

CS918: LECTURE 3

Introduction to Language Models

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LECTURE 3: CONTENTS

- Statistical language models.
 - N-grams.
 - Estimating probabilities of n-grams.
- Evaluation and perplexity.



N-GRAMS

- N-gram: sequence of n words.
- e.g. I want to go to the cinema.
 - 2-grams (bigrams): I want, want to, to go, go to, to the,...
 - 3-grams (trigrams): I want to, want to go, to go to,...
 - 4-grams: I want to go, want to go to, to go to the,...
 - ...



STATISTICAL LANGUAGE MODELLING

- Statistical language model: probability distribution over sequence of words.
- * I want to * → high probability, common sequence in English
- * want I to * → low probability or zero



STATISTICAL LANGUAGE MODELLING

- How are these probability distributions useful?
 - Machine translation: P("please, call me") > P("please, call I")
 - Spelling correction:

"its 5pm now" → correct to "it's 5pm now", higher P

Speech recognition:

P("I saw a van") >> P("eyes awe of an")

Smart compose:

You're typing "see you", the system suggests the next word: "tomorrow"



STATISTICAL LANGUAGE MODELLING

Probability of sequence of words (W):

$$P(W) = P(W_1, W_2, W_3, ..., W_n)$$

• Also, we often look at probability of upcoming word:

$$P(W_4 | W_1, W_2, W_3)$$

- Both of the above are known as language models.
 - i.e. how likely is a sequence of words?



HOW DO WE COMPUTE P(W)?

How likely is the following sequence?

P(I, found, two, pounds, in, the, library)

We can rely on the Chain Rule of Probability.



THE CHAIN RULE OF PROBABILITY

Definition of the rule:

$$P(A, B) = P(B \mid A)P(A)$$

More variables:

$$\mathrm{P}(A_1,A_2,A_3,A_4) = \mathrm{P}(A_4 \mid A_3,A_2,A_1) \cdot \mathrm{P}(A_3 \mid A_2,A_1) \cdot \mathrm{P}(A_2 \mid A_1) \cdot \mathrm{P}(A_1)$$

Generalisation of the rule:

$$\mathrm{P}(A_1\,,\ldots,A_n)=\mathrm{P}(A_n|A_{n-1},\ldots,A_1)\cdot\mathrm{P}(A_{n-1},\ldots,A_1)$$



THE CHAIN RULE OF PROBABILITY

Definition of the rule:

$$P(A, B) = P(B \mid A)P(A)$$

Let's say we have the following sentences to learn our language models:

see what I found you found a penny it has been found the book you found you came yesterday

What is the probability of "you found"?

P(you, found) = P(found | you) * P(you)



COMPUTING P(W) USING THE CHAIN RULE

• P(I, found, two, pounds, in, the, library) =

```
P(I) *
P(found | I) *
P(two | I found) *
P(pounds | I found two) *
P(in | I found two pounds) *
P(the | I found two pounds in) *
P(library | I found two pounds in the)
```



AND HOW DO WE COMPUTE PROBABILITIES?

• Diving the number of occurrences?

P(library | I found two pounds in the) =

count (I found two pounds in the library)
count (I found two pounds in the)

• **Problem:** there are so many different sequences, we won't observe enough instances in our data!



MARKOV ASSUMPTION

Approximate the probability by simplifying it:

P(library | I found two pounds in the) ≈ P(library | the)

(first-order Markov assumption)

• Or:

P(library | I found two pounds in the) ≈ P(library | in the)
(2nd-order Markov assumption)



• It's much more likely that we'll observe "the library" or "in the library" in our training data.



Which we can generalise as:

$$P(W_i \mid W_1, W_2, ..., W_{i-1}) \approx P(W_i \mid W_{i-k}, W_{i-k+1}, ..., W_{i-1})$$

i.e., we will only look at the last k words.

(kth-order Markov assumption)





COMPUTING PROBABILITIES

Let's say we have the following sentences to learn our language models:

```
see what I found
you found a penny
it has been found
the book you found
you came yesterday
```

What is the probability of "you found"?

```
With the 1st-order Markov assumption:
P(you, found) = P(found | you)
```



COMPUTING PROBABILITIES

Let's say we have the following sentences to learn our language models:

```
you found a penny it has been found the book you found you came yesterday
```

What is the probability of "you found"?

```
With the 1st-order Markov assumption:
```

```
P(you, found) = P(found | you)
```

= count(you found) / count(you) = 2/3



LANGUAGE MODELS

- We can go with bigrams, trigrams, 4-grams,...
- Note: the longer the length:
 - The more detailed our language model.
 i.e. long sequences will capture more grammar than short sequences.
 - But the more sparse our counts.
 i.e. many observations only seen once.



LANGUAGE MODELS

• Note: it's not always perfect, e.g.:

My request for using the department's supercomputer to run experiments next month is approved.

"month is approved" → unlikely "request ... approved" → more likely, not captured by n-grams

• N-grams are however often a good solution.



