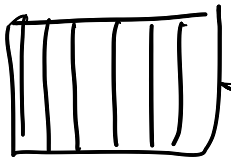
Double slit

$$I = I_0 \underbrace{\frac{\sin^2\beta}{\beta^2}}_{\text{diffraction}} \underbrace{\cos^2\gamma}_{\text{interference}}$$

$$\beta = \frac{\pi b \sin\theta}{\lambda}$$

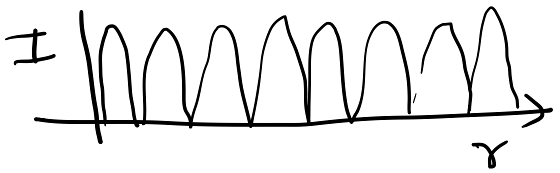
$$\gamma = \frac{\pi d \sin\theta}{\lambda}$$

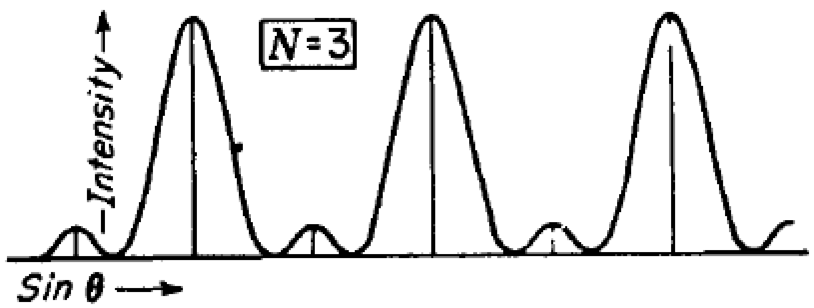
Effect of increasing # of slits.

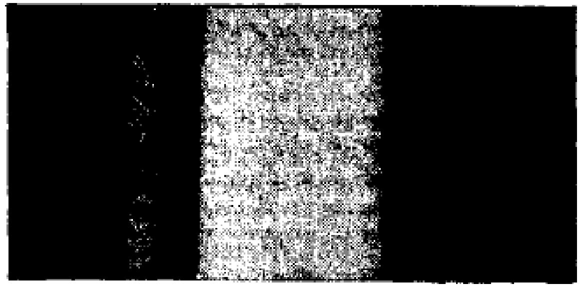
 → grating : large # of parallel equidistant slits.

→ # of slits increasing ⟶ narrowing of maxima, sharper lines

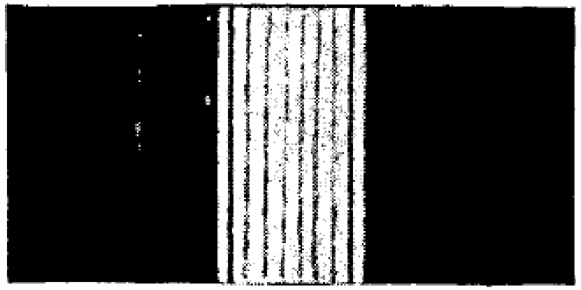
→ appearance of weak secondary maxima in between principal maxima. Recall in double slit



