Permutation & Combination.

AND → Multiply ; OR → Addition.

Combination → ^nC_r {Selection}

Permutation → ^nP_r {Arrangement}

- Factorial can also be used for arrangement.

Arrangements:

- Arrange A & B : AB , BA (2 ways)

$2! = 2$

- Arrange A, B & C : ABC, ACB, BAC, BCA, CAB, CBA

(6 ways)

$3! = 6$

$2! = 2 \times 1 = 2$; $3! = 3 \times 2 \times 1 = 6$

$4! = 4 \times 3 \times 2 \times 1 = 24$; $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

- Arrange A, B, C & D : $4! = 24$ ways.

Selection:

Select any 1 from A & B : A OR B

(2 ways)

$^2C_1 = 2$

Select A & B both : AB (1 ways)

$^2C_2 = 1$

Select any 1 from A, B & C : A or B or C.

(3 ways)

$^3C_1 = 3$

Select any 2 from A, B & C : AB, AC, BC

(3 ways)

$^3C_2 = 3$

Select any 1 from A, B, C & D : A or B or C or D

(4 ways)

$^4C_1 = 4$

Select any 2 from A, B, C & D : AB, AC, AD, BC, BD, CD

(6 ways)

$^4C_2 = 6$

Select any 3 from A, B, C & D : ABC, ABD, BCD, ACD

(4 ways)

$^4C_3 = 4$

Case 1 {non-repeated alphabets}

(A, B, C, D, E, F, G)

i) Arrange these alphabets without any restriction.

$7! = 5040$ ways.

OR

$^7P_7 = 5040$ ways.

ii) Arrange any 3 of the 7 alphabets.

$^7C_3 \times 3! = 210$ {Overall selection & then arrangement}

OR

$^7P_3 = 210$

OR

$\underline{^7C_1} \times \underline{^6C_1} \times \underline{^5C_1} = {^7C_1} \times {^6C_1} \times {^5C_1} = 210$ {Position by position arrangement}

OR

$7 \times 6 \times 5 = 210$

iii) Arrange any 4 of the 7 alphabets.

$^7C_4 \times 4! = 840$

OR

$^7P_4 = 840$

OR

$\underline{^7C_1} \times \underline{^6C_1} \times \underline{^5C_1} \times \underline{^4C_1} = {^7C_1} \times {^6C_1} \times {^5C_1} \times {^4C_1} = 840$

OR

$7 \times 6 \times 5 \times 4 = 840$

iv) Arrange the alphabets such that A & B are always together.

$\overset{2!}{\boxed{AB}}_{6}\ C_5\ D_4\ E_3\ F_2\ G_1$

$6! \times 2! = 1440$

OR

$\_\ \overbrace{C\ \_\ D\ \_\ E\ \_\ F\ \_\ G}^{5!}\ \_$ (positions 6 5 4 3 2 1) $\overset{2!}{\boxed{AB}}$

$5! \times {^6C_1} \times 2! = 1440$

v) Arrange the alphabets such that A, B & C are all always together.

$\overset{3!}{\boxed{ABC}}_{5}\ D_4\ E_3\ F_2\ G_1$

$5! \times 3! = 720$

OR

$\_\ \overbrace{D\ \_\ E\ \_\ F\ \_\ G}^{4!}\ \_$ (positions 5 4 3 2 1) $\overset{3!}{\boxed{ABC}}$

$4! \times {^5C_1} \times 3! = 720$

vi) Arrange the alphabets that they either start with A or B.

starts with A : A $\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{}$ $6!$ = 720

+

starts with B : B $\_\ \_\ \_\ \_\ \_\ \_$ $6!$ = 720

1440

OR

$\underset{2C_1}{\underline{A/B}}$ $\_\ \_\ \_\ \_\ \_\ \_$ $6!$

$2C_1 \times 6! = 1440$

vii) Arrange the alphabets that it starts & ends with a consonents.

A , E → vowels

B, C, D, F, G → consonents.

$\underset{5C_1}{\underline{C}}$ $\_\ \_\ \_\ \_\ \_$ ($5!$) $\underset{4C_1}{\underline{C}}$

$5C_1 \times 5! \times 4C_1 = 2400$

viii) Arrange the alphabets that it starts & ends with a vowel.

$\underset{2C_1}{\underline{V}}$ $\_\ \_\ \_\ \_\ \_$ ($5!$) $\underset{1C_1}{\underline{V}}$

$2C_1 \times 5! \times 1C_1 = 240$

ix) Arrange the alphabets such that it starts with a vowel & ends with a consonent or vice verra.

starts with vowel & end with consonent : $\underset{2C_1}{\underline{V}}$ $\boxed{\_\ \_\ \_\ \_\ \_}$ ($5!$) $\underset{5C_1}{\underline{C}}$

or +

Start with a consonent & end with a vowel : $\underset{5C_1}{\underline{C}}$ $\boxed{\_\ \_\ \_\ \_\ \_}$ ($5!$) $\underset{2C_1}{\underline{V}}$

$(2C_1 \times 5! \times 5C_1) + (5C_1 \times 5! \times 2C_1) = 2400$

OR

$2!$

$\underset{2C_1}{\underline{V}}$ $\boxed{\_\ \_\ \_\ \_\ \_}$ ($5!$) $\underset{5C_1}{\underline{C}}$ $2C_1 \times 5! \times 5C_1 \times 2! = 2400$

x) Arrange the alphabets such that A & B are seperated / A & B are not next to eachother.

(Tot. no. of ways w/o any restriction) − (both being together)

$7! - (6! \times 2!)$

$5040 - 1440 = 3600$ Answer.

OR

GAP Method.

$5!$

, C , D , E , F , G ,
6 5 4 3 2 1

$5! \times 6C_1 \times 5C_1 = 3600$ Answer.

