Digital Image Processing

Wavelet and Multiresolution Processing

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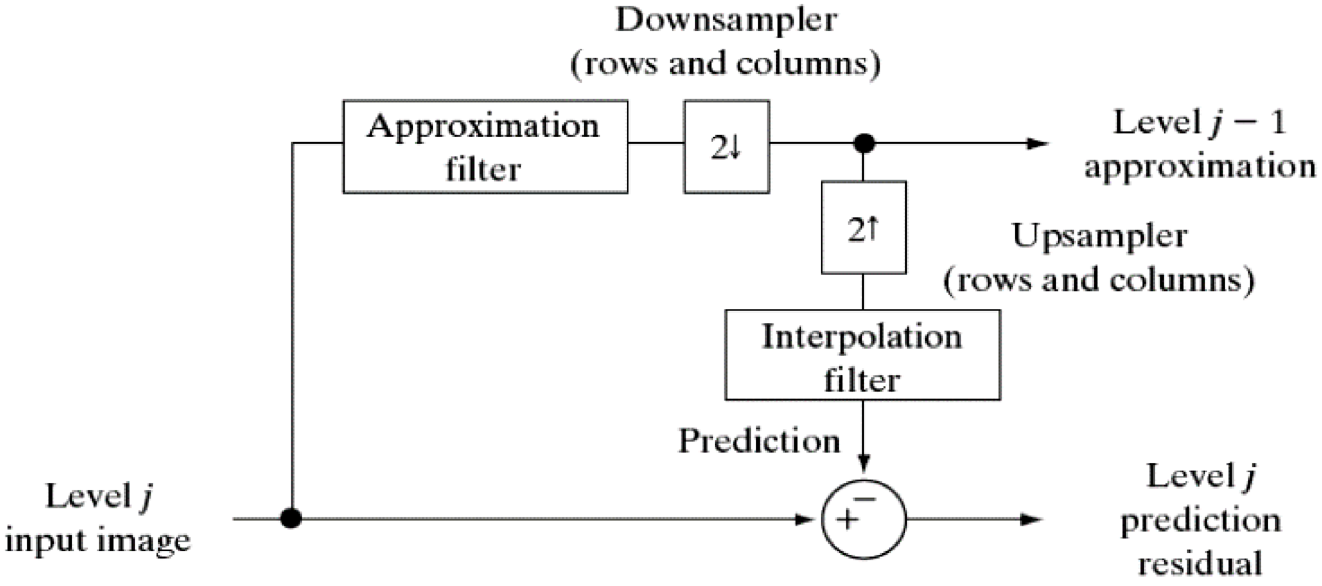
Introduction

"If you painted a picture with a sky, clouds, trees, and flowers, you would use a different size brush depending on the size of the features. Wavelets are like those brushes."

Ingrid Daubechies

Image Pyramids

› A redundant approach for image representation:



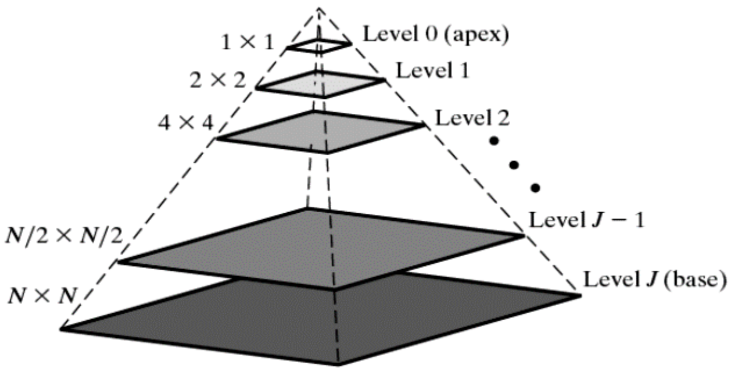
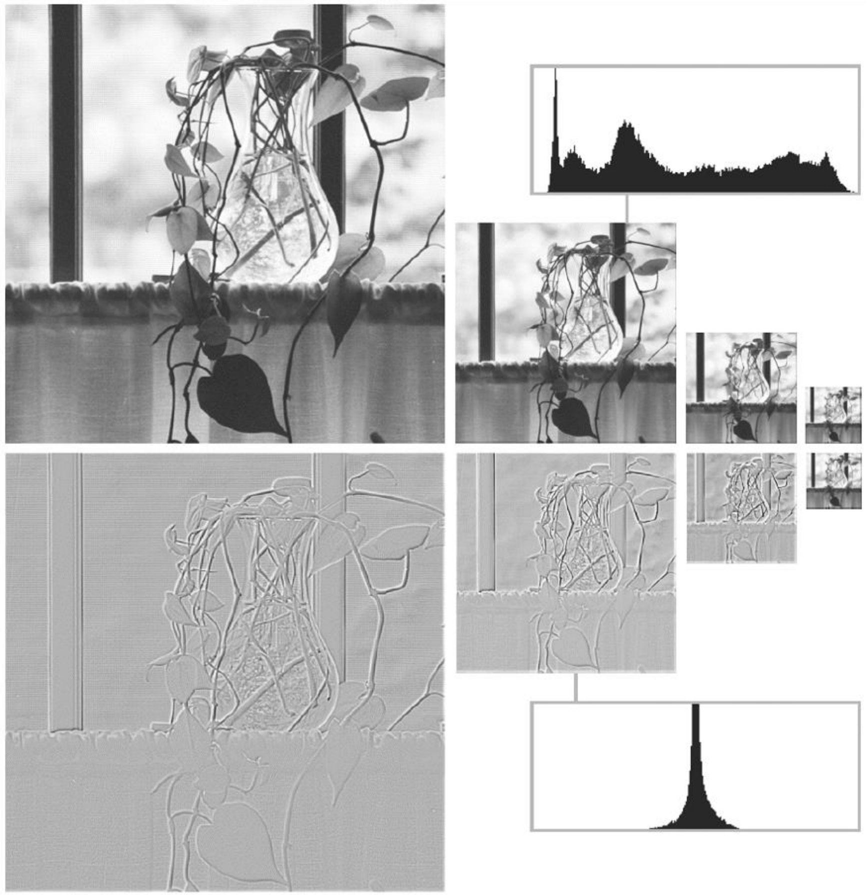


