In the game of God

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Theoretical exercise:

Question 1:

From slides we know:

$$E(u) = \int_{\Omega} L\left(u, u_{x_1}, u_{x_2}, u_{x_1x_1}, u_{x_2x_2}, u_{x_1x_2}\right) dx_1 dx_2$$

$$\frac{\partial L}{\partial u} - \left(\frac{\partial}{\partial x_1}\frac{\partial L}{\partial u_{x_1}} + \frac{\partial}{\partial x_2}\frac{\partial L}{\partial u_{x_2}}\right) + \left(\frac{\partial^2}{\partial {x_1}^2}\frac{\partial L}{\partial u_{x_1x_1}} + \frac{\partial^2}{\partial {x_2}^2}\frac{\partial L}{\partial u_{x_2x_2}} + \frac{\partial^2}{\partial x_1 x_2}\frac{\partial L}{\partial u_{x_1x_2}}\right) = 0$$

And:

$$|\nabla u| = (u_{x_1} + u_{x_2})^{1/2}$$

And in this problem we have:

$$L = \int_{\Omega} |\nabla u| + \frac{\lambda}{2}\left\{\int_{\Omega} (u - v)^2 - \sigma^2(\Omega)\right\}$$

$$E(u) = \min_{u}\{\int_{\Omega} |\nabla u| + \frac{\lambda}{2}(u - v)^2 - \frac{\lambda}{2}\sigma^2(\Omega)\}$$

In this problem that's enough to calculate this phrase:

$$\frac{\partial L}{\partial u} - \left(\frac{\partial}{\partial x_1}\frac{\partial L}{\partial u_{x_1}} + \frac{\partial}{\partial x_2}\frac{\partial L}{\partial u_{x_2}}\right) = 0$$

$$\frac{\partial L}{\partial u} = \frac{\partial}{\partial u}\left(\int_{\Omega} |\nabla u| + \frac{\lambda}{2}(u - v)^2 - \frac{\lambda}{2}\sigma^2(\Omega)\right) = \lambda(u - v)$$

$$\left(\frac{\partial}{\partial x_1}\frac{\partial L}{\partial u_{x_1}}+\frac{\partial}{\partial x_2}\frac{\partial L}{\partial u_{x_2}}\right)$$

$$=\frac{\partial}{\partial x_1}\left(\frac{\partial}{\partial u_{x_1}}\left(\left(u_{x_1}+\ u_{x_2}\right)^{\frac{1}{2}}\right)\right)+\frac{\partial}{\partial x_2}\left(\frac{\partial}{\partial u_{x_2}}\left(\left(u_{x_1}+\ u_{x_2}\right)^{\frac{1}{2}}\right)\right)$$

$$=\frac{\partial}{\partial x_1}\left(2*\frac{1}{2}*u_{x_1}\left(\left(u_{x_1}+\ u_{x_2}\right)^{\frac{-1}{2}}\right)\right)$$

$$+\frac{\partial}{\partial x_2}\left(2*\frac{1}{2}*u_{x_2}\left(\left(u_{x_1}+\ u_{x_2}\right)^{\frac{-1}{2}}\right)\right)$$

$$=\ \frac{\partial}{\partial x_1}\left(\frac{u_{x_1}}{\sqrt{u_{x_1}+\ u_{x_2}}}\right)+\frac{\partial}{\partial x_2}\left(\frac{u_{x_2}}{\sqrt{u_{x_1}+\ u_{x_2}}}\right)=\mathrm{div}(\frac{\nabla u}{|\nabla u|})$$

So in the end, we have:

$$\frac{\partial L}{\partial u}=\lambda(u-v)-\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)=0\quad\Rightarrow\quad\lambda(u-v)+\mathrm{div}\left(\frac{\nabla u}{|\nabla u|}\right)=0$$

And for a second condition, I think it comes from for boundary condition for satisfying that doesn't allows image diffuse out of the boundary of the image.

Question 2:

$$f\left(x\middle|\mu,b\right)=\frac{1}{2b}\exp(-\frac{\left|x-\mu\right|}{b})$$

Show that:

$$\min_{u}\{\int_{\Omega}\ |\nabla u|+\lambda|u-v|\}$$

As we saw in slides, we can write:

$$u_{MAP}=\arg\max\ p(u|v)=\mathrm{argmax}\frac{p(v|u)p(u)}{p(v)}=\mathrm{argmax}\,p(v|u)p(u)$$

Thus:

$$u^*_{MAP} = \operatorname{argmin}(u) - \left\{\log\left(p(u)\right) + \log\left(p(v|u)\right)\right\}$$

