Multi-Objective Portfolio Optimization

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outline

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- Multi-objective Optimization Definition
- Solving Methods
- Methodology & Approach
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Introduction

- financial portfolio selection problem
- finding the most appropriate stocks to hold, based on fluctuations of each of the stocks.
- Goal $\longrightarrow$ reaching high returns with low risks

Problem definition

we can formulate the problem in the following three different ways

1. Minimizing the risk with the constraint of having lower limit on return value:

$$\mathit{Minimize}\ F_{\Omega}(\mathrm{w})$$

$$s.t\ \mathbb{E}(\mathrm{R}(\mathrm{w})) \geq R^{\cdot}$$

$$1^{\mathrm{T}} w = 1$$

$$w_{\mathrm{j}} \geq 0,\ \forall_{\mathrm{j}} = 1, 2, \ldots, n$$

Problem definition

2. Maximizing the return with the constraint of having upper limit on risk value:

$$\textit{Maximize}\ \mathbb{E}(\,\mathrm{R}(\,\mathrm{w})\,)$$

$$s.t\ \ F_{\Omega}(\,\mathrm{w})\ \le L$$

$$1^{\mathrm{T}}w = 1$$

$$w_{\mathrm{j}} \ge 0,\ \ \forall_{\mathrm{j}} = 1, 2, \ldots, n$$

Problem definition

3. Design a multi-objective problem for maximizing return while minimizing risk level

$$\textit{Maximize } \mathbb{E}(\mathrm{R}(\mathrm{w}))$$

$$\textit{Minimize } F_{\Omega}(\mathrm{w})$$

$$s.t \;\; 1^{\mathrm{T}} w = 1$$

$$w_{\mathrm{j}} \geq 0, \;\; \forall_{\mathrm{j}} = 1, 2, \ldots, n$$

Multi-objective Optimization Definition

- are realistic models for many complex engineering optimization problems.
- A reasonable solution: is to investigate a set of solutions, each of which satisfies the objectives at an acceptable level without being dominated by any other solution
- A general multi-objective optimization problem can be described as a vector function:

$$min/ max\ x = f(x) = (f_1(x), f2(x), \ldots, fn(x))$$

$$subject\ to\ x = (x_1, x_2, \ldots, x_m) \in X$$

$$y = (y_1, y_2, \ldots, y_m) \in Y$$

Multi-objective optimization in Portfolio optimization

we write the problem in multi-objective format:

$$\mathit{Minimize}\ \left(-\mathbb{E}(\mathrm{R}(\mathrm{w})),\ F_{\Omega}(w) \right)$$

$$s.t\ \ 1^{\mathrm{T}} w = 1$$

$$x \geq 0$$

Our Novel Contribution

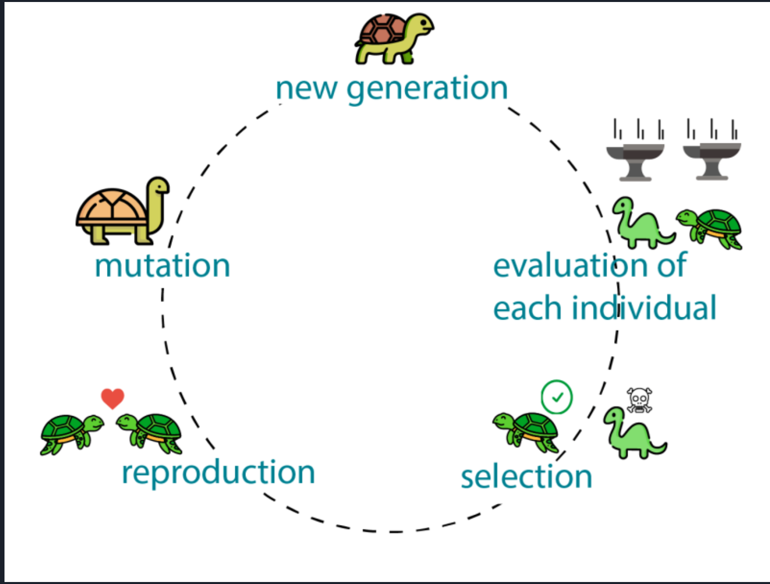
- We changed the objective function in order to have better portfolio optimization
- Added the term called "Score":

