Numeric Function

Convolution

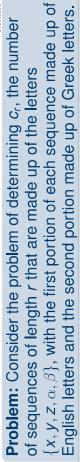
Definition

Let a and b be two numeric functions. The convolution of a and b, defined by a*b, is a numeric function c such that c=a*b, where

$$c_r = a_0b_r + a_1b_{r-1} + \dots + a_{r-1}b_1 + a_rb_0$$

= $\sum_{j=0}^{r} a_jb_{r-j}$.

Numeric Function



Solution: Let $a_r =$ the number of sequences of length r that are made up from English letters $\{x, y, z\}$;

 $b_r =$ the number of sequences of length r that are made up from Greek letters $\{\alpha,\beta\}$.

Then, we have,

$$a_r = 3', r \ge 0$$
$$b_r = 2', r \ge 0$$

Then, for c = a * b, we have,

$$c_r = \sum_{i=0}^r a_i b_{r-i} = \sum_{i=0}^r 3^i 2^{r-i}, r \ge 0.$$

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Tests for Convergence

Whether an infinite series is convergent or not, the following tests are available (see http://home.iitk.ac.in/~psraj/mth101/lecture_notes/Lecture11-13.pdf):

- Comparison Test
- Cauchy Test
- Ratio Test
 - Root Test
- Leibniz Test



Definition

For a numeric function $a=(a_0,a_1,a_2,\cdots,a_r,\cdots)$, define an infinite series

$$a_0 + a_1z + a_2z^2 + \cdots + a_rz^r + \cdots$$

which is called generating function (G.F.) of the numeric function a and denoted by

$$A(z) = \sum_{r=0}^{\infty} a_r z^r$$

provided that the series A(z) is convergent, where z is a variable.

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Properties

Let a, b, c be the numeric functions.

• If
$$a_r = z^r$$
, $r \ge 0$, then $A(z) = \frac{1}{1-z^2}$.

• If
$$b = \alpha a$$
, where α is a constant, then $B(z) = \alpha A(z)$.

• If
$$c = a + b$$
, then $C(z) = A(z) + B(z)$.

• If a is a numeric function and
$$A(z)$$
 is its generating function and $b_r = \alpha^r a_r$ for a numeric function b and α is a constant, then $B(z) = A(\alpha z)$.

• If
$$b = S^{i}.a$$
, then $B(z) = z^{i}.A(z)$

• If
$$c = S^{-i}.a$$
, then

$$C(z) = z^{-i}[A(z) - a_0 - a_1z - a_2z^2 - \dots - a_{i-1}z^{i-1}]$$

Let a, b, c be the numeric functions.

Properties

• If $b = \triangle a$, then

$$B(z) = \frac{1}{z} \Big[A(z) - a_0 \Big] - A(z)$$

• If $c = \nabla a$, then

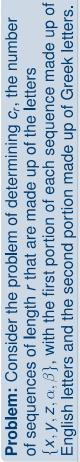
$$C(z) = (1-z)A(z)$$

• If c = a * b, that is, c is the convolution of a and b, then

$$C(z) = A(z).B(z)$$

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screte Structures



Solution: Let $a_r =$ the number of sequences of length r that are made up from English letters $\{x,y,z\}$;

 $b_r=$ the number of sequences of length r that are made up from Greek letters $\{\alpha,\beta\}$.

Then, we have,

$$a_r = 3', r \ge 0$$
$$b_r = 2', r \ge 0$$

Then, for c = a * b, we have,

$$c_r = \sum_{i=0}^r a_i b_{r-i} = \sum_{i=0}^r 3^i 2^{r-i}, r \ge 0.$$

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Now,

$$C(z) = A(z).B(z),$$

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where

$$A(z) = \sum_{r=0}^{\infty} a_r z^r = \sum_{r=0}^{\infty} 3^r z^r = \sum_{r=0}^{\infty} (3z)^r = \frac{1}{1-3z}$$

(2)

$$B(z) = \sum_{r=0}^{\infty} b_r z^r = \sum_{r=0}^{\infty} 2^r z^r = \sum_{r=0}^{\infty} (2z)^r = \frac{1}{1-2z}$$

(9)

Hence,

$$C(z) = A(z).B(z)$$

= $\frac{1}{1-3z} \cdot \frac{1}{1-2z} = \frac{\alpha}{1-2z} + \frac{\beta}{1-3z}$, say



Solving, we have, $\alpha = -2$ and $\beta = 3.$ Thus,

$$c = -2$$
 and $\beta = 3$. Thus,
$$C(z) = \sum_{r=0}^{\infty} c_r z^r$$

$$= -\frac{2}{1 - 2z} + \frac{3}{1 - 3z}$$

$$= \sum_{r=0}^{\infty} [3.3^r - 2.2^r] z^r$$

$$= \sum_{r=0}^{\infty} [3^{r+1} - 2^{r+1}] z^r$$
in:

As a result, we obtain:

$$c_r = 3^{r+1} - 2^{r+1}, r > 0$$

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Generating Function

Problem: Evaluate the sum

$$1^2 + 2^2 + 3^3 + \dots + n^2$$

using the generating function.