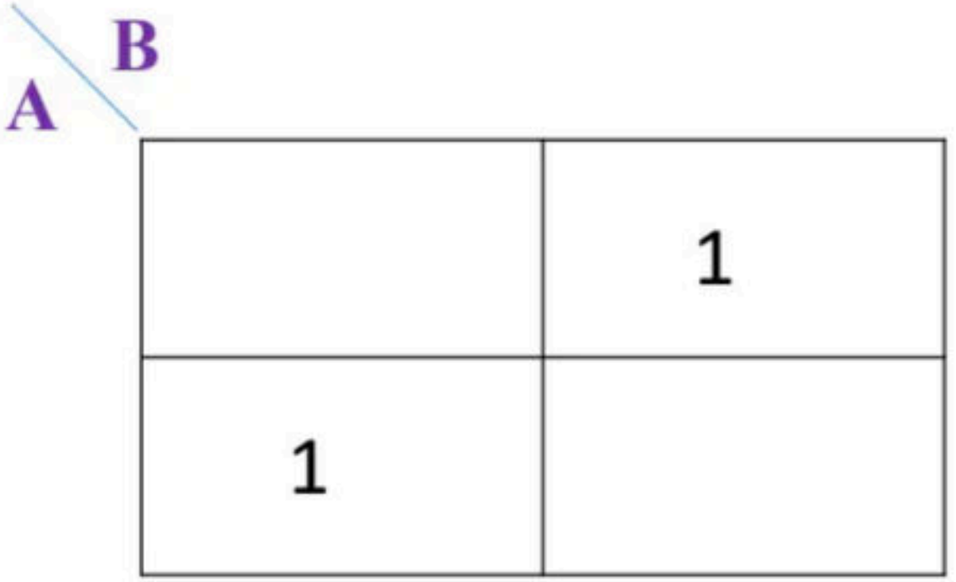


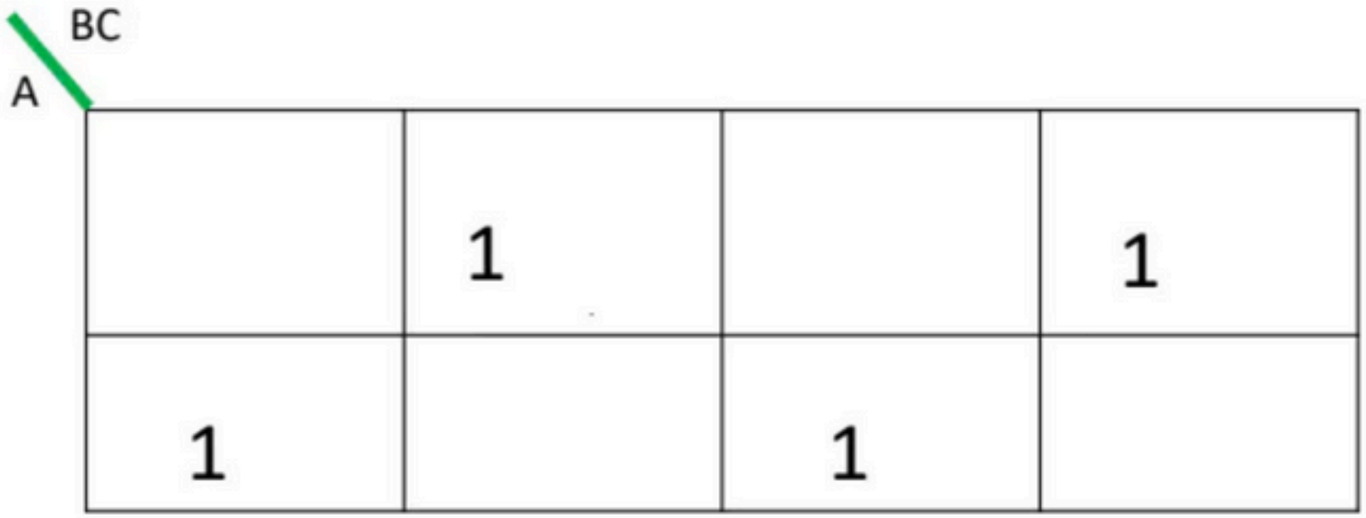
Note :

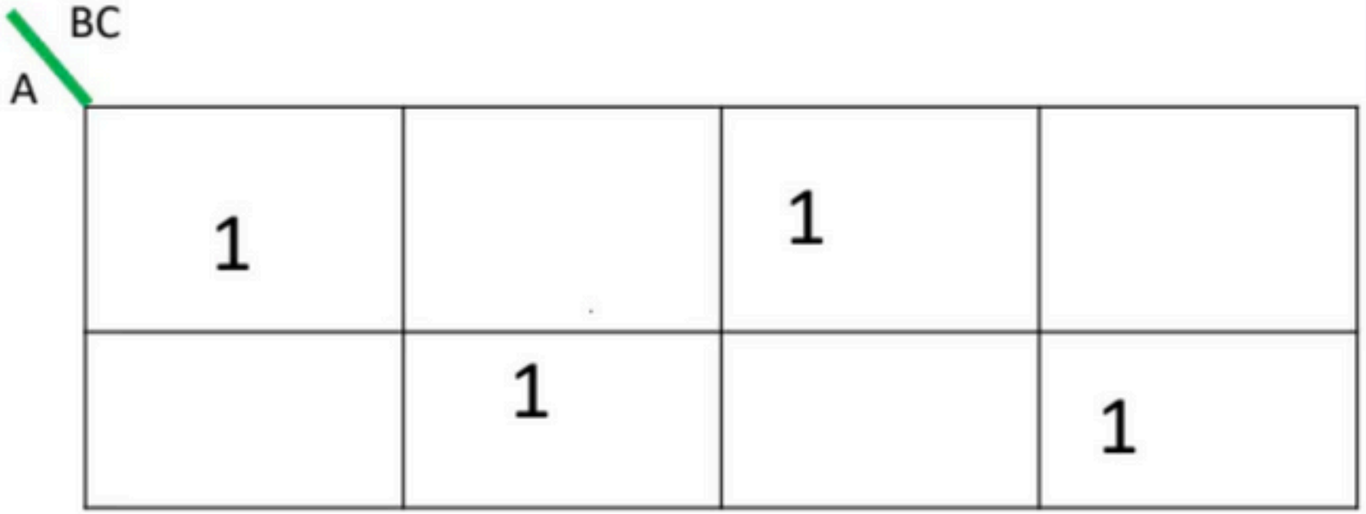
- For a n–variable Boolean expression, the maximum number of literals a minterm contains = n
- For a n – variable k-map if grouping is done by considering 2^m number of cells , then m-number of literals are eliminated and the resulting term from that group contains (n-m) number of literals

- $8\ cells\ -\ 2^3$ cells → Octet ----------> 3 variables eliminated
- $4\ cells\ -\ 2^2$ cells → Quad ----------> 2 variables eliminated
- $2\ cells\ -\ 2^1$ cells → Pair ----------> 1 variables eliminated


