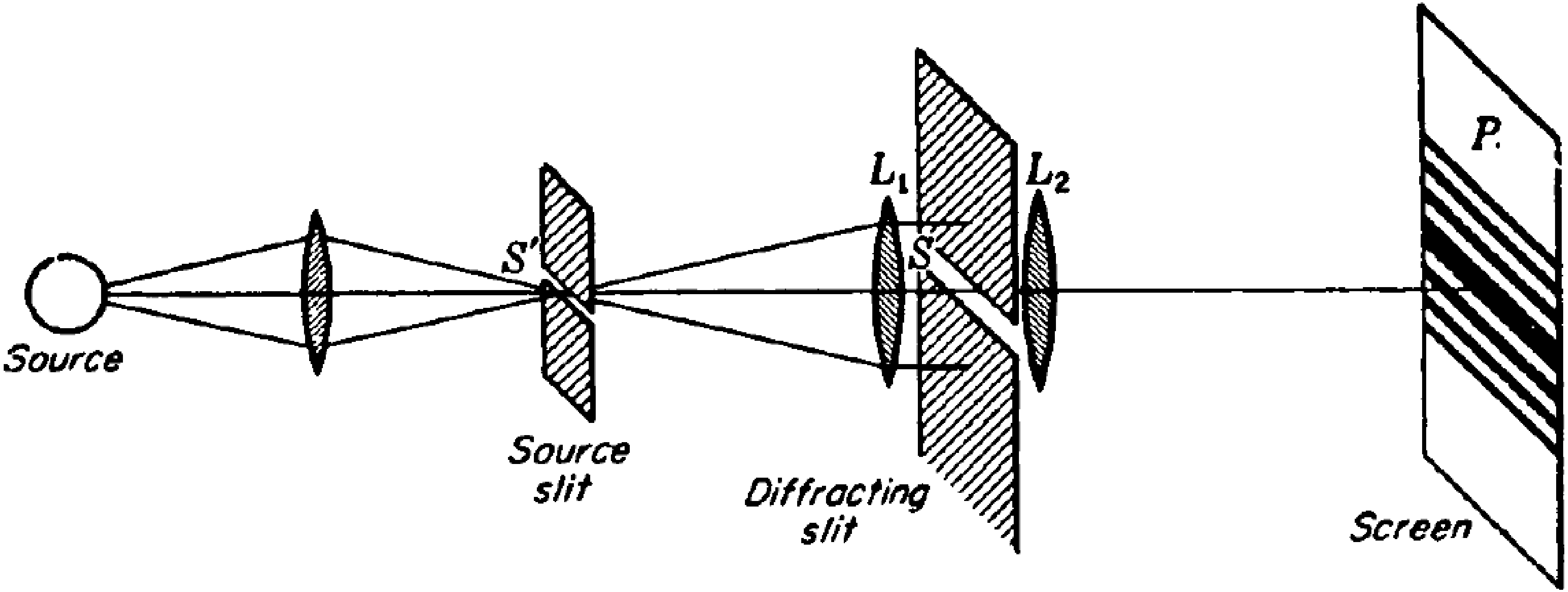
Diffraction

Diffraction → bending of light around obstacles.
spreading of light through a narrow slit into region of geometrical shadow.

Fraunhofer Diffraction : light source and screen both are at infinite distance from slit.

Fresnel diffraction : either source or screen or both are at finite distance from slit

