Operations on Binary Search Tree

Course on C-Programming & Data Structures: GATE - 2024 & 2025

Vishvadeep Gothi • Lesson 52 • Jan 31, 2023

Data Structure

Tree 7 (BST - Part 2)

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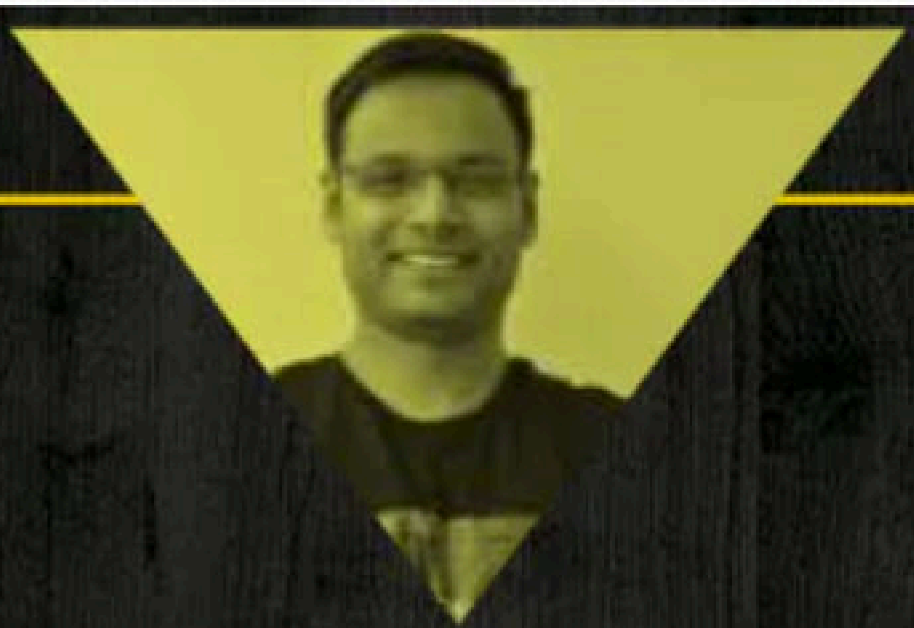


Hello!

I am Vishvadeep Gothi

I am here because I love to teach

Binary Search Tree

Question GATE-2011

We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways we can populate the tree with the given set so that it becomes a BST?

A. 0

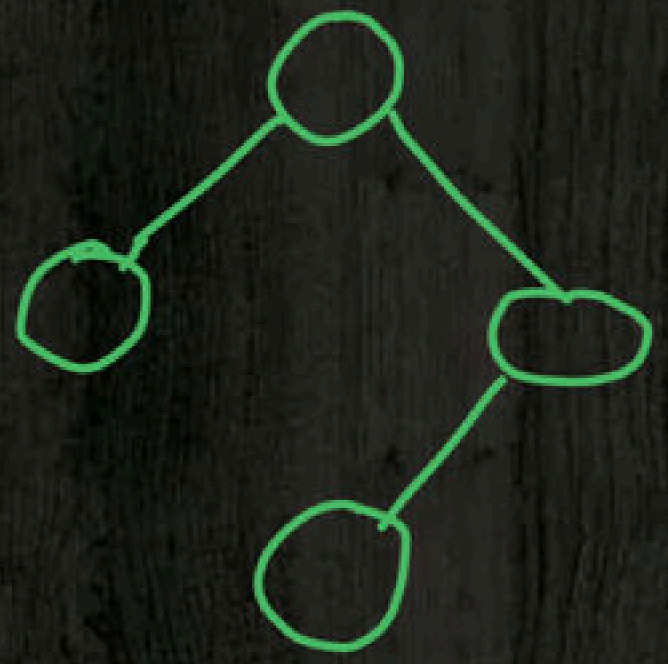
B. 1

C. n!

D. $\frac{c(2n,n)}{n+1}$

$n = 4$

1, 2, 3, 4



=>

Number of BSTs Using n-distinct Keys

	no. of BT	No. of BST
for n unlabeled node	$\frac{{}^{2n}C_n}{n+1}$	
for n destinct keys	$\frac{{}^{2n}C_n}{n+1} * n!$	$\frac{{}^{2n}C_n}{n+1} * 1$

Question GATE-2007

When searching for a key value 60 in a BST, nodes containing the key values 10, 20, 40, 50, 70, 80, 90 are traversed, not necessarily in the same order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?

10, 20, 40, 50

70, 80, 90

$= \frac{7!}{4! * 3!} = 35 = {}^7C_4$

or

${}^7C_3$

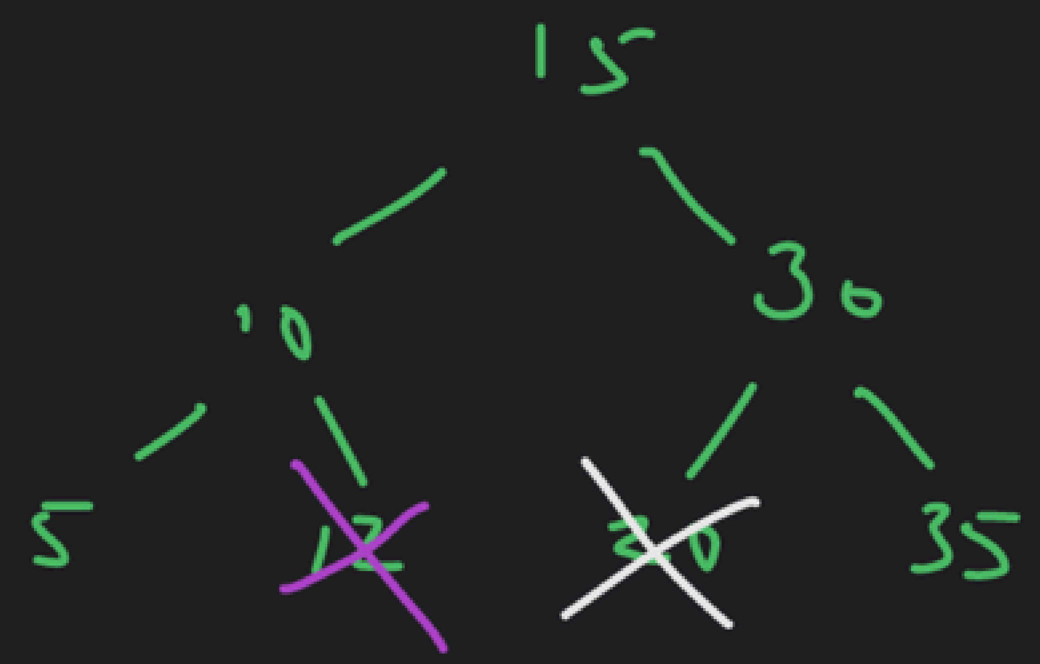
Deletion in BST

1. Search element

2. Find no. of children of node to be deleted.

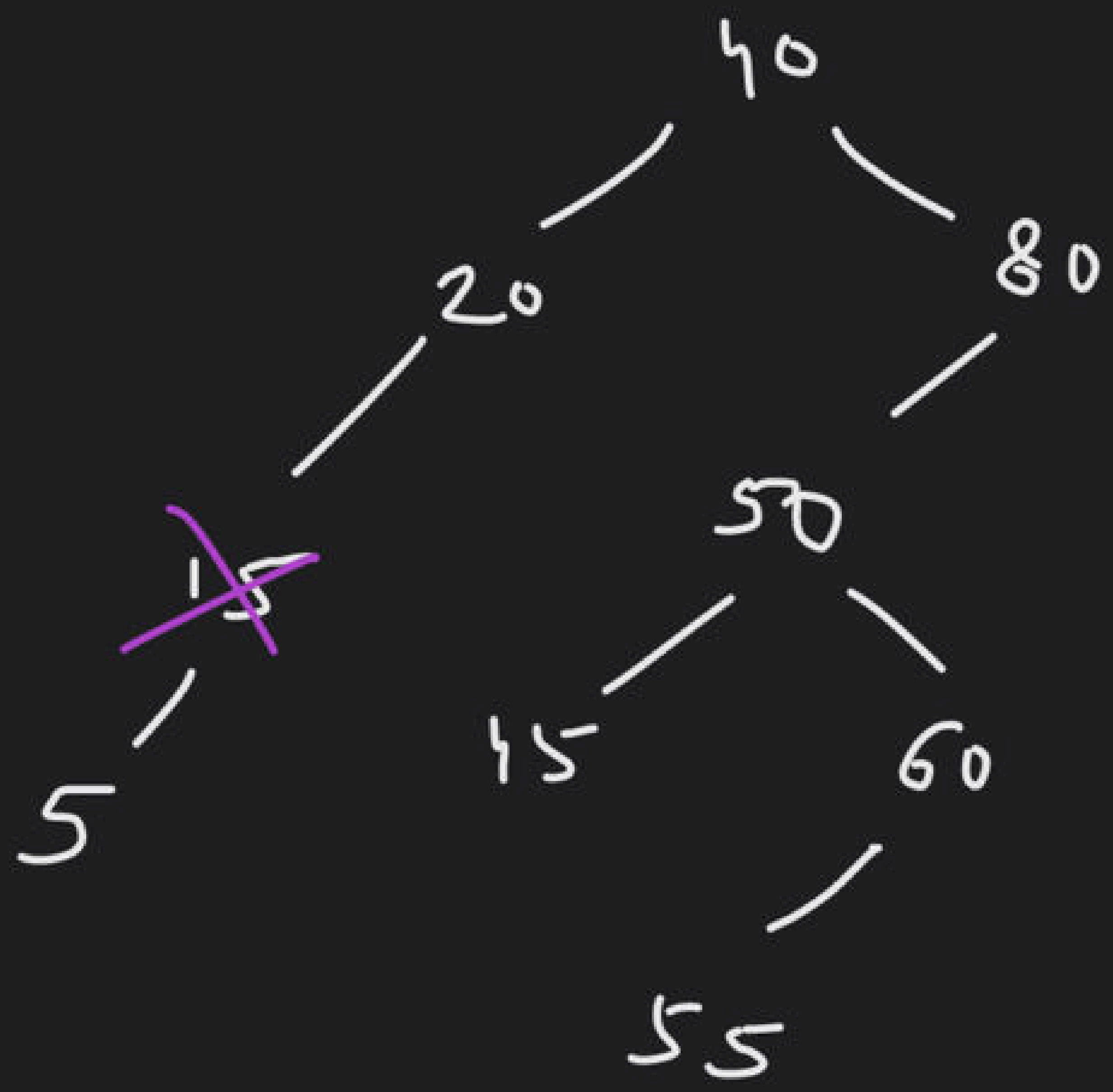
3. call one of following cases based on no. of children
   - case 0 ⇒ node has 0 child
   - Case 1 ⇒ — " — 1 — " —
   - Case 2 ⇒ — " — 2 children

