Homework 2 (due Wednesday March 20)

The solutions must be your own. Simply copying from other students will result in no credit.

Your homework should be done in LaTeX and the R code, output, graphics etc. must be embedded using Sweave. Your score will be determined by the quality of your written discussion of the results, along with the code and output.

1. Consider the Hilbert matrix H_n of order n whose elements are

$$h_{ij} = \frac{1}{i+j-1}, \quad i, j = 1, 2, \cdots, n.$$

   Consider the vector $\mathbf{b}$ with elements $b_i = \sum_{j=1}^{n} \left(j/(i+j-1) \right),\ i = 1, 2, \cdots, n$. Let $\mathbf{x}$ be the vector with elements $x_i = i,\ i = 1, 2, \cdots, n$.

   (a) Check that $H_n \mathbf{x} = \mathbf{b}$.

   (b) For $n = 1, 2, \cdots, 20$, write a program that solves $H_n \mathbf{z} = \mathbf{b}$. Use the Gaussian elimination method as done in class (including swapping entries in the index vector instead of actually swapping rows in the matrix).

   (c) Due to machine errors,the output of your program is $\mathbf{x} + \Delta\mathbf{x}$ for some $\Delta\mathbf{x}$. Let $\Delta\mathbf{b}$ be such that $H_n(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{b} + \Delta\mathbf{b}$. Create a table with the values of $\|\Delta\mathbf{b}\|_2 / \|\mathbf{b}\|_2$ and $\|\Delta\mathbf{x}\|_2 / \|\mathbf{x}\|_2$ for $n = 1, 2 \cdots, 20$. Use this to give lower bounds on the condition number of H_n, $n = 1, 2, \cdots, 20$.

2. Write a program to solve two nonlinear equations of two real variables by Newton's method. Note that in additon to subroutines that computes the two functions of the two variables, you also need subroutines to compute the two partial derivatives of each of the two functions.

   (a) Find all solutions of the system of Example 4.1 in the book by Süli and Mayers (which is in Moodle) using Newton's method. (For each of them you need to provide a suitable initial guess.) Comment on the rate of convergence of the iterations.

   (b) Rewrite the equations as in Example 4.4 and find the solution in the first quadrant by a simulatneous iteration. How does the rate of convergence compare with that of Newton's method?