Assignment 5
Of
Modelling & Simulation Lab (CS1052)

Masters of Technology in Computer Science And Engineering

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Problem 1

Problem statement 1

Find the optimal solutⁿ to the following transportatⁿ problem in which the cells contains the unit transportatⁿ cost in ruppees.

	W_1	W_2	W_3	W_4	W_5	avail.
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Req.	30	30	15	20	5	

use NWCR and LCM for initial basic feasible solutⁿ.

Problem formulatⁿ –

	W_1	W_2	W_3	W_4	W_5	Supply
F_1	x_{11} 7	x_{12} 6	x_{13} 4	x_{14} 5	x_{15} 9	40
F_2	x_{21} 8	x_{22} 5	x_{23} 6	x_{24} 7	x_{25} 8	30
F_3	x_{31} 6	x_{32} 8	x_{33} 9	x_{34} 6	x_{35} 5	20
~~Demand~~ F_4	x_{41} 5	x_{42} 7	x_{43} 7	x_{44} 8	x_{45} 6	10
Dem.	30	30	15	20	5	

The transportat^n problem is formulated as an LP model as follows —

Minimize (Total T.P cost) —

$$7x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 9x_{15} \cancel{\le 40}$$
$$+ 8x_{21} + 5x_{22} + 6x_{23} + 7x_{24} + 8x_{25} \cancel{\le 30}$$
$$+ 6x_{31} + 8x_{32} + 9x_{33} + 6x_{34} + 5x_{35} \cancel{\le 20}$$
$$+ 5x_{41} + 7x_{42} + 7x_{43} + 8x_{44} + 6x_{45} \cancel{\le 10}$$

Subject to constraint —

$$7x_{11} + 6x_{12} + 4x_{13} + 5x_{14} + 9x_{15} = 40$$
$$8x_{21} + 5x_{22} + 6x_{23} + 7x_{24} + 8x_{25} = 30$$
$$6x_{31} + 8x_{32} + 9x_{33} + 6x_{34} + 5x_{35} = 20$$
$$5x_{41} + 7x_{42} + 7x_{43} + 8x_{44} + 6x_{45} = 10$$

First we will solve it using north west corner rule.

The allocation matrix is such that.

	W_1	W_2	W_3	W_4	W_5	Sup
F_1	7 (30)	6 (10)	4	5	9	~~40~~ ~~10~~ 0
F_2	8	5 (20)	6 (10)	7	8	~~30~~ ~~10~~ 0
F_3	6	8	9 (5)	6 (15)	5	~~20~~ ~~15~~ 0
F_4	5	7	7	8 (5)	6	10 ~~5~~ 0
Demand	~~30~~ 0	~~30~~ ~~20~~ 0	~~15~~ ~~5~~ 0	~~20~~ ~~15~~ 0	~~5~~ 0	

The step by step description of this solution is given as:-

and

$$7x_{11} + 8x_{21} + 6x_{31} + 5x_{41} = 30$$

$$6x_{12} + 5x_{22} + 8x_{32} + 7x_{42} = 30$$

$$4x_{13} + 6x_{23} + 9x_{33} + 7x_{43} = 15$$

$$5x_{14} + 7x_{24} + 6x_{34} + 8x_{44} = 20$$

$$9x_{15} + 8x_{25} + 5x_{35} + 6x_{45} = 5$$

and $x_{ij} \geq 0$ for $i = 1, 2, 3$ &

$j = 1, 2, 3, 4$

Soln

Total no of supply const. = 4

Total no of demand const = 5

~~The sym val for $a_1 = 40$ & $a_2 = 30$ compared.~~

Total supply = 40 + 30 + 20 + 10

= 100

Total demand = 30 + 30 + 15 + 20 + 5

= 100

Total supply = Total demand

$\Rightarrow$ Balanced Transp. problem.

North west corner method

The min val $F_1 = 40$ & $W_1 = 30$ $\min(F_1, W_1) = 30$ is assigned to $F_1 W_1$, This meets the demand of W_1 & leaves $40 - 30 = 10$ with F_1

The min value for $F_2 = 30$ & $W_2 = 20$ are compared. The $\min(F_2, W_2) = 20$. This meets the demand of W_2 & leaves $30 - 20 = 10$ units with F_2

The min values for $F_2 = 10$ and $W_3 = 15$ are compared. The smaller of the two i.e $\min(10, 15) = 10$ is assign. to $F_2 W_3$, this exhaust

