19PHY113: Computational Engineering Mechanics

Four Bar Quick return Mechanism Design and Applications

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Acknowledgements:

We would profoundly like to thank Prof. E.A Gopalakrishnan, faculty at Amrita Vishwa Vidyapeetham, Coimbatore. He helped us understand the key concepts needed to visualise the model and perform analysis of the mechanism.

To begin with, Mithil found the types of Quick return Mechanisms. Later, Bindusri designed the mechanism based on the stroke length and timing ratio. Thus, she also did the position analysis of the mechanism. Mithil and Samhitha then analysed and derived expressions of the velocity and acceleration analysis. Pooja did the force analysis of all links in the models based on external forces and torque applied to the links of the model. Arrun coded the mechanism to computationally find out the values from the above expression. Arrun along with Aditya also did the interactive simulation for the model.

To conclude the project, Arrun and Samhitha also found practical life applications of the mechanisms in the industry. Thus the project was completed as a team within the stipulated time period.

Abstract:

Quick-return (QR) mechanisms feature different input durations for their working and return strokes. The time ratio (*TR*) of a QR mechanism is the ratio of the change in input displacement during the working stroke to its change during the return stroke. Several basic types of mechanism have a QR action. These include offset slider-crank mechanisms and the inverted slider-crank mechanisms, the crank-shaper mechanism, and the Whitworth four-bar mechanisms.

Introduction to quick return Mechanism:

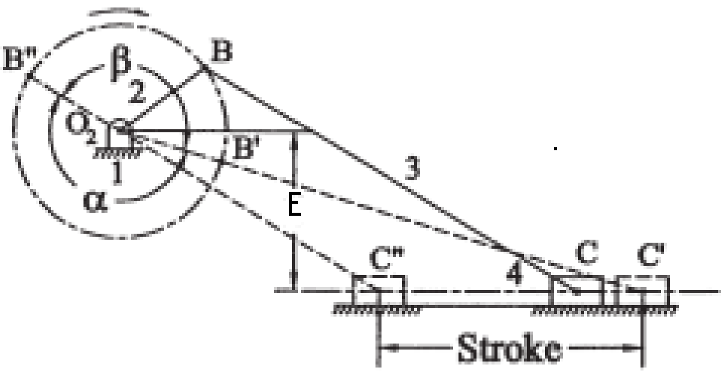
Design of a mechanism requires determining a mechanism to perform a desired task. Choosing a type of mechanism for this task (replicating a reciprocating QR device) requires determination of a mechanism to produce a desired *TR* and a necessary stroke. Moreover, there is not necessarily a unique mechanism design for a particular task: many mechanism types as mentioned before may be capable of performing it.

Several concepts of design and analysis can be illustrated by a QR mechanism project. This can be done using several mechanisms of various types and/or dimensions. Such concerns that satisfy the primary task exist such as mechanism size, minimum and maximum transmission angles, limiting accelerations, etc.. They are a few parameters considered to isolate a preferred design as per the applications.

Types of Quick Return Mechanisms:

As previously mentioned, quick return mechanisms can be synthesised from slider-crank, the inverted slider-crank, the crank-shaper and the Whitworth four-bar mechanisms.

a) Consider the offset slider-crank illustrated below. The crank (member 2) is rotating clockwise and rotates a displacement $\boldsymbol{\alpha}$ (B' to B") as the piston, C, moves from C' (top-dead-centre, TDC) to C" (bottom-dead-centre, BDC). As the piston moves from BDC to TDC the crank rotates a displacement $\boldsymbol{\beta}$ (B" to B'). The time ratio (*TR*) is given by: TR = $\boldsymbol{\alpha} \div \boldsymbol{\beta}$.



