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- The marginal PMF's p_X and p_Y can be obtained from the joint PMF as follows:
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Proof:

$$
\begin{aligned}
P_X(x) &= \mathbb{P}\{\omega \in \Omega : X(\omega) = x\} \\
&= \mathbb{P}\{\bigcup_y \{\omega \in \Omega : X(\omega) = x, Y(\omega) = y\}\} \\
&= \sum_y \mathbb{P}\{\{\omega \in \Omega : X(\omega) = x, Y(\omega) = y\}\}
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> If X and Y are independent, $E[XY] = E[X]E[Y]$.

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Does this need a proof? No if you are an unconscious statistician!

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https://en.wikipedia.org/wiki/Law_of_the_unconscious_statistician

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$E[g(X,Y)] = \sum_{xy} g(xy)p_{XY}(xy)$

The rules for more than 2 discrete random variables are similar.

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- Rules similar for more than 2 random variables.