B
A
1
1


B
A
1
1


BC
A
1
1
1
1


BC
A
1
1
1
1

AB \ CD				
	1		1	
		1		1
	1		1	
		1		1

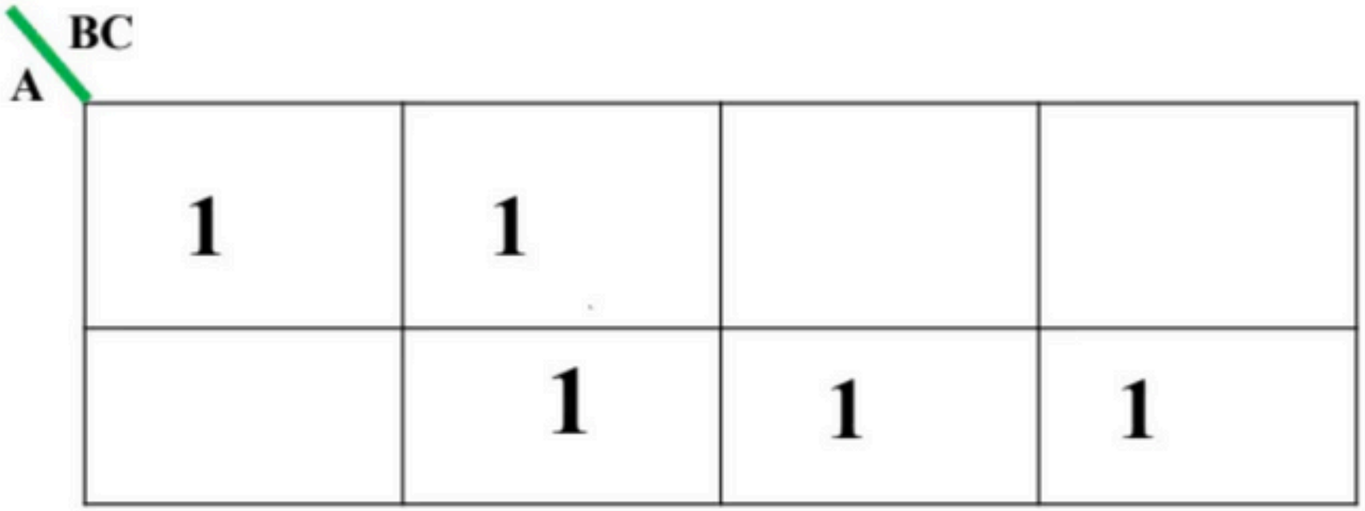
AB \ CD				
		1		1
	1		1	
		1		1
	1		1	

Q) $F(w, x, y, z) = \sum(1, 3, 4, 6, 9, 11, 12, 14)$

a) Independent of one variables
b) Independent of two variables
c) Independent of three variable's
d) Depends on all variables

Q) A logic circuit implement $F = \bar{x}y + x\bar{y}\bar{z}$, it is found that $x = y = 1$ can never occur, considering this as fact , the minimized expression of F is…….

Q) Minimize the following



NOTE:

Minimal expression may not be Unique

Q) Minimize the following

A \ BC				
	1	1	X	
		X	1	1

Minterm :

Each product term in the canonical SOP expression is called a minterm

Maxterm :

Each sum term in the canonical POS expression is called a maxterm

Implicant ,Prime Implicant , Essential Prime Implicant

Implicant : Each minterm in canonical SOP expression is known as Implicant .

Prime Implicant (PI):

Prime Implicant is a product term , obtained by combining maximum possible cells in the K- Map. While doing so make sure that a smaller group is not completely inside a bigger group .

Essential Prime Implicant (EPI) :

A prime Implicant is an EPI , if and only if it contains at least one minterm which is not covered by multiple groups .

All EPI's are PI's , but vice versa not true

$$EPI \leq PI$$

The minimal expression = (All EPI's) + (Optional PI's)

False Minterms

The maxterms are called as False Minterms

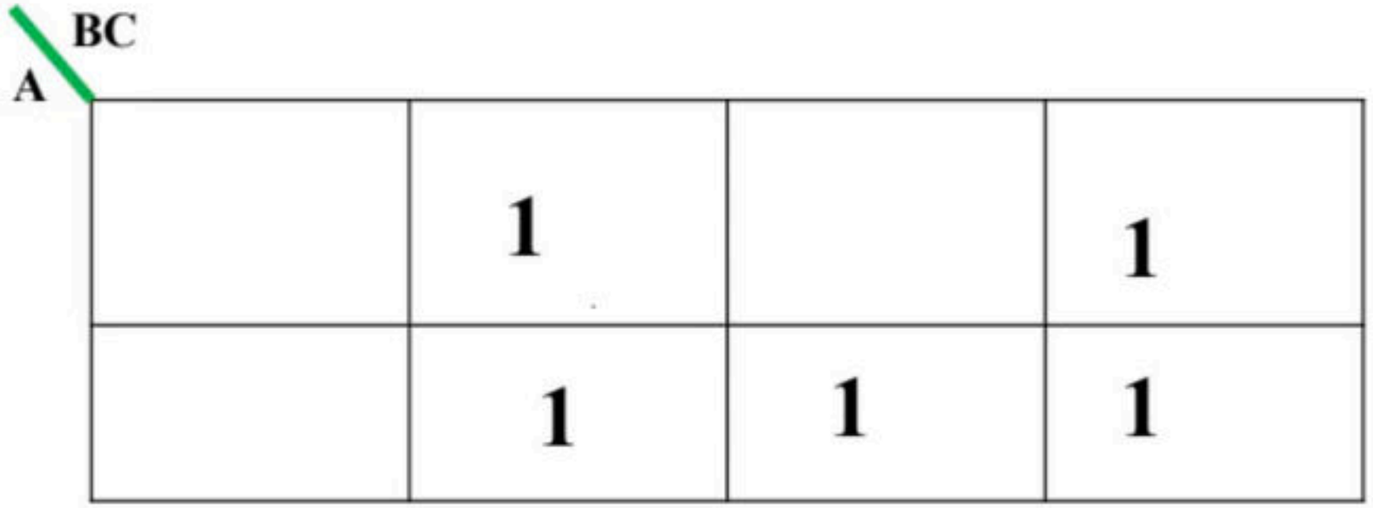
False Prime Implicants

Prime Implicants obtained using the maxterms are called as False Prime Implicants

Essential false Prime Implicants

A False Prime Implicant is said to be an Essential False Prime Implicants , if and only if it contains at least one maxterm which is not covered by multiple groups

