

Assignment 4
Of
Modelling & Simulation Lab (CS1052)

Masters of Technology in Computer Science And Engineering

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Problem statement 1

Find the optimal solution to the following transportation problem in which the cells contains the unit transportation cost in rupees.

	w_1	w_2	w_3	w_4	w_5	avail.
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Req.	30	30	15	20	5	

use NWCR and LCM for initial basic feasible solution.

Problem formulation -

	w_1	w_2	w_3	w_4	w_5	Supply
I_1	x_{11} 7	x_{12} 6	x_{13} 4	x_{14} 5	x_{15} 9	40
I_2	x_{21} 8	x_{22} 5	x_{23} 6	x_{24} 7	x_{25} 8	30
I_3	x_{31} 6	x_{32} 8	x_{33} 9	x_{34} 6	x_{35} 5	20
Dem.	x_{41} 5	x_{42} 7	x_{43} 7	x_{44} 8	x_{45} 6	10
Dem.	30	30	15	20	5	

The transportation problem is formulated as an LP model as follows -

Minimize (Total T.P cost) -

$$\begin{aligned}
 & 7x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 9x_{15} \\
 & + 8x_{21} + 5x_{22} + 6x_{23} + 7x_{24} + 8x_{25} \\
 & + 6x_{31} + 8x_{32} + 9x_{33} + 6x_{34} + 5x_{35} \\
 & + 5x_{41} + 7x_{42} + 7x_{43} + 8x_{44} + 6x_{45}
 \end{aligned}$$

Subject to constraint -

$$\begin{aligned}
 & 7x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 9x_{15} = 40 \\
 & 8x_{21} + 5x_{22} + 6x_{23} + 7x_{24} + 8x_{25} = 30 \\
 & 6x_{31} + 8x_{32} + 9x_{33} + 6x_{34} + 5x_{35} = 20 \\
 & 5x_{41} + 7x_{42} + 7x_{43} + 8x_{44} + 6x_{45} = 10
 \end{aligned}$$

First we will solve it using north west corner rule.

The allocation matrix is such that.

	w_1	w_2	w_3	w_4	w_5	exp
F_1	7 (20)	6 (10)	7	5	9	40 10
F_2	8	5 (20)	6 (10)	7	8	30 10
F_3	6	8	9 (5)	6 (15)	5	20 10
F_4	5	7	7	8 (5)	6	10 0
Demand	30 0	30 20 0	15 50	20 150	50	

The step by step description of this solution is given as :-

and

$$2x_{11} + 8x_{21} + 6x_{31} + 5x_{41} = 30$$

$$6x_{12} + 5x_{22} + 8x_{32} + 7x_{42} = 30$$

$$4x_{13} + 6x_{23} + 9x_{33} + 7x_{43} = 15$$

$$5x_{14} + 7x_{24} + 6x_{34} + 8x_{44} = 20$$

$$9x_{15} + 8x_{25} + 5x_{35} + 6x_{45} = 5$$

and $x_{ij} \geq 0$ for $i = 1, 2, 3, 4$

$j = 1, 2, 3, 4$

Soln

Total no of supply const. = 4

Total no of demand const. = 5

They sum up to 40 & 30 compared.

$$\text{Total supply} = 40 + 30 + 20 + 10 = 100$$

$$\text{Total demand} = 30 + 30 + 15 + 20 + 5 = 100$$

Total supply = Total demand
 \Rightarrow Balanced Transp. problem.

North west corner rule

The sum val $F_1 \leq 40$ & $w_1 \leq 30$
 $\min(F_1, w_1) \leq 30$ is assigned
to F_1, w_1 , this meets the
demand of w_1 & leaves $40 - 30 =$
10 with F_1 .

The sum value for $F_2 \leq 30$ &
 $w_2 \leq 20$ are compared. The
 $\min(F_2, w_2) = 20$. This meets
the demand of w_2 & leaves
 $30 - 20 = 10$ units with F_2 .

The sum values for $F_2 \leq 10$
and $w_3 \leq 15$ are compared.
The smaller of the two
i.e. $\min(10, 15) = 10$ is assign.
to F_2, w_3 , this exhaust

The capacity of F_2 and leaves $15 - 10 = 5$ units with w_3 .

The min value for $F_3 = 20$ and $w_3 = 5$ are compared. The $\min(20, 5) = 5$ is assigned to $F_3 w_3$. This meets the complete demand of w_3 and leaves $20 - 5 = 15$ units with F_3 .

The min values for $F_3 = 15$ and $w_4 = 20$ are compared.

The smaller of $15, 20 = 15$ is assigned for $F_3 w_4$, this exhaust the capacity of F_3 and leaves $20 - 15 = 5$ units with w_4 .

The min value for $F_4 = 10$ of $w_4 = 5$ are compared. The

$\min(10, 5) = 5$ is assign to $F_4 w_4$
 This meets the complete demand
 of w_4 and leaves $10 - 5 = 5$ units
 with F_4

The sum value for $F_4 = 5$ & $w_4 = 5$
 are compared.

