Medical Image Analysis and Processing

Medical Image Segmentation
Geometric Models and Deep Learning

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Contents

Active Contour Without Edge (Chan-Vese)

- The segmentation cost function is:

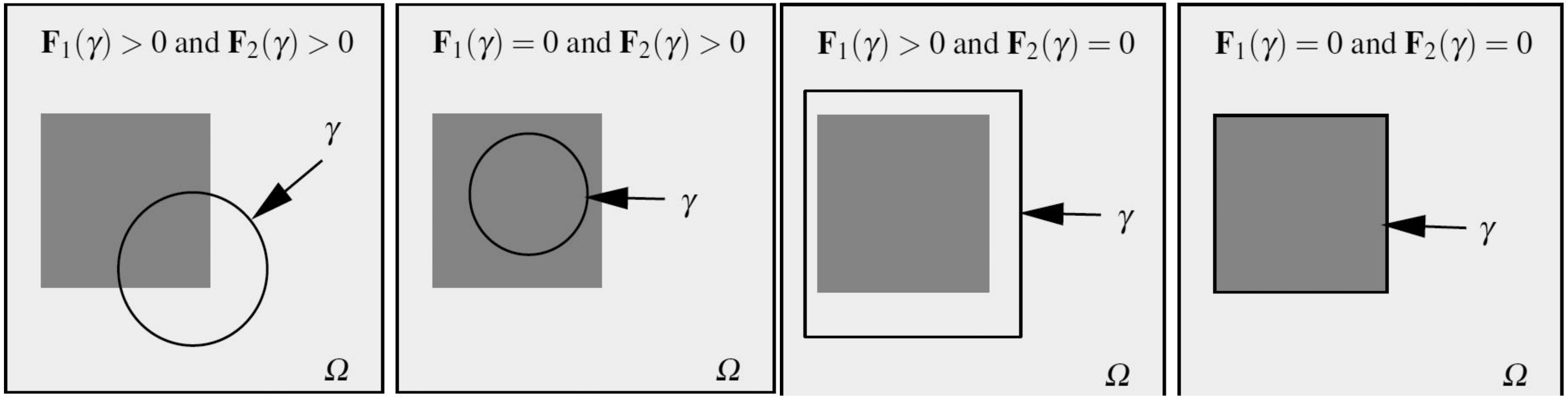
$$F(c_1, c_2, C)$$
$$= \mu Length(C) + \upsilon Area\big(inside(C)\big)$$
$$+ \int_{inside(C)} |I(x,y) - c_1|^2 dxdy + \int_{outside(C)} |I(x,y) - c_2|^2 dxdy$$

- Two first terms are smoothing factor
- c_1 and c_2 are average of input image, $I(x,y)$, inside and outside C

Active Contour Without Edge (Chan-Vese)

› Role of remaining terms:

$$F_1(C) + F_2(C) = \int_{inside(C)} |I(x,y) - c_1|^2 dxdy + \int_{outside(C)} |I(x,y) - c_2|^2 dxdy$$



Active Contour Without Edge (Chan-Vese)

- How to calculate curve *length and area*

$$Length(C) = Length(\phi = 0) = \int_{\Omega} \left|\nabla H_{\varepsilon}\big(\phi(x,y)\big)\right| dxdy$$

$$Length(C) = Length(\phi = 0) = \int_{\Omega} \delta_{\varepsilon}\big(\phi(x,y)\big)|\nabla\phi(x,y)|\, dxdy$$

$$Area(C) = Area(\phi < 0) = \int_{\Omega} \Big(1 - H_{\varepsilon}\big(\phi(x,y)\big)\Big)\, dxdy$$

Active Contour Without Edge (Chan-Vese)