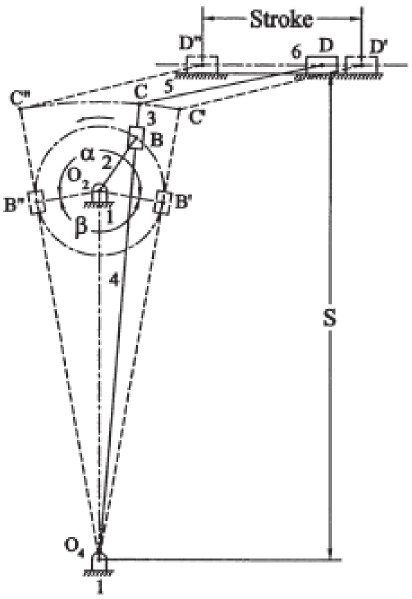
Offset Slider Crank Mechanism

If the eccentricity is E, stroke length S is defined as:

$$S = \sqrt{(L+R)^2 - E^2} - \sqrt{(L-R)^2 - E^2}$$

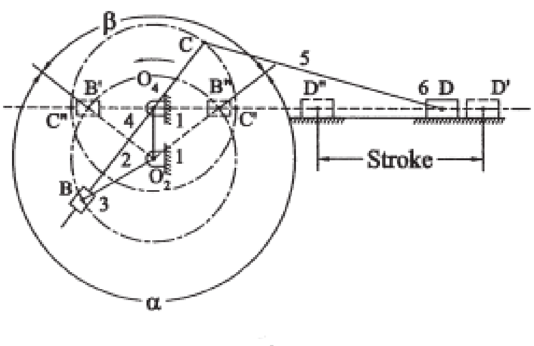
where: L is length of slider; R is length of crank

b) A crank-shaper is comprised of a tool driven by an inverted slider-crank. The crank length of a crank-shaper is less than the base length (O2 to O4) of the mechanism. A notable point here is that the crank (member 2) is rotating counter-clock wise in this case and that the follower (member 4) of the driving mechanism (the inverted slider-crank) oscillates between two extremes. The crank displacements at these extremes define the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ for the device's TR.



Crank-Shaper Mechanism

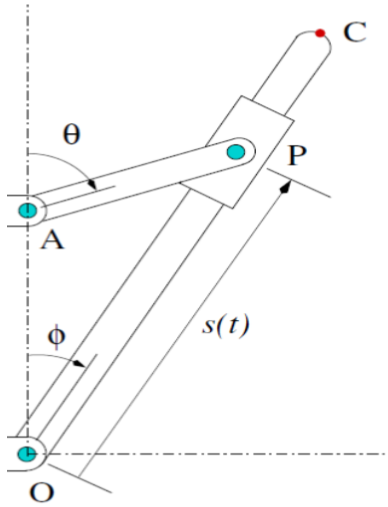
c) A Whitworth mechanism is formed when the crank of the slider-crank inversion is greater than the base distance. Here again the crank (member 2) is rotating counter-clockwise. We must note that the follower (member 4) of the Whitworth is dragged through a full rotation during a revolution of the crank. The crank displacements when the follower is parallel to the sliding direction (horizontal in Fig. 1c) define the values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$.



Whitworth Quick Return Mechanism

Design of QR Mechanisms:

The best way to design a mechanism is to derive analytical expressions for the mechanism lengths required for a desired TR. Note, however, that it is not always possible to derive a closed-form solution for link lengths as a function of a desired TR, due to the nonlinear from of the TR solution. However, if a closed-form solution for the displacements of the driving mechanism can be found, a solution of the TR for given link lengths can be found iteratively.



All the lengths are known i.e, OA , AP and OC are known and θ is the given angle and we need to find ϕ. The ratio of time of cutting stroke to time of return ratio is called as the timing ratio. Timing ratio should be calculated at the instant when phi reaches its maximum value. Stroke length is the maximum distance that a slider can move in one cycle. The formula to calculate them according to our design is:

$$TR(\text{timing ratio}) = \frac{\beta}{\alpha}$$

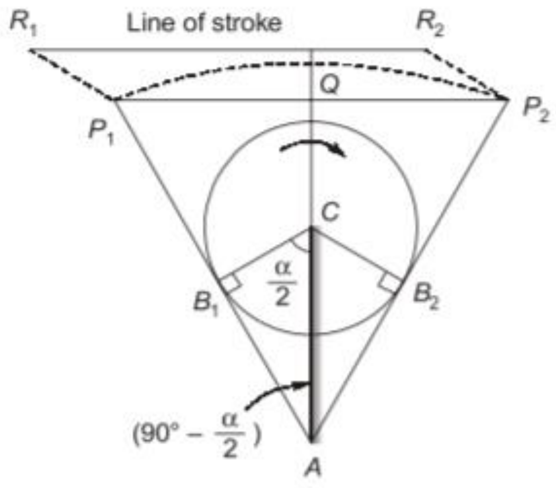
$At\ \phi_{\max}$,

$$\frac{\alpha}{2} = 90 - \phi \qquad \alpha = 180 - 2\phi$$

$$\beta = 360 - (180 - 2\phi)$$
$$= 180 + 2\phi$$

$$TR = \frac{180 + 2\phi}{180 - 2\phi}$$

$\sin ce\ \phi\ is\ always\ greater\ than\ zero$
$TR\ is\ always\ greater\ than\ 1.$



$Length\ of\ the\ stroke -$

$$P_1P_2 = 2P_1Q = 2AP_1\sin(90 - \alpha/2)$$

The link length constraints that we need to incorporate for the proper working of the mechanism is as follows:

$$r_1 + r_2 \geq r_3 (\text{limiting condition } @\theta = 0^\bullet)$$
$$r_1 - r_2 \leq r_3 (\text{limiting condition } @\theta = 180^\bullet)$$

$$r_3 < r_4$$
$$=> r_4 \text{ is the longest link}$$

$$r_1 - r_2 \geq 0 \ (\because r_3 \geq 0)$$
$$=> r_2 \leq r_1 < r_4$$

Analysis of Links during motion is as follows:

Position Analysis:

From Geometry:

$$r_1 + r_2 \cos\theta = r_3 \cos\phi \ - \ \text{I}$$
$$r_2 \sin\theta = r_3 \sin\phi \ - \ \text{II}$$
$$=> \tan\phi = \frac{r_2 \sin\theta}{r_1 + r_2 \cos\theta}$$

$$Stroke = 2 * r_4 * \sin(\phi_{\max})$$

Velocity Analysis:

Differentiating I and II, we get:

$$-r_2 \sin\theta w_2 = -r_3 \sin\phi w_3 + \dot{r}_3 \cos\phi - \ \text{III}$$
$$r_2 \cos\theta w_2 = r_3 \cos\phi w_3 + \dot{r}_3 \sin\phi - \ \text{IV}$$

From III and IV:

$$\begin{bmatrix} -r_3 \sin\phi & \cos\phi \\ r_3 \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} w_3 \\ \dot{r}_3 \end{bmatrix} = \begin{bmatrix} -r_2 \sin\theta w_2 \\ r_2 \cos\theta w_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} w_3 \\ \dot{r}_3 \end{bmatrix} = \begin{bmatrix} -r_3 \sin\phi & \cos\phi \\ r_3 \cos\phi & \sin\phi \end{bmatrix}^{-1} \begin{bmatrix} r_2 \sin\theta w_2 \\ r_2 \cos\theta w_2 \end{bmatrix}$$

Acceleration Analysis:

Differentiating III and IV, we get:

$$-r_2 \cos\theta w_2^{\,2} = -r_3 \cos\phi w_3^{\,2} - r_3 \sin\phi \alpha_3 + \ddot{r}_3 \cos\phi - 2\dot{r}_3 \sin\phi w_3 \quad -V$$

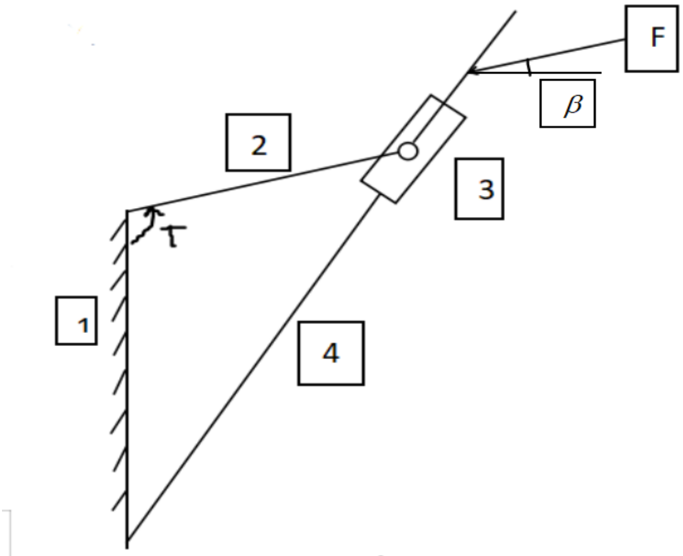
$$-r_2 \sin\theta w_2^{\,2} = -r_3 \sin\phi w_3^{\,2} + r_3 \cos\phi \alpha_3 + \ddot{r}_3 \sin\phi + 2\dot{r}_3 \cos\phi w_3 \quad -VI$$

