

**INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE**  
**B.MATH - Third Year, 2021-22**  
**Statistics - III, Test 2, November 22, 2021**

1. Consider the following model:

$$y_1 = \alpha + \beta + \gamma + \epsilon_1$$

$$y_2 = \alpha - \beta + \epsilon_2$$

$$y_3 = \alpha - 2\gamma + \epsilon_3$$

$$y_4 = \alpha + \gamma + \epsilon_4$$

where  $\alpha, \beta, \gamma$  are unknown constants and  $\epsilon_i$  are uncorrelated random variables having mean 0 and variance  $\sigma^2$ .

(a) Is  $\alpha + \beta$  is estimable? Justify.

(b) Does there exist a BLUE for  $\alpha + \beta$ ? Find it if it does.

(c) Find an unbiased estimate of  $\sigma^2$ . [15]

2. Consider the model  $\mathbf{Y} = X\beta + \epsilon$ , where  $X_{n \times p}$  has rank  $r \leq p$ ; also  $\epsilon \sim N_n(\mathbf{0}, \sigma^2 I_n)$ . Let  $(X'X)^-$  be a generalized inverse of  $X'X$ , and  $\hat{\beta}$  be a least squares solution of  $\beta$ . Suppose  $A\beta$  is estimable where the rank of  $A_{q \times p}$  is  $q$ .

(a) Show that  $A(X'X)^-X'X(X'X)^-A' = A(X'X)^-A'$ .

(b) Find the probability distribution of  $A\hat{\beta}$ . [10]