Practice Session on Calculus - Part I

Revision Course on Engineering Mathematics - GATE, CS & IT

B V Reddy • Lesson 9 • Nov 21, 2022

PROBABILITY

- Basic definitions
- Problems on Coin , Dice , Cards and Balls
- Conditional Probability
- Independent events
- Total Probability
- Bayes Theorem

PROBABILITY

- **Random Variable**

  Mean , Variance , S.D

- Probability Distributions

  Binominal ,

  Poisson,

  Uniform ,

  Exponential ,

  Gaussian (Normal)

Sample space

All the possible outcomes of a random experiment .

Eg : tossing a coin

$S = \{ \text{Head} , \text{Tail} \}$

Rolling a dice

$S = \{ 1,2,3,4,5,6\}$

Event

The outcomes of a random experiment are called as Event .Event is always subset of the sample space .

Favorable events

The outcomes which are favorable to my desired event .

Mutually exclusive events

Two events A and B are said to be mutually exclusive (disjoint or incompatible) if the occurrence of one event prevents the occurrence of other event , i.e the events does not occur simultaneously .

Equally likely events

Occurrence of any event in a random experiment are equal then the events are said to be equally likely events .

Eg :

Independent events

The occurrence of one event does not depend on another.

Eg : 1. when two dice are rolled , Getting '1' on first die does not depend on '2' on the second die .

2. when an unbiased coin is tossed two times, the event of getting a head in the first toss is independent of getting head in the second toss .

Probability

The probability of an event A is defined as

$$P(A) = \frac{number\ of\ fauourable\ events}{total\ number\ of\ possible\ events}$$

$$= \frac{n(A)}{n(S)}$$

Properties of Probability

1. $P(A) = \frac{n(A)}{n(S)}$

2. $0 \leq P(A) \leq 1$

Impossible event

Sure event
(certain event)

3. Sum of all probabilities =1

$$\sum P = 1$$

4. P(sample space) =1

5. $P(\overline{A}) = 1 - P(A)$

6. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A & B are mutually exclusive events,

$P(A \cap B) = 0$

$$7.\ P(A\cap B) \le P(A) \le P(A\cup B) \le P(A) + P(B)$$

$$8.\ P(\overline{A\cup B}) = 1 - P(A\cup B)$$

$$P(\overline{A\cup B}) = P(\overline{A}\cap\overline{B})$$

$$9.\ P(\overline{A\cap B}) = 1 - P(A\cap B)$$

$$P(\overline{A\cap B}) = P(\overline{A}\cup\overline{B})$$

$$10.\ P(\text{only } A) = P(A\cap\overline{B}) = P(A) - P(A\cap B)$$

$$11.\ P(\text{only } B) = P(\overline{A}\cap B) = P(B) - P(A\cap B)$$

12. $P(\text{Both A \& B}) = P(A \cap B)$

13. $P(\text{at least one}) = P(A \cup B)$

14. $P(\text{Either A or B}) = P(A \cup B)$

15. $P(\text{neither A nor B}) = P\left(\overline{A} \cap \overline{B}\right) = P\overline{(A \cup B)}$

16. $P(\text{exactly one}) = P(A \,\Delta\, B) = P\left(A \cap \overline{B}\right) + P\left(\overline{A} \cap B\right)$

18. $P(A \cup BUC) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

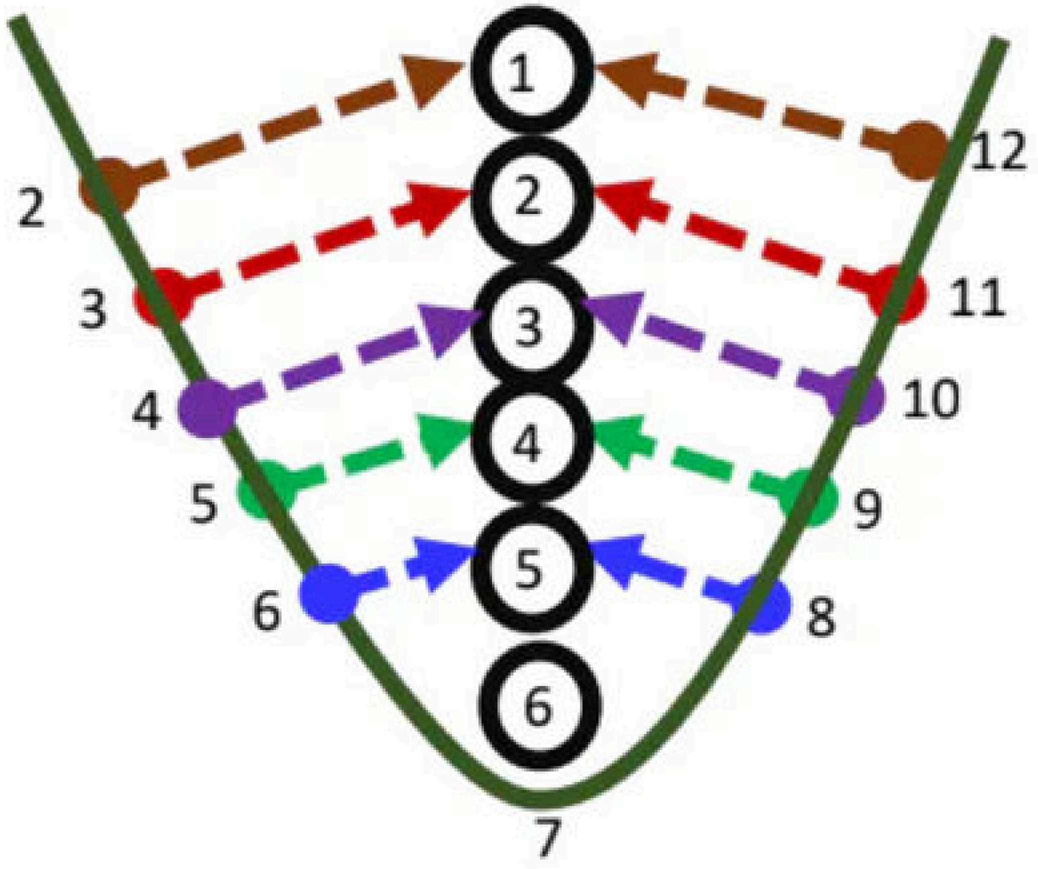
Sample Space

Rolling a dice

When two dice are rolled

n(S)=

Sum of the numbers on the dice =

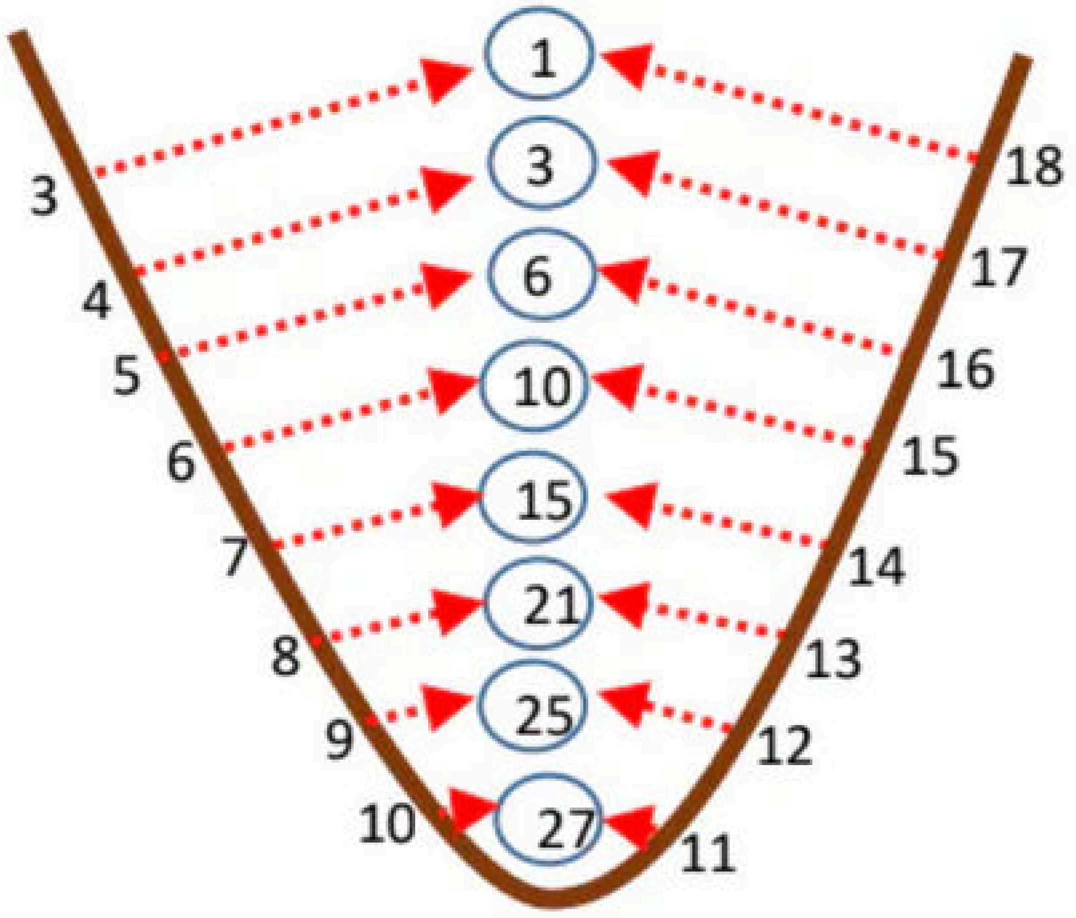


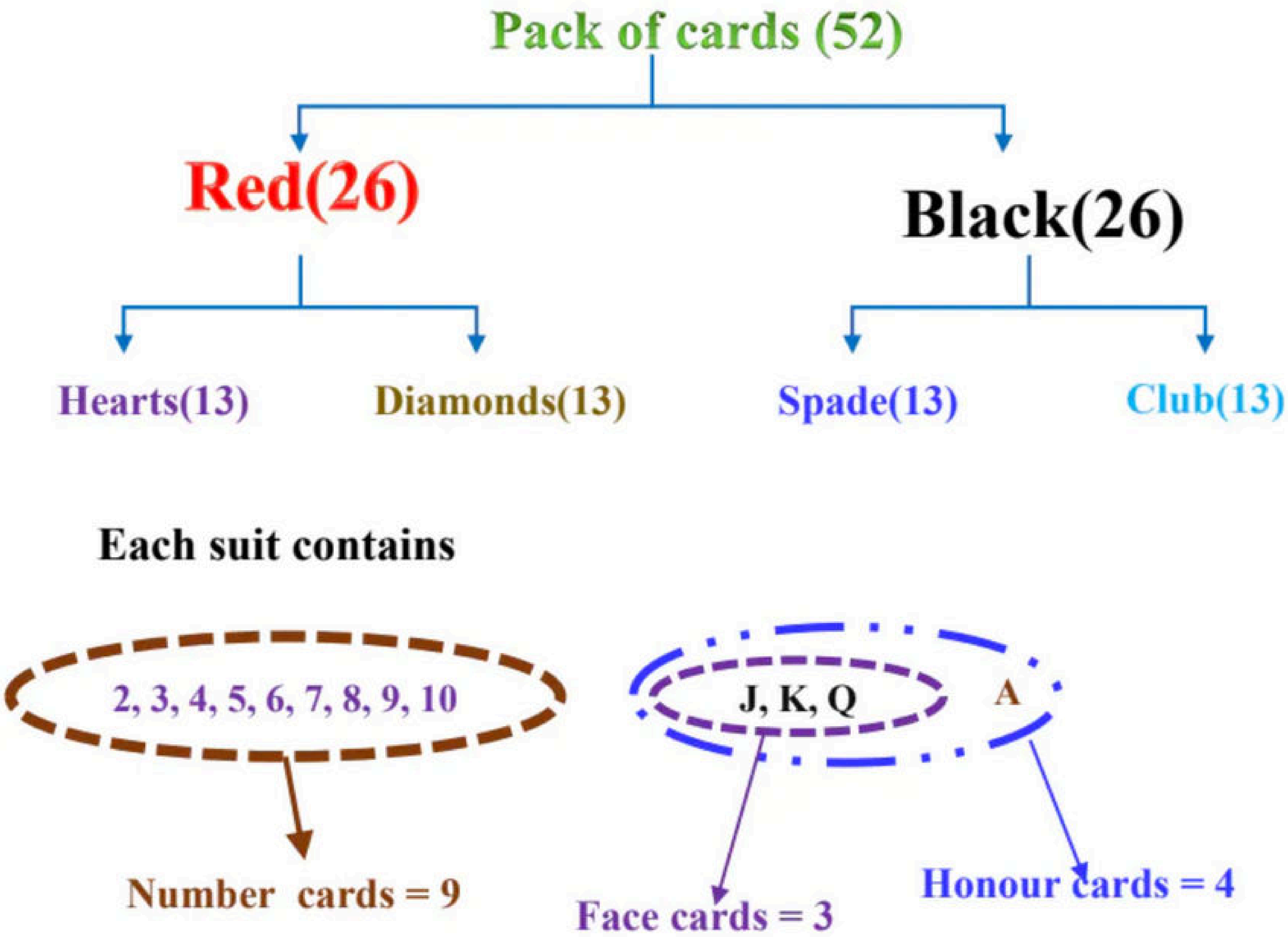
Rolling a dice

When three dice are rolled

n(S)=

Sum of the numbers on the dice = {





Pack of cards (52)
Red(26)
Black(26)
Hearts(13)
Diamonds(13)
Spade(13)
Club(13)
Each suit contains
2, 3, 4, 5, 6, 7, 8, 9, 10
J, K, Q
A
Number cards = 9
Face cards = 3
Honour cards = 4

1. Total number of face cards =
2. Total number of number cards =
3. Total number of honour cards =
4. Total number of Red Kings =
5. Total number of Spade Queen =
6. Total number of Black Diamonds =
7. Total number of Diamond Ace =
8. Total number of Black 2's =
9. Total number of Red face cards =
10. Total number of Spade King =
11. Total number of Club 9's =
12. Total number of Red Hearts =

1. When two dice are rolled , what is the probability of getting the sum

    a) sum = 4   b) sum =11   c) sum > 10

    d) sum ≤ 10   e) $4 \leq sum \leq 11$

2. A and B are playing a game of tossing a coin . One who gets head wins the game . If A starts the game , find the probabilities of their winning .

Joint Probability

If A and B are the two events in sample space S , which are not mutually exclusive then the joint probability of A and B can be denoted as $P(A \cap B)$.

Conditional Probability

If A and B are the two events in sample space S , then the conditional probability of A given B is defined as

$$P\left[\frac{A}{B}\right] =$$

the conditional probability of B given A is defined as

$$P\left[\frac{B}{A}\right] =$$

Properties of conditional probability

Pair wise independent events

If A and B are pair wise independent events

Mutually independent events

If A, B and C are Mutually independent events then

Total Probability Theorem

If sample space contains n – mutually exclusive events, then probability of event A defined on the sample space S can be expressed as a conditional probability .

$$P(A) = \sum_{n=1}^{N} P(B_n)P\left(\frac{A}{B_n}\right)$$

3. A box contains 5 red balls and 6 black balls , another box contains 6 red balls and 4 black balls. One box is chosen at random and one ball is drawn from it. Find the probability of getting

a)Red ball

b) Black ball

Bayes' theorem

If sample space S contains n- mutually exclusive events, let A is any event in the sample space , then the conditional probability of B_n given A is

$$P\left[\frac{B_n}{A}\right] = \frac{P\left(\frac{A}{B_n}\right)p(B_n)}{\sum_{n=1}^{N} P\left(\frac{A}{B_n}\right)p(B_n)}$$

4. A box contains 5 red balls and 6 white balls , another box contains 4 red ball and 6 white balls . One ball is drawn and found to be red . Find the probability that the ball is drawn from first box .

