Indian Statistical Institute

Analysis III (HW - 1) Due date: Take your sweet time Instructor : Jaydeb Sarkar

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NOTE: (i) $B_r(x_0) = \{x \in \mathbb{R}^n : d_u(x, x_0) < r\}$. (ii) $D_r(x_0) = B_r(x_0) \setminus \{x\}$. (iii) $B^n = \Pi_{i=1}^n [a_i, b_i]$. (iv) $R(B^n)$ = the set of all Riemann integrable functions on B^n. (v) $C(B^n)$ = the set of all continuous functions on B^n. (vi) $v(B^n)$ =volume of B^n, whenever $n \geq 3$. (vii) $A(B^2)$ = area of B^2.

(1) (Iterated limit) Let $f : D_r((a,b)) \to \mathbb{R}$ and let

$$\lim_{(x,y)\to(a,b)} f(x,y),$$

exists. If $f(y) := \lim_{x\to a} f(x,y)$ exists for all y in a deleted neighberhood of b, then prove that

$$\lim_{(x,y)\to(a,b)} f(x,y) = \lim_{y\to b} \lim_{x\to a} f(x,y).$$

(2) If $f \in C(B^n)$, then prove that there exists a point $a \in B^n$ such that

$$f(a) = \frac{1}{v(B^n)} \int_{B^n} f dV.$$

(3) Let $f_i \in R([a_i, b_i])$, $i = 1, \dots, n$. Suppose

$$g(x) = f_1(x_1) \cdots f_n(x_n) \qquad (x \in B^n).$$

Prove that

$$\int_{B^n} f dx = \prod_{i=1}^n \int_{a_i}^{b_i} f_i \, dx_i.$$

(4) prove that $f \notin R([0,1] \times [0,1])$, where

$$f(x,y) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q}. \end{cases}$$

(5) True of false (with justification)? "If $n > 1$ and $|f| \in R(B^n)$, then $f \in R(B^n)$."

(6) Let $f \in R(B^n)$, and let $f(x) = g(x)$ for all but finitely many $x \in B^n$. Prove that $g \in R(B^n)$ and

$$\int_{B^n} f = \int_{B^n} g.$$