The smaller of the two i.e.
 $\min(5, 5) = 5$ is assign to
 $F_4 w_5$

The TBFs =

	w_1	w_2	w_3	w_4	w_5	Supp.
F_1	7 ⁽³⁰⁾	6 ⁽¹⁰⁾	4	5	9	40
F_2	8	5 ⁽²⁰⁾	6 ⁽¹⁰⁾	7	8	30
F_3	6	8	9 ⁽⁵⁾	6 ⁽¹⁵⁾	5	20
F_4	5	7	7	8 ⁽⁵⁾	6 ⁽⁵⁾	10
Demand	30	30	15	20	5	

$$V_1 = 7 - 0 = 7$$

$$V_2 = 6 - 0 = 6$$

$$V_1 = C_{11} - u_1 = 7 - 0 \Rightarrow V_1 = 7$$

$$V_2 = C_{12} - u_1 = 6 - 0 \Rightarrow V_2 = 6$$

$$u_2 = C_{22} - V_2 = 5 - 6 \Rightarrow u_2 = -1$$

$$V_3 = C_{23} - u_2 = 6 + 1 \Rightarrow V_3 = 7$$

$$u_3 = C_{33} - V_3 = 9 - 7 \Rightarrow u_3 = 2$$

$$u_4 = C_{44} - V_4 = 8 - 4 \Rightarrow u_4 = 4$$

$$V_5 = C_{45} - u_4 = 6 - 4 \Rightarrow V_5 = 2$$

	w_1	w_2	w_3	w_4	w_5	S	u_i
F_1	7 ⁽²⁰⁾	6 ⁽¹⁰⁾	4	5	9	40	$u_1 = 0$
F_2	8	5 ⁽²⁰⁾	6 ⁽¹⁰⁾	7	8	20	$u_2 = -1$
F_3	6	8	9 ⁽⁵⁾	6 ⁽¹⁵⁾	5	20	$u_3 = 2$
F_4	5	7	7	8 ⁽⁵⁾	6 ⁽⁵⁾	10	$u_4 = 4$
F_5	30	30	15	20	5		
V_i	$V_1 = 7$	$V_2 = 6$	$V_3 = 7$	$V_4 = 4$	$V_5 = 2$		

The minimization T.P cost =

$$7 \times 30 + 6 \times 10 + 5 \times 20 + 8 \times 10 + 9 \times 5 \\ + 6 \times 15 + 8 \times 5 + 6 \times 5 = 635$$

$$\text{Total allocated cell} = 8 = m+n-1 \\ = 4+5-1 \\ = 8$$

hence the soln is non degenerate

optimality test using mooli method.

allocation table is

	w_1	w_2	w_3	w_4	w_5	Supp.
F_1	7 ⁽²⁰⁾	6 ⁽¹⁰⁾	9	5	9	40
F_2	8	5 ⁽²⁰⁾	6 ⁽¹⁰⁾	7	8	30
F_3	6	8	9 ⁽⁵⁾	6 ⁽¹⁵⁾	5	20
F_4	5	7	7	8 ⁽⁵⁾	6 ⁽⁵⁾	10
Dem.	30	30	15	20	5	

Iteration 1 of optimality test.

① since $u_i = 0$, we get

② find $d_{ij} = c_{ij} - (u_i + v_j)$

$$d_{13} = c_{13} - (u_1 + v_3) = 4 - (0 + 7) = -3$$

$$d_{14} = 5 - (0 + 4) = 1$$

$$d_{15} = 9 - (0 + 2) = 7$$

$$d_{21} = 8 - (-1 + 7) = 2$$

$$d_{24} = 7 - (-1 + 4) = 4$$

$$d_{25} = 8 - (-1 + 2) = 7$$

$$d_{31} = 6 - (2 + 7) = -3$$

$$d_{32} = 8 - (2 + 6) = 0$$

$$d_{35} = 5 - (2 + 2) = 1$$

$$d_{41} = 5 - (4 + 7) = -6$$

$$d_{42} = 7 - (4 + 6) = -3$$

$$d_{43} = 7 - (4 + 7) = -4$$

3)

	w_1	w_2	w_3	w_4	w_5	supp	u_i
f_1	$7^{(20)} (-)$	$8^{(10)} (+)$	4^{-3}	$5^{(9)} (+)$	9^7	40	0
f_2	8^2	$5^{(20)} (-)$	$6^{(10)} (+)$	7^4	8^7	30	7
f_3	6^{-3}	8^0	$9^{(5)} (-)$	$6^{(15)} (+)$	5^1	20	2
f_4	$5^{(6)} (+)$	7^{-3}	7^{-4}	$8^{(5)} (-)$	$6^{(5)}$	10	4
dem	30	30	15	20	5		
v_0	7	8	7	4	2		

$\min(\text{dis}) = 6$, $F_4 w_1$

closed path = $F_4 w_1 \rightarrow F_4 w_2 \rightarrow F_3 w_2 \rightarrow$

$F_3 w_3 \rightarrow F_2 w_3 \rightarrow F_2 w_2 \rightarrow$

$F_1 w_2 \rightarrow F_1 w_1$

4) min allocated val among all-ne is 5

	w_1	w_2	w_3	w_4	w_5	Σ
F_1	7 ⁽²⁵⁾	6 ⁽¹⁴⁾	4	5 ^(E)	9	40
F_2	8	5 ⁽¹⁵⁾	6 ⁽¹⁹⁾	7	8	30
F_3	6	8	9	6 ⁽²⁰⁾	9	20
F_4	5 ⁽⁹⁾	7	7	8	6 ⁽⁵⁾	10
D	30	30	15	20	5	

The result is degenerate since

no of alloc = 7 < 8

hence quantity ~~E~~ is assign.

to $F_1 w_4$, which has min.

t.p cost etc.

