GT - Part V

Complete Course on Algorithm for GATE - CS & IT

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```
fib(n) ==> T(n)
{
  if (n == 0 || n == 1)
      return (n)
  else
  return (fib(n-1) + fib(n-2))
          T(n-1)       T(n-2)
}
```

RR - TC

$$T(n) = \begin{cases} O(1) & \text{if } n = 0 \text{ (or) } n = 1 \\ T(n-1) + T(n-2) + C & \text{if } n > 1 \end{cases}$$

RR - Value

$$V(n) = \begin{cases} n & \text{if } n = 0 \text{ (or) } n = 1 \\ V(n-1) + V(n-2) & \text{if } n > 1 \end{cases}$$

RR - Addition

$$A(n) = \begin{cases} 0 & \text{if } n = 0 \text{ (or) } n = 1 \\ A(n-1) + A(n-2) + 1 & \text{if } n > 1 \end{cases}$$

$$T(n) = \begin{cases} O(1) & \text{if } n \le 1 \\ \boxed{T(n-1) + T(n-2)} + c & \text{if } n > 1 \end{cases}$$

more than

$1 - T - C$

we have to solve it using R.T.M

A(n)
{
if(n ≤ 1) return(n)
else
return(A(n/2) - A(n/2) - n)
}

$\underline{A(n/2)}$ → T(n/2) $\underline{A(n/2)}$ → T(n/2)

RR-TC

$$T(n) = \begin{cases} O(1) & \text{if } n \le 1 \\ \underline{T(n/2)} + \underline{T(n/2)} + C \end{cases}$$

$\Downarrow$

$2\underline{T(n/2)} + C$

$2^{\log_2 n} \Rightarrow n^{\log_2 2} \Rightarrow n^1$

$64^{\log_2 n} \Rightarrow n^{\log_2 64} \Rightarrow n^6$

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ 2T(n/2) + c & \text{if } n > 1 \end{cases}$$

cmer

$$T(50) = 2T(50/2) + c$$

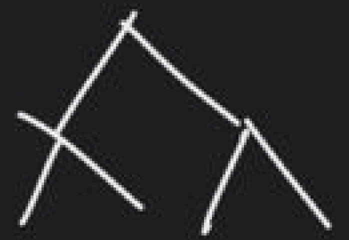
$$T(5) = 2T(5/2) + c$$

$$\frac{n}{2^k} = 1 \qquad n = 2^k \qquad \log_2 n = k$$

k- times

$\log n$

$$T(n) = 2^1 T(n/2^1) + 2^0 c$$



$$= 2\left[2T(n/2^2) + c\right] + c$$

$$= 2^2 T(n/2^2) + 2^1 c + 2^0 c$$

$$= 2^2\left[2T(n/2^3) + c\right] + 2c + c$$

$$= 2^3 T(n/2^3) + 2^2 c + 2^1 c + 2^0 c$$

$$= 2^k T(n/2^k) + 2^{k-1} c + 2^{k-2} c + \cdots + 2^1 c + 2^0 c$$

$$= n\,T(1) + c\left(2^0 + 2^1 + 2^2 + \cdots + 2^{\log n - 1}\right)$$

$$= n \cdot O(1) + c\left[\frac{2^0\left(2^{\log n} - 1\right)}{2-1}\right]$$

$$\frac{a(r^n - 1)}{r - 1}$$

$$= n + c \cdot \left[\frac{1(n-1)}{1}\right] \Rightarrow \Omega(1)$$

$$= n + c \cdot (n - 1) \Rightarrow \Theta(n) \quad //$$

$$\Rightarrow O(n^{2 \cdot 1})$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 7T(n/2) + n^2 & \text{if } n > 1 \end{cases}$$

$$T(100) = 7T\left(\frac{100}{2}\right) + 100^2$$

$$T(5) = 7T(5/2) + 5^2$$

$$T(n) = 7T(n/2) + n^2$$

$$= 7\left[7T(n/2^2) + (n/2)^2\right] + n^2$$

$$= 7^2 T(n/2^2) + \frac{7}{4} n^2 + n^2$$

$$= 7^2\left[7T(n/2^3) + \left(\frac{n}{2^2}\right)^2\right] + \frac{7}{4} n^2 + n^2$$

$$= 7^3 T(n/2^3) + \left(\frac{7}{4}\right)^2 n^2 + \left(\frac{7}{4}\right)^1 n^2 + \left(\frac{7}{4}\right)^0 n^2$$

k

$$= 7^k T\left(n/2^k\right) + n^2\left[(7/4)^{k-1} + (7/4)^{k-2} + \dots + (7/4)^1 + (7/4)^0\right]$$

$$= 7^{\log_2 n} T(1) + n^2\left[(7/4)^0 + (7/4)^1 + \dots + (7/4)^{\log_2 n - 1}\right]$$

$$= n^{\log_2 7} \cdot O(1) + n^2\left[1 \, \frac{(7/4)^{\log_2 n} - 1}{7/4 - 1}\right]$$

$$= n^{\log_2 7} + n^2\left[(7/4)^{\log_2 n}\right]$$

$$= n^{\log_2 7} + n^2 \cdot \frac{7^{\log_2 n}}{4^{\log_2 n}} \Rightarrow n^{\log_2 7} + \cancel{n^2} \cdot \frac{n^{\log_2 7}}{\cancel{n^2}}$$

$$\Rightarrow n^{\log_2 7} + n^{\log_2 7} \Rightarrow O\left(n^{\log_2 7}\right)$$

$$\Rightarrow O\left(n^{2.81}\right)$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$k = \log_2 n$$

$$T(n) = \begin{cases} 1 & \text{if } n = 0 \\ T(n-2) + n^2 & \text{if } n > 0 \end{cases}$$

$$T(100) = T(98) + 100^2$$

$$T(2) = T(0) + 2^2$$

$$1 + 2^2$$

$$T(n) = T(n-\overset{2\times 1}{2}) + (n-0)^2$$

$$\Downarrow$$

$$T(n-\overset{2\times 2}{4}) + (n-2)^2 + (n-0)^2$$

$$\Downarrow$$

$$T(n-6) + (n-4)^2 + (n-2)^2 + (n-0)^2$$

$$2\times 3$$

$$\Big\} K$$

$$T(\boxed{n-2K}) + (n-(2K-2))^2 + (n-(2K-4))^2 + \cdots + (n-2)^2 + (n-0)^2$$

$$T(0) + 2^2 + 4^2 + 6^2 + 8^2 + \cdots + (n-2)^2 + n^2$$

$$n - 2K = 0$$

$$n = 2K$$

$$K = n/2$$

$$T(n) = 1 + \left[2^2 + 4^2 + 6^2 + 8^2 + \cdots + n^2\right]$$

$$= 1 + \left[(2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + \cdots + \left(2 \times \frac{n}{2}\right)^2\right]$$

$$= 1 + 2^2\left[1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \cdots + \left(\frac{n}{2}\right)^2\right]$$

$$= 1 + 2^2\left[\frac{\frac{n}{2}\left(\frac{n}{2}+1\right)\left(2\frac{n}{2}+1\right)}{6}\right]$$

$$\Omega(n^3)$$

$$= O(n^3)$$

$$= \Theta(n^3)$$

$$= O(n^{10})$$

$$= \Omega(n)$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ T(n/2) + n & \text{if } n > 1 \end{cases}$$

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \\ \sqrt{n}\, T(\sqrt{n}) + n & \text{if } n > 2 \end{cases}$$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n-1) + n & \text{if } n > 1 \end{cases}$$