Operations on AVL Tree - Part II

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Vishvadeep Gothi • Lesson 57 • Feb 7, 2023

Data Structure
Heap 2

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Heap

A specialized tree-based data structure which is essentially an almost complete tree that satisfies the heap property.

Heap

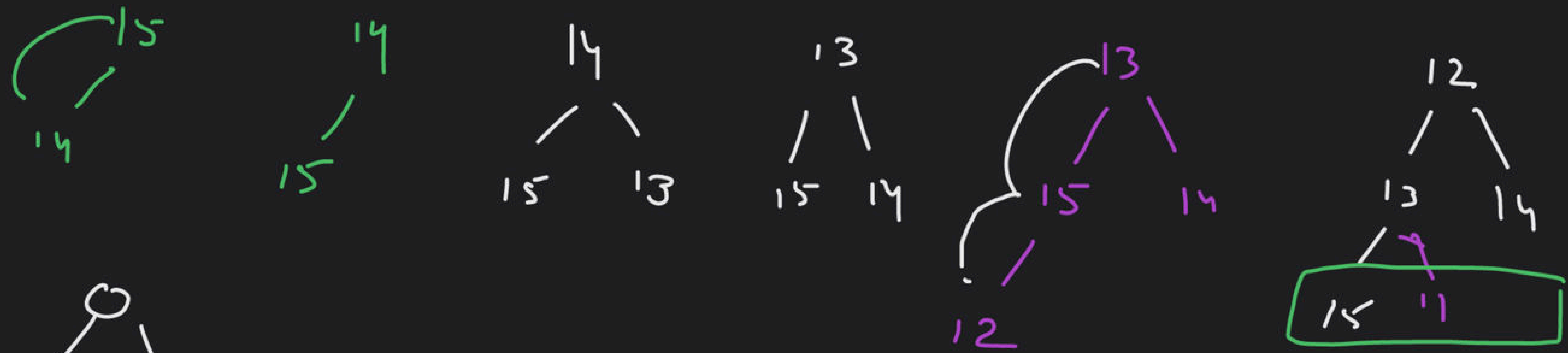
2 Properties to be satisfied:

1. It should be a complete binary tree
2. Max heap or min heap property

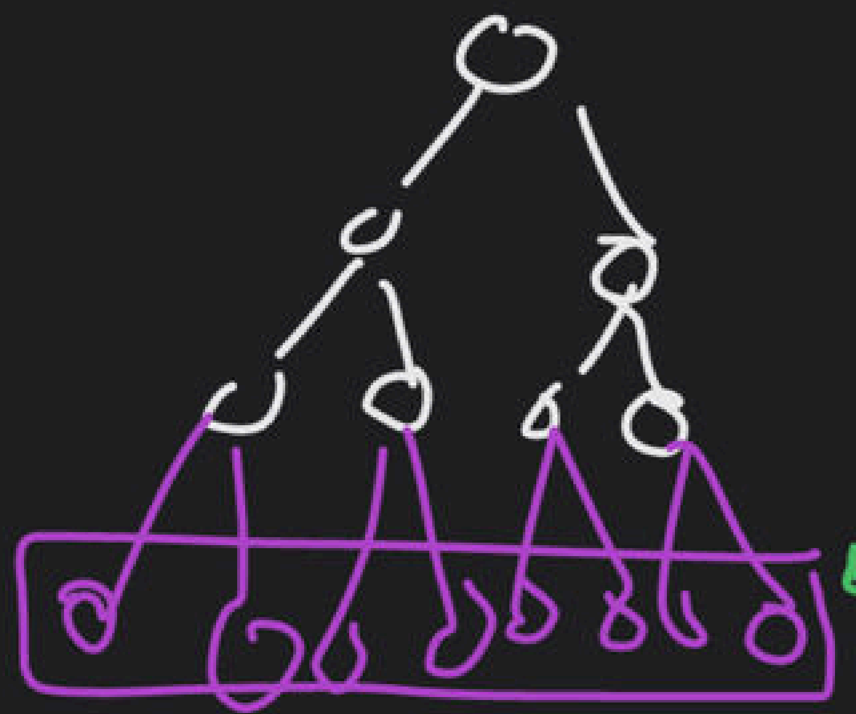
15, 14, 13, ..., 1

Inserted one-by-one in min-heap. no. of interchanges needed?

sol

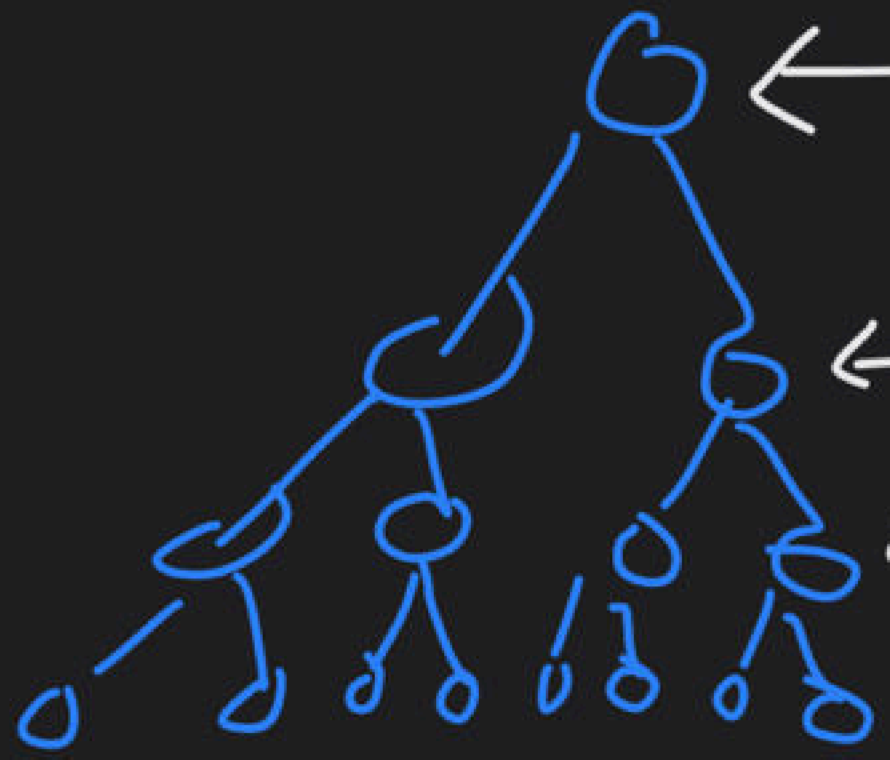


2 inters



3 interchanges

for 15 elements



0

1 => 2 * 1 = 2

2 => 4 * 2 = 8

3 => 8 * 3 = 24

34 interchanges

if in an empty min-heap, following elements inserted one-by-one. No of interchanges needed ________ ?

