Medical Image Analysis and Processing

Image Noise Filtering
Anisotropic Diffusion Filter

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Contents

› Discrete Implementation

Discrete Implementation

- Image is discrete, our needs:

$$\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}$$

- $u(x_1, x_2, t) = u(ih, jh, n\Delta t) \rightsquigarrow u_{i,j}^n$
- Fundamental approximation ($h = 1$):

$$\frac{\partial u}{\partial x_1} \cong \frac{u(x_1, x_2, t) - u(x_1 - h, x_2, t)}{h}$$

$$\frac{\partial u}{\partial x_1} \cong \frac{u(x_1 + h, x_2, t) - u(x_1, x_2, t)}{h}$$

$$\frac{\partial u}{\partial x_1} \cong \frac{u(x_1 + h, x_2, t) - u(x_1 - h, x_2, t)}{2h}$$

Discrete Implementation

- Derivatives with respect to time:

$$\frac{\partial u}{\partial t} \cong \frac{u(x_1, x_2, t + \Delta t) - u(x_1, x_2, t)}{\Delta t} = \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t}$$

- General Diffusion filter:

$$\frac{\partial u}{\partial t} = F(u, \nabla u) \Rightarrow \frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = F_{i,j}^{n}$$

$$u_{i,j}^{n+1} = u_{i,j}^{n} + \Delta t \times F_{i,j}^{n}$$

- $u_{i,j}^{0}$: Initial guess (noisy image or its gaussian smoothed)

Discrete Implementation

- We cover isotropic diffusion

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x_1}\left(g(|\nabla u|^2)u_{x_1}\right) + \frac{\partial}{\partial x_2}\left(g(|\nabla u|^2)u_{x_2}\right)$$

- Notation and fundamental approximation:

- $g(|\nabla u|^2) \cong g\left(\left(\frac{u_{i+1,j}-u_{i-1,j}}{2}\right)^2 + \left(\frac{u_{i,j+1}-u_{i,j-1}}{2}\right)^2\right) = g_{i,j}$

- $\frac{\partial u}{\partial x_1} \cong \partial^*_{x_1}(u_{i,j}) = u_{i+0.5,j} - u_{i-0.5,j}$ (Half pixel, estimated via interpolation)

- $\frac{\partial u}{\partial x_2} \cong \partial^*_{x_2}(u_{i,j}) = u_{i,j+0.5} - u_{i,j-0.5}$

Discrete Implementation

- We cover isotropic diffusion

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x_1}\left(g(|\nabla u|^2)u_{x_1}\right) + \frac{\partial}{\partial x_2}\left(g(|\nabla u|^2)u_{x_2}\right)$$

$$\frac{\partial u}{\partial t} = \delta^*_{x_1}\left(g_{i,j}\left(u_{i+0.5,j} - u_{i-0.5,j}\right)\right) + \delta^*_{x_2}\left(g_{i,j}\left(u_{i,j+0.5} - u_{i,j-0.5}\right)\right)$$

$$= g_{i+0.5,j}\left(u_{i+1,j} - u_{i,j}\right) - g_{i-0.5,j}\left(u_{i,j} - u_{i-1,j}\right) + g_{i,j+0.5}\left(u_{i,j+1} - u_{i,j}\right) - g_{i,j-0.5}\left(u_{i,j} - u_{i,j-1}\right)$$

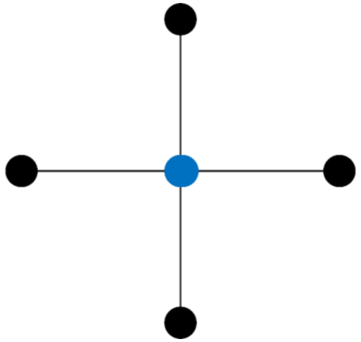
Discrete Implementation

› Half pixel estimation estimated with simple interpolation:

$$g_{i\pm0.5,j\pm0.5} = \frac{g_{i\pm1,j\pm1} + g_{i,j}}{2}$$

› After substitution and simplification:

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n}}{\Delta t} = \frac{1}{2} \sum_{(k,l)=(i\pm1,j\pm1)} \left(g_{k,l}^{n} + g_{i,j}^{n}\right)\left(u_{k,l}^{n} - u_{i,j}^{n}\right)$$