$$d_{2a} = 7 - (-1 + 9) = 3$$

$$d_{2b} = 8 - (-1 + 8) = 1$$

$$d_{31} = 6 - (1 + 7) = -2$$

$$d_{32} = 8 - (1 + 6) = 1$$

$$d_{33} = 9 - (1 + 7) = 1$$

$$d_{35} = 5 - (1 + 8) = -4$$

$$d_{42} = 7 - (-2 + 6) = 3$$

$$d_{43} = 7 - (-2 + 7) = 0$$

$$d_{44} = 8 - (-2 + 9) = 5$$

min -ve value from all $d_{ij} = -4$
and draw path from $F_3 w_5$

closed path is $F_3 w_5 \rightarrow F_3 w_4 \rightarrow$
 $F_1 w_4 \rightarrow F_1 w_1 \rightarrow F_4 w_1 \rightarrow F_4 w_5$

min value among all -ve
point on closed path = 5
hence subtract from $-f$
again the.

	w_1	w_2	w_3	w_4	w_5	supply
F_1	7 ⁽²⁰⁾	6 ⁽¹⁵⁾	4	5 ⁽⁵⁾	9	40
F_2	8	5 ⁽¹⁵⁾	6 ⁽¹⁵⁾	7	8	30
F_3	6	8	9	6 ⁽¹⁵⁾	5 ⁽⁵⁾	20
F_4	5 ⁽¹⁰⁾	7	7	8	6	10
Dem.	30	30	15	20	5	

Iteration 3

substituting $u_1 = 0$,

$$u_1 = 7 - 0 = 7$$

$$u_2 = 5 - 7 = -2$$

$$v_2 = 6 - 0 = 6$$

$$u_2 = 5 - 6 = -1$$

$$v_3 = 6 + 1 = 7$$

$$v_4 = 5 - 0 = 5$$

$$u_3 = 6 - 5 = 1$$

$$v_5 = 5 - 1 = 4$$

	w_1	w_2	w_3	w_4	w_5	s	u_i
F_1	7 ⁽²⁰⁾	6 ⁽¹⁵⁾	4 ⁽⁺⁾	5 ⁽⁵⁾	9	40	0
F_2	8	5 ⁽¹⁵⁾	6 ⁽¹⁵⁾	7 ⁽²⁾	8	30	1
F_3	6	8	9	6 ⁽¹⁵⁾	5 ⁽⁵⁾	20	1
F_4	5 ⁽¹⁰⁾	7	7	8	6	10	2
Dem.	30	30	15	20	5		
v_j	7	6	7	5	4		

$$\Delta_{13} \rightarrow d_{13} = 4 - (0 + 7) = -3$$

$$d_{15} = 9 - (0 + 1) = 8$$

$$d_{21} = 8 - (-1 + 7) = 2$$

$$d_{21} = 2 - (-1 + 5) = 2$$

$$d_{25} = 8 - (-1 + 4) = 5$$

$$d_{31} = 6 - (1 + 7) = -2$$

$$d_{32} = 8 - (1 + 6) = 1$$

$$d_{33} = 9 - (1 + 7) = 1$$

$$d_{42} = 7 - (-2 + 6) = 3$$

$$d_{43} = 7 - (-2 + 7) = 2$$

$$d_{44} = 8 - (-2 + 5) = 5$$

$$d_{45} = 6 - (-2 + 4) = 4$$

→ $d_{13} = [-13]$ minimum.

closed path = $F_1 w_3 \rightarrow F_1 w_2 \rightarrow F_2 w_2 \rightarrow F_2 w_3$

→ min value among all. $(-ve) = 15$

	w_1	w_2	w_3	w_4	w_5	Supply
F_1	7 ⁽²⁰⁾	6 ⁽⁵⁾	4 ⁽¹⁵⁾	5 ⁽⁵⁾	9	40
F_2	8	5 ⁽³⁰⁾	6	7	8	30
F_3	6	8	9	6 ⁽¹⁵⁾	5 ⁽⁵⁾	20
F_4	5 ⁽¹⁰⁾	7	7	8	6	10
Demand	30	30	15	20	5	

The solⁿ is degenerate as $7 < 8$

Hence ϵ is assigned to $F_1 w_2$

at min cost of 6.

Iteration 4

Substituting $u_1 = 0$,

$$v_1 = 7 - 0 = 7$$

$$u_2 = 9 - 7 = -2$$

$$v_2 = 6 - 0 = 6$$

$$u_2 = 9 - 6 = 3$$

$$v_3 = 4 - 0 = 4$$

$$v_4 = 9 - 0 = 9$$

$$u_3 = 6 - 9 = -3$$

$$v_5 = 5 - (-3) = 8$$

	w_1	w_2	w_3	w_4	w_5	S	u_p
F_1	7 ⁽²⁰⁾ (-)	6 ⁽¹⁰⁾	7 ⁽¹⁵⁾	5 ⁽⁵⁾ (+)	9	40	0
F_2	8 ²	5 ⁽²⁰⁾	6 ³	7 ³	8	30	-1
F_3	6 ⁻² (+)	2 ¹	9 ⁴	6 ⁽¹⁵⁾ (-)	5 ⁽⁵⁾	20	1
F_4	5 ⁽¹⁰⁾	7 ³	7 ⁵	8 ⁵	6 ⁴	10	-2
F_5	30	30	15	20	5		
F_6	7	6	4	5	4		