$n, n-1, n-2, \ldots\ldots, \underline{1}$

Ans:- we have complete BT (which occupies last level completely)

$$\sum_{L=1}^{\log_2 n} 2^L * L$$

if last level is not full

$$\sum_{L=1}^{\lfloor \log_2 n \rfloor - 1} \left(2^L * L \right)$$

$$+ (n - \log n + 1) \log n$$

L-1 level

(log n - 1)

nodes

$n - (\log n - 1) \Leftarrow$ nodes on last level

$(n - \log n + 1) * \log n$

exchanges for last level nodes

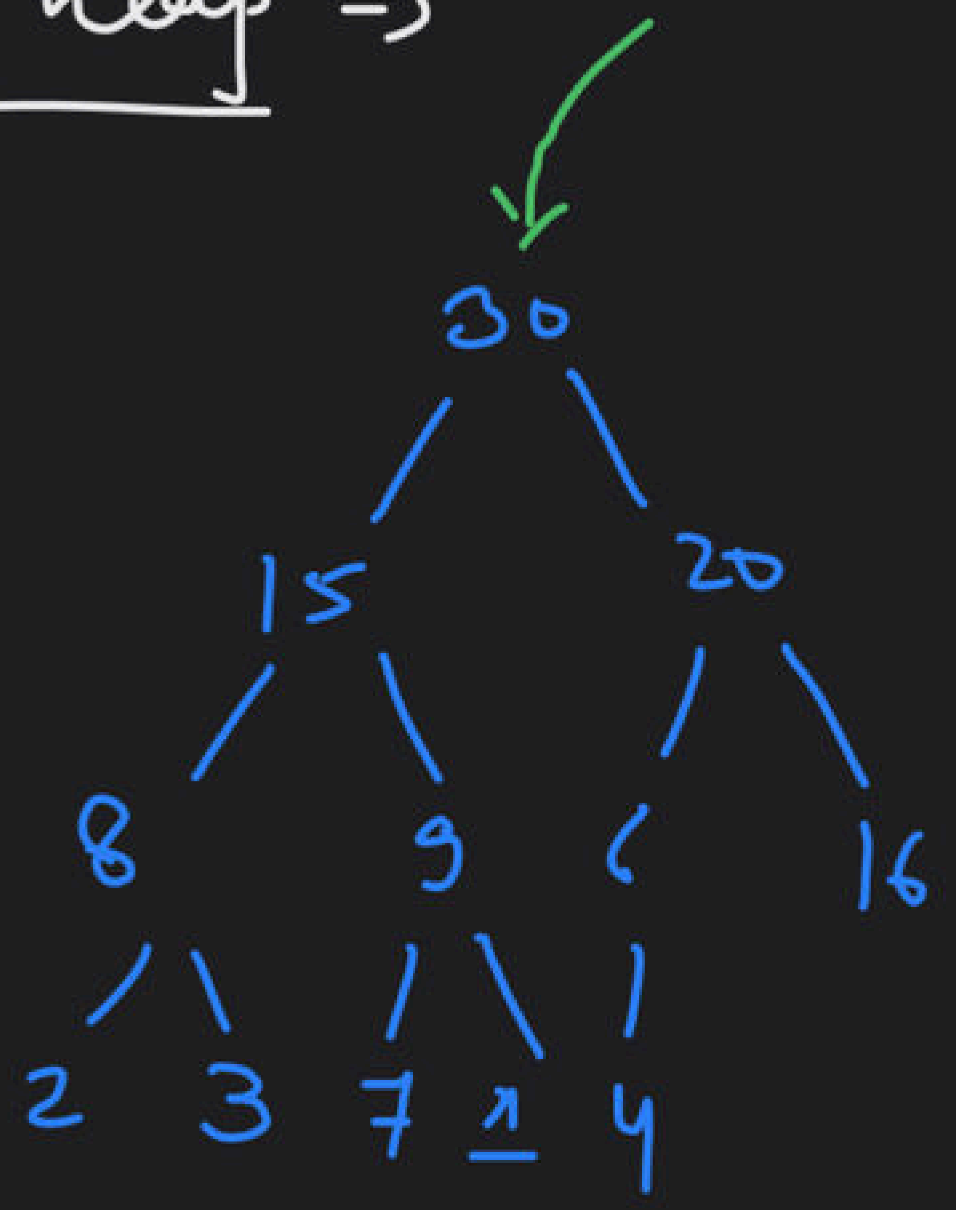
Ques) Consider a complete BT with n nodes. max level no. ________ in this tree?

Ans)

$$\lfloor \log_2 n \rfloor$$

n	1	2	3	4 - 7	8 - 15	16 - 31
L_{max}	0	1	1	2	3	4

Max heap $\Rightarrow$



Root $\Rightarrow$ max

R.T.C. to find max $\Rightarrow \Theta(1)$

R.T.C. to find min $\Rightarrow \Theta(n)$

Min heap :-

Root $\Rightarrow$ min element of heap

R.T.C to find min. $\Rightarrow \Theta(1)$

— " — max $\Rightarrow \Theta(n)$

Searching in heap :-

Through traversal

$\Downarrow$

R.T. complexity $\Rightarrow O(n)$

Ques) Consider a min-heap with n-number of integers. The run-time complexity of inserting a new integer in this heap is ________ ?

note :- Insert the element only if it is already not present in heap.

Ans:-
(A) $\Theta(1)$
(B) $O(\log n)$
✓ (C) $O(n)$
(D) $\Theta(n \log n)$

Sol^n:-
① Find element in heap $\Rightarrow O(n)$
→ insert if search unsuccessful
→ else return

Ques) Consider a max-heap with n distinct integers. The run time complexity to find second maximum element of heap is ______ ?

Ans:- $\Theta(1)$

it should be $\Rightarrow$ max(root $\rightarrow$ Lc $\rightarrow$ data, Root $\rightarrow$ RC $\rightarrow$ data)

Deletion In Heap

→ Deletⁿ always done, of root element

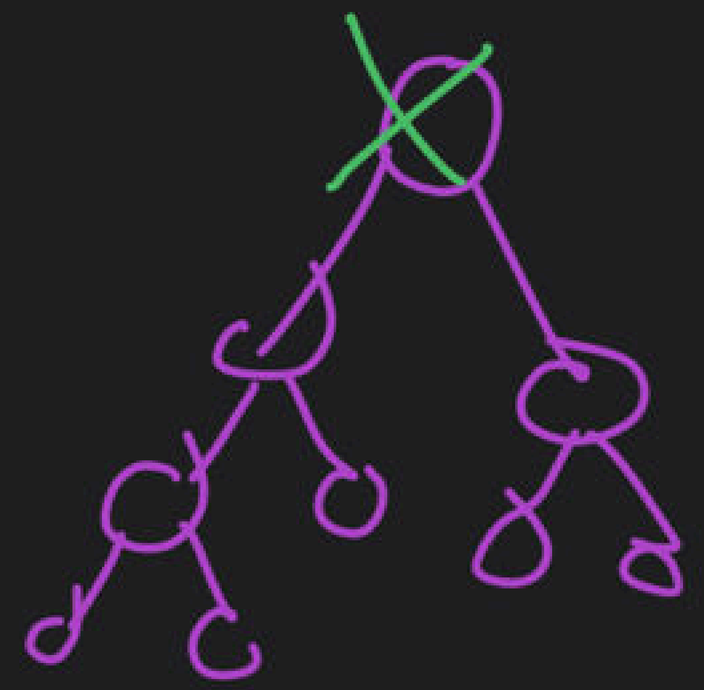
	Deletⁿ
min heap	min. element
max heap	max. element

