Lecture 1: What is algebraic geometry?

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Algebra is about 1) solving equations involving polynomials
2) Groups, rings, fields, etc. modules

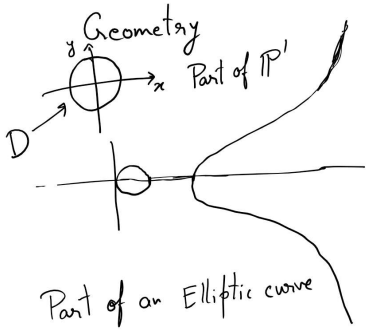
Geometry is about 1) Pictures, plane geometry, coordinate geometry, loci of equations, circle, parabola, etc.
2) manifolds, varities, schemes, etc.
Topology.

Algebra

Ex 1 $x^2+y^2=1$

2) $y^2 = x(x-1)(x-2)$

Geometry



Part of an Elliptic curve

$\left\{ \frac{\mathbb{R}[x,y]}{(x^2+y^2-1)} \right\}$ $\mathbb{R}[x,y]$

$f(a,b) = a$

$g(x,y) = x^2+x+y^2-1$

$g(a,b) = a^2+a+b^2-1$

$f|_D^{(a,b)} = a = g|_D^{(a,b)}$ where $(a,b)\in D$

$\left\{ \text{max ideals of } \frac{\mathbb{R}[x,y]}{(x^2+y^2-1)} \right\} \longleftrightarrow \left\{ \begin{matrix} \text{maximal} \\ \text{ideals in } \mathbb{R}[x,y] \\ \text{containing } (x^2+y^2-1) \end{matrix} \right\}$

$(x-1, y)$, $(x, y-1)$;

$x^2+y^2-1 = (x-1)(x+1)+y^2$

$(a,b)\in\mathbb{R}$ s.t. $a^2+b^2=1$ then

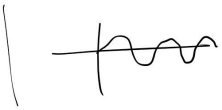
$(x-a, y-b) \Rightarrow x^2+y^2-1$

$(x-a)(x+a) + (y-b)(y+b) = x^2-a^2+y^2-b^2$
$= x^2+y^2-1$

$(a,b)\in\mathbb{R}$ be s.t. $(x-a, x-b) \Rightarrow x^2+y^2-1$

$\Rightarrow a^2+b^2=1$

3) $y = \sin(x)$



← Not algebro geometric object.

Prop: In $\mathbb{R}[x,y]$ let f be a poly then $(a,b) \in \mathbb{C}^2$ is in the locus of f iff $f \in (x-a, y-b)$.

Books: 1) Undergrad Alg geom by Miles Reid

2) " comm alg " " "

3) Intro Comm Alg Atiyah Macdonald

4) Commutative alg with a view towards alg geom — Eisenbud

Basic Alg Geom by Shafarevich

$f_1 = x^2 + y^2 = 1$

conic $x^2 = y$

$f_2 = 2x^2 + y^2 = 1$

$f_3 = x^2 - y^2 = 1$

over $\mathbb{R}$ they all look different

over $\mathbb{C}$

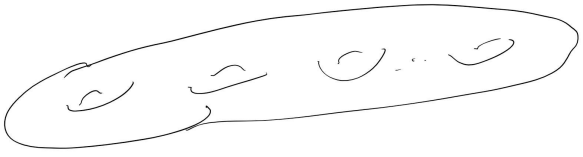
$$\frac{\mathbb{C}[x,y]}{(f_1)} \cong \frac{\mathbb{C}[x,y]}{(f_2)} \cong \frac{\mathbb{C}[x,y]}{(f_3)}$$

$\mathbb{P}^1_{\mathbb{C}}$ is one point compactification

2) E elliptic curve

$\mathbb{C}$

3) Higher genus curve



genus $g \geq 2$

$3g-3$ dim object

parametrizes all genus g compact curves.