

**Problem:** If  $R$  and  $S$  are transitive relations on a set  $A$ , then  $R \cup S$  is not transitive on  $A$ , where  $R \cap S$  is transitive on  $A$ .

**Solution:**

Let  $(x,y) \in R \cap S$  and  $(y,z) \in R \cap S$ .

Required to Prove (RTP):  $(x,z) \in R \cap S$ .

Now,  $(x,y) \in R \cap S \Rightarrow (x,y) \in R$  and  $(x,y) \in S$ .

Again,  $(y,z) \in R \cap S \Rightarrow (y,z) \in R$  and  $(y,z) \in S$ .

Since  $R$  is transitive and  $(x,y) \in R$  and  $(y,z) \in R \Rightarrow (x,z) \in R$   
 $(x, z) \in S$ .

$\Rightarrow (x,z) \in R \cap S$ .

[Counterexample]  $A = \{a, b, c\}$

$R = \{(a,a), (a,b)\}$

$S = \{(a,a), (b,b), (b,c)\}$

**$R \cup S = \{(a,a), (a,b), (b, b), (b,c)\}$  is NOT transitive on  $A$ .**