CFG and PDA - VII

Complete Course on Theory of Computation

Subbarao Lingamgunta • Lesson 31 • Mar 16, 2022

Not

① $S(a^*) = [S(a)]^*$

② $S(a+b+c) = S(a) + S(b) + S(c)$

③ $S(a \cdot b) = S(a) \cdot S(b)$

④ $S(\epsilon) = \epsilon$

⑤ $S(\phi) = \phi$

$\Sigma \rightarrow P(\Delta^*)$

R.L

ex $\Sigma = \{0,1\}$ $\Delta = \{a,b\}$ substitution s is defined as $s(0) = a$, $s(1) = b^b$

① $s(010) = s(0) \cdot s(1) \cdot s(0) = a b^b a$

② $s(0^b(0+1)1^b) = s(0^b) \cdot (s(0) + s(1)) \cdot s(1^b)$

$[s(0)]^b \cdot (s(0) + s(1)) \cdot [s(1)]^b$

$a^b (a + b^b) \cdot (b^b)^b$

$(a + a^b b^b) b^b$

$a^+ b^b$ → $a^b b^b$

$a^b b^b$

$a^b (a + b^b) \cdot b^b \Longrightarrow a^b b^b$

$\in \quad \in \quad \in$

Ex $\Sigma = \{0\}$ $\Delta = \{a, b\}$ substitution f is defined as $f(0) = (a+b)(a+b) = (a+b)^2$

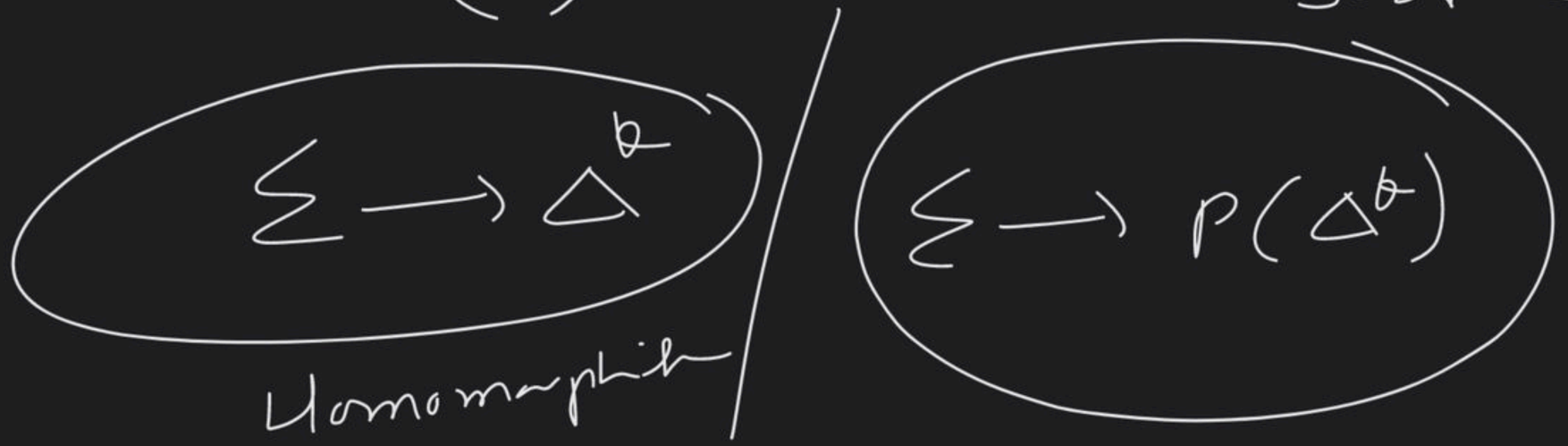
find $f(00) = f(0) \cdot f(0) = (a+b)^2 \cdot (a+b)^2 = (a+b)^4$

$f(0^b) = [f(0)]^b = [(a+b)^2]^b =$ set of all even length strings.

Homomorphism

It is a special type of substitution where every symbol in Σ mapped to one of its string over Δ

(or)



ex

$\Sigma = \{0, 1\}$ $\Delta = \{a, b, c\}$

and h is defined as

$h(0) = ab \Longrightarrow h^{-1}(ab) = 0$

$h(1) = bbc \Longrightarrow h^{-1}(bbc) = 1$

$h(010) = h(0) \cdot h(1) \cdot h(0)$

$= ab \cdot bbc \cdot ab \Rightarrow abbbcab$

$h(\epsilon) = \epsilon$

$h(\phi) = \phi$

$h(a+b) = h(a) + h(b)$

$h(a \cdot b) = h(a) \cdot h(b)$

$h(a^b) = [h(a)]^b$

Inverse Homomorphism

$h^{-1}(w) = \{ x \mid$ if $h(x) = w \}$

$h(0) = w$

$h^{-1}(w) = 0$

$h^{-1}(aa) = 0$

Homomorphism.

