Tree: Binary Tree

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Vishvadeep Gothi • Lesson 42 • Jan 19, 2023

Data Structure
Tree 1

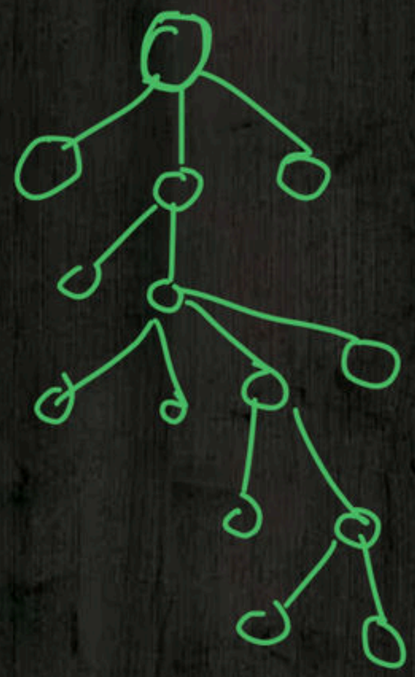
By: Vishvadeep Gothi

Tree

non-linear data structure

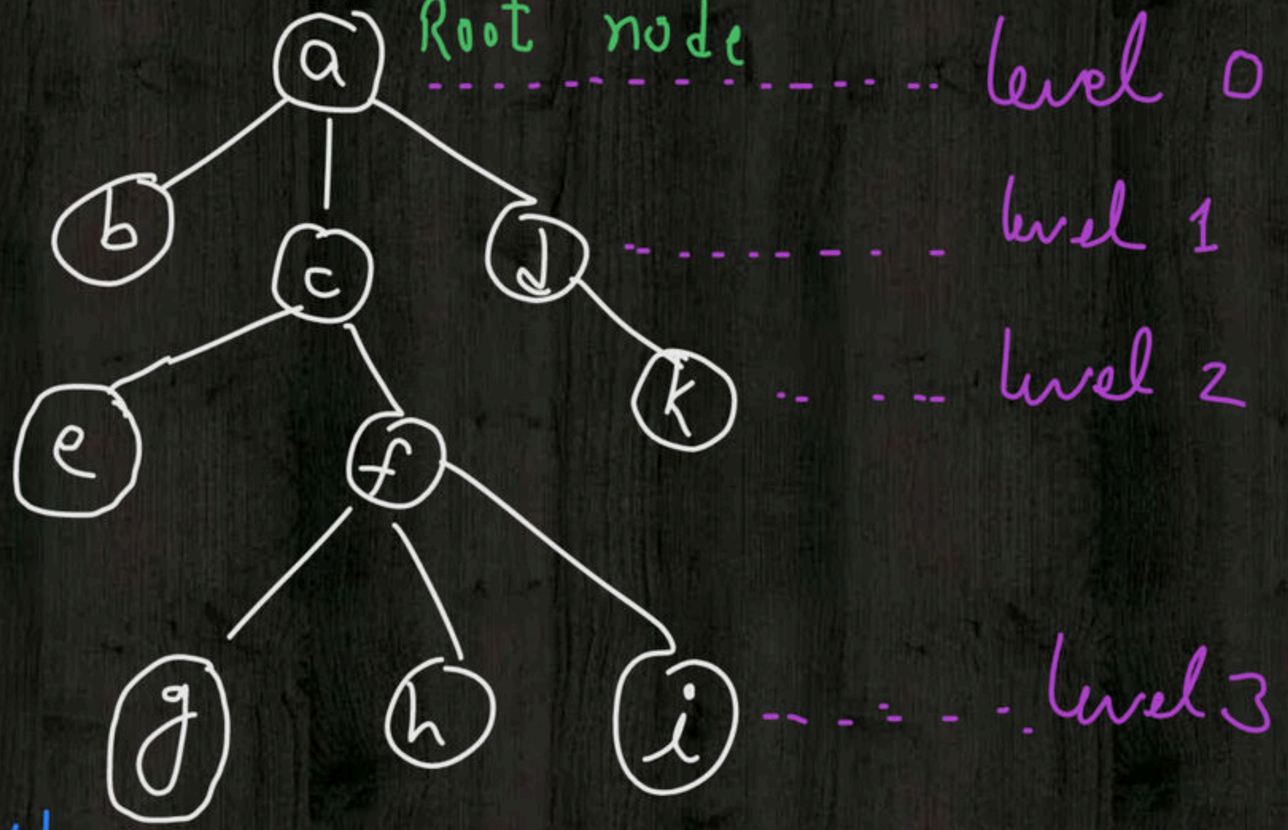
nodes

edges



Tree Terminologies

- Root
- Child
- Parent
- Sibling
- Subtree
- Internal Node => node having children => a, c, f, d
- External Node (leaf node) => nodes not having children => b, e, g, h, i, k
- Level



