اصول پردازش تصویر

Principles of Image Processing

مصطفی کمالی تبریزی

۱۴ مهر ۱۳۹۹

جلسه ششم

Image Sampling

F() = 

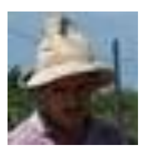
F() = 

Image Scaling

This image is too big to fit on the screen. How can we reduce it?

How to generate a half-sized version?

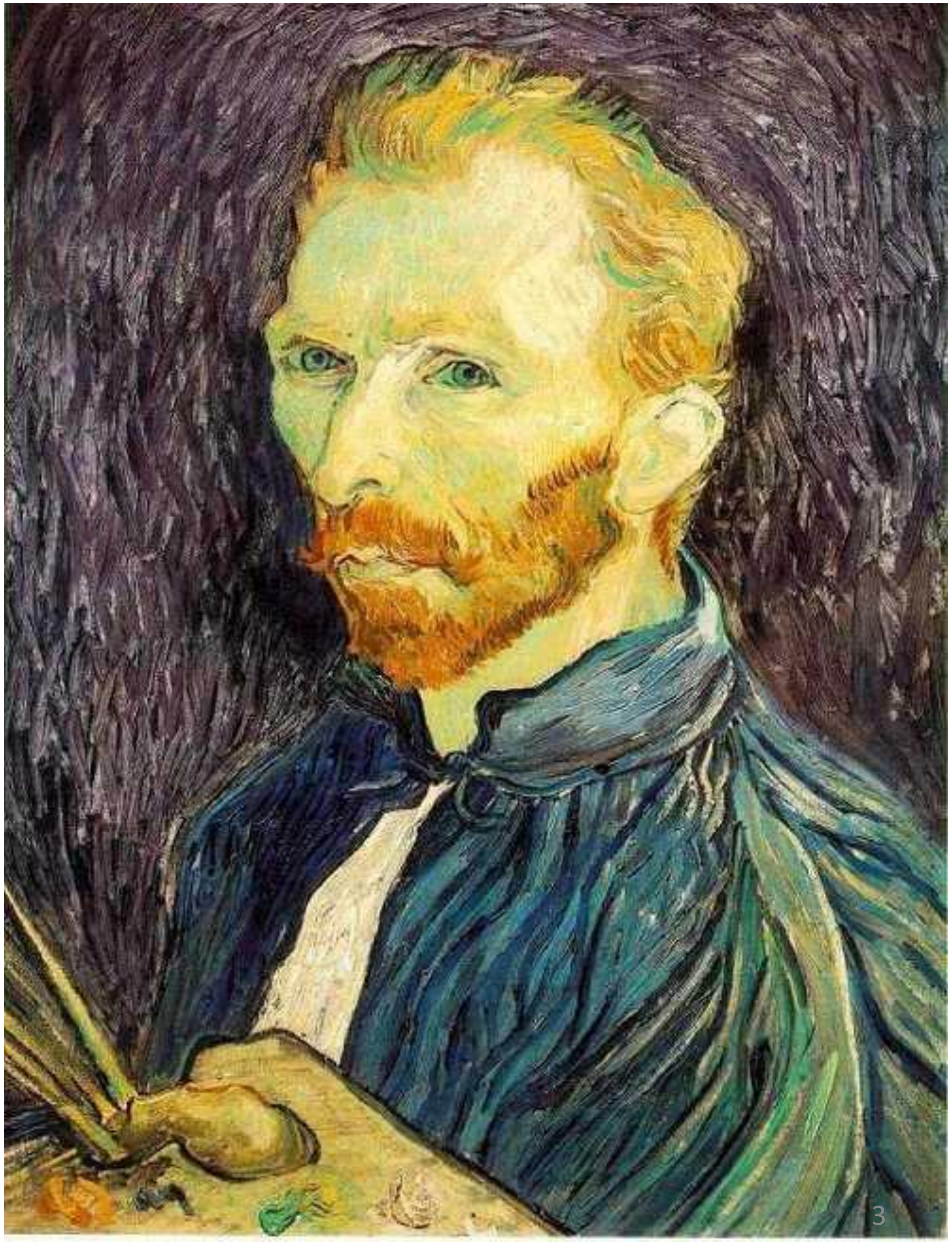
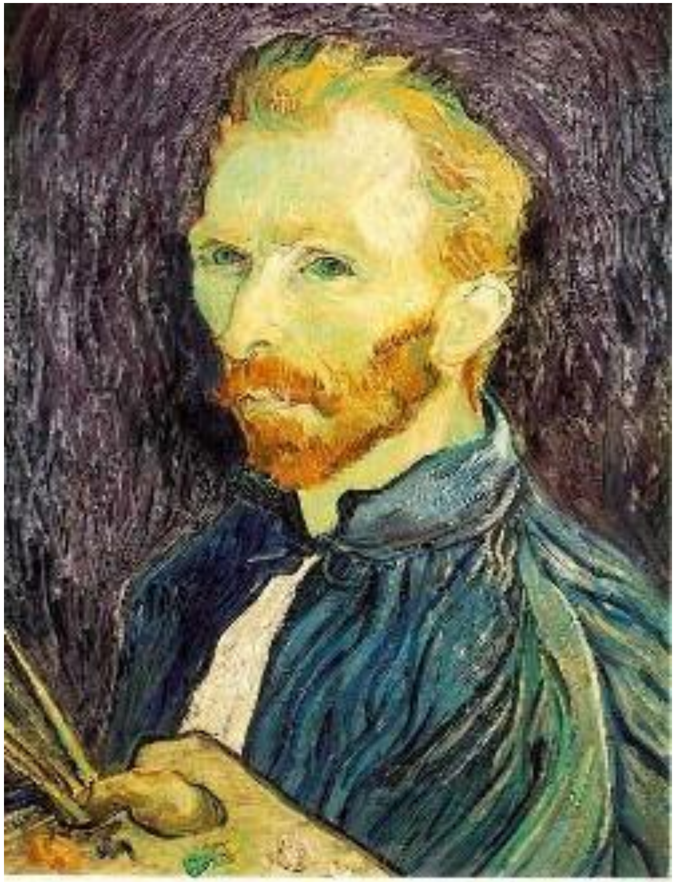


Image sub-sampling

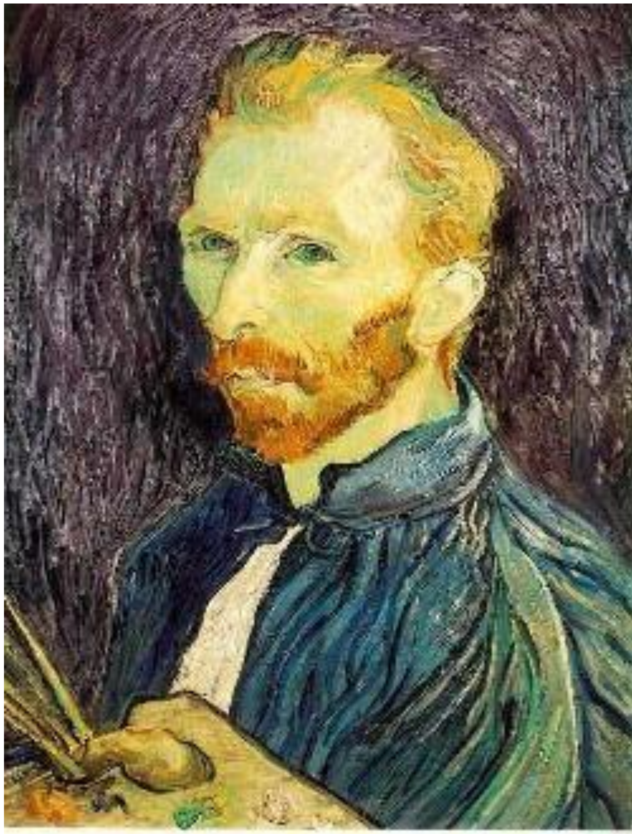


Throw away every other row and column to create a 1/2 size image (called *image sub-sampling)* with 1/4 number of pixels

1/4 size of each side
1/16 number of pixels

1/8 size of each side
1/64 number of pixels

Image sub-sampling



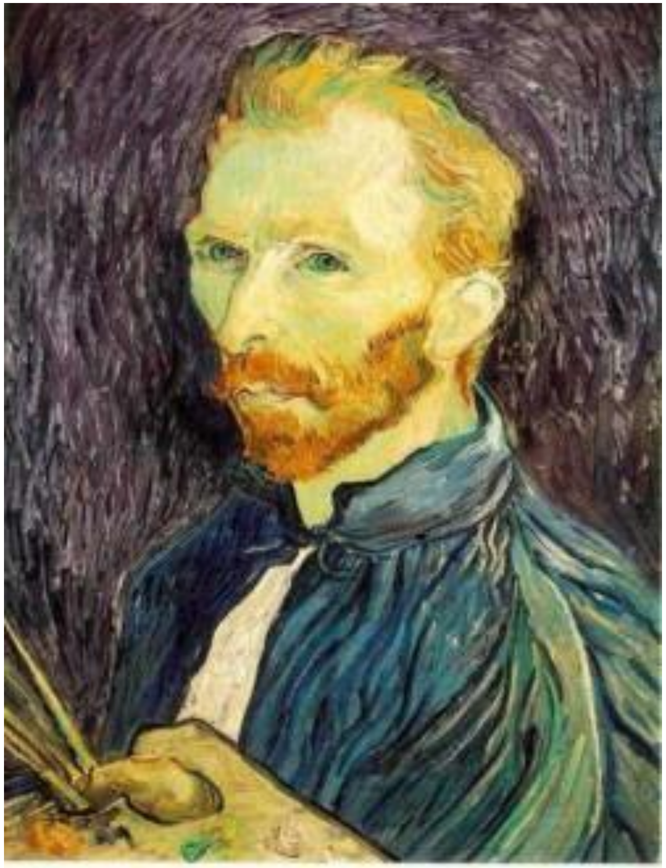
1/2

1/4 (2x zoom)

1/8 (4x zoom)

Why does this look so crufty?

