Medical Image Analysis and Processing

Medical Image Registration Methods

Emad Fatemizadeh

Distance/online Course: Session 26

Date: 30 May 2021, 9th Khordad 1400

Contents

Correlation Method

- Image correlation (for translation):

$$CC(\Delta x, \Delta y) = \frac{\sum_x \sum_y (F(x,y) - \bar{F})(M(x-\Delta x, y-\Delta y) - \bar{M})}{(\sum_u \sum_v (F(x,y) - \bar{F})^2)^{0.5} (\sum_u \sum_v (M(x,y) - \bar{M})^2)^{0.5}}$$

- Frequency Domain Implementation:

$$M(x,y) = F(x-\Delta x, y-\Delta y)$$

$$\frac{\tilde{F}(u,v)\tilde{M}^*(u,v)}{|\tilde{F}(u,v)\tilde{M}^*(u,v)|} = e^{j(u\Delta x + v\Delta y)} \xleftrightarrow{Inv\ Fourier} \delta(x-\Delta x, y-\Delta y)$$

Radial Basis Function Interpolation

- Registration is interpolation!
- General formulation in 2D:
- $T_x(x, y) = a_{00} + a_{01}x + a_{02}y + \sum_{i=1}^{N} w_{0i} g(X - X_i)$
- $T_y(x, y) = a_{10} + a_{11}x + a_{12}y + \sum_{i=1}^{N} w_{1i} g(X - X_i)$
- where g is a radial basis function:
- $g(X - X_i) = g(\|X - X_i\|) = g\left(\sqrt{(x - x_i)^2 + (y - y_i)^2 + d^2}\right)$
- Most used function in 2D is $r^2 log r^2$ (Thin Plate Spline – TPS)

Radial Basis Function Interpolation

› We need three extra equations:

$$\sum_{k=1}^{N} w_{0k} = \sum_{k=1}^{N} w_{1k} = 0$$

$$\sum_{k=1}^{N} x_k w_{0k} = \sum_{k=1}^{N} y_k w_{0k} = 0$$

$$\sum_{k=1}^{N} x_k w_{1k} = \sum_{k=1}^{N} y_k w_{1k} = 0$$

Radial Basis Function Interpolation

› TPS is energy minimizer!

$$\iint\left(\left(\frac{\partial^2 T_x}{\partial y^2}\right)^2+2\left(\frac{\partial^2 T_x}{\partial x \partial y}\right)^2+\left(\frac{\partial^2 T_x}{\partial x^2}\right)^2\right)dxdy$$

$$\iint\left(\left(\frac{\partial^2 T_y}{\partial y^2}\right)^2+2\left(\frac{\partial^2 T_y}{\partial x \partial y}\right)^2+\left(\frac{\partial^2 T_y}{\partial x^2}\right)^2\right)dxdy$$

Radial Basis Function Interpolation

- Registration is interpolation!
- General formulation in 3D:

$$T_x(x,y) = a_{00} + a_{01}x + a_{02}y + a_{03}z + \sum_{i=1}^{N} w_{0i}g(X - X_i)$$

$$T_y(x,y) = a_{10} + a_{11}x + a_{12}y + a_{13}z + \sum_{i=1}^{N} w_{1i}g(X - X_i)$$

$$T_z(x,y) = a_{20} + a_{21}x + a_{22}y + a_{23}z + \sum_{i=1}^{N} w_{2i}g(X - X_i)$$

- Most used function in 3D is r (Thin Plate Spline – TPS)

Radial Basis Function Interpolation

› We need four extra equations:

$$\sum_{k=1}^{N} w_{0k} = \sum_{k=1}^{N} w_{1k} = \sum_{k=1}^{N} w_{2k} = 0$$

$$\sum_{k=1}^{N} x_k w_{0k} = \sum_{k=1}^{N} y_k w_{0k} = \sum_{k=1}^{N} z_k w_{0k} = 0$$

$$\sum_{k=1}^{N} x_k w_{1k} = \sum_{k=1}^{N} y_k w_{1k} = \sum_{k=1}^{N} z_k w_{1k} = 0$$

$$\sum_{k=1}^{N} x_k w_{2k} = \sum_{k=1}^{N} y_k w_{2k} = \sum_{k=1}^{N} z_k w_{2k} = 0$$

