# Physics IV

Lectures 5-6

**Lorentz boost** (*x direction*)

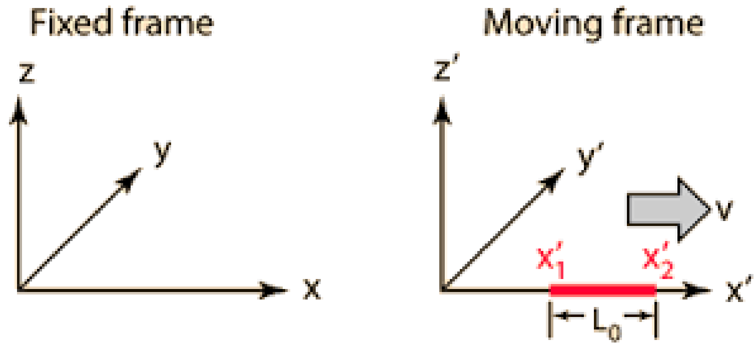
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$
$$x' = \gamma\left(x - vt\right)$$
$$y' = y$$
$$z' = z$$

**Inverse Lorentz boost** (*x direction*)

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$
$$x = \gamma\left(x' + vt'\right)$$
$$y = y'$$
$$z = z',$$

where $v$ is the relative velocity between frames in the $x$-direction, $c$ is the speed of light, and

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$




# Length Contraction

Lengths are shorter to observers who are moving relative to the object being measured.

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Length measurement of an observer moving relative to the object being measured

Rel. speed of object/observer

Speed of light

Length measurement of an observer at rest relative to the object being measured

# Time Dilation

If the time interval $T_0 = t'_2 - t'_1$ is measured in the moving reference frame, then $T = t_2 - t_1$ can be calculated using the Lorentz transformation.

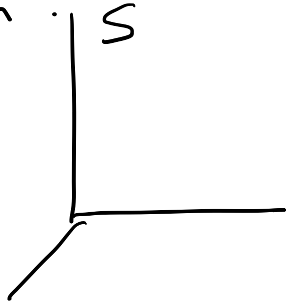
$$T = t_2 - t_1 = \frac{t'_2 + \frac{vx'_2}{c^2} - t'_1 - \frac{vx'_1}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

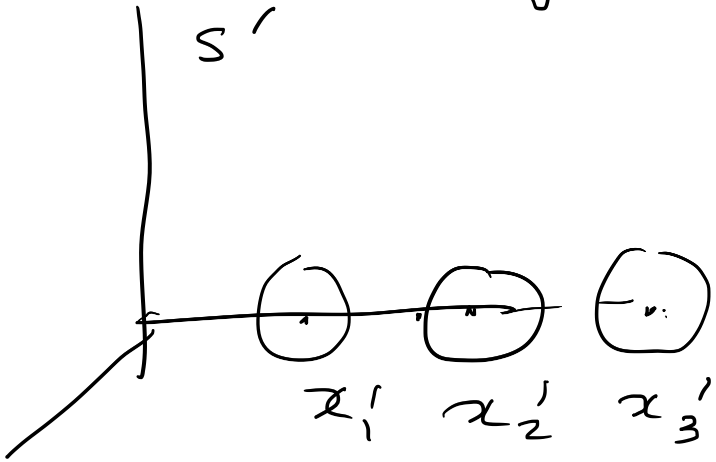
The time measurements made in the moving frame are made at the same location, so the expression reduces to:

$$T = \frac{T_0}{\sqrt{1 - \frac{v^2}{c^2}}} = T_0 \gamma$$

## ③ Clocks becoming unsynchronized

All clocks in a moving frame appear to go at the same slow rate when observed from a stationary frame, the moving clocks will appear to differ from each other in their readings depending on their location.




$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

take an instant of time in S frame $t$, then to satisfy the above,

$t' + \dfrac{vx'}{c^2}$ must have a fixed value.

greater $x'$ $\Rightarrow$ smaller $t'$.