$$\mathit{Minimize}\ \left(-\mathbb{E}(\mathrm{R}(\mathrm{w})),\ \mathrm{F}_{\Omega}(\mathrm{w}),\ -\mathrm{Score}(\mathrm{x}) \right)$$

$$s.t\ \ 1^{\mathrm{T}}w = 1$$

$$x \geq 0$$

Genetic Algorithms



Solving Methods: NGSA-II

$for\ each\ p \in P$

$S_P = \varnothing$

$n_p = 0$

$for\ each\ q \in P$

$if\ (p \prec q)\ then$ — *If p dominates q*

$S_P = S_P \cup q$ — *Add q to the set of solutions dominated*

$elseif\ (q \prec p)\ then$

$n_p = n_p + 1$ — *Increment the domination counter of p*

$if\ n_p = 0\ then$ — *p belongs to the first front*

$p_{rank} = 1$

$F_1 = F_1 \cup p$

$i = 1$ — *Initialize the front counter*

$while\ F_i \neq \varnothing$

$Q = \varnothing$ — *Used to store the members of the next fr*

$for\ each\ p \in F_i$

$for\ each\ q \in S_P$

$n_q = n_q - 1$

$if\ n_q = 0\ then$ — *q belongs to the next front*

$q_{rank} = i + 1$

$Q = Q \cup q$

$i = i + 1$

$F_i = Q$

Solving Methods: SPEA-2 Algorithms

input: N *population size*

$\overline{N}$ *archive size*

T *maximum nember of generations*

Output: $\boldsymbol{A}$ *nondominated set*

*Step*1: ***Initialization***: *Generate an initial population* $\boldsymbol{P}_0$ *and create the empty archive (external set)* $\overline{P}_0 = \varnothing$. *set* $t=0$.

*Step*2: ***Fitness assignment***: *calculate fitness values of individuals in* $\boldsymbol{P}_t$ *and* $\overline{\mathrm{P}}_{\mathrm{t}}$

*Step*3: ***Environmental selection***: *Copy all nondominated individuals in* P_t *and* $\overline{P}_t$ *to* $\overline{P_{t+1}}$.

If size of $\overline{P_{t+1}}$ *exceeds* N *then reduce* $\overline{P_{t+1}}$ *by means of the truncation operator, otherwise*

if size of $\overline{P_{t+1}}$ *is less than* $\overline{N}$ *then fill* $\overline{P_{t+1}}$ *with dominated individuals in* P_t *and* $\overline{P}_t$

Solving Methods: SPEA-2 Algorithms

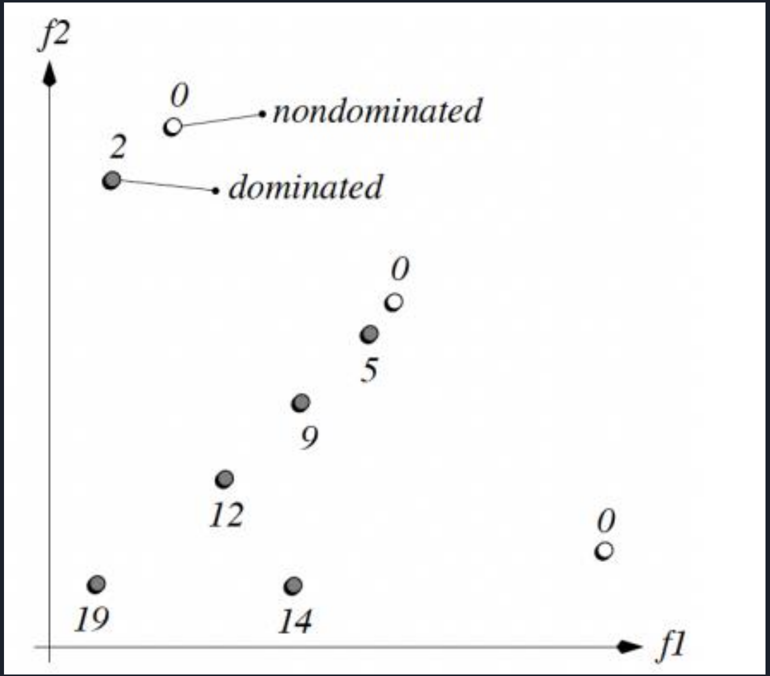
Step 4: ***Termination***: *If* $t \geq T$ *or another stopping criterion is satisfied then set* **A** *to the set of decision vectors represented by the nondominated individuals in* $\overline{P}_{t+1}$. *Stop*.

