Department of Physics
The University of Hong Kong

Quantum Monte Carlo Simulations of Dimension-tunability Phenomena in The Quantum Spin System

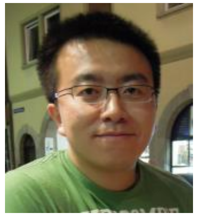
Supervisor: Dr. Zi Yang Meng

Presenter: Chengkang Zhou

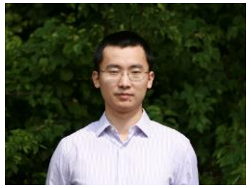
Data are generated as in: 2023.08.08

Cooperators

Zi Yang Meng	University of Hongkong	Professor
Zheng Yan	Westlake University	Professor
Peng Ye	Sun Yat-Sen University	Professor
Han-Qing Wu	Sun Yat-Sen University	Professor
Meng-Yuan Li	Sun Yat-Sen University	Ph.D Candidate
Kai Sun	University of Michigan	Professor
Oleg Starykh	University of Utah	Professor






Publication during my Ph.D. program

1. Amplitude Mode in Quantum Magnets via Dimensional Crossover, **Chengkang Zhou**, Zheng Yan, Han-Qing Wu, Kai Sun, Oleg A. Starykh, Zi Yang Meng#, Phys. Rev. Lett. 126, 227201 (2021).
2. Evolution of Dynamical Signature in the X-cube Fracton Topological Order, **Chengkang Zhou**, Meng-Yuan Li, Zheng Yan#, Peng Ye#, Zi Yang Meng, Phys. Rev. Research 4.033111 (2022).
3. Detecting Subsystem Symmetry Protected Topological Order Through Strange Correlators, **Chengkang Zhou***, Meng-Yuan Li*, Zheng Yan, Peng Ye#, Zi Yang Meng#, Phys. Rev. B. 106.214428(2022)."
4. Dynamical properties of quantum many-body systems with long-range interactions, Menghan Song, Jiarui Zhao, **Chengkang Zhou**, and Zi Yang Meng#, Phys. Rev. Research 5, 033046

Content

Amplitude mode via dimensional crossover

i. The amplitude model and coupled spin chain model

ii. Numerical method and result

Subdimensional excitation in the X-cube model

i. The type-I fracton phase in the X-cube model

ii. Numerical Result

Subsystem symmetry and Strange correlator

i. The subsystem symmetry protected topological phase

ii. Strange correlator

iii. Numerical Result

Chengkang Zhou, Zheng Yan, Han-Qing Wu, Kai Sun, Oleg A. Starykh, Zi Yang Meng, **Phys. Rev. Lett. 126, 227201 (2021)**.
Chengkang Zhou, Meng-Yuan Li, Zheng Yan, Peng Ye, Zi Yang Meng, **Phys. Rev. Research 4.033111 (2022)**.
Chengkang Zhou, Meng-Yuan Li, Zheng Yan, Peng Ye, Zi Yang Meng, **Phys. Rev. B 106, 214428 (2022)**.

Numerical Methodology

- Quantum Monte Carlo simulation with Stochastic Series Expansion Method (**SSE-QMC**)

$$Z = \sum_{S_L}\sum_{\alpha}(-1)^n \frac{\beta^n (L-n)!}{L!}\left\langle \alpha \left| \prod_{p=0}^{n-1} H_p \right| \alpha \right\rangle$$
$$= \sum_{S_L}\sum_{\alpha}(-1)^n \frac{\beta^n (L-n)!}{L!}\langle \alpha_0 | H_{p_1} | \alpha_1\rangle\langle \alpha_1 | H_{p_2} | \alpha_2\rangle .. \langle \alpha_{L-1} | H_{p_0} | \alpha_0\rangle$$

- Stochastic Analytic Continuation (**SAC**)

$$G(\tau) = \langle \hat{O}(\tau)\hat{O}(0)\rangle$$
$$A(\omega) = \frac{1}{\pi}\sum_{m,n} e^{-\beta E_n} |\langle m|\hat{O}|n\rangle|^2 \delta(\omega - [E_n - E_m])$$

$$G(\tau) = \int_{-\infty}^{\infty} K(\tau,\omega)A(\omega)d\omega$$
$$K(\tau,\omega) = \frac{1}{\pi}\left(e^{-\tau\omega} + e^{-(\beta-\tau)\omega}\right)$$

Anders W. Sandvik. **in AIP Conference Proceedings, Vol. 1297 (AIP, 2010) pp. 135–338.**
Hui Shao and Anders W. Sandvik, **Phys. Rep,1003. 1-88 (2023).**

1. Amplitude mode via dimensional crossover

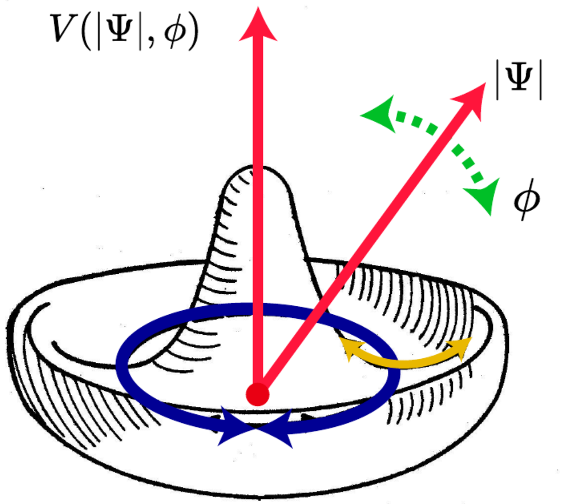
Department of Physics
The University of Hong Kong

Amplitude model and coupled spin chain model

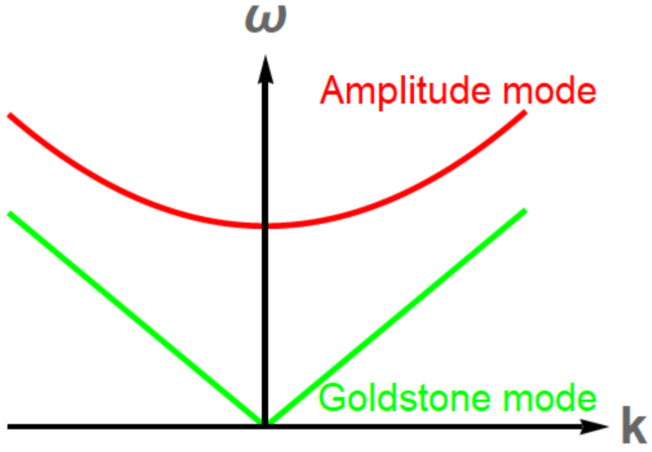
$$\Psi(\mathrm{r},t) = |\Psi(\mathrm{r},t)|e^{i\phi(\mathrm{r},t)}$$

- **Phase**/transverse/Goldstone mode:
  - Phase fluctuations
  - Massless (do not cost energy)
  - Widely observed

- **Amplitude**/longitudinal/Higgs mode:
  - Amplitude fluctuations
  - Massive (cost energy)
  - **Overdamp to gapless, hard to be observed**



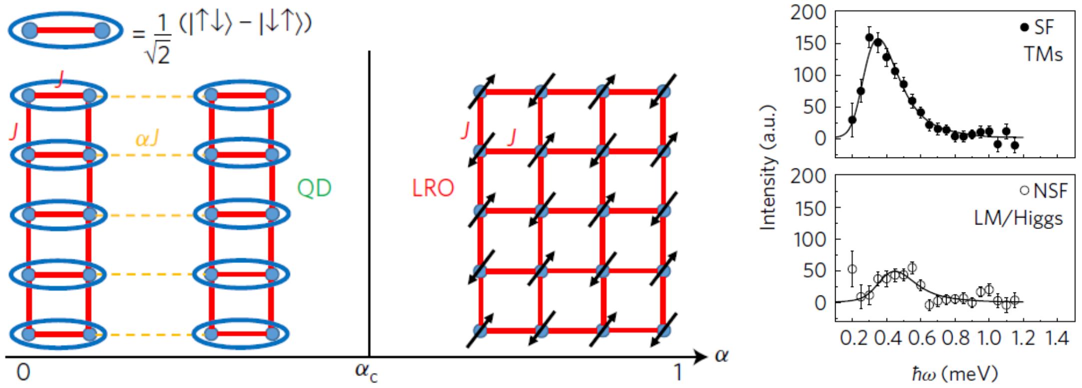
In $U(1)$ matter field, the Mexican hat potential $V(|\psi|,\phi)$ for the condensate field ψ in the long wavelength limit as a function of the amplitude $|\psi|$ and the phase ϕ. Adapted from **Annu. Rev. Condens. Matter Phys. 2015. 6:269–97**



Amplitude model and coupled spin chain model

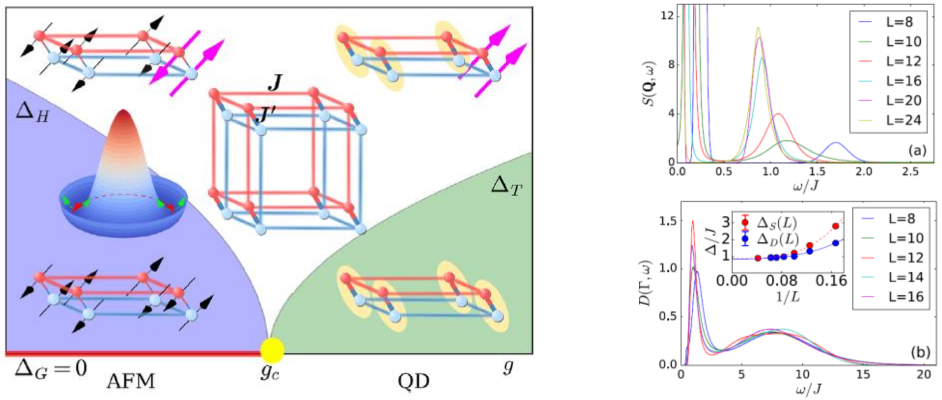
Examples that suppress the damping and see the amplitude mode in quantum magnets

- **Quantum critical points**
  - $C_9H_{18}N_2CuBr_4$



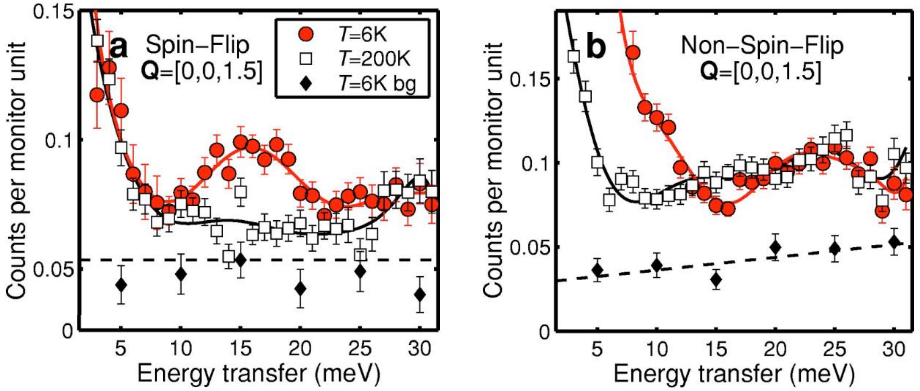
David Pekker. **Nature Physics.13.638–642 (2017)**

The 3D Dimerized Antiferromagnets



Yan Qi Qin and Zi Yang Meng, et al. **PhysRevLett.118.147207 (2017)**

- **Dimensional crossover**
  - $KCuF_3$



B. Lake, D. A. Tennant, and S. E. Nagler. **PhysRevB.71.134412 (2005)**

Department of Physics
The University of Hong Kong

Amplitude model and coupled spin chain model

$$H = J\sum_{\langle i,j\rangle_x} \mathbf{S}_i \cdot \mathbf{S}_j + J_\perp \sum_{\langle i,j\rangle_y} \mathbf{S}_i \cdot \mathbf{S}_j - h\sum_i (-1)^i S_i^z$$

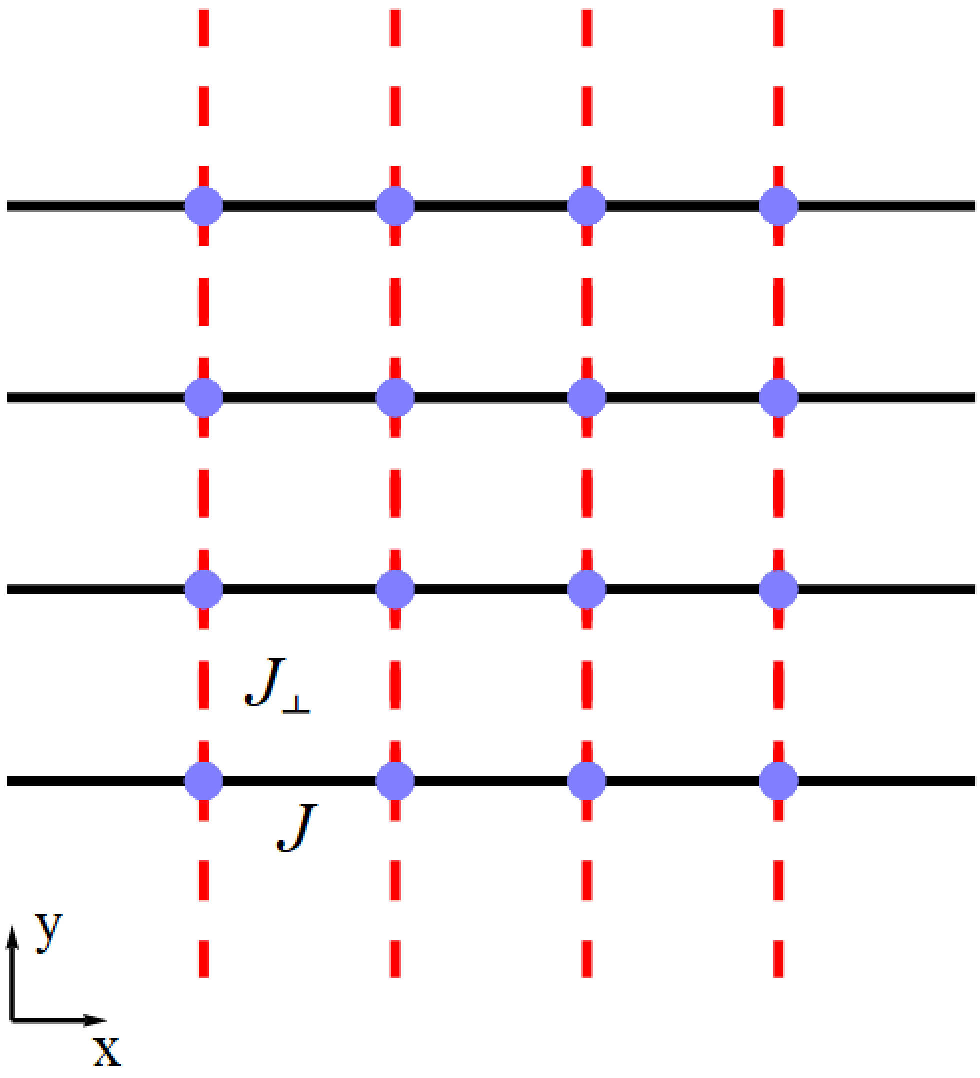
$$g = J_\perp / J$$

- The amplitude fluctuations

$$\langle S^z(\tau) S^z(0) \rangle$$

- The phase fluctuations

$$\langle S^x(\tau) S^x(0) \rangle = \langle S^y(\tau) S^y(0) \rangle$$



Amplitude model and coupled spin chain model

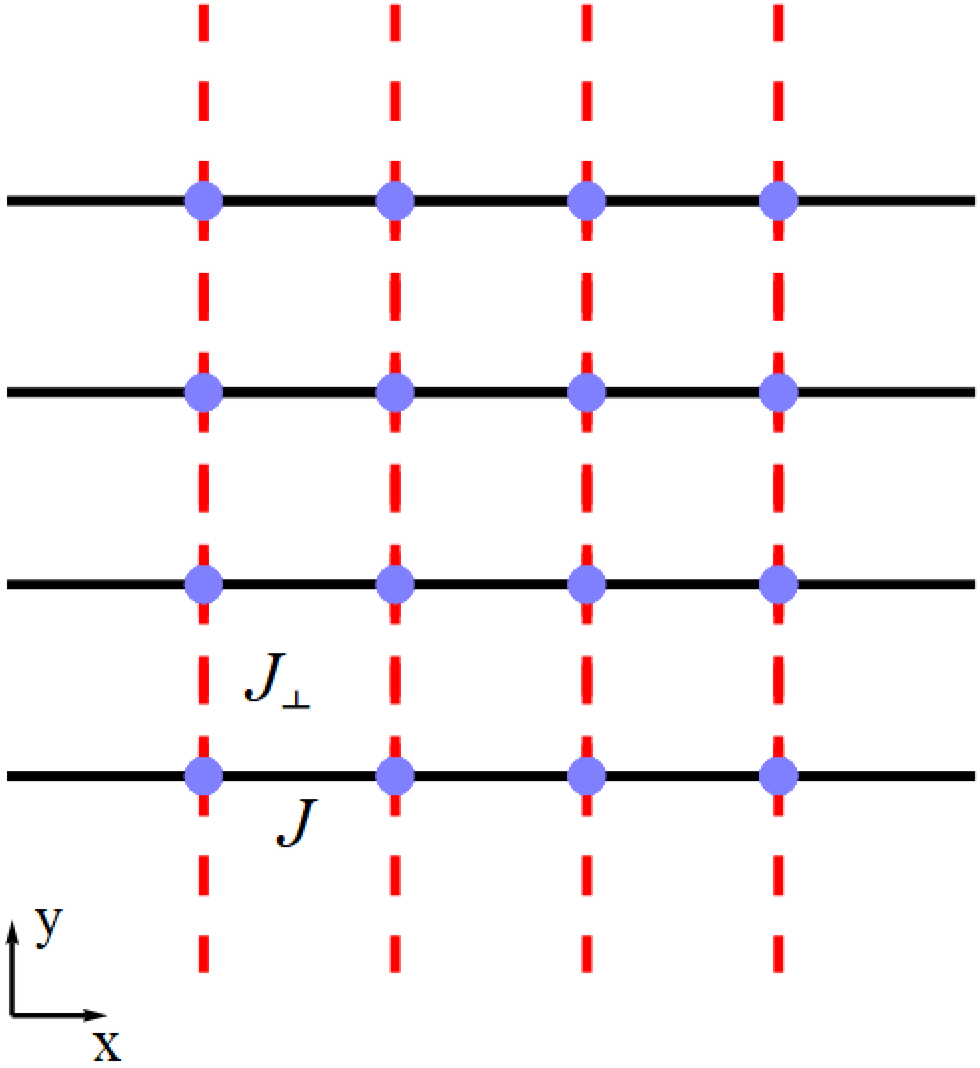
- $h = 0$
- The spin-spin correlation and its spectra function (**phase fluctuations**):

$$G_{S^x}(\mathbf{q},\tau) = \frac{1}{L^2}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\left\langle S_i^x(\tau)S_j^x(0)\right\rangle$$

