Concavity and Points of Inflections

Comprehensive Course on Engineering Mathematics

B V Reddy • Lesson 32 • Sept 3, 2023

CALCULUS

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1. Limits
2. Continuity
3. Differentiability
4. Mean value theorems
5. Taylors series
6. Maxima and minima
7. Integration

Function

The relationship between input and the outputs is called as a function.

The relationship between dependent variable and the independent variable is called as a function

Even function

A function f(x) is said to be an even function if $f(-x) = f(x)$.

Ex:

Odd Function:

A function f(x) is said to be an odd function it $f(-x) = - f(x)$.

Ex:

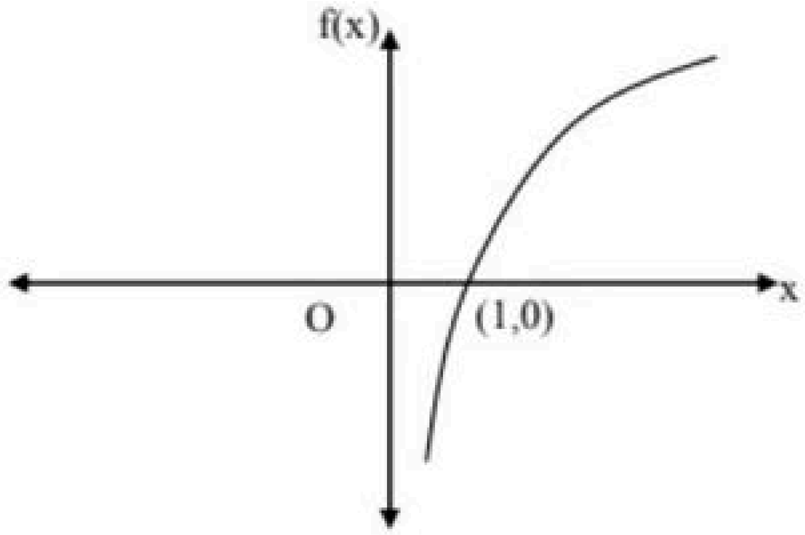
Modulus function

Step function (Greatest integer function)(Bracket function) (Floor value function)

Signum function

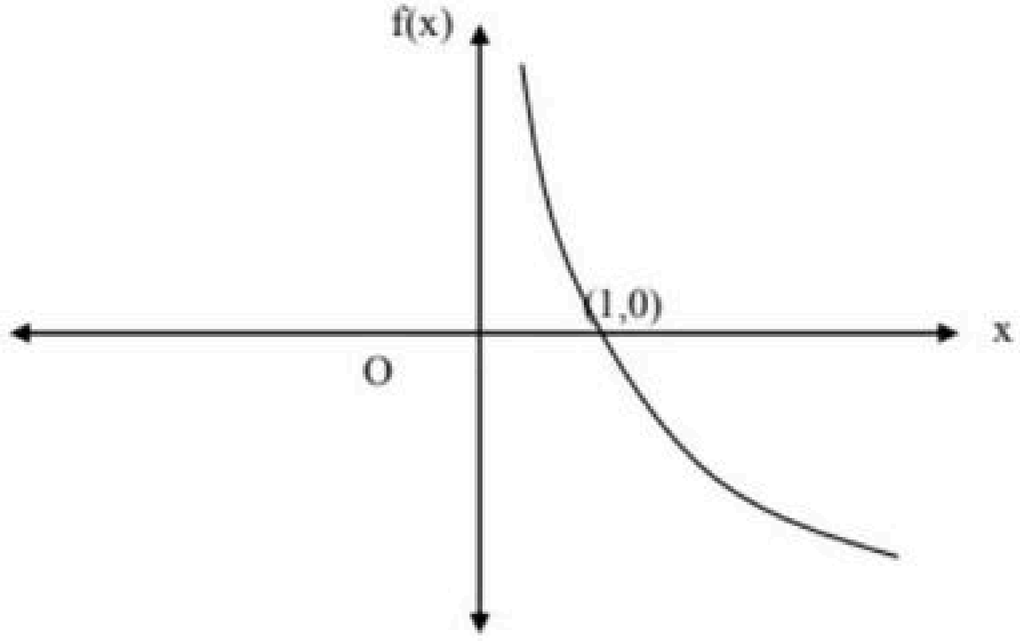
Logarithmic function

A function of the form $f(x) = \log_a x$



Case – I

For a>1



Case – II

For 0< a <1

14. The function $f(x) = e^x$ is _______ **(GATE -EC-1999)**

(a) Even (b) Odd (c) Neither even nor odd (d) None

Limit :

Limit of a function gives approximate value of the function in the neighboured of a point.

To examine the behaviour of the function $y = f(x)$, in the neighboured of a point $x = a$, when $f(x)$ is indeterminate at $x= a$.

Limit of a function

Limit of a function f(x) is said to exist at x = a , if

Difference between limit of a function and functional value at a given point

Reasons for non –existence of limit

1. If LHL or RHL or both does not exits
2. If LHL and RHL both exists but they are unequal

Indeterminate forms

Determinate forms

Algebra of limits

Evaluation of Limits

1. Find the value of function at the given limit. If it is determinate , it self is the answer .
2. If the value is indeterminate of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then apply L' Hospital rule .
3. If the value is indeterminate , but not in the $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, then convert to this form .
4. L' Hospital rule can be applied only in case of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,if can be applied any number of times, but check whether it is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

1. $\lim\limits_{x \to 0} \dfrac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} =$ ______ **(GATE -ME- 1993)**

4. $\lim\limits_{x \to \infty} \dfrac{x^3 - \cos x}{x^2 + (\sin x)^2} =$ ______ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 2 (d) Does not exist

5. $\lim_{x \to 0} x \sin \frac{1}{x} =$ ________ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 1 (d) Does not exist

8. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}$, where m is an integer, is one of the following:

(GATE -CS- 1997)

(a) m (b) $m\pi$ (c) $m\theta$ (d) 1

11. $\lim\limits_{x\to 0} \frac{1}{10} \frac{1-e^{-j5x}}{1-e^{-jx}} =$ ______ **(GATE -IN-1998)**

(a) 0 (b) 1.1 (c) 0.5 (d) 1

12. Limit of the function, $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}}$ is ________ **(GATE -EC-1999)**

(a) ½ (b) 0 (c) ∞ (d) 1

13. Value of the function $\lim_{x \to a} (x-a)^{x-a}$ is ______ **(GATE -CS-1999)**

(a) 1 (b) 0 (c) ∞ (d) a

16.Limit of the function $f(x) = \frac{1-a^4}{x^4}$ as $x \to \infty$ is given by

(GATE -CS-2000)

(a) 1 (b) e^{-a^4} (c) ∞ (d) 0

17. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} =$ ________ **(GATE -IN-2001)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

18. Limit of the following sequence as $n \to \infty$ is ______ $x_n = n^{\frac{1}{n}}$

(GATE -CE-2002)

(a) 0 (b) 1 (c) ∞ (d) $-\infty$

5

20. $\lim_{x \to 0} \frac{\sin^2 x}{x} =$ ______ **(GATE-CS-2003)**

(a) 0 (b) ∞ (c) 1 (d) -1

21. The value of the function, $f(x) = \lim_{x \to 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$ is _____ **(GATE-CS-2004)**

(a) 0 (b) $\frac{-1}{7}$ (b) $\frac{1}{7}$ (d) ∞

23. $\lim\limits_{x \to 0} \dfrac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3} =$ **(GATE-ME-2007)**

(a) 0 (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{3}$ (d) 1

25. What is the value of $\lim_{x \to \pi/4} \frac{\cos x - \sin x}{x - \pi/4}$ **(GATE-PI-2007)**

(a) $\sqrt{2}$ (b) 0 (c) $-\sqrt{2}$ (d) Limit does not exist

26. $\lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta}$ is **(GATE-EC-2007)**

(a) 0.5 (b) 1 (c) 2 (d) not defined

27. $\lim\limits_{x\to\infty} \dfrac{x - \sin x}{x + \cos x} =$ ________ **(GATE-EC-2008)**

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

30. The value of $\lim_{x \to 8} \dfrac{x^{1/3} - 2}{x - 8}$ is **(GATE-ME-2008)**

(a) $\dfrac{1}{16}$ (b) $\dfrac{1}{12}$ (c) $\dfrac{1}{8}$ (d) $\dfrac{1}{4}$

31. The value of the expression $\lim_{x \to 0} \left[\frac{\sin(x)}{e^x x} \right]$ is **(GATE-PI-2008)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{1+e}$

34. What is the value of $\lim_{n \to \alpha} \left(1 - \frac{1}{n}\right)^{2n}$? **(GATE-CS-2010)**

(a) 0 (b) e^{-2} (c) $e^{-t/2}$ (d) 1

50. The value of $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 2020}}{x + 7}$ is

GATE-2021 (CE)

(a) 1

(b) 3

(c) $\frac{7}{9}$

(d) Indeterminable

53. The value of $\lim_{x\to\infty}(1+x^2)^{e^{-x}}$ is (GATE-CS-2015)

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) ∞

55. The value of $\lim_{x\to\infty} \frac{1-\cos(x^2)}{2x^4}$ is **(GATE-ME-2015)**

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) undefined

57. The value of $\lim_{x\to 0}\left(\dfrac{-\sin x}{2\sin x - x\cos x}\right)$ is ________ **(GATE-ME-2015)**

58. The value of $\lim_{x\to\infty} \dfrac{x^2 - 5x + 4}{4x^2 + 2x}$ is

GATE-2020 (CE)

(a) 0

(b) 1

(c) $\dfrac{1}{4}$

(d) $\dfrac{1}{2}$

59. Consider the limit:

GATE-2021 (CE)

$$\lim_{x \to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$$

The limit (correct up to one decimal place) is ______

60. The value of $\lim_{x \to \infty} \frac{x \ln(x)}{1 + x^2}$ is

GATE-2021(CE)

(a) 1.0

(b) 0.5

(c) ∞

(d) 0

Continuity

Continuity

A function f(x) is said to be continuous at x=a if it satisfies the following conditions.