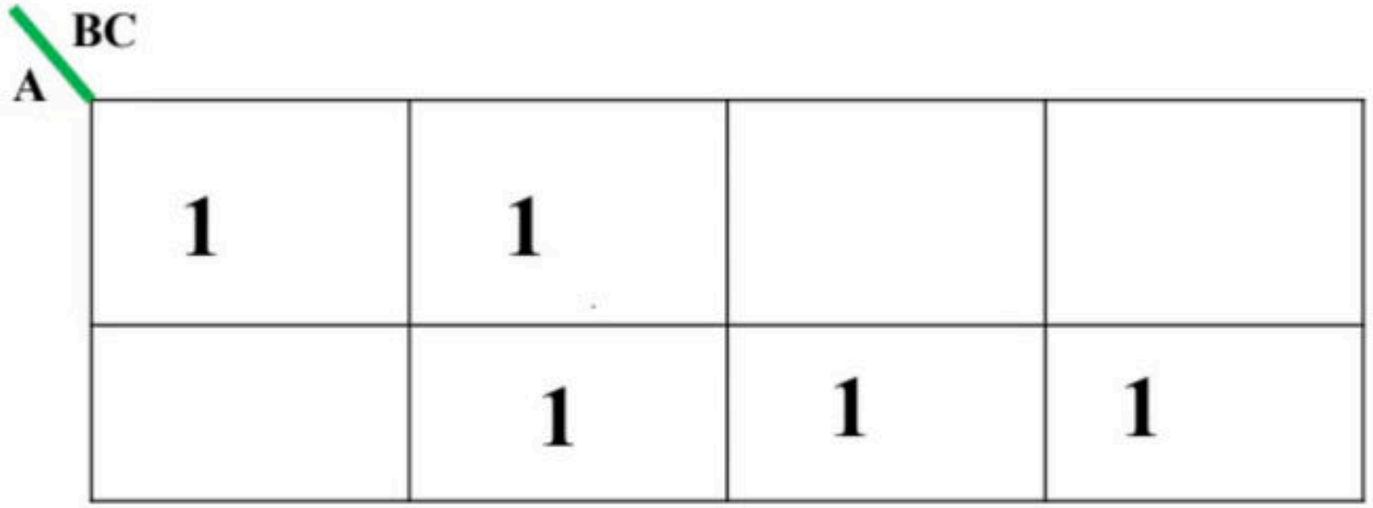
Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

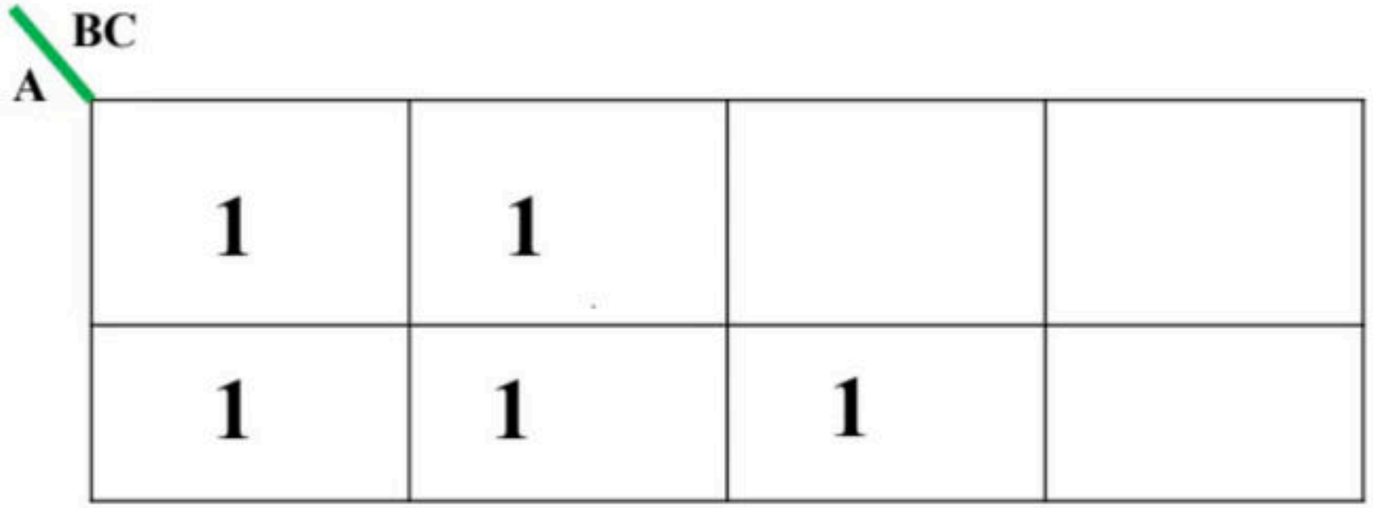
Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

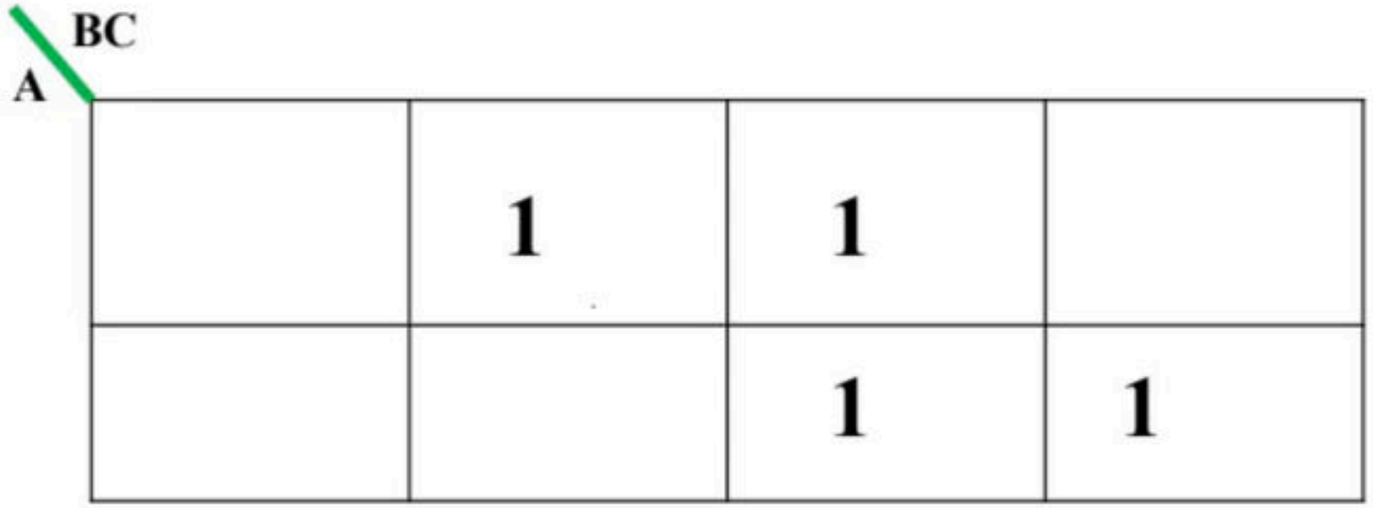
Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

