DP - Part IV

Complete Course on Algorithm for GATE - CS & IT

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Searching

Linear Search

i/p: An array of n-ele, ele - x

o/p: return position of x if it found else return (-1)

ex

A	20	50	20	10	80	90	100	5	6	200	7	400
	1	2	3	4	5	6	7	8	9	10	11	12

x=6	x=20	x=100	x=500	x=400
9	1	7	-1	12

BC: 1 (minimum Time to solve)

WC = n (maximum Time to solve the problem)

$Avg = 1 + 2 + 3 + 4 + 5 + 6 + \cdots + n-1 + n$

$= \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$

also LS – TC – $T(n)$

$T(n) = O(n)$

$= \Omega(1)$

$= \cancel{\Theta(n)}$

also LS – WC – TC – $T(n)$

$T(n) = O(n)$

$= \Omega(n)$

$= \Theta(n)$

$= \cancel{\Theta(1)}$

also LS – BC – TC – $T(n)$

$T(n) = \Theta(1)$

$\frac{0+200}{2}$

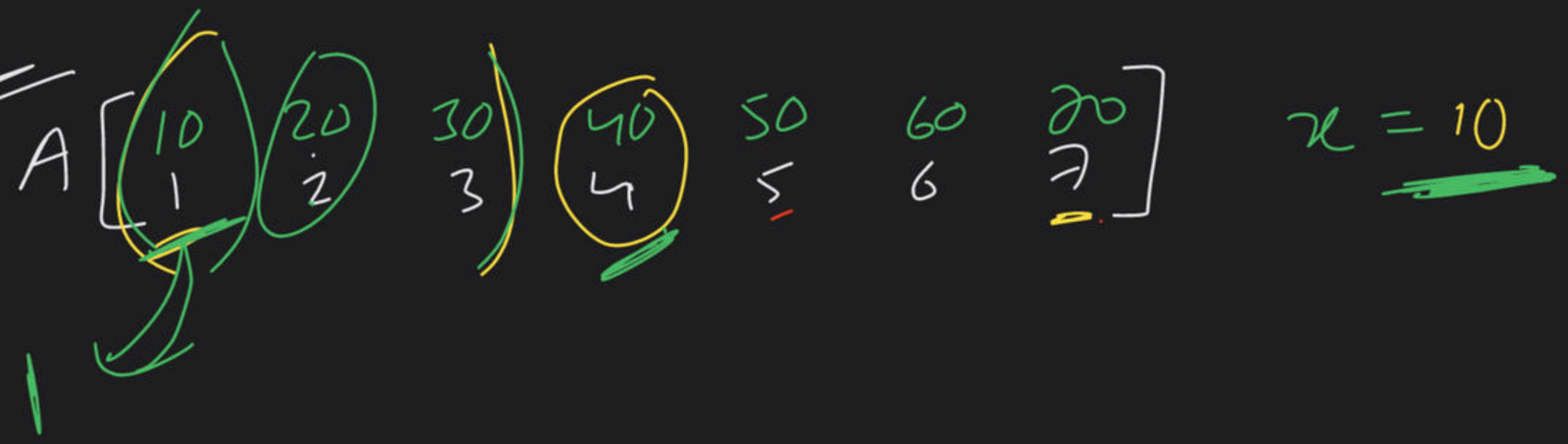
Important: $\boxed{BC \subseteq AV \subseteq WC}$

