Medical Image Analysis and Processing

Medical Image Registration Methods

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Contents

- MI Variations
- Numerical Issues
- Free Form Deformation (FFD)
- Error and Validation
- Deep Registration

Rényi Mutual Information

› Rényi Entropy:

$$E_\alpha = \frac{1}{1-\alpha}\log_2\left(\sum_{i=0}^{255} p_i^\alpha\right)$$

› Rényi Mutual Information:

$$MI_\alpha = \frac{E_\alpha^i + E_\alpha^j}{E_\alpha^{i,j}}$$

$$E_\alpha^i \sim p_i = \sum_{j=0}^{255} p_{ij}\,, E_\alpha^j \sim p_j = \sum_{j=0}^{255} p_{ij}\,, E_\alpha^{i,j} \sim p_{ij}$$

Tsallis Mutual Information

- Tsallis Entropy:

$$S_q = \frac{1}{(q-1)}\left(1 - \sum_{i=0}^{255}\sum_{j=0}^{255} p_{ij}^{q}\right)$$

- Tsallis Mutual Information:

$$R_q = S_q^i + S_q^j + (1-q)S_q^i S_q^j - S_q$$

$$S_q^i = \frac{1}{q-1}\sum_{j=0}^{255} p_{ij}\left(1 - p_{ij}^{q-1}\right)$$

$$S_q^j = \frac{1}{q-1}\sum_{i=0}^{255} p_{ij}\left(1 - p_{ij}^{q-1}\right)$$

f-information

- There are several alternatives:
- $I_\alpha = \frac{1}{\alpha(\alpha-1)}\left(\sum_{i=0}^{255}\sum_{j=0}^{255}\frac{p_{ij}^{\alpha}}{\left(p_i p_j\right)^{\alpha-1}} - 1\right), \alpha \neq \{0,1\}$
- $M_\alpha = \sum_{i=0}^{255}\sum_{j=0}^{255}\left|p_{ij}^{\alpha} - \left(p_i p_j\right)^{\alpha}\right|^{\frac{1}{\alpha}}, 0 < \alpha \leq 1$
- $\chi_\alpha = \sum_{i=0}^{255}\sum_{j=0}^{255}\frac{\left|p_{ij} - p_i p_j\right|^{\alpha}}{\left(p_i p_j\right)^{\alpha-1}}, \alpha > 1$

Numerical Issue

› MI derivative $w.r.t$ transform parameters!

› Estimate the pdf(s) using Parzen windows (kernel methods)

Kernel pdf estimation

› Parzen methods:

$$p(z) \approx P^*(z) \equiv \frac{1}{N_A} \sum_{z_j \in A} R\left(z - z_j\right)$$

› where N_A is the number of trials in the sample A, and R is kernel function (with pdf properties), like as Gaussian:

$$G_\psi(z) \equiv (2\pi)^{\frac{-n}{2}} |\psi|^{\frac{-1}{2}} \exp\left(-\frac{1}{2} z^{\mathrm{T}} \psi^{-1} z\right)$$

Entropy Estimation

› Entropy is estimated as:

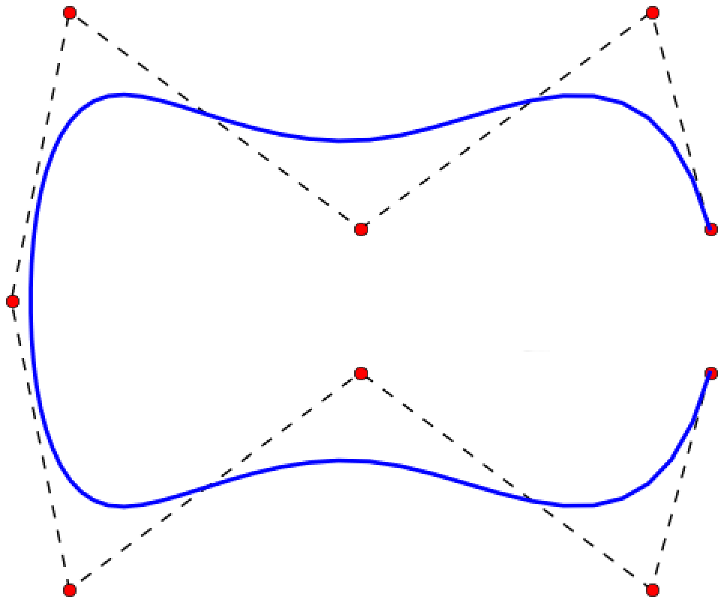
$$h(z) \approx -\frac{1}{N_B}\sum_{z_i \in B} \ln P^*(z_i)$$