ex $\Sigma = \{0,1\}$ $\Delta = \{a,b\}$ and h is defined as $h(0) = aa$, $h(1) = aba$ $\Longrightarrow h^{-1}(aba) = 1$

Let $L = (ab + ba)^* a$ $\Longrightarrow aba,$

Find $h^{-1}(L) = h^{-1}((ab + ba)^* a)$

$= h^{-1}(aba) \Rightarrow 1$

ex $\Sigma = \{0,1,2\}$ $\Delta = \{a,b\}$

$h(0) = a$, $h(1) = ab$ $h(2) = ba$

1. find $h^{-1}(ababa) = h^{-1}(a) \cdot h^{-1}(ba) \cdot h^{-1}(ba)$

$0 \quad 2 \quad 2 \Longrightarrow 022$

$h^{-1}(ab\,ab\,a) = 110$

$h^{-1}(ab\,a\,ba) = 102$

022, 110, 102

2. $h^{-1}(a(ba)^b) = h^{-1}(a) \cdot [h^{-1}(ba)]^b$

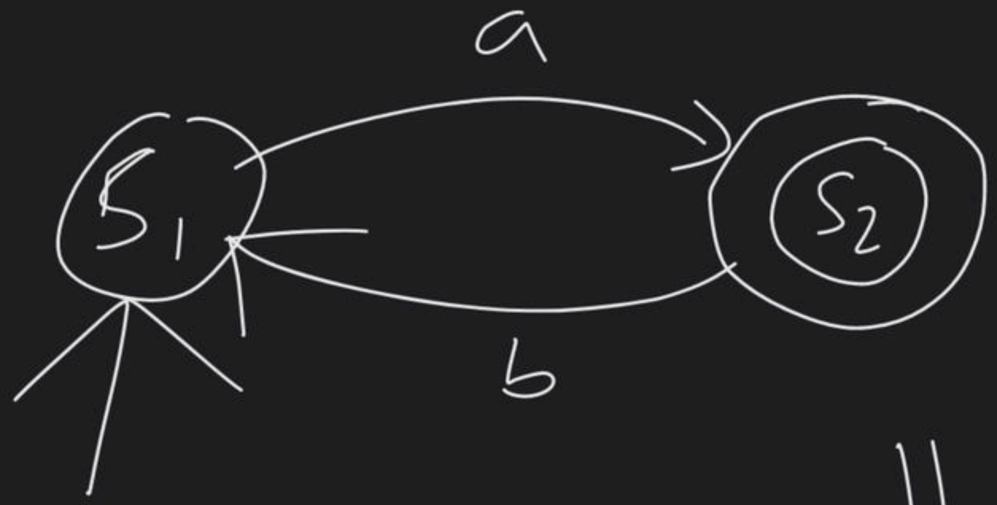
$= 0\,2^b$

$(ab)^b a$

$[h^{-1}(ab)]^b \cdot h^{-1}(a)$

$1^b \cdot 0$

$1 2^b + 1^b 0$



$$(ab)^* a \quad = \quad a(ba)^*$$

$L = \{ ab, d, babc, \ldots, bbb\ ccc \}$

half(L) (or) $\frac{1}{2}L = \{ a, ba, bbb \}$

2nd half(L) $= \{ b, bc, ccc \}$

If L is regular $\frac{1}{2}L$ also regular

So R.L closed under $\frac{1}{2}L$.

$L = \{ abc, aabbcc, aaabbbccc \}$

$\frac{1}{3}L = \{ a, aa, aaa \}$

$2^{nd} \frac{1}{3}(L) = \{ b, bb, bbb \}$

$3^{rd} \frac{1}{3}(L') = \{ c, cc, ccc \}$

$$\begin{array}{r} L - R.L \\ \frac{1}{3}L - R.L \\ \hline \frac{1}{3}L \end{array}$$

