Medical Image Analysis and Processing

Medical Image Segmentation
Pixel Classification - Clustering

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Distance/online Course: Session 20 Episode#1

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› Adaptive Fuzzy C-Means (AFCM)

› Bias Corrected FCM (BCFCM)

Pixel Classification – Image Segmentation in Presence of Intensity Inhomogeneity

- *Intensity Inhomogeneity Aware* segmentation:
- Two core article:
  - Adaptive Fuzzy C-Means (AFCM)
  - Bias Corrected FCM (BCFCM)

Adaptive Fuzzy C-Means (AFCM):

- Notation:
  - $y(i,j)$: Acquired image intensity at location (i,j),
  - C: # of segments,
  - q: fuzziness parameter of algorithm (we assume $q = 2$),
  - $u_k(i,j)$: Membership value at pixel location (i,j) for segment $\#k$,
  - v_k: Centroid of segment $\#k$
- Conventional Fuzzy C-Means (FCM) cost function:

$$J_{FCM} = \sum_{(i,j)} \sum_{k=1}^{C} u_k^2(i,j) \|y(i,j) - v_k\|_2^2, \qquad s.t. \sum_{k=1}^{C} u_k(i,j) = 1$$

Adaptive Fuzzy C-Means (AFCM):

- AFCM cost function:

$$J_{AFCM} = \sum_{(i,j)} \sum_{k=1}^{C} u_k^2(i,j) \|y(i,j) - m(i,j)v_k\|_2^2 + \cdots$$

$$\lambda_1 \sum_{(i,j)} \left((m(i,j) * D_i)^2 + \left(m(i,j) * D_j\right)^2 \right) + \cdots$$

$$\lambda_2 \sum_{(i,j)} \left((m(i,j) ** D_{ii})^2 + 2\left(m(i,j) ** D_{ij}\right)^2 + \left(m(i,j) ** D_{jj}\right)^2 \right)$$

- $m(i,j)$: unknown multiplier field (model the brightness variation), slow variation!
- D_i and D_j are 1st order finite Difference operator for partial derivatives ($\frac{\partial m}{\partial x}, \frac{\partial m}{\partial y}$)
- D_{ii}, D_{jj} and D_{ij} are 2nd order finite Difference operator for partial derivatives ($\frac{\partial^2 m}{\partial x^2}, \frac{\partial^2 m}{\partial y^2}, \frac{\partial^2 m}{\partial x \partial y}$)

Adaptive Fuzzy C-Means (AFCM):

- The second and third terms are measure of smoothness:
- For 2D continuous function f:

$$S_1 = \iint_{-\infty}^{+\infty} \left(\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \right) dxdy = \iint_{-\infty}^{+\infty} |\nabla f|^2 dxdy$$

$$S_2 = \iint_{-\infty}^{+\infty} \left(\left(\frac{\partial^2 f}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 f}{\partial y^2}\right)^2 \right) dxdy = \iint_{-\infty}^{+\infty} \|H(f)\|_F^2 dxdy$$

Adaptive Fuzzy C-Means (AFCM):

- Algorithm steps:

1. Initial guess for $\{v_k\}_{k=1}^{C}$ using k-means, FCM, … and set $m(i,j) = 1$
2. Compute membership functions:

$$u_k(i,j) = \frac{\|y(i,j) - m(i,j)v_k\|^{-2}}{\sum_{l=1}^{C}\|y(i,j) - m(i,j)v_l\|^{-2}}$$

3. Compute centroids:

$$v_k = \frac{\sum_{(i,j)} u_k^2(i,j)m(i,j)y(i,j)}{\sum_{(i,j)} u_k^2(i,j)m^2(i,j)}$$

4. Compute multiplier field:

$$y(i,j)\sum_{k=1}^{C} u_k^2(i,j)v_k = m(i,j)\sum_{k=1}^{C} u_k^2(i,j)v_k^2 + \lambda_1\big(m(i,j) ** H_1(i,j)\big) + \lambda_2\big(m(i,j) ** H_2(i,j)\big)$$

where $H_1(i,j) = D_i * \breve{D}_i + D_j * \breve{D}_j$ and $H_2(i,j) = D_{ii} * \breve{D}_{ii} + +2\big(D_{ij} * \breve{D}_{ij}\big) + D_{jj} * \breve{D}_{jj}$, $\check{f}(i) = f(-i)$

