Medical Image Analysis and Processing

Medical Image Segmentation
Deformable Model - Parametric

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Distance/online Course: Session 20 Episode#2

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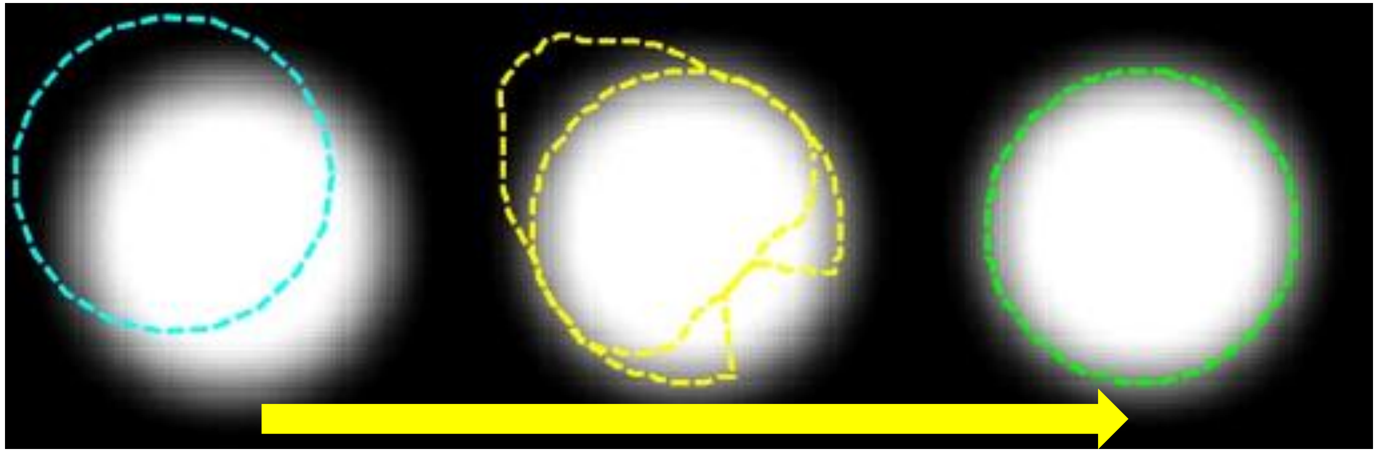
Contents

Definition

- A *deformable* model (curve or surface) is a geometric object whose shape can *change* over *time*.
- Deformable models move under the influence of the *model* itself and from the *image* data.
- The internal and external effects are defined so that the model will conform to an object boundary or other desired features within an image.

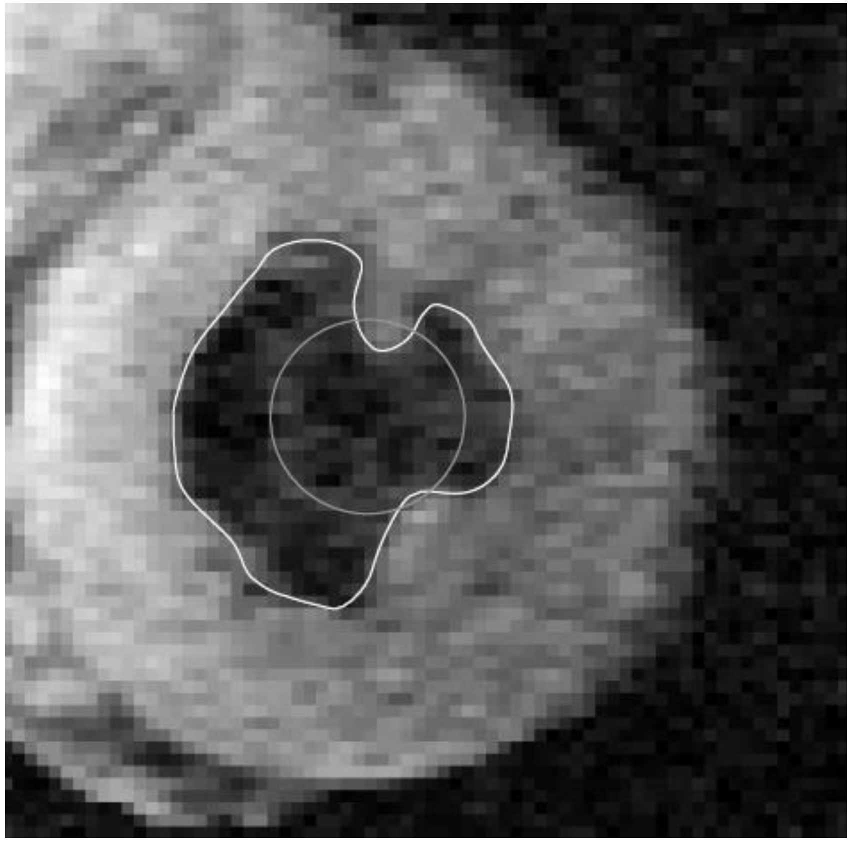
General Perspective:

- Deformable methods start with an initial contour placed in the image, either manually or automatically, which is then iteratively deformed, generating a new contour at each iteration.



