

$$(a) H_0: \mu = 11$$

$$H_a: \mu > 11$$

$$(b) \text{ test statistic: } t = \frac{\bar{x} - \mu}{s_{\bar{x}}/\sqrt{n}}$$

Sampled data:

$$\begin{array}{cccc} 12.5 & 11.9 & 13.1 & 13.8 \\ 13.0 & 10.2 & 13.6 & 14.0 \end{array}$$

$$\bar{x} = 12.7625$$

$$\frac{s_{\bar{x}}}{\sqrt{n}} = 0.44$$

$$s_{\bar{x}} = 1.2455$$

~~approx~~

$$\Rightarrow t^*_{df=7} = \frac{12.7625 - 11}{0.44} \approx 4.$$

$$(c) \text{ p-value: } P(\bar{x} > 12.7625 | H_0)$$

$$= P(t_{df=7} > 4)$$

$$\approx 0.002$$

\therefore p-value is less than $\alpha = 0.05$ level, H_0 is rejected

Hence the sample data supports alternative hypothesis at the 5% level

$$(d) CI = \bar{x} \pm t_{0.025, 7} 0.44 = 12.7625 \pm 2.365(0.44)$$

$$= (11.723, 13.803)$$