Consider the simple linear regression set-up with the assumptions stated in class.

1. Show that
$$\hat{\beta}_1 \sim \mathcal{N}\left(\beta_1, \frac{\sigma^2}{SS_{XX}}\right)$$

2. For a fixed value x of the explanatory variable, let $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$. Show that
$$\hat{y} \sim \mathcal{N}\left(\beta_0 + \beta_1 x, \left(\frac{1}{n} + \frac{(x-\bar{x})^2}{SS_{XX}}\right)\sigma^2\right)$$

Note that the variance of $\hat{y}$ is higher for x values further away from $\bar{x}$, that is the fit is more precise in the centre of the dataset.