



Practice Session on Linear Algebra - Part II

Revision Course on Engineering Mathematics - GATE, CS & IT

PROBABILITY

- Basic definitions
- Problems on Coin , Dice , Cards and Balls
- Conditional Probability
- Independent events
- Total Probability
- Bayes Theorem

PROBABILITY

- Random Variable
Mean , Variance , S.D
- Probability Distributions
 - Binomial ,
 - Poisson,
 - Uniform ,
 - Exponential ,
 - Gaussian (Normal)

Sample space

All the possible outcomes of a random experiment .

Eg : tossing a coin

$$S= \{ \text{Head , Tail} \}$$

Rolling a dice

$$S = \{ 1,2,3,4,5,6 \}$$

Event

The outcomes of a random experiment are called as Event .Event is always subset of the sample space .

Favorable events

The outcomes which are favorable to my desired event .

Mutually exclusive events

Two events A and B are said to be mutually exclusive (disjoint or incompatible) if the occurrence of one event prevents the occurrence of other event , i.e the events does not occur simultaneously .

Equally likely events

Occurrence of any event in a random experiment are equal then the events are said to be equally likely events .

Eg :

Independent events

The occurrence of one event does not depend on another.

- Eg :
1. when two dice are rolled , Getting ‘1’ on first die does not depend on ‘2’ on the second die .
 2. when an unbiased coin is tossed two times, the event of getting a head in the first toss is independent of getting head in the second toss .

Probability

The probability of an event A is defined as

$$P(A) = \frac{\text{number of favourable events}}{\text{total number of possible events}}$$
$$= \frac{n(A)}{n(S)}$$

Properties of Probability

$$1. P(A) = \frac{n(A)}{n(S)}$$

$$2. 0 \leq P(A) \leq 1$$

Impossible event

Sure event
(certain event)

3. Sum of all probabilities = 1

$$\sum P = 1$$

4. P(sample space) = 1

$$5. P(\overline{A}) = 1 - P(A)$$

$$6. P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A & B are mutually exclusive events,

$$P(A \cap B) = 0$$

$$7. P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$$

$$8. P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$P(\overline{A \cup B}) = P(\overline{A} \cap \overline{B})$$

$$9. P(\overline{A \cap B}) = 1 - P(A \cap B)$$

$$P(\overline{A \cap B}) = P(\overline{A} \cup \overline{B})$$

$$10. P(\text{only A}) = P(A \cap \overline{B}) = P(A) - P(A \cap B)$$

$$11. P(\text{only B}) = P(\overline{A} \cap B) = P(B) - P(A \cap B)$$

12. $P(\text{Both A \& B}) = P(A \cap B)$

13. $P(\text{at least one}) = P(A \cup B)$

14. $P(\text{Either A or B}) = P(A \cup B)$

15. $P(\text{neither A nor B}) = P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B})$

16. $P(\text{exactly one}) = P(A \Delta B) = P(A \cap \overline{B}) + P(\overline{A} \cap B)$

$$18. P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

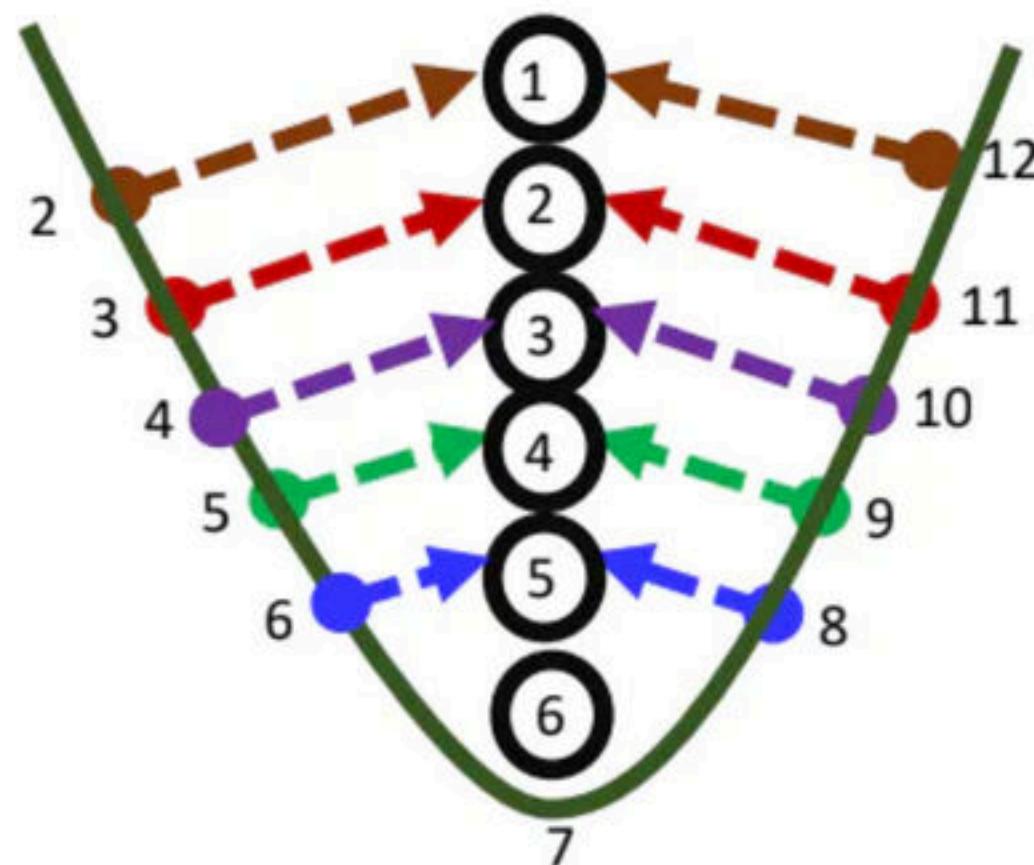
Sample Space

Rolling a dice

When two dice are rolled

$$n(S) =$$

Sum of the numbers on the dice =

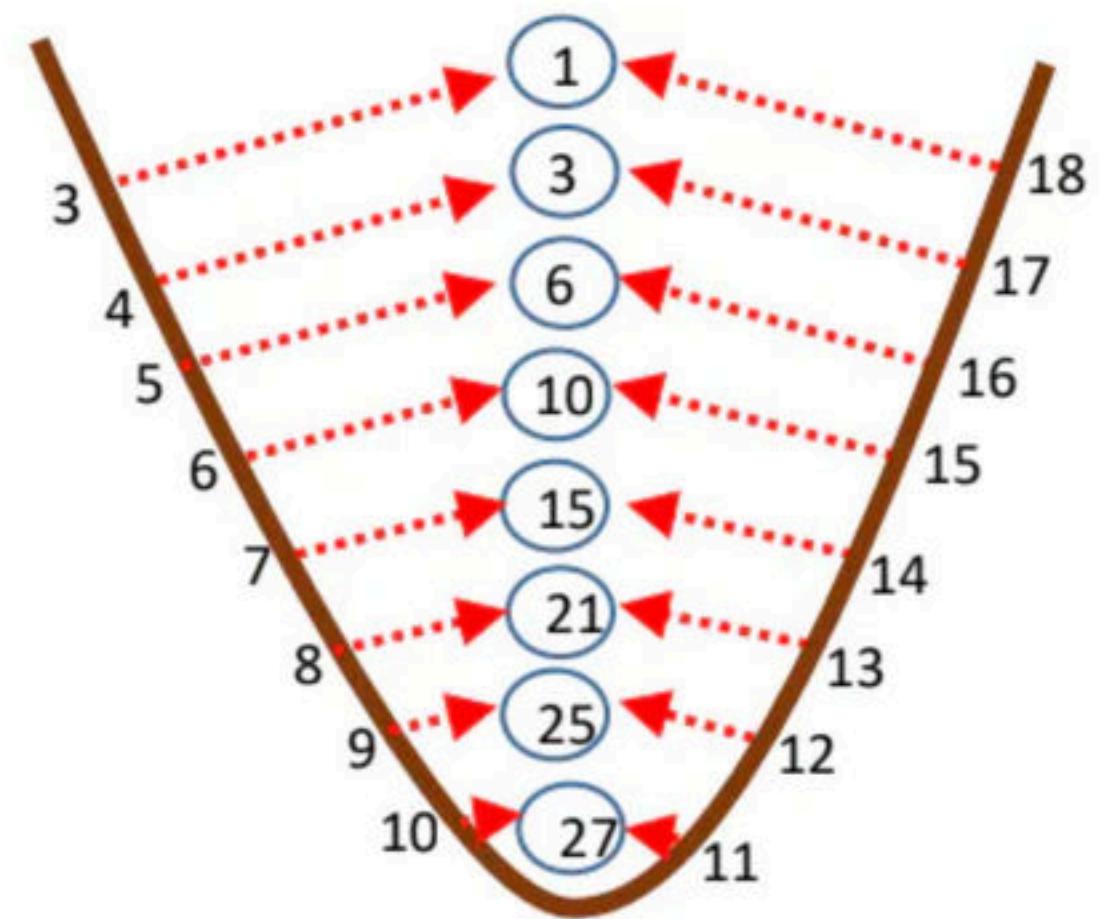


Rolling a dice

When three dice are rolled

$$n(S) =$$

Sum of the numbers on the dice = {



Pack of cards (52)