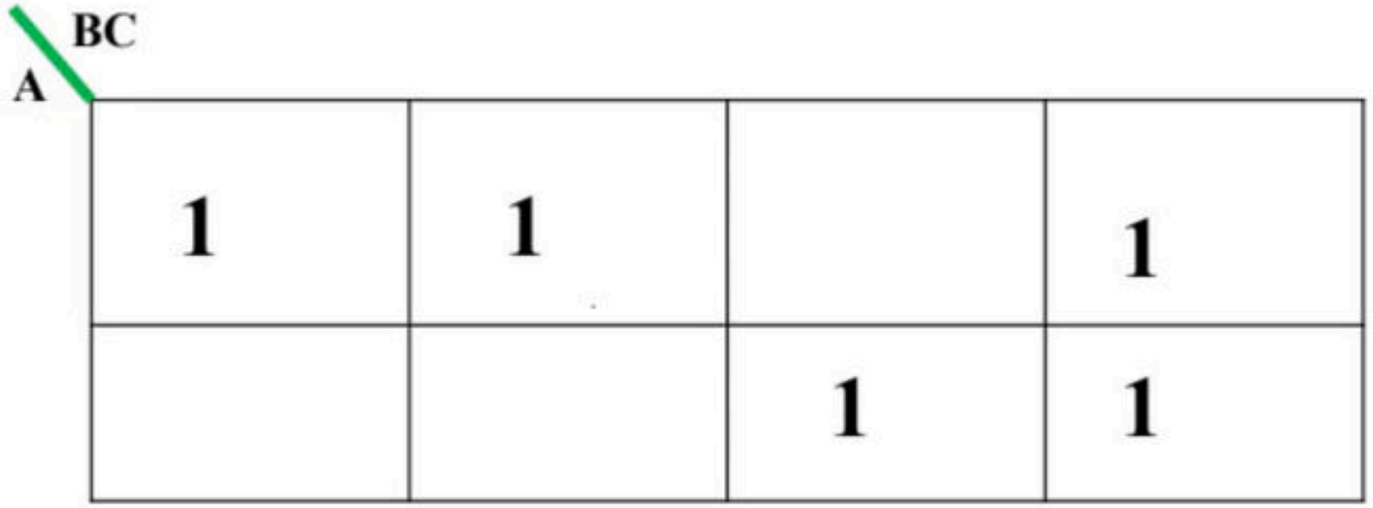
Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

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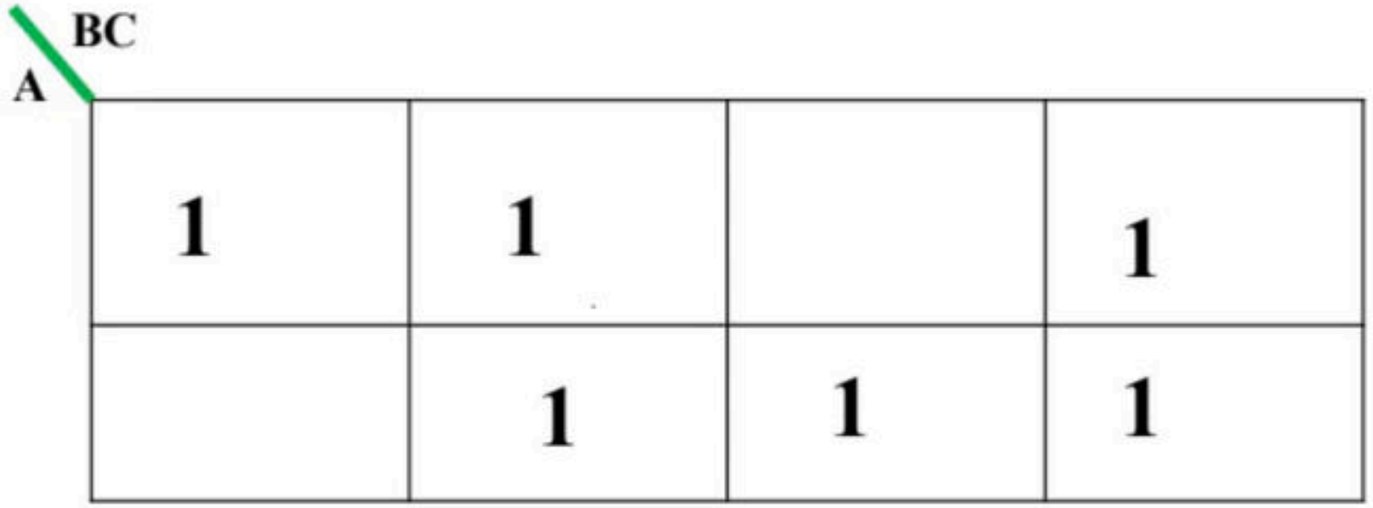
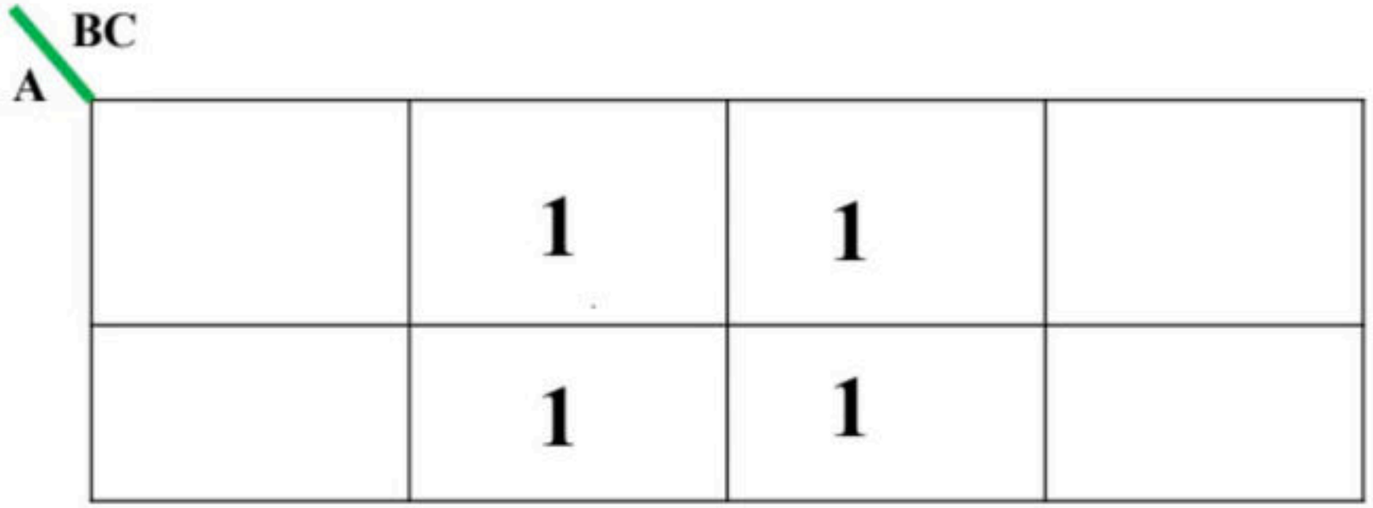