Python Code:

```
class HungarianError(Exception):
pass

try:
    import numpy as np
except ImportError:
    raise HungarianError("NumPy is not installed.")


class Hungarian:
    """
    Implementation of the Hungarian (Munkres) Algorithm using np.
    Usage:
        hungarian = Hungarian(cost_matrix)
        hungarian.calculate()
    or
        hungarian = Hungarian()
        hungarian.calculate(cost_matrix)
    Handle Profit matrix:
        hungarian = Hungarian(profit_matrix, is_profit_matrix=True)
    or
        cost_matrix = Hungarian.make_cost_matrix(profit_matrix)
    The matrix will be automatically padded if it is not square.
    For that numpy's resize function is used, which automatically adds 0's to any row/co
    Get results and total potential after calculation:
        hungarian.get_results()
        hungarian.get_total_potential()
    """

    def __init__(self, input_matrix=None, is_profit_matrix=False):
        """
        input_matrix is a List of Lists.
        input_matrix is assumed to be a cost matrix unless is_profit_matrix is True.
        """
        if input_matrix is not None:
            # Save input
            my_matrix = np.array(input_matrix)
            self._input_matrix = np.array(input_matrix)
            self._maxColumn = my_matrix.shape[1]
            self._maxRow = my_matrix.shape[0]

            # Adds 0s if any columns/rows are added. Otherwise stays unaltered
            matrix_size = max(self._maxColumn, self._maxRow)
            pad_columns = matrix_size - self._maxRow
            pad_rows = matrix_size - self._maxColumn
            my_matrix = np.pad(my_matrix, ((0, pad_columns), (0, pad_rows)), 'constant'

            # Convert matrix to profit matrix if necessary
            if is_profit_matrix:
                my_matrix = self.make_cost_matrix(my_matrix)
```

```
            self._cost_matrix = my_matrix
            self._size = len(my_matrix)
            self._shape = my_matrix.shape

            # Results from algorithm.
            self._results = []
            self._totalPotential = 0
        else:
            self._cost_matrix = None

    def get_results(self):
        """Get results after calculation."""
        return self._results

    def get_total_potential(self):
        """Returns expected value after calculation."""
        return self._totalPotential

    def calculate(self, input_matrix=None, is_profit_matrix=False):
        """
        Implementation of the Hungarian (Munkres) Algorithm.
        input_matrix is a List of Lists.
        input_matrix is assumed to be a cost matrix unless is_profit_matrix is True.
        """
        # Handle invalid and new matrix inputs.
        if input_matrix is None and self._cost_matrix is None:
            raise HungarianError("Invalid input")
        elif input_matrix is not None:
            self.__init__(input_matrix, is_profit_matrix)

        result_matrix = self._cost_matrix.copy()

        # Step 1: Subtract row mins from each row.
        for index, row in enumerate(result_matrix):
            result_matrix[index] -= row.min()

        # Step 2: Subtract column mins from each column.
        for index, column in enumerate(result_matrix.T):
            result_matrix[:, index] -= column.min()

        # Step 3: Use minimum number of lines to cover all zeros in the matrix.
        # If the total covered rows+columns is not equal to the matrix size then adjust
        total_covered = 0
        while total_covered < self._size:
            # Find minimum number of lines to cover all zeros in the matrix and find to
            cover_zeros = CoverZeros(result_matrix)
            covered_rows = cover_zeros.get_covered_rows()
            covered_columns = cover_zeros.get_covered_columns()
            total_covered = len(covered_rows) + len(covered_columns)

            # if the total covered rows+columns is not equal to the matrix size then ad
            if total_covered < self._size:
                result_matrix = self._adjust_matrix_by_min_uncovered_num(result_matrix,
```

```
        # Step 4: Starting with the top row, work your way downwards as you make assignn
        # Find single zeros in rows or columns.
        # Add them to final result and remove them and their associated row/column from
        expected_results = min(self._maxColumn, self._maxRow)
        zero_locations = (result_matrix == 0)
        while len(self._results) != expected_results:

            # If number of zeros in the matrix is zero before finding all the results tl
            if not zero_locations.any():
                raise HungarianError("Unable to find results. Algorithm has failed.")

            # Find results and mark rows and columns for deletion
            matched_rows, matched_columns = self.__find_matches(zero_locations)

            # Make arbitrary selection
            total_matched = len(matched_rows) + len(matched_columns)
            if total_matched == 0:
                matched_rows, matched_columns = self.select_arbitrary_match(zero_locatic

            # Delete rows and columns
            for row in matched_rows:
                zero_locations[row] = False
            for column in matched_columns:
                zero_locations[:, column] = False

            # Save Results
            self.__set_results(zip(matched_rows, matched_columns))

        # Calculate total potential
        value = 0
        for row, column in self._results:
            value += self._input_matrix[row, column]
        self._totalPotential = value

    @staticmethod
    def make_cost_matrix(profit_matrix):
        """
        Converts a profit matrix into a cost matrix.
        Expects NumPy objects as input.
        """
        # subtract profit matrix from a matrix made of the max value of the profit matr
        matrix_shape = profit_matrix.shape
        offset_matrix = np.ones(matrix_shape, dtype=int) * profit_matrix.max()
        cost_matrix = offset_matrix - profit_matrix
        return cost_matrix

    def _adjust_matrix_by_min_uncovered_num(self, result_matrix, covered_rows, covered_c
        """Subtract m from every uncovered number and add m to every element covered wi
        # Calculate minimum uncovered number (m)
        elements = []
        for row_index, row in enumerate(result_matrix):
            if row_index not in covered_rows:
                for index, element in enumerate(row):
                    if index not in covered_columns:
```

```
                    elements.append(element)
        min_uncovered_num = min(elements)

        # Add m to every covered element
        adjusted_matrix = result_matrix
        for row in covered_rows:
            adjusted_matrix[row] += min_uncovered_num
        for column in covered_columns:
            adjusted_matrix[:, column] += min_uncovered_num

        # Subtract m from every element
        m_matrix = np.ones(self._shape, dtype=int) * min_uncovered_num
        adjusted_matrix -= m_matrix

        return adjusted_matrix

    def __find_matches(self, zero_locations):
        """Returns rows and columns with matches in them."""
        marked_rows = np.array([], dtype=int)
        marked_columns = np.array([], dtype=int)

        # Mark rows and columns with matches
        # Iterate over rows
        for index, row in enumerate(zero_locations):
            row_index = np.array([index])
            if np.sum(row) == 1:
                column_index, = np.where(row)
                marked_rows, marked_columns = self.__mark_rows_and_columns(marked_rows,
                                                                             column_index

        # Iterate over columns
        for index, column in enumerate(zero_locations.T):
            column_index = np.array([index])
            if np.sum(column) == 1:
                row_index, = np.where(column)
                marked_rows, marked_columns = self.__mark_rows_and_columns(marked_rows,
                                                                             column_index

        return marked_rows, marked_columns

    @staticmethod
    def __mark_rows_and_columns(marked_rows, marked_columns, row_index, column_index):
        """Check if column or row is marked. If not marked then mark it."""
        new_marked_rows = marked_rows
        new_marked_columns = marked_columns
        if not (marked_rows == row_index).any() and not (marked_columns == column_index
            new_marked_rows = np.insert(marked_rows, len(marked_rows), row_index)
            new_marked_columns = np.insert(marked_columns, len(marked_columns), column_
        return new_marked_rows, new_marked_columns

    @staticmethod
    def select_arbitrary_match(zero_locations):
        """Selects row column combination with minimum number of zeros in it."""
        # Count number of zeros in row and column combinations
```

```
        rows, columns = np.where(zero_locations)
        zero_count = []
        for index, row in enumerate(rows):
            total_zeros = np.sum(zero_locations[row]) + np.sum(zero_locations[:, column
            zero_count.append(total_zeros)

        # Get the row column combination with the minimum number of zeros.
        indices = zero_count.index(min(zero_count))
        row = np.array([rows[indices]])
        column = np.array([columns[indices]])

        return row, column

    def __set_results(self, result_lists):
        """Set results during calculation."""
        # Check if results values are out of bound from input matrix (because of matrix
        # Add results to results list.
        for result in result_lists:
            row, column = result
            if row < self._maxRow and column < self._maxColumn:
                new_result = (int(row), int(column))
                self._results.append(new_result)


class CoverZeros:
    """
    Use minimum number of lines to cover all zeros in the matrix.
    Algorithm based on: http://weber.ucsd.edu/~vcrawfor/hungar.pdf
    """

    def __init__(self, matrix):
        """
        Input a matrix and save it as a boolean matrix to designate zero locations.
        Run calculation procedure to generate results.
        """
        # Find zeros in matrix
        self._zero_locations = (matrix == 0)
        self._shape = matrix.shape

        # Choices starts without any choices made.
        self._choices = np.zeros(self._shape, dtype=bool)

        self._marked_rows = []
        self._marked_columns = []

        # marks rows and columns
        self.__calculate()

        # Draw lines through all unmarked rows and all marked columns.
        self._covered_rows = list(set(range(self._shape[0])) - set(self._marked_rows))
        self._covered_columns = self._marked_columns

    def get_covered_rows(self):
        """Return list of covered rows."""
```