Case 0 :-



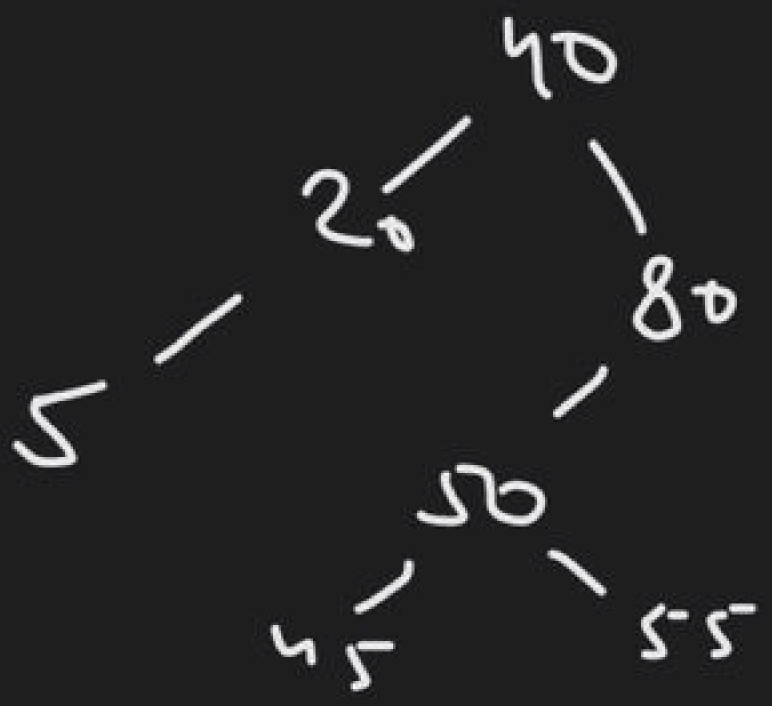
Delete ⇒ 12 , 20

Case 1:-

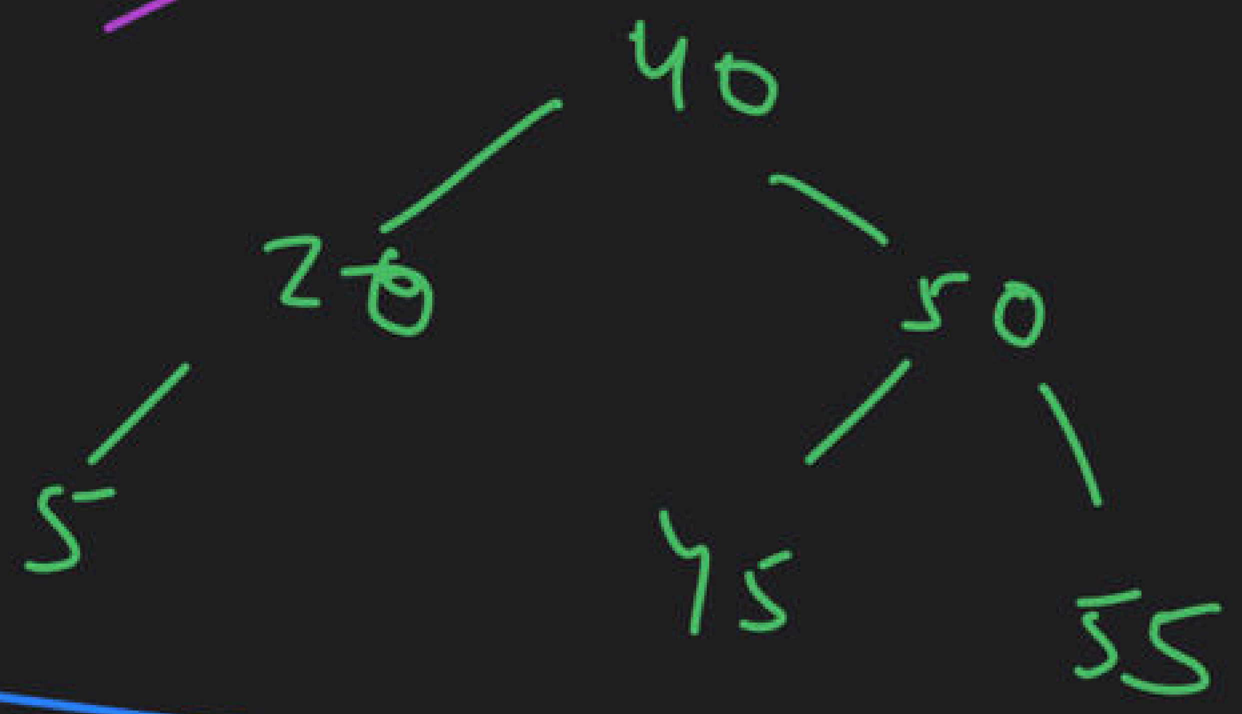


Delete:- 15

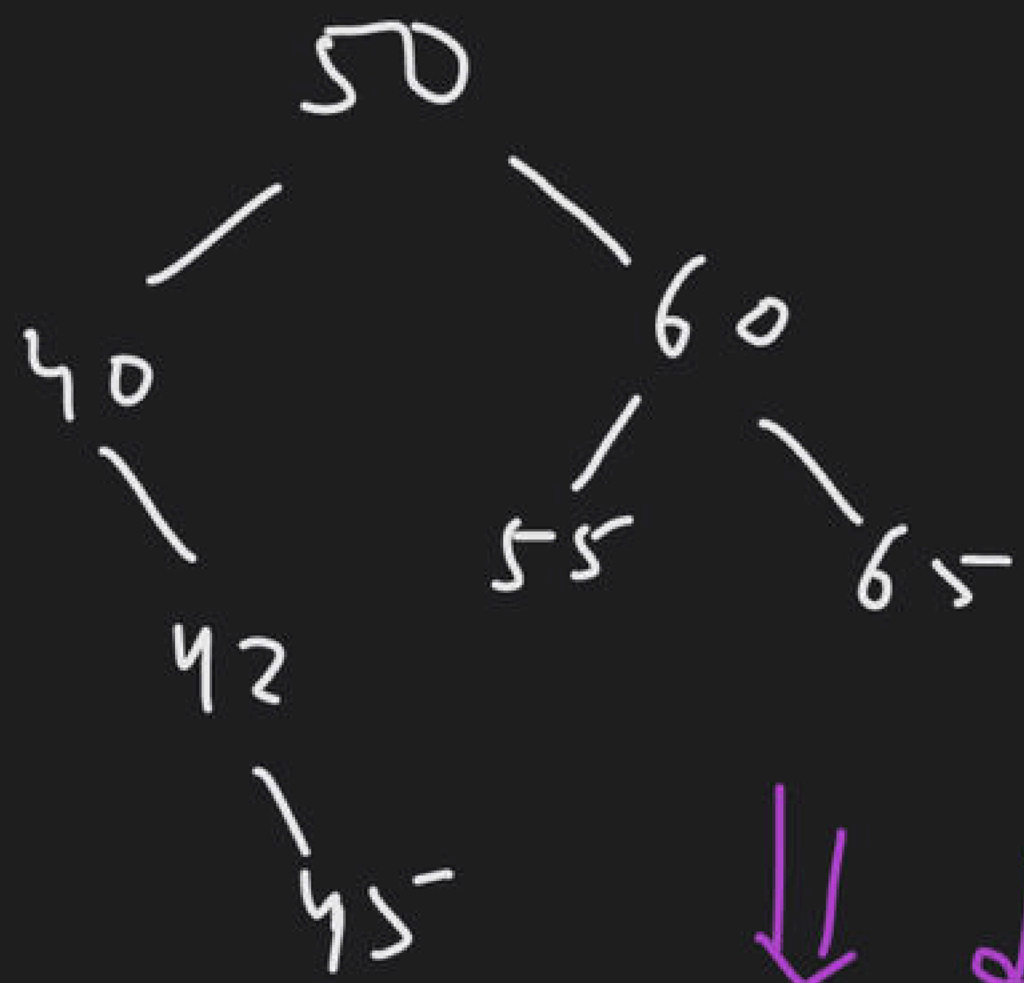
delete 60



replace deleted node by it's single child delete 80



ex.:-



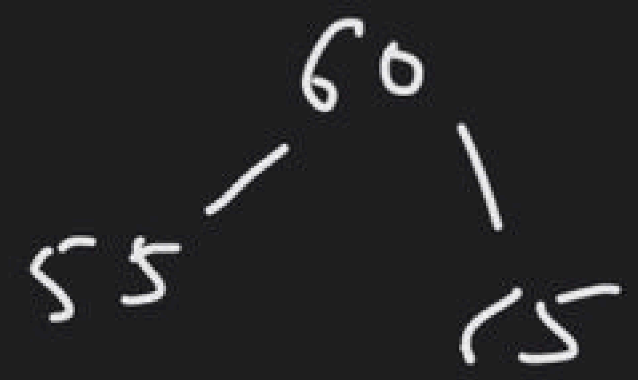
⇓ delete 40

Case 2:-

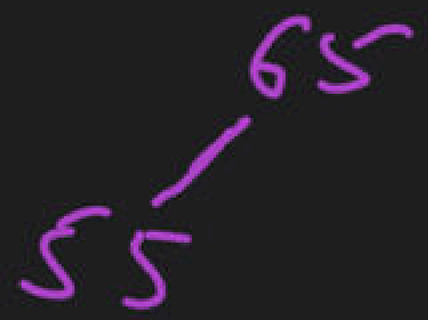
Replace deleted node by it's inorder predecessor or successor

⇒ Replacement using inorder successor :-

