Digital Image Processing

Feature Extraction

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Contents

- Fourier Descriptor
- Statistical Features
- Texture
- PCA

Fourier Descriptor

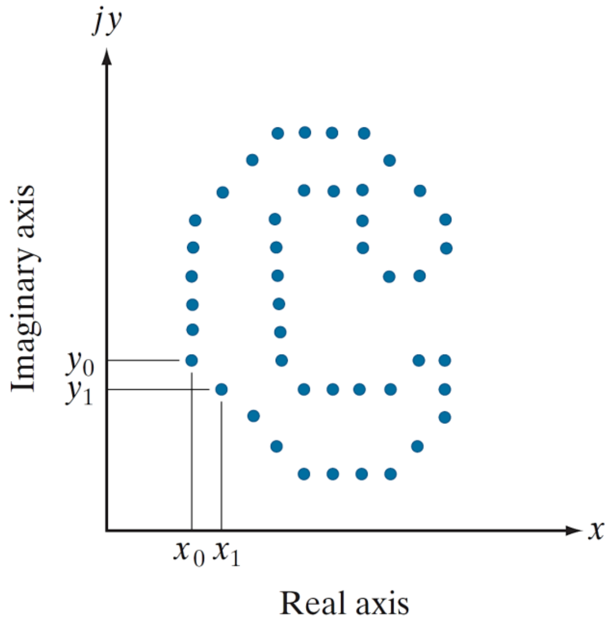
› Mathematical Formulation:

› Complex sequence: $s[k] = x[k] + jy[k]$

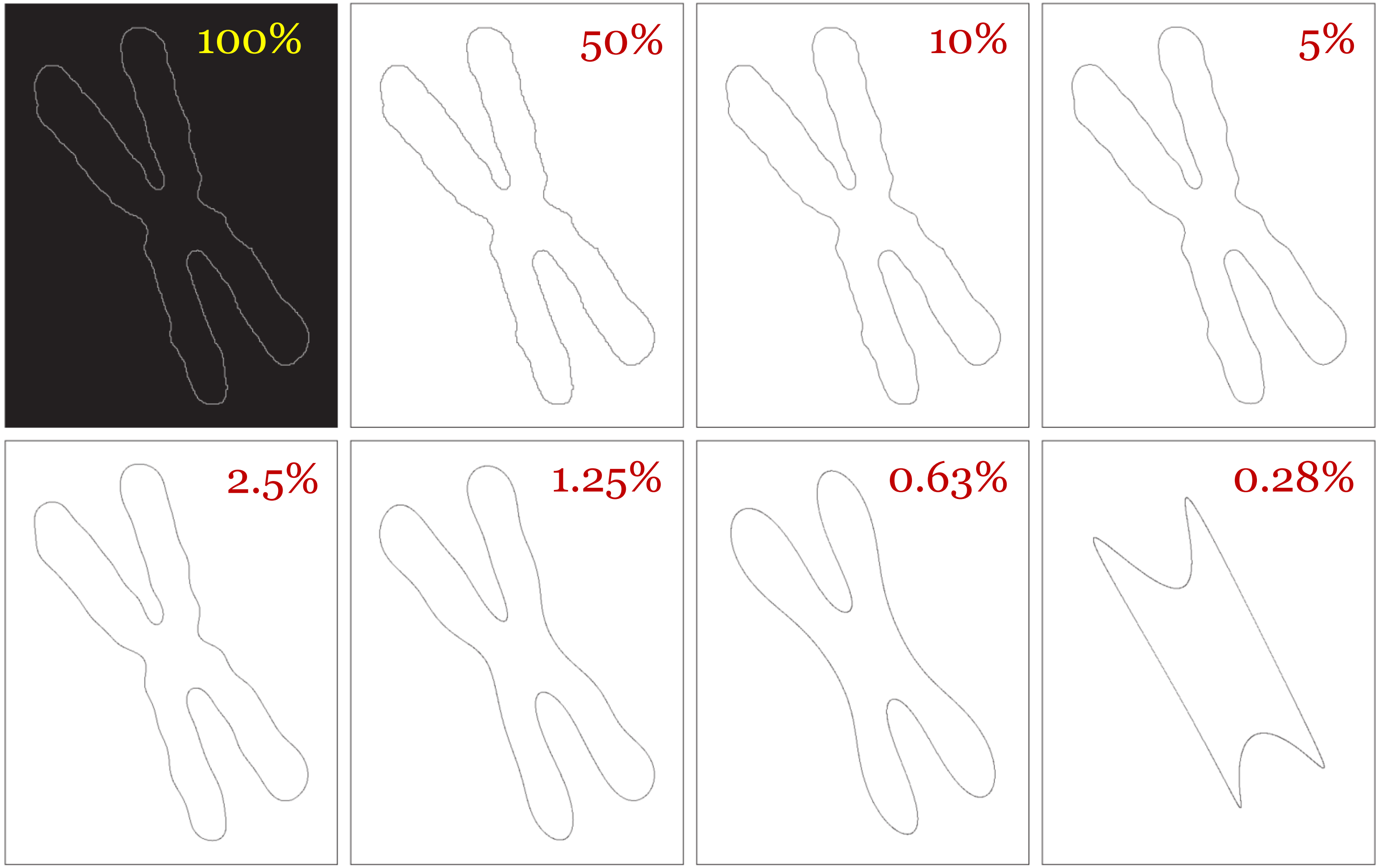
› Its DFT: $a[u] = \sum_{k=0}^{N-1} s[k]e^{-j\frac{2\pi ku}{N}}$