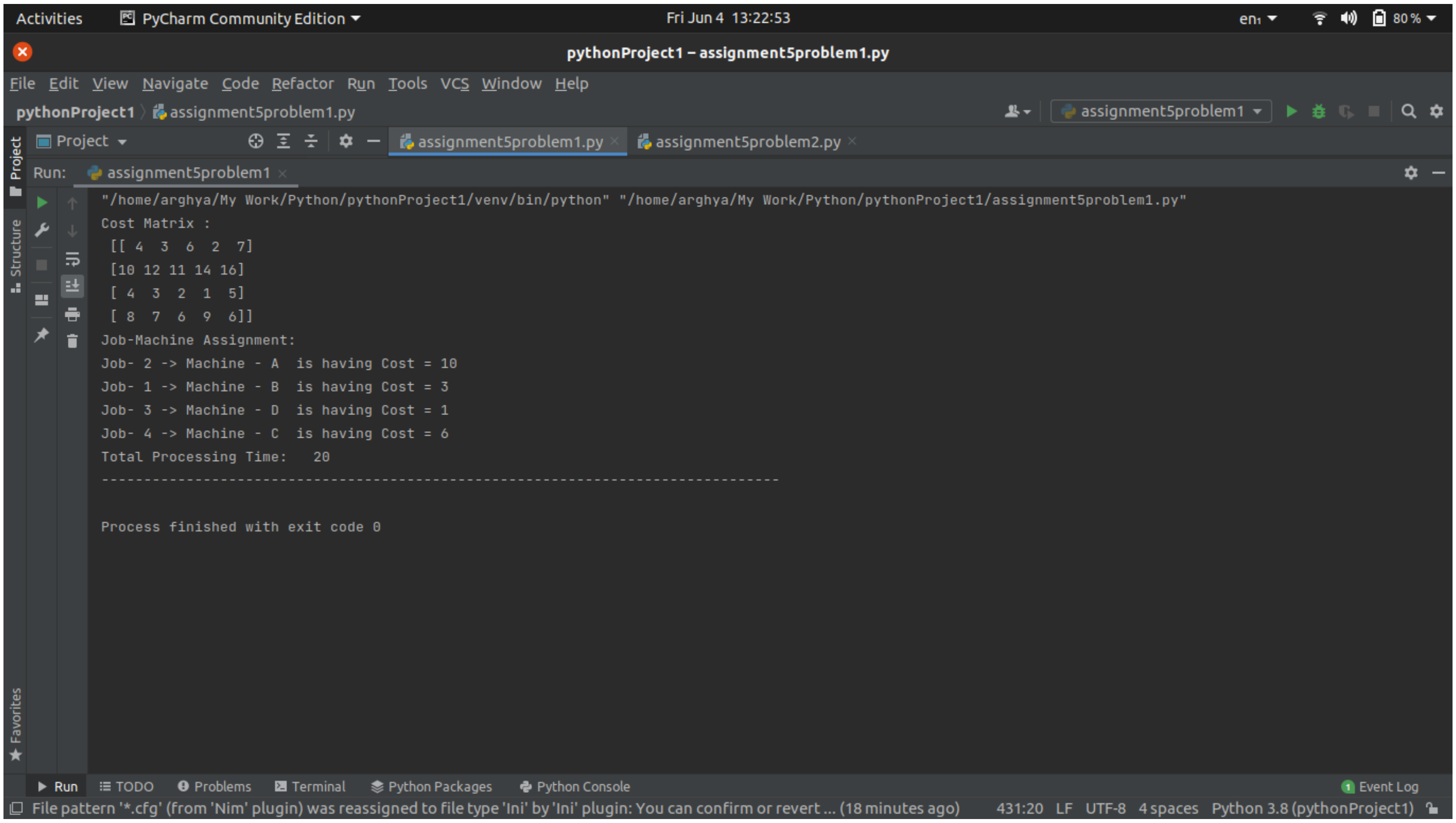
```
        return self._covered_rows

    def get_covered_columns(self):
        """Return list of covered columns."""
        return self._covered_columns

    def __calculate(self):
        """
        Calculates minimum number of lines necessary to cover all zeros in a matrix.
        Algorithm based on: http://weber.ucsd.edu/~vcrawfor/hungar.pdf
        """
        while True:
            # Erase all marks.
            self._marked_rows = []
            self._marked_columns = []

            # Mark all rows in which no choice has been made.
            for index, row in enumerate(self._choices):
                if not row.any():
                    self._marked_rows.append(index)

            # If no marked rows then finish.
            if not self._marked_rows:
                return True

            # Mark all columns not already marked which have zeros in marked rows.
            num_marked_columns = self.__mark_new_columns_with_zeros_in_marked_rows()

            # If no new marked columns then finish.
            if num_marked_columns == 0:
                return True

            # While there is some choice in every marked column.
            while self.__choice_in_all_marked_columns():
                # Some Choice in every marked column.

                # Mark all rows not already marked which have choices in marked columns
                num_marked_rows = self.__mark_new_rows_with_choices_in_marked_columns()

                # If no new marks then Finish.
                if num_marked_rows == 0:
                    return True

                # Mark all columns not already marked which have zeros in marked rows.
                num_marked_columns = self.__mark_new_columns_with_zeros_in_marked_rows(

                # If no new marked columns then finish.
                if num_marked_columns == 0:
                    return True

            # No choice in one or more marked columns.
            # Find a marked column that does not have a choice.
            choice_column_index = self.__find_marked_column_without_choice()
```

```
        while choice_column_index is not None:
            # Find a zero in the column indexed that does not have a row with a cho
            choice_row_index = self.__find_row_without_choice(choice_column_index)

            # Check if an available row was found.
            new_choice_column_index = None
            if choice_row_index is None:
                # Find a good row to accomodate swap. Find its column pair.
                choice_row_index, new_choice_column_index = self.__find_best_choice_r
                    choice_column_index)

                # Delete old choice.
                self._choices[choice_row_index, new_choice_column_index] = False

            # Set zero to choice.
            self._choices[choice_row_index, choice_column_index] = True

            # Loop again if choice is added to a row with a choice already in it.
            choice_column_index = new_choice_column_index

    def __mark_new_columns_with_zeros_in_marked_rows(self):
        """Mark all columns not already marked which have zeros in marked rows."""
        num_marked_columns = 0
        for index, column in enumerate(self._zero_locations.T):
            if index not in self._marked_columns:
                if column.any():
                    row_indices, = np.where(column)
                    zeros_in_marked_rows = (set(self._marked_rows) & set(row_indices))
                    if zeros_in_marked_rows:
                        self._marked_columns.append(index)
                        num_marked_columns += 1
        return num_marked_columns

    def __mark_new_rows_with_choices_in_marked_columns(self):
        """Mark all rows not already marked which have choices in marked columns."""
        num_marked_rows = 0
        for index, row in enumerate(self._choices):
            if index not in self._marked_rows:
                if row.any():
                    column_index, = np.where(row)
                    if column_index in self._marked_columns:
                        self._marked_rows.append(index)
                        num_marked_rows += 1
        return num_marked_rows

    def __choice_in_all_marked_columns(self):
        """Return Boolean True if there is a choice in all marked columns. Returns boole
        for column_index in self._marked_columns:
            if not self._choices[:, column_index].any():
                return False
        return True

    def __find_marked_column_without_choice(self):
        """Find a marked column that does not have a choice."""
```

```python
        for column_index in self._marked_columns:
            if not self._choices[:, column_index].any():
                return column_index

        raise HungarianError(
            "Could not find a column without a choice. Failed to cover matrix zeros. Al

    def __find_row_without_choice(self, choice_column_index):
        """Find a row without a choice in it for the column indexed. If a row does not
        row_indices, = np.where(self._zero_locations[:, choice_column_index])
        for row_index in row_indices:
            if not self._choices[row_index].any():
                return row_index

        # All rows have choices. Return None.
        return None

    def __find_best_choice_row_and_new_column(self, choice_column_index):
        """
        Find a row index to use for the choice so that the column that needs to be chan
        Return a random row and column if unable to find an optimal selection.
        """
        row_indices, = np.where(self._zero_locations[:, choice_column_index])
        for row_index in row_indices:
            column_indices, = np.where(self._choices[row_index])
            column_index = column_indices[0]
            if self.__find_row_without_choice(column_index) is not None:
                return row_index, column_index

        # Cannot find optimal row and column. Return a random row and column.
        from random import shuffle

        shuffle(row_indices)
        column_index, = np.where(self._choices[row_indices[0]])
        return row_indices[0], column_index[0]


if __name__ == '__main__':

    cost_matrix = [
        [4, 3, 6, 2, 7],
        [10, 12, 11, 14, 16],
        [4, 3, 2, 1, 5],
        [8, 7, 6, 9, 6]]

    hungarian = Hungarian(cost_matrix)
    hungarian.calculate()
    print("Cost Matrix : \n", np.array(cost_matrix))
    jobs = [1, 2, 3, 4]
    machine = ['A', 'B', 'C', 'D', 'E']
    result = hungarian.get_results()
    print("Job-Machine Assignment:")
    for i in result:
        print("Job-", jobs[i[0]], "-> Machine -", machine[i[1]], " is having Cost =", c
```

```
print("Total Processing Time:\t", hungarian.get_total_potential())
print("-" * 80)
```