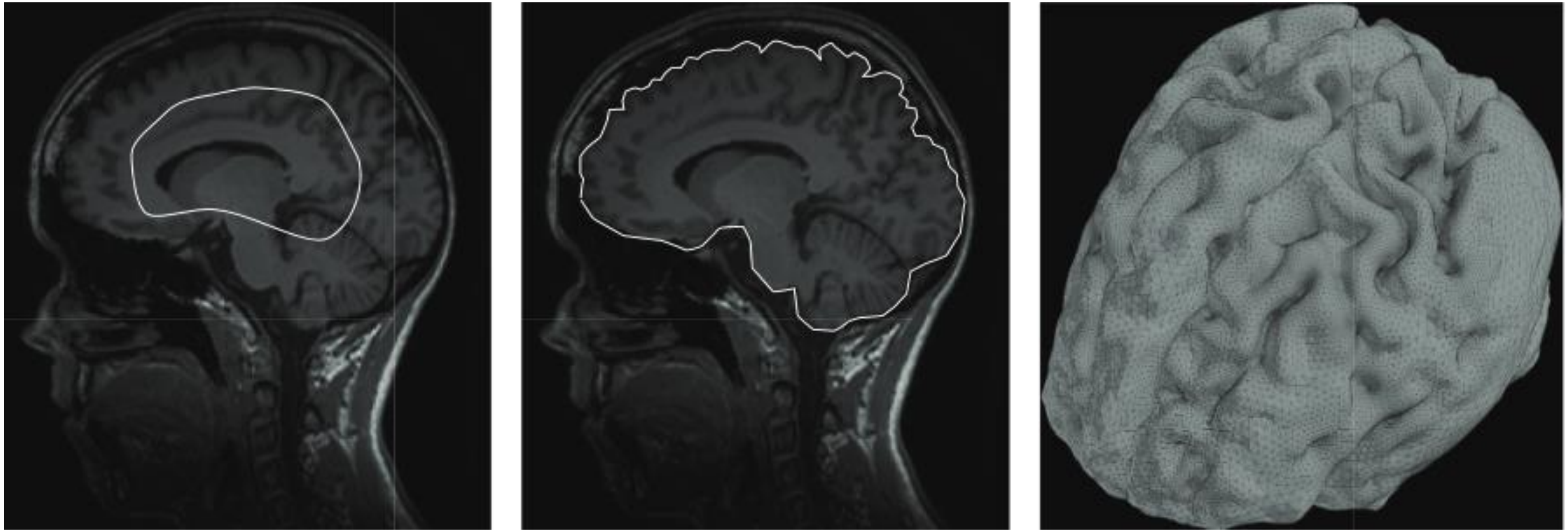
Example (1)

› Heart chamber



Example (2)

- Initial condition (Left), Final result (Middle), and 3D example (Right)



Approaches

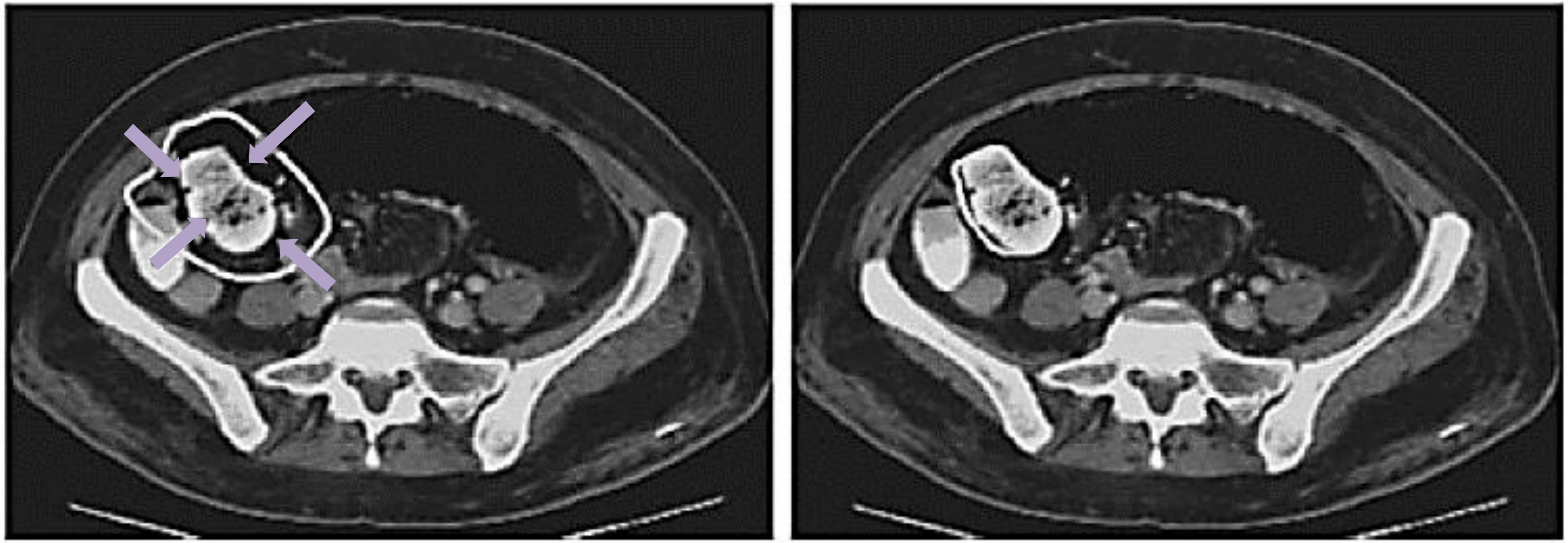
- Two main categories of deformable models:
  - Parametric (explicit) deformable model, Active Contour Models (ACM) or Snakes
  - Geometric (implicit) deformable model or level-set