Red(26)

Hearts(13)

Diamonds(13)

Black(26)

Spade(13)

Club(13)

Each suit contains

2, 3, 4, 5, 6, 7, 8, 9, 10

Number cards = 9

J, K, Q

Face cards = 3

A

Honour cards = 4

1. Total number of face cards =

2. Total number of number cards =

3. Total number of honour cards =

4. Total number of Red Kings =

5. Total number of Spade Queen =

6. Total number of Black Diamonds =

7. Total number of Diamond Ace =

8. Total number of Black 2's =

9. Total number of Red face cards =

10. Total number of Spade King =

11. Total number of Club 9's =

12. Total number of Red Hearts =

1. When two dice are rolled , what is the probability of getting the sum

- a) sum = 4
- b) sum =11
- c) sum > 10
- d) sum \leq 10
- e) $4 \leq \text{sum} \leq 11$

2. A and B are playing a game of tossing a coin . One who gets head wins the game . If A starts the game , find the probabilities of their winning .

Joint Probability

If A and B are the two events in sample space S , which are not mutually exclusive then the joint probability of A and B can be denoted as $P(A \cap B)$.

Conditional Probability

If A and B are the two events in sample space S , then the conditional probability of A given B is defined as

$$P \left[\frac{A}{B} \right] =$$

the conditional probability of B given A is defined as

$$P \left[\frac{B}{A} \right] =$$

Properties of conditional probability

Pair wise independent events

If A and B are pair wise independent events

Mutually independent events

If A, B and C are Mutually independent events
then

Total Probability Theorem

If sample space contains n – mutually exclusive events, then probability of event A defined on the sample space S can be expressed as a conditional probability .

$$P(A) = \sum_{n=1}^N P(B_n)P\left(\frac{A}{B_n}\right)$$

3. A box contains 5 red balls and 6 black balls , another box contains 6 red balls and 4 black balls. One box is chosen at random and one ball is drawn from it. Find the probability of getting

- a)Red ball
- b) Black ball

Bayes' theorem

If sample space S contains n- mutually exclusive events, let A is any event in the sample space , then the conditional probability of B_n given A is

$$P\left[\frac{B_n}{A}\right] = \frac{P\left(\frac{A}{B_n}\right)p(B_n)}{\sum_{n=1}^N P\left(\frac{A}{B_n}\right)p(B_n)}$$

4. A box contains 5 red balls and 6 white balls , another box contains 4 red ball and 6 white balls . One ball is drawn and found to be red . Find the probability that the ball is drawn from first box .

5. 25 girls out of 100 , 5 boys of 100 have color blind. One person is chosen at random and found to be colorblind . Find the probability that person is a girl .

6. A box contains 2 white and 3 black balls , a sample of size 4 is made ,what is the probability that the sample is in the order {white , black , white , black }

7. A box contains 6 red balls, 4 white balls, and 5 blue balls . Three balls are drawn successively from the box . Find the probability that they are drawn in the order red , white and blue if each ball is

- a) Replaced
- b) Not replaced

8. A box contains 52 badges numbered 1 to 52 . Suppose that the numbers 1 through 13 are considered lucky. A sample of size 2 is drawn from the box with replacement . What is the probability that

- a) Both badges drawn will be lucky
- b) Neither badges will be lucky
- c) Exactly one of the badges drawn will be lucky
- d) At least one of the badges will be lucky

9. In a experiment of drawing a card from a pack the event of getting a spade is denoted by A , and getting a face card is denoted as B , find the probabilities of $A \cup B$ and $A \cap B$

10. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that sum of the numbers is

- a) An even number
- b) Odd number

11. Find the ratio of probabilities getting sum 6 with 4 , 3 and 2 dice are rolled respectively

12. A box contains 4 point contact diodes and 6 alloy junction diodes, what is the probabilities that 3 diodes picked at random contain at least 2 point contact diodes

3.

13. A box contains 5 black , 4 white and 6 red balls . Two balls are drawn without replacement , what is the probability that the first will be white and second will be black

14. One card is drawn from a regular deck of 52 cards, what is the probability of the card being either red or king

15. One card is selected from an ordinary 52 card deck with an event A as select a king , B as select a jack or queen C as select a heart , find $P(A \cap B)$, $P(B \cap C)$ and $P(C \cap A)$

16. There are 3 black and 4 white balls in one bag, 4 black and 3 white balls in the second bag . A die is rolled and the first bag is selected if it is 1 or 3 and second bag for the remaining , find the probability of drawing a black ball from the selected bag

17. A box contains 3 coins, one is fair ,one is two headed and one coin is weighted so that the probability of heads appearing is $\frac{1}{3}$. A coin is selected at random and tossed , find the probability that head appears.

18. Three boxes numbered I II and III contain 1-white , 2 black and 3 red balls,: 2 white 1 black and 1 red ball ; 4 white 5 black and 3 red balls respectively. One box is randomly selected and a ball drawn from it . If the ball is red then find the probability that it is from box II

19. Three coins are tossed at a time find the probability of getting

- a) At most one tail
- b) At least one tail
- c) At least one head and at most one tail

20. Four coins are tossed at a time find the probability of getting at most 2 head and at most 2 tail

21. Four coins are tossed at a time find the probability of getting at most 2 head and at least 1 tail

22. Six coins are tossed at a time find the probability of getting at least 2 head and at least 2 tails

23. N- coins are tossed at a time find the probability of getting head

- a) Odd number of times
- b) Even number of times

24. Three horses A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. What is the probability that B or C wins?

- a) 2/7
- b) 3/7
- c) 4/7
- d) 6/7

25. A card is selected at random from an ordinary pack of 52 cards. Probability of selecting a Spade card or a face card is

- a) $3/32$
- b) $23/52$
- c) $22/52$
- d) $25/52$

26. Let two items be chosen from a lot containing 12 items of which 4 are defective. What is the probability that at least one item is defective?

- a) $19/33$
- b) $14/33$
- c) $1/11$
- d) $13/33$

27. A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8?

- a) $1/3$
- b) $1/4$
- c) $1/5$
- d) $2/3$

28. A point is selected at random inside a circle. Find the probability p that the point is closer to the Centre of the circle than to its circumference?

- a) $1/3$
- b) $1/4$
- c) $1/5$
- d) $2/3$

29. Let A and B be events with $P(A) = 3/8$, $P(B) = 1/2$ and $P(A \wedge B) = 1/4$ then which of the Following is false.

- a) $P(A^C \vee B^C) = 3/4$
- b) $P(A^C \wedge B^C) = 3/8$
- c) $P(A \wedge B^C) = 1/8$
- d) $P(B \wedge A^C) = 5/8$

30. Of 120 students, 60 are studying French, 50 are studying Spanish and 20 are studying French and Spanish. If a student is selected at random then which of the following is not correct.

- a) Probability that the student is studying French or Spanish is 0.75.
- b) Probability that the student is studying neither French nor Spanish is 0.25.
- c) Probability that the student is studying Spanish but not French is 0.25.
- d) Probability that the student is studying French but not Spanish is 0.3

31. In a class of 100 students, 40 failed in mathematics, 30 failed in physics, 25 failed in Chemistry, 20 failed in math's and physics, 15 failed in physics and chemistry, 10 failed in Chemistry and math's, 5 failed in math's, physics and chemistry. If a student is selected at Random then the probability that he passed in all three subjects is

- a) 0.4
- b) 0.45
- c) 0.55
- d) 0.65

32. Let a pair of dice be tossed. If the sum is 6, find the probability that one of the dice is a 2.

a) $1/5$

b) $2/5$

c) $3/5$

d) $4/5$

33. A man visits a couple who have two children. One of the children, a boy, comes in to the room . Find the probability that the other is also a boy
- a) $1/3$ b) $2/3$ c) $1/2$ d) $3/4$

34. Let A and B be events with $P(A) = 3/8$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$. Find the conditional probability $P(A|B)$

- a) $1/3$
- b) $2/5$
- c) $3/4$
- d) $1/2$

35. In certain college, 25% of the students failed mathematics, 15% of the students failed in Chemistry, and 10% of the students failed in both math's and chemistry. A student is Selected at random. If he failed chemistry, what is the probability that he failed in math's?

