Doubts Clearing Session

Comprehensive Course on Engineering Mathematics

B V Reddy • Lesson 42 • Sept 13, 2023

CALCULUS

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1. Limits
2. Continuity
3. Differentiability
4. Mean value theorems
5. Taylors series
6. Maxima and minima
7. Integration

Function

The relationship between input and the outputs is called as a function.

The relationship between dependent variable and the independent variable is called as a function

Even function

A function $f(x)$ is said to be an even function if $f(-x) = f(x)$.

Ex:

Odd Function:

A function $f(x)$ is said to be an odd function it $f(-x) = - f(x)$.

Ex:

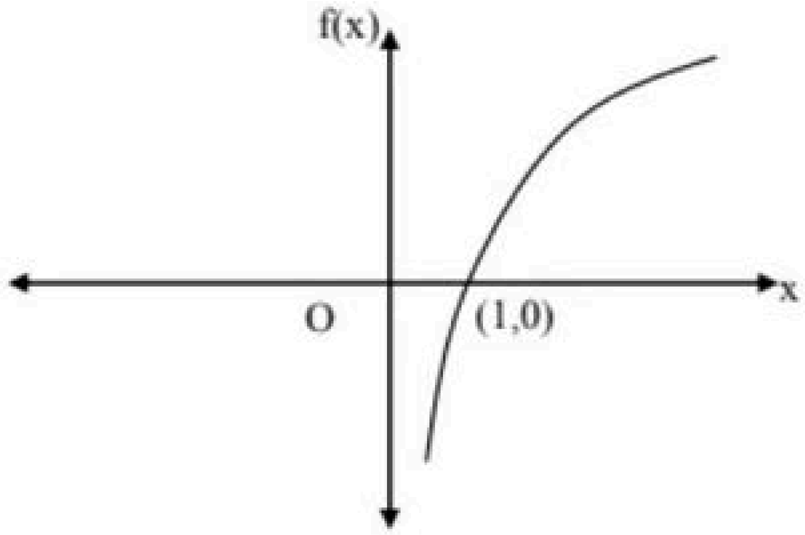
Modulus function

Step function (Greatest integer function)(Bracket function) (Floor value function)

Signum function

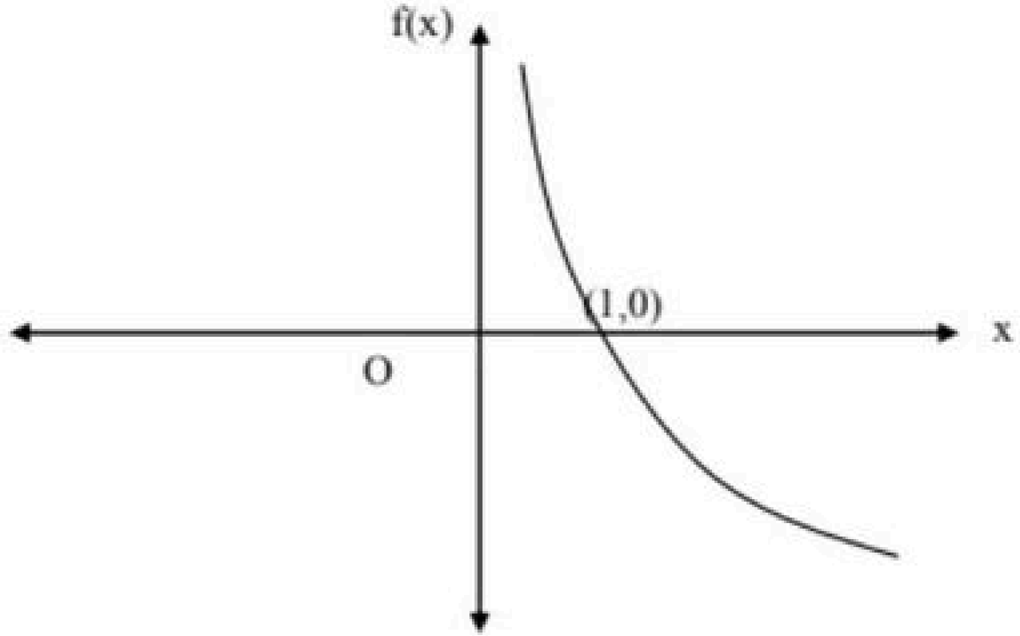
Logarithmic function

A function of the form $f(x) = \log_a x$



Case – I

For a>1



Case – II

For 0< a <1

14. The function $f(x) = e^x$ is _______ **(GATE -EC-1999)**

(a) Even (b) Odd (c) Neither even nor odd (d) None

Limit :

Limit of a function gives approximate value of the function in the neighboured of a point.

To examine the behaviour of the function $y = f(x)$, in the neighboured of a point $x = a$, when $f(x)$ is indeterminate at $x = a$.

Limit of a function

Limit of a function f(x) is said to exist at x = a , if

Difference between limit of a function and functional value at a given point

Reasons for non –existence of limit

1. If LHL or RHL or both does not exits
2. If LHL and RHL both exists but they are unequal

Indeterminate forms

Determinate forms

Algebra of limits

Evaluation of Limits

1. Find the value of function at the given limit. If it is determinate , it self is the answer .
2. If the value is indeterminate of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then apply L' Hospital rule .
3. If the value is indeterminate , but not in the $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form, then convert to this form .
4. L' Hospital rule can be applied only in case of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$,if can be applied any number of times, but check whether it is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

1. $\lim\limits_{x \to 0} \dfrac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} =$ _____ **(GATE -ME- 1993)**

4. $\lim_{x \to \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} =$ ______ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 2 (d) Does not exist

5. $\lim_{x\to 0} x \sin {}^{1}\!/_{x} =$ ________ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 1 (d) Does not exist

8. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}$, where m is an integer, is one of the following:

(GATE -CS- 1997)

(a) m (b) $m\pi$ (c) $m\theta$ (d) 1

11. $\lim_{x\to 0} \frac{1}{10} \frac{1-e^{-j5x}}{1-e^{-jx}} =$ ______ **(GATE -IN-1998)**

(a) 0 (b) 1.1 (c) 0.5 (d) 1

12. Limit of the function, $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}}$ is ________ **(GATE -EC-1999)**

(a) ½ (b) 0 (c) ∞ (d) 1

13. Value of the function $\lim_{x \to a} (x-a)^{x-a}$ is ______ **(GATE -CS-1999)**

(a) 1 (b) 0 (c) ∞ (d) a

16. Limit of the function $f(x) = \frac{1-a^4}{x^4}$ as $x \to \infty$ is given by

(GATE -CS-2000)

(a) 1 (b) e^{-a^4} (c) ∞ (d) 0

17. $\lim_{x \to \frac{\pi}{4}} \dfrac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} =$ ________ **(GATE -IN-2001)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

18. Limit of the following sequence as $n \to \infty$ is ________ $x_n = n^{\frac{1}{n}}$

(GATE -CE-2002)

(a) 0 (b) 1 (c) ∞ (d) $-\infty$

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20. $\lim_{x \to 0} \frac{\sin^2 x}{x} =$ ______ **(GATE-CS-2003)**

(a) 0 (b) ∞ (c) 1 (d) -1

21. The value of the function, $f(x) = \lim_{x \to 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$ is ______ **(GATE-CS-2004)**

(a) 0 (b) $\frac{-1}{7}$ (b) $\frac{1}{7}$ (d) ∞

23. $\lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3} =$ **(GATE-ME-2007)**

(a) 0 (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) 1

25. What is the value of $\lim_{x \to \pi/4} \frac{\cos x - \sin x}{x - \pi/4}$ **(GATE-PI-2007)**

(a) $\sqrt{2}$ (b) 0 (c) $-\sqrt{2}$ (d) Limit does not exist

26. $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta/2)}{\theta}$ is **(GATE-EC-2007)**

(a) 0.5 (b) 1 (c) 2 (d) not defined

27. $\lim_{x \to \infty} \frac{x - \operatorname{Sin} x}{x + \operatorname{Cos} x} =$ ________ **(GATE-EC-2008)**

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

30. The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{x - 8}$ is **(GATE-ME-2008)**

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

31. The value of the expression $\lim_{x \to 0} \left[\dfrac{\sin(x)}{e^x x} \right]$ is **(GATE-PI-2008)**

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) $\dfrac{1}{1+e}$

34. What is the value of $\lim\limits_{n \to \alpha} \left(1 - \frac{1}{n}\right)^{2n}$? **(GATE-CS-2010)**

(a) 0 (b) e^{-2} (c) $e^{-t/2}$ (d) 1

50. The value of $\lim_{x\to\infty} \frac{\sqrt{9x^2+2020}}{x+7}$ is

GATE-2021 (CE)

(a) 1

(b) 3

(c) $\frac{7}{9}$

(d) Indeterminable

53. The value of $\lim_{x\to\infty}(1+x^2)^{e^{-x}}$ is **(GATE-CS-2015)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) ∞

55. The value of $\lim_{x\to\infty} \frac{1-\cos(x^2)}{2x^4}$ is **(GATE-ME-2015)**

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) undefined

57. The value of $\lim_{x\to 0}\left(\dfrac{-\sin x}{2\sin x - x\cos x}\right)$ is ________ **(GATE-ME-2015)**

58. The value of $\lim_{x \to \infty} \frac{x^2 - 5x + 4}{4x^2 + 2x}$ is

GATE-2020 (CE)

(a) 0

(b) 1

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

59. Consider the limit:

GATE-2021 (CE)

$$\lim_{x \to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$$

The limit (correct up to one decimal place) is ______

60. The value of $\lim_{x \to \infty} \frac{x \ln(x)}{1+x^2}$ is

GATE-2021(CE)

(a) 1.0

(b) 0.5

(c) ∞

(d) 0

Continuity

Continuity

A function f(x) is said to be continuous at x=a if it satisfies the following conditions.

(i) f(a) is defined

(ii) $\underset{x \to a}{\mathrm{Lt}} f(x)$ exists i.e $\underset{x \to a^-}{\mathrm{Lt}} f(x) = \underset{x \to a^+}{\mathrm{Lt}} f(x)$

(iii) $\underset{x \to a}{\mathrm{Lt}} f(x) = f(a)$

Reasons of discontinuity

1.If the function is not defined at a given point.
2.If the limit of the function not exits
3. If the limit of the function exists , functional value exists but both are not equal

Left continuous (or) continuity from the left at a point

A function f(x) is said to be continuous from the left (or) left continuous at x=a if

(i) f(a) is defined

(ii) $\underset{x\to a^-}{\mathrm{Lt}} f(x) = f(a)$

Right continuous (or) continuity from the right at a point

A function f(x) is said to be continuous from the right (or) right continuous at x=a if

(i) f(a) is defined

(ii) $\underset{x\to a^+}{\mathrm{Lt}} f(x) = f(a)$

Continuity of a function in an open interval:

A function f(x) is said to be continuous in an open interval (a,b)

if f(x) is continuous $\forall \in (a, b)$

Continuity of a function on closed interval:

A function f(x) is said to be continuous on closed interval [a,b] if

(i) f(x) is continuous (a, b)

(ii) $\underset{x \to a^+}{\text{Lt}} f(x) = f(a)$

(iii) $\underset{x \to b^-}{\text{Lt}} f(x) = f(b)$

1. Sinx , cosx , e^x, a^x , |x|, polynomial functions are always continuous .

2.

Function	Points of discontinuity
Tanx , secx	
Cotx, cosecx	
$[x]$	
$\frac{1}{x}$	
Sgn(x)	

3. If $f(x)$ and $g(x)$ are two continuous functions then $f(x) + g(x)$, $f(x) - g(x)$, $f(x) \cdot g(x)$ and $\frac{f(x)}{g(x)}$ (since $g(x) \neq 0$) are also continuous.

4. Logarithmic functions are continuous in $(0, \infty)$

Types of discontinuity

1. **Discontinuity of first kind (or) Removable discontinuity**
   a. Missing point discontinuity
   b. Isolated point discontinuity

2. **Discontinuity of second kind (or) Irremovable discontinuity**
   a. Finite discontinuity(Jump discontinuity)
   b. Infinite discontinuity
   c. Oscillatory discontinuity

Discontinuity of first kind (or) Removable discontinuity

1. Missing point discontinuity

2. Isolated point discontinuity

Discontinuity of second kind (or) Irremovable discontinuity

1. Finite discontinuity

2. Infinite discontinuity

3. Oscillatory discontinuity

37. What should be the value of λ such that the function defined below is continuous at $x = \frac{\pi}{2}$? **(GATE-CE-2011)**

$$f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x}, & \text{if } x \neq \frac{\pi}{2} \\ 1, & \text{if } x = \frac{\pi}{2} \end{cases}$$

(a) 0 (b) 2π (c) 1 (d) $\frac{\pi}{2}$

42. Which one of the following functions is continuous at $x = 3$?

(GATE-CS-2013)

(a) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \\ \dfrac{x+3}{3}, & \text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8-x & \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x+3, & \text{if } x \leq 3 \\ x-4 & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3 - 27}$, if $x \neq 3$

Continuity of Composite functions

Differentiability

Derivative of a function at a point:

If a function f(x) is defined on a neighborhood of a real number 'a' and $\underset{x\to a}{\mathrm{Lt}}\frac{f(x)-f(a)}{x-a}$ exists and finite then the finite limit is called derivative or differential coefficient of f(x) at a point 'a' and it is denoted by $f^1(a)$.

$$\therefore \underset{x\to a}{Lt}\frac{f(x)-f(a)}{x-a}=f^1(a)$$

LHD

RHD

Differentiability of a function in an interval

A function f(x) is said to be differentiable in an interval [a, b]

1. f(x) is continuous in (a, b)

2. $f^1(a^-) = \lim_{x \to a^-} \frac{f(x)-f(a)}{x-a}$ **exists**

3. $f^1(a^+) = \lim_{x \to a^+} \frac{f(x)-f(a)}{x-a}$ **exists**

4. $f^1(a^-) = f^1(a^+)$

Working Procedure to check continuity & differentiability

1. Check for continuity of the given function
2. If it is discontinuous , then the function is non differentiable ,
3. If it is continuous , find the LHD and RHD .
4. If LHD and RHD exists, and LHD = RHD , then the function is differentiable .

- If f(x) and g(x) are two differentiable function then f(x)+g(x), f(x)-g(x), f(x)·g(x), $\frac{f(x)}{g(x)}$ (g(x)≠ 0) are also differentiable.
- Polynomial functions, exponential functions, sine and cosine functions are differentiable everywhere.
- Every differentiable function is continuous but a continuous function need not be differentiable.
- If the function is discontinuous , then it is not differentiable .
- |x| is continuous but not differentiable at x= 0
- |x-a| is continuous but not differentiable at x = a
- |ax-b| is continuous but not differentiable at x = $\frac{b}{a}$
- Sgn(x-a) is not differentiable at x = a
- [x] is not differentiable at all integers

Reasons for non differentiability of a function

1.Sharp Corner

2. Vertical tangent

3. Having discontinuities at $x = a$

4.Function tending to infinite at $x = a$

6. The function $f(x) = |x+1|$ on the interval $[-2, 0]$ is ________

(GATE-EC- 1995)

(a) continuous and differentiable

(b) continuous on the interval but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

9. If $y = |x|$ for $x < 0$ and $y = x$ for $x \geq 0$ then

(GATE -EC- 1997)

(a) $\frac{dy}{dx}$ is discontinuous at $x = 0$ (b) y is discontinuous at $x = 0$

(c) y is not defined at $x = 0$

(d) Both y and $\frac{dy}{dx}$ are discontinuous at $x = 0$

7. If a function is continuous at a point its first derivative **(GATE -EC- 1995)**

(a) may or may not exist

(b) exists always

(c) will not exist

(d) has a unique value

19. Which of the following functions is not differentiable in the domain [-1, 1]?

(a) $f(x) = x^2$ (b) $f(x) = x - 1$ **(GATE -EC-2002)**

(c) $f(x) = 2$ (d) $f(x) = \text{maximum } (x - x)$

22. Consider the function $f(x) = |x|^3$, where x is real.

Then the function f(x) at x = 0 is

(GATE –IN-2007)

(a) continuous but not differentiable

(b) once differentiable but not twice

(c) twice differentiable but not thrice

(d) thrice differentiable

33. If $(x) = \sin |x|$ then the value of $\frac{df}{dx}$ at $x = \frac{-\pi}{4}$ is **(GATE-PI-2010)**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) 1

36. The function $y = |2-3x|$ **(GATE-ME-2010)**

(a) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(b) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{3}{2}$

(c) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{2}{3}$

(d) is continuous $\forall x \in R$ and except at $x = 3$ and differentiable $\forall x \in R$

46. If a function is continuous at a point, **(GATE-ME-SET-3-2014)**

(a) the limit of the function may not exist at the point

(b) the function must be derivable at the point

(c) the limit of the function at the point tends to infinity

(d) the limit must exist the point and the value of limit should be same as the value of the function at the point.

Differentiability of $Y = |f(x)|$

Q. Check the differentiability of $y = |x|$

Q. Check the differentiability of $y = |e^x|$

Q. Check the differentiability of $y = |sinx|$

Q. Check the differentiability of $y = |x^3|$

Differentiability of f(x). g(x)

Q. Check the differentiability of $y = x|x|$

Q. Check the differentiability of $y = cos x|x|$

Q. Check the differentiability of $y = (x^2 - 3x + 2)|x^2 - 5x + 6|$

Mean Value Theorems

Rolle's Theorem

Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = x^2 - 1$ on [-1, 1]

Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = (x - a)^m (x - b)^n$ on [a, b]

Q. Find the value of 'C ' from Rolle's theorem for the function $f(x) = \log\left[\frac{x^2+ab}{(a+b)x}\right]$ on [a, b]

Lagrange's Mean Value Theorem

Q. Find ‘C’ of LMVT for $f(x) = \sin x - \sin 2x$ in $[0, \pi]$

Q. If $f^1(x) = \frac{1}{3-x^2}$ *and* $f(0) = 1$, find an interval in which f(1) lies

Cauchy's Mean Value Theorem

Q. The ‘C’ of Cauchy’s mean value theorem for $f(x) = e^x, g(x) = e^{-x}$ defined in [a, b] is ----------

Q. The ‘C’ of Cauchy’s mean value theorem for $f(x) = \frac{1}{x}, g(x) = \frac{1}{x^2}$ defined in [a, b] is ----------

Q. The ‘C’ of Cauchy’s mean value theorem for $f(x) = \text{Sin}x$, $g(x) = \text{Cos}x$ defined in [a, b] is ----------

Taylor's Series

Maclaurin’s Series

Q. Expand e^x by Taylor's series about $x = 0$

97. The third term in the taylor's series expansion of e^x about 'a' would be ______

(a) $e^a (x-a)$ (b) $\frac{e^a}{2}(x-a)^2$ (c) $\frac{e^a}{2}$ (d) $\frac{e^a}{6}(x-a)^3$ **GATE -1995**

98. The taylor's series expansion of sin x is __________ (GATE-EC-1998)

(a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\dots\dots\dots$

(b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\dots\dots\dots$

(c) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots\dots\dots\dots$

(d) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\dots\dots\dots$

100. The Taylor series expansion of sin x about $x = \frac{\pi}{6}$ is given by **(GATE-CE-2000)**

(a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\Pi}{6}\right)-\frac{1}{4}\left(x-\frac{\Pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\Pi}{6}\right)^3+----$

(b) $x - \frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+------$

(c) $\frac{x-\frac{\Pi}{6}}{1!}-\frac{\left(x-\frac{\Pi}{6}\right)^3}{3!}+\frac{\left(x-\frac{\Pi}{6}\right)^5}{5!}-\frac{\left(x-\frac{\Pi}{6}\right)^7}{7!}+-----$

(d) $\frac{1}{2}$

101. Limit of the following series as x approaches $\frac{\pi}{2}$ is $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \text{------}$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 1 **(GATE-CE-2001)**

102. For the function e^{-x}, the linear approximation around $x = 2$ is

(a) $(3-x)e^{-2}$

(b) $1 - x$

GATE- 2007

(c) $\left[3+2\sqrt{2}-\left(1+\sqrt{2}\right)x\right]e^{-2}$

(d) e^{-2}

103. For $|x| << 1$, $\coth(x)$ can be approximated as **(GATE-EC-2007)**

(a) x (b) x^2 (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

104. The expression $e^{\ln x}$ for $x > 0$ is equal to **(GATE-IN-2008)**

(a) $-x$ (b) x (c) x^{-1} (d) $-x^{-1}$

105. Which of the following function would have only odd powers of x in its Taylor series expansion about the point $x = 0$? **(GATE-EC-2008)**

(a) $\sin(x^3)$ (b) $\sin(x^2)$ (c) $\cos(x^3)$ (d) $\cos(x^2)$

106. In the Taylor series expansion of $e^x + \sin x$ about the point $x = \pi$, the coefficient of $(x-\pi)^2$ is **(GATE-EC-2008)**

(a) e^π (b) $0.5\, e^\pi$ (c) $e^\pi + 1$ (d) $e^\pi - 1$

107. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(GATE-ME-2008)

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$ (c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

108. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by **(GATE-EC-2010)**

(a) $1+\frac{(x-\pi)^2}{3!}+------$

(b) $-1-\frac{(x-\pi)^2}{3!}+------$

(c) $1-\frac{(x-\pi)^2}{3!}+------$

(d) $-1+\frac{(x-\pi)^2}{3!}+------$

112. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots$ corresponds to **(GATE-CE-2012)**

(a) $\sec x$ (b) e^x (c) $\cos x$ (d) $1+\sin^2 x$

113. The Taylor series expansion of $3 \sin x + 2\cos x$ is **(GATE-EC-SET-1-2014)**

(a) $2 + 3x - x^2 - \frac{x^3}{2} + ---$

(b) $2 - 3x + x^2 - \frac{x^3}{2} + ---$

(c) $2 + 3x + x^2 + \frac{x^3}{2} + ---$

(d) $2 - 3x - x^2 + \frac{x^3}{2} + ---$

Taylor's Series for functions of two variables

Q. Expand $x^2y + 3y - 2$in powers of (x-1) and (y+2) using Taylor's series

Maxima & Minima

Increasing and Decreasing functions at a point

Q. Find the set values of λ for which the function

$$f(x) = \begin{cases} x+1 & x<1 \\ \lambda & x=1 \\ x^2-x+3 & x>1 \end{cases}$$

is strictly increasing at x= 1

Increasing and Decreasing functions on an interval

Monotonic Function

Test for Monotonicity

Q. Find the interval in which $f(x) = x^3 - 3x^2 - 9x + 20$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = x^2 - 5x + 6$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = -x^2 - 5x + 6$ is increasing and decreasing

Q. Find the interval in which $f^1(x) = x(x^2 - 4)$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = (x+2)(x-1)^2(x-5)$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = \frac{(x-1)(x-5)}{(x-3)}$ is increasing or decreasing

Q. Find the interval in which $f^1(x) = \frac{(x-1)(x+5)}{(x-3)(x+4)}$ is increasing or decreasing

Stationary points

The values of x for which $f^1(x) = 0$, are called stationary points or turning points .

Critical points

The values of x for which $f^1(x) = 0$, and the points where $f^1(x)$ is not exist are called as critical points .

Local (Relative) Maxima

Local (Relative) Minimum

Extreme Points & Extreme values

The point at which the function has a maximum or a minimum is called extreme point.

The values of the function at extreme points are called extreme values(Extrema)

Methods to find Local Extremum

First derivate test

Second derivate test

62. The function $y = x^2 + \frac{250}{x}$ at $x = 5$ attains

(GATE-EC-1994)

(a) Maximum (b) Minimum (c) Neither (d) 1

63. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has

(GATE-EE-1995)

(a) a maxima at $x = 1$ and a minima at $x = 3$

(b) a maxima at $x = 3$ and a minima at $x = 1$

(c) no maxima, but a minima at $x = 3$

(d) a maxima at $x = 1$, but no minima

70. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at

(GATE-CS-2004)

(a) $x = -2$ only

(b) $x = 0$ only

(c) $x = 3$ only

(d) both $x = -2$ and $x = 3$

71. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to

(a) 2 (b) 1 (c) 0 (d) -1

72. For real x, the maximum value of $\frac{e^{\text{Sin}x}}{e^{\text{Cos}x}}$ is

(GATE-IN-2007)

(a) 1 (b) e (c) $e^{\sqrt{2}}$ (d) ∞

76. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has

(GATE-EE-2007)

(a) Only one minimum

(b) Only two minima

(c) Three minima

(d) Three maxima

77. A point on the curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is ________

(GATE-CS-2008)

(a) 0 (b) 1 (c) 2 (d) 3

78. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 is **(GATE-IN-2008)**

(a) 1 (b) 3 (c) 4 (d) 9

79. For real values of x, the minimum value of function

$f(x) = e^{x} + e^{-x}$ is

(GATE-EC-2008)

(a) 2 (b) 1 (c) 0.5 (d) 0

80. At $t = 0$, the function $f(t) = \dfrac{\sin t}{t}$ has

(GATE-EE-2010)

(a) a minimum

(b) a discontinuity

(c) a point of inflection

(d) a maximum

81. If $e^y = x^{1/x}$ then y has a

(GATE-EC-2010)

(a) maximum at $x = e$ (b) minimum at $x = e$

(c) maximum at $x = e^{-1}$ (d) minimum at $x = e^{-1}$

83. The function $f(x) = 2x - x^2 + 3$ has

(GATE-EE-2011)

(a) a maxima at $x = 1$ and a minima at $x = 5$

(b) a maxima at $x = 1$ and a minima at $x = -5$

(c) only a maxima at $x = 1$

(d) only a minima at $x = 1$

86. For $0 \leq t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$ (b) $t = \log_e 2$ (c) $t = 0$ (d) $t = \log_e 8$

87. The maximum value of the function $f(x) = \ln(1+x) - x$ (where $x > -1$) occurs at $x =$ ______________. **(GATE-EC-SET-3-2014)**

Global (Absolute) maximum

Global (Absolute) minimum

Q. Find the points of local maxima and minima if any of the following function defined in $0 \leq x \leq 6$, $f(x) = x^3 - 6x^2 + 9x + 15$.

(GATE-CS-1998)

Q. The minimum value of function $y = x^2$ in the interval [1, 5] is

(GATE-ME-2007)

(a) 0 (b) 1 (c) 25 (d) undefined

Q. Consider the function $f(x) = x^2 - x - 2$. The maximum value of f(x) in the closed interval [-4, 4] is

(GATE-EC-2007)

(a) 18 (b) 10 (c) -2.25 (d) indeterminate

Q. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(a) 21 (b) 25 (c) 41 (d) 46

GATE- 2012

Q. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is ____.

