

## B.Math II – Statistics-II, Assignment 5

**1.** Let  $X_1, X_2, \dots, X_n$  be readings on nicotine content of a random sample of cigarettes of a certain brand. If data with  $n = 9$  observations yielded a value of 1.6 mg for  $\bar{x}$  and 23.76 for  $\sum_{i=1}^n x_i^2$ , construct 95% confidence intervals for the population mean  $\mu$  and population variance  $\sigma^2$ . What assumptions did you make?

**2.** Suppose  $X_1, X_2, \dots, X_n$  is a random sample from  $\text{Poisson}(\lambda)$ . Consider testing

$$H_0 : \lambda \leq 1 \text{ versus } H_1 : \lambda > 1.$$

(a) Show that the conditions required for the existence of a UMP test are satisfied here.

(b) Derive the UMP test of level  $\alpha$ .

(c) What is the test in (b) if  $n = 5$  and  $\alpha = 0.05$ ?

**3.** Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(\mu, \sigma^2)$  and consider testing

$$H_0 : \mu \leq \mu_0 \text{ versus } H_1 : \mu > \mu_0.$$

Suppose  $n = 16$ ,  $\sigma^2 = 25$ ,  $\mu_0 = 120$  and that data yielded a value of 123 for  $\bar{x}$ . What is the form of the UMP level  $\alpha$  test? Conduct the test at  $\alpha = 0.01$ .

**4.** Suppose  $X_1, X_2, \dots, X_m$  and  $Y_1, Y_2, \dots, Y_n$  are independent random samples, respectively, from  $N(\mu_1, \sigma^2)$  and  $N(\mu_2, \tau^2)$ . Let  $\Delta = \sigma^2/\tau^2$ .

(a) Find the generalized likelihood ratio test for testing  $H_0 : \Delta = \Delta_0$  versus  $H_1 : \Delta \neq \Delta_0$ .

(b) Find a  $100(1 - \alpha)\%$  confidence set for  $\Delta$ .

(c) What is the confidence set in (b) if  $m = 10$ ,  $n = 12$  and  $\alpha = 0.05$ ?

**5.** Let  $(X_1, Y_1), \dots, (X_n, Y_n)$  be a sample from a bivariate normal distribution with zero means, variances  $\sigma_1^2$  and  $\sigma_2^2$  and correlation  $\rho$ . Consider the problem of testing  $H_0 : \rho = 0$  versus  $H_1 : \rho \neq 0$ .

(a) Show that the generalized likelihood ratio statistic is equivalent to  $|r|$  where

$$r = \frac{\sum_{i=1}^n X_i Y_i}{\sqrt{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i^2}}.$$

(b) Show that  $r^2$  has the Beta distribution with parameters  $1/2$  and  $(n - 1)/2$  when  $\rho = 0$ .

(Hint: Show that the conditional distribution of  $r^2$  given  $X_1 = x_1, \dots, X_n = x_n$  is the prescribed distribution by making a suitable orthogonal transformation of  $Y_1, \dots, Y_n$ .)