Assignment 1

Finite fields.

Instructions. Submit scanned copy of <u>handwritten</u> solutions written on a blank A4 paper with your name and roll number on top of each page. Do not submit individual pages but one consolidated pdf file.

Q.1 Prove that no finite field is algebraically closed.

Q.2 Let W be a t-dimensional subspace of $\mathbb{F}_q^n$, where $\mathbb{F}_q$ denotes the finite field of order q.

For $1 \leq k \leq n$, determine the number of k-dimensional subspaces of $\mathbb{F}_q^n$ that intersect W only in zero.

Q.3 Let p be a prime number. Let $G = GL_n(\mathbb{F}_p)$ denote the group of $n \times n$ invertible matrices with entries from $\mathbb{F}_p$. Then,

(a) find the order of G.

(b) find the order of a p-Sylow subgroup of G.

(c) give an example of a p-Sylow subgroup of G.

Q.4   Let α be a root of $x^6 + x + 1$ in $\overline{\mathbb{F}_2}$, the algebraic closure of $\mathbb{F}_2$. Then,

(a) show that $\beta^2 + \beta + 1 = 0$, where $\beta = \alpha^5 + \alpha^4 + \alpha^3 + \alpha$

(b)   Factor $x^4 + x + 1$ over $\mathbb{F}_{64}$.