Physics 4

Lecture 7 -8

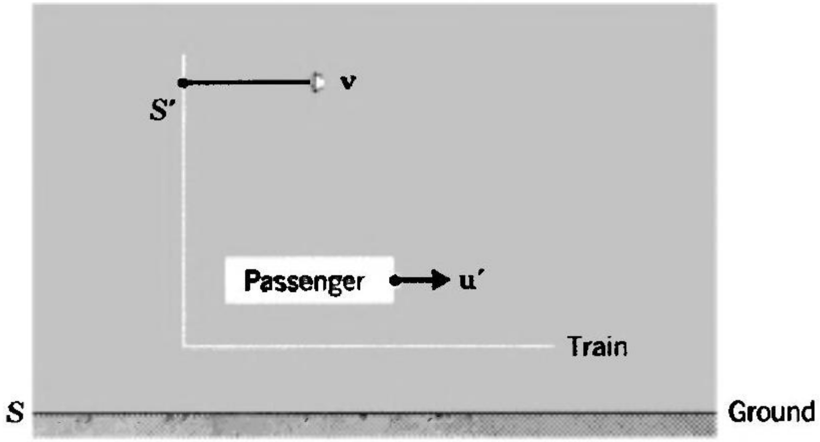


TABLE 2-2 THE RELATIVISTIC VELOCITY TRANSFORMATION EQUATIONS

$u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$	$u_x = \dfrac{u_x' + v}{1 + u_x' v/c^2}$
$u_y' = \dfrac{u_y \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$	$u_y = \dfrac{u_y' \sqrt{1 - v^2/c^2}}{1 + u_x' v/c^2}$
$u_z' = \dfrac{u_z \sqrt{1 - v^2/c^2}}{1 - u_x v/c^2}$	$u_z = \dfrac{u_z' \sqrt{1 - v^2/c^2}}{1 + u_x' v/c^2}$

Relativistic Acceleration Transformation Equation

$$a_x = \frac{du_x}{dt} \quad , \quad a_x' = \frac{du_x'}{dt'}$$

$$a_x' = \frac{a_x \left(1 - v^2/c^2\right)}{\left(1 - u_x v/c^2\right)} \qquad \text{work it out.}$$

$\downarrow$

accln. depends on ref. frame.

$a_x' = a_x$ in Galilean relativity
(v/c small).

$F = ma$ not correct eqn.

☆ In SR frames are inertial, objects can accelerate.

The invariant Interval

What is invariant in SR?

Not Δx, not Δt.

Consider the following quantity

$$(\Delta s)^2 = c^2 \Delta t^2 - (\Delta x)^2$$

[technically $(\Delta s)^2 = c \Delta t^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$]

$$c^2 t^2 - x^2$$

$$= \frac{c^2 \left(t' + \frac{v x'}{c^2} \right)^2}{1 - v^2/c^2} - \frac{(x' + v t')^2}{1 - v^2/c^2}$$

$$= \frac{t'^2 (c^2 - v^2) - x'^2 (1 - v^2/c^2)}{1 - v^2/c^2}$$

$$\therefore \quad \boxed{s^2 = c^2t^2 - x^2 = \underbrace{c^2t'^2 - x'^2}} \longrightarrow \text{INVARIANT}$$

$\longrightarrow$ invariant interval

Analogous to r^2 being invariant under rotations.

<u>Case 1</u> $s^2 > 0$ (timelike separation). Events (x, y, z, t) (x', y', z', t').

$c^2t^2 > x^2$

$\left|\frac{x}{t}\right| < c$.

L.T $x' = \gamma(x - vt)$.

Since $\left|\frac{x}{t}\right| < c$, there always exists a frame in which $x' = 0$.

If two events are timelike separated, it is always possible to find a frame where they happen in the same place.

