

**Class Exam 1: Date 3Feb2022**

**Attempt all questions. Your answers for each question should typically be just one page.**

**You need to submit ONE PDF as the answer script. PLEASE WRITE with BLACK INK.**

**Please write legibly.**

Q1. Total Marks: 6

Prove or disprove the following:

- a.)  $N^{1/2} = O(\log N)$
- b.)  $\log_a b = O(\log_b a)$  where  $a, b > 1$
- c.)

$$1+2+\dots+n \text{ is } \Theta(n^2)$$

(do not use the fact that  $1+2+3+\dots+n = n(n-1)/2$ ).

Q2. Total Marks: 6

Show that for a sequential search (e.g. for integer in an array of integers), the time complexity  $T(n)$

- a.  $T(n)$  is  $\Omega(1)$
- b.  $T(n)$  is  $\Omega(n)$
- c.  $T(n)$  is  $\Theta(n)$

Here these are all Big Omega and Big Theta.

Q3. Total Marks: 8

Show that for the recurrence relations

$$T(n) = 2 T(n/2) + n,$$

- a.)  $T(n) = O(n^3)$
  - b.)  $T(n) = O(n^2)$
- Assume  $T(1)$  is a constant.

Make sure all your proofs are rigorous.