

Assignment 3  
Of  
Modelling & Simulation Lab (CS1052)

Masters of Technology in Computer Science And Engineering

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# Problem 1

## Problem statement 1

Formulate the following

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	6	4	1	5	14
$O_2$	8	9	2	7	12
$O_3$	4	3	6	2	4
Demand	6	10	10	4	

Also determine an initial basic feasible solution of this TP using least cost method.

## Problem formulation

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	$x_{11}$ 6	$x_{12}$ 4	$x_{13}$ 1	$x_{14}$ 5	14
$O_2$	$x_{21}$ 8	$x_{22}$ 9	$x_{23}$ 2	$x_{24}$ 7	12
$O_3$	$x_{31}$ 4	$x_{32}$ 3	$x_{33}$ 6	$x_{34}$ 2	4
Demand	6	10	10	4	

The transportation problem is formulated as an LP model as follows:-

Minimize (Total transportation cost) =

$$6x_{11} + 4x_{12} + 1x_{13} + 5x_{14} + \\ 8x_{21} + 9x_{22} + 2x_{23} + 7x_{24} + \\ 4x_{31} + 3x_{32} + 6x_{33} + 2x_{34}$$

Subject to the constraints

$$x_{11} + x_{12} + x_{13} + x_{14} = 14$$

$$x_{21} + x_{22} + x_{23} + x_{24} = 12$$

$$x_{31} + x_{32} + x_{33} + x_{34} = 4$$

and

$$x_{11} + x_{21} + x_{31} = 6$$

$$x_{12} + x_{22} + x_{32} = 10$$

$$x_{13} + x_{23} + x_{33} = 10$$

$$x_{14} + x_{24} + x_{34} = 4$$

and  $x_{ij} \geq 0$  for  $i \in 1, 2, 3$  &

$$j \in 1, 2, 3, 4$$



for the above LP model, there are  $m+n = 3+12 = 15$  decision variables,  $M_{ij}$  &  $m+n = 7$  constraints, where  $m$  are the numbers of rows and  $n$  are the numbers of columns.

Existence of feasible soln

Total supply = Total demand

$$\Rightarrow \sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

$$\Rightarrow 14 + 12 + 4 = 6 + 10 + 10 + 4$$

$$\Rightarrow 30 = 30$$

The total supply is equal to total demand, hence the problem is a balanced transportation problem.

Solution :-

① Select minimum of cost matrix i.e. at  $(D_1, D_3)$  and then compare

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	6	4	1	5	<del>14</del> $a_1 = 4$
	x	x (4)	x (10)	x	
$O_2$	8	9	2	7	<del>12</del> $a_2 = 6$
	x (6)	x (6)	x	x	
$O_3$	4	3	6	2	<del>4</del> $a_3 = 0$
	x	x	x	x (4)	
Demand	<del>6</del> 0	<del>10</del> 6 0	<del>10</del> 0	<del>4</del> 0	
	$= b_1$	$= b_2$	$= b_3$	$= b_4$	

$a_1$  and  $b_3$  i.e.  $10 < 14$ , hence allocate  $x_{13} = 10$ ,  $b_3$  gets exhausted and  $a_1 = 4$  delete column  $D_3$

② Again select the minimum cost from the new cost matrix with  $D_3$  removed, i.e. 2 at  $(O_3, D_4)$  and then compare  $a_3$  &  $b_4$  where  $4 \geq 4$ , allocate  $x_{34} = 4$ , this exhausts  $a_3$  &  $b_4 = 0$ , delete  $O_3$  &  $D_4$ .



③ The next minimum is 4 at  $(O_1, D_2)$  comparing  $a_1 + b_2$  i.e.  $4 < 10$ , allocate  $x_{12} = 4$  & this exhaust  $a_1 = 0$  & leaves  $b_2 = 6$ . delete row  $O_1$ .

