TEST - II

Answer all questions. All claims must be accompanied by proper justifications. Write your names and Roll Numbers on the answer sheet. Upload your answers latest by 12:15 PM.

(1) Define the notion of a quotient map. Give an example of a quotient map that is neither an open map nor a closed map. [4]

(2) For which spaces is the whole space the only dense subset of itself. [4]

(3) Let X be a space. Assume that for each $x \in X$ there exists a continuous function $f : X \longrightarrow \mathbb{R}$ such that

$$f^{-1}(0) = \{x\}.$$

Is X Hausdorff? [4]

(4) Show that

$$d(x, y) = \frac{|x - y|}{1 + |x - y|}$$

for $x, y \in \mathbb{R}$ defines a metric on $\mathbb{R}$. In the topology induced by the metric d, is every closed and bounded subset of $\mathbb{R}$ compact? [9]

(5) Let X, Y be two spaces and K, L be non-empty compact subsets of X and Y respectively. Let $W \subseteq X \times Y$ be an open subset of $X \times Y$ such that $K \times L \subseteq W$. Show that there exist open sets $U \subseteq X$ and $V \subseteq Y$ such that $K \subseteq U$, $L \subseteq V$ and

$$K \times L \subseteq U \times V \subseteq W.$$

The topology on $X \times Y$ is the product topology. [9]