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NCIA-201: Project Report
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Exploring Bayesian Behavior in Agent Decision-Making Under Uncertainty

Introduction

In this project, an agent learns to interpret and interact with environmental stimuli by analyzing their properties and making decisions based on observed data. The agent receives measurements of the stimuli in the environment, the ground truth corresponding to the stimuli, and the history of decisions regarding whether each stimulus was accepted or rejected. The agent's learning capacity is a configurable parameter, allowing for comparisons of different training outcomes. The agent is then presented with pairs of Gaussian stimuli, each defined by a value and a noise level: it outputs a decision of "True" or "False", indicating whether it perceives stimulus 1 as greater than stimulus 2. The primary goal is to examine whether the trained agent relies on its prior beliefs when faced with uncertainty, thereby exhibiting Bayesian behavior. In a subsequent step, this behavior will be contrasted with that of less trained agents to evaluate how training levels influence perceptual decisions under uncertainty.

Hypotheses

We investigate how an agent makes perceptual decisions when faced with uncertainty. Specifically, this project examines whether the agent relies on its prior beliefs when presented with noisy stimuli, a hallmark of Bayesian behavior. The primary hypothesis posits that a trained agent will exhibit Bayesian behavior when making perceptual judgments under uncertainty by integrating prior knowledge with noisy sensory inputs. In other words, the agent's decisions will not rely solely on the immediate noisy stimulus but will also be influenced by what it has learned through training, effectively biasing its perceptions.

The second hypothesis posits that the level of training will impact the agent's prior beliefs: untrained agents, lacking a prior, will not exhibit perceptual bias, whereas agents with more training will develop increasingly defined priors and, as a result, exhibit perceptual bias. This implies that the agent's Point of Subjective Equality (PSE), the point at which two stimuli are perceived as equal, will shift differently with increasing noise in the test stimulus, depending on the training level. Specifically, a well-trained agent will exhibit a biased PSE, indicating that it is either overestimating or underestimating the test stimulus based on the mean of its prior. In contrast, an untrained agent will consistently overestimate the test stimulus.

Methods

We conducted a two-alternative forced choice (2AFC) task in which an agent, trained over 2000 trials, had to decide whether the measurement of the first stimulus was greater than that of the second stimulus ("True") or not ("False"). The first stimulus, serving as a reference, was fixed at two values: 0 and 2, each with a standard deviation of 0.1. The value of the second stimulus was systematically varied, ranging from the minimum to the maximum of the measured stimuli in the environment (determined by the environment's prior derived from the agent's training history). Additionally, the standard deviation of the second stimulus was varied in increments of 0.1, ranging from 0.1 to 0.9. For each combination of [Stimulus 1 value, Stimulus 1 standard deviation, and Stimulus 2 standard deviation], and for each discrete value of Stimulus 2 (incremented in steps of 0.1 within the measured range of the environment), 100 trials were conducted. These trials aimed to estimate the probability of the agent giving a "True" or "False" response for each pair of stimuli values under varying levels of noise (represented by the standard deviation of Stimulus 2). The collected data were then used to construct psychometric functions, representing the relationship between the test stimulus values and the agent's responses.

We also conducted the same experiment under three distinct conditions to evaluate the impact of training on performance: (1) with no prior training, (2) after 10 training trials, and (3) after 100 training trials. The experimental procedure was identical across all conditions, ensuring that the collected data could be directly compared across the different training levels.

Results

Figure (1) shows the psychometric function (raw response data) for a 2AFC task. The y-axis shows the average probability response ($P(S_{ref} > S_{test})$) as a function of the reference stimulus value ((S_{test}), x-axis). The reference (S_{ref}) is fixed at 0 with a standard deviation of 0.1, while the standard deviation of the test stimulus varies from 0.1 to 0.9 in steps of 0.1. The curve illustrates the agent's perceptual decisions across different stimulus conditions based on aggregated trial data.

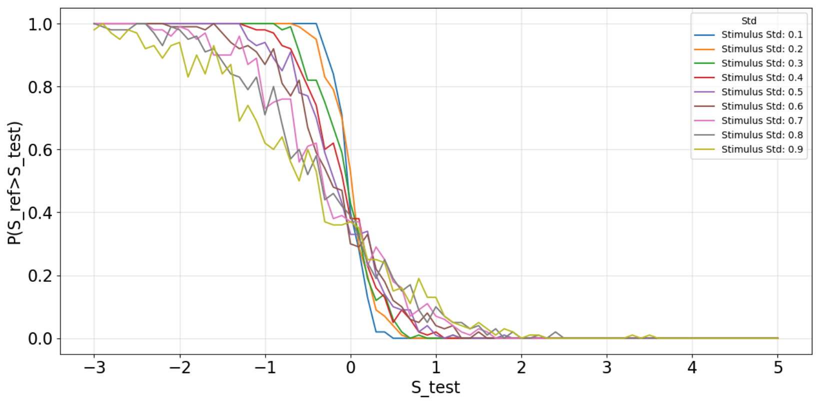


Figure 1

To enhance visualization and allow precise access to key parameters, sigmoid functions were fitted to the data. Figure 2 presents the psychometric curves for two conditions: the left panel shows the condition where the reference stimulus was fixed at 0, while the right panel represents the condition where the reference stimulus was fixed at 2.