Image Pyramids

- Mathematical Formulation:
- $\mathcal{P}^{(n+1)}_{Gaussian}(I) = \left(G_\sigma * \mathcal{P}^{(n)}_{Gaussian}(I)\right)_{\downarrow 2}$
- $\mathcal{P}^{(1)}_{Gaussian}(I) = I$
- $\mathcal{P}^{(n+1)}_{Laplacian}(I) = \mathcal{P}^{(n)}_{Laplacian}(I) - \left(\left(G_\sigma * \mathcal{P}^{(n)}_{Gaussian}(I)\right)_{\downarrow 2}\right)_{\uparrow 2} * G_\sigma$

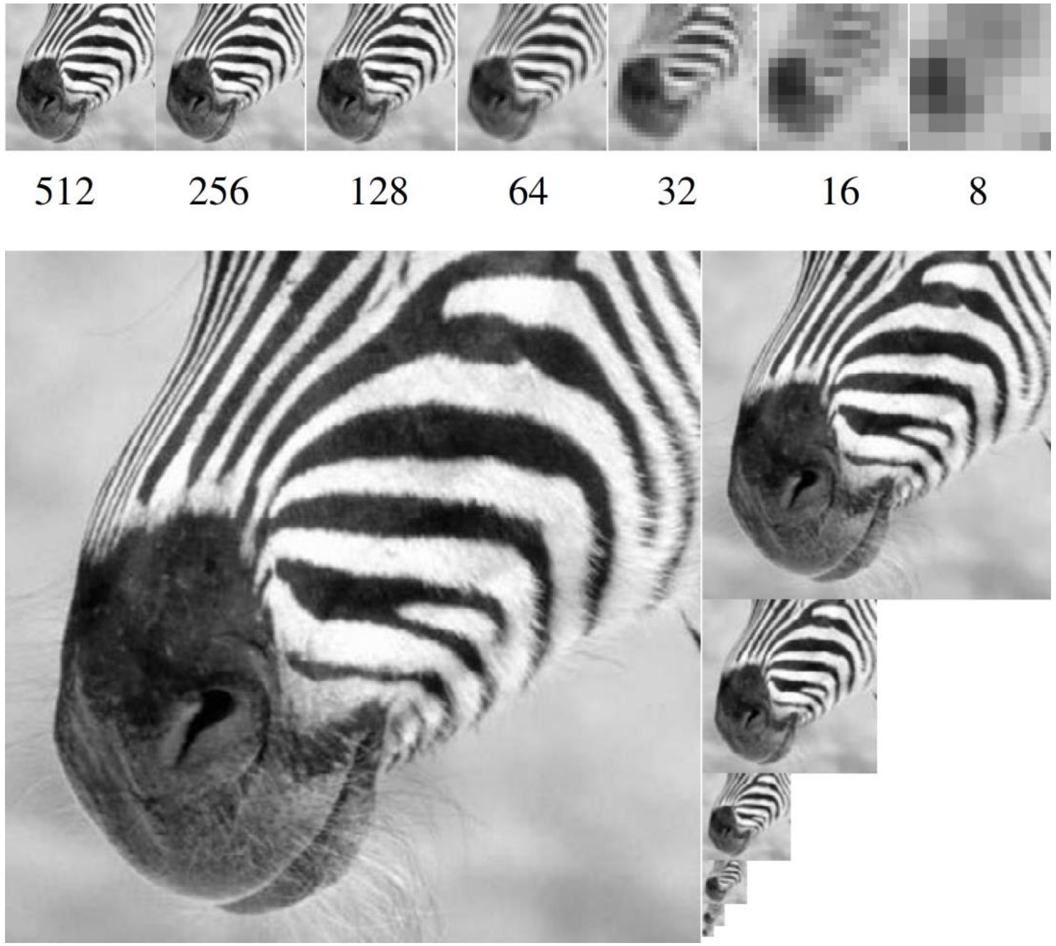
Gaussian and Laplacian Pyramids

- Example (1):
- Gaussian pyramids (top)
- Laplacian pyramids (bottom)