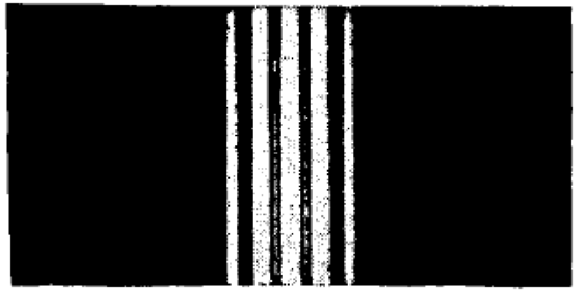




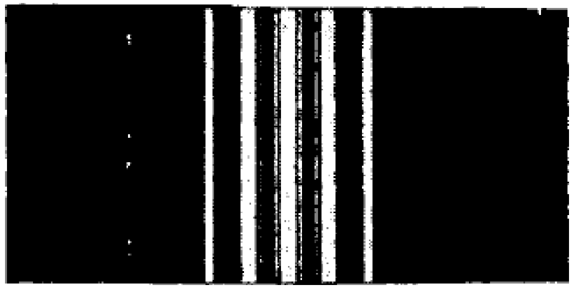
(a) 1 slit



(b) 2 slits

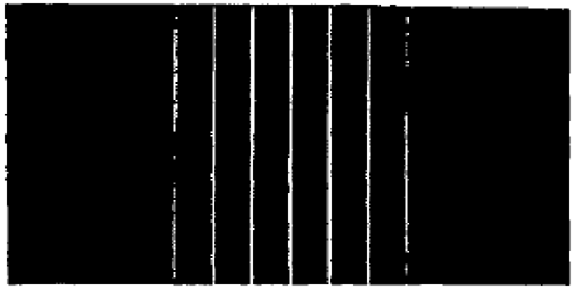


(c) 3 slits



(d) 5 slits

(e) 6 slits



(f) 20 slits

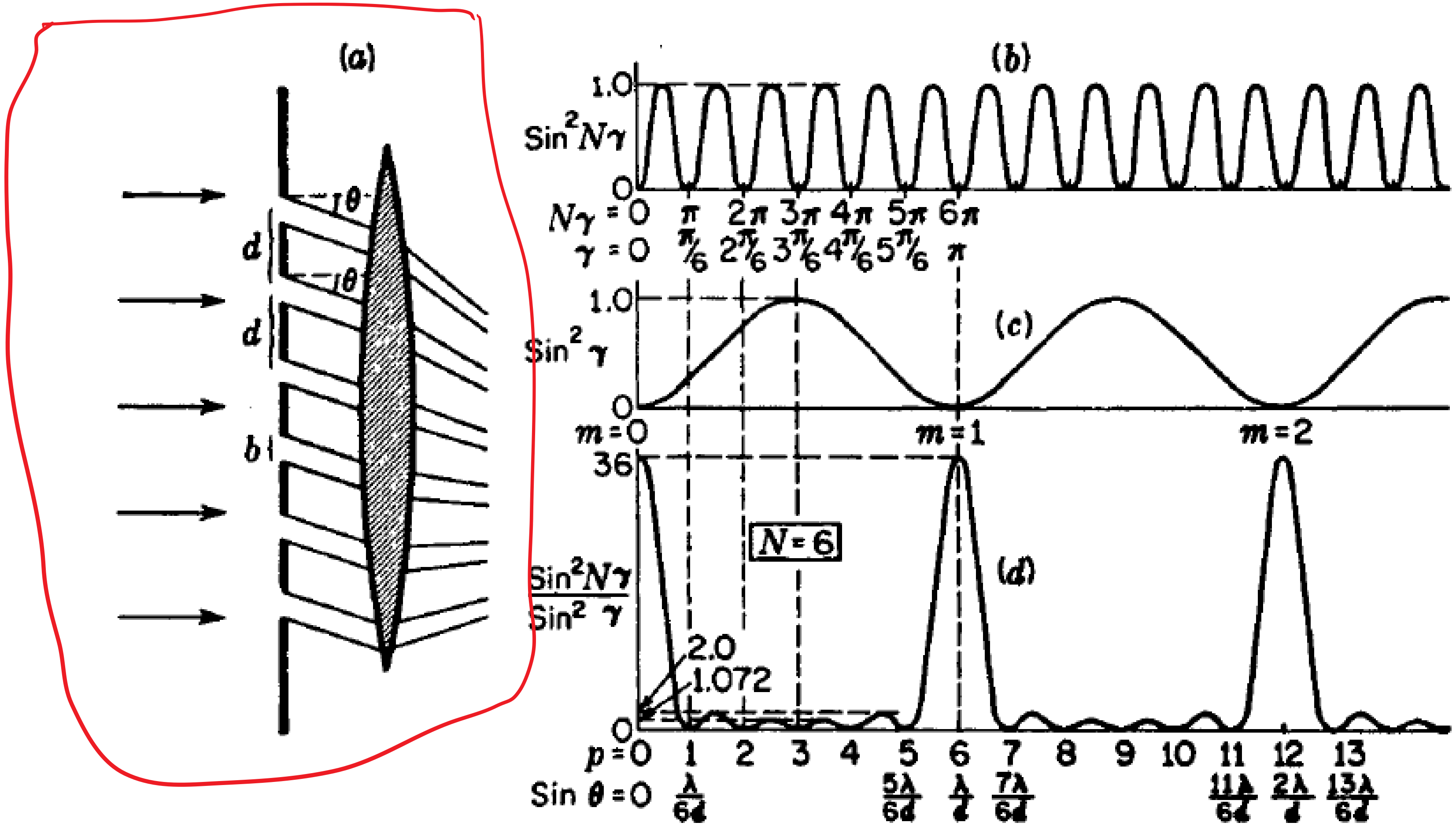


FIGURE 17C
Fraunhofer diffraction by a grating of six very narrow slits and details of the intensity pattern.

