

Theorem:  $(R^{-1})^{-1} = R$

Let  $R$  be a relation between two sets  $A$  and  $B$ .  
Then,  $R^{-1}$  will be an inverse relation from  $B$  to  $A$ .

Required to prove (RTP):

$$1) (R^{-1})^{-1} \subseteq R$$

$$2) R \subseteq (R^{-1})^{-1}.$$

$$1) \left. \begin{array}{l} \text{Let } (x, y) \in (R^{-1})^{-1}. \\ \text{Then, } (y, x) \in R^{-1} \\ \Rightarrow (x, y) \in R. \end{array} \right\} \therefore (R^{-1})^{-1} \subseteq R$$

$$2) \left. \begin{array}{l} \text{Let } (x, y) \in R \\ \text{Then, } (y, x) \in R^{-1} \\ \Rightarrow (x, y) \in (R^{-1})^{-1} \end{array} \right\} \therefore R \subseteq (R^{-1})^{-1}.$$

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$$S = \{ (x, y) \in \mathbb{N} \times \mathbb{N} \mid x + y = 5 \}$$

RTP:  $S$  is symmetric.

$$\text{Let } (x, y) \in S.$$

$$\text{Then, } x + y = 5$$

$$\Rightarrow y + x = 5$$

$$= (y, x) \in S$$

$$x S y \Rightarrow y S x, \quad \forall x, y \in \mathbb{N}.$$

$$A = \emptyset, \quad B = \{3, 4\}$$

$$A \times B = \{ (1, 3), (1,$$