

اصول پردازش تصویر

*Principles of Image Processing*

مصطفی کمالی تبریزی

۲۲ آذر ۱۳۹۹

جلسه بیست چهارم

# Image Morphing





نتیجه تمرین آقای احمد رحیمی در سال ۱۳۹۸

# Image Warping in Biology

- D'Arcy Thompson
- <http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/darcy.html>
- [http://en.wikipedia.org/wiki/D'Arcy\\_Thompson](http://en.wikipedia.org/wiki/D'Arcy_Thompson)
- Importance of shape and structure in evolution

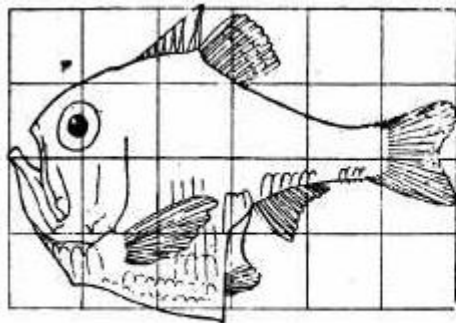
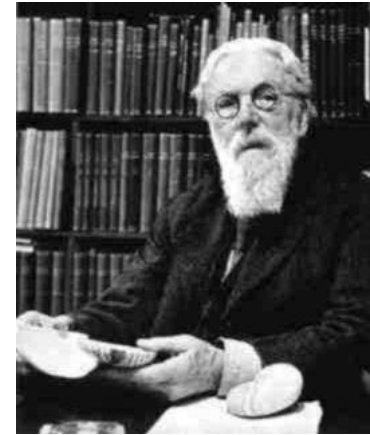
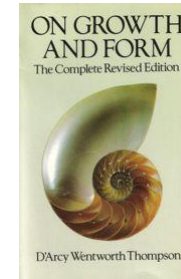


Fig. 517. *Argyropelecus Olfersi*.

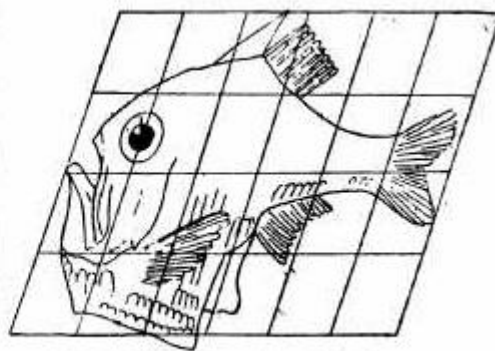
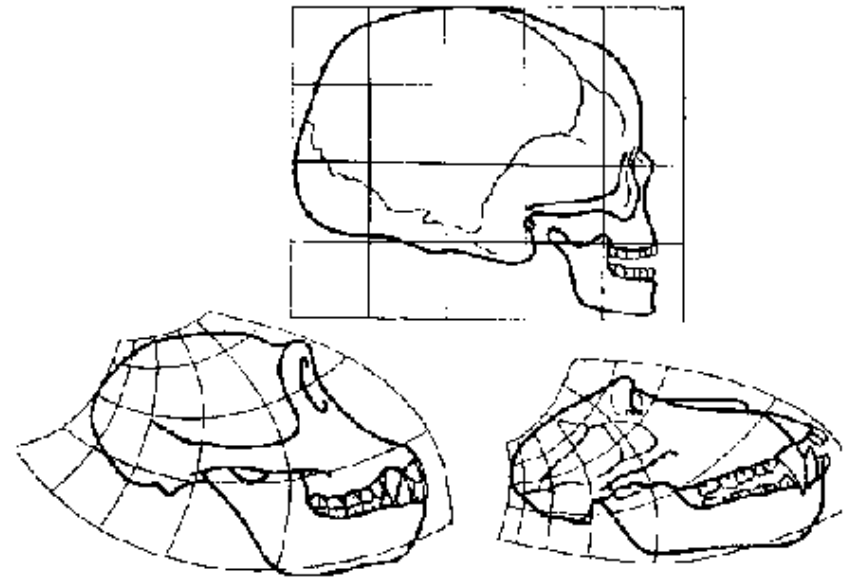
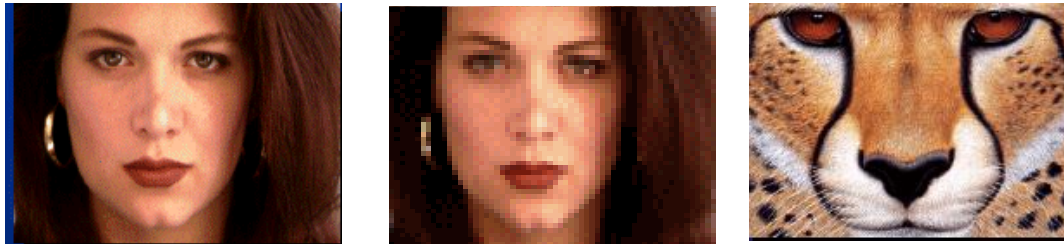


Fig. 518. *Sternoptyx diaphana*.



Skulls of a human, a chimpanzee and a baboon and transformations between them

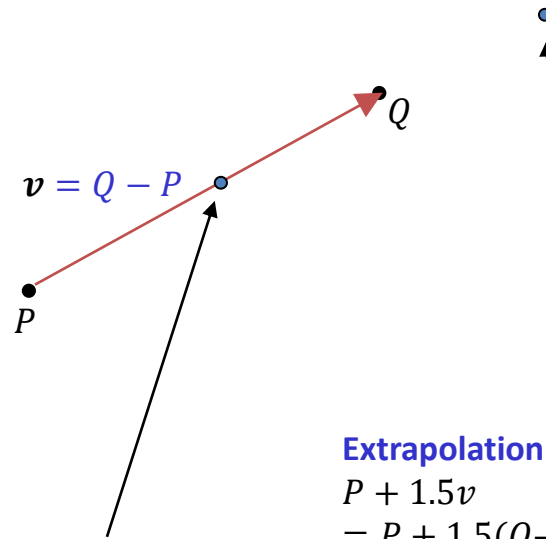
# Morphing = Object Averaging



- The aim is to find “an average” between two objects
  - Not an average of two images of objects ...
  - ... but an image of the average object!
  - How can we make a smooth transition in time?
    - Do a “weighted average” over time  $t$

# Averaging Points

What's the average  
of  $P$  and  $Q$ ?



## Linear Interpolation

New point:  $P + t * (Q - P)$

Or equivalently:  $(1 - t)P + tQ$  ( $0 < t < 1$ )

$$\begin{aligned} P + 0.5v &= P + 0.5(Q - P) \\ &= 0.5P + 0.5Q \end{aligned}$$

**Extrapolation:**  $t < 0$  or  $t > 1$

$$\begin{aligned} P + 1.5v &= P + 1.5(Q - P) \\ &= -0.5P + 1.5Q \quad (t = 1.5) \end{aligned}$$

- $P$  and  $Q$  can be anything:
  - points on a plane (2D) or in space (3D)
  - Colors in RGB (3D)
  - Whole images (m-by-n D)... etc.

# Idea #1: Cross-Dissolve



- Interpolate whole images:
- $I_{halfway} = (1 - t) I_1 + t I_2$
- This is called **cross-dissolve** in film industry
- But what if the images are not aligned?

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# Idea #2: Align, then cross-dissolve



- Align first, then cross-dissolve
  - Alignment using global warp – picture still valid