Intensity distribution from N slit grating :

Complex amplitude method

Net amplitude (complex) at P

$$\underbrace{Ae^{i\theta}}_{\tilde{A}} = a\left(1 + e^{i\delta} + e^{2i\delta} + \cdots + e^{i(N-1)\delta}\right)$$

$$= a\,\frac{1 - e^{iN\delta}}{1 - e^{i\delta}}.$$

$$I = |\tilde{A}|^2 = a^2 \frac{\left(1 - e^{iN\delta}\right)\left(1 - e^{-iN\delta}\right)}{\left(1 - e^{i\delta}\right)\left(1 - e^{-i\delta}\right)} = a^2\,\frac{1 - \cos N\delta}{1 - \cos\delta}.$$

$$I = \frac{a^2 \sin^2 \frac{N\delta}{2}}{\sin^2 \delta/2} \qquad \gamma = \frac{\delta}{2} = \frac{\pi d \sin\theta}{\lambda}.$$

a = amplitude from single slit.

$$\boxed{I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}} \qquad \beta = \frac{\pi b \sin\theta}{\lambda}$$

for $N = 2$. $\quad I = I_0 \frac{\sin^2\beta}{\beta^2} \frac{\sin^2 2\gamma}{\sin^2\gamma} = 4 I_0 \frac{\sin^2\beta}{\beta^2} \cos^2\gamma.$

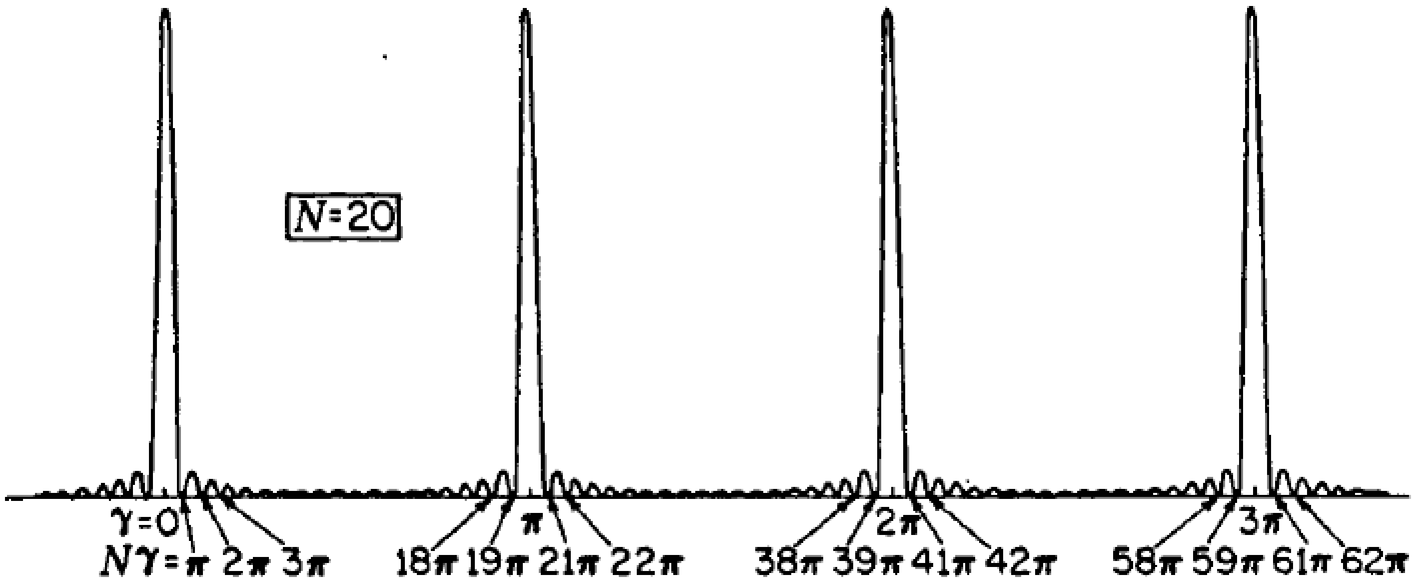


FIGURE 17D
Intensity pattern for 20 narrow slits.

