Total

State index r	Quantum nos	Magnetic moment	Energy
1.	$+ + +$	3μ	$-3\mu H$
2.	$+ + -$	μ	$-\mu H$
3.	$+ - +$	μ	$-\mu H$
4	$- + +$	$+\mu$	$-\mu H$
5.	$+ - -$	$-\mu$	μH
6.	$- + -$	$-\mu$	μH
7	$- - +$	$-\mu$	μH
8.	$- - -$	-3μ	$3\mu H$

Ex. Suppose we know that total energy $= -\mu H$

$(++-)$ $(+-+)$ $(-++)$

system can be in any of these states.

→ Do not know rel. probability of these states occurring.

Basic Postulate

$\boxed{\text{An isolated system in equilibrium is equally likely to be in any of its accessible states}}$

constant E

Examples

1. 3 spin system $E = \text{const} = -\mu H$ isolated

$(+--)$ $(+-+)$ $(-++)$

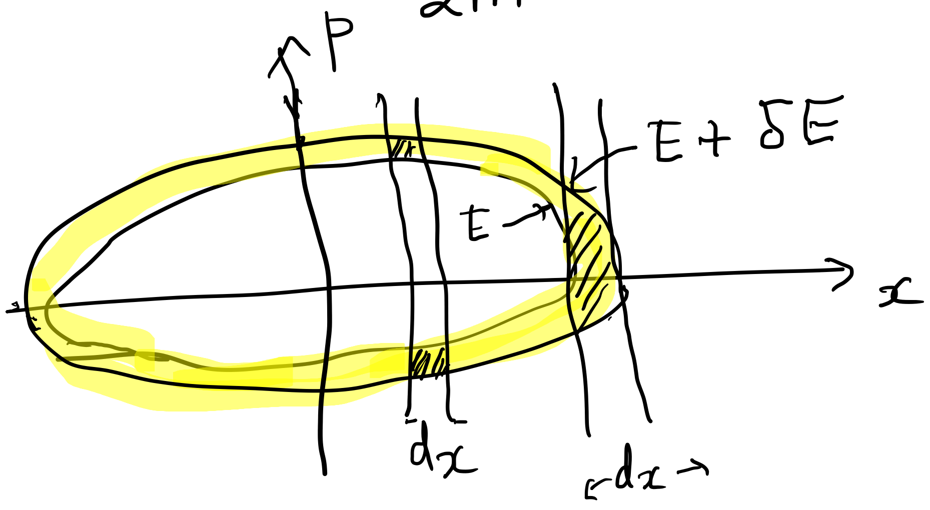
↳ + postulate

all three are equally probable.

2. N magnetic atoms placed in a mag field.
   ↳ same as 1 but very large # of states for each possible value of total energy.

3. 1 d oscillator

$E = \frac{p^2}{2m} + \frac{1}{2} k x^2$; energies between E and $E+\delta E$



Allowed region: yellow

# of microstates = # of cells in yellow region

$$\Omega = \frac{\int dp\, dx}{h_0}$$

$$\left.\begin{aligned} x &= A\cos(\omega t+\phi) \\ p &= m\dot{x} = -mA\omega\sin(\omega t+\phi) \end{aligned}\right\} \longrightarrow E = \frac{1}{2} m\omega^2 \underline{\underline{A^2}}$$

$A \to E$, ϕ is arbitrary. $0 < \phi < 2\pi$

Probability calculations

Isolated system $\quad E \longrightarrow E + \delta E$

$\downarrow$ ensemble of such systems.

$\Omega(E)$: Total # of microstates of system in this range

$\Omega(E; y_k)$: # of states in which a parameter y takes the value y_k

$$\boxed{P(y_k) = \frac{\Omega(E; y_k)}{\Omega(E)}}$$

Mean value of y

$$\boxed{\overline{y} = \frac{\sum_k \Omega(E; y_k)\, y_k}{\Omega(E)}}$$

3 spin example, $E = -\mu H$.

$(+ + -)$ $(+ - +)$ $(- + +)$.

what is the prob that first spin is up?

$P_+ = \frac{2}{3}$, $\overline{\mu_z} = \frac{1}{3}\mu + \frac{1}{3}\mu + \frac{1}{3}(-\mu) = \frac{1}{3}\mu$.

General Behavior of $\Omega(E)$

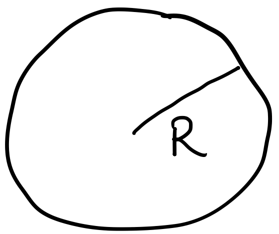
Microstate : f generalized coordinates q_i, p_i

Macrostate $E < H(p_i, q_i) < E + \delta E$

Defines R : the allowed region of phase space

$$\boxed{\Omega(E, V, N; \delta E) = \frac{1}{(h_0)^f} \int_R dp_1\, dp_2 \cdots dp_f\, dq_1 \cdots\cdots dq_f}$$

$$\int_{R \leq \sqrt{2mE}} dp_1 \cdots dp_f \qquad f = 3N.$$



$\hookrightarrow \quad C_{3N} R^{3N}.$

But volume of spherical shell $\sim \quad R^{3N-1}\,\delta R.$

$$\int dp_1 \cdots dp_{3N} \quad \propto \quad R^{3N-1} \cdot \delta R \qquad \text{N is very large}$$

$E < H < E + \delta E$

$$\sim \quad R^{3N} \cdot \delta R \qquad R = (2mE)^{1/2}$$

$$\sim \quad E^{3N/2} \frac{\delta E}{2\sqrt{E}} \quad \sim \quad E^{3N/2 - \frac{1}{2}} \, \delta E$$

$$\Omega(E) \sim V^N E^{\frac{3N}{2}}$$

$$\Downarrow$$

$\Omega(E)$ is in general is a very rapidly growing fn of E