Deletion :-

1. Delete root element

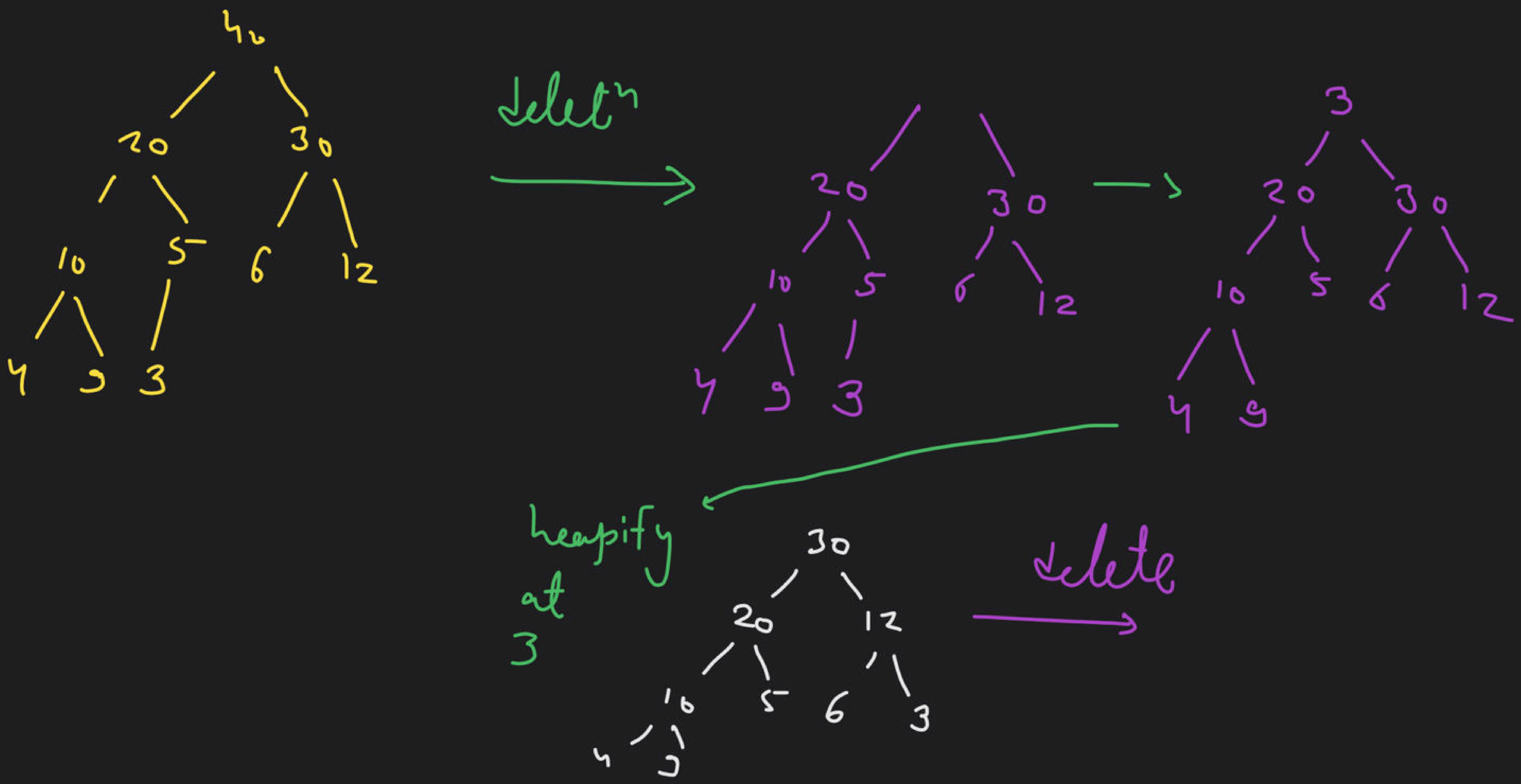
2. Remove last node of complete BT
   and place at at root.

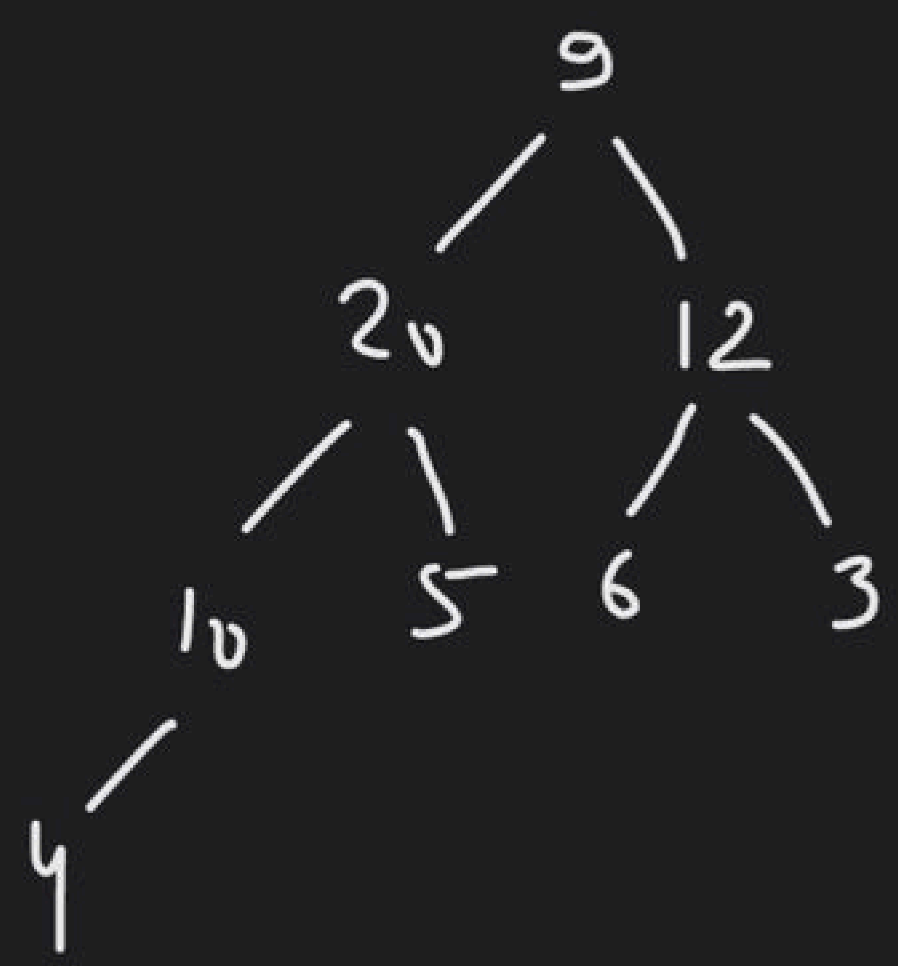
3. Heapify at root.