Analyze intensity distribution

Principal Maxima

$\frac{\sin^2 N\gamma}{\sin^2\gamma}$ ⇝ new factor represents interference term for N slits.

→ maximum f value is N^2.

$$\lim_{\gamma \to m\pi} \frac{\sin N\gamma}{\sin\gamma} = \pm N .$$

these maxima correspond to those in the double slit since for above values of γ

$$\rightarrow \boxed{d\sin\theta = m\lambda}$$

Relative intensities of diff orders of m is govened by diffraction envelope
relation between β & γ is same, condition for missing orders also same

Minima & secondary maxima

Minimum of $\frac{\sin^2 N\gamma}{\sin^2 \gamma}$: Numerator becomes becomes zero more often than denominator

$\downarrow$

$N\gamma = p\pi \longrightarrow$ minimum

except for special cases where $p = mN \longrightarrow$ principal maximum

Condn. for minimum $\gamma = \frac{p\pi}{N}$ excluding $p = mN$

path diff

$d\sin\theta = \frac{\lambda}{N}, \frac{2\lambda}{N} \; - - - - \; \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}$. Minima.

omit values $0, \frac{N\lambda}{N}, \ldots$ $\boxed{d\sin\theta = m\lambda} \rightarrow$ Principal maxima

$\boxed{\text{Between two principal maxima (N-1) point of zero intensity}}$

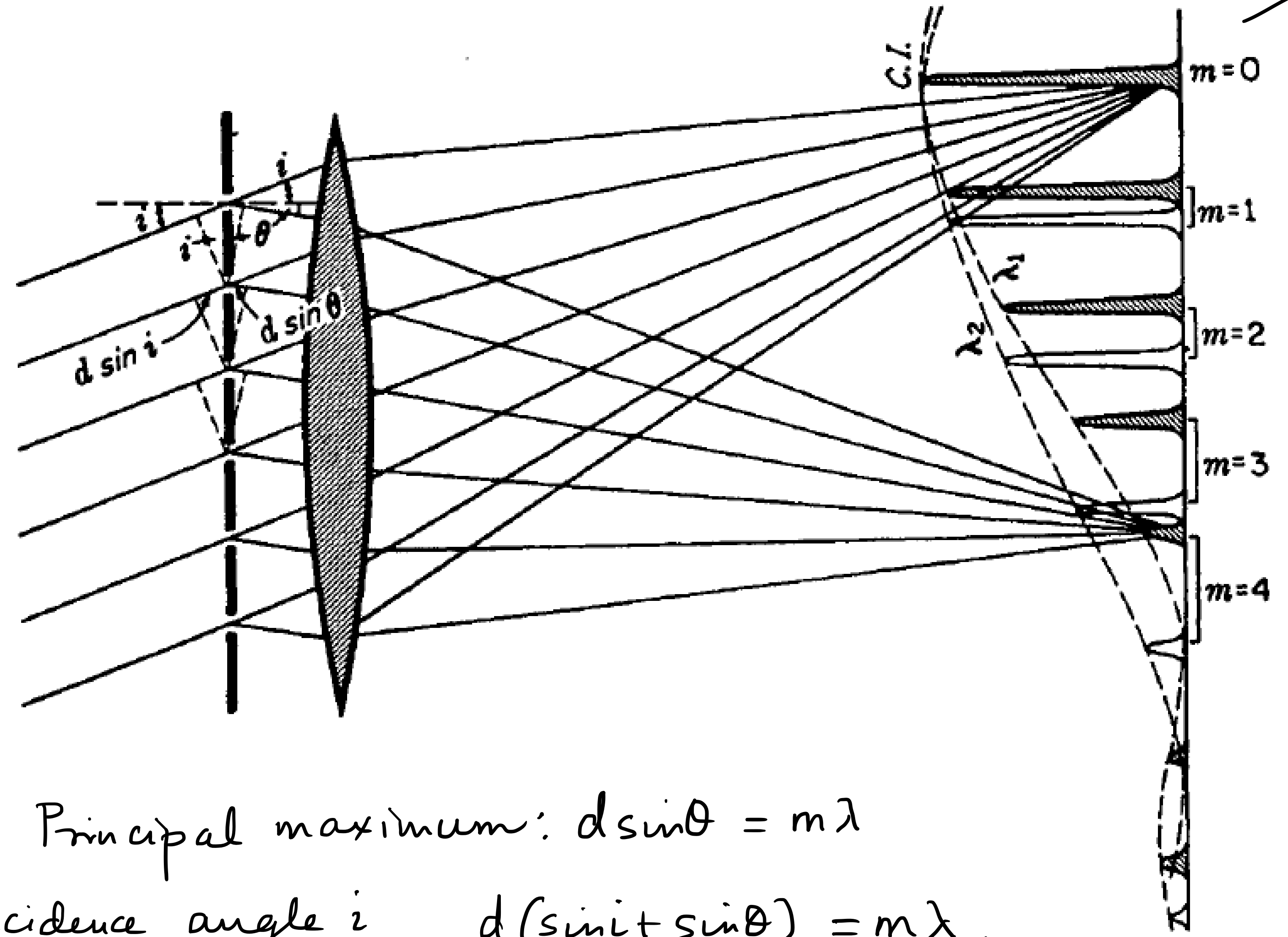
Between two minima intensity rises again $\longrightarrow$ secondary maxima

