

INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2021-22
Statistics - III, Test 1, October 27, 2021

1. Suppose (X, Y) is bivariate normal with $E(X) = 0 = E(Y)$, $Var(X) = \sigma^2 = Var(Y)$ and a correlation coefficient of $\rho = 0.4$ between X and Y .

(a) Find the distribution of

$$Q = X^2 + \frac{(Y - \rho X)^2}{1 - \rho^2}.$$

(b) Find $E(Q)$.

[10]

2. Suppose $Y \sim N_n(0, I_n)$, A_i , $i = 1, \dots, p$ are symmetric $n \times n$ matrices of rank k_i , and $A = \sum_{i=1}^p A_i$ is symmetric with rank k . Then

(i) $Y' A_i Y \sim \chi_{k_i}^2$, (ii) $Y' A_i Y$ are pairwise independent, and (iii) $Y' A Y \sim \chi_k^2$ iff any two of

(a) A_i are idempotent for all i , (b) $A_i A_j = 0$, $i \neq j$, (c) A is idempotent, are true.