Case 2 $\quad s^2 < 0 \quad$ spacelike separated.

$$s^2 = c^2t^2 - x^2.$$

$$c^2t^2 < x^2.$$

$$|t/x| < \frac{1}{c}.$$

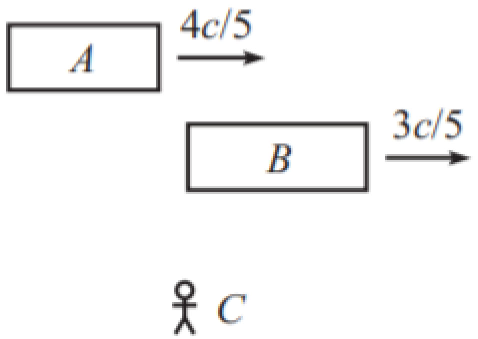
$$t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

→ Always possible to find a frame where $t' = 0 \longrightarrow$ simultaneous.

Case 3 $\quad s^2 = 0 \quad$ lightlike separated.

$$s^2 = 0 \implies c^2t^2 = x^2$$

Not possible to find any frame S' where the two events happen at the same time or the same place



Example (Passing trains): Two trains, A and B, each have proper length L and move in the same direction. A's speed is $4c/5$, and B's speed is $3c/5$. A starts behind B (see Fig. 11.16). How long, as viewed by person C on the ground, does it take for A to overtake B? By this we mean the time between the front of A passing the back of B, and the back of A passing the front of B.

$$\gamma_A = \frac{1}{\sqrt{1 - v_A^2/c^2}} = 5/3 \quad , \gamma_B = 5/4 .$$

Lengths in the ground frame $L_A = \frac{3L}{5}$, $L_B = \frac{4L}{5}$.

When overtaking B, A must travel farther than B by

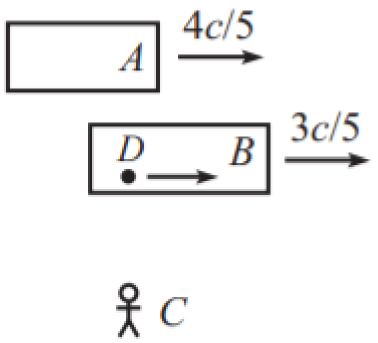
$$= L_A + L_B = \frac{7L}{5}$$

Rel. speed of the two trains as viewed by ground observer

$$C = \frac{4c}{5} - \frac{3c}{5} = \frac{c}{5}$$

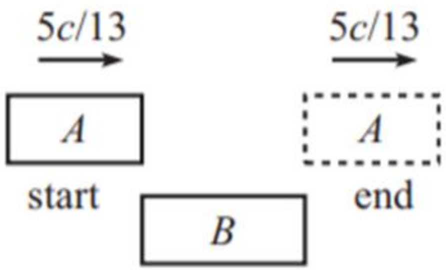
time taken to overtake B.

$$= \frac{7L/5}{c/5} = \frac{7L}{c}.$$



(a) How long, as viewed by A and as viewed by B, does it take for A to overtake B?

(b) Let event E_1 be "the front of A passing the back of B", and let event E_2 be "the back of A passing the front of B." Person D walks at constant speed from the back of B to the front (see Fig. 11.20), such that he coincides with both events E_1 and E_2. How long does the "overtaking" process take, as viewed by D?



B's frame

from B's perspective, B sees A move with speed

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}.$$

$$u = \frac{\frac{4c}{5} - \frac{3c}{5}}{1 - \frac{4}{5}\cdot\frac{3}{5}} = \frac{5c}{13}.$$

$\boxed{\gamma_u = 13/12}$ B sees A's length contracted to $\frac{12L}{13}$.

In overtaking, A must travel a dist $= \Sigma$ lengths in B's frame.

$$L + \frac{12L}{13} = \frac{25}{13}.$$

$\text{Dist} = \frac{25}{13} L$

speed of A seen by B $= \frac{5c}{13}$.

$$\boxed{\text{Total time in B's frame } t_B = \frac{25L/13}{5c/13} = \frac{5L}{c}}$$

Exact same reasoning holds from A's POV,

$$t_A = t_B = \frac{5L}{c}.$$

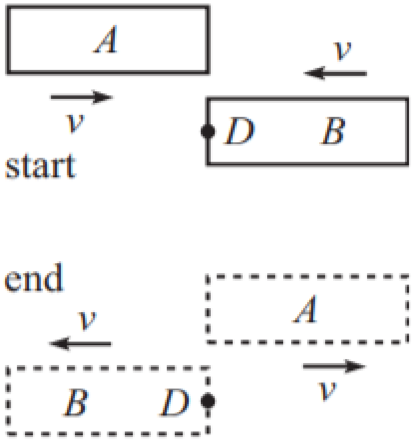


Figure 11.22

Look at things from D's POV.

