QUIZ

(1) Let α be the real 5th root of 2. Then,
  (a) $\mathbb{Q}(\alpha^2) = \mathbb{Q}(\alpha^3)$.
  (b) $\mathbb{Q}(\alpha^2)$ strictly contains $\mathbb{Q}(\alpha^3)$,
  (c) $\mathbb{Q}(\alpha^3)$ strictly contains $\mathbb{Q}(\alpha^2)$,
  (d) none of the above

  Answer: (a). It is easy to show that $\mathbb{Q}(\alpha^2)$ and $\mathbb{Q}(\alpha^3)$ are both equal to $\mathbb{Q}(\alpha)$.

(2) Let $K_1 = \mathbb{Q}(\sqrt{2})$ and $K_2 = \mathbb{Q}(\sqrt{3})$. Then
  (a) K_1 and K_2 are equal (as subfields of $\mathbb{C}$),
  (b) K_1 and K_2 are not equal but are isomorphic as fields,
  (c) K_1 and K_2 are not isomorphic as fields but are isomorphic as $\mathbb{Q}$ vector spaces,
  (d) none of the above

  Answer: (c) is true. K_1 and K_2 are not isomorphic as fields for if there existed such an isomorphism $\phi : K_1 \to K_2$, then $\phi(\sqrt{2}) = a + b\sqrt{3}$ would satisfy the equation $x^2 = 2$, which you can check is not true.

(3) Let α, β and γ be the three roots of $x^3 - 2 = 0$, where α is the real cube root of 2. Then,
  (a) $\mathbb{Q}(\alpha) = \mathbb{Q}(\beta) = \mathbb{Q}(\gamma)$ (as subfields of $\mathbb{C}$),
  (b) $\mathbb{Q}(\beta) = \mathbb{Q}(\gamma)$ but $\mathbb{Q}(\alpha)$ is not equal to either of them,
  (c) $\mathbb{Q}(\alpha)$, $\mathbb{Q}(\beta)$ and $\mathbb{Q}(\gamma)$ are not equal to one another but are all isomorphic as fields,
  (d) none of the above.

  Answer: (c) is true. Why? Also why is (b) false?

(4) Let K/F be an extension of fields, let $\alpha \in K$ be an algebraic element over F. Then, for any positive integer n,
  (a) α^n and $\alpha^{1/n}$ are algebraic over F,
  (b) α^n is algebraic over F but $\alpha^{1/n}$ may not be algebraic over F,
  (c) $\alpha^{1/n}$ is algebraic over F but α^n may not be algebraic over F,
  (d) none of the above

  Answer: (a) is the obvious answer.

(5) Let F be a field, and let $K = F[x]/(x^2)$. Then,
  (a) K is a field as well as a F vector space of dimension 2.
  (b) K is an integral domain but not a field.
  (c) K is isomorphic as a ring to a product of two copies of F (with component wise addition and multiplication).
  (d) none of the above

  Answer: (d). The first two choices are obviously false. (c) is false as there is an element of K whose square is zero (namely the coset class of x) but a product of two copies of F does not have such an element.

(6) Let t be a variable and let $F_1 = \mathbb{Q}(t)$ and $F_2 = \mathbb{Q}(t^2)$. Then the element t is
  (a) algebraic over F_1 and F_2 but not over $\mathbb{Q}$,
  (b) algebraic over F_1 but not over F_2 and $\mathbb{Q}$,

(c) algebraic over F_1, F_2 and $\mathbb{Q}$,
(d) none of the above.

Answer: (a). Clearly t is algebraic over F_1 since $t \in F_1$. Also t is not algebraic over $\mathbb{Q}$ since t is a variable. Finally t is algebraic over F_2 since it satisfies the equation $f(x) = x^2 - t^2 \in F_2[x]$.