- The bond-bond correlation and its spectra function (**scalar mode**):

$$B_i = \mathbf{S}_i\cdot\mathbf{S}_{i+\hat{x}}$$

$$G_B(\mathbf{q},\tau) = \frac{1}{L^2}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\langle B_j(\tau)B_i(0)\rangle$$



Department of Physics
The University of Hong Kong

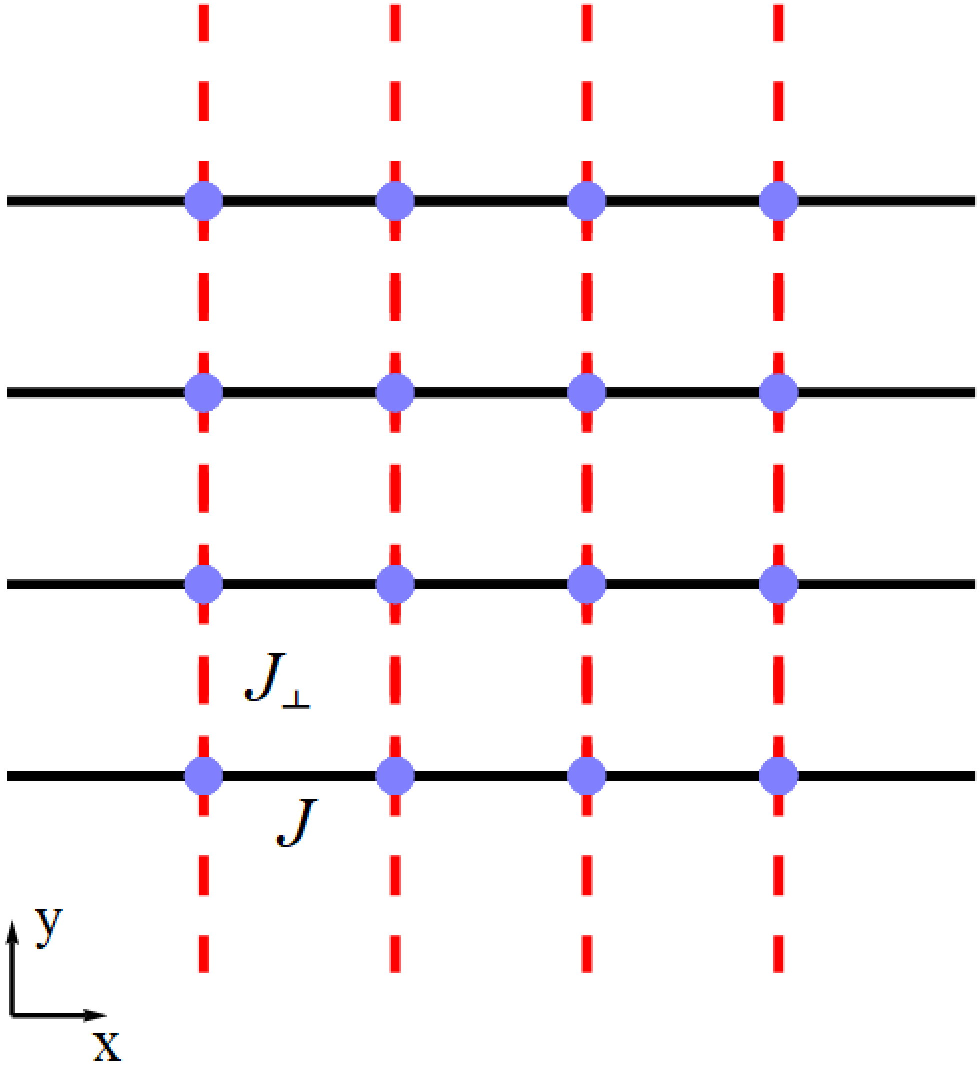
Amplitude model and coupled spin chain model

- $h \neq 0$
- The spin-spin correlation and its spectra function (**phase fluctuations**):

$$G_{S^x}(\mathbf{q},\tau) = \frac{1}{L^2}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\left\langle S_i^x(\tau)S_j^x(0)\right\rangle$$

- The spin-spin correlation and its spectra function (**amplitude fluctuations**):

$$G_{S^z}(\mathbf{q},\tau) = \frac{1}{L^2}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\left\langle S_j^z(\tau)S_i^z(0)\right\rangle$$

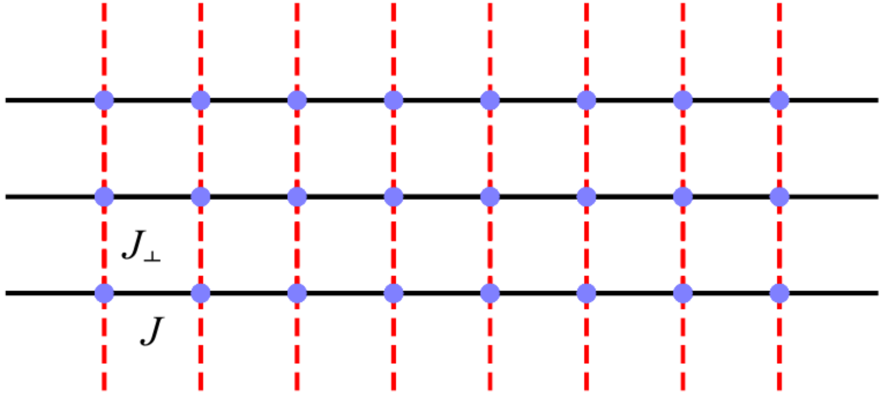


Department of Physics
The University of Hong Kong

Numerical method and result

- Mean field theory treatment and the spin chain with the effective staggered field.

$$H_{MF} = J \sum_{\langle i,j \rangle_{\mathrm{x}}} \mathbf{S}_i \cdot \mathbf{S}_j - (h + h_{\mathrm{eff}}) \sum_i (-1)^i S_i^z$$



- Following the Sine-Gordon model description, there are gapped excitations, which are soliton, the first breather and the second breather. Soliton and the first breather share the same mass. And the mass of the n-th breather is given as

$$\Delta_n = 2\Delta \sin\left(\frac{\pi \xi n}{2}\right) = 2\Delta \sin(\pi n/6)$$

$$\xi = 1/3$$

F. H. L. Essler and R. M. Konik, in **From Fields to Strings: Circumnavigating Theoretical Physics: Ian Kogan Memorial Collection** (in 3 Vols), Vol. 1, edited by M. Shifman (World Scientific Publishing Co. Pte. Ltd, 2005) pp. 684–830.

Numerical method and result

- With the random phase approximation and the Sine-Gordon model, we have

$$\omega_{S^x} = \omega_{S^y} = \Delta_0 \sqrt{1 + b_h + \cos k_y + \frac{v^2 (k_x - \pi)^2}{\Delta_0^2}}$$ Soliton

$$\omega_{S^z} = \Delta_0 \sqrt{3(1 + b_h) + \frac{Z_2}{Z_1} \cos k_y + \frac{v^2 (k_x - \pi)^2}{\Delta_0^2}}$$ The second breather

$$\omega_B = \Delta_0 \sqrt{1 + b_h + \frac{v^2 (k_x - \pi)^2}{\Delta_0^2}}$$ The first breather

in which $\frac{Z_2}{Z_1} \approx 0.49130$, $v = \pi J/2$, and $b_h = 0$ when $h = 0$.

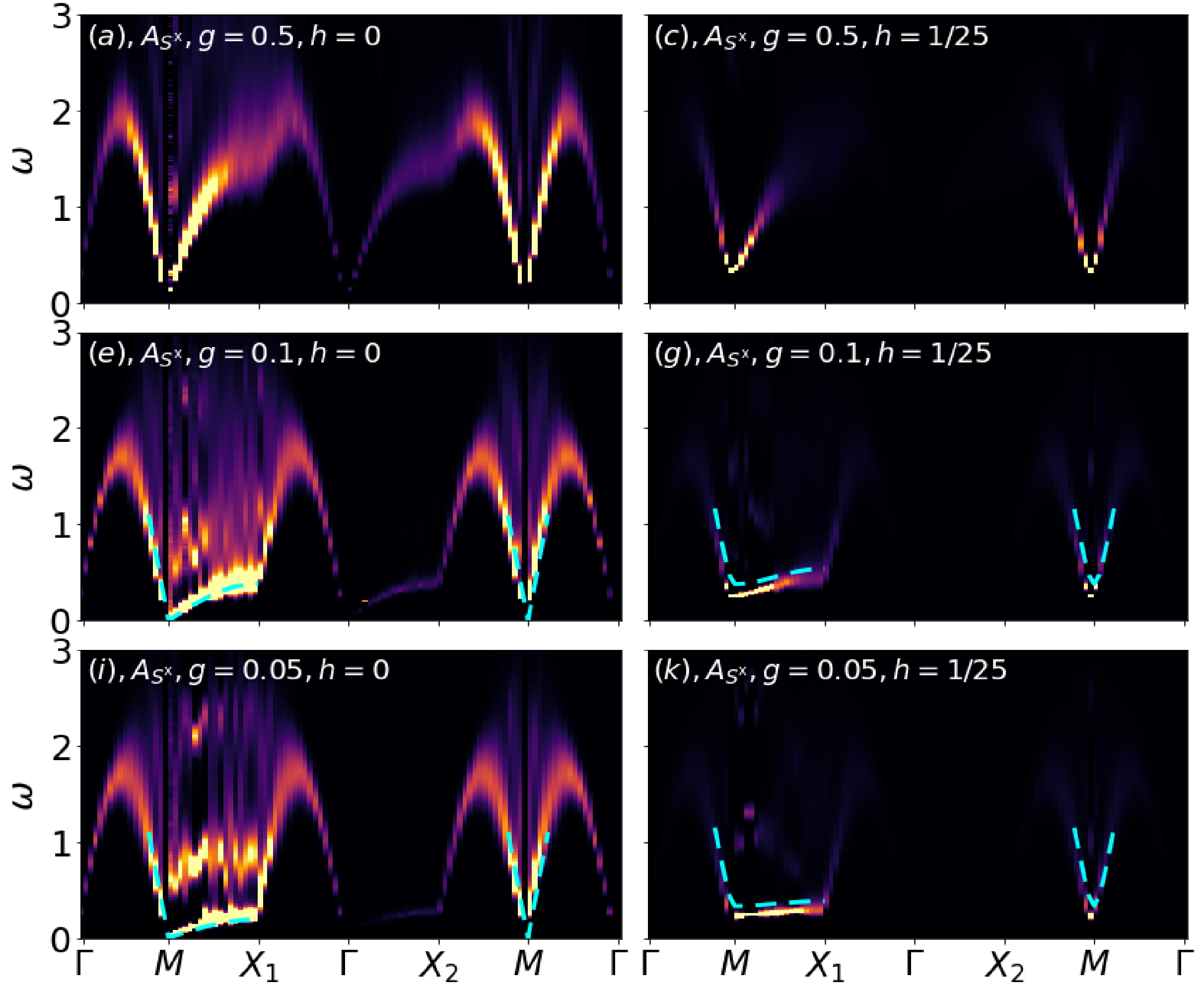
- At (k_x, k_y)= $(\pi, \pi/2)$, there should be $\omega_{S^z} = \sqrt{3}\omega_{S^x}$.

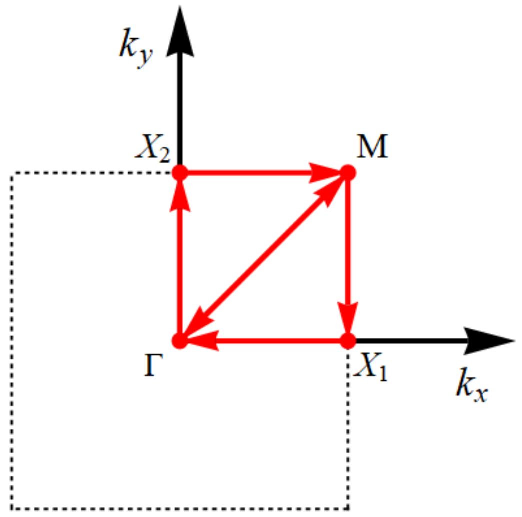
F. H. L. Essler and R. M. Konik, in **From Fields to Strings: Circumnavigating Theoretical Physics: Ian Kogan Memorial Collection** (in 3 Vols), Vol. 1, edited by M. Shifman (World Scientific Publishing Co. Pte. Ltd, 2005) pp. 684–830.

Department of Physics
The University of Hong Kong

Numerical method and result

$$\left\langle S_i^x(\tau) S_j^x(0) \right\rangle$$





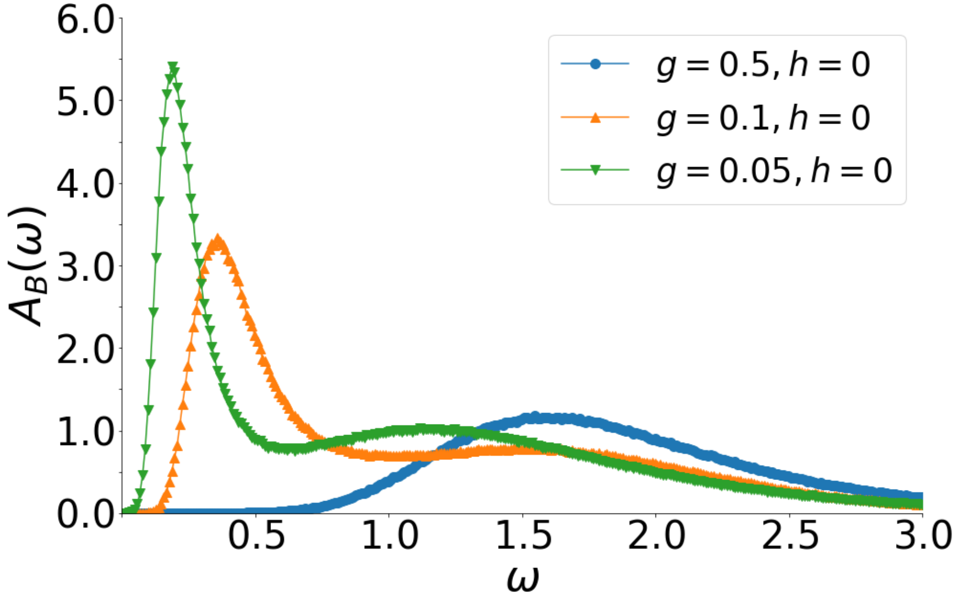
$$\omega_{S^x} = \omega_{S^y} = \Delta_0 \sqrt{1 + b_h + \cos k_y + \frac{v^2 (k_x - \pi)^2}{\Delta_0^2}}$$

$\boldsymbol{A_{S^x}(\omega)}$ is shown with g decreasing when h=0 and h=1/25, which is telling the phase mode.

Department of Physics
The University of Hong Kong

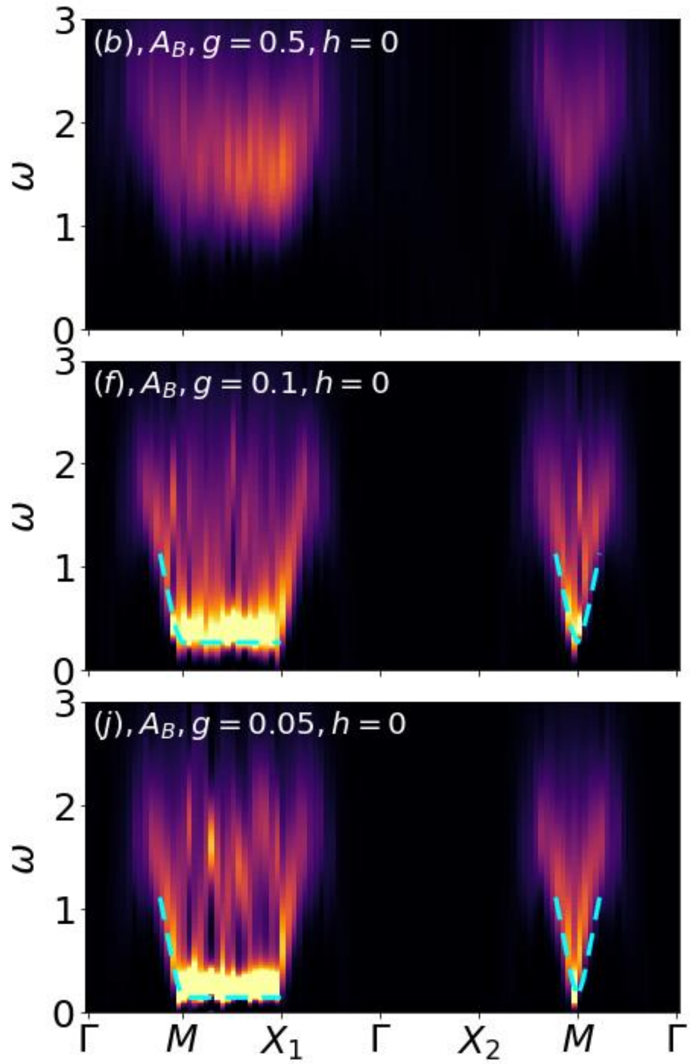
Numerical method and result

$$B_i = \mathbf{S}_i \cdot \mathbf{S}_{i+\hat{x}}$$



Frequency scan of $\boldsymbol{A_B(\omega)}$ at $(\boldsymbol{\pi, \pi/2})$ with $h = 0$ and g decreasing from 0.5 to 0.05.