Subsampling with Gaussian pre-filtering



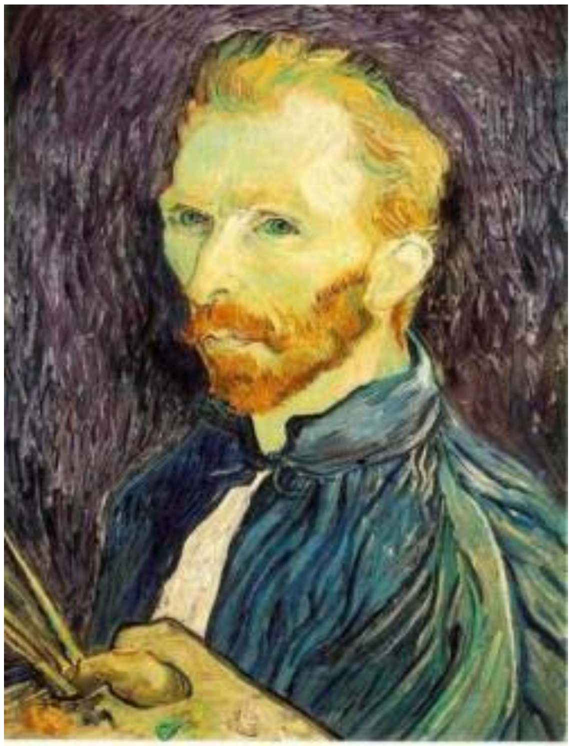
Gaussian 1/2

G 1/4

G 1/8

Solution: filter the image, *then* subsample

Subsampling with Gaussian pre-filtering



Gaussian 1/2

G 1/4 (2x zoom)

G 1/8 (4x zoom)

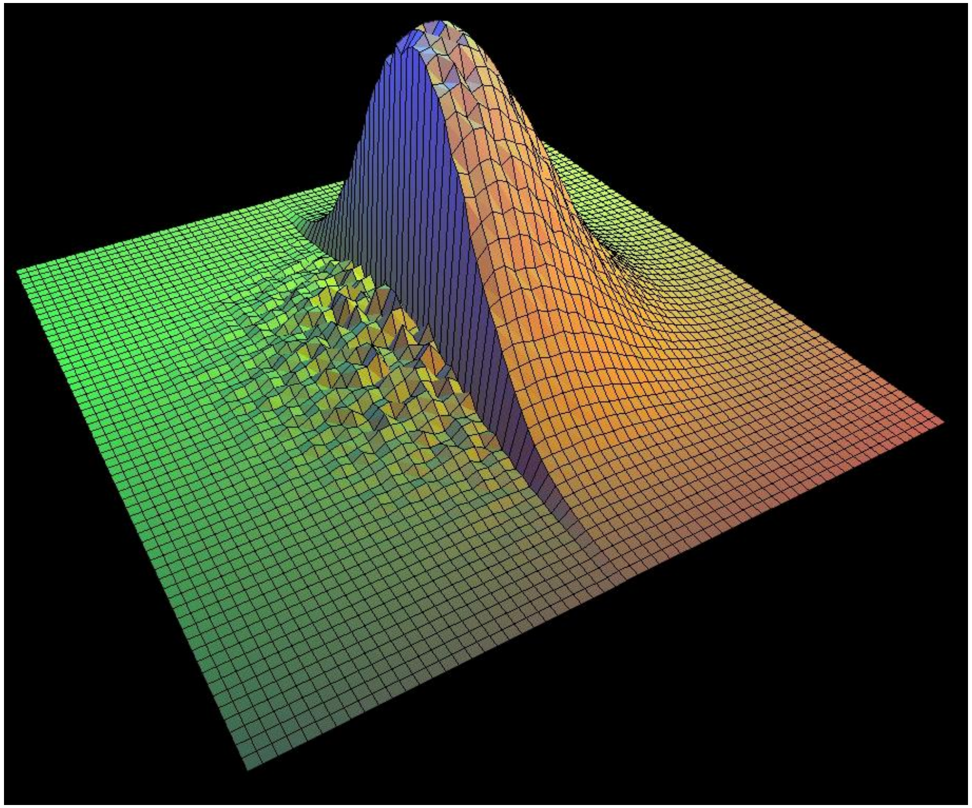
Solution: filter the image, *then* subsample

Comparing

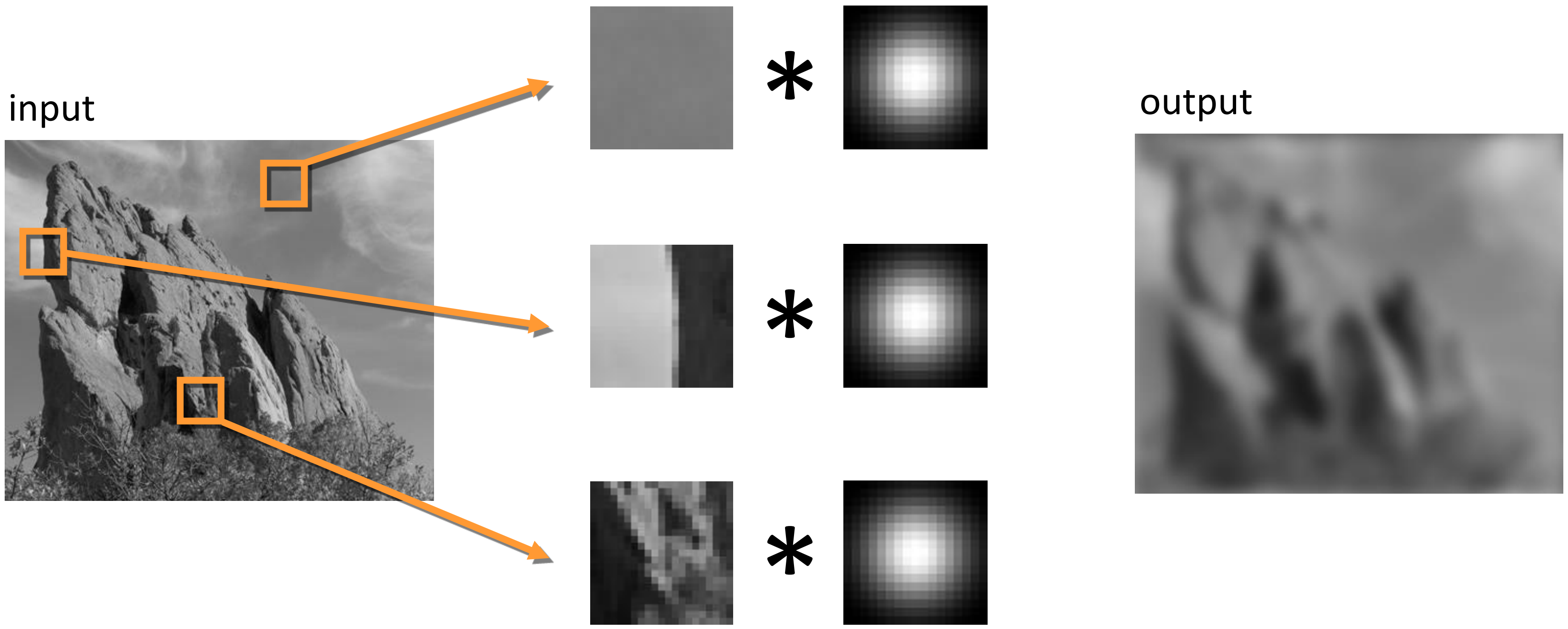
1/8 (4x zoom)

G 1/8 (4x zoom)

Bilateral Filters



Constant Blur

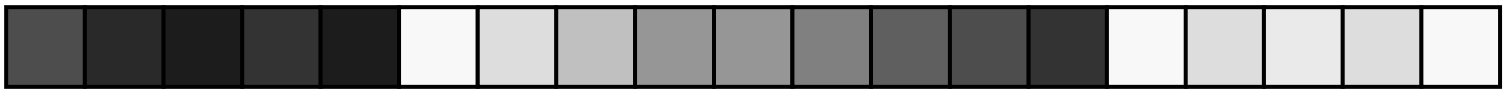


Same Gaussian kernel everywhere.

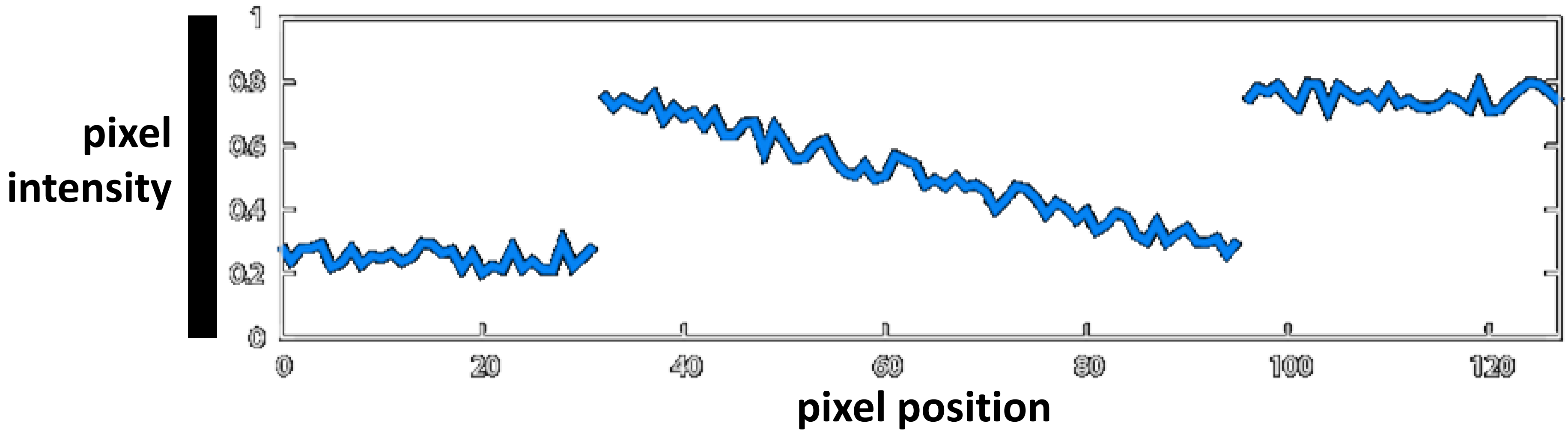
Slides courtesy of Sylvian Paris

Illustration of a 1D Image

- 1D image = line of pixels

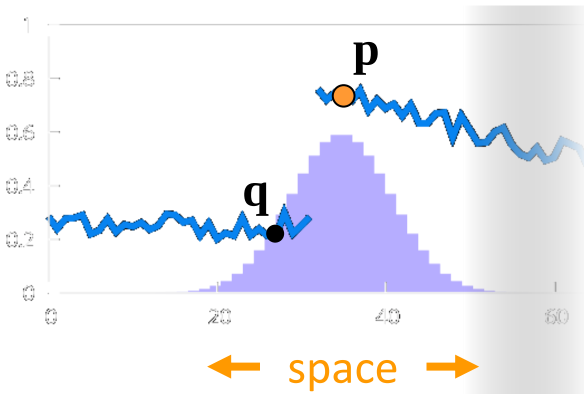


- Better visualized as a plot



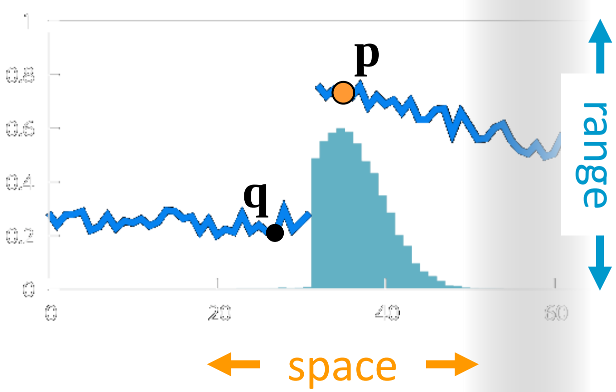
Gaussian Blur and Bilateral Filter

Gaussian blur



Bilateral filter

[Aurich 95, Smith 97, Tomasi 98]

