① $f(x)$ can be regarded as a p.d.f if

(i) $f(x) \geq 0, \ \forall x \in R$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

(i) $f(x) \geq 0, \ \forall x \in R$

$\Rightarrow x - 2 \geq 0$ or $\boxed{k \geq 2}$.

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx + \int_{2}^{\infty} f(x)\,dx = 1$$

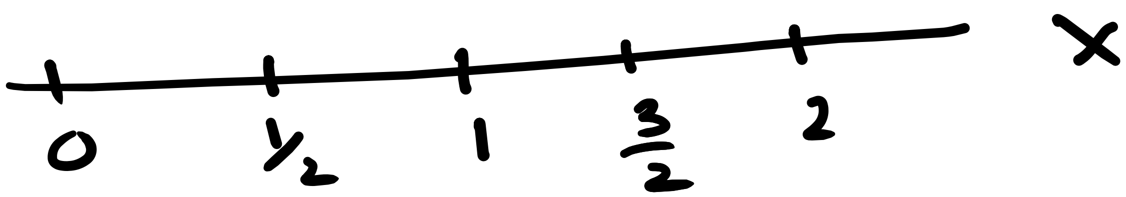
$$\Rightarrow \int_{0}^{1} x\,dx + \int_{1}^{2} (k - x)\,dx = 1$$

$$\Rightarrow \frac{1}{2} + \left[kx - \frac{x^2}{2}\right]_1^2 = 1$$

$$\Rightarrow k - 1 = 1 \Rightarrow k = 2$$

Thus,

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & \text{elsewhere.} \end{cases}$$



Required probability

$= P\left[\frac{1}{2} < X < \frac{3}{2}\right]$

$$= \int_{1/2}^{3/2} f(x)\, dx$$

$$= \int_{1/2}^{1} f(x)\, dx + \int_{1}^{3/2} f(x)\, dx$$

$$= \int_{1/2}^{1} x\, dx + \int_{1}^{3/2} (2-x)\, dx$$

$$= \frac{3}{4}.$$

(i) R.T.P. $P(a<X<b) = F(b-0) - F(a)$.

The events $(a<X<b)$ and $(X=b)$ are mutually exclusive, and

$$(a<X<b) \cup (X=b) = (a<X\le b)$$

$$\Rightarrow P(a<X<b) + P(X=b) = P(a<X\le b)$$

$$\Rightarrow P(a<X<b) = P(a<X\le b) - P(X=b)$$

$$= [F(b) - F(a)] - [F(b) - F(b-0)]$$

$$= F(b-0) - F(a).$$

(ii) R.T.P. $P(a\le X\le b) = F(b) - F(a-0)$.

The events $(X=a)$ and $(a<X\le b)$ are mutually exclusive, and

$$(X=a) \cup (a<X\le b) = (a\le X\le b)$$

$$P(X=a) + P(a<X\le b) = P(a\le X\le b)$$

$$\Rightarrow P(a\le X\le b) = [F(a) - F(a-0)] + [F(b) - F(a)]$$

$$= F(b) - F(a-0).$$

$X \sim \text{Binomial}(n, p)$

p.m.f of X

$$f_i = P(X=x_i) = P(X=i)$$

$$= {}^nC_i \, p^i q^{n-i}, \quad i = 0, 1, 2, \ldots, n$$

(*) The number of lines in operation, X, has a binomial distribution with parameters $n=20$ and $p=0.6$

Then, $q = 1-p = 0.4$

The p.m.f. of X is then given by:

$$f_i = P(X=x_i) = P(X=i)$$
$$= {}^{n}C_i \, p^i q^{n-i}$$
$$= {}^{20}C_i (0.6)^i (0.4)^{20-i}, \quad i=0,1,2,\dots,20$$

Required probability

$$= P[X \geq 10]$$
$$= \sum_{k=10}^{20} {}^{20}C_i (0.6)^i (0.4)^{20-i}$$
$$= 0.872478$$
$$\approx 87\%$$