- a) $\frac{2}{3}$
- b) $\frac{2}{5}$
- c) $\frac{3}{5}$
- d) $\frac{1}{5}$

36. A die is rolled. If the number appeared is odd, what is the probability that it is prime?

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{3}{4}$

d) 1

37. In a certain college, 4% of the men and 1% of the women are taller than 1.8m. Further more, 60% of the students are women. Now if a student is selected at random and is taller than 1.8m, what is the probability that the student is a woman ?

- a) 3/11
- b) 4/11
- c) 5/11
- d) 6/11

38. We are given three urns as follows. Urn A contains 3 red and 5 white marbles, Urn B contains 2 red and 1 white marble, Urn C contains 2 red and 3 white marbles. An urn is selected at random and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A?

- a) $45/173$
- b) $37/165$
- c) $27/109$
- d) $39/185$

39. A coin, weighted so that $P(H) = 2/3$ and $P(T) = 1/3$ is tossed. If heads appears, then a number is selected at random from the numbers 1 through 9. If tails appears, then a number is selected at random from the numbers 1 through 5. Find the probability P that an even number is selected.

- a) $67/145$
- b) $58/135$
- c) $74/157$
- d) $43/142$

40. A box contains three coins, two of them fair and one two headed. A coin is selected at random and tossed twice. If heads appears both times, what is the probability that the coin is two headed?

a) $\frac{2}{3}$

b) $\frac{1}{3}$

c) $\frac{3}{4}$

d) $\frac{1}{2}$

41. An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the opposite colour is put in to the urn. A second marble is drawn from the urn. If both marbles were of the same colour . What is the probability that they were both white?

- a) $\frac{5}{6}$
- b) $\frac{7}{8}$
- c) $\frac{8}{9}$
- d) $\frac{9}{10}$

42. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(GATE-EC-2003)

- (a) 100%
- (b) 50%
- (c) 49%
- (d) none

43. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

a) $\frac{3}{23}$

b) $\frac{6}{23}$

c) $\frac{3}{10}$

d) $\frac{3}{5}$

44. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses ?

- (a) 15 (b) 20 (c) 25 (d) 35

45. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(GATE-CS-1995)

- (a) $18/25$
- (b) $2/5$
- (c) $5/12$
- (d) $19/25$

46. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced ?

- (a) $1/26$
- (b) $1/52$
- (c) $1/169$
- (d) $1/221$

47. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is

(GATE-CS -2005)

- a) $\frac{1}{36}$
- b) $\frac{1}{6}$
- c) $\frac{1}{4}$
- d) $\frac{1}{3}$

48. Two dice are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is **(GATE-PI-2005)**

(a) $4/36$

(b) $7/36$

(c) $9/36$

(d) $10/36$

49. The probability that there are 53 Sundays in a randomly chosen leap year is
(GATE-IN-2005)
- (a) $1/7$
 - (b) $1/14$
 - (c) $1/28$
 - (d) $2/7$

50. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads ?
(GATE-EC-2009)

(a) $\left(\frac{1}{2}\right)^2$

(b) $10c_2 \left(\frac{1}{2}\right)^2$

(c) $\left(\frac{1}{2}\right)^{10}$

(d) $10c_2 \left(\frac{1}{2}\right)^{10}$

51. If three coins are tossed simultaneously, the probability of getting at least one head is
(GATE-ME-2009)

- (a) $1/8$
- (b) $3/8$
- (c) $1/2$
- (d) $7/8$

52. A fair coin is tossed independently four times. The probability of the event “The number of times heads show up is more than the number of times tails show up” is

(GATE-EC-2010)

- (a) $1/16$
- (b) $1/8$
- (c) $1/4$
- (d) $5/16$

53. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **(GATE-CE-2010)**

- (a) $1/8$
- (b) $1/6$
- (c) $1/4$
- (d) $1/2$

54. A fair dice is rolled two times. The probability that the 2nd toss results in a value that is higher than the first toss is **(GATE-EC-2011)**

- (a) 2/36
- (b) 2/6
- (c) 5/12
- (d) 1/2

55. The box 1 contains chips numbers 3, 6, 9, 12 and 15. The box 2 contains chips numbers 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is
(GATE-IN-2011)

- (a) $6/25$
- (b) $2/5$
- (c) $5/12$
- (d) $19/25$

56. A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is **(GATE-ME,PI-2012)**

- (a) $1/20$
- (b) $1/12$
- (c) $3/10$
- (d) $1/2$

57. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _____

(GATE-EC-SET-1-2014)

58. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n-3)$ is **(GATE-EE- SET-1-2014)**
- (a) 2^{-n}
 - (b) 0
 - (c) ${}^nC_{n-3}2^{-n}$
 - (d) 2^{-n+3}

59. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is

(GATE-ME- SET-2-2014)

- (a) $7/20$
- (b) $2/5$
- (c) $5/12$
- (d) $19/25$

60. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $X/1296$. The value of X is _____ **(GATE-CS- SET-1-2014)**

61. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by P . Then $100p = \underline{\hspace{2cm}}$

(GATE-CS- SET-2-2014)

62. Suppose A and B are two independent events with probabilities $P(A) = 0$ and $P(B) \neq 0$. Let \bar{A} and \bar{B} be their complements. Which one of the following statements is FALSE ?

(GATE-EC-2015)

- a) $P(A \cap B) = P(A)P(B)$
- b) $P(A|B) = P(A)$
- c) $P(A \cup B) = P(A) + P(B)$
- d) $P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B})$

63. Two coins R and S are tossed. The 4 joint events $H_R H_S$, $T_R T_S$, $H_R T_S$, $T_R H_S$ have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE ? **(GATE-EE-2015)**

- (a) The coin tosses are independent (b) R is fair, S is not
- (c) S is fair, R is not (d) The coin tosses are dependent

64. The probability that it will rain today is 0.5 the probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow ?

GATE- 97 (CS)

- (a) 0.3
- (b) 0.25
- (c) 0.35
- (d) 0.4

65. A single die is rolled two times. What is the probability that the sum is neither 8 nor 9 ?

(GATE-ME-2005)

- (a) $1/29$
- (b) $2/5$
- (c) $5/12$
- (d) $3/4$

66. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _____ **(GATE-CS- SET-2-2014)**

67. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$ denotes the probability of the event, the maximum value of $P(A) P(B)$ is _____

(GATE-CS- SET-3-2014)

68. Consider two events E₁ and E₂ such that $P(E_1) = \frac{1}{2}$
 $P(E_2) = \frac{1}{3}$ $P(E_1 \cap E_2) = \frac{1}{5}$ Which of the following statements is true ? **(GATE-EC-1999)**
- a) $P(E_1 \cup E_2) = \frac{2}{5}$
 - (b) E₁ and E₂ are independent
 - (c) E₁ and E₂ are not independent
 - (d) $P\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

69. E_1 and E_2 are events in a probability space satisfying the following constraints $P(E_1) = P(E_2)$;
 $P(E_1 \cup E_2) = 1$. E_1 & E_2 are independent then $P(E_1) =$

- (a) $6/25$
- (b) $2/5$
- (c) $5/12$
- (d) 1

70. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate1 has failed, the probability that both gates 2 and 3 will fail is

- (a) 0.240
- (b) 0.200
- (c) 0.040
- (d) 0.008

GATE-04

71. fair coin is tossed 3 times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is

GATE-05-(EE)

(a) $6/25$

(b) $2/5$

(c) $1/2$

(d) $19/25$

72. A fair dice is rolled twice. The probability that an odd number will follow an even number is
(GATE-EC-2005)

- (a) $6/25$
- (b) $2/5$
- (c) $1/4$
- (d) $19/25$

73. An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. Probability of getting at least one head is _____ **(GATE-ME-2011)**

- (a) $6/25$
- (b) $2/5$
- (c) $1/4$
- (d) $31/32$

74. Consider a die with the property that the probability of a face with ‘n’ dots showing up is proportional to ‘n’. The probability of the face with three dots showing up is _____

(GATE-EE- SET-2-2014)

75. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

76. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is **(GATE-EC- SET-3-2014)**

- (a) 0.067
- (b) 0.073
- (c) 0.082
- (d) 0.091

77. Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is _____.

(GATE-ME- SET-2-2014)

78. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?
(GATE-ME-2008)

- (a) $1/4$
- (b) $3/8$
- (c) $1/2$
- (d) $3/4$

79. The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up, 3 are defective is **(GATE-IN-2015)**

- (a) 0.001
- (b) 0.057
- (c) 0.0107
- (d) 0.3

80. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is
- a) 0.0036
 - b) 0.1937
 - c) 0.2234
 - d) 0.3874

81. The probability of an event B is P_1 . The probability of events A and B occur together is P_2 , while the probability that A or B occur together is P_3 . The probability of event A in terms of P_1 , P_2 and P_3 is

- a) P_1+P_2
- b) P_3+P_2
- c) P_3+P_1
- d) $P_3- P_1+ P_2$

82. the probability that a new Airport will get an award for its design is 0.16. The probability that it will get an award for its efficient use of materials is 0.24 and probability that it will get both the awards is 0.11. What is the probability that it will get only one of the two awards?