Radial Basis Function Interpolation

› TPS is energy minimizer!

$$\iiint \left(\left(\frac{\partial^2 T_x}{\partial x^2}\right)^2 + \left(\frac{\partial^2 T_x}{\partial y^2}\right)^2 + \left(\frac{\partial^2 T_x}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 T_x}{\partial x \partial y}\right)^2 + 2\left(\frac{\partial^2 T_x}{\partial x \partial z}\right)^2 + 2\left(\frac{\partial^2 T_x}{\partial y \partial z}\right)^2 \right) dxdydz$$

$$\iiint \left(\left(\frac{\partial^2 T_y}{\partial x^2}\right)^2 + \left(\frac{\partial^2 T_y}{\partial y^2}\right)^2 + \left(\frac{\partial^2 T_y}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 T_y}{\partial x \partial y}\right)^2 + 2\left(\frac{\partial^2 T_y}{\partial x \partial z}\right)^2 + 2\left(\frac{\partial^2 T_y}{\partial y \partial z}\right)^2 \right) dxdydz$$

$$\iiint \left(\left(\frac{\partial^2 T_z}{\partial x^2}\right)^2 + \left(\frac{\partial^2 T_z}{\partial y^2}\right)^2 + \left(\frac{\partial^2 T_z}{\partial z^2}\right)^2 + 2\left(\frac{\partial^2 T_z}{\partial x \partial y}\right)^2 + 2\left(\frac{\partial^2 T_z}{\partial x \partial z}\right)^2 + 2\left(\frac{\partial^2 T_z}{\partial y \partial z}\right)^2 \right) dxdydz$$

Registration without correspondence

› Principal Axes Registration

› Iterative Closest Point

› Using Image Information

Principal Axes Registration (PAR)

- Fixed image landmark points:

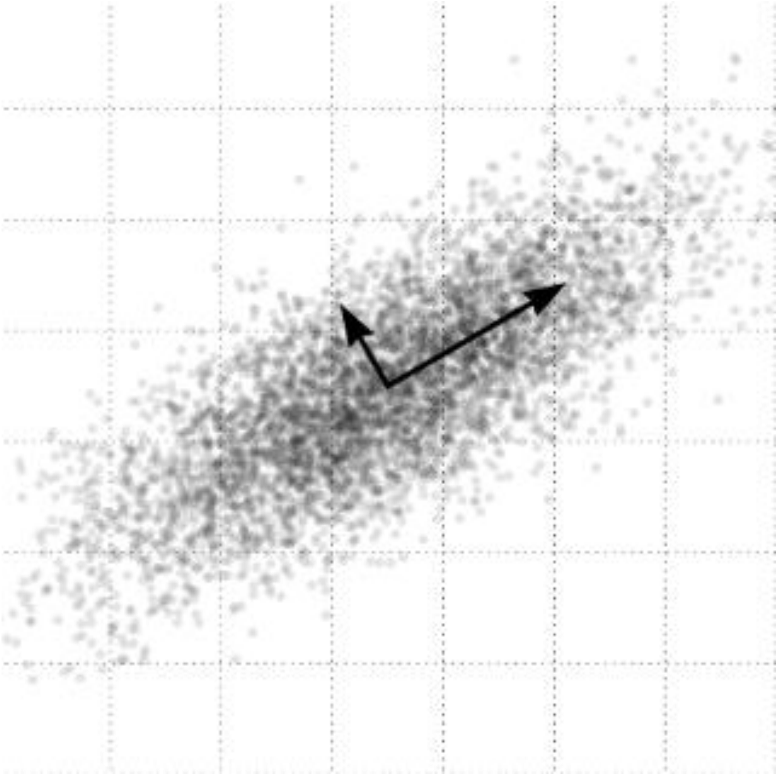
$$\{\mathbf{p}_i = (x_i, y_i) : i = 1, \dots, m\}$$

- Moving image landmark points:

$$\{\mathbf{P}_j = (X_j, Y_j) : j = 1, \dots, n\}$$

Principal Axes Registration (PAR)

› Main idea: align major axis of two point clouds (PCA)



Principal Axes Registration (PAR)

