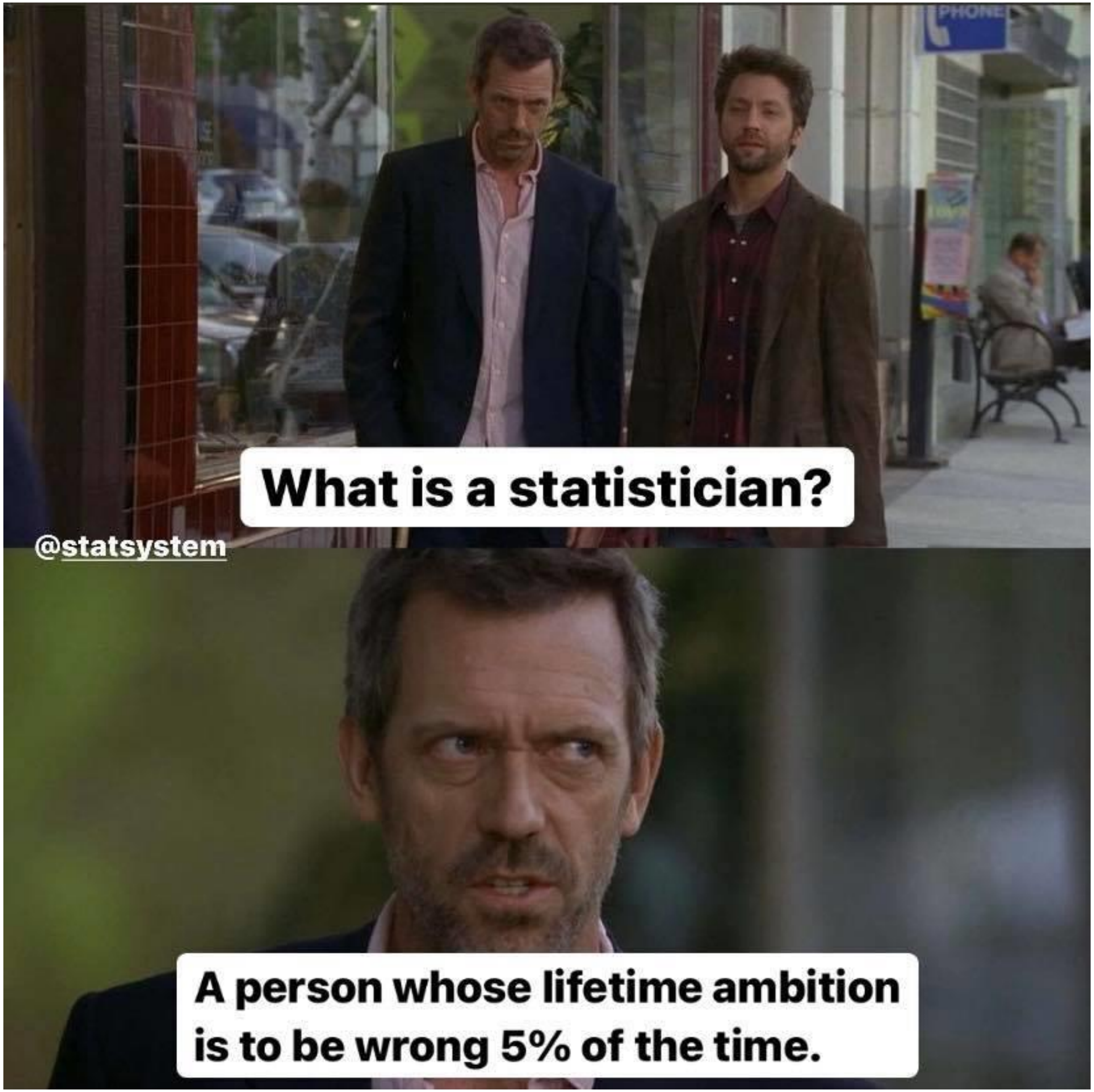
Statistics

Chapter 9: Further Testing


What is a statistician?
@statsystem
A person whose lifetime ambition is to be wrong 5% of the time.

9.1: Comparing Two Population Means: Independent Sampling

One-Tailed Test

- $H_0: (\mu_1 - \mu_2) = D_0$

$H_a: (\mu_1 - \mu_2) > D_0 \; (< D_0)$

Rejection region:

$z < -z_{\alpha} \; (> z_{\alpha})$

Two-Tailed Test

- $H_0: (\mu_1 - \mu_2) = D_0$

$H_a: (\mu_1 - \mu_2) \neq D_0$

Rejection region:

$|z| > z_{\alpha/2}$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sigma_{(\bar{x}_1 - \bar{x}_2)}}$$

$$\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \cong \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

9.1: Comparing Two Population Means: Independent Sampling

<u>Conditions Required for Valid Large-Sample Inferences about (μ_1 - μ_2)</u>

1. The two samples are randomly and independently selected from the target populations.
2. The sample sizes are both ≥ 30.

9.1: Comparing Two Population Means: Independent Sampling

We want to test the hypothesis that there is no significant difference in retention at privates and publics.