5. 25 girls out of 100 , 5 boys of 100 have color blind. One person is chosen at random and found to be colorblind . Find the probability that person is a girl .

6. A box contains 2 white and 3 black balls , a sample of size 4 is made ,what is the probability that the sample is in the order {white , black , white , black }

7. A box contains 6 red balls, 4 white balls, and 5 blue balls . Three balls are drawn successively from the box . Find the probability that they are drawn in the order red , white and blue if each ball is

a) Replaced

b) Not replaced

8. A box contains 52 badges numbered 1 to 52 . Suppose that the numbers 1 through 13 are considered lucky. A sample of size 2 is drawn from the box with replacement . What is the probability that

a) Both badges drawn will be lucky

b) Nether badges will be lucky

c) Exactly one of the badges drawn will be lucky

d) At least one of the badges will be lucky

9. In a experiment of drawing a card from a pack the event of getting a spade is denoted by A , and getting a face card is denoted as B, find the probabilities of $A \cup B$ and $A \cap B$

10. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that sum of the numbers is

a) An even number

b) Odd number

11. Find the ratio of probabilities getting sum 6 with 4 , 3 and 2 dice are rolled respectively

12. A box contains 4 point contact diodes and 6 alloy junction diodes, what is the probabilities that 3 diodes picked at random contain at least 2 point contact diodes

3.

13. A box contains 5 black , 4 white and 6 red balls . Two balls are drawn without replacement , what is the probability that the first will be white and second will be black

14. One card is drawn from a regular deck of 52 cards, what is the probability of the card being either red or king

15. One card is selected from an ordinary 52 card deck with an event A as select a king , B as select a jack or queen C as select a heart , find P(A$\cap B$) , P(B$\cap C$) and P(C $\cap A$)

16. There are 3 black and 4 white balls in one bag, 4 black and 3 white balls in the second bag . A die is rolled and the first bag is selected if it is 1 or 3 and second bag for the remaining , find the probability of drawing a black ball from the selected bag

17. A box contains 3 coins, one is fair ,one is two headed and one coin is weighted so that the probability of heads appearing is 1/3 . A coin is selected at random and tossed , find the probability that head appears.

18. Three boxes numbered I II and III contain 1-white , 2 black and 3 red balls,: 2 white 1 black and 1 red ball ; 4 white 5 black and 3 red balls respectively. One box is randomly selected and a balls drawn from it . If the ball is red then find the probability that it is from box II

19. Three coins are tossed at a time find the probability of getting

a) At most one tail

b) At least one tail

c) At least one head and at most one tail

20. Four coins are tossed at a time find the probability of getting at most 2 head and at most 2 tail

21. Four coins are tossed at a time find the probability of getting at most 2 head and at least 1 tail

22. Six coins are tossed at a time find the probability of getting at least 2 head and at least 2 tails

23. N- coins are tossed at a time find the probability of getting head

a) Odd number of times

b) Even number of times

24. Three horses A, B and C are in a race. A is twice as likely to win as B and B is twice as likely to win as C. What is the probability that B or C wins?

a) 2/7 b) 3/7 c) 4/7 d) 6/7

25. A card is selected at random from an ordinary pack of 52 cards. Probability of selecting a Spade card or a face card is

a) 3/32 b) 23/52 c) 22/52 d) 25/52

26. Let two items be chosen from a lot containing 12 items of which 4 are defective. What is the probability that at least one item is defective?

a) 19/33   b) 14/33   c) 1/11   d) 13/33

27. A number is selected at random from first 200 natural numbers. Find the probability that the number is divisible by 6 or 8?

a) 1/3 b) 1/4 c) 1/5 d)2/3

28. A point is selected at random inside a circle. Find the probability p that the point is closer to the Centre of the circle than to its circumference?

a) 1/3 b) 1/4 c)1/5 d)2/3

29. Let A and B be events with $p(A) = 3/8$, $P(B) = 1/2$ and $P(A \wedge B) = 1/4$ then which of the Following is false.

a) $P(A^C \vee B^C) = 3/4$ b) $P(A^C \wedge B^C) = 3/8$ c) $P(A \wedge B^C) = 1/8$ d) $P(B \wedge A^C) = 5/8$

30. Of 120 students, 60 are studying French, 50 are studying Spanish and 20 are studying French and Spanish. If a student is selected at random then which of the following is not correct.

a) Probability that the student is studying French or Spanish is 0.75.

b) Probability that the student is studying neither French nor Spanish is 0.25.

c) Probability that the student is studying Spanish but not French is 0.25.

d) Probability that the student is studying French but not Spanish is 0.3

31. In a class of 100 students, 40 failed in mathematics, 30 failed in physics, 25 failed in Chemistry, 20 failed in math's and physics, 15 failed in physics and chemistry, 10 failed in Chemistry and math's, 5 failed in math's, physics and chemistry. If a student is selected at Random then the probability that he passed in all three subjects is

a) 0.4 b) 0.45 c) 0.55 d) 0.65

32. Let a pair of dice be tossed. If the sum is 6, find the probability that one of the dice is a 2.

a) 1/5 b) 2/5 c) 3/5 d) 4/5

33. A man visits a couple who have two children. One of the children, a boy, comes in to the room . Find the probability that the other is also a boy

a) 1/3　　b) 2/3　　c) 1/2　　d) 3/4

34. Let A and B be events with $P(A) = 3/8$, $P(B) = 5/8$ and $P(A \cup B) = 3/4$. Find the conditional probability $P(A/B)$

a) 1/3 b) 2/5 c) 3/4 d) 1/2

35. In certain college, 25% of the students failed mathematics, 15% of the students failed in Chemistry, and 10% of the students failed in both math's and chemistry. A student is Selected at random. If he failed chemistry, what is the probability that he failed in math's?

a) 2/3 b)2/5 c) 3/5 d)1/5

36. A die is rolled. If the number appeared is odd, what is the probability that it is prime?

a) 1/3 b) 2/3 c) 3/4 d)1

37. In a certain college, 4% of the men and 1% of the women are taller than 1.8m. Further more, 60% of the students are women. Now if a student is selected at random and is taller than 1.8m, what is the probability that the student is a woman ?

a)3/11 b) 4/11 c) 5/11 d) 6/11

38. We are given three urns as follows. Urn A contains 3 red and 5 white marbles, Urn B contains 2 red and 1 white marble, Urn C contains 2 red and 3 white marbles. An urn is selected at random and a marble is drawn from the urn. If the marble is red, what is the probability that it came from urn A?

a) 45/173 b) 37/165 c) 27/109 d) 39/185

39. A coin, weighted so that $P(H) = 2/3$ and $P(T) = 1/3$ is tossed. If heads appears, then a number is selected at random from the numbers 1 through 9. If tails appears, then a number is selected at random from the numbers 1 through 5. Find the probability P that an even number is selected.

a) 67/145 b) 58/135 c)74/157 d)43/142

40. A box contains three coins, two of them fair and one two headed. A coin is selected at random and tossed twice. If heads appears both times, what is the probability that the coin is two headed?

a) 2/3 b) 1/3 c) 3/4 d) 1/2

41. An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the opposite colour is put in to the urn. A second marble is drawn from the urn. If both marbles where of the same colour . What is the probability that they were both white?

a) 5/6 b)7/8 c)8/9 d)9/10

42. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(GATE-EC-2003)

(a) 100% (b) 50% (c) 49% (d) none

43. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children?

$a)\frac{3}{23}$ $b)\frac{6}{23}$ $c)\frac{3}{10}$ $d)\frac{3}{5}$

44. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students Have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses ?

(a) 15 (b) 20 (c) 25 (d) 35

45. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is

(GATE-CS-1995)

(a) 18/25 (b) 2/5 (c) 5/12 (d) 19/25

46. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced ?

(a) 1/26 (b) 1/52 (c) 1/169 (d) 1/221

47. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is

(GATE-CS -2005)

a) $\frac{1}{36}$ b) $\frac{1}{6}$ c) $\frac{1}{4}$ d) $\frac{1}{3}$

48. Two dice are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is **(GATE-PI-2005)**

(a) 4/36 (b) 7/36 (c) 9/36 (d) 10/36

49. The probability that there are 53 Sundays in a randomly chosen leap year is

(GATE-IN-2005)

(a) 1/7 (b) 1/14 (c) 1/28 (d) 2/7

50. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads ? **(GATE-EC-2009)**

(a) $\left(\frac{1}{2}\right)^2$ (b) $10c_2\left(\frac{1}{2}\right)^2$ (c) $\left(\frac{1}{2}\right)^{10}$ (d) $10c_2\left(\frac{1}{2}\right)^{10}$

51. If three coins are tossed simultaneously, the probability of getting at least one head is

(GATE-ME-2009)

(a) 1/8 (b) 3/8 (c) 1/2 (d) 7/8

52. A fair coin is tossed independently four times. The probability of the event "The number of times heads show up is more than the number of times tails show up" is

(GATE-EC-2010)

(a) 1/16 (b) 1/8 (c) 1/4 (d) 5/16

53. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **(GATE-CE-2010)**

(a) 1/8 (b) 1/6 (c) 1/4 (d) 1/2

54. A fair dice is rolled two times. The probability that the 2nd toss results in a value that is higher than the first toss is **(GATE-EC-2011)**

(a) 2/36 (b) 2/6 (c) 5/12 (d) 1/2

55. The box 1 contains chips numbers 3, 6, 9, 12 and 15. The box 2 contains chips numbers 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is

(GATE-IN-2011)

(a) 6/25 (b) 2/5 (c) 5/12 (d) 19/25

56. A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is **(GATE-ME,PI-2012)**

(a) 1/20 (b) 1/12 (c) 3/10 (d) ½

57. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is _______ **(GATE-EC-SET-1-2014)**

58. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is (n-3) is **(GATE-EE- SET-1-2014)**

(a) 2^{-n} (b) 0 (c) ${}^{n}C_{n-3}2^{-n}$ (d) 2^{-n+3}

59. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is

(GATE-ME- SET-2-2014)

(a) 7/20 (b) 2/5 (c) 5/12 (d) 19/25

60. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is X/1296. The value of X is _______ **(GATE-CS- SET-1-2014)**

61. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by P. Then 100p = __________ **(GATE-CS- SET-2-2014)**

62. Suppose A and B are two independent events with probabilities P(A) = 0 and P(B) $\neq 0$. Let $\bar{A}$ and $\bar{B}$ be their complements. Which one of the following statements is FALSE ?

(GATE-EC-2015)

a) $P(A\cap B) = P(A)P(B)$ b) $P(A/B) = P(A)$ c) $P(A\cup B) = P(A) + P(B)$ d) $P(\bar{A}\cap \bar{B}) = P(\bar{A})P(\overline{B})$

63. Two coins R and S are tossed. The 4 joint events H_RH_S, T_RT_S, H_RT_S, T_RH_S have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE ? **(GATE-EE-2015)**

(a) The coin tosses are independent
(b) R is fair, S is not
(c) S is fair, R is not
(d) The coin tosses are dependent

64. The probability that it will rain today is 0.5 the probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow ? **GATE- 97 (CS)**

(a) 0.3 (b) 0.25 (c) 0.35 (d) 0.4

65. A single die is rolled two times. What is the probability that the sum is neither 8 nor 9 ?

(GATE-ME-2005)

(a) 1/29 (b) 2/5 (c) 5/12 (d) 3/4

66. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is _____ **(GATE-CS- SET-2-2014)**

67. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If P(.) denotes the probability of the event, the maximum value of P(A) P(B) is ______

(GATE-CS- SET-3-2014)

68. Consider two events E1 and E2 such that $P(E_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{3}$ $\quad P(E_1 \cap E_2) = \frac{1}{5}$ Which of the following statements is true ? **(GATE-EC-1999)**

$a)$ $P(E_1 \cup E_2) = \frac{2}{5}$

(b) E_1 and E_2 are independent

(c) E_1 and E_2 are not independent (d) $P\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

69. E_1 and E_2 are events in a probability space satisfying the following constraints $P(E_1) = P(E_2)$; $P(E_1 \cup E_2) = 1$. E_1 & E_2 are independent then $P(E_1) =$

(a) 6/25 (b) 2/5 (c) 5/12 (d) 1

70. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate1 has failed, the probability that both gates 2 and 3 will fail is **GATE-04**

(a) 0.240 (b) 0.200 (c) 0.040 (d) 0.008

71. fair coin is tossed 3 times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is

GATE-05-(EE)

(a) 6/25 (b) 2/5 (c) 1/2 (d) 19/25

72. A fair dice is rolled twice. The probability that an odd number will follow an even number is

(GATE-EC-2005)

(a) 6/25 (b) 2/5 (c) 1/4 (d) 19/25

73. An unbiased coin is tossed five times. The outcome of each toss is either a head or a tail. Probability of getting at least one head is _______ **(GATE-ME-2011)**

(a) 6/25 (b) 2/5 (c) 1/4 (d) 31/32

74. Consider a die with the property that the probability of a face with ‘n’ dots showing up is proportional to ‘n’. The probability of the face with three dots showing up is _______

(GATE-EE- SET-2-2014)

75. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

76. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is **(GATE-EC- SET-3-2014)**

(a) 0.067 (b) 0.073 (c) 0.082 (d) 0.091

77. Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is ______. **(GATE-ME- SET-2-2014)**

78. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times ?

(GATE-ME-2008)

(a) 1/4 (b) 3/8 (c) 1/2 (d) ¾

79. The probability that a thermistor randomly picked up from a production unit is defective is 0.1. The probability that out of 10 thermistors randomly picked up, 3 are defective is

(GATE-IN-2015)

(a) 0.001 (b) 0.057 (c) 0.0107 (d) 0.3

80. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is

a) 0.0036 b) 0.1937 c) 0.2234 d) 0.3874

81. The probability of an event B is P1 . The probability of events A and B occur together is P2 , while the probability that A or B occur together is P3 . The probability of event A in terms of P1, P2 and P3 is

a) P1+P2

b) P3+P2

c) P3+P1

d) P3- P1+ P2

82. the probability that a new Airport will get an award for its design is 0.16. The probability that it will get an award for its efficient use of materials is 0.24 and probability that it will get both the awards is 0.11. What is the probability that it will get only one of the two awards?

a) 0.29 b) 0.18 c) 0.21 d) 0.19

83.A jar has 5 marbles, one of each of the colors, red, white, blue, green and yellow. If 1 Marble is removed from the jar, what is the probability that the yellow one is removed?

a) 1/5 b) ½ c) 4/5 d) ¾

84. A jar contains 4 marbles. 2 red and 2 white. Two marbles are chosen at random. If p_1 is the probability that the marbles chosen are of same color and p_2 is the probability that the marbles chosen be of different colors, then which of the following is true?

a) $p_1 = p_2$ b) $p_1 = 2p_2$ c) $p_2 = 2p_1$ d) $2p_1 = 3p_2$

85. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day ?

a) $1/7^7$ b) $1/7^6$ c) $1/2^7$ d) $7/2^7$

86. If P and Q are two random events, then the following is true

a) Independence of P and Q implies that Probability($P \cap Q$) = 0

b) Probability ($P \cap Q$) $\geq$ Probability(P) + Probability(Q)

c) If P and Q are mutually exclusive then they must be independent

d) Probability($P \cap Q$) $\leq$ Probability(P)

87. Find the probability of not getting a total of 7 or 11 on either of two tosses of a pair of fair dice?

a) 25/36 b) 21/49 c) 7/9 d) 49/81

88. Find the probability of a 4 turning up at least once in two tosses of a fair dice?

a) 8/32 b) 10/36 c) 11/36 d) 13/36

89. Candidates were asked to come to an interview with 3 pens each. Black, blue, green and red were the permitted pen colours that the candidate could bring. The probability that a candidate comes with all 3 pens having the same colour is _________.