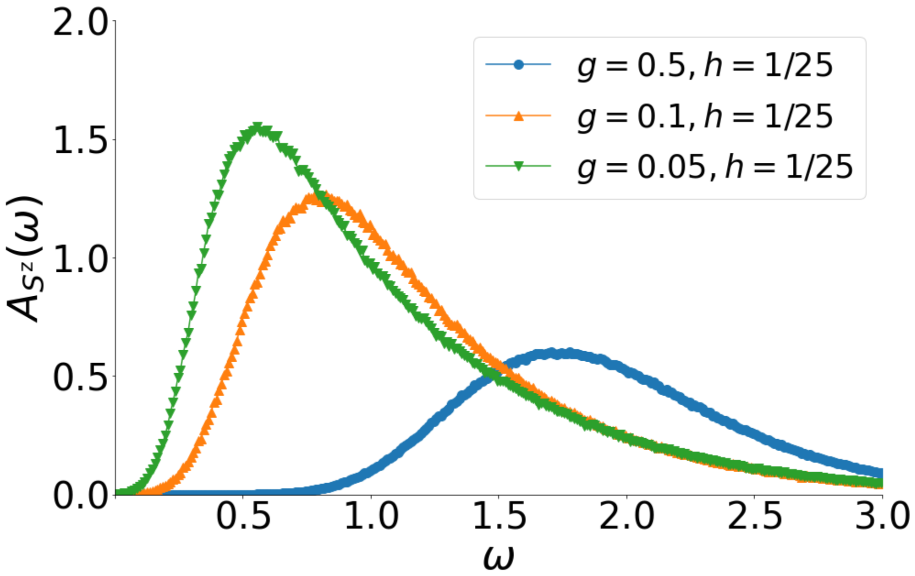
$$\langle B_j(\tau) B_i(0) \rangle$$



$\boldsymbol{A_B(\omega)}$ is shown with g decreasing when h=0, which corresponds to the first breather.

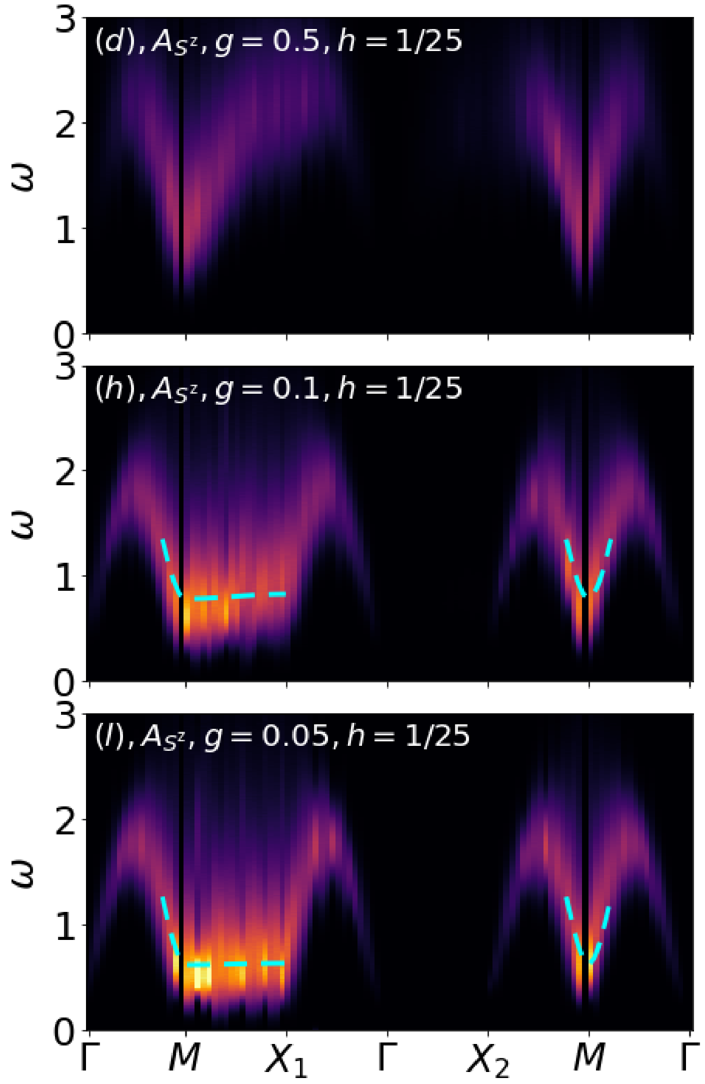
Department of Physics
The University of Hong Kong

Numerical method and result



Frequency scan of $A_{S^z}(\omega)$ at $(\pi, \pi/2)$ with $h = 1/25$ and g decreasing from 0.5 to 0.05.

$$\langle S_i^z(\tau) S_j^z(0) \rangle$$



$A_{S^z}(\omega)$ is shown with g decreasing when h=1/25, which corresponds to the first breather.

Department of Physics
The University of Hong Kong

Numerical method and result

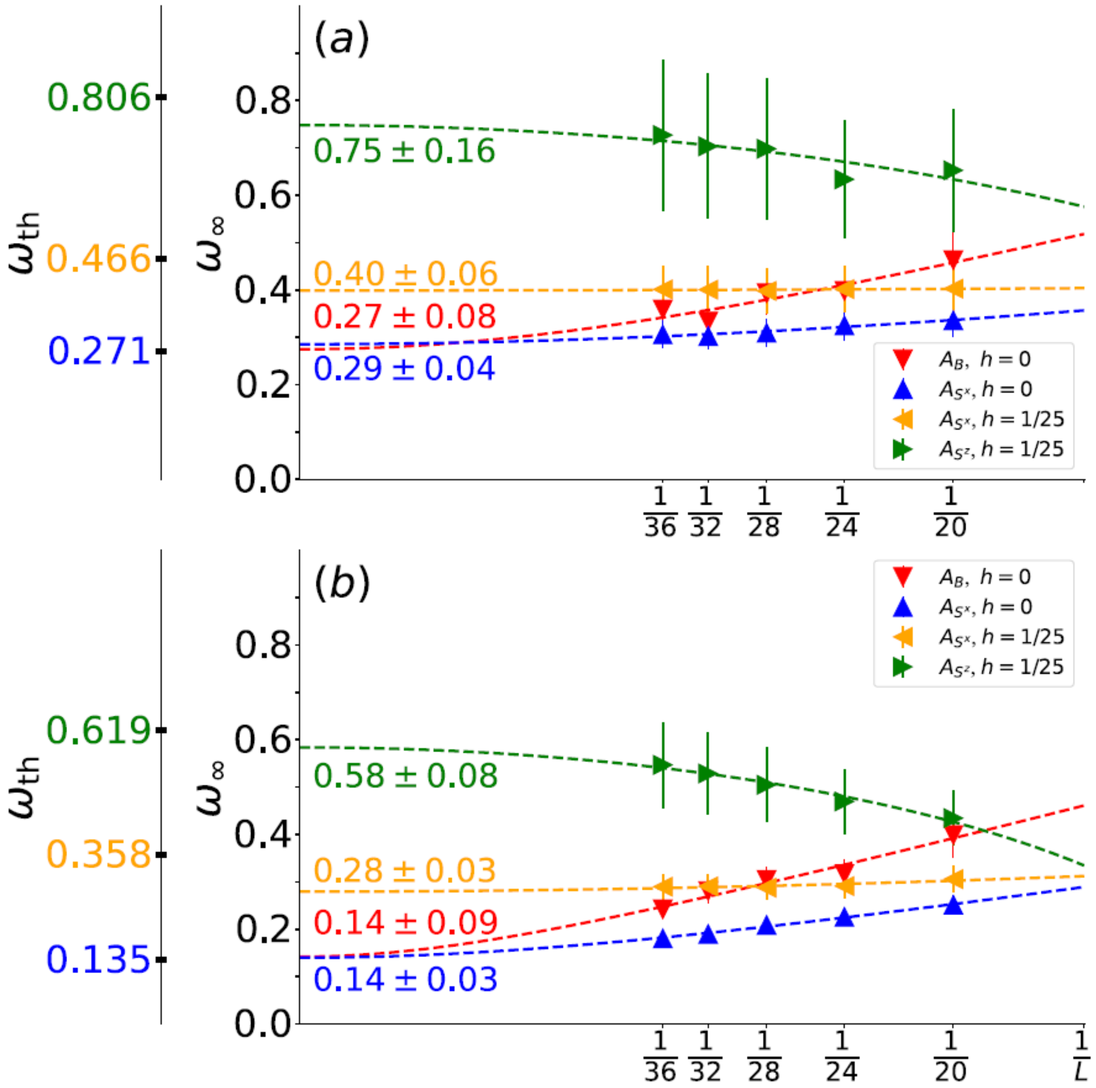
We take the following functional form for the extrapolation to the infinite size at $(\pi,\pi/2)$.

$$\omega_L = \sqrt{\omega_\infty^2 + L_0^2/L^2}$$

$$\omega_{S^x} = \omega_{S^y} = \Delta_0\sqrt{1 + b_h + \cos k_y + \frac{v^2(k_x-\pi)^2}{\Delta_0^2}}$$

$$\omega_{S^z} = \Delta_0\sqrt{3(1 + b_h) + \frac{Z_2}{Z_1}\cos k_y + \frac{v^2(k_x-\pi)^2}{\Delta_0^2}}$$

$$\omega_B = \Delta_0\sqrt{1 + b_h + \frac{v^2(k_x-\pi)^2}{\Delta_0^2}}$$



Finite-size analysis for (a) **g=0.1** and (b) **g=0.05** at the momentum point $(\pi,\pi/2)$.

2. Subdimensional Excitation In The X-cube Model

Department of Physics
The University of Hong Kong

The Type-I Fracton Phase In The X-cube Model

- The X-cube model
  - Hamiltonian of the X-cube model

$$H = -K\sum_{i} A_{c,i} - \Gamma \sum_{i,v} B_{v,i}$$

  - The ground state (Cage-net state)

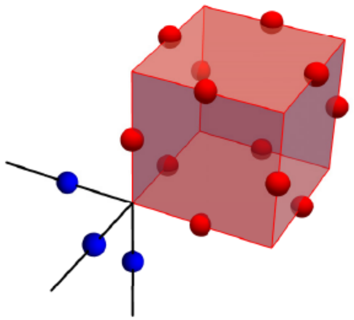
$$|GS\rangle = \prod \frac{1+B_{v,i}}{\sqrt{2}} |\uparrow\uparrow\ldots\uparrow\rangle$$

    - 1, The equal-weight superposition of all allowed cage configurations.
    - 2, All terms have

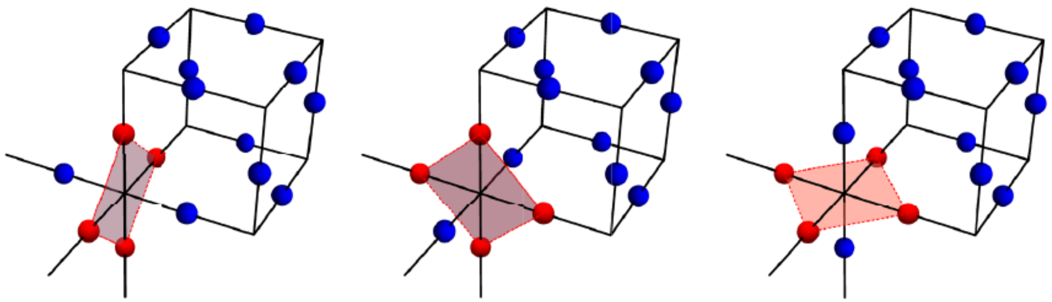
$$\langle GS|A_{c,i}|GS\rangle = 1$$

$$\langle GS|B_{v,i}|GS\rangle = 1$$

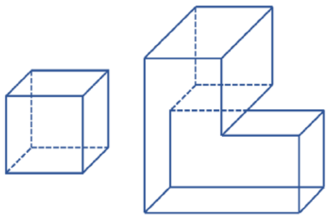
    - 3, **Vacuum state**



$$A_{c,i} = \prod_{k\in c} \sigma_k^x$$



$$B_{v_x,i} = \prod_{k\in v_x} \sigma_k^z \qquad B_{v_y,i} = \prod_{k\in v_y} \sigma_k^z \qquad B_{v_z,i} = \prod_{k\in v_z} \sigma_k^z$$

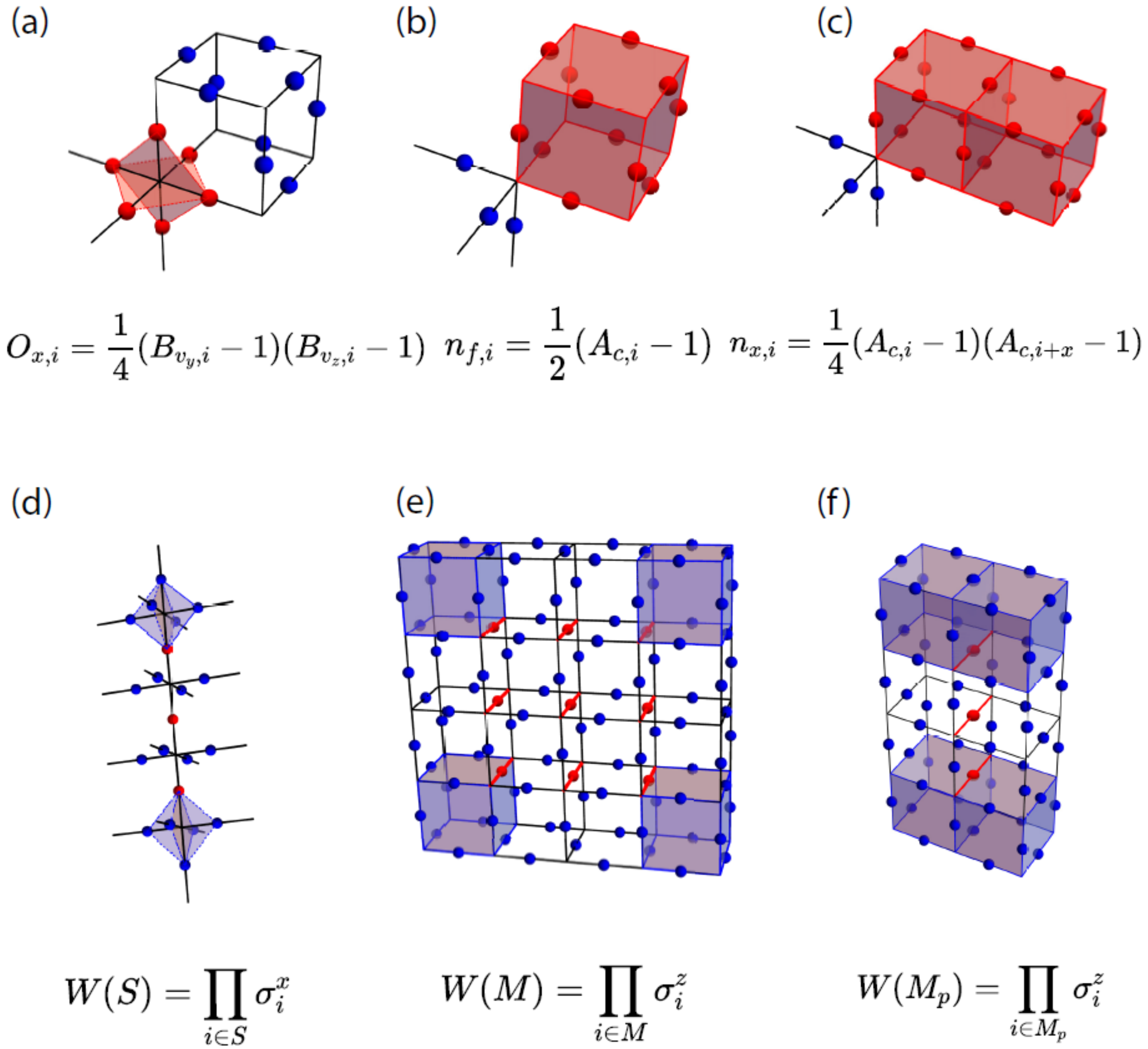


Sagar Vijay, Jeongwan Haah, and Liang Fu, **Phys. Rev. B 94, 235157 (2016)**.
Prem, Abhinav and Huang, Sheng-Jie and Song, Hao and Hermele, Michael, **Phys. Rev. X 9.021010 (2019)**.