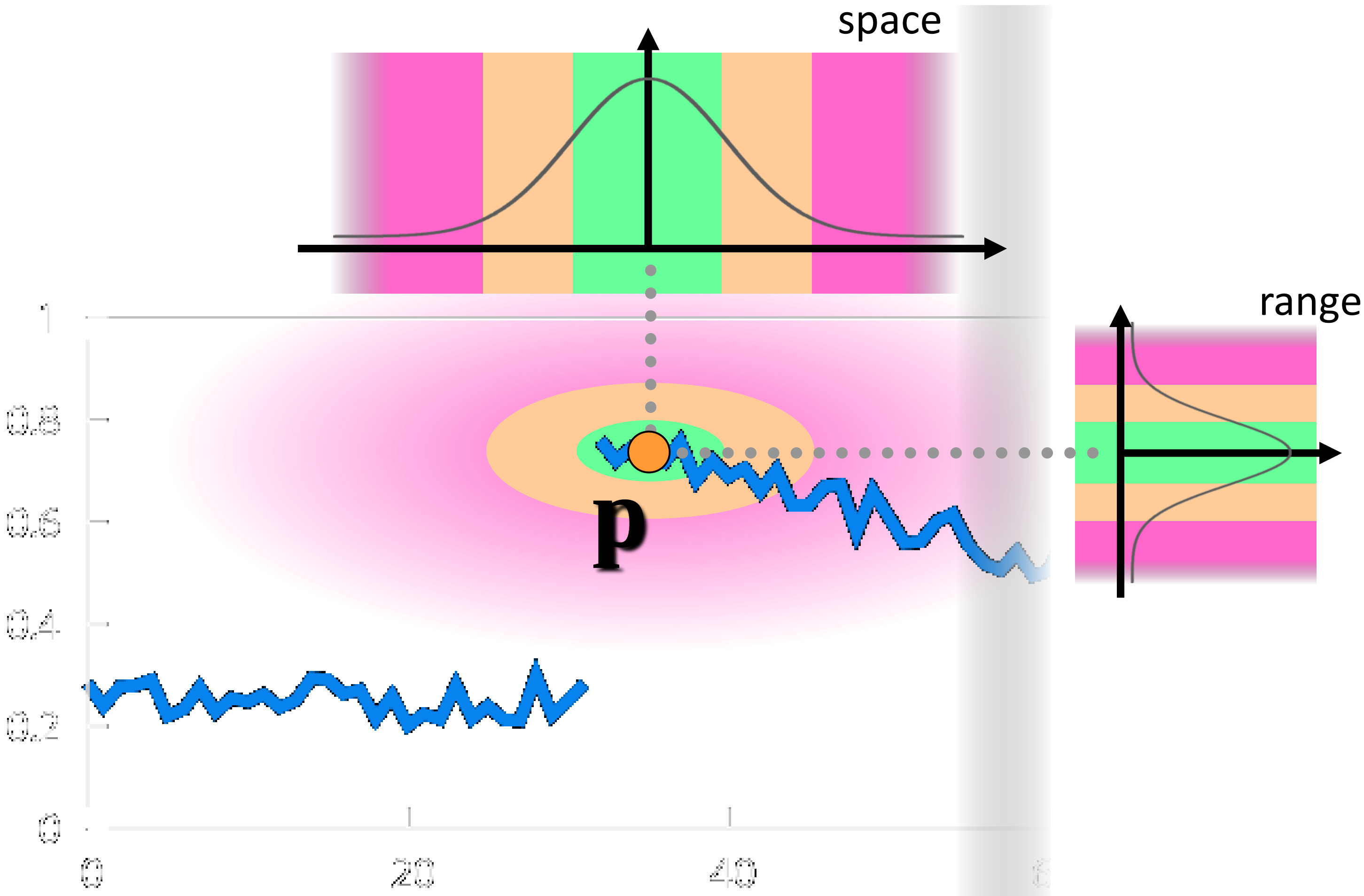


$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} I_{\mathbf{q}}$$

$$BF[I]_{\mathbf{p}} = \underbrace{\frac{1}{W_{\mathbf{p}}}}_{\text{normalization}} \sum_{\mathbf{q}\in S} \underbrace{G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)}_{\text{space}} \underbrace{G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)}_{\text{range}} I_{\mathbf{q}}$$

Influence of Pixels

Only pixels close in space and in range are considered.



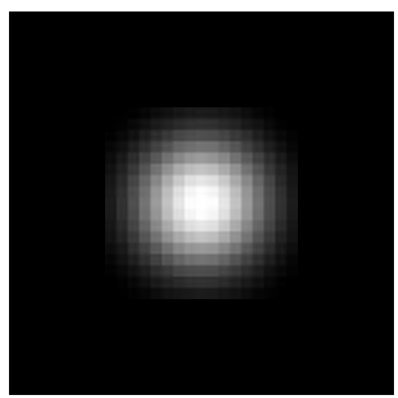
Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels**.

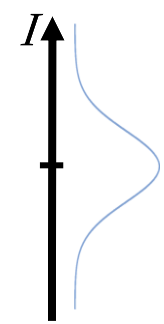
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

normalization factor

space weight

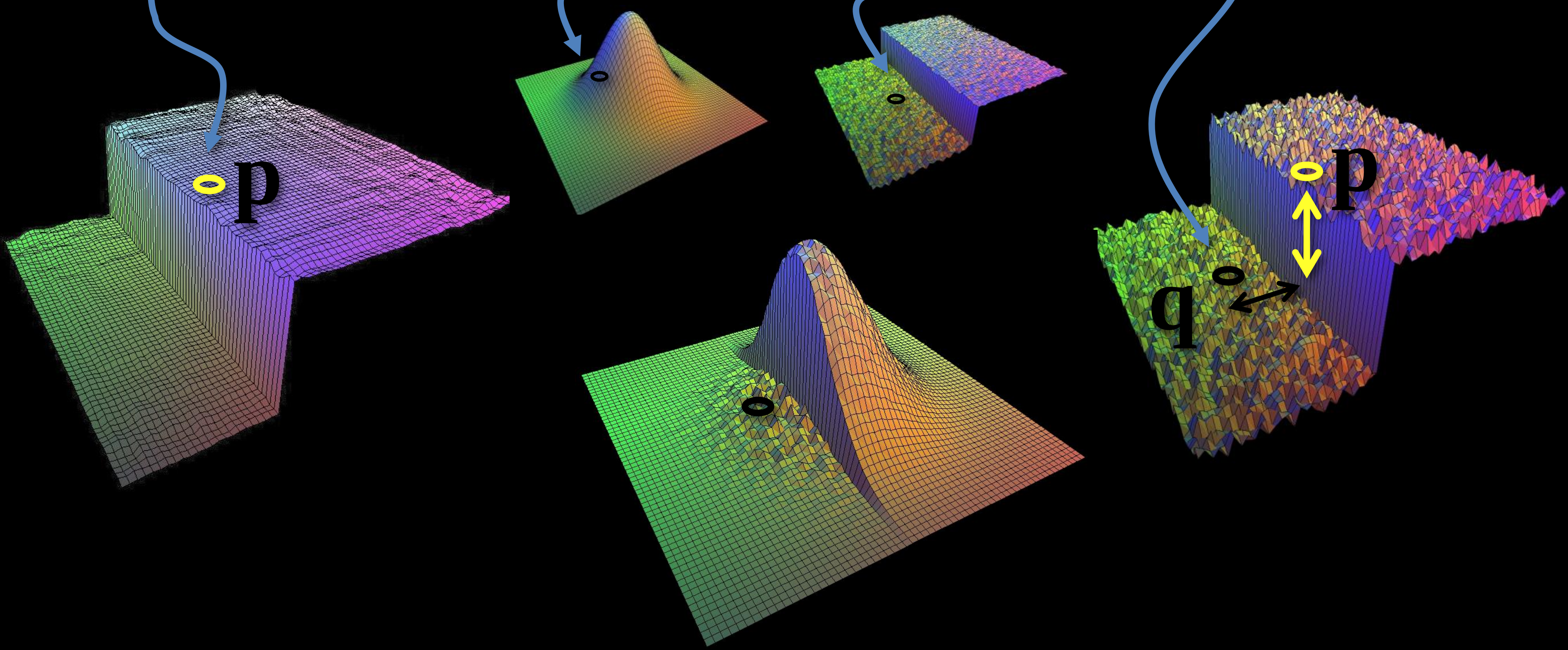


range weight



Bilateral Filter on a Height Field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p} - \mathbf{q}\|)\; G_{\sigma_r}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\; I_{\mathbf{q}}$$



Reproduced from [Durand 02]

