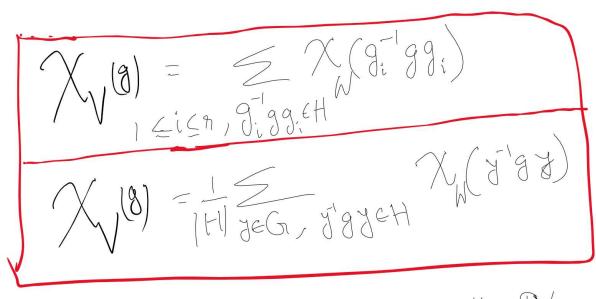
Lecture 21: Frobenius reciprocity

Let G be a group & H & G. Let V be a G-sepr & W & V an H-stable subspace. We say V = Ind H W if V= 9.W where 9,,,,g, EG are representatives of the left cosets of H in G/H. Note n=[G:H]

Q Let p': a > GL(V') be another repr. Let f: W- V' be an H-equivarion mat. Then f extends uniquely to a G-equinariant map F: V -> V'. $Hom(W,V') \cong Hom(V,V')$ For W, W' H-rept (-Hom (W, V)H) Inda W & Inda W' = Inda (WOW) - - - 0 W' & W subrefer Ind W' & Ind W as subrefer. -.. 2 (Ind GW) & V' = Ind G(W&V') here V' is a Gr-repr.

Ind K[H] = k[G]. $\text{Ind}_{H}^{G}W\cong k[G]\otimes W$

Det Glegroup. H ≤ G & K ≤ H. Let Whea K-repa-Ind (Ind W) = Ind K W



Define

Let f be a class function on H. Define $f' = \text{Ind}_{H} f : G_{h} \longrightarrow \mathbb{C}$ $f'(g) = \frac{1}{|H|} \underbrace{\sum_{y \in G_{h}} f(y^{\dagger}gy)}_{y^{\dagger}gy \in H} \underbrace{\sum_{g \in G_{h}} f(g)}_{|H|} \underbrace{\sum_{g \in G_{h}} f(g)$

DINGH XW = X INGHW

3 f'= IndH is a class function G.

Note that class functions, are C-linear combinations of characters of H (& G respectively)

If $f_1 \& f_2$ are class functions on $H \& a_1, a_2 \notin C$ Then $T_{nd}(a_1f_1 + a_2f_2) = a_1 T_{nd}f_1 + a_2 T_{nd}f_2$

Det f' be a class function on G then

If = Resf' is a class function G.

Recall $\langle P_1, P_2 \rangle_{G} = \frac{1}{|G|} \underbrace{\sum_{g \in G_1} P_1(g) P_2(g^{-1})}_{g \in G_1} = \underbrace{(P_1 | P_2)}_{a \text{ chancey}} \underbrace{ig P_2 is}_{a \text{ chancey}}$ Let V, V' be G-repr. P. class function on G. (V, V') = din Hom (V, V') = din Hom (V, V') $\langle V, V' \rangle_{G} = \langle \chi_{V}, \chi_{V'} \rangle_{G}$ $V = N_1 W_1 + \cdots + N_n W_n$ where 12001 din Hom (Fabrenius reciprocity):

We are isen

We are isen

We have isen

We have isen

The Will of Whi is the way in the condity of the conditions of the LP, nes Y) = (IndP, Y)G where P is a class function on H & Y is a class function on G.

Pf: Since (1) is bilinear & the characters generate the space of class function & as a vector space. We may assume $p = X_W$ & Y=Xv where Wis an Hreps & V' is a G-reph. Then $\langle \mathcal{P}, \text{Res } \mathcal{Y} \rangle = \langle \mathcal{W}, \mathcal{V}' \rangle_{H}$ Hom H(N, V') = Hom (Ind W, V') = di- Hom (W, V') = dix Hom (Ind W, V') CW/V'>n (Ind W, V')