ESTIMATING N-GRAM PROBABILITIES



ESTIMATING BIGRAM PROBABILITIES

Maximum Likelihood Estimate (MLE):

$$P(W_i | W_{i-1}) = \frac{count(W_{i-1}, W_i)}{count(W_{i-1})}$$

e.g.

$$P(the \mid in) = \frac{count("in the")}{count("in")}$$



- For the following sentences:
 - <s> Yes, I am watching the TV right now </s>
 - <s> The laptop I gave you is very good </s>
 - <s> I am very happy to be here </s>



• For the following sentences:

```
<s> Yes, I am watching the TV right now </s>
```

<s> The laptop I gave you is very good </s>

<s> | am very happy to be here </s>

P(am | I) = count("I am") / count("I") = 2 / 3 = 0.667



- For the following sentences:
 - Yes, I am watching the TV right now </s>
 - <s> The laptop I gave you is very good </s>
 - <s> I am very happy to be here </s>
- P(I | <s>) = count("<s> I") / count("<s>") = 1 / 3 = 0.333



- For the following sentences:
 - <s> Yes, I am watching the TV right now </s>
 - <s> The laptop I gave you is very good </s>
 - <s> I am very happy to be here </s>
- P(now | right) = count("right now") / count("right") = 1 / 1 = 1

Note: we have a very small corpus, so our system is learning that "right" is ALWAYS followed by "now"



EXAMPLE OF BIGRAM COUNTS

Sample of bigrams counted in a corpus of 9,222 sentences.

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	1	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

"i" is followed by another "i" on 5 occasions

"to" is followed by "eat" on 686 occasions



INFERRING BIGRAM PROBABILITIES

We also have the counts of words:

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

• To infer probabilities of bigrams, we'll divide bigram counts by word counts.

(remember: $P(w_1, w_2) = count(w_1 w_2) / count(w_1)$)



INFERRING BIGRAM PROBABILITIES

Having the following two:

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

• We infer the following probabilities:

5	2533	

	i	want	to	eat	chinese	food	lunch	spend
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0



BIGRAM ESTIMATE OF SENTENCE PROBABILITY

Now we want to estimate the probability of a given sentences:

I want to eat chinese food.

```
    P(I want to eat chinese food) =
        P(want | I) [0.33]
        * P(to | want) [0.66]
        * P(eat | to) [0.28]
        * P(chinese | eat) [0.021]
        * P(food | chinese) [0.52]
```



BIGRAM ESTIMATE OF SENTENCE PROBABILITY

Now we want to estimate the probability of a given sentences:

I want to eat chinese food.

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    P(I want to eat chinese food) =
        P(want | I) [0.33]
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        * P(chinese | eat) [0.021]
        * P(food | chinese) [0.52]
```

= 0.000665945



PRACTICAL SOLUTION

- 0.000665945 is a very low probability.
 - We may end up with a **floating point underflow!**
- Solution:
 - Use **log space** instead.
 - Sum instead of multiplication.

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$



PRACTICAL SOLUTION

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

I want to eat chinese food.

$$log(0.33) + log(0.66) + log(0.28) + log(0.021) + log(0.52) = -3.1766$$

want I eat food to lunch.

$$log(0.0022) + log(0.0036) + log(0.0027) + log(0.014) + log(0.0025) = -12.1258$$

Log scale is leading to **negative values**, **but** we just want them to be **comparable**. i.e. in English, sentence 1 much more likely than sentence 2.



COLLECTIONS OF LANGUAGE MODELS

Collection of Google N-grams:

```
File sizes: approx. 24 GB compressed (gzip'ed) text files
Number of tokens: 1,024,908,267,229
Number of sentences:
                      95,119,665,584
                     13,588,391
Number of unigrams:
Number of bigrams:
                        314,843,401
                        977,069,902
Number of trigrams:
Number of fourgrams:
                      1,313,818,354
Number of fivegrams:
                      1,176,470,663
```



COLLECTIONS OF LANGUAGE MODELS

Collection of Google N-grams:

