Numerical Methods

- Using R
- Download R & R Studio

40% Final, 60% Assignments, quizzes

Will consider: Solving "on computer"

- $f(x) = 0$ $\qquad f: \mathbb{R} \to \mathbb{R}$ some fn
- $\underset{n\times n}{A}\ \overset{n\times 1}{x} = b_{n\times 1}$
- $\displaystyle\int_a^b f(x)\,dx$ $\qquad f: \mathbb{R} \to \mathbb{R}$ bounded continuous
- $y: [0,T] \to \mathbb{R}$ $\qquad f: \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$

$$\frac{dy}{dt} = f(t,y)$$

- often not able to find exact solns
- Need approximations for practical purposes
- How good is approximation?

  What is good enough?

Efficiency. How much work to reach soln.

Accuracy: How close to true value

Precision: Level of detail in our solution.

$$\pi = 3.14159265 35 \ldots$$

$$\frac{22}{7} = 3.14285714 \ldots$$

3.1416 is more accurate approximation to π

Computer softwares work with floating point numbers

Floating point numbers in R

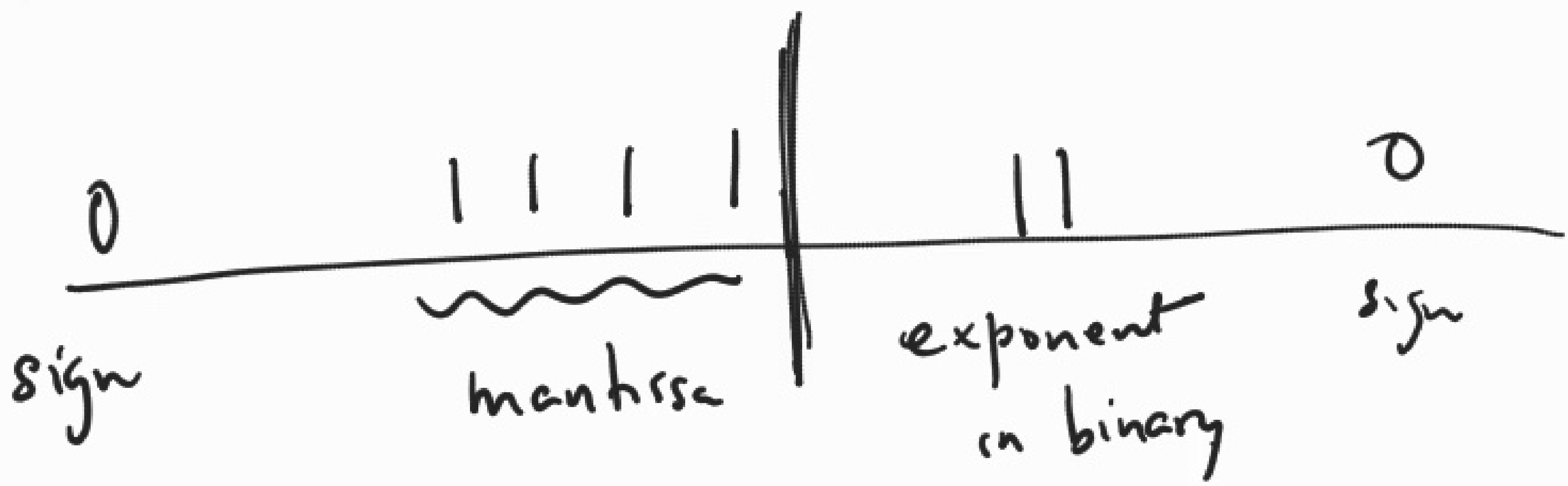
64 bits

1 bit	52 bits	1bit	10 bits
sign	mantissa	sign	exponent

$$4 = \underbrace{1}_{\text{mantissa}} \times 2^{\overbrace{2}^{\text{exponent}}}$$

$$15 = 1111 = 1\times2^3 + 1\times2^2 + 1\times2 + 1\times2^0$$

Sign 0 is +1   1 is -1



Binary 1.111×2^3

Largest number that R can work with = ?

Max Exponent = $\underbrace{111\cdots\cdots1}_{10} = 2^{10}-1$
$= 1023$

Max Mantissa = $\underbrace{1}_{1}.\underbrace{11\cdots\cdots1}_{51}$ = $1 + 1\cdot\frac{1}{2} + 1\cdot\frac{1}{4} + \cdots + 1\cdot\frac{1}{2^{51}}$

Binary   Decimal

$\left(1+\frac{1}{2}+\cdots+\frac{1}{2^{51}}\right) \times 2^{1023}$ ← Max # R can handle

$> 2 \times 2^{1023}$ NA

$> 1.5 \times 2^{1023}$ ✓

Smallest positive number R can handle : $\frac{1}{2^{51}} \times 2^{-1023}$

$> 20.55 - 19.2 - 1.35$

$1.332268\ e^{-15}$

$> 20.55 - 1.35 - 19.2$

0