Physics 4

Lecture 9 -10

Geometric Representation of Space-Time : Minkowski Diagrams

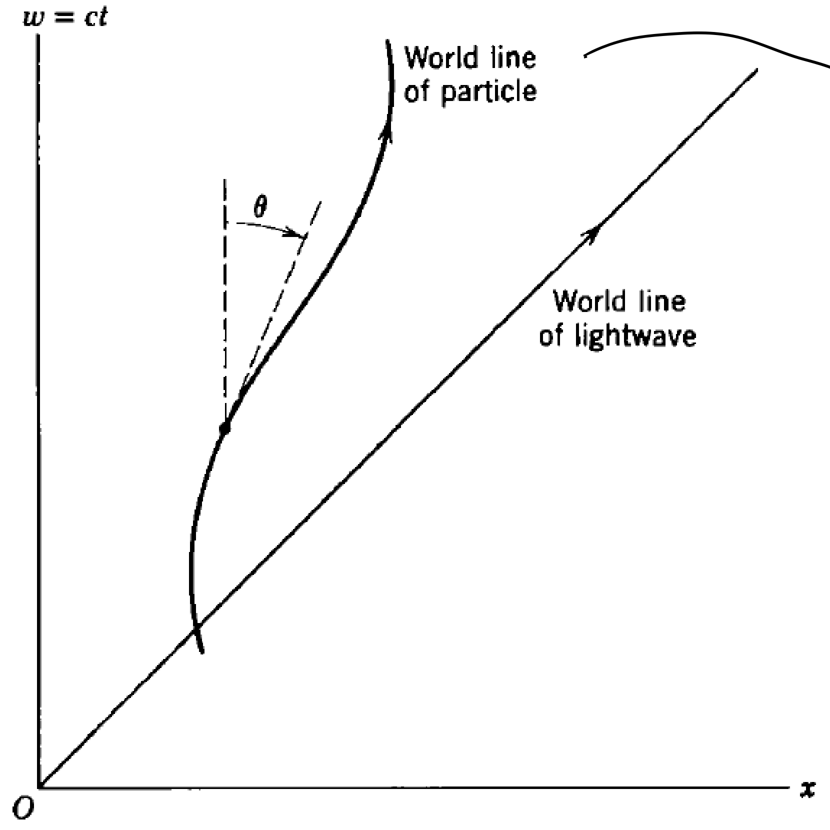
Symmetric way of writing L.T.

2D

$$x' = \frac{x - \beta ct}{\sqrt{1-\beta^2}}, \qquad x = \frac{x' + \beta ct'}{\sqrt{1-\beta^2}}.$$

$$ct' = \frac{ct - \beta x}{\sqrt{1-\beta^2}} \qquad ct = \frac{ct' + \beta x'}{\sqrt{1-\beta^2}}.$$

Events (x, t)



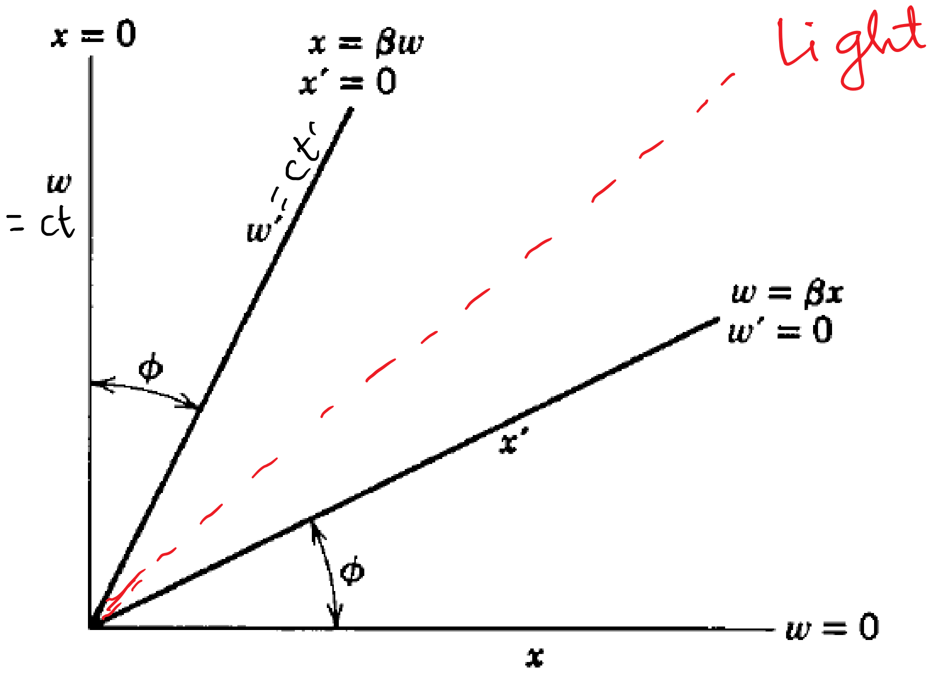
points ≡ events.

