Tutorial Exercise 6

MA2.101: Linear Algebra (Spring 2022)

May 11, 2022

1 Matrix Representation of Linear Transformation

Problem 1

Let $T : R^3 \to P_2$ be a linear transformation, where P_2 is the vector space of polynomials in x with real coefficients having degree at most 2, given by

$$T(\begin{bmatrix} a \\ b \\ c \end{bmatrix}) = (a-b)x^2 + cx + (a+b+c)$$

Let $\tau = (\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix})$ and $\Omega = (x+1, x^2-x, x^2+x-1)$ be the respective bases. Find $[T]_\tau^\Omega$

Problem 2

Let $T : R^2 \to R^2$ be a linear transformation. Let $\tau = (\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix})$ and $\Omega = (\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix})$ be ordered basis for R^2. Suppose $T(\begin{bmatrix} 1 \\ 0 \end{bmatrix}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix})$ and $T(\begin{bmatrix} 0 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix})$. Find $[T]_\tau^\Omega$.

2 Isomorphism

Problem 3

let V be he set of complex numbers and let F be the field of real numbers. With the usual operators V is a vector space over F. Describe explicitly an Isomorphism of this space onto R^2.

Problem 4

Let V be the set of complex numbers regarded as a vector space over the field of real numbers. We define a function T from V into the space of 2 x 2 real matrices as follows, If $z = x + yi$ with x and y real numbers then,

$$T(z) = \begin{bmatrix} x+7y & 5y \\ -10y & x-7y \end{bmatrix}$$

1. Verify that T is a one-one linear transformation of V into the space of 2 x 2 matrices.
2. Verify that $T(z_1 z_2) = T(z_1)T(z_2)$.