

Errata for “Field and Galois Theory”

June 10, 2018

The following list enumerates typographical errors and other mistakes in my book, found mostly by readers of the book. Thanks to all who sent me errors they noticed.

1. Page 3, second displayed equation: a_n should be a_k after the equal sign.
2. Page 10, line -14: $(b/a)^4$ should be $(b/a)^2$.
3. Page 11, line 3 of the proof of 1.24: Corollary 1.22 should be Proposition 1.23.
4. Page 12, Example 1.27: on the last line, $\sqrt{2}$ should be $\sqrt[3]{2}$.
5. Page 13, Problem 6: the rings need to be assumed commutative and the maps should be F -homomorphisms.
6. Page 20, first line of the last displayed equation: τ_j should be τ_i .
7. Page 32, displayed equation: $\{|F|, |\mathbb{N}\}$ should be $\max\{|F|, |\mathbb{N}\}$.
8. Page 32, fourth line of the proof of Theorem 3.14. It should say that M is an algebraic closure of F (not of M).
9. Page 34, line -5: $\min(F, a)$ should be $\min(F, \alpha)$.
10. Page 36, Example 3.26, line 7: $\mathbb{Q}(x)$ should be $\mathbb{Q}[x]$.
11. Page 36, third line of the proof of Proposition 3.28: Lemma 3.17 should be Lemma 3.18.
12. Page 37, proof of (3) implies (4): Two instances of M should both be N .
13. Page 42, last line of prof of 4.6: statement 2 should be statement 1.
14. Page 42, definition 4.7: α should be assumed algebraic over F .
15. Page 44, line -4: b_i should be a_i .
16. Page 44, line -4: first m_i should be mi and $m - 1_i$ should be $(m - 1)i$.

17. Page 44, line -3: $m = 1$ should be $m = 0$.
18. Page 45, line -4 of proof of 4.16: b_i should be b_1 .
19. Page 47, line 10/11: Lemma 4.10 should be Corollary 4.10.
20. Page 48, line 3 of the proof: separable over S should be separable over F .
21. Page 50, Problem 12: The displayed equation should read

$$F \subset F(a_1) \subset F(a_1, a_2) \subset \cdots \subset F(a_1, \dots, a_n) = K.$$

22. Page 55, line -3: $\alpha \in F$ should be $\alpha \in K$.
23. Page 57, last line of Example 5.11: $\mathbb{Q}(\sqrt[3]{2}, \omega)$ is the normal closure of $\mathbb{Q}(\sqrt[3]{2})/\mathbb{Q}$, not of $\mathbb{Q}(\omega)/\mathbb{Q}$.
24. Page 59, line -5 before Problems : $E = F$ should be $E = \mathbb{R}$.
25. Page 59, line -3 before Problems: M should be P .
26. Page 59, line -2 before Problems: $|G| = 1$ should be $|G| = 2$.
27. Page 60, Problem 6: it should say: Show that there is no intermediate field L of N/F with $[L : F] = 2$.
28. Page 60, Problem 10: K/F should be assumed to be a finite Galois extension.
29. Page 62, Problem 21(b): K/F should be a finite extension but F is infinite.
30. Page 65, line -8: should be F and \mathbb{F}_p^n are isomorphic as \mathbb{F}_p -vector spaces.
31. Page 65, line -4: “orders of” should precede elements.
32. Page 69, line 1 first f should be F .
33. Page 76, line 17: σ should be σ_2 .
34. Page 80, line 10: It should say $N_{K/F}(\alpha) = (-1)^{p-1}a$.
35. Page 82, line 8: $f(a) = f_i(a)$ should be $\sigma_i(a) = a_i$.
36. Page 85, Example 8.15, line 7 and 11: $(-1)^n$ should be $(-1)^{n+1}a$.
37. Page 90, line 8: $\tau^{m'}(a)$ should be $\tau(a^{m'})$.
38. Page 90, line -4: $\wp^{-1}(a)$ should be $\wp^{-1}(b)$.
39. Page 93, line -3: $\sigma \in G$ should be $\tau \in G$.

