Assignment 2

Physics III: Electricity and Magnetism
B. Math. Year 3,
September - December 2021.

Due on: November 19^{th}, 2021.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Please feel free to discuss amongst yourselves; however, copying the assignment solutions from someone else is strictly prohibited and both persons involved will be penalized. Each one of you must submit your own answers. Total: 50 points.

1. Suppose that a closed volume without any charge density inside is bounded by n conducting planes which form a regular convex polyhedron (n can be one of five integers in three dimensions). Each surface i is held at a different potential Φ_i, $i = 1 \dots n$. Find the potential at the center of the polyhedron [10].

2. (a) Problem 3.9, Modern Electrodynamics by Andrew Zangwill, 2013 edition. [3]

   (b) Suppose a total charge Q is contained inside the volume $\mathbb{V}$. Let the potential take the constant value φ_0 on the surface $\partial\mathbb{V}$. Show that the total electrostatics energy contained *outside* the sphere is

   $$U_E = \frac{1}{2} Q \varphi_0$$

   [7]

3. Prove the Reciprocity theorem when both volume and surface charge densities are present. Suppose a certain volume $\mathbb{V}$ and surface $\partial\mathbb{V}$ are present. Let $(\rho_1, \sigma_1, \Phi_1)$ be the charge density (in $\mathbb{V}$), surface charge density (on $\partial\mathbb{V}$) and the potential (caused by these densities) respectively in a given electrostatic configuration. On the same volume(s) and surface(s) let another configuration have $(\rho_2, \sigma_2, \Phi_2)$ (same $\mathbb{V}$ and $\partial\mathbb{V}$). Derive Green's reciprocity theorem involving volumes and surfaces. *Hint*: You may find Green's second identity useful. [10]

4. Two infinite conducting grounded (at zero potential) parallel plates are held at a distance d apart. A point charge of strength q is held between the two plates at a distance λ from one of the plates ($\lambda < d$). Find the surface charge induced on the two plates by q. You must show the reasoning you used to arrive at the solution clearly. [10]

5. Two infinite grounded conducting plates are held at $x = \pm d$. An infinite conducting sheet of uniform charge density σ are inserted between (and parallel to) the two plates at a position x (arbitrary, except $|x| < d$). Find

   (a) The induced charges on the two plates. [4]

   (b) The potential at the position of the charged conducting sheet. [4]

   (c) The force per unit area on the charged sheet. [2]