From V and VI:

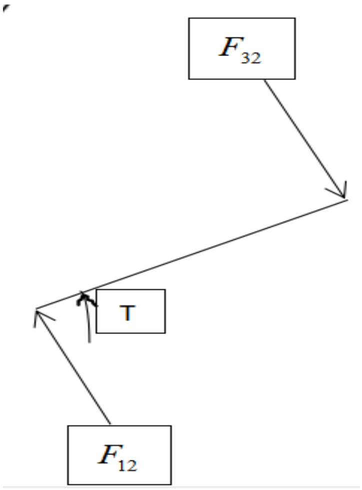
$$\begin{bmatrix} -r_3 \sin\phi & \cos\phi \\ r_3 \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \ddot{r}_3 \end{bmatrix} = \begin{bmatrix} -r_2 \cos\theta w_2^{\,2} + r_3 \cos\phi w_3^{\,2} + 2\dot{r}_3 \sin\phi w_3 \\ -r_2 \sin\theta w_2^{\,2} + r_3 \sin\phi w_3^{\,2} - 2\dot{r}_3 \cos\phi w_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha_3 \\ \ddot{r}_3 \end{bmatrix} = \begin{bmatrix} -r_3 \sin\phi & \cos\phi \\ r_3 \cos\phi & \sin\phi \end{bmatrix}^{-1} \begin{bmatrix} -r_2 \cos\theta w_2^{\,2} + r_3 \cos\phi w_3^{\,2} + 2\dot{r}_3 \sin\phi w_3 \\ -r_2 \sin\theta w_2^{\,2} + r_3 \sin\phi w_3^{\,2} - 2\dot{r}_3 \cos\phi w_3 \end{bmatrix}$$

Force Analysis of our model:

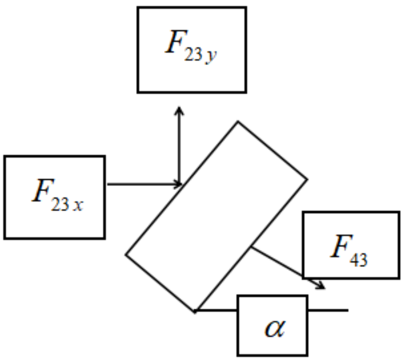


FREE BODY DIAGRAMS



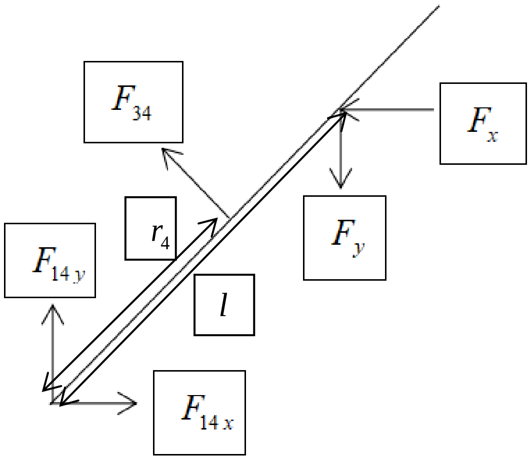
Link 2:

$$F_{12x} + F_{32x} = 0$$
$$F_{12y} + F_{32y} = 0$$
$$r_2 \times F_{32} = T$$
$$T \;\; \alpha \;\; F_{32}$$



