

$Z_p = \{0, 1, 2, \dots, p-1\}$ = set of residues (integers) that are divisible by p

$$5P = \mathbf{P + P} + P + P + P = (\mathbf{2P + P}) + P + P = (\mathbf{3P + P}) + P = 4P + P = 5P$$

$$k.P = P + P + \dots + P \text{ (k times)}$$

Optimization Method:

$$k = 7 = (111)_2 = 1 + 2 + 4$$

$$7.P = (1 + 2 + 4).P = P + 2P + 4P = P + 2(P + 2P) \rightarrow 4 \text{ point additions}$$

$$[2P = P + P \rightarrow 1 \text{ point addition}$$

$$P + 2P = 3P \rightarrow 1 \text{ point addition}$$

$$2(3P) = 2Q = Q + Q \rightarrow 1 \text{ point addition}$$

$$P + 6P \rightarrow 1 \text{ point addition}$$

$$\text{Total point additions} = 4]$$

$$\text{No. of point additions: } \log_2(k)$$

$$P, K_1.P, K_2.P$$

$$\text{ECDLP} \Rightarrow K_1$$

$$K_1 K_2.P = K_1. (K_2.P)$$

$a \pmod{p} \Rightarrow$ when a is divided by p , the remainder will be considered

$$a = b \pmod{p} \Rightarrow p \mid (a-b) \Rightarrow a \pmod{p} = b \pmod{p}$$

$$-5 = 10 \pmod{5} \Rightarrow 5 \mid [-10-5] = -15$$

$$y^2 = a \pmod{p}$$

$$P + (-P) = O = (-P) + P$$

$$4a^3 + 27b^2 = 4 + 27 = 31 \pmod{23} = 8 \neq 0$$