Arghavan Aslani, Maiko Keramidas, Minji Jeong, Sihan Xue Code Repository
NCIA-201: Project Report
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Exploring Bayesian Behavior in Agent Decision-Making Under Uncertainty

Introduction

In this project, an agent learns to interpret and interact with environmental stimuli by analyzing their properties and making decisions based on observed data. The agent receives measurements of the stimuli in the environment, the ground truth of the stimuli, and decisions made to accept or reject each stimulus. The agent's learning capacity is a configurable parameter, allowing for comparisons of different training outcomes. The agent is presented with pairs of Gaussian stimuli, each characterized by a value and a noise level, and it outputs a decision of "True" or "False," indicating whether stimulus 1 was preferred over stimulus 2. The core aim is to investigate if the trained agent, when faced with uncertainty, relies on its prior beliefs, thus demonstrating Bayesian behavior. This behavior will be contrasted with that of less trained agents.

Research Question and Objectives

We address the problem of understanding how an agent makes decisions when faced with uncertainty. Specifically, the project investigates whether an agent, when presented with noisy stimuli, relies on its prior beliefs, which is a characteristic of Bayesian behavior. The primary hypothesis is that a trained agent, when making perceptual judgments under uncertainty, will exhibit Bayesian behavior by integrating prior knowledge with noisy sensory inputs. This means the agent's decisions will not solely depend on the immediate noisy stimulus but will be influenced by what it has learned through training, effectively biasing the agent's perception.

As an additional hypothesis, it is expected that the level of training will impact the agent's prior beliefs, with untrained agents lacking a prior, and thus not showing Bayesian bias, whereas agents with increasing amounts of training will develop a more defined prior. This implies that the agent's Point of Subjective Equality (PSE), which is the point at which two stimuli are perceived as equal, will shift differently with increasing noise in the test stimulus for agents with different training levels. Specifically, a well-trained agent will show a bias in their PSE, indicating they are either overestimating or underestimating the test stimulus based on the mean of their prior, while an untrained agent will consistently overestimate the test stimulus.

Methods

We conducted a two-alternative forced choice (2AFC) task where an agent, trained on 2000 trials, was required to decide whether the measurement of the first stimulus was greater than that of the second stimulus (True) or not (False). The first stimulus, used as a reference, was fixed at two values: 0 and 2, with a standard deviation of 0.1 for both cases. The value of the second stimulus was varied systematically, ranging from the minimum to the maximum of the measured stimuli in the environment (derived from the prior in the environment based on the agent's training history). Additionally, the standard deviation of the second stimulus was varied in steps of 0.1, from 0.1 to 0.9. For each combination of [Stimulus 1 value, standard deviation of Stimulus 1, and standard deviation of Stimulus 2], and for each discrete value of

Stimulus 2 (incremented in steps of 0.1 within the measured range of the environment), 100 trials were conducted. These trials aimed to estimate the probability of obtaining a "True" or "False" response. Finally, the collected data were used to construct psychometric functions, representing the relationship between the test stimulus values and the agent's responses.

We also conducted the same experiments under three distinct conditions to evaluate the effects of training on performance: (1) without any prior training, (2) after 10 training trials, and (3) after 100 training trials. For each condition, the experimental procedure remained consistent, ensuring that the data collected could be directly compared across the training levels.

Results

Figure (1) shows the psychometric function for a 2AFC task. The y-axis shows the probability ($P(S_{ref} > S_{test})$) as a function of the reference stimulus value ((S_{ref}), x-axis). The reference (S_{ref}) is fixed at 0 with a standard deviation of 0.1, while the standard deviation of the test stimulus varies from 0.1 to 0.9 in steps of 0.1. The curve illustrates the agent's decision-making across different stimulus conditions based on aggregated trial data.

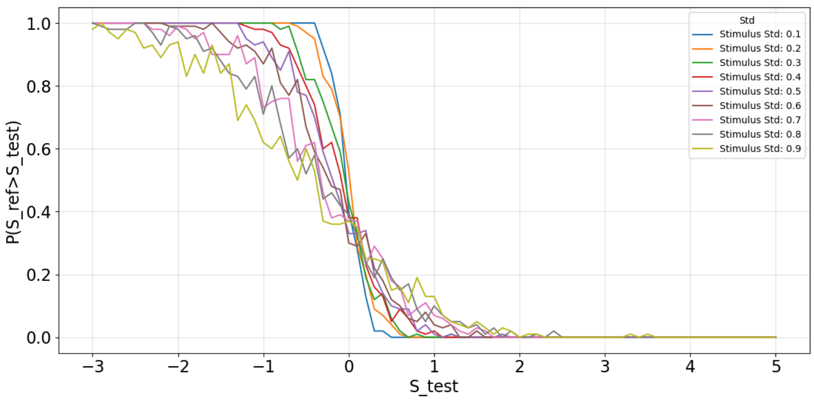

Figure 1

To enhance visualization and allow precise access to specific points on the psychometric functions, sigmoid curves were fitted to the data. Figure 2 presents the fitted sigmoid functions overlaid on the psychometric functions for two conditions: the left panel shows the condition where the reference stimulus was fixed at 0, while the right panel represents the condition where the reference stimulus was fixed at 2. The sigmoid model used to fit the psychometric data is defined as: $y = \frac{1}{1+e^{-c(x-d)}} + b$.