(GATE-16-EE)

90. An urn contains 5 red and 7 green balls, A ball is drawn at random and its colour is noted. The ball is placed back into the urn along with another ball of the same colour. The probability of getting a red ball in the next drawn is

(GATE- 16 – IN)

(a) $\frac{65}{156}$ (b) $\frac{67}{156}$

(c) $\frac{79}{156}$ (d) $\frac{89}{156}$

91. The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is ____________.

(GATE- 16 – EC – SET3)

92. The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing and gives 5 screws guarantee of a replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to replaced is________________.

(GATE-16-ME-SET2)

93. Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(GATE-16-ME-SET3)

(a) $\frac{16}{5525}$ (b) $\frac{64}{2197}$

(c) $\frac{3}{13}$ (d) $\frac{8}{16575}$

94. A fair coin is tossed N times. The probability that head does not turn up in any of the tosses is

(GATE-16-PI-SET1)

(a) $\left(\frac{1}{2}\right)^{N-1}$

(b) $1-\left(\frac{1}{2}\right)^{N-1}$

(c) $\left(\frac{1}{2}\right)^{N}$

(d) $1-\left(\frac{1}{2}\right)^{N}$

95. X and Y are two random independent events. It is known that $P(X) = 0.40$ and $P(X \cup Y^C) = 0.7$. Which of the following is the value of $P(X \cup Y)$?

(a) 0.7 (b) 0.5 **(GATE-16-CE-SET2)**

(c) 0.4 (d) 0.3

96. Consider the following experiment.

Step1: Flip a fair coin twice.

Step2: If the outcomes are (TAILS, HEADS) then output Y and stop.

Step3: If the outcomes are either (HEADS, HEADS) or (HEADS, TAILS), then output N and stop.

Step4: If the outcomes are (TAILS, TAILS), then go to step 1.

The probability that the output of the experiment is Y is (up to two decimal places)________.

(GATE – 16 -CSE-SETI)

97. Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is________.

(GATE – 16 -CSE-SET2)

98. The probability that a communication system will have high fidelity is 0.81, The probability that the system will have both high fidelity and high selectivity is 0.18. The probability that a given system with high fidelity will have high selectivity is

(a) 0.181 (b) 0.191 (c) 0.222 (d) 0.826 **(GATE-17-IN)**

99. 500 students are taking one or more courses out of chemistry, physics and Mathematics. Registration enrolment as follows: chemistry (329), physics(186), Mathematics (295), chemistry and physics(83), chemistry and Mathematics (217), and physics and Mathematics(63), How many students are taking all 3 subjects?

(a) 37 (b) 43 (c) 47 (d) 53 **(GATE-17-EC)**

100. Three fair cubical dicę are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place)_______.

(GATE-17-EC)

101. An urn contains 5 red balls and 5 black balls. In the first draw, one ball is picked at random and discarded without noticing its colour. The probability to get a red ball the second draw is

(GATE-17-EE)

(a) $\frac{1}{2}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{6}{9}$

102. A two-faced fair coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes: H,H,H. If the coin is tossed one more time, the probability (up to one decimal place) of obtaining H again, given the previous realizations of H, H and H, would be________.

(GATE-17-CE)

103. P and Q are considering to apply for a job. The probability that P applies for the job is $\frac{1}{4}$, the probability that P applies for the job given that Q applies for the job is $\frac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\frac{1}{3}$. Then the probability that P does not apply for the job given that Q does not apply for the job is

(a) $\frac{4}{5}$ (b) $\frac{5}{6}$ **(GATE-17-CSIT)**

(b) (c) $\frac{7}{8}$ (d) $\frac{11}{12}$.

104. Probability (up to one decimal place) of consecutively picking three red balls with out replacement from a box containing 5 red balls and 1 white balls is __________

(GATE-18-CE)

105. Two people, P and Q, decide to independently roll two identical dice, each with six faces, numbered 1 to 6. The person with lower number wins. In case of a tie, they roll dice repeatedly until there is no tie. Define trial as a throw of dice by P and Q. assume all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is ___________

(GATE-18-CSIT)

106. A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than the boys?

(GATE-18-EE)

(a) 0

(b) $\frac{325}{864}$

(c) $\frac{525}{864}$

(b) $\frac{4}{11}$

107. Consider a sequence of tossing a fair coin where the outcomes of the tosses are independent. The probability of getting the head for the third time in the fifth toss is

(GATE-18-IN)

(a) $\frac{5}{16}$ (b) $\frac{3}{16}$

(c) $\frac{3}{5}$ (d) $\frac{9}{16}$

108. Two bags A and B have equal number of balls. Bag A have 20% red balls and 80% green balls. Bag B has 30% red balls, 60% green balls and 10% yellow balls. Contents of bag A and B are mixed thoroughly, and a ball is randomly picked from the mixture. What is the chance that the ball picked is red?

(a) 20% (b) 25% **(GATE-18-IN)**

(c) 30% (d) 40%

109. Four red balls, four green balls, four blue balls are put in box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

(GATE-18-ME)

(a) $\frac{1}{72}$ (b) $\frac{1}{55}$

(c) $\frac{1}{36}$ (d) $\frac{1}{27}$

110. A six faced fair dice is rolled five times. The probability (in %) of obtaining "ONE" at least four times is

(a) 33.33 (b) 3.33 **(GATE-18-ME)**

(c) 0.33 (d) 0.0033

111.an unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials

(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT_ _

Which statement describing last two coin tosses of the fourth trial has the highest probability of being correct?

(GATE-18-ME)

(a) Two T will occur

(b) One H and one T will occur

(c) Two H will occur

(d) One H will be followed by one T

112. The probabilities of occurrence of events F and G are $P(F) = 0.3$ and $P(G) = 0.4$, respectively. The probability that both events occur simultaneously is $P(F \cap G) = 0.2$. The probability of occurrence of at least one event $P(F \cup G)$ is__________

(a) 0.5 (b) 0.2

(c) 0.7 (d) 1

113. A box has 8 red balls and 8 green balls. Two balls are drawn randomly in succession from the box without replacement. The probability that first ball drawn is red and second ball drawn is green is

(GATE-19-IN)

(a) 4/15 (b) 7/16

(c) 1/2 (d) 8/15

114. A fair coin is tossed 20 times. The probability that 'head' will appear exactly 4 times in the first ten tosses, and 'tail' will appear exactly 4 times in the next 10 tosses is ______ (round off to 3 decimal places). **(GATE-20-ME)**

115. A bag contains 7 red and 4 white balls. Two balls are drawn at random. What is the probability that both balls are red?

(a) $\frac{28}{55}$ (b) $\frac{21}{55}$

(c) $\frac{7}{55}$ (d) $\frac{4}{55}$

116. A company is hiring to fill four managerial vacancies. The candidates are five men and three women. If every candidate is equally likely to be chosen then the probability that at least one woman will be selected is ______ (round off to 2 decimal places)

(GATE-20-ME)

117. If P, Q, R, S are four individuals How many teams of size exceeding one can be formed, with Q as a member?

(GATE-2020-EE)

(a) 8 (b) 6 (c) 5 (d) 7

118. In a school of 1000 student, 300 student play chess and 600 student play football. If 50 students play both chess and football, the number of students who play neither is__________.

(a) 150 (b) 50 **(GATE-2020(CE))**

(c) 100 (d) 200

119. A fair (unbiased) coin is tossed 15 times. The probability of getting exactly 8 Heads (round off to three decimal places), is________.

(GATE-2020(CE))

120. Consider two identical bags B_1, and B_2, each containing 10 balls of identical shapes and sizes. Bag B_1, contains 7 Red and 3 Green balls, while bag B_2, contains 3 Red and 7 Green balls. A bag is picked at random and a ball is drawn from it, which was found to be Red. The probability that the Red ball came from bag B_1 (rounded off to one decimal place) is____________.

(GATE- 2020(IN))

121. In a company, 35% of the employees drink coffee, 40% of the employees drink tea and 10% of the employees drink both tea and coffee. What % of employees drink neither tea nor coffee?

(GATE-21-CE)

(a) 15 (b) 35

(c) 25 (d) 40

122. Two identical cube shaped dice each with faces numbered 1 to 6 are rolled simultaneously. The probability that an even number is rolled out on each dice is :

(GATE-21-CE)

(a) $\frac{1}{8}$ (b) $\frac{1}{36}$

(c) $\frac{1}{4}$ (d) $\frac{1}{12}$

123. A digital watch X beeps every 30 seconds while watch Y beeps every 32 seconds. They beeped together at 10 AM. The immediate next time that they will beep together is ______. **(GATE-21-ME)**

(a) 11 AM

(a) 10.08 AM

(c) 10.42AM

(d) 10.00PM

124. A bag has r red balls and b black balls . All balls are identical except for their colours . In a trial , a ball is randomly drawn from the bag , its colour is noted and the ball is placed back into the bag along with another ball of the same colour . Note that the number of balls in the bag will increase by one , after the trial . A sequence of four such trials is conducted . Which one of the following choices gives the probability of drawing a red ball in the fourth trial ?

(GATE-2021-cs)

(a) $\frac{r}{r+b}$

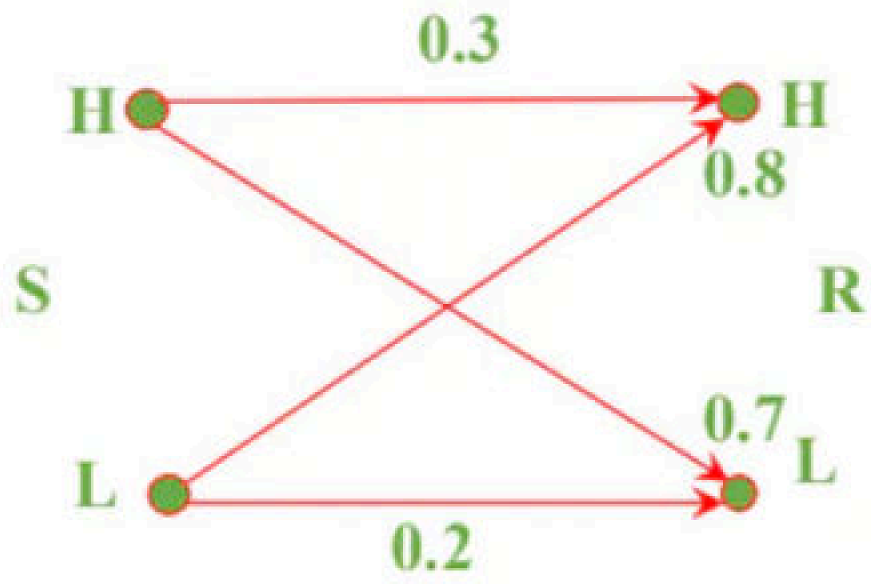
(b) $\frac{r}{r+b+3}$

(c) $\left(\frac{r}{r+b}\right)\left(\frac{r+1}{r+b+1}\right)\left(\frac{r+2}{r+b+2}\right)\left(\frac{r+3}{r+b+3}\right)$

(d) $\frac{r+3}{r+b+3}$

125. A sender (S) transmits a signal , which can be one of the two kinds : H and L with probabilities 0.1 and 0.9 respectively , to a receiver (R) .

In the graph below , the weight of edge (u , v) is the probability of receiving *v* when *u* is transmitted , where *u* , v ∈(H , L) . For example , the probability that the received signal is L given the transmitted signal was H , is 0.7 .



If the received signal is H , the probability that the transmitted signal was H (rounded to 2 decimal places) is________________.

(GATE-2021-cs)

126. There are 6 jobs with distinct difficulty levels , and 3 computers with distinct processing speeds . Each job is assigned to a computer such that ,The fastest computer gets the toughest job and the slowest computer gets the easiest job. Every computer gets at least one job .

The number of ways in which this can be done is_______________

(GATE-2021-cs)

127. A box contains 15 blue balls and 45 black balls , if 2 balls are selected randomly, without replacement, the probability of an outcome in which the first selected is a blue ball and the second selected is a black ball ….

a)45/236
b)1/4
c)3/16
d)3/4

128. A box contains the following three coins ,

I. A fair coin with head on one face and tail on the other face .

II . A coin with heads on both the faces .

III . A coin with tails on both the faces.

A coin is picked randomly from the box and tossed . Out of the two remaining coins in the box , one coin is then picked randomly and tossed . If the first toss results in a head , the probability of getting a head in the second toss is

(GATE – 2021 – EC)

(a) $\frac{1}{2}$ (b) $\frac{2}{5}$

(c) $\frac{1}{3}$ (d) $\frac{2}{3}$

129. There are five bags each containing identical sets of ten distinct chocolates . One chocolate is picked from each bag . The probability that at least two chocolates are identical is____________.

(GATE-2021-cs)

(a) 0.6976 (b) 0.3024

(c) 0.4235 (d) 0.8125

130. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is **(GATE-CS-1998)**

a) 1/6
b) 3/8
c) 1/8
d) 1/2

131. Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is **(GATE-CS-2002)**

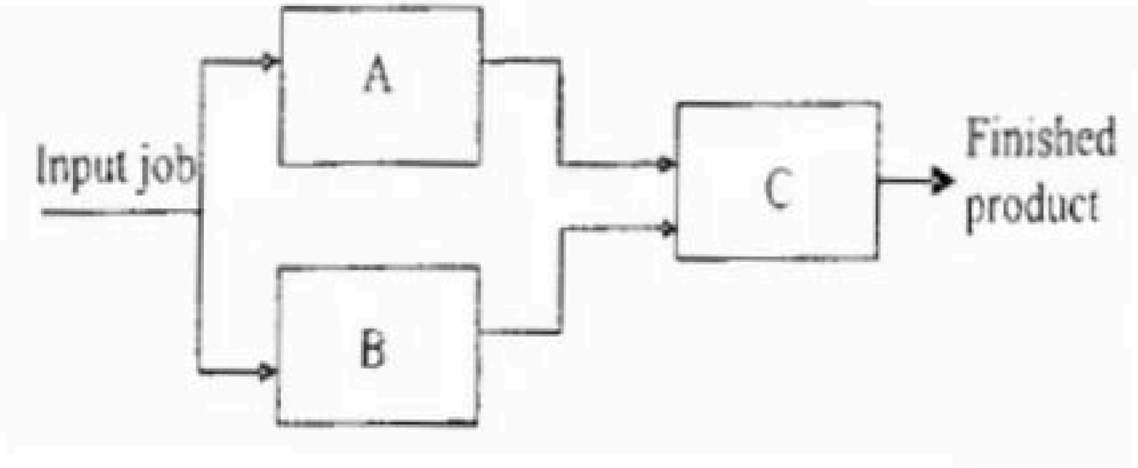
(a) 1/16 (b) 1/8 (c) 7/8 (d) 15/16

132. Assume for simplicity that N people, all born in April (a month of 30 days) are collected in a room, consider the event of at least two people in the room being born on the same date of the month (even if in different years e.g. 1980 and 1985). What is the smallest N so that the probability of this exceeds 0.5 is ?