④ select 8 at  $(O_2, D_1)$ . compare  $a_2 + b_1$  i.e.  $6 < 12$ , allocate  $x_{21} = 6$  & exhaust  $b_1$  & leave  $b_2 = 6$ , then select 9 i.e. at  $(O_2, D_2)$  & allocate remaining value i.e.  $x_{22} = 6$ . this exhausts all the supply & demand.

Allocation matrix -

	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	0	4	10	0	14
$O_2$	6	6	0	0	12
$O_3$	0	0	0	4	4
Demand	6	10	10	4	30

$$\begin{aligned}
 \text{Total cost} &= 4 \times 4 + 1 \times 10 + 8 \times 6 + 9 \times 6 + 2 \times 9 \\
 &= 16 + 10 + 48 + 54 + 18 \\
 &= 136
 \end{aligned}$$

The number of allocations  $= 5$ ,  
 $f(m+n)-1 = 6 \Rightarrow 5 \neq 6$  hence  
 the solution is degenerate

To solve degeneracy problem we  
 have to select a position in the  
 matrix which is unallocated &  
 also have maximum cost &  
 do not create a loop.

So if we put the value epsilon  
 at  $(O_1, D_1)$  or  $(O_2, D_3)$ , it will  
 create loop whether rest of the  
 position is independent ~~of~~  
 and we can put epsilon on  
 the position with minimum  
 cost i.e 3 at  $(O_3, D_2)$



	$D_1$	$D_2$	$D_3$	$D_4$	Supply
$O_1$	0	4	10	0	14
$O_2$	6	6	0	0	12
$O_3$	0	$\epsilon$	0	4	4
Demand	6	10	10	4	0

$$\begin{aligned}
 \text{Total cost} &= 4 \times 4 + 10 \times 1 + 6 \times 8 + 6 \times 9 \\
 &\quad + 3 \times \epsilon + 4 \times 2 \\
 &= 16 + 10 + 48 + 54 + 8 + 3\epsilon \\
 &= 136 + 3\epsilon \\
 &\approx 136 \quad \text{where } \epsilon = 0 \\
 &\quad \quad \quad \underline{\quad}
 \end{aligned}$$



Python Code:

```
import numpy as np

def check_loop(p, row, column):
    p[row, column] = -1
    flag = 1
    while flag != 0:
        flag = 0
        if p.size != 0:
            row = np.count_nonzero(p, axis=1)
            f = 0
            for index in range(len(row)):
                if row[index] < 2:
                    flag = 1
                    p = np.delete(p, (index - f), axis=0)
                    f += 1
            if p.size != 0:
                e = 0
                col = np.count_nonzero(p, axis=0)
                for index in range(len(col)):
                    if col[index] < 2:
                        flag = 1
                        p = np.delete(p, (index - e), axis=1)
                        e += 1
        if p.size != 0:
            return 0
    else:
        return 1

if __name__ == '__main__':
    cm = np.array([[6.0, 4.0, 1.0, 5.0], [8.0, 9.0, 2.0, 7.0], [4.0, 3.0, 6.0, 2.0]])
    s = np.array([14.0, 12.0, 4.0])
    d = np.array([6.0, 10.0, 10.0, 4.0])
    c = cm.copy()
    print("The Cost Matrix is: ")
    print(c)
    print("The Supply is: ", s)
    print("The Demand is: ", d)
    m, n = c.shape
    print("No of Rows & No of Columns: (", m, ", ", n, ")")
    total_cost = 0
    no_alloc = 0
    total_demand = np.sum(d)
    total_supply = np.sum(s)
    alloc = []
    if total_demand == total_supply:
        print("It is a Balanced Transportation Problem")
    else:
        print("It is an UnBalanced Transportation Problem")
        if total_demand > total_supply:
            new = np.array(np.zeros(n))
            c = np.row_stack((c, new))
            s = np.append(s, total_demand - total_supply)
            m = m + 1
        else:
            new = np.array(np.zeros(m))
            c = np.column_stack((c, new))
            d = np.append(d, total_supply - total_demand)
            n = n + 1
    print("The New Balanced Cost Matrix is: ")
    print(c)
    print("The Supply is: ", s)
```

```

print("The Demand is: ", d)
a = np.zeros(c.shape)
min_cost = np.amin(c)
while min_cost != np.inf:
    indexes = np.where(c == min_cost)
    i = indexes[0][0]
    j = indexes[1][0]
    x = min(s[i], d[j])
    s[i] -= x
    d[j] -= x
    total_cost += (x * c[i, j])
    no_alloc += 1
    a[i, j] = x
    alloc.append((i, j))
    if s[i] < d[j]:
        x = 0
        while x < n:
            c[i, x] = np.inf
            x += 1
    elif s[i] > d[j]:
        y = 0
        while y < m:
            c[y, j] = np.inf
            y += 1
    else:
        x = 0
        while x < n:
            c[i, x] = np.inf
            x += 1
        y = 0
        while y < m:
            c[y, j] = np.inf
            y += 1
    min_cost = np.amin(c)
print("Total Cost: ", total_cost)
unalloc = []
for i in range(m):
    for j in range(n):
        if not (i, j) in alloc:
            unalloc.append((i, j))
print("List of Allocated Positions: ", alloc)
print("List of Unallocated Positions: ", unalloc)
print("Allocation Matrix: ")
print(a)
no_loop = []
if no_alloc == m + n - 1:
    print("Non Degeneracy")
else:
    print("Degeneracy")

    for i in unalloc:
        if check_loop(a.copy(), i[0], i[1]) == 1:
            no_loop.append(i)

    min_epi_list = []
    for i in no_loop:
        min_epi_list.append(cm[i[0], i[1]])

    min_epi = min(min_epi_list)
    ind = min_epi_list.index(min_epi)
    loc = no_loop[ind]
    a[loc[0], loc[1]] = -1
print("Allocation Matrix After Converting Degeneracy to Non-Degeneracy is : ")
print(a)