④ **Simultaneity not absolute**

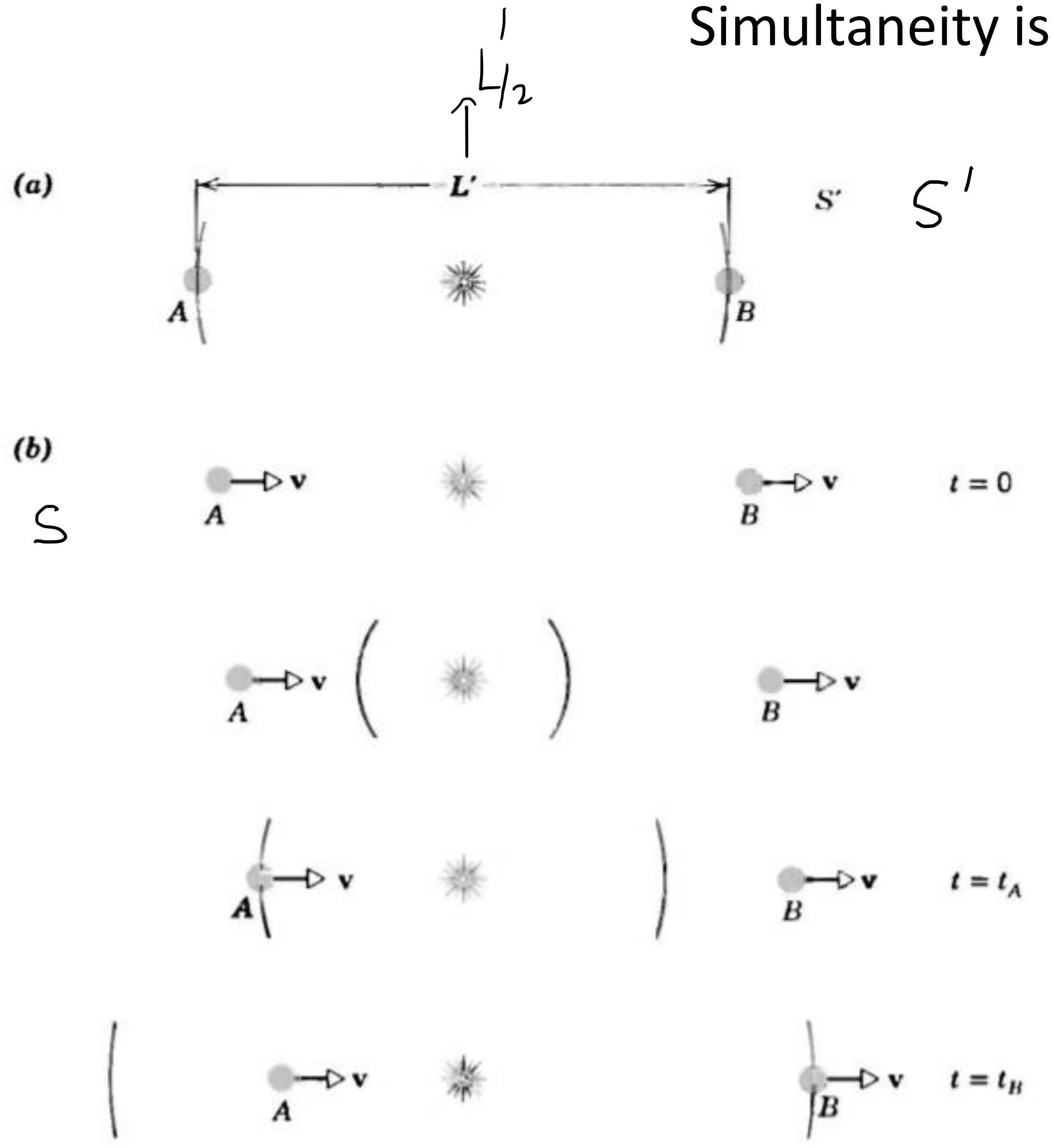
$$t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right)$$

$$t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right).$$

$$t_1 - t_2 = \gamma\left[(t_1' - t_2') + \frac{v}{c^2}(x_1' - x_2')\right].$$

$t_1' = t_2 \implies$ does not imply $t_1 = t_2$, if $x_1' \neq x_2'$

# Simultaneity is Relative .....




A, B synchoonize clocks.
A, B set clocks to $t = 0$, when they recive light signal.

