Graph Theory

Assignment 1

Instructions: All questions carry equal marks. All graphs are assumed to be simple.

1. Let G be a graph whose vertex set is the set of k-tuples with either 0 or 1 as entries. Two such tuples are adjacent if and only if they differ in exactly one co-ordinate. Determine whether this graph is connected, Bipartite, Eulerian, Hamiltonian.

2. The *girth* of a graph with a cycle is the length of its shortest cycle. (A graph with no cycle is said to have infinite girth). Let G be a graph with girth 5 and minimum degree k. Prove that G has at least $k^2 + 1$ vertices.

3. For each $k > 1$, prove that a $2k+1$-regular graph with at least one cut-edge must contain at least $4k + 6$ vertices. Can equality be attained? Justify your answer.

4. For $n \geq 3$, prove that the smallest number of edges in a connected graph whose every edge belongs to a triangle is $\frac{3(n-1)}{2}$. Show that equality is achieved by constructing an example.