(a) 20 (b) 7 (c) 15 (d) 16 **GATE- 2009 EE**

133. The figure shown the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as : $P_A = 0.15$, $P_B = 0.05$ & $Pc = 0.1$. Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is

GATE- 2014 IN



134. An examination consists of two papers, paper 1 and paper 2. The probability of failing in paper 1 is 0.3 and that in paper 2 is 0.2. Given that a student has failed in paper 2, the probability of failing in paper 1 is 0.6. The probability of a student failing in both the papers is **(GATE-EC-2007)**

(a) 0.5 (b) 0.18 (c) 0.12 (d) 0.06

135. Consider two independent random variables X and Y with identical distribution's. The variables X and Y take values 0, 1, and 2 with probability 1/2, 1/4 and 1/4 respectively. What is the conditional probability $P(X + Y = 2/X - Y = 0)$? **GATE- 2014 EC**

(a) 0 (b) 1/16 (c) 1/6 (d) 1

136. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that first removed ball is white, the probability that the 2^{nd} removed ball is red is **(GATE-EE-2010)**

(a) 1/3 (b) 3/7 (c) 1/2 (d) 4/7

137. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is

GATE- 2010 ME

(a) 2/315 (b) 1/630 (c) 1/1260 (d) 1/2520

138. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads ?

GATE- 2011 CS

a) 1/3
b) ¼
c) ½
d) 2/3

139. Parcels from sender S to receiver R pass sequentially through two post-office. Each post-office has a probability 1/5 of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post-office is _______ **(GATE-EC- SET-4-2014)**

140. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting positive report is 0.01, the probability that a person tested gets an incorrect report is **(GATE-IN-2009)**

(a) 0.0027 (b) 0.0173 (c) 0.1497 (d) 0.2100

141. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty ? **(GATE-CS-2010)**

(a) $pq + (1 - p)(1 - q)$ (b) $(1 - q)\,p$ (c) $(1 - p)q$ (d) pq

142. An automobile plant contacted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to reliable, is made by Y is **(GATE-CE-2012)**

(a) 0.288 (b) 0.334 (c) 0.667 (d) 0.720

143. The probability that student knows the correct answer to a multiple choice question is 2/3 . If the student does not known the answer, then the student guesses the answer. The probability of the guessed answer being correct is 1/4 . Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is **GATE- 2013 ME**

(a) 2/3 (b) 3/4 (c) 5/6 (d) 8/9

144.. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is

(GATE-ME- SET-3-2014)

145. In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day ?

(GATE-PI- SET-1-2014)

(a) 3/10 (b) 9/11 (c) 14/17 (d) 27/41

146. A committee of 4 is to be formed from among 4 girls and 5 boys. What is the probability that the committee will have number of boys less than number of girls?

(a) 2/9 (b) 4/9

(c) 4/5 (d) 1/6

147. A bag contains 4 white and 2 black balls and another bag contains 3 of each colour. A bag is selected at random and a ball is drawn at random from the bag chosen. The probability of white ball drawn is

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{5}{12}$ (a) $\frac{7}{12}$

Random Variable

Cumulative Distribution Function (CDF)

Properties of CDF

Probability Density Function (PDF)

Properties of PDF

Mean (Average Value) (Expectation) (1st Moment)

Properties of Mean

Variance

148. If the probability density function of a random variable x is given by

$$f(x) = \begin{cases} \frac{kx^2}{0} & -1 \leq x \leq 1 \\ 0, & elesewhere \end{cases}$$

the value of k is_______________.

(GATE- 2020(PI)

149. The constant C such that the function

$$f(x) = Cx^2, \quad 0 < x < 3.$$
$$= 0. \quad \text{Otherwise}$$

is a probability density function.

a)1

b)1/9

c)1/8

d)1/3

150. A random variable x has density function $f(x) = C/(x^2 + 1)$ where $-\infty < x < \infty$. then the value of the constant C

a) $\frac{1}{\pi}$ b) $\frac{2}{\pi}$ c) $\frac{1}{2\pi}$ d) $\frac{-1}{\pi}$

151. $P_x(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relation M and N is **(GATE-IN-2008)**

a) $M+\frac{2}{3}N = 1$ b) $2M+N= 3$

(c) $M + N = 1$ (d) $M + N = 3$

152. A continuous random variable X has a probability density function

$f(x) = e^{-x}$, $0 < x < \infty$. Then $P\{X > 1\}$ is **(GATE-EE-2013)**

(a) 0.368 (b) 0.5 (c) 0.632 (d) 1.0

153. Find the value of λ such that the function f(x) is a valid probability density function

$f(x) = \lambda(x-1)(2-x)$ for $1 < x < 2$ ____ **(GATE-CE-2013)**

a) 5
b) 6
c) 2
d) 1

154. the density function of a random variable X is given by

$$f(x) = x/2 \quad 0 < x < 2$$
$$= 0 \text{ otherwise}$$

Then the mean and variance of X are

a) 4/3, 2/9 b) 2/3, 4/9 c) 4/3, 4/9 d) 2/3, 2/9

155. The expectation of discrete random variable X whose probability function is given by

$f(x) = (1/2)^x, \; x = 1, 2, 3, \ldots\ldots\ldots$ is

a) 1 b) 2 c) 3 d) 4

156. The random variable X takes on the values 1, 2 (or) 3 with probabilities **GATE- 2007(PI)**

$\frac{2+5P}{5}, \frac{1+3P}{5}, and \ \frac{1.5+2P}{5}$ respectively the values of P and E(X) are respectively

(a) 0.05, 1.87 (b) 1.90, 5.87 (c) 0.05, 1.10 (d) 0.25, 1.40

157. If E denotes expectation, the variance of a random variable X is given by **(2007)**

(a) $E(X^2) - E^2(X)$

(b) $E(X^2) + E^2(X)$

(c) $E(X^2)$

(d) $E^2(X)$

158. discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true?

GATE- 2009 (EC)

K	1	2	3	4	5
P(X=K)	0.1	0.2	0.4	0.2	0.1

a) Both the student and the teacher are right
b) Both the student and the teacher are wrong
c) The student is wrong but the teacher is right
d) The student is right but the teacher is wrong

159. If the difference between the expectation of the square of a random variable $[E(X^2)]$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then, **(GATE-CS-2011)**

(a) $R = 0$ (b) $R < 0$ (c) $R \geq 0$ (d) $R > 0$

160. Let X be a real-valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds true is **(GATE-EC-2014)**

(a) $(E[X])^2 > E[X^2]$ (b) $E[X^2] \geq (E[X])^2$ (c) $E[X^2] = (E[X])^2$ d) $E[X^2] > (E[X])^2$

161. Let the probability density function of a random variable X, be given as:

$f_x(x) = \frac{3}{2}e^{-3x}u(x) + \mathrm{a}e^{4x}u(-x)$ where u(x) is the unit step function.

Then the value of 'a' and prob $\{X \leq 0\}$, respectively, are

(GATE-16-EE)

(a) $2, \frac{1}{2}$

(b) $4, \frac{1}{2}$

(c) $2, \frac{1}{4}$

(d) $4, \frac{1}{4}$

162. A probability density function this interval [a,1] is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is_____________.

(GATE-16-CSE-SETI)

163. If f(x) and g(x) are two probability density functions,

$$f(x) = \begin{cases} \dfrac{X}{a} + 1 & : \ -a \le x < 0 \\ -\dfrac{X}{a} + 1 & : \ 0 \le x \le a \\ 0 & : \ otherwise \end{cases} \qquad g(x) = \begin{cases} -\dfrac{X}{a} & : -a \le x < 0 \\ \dfrac{X}{a} & : 0 \le x \le a \\ 0 & : \text{otherwise} \end{cases}$$

Which of the following statements is true?

(GATE-16-CE-SET2)

(a) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are same.

(b) Mean of f(x) and g(x) are same; Variance of f(x) and g(x) are different.

(c) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are same.

(d) Mean of f(x) and g(x) are different; Variance of f(x) and g(x) are different.

Probability Distributions

Binomial ,

Poisson,

Uniform ,

Exponential ,

Gaussian (Normal)

Binomial Distribution

Poisson’s Distribution

Uniform Distribution

Exponential Distribution

164. Consider a Poisson distribution for the tossing of a biased coin. The mean for this distribution is μ. The standard deviation for this distribution is given by

(GATE-16-ME-SET1)

(a) $\sqrt{\mu}$ (b) μ^2 (c) μ (d) $1/\mu$

165. Mr.Asmit try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by Mr. Asmit, then the average number of attempts that passengers need to make to get a seat reserved is

(GATE-17-EC)

166. Assume that in a traffic junction, the cycle of the traffic signal lights is 2 minutes of green (vehicle does not stop) and 3 minutes of red (vehicle stops). Consider that the arrival time of Ms. Jyoti with BMW at the junction is uniformly distributed over 5minute cycle. The expected waiting time (in minutes) of the Ms. Jyoti at the junction is

(GATE-17-EE)

167. Mr. Adwaith rolled six-face fair dice a large number of times. The mean value of the outcomes is

(GATE-17-ME)

168. For the function $f(x) = a + bx$, $0 \leq x \leq 1$, to be a valid probability density function, which one of the following statements is correct?

(GATE-17-CE)

(a) $a = 1, b = 4$

(b) $a = 0.5, b = 1$

(c) $a = 0, b = 1$

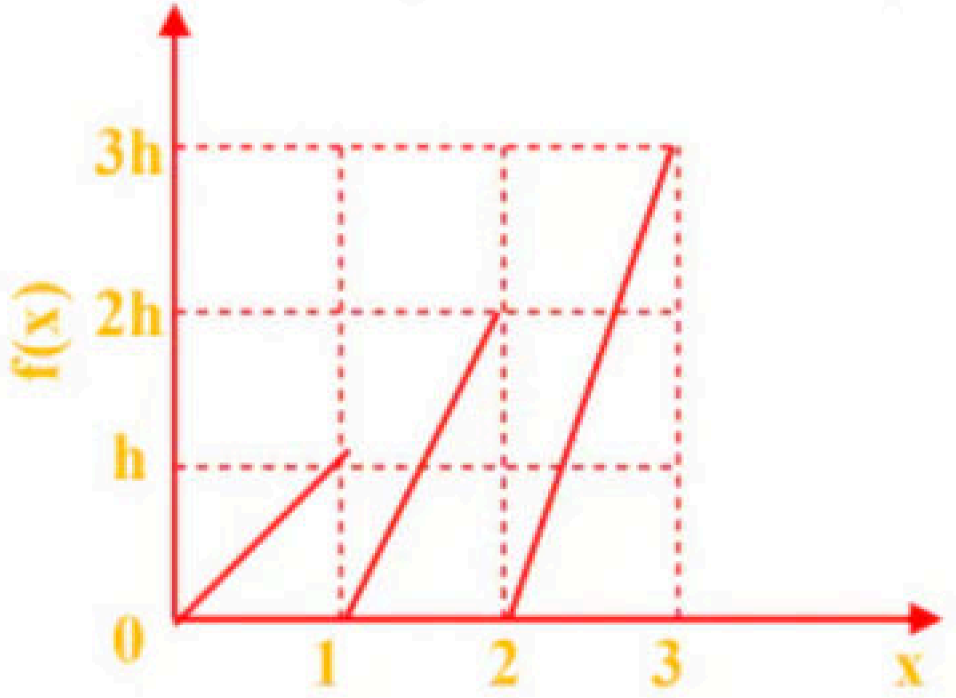
(d) $a = 1, b = -1$

169. If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X+2)^2]$ equals__________.

(GATE-17-CSIT)

170. The graph of a function f(x) is shown in the figure. For f(x) to be valid probability density function, the value of h is

(GATE-18-CE)



(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 1

(d) 3

171. X and Y are two independent random variables with variances one and two respectively. Let Z =X − Y. The variance of Z is

(GATE-18-IN)

(a) 0 (b) 1

(c) 2 (d) 3

172. The time to pass through a security screening at an airport follows an exponential distribution. The mean time to pass through the security screening is 15 minutes. To catch the flight, Ms. Anmol must clear the security screening within 15 minutes. The probability that the Ms. Anmol will miss the flight is _____.[round off to 3 decimal places]

(GATE-21-PI)

173. For a given biased coin, the probability that the outcome of a toss is a head is 0.4. This coin is tossed 1,000 times. Let X denote the random variable whose value is the number of times that head appeared in these 1,000 tosses. The standard deviation of X (rounded to 2 decimal places) is____________________.

(GATE-2021-cs)

174. In an examination, Mr. Abhi can choose the order in which two questions (QuesA and QuesB) must be attempted.

- If the first question is answered wrong , Mr. Abhi gets zero marks .
- If the first questions is answered correctly and the second question is not answered correctly , the Mr. Abhi gets the marks only for the first question .
- If both the questions are answered correctly , the Mr. Abhi gets the sum of the marks of the two questions .

The following table shows the probability of correctly answering a question and the marks of the question respectively.

Question	Probability of answering correctly	Marks
Question A	0.8	10
Question B	0.5	20

Assuming that the Mr. Abhi always wants to maximize his expected marks in the examination , in which order should he attempt the questions and what is the expected marks for that order (assume that the questions are independent) ?

(GATE-2021-CS)

(a) First QuesA and then QuesB . Expected marks 14 .

(b) First QuesB and then QuesA . Expected marks 22 .

(c) First QuesB and then QuesA . Expected marks 14 .

(d) First QuesA and then QuesB . Expected marks 16 .

175. Two continuous random variables X and Y are related as

$$Y = 2X + 3$$

Let σ_X^2 and σ_Y^2 of denote the variance of X and Y , respectively . The variances are related as

(GATE – 2021 – EC)

(a) $\sigma_Y^2 = 25\sigma^2 X$

(b) $\sigma_Y^2 = 5\sigma_X^2$

(c) $\sigma_Y^2 = 2\sigma_X^2$

(d) $\sigma_Y^2 = 4\sigma_X^2$

176. Consider that X and Y are independent continuous valued random variables with uniform PDF given by $X \sim U(2, 3)$ and $Y \sim U(1, 4)$. Then $P(Y \leq X)$ is equal to________ (rounded off to two decimal places).

(GATE – 2021 – IN)

177. Two independent random variables X and Y are uniformly distributed in the interval [-1, 1]. The probability that max[X, Y] is less than 1/2 is

GATE- 12 (EC)

(a) 3/4 (b) 9/16 (c) 1/4 (d) 2/3

178. let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, E[X], is _**(GATE-EC- SET-2-2014)**

179. in the following table, x is a discrete random variable and p(x) is the probability density. The standard deviation of x is

x	1	2	3
p(x)	0.3	0.6	0.1

(a) 0.18 (b) 0.36 (c) 0.54 (d) 0.6

180. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of $\frac{1}{6}, \frac{2}{3}$ *and* $\frac{1}{6}$ and, respectively. Then mean value and the variance of the number of defective pieces produced by **(GATE-ME- SET-3-2014)**

(a) 1 and 1/3 (b) 1/3 and 1 (c) 1 and 4/3 (d) 1/3 and 4/3

181. Each of the seven words in the sentence “Anupam and Vishwajith are good enemies” is written on a separate piece of paper. These seven pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is ________. (The answer should be rounded to one decimal place.)

