

$f: A \rightarrow B \Rightarrow f\text{-image of } a = f(a)$

$g: B \rightarrow C$

$g \circ f: A \rightarrow C$  provided that  $\text{range } f = \text{dom } g$

$g(f(a)) = g[f(a)]$ , for all  $a$  in  $A$ .

$(a_1, a_2, \dots, a_r)$

$f(a_1) = a_2$

$f(a_2) = a_3 \Rightarrow f(f(a_1)) = a_3 \Rightarrow f^2(a_1) = a_3 \dots$

$f(a_3) = a_4 \Rightarrow f^3(a_1) = a_4 \dots$

$\dots$

$f^{r-1}(a_1) = a_r$

$f(a_r) = a_1 \Rightarrow f^r(a_1) = a_1$ .

$f^m = f \circ f \circ \dots \circ f$  (m times)

$f^m \circ g^m = f \circ f \circ \dots \circ f \circ g \circ g \circ \dots \circ g$   
 $= f \circ f \circ \dots \circ (f \circ g) \circ g \circ \dots \circ g$   
 $= h_1$

$(fg)^m = fg \circ fg \circ \dots \circ fg$   
 $= f \circ (g \circ f) \circ g \circ \dots \circ fg = h_2$   
 $= f \circ f \circ g \circ g \circ \dots$

$(1 \ 6) (1 \ 4) (1 \ 2) = (1 \ 2 \ 4 \ 6)$

LHS =  $(1 \ 2 \ 3 \ 4 \ 5 \ 6) \rightarrow f$   
 $6 \ 2 \ 3 \ 4 \ 5 \ 1$

$\cdot (1 \ 2 \ 3 \ 4 \ 5 \ 6) \rightarrow g$   
 $4 \ 2 \ 3 \ 1 \ 5 \ 6$

$\cdot (1 \ 2 \ 3 \ 4 \ 5 \ 6)$   
 $2 \ 1 \ 3 \ 4 \ 5 \ 6$

$= (1 \ 2 \ 3 \ 4 \ 5 \ 6) \rightarrow f[g(a_i)] \ a_1 = 1, g(a_1) = g(1) = 4 \Rightarrow f[g(1)] = f(4) = 4$   
 $4 \ 2 \ 3 \ 6 \ 5 \ 1$

$\cdot (1 \ 2 \ 3 \ 4 \ 5 \ 6)$   
 $2 \ 1 \ 3 \ 4 \ 5 \ 6$

$= (1 \ 2 \ 3 \ 4 \ 5 \ 6)$   
 $2 \ 4 \ 3 \ 6 \ 5 \ 1$

$= (1 \ 2 \ 4 \ 6) \cdot (3) \cdot (5)$   
 $2 \ 4 \ 6 \ 1 \ 3 \ 5$

$= (1 \ 2 \ 4 \ 6) = \text{RHS}$

$p = (1 \ 2 \ 3 \ 4 \ 6) = (1 \ 6) (1 \ 4) (1 \ 3) (1 \ 2) = \text{even permutation} \rightarrow A_n$

$q_0 \cdot p = (2 \ 3) \cdot (1 \ 6) (1 \ 4) (1 \ 3) (1 \ 2) = \text{odd permutation} \rightarrow B_n$

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$f(x) = x$

$g(x) = x+1$ ,  $x$  is real

$g \circ f: \mathbb{R} \rightarrow \mathbb{R}$

$$\begin{aligned} g \cdot f(x) &= g[f(x)] = g(x) = x+1 \\ f \cdot g(x) &= f[g(x)] = f(x+1) = x+1 \end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 4 & 6 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ & 2 & 4 & 3 & 6 & 5 & 1 \end{pmatrix}$$

$$\begin{aligned} (1) \cdot (2) &= (\textcolor{red}{1} \ 2 \ 3 \ 4 \ 5) \\ &\quad \textcolor{red}{1} \ 2 \ 3 \ 4 \ 5 \\ &\quad \cdot (1 \ \textcolor{blue}{2} \ 3 \ 4 \ 5) \\ &\quad \quad 1 \ \textcolor{blue}{2} \ 3 \ 4 \ 5 \\ &= (1 \ 2 \ 3 \ 4 \ 5) \\ &\quad 1 \ 2 \ 3 \ 4 \ 5 \end{aligned}$$

$$f = (1 \ 2 \ 3) = \begin{pmatrix} 1 & 2 & 3 \\ & 2 & 3 & 1 \end{pmatrix}$$

$$\begin{aligned} f^{-1} \cdot g &= (2 \ 3 \ 1) \cdot (1 \ 3 \ 2) \\ &\quad 1 \ 2 \ 3 \quad 3 \ 2 \ 1 \\ &= \begin{pmatrix} 1 & 2 & 3 \\ & 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ & 2 & 3 & 1 \end{pmatrix} \end{aligned}$$