```
ceramics collection , 144
ceramics collection . 247
ceramics collection  120
ceramics collection and 43
ceramics collection at 52
ceramics collection is 68
ceramics collection of 76
ceramics collection | 59
ceramics collections , 66
ceramics collections . 60
ceramics combined with 46
ceramics come from 69
ceramics comes from 660
```



COLLECTIONS OF LANGUAGE MODELS

Google N-grams is the best collection of language models today.

• Caveat: it has a cost, \$150.

https://catalog.ldc.upenn.edu/LDC2006T13



COLLECTIONS: FREE ALTERNATIVE

Google Books N-grams:

1-grams 0 1 2 3 4 5 6 7 8 9 a b c d e f g h i j k l m n o other p pos punctuation g r s t u v w x y z

4-grams 0 1 2 3 4 5 6 7 8 9 ADJ ADP ADV CONJ DET NOUN NUM PRON PRT VERB a aa ab ac ad ae af ag ah ai aj ak al am an ao ap aq ar as at au av aw ax ay az b ba bb bc bd be

http://storage.googleapis.com/books/ngrams/books/datasetsv2.html



EVALUATION AND PERPLEXITY



EVALUATION OF OUR MODEL

- We want to evaluate whether our language model is good.
 i.e. does our language model prefer good sentences to bad ones?
- i.e. does it assign higher probability:
 - to "real" or "frequent" sentences (e.g. I want to)
 - than "ungrammatical" or "rarely observed" sentences? (e.g. want I to)



EVALUATION OF OUR MODEL

Evaluation:

- Is our language model good in giving high probabilities to sentences in our corpus?
- Usually done in a comparative way:
 - Train language model 1 (LM1) from corpus 1.
 - Train language model 2 (LM2) from corpus 2.
 - For sentences in corpus 3, which of LM1 and LM2 is giving me higher probabilities?
- We need an evaluation metric to determine which of LM1 or LM2 is best.



EVALUATION APPROACHES

- Two different evaluation approaches:
 - Extrinsic or in-vivo evaluation.
 i.e. test externally on another task.
 - Intrinsic or in-vitro evaluation.
 i.e. evaluate the models directly.



EXTRINSIC EVALUATION

- Test LM1 and LM2 in some NLP task (sentiment analysis, MT, spell corrector, etc.).
- Which of them leads to better performance?

i.e. what it the accuracy of my sentiment analysis tool by using LM1 and LM2?



DISADVANTAGE OF EXTRINSIC EVALUATION

- Extrinsic evaluation is ideal but can take very long to run.
- Alternatively intrinsic evaluation.
 - Consists in computing perplexity.
 - Rough approximation.
 - Only valid if tested in similarly looking data.

i.e. don't train a LM from news articles, and test it on social media.



• Perplexity:

given a language model, on average: how difficult is it to **predict the next word**?

e.g. I always order pizza with cheese and \longrightarrow ???



- The Shannon Game:
 - How well can we predict the next word?

pizza with cheese and

mushrooms 0.1 pepperoni 0.1 jalapeños 0.01

. . . .

biscuits 0.000001

A good model:

the one that gives higher probability to the actual next word.

- If the actual sentence is "pizza with cheese and biscuits", my model is quite bad.
- If the actual sentence is "pizza with cheese and mushrooms", my model is better.



 To compute the perplexity, we will use an unseen test set (held out corpus).

(different from the training data used to build the model)

• While we want to maximise P(sentence) we aim to get a low perplexity, it's inverse to probability.



• Perplexity of a sequence of N words W, PP(W):

$$PP(W) = P(W_1 W_2 ... W_N)^{-\frac{1}{N}}$$
$$= \sqrt[N]{\frac{1}{P(W_1 W_2 ... W_N)}}$$



$$PP(W) = P(w_1w_2...w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1w_2...w_N)}}$$
Chain rule
$$PP(W) = \sqrt[N]{\frac{1}{P(w_i|w_1...w_{i-1})}}$$

$$PP(W) = \sqrt[N]{\frac{1}{P(w_i|w_1...w_{i-1})}}$$

$$PP(W) = \sqrt[N]{\frac{1}{P(w_i|w_{i-1})}}$$



PERPLEXITY AS A BRANCHING FACTOR

- Suppose we have a sentence consisting of random digits [0-9].
 - All digits have the same probability: 1/10.
- What is the perplexity?

$$PP(W) = P(w_1 w_2 ... w_N)^{-\frac{1}{N}}$$

$$= (\frac{1}{10}^N)^{-\frac{1}{N}}$$

$$= \frac{1}{10}^{-1}$$

$$= 10$$



INTERPRETING PERPLEXITY

- Lower perplexity (i.e. higher probability), better model!
- e.g. WSJ corpus: 38 million words for training, 1.5 million for testing.

	Unigram	Bigram	Trigram
Perplexity	962	170	109



LANGUAGE MODELS

• Remember: language models will work best on similarly looking data.

- If we train on social media data, that may not work for novels.
 - OMG I'm ROTFL! → may be frequent in SM, unlikely in novels!



LANGUAGE MODELS

- **Limitation:** We're assuming that we have computed all probabilities for n-grams in the test data.
 - Is this the reality though?
- I may train with the following n-grams:

I found a penny.

I found a fiver.

I found a book.

• If my test data has "I found a tenner", is the probability of this sentence really 0? just because I haven't seen it in my training data?



LANGUAGE MODELS

- There are different approaches for **dealing with zeros** and to enable **generalisation of trained models**.
 - In the next lecture.



ASSOCIATED READING

• Jurafsky, Daniel, and James H. Martin. 2009. Speech and Language Processing: An Introduction to Natural Language Processing, Speech Recognition, and Computational Linguistics. 3rd edition. **Chapters 3.1-3.2**.

• Bird Steven, Ewan Klein, and Edward Loper. Natural Language Processing with Python. O'Reilly Media, Inc., 2009. **Chapter 2, Section 2**.