Parametric Deformable Model

- Track the boundary by matching the deformable model to the image curve, influenced by *external* and *internal* forces, in order to minimize the defined energy functional.
- Internal force: include curve elasticity and rigidity and force resist deformation.
- External force: Push the contour towards object true contours and forces resist deformation

Parametric Deformable Model

› Example:

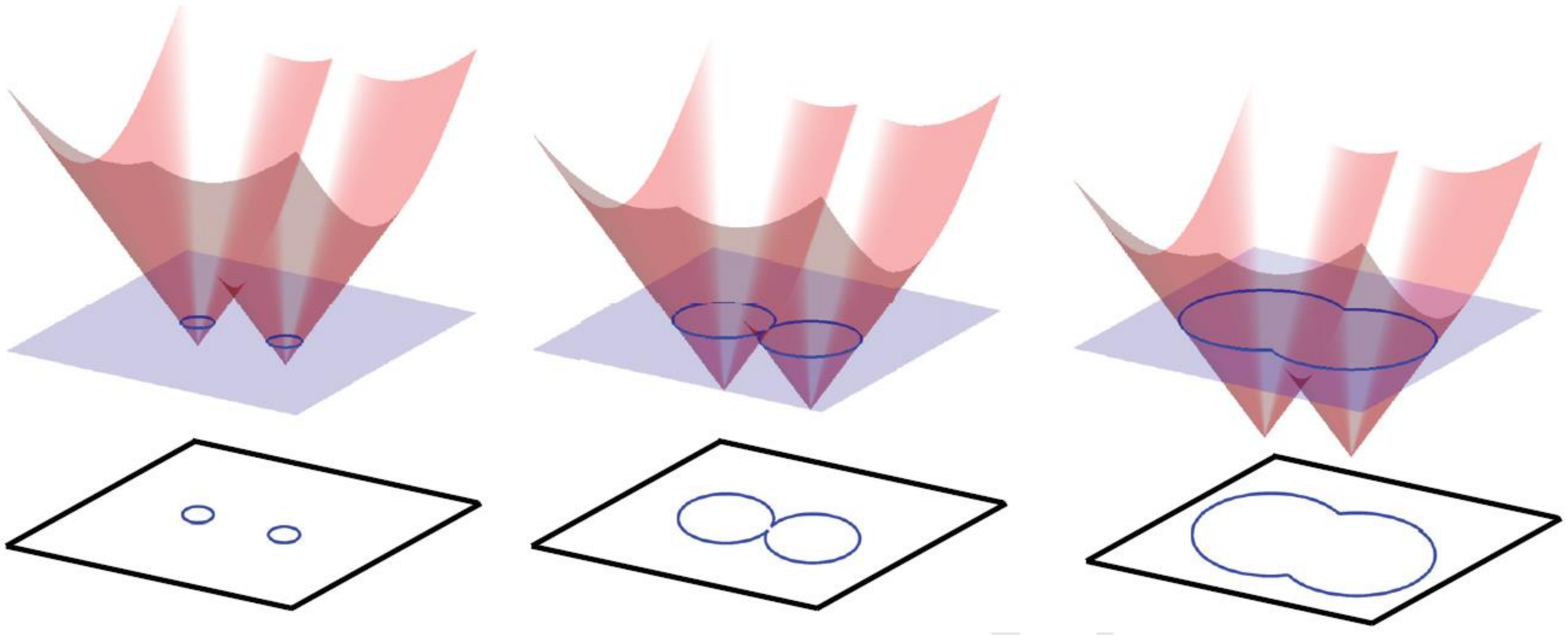


Geometric Deformable Model

- Using curve evolution theory and level set methods. Instead of parameterizing a curve, it is depicted as a zero level set of a higher dimensional function whose evolution defines the initial value problem.
- Contour evolution is associated with speed function of level sets. Here, the contour is independent of the curves parameterization.