$t = 0$

Not simultaneous in S.

How much do the $S'$ clocks differ in readings according to the S observer?

Let $t=0$ be the time S sees the flash go off.

at $t=t_A$, $\quad ct_A = \frac{L'}{2}\sqrt{1-v^2/c^2} - vt_A$

$t=t_B$ $\quad ct_B = \frac{L'}{2}\sqrt{1-v^2/c^2} + vt_B$.

$$\Delta t = t_B - t_A = \frac{L'}{2}\frac{\sqrt{1-v^2/c^2}}{c-v} - \frac{L'}{2}\frac{\sqrt{1-v^2/c^2}}{c+v}$$

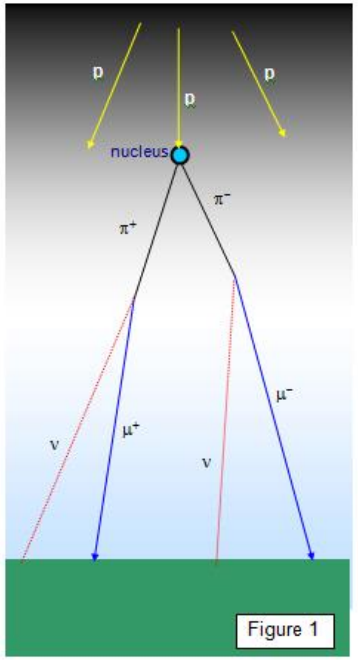
$$\Delta t = \frac{L' v\sqrt{1-v^2/c^2}}{c^2-v^2}.$$

During this interval, S observes clock A to run slow by a factor $\sqrt{1-v^2/c^2}$, so to observer S it will read

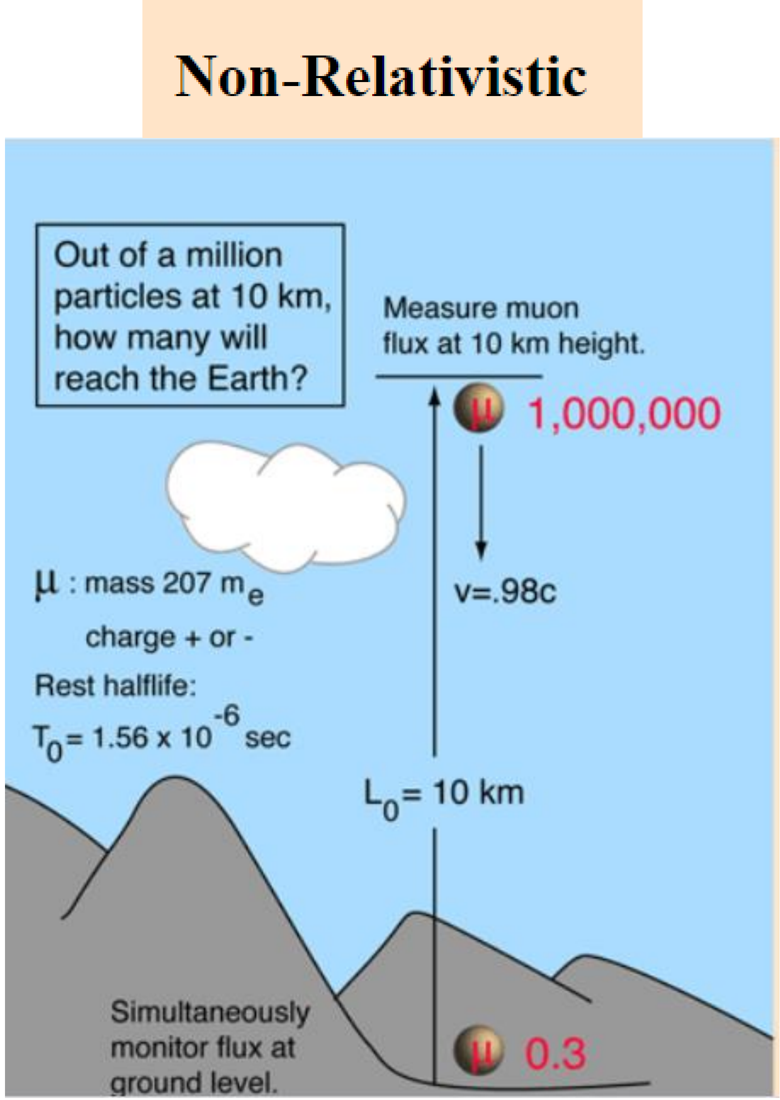
$$\boxed{\Delta t' = \Delta t\sqrt{1-v^2/c^2} = \frac{L'v}{c^2}}$$

