

SET THEORY

Problem: For any two sets A and B , prove that $A \cap B = A$ if and only if $A \subseteq B$.

We first prove that $A \cap B = A \Rightarrow A \subseteq B$.

Since $A \cap B \subseteq B$ by definition, $A \subseteq B$.

Conversely, we prove that $A \subseteq B \Rightarrow A \cap B = A$.

By definition, $A \cap B \subseteq A$.

If $x \in A$, then $x \in B$, since $A \subseteq B$.

Hence, $x \in A \cap B$ and $A \subseteq A \cap B$.

Thus, $A \cap B = A$.

SET THEORY

Problem: A survey of 1000 smokers reported that 850 smoked cigarettes, 200 smoked pipes, and 300 smoked bidis, whereas 130 smoked cigarettes and pipes, 220 smoked cigarettes and bidis, 30 smoked pipes and bidis, and 20 smoked all three. Are these figures consistent? Justify your answer.

- Assume A = the set of smokers who smoked cigarettes
 B = the set of smokers who smoked pipes
 C = the set of smokers who smoked bidis
- By the condition, we have:

$$\begin{aligned} |A| &= 850, |B| = 200, |C| = 300 \\ |A \cup B \cup C| &= 1000 \\ |A \cap B| &= 130 \\ |A \cap C| &= 220 \\ |B \cap C| &= 30 \\ |A \cap B \cap C| &= 20 \end{aligned}$$

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Problem: A survey of 1000 smokers reported that 850 smoked cigarettes, 200 smoked pipes, and 300 smoked bidis, whereas 130 smoked cigarettes and pipes, 220 smoked cigarettes and bidis, 30 smoked pipes and bidis, and 20 smoked all three. Are these figures consistent? Justify your answer (Continued...)

- We know,

$$\begin{aligned}|A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| \\&= 850 + 200 + 300 - 130 - 30 - 220 + 20 \\&= 990 \\&\neq 1000.\end{aligned}$$

- Hence, the given figures in the survey report are NOT consistent.

End of this lecture