(GATE-CS- SET-2-2014)

182. A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denote the number of times the die is thrown. The expected value of X is ___

(GATE-EC-2015)

183. Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is _______

(GATE-EC-2015)

165. Mr.Asmit try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by Mr. Asmit, then the average number of attempts that passengers need to make to get a seat reserved is

(GATE-17-EC)

184. the variance of the random variable X with probability density function

$f(x) = \frac{1}{2}|x|e^{-|x|}$ is __________ **(GATE-EC-2015)**

185. continuous random variable X has probability density given by $f(x) = \begin{cases} 2\,e^{-2x}, & x > 0 \\ 0\ , & x \leq 0 \end{cases}$

Then expectation of X

a) 0.1 b) 0.25 c) 0.5 d) 0.75

186. On a rainy day an umbrella salesman can earn Rs. 300, and on a fair day (no rain) he loses Rs. 60. What is his expected income per day, if the probability for a rainy day is 0.3

a) Rs 24 b) Rs 36 c) Rs 48 d) Rs 64

187. Let X be a random variable with probability density function

$$f(x) = 0.2 \quad |x| < 1$$
$$= 0.1 \quad 1 < |x| < 4$$
$$= 0 \quad \text{otherwise}$$

The probability P(0.5 < x < 5) is ______ **(GATE-EE- SET-2-2014)**

188. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is ______**(GATE-EE- SET-3-2014)**

189. Given that x is random variable in the range [0, ∞] with a probability density function $f(x) = \frac{e}{k}^{-0.5x}$, the value of the constant K is______ **(GATE-IN- SET-1-2014)**

190. The probability density function of evaporation E on any day during a year in a watershed is given by f(E) = $\frac{1}{5}$ $0 < E < 5mm/day$

The probability that E lies in between 2 and 4 mm/day in the watershed is (in decimal)

(GATE-CE- SET-1-2014)

191. The probability density function of a random variable X is $p_x(x) = e^{-x}$ $x \geq 0$ and 0 otherwise. The expected value of the function $g_x(x) = e^{3x/4}$ is _____

192. A random variable X has the following probability function

x:	0	1	2	3	4	5	6
P (x):	k	3k	5k	7k	9k	11k	13k

Then $P(3 < x \leq 6) =$

a) 11/109 b) 22/49 c) 33/49 d) 44/49

193. X is uniformly distributed random variable that take values between 0 and 1. The value of $E(X^2)$ will be **(GATE-EE-2008)**

(a) 0 (b) 1/8 (c) 1/4 (d) 1/2

194. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be

(GATE-IN-2008)

(a) 16/3 (b) 6 (c) 256/9 (d) 36

195. The standard deviation of a uniformly distributed random variable b/w 0 and 1 is

(GATE-ME-2009)

196. The life of a bulb (in hours) is a random variable with an exponential distribution

$$f(t) = \alpha e^{-\alpha t} \quad 0 \leq t \leq \infty$$

The probability that its value lies b/w 100 and 200 hours is **(GATE-PI-2005)**

197. assume that the duration in minutes of a telephone conversion follows the exponential distribution $f(x) = \frac{1}{5} e^{\frac{-x}{5}} \qquad x > 0$. The probability that the conversation will exceed five minutes is **(GATE-IN-2007)**

198. If the probability of a defective bolt is 0.1, then the mean and standard deviation for the Number of defective bolts in a total of 400 bolts are------ and -------

a) 40, 6　　b) 36, 9　　c) 36, 6　　d) 40, 9

199.the probability that an individual suffers a bad reaction from injection of a serum is 0.001. Determine the probability that out of 2000 individuals, exactly 3 individuals suffer a bad reaction.

a) 0.12 b) 0.08 c) 0.18 d) 0.003

200. If X follows poisson distribution such that $P(X = 1) = P(X = 2)$ then $P(X = 0) =$

a) e^{-1} b) e^{-2} c) e^{-3} d) e^{-4}

201. In a certain factory of turning razor blades, there is a small chance (1/500) for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing at least one defective blade in a consignment of 10,000 packets.

a) 9802 b) 198 c) 2 d) 196

$$\frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$p = \frac{1}{500}$$

$$np = \lambda = \frac{10}{500} = \frac{1}{50}$$

$$P(\text{at least one}) = 1 - P(\text{none})$$

$$= 1 - \frac{e^{-\frac{1}{50}}\left(\frac{1}{50}\right)^{0}}{0!} = 0.0198$$

$$= 10000 \times 0.0198$$

$$= \underline{\underline{198}}$$

202. a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

(GATE-EE-2000)

(a) 90 and 9 (b) 9 and 90 (c) 81 and 9 (d) 9 and 81

$p = 0.1$

$q = 0.9$

mean $= np = 90$

$SD = \sqrt{npq} = 9.$

203. Suppose p is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and p has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval ?
(GATE-CS-2013)

(a) $8/(2e^3)$ (b) $9/(2e^3)$ (c) $17/(2e^3)$ (d) $26/(2e^3)$

$\lambda = 3$

$$P(x < 3) = P(x=0) + P(x=1) + P(x=2)$$

$$= e^{-3}\left[1 + \frac{3^1}{1} + \frac{3^2}{2}\right] = \frac{17}{2}e^{-3}.$$

204. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is **(ME- SET-4-2014)**

(a) 0.029 (b) 0.034 (c) 0.039 (d) 0.044

$\lambda = 5.2$

$p(x < 2) = p(x=0) + p(x=1)$

$= e^{-5.2}[1 + 5.2] = 6.2\,e^{-5.2}$

205. A traffic office imposes on an average 5 number of penalties daily on traffic violators. Assume that the number of penalties on different days in independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ________

(GATE-CE- SET-2-2014)

$$\lambda = 5$$

$$P(x < 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= e^{-5}\left[1 + 5 + \frac{5^2}{2} + \frac{5^3}{6}\right]$$

$$= \underline{\underline{0.265}}$$

206. An observer counts 240veh/hr at a specific highway location. Assume that the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is ____ **(CE- SET-2-2014)**

$$\frac{\overset{4}{\cancel{240}}}{60 \times \cancel{60}} \text{ veh/sec.}$$

$$\frac{2}{30} \times 30 = 2$$

$$\lambda = 2.$$

$$P(x=1) = \frac{e^{-2}\, 2^{1}}{1!}$$

$$= 0.270$$

207. suppose that the expectation of a random variable X is 5. Which of the following statements is true?

a) There is a sample point at which X has the value 5

b) There is a sample point at which X has the value > 5

c) There is a sample point at which X has a value ≥ 5

d) none of the above

x	x_1	x_2	x_3	x_4
$P(x)$	P_1	P_2	P_3	P_4

$$E[x] = \sum x \, P(x).$$

$$= x_1 P_1 + x_2 P_2 + x_3 P_3 + x_4 P_4$$

208. Let X and Y be two independent random variables. Which one of the relations b/w expectation (E), variance (V_{ar}) and covariance (C_{ov}) given below is FALSE?

(a) $E(XY) = E(X)\,E(Y)$ (b) $\mathrm{cov}(X, Y) = 0$

(c) $\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$ (d) $E(X^2 Y^2) = (E(X))^2 (E(Y))^2$

$\mathrm{var}(x)$.

$E[xy] = E[x]\,E[y]$

$\mathrm{cov}(x, y) = E[xy] - E[x]\,E[y] = 0$

$\mathrm{var}(x+y) = \mathrm{var}(x) + \mathrm{var}(y)$.

209. A player tosses a fair die. If a prime number occurs he wins that number of rupees, but if a non prime number occurs he loses that number of rupees. His expectation in rupees for each tossing is

a) 1/6 b) 1/2 c) –1/2 d) –1/6

x	-1	2	3	-4	5	-6.
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E[x] = \frac{-1}{6} + \frac{2}{6} + \frac{3}{6} - \frac{4}{6} + \frac{5}{6} - \frac{6}{6} = \frac{-1}{6}.$$

210. If a random variable X satisfies the Poission's distribution with a mean value of 2, then the probability that $X \geq 2$ is **(GATE-PI-2010)**

(a) $2e^{-2}$ (b) $1-2e^{-2}$ (c) $3e^{-2}$ (d) $1-3e^{-2}$

$\lambda = 2.$

$$P(x \geq 2) = 1 - P(x < 2).$$

$$= 1 - e^{-2}\left[1 + 2\right]$$

$$= 1 - e^{-2}\left[3\right].$$

Normal Distribution

pdf

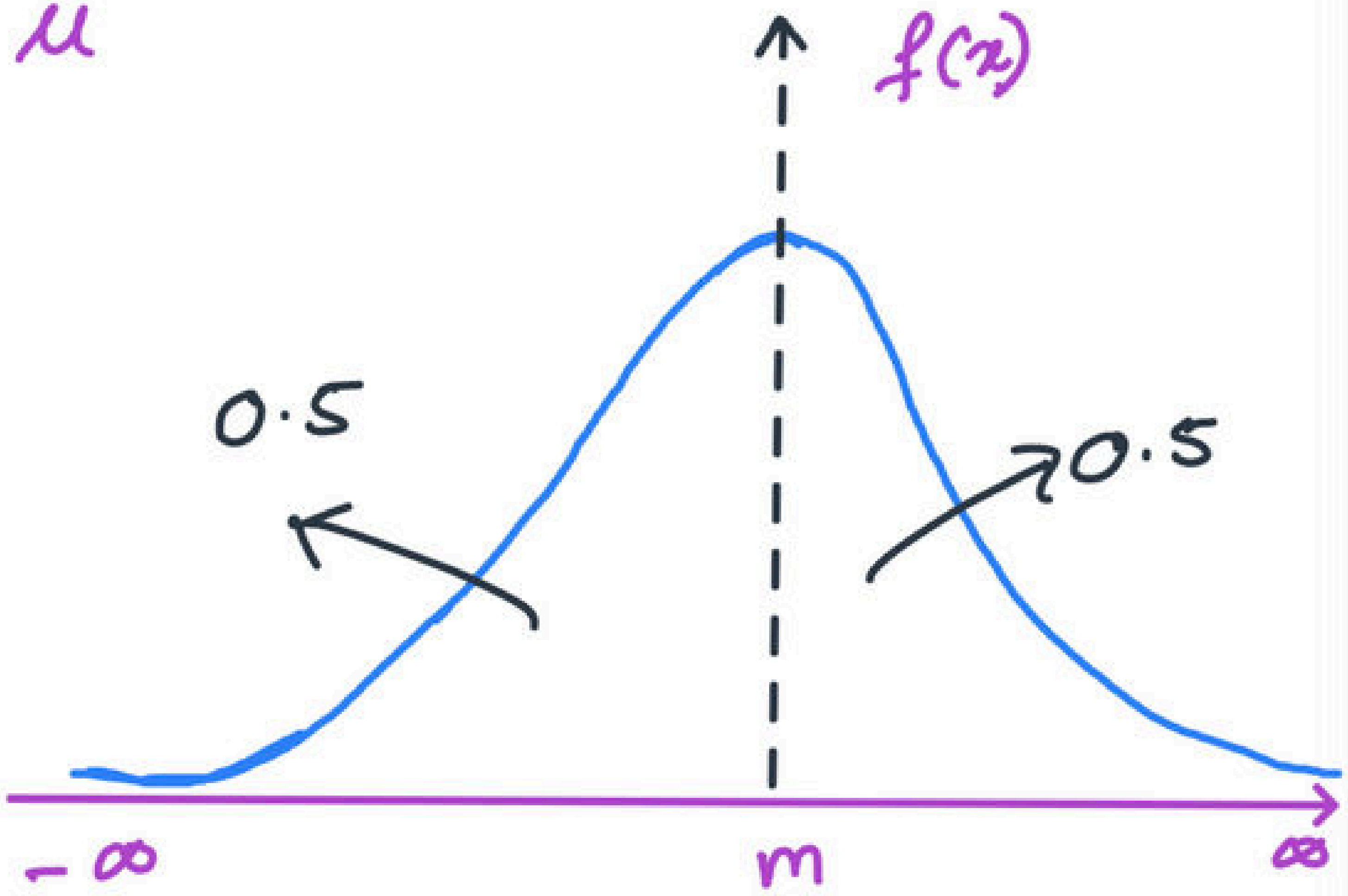
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}} \qquad -\infty < x < \infty$$

$m \rightarrow$ mean $= \mu$

$\sigma \rightarrow$ S.D

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{m} f(x)\,dx = 0.5 = \int_{m}^{\infty} f(x)\,dx.$$



$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}.$$

$$z = \frac{x-m}{\sigma}$$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \longrightarrow$$ Standard normal density function.

$$\boxed{\begin{aligned} \text{mean} &= 0 \\ \text{S.D} &= 1 \end{aligned}}$$

Normal Density function

1.The curve is smooth, regular, bell shaped and symmetrical about mean

2. The area under the normal density function is unity .

3.The maximum value of density function occurs at $x = \mu$

Its maximum value of density function is $= \frac{1}{\sigma\sqrt{2\pi}}$

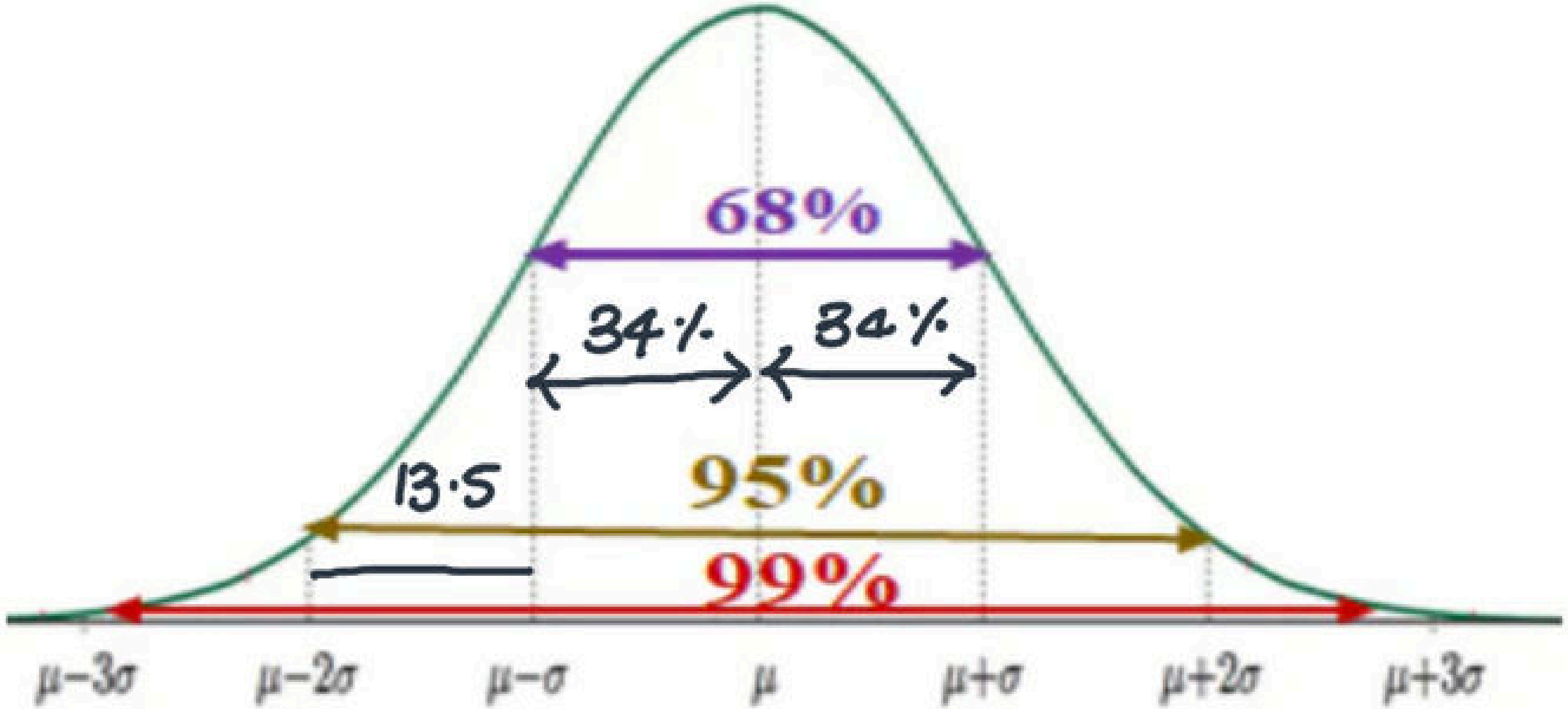
4. If σ – increases then normal density function decreases and curve tends to be flat