They are much smaller intensity than principal maxima.

- The secondary maxima are not of equal intensity and they fall off in intensity on either side of principal maxima. Not equally spaced either

$d = 5b$.

5th order missing.



Principal maximum: $d \sin\theta = m\lambda$

at incidence angle i $\quad d(\sin i + \sin\theta) = m\lambda$.

$n=0$ central image.

maximum irrespective of λ.

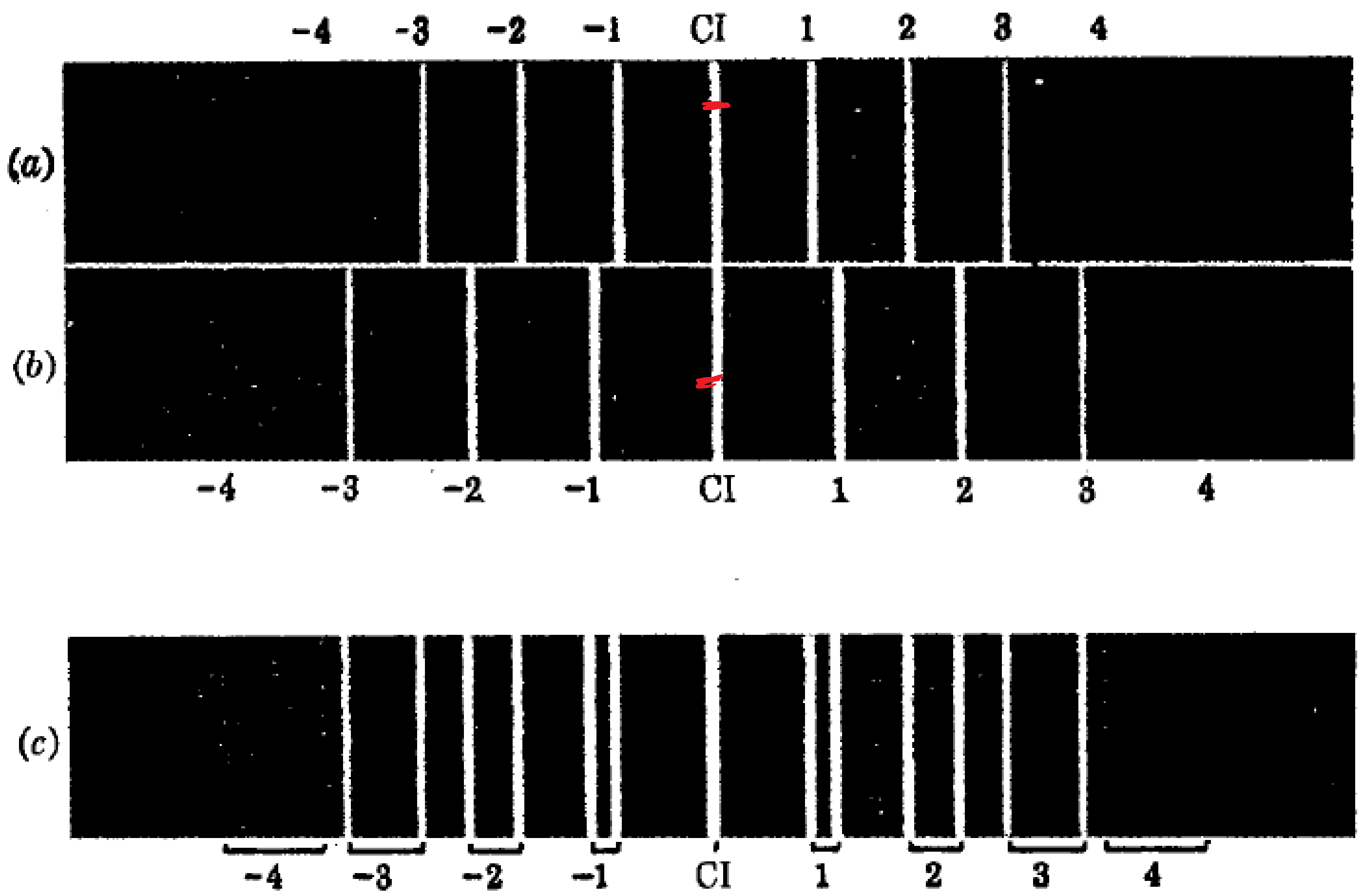
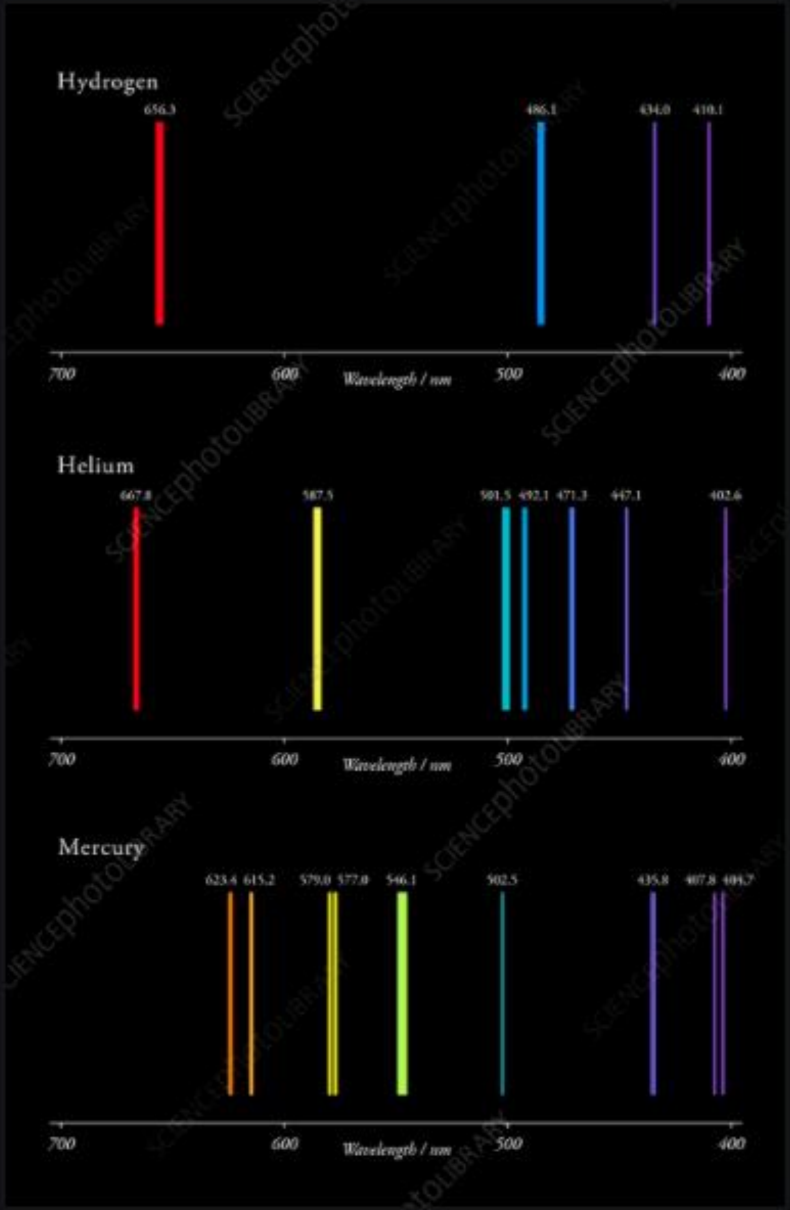
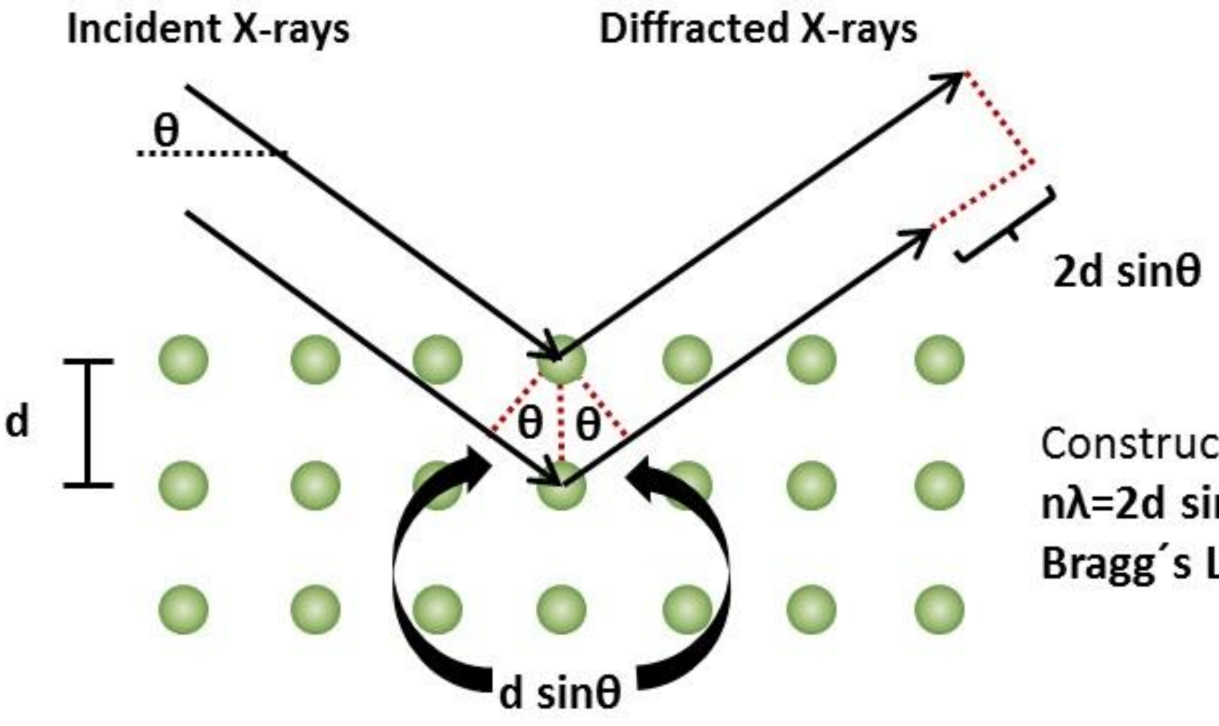
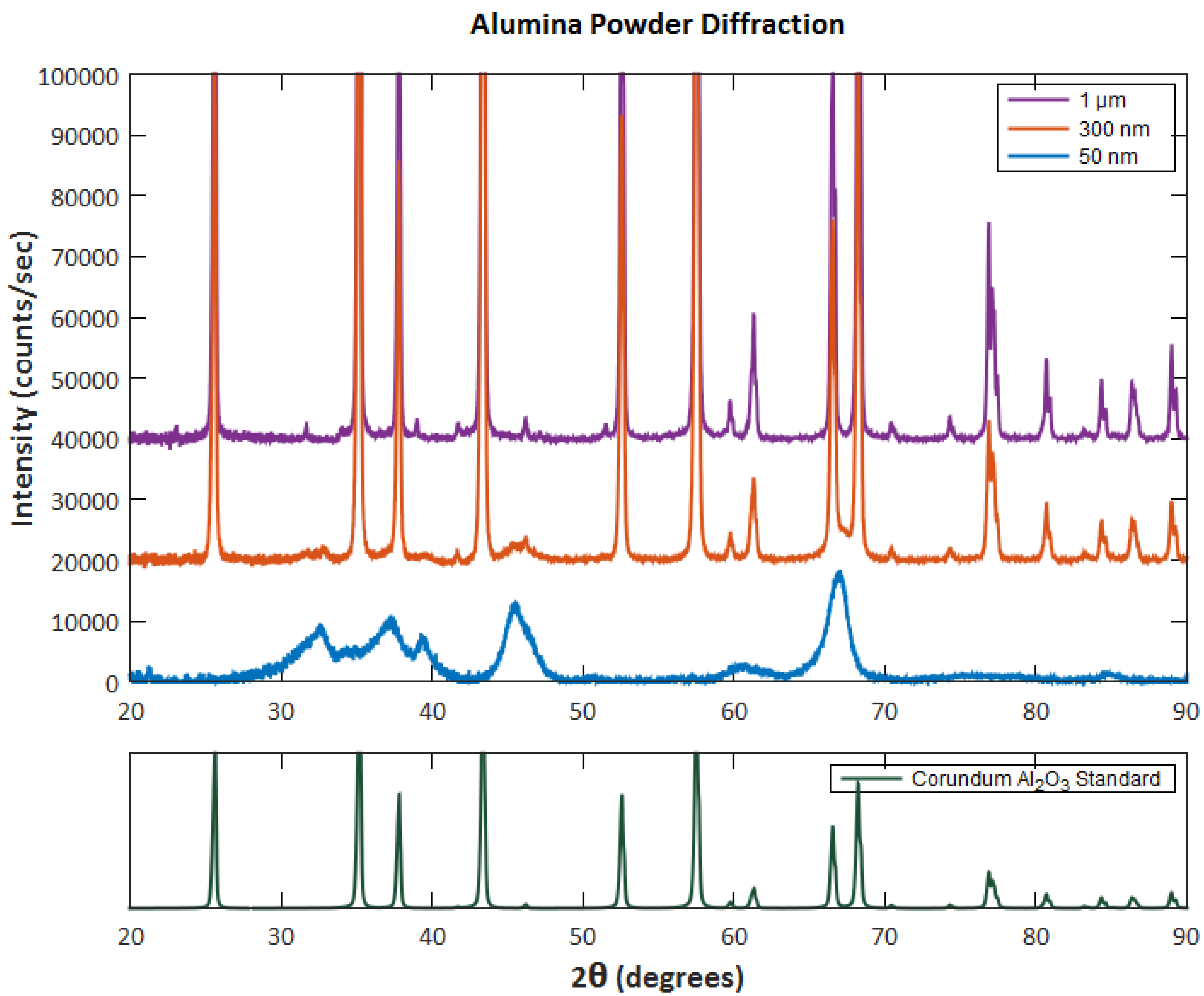


FIGURE 17F
Grating spectra of two wavelengths: (*a*) $\lambda_1 = 4000$ Å; (*b*) $\lambda_2 = 5000$ Å; (*c*) λ_1 and λ_2 together.


Hydrogen
656.3
486.1
434.0
410.1
700
600
Wavelength / nm
500
400
Helium
667.8
587.5
501.5
492.1
471.3
447.1
402.6
700
600
Wavelength / nm
500
400
Mercury
623.4
615.2
579.0
577.0
546.1
502.5
435.8
407.8
404.7
700
600
Wavelength / nm
500
400



Constructive interference when
$n\lambda = 2d \sin\theta$
Bragg´s Law


Alumina Powder Diffraction
Intensity (counts/sec)
100000
90000
80000
70000
60000
50000
40000
30000
20000
10000
0
20
30
40
50
60
70
80
90
1 μm
300 nm
50 nm
Corundum Al_2O_3 Standard
2θ (degrees)