Ex.:- 1



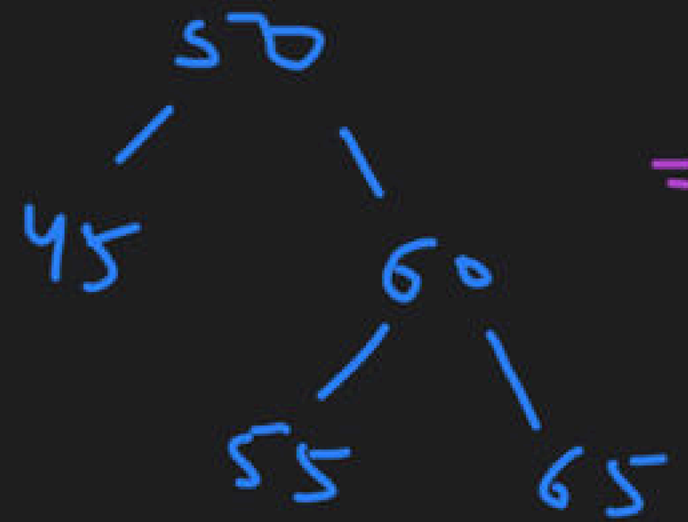
⇒



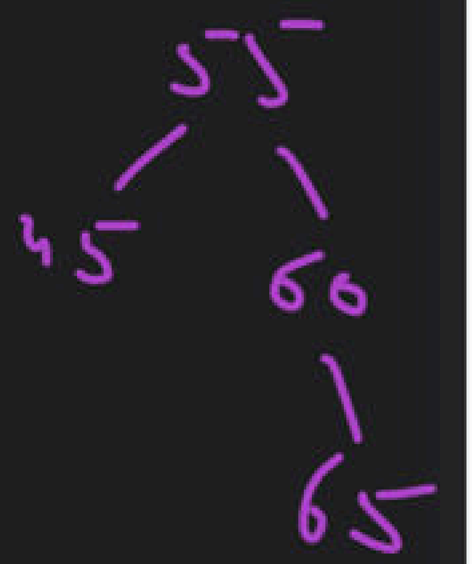
delete 60 :-

Inorder successor ⇒ 65

ex:- 2



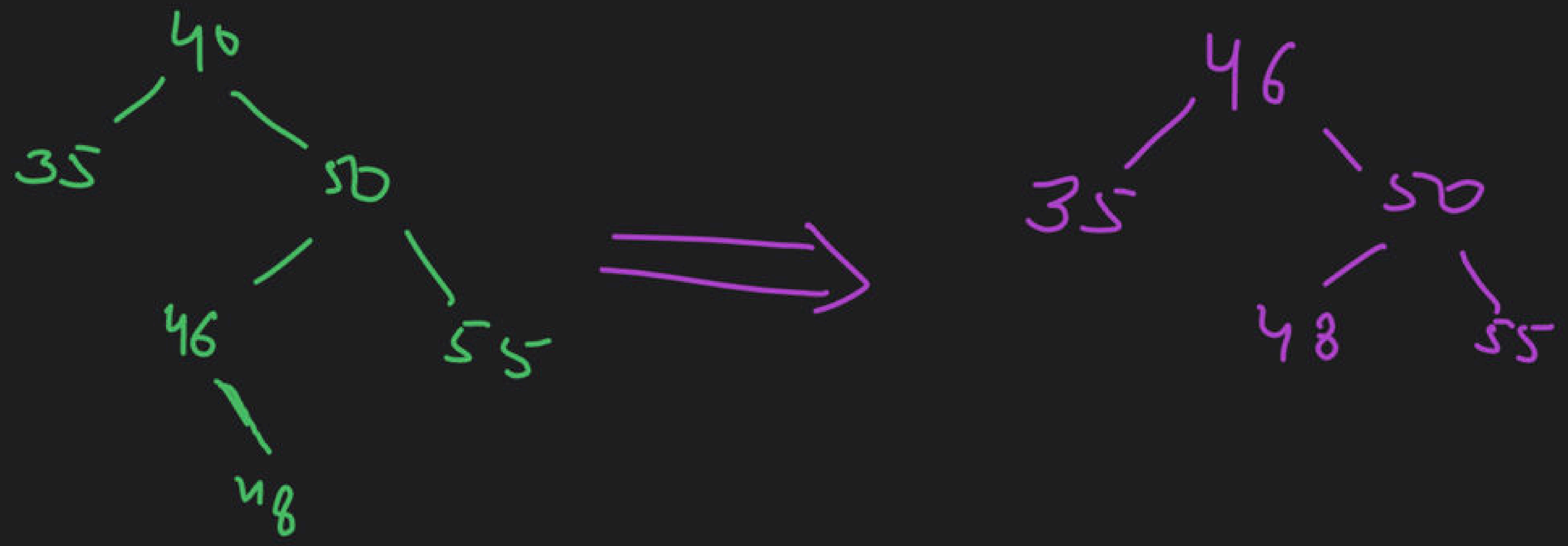
⇒



delete ⇒ 50

Inorder successor ⇒ 55

Ex 3 :-

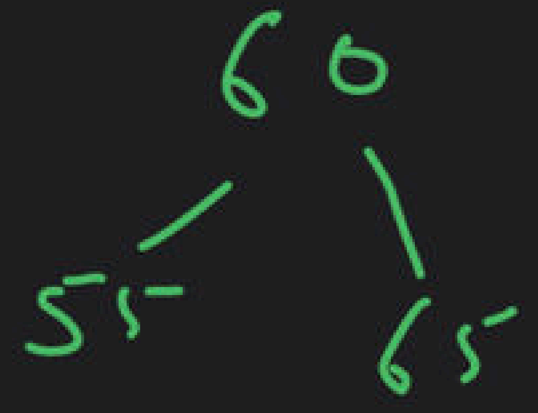


delete ⇒ 40

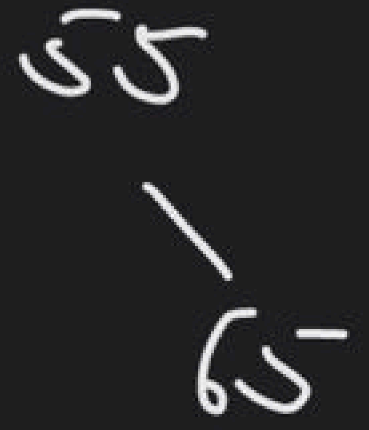
Inorder successor ⇒ 46

⇒ Replace deleted node by inorder predecessor :-

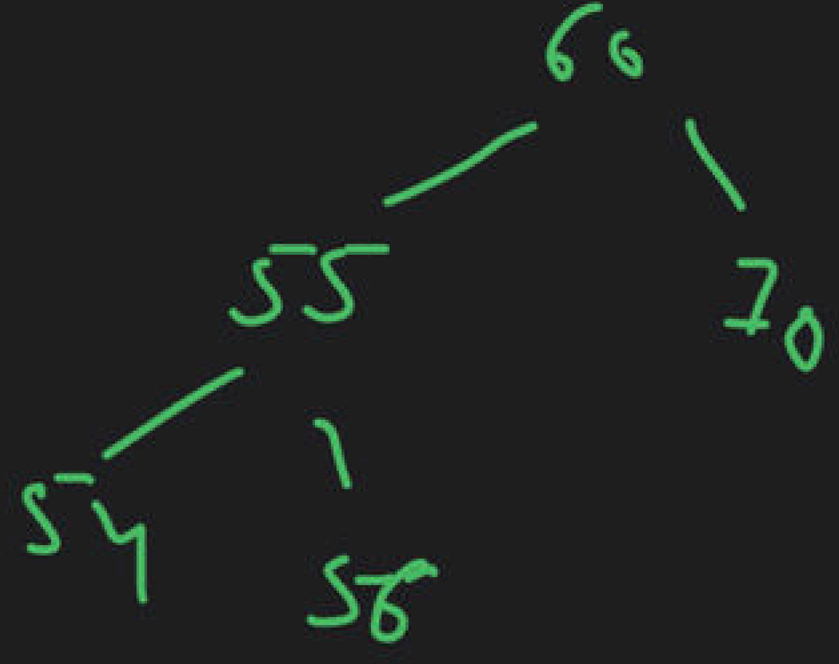
(1)