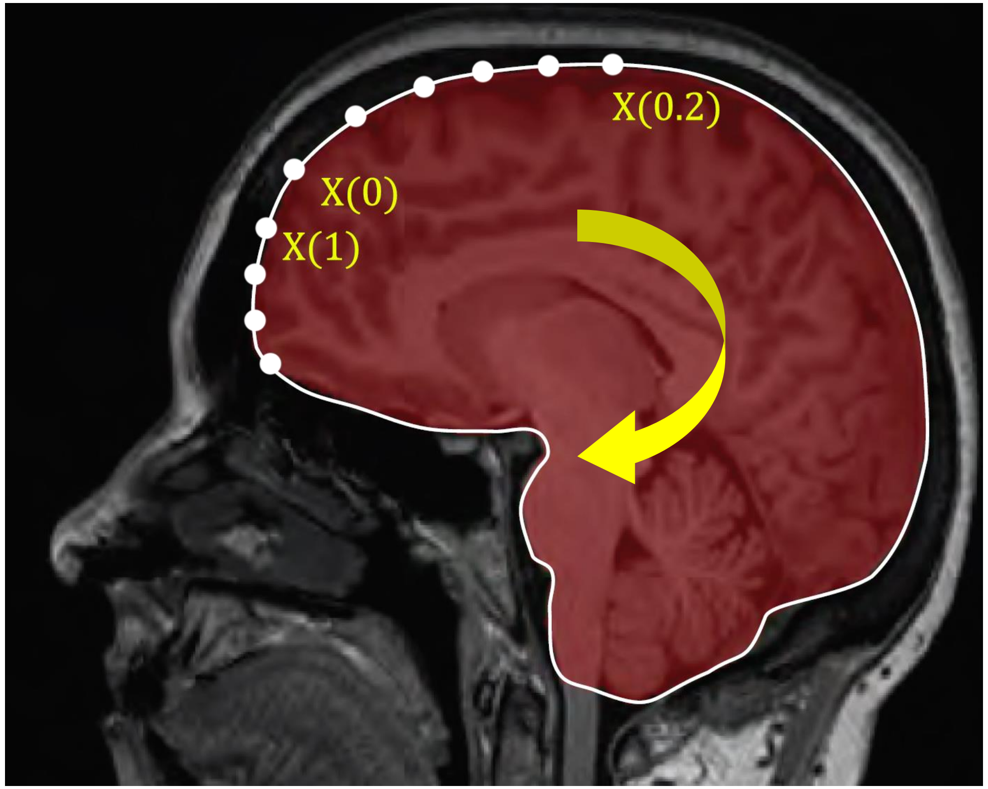
Geometric Deformable Model

› Illustration:



Parametric Deformable Model

- Parametric Curve:
- $X(s) = \begin{pmatrix} x(s) \\ y(s) \end{pmatrix}, \ 0 \leq s \leq 1$
- Example, circle:
  - $x(s) = r cos(2\pi s)$
  - $y(s) = r sin(2\pi s)$



Elasticity Energy

- First derivative of curve is:

$$\frac{\partial X(s)}{\partial s} = X'(s) = \begin{pmatrix} x'(s) \\ y'(s) \end{pmatrix}$$

- Elasticity Energy is defined as:

$$\int_0^1 \left|\frac{\partial X(s)}{\partial s}\right|^2 ds$$

- Discrete Approximation: $\left|\frac{\partial X(s)}{\partial s}\right|^2 \propto |X(s+\Delta s) - X(s)|^2$
- The distance between the successive points on the curve

Rigidity/Bending Energy

- Second derivative of curve is:

$$\frac{\partial X^2(s)}{\partial s^2} = X''(s) = \begin{pmatrix} x''(s) \\ y''(s) \end{pmatrix}$$