When clock B is set to read $t' = 0$, S observer finds S' clocks to be out of sync. with clock A reading ahead of time by $\boxed{\frac{L'v}{c^2}}$

# Time Dilation is real ! : Muon lifetime




Expt shows that about 49,000 in a million survive!




Rest half life $= 1.56 \times 10^{-6}$ s.

$L_0 = 10^4$ m.

Time : $T = \dfrac{10^4 \text{ m}}{(0.98)(3\times10^8) \text{ m/s}}$

$= 34 \times 10^{-6}$ s

$= \dfrac{34 \times 10^{-6}}{1.56 \times 10^{-6}}$ half lives.

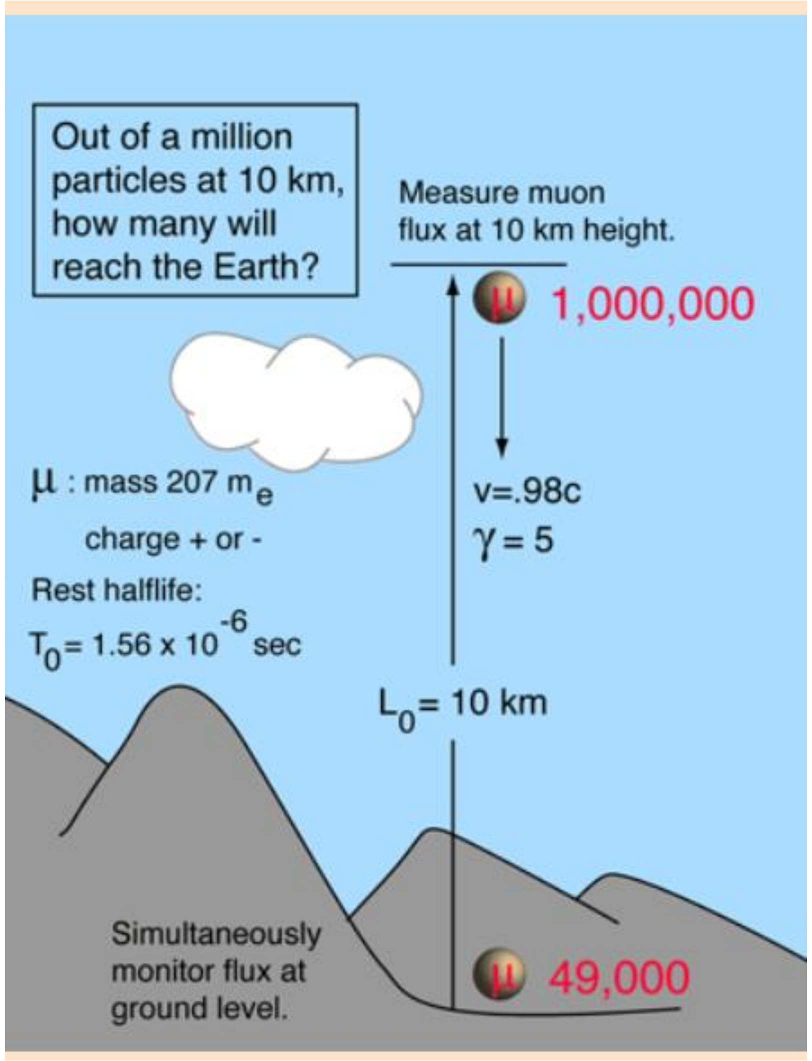
$= 21.8$ half lives

Survival rate

