$$\begin{cases} F_1 = 1, \quad F_2 = 1, \quad F_{k+1} = F_k + F_{k-1} \\ F_3 = F_2 + F_1 = 1+1 = 2 \\ F_4 = F_3 + F_2 = 2+1 = 3 \\ F_5 = F_4 + F_3 = 3+2 = 5, \dots \\ \vdots \end{cases}$$

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a) Let $b = s^i \cdot a = s^2 \cdot a, \quad i = 2$

$$b_r = \begin{cases} 0, & 0 \leq r \leq i-1 = 1 \\ a_{r-i}, & r \geq i = 2 \end{cases}$$

$$b_r = \begin{cases} 0, & 0 \leq r \leq 1 \\ 2, & 2 \leq r \leq 5 \\ 2^{-(r-2)} + 5, & r \geq 6 \end{cases}$$

$$\therefore \boxed{b_r = \begin{cases} 0, & 0 \leq r \leq 1 \\ 2, & 2 \leq r \leq 5 \\ 4 \cdot 2^{-r} + 5, & r \geq 6 \end{cases}}$$

$$c = 5^{-2} \cdot a \quad (i=2)$$

$$c_r = \begin{cases} 2, & 0 \le r \le 1 \\ \frac{1}{4} \cdot 2^{-r} + 5, & r \ge 2 \end{cases}$$

b) Let $d = \Delta a$

$\therefore \; d_r = a_{r+1} - a_r, \quad r \ge 0$

$d_0 = a_1 - a_0 = 0$

$d_1 = a_2 - a_1 = 0$

$d_2 = a_3 - a_2 = 0$

$d_3 = a_4 - a_3 = 2^{-4} + 5 - 2 = 2^{-4} + 3 = \frac{49}{16}$

$d_4 = a_5 - a_4 = (2^{-5} + 5) - (2^{-4} + 5)$

$= 2^{-5} - 2^{-4} = 2^{-4}(2^{-1} - 1) = -2^{-5}$

$= -2^{-(4+1)}$

$\vdots$

$$\therefore \; d_r = \begin{cases} 0, & 0 \le r \le 2 \\ \frac{49}{16}, & r = 3 \\ -2^{-(r+1)}, & r \ge 4. \end{cases}$$

Let $e = \nabla a$

$$e_r = \begin{cases} 2, & r = 0 \\ 0, & 1 \le r \le 3 \\ \frac{49}{16}, & r = 4 \\ -2^{-r}, & r \geq 4 \end{cases}.$$