Part 1.

(i) we show that $[R', \oslash]$ is an abelian group.

(a) closure: it holds from defn of $\oslash$,

$a \oslash b = a+b+1, \ \forall a, b \in R$ (A1)

(b) Associativity: ~~Let~~ $a, b, c \in R'$.

$(a \oslash b) \oslash c = (a+b+1) \oslash c = (a+b+1)+c+1$
$= a+b+c+2$

$a \oslash (b \oslash c) = a \oslash (b+c+1)$
$= a+(b+c+1)+1$ (A2)
$= a+b+c+2$

$\therefore (a \oslash b) \oslash c = a \oslash (b \oslash c), \ \forall a, b, c \in R'$

(c) Existence of Identity:

~~Let~~ $e \in R'$ be the additive identity in R' w.r.to. $\oslash$.

Then, $e \oslash a = a \oslash e = a, \ \forall a \in R'$

(A3)

$\therefore e \oslash a = a$

$\Rightarrow e+a+1 = a \Rightarrow e+1=0 \Rightarrow e=-1$

$\therefore e=-1 \in R'$ is the additive identity

(d) Existence of Inverse:

~~Let~~ $i \in R'$ be the additive inverse of $a \in R'$ w.r.to. $\oslash$.

Then, $i \oslash a = a \oslash i = e = -1$

$\therefore i \oslash a = -1 \Rightarrow i+a+1 = -1$

$\Rightarrow i = -a-2 = -(a+2)$

is the additive inverse of $a \in R'$. (A4)

(e) Commutativity: (A5)

~~Let~~ $a, b \in R'$

Then, $a \oslash b = a+b+1 = b+a+1 = b \oslash a$

$\therefore [R', \oslash]$ is an abelian group.

(ii) we show that $\langle R', \otimes \rangle$ is a semigroup.

(a) [M1] closure: holds from defn of $\otimes$
where $a \otimes b = a \times b + a + b,\ \forall a, b \in R'$.

(b) [M2] Associativity: ~~Let~~ $a, b, c \in R'$

Then, $(a \otimes b) \otimes c = (ab + a + b) \otimes c$

$= (ab + b + c)c + (ab + a + b) + c$

$= abc + ac + bc + ab + a + b + c$

$a \otimes (b \otimes c) = a \otimes (bc + b + c)$

$= a(bc + b + c) + a + (bc + b + c)$

$= abc + ab + ac + bc + a + b + c$

$\therefore (a \otimes b) \otimes c = a \otimes (b \otimes c)$.

(iii) [M3] $\otimes$ distributes over $\oslash$

ie, $a \otimes (b \oslash c) = (a \otimes b) \oslash (a \otimes c)$

$(b \oslash c) \otimes a = (b \otimes a) \oslash (c \otimes a)$

$\therefore \langle R', \oslash, \otimes \rangle$ forms a ring.

PART 2: If $\langle R', \oslash, \otimes \rangle$ is a ring with identity, then there exists an identity e in $\langle R', \otimes \rangle$ such that

$e \otimes x = x \otimes e = x,\ \forall x \in R'$

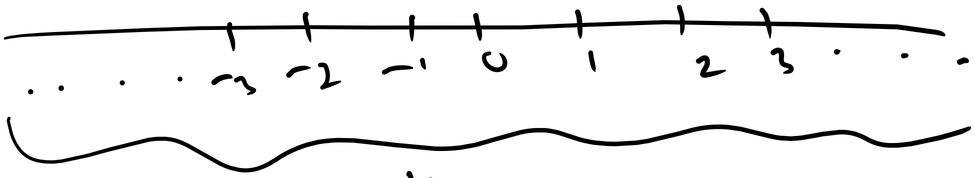
Now, $e \otimes x = x$

$\Rightarrow ex + e + x = x \Rightarrow e(x+1) = 0$

$\Rightarrow e = 0$, since $x + 1 \neq 0$.

$e = 0$ is the identity in $(R', \otimes)$.

$\mathbb{Z}$



$\Downarrow$

$\boxed{\mathbb{Z}_n} = \{0, 1, 2, \dots, n-1\}$

$\boxed{0, 1, 2, 3, \dots, n-1}$

Galois $\boxed{\mathbb{Z}_n}$

Given $a_1, a_2, \dots, a_n$:

$gcd(a_1, a_2, a_3, \dots, a_n)$

$= \boxed{gcd(\underline{gcd(a_1, a_2, \dots, a_{n-1})}, \underline{a_n})}$

$=$ ⋮

$= gcd[\underline{gcd}(a_1, gcd(a_2, a_3), \dots a_{n-1}, a_n]$