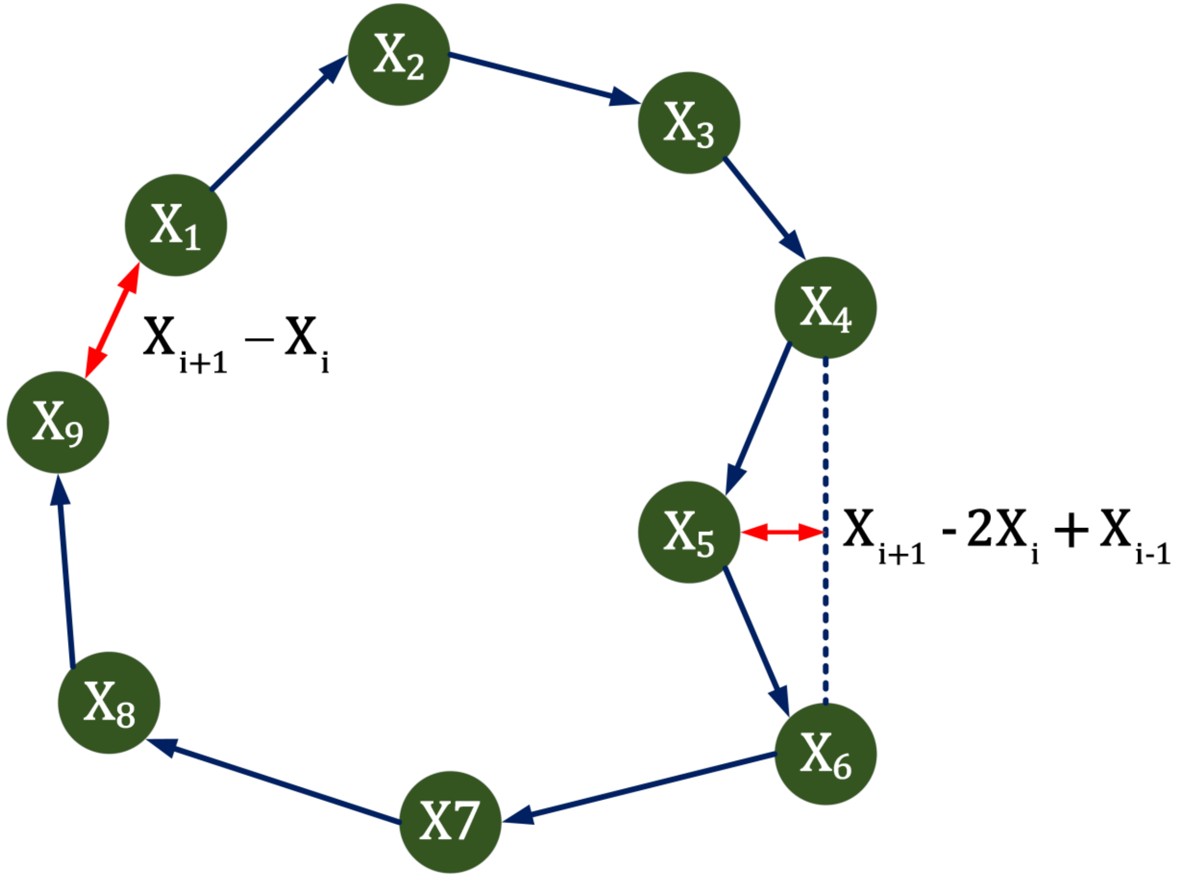
- Rigidity/Bending Energy or Stiffness is defined as:

$$\int_0^1 \left| \frac{\partial^2 X(s)}{\partial s^2} \right|^2 ds$$

- Discrete Approximation: $\left| \frac{\partial^2 X(s)}{\partial s^2} \right|^2 \propto |X(s + \Delta s) - 2X(s) + X(s - \Delta s)|^2$
- The deviation from straight line in three successive points

Elasticity and Rigidity Illustration

› Elasticity and rigidity discrete illustration:



Energy *minimization* formulation:

› A deformable contour is a curve, $X(s) = \big(x(s), y(s)\big)'$, $0 \le s \le 1$

which moves through the spatial domain of an image to minimize the following energy functional:

$$\mathcal{E}(X) = \frac{1}{2}\int_0^1 \{E_{internal}\big(X(s)\big) + E_{image}\big(X(s)\big) + E_{user}\big(X(s)\big)\}ds$$

$$\mathcal{E}(X) = \mathcal{E}_{internal}(X) + \underbrace{\mathcal{E}_{image}(X) + \mathcal{E}_{user}(X)}_{\mathcal{E}_{external}(X)}$$

› In what following we explain each term

Internal (Contour) energy - Elasticity:

- Internal (Contour) energy:

$$E_{internal}\big(X(s)\big) = \alpha(s)E_{Elasticity}\big(X(s)\big) + \beta(s)E_{Rigidity}\big(X(s)\big)$$

$$E_{internal}\big(X(s)\big) = \alpha(s)\left|\frac{\partial X(s)}{\partial s}\right|^2 + \beta(s)\left|\frac{\partial^2 X(s)}{\partial s^2}\right|^2$$

- The elasticity term discourages stretching and makes the model behave like an *elastic string*.
- Decreasing elasticity allows the contour to increase in length, while increasing elasticity increases the tension of the model by reducing its length.

Internal (Contour) energy - Rigidity:

- Internal (Contour) energy:

$$E_{internal}\big(X(s)\big) = \alpha(s)E_{Elasticity}\big(X(s)\big) + \beta(s)E_{Rigidity}\big(X(s)\big)$$

$$E_{internal}\big(X(s)\big) = \alpha(s)\left|\frac{\partial X(s)}{\partial s}\right|^2 + \beta(s)\left|\frac{\partial^2 X(s)}{\partial s^2}\right|^2$$

- The rigidity term discourages bending and makes the model behave like a *rigid rod*.
- Decreasing rigidity allows the active contour model to develop corners, while increasing rigidity makes the model smoother and less flexible.