Space and Range Parameters

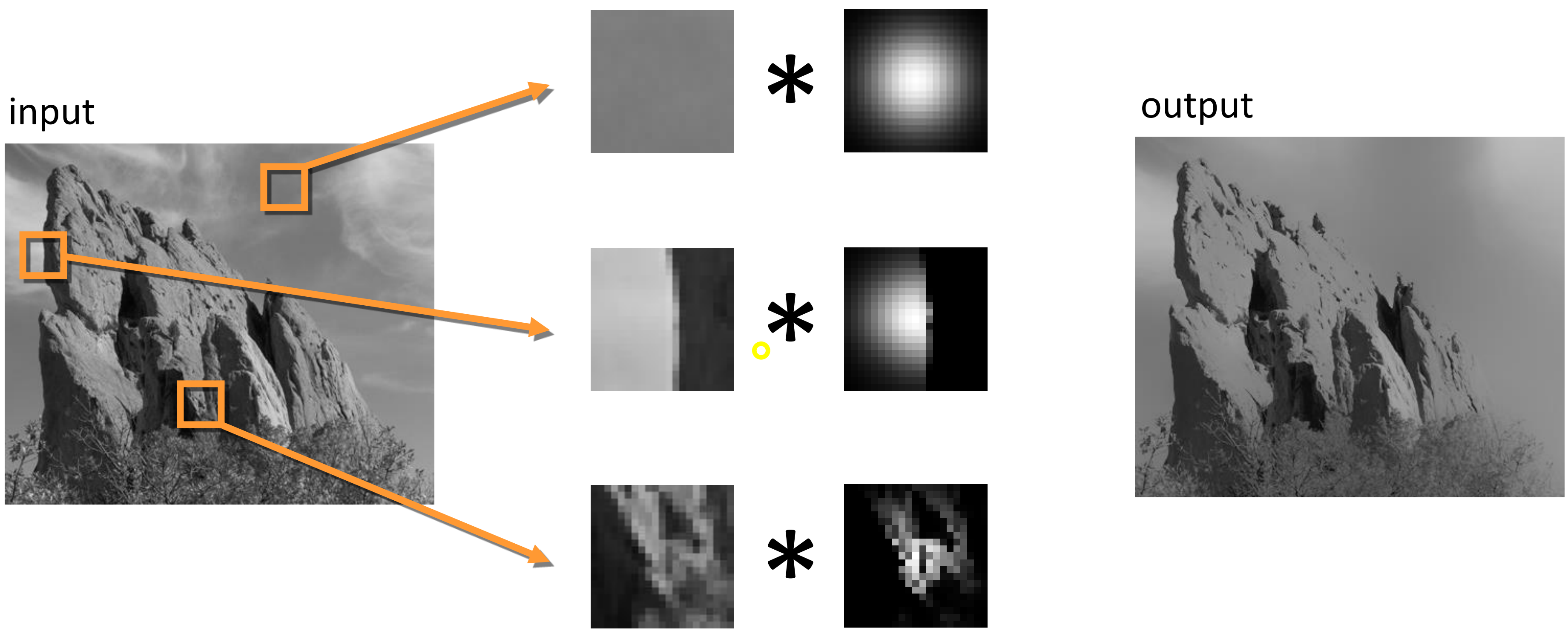
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_{\mathrm{s}}}(\|\mathbf{p}-\mathbf{q}\|)\, G_{\sigma_{\mathrm{r}}}(|I_{\mathbf{p}} - I_{\mathbf{q}}|)\, I_{\mathbf{q}}$$

- space σ_{s} : spatial extent of the kernel, size of the considered neighborhood.

- range σ_{r} : "minimum" amplitude of an edge

Bilateral filter

Maintains edges when blurring!



The kernel shape depends on the image content.

Slides courtesy of Sylvian Paris

Comparing

Original Image

Constant Filter

Bilateral Filter

References

- Bilateral Filters
  Szeliski, section 3.3

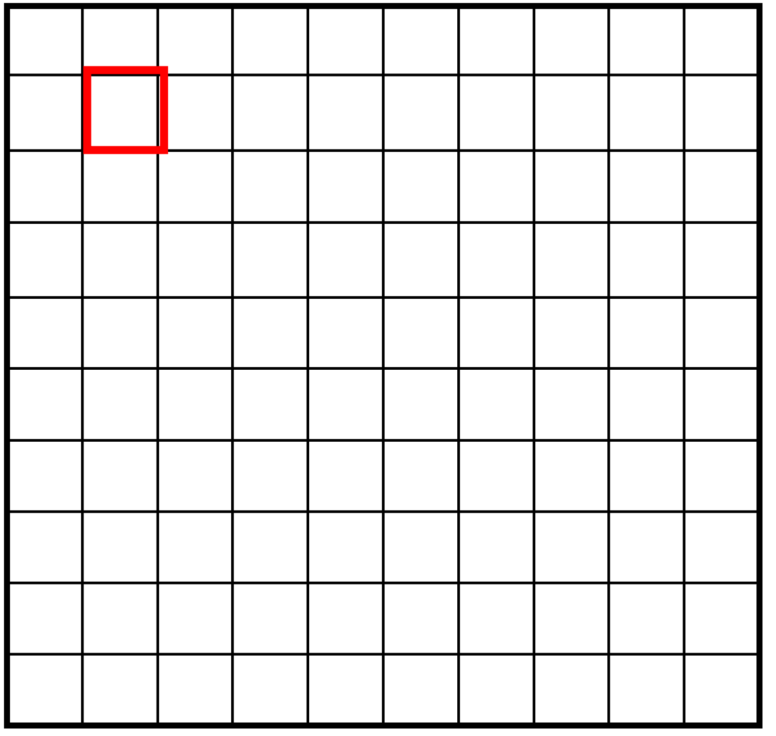
Median Filter

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$



Credit: S. Seitz

Image filtering
Median Filter

$$f[.,.]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$h[.,.]$$

	0	0							

Credit: S. Seitz

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	0	0						

Credit: S. Seitz

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	0	0	0					

Credit: S. Seitz

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	0	0	0	0				

Credit: S. Seitz

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	0	0	0	0				
				?					

Credit: S. Seitz

Image filtering
Median Filter

$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	0	0	0	0				
						?			
				90					

Credit: S. Seitz

Image filtering
Median Filter

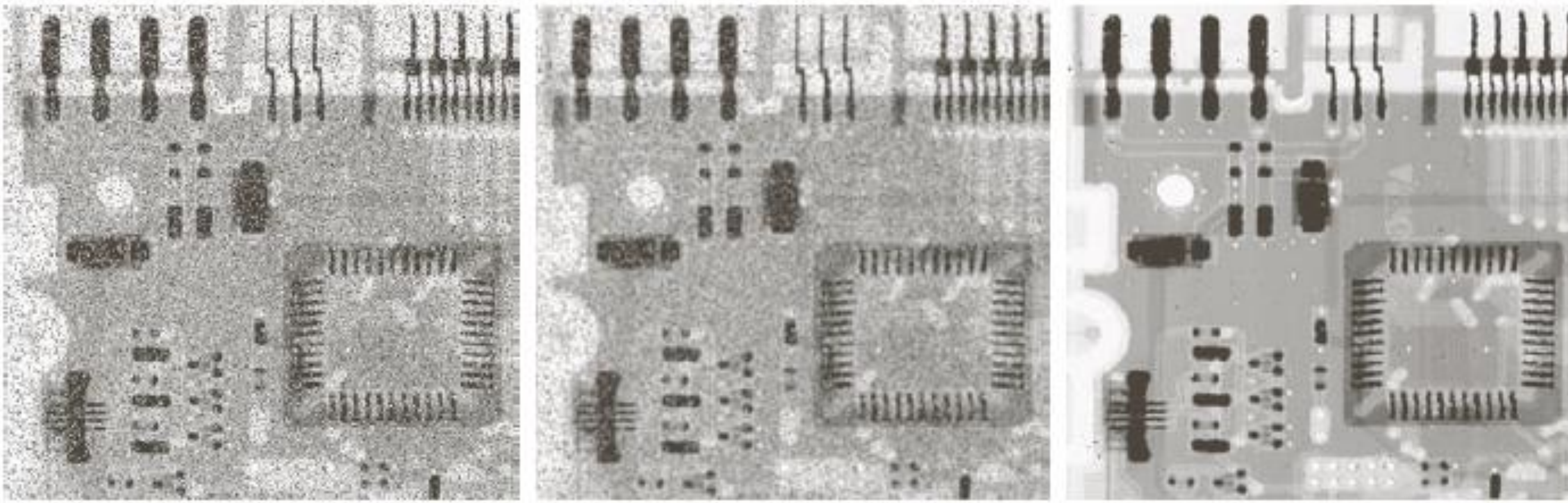
$f[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Credit: S. Seitz



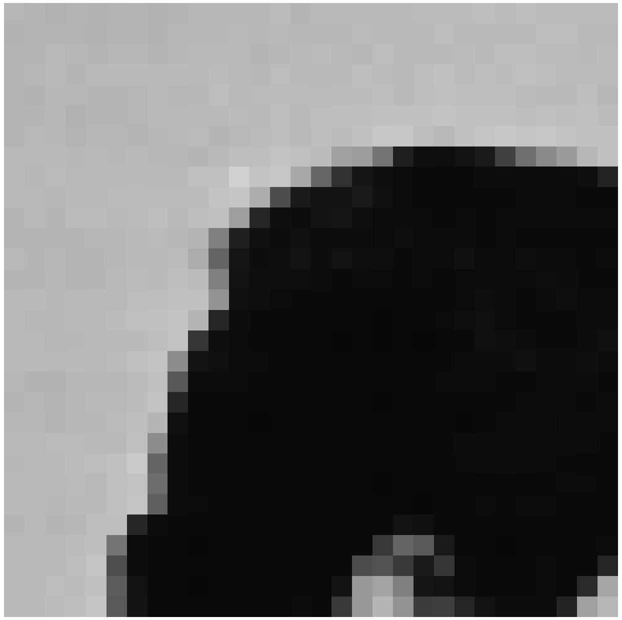
a b c

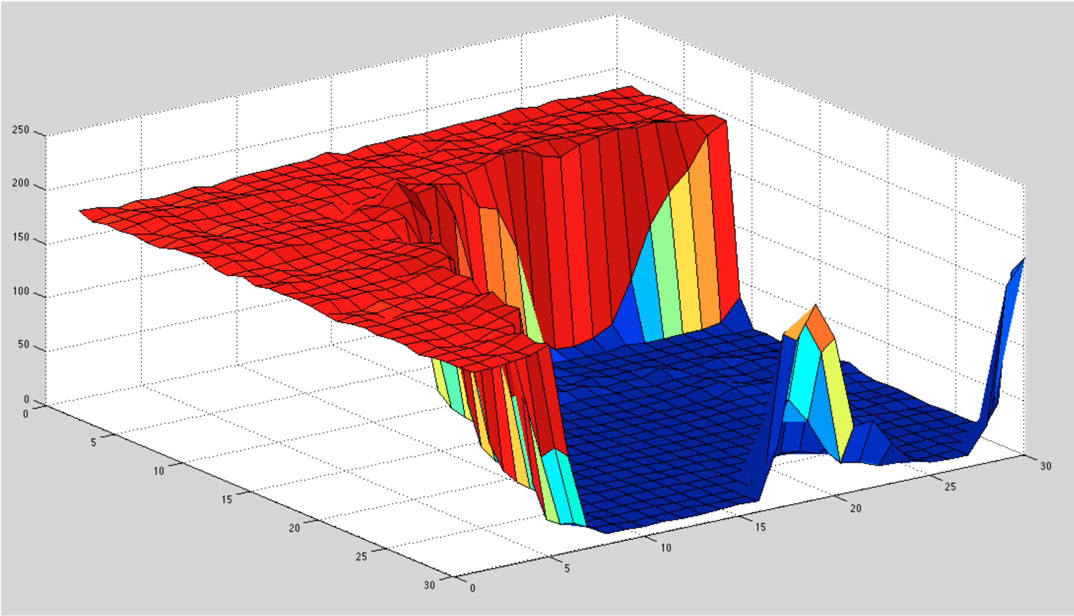
FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