GATE-2014

Q. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

(a) e^{-1} (b) e (c) $1 - e^{-1}$ (d) $1 + e^{-1}$

GATE-2014

Q. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval $[-3, 3]$ is **(GATE-EC-SET-2-2014)**

(a) 20 (b) 28 (c) 16 (d) 32

Convex and Concave functions

Point of Inflection

For a continuous function f(x) said to have a point of inflection at $x = x_o$ if

1.

2.

Q. Number of inflection points for the curve $y = x + 2x^4$ is ______

(GATE-CE-1999)

(a) 3 (b) 1 (c) 0 (d) 2

2

Q. At $x = 0$, the function $f(x) = x^3 + 1$ has

(GATE-ME,PI-2012)

(a) a maximum value

(b) a minimum value

(c) a singularity

(d) a point of inflection

Q. Find the points of inflections for $f(x) = x^3 - 3x^2 - 7x + 8$

Q. Find the points of inflections of $f(x) = e^{-x^2}$

Maxima and minima for functions of two variables

Let $z = f(x,y)$ be the function of two variables for which maxima or minima is to be obtained.

Working rule

Step 1: find p, q, r, s and t

Step 2: equate p and q to zero for obtaining stationary points.

Step 3: find r, s and t at each stationary point.

(a)If $rt - s^2 > 0$ and $r > 0$ then f(x, y) has a minimum at that stationary point.

(b)If $rt - s^2 > 0$ and $r < 0$ then f(x, y) has a maximum at that stationary point.

(c)If $rt - s^2 < 0$ then f(x, y) has no extremum at that stationary point and such points are called saddle points.

(d)If $rt - s^2 = 0$ then the case is undecided.

Q. Given a function $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$, the optimal values of $f(x, y)$ is

(GATE-CE-2010)

(a) a minimum equal to $\frac{10}{3}$

(b) a maximum equal to $\frac{10}{3}$

(c) a minimum equal to $\frac{8}{3}$

(d) a maximum equal to $\frac{8}{3}$

Q. The function $f(x,y) = 2x^2 + 2xy - y^3$ has **(GATE-EC-2000)**

(a) Only one stationary point at (0, 0)

(b) Two stationary points at (0, 0) and (1/6, -1/3)

(c) Two stationary points at (0, 0) and (1, -1)

(d) No stationary point

Q. The function $f(x) = 8 \log x - x^2 + 3$ attains its minimum over the interval $[1,e]$ at $x =$________________ (Here $\log_e x$ is the natural logarithm of x.)

(GATE-2022-ECE)

(a) 2 (b) 1

(c) e (d) $\frac{1+e}{2}$

Q. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

a) 8 m b)10m c)12m d)14m

Constrained maxima and minima

Sometimes it is required to find the extremum of a function subject to some other conditions involving the variables. Such problems are called constrained maxima and minima problems

Q. If the sum of the two positive numbers is 18 , then the maximum value of their product is

a)81 b)85 c)72 d)80

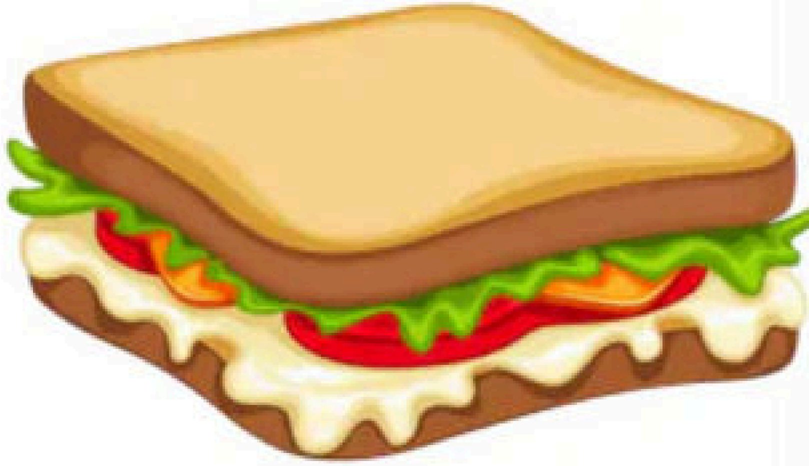
Q. If $x^2 + y^2 = 1$ then the maximum value of x+y

Lagrange's Method

Q. Find the maximum and minimum values of $x + y + z$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$

Q. Find the point on the plane $x + 2y + 3z = 4$, that is closest to the origin.

Sandwich theorem



Q. If $3 - \frac{x^2}{12} \leq f(x) \leq 3 + \frac{x^3}{9}$, then the value of $\underset{x \to 0}{Lt} \; f(x)$

Q. Find $\underset{n\to\infty}{Lt} \dfrac{[x]+[2x]+[3x]+-\cdots+[nx]}{n^2}$, where [.] is the greatest integer function.

Double Limit

Q. Find $\underset{\substack{x \to 0 \\ y \to 0}}{Lt} (x^3 + y^3)$

Q. Find $\underset{\substack{x \to 1 \\ y \to 2}}{Lt} \dfrac{3x^2y}{x^2+y^2+5}$

Q. Find $\underset{\substack{x \to 1 \\ y \to 2}}{Lt} \frac{2x^2+y^2}{2xy}$

Q. Find $\underset{\substack{x \to \infty \\ y \to 2}}{Lt} \dfrac{2x-3}{x^3+4y^3}$

Q. Find $\underset{\substack{x \to 0 \\ y \to 0}}{Lt} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

Definite Integrals

Multiple Integral's

Double Integrals

Concept of Strip

Vertical Strip

Horizontal Strip

Q: $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ (GATE-EC-2000)

(a) 0 (b) π (c) $\pi/2$ (d) 2

Q. Evaluate $\int_1^2 \int_3^4 (xy + e^y)\, dy\, dx$

249. $\int_0^2 \int_0^3 xy \, dxdy$

(A) 0 (B) 9 (C) 8 (D) 1

250. $\int_0^{\frac{\pi}{2}}\int_{-1}^{1} x^2y^2dxdy$

(A) $\frac{\pi^3}{36}$ (B) $\frac{\pi}{0}$ (C) -1 (D) 0

251. Evaluate $\int_{-1}^{2}(1+|x|)\,dx$

(A) 3.5

(B) 5.5

(C) 4

(D) None of these

252. $\int_0^{\pi} \sin^5 x \cos^9 x dx =$ ________

(A) 0

(B) 1

(C) ∞

(D) None of these

253. Let $f(x)$ be any bounded real valued function in the interval $[a, b]$.

Consider the following statements:

A: $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} \{f(x) + f(-x)\}\,dx$

B: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

Then which of the following is appropriate?

(A) A and B both are true and they are interdependent

(B) A and B are true independently

(C) A is true and B is false always

(D) A is true and B is true in special case

254. For which value of n, $\int_0^{\frac{\pi}{2}} \frac{dx}{16\cos^2 x + 25\sin^2 x}$ becomes equal to $n \times \pi$.

(A) $\frac{1}{40}$ (B) $\frac{1}{50}$ (C) $\frac{1}{20}$ (D) $\frac{1}{30}$

255. Evaluate $\int_0^1 \int_1^2 (x^2 + y^2) dx dy$

(A) $\frac{-8}{3}$ (B) $\frac{8}{3}$ (C) 0 (D) 1

256. The value of $\int_{-4}^{7} |x|\, dx$ is

(a) 30.5

(b) 30

(c) 32.5

(d) 32

257. The value of $\int_{0}^{1.5} x[x^2]\,dx$, where $[x]$ is a step function, is

(a) $\frac{4}{3}$

(b) $\frac{1}{2}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

258. The value of $\int_0^{\pi} x \sin^8(x) \cos^6(x)\, dx$ is

(a) $\dfrac{\pi^2}{512}$

(b) $\dfrac{105\pi^2}{512}$

(c) $\dfrac{105\pi}{86016}$

(d) $\dfrac{5\pi^2}{4096}$

259. The value of $\int\limits_{x=1}^{a}\int\limits_{y=1}^{b}\frac{1}{xy}\,dx\,dy$ is ______.

(a) (log a) (log b)

(b) log (ab)

(c) log a – log b

(d) log (a + b)

260. $\int_1^3 \int_1^2 xy^2 \, dx \, dy =$

(a) 10

(b) 11

(c) 13

(d) 12

261. $\int_0^1 \int_0^2 \int_1^2 x^2 yz \, dx \, dy \, dz =$

(a) $-\frac{7}{3}$

(b) $\frac{7}{3}$

(c) $\frac{7}{2}$

(d) $-\frac{7}{2}$

262. The value of $\int_{x=0}^{1} \int_{y=0}^{2} xy \, dx \, dy$ is ________.

263. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right) dx =$ ______

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) none of these

264. $\int_{-\pi}^{\pi} \sin^4 x . dx =$

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$

(d) 0

265. $\int_{-1}^{2} \frac{|x|}{x}\, dx = ------$

266. $\int_{0}^{\pi} |\cos x| \, dx =$

267. $\int_0^n [x]\,dx =$ ____ , where [x] is a step functi

and 'n' is an integer.

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n-1)}{2}$

(c) $\frac{n}{2}$ (d) $\frac{n+1}{2}$

268. $\int_{0}^{\frac{\pi}{2}} \log(\tan x)\,dx =$

(a) 0

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) π

269. Let $\frac{d}{dx}[F(x)] = \frac{e^{\sin x}}{x}, x > 0.$

If $\int_1^4 \left(\frac{2e^{\sin x^2}}{x}\right) dx = F(k) - F(1)$

then k =__________.

270. $\int_{0}^{\pi/4} \log(1+\tan x)\,dx =$

(a) 0

(b) $(\pi/2) \log 2$

(c) $(\pi/8) \log 2$

(d) $(-\pi/4) \log 2$

271. $\int_0^{\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0

(b) $3\pi/256$

(c) $3\pi/128$

(d) $5\pi/128$

272. $\int_{0}^{2\pi} \sin^4 x \cos^6 x \, dx =$

(a) $3\pi/128$ (b) $3\pi/256$

(c) $3\pi/64$ (d) 0

273. $\int_0^{2\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0

(b) $3\pi/128$

(c) $5\pi/128$

(d) $3\pi/256$

274. $\int_{0}^{\pi/2}\int_{0}^{\pi/2} \sin(x+y)\,dx\,dy$ **(GATE-EC-2000)**

(a) 0 (b) π (c) $\pi/2$ (d) 2

275. The value of the integral is $I = \int_0^{\pi/4} \cos^2 x \, dx$ **(GATE-CE-2001)**

(a) $\frac{\pi}{8}+\frac{1}{4}$ (b) $\frac{\pi}{8}-\frac{1}{4}$ (c) $\frac{-\pi}{8}-\frac{1}{4}$ (d) $\frac{-\pi}{8}+\frac{1}{4}$

276. The value of the integral $\int_{-\pi/2}^{\pi/2} (x \cos x)\, dx$ is **(GATE-PI-2008)**

(a) 0 (b) $\pi - 2$ (c) π (d) $\pi + 2$

277. The value of the following definite integral in $\int_{-\pi/2}^{\pi/2} \frac{\text{Sin}2x}{1+\cos x}dx$ = **(GATE-ME-2002)**

(a) -2log 2 (b) 2 (c) 0 (d) None

278. The value of the following improper integral is $\int_0^1 x \log x \, dx$ =

(GATE-ME-2002)

(a) 1/4 (b) 0 (c) -1/4 (d) 1

279. $\int_{-a}^{a}\left[\sin^6 x + \sin^7 x\right]dx$ is equal to **(GATE-ME-2004)**

(a) $2\int_{0}^{a} \text{Sin}^6 x\, dx$ (b) $2\int_{0}^{a} \text{Sin}^7 x\, dx$ (c) $2\int_{0}^{a}\left(\text{Sin}^6 x + \text{Sin}^7 x\right)dx$ (d) zero

280. The value of $\int_0^3 \int_0^x (6 - x - y)\, dxdy$ is ________ **(GATE-CS-2008)**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

281. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\,dx\,dy\,dz$

(A) $\dfrac{e^{4a}}{8} - \dfrac{3e^{2a}}{4} + e^{a} - \dfrac{3}{8}$

(B) $\dfrac{e^{4a}}{4} - \dfrac{3e^{2a}}{4}$

(C) $\dfrac{e^{4a}}{8} - \dfrac{3e^{2a}}{4} - \dfrac{3}{8}$

(D) None

282. $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dx\, dy =$

(a) $\frac{2}{35}$

(b) $-\frac{3}{35}$

(c) $\frac{3}{35}$

(d) $-\frac{2}{35}$

283. $\int_{0}^{1}\int_{4y}^{4} e^{x^2}\, dx\, dy =$

(a) $\frac{\left(e^{16}-1\right)}{8}$

(b) $-\frac{\left(e^{16}+1\right)}{8}$

(c) 0

(d) $-\frac{\left(e^{16}-1\right)}{8}$

284. $$\int_0^4 \int_0^{x^2} e^{y/x}\, dy\, dx =$$

(a) $3e^4$

(b) $3e^4 + 7$

(c) $-3e^4$

(d) $3e^4 - 7$

286. $$\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz=$$

(a) 1

(b) 2

(c) 3

(d) 0

287. The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to ________.

(GATE-16-EC)

288. $\int_{1/\pi}^{2/\pi} \frac{\cos(1/x)}{x^2} dx =$ ________ **(GATE-CS-2015)**

289. Given the following statements about a function $f: R \rightarrow R$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(GATE-16-EC)

(a) P is true, Q is false, R is false

(b) P is false, Q is true, R is true

(c) P is false, Q is true, R is false

(d) P is true, Q is false, R is true

290. The value of $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r \sin\theta \, dr \, d\theta$ is ________.

(a) $\frac{a^2}{2}$ (b) $2a^2$

(c) $\frac{2a^2}{3}$ (d) $4a^2$

292. $\int_0^1 \int_0^{\sqrt{(1-x^2)}} y^2 \, dx dy =$

(a) $-\frac{\pi}{16}$

(b) $\frac{\pi}{16}$

(c) $\frac{\pi}{8}$

(d) $-\frac{\pi}{8}$

301. The value of $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$ is ______________

(A) $\frac{13}{9} - \frac{ln\ 3}{6}$

(B) $\frac{7}{6} - \frac{ln\ 3}{6}$

(C) $\frac{1}{6} - ln\ 3$

(D) $\frac{3}{2} - ln\ 3$

302. The value of $\int_0^1 \int_0^2 \int_1^2 x^2 yz dz dy dx$ is __________

(A) 0 (B) 1

(C) 2 (D) 3

304. The value of $\int_{-1}^{2}\int_{x^2}^{x+2} dydx =$ ________

(A) $\frac{7}{2}$

(B) $\frac{9}{2}$

(C) $\frac{11}{2}$

(D) $\frac{5}{2}$

305. The value of $\int_0^1 \int_0^1 \frac{dydx}{\sqrt{1-x^2}\sqrt{1-y^2}} =$ ________

`(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{16}$

306. The value of $\int_0^{100\pi} |sin\ x| dx$ is ____________

(A) 100 (B) 100π

(C) 200π (D) 200

307. The value of integral $\int_{-1}^{1} ln\left(\frac{2-x\cos x}{2+x\cos x}\right)dx$ is ________

(A) $xln(2 + x\cos x)$ (B) $xln(2 - x\cos x)$

(C) $x\cos x$ (D) 0

308. If $f(x) = \int_x^0 sin\, t^2 dt$ then $f'(x)$ is ________

(A) $2x sin\, x^2$ (B) $-sin\, x^2$

(C) $2x cos\, x^2$ (D) $cos\, x^2$

Q. Evaluate $\int_0^5 \int_0^{x^2} x(x^2 + y^2)\,\mathrm{d}x\,\mathrm{d}y$

Q. The value of $\int_0^3 \int_0^x (6 - x - y)\, dx dy$ is ________ **(GATE-CS-2008)**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

Q. The value of the double integral $\int_0^1 \int_x^{1/x} \frac{x}{1+y^2} \, dx \, dy =$ ______ **(GATE-EC-1993)**

Change of order of integration

Q. Evaluate $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$

Q. Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dy\, dx$

Q. By reversing the order of integration $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$ may be represented as

(a) $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$ (b) $\int_0^2 \int_y^{\sqrt{y}} f(x, y)\, dx\, dy$ **(GATE-EC-1995)**

(c) $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y)\, dx\, dy$ (d) $\int_{x^2}^{2x} \int_0^2 f(x, y)\, dy\, dx$

Q. Changing the order of integration in double integral $I = \int_0^8 \int_{x/4}^2 f(x,y)\,dy\,dx$ leads to $I = \int_r^s \int_p^q f(x,y)\,dx\,dy$. What is q?

(GATE-EC-2005)

(a) $4y$

(b) $16y^2$

(c) x

(d) 8

Triple integrals

Q. Evaluate $\int\int\int_R (x + y + z)\, dx\, dy\, dz$ where $R: 0 \le x \le 1,\ \ 1 \le y \le 2\ ,\ \ 2 \le z \le 3$

Q. Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z)\, dx\, dy\, dz$

Change of variables

Q. To evaluate the double integral $\int_0^8 \left(\int_{y/2}^{(y/2)+1} \left(\frac{2x-y}{2} \right) dx \right) dy$, we make the substitution $u = \left(\frac{2x-y}{2} \right)$ and $v = \frac{y}{2}$. The integral will reduce to **(GATE-EE-SET-2-2014)**

(a) $\int_0^4 \left(\int_0^2 2u\,du \right) dv$ (b) $\int_0^4 \left(\int_0^1 2u\,du \right) dv$ (c) $\int_0^4 \left(\int_0^1 u\,du \right) dv$ (d) $\int_0^4 \left(\int_0^{21} 2u\,du \right) dv$

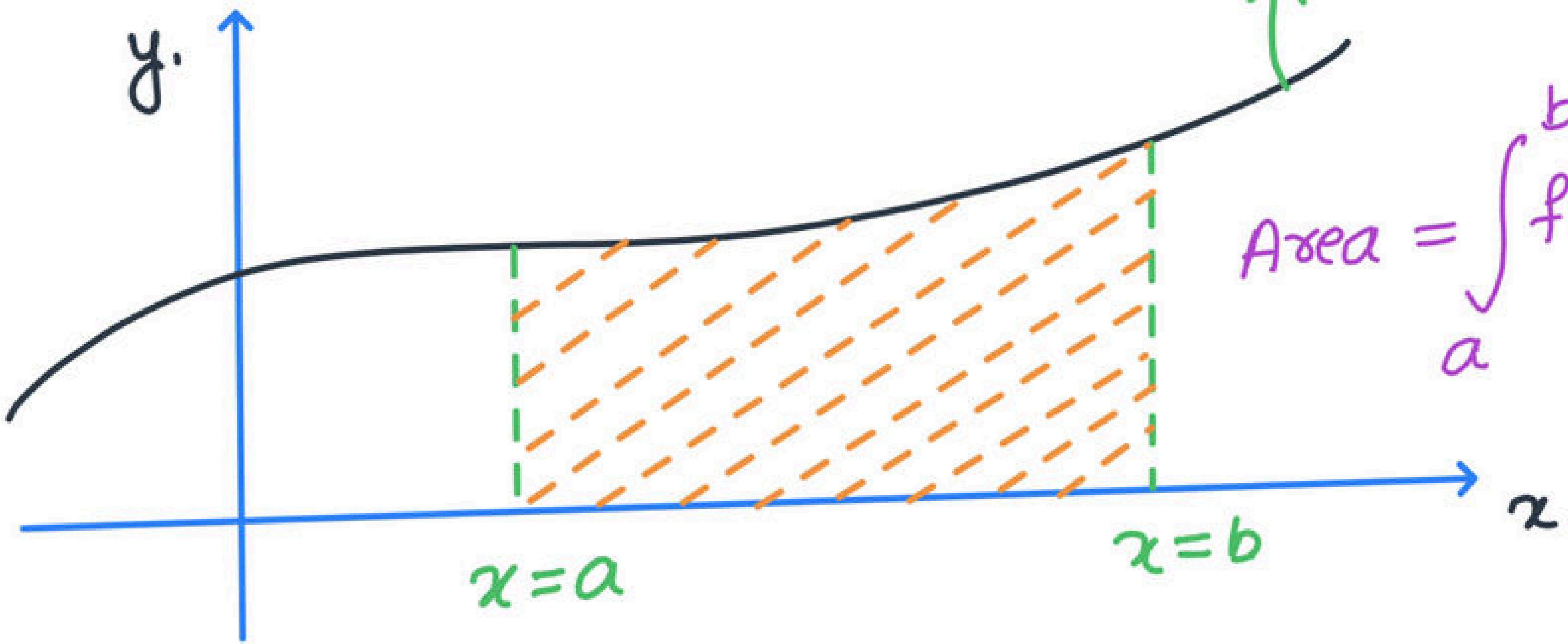
Q. By a change of variables $x(u, v) = uv$, $y(u,v) = v/u$ in a double integral, the integral $f(x,y)$ changes to $f\left(uv, \frac{u}{v}\right)\phi(u,v)$ Then $\phi(u,v)$ is ________ **(GATE-EE-2005)**

(a) $\frac{2v}{u}$ (b) $2uv$ (c) v^2 (d) 1

Area bounded by the curves

1. If $y = f(x)$

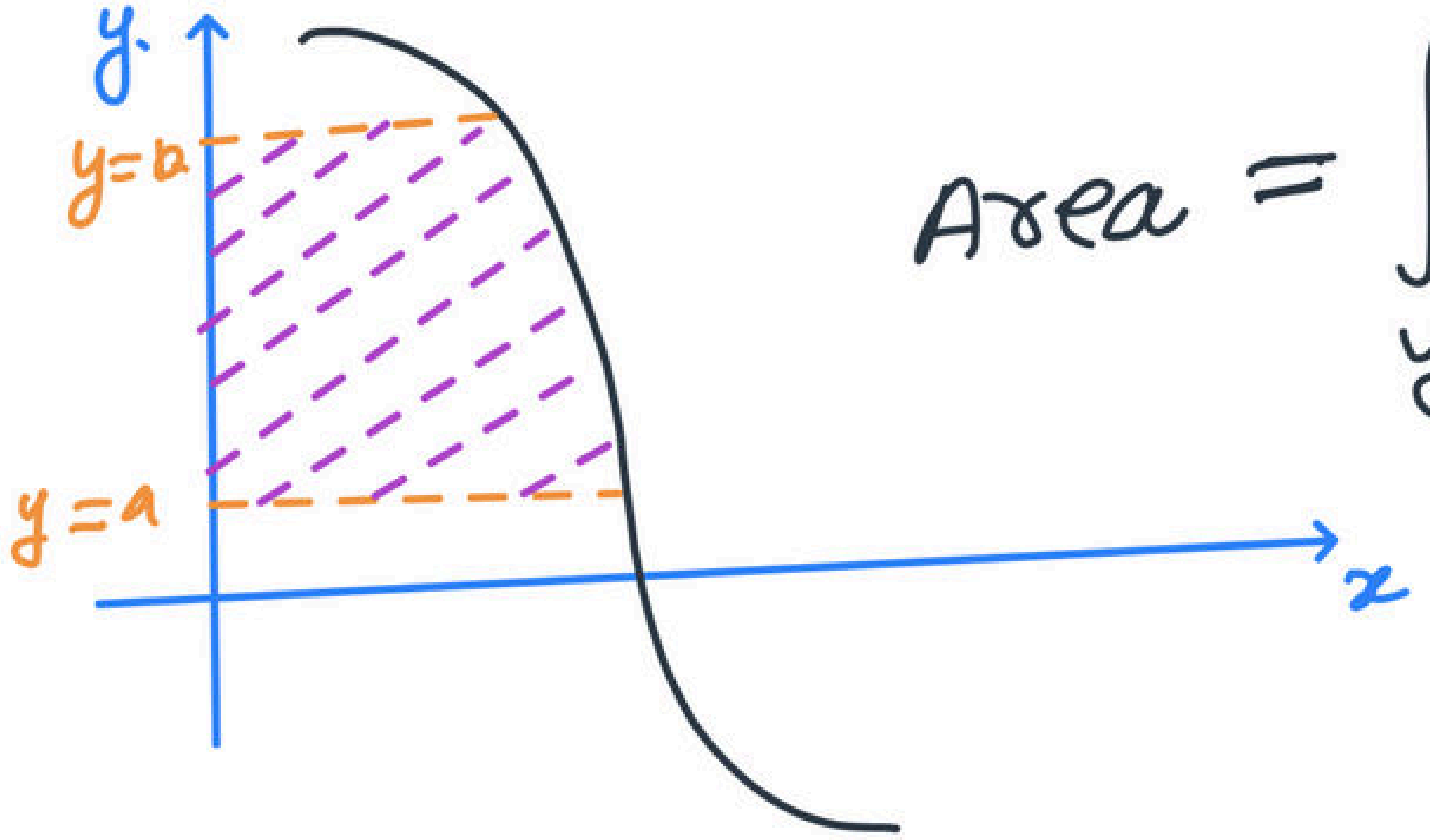
Area bounded by the curve $y = f(x)$ and the x-axis $x = a$ to $x = b$



$$\text{Area} = \int_a^b f(x)\,dx.$$

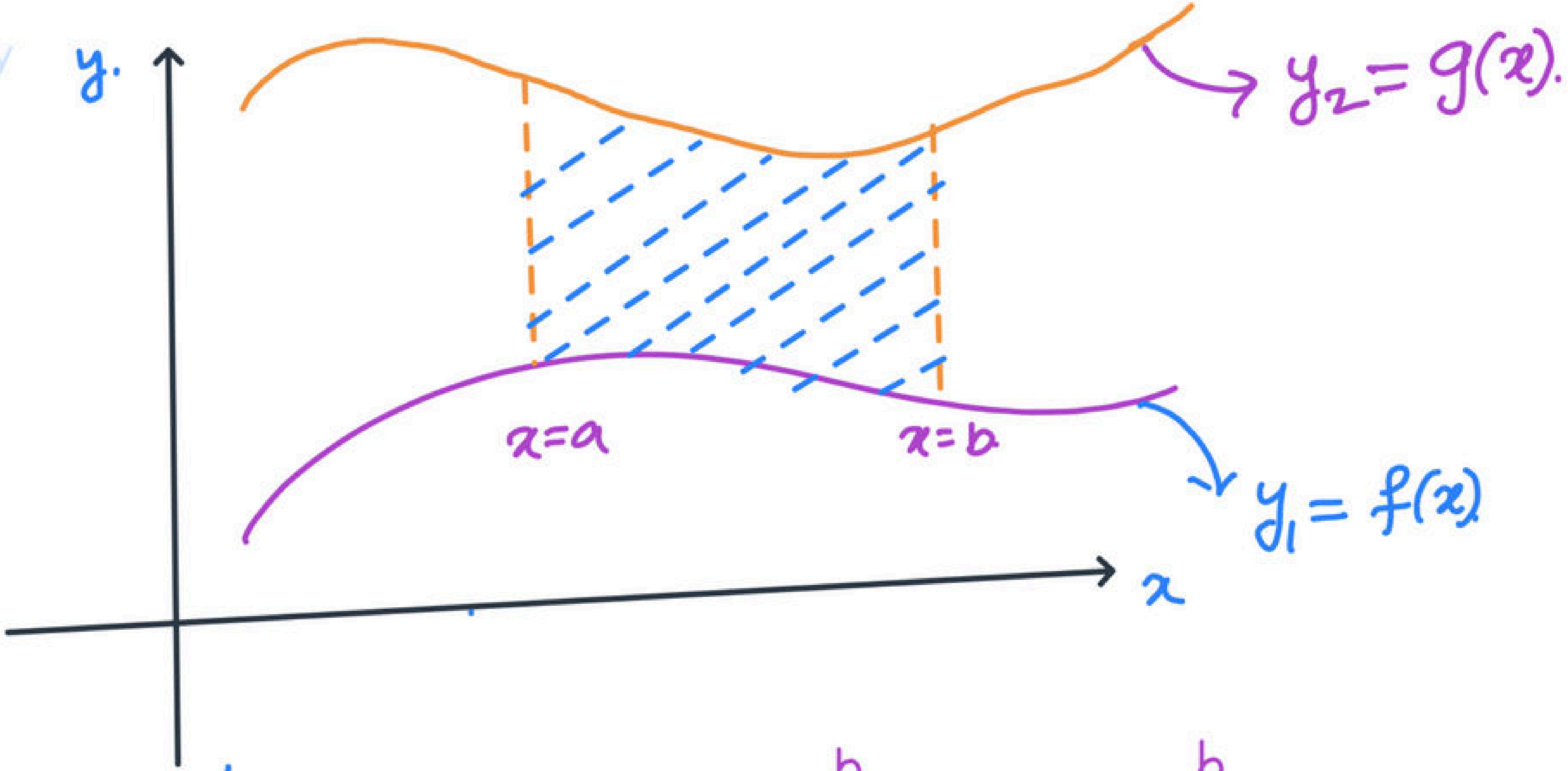
2. $x = f(y)$

Area bounded by the curve $x = f(y)$ and the y-axis from $y=a$ to $y=b$.