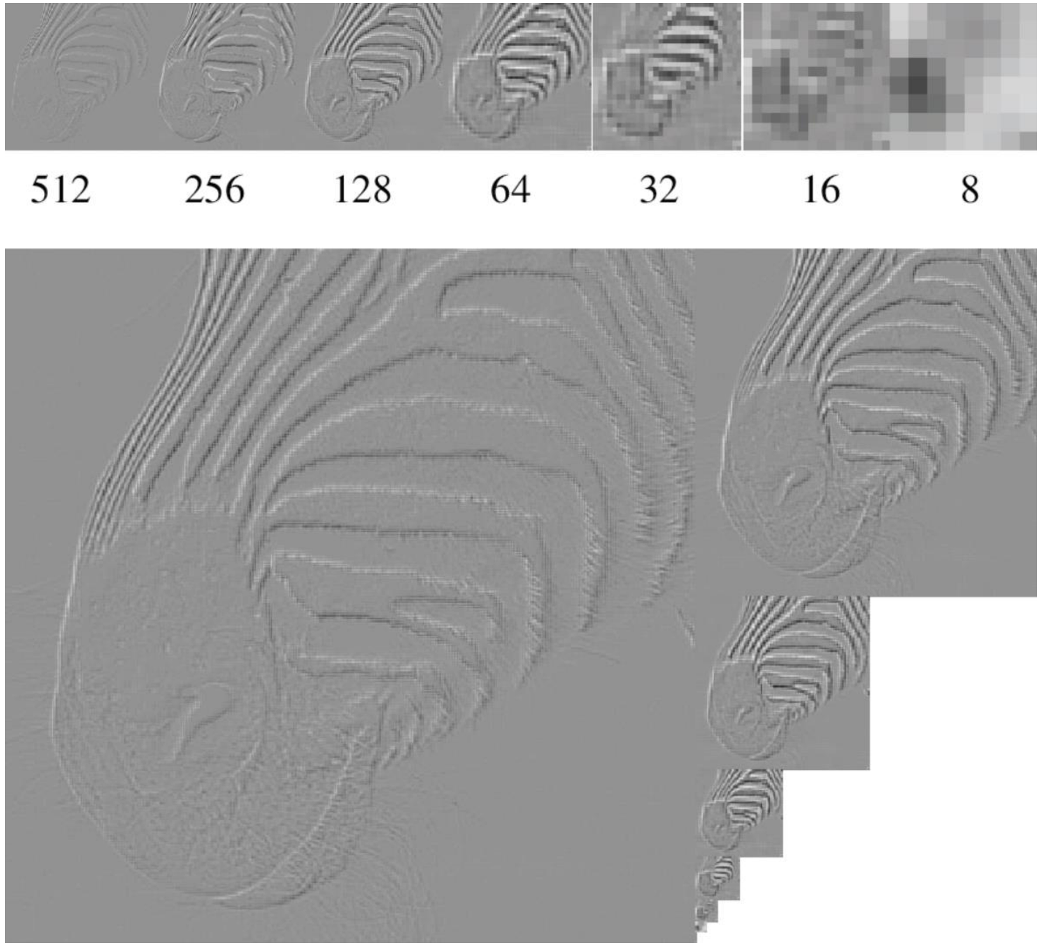
Gaussian and Laplacian Pyramids

- Example (2):
- Gaussian pyramid



Gaussian and Laplacian Pyramids

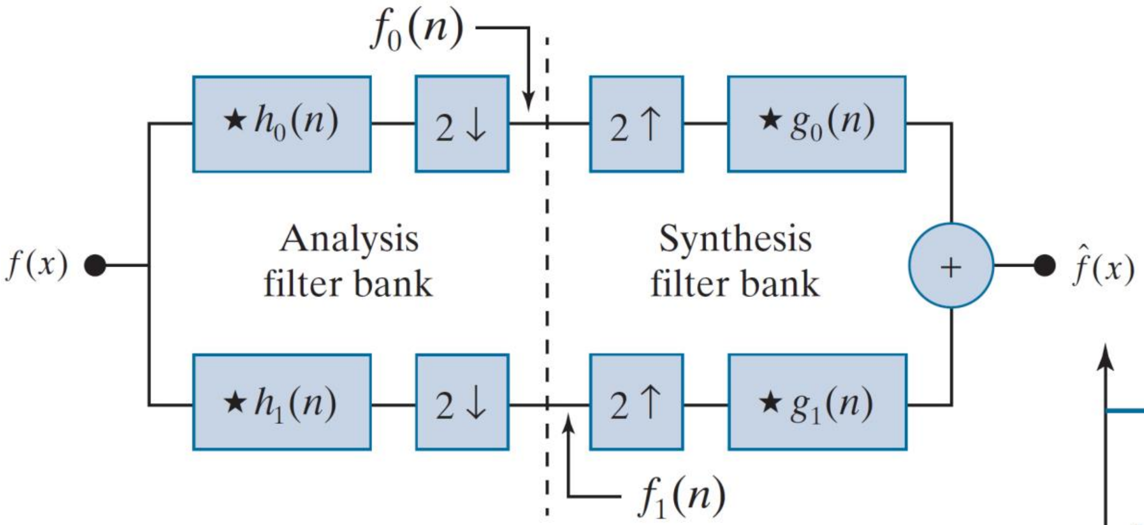
- Example (2):
- Laplacian pyramid

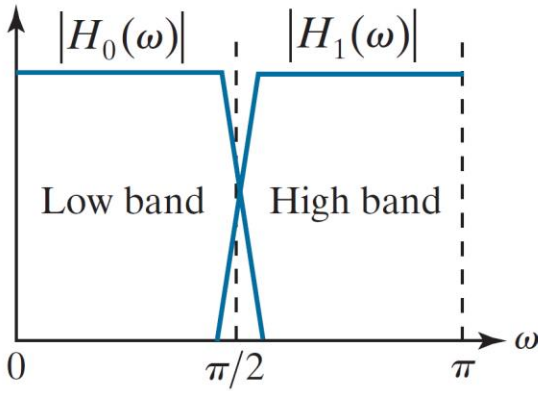


Sub-band Coding

› Consider 1-D Decomposition-Reconstruction Scheme:

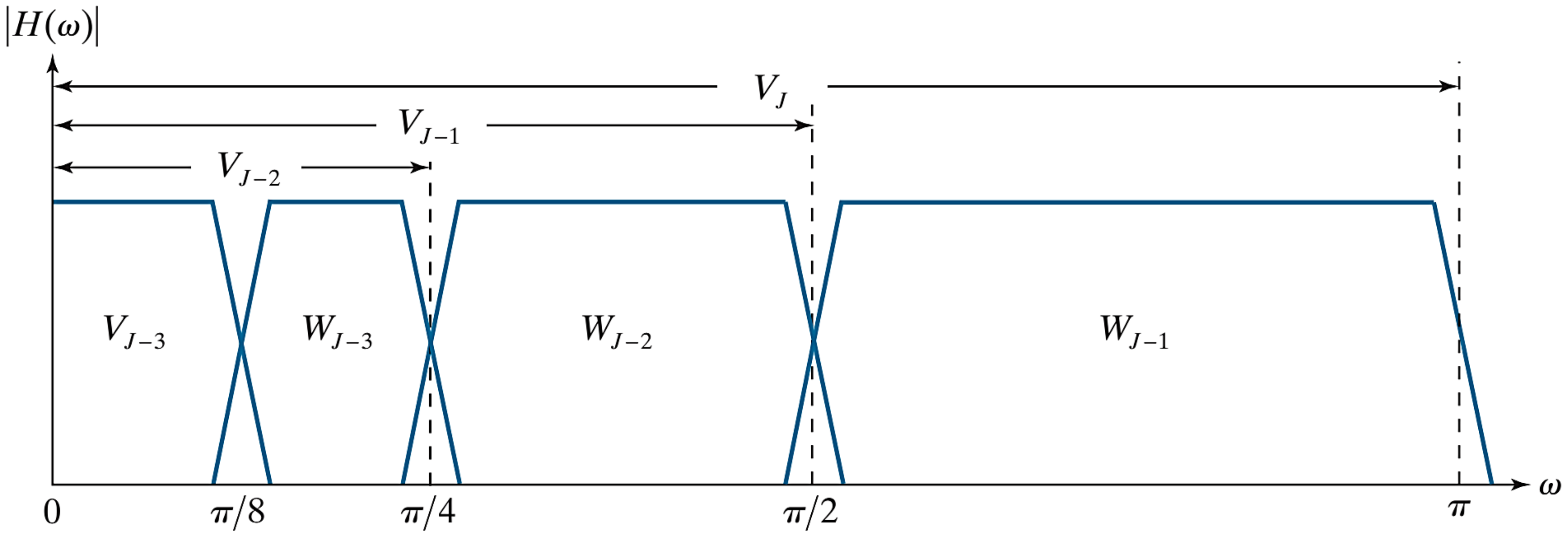
› Goal: $f(x) = \hat{f}(x)$, Perfect Reconstruction (PR)