References

- Median Filtering
  Gonzalez, section 3.6
  Szeliski, section 3.3

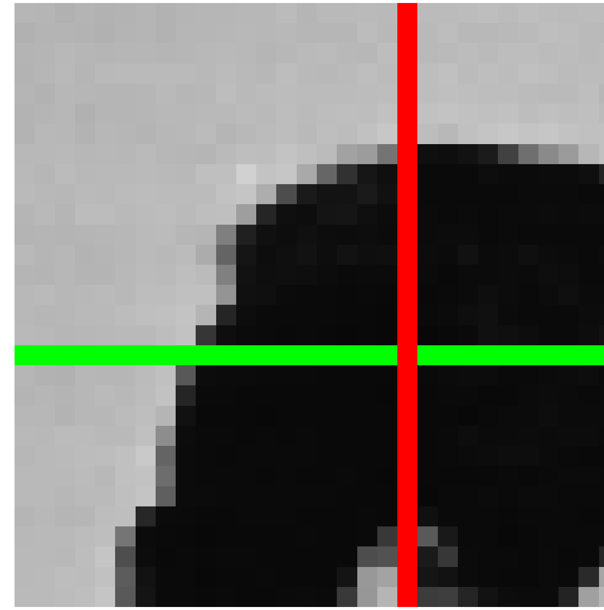
Edge Detection

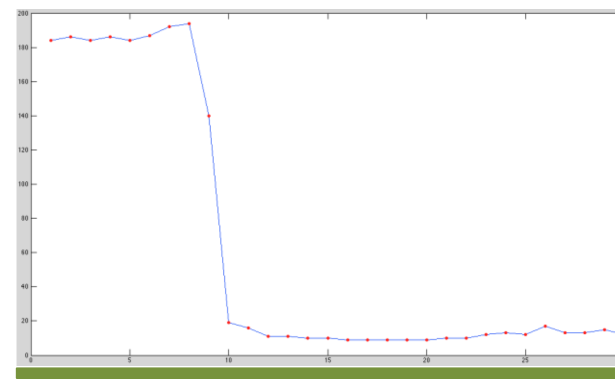


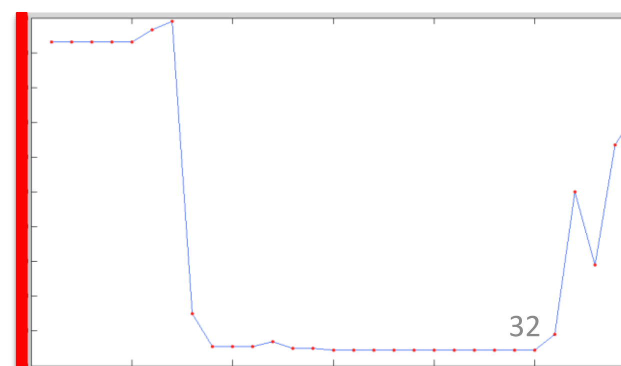


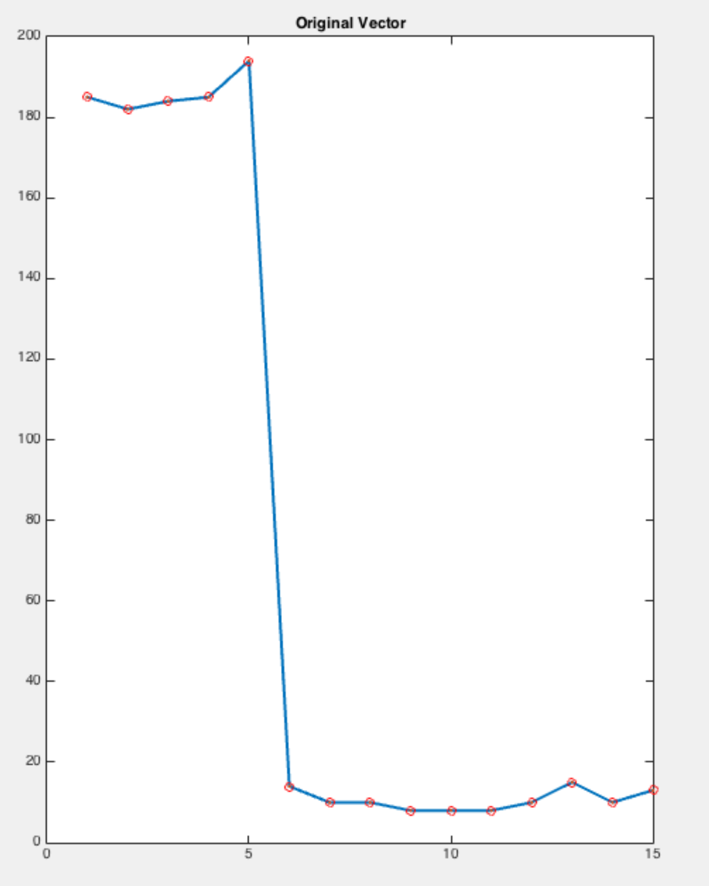
How can we find edges?

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	182	177	178	181	181	182	183	181	185	185	187	188	187	188	188
2	180	184	181	182	182	185	181	186	184	189	185	188	189	187	186
3	181	183	181	182	182	189	187	187	187	189	189	190	189	186	190
4	179	183	183	180	184	181	187	185	188	199	189	192	199	198	192
5	185	187	185	186	184	191	195	169	49	15	10	10	11	12	15
6	181	181	188	187	186	191	37	13	21	12	11	11	11	12	12
7	185	183	181	186	187	100	13	18	18	15	12	17	12	12	10
8	184	183	186	189	192	148	15	10	9	9	9	11	12	12	11
9	185	182	184	185	194	14	10	10	8	8	8	10	15	10	13
10	182	177	182	187	88	11	10	10	9	9	10	12	10	11	13
11	183	179	183	190	17	9	8	9	9	9	9	8	11	13	11
12	183	186	189	201	11	9	10	10	9	9	9	11	13	11	9
13	185	183	186	196	11	10	10	10	9	9	8	10	10	11	10
14	184	185	190	11	9	9	9	10	9	56	89	10	8	10	10
15	185	189	193	18	10	9	10	9	20	163	21	11	9	11	42

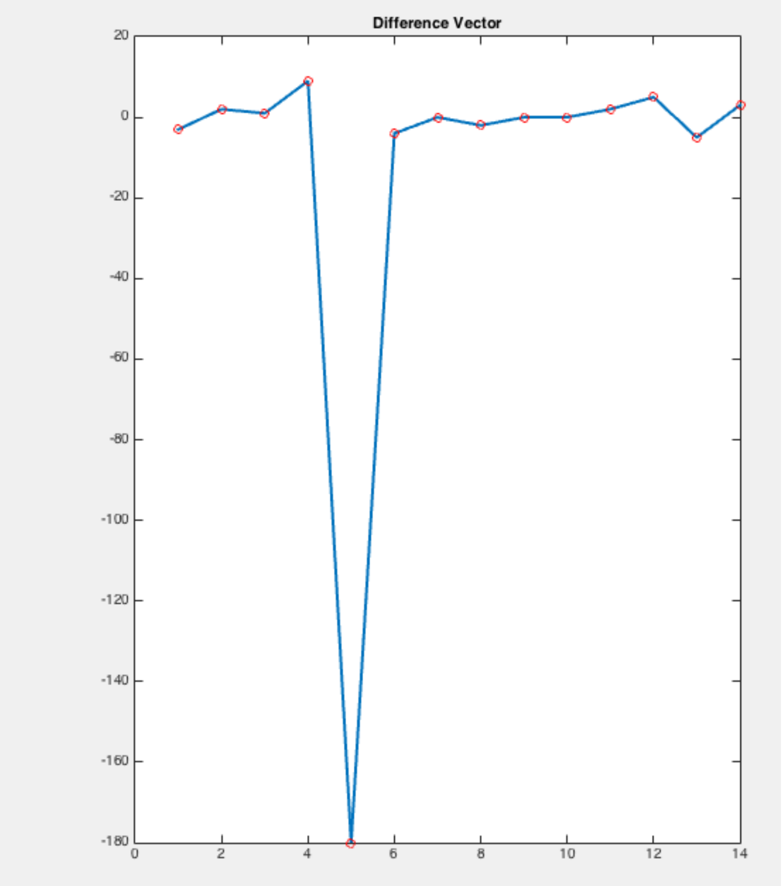








Original Vector



Difference Vector

Filtering:

-1	1

Basic Gradient Filters

Horizontal Gradient

-1	1

Vertical Gradient

-1
1

Taylor Expansion:

$$f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2!} f''(x) + \frac{\Delta x^3}{3!} f'''(x) + \ldots$$

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\Delta x = 1$$

$$f'(x) = f(x+1) - f(x)$$

Basic Gradient Filters

Horizontal Gradient

0	0	0
-1	0	1
0	0	0

or

-1	0	1

Vertical Gradient

0	-1	0
0	0	0
0	1	0

or

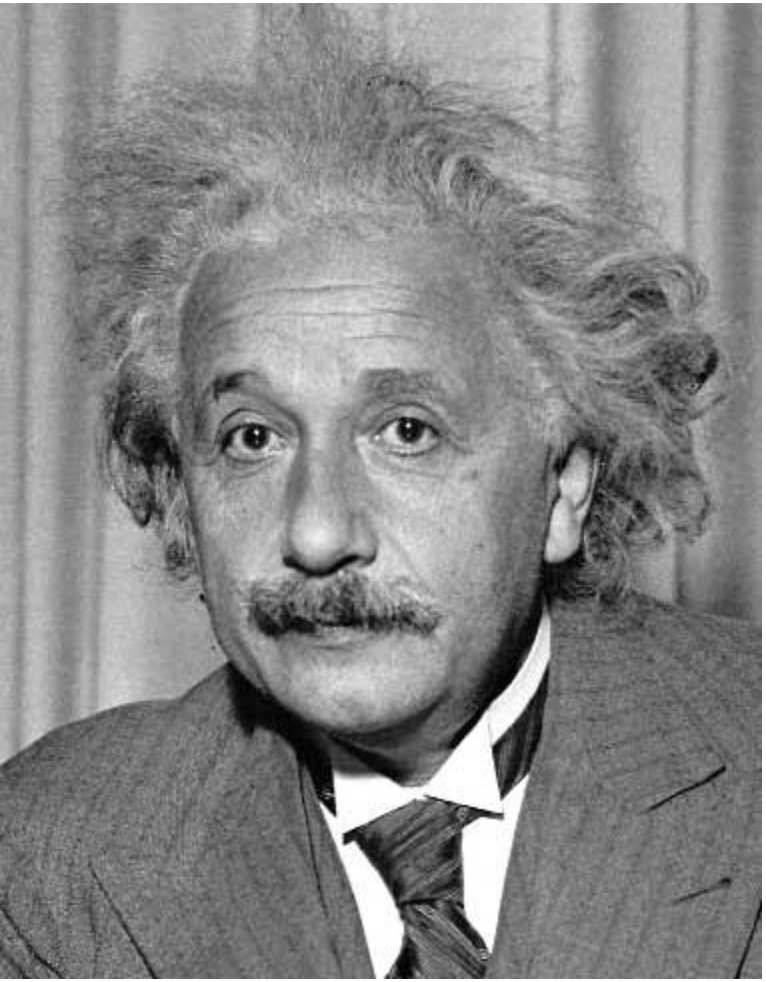
-1
0
1

$$f(x+1,y) = f(x,y) + f'(x,y) + \frac{1}{2!} f''(x,y) + \frac{1}{3!} f'''(x,y) + \ldots$$

