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$F_X : \mathbb{R} \to [0, 1]$ is non-decreasing and right continuous.

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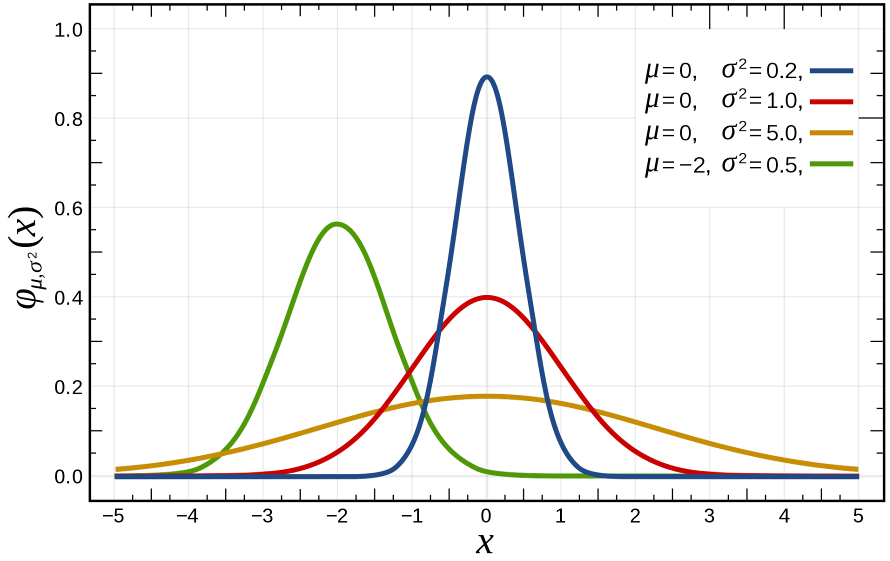
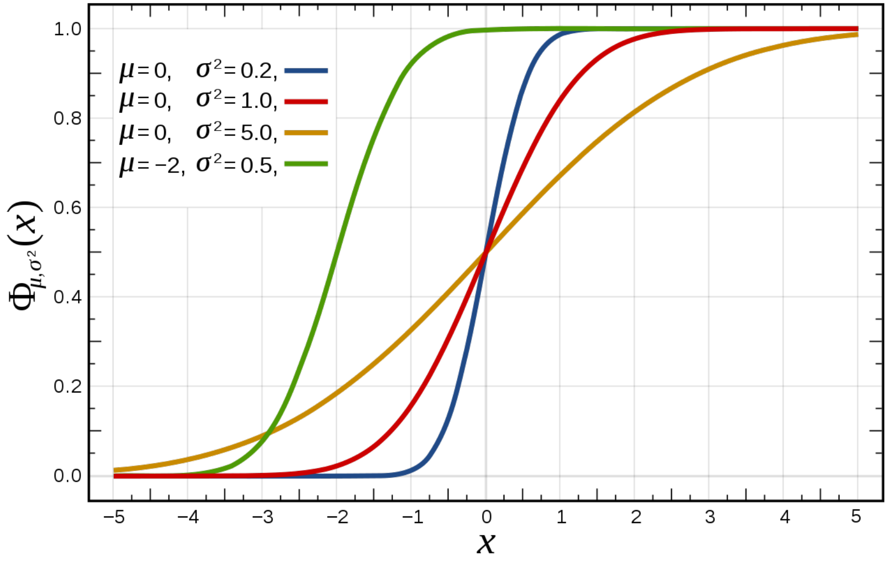
- Verify: $\int_{-\infty}^{\infty} f_X(x)dx$ $E[X] = \mu$ and $Var(X) = \sigma^2$.





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- GP Regression, Gaussian mixture models, used widely in ML.

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Important ones are Beta, Gamma, Erlang, Logistic, Weibull ....

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- HW: Find the MGF for a random variable X that has the following distributions: Binomial(n,p), Normal $\mathcal{N}(0, 1)$, Poisson(λ)

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- HW: Find MGF of all random variables seen till now and use it to obtain moments.