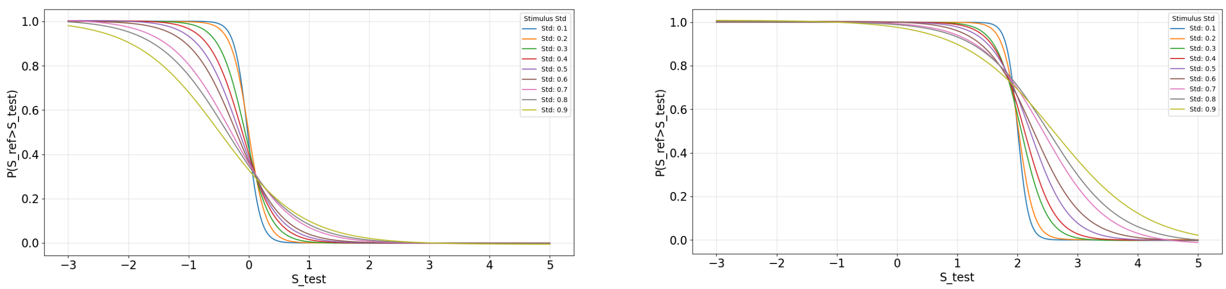

Figure 2

The Point of Subjective Equality (PSE) is the test stimulus value where the agent perceives the reference and test stimuli as equal, with a 50% choice probability. Figure 2 shows that in both conditions, the PSE shifts in the presence of noise. As test stimulus noise increases, the PSE bias becomes larger. When the reference stimulus (S_{ref}) is 0, the PSE shifts toward negative values, indicating that the agent overestimates the noisy test stimulus, resulting in lower PSE values. Conversely, when (S_{ref}) is 2, the PSE shifts toward positive values, as the agent underestimates the noisy test stimulus, leading to higher PSE values. This indicates that the mean of the prior lies between 0 and 2. To determine the reference stimulus value where the PSE bias is minimized, we performed an optimization procedure. The process interpolates between the PSE and slope values for two conditions (S_{ref}= 0) and (S_{ref}= 2) and minimizes the absolute PSE. The search for the optimal (μ) starts at 1 and is bounded between 0 and 2. The optimization identifies (μ) where the absolute PSE is minimal. The optimization algorithm determined the optimal reference stimulus value to be (μ = 0.967). Figure 3 presents the results of the same experiment conducted with the reference stimulus set to this optimal μ. As shown, the PSE no longer exhibits a noticeable bias as the noise in the test stimulus increases, confirming that the prior mean has been accurately identified as the point where PSE bias is minimized.

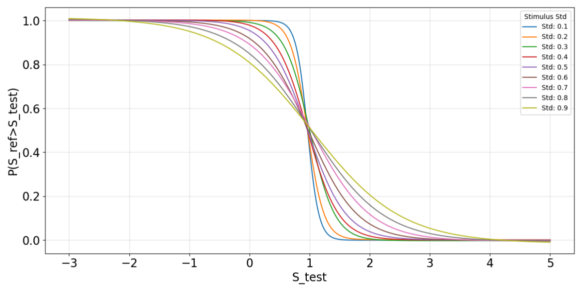

Figure 3

To further estimate the prior, we introduced a variable Z, defined as: $Z = X_1 - X_2$

The psychometric function serves as the cumulative distribution function (CDF) of the posterior distribution. The mean of the posterior distribution aligns with the PSE, and the variance of the posterior is given by:

$\sigma_{posterior} = (\frac{1}{\sqrt{2\pi}\,.\,(slope\ at\ x=\mu)})$;

and $\sigma_Z^2 = \sigma_1^2 + \sigma_2^2$; $\mu_Z = \mu_1 - \mu_2$.

Posterior mean: $\mu_{post} = \frac{\mu_Z/\sigma_Z^2 + \mu_{prior}/\sigma_{prior}^2}{1/\sigma_Z^2 + 1/\sigma_{prior}^2}$

Posterior variance: $\sigma_{post}^2 = \left(\frac{1}{\sigma_Z^2} + \frac{1}{\sigma_{prior}^2}\right)^{-1}$

Using the calculated values for the posterior mean and variance, as well as for (Z), we derived the optimal parameters for the prior distribution. The optimization yielded the following results:

μ_{prior}: 1.012 ; σ_{prior}^2: $1e^{-06}$.

To evaluate the learning process and the influence of training on the agent, we repeated the same experiments with varying numbers of training trials (figure (4)). In the untrained agent, regardless of whether the reference stimulus was 0 or 2, an increase in uncertainty led to a consistent shift in the PSE toward negative values, indicating an overestimation of the test stimulus. This behavior demonstrates that the untrained agent lacks a prior, meaning its prior knowledge is not constrained to any specific value. However, as the number of training trials increased, a bounded window began to emerge, representing the limits within which the mean of the agent's prior is shaped. This window reflects the agent's gradual acquisition of prior knowledge as a product of training, which restricts its interpretation of the stimuli to values within this range.