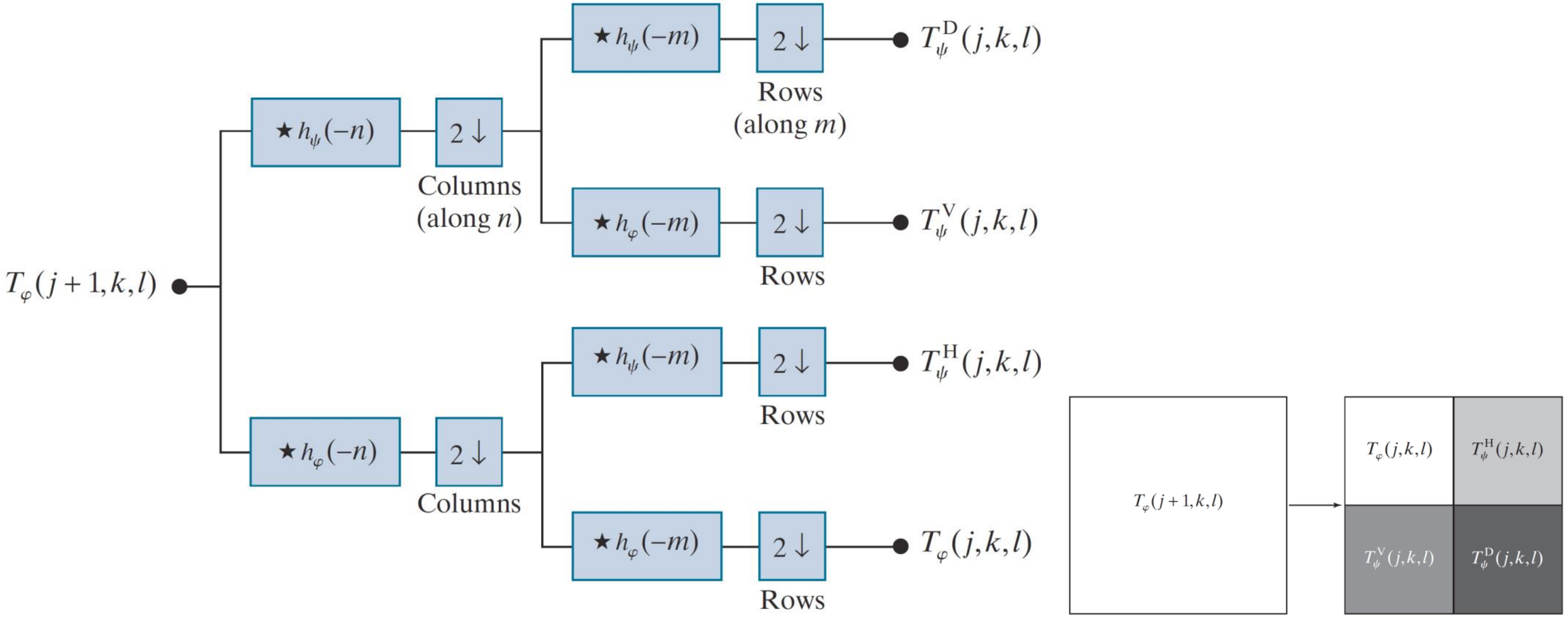
Multistage Sub-band Coding

› Successive Decomposition (and Reconstruction):



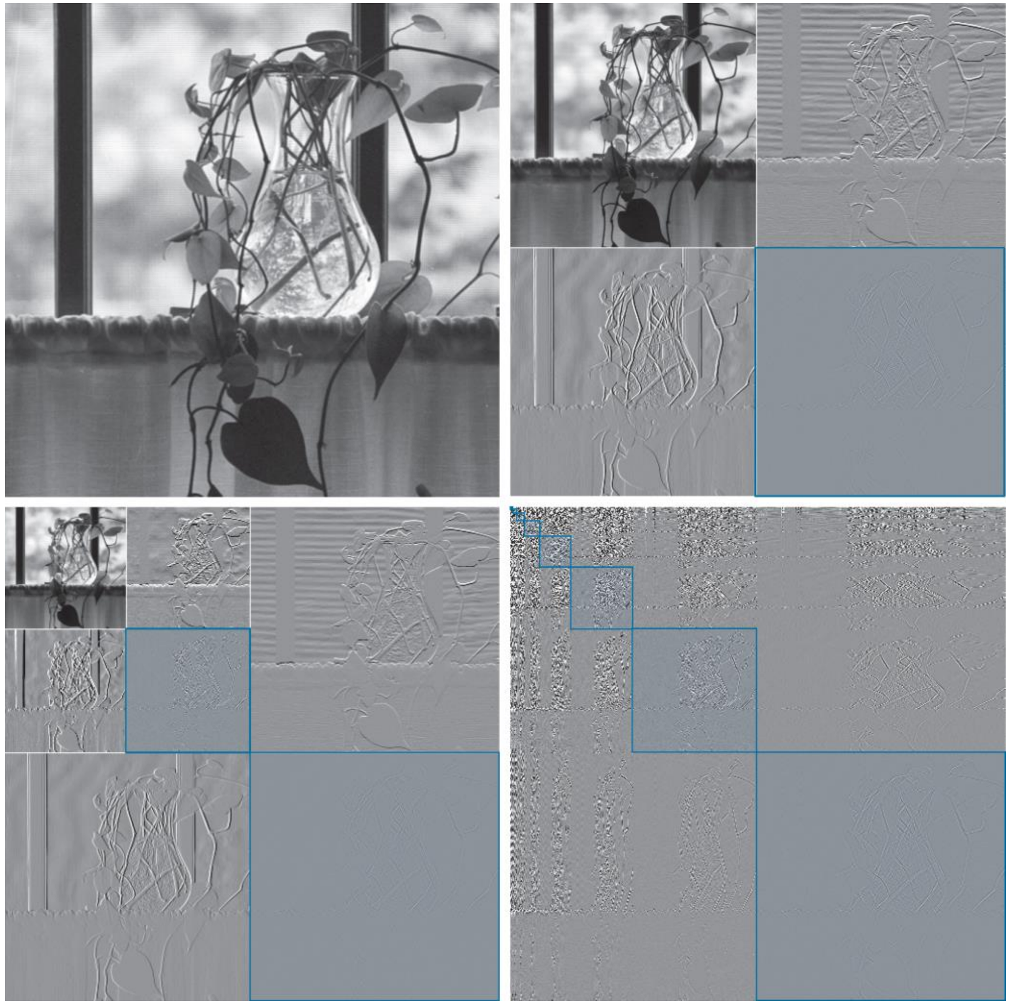
2D Wavelet

› Two direction → Four sub-band (Decomposition scheme)