D is at rest

The two trains are coming at him with equal and opposite speed v.

why equal and opposite?

otherwise would not coincide with E_2.

The rel. addition of vel. of v with itself is the speed of A as viewed by B; which we have calculated as $\frac{5c}{13}$.

addition of vel.

$$\frac{2v}{1+v^2/c^2} = \frac{5c}{13} \Rightarrow v = c/5 \quad \left(\text{ignored soln. } v = 5c\right)$$

Next step find γ_v

$$\gamma_v = \frac{1}{\sqrt{1 - \frac{1}{25}}} = \frac{5}{2\sqrt{6}}.$$

D will see both A, B contracted with same γ_v.

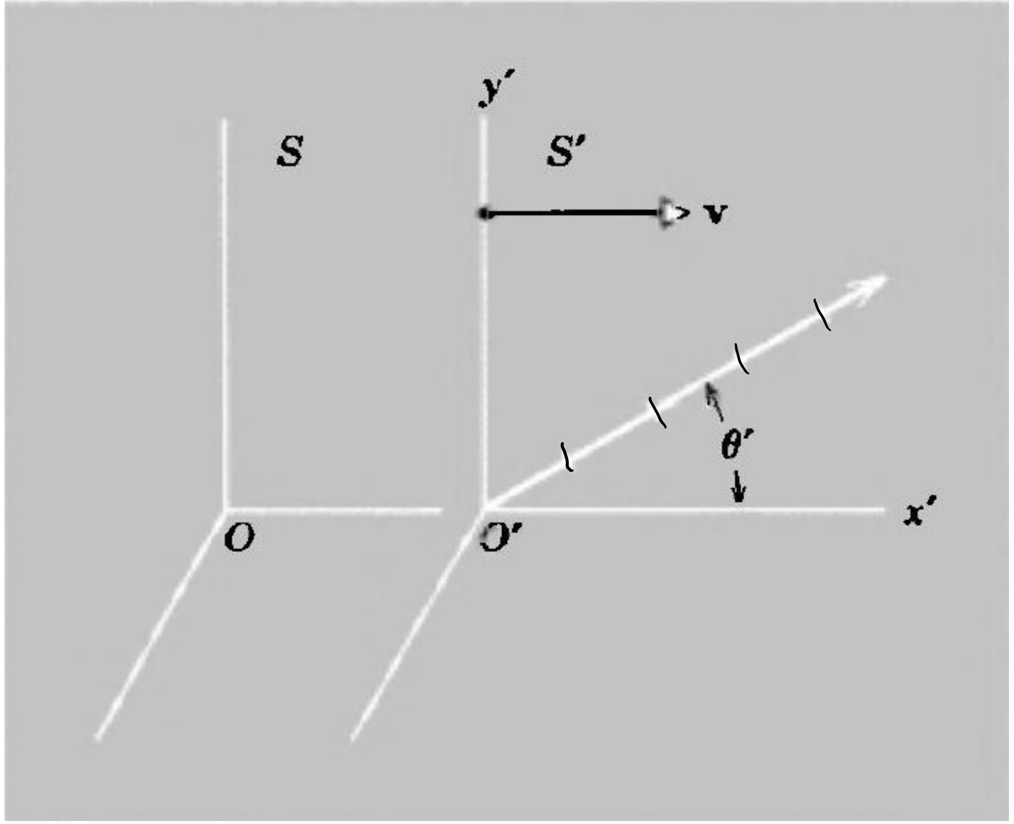
$$L_A = L_B = \frac{2\sqrt{6}\,L}{5}.$$

During the overtaking, each train has to travel a distance = its own length, because E_1 and E_2 take place at D.

