Recap

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- Continuity of set-function $\mathbb{P}$.

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- Conditional Probability.

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- Conditional Probability.
- Law of Total Probability.

This class

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- Independence and Correlation between sets.

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- Conditional independence.

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- Independence and Correlation between sets.
- Conditional independence.
- Principles of counting.

Bayes rule revisited

Let $B_1, B_2, \ldots B_n$ be the partition of the sample space Ω. Then for any event A with $P(A) > 0$ we have

$$P(B_j/A) = \frac{P(A/B_j)P(B_j)}{\sum_{i=1}^{n} P(A/B_i)P(B_i)}.$$

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> - Two events A, B are independent iff $P(A/B) = P(A)$ and $P(B/A) = P(B)$.
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- If A and B are independent, then so are A^c and B^c.
- What about A and B^c? Are they independent?
- If $A_1, A_2, \ldots, A_n$ are independent, then prove that

$$P(\cup_{i=1}^{n} A_i) = 1 - \prod_{i=1}^{n} [1 - P(A_i)]$$

Mutual and Pairwise Independence

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- HW: Find an example where pairwise independence does not imply mutual independence.

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- Are the events mutually independent?
- Which pair of event is independent?

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- A and B have the opposite correlation as A and B^c.

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- $P(A \cap B) = 0$
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- $P(A/B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A)}{P(B^c)}$

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- If $A \subseteq B$, then two events are neither mutually exclusive nor independent.

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Two events A and B are said to be conditionally independent of event C ($P(C) > 0$) if $P((AB)/C) = P(A/C).P(B/C)$

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HW: Verify if events A and B are conditionally independent of event C (in the experiment of picking number randomly in $\{1, .., 10\}$)

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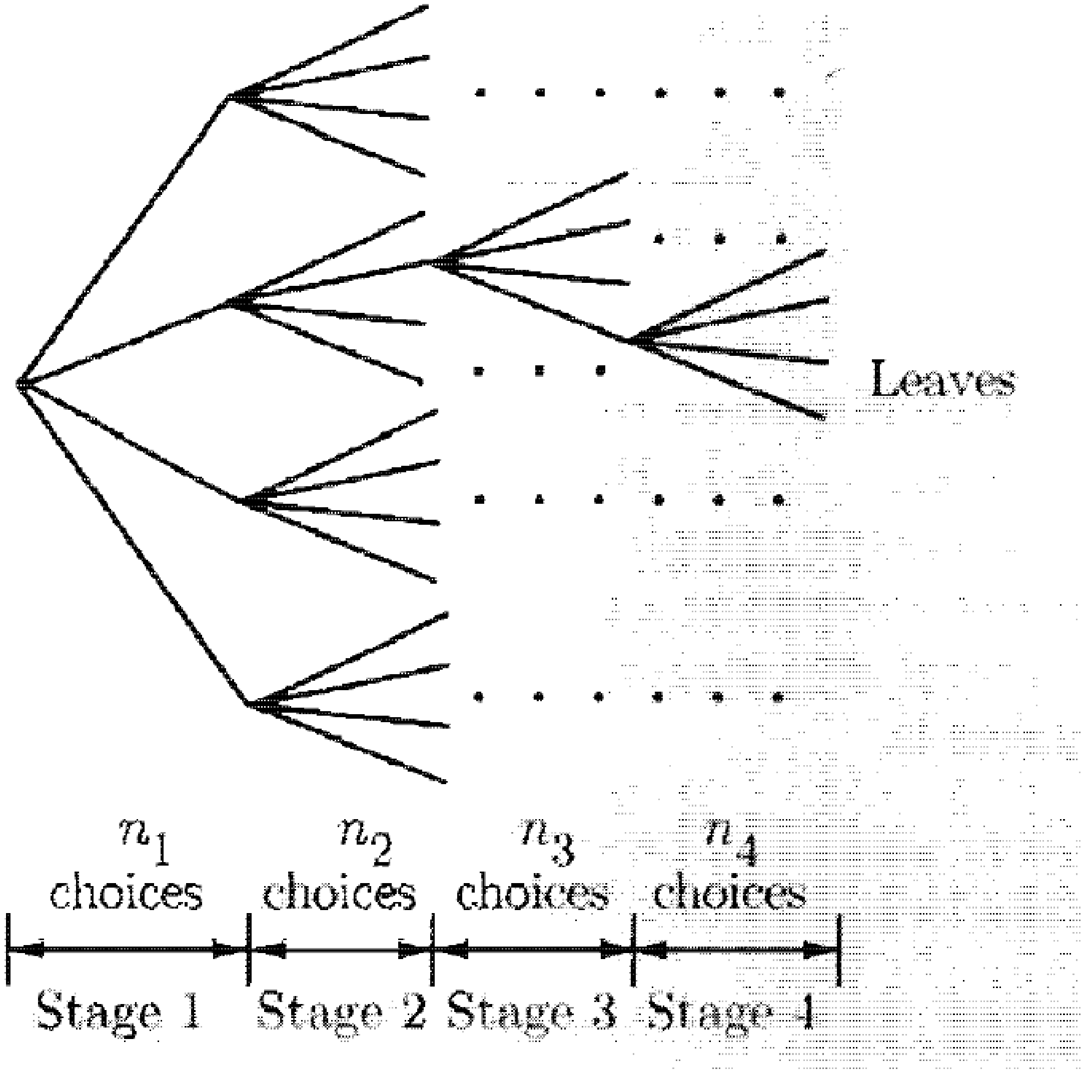
- Are A and B independent? HW

First principle of counting



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- ${}^{n}P_k = {}^{n}C_k \times k!$

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- Sampling can be ordered or unordered.
- In ordered sampling, $(a, b, c) \neq (c, b, a)$.
- This leaves us with 4 combinations.
  1. Ordered sampling with replacement
  2. Ordered sampling without replacement
  3. Unordered sampling with replacement
  4. Unordered sampling without replacement

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- Total n^k.

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- Essentially we want to count distinct k sized subsets from n objects without caring for ordering.

Unordered sampling without replacement

- Here you want to sample k out of n objects without replacement and the ordering of the k objects does not matters.
- Because we sample without replacement, repetition is not allowed.
- How many ways can you choose k objects out of n this way?
- a) nk? b) $\binom{n}{k}$ c) k^n d) none ?
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- ${}^nC_k$.

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- $\binom{n+k-1}{k}$ Think(HW).