Optics :

Jenkins & White : Fundamentals of Optics

Ghatak : Optics.

Geometrical Optics

- rectilinear propagation → shadows.
- reflection — law of reflection
- refraction — Snell's Law
- Dispersion

finite speed of propagation

framework was corpuscular theory of light

Optiks (Newton)

$\downarrow$ light is made up of particles obeying Newtonian dynamics.

rectilinear prop ✓

reflection ✓

refraction ✓ Snell's Law could be derived.

but he said that if refracted ray moves towards normal, <u>speed becomes higher</u>

- shadows were not completely dark (diffraction)

- Newton analyzed and observation of Newton's Rings $\longrightarrow$ wrong explanation

1801 Young's double slit $\longrightarrow$ interference fringes.

$\longrightarrow$ could not be explained with corpuscular theory

Wave Optics

- Interference ✓
- Diffraction ✓
- Electromagnetic Character Maxwell (1861) $\rightarrow \vec{E}, \vec{B}$

  $\hookrightarrow$ wave solutions

  out pops velocity of wave

  $v = c$!!
- Polarization

Photoelectric Effect (1905). $\longrightarrow$ could not be explained wave theory of light.

$\downarrow$

light composed of photons (particles of light discrete energies).

$E = h\nu$

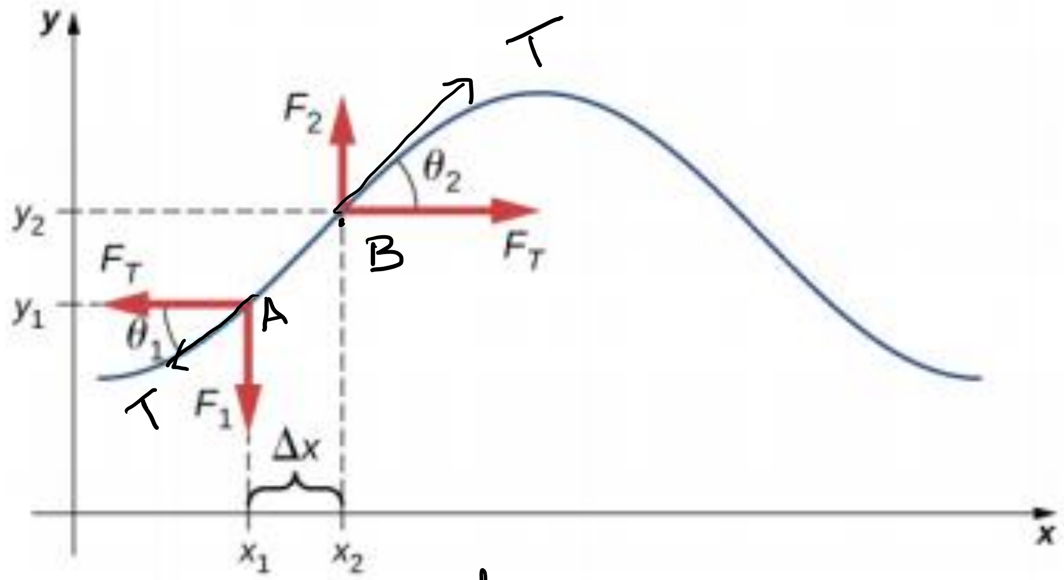
Wave - particle duality.

Quantum Optics

Lasers.

$\longrightarrow$ not deal with it in this course.

✓ string.



$x \quad x + dx$.

Equilibrium position of string along the x-axis

Upward force at A $= -T\sin\theta_1 \simeq -T\tan\theta_1 \simeq -T\left.\dfrac{\partial y}{\partial x}\right|_x$.

Upward force at B $= T\sin\theta_2 \simeq T\tan\theta_2 \simeq T\left.\dfrac{\partial y}{\partial x}\right|_{x+dx}$

Net transverse force.

$$T\left[\left(\frac{\partial y}{\partial x}\right)_{x+dx} - \left(\frac{\partial y}{\partial x}\right)_{x}\right] = T\frac{\partial^2 y}{\partial x^2}\,dx.$$

$$\because \left(\frac{\partial y}{\partial x}\right)_{x+dx} = \left(\frac{\partial y}{\partial x}\right)_{x} + \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)\Big|_{x} dx$$

Newton's 2nd Law

$$\Delta m\frac{\partial^2 y}{\partial t^2} = T\frac{\partial^2 y}{\partial x^2}\,dx.$$

$$\Delta m = \mu\, dx$$

$\mu \rightarrow$ mass/length

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{T/\mu}\frac{\partial^2 y}{\partial t^2}$$

$$v = \sqrt{\frac{T}{\mu}}\,.$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}} \longrightarrow \text{One dimensional wave equation}$$

Generalized to 3D.

$$\nabla^2 \phi = \frac{1}{v^2}\frac{\partial^2 \phi}{\partial t^2}$$

$$\phi = \phi(x, y, z, t) \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

General soln. of wave eqn.

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0 .$$

Change variables

$$\left.\begin{aligned} \xi &= x + vt \\ \eta &= x - vt . \end{aligned}\right\} \qquad \begin{aligned} x &= \tfrac{1}{2}(\xi+\eta) \\ vt &= \tfrac{1}{2}(\xi-\eta) . \end{aligned}$$

$$\frac{\partial y}{\partial x} = \frac{\partial y}{\partial \xi}\frac{\partial \xi}{\partial x} + \frac{\partial y}{\partial \eta}\frac{\partial \eta}{\partial x} .$$

$$= \frac{\partial y}{\partial \xi} + \frac{\partial y}{\partial \eta}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}\left[\frac{\partial y}{\partial \xi} + \frac{\partial y}{\partial \eta}\right]$$

$$= \frac{\partial}{\partial \xi}\left(\frac{\partial y}{\partial \xi}\right)\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\left(\frac{\partial y}{\partial \xi}\right)\frac{\partial \eta}{\partial x}$$

$$+ \frac{\partial}{\partial \xi}\left(\frac{\partial y}{\partial \eta}\right)\frac{\partial \xi}{\partial x} + \frac{\partial}{\partial \eta}\left(\frac{\partial y}{\partial \eta}\right)\frac{\partial \eta}{\partial x}.$$

$$= \frac{\partial^2 y}{\partial \xi^2} + 2\frac{\partial^2 y}{\partial \xi \partial \eta} + \frac{\partial^2 y}{\partial \eta^2}.$$

Similarly

$$\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial \xi^2} + \frac{\partial^2 y}{\partial \eta^2} - 2\frac{\partial^2 y}{\partial \xi \partial \eta}.$$

Putting all together

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

$$\Rightarrow \boxed{4\frac{\partial^2 y}{\partial \xi \partial \eta} = 0}$$

$$\frac{\partial^2 y}{\partial \xi \, \partial \eta} = 0 .$$

$$\frac{\partial}{\partial \xi}\left(\frac{\partial y}{\partial \eta}\right) = 0$$

$$\Rightarrow \quad \frac{\partial y}{\partial \eta} = \tilde{f}(\eta) .$$

$$\Rightarrow \quad y = \underbrace{\int \tilde{f}(\eta)\, d\eta}_{f(\eta)} + g(\xi) .$$

$$\Rightarrow \quad \boxed{y = f(\eta) + g(\xi) = f(x - vt) + g(x + vt)}$$