OR

$5! \times 6C_2 \times 2! = 3600$ Answer

xi) Arrange the alphabets such that A, B & C are all seperated from eachother.

$4!$

, D , E , F , G ,
5 4 3 2 1

$4! \times 5C_1 \times 4C_1 \times 3C_1 = 1440$ Answer

OR

$4! \times 5C_3 \times 3! = 1440$ Answer

xii) Arrange the alphabets such that A, B, C & D are all seperated from eachother.

$3!$

, E , F , G ,
4 3 2 1

$3! \times 4C_1 \times 3C_1 \times 2C_1 \times 1C_1 = 144$

OR

$3! \times 4C_4 \times 4! = 144$

xiii) Arrange the alphabets such that A, B & C are not next to eachother.

(Tot. no. of ways w/o Restriction) − (All 3 being together)

$7! - (5! \times 3!) = 4320$ Answer

OR

A, B & C all seperated = $\overset{4!}{\_D\_E\_F\_G\_}$ $= 4! \times 5C_1 \times 4C_1 \times 3C_1$ $= 1440$

+

AB tog. but C seperated = $\overset{2!}{\boxed{AB}}$, C $\overset{4!}{\_D\_E\_F\_G\_}$ $= 4! \times 5C_1 \times 2! \times 4C_1$ $= 960$

+

AC tog. but B seperated = $\overset{2!}{\boxed{AC}}$, B $\overset{4!}{\_D\_E\_F\_G\_}$ $= 4! \times 5C_1 \times 2! \times 4C_1$ $= 960$

+

BC tog. but A seperated = $\overset{2!}{\boxed{BC}}$, A $\overset{4!}{\_D\_E\_F\_G\_}$ $= 4! \times 5C_1 \times 2! \times 4C_1$ $= 960$

$1440 + 960 + 960 + 960 = 4320$ Answer.

xiv) Arrange the alphabets such that A, B, C & D are not all next to eachother.

(Tot. no. of ways w/o Restriction) − (A B C D all together)

$(7!) - (\overset{4!}{ABCD}\ E\ F\ G)$

$(7!) - (4! \times 4!) = 5040 - 576$

$= 4464$ Answer

Case II: (Simple Selection)

1 President, 2 Sec, 8 members.

i) Select a team of 4 people w/o Restriction.

$11C_4 = 330$ Answer

ii) Select a team which must include the President.

$1C_1 \times 10C_3 = 120$ Answer

iii) Select a team which must include the President & any 1 of his secretary.

$1C_1 \times 2C_1 \times 8C_2 = 56$ Answer

iv) Select a team which must include both of the sec & the President.

$1C_1 \times 2C_2 \times 8C_1 = 8$ Answer.

v) Select a team which must include the President but not any one his sec.

$1C_1 \times 8C_3 = 56$ Answer

Case III: (Team member Selection)

6 Men & 4 Women

i) Select a team of 4 people without any restriction.

$10C_4 = 210$ Answer

OR

$(4M, 0W) + (3M, 1W) + (2M, 2W) + (1M, 3W) + (0M, 4W)$

$(6C_4 \times 4C_0) + (6C_3 \times 4C_1) + (6C_2 \times 4C_2) + (6C_1 \times 4C_3) + (6C_0 \times 4C_4)$

$15 + 80 + 90 + 24 + 1$

$= 210$ Answer.

Tot. no. of Selections w/o any restriction = Sum of selections with all the restrictions.

ii) Select a team of 4 people which consists of atleast 1 women.

Atleast 1 women = $(W \geq 1)$

$(1W, 3M) + (2W, 2M) + (3W, 1M) + (4W, 0M)$

$({}^4C_1 \times {}^6C_3) + ({}^4C_2 \times {}^6C_2) + ({}^4C_3 \times {}^6C_1) + ({}^4C_4 \times {}^6C_0)$

$80 + 90 + 24 + 1$

$= 195$ Answer.

OR

$\left(\begin{array}{c}\text{Tot. no. of selection} \\ \text{w/o Restriction}\end{array}\right) - (W < 1)$

$({}^{10}C_4) - (0W, 4M)$

$210 - ({}^4C_0 \times {}^6C_4) = 210 - 15$

$= 195$ Answer.

iii) Select a team which consists of atmost 3 women.

(Women ≤ 3)

$(0W, 4M) + (1W, 3M) + (2W, 2M) + (3W, 1M)$

$({}^4C_0 \times {}^6C_4) + ({}^4C_1 \times {}^6C_3) + ({}^4C_2 \times {}^6C_2) + ({}^4C_3 \times {}^6C_1)$

$15 + 80 + 90 + 24$

$= 209$ Answer

OR

$\left(\begin{array}{c}\text{Tot. no. of Selections} \\ \text{w/o Restriction}\end{array}\right) - (\text{Women} > 3)$

$({}^{10}C_4) - (4W, 0M)$

$210 - ({}^4C_4 \times {}^6C_0) = 210 - 1$

$= 209$ Answer

Case IV: (Complicated Seating Arrangement)

i) Arrange 3 people in a row of 7 chairs without any restriction.

$\underline{{}^7C_1} \times \underline{{}^6C_1} \times \underline{{}^5C_1}$

${}^7C_1 \times {}^6C_1 \times {}^5C_1 = 210$ Answer.

OR

${}^7C_3 \times 3! = 210$ Answer.

ii) find the no. of ways of seating a family of 3 members together on these 7 chairs.

$\overset{3!}{\boxed{M\ W\ C}}$ — — — —

— $\overset{3!}{\boxed{M\ W\ C}}$ — — —

— — $\overset{3!}{\boxed{M\ W\ C}}$ — —

— — — $\overset{3!}{\boxed{M\ W\ C}}$ —

— — — — $\overset{3!}{\boxed{M\ W\ C}}$

$3! + 3! + 3! + 3! + 3! = 30$ Answer.

