QUIZ

(1) Consider the finite Galois extension $\mathbb{Q}(\sqrt{5}, i)/\mathbb{Q}$ with Galois group G. Consider the elements $\sigma, \tau \in G$ defined by $\sigma(\sqrt{5}) = \sqrt{5}$, $\sigma(i) = -i$, and $\tau(\sqrt{5}) = -\sqrt{5}$, $\tau(i) = i$. Then the fixed field corresponding to the subgroup of G generated by $\sigma \circ \tau$ is

(a) $\mathbb{Q}(\sqrt{5})$.
(b) $\mathbb{Q}(i)$.
(c) $\mathbb{Q}(\sqrt{-5})$.
(d) none of the above.

Answer: (c)

(2) Writing the finite separable extension $\mathbb{R}(x, y)/\mathbb{R}(x^2, y^2)$ (where x, y are variables) as a simple extension $\mathbb{R}(x, y) = \mathbb{R}(x^2, y^2)(\alpha)$, a choice for α can be

(a) x
(b) y
(c) xy
(d) $x + y$

Answer: (d)

(3) Let K be the splitting field over $\mathbb{Q}$ of $f(x) = x^4 - 2$, and let $G = Gal(K/\mathbb{Q})$. Let $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots $\sqrt[4]{2}, -\sqrt[4]{2}, i\sqrt[4]{2}, -i\sqrt[4]{2}$ respectively of $f(x)$. Consider the following maps from K to K (fixing F),

(a) σ_1 such that $\sigma_1(\alpha_1) = \alpha_3$ and $\sigma_1(\alpha_2) = \alpha_1$
(b) σ_2 such that $\sigma_2(\alpha_1) = \alpha_3$ and $\sigma_2(\alpha_2) = \alpha_4$
(c) σ_3 such that $\sigma_3(\alpha_1) = \alpha_3$ and $\sigma_3(\alpha_3) = \alpha_2$
(d) σ_4 such that $\sigma_4(\alpha_1) = \alpha_3$ and $\sigma_4(\alpha_3) = \alpha_1$

How many of them are NOT elements of G?

(a) None
(b) One
(c) Two
(d) Three

Answer: (b)