Output :


Activities
PyCharm Community Edition
Fri Jun 4 13:22:53
80 %
pythonProject1 – assignment5problem1.py
File Edit View Navigate Code Refactor Run Tools VCS Window Help
pythonProject1 assignment5problem1.py
assignment5problem1
Project
assignment5problem1.py
assignment5problem2.py
Run: assignment5problem1
"/home/arghya/My Work/Python/pythonProject1/venv/bin/python" "/home/arghya/My Work/Python/pythonProject1/assignment5problem1.py"
Cost Matrix :
[[4 3 6 2 7]
[10 12 11 14 16]
[4 3 2 1 5]
[8 7 6 9 6]]
Job-Machine Assignment:
Job- 2 -> Machine - A is having Cost = 10
Job- 1 -> Machine - B is having Cost = 3
Job- 3 -> Machine - D is having Cost = 1
Job- 4 -> Machine - C is having Cost = 6
Total Processing Time: 20

Process finished with exit code 0
Run TODO Problems Terminal Python Packages Python Console
Event Log
File pattern '*.cfg' (from 'Nim' plugin) was reassigned to file type 'Ini' by 'Ini' plugin: You can confirm or revert ... (18 minutes ago)
431:20 LF UTF-8 4 spaces Python 3.8 (pythonProject1)

Problem 2

Problem statement 1

Find the optimal solutn to the following transportatn problem in which the cells contains the unit transportatn cost in ruppees.

	W_1	W_2	W_3	W_4	W_5	avail.
F_1	7	6	4	5	9	40
F_2	8	5	6	7	8	30
F_3	6	8	9	6	5	20
F_4	5	7	7	8	6	10
Req.	30	30	15	20	5	

use NWCR and LCM for initial Basic feasible solutn.

