Doubt Clearing Session

Complete Course on Algorithm for GATE - CS & IT

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$$T(n) = \begin{cases} 10 & \text{if } n = 10 \\ T(n/2) + n & \text{if } n > 1 \end{cases}$$

$$\frac{n}{2^k} = 10, 1 \quad n = 2^k \cdot 10, 1 \quad k = \log_2 n$$

$$T(n) = T(n/2^1) + \frac{n}{2^0}$$

$$= T(n/2^2) + \frac{n}{2^1} + \frac{n}{2^0}$$

$$= T(n/2^3) + \frac{n}{2^2} + \frac{n}{2^1} + \frac{n}{2^0}$$

$$k = \log n$$

$$\cancel{n^2} \log n \qquad \cancel{n^{2.81}} \qquad \log n \qquad n^{0.81}$$

$$= T(n/2^k) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + \dots + \frac{n}{2^1} + \frac{n}{2^0}$$

$$O(1)$$

$$= T(1) + n\left[\left(\tfrac{1}{2}\right)^0 + \left(\tfrac{1}{2}\right)^1 + \dots + \left(\tfrac{1}{2}\right)^{\log n - 1}\right]$$

$$= 10 + n\left[\frac{1\left(1 - \left(\tfrac{1}{2}\right)^{\log n}\right)}{1 - \tfrac{1}{2}}\right]$$

$$= 10 + n\left[\frac{(1-0)}{\tfrac{1}{2}}\right]$$

$$10 + n[2] \Longrightarrow O(n)$$

$$\Longrightarrow \Theta(n)$$

$$\Longrightarrow O(n \log n)$$

$$T(n) = \begin{cases} 2 & \text{if } n = 2 \\ \sqrt{n}\, T(\sqrt{n}) + n & \text{if } n > 2 \end{cases}$$

$$T(100) = 10\,T(10) + 100$$

$2^1 - 1$

$$T(n) = n^{1/2^1}\, T(n^{1/2^1}) + n$$

$$= n^{1/2}\left[n^{1/2^2}\, T(n^{1/2^2}) + n^{1/2} \right] + n$$

$2^2 - 1$

$$= n^{3/2^2}\, T(n^{1/2^2}) + \underbrace{n + n}_{2}$$

$$= n^{3/2^2}\left[n^{1/2^3}\, T(n^{1/2^3}) + n^{1/2^2} \right] + n + n$$

$2^3 - 1$

$$= n^{7/2^3}\, T(n^{1/2^3}) + \underbrace{n + n + n}_{3}$$

K

$$= n^{\frac{2^K - 1}{2^K}}\, T\left(n^{\frac{1}{2^K}}\right) + K \cdot n$$

$$= n^{1 - \frac{1}{2^K}}\, T\left(n^{\frac{1}{2^K}}\right) + Kn$$

$$= \frac{n^1}{n^{1/2^K}}\, T\left(n^{\frac{1}{2^K}}\right) + Kn$$

$$= \frac{n}{2}\, T(2) + n \cdot \log\log n$$

$$= \frac{n}{\cancel{2}} \cdot \cancel{2} + n \cdot \log n$$

$$= n + n \cdot \log n = \Theta(n \log n)$$

$$n^{\frac{1}{2^k}} = 2$$

$$\frac{1}{2^k} \log_2 n = 1$$

$$\log_2 n = 2^k$$

$$\log_2 \log_2 n = k$$

$$2 = 2^{\log_2 n \cdot \frac{1}{\log n}}$$

$$n^{\frac{1}{2^k}} = n^{\frac{1}{2^{\log_2(\log_2 n)}}} = n^{\frac{1}{\log_2 n}} = \left(2^{\log_2 n}\right)^{\frac{1}{\log n}}$$

$n^{\frac{1}{\log n}} \Longrightarrow \left(2^{\log_2 n}\right)^{\frac{1}{\log n}} \Longrightarrow 2$