```



Output :

The Cost Matrix is:

```
[[6. 4. 1. 5.]
```

```
 [8. 9. 2. 7.]
```

```
 [4. 3. 6. 2.]]
```

The Supply is: [14. 12. 4.]

The Demand is: [ 6. 10. 10. 4.]

No of Rows & No of Columns: ( 3 , 4 )

It is a Balanced Transportation Problem

Total Cost: 136.0

List of Allocated Positions: [(0, 2), (2, 3), (0, 1), (1, 0), (1, 1)]

List of Unallocated Positions: [(0, 0), (0, 3), (1, 2), (1, 3), (2, 0), (2, 1), (2, 2)]

Allocation Matrix:

```
[[ 0.  4. 10.  0.]
```

```
 [ 6.  6.  0.  0.]
```

```
 [ 0.  0.  0.  4.]]
```

Degeneracy

Allocation Matrix After Converting Degeneracy to Non-Degeneracy is :

```
[[ 0.  4. 10.  0.]
```

```
 [ 6.  6.  0.  0.]
```

```
 [ 0. -1.  0.  4.]]
```

## Problem statement

Formulate the following TP & find IBFS by LCM.

	1	2	3	4	5	Available
A	4	3	1	2	6	80
B	5	2	3	4	5	40
C	3	5	6	3	2	40
D	2	4	4	5	3	20
Required	60	60	30	40	10	200/180

## Problem formulation

The transportation problem is formulated as an LP as follows:-

	1	2	3	4	5	Available
A	$x_{11}$ 4	$x_{12}$ 3	$x_{13}$ 1	$x_{14}$ 2	$x_{15}$ 6	80
B	$x_{21}$ 5	$x_{22}$ 2	$x_{23}$ 3	$x_{24}$ 4	$x_{25}$ 5	40
C	$x_{31}$ 3	$x_{32}$ 5	$x_{33}$ 6	$x_{34}$ 3	$x_{35}$ 2	40
D	$x_{41}$ 2	$x_{42}$ 4	$x_{43}$ 4	$x_{44}$ 5	$x_{45}$ 3	20
Required	60	60	30	40	10	200/180