› Exact IDFT: $s[k] = \frac{1}{N}\sum_{u=0}^{N-1} a[u]e^{j\frac{2\pi ku}{N}}$

› Approximately IDFT: $\hat{s}[k] = \frac{1}{N}\sum_{u=0}^{P-1} a[u]e^{j\frac{2\pi ku}{N}}$



Fourier Descriptor Example



Fourier Descriptor Properties

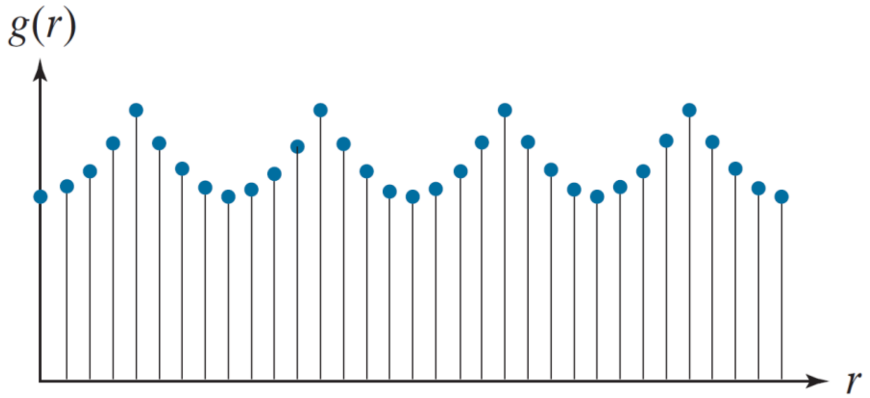
- Is it invariant to:
  - Rotation?
  - Scaling?
  - Translation?
  - Starting point?

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

  - Invariant descriptor: $\frac{|a(u)|}{\sum_{k=0}^{N-1}|a(k)|}$

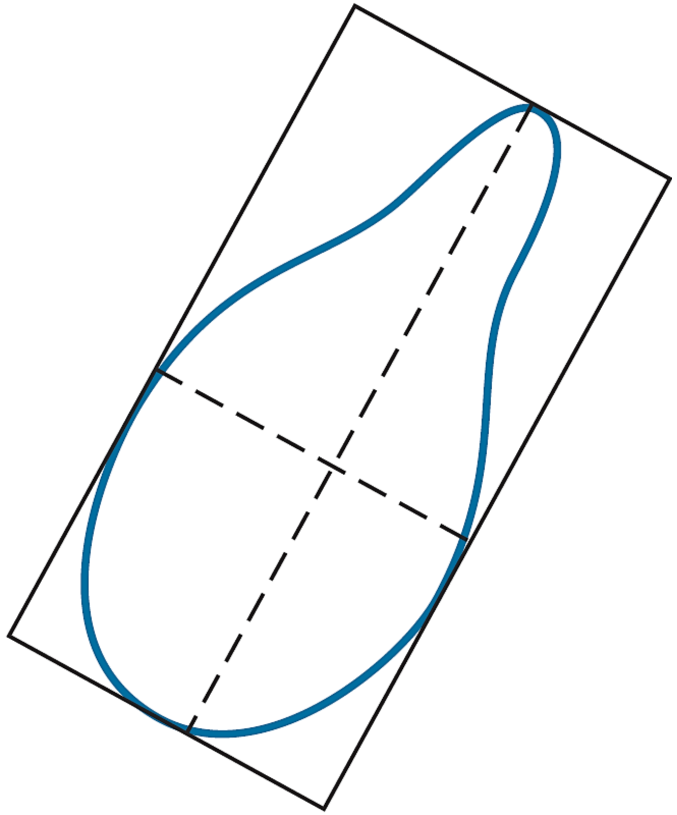
Statistical Feature from 1-D representation

- Suppose we have 1-D representation (Signature or Fourier transform), using raw data or data pdf:
- Data pdf:
  - $\mu_n(z) = \sum_{i=0}^{A-1}(z_i - m)^n p(z_i)$
  - $m = \sum_{i=0}^{A-1} z_i p(z_i)$
- Raw data:
  - $\mu_n(r) = \sum_{i=0}^{K-1}(r_i - m)^n g(r_i)$
  - $m = \sum_{i=0}^{K-1} r_i g(r_i)$



Region Feature Descriptor (1)

› Major and minor axis, bounding box:



Region Feature Descriptor (2)

- P: The *perimeter* of a region is the length of its boundary.
- A: The *area* of a region is number of pixels in the region.
- Compactness:

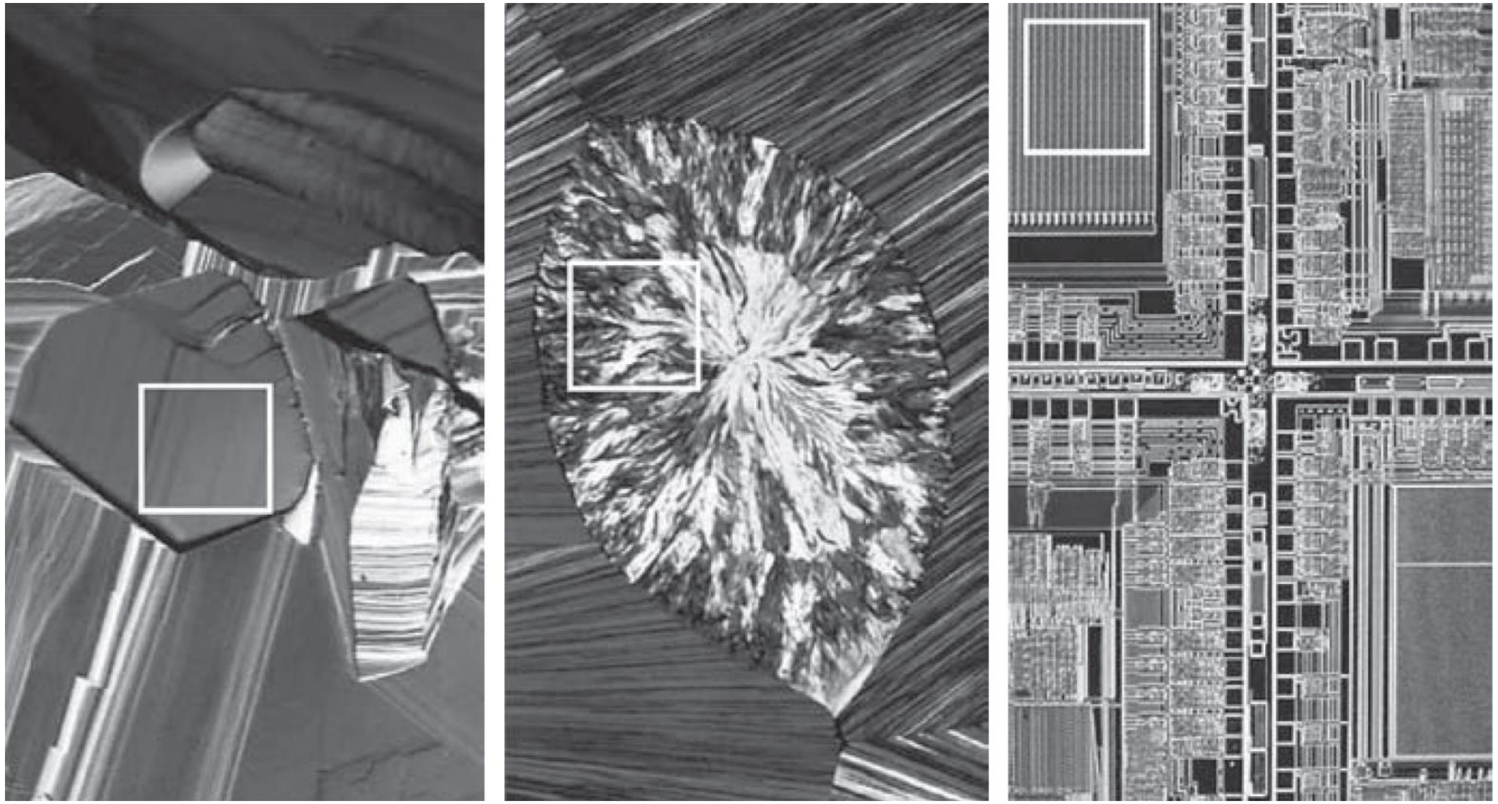
$$compactness = \frac{P^2}{A}$$

- Circularity:

$$circularity = \frac{4\pi A}{P^2}$$

Texture

› Examples: smooth (left), Coarse (middle), and regular (right)



