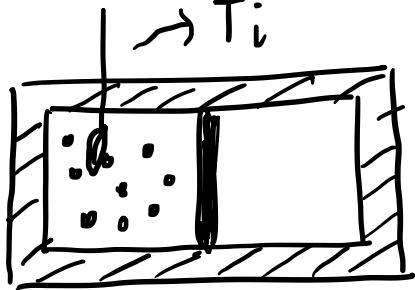
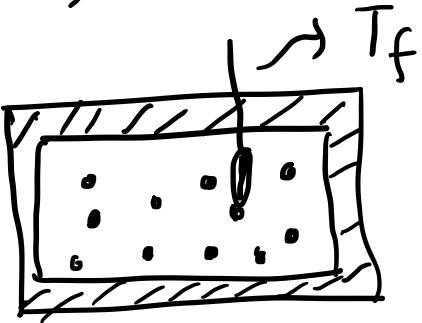


Joule's experiment (1843)



Before T_i



After $T_f = ?$

Adiabatic free expansion

$$\delta Q = dU + \delta W \Rightarrow dU = 0$$

"0

"0

$$U_i = U_f$$

Joule's result :

$$T_i = T_f$$

$$dU = \left(\frac{\partial U}{\partial V}\right)_T dV + \left(\frac{\partial U}{\partial T}\right)_V dT$$

$$\left(\frac{\partial U}{\partial V}\right)_T = 0$$

$$V = U(T)$$

Adiabatic quasistatic changes (expansion/compression) \rightarrow ideal gas

$$dQ = C_V dT + PdV$$

$$dQ = 0$$

$$C_V dT + PdV = 0$$

$$C_V dT + \frac{nRT}{V} dV = 0$$

$$\frac{dT}{T} + \frac{nR}{C_V} \frac{dV}{V} = 0$$

$$\ln T + \frac{C_P - C_V}{C_V} \ln V = 0$$

$$\ln T + (\gamma - 1) \ln V = 0$$

$$TV^{\gamma-1} = \text{const}$$

$$PV^\gamma = \text{const}$$

$$TP^{\frac{1-\gamma}{\gamma}} = \text{const}$$

Application

$T(h) \rightarrow$ Dependence of T on height of atmosphere.

$$dh \left\{ \begin{array}{l} p + dp \\ p \end{array} \right.$$

weight of air in the slice = $\rho g dh$

$$dp = -\rho g dh$$

$$PV = nRT$$

ideal gas

$$= \frac{m}{M} RT$$

M = mol. wt

$$\boxed{\rho = \frac{PM}{RT}}$$

$$dp = -\frac{PM}{RT} gdh$$

$$dP = -\frac{PM}{RT} g dh$$

$$= -\frac{gM}{R} \frac{P}{T} dh$$

$$-\frac{P}{T} \frac{\gamma}{1-\gamma} dT = -\frac{gM}{R} \frac{P}{T} dh$$

$$\boxed{\frac{dT}{dh} = \frac{gM}{R} \left(\frac{1-\gamma}{\gamma} \right)}$$

$$\simeq -9.8 \text{ degrees/km}$$

$$TP^{\frac{1-\gamma}{\gamma}} = \text{const}$$

↑ adiabatic

$$dT P^{\frac{1-\gamma}{\gamma}} + \frac{1-\gamma}{\gamma} T \frac{dp}{P} P^{\frac{1-\gamma}{\gamma}} = 0$$

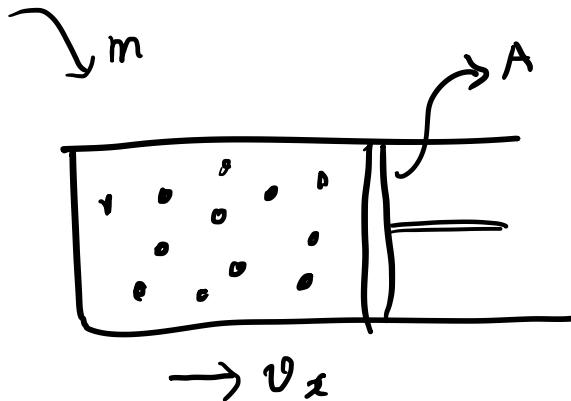
$$\boxed{dP = -\frac{P}{T} \frac{\gamma}{1-\gamma} dT}$$

$$M = 28.8 \quad \gamma = \frac{C_p}{C_v} = \frac{7}{5}$$

g, R,
↑ diatomic

Detour - Kinetic theory of gases

- atoms are hard spheres
- no interaction between atoms
- collisions are elastic
- No external field \rightarrow atoms are distributed uniformly
- No preferred direction for velocity



Momentum delivered to the piston
 $= 2mv_x$

of collisions with piston in Δt = ?

N atoms in volume V , $\frac{N}{V} = n$

$$= n v_x \Delta t A$$

$$F = 2mv_x \times n v_x A$$

$$P = 2mn v_x^2 \times$$

- Only $\frac{1}{2}$ will have $+ve v_x$
- factor of 2 must go

- Not all particles will have same v_x

$$P = mn \langle v_x^2 \rangle$$

$$\text{Isotropy} \rightarrow \langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle$$

$$P = \frac{1}{3} mn \langle v^2 \rangle$$

$$\boxed{P = \frac{2n}{3} \langle \frac{1}{2} m v^2 \rangle}$$

$$P = \frac{2}{3} \frac{N}{V} \langle \frac{1}{2} m v^2 \rangle$$

$$\boxed{PV = \frac{2}{3} U}$$

Combining with equation of state

$$PV = nRT$$

$$T \propto \left\langle \frac{1}{2}mv^2 \right\rangle$$

$$\frac{2}{3}U = nRT$$

$$U = \frac{3}{2}nRT = \frac{3}{2}NkT$$

$$C_V = \frac{\partial U}{\partial T} = \frac{3}{2}nR$$

$$C_P = \frac{5}{2}nR$$