$\dfrac{I}{I_0} = 2^{-21.8} \simeq 0.27 \times 10^{-6}$

0.3 in a million survive!

# Relativistic, Earth-Frame Observer




Distance

$$L_0 = 10^4 \text{ m}.$$

$$\text{time } T = \frac{10^4 \text{m}}{(0.98)(3 \times 10^8 \text{ m/s})}$$

$\gamma = 5$.

$$= 34 \times 10^{-6} \text{s}$$

Time dilated half life

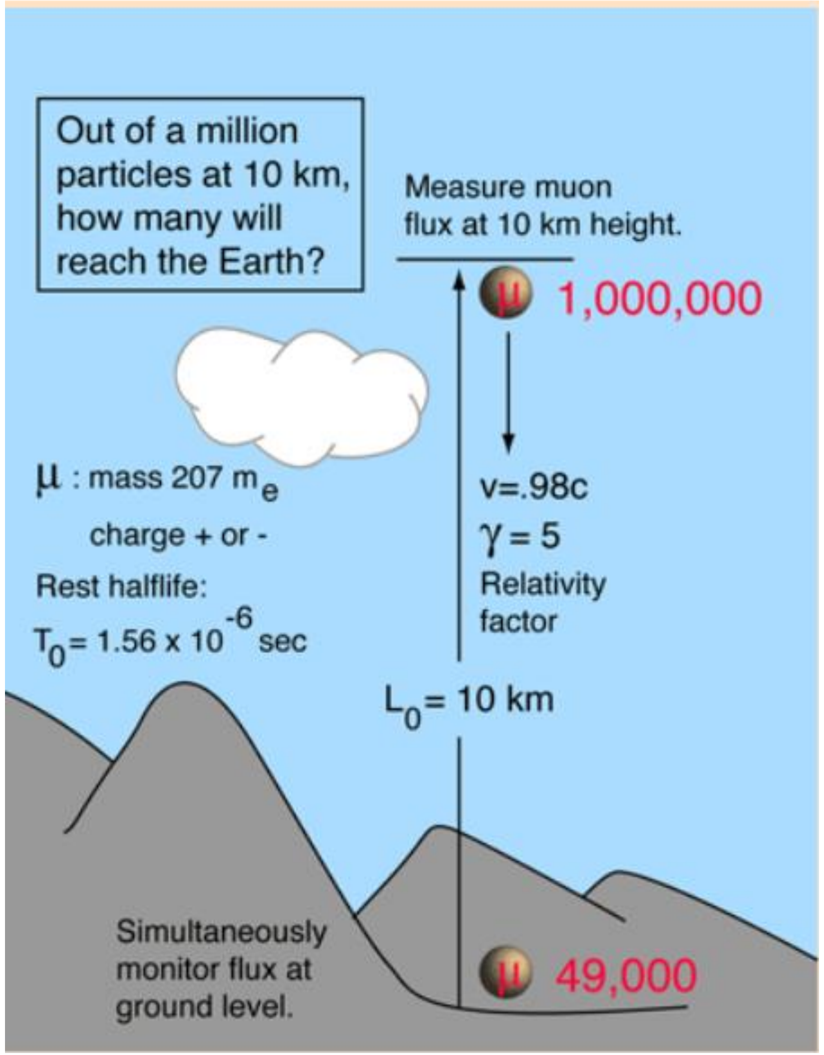
$$= 5 \times 1.56 \times 10^{-6} \text{s} = 7.8 \, \mu\text{s}.$$

$$T = \frac{34}{7.8} \text{ half lives} = 4.36 \text{ h.l}$$

$$\frac{I}{I_0} = 2^{-4.36} = 0.049.$$

49,000 per million.

