

اصول پردازش تصویر

*Principles of Image Processing*

مصطفی کمالی تبریزی

۱۷ آذر ۱۳۹۹

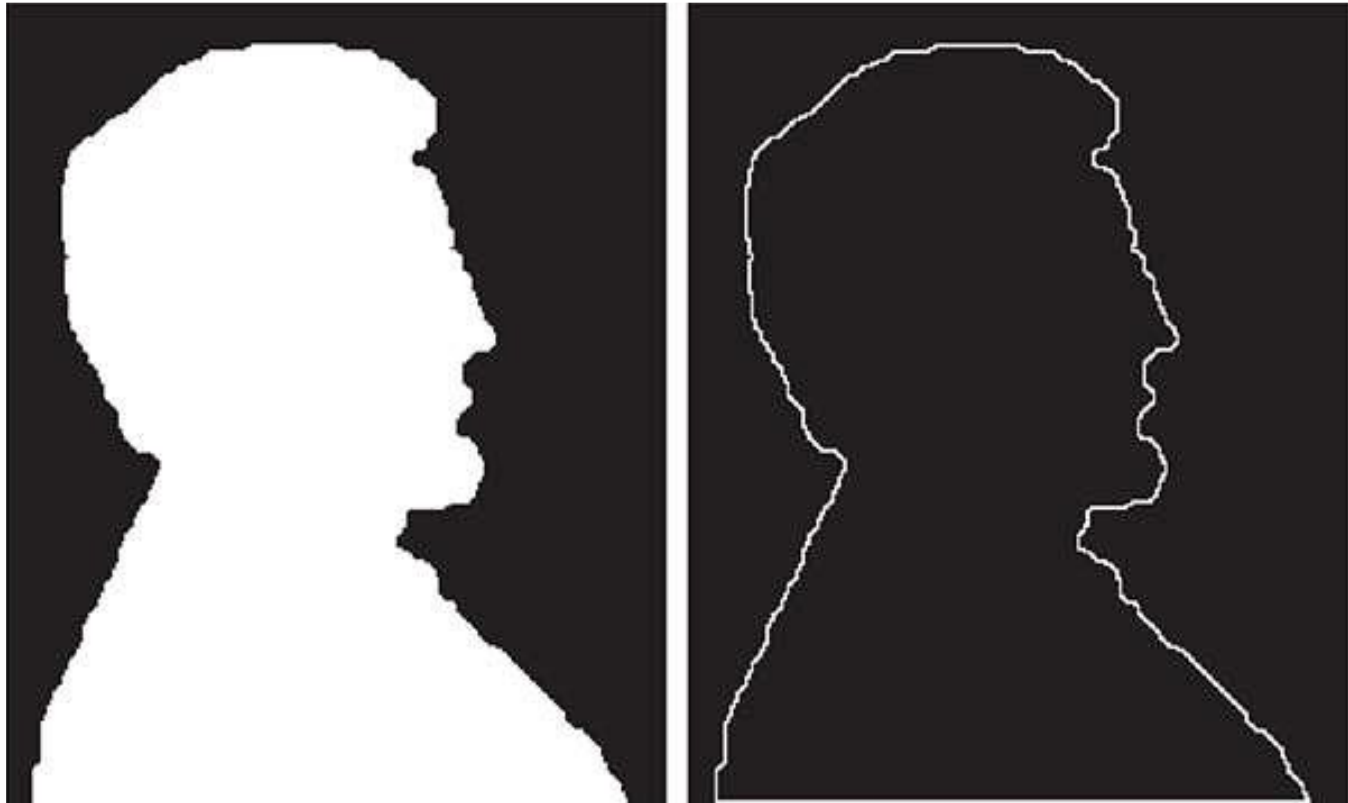
جلسه بیست سوم

# *Morphology*

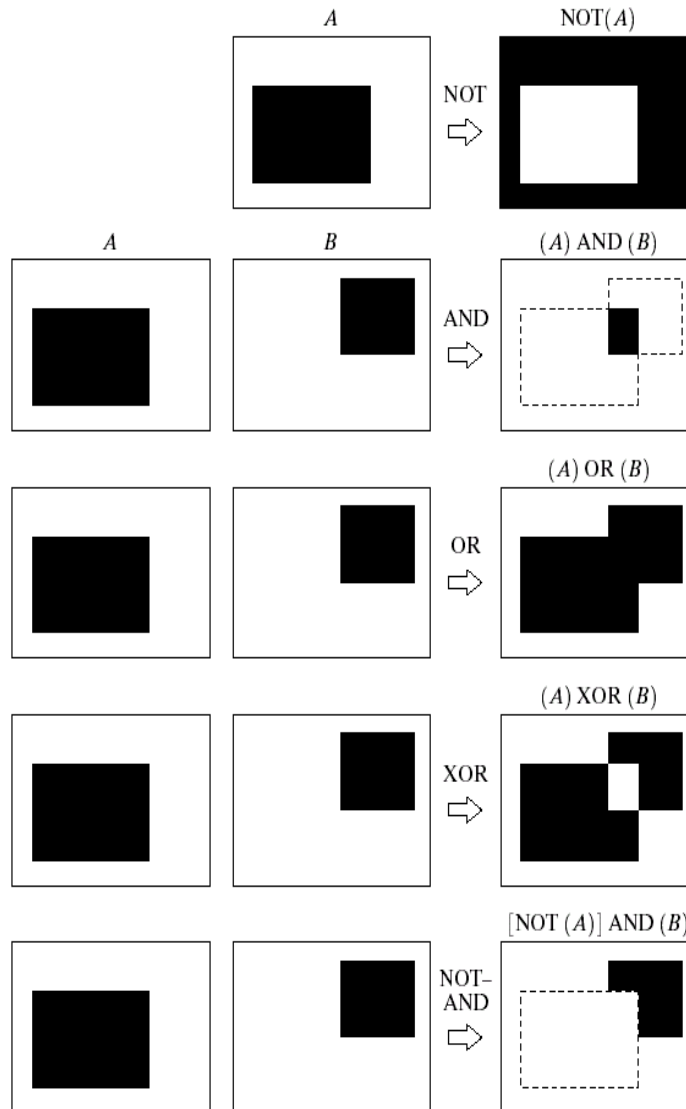
ریخت شناسی

# Morphological Operation

- What if your images are binary masks?
- Binary image processing is a well-studied field, based on set theory, called Mathematical Morphology



# Preliminaries



**FIGURE 9.4**

Some logic operations between binary images. Black represents binary 1s and white represents binary 0s in this example.

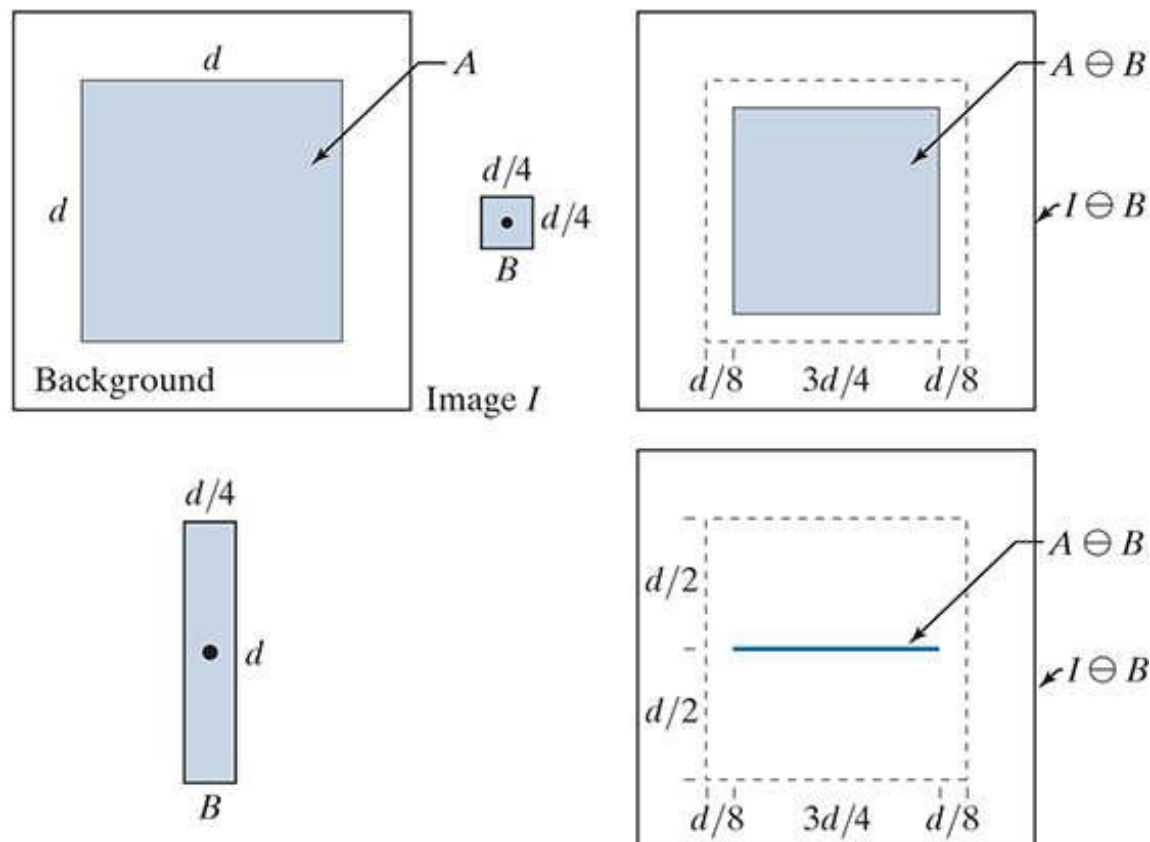
# Erosion فرسایش

Image

Structuring Element (SE)

