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2019201011

PAGE No.
DATE / /

OM - HW - 3

Q-ID : 101929 - Solve using Bala's algorithm

$$\max_{x_i} z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

subject to :

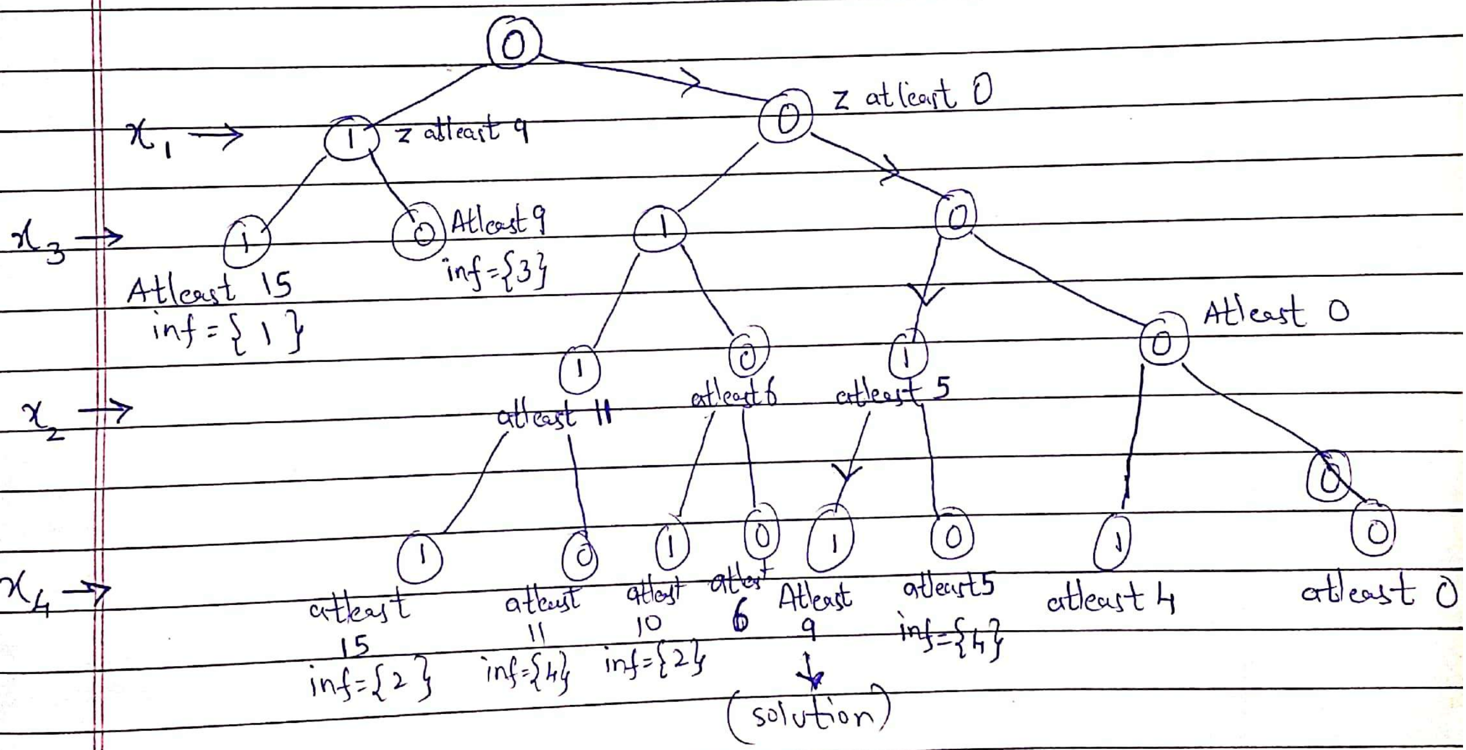
$$6x_1 + 3x_2 + 5x_3 + 2x_4 \le 10 \quad - (1)$$

$$x_3 + x_4 \le 1 \quad - (2)$$

$$x_3 \le x_1 \text{ or } x_3 - x_1 \le 0 \quad - (3)$$

$$x_4 \le x_2 \text{ or } x_4 - x_2 \le 0 \quad - (4)$$

x_j is binary for $j = 1 \dots 4$



Hence using Bala's algorithm the optimal $z^* = 9$ for values of x_i's as $x_1 = 0$, $x_2 = 1$, $x_3 = 0$, $x_4 = 1$.

