



GT - Part IV

Complete Course on Algorithm for GATE - CS & IT

fact(n)

if (n ≤ 1) return 1
else
return (n * fact(n-1))

RR-Value

$$V(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ n * V(n-1) & \text{if } n > 1 \end{cases}$$

RR-No. of multiplication

$$m(n) = \begin{cases} 0 & \text{if } n \leq 1 \\ m(n-1) + 1 & \text{if } n > 1 \end{cases}$$

RR-Time

$$T(n) = \begin{cases} O(1) & \text{if } n \leq 1 \\ T(n-1) + c & \text{if } n > 1 \end{cases}$$

Recurrence Relation solving

- ① Substitution method
- ② Recursive Tree method
- ③ Master Theorem

Substitution method

$$T(n) = \begin{cases} O(1) & \text{if } n \leq 1 \\ T(n-1) + C & \text{if } n > 1 \end{cases}$$

const

$$\begin{aligned} \checkmark T(50) &= T(49) + C \\ \checkmark T(200) &= T(199) + C \end{aligned}$$

$$\begin{aligned} T(n) &= T(n-1) + C \\ &\Downarrow \\ T(n-2) + C + C \\ &\Downarrow \\ T(n-3) + C + C + C \end{aligned}$$

$$\begin{aligned} n-k &= 1 \\ n-1 &= k \end{aligned}$$

$\underbrace{\hspace{1cm}}_{n-1}$ $\left\{ \begin{array}{l} K \text{ times} \end{array} \right.$

$$\begin{aligned} &= T(n-k) + K \cdot C \\ &= T(n - (n-1)) + (n-1) \cdot C \\ &= T(1) + (n-1) \cdot C \\ &= O(1) + (n-1) \cdot C = \underline{\underline{O(n)}} \end{aligned}$$

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(n-1) * n & \text{if } n > 1 \end{cases}$$

$$T(50) = T(49) * 50$$

$$T(200) = T(199) * 200$$

$$T(2) = T(1) * 2$$

$$T(1) = 1$$



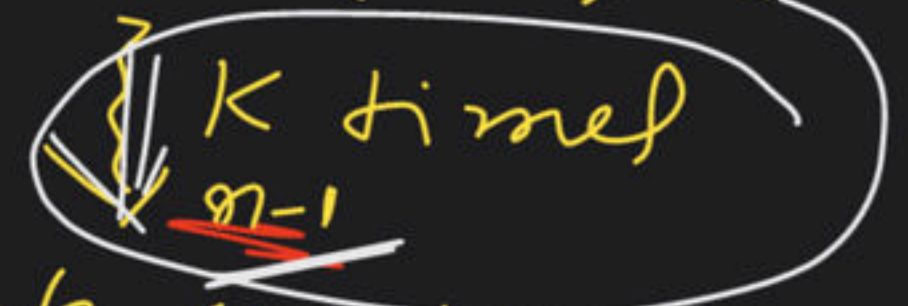
$$T(n) = T(n-1) * (n-0)$$

$$= T(n-2) * (n-1) * (n-0)$$

$$= T(n-3) * (n-2) * (n-1) * (n-0)$$



$$= T(n-100) * (n-99) * (n-98) * \dots * (n-0)$$



$$= T(n-K) * (n-(K-1)) * (n-(K-2)) * \dots * (n-0)$$

$$= T(n-(n-1)) * (n-[(n-1)-1]) * (n-[(n-1)-2]) * \dots * n$$

$$= T(1) * 2 * 3 * 4 * 5 * 6 * \dots * (n-1) * n$$

$$= 1 * 2 * 3 * 4 * \dots * n = n! = O(n^n)$$

$$n-K=1$$

$$K=n-1$$

Fibonacci Series

n	0	1	2	3	4	5	6	7	8	9	10
fib(n)	0	1	1	2	3	5	8	13	21	34	55

$$\text{fib}(5) = 5 + 3 = 8$$

$$f(n) = f(n-1) + f(n-2)$$

Value

Time

Addition

fib(n)

if (n == 1 || n == 0)

return n

else

return (fib(n-1) + fib(n-2))

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