We consider $p(u) \propto e^{-f(u)}$ so we have:

$$u^*_{MAP} = \operatorname{argmin}(u)\{f(u) + -\frac{|v-u|}{2b^2}\}$$

So if we assume $\lambda = -\frac{1}{2b^2}$ and $f(u) = |\nabla u|$ we have:

$$u^*_{MAP} = \min_u\{\int_\Omega |\nabla u| + \lambda|u-v|\}$$

Question 3:

The variation of PFOM, SSIM, PSNR and EPI, with respect to the number of iterations are compared in terms of their ability to quantify blurring of edge pixels during restoration, with the support of qualitative evaluation of the restored images and the binary edge map extracted from them.

$$\text{EPI}=\frac{\Gamma\left(\Delta s-\overline{\Delta s}\,,\ \widehat{\Delta s}-\overline{\widehat{\Delta s}}\right)}{\sqrt{\Gamma\left(\Delta s-\overline{\Delta s},\Delta s-\overline{\Delta s}\right).\Gamma\left(\widehat{\Delta s}-\overline{\widehat{\Delta s}},\widehat{\Delta s}-\overline{\widehat{\Delta s}}\right)}} \quad (8)$$

$$\Gamma(s_1,s_2)=\sum_{i,j\in ROI} s_1(i,j).s_2(i,j)(9)$$

Where $\Delta s(i,j)$and $\widehat{\Delta s(\iota,\jmath)}$are the high pass filtered version of the Region of Interest (ROI) in the reference s(i,j) and its degraded or its transformed version $\widehat{\Delta s(\iota,\jmath)}$, obtained with a standard approximation of 3*3 Laplacian nel. $\overline{\Delta s}$ and $\overline{\Delta s}$ are the mean of Laplacian filtered ROI in the reference and the transformed images, respectively.

Question 4:

The Trilateral Filter for High Contrast Images and Meshes:

Edges are perceived discontinuities that are not always matched to a reliable mathematical discontinuity. In the past six years, several edge-preserving smoothing methods have addressed the stubborn problem of visual appearance-preservin

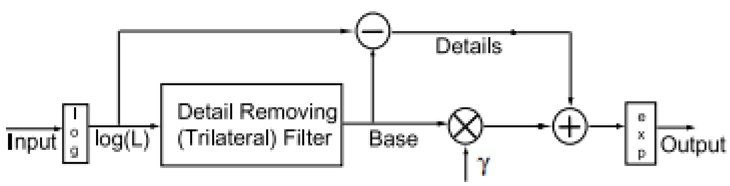


Figure 3: *Contrast reduction method based on edge-preserving filters.*

This frequently recurring contrast reduction scheme in Figure 3 is homomorphic; it filters the logarithm of intensity as Stockham, using a "detail removing filter" of some sort to smooth away the small and complex variations presumably due to reflectance changes. The large simplified illumination-related filter output "Base" is compressed, usually by a scale factor γ (and sometimes offset by a scale factor log(M), and then added back to the complex details that the filter removed. Conversion from logarithmic back to a linear signal produces the displayed image result. A few papers such as \extended this idea by using multiple filters to refine compression amount.

With few exceptions, edge-preserving filters useful for contrast reduction fall into two broad classes of (a) iterative solvers and (b) nonlinear filter.

The trilateral filter is a substantially improved "detailremoving filter" for Figure 3 because it:

- better approximates scene illumination as a sharplybounded, piecewise smooth signal with locally constant gradient,
- works in one pass, without an iterative PDE solver,
- forms sharp boundaries and corners much like shock forming PDEs,
- self-adjusts to the image, requiring one user-supplied parameter,
- extends easily to N-dimensional signals, both discrete and continuous-valued

Refrence:

- https://users.cs.northwestern.edu/~jet/Publications/Tumblin_EGSR2003paper.pdf

Simulation exercises:

Question 1:

a) We create a phantom and then apply Gaussian noise with variance (0.05)^2=0.0025 on it. You can see these images below:

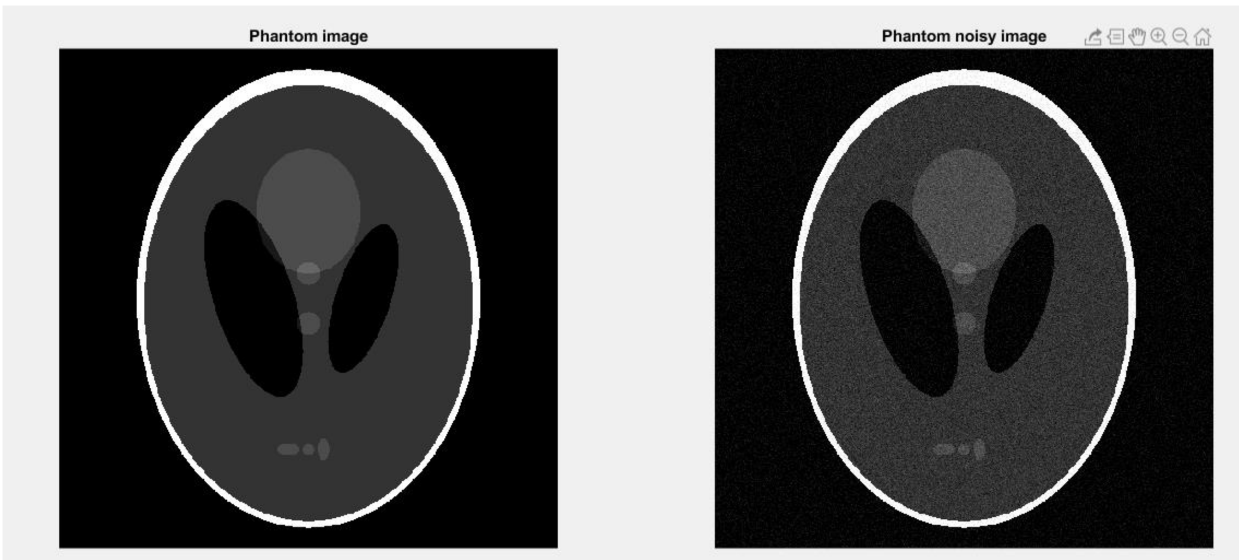


Figure 1: original & noisy image

b) now we apply NLM filter to noisy image. We have this result:

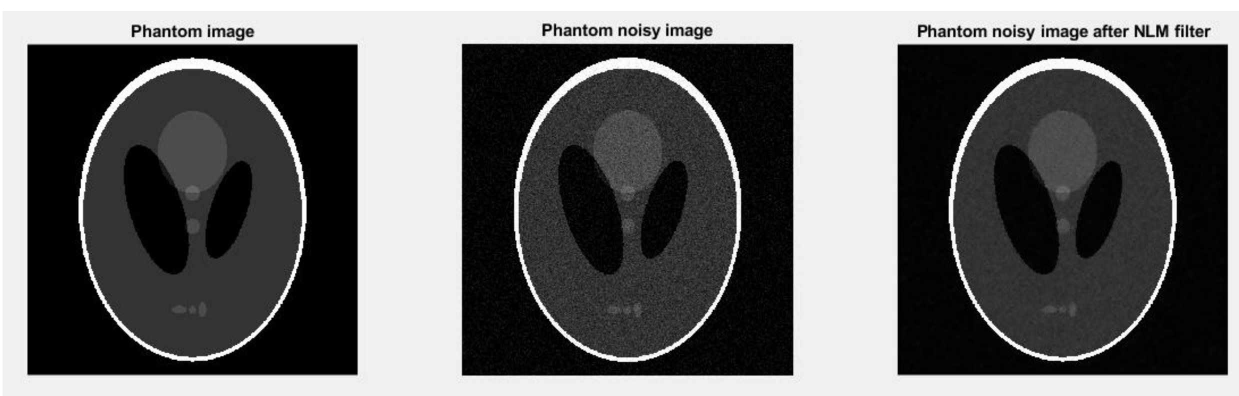


Figure 2: image after NLM filtering

c)at the end we can calculate SNR and EPI after filtering.

	SNR	EPI
Noisy Image	15.4977	0.7326
Noisy Image after NLM	22.6506	0.9655

Question 2:

a) we know we use bilateral filter because do not hurt to edges. So $h_g$controls edge preserving capability and $h_x$controls the smoothness of details.
If we have an image with high contrast, h_g should be high.
If we have an image with large size, h_x should be high.

b) now we apply bilateral filter to noisy image. We have this result:

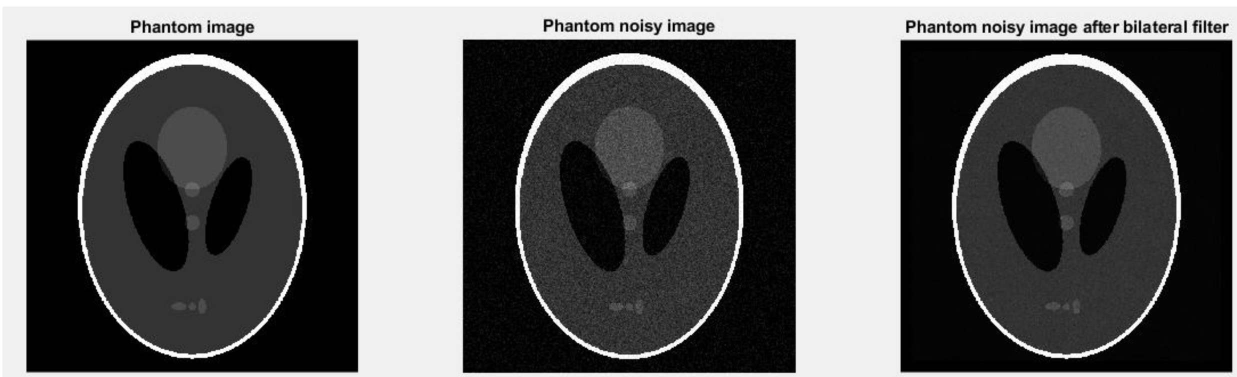


Figure 3: image after apply bilateral filter

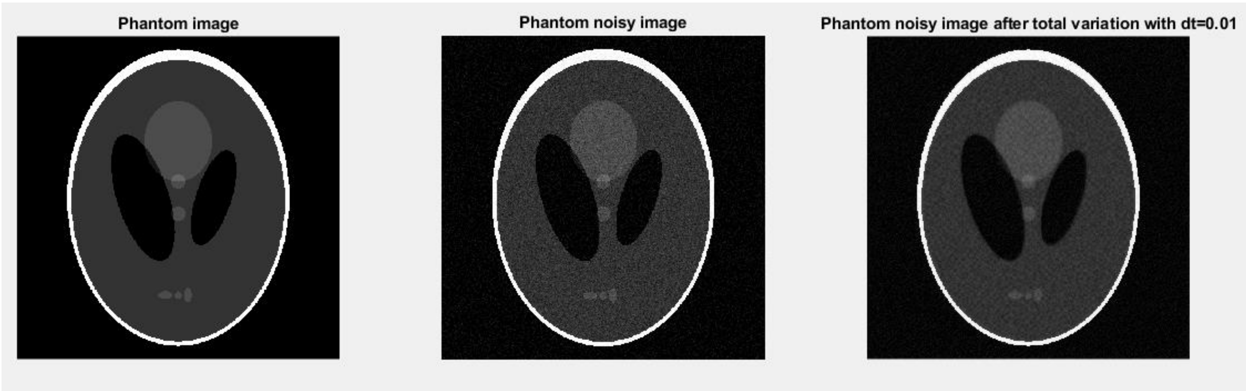
c) at the end we can calculate SNR and EPI after filtering.

	SNR	EPI
Noisy Image	15.5023	0.7316
Noisy Image after bilateral filter	22.7276	0.9536

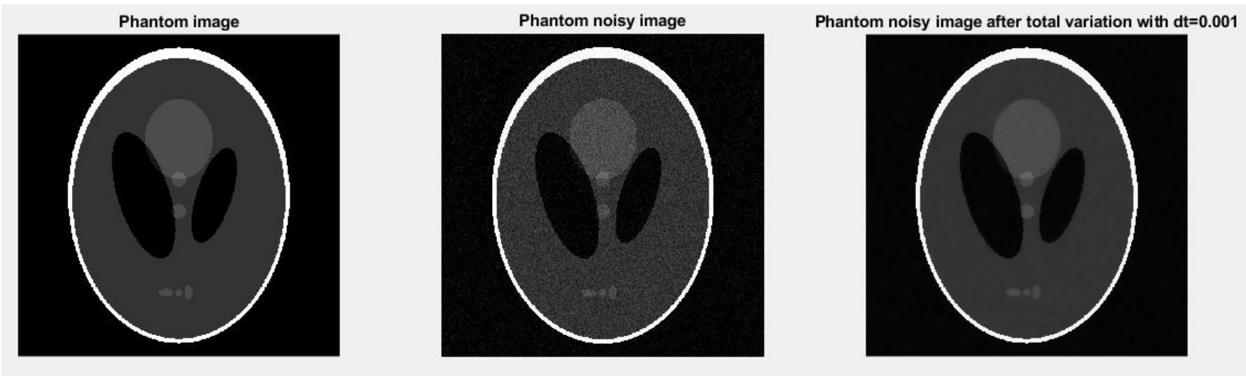
Question 3:

a) in this section we should write TV code.

if we apply TV with dt=0.01, we didn't get a good result. You could see the result below figure:



But, if we apply TV with dt=0.001, we get a good result.



b) at the end we can calculate SNR and EPI after filtering.

	SNR	EPI
Noisy Image	15.5023	0.7316
Noisy Image after total variation with dt=0.01	15.5011	0.6798
Noisy Image after total variation with dt=0.001	22.1156	0.9919

Question 4:

Now we can compare the previous results together.

	SNR	EPI
Noisy Image	15.5023	0.7316
Noisy Image after NLM	22.6506	0.9655
Noisy Image after bilateral filter	22.7276	0.9536
Noisy Image after total variation with dt=0.01	15.5011	0.6798
Noisy Image after total variation with dt=0.001	22.1156	0.9919

Base on the above table, we can see bilateral filter have the best performance for noise reduction between others because have maximum SNR. and if we compare EPI column we can see total variation with dt=0.001 have the good effect for edge preserving .