QUIZ

(1) The field extension $k(t^{1/p})/k(t)$ (where k is a field of characteristic $p > 0$ and t is a variable)
  (a) is both normal and Galois,
  (b) is neither normal nor Galois,
  (c) is Galois but not normal,
  (d) is normal but not Galois.

  Answer: (d), it is a splitting field of $x^p - t$ over $k(t^{1/p})$ but it is not Galois as the Galois group is the trivial group but the extension is of degree p.

(2) The polynomial $x^7 - 2$ is
  (a) irreducible over both $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt[3]{2})$,
  (b) reducible over both $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt[3]{2})$,
  (c) irreducible over $\mathbb{Q}(\sqrt{2})$ and reducible over $\mathbb{Q}(\sqrt[3]{2})$,
  (d) reducible over $\mathbb{Q}(\sqrt{2})$ and irreducible over $\mathbb{Q}(\sqrt[3]{2})$.

  Answer: (a) is true (see Problem 6, page 37, Morandi).

(3) The cardinality of the splitting field of $x^p - x - 1$ over $\mathbb{F}_p$ is
  (a) p,
  (b) p^2,
  (c) p^p,
  (d) none of the above.

  Answer: (c) is true, $x^p - x - 1$ is irreducible over $\mathbb{F}_p$ (see page 38 of Morandi, problems 10 and 11), and if one attaches one root of this equation to $\mathbb{F}_p$ then all roots belong to that extension.