

## QUIZ

- (1) Let  $K$  and  $L$  be algebraic field extensions of  $F$  such that both are contained in a common field  $E$ . Then the following is true
  - (a) if  $K/F$  and  $L/F$  are both separable then so is  $KL/F$ , but not conversely.
  - (b) if  $KL$  is separable over  $F$ , then so are  $K/F$  and  $L/F$ , but not conversely.
  - (c)  $K/F$  and  $L/F$  are separable, if and only if,  $KL/F$  is separable.
  - (d) none of the above.

Answer: (c)
- (2) Let  $K$  and  $L$  be algebraic field extensions of  $F$  such that both are contained in a common field  $E$ . Then the following is true
  - (a) if  $K/F$  and  $L/F$  are both normal then so is  $KL/F$ , but not conversely.
  - (b) if  $KL$  is normal over  $F$ , then so are  $K/F$  and  $L/F$ , but not conversely.
  - (c)  $K/F$  and  $L/F$  are normal, if and only if,  $KL/F$  is normal.
  - (d) none of the above.

Answer: (a)
- (3) Let  $K$  and  $L$  be algebraic field extensions of  $F$  such that both are contained in a common field  $E$ . Then the following is true
  - (a) if  $K/F$  and  $L/F$  are both purely inseparable then so is  $KL/F$ , but not conversely.
  - (b) if  $KL$  is purely inseparable over  $F$ , then so are  $K/F$  and  $L/F$ , but not conversely.
  - (c)  $K/F$  and  $L/F$  are purely inseparable, if and only if,  $KL/F$  is purely inseparable.
  - (d) none of the above.

Answer: (c)
- (4) Let  $K$  and  $L$  be algebraic field extensions of  $F$  such that both are contained in a common field  $E$ . Then the following is true
  - (a) if  $K/F$  and  $L/F$  are both Galois then so is  $KL/F$ , but not conversely.
  - (b) if  $KL$  is Galois over  $F$ , then so are  $K/F$  and  $L/F$ , but not conversely.
  - (c)  $K/F$  and  $L/F$  are Galois, if and only if,  $KL/F$  is Galois.
  - (d) none of the above.

Answer: (a)
- (5) Let  $p$  be a prime number and consider the finite extension  $\mathbb{Q}(\omega)/\mathbb{Q}$  where  $\omega = e^{2i\pi/p}$ . Then,
  - (a)  $\mathbb{Q}(\omega)/\mathbb{Q}$  is a Galois extension of order  $p$  with cyclic Galois group,
  - (b)  $\mathbb{Q}(\omega)/\mathbb{Q}$  is a Galois extension of order  $p - 1$  with cyclic Galois group,
  - (c)  $\mathbb{Q}(\omega)/\mathbb{Q}$  is an extension of order  $p$  which is not Galois with trivial Galois group
  - (d)  $\mathbb{Q}(\omega)/\mathbb{Q}$  is a Galois extension with abelian but not cyclic Galois group

Answer: (b)
- (6) Let  $t$  be a variable, and consider the two field extensions  $\mathbb{F}_2(t)/\mathbb{F}_2(t^2 + t)$  and  $\mathbb{R}(t)/\mathbb{R}(t^4)$ . Then,

- (a) both extensions are Galois.
- (b) neither is Galois.
- (c) the first is not Galois but the second is Galois.
- (d) the first is Galois but the second is not Galois.

Answer: (d)