(i) f(a) is defined

(ii) $\underset{x\to a}{\mathrm{Lt}} f(x)$ exists i.e $\underset{x\to a^-}{\mathrm{Lt}} f(x) = \underset{x\to a^+}{\mathrm{Lt}} f(x)$

(iii) $\underset{x\to a}{\mathrm{Lt}} f(x) = f(a)$

Reasons of discontinuity

1. If the function is not defined at a given point.
2. If the limit of the function not exits
3. If the limit of the function exists , functional value exists but both are not equal

Left continuous (or) continuity from the left at a point

A function f(x) is said to be continuous from the left (or) left continuous at x=a if

(i) f(a) is defined

(ii) $\underset{x\to a^-}{\mathrm{Lt}} f(x) = f(a)$

Right continuous (or) continuity from the right at a point

A function f(x) is said to be continuous from the right (or) right continuous at x=a if

(i) f(a) is defined

(ii) $\underset{x\to a^+}{\mathrm{Lt}} f(x) = f(a)$

Continuity of a function in an open interval:

A function f(x) is said to be continuous in an open interval (a,b)

if f(x) is continuous $\forall \in (a, b)$

Continuity of a function on closed interval:

A function f(x) is said to be continuous on closed interval [a,b] if

(i) f(x) is continuous (a, b)

(ii) $\operatorname{Lt}_{x \to a^+} f(x) = f(a)$

(iii) $\operatorname{Lt}_{x \to b^-} f(x) = f(b)$

1. Sinx , cosx , e^x, a^x , |x|, polynomial functions are always continuous .

2.

Function	Points of discontinuity
Tanx , secx	
Cotx, cosecx	
$[x]$	
$\frac{1}{x}$	
Sgn(x)	

3. If f(x) and g(x) are two continuous functions then $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$ and

$\frac{f(x)}{g(x)}$ (since $g(x) \neq 0$) are also continuous.

4. Logarithmic functions are continuous in $(0, \infty)$

Types of discontinuity

1. Discontinuity of first kind (or) Removable discontinuity

a. Missing point discontinuity

b. Isolated point discontinuity

2. Discontinuity of second kind (or) Irremovable discontinuity

a. Finite discontinuity(Jump discontinuity)

b. Infinite discontinuity

c. Oscillatory discontinuity

Discontinuity of first kind (or) Removable discontinuity

1. Missing point discontinuity

2. Isolated point discontinuity

Discontinuity of second kind (or) Irremovable discontinuity

1. Finite discontinuity

2. Infinite discontinuity

3. Oscillatory discontinuity

37. What should be the value of λ such that the function defined below is continuous at $x = \frac{\pi}{2}$? **(GATE-CE-2011)**

$$f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x}, & \text{if } x \neq \frac{\pi}{2} \\ 1, & \text{if } x = \frac{\pi}{2} \end{cases}$$

(a) 0 (b) 2π (c) 1 (d) $\frac{\pi}{2}$

42. Which one of the following functions is continuous at $x = 3$?

(GATE-CS-2013)

(a) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x - 1 & \text{if } x > 3 \\ \dfrac{x+3}{3}, & \text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8 - x & \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x + 3, & \text{if } x \leq 3 \\ x - 4 & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3 - 27}$, if $x \neq 3$

Continuity of Composite functions

Differentiability

Derivative of a function at a point:

If a function f(x) is defined on a neighborhood of a real number 'a' and $\underset{x\to a}{Lt}\frac{f(x)-f(a)}{x-a}$ exists and finite then the finite limit is called derivative or differential coefficient of f(x) at a point 'a' and it is denoted by $f^1(a)$.

$$\therefore \underset{x\to a}{Lt}\frac{f(x)-f(a)}{x-a}=f^1(a)$$

LHD

RHD

Differentiability of a function in an interval

A function f(x) is said to be differentiable in an interval [a, b]

1. f(x) is continuous in (a, b)

2. $f^1(a-) = \lim\limits_{x \to a^-} \frac{f(x)-f(a)}{x-a}$ **exists**

3. $f^1(a^+) = \lim\limits_{x \to a^+} \frac{f(x)-f(a)}{x-a}$ **exists**

4. $f^1(a-) = f^1(a^+)$

Working Procedure to check continuity & differentiability

1. Check for continuity of the given function
2. If it is discontinuous , then the function is non differentiable ,
3. If it is continuous , find the LHD and RHD .
4. If LHD and RHD exists, and LHD = RHD , then the function is differentiable .

- If f(x) and g(x) are two differentiable function then f(x)+g(x), f(x)-g(x), f(x)·g(x), $\frac{f(x)}{g(x)}$ (g(x)≠ 0) are also differentiable.
- Polynomial functions, exponential functions, sine and cosine functions are differentiable everywhere.
- Every differentiable function is continuous but a continuous function need not be differentiable.
- If the function is discontinuous , then it is not differentiable .
- |x| is continuous but not differentiable at x= 0
- |x-a| is continuous but not differentiable at x = a
- |ax-b| is continuous but not differentiable at x = $\frac{b}{a}$
- Sgn(x-a) is not differentiable at x = a
- [x] is not differentiable at all integers

Reasons for non differentiability of a function

1. Sharp Corner

2. Vertical tangent

3. Having discontinuities at $x = a$

4.Function tending to infinite at $x = a$

6. The function $f(x) = |x+1|$ on the interval $[-2, 0]$ is ________

(GATE-EC- 1995)

(a) continuous and differentiable

(b) continuous on the interval but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

9. If $y = |x|$ for $x < 0$ and $y = x$ for $x \geq 0$ then

(GATE -EC- 1997)

(a) $\frac{dy}{dx}$ is discontinuous at $x = 0$ (b) y is discontinuous at $x = 0$

(c) y is not defined at $x = 0$

(d) Both y and $\frac{dy}{dx}$ are discontinuous at $x = 0$

7. If a function is continuous at a point its first derivative **(GATE -EC- 1995)**

(a) may or may not exist (b) exists always

(c) will not exist (d) has a unique value

19. Which of the following functions is not differentiable in the domain [-1, 1]?

(a) $f(x) = x^2$

(b) $f(x) = x - 1$

(GATE -EC-2002)

(c) $f(x) = 2$

(d) $f(x) = \text{maximum } (x - x)$

22. Consider the function $f(x) = |x|^3$, where x is real.

Then the function f(x) at x = 0 is

(GATE –IN-2007)

(a) continuous but not differentiable (b) once differentiable but not twice
(c) twice differentiable but not thrice (d) thrice differentiable

33. If $f(x) = \sin|x|$ then the value of $\frac{df}{dx}$ at $x = \frac{-\pi}{4}$ is **(GATE-PI-2010)**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) 1

36. The function $y = |2-3x|$ **(GATE-ME-2010)**

(a) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(b) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{3}{2}$

(c) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{2}{3}$

(d) is continuous $\forall x \in R$ and except at $x = 3$ and differentiable $\forall x \in R$

$2 - 3x = 0$

$x = \frac{2}{3}$.

46. If a function is continuous at a point, **(GATE-ME-SET-3-2014)**

(a) the limit of the function may not exist at the point

(b) the function must be derivable at the point

(c) the limit of the function at the point tends to infinity

(d) the limit must exist the point and the value of limit should be same as the value of the function at the point.

$\boxed{L \quad \boxed{C} \quad D}$

Differentiability of Y= |f(x)|

1. Check for continuity & differentiability of $f(x)$

2. If $f(x)$ is not differentiable, then $|f(x)|$ also not differential

3. If $f(x)$ is differentiable,

then $f(x) = 0$

$x = \alpha, \beta$

if $f'(x)\Big|_{x=\alpha} = 0$, then $|f(x)|$ is differentiable at $x = \alpha$.

if $f'(x)\Big|_{x=\beta} \neq 0$

then $|f(x)|$ is not differentiable at $x=\beta$.