R. T. complexity $\Rightarrow O(\log n)$

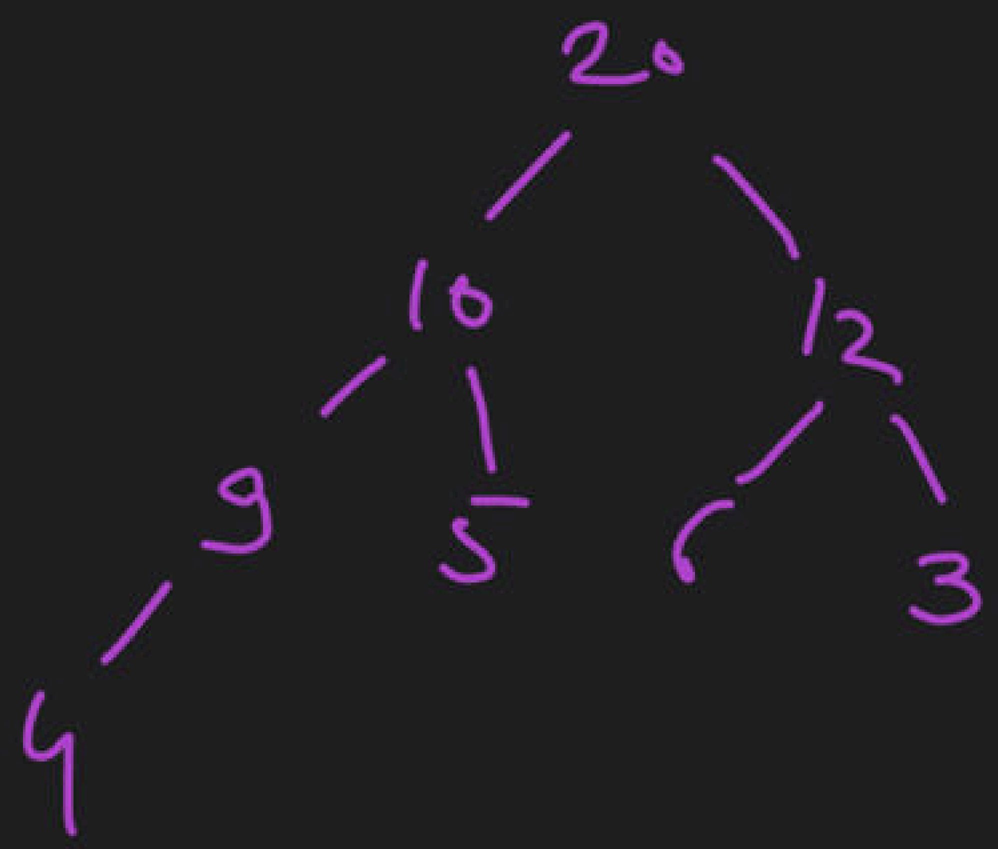
ex:- max heap





heapify

at 9

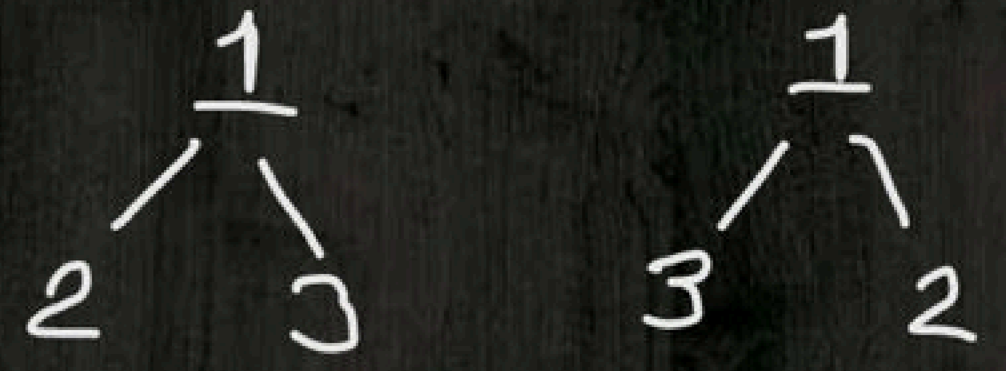


Number of Heaps Using N keys

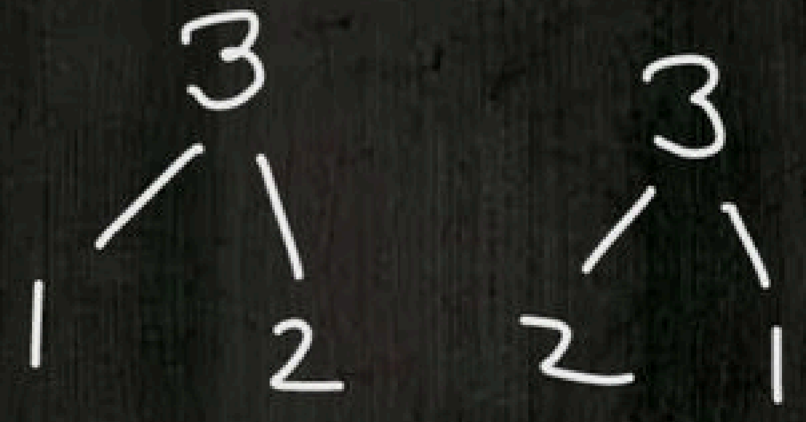
$n = 3$

no. of min heaps using 3 distinct keys

keys:- 1, 2, 3

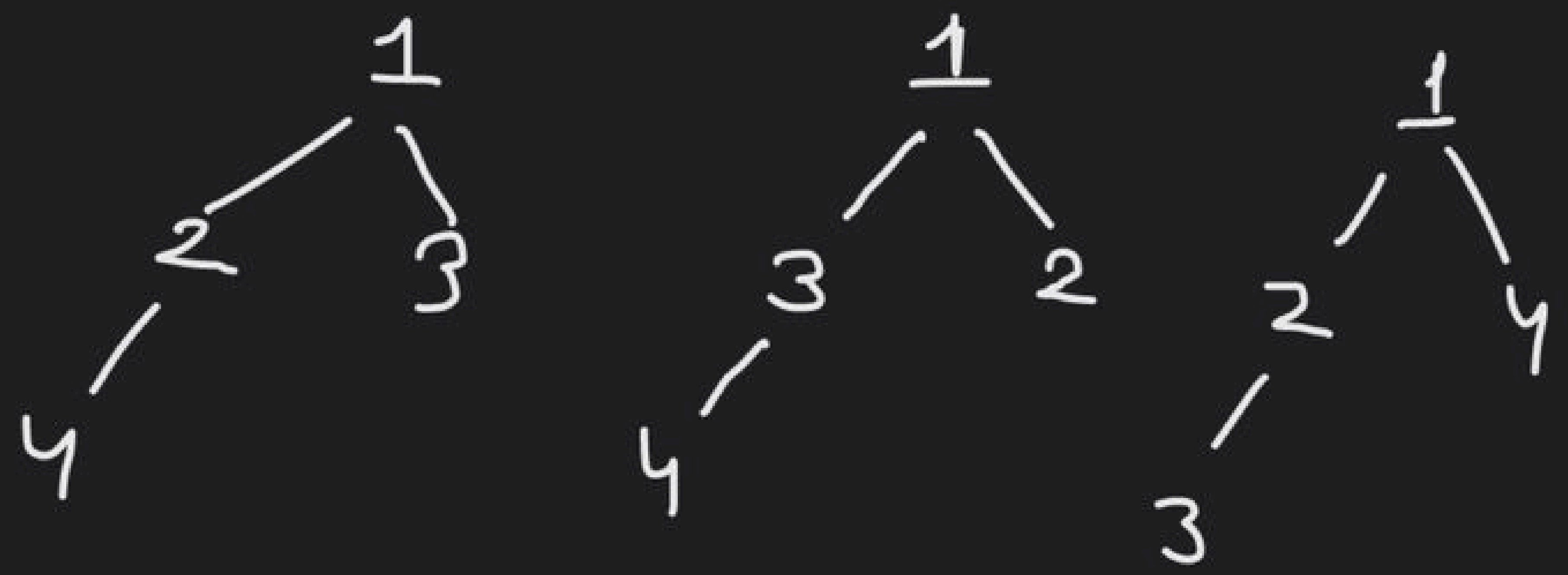


max heaps



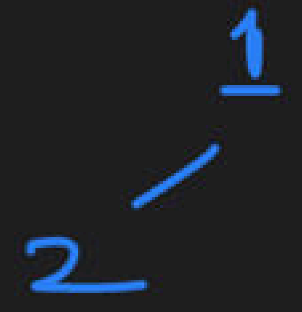
no. of min heap with 4 distinct keys

keys:- 1, 2, 3, 4



$n = 2$

keys = 1, 2



max heap

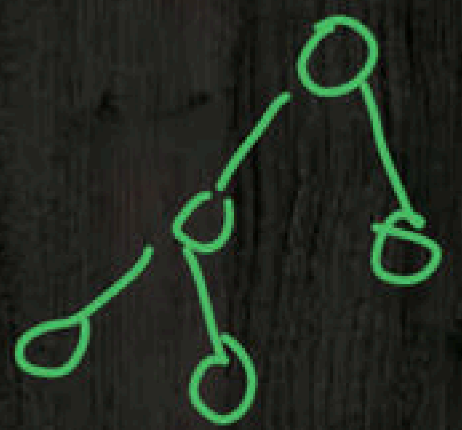
Number of Heaps Using N keys

$$T(N) = (N-1)Ck * T(k) * T(N-k-1)$$

Where k = number of nodes on left subtree of root

$N = 5$, $k = 3$

CBT with 5 nodes



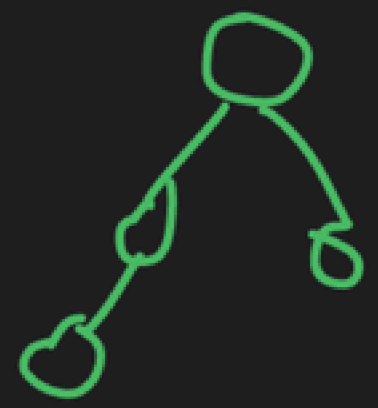
$$T(5) = {}^{4}C_{3} * T(3) * T(1)$$

$$= 4 * 2 * 1 = 8$$

N	1	2	3	4	5	6	7
T(N)	1	1	2	3	8		

$n = 4$

$k = 2$



$$T(4) = {}^{3}C_{2} * T(2) * T(1)$$

$$= 3 * 1 * 1$$

$$= 3$$

GATE — 2018

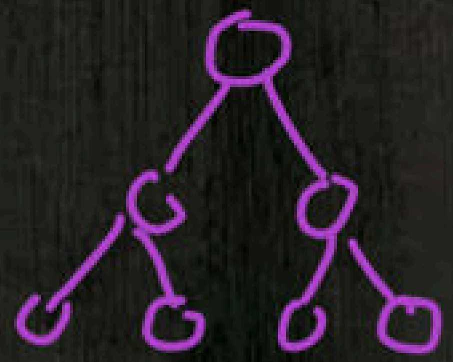
Question

a, - - - - - 97

The number of possible min-heaps containing each value from {1,2,3,4,5,6,7} exactly once is ?

$n = 7$

$k = 3$



$$T(7) = {}^{6}C_{3} * T(3) * T(3)$$

$$= \frac{6 * 5 * 4}{3 * 2 * 1} * 2 * 2$$

$$= 80$$

Ans

BST PYQs

Question GATE-1996

A binary search tree is generated by inserting in order the following integers:

$$50, 15, 62, 5, 20, 58, 91, 3, 8, 37, 60, 24$$

The number of nodes in the left subtree and right subtree of the root respectively is

A. $(4,7)$ B. $(7,4)$ C. $(8,3)$ D. $(3,8)$

Question GATE-1996

A binary search tree is used to locate the number 43. Which of the following probe sequences are possible and which are not? Explain.

(a)	61	52	14	17	40	43
(b)	2	3	50	40	60	43
(c)	10	65	31	48	37	43
(d)	81	61	52	14	41	43
(e)	17	77	27	66	18	43

Question GATE-2001

A. Insert the following keys one by one into a binary search tree in the order specified.

$$15, 32, 20, 9, 3, 25, 12, 1$$

Show the final binary search tree after the insertions.

B. Draw the binary search tree after deleting 15 from it.

Question GATE-2003

A data structure is required for storing a set of integers such that each of the following operations can be done in $O(\log n)$ time, where n is the number of elements in the set.

I. Deletion of the smallest element

II. Insertion of an element if it is not already present in the set

Which of the following data structures can be used for this purpose?

A. A heap can be used but not a balanced binary search tree
B. A balanced binary search tree can be used but not a heap
C. Both balanced binary search tree and heap can be used
D. Neither balanced search tree nor heap can be used

Question GATE-2003

Let $T(n)$ be the number of different binary search trees on n distinct elements.

Then $T(n) = \sum_{k=1}^{n} T(k-1)T(x)$, where x is

A. $n-k+1$ B. $n-k$ C. $n-k-1$ D. $n-k-2$