OR

$\overset{3!}{\boxed{M\ W\ C}}$

| — | — | — | — |
5 4 3 2 1

${}^5C_1 \times 3! = 30$ Answer.

iii) Adjust the family in such a way that they are together and child sits in between them

$\boxed{M\ C\ W}$ $2!$

| 5 | 4 | 3 | 2 | 1 |

$5C_1 \times 2! = 10$ ways Answer

iv) Adjust the family such that child sits next to his mother but man cannot sit next to any of them.

$\boxed{W\ C}^{2!}$ _ 4 3 2 1 $= 2! \times 4C_1 = 8$

_ $\boxed{W\ C}^{2!}$ _ 3 2 1 $= 2! \times 3C_1 = 6$

3 _ $\boxed{W\ C}^{2!}$ _ 2 1 $= 2! \times 3C_1 = 6$

3 2 _ $\boxed{W\ C}^{2!}$ _ 1 $= 2! \times 3C_1 = 6$

3 2 1 _ $\boxed{W\ C}^{2!}$ _ $= 2! \times 3C_1 = 6$

4 3 2 1 _ $\boxed{W\ C}^{2!}$ $= 2! \times 4C_1 = 8$

40 ways

Answer

OR

$\boxed{W\ C}^{2!}$ M

| 5 | 4 | 3 | 2 | |

$5C_1 \times 2! \times 4C_1 = 40$ ways Answer.

Case V: (Applications of Arrangements)

5 books by Dan Brown

4 books by John Grisham

3 books by Charles Dickens

i) Arrange these books without any restriction

$12! = 479001600$ Answer

ii) books by John Grisham are kept together.

$\boxed{\_\ \_\ \_\ \_}^{4!}$ 9 8 7 6 5 4 3 2 1

$9! \times 4! = 8709120$ Answer.

iii) Books by each author are kept together.

$\underbrace{\boxed{DB}^{5!}\ \boxed{JG}^{4!}\ \boxed{CD}^{3!}}_{3!}$

$5! \times 4! \times 3! \times 3! = 103680$ Answer

iv) All the books by Charles Dickens must be seperated from eachother.

$9!$

| 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |

$9! \times 10C_1 \times 9C_1 \times 8C_1 = 261273600$ Answer

v) All the books by Charles dickens are not next to eachother.

$\left(\begin{array}{c}\text{Total no. of} \\ \text{Arrange. w/o Rest}\end{array}\right) - \left(\begin{array}{c}\text{All 3 being} \\ \text{together}\end{array}\right)$

$(12!) - (\boxed{CD}^{3!}$ 10 9 8 7 6 5 4 3 2 1 $)$

$(12!) - (3! \times 10!)$

$= 457228800$ Answer.

Case VI: (Repeated letter Arrangement)

- AA → $1 = \frac{2!}{2!} = 1$
- AAA → $1 = \frac{3!}{3!} = 1$
- AAB, ABA, BAA → $3 = \frac{3!}{2!} = 3$
- AAAB, AABA, ABAA, BAAA → $4 = \frac{4!}{3!} = 4$
- AABB, BBAA, BABA, ABAB, ABBA, BAAB, → $6 = \frac{4!}{2!2!} = 6$

Example 1 : EVERMORE.

i) find the no. of arrangements w/o any restriction.

3Es, 2Rs

$\frac{8!}{3!\,2!} = 3360$ Answer.

ii) find the no. of arrangements which start & end with R.

R $\boxed{\underset{3Es}{\_\ \_\ \_\ \_\ \_\ \_}}$ R {fixed positioning method}

$\frac{6!}{3!} = 120$ Answer

OR

$\underset{2C_1}{R}\ \underset{6!}{\boxed{\underset{3Es}{\_\ \_\ \_\ \_\ \_\ \_}}}\ \underset{1C_1}{R}$ {Overall arrangement}

$\frac{2C_1 \times 6! \times 1C_1}{3! \times 2!} = \frac{\cancel{2C_1 \times 1C_1} \times 6!}{\cancel{2!} \times 3!} = \frac{6!}{3!} = 120$ Answer

i) find the no. of arrangements that start & end with an E.

E $\boxed{\underset{2Rs}{\_\ \_\ \_\ \_\ \_\ \_}}$ E {fixed positioning method}

$\frac{6!}{2!} = 360$ Answer

OR

$\underset{3C_1}{E}\ \underset{6!}{\boxed{\underset{2Rs}{\_\ \_\ \_\ \_\ \_\ \_}}}\ \underset{2C_1}{E}$

$\frac{3C_1 \times 6! \times 2C_1}{2! \times 3!} = \frac{\cancel{3C_1 \times 2C_1} \times 6!}{\cancel{3!} \times 2!} = \frac{6!}{2!} = 360$ Answer.

Example 2 : INFINITE

i) Arrange the alphabets w/o any restriction

3 I's, 2 N's

$\frac{8!}{3! \times 2!} = 3360$ Answer.

ii) Arrange the alphabets such that all I's lie together.

$\boxed{III}$ N N F T E

$\frac{3!}{3!}$ 6 5 4 3 2 1

$\frac{6!}{2!} \times \frac{3!}{3!} = 360$ Answer.

iii) Arrange the alphabets such that they start from III.

$\boxed{III}$ N N F T E

$\frac{3!}{3!}$ 5 4 3 2 1

$\frac{5!}{2!} \times \frac{3!}{3!} = 60$ Answer

Example 3 : MAXIMUM

i) Arrange the alphabets without any restriction.

3Ms ; $\frac{7!}{3!} = 840$ Answer

ii) Arrange the alphabets such that all vowels are together.

vowels : A I U ; consonents : M X M M

$\boxed{A\ I\ U}$ M X M M

3! 5 4 3 2 1

$\frac{5!}{3!} \times 3! = 120$ Answer.

iii) Arrange the alphabets such that all consonents are together.