$$- \quad f(x-1,y) = f(x,y) - f'(x,y) + \frac{1}{2!} f''(x,y) - \frac{1}{3!} f'''(x,y) + \ldots$$

$$f'(x,y) = \frac{f(x+1,y) - f(x-1,y)}{2}$$

Sobel Filters

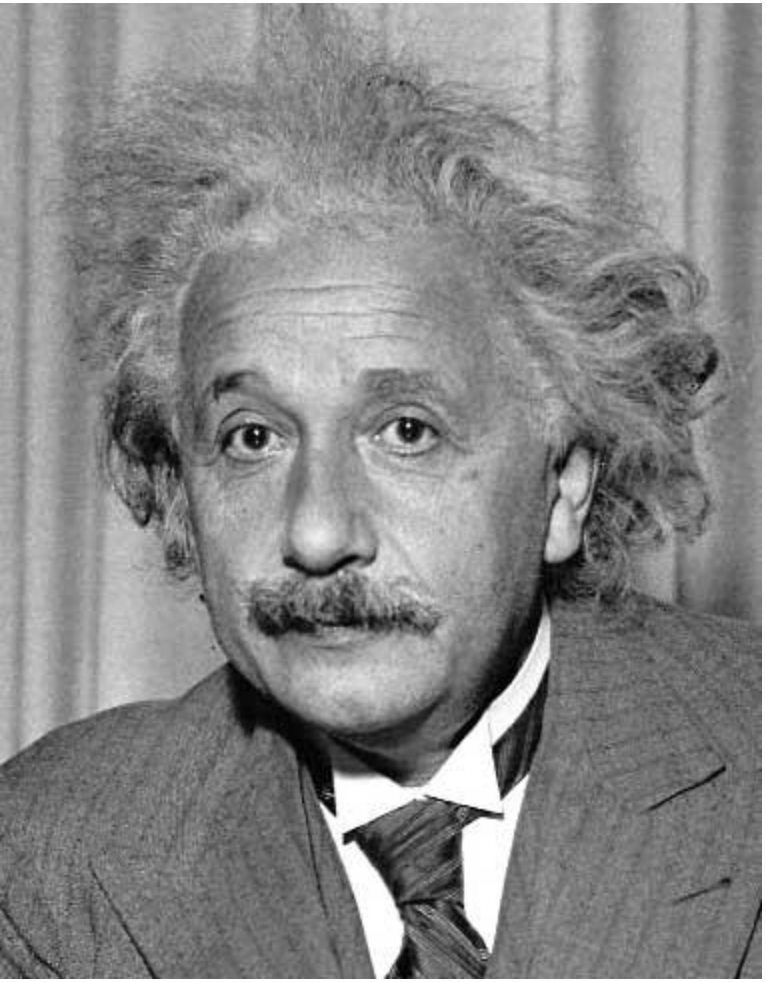


1	0	-1
2	0	-2
1	0	-1

Sobel

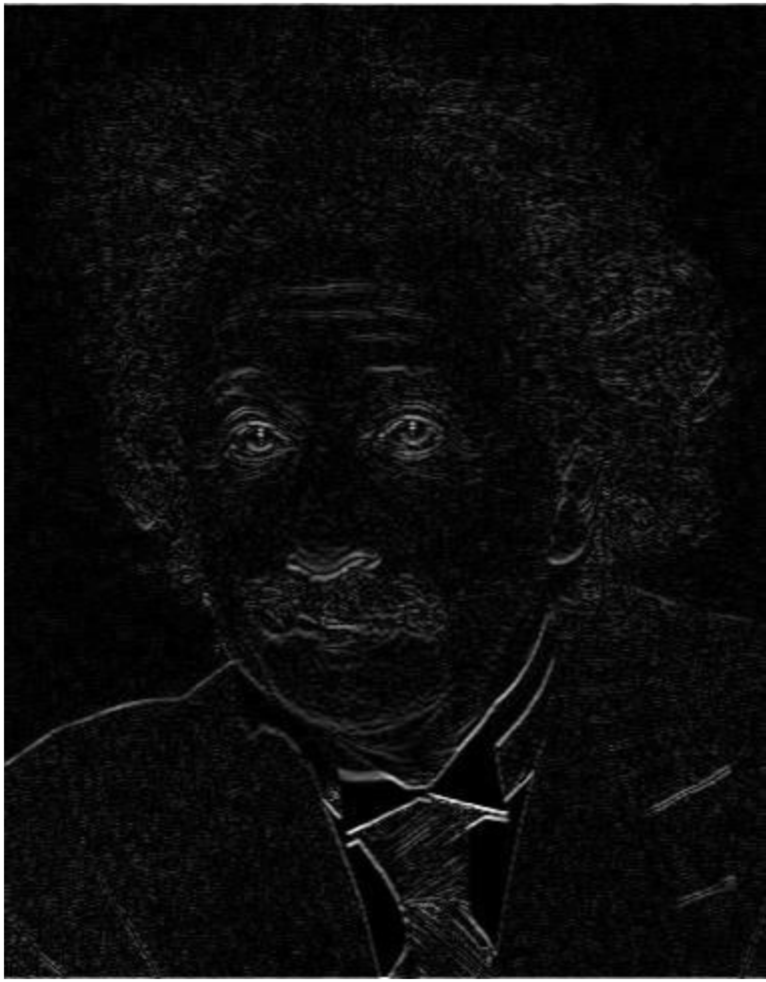
Vertical Edge
(absolute value)

Sobel Filters



1	2	1
0	0	0
-1	-2	-1

Sobel



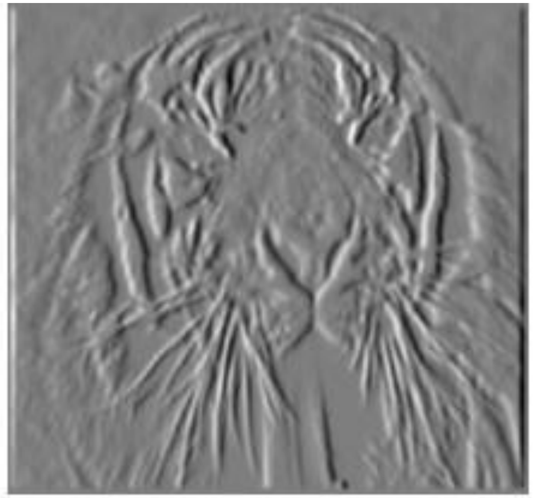
Horizontal Edge
(absolute value)

Sobel Filters: Separable

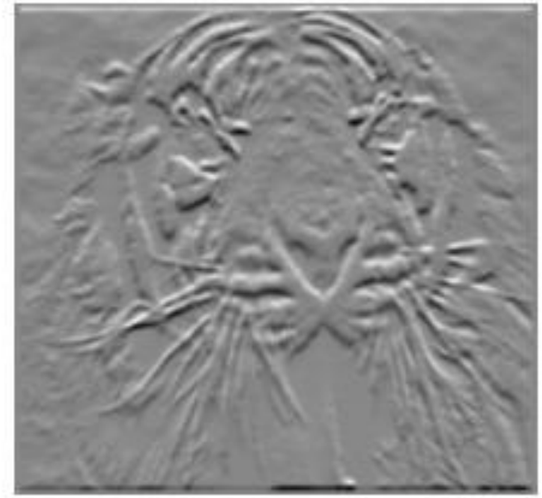
$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \star \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \star \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Image Gradient



$$\frac{\partial f(x,y)}{\partial x}$$



$$\frac{\partial f(x,y)}{\partial y}$$

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradient: $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right)$

Magnitude: $\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2}$

$$\frac{\partial f}{\partial x} \quad \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \qquad\qquad \frac{\partial f}{\partial y} \quad \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 0 & -2 \\ 0 & -2 & -2 \end{bmatrix}$$

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \neq \text{gradient}$$

Gradient: $\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

Magnitude: $\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

$$\frac{\P f}{\P x} \quad \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array}$$

$$\frac{\P f}{\P y} \quad \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 0 & 0 & 0 \\ \hline -1 & -2 & -1 \\ \hline \end{array} \star \begin{array}{|c|c|c|} \hline 1 & 0 & -1 \\ \hline 2 & 0 & -2 \\ \hline 1 & 0 & -1 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline 4 & 0 & -4 \\ \hline 0 & 0 & 0 \\ \hline -4 & 0 & 4 \\ \hline \end{array}$$

