Tutorial Exercise 7

MA2.101: Linear Algebra (Spring 2022)

May 26, 2022

1 Recap: Linear Transformations

1.1

Give an example of a function $f : R^2 \mapsto R$ such that $f(av) = af(v)$ for all $a \in R$ and all $v \in R^2$ but f is not linear.

Note: This shows that homogeneity alone is not enough to imply that a function is a linear map. Additivity alone is also not enough to imply that a function is a linear map.

1.2

Suppose that T is a linear map from V to $\mathbf{F}$. Prove that if $u \in V$ is not in $null(T)$, then

$$V = \text{null}(T) \oplus \{au : a \in F\}$$

2 Inner Product Spaces

2.1

Let A, B $\in C^{n\times n}$. Define the Frobenius inner product as $\langle A, B\rangle = tr(\text{B}^*\text{A})$.
Prove that this is an inner product?

2.2

$\langle X, Y\rangle = |x_1y_1| + |x_2y_2|$ where X,Y $\in R^2$. Verify if this is an inner product?

3 Linear Functionals

3.1

In R^3,let α_1=(1, 0, 1), α_2=(0, 1, -2), α_3=(-1, -1, 0). If f is a linear functional on R^3 such that f(α_1)=1, f(α_2)=-1, f(α_3)=3 and if α = (a,b,c) then find f(α).

3.2

Let B=$\{\alpha_1$,α_2,$\alpha_3\}$ be the ordered basis for C^3 defined by α_1=(1, 0, -1), α_2=(1, 1, 1),
α_3=(2, 2, 0). Find the dual basis of B.