Tutorial - 1

MA2.101: Linear Algebra (Spring 2022)

March 31, 2022

1 Fields and sub-fields

1.1

Let F be a set which contains exactly two elements, 0 and 1. We define addition and multiplication operations $(+, \cdot)$ by the tables:

+	0	1
0	0	1
1	1	0

$\cdot$	0	1
0	0	0
1	0	1

Verify that $(F, +, \cdot)$ is a field.

1.2

Which of the following sets of real numbers are subfields of the field $\mathbb{R}$?

- $\mathbb{Q}$, the set of rational numbers.

- $\mathbb{Z}$, the set of integers.

- $\{\cdots, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \cdots\}$, the set of integer powers of 2.

- $\mathbb{R}$, the set of all real numbers.

- $\{0\}$, the singleton set consisting of the number zero.

- $\{0, 1\}$

2 Systems of Equations

Let F be the field of complex numbers. In each of the following examples, are the two systems of linear equations equivalent? If so, express each equation in each system as a linear combination of the equations in the other system.

2.1 Example 1

$$x_1 - x_2 = 0 \qquad\qquad 3x_1 + x_2 = 0$$
$$2x_1 + x_2 = 0 \qquad\qquad x_1 + x_2 = 0$$

2.2 Example 2

$$-x_1 + x_2 + 4x_3 = 0 \qquad\qquad x_1 - x_3 = 0$$
$$x_1 + 3x_2 + 8x_3 = 0 \qquad\qquad x_2 + 3x_3 = 0$$
$$\frac{1}{2}x_1 + x_2 + \frac{5}{2}x_3 = 0$$