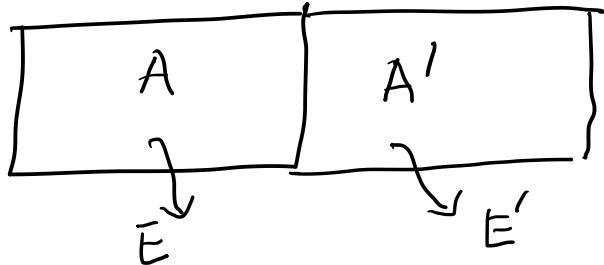


$$\textcircled{2} \quad \Omega(E) \sim E^f \quad f : \text{no. of degrees of freedom}$$

\textcircled{3} Isolated system : all microstates are equally probable

Thermal interaction between macroscopic systems



$$\begin{array}{ll}\Omega(E) & \Omega'(E') \\ \text{\# of states} & \text{\# of states of } A' \\ \text{between} & \text{between} \\ E & E' \\ \text{and } E + \delta E & E' & E' + \delta E'\end{array}$$

Suppose A has energy E

↓ A' must have $E' = (\bar{E}^{(0)} - E)$

of states accessible to $A^{(0)}$ can be regarded as a fn. of a single parameter \bar{E} , energy of A . $\Omega^{(0)}(E)$

$$\begin{aligned}A^{(0)} &= A + A' \rightarrow \text{isolated} . \\ \bar{E}^{(0)} &= E + E' \text{ (weakly interacting)}\end{aligned}$$

Fundamental Postulate applied to $A^{(0)}$

↓ equally likely to be found in any of the states of $\Omega^{(0)}(E)$

Prob $P(E)$ of finding the system $A^{(0)}$ in a configuration where A has energy between E & $E + \delta E$ $\propto \Omega^{(0)}(E)$

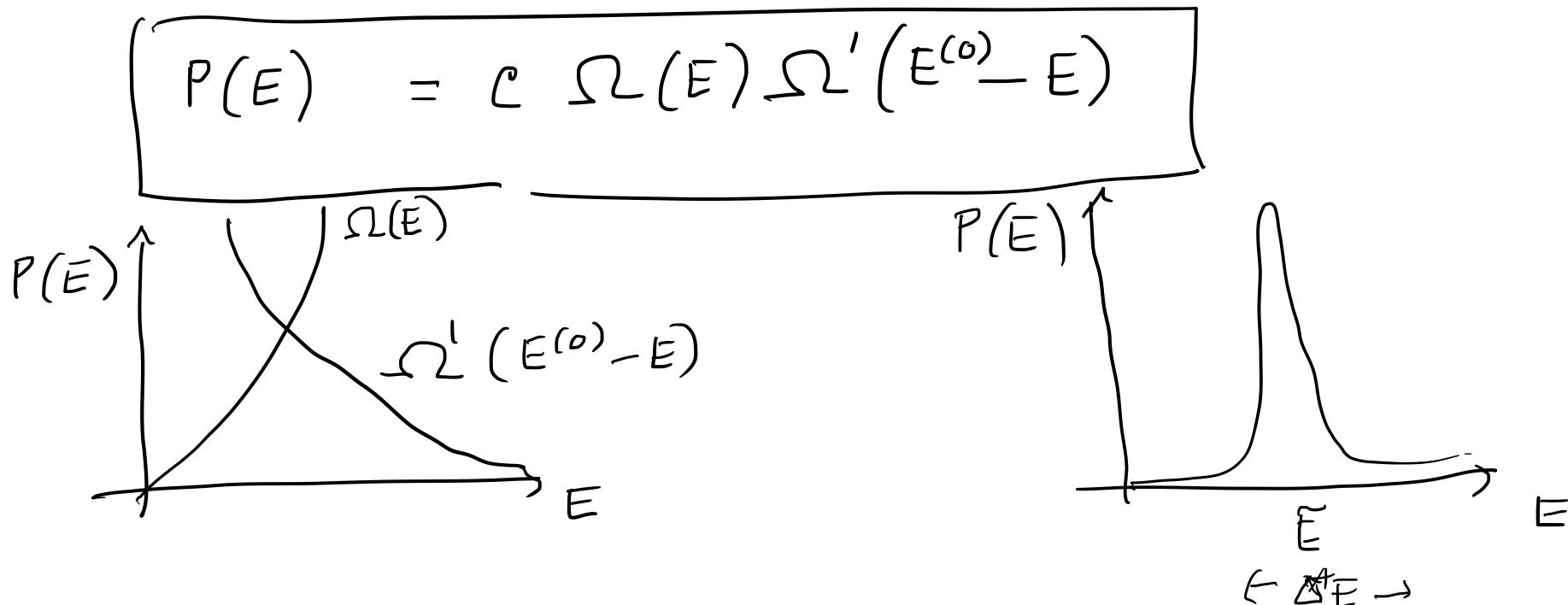
$$P(E) = C \Omega^{(0)}(E)$$

$$\boxed{P(E) = \frac{\Omega^{(0)}(E)}{\Omega^{(0)}_{\text{tot}}}}$$

$$C^{-1} = \Omega^{(0)}_{\text{tot}} = \sum_E \Omega^{(0)}(E)$$

But when A has energy E it must be in one of the $\Omega(E)$ states. At the same A' must be in one of its $\Omega'(E')$ states, ie, $\Omega'(E^{(0)} - E)$ states

