INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Second Year, 2020-21
Statistics - II, Test 1, March 10, 2021
Time: 45 minutes; submission must be complete by 4:15 pm
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You may freely consult the lecture notes, but no other books or resources may be consulted. You may use any of the results stated and discussed in the lecture notes, by stating them explicitly. Results from the assignments may not be used without establishing them.

1. Suppose $X \sim \text{Poisson}(\lambda)$, $Y \sim \text{Poisson}(2\lambda)$ and they are independently distributed where $\lambda > 0$ is unknown.
(a) Find the minimal sufficient statistic $T = T(X, Y)$ for λ.
(b) Find the conditional distribution of (X, Y) given T. [10]

2. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples, respectively, from $N(\mu, \sigma^2)$ and $N(2\mu, \sigma^2)$, where $-\infty < \mu < \infty$, $\sigma^2 > 0$.
(a) Does this model belong to the exponential family of distributions? Justify.
(b) Find the UMVUE of μ. [15]