degree of a node $\Rightarrow$ no. of children

Total no. of nodes = total no of internal (non-leaf) nodes (I)

+

total no. of external nodes (L)

N

$$N = I + L$$

Ques) Consider a tree which has 30 leaf nodes and 15 such nodes which have atleast one child. Total no. of nodes in the tree is ?

Ans:-

$$L = 30$$

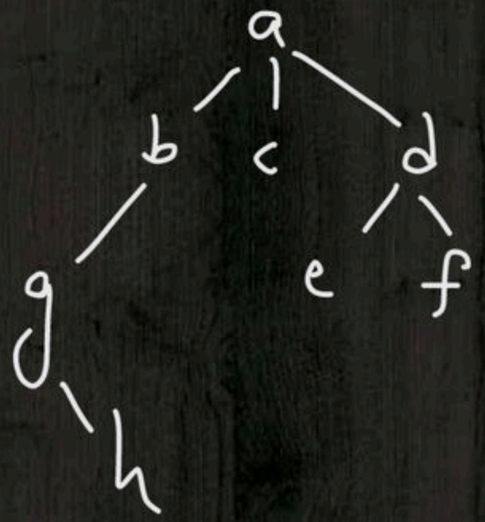
$$I = 15$$

$$N = 30 + 15$$

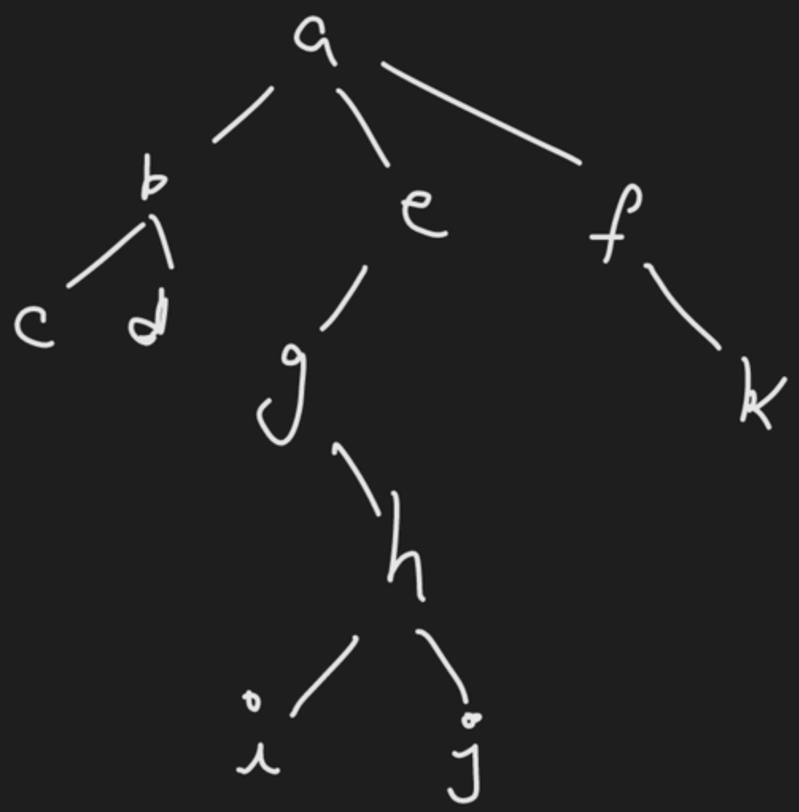
$$\boxed{N = 45}$$

Parenthesis Representation

Root (child 1 , child 2 , ... , child n)



a (b (g (h)) , c , d (e , f))