$$\Omega^{(0)}(E) = \Omega(E) \Omega'(E^{(0)} - E)$$



$P(E)$ will have a maximum

$\ln P(E)$ will have a maximum

Condition for a maximum

$$\frac{\partial \ln P}{\partial E} = 0$$

$$\ln P = \ln C + \ln \Omega(E) + \ln \Omega'(E') ; E' = E^{(o)} - E$$

$$\frac{\partial \ln P}{\partial E} = \frac{\partial \ln \Omega(E)}{\partial E} + \frac{\partial \ln \Omega'(E')}{\partial E'} (-1) = 0$$

$$\frac{\partial \ln \Omega(E)}{\partial E} = \frac{\partial \ln \Omega'(E')}{\partial E'}$$

$$\beta(\tilde{E}) = \beta'(E')$$

\tilde{E} determined by $P(E) \max.$

$$\boxed{\beta = \frac{\partial \ln \Omega}{\partial E}}$$

β has dimensions $\frac{1}{E}$

define another dimensionless parameter T

$$\left. \begin{array}{l} kT = \frac{1}{\beta} \\ k: \text{const} \end{array} \right\}$$

Also define

$$\boxed{S \equiv k \ln \Omega}$$

$$\boxed{\frac{1}{T} = \frac{\partial S}{\partial E}}$$

entropy .

Max probability \rightarrow Total entropy $= S + S' = \max$.

\downarrow condition

$$\boxed{T = T'}$$

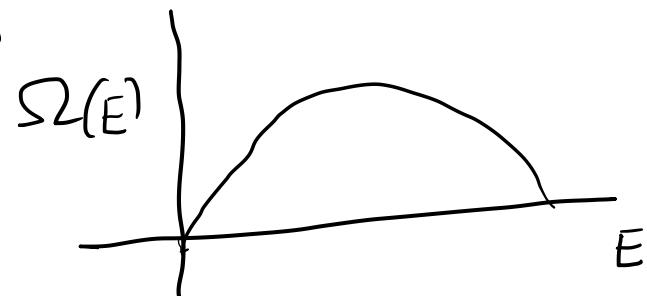
$$\frac{1}{kT} = \beta = \frac{\partial \ln \Omega}{\partial E} , \text{ since } \Omega \sim E^f$$

$\beta > 0, T > 0$

systems do not have upper bound of energy

spin systems $\rightarrow E$ bounded on both ends

$$T < 0$$



Ensembles of physical interest

1. Isolated systems.

$E = \text{const}$, N particles in vol V

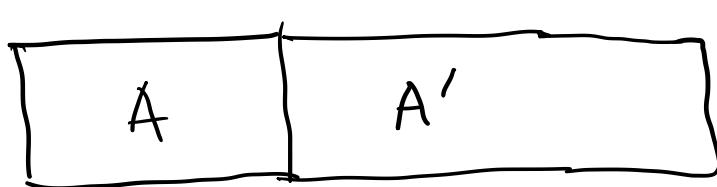
Ensemble: many such systems

energy of a system in microstate r is \bar{E}_r

$$P_r = \begin{cases} c & \text{if } E < \bar{E}_r < E + \delta E \\ 0 & \text{otherwise} \end{cases} \quad \sum P_r = 1 \quad \text{determines } c$$

microcanonical ensemble

2. System in contact with reservoir



$$A \ll A'$$

Q: Under conditions of equilibrium what is the prob.
Pr of finding A in one of its microstates & of energy
 E_r ?

• weak int. $E^{(0)} = E + E'$

• $A + A' \rightarrow$ isolated .

$$E_r + E' = E^{(0)}$$

If A is in one definite state r, E_r ,

A' must have energy $E^{(0)} - E_r$.

of states accessible to A⁽⁰⁾ = $\Omega'(E^{(0)} - E_r)$

Applying fundamental postulate to A⁽⁰⁾

$$P_r = C' \Omega'(E^{(0)} - E_r) , [C' \text{ determined} \rightarrow \sum_r P_r = 1]$$

Now consider $A \ll A'$, $E_r \ll E^{(0)}$.

$$\ln \Omega'(E^{(0)} - E_r) = \ln \Omega'(E^{(0)}) - \underbrace{\left[\frac{\partial \ln \Omega'}{\partial E'} \right]_0}_{\beta \approx \frac{1}{kT}} E_r + \dots$$

$\beta \approx \frac{1}{kT} \rightarrow \text{temp. of reservoir}$

$$\ln \Omega'(E^{(0)} - E_r) \approx \ln \Omega'(E^{(0)}) - \beta \bar{E}_r .$$

$$\Omega'(E^{(0)} - E_r) = \underbrace{\Omega'(E^{(0)})}_{\text{const ind of } \bar{E}_r} e^{-\beta \bar{E}_r}$$

→

$$P_r = C e^{-\beta \bar{E}_r}$$

$$C^{-1} = \sum_r e^{-\beta \bar{E}_r}$$

Boltzmann factor

$$P_r = \frac{e^{-\beta \bar{E}_r}}{\sum_r e^{-\beta \bar{E}_r}}$$

ensemble of systems in contact with reservoir of fixed T
 → Canonical Ensemble