world line.

tangent to world line

$$\frac{dx}{d(ct)} = \frac{1}{c}\frac{dx}{dt} < 45^\circ$$

to the time



Represent S and S' on same diagram.

$x' = 0 \quad [x' = \gamma(x - \beta ct)]$

$x = \beta ct$.

$ct' = 0$.

$ct = \beta x$.

$\boxed{\tan\phi = \beta}$

L.T transforms an orthogonal $\longrightarrow$ non-orthogonal coordinate systems.

at $x = 0$
$ct = 1$,
unit time in S.
at any other pt

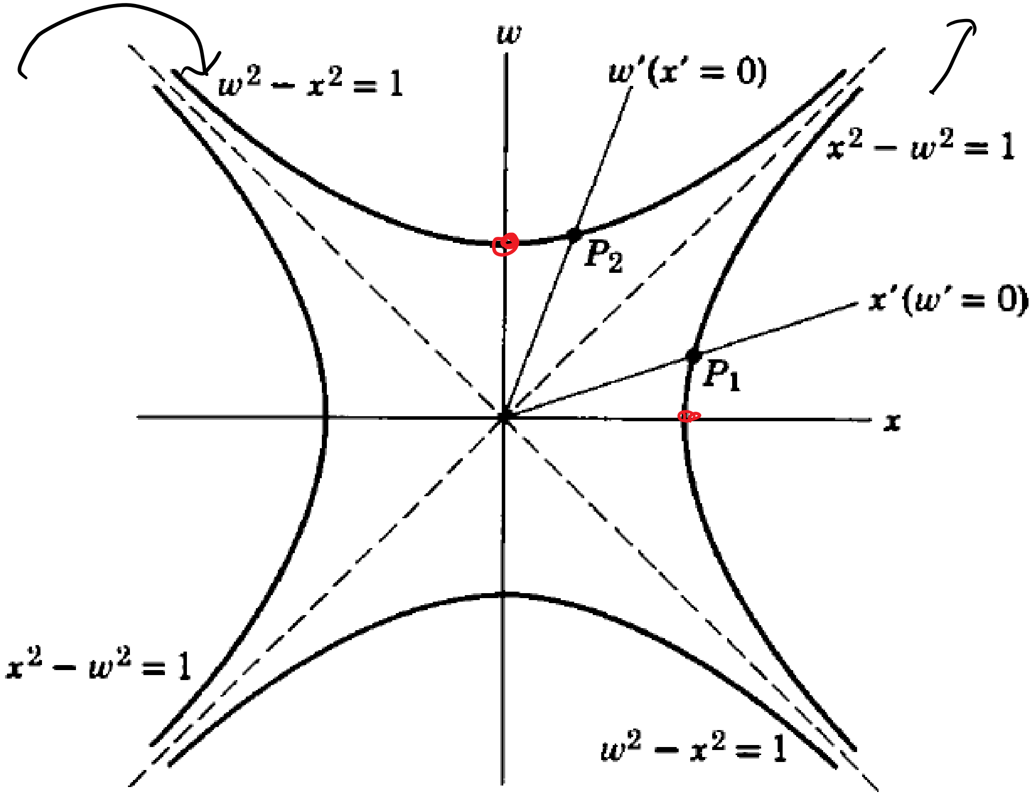


Fig A-3.

$x = 1; w = 0$

$w^2 - x^2 = 1 = w'^2 - x'^2 = 1$
↳ invariant hyperbola
$x^2 - w^2 = 1$.