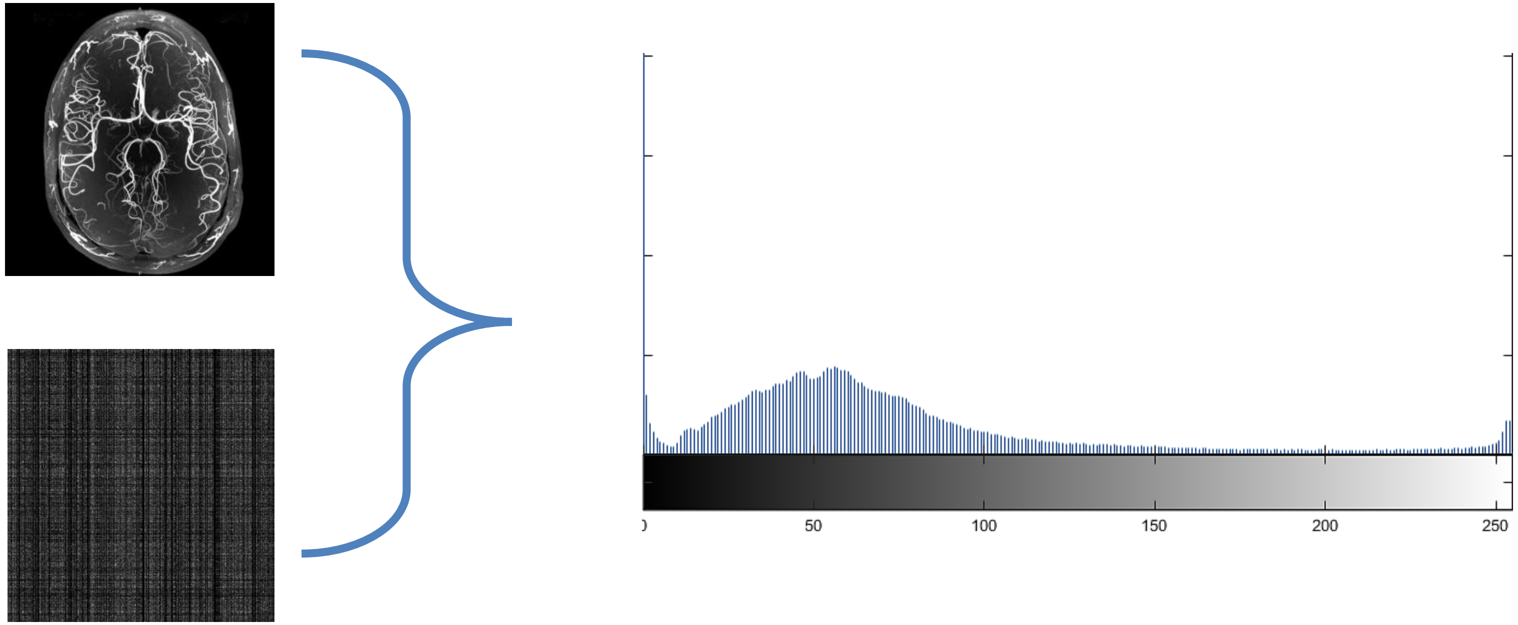
Texture – First Order Statistics

› Based on statistic of normalized histogram (pdf):

- $m = \sum_{i=0}^{L-1} z_i p(z_i)$
- Variance: $\mu_2(z) = \sigma^2(z) = \sum_{i=0}^{L-1}(z_i - m)^2 p(z_i)$
- Skewness: $\mu_3(z) = \frac{\sum_{i=0}^{L-1}(z_i-m)^3 p(z_i)}{\sigma^3(z)}$,
- Kurtosis: $\mu_4(z) = \frac{\sum_{i=0}^{L-1}(z_i-m)^4 p(z_i)}{\sigma^4(z)}$,
- Uniformity: $U(z) = \sum_{i=0}^{L-1} p^2(z_i)$
- Entropy: $E(z) = -\sum_{i=0}^{L-1} p(z_i) log_2\big(p(z_i)\big)$

Texture – First Order Statistics

- First order histogram is NOT unique!
- Too many images have same normalized histogram



Texture – Second Order Statistics

- Gray Level Co-occurrence Matrix (GLCM):
- Let Q be an operator that defines the position of two pixels relative to each other, and consider an image, f , with L possible intensity levels.
- $GLCM(i,j) = G_{ij}$ is the number of times that pixel pairs with intensities z_i and z_j occur in image f in the position specified by Q

GLCM Matrix

- Example:
- Q: one pixel immediately to the right:
- $p_{ij} = \frac{G_{ij}}{\sum_{i=0}^{L-1} \sum_{j=0}^{L-1} G_{ij}}$
- Problems with GLCM!

1	1	7	5	3	2
5	1	6	1	2	5
8	8	6	8	1	2
4	3	4	5	5	1
8	7	8	7	6	2
7	8	6	2	6	2

Image f

	1	2	3	4	5	6	7	8
1	1	2	0	0	0	1	1	0
2	0	0	0	0	1	1	0	0
3	0	1	0	1	0	0	0	0
4	0	0	1	0	1	0	0	0
5	2	0	1	0	1	0	0	0
6	1	3	0	0	0	0	0	1
7	0	0	0	0	1	1	0	2
8	1	0	0	0	0	2	2	1

Co-occurrence matrix **G**

Descriptor based on GLCM Matrix

- Some useful descriptor (k: size of GLCM matrix):
- Contrast: $\sum_{i=1}^{K}\sum_{j=1}^{K}(i-j)^2 p_{ij}$
- Uniformity: $\sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij}^2$
- Homogeneity: $\sum_{i=1}^{K}\sum_{j=1}^{K}\frac{p_{ij}}{1+|i-j|}$
- Entropy: $-\sum_{i=1}^{K}\sum_{j=1}^{K} p_{ij} log_2 p_{ij}$
- …

Moment of Image

- Consider image f as 2D joint probability distribution
- The 2-D moment of order $(p+q)$ of an $M \times N$ digital image, $f(x,y)$, is defined as:

$$m_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} x^p y^q f(x,y)$$

- Central moment of order $(p+q)$, is defined as

$$\mu_{pq} = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

- Where:

$$\bar{x} = \frac{m_{10}}{m_{00}}, \bar{y} = \frac{m_{01}}{m_{00}}$$

Moment of Image

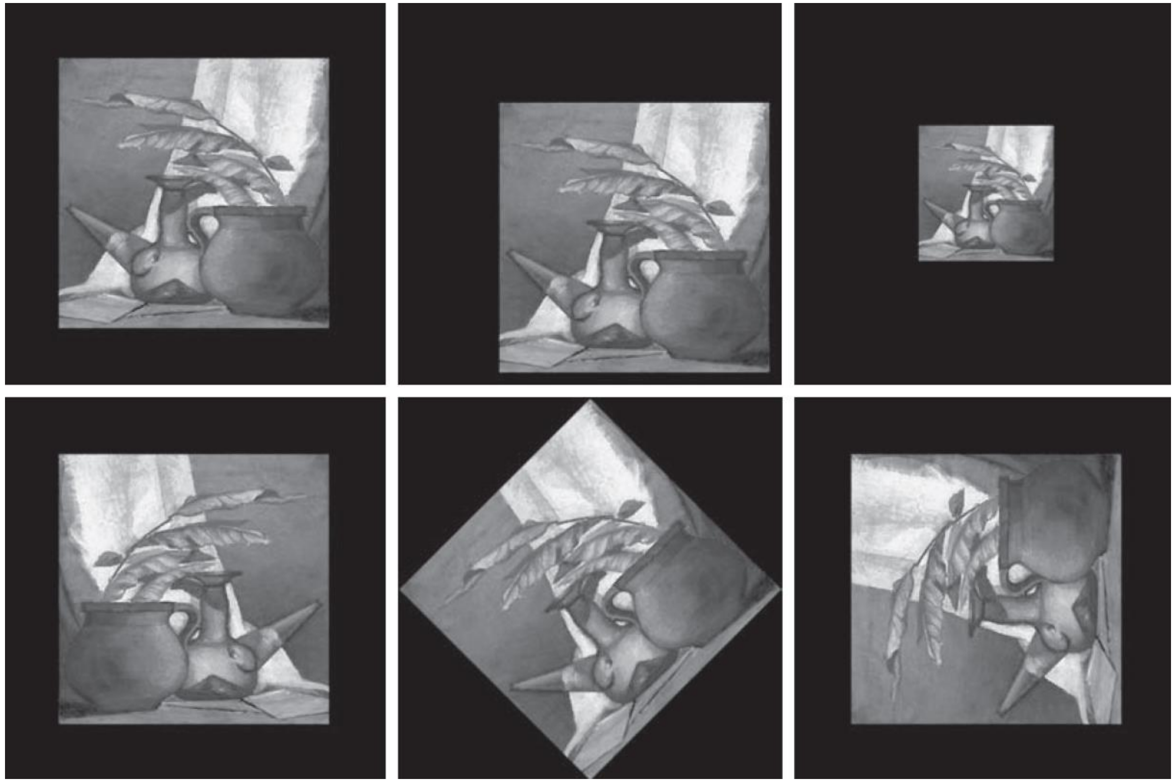
- Normalized central moment of order $(p + q)$, is defined as

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^{\gamma}}, \qquad \gamma = \frac{p+q}{2} + 1$$

- For $(p + q) = 2, 3, \ldots$ A set of seven, 2-D moment invariants can be derived from the second and third normalized central moments (*Hu* moments).
  - $\phi_1 = \eta_{20} + \eta_{02}$
  - $\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$
  - ....

HU moments - Example

› Consider an image ant its variations:



HU moments - Example

› Consider an image ant its variations:

Moment Invariant	Original Image	Translated	Half Size	Mirrored	Rotated 45°	Rotated 90°
ϕ_1	2.8662	2.8662	2.8664	2.8662	2.8661	2.8662
ϕ_2	7.1265	7.1265	7.1257	7.1265	7.1266	7.1265
ϕ_3	10.4109	10.4109	10.4047	10.4109	10.4115	10.4109
ϕ_4	10.3742	10.3742	10.3719	10.3742	10.3742	10.3742
ϕ_5	21.3674	21.3674	21.3924	21.3674	21.3663	21.3674
ϕ_6	13.9417	13.9417	13.9383	13.9417	13.9417	13.9417
ϕ_7	–20.7809	–20.7809	–20.7724	20.7809	–20.7813	–20.7809

PCA – Principal Component Analysis

- Assume we have K observations of n-dimensional feature vector, $\boldsymbol{x} \in \mathbb{R}^n$:
- Mean vector:

$$m_{\boldsymbol{x}} = E\{\boldsymbol{x}\} \cong \frac{1}{K}\sum_{i=1}^{K}\boldsymbol{x}_i\,, \quad m_{\boldsymbol{x}} \in \mathbb{R}^n$$

- Covariance Matrix:

$$C_{\boldsymbol{x}} = E\{(\boldsymbol{x} - m_{\boldsymbol{x}})(\boldsymbol{x} - m_{\boldsymbol{x}})^T\} \cong \frac{1}{K-1}\sum_{i=1}^{K}(\boldsymbol{x}_i - m_{\boldsymbol{x}})(\boldsymbol{x}_i - m_{\boldsymbol{x}})^T\,, \quad C_{\boldsymbol{x}} \in \mathbb{R}^{n\times n}$$

PCA – Principal Component Analysis

- Covariance Matrix Properties:
  - Real and symmetric,
  - Semipositive definite Matrix: n orthonormal eigenvectors and n non-negative eigenvalues.

$$C_x\boldsymbol{v} = \lambda\boldsymbol{v} \Rightarrow \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0, \qquad \boldsymbol{v}_i \perp \boldsymbol{v}_j, \qquad \|\boldsymbol{v}_i\| = 1$$

  - For uncorrelated features, C_x is diagonal!

