Tutorial Questions

1. In Z, we define a * b = a + b + 1, show that (Z, *) is an abelian group
2. Show that cube root of unity is an abelian group under multiplication
3. If G = {f_1, f_2, f_3, f_4} of four functions defined by f_1 = x, f_2 = -x, f_3 = 1/x, f_4 = -1/x for all x ϵ R - {0} is an abelian group
4. Show that H = {0, 2, 4} is a subgroup of the group G = {0, 1, 2, 3, 4, 5} under addition modulo 6
5. Prove that H = {a+ib | a, b ϵ Q} is a subgroup of the group (C, +)
6. If a is an element of a group G, then prove that its normalizer N(a) = {x ϵ G | ax = xa} is a subgroup of G
7. Find all the cosets of 3Z in the group (Z, +)
8. Find all the cosets of H = {0, 4} in the group G = (Z_8, $+_8$)
9. If H is a subgroup of a group G and g ϵ G, then prove that:
    a. gHg^{-1} = {ghg^{-1} | h ϵ H} is a subgroup of G
    b. If H is finite, then O(H) = O(gHg^{-1})
10. Show that the additive group (R, +) of real numbers is isomorphic to the multiplicative group (R^+, X) of positive real numbers
11. Prove that the set G = {x | x^n = 1} of n^{th} roots of unity is a finite multiplicative tcyclic group of order n
12. Find all the generators of the cyclic group (G = {0, 1, 2, 3, 4, 5}, $+_6$)
13. Find all the generators of the cyclic group (G = {1, 2, 3, 4}, x_5)
14. Let H be a set of all 2x2 matrices of the form [[a, b], [0, d]], with a, b, d ϵ R and ad ≠ 0
    a. Show that H is a subgroup of GL_2(R)
    b. Is H a normal subgroup of GL_2(R)?
15. Let the mapping f be f: (C_0, x) -> (R_0, x), f(z) = |z| for all z ϵ C_0.
    a. Is mapping f a homomorphism?
    b. If yes, what is the Ker (f)