Akshay Bankar
2019201011

PAGE No.
DATE / /

OM - HomeWork Set-3

Q-ID- 101930

(a) Let x^* be solution of ILP and y^* be solution of LP relaxed version.

Then y^* satisfies the edge constraint of ILP i.e

$$x_i + x_j \geq 1 \quad \forall \{i,j\} \in E$$

and,

x_i's are real number between 0 and 1 for LP relax version.

~~given by the integre~~

Hence the feasible region of LP relax version is always inside the feasible region of ILP.

Hence, $OPT_{LP} \leq OPT_{ILP}$ i.e $y^* \leq x^*$.

Also, we know that the integer programming solution is always inferior to the linear programming problem i.e for a minimization problem, $OPT_{LP} \leq OPT_{ILP}$.

(b) For the given LP relaxed formulation, $x_u + x_v \geq 1$ and therefore, at least one of x_u and x_v is at least $1/2$ and is picked in the vertex cover. i.e one of u or v belongs to S. Hence LPR formulation gives a valid vertex cover.

(c) To find the approximation factor of LP relaxation, given

$$OPT_{Round} = \sum_{i \in V} x_i^*, \quad x_i^* = \begin{cases} 1 & \text{if } x_i \geq 1/2 \\ 0 & \text{if } x_i < 1/2 \end{cases}$$

$$\Rightarrow \quad \leq \sum_{i \in V} 2x_i, \text{ where } x_i \text{ is the fractional value in LPR.}$$

This is valid because $0 \leq x_i \leq 1 \ \forall i \in V$ and $x_i + x_j \geq 1 \ \forall \{i,j\} \in E$.

$$\therefore \quad OPT_{Round} \leq 2 \sum_{i \in V} x_i$$

$$\Rightarrow \quad = 2 OPT_{LP} \quad \text{[solution of LP relaxed version]}$$

$$\leq 2\, OPT_{ILP} \quad \text{[Proved in (a)]}$$

$$\therefore \frac{OPT_{Round}}{OPT_{ILP}} \leq 2, \quad \text{Hence } \max \frac{OPT_{Round}}{OPT_{ILP}} = 2.$$

Hence LPR version is 2-approximation of ILP.

OM- Homework Set-3

Akshay Bankar
2019201011

PAGE No. DATE / /

Q-ID- 101931

Given system of linear equations-

$$2x_1 + 2x_2 + 7x_3 = 7$$
$$3x_1 + (\lambda+9)x_2 + (\lambda+11)x_3 = 10$$
$$3x_1 + (\lambda+4)x_2 + 11x_3 = 10$$

If the linear system is represent as $Ax = B$, then for the system to be inconsistent i.e having no solution. the $\text{rank}(A) < \text{rank}(A|B)$, where $(A|B)$ is the augmented matrix

$$[A|B] = \left[\begin{array}{ccc|c} 2 & 2 & 7 & 7 \\ 3 & (\lambda+9) & (\lambda+11) & 10 \\ 3 & (\lambda+4) & 11 & 10 \end{array}\right]$$

→ Aim is to make one of the rows of A zero such that ~~rank(A) <~~ rank (A) < rank (A|B) by row transformations

$$\Rightarrow R_2 \to R_2 - R_3 \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 7 & 7 \\ 0 & 5 & \lambda & 0 \\ 3 & \lambda+4 & 11 & 10 \end{array}\right]$$

$$R_3 \to R_3 - \frac{3}{2}R_1 \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 7 & 7 \\ 0 & 5 & \lambda & 0 \\ 0 & \lambda+4 & 1/2 & -1/2 \end{array}\right]$$

$$R_2 \to R_2 - \left(\frac{5}{\lambda+4}\right)R_3 \Rightarrow \left[\begin{array}{ccc|c} 2 & 2 & 7 & 7 \\ 0 & 0 & \lambda - \frac{5}{2(\lambda+1)} & \frac{5}{2(\lambda+1)} \\ 0 & \lambda+1 & 1/2 & -1/2 \end{array}\right]$$

Therefore, for $\text{rank}(A) < \text{rank}(A|B)$,

$$\lambda - \frac{5}{2(\lambda+1)} = 0 \Rightarrow 2\lambda^2 + 2\lambda - 5 = 0$$

or $$\lambda = \frac{-2 \pm \sqrt{4+40}}{4} \Rightarrow \lambda = \frac{-1 \pm \sqrt{11}}{2}$$

$\therefore$ for $\lambda = \frac{-1+\sqrt{11}}{2}$ or $\lambda = \frac{-1-\sqrt{11}}{2}$, the given system of linear equations will have no solution.