$\boxed{M\ X\ M\ M}$ A I U

$\frac{4!}{3!}$ 4 3 2 1

$4! \times \frac{4!}{3!} = 96$ Answer

Example 4 : SELECTION

i) Arrange the alphabets w/o any restriction

2 Es ; $\frac{9!}{2!} = 181440$ Answer

ii) Arrange the alphabets such that both E's are seperated.

$7!$

$_{8}|S_{7}|L_{6}|C_{5}|T_{4}|I_{3}|O_{2}|N_{1}|$

$7! \times \frac{{}^8C_1 \times {}^7C_1}{2!} = 141{,}120$ Answer

iii) Arrange the alphabets such that both E's are together.

$\boxed{EE}$ S L C T I O N

$\frac{2!}{2!}$ 8 7 6 5 4 3 2 1

$8! \times \frac{2!}{2!} = 40320$ Answer

Case $\overline{\text{VII}}$: (CODE GENERATION)

- no. of possible ATM pincodes:

i) no digit is repeated.

${}^{10}C_4 \times 4! = 5040$ Answer

OR

${}^{10}C_1 \times {}^9C_1 \times {}^8C_1 \times {}^7C_1 = 5040$ Answer

ii) Repetition of digits is allowed.

${}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1$

$({}^{10}C_1)^4 = (10)^4 = 10{,}000$ Answer.

- MSN chatrooms passwords
  - 6 alphabet password

i) no. repetition is allowed.

${}^{26}C_6 \times 6! = 165{,}765{,}600$ Answer.

OR

${}^{26}C_1 \times {}^{25}C_1 \times {}^{24}C_1 \times {}^{23}C_1 \times {}^{22}C_1 \times {}^{21}C_1 = 165{,}765{,}600$

ii) Repetition of alphabets is allowed.

${}^{26}C_1$ ${}^{26}C_1$ ${}^{26}C_1$ ${}^{26}C_1$ ${}^{26}C_1$ ${}^{26}C_1$

$({}^{26}C_1)^6 = (26)^6 = 308{,}915{,}776$ Answer

- JAZZ, PAKISTAN KA SUBSE BARA NETWORK

$0300 = {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1 = 10^7$

+

$0301 = ({}^{10}C_1)^7 = 10^7$

+

$0302 = ({}^{10}C_1)^7 = 10^7$

+

$0305 = ({}^{10}C_1)^7 = 10^7$

+

$0306 = ({}^{10}C_1)^7 = 10^7$

+

$0307 = ({}^{10}C_1)^7 = 10^7$

+

$0308 = ({}^{10}C_1)^7 = 10^7$

+

$0309 = ({}^{10}C_1)^7 = 10^7$

+

$0310 = ({}^{10}C_1)^7 = 10^7$

$9 \times 10^7 = 90000000$ Answer

- Bike Number Plate:

3 alphabet & 4 numbers.

${}^{26}C_1\ {}^{26}C_1\ {}^{26}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1$

$(26)^3 \times (10)^4 = 17576000$ Answer

- Car Number Plate:
  - 1 Alphabet & 4 digit:

${}^{26}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1 = 26 \times (10)^4 = 260{,}000$ Answer

  - 2 Alphabet & 3 digit:

${}^{26}C_1\ {}^{26}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1 = (26)^2 \times ({}^{10}C_1)^3 = 676{,}000$ Answer

  - 3 Alphabet & 3 digit:

${}^{26}C_1\ {}^{26}C_1\ {}^{26}C_1\ {}^{10}C_1\ {}^{10}C_1\ {}^{10}C_1 = (26)^3 \times (10)^3 = 17{,}576{,}000$ Answer

Example 5: EXAMINATION , 2A's, 2N's, 2I's

i) find the no. of 4 alphabet code words using all distinct alphabets

E X AA M II NN T O

1 2 3 4 5 6 7 8

$^8C_4 \times 4! = 1680$ Answer

ii) find the no. of 4 alphabet code words using 2 same & 2 distinct alphabets.

(2A's, 2dist) + (2I's, 2 dist) + (2N's, 2 dist)

AA $^7C_2 \times \frac{4!}{2!}$ II $^7C_2 \times \frac{4!}{2!}$ NN $^7C_2 \times \frac{4!}{2!}$

$$\left(^7C_2 \times \frac{4!}{2!}\right) + \left(^7C_2 \times \frac{4!}{2!}\right) + \left(^7C_2 \times \frac{4!}{2!}\right)$$

$252 + 252 + 252$

$= 756$ Answer

Case VIII : (Number Arrangement)

Ex1 1, 2, 3, 4, 5, 6.

i) 4 digit number

a) No repetition is allowed.

$^6C_4 \times 4! = 360$ Answer

b) Repetition is allowed

$\underline{^6C_1}\ \underline{^6C_1}\ \underline{^6C_1}\ \underline{^6C_1} = (^6C_1)^4$

$= 1296$ Answer

ii) 4-digit number which has to be greater than 4000

a) Rep. is not allowed

$\underset{^3C_1}{\underline{4/5/6}}$ ___ ___ ___ $^5C_3 \times 3!$

$^3C_1 \times {}^5C_3 \times 3! = 180$ Answer

b) Rep. is allowed

$\underset{^3C_1}{\underline{4/5/6}} \times \underline{^6C_1} \times \underline{^6C_1} \times \underline{^6C_1}$

$^3C_1 \times (^6C_1)^3 = 648$ Answer

iii) Number has to be in between 2500 & 5000 & repetition is not allowed.

