B.Math II, Statistics-II – Assignment 4

1. Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be independent random samples from $N(\xi, \sigma^2)$ and $N(\eta, \tau^2)$ respectively. Find minimal sufficient statistics and MLE for the following cases:
(a) ξ, η, σ, τ are arbitrary; $-\infty < \xi, \eta < \infty$, $0 < \sigma, \tau$.
(b) $\sigma = \tau$, but otherwise as in (a).
(c) $\xi = \eta$, but otherwise as in (a).

2. Let $X_1, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2 both are known to be nonnegative but otherwise unspecified. Find MLE of μ and σ^2.

3. Let $X_1, \ldots, X_n$ be a random sample from $N(0, \theta^2)$, $\theta > 0$. Let $\delta = \Phi(1/\theta)$, where Φ is the standard normal cdf.
(a) Find the method of moments estimators of θ and δ.
(b) Find the MLE of θ and δ.
(c) Find the UMVUE of θ and δ.

4. The three-parameter Weibull distribution is widely used in reliability study as a model for lifetimes. The density of this distribution is

$$f(x; a, b, c) = \begin{cases} (a/b)((x-c)/b)^{a-1} \exp\left(-((x-c)/b)^a\right), & \text{if } x > c, \\ 0 & \text{otherwise,} \end{cases}$$

where $-\infty < c < \infty$, $a > 0$, $b > 0$. Let $X_1, X_2, \ldots, X_n$ be a random sample from this distribution. Suppose that a and c are known.
(a) Find the information number of b.
(b) Find the Cramer-Rao lower bound for the variance of an unbiased estimator of b^{2a}.
(c) Find the UMVUE of b^{2a}. Does it attain the lower bound given in (b) above?