$$I \ominus B$$

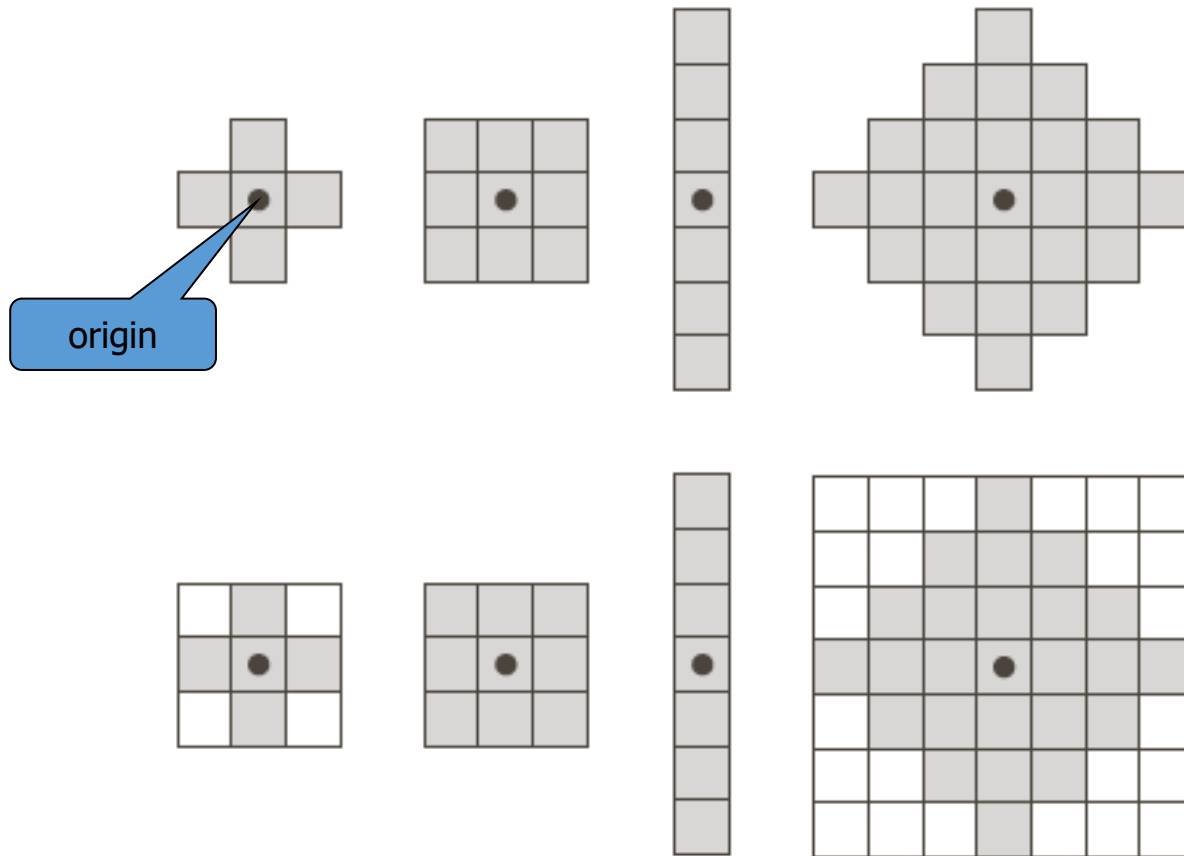
a b c  
d e



**FIGURE 9.4**

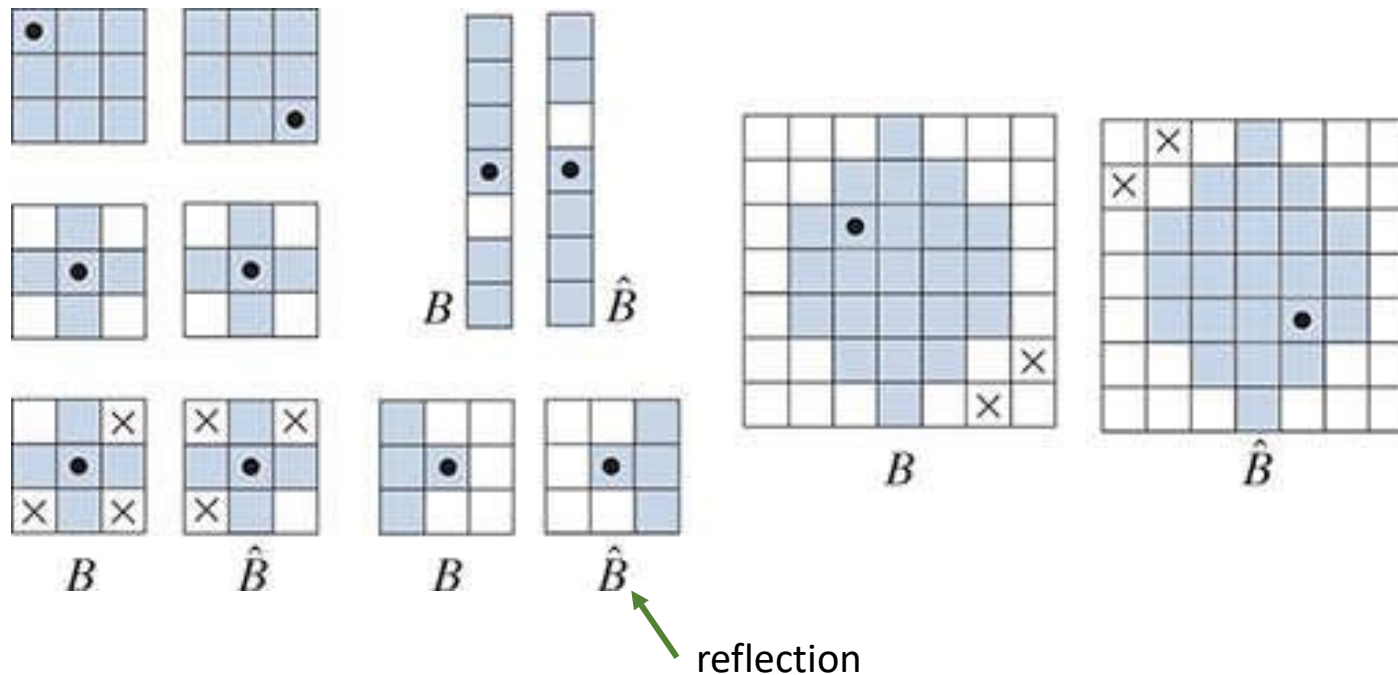
(a) Image  $I$ , consisting of a set (object)  $A$ , and background. (b) Square SE,  $B$  (the dot is the origin). (c) Erosion of  $A$  by  $B$  (shown shaded in the resulting image). (d) Elongated SE. (e) Erosion of  $A$  by  $B$ . (The erosion is a line.) The dotted border in (c) and (e) is the boundary of  $A$ , shown for reference.

# Structuring Elements (SE)



**FIGURE 9.2** First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

# Structuring Elements (SE)

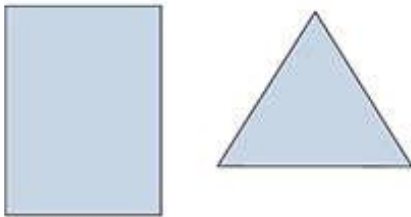


**FIGURE 9.2**

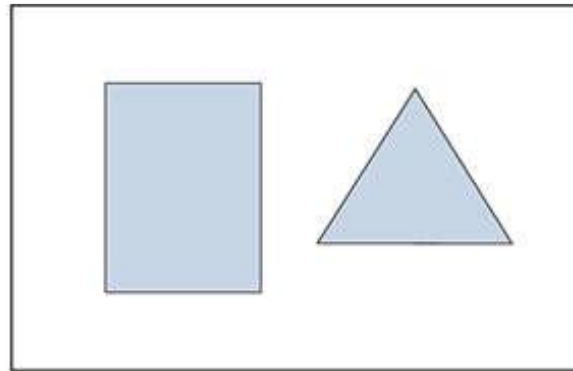
Structuring elements and their reflections about the origin (the x's are don't care elements, and the dots denote the origin). Reflection is rotation by 180 degrees of an SE about its origin.



