Tutorial Exercise 4

MA2.101: Linear Algebra (Spring 2022)

April 19, 2022

Problem 1

Determine which of the following sets are subspaces of R^3

1. $\{(x, y, z) \in R^3 | x + 2y = 3z\}$
   Answer: $\{(x, y, z) \in R^3 | x + 2y - 3z = 0\}$
   This is a subspace since it is the set of solutions to a homogeneous linear equation

2. $\{(x, y, z) \in R^3 | Ax = 0$, A is a 2×3 matrix$\}$
   Answer: Consider A, B and C $\in Field$
   $Now,(cA + B)x = (cA)x + Bx = c(Ax) + Bx = c(0) + 0 = 0 \forall A, Binset, C \in Field.cA + B \in set.$
   $\therefore Givensetisasubspaceof \mathrm{R}^3$

3. $\{(0, a, a + 1) | a \in R\}$
   Answer: If (0,0,0) = (0, α, $\alpha + 1$) then $\alpha = 0$ and $\alpha + 1 = 0$.
   No solutions for α, so (0,0,0) is not in the set
   $\therefore$ The set is <u>not</u> a subspace.

4. $\{a(1, 0, 2) + b(5, 5, 7) | a, b \in R\}$
   Answer: $\{a(1, 0, 2) + b(5, 5, 7) | a, b \in R\} = \text{Span}\{(1, 0, -2), (5, 5, 7)\}$
   $\therefore$ The set is a subspace

5. $\{(k, m, n) \in R^3 | k^2 = n^2\}$ **Answer:** (1,0,1) is in the set since $1^2 = 1^2$
   (1,0,-1) is in the set since $1^2 = (-1)^2$
   but (1,0,1) + (1,0,-1) = (2,0,0(is not in the set since $2^2 \neq 0^2$. So the set is <u>not</u> a subspace since it is not closed under vector addition

Problem 2

Let $\{v_1, v_2, v_3, ...v_n\}$ be a set of nonzero vectors in $\mathbb{R}^n$ such that the dot product $v_i \cdot v_j$=0 $\forall i \neq j$. Prove that the set is linearly independent.
Answer: $\{v_1, v_2, v_3, ...v_n\}$ non zero vectors in $\mathbb{R}^n$ such that $v_i, v_j = 0 \forall i \neq j$
Assume $\{v_1, v_2, v_3, ...v_n\}$ are linearly dependent
For some non zero scalars $c_1, c - 2, \ldots, c_n$

$$c_1v_1 + c_2v_2 + \ldots + c_nv_n = 0$$

Dot Products with v_1

$$c_1|v_1|^2 + c_2(0) + \ldots + c_n(0) = 0c_1|v_1|^2 = 0|v_1|^2 = 0, c_1 \neq 0$$

But v_1 is non zero. Contradicting initial conditions because of wrong assumptions.
$\therefore \{v_1, v_2, v_3, ...v_n\}$ are linearly independent.

Problem 3

Let V be a vector space:

1. Suppose $v \in V$, is the set $\{0, v\}$ linearly dependent? Explain.

   Answer: Yes. Let λ be any nonzero scalar. Then

   $$\lambda 0 + 0v = 0$$

   is a nontrivial linear combination of 0 and v that yields 0 , so this set is linearly dependent.

2. Suppose $\{v_1, v_2, \ldots, v_n\}$ is a set of vectors and $u \in span(v_1, v_2, \ldots, v_n)$. Show that $\{v_1, v_2, \ldots, v_n, u\}$ is linearly dependent.
   Answer: Since $u \in \operatorname{span}(v_1, \ldots, v_n)$, we know there exist numbers $\lambda_1, \ldots, \lambda_n$ so that

   $$u = \lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n.$$

   But then

   $$\lambda_1 v_1 + \lambda_2 v_2 + \ldots + \lambda_n v_n - u = u - u = 0$$

   is a nontrivial linear combination of the vectors $\{v_1, \ldots, v_n, u\}$ (since the coefficient of u is $-1 \neq 0$) that produces 0 , so this set is linearly dependent.

Problem 4

If A is a subspace of V, must its complement be a subspace?

Answer: If A is a subspace of V then A contains the zero vector.V-A(complement of A) does not contain the zero vector so the complement of A is not a subspace of V.