DATE Qn: For what values of $\alpha, \beta, \gamma, \delta, \eta$ is the current soln optimal & there are multiple solns?

	x_1	x_2	x_3	x_4	x_5
-10	δ	-2	0	0	0
4	-1	η	1	0	0
1	α	-4	0	1	0
β	γ	3	0	0	1

$\bar{c}_2 = -2 < 0$ so 2^{nd} basic direction has cost decrease. Use as pivot column.

For the point to be optimal, should not be able to go in this direction without leaving polyhedron.

So $\boxed{\beta = 0}$ 3^{rd} row is pivot column

-10	$\delta + 2\gamma/3$	0	0	0	$2/3$
4	$-1 - \frac{\eta\gamma}{3}$	0	1	0	$-\eta/3$
1	$\alpha + \frac{4\gamma}{3}$	0	0	1	$4/3$
0	$\gamma/3$	1	0	0	$\frac{1}{3}$

— (*)

If $\delta + \frac{2\gamma}{3} > 0$ then the current point is unique optimal soln.
(Check Exs Sheet 2.)

Therefore $\delta + \frac{2\gamma}{3} \leq 0$

~~Consider now the case $\delta + \frac{2\gamma}{3}$~~

Note that since the current point is degenerate we cannot conclude that $\bar{c} = \underset{\sim}{0}$

(A Thm in class stated optimal & nondegenerate implies $\underset{\sim}{\bar{c}} = \underset{\sim}{0}$)

Consider now the case $\delta + \frac{2\gamma}{3} < 0$. Use 1st column as pivot column

In this case need $\gamma > 0$ so that we cannot move in this direction. Use 3rd row as pivot row.

-10	0	a	0	0	b
4	0	$\left(1+\frac{\eta\gamma}{3}\right)\cdot\frac{3}{\gamma}$	1	0	$-\frac{\eta}{3}+\frac{1}{\gamma}\left(1+\frac{\eta\gamma}{3}\right)$
1	0	$-\left(\alpha+\frac{4\gamma}{3}\right)\frac{3}{\gamma}$	0	1	$\frac{4}{3}-\left(\alpha+\frac{4\gamma}{3}\right)\frac{1}{\gamma}$
0	1	$3/\gamma$	0	0	$1/\gamma$

$$a = -\left(\delta+\frac{2\gamma}{3}\right)\cdot\frac{3}{\gamma} = -\frac{3\delta}{\gamma} - 2 > 0$$

$$b = \frac{2}{3} - \left(\delta + \frac{2\gamma}{3}\right)\cdot\frac{1}{\gamma} = -\frac{\delta}{\gamma} > 0$$

Then the point becomes unique optimum if $\delta + \frac{2\gamma}{3} < 0$

Therefore we must $\boxed{\delta + \frac{2\gamma}{3} = 0}$

Then ~~$a \geq 0$ & $b \geq 0$~~. Return to (*)

Clearly current point is optimal since $\bar{c} \geq 0$

Need to be able to move along 1st basic direction with no change in cost.

Therefore $\boxed{\gamma \leq 0}$ No restriction on $-1-\frac{\eta\gamma}{3}$, $\alpha + \frac{4\gamma}{3}$

ANSWER: $\beta = 0$, $\gamma \leq 0$, $\delta + \frac{2\gamma}{3} = 0$, $\alpha \in \mathbb{R}$, $\eta \in \mathbb{R}$