Q. Check the differentiability of $y = |x|$

$f(x) = x$

↳ continous & differentiable.

$f(x) = 0$

$x = \underline{0}$

$f'(x) = 1$

$f'(x)\Big|_{x=0} = 1$

$f'(x)\Big|_{x=0} \neq 0$

$|x|$ is not differentiable at $x = 0$.

Q. Check the differentiability of $y = |e^x|$

$f(x) = e^x$

$\hookrightarrow$ continuous & differentiable.

$f(x) = 0$

$e^x = 0 = e^{-\infty}$

$x = -\infty$

$f'(x)\Big|_{x=-\infty} = e^x = e^{-\infty} = 0.$

$|e^x|$ is differentiabl

Q. Check the differentiability of $y = |sin x|$

$f(x) = \sin x.$

$\hookrightarrow$ continuous & differentiable.

$f(x) = 0$

$\sin x = 0$

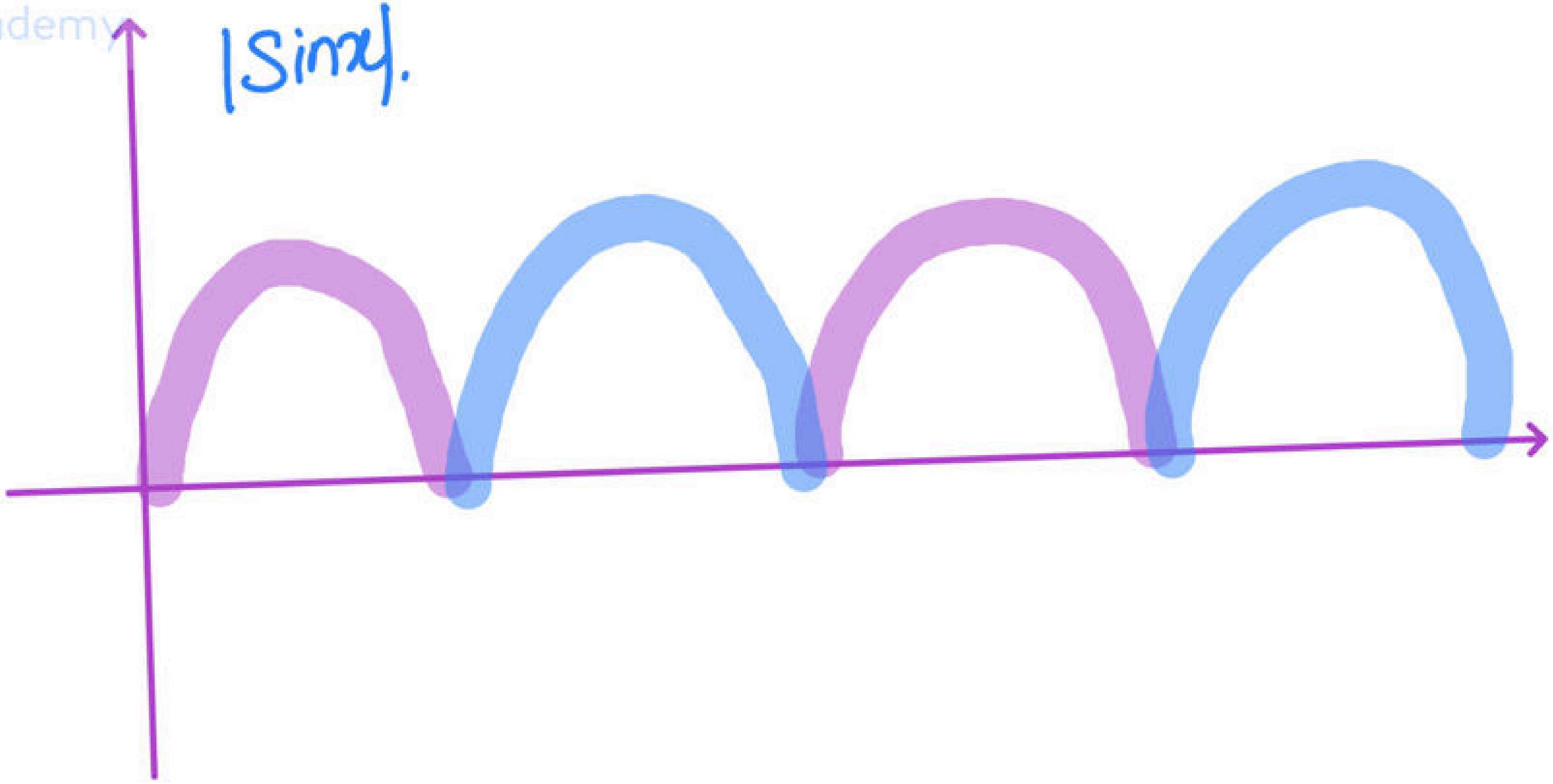
$x = n\pi$

$f'(x) = \cos x.$

$f'(x)\Big|_{x=n\pi} = \cos n\pi = (-1)^n \neq 0.$

$|\sin x|$ is not differentiable

$|\sin x|$.



Q. Check the differentiability of $y = |x^3|$

Differentiability of f(x). g(x)

$$h(x) = f(x)\, g(x)$$

1. If $f(x)$ is continuous & differentiable at $x=a$
2. If $g(x)$ is continuous but not differentiable at $x=a$.
3. if $f(x)\big|_{x=a} = 0$, then $h(x)$ is differentiable at $x=a$.
4. If $f(x)\big|_{x=a} \neq 0$, then $h(x)$ is not differentiable at $x=a$.

Q. Check the differentiability of $y = x|x|$

$y = x|x|$

$f(x) = x \rightarrow$ continuous & differentiable.

$g(x) = |x| \rightarrow$ continuous but not differentiable at $x=0$.

$f(x)\Big|_{x=0} = 0$

So, $y = x|x|$ is differentiable.

$$y = x(-x) \quad , \quad x < 0.$$

$$= x(+x) \quad , \quad x > 0.$$

$$y' = -2x, \quad x < 0.$$

$$= 2x, \quad x > 0.$$

LHD $= 0$

RHD $= 0.$

$y = x|x|$ is differentiable

Q. Check the differentiability of $y = cosx|x|$

$f(x) = \cos x$

$g(x) = |x|$, not differentiable at $\underline{x=0}$.

$f'(x)\Big|_{x=0} = \cos 0 = 1 \neq 0$.

So, $y = \cos x|x|$, is not differentiable at $x=0$.

Q. Check the differentiability of $y = (x^2 - 3x + 2)|x^2 - 5x + 6|$

$f(x) = x^2 - 3x + 2.$

$g(x) = x^2 - 5x + 6$

$g(x) = (x-2)(x-3)$

$x = 2, 3.$

$f(x)\Big|_{x=2} = 4 - 6 + 2 = 0$ ✓

$f(x)\Big|_{x=3} = 9 - 9 + 2 \neq 0$

$y = (x^2 - 3x + 2)|x^2 - 5x + 6|$

is differentiable at $x = 2$ and not differentiable at $x = 3$.