(2500 – 3000) + (3000 – 5000)

$\underset{^1C_1}{\underline{2}}\ \underset{^2C_1}{\underline{5/6}}\ \underset{^4C_2 \times 2!}{\underline{\quad}\ \underline{\quad}} + \underset{^2C_1}{\underline{3/4}}\ \underset{^5C_3 \times 3!}{\underline{\quad}\ \underline{\quad}\ \underline{\quad}} = 24 + 120 = 144$

iv) Number has to be in between 2500 & 5000 & repetition is allowed.

(2500 – 3000) + (3000 – 5000)

$\underset{^1C_1}{\underline{2}}\ \underset{^2C_1}{\underline{5/6}}\ \underset{^6C_1}{\underline{\quad}}\ \underset{^6C_1}{\underline{\quad}} \quad \underset{^2C_1}{\underline{3/4}}\ \underset{^6C_1}{\underline{\quad}}\ \underset{^6C_1}{\underline{\quad}}\ \underset{^6C_1}{\underline{\quad}} = 72 + 432 = 504$

Ex 2 (1, 2, 2, 3, 6, 7, 8)

i) find the no. of arrangements w/o any restriction.

$\frac{7!}{2!} = 2520$ Answer

ii) find the no. of arrangements in which 1, 3 & 7 are next to eachother

$\underset{3!\ 5}{\boxed{1\ 3\ 7}}$ $\underset{4}{\underline{\quad}}\ \underset{3}{\underline{\quad}}\ \underset{2}{\underline{\quad}}\ \underset{1}{\underline{\quad}}$ two 2's

$3! \times \frac{5!}{2!} = 360$ Answer

iii) Overall 7 digit is an even number

ending with 2: $\boxed{\overset{6!}{\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}}}$ 2

+

ending with 6: $\boxed{\overset{6!/2!}{\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}}}$ 6

+

ending with 8: $\boxed{\overset{6!/2!}{\underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}}}$ 8

$6! + \frac{6!}{2!} + \frac{6!}{2!} = 1440$ Answer.

Question 1:

A committee of 5 people is to be chosen from 6 men and 4 women. In how many ways can this be done

(i) if there must be 3 men and 2 women on the committee, [2]

(ii) if there must be more men than women on the committee, [3]

(iii) if there must be 3 men and 2 women, and one particular woman refuses to be on the committee with one particular man? [3]

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i) 6M, 4W ; $(3M, 2W) = {}^6C_3 \times {}^4C_2 = 120$ Answer

ii) $(3M, 2W) + (4M, 1W) + (5M, 0W)$

$({}^6C_3 \times {}^4C_2) + ({}^6C_4 \times {}^4C_1) + ({}^6C_5 \times {}^4C_0)$

$120 + 60 + 6 = 186$ Answer

iii) $(3M, 2W)$

$(Z_{in}, N_{out}) + (Z_{out}, N_{in/out})$

$({}^1C_1 \times {}^3C_1 \times {}^5C_3) + ({}^3C_2 \times {}^6C_3)$

$30 + 60 = 90$ Answer

OR

$(Z_{in}, N_{out}) + (Z_{out}, N_{in}) + (Z_{out}, N_{out})$

$({}^1C_1 \times {}^3C_1 \times {}^5C_3) + ({}^3C_2 \times {}^1C_1 \times {}^5C_2) + ({}^3C_2 \times {}^5C_3)$

$30 + 30 + 30 = 90$ Answer

Question 2:

(a) A collection of 18 books contains one Harry Potter book. Linda is going to choose 6 of these books to take on holiday.

(i) In how many ways can she choose 6 books? [1]

(ii) How many of these choices will include the Harry Potter book? [2]

(b) In how many ways can 5 boys and 3 girls stand in a straight line

(i) if there are no restrictions, [1]

(ii) if the boys stand next to each other? [4]

Nov 2003, P6, Q6

a) i) ${}^{18}C_6 = 18,564$ Answer

ii) ${}^1C_1 \times {}^{17}C_5 = 6188$ Answer

b) i) $8! = 40320$ Answer

ii) $\boxed{BBBBB}$ G G G

5! 4 3 2 1

$5! \times 4! = 2880$ Answer

Question 3:

The word ARGENTINA includes the four consonants R, G, N, T and the three vowels A, E, I.

(i) Find the number of different arrangements using all nine letters. [2]

(ii) How many of these arrangements have a consonant at the beginning, then a vowel, then another consonant, and so on alternately? [3]

Nov 2004, P6, Q1

i) R G N N T A A E I

$\frac{9!}{2! \times 2!} = 90720$ Answer

ii) C V C V C V C V C

${}^5C_1$ ${}^4C_1$ ${}^4C_1$ ${}^3C_1$ ${}^3C_1$ ${}^2C_1$ ${}^2C_1$ ${}^1C_1$ ${}^1C_1$

$\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1}{2!} \times \frac{{}^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1}{2!}$

$\frac{5!}{2!} \times \frac{4!}{2!} = 720$ Answer

Question 4:

(a) A football team consists of 3 players who play in a defence position, 3 players who play in a midfield position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfield player, must be included, together with one defence and one forward player, [2]

(ii) if exactly one forward player must be included, together with any two others. [2]

(b) Find how many different arrangements there are of the nine letters in the words GOLD MEDAL

(i) if there are no restrictions on the order of the letters, [2]

(ii) if the two letters D come first and the two letters L come last. [2]

June 2005, P6, Q7

a) i) 3D, 3M, 5F , 3 to be chosen.