$$\rightarrow d_{15} = 9 - (0 + 4) = 5$$

$$d_{21} = 8 - (-1 + 7) = 2$$

$$d_{23} = 6 - (-1 + 4) = 3$$

Iteration 5

~~simultaneous~~ by putting $u_4 = 0$

$$v_1 = 7 - 0 = 7$$

$$u_3 = 6 - 7 = -1$$

$$v_5 = 9 + 1 = 10$$

$$u_2 = 5 - 7 = -2$$

$$v_2 = 6 - 0 = 6$$

$$u_2 = 5 - 6 = -1$$

$$v_3 = 4 - 0 = 4$$

$$v_4 = 5 - 0 = 5$$

	w_1	w_2	w_3	w_4	w_5	c.	up
F_1	7 ⁽⁷⁾	6 ⁽⁶⁾	4 ⁽¹⁵⁾	5 ⁽²⁰⁾	9 ⁽⁹⁾	40	0
F_2	8 ²	5 ⁽²⁰⁾	6 ³	7 ³	8 ³	30	-1
F_3	6 ⁽¹⁵⁾	8 ³	9 ⁶	6 ²	5 ⁽⁵⁾	20	-1
F_4	5 ⁽¹⁰⁾	7 ³	7 ⁵	8 ⁵	6 ²	10	-2
D	30	30	15	20	5		
v_j	7	6	4	5	6		

$$d_{15} = 9 - (0 + 6) = 3$$

$$d_{21} = 8 - (-1 + 7) = 2$$

$$d_{23} = 6 - (-1 + 4) = 3$$

$$d_{24} = 7 - (-1 + 5) = 3$$

$$d_{25} = 8 - (-1 + 6) = 3$$

$$d_{32} = 8 - (-1 + 6) = 3$$

$$d_{33} = 9 - (-1 + 4) = 6$$

$$d_{34} = 6 - (-1 + 5) = 2$$

$$d_{42} = 7 - (-2 + 6) = 3$$

$$d_{43} = 7 - (-2 + 6) = 5$$

$$d_{44} = 8 - (-2 + 5) = 5$$

$$d_{45} = 6 - (-2 + 6) = 2$$

since all $d_{ij} \geq 0$

so final ans \rightarrow

	w_1	w_2	w_3	w_4	w_5	Supp.
F_1	7 ⁵	6 ¹⁵	4 ¹⁵	5 ²⁰	9	40
F_2	8	5 ³⁰	6	7	8	30
F_3	6 ¹⁵	8	9	6	5 ⁵	20
F_4	5 ¹⁰	7	7	8	6	10
D.	30	30	15	20	5	

min Tot. TPC $= 7 \times 5 + 4 \times 15 + 5 \times 30 + 9 \times 30 + 6 \times 15 + 5 \times 5 + 9 \times 10 = 510$.

Least cost method -

	w_1	w_2	w_3	w_4	w_5	supply
F_1	7⁽⁵⁾	6	4⁽¹⁵⁾	5⁽²⁰⁾	9	40 ²⁵ 5
F_2	8	5⁽³⁰⁾	6	7	8	30 0
F_3	6⁽¹⁵⁾	8	9	6	5⁽⁵⁾	20 ¹⁵ 5
F_4	5⁽¹⁰⁾	7	7	8	6	10 0
Dem.	30	30	15	20	5	
	20	0	0	0	0	
	10					

→ The smallest TP cost is 4 in $F_1 w_3$,
 The alloc to $F_1 w_3 \geq \min(40, 15)$
 $= 15$

This satisf. entire dem. of w_3 & leaves
 $40 - 15 = 25$ with F_1

→ The next TP cost is 5 in $F_2 w_2$
 the alloc to $F_2 w_2 \geq \min(30, 30)$
 $= 30$
 $w_2 \geq 30$, $F_2 \geq 0$

→ The next TP is 5 in $F_1 W_4$
the alloc to $F_1 W_4 = \min(20, 20)$
 $W_4 = 0, F_1 = 5$

→ The next TP is 5 in $F_4 W_1$
alloc of $F_4 W_1 = \min(10, 20) = 10$
 $W_1 = 20, F_4 = 0$

→ The next TP is 5 in $F_3 W_5$
alloc of $F_3 W_5 = \min(20, 5) = 5$
 $W_5 = 0, F_3 = 15$.

→ next min TP is 6 in $F_3 W_1$
the alloc to $F_3 W_1 = \min(15, 20)$
 $= 15$

This exhaust $F_3 = 0$ & $W_1 = 5$

→ next min TP is 7 in $F_1 W_1$
alloc to $F_1 W_1 = \min(5, 5) = 5$
 $F_1 = 0, W_1 = 0$.

Hence the IBFS is

	w_1	w_2	w_3	w_4	w_5	S.
F_1	7 (1)	6 (E)	4 (16)	5 (20)	9	40
F_2	8	5 (20)	6	7	8	30
F_3	6 (5)	8	9	6	5 (5)	20
F_4	5 (10)	7	7	8	6	10
Δ	30	30	15	20	5	

$$\begin{aligned} \min \text{ TPC} &= 7 \times 5 + 4 \times 15 + 5 \times 20 + 5 \times 30 + \\ & 6 \times 5 + 5 \times 5 + 5 \times 10 \\ &= 510 \end{aligned}$$

here no. of allocⁿ is 7
no. of maxⁿ is 4 + 5 = 9

hence it is degenerate.

to resolve degeneracy, if we add E into F_1, w_2 at cost 6.