Department of Physics
The University of Hong Kong

The Type-I Fracton Phase In The X-cube Model

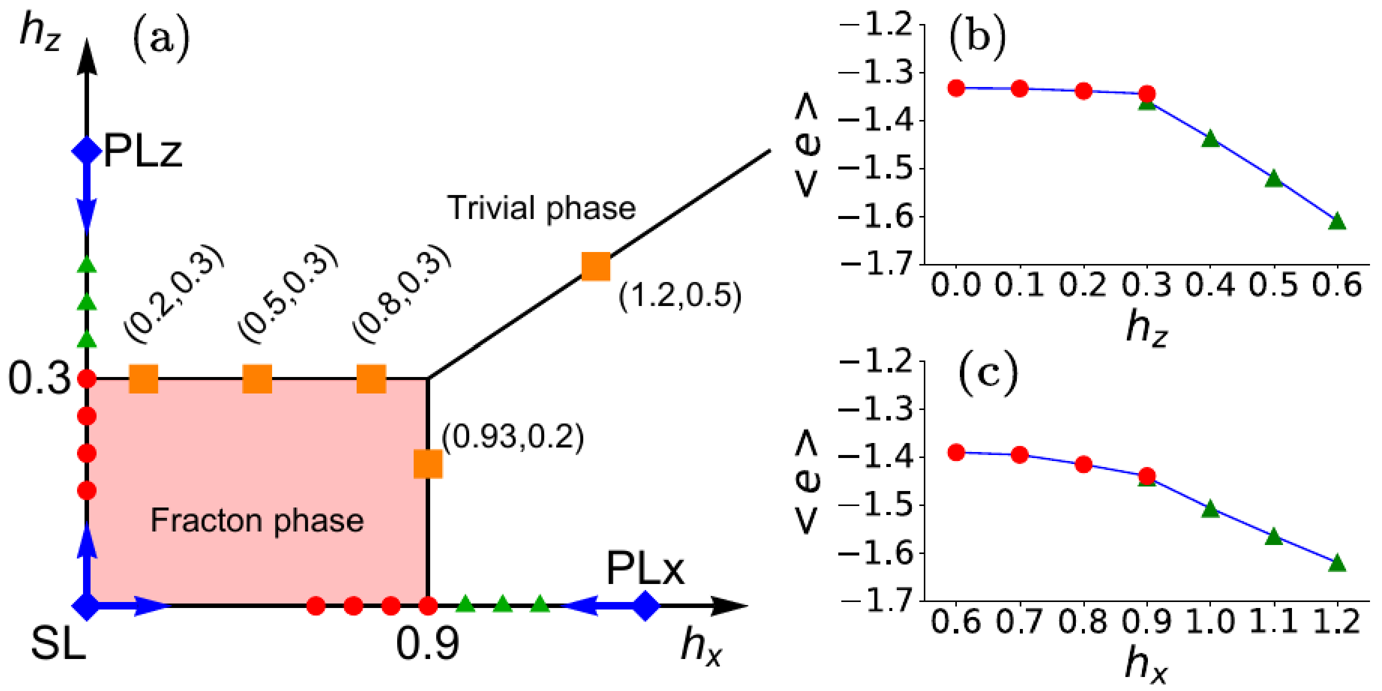
- The X-cube model
  - The subdimsional excitations
    - fractons (0D) :
      - **0-dimensional** Completely immobile;
      - Created by a membrane operator.
    - lineons (1D) :
      - **1-dimensional** mobile along straight lines;
      - Created by a string operator.
    - planons (2D) :
      - **2-dimensional** mobile within flat planes;
      - Two neighboring fractons.



Department of Physics
The University of Hong Kong

The Type-I Fracton Phase In The X-cube Model

- The X-cube model
  - Inserting Zeeman-like fields to introduce a fluctuation.
  - Phase diagram $$H = -K\sum_i A_{c,i} - \Gamma \sum_{i,v} B_{v,i} - h_x \sum_i \sigma_i^x - h_z \sum_i \sigma_i^z$$



Devakul, Trithep and Parameswaran, S. A. and Sondhi, S. L., **Phys. Rev. B 97.041110(R) (2018)**.
Chengkang Zhou, Meng-Yuan Li, Zheng Yan, Peng Ye, Zi Yang Meng, **Phys. Rev. Research 4.033111 (2022)**.

Department of Physics
The University of Hong Kong

Numerical Result

$$H = -K\sum_i A_{c,i} - \Gamma\sum_{i,v} B_{v,i} - h_x\sum_i \sigma_i^x - h_z\sum_i \sigma_i^z$$

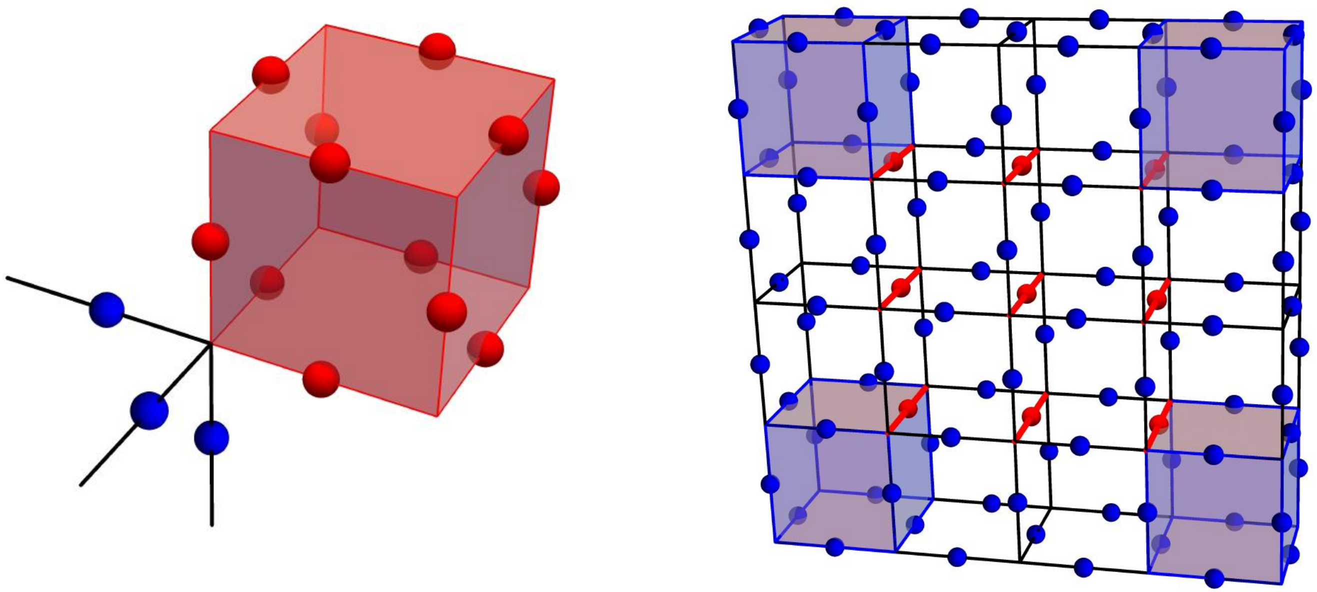
- Fractons
  - Created by

$$W(M) = \prod_{i\in M}\sigma_i^z$$

  - Density operator:

$$n_{f,i} = \frac{1}{2}(A_{c,i} - 1)$$

  - Mobility constraints:
    - Completely immobile
    - Costing $8K$ energy

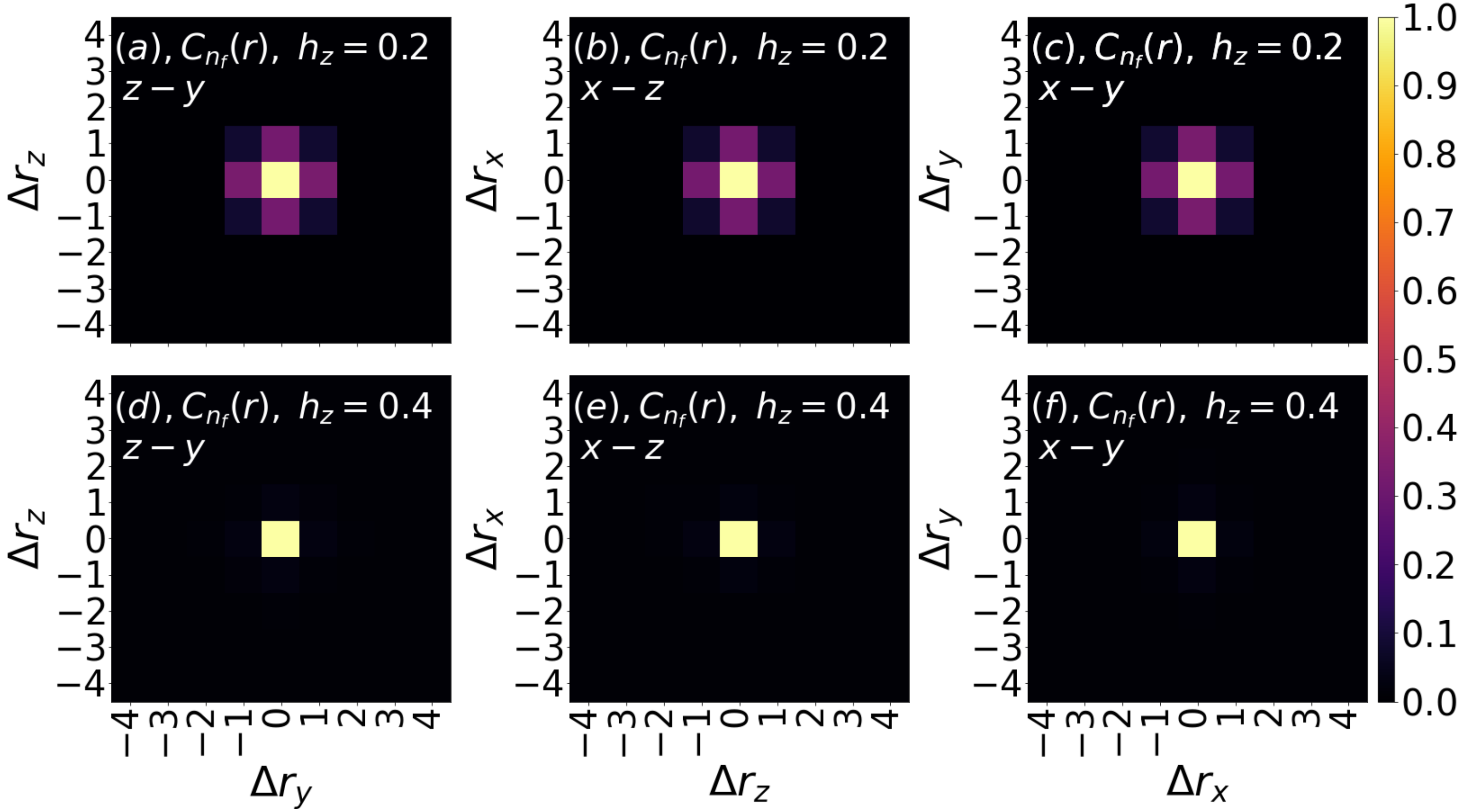


Rahul M. Nandkishore and Michael Hermele **Annu. Rev. Condens. Matter Phys. 2019. 10:295–313**

Numerical Result

- Fractons

$$C_{n_f}(\mathbf{r}) = \frac{\langle n_{f,i} n_{f,i+\mathbf{r}} \rangle - \langle n_{f,i} \rangle^2}{\langle n_{f,i}^2 \rangle - \langle n_{f,i} \rangle^2}$$

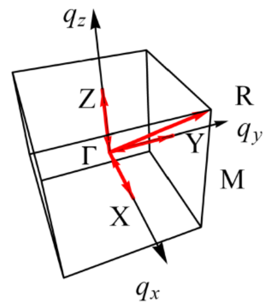


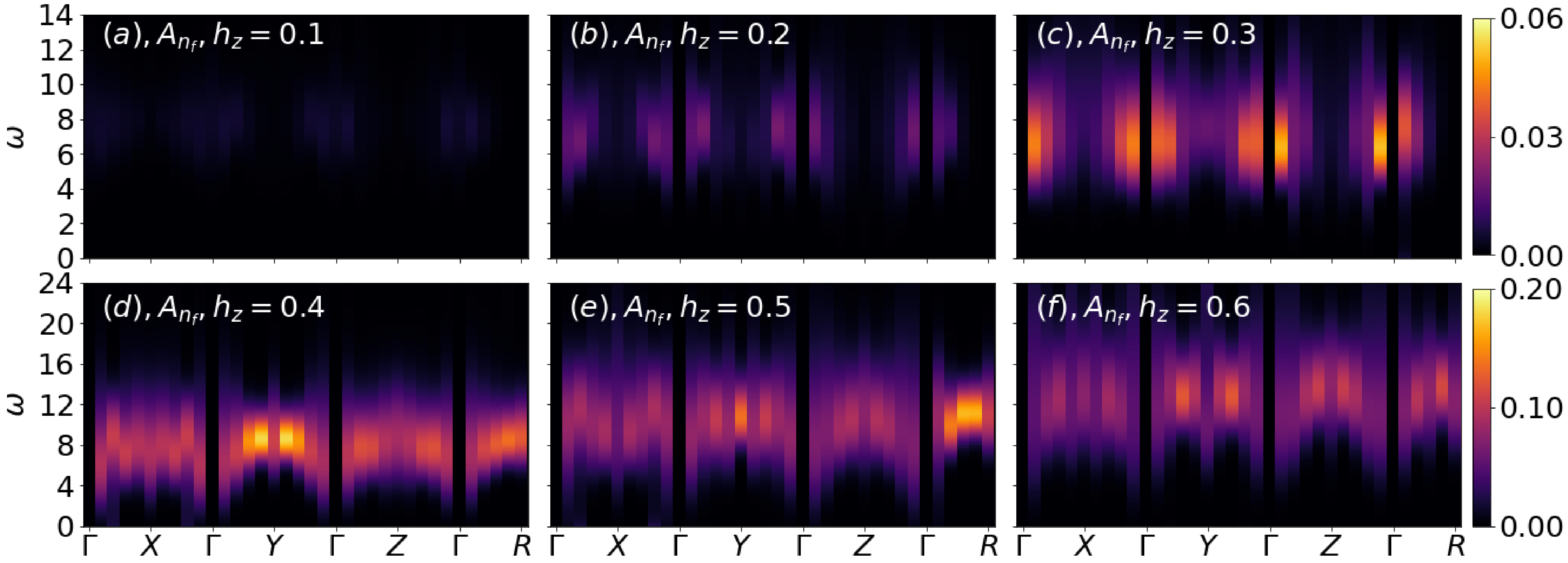
Department of Physics
The University of Hong Kong

Numerical Result

- Fractons

$$G_{n_f}(\mathbf{q},\tau) = \frac{1}{L^3}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\langle n_{f,i}(\tau)n_{f,j}(0)\rangle$$





Numerical Result

- Lineons
  - Created by

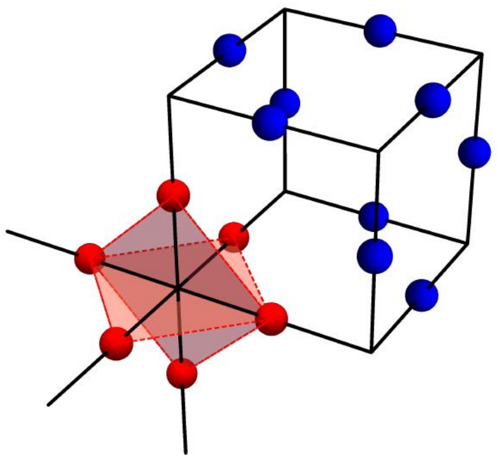
$$W(S) = \prod_{i \in S} \sigma_i^x$$

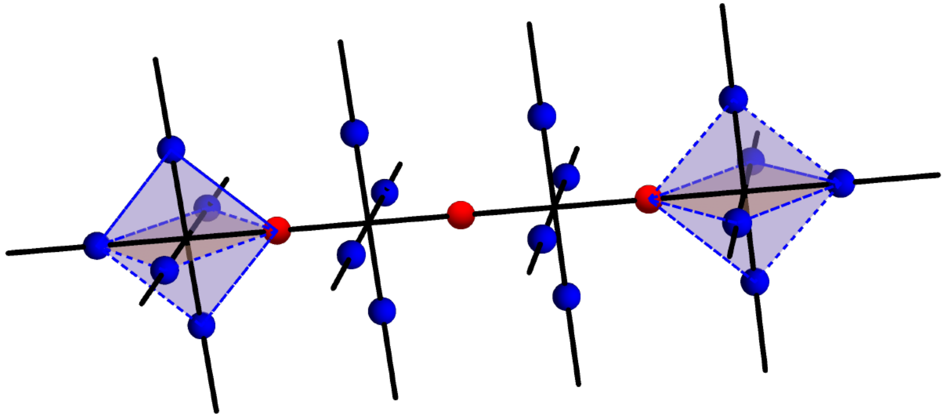
$$H = -K \sum_i A_{c,i} - \Gamma \sum_{i,v} B_{v,i} - h_x \sum_i \sigma_i^x - h_z \sum_i \sigma_i^z$$

  - Density operator:

$$O_{x,i} = \frac{1}{4}(B_{v_y,i} - 1)(B_{v_z,i} - 1)$$

  - Mobility constraints:
    - Mobile along a straight line
    - Costing 8Γ energy