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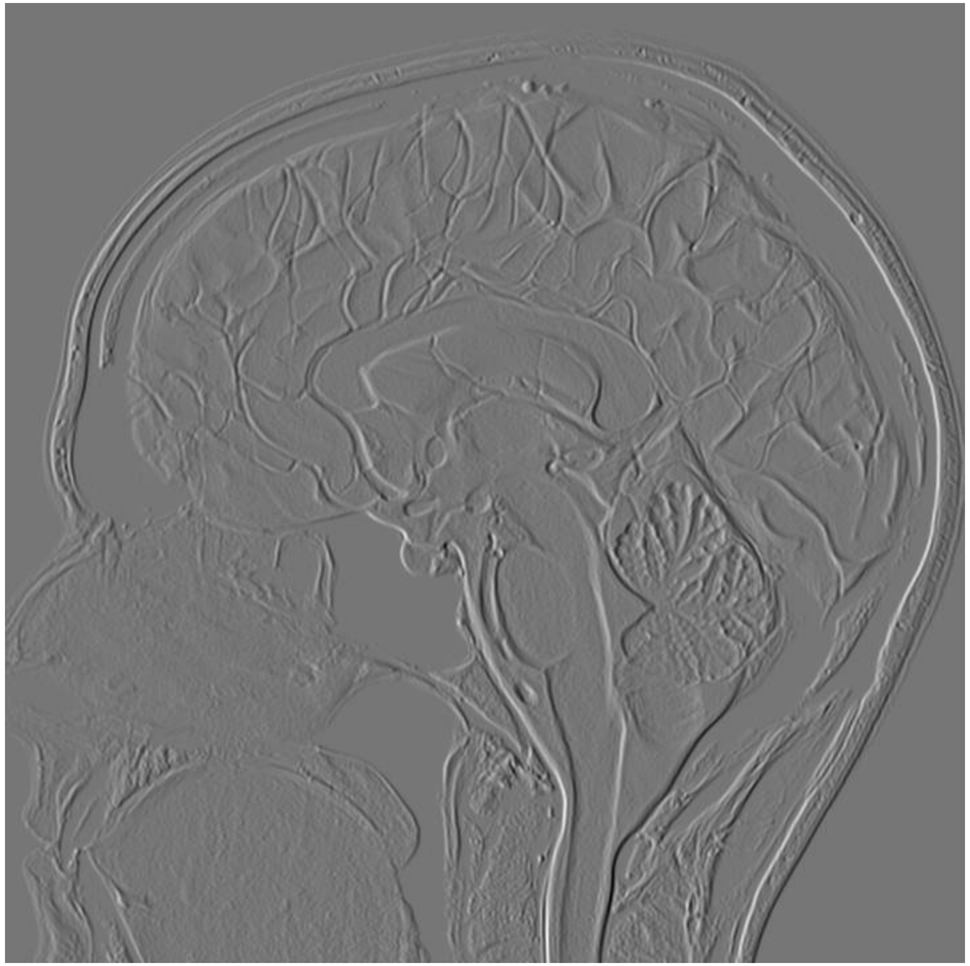
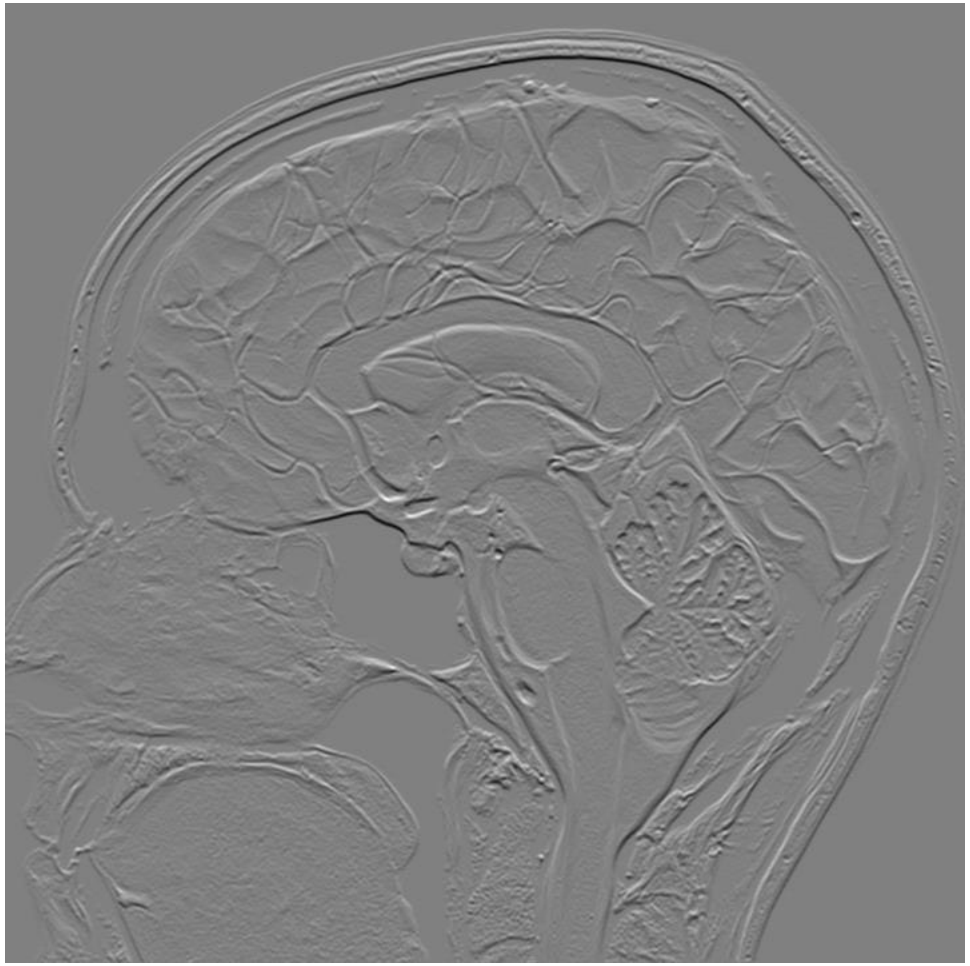
Example – Input Image

