Assignment 3

Combinatorial Geometries.

1. Let $(X, \mathcal{L})$ be a linear space formed by points and lines of a finite <u>modular</u> geometry of rank 3. Prove that $(X, \mathcal{B})$ consists of one line of size $|X|-1$ & all other lines of size 2 OR $(X, \mathcal{L})$ is a projective plane.

2. Let $(X, \mathcal{F}_S)$ be a geometry & $F \in \mathcal{F}_S$. Prove that $\exists\ E \in \mathcal{F}_S$ such that $E \cap F = \Phi$, $E \vee F = X$ and $\text{rank}(E) + \text{rank}(F) = \text{rank}(X)$

(generalization of complement of a subspace in a v.space!)

3. Write down the proof of the "exchange lemma".