

### Homework 1 (due Friday March 5)

*The solutions must be your own. Simply copying from other students will result in no credit.*

Your homework should be done in LaTeX and the R code, output, graphics etc. must be embedded using Sweave. Your score will be determined by the quality of your written discussion of the results, along with the code and output.

1. Write a program that implements the bisection method to find a root of  $f(x) = 0$ . Note that two initial values  $x_0$  and  $x_1$  are required such that  $f(x_0)f(x_1) \leq 0$ .
2. Write a program that implements the secant method to find a root of  $f(x) = 0$ . Two initial values  $x_0$  and  $x_1$  are required but we do not require a change of sign of  $f(x)$  between them.
3. Write a program that implements the Newton method to find a root of  $f(x) = 0$ .
4. Write a program that implements the “Illinois” method to find a root of  $f(x) = 0$ . Here is a description of the method.

We use the notation  $f_n := f(x_n)$ . We start with two points  $x_0$  and  $x_1$  for which  $f$  has opposite sign. Now assume that for a certain  $n$ ,  $f_{n-1}f_n < 0$ . Then  $x_{n+1}$  is defined by the secant approximation. If  $f_nf_{n+1} < 0$  we can proceed to the next step using  $x_n$  and  $x_{n+1}$ . Otherwise, a modified formula is used: In this second case, there is a change of sign between  $x_{n-1}$  and  $x_{n+1}$ , since  $f_{n-1}f_n < 0$  and  $f_nf_{n+1} > 0$ . Find the intersection of the straight line through  $(x_{n+1}, f(x_{n+1}))$  and  $(x_{n-1}, f(x_{n-1})/2)$  with the  $x$ -axis. This point is chosen as  $x_{n+2}$  if there is a change of sign between it and  $x_{n+1}$ . Otherwise, find the intersection of the straight line through  $(x_{n+1}, f(x_{n+1}))$  and  $(x_{n-1}, f(x_{n-1})/4)$  with the  $x$ -axis, and test for a change of sign. If necessary, additional points are tested after replacing  $f(x_{n-1})/4$  with  $f(x_{n-1})/8$ , etc. (it can be established that we eventually get a change of sign and so we do not get stuck in an infinite loop). Note that once we succeed the assumption  $f_{n+1}f_{n+2} < 0$  is again valid and we proceed using  $x_{n+1}$  and  $x_{n+2}$ . We can continue the iteration using the same recipe.

After trying your programs for some simple functions, e.g.  $x^2 - 2 = 0$ , use them all to find an accurate root of

$$f(x) = \sin(x^2) + 1.02 - \exp(-x) = 0.$$

If there is more than one root, find the *one furthest to the left*. In order to get started make a plot of the function using R. Then find as many additional roots as you can. Is there any difference in the accuracy with which you can find the different roots?

Compare the performance of the four methods. How many steps are required for each to get the best accuracy possible; there is no point continuing an iteration if  $x_{k+1} = x_k$ . How many steps are required to reach an error below  $10^{-6}$ ?