External (image and user) energy:

- External (image and user) energy:

$$E_{external}\big(X(s)\big) = E_{image}\big(X(s)\big) + E_{user}\big(X(s)\big)$$

- There are many choices for $E_{image}\big(X(s)\big)$ and $E_{user}\big(X(s)\big)$
- A typical potential energy function designed to lead a deformable contour toward step edges is:

$$E_{image}\big(X(s)\big) = -w_e|\nabla[G_\sigma(x,y) * I(x,y)]|^2$$

- We will discuss more about this.

This energy minimization problem

- This energy minimization problem:

$$\mathcal{E}(X) = \frac{1}{2}\int_0^1 \left\{\alpha(s)\left|\frac{\partial X(s)}{\partial s}\right|^2 + \beta(s)\left|\frac{\partial^2 X(s)}{\partial s^2}\right|^2 + E_{external}\big(X(s)\big)\right\} ds$$

- may be solved by calculus of variation:

$$\frac{\partial}{\partial s}\left(\alpha(s)\frac{\partial X(s)}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\beta(s)\frac{\partial^2 X(s)}{\partial s^2}\right) - \nabla\Big(E_{external}\big(X(s)\big)\Big) = 0$$

- To gain some insight about the physical behavior of deformable contours

$$F_{internal}(X) + F_{external}(X)=0$$

How to solve Euler-Lagrange equation?

- The deformable contour is made dynamic by treating $X(s)$ as a function of t as well as s, $X(s,t)$

$$\lambda \frac{\partial X}{\partial t} = \frac{\partial}{\partial s}\left(\alpha \frac{\partial X}{\partial s}\right) - \frac{\partial^2}{\partial s^2}\left(\beta \frac{\partial^2 X}{\partial s^2}\right) - \nabla E_{external}(X)$$

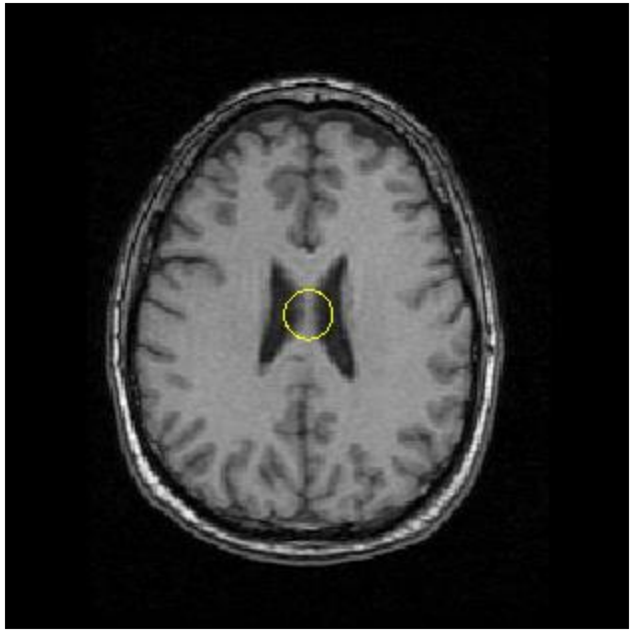
$$\lambda \frac{\partial X}{\partial t} = F_{internal}(X) + F_{external}(X) = F_{total}(X)$$

- Steady state solution of above equation is solution of static equation

$$X(s, t+\Delta t) = X(s,t) + \frac{\Delta t}{\lambda} F_{total}\big(X(s,t)\big), \qquad X(s,0) = X_0(s)$$

Illustration

› How it works:



General Formulation

- Directly force formulation permits the use of more general types of external forces that are not potential forces, i.e., *forces that cannot be written as the negative gradient of potential energy functions*, using Newton's 2nd law:

$$\mu\frac{\partial^2 X}{\partial t^2}+\lambda\frac{\partial X}{\partial t}-\frac{\partial}{\partial s}\left(\alpha\frac{\partial X}{\partial s}\right)+\frac{\partial^2}{\partial s^2}\left(\beta\frac{\partial^2 X}{\partial s^2}\right)=-\nabla E_{external}(X)$$

- In image segmentation applications we set μ equal to zero, hence:

$$\lambda\frac{\partial X}{\partial t}=F_{internal}(X)+F_{external}(X),\quad F_{external}(X)=\sum_{i=1}^{N}F_N(X)$$

Multiscale Gaussian potential force (image)