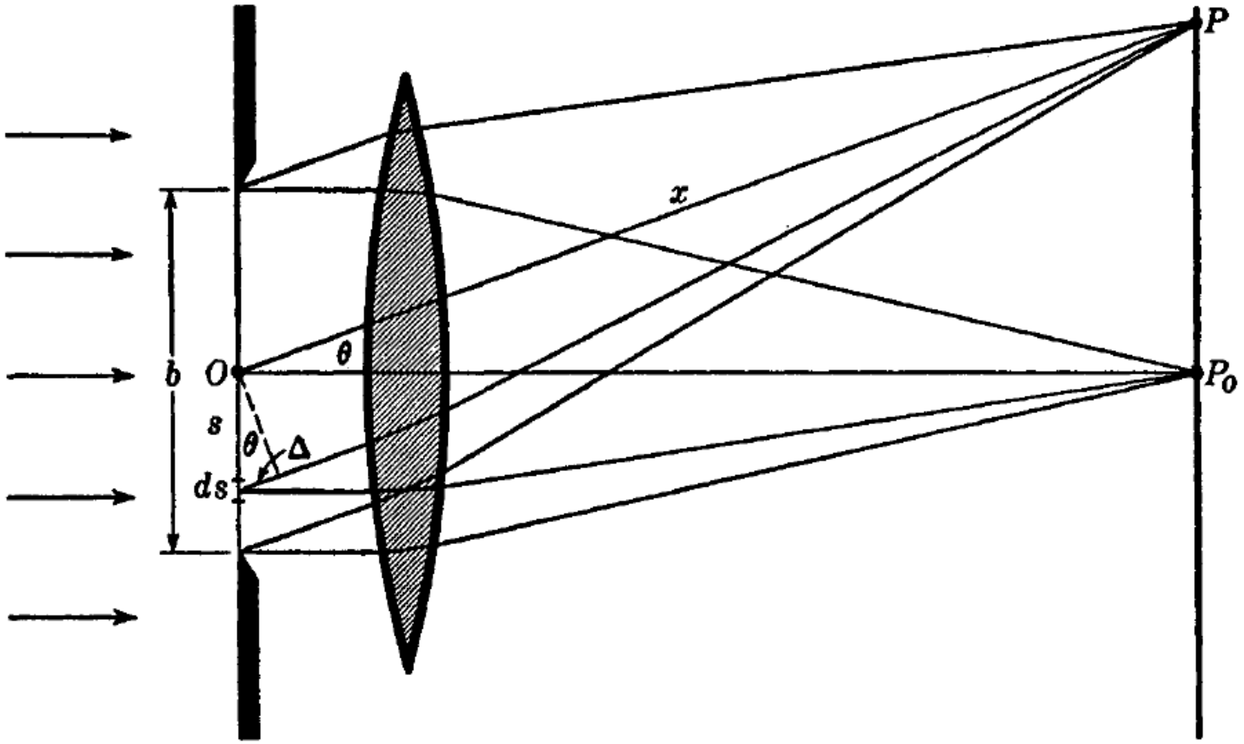
→ mathematically messy



Source
S'
Source
slit
L_1
S
Diffracting
slit
L_2
P.
Screen

slit width
= b

OP = x



ds
= element of wavefront in the plane of the slit at dist s from O

$$dy_0 = a\,ds \sin(\omega t - kx).$$

$$dy_s = a\,ds \sin(\omega t - k(x+\Delta))$$

$$\Delta = s\sin\theta.$$

$$= a\,ds \sin(\omega t - kx - ks\sin\theta)$$

our task

sum all elements from one edge of slit to another

integrate from $-b/2$ to $+b/2$.

Integrate contribution from symmetrical pairs

$$dy = dy_{-s} + dy_s$$

$$= a\,ds\,[\sin(\omega t - kx - ks\sin\theta) + \sin(\omega t - kx + ks\sin\theta)]$$

$$= 2a\,ds\,\cos(ks\sin\theta)\,\sin(\omega t - kx).$$

Integrate from 0 to $b/2$

$$y = 2a \sin(\omega t - kx) \int_0^{b/2} \cos(ks\sin\theta)\, ds \,.$$

$$y = 2a \sin(\omega t - x) \left[\frac{\sin(ks\sin\theta)}{k\sin\theta} \right]_0^{b/2}$$

$$y = \boxed{\frac{ab \sin \frac{kb}{2}\sin\theta}{\frac{kb}{2}\sin\theta}} \sin(\omega t - x)$$

→ new amplitude.

$$\boxed{y = A_0 \frac{\sin\beta}{\beta} \sin(\omega t - kx)}$$

$A_0 = ab$

$\beta = \frac{1}{2} kb \sin\theta$

$= \frac{\pi b \sin\theta}{\lambda}$

$\frac{1}{2}$ the phase diff coming from opp ends of slit.