- a) 0.29
- b) 0.18
- c) 0.21
- d) 0.19

83. A jar has 5 marbles, one of each of the colors, red, white, blue, green and yellow. If 1 Marble is removed from the jar, what is the probability that the yellow one is removed?

- a) $\frac{1}{5}$
- b) $\frac{1}{2}$
- c) $\frac{4}{5}$
- d) $\frac{3}{4}$

84. A jar contains 4 marbles. 2 red and 2 white. Two marbles are chosen at random. If p_1 is the probability that the marbles chosen are of same color and p_2 is the probability that the marbles chosen be of different colors, then which of the following is true?

- a) $p_1 = p_2$
- b) $p_1 = 2p_2$
- c) $p_2 = 2p_1$
- d) $2p_1 = 3p_2$

85. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day ?

- a) $1/7^7$
- b) $1/7^6$
- c) $1/2^7$
- d) $7/2^7$

86. If P and Q are two random events, then the following is true

- a) Independence of P and Q implies that $\text{Probability}(P \cap Q) = 0$
- b) $\text{Probability}(P \cap Q) \geq \text{Probability}(P) + \text{Probability}(Q)$
- c) If P and Q are mutually exclusive then they must be independent
- d) $\text{Probability}(P \cap Q) \leq \text{Probability}(P)$

87. Find the probability of not getting a total of 7 or 11 on either of two tosses of a pair of fair dice?

- a) $25/36$
- b) $21/49$
- c) $7/9$
- d) $49/81$

88. Find the probability of a 4 turning up at least once in two tosses of a fair dice?

a) $8/32$

b) $10/36$

c) $11/36$

d) $13/36$

89.Candidates were asked to come to an interview with 3 pens each. Black, blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is _____.

(GATE-16-EE)

90. An urn contains 5 red and 7 green balls, A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next drawn is

(a) $\frac{65}{156}$

(b) $\frac{67}{156}$

(c) $\frac{79}{156}$

(d) $\frac{89}{156}$

(GATE- 16 – IN)

91. The probability of getting a “head” in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a “head” is obtained. If the tosses are independent, then the probability of getting “head” for the first time in the fifth toss is _____.

(GATE- 16 – EC – SET3)

92. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of a replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is _____.

(GATE-16-ME-SET2)

93. Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(GATE-16-ME-SET3)

(a) $\frac{16}{5525}$

(b) $\frac{64}{2197}$

(c) $\frac{3}{13}$

(d) $\frac{8}{16575}$

94. A fair coin is tossed N times. The probability that head does not turn up in any of the tosses is

(GATE-16-PI-SET1)

(a) $\left(\frac{1}{2}\right)^{N-1}$

(b) $1 - \left(\frac{1}{2}\right)^{N-1}$

(c) $\left(\frac{1}{2}\right)^N$

(d) $1 - \left(\frac{1}{2}\right)^N$

95. X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which of the following is the value of $P(X \cup Y)$?

- (a) 0.7
- (b) 0.5
- (c) 0.4
- (d) 0.3

(GATE-16-CE-SET2)

96. Consider the following experiment.

Step1: Flip a fair coin twice.

Step2: If the outcomes are (TAILS, HEADS) then output Y and stop.

Step3: If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step4: If the outcomes are (TAILS, TAILS), then go to step 1.

The probability that the output of the experiment is Y is (up to two decimal places)_____.

(GATE – 16 -CSE-SETI)

97. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is _____.

(GATE – 16 -CSE-SET2)

98. The probability that a communication system will have high fidelity is 0.81, The probability that the system will have both high fidelity and high selectivity is 0.18. The probability that a given system with high fidelity will have high selectivity is

- (a) 0.181
- (b) 0.191
- (c) 0.222
- (d) 0.826

(GATE-17-IN)

99. 500 students are taking one or more courses out of chemistry, physics and Mathematics. Registration enrolment as follows: chemistry (329), physics(186), Mathematics (295), chemistry and physics(83), chemistry and Mathematics (217), and physics and Mathematics(63), How many students are taking all 3 subjects?

(a) 37

(b) 43

(c) 47

(d) 53 **(GATE-17-EC)**

100. Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place)_____.

(GATE-17-EC)

101. An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball the second draw is

(a) $\frac{1}{2}$

(b) $\frac{4}{9}$

(c) $\frac{5}{9}$

(d) $\frac{6}{9}$

(GATE-17-EE)

102. A two-faced fair coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes: H,H,H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realizations of H, H and H, would be _____.

(GATE-17-CE)

103. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(a) $\frac{4}{5}$

(b) $\frac{5}{6}$

(b) (c) $\frac{7}{8}$

(d) $\frac{11}{12}$.

(GATE-17-CSIT)

104. Probability (up to one decimal place) of consecutively picking three red balls with out replacement from a box containing 5 red balls and 1 white balls is _____
(GATE-18-CE)

105. Two people, P and Q, decide to independently roll two identical dice, each with six faces, numbered 1 to 6. The person with lower number wins. In case of a tie, they roll dice repeatedly until there is no tie. Define trial as a throw of dice by P and Q. assume all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is _____

(GATE-18-CSIT)

106. A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than the boys?

(a) 0

(b) $\frac{325}{864}$

(GATE-18-EE)

(c) $\frac{525}{864}$

(b) $\frac{4}{11}$

107. Consider a sequence of tossing a fair coin where the outcomes of the tosses are independent. The probability of getting the head for the third time in the fifth toss is

(GATE-18-IN)

- (a) $\frac{5}{16}$
- (b) $\frac{3}{16}$
- (c) $\frac{3}{5}$
- (d) $\frac{9}{16}$

108. Two bags A and B have equal number of balls. Bag A have 20% red balls and 80% green balls. Bag B has 30% red balls, 60% green balls and 10% yellow balls. Contents of bag A and B are mixed thoroughly, and a ball is randomly picked from the mixture. What is the chance that the ball picked is red?

- (a) 20%
- (c) 30%

- (b) 25%
- (d) 40%

(GATE-18-IN)

109. Four red balls, four green balls, four blue balls are put in box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

(a) $\frac{1}{72}$

(c) $\frac{1}{36}$

(b) $\frac{1}{55}$

(d) $\frac{1}{27}$

(GATE-18-ME)

110. A six faced fair dice is rolled five times. The probability (in %) of obtaining “ONE” at least four times is

- (a) 33.33
- (c) 0.33

- (b) 3.33
- (d) 0.0033

(GATE-18-ME)

111.an unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials

- (1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__

Which statement describing last two coin tosses of the fourth trial has the highest probability of being correct?

(GATE-18-ME)

- (a) Two T will occur
- (b) One H and one T will occur
- (c) Two H will occur
- (d) One H will be followed by one T

112. The probabilities of occurrence of events F and G are $P(F) = 0.3$ and $P(G) = 0.4$, respectively. The probability that both events occur simultaneously is $P(F \cap G) = 0.2$. The probability of occurrence of at least one event $P(F \cup G)$ is _____

- (a) 0.5
- (b) 0.2
- (c) 0.7
- (d) 1

113. A box has 8 red balls and 8 green balls. Two balls are drawn randomly in succession from the box without replacement. The probability that first ball drawn is red and second ball drawn is green is

- (a) $4/15$
- (b) $7/16$
- (c) $1/2$
- (d) $8/15$

(GATE-19-IN)

114. A fair coin is tossed 20 times. The probability that ‘head’ will appear exactly 4 times in the first ten tosses, and ‘tail’ will appear exactly 4 times in the next 10 tosses is _____ (round off to 3 decimal places). **(GATE-20-ME)**

115. A bag contains 7 red and 4 white balls. Two balls are drawn at random. What is the probability that both balls are red?

(a) $\frac{28}{55}$

(b) $\frac{21}{55}$

(c) $\frac{7}{55}$

(d) $\frac{4}{55}$

116. A company is hiring to fill four managerial vacancies. The candidates are five men and three women. If every candidate is equally likely to be chosen then the probability that at least one woman will be selected is _____ (round off to 2 decimal places)

(GATE-20-ME)

117. If P, Q, R, S are four individuals How many teams of size exceeding one can be formed, with Q as a member?

(a) 8

(b) 6

(c) 5

(d) 7

(GATE-2020-EE)

118. In a school of 1000 student, 300 student play chess and 600 student play football. If 50 students play both chess and football, the number of students who play neither is _____.

(a) 150

(b) 50

(c) 100

(d) 200

(GATE-2020(CE))

119. A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 Heads (round off to three decimal places), is _____.

(GATE-2020(CE))

120. Consider two identical bags B_1 , and B_2 , each containing 10 balls of identical shapes and sizes. Bag B_1 , contains 7 Red and 3 Green balls, while bag B_2 , contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B_1 (rounded off to one decimal place) is _____.