## Existence of feasible solution

$$\Rightarrow \sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

$$\Rightarrow 80 + 40 + 40 + 20 = 60 + 60 + 30 + 40 + 10$$

$$180 \neq 200$$

Hence the ~~problem~~ <sup>problem</sup> is unbalanced

transportation problem, so to solve this question, we need to insert a dummy row E, with cost (0) to all the cells and availability = |Total demand - Total supply| = 20.

hence the new cost matrix =

	1	2	3	4	5	Avail.
A	$x_{11}$ 4	$x_{12}$ 3	$x_{13}$ 1	$x_{14}$ 2	$x_{15}$ 6	80
B	$x_{21}$ 5	$x_{22}$ 2	$x_{23}$ 3	$x_{24}$ 4	$x_{25}$ 5	40
C	$x_{31}$ 3	$x_{32}$ 5	$x_{33}$ 6	$x_{34}$ 3	$x_{35}$ 2	40
D	$x_{41}$ 2	$x_{42}$ 4	$x_{43}$ 1	$x_{44}$ 5	$x_{45}$ 3	20
E	$x_{51}$ 0	$x_{52}$ 0	$x_{53}$ 0	$x_{54}$ 0	$x_{55}$ 0	20
Required	60	60	30	40	10	200

$$\begin{aligned} \text{Total cost} = & 4x_{11} + 3x_{12} + 1x_{13} + 2x_{14} + 6x_{15} + \\ & 5x_{21} + 2x_{22} + 3x_{23} + 4x_{24} + 5x_{25} + \\ & 3x_{31} + 5x_{32} + 6x_{33} + 3x_{34} + 2x_{35} + \\ & 2x_{41} + 4x_{42} + 4x_{43} + 5x_{44} + 3x_{45} + \\ & 0x_{51} + 0x_{52} + 0x_{53} + 0x_{54} + 0x_{55} \end{aligned}$$

Subject to constraint

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 80$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 40$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 40$$

$$x_{41} + x_{42} + x_{43} + x_{44} + x_{45} = 20$$

$$x_{51} + x_{52} + x_{53} + x_{54} + x_{55} = 20$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 60$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 30$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 40$$

$$x_{15} + x_{25} + x_{35} + x_{45} + x_{55} = 10$$

and  $x_{ij} \geq 0$  for  $i = 1, 2, 3, 4, 5$  &  
 $j = 1, 2, 3, 4, 5$



for the above model, there are  $5 \times 5 = 25$  decision variables &  $m+n = 5+5 = 10$  constraints, where  $m = \text{rows}$  &  $n = \text{columns}$ .

### Solution

	1	2	3	4	5	avail.
A	4 $\times$	3 $\times$ (10)	1 $\times$ (30)	2 $\times$ (40)	6 $\times$	80 50 10 0 $a_1$
B	5 $\times$	2 $\times$ (40)	3 $\times$	4 $\times$	5 $\times$	40 0 $a_2$
C	3 $\times$ (20)	5 $\times$ (10)	6 $\times$	3 $\times$	2 $\times$ (10)	40 30 10 0 $a_3$
D	2 $\times$ (20)	4 $\times$	4 $\times$	5 $\times$	3 $\times$	20 0 $a_4$
E	0 $\times$ (20)	0 $\times$	0 $\times$	0 $\times$	0 $\times$	20 0 $a_5$
Required	60 40 20 20 0 $\geq b_1$	60 30 10 20 0 $\geq b_2$	30 0 20 0 $\geq b_3$	40 0 20 0 $\geq b_4$	10 0 20 0 $\geq b_5$	

① The minimum of the cost material i.e. 0 at  $(E, 1)$  and then compare  $a_5$  of  $b_1 \Rightarrow b_1 > a_5$  hence allocate  $a_5 = 20$  which exhausts  $a_5$  hence delete row E.



② Similarly select 1 at  $(A, 3)$   
allocate  $x_{13} = 30$   $\begin{smallmatrix} \infty & \infty \\ 0 & 0 \end{smallmatrix}$   $b_3 < a_1$ .  
this exhausts  $b_3$  hence delete  
column 3.

