

Graph Theory

Assignment 4

Planar Graphs

Instructions: All questions carry equal marks. All graphs are assumed to be simple.

1. Give an example of a strictly increasing sequence a_1, \dots, a_n, \dots of natural numbers such that for all i , there exists a connected 5-regular planar graph on a_i vertices.
2. Let G be planar graph on n vertices with girth k . Prove that G has at most $\frac{k(n-2)}{(k-2)}$ edges. (Note that this proves that the Petersen graph is non-planar!)
3. (An example of planar graph with maximal edges) Let G be a graph on vertices v_1, v_2, \dots, v_n such that $v_i v_j$ is an edge iff $0 < |i - j| \leq 3$. Prove that G is planar having $3n - 6$ edges.
4. Let G be a planar graph with at least 11 vertices, prove that \bar{G} can not be planar. (Actually, the same statement is true even for a graph with nine vertices, but much harder to prove.) Find a planar graph on 8 vertices whose complement is also planar.