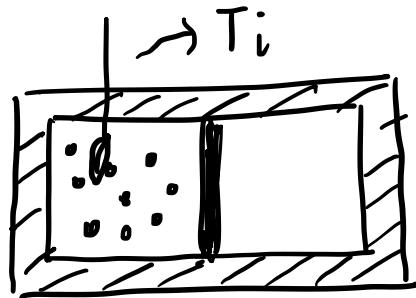
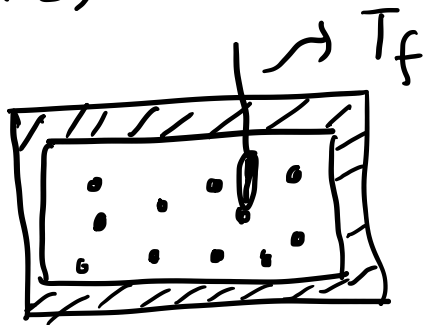


Joule's experiment (1843)



Before T_i



After $T_f = ?$

Adiabatic free expansion

$$\underset{=0}{dQ} = \underset{=0}{dU} + dW \Rightarrow dU = 0$$

$$\boxed{U_i = U_f}$$

Joule's result: $\boxed{T_i = T_f}$

$$dU = \left(\frac{\partial U}{\partial V} \right)_T dV + \left(\frac{\partial U}{\partial T} \right)_V dT$$

$$\boxed{\left(\frac{\partial U}{\partial V} \right)_T = 0}, \quad U = U(T)$$

Adiabatic quasistatic changes (expansion/compression)
ideal gas

$$dQ = C_v dT + p dV$$

$$dQ = 0$$

$$C_v dT + p dV = 0$$

$$C_v dT + \frac{nRT}{V} dV = 0$$

$$\frac{dT}{T} + \frac{nR}{C_v} \frac{dV}{V} = 0$$

$$\ln T + \frac{C_p - C_v}{C_v} \ln V = 0$$

$$\ln T + (\gamma - 1) \ln V = 0$$

$$\begin{aligned} TV^{\gamma-1} &= \text{const} \\ PV^{\gamma} &= \text{const} \\ TP^{\frac{1-\gamma}{\gamma}} &= \text{const} \end{aligned}$$

Application

$T(h) \rightarrow$ Dependence of T on height of atmosphere.

$$dh \left\{ \begin{array}{l} \text{---} p + dp \\ \text{---} p \end{array} \right.$$

weight of air in the slice = $\rho g dh$

$$dp = -\rho g dh$$

$$PV = nRT$$

$$= \frac{m}{M} RT$$

ideal gas

$M = \text{mol. wt}$

$$\rho = \frac{PM}{RT}$$

$$dp = -\frac{PM}{RT} g dh$$

$$dp = - \frac{PM}{RT} g dh$$

$$= - \frac{gM}{R} \frac{P}{T} dh$$

}

$$T P^{\frac{1-\gamma}{\gamma}} = \text{const} \quad \sim \text{adiabatic}$$

$$dT P^{\frac{1-\gamma}{\gamma}} + \frac{1-\gamma}{\gamma} T \frac{dp}{P} P^{\frac{1-\gamma}{\gamma}} = 0$$

$$\boxed{dp = - \frac{P}{T} \frac{\gamma}{1-\gamma} dT}$$

$$-\frac{P}{T} \frac{\gamma}{1-\gamma} dT = - \frac{gM}{R} \frac{P}{T} dh$$

$$\boxed{\frac{dT}{dh} = \frac{gM}{R} \left(\frac{1-\gamma}{\gamma} \right)}$$

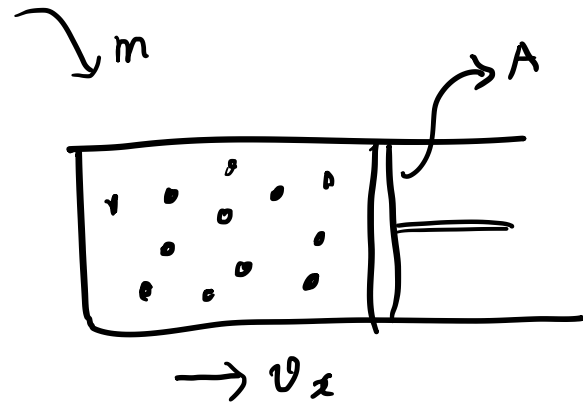
$$\approx -9.8 \text{ degrees/km}$$

$$M = 28.8 \quad \gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

$g, R,$ \sim diatomic

Detour - Kinetic theory of gases

- atoms are hard spheres
- no interaction between atoms
- collisions are elastic
- No external field \longrightarrow atoms are distributed uniformly
- No preferred direction for velocity



Momentum delivered to the piston

$$= 2mv_x$$

of collisions with piston in $\Delta t = ?$

N atoms in volume V , $\frac{N}{V} = n$

$$\rightarrow = n v_x \Delta t A$$

$$F = 2mv_x \times n v_x A$$

$$P = 2m n v_x^2 \quad \text{X}$$

- only $\frac{1}{2}$ will have +ve v_x
factor of 2 must go

- Not all particles will have same v_x

$$P = mn \langle v_x^2 \rangle$$

$$\text{Isotropy} \rightarrow \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$P = \frac{1}{3} mn \langle v^2 \rangle$$

$$P = \frac{2n}{3} \left\langle \frac{1}{2} m v^2 \right\rangle$$

$$P = \frac{2}{3} \frac{N}{V} \left\langle \frac{1}{2} m v^2 \right\rangle$$

$$PV = \frac{2}{3} U$$

Combining with equation of state

$$PV = nRT$$

$$\frac{2}{3}U = nRT \rightarrow T \propto \left\langle \frac{1}{2}mv^2 \right\rangle$$

$$U = \frac{3}{2}nRT = \frac{3}{2}NkT$$

$$C_V = \frac{\partial U}{\partial T} = \frac{3}{2}nR$$

$$C_P = \frac{5}{2}nR$$