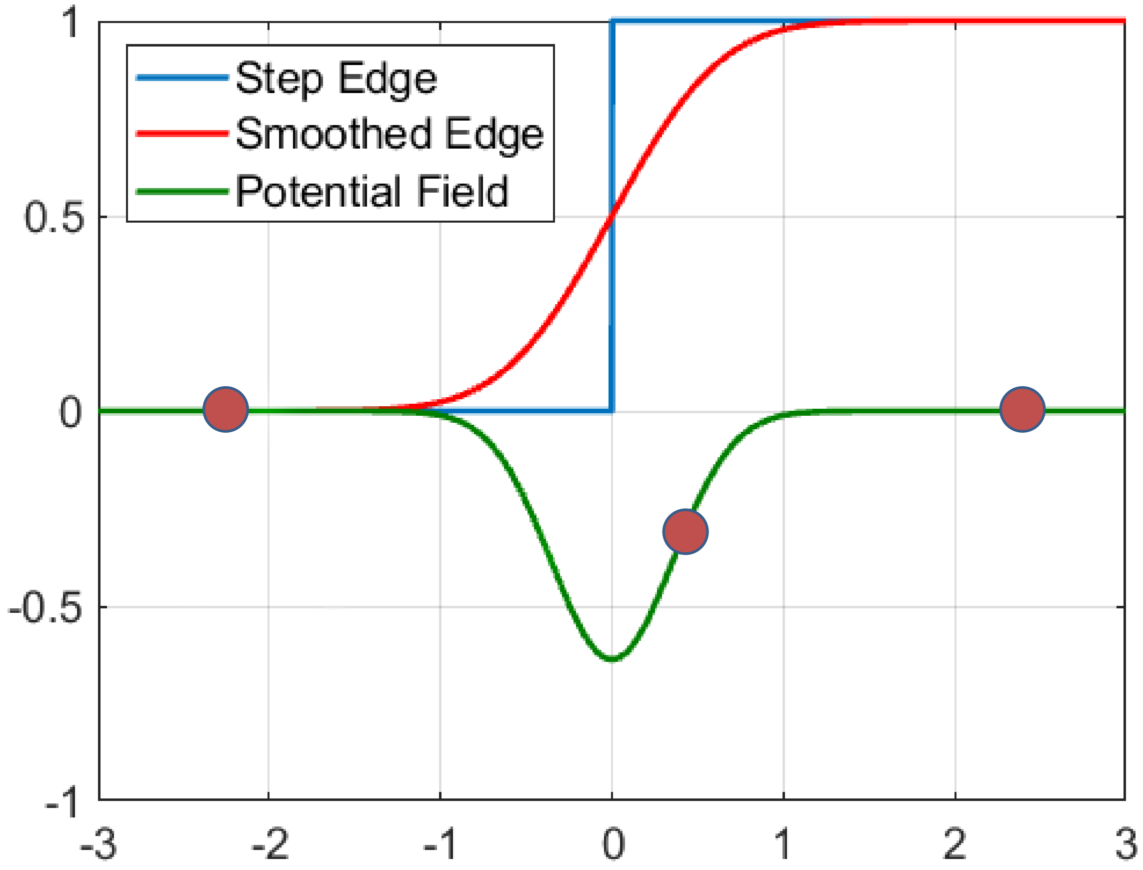
- Definition:

$$E_{image}\big(X(s)\big) = -w_e|\nabla[G_\sigma(x,y) * I(x,y)]|^2, w_e > 0$$

- Which designed to lead a deformable contour toward step edges.
- Effect of $G_\sigma(x,y)$:
  - Noise reduction
  - Broaden potential wall

Multiscale Gaussian potential force (image)

› Multiscale Gaussian potential force:



Multiscale Gaussian potential force (image)

- How to select Gaussian filter parameter (σ):
- Small σ:
  - Proc: Follow the boundary accurately,
  - Cons: Gaussian potential force can only attract the model toward the boundary when it is initialized *nearby*.
- Large σ:
  - Proc: Broaden attraction rang
  - Cons: Steady state solution will be far from true edge point
- Trade off strategy:
  - Multiscale (coarse-to-fine), monotonically decreasing $\sigma(t)$

Pressure force or Balloon force (user):

- Definition:

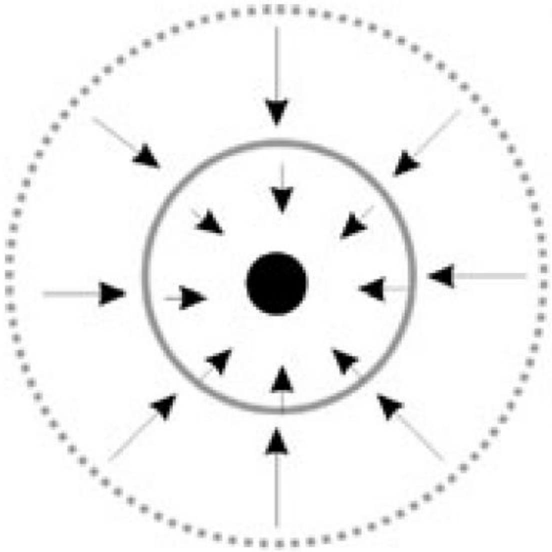
$$F_{Balloon}\big(X(s)\big) = k\,\frac{\vec{N}(X)}{\left|\vec{N}(X)\right|}$$

- Where $\vec{N}(X)$ is inward normal vector and sign of k determine whether to inflate or deflate the model and is typically chosen by the user.
- The value of k determines the strength of the pressure force. It must be carefully selected so that the pressure force is *slightly smaller* than the Gaussian potential force at *significant* edges, but *large* enough to pass through *weak* or spurious edges.