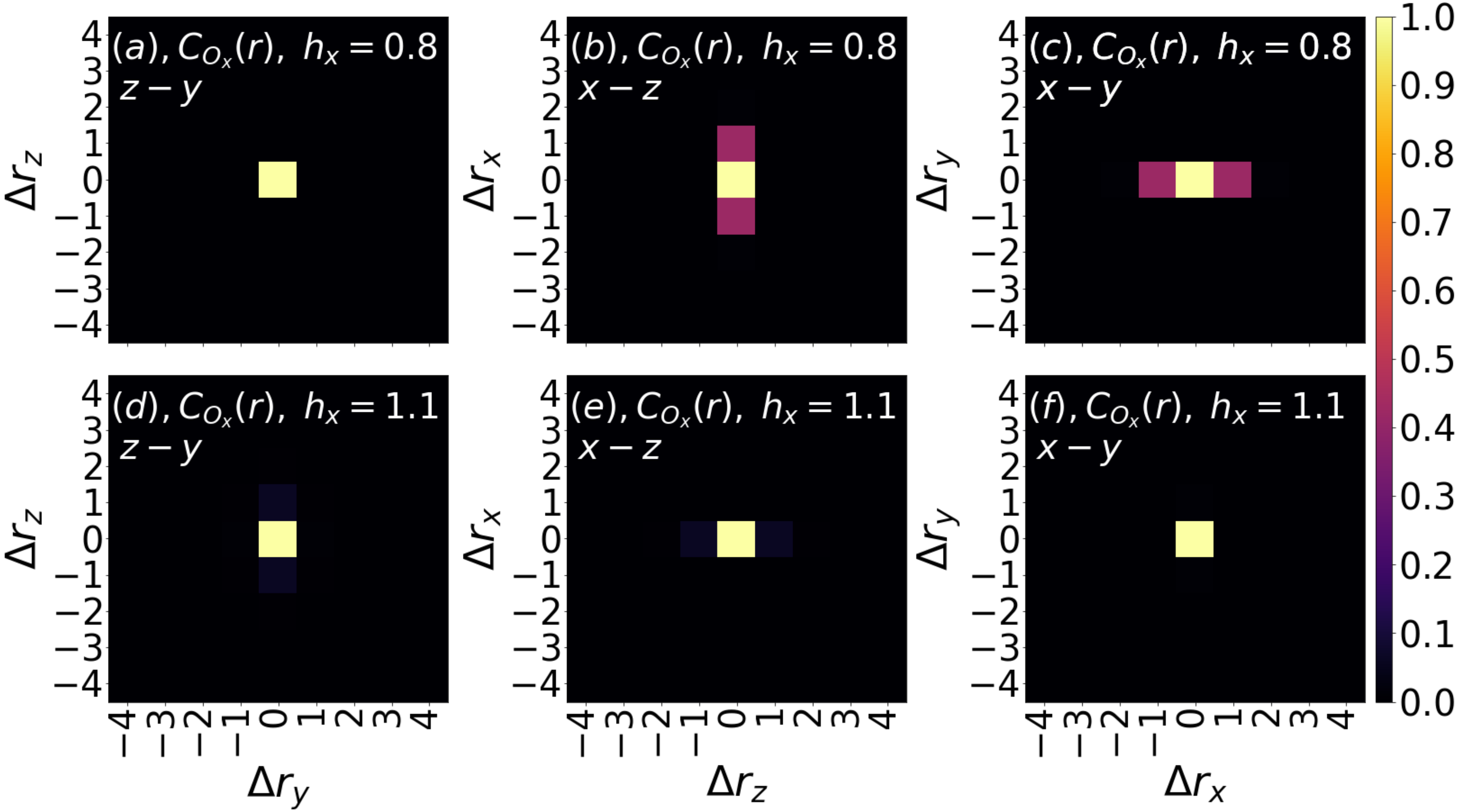
Rahul M. Nandkishore and Michael Hermele **Annu. Rev. Condens. Matter Phys. 2019. 10:295–313**

Department of Physics
The University of Hong Kong

Numerical Result

- Lineons

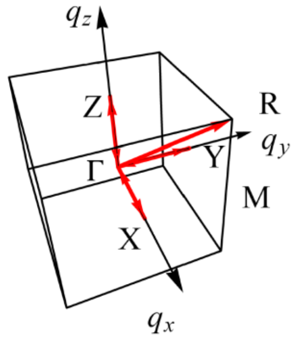
$$C_{O_x}(\mathbf{r}) = \frac{\langle O_{x,i} O_{x,i+\mathbf{r}} \rangle - \langle O_{x,i} \rangle^2}{\langle O_{x,i}^2 \rangle - \langle O_{x,i} \rangle^2}$$

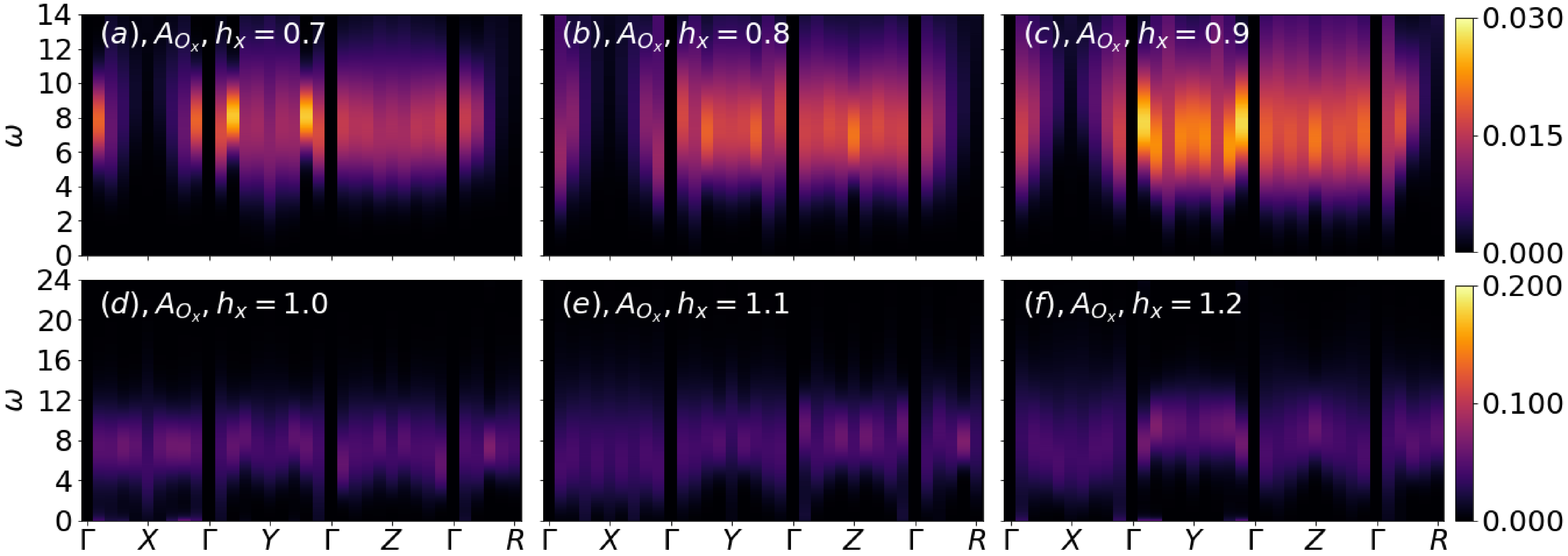


Numerical Result

- Lineons

$$G_{O_x}(\mathbf{q},\tau) = \frac{1}{L^3}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\langle O_{x,i}(\tau)O_{x,j}(0)\rangle$$

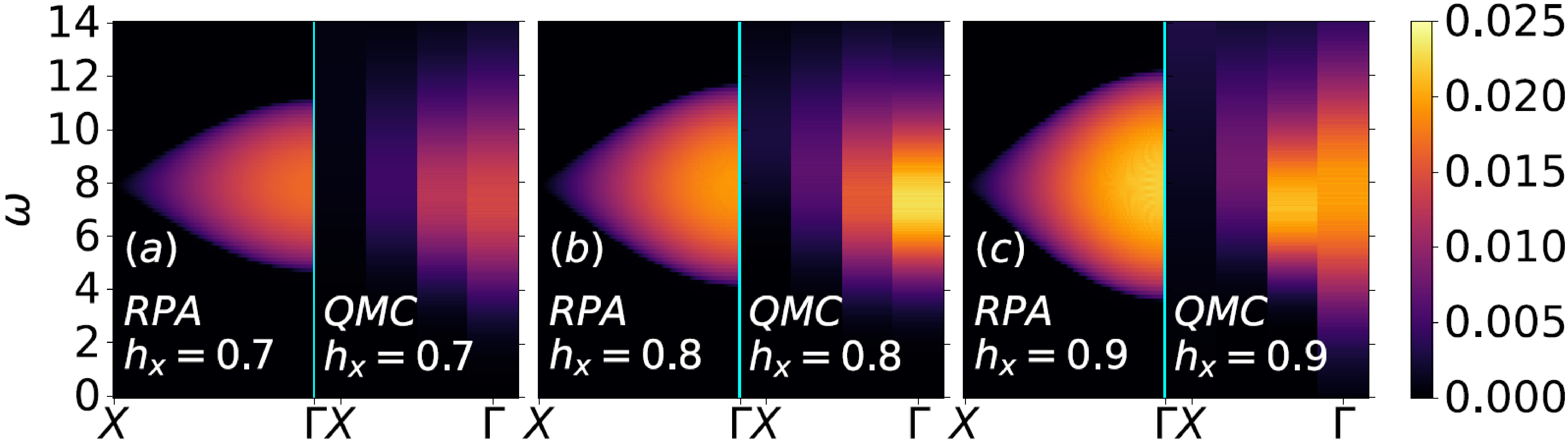




Numerical Result

- Lineons

$$\chi_0(q,\omega) = \sum_{\nu,\nu'} \sum_k \frac{1}{V} \frac{n_F(\xi^{\nu'}_{k+q}) - n_F(\xi^{\nu}_k)}{\omega + i\eta - (\xi^{\nu}_k - \xi^{\nu'}_{k+q})} \qquad \chi_{RPA}(q,\omega) = \frac{\chi_0(q,\omega)}{1 + J\chi_0(q,\omega)}$$



M. Mühlhauser, M. R. Walther, D. A. Reiss, and K. P. Schmidt, **Phys. Rev. B 101, 054426 (2020)**.

3. Subsystem symmetry and Strange correaltor

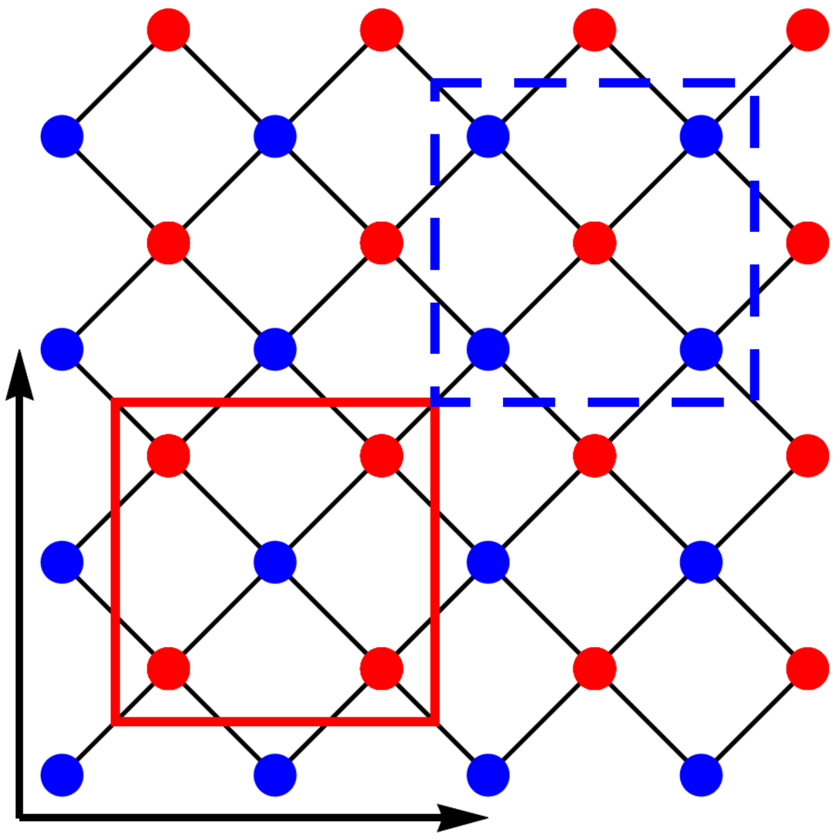
The Subsystem Symmetry Protected Topological Phase

- What is subsystem symmetry?
  - $\mathbf{0 < d < D}$ ($\boldsymbol{D}$ for system dimension)
  - 2D Cluster model

$$H = -K \sum_i X_i \prod_j Z_j$$

$$= -K \sum_i A_i - K \sum_i B_i$$

$$A_i \equiv \tau_i^x \prod_j \sigma_j^z \quad B_i \equiv \sigma_i^x \prod_j \tau_j^z$$



$$\tau_i^x \prod_j \sigma_j^z \qquad \sigma_i^x \prod_j \tau_i^z$$

Yizhi You, Trithep Devakul, F. J. Burnell, and S. L. Sondhi, **Phys. Rev. B 98, 035112 (2018)**.

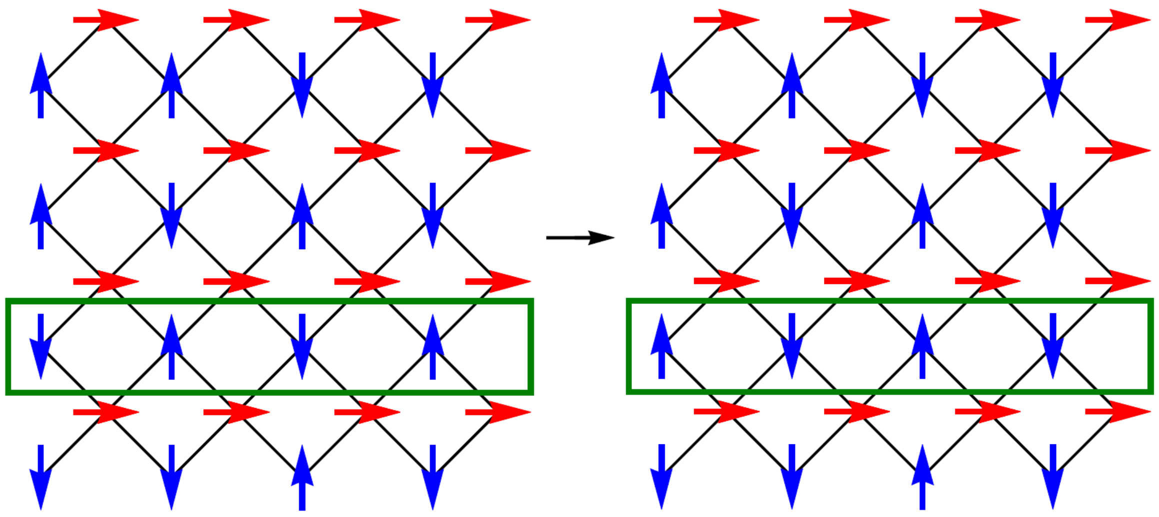
The Subsystem Symmetry Protected Topological Phase

- What is subsystem symmetry?
  - Subsystem symmetry transformations
    - Flipping all the spin along a line

$$H = -K\sum_i A_i - K\sum_i B_i$$

$$A_i \equiv \tau_i^x \prod_j \sigma_j^z \quad B_i \equiv \sigma_i^x \prod_j \tau_j^z$$

$$\prod_{i\in l_k} \tau_i^x \qquad \prod_{i\in l_k} \sigma_i^x$$



Yizhi You, Trithep Devakul, F. J. Burnell, and S. L. Sondhi, **Phys. Rev. B 98, 035112 (2018)**.

Department of Physics
The University of Hong Kong

The Subsystem Symmetry Protected Topological Phase

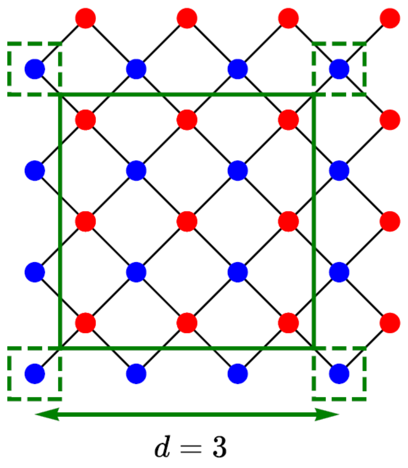
- 2D Cluster model
  - Perturbation
    - **Transverse field (symmetry respecting)**
    - Longitude field (symmetry breaking)

$$H = -K\sum_i A_i - K\sum_i B_i - h_x\sum_i \tau_i^x - h_x\sum_i \sigma_i^x$$

$$H_l = -K\sum_i A_i - K\sum_i B_i - h_z\sum_i \tau_i^z - h_z\sum_i \sigma_i^z$$

  - Observable
    - Energy per site $\langle e \rangle$
    - the magnetization of spin σ along x-direction $\langle m_{\sigma^x}\rangle = \langle \sum \sigma_i^x \rangle / N_s$
    - the magnetization of spin τ along z-direction $\langle m_{\tau^z}\rangle = \langle \sum_i \tau_i^z \rangle / N_s$
    - the membrane order parameter O_d