› or:

$$h(z) \approx h^*(z) \equiv \frac{-1}{N_B}\sum_{z_i \in B} \ln \frac{1}{N_A}\sum_{z_j \in A} G_\psi\left(z_i - z_j\right)$$

Free From Deformation (FFD):

- Cubic B-Spline: A method of curve interpolation based on control points
- $s(x) = \sum_{m=0}^{3} B_m(u)\phi_{j+m}$
- $j = \left\lfloor \frac{x}{n_x} \right\rfloor - 1, \quad u = \frac{x}{n_x} - \left\lfloor \frac{x}{n_x} \right\rfloor, \quad 0 \leq x < X, \phi_j:$ Control Point
- $B_0(x) = \frac{(1-u)^3}{6}$
- $B_1(x) = \frac{(3u^3-6u^2+4)}{6}$
- $B_2(x) = \frac{(-3u^3+3u^2+3u+1)}{6}$
- $B_0(x) = \frac{u^3}{6}$



Free From Deformation (FFD):

- Definition:

$$\Omega = \{(x, y, z) | 0 \le x < X,\ 0 \le y < Y,\ 0 \le z < Z\}$$

Φ: $n_x \times n_y \times n_z$: Mesh of Control Points, $\phi_{i,j,k}$ with uniform spacing δ

- Cubic B-Spline in 3D (Tensor Product):

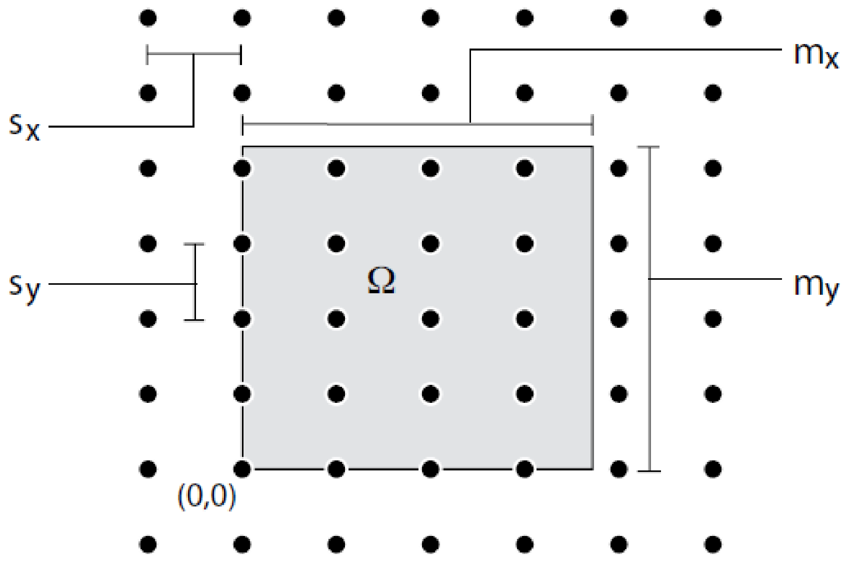
$$T(x, y, z) = \sum_{l=0}^{3} \sum_{m=0}^{3} \sum_{n=0}^{3} B_l(u) B_m(v) B_n(w) \phi_{i+l, j+m, k+n}$$