Optimal test

Substituting $u_i \geq 0$, we get.

$$V_1 = 7 - 0 = 7$$

$$U_2 = 6 - 7 = -1$$

$$V_5 = 5 + 1 = 6$$

$$U_4 = 9 - 7 = -2$$

$$V_2 = 6 - 0 = 6$$

$$U_2 = 9 - 6 = 3$$

$$V_3 = 4 - 0 = 4$$

$$V_4 = 9 - 0 = 9$$

$$\Rightarrow d_{15} = 9 - (0 + 6) = 3$$

$$d_{21} = 8 - (-1 + 7) = 2$$

$$d_{23} = 6 - (-1 + 4) = 3$$

$$d_{24} = 7 - (-1 + 9) = 3$$

$$d_{25} = 8 - (-1 + 6) = 3$$

$$d_{32} = 8 - (-1 + 6) = 3$$

$$d_{33} = 9 - (-1 + 4) = 6$$

$$d_{34} = 6 - (-1 + 9) = 2$$

$$d_{42} = 7 - (-2 + 6) = 3$$

$$d_{43} = 7 - (-2 + 4) = 5$$

$$d_{44} = 8 - (-2 + 9) = 5$$

$$d_{45} = 6 - (-2 + 6) = 2$$

Since all $d_{ij} > 0$,

so the solⁿ is optimal & unique

The min total transp. cost =

$$7 \times 5 + 4 \times 15 + 5 \times 20 + 5 \times 30 +$$

$$6 \times 15 + 5 \times 5 + 5 \times 10$$

$$= 510.$$

Python Code:

```
import numpy as np
from collections import Counter
import math

def check_loop(p, row, column):
    p[row, column] = -1
    flag = 1
    while flag != 0:
        flag = 0
        if p.size != 0:
            row = np.count_nonzero(p, axis=1)
            f = 0
            for index in range(len(row)):
                if row[index] < 2:
                    flag = 1
                    p = np.delete(p, (index - f), axis=0)
                    f += 1
        if p.size != 0:
            e = 0
            col = np.count_nonzero(p, axis=0)
            for index in range(len(col)):
                if col[index] < 2:
                    flag = 1
                    p = np.delete(p, (index - e), axis=1)
                    e += 1
    if p.size != 0:
        return 0
    else:
        return 1

def max_allocation_row(non_zero_list):
    max_values = np.zeros(len(non_zero_list))
    for val in non_zero_list:
        max_values[val[0]] += 1.0
    return np.where(max_values == np.amax(max_values))

def modi(c_modi, a_modi, m_modi, n_modi):
    iteration_count = 1
    while True:
        print()
        print("Iteration - ", iteration_count)
        print("Start AL \n", a_modi)
        u = np.array([np.nan] * m_modi)
        v = np.array([np.nan] * n_modi)
        p = np.zeros((m_modi, n_modi))

        _x, _y = np.where(a_modi > 0)
        nonzero = list(zip(_x, _y))
        _x1, _y1 = np.where(a_modi == -1)
        if -1 in a_modi:
            nz_a = np.where(a_modi == -1)
            nonzero.append((min(nz_a[0]), min(nz_a[1])))
        f = max_allocation_row(nonzero)
        u[f[0][0]] = 0
        for i, j in nonzero:
            if i == f[0][0]:
                v[j] = c_modi[i, j] - u[i]
        print("U = ", u)
        print("V = ", v)
        while any(np.isnan(u)) or any(np.isnan(v)):
```

```

for i, j in nonzero:
    for j2 in range(0, len(v)):
        if j2 == j and not math.isnan(v[j]) and math.isnan(u[i]):
            u[i] = c_modi[i, j] - v[j]
for i, j in nonzero:
    for j2 in range(0, len(u)):
        if j2 == i and not math.isnan(u[i]) and math.isnan(v[j]):
            v[j] = c_modi[i, j] - u[i]
print("U = ", u)
print("V = ", v)

# Finding P-matrix
for i in range(m_modi):
    for j in range(n_modi):
        if not nonzero.__contains__((i, j)):
            p[i, j] = c_modi[i, j] - u[i] - v[j]
print("P-matrix")
print(p)
# Stop condition
small_val = np.min(p)
if small_val >= 0:
    break
i, j = np.argwhere(p == small_val)[0]
start = (i, j)
print("Start : ", start)
# Finding cycle elements
t = np.copy(a_modi)
t[start] = 1
while True:
    _xs, _ys = np.nonzero(t)
    xcount, ycount = Counter(_xs), Counter(_ys)
    for x, count in xcount.items():
        if count <= 1:
            t[x, :] = 0
    for y, count in ycount.items():
        if count <= 1:
            t[:, y] = 0
    if all(x > 1 for x in xcount.values()) and all(y > 1 for y in ycount.values()):
        break

# Finding cycle chain order
def dist(x1, y1, x2, y2):
    if x1 == x2 or y1 == y2:
        return abs(x1 - x2) + abs(y1 - y2)
    else:
        return np.inf

fringe = set(tuple(p) for p in np.argwhere(t != 0))
alloc_modi = fringe
size = len(fringe)
path = [start]
while len(path) < size:
    last = path[-1]
    if last in fringe:
        fringe.remove(last)
    next_val = min(fringe, key=lambda xy: dist(last[0], last[1], xy[0], xy[1]))
    path.append(next_val)

# Improving solution on cycle elements
neg = path[1::2]
pos = path[::2]
print("Negative Value:", neg)
print("Positive Value:", pos)
ql = []
for row in neg:
    ql.append(a_modi[row[0], row[1]])

