

B.Math II, Statistics-II – Assignment 2

- 1.** Let X_1, \dots, X_n be a random sample from the distribution with density $f(x|\theta) = \exp(-(x - \theta)^2/2)/\sqrt{2\pi}$, $-\infty < x < \infty$, $-\infty < \theta < \infty$. Find a minimal sufficient statistic for θ .
- 2.** A random variable X has the $P(c, k)$ distribution if its cdf is $1 - (k/x)^c$, $x > k > 0$, $c > 0$.
 - (a) Show that the distributions $\{P(c, 1), c > 0\}$ constitute a one-parameter exponential family.
 - (b) If you have a random sample from this family can you find a minimal sufficient statistic for c ?
- 3.** Show that each of the following families is an exponential family.
 - (a) $X|X > 0$, where $X \sim \text{Binomial}(n, p)$, $0 < p < 1$, n known.
 - (b) Gamma family with either one of the parameters is known, or both unknown.
 - (c) Beta family with either one of the parameters is known, or both unknown.
- 4.** Let N be a positive random variable with $P(N = n) = p_n$, such that $\sum_{n=1}^{\infty} p_n = 1$. Having observed $N = n$, perform n Bernoulli(θ) trials, getting X successes. Prove that (N, X) is minimal sufficient but N is ancillary.