Akshay Bankar
2019201011

OM- Homework Set - 3

PAGE No.	
DATE	/ /

Q ID - 101932

For the given constraints, introduce basic variables and convert them to equalities.

$$5x_1 - 6x_2 - x_3 + x_4 = -9 \quad - \quad (1)$$
$$-2x_1 + x_2 + 4x_3 = 3 \quad - \quad (2)$$
$$13x_1 - 8x_3 + x_5 = 0 \quad - \quad (3)$$

~~First, b~~

First, basic variable, $B = \{x_2, x_4, x_5\}$

for (1) $b_1 = -9 < 0$

$\therefore$ multiply by -1

$$-5x_1 + 6x_2 + x_3 \geq 9$$
$$-2x_1 + x_2 + 4x_3 = 3$$
$$13x_1 - 8x_3 \leq 0$$

	x_1	x_2	x_3	x_4	x_5	21
	6	7	-3	0	0	
x_2	-2	1	4	0	0	3
x_4	5	-6	-1	1	0	-9
x_5	13	0	-8	0	1	0

add x_3, remove x_2

~~x_2~~	x_1	x_2	x_3	x_4	x_5	-2.25
~~x_2~~	4.5	7.75	0	0	0	
x_3	-2	1	4	0	0	3
x_4	4.5	-5.75	0	1	0	-8.25
x_5	9	2	0	0	1	6

$B = \{x_3, x_4, x_5\}$

Hence we get $z = -2.25$ with

$x_1 = 0$, $x_2 = 0$, $x_3 = 0.75$, $x_4 = -8.25$, $x_5 = 6$

OM Homework Set - 3

Akshay Banker
2019201011

PAGE No.
DATE / /

Q. ID - 101933

LU factorization of the matrix,

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \\ 2 & 6 & 5 \end{bmatrix}$$

$$A = LU \Rightarrow \begin{bmatrix} a_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} l_{11}^{(1)} & 0 \\ L_{21}^{(1)} & L_{22}^{(1)} \end{bmatrix} \begin{bmatrix} u_{11}^{(1)} & U_{12}^{(1)} \\ 0 & U_{22}^{(1)} \end{bmatrix}$$

where the super-script denotes the step number.

Therefore,

$\cancel{l_{11} = 1}$, $\cancel{l_{11} u_{11} = a_{11}} \Rightarrow \cancel{u_{11} =}$

Step-1:

$$\left[\begin{array}{c|cc} 3 & 1 & 4 \\ \hline 1 & 5 & 9 \\ 2 & 6 & 5 \end{array}\right] = \begin{pmatrix} l_{11}^{(1)} & 0 \\ L_{21}^{(1)} & L_{22}^{(1)} \end{pmatrix} \begin{pmatrix} u_{11}^{(1)} & U_{12}^{(1)} \\ 0 & U_{22}^{(1)} \end{pmatrix}$$

$$\Rightarrow \boxed{l_{11}^{(1)} = 1}, \quad l_{11}^{(1)} u_{11}^{(1)} = a_{11} \Rightarrow \boxed{u_{11}^{(1)} = 3}$$

$$L_{21}^{(1)} = \frac{1}{a_{11}} A_{21} = \begin{pmatrix} 1/3 \\ 2/3 \end{pmatrix}, \quad U_{12}^{(1)} = \begin{pmatrix} 1 & 4 \end{pmatrix}$$

~~Step-2~~ $L_{22}^{(1)} U_{22}^{(1)} = A_{22} - \frac{1}{a_{11}} A_{21} A_{12} = \begin{pmatrix} 5 & 9 \\ 6 & 5 \end{pmatrix} - \frac{1}{3}\begin{pmatrix} 1 \\ 2 \end{pmatrix}\begin{pmatrix} 1 & 4 \end{pmatrix} = \begin{pmatrix} 14/3 & 23/3 \\ 16/3 & 7/3 \end{pmatrix}$