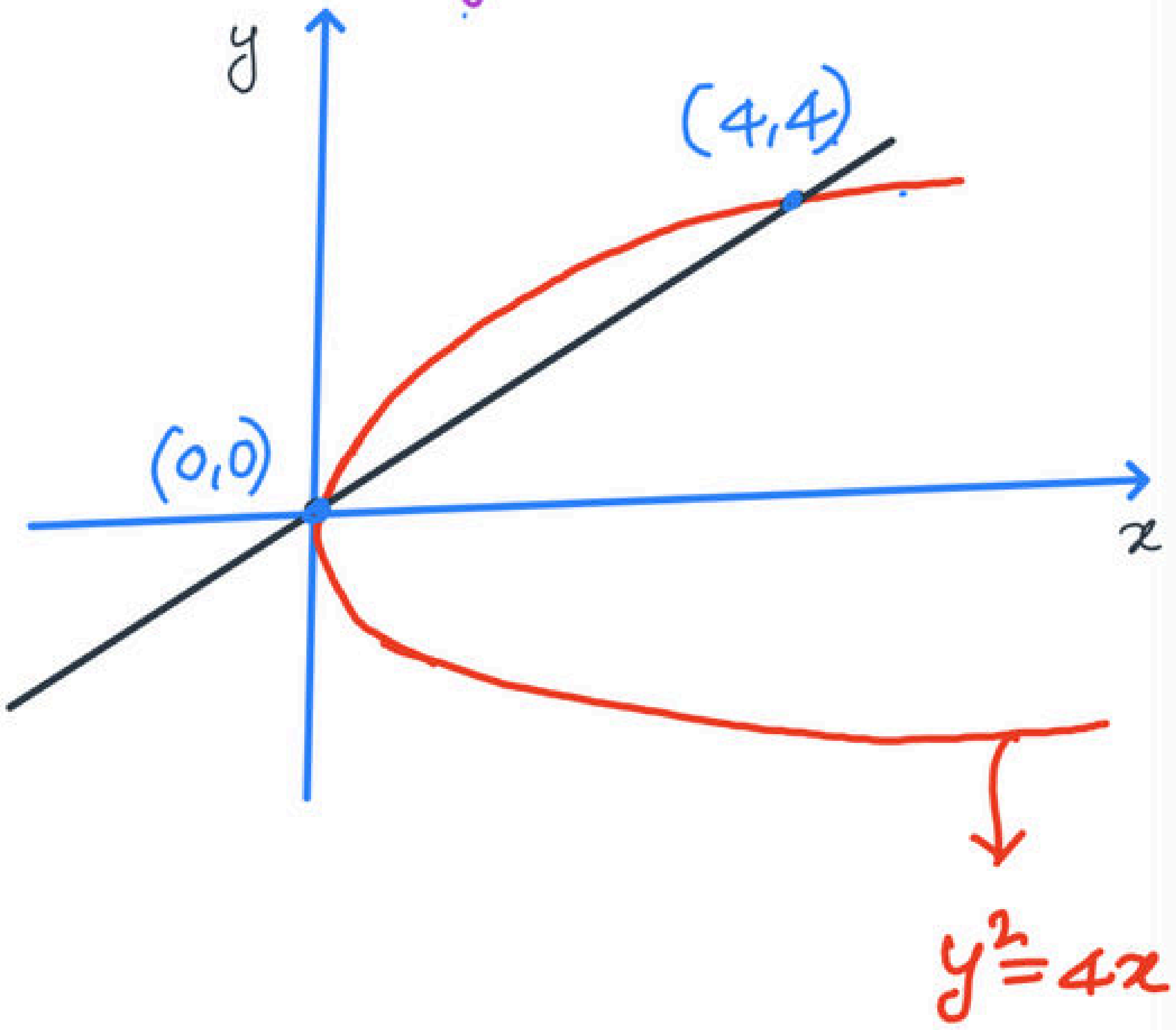
$$\text{Area} = \int_{y=a}^{b} f(y)\,dy.$$

3.



$$\text{Area} = \int_a^b (y_2 - y_1)\,dx = \int_a^b g(x)\,dx - \int_a^b f(x)\,dx.$$

Q) Find the area bounded by the curves $y^2 = 4x$ and $y = x$.



$y^2 = 4x$

$x = \frac{y^2}{4}$

$y^2 = 4x$ $\quad$ $y = x$

$y^2 = 4y$

$y(y-4) = 0$

$y = 0, \quad y = 4$

$x = 0 \quad , \quad x = 4$

$$\text{Area} = \int_{x} (y_2 - y_1)\,dx.$$

$$= \int_{x=0}^{4} (\sqrt{4x} - x)\,dx.$$

$$= 2\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\Bigg|_0^4$$

$$\text{Area} = \frac{4}{3}\left(4^{\frac{3}{2}}\right) - \frac{1}{2}\left(4^2\right)$$

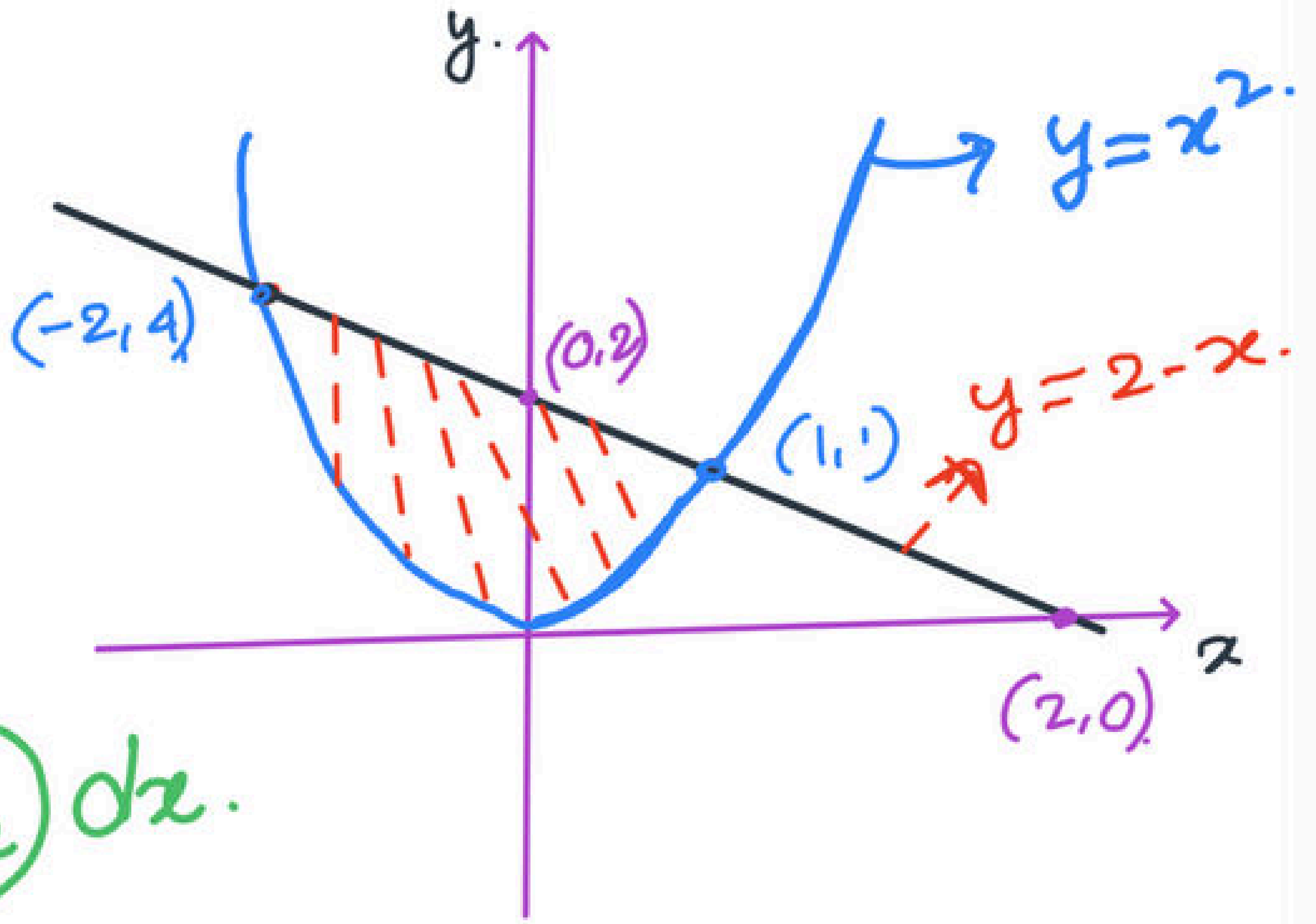
$$= \frac{4}{3}(8) - 8$$

$$= 8\left(\frac{4}{3} - 1\right)$$

$$= 8\left(\frac{1}{3}\right) = \frac{8}{3}.$$

326. The area of the region enclosed by the curve $y = x^2$ and the straight-line $x + y = 2$ is

(A) 3
(B) 27/2
(C) 9/2
(D) 9



$\frac{x}{2} + \frac{y}{2} = 1$

$y = x^2$

$2 - x = x^2$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1, -2$

(1, 1) (-2, 4)

$\text{Area} = \int_x (y_1 - y_2)\,dx$

$= \int_{-2}^{1} [2 - x - x^2]\,dx$

Area $= \int_{-2}^{1} [2 - x - x^2]\, dx.$

$\frac{1}{2}[1 - (-2)^2].$

$= \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right|_{-2}^{1}$

$= 2(1+2) - \frac{1}{2}(1-4) - \frac{1}{3}(1+8)$

$= \frac{9}{2}$

327. The area of the region bounded by the curve $x^2 = 2y$ and $y^2 = 2x$ is

(A) 1/3 (B) 2/3

(C) 4/3 (D) 4

$x^2 = 2y$

$x^2 = 2\sqrt{2x}$

$x^4 = 8x$

$x(x^3 - 8) = 0$

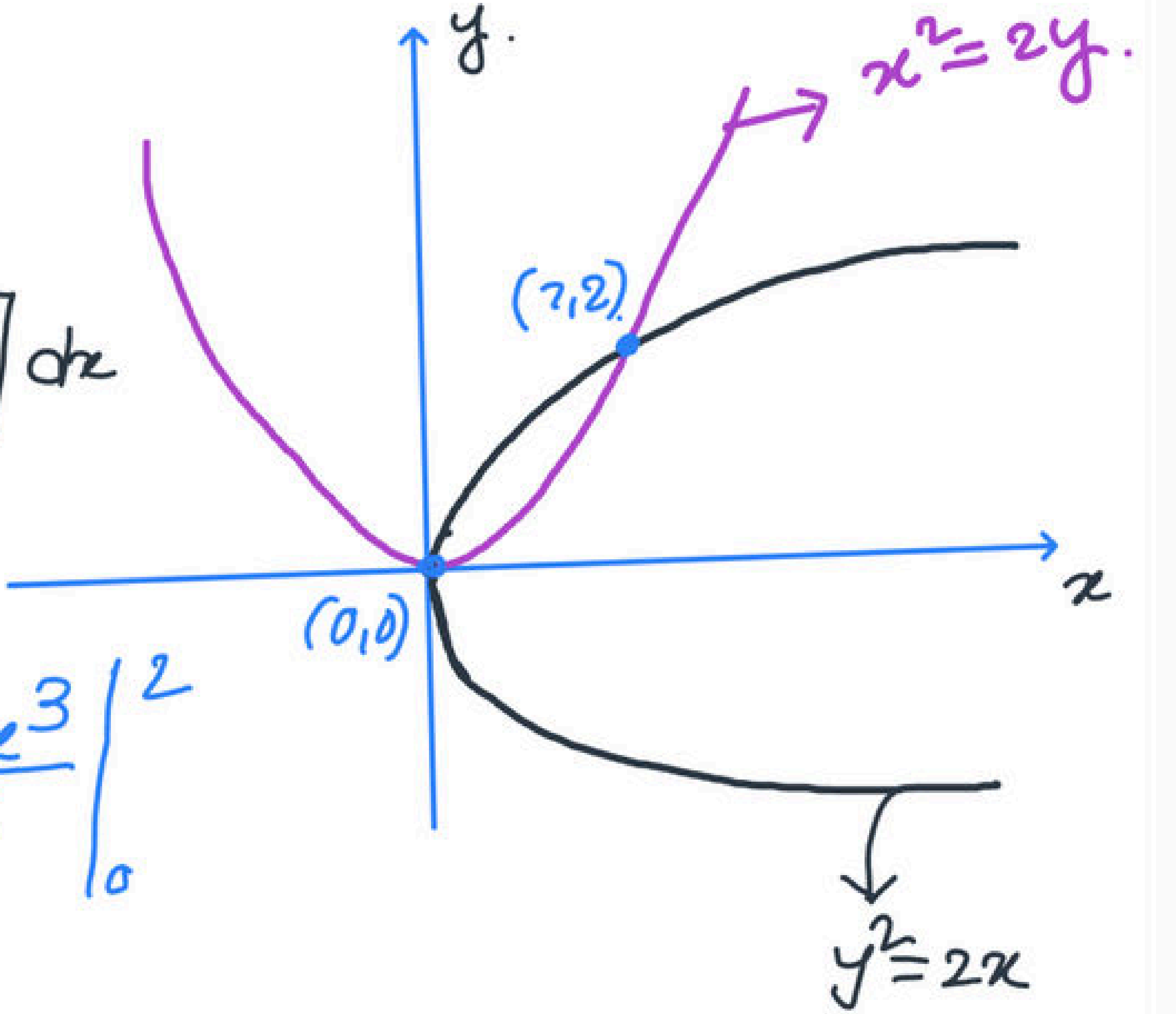
$x = 0, \quad x = 2$

$y = 0, \quad y = 2$

$$\text{Area} = \int_{x=0}^{2} \left[\sqrt{2x} - \frac{x^2}{2}\right] dx$$

$$= \left.\frac{\sqrt{2}\, x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^2 - \left.\frac{x^3}{6}\right|_0^2$$

$$= \frac{4}{3}$$



328. Area enclosed by the curves $y^2 = x$ and $y^2 = 2x-1$ lying in the first quadrant is

(A) 1/6 (B) 1/4

(C) 1/2 (D) 1/3

329. The value of $\int\int xy(x+y)dxdy$ over the area between y = x^2 and y = x

(A) 1/90 (B) 1/45

(C) 3/56 (D) 1/15

330. The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to

(a) 6 (b) 18 (c) ∞ (d) None **(GATE-ME-1995)**

$x = y - 4$

$2[x+4] = x^2$

$y = x + 4$

$x^2 - 2x - 8 = 0$

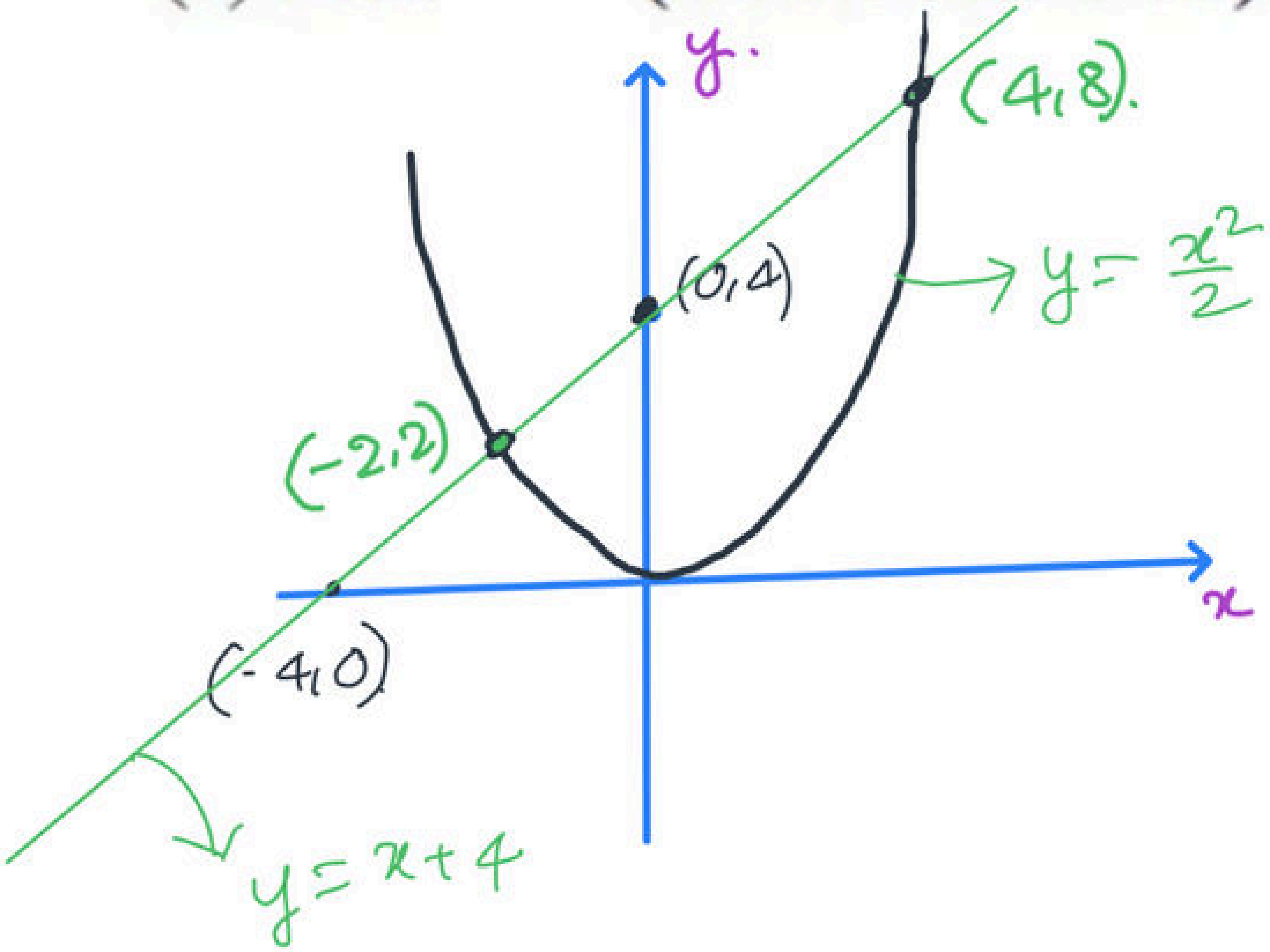
$(x-4)(x+2) = 0$

$(-2, 2)$

$(4, 8)$

$x = -2, 4$

$y = 2, 8$



$$\text{Area} = \int_{x=-2}^{4} \left[x + 4 - \frac{x^2}{2}\right] dx$$

$$\text{Area} = \int_{x=-2}^{4} \left[x + 4 - \frac{x^2}{2} \right] dx$$

$$= \frac{x^2}{2} + 4x - \frac{x^3}{6} \Big|_{-2}^{4}$$

$$= \underline{\underline{18}}$$

B. V. Reddy

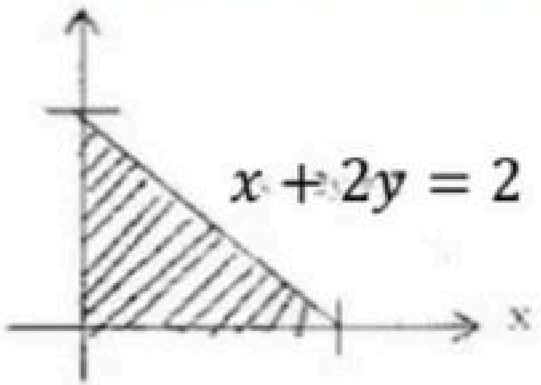
331. Area bounded by the curve $y = x^2$ and the lines $x = 4$ and $y = 0$ is given by

(a) 64 (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{128}{4}$ **(GATE-EE-1997)**

332. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is

(a) 1/8 (b) 1/6 (c) 1/3 (d) 1/2

333. Consider the shaded triangular region P shown in the figure. What is $\iint_P xy \, dx \, dy$?



(GATE-ME-2008)

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{7}{16}$ (d) 1

336. The volume under the surface $z(x, y) = x + y$ and above the triangle in the xy plane defined by $\{0 \le y \le x \text{ and } 0 \le x \le 12\}$ is ______. **(GATE-EC-SET-1-2014)**

337. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y-axis is **(GATE-EE-1994)**

(a) $\frac{128\pi}{5}$ (b) $\frac{5}{128\pi}$ (c) $\frac{127}{5\pi}$ (d) None of the above

338. The volume of an object expressed in spherical co-ordinates is given by

$V = \int_0^{2\pi} \int_0^{\pi/3} \int_0^{1} r^2 \operatorname{Sin}\phi \, dr d\phi \, d\theta$. The value of the integral **(GATE-EE-2004)**

(a) $\pi/3$ (b) $\pi/6$ (c) $2\pi/3$ (d) $\pi/4$

339. The volume enclosed by the surface $f(x, y) = \mathbf{e}^x$ over the triangle bounded by the line $x = y$, $x = 0$, $y = 1$ in the xy plane is ______ **(GATE-EE-2015)**

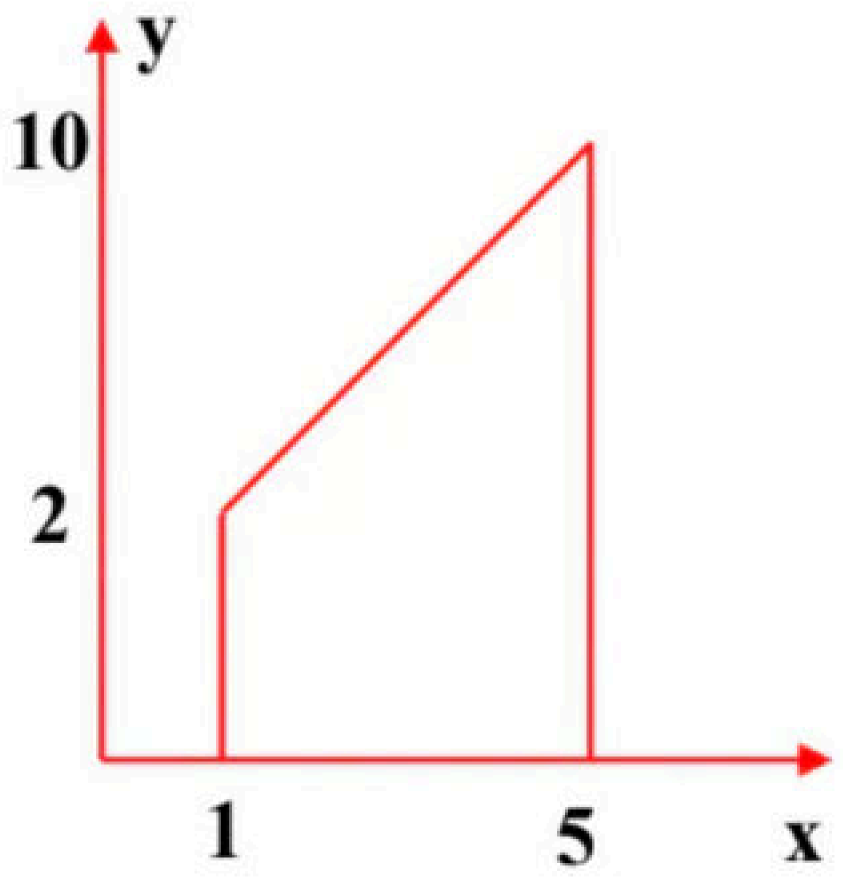
342. A triangle in the x-y plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is ________.

(GATE-16-EC)

343. Let $I = c \iint_R xy^2 dxdy$, where R is the region shown in the figure and $C = 6 \times 10^{-4}$. The value of I equals ________.