› Centroid of fixed (left) and moving (right) images:

$$\bar{x} = \frac{1}{m}\sum_{i=1}^{m} x_i\,, \qquad \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\bar{y} = \frac{1}{m}\sum_{i=1}^{m} y_i\,, \qquad \bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

Principal Axes Registration (PAR)

- First shift to centroid:

$$\mathbf{T}_1 = \begin{bmatrix} 1 & 0 & -\bar{x} \\ 0 & 1 & -\bar{y} \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{p}_i = \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix},$$

$$\mathbf{T}_2 = \begin{bmatrix} 1 & 0 & -\bar{X} \\ 0 & 1 & -\bar{Y} \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{P}_j = \begin{bmatrix} X_j \\ Y_j \\ 1 \end{bmatrix}$$

Principal Axes Registration (PAR)

› The angle between the major axis of the points and the x-axis is:

$$\alpha = 0.5\tan^{-1}\left\{\frac{2\sum_{i=1}^{m}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{m}(x_i - \bar{x})^2 - \sum_{i=1}^{m}(y_i - \bar{y})^2}\right\}$$

$$\beta = 0.5\tan^{-1}\left\{\frac{2\sum_{j=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{j=1}^{n}(X_i - \bar{X})^2 - \sum_{j=1}^{n}(Y_i - \bar{Y})^2}\right\}$$

› Rotational difference between two images is: $\theta = \alpha - \beta$

Principal Axes Registration (PAR)

- Another approach is to whiten both points cloud, $\tilde{p} = A(p - \bar{p})$
- where $A = [v_1|v_2|\ldots|v_D]$ and $\{v_i\}_{i=1}^{D}$ are eigenvectors of covariance matrix of points cloud

$$\boldsymbol{c} = \frac{1}{m}\begin{bmatrix} \sum_{i=1}^{m}(x_i - \bar{x})^2 & \sum_{i=1}^{m}(x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{i=1}^{m}(x_i - \bar{x})(y_i - \bar{y}) & \sum_{i=1}^{m}(y_i - \bar{y})^2 \end{bmatrix}$$

$$\mathbf{C} = \frac{1}{n}\begin{bmatrix} \sum_{j=1}^{n}\left(X_j - \bar{X}\right)^2 & \sum_{j=1}^{n}\left(X_j - \bar{X}\right)\left(Y_j - \bar{Y}\right) \\ \sum_{j=1}^{n}\left(X_j - \bar{X}\right)\left(Y_j - \bar{Y}\right) & \sum_{j=1}^{n}\left(Y_j - \bar{Y}\right)^2 \end{bmatrix}$$

Iterative Closest Points (ICP)

- The basic idea behind ICP is that, if we somehow knew correspondences, we could solve for the translation that minimizes pairwise distance.
- How to find correspondence?

1) User input

2) Feature detection and Signature (SIFT/SURF/ORB/…)

3) ICP assumes closest point are correspond! (after an initial registration, like as PAR