Step 5: ***Mating selection***: *Perform binary tournament selection with replacement on* $\overline{P}_{t+1}$ *in order to fill the mating pool*.

Step 6: ***Variation***: *Apply recombination and mutation operators to the mating pool and set* P_{t+1}. *to the resulting population*. *Increment generation counter* ($t = t + 1$) *and go to Step* 2.

Solving Methods: SPEA-2

the fitness assignment in a maximization problem:



Methodology & Approach: Data

- monthly return of 10 shares in the Iran stock market:
    1. Mellat bank (MLT)
    2. hiweb (HW),
    3. Tondgooyan Petrochemical (TOP)
    4. Iran national copper industry (NCI)
    5. Persian Golf Petrochemical (PGP)
    6. Iran khodro Investigation (IKHI)
    7. Ghadir Investigation (GHI)
    8. Shooyandeh Industry (SHI)
    9. Bandar Abbas oil refining(BAOR)
    10. Barekat Pharmacy (BPH)
- the data collected for 12 months.

2 cases with different objective functions

Case 1:

$$Minimize\ (-\mathbb{E}(\mathrm{R}(\mathrm{w})),\ F_{\Omega}(w))$$

$$s.t\ \ 1^{\mathrm{T}}w=1$$

$$x \geq 0$$

Case 2:

$$Minimize\ (-\mathbb{E}(\mathrm{R}(\mathrm{w})),\ \mathrm{F}_{\Omega}(\mathrm{w}),\ -\mathrm{Score}(\mathrm{x}))$$