(Give the answer up to two decimal places)

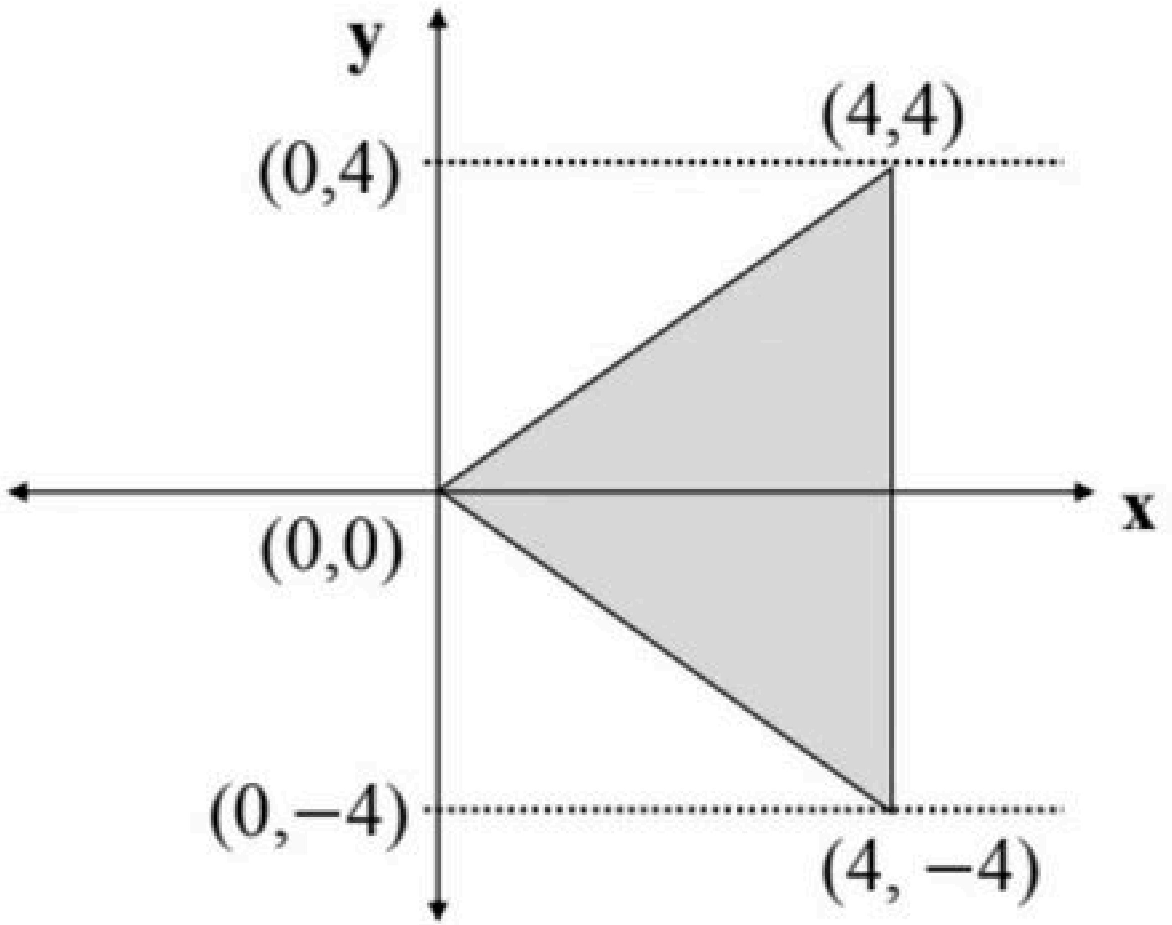
(GATE-17-EE)



349. The value the of integral $\iint_D 3(x^2 + y^2)dxdy$

where D is the shaded triangular region shown in the diagram is ________________(rounded off nearest integer).

(GATE-2022-ECE)



CALCULUS DPP

S. No	Topics	link
1	**Calculus DPP Discussion**	Class -1
2		Class -2
3		Class -3
4		Class -4
5		Class -5
6		Class -6
7		Class -7
8		Class -8
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10		Class -10
11		Class -11
12		Class -12
13		Class -13
14		Class -14
15		Class -15
16		Class -16
17		Class -17
18		Class -18
19		Class -19
20		Class -20
24		Class -21
25		Class -22
26		Class -23
27		Class -24
28		Class -25
29		Class -26
30		Class -27
31		Class -28
32		Class -29
33		Class -30
34		Class -31
35		Class -32
36		Class -33
37		Class -34
38		Class -35
39		Class -36
40		Class -37

1. $\lim_{x \to 0} \dfrac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} =$ ______ **(GATE -ME- 1993)**

2. The integration of $\int \log x \, dx$ has the value **(GATE -EC- 1994)**

(a) $(x \log x - 1)$

(b) $\log x - x$

(c) $x (\log x - 1)$

(d) None of the above

3. $\lim_{n\to\infty}\left(\sqrt{n^2+n}\right)-\sqrt{n^2+1}$ is ________ .

(GATE-16-IN)

4. $\lim_{x\to\infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} =$ ______ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 2 (d) Does not exist

5. $\lim_{x \to 0} x \sin \frac{1}{x} =$ ________ **(GATE -CS- 1995)**

(a) ∞ (b) 0 (c) 1 (d) Does not exist

6. The function $f(x) = |x+1|$ on the interval $[-2, 0]$ is ________

(GATE-EC- 1995)

(a) continuous and differentiable

(b) continuous on the interval but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

7. If a function is continuous at a point its first derivative **(GATE -EC- 1995)**

(a) may or may not exist

(b) exists always

(c) will not exist

(d) has a unique value

8. $\lim\limits_{\theta \to 0} \dfrac{\sin m\theta}{\theta}$, where m is an integer, is one of the following:

(GATE -CS- 1997)

(a) m (b) $m\pi$ (c) $m\theta$ (d) 1

9. If $y = |x|$ for $x < 0$ and $y = x$ for $x \geq 0$ then

(GATE -EC- 1997)

(a) $\frac{dy}{dx}$ is discontinuous at $x = 0$ (b) y is discontinuous at $x = 0$

(c) y is not defined at $x = 0$

(d) Both y and $\frac{dy}{dx}$ are discontinuous at $x = 0$

10. The value of

$$\lim_{x \to 1} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1}$$

Is________________________[round off to one decimal place]

(GATE-2022-PI)

11. $\lim\limits_{x\to 0} \frac{1}{10} \frac{1-e^{-j5x}}{1-e^{-jx}} =$ ______ (GATE -IN-1998)

(a) 0 (b) 1.1 (c) 0.5 (d) 1

12. Limit of the function, $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+n}}$ is ________ **(GATE -EC-1999)**

(a) ½ (b) 0 (c) ∞ (d) 1

13. Value of the function $\lim_{x \to a} (x-a)^{x-a}$ is ______ **(GATE -CS-1999)**

(a) 1 (b) 0 (c) ∞ (d) a

14. The function $f(x) = e^x$ is ________ **(GATE -EC-1999)**

(a) Even (b) Odd (c) Neither even nor odd (d) None

15. Consider the following integral $\lim_{a \to \infty} \int_1^a x^{-4}\, dx$ ______ **(GATE -CS-2000)**

(a) diverges

(b) converges to 1/3

(c) converges to $-1/a^3$

(d) converges to 0

16. Limit of the function $f(x) = \frac{1-a^4}{x^4}$ as $x \to \infty$ is given by

(GATE -CS-2000)

(a) 1 (b) e^{-a^4} (c) ∞ (d) 0

17. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sin 2\left(x - \frac{\pi}{4}\right)}{x - \frac{\pi}{4}} =$ ______ **(GATE -IN-2001)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

18. Limit of the following sequence as $n \to \infty$ is ______ $x_n = n^{\frac{1}{n}}$

(GATE -CE-2002)

(a) 0 (b) 1 (c) ∞ (d) $-\infty$

5

19. Which of the following functions is not differentiable in the domain [-1, 1]?

(a) $f(x) = x^2$ (b) $f(x) = x - 1$ **(GATE -EC-2002)**

(c) $f(x) = 2$ (d) $f(x) = \text{maximum}\ (x - x)$

20. $\lim_{x \to 0} \frac{\sin^2 x}{x} =$ ______ **(GATE-CS-2003)**

(a) 0 (b) ∞ (c) 1 (d) -1

21. The value of the function, $f(x) = \lim_{x \to 0} \frac{x^3 + x^2}{2x^3 - 7x^2}$ is _____ **(GATE-CS-2004)**

(a) 0 (b) $\frac{-1}{7}$ (b) $\frac{1}{7}$ (d) ∞

22. Consider the function $f(x) = |x|^3$, where x is real.

Then the function f(x) at x = 0 is

(GATE –IN-2007)

(a) continuous but not differentiable

(b) once differentiable but not twice

(c) twice differentiable but not thrice

(d) thrice differentiable

23. $\lim\limits_{x \to 0} \dfrac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3} =$ **(GATE-ME-2007)**

(a) 0 (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{3}$ (d) 1

24. If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots\ldots\alpha}}}$ then $y(2) =$ ------ (GATE-ME-2007)

(a) 4 (or) 1

(b) 4 only

(c) 1 only

(d) Undefined

25. What is the value of $\lim_{x \to \pi/4} \frac{\cos x - \sin x}{x - \pi/4}$ **(GATE-PI-2007)**

(a) $\sqrt{2}$ (b) 0 (c) $-\sqrt{2}$ (d) Limit does not exist

26. $\lim_{\theta \to 0} \frac{\sin(\theta/2)}{\theta}$ is **(GATE-EC-2007)**

(a) 0.5 (b) 1 (c) 2 (d) not defined

27. $\lim_{x\to\infty} \frac{x - \text{Sin}\,x}{x + \text{Cos}\,x} =$ ________ **(GATE-EC-2008)**

(a) 1 (b) -1 (c) ∞ (d) $-\infty$

28. Given $y = x^2 + 2x + 10$ the value of $\left.\frac{dy}{dx}\right|_{x=1}$ is equal to **(GATE-IN-2008)**

(a) 0 (b) 4 (c) 12 (d) 13

29. $\lim_{x \to 0} \frac{\sin x}{x}$ is **(GATE-IN-2008)**

(a) indeterminate (b) 0 (c) 1 (d) ∞

30. The value of $\lim_{x \to 8} \frac{x^{1/3} - 2}{x - 8}$ is **(GATE-ME-2008)**

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

31. The value of the expression $\lim_{x \to 0}\left[\frac{\sin(x)}{e^x x}\right]$ is **(GATE-PI-2008)**

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{1}{1+e}$

32. The total derivative of the function 'xy' is **(GATE-PI-2009)**

(a) $x\,dy + y\,dx$ (b) $x\,dx + y\,dy$ (c) $dx + dy$ (d) $dx\,dy$

33. If (x) = sin |x| then the value of $\frac{df}{dx}$ at $x = \frac{-\pi}{4}$ is **(GATE-PI-2010)**

(a) 0 (b) $\frac{1}{\sqrt{2}}$ (c) $-\frac{1}{\sqrt{2}}$ (d) 1

34. What is the value of $\lim_{n \to \alpha} \left(1 - \frac{1}{n}\right)^{2n}$? **(GATE-CS-2010)**

(a) 0 (b) e^{-2} (c) $e^{-t/2}$ (d) 1

35. The $\lim_{x\to 0}\dfrac{\sin\left(\frac{2}{3}x\right)}{x}$ is (GATE-CE-2010)

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{3}{2}$ (d) ∞

36. The function $y = |2-3x|$ **(GATE-ME-2010)**

(a) is continuous $\forall x \in R$ and differentiable $\forall x \in R$

(b) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{3}{2}$

(c) is continuous $\forall x \in R$ and differentiable $\forall x \in R$ except at $x = \frac{2}{3}$

(d) is continuous $\forall x \in R$ and except at $x = 3$ and differentiable $\forall x \in R$

37. What should be the value of λ such that the function defined below is continuous at $x = \frac{\pi}{2}$? **(GATE-CE-2011)**

$$f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x}, & \text{if } x \neq \frac{\pi}{2} \\ 1, & \text{if } x = \frac{\pi}{2} \end{cases}$$

(a) 0 (b) 2π (c) 1 (d) $\frac{\pi}{2}$

38. What is $\operatorname*{Lim}_{\theta \to 0} \frac{\operatorname{Sin}\theta}{\theta}$ equal to ? **(GATE-ME-2011)**

(a) θ (b) $\operatorname{Sin}\theta$ (c) 0 (d) 1

39. Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0$, $f(x)$ is **(GATE-ME,PI-2012)**

(a) continuous and differentiable (b) non-continuous and differentiable

(c) continuous and non-differentiable

(d) neither continuous nor differentiable

40. $\lim_{x \to 0}\left(\frac{1-\cos x}{x^2}\right)$ is **(GATE-ME,PI-2012)**

(a) 1/4 (b) 1/2 (c) 1 (d) 2

41. A function $y = 5x^2 + 10x$ is defined over an open interval $x = (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly **(GATE-EE-2013)**

(a) 20 (b) 25 (c) 30 (d) 35

42. Which one of the following functions is continuous at $x = 3$?

(GATE-CS-2013)

(a) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \\ \dfrac{x+3}{3}, & \text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8-x & \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x+3, & \text{if } x \leq 3 \\ x-4 & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3-27}$, if $x \neq 3$

43. The value of $\lim_{x \to \infty}\left(1+\frac{1}{x}\right)^{x}$ is (GATE-EC-SET-2-2014)

(a) $\ln 2$ (b) 1.0 (c) e (d) ∞

44. $\underset{x \to 0}{\text{Lt}} \dfrac{x - \sin x}{1 - \cos x}$ is (GATE-ME-SET-1-2014)

(a) 0 (b) 1 (c) 3 (d) not defined

45. $\underset{x \to 0}{\mathrm{Lt}} \left(\dfrac{e^{2x} - 1}{\sin(4x)} \right)$ is equal to **(GATE-ME-SET-2-2014)**

(a) 0 (b) 0.5 (c) 1 (d) 2

46.If a function is continuous at a point, **(GATE-ME-SET-3-2014)**

(a) the limit of the function may not exist at the point

(b) the function must be derivable at the point

(c) the limit of the function at the point tends to infinity

(d) the limit must exist the point and the value of limit should be same as the value of the function at the point.

47. $\lim_{x \to \infty} \left(\frac{x + \sin x}{x} \right)$ equal to **(GATE-CE-SET-1-2014)**

(a) $-\infty$ (b) 0 (c) 1 (d) ∞

48. The expression $\lim_{a\to 0} \frac{x^a - 1}{a}$ is equal to **(GATE-CE-SET-2-2014)**

(a) $\log x$ (b) 0 (c) $x \log x$ (d) ∞

49. The function $f(x) = x \sin x$ satisfies the following equation:

$f''(x) + f(x) + t \cos x = 0$. The value of t is ______.

(GATE-CS-SET-1-2014)

50. The value of $\lim_{x\to\infty} \frac{\sqrt{9x^2+2020}}{x+7}$ is GATE-2021 (CE)

(a) 1

(b) 3

(c) $\frac{7}{9}$

(d) Indeterminable

51. $\lim_{x \to \infty} x^{1/x}$ is (GATE-CS-2015)

(a) ∞ (b) 0 (c) 1 (d) Not defined

52. The limit $p = \lim_{x \to \pi} \left(\frac{x^2 + \alpha x + 2\pi^2}{x - \pi + 2\sin x} \right)$

has a finite value of real α. The value of α and the corresponding limit p are

(GATE-2022-ME)

(a) $\alpha = -3\pi$, and $p = \pi$

(b) $\alpha = -2\pi$, and $p = 2\pi$

(c) $\alpha = \pi$, and $p = \pi$

(d) $\alpha = 2\pi$, and $p = 3\pi$

53. The value of $\lim_{x\to\infty}(1+x^2)^{e^{-x}}$ is (GATE-CS-2015)

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) ∞

54. $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x}$ is equal to **(GATE-CE-2015)**

(a) e^{-2} (b) e (c) 1 (d) e^{2}

55. The value of $\lim_{x\to\infty} \frac{1-\cos(x^2)}{2x^4}$ is (GATE-ME-2015)

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) undefined

56. At $x = 0$, the function $f(x) = |x|$ has **(GATE-ME-2015)**

(a) A minimum (b) A maximum

(c) A point of inflextion (d) neither a maximum nor minimum

57. The value of $\lim_{x\to 0}\left(\dfrac{-\sin x}{2\sin x - x\cos x}\right)$ is ______ **(GATE-ME-2015)**

58. The value of $\lim_{x\to\infty} \frac{x^2-5x+4}{4x^2+2x}$ is

GATE-2020 (CE)

(a) 0

(b) 1

(c) $\frac{1}{4}$

(d) $\frac{1}{2}$

59. Consider the limit:

GATE-2021 (CE)

$$\lim_{x \to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$$

The limit (correct up to one decimal place) is ______

60. The value of $\lim_{x \to \infty} \frac{x \ln(x)}{1+x^2}$ is

GATE-20201(CE)

(a) 1.0

(b) 0.5

(c) ∞

(d) 0

61. The function $f(x,y) = x^2y - 3xy + 2y + x$ has

(GATE-ME-1994)

(a) No local extreme

(b) One local maximum but no local minimum

(c) One local minimum but no local maximum

(d) One local minimum and one local maximum

62. The function $y = x^2 + \frac{250}{x}$ at $x = 5$ attains

(GATE-EC-1994)

(a) Maximum (b) Minimum (c) Neither (d) 1

63. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has

(GATE-EE-1995)

(a) a maxima at $x = 1$ and a minima at $x = 3$

(b) a maxima at $x = 3$ and a minima at $x = 1$

(c) no maxima, but a minima at $x = 3$

(d) a maxima at $x = 1$, but no minima

64. Find the points of local maxima and minima if any of the following function defined in

$0 \le x \le 6$, $f(x) = x^3 - 6x^2 + 9x + 15$.

(GATE-CS-1998)

65. The continuous function f(x,y) is said to have saddle point at (a, b) if

(a) $f_x(a, b) = f_y(a, b) = 0$, $f_{xy}^2 - f_{xx}f_{yy} < 0$ at (a, b)

(GATE-EE-1998)

(b) $f_x(a, b) = 0$, $f_y(a, b) = 0$, $f_{xy}^2 - f_{xx}f_{yy} > 0$ at (a, b)

(c) $f_x(a, b) = 0$, $f_y(a, b) = 0$, f_{xx} and $f_{yy} < 0$ at (a, b)

(d) $f_x(a, b) = 0$, $f_y(a, b) = 0$, $f_{xy}^2 - f_{xx}f_{yy} = 0$ at (a, b)

66. Number of inflection points for the curve $y = x + 2x^4$ is _____

(GATE-CE-1999)

(a) 3 (b) 1 (c) 0 (d) 2

67. If $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$, $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to ____

(GATE-EC-2000)

(a) 0 (b) 1 (c) 2 (d) $-3(x^2 + y^2 + z^2)^{-5/2}$

68. The following function has local minima at which value of x,

$$f(x) = x\sqrt{5 - x^2}$$

(a) $\frac{-\sqrt{5}}{2}$ (b) $\sqrt{5}$ (c) $\sqrt{\frac{5}{2}}$ (d) $-\sqrt{\frac{5}{2}}$

69. The function $f(x,y) = 2x^2 + 2xy - y^3$ has **(GATE-EC-2000)**

(a) Only one stationary point at (0, 0)

(b) Two stationary points at (0, 0) and (1/6, -1/3)

(c) Two stationary points at (0, 0) and (1, -1)

(d) No stationary point

70. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at

(GATE-CS-2004)

(a) $x = -2$ only

(b) $x = 0$ only

(c) $x = 3$ only

(d) both $x = -2$ and $x = 3$

71. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to

(a) 2 (b) 1 (c) 0 (d) -1

72. For real x, the maximum value of $\frac{e^{\text{Sinx}}}{e^{\text{Cosx}}}$ is

(GATE-IN-2007)

(a) 1 (b) e (c) $e^{\sqrt{2}}$ (d) ∞

73. The minimum value of function $y = x^2$ in the interval [1, 5] is

(GATE-ME-2007)

(a) 0 (b) 1 (c) 25 (d) undefined

74. For the function $f(x,y) = x^2 - y^2$ defined on R^2, the point (0, 0) is **(GATE-PI-2007)**

(a) a local minimum

(b) Neither a local minimum (nor) a local maximum

(c) a local maximum

(d) Both a local minimum and a local maximum

75. Consider the function $f(x) = x^2 - x - 2$. The maximum value of f(x) in the closed interval [-4, 4] is

(GATE-EC-2007)

(a) 18 (b) 10 (c) -2.25 (d) indeterminate

76. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has

(GATE-EE-2007)

(a) Only one minimum

(b) Only two minima

(c) Three minima

(d) Three maxima

77. A point on the curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is ________

(GATE-CS-2008)

(a) 0 (b) 1 (c) 2 (d) 3

78. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 is **(GATE-IN-2008)**

(a) 1 (b) 3 (c) 4 (d) 9

79. For real values of x, the minimum value of function

$f(x) = e^x + e^{-x}$ is

(GATE-EC-2008)

(a) 2 (b) 1 (c) 0.5 (d) 0

80. At $t = 0$, the function $f(t) = \frac{\sin t}{t}$ has

(GATE-EE-2010)

(a) a minimum

(b) a discontinuity

(c) a point of inflection

(d) a maximum

81. If $e^y = x^{1/x}$ then y has a

(GATE-EC-2010)

(a) maximum at $x = e$ (b) minimum at $x = e$

(c) maximum at $x = e^{-1}$ (d) minimum at $x = e^{-1}$

82. Given a function $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$, the optimal values of $f(x, y)$ is

(GATE-CE-2010)

(a) a minimum equal to $\frac{10}{3}$

(b) a maximum equal to $\frac{10}{3}$

(c) a minimum equal to $\frac{8}{3}$

(d) a maximum equal to $\frac{8}{3}$

83. The function $f(x) = 2x - x^2 + 3$ has

(GATE-EE-2011)

(a) a maxima at $x = 1$ and a minima at $x = 5$

(b) a maxima at $x = 1$ and a minima at $x = -5$

(c) only a maxima at $x = 1$

(d) only a minima at $x = 1$

84. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is

(a) 21 (b) 25 (c) 41 (d) 46

GATE- 2012

85. At $x = 0$, the function $f(x) = x^3 + 1$ has

(GATE-ME,PI-2012)

(a) a maximum value

(b) a minimum value

(c) a singularity

(d) a point of inflection

86. For $0 \leq t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at

(a) $t = \log_e 4$ (b) $t = \log_e 2$ (c) $t = 0$ (d) $t = \log_e 8$

87. The maximum value of the function $f(x) = \ln(1+x) - x$ (where $x > -1$) occurs at $x =$ ____________. **(GATE-EC-SET-3-2014)**

88. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is ____.

GATE-2014

2

89. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is

(a) e^{-1} (b) e (c) $1 - e^{-1}$ (d) $1 + e^{-1}$

GATE-2014

90. Minimum of the real valued function $f(x) = (x-1)^{2/3}$ occurs at x equal to

(a) $-\infty$ (b) 0 (c) 1 (d) ∞

GATE-2014

91. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval $[-3, 3]$ is **(GATE-EC-SET-2-2014)**

(a) 20 (b) 28 (c) 16 (d) 32

0

92. While minimizing the function $f(x)$, necessary and sufficient conditions for a point, x_0 to be a minima are : **(GATE-CE-2015)**

(a) $f'(x_0) > 0$ and $f'(x_0) = 0$
(b) $f'(x_0) < 0$ and $f''(x_0) = 0$
(c) $f'(x_0) = 0$ and $f''(x_0) < 0$
(d) $f'(x_0) = 0$ and $f''(x_0) > 0$

93. The value of ε in the mean value theorem of $f(b) - f(a) = (b-a) f'(\varepsilon)$ for $f(x) = Ax^2 + Bx + C$ in (a, b) is **(GATE-EC-1994)**

(a) $b + a$ (b) $b - a$ (c) $\frac{b+a}{2}$ (d) $\frac{b-a}{2}$

94. If $f(0) = 2$ and $f'(x) = \frac{1}{5 - x^2}$, then the lower and upper bounds of f(1) estimated by the mean value theorem are ________ f(x) is defined in [0 ,1] **(GATE-EC-1995)**

(a) 1.9, 2.2 (b) 2.2, 2.25 (c) 2.25, 2.5 (d) None of the above

95. A function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval [-1, 1]. The value of x, in the open interval (-1, 1) for which the mean value theorem is satisfied, is

(GATE-EC-2015)

(a) -1/2 (b) -1/3 (c) 1/3 (d) 1/2

96. If a continuous function f(x) does not have a root in the interval [a, b], then which one of the following statements is TRUE ? **(GATE-EE-2015)**

(a) $f(a).f(b) = 0$ (b) $f(a).f(b) < 0$ (c) $f(a).f(b) > 0$ (d) $f(a)/f(b) \leq 0$

97. The third term in the taylor's series expansion of e^x about 'a' would be ______

(a) $e^a (x-a)$ (b) $\frac{e^a}{2}(x-a)^2$ (c) $\frac{e^a}{2}$ (d) $\frac{e^a}{6}(x-a)^3$ GATE -1995

98. The taylor's series expansion of sin x is __________ **(GATE-EC-1998)**

(a) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\ldots\ldots\ldots$

(b) $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\ldots\ldots\ldots$

(c) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots\ldots\ldots$

(d) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots\ldots$

99. A discontinuous real function can be expressed as **(GATE-CE-1998)**

(a) Taylor's series and Fourier's series

(b) Taylor's series and not by Fourier's series

(c) neither Taylor's series nor Fourier's series

(d) not by Taylor's series, but by Fourier's series

100. The Taylor series expansion of sin x about $x = \frac{\pi}{6}$ is given by **(GATE-CE-2000)**

(a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\Pi}{6}\right)-\frac{1}{4}\left(x-\frac{\Pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\Pi}{6}\right)^3+----$

(b) $x - \frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+------$

(c) $\frac{x-\frac{\Pi}{6}}{1!}-\frac{\left(x-\frac{\Pi}{6}\right)^3}{3!}+\frac{\left(x-\frac{\Pi}{6}\right)^5}{5!}-\frac{\left(x-\frac{\Pi}{6}\right)^7}{7!}+-----$

(d) $\frac{1}{2}$

101. Limit of the following series as x approaches

$\frac{\pi}{2}$ is $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + - - - - - -$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) 1 **(GATE-CE-2001)**

102. For the function e^{-x}, the linear approximation around $x = 2$ is

(a) $(3-x)e^{-2}$

(b) $1 - x$

GATE- 2007

(c) $\left[3 + 2\sqrt{2} - \left(1 + \sqrt{2}\right)x\right]e^{-2}$

(d) e^{-2}

103. For $|x| << 1$, $\coth(x)$ can be approximated as **(GATE-EC-2007)**

(a) x (b) x^2 (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

104. The expression $e^{\ln x}$ for $x > 0$ is equal to **(GATE-IN-2008)**

(a) $-x$ (b) x (c) x^{-1} (d) $-x^{-1}$

105. Which of the following function would have only odd powers of x in its Taylor series expansion about the point $x = 0$? **(GATE-EC-2008)**

(a) $\sin(x^3)$ (b) $\sin(x^2)$ (c) $\cos(x^3)$ (d) $\cos(x^2)$

106. In the Taylor series expansion of $e^x + \sin x$ about the point $x = \pi$, the coefficient of $(x-\pi)^2$ is **(GATE-EC-2008)**

(a) e^π (b) $0.5\ e^\pi$ (c) $e^\pi + 1$ (d) $e^\pi - 1$

107. In the Taylor series expansion of e^x about $x = 2$, the coefficient of $(x-2)^4$ is

(GATE-ME-2008)

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$ (c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

108. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by **(GATE-EC-2010)**

(a) $1+\frac{(x-\pi)^2}{3!}+------$

(b) $-1-\frac{(x-\pi)^2}{3!}+------$

(c) $1-\frac{(x-\pi)^2}{3!}+------$

(d) $-1+\frac{(x-\pi)^2}{3!}+------$

109. The infinite series $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots\ldots$ Converges to **(ME-2010)**

(a) $\cos(x)$ (b) $\sin(x)$ (c) $\sinh(x)$ (d) e^x

110. A series expansion for the function $\sin\theta$ is ________ (GATE-ME-2011)

(a) $1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}-\ldots\ldots\ldots$

(b) $\theta-\frac{\theta^3}{3!}+\frac{\theta^5}{5!}-\ldots\ldots\ldots\ldots\ldots$

(c) $1+\theta+\frac{\theta^2}{2!}+\frac{\theta^3}{3!}+\ldots\ldots$

(d) $\theta+\frac{\theta^3}{3!}+\frac{\theta^5}{5!}+\ldots\ldots\ldots\ldots$

111. Consider the following inequalities.

(i) $3p - q < 4$

(ii) $3q - p < 12$

Which one of the following expressions below satisfies the above two inequalities?