Private Colleges

- *n*: 71
- Mean: 79.37
- Standard Deviation: 9.45
- Variance: 91.17

Public Universities

- *n*: 32
- Mean: 84
- Standard Deviation: 9.88
- Variance: 99.44

$$\sigma_{(\bar{x}_1 - \bar{x}_2)} \cong \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{91.17}{71} + \frac{97.64}{32}} \cong 2.08$$

9.1: Comparing Two Population Means: Independent Sampling

$$H_0 : (\mu_1 - \mu_2) = 0$$

$$H_a : (\mu_1 - \mu_2) \neq 0$$

$$\alpha = .05$$

Test statistic: $$z = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sigma_{(\bar{x}_1 - \bar{x}_2)}} = \frac{-5.83}{2.08} = -2.799$$

Reject the null hypothesis: $$|z| > z_{\alpha/2}$$

$$2.799 > 1.96$$

9.1: Comparing Two Population Means: Independent Sampling

For small samples, the *t*-distribution can be used with a **pooled sample estimator of σ²**, s_p^2

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

9.1: Comparing Two Population Means: Independent Sampling

Small Sample Confidence Interval for (μ_1 - μ_2)

$$(\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sigma_{(\bar{x}_1 - \bar{x}_2)} = (\bar{x}_1 - \bar{x}_2) \pm t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

The value of t is based on (n_1 + n_2 -2) degrees of freedom.

9.1: Comparing Two Population Means: Independent Sampling

One-Tailed Test

- H_0: $(\mu_1 - \mu_2) = D_0$

H_a: $(\mu_1 - \mu_2) > D_0$ ($< D_0$)

Rejection region:

$t < -t_\alpha$ ($> t_\alpha$)

Two-Tailed Test

- H_0: $(\mu_1 - \mu_2) = D_0$*

H_a: $(\mu_1 - \mu_2) \neq D_0$

Rejection region:

$|t| > t_{\alpha/2}$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

9.1: Comparing Two Population Means: Independent Sampling

Conditions Required for Valid Small-Sample Inferences about (μ_1 - μ_2)

1. The two samples are randomly and independently selected from the target populations.
2. Both sampled populations have distributions that are approximately normal.
3. The population variances are equal.

9.1: Comparing Two Population Means: Independent Sampling

- Does class time affect performance?
  - The test performance of students in two sections of international trade, meeting at different times, were compared.

8:00 a.m. Class	9:30 a.m. Class
Mean: 78	Mean: 82
Standard Deviation: 14	Standard Deviation: 17
Variance: 196	Variance: 289
n: 21	n: 21

With $\alpha = .05$, test $H_0 : \mu_1 = \mu_2$

9.1: Comparing Two Population Means: Independent Sampling

8:00 a.m. Class	9:30 a.m. Class
Mean: 78	Mean: 82
Variance: 196	Variance: 289
n: 21	n: 21

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 \neq 0$$

$$\alpha = .05$$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(21-1)196 + (21-1)289}{21+21-2} = 242.5$$

$$\text{Test Statistic} : t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(78-82) - 0}{\sqrt{242.5\left(\frac{1}{21} + \frac{1}{21}\right)}} = -.832$$

9.1: Comparing Two Population Means: Independent Sampling

<u>8:00 a.m. Class</u>	<u>9:30 a.m. Class</u>
Mean: 78	Mean: 82
Variance: 196	Variance: 289
n: 21	n: 21

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 \neq 0$$

$$\alpha = .05$$

With df = 18 + 24 – 2 = 40, $t_{\alpha/2} = t_{.025} = 2.021$.

Since out test statistic t = -.812. $|t| < t_{.025}$.

<u>Do not reject the null hypothesis</u>

$$s_p^2 = \frac{(n_1 \ldots}{\ldots} \ldots = 242.5$$

$$\text{Test Statistic} : t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(78 - 82) - 0}{\sqrt{242.5\left(\frac{1}{21} + \frac{1}{21}\right)}} = -.832$$

9.2: Comparing Two Population Means: Paired Difference Experiments

$$\underline{\text{Paired Difference Confidence Interval for } \mu_{\mathrm{d}} = \mu_1 - \mu_2}$$

$$\text{Large Sample: } \bar{x}_d \pm z_{\alpha/2} \frac{\sigma_d}{\sqrt{n_d}} \cong \bar{x}_d \pm z_{\alpha/2} \frac{s_d}{\sqrt{n_d}}$$

$$\text{Small Sample: } \bar{x}_d \pm t_{\alpha/2} \frac{s_d}{\sqrt{n_d}} \text{ with } (n_d - 1) \text{ degrees of freedom,}$$

where $\bar{x}_d$ = sample mean difference

where s_d = sample standard deviation of differences

where n_d = number of pairs observed

9.2: Comparing Two Population Means: Paired Difference Experiments

Paired Difference

Hypothesis Tests for

$\mu_{\mathrm{d}} = \mu_1 - \mu_2$

Large sample test statistic

$$z = \frac{\bar{x}_d - D_0}{\sigma_d / \sqrt{n_d}} \cong \frac{\bar{x}_d - D_0}{s_d / \sqrt{n_d}}$$

