

INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE

B.MATH - Second Year, 2020-21

Statistics - II, Test 2, April 16, 2021

Time: 1 hour; submission must be complete by 4:30 pm

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**You may freely consult the lecture notes, but no other books or resources may be consulted. You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them.**

**1.**  $X_1, X_2, \dots, X_n$  are waiting times which are modelled as i.i.d.  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ , with density  $f_X(x|\lambda) = \lambda \exp(-\lambda x)$ ,  $x > 0$ . It is of interest to test  $H_0 : \lambda = 0.5$  versus  $H_1 : \lambda > 0.5$  at the significance level of  $\alpha = 0.01$ .

Consider the test  $\phi_1$  which rejects  $H_0$  when  $X_1 < C$  where  $C$  is the 0.01 quantile of  $\chi^2_2$ . i.e.,  $P(T < C) = 0.01$  with  $T \sim \chi^2_2$ .

(a) Is  $\phi_1$  a level  $\alpha$  test for  $H_0$  versus  $H_1$ ? Justify.

(b) Show that UMP level  $\alpha$  test for  $H_0$  versus  $H_1$  exists.

(c) Derive the UMP level  $\alpha$  test for  $H_0$  versus  $H_1$ . [3+3+8]

**2.** Let  $X|\lambda \sim \text{Poisson}(\lambda)$ ,  $\lambda > 0$ . Consider the Gamma prior on  $\lambda$  with mean 2 and variance 7.

(a) Find the posterior distribution of  $\lambda$  if  $X = x$  is observed.

(b) Find the highest posterior density estimate of  $\lambda$ . [6+5]