Step-2:

$$\left[\begin{array}{c|c} 14/3 & 23/3 \\ 16/3 & 7/3 \end{array}\right] = \begin{pmatrix} l_{11}^{(2)} & 0 \\ L_{21}^{(2)} & L_{22}^{(2)} \end{pmatrix} \begin{pmatrix} u_{11}^{(2)} & U_{12}^{(2)} \\ & U_{22}^{(2)} \end{pmatrix}$$

$$\boxed{l_{11}^{(2)} = 1}, \quad \boxed{u_{11}^{(2)} = \frac{14}{3}}$$

$$L_{21}^{(2)} = \frac{1}{a_{11}} A_{21} = \frac{3}{14} \times \frac{16}{3} \Rightarrow \boxed{L_{21}^{(2)} = \frac{8}{7}}, \quad \boxed{U_{12}^{(2)} = \frac{23}{3}}$$

$$L_{22}^{(2)} U_{22}^{(2)} = \frac{7}{3} - \frac{3}{14} \times \frac{16}{3} \times \frac{23}{3}, \quad \boxed{L_{22}^{(2)} = 1}$$

$$\Rightarrow U_{22}^{(2)} = \frac{7 \times 14 - 16 \times 23}{14 \times 3} \Rightarrow \boxed{U_{22}^{(2)} = \frac{-45}{7}}$$

$$\therefore \begin{bmatrix} 3 & 1 & 4 \\ 1 & 5 & 9 \\ 2 & 6 & 5 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1 & 0 \\ 2/3 & 8/7 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 3 & 1 & 4 \\ 0 & 14/3 & 23/3 \\ 0 & 0 & -45/7 \end{bmatrix}}_{U}$$

OM Homework Set-3

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Q-ID 101934

Recursive method of QR decomposition -
Factorize A, Q and R in the following manner -

$$A = [a_1 \quad A_2], \quad Q = [q_1 \quad Q_2], \quad R = \begin{bmatrix} r_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix}$$

Q is orthogonal, $Q^T Q = \begin{pmatrix} q_1^T \\ Q_2^T \end{pmatrix} (q_1 \quad Q_2) = \begin{pmatrix} q_1^T q_1 & q_1^T Q_2 \\ Q_2^T q_1 & Q_2^T Q_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & I \end{bmatrix}$

Therefore, $q_1^T q_1 = 1$, $q_1^T Q_2 = 0$, $Q_2^T Q_2 = I$

Also, $r_{11} > 0$ & R_{22} is upper triangular matrix with +ve diagonals.

$\therefore \quad (a_1 \quad A_2) = (q_1 \quad Q_2) \begin{pmatrix} r_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix}$

$\Rightarrow \quad a_1 = q_1 r_{11} \Rightarrow \boxed{q_1 = \frac{a_1}{r_{11}}}$ and, $A_2 = q_1 R_{12} + Q_2 R_{22}$

$\Rightarrow \quad q_1^T A_2 = q_1^T q_1 R_{12} + q_1^T Q_2 R_{22}$

$\Rightarrow \quad \boxed{R_{12} = q_1^T A_2}$

Compute QR of $[A_2 - q_1 R_{12}] = Q_{12} R_{22}$

Given $A = \begin{pmatrix} 4 & 6 & 2 & -6 \\ 6 & 34 & 3 & -9 \\ 2 & 3 & 2 & -1 \\ -6 & -9 & -1 & 8 \end{pmatrix}$, $q_1' = a_1 = \begin{pmatrix} 4 \\ 6 \\ 2 \\ -6 \end{pmatrix}$

$r_{11} = \|a_1\| = \sqrt{4^2+6^2+2^2+(-6)^2} = \sqrt{92} = 9.5917$

$\Rightarrow q_1 = \frac{1}{\|a_1\|} a_1 = (0.417 \quad 0.6255 \quad 0.2085 \quad -0.6255)^T$

$R_{12} = (0.417 \quad 0.6255 \quad 0.2085 \quad -0.6255) \times \begin{pmatrix} 6 \\ 34 \\ 3 \\ -9 \end{pmatrix} = 30.0261$