Point P_1:
intersection of x' axis, $w' = 0$
with $x^2 - w^2 = 1 = x'^2 - w'^2$
$P_1 \equiv x' = 1,\ w' = 0$.
marks off unit dist ~~in~~ along x' axis

P_1 coordinates S

$ct = \gamma(ct' + \beta x') = \dfrac{\beta}{\sqrt{1-\beta^2}}$.

$x = \dfrac{1}{\sqrt{1-\beta^2}}$.

Similarly P_2 marks 1 unit of $w' = ct'$ along w' axis.

Invariant hyperbolae are used to calibrate the x', ct' axes.

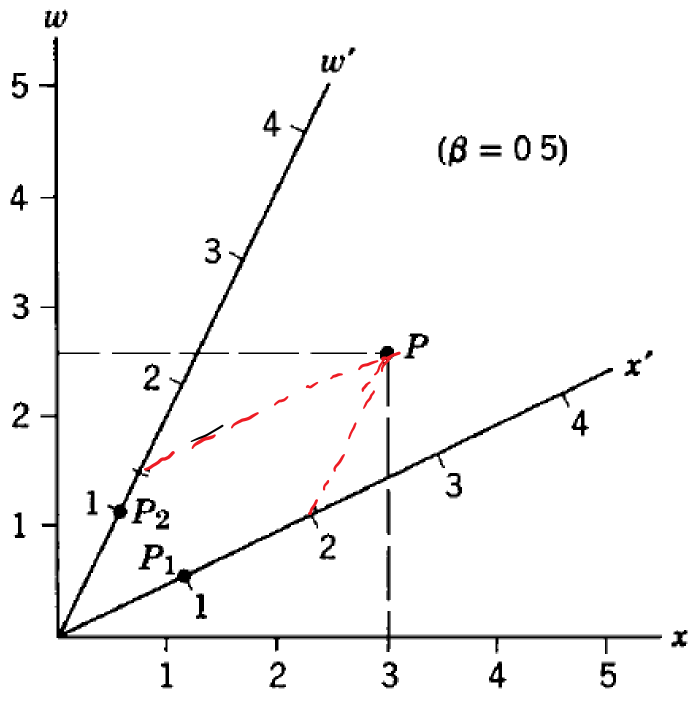


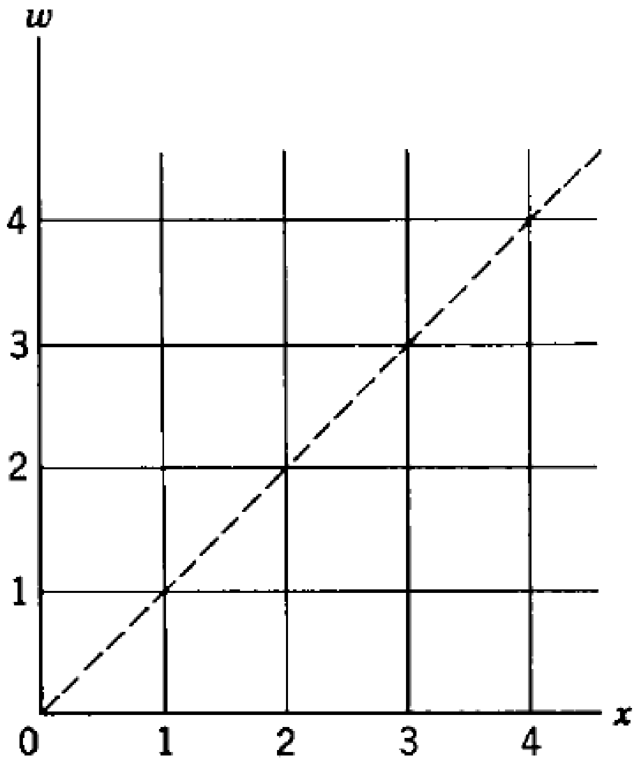
Fig. A-4.

Once calibration is done, can get rid of the hyperbolae.

