

Tutorial Exercise 8

MA2.101: Linear Algebra (Spring 2022)

June 1, 2022

1 Orthogonal vectors

Which of the following statements are false?

- Not every linearly independent set in R^n is an orthogonal set.
- Not every orthogonal set in R^n is linearly independent.
- An invertible matrix with orthogonal columns is an orthogonal matrix.
- The orthogonal projection of y onto v is the same as the orthogonal projection of y onto cv whenever $c \neq 0$.

2 Gram-Schmidt orthogonalization

Find an orthonormal basis of $P_2(R)$, where the inner product is given by $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$.

3 Inequalities

- Prove that the Cauchy-schwarz inequality is an equality if and only if one of u and v is a scalar multiple of the other.

$$|\langle u, v \rangle| \leq \|u\| \|v\|$$

- Let $u = \begin{bmatrix} a \\ b \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Use the Cauchy-Schwarz inequality to show that

$$\left(\frac{a+b}{2}\right)^2 \leq \frac{a^2+b^2}{2}$$