National University of Computer and Emerging Sciences, Karachi.
FAST School of Computing
Assignment # 1, Fall 2023 CS1005-Discrete Structures

Instructions: **Max. Points: 100**

1- **This is hand written assignment. You have to submit the hard copy on 20 September 2023 in your class as well scan copy on Google classroom.**
2- **Just write the question number instead of writing the whole question.**
3- **You can only use A4 size paper for solving the assignment.**
4- **You have to write your student ID and Section on the top of each page.**

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1. Which of the these are propositions? What are the truth values of those that are propositions?
   a. 2+x>8
   b. Napoleon won the battle of Waterloo.
   c. 2+3
   d. In 1492 Columbus sailed the ocean blue.
   e. Does John love Discrete Mathematics?

2. Express the given statements using logical connectives.
   a. A student gets A in Discrete if and only if his weight total is $\geq 95\%$
   b. Either Alice is smart, or she is not smart but honest.
   c. $\sqrt{30}$ is greater than 6 or $\sqrt{30}$ is less than 5.
   d. Sam had pizza last night and Chris finished her homework.
   e. Either Chris finished her homework or Pat watched the news this morning, but not both.

3. Let $p = "2 \leq 5"$, $q = "8\ is\ an\ even\ integer"$, and $r = "11\ is\ a\ prime\ number"$. Express the following as statement in English and determine whether the statement is true or false.
   a. $\neg p \wedge q$
   b. $(p \wedge q) \rightarrow r$
   c. $p \rightarrow (q \vee (\neg r))$
   d. $(\neg p) \rightarrow (\neg q)$

4. Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP.
   Express the given statements using Logical Connectives. Also determine the truth value of each of these propositions.
   a) Smartphone B has the most RAM of these three smartphones.
   b) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
   c) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
   d) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
   e) Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

5. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Express the given statements using Logical Connectives. Also determine the truth value of each of these propositions for the most recent fiscal year.
   a) Quixote Media had the largest annual revenue.
   b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
   c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.
   d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
   e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue

6. Consider the statement: "if you keep your textbook, it will be useful reference in your future course."
   a. Describe at least five different ways(cases) to write the conditional statement $p \rightarrow q$ in English.
   b. State the converse, inverse, and contrapositive of conditional statement.
   c. Given a conditional statement $p \rightarrow q$. Find the inverse of its inverse, the inverse of its converse, and the inverse of its contrapositive.

7. Use De Morgan's law to find the negation of each of the following statements.
   a. $0 \leq x \leq 4$
   b. $-2 < x \leq 5$
   c. The fan is slow, or it Is very hot.
   d. Akram is unfit and Ahmed is injured.
   e. Zia is neither healthy, wealthy nor wise.

8. Use De Morgan's laws to find the negation of each of the following statements.
   a) Jan is rich and happy. b) Carlos will bicycle or run tomorrow. c) The fan is slow or it is very hot. d) Akram is unfit and Saleem is injured.

9. Prove the following equivalences by using laws of logic.
   a. $(p \wedge q) \vee (\neg p \wedge q) \Leftrightarrow q$
   b. $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$
   c. $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$
   d. $p(\wedge q) \rightarrow (p \rightarrow q) \equiv T$

10. Using Truth Table, show that these compound propositions are logically equivalent or not.
    a. $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$
    b. $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$

11. Given the following logical propositions a premises. Using rule of inference prove that conclusion is $\neg r$.
    $(t \rightarrow (r \vee p)) \rightarrow ((\neg r \vee k) \wedge \neg k)$

12. What rule of inference is used in each of these arguments?
    a. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
    b. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
    c. If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
    d. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
    e. If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will get sunburned.

13. By using Law if inference, show that the following statement is valid.
    a. If today is Tuesday, I have a test in Mathematics or Economics. If my Economics professor is sick, I will not have a test in Economics. Today is Tuesday, and my Economics professor is sick. Therefore, I will have a test in Mathematics.
    b. If Ali is a lawyer, then he is ambitious. If Ali is an early riser, then he does not like chocolates. If Ali is ambitious, then he is an early riser. Therefore, if Ali is a lawyer, then he does not like chocolates.

14. Given propositional function $q(x, y): x + y = 1$, which of the following are propositions; which are not? For those that are, determine their truth values.
    a. $q(x, y)$
    b. $q(-6,7)$
    c. $q(x + 1, -x)$
    d. $q(x, 3)$
    e. $q(1,1)$
    f. $q(5, -4)$

15. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
    a. $\exists x(x^2 = 2)$
    b. $\exists x(x^2 = -1)$
    c. $\exists x(x^2 = 2)$
    d. $\forall x(x^2 + 2 \geq 1$

16. Let P(x) be the statement "x can speak Russian" and let Q(x) be the statement "x knows the computer language C++." Express each of these sentences in terms of P(x), Q(x), quantifiers, and logical connectives. The domain for quantifiers consists of all students at your school.
    a. There is a student at your school who can speak Russian and who knows C++.
    b. There is a student at your school who can speak Russian but who doesn't know C++.
    c. Every student at your school either can speak Russian or knows C++.
    d. No student at your school can speak Russian or knows C++.