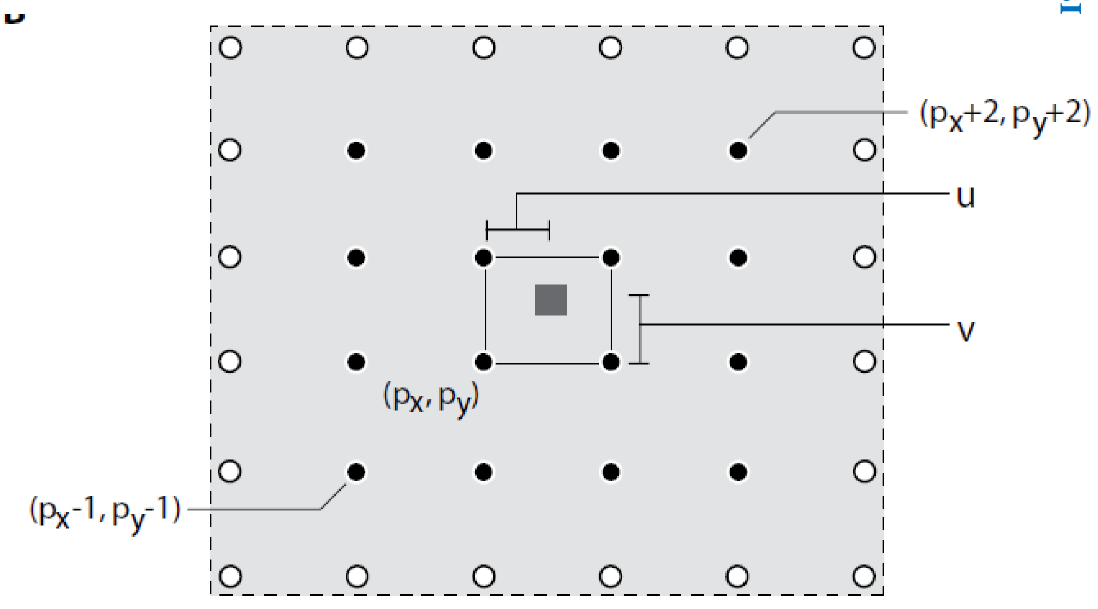
$$i = \left\lfloor \frac{x}{n_x} \right\rfloor - 1,\ j = \left\lfloor \frac{y}{n_y} \right\rfloor - 1,\ k = \left\lfloor \frac{z}{n_z} \right\rfloor - 1$$

$$u = \frac{x}{n_x} - \left\lfloor \frac{x}{n_x} \right\rfloor,\ v = \frac{y}{n_y} - \left\lfloor \frac{y}{n_y} \right\rfloor,\ w = \frac{z}{n_z} - \left\lfloor \frac{z}{n_z} \right\rfloor$$

Free From Deformation (FFD):

› Control Points in 2D





Registration Using FFD

- Find best matching Control Points ($\phi(i,j,k)$)
- Define Optimization Criteria (Similarity + Smoothness):

$$C(\Phi) = -C_{Similarity}(F(x,y,x,), M(\boldsymbol{T}(x,y,z))) + \lambda C_{Smoothness}(\boldsymbol{T})$$

- F : Fixed image
- M : Moving image
- $\boldsymbol{T}$: transform

Registration Using FFD - Smoothness

- Any smoothness criteria (on transform or control points movement):

$$C_{smoothness} = \frac{1}{Volume}\int_0^X\int_0^Y\int_0^Z\left[\left(\frac{\partial^2 T}{\partial x^2}\right)^2 + \left(\frac{\partial^2 T}{\partial x^2}\right)^2 + \left(\frac{\partial^2 T}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 T}{\partial x \partial y}\right)^2 + 2\left(\frac{\partial^2 T}{\partial x \partial z}\right)^2 + 2\left(\frac{\partial^2 T}{\partial y \partial z}\right)^2\right] dxdydz$$

- or

$$C_{smoothness} = \frac{1}{N}\sum_{i,j,k}\|\nabla T_x(i,j,k)\|^2 + \|\nabla T_y(i,j,k)\|^2 + \|\nabla T_z(i,j,k)\|^2$$