$q_2' = A_2 - q_1 R_{12} = \begin{pmatrix} 6 \\ 34 \\ 3 \\ -9 \end{pmatrix} - 30.0261 \begin{pmatrix} 0.417 \\ 0.6255 \\ 0.2085 \\ -0.6255 \end{pmatrix} = \begin{pmatrix} -6.5217 \\ 15.2174 \\ -3.2609 \\ 9.7826 \end{pmatrix}$

$r_{22} = \|q_2'\| = \sqrt{(-6.5217)^2 + (15.2174)^2 + (-3.2609)^2 + (9.7826)^2} = 19.5047$

$$q_2 = \frac{1}{\|q_2'\|} \cdot q_2' = \frac{1}{19.5047} \begin{pmatrix} -6.5217 \\ 15.2174 \\ -3.2609 \\ 9.7826 \end{pmatrix} = \begin{pmatrix} -0.3344 \\ 0.7802 \\ -0.1672 \\ 0.5016 \end{pmatrix}$$

$$r_{13} = q_1^T \cdot q_3 = \begin{bmatrix} 0.417 & 0.6255 & 0.2085 & -0.6255 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 2 \\ -1 \end{bmatrix} = 0.8359$$

$$q_3' = q_3 - r_{13} q_1 - r_{13} q_2 = \begin{pmatrix} 2 \\ 3 \\ 2 \\ -1 \end{pmatrix} - 3.7533 \begin{pmatrix} 0.417 \\ 0.6255 \\ 0.2085 \\ -0.6255 \end{pmatrix} - 0.8359 \begin{pmatrix} -0.3344 \\ 0.7802 \\ -0.1672 \\ 0.5016 \end{pmatrix} = \begin{pmatrix} 0.7143 \\ 0 \\ 1.3571 \\ 0.9286 \end{pmatrix}$$

$$r_{33} = \|q_3'\| = \sqrt{(0.7143)^2 + (0)^2 + (1.3571)^2 + (0.9286)^2} = \sqrt{3.2143} = 1.7928$$

$$q_3 = \frac{1}{\|q_3'\|} \cdot q_3' = \frac{1}{1.7928} \begin{pmatrix} 0.7143 \\ 0 \\ 1.3571 \\ 0.9286 \end{pmatrix} = \begin{pmatrix} 0.3984 \\ 0 \\ 0.757 \\ 0.5179 \end{pmatrix}$$

$$r_{24} = q_2^T \cdot q_4 = \begin{pmatrix} -0.3344 & 0.7802 & -0.1672 & 0.5016 \end{pmatrix} \begin{pmatrix} -6 \\ -9 \\ -1 \\ 8 \end{pmatrix} = \cancel{0.996} \; \cancel{-13.3449}$$
$$= -0.8359$$

$$r_{14} = q_1^T \cdot q_4 = \begin{pmatrix} \cancel{-0.3344} \; 0.417 & 0.6255 & 0.2085 & -0.6255 \end{pmatrix} \begin{pmatrix} -6 \\ -9 \\ -1 \\ 8 \end{pmatrix} = -13.3449.$$

$$r_{34} = q_3^T \cdot q_4 = \begin{pmatrix} 0.3984 & 0 & 0.757 & 0.5179 \end{pmatrix} \begin{pmatrix} -6 \\ -9 \\ -1 \\ 8 \end{pmatrix} = 0.996$$

$$q_4' = q_4 - r_{14} q_1 - r_{24} q_2 - r_{34} q_3 = \begin{pmatrix} -6 \\ -9 \\ -1 \\ 8 \end{pmatrix} + 13.3449 \begin{pmatrix} 0.417 \\ 0.6255 \\ 0.2085 \\ -0.6255 \end{pmatrix} + 0.8359 \begin{pmatrix} -0.3344 \\ 0.7802 \\ -0.1672 \\ 0.5016 \end{pmatrix} - 0.996 \begin{pmatrix} 0.3984 \\ 0 \\ 0.757 \\ 0.5179 \end{pmatrix}$$