Mean Value Theorems

Rolle's Theorem

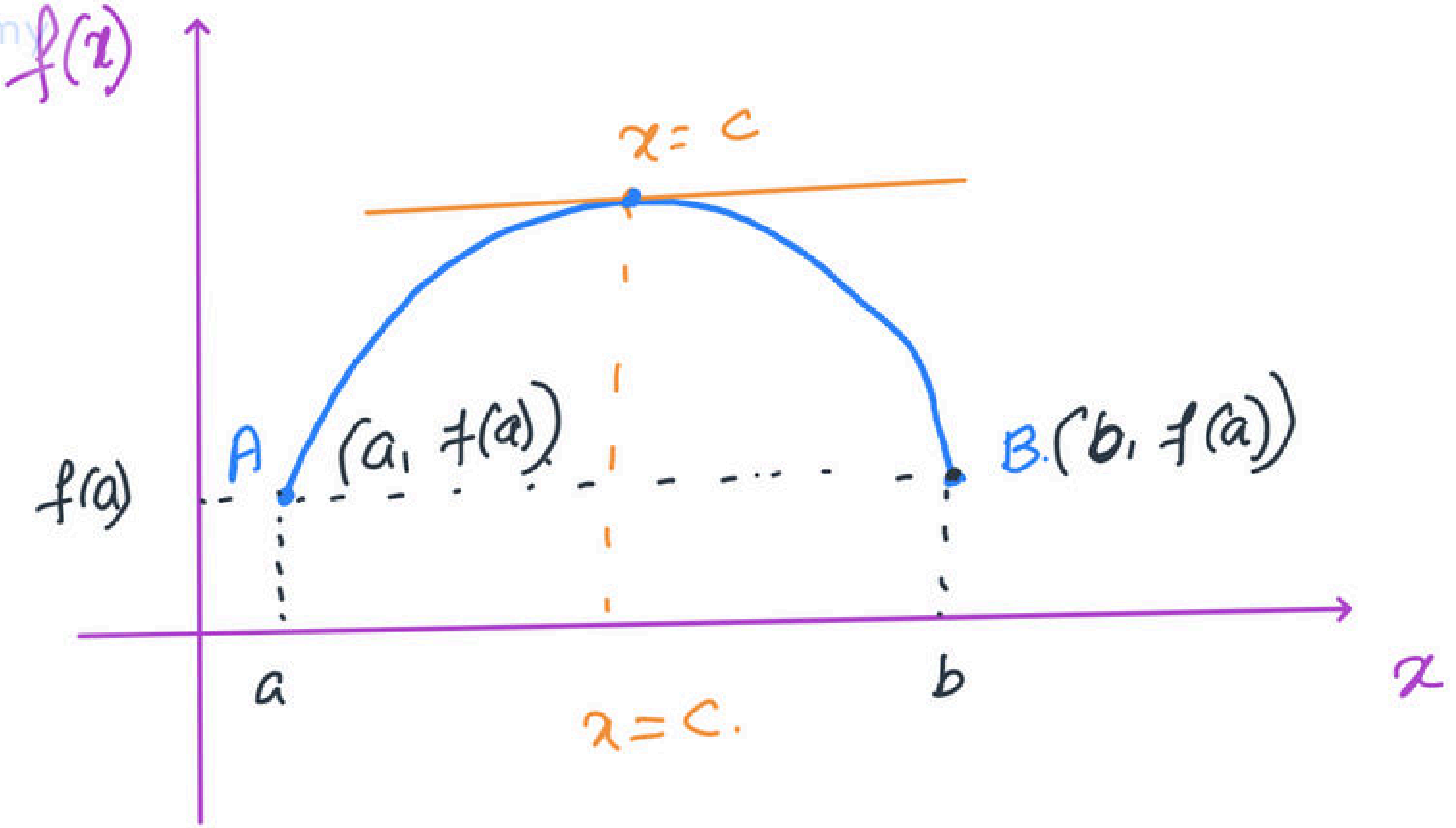
If $f(x)$ is defined in $[a, b]$ such that

1. $f(x)$ is continuous in $[a, b]$

2. $f(x)$ is differentiable in (a, b)

3. if $f(a) = f(b)$

then there exist at least one point $c \in (a, b)$ at which tangent is parallel to x-axis.



$$\boxed{f'(c) = 0.}$$

Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = x^2 - 1$ on [-1, 1]

$f(x) = x^2 - 1.$

$\hookrightarrow$ continuous & differentiable.

$f'(c) = 0$

$2C - 0 = 0$

$C = 0$

Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = (x-a)^m (x-b)^n$ on [a, b]

$$f(x) = (x-a)^m (x-b)^n.$$

$$f'(x) = m(x-a)^{m-1}(x-b)^n + n(x-a)^m (x-b)^{n-1}$$

$$f'(x) = 0$$

$$m(x-a)^{m-1}(x-b)^n + n(x-a)^m(x-b)^{n-1} = 0$$

$$m(x-a)^{-1} + n(x-b)^{-1} = 0.$$

$$m(x-a)^{-1} + n(x-b)^{-1} = 0.$$

$$\frac{m}{x-a} = -\frac{n}{x-b}$$

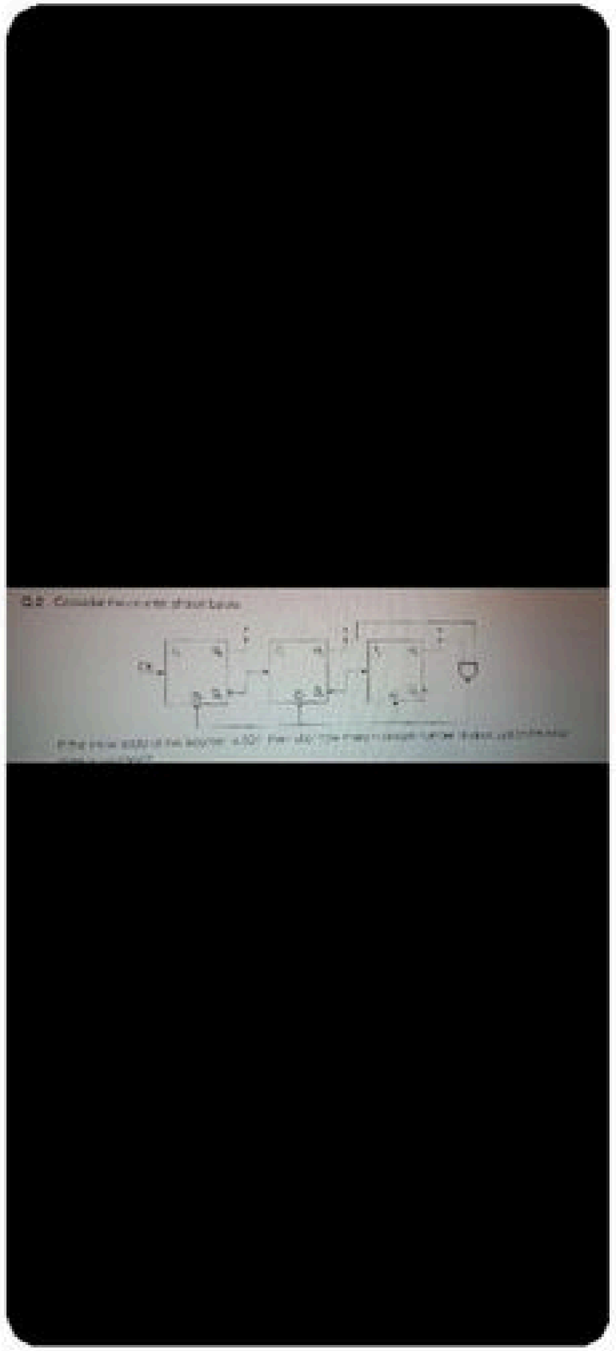
$$mx - mb = -nx + na.$$

$$x(m+n) = na + mb.$$

$$\boxed{x = \frac{mb+na}{m+n}.}$$

▲ 1 • Asked by Divyanshu

Please help me with this doubt



Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = \log\left[\frac{x^2+ab}{(a+b)x}\right]$ on [a, b]

$$f(x) = \log\left[\frac{x^2+ab}{(a+b)x}\right]$$

$$f'(x) = \frac{1}{\frac{x^2+ab}{(a+b)x}}\left[\frac{(a+b)x(2x) - (x^2+ab)(a+b)}{(a+b)^2x^2}\right] = 0$$

$$(a+b)\,2x^2 = (x^2+ab)(a+b)$$

$$2x^2 = x^2+ab$$

$$x^2 = ab$$

$$\boxed{x = \sqrt{ab}}$$

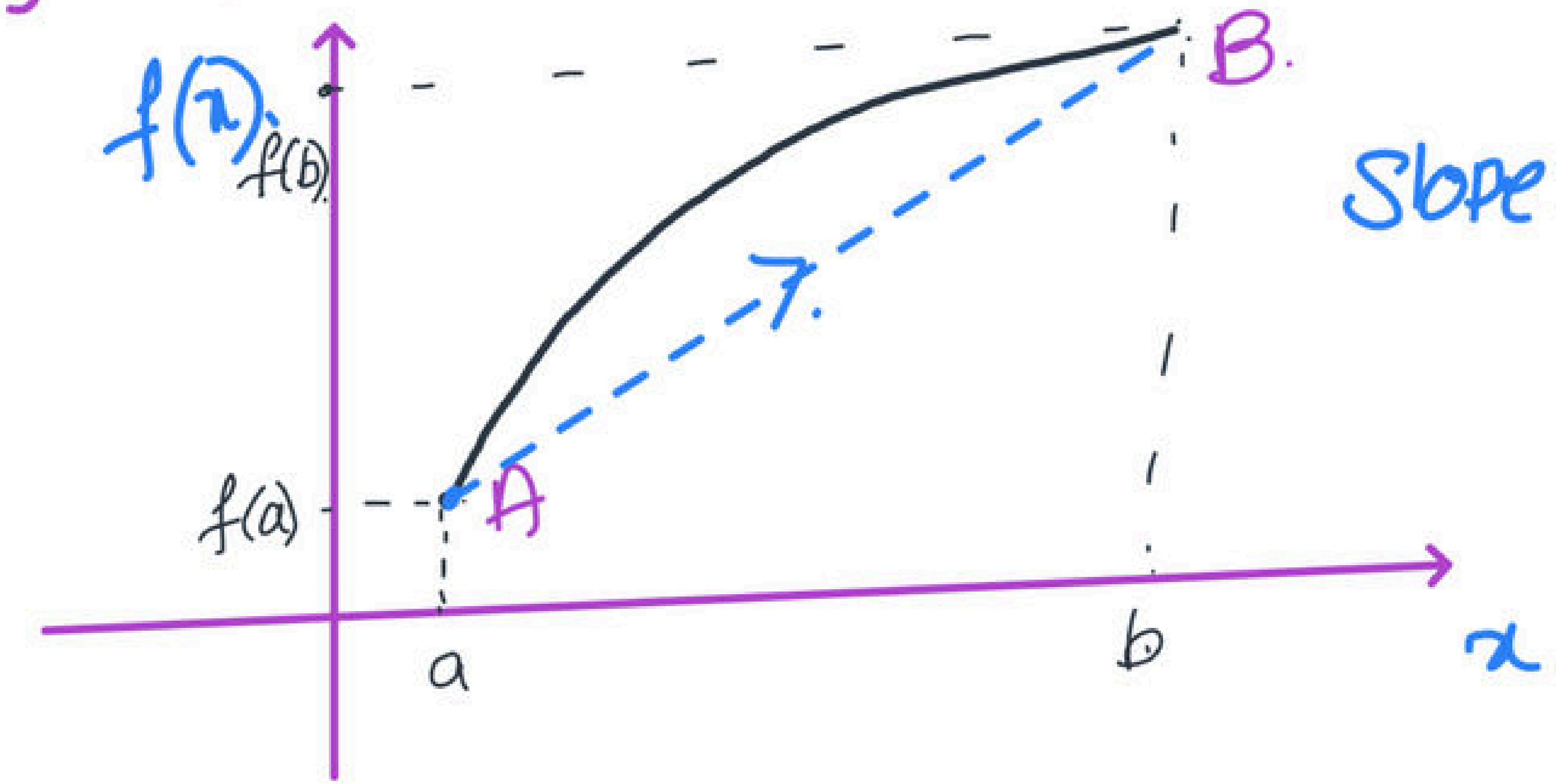
$$\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

