Assignment 4

Physics III: Electricity and Magnetism
B. Math. Year 3,
September - December 2021.

Due on: December 20^{th}, 2021.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Please feel free to discuss amongst yourselves; however, copying the assignment solutions from someone else is strictly prohibited and both persons involved will be penalized. Each one of you must submit your own answers. Total: 60 points.

1. (a) i. Consider a region in space which is bounded by two infinite conducting half-planes (like the back-rest and the sitting surface of a bench in the park, which intersect at a straight line, except both surfaces are semi-infinitely extended). Each of these planes are grounded. A charge $+Q$ is placed at the same perpendicular distance d from each plane. Find the electric potential and the electric field in any region not shielded from the charge. [5]

   ii. Find the total interaction potential energy of the system. [5]

   (b) Find the capacitance per unit length of a infinitely long cylinder of radius a situated a distance d from a infinite conducting plane ($a < d$, and d is measured between the plane and the cylindrical axis, which is parallel to the plane). [5]

2. One can construct a co-axial capacitor by using two co-axial infinite conducting cylindrical surfaces, with radii a and b, respectively ($a < b$). The insulating region between the two surfaces is equally divided in two parts, such that the region $0 \leq \varphi \leq \pi$ is filled with a dielectric material ε_1 and the region $\pi \leq \varphi \leq 2\pi$ is filled with a dielectric material ε_2. Find the capacitance per unit (axial) length of such a capacitor. [10].

3. (a) Yet another co-axial problem! There are two co-axial cylindrical conductors, one being a solid cylinder of radius a_1 of infinite length, through which flows a uniform current I parallel to the axis; the other being a cylindrical shell, of inner radius a_2 and outer radius a_3, through which the same current returns, in an azimuthally symmetric fashion. Obviously $a_1 < a_2 < a_3$. Find the magnetic field everywhere. [7]

   (b) Here is another expression for a magnetic field subtended at a generic point P by a closed loop carrying a current I. Using Biot-Savart Law, relate the magnetic field at P due to the loop to Ω, the solid angle subtended by the loop at P, and the current I. [8]

(c) Since at P, $\vec{j} = 0$, this means the magnetic field can be expressed as a pure gradient of a scalar potential (just like the electrostatic electric field), Ψ_M. For the problem above, find Ψ_M. [5]

4. The dielectric constant of a sphere of radius R depends on its location and is given by

$$\varepsilon(r) = \begin{cases} \varepsilon_0 \left(\frac{r}{R}\right)^2 & r < R \\ \varepsilon_0 & r \geq R \end{cases}$$

The sphere is embedded in a uniform external electric field $\vec{E}_0$.

(a) Find the differential equation satisfied by the potential $\phi(\vec{r})$. [5]

(b) Find the potential everywhere using any method you choose, using the appropriate boundary conditions. Please give arguments for every relevant interim step you take. [7]

(c) Find the net polarization of the sphere. [3]