› How to calculate *inside/outside* integral

$$\int_{inside(C)} |I(x,y) - c_1|^2 dxdy = \int_\Omega \left(1 - H_\varepsilon\big(\phi(x,y)\big)\right) |I(x,y) - c_1|^2 dxdy$$

$$\int_{outside(C)} |I(x,y) - c_1|^2 dxdy = \int_\Omega H_\varepsilon\big(\phi(x,y)\big) |I(x,y) - c_2|^2 dxdy$$

$$c_1(\phi) = \frac{\int_\Omega \left(1 - H_\varepsilon\big(\phi(x,y)\big)\right) I(x,y) dxdy}{\int_\Omega \left(1 - H_\varepsilon\big(\phi(x,y)\big)\right) dxdy}$$

$$c_2(\phi) = \frac{\int_\Omega H_\varepsilon\big(\phi(x,y)\big) I(x,y) dxdy}{\int_\Omega H_\varepsilon\big(\phi(x,y)\big) dxdy}$$

Active Contour Without Edge (Chan-Vese)

› Level-set formulation (Using Calculus of Variation):

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\{\mu\kappa - \upsilon + |I - c_1|^2 - |I - c_2|^2\}$$

Geometric Deformable Model

- More and more variation:
- Distance Regularized Level Set Evolution (DRLSE)
- Localizing Region-Based Active Contours (LRBAC)
- ….

Machine Learning Approaches

- Recalls from introductory:
- Unsupervised algorithm: Clustering (GMM, FCM, and ...)
- Supervised algorithm: Bayes, SVM, MLP
- We focus on Deep Convolutional Neural Networks

Deep CNN for image Segmentation

› Problem definition:

Segment a C channel image to S segments

› Network input: A C-channel full size image/volume (or patches)

$$input \in \mathbb{R}^{D\times H\times W\times C}$$

› Network output: A S-channel (binary or probability map)

$$output \in \mathbb{R}^{D\times H\times W\times S}$$

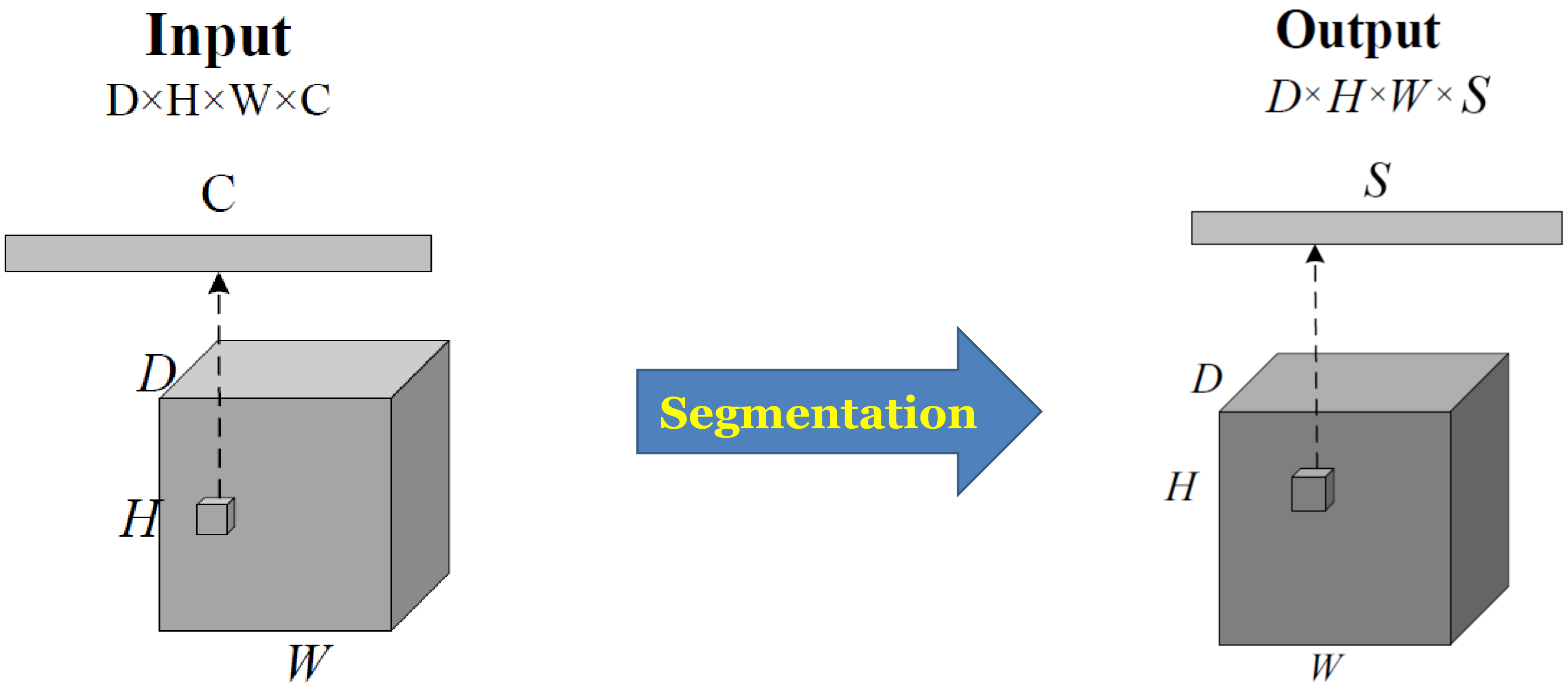
› In some applications output dimension is smaller than input dimension