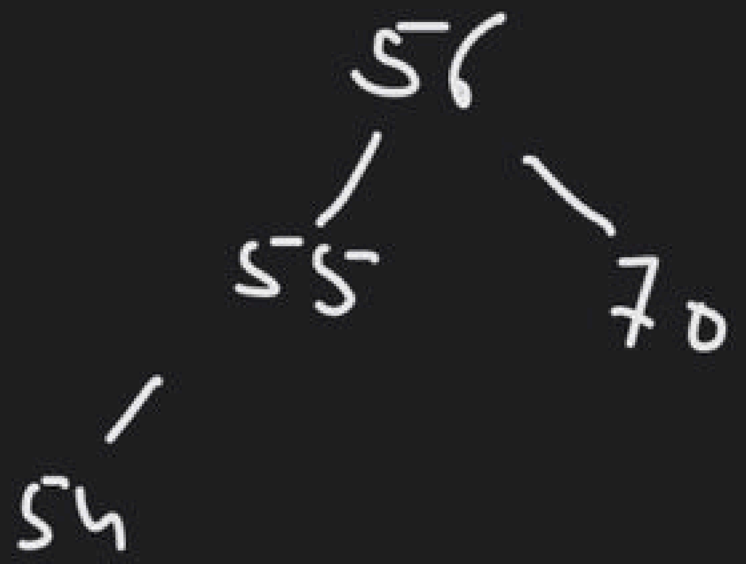
delete 60 ⟶ inorder predecessor ⇒ 55



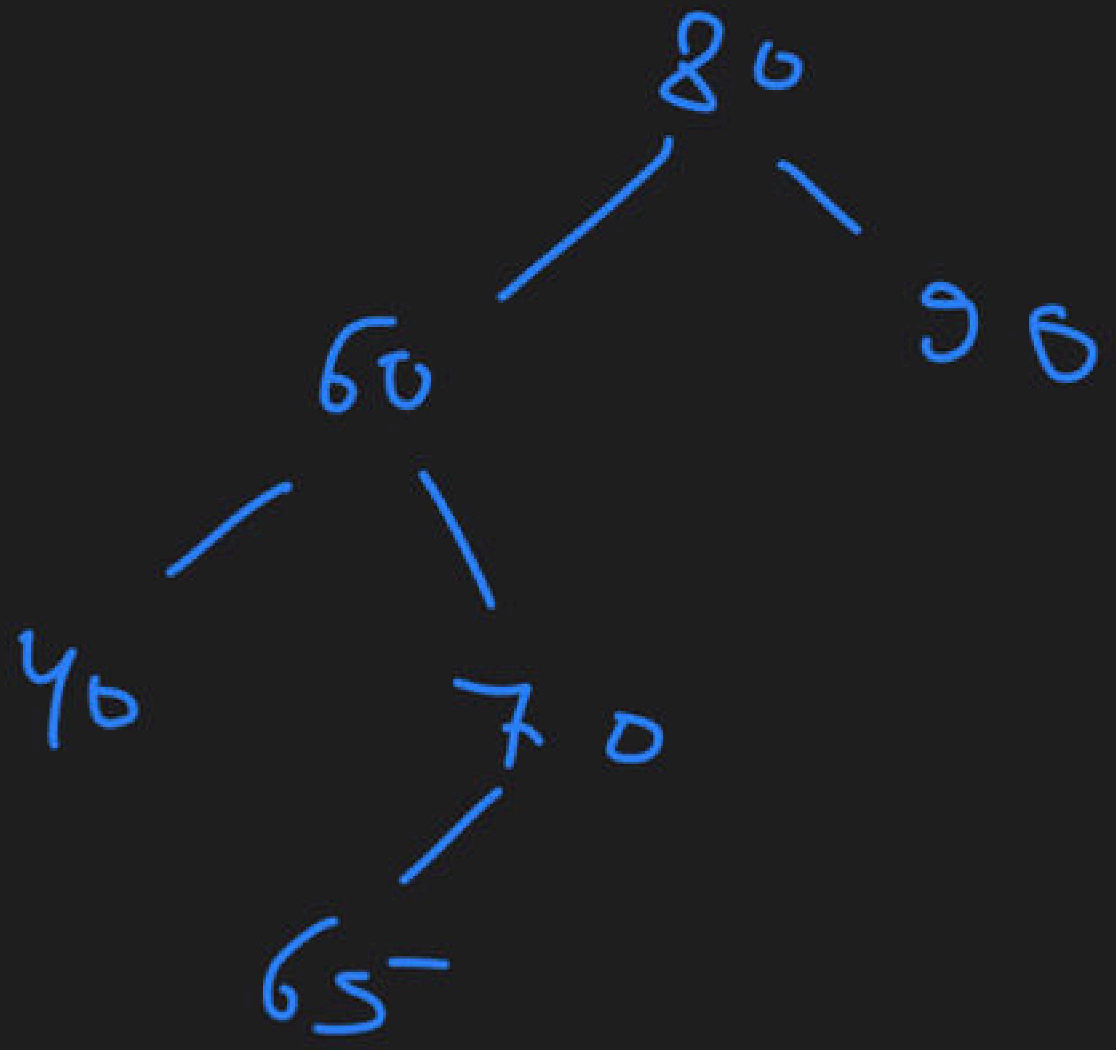
(2)



delete 60 ⟶ inorder predecessor = 56



3



Delete 80

Inorder
Predecessor ⇒ 70

Complexity

	Searching	Insertion	Deletion
best/	$O(h)$	$O(h)$	$O(h)$
avg. case	$O(\log_2 h)$	$O(\log_2 n)$	$O(\log_2 n)$
worst case	$O(n)$	$O(n)$	$O(n)$

Searching in Tree

Balanced Tree

AVL Tree

AVL Tree

$$Balance\ factor = h_L - h_R = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

DPP

Question 1

When searching for a key value 60 in a BST, nodes containing the key values 5, 10, 20, 30, 40, 50 are traversed, not necessarily in the same order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 60?

Question 2

When searching for a key value 50 in a BST, nodes containing the key values 12, 18, 27, 43, 64, 78, 81, 90 are traversed, not necessarily in the same order given. How many different orders are possible in which these key values can occur on the search path from the root to the node containing the value 50?

Question 3 GATE-2005

The numbers $1, 2, \ldots . n$ are inserted in a binary search tree in some order. In the resulting tree, the right subtree of the root contains p nodes. The first number to be inserted in the tree must be

A. p

B. $p+1$

C. $n-p$

D. $n-p+1$

Question 4 GATE-2008

You are given the postorder traversal, P, of a binary search tree on the n elements $1, 2, \ldots, n$. You have to determine the unique binary search tree that has P as its postorder traversal. What is the time complexity of the most efficient algorithm for doing this?