Small sample test statistic

$$t_{\alpha,\, n_d - 1} = \frac{\bar{x}_d - D_0}{s_d / \sqrt{n_d}}$$

One-Tailed Test

$H_0 : \mu_d = D_0$

$H_a : \mu_d < (\text{or} >) D_0$

Large sample rejection region: $z < (\text{or} >) - z_\alpha$

Small sample rejection region: $t < (\text{or} >) - t_\alpha$

Two-Tailed Test

$H_0 : \mu_d = D_0$

$H_a : \mu_d \neq D_0$

Large Sample rejection region: $|z| > z_{\alpha/2}$

Small sample rejection region: $|t| > t_{\alpha/2}$

9.2: Comparing Two Population Means: Paired Difference Experiments

- Suppose 150 items were priced at two online stores, “cport” and “warriorwoman.”
  - Mean difference: $1.75
  - Standard Deviation: $10.35
  - Test at the 95% level that the difference in the two stores is zero.

9.2: Comparing Two Population Means: Paired Difference Experiments

Suppose 150 items were priced at two online stores, "cport" and "warriorwoman."

- Mean difference: \$1.75
- Standard Deviation: \$10.35
- α = .05

$$H_0 : \mu_d = 0$$

$$H_a : \mu_d \neq 0$$

$$z = \frac{\bar{x}_d - D_0}{\sigma_d / \sqrt{n_d}}$$

$$z = \frac{1.75 - 0}{10.35 / \sqrt{150}}$$

$$z = 2.07$$

9.2: Comparing Two Population Means: Paired Difference Experiments

Suppose 150 items were priced at two online stores, "cport" and "warriorwoma

- Mean differenc $1.75
- Standard Devia $10.35
- ⍰ = .05

The critical value of $z_{.05}$ is 1.96, so we would reject this null hypothesis.

$$H_0 : \mu_d = 0$$

$$H_a : \mu_d \neq 0$$

$$\frac{\bar{x}_d - D_0}{\sigma_d / \sqrt{n_d}}$$

$$= \frac{1.75 - 0}{10.35 / \sqrt{150}}$$

$$z = 2.07$$

9.3: Comparing Two Population Proportions: Independent Sampling

Large-Sample Test of Hypothesis about $p_1 - p_2$

One-Tailed Test	Two-Tailed Test
$H_0: (p_1 - p_2) = D_0$ (very often 0)	$H_0: (p_1 - p_2) = D_0$
$H_a: (p_1 - p_2) > (\text{or} <) D_0$	$H_a: (p_1 - p_2) \neq D_0$
Rejection region: $\lvert z \rvert > z_\alpha$	Rejection region: $\lvert z \rvert > z_{\alpha/2}$

$$\text{Test Statistic}: z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sigma_{(\hat{p}_1 - \hat{p}_2)}}$$

$$\text{where } \hat{p} = \frac{x_1 + x_2}{n_1 + n_2} \text{ and } \sigma_{(\hat{p}_1 - \hat{p}_2)} \cong \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

9.3: Comparing Two Population Proportions: Independent Sampling

- Randy Stinchfield of the University of Minnesota studied the gambling activities of public school students in 1992 and 1998 (*Journal of Gambling Studies*, Winter 2001). His results are reported below:

	1992	**1998**
Survey *n*	21,484	23,199
Number who gambled	4.684	5,313
Proportion who gambled	.218	.229

- Do these results represent a statistically significant difference at the $\alpha = .01$ level?

9.3: Comparing Two Population Proportions: Independent Sampling

	1992	1998
Survey *n*	21,484	23,199
Number who gambled	4.684	5,313
Proportion who gambled	.218	.229

Two-Tailed Test

$$H_0 : (p_1 - p_2) = D_0$$

$$H_a : (p_1 - p_2) \neq D_0$$

Rejection region:

$$|z| > z_{\alpha/2} > 2.576$$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{4,684 + 5,313}{21,484 + 23,199} = .224$$

$$\sigma_{(\hat{p}_1 - \hat{p}_2)} \cong \sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$= \sqrt{(.224)(.776)\left(\frac{1}{21,484} + \frac{1}{23,199}\right)} = .00395$$

$$\text{Test Statistic: } z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sigma_{(\hat{p}_1 - \hat{p}_2)}} = \frac{(.218 - .229)}{.00395}$$

$$= -2.786$$

9.3: Comparing Two Population Proportions: Independent Sampling

	1992	1998
Survey *n*	21,484	23,199
Number who gambled	4,68	5,313
Proportion who gambled	.21	

Since the computed value of *z*, -2.786, is of greater magnitude than the critical value, 2.576, we can reject the null hypothesis at the α = .01 level.