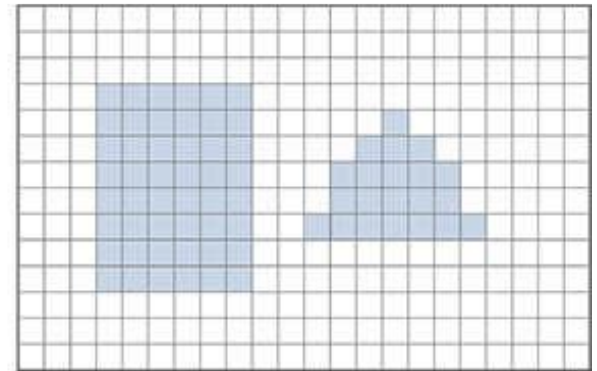
# Rectangular Arrays of Shapes



Objects represented  
as sets



Objects represented as  
a graphical image



Digital image



Structuring element  
represented as a set



Structuring element  
represented as a graphical image

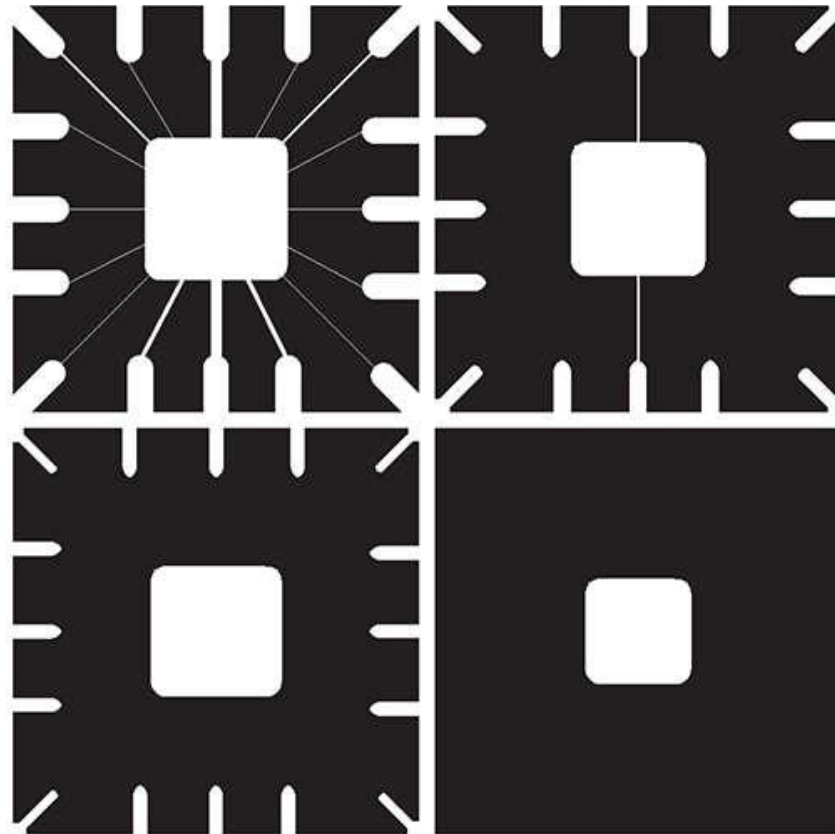


Digital  
structuring element

# Erosion فرسایش

$$I \ominus B$$

a b  
c d



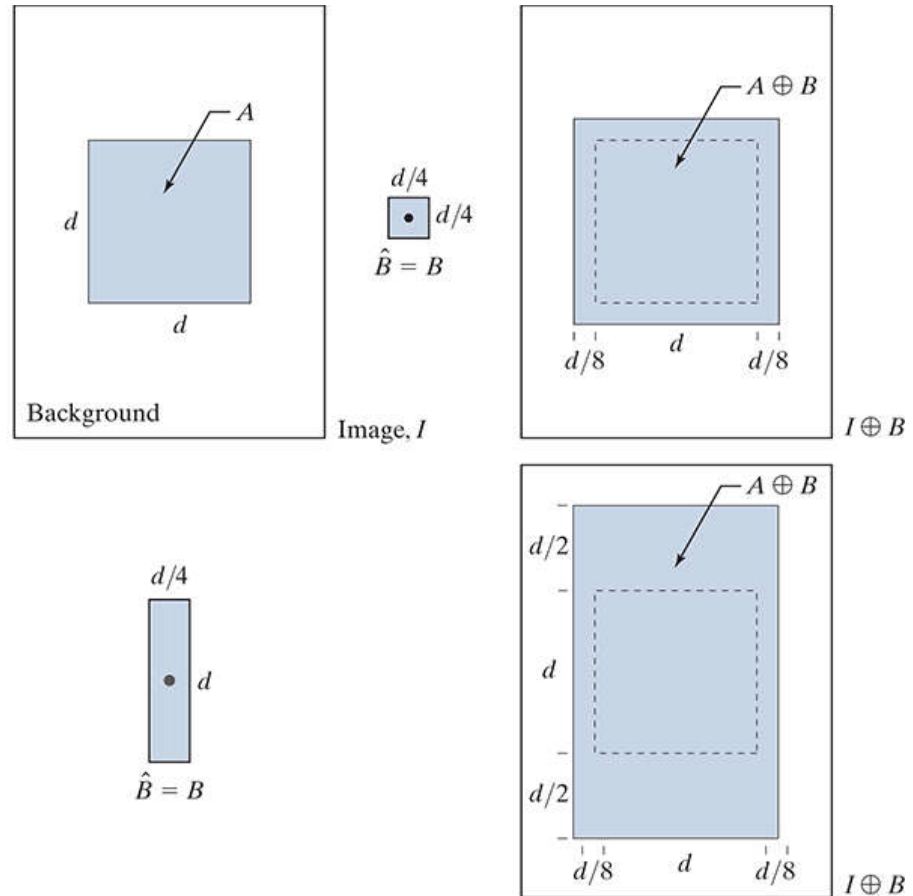
**FIGURE 9.5**

Using erosion to remove image components. (a) A  $486 \times 486$  binary image of a wire-bond mask in which foreground pixels are shown in white. (b)–(d) Image eroded using square structuring elements of sizes  $11 \times 11$ ,  $15 \times 15$  and  $45 \times 45$  elements, respectively, all valued 1.

# Dilation گسترش

$$I \oplus B$$

a b c  
d e

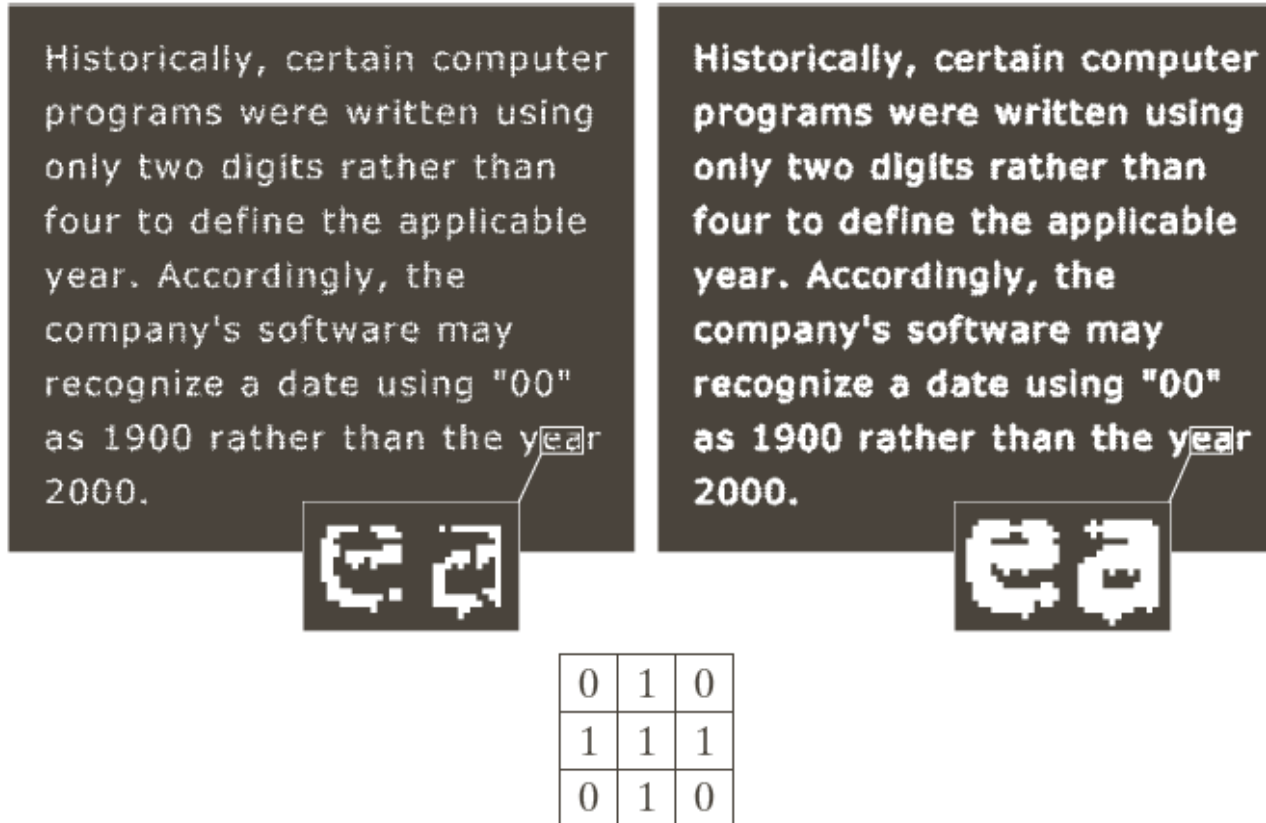


**FIGURE 9.6**

(a) Image  $I$ , composed of set (object)  $A$  and background. (b) Square SE (the dot is the origin). (c) Dilation of  $A$  by  $B$  (shown shaded). (d) Elongated SE. (e) Dilation of  $A$  by this element. The dotted line in (c) and (e) is the boundary of  $A$ , shown for reference.