Link 3:

$$\alpha = 90 - \phi$$
$$F_{23x} + F_{43}\cos\alpha = 0$$
$$F_{23y} + F_{43}\sin\alpha = 0$$



Link 4:

$$F_{14x} + F_{34}\cos\phi - F_x = 0$$
$$F_{14y} + F_{34}\sin\phi - F_y = 0$$
$$r_4 \times F_{34} + l \times F = 0$$

$$r_4 \times F_{34} + l \times F = 0$$

from the above equation we get F_{34}

substuting it in $F_{23x} + F_{43}\cos\alpha = 0$ *and* $F_{23y} + F_{43}\sin\alpha = 0$ *we get* F_{23}

thus we know F_{32}

and from $r_2 \times F_{32} = T$ *we get* T

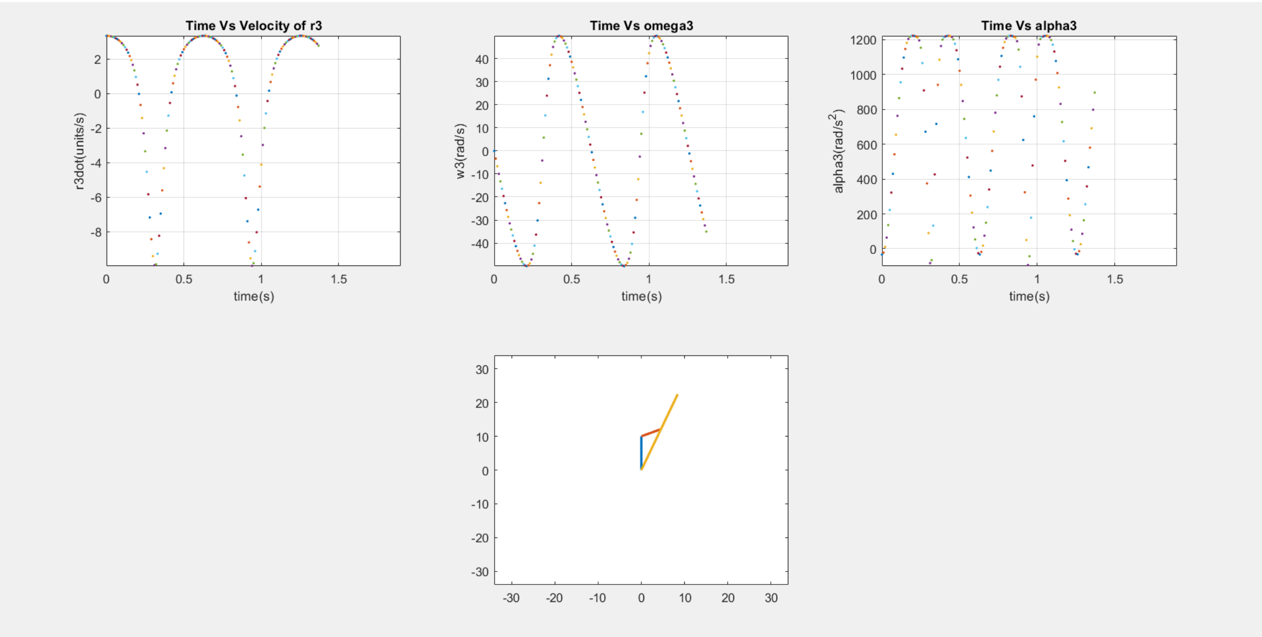
Code to Execute in MATLAB is as follows:

```
clc;clear all;close all;
l1=input("Enter length of link 1\n");
l2=input("Enter length of link 2\n");
l4=input("Enter length of link 4\n");
lengths=sort([l1 l2 l4]);
r2=lengths(1);r1=lengths(2);r4=lengths(3);
grLim=max([r1 r2 r4])+10;
if(r2<=r1 && r2+r1<=r4)
    w2=input("Enter angular velocity of link r2\n");
    rev=input("Enter no. of revolutions to demonstrate\n");
    delt=0.01;
    theta(1)=0;phi=0;t(1)=0;
    totalTime=2*pi*rev/w2;
    for k=1:totalTime/delt+1
        t(k+1)=t(k)+delt;
        theta(k+1)=theta(k)+w2*delt;
        phi(k)=atan(r2*sin(theta(k))/(r1+r2*cos(theta(k))));
        r3(k)=sqrt(r1^2+r2^2+2*r1*r2*cos(theta(k)));
        velA=[-r3(k)*sin(phi(k)) cos(phi(k));r3(k)*cos(phi(k))
sin(phi(k))];
        velB=[-r2*sin(theta(k))*w2;r2*cos(theta(k))*w2];
        velX=velA\velB;
```

```
        r3dot(k)=velX(1);w3(k)=velX(2);
        accA=velA;
        accB=[-
r2*sin(phi(k))*w2^2+r3(k)*sin(phi(k))*w3(k)^2+2*r3(k)*cos(phi(k))*w3
(k);-
r2*cos(phi(k))*w2^2+r3(k)*cos(phi(k))*w3(k)^2+2*r3(k)*sin(phi(k))*w3
(k)];
        accX=accA\accB;
        alpha3(k)=accX(1);r3ddot(k)=accX(2);
    end
    phiMin=min(phi);phiMax=max(phi);
    stroke=2*r4*sin(phiMax);
    disp("Stroke Length of the tool is "+stroke);
    phiTR=asin(r2/r1);
    TR_th=(pi+2*phiTR)/(pi-2*phiTR);
    TR_pr=(pi+2*phiMax)/(pi-2*phiMax);
    disp("Timing Ratio of the Mechanism is "+TR_pr);

    %simulation begins
    r3dotMin=min(r3dot);r3dotMax=max(r3dot);
    w3Min=min(w3);w3Max=max(w3);
    alpha3Min=min(alpha3);alpha3Max=max(alpha3);
    figure
    for i=1:length(t)-1
        drawnow()
        subplot(2,3,1)
        grid on;
        plot(t(i),r3dot(i),'.');