Problem formulatn –

the capacity of F_2 and leaves 15 $- 10 = 5$ units with W_3.

The min value for $F_3 = 20$ and $W_3 = 5$ are compared. The min $(20, 5) = 5$ is assigned to $F_3 W_3$ This meets the complete demand of W_3 and leaves $20 - 5 = 15$ units with F_3

The min values for $F_3 = 15$ and $W_4 = 20$ are compared.
The smaller of $15, 20 = 15$ is assigned for $F_3 W_4$, this exhaust the capacity of F_3 and leaves $20 - 15 = 5$ with W_4.

The min value for $F_4 = 10$ & $W_4 = 5$ are compared. The

min (10, 5) = 5 is assign to $F_4 W_4$
This meets the complete demand of W_4 and leaves 10 - 5 = 5 units with F_4

The sum value for $F_4 = 5$ & $W_5 = 5$ are compared.
The smaller of the two i.e min (5, 5) = 5) is assign to $F_4 W_5$
The IBFS =

	W_1	W_2	W_3	W_4	W_5	Supp.
F_1	7 (30)	6 (10)	4	5	9	40
F_2	8	5 (20)	6 (10)	7	8	30
F_3	6	8	9 (5)	6 (15)	5	20
F_4	5	7	7	8 (5)	6 (5)	10
Demand	30	30	15	20	5	

$u_1 = 7 - 0 = 7$

$v_2 = 6 - 2 =$

$v_1 = c_{11} - u_1 = 7 - 0 \Rightarrow v_1 = 7$

$v_2 = c_{12} - u_1 = 6 - 0 \Rightarrow v_2 = 6$

$u_2 = c_{22} - v_2 = 5 - 6 \Rightarrow u_2 = -1$

$v_3 = c_{23} - u_2 = 6 + 1 \Rightarrow v_3 = 7$

$u_3 = c_{33} - v_3 = 9 - 7 \Rightarrow u_3 = 2$

$u_4 = c_{44} - v_4 = 8 - 4 \Rightarrow u_4 = 4$

$v_5 = c_{45} - u_4 = 6 - 4 = v_5 = 2$

	w_1	w_2	w_3	w_4	w_5	S	u_i
F_1	7 (30)	6 (10)	4	5	9	40	$u_1 = 0$
F_2	8	5 (20)	6 (10)	7	8	20	$u_2 = -1$
F_3	6	8	9 (5)	6 (15)	5	20	$u_3 = 2$
[illegible]	5	7	7	8 (5)	6 (5)	10	$u_4 = 4$
[illegible]	30	30	15	20	5		
v_j	$v_1 = 7$	$v_2 = 6$	$v_3 = 7$	$v_4 = 4$	$v_5 = 2$		

The minimization T.P cost =

$7\times30+6\times10+5\times20+6\times10+9\times5$
$+6\times15+8\times5+6\times5=635$

Total allocated cell $=8=m+n-1$
$=4+5-1$
$=8$

hence the solⁿ is non degenerate

optimality test using modi method.

Allocatⁿ table is

	W_1	W_2	W_3	W_4	W_5	Supp.
F_1	7 (30)	6 (10)	9	5	9	40
F_2	8	5 (20)	6 (10)	7	8	30
F_3	6	8	9 (5)	6 (15)	5	20
F_4	5	7	7	8 (5)	6 (5)	10
Dem.	30	30	15	20	5	

Iteratⁿ 1 of optimality test.

(1) Subs. $u_1=0$, we get

② find $d_{ij} = c_{ij} - (u_i + v_j)$

$d_{13} = c_{13} - (u_1 + v_3) = 4 - (0+7) = -3$

$d_{14} = 5 - (0+4) = 1$

$d_{15} = 9 - (0+2) = 7$

$d_{21} = 8 - (-1+7) = 2$

$d_{24} = 7 - (-1+4) = 4$

$d_{25} = 8 - (-1+2) = 7$

$d_{31} = 6 - (2+7) = -3$

$d_{32} = 8 - (2+6) = 0$

$d_{35} = 5 - (2+2) = 1$

$d_{41} = 5 - (4+7) = -6$

$d_{42} = 7 - (4+6) = -3$

$d_{43} = 7 - (4+7) = -4$

3)

	w_1	w_2	w_3	w_4	w_5	supp	u_i
t_1	7 (30) (−)	6 (10) (+)	4^{-3}	5^{1}	9^{7}	40	0
t_2	8^{2}	5 (20) (−)	6 (10) (+)	7^{4}	8^{7}	30	−1
t_3	6^{-3}	8^{0}	9 (5) (−)	6 (15) (+)	5^{1}	20	2
t_4	5^{-6} (+)	7^{-3}	7^{-4}	8 (5) (−)	6 (5)	10	4
dem	30	30	15	20	5		
v_j	7	6	7	4	2		