$$q_4' = \begin{pmatrix} -1.1111 \\ 0 \\ 0.8889 \\ -0.4444 \end{pmatrix}, \quad r_{44} = \|q_4'\| = \sqrt{(-1.1111)^2 + (0)^2 + (0.8889)^2 + (-0.444)^2}$$

$$r_{44} = \sqrt{2.222} = 1.4907$$

$$q_4 = \frac{1}{\|q_4'\|} q_4' = \frac{1}{1.4907}\begin{pmatrix} -1.1111 \\ 0 \\ 0.8889 \\ -0.4444 \end{pmatrix} = \begin{pmatrix} -0.7454 \\ 0 \\ 0.5963 \\ -0.2981 \end{pmatrix}$$

$$\therefore Q = (q_1, q_2, q_3, q_4) = \begin{pmatrix} 0.417 & -0.3344 & 0.3984 & -0.7454 \\ 0.6255 & 0.7802 & 0 & 0 \\ 0.2085 & -0.1672 & 0.757 & 0.5963 \\ -0.6255 & 0.5016 & 0.5179 & -0.2981 \end{pmatrix}$$

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{pmatrix} = \begin{pmatrix} 9.5917 & 30.0261 & 3.7533 & -13.3449 \\ 0 & 19.5047 & 0.8359 & -0.8359 \\ 0 & 0 & 1.7928 & 0.996 \\ 0 & 0 & 0 & 1.4907 \end{pmatrix}$$

OM - HomeWork set-3

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Q-ID 101935

Given matrix $A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & d & c \\ 0 & 5 & 3 \end{pmatrix}$

(a) To find c, d such that A has real eigenvalues and orthogonal eigenvectors we use following two properties :-

(i) Trace of A, tr(A) = sum of eigenvalues i.e $tr(A) = (1 + d + 3)$ must be real.

Hence $d \in \mathbb{R}$.

(ii) for a matrix to have orthogonal eigenvectors a necessary and sufficient condition is that it be normal i.e

$$AA^T = A^T A$$

$$\Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 2 & d & c \\ 0 & 5 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & d & 5 \\ 0 & c & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 0 \\ 2 & d & 5 \\ 0 & c & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & d & c \\ 0 & 5 & 3 \end{pmatrix}$$

$$LHS = \begin{pmatrix} 5 & 2+2d+c & 10 \\ 2+2d & 4+d^2+c^2 & 5d+3c \\ 10 & 5d+3c & 34 \end{pmatrix}, \quad RHS = \begin{pmatrix} 5 & 2+2d & 2c \\ 2+2d & 4+d^2+25 & cd+15 \\ 2c & cd+15 & c^2+9 \end{pmatrix}$$

Equating elements of LHS & RHS, we get

$2c = 10$ or $\boxed{c = 5}$ and $\boxed{d \in \mathbb{R}}$ i.e any real number

~~$4+d$~~ ~~$5d + 3c = cd+15 \Rightarrow d =$~~

(b) The orthonormal vectors which are linear combination of columns of A can be found by Gram-Schmidt procedure.
Given independent vectors c_1, c_2, c_3 which are columns of A, it finds orthonormal vectors q_1, q_2, q_3 such that
$$\text{span}(c_1, c_2, c_3) = \text{span}(q_1, q_2, q_3)$$

Step 1: $\hat{q}_1 = \frac{c_1}{\|c_1\|} \Rightarrow \|c_1\| = \sqrt{1^2+2^2+0} = \sqrt{5} \Rightarrow q_1 = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\\0\end{pmatrix}$

Step 2: $\bar{q}_2 = c_2 - (q_1^T c_1) q_1$ i.e remove q_1 component from c_2
$$= \begin{pmatrix}2\\d\\5\end{pmatrix} - \begin{pmatrix}1/\sqrt{5} & 2/\sqrt{5} & 0\end{pmatrix}\begin{pmatrix}2\\d\\5\end{pmatrix}\begin{pmatrix}1/\sqrt{5}\\2/\sqrt{5}\\0\end{pmatrix}$$

$$\bar{q}_2 = \begin{bmatrix}(8+2d)/5\\d-4/5\\5\end{bmatrix} \Rightarrow \hat{q}_2 = \frac{\bar{q}_2}{\|\bar{q}_2\|} = \frac{5}{\sqrt{5d^2+24d+189}}\begin{bmatrix}(8+2d)/5\\(d-4)/5\\5\end{bmatrix}$$

