$X \sim N(\mu, \sigma^2)$. $I(\mu, \sigma^2) = ((I_{ij}(\mu, \sigma^2)))$, where

$$
\begin{aligned}
I_{11}(\mu, \sigma^2) &= E_{\mu,\sigma^2}\left[\frac{\partial}{\partial \mu} \log f(X|\mu, \sigma^2)\right]^2 \\
I_{22}(\mu, \sigma^2) &= E_{\mu,\sigma^2}\left[\frac{\partial}{\partial \sigma^2} \log f(X|\mu, \sigma^2)\right]^2 \\
I_{12}(\mu, \sigma^2) &= E_{\mu,\sigma^2}\left[\frac{\partial}{\partial \mu} \log f(X|\mu, \sigma^2) \frac{\partial}{\partial \sigma^2} \log f(X|\mu, \sigma^2)\right].
\end{aligned}
$$

$$
\begin{aligned}
\log f(x|\mu, \sigma^2) &= -\frac{1}{2} \log \sigma^2 - \frac{1}{2\sigma^2}(x-\mu)^2, \\
\frac{\partial}{\partial \mu} \log f(x|\mu, \sigma^2) &= -\frac{1}{2\sigma^2} 2(x-\mu)(-1), \\
\frac{\partial}{\partial \sigma^2} \log f(x|\mu, \sigma^2) &= -\frac{1}{2}\frac{1}{\sigma^2} + \frac{1}{2\sigma^4}(x-\mu)^2.
\end{aligned}
$$

$$
\begin{aligned}
I_{11}(\mu,\sigma^2) &= E_{\mu,\sigma^2}\left[\frac{(X-\mu)^2}{\sigma^4}\right] = \frac{1}{\sigma^2}, \\
I_{22}(\mu,\sigma^2) &= \frac{1}{4\sigma^8}E_{\mu,\sigma^2}\left[(X-\mu)^2-\sigma^2\right]^2 = \frac{2\sigma^4}{4\sigma^8} = \frac{1}{2\sigma^4}. \\
I_{12}(\mu,\sigma^2) &= \frac{1}{2}E_{\mu,\sigma^2}\left[\left(\frac{X-\mu}{\sigma^2}\right)\left(\frac{(X-\mu)^2-\sigma^2}{\sigma^4}\right)\right] = 0.
\end{aligned}
$$

$$
I(\mu,\sigma^2) = \begin{pmatrix} \frac{1}{\sigma^2} & 0 \\ 0 & \frac{1}{2\sigma^4} \end{pmatrix}.
$$

$$
|I(\mu,\sigma^2)| \propto (\sigma^2)^{-3}.
$$

Jeffreys (formal) prior is

$$\pi(\mu, \sigma^2)\, d\mu\, d\sigma^2 = |I(\mu, \sigma^2)|^{1/2}\, d\mu\, d\sigma^2 \propto (\sigma^2)^{-3/2}\, d\mu\, d\sigma^2.$$

Why is this not the same as the left invariant Haar measure on the affine group, which is

$$\sigma^{-2}?$$

Since $d\sigma^2 = 2\sigma\, d\sigma$,

$$\pi(\mu, \sigma)\, d\mu\, d\sigma \propto (\sigma^2)^{-3/2}\, d\mu\, \sigma\, d\sigma = \sigma^{-2}\, d\mu\, d\sigma.$$

$$\frac{P^{\pi}(\Theta_0|x)}{P^{\pi}(\Theta_1|x)} = \frac{P^{\pi}(\Theta_0|x)}{1 - P^{\pi}(\Theta_0|x)} = \frac{\pi_0}{1-\pi_0} \times \ \mathrm{BF}_{01}(x).$$

If $\Theta_0 = \{\theta_0\}$, then

$$\begin{aligned}
\frac{\pi(\theta_0|x)}{1-\pi(\theta_0|x)} &= \frac{\pi_0}{1-\pi_0} \times \ \mathrm{BF}_{01}(x) \\
\frac{1-\pi(\theta_0|x)}{\pi(\theta_0|x)} &= \frac{1-\pi_0}{\pi_0} \times \ \mathrm{BF}_{01}^{-1}(x) \\
\frac{1}{\pi(\theta_0|x)} - 1 &= \frac{1-\pi_0}{\pi_0} \times \ \mathrm{BF}_{01}^{-1}(x) \\
\frac{1}{\pi(\theta_0|x)} &= 1 + \frac{1-\pi_0}{\pi_0} \times \ \mathrm{BF}_{01}^{-1}(x)
\end{aligned}$$