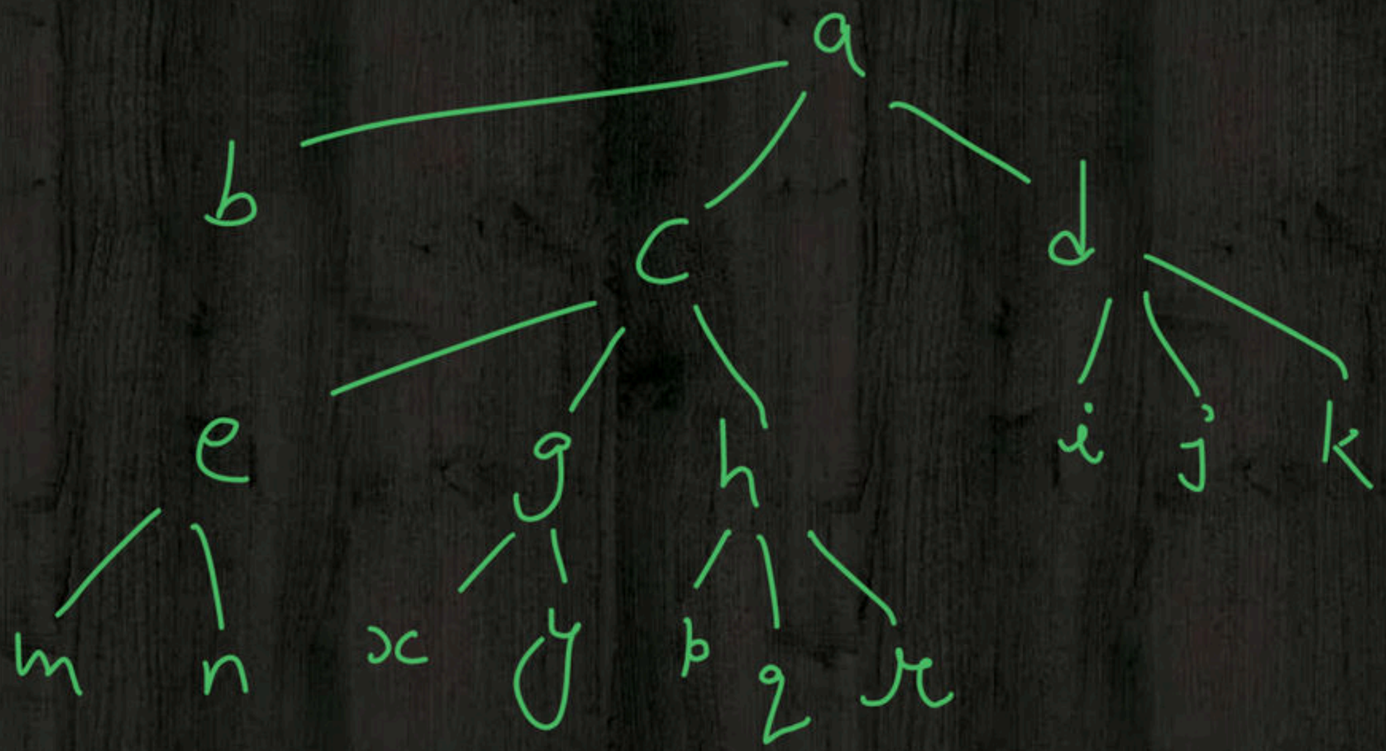


a(b (c, d) , e(g(h (i, j))), f(k))