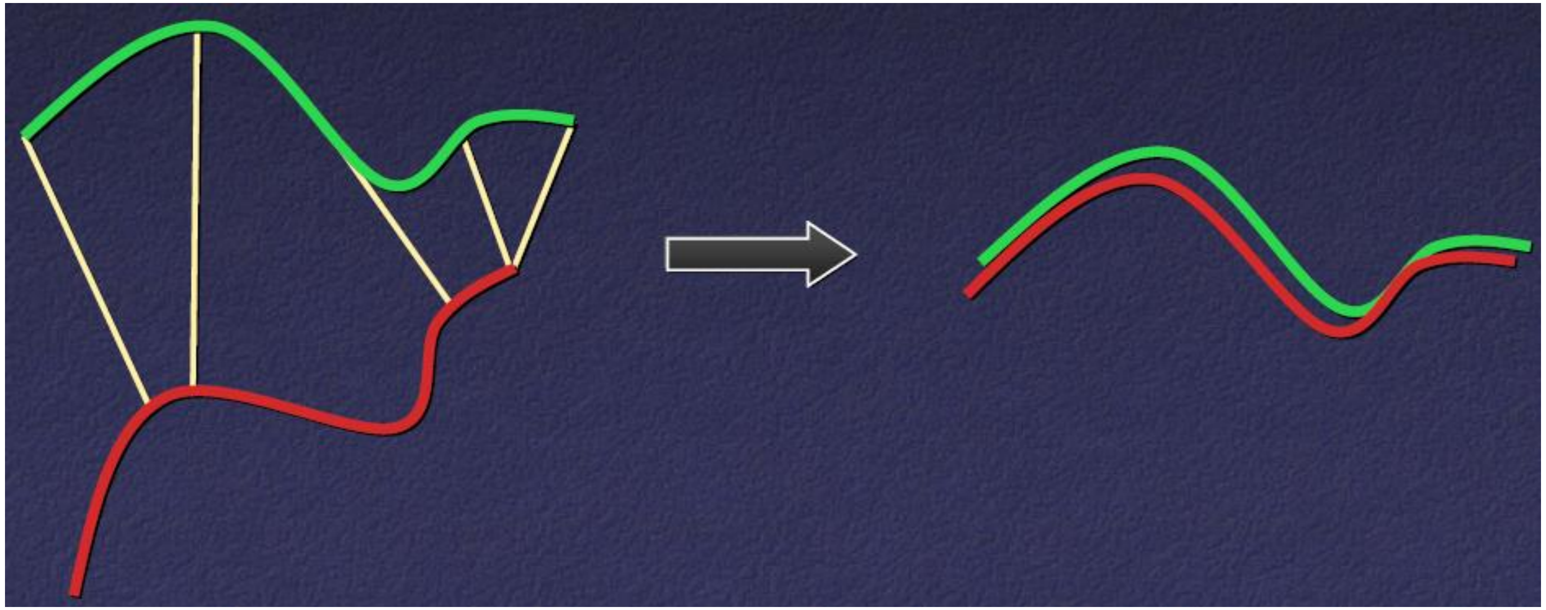
Iterative Closest Points (ICP)

› ICP steps:

1. Initial registration (principal axes registration)
2. Selecting source (from fixed image) landmarks (all or random pick-up)
3. Matching to the landmarks in destination (moving) images
4. Rejecting outlier matches (based on distances)
5. Optimize and apply model (rigid body transform) to moving landmarks
6. Repeat from step #2 until convergence

Iterative Closest Points (ICP)

- ICP start to stop:



- There are many variation for each step

Using Image Information

- We need an similarity/distance measure between two images Fixed (A) and moving (B'), then optimize the measure with respect to transformation parameters

Similarity Measure - SSD

- Sum of Squares of Differences (SSD):

$$\mathrm{SSD} = \sum_i |A(i) - B'(i)|^2 \quad \forall i \in A \cap B'$$

- SSD is the optimum measure when A and B' differ only by Gaussian noise.
- In MRI, noise is not Gaussian and SSD is too sensitive to outlier (for example tumors)

Similarity Measure - SAD

- Sum of Absolute of Differences (SAD):

$$SAD = \sum_i |A(i) - B'(i)| \quad \forall i \in A \cap B'.$$

- SAD is the optimum measure when A and B' differ only by Laplacian noise.
- SAD problem is differentiability.

Similarity Measure - CC

- Correlation Coefficient (CC):

$$\mathrm{CC} = \frac{\sum_i (A(i) - \bar{A})(B'(i) - \bar{B}')}{\{\sum_i (A(i) - \bar{A})^2 \sum_i (B'(i) - \bar{B}')^2\}^{0.5}} \quad \forall i \in A \cap B'$$

- Invariant to bias (DC level) and scale in brightness

Similarity Measure - RIU

› Ratio-Image Uniformity (RIU):

a. Smooth and interpolate the images to have cubic voxels (optional), and determine the number of voxels N in B' that lie within $A \cap B'$.

b. Calculate the ratio image $R(i) = B'(i)/A(i)$.

c. Calculate the mean of R: $\mu_R = \frac{1}{N}\sum_i R(i)$.

d. Calculate the standard deviation of R: $\sigma_R^2 = \frac{1}{N}\sum_i (R(i) - \mu_R)^2$.

e. RIU = the normalized standard deviation: σ_R/μ_R.