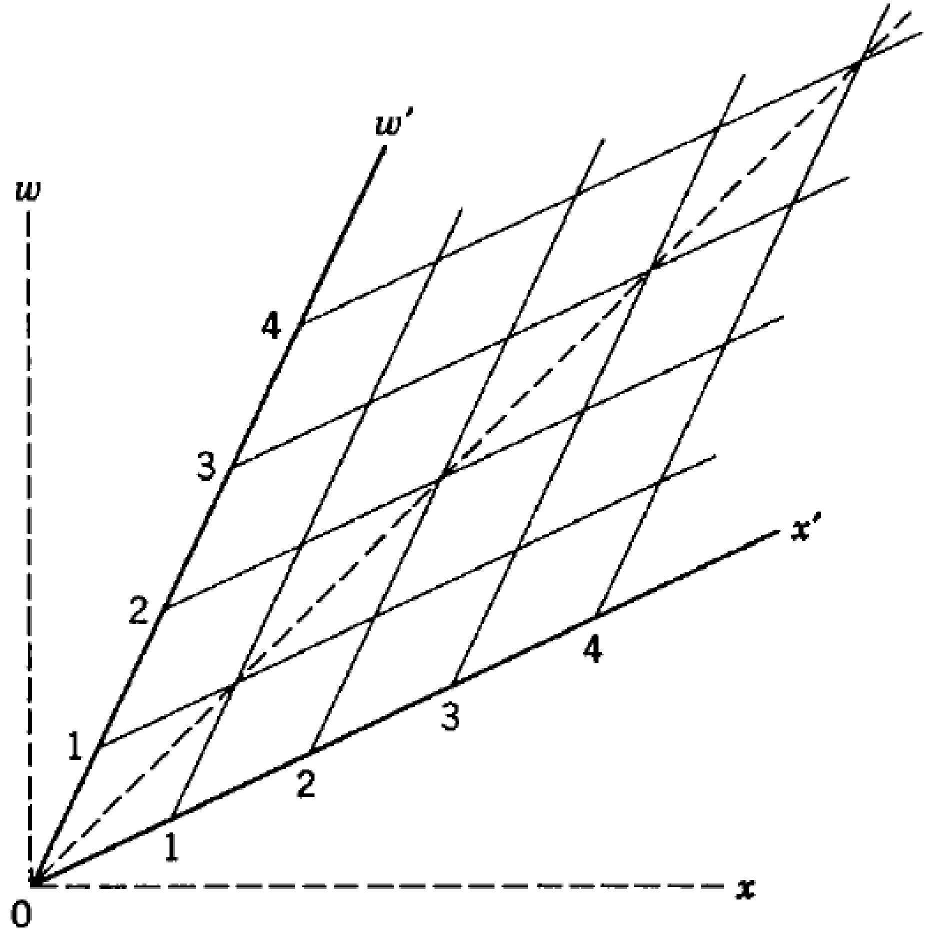
coordinates of P in S $(x = 3, w = 2.5)$

coordinates of P in S'.

$(x' = 2, w' = 1.5)$



(a)



(b)

S

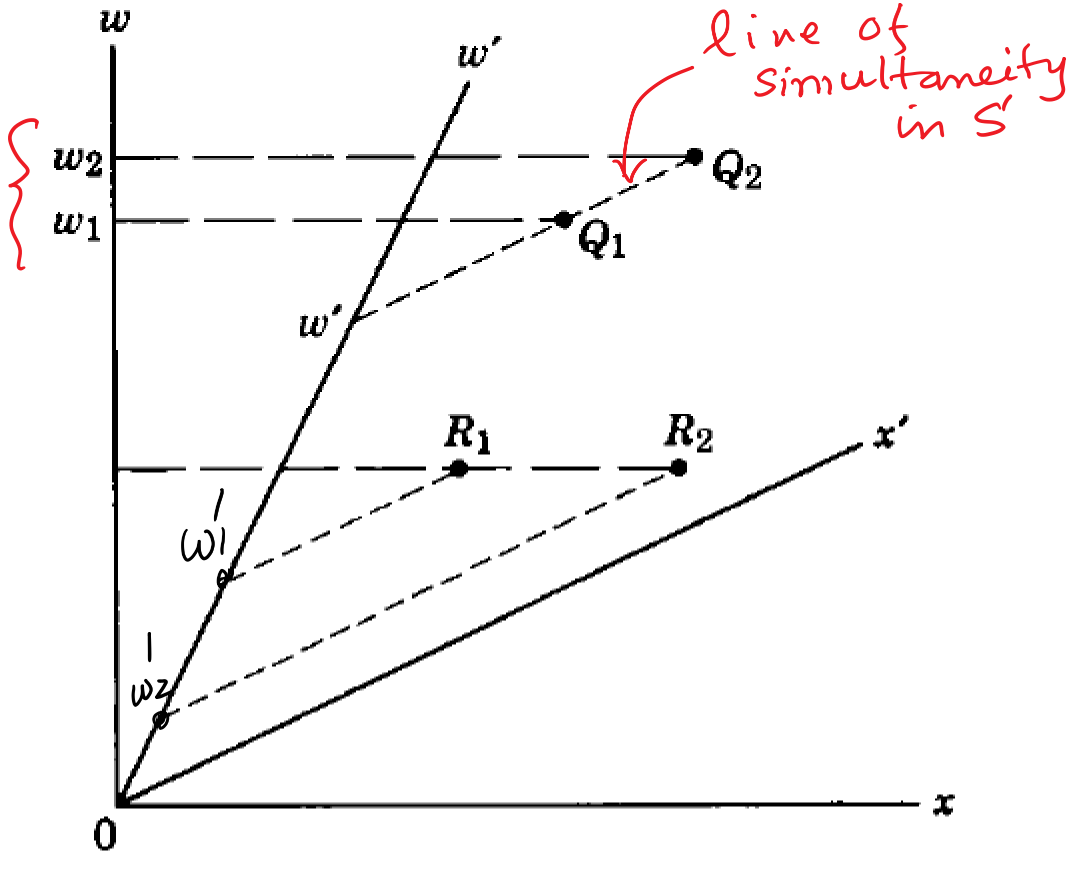
S - S'