# Relativistic, Muon-Frame Observer




$L_0 = 10\,km$

muon sees length contracted

$$L = \frac{10}{5} = 2\,km.$$

$$T = \frac{2000\,m}{(0.98)(3\times10^{8}\,m/s)}$$

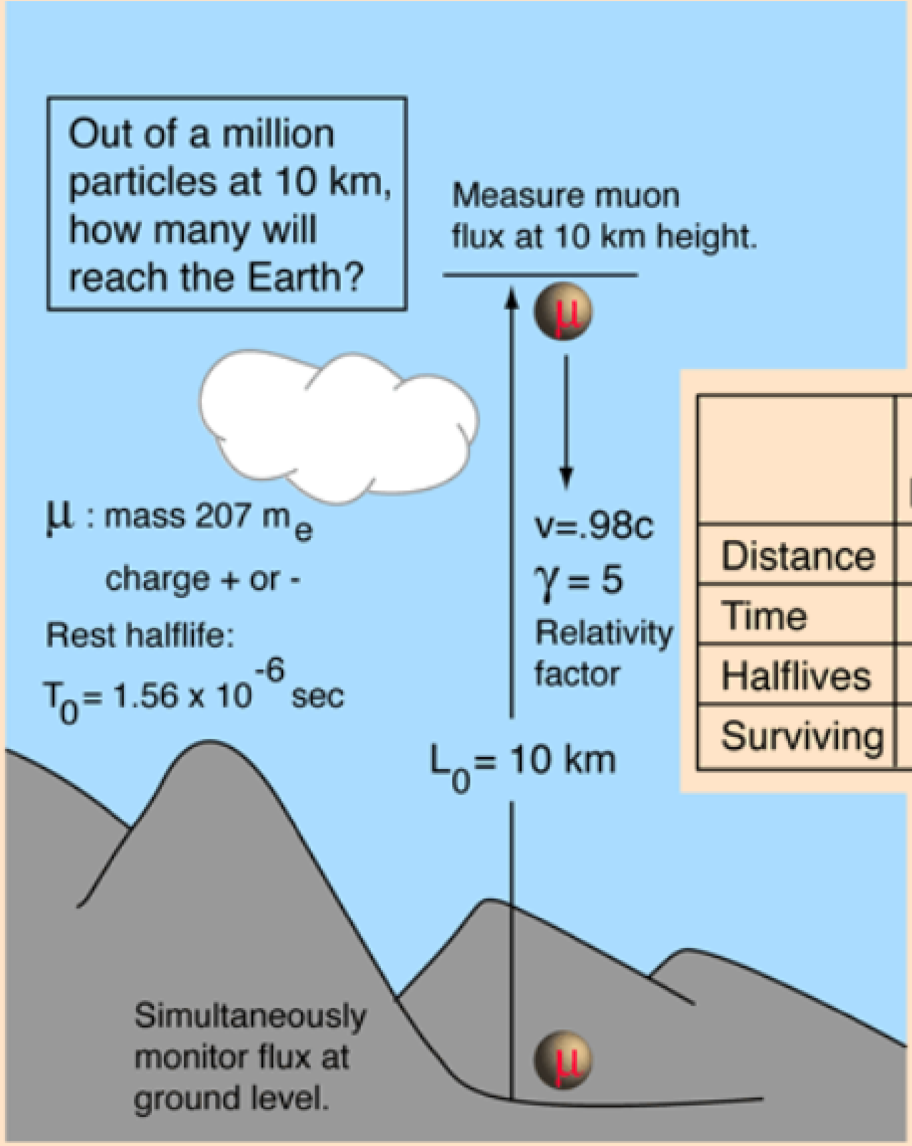
$$= 6.8\times10^{-6}$$

$$= \frac{6.8}{1.56} = 4.36.$$

$$\frac{I}{I_0} = 2^{-4.36} = 0.049.$$

# Muon Experiment

## Comparison of Reference Frames




By the basic principle of relativity, all valid descriptions must agree on the final result.