(GATE-2022-PI)

(a) $8 \leq p + q < 16$ (b) $p + q = 8$

(c) $p + q \geq 16$ (d) $p + q < 8$

112. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots$ corresponds to **(GATE-CE-2012)**

(a) $\sec x$ (b) e^x (c) $\cos x$ (d) $1+\sin^2 x$

113. The Taylor series expansion of 3 sin x + 2cos x is **(GATE-EC-SET-1-2014)**

(a) $2 + 3x - x^2 - \frac{x^3}{2} + ---$

(b) $2 - 3x + x^2 - \frac{x^3}{2} + ---$

(c) $2 + 3x + x^2 + \frac{x^3}{2} + ---$

(d) $2 - 3x - x^2 + \frac{x^3}{2} + ---$

114. In which of the following functions Mean Value theorem is not applicable?

(i) $y = \frac{1}{x}, x \in [-1, 1]$

(ii) $y = |x|, x \in [-1, 1]$

(iii) $y = x \sin\frac{1}{x}, x \in \left[+\frac{\pi}{4}, \frac{\pi}{2}\right]$

(iv) $y = \sin k \left[0, \frac{\pi}{2}\right]$

(A) (i), (ii), (iii) (B) (i), (ii)

(C) (i), (iii) (D) (i), (ii), (iii), (iv)

115. Which one of the following not the correct statement?

(GATE-19-CE)

(a) The function $\sqrt[x]{x}$, $(x > 0)$, has the global minimum at $x = e$

(b) The function $|x|$ has the global minima at $x = 0$

(c) The function x^3 has neither global minima nor global maxima

(d) The function $\sqrt[x]{x}$, $(x > 0)$, has the global maxima at $x = e$

116. The following inequality is true for all x close to zero. $\left(2-\frac{x^2}{3}\right) < \frac{x\sin x}{1-\cos x} < 2$, what is the value $\lim_{x\to 0}\frac{x\sin x}{1-\cos x}$?

(GATE-19-CE)

(a) 2 (b) 1 (c) 0 (a) $\frac{1}{2}$

117. If f(x) satisfies Rolles theorem on [a,b], then the value of $\int_a^b f'(x)\,dx$ is ____.

(A) f(b)-f(a)
(B) f(a)
(C) f(b)
(D) 0

118. Consider the polynomial $f(x) = x^3 - 6x^2 + 11x - 6$ on the domain S given by $1 \le x \le 3$. the first and second derivatives are $f'(x)$ and $f''(x)$.

Consider the following statements.

I. The given polynomial is zero at the boundary points $x = 1$ and $x = 3$

II. There exists one local maxima of f(x) within the domain S.

III. The second derivative $f''(x) > 0$ throughout the domain S.

IV. There exists one local minima of f(x) within the domain S.

The correct option is.

(GATE-2022-CE)

(a) Only statements I, II and III are correct

(b) Only statements I, II and IV are correct

(c) Only statements I and IV are correct

(d) Only statements II and IV are correct

119. The value of the following limit is ______________

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{1-e^{2\sqrt{x}}}$$

(GATE-2022-CSE)

120. A function y(x) is defined in the interval [0, 1] on the x-axis as

$$y(x) = \begin{cases} 2 \text{ if } 0 \le x < \frac{1}{3} \\ 3 \text{ if} \frac{1}{3} \le x \le \frac{3}{4} \\ 1 \text{ if} \frac{3}{4} \le x \le 1 \end{cases}$$

Which one of the following is the area under the curve for the interval [0,1] on the x-axis?

(GATE-2022-CSE)

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$

(c) $\frac{13}{6}$ (d) $\frac{6}{13}$

121. Let 'r' be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is?

(GATE-2022-CSE)

(a) 51
(b) −51
(c) 126
(d) −126

122. Define $[x]$ as the greatest integer less than or equal to x, for each $x \in (-\infty, \infty)$. If $y = [x]$, then area under y for $x \in [1, 4]$ is ______.

GATE- 2020 (ME)

(a) 3 (b) 1

(c) 6 (d) 4

123. The value of $\lim_{x\to 1}\left(\frac{1-e^{-c(1-x)}}{1-xe^{-c(1-x)}}\right)$

GATE- 2020 (ME)

(a) $c + 1$

(b) $\frac{c+1}{c}$

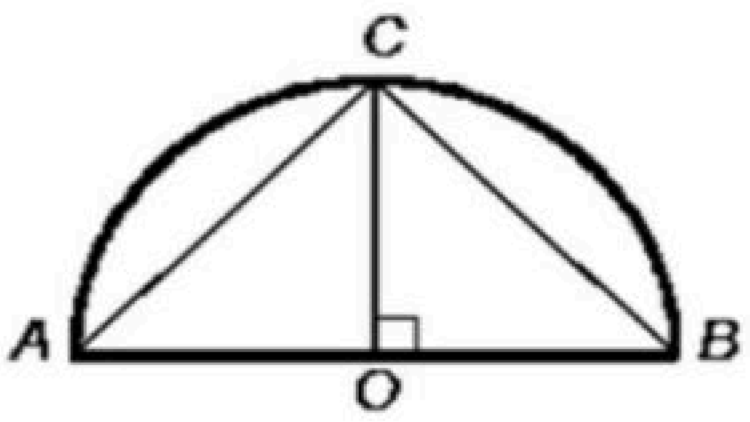
(c) c

(d) $\frac{c}{c+1}$

124. Given a semicircle with O as the centre; as shown in the figure, the ratio $\frac{\overline{AC}+\overline{CB}}{\overline{AB}}$ is

________ . Where $\overline{AC}$, $\overline{CB}$ and $\overline{AB}$ are chords.

GATE- 2020 (EE)



(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) 2

(d) 3

125. The real numbers, x and y with $y = 3x^2 + 3x + 1$, the maximum and minimum value of y for $x \in [-2, 0]$ are respectively ________ .

GATE- 2020 (EE)

(a) 7 and $\frac{1}{4}$

(b) 7 and 1

(c) -2 and $\frac{-1}{2}$

(d) 1 and $\frac{1}{4}$

126. If $f(x) = x^2$ for each $x \in (-\infty, \infty)$, then $\frac{f(f(f(x)))}{f(x)}$ is equal to ________.

GATE- 2020 (CE)

(a) $f(x)$

(b) $(f(x))^4$

(c) $(f(x))^2$

(d) $(f(x))^3$

127. Consider the functions:

I. e^{-x}

II. $x^2 - \sin x$

III. $\sqrt{x^3 + 1}$

Which of the above functions is/are increasing everywhere in [0,1]?

(a) I and III only (b) II and III only

(c) III only (d) II only

GATE- 2020 (CS)

128. Consider the function $f(x, y) = x^2 + y^2$. The minimum value of the function attains on the line $x + y = 1$ (rounded off to two decimal places) is _____.

GATE- 2020 (IN)

129. Let f(x) be a real-valued function such that $f'(x_0)=0$ for some $x_0 \in (0,1)$, and $f''(x) > 0$ for all $x \in (0,1)$. Then f(x) has

(a) exactly one local minimum in (0,1)

(b) one local maximum in (0,1)

(c) no local minimum in (0,1)

(d) two distinct local minima in (0,1)

GATE- 2021 (EE)

130. A function, λ, is defined by

GATE- 2021 (CE)

$$\lambda(p,q)=\begin{Bmatrix}(p-q)^2, & \text{if } p \geq q \\ p+q, & \text{if } p < q\end{Bmatrix}$$

The value of the expression $\dfrac{\lambda(-(-3+2),(-2+3))}{(-(2+1))}$ is

(a) $\dfrac{16}{3}$

(b) -1

(c) 0

(d) 16

131. If $\left(x-\frac{1}{2}\right)^2-\left(x-\frac{3}{2}\right)^2=x+2$, then the value of x is :

GATE- 2021 (CS)

(a) 2 (b) 8

(c) 4 (d) 6

132. Suppose that $f : R \rightarrow R$ is a continuous function on the interval $[-3, 3]$ and a differentiable function in the interval $(-3, 3)$ such that for every x in the interval. $f'(x) \leq 2$. If $f(-3) = 7$, then then $f(3)$ is at most ________.

GATE- 2021 (CS)

133. Consider the following expression:

$$\lim_{x \to -3} \frac{\sqrt{2x+22}-4}{x+3}$$

GATE- 2021 (CS)

The value of the above expression (rounded to 2 decimal places) is ______.

134. p and q are positive integers and $\frac{p}{q}+\frac{q}{p}=3$,

GATE- 2021 (CS)

then, $\frac{p^2}{q^2}+\frac{q^2}{p^2}=$

(a) 3
(b) 9
(c) 11
(d) 7

135. A straight line of the form $y = mx + c$ passes through the origin and the point $(x, y) = (2,6)$. The value of m is ________.

(GATE-16-EC)

136. Consider the function $f(x) = -x^2 + 10x + 100$. The minimum value of the function in the interval [5, 10] is ______.

GATE- 2021 (CS)

137. Let $f: [-1,1] \rightarrow R$, where $f(x) = 2x^3 - x^4 - 10$. The minimum value of $f(x)$ is ________.

(GATE-16-IN)

138. Consider the function $f(x) = 2x^3 - 3x^2$ in the domain [-1, 2]. The global minimum of f(x) is ________.

(GATE-16-ME)

139. The values of x for which the function $f(x) = \frac{x^2-3x-4}{x^2+3x-4}$ is NOT continuous are

(GATE-16-ME)

(a) 4 and -1 (b) 4 and 1

(c) -4 and1 (d) -4 and -1

140. $\lim_{x \to 0} \frac{\log_e(1+4x)}{e^{3x}-1}$ is equal to

(GATE-16-ME)

(a) 0 (b) $\frac{1}{12}$

(c) $\frac{4}{3}$ (d) 1

141. $\lim_{x \to \infty} \left(\sqrt{x^2 + x - 1} - x \right)$ is

(GATE-16-ME)

(a) 0

(b) ∞

(c) $\frac{1}{2}$

(d) $-\infty$

142. At x = 0, the function is

$$f(x) = \left|\sin\frac{2\pi x}{L}\right| (-\infty < x < \infty, L > 0)$$

(GATE-16-PI)

(a) continuous and differentiable.

(b) not continuous and not differentiable.

(c) not continuous but differentiable.

(d) continuous but not differentiable

143. Absolute maxima or minima of a function $f(x)$ occur

(A) Only at the end point of the curve

(B) Only at the critical points of the curve

(C) Both at the end point and critical points of the curve

(D) Either at the end point or at the critical point of the curve

144. The range of values of k for which the function $f(x) = (k^2 - 4)x^2 + 6x^3 + 8x^4$ has a local maxima at point $x = 0$ is **(GATE-16-PI)**

(a) $k < -2$ or $k > 2$ (b) $k \leq -2$ or $k \geq 2$

(c) $-2 < k < 2$ (d) $-2 \leq k \leq 2$

145. $\lim_{x \to 0} \left(\frac{e^{5x}-1}{x}\right)^2$ is equal to ________.

(GATE-16-PI)

146. $\lim_{x \to 4} \frac{\sin(x-4)}{x-4} =$ ________.

(GATE-16-CSE)

147. The quadratic approximation of $f(x) = x^3 - 3x^2 - 5$ at the point $x = 0$ is

(GATE-16-PI)

(a) $3x^2 - 6x + 5$

(b) $-3x^2 - 5$

(c) $-3x^2 + 6x - 5$

(d) $3x^2 - 5$

148. The minimum value of the function $f(x) = \frac{1}{3}x(x^2 - 3)$ in the interval $-100 \le x \le 100$ occurs at $x =$ __________.

(GATE-17-EC)

149. Let $f(x) = e^{x+x^2}$ for real x. From among the following. Choose the Taylor series approximation of f(x) around x = 0, which includes all powers of x less than or equal to 3.

(GATE-17-EC)

(a) $1 + x + x^2 + x^3$

(b) $1 + x + \frac{3}{2}x^2 + x^3$

(c) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$

(d) $1 + x + 3x^2 + 7x^3$

150. A function f(x) is defined as $f(x) = \begin{cases} e^x, & x < 1 \\ \ln x + ax^2 + bx, & x \geq 1 \end{cases}$, where $x \in R$. Which one of the following statements is TRUE?

(GATE-17-EE)

(a) f(x) is NOT differentiable at x = 1 for any values of a and b.

(b) f(x) is differentiable at x = 1 for the unique values of a and b.

(c) f(x) is differentiable at x = 1 for all values of a and b such that a + b = e.

(d) f(x) is differentiable at x = 1 for all values of a and b.

151. Consider the following inequalities.

(i) $2x-1>7$ (ii) $2x-9<1$

Which one of the following expressions below satisfies the above two inequalities?

(GATE-2022-ECE)

(a) $x \leq -4$ (b) $-4 < x \leq 4$

(c) $4 < x < 5$ (d) $x \geq 5$

152. The function $f(x) = 8 \log x - x^2 + 3$ attains its minimum over the interval $[1,e]$ at $x =$__________________ (Here $\log_e x$ is the natural logarithm of x.)

(GATE-2022-ECE)

(a) 2 (b) 1

(c) e (d) $\frac{1+e}{2}$

153. In the open interval (0,1), the polynomial $p(x) = x^4 - 4x^3 + 2$ has

(a) no real roots

(b) two real roots

(c) one real root

(d) three real roots

154. Let $y^2 - 2y + 1 = x$ and $\sqrt{x} + y = 5$. The value of $x + \sqrt{y}$ equals ________. (Given the answer up to three decimal places)

(GATE-17-EE)

155. Rolle's theorem cannot be applicable for

(A) $f(x)=\sqrt{4-x^2}$ in $[-2, 2]$

(B) $f(x)=[x]$ in $[-1,1]$

(C) $f(x)=x^2+3x-4$ in $[-4,1]$

(D) $f(x)=\cos 2x$ in $[0, \pi]$

156. The number C that satisfy the conclusion of mean value theorem for $f(x) = x + (4/x)$ in the interval [1, 8] is

(a) 4.5
(b) 3.5
(c) $2\sqrt{2}$
(d) 5

157. If $f'(x) = \frac{1}{1+x^2}$ for all x & $f(0) = 0$ then an interval in which $f(2)$ lies, is

(a) (2, 4)

(b) (0.4, 2)

(c) (0, 0.4)

(d) (0.1, 0.2)

158. The value C of Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in the interval (2, 3) is ________.

159. The Taylor's series expansion of $f(x) = e^{\sin x}$ about $x = 0$, is

(a) $f(x) = 1 + x + \frac{x^2}{2} + \ldots\ldots$

(b) $f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots\ldots$

(c) $f(x) = 1 - x + \frac{x^2}{2} - \ldots\ldots$

(d) $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots\ldots$

160. In the Taylor's series expansion of $f(x) = \log \sec x$ about $x = 0$, coefficient of $x^4 = $ ___

(a) $\frac{1}{12}$

(b) $\frac{1}{14}$

(c) 12

(d) 14

161. The maximum value of the function $f(x) = \frac{x^3}{3} - x$ occurs at

(a) 1

(b) -1

(c) $\frac{1}{\sqrt{3}}$

(d) 0

162. The maximum value of the function

$f(x) = x^3 - 6x^2 + 9x + 1$ in $[0, 2]$ is _____.

163. The maximum value of the function

$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$ is ______.

164. The maximum value of the determinant among all 2×2 real symmetric matrices with trace 10 is ______.

165. Find 'C' of Rolle's Theorem for

$f(x) = e^x (\sin x - \cos x)$ in $[\pi/4, 5\pi/4]$

(a) $\pi / 2$

(b) $3\pi / 4$

(c) π

(d) does not exist

166. The mean value C of Lagrange's Theorem for the function $f(x) = 3x^2 + 5x + 8$ in $\left[\frac{11}{2}, \frac{13}{2}\right]$ is ______.

167. If $f(x)=\frac{1}{x}, g(x)=\frac{1}{x^2}$ in $[1,2]$ then the mean value C of Cauchy's mean value theorem is

(a) $\frac{4}{3}$ (b) $\frac{5}{4}$

(c) $\frac{5}{3}$ (d) none of these

168. How many of the following functions satisfy Lagrange's mean value theorem in the given interval?

$f(x) = |x + 2|$ in $[-2, 0]$

$g(x) = 2 + (4 - x)^{1/3}$ in $[1, 6]$

$h(x) = \log(1 + x^3)$ in $[0, 3]$

$$p(x) = \begin{cases} 1 + x^2, & 0 \le x < 1 \\ 1, & x = 1 \end{cases}$$

(a) 0 (b) 1

(c) 2 (d) 3

169. Let f(x) and g(x) be differentiable for $0 \le x \le 2$ such that $f(0) = 4$, $f(2) = 8$, $g(0) = 0$ and $f^1(x) = g^1(x)$ for all x in [0, 2] then the value of g(2) must be________.

170. The function $f(x) = 3x^4 - 4x^3 + 10$ has a minimum value at $x =$ ______.

171. The maximum value of the function $f(x) = x^3 - 9x^2 + 24x + 5$ in $[1,6]$ is

172. If $f(x) = a \log x + bx^2 - x$ has its extreme values at $x = -1$ and $x = 2$ then

(a) $a = 2, b = -1$

(b) $a = 2, b = -1/2$

(c) $a = -2, b = 2$

(d) $a = -2, b = 1/2$

173. The function $f(x, y) = x^3 - 3x^2 + 4y^2 - 10$ at $(2,0)$ has

(a) a maximum

(b) a minimum

(c) a saddle point

(d) both (a) & (b)

174. The function

$f(x, y) = x^2y - 3xy + 2y + x$ has

(a) no local extremum

(b) one local minimum but no local maximum

(c) one local maximum but no local minimum

(d) one local minimum and one local maximum

175. If $f(x,y) = xy+(x-y)$ then the saddle point of $f(x, y)$ is

(a) $(1, -1)$
(b) $(-1, 1)$
(c) $(-1,-1)$
(d) $(1,1)$

176. The distance between origin and a point nearest to it on the surface $z^2 = 1 + xy$ is

(a) $\sqrt{3}$ (b) $\sqrt{2}$

(c) 1 (d) none of these

177. If $f(x) = x^2\sin\left(\frac{1}{x}\right)$ then which of the following is true?

(a) $f^{1}(0)$ exists but $f^{11}(0)$ does not exist

(b) both $f^{1}(0)$ and $f^{11}(0)$ exist

(c) neither $f^{1}(0)$ nor $f^{11}(0)$ does not exist

(d) $f^{1}(0)$ does not exist but $f^{11}(0)$ exists

178. If $f(x)=x\left(1+\left(\frac{1}{3}\right)\sin(\log x)\right)$ then f(x) is

(a) continuous at $x = 0$ but not differentiable at $x=0$

(b) differentiable at $x=0$ but not continuous at $x=0$

(c) continuous and differentiable at $x=0$

(d) neither continuous nor differentiable at $x=0$

179. The function $f(x) = |x| + |x + 1| + |x - 2|$ is differentiable at $x =$

(a) 1 (b) -1

(c) 0 (d) 2

180. Find C of the Rolle's theorem for

$f(x) = x(x-1)(x-2)$ in $[1, 2]$

(a) 1.5

(b) $1 - (1/\sqrt{3})$

(c) $1 + (1/\sqrt{3})$

(d) 1.25

181. Find C of Rolle's theorem for

$f(x) = (x + 2)^3 (x - 3)^4$ in $[-2, 3]$

(a) 1/7

(b) 2/7

(c) 1/2

(d) 3/2

182. Find C of the Rolle's theorem for

$f(x) = e^x \sin x$ in $[0, \pi]$

(a) $\pi/4$

(b) $\pi/2$

(c) $3\pi/4$

(d) does not exist

183. $\underset{x\to 0}{\mathrm{Lt}}[(e^{2x}-1)\cot 3x] =$

(a) 0

(b) $\frac{3}{2}$

(c) 1

(d) $\frac{2}{3}$

184. Find C of Rolle's theorem for

$f(x) = \log[(x^2 + ab) / (a+b)x]$ in $[a,b]$

(a) $(a+b)/2$

(b) $\sqrt{ab}$

(c) $2ab / (a + b)$

(d) $(b - a)/2$

185. Rolle's theorem cannot be applied for the function $f(x) = |x|$ in $[-2, 2]$ because

(a) $f(x)$ is not continuous in $[-2, 2]$

(b) $f(x)$ is not differentiable in $(-2, 2)$

(c) $f(-2) \neq f(2)$

(d) none of the above

186. Find C of Lagrange's mean value theorem for $f(x) = \log x$ in $[1, e]$

(a) $e - 2$ (b) $e - 1$

(c) $(e + 1) / 2$ (d) $(e - 1) / 2$

187. Find C of Lagrange's mean value theorem for $f(x) = lx^2 + mx + n$ in $[a, b]$

(a) $(a + b) / 2$

(b) $\sqrt{ab}$

(c) $2ab / (a + b)$

(d) $(b - a)/2$

188. Find C of Lagrange's theorem mean value theorem for $f(x) = 7x^2 - 13x - 19$ in $[-11/7, 13/7]$

(a) 1/7 (b) 2/7

(c) 3/7 (d) 4/7

189. Find C of Lagrange's mean value theorem for $f(x) = e^x$ in $[0, 1]$

(a) 0.5

(b) $\log(e - 1)$

(c) $\log(e + 1)$

(d) $\log[(e + 1)/ (e - 1)]$

190. Find C of Cauchy's mean value theorem fo

the functions $1/x$ and $1/x^2$ in $[a, b]$

(a) $(a + b)/2$

(b) $\sqrt{ab}$

(c) $2ab / (a + b)$

(d) $(b - a)/2$

191. The function $f(x) = 2x^3 - 3x^2 - 36x + 10$ has a maximum at $x =$

(a) 3 (b) 2

(c) -3 (d) -2

192. The minimum value of

$f(x) = 2x^3 - 3x^2 - 36x + 10$ is

(a) 0 (b) -13

(c) -17 (d) -71

193. A maximum value of $f(x) = (\log x / x)$ is

(a) e

(b) e^{-1}

(c) $e - 1$

(d) $e + 1$

194. The function $f(x) = x^x$ has a minimum at x

(a) e (b) e^{-1}

(c) 0 (d) $e + 1$

195. The maximum value of $x \cdot e^{-x}$ is

(a) e

(b) e^{-1}

(c) 1

(d) $-e$

196. At (a, a), $f(x, y) = xy + a^3/x + a^3/y$ has

(a) a maximum

(b) a minimum

(c) a maximum if $a > 0$

(d) neither maximum nor minimum

197. If $f^1(x) = (x + 2)(x - 1)^2(2x - 1)(x - 3)$ the

at $x = ½$, f(x) has

(a) a maximum

(b) a minimum

(c) neither maximum nor minimum

(d) no stationary point

198. Find the maximum value of $x^2 + y^2 + z^2$ so that $x + y + z = 1$

(a) 1
(b) 1/2
(c) 1/3
(d) 1/4

199. If $f(x) = \begin{cases} 4(3^x), \text{ if } x < 0 \\ 2a + x, \text{ if } x \geq 0 \end{cases}$

is continues at $x = 0$, then $a =$ _____.

200. A function f(x) differentiable in the interval $0 \leq x \leq 5$, is such that $f(0) = 4$ and $f(5) = -1$.

If $g(x) = \frac{f(x)}{x+1}$ then there exists a constant $c \in (0, 5)$ such that $g^{1}(c) =$

(a) $\frac{-2}{5}$ (b) $\frac{2}{5}$

(c) $\frac{-3}{5}$ (d) $\frac{-5}{6}$

201. If Rolle's Theorem hold for the function $f(x) = x^3 + ax^2 + bx$ in the interval $1 \le x \le 2$ at the point $x = \frac{4}{3}$ then $(a, b) =$ _____.

(a) $(-5, 8)$

(b) $(-8, -5)$

(c) $(5, 8)$

(d) None of the above

202. If $f(x) = \dfrac{x^3+1}{x+1}$ is continuous at $x = -1$, then

$f(-1) =$ ________.

203. If $f(x) = \dfrac{ax+b}{cx+d}$ then f(x) has

(a) a maximum

(b) a minimum

(c) no extremum

(d) an extremum, if $ad = bc$

204. The function $f(x) = x^{\frac{1}{x}}$ has ______.

(a) a maximum at $x = e^{-1}$

(b) a minimum at $x = e^{-1}$

(c) a maximum at $x = e$

(d) a minimum at $x = e$

205. $\underset{x \to 0}{\mathrm{Lt}} \frac{|x|}{x} = \_\_\_\_\_$, where | x | is a modulus function

(a) 0

(b) 1

(c) –1

(d) limit does not exist

206. $\underset{x \to 6}{\mathrm{Lt}} [x] =$ ______, where [x] is a step function

(a) –6

(b) 5

(c) 0

(d) limit does not exist

207. $\underset{x \to 0}{\text{Lt}} \dfrac{x - \sin x}{1 - \cos x} =$ ______.

(a) 0 (b) 4

(c) –3 (d) 1

208. $\underset{x \to \frac{\pi}{2}}{\text{Lt}} \left(\sec x - \dfrac{1}{1 - \sin x} \right) = \_\_\_\_\_\_$.