Two-Tailed

$H_0 : (p_1 - p_2$

$H_a : (p_1 - p_2) \neq D_0$

Rejection region:

$|z| > z_{\alpha/2} > 2.576$

$$\hat{p} = \frac{x_1 + x_2}{} \quad \frac{4,684 + 5,313}{4 + 23,199} = .224$$

$$\frac{1}{n_2}$$

$$\frac{1}{,484} + \frac{1}{23,199} = .00395$$

$$\text{Test Statistic: } z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sigma_{(\hat{p}_1 - \hat{p}_2)}} = \frac{(.218 - .229)}{.00395}$$

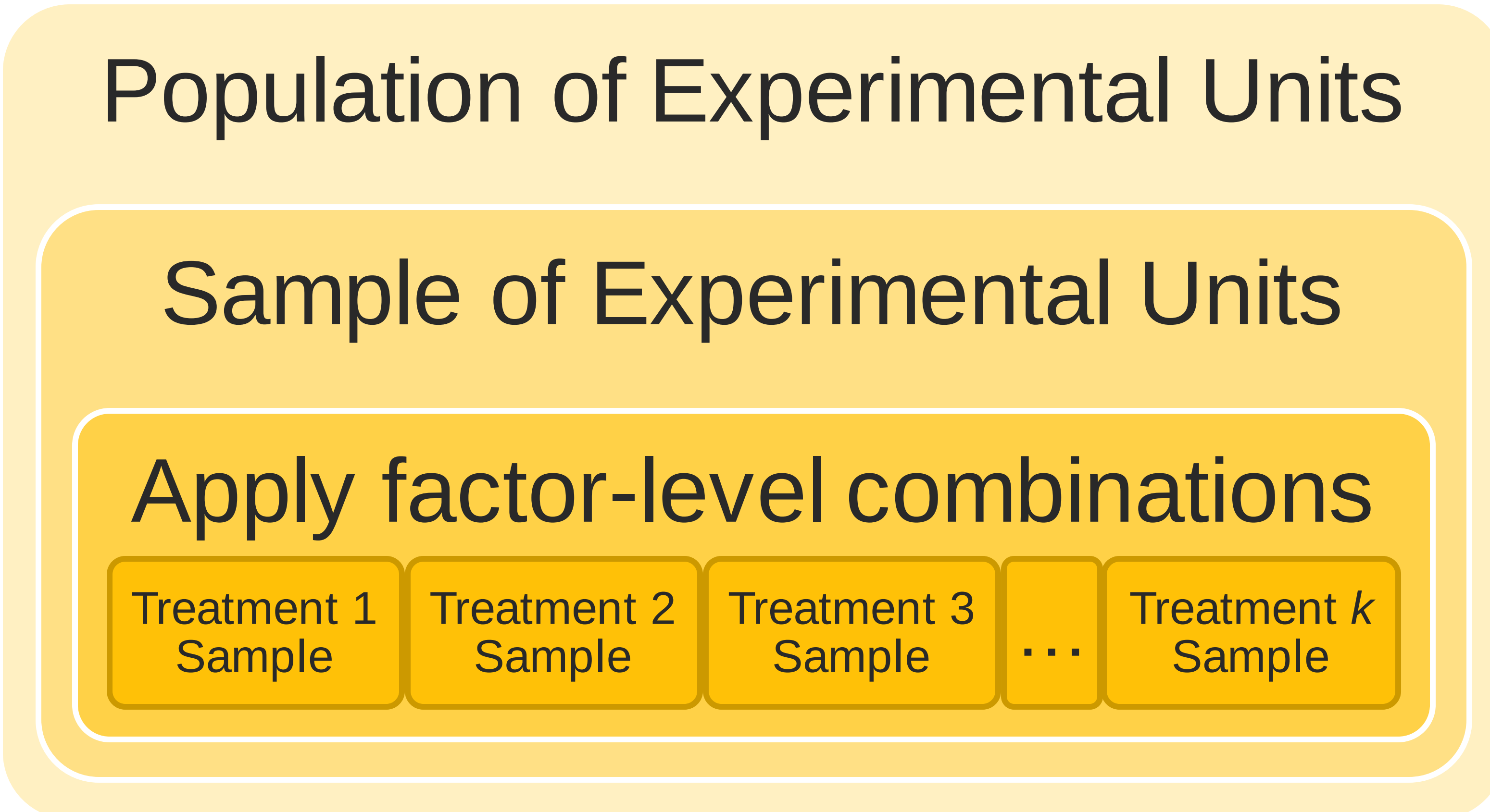
$$= -2.786$$

9.3: Comparing Two Population Proportions: Independent Sampling

- For valid inferences
  - The two samples must be independent
  - The two sample sizes must be large:

$$n_1\hat{p}_1 \geq 15 \text{ and } n_1\hat{q}_1 \geq 15$$
$$n_2\hat{p}_2 \geq 15 \text{ and } n_2\hat{q}_2 \geq 15$$

9.4: Comparing Three or More Population Means: Analysis of Variance



9.4: Comparing Three or More Population Means: Analysis of Variance

- Very often the object is to determine whether the varying treatments result in different means:

  H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \cdots = \mu_k$

  H_a: At least two of the *k* treatment means differ

9.4: Comparing Three or More Population Means: Analysis of Variance

- Testing the equity of the means involves comparing the variability *among* the different treatments as well as *within* the treatments, adjusted for degrees of freedom.

