(iii) $BX + AO > AX$

$\Rightarrow (a-R) + a > a+R$

$\Rightarrow 2a - R > a+R$

$\Rightarrow a > 2R$

$\Rightarrow \boxed{R < \frac{a}{2}}$

Thus, AX, BX and AO can form the sides of a triangle if (i), (ii) & (iii) hold ~~simultaneously~~ simultaneously

ie, $R > -\frac{a}{2}$ and $R < \frac{a}{2}$

ie, $\boxed{-\frac{a}{2} < R < \frac{a}{2}}$

$\therefore$ Required probability

$$= P\left[-\frac{a}{2} < R < \frac{a}{2}\right]$$

$$= \int_{-\frac{a}{2}}^{a/2} f(r)\, dr$$

$$= \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{2a}\, dr$$

$$= \frac{1}{2a} \int_{-\frac{a}{2}}^{a/2} dr = \frac{1}{2a} \left[r \right]_{-\frac{a}{2}}^{a/2}$$

$$= \frac{1}{2a} \times a = \frac{1}{2}$$