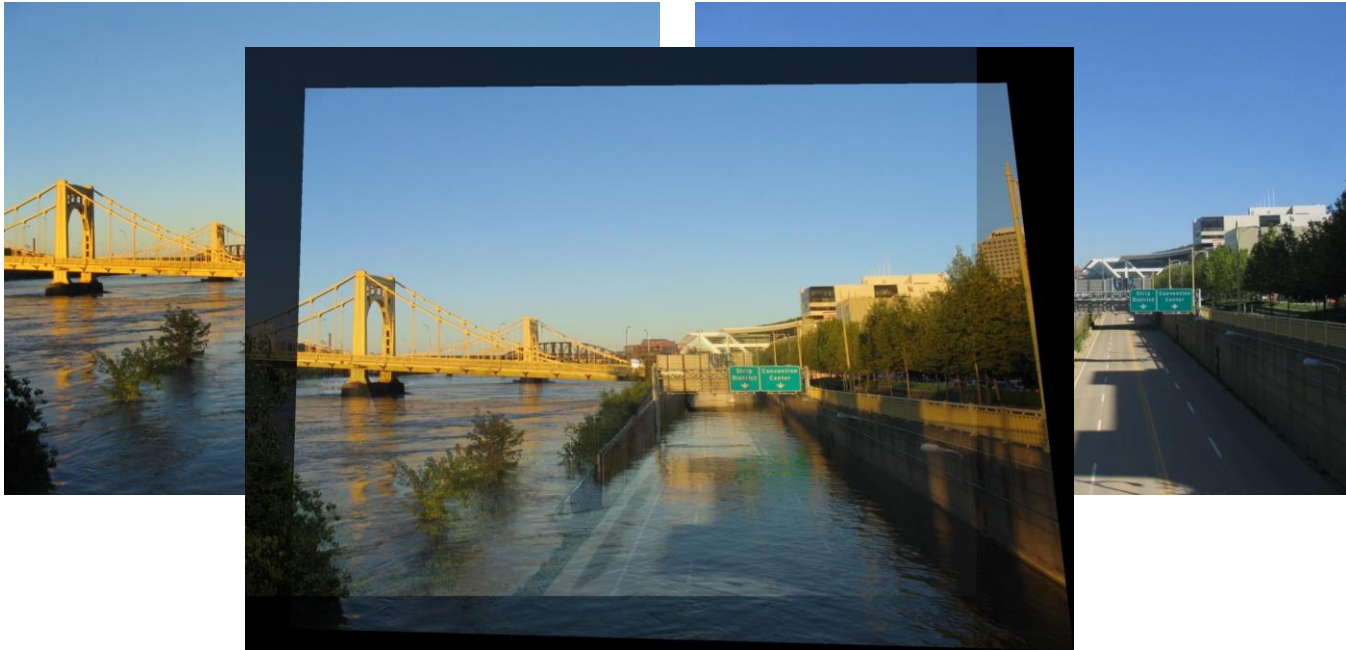
## Idea #2: Align, then cross-dissolve



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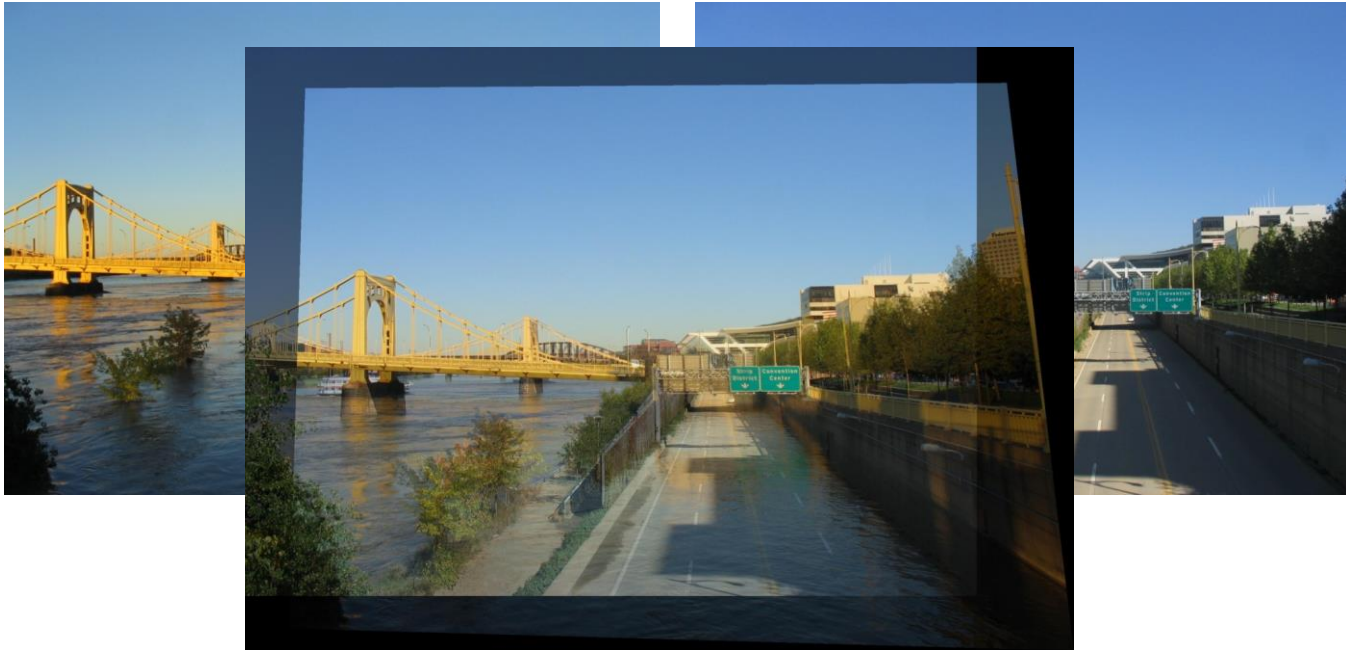


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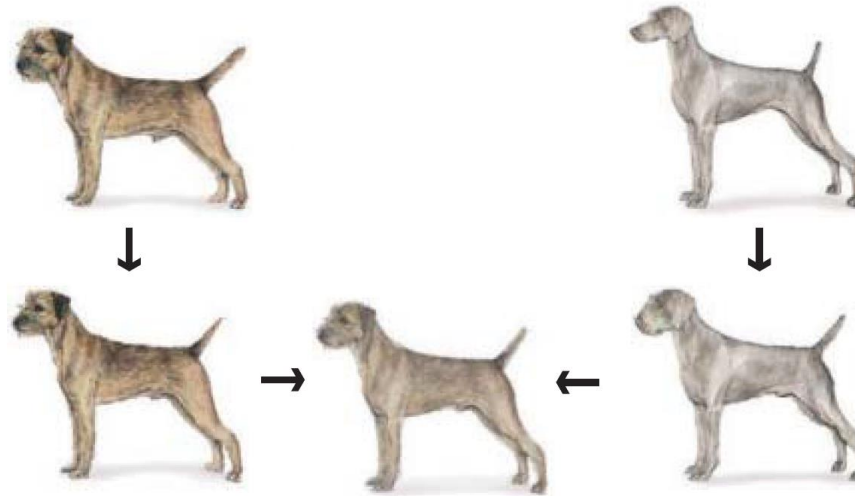
- Align first, then cross-dissolve
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# Dog Averaging



- What to do?
  - Cross-dissolve doesn't work
  - Global alignment doesn't work
    - Cannot be done with a global transformation (e.g. affine)
  - Any ideas?
- Feature matching!
  - Nose to nose, tail to tail, etc.
  - This is a local (non-parametric) warp

# Idea #3: Local warp, then cross-dissolve



- **Morphing procedure**
- *For every frame  $t$ ,*
  1. Find the average shape (the “mean dog” 😊)
    - local warping
  2. Find the average color
    - Cross-dissolve the warped images

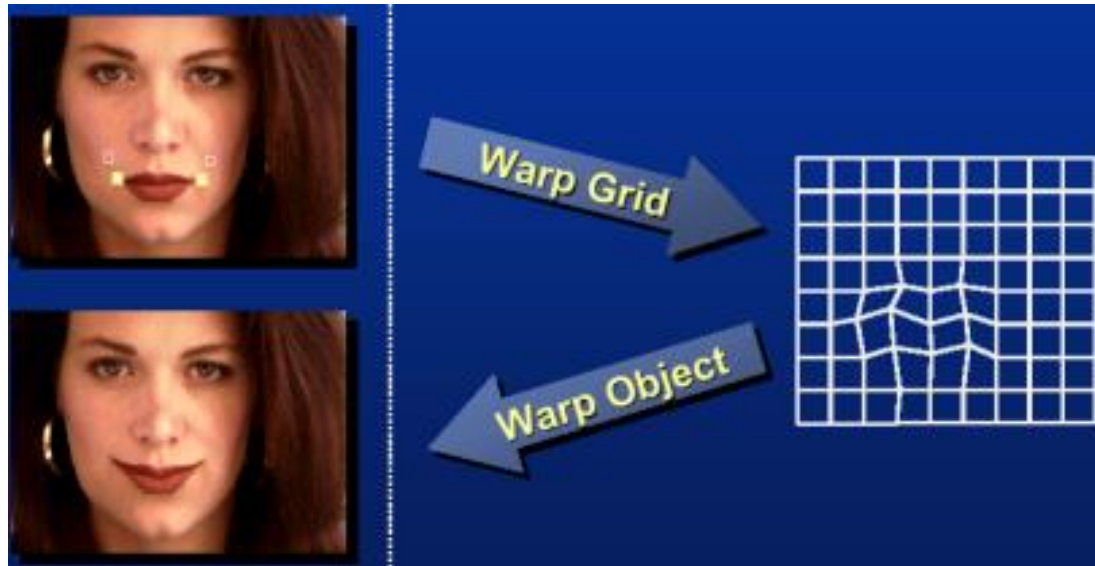
# Local (non-parametric) Image Warping



- Need to specify a more detailed warp function
  - Global warps were functions of a few (2, 4, 8) parameters
  - Non-parametric warps  $u(x, y)$  and  $v(x, y)$  can be defined independently for every single location  $(x, y)$ !
  - Once we know vector field  $(u, v)$  we can easily warp each pixel (use backward warping with interpolation)