Variation between the treatment means:

Sum of Squares for Treatments (SST)

$$SST = \sum_{i=1}^{k} n_i (\bar{x}_i - \bar{x})^2$$

Variation within the treatments:

Sum of Squares for Error (SSE)

$$SSE = \sum_{j=1}^{n_1} (x_{1j} - \bar{x}_1)^2 + \sum_{j=1}^{n_2} (x_{2j} - \bar{x}_2)^2 + \cdots + \sum_{j=1}^{n_k} (x_{kj} - \bar{x}_k)^2$$

$$= (n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \cdots + (n_k - 1)s_k^2$$

9.4: Comparing Three or More Population Means: Analysis of Variance

- Adjusting for degrees of freedom produces comparable measures of variability

Mean Square for Treatments (MST) $MST = \frac{SST}{k-1}$
Mean Square for Error (MSE) $MSE = \frac{SSE}{n-k}$

9.4: Comparing Three or More Population Means: Analysis of Variance

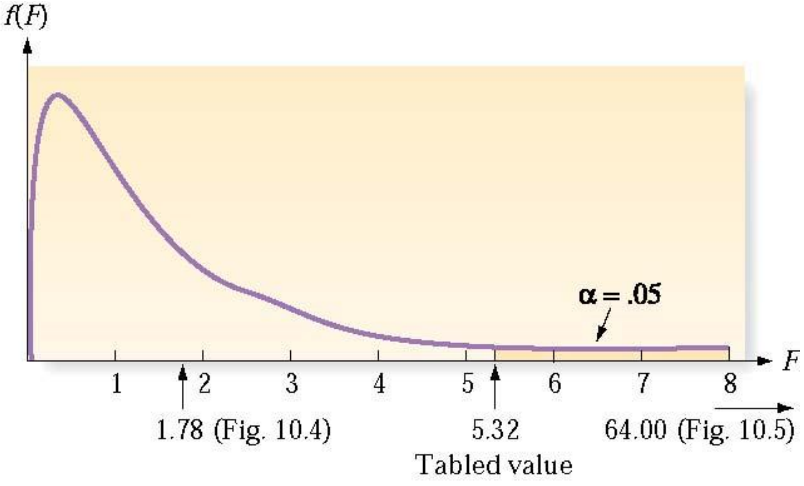
- The ratio of the variability among the treatment means to that within the treatment means is an *F* -statistic:

$$F_{k-1,n-k} = \frac{MST}{MSE}$$

with *k-1* numerator and *n-k* denominator degrees of freedom.

9.4: Comparing Three or More Population Means: Analysis of Variance

If *F < 1, the difference between the treatment means may be attributable to sampling error.**



If *F > 1 (significantly), there is support for the alternative hypothesis that the treatments themselves produce different results.**

9.4: Comparing Three or More Population Means: Analysis of Variance

ANOVA F-Test to Compare k Treatment Means:
Completely Randomized Design

$H_0 : \mu_1 = \mu_2 = \cdots = \mu_k$

$H_a :$ At least two treatment means differ.

Test Statistic: $F = \dfrac{MST}{MSE}$

Rejection region: $F^* > F_\alpha$, with $k - 1$ numerator and $n - k$ denominator degrees of freedom.

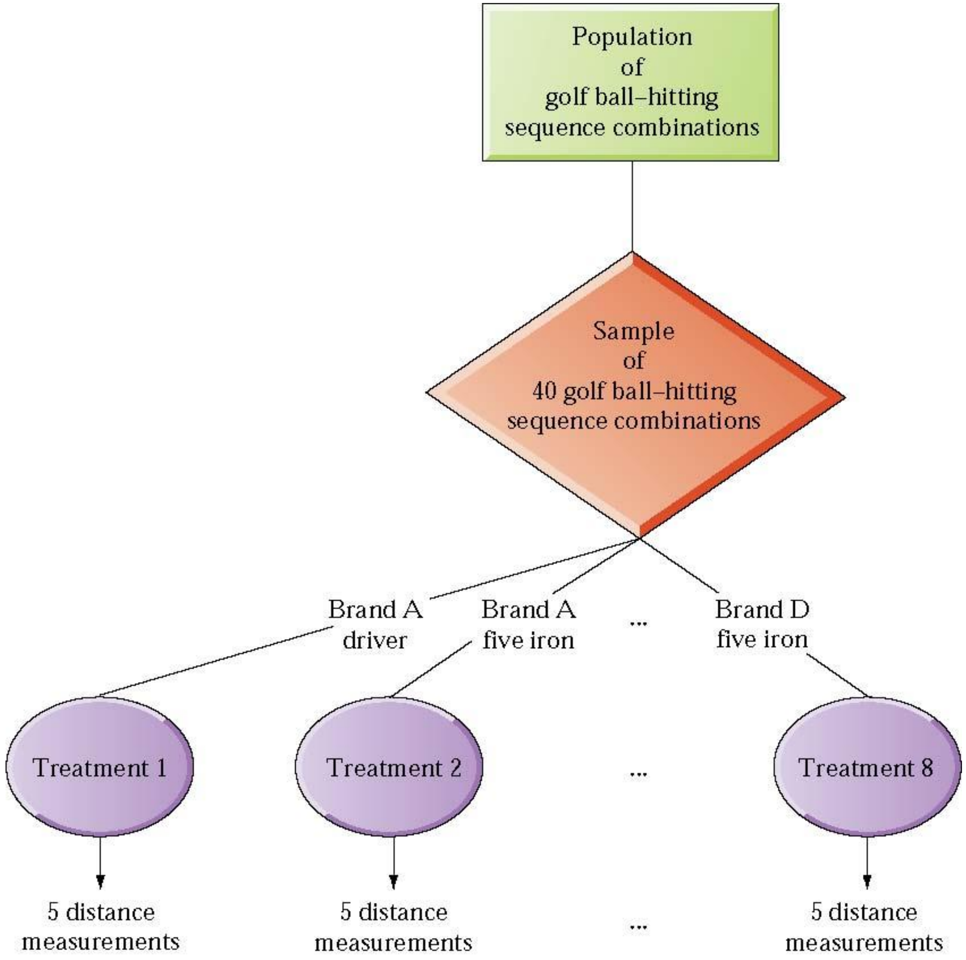
9.4: Comparing Three or More Population Means: Analysis of Variance