5. Step over 2-3-4 until convergence

Adaptive Fuzzy C-Means (AFCM):

› Step (4):

$$y(i,j)\sum_{k=1}^{C} u_k^2(i,j)v_k = m(i,j)\sum_{k=1}^{C} u_k^2(i,j)v_k^2 + \lambda_1\big(m(i,j) ** H_1(i,j)\big) + \lambda_2\big(m(i,j) ** H_2(i,j)\big)$$

› may be rewrite as: $\boldsymbol{f} = \boldsymbol{A}\boldsymbol{m}$

› For $y \in \mathbb{R}^{M\times N}$ then $\boldsymbol{f} \in \mathbb{R}^{MN}, \boldsymbol{m} \in \mathbb{R}^{MN}$ and $\boldsymbol{A} \in \mathbb{R}^{MN\times MN}$

› Huge! systems of linear equations

› It is possible to solve via weighted Jacobi iteration:

$$\boldsymbol{m}^{(i+1)} = [(1-\omega)\boldsymbol{I} - \omega\boldsymbol{D}^{-1}(\boldsymbol{L}+\boldsymbol{U})]\boldsymbol{m}^{(i)} + \omega\boldsymbol{D}^{-1}\boldsymbol{f}, \qquad \boldsymbol{A} = \boldsymbol{D}+\boldsymbol{L}+\boldsymbol{U}, \omega^* = \frac{2}{3}$$

and multiresolution (multigrid) scheme.

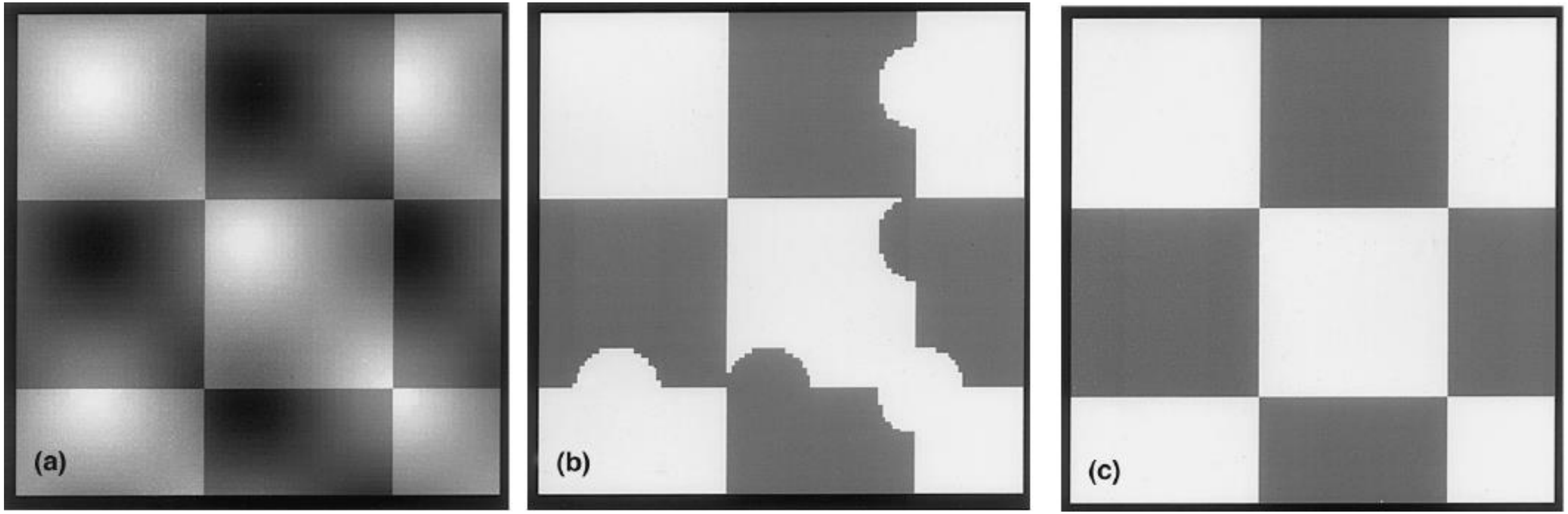
Adaptive Fuzzy C-Means (AFCM):