③ select 2 at  $(A, 4)$ , allocate  
 $x_{14} = 40$   $\begin{smallmatrix} \infty & \infty \\ 0 & 0 \end{smallmatrix}$   $b_4 < a_1$ , this  
exhausts  $b_4$  hence delete column  
4.

④ select 2 at  $(B, 2)$  allocate  $x_{22} = 40$   
 $\begin{smallmatrix} \infty & \infty \\ 0 & 0 \end{smallmatrix}$   $b_2 > a_2$ , this exhausts  $a_2$   
hence delete row B.

⑤ select 2 at  $(C, 5)$ ,  $x_{35} = 10$ ,  
delete column 5<sup>th</sup> since  
 $b_5 < a_3$  and  $b_5$  gets exhausted

⑥ select 2 at  $(D, 1)$ ,  $x_{41} = 20$ ,  
delete row D  $\begin{smallmatrix} \infty & \infty \\ 0 & 0 \end{smallmatrix}$   $b_1 > a_4$  and  
 $a_4$  gets exhausted.

⑦ select 3 at  $(A, 2)$   $x_{12} = 10$ ,  
delete row A  $\begin{smallmatrix} \infty & \infty \\ 0 & 0 \end{smallmatrix}$   $b_2 > a_1$  and  
 $a_1$  gets exhausted.

⑤ At last select  $\min = 3$  at  $(G, 1)$   
 $M_{31} = 20$  delete column 1 and  
 left with  $\min = 5$  at  $(C, 2)$   
 $M_{32} = 10$ , & delete 2nd column  
 & all the available & required  
 gets exhausted.

allocation matrix

	1	2	3	4	5	available
A	0	10	30	40	0	80
B	0	40	0	0	0	40
C	20	10	0	0	10	40
D	20	0	0	0	0	20
E	20	0	0	0	0	20
Required	60	60	30	40	10	200

No of allocation  $= g + m + n - 1 = 5 + 5 - 1 = 9$  hence  
 the sol<sup>n</sup> is non degenerate.

$$\begin{aligned}
 \text{Total cost} &= (10 \times 3) + (30 \times 1) + (40 \times 2) + (40 \times 2) \\
 &\quad + (20 \times 3) + (10 \times 5) + (10 \times 2) + \\
 &\quad + (20 \times 2) + (20 \times 0) \\
 &= 30 + 30 + 80 + 80 + 60 + 50 + 20 + 40 + 0 \\
 &= 390
 \end{aligned}$$