Python Code:

```
class HungarianError(Exception):
pass

try:
    import numpy as np
except ImportError:
    raise HungarianError("NumPy is not installed.")


class Hungarian:
    """
    Implementation of the Hungarian (Munkres) Algorithm using np.
    Usage:
        hungarian = Hungarian(cost_matrix)
        hungarian.calculate()
    or
        hungarian = Hungarian()
        hungarian.calculate(cost_matrix)
    Handle Profit matrix:
        hungarian = Hungarian(profit_matrix, is_profit_matrix=True)
    or
        cost_matrix = Hungarian.make_cost_matrix(profit_matrix)
    The matrix will be automatically padded if it is not square.
    For that numpy's resize function is used, which automatically adds 0's to any row/co
    Get results and total potential after calculation:
        hungarian.get_results()
        hungarian.get_total_potential()
    """

    def __init__(self, input_matrix=None, is_profit_matrix=False):
        """
        input_matrix is a List of Lists.
        input_matrix is assumed to be a cost matrix unless is_profit_matrix is True.
        """
        if input_matrix is not None:
            # Save input
            my_matrix = np.array(input_matrix)
            self._input_matrix = np.array(input_matrix)
            self._maxColumn = my_matrix.shape[1]
            self._maxRow = my_matrix.shape[0]

            # Adds 0s if any columns/rows are added. Otherwise stays unaltered
            matrix_size = max(self._maxColumn, self._maxRow)
            pad_columns = matrix_size - self._maxRow
            pad_rows = matrix_size - self._maxColumn
            my_matrix = np.pad(my_matrix, ((0, pad_columns), (0, pad_rows)), 'constant'

            # Convert matrix to profit matrix if necessary
            if is_profit_matrix:
                my_matrix = self.make_cost_matrix(my_matrix)
```

```
            self._cost_matrix = my_matrix
            self._size = len(my_matrix)
            self._shape = my_matrix.shape

            # Results from algorithm.
            self._results = []
            self._totalPotential = 0
        else:
            self._cost_matrix = None

    def get_results(self):
        """Get results after calculation."""
        return self._results

    def get_total_potential(self):
        """Returns expected value after calculation."""
        return self._totalPotential

    def calculate(self, input_matrix=None, is_profit_matrix=False):
        """
        Implementation of the Hungarian (Munkres) Algorithm.
        input_matrix is a List of Lists.
        input_matrix is assumed to be a cost matrix unless is_profit_matrix is True.
        """
        # Handle invalid and new matrix inputs.
        if input_matrix is None and self._cost_matrix is None:
            raise HungarianError("Invalid input")
        elif input_matrix is not None:
            self.__init__(input_matrix, is_profit_matrix)

        result_matrix = self._cost_matrix.copy()

        # Step 1: Subtract row mins from each row.
        for index, row in enumerate(result_matrix):
            result_matrix[index] -= row.min()

        # Step 2: Subtract column mins from each column.
        for index, column in enumerate(result_matrix.T):
            result_matrix[:, index] -= column.min()

        # Step 3: Use minimum number of lines to cover all zeros in the matrix.
        # If the total covered rows+columns is not equal to the matrix size then adjust
        total_covered = 0
        while total_covered < self._size:
            # Find minimum number of lines to cover all zeros in the matrix and find to
            cover_zeros = CoverZeros(result_matrix)
            covered_rows = cover_zeros.get_covered_rows()
            covered_columns = cover_zeros.get_covered_columns()
            total_covered = len(covered_rows) + len(covered_columns)

            # if the total covered rows+columns is not equal to the matrix size then ad
            if total_covered < self._size:
                result_matrix = self._adjust_matrix_by_min_uncovered_num(result_matrix,
```

```
        # Step 4: Starting with the top row, work your way downwards as you make assignn
        # Find single zeros in rows or columns.
        # Add them to final result and remove them and their associated row/column from
        expected_results = min(self._maxColumn, self._maxRow)
        zero_locations = (result_matrix == 0)
        while len(self._results) != expected_results:

            # If number of zeros in the matrix is zero before finding all the results tl
            if not zero_locations.any():
                raise HungarianError("Unable to find results. Algorithm has failed.")

            # Find results and mark rows and columns for deletion
            matched_rows, matched_columns = self.__find_matches(zero_locations)

            # Make arbitrary selection
            total_matched = len(matched_rows) + len(matched_columns)
            if total_matched == 0:
                matched_rows, matched_columns = self.select_arbitrary_match(zero_locatic

            # Delete rows and columns
            for row in matched_rows:
                zero_locations[row] = False
            for column in matched_columns:
                zero_locations[:, column] = False

            # Save Results
            self.__set_results(zip(matched_rows, matched_columns))

        # Calculate total potential
        value = 0
        for row, column in self._results:
            value += self._input_matrix[row, column]
        self._totalPotential = value

    @staticmethod
    def make_cost_matrix(profit_matrix):
        """
        Converts a profit matrix into a cost matrix.
        Expects NumPy objects as input.
        """
        # subtract profit matrix from a matrix made of the max value of the profit matr
        matrix_shape = profit_matrix.shape
        offset_matrix = np.ones(matrix_shape, dtype=int) * profit_matrix.max()
        cost_matrix = offset_matrix - profit_matrix
        return cost_matrix

    def _adjust_matrix_by_min_uncovered_num(self, result_matrix, covered_rows, covered_c
        """Subtract m from every uncovered number and add m to every element covered wi
        # Calculate minimum uncovered number (m)
        elements = []
        for row_index, row in enumerate(result_matrix):
            if row_index not in covered_rows:
                for index, element in enumerate(row):
                    if index not in covered_columns:
```

```
                    elements.append(element)
        min_uncovered_num = min(elements)

        # Add m to every covered element
        adjusted_matrix = result_matrix
        for row in covered_rows:
            adjusted_matrix[row] += min_uncovered_num
        for column in covered_columns:
            adjusted_matrix[:, column] += min_uncovered_num

        # Subtract m from every element
        m_matrix = np.ones(self._shape, dtype=int) * min_uncovered_num
        adjusted_matrix -= m_matrix

        return adjusted_matrix

    def __find_matches(self, zero_locations):
        """Returns rows and columns with matches in them."""
        marked_rows = np.array([], dtype=int)
        marked_columns = np.array([], dtype=int)

        # Mark rows and columns with matches
        # Iterate over rows
        for index, row in enumerate(zero_locations):
            row_index = np.array([index])
            if np.sum(row) == 1:
                column_index, = np.where(row)
                marked_rows, marked_columns = self.__mark_rows_and_columns(marked_rows,
                                                                           column_index

        # Iterate over columns
        for index, column in enumerate(zero_locations.T):
            column_index = np.array([index])
            if np.sum(column) == 1:
                row_index, = np.where(column)
                marked_rows, marked_columns = self.__mark_rows_and_columns(marked_rows,
                                                                           column_index

        return marked_rows, marked_columns

    @staticmethod
    def __mark_rows_and_columns(marked_rows, marked_columns, row_index, column_index):
        """Check if column or row is marked. If not marked then mark it."""
        new_marked_rows = marked_rows
        new_marked_columns = marked_columns
        if not (marked_rows == row_index).any() and not (marked_columns == column_index
            new_marked_rows = np.insert(marked_rows, len(marked_rows), row_index)
            new_marked_columns = np.insert(marked_columns, len(marked_columns), column_
        return new_marked_rows, new_marked_columns

    @staticmethod
    def select_arbitrary_match(zero_locations):
        """Selects row column combination with minimum number of zeros in it."""
        # Count number of zeros in row and column combinations
```

```
        rows, columns = np.where(zero_locations)
        zero_count = []
        for index, row in enumerate(rows):
            total_zeros = np.sum(zero_locations[row]) + np.sum(zero_locations[:, column
            zero_count.append(total_zeros)

        # Get the row column combination with the minimum number of zeros.
        indices = zero_count.index(min(zero_count))
        row = np.array([rows[indices]])
        column = np.array([columns[indices]])

        return row, column

    def __set_results(self, result_lists):
        """Set results during calculation."""
        # Check if results values are out of bound from input matrix (because of matrix
        # Add results to results list.
        for result in result_lists:
            row, column = result
            if row < self._maxRow and column < self._maxColumn:
                new_result = (int(row), int(column))
                self._results.append(new_result)


class CoverZeros:
    """
    Use minimum number of lines to cover all zeros in the matrix.
    Algorithm based on: http://weber.ucsd.edu/~vcrawfor/hungar.pdf
    """

    def __init__(self, matrix):
        """
        Input a matrix and save it as a boolean matrix to designate zero locations.
        Run calculation procedure to generate results.
        """
        # Find zeros in matrix
        self._zero_locations = (matrix == 0)
        self._shape = matrix.shape

        # Choices starts without any choices made.
        self._choices = np.zeros(self._shape, dtype=bool)

        self._marked_rows = []
        self._marked_columns = []

        # marks rows and columns
        self.__calculate()

        # Draw lines through all unmarked rows and all marked columns.
        self._covered_rows = list(set(range(self._shape[0])) - set(self._marked_rows))
        self._covered_columns = self._marked_columns

    def get_covered_rows(self):
        """Return list of covered rows."""
```