PI

EPI

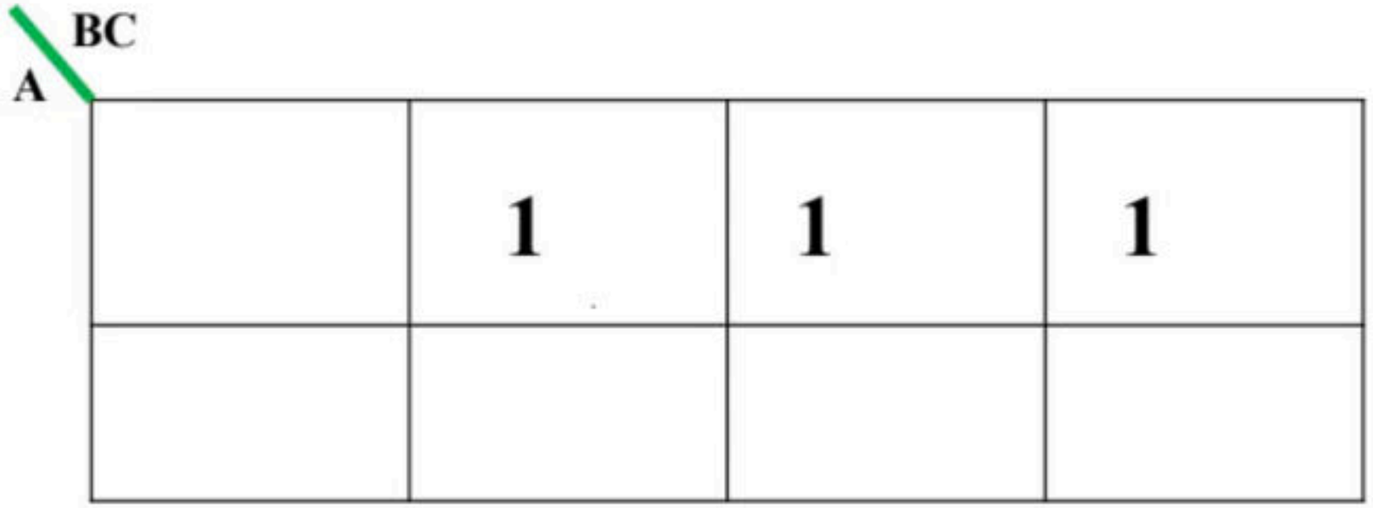
Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

Q) Find the number of Prime Implicants and Essential Prime Implicants



PI

EPI

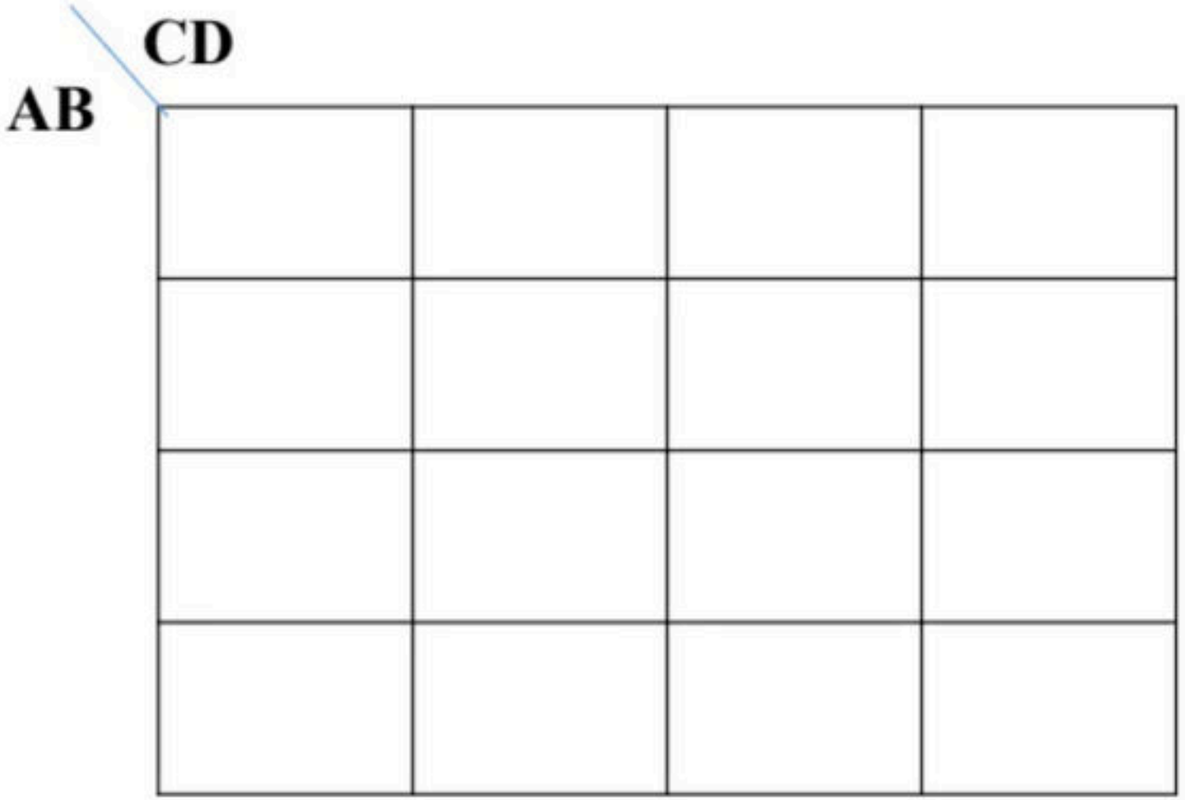
Q) Find the number of Prime Implicants and Essential Prime Implicants

AB \ CD			
1	1		1
			1
1			
1			1

PI

EPI

Q) F (A, B,C,D) = $\sum m(0, 2, 3, 4, 5, 6, 10)$



PI

EPI

NOTE:

- The minimal expression = (All EPI's) + (Optional PI's)
- If all PI's are EPI's , then the minimal expression is unique
- The sufficient condition for a K-map to have unique solution is the number of PI's = number of EPI's