BS

i/p: Sorted array of n-ele, ele-x

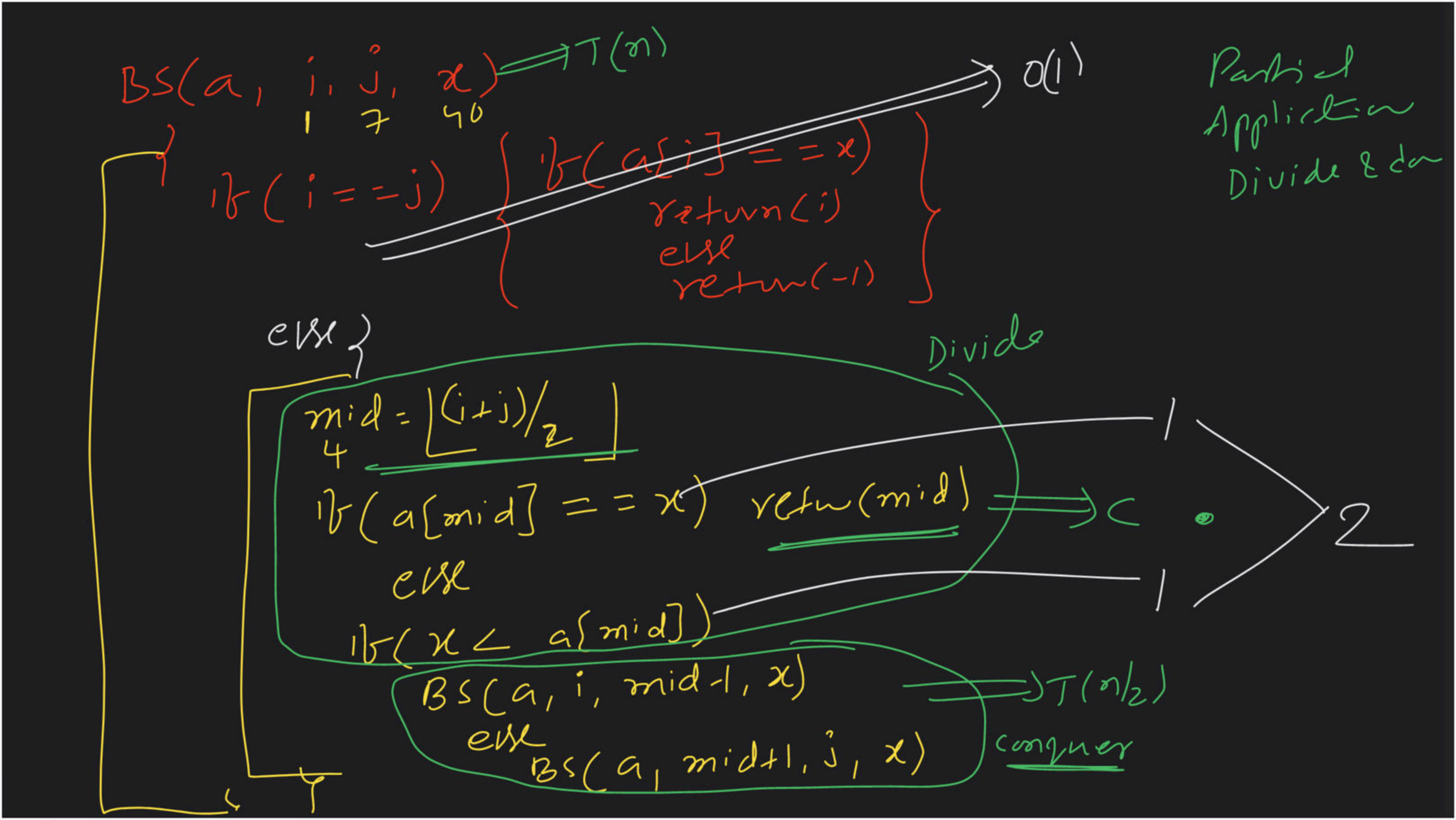
o/p: return position of x if found else (-1)

ex



$$mid = \frac{1+7}{2} = 4$$

$$= \frac{1+3}{2} = 2$$


BS(a, i, j, x)
1 7 40
T(n)
O(1)
Partial Application
Divide & Conquer
if(i==j)
if(a[i]==x)
return(i)
else
return(-1)
else
Divide
mid = ⌊(i+j)/2⌋
4
if(a[mid]==x) return(mid)
else
if(x < a[mid])
BS(a, i, mid-1, x)
else
BS(a, mid+1, j, x)
T(n/2)
conquer
1
1
c
2

Let $T(n)$ be the TC of above algo.

RR-TC { Worst case, Avg case }

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ c + T(n/2) & \text{if } n > 1 \end{cases}$$

$$T(n) = T(n/2) + c$$

$$= T(n/2^2) + c + c$$

$$= T(n/2^3) + c + c + c$$

$\vdots$ $\log n$

$$= T(1) + c \cdot \log n$$

$$= O(1) + c \cdot \log n \Rightarrow O(\log n)$$

Recursion

$T(n) = O(1)$

Static Space (WC, AC)

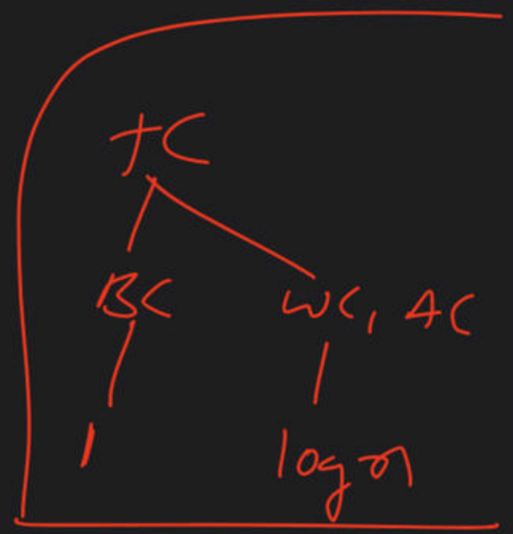
$T(n) = O(\log n)$

$= \Omega(1)$

$= \cancel{O(\log n)}$

$= \cancel{O(1)}$

```
BS(a, i, j, x)
{
    while(i <= j)
    {
        if(i == j) { if(a[i] == x) return(i) else return(-1) }
        else {
            mid = ⌊(i+j)/2⌋
            if(a[mid] == x) return(mid)
            else
                if(x < a[mid]) j = mid-1
                else i = mid+1
        }
    }
}
```



Space
$\Downarrow$
$O(1)$