```

```

q = int(min(q1))
for row in neg:
    if a_modi[row[0], row[1]] == -1:
        a_modi[row[0], row[1]] = 0 - q
    else:
        a_modi[row[0], row[1]] -= q
for row in pos:
    if a_modi[row[0], row[1]] == -1:
        a_modi[row[0], row[1]] = 0 + q
    else:
        a_modi[row[0], row[1]] += q

x_al = np.nonzero(a_modi)[0]
y_al = np.nonzero(a_modi)[1]
for i in range(len(x_al)):
    alloc_modi.add((x_al[i], y_al[i]))
print("Mid Allocation Table : \n", a_modi)

_x2, _y2 = np.where(a_modi > 0)
# alloc_modi = list(zip(_x2, _y2))
no_alloc_modi = np.count_nonzero(a_modi)
unalloc_modi = []
for i1 in range(m_modi):
    for j1 in range(n_modi):
        if not (i1, j1) in alloc_modi:
            unalloc_modi.append((i1, j1))
print(a_modi)
no_loop_modi = []
if no_alloc_modi == m_modi + n_modi - 1:
    print("Non Degeneracy")
else:
    print("Degeneracy")
    print("Values of epsilon is -1")
    for i1 in unalloc_modi:
        if check_loop(a_modi.copy(), i1[0], i1[1]) == 1:
            no_loop_modi.append(i1)
    min_epi_list = []
    for i1 in no_loop_modi:
        min_epi_list.append(c_modi[i1[0], i1[1]])
    min_epi = min(min_epi_list)
    ind = min_epi_list.index(min_epi)
    loc = no_loop_modi[ind]
    a_modi[loc[0], loc[1]] = -1
    print("END AL : \n", a_modi)
    print()
iteration_count += 1

return a_modi

```

```

def nwcr(cm_nwcr, m_nwcr, n_nwcr, s_nwcr, d_nwcr):
    c_nwcr = cm_nwcr.copy()
    a = np.zeros(c_nwcr.shape)
    total_cost_nwcr = 0
    no_alloc_nwcr = 0
    alloc_nwcr = []
    i = 0
    j = 0
    while (i < m_main) and (j < n_main):
        x = min(s_nwcr[i], d_nwcr[j])
        s_nwcr[i] = s_nwcr[i] - x
        d_nwcr[j] = d_nwcr[j] - x
        total_cost_nwcr = total_cost_nwcr + x * c_nwcr[i, j]
        no_alloc_nwcr += 1
        alloc_nwcr.append((i, j))
        a[i, j] = x

```



```

    if s_nwcr[i] < d_nwcr[j]:
        i = i + 1
    elif s_nwcr[i] > d_nwcr[j]:
        j = j + 1
    else:
        i = i + 1
        j = j + 1
print("Total Cost: ", total_cost_nwcr)
unalloc_nwcr = []
for i1 in range(m_main):
    for j1 in range(n_main):
        if not (i1, j1) in alloc_nwcr:
            unalloc_nwcr.append((i1, j1))
print("Allocated Positions: ", alloc_nwcr)
print("Unallocated Positions: ", unalloc_nwcr)
print("Allocation Matrix: ")
print(a)
no_loop_nwcr = []
if no_alloc_nwcr == m_nwcr + n_nwcr - 1:
    print("Non Degeneracy")
else:
    print("Degeneracy")
    print("Values of epsilon is -1")
    for i1 in unalloc_nwcr:
        if check_loop(a.copy(), i1[0], i1[1]) == 1:
            no_loop_nwcr.append(i1)
    min_epi_list = []
    for i1 in no_loop_nwcr:
        min_epi_list.append(cm_nwcr[i1[0], i1[1]])

    min_epi = min(min_epi_list)
    ind = min_epi_list.index(min_epi)
    loc = no_loop_nwcr[ind]
    a[loc[0], loc[1]] = -1
    print("Allocation Matrix After Converting Degeneracy to Non-Degeneracy is : ")
    print(a)
    opt1 = modi(cm_nwcr.copy(), a.copy(), m_nwcr, n_nwcr, )
    print('optimised Allocation Matrix: ')
    print(opt1)
    for row in range(0, m_nwcr):
        for column in range(0, n_nwcr):
            if opt1[row][column] < 0:
                opt1[row][column] = 0
    print("Total Optimal Cost = ", np.sum(opt1 * cm_nwcr))

```

```

def lcm(cm_lcm, m_lcm, n_lcm, s_lcm, d_lcm):
    c_lcm = cm_lcm.copy()
    total_cost_lcm = 0
    no_alloc_lcm = 0
    alloc_lcm = []
    a = np.zeros(c_lcm.shape)
    min_cost = np.amin(c_lcm)
    while min_cost != np.inf:
        indexes = np.where(c_lcm == min_cost)
        i = indexes[0][0]
        j = indexes[1][0]
        x = min(s_lcm[i], d_lcm[j])
        s_lcm[i] -= x
        d_lcm[j] -= x
        total_cost_lcm += (x * c_lcm[i, j])
        no_alloc_lcm += 1
        a[i, j] = x
        alloc_lcm.append((i, j))
        if s_lcm[i] < d_lcm[j]:
            x = 0