PCA – Principal Component Analysis

- Let A be a matrix whose rows are formed from the eigenvectors of C_x , arranged in descending value of their eigenvalues, so that the first row of A is the eigenvector corresponding to the largest eigenvalue.

$$A^T = [\boldsymbol{v}_1 | \boldsymbol{v}_2 | \dots | \boldsymbol{v}_n] \Rightarrow A^T = A^{-1}$$

- Then for transformation, $\boldsymbol{y} = A(\boldsymbol{x} - m_{\boldsymbol{x}})$, we have:

$$m_{\boldsymbol{y}} = 0, C_{\boldsymbol{y}} = diag(\lambda_1, \lambda_2, \dots, \lambda_n)$$

- $\boldsymbol{y}$ is white random vector!
- This transform known as *Hotelling* or *KL* transform

PCA – Principal Component Analysis

- Reconstruction $\boldsymbol{x}$ from $\boldsymbol{y}$:

$$\boldsymbol{y} = A(\boldsymbol{x} - m_{\boldsymbol{x}}) \Rightarrow \boldsymbol{x} = A^T\boldsymbol{y} + m_{\boldsymbol{x}} = [\boldsymbol{v}_1|\boldsymbol{v}_2| \dots |\boldsymbol{v}_n]\boldsymbol{y} + m_{\boldsymbol{x}}$$

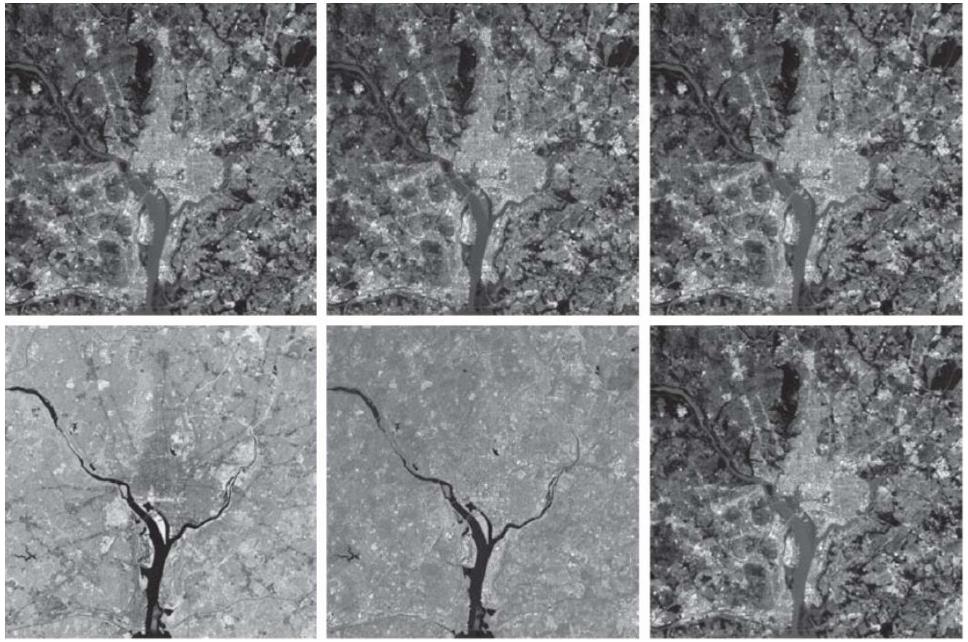
- Approximate reconstruction, using first m-columns of A^T (A_m^T) and first m-element of $\boldsymbol{y}$
- $A_m^T = [\boldsymbol{v}_1|\boldsymbol{v}_2|\dots|\boldsymbol{v}_m], \boldsymbol{y}_m = (y_1, y_2, \dots, y_m)^T$
- $\widetilde{\boldsymbol{x}}_{\boldsymbol{m}} = A_m^T\boldsymbol{y}_m + m_{\boldsymbol{x}}$
- It can be shown, the reconstruction error (lowest one) is:

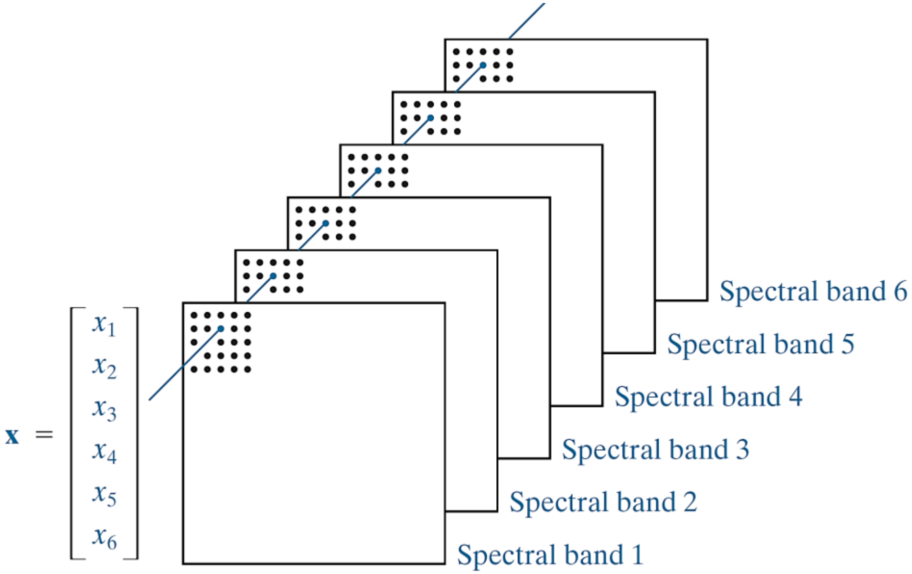
$$E\{\|\boldsymbol{x} - \widetilde{\boldsymbol{x}}_{\boldsymbol{m}}\|^2\} = \sum_{i=m+1}^{n} \lambda_i$$

