QUIZ

(1) Consider the two statements: (P) K/F is a finite Galois extension, (Q) K is a splitting field of a separable polynomial $f(x) \in F[x]$ over F. Then,
  (a) (P) is equivalent to (Q),
  (b) (P) implies (Q) but not conversely,
  (c) (Q) implies (P) but not conversely,
  (d) none of the above.

  Answer: (a), consider the product of finitely many separable polynomials to get one separable polynomial.

(2) The degree of the splitting field of $x^8 + x^4 + 1$ over the finite field $\mathbb{F}_2$ is
  (a) 2,
  (b) 4,
  (c) 8,
  (d) 16.

  Answer: (a) is true. If $f(x) = x^8 + x^4 + 1$, then $f(x) = (x^2 + x + 1)^4$ (since we are in char 2). Now $x^2 + x + 1$ is irreducible over $\mathbb{F}_2$, hence the splitting field of $x^8 + x^4 + 1$ is the same as the splitting field of $x^2 + x + 1$, which is a degree 2 extension of $\mathbb{F}_2$ (a field with 4 elements).

(3) The Galois group of the splitting field of $x^3 - 2$ over $\mathbb{Q}$ is
  (a) $\mathbb{Z}/2\mathbb{Z}$,
  (b) $\mathbb{Z}/3\mathbb{Z}$
  (c) $\mathbb{Z}/6\mathbb{Z}$
  (d) S_3.

  Answer: (d) is true, the splitting field is $\mathbb{Q}(\sqrt[3]{2}, \omega)$ where ω is a nontrivial cube root of unity. Hence the Galois group has order 6 (why?), and it is non-abelian (why?), thus it must be isomorphic to S_3.