- **Inner product**
    - $(|v\rangle, \lambda|w\rangle) = \lambda(|v\rangle, |w\rangle)$
    - $(\lambda|v\rangle, |w\rangle) = \lambda*(|v\rangle, |w\rangle)$
    - $(|v\rangle, |w\rangle) = (|w\rangle, |v\rangle)*$
- Dual vector is a linear operator from $V \to \mathbb{C}$ since $(\langle v|)|w\rangle = \langle v|w\rangle$. It's a row vector. **So, row vector is an operator?**
- **Orthogonal vectors**: $(|v\rangle, |w\rangle) = 0$
- **Orthonormal basis set** is complete
    - $\sum_i |i\rangle\langle i| = I$
- **Gram–Schmidt method** makes a basis set orthonormal.
- **Outer-product representation**
    - $A = I_W A I_V = \sum_{ij} |w_j\rangle\langle w_j|A|v_i\rangle\langle v_i| = \sum_{ij} \langle w_j|A|v_i\rangle |w_j\rangle\langle v_i|$
- Outer product representation reduces to **diagonal representation** (eigendecomposition) when $|v_i\rangle, |w_j\rangle$ are orthonormal eigenvalues of A (do they need to form a basis?)
    - $A = \sum_i \lambda_i |i\rangle\langle i|$. Example: **Hamiltonian** operator! $H = \sum_E E|E\rangle\langle E|$
- **Adjoint vector** $|v\rangle^\dagger \equiv \langle v|$
- **Adjoint operator**
    - $(|v\rangle, A|w\rangle) = (A^\dagger|v\rangle, |w\rangle)$.
    - Some properties:
        - For matrix rep. $A^\dagger = (A*)^{\mathrm{T}}$
        - $(AB)^\dagger = B^\dagger A^\dagger$
        - $(A^\dagger)^\dagger = A$
        - $\left(\sum_i a_i A_i\right)^\dagger = \left(\sum_i a_i^* A_i^\dagger\right)$
- **Self-adjoint/Hermitian operator:** $A^\dagger = A$
- **Simultaneous diagonalization theorem**: $[A, B] = 0$ iff there exists an orthonormal basis in which A, B are both diagonal. Important for **uncertainty relation**. (***Check***: is this the **complete set of commutating operators** Cohen Tannoudji keeps mentioning?)
- **Projector operator**: Hermitian operator that projects to a subspace
    - $P = \sum_{i=1}^{k} |i\rangle\langle i|$, $P^\dagger = P$, $P^2 = P$
    - Special case: **Measurement operator** $M = \sum_i \lambda_i P_i$
-
- Pauli matrices
    - $\sigma_1 \equiv \sigma_x \equiv X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\sigma_1 \equiv \sigma_x \equiv X \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
    - $\sigma_2 \equiv \sigma_y \equiv Y \equiv \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\sigma_3 \equiv \sigma_z \equiv Z \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$