(GATE- 2020(IN))

121. In a company, 35% of the employees drink coffee, 40% of the employees drink tea and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

- (a) 15
- (c) 25

- (b) 35
- (d) 40

(GATE-21-CE)

122. Two identical cube shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is :

(a) $\frac{1}{8}$

(c) $\frac{1}{4}$

(b) $\frac{1}{36}$

(d) $\frac{1}{12}$

(GATE-21-CE)

123.A digital watch X beeps every 30 seconds while watch Y beeps every 32 seconds. They beeped together at 10 AM. The immediate next time that they will beep together is _____.

(GATE-21-ME)

- (a) 11 AM
- (a) 10.08 AM
- (c) 10.42AM
- (d) 10.00PM

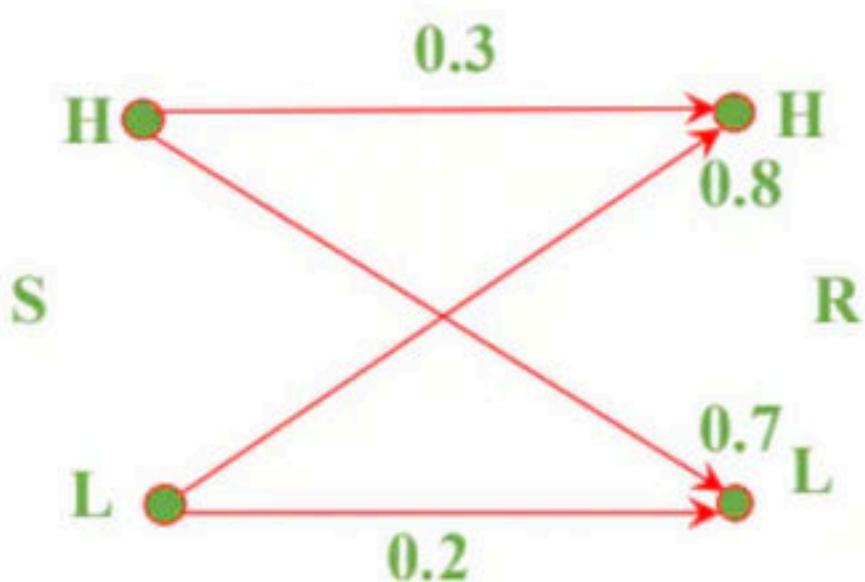
124. A bag has r red balls and b black balls . All balls are identical except for their colours . In a trial , a ball is randomly drawn from the bag , its colour is noted and the ball is placed back into the bag along with another ball of the same colour . Note that the number of balls in the bag will increase by one , after the trial . A sequence of four such trials is conducted . Which one of the following choices gives the probability of drawing a red ball in the fourth trial ?

(GATE-2021-cs)

- (a) $\frac{r}{r+b}$
- (b) $\frac{r}{r+b+3}$
- (c) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$
- (d) $\frac{r+3}{r+b+3}$

125. A sender (S) transmits a signal , which can be one of the two kinds : H and L with probabilities 0.1 and 0.9 respectively , to a receiver (R) .

In the graph below , the weight of edge (u , v) is the probability of receiving v when u is transmitted , where $u, v \in \{H, L\}$. For example , the probability that the received signal is L given the transmitted signal was H , is 0.7 .



If the received signal is H , the probability that the transmitted signal was H (rounded to 2 decimal places) is _____.

(GATE-2021-cs)

126. There are 6 jobs with distinct difficulty levels , and 3 computers with distinct processing speeds . Each job is assigned to a computer such that ,The fastest computer gets the toughest job and the slowest computer gets the easiest job. Every computer gets at least one job .
The number of ways in which this can be done is _____

(GATE-2021-cs)

▲ 1 • Asked by Shashank

Please help me with this doubt

A and B decided to meet on February 14th, Sunday 2021, between 6 pm and 7 pm. The probability that they will meet, provided one cannot wait for the other for more than 12 minutes, is:

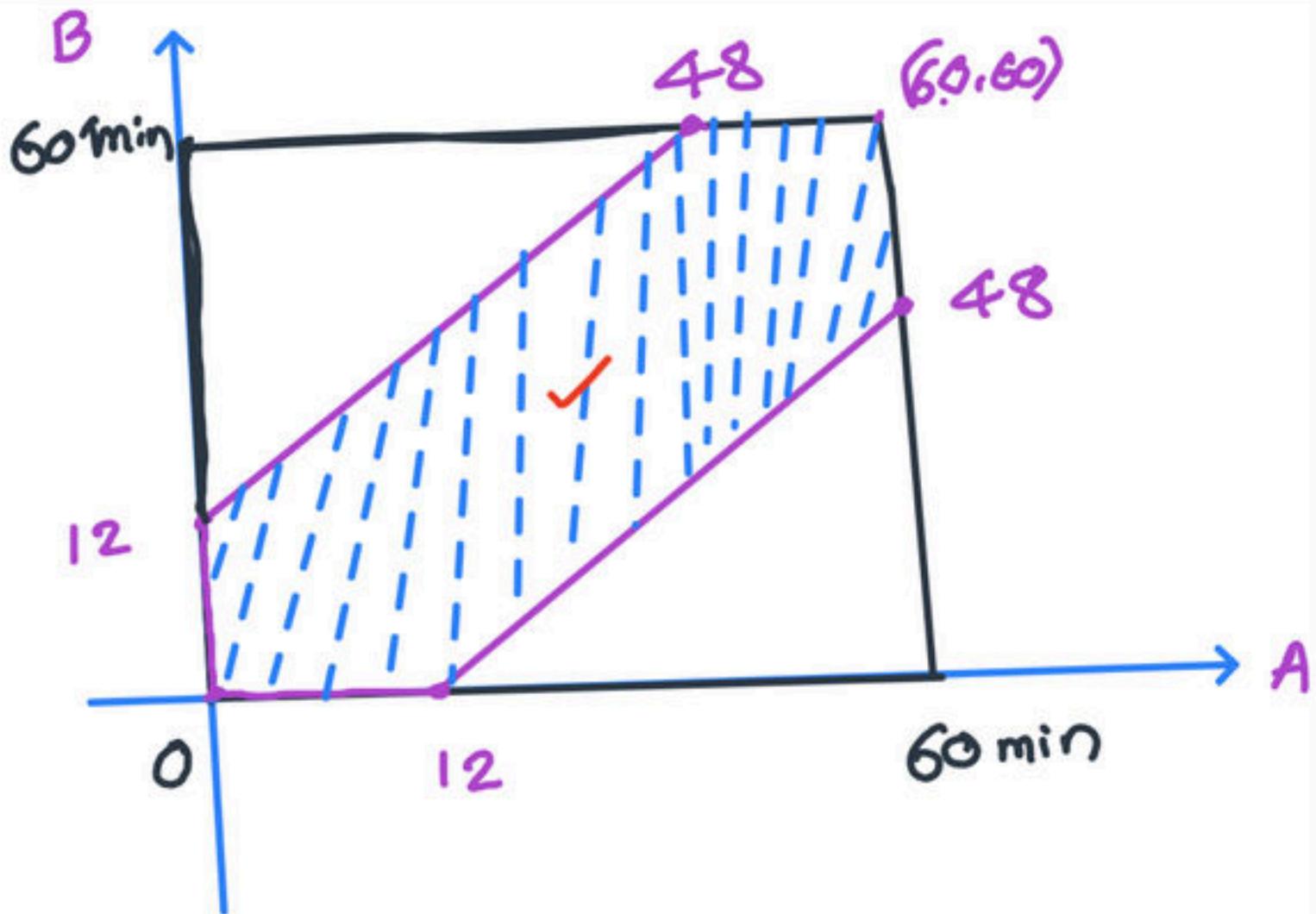
Select your answer

A $\frac{5}{9}$

B $\frac{7}{16}$

C $\frac{9}{25}$

D $\frac{11}{36}$



$$x \quad y \quad -12 \leq x-y \leq 12$$

$$|x-y| \leq 12$$

$$P = \frac{60 \times 60 - (48)(48)}{60 \times 60}.$$

$$P = 1 - \frac{48 \times 48}{60 \times 60} = \frac{9}{25}$$

127. A box contains 15 blue balls and 45 black balls, if 2 balls are selected randomly, without replacement, the probability of an outcome in which the first selected is a blue ball and the second selected is a black ball

~~a) $\frac{45}{236}$~~

b) $\frac{1}{4}$

c) $\frac{3}{16}$

d) $\frac{3}{4}$

$$\frac{15}{60} \cdot \frac{45}{59}$$

128. A box contains the following three coins ,

- I. A fair coin with head on one face and tail on the other face .
- II . A coin with heads on both the faces .
- III . A coin with tails on both the faces.

