A set S is called infinite, if there exists a bijection $f: S \to S'$, where $S' \subset S$.

* Prove that the set of real numbers, $\mathbb{R}$, is <u>infinite</u>.

<u>Proof.</u> $S = \mathbb{R}$

$S' = (-1, 1) = \{x \mid -1 < x < 1, x \in \mathbb{R}\}$.

Construct a mapping:

$$f: S \to S'$$

$$f(x) = \begin{cases} \dfrac{x}{1+x}, & \text{if } x \geq 0 \\ \dfrac{x}{1-x}, & \text{if } x < 0 \end{cases}$$

$$\text{range of } f = \begin{cases} [0, 1) & \text{when } x \geq 0 \\ (-1, 0) & \text{when } x < 0 \end{cases}$$

$A - (B \cup C) = (A - B) \cap (A - C)$

$$\begin{aligned} LHS &= A - (B \cup C) & [A - B = A \cap B'] \\ &= A \cap (B \cup C)' & [(B \cup C)' = B' \cap C'] \\ &= A \cap (B' \cap C') \\ &= (A \cap A) \cap (B' \cap C') \\ &= \underline{A \cap (A \cap B')} \cap C' \\ &= (A \cap B') \cap (A \cap C') \\ &= (A - B) \cap (A - C) = RHS. \end{aligned}$$

$$\left.\begin{aligned} &(i)\ A - (B \cup C) \subseteq (A - B) \cap (A - C) \\ &(ii)\ (A - B) \cap (A - C) \subseteq A - (B \cup C) \end{aligned}\right\}.$$