(a) 0 (b) ∞ (c) -1 (d) 100

209. If $f(x) = \left(\frac{1-x}{x+1}\right)^{\frac{1}{x}}$ is continuous at $x = 0$

then $f(0) =$

(a) e^{-2} (b) e^{2}

(c) $\sqrt{e}$ (d) $e^{-1/2}$

210. If $f(x)=\begin{cases} ax^2+1, & x\le 1 \\ x^2+ax+b, & x>1 \end{cases}$ is differentiable at $x = 1$ then

(a) $a = 1, b = 1$ (b) $a = 1, b = 0$

(c) $a = 2, b = 0$ (d) $a = 2, b = 1$

211. The function $f(x) = |x - 4|$ on the interval $[0,5]$ is

(a) continuous and differentiable

(b) neither continuous nor differentiable

(c) differentiable but not continuous

(d) continuous on the interval but not differentiable

212. Lagrange's mean value theorem does not hold for $f(x) = x^{\frac{-2}{3}}$ in $[-1, 1]$, because

(a) not continuous in $(-1, 1)$

(b) not differentiable in $(-1, 1)$

(c) continuous but not differentiable in $(-1, 1)$

(d) neither continuous nor differentiable in the given interval

213. The value of 'c' of Cauchy's mean value theorem for $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ in $[1, 3]$ is ______.

(a) 1.732 (b) 2.732

(c) 3.732 (d) –1.732

214. The coefficient of $(x-2)^3$ in the Taylor series expansion of the function $f(x) = \log x$ about the point 2 is

(a) $\frac{1}{4}$ (b) $\frac{1}{12}$ (c) $\frac{1}{24}$ (d) $\frac{1}{36}$

215. $f(x)=\int_0^x (t-2)^2(t-1)dt$ has a

(a) maximum at $x=1$

(b) minimum at $x=1$

(c) maximum at $x=2$

(d) minimum at $x=2$

216. Suppose that the function 'f' attains a maximum at $x = x_1$ and a minimum at $x = x_2$ such that $x_2 = x_1^2$.

If $f(x) = 2x^3 - 9ax^2 + 12a^2 x + 1, a > 0$ then the value of a = ______.

217. If α, β are the roots of the equation $x^2 - (a - 2)x - (a + 1) = 0$, where 'a' is a variable then the minimum value of $\alpha^2 + \beta^2 =$ ________.

218. Let x and y be integers satisfying the following equations

$$2x^2 + y^2 = 34$$

$$x + 2y = 11$$

The value of $(x + y)$ is ________.

(GATE-17-EE)

219. Let $g(x) = \begin{cases} -x, & x \le 1 \\ x+1, & x > 1 \end{cases}$ and

$$f(x) = \begin{cases} 1-x, & x \le 0 \\ x^2, & x > 0 \end{cases}$$

Consider the composition of f and g, i.e., $(f \circ g)(x) = f(g(x))$. The number of discontinuities in $(f \circ g)(x)$ present in the interval $(-\infty, 0)$ is

(GATE-17-EE)

(a) 0 (b) 1

(c) 2 (d) 4

220. The value of $\lim_{x \to 0} \left(\frac{x^3 - \sin(x)}{x} \right)$ is

(GATE-17-ME)

(a) 0 (b) 0

(c) 1 (d) -1

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221. $\lim_{x\to 0}\left(\frac{\tan x}{x^2-x}\right)$ is equal to ____________ (GATE-17-CE)

222. The value of $\lim_{x \to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$ (GATE-17-CSIT)

(a) is 0 (b) is -1

(c) is 1 (d) does not exists

223. At the point x = 0, the function $f(x) = x^3$ has

(a) local maximum **(GATE-18-CE)**

(b) local minimum

(c) both local maximum and local minimum

(d) Neither local maximum nor local minimum

224. Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval $[0, 2]$ is __________ (up to 2 decimal places)

(GATE-18-EE)

225. Consider two functions $f(x) = (x - 2)^2$ and $g(x) = 2x - 1$, where x is real. The smaller value of x for which $f(x) = g(x)$ is ____________

(GATE-18-IN)

226. For $0 \le x \le 2\pi$, sinx and cosx are both decreasing functions in the interval__________

(GATE-18-IN)

(a) $\left(0, \frac{\pi}{2}\right)$ (a) $\left(\frac{\pi}{2}, 0\right)$

(c) $\left(\pi, \frac{3\pi}{2}\right)$ (c) $\left(\frac{\pi}{2}, \pi\right)$

227. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is

(GATE-ME,PI-2012)

(a) 8 meters (b) 10 meters (c) 12 meters (d) 14 meters

228. A real-valued function y of real variable x is such that $y = 5|x|$. At $x = 0$, the function is

(GATE-18-PI)

(a) discontinuous but differentiable

(b) both continuous and differentiable

(c) discontinuous and not differentiable

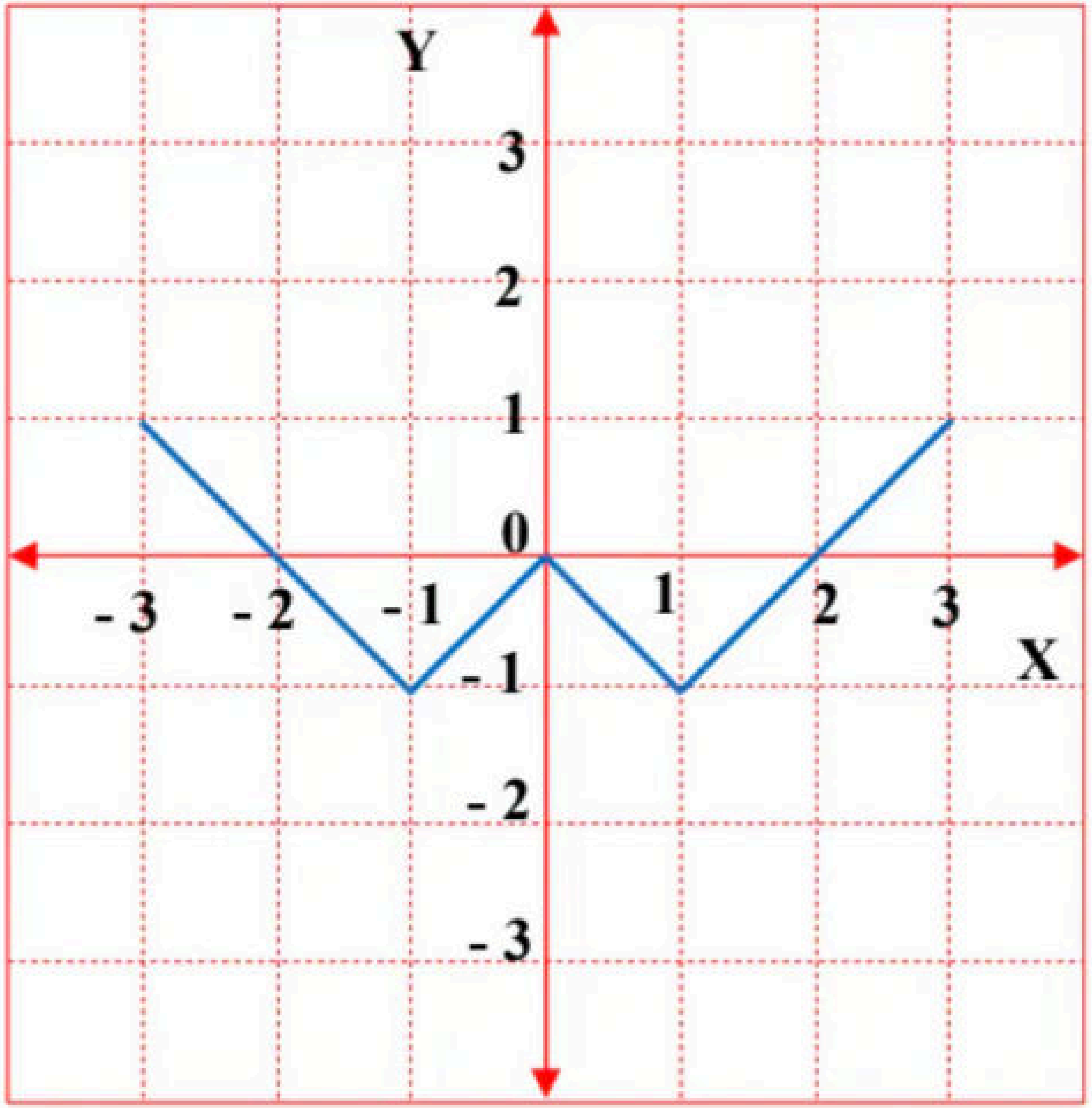
(d) continuous but not differentiable

229. Which of the following functions describe the graph shown in the below figure?

(GATE-18-PI)

(a) $y = ||x| + 1| - 2$ (b) $y = ||x| - 1| - 1$

(c) $y = ||x| + 1| - 1$ (d) $y = |x - 1| - 1$



230. Let f be the real-valued function of real variable defined as $f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x<0$. Which of the following statements is true?

(GATE-18-EE)

a) f(x) is discontinuous at $x = 0$

(b) f(x) is continuous but not differentiable at $x = 0$

(c) f(x) is differentiable, but its first derivative is not continuous at $x = 0$

(d) f(x) is differentiable, but its first derivative is not differentiable at $x = 0$

231. Let $f(x) = x^{-(1/3)}$ and A denote the area of the region bounded by f(x) and the X-axs, when x varies from – 1to 1. Which of the following statements is/are TRUE ?

(GATE-CS-2015)

(I) f is continuous in [-1, 1] (II) f is not bounded in [-1, 1]

(III) A is nonzero and finite

(a) II only (b) III only (c) II and III only (d) I, II and III

232. Compute $\lim_{x \to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

(GATE-19-CSIT)

(a) 1 (b) Limit does not exist

(c) 53/12 (d) 108/7

233. Which of the following is correct?

(GATE-19-CE)

(a) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(b) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = \infty$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(c) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 1$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = 1$

(b) $\lim_{x\to 0}\left(\frac{\sin 4x}{\sin 2x}\right) = 2$ and $\lim_{x\to 0}\left(\frac{\tan x}{x}\right) = \infty$

234. for a small value of h the Taylor series expansion for $f(x+h)$ is

(GATE-19-CE)

(a) $f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\cdots\cdots\infty$

(b) $f(x)-hf'(x)+\frac{h^2}{2}f''(x)-\frac{h^3}{3}f'''(x)+\cdots\cdots\infty$

(c) $f(x)+hf'(x)+\frac{h^2}{2}f''(x)+\frac{h^3}{3}f'''(x)+\cdots\cdots\infty$

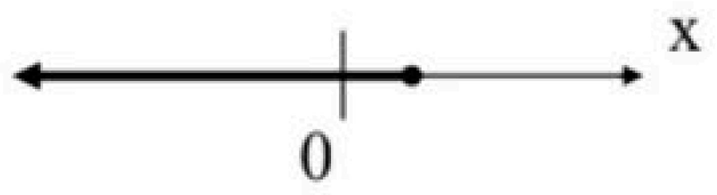
(d) $f(x)-hf'(x)+\frac{h^2}{2!}f''(x)-\frac{h^3}{3!}f'''(x)+\cdots\cdots\infty$

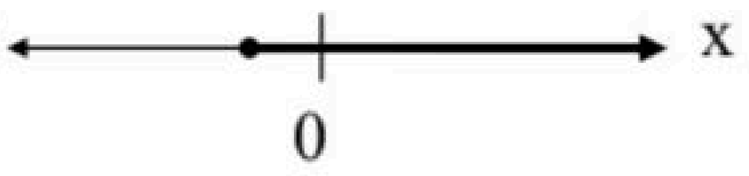
235. The global minimum of $x^3 e^{-|x|}$ for $x \in (-\infty, \infty)$ occurs at x = ____________(round off to one decimal place)

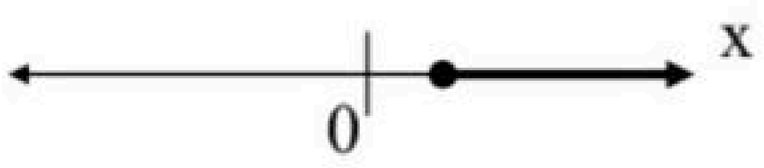
(GATE-2022-IN)

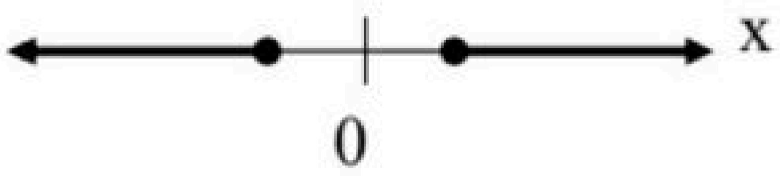
236. Which one of the following is a representation (not to scale and in bold) of all values of x satisfying the inequality $2 - 5x \leq -\left(\frac{6x-5}{3}\right)$ on the real number line?

(GATE-2022-ME)

(a) 

(b) 

(c) 

(d) 

237. The minimum value of $2x+3y$, when $xy=6$ is

(A) 12 (B) 9 (C) 8 (D) 6

238. $f(x)=\dfrac{1}{4x^2+2x+1}$, then its max value is

(A) $\dfrac{4}{3}$ (B) $\dfrac{2}{3}$ (C) 1 (D) $\dfrac{3}{4}$

239. The series $\sum_{m=0}^{\alpha} \frac{1}{4^m}(x-1)^{2m}$ converges for (GATE-IN-2011)

(a) -2< x < 2 (b) -1 < x < 3 (c) -3 < x < 1 (d) x < 3

240. The series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges to **(GATE-EC-SET-4-2014)**

(a) $2 \ln 2$ (b) $\sqrt{2}$ (c) 2 (d) e

241. The value of $\sum_{n-0}^{\infty} n\left(\frac{1}{2}\right)^{n}$ is ______ **(GATE-EC-2015)**

242. Consider the sequence, $x_n = 0.5x_{n-1} + 1$, $n = 1, 2, \ldots$ with $x_0 = 0$. Then $\lim_{n\to\infty} x_n$ is

GATE- 2021 (CS)

(a) 2 (b) 1

(c) 0 (d) ∞

243. Let $S = \sum_{n=0}^{\infty} n\alpha^n$ where $|\alpha| < 1$. The value of α in the range $0 < \alpha < 1$, such that $S = 2\alpha$ is

____

(GATE-16-EE)

244. $f(z) = (z-1)^{-1} - 1 + (z-1) - (z-1)^2 + \ldots$.. is the series expansion of

(a) $\dfrac{1}{(z-1)^2}$ for $|z-1|<1$

(b) $\dfrac{1}{z(z-1)}$ for $|z-1|<1$

GATE- 2021 (CS)

(c) $\dfrac{-1}{z(z-1)}$ for $|z-1|<1$

(d) $\dfrac{-1}{(z-1)}$ for $|z-1|<1$

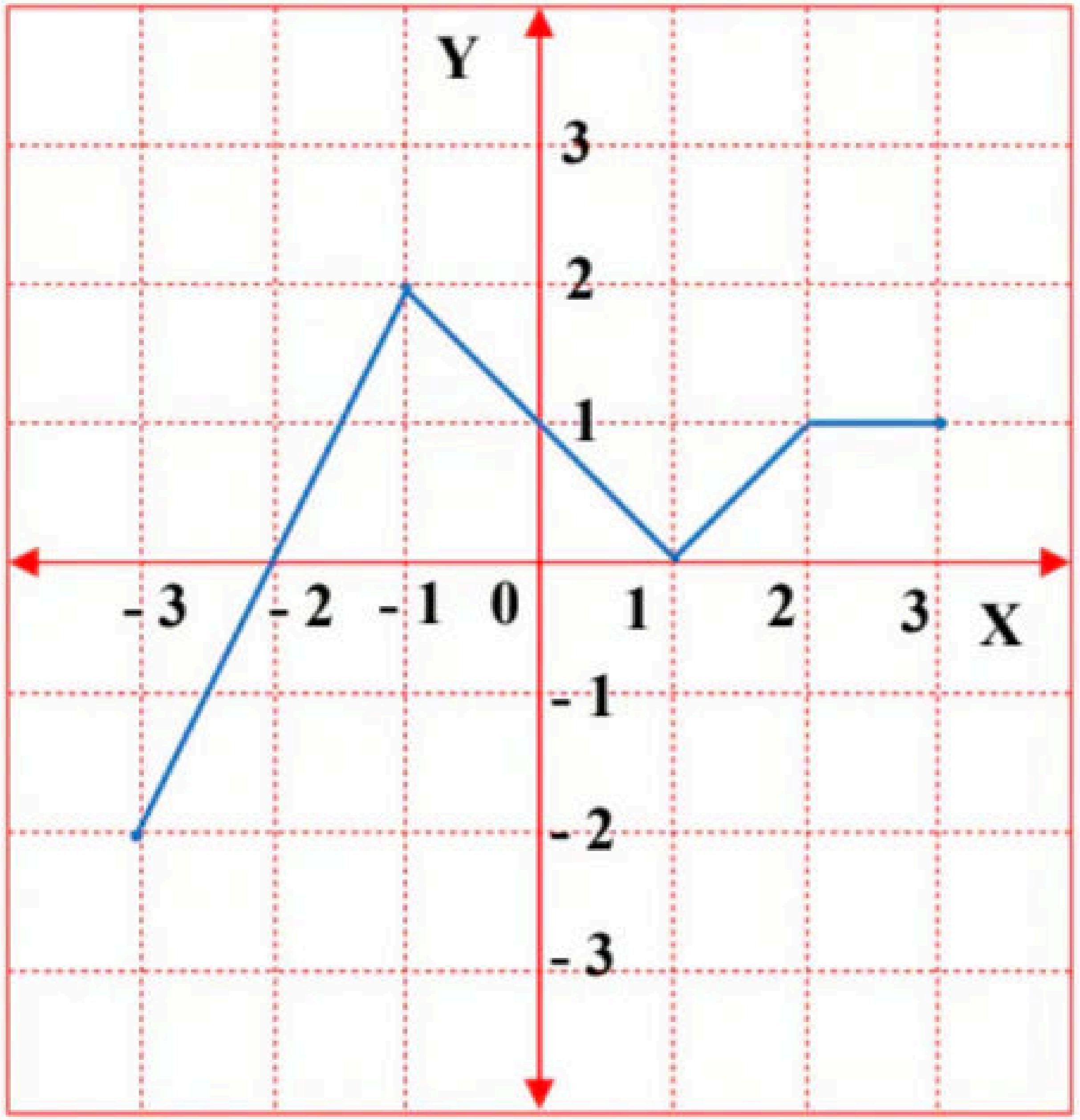
245. Which of the following function(s) is an accurate description of the graph for the range(s) indicated?

(i) $y = 2x + 4$ for $-3 \le x \le 1$

(ii) $y = |x - 1|$ for $-1 \le x \le 2$

(iii) $y = \big||x| - 1\big|$ for $-1 \le x \le 2$

(iv) $y = 1$ for $2 \le x \le 3$



(a) (i), (ii) and (iii) only

(b) (i), (ii) and (iv) only

(c) (i) and (iv) only

(d) (ii) and (iv) only

246. Consider the following series:

$$\sum_{n=1}^{\infty} \frac{n^d}{c^n}$$

For which of the following combinations of c, d values does this series converge?

(GATE-2022-ECE)

(a) $c = 1, d = -1$ (b) $c=2, d=1$

(c) $c = 0.5, d = -10$ (d) $c = 1, d = -2$

247. Let f be differentiable for all x, if f(1)=-2, and $f'(x) \geq 2$ for all $x \in [1,6]$ thus

(A) $f(6) < 8$

(B) $f(6) \geq 8$

(C) $f(6) \geq 5$

(D) $f(6) \leq 5$

248. The quadratic equation $3ax^2 + 2bx + c = 0$ has at least one root between 0 and 1, if

(A) $a+b+c=0$ (B) $c=0$

(C) $3a+2b+c=0$ (D) $a+b=c$

249. $\int_0^2 \int_0^3 xy\, dxdy$

(A) 0 (B) 9 (C) 8 (D) 1

250. $\int_0^{\frac{\pi}{2}} \int_{-1}^{1} x^2 y^2 dxdy$

(A) $\frac{\pi^3}{36}$ (B) $\frac{\pi}{0}$ (C) -1 (D) 0

251. Evaluate $\int_{-1}^{2}(1+|x|)\,dx$

(A) 3.5 (B) 5.5

(C) 4 (D) None of these

252. $\int_{0}^{\pi} \sin^5 x \cos^9 x dx =$ ________

(A) 0

(B) 1

(C) ∞

(D) None of these

253. Let $f(x)$ be any bounded real valued function in the interval $[a, b]$.

Consider the following statements:

A: $\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} \{f(x) + f(-x)\}\,dx$

B: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

Then which of the following is appropriate?

(A) A and B both are true and they are interdependent

(B) A and B are true independently

(C) A is true and B is false always

(D) A is true and B is true in special case

254. For which value of n, $\int_0^{\frac{\pi}{2}} \frac{dx}{16\cos^2 x + 25\sin^2 x}$ becomes equal to $n \times \pi$.

(A) $\frac{1}{40}$ (B) $\frac{1}{50}$ (C) $\frac{1}{20}$ (D) $\frac{1}{30}$

255. Evaluate $\int_0^1 \int_1^2 (x^2 + y^2) dx dy$

(A) $\frac{-8}{3}$ (B) $\frac{8}{3}$ (C) 0 (D) 1

256. The value of $\int_{-4}^{7} |x|\, dx$ is

(a) 30.5

(b) 30

(c) 32.5

(d) 32

257. The value of $\int_0^{1.5} x[x^2]\,dx$, where $[x]$ is a step function, is

(a) $\frac{4}{3}$
(b) $\frac{1}{2}$
(c) $\frac{2}{3}$
(d) $\frac{3}{4}$

258. The value of $\int_0^{\pi} x \sin^8(x) \cos^6(x)\, dx$ is

(a) $\frac{\pi^2}{512}$

(b) $\frac{105\pi^2}{512}$

(c) $\frac{105\pi}{86016}$

(d) $\frac{5\pi^2}{4096}$

259. The value of $\int_{x=1}^{a} \int_{y=1}^{b} \frac{1}{xy} \, dx \, dy$ is ______.

(a) (log a) (log b)

(b) log (ab)

(c) log a – log b

(d) log (a + b)

260. $\int_1^3 \int_1^2 xy^2 \, dx \, dy =$

(a) 10
(b) 11
(c) 13
(d) 12

261. $\int_0^1 \int_0^2 \int_1^2 x^2 yz \, dx \, dy \, dz =$

(a) $-\frac{7}{3}$

(b) $\frac{7}{3}$

(c) $\frac{7}{2}$

(d) $-\frac{7}{2}$

262. The value of $\int\limits_{x=0}^{1} \int\limits_{y=0}^{2} xy \, dx \, dy$ is ________.

263. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log\left(\frac{1+x}{1-x}\right) dx =$ ______

(a) 0

(b) 1

(c) $\frac{\pi}{2}$

(d) none of these

264. $\int_{-\pi}^{\pi} \sin^4 x.dx =$

(a) π

(b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$

(d) 0

265. $\int_{-1}^{2} \frac{|x|}{x}\,dx = ------$

266. $\int_{0}^{\pi} |\cos x| \, dx =$

267. $\int_{0}^{n} [x]\, dx =$ ____ , where [x] is a step functi

and 'n' is an integer.

(a) $\dfrac{n(n+1)}{2}$ (b) $\dfrac{n(n-1)}{2}$

(c) $\dfrac{n}{2}$ (d) $\dfrac{n+1}{2}$

268. $\int_{0}^{\frac{\pi}{2}} \log(\tan x)\, dx =$

(a) 0

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) π

269. Let $\frac{d}{dx}[F(x)] = \frac{e^{\sin x}}{x}, x > 0.$

If $\int_1^4 \left(\frac{2e^{\sin x^2}}{x}\right)dx = F(k) - F(1)$

then k =________.

