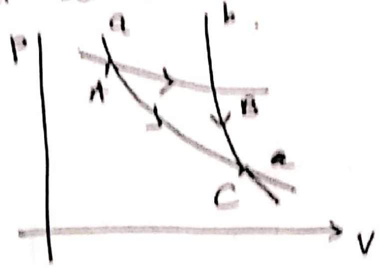
Solution Set for HW 2

1. Assume that two reversible adiabatic lines a, b
(5) intersect at C.



Consider an isothermal line that intersects with a, b at A, B. [Convince yourself that such a line can always be found.

Then the cycle $A \to B \to C \to A$ can be considered as a reversible cycle in which the system takes a positive amount of heat Q from the environment only during the leg $A \to B$ and does positive work $W = $ (area ABC). By first law $Q = W$ since $\Delta U = 0$.

So in this cycle the system has taken a positive amount of heat Q from a source and converted it entirely into work without any other change $\Rightarrow$ Contradicts Kelvin-Planck statement.

$\Rightarrow$ a, b cannot intersect.

Note: As brought up in class, this does not hold true for a reversible and irreversible adiabatic curve. e.g. You can start out an adiabatic quasistatic process and an adiabatic free expansion from the same initial conditions. Of course a free expansion curve cannot be plotted on a PV diagram.

2 (a) From Maxwell relations.
(5)

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V = \frac{P}{T}, \quad P, T > 0$$

Here $P = kT$.

$$\left(\frac{\partial P}{\partial T}\right)_V = k = \frac{P}{T} \quad \Bigg\} \quad \left(\frac{\partial S}{\partial V}\right)_T > 0.$$

entropy increases with volume.

2 (b). $\left(\frac{\partial H}{\partial P}\right)_T$

(5)

$$dH = TdS + VdP.$$

$\frac{\partial H}{\partial P}$ Isenthalpic process. (J-T).

$$dH = 0$$

Joule Thomson coeff.

$\left(\frac{\partial T}{\partial P}\right)_H > 0$ for cooling.

Now; $$\left(\frac{\partial H}{\partial P}\right)_T = -\frac{(\partial H/\partial T)_P}{\left(\frac{\partial P}{\partial T}\right)_H.} = -\frac{C_P}{\left(\frac{\partial P}{\partial T}\right)_H}.$$

$$C_P > 0.$$

$$\therefore \left(\frac{\partial H}{\partial P}\right)_T = -\frac{C_P}{\left(\frac{\partial P}{\partial T}\right)_H} < 0$$

3. (i). Equation of state.

(5)

$$PV = \frac{1}{3}U \quad ; \quad p = \frac{1}{3}u. \quad \text{—— (1)}$$

we have derived in class

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P \quad \text{—— (2)}$$

[Using (1) in (2)

$$u = \frac{1}{3}T\frac{du}{dT} - \frac{1}{3}u.$$

$$4\frac{dT}{T} = \frac{du}{u}.$$

$$\boxed{u = cT^4}.$$

(ii) $$TdS = dU + pdV.$$

3 (ii) entropy density.

(5)

$$Tds = dU + pdV. \quad \Bigg| \quad \begin{aligned} U &= cT^4 V. \\ dU &= cT^4 dV + 4cT^3 V dT \end{aligned}$$

$$Tds = cT^4 dV + 4cT^3 V dT + \frac{c}{3} T^4 dV$$

$$dS = \frac{4}{3} c T^3 dV + 4cT^2 V dT.$$

$$dS = d\left(\frac{4}{3} c T^3 V\right).$$

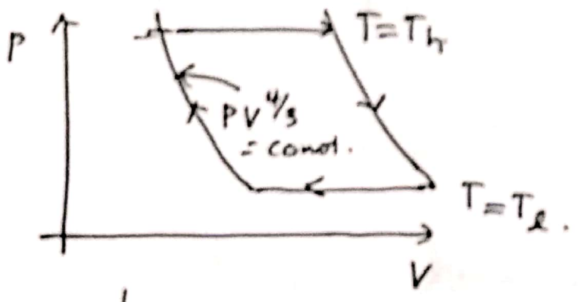
$$S = \frac{4}{3} c T^3 V + \underbrace{S_0}_{=0 \text{ by given bdy condn.}}$$

$$\boxed{s = \frac{S}{V} = \frac{4}{3} c T^3}$$

(iii). Adiabatics for this gas. $\Rightarrow$ $VT^3 = \text{const.}$ $\rightarrow$ $PV^{4/3} = \text{const}$

isothermal $P = \text{const}$

(5)



↳ Carnot cycle.

4.
(10)

Heat Capacity $C = aT^3 = \left(\frac{\partial U}{\partial T}\right)_V$.

$$dQ_l = aT_l^3 \, dT_l.$$

For the refrigerator

$$Q_l + W = Q_h.$$

$$dW = \left(1 - \frac{T_l}{T_h}\right) Q_h. = \left(T_h - T_l\right) \frac{Q_l}{T_l}.$$

$$dW = \left(T_h - T_l\right) \frac{dQ_l}{T_l}.$$

$$= \left(\frac{T_h}{T_l} - 1\right) a \, T_l^3 dT_l \quad \} \quad T_h = T_i, \text{ if } T_l \; dT_l$$

$$W = T_h \int_{T_i}^{0} a \, T_l^2 dT_l - \int_{T_i}^{0} a \, T_l^3 dT_l.$$

$$W = \frac{a}{3} \left(-T_i^3 T_h\right) + a \frac{T_i^4}{4}. \qquad T_h = T_i$$

$$W = -\frac{a}{3} T_h^4 + a \frac{T_h^4}{4}.$$

$$= -\frac{a T_h^4}{12}.$$

work done by the system

Electrical energy required $= \dfrac{a T_h^4}{12}$.

5. T_h and T_i

i(b) $W = (T_h - T_l)\frac{Q_l}{T_l}$.

$= \left(\frac{T_h}{T_l} - 1\right) Q_l$.

$P = \left(\frac{T_h}{T_l} - 1\right)\frac{dQ_l}{dt}$.

$= \left(\frac{T_h}{T_l} - 1\right) A (T_h - T_l)$.

$\Rightarrow \frac{P}{A} T_l = (T_h - T_l)^2$.

$\Rightarrow T_l^2 - 2 T_l T_h - \frac{P}{A} T_l + T_h^2 = 0$.

$$\boxed{T_l = T_h + \frac{P}{2A} - \sqrt{\left(T_h + \frac{P}{2A}\right)^2 - T_h^2}}$$

minus sign taken since
$T_l < T_h$