- Quiz has been added on Moodle under the Topic Heading Quizzes, right below the Statistical Mechanics Section
- Opens at 10:20 , closes at 11:00
- Duration 30min
- Only one attempt allowed
- Review accessible after closing

Equipartition Theorem

$$E = E(q_1, q_2 \cdots q_f, p_1 \cdots p_f).$$

a. $$E = \epsilon_i(p_i) + E'\overbrace{(q_1, \dots q_f, p_1 \dots\, p_f)}^{\text{does not depend on } p_i}.$$

no p_i

b. $$\epsilon_i(p_i) = b p_i^2 .$$

- p_i = momentum $\quad K.E = \sum_{i=1}^{3N} \frac{p_i^2}{2m}$ $\quad$ ideal monatomic gas.

  $U(q_1, \cdots q_f) \longrightarrow$ does not depend on p_i

- a, b satisfied for q_i instead of p_i, e.g. harmonic oscillator $U(q_1 \cdots q_f) = k \sum q_i^2$

$$\overline{\epsilon_i} = \frac{\displaystyle\int_{-\infty}^{+\infty} e^{-\beta E(q_1,\dots q_f, p_1 \dots p_f)} \epsilon_i \, dq_1 \dots dq_f \dots dp_f}{\displaystyle\int_{-\infty}^{+\infty} e^{-\beta E} \, dq_1 \dots dq_f \, dp_1 \dots dp_f} .$$

cond (a)

$$\overline{\epsilon_i} = \frac{\displaystyle\int_{-\infty}^{+\infty} e^{-\beta(E' + \epsilon_i)} \epsilon_i \, dq_1 \dots dp_f}{\displaystyle\int_{-\infty}^{+\infty} e^{-\beta(\epsilon_i + E')} \, dq_1 \dots dp_f}$$

$$\overline{\epsilon_i} = \frac{\int e^{-\beta\epsilon_i}\epsilon_i\, dp_i \int e^{-\beta E'}\, dq_1 \cdots dp_f}{\int e^{-\beta\epsilon_i}\, dp_i \int e^{-\beta E'}\, dq_1 \cdots dp_f}$$

$$\overline{\epsilon_i} = \frac{\int e^{-\beta\epsilon_i}\epsilon_i\, dp_i}{\int e^{-\beta\epsilon_i}\, d\phi_i} = -\frac{\partial}{\partial\beta}\ln\int_{-\infty}^{+\infty} e^{-\beta\epsilon_i}\, dp_i$$

Assumption b : $\epsilon_i = b p_i^2$

$$\int_{-\infty}^{+\infty} e^{-\beta \epsilon_i}\, dp_i = \int_{-\infty}^{+\infty} e^{-\beta b p_i^2}\, dp_i$$

$$= \beta^{-1/2} \int_{-\infty}^{+\infty} e^{-b y^2}\, dy \qquad y = \beta^{1/2} p_i$$

$$\ln \int_{-\infty}^{+\infty} e^{-\beta \epsilon_i}\, dp_i = -\frac{1}{2} \ln \beta + \underbrace{\ln \int_{-\infty}^{+\infty} e^{-b y^2}\, dy}_{\text{independent of } \beta}$$

$$\boxed{\overline{\epsilon_i} = \frac{-\partial}{\partial \beta}\left(-\frac{1}{2} \ln \beta\right) = \frac{1}{2\beta} = \frac{1}{2} kT}$$

$\epsilon_i = b p_i^2$ or $b q_i^2$ will give same result.