40. Page 94, line 13: second $i - 1$ exponent should be $j - 1$.
41. Page 98, line 3 of Example 10.7: $\langle \sigma \rangle$ should be $\langle \tau \rangle$.
42. Page 107 l -4: $h \in H$ should be $g \in G$ at the end of the set description.
43. Page 107, l -3: ϕ is nondegenerate should be B is nondegenerate.
44. Page 108, end of the proof. The argument shows H is isomorphic to a subgroup of G . Symmetry shows the reverse, so $H \cong G$.
45. Page 108 line 1: $|G|$ should be $|C|$.
46. Page 109, line 19: should be $F(G) = F(\{\sqrt[n]{a} : aF^{*n} \in G\})$.
47. Page 115, line 5 of proof: a_1 should be s_1 .
48. Page 124, line -12: $4u^6$ should be u^6 ,
49. Page 119, Proposition 12.13: displayed equation makes rows dependent, not columns.
50. Page 123, Problem 8(c): n should be p , a prime.
51. Page 129, displayed equation calculating $v(')^2$: $u - a$ should be $u + a$.
52. Page 130, line -3. v' should have $\frac{i}{2}\sqrt{10 + 2\sqrt{5}}$.
53. Page 130, Example 13.5, line 3: x_3 and x_2 should be x^3 and x^2 , respectively.
54. Page 132, line -3: $f(x) = 0$ should be $r(y) = 0$.
55. Page 132, Problem 1: f should be g in line 3.
56. Page 149, line -8: The reference to Proposition 3.28 shouldn't be here; it should be in the next sentence.
57. Page 149, line -7: γ_i is an n th root, not an n th power.
58. Page 159, statement of Theorem 17.8: N should be H (twice).
59. Page 160, second displayed equation: $\sigma\tau\sigma^{-1}(a))$ should be $\sigma\tau\sigma^{-1}(b))$.
60. Page 162: The argument for why no t exists isn't clear. Instead, for $r = p^m$, the condition $n_r = t \pmod{p^m}$ becomes $-1 = t(p-1) \pmod{p^m}$, so p^m divides $t(p-1)+1$ for all m , which is impossible.
61. Page 163, Exercise 13: $\varphi_{\beta\gamma} \circ \varphi_{\alpha\beta}$ should be $\varphi_{\alpha\beta} \circ \varphi_{\beta\gamma}$.
62. Page 171, Problem 5. In the second to last line, I should be K .

63. Page 174, Example 19.2, line -4: independent should be dependent.
64. Page 174, Example 19.4, line 2: F should be K ; line 3: K should be F ; line 4: F should be K .
65. Page 175, proof of Lemma 19.7, line -5: $g(t_1, \dots, t_{m-1})$ should be $g_j(t_1, \dots, t_{m-1})$.
66. Page 175, proof of Lemma 19.7, line -4: $F(t_1, \dots, t_m)$ should be $F(t_1, \dots, t_{m-1})$.
67. Page 177, proof of Lemma 19.13, line 4: Need to state that $g_m \neq 0$ when the g_j are defined.
68. Page 177, Theorem 19.14, proof: X in the last line should be T , while all M in the proof should preferably be X .
69. Page 181, line -4: the first $x \prec S$ should be $x \prec S_0$.
70. Page 185: It is unstated that $a_i = sk_i$ and $b_i = tl_i$. The map on line -4 of the proof should go from $A \otimes_F B$ to $K \otimes -FL$.
71. Page 186: The argument for why f is an isomorphism only works in the finite dimensional case. In general one can see it by using the UMP for tensor products to produce an inverse.
72. Page 223, Problem 3: Problem 19 of appendix D does not exist.
73. Page 233, Theorem 4.4: f should be assumed non-constant.
74. Page 237, Example 5.9, last sentence: Problem 3 of Section 10 should be Problem 10 of Section 3.
75. Page 239, Exercise 18. Part (b) is wrong as stated. If A and B are UFDs and a is assumed irreducible in B , then I think it is correct (and is what is needed in the proof of Theorem 4.5).
76. Page 247, Theorem 2.3: p divides $|G|$ should be n divides $|G|$.
77. Page 247, Theorem 2.2: $P_2 = xP_2^{-1}$ should be $P_2 = xP_1x^{-1}$.
78. Page 250, line 2: $\varphi_{j,k} \circ \varphi_{i,j}$ should be $\varphi_{i,j} \circ \varphi_{j,k}$.
79. Page 250, Definition 4.1: the arrows τ and τ_i are backwards.
80. Page 263, Line 11: $v_i \otimes \widehat{u}_j$ should be $v_j \otimes \widehat{u}_i$.
81. Page 268, bottom of the page: Need to assume that X is the union of \mathcal{C} .

82. Page 269, Example 1.7, line 6, 7: R should be X and $\bigcup_i Z(S_i) = Z(\bigcap_i S_i)$ should be $\bigcap_i Z(S_i) = Z(\bigcup_i S_i)$.
83. Page 272, Problem 1. Need to assume that X is the union of \mathcal{C} .