Example 3.6 This example shows that the simplex method can indeed cycle. We consider a problem described in terms of the following initial tableau.

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	−3/4	20	−1/2	6	0	0	0
$x_5 =$	0	1/4*	−8	−1	9	1	0	0
$x_6 =$	0	1/2	−12	−1/2	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

We use the following pivoting rules:

(a) We select a nonbasic variable with the most negative reduced cost $\overline{c}_j$ to be the one that enters the basis.

(b) Out of all basic variables that are eligible to exit the basis, we select the one with the smallest subscript.

We then obtain the following sequence of tableaux (the pivot element is indicated by an asterisk):

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	−4	−7/2	33	3	0	0
$x_1 =$	0	1	−32	−4	36	4	0	0
$x_6 =$	0	0	4*	3/2	−15	−2	1	0
$x_7 =$	1	0	0	1	0	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	0	0	−2	18	1	1	0
$x_1 =$	0	1	0	8*	−84	−12	8	0
$x_2 =$	0	0	1	3/8	−15/4	−1/2	1/4	0
$x_7 =$	1	0	0	1	0	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	1/4	0	0	−3	−2	3	0
$x_3 =$	0	1/8	0	1	−21/2	−3/2	1	0
$x_2 =$	0	−3/64	1	0	3/16*	1/16	−1/8	0
$x_7 =$	1	−1/8	0	0	21/2	3/2	−1	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	−1/2	16	0	0	−1	1	0
$x_3 =$	0	−5/2	56	1	0	2*	−6	0
$x_4 =$	0	−1/4	16/3	0	1	1/3	−2/3	0
$x_7 =$	1	5/2	−56	0	0	−2	6	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	−7/4	44	1/2	0	0	−2	0
$x_5 =$	0	−5/4	28	1/2	0	1	−3	0
$x_4 =$	0	1/6	−4	−1/6	1	0	1/3*	0
$x_7 =$	1	0	0	1	0	0	0	1

		x_1	x_2	x_3	x_4	x_5	x_6	x_7
	3	−3/4	20	−1/2	6	0	0	0
$x_5 =$	0	1/4	−8	−1	9	1	0	0
$x_6 =$	0	1/2	−12	−1/2	3	0	1	0
$x_7 =$	1	0	0	1	0	0	0	1

After six pivots, we have the same basis and the same tableau that we started with. At each basis change, we had $\theta^* = 0$. In particular, for each intermediate tableau, we had the same feasible solution and the same cost. The same sequence of pivots can be repeated over and over, and the simplex method never terminates.

Comparison of the full tableau and the revised simplex methods

Let us pretend that the problem is changed to

$$\begin{aligned} \text{minimize} \quad & \mathbf{c}'\mathbf{x} + \mathbf{0}'\mathbf{y} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} + \mathbf{I}\mathbf{y} = \mathbf{b} \\ & \mathbf{x}, \mathbf{y} \geq \mathbf{0}. \end{aligned}$$

We implement the simplex method on this new problem, except that we never allow any of the components of the vector $\mathbf{y}$ to become basic. Then, the simplex method performs basis changes as if the vector $\mathbf{y}$ were entirely