Similarity Measure - PIU

- Partitioned Intensity Uniformity (PIU):

$$\text{PIU} = \sum_{a\in\{a\}} \frac{n_A(a)}{N}\frac{\sigma_{B'}(a)}{\mu_{B'}(a)}$$

- $n_A(a)$ is the number of voxels in image A with intensity a, and $\mu_{B'}(A)$ and $\sigma_{B'}(A)$ are the mean and standard deviation of the voxels in image B' that co-occur with voxels whose intensities lie in partition $\boldsymbol{a}$ in image A.

Similarity Measure - RC

- Given two Image I and J, Intensity Correction field (S), AWG (η), a basis for transform ($\boldsymbol{q}_n$), $I = J(T) + S + \eta$
- Residual Complexity (RC):

$$\mathrm{RC}(T) = \sum_{n=1}^{N} log\left(\frac{(\boldsymbol{q}_n \boldsymbol{r})^2}{\alpha} + 1\right)$$

- where:

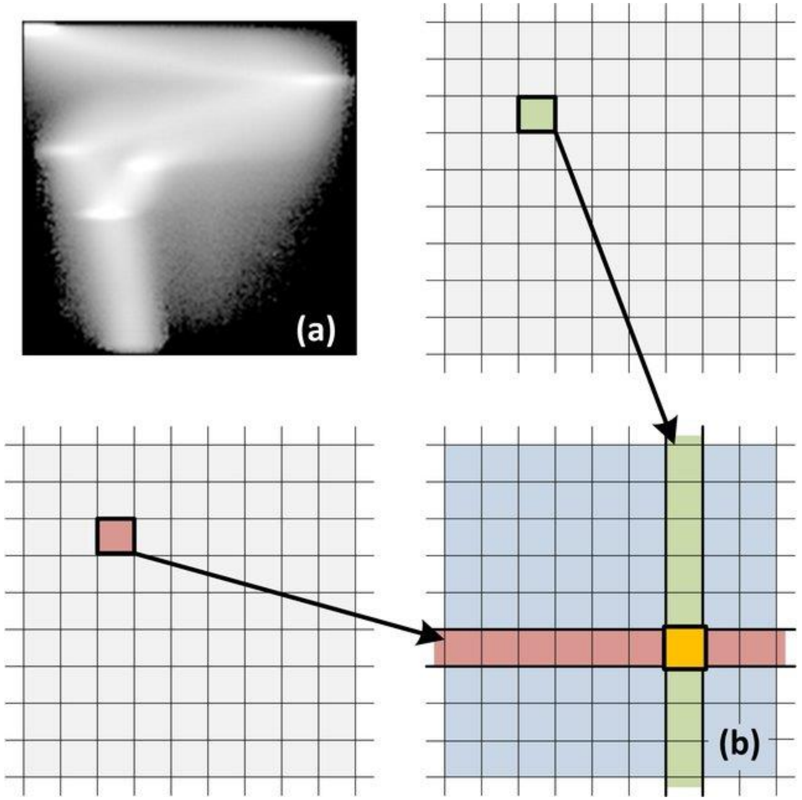
$$\boldsymbol{r} = I - J(T)$$

Joint Probability Distributions

- For two images A and B, the joint histogram is two-dimensional and is constructed by plotting the intensity a of each voxel in image A against the intensity b of the corresponding voxel in image B.
- The value of each histogram location $h(a, b)$ will therefore correspond to the number of image voxels with intensity a in modality A and intensity b in modality B.