Question

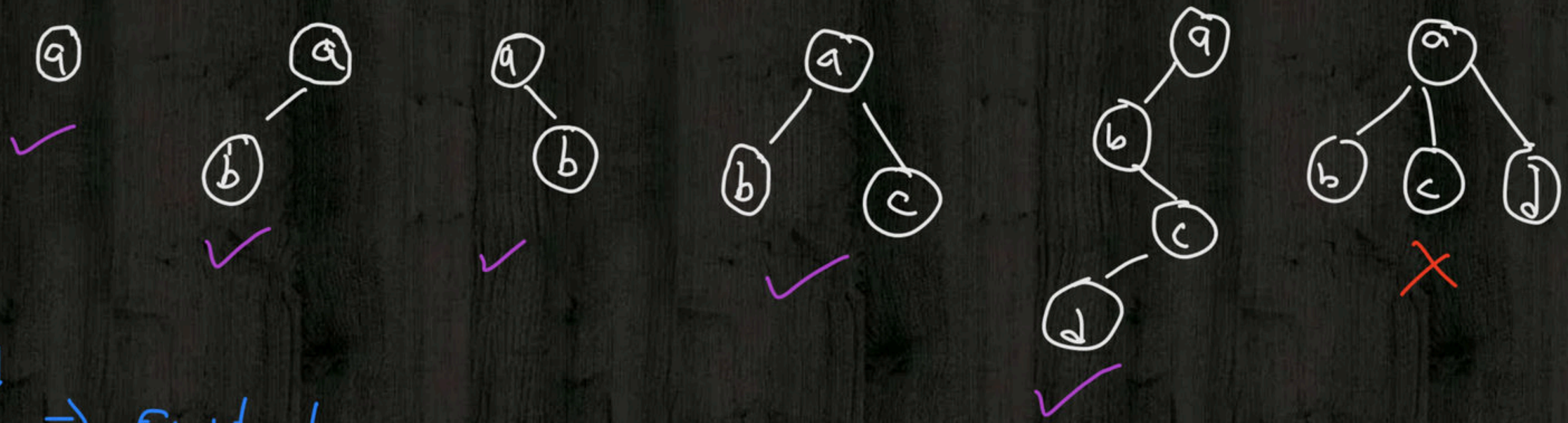
Draw the tree for:

a(b, c(e(m, n), g(x, y), h(p, q, r)), d (i, j, k))

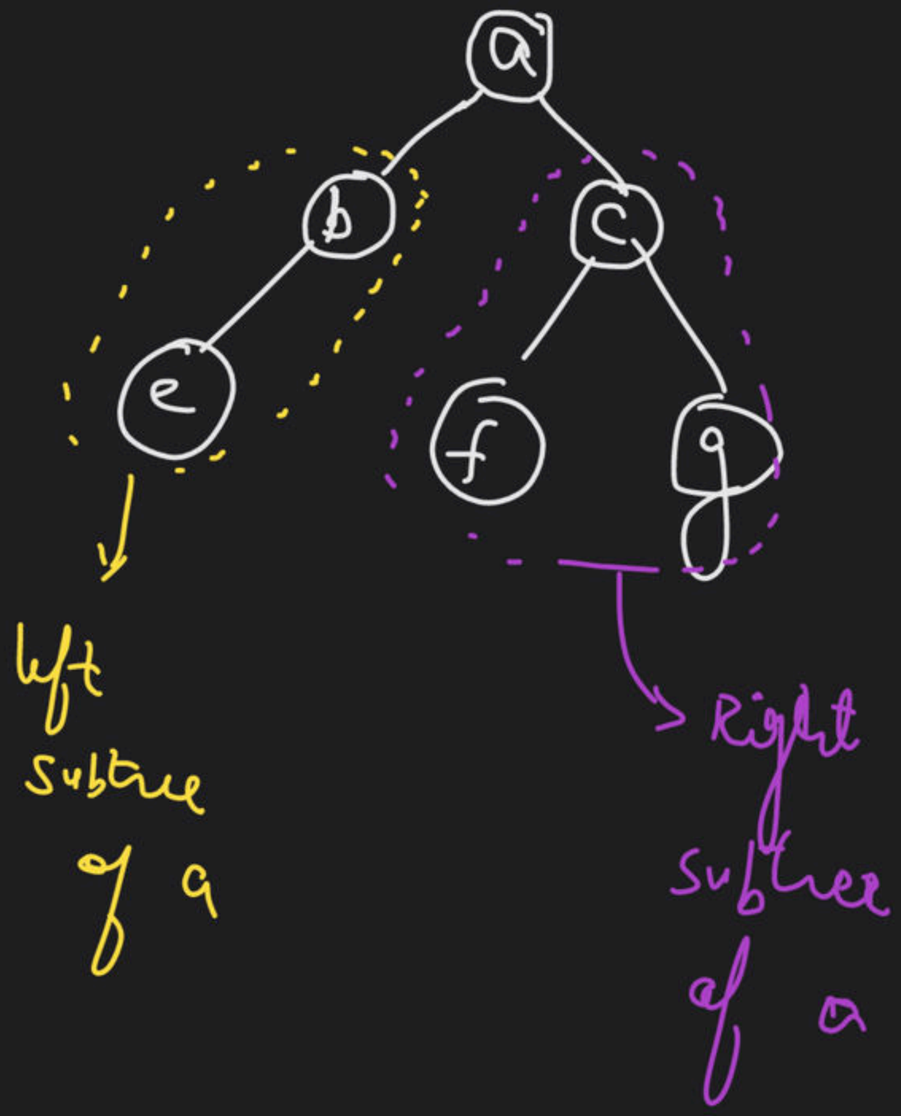


Binary Tree

A tree in which each node has max. 2 children.



