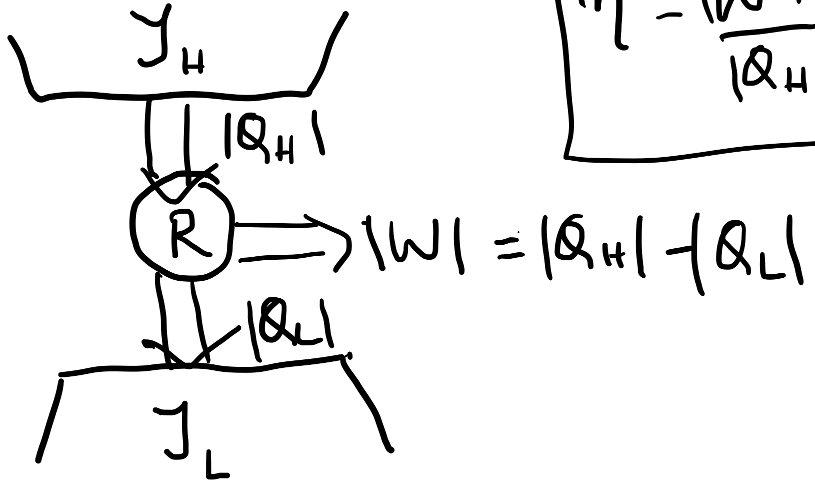
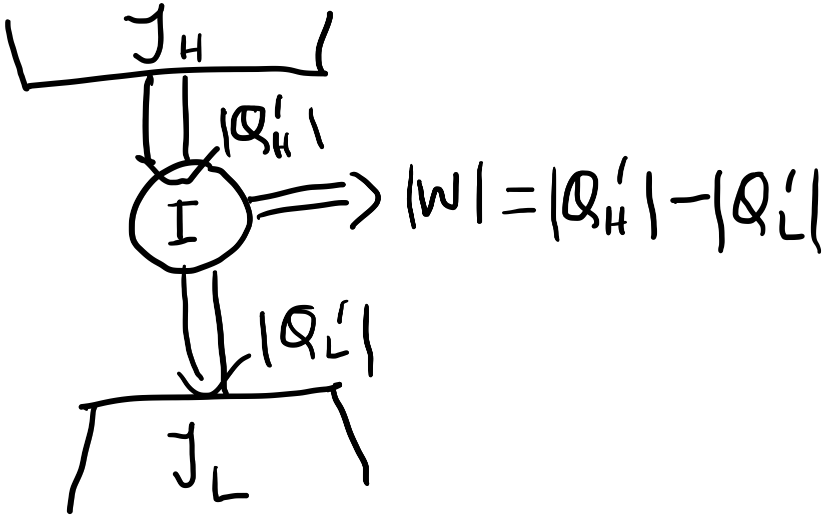
Carnot's Theorem and Corollary

Carnot's Theorem : No heat engine operating between two reservoirs at given temperatures can be more efficient than a Carnot engine working between the same two temperatures.