Regular Languages closed under

⇓

(1) Union (2) Intersection (3) Kleene star

(4) Concatenation (5) prefix (6) Suffix (7) Substring

(8) Substitution (9) Homomorphism (10) Inverse Homomorphism

(11) complement (12) Difference (13) Reversal

(14) Quotient operation (15) $\frac{1}{2}L$ (16) $\frac{1}{3}L$

Subset not closed over any Lang

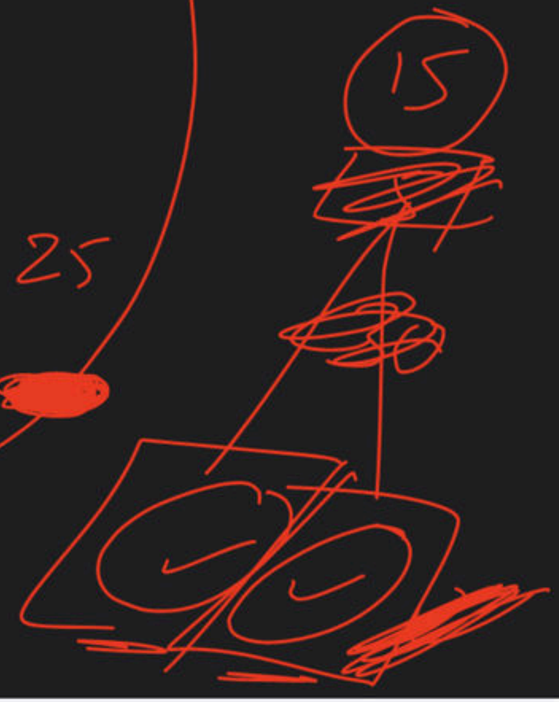
$L = \{ ab, bc \}$

$ab + bc$

Finite Automata

&

Regular Language

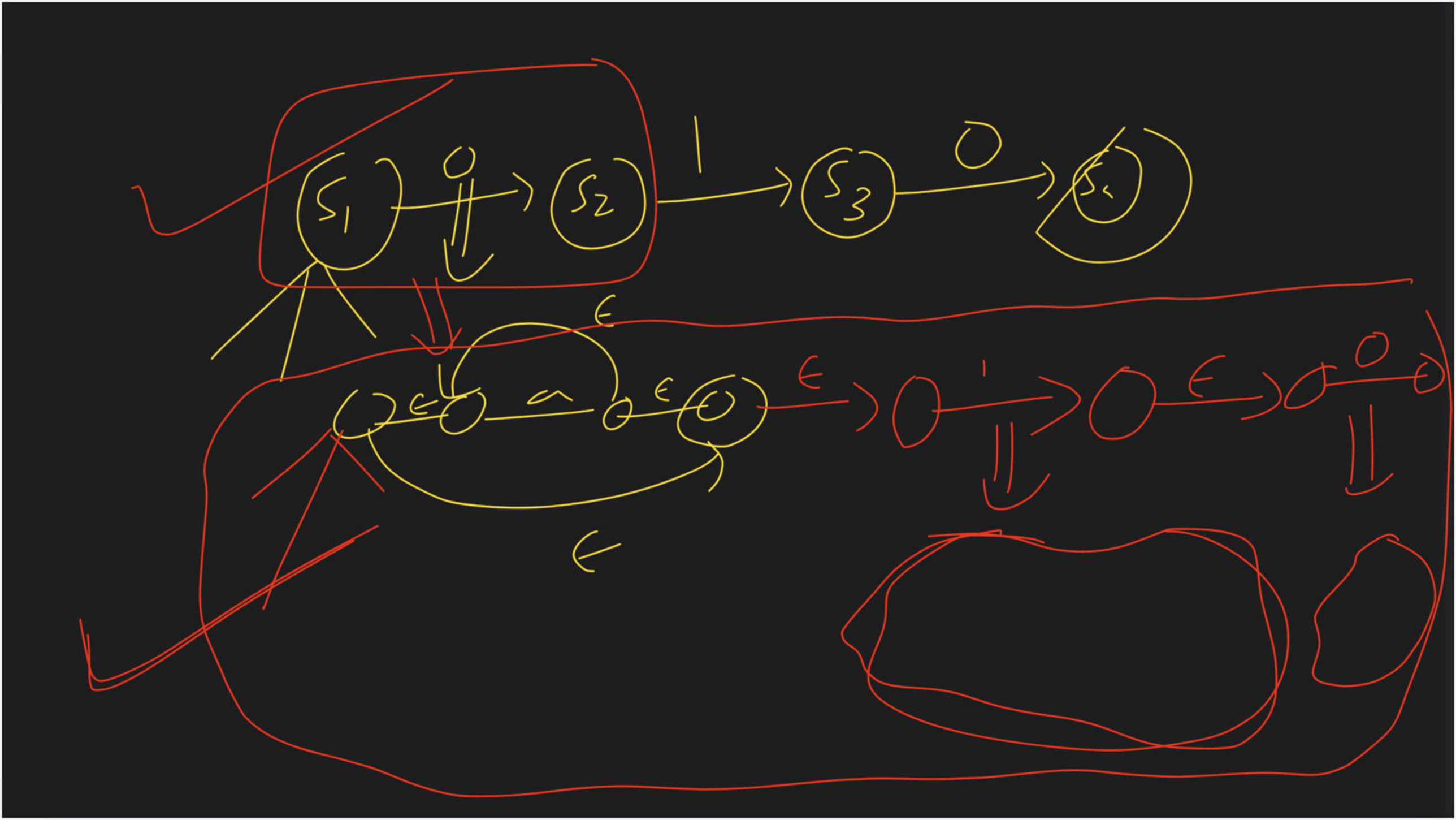


Context Free Languages & Push Down Automata

Note

In the given R.L if every symbol substituted by another R.L then result also R.L.

So R.L are closed under substitution operation.


S1
0
S2
1
S3
0
S4
ε
ε
ε
ε
ε
ε
ε
0
1
0
0
1