Physics 4

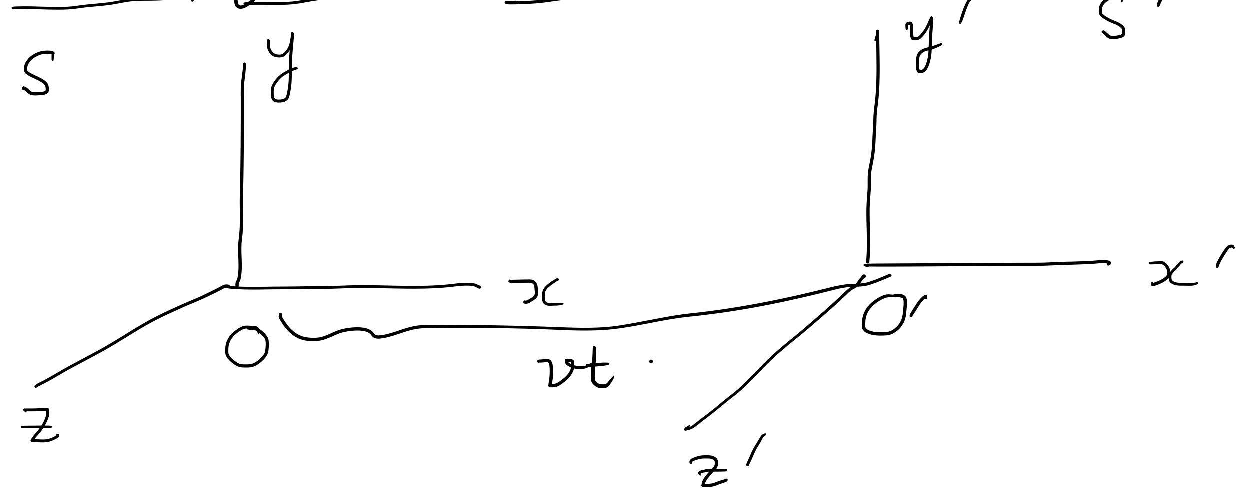
Lecture 3-4

Einstein's Postulate

1. The laws of physics are the same in all inertial frames.

2. The speed of light has the same value c in free space in ALL inertial frames.

Lorentz Transformations :



$S' \rightarrow v$

S' moves with vel v rel. to S along the common $x-x'$

$$\left.\begin{aligned} x' &= x'(x,y,z,t) \\ y' &= y'(x,y,z,t) \\ z' &= z'(x,y,z,t) \\ t' &= t'(x,y,z,t) \end{aligned}\right\}$$

Inputs

- Postulates of relativity
- homogeneity and isotropy of space time

- Homogeneity $\longrightarrow$ linear transfms.

$x \longrightarrow \quad x' = f(x)$

$x \longrightarrow \quad x + \epsilon \qquad x' \cong f(x) + \frac{\partial f}{\partial x} \epsilon + h.o.t.$

Must be independent of ϵ. $\frac{\partial f}{\partial x} = 0$.

$x' = a x^2$.

$x_2' - x_1' = a(x_2^2 - x_1^2)$.

shift origin, length will depend on it

$$x' = a_{11} x + a_{12} y + a_{13} z + a_{14} t \quad (1)$$

$$y' = a_{21} x + a_{22} y + a_{23} z + a_{24} t \quad (2)$$

$$z' = a_{31} x + a_{32} y + a_{33} z + a_{34} t \quad (3)$$

$$t' = a_{41} x + a_{42} y + a_{43} z + a_{44} t \quad (4)$$

$$x'_i = \sum_i a_{ij} x_j$$

- a_{ij} depend on v
- x-axis is continuously coinciding with x' axis

$y = z = 0 \Rightarrow y' = z' = 0$

$a_{21}, a_{24}, a_{31}, a_{34} = 0$

$y' = a_{22} y + a_{23} z \quad ; \quad z' = a_{32} y + a_{33} z$

- $z = 0$ plane $\longrightarrow$ $x' - y'$ plane $(z' = 0)$

$\Rightarrow y' = a_{22} y \quad , \quad z' = a_{33} z$

- Rod, of unit length lying at rest along the y-axis of S frame

$y' = a_{22} \quad (y = 1)$. Same rod at rest in S' frame along y' axis

S observer sees the length of rod as of length

$$\left(y' = a_{22} y \right) \qquad y = \frac{1}{a_{22}}.$$

equivalence of frames, lengths measured should

$$\frac{1}{a_{22}} = a_{22} \implies a_{22} = 1.$$

$$\boxed{y' = y, \quad z' = z}$$

$$x' = a_{11}x + a_{12}y + a_{13}z + a_{14}t \quad \text{—} (1)$$

$$t' = a_{41}x + a_{42}y + a_{43}z + a_{44}t \quad \text{—} (4)$$

- Symmetry $\rightarrow$ t' independent of y, z
  place two clocks symmetrically in $y-z$ plane (e.g $+y$ & $-y$, $+z$, $-z$) about the x-axis would in general disagree as observed from S' violates isotropy, $a_{42} = a_{43} = 0$

$$t' = a_{41}x + a_{44}t \quad \text{—} (6)$$

eqn. (1)

$$x' = a_{11}x + a_{12}y + a_{13}z + a_{14}t.$$

we must satisfy

$$x' = 0, \quad x = vt, \quad a_{12} = a_{13} = 0$$

$$x' = a_{11}(x - vt) \quad \text{— (5)}$$

$$t' = a_{41}x + a_{44}t \quad \text{— (6)}$$

At $t = 0$, a spherical light wave leaves origin O of S, which coincides with O' at $t = 0$.