Question GATE-2004

The following numbers are inserted into an empty binary search tree in the given order: $10, 1, 3, 5, 15, 12, 16$. What is the height of the binary search tree (the height is the maximum distance of a leaf node from the root)?

A. 2 B. 3 C. 4 D. 6

Question GATE-2004

A program takes as input a balanced binary search tree with n leaf nodes and computes the value of a function $g(x)$ for each node x. If the cost of computing $g(x)$ is:

$$\min\left(\begin{matrix}\text{number of leaf-nodes}\\ \text{in left-subtree of } x\end{matrix}, \begin{matrix}\text{number of leaf-nodes}\\ \text{in right-subtree of } x\end{matrix}\right)$$

Then the worst-case time complexity of the program is?

A. $\Theta(n)$

B. $\Theta(n \log n)$

C. $\Theta(n^2)$

D. $\Theta(n^2 \log n)$

Question GATE-2005

The numbers $1, 2, \ldots. n$ are inserted in a binary search tree in some order. In the resulting tree, the right subtree of the root contains p nodes. The first number to be inserted in the tree must be

A. p B. $p+1$ C. $n-p$ D. $n-p+1$

Question GATE-2005

A binary search tree contains the numbers $1, 2, 3, 4, 5, 6, 7, 8$. When the tree is traversed in pre-order and the values in each node printed out, the sequence of values obtained is $5, 3, 1, 2, 4, 6, 8, 7$. If the tree is traversed in post-order, the sequence obtained would be

A. $8, 7, 6, 5, 4, 3, 2, 1$

B. $1, 2, 3, 4, 8, 7, 6, 5$

C. $2, 1, 4, 3, 6, 7, 8, 5$

D. $2, 1, 4, 3, 7, 8, 6, 5$

Question GATE-2006

Suppose that we have numbers between 1 and 100 in a binary search tree and want to search for the number 55. Which of the following sequences CANNOT be the sequence of nodes examined?

A. $\{10, 75, 64, 43, 60, 57, 55\}$

B. $\{90, 12, 68, 34, 62, 45, 55\}$

C. $\{9, 85, 47, 68, 43, 57, 55\}$

D. $\{79, 14, 72, 56, 16, 53, 55\}$

Question GATE-2007

When searching for the key value 60 in a binary search tree, nodes containing the key values 10, 20, 40, 50, 70, 80, 90 are traversed, not necessarily in the order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?

A. 35 B. 64 C. 128 D. 5040

Question GATE-2008

A Binary Search Tree (BST) stores values in the range 37 to 573. Consider the following sequence of keys.

I. $81, 537, 102, 439, 285, 376, 305$
II. $52, 97, 121, 195, 242, 381, 472$
III. $142, 248, 520, 386, 345, 270, 307$
IV. $550, 149, 507, 395, 463, 402, 270$

Suppose the BST has been unsuccessfully searched for key 273. Which all of the above sequences list nodes in the order in which we could have encountered them in the search?

A. II and III only B. I and III only C. III and IV only D. III only

Question GATE-2008

How many distinct BSTs can be constructed with 3 distinct keys?

A. 4 B. 5 C. 6 D. 9

Question GATE-2008

Which of the following is TRUE?

A. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a binary search tree is $O(n)$
B. The cost of searching an AVL tree is $\Theta(\log n)$ but that of a complete binary tree is $\Theta(n \log n)$
C. The cost of searching a binary search tree is $O(\log n)$ but that of an AVL tree is $\Theta(n)$
D. The cost of searching an AVL tree is $\Theta(n \log n)$ but that of a binary search tree is $O(n)$

Question GATE-2008

You are given the postorder traversal, P, of a binary search tree on the n elements $1, 2, \ldots, n$. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

D. None of the above, as the tree cannot be uniquely determined

Question GATE-2009

Suppose the numbers $7, 5, 1, 8, 3, 6, 0, 9, 4, 2$ are inserted in that order into an initially empty binary search tree. The binary search tree uses the usual ordering on natural numbers. What is the in-order traversal sequence of the resultant tree?

A. $7\ 5\ 1\ 0\ 3\ 2\ 4\ 6\ 8\ 9$

B. $0\ 2\ 4\ 3\ 1\ 6\ 5\ 9\ 8\ 7$

C. $0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9$

D. $9\ 8\ 6\ 4\ 2\ 3\ 0\ 1\ 5\ 7$

Question GATE-2012

The worst case running time to search for an element in a balanced binary search tree with $n2^n$ elements is

A. $\Theta(n \log n)$

B. $\Theta(n2^n)$

C. $\Theta(n)$

D. $\Theta(\log n)$

Question GATE-2013

The preorder traversal sequence of a binary search tree is $30, 20, 10, 15, 25, 23, 39, 35, 42$. Which one of the following is the postorder traversal sequence of the same tree?

A. 10, 20, 15, 23, 25, 35, 42, 39, 30
B. 15, 10, 25, 23, 20, 42, 35, 39, 30
C. 15, 20, 10, 23, 25, 42, 35, 39, 30
D. 15, 10, 23, 25, 20, 35, 42, 39, 30

Question GATE-2013

Which one of the following is the tightest upper bound that represents the time complexity of inserting an object into a binary search tree of n nodes?

A. $O(1)$ B. $O(\log n)$ C. $O(n)$ D. $O(n \log n)$

Question GATE-2014

Suppose we have a balanced binary search tree T holding n numbers. We are given two numbers L and H and wish to sum up all the numbers in T that lie between L and H. Suppose there are m such numbers in T. If the tightest upper bound on the time to compute the sum is $O(n^a \log^b n + m^c \log^d n)$, the value of $a + 10b + 100c + 1000d$ is ______.

Question GATE-2015

While inserting the elements 71, 65, 84, 69, 67, 83 in an empty binary search tree (BST) in the sequence shown, the element in the lowest level is

(A) 65

(B) 67

(C) 69

(D) 83

Question GATE-2015

Which of the following is/are correct in order traversal sequence(s) of binary search tree(s)?

I. 3, 5, 7, 8, 15, 19, 25
II. 5, 8, 9, 12, 10, 15, 25
III. 2, 7, 10, 8, 14, 16, 20
IV. 4, 6, 7, 9, 18, 20, 25

A. I and IV only B. II and III only C. II and IV only D. II only

Question GATE-2015

What are the worst-case complexities of insertion and deletion of a key in a binary search tree?

A. $\Theta(\log n)$ for both insertion and deletion
B. $\Theta(n)$ for both insertion and deletion
C. $\Theta(n)$ for insertion and $\Theta(\log n)$ for deletion
D. $\Theta(\log n)$ for insertion and $\Theta(n)$ for deletion

Question GATE-2017

Let T be a binary search tree with 15 nodes. The minimum and maximum possible heights of T are:

Note: The height of a tree with a single node is 0.

(A) 4 and 15 respectively

(B) 3 and 14 respectively

(C) 4 and 14 respectively

(D) 3 and 15 respectively

Question GATE-2017

The pre-order traversal of a binary search tree is given by 12, 8, 6, 2, 7, 9, 10, 16, 15, 19, 17, 20.

Then the post-order traversal of this tree is:

(A) 2, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 20

(B) 2, 7, 6, 10, 9, 8, 15, 17, 20, 19, 16, 12

(C) 7, 2, 6, 8, 9, 10, 20, 17, 19, 15, 16, 12

(D) 7, 6, 2, 10, 9, 8, 15, 16, 17, 20, 19, 12

Question GATE-2020

The preorder traversal of a binary search tree is $15, 10, 12, 11, 20, 18, 16, 19$.

Which one of the following is the postorder traversal of the tree?

A. $10, 11, 12, 15, 16, 18, 19, 20$

B. $11, 12, 10, 16, 19, 18, 20, 15$

C. $20, 19, 18, 16, 15, 12, 11, 10$

D. $19, 16, 18, 20, 11, 12, 10, 15$

Question GATE-2020

In a balanced binary search tree with n elements, what is the worst case time complexity of reporting all elements in range $[a, b]$? Assume that the number of reported elements is k.

A. $\Theta(\log n)$
B. $\Theta(\log n + k)$
C. $\Theta(k \log n)$
D. $\Theta(n \log k)$

Question GATE-2021

A binary search tree T contains n distinct elements. What is the time complexity of picking an element in T that is smaller than the maximum element in T?

A. $\Theta(n \log n)$
B. $\Theta(n)$
C. $\Theta(\log n)$
D. $\Theta(1)$

Question GATE-2022

Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3^{rd} largest element of the tree is stored at index______________.

Heap PYQs

Question GATE-1996

The minimum number of interchanges needed to convert the array into a max-heap is

$89, 19, 40, 17, 12, 10, 2, 5, 7, 11, 6, 9, 70$

A. 0 B. 1 C. 2 D. 3

Question GATE-1999

Draw the min-heap that results from insertion of the following elements in order into an initially empty min-heap: $7, 6, 5, 4, 3, 2, 1$. Show the result after the deletion of the root of this heap.

Question GATE-2001

Consider any array representation of an n element binary heap where the elements are stored from index 1 to index n of the array. For the element stored at index i of the array $(i \leq n)$, the index of the parent is

A. $i-1$ B. $\lfloor \frac{i}{2} \rfloor$ C. $\lceil \frac{i}{2} \rceil$ D. $\frac{(i+1)}{2}$

Question GATE-2003

In a min-heap with n elements with the smallest element at the root, the 7^{th} smallest element can be found in time

A. $\Theta(n \log n)$

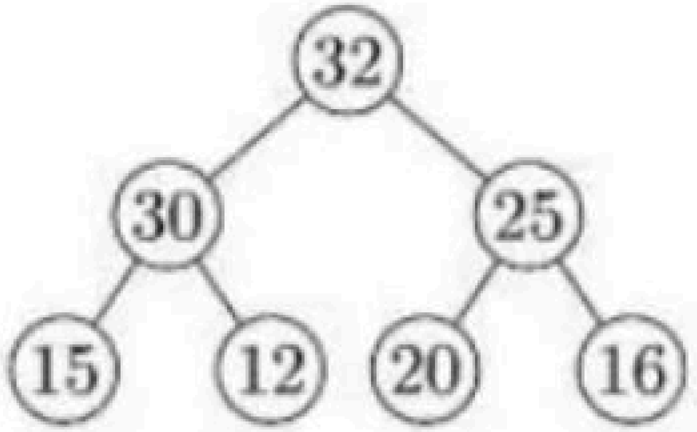
B. $\Theta(n)$

C. $\Theta(\log n)$

D. $\Theta(1)$

Question GATE-2004

The elements $32, 15, 20, 30, 12, 25, 16,$ are inserted one by one in the given order into a maxHeap. The resultant maxHeap is

A.