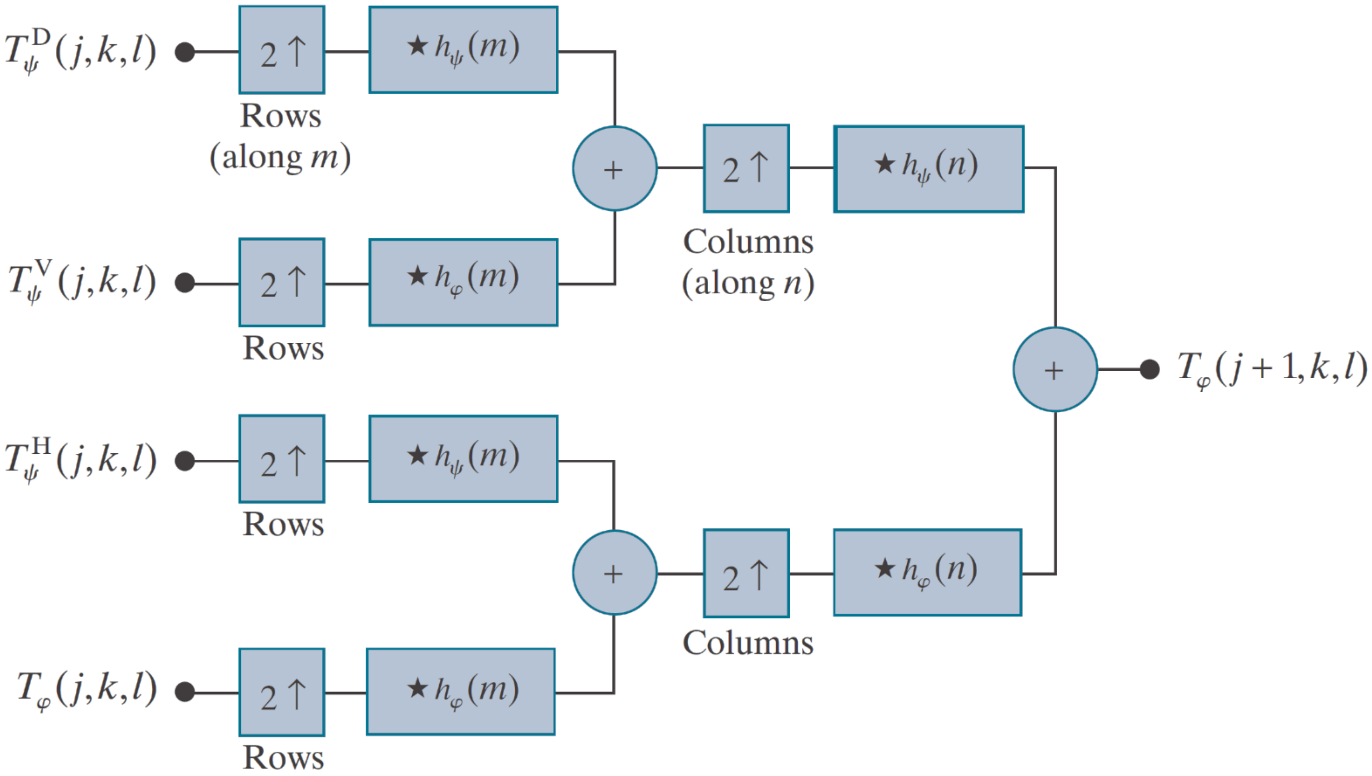
2D Wavelet

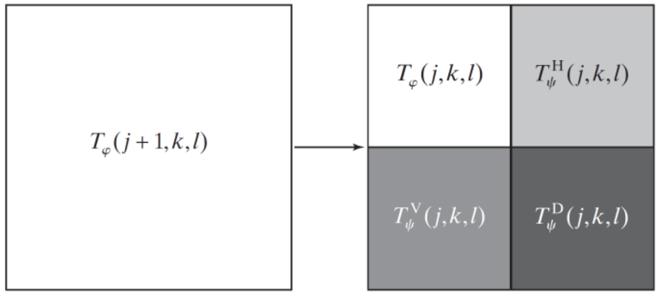
› Example:



2D Wavelet

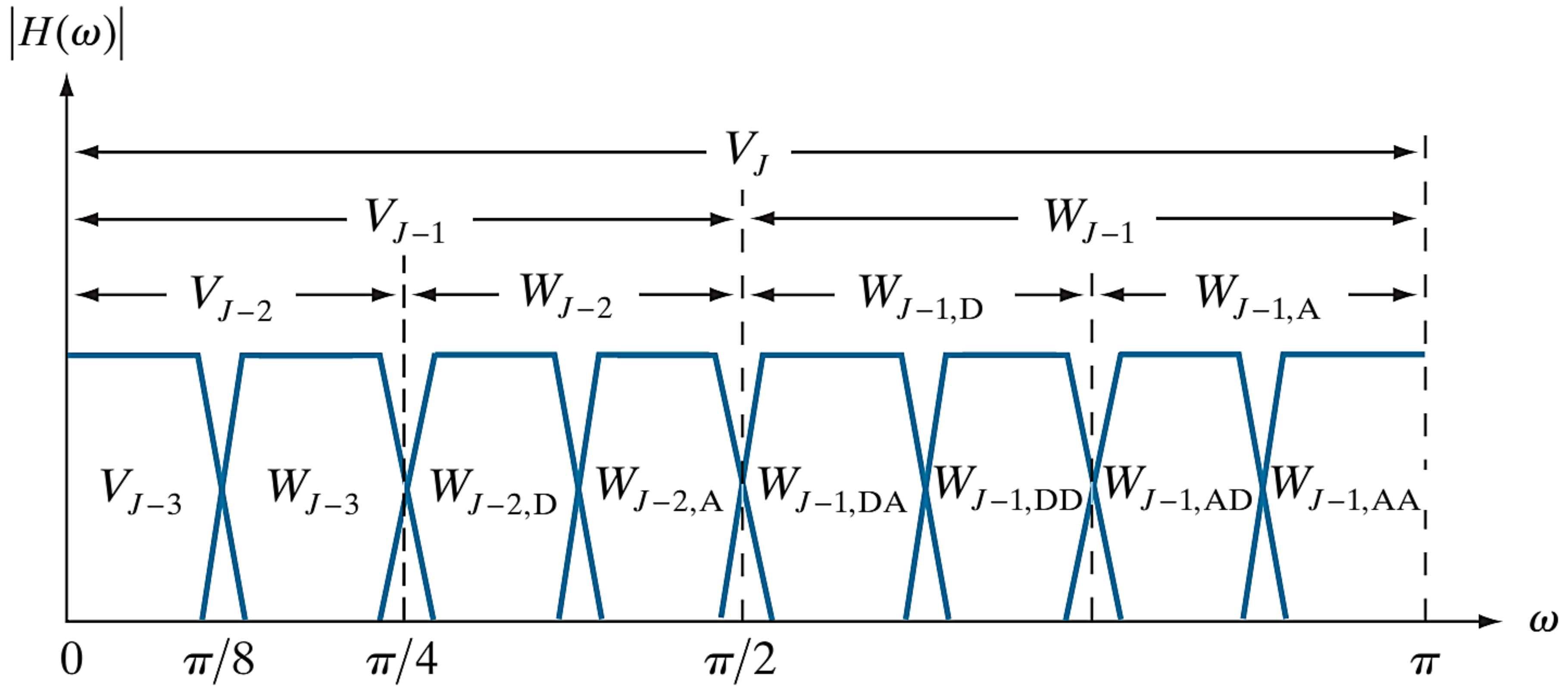
› Two direction → Four sub-band (Reconstruction scheme)





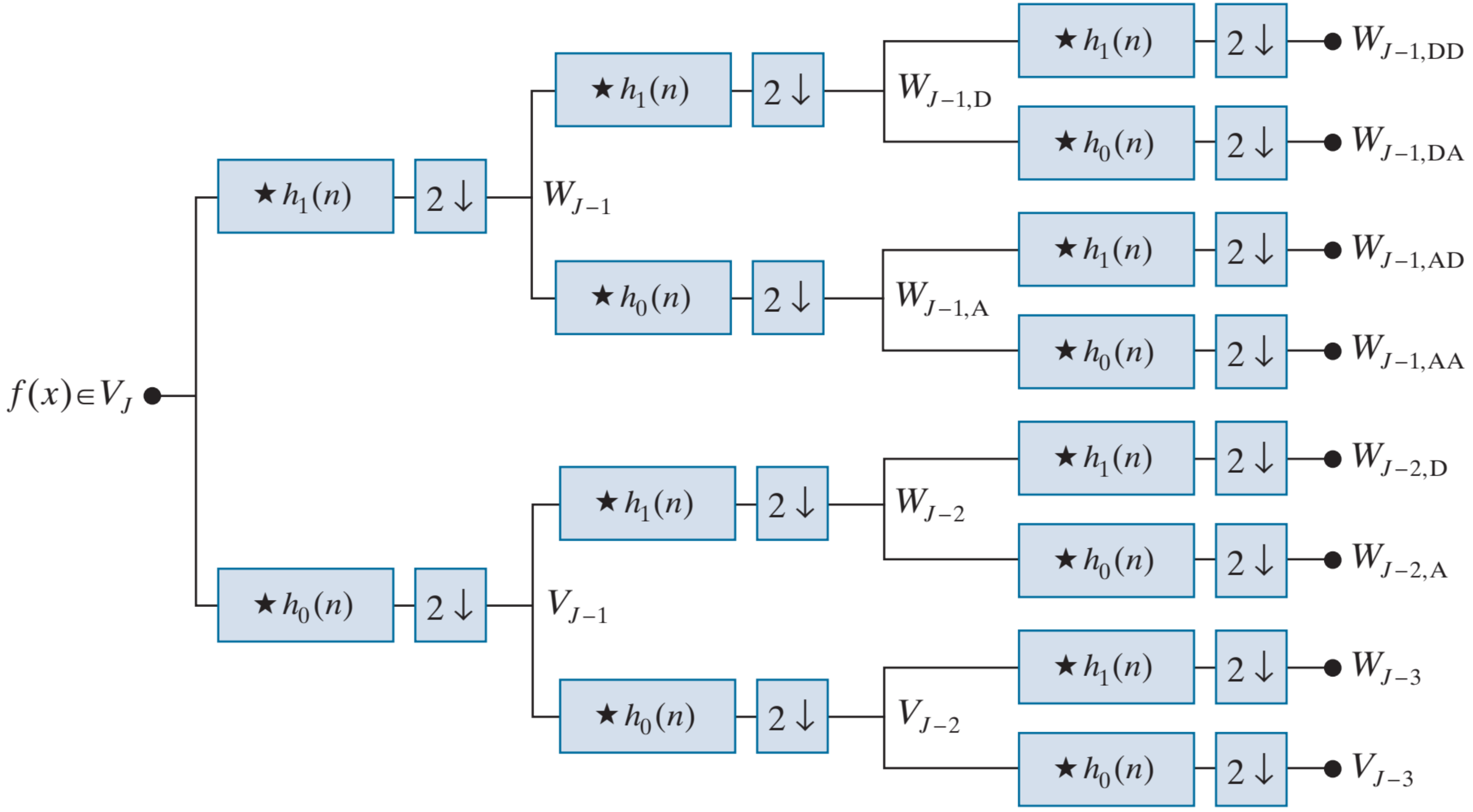
Wavelet Packet

› Full Partitioning (3 stage decomposition):



