Wave eqn:

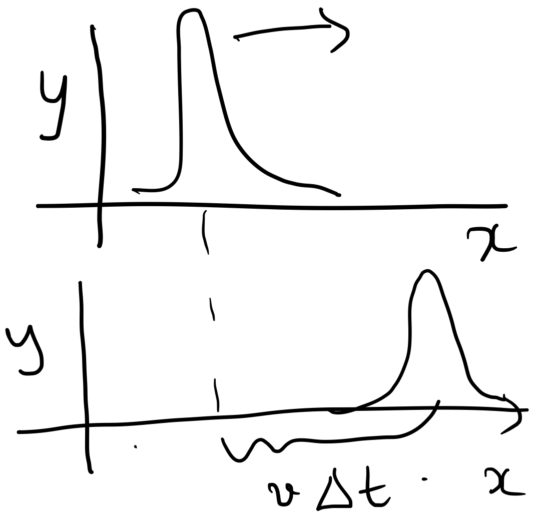
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial y}{\partial t^2}.$$

Soln $\Rightarrow$ $y = f(x+ct) + g(x-ct)$.

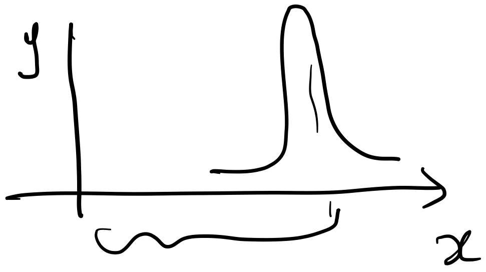
$f(x+ct)$ $\downarrow$ wave prop in $-x$ direction

$g(x-ct)$ $\searrow$ wave prop in $+x$ direction

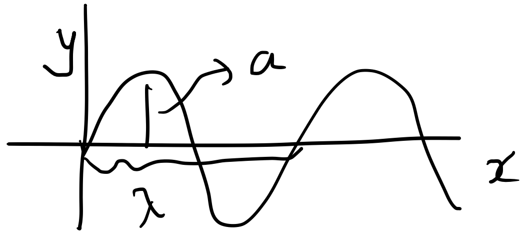
$t=0$



$t = \Delta t$



Sinusoidal waves



$$y(x,t) = a \sin\left[k(x \mp ct) + \phi\right]$$

a → amplitude

ϕ → phase

$k = \frac{2\pi}{\lambda}$

k → wave #

$$y = a\sin(kx - \omega t)$$

$$\omega = \frac{2\pi}{T} = 2\pi\nu.$$

$$\nu\lambda = c.$$

Energy transport in wave motion

transverse vibration of a string.

Time variation of a displacement of a particle

$$y = a\sin(\omega t + \phi)$$

Instantaneous velocity

$$\dot{y} = a\omega\cos(\omega t + \phi).$$

Kinetic energy

$$T = \frac{1}{2}m\dot{y}^2 = \frac{1}{2}m a^2\omega^2\cos^2(\omega t + \phi).$$

Total energy = max value of $T = \frac{1}{2}ma^2\omega^2$

$$E = \frac{1}{2} m\omega^2 a^2 .$$

$$\text{Intensity} = \frac{\text{energy}}{\text{area} \times \text{time}} = \underbrace{\rho_E}_{\text{energy/vol}} \times v = \varepsilon v .$$

Sound wave propagating through gas

$$\varepsilon = \frac{1}{2} m n a^2 \omega^2 \qquad n = \text{molecules/volume} .$$

$$= \frac{1}{2} \rho a^2 \omega^2$$

$$I = \varepsilon v = \frac{1}{2} \rho v a^2 \omega^2 \rightarrow \boxed{I \propto a^2}$$