${}^1C_1 \times {}^3C_1 \times {}^5C_1 = 15$ Answer

ii) ${}^5C_1 \times {}^6C_2 = 75$ Answer

OR

$(1F, 2D) + (1F, 2M) + (1F, 1M, 1D)$

${}^5C_1 \times {}^3C_2 + {}^5C_1 \times {}^3C_2 + {}^5C_1 \times {}^3C_1 \times {}^3C_1$

$15 + 15 + 45 = 75$ Answer

b) i) G O L L D D M E A

$\frac{9!}{2! \times 2!} = 90720$ Answer

ii) D D $\boxed{\overset{5!}{\_\ \_\ \_\ \_\ \_}}$ L L

$5! = 120$ Answer

Question 5:

A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces? [2]

(ii) How many different arrangements are there if the 4 empty spaces are next to each other? [3]

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to each other. [2]

Nov 2005, P6, Q3

i) ${}^{13}C_9 \times 9! = 259459200$ Answer

ii) $\boxed{\_\ \_\ \_\ \_}$ _ _ _ _ _ _ _ _ _

10 9 8 7 6 5 4 3 2 1

$10! = 3628800$ Answer

iii) $\frac{(\text{Tot. no. of arr. w/o Rest.}) - (\text{All 4 being together})}{\text{Tot. no. of arr. w/o Res.}}$

$\frac{({}^{13}C_9 \times 9!) - 10!}{{}^{13}C_9 \times 9!} = \frac{141}{143} = 0.986$ Answer

Question 8:

The six digits 4, 5, 6, 7, 7, 7 can be arranged to give many different 6-digit numbers.

(i) How many different 6-digit numbers can be made? [2]

(ii) How many of these 6-digit numbers start with an odd digit and end with an odd digit? [4]

Nov 2007, P6, Q3

i) $\frac{6!}{3!} = 120$ Answer

ii) start with 5 end with 7 : 5 $\boxed{\overset{4!/2!}{\_\ \_\ \_\ \_}}$ 7 $= \frac{4!}{2!} = 12$

+

start with 7 end with 5 : 7 $\boxed{\overset{4!/2!}{\_\ \_\ \_\ \_}}$ 5 $= \frac{4!}{2!} = 12$

+

start & end with 7 : 7 $\boxed{\overset{4!}{\_\ \_\ \_\ \_}}$ 7 $= 4! = \underline{24}$

$\underline{48}$ Answer

Question 10:

A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopranos, 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen? [2]

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next to each other? [3]

(iii) The 4 tenors and 4 basses in the group stand in a single line with all the tenors next to each other and all the basses next to each other. How many possible arrangements are there if three of the tenors refuse to stand next to any of the basses? [3]

i) 13S, 12A, 6T, 7B $\longrightarrow$ 10S, 9A, 4T, 4B

$^{13}C_{10} \times {}^{12}C_9 \times {}^6C_4 \times {}^7C_4 = 33033000$ Answer

ii) $\boxed{6!}$ _ _ _ _

5 4 3 2 1

$6! \times 5! = 86400$ Answer

iii) $\underbrace{\boxed{T\ T\ T}}_{3!}\ T_G\ \underbrace{\boxed{B\ B\ B\ B}}_{4!}$ 2!

$3! \times 2! \times 4! = 288$ ways Answer.

Question 12:

(a) (i) Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and 6 with no digit being repeated. [1]

(ii) Find how many different odd numbers greater than 500 can be made using some or all of the digits 1, 3, 5 and 6 with no digit being repeated. [4]

(b) Six cards numbered 1, 2, 3, 4, 5, 6 are arranged randomly in a line. Find the probability that the cards numbered 4 and 5 are **not** next to each other. [3]

Nov 2009, P62, Q4

a) i) $^4C_4 \times 4! = 24$ OR $\underline{^4C_1}\ \underline{^3C_1}\ \underline{^2C_1}\ \underline{^1C_1}$

$^4C_1 \times {}^3C_1 \times {}^2C_1 \times {}^1C_1 = 24$

ii) $500 - 600 =$ $\underline{5}\ \underset{2C_1}{\underline{\ \ }}\ \underset{2C_1}{\underline{1/3}} = {}^2C_1 \times {}^2C_1 = 4$

\+

$600 - 700 =$ $\underline{6}\ \underset{2C_1}{\underline{\ \ }}\ \underset{3C_1}{\underline{1/3/5}} = {}^2C_1 \times {}^3C_1 = 6$

\+

$1000 - 1999 =$ $\underline{1}\ \underset{2C_1}{\underline{\ \ }}\ \underset{1C_1}{\underline{\ \ }}\ \underset{2C_1}{\underline{3/5}} = {}^2C_1 \times {}^1C_1 \times {}^2C_1 = 4$

\+

$3000 - 3999 =$ $3\ \underset{2C_1}{\underline{\ \ }}\ \underset{1C_1}{\underline{\ \ }}\ \underset{2C_1}{\underline{1/5}} = {}^2C_1 \times {}^1C_1 \times {}^2C_1 = 4$

\+

$5000 - 5999 =$ $5\ \underset{2C_1}{\underline{\ \ }}\ \underset{1C_1}{\underline{\ \ }}\ \underset{2C_1}{\underline{1/3}} = {}^2C_1 \times {}^1C_1 \times {}^2C_1 = 4$

\+

$6000 - 6999 =$ $6\ \underset{2C_1}{\underline{\ \ }}\ \underset{1C_1}{\underline{\ \ }}\ \underset{3C_1}{\underline{1/3/5}} = {}^2C_1 \times {}^1C_1 \times {}^3C_1 = \underline{6}$

$\underline{28}$ ways.

b) 4!

$_5|\,1\,|_4\,2\,|_3\,3\,|_2\,6\,|_1$

$\dfrac{4! \times {}^5C_1 \times {}^4C_1}{6!} = \dfrac{48\cancel{0}}{72\cancel{0}}$

$= \dfrac{2}{3}$ Answer

Question 14:

Nine cards, each of a different colour, are to be arranged in a line.

(i) How many different arrangements of the 9 cards are possible? [1]

The 9 cards include a pink card and a green card.

