

B.Math II – Statistics-II – Assignment 6

1. A large shipment of parts is received, out of which 5 are tested for defects. The number of defective parts in the sample, X is assumed to be Binomial with parameter θ . From past shipments it is known that θ has a Beta(1, 9) distribution.

- (a) Find the HPD estimate of θ if $x = 0$ is observed.
- (b) Find a 95% credible set for θ if $x = 0$ is observed.
- (c) Interpret the results of both (a) and (b).
- (d) For testing $H_0 : \theta \leq 0.10$ versus $H_1 : \theta > 0.10$, find the posterior odds ratio.

2. Let X_1, \dots, X_n be a random sample from $U(0, \theta)$. Let

$$\pi(\theta) \propto \theta^{-(a+1)}, \quad \theta > \theta_0,$$

where $a > 0$ and $\theta_0 > 0$ are given. Find the HPD estimate of θ and the $100(1-\alpha)\%$ HPD credible set for θ . Also find the posterior mean and the posterior variance of θ .

3. Let X_1, X_2, \dots be i.i.d. observations from some population with mean μ and finite variance $\sigma^2 > 0$. Let $Y_i = X_{2i-1}$, $i = 1, 2, \dots$ and $Z_i = X_{2i}$, $i = 1, 2, \dots$. For $n \geq 2$, define

$$T_n^{(1)} = \frac{1}{n} \sum_{i=1}^n X_i, \quad T_n^{(2)} = \frac{1}{[m]} \sum_{i=1}^{[m]} Y_i, \quad T_n^{(3)} = \frac{1}{[m]} \sum_{i=1}^{[m]} Z_i,$$

where $[m]$ is the integer part of $n/2$.

- (a) Show that $T_n^{(j)}$, $j = 1, 2, 3$ are all consistent estimators of μ .
- (b) Show that these are asymptotically normal.
- (c) Find the relative efficiencies of these estimators.

4. Let $X_{ij} = \mu_i + \epsilon_{ij}$, $j = 1, 2$; $i = 1, 2, \dots$, where $\epsilon_{ij} \sim N(0, \sigma^2)$ are i.i.d. Here σ^2 and μ_1, μ_2, \dots are unknown.

- (a) Find MLE of $(\sigma^2, \mu_1, \mu_2, \dots)$.
- (b) Show that the MLE of σ^2 is not consistent.
- (c) Find a consistent estimator of σ^2 .