A. $\Theta(\log n)$

B. $\Theta(n)$

C. $\Theta(n \log n)$

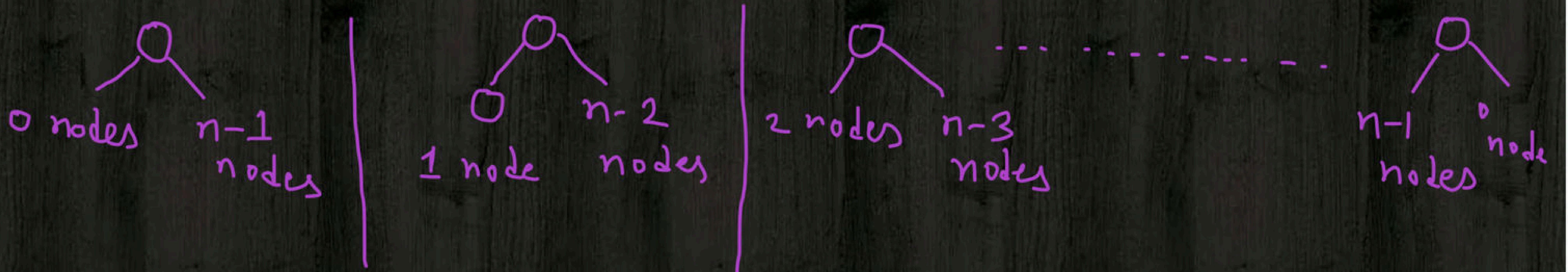
D. None of the above, as the tree cannot be uniquely determined

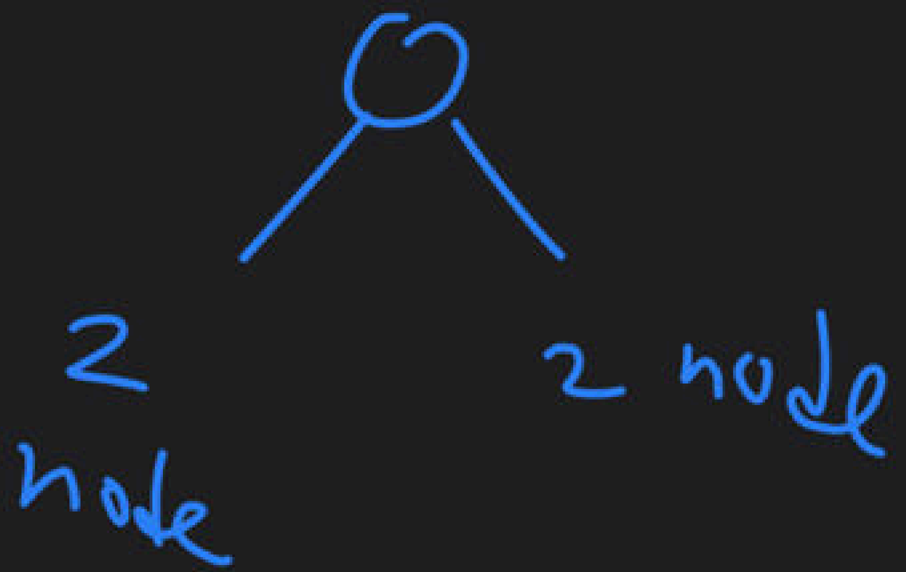
Question 5 GATE-2003

Let $T(n)$ be the number of different binary search trees on n distinct elements. Then $T(n) = \sum_{k=1}^{n} T(k-1)T(x)$, where x is

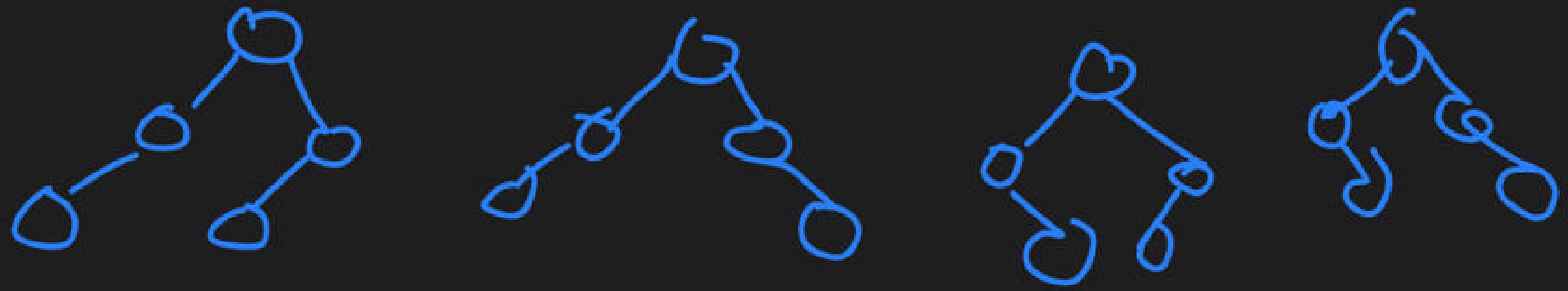
A. $n-k+1$ B. $n-k$ C. $n-k-1$ D. $n-k-2$

n number of nodes => let's make diff-2 BTS





$T(2) * T(2)$



4 trees

$$\sum_{k=0}^{n-1} T(k) * T(n-k-1)$$

$$\sum_{k=1}^{n} T(k-1)\, T(n-k)$$

Tree PYQS

Question GATE-1987

Construct a binary tree whose preorder traversal is

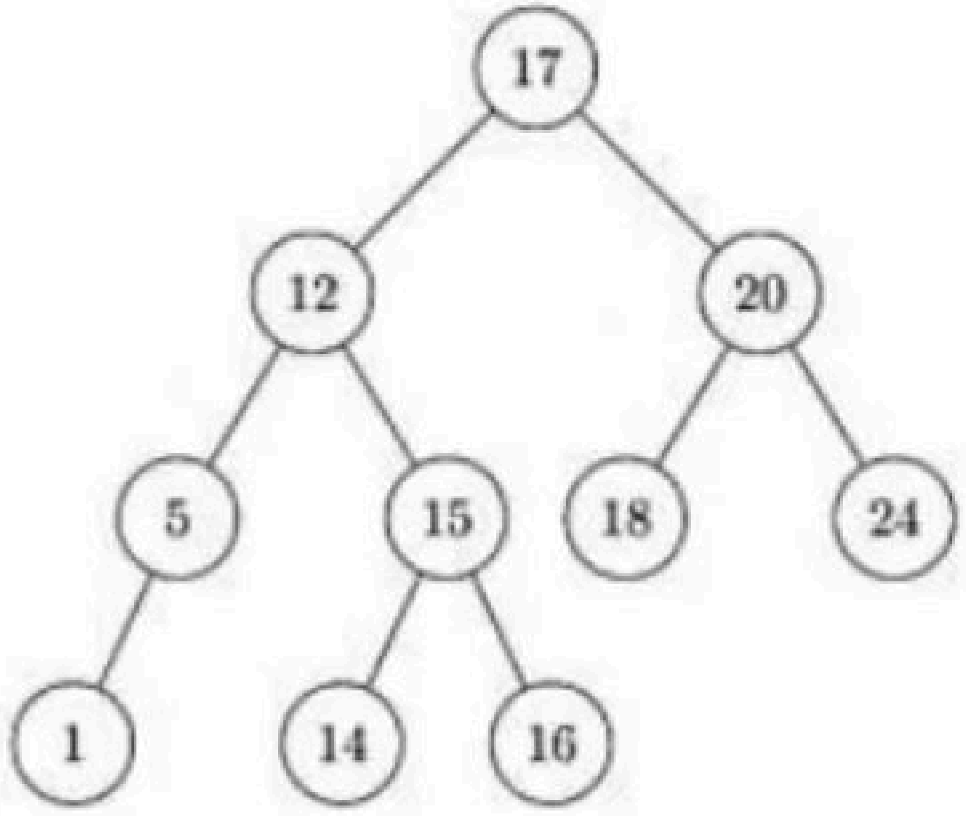
- K L N M P R Q S T

and inorder traversal is

- N L K P R M S Q T

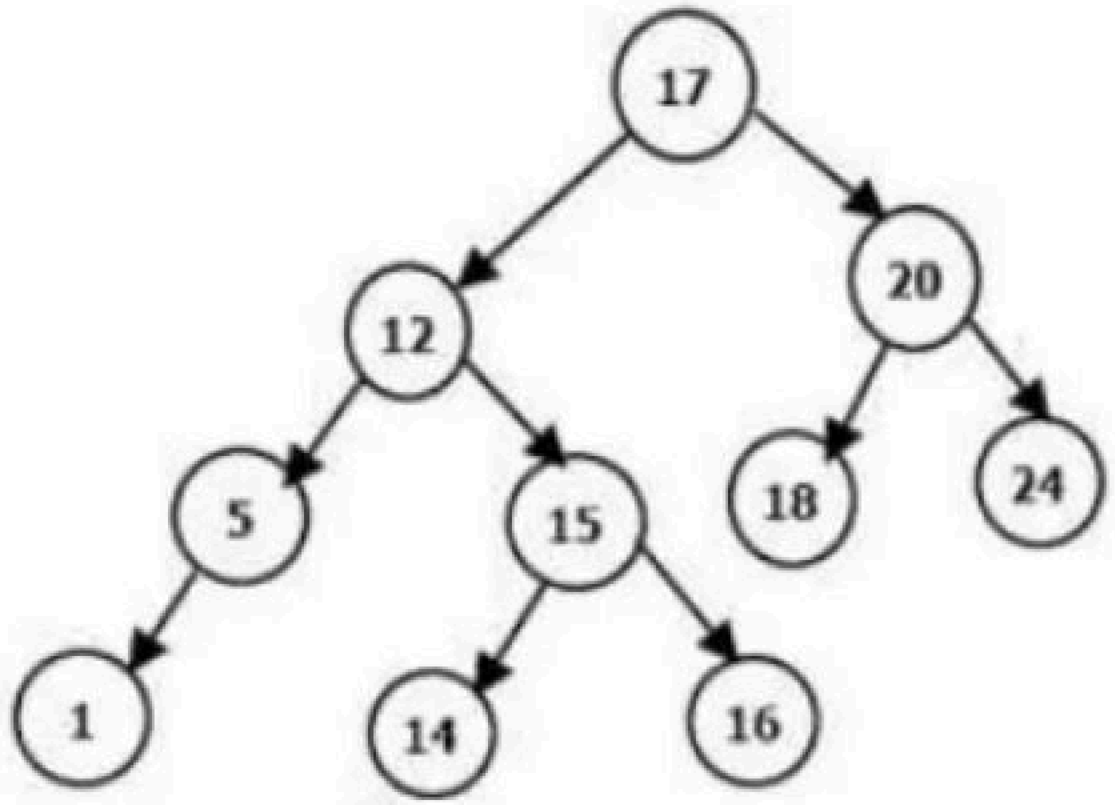
Question GATE-1988

Mark the balance factor of each on the tree given on the below figure and state whether it is height-balanced.