# Dilation گسترش

$$I \oplus B$$



a c  
b

**FIGURE 9.7**

(a) Sample text of poor resolution with broken characters (see magnified view).  
(b) Structuring element.  
(c) Dilation of (a) by (b). Broken segments were joined.

# Duality of Dilation and Erosion

$$\begin{aligned}(I \ominus B)^c &= I^c \oplus \hat{B} \\ (I \oplus B)^c &= I^c \ominus \hat{B}\end{aligned}$$

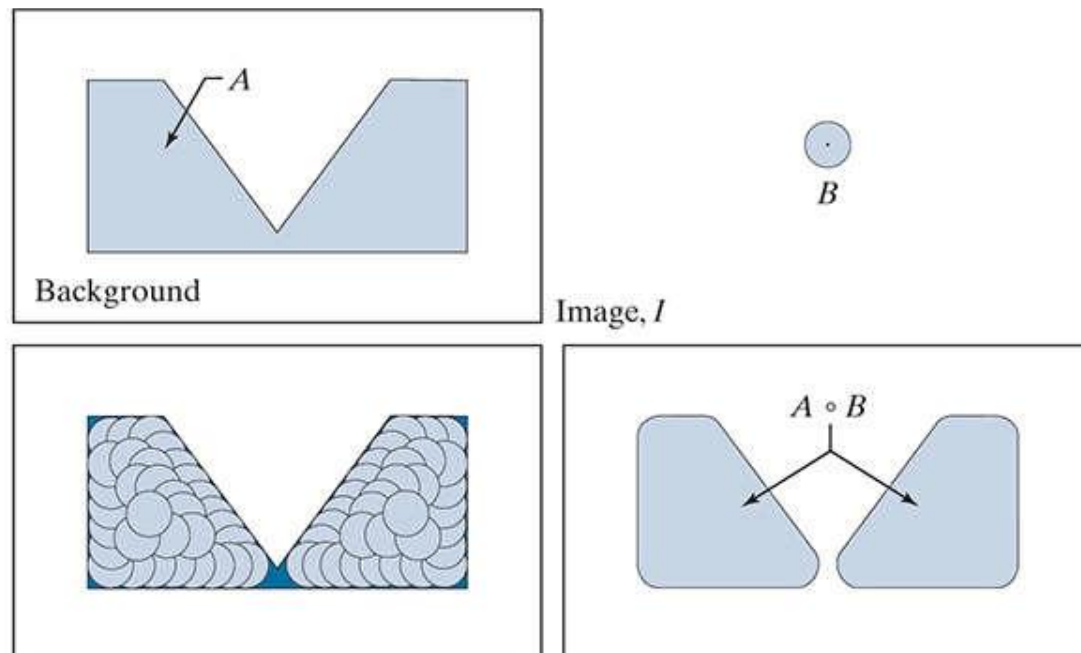
*B is usually symmetric, then:*

$$\begin{aligned}(I \ominus B)^c &= I^c \oplus B \\ (I \oplus B)^c &= I^c \ominus B\end{aligned}$$

# Opening گشودن

$$I \circ B = (I \ominus B) \oplus B$$

a	b
c	d



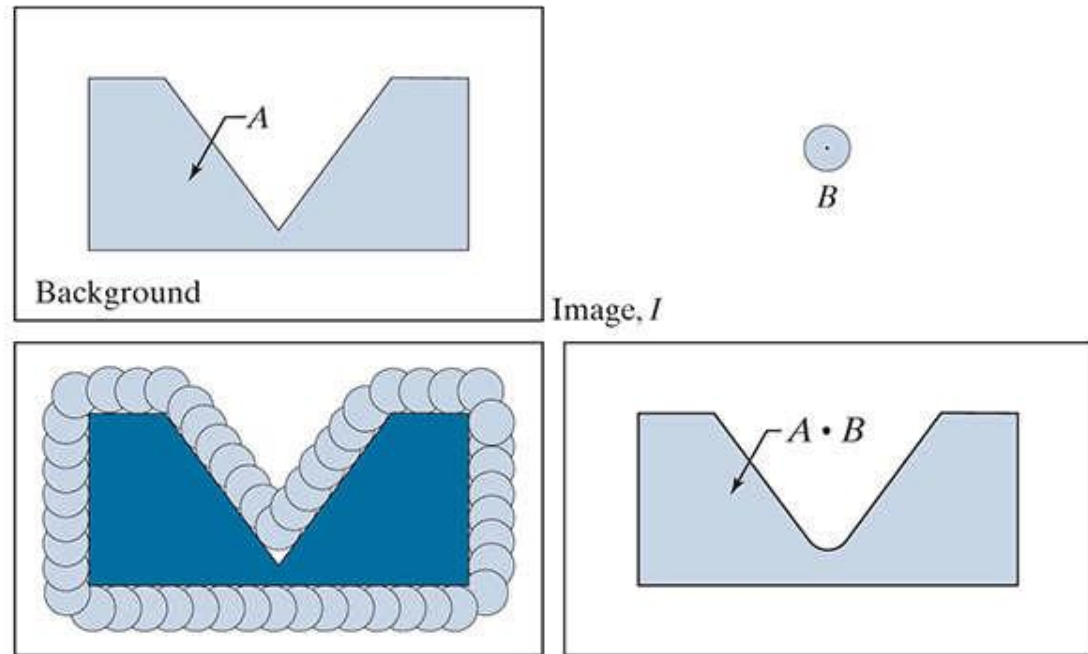
**FIGURE 9.8**

(a) Image  $I$ , composed of set (object)  $A$  and background. (b) Structuring element,  $B$ . (c) Translations of  $B$  while being contained in  $A$ . ( $A$  is shown dark for clarity.) (d) Opening of  $A$  by  $B$ .

# Closing بستن

$$I \cdot B = (I \oplus B) \ominus B$$

a b  
c d



**FIGURE 9.9**

(a) Image  $I$ , composed of set (object)  $A$ , and background. (b) Structuring element  $B$ . (c) Translations of  $B$  such that  $B$  does not overlap any part of  $A$ . ( $A$  is shown dark for clarity.) (d) Closing of  $A$  by  $B$ .

# Duality of Opening and Closing

$$\begin{aligned}(A \circ B)^c &= A^c \cdot \hat{B} \\ (A \cdot B)^c &= A^c \circ \hat{B}\end{aligned}$$

*B is usually symmetric, then:*

$$\begin{aligned}(A \circ B)^c &= A^c \cdot B \\ (A \cdot B)^c &= A^c \circ B\end{aligned}$$