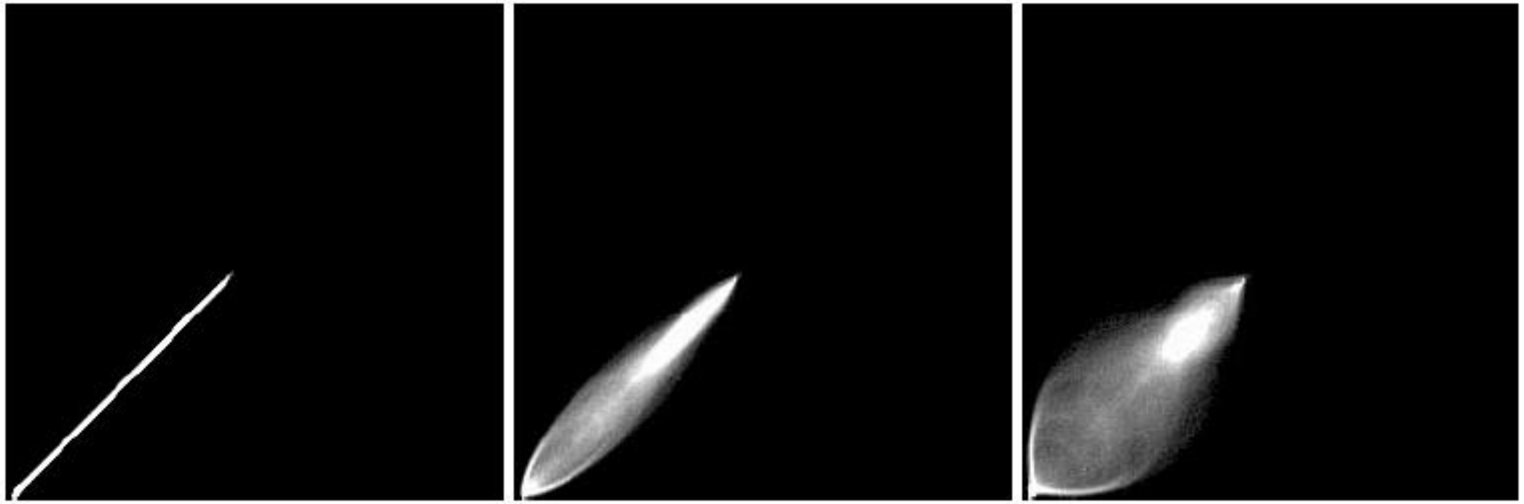
Joint Probability Distributions

› Joint histograms and Joint probability distributions:



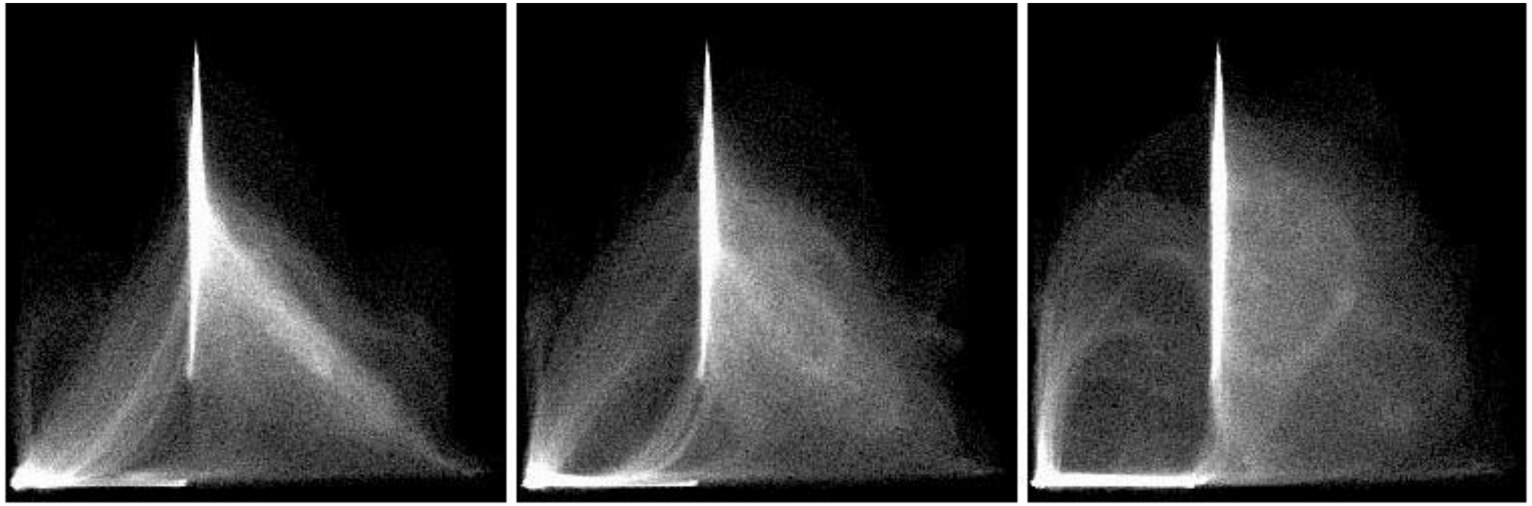
Joint pdf Example (1)

- MRI-MRI (same image), MRI-CT (same subject), MRI-PET (same subject) all zero translation



Joint pdf Example (2)

› MRI-MRI (same image), MRI-CT (same subject), MRI-PET (same subject) all 2mm lateral translation.