eqn. of wavefront eqn. of a sphere whose radius
/pands at rate c in both S and S'

$$x^2 + y^2 + z^2 = c^2 t^2 \quad — (7)$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2 \quad — (8)$$

Substitute into (8) transfm. eqn.s $y = y'$, $z = z'$
and (5) & (6)

$$a_{11}^2 (x - vt)^2 + y^2 + z^2 = c^2 (a_{41} x + a_{44} t)^2 .$$

$$(a_{11}^2 - c^2 a_{41}^2) x^2 + y^2 + z^2 - 2(v a_{11}^2 + c^2 a_{41} a_{42}) \cancel{x} t$$
$$= (c^2 a_{44}^2 - v^2 a_{11}^2) t^2$$

→ equivalent to (7)

$$c^2 a_{44}^2 - v^2 a_{11}^2 = c^2 \qquad — (9)$$

$$a_{11}^2 - c^2 a_{41}^2 = 1 \qquad — (10)$$

$$v^2 a_{11}^2 + c^2 a_{41} a_{44} = 0 \qquad — (11)$$

$$a_{44} = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad — (12)$$

$$a_{11} = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad — (13)$$

$$a_{41} = -v/c^2 \sqrt{1 - v^2/c^2} \qquad — (14)$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{y' = y}$$

$$z' = z$$

$$t' = \frac{t - \dfrac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

Lorentz transformation
(Boosts).

- If S and S' were switched $v \to -v$

$$\boxed{\begin{aligned} x &= \frac{x' + vt}{\sqrt{1-v^2/c^2}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + \frac{vx}{c^2}}{\sqrt{1-v^2/c^2}} \end{aligned}}$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}.$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

In the limit $v/c \ll$

$$\Longrightarrow \left.\begin{aligned} x' &= x - vt \\ t' &= t \end{aligned}\right\} \text{Recover Galilean transformations.}$$

- What happens if $v > c$, things become unphysical.

Rewrite L.T. in the following form.

$$\beta = v/c .$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} .$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} = \gamma (x - \beta ct) .$$

$$t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} = \gamma \left(t - \frac{\beta x}{c} \right)$$

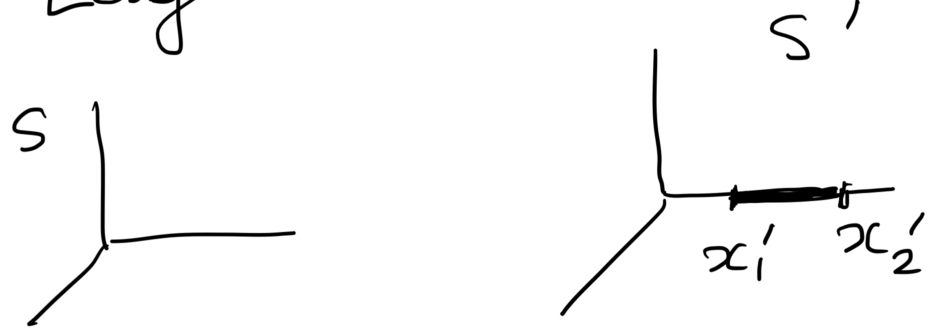
$$ct' = \gamma (ct - \beta x)$$

$$\boxed{\begin{aligned} x' &= \gamma(x - \beta ct) \\ ct' &= \gamma(ct - \beta x) \end{aligned}} \longrightarrow \text{Note the symmetry}$$

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \cdot \begin{pmatrix} x \\ ct \end{pmatrix}.$$

Consequences of L.T

1) Length contraction :



Rod lying at rest along x' axis of S' frame

Rest length $= x_2' - x_1'$.

Q: What is the rod's length measured by S.

$$x_2' = \frac{x_2 - v t_2}{\sqrt{1 - v^2/c^2}} = \gamma(x_2 - v t_2).$$

$$x_1' = \gamma(x_1 - v t_1).$$

Length of rod in S : Distance between the end points measured at the same time in that frame ($t_2 = t_1$).