$$= \frac{1}{2} \rho v \omega^2 a^2 .$$

Monochromatic
$\omega = \text{const.}$

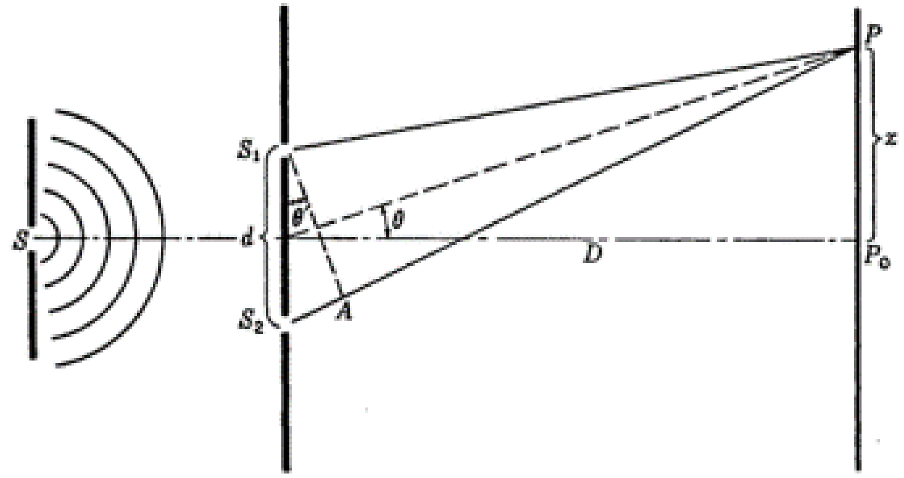


FIGURE 13F
Path difference in Young's experiment.

y at P : superposition of two waves of same freq and amplitude but with a phase diff

$$y_1 = a \sin(kx - \omega t) \quad \text{—} \ (1)$$

$$y_2 = a \sin(k(x + \Delta) - \omega t) \quad \text{—} \ (2)$$

Superposition

$$y = y_1 + y_2$$

$$= a\sin(kx - \omega t) + a\sin(kx + k\Delta - \omega t)$$

$$\boxed{y = \underbrace{2a\cos\frac{k\Delta}{2}}_{A}\ \sin\left(k\left(x + \frac{\Delta}{2}\right) - \omega t\right)}$$ At P.

$\boxed{k\Delta = \delta}$

Path diff $= (S_2P - S_1P) = \Delta$

Phase diff $= \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(S_2P - S_1P) = \delta$

$\theta = \theta'$

$PA = S_1P$.

$\sin\theta \simeq \tan\theta$

$\theta \cong \theta' \simeq$ small

$= \dfrac{x}{D}$.

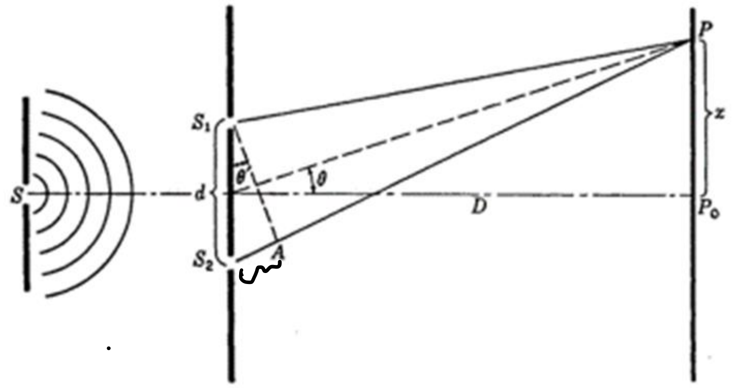


FIGURE 13F
Path difference in Young's experiment.

path difference

$= d\sin\theta$.

$$\boxed{I \simeq A^2 = 4a^2\cos^2 \delta/2} \rightarrow \text{Interference!}$$

Condn. Maxima $\quad \delta = 2m\pi \quad$ $m = 0, 1, 2, 3$. bright fringe.

path diff $\rightarrow$ $\boxed{d\sin\theta = m\lambda}$

$$\sin\theta = \frac{x}{D}.$$

$$\frac{d\,x_m}{D} = m\lambda.$$

$$\boxed{x_m = m\frac{\lambda D}{d}}$$ → location of m^{th} ~~bring~~ bright fringe.

For dark fringe/minima.

$$\frac{\delta}{2} = (2m+1)\pi/2.$$

$$\boxed{x_m = \left(m+\frac{1}{2}\right)\frac{\lambda D}{d}}$$ → dark fringe.