$$O_d = \prod_{i\in C} \tau_i^z \prod_{j\in M} \sigma_j^x$$

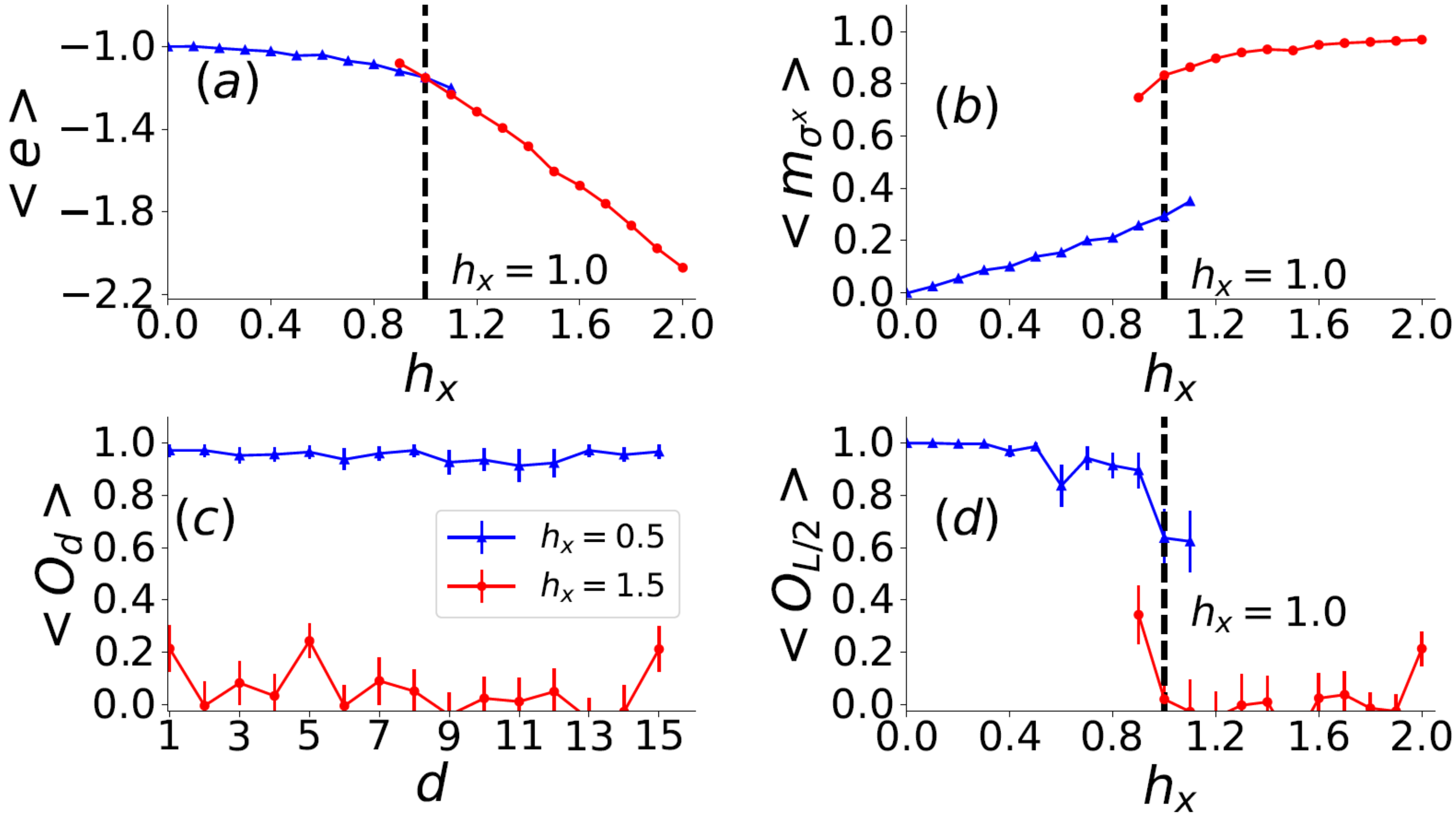


Department of Physics
The University of Hong Kong

The Subsystem Symmetry Protected Topological Phase

- 2D Cluster model

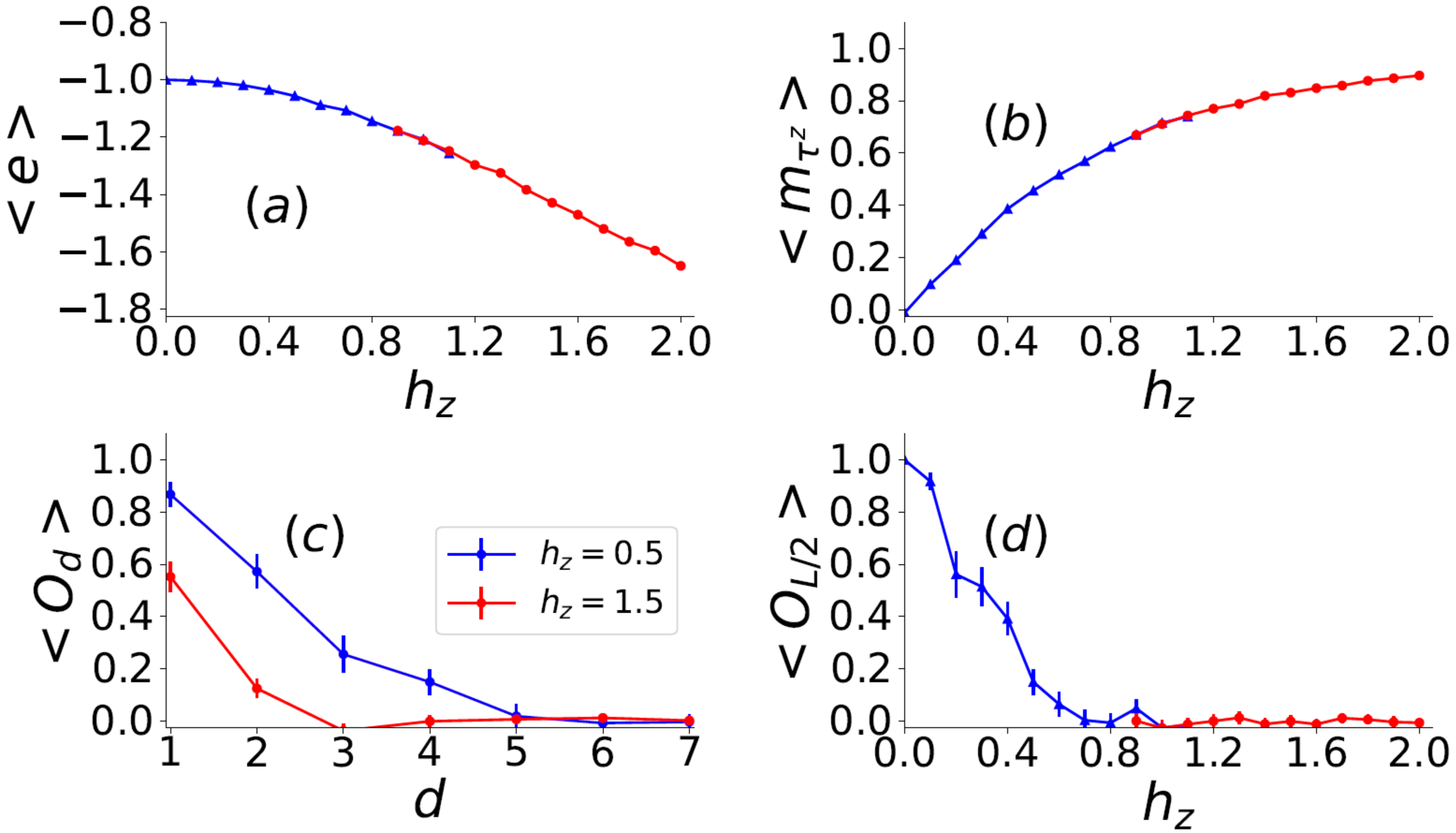
$$H = -K\sum_i A_i - K\sum_i B_i - h_x\sum_i \tau_i^x - h_x\sum_i \sigma_i^x$$



Department of Physics
The University of Hong Kong

The Subsystem Symmetry Protected Topological Phase

- 2D Cluster model $$H_l = -K\sum_i A_i - K\sum_i B_i - h_z\sum_i \tau_i^z - h_z\sum_i \sigma_i^z$$



Strange Correlator

- **Strange correlator**

$$C_\phi(\Delta r) = \frac{\langle \Omega | \phi(i + \Delta r)\phi(i) | \Psi \rangle}{\langle \Omega | \Psi \rangle}$$

- Here, we consider the 2D Cluster model perturbated by transverse field,

$$H = -K \sum_i A_i - K \sum_i B_i - h_x \sum_i \tau_i^x - h_x \sum_i \sigma_i^x$$

- The ground state of the Hamiltonian is projected out via an opened operator string with n bond operators, $|\Psi\rangle = (-H)^n |\Psi(0)\rangle$
- Meanwhile the symmetric trivial direct-product state $|\Omega\rangle$ is taking as

$$\begin{aligned} |\Omega\rangle &= \prod_i |\tau_{i,+}^x\rangle \otimes |\sigma_{i,+}^x\rangle \\ &= \prod_i \frac{1}{\sqrt{2}} [|\tau_{i,+}^z\rangle + |\tau_{i,-}^z\rangle] \otimes |\sigma_{i,+}^x\rangle \end{aligned}$$

Yi-Zhuang You, Zhen Bi, Alex Rasmussen, Kevin Slagle, and Cenke Xu, **Phys. Rev. Lett. 112, 247202 (2014)**.

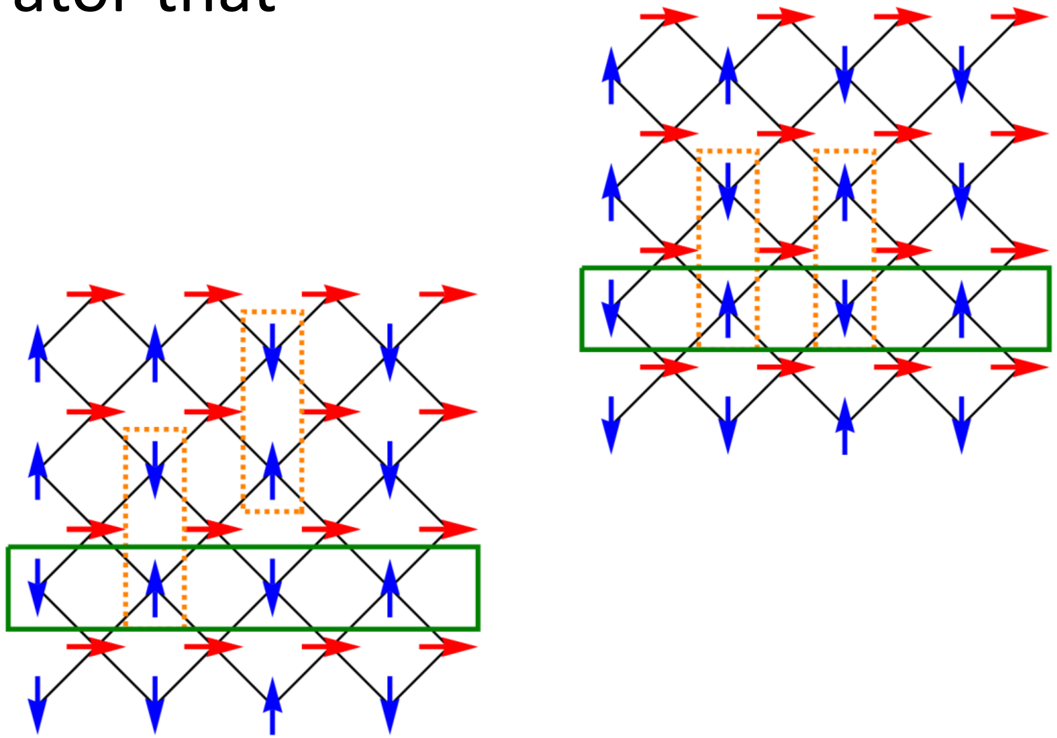
Department of Physics
The University of Hong Kong

Numerical Result

- Firstly, we consider the local dimer operator that

$$\phi(i) = D_i = \tau_i^z \tau_{i+\hat{y}}^z$$

$$C_D(\Delta r) = \frac{\langle \Omega | D_{i+\Delta r} D_i (-H)^{2n} | \Psi(0) \rangle}{\langle \Omega | (-H)^{2n} | \Psi(0) \rangle}.$$



- For any $\Delta r_y = 0$, C_D is

$$\langle \Omega | D_{i+\Delta r} D_i | \Psi \rangle = \langle \Omega | \prod_{k \in S} B_k | \Psi \rangle = \langle \Omega | \Psi \rangle$$

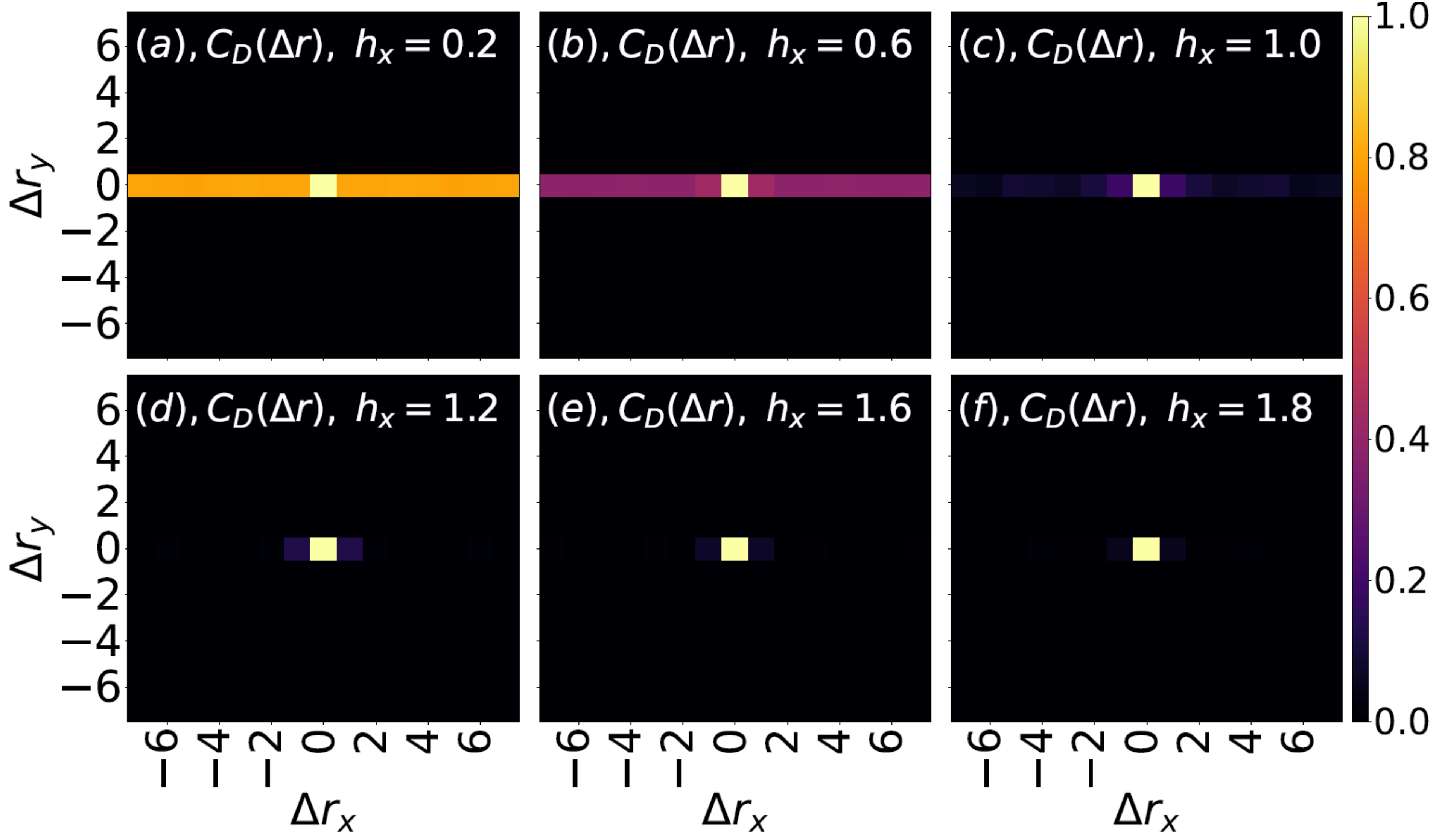
- For any $\Delta r_y \neq 0$, C_D is zero that

$$\langle \Omega | D_{i+\Delta r} D_i | \Psi \rangle = \langle \Omega | \mathcal{U}_x D_{i+\Delta r} D_i \mathcal{U}_x^\dagger | \Psi \rangle = - \langle \Omega | D_{i+\Delta r} D_i | \Psi \rangle \qquad \mathcal{U}_x = \prod_{i \in l_x} \tau_i^x$$

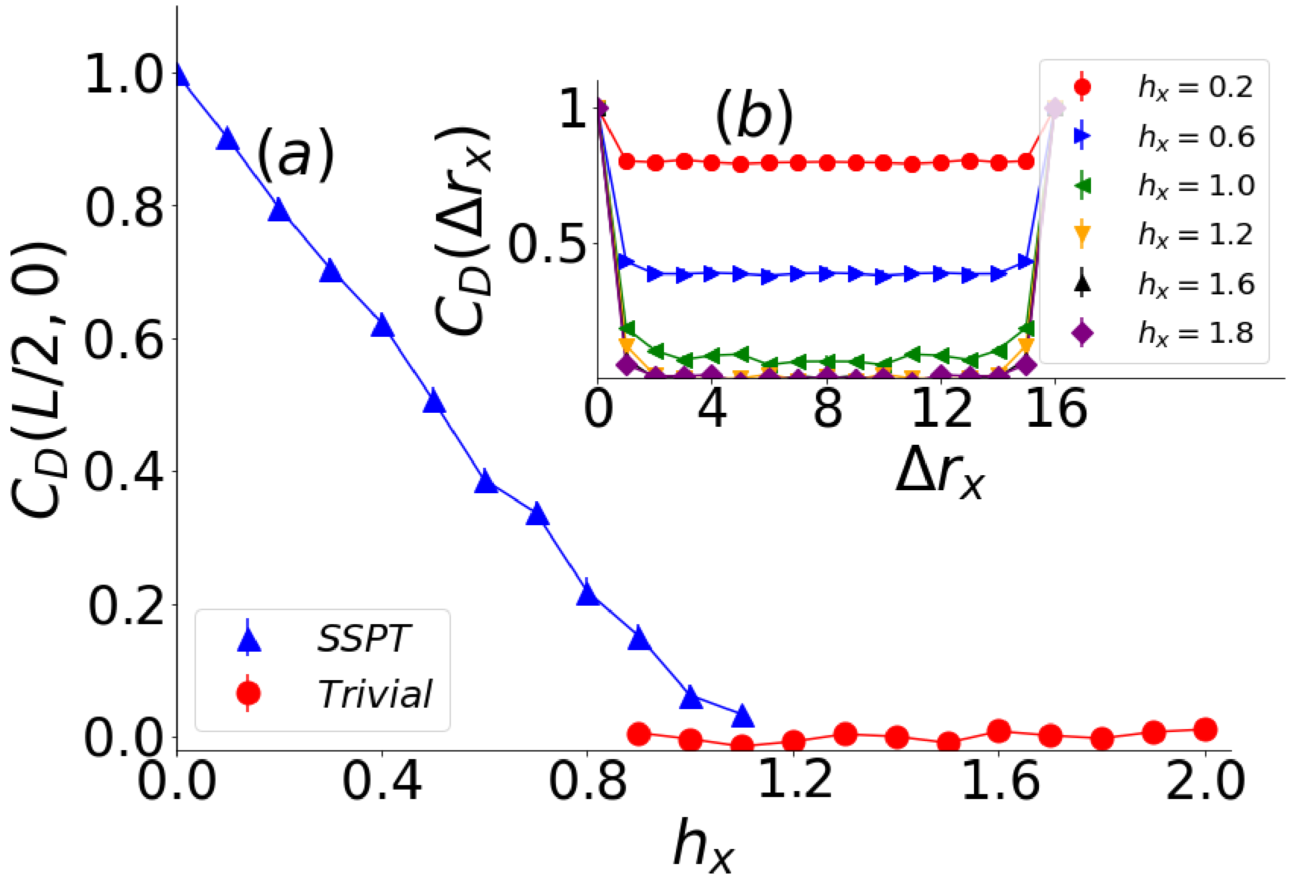
- We can use $C_D(L/2,0)$ as an order parameter.

