Physics 2 HW1 solns

ans.

1. Adiabatic compressibility

$PV^{\gamma} = \text{const}$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0.$$

$$\boxed{B_{ad} = -V\left(\frac{\partial P}{\partial V}\right)_{ad} = \gamma P} \quad \boxed{K_{ad} = \frac{1}{\gamma P}} = \boxed{\frac{1}{\rho c^2}},$$

2. $c = \sqrt{\left(\frac{dP}{d\rho}\right)_{ad}}$.

$$c^2 = \left(\frac{dP}{d\rho}\right)_{ad}.$$

Now, $\frac{d\rho}{\rho} = -\frac{dV}{V}$.

$$\therefore \left(\frac{dP}{d\rho}\right)_{ad} = \left(\frac{\partial P}{\partial V}\right)_{ad}\left(\frac{\partial V}{\partial \rho}\right).$$

$$= -\frac{V}{\rho}\left(\frac{\partial P}{\partial V}\right)_{ad}.$$

from problem 1

$$\boxed{\left(\frac{dP}{d\rho}\right)_{ad} = \frac{\gamma P}{\rho}}$$

$$\frac{P}{\rho} = \frac{RT}{M} = \frac{(C_p - C_v)T}{M} = (c_p - c_v)T = (\gamma - 1)c_v T.$$

we have, $c^2 = \gamma(\gamma-1) c_v T$

or $c_v T = \frac{c^2}{\gamma(\gamma-1)}$.

Now, ~~u~~ we know

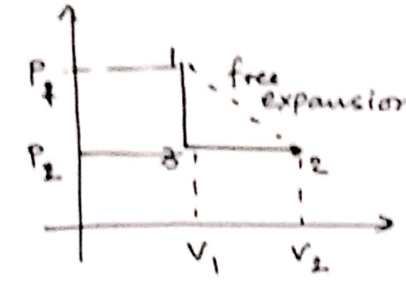
$$u = c_v T + \text{const} = \frac{c^2}{\gamma(\gamma-1)} + \text{const}$$

$$h = u + PV = c_v T + (c_p - c_v)T + \text{const}$$

$$= c_v T + (\gamma - 1) c_v T + \text{const}$$

$$= \frac{c^2}{\gamma(\gamma-1)}[\cancel{1} + \gamma - \cancel{1}] + \text{const} = \frac{c^2}{\gamma - 1} + \text{const}.$$

3.



$W_{12} = 0$
$Q_{12} = 0$ } free expansion

$U_1 = U_2$

$T_1 = T_2$.

$W_{2\to 3} = -\int_{V_1}^{V_2} P_2\, dV = P_2 (V_2 - V_1)$. } quasi static compression.

The temp changes from T_2 to T_3 in the process

Heat absorbed by gas.

$Q_{2\to 3} = \int_{T_2}^{T_3} C_p\, dT = C_p (T_3 - T_2)$.

$3 \to 1$ quasistatic isochoric change. $W_{31} = 0$.

$Q_{3\to 1} = \int_{T_3}^{T_1} C_v\, dT = C_v (T_1 - T_3)$.

Over the cycle $\Delta U = \Delta Q + \Delta W = 0$. (1st Law).

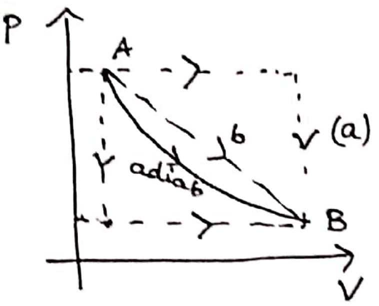
$Q_{1\to 2} + Q_{2\to 3} + Q_{3\to 1} + W_{1\to 2} + W_{2\to 3} + W_{3\to 1} = 0$.

$\because T_2 = T_1 \to$ $C_p (T_3 - T_1) - C_v (T_3 - T_1) + P_2 (V_2 - V_1) = 0$.

$(C_p - C_v)(T_3 - T_1) = P_2 V_1 - P_2 V_2 = R(T_3 - T_1)$

$\boxed{C_p - C_v = R}$ proved.

4.