Lagrange's Mean Value Theorem (LMVT)

If $f(x)$ is defined in $[a, b]$ such that

1. $f(x)$ is continuous on $[a, b]$

2. $f(x)$ is differentiable on (a, b)



$$\text{Slope} = \frac{f(b) - f(a)}{b - a}.$$

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}.}$$

Q. Find 'C' of LMVT for $f(x) = \sin x - \sin 2x$ in $[0, \pi]$

$$f(x) = \sin x - \sin 2x$$

$$f'(x) = \cos x - 2\cos 2x.$$

$$f'(x) = \frac{f(b) - f(a)}{b-a}.$$

$$\cos x - 2\cos 2x = \frac{(\sin\pi - \sin 2\pi) - (\sin 0 - \sin 0)}{\pi - 0}$$

$$\cos x - 2\cos 2x = 0$$

$$\cos x = 2\cos 2x$$

$$\cos x = 2(2\cos^2 x - 1)$$

$$4\cos^2 x - \cos x - 2 = 0$$

$$\cos x = \frac{+1 \pm \sqrt{1+32}}{8}$$

$$\cos x = \frac{1 \pm \sqrt{33}}{8}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos x = \frac{1+\sqrt{33}}{8}, \quad x = 32.54^\circ$$

$$\cos x = \frac{1-\sqrt{33}}{8}, \quad x = 126.3$$

Q. If $f^1(x) = \frac{1}{3-x^2}$ *and* $f(0) = 1$, find an interval in which f(1) lies

$[0, 1]$

$$f'(x) = \frac{f(b) - f(a)}{b-a}.$$

$$f'(1) = \frac{f(1) - f(0)}{1-0}$$

$$f'(1) = f(1) - 1$$

$$f'(a) \le f(1) \le f'(1)$$

$$\frac{1}{3} \le f(1) \le \frac{1}{2}.$$

Cauchy's Mean Value Theorem

Q. The ‘C’ of Cauchy’s mean value theorem for $f(x) = e^x, g(x) = e^{-x}$ defined in [a, b] is ----------

Q. The ‘C’ of Cauchy’s mean value theorem for $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$ defined in [a, b] is ----------

Q. The 'C' of Cauchy's mean value theorem for $f(x) = \text{Sin}x$, $g(x) = \text{Cos}x$ defined in [a, b] is ----------

Taylor's Series

Maclaurin's Series

Q. Expand e^x by Taylor's series about $x = 0$

97. The third term in the taylor's series expansion of e^x about 'a' would be ______

(a) $e^a(x-a)$ (b) $\frac{e^a}{2}(x-a)^2$ (c) $\frac{e^a}{2}$ (d) $\frac{e^a}{6}(x-a)^3$ **GATE -1995**

98. The taylor's series expansion of sin x is __________ **(GATE-EC-1998)**

(a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots$

(b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots\ldots$

(c) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots\ldots$

(d) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots$

100. The Taylor series expansion of sin x about $x = \frac{\pi}{6}$ is given by **(GATE-CE-2000)**

(a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\Pi}{6}\right)-\frac{1}{4}\left(x-\frac{\Pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\Pi}{6}\right)^3+----$

(b) $x - \frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+------$

(c) $\frac{x-\frac{\Pi}{6}}{1!}-\frac{\left(x-\frac{\Pi}{6}\right)^3}{3!}+\frac{\left(x-\frac{\Pi}{6}\right)^5}{5!}-\frac{\left(x-\frac{\Pi}{6}\right)^7}{7!}+------$

(d) $\frac{1}{2}$

101. Limit of the following series as x approaches

$\frac{\pi}{2}$ is $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + -----$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 1 **(GATE-CE-2001)**

102. For the function e^{-x}, the linear approximation around $x = 2$ is

GATE- 2007

(a) $(3-x)e^{-2}$

(b) $1 - x$

(c) $\left[3+2\sqrt{2}-\left(1+\sqrt{2}\right)x\right]e^{-2}$

(d) e^{-2}

103. For $|x| << 1$, $\coth(x)$ can be approximated as **(GATE-EC-2007)**

(a) x (b) x^2 (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

104. The expression $e^{\ln x}$ for $x > 0$ is equal to **(GATE-IN-2008)**

(a) $-x$ (b) x (c) x^{-1} (d) $-x^{-1}$

105. Which of the following function would have only odd powers of x in its Taylor series expansion about the point $x = 0$? **(GATE-EC-2008)**

(a) $\sin(x^3)$ (b) $\sin(x^2)$ (c) $\cos(x^3)$ (d) $\cos(x^2)$

106. In the Taylor series expansion of $e^x + \sin x$ about the point $x = \pi$, the coefficient of $(x-\pi)^2$ is **(GATE-EC-2008)**

(a) e^π (b) $0.5\ e^\pi$ (c) $e^\pi + 1$ (d) $e^\pi - 1$

107. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(GATE-ME-2008)

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$ (c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

108. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by **(GATE-EC-2010)**

(a) $1+\frac{(x-\pi)^2}{3!}+------$

(b) $-1-\frac{(x-\pi)^2}{3!}+------$

(c) $1-\frac{(x-\pi)^2}{3!}+------$

(d) $-1+\frac{(x-\pi)^2}{3!}+------$

112. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots$ corresponds to **(GATE-CE-2012)**

(a) $\sec x$ (b) e^x (c) $\cos x$ (d) $1+\sin^2 x$

113. The Taylor series expansion of 3 sin x + 2cos x is **(GATE-EC-SET-1-2014)**

(a) $2 + 3x - x^2 - \frac{x^3}{2} + ---$

(b) $2 - 3x + x^2 - \frac{x^3}{2} + ---$

(c) $2 + 3x + x^2 + \frac{x^3}{2} + ---$

(d) $2 - 3x - x^2 + \frac{x^3}{2} + ---$

Taylor’s Series for functions of two variables

Q. Expand $x^2y + 3y - 2$in powers of (x-1) and (y+2) using Taylor's series

Maxima & Minima

Increasing and Decreasing functions at a point

Q. Find the set values of λ for which the function

$$f(x) = \begin{cases} x+1 & x<1 \\ \lambda & x=1 \\ x^2-x+3 & x>1 \end{cases}$$

is strictly increasing at x= 1

Increasing and Decreasing functions on an interval

Monotonic Function

Test for Monotonicity

Q. Find the interval in which $f(x) = x^3 - 3x^2 - 9x + 20$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = x^2 - 5x + 6$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = -x^2 - 5x + 6$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = x(x^2 - 4)$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = (x+2)(x-1)^2(x-5)$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = \frac{(x-1)(x-5)}{(x-3)}$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = \frac{(x-1)(x+5)}{(x-3)(x+4)}$ is increasing or decreasing

Stationary points

The values of x for which $f^1(x) = 0$, are called stationary points or turning points .

Critical points

The values of x for which $f^1(x) = 0$, and the points where $f^1(x)$ is not exist are called as critical points .

Local (Relative) Maxima

Local (Relative) Minimum

Extreme Points & Extreme values

The point at which the function has a maximum or a minimum is called extreme point.