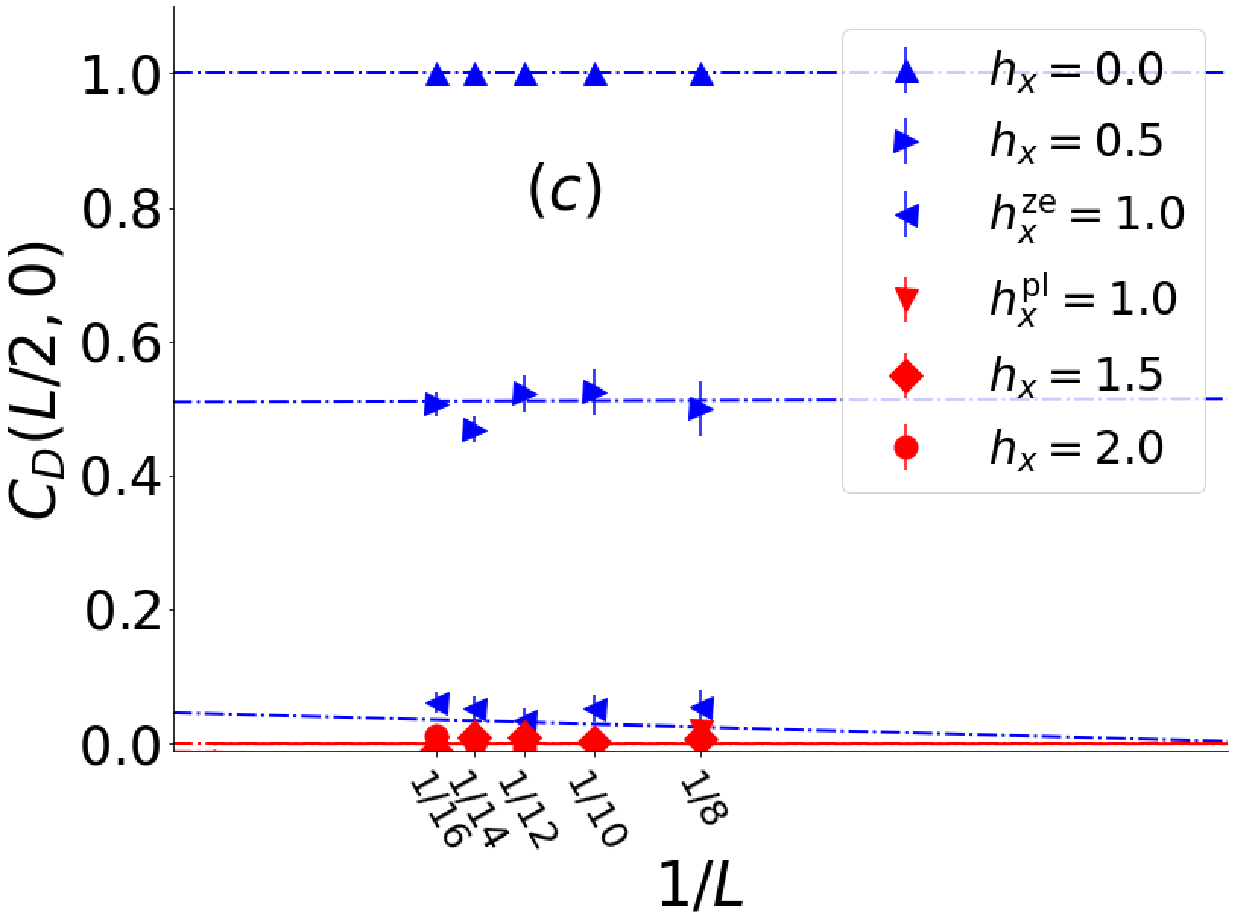
Department of Physics
The University of Hong Kong

Numerical Result



Numerical Result





Conclusion

- Firstly, our results suggest a useful way to identify the amplitude mode in quantum magnet materials through **dimensional crossover**.

- Secondly, we find that the dynamic spectra in the X-cube model reflect the mobility restrictions of each **subdimensional excitation**.

- Finally, via the strange correlator, we characterize the non-trivial to trivial phase transition for the 2D Cluster model with a local dimer at long distances, which plays a role as a **strange order parameter**.

Thanks for your attention

Department of Physics
The University of Hong Kong

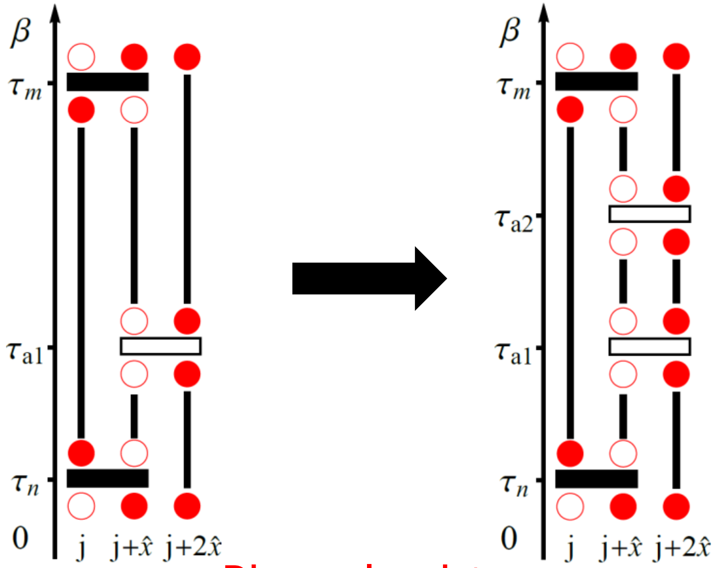
Numerical Methodology

Quantum Monte Carlo

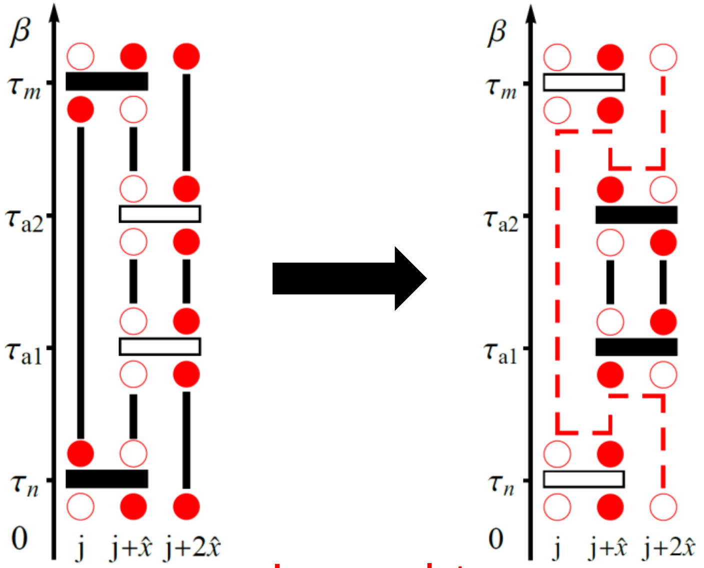
- In our quantum Monte Carlo simulation with Stochastic Series Expansion Method (SSE-QMC), we construct a **configuration space** by expanding the partition function **in a chosen base**,

$$Z = \sum_{S_L}\sum_{\alpha}(-1)^n \frac{\beta^n (L-n)!}{L!}\left\langle \alpha \middle| \prod_{p=0}^{n-1} H_p \middle| \alpha \right\rangle$$
$$= \sum_{S_L}\sum_{\alpha}(-1)^n \frac{\beta^n (L-n)!}{L!}\langle \alpha_0|H_{p_1}|\alpha_1\rangle\langle \alpha_1|H_{p_2}|\alpha_2\rangle..\langle \alpha_{L-1}|H_{p_0}|\alpha_0\rangle$$

- Then, we sample the configuration space by applying the following updates



Diagonal update



Loop update

Anders W. Sandvik. **in AIP Conference Proceedings, Vol. 1297 (AIP, 2010) pp. 135–338.**

Department of Physics
The University of Hong Kong

Numerical Methodology

Stochastic Analytic Continuation

$$G(\tau) = \langle \hat{O}(\tau)\hat{O}(0)\rangle$$

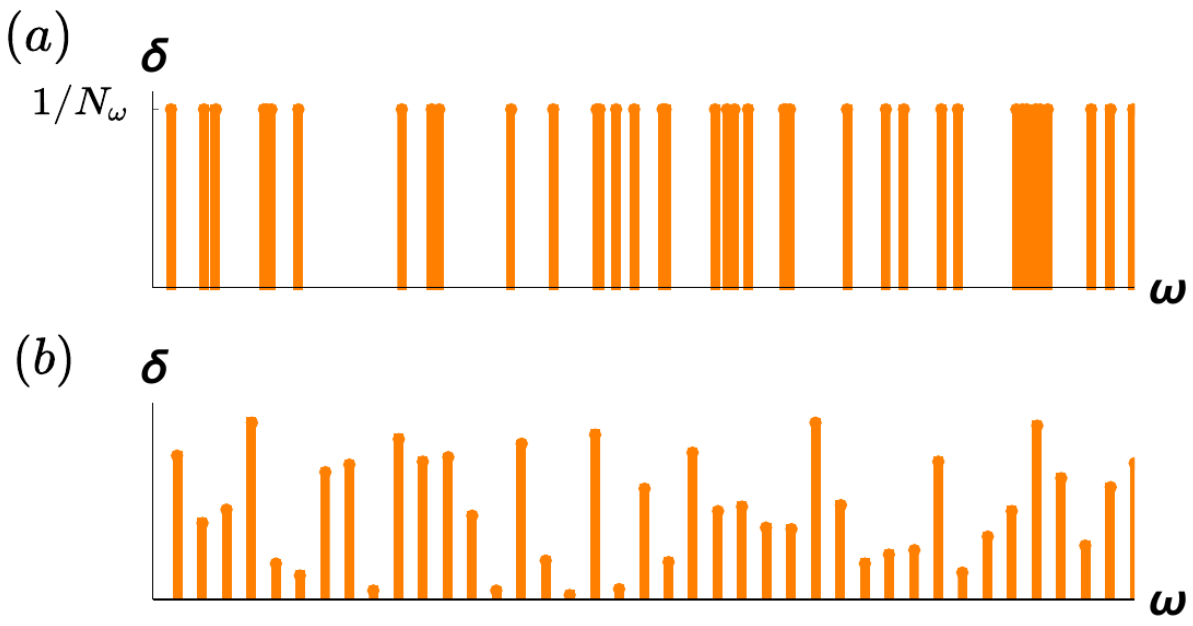
$$A(\omega) = \frac{1}{\pi}\sum_{m,n} e^{-\beta E_n}|\langle m|\hat{O}|n\rangle|^2\delta(\omega - [E_n - E_m])$$

$$G(\tau) = \int_{-\infty}^{\infty} K(\tau,\omega)A(\omega)d\omega$$

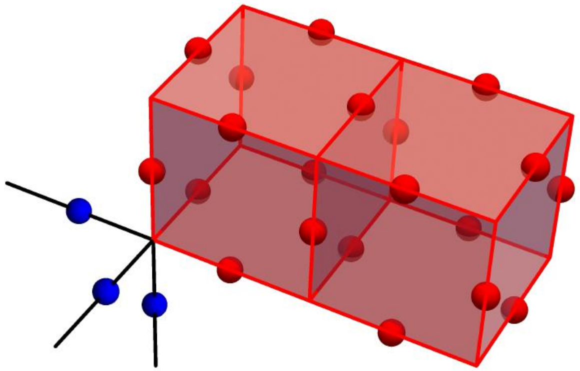
$$K(\tau,\omega) = \frac{1}{\pi}\left(e^{-\tau\omega} + e^{-(\beta-\tau)\omega}\right)$$

$$P(A) \propto \exp\left(-\frac{\chi^2}{2\Theta}\right)$$

$$\chi^2 = \sum_{i,j}[G'(\tau_i) - \bar{G}(\tau_i)]C_{ij}^{-1}[G'(\tau_j) - \bar{G}(\tau_j)]$$



Planon



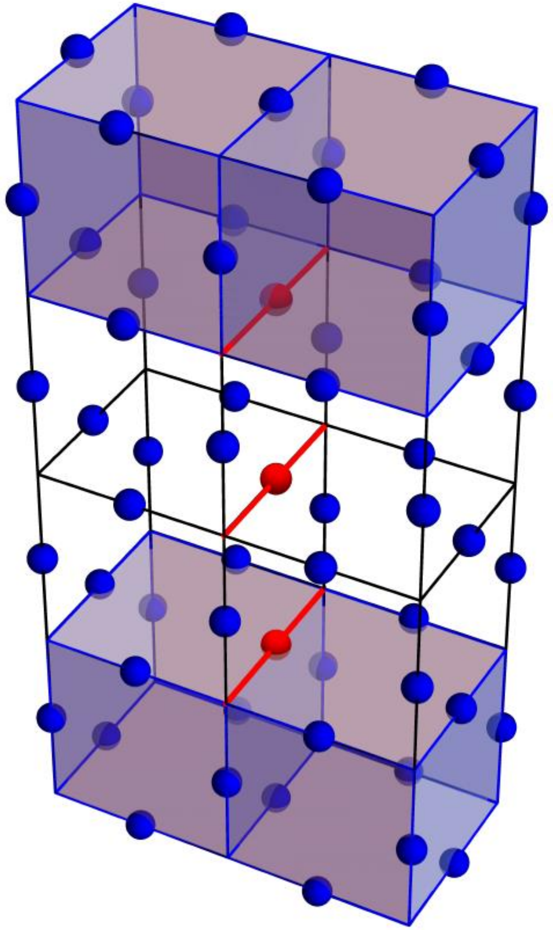
- Created by

$$W(M_p) = \prod_{i \in M_p} \sigma_i^z$$

- Density operator:

$$n_{x,i} = \frac{1}{4}(A_{c,i} - 1)(A_{c,i+x} - 1)$$

- Mobility constraints:
  - Mobile within flat planes
  - Two neighboring fractons
  - Costing $8K$ energy

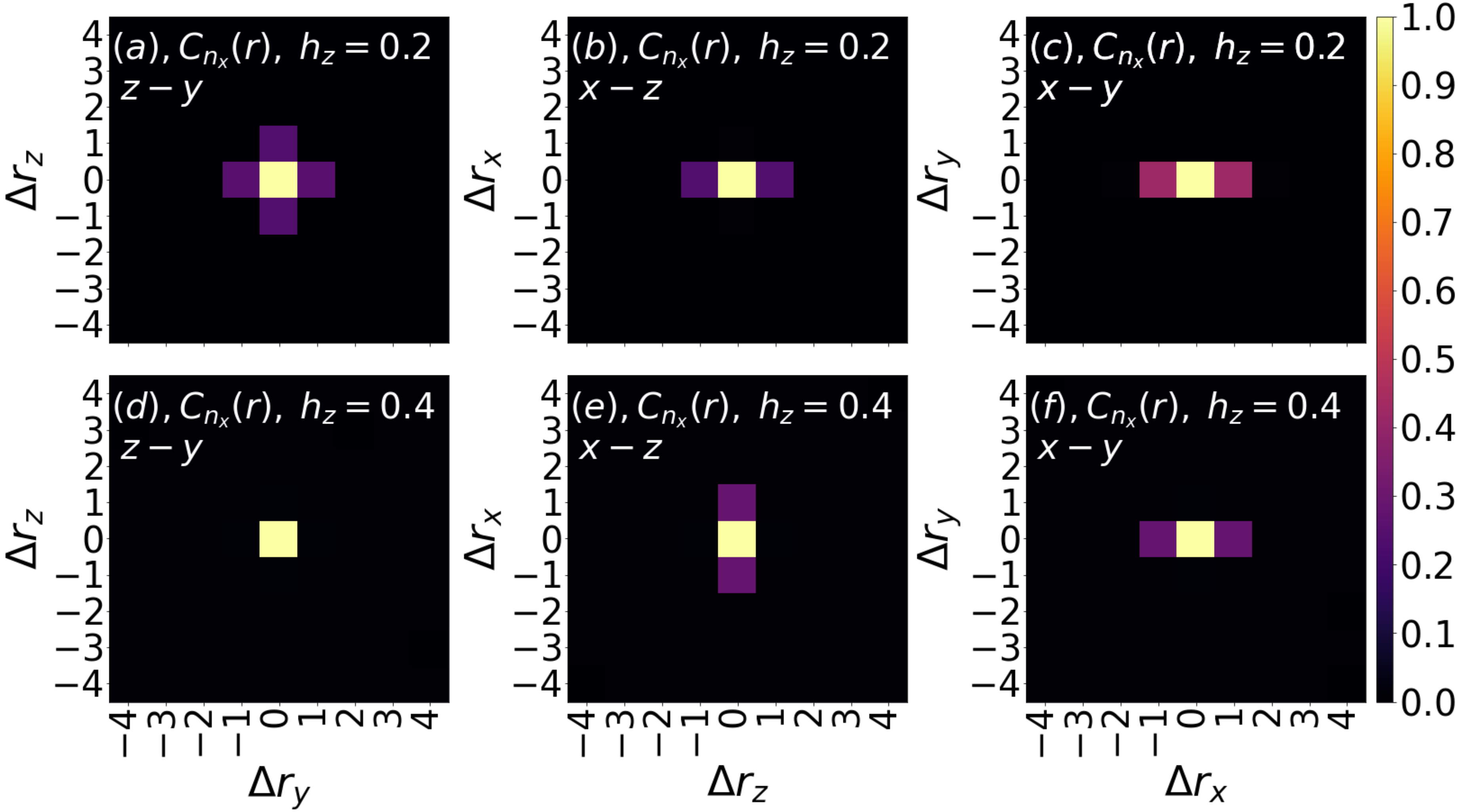


Rahul M. Nandkishore and Michael Hermele **Annu. Rev. Condens. Matter Phys. 2019. 10:295–313**