A coin is picked randomly from the box and tossed . Out of the two remaining coins in the box , one coin is then picked randomly and tossed . If the first toss results in a head , the probability of getting a head in the second toss is

F	2H	2T
---	----	----

(GATE - 2021 - EC)

(a) $\frac{1}{2}$
~~(c) $\frac{1}{3}$~~

(b) $\frac{2}{5}$
(d) $\frac{2}{3}$

$$P(H_2 | H_1) = \frac{P(H_2 \cap H_1)}{P(H_1)} = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{3}$$

F	2H	2T
---	----	----

$$P(H_1) = \frac{1}{3} \frac{1}{2} + \frac{1}{3}(1) + \frac{1}{3}(0) = \frac{1}{3} \frac{1}{2} + \frac{1}{3} = \frac{1}{2}$$

$$\begin{aligned} P(H_1 \cap H_2) &= \frac{1}{3} \frac{1}{2} \frac{1}{2}(1) + \frac{1}{3} \frac{1}{2} \frac{1}{2}(0) \\ &\quad + \frac{1}{3}(1) \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3}(1) \frac{1}{2}(0) \end{aligned}$$

$$P(H_1 \cap H_2) = + 0 + 0$$

$$P(H_1 \cap H_2) = 1/6$$

F	2H	✓
F	<u>2T</u>	✓
2H	F	✓
2H	2T	.
<u>2T</u>	F	.
<u>2T</u>	2H	.

129. There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag. The probability that at least two chocolates are identical is _____.

(GATE-2021-cs)

(a) 0.6976
(c) 0.4235

(b) 0.3024
(d) 0.8125

1, 2, 3, 4, 5
6, 7, 8, 9, 10

1, 2, 3, 4, 5
6, 7, 8, 9, 10

1, 2, 3, 4, 5
6, 7, 8, 9, 10

1, 2, 3, 4, 5
6, 7, 8, 9, 10

$$P = 1 - \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10} \cdot \frac{7}{10} \cdot \frac{6}{10}$$

P = 0.6976

1, 2, 3, 4, 5
6, 7, 8, 9, 10

130. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is (GATE-CS-1998)

- a) $1/6$
- ~~b) $3/8$~~
- c) $1/8$
- d) $1/2$

$$E \ E \ O \rightarrow \frac{3!}{2!} = 3.$$

$$P = 3 \left[\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \right] = \frac{3}{8}$$

131. Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is
(a) $1/16$ (b) $1/8$ (c) $7/8$ (d) $15/16$
- ~~(GATE-CS-2002)~~

$$\underline{H} \quad \underline{H} \quad \underline{H} \quad \underline{T} \rightarrow \frac{4!}{3!} = 4$$

$$H \quad H \quad T \quad T \rightarrow \frac{4!}{2! 2!} = 6.$$

$$\frac{14}{2^4}$$

$$H \quad T \quad T \quad T \rightarrow \frac{4!}{3!} = 4$$

$$\frac{14}{16}.$$

132. Assume for simplicity that N people, all born in April (a month of 30 days) are collected in a room, consider the event of at least two people in the room being born on the same date of the month (even if in different years e.g. 1980 and 1985). What is the smallest N so that the probability of this exceeds 0.5 is ?

(a) 20

~~(b) 7~~

(c) 15

(d) 16

GATE- 2009 EE

$$\underline{N=2} \checkmark$$

$$P = 1 - \frac{30}{30} \cdot \frac{29}{30} = 0.033$$

30-days.

$$\underline{N=7}$$

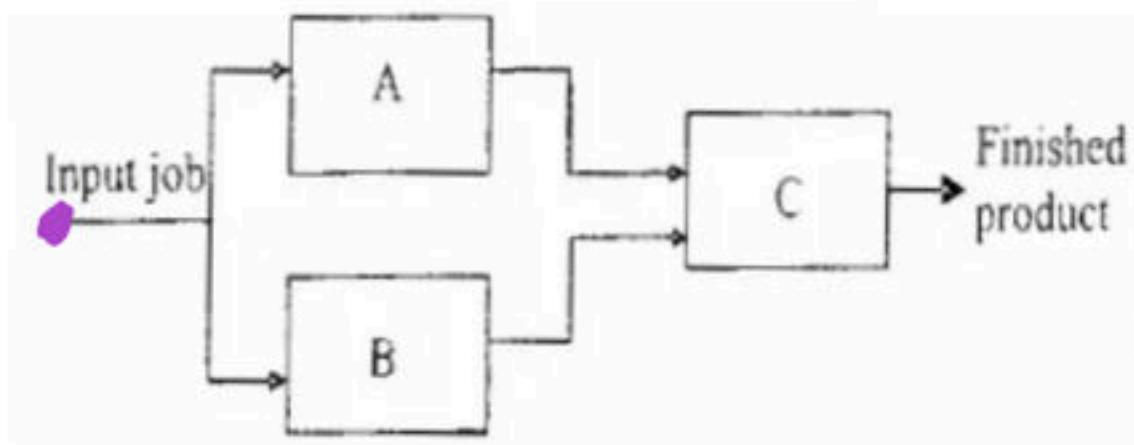
$$P = 1 - \frac{30}{30} \cdot \frac{29}{30} \cdot \frac{28}{30} \cdot \frac{27}{30} \cdot \frac{26}{30} \cdot \frac{25}{30} \cdot \frac{24}{30}.$$

$$P = 0.53$$

133. The figure shown the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as : $P_A = 0.15$, $P_B = 0.05$ & $P_C = 0.1$. Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is

GATE- 2014 IN

(A)



$$ABC + \bar{A}BC + A\bar{B}C$$

$$P(A \cap B \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap \bar{B} \cap C)$$

$$P(A)P(B)P(C) + P(\bar{A})P(B)P(C) + P(A)P(\bar{B})P(C)$$

$$= 0.893$$

$$Y = (\bar{A} + B)C$$

$$Y = AC + BC$$

$$Y = \underline{\bar{A}\bar{B}C + ABC + \bar{A}BC + ABC}$$

134. An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is

(GATE-EC-2007)

- (a) 0.5 (b) 0.18 (c) 0.12 (d) 0.06

$$P(R_1) = 0.3$$

$$P(R_2) = 0.2$$

$$P(R_1 | R_2) = 0.6$$

$$P(R_1 \cap R_2) = ?$$

$$P(R_1 | R_2) = \frac{P(R_1 \cap R_2)}{P(R_2)}$$

$$\begin{aligned} P(R_1 \cap R_2) &= P(R_1 | R_2) P(R_2) \\ &= (0.6)(0.2) = 0.12. \end{aligned}$$

135. Consider two independent random variables X and Y with identical distribution's. The variables X and Y take values 0, 1, and 2 with probability 1/2, 1/4 and 1/4 respectively. What is the conditional probability $P(X + Y = 2/X - Y = 0)$? **GATE- 2014 EC**
- (a) 0 (b) 1/16 (c) 1/6 (d) 1

$$P\left(\frac{X+Y=2}{X-Y=0}\right) = \frac{P[(X+Y=2) \cap (X-Y=0)]}{P(X-Y=0)} =$$

$X = 0, 1, 2$	$(0,0)$	$(1,1)$	$\checkmark (2,2)$
$Y = 0, 1, 2$			

$$= \frac{P(x=1, y=1)}{P(x=0, y=0) + P(x=1, y=1) + P(x=2, y=2)}$$

$$= \frac{P(x=1) P(y=1)}{P(x=0) P(y=0) + P(x=1) P(y=1) + P(x=2) P(y=2)}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{4}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{1}{4}} = \frac{1}{6}$$

136. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that first removed ball is white, the probability that the 2nd removed ball is red is

- (a) $1/3$ (b) $3/7$

(GATE-EE-2010)

- ~~(c) $1/2$~~ (d) $4/7$

$$\frac{3}{6} = \frac{1}{2}$$

137. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

- (a) $\frac{2}{315}$ (b) $\frac{1}{630}$

- ~~(c) $\frac{1}{1260}$~~

- GATE- 2010 ME**
(d) $\frac{1}{2520}$

$\omega\omega \text{ NNN BBBB.}$

$$\frac{2}{9} \cdot \frac{1}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} \cdot \frac{1}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} \cdot \frac{2}{2} \cdot \frac{1}{1}.$$

138. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads ?

