ISI B.Math Physics II
Problem Set 3

1. **(Marks : 5 + 5 = 10)**

The energy levels of a quantum mechanical oscillator with frequency ν is given by

$$\epsilon_n = (n + \frac{1}{2})h\nu$$

where $n = 0, 1, 2, \cdots$ and h is Planck's constant. When a system consisting of N independent oscillators has the total energy

$$E = \frac{1}{2}Nh\nu + Mh\nu$$

where M is an integer,
i) Find the number of states Ω_M corresponding to the energy E
ii) Find the energy E as a function of the temperature T of the system.

2. **(Marks : 6 + 6 + 8 = 20)**

Consider a system of N atoms with spin such that when placed in a magnetic field, the spin (or intrinsic magnetic moment) can align itself either parallel or antiparallel to the magnetic field. When the spin is aligned along the magnetic field the energy of the atom is $-\epsilon$ and when it is aligned opposite to the magnetic field the energy of the atom is ϵ.

i) Find the entropy $S(N, E)$ of a system of N such atoms with total energy E.
ii) Find the temperature $T(E)$ of the system. Can the temperature be negative? If so, under what conditions ?
iii) Find the heat capacity C as a function of T and make a rough plot of C vs T. Does the behaviour of heat capacity at high and low temperature limits match your experience from other systems that you have encountered ?

3. **(Marks : 5 + 3 + 2 = 10)**

An ideal gas consisting of N particles of mass m is placed in an infinitely tall cylindrical container placed in a uniform gravitational field, and is in thermal equilibrium at a temperature T. Find the Helmholtz free energy, mean energy and heat capacity of the system.

In all the above problems you may use Stirling's approximation for large N

$$\log N! \simeq N \log N - N$$