› Example for A decomposition:

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{\boldsymbol{A}} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}}_{\boldsymbol{D}} + \underbrace{\begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}}_{\boldsymbol{L}} + \underbrace{\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}}_{\boldsymbol{U}}$$

Adaptive Fuzzy C-Means (AFCM):

› Results:



(a) original (b) FCM (c) AFCM

Bias Corrected FCM (BCFCM)

- Notation:
  - $y(i,j)$: Log-transferred *acquired* image intensity at location (i,j),
  - $x(i,j)$: Log-transferred *true* image intensity at location (i,j),
  - $\beta(i,j)$: Log-transferred *bias field* at location (i,j),

$$y(i,j) = x(i,j) + \beta(i,j)$$

  - C: # of segments,
  - q: fuzziness parameter of algorithm (we assume $q = 2$),
  - $u_k(i,j)$: Membership value at pixel location (i,j) for segment $\#k$,
  - v_k: Centroid of segment $\#k$

Bias Corrected FCM (BCFCM)

- Bias Corrected Fuzzy C-Means (AFCM) cost function:

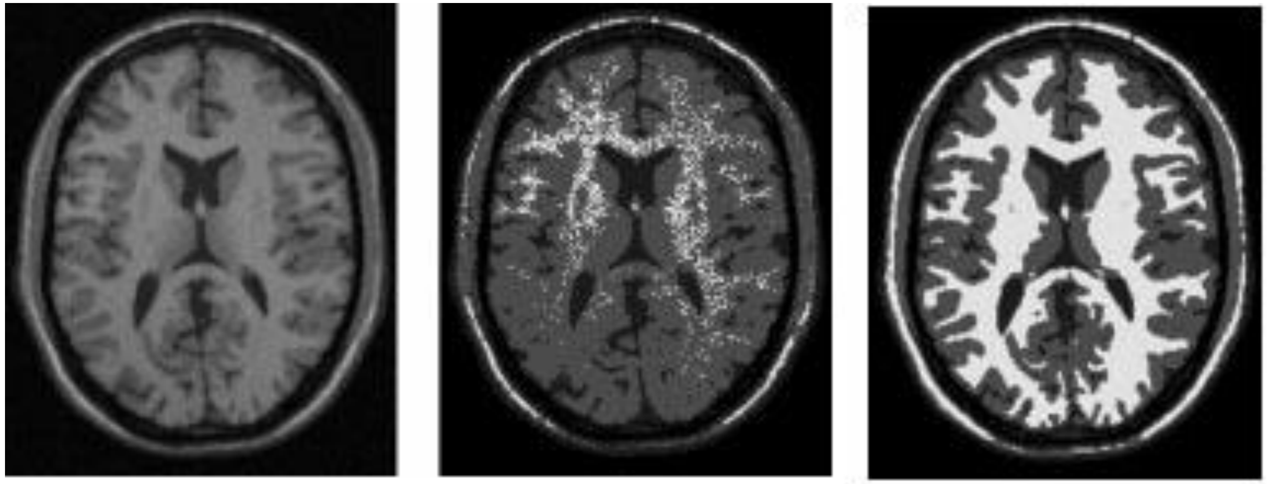
$$J_{BCFCM}=\sum_{(i,j)}\sum_{k=1}^{C}u_k^2(i,j)\|y(i,j)-\beta(i,j)-v_k\|_2^2+$$

$$\frac{\alpha}{N_R}\sum_{(i,j)}\sum_{k=1}^{C}\left(u_k^2(i,j)\sum_{y(p,q)\in N_{(i,j)}}\|y(p,q)-\beta(p,q)-v_k\|_2^2\right)$$

- where $N_{(i,j)}$ stands for the set of neighbors that exist in a window around location (i,j) and N_R is its cardinality.
- The new term the labeling of a pixel (voxel) to be influenced by the labels in its immediate neighborhood, the neighborhood effect acts as a regularizer and biases the solution toward piecewise-homogeneous labeling.

Bias Corrected FCM (BCFCM)

- Results (5% Gaussian noise and 20% intensity inhomogeneity):
- Left (simulated data), Center (FCM), Right (BCFCM)



The End

- AnY QuEsTiOn?