```

```

        while x < n_lcm:
            c_lcm[i, x] = np.inf
            x += 1
    elif s_lcm[i] > d_lcm[j]:
        y = 0
        while y < m_lcm:
            c_lcm[y, j] = np.inf
            y += 1
    else:
        x = 0
        while x < n_lcm:
            c_lcm[i, x] = np.inf
            x += 1
        y = 0
        while y < m_lcm:
            c_lcm[y, j] = np.inf
            y += 1
    min_cost = np.amin(c_lcm)
print("Total Cost: ", total_cost_lcm)
unalloc_lcm = []
for i in range(m_lcm):
    for j in range(n_lcm):
        if not (i, j) in alloc_lcm:
            unalloc_lcm.append((i, j))
print("List of Allocated Positions: ", alloc_lcm)
print("List of Unallocated Positions: ", unalloc_lcm)
print("Allocation Matrix: ")
print(a)
no_loop = []
if no_alloc_lcm == m_lcm + n_lcm - 1:
    print("Non Degeneracy")
else:
    print("Degeneracy")
    print("Values of epsilon is -1")
    for i in unalloc_lcm:
        if check_loop(a.copy(), i[0], i[1]) == 1:
            no_loop.append(i)

    min_epi_list = []
    for i in no_loop:
        min_epi_list.append(cm_lcm[i[0], i[1]])

    min_epi = min(min_epi_list)
    ind = min_epi_list.index(min_epi)
    loc = no_loop[ind]
    a[loc[0], loc[1]] = -1
    print("Allocation Matrix After Converting Degeneracy to Non-Degeneracy is : ")
    print(a)
    opt1 = modi(cm_lcm.copy(), a.copy(), m_lcm, n_lcm, )
    print('optimised Allocation Matrix: ')
    print(opt1)
    for row in range(0, m_lcm):
        for column in range(0, n_lcm):
            if opt1[row][column] < 0:
                opt1[row][column] = 0
    print("Total Optimal Cost = ", np.sum(opt1 * cm_lcm))

if __name__ == '__main__':
    cm_main = np.array([[7.0, 6, 4, 5, 9],
                        [8, 5, 6, 7, 8],
                        [6, 8, 9, 6, 5],
                        [5, 7, 7, 8, 6]])
    s_main = np.array([40.0, 30, 20, 10])
    d_main = np.array([30.0, 30, 15, 20, 5])
    c_main = cm_main.copy()

```

```

print("The Cost Matrix is: ")
print(c_main)
print("The Supply is: ", s_main)
print("The Demand is: ", d_main)
m_main, n_main = c_main.shape
print("No of Rows & No of Columns: (", m_main, ", ", n_main, ")")
total_demand = np.sum(d_main)
total_supply = np.sum(s_main)
if total_demand == total_supply:
    print("It is a Balanced Transportation Problem")
else:
    print("It is an UnBalanced Transportation Problem")
    if total_demand > total_supply:
        new = np.array(np.zeros(n_main))
        c_main = np.row_stack((c_main, new))
        s_main = np.append(s_main, total_demand - total_supply)
        m_main = m_main + 1
    else:
        new = np.array(np.zeros(m_main))
        c_main = np.column_stack((c_main, new))
        d_main = np.append(d_main, total_supply - total_demand)
        n_main = n_main + 1
    print("The New Balanced Cost Matrix is: ")
    print(c_main)
    print("The Supply is: ", s_main)
    print("The Demand is: ", d_main)
print("Northwest corner method")
nwcr(c_main.copy(), m_main, n_main, s_main.copy(), d_main.copy())
print()
print("Least Cost Method")
lcm(c_main.copy(), m_main, n_main, s_main.copy(), d_main.copy())
print()

```


Output :

```
"/home/arghya/My Work/Python/pythonProject1/venv/bin/python" "/home/arghya/My Work/Python/pythonProject1/assignment4problem1.py"
The Cost Matrix is:
[[7. 6. 4. 5. 9.]
 [8. 5. 6. 7. 8.]
 [6. 8. 9. 6. 5.]
 [5. 7. 7. 8. 6.]]
The Supply is: [40. 30. 20. 10.]
The Demand is: [30. 30. 15. 20. 5.]
No of Rows & No of Columns: ( 4 , 5 )
It is a Balanced Transportation Problem
Northwest corner method
Total Cost: 635.0
Allocated Positions: [(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), (2, 3), (3, 3), (3, 4)]
Unallocated Positions: [(0, 2), (0, 3), (0, 4), (1, 0), (1, 3), (1, 4), (2, 0), (2, 1), (2, 4), (3, 0), (3, 1), (3, 2)]
Allocation Matrix:
[[30. 10. 0. 0. 0.]
 [ 0. 20. 10. 0. 0.]
 [ 0. 0. 5. 15. 0.]
 [ 0. 0. 0. 5. 5.]]
Non Degeneracy

Iteration - 1
Start AL
[[30. 10. 0. 0. 0.]
 [ 0. 20. 10. 0. 0.]]
Start AL
[[30. 10. 0. 0. 0.]
 [ 0. 20. 10. 0. 0.]]
[ 0. 0. 5. 15. 0.]
[ 0. 0. 0. 5. 5.]]
U = [ 0. nan nan nan]
V = [ 7. 6. nan nan nan]
U = [ 0. -1. nan nan]
V = [ 7. 6. 7. nan nan]
U = [ 0. -1. 2. nan]
V = [ 7. 6. 7. 4. nan]
U = [ 0. -1. 2. 4.]
V = [7. 6. 7. 4. 2.]
P-matrix
[[ 0. 0. -3. 1. 7.]
 [ 2. 0. 0. 4. 7.]
 [-3. 0. 0. 0. 1.]
 [-6. -3. -4. 0. 0.]]
Start : (3, 0)
Negative Value: [(0, 0), (1, 1), (2, 2), (3, 3)]
Positive Value: [(3, 0), (0, 1), (1, 2), (2, 3)]
Mid Allocation Table :
[[25. 15. 0. 0. 0.]
 [ 0. 15. 15. 0. 0.]
 [ 0. 0. 0. 20. 0.]
```

Mid Allocation Table :

```
[[25. 15.  0.  0.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 20.  0.]
 [ 5.  0.  0.  0.  5.]]
[[25. 15.  0.  0.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 20.  0.]
 [ 5.  0.  0.  0.  5.]]
```

Degeneracy

Values of epsilon is -1

END AL :

```
[[25. 15.  0. -1.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 20.  0.]
 [ 5.  0.  0.  0.  5.]]
```

Iteration - 2

Start AL

```
[[25. 15.  0. -1.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 20.  0.]
 [ 5.  0.  0.  0.  5.]]
```

U = [0. nan nan nan]

U = [0. nan nan nan]

V = [7. 6. nan 5. nan]

U = [0. -1. 1. -2.]

