Discrete Structures (MA5.101)

Quiz - 2 (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 60 Minutes Total Marks: 30

Instructions: This is online examination.

Write at the top of your answer book the following:

Discrete Structures (MA5.101)

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Date: 27-Dec-2021

Name:

Roll Number:

Submit your scanned hand-written answer script in the moodle with the file name: RollNo_Quiz2_SecNo_27Dec2021.pdf

December 30, 2021

1. Choose the correct option for the following questions:

(a) Let a set A have 11 district elements, then the number of binary relations on A are ________.

(A) 121
(B) 2^{11}
(C) 121^2
(D) 2^{121}

Answer: (D)
Explanation: A is the set with 11 elements. A relation on A is defined as $A \times A$. There are 11^2 number of ordered pairs in relation. So, the number of binary relations is $2^(11 * 11) = 2^121$.

(b) The rank of smallest equivalence relation on a set S, where S contains 12 distinct elements is ________.

(A) 12
(B) 144
(C) 132
(D) 156

Answer: (A)
Explanation: In the case of smallest equivalence relation, each element is in one equivalence class like $\{a_1\}$, $\{a_2\}$, $\cdots$ are equivalence classes. So, the rank or number of equivalence classes is n for a set with n elements and so the answer is 12.

(c) Let S be a set of the elements $\{1, 2, 3, 4, 5\}$. The transitive closure of the relation $\{(0, 1)$, $(1, 2)$, $(2, 2)$, $(3, 4)$, $(5, 3)$, $(5, 4)\}$ on the set S is ________.

(A) $\{(0,1),(1,2),(2,2),(3,4)\}$

(B) $\{(0,0),(1,1),(2,2),(3,3),(4,4),(5,5)\}$

(C) $\{(0,1),(1,1),(2,2),(5,3),(5,4)\}$

(D) $\{(0,1),(0,2),(1,2),(2,2),(3,4),(5,3),(5,4)\}$

Answer: (D)

Explanation: Let R be a relation on a set A. The connectivity relation on R^* consists of pairs (a,b) such that there is a path of length at least one from a to b in R. Mathematically, $R^* = R^1 \cup R^2 \cup R^3 \cup \cdots \cup R^n$. Hence the answer is $\{(0,1),(0,2),(1,2),(2,2),(3,4),(5,3),(5,4)\}$.

(d) Let a relation R is defined on $\mathbb{Z}$ (set of integers) as ${}_xR_y$ iff $x+y$ is even and R is known as __________.

(A) an equivalence relation with one equivalence class

(B) an equivalence relation with three equivalence classes

(C) an equivalence relation with two equivalence classes

(D) an equivalence relation

Answer: (C)

Explanation: R is reflexive as (a+b) is even for any integer; R is symmetric as if (a+b) is even (b+a) is also even; R is transitive as if ((a+b)+c) is even, then (a+(b+c)) is also even. So, R is an equivalence relation. For set of natural numbers, sum of even numbers always give even, sum of odd numbers always give even and sum of any even and any odd number always give odd. So, must have two equivalence classes -¿ one for even and one for odd. $\ldots$, -4, -2, 0, 2, $\ldots$ and $\ldots$, -3, -1, 1, 3, $\ldots$.

(e) Let S be a set have n elements and R be a binary relation on the set S. Then, the time complexity for computing the transitive closure of R should be __________.

(A) $O(n)$

(B) $O(n^3)$

(C) $O(n^{(n+3/2)})$

(D) $O(\log n)$

Answer: (B)

Explanation: Calculation of transitive closure results into matrix multiplication. We can do matrix multiplication in $O(n^3)$ time. There are better algorithms that do less than cubic time.

(f) Let a relation R is defined as $R = \{(x,y)|y = x-1,\ \&\ x,y \in \{1,2,3\}\}$. Then, the reflexive transitive closure of R is __________.