# Image Warping – non-parametric

- Move control points to specify a spline warp
- Spline produces a smooth vector field

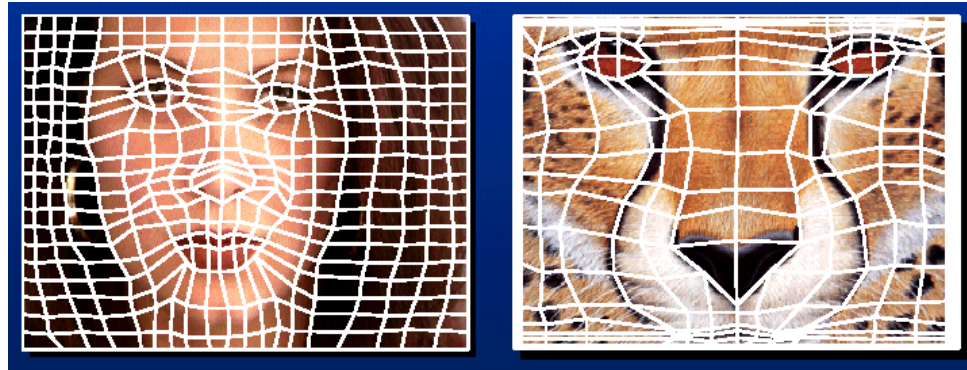


# Warp specification - dense

- How can we specify the warp?

Specify corresponding *spline control points*

- *interpolate* to a complete warping function



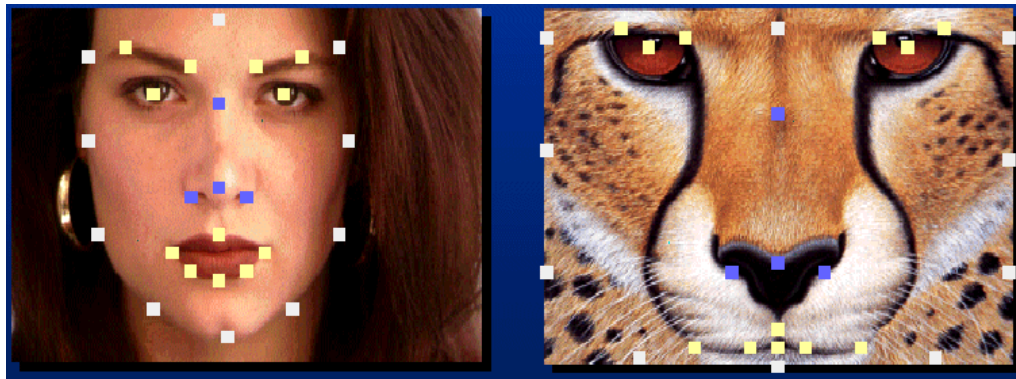
But we want to specify only a few points, not a grid

# Warp specification - sparse

- How can we specify the warp?

Specify corresponding *points*

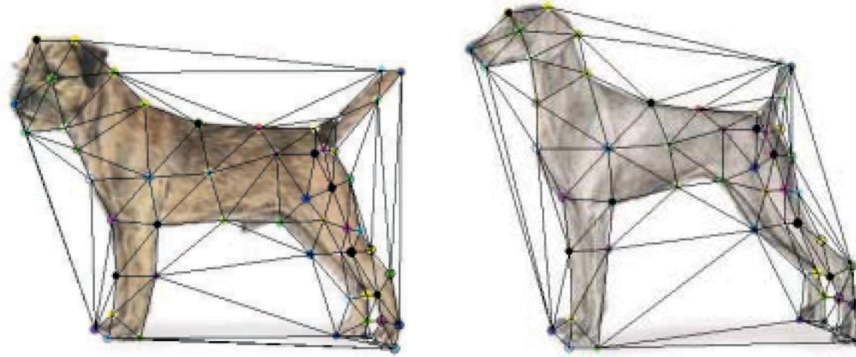
- *interpolate* to a complete warping function
- How do we do it?



How do we go from feature points to pixels?



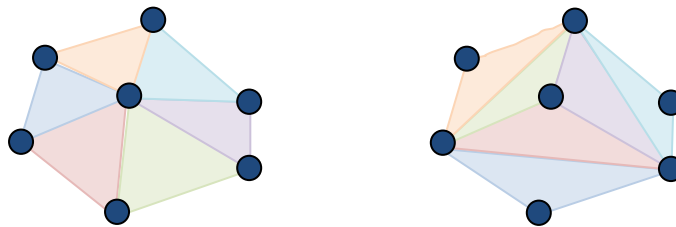
# Triangular Mesh



1. Input correspondences at key feature points
2. Define a triangular mesh over the points
  - Same mesh (triangulation) in both images!
  - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
  - Affine warp with three corresponding points (just like take-home question)

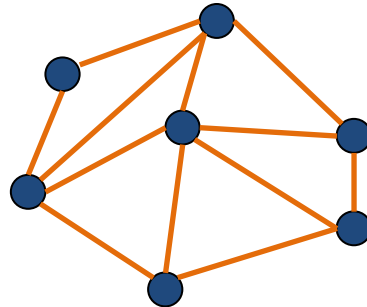
# Triangulations

- A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.
- There are an exponential number of triangulations of a point set.



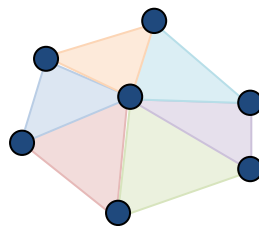
# An $O(n^3)$ Triangulation Algorithm

- Repeat until impossible:
  - Select two sites.
  - If the edge connecting them does not intersect previous edges, keep it.

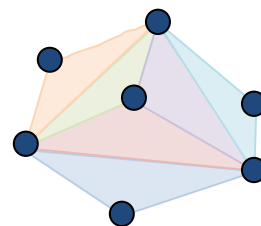


# “Quality” Triangulations

- Let  $\alpha(T_i) = (\alpha_{i1}, \alpha_{i2}, \dots, \alpha_{i3})$  be the vector of angles in the triangulation  $T$  in increasing order:
  - A triangulation  $T_1$  is “better” than  $T_2$  if the smallest angle of  $T_1$  is larger than the smallest angle of  $T_2$
  - Delaunay triangulation is the “best” (maximizes the smallest angles)



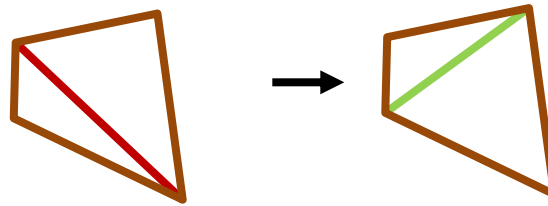
good



bad

# Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.



If an edge flip improves the triangulation, the first edge is called “*illegal*”.

# Illegal Edges

An edge  $pq$  is “illegal” *iff* one of its opposite vertices is inside the circle defined by the other three vertices (see Thale’s theorem)

- A triangle is Delaunay *iff* no other points are inside the circle through the triangle’s vertices
- The Delaunay triangulation is not unique if more than three nearby points are co-circular
- The Delaunay triangulation does not exist if three nearby points are colinear



# Naïve Delaunay Algorithm

Start with an arbitrary triangulation.

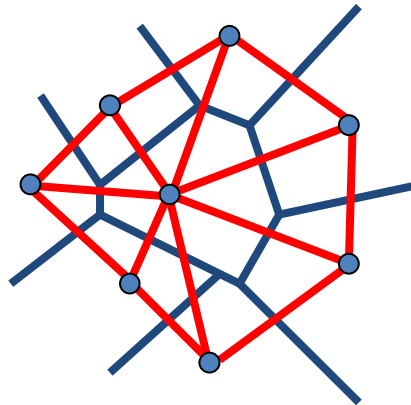
Flip any illegal edge until no more exist.

Could take a long time to terminate.

# Delaunay Triangulation by Duality

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

The DT may be constructed in  $O(n \log n)$  time.