V = [7. 6. 7. 5. 8.]

P-matrix

```
[[ 0.  0. -3.  0.  1.]
 [ 2.  0.  0.  3.  1.]
 [-2.  1.  1.  0. -4.]
 [ 0.  3.  2.  5.  0.]]
```

Start : (2, 4)

Negative Value: [(3, 4), (0, 0), (2, 3)]

Positive Value: [(2, 4), (3, 0), (0, 3)]

Mid Allocation Table :

```
[[20. 15.  0.  5.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 15.  5.]
 [10.  0.  0.  0.  0.]]
[[20. 15.  0.  5.  0.]
 [ 0. 15. 15.  0.  0.]
 [ 0.  0.  0. 15.  5.]
 [10.  0.  0.  0.  0.]]
```

Non Degeneracy

Iteration - 3

Start AL

```
Iteration - 3
Start AL
[[20. 15. 0. 5. 0.]
 [ 0. 15. 15. 0. 0.]
 [ 0. 0. 0. 15. 5.]
 [10. 0. 0. 0. 0.]]
U = [ 0. nan nan nan]
V = [ 7. 6. nan 5. nan]
U = [ 0. -1. 1. -2.]
V = [7. 6. 7. 5. 4.]
P-matrix
[[ 0. 0. -3. 0. 5.]
 [ 2. 0. 0. 3. 5.]
 [-2. 1. 1. 0. 0.]
 [ 0. 3. 2. 5. 4.]]
Start : (0, 2)
Negative Value: [(0, 1), (1, 2)]
Positive Value: [(0, 2), (1, 1)]
Mid Allocation Table :
[[20. 0. 15. 5. 0.]
 [ 0. 30. 0. 0. 0.]
 [ 0. 0. 0. 15. 5.]
 [10. 0. 0. 0. 0.]]
[[20. 0. 15. 5. 0.]
 [ 0. 30. 0. 0. 0.]
```

```
[[20. 0. 15. 5. 0.]
 [ 0. 30. 0. 0. 0.]
 [ 0. 0. 0. 15. 5.]
 [10. 0. 0. 0. 0.]]
```

```
Degeneracy
Values of epsilon is -1
END AL :
[[20. -1. 15. 5. 0.]
 [ 0. 30. 0. 0. 0.]
 [ 0. 0. 0. 15. 5.]
 [10. 0. 0. 0. 0.]]
```

```
Iteration - 4
Start AL
[[20. -1. 15. 5. 0.]
 [ 0. 30. 0. 0. 0.]
 [ 0. 0. 0. 15. 5.]
 [10. 0. 0. 0. 0.]]
U = [ 0. nan nan nan]
V = [ 7. 6. 4. 5. nan]
U = [ 0. -1. 1. -2.]
V = [7. 6. 4. 5. 4.]
P-matrix
[[ 0. 0. 0. 0. 5.]
```



```
P-matrix
[[ 0.  0.  0.  0.  5.]
 [ 2.  0.  3.  3.  5.]
 [-2.  1.  4.  0.  0.]
 [ 0.  3.  5.  5.  4.]]

Start : (2, 0)
Negative Value: [(0, 0), (2, 3)]
Positive Value: [(2, 0), (0, 3)]
```

```
Mid Allocation Table :
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]

[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
```

Non Degeneracy

```
Iteration - 5
Start AL
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
```

```
Iteration - 5
Start AL
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]

U = [ 0. nan nan nan]
V = [ 7.  6.  4.  5. nan]
U = [ 0. -1. -1. -2.]
V = [7. 6. 4. 5. 6.]
```

```
P-matrix
[[0. 0. 0. 0. 3.]
 [2. 0. 3. 3. 3.]
 [0. 3. 6. 2. 0.]
 [0. 3. 5. 5. 2.]]
```

```
optimised Allocation Matrix:
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]

Total Optimal Cost = 510.0
```

```
Least Cost Method
Total Cost: 510.0
List of Allocated Positions: [(0, 2), (0, 3), (1, 1), (2, 4), (3, 0), (2, 0), (0, 0)]
```

```
Least Cost Method
Total Cost: 510.0
List of Allocated Positions: [(0, 2), (0, 3), (1, 1), (2, 4), (3, 0), (2, 0), (0, 0)]
List of Unallocated Positions: [(0, 1), (0, 4), (1, 0), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)]
Allocation Matrix:
[[ 5.  0. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
Degeneracy
Values of epsilon is -1
Allocation Matrix After Converting Degeneracy to Non-Degeneracy is :
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]

Iteration - 1
Start AL
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
U = [ 0. nan nan nan]
V = [ 7.  6.  4.  5. nan]

Iteration - 1
Start AL
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
U = [ 0. nan nan nan]
V = [ 7.  6.  4.  5. nan]
U = [ 0. -1. -1. -2.]
V = [7. 6. 4. 5. 6.]
P-matrix
[[0. 0. 0. 0. 3.]
 [2. 0. 3. 3. 3.]
 [0. 3. 6. 2. 0.]
 [0. 3. 5. 5. 2.]]
optimised Allocation Matrix:
[[ 5. -1. 15. 20.  0.]
 [ 0. 30.  0.  0.  0.]
 [15.  0.  0.  0.  5.]
 [10.  0.  0.  0.  0.]]
Total Optimal Cost = 510.0

Process finished with exit code 0
```