$(1-z)^{-1} = \frac{1}{1-z} = 1 + z + z^2 + \cdots + z^r + \cdots \infty$

converges if $|z| < 1$ i.e., $\boxed{-1 < z < 1}$.

(1) If $a_r = z^r$, then

$A(z)$ = G.f. of n.f. $a = (a_0, a_1, \ldots, a_r, \ldots)$

$$= \sum_{r=0}^{\infty} a_r z^r = \sum_{r=0}^{\infty} z^r \cdot z^r = \sum_{r=0}^{\infty} (z^2)^r$$

$$= 1 + z^2 + (z^2)^2 + \cdots + (z^2)^r + \cdots$$

$$= \frac{1}{1-z^2}.$$

(6) Let $c = S^{-i} \cdot a$. Then, $c_r = a_{r+i}, r \geq 0$

$\therefore$ $C(z)$ = G.f. of n.f. C

$$= \sum_{r=0}^{\infty} c_r z^r$$

$$= \sum_{r=0}^{\infty} a_{r+i} \cdot z^r$$

$$= \sum_{r=0}^{\infty} \left(a_{r+i} z^{r+i}\right) \cdot z^{-i}$$

$$= z^{-i} \sum_{r=0}^{\infty} a_{r+i} z^{r+i}$$

$$= z^{-i} \left[a_i z^i + a_{i+1} z^{i+1} + \cdots\right]$$

$$= z^{-i} \left[(a_0 + a_1 z + a_2 z^2 + \cdots + a_i z^i + \cdots + a_r z^r + \cdots) - (a_0 + a_1 z + a_2 z^2 + \cdots + a_{i-1} z^{i-1})\right]$$

$$= z^{-i} \left[\sum_{r=0}^{\infty} a_r z^r - (a_0 + a_1 z + a_2 z^2 + \cdots + a_{i-1} z^{i-1})\right]$$

$$= z^{-i} \left[A(z) - (a_0 + a_1 z + a_2 z^2 + \cdots + a_{i-1} z^{i-1})\right]$$