valid BT ⇒ Empty tree



b is left child of a

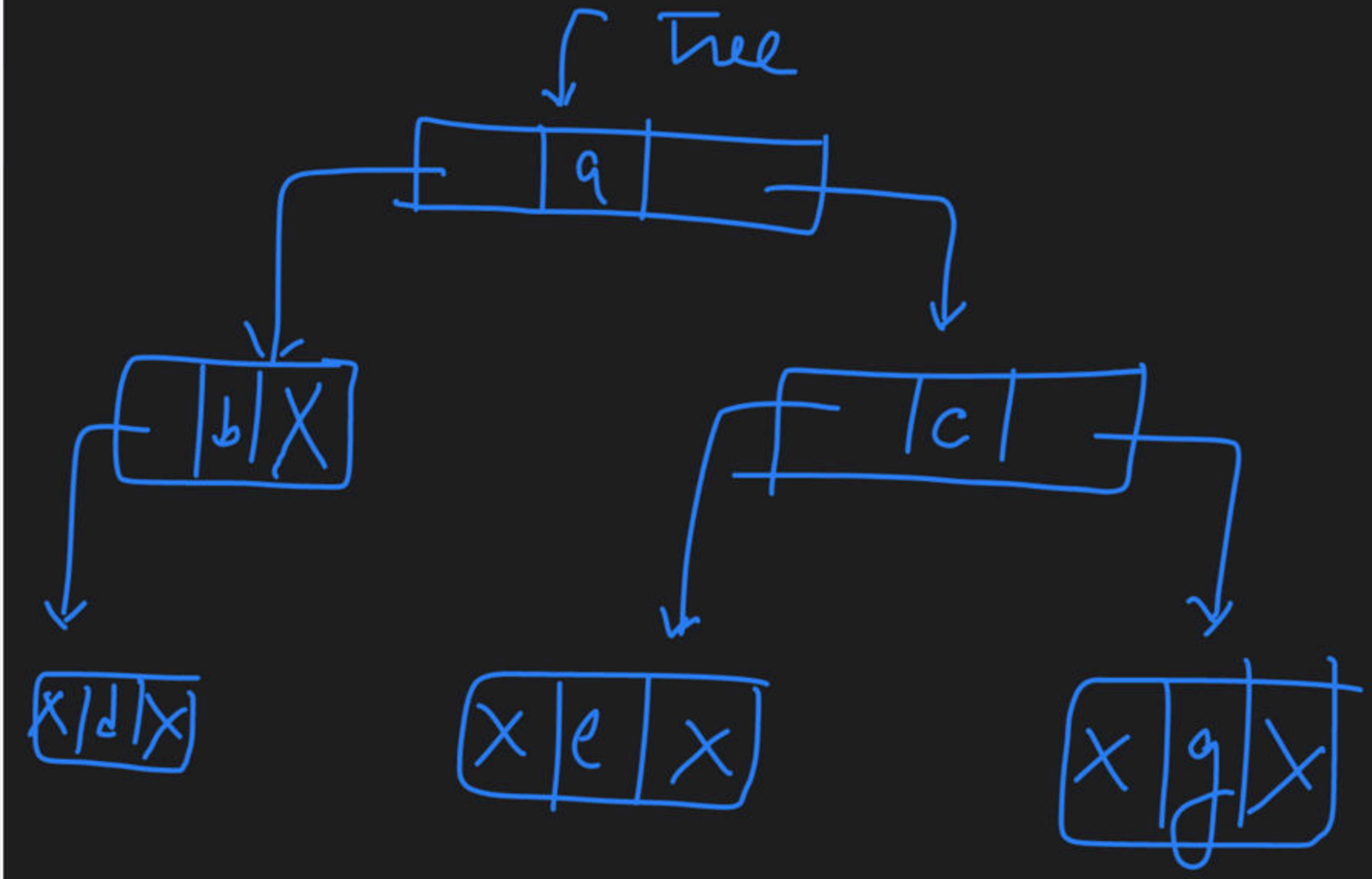
c is Right —11— a

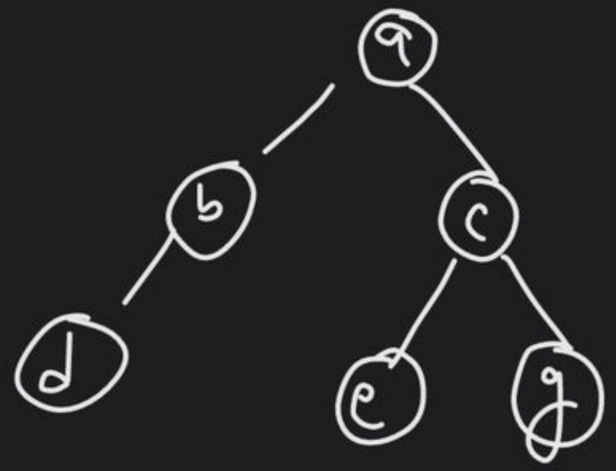
Linked Representation on Binary Tree

node

Left Child	Key	Right Child

```
struct BTnode
{
  char key;
  struct BTnode *Leftchild;
  struct BTnode *Rightchild;
};
```





empty tree condition =>

if (Tree == NULL) empty tree

or

if (! Tree)

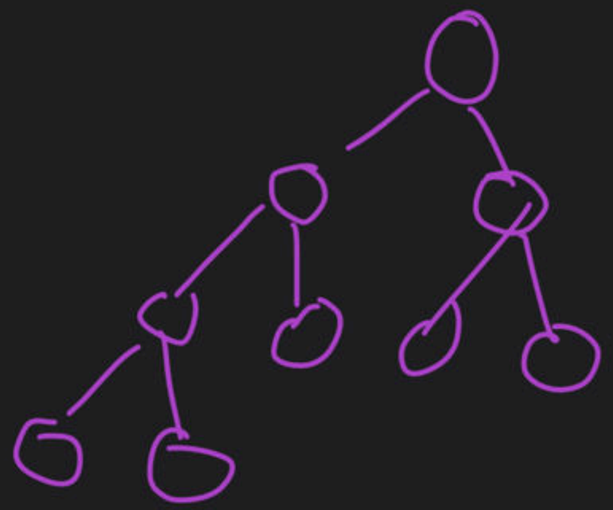
assume pointer p points to a node in Binary tree.

```
if (p -> leftchild == NULL && p -> rightchild == NULL)
```