Simultaneity represented on Minkowski diagrams

Not simultaneous in S frame

$w_2 \neq w_1$

$t_2 \neq t_1$



In S' two events will be simultaneous if they have the same w'. ∴ These events have to lie on a line parallel to x'-axis

R_1, R_2 are simultaneous in S frame.

But not so in S' frame

$w_1' = w_2'$

Length Contraction

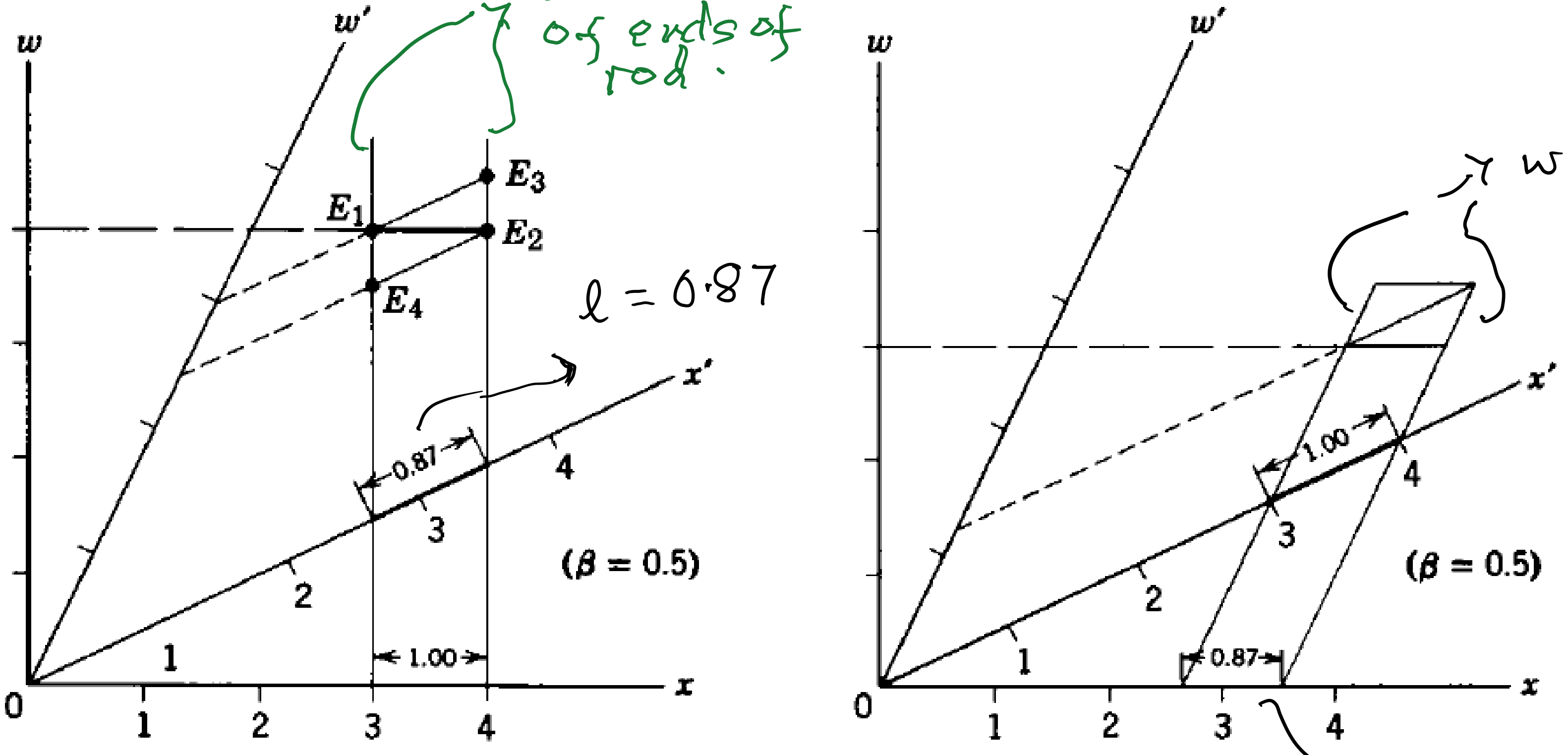
rod is at rest in S′.