# Properties of Opening and Closing

- Opening

1.  $A \circ B \subseteq A$

2.  $C \subseteq D \Rightarrow C \circ B \subseteq D \circ B$

3.  $(A \circ B) \circ B = A \circ B$

- Closing

1.  $A \subseteq A \cdot B$

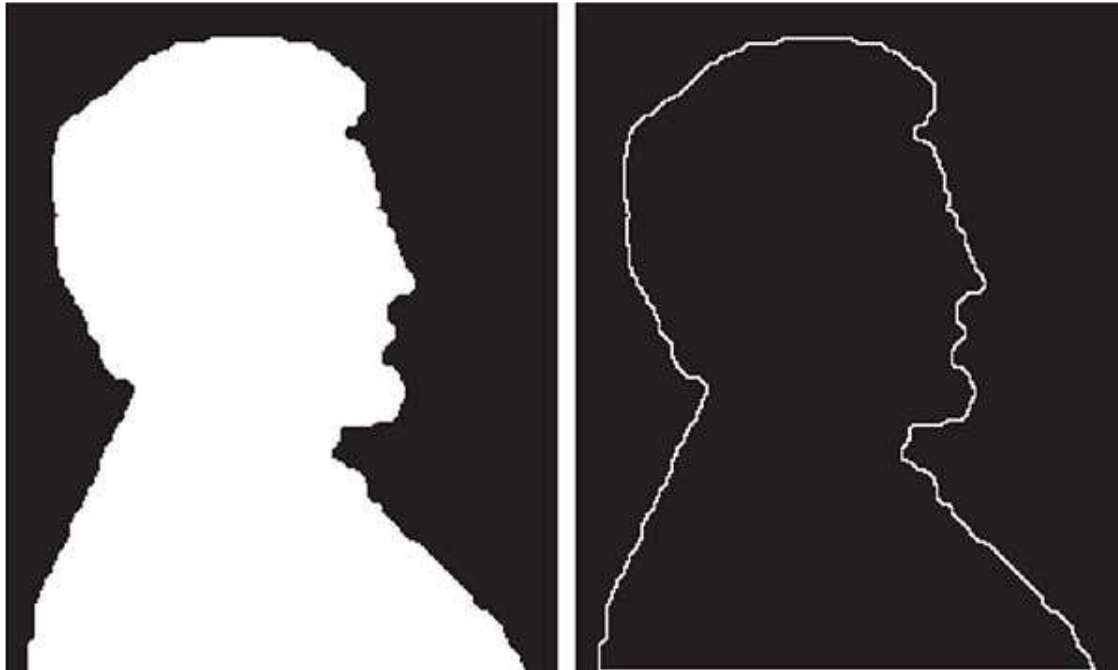
2.  $C \subseteq D \Rightarrow C \cdot B \subseteq D \cdot B$

3.  $(A \cdot B) \cdot B = A \cdot B$

# Boundary Extraction استخراج مرز

$$\beta_{inner}(A) = A - (A \ominus B)$$

$$\beta_{outer}(A) = (A \oplus B) - A$$



# Connected Components مؤلفه های همبند

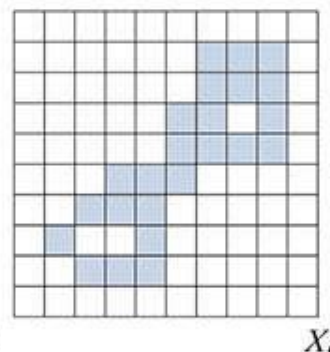
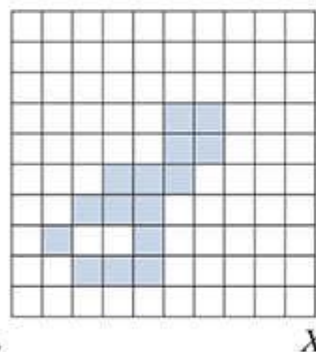
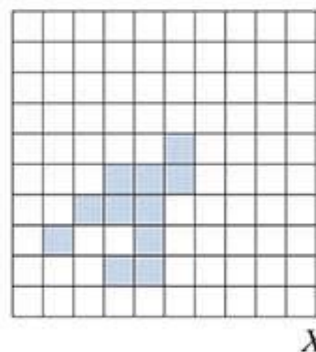
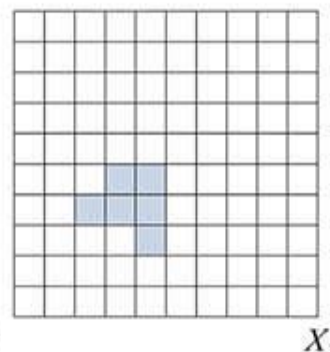
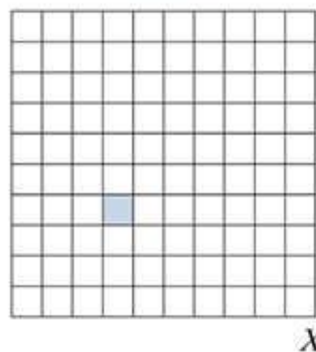
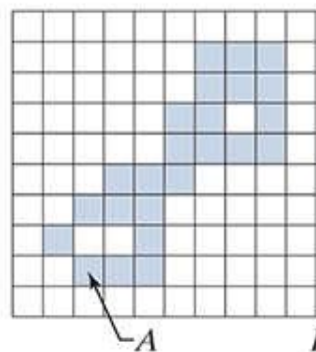
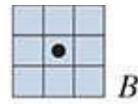
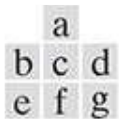
$$X_0 = p$$

A point in the component

$$X_k = (X_{k-1} \oplus B) \cap A$$

Until  $X_k = X_{k-1}$

$X_k$  is a CC.



**FIGURE 9.19**

(a) Structuring element. (b) Image containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9-20)

For each hole, a point in the hole

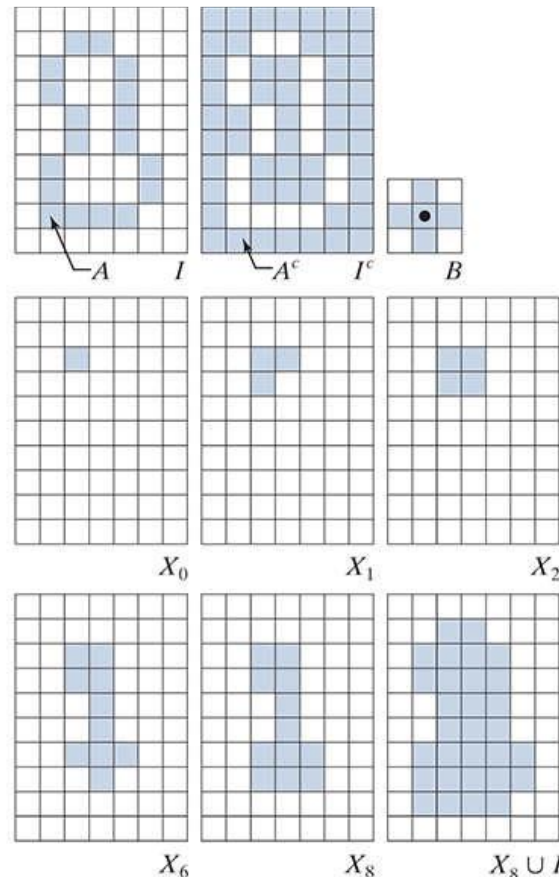
# Hole Filling

$$\begin{aligned}
 X_0 &= p \\
 X_k &= (X_{k-1} \oplus B) \cap A^c \\
 \text{Until } X_k &= X_{k-1} \\
 I &= I \cup X_k
 \end{aligned}$$

a	b	c
d	e	f
g	h	i

**FIGURE 9.17**

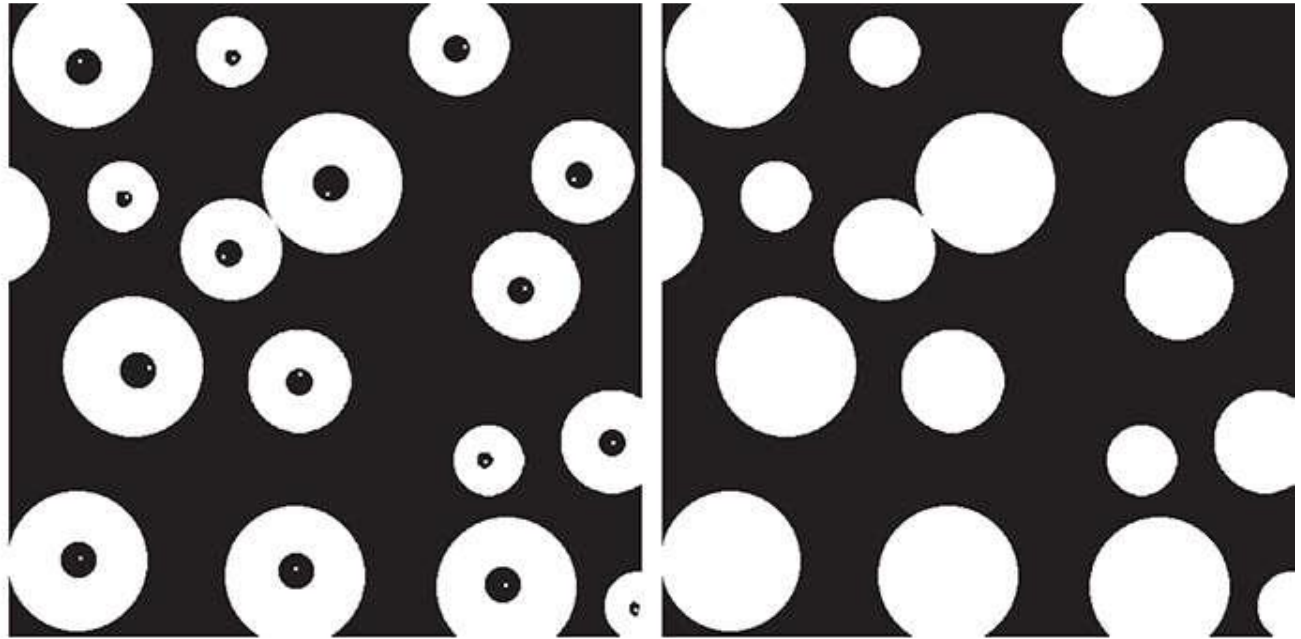
Hole filling. (a) Set  $A$  (shown shaded) contained in image  $I$ . (b) Complement of  $I$ . (c) Structuring element  $B$ . *Only* the foreground elements are used in computations (d) Initial point inside hole, set to 1. (e)–(h) Various steps of **Eq. (9-19)**. (i) Final result [union of (a) and (h)].



