Graph Theory

B. Math. II

Semestral Examination

Instructions: All questions carry ten marks. All graphs are assumed to be simple. Results proved in the course can be used without proof.

1. Prove that a graph G with n vertices and $e \leq n-1$ edges has $n-e$ connected components if and only if it contains no cycle.

2. Prove that every simple planar graph with at least four vertices have at least four vertices of degree less than 6.

3. Let $n \geq 3$ be a natural number and let S be a subset of n points in the plane such that the distance between any two distinct points of S is at least one. Then, prove that there are at most $3n-6$ pairs u, v in S such that $d(u,v) = 1$.

4. Which of the following can be the characteristic polynomial of a tree? Justify your answer.

   (a) $X^9 - 8X^7$

   (b) $X^{10} - 9X^7 + 19X^5$

   (c) $X^{13} - 27X^{11} + 14X^5 + X$