The values of the function at extreme points are called extreme values(Extrema)

Methods to find Local Extremum

First derivate test

Second derivate test

62. The function $y = x^2 + \dfrac{250}{x}$ at $x = 5$ attains

(GATE-EC-1994)

(a) Maximum (b) Minimum (c) Neither (d) 1

63. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has

(GATE-EE-1995)

(a) a maxima at $x = 1$ and a minima at $x = 3$

(b) a maxima at $x = 3$ and a minima at $x = 1$

(c) no maxima, but a minima at $x = 3$

(d) a maxima at $x = 1$, but no minima

70. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at

(GATE-CS-2004)

(a) $x = -2$ only

(b) $x = 0$ only

(c) $x = 3$ only

(d) both $x = -2$ and $x = 3$

71. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to

(a) 2 (b) 1 (c) 0 (d) -1

72. For real x, the maximum value of $\frac{e^{\operatorname{Sin} x}}{e^{\operatorname{Cos} x}}$ is

(GATE-IN-2007)

(a) 1 (b) e (c) $e^{\sqrt{2}}$ (d) ∞

76. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has

(GATE-EE-2007)

(a) Only one minimum

(b) Only two minima

(c) Three minima

(d) Three maxima

77. A point on the curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is ________ **(GATE-CS-2008)**

(a) 0 (b) 1 (c) 2 (d) 3

78. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 is **(GATE-IN-2008)**

(a) 1 (b) 3 (c) 4 (d) 9

79. For real values of x, the minimum value of function

$f(x) = e^{x} + e^{-x}$ is

(GATE-EC-2008)

(a) 2 (b) 1 (c) 0.5 (d) 0

80. At $t = 0$, the function $f(t) = \dfrac{\sin t}{t}$ has

(GATE-EE-2010)

(a) a minimum

(b) a discontinuity

(c) a point of inflection

(d) a maximum

81. If $e^y = x^{1/x}$ then y has a

(GATE-EC-2010)

(a) maximum at $x = e$

(b) minimum at $x = e$

(c) maximum at $x = e^{-1}$

(d) minimum at $x = e^{-1}$

83. The function $f(x) = 2x - x^2 + 3$ has

(GATE-EE-2011)

(a) a maxima at $x = 1$ and a minima at $x = 5$

(b) a maxima at $x = 1$ and a minima at $x = -5$

(c) only a maxima at $x = 1$

(d) only a minima at $x = 1$

86. For $0 \le t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$ (b) $t = \log_e 2$ (c) $t = 0$ (d) $t = \log_e 8$

87. The maximum value of the function $f(x) = \ln(1+x) - x$ (where $x > -1$) occurs at $x =$ ______________.

(GATE-EC-SET-3-2014)

Global (Absolute) maximum

Global (Absolute) minimum

Q. Find the points of local maxima and minima if any of the following function defined in $0 \le x \le 6$, $f(x) = x^3 - 6x^2 + 9x + 15$.

(GATE-CS-1998)

Q. The minimum value of function $y = x^2$ in the interval $[1, 5]$ is

(GATE-ME-2007)

(a) 0 (b) 1 (c) 25 (d) undefined

Q. Consider the function $f(x) = x^2 - x - 2$. The maximum value of $f(x)$ in the closed interval $[-4, 4]$ is

(GATE-EC-2007)

(a) 18 (b) 10 (c) -2.25 (d) indeterminate

Q. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval $[1, 6]$ is

(a) 21 (b) 25 (c) 41 (d) 46

GATE- 2012

Q. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is ____.

GATE-2014

Q. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

(a) e^{-1} (b) e (c) $1 - e^{-1}$ (d) $1 + e^{-1}$

GATE-2014

Q. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval [-3, 3] is **(GATE-EC-SET-2-2014)**

(a) 20 (b) 28 (c) 16 (d) 32

Convex and Concave functions

Point of Inflection

For a continuous function f(x) said to have a point of inflection at $x = x_0$ if

1.

2.

Q. Number of inflection points for the curve $y = x + 2x^4$ is ______

(GATE-CE-1999)

(a) 3 (b) 1 (c) 0 (d) 2

2

Q. At $x = 0$, the function $f(x)= x^3 + 1$ has

(GATE-ME,PI-2012)

(a) a maximum value

(b) a minimum value

(c) a singularity

(d) a point of inflection

Q. Find the points of inflections for $f(x) = x^3 - 3x^2 - 7x + 8$

Q. Find the points of inflections of $f(x) = e^{-x^2}$

Maxima and minima for functions of two variables

Let $z = f(x,y)$ be the function of two variables for which maxima or minima is to be obtained.

Working rule

Step 1: find p, q, r, s and t

Step 2: equate p and q to zero for obtaining stationary points.

Step 3: find r, s and t at each stationary point.

(a)If $rt - s^2 > 0$ and $r > 0$ then f(x, y) has a minimum at that stationary point.

(b)If $rt - s^2 > 0$ and $r < 0$ then f(x, y) has a maximum at that stationary point.

(c)If $rt - s^2 < 0$ then f(x, y) has no extremum at that stationary point and such points are called saddle points.

(d)If $rt - s^2 = 0$ then the case is undecided.

Q. Given a function $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$, the optimal values of $f(x, y)$ is

(GATE-CE-2010)

(a) a minimum equal to $\frac{10}{3}$

(b) a maximum equal to $\frac{10}{3}$

(c) a minimum equal to $\frac{8}{3}$

(d) a maximum equal to $\frac{8}{3}$

Q. The function $f(x,y) = 2x^2 + 2xy - y^3$ has **(GATE-EC-2000)**

(a) Only one stationary point at (0, 0)

(b) Two stationary points at (0, 0) and (1/6, -1/3)

(c) Two stationary points at (0, 0) and (1, -1)

(d) No stationary point

Q. The function $f(x) = 8 \log x - x^2 + 3$ attains its minimum over the interval [1,e] at $x =$_______________ (Here $\log_e x$ is the natural logarithm of x.)

(GATE-2022-ECE)

(a) 2 (b) 1

(c) e (d) $\frac{1+e}{2}$

Q. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

a) 8 m b)10m c)12m d)14m

Constrained maxima and minima

Sometimes it is required to find the extremum of a function subject to some other conditions involving the variables. Such problems are called constrained maxima and minima problems

Q. If the sum of the two positive numbers is 18 , then the maximum value of their product is

a)81 b)85 c)72 d)80

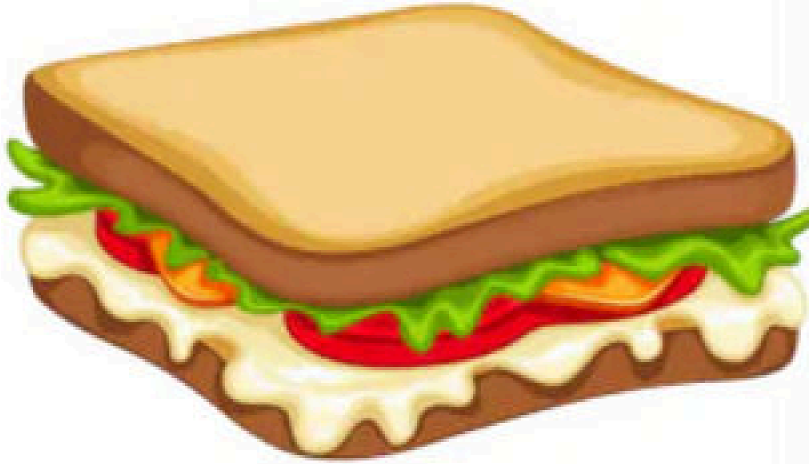
Q. If $x^2 + y^2 = 1$ then the maximum value of x+y

Lagrange's Method

Q. Find the maximum and minimum values of $x + y + z$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Q. Find the point on the plane $x + 2y + 3z = 4$, that is closest to the origin.

Sandwich theorem



Q. If $3-\frac{x^2}{12} \le f(x) \le 3+\frac{x^3}{9}$, then the value of $\underset{x \to 0}{Lt}\ f(x)$

Q. Find $\underset{n \to \infty}{Lt} \dfrac{[x]+[2x]+[3x]+-\cdots+[nx]}{n^{2}}$, where [.] is the greatest integer function.

Double Limit

Q. Find $\underset{\substack{x \to 0 \\ y \to 0}}{Lt} (x^3 + y^3)$

Q. Find $\underset{\substack{x \to 1 \\ y \to 2}}{Lt} \frac{3x^2y}{x^2+y^2+5}$

Q. Find $\underset{\substack{x \to 1 \\ y \to 2}}{Lt} \frac{2x^2+y^2}{2xy}$

Q. Find $\underset{\substack{x \to \infty \\ y \to 2}}{Lt} \dfrac{2x-3}{x^3+4y^3}$

Q. Find $\underset{\substack{x \to 0 \\ y \to 0}}{Lt} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

Definite Integrals

Multiple Integral's

Double Integrals

Concept of Strip

Vertical Strip

Horizontal Strip

Q: $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ **(GATE-EC-2000)**

(a) 0 (b) π (c) $\pi/2$ (d) 2

Q. Evaluate $\int_{1}^{2} \int_{3}^{4} (xy + e^{y})\, dy\, dx$

249. $\int_0^2 \int_0^3 xy \, dxdy$

(A) 0 (B) 9 (C) 8 (D) 1

250. $\int_0^{\frac{\pi}{2}}\int_{-1}^{1} x^2y^2\,dx\,dy$

(A) $\frac{\pi^3}{36}$ (B) $\frac{\pi}{0}$ (C) -1 (D) 0

251. Evaluate $\int_{-1}^{2}(1+|x|)\,dx$

(A) 3.5

(B) 5.5

(C) 4

(D) None of these

252. $\int_0^{\pi} \sin^5 x \cos^9 x dx =$ ________

(A) 0 (B) 1

(C) ∞ (D) None of these

253. Let $f(x)$ be any bounded real valued function in the interval $[a, b]$.

Consider the following statements:

A: $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} \{f(x) + f(-x)\}\,dx$

B: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

Then which of the following is appropriate?