p points to a leaf node

Ques) consider a binary tree which has 5 leaf nodes. No. of nodes having 2 children in the tree is__?

Ans) 4



no. of leaf nodes in Binary tree (L) = no. of nodes with 2 children (I_2) + 1

$$L = I_2 + 1$$

Total internal node = no. of nodes with 2 children + no. of nodes with 1 child

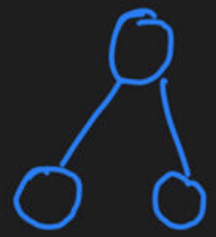
$$I = I_2 + I_1$$

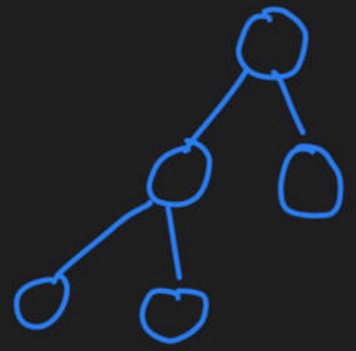
$$N = L + I$$

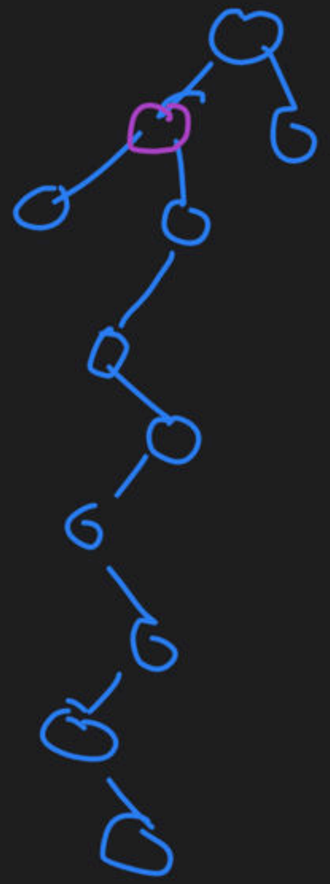
$$N = (I_2 + 1) + (I_2 + I_1)$$

$$\boxed{N = 2I_2 + I_1 + 1}$$

(9)