$$\boxed{I \approx I_0 \frac{\sin^2\beta}{\beta^2} \approx A}$$

Maximum intensity at P_0. $\beta = 0$.

$$\boxed{I = I_0} \longrightarrow \text{principal maximum.}$$

- minima $\beta \neq 0$, $\sin\beta = 0$

$$\beta = m\pi \qquad\qquad \beta = \frac{\pi b \sin\theta}{\lambda}$$

$$\boxed{b\sin\theta = m\lambda}$$

Between two successive minima, but not exactly halfway there will be <u>secondary maxima</u>

Location of secondary maxima.

$$A = A_0 \frac{\sin\beta}{\beta}$$

$$\frac{dA}{d\beta} = 0 \implies \frac{\cos\beta}{\beta} - \frac{\sin\beta}{\beta^2} = 0$$

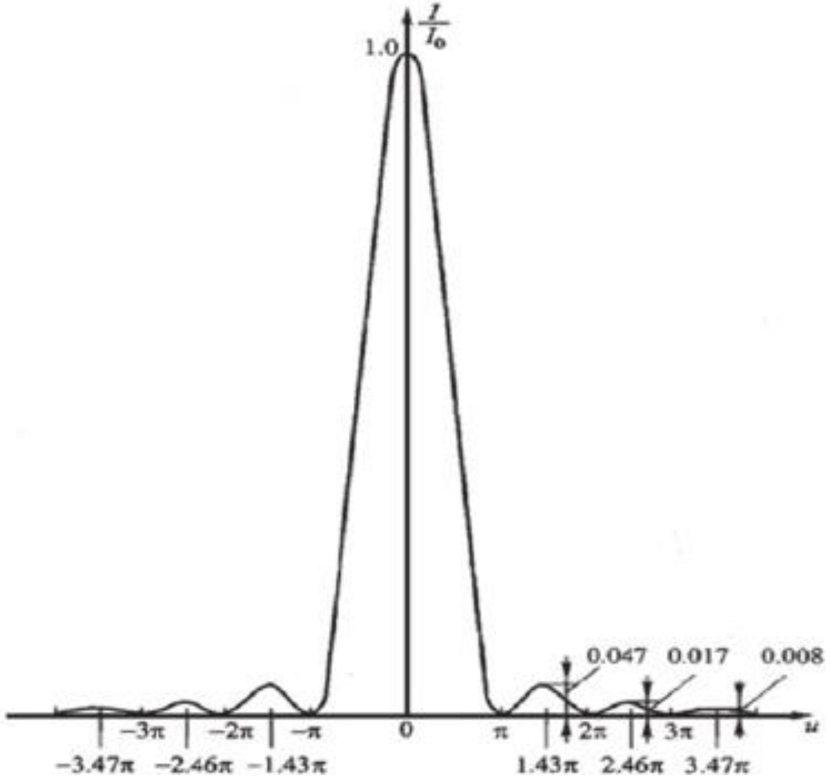
