Analysis III (HW - 5)   Due date: Take your time   Instructor : Jaydeb Sarkar

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NOTE: (i) $B^n = \Pi_{i=1}^n [a_i, b_i]$. (ii) $R(B^n)$ = the set of all Riemann integrable functions on B^n. (iii) $v(\Omega)$ =volume of $\Omega(\subseteq \mathbb{R}^n)$, whenever $n \geq 3$. (iv) $A(\Omega)$ = area of $\Omega(\subseteq \mathbb{R}^2)$.

(1) Evaluate the line integral of the function $f(x, y, z) = xy + y + z$ along the curve $\gamma(t) = \langle 2t, t, 2 - 2t \rangle$, $t \in [0, 1]$.

(2) Evaluate the line integral of $f(x, y, z) = \sqrt{x^2 + z^2}$ along the curve $\gamma(t) = \langle 0, \cos t, \sin t \rangle$, $t \in [0, \frac{\pi}{2}]$.

(3) Compute $\int_C (x + \sqrt{y} - z^2)\, ds$, where $C = C_1 \cup C_2$, and parametrizations of C_1 and C_2 are given by $\gamma_1(t) = \langle t, t^2, 0 \rangle$ and $\gamma_1(t) = \langle 1, 1, t \rangle$, $t \in [0, 1]$, respectively.

(4) Find the work done by the force field $F(x, y, z) = \langle -\frac{1}{2}x, -\frac{1}{2}y, -\frac{1}{4} \rangle$ on a particle as it moves along the helix $\gamma(t) = \langle \cos t, \sin t, t \rangle$ from $(1, 0, 0)$ to $(-1, 0, 3\pi)$.

(5) Evaluate the line integral $\int_C x^2 y\, dx + (x - 2y)\, dy$, where C is the part of the parabola $y = x^2$ from $(0, 0)$ to $(1, 1)$.

   [Remark: Let $F = \langle f, g, h \rangle$ be a vector field, and let $\gamma(t) = \langle x(t), y(t), z(t) \rangle = x(t)i + y(t)j + z(t)k$, $t \in [a, b]$, be a parametrization of C. We know that $\int_C F \cdot dr = \int_a^b F(\gamma(t)) \cdot \gamma'(t) dt$. However, note that

$$\int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt = \int_a^b \langle f(\gamma(t)), g(\gamma(t)), h(\gamma(t)) \rangle \cdot \langle dx/dt, dy/dt, dz/dt \rangle\, dt,$$

and so $\int_C F \cdot dr = \int_C f dx + g dy + h dz$.]

(6) Let C be the line segment joining the points (x_1, y_1) to (x_2, y_2). Prove that

$$\int_C x\, dy - y\, dx = \det \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}.$$

(7) Find a parametric representation for the curve resulting by the intersection of the plane $3x + y + z = 1$ and the cylinder $x^2 + 2y^2 = 1$ in $\mathbb{R}^3$.

(8) Find a parametrization for the solid triangle $\Delta := \Delta ABC$, where $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $C = (0, 0, 1)$. Also find a parametrization for the curve $\partial \Delta$.

(9) Compute the equation of the surface of revolution generated by revolving the hyperbola $x^2 - 4z^2 = 1$ about the z-axis.

(10) Compute the area of the piece of the paraboloid $z = x^2 + y^2$ which is cut out by the region between the cylinder $x^2 + y^2 = 2$ and the cylinder $x^2 + y^2 = 6$.

(11) Find a unit vector normal to the surface $z = 4 - x^3 - y^3$ at the point $(1, 1, 2)$. Also compute the tangent place to the surface at $(1, 1, 2)$.

(12) Find the equation of the tangent plane to the surface $z = x \exp(-2y)$ at the point $(1, 0, 1)$.

(13) Set up the iterated integral for the surface area of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(14) If $a > b > 0$, then find the area of the surface obtained by rotating the circle $z^2 + (x - a)^2 = b^2$ about the z axis.