U is a state fn. so $U(B) - U(A)$ does not ~~b~~ depend on path. Let us calculate along adiabatic

$$đQ = dU + p\,dV .$$

adiabatic path $đQ = 0$

$$dU = -p\,dV$$

$$U(B) - U(A) = -\int_A^B P\,dV$$

$$= -\alpha \int_A^B V^{-5/3}\,dV .$$

$$= \frac{3}{2}\alpha \left(V_B^{-2/3} - V_A^{-2/3}\right)$$

$$= \frac{3}{2}\left(P_B V_B^{5/3} V_B^{-2/3} - P_A V_A^{5/3} V_A^{-2/3}\right)$$

$$\boxed{U(B) - U(A) = \frac{3}{2}\left(P_B V_B - P_A V_A\right)}$$

(a) $(W_{AB})_{(a)} = -P_A (V_B - V_A)$

$$Q_{AB} = \frac{3}{2}\left(P_B V_B - P_A V_A\right) + P_A (V_B - V_A)$$

(b) $(W_{AB})_{(b)} = -\frac{1}{2}(V_B - V_A)(P_A - P_B) - (V_B - V_A) P_B .$

$$Q_{AB} = \frac{3}{2}\left(P_B V_B - P_A V_A\right) - (W_{AB})_{(b)}$$

(c) $(W_{AB})_{(c)} = -P_B (V_B - V_A) .$

$$Q_{AB} = \frac{3}{2}\left(P_B V_B - P_A V_A\right) + P_B (V_B - V_A) .$$

5.

~~1. $dQ = dU$~~

1. $dQ = dU + pdV$.

$$dQ = \left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial P}\right)_T dP + pdV$$

$$\left(\frac{dQ}{dT}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P.$$

$$C_P = \left(\frac{\partial U}{\partial T}\right)_P + PV\beta.$$

$$\boxed{\left(\frac{\partial U}{\partial T}\right)_P = C_P - PV\beta} \quad \text{--- (1)}$$

Now, $dQ = dU + pdV$.

$$= \left(\frac{\partial U}{\partial P}\right)_T dP + \left(\frac{\partial U}{\partial T}\right)_P dT + pdV.$$

$$\left(\frac{dQ}{dT}\right)_V = \left(\frac{\partial U}{\partial P}\right)_T\left(\frac{\partial P}{\partial T}\right)_V + \left(\frac{\partial U}{\partial T}\right)_P + \cancel{P\left(\frac{\partial V}{\partial T}\right)_P}. \quad \left[\begin{matrix} dV = 0 \\ \text{const } V \end{matrix}\right.$$

$$C_V = \left(\frac{\partial U}{\partial P}\right)_T\left(\frac{\partial P}{\partial T}\right)_V + \underbrace{C_P - PV\beta}_{\text{from (1)}} + \cancel{PV\beta}$$

Now, $\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial V}{\partial P}\right)_T = -1.$

$$\therefore \left(\frac{\partial P}{\partial T}\right)_V = -\frac{\left(\frac{\partial V}{\partial T}\right)_P}{\left(\frac{\partial V}{\partial P}\right)_T} = \frac{-\beta \cancel{V}}{-\kappa \cancel{V}} = \frac{\beta}{\kappa}$$

$$\therefore \left(\frac{\partial U}{\partial P}\right)_T = -\frac{(C_P - C_V)}{\beta/\kappa} + \frac{PV\beta}{\beta/\kappa}.$$

$$\Rightarrow \boxed{\left(\frac{\partial U}{\partial P}\right)_T = PV\kappa - \frac{(C_P - C_V)}{\beta/\kappa}}$$

Proved.

6.

$$\frac{U}{V} = cT^4.$$

$$P = \frac{1}{3}\frac{U}{V}$$

a)

$$đQ = dU + p\,dV.$$

$$U = cVT^4.$$

$$dU = c\,dV\,T^4 + 4cVT^3 dT.$$

adiabatic.

$$đQ = c\,dV\,T^4 + 4cVT^3 dT + \frac{c}{3}T^4 dV = 0$$

$$\frac{4}{3}T^4 dV + 4VT^3 dT = 0.$$

$$T^4 dV + 3VT^3 dT = 0.$$

$$\frac{dV}{V} + 3\frac{dT}{T} = 0.$$

$$\boxed{VT^3 = \text{const}}$$

b)

$$U = cVT^4.$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = 4cVT^3$$

c)

$$đQ = dU + p\,dV.$$

The eqn. of state is $p = \frac{c}{3}T^4$.

$C_p = \left(\frac{đQ}{dT}\right)_p$ → it is not possible to change temp while keeping P const

So, at const. pr, adding heat does not change temp,

so C_p is formally infinite