Class Assignment - 2a

MA2.101: Linear Algebra (Spring 2022)

Deadline: April 11, 2022

Instructions

- All questions are compulsory.
- Upload scanned copies of handwritten solutions on Moodle.

Question 1

Prove that if A is a matrix of size $m \times n$ and $m < n$, then the homogeneous system of linear equations $Ax = 0$ has a non-trivial solution.

Question 2

Prove that if A is a square matrix of size $n \times n$ then A is row equivalent to $\mathbb{I}_{n\times n}$ iff *(if and only if)* the system of linear equations $Ax = 0$ only has the trivial solution.

Question 3

If A, B, C are matrices over a field $\mathbb{F}$ such that the products BC and $A(BC)$ are well defined, and so are the products AB and $(AB)C$, then prove

$$A(BC) = (AB)C$$

Question 4

Let e be an elementary row operation and let E be an elementary matrix of size $m \times m$ such that $E = e(\mathbb{I}_{m\times m})$ then prove that

$$e(A) = EA$$

holds $\forall$ matrices A of size $m \times n$.