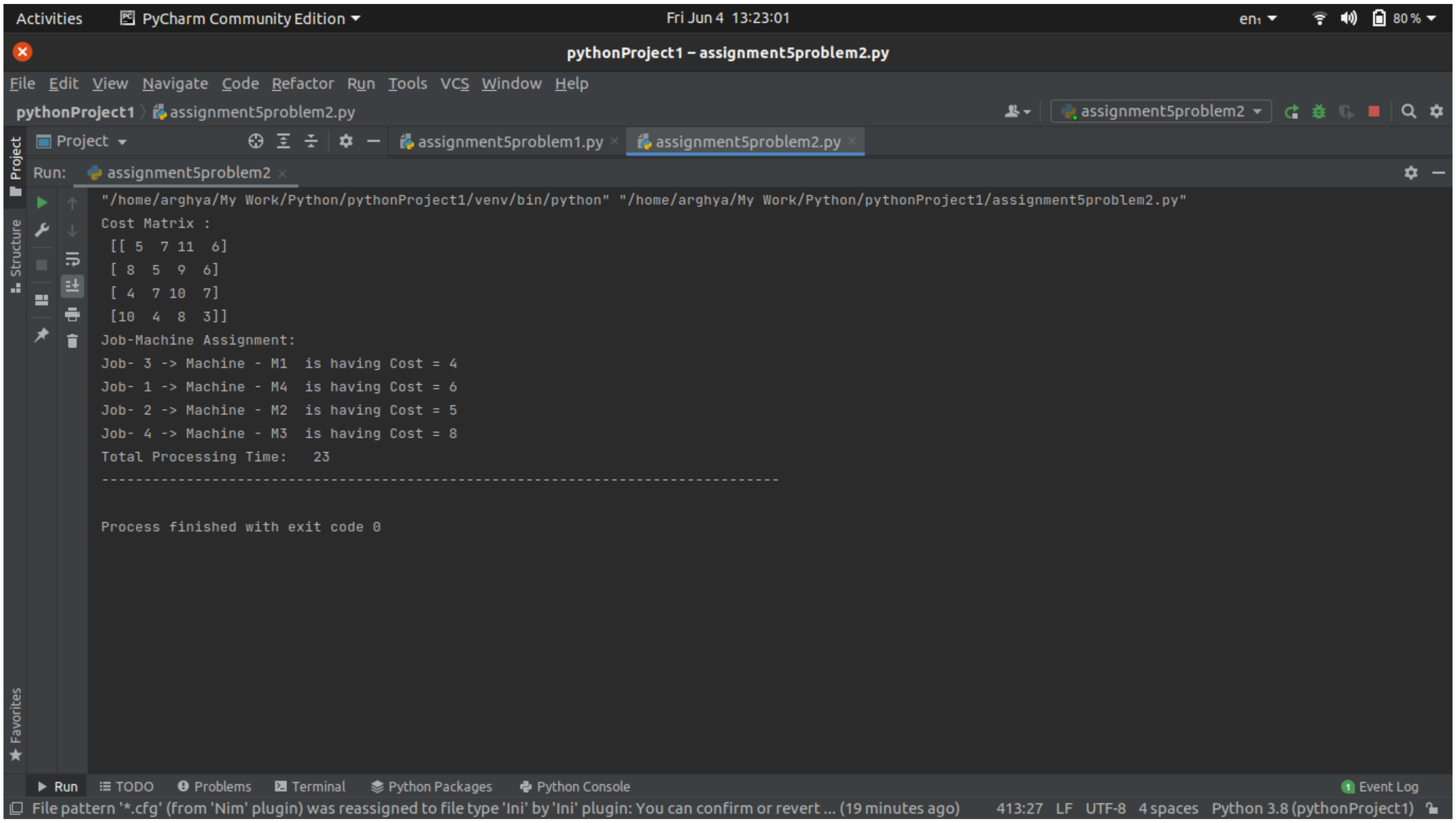
```
        return self._covered_rows

    def get_covered_columns(self):
        """Return list of covered columns."""
        return self._covered_columns

    def __calculate(self):
        """
        Calculates minimum number of lines necessary to cover all zeros in a matrix.
        Algorithm based on: http://weber.ucsd.edu/~vcrawfor/hungar.pdf
        """
        while True:
            # Erase all marks.
            self._marked_rows = []
            self._marked_columns = []

            # Mark all rows in which no choice has been made.
            for index, row in enumerate(self._choices):
                if not row.any():
                    self._marked_rows.append(index)

            # If no marked rows then finish.
            if not self._marked_rows:
                return True

            # Mark all columns not already marked which have zeros in marked rows.
            num_marked_columns = self.__mark_new_columns_with_zeros_in_marked_rows()

            # If no new marked columns then finish.
            if num_marked_columns == 0:
                return True

            # While there is some choice in every marked column.
            while self.__choice_in_all_marked_columns():
                # Some Choice in every marked column.

                # Mark all rows not already marked which have choices in marked columns
                num_marked_rows = self.__mark_new_rows_with_choices_in_marked_columns()

                # If no new marks then Finish.
                if num_marked_rows == 0:
                    return True

                # Mark all columns not already marked which have zeros in marked rows.
                num_marked_columns = self.__mark_new_columns_with_zeros_in_marked_rows(

                # If no new marked columns then finish.
                if num_marked_columns == 0:
                    return True

            # No choice in one or more marked columns.
            # Find a marked column that does not have a choice.
            choice_column_index = self.__find_marked_column_without_choice()
```

```
            while choice_column_index is not None:
                # Find a zero in the column indexed that does not have a row with a cho
                choice_row_index = self.__find_row_without_choice(choice_column_index)

                # Check if an available row was found.
                new_choice_column_index = None
                if choice_row_index is None:
                    # Find a good row to accomodate swap. Find its column pair.
                    choice_row_index, new_choice_column_index = self.__find_best_choice_r
                        choice_column_index)

                    # Delete old choice.
                    self._choices[choice_row_index, new_choice_column_index] = False

                # Set zero to choice.
                self._choices[choice_row_index, choice_column_index] = True

                # Loop again if choice is added to a row with a choice already in it.
                choice_column_index = new_choice_column_index

    def __mark_new_columns_with_zeros_in_marked_rows(self):
        """Mark all columns not already marked which have zeros in marked rows."""
        num_marked_columns = 0
        for index, column in enumerate(self._zero_locations.T):
            if index not in self._marked_columns:
                if column.any():
                    row_indices, = np.where(column)
                    zeros_in_marked_rows = (set(self._marked_rows) & set(row_indices))
                    if zeros_in_marked_rows:
                        self._marked_columns.append(index)
                        num_marked_columns += 1
        return num_marked_columns

    def __mark_new_rows_with_choices_in_marked_columns(self):
        """Mark all rows not already marked which have choices in marked columns."""
        num_marked_rows = 0
        for index, row in enumerate(self._choices):
            if index not in self._marked_rows:
                if row.any():
                    column_index, = np.where(row)
                    if column_index in self._marked_columns:
                        self._marked_rows.append(index)
                        num_marked_rows += 1
        return num_marked_rows

    def __choice_in_all_marked_columns(self):
        """Return Boolean True if there is a choice in all marked columns. Returns boole
        for column_index in self._marked_columns:
            if not self._choices[:, column_index].any():
                return False
        return True

    def __find_marked_column_without_choice(self):
        """Find a marked column that does not have a choice."""
```

```
        for column_index in self._marked_columns:
            if not self._choices[:, column_index].any():
                return column_index

        raise HungarianError(
            "Could not find a column without a choice. Failed to cover matrix zeros. Al

    def __find_row_without_choice(self, choice_column_index):
        """Find a row without a choice in it for the column indexed. If a row does not
        row_indices, = np.where(self._zero_locations[:, choice_column_index])
        for row_index in row_indices:
            if not self._choices[row_index].any():
                return row_index

        # All rows have choices. Return None.
        return None

    def __find_best_choice_row_and_new_column(self, choice_column_index):
        """
        Find a row index to use for the choice so that the column that needs to be chan
        Return a random row and column if unable to find an optimal selection.
        """
        row_indices, = np.where(self._zero_locations[:, choice_column_index])
        for row_index in row_indices:
            column_indices, = np.where(self._choices[row_index])
            column_index = column_indices[0]
            if self.__find_row_without_choice(column_index) is not None:
                return row_index, column_index

        # Cannot find optimal row and column. Return a random row and column.
        from random import shuffle

        shuffle(row_indices)
        column_index, = np.where(self._choices[row_indices[0]])
        return row_indices[0], column_index[0]


if __name__ == '__main__':
    cost_matrix = [
        [5, 7, 11, 6],
        [8, 5, 9, 6],
        [4, 7, 10, 7],
        [10, 4, 8, 3]]

    hungarian = Hungarian(cost_matrix)
    hungarian.calculate()
    print("Cost Matrix : \n", np.array(cost_matrix))
    jobs = [1, 2, 3, 4]
    machine = ['M1', 'M2', 'M3', 'M4']
    result = hungarian.get_results()
    print("Job-Machine Assignment:")
    for i in result:
        print("Job-", jobs[i[0]], "-> Machine -", machine[i[1]], " is having Cost =", c
    print("Total Processing Time:\t", hungarian.get_total_potential())
```

```
print("-" * 80)
```