# Hole Filling

$$\begin{aligned} X_0 &= p \\ X_k &= (X_{k-1} \oplus B) \cap A^c \\ \text{Until } X_k &= X_{k-1} \\ I &= I \cup X_k \end{aligned}$$

a b



**FIGURE 9.18**

(a) Binary image. The white dots inside the regions (shown enlarged for clarity) are the starting points for the hole-filling algorithm. (b) Result of filling all holes.

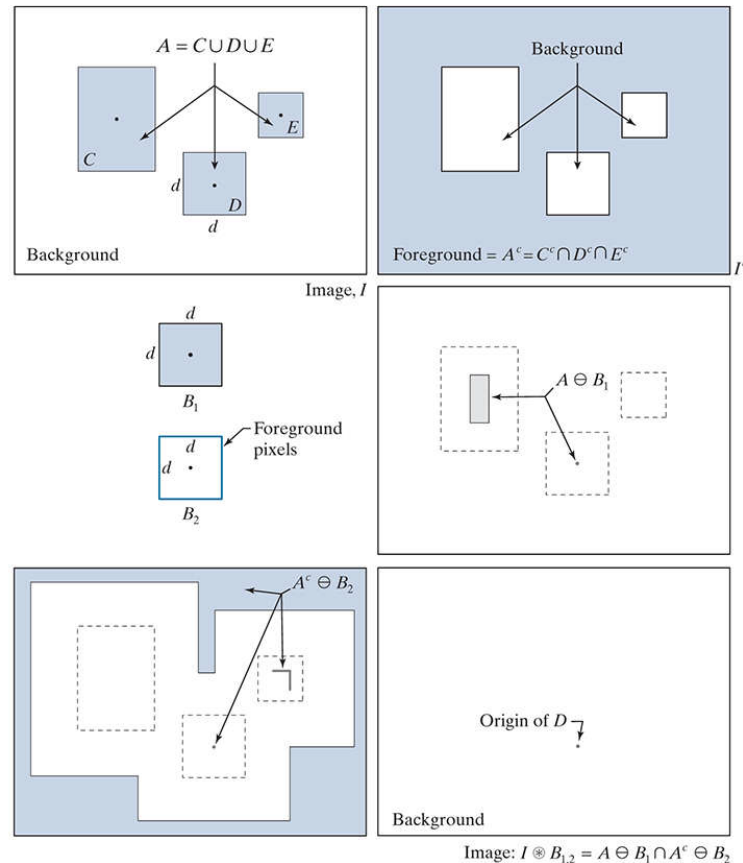
# Hit-or-Miss Transformation (HMT)

$$A \circledast \{B_1, B_2\} = (A \ominus B_1) \cap (A^c \ominus B_2)$$

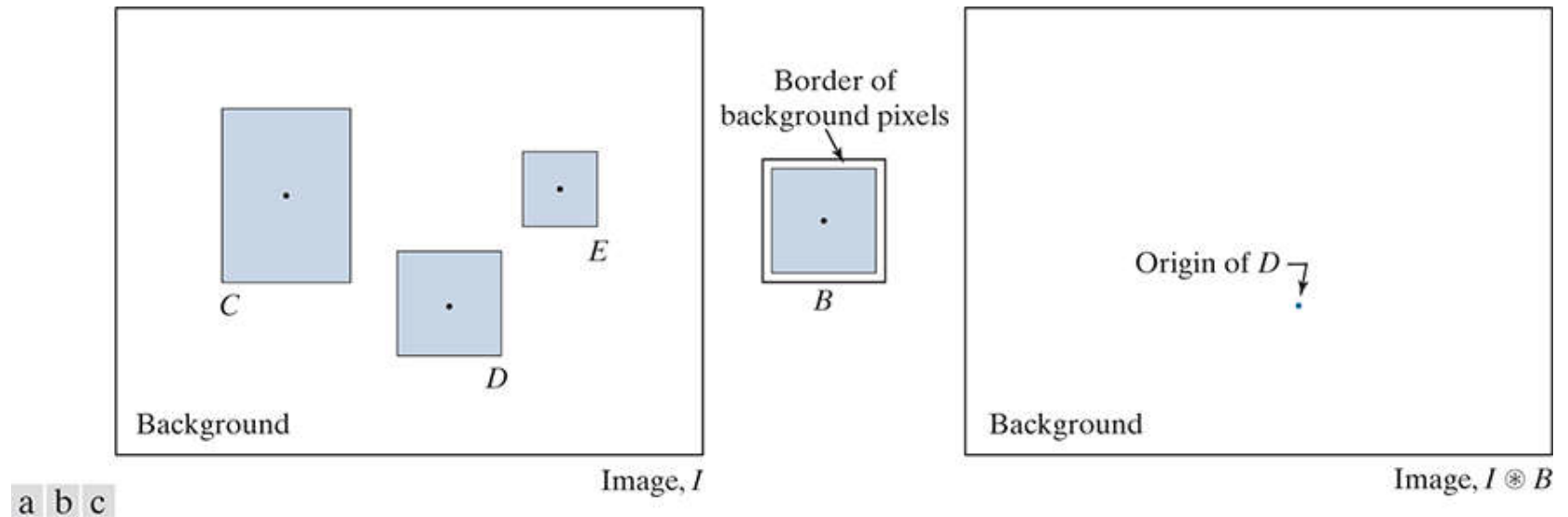
a	b
c	d
e	f

**FIGURE 9.12**

(a) Image consisting of a foreground (1's) equal to the union,  $A$ , of set of objects, and a background of 0's. (b) Image with its foreground defined as  $A^c$ . (c) Structuring elements designed to detect object  $D$ . (d) Erosion of  $A$  by  $B_1$ . (e) Erosion of  $A^c$  by  $B_2$ . (f) Intersection of (d) and (e), showing the location of the origin of  $D$ , as desired. The dots indicate the origin of their respective components. Each dot is a single pixel.

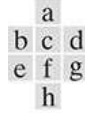


# Hit-or-Miss Transformation (HMT)

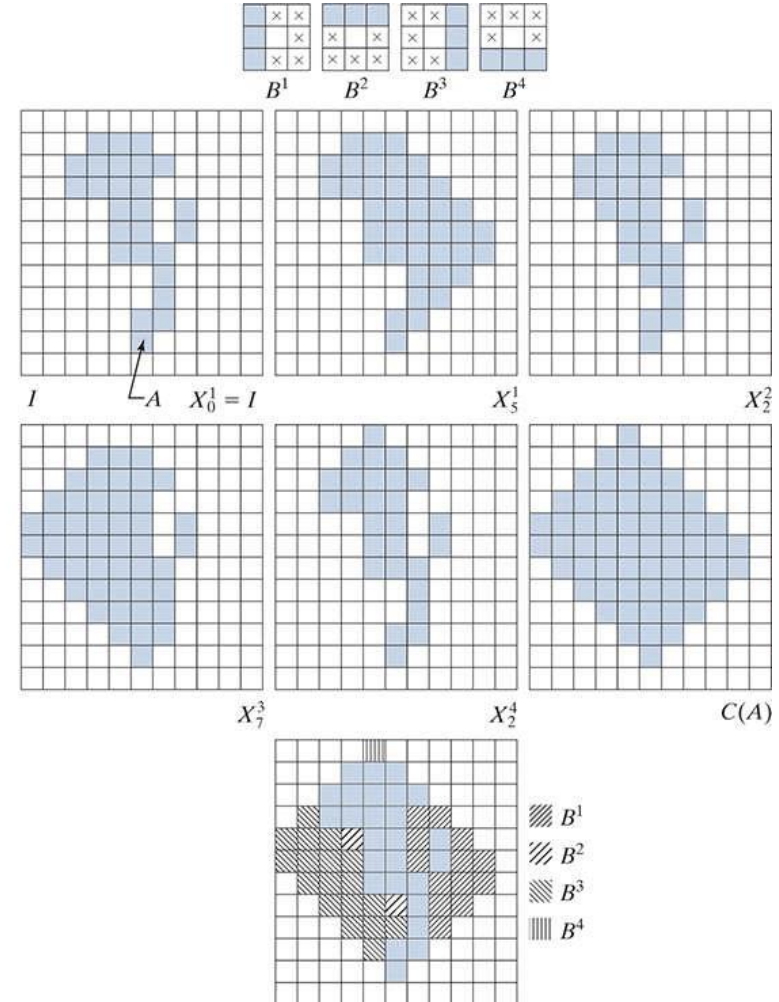


# پوشش محدب Convex Hull

$$\begin{aligned}
 X_0^i &= I \\
 X_k^i &= (X_{k-1}^i \circledast B^i) \cup X_{k-1}^i \\
 i &= 1, 2, 3, 4 \\
 k &= 1, 2, 3, \dots \\
 \text{Until } X_k^i &= X_{k-1}^i \\
 \text{then } D^i &= X_k^i \\
 CH &= \bigcup_{i=1}^4 D^i
 \end{aligned}$$