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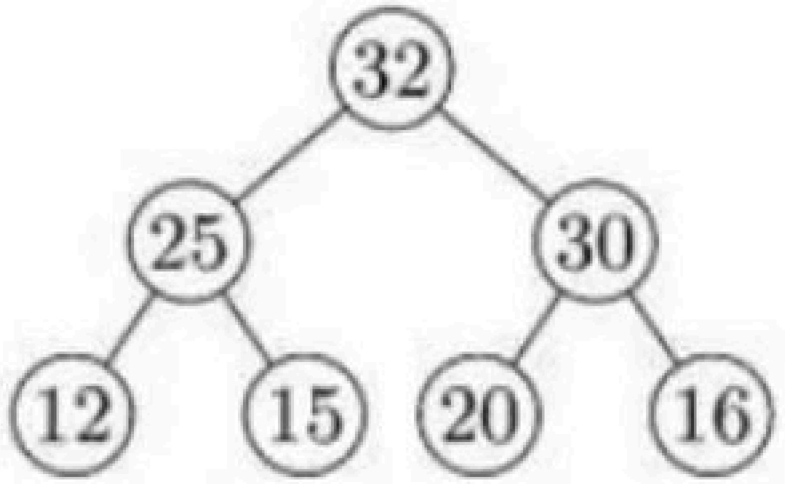
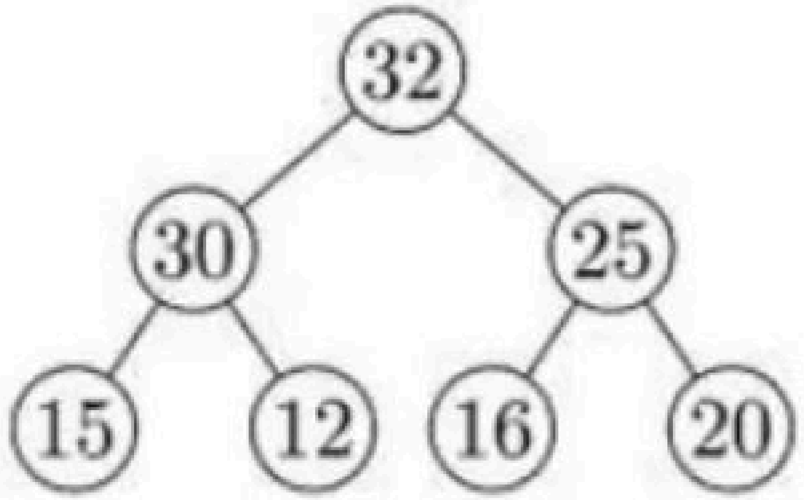
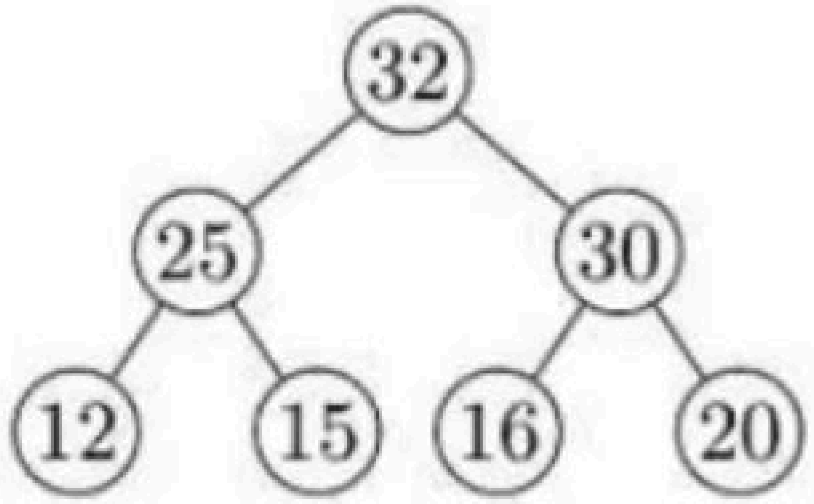


C.


D.


Question GATE-2004

An array of integers of size n can be converted into a heap by adjusting the heaps rooted at each internal node of the complete binary tree starting at the node $\lfloor (n-1)/2 \rfloor$, and doing this adjustment up to the root node (root node is at index 0) in the order $\lfloor (n-1)/2 \rfloor$, $\lfloor (n-3)/2 \rfloor$, ..., 0. The time required to construct a heap in this manner is

A. $O(\log n)$ B. $O(n)$ C. $O(n \log \log n)$ D. $O(n \log n)$

Question GATE-2005

A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is: $10, 8, 5, 3, 2$. Two new elements 1 and 7 are inserted into the heap in that order. The level-order traversal of the heap after the insertion of the elements is:

A. $10, 8, 7, 5, 3, 2, 1$

B. $10, 8, 7, 2, 3, 1, 5$

C. $10, 8, 7, 1, 2, 3, 5$

D. $10, 8, 7, 3, 2, 1, 5$

Question GATE-2006

In a binary max heap containing n numbers, the smallest element can be found in time

A. $O(n)$
B. $O(\log n)$
C. $O(\log \log n)$
D. $O(1)$

Question GATE-2006

A 3-ary max heap is like a binary max heap, but instead of 2 children, nodes have 3 children. A 3-ary heap can be represented by an array as follows: The root is stored in the first location, $a[0]$, nodes in the next level, from left to right, is stored from $a[1]$ to $a[3]$. The nodes from the second level of the tree from left to right are stored from $a[4]$ location onward. An item x can be inserted into a 3-ary heap containing n items by placing x in the location $a[n]$ and pushing it up the tree to satisfy the heap property.

Which one of the following is a valid sequence of elements in an array representing 3-ary max heap?

A. $1, 3, 5, 6, 8, 9$
B. $9, 6, 3, 1, 8, 5$
C. $9, 3, 6, 8, 5, 1$
D. $9, 5, 6, 8, 3, 1$

Suppose the elements $7, 2, 10$ and 4 are inserted, in that order, into the valid 3-ary max heap found in the previous question, Q.76. Which one of the following is the sequence of items in the array representing the resultant heap?

A. $10, 7, 9, 8, 3, 1, 5, 2, 6, 4$
B. $10, 9, 8, 7, 6, 5, 4, 3, 2, 1$
C. $10, 9, 4, 5, 7, 6, 8, 2, 1, 3$
D. $10, 8, 6, 9, 7, 2, 3, 4, 1, 5$

Question GATE-2006

Which of the following sequences of array elements forms a heap?

A. $\{23, 17, 14, 6, 13, 10, 1, 12, 7, 5\}$

B. $\{23, 17, 14, 6, 13, 10, 1, 5, 7, 12\}$

C. $\{23, 17, 14, 7, 13, 10, 1, 5, 6, 12\}$

D. $\{23, 17, 14, 7, 13, 10, 1, 12, 5, 7\}$

Question GATE-2006

An array X of n distinct integers is interpreted as a complete binary tree. The index of the first element of the array is 0. If only the root node does not satisfy the heap property, the algorithm to convert the complete binary tree into a heap has the best asymptotic time complexity of

A. $O(n)$ B. $O(\log n)$ C. $O(n \log n)$ D. $O(n \log \log n)$

Question GATE-2007

Consider the process of inserting an element into a *Max Heap*, where the *Max Heap* is represented by an *array*. Suppose we perform a binary search on the path from the new leaf to the root to find the position for the newly inserted element, the number of *comparisons* performed is:

A. $\Theta(\log_2 n)$

B. $\Theta(\log_2 \log_2 n)$

C. $\Theta(n)$

D. $\Theta(n \log_2 n)$

Question GATE-2009

Consider a binary max-heap implemented using an array.
Which one of the following array represents a binary max-heap?

