

B.Math II – Statistics-II, Assignment 5

1. Let X_1, X_2, \dots, X_n be readings on nicotine content of a random sample of cigarettes of a certain brand. If data with $n = 9$ observations yielded a value of 1.6 mg for \bar{x} and 23.76 for $\sum_{i=1}^n x_i^2$, construct 95% confidence intervals for the population mean μ and population variance σ^2 . What assumptions did you make?

2. Suppose X_1, X_2, \dots, X_n is a random sample from $\text{Poisson}(\lambda)$. Consider testing

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.

(b) Derive the UMP test of level α .

(c) What is the test in (b) if $n = 5$ and $\alpha = 0.05$?

3. Let X_1, X_2, \dots, X_n be a random sample from $N(\mu, \sigma^2)$ and consider testing

$$H_0 : \mu \leq \mu_0 \text{ versus } H_1 : \mu > \mu_0.$$

Suppose $n = 16$, $\sigma^2 = 25$, $\mu_0 = 120$ and that data yielded a value of 123 for \bar{x} . What is the form of the UMP level α test? Conduct the test at $\alpha = 0.01$.

4. Suppose X_1, X_2, \dots, X_m and Y_1, Y_2, \dots, Y_n are independent random samples, respectively, from $N(\mu_1, \sigma^2)$ and $N(\mu_2, \tau^2)$. Let $\Delta = \sigma^2/\tau^2$.

(a) Find the generalized likelihood ratio test for testing $H_0 : \Delta = \Delta_0$ versus $H_1 : \Delta \neq \Delta_0$.

(b) Find a $100(1 - \alpha)\%$ confidence set for Δ .

(c) What is the confidence set in (b) if $m = 10$, $n = 12$ and $\alpha = 0.05$?

5. Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be a sample from a bivariate normal distribution with zero means, variances σ_1^2 and σ_2^2 and correlation ρ . Consider the problem of testing $H_0 : \rho = 0$ versus $H_1 : \rho \neq 0$.

(a) Show that the generalized likelihood ratio statistic is equivalent to $|r|$ where

$$r = \frac{\sum_{i=1}^n X_i Y_i}{\sqrt{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i^2}}.$$

(b) Show that r^2 has the Beta distribution with parameters $1/2$ and $(n - 1)/2$ when $\rho = 0$.

(Hint: Show that the conditional distribution of r^2 given $X_1 = x_1, \dots, X_n = x_n$ is the prescribed distribution by making a suitable orthogonal transformation of Y_1, \dots, Y_n .)