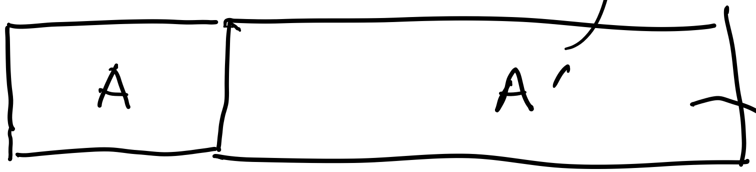
Recap

reservoir



reservoir at a given temp T

$A^{(0)} = A + A' \Rightarrow$ isolated

canonical ensemble

$$\boxed{P_r = \frac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}}$$

Boltzmann distribution

$$\beta = \frac{1}{kT}$$

P_r : prob. of finding A in one particular microstate r

$P(E)$: prob. of finding A in an energy range E to $E+\delta E$

$$P(E) = \sum_r P_r$$

r → over all states in given energy range

$$E < E_r < E + \delta E$$

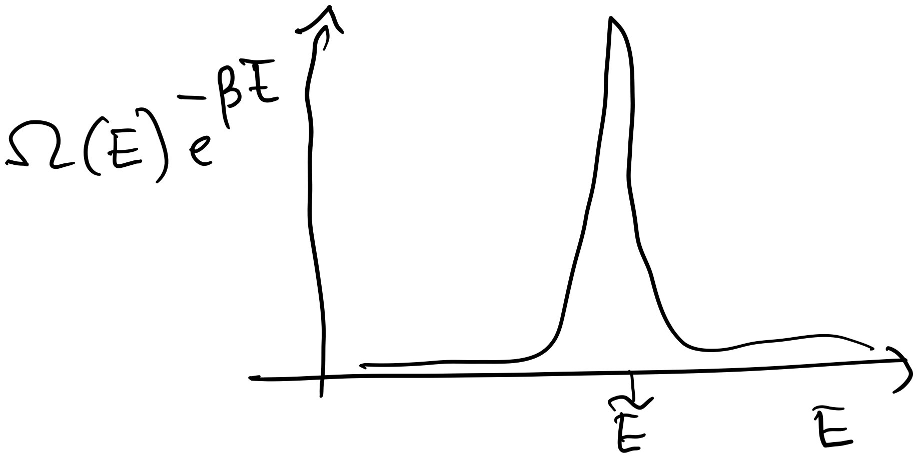
$\Omega(E)$: # of states in this energy range

→ all have equall prob. ~ $e^{-\beta E}$

$$P(E) = C\,\Omega(E)\,e^{-\beta E}$$

⇒ valid no matter how small A is

$\Omega(E) \sim$ rapidly rising fn. of E



$P(E)$ has a peak.

say y is some parameter characterizing the system

$$\boxed{\bar{y} = \frac{\sum_r y_r e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}}$$

Applications of the canonical distribution

Paramagnetism :

N_0 magnetic atoms per unit volume

Placed in an external mag field H.

- Each atom has spin $\frac{1}{2}$ and an intrinsic mag moment μ.
- can point only either parallel or antiparallel to H.

Q: Substance at temp T, what is the mean mag moment μ_H (in the direction of H) of such an atom?

System A : single atom

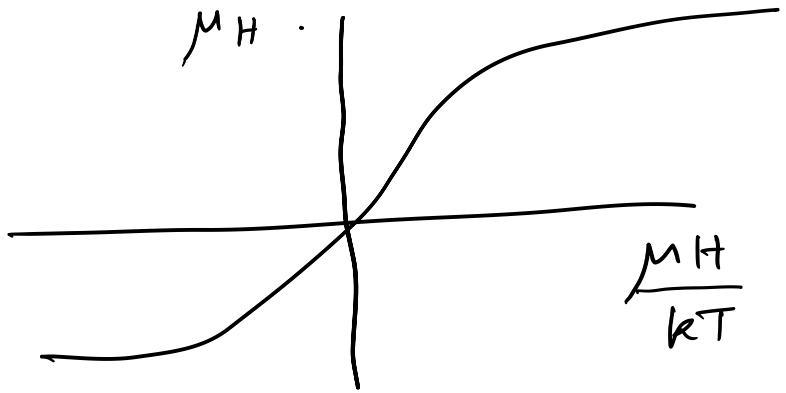
reservoir A' : rest of the atoms in material.

