$a \mid b$ and $b \mid a \Rightarrow a = \pm b$

$\boxed{a = b}$

No. of equivalence relations on A, $|A| = n$,

= No. of partitions of A

$= \sum_{r=1}^{n} S(n, r)$

Stirling number: Recurrence Relation

$S(n,1) = 1, \quad S(n,n) = 1$

$S(n,r) = S(n-1, r-1) + r \cdot S(n-1, r), \quad 1 < r < n.$

$n = 5$

No. of equivalence relations $= \sum_{r=1}^{5} S(5, r)$

$= \underbrace{S(5,1)}_{1} + \underbrace{S(5,2)}_{15} + \underbrace{S(5,3)}_{25} + \underbrace{S(5,4)}_{10} + \underbrace{S(5,5)}_{1}$

$= 52$

Now, $S(5,2) = \overset{1}{S(4,1)} + 2 \cdot S(4,\overset{7}{2}) \rightarrow (1)$

$S(5,3) = S(4,\overset{7}{2}) + 3 \cdot S(4,\overset{6}{3}) \rightarrow (2)$

$S(5,4) = S(4,\underset{6}{3}) + 4 \cdot \underbrace{S(4,4)}_{1} \rightarrow (3)$

$S(4,2) = S(3,1) + 2 \cdot S(3,2)$

$= 1 + 2 [S(2,1) + 2 \cdot S(2,2)]$

$= 1 + 2(1 + 2 \cdot 1) = 7$

$S(4,3) = S(3,2) + 3 \cdot S(3,3)$

$= 3 + 3 \cdot 1 = 6$

$a \mid b \Rightarrow b = ax$

$b \mid c \Rightarrow c = by$

$c = (ax)y = a \cdot z, \quad z = xy$

$\Rightarrow a \mid c$