- Conditions required for a Valid ANOVA *F*-Test: Completely Randomized Design
  1. The samples are randomly selected in an independent manner from the *k* treatment populations.
  2. All *k* sampled populations have distributions that are approximately normal.
  3. The *k* population variances are equal.

9.4: Comparing Three or More Population Means: Analysis of Variance

- The USGA compares the driving distance of four brands of golf balls.
  - H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
  - H_a: The mean distances differ for at least two of the brands
  - $\alpha = .10$
  - Test Statistic: F = MST/MSE
  - Rejection region: $F > 2.25 = F_{.10 \text{ with } v_1 = 3 \text{ and } v_2 = 36}$

9.4: Comparing Three or More Population Means: Analysis of Variance



9.4: Comparing Three or More Population Means: Analysis of Variance

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  - $\alpha = .10$ Test Statistic: F = MST/MSE
  - Rejection region: $F > 2.25 = F_{.10 \text{ with } v_1 = 3 \text{ and } v_2 = 36}$

Source	Degrees of Freedom	Sum of Squares	Mean Square	*F*	*p*-value
Brands	3	2,794.39	931.46	43.99	.000
Error	36	762.30	21.18		

9.4: Comparing Three or More Population Means: Analysis of Variance

- The USGA compares the driving distance of four brands of golf balls.
  - H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
  - H_a: The mean distances differ for at least two of the brands
  - $\alpha = .10$ Test Statistic: F = MST/MSE
  - Rejection region: $F > 2.25 = F_{.10 \text{ with } v_1 = 3 \text{ and } v_2 = 36}$

Source	Degre Free	f	Mean Square	*F*	*p*-value
Brands	3		931.46	43.99	.000
Error	36		21.18		

Since the calculated $F > 2.25$, we reject the null hypothesis.

9.5 Testing Categorical Probabilities:Multinomial Experiment

- Properties of the Multinomial Experiment

1. The experiment consists of n identical trials.
2. There are k possible outcomes (called **classes**, **categories** or **cells**) to each trial.
3. The probabilities of the k outcomes, denoted by p_1, p_2, …, p_k, where $p_1 + p_2 + \ldots + p_k = 1$, remain the same from trial to trial.
4. The trials are independent.
5. The random variables of interest are the **cell counts** n_1, n_2, …, n_k of the number of observations that fall into each of the k categories.

9.5: Testing Categorical Probabilities: Multinomial Experiments

- Suppose three candidates are running for office, and 150 voters are asked their preferences.
  - Candidate 1 is the choice of 61 voters.
  - Candidate 2 is the choice of 53 voters.
  - Candidate 3 is the choice of 36 voters.
- Do these data suggest the population may prefer one candidate over the others?

9.5: Testing Categorical Probabilities: Multinomial Experiments

Candidate 1 is the choice of 61 voters.

Candidate 2 is the choice of 53 voters.

Candidate 3 is the choice of 36 voters.

