

$$\begin{aligned}
 x^8 \pmod{m(x)} &= [m(x) - x^8] \pmod{2} \\
 &= m(x) \text{ xor } x^8 \\
 &= (x^8 + x^4 + x^3 + x + 1) \text{ xor } x^8 \\
 &= x^4 + x^3 + x + 1 \\
 &= (0001 \ 1011)
 \end{aligned}$$

$$x^8 \bmod (x^8 + x^4 + x^3 + x + 1) = ?$$

1 <- Quotient

$$\begin{array}{r}
 x^8 + x^4 + x^3 + x + 1 \mid x^8 \\
 \underline{x^8 + x^4 + x^3 + x + 1} \\
 x^4 + x^3 + x + 1
 \end{array}$$

$$x-y \pmod{2} = x \text{ xor } y$$

$$\begin{aligned}
 g(x) + h(x) \pmod{m(x)} &= g(x) \pmod{m(x)} \text{ xor } h(x) \pmod{m(x)} \\
 g(x) &= 1.x^8 = x^8 \\
 h(x) &= (b_6 \ b_5 \dots b_1 \ b_0 \ 0) \\
 h(x) \bmod [m(x)] &= h(x) = (b_6 \ b_5 \dots b_1 \ b_0 \ 0) \\
 g(x) \bmod m(x) &= x^8 \bmod (m(x)) = (0001 \ 1011)
 \end{aligned}$$

$$g(x) + h(x) \pmod{m(x)} = (b_6 \ b_5 \dots b_1 \ b_0 \ 0) \text{ xor } (0001 \ 1011)$$

Product of two polynomials $f(x)$ and $g(x)$ in $GF(2^8)$:

$$\text{Let } f(x) = b_7 x^7 + b_6 x^6 + \dots + b_1 x + b_0$$

$$\begin{aligned}
 &\mathbf{f(x) \cdot g(x) \bmod m(x)} \\
 &= \mathbf{b_7 [x^7 \cdot x g(x)] \text{ xor } b_6 [x^6 \cdot x g(x)] \text{ xor } \dots \text{ xor } b_1 [x \cdot x g(x)] \text{ xor } b_0 g(x)}
 \end{aligned}$$

$$\begin{array}{r}
 H = 1 \ 0 \ 1 \ 0 \ 1 \ 1 \ 0 \\
 \quad 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1 \\
 \quad 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1
 \end{array}$$

$$x = \langle 1 \ 0 \ 0 \ 1 \rangle = \langle x^3 \ x^5 \ x^6 \ x^7 \rangle$$

$$y = \langle y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7 \rangle$$

$$\begin{aligned}
 (y_3 \ y_5 \ y_6 \ y_7) &= (x^3 \ x^5 \ x^6 \ x^7) \\
 \text{error detecting code} &= (y_1 \ y_2 \ y_4)
 \end{aligned}$$