1. Suppose (X, Y) is bivariate normal with $E(X) = 0 = E(Y)$, $Var(X) = \sigma^2 = Var(Y)$ and a correlation coefficient of $\rho = 0.4$ between X and Y.
(a) Find the distribution of

$$Q = X^2 + \frac{(Y - \rho X)^2}{1 - \rho^2}.$$

(b) Find $E(Q)$. [10]

2. Suppose $Y \sim N_n(0, I_n)$, A_i, $i = 1, \ldots, p$ are symmetric $n \times n$ matrices of rank k_i, and $A = \sum_{i=1}^{p} A_i$ is symmteric with rank k. Then
(i) $Y'A_iY \sim \chi^2_{k_i}$, (ii) $Y'A_iY$ are pairwise independent, and (iii) $Y'AY \sim \chi^2_k$ iff any two of
(a) A_i are idempotent for all i, (b) $A_iA_j = 0$, $i \neq j$, (c) A is idempotent, are true.