n =150

$$H_0: p_1 = p_2 = p_3 = 1/3 = \text{ No preference}$$

$$H_a: \text{ At least one of the proprtions exceeds } 1/3$$

$$E(\text{Number of votes for each candidate}|\, H_0) = 150/3 = 50$$

$$E_1 = E_2 = E_3 = 50$$

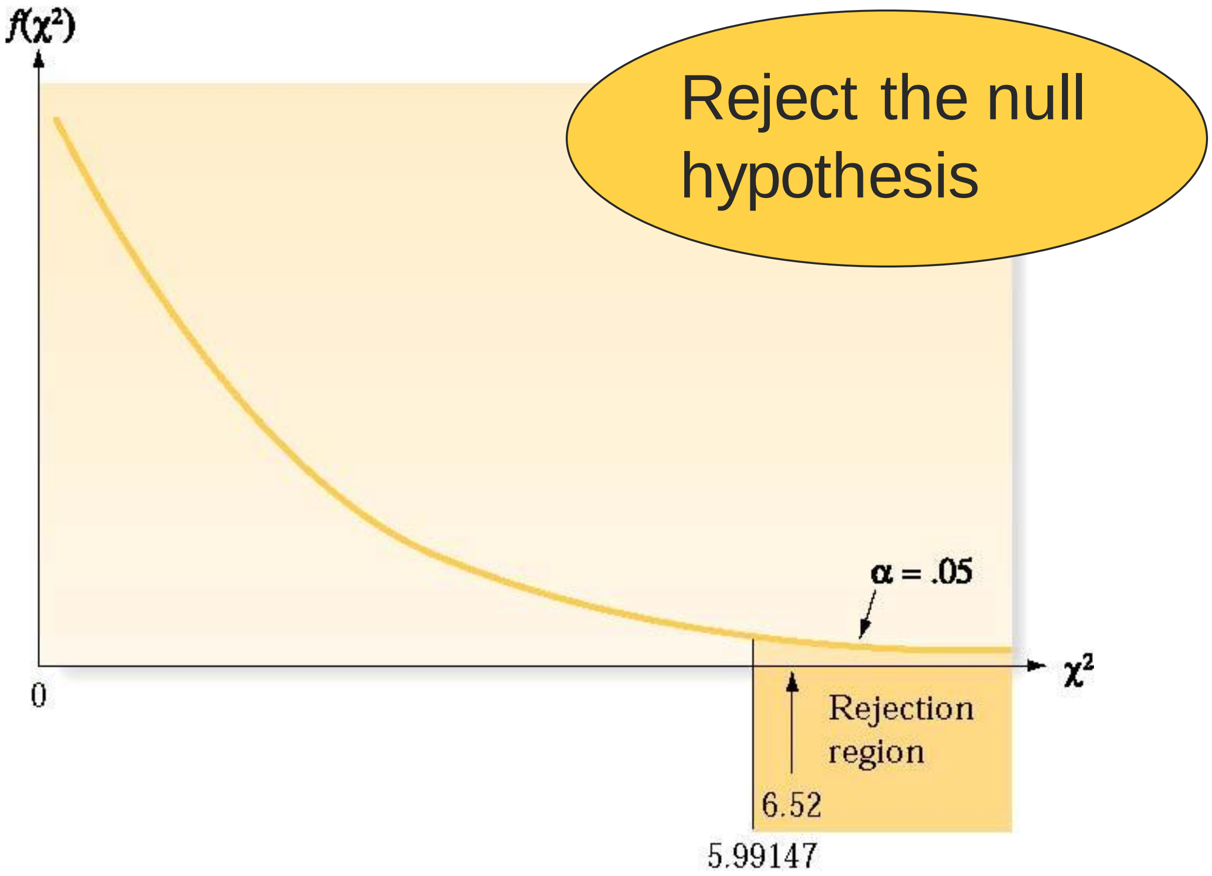
A chi-square (χ^2) test is used to test H_0.

$$\chi^2 = \frac{[n_1 - E_1]^2}{E_1} + \frac{[n_2 - E_2]^2}{E_2} + \frac{[n_3 - E_3]^2}{E_3}$$

$$\chi^2 = \frac{[61-50]^2}{50} + \frac{[53-50]^2}{50} + \frac{[36-50]^2}{50} = 6.52$$

$$\chi^2_{.05, df=2} = 5.99147$$

9.5: Testing Categorical Probabilities: Multinomial Experiments



9.5: Testing Categorical Probabilities: Multinomial Experiments

Test of a Hypothesis about Multinomial Probabilities:

One-Way Table

H_0: $p_1 = p_{1,0}$, $p_2 = p_{2,0}$, … , $p_k = p_{k,0}$

where $p_{1,0}$, $p_{2,0}$, …, $p_{k,0}$ represent the hypothesized values of the multinomial probabilities

H_a: At least one of the multinomial probabilities does not equal its hypothesized value

Test statistic: $\chi^2 = \sum \frac{[n_i - E_i]^2}{E_i}$

Rejection region: $\chi^2 > \chi^2_\alpha$, with (k-1) df.

where $E_i = np_{1,0}$, is the **expected cell count** given the null hypothesis.

9.5: Testing Categorical Probabilities: Multinomial Experiments

- Conditions Required for a Valid χ^2 Test: One-Way Table

1. A multinomial experiment has been conducted.
2. The sample size n will be large enough so that, for every cell, the expected cell count $E(n_i)$ will be equal to 5 or more.

9.5: Testing Categorical Probabilities: Multinomial Experiments

Example 9.5: Distribution of Opinions About Marijuana Possession **Before** Television Series has Aired

Legalization	Decriminalization	Existing Law	No Opinion
7%	18%	65%	10%

Table 9.5: Distribution of Opinions About Marijuana Possession **After** Television Series has Aired

Legalization	Decriminalization	Existing Law	No Opinion
39	99	336	26

9.5: Testing Categorical Probabilities: Multinomial Experiments

Expected Distribution of 500 Opinions About Marijuana Possession After Television Series has Aired

Legalization	Decriminalization	Existing Law	No Opinion
500(.07)=35	500(.18)=90	500(.65)=325	500(.10)=50

$H_0 : p_1 = .07, p_2 = .18, p_3 = .65, p_4 = .10$

$H_a :$ At least one of the proportions differs from its null hypothesis value.

