Assignment 1

Physics III: Electricity and Magnetism
B. Math. Year 3,
September - December 2021.

Due on: October 28^{th}, 2021.

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Please feel free to discuss amongst yourselves; however, copying the assignment solutions from someone else is strictly prohibited and both persons involved will be penalized. Each one of you must submit your own answers. Total: 60 points.

1. (a) Total charge Q is uniformly distributed inside a sphere of radius R. Find the total force the southern hemisphere exerts on the northern hemisphere of the sphere. [5]

   (b) A uniform surface charge density σ is smeared over a hemispherical bowl of radius R. What is the potential difference between the north pole and the center of the sphere. Ignore the flat surface at the equator in your calculations. [5]

   (c) The electric potential of a charge configuration is given by

   $$\Phi\left(\vec{r}\right) = A\frac{e^{-\lambda r}}{r}$$

   Find the electric field $\vec{E}\left(\vec{r}\right)$, the charge density $\rho\left(\vec{r}\right)$ and the total charge Q that can lead to this configuration potential. A and λ are constants. [5]

2. Imagine two infinitely long and infinitely thin wires running parallel to the x-axis at a distance $y = \pm d_0$ from the x-axis, lying on the xy-plane. One (at $y = +d_0$) of them carries a uniform charge density λ and the other a uniform charge density $-\lambda$.

   (a) Find the potential $\Phi\left(\vec{r}\right)$ at any point $\vec{r}$, using the origin as reference. [7]

   (b) Show that the equipotential surfaces are circular cylinders. [4]

   (c) For a given potential Φ_0, find the location and orientation of the axis and radius of the equipotential cylinder. [4]

3. Imagine that the actual force between two charges q_1 and q_2 is not quite Coulomb's Law, but has a small correction. The precise form is (for the force on q_2 due to q_1):

$$\vec{F_{12}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\left(1+\frac{r_{12}}{\lambda}\right)e^{-\frac{r_{12}}{\lambda}}\hat{e}_{12}$$

where λ is a new constant of nature, having the dimension of length, r_{12} is the distance between the two charges, and $\hat{e}_{12}$ is the unit vector from q_1 to q_2, ensuring that the force acts along the line joining the two charges, and it is repulsive (attractive) when the two charges are of the same (opposite) signs.

(a) Prove that one can attach a potential $\Phi(\vec{r})$ which can generate the corrected Coulomb force as above, without actually evaluating the potential. [3]

(b) Evaluate the potential $\Phi(\vec{r})$. [4]

(c) For a point charge q at the origin, derive the modified integral form of Gauss'Law for "electrostatics" with the new force law for any sphere centred at the origin. As a reminder, in the limit $\lambda \to \infty$, the force law reduces to our well-known $\frac{1}{r^2}$ Coulomb's Law, and the Gauss' Law, in integral form, reads

$$\oint_{\mathcal{S}} \vec{E}.d\vec{S} = \frac{q}{\epsilon_0}$$

where all the symbols have the usual meanings. [5]

(d) Assume that the integral form of Gauss's Law not only holds for a single charge at the origin of an arbitrary sphere, but for any continuous charge distribution $\rho(\vec{r})$ over any closed volume V, enclosed by the surface $\mathcal{S}$. Derive the modified Poisson's Equation in such a scenario. [3]

4. Modern Electrodynamics by Andrew Zangwill, 2013 Edition, Cambridge University Press, **Problem 3.19, parts (a) and (b)**. Part (a) is worth 8 points, while part (b) is worth 7 points. In part (b), the word *localized* should be interpreted as: a surface $\partial\mathbb{V}$ (with well-defined outside and inside) can always be found completely enclosing a finite volume $\mathbb{V}$ such that there is no charge on $\partial\mathbb{V}$ or outside it.