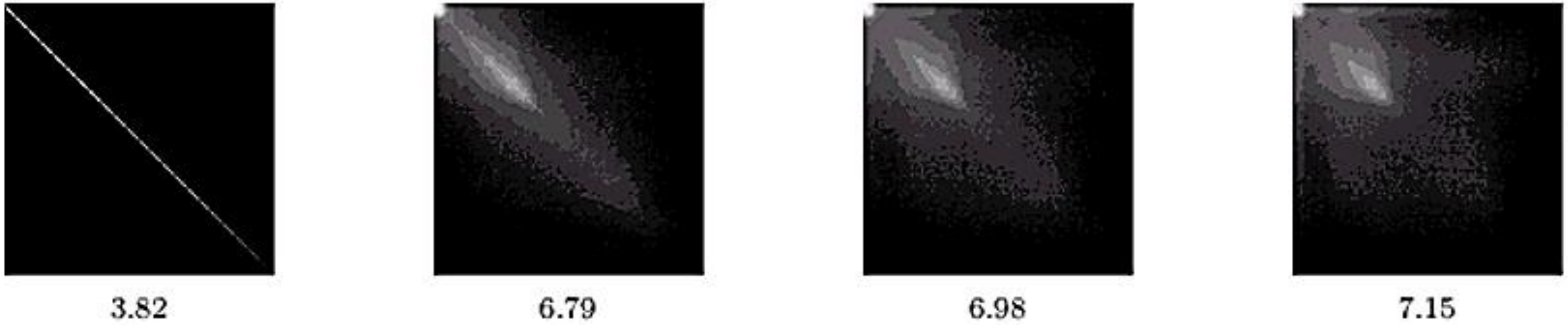
Joint pdf Example (3)

- MRI-MRI (same image), MRI-CT (same subject), MRI-PET (same subject) all 5mm lateral translation.

Joint Entropy

› Definition:

$$H(A, B') = -\sum_{i,j} p(i,j) \cdot \log[p(i,j)]$$



Mutual Information (MI)

- Definition(s):
  - $I(A,B) = H(B) - H(B|A) = H(A) - H(A|B)$
  - $I(A,B) = H(A) + H(B) - H(A,B)$
  - $I(A,B) = \sum_{a,b} p(a,b) log\left(\frac{p(a,b)}{p(a)p(b)}\right)$
- Mutual information is the amount that the uncertainty in B (or A) is reduced when A (or B) is known.

Mutual Information (MI)

- Mutual information is the amount that the uncertainty in B (or A) is reduced when A (or B) is known.
- Maximizing the mutual info is equivalent to minimizing the joint entropy (last term)
- Advantage in using mutual info over joint entropy is it includes the individual input's entropy
- Works better than simply joint entropy in regions of image background (low contrast) where there will be low joint entropy but this is offset by low individual entropies as well so the overall mutual information will be low.

Mutual Information (MI)

- This definition is related to the Kullback-Leibler distance between two distributions
- Measures the dependence of the two distributions
- In image registration $I(A, B)$ will be maximized when the images are aligned
- In feature selection choose the features that minimize $I(A, B)$ to ensure they are not related.

MI Properties

- It can be shown:
  - $I(A, B) = I(B, A)$
  - $I(A, A) = H(A)$
  - $I(A, B) < H(A), I(A, B) < H(B)$
  - $I(A, B) \geq 0$
  - $I(A, B) = 0$ for independent variables

Related Measure

› Normalized Mutual Information (NMI):

$$NMI(A,B) = \frac{H(A) + H(B)}{H(A,B)}$$

› Entropy Correlation Coefficient (ECC):

$$ECC(A,B) = 2 - \frac{2}{NMI(A,B)}$$

$$ECC(A,B) = \sqrt{2 - \frac{2}{NMI(A,B)}}$$

The End

› AnY QuEsTiOn?