› C : 1 (CT), 3 (RGB images), 3 or more (MRI), ...,

› S : 2 (foreground and background), more (organ segmentation)

Input-Output Illustration

› Input-output dimension:



Loss/Cost Functions

› We formulate for two segments (classes), generalization to S segments is trivial!

› Cross Entropy Loss:

$$CE(p, \hat{p}) = -(p\log(\hat{p}) + (1-p)\log(1-\hat{p}))$$

Loss/Cost Functions

- Network optimization algorithm minimize/maximize Cost function
- We formulate for two segments (classes), generalization to S segments is trivial!

Definitions

› Network Target (desired value):

$$Prob(Y = 1) = p, \qquad Prob(Y = 0) = 1 - p,$$

› Network Output (actual value):

$$Prob(\hat{Y} = 1) = \hat{p}, \qquad Prob(\hat{Y} = 0) = 1 - \hat{p},$$

› Output activation function is usually sigmoid or softmax:

› $Prob(\hat{Y} = 1) = \sigma(x) = \frac{1}{1+e^{-x}},$

› $Prob(\hat{Y} = 1) = softmax(x) = \frac{e^{z_1}}{e^{z_1}+e^{z_2}}$ (for $S > 2$ segmentation tasks)

Loss Function – Cross Entropy

- The cross entropy (CE) is one of the most popular loss functions.
- The CE compares pixel-wisely the predicted category vector with the target segmentation result vector.
- For binary (two segments):

$$CE(p,\hat{p}) = -(p\log(\hat{p}) + (1-p)\log(1-\hat{p}))$$

Loss Function – Weighted Cross Entropy

- The WCE address the problem of class imbalance (segments with small areas)
- Weighted Cross Entropy (Two Classes):

$$WCE(p, \hat{p}) = -(\beta p \log(\hat{p}) + (1-p)\log(1-\hat{p}))$$

- Balanced Cross Entropy (Two Classes):

$$BCE(p, \hat{p}) = -(\beta p \log(\hat{p}) + (1-\beta)(1-p)\log(1-\hat{p}))$$

Loss Function – Dice Loss

- The Dice is a popular performance metric for the *evaluation* of medical image segmentation.
- Dice Loss (1):

$$\mathrm{Dice}(A,B) = 1 - \frac{|A \cap B|}{|A \cup B|}, or\ \mathrm{Dice}(A,B) = 1 - \frac{2|A \cap B|}{|A| + |B|}$$

- Dice Loss (2):

$$DL(p,\hat{p}) = 1 - \frac{2\langle p,\hat{p}\rangle}{\|p\|_1 + \|\hat{p}\|_1}$$

- where $\langle p,\hat{p}\rangle$ is the dot product of the ground truth (each channel) and the prediction results matrix.