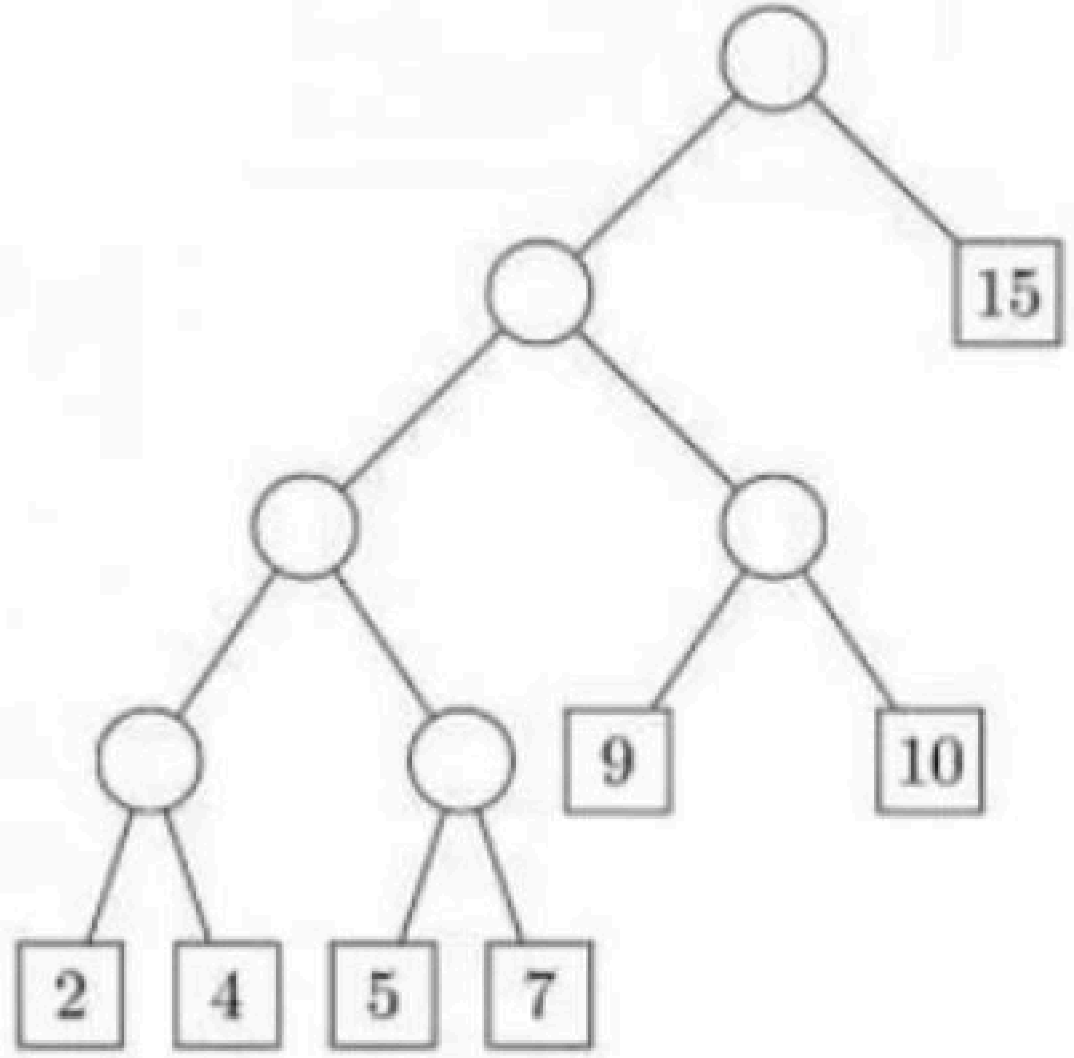
Question GATE-1988

Consider the tree given in the below figure, insert 13 and show the new balance factors that would arise if the tree is not rebalanced. Finally, carry out the required rebalancing of the tree and show the new tree with the balance factors on each mode.



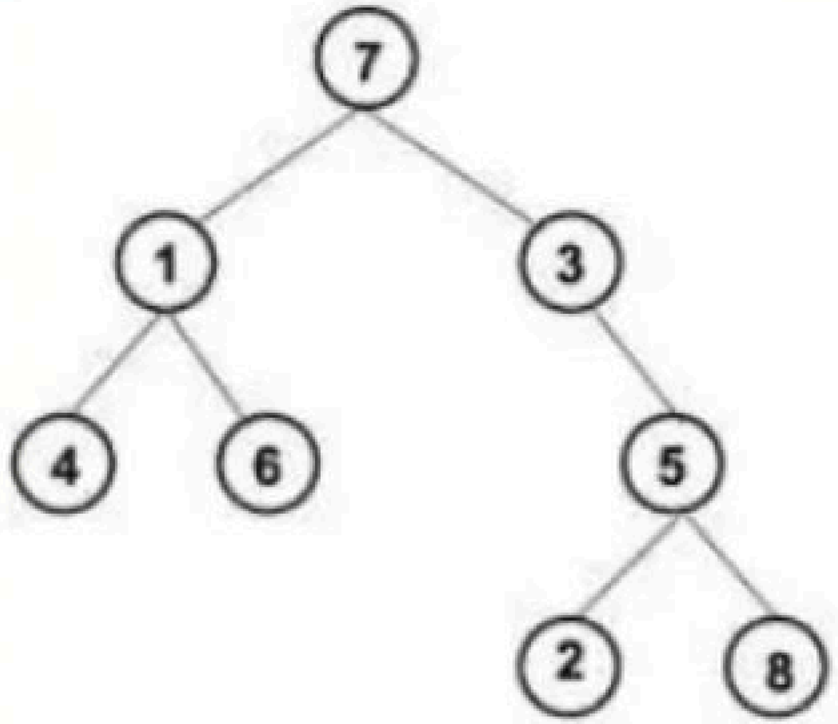
Question GATE-1991

The weighted external path length of the binary tree in figure is ______



Question GATE–1991

If the binary tree in figure is traversed in inorder, then the order in which the nodes will be visited is ______



Question GATE-1994

A rooted tree with 12 nodes has its nodes numbered 1 to 12 in pre-order. When the tree is traversed in post-order, the nodes are visited in the order $3, 5, 4, 2, 7, 8, 6, 10, 11, 12, 9, 1$.

Reconstruct the original tree from this information, that is, find the parent of each node, and show the tree diagrammatically.

Question GATE-1995

A binary tree T has n leaf nodes. The number of nodes of degree 2 in T is

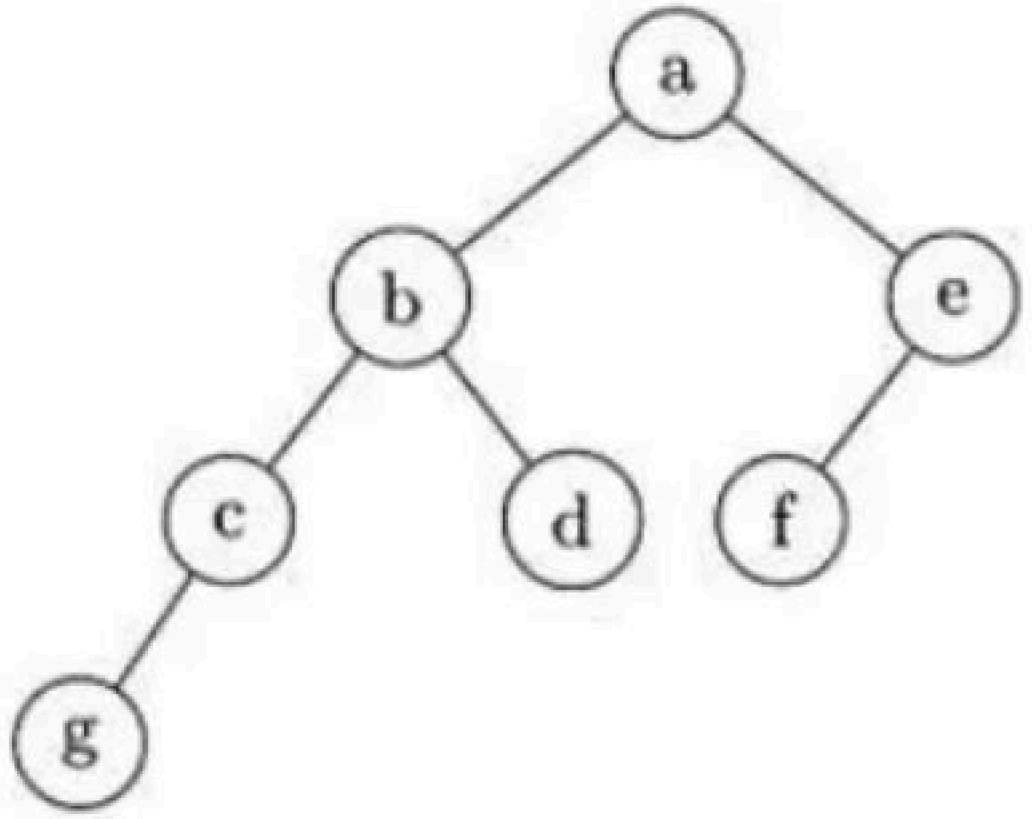
A. $\log_2 n$ B. $n-1$ C. n D. 2^n

Question GATE-1995

What is the number of binary trees with 3 nodes which when traversed in post-order give the sequence A, B, C? Draw all these binary trees.