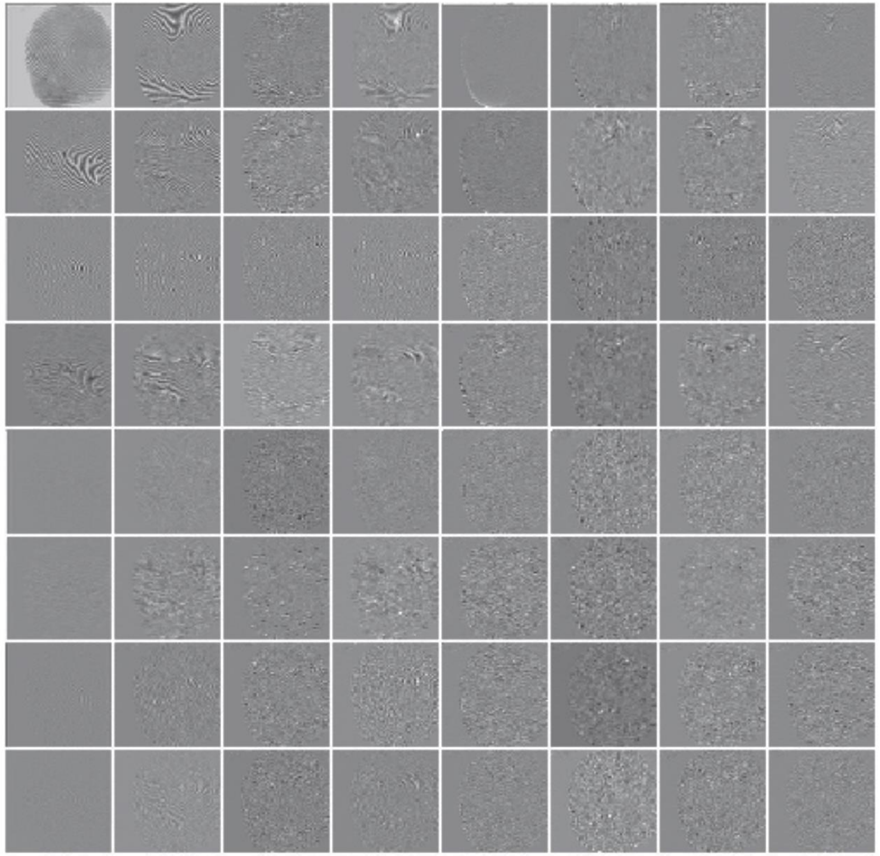
Wavelet Packet

- Full Partitioning (3 stage decomposition):



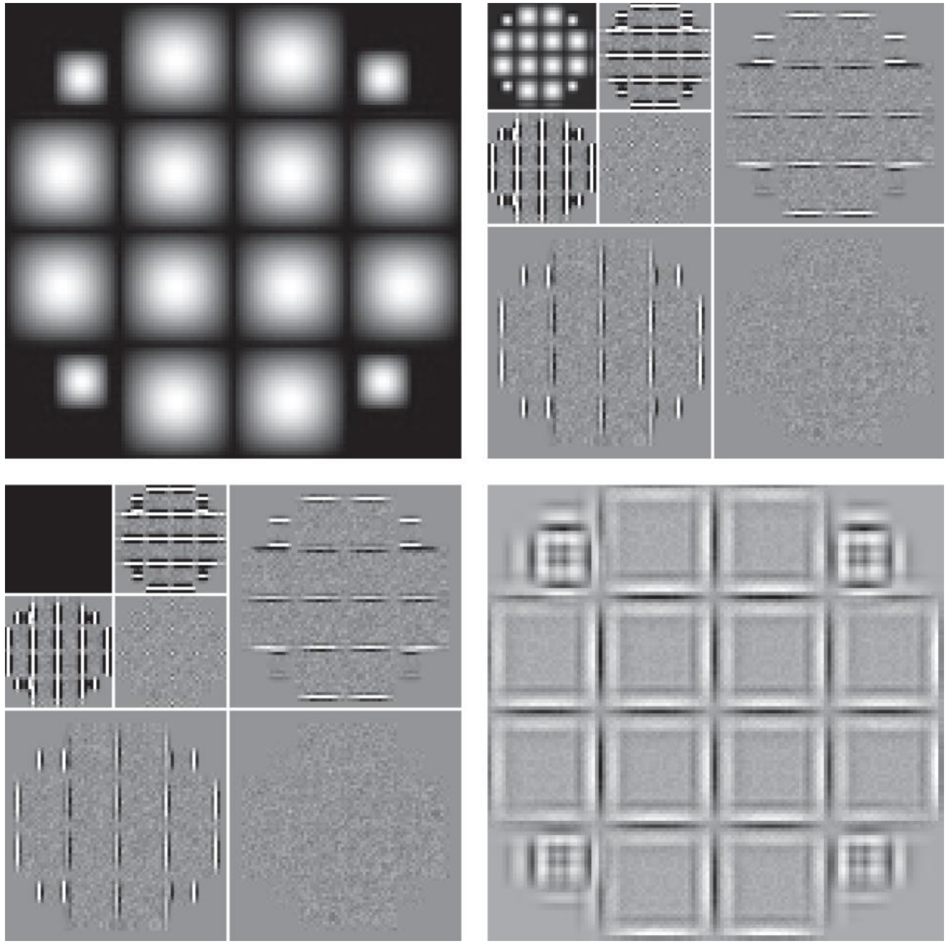
Wavelet Packet - Example

› Full Partitioning (3 stage decomposition):



