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Towards a formal definition of $\mathbb{P}$

Probability measure $\mathbb{P}$ can be defined as a set-function $\mathbb{P} : \mathcal{P}(\Omega) \to [0, 1]$ that satisfies the following 3 axioms.
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- The following counter-example will construct a set-function $\mathbb{P}$ for which you cannot assign valid probabilities to every subsets in Ω without violating these axioms.

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- $\mathcal{P}(\mathbb{R})$ is unimaginably complex!

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- $\mathcal{F} = \{\Phi, \Omega, \mathbb{R}_-, \mathbb{R}_+\}$. This is all that this $\mathbb{P}$ can measure!

Towards sigma-algebra

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- For example, if $B \in \mathcal{F}$, then $B^c \in \mathcal{F}$. Also, $\emptyset$ and Ω in $\mathcal{F}$.
- A domain with such nice properties is called as a *sigma-algebra*.

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When Ω is countable and finite, we will consider power-set $\mathcal{P}(\Omega)$ as the domain.

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- For a collection of sets $\mathcal{C} \subset 2^{\Omega}$, the $\sigma-$*algebra* generated by $\mathcal{C}$, denoted by $\sigma(\mathcal{C})$ is the smallest $\sigma-$*algebra* containing $\mathcal{C}$. Here $\sigma(\mathcal{C}) = \{\cap \mathcal{F} : \mathcal{C} \subseteq \mathcal{F}\}$.

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- Recall that when $|\Omega| < \infty$, we consider $\mathcal{F} = 2^{\Omega}$.

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Probability space for $U[0, 1]$

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- Lets call it $\mathcal{F}^{++} = \{\Phi, [0,1], [0,.5), [.5,1], [.25,.75], [.25,.5), [.5,.75], [.25,1], [0,0.75], [0,.25) \cup [.5,1], [0,.5) \cup (.75,1], [0,.25), (0.75,1]\}$

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- But $\mathcal{F}^{++}$ is still not a sigma-algebra as each red set will demand a furthermore sets to be added.
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- Continuing on these lines, the resulting sigma algebra is called a borel-sigma algebra $\mathcal{B}[0,1]$.

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- How would you define $\mathcal{B}(\mathbb{R}^2)$?

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- State and prove the inclusion-exclusion principle for $P(\cup_{i=1}^{n} A_i)$

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- $P([0,.25] \cap [.75,1]) = P(\emptyset) = 0$ This event will never happen.

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- $\bigcup_{\omega \in \Omega} \{\omega\}$ is an uncountable disjoint union!