A. $\{25, 12, 16, 13, 10, 8, 14\}$

B. $\{25, 14, 13, 16, 10, 8, 12\}$

C. $\{25, 14, 16, 13, 10, 8, 12\}$

D. $\{25, 14, 12, 13, 10, 8, 16\}$

Question GATE-2009

Consider a binary max-heap implemented using an array.
What is the content of the array after two delete operations on $\{25, 14, 16, 13, 10, 8, 12\}$

A. $\{14, 13, 12, 10, 8\}$

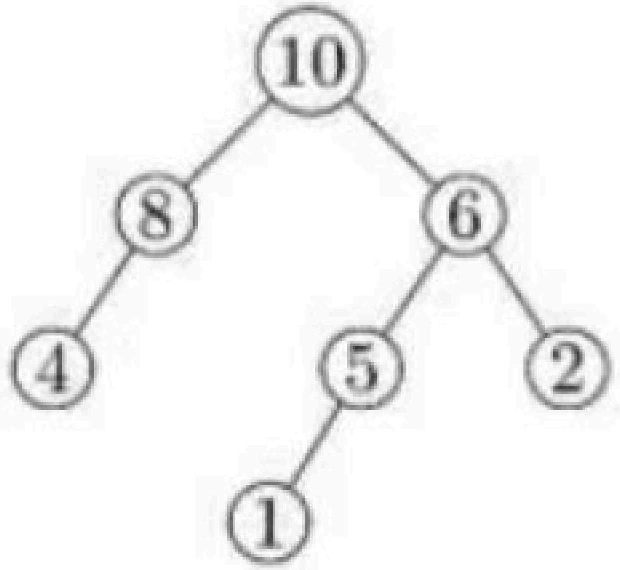
B. $\{14, 12, 13, 8, 10\}$

C. $\{14, 13, 8, 12, 10\}$

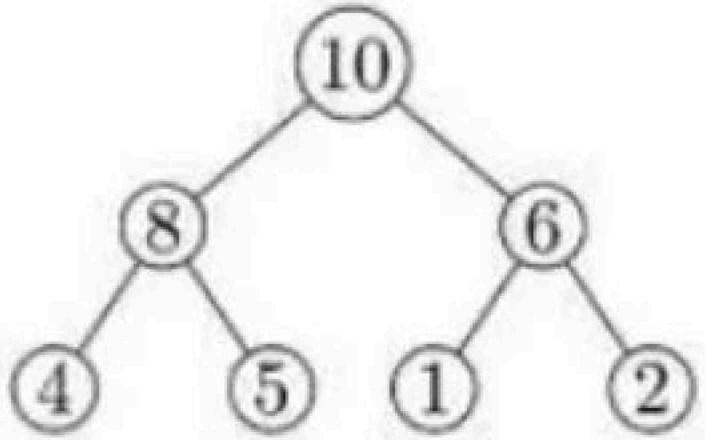
D. $\{14, 13, 12, 8, 10\}$

Question GATE-2011

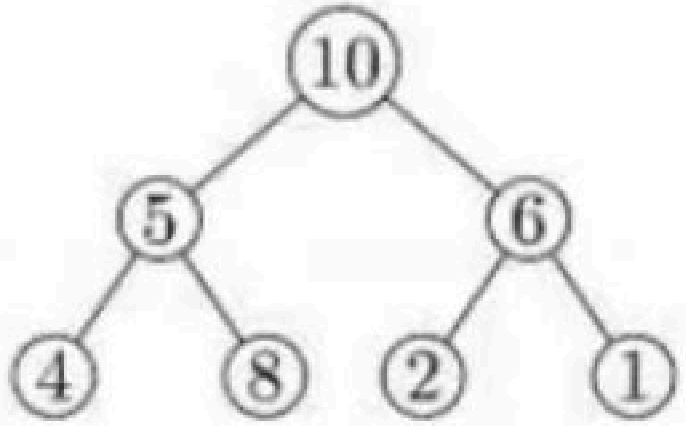
A max-heap is a heap where the value of each parent is greater than or equal to the value of its children. Which of the following is a max-heap?



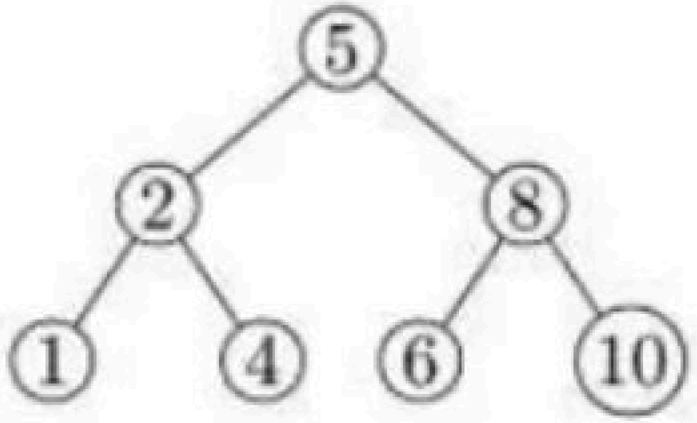
A.



B.



C.



D.

Question GATE-2015

Consider a max heap, represented by the array: $40, 30, 20, 10, 15, 16, 17, 8, 4$.

Array index	1	2	3	4	5	6	7	8	9
Value	40	30	20	10	15	16	17	8	4

Now consider that a value 35 is inserted into this heap. After insertion, the new heap is

A. $40, 30, 20, 10, 15, 16, 17, 8, 4, 35$

B. $40, 35, 20, 10, 30, 16, 17, 8, 4, 15$

C. $40, 30, 20, 10, 35, 16, 17, 8, 4, 15$

D. $40, 35, 20, 10, 15, 16, 17, 8, 4, 30$

Question GATE-2015

Consider a complete binary tree where the left and right subtrees of the root are max-heaps. The lower bound for the number of operations to convert the tree to a heap is

A. $\Omega(\log n)$

B. $\Omega(n)$

C. $\Omega(n \log n)$

D. $\Omega(n^2)$

Question GATE-2015

Consider the following array of elements.

$\langle 89, 19, 50, 17, 12, 15, 2, 5, 7, 11, 6, 9, 100 \rangle$

The minimum number of interchanges needed to convert it into a max-heap is

A. 4   B. 5   C. 2   D. 3

Question GATE-2016

An operator $delete(i)$ for a binary heap data structure is to be designed to delete the item in the i-th node. Assume that the heap is implemented in an array and i refers to the i-th index of the array. If the heap tree has depth d (number of edges on the path from the root to the farthest leaf), then what is the time complexity to re-fix the heap efficiently after the removal of the element?

A. $O(1)$

B. $O(d)$ but not $O(1)$

C. $O(2^d)$ but not $O(d)$

D. $O(d\,2^d)$ but not $O(2^d)$

Question GATE-2016

A complete binary min-heap is made by including each integer in $[1, 1023]$ exactly once. The depth of a node in the heap is the length of the path from the root of the heap to that node. Thus, the root is at depth 0. The maximum depth at which integer 9 can appear is _________.

Question GATE-2018

The number of possible min-heaps containing each value from $\{1, 2, 3, 4, 5, 6, 7\}$ exactly once is ______

Question GATE-2019

Consider the following statements:

I. The smallest element in a max-heap is always at a leaf node
II. The second largest element in a max-heap is always a child of a root node
III. A max-heap can be constructed from a binary search tree in $\theta(n)$ time
IV. A binary search tree can be constructed from a max-heap in $\theta(n)$ time

Which of te above statements are TRUE?

A. I, II and III
B. I, II and IV
C. I, III and IV
D. II, III and IV

Question GATE-2020

Consider the array representation of a binary min-heap containing 1023 elements. The minimum number of comparisons required to find the maximum in the heap is ________ .

Happy Learning

Graph → DFS

→ DFS

Hashing