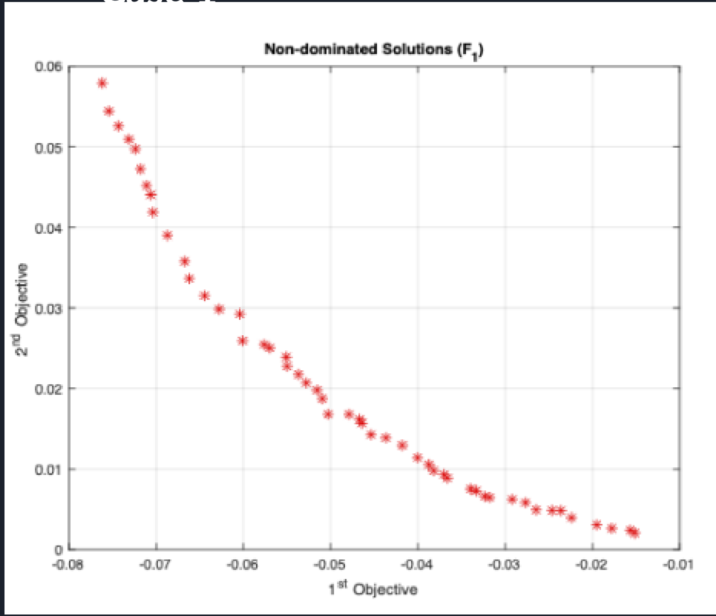
$$s.t\ \ 1^{\mathrm{T}}w=1$$

$$x \geq 0$$

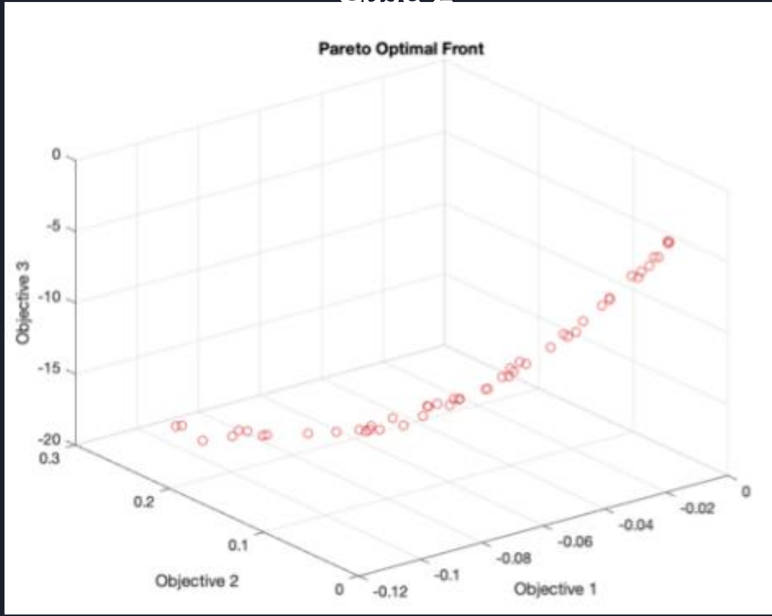
Methodology & Approach: Result

NGSA-2 algorithm

Case 1



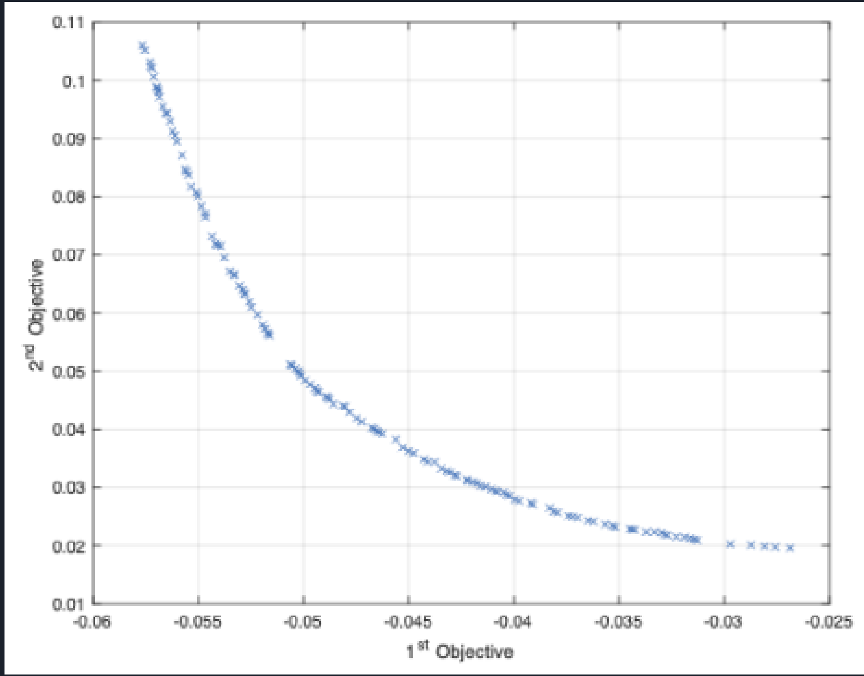
Case 2



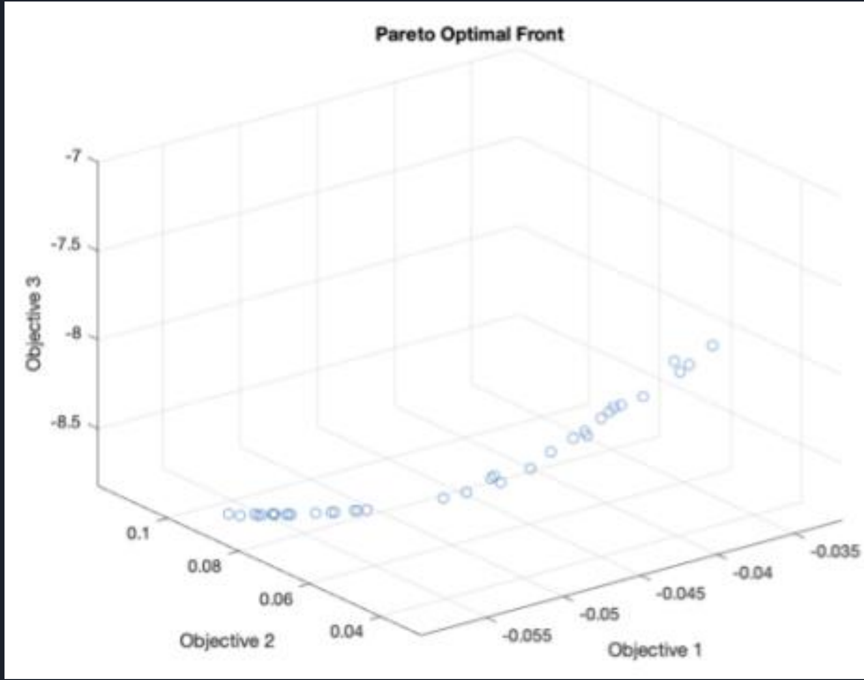
Methodology & Approach: Result

SPEA-2 algorithm

Case 1



Case 2



Methodology & Approach: Result

Example of normal case using NGSA-II:

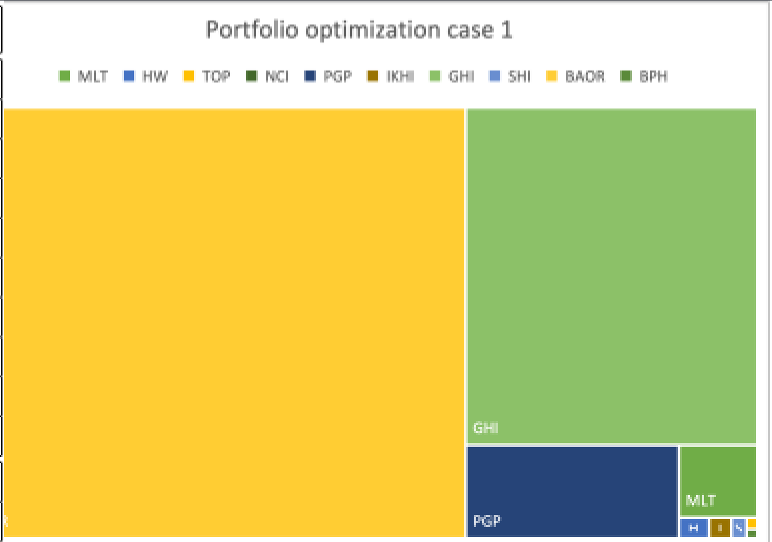
	Solution
Retrun	0.04809
Risk	0.021445

Portfolio in case 1

using SPEA-2

algorithm:

	Solution
MLT	0.0167
HW	0.0018
TOP	0.0004
NCI	0.0000
PGP	0.0581
IKHI	0.0014
GHI	0.2871
SHI	0.0009
BAOR	0.6333
BPH	0.0003
Retrun	0.046179
Risk	0.050178



Methodology & Approach: Result

Example of normal case using NGSA-II:

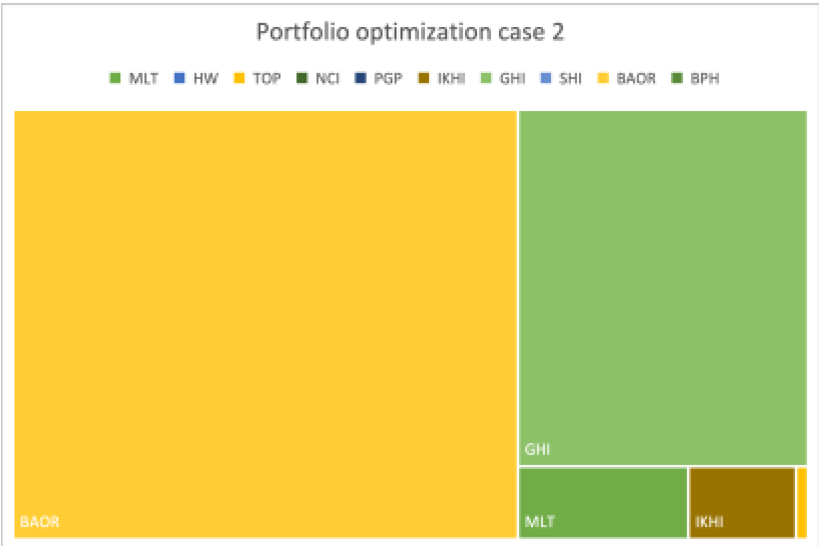
	Solution
Retrun	0.061405
Risk	0.078709
Score	11.345

Portfolio in case 2

using SPEA-2

algorithm:

	Solution
MLT	0.0362
HW	0.0000
TOP	0.0026
NCI	0.0000
PGP	0.0000
IKHI	0.0229
GHI	0.3034
SHI	0.0000
BAOR	0.6348
BPH	0.0000
Retrun	0.045311
Risk	0.046755
Score	8.007



Future Work

Add term liquidity to objective func

$$\mathit{Minimize}\ \left(-\mathbb{E}(\mathrm{R}(\mathrm{w}))\ ,\ \mathrm{F}_{\Omega}(\mathrm{w})\ ,\ -\mathrm{Score}(\mathrm{x})\ ,\ -\mathrm{Liquidity}(\mathrm{x}) \right)$$

$$s.t\ \ 1^{\mathrm{T}}w = 1$$

$$x \geq 0$$

Thanks for your attention!