Question GATE-1996

In the balanced binary tree in the below figure, how many nodes will become unbalanced when a node is inserted as a child of the node "g"?



A. 1

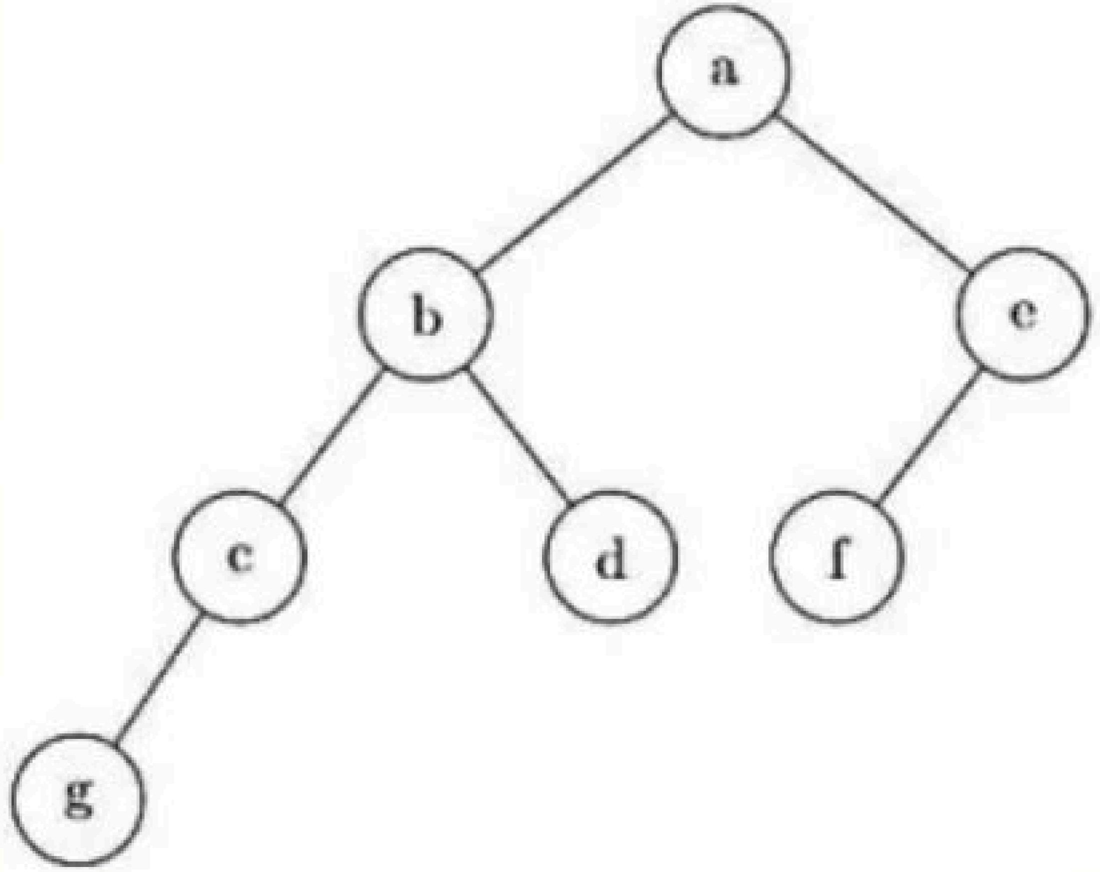
B. 3

C. 7

D. 8

Question GATE-1996

Which of the following sequences denotes the post order traversal sequence of the below tree?



A. $f\ e\ g\ c\ d\ b\ a$

B. $g\ c\ b\ d\ a\ f\ e$

C. $g\ c\ d\ b\ f\ e\ a$

D. $f\ e\ d\ g\ c\ b\ a$

Question GATE-1998

Draw the binary tree with node labels a, b, c, d, e, f and g for which the inorder and postorder traversals result in the following sequences:

Inorder: a f b c d g e

Postorder: a f c g e d b

Question GATE-2000

Consider the following nested representation of binary trees: $(X\,Y\,Z)$ indicates Y and Z are the left and right subtrees, respectively, of node X. Note that Y and Z may be $NULL$, or further nested. Which of the following represents a valid binary tree?

A. $(1\,2\,(4\,5\,6\,7))$
B. $(1\,(2\,3\,4)\,5\,6)\,7)$
C. $(1\,(2\,3\,4)\,(5\,6\,7))$
D. $(1\,(2\,3\,NULL)\,(4\,5))$

Question GATE-2000

Let LASTPOST, LASTIN and LASTPRE denote the last vertex visited `in a postorder, inorder and preorder traversal respectively, of a complete binary tree. Which of the following is always true?

A. LASTIN = LASTPOST

B. LASTIN = LASTPRE

C. LASTPRE = LASTPOST

D. None of the above

Question GATE-2002

A weight-balanced tree is a binary tree in which for each node, the number of nodes in the left sub tree is at least half and at most twice the number of nodes in the right sub tree. The maximum possible height (number of nodes on the path from the root to the furthest leaf) of such a tree on n nodes is best described by which of the following?

A. $\log_2 n$ B. $\log_{\frac{4}{3}} n$ C. $\log_3 n$ D. $\log_{\frac{3}{2}} n$

Question GATE-2004

Consider the label sequences obtained by the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely?

I. preorder and postorder
II. inorder and postorder
III. preorder and inorder
IV. level order and postorder

A. I only B. II, III C. III only D. IV only

Question GATE-2004

Consider the label sequences obtained by the following pairs of traversals on a labeled binary tree. Which of these pairs identify a tree uniquely?

I. preorder and postorder
II. inorder and postorder
III. preorder and inorder
IV. level order and postorder

A. I only   B. II, III   C. III only   D. IV only

Question GATE-2004

Consider the following C program segment

```
struct CellNode{
    struct CellNode *leftChild
    int element;
    struct CellNode *rightChild;
    };

int Dosomething (struct CellNode *ptr)
{
    int value = 0;
    if(ptr != NULL)
    {
        if (ptr -> leftChild != NULL)
            value = 1 + DoSomething (ptr -> leftChild);
        if (ptr -> rightChild != NULL)
            value = max(value, 1 + Dosomething (ptr -> rightChild));
    }
    return(value);
```

The value returned by the function DoSomething when a pointer to the root of a non-empty tree is passed as argument is

A. The number of leaf nodes in the tree

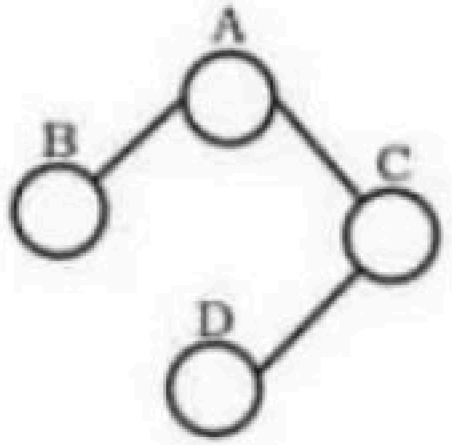
B. The number of nodes in the tree

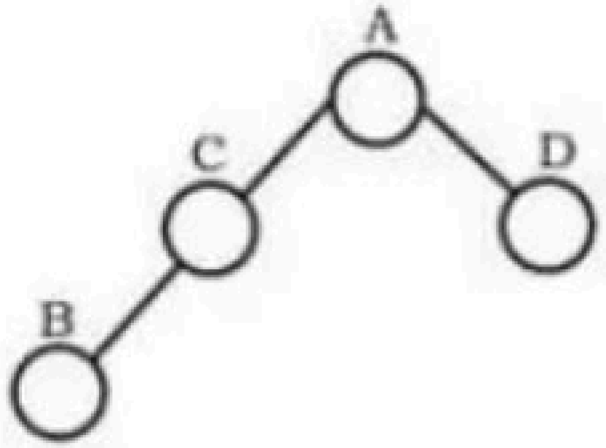
C. The number of internal nodes in the tree

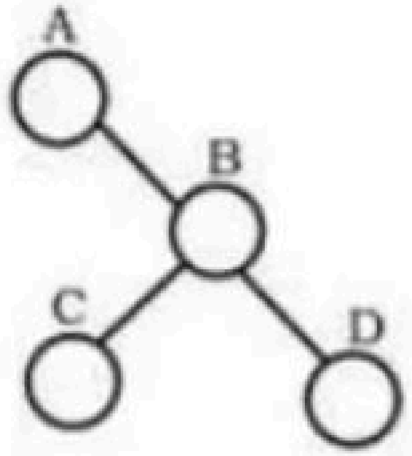
D. The height of the tree

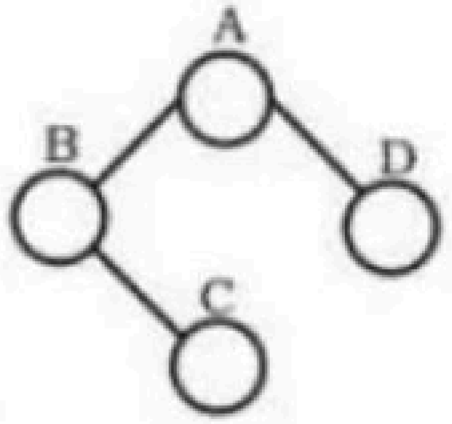
Question GATE-2004

Which one of the following binary trees has its inorder and preorder traversals as $BCAD$ and $ABCD$, respectively?