- It is method of feature reduction, $\boldsymbol{y}_m$

PCA – Example (1)

› Consider 6-channels images ($M \times N$ observation (each one 6-dimensional)

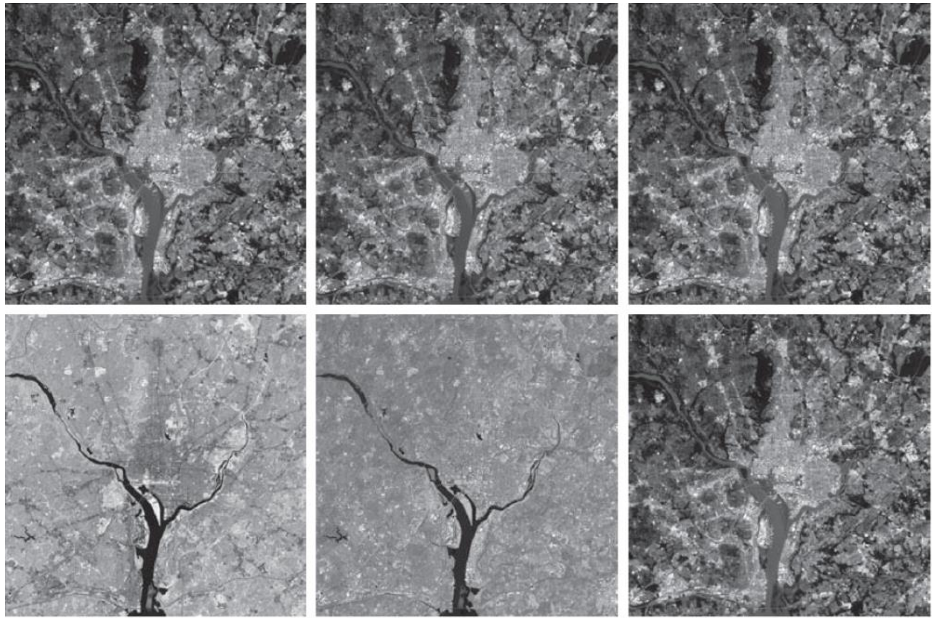


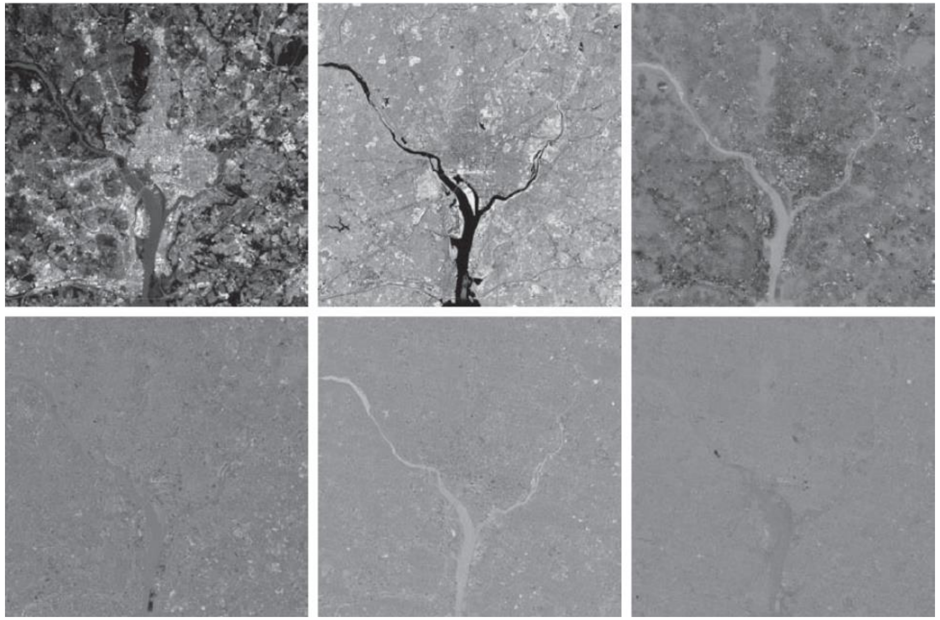


λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
10344	2966	1401	203	94	31

PCA – Example (1)

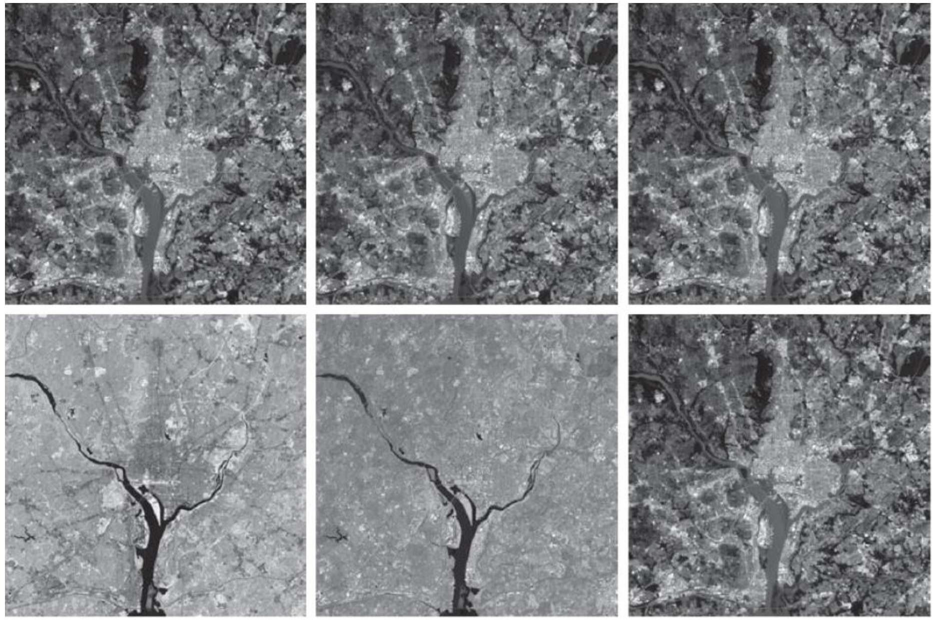
› Raw data (left) and six principal components, $\boldsymbol{y}$, (right)