Demos:

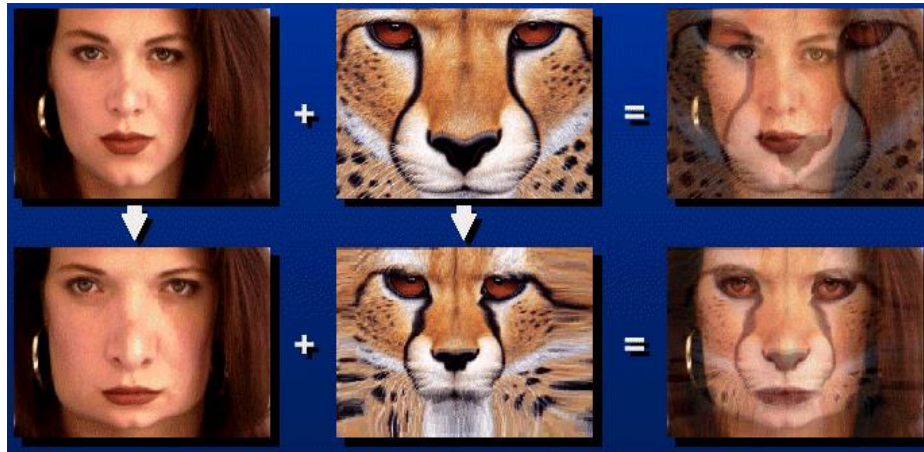
<http://www.cs.cornell.edu/home/chew/Delaunay.html>

<http://alexbeutel.com/webgl/voronoi.html>



# Image Morphing

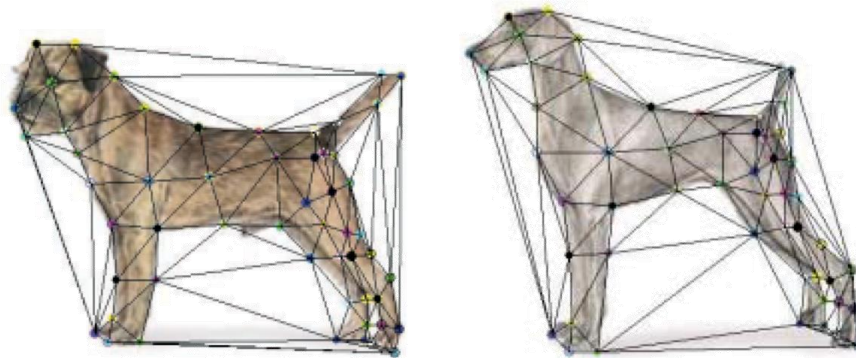
- How do we create a morphing sequence?
  1. Create an intermediate shape (by interpolation)
  2. Warp both images towards it
  3. Cross-dissolve the colors in the newly warped images



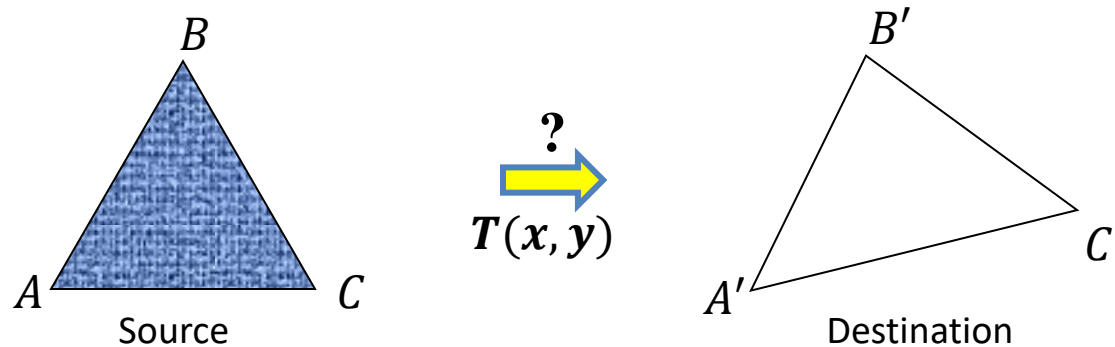
# Warp interpolation

How do we create an intermediate shape at time  $t$ ?

- Assume  $t \in [0,1]$
- Simple linear interpolation of each feature pair
  - $t * p_0 + (1 - t) * p_1$  for corresponding features  $p_0$  and  $p_1$

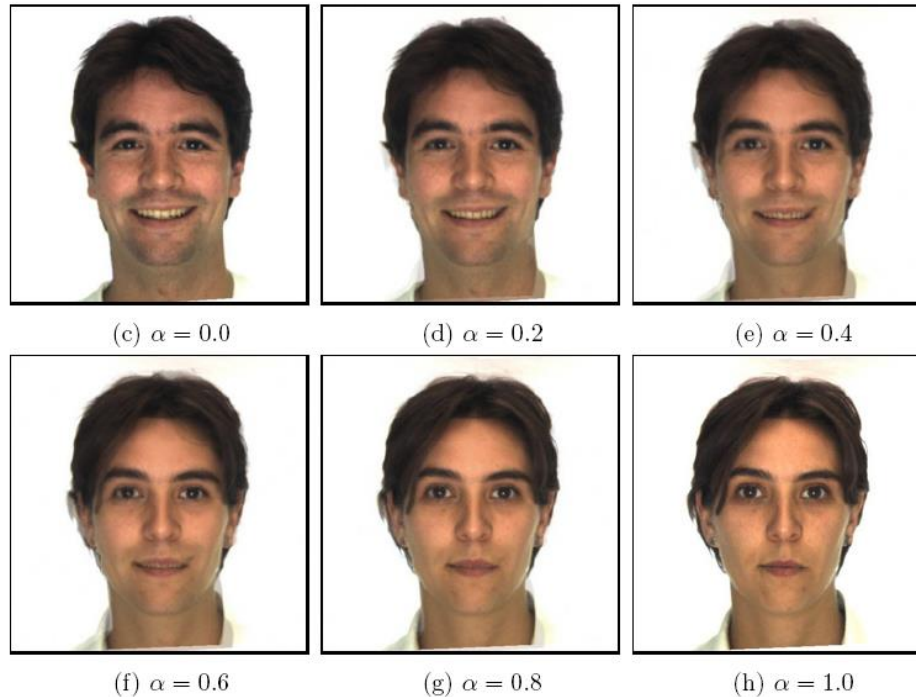


Suppose we have two triangles:  $ABC$  and  $A'B'C'$ .  
What transformation will map  $A$  to  $A'$ ,  $B$  to  $B'$ , and  $C$  to  $C'$ ? How can we get the parameters?



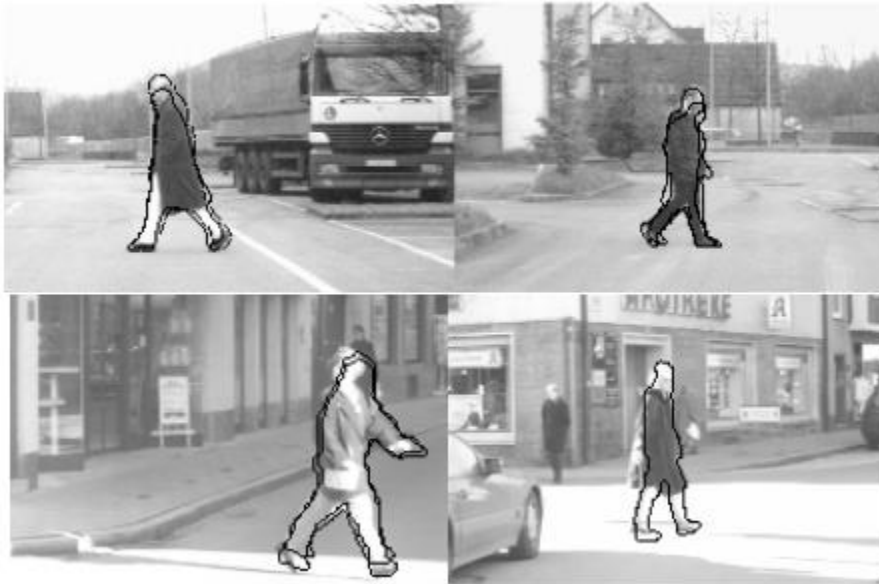
# Morphing & Matting

- Extract foreground first to avoid artifacts in the background



# Summary of Morphing

1. Define corresponding points
2. Define triangulation on points
  - Use same triangulation for both images
3. For each  $t = 0 : step : 1$ 
  - a. Compute the average shape (weighted average of points)
  - b. For each triangle in the average shape
    - Get the affine projection to the corresponding triangles in each image
    - For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (optionally use interpolation)
  - c. Save the image as the next frame of the sequence



