MINIMIZING TOTAL COST FUNCTION USING THE PRINCIPLES OF BLACK BOX OPTIMIZATION

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TABLE OF CONTENT

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Abstract

"Black Box optimization" refers to a problem setup in which an optimization algorithm is supposed to optimize (e.g., minimize) an objective function through a so-called black-box interface: the algorithm may query the value **f(x)** for a point **x**, but it does not obtain gradient information, and in particular, it cannot make any assumptions on the analytic form of **f** (e.g., being linear or quadratic). We proposed a unique approach to solving this problem which uses the concept of compass search coupled with the principle of the Moore-Penrose inverse.

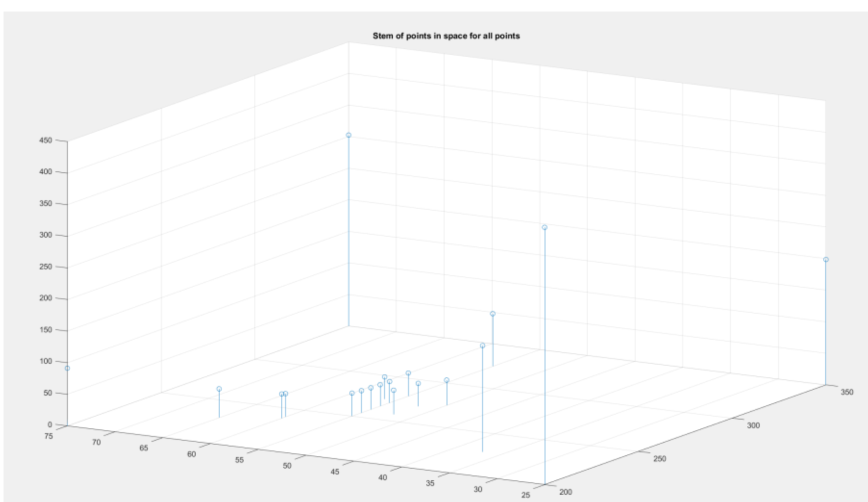

Figure 1: Stem plot of all points gathered in Space

Method

I. Choice of points:

The initial guess of the points were the 4 extreme bounds of the range of values provided which were (200,25); (200,75); (350,25) and (350,75). The advantage of compass search[0] is that it is a derivative-free approach. We noticed that the minimum value of the 4 points is at (200, 75). Thus, from there, we took points which were North, South, East, and West of the above points. Later we found the new minimum point for that iteration (least value of the four points) and we took that improved point as our new (as shown in the contour plot in **Figure 2**).

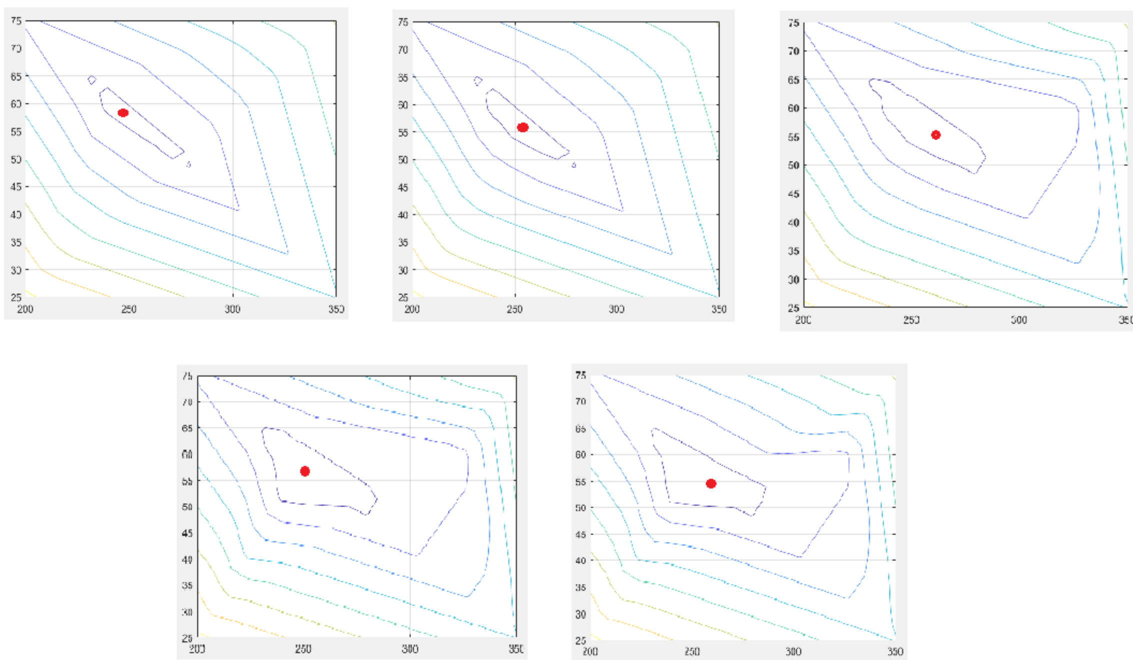

Figure 2: Nature of contour descent approach used in compass method

II. Moore-Penrose equation (Pseudoinverse)

The Moore-Penrose pseudo-inverse[2] is a way to calculate the solution of the system of linear equations expressed as:

$$AX \approx f$$

Where A is a matrix of coefficients of X and X is linearly independent terms of both c and r like (c, r^2, r^3, sin(c), r, cos(c), r^2, sin(c^2) etc.) where c and r stand for the cost variable and risk variable respectively and f is the output of function at those values. Since the matrix is not a square matrix and has rank less than total columns, we have to opt for the

Thus, the above equation can be written as:

$$AX + e = f$$
$$A^T AX + A^T e = A^T f$$
$$X = (A^T A)^{-1} A^T f$$
$$X = pinv(A) * f$$

But The voice of Using the above principle we generated a function called polyfit for 2 variable black box optimization using python. In the function, we set convergence of coefficients if the L2-norm of predicted and true value is below a threshold score or if coefficients due to which predictions occur don't change.

III. Our Approach to choose functions

From the Fourier transform, we know that any signal can be expressed as the linear combination of sines and cosines. Thus, we took sines and cosines and found that points on the extremes don't satisfy the curve equation. Also, there were rapid sinusoid fluctuations in derivatives indicating there were multiple local minima. The plot in **Figure 3** also shows that it is an exploding sine or cosine curve under an exponential envelope.

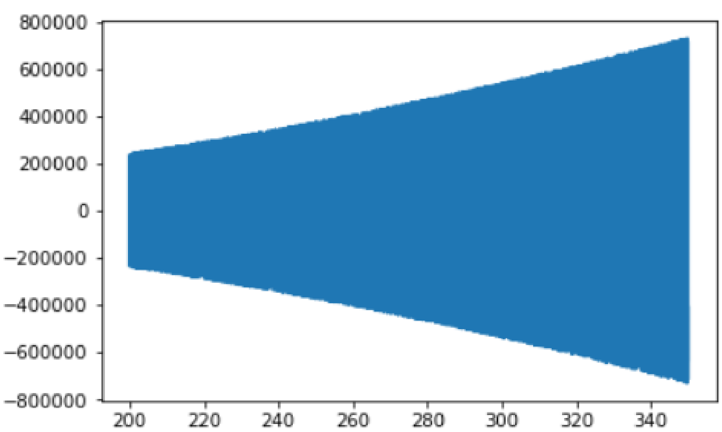

Figure 3: Exploding derivative in exponential envelope

Thus, we decided to consider a logarithmic component too (since heavy exponentials lead to failure of convergence of SVD). The fit was found to be accurate, but new points don't necessarily satisfy the curve equation and yield a minimal error. While doing the literature survey, we were inspired to include a logarithmic part as well to consider the absolute value of the resultant function as a part of the combination.

For finding the global minima, the initial approach was to take the partial derivative of the function wrt. C and R and then find the values, but it didn't ensure convergence at global minima. Now that we also knew the function has multiple local minima, we wrote a function that computed the global minima in a Brute force approach.

IV. Our final answer

We queried 19 points (**Table 1**) and we believe that the global minima lie at:

Total Cost Value: 33.62

Corr. Cost variable Value: 270 (+- 2.5)

Corr. Risk variable Value: 55 (+- 1)

Appendix

S. No	Cost (C)	Risk(R)	Total Risk (T)
1	200	75	91.171
2	350	25	198.4252
3	200	25	407.3497
4	350	75	302.3131
5	275	50	39.9434
6	238	60	38.4335
7	230	65	45.596
8	250	55	36.368
9	325	55	83.132
10	240	60	36.8001
11	270	52	36.2909
12	260	55	34.2857
13	255	55	35
14	228	37	168.304
15	265	55	33.9
16	273.8	56.2	35.04
17	257.65	52.15	38.2711
18	270	55	33.62
19	280	55	36.61

Table 1: The list of values queried using the Optimizer App

Reference

[1] Kolda, Tamara G., et al. “Optimization by Direct Search: New Perspectives on Some Classical and Modern Methods.” *SIAM Review*, vol. 45, no. 3, Society for Industrial and Applied Mathematics, 2003, pp. 385–482, http://www.jstor.org/stable/25054427.

[2] Saunders, B. David. "Black box methods for least squares problems." *Proceedings of the 2001 international symposium on Symbolic and algebraic computation*. 2001.

[3] Rakha, Medhat A. "On the Moore–Penrose generalized inverse matrix." *Applied Mathematics and Computation* 158.1 (2004): 185-200.