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# Discrete Structures (Monsoon 2021)


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# Discrete Numeric Functions and Generating Functions

# Numeric Function

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## Definition

A numeric function $a$ is written as $a_0, a_1, a_2, \cdots, a_r, \cdots$ to denote the values of the function at $0, 1, 2, \cdots, r, \cdots$

**Example:** $a_r = 7r^3 + 1, r \geq 0$.
Then, $a = (1, 8, 57, 190, 449, 876, 1513, 2402, 3585, 5104, 7001, \cdots)$

# Numeric Function

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## Right Shift

- Let $a = (a_0, a_1, a_2, \cdots, a_r, \cdots)$ be a numeric function and $i$ be a positive integer.
- $S^i.a$ denotes a numeric function such that its value at $r$ is 0 for $r = 0, 1, 2, \cdots, i-1$; and is $a_{r-i}$ for $r \geq i$.
- If $b = S^i.a$, then

$$b_r = \begin{cases} 0, & 0 \leq r \leq i-1 \\ a_{r-i}, & r \geq i \end{cases}$$

- $S$ = shift; $S^i \leftarrow$ right shift

# Numeric Function

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## Left Shift

- Let $a = (a_0, a_1, a_2, \cdots, a_r, \cdots)$ be a numeric function and $i$ be a positive integer.
- $S^{-i}.a$ denotes a numeric function such that its value at $r$ is $a_{r+i}$ for $r \geq 0$.
- If $c = S^{-i}.a$, then

$$c_r = a_{r+i}, r \geq 0.$$

# Numeric Function

## Forward Difference

- Let $a = (a_0, a_1, a_2, \cdots, a_r, \cdots)$ be a numeric function.
- The forward difference of $a$ is defined as $\triangle a$.
- If $b = \triangle a$, then

$$b_r = a_{r+1} - a_r, r \geq 0.$$

Thus, we have:

$$\begin{aligned} b_0 &= a_1 - a_0 \\ b_1 &= a_2 - a_1 \\ b_2 &= a_3 - a_2 \\ \vdots & \quad \vdots \end{aligned}$$

# Numeric Function

## Backward Difference

- Let $a = (a_0, a_1, a_2, \cdots, a_r, \cdots)$ be a numeric function.
- The backward difference of $a$ is defined as $\nabla a$.
- If $c = \nabla a$, then

$$c_r = \begin{cases} a_0, & r = 0 \\ a_r - a_{r-1}, & r \geq 1 \end{cases}$$

Thus, we have:

$$\begin{aligned} c_0 &= a_0 \\ c_1 &= a_1 - a_0 \\ c_2 &= a_2 - a_1 \\ \vdots & \quad \vdots \end{aligned}$$

# Numeric Function

**Problem:** Let $a$ be a numeric function such that

$$a_r = \begin{cases} 2, & 0 \leq r \leq 3 \\ 2^{-r} + 5, & r \geq 4 \end{cases}$$

(a) Determine $S^2 a$ and $S^{-2} a$.
(b) Determine $\triangle a$ and $\nabla a$.