ISI B.Math Physics II
Problem Set I Total Marks = 50

Marks = 5

1. Derive the adiabatic compressibility κ_{ad}, when an ideal gas is quasi-statically and adiabatically compressed. The speed of sound is given by $c = \sqrt{(\frac{\partial P}{\partial \rho})_{ad}}$.

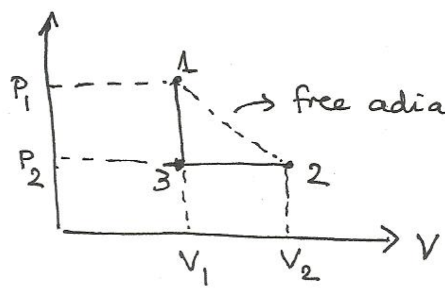
Marks = 5 + 5

2. If c is the speed of sound given by $c = \sqrt{(\frac{\partial P}{\partial \rho})_{ad}}$, and γ is the ratio of the specific heats at constant pressure and constant volume, show that the internal energy u, and the enthalpy $h = u + pv$ per unit mass of an ideal gas may be expressed by the following expressions

$$u = \frac{c^2}{\gamma(\gamma - 1)} + constant$$

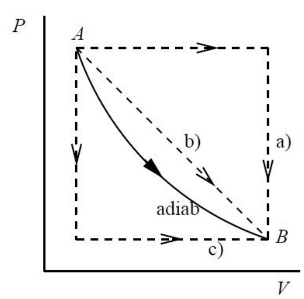
$$h = \frac{c^2}{\gamma - 1} + constant$$

Marks = 5



3. A mole of ideal gas with pressure P_1 and volume V_1 is freely and adiabatically expanded to V_2 while maintaining pressure at P_2. Finally the gas is heated quasi-statically until the pressure returns to P_1 while the volume remains V_1. This cycle is called Mayer's cycle. Prove Mayer's relation $c_p - c_v = R$ using this cycle. Assume that the molar specific heat is constant.

Marks = 4 + 4 + 2



4. In a quasi-static process $A \rightarrow B$(see diagram) in which no heat is exchanged with the environment, the mean pressure $\bar{P}$ of a certain amount of gas is found to change with its volume V according to the relation.

$$\bar{P} = \alpha V^{-\frac{5}{3}}$$

where α is a constant. Find the quasistatic , reversible work done on the system and the net heat absorbed by the gas in each of the following processes, all of which take the system from macrostate A to macrostate B. Express your results in terms of P_A, P_B and V_A, V_B (The constant α should not appear in your results.
(a) The system is expanded from its original to final volume, heat being added to keep the pressure constant. The volume is then kept constant and heat is extracted to reduce the pressure to its final value
(b) The volume is increased and heat is supplied to cause the pressure to decrease linearly with volume.
(c) The two steps of process (a) are performed in the opposite order.

Marks = (3+ 7)
5. Regarding the internal energy of a hydrostatic system to be a function of T and P, derive the following equations

$$\left(\frac{\partial U}{\partial T}\right)_P = C_P - PV\beta$$

$$\left(\frac{\partial U}{\partial P}\right)_T = PV\kappa - (C_P - C_V)\frac{\kappa}{\beta}$$

Marks = (6 + 2 +2)
6. In this problem we consider a photon gas in thermal equilibrium at temperature T. Such a thermodynamic system is usually called black body radiation. It is known that the internal energy U of a photon gas is related to its temperature T and the volume V through the relation

$$\frac{U}{V} = cT^4$$

where c is a constant. It is also known that its equation of state is given by

$$P = \frac{1}{3}\frac{U}{V}$$

(a) Calculate how the temperature of the temperature of a photon gas varies with temperature during a quasistatic, reversible, adiabatic compression of the photon gas.

(b)What is C_V for a photon gas ?

(c) What is C_P for a photon gas ?