T_2	0	1	2	...	15
L	1	2	3		16

Question GATE-2000

Consider the following nested representation of binary trees: (X Y Z) indicates Y and Z are the left and right subtrees respectively of node X. Note that Y and Z may be NULL or further nested. Which of the following represents a valid binary tree?

A. (1 2 (4 5 6 7))
B. (1 ((2 3 4) 5 6) 7)
C. (1 (2 3 4)(5 6 7))
D. (1 (2 3 NULL)(4 5))

Question

A binary tree T has 29 leaves. The number of nodes in T having 2 children is ____ ?

Ans => 28

Ans = 39

Question

In a binary tree, the number of internal nodes of degree 1 is 8, and the number of internal nodes of degree 2 is 15. Total number of nodes in the binary tree is?

$I_1 = 8$

$I_2 = 15$

$N = 2I_2 + I_1 + 1$

$= 2 * 15 + 8 + 1$

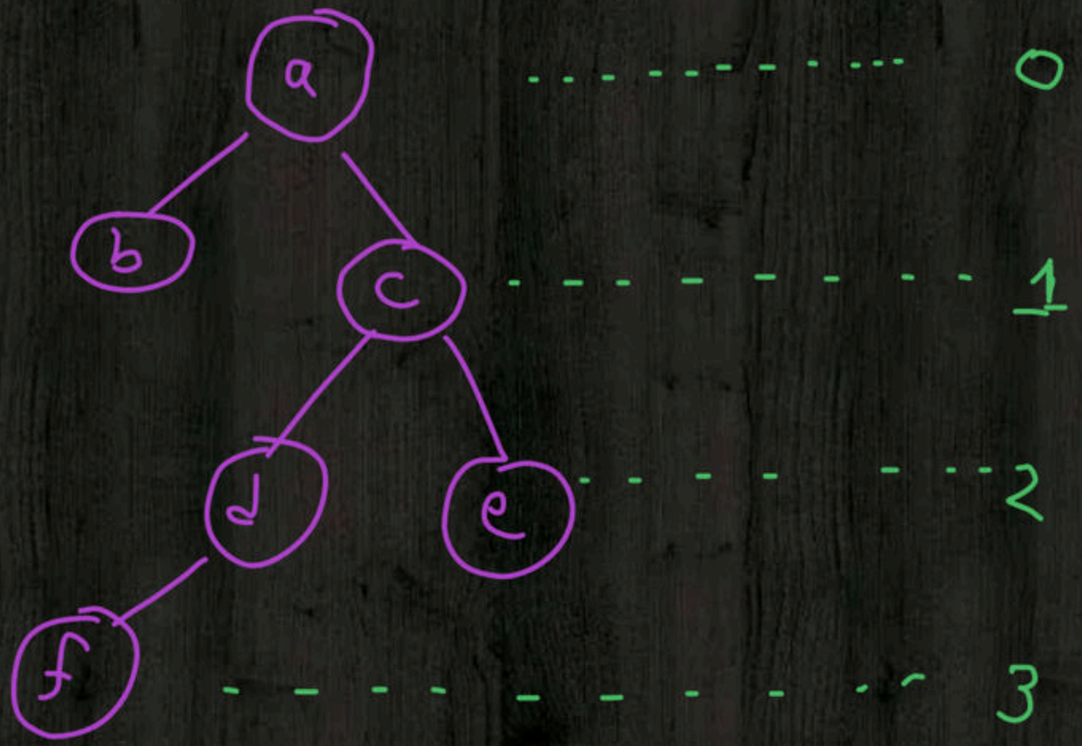
$= 39$

$I_1 = 8 + 15 = 23$

$L = 15 + 1 = 16$

$23 + 16 = 39$

Height of the Tree



Height of the Tree

Def. 1	Def. 2
1. H(Tree) = max level no.	2. H(tree) = max level no. + 1
2. H(Tree) with single node is zero.	2. H(tree) with single node is 1
3. Height of an empty tree is -1	3. Height of an empty tree is 0
4. H(Tree) = Distance of farthest leaf node from root	4. — " — +1
5. H(Tree) = no. of edges in path from root to farthest leaf node	5. H(tree) = no. of nodes in the path from root to farthest leaf node