5. If σ – decreases then normal density function increases and curve tends to be more peaked at mean .

6. $P(\mu - \sigma < x < \mu + \sigma) = 0.6826$

$P(\mu - 2\sigma < x < \mu + 2\sigma) = 0.9544$

$P(\mu - 3\sigma < x < \mu + 3\sigma) = 0.9973$

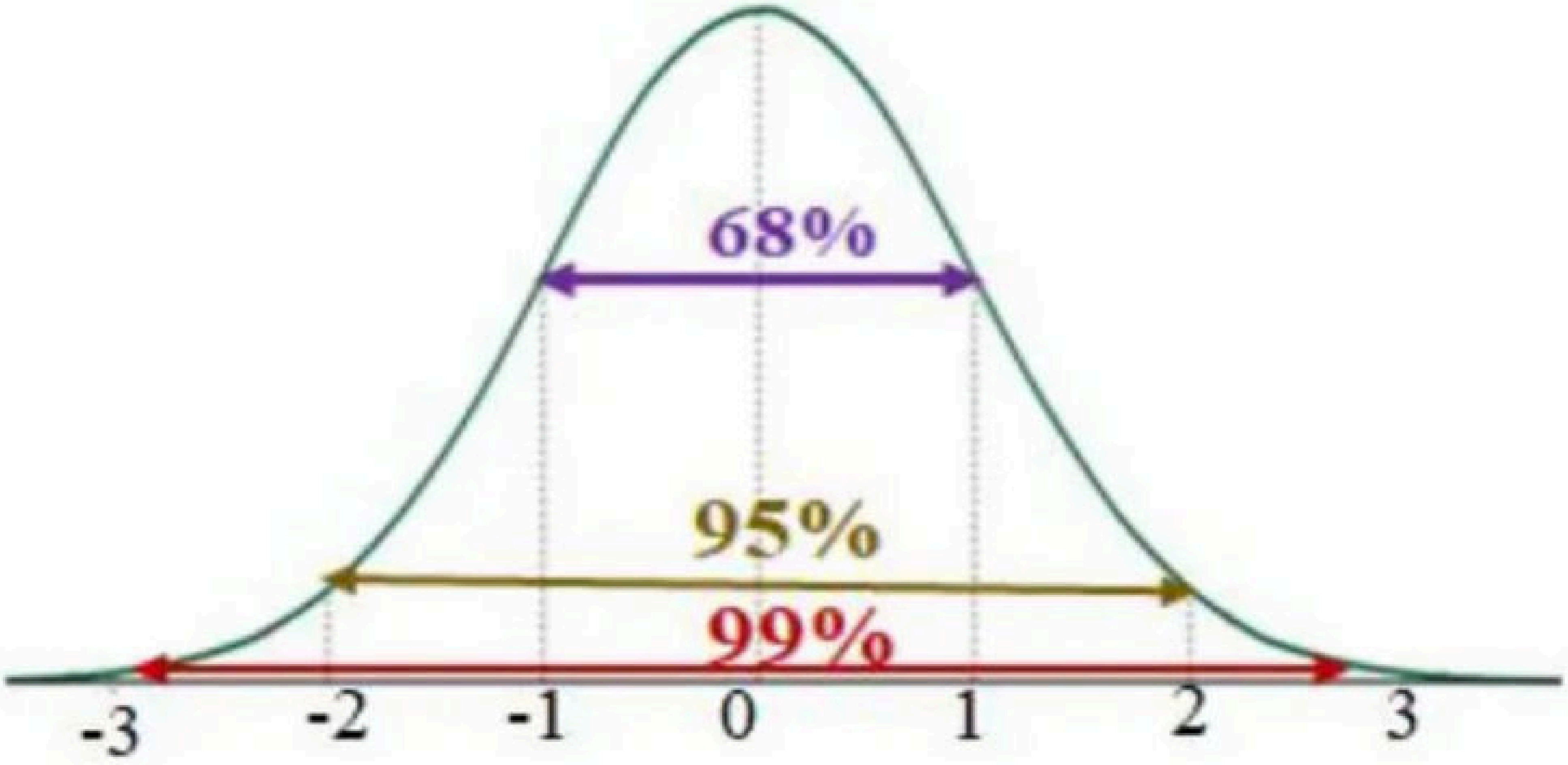


7. Standard Normal density function

$P(-1 \leq Z \leq 1) = 0.6826$

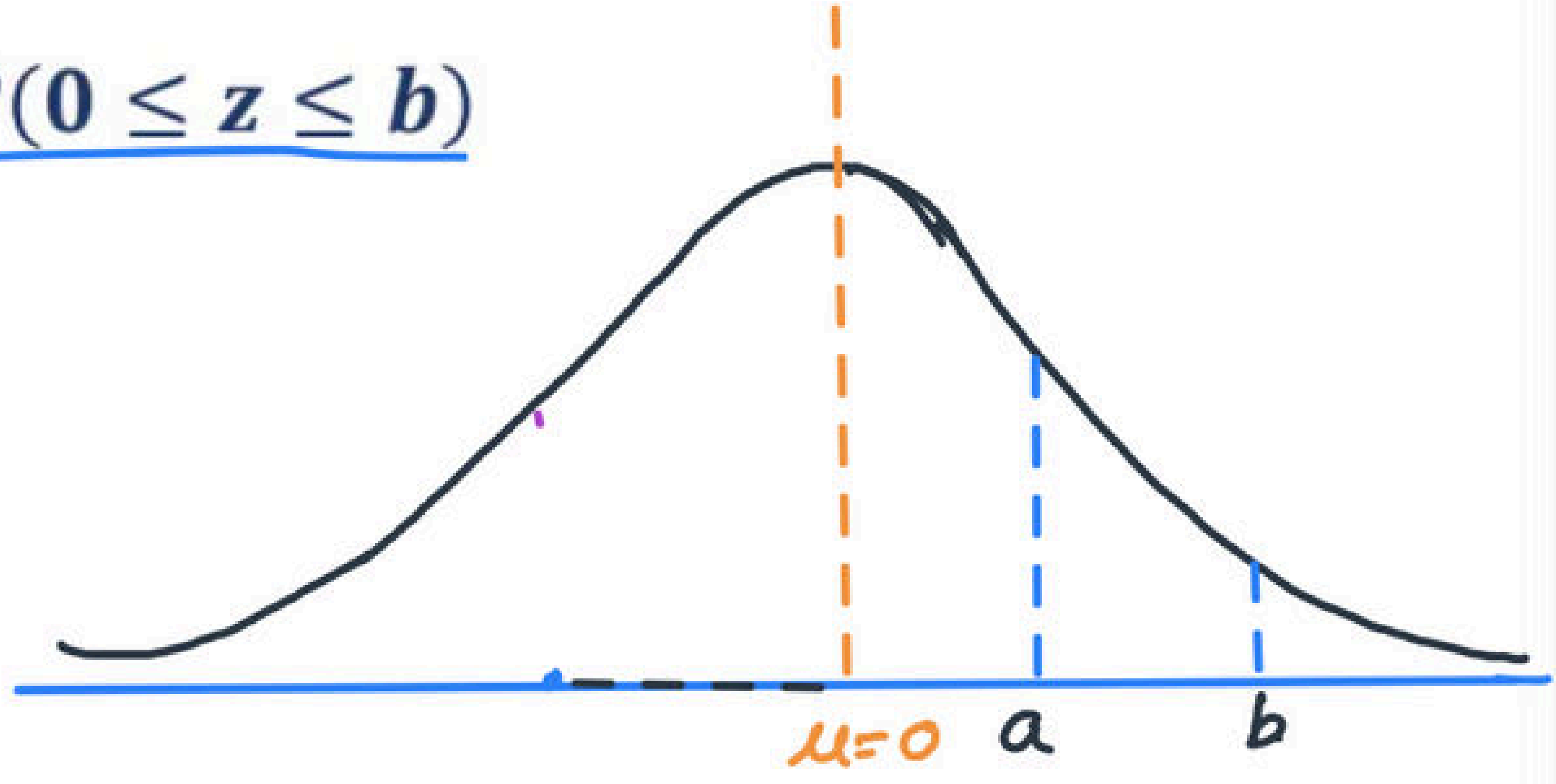
$P(-2 \leq Z \leq 2) = 0.9544$

$P(-3 \leq Z \leq 3) = 0.9973$



8. $P(-a \le Z \le a) = 2P(0 \le z \le a)$

9. $P(-a \le Z \le b) = P(0 \le z \le a) + P(0 \le z \le b)$



10. $P(Z \ge a) = 0 \cdot 5 - P(0 \le z \le a)$

11. $P(Z \le -a) = P(z \ge a)$

12. $P(Z \ge -a) = P(z \le a)$

13. $P(Z \le a) = 0 \cdot 5 + P(0 \le z \le a)$

$P(0 < z < a) + 0.5$

14. If $a < b$

$P(a \le Z \le b) = p(0 \le z \le b) - P(0 \le z \le a)$.

211. Area under normal curve between Z = 0and Z = 1.2 is 0. 3849. Which of the following statements is false.

a) P(Z >1.2) = 0.1151

b) P(Z <1.2) = 0.8849

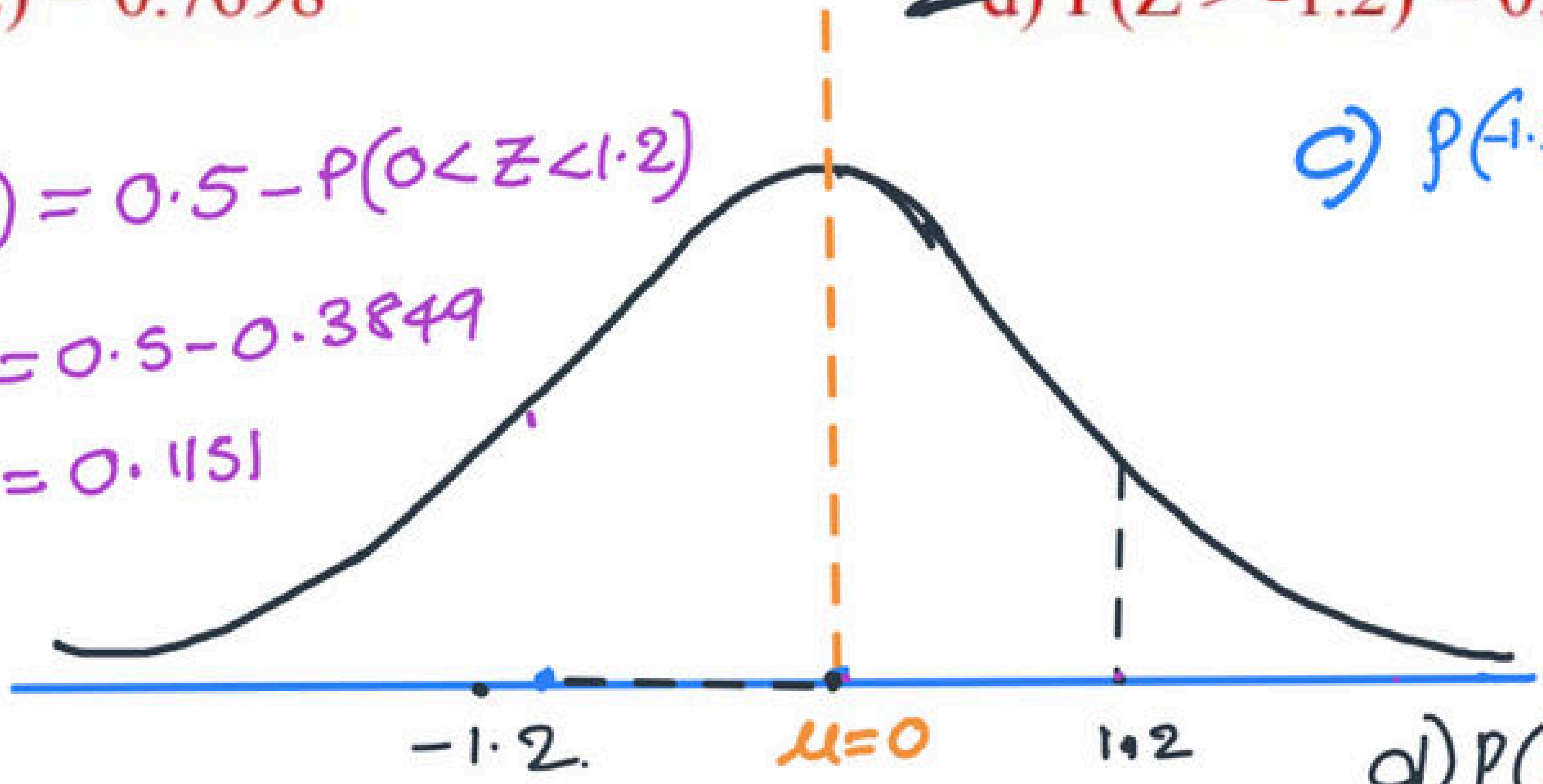
c) P(-1.2 <Z <1.2) = 0.7698

~~d) P(Z > -1.2) = 0.1151~~

a) $P(Z>1.2) = 0.5 - P(0<Z<1.2)$

$= 0.5 - 0.3849$

$= 0.1151$



c) $P(-1.2 < Z < 1.2) = 2P(0<Z<1.2)$

$= 2(0.3849)$

$= 0.7698$

d) $P(Z>-1.2) = 0.5 + 0.3849$

$= \underline{\underline{0.8849}}$

b) $P(Z<1.2) = P(-\infty < Z < 1.2)$

$= P(-\infty < Z < 0) + P(0<Z<1.2)$

$= 0.5 + 0.3849 = 0.8849$

212. Suppose that the temperature during june is normally distributed with mean 20^0 c and standard deviation 3.33^0. Find the probability P that the temperature is between 21.11^0c and 26^0c (Area under the normal curve between Z = 0 and Z = 1.8 is 0.4772 and between Z = 0 and Z = 0.33 is 0. 1293)

a) 0.3479 b) 0.6065 c) 0.8479 d) 0.1065

$$Z = \frac{x - m}{\sigma}$$

$x = 21.11$

$$Z = \frac{21.11 - 20}{3.33} = 0.33$$

$x = 26$

$$Z = \frac{26 - 20}{3.33} = 1.8$$

$$P(21.11 < x < 26) = P(0.33 < Z < 1.8)$$

$$= P(0 < Z < 1.8) - P(0 < Z < 0.33)$$

$$= 0.4772 - 0.1293$$

$$= 0.3479$$

213. Suppose the waist measurements of 500 boys are normally distributed with mean 66cm and standard deviation 5cm. Find the number of boys with waists ≤ 70cm

(Area under the normal curve between z = 0 and Z = 0.8 is 0.2881)

~~a) 394~~ b) 288 c) 788 d) 112

$$Z = \frac{x - m}{\sigma}$$

$x = 70$

$$Z = \frac{70 - 66}{5} = 0.8$$

$$P(x \le 70) = P(Z \le 0.8)$$

$$= 0.5 + 0.2881$$

$$= 0.7881.$$

$$N = (0.7881)(500)$$

$$= 394$$

214. A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it replaced with along another orange ball. First ball is drawn. What is the probability of getting an orange ball in the next draw?

