Graph Theory

Assignment 3

Matchings

Instructions: All questions carry equal marks. All graphs are assumed to be simple.

1. For all $n \geq 10$, give an example of a graph G without any 1-factor but for which Hall's condition holds.

2. Let G be a bipartite graph with partition X and Y. Assume that G has a matching of the size $|X|$. Prove that there is a vertex $x \in X$ such that every edge incident with x can be extended to a matching of maximum size.

3. Let G be a connected graph. Two players play a game on G by alternately choosing vertices such that each choice is adjacent to the previously chosen vertex with the condition that no vertex can be chosen twice. The last player able to choose wins.

   Prove that the person who chooses second in this game has a winning strategy (no matter what the first choice was made) if and only if G has a perfect matching.

4. Let G be a bipartite graph with partitions of equal size, say n. If $|E(G)| > (k-1)n$, then prove that G has a matching of size k.