Graph Theory

Assignment 2

Connectivity

Instructions: All questions carry equal marks. All graphs are assumed to be simple.

1. For every $0 < k \leq m$, construct a graph G whose connectivity is k and $\delta(G) = m$.

2. Construct a smallest graph (i.e. with least number of edges) on 6 vertices having connectivity 3, and a pair of vertices $x \neq y$ with *four* internally disjoint $x - y$ paths. Justify your answer.

3. A graph is said to be *minimally k-connected* is $G \setminus e$ is not k-connected for any edge e. Prove that a minimally 2-connected graph must have a vertex of degree 2.

4. Prove that if G is 2-connected, then $G \setminus xy$ is also 2-connected if and only if x and y lie on a cycle in $G \setminus xy$.