Total time in D's frame

$$\boxed{t_D = \frac{2\sqrt{6}\,L/5}{c/5} = \frac{2\sqrt{6}\,L}{c}}$$

Relativistic Doppler Effect



From origin O′ of S′ sent out plane monochromatic light wave of unit amplitude. The rays are chosen to be in $x'-y'$ plane, making angle θ' with x'-axis

Representation of plane wave.

$\sim A\cos(\vec{k}'\cdot\vec{r}' - \omega' t')$. $A = 1$.

$$\sim \cos 2\pi\left[\frac{x'\cos\theta' + y'\sin\theta}{\lambda'} - \nu' t'\right] \quad \text{— (1)}$$

Moving with c in θ' direction

$$\boxed{c = \nu'\lambda' = \nu\lambda}$$

How will the S observer see the wave?

S frame wavefronts will still be planes since L.T is linear $\rightarrow$ transforms planes $\rightarrow$ planes.

S observer

$$\cos 2\pi \left[\frac{x\cos\theta + y\sin\theta}{\lambda} - \nu t \right] \quad \text{—} \ (2)$$

We are seeking transfors between θ an θ', ν and ν'; λ, λ'

Apply L.T to (1).

$$\cos 2\pi \left[\frac{1}{\lambda'} \frac{(x - vt)}{\sqrt{1-\beta^2}} \cos\theta' + y \frac{\sin\theta'}{\lambda'} - \nu' \frac{\left(t - \frac{vx}{c^2}\right)}{\sqrt{1-\beta^2}} \right]$$

$$\cos 2\pi \left[\frac{\cos\theta' + \beta}{\lambda'\sqrt{1-\beta^2}} x + \frac{\sin\theta'}{\lambda'} y - \frac{(\beta\cos\theta' + 1)}{\sqrt{1-\beta^2}} t \right] \ (3)$$

Compare (2) & (3)

We get

$$\frac{\cos\theta}{\lambda} = \frac{\cos\theta' + \beta}{\lambda'\sqrt{1-\beta^2}} \quad \text{—} \quad 3$$

$$\frac{\sin\theta}{\lambda} = \frac{\sin\theta'}{\lambda'} \quad \text{—} \quad 4 .$$

$$\nu = \nu' \frac{(1+\beta\cos\theta')}{\sqrt{1-\beta^2}} \quad \text{—} \quad 5 .$$

Also

$$\nu\lambda = \nu'\lambda' = c \quad \text{—} \quad 6 .$$

$\lambda', \theta', \nu' \longrightarrow$ four eqns

some redundancy, overdetermined.

Taking ratio of 3, 4

$$\boxed{\tan\theta = \frac{\sin\theta'\sqrt{1-\beta^2}}{\cos\theta + \beta}} \quad \text{— } 7$$

$$\boxed{\nu = \frac{\nu'(1+\beta\cos\theta')}{\sqrt{1-\beta^2}}} \quad \text{— } 5$$

Rel. formula reduces to class. form.

$$\nu' = \frac{\nu(1-\beta\cos\theta)}{\sqrt{1-\beta^2}}.$$

$$\nu = \frac{\nu'\sqrt{1-\beta^2}}{1-\beta\cos\theta} \simeq \frac{\nu'}{1-\beta\cos\theta} \simeq \underline{\underline{\nu'(1+\beta\cos\theta)}}.$$

classical result ✓.

$\theta = 0$.

$$\nu = \nu'(1+\beta) = \nu'\left(1+\frac{v}{c}\right).$$

$\theta = 180^\circ$

$$\nu = \nu'(1 - v/c).$$

$\boxed{\theta = 90^\circ, \quad \nu = \nu'}$ ⟶ No transvere Doppler

$$\nu = \frac{\nu' \sqrt{1-\beta^2}}{1 - \beta \cos\theta}$$

$\theta = 0$, $\theta = 180^\circ$

$$\nu = \nu' \sqrt{\frac{1+\beta}{1-\beta}} \quad \theta = 0$$

$\theta = 180^\circ$

$$\nu = \nu' \sqrt{\frac{1-\beta}{1+\beta}}.$$

$\underline{\theta = 90^\circ}$

$$\boxed{\nu = \nu' \sqrt{1-\beta^2}}$$ transverse Doppler effect is pure relativistic effect