Loss Function – Tversky Loss

› This is a regularized version of Dice Loss to control the contribution of both false positive and false negative to the loss function.

› Tversky Loss (TL):

$$TL(p,\hat{p}) = 1 - \frac{\langle p,\hat{p}\,\rangle}{\langle p,\hat{p}\,\rangle + \alpha\langle 1-p,\hat{p}\,\rangle + \beta\langle p,1-\hat{p}\,\rangle}$$

› A most used parameter selection: $\beta = 1 - \alpha$

› Focused TI: $\mathrm{F}TL(p,\hat{p}) = \left(TL(p,\hat{p})\right)^{\gamma}, \alpha = 0.7, \beta = 0.3, \gamma = 0.75$

Loss Function – Others

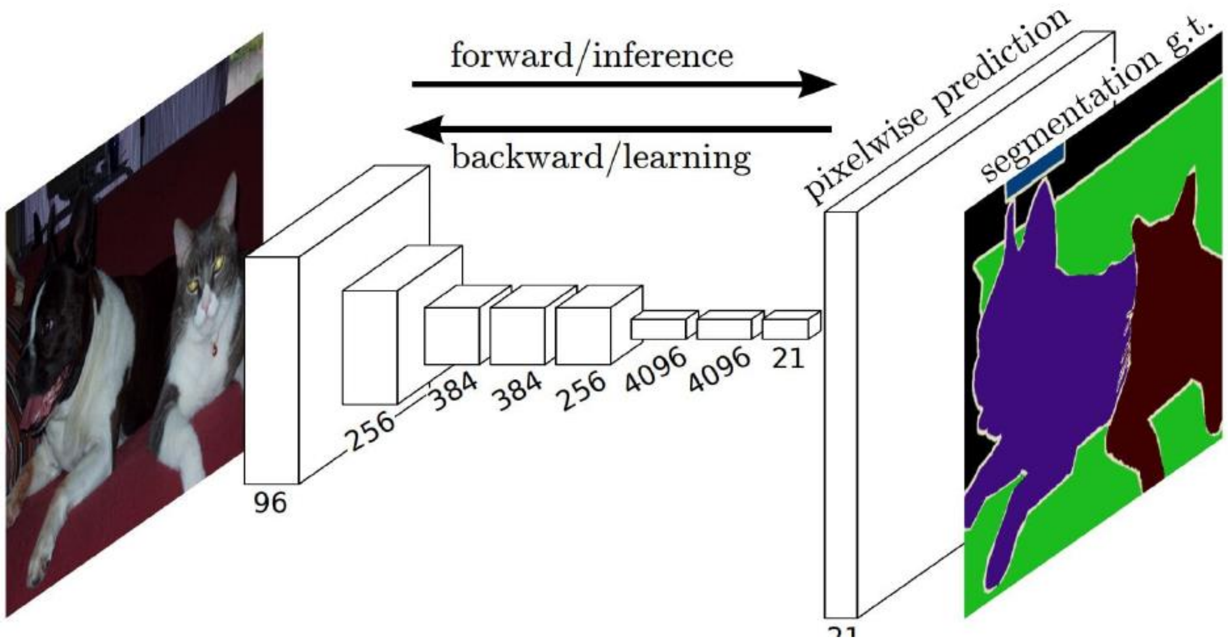
- There are many proposal for loss function to address imbalance problem:
  - Generalized Dice Loss (GDL)
  - Boundary Loss (BL)
  - Exponential Logarithmic Loss (EXP Loss)
  - …