        xlabel('time(s)');
        ylabel('r3dot(units/s)');
        title("Time Vs Velocity of r3");
        axis([0 length(t)*delt r3dotMin r3dotMax ]);
        hold on

        subplot(2,3,2)
        grid on;
        plot(t(i),w3(i),'.');
        xlabel('time(s)');
        ylabel('w3(rad/s)');
        title("Time Vs omega3");
        axis([0 length(t)*delt w3Min w3Max]);
        hold on

        subplot(2,3,3)
        grid on;
        plot(t(i),alpha3(i),'.');
        xlabel('time(s)');
        ylabel('alpha3(rad/s^2)');
        title("Time Vs alpha3");
        axis([0 length(t)*delt alpha3Min alpha3Max]);
        hold on

        subplot(2,3,5)
        th=theta(i);ph=phi(i);
        grid on;
        plot([0 0],[0 r1],'LineWidth',2)
```

```
        axis([-grLim grLim -grLim grLim])
        hold on;
        plot([0 r2*sin(th)],[r1 r1+r2*cos(th)],'LineWidth',2)
        hold on;
        plot([0 r4*sin(ph)],[0 r4*cos(ph)],'LineWidth',2)
        hold off;
        pause(0.01);
    end
else
    disp("Invalid Link lengths to build a Quick return Mechanism");
end
```

Output plot is as follows:



Applications of Quick Return Mechanisms:

Using kinematics, the force and geometry of the rotating joint affects the force and motion of the connected arm. From an engineering standpoint, the quick return mechanism impacted the technology of the Industrial Revolution by minimizing the duration of a full revolution, thus reducing the amount of time needed for a cut or press. Quick return mechanisms are found throughout the engineering industry in different machines: Shaper , Screw press, Power-driven saws, Mechanical actuators etc.

Hydraulic drive mechanism is one of the mechanism used in shaper machine. In this mechanism, the ram is moved forward and backward by a piston moving in a cylinder placed under the ram. Hydraulic fluid is used in hydraulic quick return mechanism for the movement of ram.

The Whitworth quick return mechanism can been modified and used for constructing high-velocity impacting press. The impacting press drive comprises a Whitworth quick return mechanism consisting of a crank and a drive arm together with a variable speed DC motor, a flywheel, bearings, etc.