shape matching

# Shape Matching

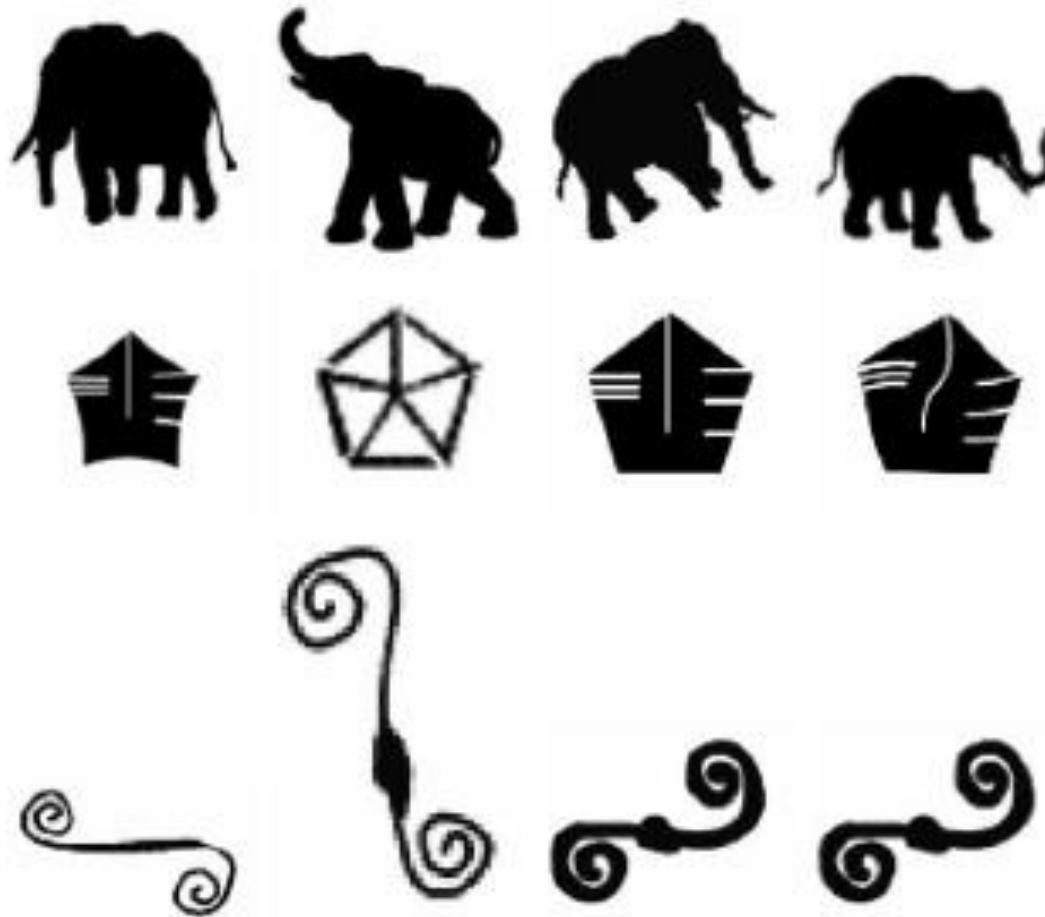


Fig. 11. Examples of shapes in the MPEG7 database for three different categories.

# Questions

- What features?
- How to compare shapes?

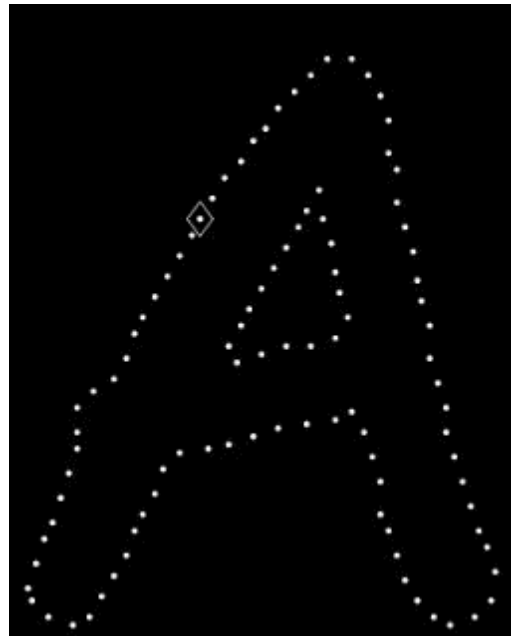


# Challenging!

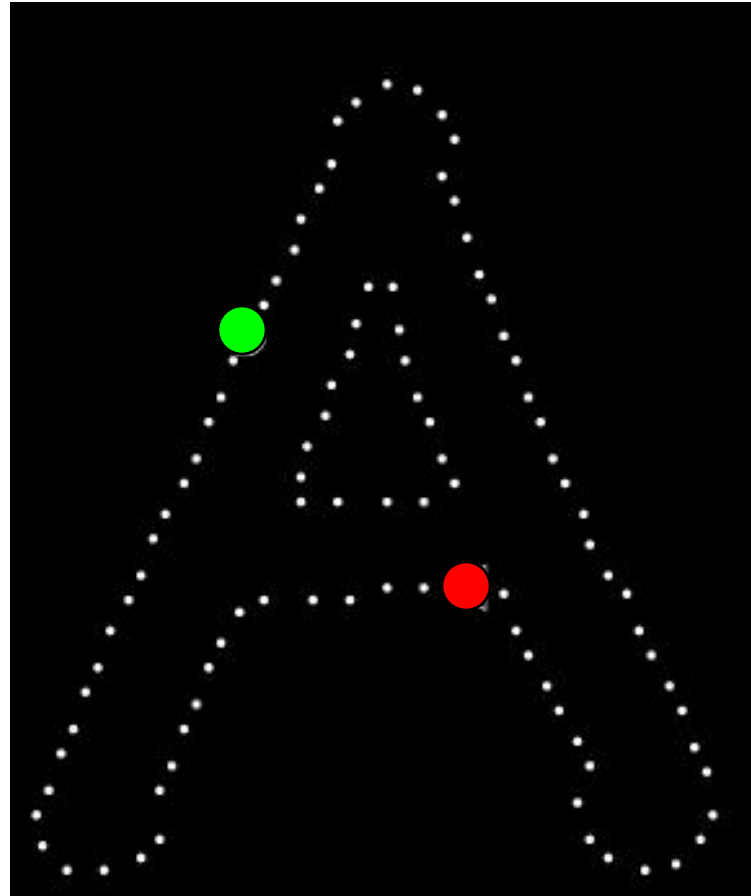
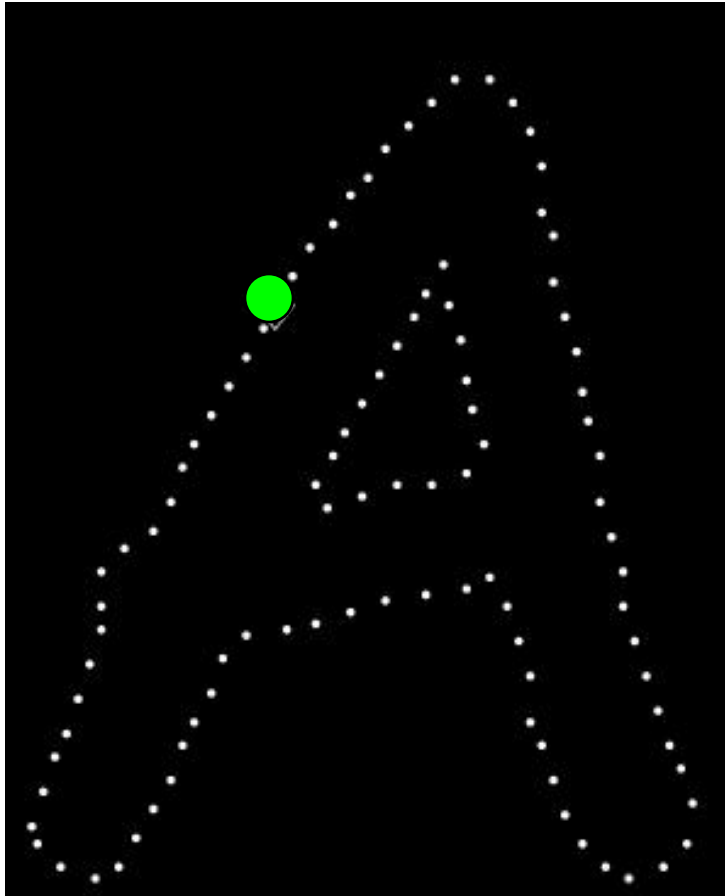


Fig. 1. Examples of two handwritten digits. In terms of pixel-to-pixel comparisons, these two images are quite different, but to the human observer, the shapes appear to be similar.

- What limitations might we have using only edge points to represent a shape?
- How descriptive is a point?