17. Let Q(x, y) be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English
    a. $\exists x \exists y Q(x, y)$
    b. $\exists x \forall y Q(x, y)$
    c. $\forall x \exists y Q(x, y)$
    d. $\forall y \exists x Q(x, y)$
    e. $\forall x \forall y Q(x, y)$

18. Let A= {1,2,3} and B={1,2,3,4}. Determine whether A is a subset of B, and whether A is a proper subset of B.

a) Given the set C= {x|x is a prime number less than 10), list all the proper subsets of C.

b) Given L= {1,2,3}, list all the subsets of L and then find the power set of L.

c) If M is a set with k elements, how many elements are in its power set? Express your answer in terms of k.

d) Define the set N= {x|x is a prime number between 10 and 20). What is the cardinality of the power set of N?

19. Let E={1,2,4} and F={a,b,c,d}. Find E×F (the Cartesian product of E and F).

Given the sets U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {1, 2, 4, 5}, B = {2, 3, 5, 6}, and C = {4, 5, 6, 7}, let's analyze the expressions:
a) Prove or disprove: (A - (A ∩ B)) ∩ (B - (A ∩ B)) = Ø
b) Prove or disprove: (A - B) ∪ (A ∩ B) = A

c) Prove or disprove: (A - B) - C = (A - C) - B
d) Prove or disprove: (B̄ ∪ (B̄ - A)) = B

20. Prove or disprove the following expression by using the set identities:
a) (A − (A B)) (B − (A B)) = Φ b) (A − B) ∪ (A B) = A c) (A − B) − C = (A − C) − B d) (B̄ ∪ (B̄ − A)) = B

21.
a. In a bushel of 200 potatoes, 40 have worms in them, and 30 have bruises. Only those potatoes with neither worms nor bruises can be sold. If there are 20 bruised potatoes that have worms in them, how many of the 200 potatoes can be sold?

b. A famous Ice cream company wants to launch exciting deals for their new store in Karachi. The survey of 1,000 customers shows 250 of them like mixed berry, 400 like Irish cream, and 100 of them like both. How many of them like either of the flavors and how many of them like none?

c. The Computer Science Department of XYZ College is planning to offer elective courses to the sophomores. In a survey on the course preferences of students, the following data was obtained: 44 like Accounting, 35 like Psychology, 38 like Business Analytics, 13 like both Accounting and Psychology, 21 like both Psychology and Business Analytics, 16 like both Business Analytics and Accounting, 19 like all three subjects, and 14 like none of these three subjects. How many students were surveyed?

22. Use set-builder notation and logical equivalences to prove the following. A X (B ∩ C) = (A X B) ∩ (A X C).

23. Let A= {1, 3, 5, 7 } and B = {2, 4, 6, 8}. Consider the following functions:
a) f(1) = 4, f(3) = 2, f(5) = 6, f(7) = 8
b) f(1) = 4, f(3) = 4, f(5) = 8, f(7) = 6
c) f(1) = 8, f(3) = 4, f(5) = 6, f(7) = 8
d) f(1) = 6, f(3) = 2, f(5) = 4, f(7) = 8

(i) Determine the Domain, Co-domain and Range of the functions.
(ii) Determine whether the functions are Injective, Surjective and Bijective or not?
(iii) Determine the   inverse of function if exists.

24. Let f and g be the functions from the set of integers to the set of integers defined by $f(x) = 4x + 3$ and g(x) = 3x + 4.
(a) What is the composition of f and g? What is the   composition of g and f?
(b) Which type of function f and g are?
(c) Are f and g invertible?

25 . a) Let $f(x) = \lfloor x^2 / 2 \rfloor$, Find $f(x)$ if:
(i) S = {-2,-1,0,1,2,3}
(ii) S = {0,1,2,3,4,5}
(iii) S = {1,5,7,11}
(iv) S = {2,6,10,14}

(b)
(i)   ⌈ 3 4 ⌉
(ii) ⌊ 7 8 ⌋
(iii) ⌈− 3 4 ⌉
(iv) ⌊− 7 8 ⌋
(v)   ⌈3 ⌉
(vi) ⌊− 1 ⌋
(vii) ⌊ 1 2 + ⌈ 3 2 ⌉ ⌋
    (viii)⌊ 1 2 .⌊ 5 2 ⌋⌋

(c) Prove or disproof that if x is a real number, then $\lfloor - x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = - \lfloor x \rfloor$.