Multi-Objective Portfolio Optimization

Parastoo Azizeddin 97106913
Amirreza Hatamipour 97101507

I. INTRODUCTION

Financial portfolio selection has been one of the most critical decision-making fields studied in modern finance since the 1950s. One of the significant concerns in Financial portfolio selection is the problem of finding the most appropriate stocks to hold based on fluctuations of each of the stocks. This problem can be achieved as a two-stage process: (a) evaluating the available stocks to select the ones that best meet the investor's preferences, (b) specification of the amount of capital budget to be invested in each of the stocks selected in the first stage. Investors are always interested in managing their portfolios, reaching high returns with low risks. Risk is a criterion indicating uncertainty about the oncoming return. In the financial portfolio selection problem, the investor usually considers several conflicting objectives such as return, risk, and liquidity. Return in the stock market is the aggregate of company dividend and its stock price changes. By diversification of risk and yield maximization, a desirable investment occurs. In this project, we tend to reach an optimal portfolio, and we use Iran's Stock Market data as a case study.

II. Literature Review

A. Portfolio Optimization

Markowitz was a pioneer in the field of portfolio optimization. Markowitz introduced the mean-variance (MV) model for PO, where simultaneously returns and risks are optimized. He formed an optimization problem to divide resources to minimize the risk at any level of expected return. He measured risk through the second-order deviation of returns from the mean (a quadratic of the co-variance matrix) [1]. However, due to the difficulties in PO, the original form of this model has not been used extensively to construct a large-scale portfolio.

More recent studies have focused on considering additional constraints and alternative decision criteria such as downside risk measures, cardinality, and rebalancing [2].

Although previous studies have tried to solve some of the problems in the PO field, the issue of tradeoffs between investors' preferences regarding several objectives has been partially addressed in the existing portfolio literature.

B. Multi-objective Optimization

An economist, V. Pareto, developed Multi-objective optimization, which is an alternative approach to the portfolio optimization problem [3]. The multi-objective approach combines multiple objectives $f_1(\vec{x}), f_2(\vec{x}), ..., f_n(\vec{x})$ into one objective function by assigning a weighting coefficient to each objective. The standard solution technique is to minimize a weighted convex sum of the objectives using a single-objective method, shown below:

$$\text{Minimize } F(\vec{x}) = \sum_{i=1}^{n} a_i f_i(\vec{x})$$

III. Problem Definition

Based on their preferences and wealth, investors choose how to maximize their utility while lowering the level of risk. Also, due to the difference in risk aversion among investors, we can formulate the problem in the following three different ways:

- **Form 1:** Minimizing the risk with the constraint of having lower limit on return value [4], [5]:

$$\begin{aligned} \text{Minimize} \quad & F_\Omega(w) \\ \text{s.t} \quad & \mathbb{E}(R(w)) \geq R^* \\ & 1^T w = 1 \\ & w_j \geq 0, \forall_j = 1,2,...,n \end{aligned}$$

- **Form 2:** Maximizing the return with the constraint of having upper limit on risk value [5]:

$$\begin{aligned} \text{Maximize} \quad & \mathbb{E}(R(w)) \\ \text{s.t} \quad & F_\Omega(w) \leq L^* \\ & 1^T w = 1 \\ & w_j \geq 0, \forall_j = 1,2,...,n \end{aligned}$$

- **Form 3:** Design a multi-objective problem for maximizing return while minimizing risk level [6]:

$$\begin{aligned} \text{Maximize} \quad & \mathbb{E}(R(w)) \\ \text{Minimize} \quad & F_\Omega(w) \\ \text{s.t} \quad & 1^T w = 1 \\ & w_j \geq 0, \forall_j = 1,2,...,n \end{aligned}$$

where $F_\Omega(w)$ is a function of risk level measurements, L^* is the highest acceptable risk level, and $\mathbb{E}(R(w))$ is expected return of investment.

IV. Methodology & Approach

This project aims to build an optimal portfolio of different company shares, maximizing the whole return while minimizing risk factors using Tehran Stock Exchange data. We tend to find a solution for form 3 using a multi-objective approach based on the Tehran Stock Exchange. We are planning to test different algorithms for solving this problem, such as Genetic Programming, Particle Swarm Optimization,....

Also, we are planning to add some more new elements to our objective function and additional constraints to build a better portfolio.

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