270. $\int_0^{\pi/4} \log(1+\tan x)\,dx =$

(a) 0

(b) $(\pi/2)\log 2$

(c) $(\pi/8)\log 2$

(d) $(-\pi/4)\log 2$

271. $\int_{0}^{\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0 (b) $3\pi/256$

(c) $3\pi/128$ (d) $5\pi/128$

272. $\int_0^{2\pi} \sin^4 x \cos^6 x \, dx =$

(a) $3\pi/128$ (b) $3\pi/256$

(c) $3\pi/64$ (d) 0

273. $\int_{0}^{2\pi} \sin^4 x \cos^5 x \, dx =$

(a) 0 (b) $3\pi/128$

(c) $5\pi/128$ (d) $3\pi/256$

274. $\int_0^{\pi/2}\int_0^{\pi/2} \sin(x+y)\,dx\,dy$ **(GATE-EC-2000)**

(a) 0 (b) π (c) $\pi/2$ (d) 2

275. The value of the integral is $I = \int_0^{\pi/4} \cos^2 x \, dx$ (GATE-CE-2001)

(a) $\frac{\pi}{8}+\frac{1}{4}$ (b) $\frac{\pi}{8}-\frac{1}{4}$ (c) $\frac{-\pi}{8}-\frac{1}{4}$ (d) $\frac{-\pi}{8}+\frac{1}{4}$

276. The value of the integral $\int_{-\pi/2}^{\pi/2} (x \cos x)\, dx$ is **(GATE-PI-2008)**

(a) 0 (b) $\pi - 2$ (c) π (d) $\pi + 2$

277. The value of the following definite integral in $\int_{-\pi/2}^{\pi/2} \frac{\text{Sin}2x}{1+\cos x} dx$ = **(GATE-ME-2002)**

(a) -2log 2 (b) 2 (c) 0 (d) None

278. The value of the following improper integral is $\int_0^1 x \log x \, dx =$

(GATE-ME-2002)

(a) 1/4 (b) 0 (c) -1/4 (d) 1

279. $\int_{-a}^{a}\left[\sin^6 x+\sin^7 x\right]dx$ is equal to **(GATE-ME-2004)**

(a) $2\int_0^a \text{Sin}^6 x\,dx$ (b) $2\int_0^a \text{Sin}^7 x\,dx$ (c) $2\int_0^a \left(\text{Sin}^6 x+\text{Sin}^7 x\right)dx$ (d) zero

280. The value of $\int_0^3 \int_0^x (6 - x - y)\, dx dy$ is ________ **(GATE-CS-2008)**

(a) 13.5 (b) 27.0 (c) 40.5 (d) 54.0

281. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} dx\, dy\, dz$

(A) $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} + e^a - \frac{3}{8}$

(B) $\frac{e^{4a}}{4} - \frac{3e^{2a}}{4}$

(C) $\frac{e^{4a}}{8} - \frac{3e^{2a}}{4} - \frac{3}{8}$

(D) None

282. $\int_{0}^{1}\int_{x}^{\sqrt{x}}\left(x^{2}+y^{2}\right)dx\,dy =$

(a) $\frac{2}{35}$

(b) $-\frac{3}{35}$

(c) $\frac{3}{35}$

(d) $-\frac{2}{35}$

283. $\int_0^1 \int_{4y}^4 e^{x^2}\, dx\, dy =$

(a) $\frac{(e^{16}-1)}{8}$

(b) $-\frac{(e^{16}+1)}{8}$

(c) 0

(d) $-\frac{(e^{16}-1)}{8}$

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284. $\int_0^4 \int_0^{x^2} e^{y/x}\, dy\, dx =$

(a) $3e^4$

(b) $3e^4 + 7$

(c) $-3e^4$

(d) $3e^4 - 7$

285. $\int_{0}^{\infty}\int_{x}^{\infty}\left(\frac{e^{-y}}{y}\right)dy\,dx =$

(a) 0 (b) 2

(c) 3 (d) 1

286. $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dx\,dy\,dz =$

(a) 1

(b) 2

(c) 3

(d) 0

287. The integral $\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to ________.

(GATE-16-EC)

288. $\displaystyle\int_{1/\pi}^{2/\pi} \frac{\cos(1/x)}{x^2}\,dx =$ ________ **(GATE-CS-2015)**

289. Given the following statements about a function $f: R \rightarrow R$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(GATE-16-EC)

(a) P is true, Q is false, R is false

(b) P is false, Q is true, R is true

(c) P is false, Q is true, R is false

(d) P is true, Q is false, R is true

290. The value of $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r \sin\theta \, dr \, d\theta$ is ______.

(a) $\frac{a^2}{2}$

(b) $2a^2$

(c) $\frac{2a^2}{3}$

(d) $4a^2$

291. $$\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^4 x + \sin^4 x}\,dx = \_\_\_\_\_\_.$$

292. $\int_{0}^{1}\int_{0}^{\sqrt{(1-x^2)}} y^2\,dxdy =$

(a) $-\frac{\pi}{16}$

(b) $\frac{\pi}{16}$

(c) $\frac{\pi}{8}$

(d) $-\frac{\pi}{8}$

293. If [x] stands for greatest integer not exceeding 'x', then $\int_4^{10}[x]dx =$ ______.

294. Change the order of integration in the integral $I=\int_{-a}^{a}\int_{0}^{\sqrt{(a^2-y^2)}} f(x,y)\,dx\,dy$

(a) $I=\int_{0}^{a}\int_{-\sqrt{(a^2-x^2)}}^{\sqrt{(a^2-x^2)}} f(x,y)\,dy\,dx$

(b) $I=\int_{-a}^{a}\int_{-\sqrt{(a^2-x^2)}}^{\sqrt{(a^2-x^2)}} f(x,y)\,dy\,dx$

(c) $I=\int_{0}^{a}\int_{-a}^{a} f(x,y)\,dy\,dx$

(d) None

295. By reversing the order of integration, the double integral $\int_0^a \int_{\sqrt{ax}}^a \phi(x, y)\, dy dx$ is represented as $\int_p^q \int_r^s \phi(x, y)\, dx dy$ then the product of q and s is ______.

(a) y^2/a (b) ay^2

(c) y^2 (d) 0

296. Changing the order of integration in double integral $I = \int_0^8 \int_{x/4}^2 f(x, y)\,dy\,dx$ leads to $I = \int_r^s \int_p^q f(x, y)\,dy\,dx$. What is q? (GATE-EC-2005)

(a) $4y$ (b) $16y^2$ (c) x (d) 8

297. By reversing the order of integration $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$ may be represented as

(a) $\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx$ (b) $\int_0^2 \int_y^{\sqrt{y}} f(x, y)\, dx\, dy$ **(GATE-EC-1995)**

(c) $\int_0^4 \int_{y/2}^{\sqrt{y}} f(x, y)\, dx\, dy$ (d) $\int_{x^2}^{2x} \int_0^2 f(x, y)\, dy\, dx$

298. By a change of variables x (u, v) = uv, y(u,v) = v/u in a double integral, the integral f(x,y) changes to $f\left(uv, \frac{u}{v}\right)\phi(u,v)$ Then $\phi(u,v)$ is ________ **(GATE-EE-2005)**

(a) $\frac{2v}{u}$ (b) 2 uv (c) v^2 (d) 1

299. To evaluate the double integral $\int_0^8 \left(\int_{y/2}^{\left(\frac{y}{2}\right)+1} \left(\frac{2x-y}{2}\right) dx \right) dy$, we make the substitution $u = \left(\frac{2x-y}{2}\right)$ and $v = \frac{y}{2}$. The integral will reduce to

(A) $\int_0^4 \left(\int_0^2 2u du \right) dv$

(B) $\int_0^4 \left(\int_0^1 2u du \right) dv$

(C) $\int_0^4 \left(\int_0^1 u du \right) dv$

(D) $\int_0^4 \left(\int_0^{21} 2u du \right) dv$

300. The values of the integrals $\int_0^1 \left[\int_0^1 \frac{x-y}{(x+y)^3} dy\right] dx$ and $\int_0^1 \left[\int_0^1 \frac{x-y}{(x+y)^3} dx\right] dy$ are

(GATE-17-EC)

(a) same and equal to 0.5

(b) same and equal to -0.5

(c) 0.5 and -0.5, respectively

(d) -0.5 and -0.5, respectively

301. The value of $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xyz\,dz\,dy\,dx$ is ____________

(A) $\frac{13}{9} - \frac{ln\,3}{6}$

(B) $\frac{7}{6} - \frac{ln\,3}{6}$

(C) $\frac{1}{6} - ln\,3$

(D) $\frac{3}{2} - ln\,3$

302. The value of $\int_0^1 \int_0^2 \int_1^2 x^2 yz\,dz\,dy\,dx$ is __________

(A) 0 (B) 1

(C) 2 (D) 3

303. The Value of the integral $\int_0^1 \int_y^1 y\sqrt{1+x^3}\,dxdy =$ ______

(A) $2\sqrt{2}$

(B) $\frac{2\sqrt{2}-1}{2}$

(C) $\frac{2\sqrt{2}-1}{8}$

(D) $\frac{2\sqrt{2}-1}{9}$

304. The value of $\int_{-1}^{2}\int_{x^2}^{x+2} dydx =$ ________

(A) $\frac{7}{2}$

(B) $\frac{9}{2}$

(C) $\frac{11}{2}$

(D) $\frac{5}{2}$

305. The value of $\int_0^1 \int_0^1 \frac{dydx}{\sqrt{1-x^2}\,\sqrt{1-y^2}} =$ ________

`(A) $\frac{\pi^2}{4}$

(B) $\frac{\pi^2}{2}$

(C) $\frac{\pi^2}{8}$

(D) $\frac{\pi^2}{16}$

306. The value of $\int_0^{100\pi} |sin\, x| dx$ is ____________

(A) 100 (B) 100π

(C) 200π (D) 200

307. The value of integral $\int_{-1}^{1} ln\left(\frac{2-x\cos x}{2+x\cos x}\right)dx$ is ________

(A) $xln(2 + x\cos x)$

(B) $xln(2 - x\cos x)$

(C) $x\cos x$

(D) 0

308. If $f(x) = \int_x^0 \sin t^2 dt$ then $f'(x)$ is ________

(A) $2x\sin x^2$
(B) $-\sin x^2$
(C) $2x\cos x^2$
(D) $\cos x^2$

309. $\lim\limits_{x\to 0}\frac{1}{x}\int_x^{2x} e^{-t^2}\,dt$ equals ______.

(A) does not exists (B) infinite

(C) exists and equals to 1 (D) exists and equals to 0

310. $\int_0^{\frac{\pi}{4}} \frac{1}{a^2\cos^2 x+b^2\sin^2 x}dx =$ ________ $(a>0)(b>0)$

(A) $\frac{1}{ab}$

(B) $\frac{1}{ab}tan^{-1}\left(\frac{b}{a}\right)$

(B) $\frac{1}{ab}tan^{-1}\left(\frac{a}{b}\right)$

(D) 0

311. $\int_0^1 \frac{ln\,(1+x)}{1+x^2}\,dx$ is

(A) $\frac{\pi}{4} ln\, 2$

(B) $\frac{\pi}{2} ln\, 2$

(C) $\frac{\pi}{8} ln\, 2$

(D) 0

312. Let D be the determinant

$$\begin{bmatrix} \boldsymbol{\cos\theta} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{2\cos\theta} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \boldsymbol{2\cos\theta} \end{bmatrix}$$

Then $\int_0^{\frac{\pi}{6}} D d\theta$

(A) 1 (B) 1/3

(C) 4/3 (D) 3/2

313. The integral $\int_0^{\frac{\pi}{2}} min(sin\, x, cos\, x)dx$ equals

(A) $\sqrt{2} - 2$

(B) $2 - \sqrt{2}$

(C) $2\sqrt{2}$

(D) $2 + \sqrt{2}$

314. The value of $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ is

(A) 0 (B) $\frac{\pi}{2}$

(C) π (D) 2π

315. The value of integral $\int_0^9 \frac{dy}{\sqrt{y}\sqrt{1+\sqrt{y}}}$ is

(A) 4

(B) $4\left(\sqrt{10}-1\right)$

(C) 8

(D) 12

316. The value of $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ is ________

(A) $\frac{\pi a}{4}$

(B) $\frac{\pi a}{8}$

(C) $\frac{\pi a}{2}$

(D) πa

317. $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}} dy dx =$ ________

(A) 1
(B) 1/2
(C) 1/3
(D) ¼

318. The value of $\int_0^\infty \int_x^\infty \frac{1}{y} e^{-\frac{y}{2}} dy dx =$ ________

(A) 1
(B) 2
(C) 3
(D) 4

319. The value of $\int_0^a \int_0^x \int_0^y xyz\,dz\,dy\,dx$ is

(A) $\frac{a^4}{16}$

(B) $\frac{a^4}{12}$

(C) $\frac{a^6}{48}$

(D) $\frac{a^4}{4}$

320. The value of $\int_0^1 x^6\sqrt{1-x^2}dx$ is

(A) $\frac{5\pi}{256}$

(B) $\frac{5\pi}{128}$

(C) $\frac{5\pi}{512}$

(D) $\frac{3\pi}{512}$

321. The value of $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y+2z)dzdydx$ is

(A) $\frac{1}{53}$ (B) $\frac{2}{21}$

(C) $\frac{1}{6}$ (D) $\frac{5}{3}$

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322. Let $E = \{(x,y) \in R^2, 0<x<y, 0<y<\infty\}$ then $\int\int_E ye^{-(x+y)}dxdy =$ ________

323. Let $\int_0^1 \int_y^1$ xysin (xy)dxdy $= \int_0^1 \int_a^b$ xysin xydydx then

(A) $a = 0, b = x$

(B) a = 1, b = x

(C) a = 0, b = 1

(D) a = -1, b = x

324. The integral $\int_0^1 \int_{x^2}^{x} \left(\frac{x}{y}\right) e^{-\frac{x^2}{y}} dydx$ equals

(A) $(e-2)/e$
(B) $(e-1)/2e$
(C) $(e-1)/2$
(D) $(e-2)/2e$

325. Differentiate with respect to t. $f(t) = \int_{-t^2}^{\alpha} e^{-x^2} dx$

(A) $2te^{-t^4}$ (B) $2te^{t^4}$

(C) $-2te^{t^4}$ (D) $-2te^{-t^4}$

326. The area of the region enclosed by the curve $y = x^2$ and the straight-line $x + y = 2$ is

(A) 3
(B) 27/2
(C) 9/2
(D) 9

327. The area of the region bounded by the curve $x^2 = 2y$ and $y^2 = 2x$ is

(A) 1/3 (B) 2/3

(C) 4/3 (D) 4

328. Area enclosed by the curves $y^2 = x$ and $y^2 = 2x-1$ lying in the first quadrant is

(A) 1/6　　(B) 1/4

(C) 1/2　　(D) 1/3

329. The value of $\int\int xy(x+y)dxdy$ over the area between y = x^2 and y = x

(A) 1/90 (B) 1/45

(C) 3/56 (D) 1/15

330. The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to

(a) 6 (b) 18 (c) ∞ (d) None **(GATE-ME-1995)**

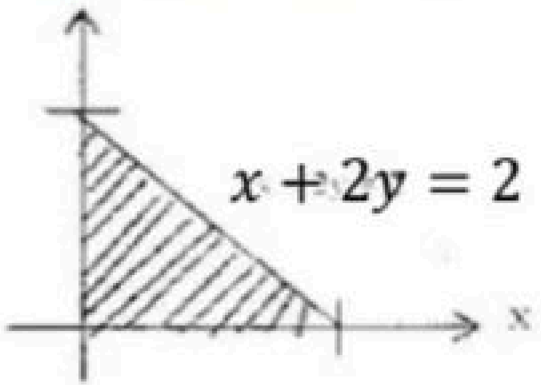
331. Area bounded by the curve $y = x^2$ and the lines $x = 4$ and $y = 0$ is given by

(a) 64 (b) $\frac{64}{3}$ (c) $\frac{128}{3}$ (d) $\frac{128}{4}$ **(GATE-EE-1997)**

332. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is

(a) 1/8 (b) 1/6 (c) 1/3 (d) 1/2

333. Consider the shaded triangular region P shown in the figure. What is $\iint_P xy\, dx\, dy$?



(GATE-ME-2008)

(a) $\frac{1}{6}$ (b) $\frac{2}{9}$ (c) $\frac{7}{16}$ (d) 1

334. Consider the following definite integral

$$I = \int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$$

The value of the integral is

(GATE-17-CE)

(a) $\frac{\pi^3}{24}$ (b) $\frac{\pi^3}{12}$

(c) $\frac{\pi^3}{48}$ (d) $\frac{\pi^3}{64}$

335. Let x be a continuous variable defined over the interval $(-\infty, \infty)$ and $f(x) = e^{-x-e^{-x}}$. The integral $g(x) = \int f(x)dx$ is equal to

(GATE-17-CE)

(a) e^{e-x} (b) $e^{-e^{-x}}$

(c) $e^{-e^{x}}$ (d) e^{-x}

336. The volume under the surface $z(x, y) = x + y$ and above the triangle in the xy plane defined by $\{0 \leq y \leq x \text{ and } 0 \leq x \leq 12\}$ is ______. **(GATE-EC-SET-1-2014)**

337. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y-axis is **(GATE-EE-1994)**

(a) $\frac{128\pi}{5}$ (b) $\frac{5}{128\pi}$ (c) $\frac{127}{5\pi}$ (d) None of the above

338. The volume of an object expressed in spherical co-ordinates is given by

$$V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^{1} r^2 \, \mathrm{Sin}\phi \, dr\, d\phi \, d\theta.$$ The value of the integral **(GATE-EE-2004)**

(a) $\pi/3$ (b) $\pi/6$ (c) $2\pi/3$ (d) $\pi/4$

339. The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the line $x = y$, $x = 0$, $y = 1$ in the xy plane is ______ **(GATE-EE-2015)**

340. The region specified by

$$\left\{(\rho, \varphi, z): 3 \leq \rho \leq 5, \frac{\pi}{8} \leq \varphi \leq \frac{\pi}{4}, 3 \leq z \leq 4.5\right\}$$

in cylindrical coordinates has volume of ________.

(GATE-16-EC)

341. How many distinct values of x satisfy the equation $\sin x = \frac{x}{2}$, where x is in radians?

(GATE-16-EC)

(a) 1

(b) 2

(c) 3

(d) 4 or more

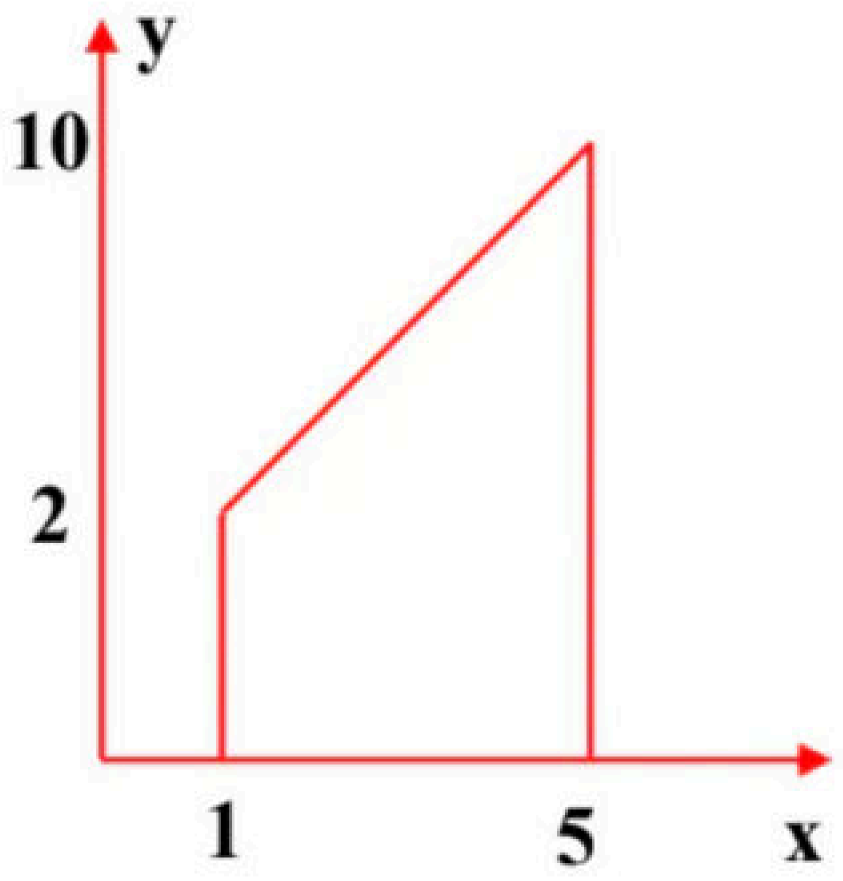
342. A triangle in the x-y plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is ________.

(GATE-16-EC)

343. Let $I = c \iint_R xy^2 dxdy$, where R is the region shown in the figure and $C = 6 \times 10^{-4}$. The value of I equals __________.

(Give the answer up to two decimal places)

(GATE-17-EE)



344. if $f(x) = R\sin\left(\frac{\pi x}{2}\right) + S, f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2R}{\pi}$, then constants R and S are respectively.

(GATE-17-CSIT)

(a) $\frac{2}{\pi}$ and $\frac{16}{\pi}$ (b) $\frac{2}{\pi}$ and 0

(c) $\frac{4}{\pi}$ and 0 (b) $\frac{4}{\pi}$ and $\frac{16}{\pi}$

345. The value of the integral $\int_0^{\pi} x\cos^2 x dx$ is

(GATE-18-CE)

(a) $\frac{\pi^2}{8}$

(b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{2}$

(d) π^2

346. The value of $\int_0^{\frac{\pi}{4}} x\cos(x^2)dx$ correct to 3 decimal places (assuming $\pi = 3.14$) is__________

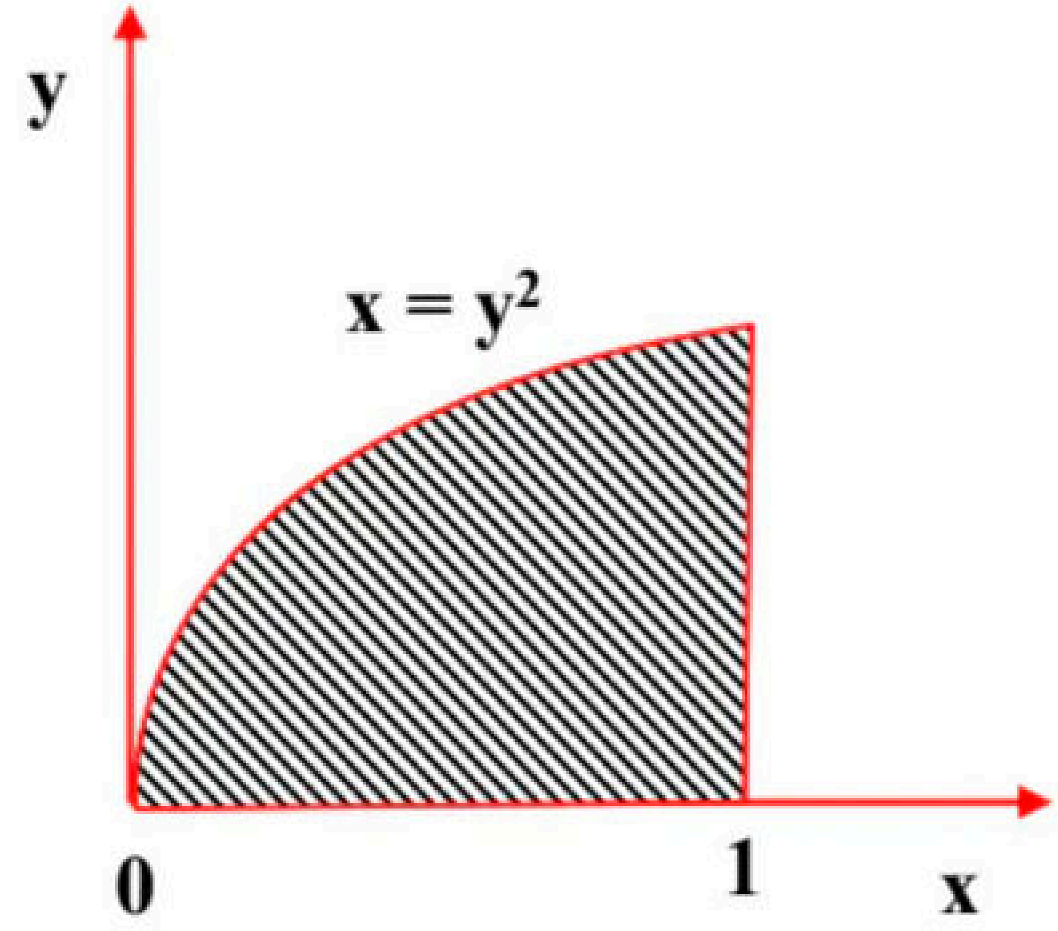
(GATE-18-CSIT)

347. The value of the integral $\int_0^{\pi}\int_y^{\pi}\frac{\sin x}{x}\,dxdy$ is equal **(GATE-19-CSIT)**

348. A parabola $x = y^2$ with $0 \le x \le 1$ is shown in the figure. The volume of the solid of rotation obtained by rotating the shaded area by 360^0 around the x-axis is

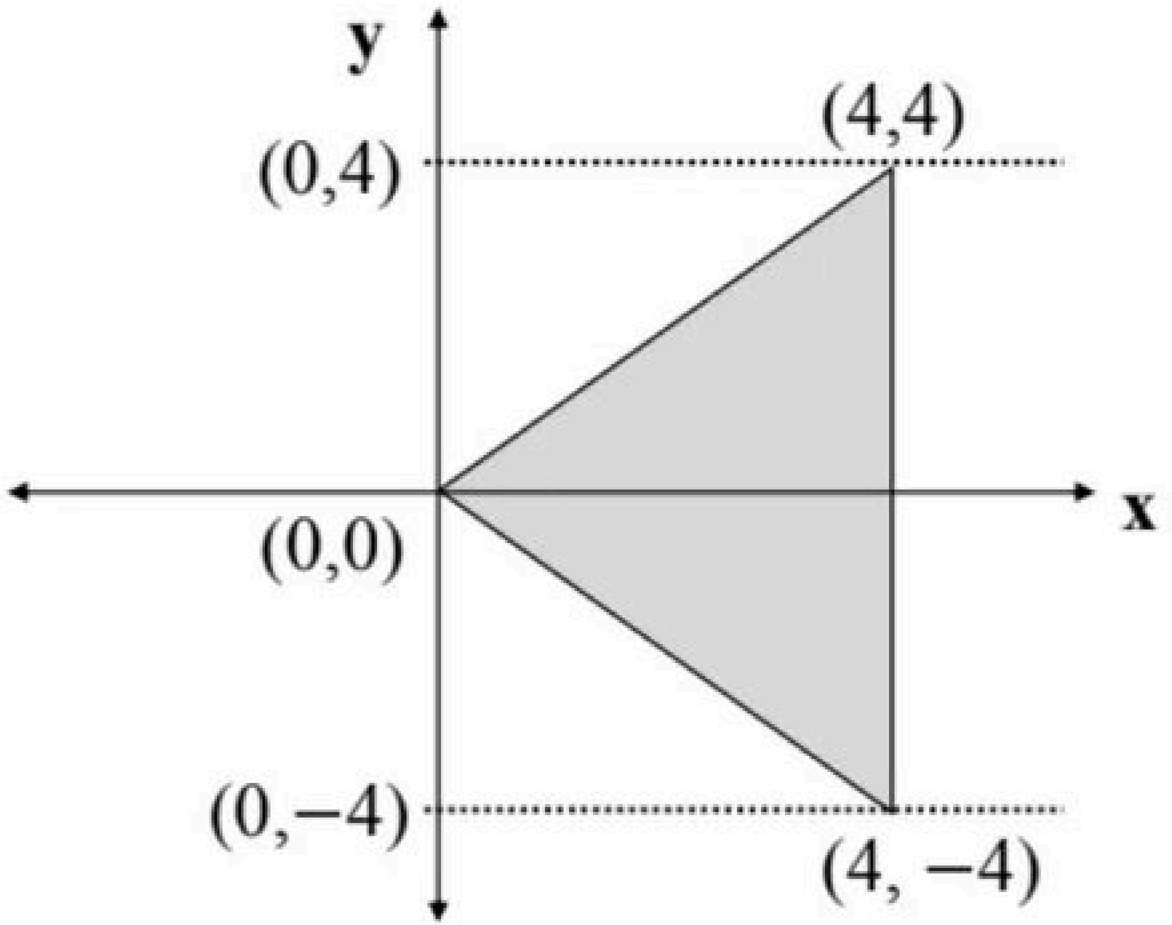
(GATE-19-ME)

(a) π (b) $\frac{\pi}{4}$ (c) 2π (d) $\frac{\pi}{2}$



349. The value the of integral $\iint_D 3(x^2 + y^2)dxdy$ where D is the shaded triangular region shown in the diagram is ______________(rounded off nearest integer).