**FIGURE 9.21**  
 (a) Structuring elements.  
 (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



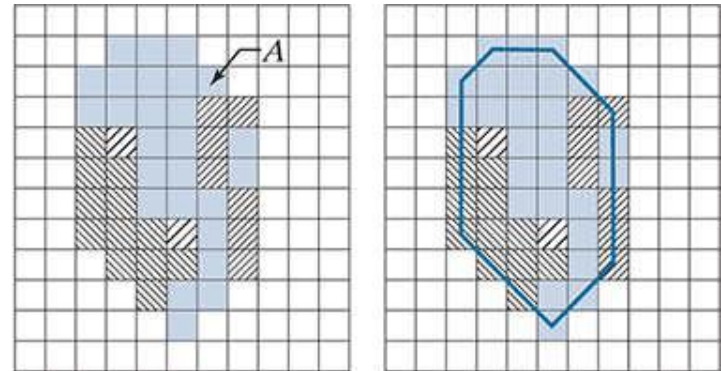


# پوشش محدب Convex Hull

a b

**FIGURE 9.22**

(a) Result of limiting growth of the convex hull algorithm. (b) Straight lines connecting the boundary points show that the new set is convex also.



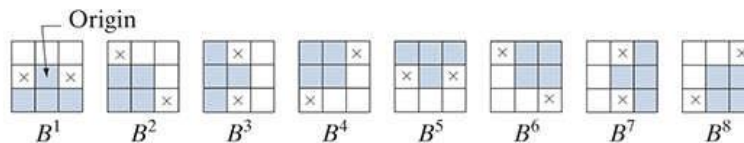
# Thinning

نازک سازی

$$A \otimes B = A \cap (A * B)^c$$

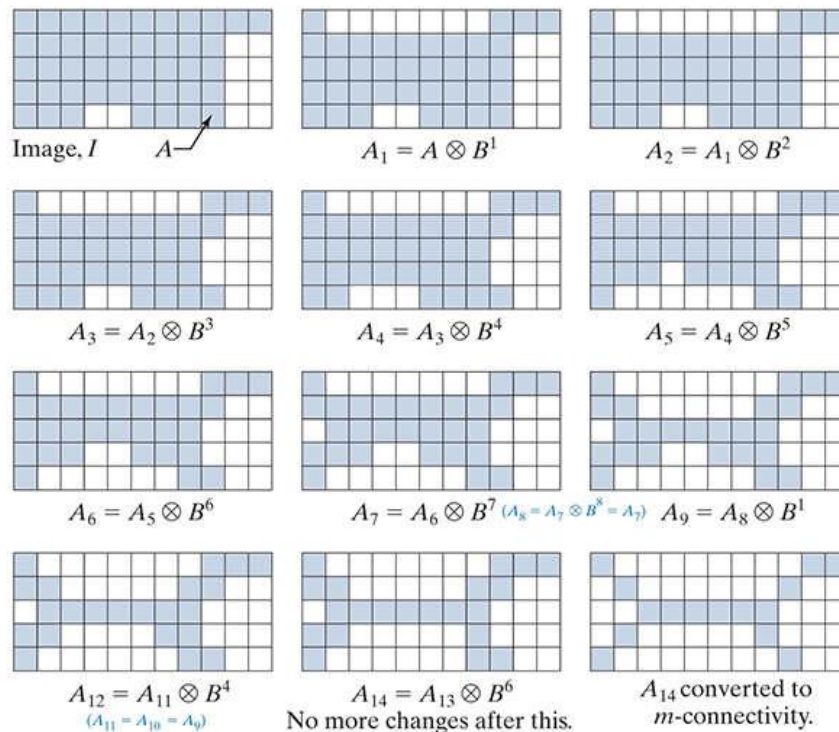
$$A \otimes \{B\} = ((... ((A \otimes B^1) \otimes B^2) ...) \otimes B^n)$$

$$\{B\} = \{B^1, B^2, \dots, B^n\}$$



**FIGURE 9.23**

(a) Structuring elements. (b) Set  $A$ . (c) Result of thinning  $A$  with  $B^1$  (shaded). (d) Result of thinning  $A_1$  with  $B_2$  (e)–(i) Results of thinning with the next six SEs. (There was no change between  $A_7$  and  $A_8$ ) (j)–(k) Result of using the first four elements again. (l) Result after convergence. (m) Result converted to  $m$ -connectivity.



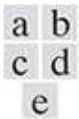
# Thickening

$$A \odot B = A \cup (A * B)$$

$$A \odot \{B\} = ((\dots ((A \odot B^1) \odot B^2) \dots) \odot B^n)$$

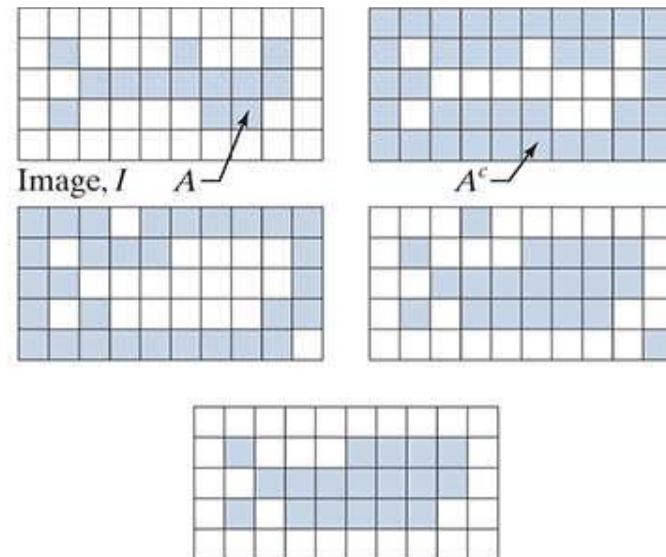
$$\{B\} = \{B^1, B^2, \dots, B^n\}$$

$$A \odot \{B\} = (A^c * \{B\})^c$$



**FIGURE 9.24**

(a) Set  $A$ . (b) Complement of  $A$ . (c) Result of thinning the complement. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.



# Skeleton

$$A \ominus kB = ((... ((A \ominus B) \ominus B) ...) \ominus B)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

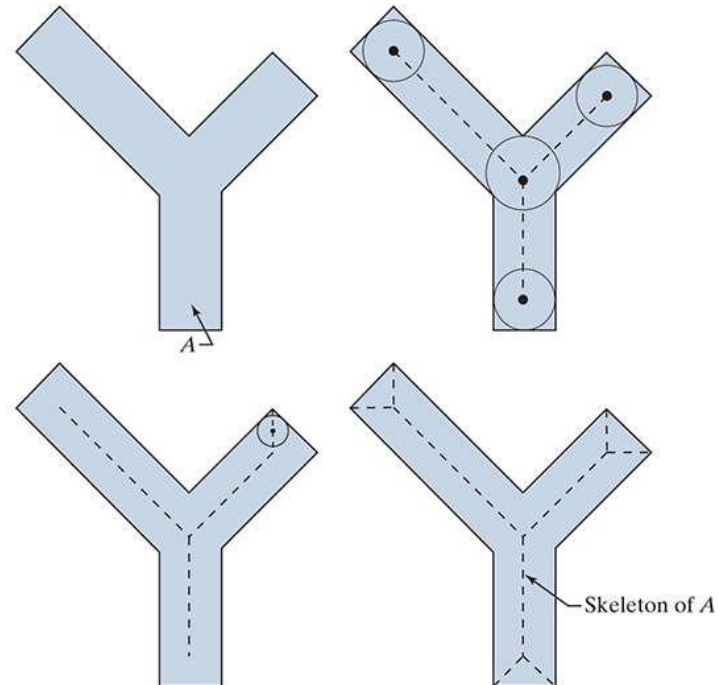
$$S(A) = \bigcup_{k=0}^K S_k(A)$$

$$K = \max\{k \mid (A \ominus kB) \neq \emptyset\}$$

a b  
c d

**FIGURE 9.25**

(a) Set  $A$ . (b) Various positions of maximum disks whose centers partially define the skeleton of  $A$ . (c) Another maximum disk, whose center defines a different segment of the skeleton of  $A$ . (d) Complete skeleton (dashed).