Department of Physics
The University of Hong Kong

Planon

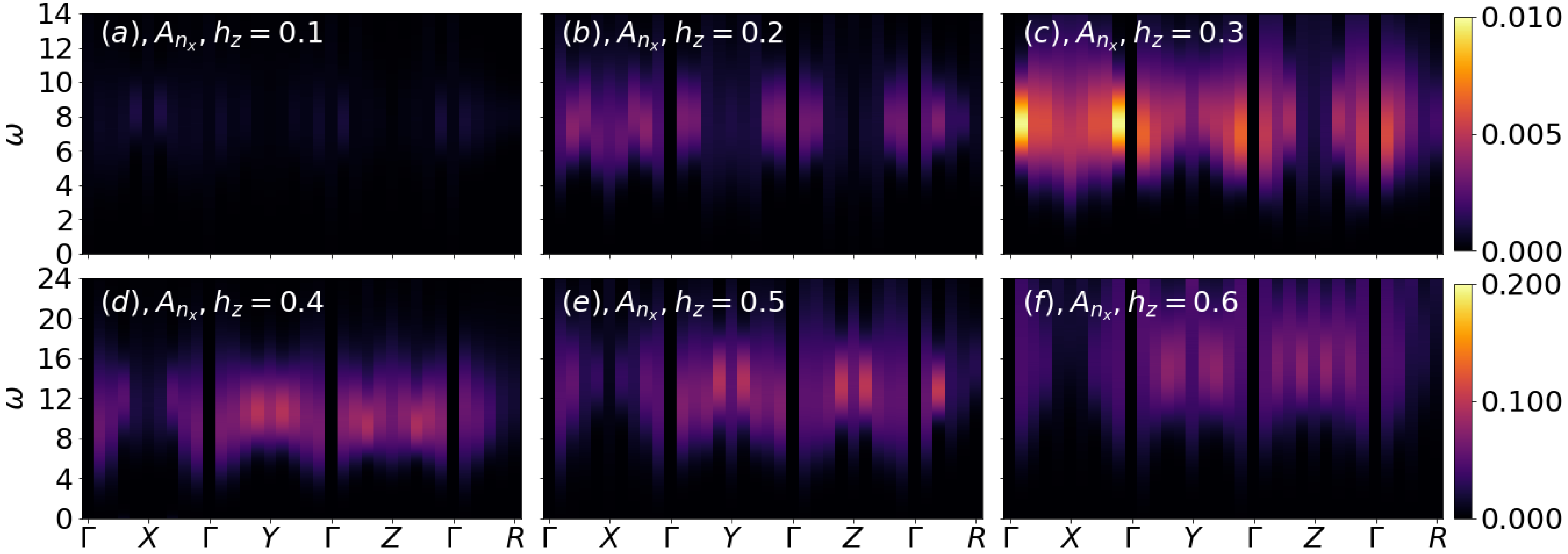
$$C_{n_x}(\mathbf{r}) = \frac{\langle n_{x,i} n_{x,i+\mathbf{r}} \rangle - \langle n_{x,i} \rangle^2}{\langle n_{x,i}^2 \rangle - \langle n_{x,i} \rangle^2}$$



Department of Physics
The University of Hong Kong

Planon

$$G_{n_x}(\mathbf{q},\tau) = \frac{1}{L^3}\sum_{i,j} e^{-i\mathbf{q}\cdot(\mathbf{r}_i-\mathbf{r}_j)}\langle n_{x,i}(\tau)n_{x,j}(0)\rangle$$



Department of Physics
The University of Hong Kong

Choice of local operator

- When it comes to the local operator ϕ, we start from the exactly solvable point $h_x = 0$.

ϕ	$C_\phi(\Delta r)$
τ_i^z, σ_i^z	0
σ_i^x, τ_i^x	1
$D_i = \tau_i^z \tau_{i+\hat{y}}^z, D_i' = \sigma_i^z \sigma_{i+\hat{y}}^z$	1 for $\Delta r_y = 0$, 0 for $\Delta r_y \neq 0$
B_i, A_i	1

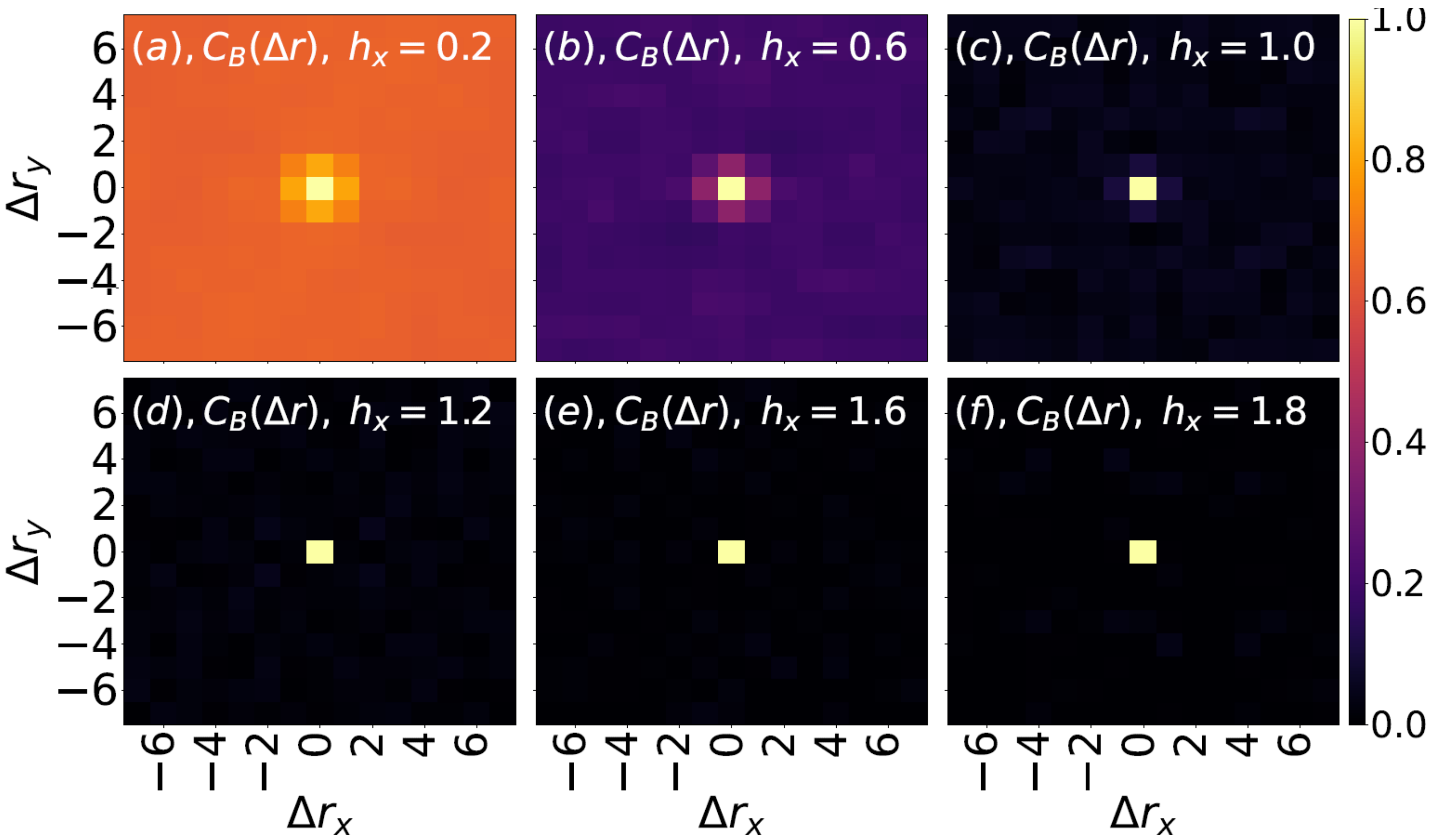
- In the 2D cluster model, the strange correlator of the single spin are independent of the transverse field h_x. For instance, the strange correlator about σ_i^x is always 1 regardless of h_x, so such strange correlators are not useful in detecting SSPT order.

Department of Physics
The University of Hong Kong

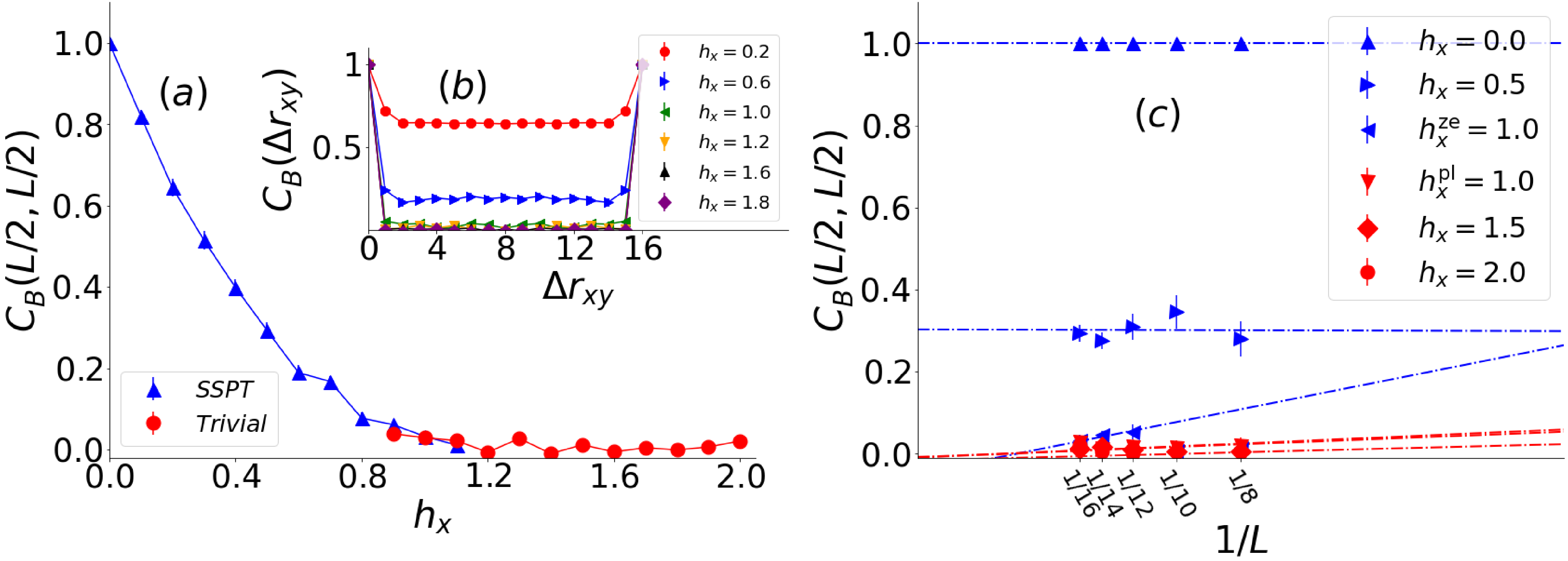
Choice of local operator

$$\phi(i) = B_i$$

$$C_B(\Delta r) = \frac{\langle \Omega | B_{i+\Delta r} B_i (-H)^{2n} | \Psi(0) \rangle}{\langle \Omega | (-H)^{2n} | \Psi(0) \rangle}.$$

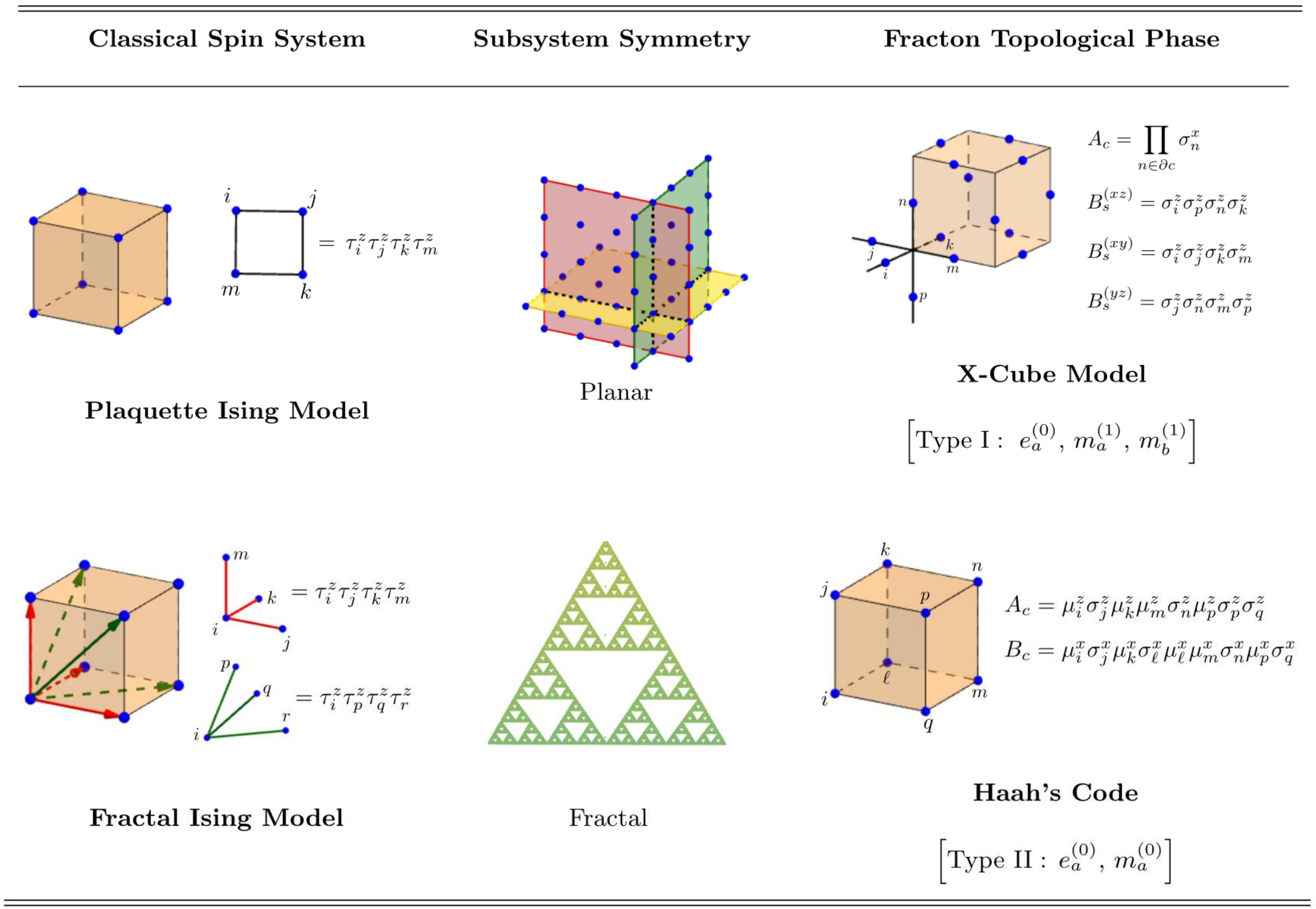


Choice of local operator



Department of Physics
The University of Hong Kong

Relation between Fracton and Subsystem symmetry



Sagar Vijay, Jeongwan Haah, and Liang Fu, **Phys. Rev. B 94, 235157 (2016)**.

Quantum memory and Calderbank-Shor-Steane (CSS) code

$$H = -\sum A - \sum B$$
$$A = \prod X_i$$
$$B = \prod Z_i$$

R. Calderbank and Peter W. Shor, **Phys. Rev. A 54, 1098 (1996)**.
Andrew Steane, **Proc. R. Soc. Lond. A.4522551–2577 (1996)**

Good quantum error-correcting codes exist

A. R. Calderbank and Peter W. Shor
AT&T Research, 600 Mountain Avenue, Murray Hill, New Jersey 07974
(Received 12 September 1995)

A quantum error-correcting code is defined to be a unitary mapping (encoding) of k qubits (two-state quantum systems) into a subspace of the quantum state space of n qubits such that if any t of the qubits undergo arbitrary decoherence, not necessarily independently, the resulting n qubits can be used to faithfully reconstruct the original quantum state of the k encoded qubits. Quantum error-correcting codes are shown to exist with asymptotic rate $k/n = 1 - 2H_2(2t/n)$ where $H_2(p)$ is the binary entropy function $-p\log_2 p - (1-p)\log_2(1-p)$. Upper bounds on this asymptotic rate are given. [S1050-2947(96)00708-1]

PACS number(s): 03.65.Bz, 89.70.+c

Multiple-particle interference and quantum error correction

By Andrew Steane
Department of Atomic and Laser Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK
a.steane@physics.oxford.ac.uk

The concept of *multiple-particle interference* is discussed, using insights provided by the classical theory of error correcting codes. This leads to a discussion of error correction in a quantum communication channel or a quantum computer. Methods of error correction in the quantum regime are presented, and their limitations assessed. A quantum channel can recover from arbitrary decoherence of x qubits if K bits of quantum information are encoded using n quantum bits, where K/n can be greater than $1 - 2H(2x/n)$, but must be less than $1 - 2H(x/n)$. This implies exponential reduction of decoherence with only a polynomial increase in the computing resources required. Therefore quantum computation can be made free of errors in the presence of physically realistic levels of decoherence. The methods also allow isolation of quantum communication from noise and evesdropping (quantum privacy amplification).