Tutorial Exercise 5

MA2.101: Linear Algebra (Spring 2022)

May 2, 2022

1 Bases and Dimensions

Problem 1

Let V be a vector space with $\mathbf{v} = (v_1, v_2, \cdots, v_n)$ as one of its basis. Now we define a new list $\tilde{\mathbf{v}}$ by subtracting from each vector of $\mathbf{v}$ (except the first one) its preceding vector, i.e.

$$\tilde{\mathbf{v}} = (v_1, v_2 - v_1, v_2 - v_3 \cdots, v_n - v_{n-1})$$

Prove that $\tilde{\mathbf{v}}$ is also a basis of V.

Hint: Using the properties of $\mathbf{v}$ as a basis, you have to show the two properties satisfy for $\tilde{\mathbf{v}}$ — it is linearly independent, and it spans V.

Problem 2

Let $\mathcal{P}_4(\mathbf{F})$ be the vector space of all polynomials of degree at most 4 over field ($\mathbf{F}$). And let $\mathbf{p}$ denote a basis of this vector space.

- Find the dimension of $\mathcal{P}_4(\mathbf{F})$.
- Prove or refute: There is a valid basis $\mathbf{p} = (p_0, p_1, p_2, p_3, p_4)$ of $\mathcal{P}_4(\mathbf{F})$ such that none of the polynomials p_0, p_1, p_2, p_3, p_4 has degree 3.

2 Coordinates

Problem 3

Let B=$\{\alpha_1,\alpha_2,\alpha_3\}$ be the ordered basis for R^3 consisting of α_1=(1, 0, -1), α_2=(1, 1, 1), α_3=(1, 0, 0). What are the coordinates of the vector (a,b,c) in the ordered basis B.

Problem 4

Consider A=$\left\{ \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\2\\-3\end{bmatrix} \right\}$ and B=$\left\{ \begin{bmatrix}3\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\2\\0\end{bmatrix} \right\}$. Both A & B are bases for R^3

1. Find $P_{B->A}$
2. Find $P_{A->B}$
3. Let X=$2a_1$ - a_2 + $3a_3$. Find $[X]_B$