GATE- 2011 CS

- a) $\frac{1}{3}$
- b) $\frac{1}{4}$
- c) $\frac{1}{2}$
- d) $\frac{2}{3}$

$\left\{ H \ H \quad H \ T \quad T \ H \right\}$

$$P = \frac{1}{3}.$$

139. Parcels from sender S to receiver R pass sequentially through two post-office. Each post-office has a probability $1/5$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is _____

(GATE-EC- SET-4-2014)

J

P₁

P₂

BVRreddy

$$P(L|P_1) = \frac{1}{5}$$

$$P(F|P_1) = \frac{4}{5}$$

$$P(L|P_2) = \frac{1}{5}$$

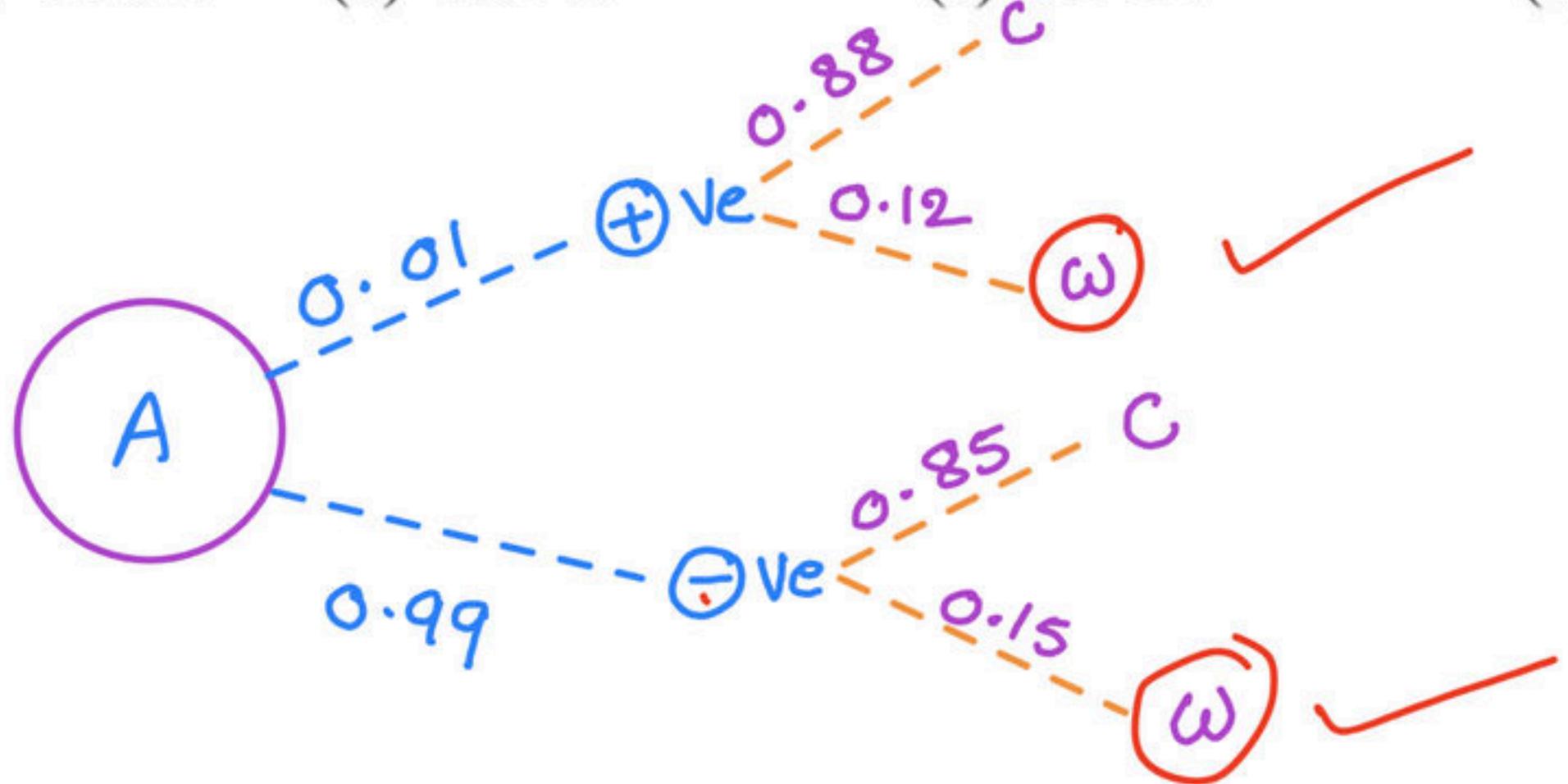
$$P(F|P_2) = \frac{4}{5}$$

$$P = \frac{\frac{4}{5} \cdot \frac{1}{5}}{\frac{1}{5} + \frac{4}{5} \cdot \frac{1}{5}}$$

$$P = \frac{4}{9}$$

140. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting positive report is 0.01, the probability that a person tested gets an incorrect report is (GATE-IN-2009)

- (a) 0.0027 (b) 0.0173 (c) 0.1497 (d) 0.2100



$$P = 0.01(0.12) + (0.99)(0.15) = \underline{\underline{0.1497}}$$

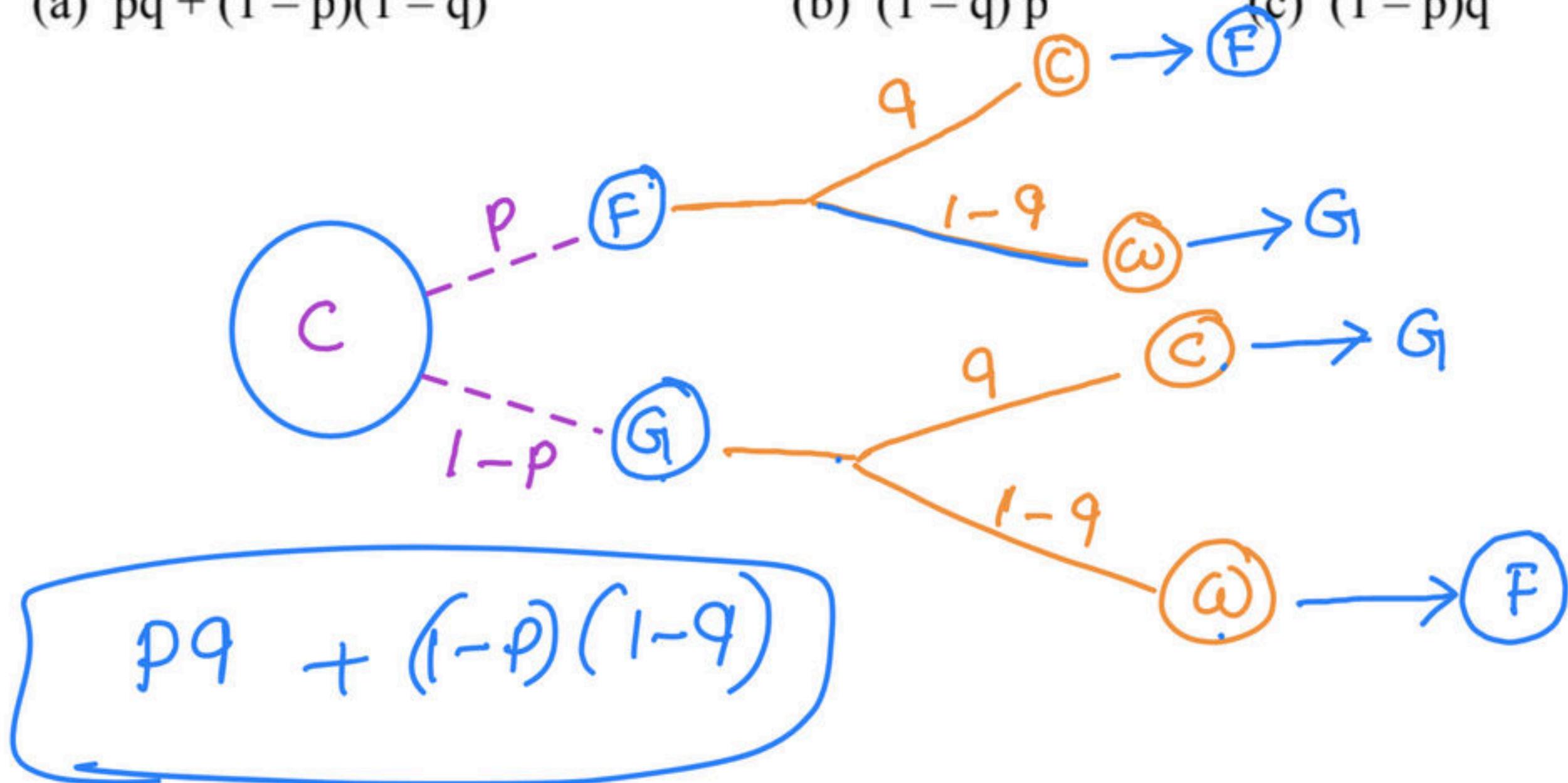
141. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p . The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q . What is the probability of a computer being declared faulty? **(GATE-CS-2010)**

(a) $pq + (1 - p)(1 - q)$

(b) $(1 - q)p$

(c) $(1 - p)q$

(d) pq



142. An automobile plant contacted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to reliable, is made by Y is

(GATE-CE-2012)