$$\boxed{\tan\beta = \beta}$$

graphically found as intersection

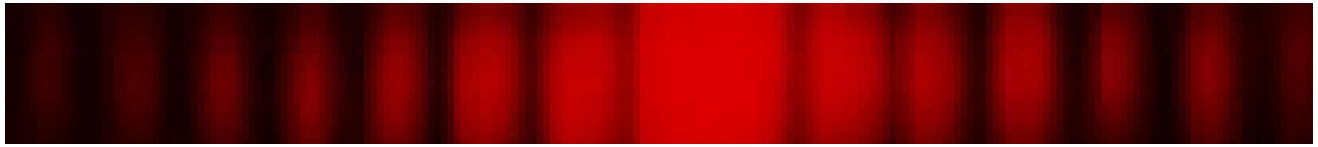
$y = \tan\beta$

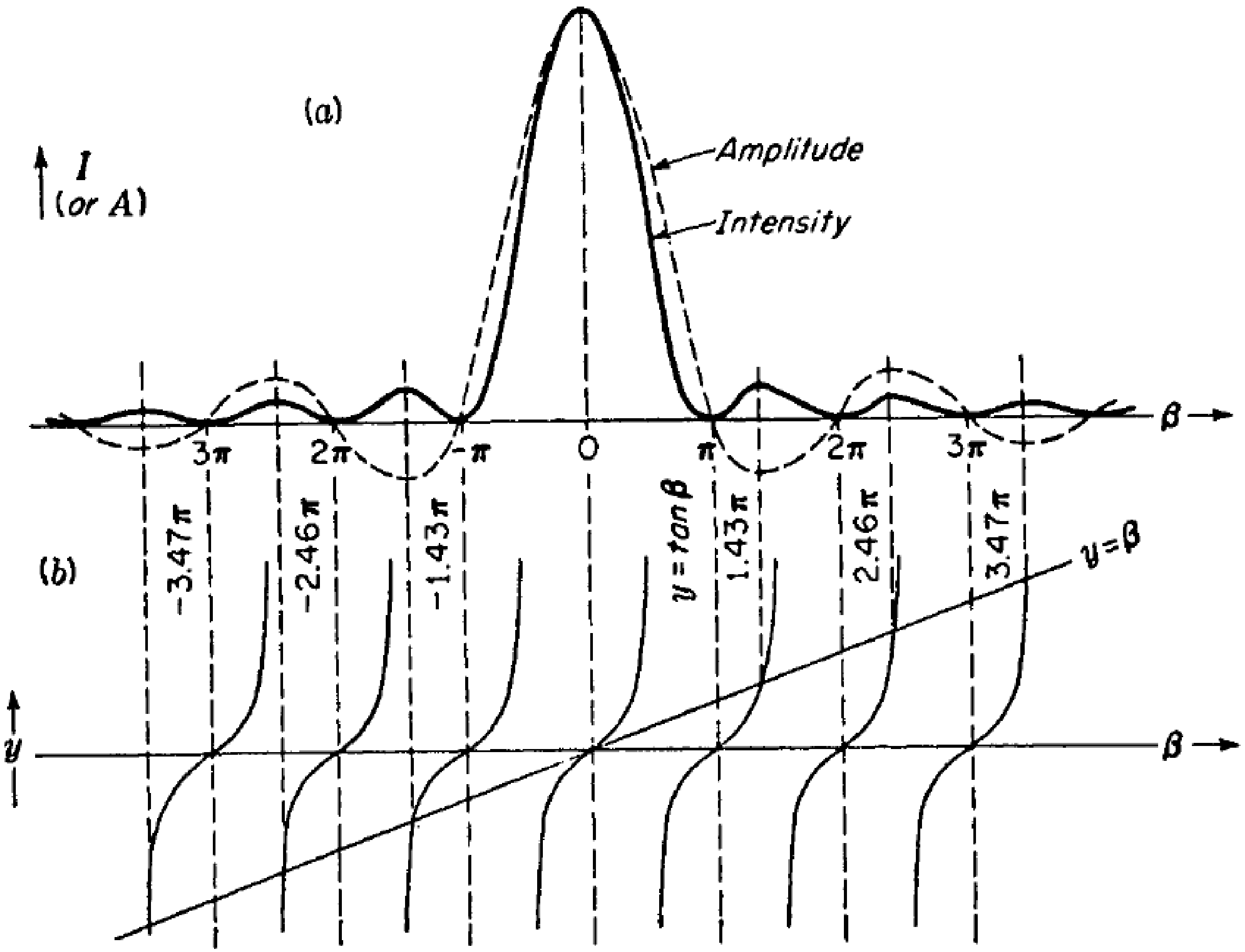
$y = \beta$.

- the intensities of the secondary maxima can be approximated well by values of $\left(\frac{\sin\beta}{\beta}\right)^2$ at the halfway pts. $\beta = 3\pi/2,\ 5\pi/2 \ldots\ldots$



$\frac{I}{I_0}$
1.0
0.047
0.017
0.008
-3π
-2π
$-\pi$
0
π
2π
3π
u
-3.47π
-2.46π
-1.43π
1.43π
2.46π
3.47π




(a)
I
(or A)
Amplitude
Intensity
β
3π
2π
-π
0
π
2π
3π
(b)
-3.47π
-2.46π
-1.43π
y = tan β
1.43π
2.46π
3.47π
y=β
y
β