(GATE-2022-ECE)



350. If $f(x) = 2\ln(\sqrt{e^x})$, what is the area bounded by f(x) for the interval [0, 2] on the x-axis **(GATE-2022-ME)**

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 4

351. The volume of the solid revolution generated by revolving the area bounded by the curve $y = \sqrt{x}$ and the straight lines x=4, y=0 about the x-axis, is _____

(A) 2π (B) 4π

(C) 8π (D) 12π

352. The volume of the revolution of $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$ about x axis between x = 0 and x = b is

(A) $\frac{\pi a^3}{8}\left(e^{\frac{2b}{a}} - e^{-\frac{2b}{a}}\right) - \frac{\pi a^2 b}{2}$

(B) $-\frac{\pi a^3}{8}\left(e^{\frac{2b}{a}} - e^{-\frac{2b}{a}}\right) + \frac{\pi a^2 b}{2}$

(C) $-\frac{\pi a^3}{8}\left(e^{\frac{2b}{a}} - e^{-\frac{2b}{a}}\right) - \frac{\pi a^2 b}{2}$

(D) $\frac{\pi a^3}{8}\left(e^{\frac{2b}{a}} - e^{-\frac{2b}{a}}\right) + \frac{\pi a^2 b}{2}$

353. Let V be the region bounded by the planes x = 0, x = 2, y = 0, z = 0 and y + z = 1. Then the value of the integral $\int\int_V\int y\,dx\,dy\,dz$ is

(A) 1/2 (B) 4/3

(C) 1 (D) 1/3

354. Find the volume under the plane $z = 8x + 6y$ over the region $R = \{(x,y): 0 \leq x \leq \mathbf{1}, 0 \leq y \leq 2x^2\}$ is ________

(A) $\frac{16}{5}$ (B) $\frac{32}{5}$

(C) 16 (D) 32

355. The value of $\int_0^\infty e^{-y^3}.y^{1/2}\,dy$ is ________ (GATE-ME-1994)

356. For $\lambda > \mathbf{0}$, the value of integral $\int_0^\infty e^{-\lambda x^2} dx$ equals

(A) $\frac{1}{2}\sqrt{\frac{\pi}{\lambda}}$

(B) $\sqrt{\frac{\pi}{2\lambda}}$

(C) $\sqrt{\frac{2\pi}{\lambda}}$

(D) $2\sqrt{\frac{\pi}{\lambda}}$

357. The value of the double integral $\int_{0}^{1}\int_{x}^{1/x}\frac{x}{1+y^2}\,dx\,dy =$ ______ **(GATE-EC-1993)**

358. Given $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$. If a and b are integers, the value of $\int_{-\infty}^{\infty} e^{-a(x+b)^2} dx$ is______

(GATE-2022-ME)

(a) $\sqrt{a\pi}$ (b) $\sqrt{\frac{\pi}{a}}$

(c) $b\sqrt{\pi a}$ (d) $b\sqrt{\frac{\pi}{a}}$

359. The value of the integral $\int_{-\infty}^{0} e^{-\left(\frac{x^2}{20}\right)} dx$ is ________

(A) $1/2$ (B) $-\sqrt{5\pi}$

(C) $\sqrt{10}$ (D) π

360. Evaluate: $\lim_{x \to 0} \frac{1}{x^{191}}$

(A) ∞

(B) 0

(C) $-\infty$

(D) None of these

361. Which of the following is true?

(A) Every continuous function has derivative at every point

(B) Every differentiable function may not be continuous everywhere

(C) Every differentiable function is automatically continuous

(D) A function is continuous iff it is differentiable

362. Which of the following options are necessary so that a function is differentiable at a particular point x?

(i) The existence of function at x

(ii) Existence of either right-handed or left-handed derivative or both

(iii) Continuity of the function at that particular point, x

(iv) Existence of both right-handed and left-handed derivative at x

(v) Existence and equality of both right-handed and left-handed derivative at x

(A) (i), (v) (B) (iii), (iv)

(C) (i), (iii) (D) (i), (ii)

363. The function $f(x) = |x - 4|$ on the interval $[0, 5]$ is

(a) continuous and differentiable

(b) neither continuous nor differentiable

(c) differentiable but not continuous

(d) continuous on the interval but not differentiable

364. Evaluate: $\lim_{x \to 0} \frac{1-\cos x}{x}$

(A) 0 (B) 1 (C) 2 (D) ∞

365. Consider the following statements:

S1: $f(x)=\cos|x|$ is continuous at $x=0$

S2: $f(x)=\cos|x|$ is differentiable at $x=0$

(A) S1 and S2 both are true

(B) S1 is true, S2 is false

(C) S1 is false, S2 is true

(D) Both are false

366. Consider the following function:

$$f(x) = e^{x} \sin x \,(x \neq 0)$$

$$= 0 \,(x = 0)$$

Which of the following is true?

(A) f is differentiable and hence continuous at $x = 0$

(B) f is continuous but not differentiable at $x = 0$

(C) f is neither continuous nor differentiable at $x = 0$

(D) f is differentiable but not continuous at $x = 0$

367. Consider the function: $f(x)=[x]+1$ over positive integer including 0.

Then the total number of point of discontinuities of $f(x)$ are,

(A) 1 (B) Infinite

(C) 0 (D) None of these

368. Consider the following function:

$f(x) = x + |x| + 5$

Which of the following is true?

(A) f is continuous and differentiable at $x = 0$

(B) f is continuous but not differentiable at $x = 0$

(C) f is differentiable but not continuous at $x = 0$

(D) f is neither differentiable nor continuous at $x = 0$

369. The derivative of nth – root function $f(x)=\sqrt[n]{x}$ is,

(A) $\frac{1}{n}x^{\frac{1}{n}-1}$

(B) nx^{n-1}

(C) $\frac{1}{n}x^{n-1}$

(D) $\sqrt{n}x^{\sqrt{n}-1}$

370. Which of the following points are least essential so that Mean Value theorem will be valid for the function $f(x)$?

(A) Continuity and differentiability of $f(x)$ in the interval including end-points

(B) Continuity and differentiability of $f(x)$ in the interval excluding the end points

(C) Continuity in closed interval and differentiability in the open interval

(D) Only differentiability in closed interval

371. $\lim\limits_{x\to 0}\dfrac{\sqrt[k]{1+x}-1}{x}$ (k is a positive integer)

(A) k (B) -k (C) $\dfrac{1}{k}$ (D) $-\dfrac{1}{k}$

372. $\lim_{x \to 1} \frac{x^2 - \sqrt{x}}{\sqrt{x} - 1}$

(A) 1 (B) 2 (C) 3 (D) 4

373. $\lim_{x \to 0} \frac{\sqrt[3]{1+x} - \sqrt[3]{1-x}}{x} =$

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 0 (D) 1

374. $\lim_{x \to a} \frac{x^{\frac{5}{8}} - a^{\frac{5}{8}}}{x^{\frac{1}{3}} - a^{\frac{1}{3}}} =$

(A) $\frac{15}{8} a^{\frac{7}{24}}$

(B) $\frac{15}{4} a^{\frac{7}{24}}$

(C) $\frac{-15}{8} a^{\frac{7}{24}}$

(D) $\frac{15}{4} a^{\frac{-7}{24}}$

375. $\lim\limits_{x\to 0} \dfrac{\sin 3x \tan 4x}{x \sin 5x}$

(A) 1 (B) $\dfrac{5}{12}$ (C) 0 (D) $\dfrac{12}{5}$

376. $\underset{x \to 0}{\text{lt}} \dfrac{(1-e^{x})\sin x}{x^{2}+x^{3}} =$

(A) -1 (B) 0 (C) 1 (D) 2

377. $$\lim_{x \to \frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right)\sec x}{\operatorname{cosec} x} =$$

(A) 1 (B) 0 (C) -1 (D) $\frac{1}{\pi}$

378. $\lim\limits_{x\to 0} \dfrac{4^x - 9^x}{x(4^x + 9^x)} =$ ______

(A) $\ln\left(\dfrac{3}{2}\right)$ (B) $\ln\left(\dfrac{2}{3}\right)$

(C) $\ln\left(\dfrac{4}{3}\right)$ (D) $\ln 2$

379. $\lim_{x \to 0} \dfrac{a^{\tan x} - 1}{x} =$

(A) a (B) 1 (C) 2 ln a (D) ln a

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380. The function $f(x)=(1+x)^{\frac{5}{x}}$ for $x \neq 0$

$$=e^5 \quad \text{for } x=0$$

then

(A) f(x) is continuous at x=0

(B) right continuous at x=0

(C) left continuous at x=0

(D) cannot be determined

381. Evaluate: $\lim_{n \to \infty} \frac{n^2 + 1}{n^2 + n + 1}$

(A) Undefined
(B) 1
(C) 0
(D) None of these

382. Evaluate: $\lim_{x \to 7}\left[\frac{x^6 - 7^6}{x - 7} + \frac{x^2 - 7^2}{x - 7}\right]$

(A) 327040 (B) 100856

(C) ∞ (D) None of these

383. Evaluate: $\lim_{x\to 0}\dfrac{2\sin^4\dfrac{x}{2}}{\dfrac{\left(x^4\right)}{2}}$

(A) 16

(B) $\dfrac{1}{2}$

(C) not exist

(D) $\dfrac{1}{4}$

384. Consider the statements:

S1: $f(x) = x + [x], x \in Z$ is not continuous at $x = 0$

S2: $\lim_{x \to 0+} f'(x) = \lim_{x \to 0-} f'(x)$

(A) S1 and S2 both are false

(B) S1 is true, S2 is false

(C) S1 is false, S2 is true

(D) S1 and S2 both are true but S2 is not the correct explanation of S1

385. Evaluate: $\lim_{x \to 0} \frac{e^{2x} - 1}{\log(1+x)}$

(A) 0 (B) 2 (C) 4 (D) $\frac{1}{2}$

386. Evaluate: $\lim_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$

(A) 0 (B) ∞ (C) 1 (D) $\frac{\pi}{2}$

387. $f(x)=\frac{x^2-4}{x-2},(x\neq 2)$ and $f(x)=a$ at $x=2$

Then, which of the following is true?

(A) $f(x)$ is continuous at $x=2$ if $a=1$

(B) $f(x)$ is continuous at $x=2$ if $a=4$

(C) $f(x)$ is continuous at $x=2$ if $a=2$

(D) $f(x)$ is continuous at $x=2$ if $a=0$

388. Evaluate : $\lim\limits_{x\to 0}\dfrac{5e}{e^2+e^{\frac{1}{x}}}$

(A) Doesn't exist (B) 0

(C) $\dfrac{5}{e}$ (D) None of these

389. The value of $\lim_{x \to 2a} \frac{\sqrt{x-2a}+\sqrt{x}-\sqrt{2a}}{\sqrt{x^2-4a^2}}$

(A) $\frac{1}{\sqrt{a}}$ (B) $\frac{1}{2\sqrt{a}}$ (C) $\frac{\sqrt{a}}{2}$ (D) $2\sqrt{a}$

390. The value $\lim_{x \to a} \frac{x - a}{|x - a|}$

(A) 0 (B) 1

(C) -1 (D) does not exist

891. $\lim\limits_{x \to 5} \dfrac{\sin^2(x-5)\tan(x-5)}{(x^2-25)(x-5)}$

(A) 1 (B) $\dfrac{1}{10}$ (C) 0 (D) -6

392. $\lim_{x \to 0} \frac{\text{Tanx} - \sin x}{x^3}$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{-1}{3}$

393. If $\underset{x \to 0}{\text{lt}} \dfrac{x(1 + a\cos x) - b\sin x}{x^3} = 1$ then a and b are

(A) $\dfrac{1}{2}, \dfrac{-3}{2}$ (B) $\dfrac{5}{2}, \dfrac{3}{2}$

(C) $\dfrac{-5}{2}, \dfrac{-3}{2}$ (D) $\dfrac{5}{2}, \dfrac{-3}{2}$

394. $\lim\limits_{x \to \infty}\left(\dfrac{x+a}{x+b}\right)^{x+b}$

(A) 1 (B) e^{b-a} (C) e^{a-b} (D) e^{b}

395. The function $f(x) = \dfrac{x\text{Tan}2x}{\sin 3x \sin 5x}$ for $x \neq 0$

$= k$ for $x=0$

is continuous at $x=0$ then $f(0)=$_____.

(A) $\dfrac{2}{13}$ (B) $\dfrac{2}{17}$ (C) $\dfrac{2}{11}$ (D) $\dfrac{2}{15}$

396. $\underset{x\to\infty}{\text{lt}}\left(\dfrac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} =$

(A) $e^{\frac{-2}{3}}$ (B) $e^{\frac{3}{2}}$ (C) $e^{\frac{2}{3}}$ (D) e

397. If the function $f(x)=\dfrac{\sin^2 ax}{x^2}$ for $x \neq 0$

$=1$ for $x=0$

is continuous at x=0 then a = ____ .

(A) ± 1 (B) 0 (C) $\pm\dfrac{1}{2}$ (D) $\pm\dfrac{1}{3}$

398. A function f(x) is defined as

$$f(x)=\begin{cases} ax-b; & x \le 1 \\ 3x; & 1 < x < 2 \\ b\,x^2-a; & x \ge 2 \end{cases}$$

is continuous at x=1, 2 then

(A) a=5, b=2 (B) a=6, b=3

(C) a=7, b=4 (D) a=8, b=5

899. Let $f(x) = x$ for $x < 1$

$$= 2 - x \quad \text{for } 1 \le x \le 2$$

$$= -2 + 3x - x^2 \quad \text{for } x > 2$$

then $f(x)$ is

(A) differentiable at $x=1$,

(B) differentiable at $x=2$

(C) differentiable at $x=1$ and $x=2$

(D) differentiable at $x=0$

400. $\underset{x \to \frac{5}{4}}{Lt}\left(x-[x]\right) =$ ______, where [x] is a step function

(a) $\frac{1}{4}$ (b) $-\frac{1}{4}$

(c) $\frac{1}{3}$ (d) Doesn't exist

401. $\underset{x \to 2}{\text{Lt}} \dfrac{|x-2|}{x-2} =$ ______, where $|x - 2|$ is a modulus function.

(a) 1 (b) 2

(c) –1 (d) Doesn't exist

402. $\underset{x \to 4}{\mathrm{Lt}} [x] =$ ____, where [x] is a step function

(a) -4 (b) 4

(c) 1 (d) Doesn't exist

403. $\underset{x \to 0}{\mathrm{Lt}} \dfrac{x\sin(x)}{1-\cos(x)} =$ ______.

404. $\underset{x \to \frac{\pi}{2}}{\mathrm{Lt}} \left(\tan x - \sec x\right) =$ ______.

405. $\underset{x \to \infty}{Lt} (1 + x^2)^{e^{-x}} =$ ______

(a) 2 (b) 0 (c) 1 (d) –1

406. $\underset{x \to 0}{\mathrm{Lt}}\, x^x =$ ______.

(a) 0 (b) 1 (c) –1 (d) e

407. $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{a+x} - \sqrt{a-x}}{x} =$

(a) $\dfrac{1}{\sqrt{a}}$

(b) $\sqrt{a}$

(c) $\dfrac{1}{2\sqrt{a}}$

(d) $2\sqrt{a}$

408. Which of the following is continuous at $x = 2$?

(a) $f(x)=\begin{cases} 3 & ,x=2 \\ 2x-1, & x>2 \\ \dfrac{x+7}{3}, & x<2 \end{cases}$

(b) $f(x)=\begin{cases} 2 & ,x=2 \\ 8-x, & x\neq 2 \end{cases}$

(c) $f(x)=\begin{cases} x+2, & x\leq 2 \\ x-4, & x>2 \end{cases}$

(d) $f(x)=\dfrac{1}{x^3-8}, x\neq 2$

409. Let $g(x) = \begin{cases} -x, & x \le 1 \\ x+1 & x \ge 1 \end{cases}$ and

$$f(x) = \begin{cases} 1-x, & x \le 0 \\ x^{2.} & x > 0 \end{cases}.$$

Consider the composition of f and g, i.e. $(f \circ g)(x) = f(g(x))$.

The number of discontinuities in $(f \circ g)(x$ present in the interval $(-\infty, 0)$ is

(a) 0 (b) 1

(c) 2 (d) 4

410. If $f(x)=\begin{cases} x & , \quad x \le 1 \\ 2x-1, & \text{when } x > 1 \end{cases}$

then at $x = 1$ which of the following is **true**?

(a) f(x) is continuous but not differentiate

(b) f(x) is continuous and differentiable

(c) f(x) is neither continuous nor differentiable

(d) f(x) is differentiable but not continuous

411. Let $f(x)=\begin{cases} x^2 & , \text{ if } x \le 2 \\ mx+b, & \text{if } x > 2 \end{cases}$.

If $f(x)$ is differentiable every where then

(a) $m = 4$ and $b = -4$

(b) $m = 4$ and $b = 4$

(c) $m = -4$ and $b = -4$

(d) $m = -4$ and $b = 4$

412. Which of the following functions is differentiable in the domain [–1, 1]?

(a) $f(x) = |x|$ (b) $f(x) = \cot x$

(c) $f(x) = \sec x$ (d) $f(x) = \operatorname{cosec} x$

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413. $\underset{x \to 0}{\mathrm{Lt}} \dfrac{1 - \cos x}{x \sin x} =$

(a) 0

(b) 1

(c) $\dfrac{1}{2}$

(d) does not exist

414. If $\underset{x \to 0}{\mathrm{Lt}} \dfrac{\sin 2x + a \sin x}{x^3} = b$ where b is finite value then find a and

(a) $a = -2, b = -1$

(b) $a = 2, b = -1$

(c) $a = 0, b = 3$

(d) $a = -2, b = 1$

415. $$\lim_{x\to 0}\frac{\log(\sin 2x)}{\log(\sin x)} = \underline{\qquad\qquad}.$$

416. $\underset{x \to a}{\mathrm{Lt}} (a - x) \tan\left(\frac{\pi x}{2a}\right) =$ ________.

417. $\underset{x \to 0}{\mathrm{Lt}}\, x^{\sin x} =$ __________.

418. $\underset{x \to 0}{\mathrm{Lt}} \left(\dfrac{1}{x}\right)^{\tan x} =$ __________.

419. $\underset{x \to \pi}{\mathrm{Lt}} \cot(x) =$

(a) 0

(b) 1

(c) −1

(d) does not exist

420. $\underset{x \to a}{\mathrm{Lt}} [x] =$, where [x] is step function and 'a' is an integer

(a) a (b) $a - 1$

(c) 0 (d) does not exist

421.

$$\underset{x\to\infty}{\mathrm{Lt}}\ x\tan\left(\frac{1}{x}\right) = \_\_\_\_\_\_\_\_.$$

422. If $f(x) = (x+1)^{\cot x}$ is continuous at $x = 0$ then $f(0) =$ ______

(a) 0

(b) 1

(c) e

(d) none of these

423. Let $f(x)=\begin{cases} 0 & \text{for } x=0 \\ \frac{1}{2}-x & \text{for } 0<x<\frac{1}{2} \\ \frac{1}{2} & \text{for } x=\frac{1}{2} \\ \frac{3}{2}-x & \text{for } \frac{1}{2}<x<1 \\ 1 & \text{for } x\geq 1 \end{cases}$

then which of the following is true?

(a) $f(x)$ is right continuous at $x=0$

(b) $f(x)$ is discontinuous at $x=\frac{1}{2}$

(c) $f(x)$ is continuous at $x=1$

(d) All are true

424. If $f(x) = 3 + x$ when $x \geq 0$

$= 3 - x$ when $x < 0$

then $f(x)$ at $x = 0$ is

(a) continuous but not differentiable

(b) continuous and differentiable

(c) neither continuous nor differentiable

(d) differentiable but not continuous

425. If (f) = x |x| then f (x) at x = 0 is

(a) continuous and differentiable

(b) continuous but not differentiable

(c) differentiable but not continuous

(d) neither continuous nor differentiable

426. The function $f(x) = |x+1|$ on the interval $[-2,0]$ is

(a) continuous and differentiable

(b) continuous on the interval but not differentiable

(c) neither continuous nor differentiable

(d) differentiable but not continuous

427. The function

$$f(x) = \frac{x e^{1/x}}{1+e^{1/x}}, x \neq 0$$
$$= 0 \qquad , x = 0 \text{ is}$$

(a) differentiable but not continuous at x= 0

(b) not differentiable at x = 0

(c) differentiable and continuous at x = 0

(d) not continuous at x = 0

428. $\underset{x \to 0}{\mathrm{Lt}} \sin\left(\frac{1}{x}\right) =$

(a) ∞

(b) 0

(c) 1

(d) does not exist

429. $\underset{x \to 0}{\mathrm{Lt}}\, x.\sin\left(\frac{1}{x}\right) =$

(a) 0

(b) 1

(c) ∞

(d) does not exist

430. $\underset{x \to 2}{\mathrm{Lt}} \sqrt{4 - x^2} =$

(a) 0

(b) imaginary

(c) does not exist

(d) indeterminate

431. $\underset{x \to 0^+}{\mathrm{Lt}} \log x =$

(a) ∞ (b) $-\infty$

(c) 0 (d) does not exist

432. If $\underset{x \to 0}{\text{Lt}} \left\{ \dfrac{x(1 + a\cos x) - b\sin x}{x^3} \right\} = 1$

then $(a, b) =$

(a) –5/2, 3/2
(b) 5/2, –3/2
(c) 5/2, 3/2
(d) –5/2, –3/2

433. $\underset{x \to 0}{\mathrm{Lt}} \left(\dfrac{\log x}{\log \operatorname{cosec} x} \right) =$

(a) 1 (b) – 1

(c) 0 (d) does not exist

434. $\underset{x \to \infty}{\mathrm{Lt}} \left(\frac{x^n}{e^x} \right) =$

(a) 0

(b) n!

(c) 1

(d) ∞

435. $$\underset{x \to \infty}{\text{Lt}} \left(\frac{\log x}{x^n} \right) =$$

(a) 0

(b) 1/n

(c) 1

(d) –1/n

436. If $f(x) = \{\sin^2(ax) / x^2\}, \ x \neq 0$

$= 1 \qquad , \ x = 0$

is continuous then a =

(a) 0, 1

(b) –1, 1

(c) 0, –1

(d) none of these

437. If $f(x) = 1$ when $x < 0$

$= 1 + \sin x$ when $0 \le x \le \pi/2$

$= 2 + (x - \pi/2)^2$ when $x > \pi/2$

then f(x) is

(a) continuous at $x = 0$ but discontinuous at $x = \pi/2$

(b) continuous at $x = \pi/2$ but discontinuous at $x = 0$

(c) continuous for all values of x

(d) discontinuous at $x = 0$ and at $x = \pi/2$

438. If $f(x) = x^2\sin\left(\frac{1}{x}\right)$ then which of the following is true?

(a) $f^{1}(0)$ exists but $f^{11}(0)$ does not exist

(b) both $f^{1}(0)$ and $f^{11}(0)$ does not exist

(c) neither $f^{1}(0)$ nor $f^{11}(0)$ does not exist

(d) $f^{1}(0)$ does not exist but $f^{11}(0)$ exists

439. If $f(x)=x\left(1+\left(\frac{1}{3}\right)\sin(\log x)\right)$ then f(x) is

(a) continuous at $x = 0$ but not differentiable at $x = 0$

(b) differentiable at $x=0$ but not continuous at $x = 0$

(c) continuous and differentiable at $x = 0$

(d) neither continuous nor differentiable at $x = 0$

440. The function $f(x) = |x| + |x + 1| + |x - 2|$ is differentiable at $x =$

(a) 1 (b) -1

(c) 0 (d) 2

441. If $f(x) = 2 + x$ when $x \geq 0$

$= 2 - x$ when $x < 0$

then $f(x)$ at $x = 0$ is

(a) continuous but not differentiable

(b) continuous and differentiable

(c) neither continuous nor differentiable

(d) differentiable at $x = 0$ but not continuous

442. Let $f(x)=\begin{cases} x^2.\sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

At $x = 0$, which of the following is ***true*** ?

(a) f(x) is continuous but not differentiable

(b) f(x) is differentiable and continuous

(c) f(x) is neither continuous nor differentiable

(d) f(x) is differentiable but not continuous

443. If $f(x) = x.|x|$ then at $x = 0$ which of the following statements is ***true***?

(a) f(x) is continuous and differentiable

(b) f(x) is not continuous and not differentiable

(c) f(x) is continuous but not differentiable

(d) f(x) is differentiable but not continuous

444. Let $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ -x, & \text{if } x \text{ is irrational} \end{cases}$

Which of the following is ***true***?

(a) f(x) is continuous every where

(b) f(x) is discontinuous every where

(c) f(x) is discontinuous only at x = 0

(d) f(x) is continuous only at x = 0

445. $\underset{x\to 0}{\text{Lt}} \dfrac{|x|}{x} =$ ______, where $|x|$ is a modulus function

(a) 0

(b) 1

(c) –1

(d) limit does not exist

446. $\underset{x \to 6}{\text{Lt}} [x] =$ ______, where [x] is a step function

(a) –6

(b) 5

(c) 0

(d) limit does not exist

447. $\underset{x \to 0}{\mathrm{Lt}} \dfrac{x - \sin x}{1 - \cos x} =$ ________.

(a) 0 (b) 4

(c) –3 (d) 1

448. $\underset{x \to \frac{\pi}{2}}{\text{Lt}} \left(\sec x - \dfrac{1}{1 - \sin x} \right) = \_\_\_\_\_\_.$

(a) 0 (b) ∞ (c) -1 (d) 100

449. $\underset{n\to\infty}{\mathrm{Lt}} \left(7^n + 5^n\right)^{\frac{1}{n}} =$ ________.

(a) 7 (b) –7

(c) 5 (d) 0

450. If $f(x) = \left(\frac{1-x}{x+1}\right)^{\frac{1}{x}}$ is continuous at $x = 0$

then $f(0) =$

(a) e^{-2} (b) e^{2}

(c) $\sqrt{e}$ (d) $e^{-1/2}$

451. Which one of the following functions is continuous at $x = 3$?

(a) $f(x)=\begin{cases} 5, & \text{if} \quad x=3 \\ 2x-1, & \text{if} \quad x>3 \\ \dfrac{x+7}{2}, & \text{if} \quad x<3 \end{cases}$

(b) $f(x)=\begin{cases} 4, & \text{if} \quad x=3 \\ 8-x & \text{if} \quad x\neq 3 \end{cases}$

(c) $f(x)=\begin{cases} x+3, & \text{if} \quad x\leq 3 \\ x-4 & \text{if} \quad x>3 \end{cases}$

(d) $f(x)=\dfrac{1}{x^3-27}, \quad \text{if} \quad x\neq 3$

452. Let f be a real-valued function of a real variable defined as

$f(x) = x^2$ for $x \geq 0$, and $f(x) = -x^2$ for $x < 0$.

Which one of the following statements is true?

(a) $f(x)$ is discontinuous at $x = 0$

(b) $f(x)$ is continuous but not differentiable at $x = 0$

(c) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$

(d) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$

453. If $f(x)=\begin{cases} ax^2+1, & x\le 1 \\ x^2+ax+b, & x>1 \end{cases}$ is differentiable

at $x=1$ then

(a) $a=1, b=1$ (b) $a=1, b=0$

(c) $a=2, b=0$ (d) $a=2, b=1$