Discrete Structures (MA5.101)

Quiz - 4 (Monsoon 2021)

International Institute of Information Technology, Hyderabad

Time: 90 Minutes Total Marks: 40

Instructions: This is online examination.

Write at the top of your answer book the following:

Discrete Structures (MA5.101)

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Date: 23-Feb-2022

Name:

Roll Number:

Submit your scanned hand-written answer script in the moodle with the file name: RollNo_Quiz4_SecNo_23Feb2022.pdf

1. Let the null space of an $r \times n$ canonical parity check matrix be a group code that satisfies the following conditions:

   - for each coordinate there is some code word with a 1 in that position
   - for each pair of coordinates there is some code word that has different values in those two positions

   Prove that the set of code words with a 0 in the i^{th} coordinate is a subgroup of that code.

   [6]

2. Given the following parity-check matrix, H:

$$H = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{pmatrix}.$$

   (i) Encode the message $\langle\ 1\ 0\ 1\ 0\ \rangle$ using H.
   (ii) Decode the received tuple $\langle\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ \rangle$ assuming that error, if any, is a single-error.

   [6 + 3 = 9]

3. Exhibit a structure having left identity and no right identity. Examine whether the structure is a monoid.

   [5]

4. Let H and K be two subgroups of a group G. The product set of H and K, written as $H.K$, is the set of all products of the form: $h.k$, for h in H and k in K. Show that the product set $H.K$ is a normal subgroup of G, if H and K are normal in G.

   [10]

5. Let H be a normal subgroup in a group $\langle G, . \rangle$. Prove that $\langle G/H, \circ \rangle$ is abelian if and only if $g_1.g_2.g_1^{-1}.g_2^{-1}$ is in H for all g_1 and g_2 in G.

[10]

********** **End of Question Paper** **********