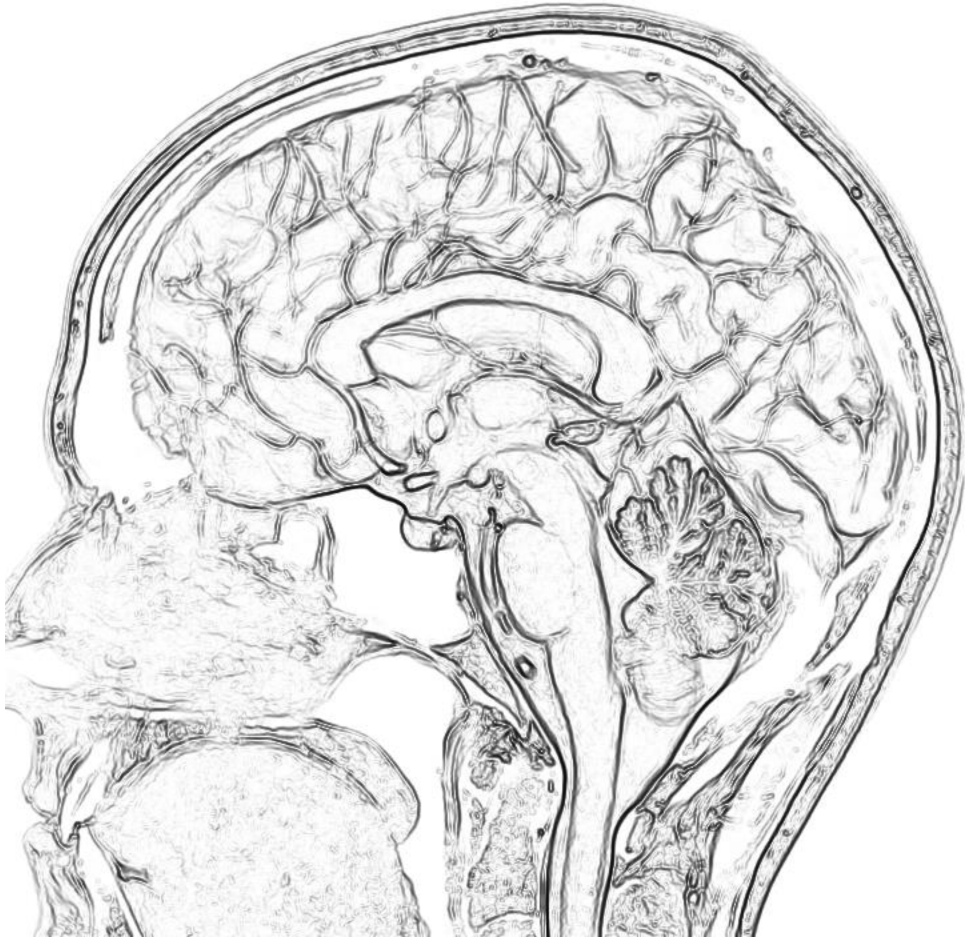
Example: ∇u: $(\partial u / \partial x_1,\ \partial u / \partial x_2)$





