(1) Find the equation of the tangent plane to $x^2+y^2=z^2$ at the point (m,n,p), and then show that it must pass through the line $mx+ny=0$ in the xy-plane.

(2) Compute the area of the portion of paraboloid of rotation $z=x^2+y^2$ under the plane $z=4$.

(3) Compute (i) $\int_C xe^y\,dx + x^2y\,dy$ along the curve $x=3t$, $y=t^2$, $0\le t\le 1$; (ii) $\int_C yz\,dx+xz\,dy+xy\,dz$ along the curve $r(t)=\langle t,t^2,t^3\rangle$, $0\le t\le 1$.

(4) Find the area of the surface $\langle v\cos u, v\sin u, u\rangle$, for $0\le u\le\pi$ and $0\le v\le 1$.

(5) Find the area of (i) the portion of $2x+4y+z=0$ inside $x^2+y^2=1$; (ii) $z=\sqrt{x^2+y^2}$ that lies below $z=2$; (iii) the cone $z=k\sqrt{x^2+y^2}$, $k>0$, that lies over a region R with area a.

(6) An object moves from $(1,0,0)$ to $(-1,0,\pi)$ along the curve $r(t)=\langle\cos t,\sin t,t\rangle$, subject to the force field $\langle y^2,y^2,xz\rangle$. Find the work done.

(7) Evaluate the surface integral $\int_S xy\,dS$, where S is the triangular region with vertices $(1,0,0)$, $(0,2,0)$ and $(0,0,2)$. [Ans: $\frac{1}{\sqrt{6}}$. Here the region of parametrization is given by the plane $z=2-2x-y$.]

(8) Let F and G be two C^1-vector fields on $\mathbb{R}^3$. Prove that

$$div(F\times G)=curl(F)\cdot G+F\cdot curl(G).$$

(9) Use Green's theorem to prove that

$$\int_C -x^2y\,dx+xy^2\,dy=8\pi,$$

where C is the circle $x^2+y^2=4$ (with counterclockwise orientation).

(10) Prove that in two dimensions the divergence theorem and Stokes' theorem are identical.

(11) Recall that the area of a (suitable) region R is given by

$$\text{area}(R)=\frac{1}{2}\int_C\langle -y,x\rangle\cdot dr,$$

where C is the boundary of R. Explain this relation geometrically.

(12) For the field $F=\langle x,y,z\rangle$ and the cylinder V given by $x^2+y^2\le 1$, $0\le z\le 1$;
   (i) Compute $\nabla\cdot F$ and $\int_V\nabla\cdot F$.
   (ii) Compute $\int_{\partial V}F\cdot dS$ (outward normal direction). Is this supposed to match (i)?

(13) Determine whether or not $F=\langle z\sec^2 x, z, y+\tan x\rangle$ is conservative. If F is conservative, find a potential function for F.

(14) For the gravitational field $F(x)=-k\frac{x}{\|x\|}$ in $\mathbb{R}^3$ and the sphere $V=\{x\in\mathbb{R}^3:\|x\|\le 1\}$;
   (i) Compute $\nabla\cdot F$ and $\int_V\nabla\cdot F$.
   (ii) Compute $\int_{\partial V}F\cdot dS$ (outward normal direction). Is this supposed to match (i)?

(15) Find the flux of $F(x,y,z)=(x,y,3)$ out of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$.

(16) (i) Prove that $\text{curl}\circ\nabla=0$ and $\text{div}\circ\text{curl}=0$. (ii) Prove that the curl through all closed surfaces in $\mathbb{R}^3$ has zero flux. That is, if S is a closed surface in the domain of a field F, then

$$\int_S(\nabla\times F)\cdot dS=0.$$

(17) Explain why the surface $z = x^2 \cos y$ is orientable.

(18) If $R = \{(x, y) : |x| \leq 2, |y| \leq 1\}$, then compute $\int_{\partial R} e^{2x+3y}\, dx + e^{xy}\, dy$.

(19) If $R = \{(x, y) : |x| \leq 1, x^2 \leq y \leq 1\}$, then compute $\int_{\partial R} \sqrt{1 + x^2}\, dy$.

(20) Let $F(x, y, z) = \langle y, z, -2x \rangle$, and let C be a simple closed curve contained in the plane $x + y + z = 1$. Prove that $\int_C F \cdot dr = 0$.

(21) Verify Stokes's theorem for the field $F = -yi + xj + e^z \ln(1 + z)k$ and the surface $z = \sqrt{4 - x^2 + y^2}$. (Find separately the path integral around the edge and the flux of the curl. You must decide which sense of the normal to pick.)

(22) Compute the outward flux $\int_S F \cdot dS$, where S is the sphere $x^2 + y^2 + z^2 = 4$ and $F(x, y, z) = (x^2 + y^2 + z^2)(xi + yj + zk)$. [Hint: Divergence theorem]