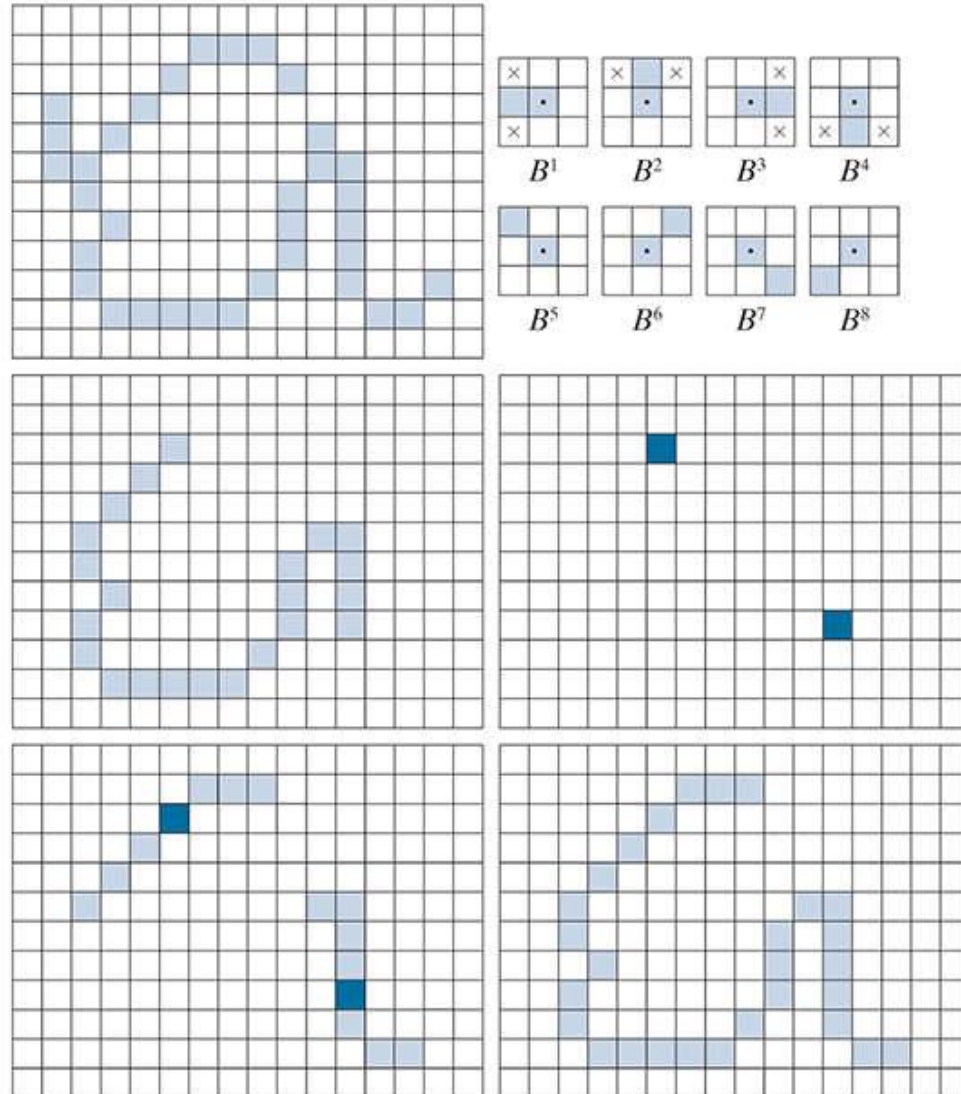


# Pruning

a b  
c d  
e f

**FIGURE 9.27**

(a) Set  $A$  of foreground pixels (shaded). (b) SEs used for deleting end points. (c) Result of three cycles of thinning. (d) End points of (c). (e) Dilation of end points conditioned on (a). (f) Pruned image.



# Grayscale Erosion and Dilation

$$\begin{aligned} [f \ominus b](x, y) &= \min_{(s,t) \in b} \{f(x + s, y + t)\} \\ [f \oplus b](x, y) &= \max_{(s,t) \in \hat{b}} \{f(x - s, y - t)\} \end{aligned}$$

# Grayscale Opening and Closing

$$\begin{aligned} f \circ b &= (f \ominus b) \oplus b \\ f \cdot b &= (f \oplus b) \ominus b \end{aligned}$$

# Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$

# *References*

- Morphological Image Processing
- Gonzalez: Chapter 9  
Szeliski: Section 3.3.2



# *Background Subtraction*

# *Background Subtraction – Input 1*



# *Background Subtraction – Input 2*



# *Background Subtraction – Input 3*





# *Background Subtraction – Input 4*



# *Background Subtraction – Input 5*



# *Background Subtraction – Background*





# *Background Subtraction*





# Background Subtraction – input 1



# Background Subtraction – input 2



# Background Subtraction – input 3





# Background Subtraction – input 4



# Background Subtraction – Background



# Background Subtraction





# Foreground



# Foreground





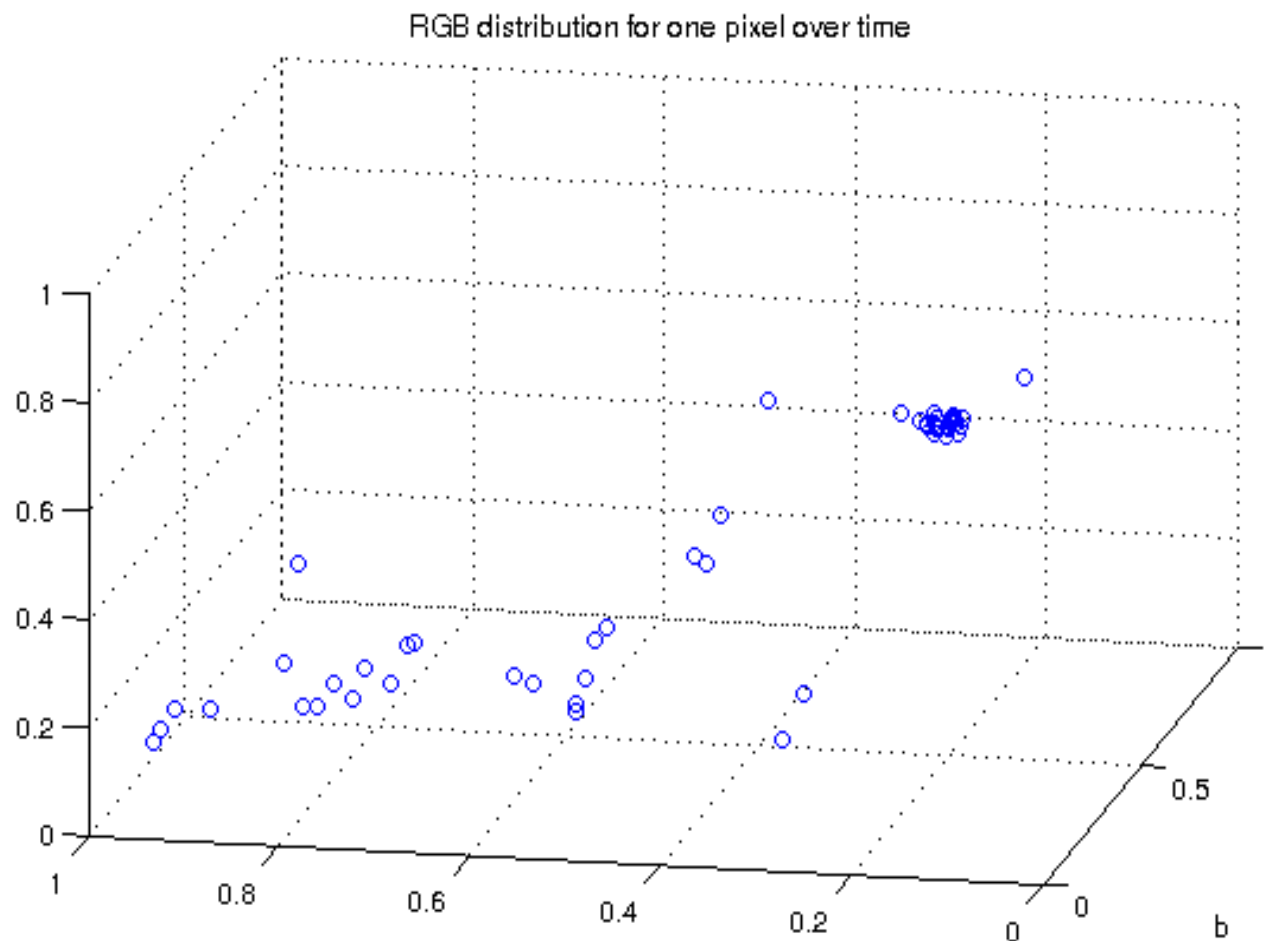
# Method: Median

`size(frames): [640, 480, 3, 100]`

`Background = median(frames, 4);`

- ❖ Pros: quick, simple, fast
- ❖ Cons: Each pixel needs to be foreground in more than 50% of frames

# Method: RANSAC



# Method: RANSAC

- Sample several RGB values
- Pick the one with the most near neighbours
- ❖ Pros: Accurate, 30% foreground is often enough
- ❖ Cons: Slow, complex

# Other ideas to improve background

## Spatial Clues

Neighboring pixels need to agree

## Mean-shift

Take the mean value of the concentrated points to reduce noise

Hole filling techniques for uncertain pixels