① If A and B be two events,
$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$.

<u>Proof.</u> By De Morgan's law,

$$\bar{A} \cup \bar{B} = \overline{A \cap B}$$

$$\therefore P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) \qquad \left[P(\bar{A}) = 1 - P(A) \right]$$
$$= 1 - P(A \cap B).$$

Theorem: For events A and B in a sample space,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

For three events A, B and C in a sample space,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C).$$

Generalizing for n events, say $A_1, A_2, \cdots A_n$, in a sample space,

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{\substack{i,j=1 \\ i \neq j}}^{n} P(A_i \cap A_j) + \sum_{\substack{i,j,k=1 \\ i \neq j \neq k}}^{n} P(A_i \cap A_j \cap A_k) - \cdots + (-1)^{n-1} P(A_1 \cap A_2 \cap \cdots \cap A_n).$$

[Proof by Mathematical Induction]

[Boole's Inequality]

for any n events $A_1, A_2, \dots, A_n$

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) \leq P(A_1) + P(A_2) + \cdots + P(A_n)$$

Proof [by Mathematical Induction]

[Basis] $n = 2$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$[P(A_i) \geq 0]$

$$\leq P(A_1) + P(A_2)$$

The theorem holds for $n = 2$.

[Hypothesis] Assume that the theorem holds for $n = k$, $n > 2$.

$\therefore$ By assumption,

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) \leq P(A_1) + P(A_2) + \cdots + P(A_k)$$

[Induction] R.T.P. theorem holds for $n = k+1$.

Now, $P(A_1 \cup A_2 \cup \cdots \cup A_k \cup A_{k+1})$

$$= P[(A_1 \cup A_2 \cup \cdots \cup A_k) \cup A_{k+1}]$$

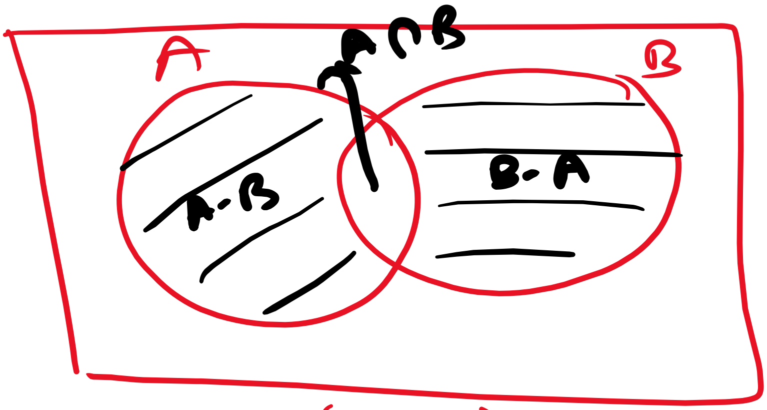
$$= P(A_1 \cup A_2 \cup \cdots \cup A_k) + P(A_{k+1}) - P[(A_1 \cup A_2 \cup \cdots \cup A_k) \cap A_{k+1}]$$

$$\leq \underline{P(A_1 \cup A_2 \cup \cdots \cup A_k)} + P(A_{k+1})$$

$$\leq P(A_1) + P(A_2) + \cdots + P(A_k) + P(A_{k+1})$$

by hypothesis.

Problem: Find the probability of occurrence of only one of the events A and B.



$A \cup B = (A-B) \cup (A \cap B) \cup (B-A)$

and $(A-B)$, $(A \cap B)$ and $(B-A)$ are pairwise mutually exclusive events.

Required probability $= P[(A-B) \cup (B-A)]$

Now, $P(A \cup B) = P(A-B) + P(A \cap B) + P(B-A)$

$\Rightarrow \underline{P(A-B) + P(B-A)} = P(A \cup B) - P(A \cap B)$

$\Rightarrow P[(A-B) \cup (B-A)]$

$= P(A) + P(B) - P(A \cap B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$