(a) 0.288

(b) 0.334

(c) 0.667

(d) 0.720

$$P(X) = 0.6$$

$$P(Y) = 0.4$$

$$P(R/X) = 0.96$$

$$P(R/Y) = 0.72$$

$$P = \frac{(0.4)(0.72)}{(0.6)(0.96) + (0.4)(0.72)}$$

$$\underline{\underline{P = 0.334}}$$

143. The probability that student knows the correct answer to a multiple choice question is $\frac{2}{3}$.

If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is $\frac{1}{4}$. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is

GATE- 2013 ME

- (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{5}{6}$ (d) $\frac{8}{9}$

$$P = \frac{\frac{2}{3} (1) + \frac{1}{3} \frac{1}{4}}{k}$$

$$= \frac{8}{9}.$$

144.. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is

(GATE-ME- SET-3-2014)

$$P(E) = \frac{1}{2} (0.8) + \frac{1}{2} (0.5)$$

$$P(E) = 0.65$$

145. In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day?

(GATE-PI- SET-1-2014)

(a) $\frac{3}{10}$

(b) $\frac{9}{11}$

(c) $\frac{14}{17}$

(d) $\frac{27}{41}$

$$P(R) = 0.7$$

$$P(\bar{R}) = 0.3.$$

$$P(L|R) = 0.8$$

$$P(F|R) = 0.2$$

$$P(L|\bar{R}) = 0.1$$

$$P(F|\bar{R}) = 0.9.$$

$$P = \frac{(0.3)(0.9)}{(0.7)(0.2) + 0.3(0.9)}$$

146. A committee of 4 is to be formed from among 4 girls and 5 boys. What is the probability that the committee will have number of boys less than number of girls?

- (a) $\frac{2}{9}$
- (b) $\frac{4}{9}$
- (c) $\frac{4}{5}$
- (d) $\frac{1}{6}$

$$\begin{aligned} G_1 G_1 G_1 G_1 &\rightarrow \frac{4C_4}{9C_4} \\ G_1 G_1 G_1 B &\rightarrow \frac{4C_3 \cdot 5C_1}{9C_4} \end{aligned}$$

$$P = \frac{4C_4 + 4C_3 \cdot 5C_1}{9C_4}$$

147. A bag contains 4 white and 2 black balls and another bag contains 3 of each colour. A bag is selected at random and a ball is drawn at random from the bag chosen. The probability of white ball drawn is

(a) $\frac{1}{3}$

(b) $\frac{1}{4}$

(c) $\frac{5}{12}$

~~(a) $\frac{7}{12}$~~

$$P = \frac{1}{2} \cdot \frac{4}{6} + \frac{1}{2} \cdot \frac{3}{6}.$$

Random Variable

Cumulative Distribution Function (CDF)

$$F(x) = P(X \leq x)$$

X → Random variable (RV)

$$F(i) = P(X \leq i)$$

$$F(2) = P(X \leq 2)$$

Eg:

Tossing 2-coins.

$$S = \left\{ \begin{array}{ll} HH, & H T \\ x=2 & x=1 \end{array} \right. \quad \left. \begin{array}{ll} T H, & TT \\ x=1 & x=0 \end{array} \right\}$$

$X \rightarrow$ No. of heads in the odd come.

x	0	1	2
$P(x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Properties of CDF

Probability Density Function (PDF)

Properties of PDF

Mean (Average Value) (Expectation) (1st Moment)

Properties of Mean

Variance

148. If the probability density function of a random variable x is given by

$$f(x) = \begin{cases} \frac{kx^2}{0} & -1 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

the value of k is _____.

(GATE- 2020(PI))

149. The constant C such that the function

$$f(x) = \begin{cases} Cx^2, & 0 < x < 3. \\ 0, & \text{Otherwise} \end{cases}$$

is a probability density function.

- a) 1
- b) 1/9
- c) 1/8
- d) 1/3

150. A random variable x has density function $f(x) = C/(x^2 + 1)$ where $-\infty < x < \infty$.
then the value of the constant C

a) $\frac{1}{\pi}$

b) $\frac{2}{\pi}$

c) $\frac{1}{2\pi}$

d) $\frac{-1}{\pi}$

151. $P_x(X) = Me^{-2|x|} + Ne^{-3|x|}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relation M and N is **(GATE-IN-2008)**

- a) $M + \frac{2}{3}N = 1$
- b) $2M + N = 3$
- c) $M + N = 1$
- d) $M + N = 3$

152. A continuous random variable X has a probability density function

$f(x) = e^{-x}$, $0 < x < \infty$. Then $P\{X > 1\}$ is

(a) 0.368 (b) 0.5

(c) 0.632

(d) 1.0

(GATE-EE-2013)

153. Find the value of λ such that the function $f(x)$ is a valid probability density function
 $f(x) = \lambda(x - 1)(2 - x)$ for $1 < x < 2$ _____ (GATE-CE-2013)

- a) 5
- b) 6
- c) 2
- d) 1

154. the density function of a random variable X is given by

$$f(x) = x/2 \quad 0 < x < 2$$

= 0 otherwise

Then the mean and variance of X are

- a) 4/3, 2/9
- b) 2/3, 4/9
- c) 4/3, 4/9
- d) 2/3, 2/9

155. The expectation of discrete random variable X whose probability function is given by

$$f(x) = (1/2)^x, \quad x = 1, 2, 3, \dots \text{ is}$$

- a) 1
- b) 2
- c) 3
- d) 4

156. The random variable X takes on the values 1, 2 (or) 3 with probabilities **GATE- 2007(PI)**
 $\frac{2+5P}{5}$, $\frac{1+3P}{5}$, and $\frac{1.5+2P}{5}$ respectively the values of P and E(X) are respectively

- (a) 0.05, 1.87 (b) 1.90, 5.87 (c) 0.05, 1.10 (d) 0.25, 1.40

157. If E denotes expectation, the variance of a random variable X is given by (2007)

(a) $E(X^2) - E^2(X)$

(c) $E(X^2)$

(b) $E(X^2) + E^2(X)$

(d) $E^2(X)$

158. discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true?

K	1	2	3	4	5
P(X=K)	0.1	0.2	0.4	0.2	0.1

GATE- 2009 (EC)

- a) Both the student and the teacher are right
- b) Both the student and the teacher are wrong
- c) The student is wrong but the teacher is right
- d) The student is right but the teacher is wrong

159. If the difference between the expectation of the square of a random variable [$E(X^2)$] and the square of the expectation of the random variable [$E(X)$] 2 is denoted by R, then,

(GATE-CS-2011)

- (a) $R = 0$
- (b) $R < 0$
- (c) $R \geq 0$
- (d) $R > 0$

160. Let X be a real-valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2 , respectively. The relation which always holds true is (GATE-EC-2014)

- (a) $(E[X])^2 > E[X^2]$
- (b) $E[X^2] \geq (E[X])^2$
- (c) $E[X^2] = (E[X])^2$
- (d) $E[X^2] > (E[X])^2$

161. Let the probability density function of a random variable X, be given as:

$$f_x(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x) \text{ where } u(x) \text{ is the unit step function.}$$

Then the value of ‘a’ and prob $\{X \leq 0\}$, respectively, are

(GATE-16-EE)

(a) $2, \frac{1}{2}$

(b) $4, \frac{1}{2}$

(c) $2, \frac{1}{4}$

(d) $4, \frac{1}{4}$

162. A probability density function this interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is _____.

(GATE-16-CSE-SETI)

/

163. If $f(x)$ and $g(x)$ are two probability density functions,

$$f(x) = \begin{cases} \frac{x}{a} + 1 & : -a \leq x < 0 \\ -\frac{x}{a} + 1 & : 0 \leq x \leq a \\ 0 & : 0 \leq x \leq a \end{cases}$$

$$g(x) = \begin{cases} -\frac{x}{a} & : -a \leq x < 0 \\ \frac{x}{a} & : 0 \leq x \leq a \\ 0 & : \text{otherwise} \end{cases}$$

Which of the following statements is true?

- (a) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are same. **(GATE-16-CE-SET2)**
- (b) Mean of $f(x)$ and $g(x)$ are same; Variance of $f(x)$ and $g(x)$ are different.
- (c) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are same.
- (d) Mean of $f(x)$ and $g(x)$ are different; Variance of $f(x)$ and $g(x)$ are different.

Probability Distributions

Binomial ,

Poisson,

Uniform ,

Exponential ,

Gaussian (Normal)

164. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ . The standard deviation for this distribution is given by

- (a) $\sqrt{\mu}$
- (b) μ^2
- (c) μ
- (d) $1/\mu$

(GATE-16-ME-SET1)

165. Mr.Aishwary try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by Mr. Aishwary, then the average number of attempts that passengers need to make to get a seat reserved is

(GATE-17-EC)

166. Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of Ms. Jyoti with BMW at the junction is uniformly distributed over 5 minute cycle. The expected waiting time (in minutes) of the Ms. Jyoti at the junction is

(GATE-17-EE)