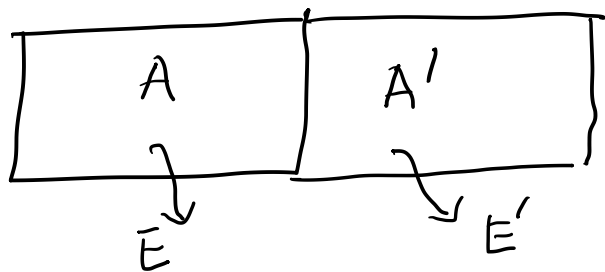


② $\Omega(E) \sim E^f$ f : no. of degrees of freedom

③ Isolated system : all microstates are equally probable

Thermal interaction between macroscopic systems



$\Omega(E)$
of states
between
 E & $E + \delta E$

$\Omega'(E')$
of states of A'
between
 E' & $E' + \delta E'$

$$A^{(0)} = A + A' \rightarrow \text{isolated}.$$

$$E^{(0)} = E + E' \text{ (weakly interacting)}$$

Suppose A has energy E

$$\downarrow A' \text{ must have } E' = (E^{(0)} - E)$$

of states accessible to $A^{(0)}$ can be regarded as a fn. of a single parameter E , energy of A . $\Omega^{(0)}(E)$

Fundamental Postulate applied to $A^{(0)}$

↓ equally likely to be found in any of the states of $\Omega^{(0)}(E)$

Prob $P(E)$ of finding the system $A^{(0)}$ in a configuration where A has energy between E & $E + \delta E \propto \Omega^{(0)}(E)$

$$P(E) = C \Omega^{(0)}(E)$$

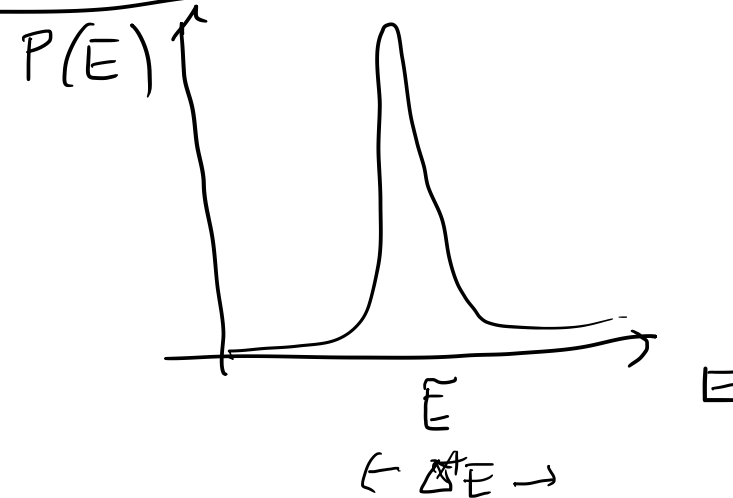
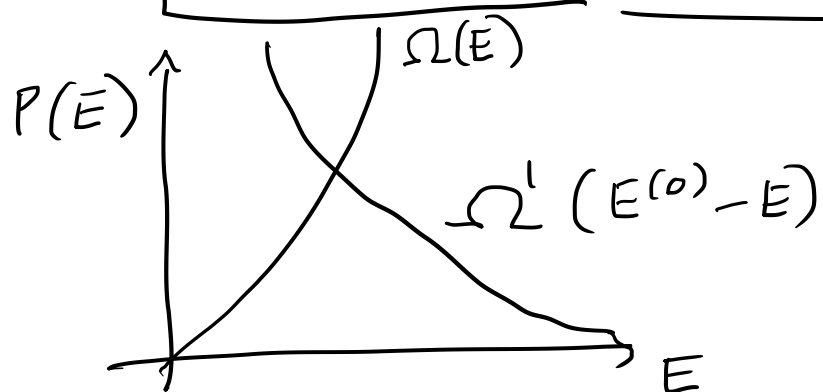
$$P(E) = \frac{\Omega^{(0)}(E)}{\Omega^{(0)}_{\text{tot}}}$$

$$C^{-1} = \Omega^{(0)}_{\text{tot}} = \sum_E \Omega^{(0)}(E)$$

But when A has energy E it must be in one of the $\Omega(E)$ states. At the same time A' must be in one of its $\Omega'(E')$ states, i.e., $\Omega'(E^{(0)} - E)$ states.

