QUIZ

(1) Let R be a principal ideal domain. Then
   (a) an irreducible element of R is a prime element of R but not conversely
   (b) a prime element of R is an irreducible element of R but not conversely
   (c) irreducible elements and prime elements of R are one and the same
   (d) none of the above
   Answer: (c), this is true for all UFD's (and PID's are UFD's).

(2) Which of the following statements is true?
   (a) Any two elements of a unique factorization domain has a gcd
   (b) Every commutative ring R with 1 has a prime ideal which is not maximal
   (c) The ring $\mathbb{Z}[x, y]$ is not a unique factorization domain.
   (d) Any prime ideal of a PID is generated by an irreducible polynomial.
   Answer: (a) is true, the other 3 statements are false. Why are they false?

(3) Let $I = (x^3, x - 1)$ and $J = (x^2, x + 7)$ be two ideals of $\mathbb{Z}[x]$. Then
   (a) I and J are both prime ideals which are not maximal,
   (b) I is maximal but J is a prime which is not maximal,
   (c) I is maximal but J is not a prime ideal,
   (d) None of the above
   Answer: (d). Actually $I = \mathbb{Z}[x]$, and $\mathbb{Z}[x]/J = \mathbb{Z}/49\mathbb{Z}$. Can you prove them?