(GATE-2022-CSE)

(a) $\cancel{\frac{1}{2}}$ (b) $\frac{8}{25}$

(c) $\frac{19}{50}$ (d) $\frac{23}{50}$

3-G

2-O

$$P = \frac{3}{5}\,\frac{2}{4} + \frac{2}{5}\,\frac{3}{6}.$$

$$P = \frac{1}{2}.$$

215. The mean inside diameter of a sample of 200 washers produced by a machine is 12mm and the standard deviation is 0.02mm. The purpose for which these washers are intended allows a maximum tolerance in the diameter of 11.97 to 12.03mm. Other wise the washers are considered to be defective. Determine the percentage of non defective washers produced by the machine, assuming the diameters are normally distributed.(Area under the normal curve between Z = 0 and Z = 1.5 is 0.4332)

a) 43.32% b) 86.64% c) 93.32% d) 54.68%

$n = 200$

$m = 12$

$\sigma = 0.02$

$x = 11.97$

$$Z = \frac{x - m}{\sigma} = \frac{11.97 - 12}{0.02} = -1.5$$

$x = 12.03$

$$Z = \frac{12.03 - 12}{0.02} = 1.5$$

$$P(-1.5 < Z < 1.5) = 2P(0 < Z < 1.5)$$
$$= 2(0.4332)$$
$$= 0.8664$$

216. Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two independent binary random variables. If $P(X = 0) = p$ and $P(Y = 0) = q$, then $P(X + Y \geq 1)$ is equal to **(GATE-EC-2015)**

(a) $pq + (1 - p)(1 - q)$ (b) pq

(c) $p(1 - q)$ (d) $1 - pq$

$$P(x + y \geq 1) = 1 - P(x + y < 1)$$

$$= 1 - P(x = 0, y = 0)$$

$$= 1 - P(x = 0) \cdot P(y = 0)$$

$$= \underline{\underline{1 - pq}}$$

217. A normal random variable X has the following probability density function

$$f_x(x) =. \frac{1}{\sqrt{8\pi}} e^{-\left\{\frac{(X-1)^2}{8}\right\}}, -\infty < x < \infty$$

Then $\int_1^{\infty} f_x(x)dx =$

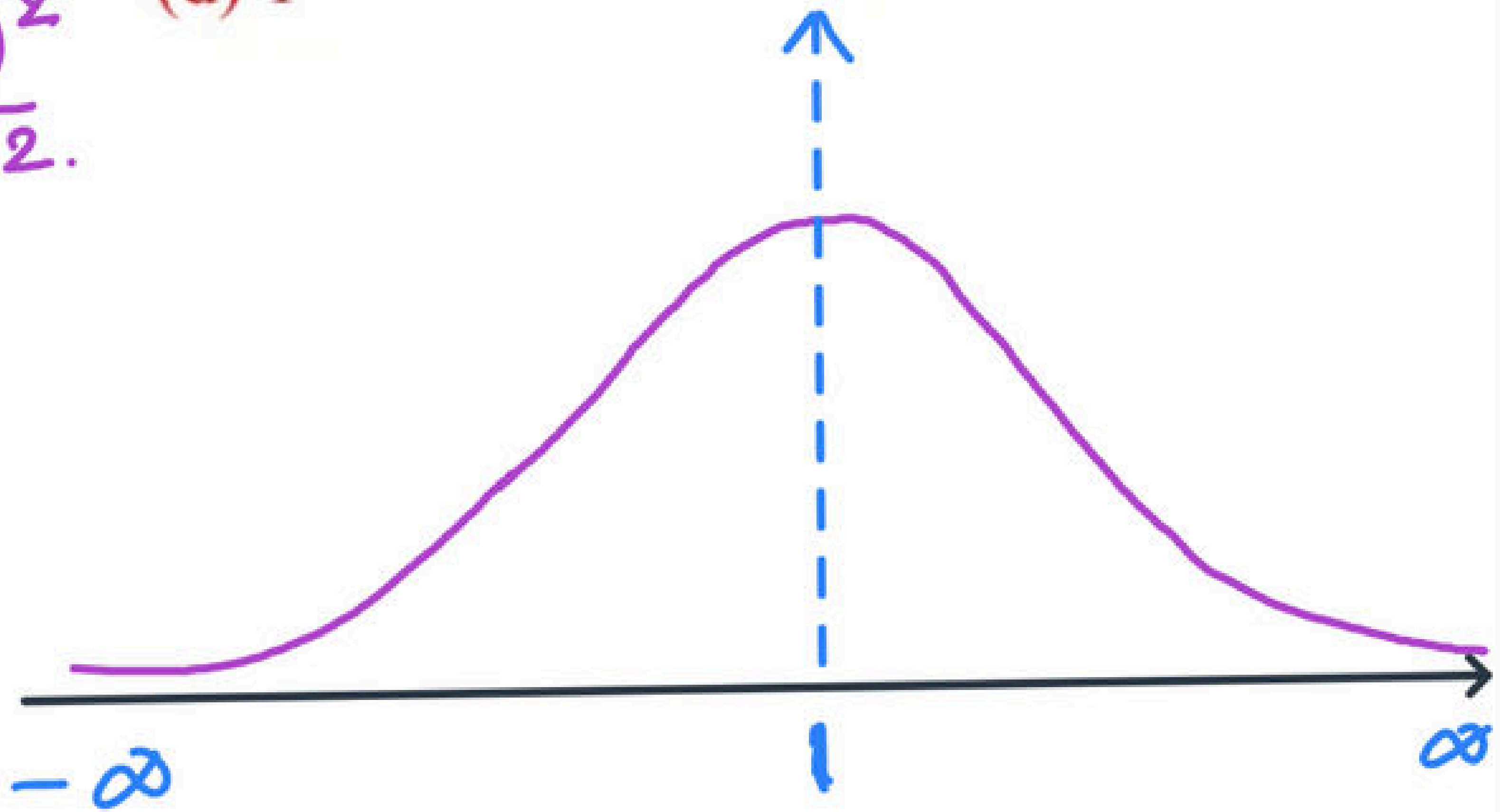
(GATE-16-PI-SET1)

(a) 0 ~~(b)~~ $\cancel{\frac{1}{2}}$ (c) $1 - \frac{1}{e}$ (d) 1

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

$m = 1$

$\sigma = 2.$



218. Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will

(a) 0 (b) 0.5 (c) 1 (d) 10σ **(IN-2008)**

$$F(x) = P(X \leq x)$$

$$F(0) = P(X \leq 0)$$

$$= P(-\infty < x \leq 0)$$

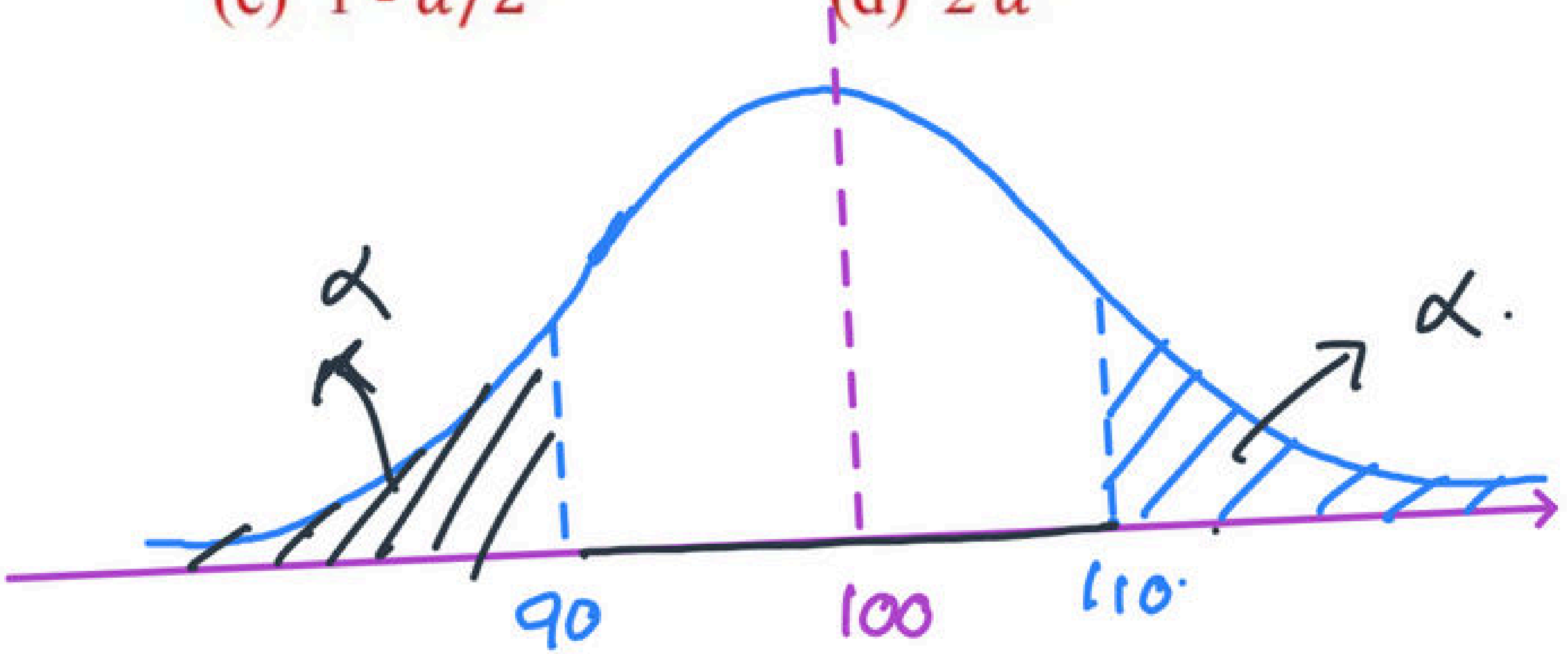
$$= 0.5$$

219. For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$ if the probability is $P = \alpha$ for $x \geq 110$. Then the probability of x laying b/w 90 and 110 i.e., $P(90 \leq x \leq 110)$ and equal to **(GATE-PI-2008)**

(a) $1 - 2\alpha$ (b) $1 - \alpha$ (c) $1 - \alpha/2$ (d) 2α

$$P(90 \leq x \leq 110)$$

$$= 1 - 2\alpha.$$



220. The standard normal cumulative probability distribution function can be approximated as $F(X_N) = \frac{1}{1+\exp(-1.7255 X_N |X_N|^{0.12})}$ where X_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be b/w 90 cm and 102 cm is **(GATE-CE-2009)**

(a) 66.7% (b) 50.0% (c) 33.3% (d) 16.7%

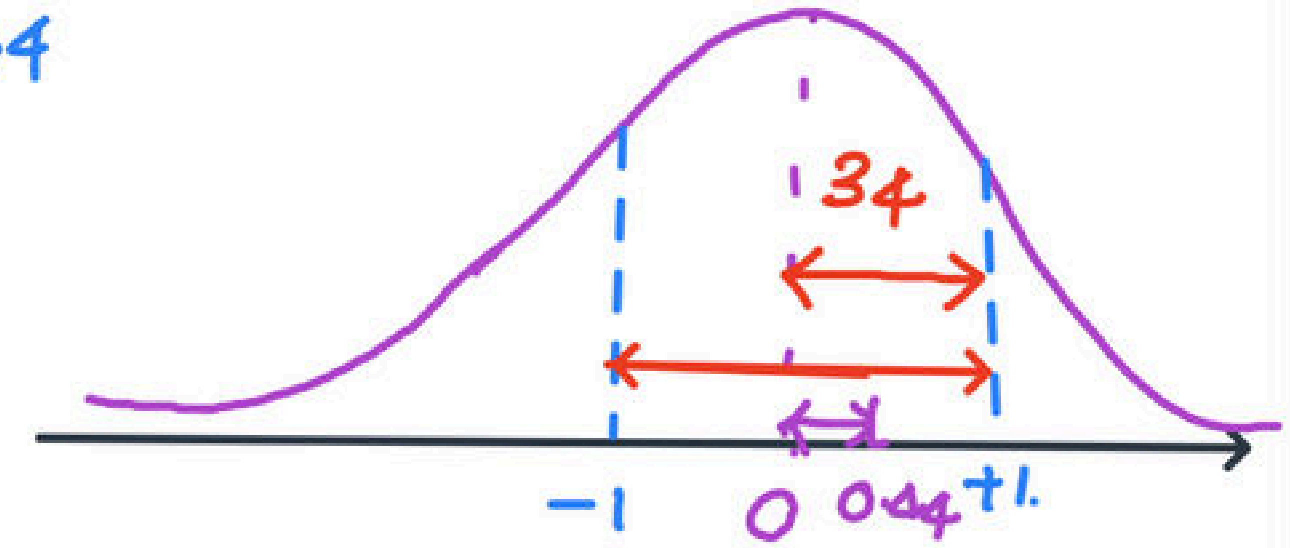
$P(-0.44 < Z < 0) = P(0 \leq Z \leq 0.44)$

$m = 102$

$\sigma = 27$

$Z = \frac{x-m}{\sigma} = \frac{90-102}{27} = -0.44$

$= 0$

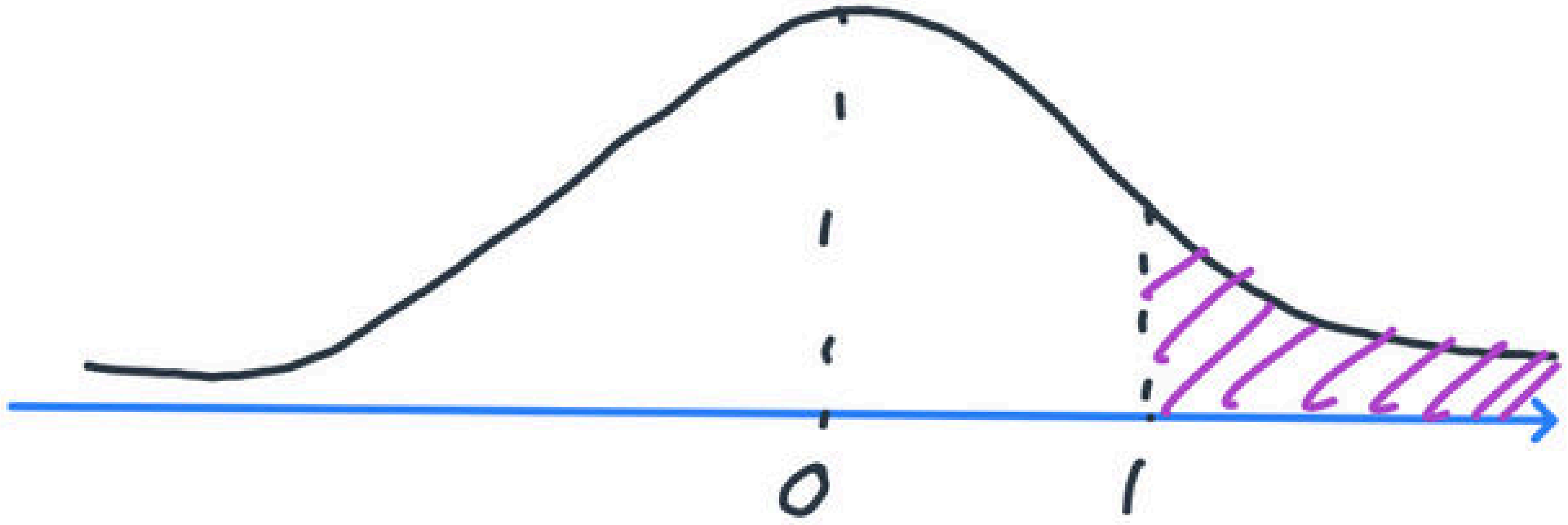


221. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000mm and 200mm, respectively. The probability that the annual precipitation will be more than 1200mm is **(GATE-CE-2012)**

~~(a) <50%~~ (b) 50% (c) 75% (d) 100%

$$Z = \frac{1200 - 1000}{200} = 1.$$

$$P(Z \geq 1) =$$



222. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geq 2U)$ is **(EC-2013)**