Q. What is the minimized logic expression corresponding to the given Karnaugh-map?

(a) XZ

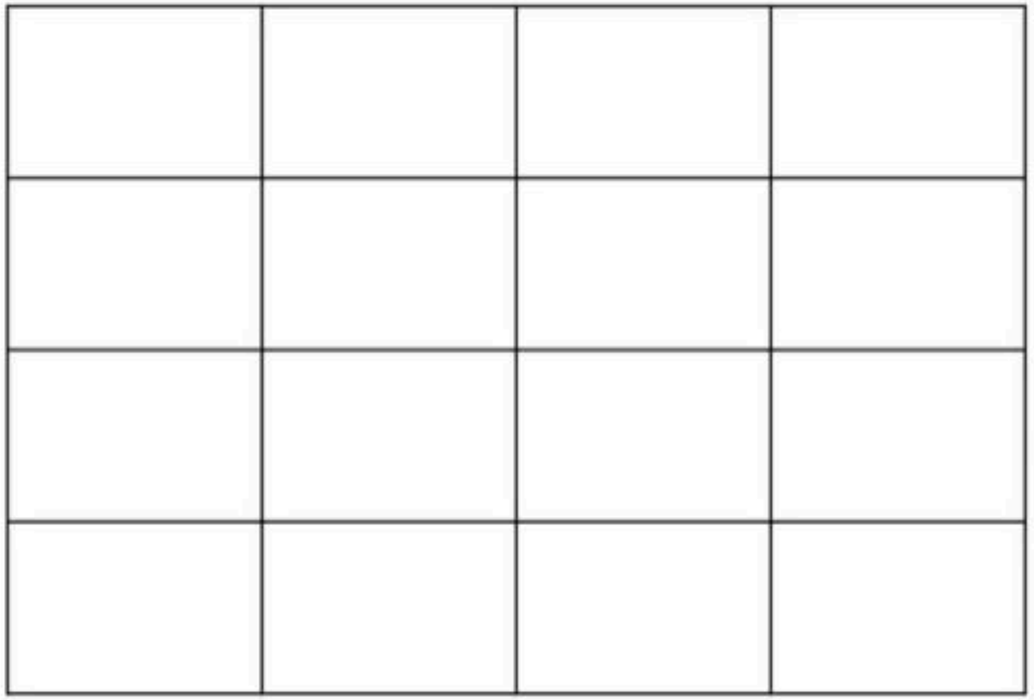
(b) $\overline{W}\,X\,\overline{Y} + \overline{W}\,Y\,Z + W\,\overline{Y}\,Z + W\,X\,Y$

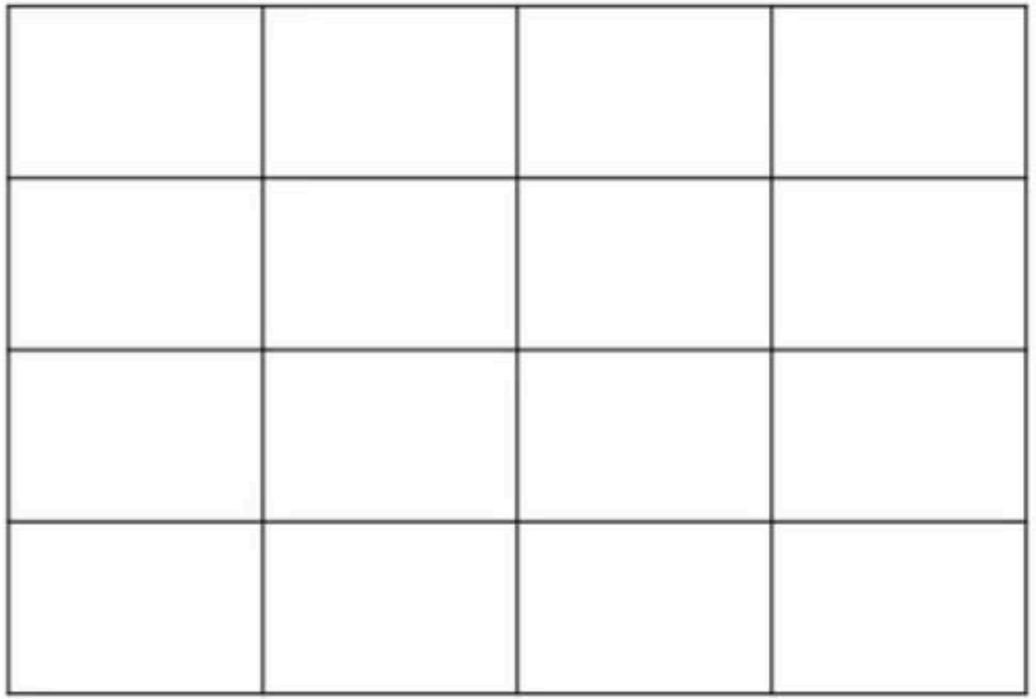
(c) $\overline{W}\,X\,\overline{Y} + \overline{W}\,Y\,Z + W\,\overline{Y}\,\overline{Z} + W\,X\,\overline{Y}$

(d) $XZ + \overline{W}\,Y\,Z + \overline{W}\,X\,\overline{Y} + W\,X\,Y + W\,\overline{Y}\,Z$

WX \ YZ	00	01	11	10
00			1	
01	1	1	1	
11		1	1	1
10		1		

Q) $F(A, B, C, D, E) = \sum m(0, 2, 4, 7, 10, 12, 13, 18, 23, 26, 28, 29)$





Q. How many min terms (excluding redundant terms) does the minimal switching function

$f(v, w, x, y, z) = x + \bar{y}\, z$ originally have

(a)16 (b) 20

(c)24 (d) 32

Q. Consider the Karnaugh map given below:

The function represented by this map can be simplified to the minimal form as

(a) $X_1 \overline{X}_2 \overline{X}_4 + X_2 X_4 + X_1 \overline{X}_3$

(b) $X_1 X_2 X_4 + X_2 X_4 + X_1 \overline{X}_2 \overline{X}_3 \overline{X}_4$

(c) $X_2 X_4 + \overline{X}_2 \overline{X}_4 + X_3 \overline{X}_1$

(d) $X_1 \overline{X}_2 \overline{X}_4 + \overline{X}_1 X_2 \overline{X}_3 X_4 + X_1 X_2$

X_1X_2 \ X_3X_4	00	01	11	10
00	1		d	D
01		1	d	1
11		d	1	
10	1	d		D

Q. Find the number of EPI's in the following function F

$F(W,X,Y,Z)=\sum m(0, 2, 5, 7, 8, 10, 12, 14, 15)$

(a) 5

(b) 4

(c) 3

(d) 2

Q. For the following function $F(A, B, C, D) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$ the number of essential prime implicants and non-essential prime implicants respectively are?

(a) 1,3
(b) 2,2
(c) 3, 1
(d) 2,3

Q. Consider the Boolean function, $F(w,x,y,z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$. Which one of the following is the complete set of essential prime implicants?

(a) $w,\ y,\ xz,\ \bar{x}\bar{z}$

(b) $w,\ y,\ xz$

(c) $y,\ \bar{x}\bar{y}\bar{z}$

(d) $y,\ xz,\ \bar{x}\bar{z}$

Q. The number of essential prime Implicants of the given minterms $F(A,B,C.,D) = \sum m(0,1,4,5,8,11,12,13,15)$ is _____.