(ii) How many different arrangements do not have the pink card next to the green card? [3]

Consider all possible choices of 3 cards from the 9 cards with the 3 cards being arranged in a line.

(iii) How many different arrangements in total of 3 cards are possible? [2]

(iv) How many of the arrangements of 3 cards in part (iii) contain the pink card? [2]

(v) How many of the arrangements of 3 cards in part (iii) do not have the pink card next to the green card? [2]

June 2010, P62, Q7

i) $9! = 362,880$ Answer

ii) 7!

$|_8 - |_7 - |_6 - |_5 - |_4 - |_3 - |_2 - |_1$

$7! \times {}^8C_1 \times {}^7C_1 = 282240$ Answer

OR

2!

$\boxed{PG}$ _ _ _ _ _ _ _

8 7 6 5 4 3 2 1

(Tot. no. of arr. w/o Res) − (both being together)

$9! - (8! \times 2!) = 282240$ Answer

iii) $^9C_3 \times 3! = 504$ Answer

iv) $^1C_1 \times {}^8C_2 \times 3! = 168$ Answer

v) (Tot. no. of ways w/o Res) − (P & G tog.)

$({}^9C_3 \times 3!) - \underbrace{\boxed{PG}}_{2!}\ {}^7C_1$ 2!

$({}^9C_3 \times 3!) - (2! \times {}^7C_1 \times 2!)$

$504 - 28 = 476$ Answer

OR

P & G not selected : $^7C_3 \times 3! =$ 210

\+

P selected but G not selected : $^1C_1 \times {}^7C_2 \times 3! =$ 126

\+

G selected but P not selected : $^1C_1 \times {}^7C_2 \times 3! =$ 126

\+

P & G both selected but sep. : P ${}^7C_1$ G $= {}^7C_1 \times 2! =$ $\underline{14}$ 2!

$\underline{476}$ Answer

Question 27:

(a) In a sweet shop 5 identical packets of toffees, 4 identical packets of fruit gums and 9 identical packets of chocolates are arranged in a line on a shelf. Find the number of different arrangements of the packets that are possible if the packets of chocolates are kept together. [2]

(b) Jessica buys 8 different packets of biscuits. She then chooses 4 of these packets.

(i) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account? [2]

The 8 packets include 1 packet of chocolate biscuits and 1 packet of custard creams.

(ii) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account and the packet of chocolate biscuits and the packet of custard creams are both chosen? [3]

(c) 9 different fruit pies are to be divided between 3 people so that each person gets an odd number of pies. Find the number of ways this can be done. [5]

Nov 2012, P61, Q7

a) 5T, 4F, 9C

10 9 8 7 6 5 4 3 2 1

$\frac{10!}{4!\times 5!}\times\frac{9!}{9!} = 1260$ Answer

b) i) $^8C_4\times 4! = 1680$ Answer

ii) $^1C_1\times{}^1C_1\times{}^6C_2\times 4! = 360$ Answer

c) (W, N, A)

$(7,1,1) = {}^9C_7\times{}^2C_1\times{}^1C_1\times\frac{3!}{2!} = 216$

+

$(5,1,3) = {}^9C_5\times{}^4C_1\times{}^3C_3\times 3! = 3024$

+

$(3,3,3) = {}^9C_3\times{}^6C_3\times{}^3C_3\times\frac{3!}{3!} = 1680$

4920 Answer

Question 28:

(a) A team of 3 boys and 3 girls is to be chosen from a group of 12 boys and 9 girls to enter a competition. Tom and Henry are two of the boys in the group. Find the number of ways in which the team can be chosen if Tom and Henry are either both in the team or both not in the team. [3]

(b) The back row of a cinema has 12 seats, all of which are empty. A group of 8 people, including Mary and Frances, sit in this row. Find the number of different ways they can sit in these 12 seats if

(i) there are no restrictions, [1]

(ii) Mary and Frances do not sit in seats which are next to each other, [3]

(iii) all 8 people sit together with no empty seats between them. [3]

Nov 2012, P62, Q5

a) (T & H both in) + (T & H both out)

$(^1C_1\times{}^1C_1\times{}^{10}C_1\times{}^9C_3) + (^9C_3\times{}^{10}C_3)$

840 + 10080 = 10920 Answer

b) i) $^{12}C_8\times 8! = 19,958,400$ Answer

ii) (Tot no. of ways w/o Rest.) − (M & F both together)

$(^{12}C_8\times 8!) - (\boxed{MF}\ \_\ \_\ \_\ \_\ \_\ \_\ \_)$

2! 7 6 5 4 3 2 1

$(^{12}C_8\times 8!) - (^{11}C_7\times 7!\times 2!)$

19958400 − 3326400 = 16,632,000 Answer

OR

$^{10}C_6\times 6!\times{}^{11}C_1\times{}^{10}C_1 = 16,632,000$ Answer

iii) 5 4 3 2 1

$^5C_1\times 8! = 201,600$ Answer

Question 29:

(a) A chess team of 2 girls and 2 boys is to be chosen from the 7 girls and 6 boys in the chess club. Find the number of ways this can be done if 2 of the girls are twins and are either both in the team or both not in the team. [3]

(b) **(i)** The digits of the number 1 244 687 can be rearranged to give many different 7-digit numbers. How many of these 7-digit numbers are even? [4]

(ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 4, 6, 7, 8? [2]

(c) Helen has some black tiles, some white tiles and some grey tiles. She places a single row of 8 tiles above her washbasin. Each tile she places is equally likely to be black, white or grey. Find the probability that there are no tiles of the same colour next to each other. [3]

Nov 2012, P63, Q6

a) (both twins in) + both twins out.