| | Relativistic Muon | Relativistic Ground | Non-Relativistic |
|---|---|---|---|
| Distance | 2 km | 10 km | 10 km |
| Time | 6.8μs | 34μs | 34μs |
| Halflives | 4.36 | 4.36 | 21.8 |
| Surviving | 49000 | 49000 | 0.3 |

Comparison of the three approaches to the muon survival rate.

In the muon experiment, the relativistic approach yields agreement with experiment and is greatly different from the non-relativistic result. Note that the muon and ground frames do not agree on the distance and time, but they agree on the final result. One observer sees time dilation, the other sees length contraction, but neither sees both.

# Relativistic snake

A relativistic snake of proper length 100 cm is moving at 0.6c to the right on a table. A naughty boy to tease the snake takes two hatchets 100 cm apart and plans to bounce them simultaneously on the table so that the left hatchet lands immediately behind the snake's tail.

$\boxed{\text{Will the snake be unharmed?}}$

- Boy's argument: snake moves at 0.6c. length is contracted by $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.36}} = \frac{5}{4}$.

Length in boy's frame $= \frac{100 \times 4}{5} = 80$ cm

$\Rightarrow$ right hatchet will fall 20 cm ahead of snake's head unharmed!

## Snake's story

$\Downarrow$ Separation between hatchets will be contracted to 80 cm, Since I am 100 cm long, my head will be cut off!

Paradox?

No: Simultaneity applied incorrectly.

Snake's frame $S'$, origin at $\underbrace{x' = 0}_{\text{tail}}$, head at $x' = 100$ cm.

Two hatchets at rest in $S$, $x = 0$ left and $x = 100$ right.

As observed in $S$ both hatchets bounce simultaneously at $t = 0$, at this time, tail is at $x = 0$, head must be at 80 cm

$$x = \gamma(x' + vt') \longrightarrow \text{can check.}$$

snake's argument is wrong.

Let us examine the coordinates of the bounce as observed in $S'$. Left hatchet falls at $t_L = 0$ and $x_L = 0$.

As seen in $S'$

$$t_L' = \gamma\left(t_L - \frac{v x_L}{c^2}\right) = 0$$
$$x_L' = \gamma\left(x_L - v t_L\right) = 0$$

falls immediatly behind tail

What about right hatchet

$t_R = 0$, $x_R = 100\ cm$.

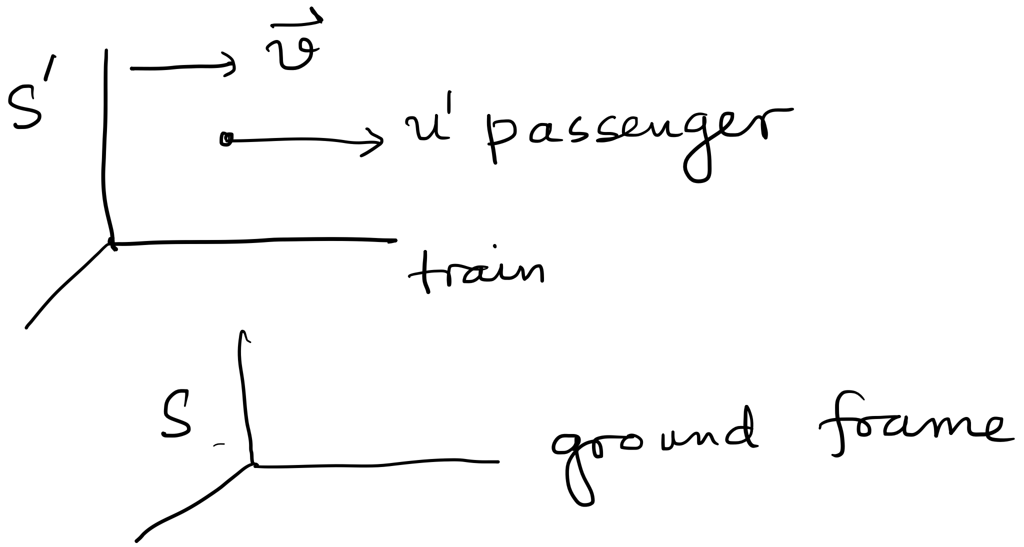
$$t_R' = \gamma\left(t_R - \frac{v x_R}{c^2}\right)$$
$$= \gamma\left(0 - \frac{0.6c \times 100\ cm}{c^2}\right) = -2.5\ ns$$