$\log n^{0}$ $\frac{1}{\log n}$

$\Longrightarrow 2$

$n = 2^{\log_2 n}$

$\frac{1}{2^{K}}$ $\frac{1}{2^{\log(\log_2 n)}}$ $\frac{1}{\log_2 n^2}$

$n \Longrightarrow n \Longrightarrow n^{1/\ldots}$ $n^{\frac{1}{\log n}}$

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n-1) + n & \text{if } n > 1 \end{cases}$$

$T(100) = 2T(99) + 100$

$n - K = 1$

$\boxed{n - 1 = K}$

$$T(n) = 2T(n-1) + n$$

$$= 2\left[2T(n-2) + (n-1)\right] + n$$

$$= 2^2 T(n-2) + 2(n-1) + n$$

$$= 2^2\left[2T(n-3) + (n-2)\right] + 2(n-1) + n$$

$$= 2^3 T(n-3) + 2^2(n-2) + 2^1(n-1) + 2^0(n-0)$$

$K = n - 1$

$$= 2^{n-1} T(\cancel{n} - (\cancel{n} - 1)) + 2^{n-2}(\cancel{n} - (\cancel{n} - 2)) + 2^{n-3}(\cancel{n} - (\cancel{n} - 3)) + \dots + 2^1(n-1) + 2^0(n-0)$$

$$= 2^{n-1}\cdot 1 + 2^{n-2}\cdot 2 + 2^{n-3}\cdot 3 + \cdots + 2^{1}\cdot(n-1) + 2^{0}(n-0)$$

$$T(n) = 2^{0}(n-0) + 2^{1}(n-1) + 2^{2}(n-2) + \cdots + 2^{n-2}\cdot 2 + 2^{n-1}\cdot 1$$

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$$T(n) = 2^{0}(n-0) + 2^{1}(n-1) + 2^{2}(n-2) + \cdots + 2^{n-2}\cdot 2 + 2^{n-1}\cdot 1 + 0$$

$$2\cdot T(n) = 0 + 2^{1}(n-0) + 2^{2}(n-1) \qquad + 2^{n-2}\cdot 3 + 2^{n-1}\cdot 2 + 2^{n}\cdot 1$$

$$T(n) - 2T(n) = 2^{0}(n-0) - 2^{1} - 2^{2} - 2^{3} - 2^{4} \cdots - 2^{n-2} - 2^{n-1} - 2^{n}$$

$$-T(n) = n - [2^1 + 2^2 + 2^3 + \cdots + 2^n]$$

$$T(n) = -n + \{2^1 + 2^2 + \cdots + 2^n\}$$

$$\Rightarrow \frac{2(2^n - 1)}{2-1} - n$$

$$\Rightarrow 2^{n+1} - 2 - n$$

$$T(n) = 2^{n+1} - 2 - n \Rightarrow \Theta(2^n)$$

$$T(n) = 2T(n/2) + n\log n$$

$$T(n) = 10 \cdot 2^{9} + 20 \cdot 2^{11} + 30 \cdot 2^{13} + 40 \cdot 2^{15} + \cdots$$

$$2^2 \cdot T(n) = 0 + 10 \cdot 2^{11} + 20 \cdot 2^{13} + 30 \cdot 2^{15} + \cdots$$

$$T(n) - 2^2 \cdot T(n) = 10 \cdot 2^9 + 10 \cdot 2^{11} + 10 \cdot 2^{13} + 10 \cdot 2^{15}$$

$$-3T(n) = 10\left[2^9 + 2^{11} + 2^{13} + 2^{15} + \cdots\right]$$

$$T(n) = \frac{10\left[\quad\right]}{-3}$$

$$\frac{a(r^n-1)}{r-1} \quad \text{if } r>1$$

$$\frac{a(1-r^n)}{1-r} \quad \text{if } r<1$$

$$\left(\frac{1}{2}\right)^{2} = (0.25)^{2}$$

0.0625