$$\Omega^{(0)}(E) = \Omega(E) \Omega'(E^{(0)} - E)$$

$$P(E) = C \Omega(E) \Omega'(E^{(0)} - E)$$



$P(E)$ will have a maximum

$\ln P(E)$ will have a maximum

Condition for a maximum

$$\frac{\partial \ln P}{\partial E} = 0$$

$$\ln P = \ln C + \ln \Omega(E) + \ln \Omega'(E') \quad ; \quad E' = E^{(0)} - E$$

$$\frac{\partial \ln P}{\partial E} = \frac{\partial \ln \Omega(E)}{\partial E} + \frac{\partial \ln \Omega'(E')}{\partial E'} (-1) = 0$$

$$\frac{\partial \ln \Omega(E)}{\partial E} = \frac{\partial \ln \Omega'(E')}{\partial E'}$$

$$\beta(\tilde{E}) = \beta'(\tilde{E}')$$

\tilde{E} determined by $P(E)$ max.

$$\beta = \frac{\partial \ln \Omega}{\partial E}$$

β has dimensions $\frac{1}{E}$

define another dimensionless parameter T

$$kT = \frac{1}{\beta}$$

k : const

Also define

$$S \equiv k \ln \Omega$$

entropy.

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

Max probability \Rightarrow Total entropy = $S + S' = \max$.

\downarrow condition

$$\boxed{T = T'}$$

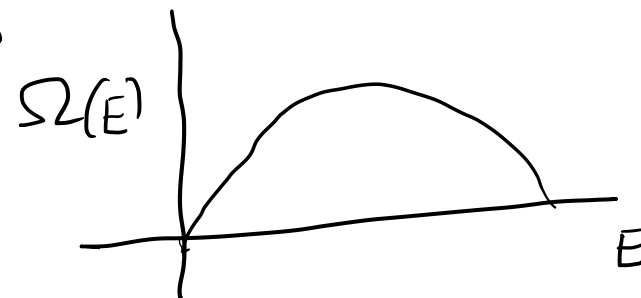
$$\frac{1}{kT} = \beta = \frac{\partial \ln \Omega}{\partial E}, \quad \text{since } \Omega \sim E^f$$

$\beta > 0, T > 0$.

systems do not have upper bound of energy

spin systems $\rightarrow E$ bounded on both ends

$$T < 0$$



Ensembles of physical interest

1. Isolated systems.

$E = \text{const}$, N particles in vol V .

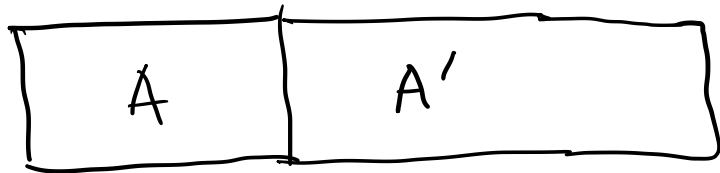
Ensemble : many such systems .

energy of a system in microstate r is E_r

$$P_r = \begin{cases} C & \text{if } E < E_r < E + \delta E \\ 0 & \text{otherwise} \end{cases} \quad \sum P_r = 1 \quad \text{determines } C$$

microcanonical ensemble .

2. System in contact with reservoir



$$A \ll A'$$

Q: Under conditions of equilibrium what is the prob.
Pr of finding A in one of its microstates r of energy
 E_r ?

- weak int. $E^{(0)} = E + E'$

- $A + A' \rightarrow \text{isolated}$

$$E_r + E' = E^{(0)}$$

If A is in one definite state r , E_r ,

A' must have energy $E^{(0)} - E_r$.

of States accessible to $A^{(0)} = \Omega'(E^{(0)} - E_r)$

Applying fundamental postulate to $A^{(0)}$

$$P_r = C' \Omega'(E^{(0)} - E_r) \quad , \quad [C' \text{ determined} \rightarrow \sum_r P_r = 1]$$

Now consider $A \ll A'$, $E_r \ll E^{(0)}$.

$$\ln \Omega'(E^{(0)} - E_r) = \ln \Omega'(E^{(0)}) - \underbrace{\left[\frac{\partial \ln \Omega'}{\partial E'} \right]_0}_{\beta \sim \frac{1}{kT}} E_r + \dots$$

$\beta \sim \frac{1}{kT} \rightarrow \text{temp. of reservoir.}$

$$\ln \Omega'(E^{(0)} - E_r) \approx \ln \Omega'(E^{(0)}) - \beta E_r.$$

$$\Omega'(E^{(0)} - E_r) = \underbrace{\Omega'(E^{(0)})}_{\text{const ind of } E_r} e^{-\beta E_r}$$

Boltzmann
factor



$$P_r = C e^{-\beta E_r}$$

$$C^{-1} = \sum_r e^{-\beta E_r}$$

$$P_r = \frac{e^{-\beta E_r}}{\sum_r e^{-\beta E_r}}$$

ensemble of systems in contact with reservoir of fixed T
 Canonical Ensemble