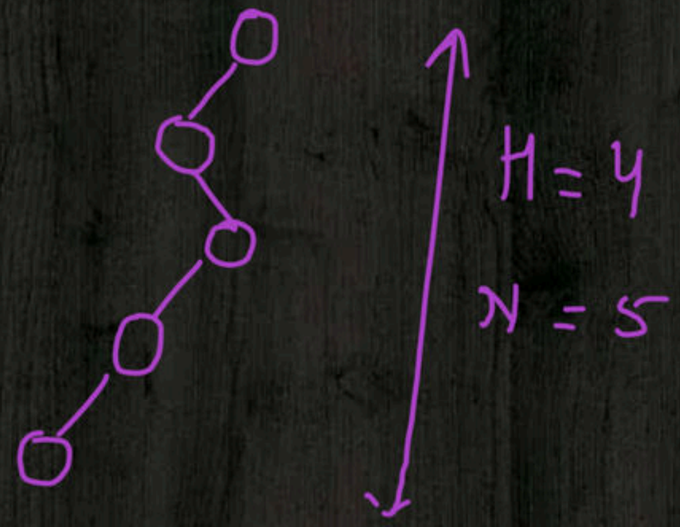
Height of the Tree: Recursuve Approach

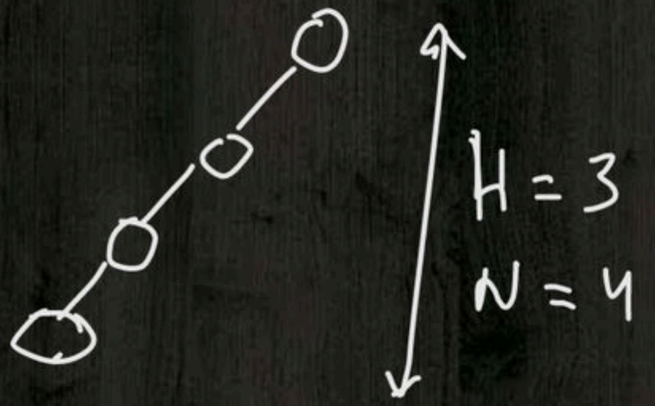
Question

binary

Maximum and minimum nodes in a tree with height H?
Note: Height of a tree with single node is 0

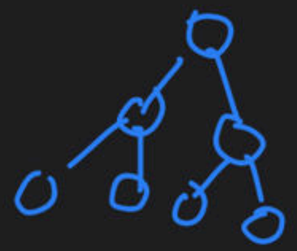
Minimum :-





$$N_{min}(H) = H + 1$$

max:-



$h = 0$
$N = 1$

$h = 1$
$N = 3$

$h = 2$
$N = 7$

$h = 3$
$N = 15$

$$N_{max}(H) = 2^{H+1} - 1$$

Question

How many binary tree can be constructed using 3 unlabeled nodes?

Question

How many binary tree can be constructed using 3 distinct keys?

DPP

Question

Draw the tree for:

I. A(B, C(E, F), D(G, H, K))

II. A(C(B, F), D ,E(X, Y(P, Q), Z))

III. a(b, c(e, g(x, y, z), h(p, q, r)), d (i, j, k), f)

IV. 1(2, 3(7, 8, 9), 4, 5(10, 11, 12, 13, 14), 6(21, 23, 34, 35))

Question

Calculate following for above all tree in previous question:

1. Get the height of the trees
2. Total number of leaf nodes
3. Total number of non-leaf nodes

Question

Maximum and minimum nodes in a tree at a level L?

Question

Consider a binary tree with n nodes. If it has L leaf nodes then:

1. Calculate number of internal nodes with 2 children L2?
2. Calculate total number of internal nodes?

Question

Consider a binary tree which has 50 nodes with degree 1 and total 80 leaf nodes. Calculate:

1. Number of internal nodes with 2 children L2
2. Total number of internal nodes?
3. Total number of nodes in tree?

Question

Minimum and maximum possible height of a tree with n nodes?
Note: H(tree) with single node is 0

Happy Learning