Statistics-III – Assignment 1

1. Let $Y_1, \ldots, Y_n$ be independent random variables with unit variance, and let $X_1 = Y_1$, $X_i = Y_i - Y_{i-1}$ for $1 < i \leq n$. Find the covariance matrix of $\mathbf{X} = (X_1, X_2, \ldots, X_n)'$.

2. Suppose $\Sigma = \text{ Cov } (X) = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}$. Show that $-1/2 \leq \rho \leq 1$.

3. Let $X_1, \ldots, X_n$ be i.i.d Exponential with mean 1. Define $Y_1 = nX_{(1)}$, $Y_2 = (n-1)(X_{(2)} - X_{(1)})$, $Y_i = (n-i+1)(X_{(i)} - X_{(i-1)})$ for $3 \leq i \leq n$, where $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$ are the order statistics. Show that $Y_1, \ldots, Y_n$ are i.i.d Exponential with mean 1.
(Hint. Note, in the joint density, $\sum_{i=1}^n x_{(i)} = \sum_{i=1}^n y_i$.)