Python Code:

```
import numpy as np

def check_loop(p, row, column):
    p[row, column] = -1
    flag = 1
    while flag != 0:
        flag = 0
        if p.size != 0:
            row = np.count_nonzero(p, axis=1)
            f = 0
            for index in range(len(row)):
                if row[index] < 2:
                    flag = 1
                    p = np.delete(p, (index - f), axis=0)
                    f += 1
            if p.size != 0:
                e = 0
                col = np.count_nonzero(p, axis=0)
                for index in range(len(col)):
                    if col[index] < 2:
                        flag = 1
                        p = np.delete(p, (index - e), axis=1)
                        e += 1
        if p.size != 0:
            return 0
    else:
        return 1

if __name__ == '__main__':
    cm = np.array([
        [4.0, 3.0, 1.0, 2.0, 6.0],
        [5.0, 2.0, 3.0, 4.0, 5.0],
        [3.0, 5.0, 6.0, 3.0, 2.0],
        [2.0, 4.0, 4.0, 5.0, 3.0]])
    s = np.array([80.0, 40.0, 40.0, 20.0])
    d = np.array([60.0, 60.0, 30.0, 40.0, 10.0])
    c = cm.copy()
    print("The Cost Matrix is: ")
    print(c)
    print("The Supply is: ", s)
    print("The Demand is: ", d)
    m, n = c.shape
    print("No of Rows & No of Columns: (", m, ", ", n, ")")
    total_cost = 0
    no_alloc = 0
    total_demand = np.sum(d)
    total_supply = np.sum(s)
    alloc = []
    if total_demand == total_supply:
        print("It is a Balanced Transportation Problem")
    else:
        print("It is an UnBalanced Transportation Problem")
        if total_demand > total_supply:
            new = np.array(np.zeros(n))
            c = np.row_stack((c, new))
            s = np.append(s, total_demand - total_supply)
            m = m + 1
        else:
            new = np.array(np.zeros(m))
            c = np.column_stack((c, new))
            d = np.append(d, total_supply - total_demand)
```

```

        n = n + 1
    print("The New Balanced Cost Matrix is: ")
    print(c)
    print("The Supply is: ", s)
    print("The Demand is: ", d)
a = np.zeros(c.shape)
min_cost = np.amin(c)
while min_cost != np.inf:
    indexes = np.where(c == min_cost)
    i = indexes[0][0]
    j = indexes[1][0]
    x = min(s[i], d[j])
    s[i] -= x
    d[j] -= x
    total_cost += (x * c[i, j])
    no_alloc += 1
    a[i, j] = x
    alloc.append((i, j))
    if s[i] < d[j]:
        x = 0
        while x < n:
            c[i, x] = np.inf
            x += 1
    elif s[i] > d[j]:
        y = 0
        while y < m:
            c[y, j] = np.inf
            y += 1
    else:
        x = 0
        while x < n:
            c[i, x] = np.inf
            x += 1
        y = 0
        while y < m:
            c[y, j] = np.inf
            y += 1
    min_cost = np.amin(c)
print("Total Cost: ", total_cost)
unalloc = []
for i in range(m):
    for j in range(n):
        if not (i, j) in alloc:
            unalloc.append((i, j))
print("List of Allocated Positions: ", alloc)
print("List of Unallocated Positions: ", unalloc)
print("Allocation Matrix: ")
print(a)
no_loop = []
if no_alloc == m + n - 1:
    print("Non Degeneracy")
else:
    print("Degeneracy")

    for i in unalloc:
        if check_loop(a.copy(), i[0], i[1]) == 1:
            no_loop.append(i)

    min_epi_list = []
    for i in no_loop:
        min_epi_list.append(cm[i[0], i[1]])

    min_epi = min(min_epi_list)
    ind = min_epi_list.index(min_epi)
    loc = no_loop[ind]
    a[loc[0], loc[1]] = -1

```

```
print("Allocation Matrix After Converting Degeneracy to Non-Degeneracy is : ")  
print(a)
```



Output :

The Cost Matrix is:

```
[[4. 3. 1. 2. 6.]  
 [5. 2. 3. 4. 5.]  
 [3. 5. 6. 3. 2.]  
 [2. 4. 4. 5. 3.]]
```

The Supply is: [80. 40. 40. 20.]

The Demand is: [60. 60. 30. 40. 10.]

No of Rows & No of Columns: ( 4 , 5 )

It is an UnBalanced Transportation Problem

The New Balanced Cost Matrix is:

```
[[4. 3. 1. 2. 6.]  
 [5. 2. 3. 4. 5.]  
 [3. 5. 6. 3. 2.]  
 [2. 4. 4. 5. 3.]  
 [0. 0. 0. 0. 0.]]
```

The Supply is: [80. 40. 40. 20. 20.]

The Demand is: [60. 60. 30. 40. 10.]

Total Cost: 390.0

List of Allocated Positions: [(4, 0), (0, 2), (0, 3), (1, 1), (2, 4), (3, 0), (0, 1), (2, 0), (2, 1)]

List of Unallocated Positions: [(0, 0), (0, 4), (1, 0), (1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)]

Allocation Matrix:

```
[[ 0. 10. 30. 40.  0.]  
 [ 0. 40.  0.  0.  0.]  
 [20. 10.  0.  0. 10.]  
 [20.  0.  0.  0.  0.]  
 [20.  0.  0.  0.  0.]]
```

Non Degeneracy