The sigmoid model used to fit the data is defined as: $y = \frac{1}{1+e^{-c(x-d)}} + b$.

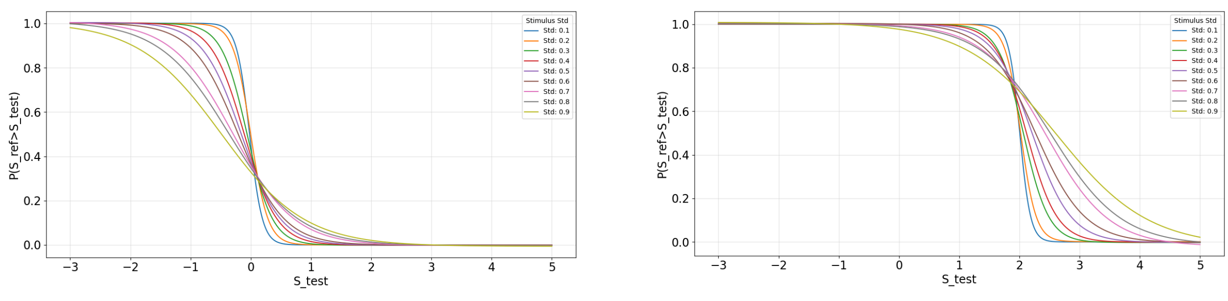


Figure 2

The Point of Subjective Equality (PSE) represents the test stimulus value at which the agent perceives the reference and test stimuli as equal, corresponding to a 50% probability of choosing either response option. Figure 2 shows that in both conditions, the PSE shifts in the presence of noise. As test stimulus noise increases, the PSE bias becomes larger. When the reference stimulus (S_{ref}) is 0, the PSE shifts toward negative values, indicating that the agent overestimates the noisy test stimulus, leading to lower PSE values. Conversely, when (S_{ref}) is 2, the PSE shifts toward positive values, indicating that the agent underestimates the noisy test stimulus, leading to higher PSE values. These patterns suggest that the mean of the prior lies between 0 and 2.

To determine the reference stimulus value (μ) at which the PSE bias is minimized, we performed an optimization procedure. The process interpolates between the PSE and slope values for two conditions (S_{ref}= 0) and (S_{ref}= 2) and minimizes the absolute PSE. The search for the optimal (μ) starts at 1 and is bounded between 0 and 2. The optimization procedure identifies the value of (μ) at which the absolute PSE is minimal. The optimization algorithm determined the optimal reference stimulus value to be μ = 0.967. Figure 3 presents the results of the same experiment conducted with the reference stimulus set to this optimal (μ). As shown, the PSE no longer exhibits a noticeable bias as the noise in the test stimulus increases, confirming that the prior mean has been accurately identified as the point where PSE bias is minimized.

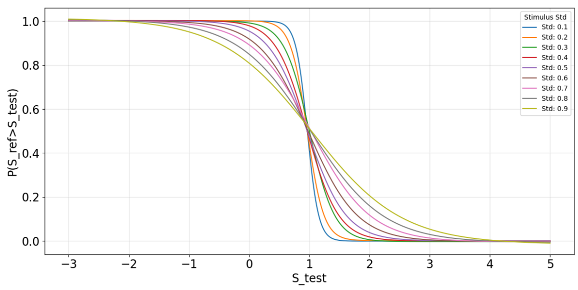


Figure 3

To further estimate the prior, we introduced a variable Z, defined as: $Z = X_1 - X_2$

The psychometric function serves as the cumulative distribution function (CDF) of the posterior distribution. The mean of the posterior distribution aligns with the PSE, and the variance of the posterior is given by:

$\sigma_{posterior} = (\frac{1}{\sqrt{2\pi}\,.\,(slope\ at\ x=\mu)})$;

and $\sigma_Z^2 = \sigma_1^2 + \sigma_2^2$; $\mu_Z = \mu_1 - \mu_2$.

Posterior mean: $\mu_{post} = \frac{\mu_Z/\sigma_Z^2 + \mu_{prior}/\sigma_{prior}^2}{1/\sigma_Z^2 + 1/\sigma_{prior}^2}$

Posterior variance: $\sigma_{post}^2 = \left(\frac{1}{\sigma_Z^2} + \frac{1}{\sigma_{prior}^2}\right)^{-1}$

Using the calculated values for the posterior mean and variance, as well as for (Z), we derived the optimal parameters for the prior distribution. The optimization yielded the following results:

μ_{prior}: 1.012 ; σ_{prior}^2: $1e^{-06}$.