Registration Using FFD - Similarity

- Any Similarity/distance measure:
- $C^{MI}_{Similarity}(A, B) = H(A) + H(B) - H(A, B)$
- $C^{NMI}_{Similarity}(A, B) = \frac{H(A)+H(B)}{H(A,B)}$
- $C^{SSD}_{Similarity}(A, B) = \frac{1}{n}\sqrt{\sum(A - B)^2}$
- $C^{SAD}_{Similarity}(A, B) = \frac{1}{n}\sum|A - B|$
- $C^{CC}_{Similarity}(A, B) = \frac{\sum(A-\bar{A})(B-\bar{B})}{\sqrt{\sum(A-\bar{A})^2}\sqrt{\sum(B-\bar{B})^2}}$

Registration Using FFD - transform

› Transformation (Global+Local):

› $T_{Global}(x, y, z) = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} \theta_{14} \\ \theta_{24} \\ \theta_{34} \end{bmatrix}$

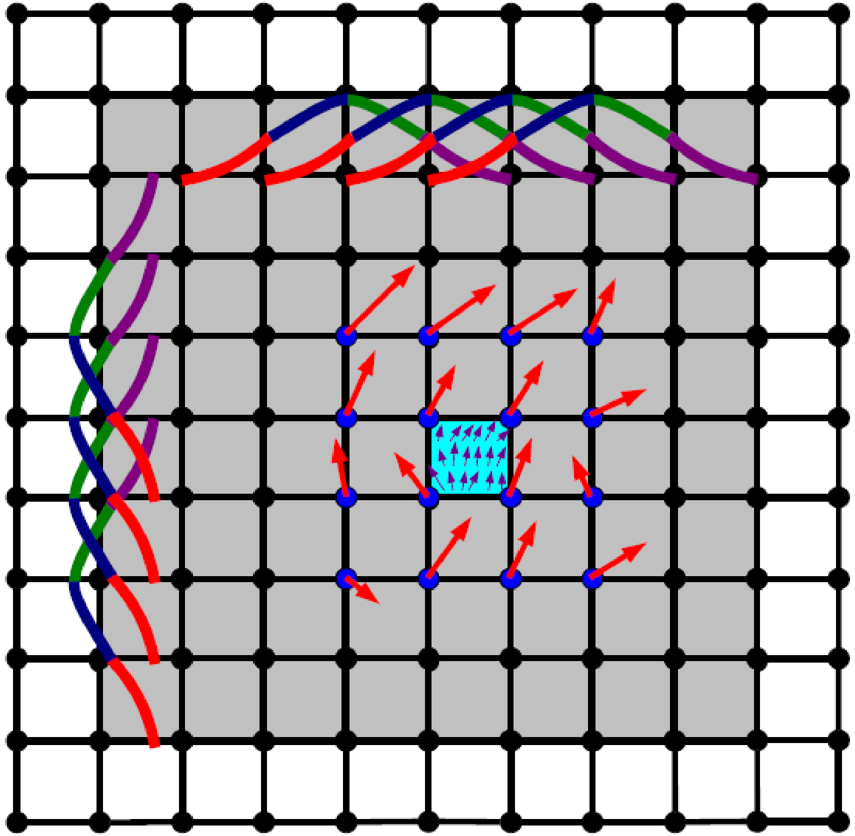
› $T_{Local}\,(x, y, z) = \sum_{l=0}^{3} \sum_{m=0}^{3} \sum_{n=0}^{3} B_l(u) B_m(v) B_n(w) \phi_{i+l, j+m, k+n}$

› $T(x, y, z) = T_{Global}(x, y, z) + T_{Local}(x, y, z)$

› $C(\Phi, \Theta) = -C_{Similarity}(F(x, y, z), M(T(x, y, z))) + \lambda C_{Smoothness}(T)$

FFD – Vector Field

› Vector Field:



Error and Validation

› Accuracy of registration?

› The main PROBLEM is lack of ground truth!

Errors in Point Based Registration

- Fiducial Localization Error (FLE):
  - Physical Landmark Uncertainty.
- Fiducial Registration Error (FRE):
  - Distance between correspondence Landmark after registration phase.
- Target Registration Error (TRE):
  - Distance between correspondence points after registration phase.