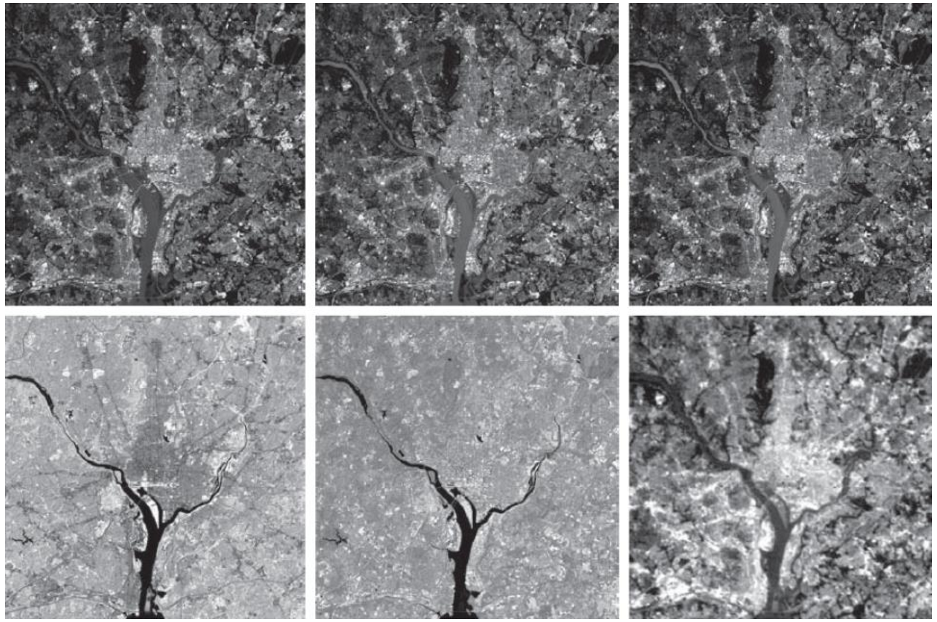




PCA – Example (1)

- Raw data (left) and reconstruction from two principal corresponding to the two largest eigenvalues (right)

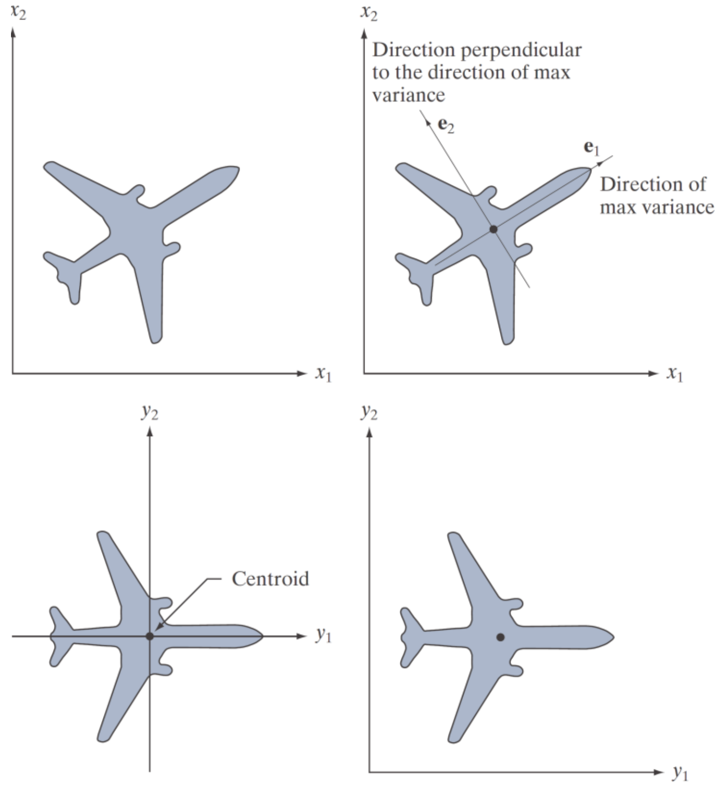




PCA – Example (2)

› Consider two-dimensional points cloud (airplane):

› Object normalization!



Whole image features

- There are several feature extraction methods:
  - SIFT
  - SURF
  - ORB
  - BRISK
  - KAZE
  - AKAZE
- See https://doi.org/10.1109/ICOMET.2018.8346440 for comparison

The End

- AnY QuEsTiOn?