Discrete Structures (MA5.101)

Quiz - 3 (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 60 Minutes Total Marks: 30

Instructions: This is online examination.

Write at the top of your answer book the following:

Discrete Structures (MA5.101)

Quiz - 3 (Monsoon 2021)

Date: 7-Feb-2022

Name:

Roll Number:

Submit your scanned hand-written answer script in the moodle with the file name: RollNo_Quiz3_SecNo_7Feb2022.pdf

1. Answer the following questions:

   (a) Let $\alpha = \frac{1+\sqrt{5}}{2}$ and F_n the n^{th} Fibonacci's number. Then $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} =$ _____.

   (b) Let S be an infinite set and $x \notin S$. Then, the connection between the cardinalities of S and $S \cup \{x\}$ is _____.

   (c) If a is a discrete numeric function, then $S^{-1}(\nabla a) =$ _____.

   (d) The permutation

   $$p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

   is _____ (even/odd).

   (e) The generating function of the recurrence relation $a_k - 7a_{k-1} + 10a_{k-2} = 0$ is _____.

   [$5 \times 1 = 5$]

2. (a) **(Countable Sets)** Given a set of $(n+1)$ positive integers, none of which exceeds $2n$, show that at least one number of the set must divide another member of the set.

   [Hint: Any positive integer p can be expressed uniquely as $p = 2^k.m$ where $k \geq 0$ and m is an odd positive integer]

   (b) **(Mathematical Induction)** A rubber costs Rs. 5 and a ball pen costs Rs. 9. Show by using the principle of mathematical induction that any amount, in exact rupees, exceeding Rs. 31 can be spent in buying rubbers and ball pens.

   (c) **(Permutations)** Show that, if p is an arbitrary permutation and q is the cycle $(1\ 2 \cdots\ i)$, then the permutation $q^{-1} \circ p \circ q$ has the same cycle structure as p.

   [5 + 5 + 5 = 15]

3. **(Recurrence Relations and Generating Functions)**

(a) Let a, b be the numeric functions such that $b = \triangle a$. Derive the generating function of b.

(b) Every particle inside a nuclear reactor splits into two particles in each second. Suppose one particle is injected into the reactor every second beginning at time $t = 0$.
(i) Express the number of particles a_n in the reactor at the n^{th} second as a discrete numeric function.
(ii) Derive a_n using the generating function.

[3 + (2 + 5) = 10]

********** **End of Question Paper** **********