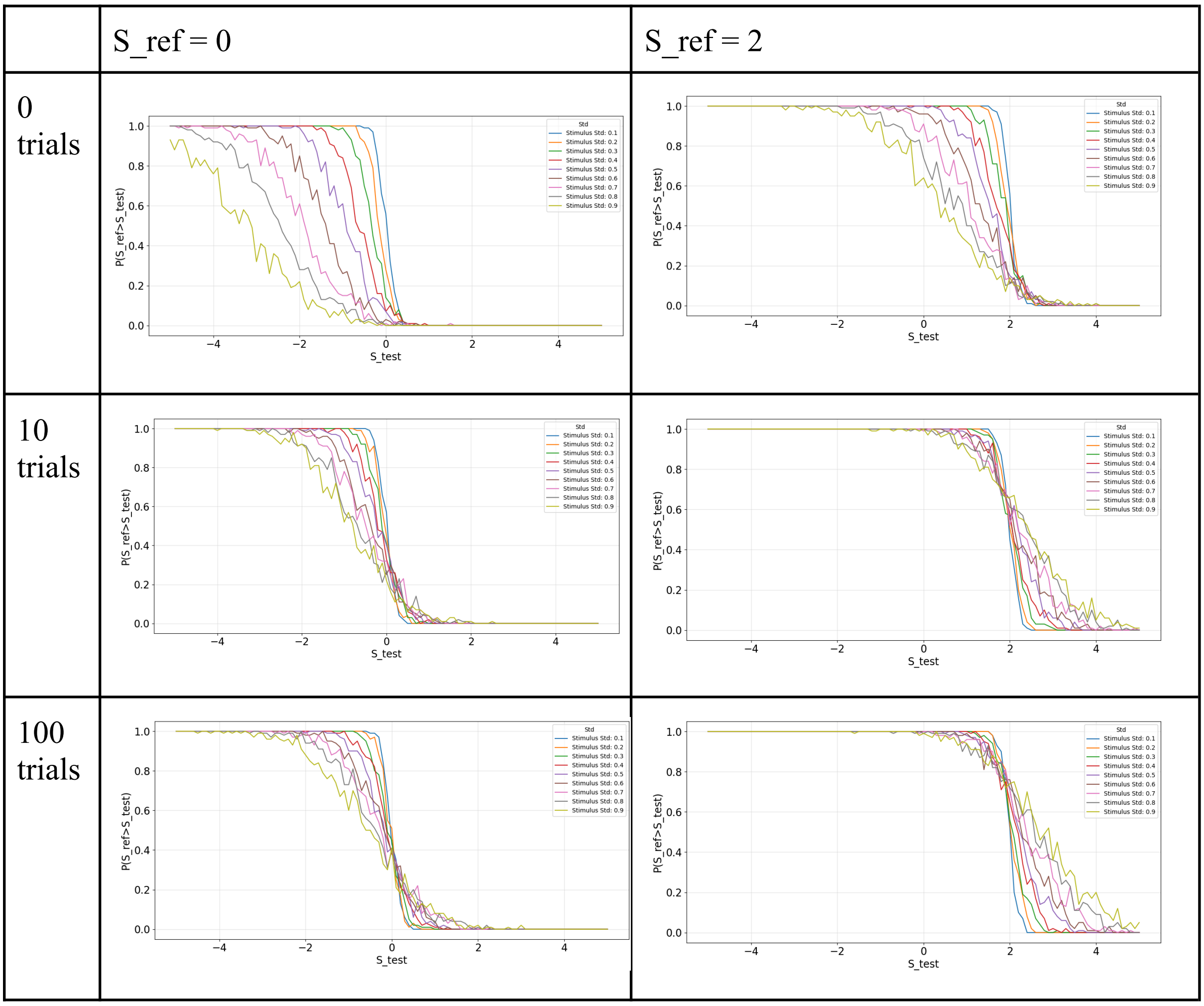

Figure 4

Discussion, Conclusion and Perspectives

The results provide evidence that training significantly influences the decision-making process of the agent under uncertainty. The analyses highlight that as the number of training trials increases, the agent's prior becomes progressively more defined, resulting in more accurate and consistent interpretations of noisy stimuli.

In this project, having access to the range of measured stimuli in the environment significantly simplified the process of identifying the prior. Also, determining the Point of Subjective Equality (PSE) from the psychometric function was made much easier under these controlled conditions. In more realistic settings, running 100 trials for each condition would be impractical, requiring the development and application of alternative algorithms to construct a psychometric function efficiently. Additionally, we operated under the assumption that the prior, likelihood, and posterior distributions were Gaussian, which allowed us to simplify the calculations and derive the prior accordingly. In more complex or realistic scenarios, however, priors may not adhere to such simple distributions or be as easily modeled, posing additional challenges for analysis and interpretation.