# QUIZ

(1) Consider the two statements: (P) $x^3 - 2$ is irreducible over $\mathbb{Q}(\sqrt{2})$, (Q) $x^2 - 2$ is irreducible over $\mathbb{Q}(\sqrt[3]{2})$

(a) Both statements are true,
(b) (P) is true and (Q) is false,
(c) (P) is false and (Q) is true,
(d) none of the above

Answer: (a). If $x^3 - 2$ is reducible over $\mathbb{Q}(\sqrt{2})$, then $\mathbb{Q}(\sqrt[3]{2}) \subset \mathbb{Q}(\sqrt{2})$ (why?), which is not possible (why?). Similarly if $x^2 - 2$ is reducible over $\mathbb{Q}(\sqrt[3]{2})$, then $\mathbb{Q}(\sqrt{2}) \subset \mathbb{Q}(\sqrt[3]{2})$, which is also not possible (why?).

(2) The number of intermediate fields $L$ such that $\mathbb{Q} \subset L \subset \mathbb{Q}(\sqrt[5]{3})$ with $L \neq \mathbb{Q}$ and $L \neq \mathbb{Q}(\sqrt[5]{3})$ is

(a) 0,
(b) 1,
(c) infinitely many,
(d) none of the above

Answer: (a) is true, since any such $L$ would imply $[\mathbb{Q}(\sqrt[5]{3}) : L][L : \mathbb{Q}] = 5$ by the product formula.

(3) The cardinality of the Galois group $G(\mathbb{Q}(\sqrt[3]{2}, i)/\mathbb{Q})$ is

(a) 1,
(b) 2,
(c) 4,
(d) none of the above.

Answer: (b) is true. The two elements of $G(\mathbb{Q}(\sqrt[3]{2}, i)/\mathbb{Q})$ are the identity and an element $\sigma$ which satisfies $\sigma(i) = -i$ and $\sigma(\sqrt[3]{2}) = \sqrt[3]{2}$ (why?).