$(^2C_2\times{}^6C_2) + (^5C_2\times{}^6C_2) = 15 + 150 = 165$ Answer

b) i) ending with 2: $\boxed{\text{two 4's}}\,2 = \frac{6!}{2!} = 360$

+

ending with 4: $\boxed{\text{one 4}}\,4 = 6! = 720$

+

ending with 6: $\boxed{\text{two 4's}}\,6 = \frac{6!}{2!} = 360$

+

ending with 8: $\boxed{\text{two 4's}}\,8 = \frac{6!}{2!} = 360$

1800 Answer

ii) 1, 2, 4, 6, 7, 8 ; 20,000 —— 30,000

$2\,\boxed{^5C_4\times 4!}$

$^1C_1\times{}^5C_4\times 4! = 120$ Answer

c) B/W/G

1 2/3 2/3 2/3 2/3 2/3 2/3 2/3

$1\times\left(\frac{2}{3}\right)^7 = \frac{128}{2187}$ Answer

OR

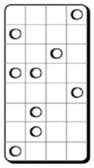
$^3C_1$ $^2C_1$ $^2C_1$ $^2C_1$ $^2C_1$ $^2C_1$ $^2C_1$ $^2C_1$

prob = $\frac{\text{No. of desired}}{\text{Tot. no. of possible}} = \frac{^3C_1\times(^2C_1)^7}{(^3C_1)^8}$

$= \frac{128}{2187}$ Answer

Question 36:

In a certain hotel, the lock on the door to each room can be opened by inserting a key card. The key card can be inserted only one way round. The card has a pattern of holes punched in it. The card has 4 columns, and each column can have either 1 hole, 2 holes, 3 holes or 4 holes punched in it. Each column has 8 different positions for the holes. The diagram illustrates one particular key card with 3 holes punched in the first column, 3 in the second, 1 in the third and 2 in the fourth.



(i) Show that the number of different ways in which a column could have exactly 2 holes is 28. [1]

(ii) Find how many different patterns of holes can be punched in a column. [4]

(iii) How many different possible key cards are there? [2]

Nov 2002, P6, Q4

i) $^8C_2 = 28$ ways

ii) one hole + two holes + three holes + four holes

$8C_1 + 8C_2 + 8C_3 + 8C_4 = 8 + 28 + 56 + 70$

$= 162$ Answer

iii) $162 \times 162 \times 162 \times 162 = (162)^4$

$= 688747536$ Answer

Question 37:

(a) The menu for a meal in a restaurant is as follows.

Starter Course

Melon
or
Soup
or
Smoked Salmon

Main Course

Chicken
or
Steak
or
Lamb Cutlets
or
Vegetable Curry
or
Fish

Dessert Course

Cheesecake
or
Ice Cream
or
Apple Pie

All the main courses are served with salad and either new potatoes or french fries.

(i) How many different three-course meals are there? [2]

(ii) How many different choices are there if customers may choose only two of the three courses? [3]

(b) In how many ways can a group of 14 people eating at the restaurant be divided between three tables seating 5, 5 and 4? [3]

June 2004, P6, Q5

a) i) (S, M, D)

$(3C_1) \times (5C_1 \times 1C_1 \times 2C_1) \times (3C_1) = 90$ Answer

ii) (S, M) + (S, D) + (M, D)

$(3C_1 \times 5C_1 \times 1C_1 \times 2C_1) + (3C_1 \times 3C_1) + (5C_1 \times 1C_1 \times 2C_1 \times 3C_1)$

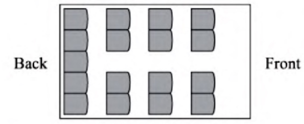
$30 + 9 + 30 = 69$ Answer

b) 5, 5, 4 OR 4, 5, 5 OR 5, 4, 5

$(14C_5 \times 9C_5 \times 4C_4)$ $14C_4 \times 10C_5 \times 5C_5$ $14C_5 \times 9C_4 \times 5C_5$

$= 252252$ $= 252252$ $= 252252$

Question 38:



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.

(i) How many possible seating arrangements are there for the 11 passengers? [2]

(ii) How many possible seating arrangements are there if 5 particular people sit in the back row? [3]

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

(iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 married couples and 1 other person, who may or may not be married? [3]

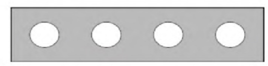
June 2006, P6, Q4

i) $17C_{11} \times 11! = 4.94 \times 10^{11}$

ii) $5C_5 \times 5! \times 12C_6 \times 6! = 79833600$

iii) $3C_2 \times 7C_1 = 21$ Answer

Question 40:



Pegs are to be placed in the four holes shown, one in each hole. The pegs come in different colours and pegs of the same colour are identical. Calculate how many different arrangements of coloured pegs in the four holes can be made using

(i) 6 pegs, all of different colours, [1]

(ii) 4 pegs consisting of 2 blue pegs, 1 orange peg and 1 yellow peg. [1]

Beryl has 12 pegs consisting of 2 red, 2 blue, 2 green, 2 orange, 2 yellow and 2 black pegs. Calculate how many different arrangements of coloured pegs in the 4 holes Beryl can make using

(iii) 4 different colours, [1]

(iv) 3 different colours, [3]

(v) any of her 12 pegs. [3]

Nov 2010, P61, Q6

i) $6C_4 \times 4! = 360$ Answer

ii) $4C_4 \times \frac{4!}{2!} = 12$ Answer

iii) $6C_4 \times 4! = 360$ Answer

iv) $6C_3 \times \left\{ \frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} \right\} = 6C_3 \times (12 + 12 + 12)$

$= 20 \times 36 = 720$ Answer

v) All diff. + 3 diff. + 2 diff.

$(6C_4 \times 4!) + \left\{ 6C_3 \times \left(\frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} \right) \right\} + 6C_2 \times \left\{ \frac{4!}{2!\,2!} \right\}$

$360 + 720 + (15 \times 6) = 1170$ Answer