Deep Architecture for Image Segmentation

- Deep Policy for image segmentation:
  - Convolutional Neural Networks (CNNs)
  - Recurrent Neural Networks (RNNs) and the LSTM
  - Encoder-Decoder and Auto-Encoder Models
  - Generative Adversarial Networks (GANs)

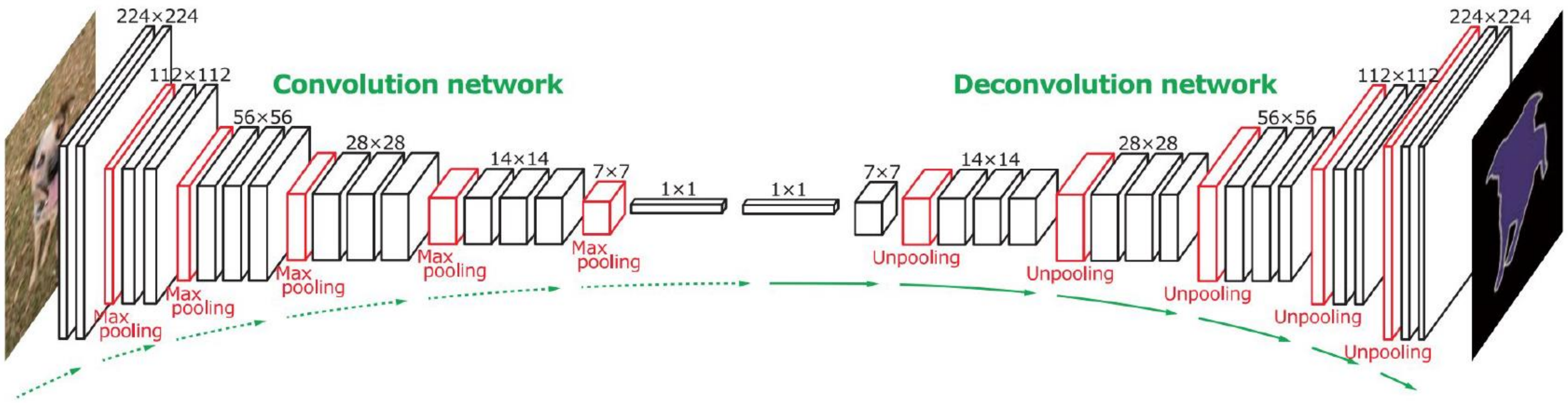
Deep Models for Segmentation

› Fully Convolutional Networks (FCNs)