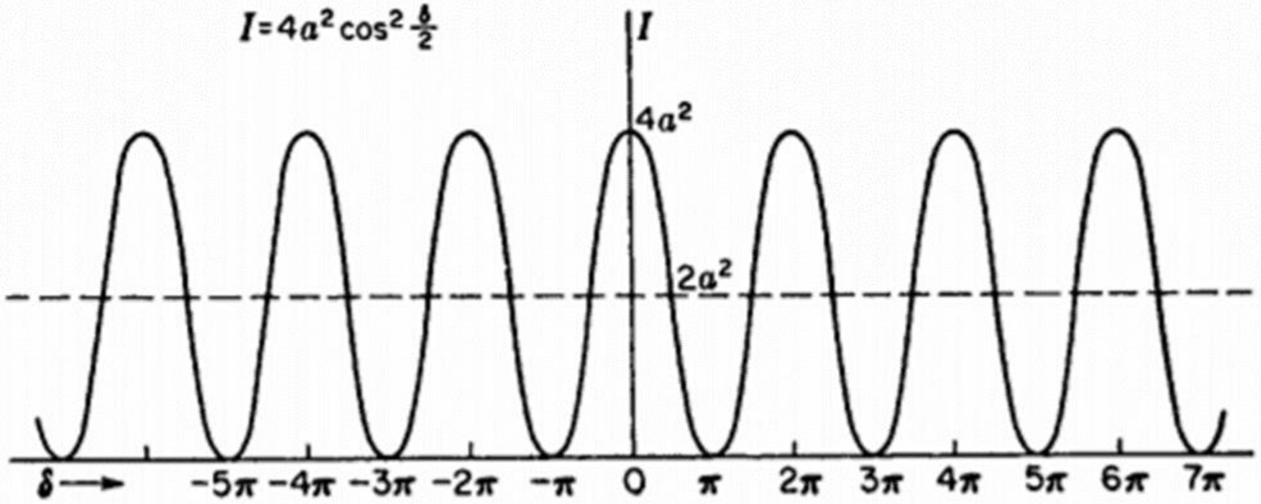
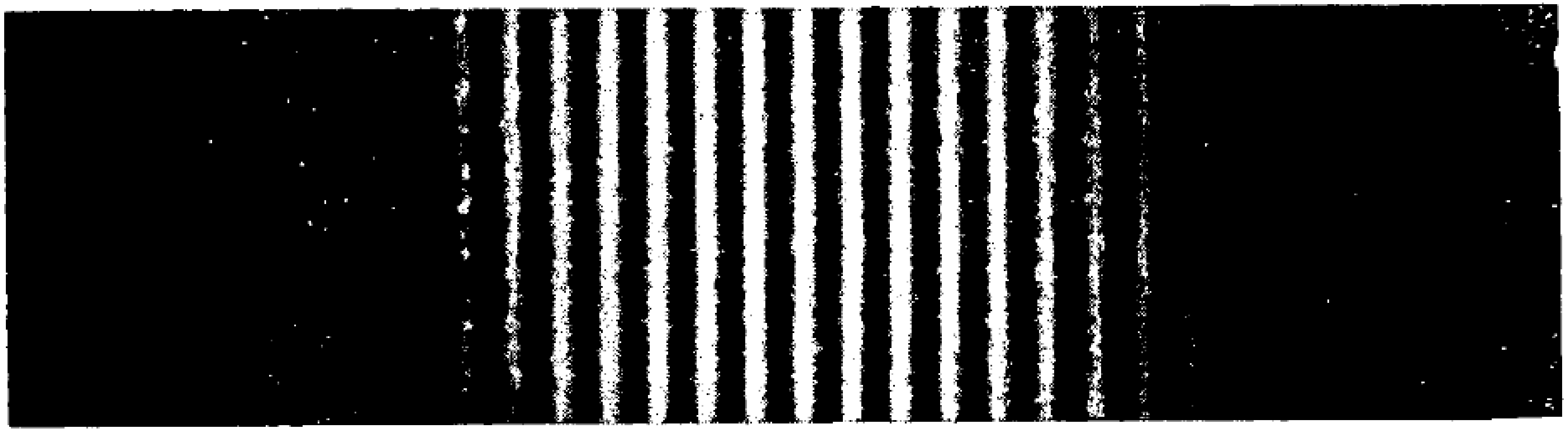


FIGURE 13H
Intensity distribution for the interference fringes from two waves of the same frequency.



Two slit experiment $\longrightarrow$ interference by division of wavefront.

$\downarrow$

Two sources at two slits.

what if we used two filament light bulbs placed behind each slit? Would we see fringes?

$\longrightarrow$ no stable interference pattern

$\rightarrow$ COHERENT SOURCES; the phase diff between waves from S_1 and S_2 is constant and stationary.