To evaluate the learning process and the impact of training on the agent's perceptual decisions, we repeated the same experiment with varying numbers of training trials (Figure 4). In the untrained agent, an increase in uncertainty consistently shifted the PSE toward negative values, regardless of whether the reference stimulus was 0 or 2. This indicates an overestimation of the test stimulus, demonstrating that the untrained agent lacks a prior, meaning that its prior knowledge is not constrained to any specific value. However, as the number of training trials increased, a bounded window began to emerge, representing the limits within which the mean of the agent's prior is shaped. This window reflects the agent’s gradual acquisition of prior knowledge through training, which restricts its interpretation of the stimuli to values within this range.

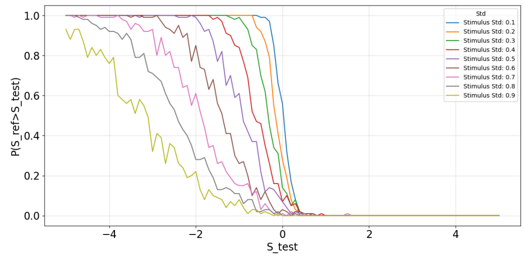
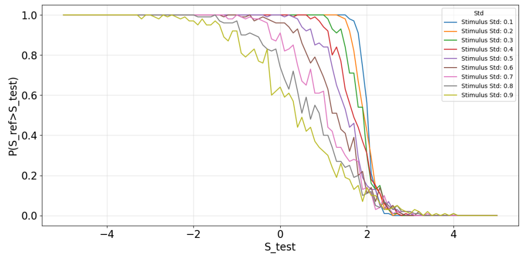
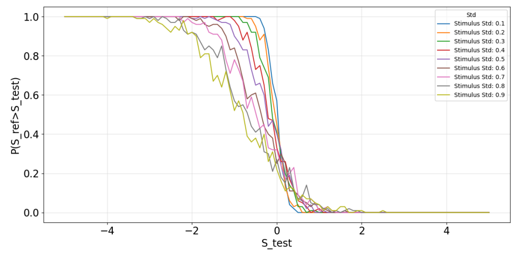
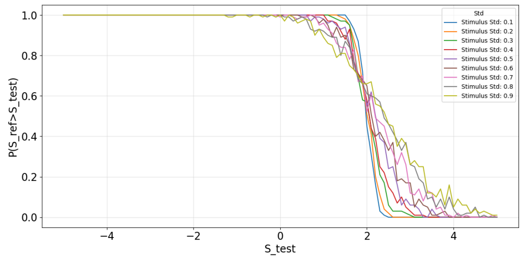
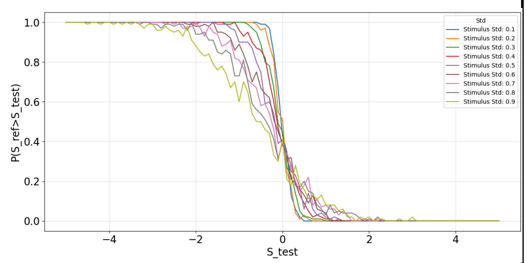
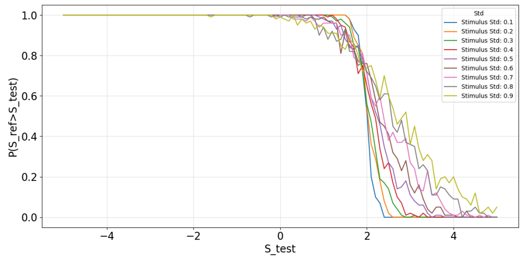
	S_ref = 0	S_ref = 2
0 trials		
10 trials		
100 trials		

Figure 4

Discussion

The results align with Bayesian behavior, providing evidence that the agent incorporates a prior belief about the stimulus value when making perceptual decisions and increasingly relies on the prior under noisier conditions, leading to biased perceptions. Moreover, the results support that training plays a significant role in shaping the agent’s perceptual decision-making process under uncertainty. The analyses highlight that, with an increasing number of training trials, the agent's prior becomes progressively more defined, resulting in interpretations of noisy stimuli that are increasingly biased toward the prior.

In this project, having access to the range of measured stimuli in the environment significantly simplified the identification of the prior. Similarly, determining the Point of Subjective Equality (PSE) from the psychometric function was made much easier under these controlled conditions. However, in more realistic settings, running 100 trials for each condition would be impractical, requiring the development and application of alternative algorithms to construct psychometric functions efficiently. Additionally, we operated under the assumption that the prior, likelihood, and posterior distributions were Gaussian, which simplified the calculations and enabled the derivation of the prior. However, in more complex or realistic scenarios, priors may not adhere to such simple distributions, making them harder to model and introducing additional challenges for analysis and interpretation.