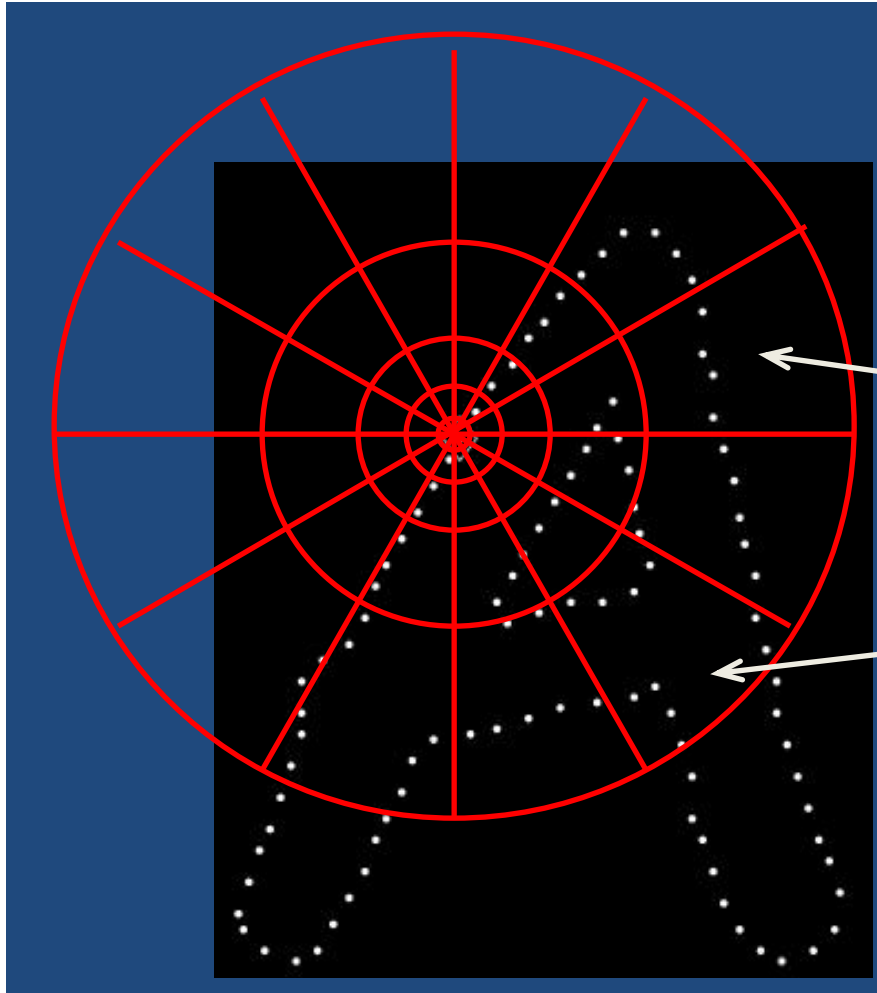


# Comparing Shapes



What points on these two sampled contours are most similar? How do you know?

# Shape Context Descriptor



Count the number of points inside each bin, e.g.:

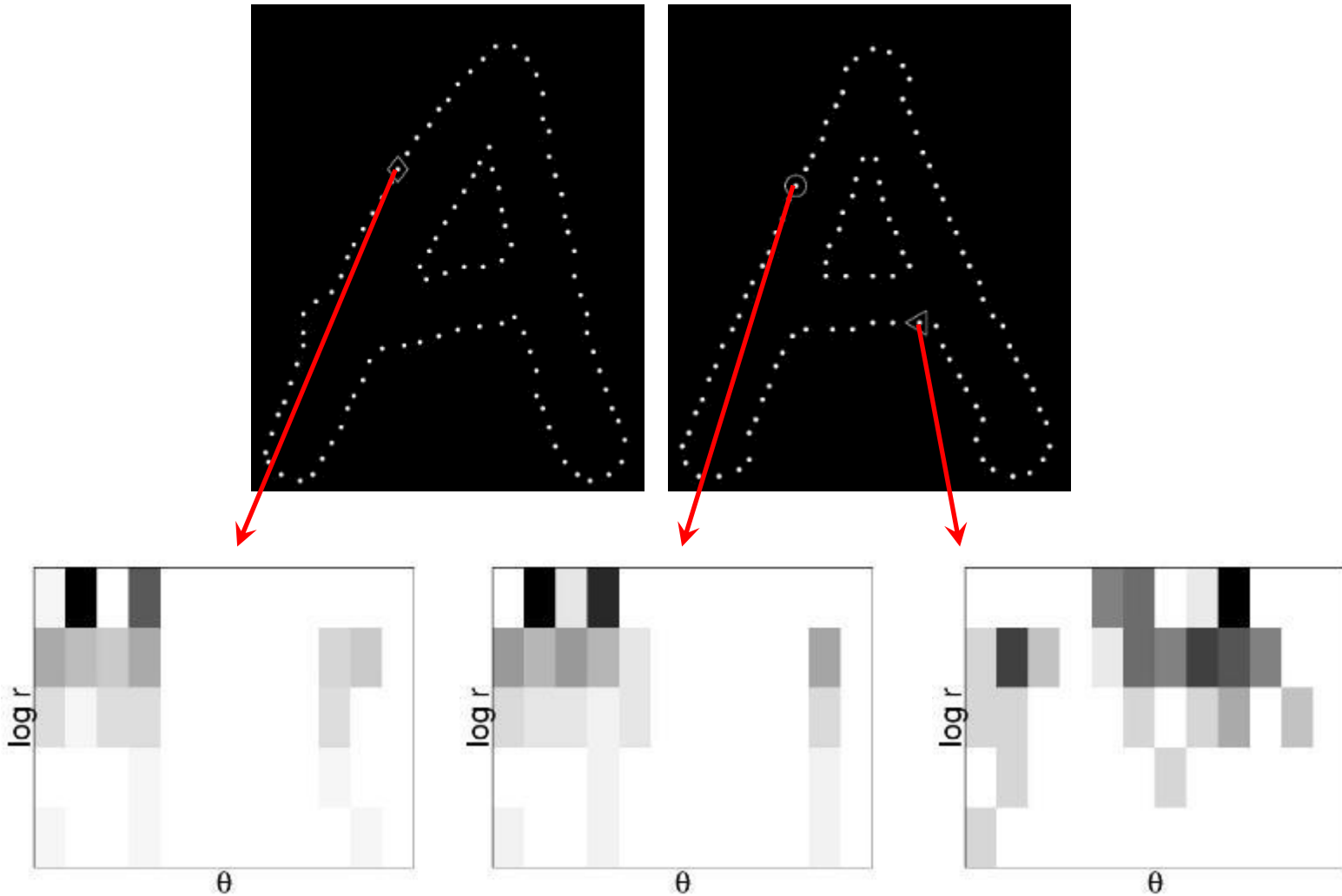
Count = 4

⋮

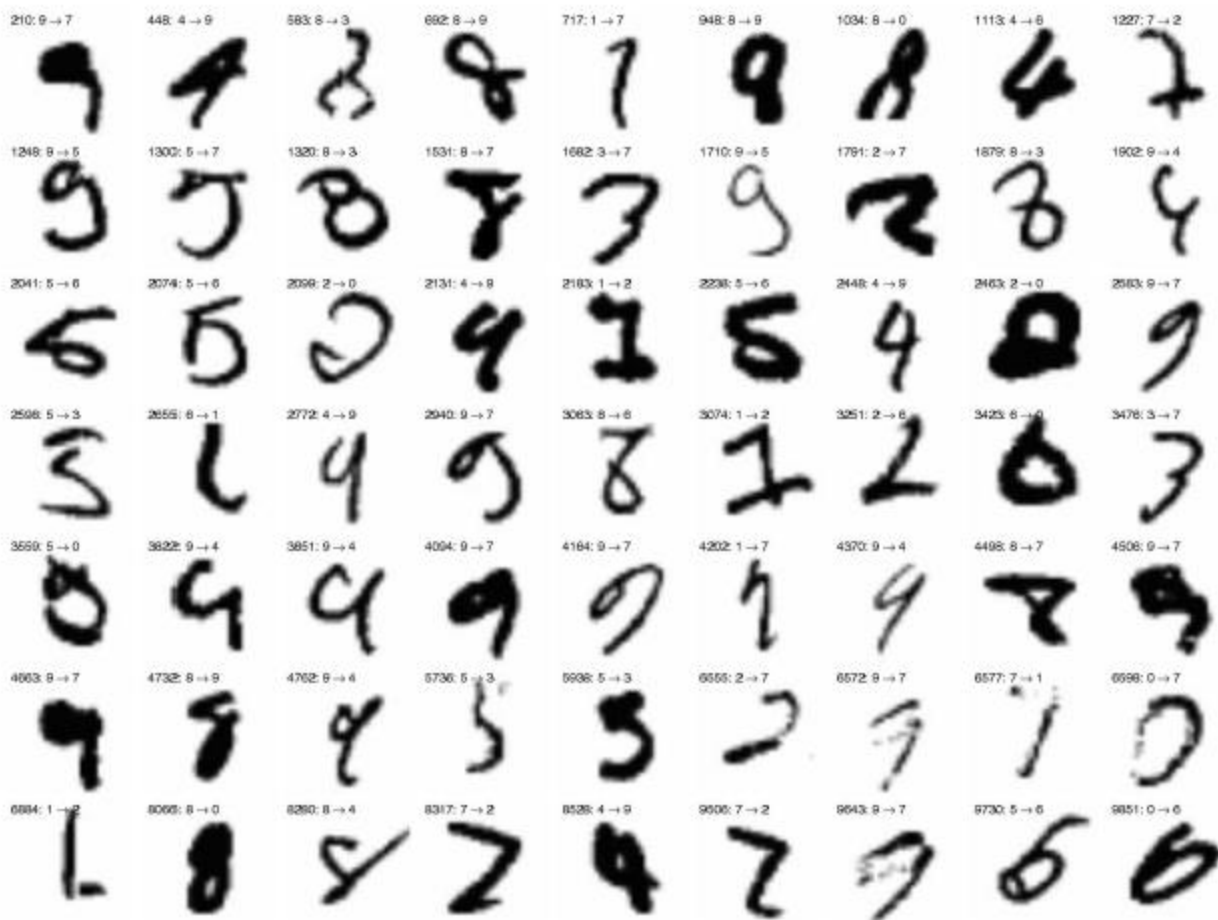
Count = 10

Compact representation of distribution of points relative to each point

# Shape Context Descriptor



# Shape context matching with handwritten digits



Only errors made out of 10,000 test examples

# Shape matching application: CAPTCHA's

## CAPTCHA:

*“Completely Automated Public Turing Test To Tell Computers and Humans Apart”*

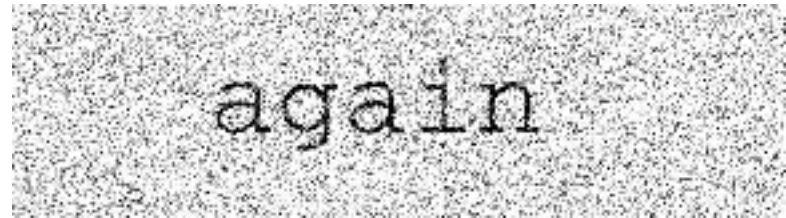
[www.captcha.net](http://www.captcha.net)

Luis von Ahn, Manuel Blum, Nicholas Hopper, and John Langford

CMU 2000

# Shape matching application: breaking a visual CAPTCHA

- Use shape matching to recognize characters, words in spite of clutter, warping, etc.

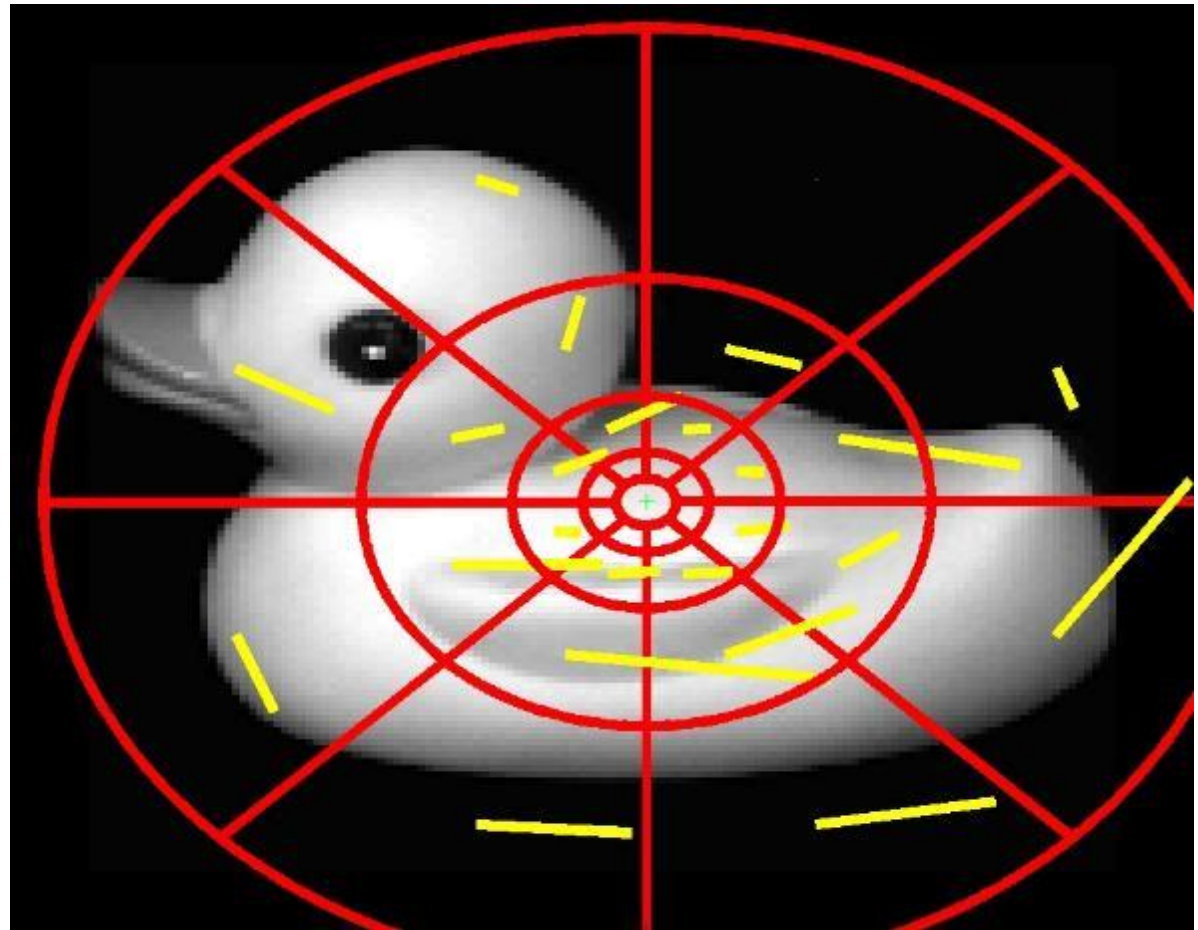


G. Mori and J. Malik, "Recognizing Objects in Adversarial Clutter: Breaking a Visual CAPTCHA", CVPR 2003



# Features: Generalized Shape Contexts

- Can put more than just point counts in bins
  - Oriented Energy
  - Colour info
  - Optical flow



# Fast Pruning: Representative Shape Contexts



- Pick  $k$  points in the image at random
  - Compare to all shape contexts for all known letters
  - Vote for closely matching letters
- Keep all letters with scores under threshold

# Algorithm A: bottom-up

- Look for letters
  - Representative Shape Contexts
- Find pairs of letters that are “consistent”
  - Letters nearby in space
- Search for valid words
- Give scores to the words

profit

Input

p o u r f o q f l i t

Locations of possible letters

p o u r f o q f l i t

Possible strings of letters

ROLL:11.94  
PROFIT:9.42

Matching words

# EZ-Gimpy Results with Algorithm A

- 158 of 191 images correctly identified: 83%
  - Running time: ~10 sec. per image (MATLAB, 1 Ghz P3)



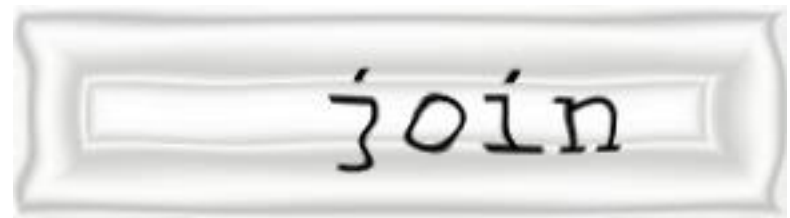
horse



spade



smile



join



canvas



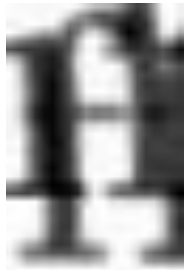
here

# Gimpy



- Multiple words, task is to find 3 words in the image
- Clutter is other objects, not texture

# Algorithm B: Letters are not enough



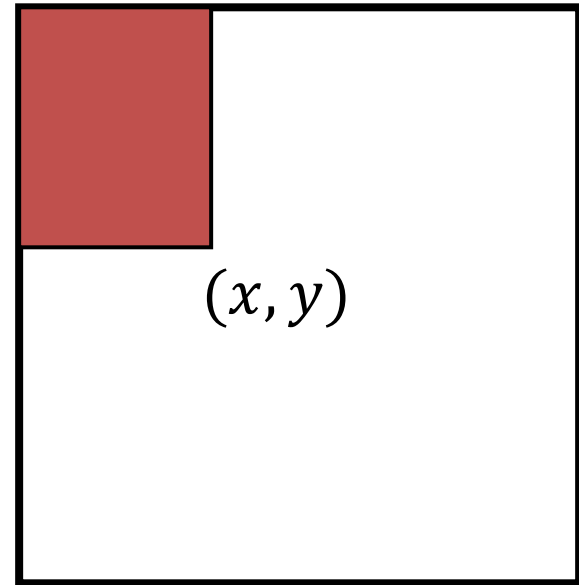
- Hard to distinguish single letters with so much clutter
- Find words instead of letters
  - Use long range info over entire word
  - Stretch shape contexts into ellipses
- Search problem becomes huge
  - # of words 600 vs. # of letters 26
  - Prune set of words using opening/closing bigrams