Test statistic: $\chi^2 = \sum \frac{[n_i - E_i]^2}{E_i}$

Rejection region: $\chi^2 > \chi^2_{\alpha=.01, df=3} = 11.3449$

9.5: Testing Categorical Probabilities: Multinomial Experiments

Expected Distribution of 500 Opinions About Marijuana Possession After Television Series has Aired

Legalization	Decriminalization	Existing Law	No Opinion
500(.07)=35	500(.18)=90	500(.65)=325	500(.10)=50

$$\text{Rejection region: } \chi^2 > \chi^2_{\alpha=.01,df=3} = 11.3449$$

$$\chi^2 = \frac{(39-35)^2}{35} + \frac{(99-90)^2}{90} + \frac{(336-325)^2}{325} + \frac{(26-50)^2}{50}$$

$$\chi^2 = 13.249$$

9.5: Testing Categorical Probabilities: Multinomial Experiments

Expected Distribution of 500 Opinions About Marijuana Possession After Television Series has Aired

Legalization	Decriminalization	Existing Law	No Opinion
500(.07)=35	500(.18)=90	500(.65)=325	500(.10)=50

$$\text{Rejection region: } \chi^2 > \chi^2_{\alpha=.01,df=3} = 11.3449$$

$$\chi^2 = \frac{(39-35)^2}{35} + \frac{(99-90)^2}{90} + \frac{(336-325)^2}{325} + \frac{(26-50)^2}{50}$$

$$\chi^2 = 13.249$$

Reject the null hypothesis

9.5: Testing Categorical Probabilities: Multinomial Experiments

- Inferences can be made on any single proportion as well:
  - 95% confidence interval on the proportion of citizens in the viewing area with no opinion is

$$\hat{p}_4 \pm 1.96\sigma_{\hat{p}_4}$$

$$\text{where } \hat{p}_4 = \frac{n_4}{n} = \frac{26}{500} = .052$$

$$\text{and } \sigma_{\hat{p}_4} \approx \sqrt{\frac{\hat{p}_4(1-\hat{p}_4)}{n}} \approx \sqrt{\frac{.052(.948)}{500}} \approx .0099$$

$$\hat{p}_4 \pm 1.96\sigma_{\hat{p}_4} = .052 \pm 1.96(.0099) = .052 \pm .019$$

9.6: Testing Categorical Probabilities: Two-Way Table

- Chi-square analysis can also be used to investigate studies based on qualitative factors.
  - Does having one characteristic make it more/less likely to exhibit another characteristic?

9.6: Testing Categorical Probabilities: Two-Way Table

The columns are divided according to the subcategories for one qualitative variable and the rows for the other qualitative variable.

		Column				
		1	2	$\dashrightarrow$	c	**Row Totals**
	1	n_{11}	n_{12}	$\dashrightarrow$	n_{1c}	R_1
Row	2	n_{21}	n_{22}	$\dashrightarrow$	n_{2c}	R_2
	$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$
	r	n_{r1}	n_{r2}	$\dashrightarrow$	n_{rc}	R_r
Column Totals		C_1	C_1		C_1	n

9.6: Testing Categorical Probabilities: Two-Way Table

General Form of a Two-way (Contigency) Table Analysis:

A Test for Independence

H_0 : The two classifications are independent

H_a : The two classifications are dependent

Test statistic: $\chi^2 = \sum \frac{[n_{ij} - E_{ij}]^2}{\bar{E}_{ij}}$

where $E_{ij} = \frac{R_i C_j}{n}$

and $R_i =$ total for row $i, C_j =$ total for row $j, n =$ sample size

Rejection region: $\chi^2 > \chi^2_{\alpha}$, df = $(r-1)(c-1)$

9.6: Testing Categorical Probabilities: Two-Way Table

- The results of a survey regarding marital status and religious affiliation are reported below.

		Religious Affiliation					
		A	B	C	D	None	Totals
Marital Status	Divorced	39	19	12	28	18	116
	Married, never divorced	172	61	44	70	37	384
	Totals	211	80	56	98	55	500

H_0: Marital status and religious affiliation are independent
H_a: Marital status and religious affiliation are dependent

9.6: Testing Categorical Probabilities: Two-Way Table

- The expected frequencies are included below:

Marital Status / Religious Affiliation

	A	B	C	D	None	Totals
Divorced	39 (48.95)	19 (18.56)	12 (12.99)	28 (27.74)	18 (12.76)	116
Married, never divorced	172 (162.05)	61 (61.44)	44 (43.01)	70 (75.26)	37 (42.24)	384
Totals	211	80	56	98	55	500

The *chi-square* value computed with SAS is 7.1355, with *p*-value = .1289. Even at the $\alpha = .10$ level, we cannot reject the null hypothesis.