Q. The minimized SOP expression of the given minterms $F(A, B, C, D) = \sum m(0, 1, 4, 5, 7, 8, 9, 12, 13, 14, 15)$ is

(a) $F = \bar{C} + AB + BD$

(b) $F = \bar{C} + AB$

(c) $F = (A + D).B$

(d) $F = \bar{C} + BD$

Q. Given the Boolean function F in the three variables A,B and C as $F(A,B,C) = \sum m(2,5,7)$ then the minimized POS expression is

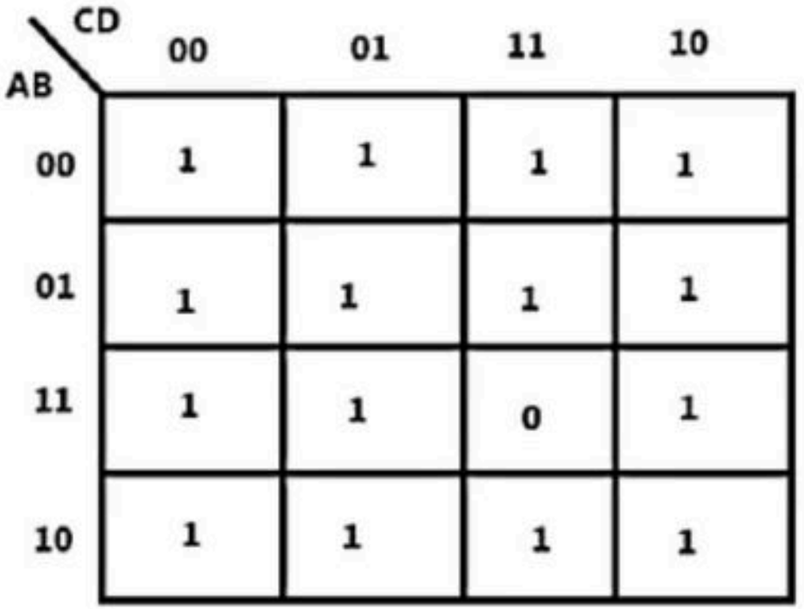
(a) $F = \left(\overline{A} + C\right)\left(A + B\right)\left(A + \overline{C}\right)$

(b) $F = \left(\overline{A} + C\right)\left(B + C\right)\left(A + \overline{C}\right)$

(c) Both (a) and (b)

(d) None

Q. K-map of a Boolean function is given below, after simplification choose the correct option.



AB \ CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

(a) $\bar{A}+\bar{B}$

(b) $\bar{A}+\bar{B}+\bar{C}$

(c) $A\bar{B}+B\bar{C}+\bar{A}C$

(d) $\bar{A}+\bar{B}+\bar{C}+\bar{D}$

Q. Simplify $f(A,B,C,D) = \sum m(0,1,4,5,9,11,14,15) + \sum d(10,13)$

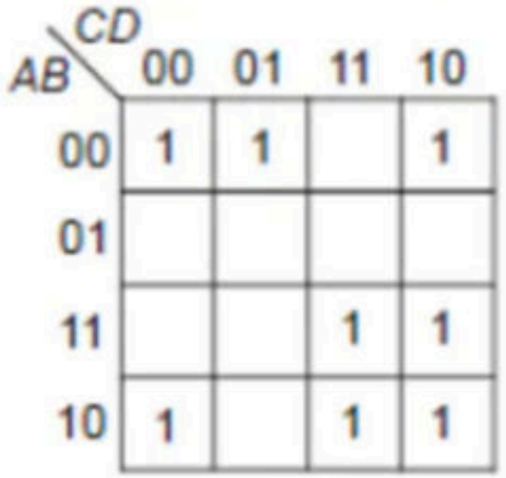
(a) $\overline{A}\,\overline{C} + AC$

(b) $\overline{A}\,\overline{C} + \overline{C}D$

(c) $\overline{A}\,\overline{C} + \overline{B}C + \overline{C}D$

(d) $\overline{A}\,\overline{C} + AC + \overline{C}D$

Q. For the *k* -map shown below, the minimized logical expression in SOP form is



a. $\bar{A}\bar{B}\bar{C} + AC + \bar{B}CD + \bar{A}\bar{B}\bar{D}$

b. $\bar{A}\bar{B}\bar{C} + \bar{B}\bar{D} + AC$

c. $\bar{A}\bar{B}\bar{C} + AC + \bar{A}\bar{B}CD + A\bar{B}\bar{C}D$

d. $\bar{A}\bar{B}\bar{C} + AC + \bar{A}\bar{B}\bar{D}$

Q. Consider a function $F = A \odot B \odot C \odot D$. The total number of essential prime implicants of the given function will be _____.

Q. A logical function is given as $F(A, B, C, D) = \Sigma m\ (0, 4, 5, 10, 11, 13, 15)$.

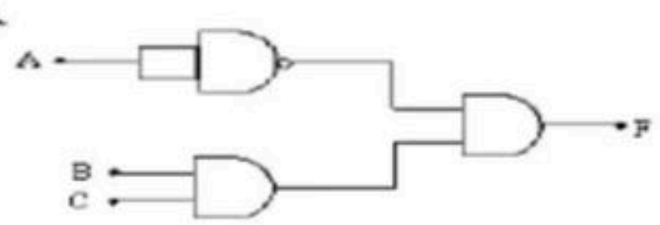
The number of Essential Prime Implicants in the minimized expression will be

_____.

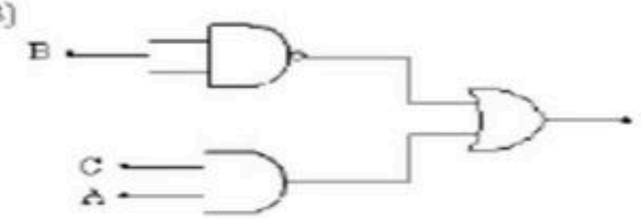
Which of the following logic circuit is realization of the function F whose Karnaugh map is as shown below

C \ AB	00	01	11	10
0	1	1		
1	1			1

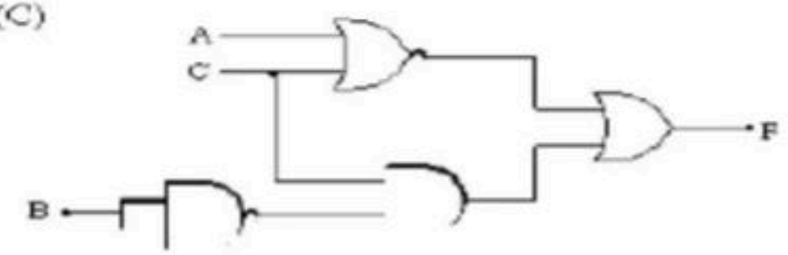
(A



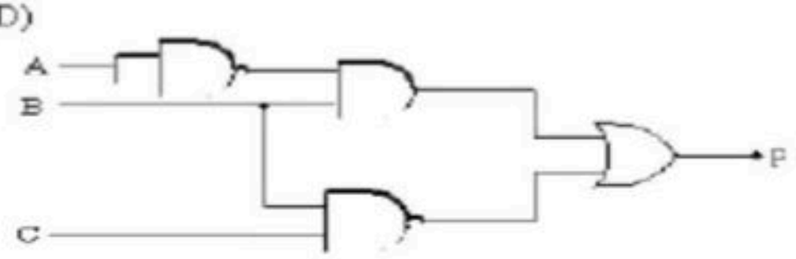
(B)



(C)



(D)



The total number of possible minimal expressions for the four variable Boolean function:

$F(A,B,C,D) = \Sigma m(0,2,3,5,7,8,9,10,11,13,15)$ is _____

Simplified form of the Boolean function

$F(A, B, C, D, E) = \Sigma m(0, 2, 4, 6, 9, 13, 21, 23, 25, 29, 31)$ is

(A) $F = \overline{A}\,\overline{B}\,\overline{E} + \overline{B}D\overline{E} + ADE$