Pressure force or Balloon force (user):

› Illustration:

$$F_{Balloon}\big(X(s)\big) = k\,\frac{\vec{N}(X)}{\left|\vec{N}(X)\right|}$$



Distance potential force (image):

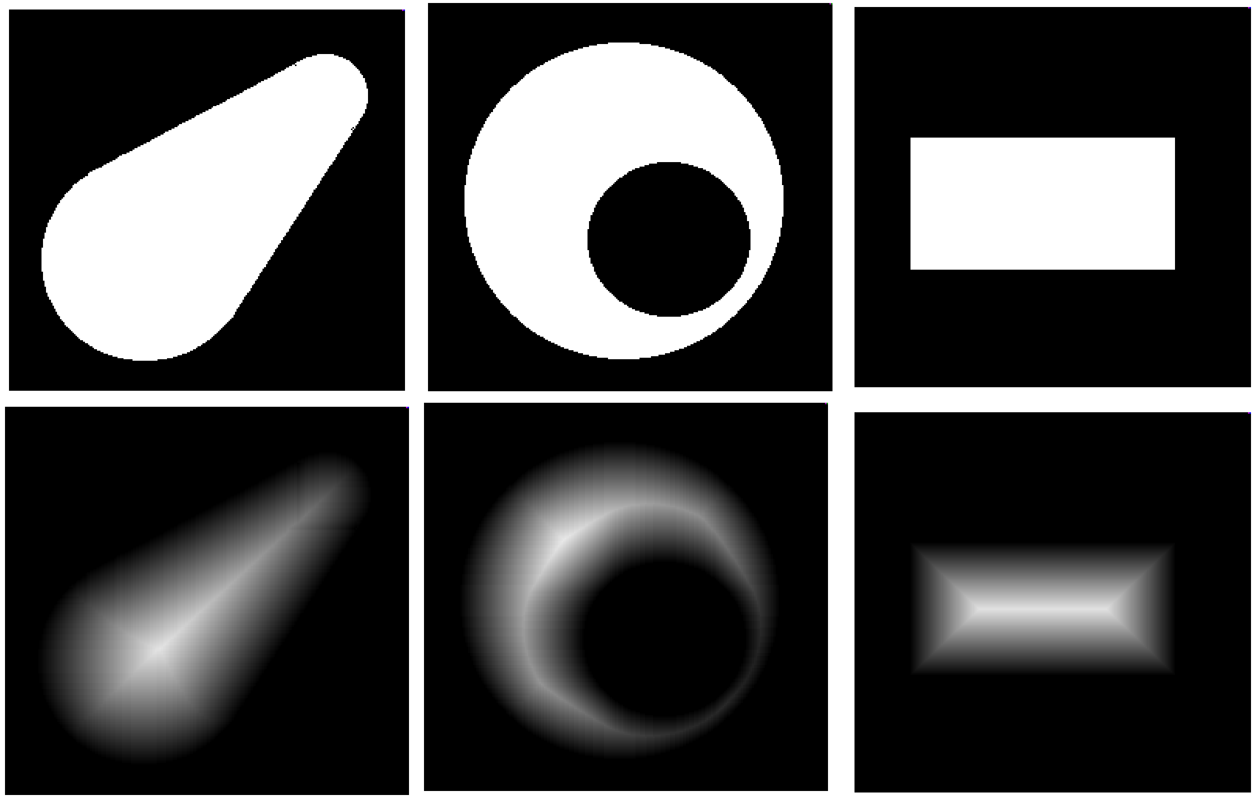
- We assume a prior knowledge about good local edge points
- We construct a distance map, $d(x, y)$, at each pixel obtained by calculating the distance between the pixel $(x(s), y(s))$ and the closest boundary point (prior knowledge), based on any distance function:

$$E_{distance}(x, y) = -w_d exp\left(-d(x, y)\right)$$

$$E_{distance}(x, y) = -\frac{1}{max(d(x, y), 1)}$$

Distance potential force (image):

› Distance map:



Dynamic Distance Force (image):

- We assume a prior knowledge about good local edge points
- We construct a *signed* distance map, $D(x, y)$, at each pixel obtained by calculating the distance between the pixel $\big(x(s), y(s)\big)$ and the closest boundary point (prior knowledge), based on any distance:

$$F_D(x, y) = w_D \frac{D(x, y)}{D_{max}} \vec{N}(X)$$

The End

- AnY QuEsTiOn?