Let $\frac{5}{\sqrt{5d^2+24d+189}} = a_1 \Rightarrow \hat{q}_2 = a_1\begin{pmatrix}(8+2d)/5\\(d-4)/5\\5\end{pmatrix}$

Step 3: $\bar{q}_3 = c_3 - (q_1^T c_3) q_1 - (q_2^T c_3) q_2$
$$\Rightarrow \bar{q}_3 = \begin{pmatrix}0\\c\\3\end{pmatrix} - \begin{pmatrix}2c/5\\4c/5\\0\end{pmatrix} - \left(\frac{c(d-4)}{5} a_1 + 15 a_1\right) a_1 \begin{pmatrix}(8+2d)/5\\(d-4)/5\\0\end{pmatrix}$$

$$\Rightarrow \bar{q}_3 = \begin{bmatrix} -2c/5 - \left[(c(d-4)+75) a_1^2 (8+2d)\right]/25 \\ c - 4c/5 - \cancel{\frac{(d-4)}{5}} \frac{(c(d-4)+75)(d-4)}{5d^2+24d+189} \\ 3 - 5a_1 \end{bmatrix}$$

Let $a_2 = \|\bar{q}_3\| \Rightarrow \hat{q}_3 = \frac{\bar{q}_3}{a_2}$

Now, these q_1, q_2, q_3 are orthonormal. Hence there dot product is zero.

$\therefore\ \bar{q}_1 \cdot \bar{q}_2 = 0 \Rightarrow \begin{pmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \end{pmatrix} \begin{pmatrix} (8+2d)/5 \\ (d-4)/5 \\ 5 \end{pmatrix} = 0$

$\Rightarrow \quad 8+2d + 2(d-4) = 0$

$\Rightarrow \quad 4d + 4 = 0$

$\Rightarrow \quad \boxed{d = -1}$

Also, $\bar{q}_1 \cdot \bar{q}_3 = 0$ and putting $d = -1$ in $\bar{q}_3$

$$\Rightarrow \begin{pmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \end{pmatrix} \begin{pmatrix} -2c/5 - 6a_1^2(75-5c)/25 \\ c - 4c/5 + a_1 \dfrac{75-25c}{74} \\ 3 - 5a_1 \end{pmatrix} = 0$$

$$a_1 = \frac{5}{\sqrt{5d^2+24d+189}} = \frac{5}{\sqrt{74}}$$

$$\Rightarrow \frac{-2c}{5} - \frac{6 \times \cancel{125}^{\,5}(75-5c)}{74 \times \cancel{25}} + \frac{2c+8c}{5} + \frac{150-50c}{74} = 0$$

$$\cancel{\frac{6c}{5}}\ \frac{16c}{5} - \frac{\cancel{60(75-5c)}\ 30(75-5c)+150-50c}{74} = 0$$

$$\frac{16c}{5} - \frac{2400-200c}{74} = 0$$

$\Rightarrow \quad 1184 - 12000 + 1000c = 0$

$\Rightarrow \quad c = \dfrac{10816}{1000}$

$\Rightarrow \quad \boxed{c = 10.816}$

$\therefore \bar{q}_1 \cdot \bar{q}_2 = 0 \Rightarrow \begin{pmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \end{pmatrix} \begin{pmatrix} (8+2d)/5 \\ (d-4)/5 \\ 5 \end{pmatrix} = 0$

$\Rightarrow 8+2d + 2(d-4) = 0$

$\Rightarrow 8+4d \cancel{+8} = 0$

$\Rightarrow \cancel{\boxed{d=-1}} \quad \boxed{d=0}$

Also, $\bar{q}_1 \cdot \bar{q}_3 = 0$ and putting $d = 0$ in $\bar{q}_3$,

$\Rightarrow \begin{pmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \end{pmatrix} \begin{pmatrix} -2c/5 - \cancel{8}q_1^2 \dfrac{(-4c+75)\,\cancel{(75-5c)}}{25} \\ c - 4c/5 + q_1 \dfrac{\cancel{75-25c}(-4c+75)}{\cancel{74}\ 5d^2+24d+189} \\ 3 - 5q_1 \end{pmatrix} = 0$