Errors in Point Based Registration

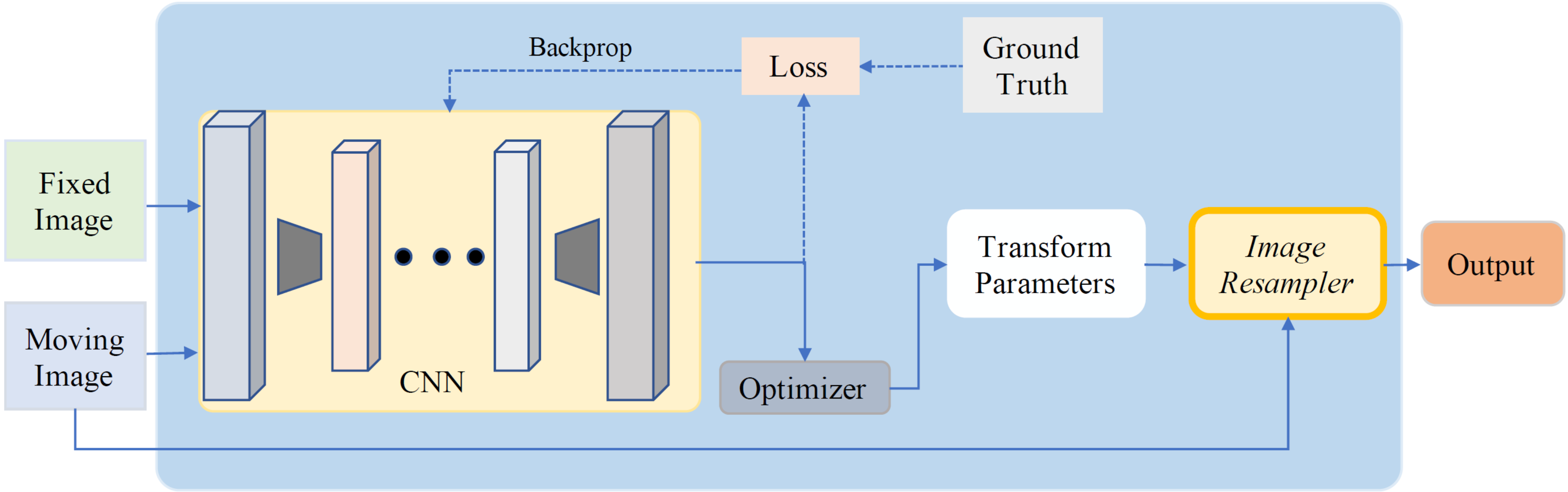
- FLE:
  - Operator and Algorithm.
- FRE:
  - Rigid transform (large sets of points)
- TRE:
  - Hard to estimation: Training and test sets.

Errors in Image Based Registration

- Similarity measure!