$$\boxed{\eta = \frac{|W|}{|Q_H|}}$$





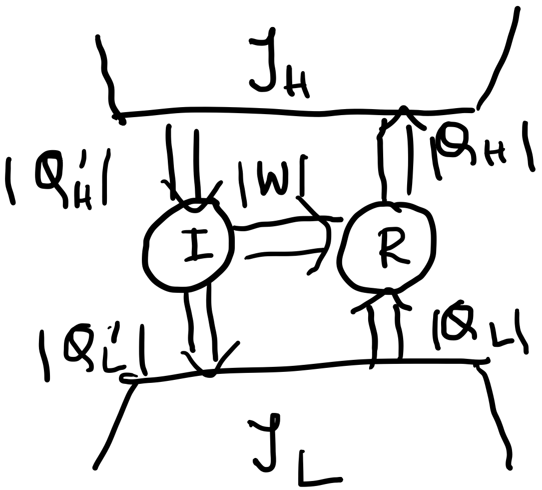
Let us assume $\eta_I > \eta_R$

$$\eta_I > \eta_R$$

$$\frac{|W|}{|Q_H'|} > \frac{|W|}{|Q_H|}$$

$$\Rightarrow \boxed{|Q_H| > |Q_H'|}$$

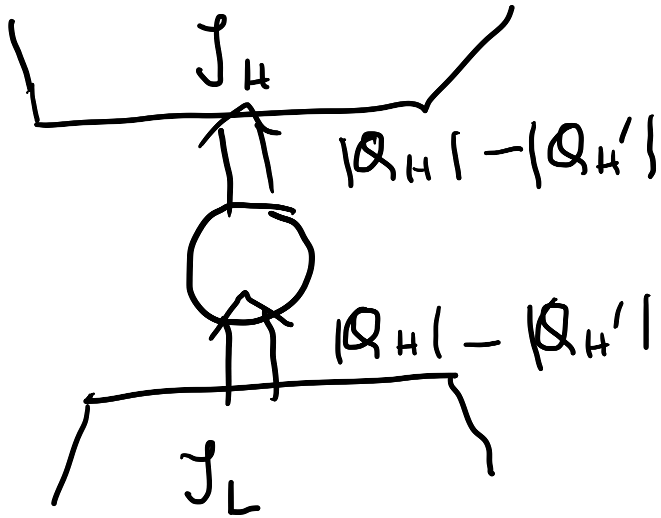
Now let I drive R backwards as a refrigerator



$$W = |Q_H| - |Q_L|$$

$$W = |Q_H'| - |Q_L'|$$

$$|Q_H'| - |Q_H| = |Q_L| - |Q_L'|$$



We know $|Q_H| > |Q_H'|$

$$|Q_H| - |Q_H'| > 0$$

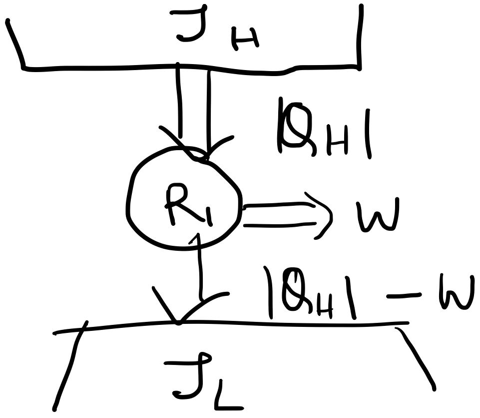
violates Clausius

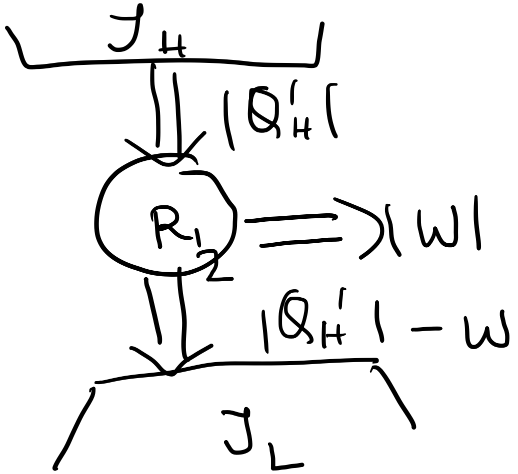
{Net effect transfer $|Q_H| - |Q_H'|$ units of heat from a low temp source to a high temp source, without doing work on surroundings}

$\eta_I > \eta_R$ must be wrong

$$\boxed{\eta_I \leq \eta_R}$$

Corollary : All Carnot engines operating between the same two temperatures have equal efficiency.





- R_1 drives R_2 backwards , Carnot's thm $\Rightarrow$ $\eta_{R_1} \leq \eta_{R_2}$
- R_2 drives R_1 backwards $\Rightarrow$ $\eta_{R_2} \leq \eta_{R_1}$

