Physics IV
ISI B.Math
HW set 3

1. A particle in the infinite square well as done in class has as its initial wave function an even mixture of the first two stationary states

$$\Psi(x,0) = A[\psi_1(x) + \psi_2(x)]$$

(a) Normalize $\Psi(x,0)$

(b) Find $\Psi(x,t)$ and $|\Psi(x,t)|^2$. Express the latter as a sinusoidal function of time. To simplify the result, let $\omega = \pi^2\hbar/2ma^2$.

(c) Compute $< x >$. Notice that it oscillates in time. What is the frequency of oscillation ? What is the amplitude of oscillation ?

(d) Compute $< p >$

(e) If you measured the energy of this particle, what values might you get, and what is the probability of getting them ? Find the expectation value of H. How does it compare with E_1 and E_2 ?

2. A particle of mass m is confined to a one dimensional region $0 \leq x \leq a$ with infinite potential walls. At $t = 0$, its normalized wave function is

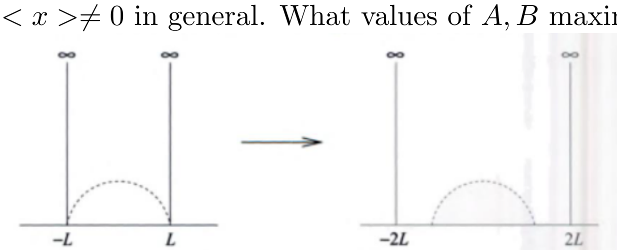
$$\Psi(x,0) = \sqrt{\frac{8}{5a}}\left[1 + \cos\left(\frac{\pi x}{a}\right)\right]\sin\left(\frac{\pi x}{a}\right)$$

(a) What is the wave function at a later time $t = t_0$?

(b) What is the average energy of the system at $t = 0$ and $t = t_0$?

(c) What is the probability that the particle is found in the left half of the box (i.e, in the region $0 \leq x \leq \frac{a}{2}$) at $t = t_0$?

3. Consider a linear harmonic oscillator and let ψ_0 and ψ_1 be its normalized ground and first excited states. Let $A\psi_0 + B\psi_1$ be the wave function at a certain instant of time A, B real. Show that $< x > \neq 0$ in general. What values of A, B maximize and minimize $< x >$?



4. A particle of mass m is in an infinite one-dimensional box with walls at $x = -L$ and $x = L$ and is in its ground state $\psi_0(x)$ at $t = 0$. Assume now that at $t = 0$ the walls of the box move instantaneously so that its width doubles ($-2L < x < 2L$). This change does not affect the state

of the particle which remains the same before and after (i.e, $\psi_0(x)$ of the box of width $2L$.)

(a) Write down the wave function of the particle at time $t > 0$.

(b) Calculate the probability P_n of finding the particle in an arbitrary stationary state $\tilde{\psi_n}(x)$ of the modified system. What is the probability of finding the system in an odd state ?

(c) What is $< H >$ = the expectation value of the energy at time $t > 0$?

[You can use : $\sum_{n=0}^{\infty} \frac{(2n+1)^2}{[(2n+1)^2-4]^2} = \frac{\pi^2}{16}$]

5. A particle of mass m in the infinite square well(of width a) starts out in the left half of the well, and is at $(t = 0)$ equally likely to be found at any point in that region.

(a) What is the initial wave function $\Psi(x, 0)$? (Assume it is real)

(b) What is the probability that a measurement of energy would yield the value $\frac{\pi^2\hbar^2}{2ma^2}$?