Homework 4 (due Wednesday May 5)

The solutions must be your own. Simply copying from other students will result in no credit.

Your homework should be done in LaTeX and the R code, output, graphics etc. must be embedded using Sweave. Your score will be determined by the quality of your written discussion of the results, along with the code and output.

1. Write a program that inputs a function on $[0, 1]$ and a positive integer n, and then computes the natural cubic spline interpolant of the function with n subintervals. The program should also plot the function as well as the interpolant. Try it on the function $(1 + x^2)^{-1}$ with $n = 10$.

2. Write a program that inputs a function on $[0, 1]$ and a positive integer n, and then computes the piecewise cubic Hermite interpolant of the function with n subintervals. The program should also plot the function as well as the interpolant. Try it on the function $(1 + x^2)^{-1}$ with $n = 10$.

3. Our goal is to solve the differential equation

$$\frac{dx(t)}{dt} = f\left(t, x(t)\right), \qquad 0 \le t \le T \tag{0.1}$$

$$x(0) = x_0. \tag{0.2}$$

   Here f is continuosly differentiable with $\frac{\partial f}{\partial t}$ and $\frac{\partial f}{\partial x}$ bounded in $[0, T] \times \mathbb{R}$.

   Write a program that takes as input f, T, x_0 and n, the number of subintervals to consider. The program applies the Rectangle Rule, Explicit Midpoint rule and Implicit Midpoint Rule to find approximations to the differential equation. The program also plots all the approximate solutions in one plot.

   Try the above on $f(t, x) = t^2 \sin x$ with $T = 1$ and $x_0 = 1$.