world lines of ends of rod.

world lines of ends in S′ frame.

$t = t_0$

$l = 0.87$



(a) (b)

S measures 0.87.

In (a) rod of unit length is at rest along x axis, with ends at 3 and 4.

S uses E_1 & E_2 to measure length.

S′ will use either E_1, E_3 or E_2, E_4 to measure length

Time Dilation

In S', the clock is moving.

In S' to measure time interval, use 2 clocks, one at location of T_1 and other at T_2 from the pt. of view of S', clock has slowed down

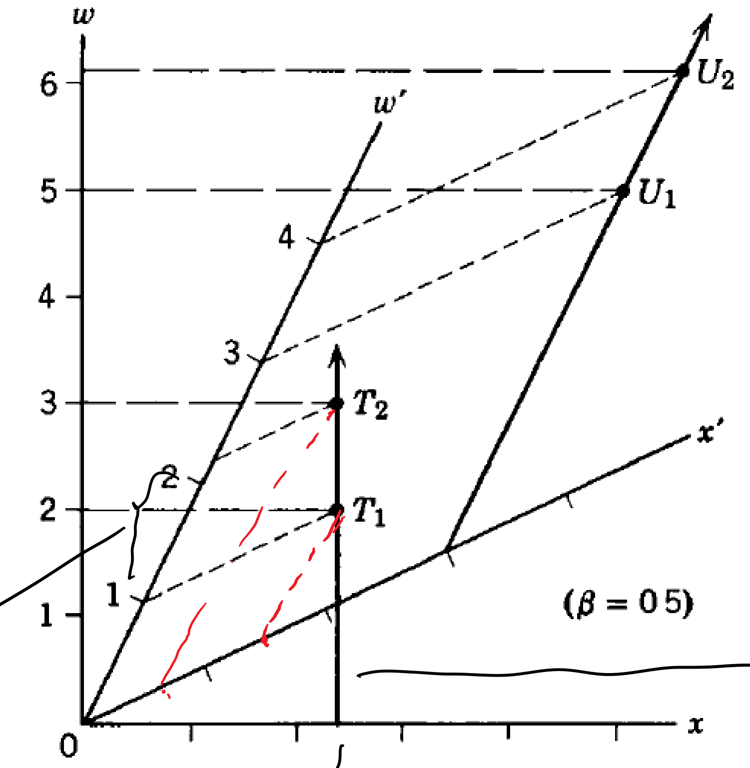
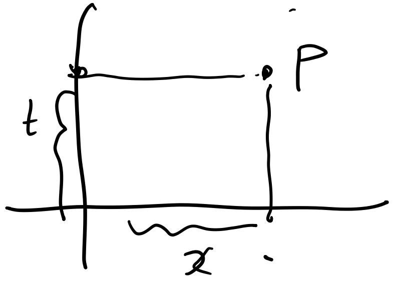


Fig. A-8.

greater than 1

world line of clock.