$$\frac{\P f}{\P x} \star \frac{\P f}{\P y} \neq \text{gradient}$$

Other Edge Detectors

- Roberts Cross

1	0
0	-1

0	1
-1	0

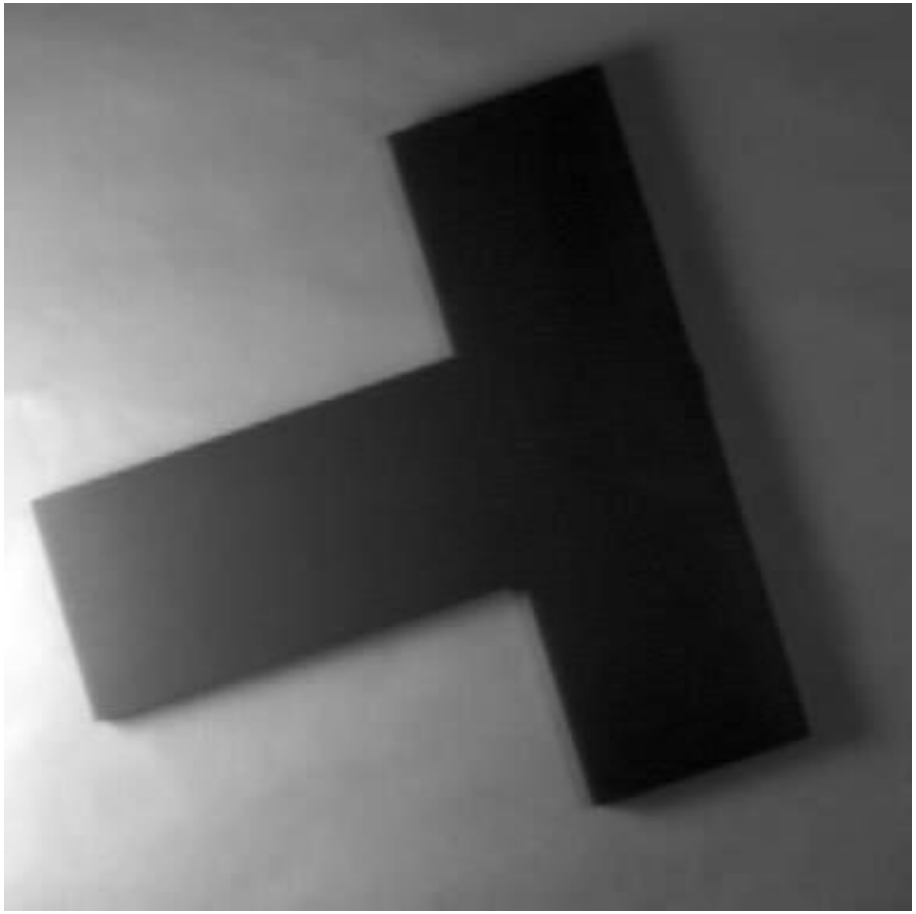
- Prewitt

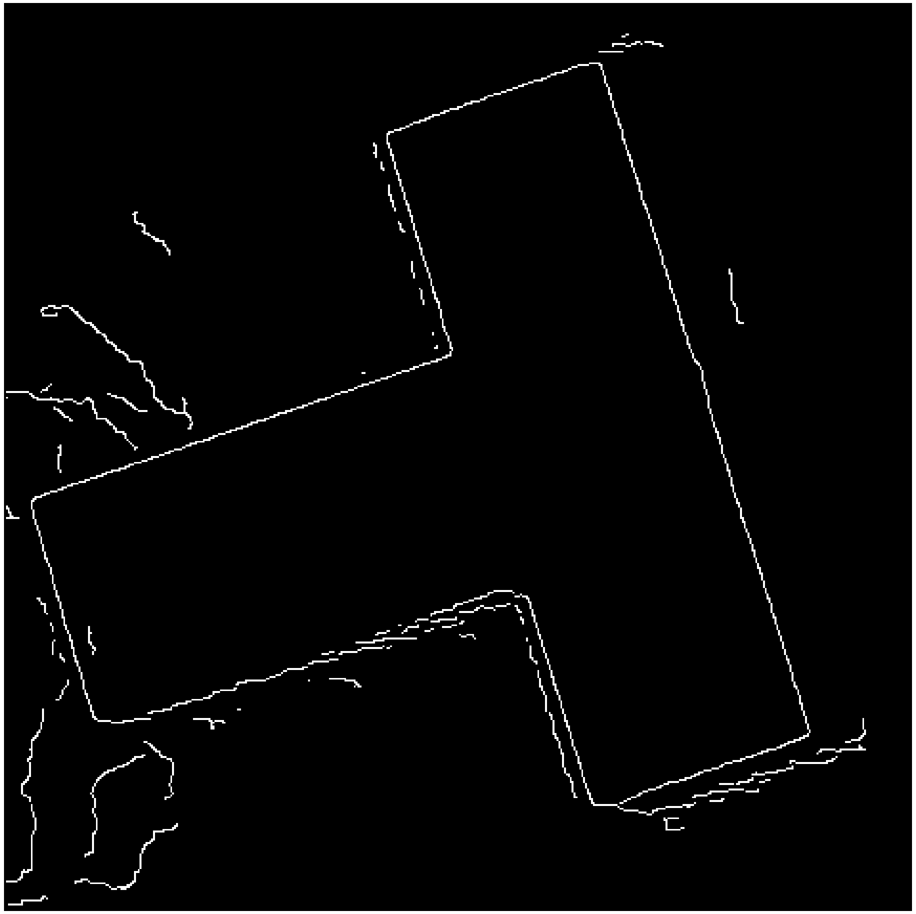
-1	0	1
-1	0	1
-1	0	1

1	1	1
0	0	0
-1	-1	-1

- Canny

Example



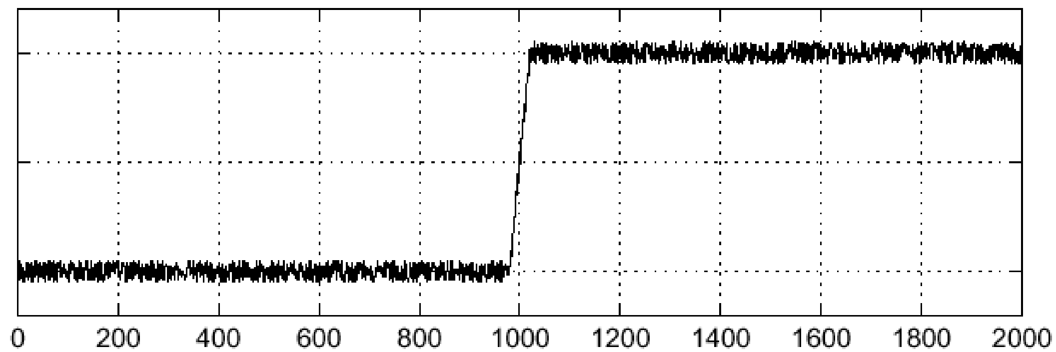


Edge Detector: Canny, Threshold: 0.1

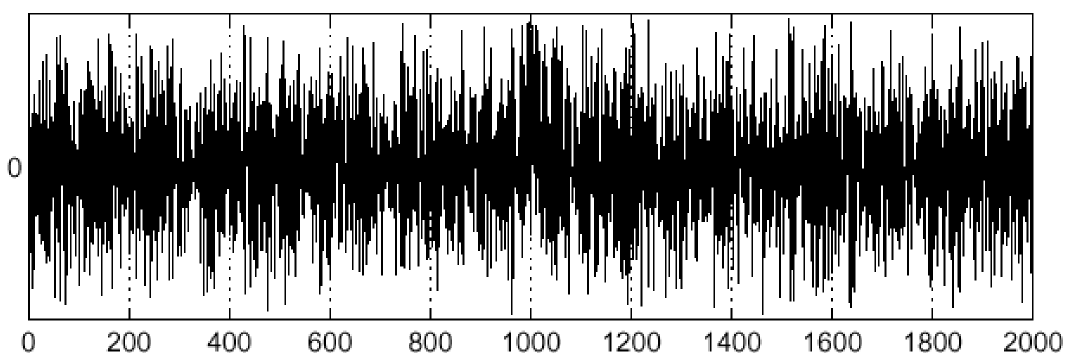
Effects of Noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal

$f(x)$



$\frac{\partial}{\partial x} f(x)$



Where is the edge?

