B.Math II, Statistics-II – Assignment 1

**1.** Let $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$, $Y \sim \text{Poisson}(\mu)$, $\mu > 0$ be independent. If $X$ and $Y$ are not observable directly and only $Z = X + Y$ is observable, show that $(\lambda, \mu)$ is not identifiable.

**2.** Let $X$ have uniform distribution over $(\theta_1, \theta_2)$. Find the reparameterization with which this a location-scale distribution.

**3.** Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x|\theta) = \begin{cases} \exp(i\theta - x) & x \geq i\theta \\ 0 & x < i\theta, \end{cases} \qquad i = 1, 2, \ldots, n.$$

Prove that $T = \min_i\{X_i/i\}$ is a sufficient statistics for $\theta$.

**4.** Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x_i|\theta) = \begin{cases} \frac{1}{2i\theta} & -i(\theta - 1) < x_i < i(\theta + 1) \\ 0 & \text{otherwise,} \end{cases}$$

where $\theta > 0$. Find a two-dimensional sufficient statistic for $\theta$.

**5.** For $n > 1$, which of the following statistics are equivalent? Prove.
(a) $\sum_{i=1}^n x_i$ and $\sum_{i=1}^n \log x_i$, $x_i > 0$
(b) $(\sum_{i=1}^n x_i, \sum_{i=1}^n x_i^2)$ and $(\overline{x}, \sum_{i=1}^n (x_i - \overline{x})^2)$
(c) $(\sum_{i=1}^n x_i, \sum_{i=1}^n x_i^3)$ and $(\overline{x}, \sum_{i=1}^n (x_i - \overline{x})^3)$.