B.Math II, Statistics-II – Assignment 3

1. A random variable X has the $P(c,k)$ distribution if its cdf if $1-(k/x)^c$, $x > k > 0$, $c > 0$.
If you have a random sample from $\{P(c,1), c > 0\}$, can you find a minimal sufficient statistic for c which is complete also?

2. Let $X =$ the tensile strength at $-200°$F of a type of steel that exhibits "cold brittleness" at low temperatures. Suppose that X has the Weibull distribution with density

$$f(x|a,b) = ab^{-a}x^{a-1}\exp(-(x/b)^a), \qquad x > 0, a > 0, b > 0.$$

Five different specimens of this type of steel are tested and the observed ultimate tensile strengths are 15, 20, 10, 16 and 18. Find the maximum likelihood estimates of a and b.

3. Let $X_1, \ldots, X_n$ be a random sample from $U[\theta, \theta+1]$, $-\infty < \theta < \infty$. Show that the two dimensional minimal sufficient statistics is not complete.

4. Let $X_1, \ldots, X_n$ be independent random variables with density

$$f(x|\theta) = \begin{cases} \theta x^{-2} & x \geq \theta \\ 0 & x < \theta, \end{cases}$$

where $\theta > 0$.
(a) What is sufficient for θ?
(b) Find MLE of θ.