(A) A and B both are true and they are interdependent

(B) A and B are true independently

(C) A is true and B is false always

(D) A is true and B is true in special case

254. For which value of n, $\int_0^{\frac{\pi}{2}} \frac{dx}{16\cos^2 x + 25\sin^2 x}$ becomes equal to $n \times \pi$.

(A) $\frac{1}{40}$ (B) $\frac{1}{50}$ (C) $\frac{1}{20}$ (D) $\frac{1}{30}$

255. Evaluate $\int_0^1 \int_1^2 \left(x^2 + y^2\right) dxdy$

(A) $\frac{-8}{3}$ (B) $\frac{8}{3}$ (C) 0 (D) 1

256. The value of $\int_{-4}^{7} |x|\,dx$ is

(a) 30.5

(b) 30

(c) 32.5

(d) 32

257. The value of $\int_{0}^{1.5} x[x^2]\,dx$, where $[x]$ is a step function, is

(a) $\frac{4}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

258. The value of $\int_{0}^{\pi} x \sin^{8}(x) \cos^{6}(x)\, dx$ is

(a) $\frac{\pi^2}{512}$

(b) $\frac{105\pi^2}{512}$

(c) $\frac{105\pi}{86016}$

(d) $\frac{5\pi^2}{4096}$

259. The value of $\int_{x=1}^{a} \int_{y=1}^{b} \frac{1}{xy} \, dx \, dy$ is ______.

(a) (log a) (log b)

(b) log (ab)

(c) log a – log b

(d) log (a + b)

260. $\int_{1}^{3}\int_{1}^{2} xy^2 \, dx \, dy =$

(a) 10 (b) 11

(c) 13 (d) 12

261. $\int_0^1 \int_0^2 \int_1^2 x^2 yz \, dx \, dy \, dz =$

(a) $-\frac{7}{3}$

(b) $\frac{7}{3}$

(c) $\frac{7}{2}$

(d) $-\frac{7}{2}$

262. The value of $\int_{x=0}^{1} \int_{y=0}^{2} xy \, dx \, dy$ is ________.

263. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right) dx =$ ______

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) none of these

264. $\int_{-\pi}^{\pi} \sin^4 x.dx =$

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$

(d) 0

265. $\int_{-1}^{2} \frac{|x|}{x}\, dx = ------$

266. $\int_{0}^{\pi} |\cos x| \, dx =$

267. $\int_0^n [x]\,dx =$ ____ , where [x] is a step functi

and 'n' is an integer.

(a) $\frac{n(n+1)}{2}$

(b) $\frac{n(n-1)}{2}$

(c) $\frac{n}{2}$

(d) $\frac{n+1}{2}$

268. $\int_{0}^{\frac{\pi}{2}} \log(\tan x)\,dx =$

(a) 0

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) π

269. Let $\frac{d}{dx}[F(x)] = \frac{e^{\sin x}}{x}, x > 0.$

If $\int_1^4 \left(\frac{2e^{\sin x^2}}{x}\right)dx = F(k) - F(1)$

then k = __________.

270. $\int_{0}^{\pi/4} \log(1+\tan x)\, dx =$

(a) 0

(b) $(\pi/2) \log 2$

(c) $(\pi/8) \log 2$

(d) $(-\pi/4) \log 2$

271. $\int_0^{\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0

(b) $3\pi/256$

(c) $3\pi/128$

(d) $5\pi/128$

272. $\int_{0}^{2\pi} \sin^4 x \cos^6 x \, dx =$

(a) $3\pi/128$

(b) $3\pi/256$

(c) $3\pi/64$

(d) 0

273. $\int_{0}^{2\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0 (b) $3\pi/128$

(c) $5\pi/128$ (d) $3\pi/256$

274. $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ **(GATE-EC-2000)**

(a) 0 (b) π (c) $\pi/2$ (d) 2

275. The value of the integral is $I = \int_0^{\pi/4} \cos^2 x \, dx$ (GATE-CE-2001)

(a) $\frac{\pi}{8}+\frac{1}{4}$ (b) $\frac{\pi}{8}-\frac{1}{4}$ (c) $\frac{-\pi}{8}-\frac{1}{4}$ (d) $\frac{-\pi}{8}+\frac{1}{4}$

276. The value of the integral $\int_{-\pi/2}^{\pi/2}(x \cos x)\,dx$ is **(GATE-PI-2008)**

(a) 0 (b) $\pi - 2$ (c) π (d) $\pi + 2$

277. The value of the following definite integral in $\int_{-\pi/2}^{\pi/2} \frac{\text{Sin}2x}{1+\cos x} dx$ = **(GATE-ME-2002)**

(a) -2log 2 (b) 2 (c) 0 (d) None

278. The value of the following improper integral is $\int_0^1 x \log x \, dx =$

(GATE-ME-2002)

(a) 1/4 (b) 0 (c) -1/4 (d) 1

279. $\int_{-a}^{a}\left[\sin^6 x+\sin^7 x\right]dx$ is equal to **(GATE-ME-2004)**

(a) $2\int_{0}^{a}\text{Sin}^6 x\,dx$ (b) $2\int_{0}^{a}\text{Sin}^7 x\,dx$ (c) $2\int_{0}^{a}\left(\text{Sin}^6 x+\text{Sin}^7 x\right)dx$ (d) zero

280. The value of $\int_0^3 \int_0^x (6-x-y)\, dx dy$ is ________ **(GATE-CS-2008)**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

281. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx\, dy\, dz$

(A) $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \frac{3}{8}$

(B) $\frac{e^{4a}}{4} - \frac{3e^{2a}}{4}$

(C) $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} - \frac{3}{8}$

(D) None

282. $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dx\, dy =$

(a) $\frac{2}{35}$

(b) $-\frac{3}{35}$

(c) $\frac{3}{35}$

(d) $-\frac{2}{35}$

283. $\int_0^1 \int_{4y}^4 e^{x^2}\, dx\, dy =$

(a) $\frac{\left(e^{16}-1\right)}{8}$

(b) $-\frac{\left(e^{16}+1\right)}{8}$

(c) 0

(d) $-\frac{\left(e^{16}-1\right)}{8}$

284. $$\int_0^4 \int_0^{x^2} e^{y/x}\, dy\, dx =$$

(a) $3e^4$

(b) $3e^4 + 7$

(c) $-3e^4$

(d) $3e^4 - 7$

286. $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz =$

(a) 1

(b) 2

(c) 3

(d) 0

287. The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to ________.

(GATE-16-EC)

288. $\int_{1/\pi}^{2/\pi} \frac{\cos(1/x)}{x^2} dx =$ ________ **(GATE-CS-2015)**

289. Given the following statements about a function $f: R \to R$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(GATE-16-EC)

(a) P is true, Q is false, R is false

(b) P is false, Q is true, R is true

(c) P is false, Q is true, R is false

(d) P is true, Q is false, R is true

290. The value of $\displaystyle\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r \sin\theta \, dr \, d\theta$ is ________.