A. 

B. 

C. 

D. 

Question GATE-2005

In a binary tree, for every node the difference between the number of nodes in the left and right subtrees is at most 2. If the height of the tree is $h > 0$, then the minimum number of nodes in the tree is

A. 2^{h-1} B. $2^{h-1}+1$ C. $2^h - 1$ D. 2^h

Question GATE-2006

A scheme for storing binary trees in an array X is as follows. Indexing of X starts at 1 instead of 0. the root is stored at $X[1]$. For a node stored at $X[i]$, the left child, if any, is stored in $X[2i]$ and the right child, if any, in $X[2i+1]$. To be able to store any binary tree on n vertices the minimum size of X should be

A. $\log_2 n$ B. n C. $2n+1$ D. 2^n-1

Question GATE-2006

An array X of n distinct integers is interpreted as a complete binary tree. The index of the first element of the array is 0. The index of the parent of element $X[i], i \neq 0$, is?

A. $\left\lfloor \frac{i}{2} \right\rfloor$

B. $\left\lceil \frac{i-1}{2} \right\rceil$

C. $\left\lceil \frac{i}{2} \right\rceil$

D. $\left\lceil \frac{i}{2} \right\rceil - 1$

Question GATE-2006

An array X of n distinct integers is interpreted as a complete binary tree. The index of the first element of the array is 0. If the root node is at level 0, the level of element $X[i]$, $i \neq 0$, is

A. $\lfloor \log_2 i \rfloor$
B. $\lceil \log_2(i+1) \rceil$
C. $\lfloor \log_2(i+1) \rfloor$
D. $\lceil \log_2 i \rceil$

Question GATE-2006

In a binary tree, the number of internal nodes of degree 1 is 5, and the number of internal nodes of degree 2 is 10. The number of leaf nodes in the binary tree is

A. 10 B. 11 C. 12 D. 15

Question GATE-2007

The height of a binary tree is the maximum number of edges in any root to leaf path. The maximum number of nodes in a binary tree of height h is:

A. $2^h - 1$ B. $2^{h-1} - 1$ C. $2^{h+1} - 1$ D. 2^{h+1}

Question GATE-2007

The maximum number of binary trees that can be formed with three unlabeled nodes is:

A. 1 B. 5 C. 4 D. 3

Question GATE-2007

The inorder and preorder traversal of a binary tree are

d b e a f c g and a b d e c f g, respectively

The postorder traversal of the binary tree is:

A. d e b f g c a

B. e d b g f c a

C. e d b f g c a

D. d e f g b c a

Question GATE-2007

Consider the following C program segment where *CellNode* represents a node in a binary tree:

```
struct CellNode {
    struct CellNode *leftChild;
    int element;
    struct CellNode *rightChild;
};

int Getvalue (struct CellNode *ptr) {
    int value = 0;
    if (ptr != NULL) {
        if ((ptr->leftChild == NULL) &&
            (ptr->rightChild == NULL))
            value = 1;
        else
            value = value + GetValue(ptr->leftChild)
                      + GetValue(ptr->rightChild);
    }
    return(value);
}
```

The value returned by *GetValue* when a pointer to the root of a binary tree is passed as its argument is:

A. the number of nodes in the tree

B. the number of internal nodes in the tree

C. the number of leaf nodes in the tree

D. the height of the tree

Question GATE-2008

The following three are known to be the preorder, inorder and postorder sequences of a binary tree. But it is not known which is which.

I. $MBCAFHPYK$
II. $KAMCBYPFH$
III. $MABCKYFPH$

Pick the true statement from the following.

A. I and II are preorder and inorder sequences, respectively
B. I and III are preorder and postorder sequences, respectively
C. II is the inorder sequence, but nothing more can be said about the other two sequences
D. II and III are the preorder and inorder sequences, respectively

Question GATE-2008

A binary tree with $n > 1$ nodes has n_1, n_2 and n_3 nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbours.

n_3 can be expressed as

A. $n_1 + n_2 - 1$

B. $n_1 - 2$

C. $[((n_1 + n_2)/2)]$

D. $n_2 - 1$

Question GATE-2008

A binary tree with $n > 1$ nodes has n_1, n_2 and n_3 nodes of degree one, two and three respectively. The degree of a node is defined as the number of its neighbours.

Starting with the above tree, while there remains a node v of degree two in the tree, add an edge between the two neighbours of v and then remove v from the tree. How many edges will remain at the end of the process?

A. $2*n_1-3$

B. n_2+2*n_1-2

C. n_3-n_2

D. n_2+n_1-2

Question GATE-2010

In a binary tree with n nodes, every node has an odd number of descendants. Every node is considered to be its own descendant. What is the number of nodes in the tree that have exactly one child?

A. 0 B. 1 C. $\frac{(n-1)}{2}$ D. $n-1$

Question GATE-2011

We are given a set of n distinct elements and an unlabeled binary tree with n nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?

A. 0

B. 1

C. $n!$

D. $\frac{1}{n+1}.^{2n}C_n$

Question GATE-2012

The height of a tree is defined as the number of edges on the longest path in the tree. The function shown in the pseudo-code below is invoked as height (root) to compute the height of a binary tree rooted at the tree pointer root.

```
int height(treeptr n)
{ if(n == NULL) return -1;

  if(n -> left == NULL)
     if(n -> right == NULL) return 0;
     else return B1; // Box 1

  else{h1 = height(n -> left);
       if(n -> right == NULL) return (1+h1);
       else{h2 = height(n -> right);
            return B2; // Box 2
            }
       }
}
```

The appropriate expressions for the two boxes **B1** and **B2** are:

A. **B1**: $(1 + \text{height}(n \rightarrow \text{right}))$; **B2**: $(1 + \max(h1, h2))$
B. **B1**: $(\text{height}(n \rightarrow \text{right}))$; **B2**: $(1 + \max(h1, h2))$
C. **B1**: $\text{height}(n \rightarrow \text{right})$; **B2**: $\max(h1, h2)$
D. **B1**: $(1 + \text{height}(n \rightarrow \text{right}))$; **B2**: $\max(h1, h2)$

Question GATE-2014

Consider a rooted n node binary tree represented using pointers. The best upper bound on the time required to determine the number of subtrees having exactly 4 nodes is $O(n^a \log^b n)$. Then the value of $a + 10b$ is __________.

Question GATE-2015

The height of a tree is the length of the longest root-to-leaf path in it. The maximum and minimum number of nodes in a binary tree of height 5 are

A. 63 and 6, respectively

B. 64 and 5, respectively

C. 32 and 6, respectively

D. 31 and 5, respectively

Question GATE-2015

A binary tree T has 20 leaves. The number of nodes in T having two children is ______.

Question GATE-2015

Consider a binary tree T that has 200 leaf nodes. Then the number of nodes in T that have exactly two children are ________.

Question GATE-2016

Consider the following New-order strategy for traversing a binary tree:

- Visit the root;
- Visit the right subtree using New-order;
- Visit the left subtree using New-order;

The New-order traversal of the expression tree corresponding to the reverse polish expression

```
3 4 * 5 - 2 ^ 6 7 * 1 + -
```

is given by:

A. $+ - 1 6 7 * 2 \wedge 5 - 3 4 *$
B. $- + 1 * 6 7 \wedge 2 - 5 * 3 4$
C. $- + 1 * 7 6 \wedge 2 - 5 * 4 3$
D. $1 7 6 * + 2 5 4 3 * - \wedge -$

Question GATE-2019

Let T be a full binary tree with 8 leaves. (A full binary tree has every level full.) Suppose two leaves a and b of T are chosen uniformly and independently at random. The expected value of the distance between a and b in T (ie., the number of edges in the unique path between a and b) is (rounded off to 2 decimal places) __________.

Question GATE-2021

Consider a complete binary tree with 7 nodes. Let A denote the set of first 3 elements obtained by performing Breadth-First Search (BFS) starting from the root. Let B denote the set of first 3 elements obtained by performing Depth-First Search (DFS) starting from the root.

The value of $|A-B|$ is ______________ .

Happy Learning