(A) $\{(x,y)|x \geq y\ \&\ x,y \in \{1,2,3\}\}$

(B) $\{(x,y)|x = y\ \&\ x,y \in \{1,2,3\}\}$

(C) $\{(x,y)|x > y\ \&\ x,y \in \{1,2,3\}\}$

(C) $\{(x,y)|x \leq y\ \&\ x,y \in \{1,2,3\}\}$

Answer: (A)

(g) A partial order $\leq$ is defined on the set $S = \{x, b_1, b_2, \cdots b_n, y\}$ as $x \leq b_i$ for all i and $b_i \leq y$ for all i, where $n \geq 1$. The number of total orders on the set S which contain the partial order $\leq$ is __________.

(A) $n+4$

(B) $n!$

(C) n^2

(D) n^3

Answer: (B)

(h) Let a relation R on $\mathbb{Z}$ and define as $(a, b) \in R | a \geq b^2$. Then R is __________.

(A) Not transitive

(B) Antisymmetric

(C) Symmetric

(D) Not reflexive

Answer: (B), (D)
(D) (Not reflexive because we can't have (2,2).)
Not symmetric because if we have (9,3), we can't have (3,9).
(B) Antisymmetric, because each integer will map to another integer but not in reverse (besides 0 and 1).
Is transitive because if $a \geq b^2$ and $b \geq c^2$, then $a \geq c^2$

(i) For $x, y \in \mathbb{Z}$ defined as $x|y$, which means that x divides y is a relation which does not satisfy __________.

(A) reflexive and symmetric relations

(B) symmetric relation

(C) transitive relation

(D) irreflexive and symmetric relation

Answer: (A), (B)

(j) For $x, y \in R$ defined as $x = y$, which means that $|x| = |y|$. If $[x]$ is an equivalence relation in R, then the equivalence relation for $[17]$ is __________.

(A) $\{, \cdots, -11, -7, 0, 7, 11, \cdots\}$

(B) $\{, \cdots, -17, 0, 17, \cdots\}$

(C) $\{2, 4, 9, 11, 15, \cdots\}$

(D) $\{-17, 17\}$

Answer: (D)
Explanation: We can find that $[17] = \{a \in R | a = 17\} = \{a \in R || a| = |17|\} = \{-17, 17\}$ and $[-17] = \{a \in R | a = -17\} = \{a \in R || a| = | - 17|\} = \{-17, 17\}$. Hence, the required equivalence relation is $\{-17, 17\}$.

[$10 \times 1 = 10$]

2. Consider the set $S = \{1, 2, 3, 4\}$ and a relation R defined in it as $R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$.

(i) If the symmetric closure of R is exist then find out, else why not.
Answer: $\{(1, 1), (1, 3), (1, 4), (2, 3), (3, 1), (3, 2), (3, 4), (4, 1), (4, 3)\}$

(ii) What about the transitive closure of R.
Answer: $\{(1, 1), (1, 4), (2, 1), (2, 3), (2, 4), (3, 1), (3, 4)\}$

[5 + 5 = 10]

3. (a) Prove that a relation R defined on a set A is an equivalence relation, if and only if R is reflexive and such that ${}_aR_b$ and ${}_bR_c$ imply ${}_cR_a$, for $a, b, c \in A$.

(b) Let S be a set and let R be a binary relation on S. Then, prove or disprove $R \cup R^{-1}$ is the smallest symmetric relation containing the relation R.

Answer:

(b) We must prove (i) that $R \cup R^{-1}$ is symmetric and (ii) that if S is a symmetric relation on X and $R \subseteq S$, then $R \cup R^{-1} \subseteq S$.

(i) It is enough to show that $(R \cup R^{-1})^{-1} = R \cup R^{-1}$ since the result then follows from Theorem 3 in Section 3.4.2.

$$\begin{aligned}(R \cup R^{-1})^{-1} &= R^{-1} \cup (R^{-1})^{-1} \\ &= R^{-1} \cup R \\ &= R \cup R^{-1}\end{aligned}$$

(ii) Suppose S is a symmetric relation on X and $R \subseteq S$. We must show that $R^{-1} \subseteq S$. By Theorem 2 (c) in Section 3.2.1, $R^{-1} \subseteq S^{-1}$, and by Theorem 3 in Section 3.4.2, $S^{-1} = S$. So, $R^{-1} \subseteq S$.

[5 + 5 = 10]

******************* End of Question Paper **********************