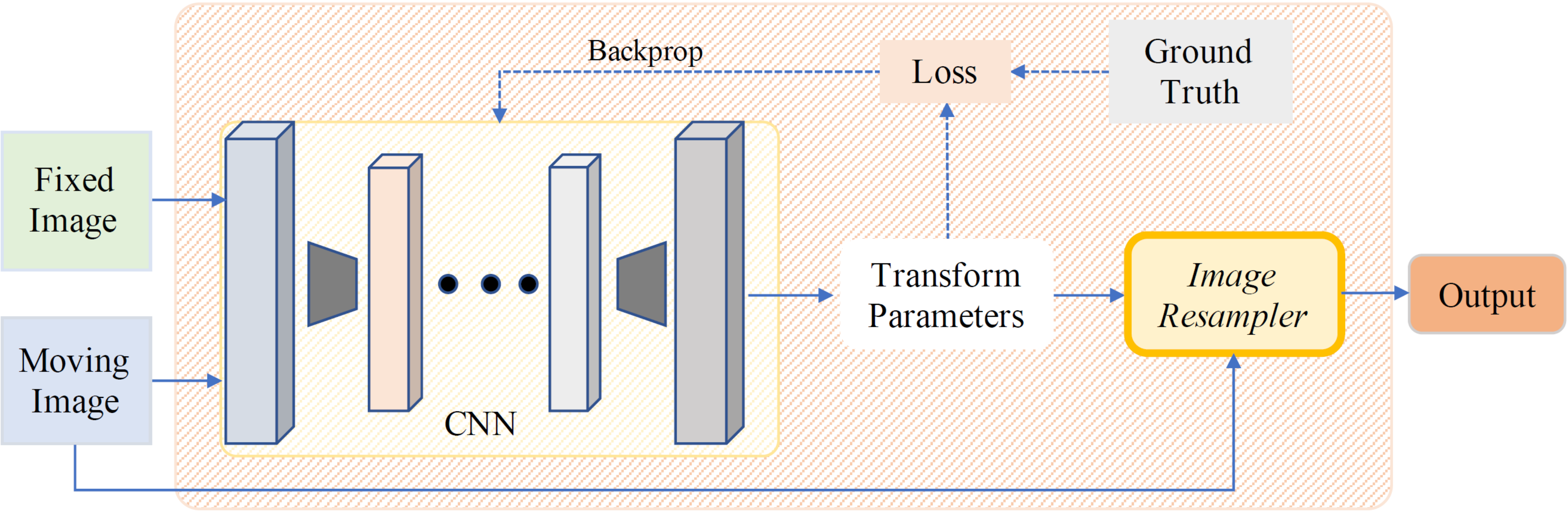
Registration Using Deep Networks

› Deep Registration Pipeline:



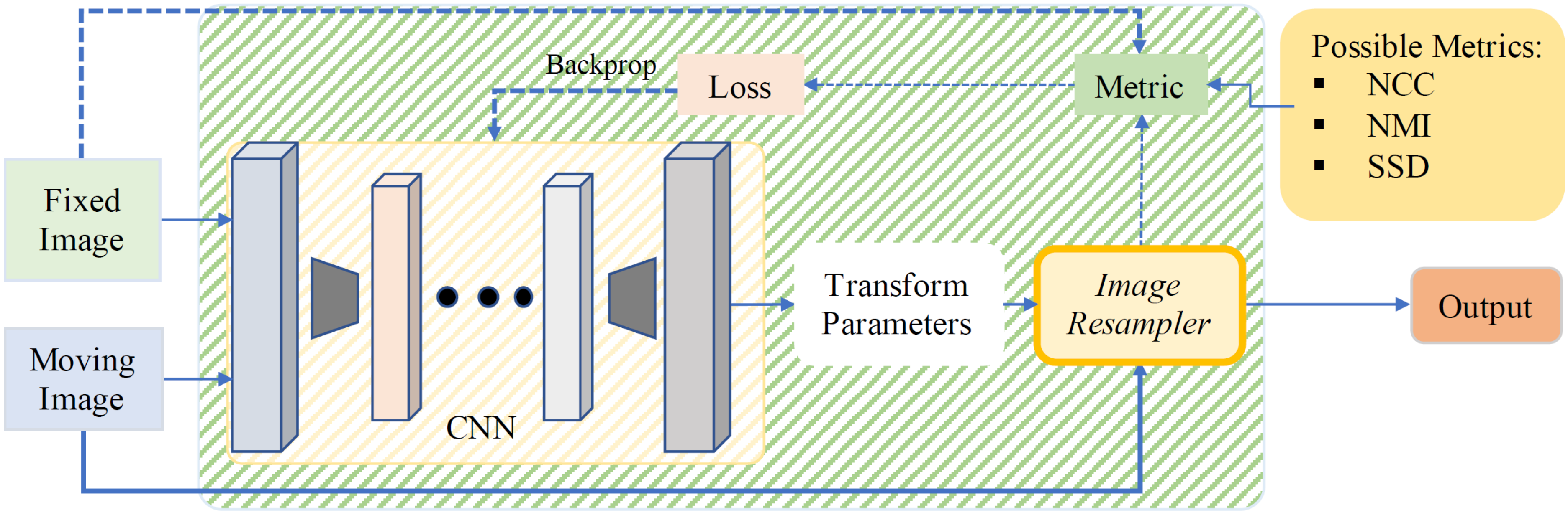
Registration Using Deep Networks

› Supervised registration.



Registration Using Deep Networks

› Unsupervised registration.



The End

› AnY QuEsTiOn?