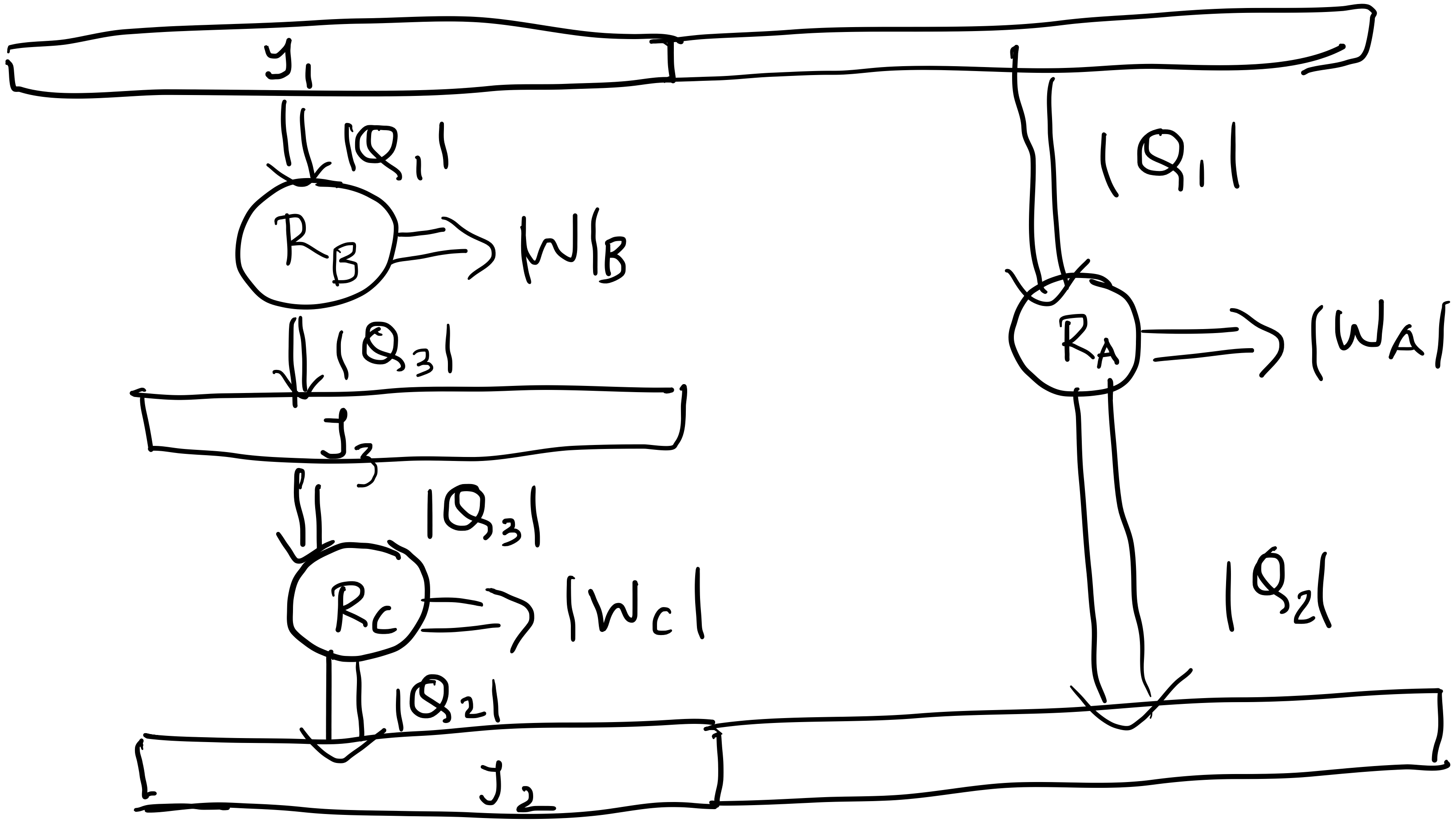
$\boxed{\eta_{R_1} = \eta_{R_2}}$ $\Rightarrow$ independent of working substance

Thermodynamic Temperature Scale

Zeroth law was the basis of temperature, but an empirical scale must be defined in terms of a thermometric property of a specific substance and thermometer, such as the ideal gas scale using a constant volume gas thermometer.

Carnot engine provides a thermodynamic scale independent of working substance !!

$J_1 > J_3 > J_2$



$$\eta_R = 1 - \frac{|Q_L|}{|Q_H|}$$

efficiency depends only on $\mathcal{T}_H, \mathcal{T}_L$

$$\eta_R = \phi(\mathcal{T}_H, \mathcal{T}_L) \longrightarrow \text{unknown fn.}$$

$$\frac{|Q_H|}{|Q_L|} = \frac{1}{1 - \phi(\mathcal{T}_H, \mathcal{T}_L)} = f(\mathcal{T}_H, \mathcal{T}_L)$$