- each atom in two possible states (+) or (−).
- (+) is ∥ to H , $\mu_H = \mu$, $\epsilon_+ = -\mu H$.
- (−) is anti ∥ to H , $\mu_H = -\mu$, $\epsilon_- = \mu H$.

$$P_+ = C e^{-\beta \epsilon_+} = C e^{\beta \mu H}$$

$$P_- = C e^{-\beta \epsilon_-} = C e^{-\beta \mu H}$$

$$\bar{\mu}_H = ?$$



$$\bar{\mu}_H = \frac{P_+ \mu + P_-(-\mu)}{P_+ + P_-}$$

$$= \mu \frac{e^{\beta \mu H} - e^{-\beta \mu H}}{e^{\beta \mu H} + e^{-\beta \mu H}}$$

$$\boxed{\bar{\mu}_H = \mu \tanh \frac{\mu H}{kT}}$$

$T \to \infty$

$\bar{\mu}_H = 0$

$T \to 0$

$\bar{\mu}_H = \mu$

Magnetization $\overline{M_0}$ = mean magnetic moment per unit volume in the direction of H

$$\boxed{\overline{M_0} = N_0 \overline{\mu_H}}$$

$$= N_0 \mu \tanh \frac{\mu H}{kT}$$

$$\frac{\mu H}{kT} = y$$

$$\tanh y = \frac{e^{y} - e^{-y}}{e^{y} + e^{-y}}$$

$y \ll 1$.
$$\simeq \frac{1 + y - 1 + y}{1 + y + 1 - y}$$

$$\simeq y.$$

$\frac{\mu H}{kT} \ll 1$.

$y \gg 1$

$\tanh y = 1$

$\frac{\mu H}{kT} \ll 1$ $\quad \overline{M_0} \simeq N_0 \mu \frac{\mu H}{kT}$

$$\simeq \frac{N_0 \mu^2 H}{kT}$$

$$\boxed{\overline{M_0} \simeq \chi H}$$

χ → susceptibility.

$$\bar{M}_0 = \chi H$$

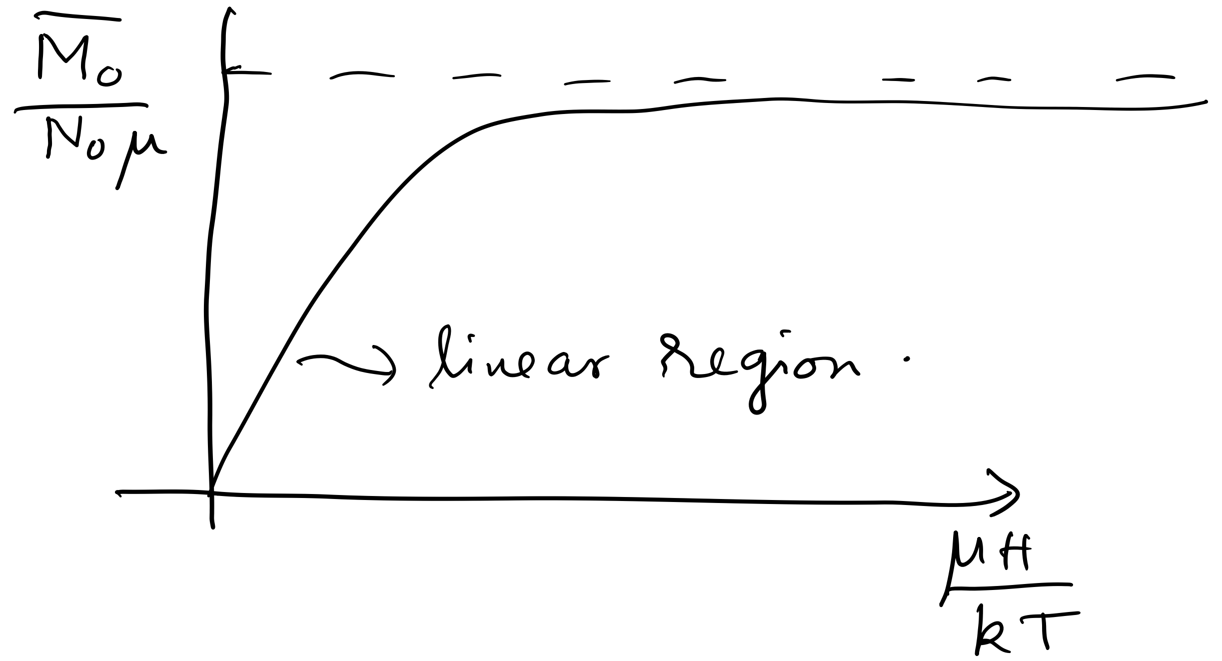
$$\chi = \frac{N_0 \mu^2}{kT}$$

$$\boxed{\chi \propto \frac{1}{T}}$$ → Curies' Law.

On the other hand

$\bar{M}_0 \longrightarrow N_0 \mu \qquad \frac{\mu H}{kT} >> 1$.

→ independent of H → equal to the saturation magnetization



non-interacting mag atoms of spin half & mag moment μ.

Example 2 : Molecule in an ideal gas.

molecule $\rightarrow$ system

other molecules $\rightarrow$ reservoir.

$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m}.$$

Boltzmann factor $\sim e^{-E/kT}$

$$\sim e^{-\frac{mv^2}{2kT}}.$$