Output :


Activities
PyCharm Community Edition
Fri Jun 4 13:23:01
80 %
pythonProject1 – assignment5problem2.py
File Edit View Navigate Code Refactor Run Tools VCS Window Help
pythonProject1 assignment5problem2.py
assignment5problem2
Project
assignment5problem1.py
assignment5problem2.py
Run: assignment5problem2
"/home/arghya/My Work/Python/pythonProject1/venv/bin/python" "/home/arghya/My Work/Python/pythonProject1/assignment5problem2.py"
Cost Matrix :
[[5 7 11 6]
[8 5 9 6]
[4 7 10 7]
[10 4 8 3]]
Job-Machine Assignment:
Job- 3 -> Machine - M1 is having Cost = 4
Job- 1 -> Machine - M4 is having Cost = 6
Job- 2 -> Machine - M2 is having Cost = 5
Job- 4 -> Machine - M3 is having Cost = 8
Total Processing Time: 23

Process finished with exit code 0
Structure
Favorites
Run
TODO
Problems
Terminal
Python Packages
Python Console
Event Log
File pattern '*.cfg' (from 'Nim' plugin) was reassigned to file type 'Ini' by 'Ini' plugin: You can confirm or revert ... (19 minutes ago)
413:27 LF UTF-8 4 spaces Python 3.8 (pythonProject1)