(B) $F = \overline{A}\,\overline{B}\,\overline{C} + B\overline{D}E + ACE$

(C) $F = \overline{A}\,\overline{B}\,\overline{E} + BCE + ACE$

(D) $F = \overline{A}\,\overline{B}\,\overline{E} + B\overline{D}E + ACE$

A logic circuit implements the following Boolean function $F(W, X, Y, Z) = \Sigma m(2, 3, 6, 7, 8, 12)$ It is found that the input combination for which $W = Y$ can never occur. Then minimal expression for F is

(A) $WY + \overline{Y}Z$

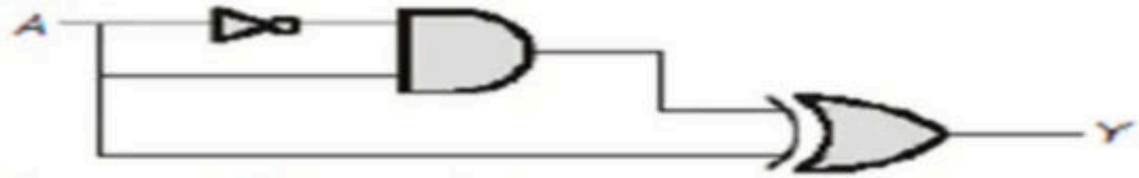
(B) $\overline{Y} + Z$

(C) $W\overline{Y} + Z$

(D) $Y + \overline{Z}$

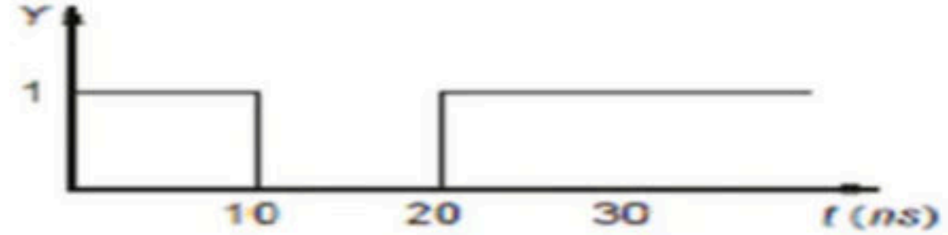
Answer : D

Consider the circuit shown in figure below



If propagation delay of NOT gate is 10 nsec, AND gate is 20 nsec and *X*-*OR* gate is 10 nsec. If *A* is connected to V_{CC} at t = 0, then waveform for output *Y* is

a.



b.

c.

d.

Five soldiers *A*, *B*, *C*, *D* and *E* volunteer to perform an important military task if their following conditions are satisfied

(i) either *A* or *B* or both must go

(ii) either *C* or *E* but both must not go

(iii) either both *A* and *C* go or neither goes

(iv) If '*D*' goes, then '*E* ' must also go

(v) If '*B*' goes, then *A* and *C* must also go

The minimal combination of soldiers who can get the arrangement will be

Ans : $AC\bar{D}\bar{E}$

How many Boolean functions of the type $f(x, y, z) = f(\bar{x}, \bar{y}, \bar{z})$ are available with three variables ?

(a) 4
(b) 8
(c) 32
(d) 16

The logic expression $Y = \Sigma m (0, 3, 6, 7, 10, 12, 15)$ is equivalent to

(a) $Y = \Pi M (0, 3, 6, 7, 10, 12, 15)$

(b) $Y = \Pi M (1, 2, 4, 5, 8, 9, 11, 13, 14)$

(c) $Y = \Sigma m (1, 2, 4, 5, 8, 9, 11, 13, 14)$

(d) $Y = \Sigma m (3, 0, 10, 12)$

Pick up correct statements from the following:

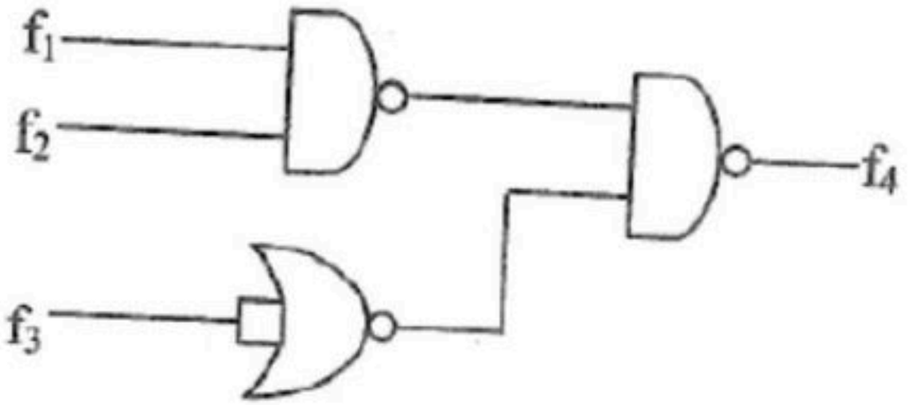
1. In an n – variable K – map combining 16 adjacent cells containing 1's as a single group will generate a prime implicant which is independent of four variables.
2. D – latch cannot be implemented with multiplexer circuit.
3. Lockout problem occurs only in ripple counters.
4. The minimum size of ROM required to implement 3 – bit binary to gray / gray to binary code converter is $2^4 \times 3$.

5 Minimum number of NOR gates (2 - input) required to implement half subtractor circuit are five.

(a) 1, 2, 3 (b) 2, 3, 4

(c) 1, 4, 5 (d) 3, 4, 5

Consider the logic circuit shown in figure.



The functions f_1, f_2 and f_4 are

$f_1(w, x, y, z) = \sum m(8, 9, 10)$

$f_2(w, x, y, z) = \sum m(7, 8, 12, 13, 14, 15)$

$f_4(w, x, y, z) = \sum m(8,9)$

Then $f_3(w, x, y, z)$ is

(a) $\sum m(9, 10)$

(b) $\sum m(9)$

(c) $\sum m(1, 8, 9)$

(d) $\sum m(8, 10, 15)$

How many minimum number of NAND gates are required to implement the following Boolean equation, by taking inputs as A, B, C, D, E, F

$$F = \overline{A}C + \overline{D}F + \overline{B}C + \overline{E}F$$

(a) 4
(b) 5
(c) 6
(d) 7

Which of the following statements are correct?

1. The Dual of i^{th} minterms is maxterm $(2^n - i - 1)$.
2. Sum of all minterms is always 0.
3. Minterms and its corresponding Maxterms form a complement pair.
4. Minterms are standard product terms.

(a) 1, 3, 4
(b) 3, 4
(c) 2, 3, 4
(d) 1, 2, 4

Match List I with List II and select the correct answer using the codes given below the lists:

List - I	List - II
A. Boolean Algebra	1. Minimization
B. K-Map	2. Synchronous circuits
C. Clock	3. Error correction
D. Parity	4. DeMorgan's theorem

Codes:

	A	B	C	D
(a)	4	1	2	3
(b)	4	3	2	1
(c)	3	4	2	1
(d)	4	2	1	3