Physics IV
ISI B.Math
HW set 4

1. Show that in the nth stationary state of the harmonic oscillator, the average (expectation value) of the kinetic energy $< T >$ is equal to the average potential energy $< V >$ (Virial theorem). What is the lowest average kinetic energy a quantum harmonic oscillator can have ?

2. For a harmonic oscillator of mass m and angular frequency ω, show that $< x > (t) = A\cos\omega t + B\sin\omega t$ where A and B are constants.

3. For a particle moving in one dimension, show that the operator $\hat{x}\hat{p}$ is not Hermitian. Construct an operator which corresponds to this physically observable product that *is* Hermitian.

4. Consider an operator $\hat{A}$ whose commutator with the Hamiltonian $\hat{H}$ is a constant c , i.e, $[\hat{H}, \hat{A}] = c$. Find $< \hat{A} >$ at $t > 0$, given that the system is in a normalized eigenstate of $\hat{A}$ at $t = 0$ corresponding to the eigenvalue a.

5. A free particle of mass m in one dimension is known to be in the initial state

$$\psi(x, 0) = \sin k_0 x$$

(a) What is $\psi(x, t)$?

(b) What values of p will a measurement yield at time t and with what probabilities do these values occur ?

(c) Suppose that p is measured at $t = 3s$ and the value $\hbar k_0$ is found. What is $\psi(x, t)$ at $t > 3s$.?

6. Show that $[f(x), \hat{p}] = i\hbar\frac{df}{dx}$ for any function f.