Do not fall simultaneously.

$$x_R' = \gamma(x_R - v t_R).$$
$$= \frac{5}{4}(100\,\text{cm} - 0)$$
$$= 125\,\text{cm}.$$

Hatchet definitely misses the snake!!

# Relativistic addition of velocity




passenger's vel. $\vec{u}$ w.r.t ground.

Galilean relativity.

$$\vec{u} = \vec{u}' + \vec{v}.$$

all vel. are along $x - x'$ direction.

$x' = u't'$ ⟶ in $S'$

$$x' = \gamma(x - vt) \quad , \quad t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

$$x' = u'\gamma\left(t - \frac{vx}{c^2}\right) = \gamma(x - vt).$$

Rearranging.

$$x - vt = u'\left(t - \frac{vx}{c^2}\right).$$

$$x = \frac{(u'+v)}{\left(1 + \frac{u'v}{c^2}\right)}\, t$$

Passenger's speed rel. to ground : $u$ ; $x = ut$.

$$ut = \frac{u'+v}{1 + \frac{u'v}{c^2}}\, t.$$

$$\boxed{u = \frac{u'+v}{1 + \frac{u'v}{c^2}}}$$

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

- symmetric w.r.t $u', v$
- $u'v \ll c^2$, $u \simeq u' + v$ as it should.
- If $u' = c$; $u = \frac{c + v}{1 + \frac{\cancel{c}v}{c^{\cancel{2}}}} = c$

Extend to 3D; velocity of frame S w.r.t S' is still along x axis

$$\boxed{u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}}$$

## Transverse vel. addition

Imagine object $\parallel$ to $y'$ axis in $S'$,

at $y_1'$ at $t_1'$, at $y_2'$ at $t_2'$.

vel. in $S'$ $= u_y' = \dfrac{\Delta y'}{\Delta t'} = \dfrac{y_2' - y_1'}{t_2' - t_1'}$.

Use L.T to find vel. in S.

$$\Delta y' = \Delta y$$

$$\Delta t' = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right).$$

$$\frac{\Delta y'}{\Delta t'} = \frac{\Delta y \sqrt{1 - v^2/c^2}}{\Delta t - \Delta x\, v^2/c^2} = \frac{\dfrac{\Delta y}{\Delta t}\sqrt{1 - v^2/c^2}}{1 - \left(\dfrac{\Delta x}{\Delta t}\right)\dfrac{v}{c^2}}$$

$$\frac{\Delta y'}{\Delta t'} = \frac{\dfrac{\Delta y}{\Delta t}\sqrt{1-v^2/c^2}}{1 - \left(\dfrac{\Delta x}{\Delta t}\right)\dfrac{v}{c^2}} .$$

$$\boxed{u'_y = \frac{u_y}{\gamma\left(1 - \dfrac{u_x v}{c^2}\right)}}$$

Inverse transfn:

$v \rightarrow -v$.

$$\boxed{u_y = \frac{u'_y}{\gamma\left(1 + \dfrac{u_x v}{c^2}\right)}}$$

$$\boxed{u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}}$$

$$\boxed{u_z = \frac{u_z'}{\gamma\left(1 + u_x' v / c^2\right)}}$$