Tutorial-7 Questions

1. Show that {1, 2, 3} under multiplication modulo 4 is not a group but that {1, 2, 3, 4} under multiplication modulo 5 is a group.
2. In Z^*_9 , find the inverse of 2, 7, and 8.
3. Prove that a group of even order must have an element of order 2.
4. Let G be a group with the property that for any x, y, z in the group, xy = zx implies y = z. Prove that G is Abelian.
5. Suppose that G is an Abelian group with an odd number of elements. Show that the product of all of the elements of G is the identity.
6. If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G.
7. Let G be an Abelian group and $H = \{x \in G \mid |x| \text{ is odd}\}$. Prove that H is a subgroup of G.
8. Show that Z^*_8 is isomorphic to Z^*_{12}.
9. Let $H = \{0, \pm1, \pm3, \pm6, \ldots\}$. Find all the left cosets of H in Z.
10. Prove that a group of order 3 must be cyclic.