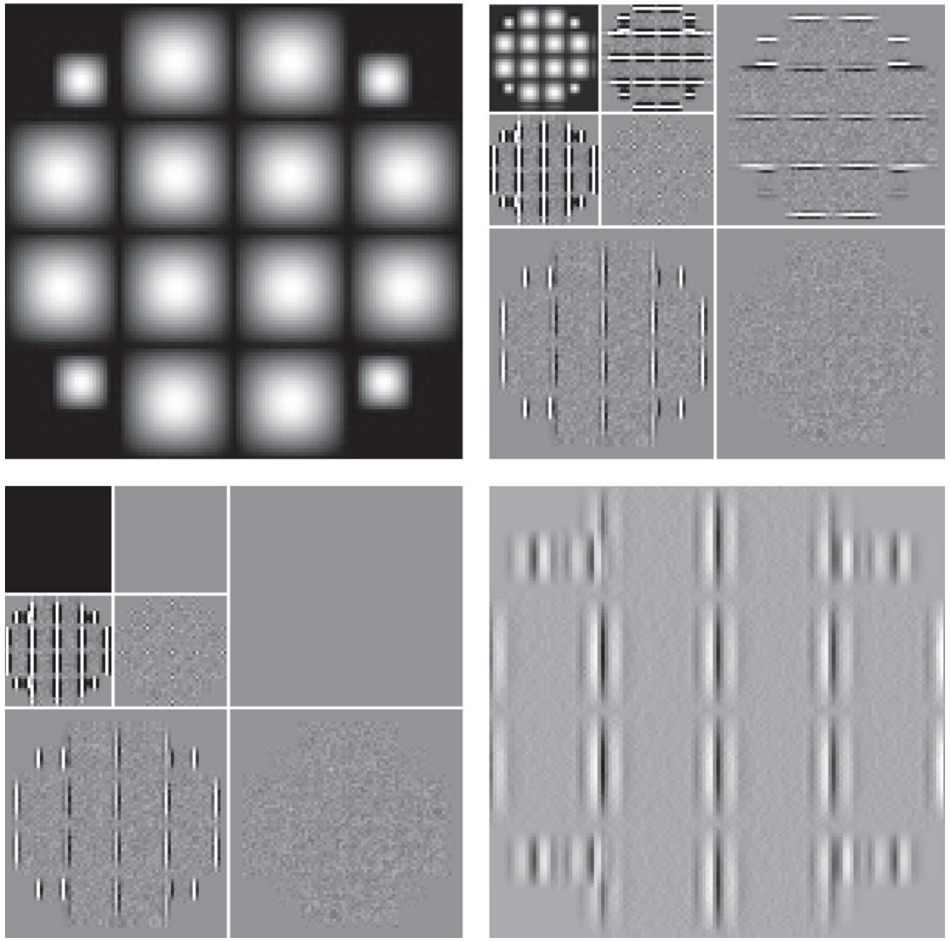
Wavelet-based Edge Detection

› Idea: sub-band manipulation:



Wavelet-based Edge Detection

› Idea: sub-band manipulation:



Wavelet-based Denoising

- Noisy image/signal model:

$$X(t) = S(t) + N(t)$$

  - $X(t)$: Corrupted Signal
  - $S(t)$: Uncorrupted Signal
  - $N(t)$: Additive Noise
- Wavelet Denoising Scheme

$$Y = W(X)$$
$$Z = D(Y, \lambda)$$
$$\hat{S} = W^{-1}(Z)$$

- W, W^{-1} : Forward and Inverse wavelet transform
- $D\ (., \lambda)$: Thresholding operator (λ being the threshold)

Wavelet-based Denoising

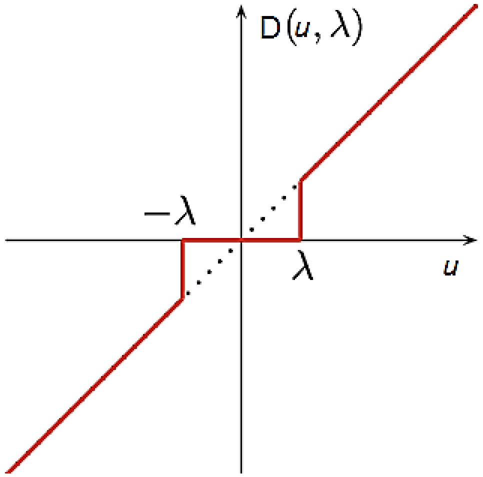
- Motivation for Thresholding:
  - Small coefficients: Dominated by noise.
  - Large coefficients: Dominated by signal.
  - Replacing small coefficients with zero!
- Some Assumption:
  - Wavelet de-correlating property generate a sparse signal.
  - Noise spreads out equally along all coefficients.
  - The noise level is NOT too high.

Wavelet-based Denoising

› Hard and Soft Thresholding:

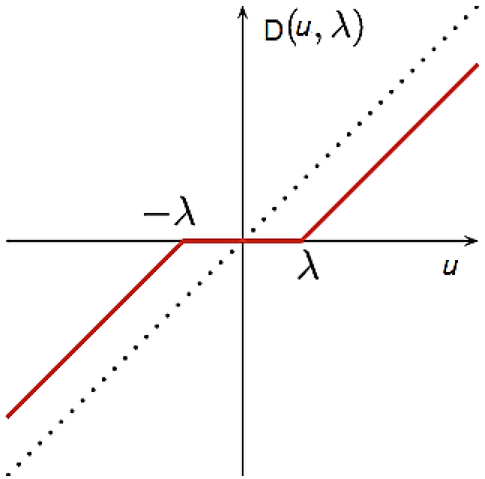
–Hard:

$$D(u,\lambda)=\begin{cases} u & |u|>\lambda \\ 0 & |u|\le\lambda \end{cases}$$



–Soft:

$$D(u,\lambda)=\begin{cases} u-\lambda & u>\lambda \\ 0 & -\lambda\le|u|\le\lambda \\ u+\lambda & u<-\lambda \end{cases}$$



Wavelet-based Denoising

- The most important question:
  - Very Low threshold: Noisy-Like result
  - Very High Threshold: Too smooth result.
- Several methods proposed:
  - VisuShrink
  - SureShrink
  - Bayesshrink
  - NormalShrink

Wavelet-based Denoising

- VisuShrink (Universal Thresholding):

$$\lambda = \sigma_N \sqrt{2Ln(N)}$$

- N: Sample (Signal/Image) size (# of pixels in image)
- σ_N^2: Noise variance
- Which sub-band:
  - All
  - Details (HL,LH, HH)
- Noise Estimation:

$$\hat{\sigma}_N = \frac{median(|Y_{ij}|)}{0.6745}, \ Y_{ij} \in \ HH_{Finest\ Level}$$

Challenges:

- Wavelet base,
- Threshold Selection,
- Threshold function,
- Decomposition Scheme.

Matlab Command

- *dwt2*: Single-level discrete 2-D wavelet transform
- *idwt2*: Single-level inverse discrete 2-D wavelet transform
- *wavedec2*: Multilevel 2-D wavelet decomposition
- *waverec2*: Multilevel 2-D wavelet reconstruction
- *upcoef2*: Direct reconstruction from 2-D wavelet coefficients
- *detcoef2*: 2-D detail coefficients
- *appcoef2*: 2-D approximation coefficients
- *upwlev2*: Single-level reconstruction of 2-D wavelet decomposition
- *wthresh*: Soft or hard thresholding
- *Wthcoef2*: Wavelet coefficient thresholding 2-D

The End

- AnY QuEsTiOn?