$$x_2' - x_1' = \gamma(x_2 - x_1) - \gamma v\underbrace{(t_2 - t_1)}_{=0}$$

$$= \gamma(x_2 - x_1).$$

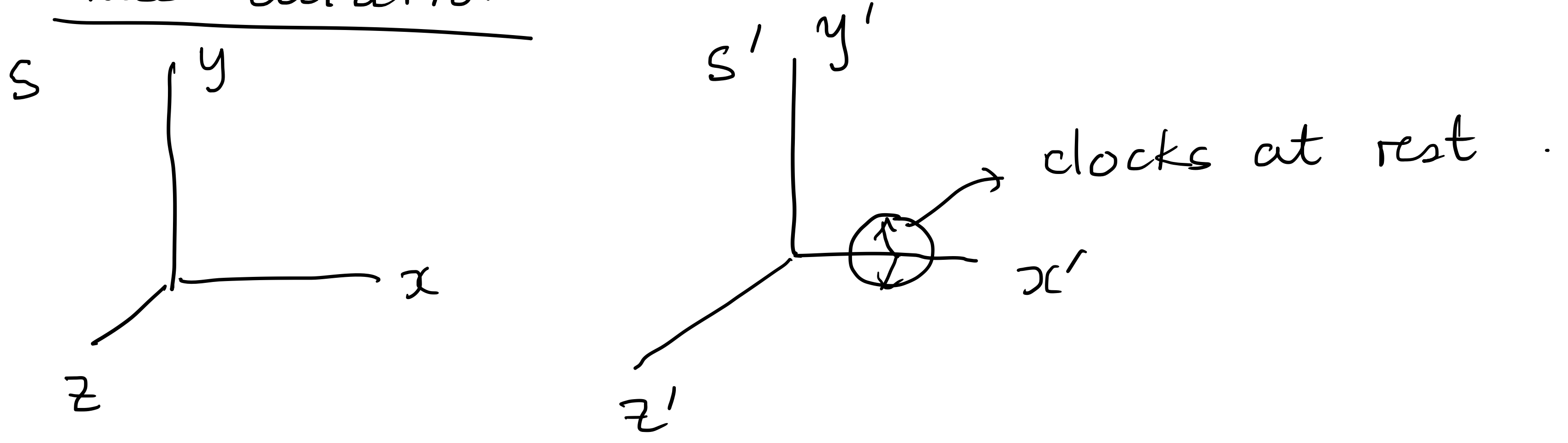
$$x_2 - x_1 = \frac{x_2' - x_1'}{\gamma}$$

Rest length $= L_0$, L : length seen from moving frame.

$$\boxed{L = \frac{L_0}{\gamma}} \qquad \gamma > 1 \quad \therefore \boxed{L < L_0}$$

$y = y'$, $z = z'$ $\rightarrow$ No length contraction in $\perp$ direction

Time dilation



time interval $t_2' - t_1'$ $\Rightarrow$ measured in S'.

what is the corresponding time interval in S.

$$t_2 = \gamma \left(t_2' + \frac{v x'}{c^2} \right)$$

x': same clock at rest

$$t_1 = \gamma \left(t_1' + \frac{v x'}{c^2} \right)$$

$$\boxed{t_2 - t_1 = \gamma (t_2' - t_1')} \Rightarrow \text{time intervals are dilated.}$$

Terminology

Frame in which body is at rest : proper frame

Length in this frame → proper length

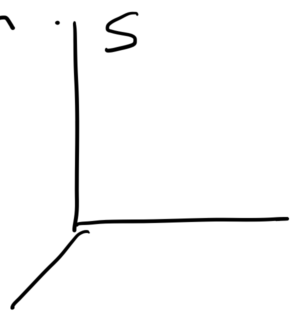
Time interval in this frame → proper time interval.

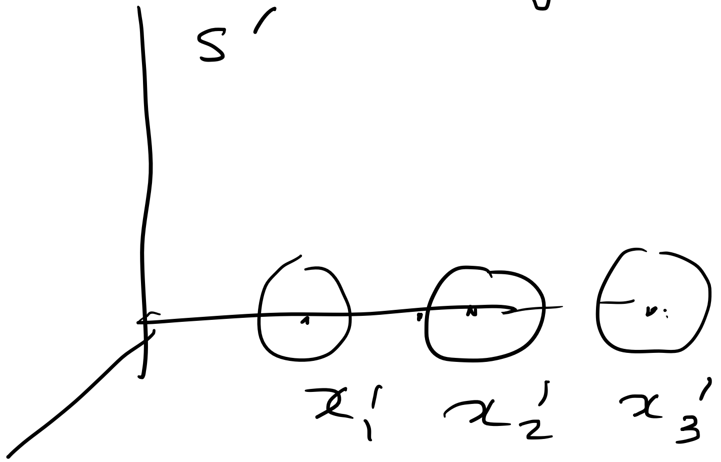
Infinitesimal version

$$dt = \gamma \, \underset{\sim}{d\tau} \rightarrow \text{proper time interval.}$$

(3) Clocks becoming unsynchronized

All clocks in a moving frame appear to go at the same slow rate when observed from a stationary frame, the moving clocks will appear to differ from each other in their readings depending on their location.





$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

take an instant of time in S frame t, then to satisfy the above

$t' + \dfrac{vx'}{c^2}$ must have a fixed value.

greater x' $\Rightarrow$ smaller t'.

④ <u>Simultaneity not absolute</u>

$$t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right)$$

$$t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right).$$

$$t_1 - t_2 = \gamma\left[(t_1' - t_2') + \frac{v}{c^2}(x_1' - x_2')\right].$$

$t_1' = t_2 \implies$ does not imply $t_1 = t_2$, if $x_1' \neq x_2'$