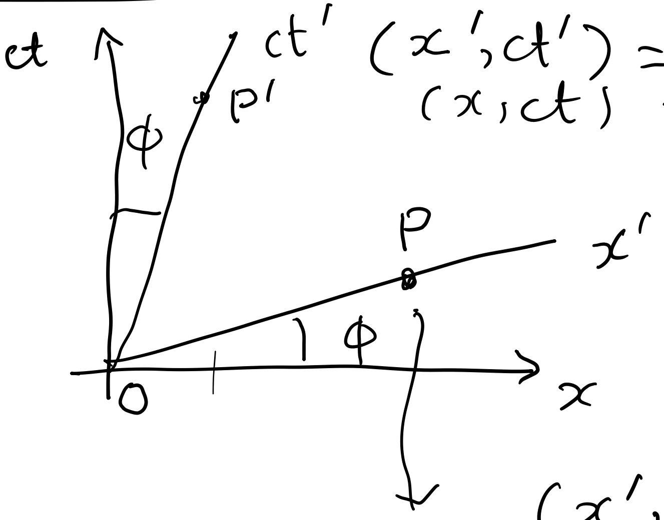
Let the clock be at rest in frame S.



stationary clock.

$x = 2.3$, . T_1, T_2 are events of ticking at $w\ (=ct) = 2$, $w = 3$. time interval in $S = 1$.

ct'-axis and unit size



$(x', ct') = (0, 1)$

$(x, ct) = (\beta\gamma, \gamma)$

$\tan\phi = \beta$.

$(x', ct') = (1, 0)$.

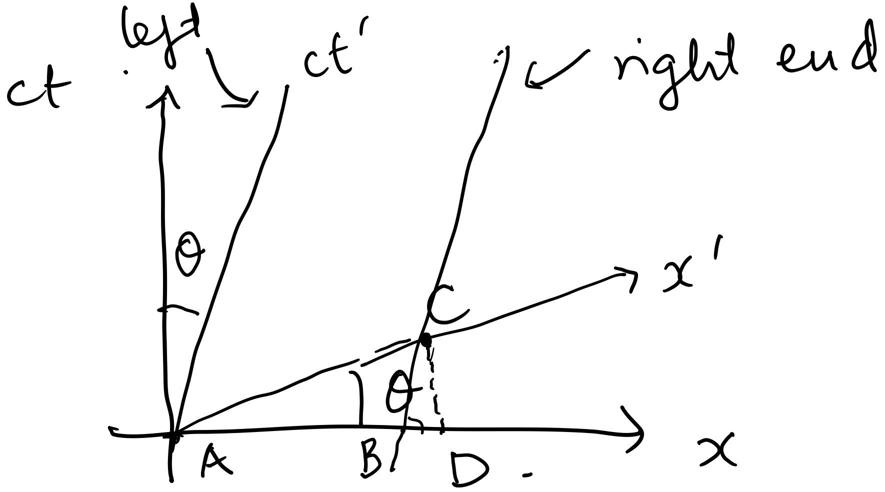
$(x, ct) = (\gamma, \beta\gamma)$

similarly

$$\frac{\text{one } x' \text{ unit}}{\text{one } x \text{ unit}} = \sqrt{\frac{1+\beta^2}{1-\beta^2}}.$$

distance $OP' = \gamma\sqrt{1+\beta^2}$.

$$\boxed{\frac{\text{one } ct' \text{ unit}}{\text{one } ct \text{ unit}} = \gamma\sqrt{1+\beta^2} = \frac{\sqrt{1+\beta^2}}{\sqrt{1-\beta^2}}}$$



$\tan\theta = \beta$

Length contraction

Meter stick at rest along x' axis is in S' one end at A, other at C.

for S' $AC = 1$.

World line of left end is the ct' axis

How does S measure metre stick?

location of right end for S will be at B. ; measures both ends simultaneously at $t=0$; $AB = ?$.

$$CD = AC\sin\theta \;, \quad \angle BCD = \theta \;, \quad BD = CD\tan\theta = AC\sin\theta\tan\theta$$

$$AB = AD - BD = AC\cos\theta - AC\sin\theta\tan\theta$$
$$= AC\cos\theta\,(1 - \tan^2\theta)$$
$$= \sqrt{\frac{1+\beta^2}{1-\beta^2}}\;\frac{1}{\sqrt{1+\beta^2}}(1-\beta^2) = \sqrt{1-\beta^2}.$$