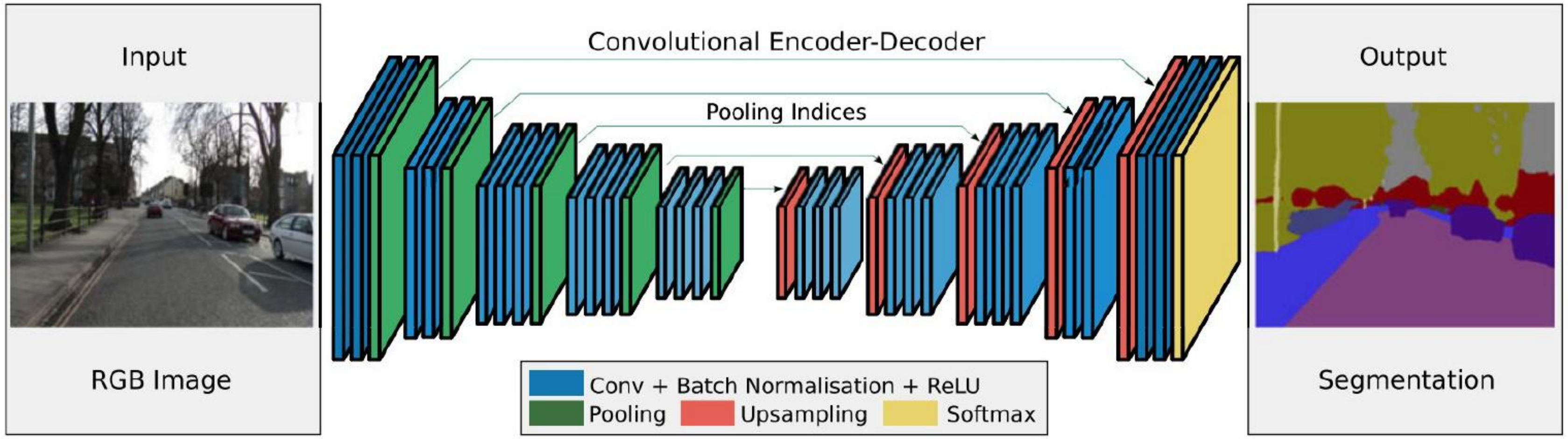
Deep Models for Segmentation

› DeConvNet:



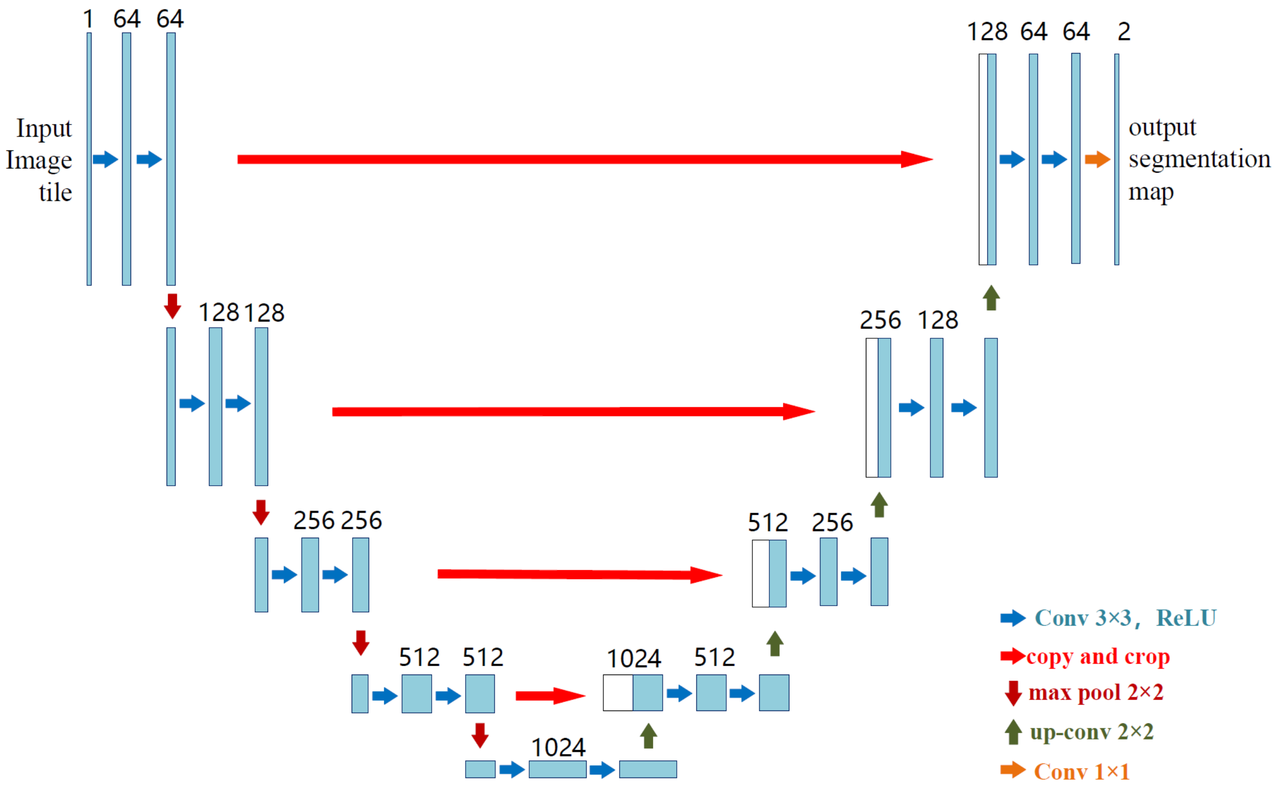
Deep Models for Segmentation

› SegNet



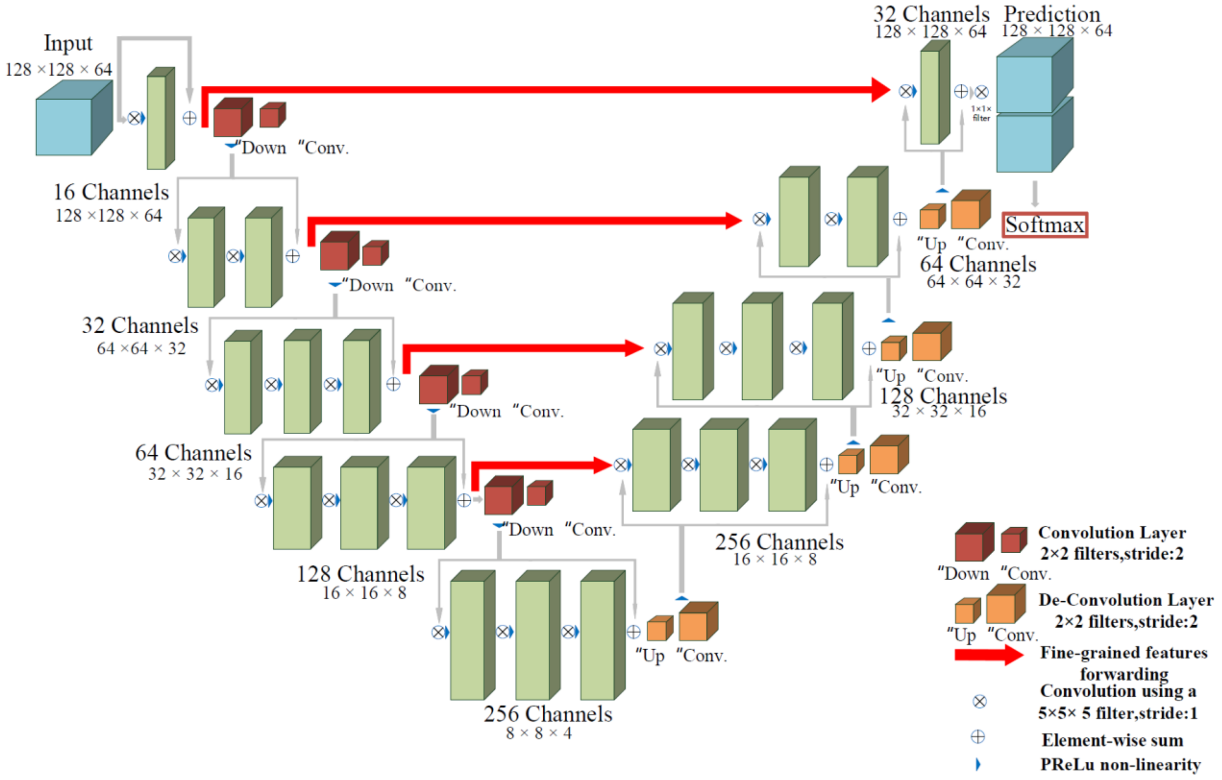
Deep Models for Segmentation

› U-Net (First Medical Segmentation Model):



Deep Models for Segmentation

› V-Net (3D U-Net):



Deep Models for Segmentation

- New aspects:
- Weakly Supervised
- Attention Based
- R-CNN
- Deep + Active Contour
- Deep + Non Local Means
- ….

The End

› AnY QuEsTiOn?