(a) $\dfrac{a^2}{2}$ (b) $2a^2$

(c) $\dfrac{2a^2}{3}$ (d) $4a^2$

292. $\int_0^1 \int_0^{\sqrt{(1-x^2)}} y^2 \, dx dy =$

(a) $-\frac{\pi}{16}$ (b) $\frac{\pi}{16}$

(c) $\frac{\pi}{8}$ (d) $-\frac{\pi}{8}$

301. The value of $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$ is ______________

(A) $\frac{13}{9} - \frac{ln\,3}{6}$

(B) $\frac{7}{6} - \frac{ln\,3}{6}$

(C) $\frac{1}{6} - ln\,3$

(D) $\frac{3}{2} - ln\,3$

302. The value of $\int_0^1 \int_0^2 \int_1^2 x^2 yz\,dz\,dy\,dx$ is ___________

(A) 0 (B) 1

(C) 2 (D) 3

304. The value of $\int_{-1}^{2}\int_{x^2}^{x+2} dydx =$ ________

(A) $\frac{7}{2}$

(B) $\frac{9}{2}$

(C) $\frac{11}{2}$

(D) $\frac{5}{2}$

305. The value of $\int_0^1 \int_0^1 \frac{dydx}{\sqrt{1-x^2}\sqrt{1-y^2}} =$ __________

`(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{16}$

306. The value of $\int_0^{100\pi} |sin\ x| dx$ is ____________

(A) 100 (B) 100π

(C) 200π (D) 200

307. The value of integral $\int_{-1}^{1} ln\left(\frac{2-x\cos x}{2+x\cos x}\right) dx$ is ________

(A) $xln(2 + x\cos x)$

(B) $xln(2 - x\cos x)$

(C) $x\cos x$

(D) 0

308. If $f(x) = \int_{x}^{0} \sin t^2 dt$ then $f'(x)$ is ________

(A) $2x\sin x^2$ (B) $-\sin x^2$

(C) $2x\cos x^2$ (D) $\cos x^2$

Q. Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\,dx\,dy$

Q. The value of $\int_0^3 \int_0^x (6 - x - y)\, dx dy$ is ________ **(GATE-CS-2008)**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

Q. The value of the double integral $\int_0^1 \int_x^{1/x} \frac{x}{1+y^2}\, dx\, dy =$ ______ **(GATE-EC-1993)**

Change of order of integration

Q. Evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} \, dy \, dx$

Q. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dy\, dx$

Q. By reversing the order of integration $\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx$ may be represented as

(a) $\int_0^2 \int_{x^2}^{2x} f(x,y)\,dy\,dx$

(b) $\int_0^2 \int_y^{\sqrt{y}} f(x,y)\,dx\,dy$

(GATE-EC-1995)

(c) $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy$

(d) $\int_{x^2}^{2x} \int_0^2 f(x,y)\,dy\,dx$

Q. Changing the order of integration in double integral $I = \int_0^8 \int_{x/4}^2 f(x, y)\,dy\,dx$ leads to $I = \int_r^s \int_p^q f(x, y)\,dx\,dy$. What is q?

(GATE-EC-2005)

(a) $4y$

(b) $16y^2$

(c) x

(d) 8

Triple integrals

Q. Evaluate $\int\int\int_R (x + y + z)\, dx\, dy\, dz$ where $R: 0 \leq x \leq 1,\ \ 1 \leq y \leq 2\ ,\ \ 2 \leq z \leq 3$

Q. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz$

Change of variables

Q. To evaluate the double integral $\int_0^8\left(\int_{y/2}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$, we make the substitution $u=\left(\frac{2x-y}{2}\right)$ and $v=\frac{y}{2}$. The integral will reduce to **(GATE-EE-SET-2-2014)**

(a) $\int_0^4\left(\int_0^2 2u\,du\right)dv$ (b) $\int_0^4\left(\int_0^1 2u\,du\right)dv$ (c) $\int_0^4\left(\int_0^1 u\,du\right)dv$ (d) $\int_0^4\left(\int_0^{21} 2u\,du\right)dv$

Q. By a change of variables $x(u, v) = uv$, $y(u,v) = v/u$ in a double integral, the integral $f(x,y)$ changes to $f\left(uv, {}^{u}\!/_{v}\right)\phi(u,v)$ Then $\phi(u,v)$ is ________ **(GATE-EE-2005)**

(a) $\dfrac{2v}{u}$ (b) $2uv$ (c) v^2 (d) 1

Area bounded by the curves

326. The area of the region enclosed by the curve $y = x^2$ and the straight-line $x + y = 2$ is

(A) 3

(B) 27/2

(C) 9/2

(D) 9

327. The area of the region bounded by the curve $x^2 = 2y$ and $y^2 = 2x$ is

(A) 1/3
(B) 2/3
(C) 4/3
(D) 4

328. Area enclosed by the curves $y^2 = x$ and $y^2 = 2x-1$ lying in the first quadrant is

(A) 1/6 (B) 1/4

(C)1/2 (D) 1/3

329. The value of $\int\int xy(x+y)dxdy$ over the area between y = x^2 and y = x

(A) 1/90 (B) 1/45

(C) 3/56 (D) 1/15

330. The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to

(a) 6 (b) 18 (c) ∞ (d) None **(GATE-ME-1995)**

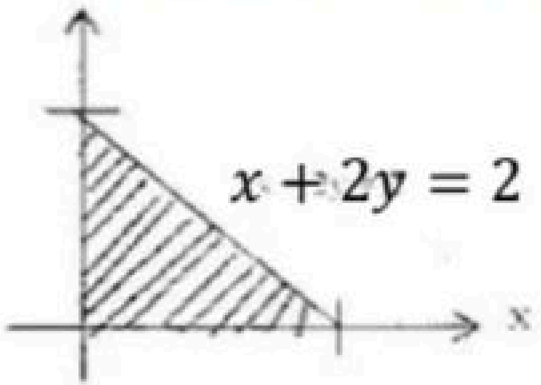
331. Area bounded by the curve $y = x^2$ and the lines $x = 4$ and $y = 0$ is given by

(a) 64 (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{128}{4}$ **(GATE-EE-1997)**

332. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is

(a) 1/8 (b) 1/6 (c) 1/3 (d) 1/2

333. Consider the shaded triangular region P shown in the figure. What is $\iint_P xy \, dx \, dy$?



(GATE-ME-2008)

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{7}{16}$ (d) 1

336. The volume under the surface $z(x, y) = x + y$ and above the triangle in the xy plane defined by $\{0 \le y \le x \text{ and } 0 \le x \le 12\}$ is ______. **(GATE-EC-SET-1-2014)**

337. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y-axis is **(GATE-EE-1994)**

(a) $\frac{128\pi}{5}$ (b) $\frac{5}{128\pi}$ (c) $\frac{127}{5\pi}$ (d) None of the above

338. The volume of an object expressed in spherical co-ordinates is given by

$$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} r^2 \, \text{Sin}\phi \; dr d\phi \; d\theta.$$ The value of the integral **(GATE-EE-2004)**

(a) $\pi/3$ (b) $\pi/6$ (c) $2\pi/3$ (d) $\pi/4$

339. The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the line $x = y$, $x = 0$, $y = 1$ in the xy plane is ______ (GATE-EE-2015)

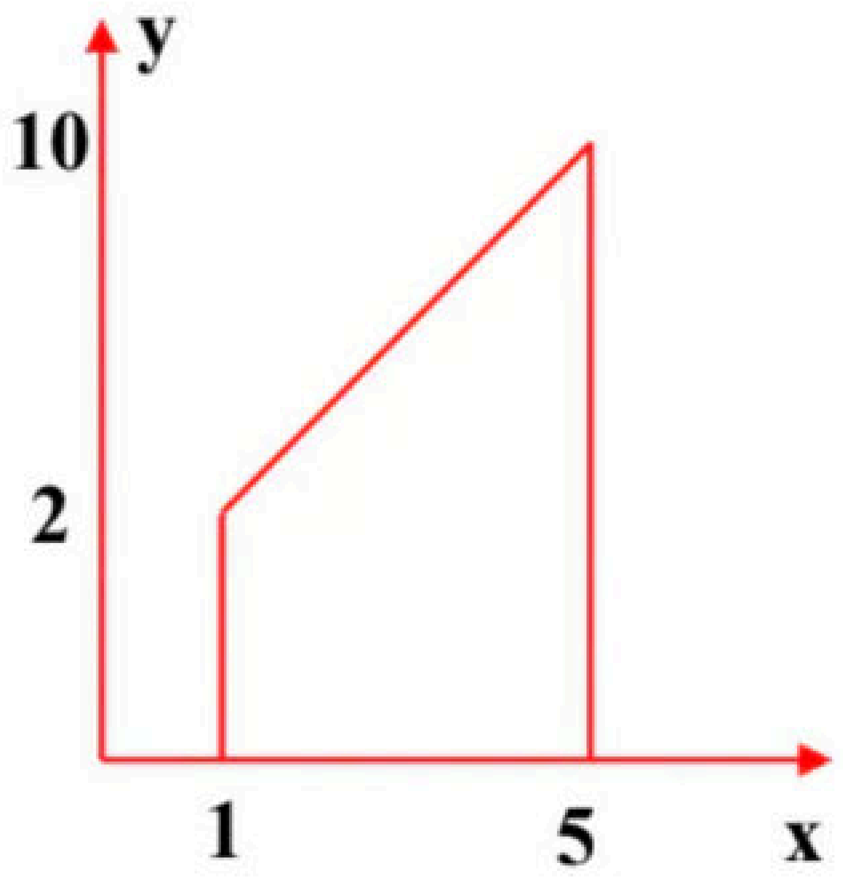
342. A triangle in the x-y plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is ________.

(GATE-16-EC)

343. Let $I = c \iint_R xy^2 dxdy$, where R is the region shown in the figure and $C = 6 \times 10^{-4}$. The value of I equals __________.

(Give the answer up to two decimal places)

(GATE-17-EE)



349. The value the of integral $\iint_D 3(x^2 + y^2)dxdy$

where D is the shaded triangular region shown in the diagram is ________________(rounded off nearest integer).

(GATE-2022-ECE)