(a) 4/9 (b) 1/2 (c) 2/3 (d) 5/9

$Z = 3V - 2U$

$E[Z] = 3E[V] - 2E[U] = 0$

$\text{Var}(Z) = 3^2 \text{var}(V) + 2^2 \text{var}(U)$

$\text{Var}(Z) = 9\frac{1}{9} + 4\frac{1}{4} = 2.$

$Z.$

$P(Z \geq 0) = 0.5$

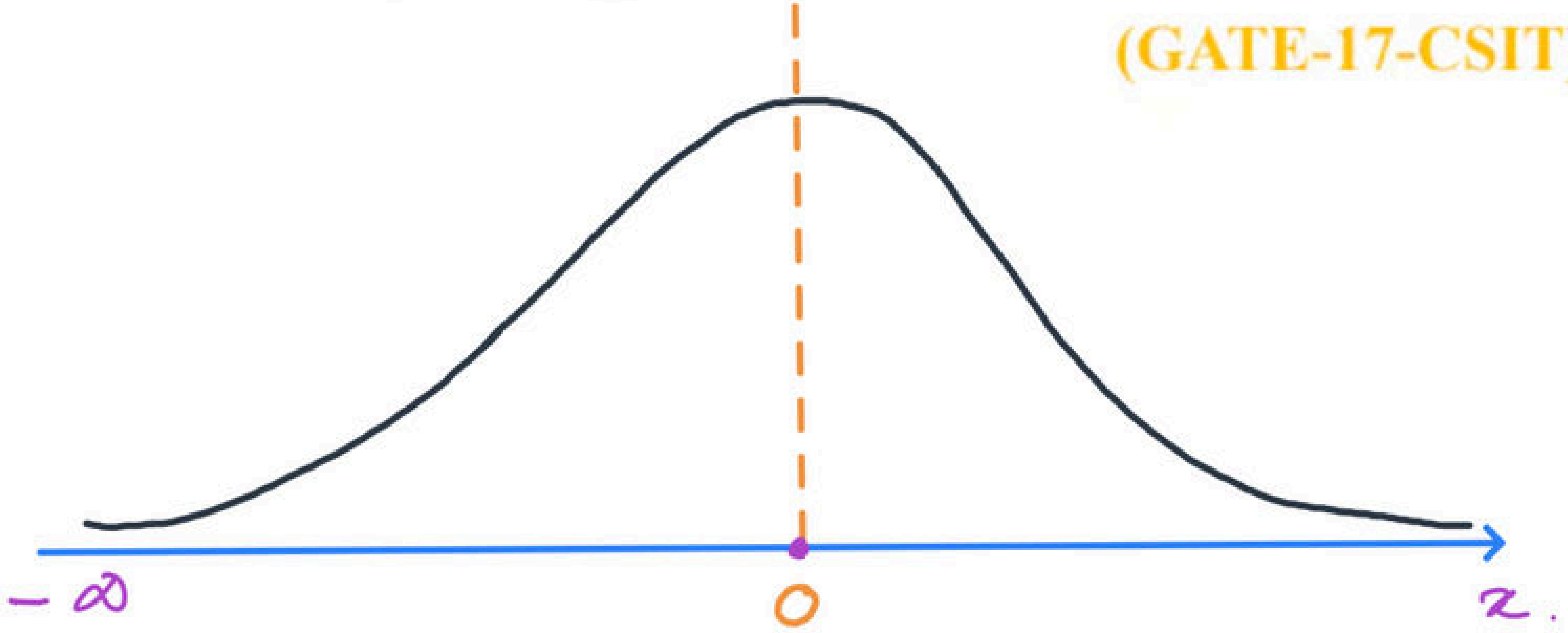
$P(3V \geq 2U)$

$P(\underbrace{3V - 2U}_{Z.} \geq 0)$

223. Let X be a Gaussian random variable with mean 0 and variance σ^2. Let Y = max(X, 0) where max(a, b) is the maximum of a and b. The median of Y is______________.

(GATE-17-CSIT)

$y = \max(x, 0)$



$$y = 0, \quad x < 0$$
$$= x, \quad x > 0.$$

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-m)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} \quad 0 < y < \infty$$

$$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}}. \quad 0 < y < \infty$$

$$\int_0^{y_1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{y^2}{2\sigma^2}} dy = \frac{1}{2\sqrt{2\pi\sigma^2}}.$$

$$\int_0^{y_1} e^{-\frac{y_1^2}{2\sigma^2}} dy. = \frac{1}{2}$$

$$\int_0^{y_1} e^{-\frac{y^2}{2\sigma^2}}\, dy = \frac{1}{2}.$$

$$\int^{\frac{y_1^2}{2\sigma^2}} e^{-t} \frac{1}{\sqrt{2}} \sigma\, t^{-\frac{1}{2}} dt = \frac{1}{2}.$$

$t = 0$

$$\frac{y^2}{2\sigma^2} = t$$

$$\frac{\cancel{2} y\, dy}{\cancel{2}\sigma^2} = dt.$$

$$dy = \frac{\sigma^2 dt}{y}.$$

$$dy = \frac{\sigma^2 dt}{\sqrt{2}\sigma \sqrt{t}}.$$

$$dy = \frac{1}{\sqrt{2}} \sigma\, t^{-\frac{1}{2}}\, dt$$

224. sample of 15 data is as follows: 17,18,17,17,13,18,5,5,6,7,8,9,20,17,3. The mode of the data is

(a) 4 (b) 13 (c) 17 (d) 20

(GATE-17-ME)

225. Weights (in kg) of six products are 3, 7, 6, 2, 3 and 4. The median weight (in kg up to one decimal place is ______)

(GATE-18-PI)

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$

(c) $\frac{7}{2}$ (d) 1

2 3 3 4 6 7.

$$\frac{3+4}{2} = \frac{7}{2}.$$

226. The number of parameters in the univariate exponential and Gaussian distributions, respectively, are

(a) 2 and 2 (b) 1 and 2 **(GATE-17-CE)**

(c) 2 and 1 (d) 1 and 1

227. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and 2V be F(x) and G(x) respectively. Then, for all values of x **(GATE-EC-2013)**

(a) $F(x) - G(x) \leq 0$ ✗

(b) $F(x) - G(x) \geq 0$ ✗

(c) $(F(x) - G(x)).x \leq 0$

(d) $(F(x) - G(x)).x \geq 0$ ✓

$F(x) = P(U \leq x)$

$G(x) = P(2V \leq x)$

$G(x) = P(V \leq x/2)$

if $x = +10$

$F(10) = P(U \leq 10)$

$G(10) = P(V \leq 5)$

$\boxed{F(x) - G(x) \geq 0}$

$x.\left[F(x) - G(x)\right] \geq 0$

$$x = -10$$

$$F(-10) = P(U \leq -10)$$

$$G(-10) = P(V \leq -5)$$

$$\boxed{F(x) - G(x) \leq 0}$$

$$x \cdot \left[F(x) - G(x) \right] \geq 0$$

228. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X<0\}$ is

(GATE-ME-2013)

(a) 0.5 (b) greater than zero and less than 0.5

(c) greater than 0.5 and less than 1.0 (d) 1.0

229. The area (in percentage) under standard normal distribution curve of variable within limits from -3 to + 3 is

(GATE-16-ME- SET3)

230. Consider a binomial random variable X. If $X_1, X_2, X_3 \ldots \ldots X_n$ are independent and identically distributed samples from the distribution of X with sum $Y = \sum_{i=1}^{n} X_i$, then the distribution of Y as n→∞ can be approximated as

(GATE-21-PI)

(a) Exponential (b) Bernoulli

(c) Binomial (d) Normal

231. There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random.

What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?

(GATE-2022-EEE)

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{6}$ (d) $\frac{5}{36}$

232. The probability density function of a random variable x be given as $f(x) = ae^{-2|x|}$; the value of a is_______________

(GATE-2022-EEE)

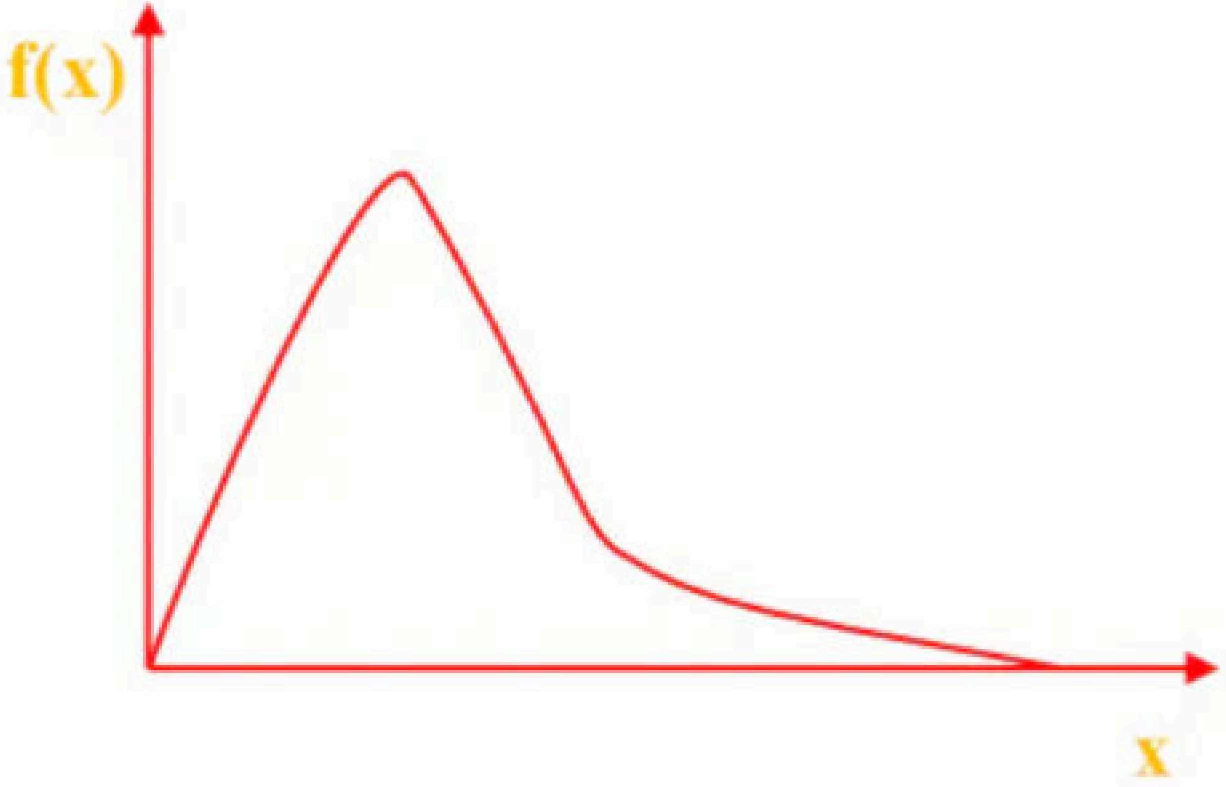
233. A probability distribution with right skew is shown in the figure. The correct statement for the probability distribution is **(GATE-18-CE)**

(a) Mean is equal to mode

(b) Mean is greater than median but less than mode

(c) Mean is greater than median and mode

(d) Mode is greater than median



234. A bag contains six red balls and four blue balls. If three balls are drawn in succession without replacement, the probability that the second and third balls drawn are red is ___________(round off to two decimal places)

•

(GATE-2022-IN)

235. A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of Rs.500 and a standard deviation of Rs.50. The percentage of savings account holders, who maintain an average daily balance more than Rs.500 is______ **(GATE-ME- SET-4-2014)**

236. f (x) is a continuous, real valued random variable defined over the interval $(-\infty, \infty)$ and its occurrence is defined by the density function given as :

$f(x) = \frac{1}{\sqrt{2\pi b^2}} e^{\frac{1}{2}(\frac{x-a}{b})^2}$ where 'a' and 'b' are the statistical attributes of the random {x}.

The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi b^2}} e^{\frac{1}{2}(\frac{x-a}{b})^2} dx$ is **(GATE-CE- SET-2-2014)**

(a) 1 (b) 0.5 (c) π (d) $\pi/2$

237. Which one of the following statements is not true

a) The measure of skew ness depends upon the amount of dispersion

b) In a symmetric distribution the values of mean, mode and median are the same

c) In a positively skewed distribution: mean > median > mode

d) In a negatively skewed distribution: mode > mean > median

238. The average grade for an examination is 74 and the standard deviation is 7. If 12% of the class are given A's and the grades are curved to follow normal distribution then what is the lowest possible A?

[The area under the standard normal curve to the left of $Z = 1.175$ is 0.88]

a) 79 b) 81 c) 83 d) 85

239. If the interarrival time is exponential and 8 customers per hour arrive in a bank, then the probability of no arrival of customer during a period of 15 minutes is________ [round off to two decimal places]

(GATE-2022-PI)

240. The spot speeds (expressed in km/hr) observed at a road section are 66, 62, 45, 79, 32, 51, 56, 60, 53 and 49. The median speed (expressed in km/hr) is

(GATE-16-CE-SET2)

(Note: answer with one decimal accuracy)

241. A cab was involved in hit and run accident at night. You are given following data about cabs in the city and the accident

(i) 85% of the cabs in the city are green and remaining cabs are blue.

(ii) A witness identified the cab involved in the accident as blue

(iii) it is known than the witness can correctly identify the cab colour only 80% of the time

Which of the following options is closest to the probability that the accident was caused by the blue cab?

(GATE-18-EC)

(a) 12% (b) 15%

(c) 41% (b) 80%

242. Let X1, X2 be two independent normal random variables with means μ_1, μ_2 and standard deviations σ_1 and σ_2 respectively. Consider $Y = X_1 - X_2$, $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1, \sigma_2 = 2$, then, **(GATE-18-ME)**

(a) Y is normally distributed with mean 0 and variance 1

(b) Y is normally distributed with mean 0 and variance 5

(c) Y has mean 0 and variance 5, but not normally distributed

(d) Y has mean 0 and variance 1, but not normally distributed

243. Suppose Y is distributed uniformly in the open interval (1,6). The probability that the polynomial $3x^2+6xy+3y+6$ has only real roots is (rounded off to one decimal place)_______

(GATE-19-CSIT)

244. The lengths of a large tank of titanium rods follow a normal distribution with a mean of (μ) of 440 mm and standard deviation (σ) of 1 mm. What is the percentage of rods whose lengths lie between 438 mm and 441mm?

(GATE-19-ME)

(a) 81.85% (b) 68.4% (c) 99.75% (d) 86.64%

245. If x is the mean of data 3, x, 2 and 4, then the mode is _________

(GATE-19-ME)

246. The sum of two normally distributed random variables X and Y is

(GATE-20-ME)

(a) Normally distributed, only if X and Y have the same standard deviation
(b) Always normally distributed
(c) Normally distributed, only if X and Y have the same mean
(d) Normally distributed, only if X and Y are independent

247. Suppose the probability that a coin toss shows "head" is p, where $0 < p < 1$. The coin is tossed repeatedly until the first "head" appears. The expected number of tosses required is

(GATE-21-EE)

(a) $\frac{(1-p)}{p}$ (b) $\frac{1}{p}$

(c) $\frac{1}{p^2}$ (d) $\frac{p}{(1-p)}$

248. Let X be a continuous random variable denoting the temperature measured. The range of temperature is (0, 100) degree Celsius and let the probability density function of X be $f(x) = 0.01$ for $0 \leq X \leq 100$. The mean of X is _______.

(GATE-21-EE)

(a) 5.0 (b) 50.0

(c) 25.0 (d) 2.5

249. The mean and variance, respectively, of a binomial distribution for n independent trials with the probability of success as p, are

(GATE-21-ME)

(a) $\sqrt{np}, np(1-2p)$

(b) $\sqrt{np}, \sqrt{np(1-p)}$

(c) np, np

(d) $np, np(1-p)$

250. If x is a random variable with the expected value of 5 and the variance of 1, then the expected value of x^2 is

(a) 36 (b) 26 **(GATE- 2020(PI)**

(c) 25 (d) 24