$q_1 = \dfrac{5}{\sqrt{5d^2+24d+189}} = \dfrac{\cancel{5}}{\sqrt{74}} \quad \dfrac{5}{\sqrt{189}}$

$\Rightarrow -\dfrac{\cancel{2}c}{5} - 6 \times \dfrac{\cancel{125}\,5}{74 \times \cancel{25}}(75-5c) + 2c + \dfrac{8c}{5} + \dfrac{150-50c}{74} = 0$

$\cancel{\dfrac{6c}{5}} \ \dfrac{16c}{5} - \cancel{60(75-5c)} \ \dfrac{30(75-5c)+150-50c}{74} = 0$

$\dfrac{16c}{5} - \dfrac{2400-200c}{74} = 0$

$\Rightarrow 1184 - 12000 + 1000c = 0$

$\Rightarrow c = \dfrac{10816}{1000}$

$\Rightarrow \boxed{c = 10.816}$

$-\dfrac{2c}{5} - \dfrac{(-4c+75)8}{\cancel{\sqrt{189}}\,189} + 2c - \dfrac{8c}{5} + \dfrac{2 \times (-4c+75)8}{189} = 0$

The $\cancel{\Rightarrow}$ equation satisfies for any value of c.

OM - Homework Set-3 Akshay Bankar
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Q-ID 101936

To find solution of $Ax = b$ with smallest value of $\|x - x_0\|_2$ i.e

$$\text{minimize } \|x - x_0\|_2$$
$$\text{subject to } Ax = b$$

The constraint from the optimization problem can be removed by introducing Lagrangian λ.

$$\therefore \quad L(\lambda) = (x - x_0)^T (x - x_0) + \lambda (Ax - b) \quad - (1)$$

Differentiate ~~the~~ L w.r.t x ~~we get~~, & equate to zero,

$$\frac{\partial L}{\partial x} = 2(x - x_0) + A^T \lambda = 0$$

$$\Rightarrow \quad x = -\frac{A^T \lambda}{2} + x_0 \quad - (2)$$

Differentiate L i.e (1) w.r.t λ & equate it to zero,

$$\frac{\partial L}{\partial \lambda} = Ax - b = 0$$

$$\Rightarrow \quad Ax = b$$

Substitute x in this from (2).

$$\Rightarrow \quad A\left(-\frac{A^T \lambda}{2} + x_0\right) = b$$

$$\Rightarrow \quad \lambda = -2(AA^T)^{-1}(b - x_0) \quad - (3)$$

Substitute λ in (2) to get solution of x.

$$\Rightarrow \quad \boxed{x = A^T (AA^T)^{-1} (b - x_0) + x_0}$$

Solution

OM- Homework Set-3

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To find dual of : $\min\ z = 6x_1 + 7x_2 - 3x_3$

subject to : $5x_1 - 6x_2 - x_3 \le -9$

$-2x_1 + x_2 + 4x_3 = 3$

$13x_1 - 8x_3 \le 0$

$x_1, x_2, x_3 \geq 0$

Comparing this with standard form :

$\min\ c^T x$ s.t $Ax \le b$, $x_i \ge \forall\ i = 1, 2, 3$

we have, $c = \begin{pmatrix} 6 \\ 7 \\ -3 \end{pmatrix}$, $A = \begin{bmatrix} 5 & -6 & -3 \\ -2 & 1 & 4 \\ 13 & 0 & -8 \end{bmatrix}$, $b = \begin{bmatrix} -9 \\ 3 \\ 0 \end{bmatrix}$

Dual form is given by,

$\max\ b^T y$ s.t $-Ay \le c$

Since 2^{nd} constraint in the primal is equality, the corresponding dual variable y_2 will be unconstrained in sign.

$\therefore$ Dual : $\max\ b^T y$ s.t $-Ay \le c$, $y_1, y_3 \ge 0$; y_2 unrestricted in sign

$\therefore$

$\max\ z_y = 9y_1 + 3y_2$

s.t $-5y_1 - 2y_2 - 13y_3 \le 6$

$6y_1 + y_2 \le 7$

$y_1 + 4y_2 + 8y_3 \le -3$ and $y_1, y_3 \ge 0$; y_2 unrestricted in sign.