Example: $|\nabla u|^2$

Example: $G(|\nabla u|^2)$

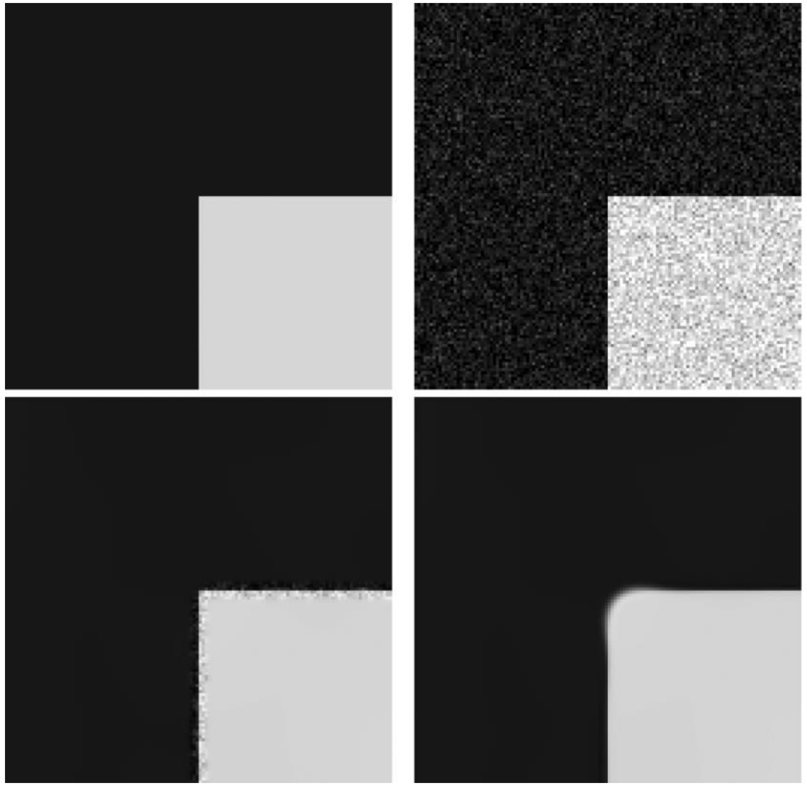


Example $G\nabla u$: $(G \times (\partial u/\partial x_1), G \times (\partial u/\partial x_2))$

Example: $div(G(|\nabla u|^2)\nabla u)$

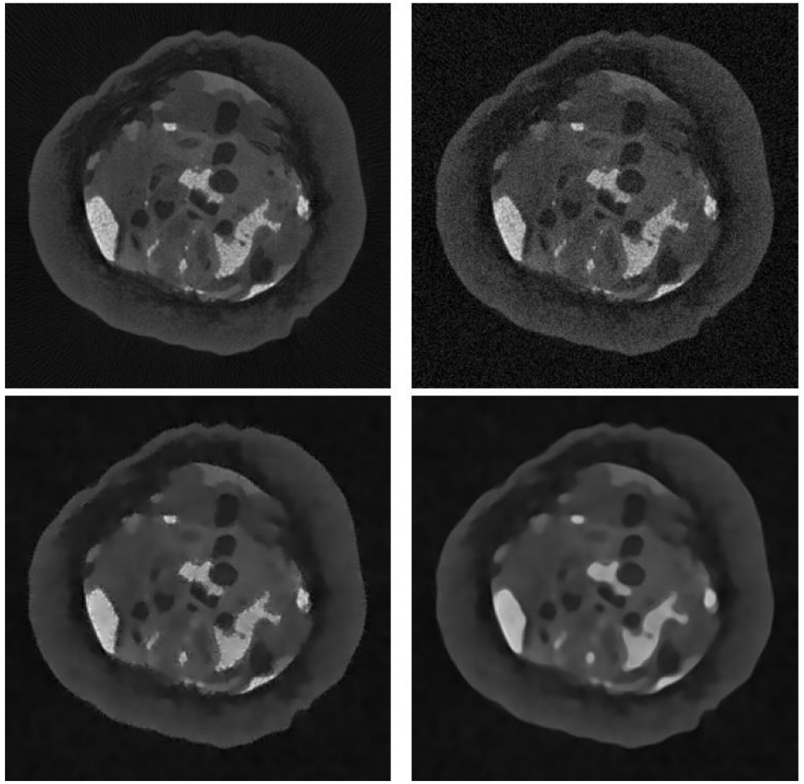
Example – Denoising (Phantom)

- Top Left: Original
- Top Right: Noisy
- Bottom Left: Isotropic diffusion
- Bottom Right: Anisotropic diffusion



Example – Denoising (Real)

- Top Left: Original
- Top Right: Noisy
- Bottom Left: Isotropic diffusion
- Bottom Right: Anisotropic diffusion



Example – Artistic Effect

The End

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