Source: S. Seitz

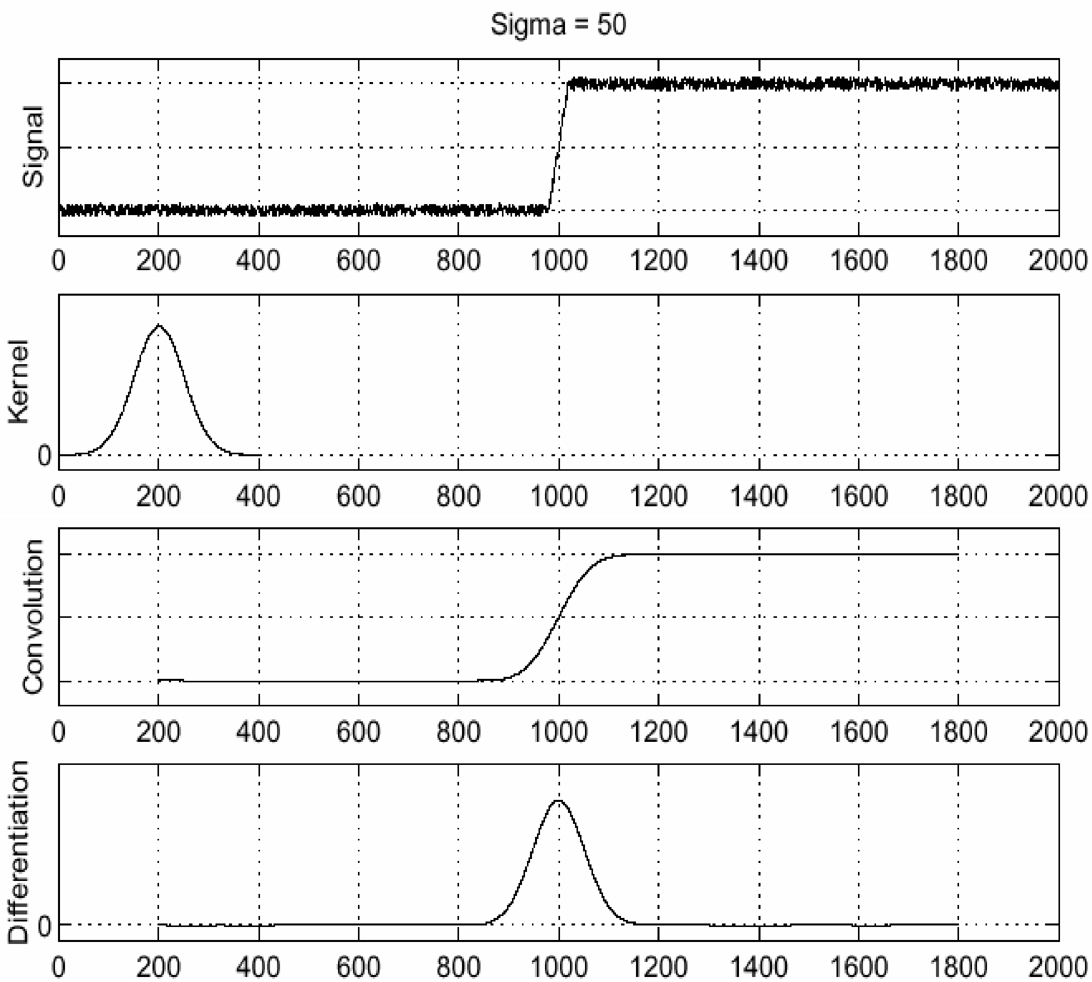
Solution: Smooth First

f

g

$f * g$

$\frac{\P}{\P x}(f * g)$



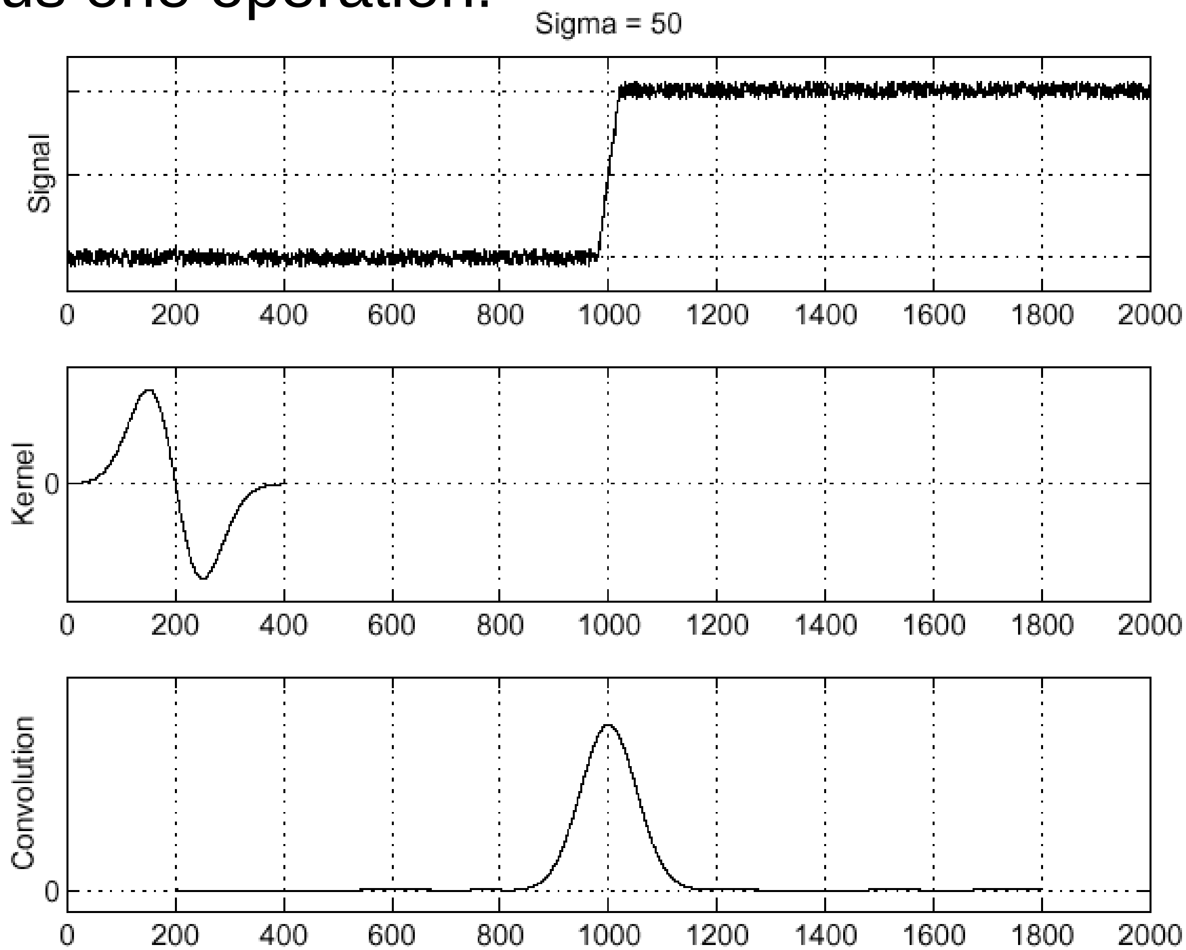
- To find edges, look for peaks in $\frac{\P}{\P x}(f * g)$

Source: S. Seitz

Derivative theorem of convolution

$$\frac{\partial}{\partial x}(g * f) = \frac{\partial g}{\partial x} * f$$

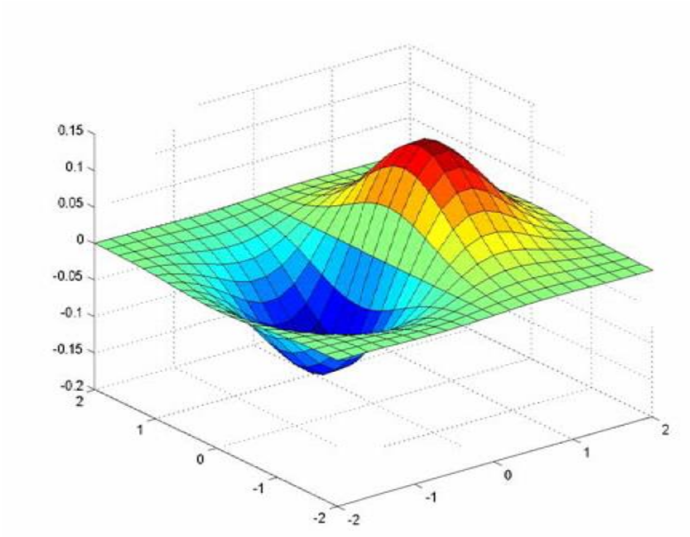
- This saves us one operation:



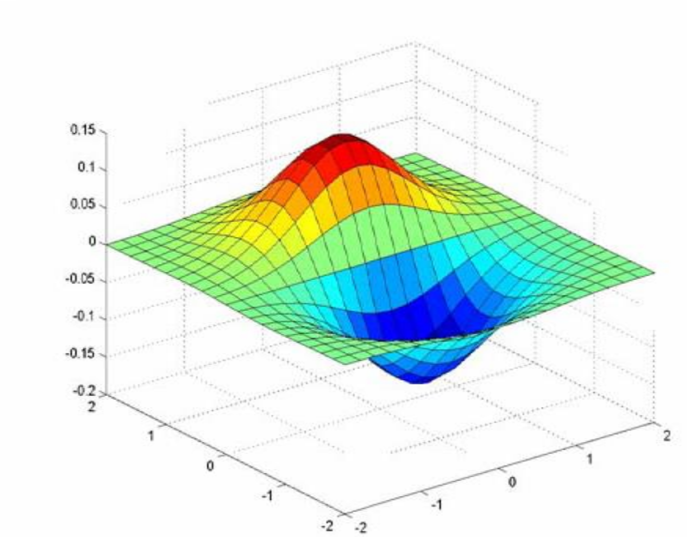
$$\frac{\partial g}{\partial x}$$

$$\frac{\partial g}{\partial x} * f$$

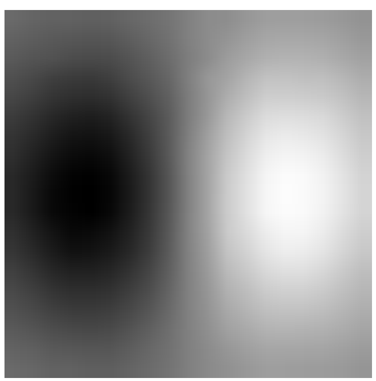
Derivative of Gaussian Filter

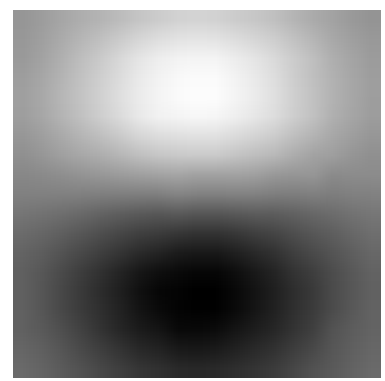


y-direction



x-direction

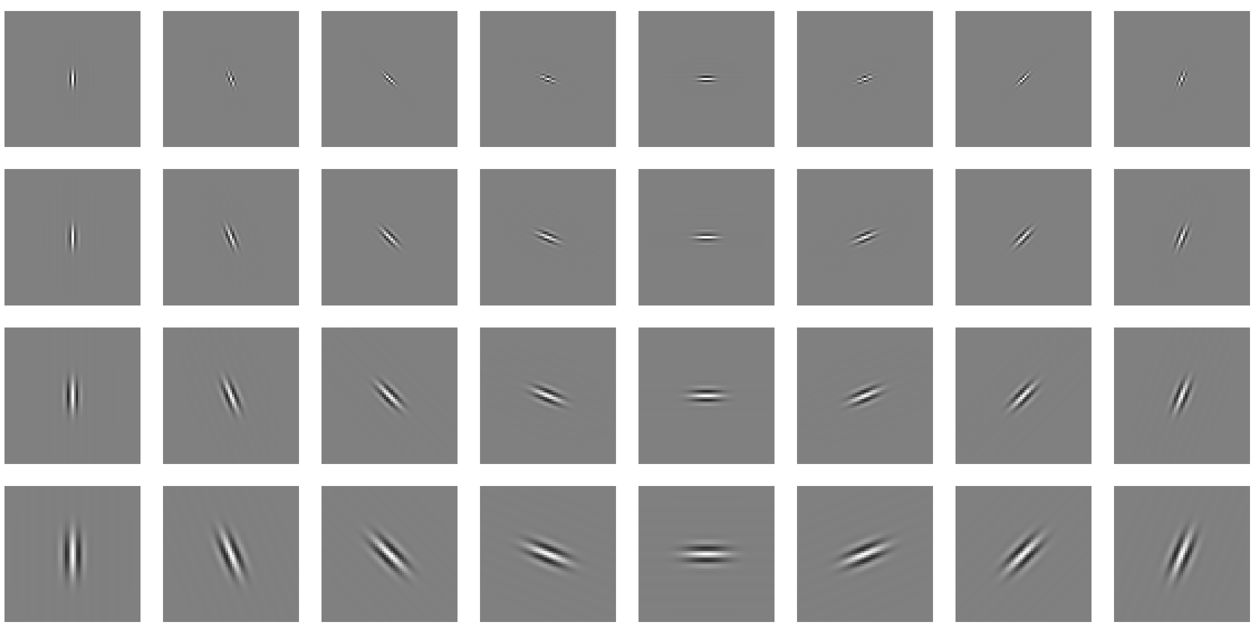




- Which one finds horizontal/vertical edges?

Clues from Human Perception

- Early processing in humans filters for various orientations and scales of frequency
- Perceptual cues in the mid frequencies dominate perception
- When we see an image from far away, we are effectively subsampling it



Early Visual Processing: Multi-scale edge and blob filters

References

- Edge Detection
  Gonzalez, section 10.2
  Szeliski, section 4.2