# *Integral Images*

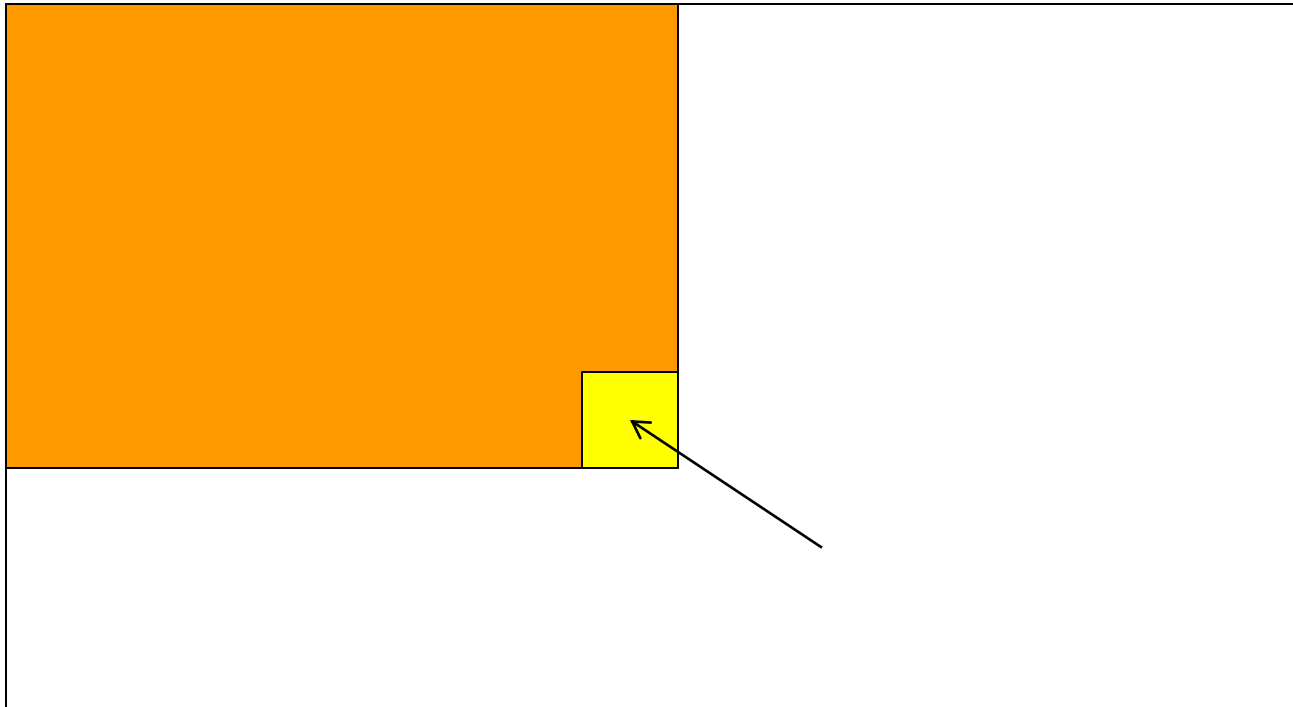
# *Fast computation of convolution with box filters with integral images*

- The *integral image* computes a value at each pixel  $(x, y)$  that is the sum of the pixel values above and to the left of  $(x, y)$ , inclusive.
- This can quickly be computed in one pass through the image.

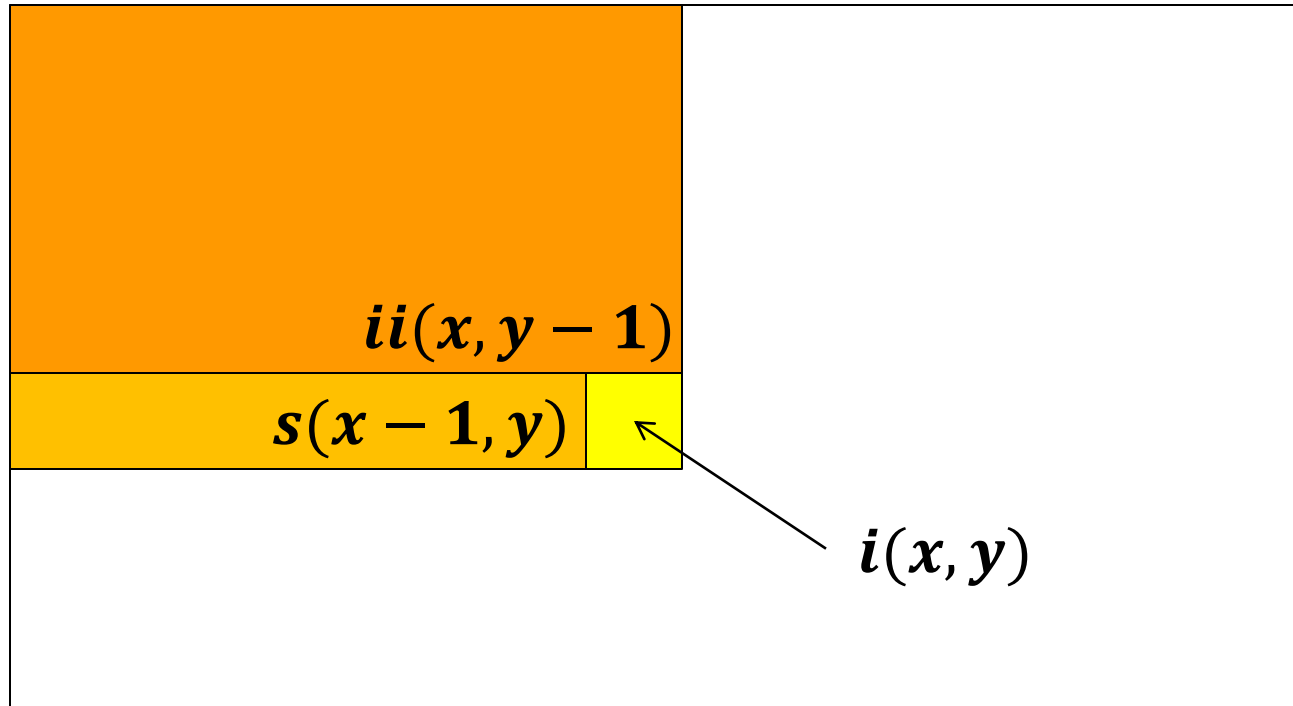




# *Computing the integral image*



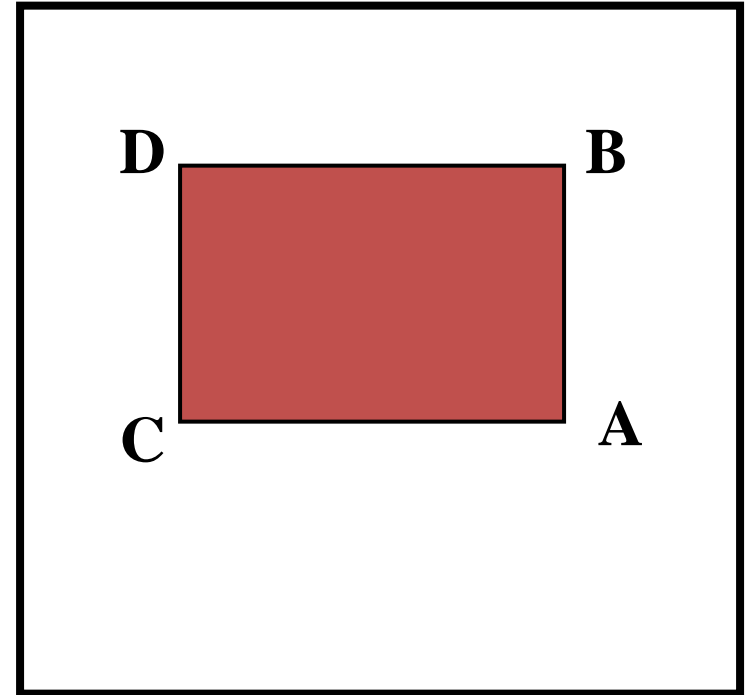
# Computing the integral image



- Cumulative row sum:  $s(x, y) = s(x - 1, y) + i(x, y)$
- Integral image:  $ii(x, y) = ii(x, y - 1) + s(x, y)$
- In MATLAB:  $ii = \text{cumsum}(\text{cumsum}(\text{double}(i)), 2)$
- In Python:  $ii = \text{cumsum}(\text{cumsum}(\text{double}(i), 0), 1)$

## *Computing sum within a rectangle*

- Let A, B, C, and D be the values of the integral image at the corners of a rectangle.
- Then the sum of original image values within the rectangle can be computed as:  
$$\text{sum} = A - B - C + D$$
- Only 3 additions are required for any size of rectangle!



# *Example*