for R_A $\quad \dfrac{|Q_1|}{|Q_2|} = f(J_1, J_2)$

R_B $\quad \dfrac{|Q_1|}{|Q_3|} = f(J_1, J_3)$

R_C $\quad \dfrac{|Q_3|}{|Q_2|} = f(J_3, J_2)$

Since $|Q_1|/|Q_2| = \dfrac{|Q_1|/|Q|_3}{|Q_2|/|Q_3|}$

$$\frac{|Q_1|}{|Q_2|} = \frac{|Q_1|/|Q_3|}{|Q_2|/|Q_3|}$$

$$\Rightarrow \quad f(T_1, T_2) = \frac{f(T_1, T_3)}{f(T_2, T_3)}$$

T_3 is arbitrarily chosen and drops out of ratio

$$\frac{|Q_1|}{|Q_2|} = \frac{\psi(T_1)}{\psi(T_2)}.$$

absolute scale

Define

$$\boxed{\frac{|Q_1|}{|Q_2|} = \frac{T_1}{T_2}}$$

→ Thermodynamic temp T

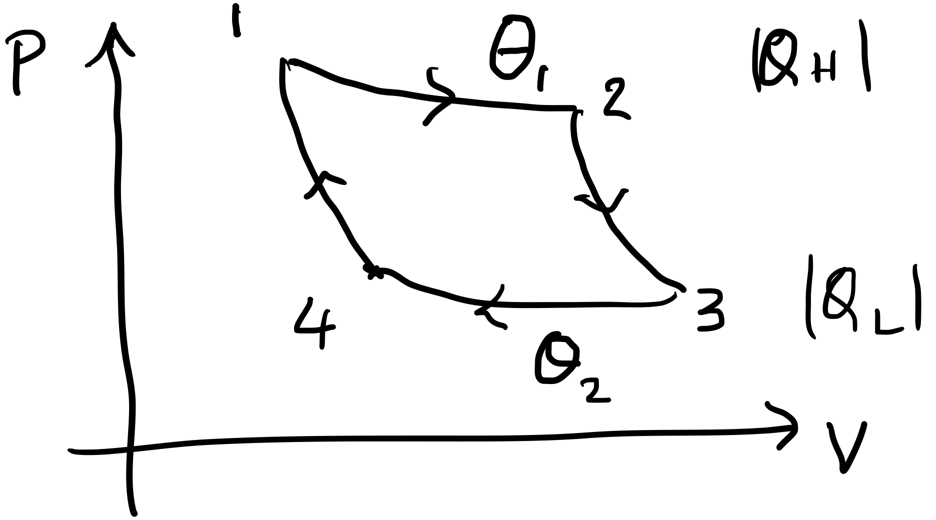
→ independent of working substance

Define $T_{TP} = 273.16$

$$\frac{|Q|}{|Q_{TP}|} = \frac{T}{T_{TP}}$$

$$\boxed{T = 273.16 \frac{|Q|}{|Q_{TP}|}}$$

Equality of Thermo & Ideal gas scales



$$1 \to 2: \quad dQ = C_v \cancel{d\theta}^{\,0} + p\,dV$$

$$\theta = \text{const} = \theta_1$$

$$|Q_H| = \int_{V_1}^{V_2} p\,dV = nR\theta_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$|Q_L| = nR\theta_2 \ln\left(\frac{V_3}{V_4}\right)$$

For adiabatic legs

$$\theta V^{\gamma - 1} = \text{const}.$$

for $2 \to 3$

$$\theta_1 V_2^{\gamma - 1} = \theta_2 V_3^{\gamma - 1}$$

for $4 \to 1$

$$\left.\begin{aligned} \frac{\theta_1}{\theta_2} &= \left(\frac{V_3}{V_2}\right)^{\gamma - 1} \\ \frac{\theta_1}{\theta_2} &= \left(\frac{V_4}{V_1}\right)^{\gamma - 1} \end{aligned}\right\} \left(\frac{V_1}{V_2}\right) = \left(\frac{V_4}{V_3}\right)$$

$$\eta = \frac{|Q_H| - |Q_L|}{|Q_H|}$$

$$= 1 - \frac{|Q_L|}{|Q_H|}$$

$$= 1 - \frac{nR\theta_2 \ln\left(\frac{V_3}{V_4}\right)}{nR\theta_1 \ln\left(\frac{V_2}{V_1}\right)}$$

$$\boxed{\theta \equiv T}$$

$$\boxed{\eta = 1 - \frac{\theta_2}{\theta_1}}$$

$$\frac{|Q_H|}{|Q_L|} = \frac{\theta_1}{\theta_2} = \frac{T_1}{T_2}$$