Infinitesimal Calculus 3

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Notice that if we have $y = (y_1, \dots, y_s)$ and $x = (x_1, \dots, x_k)$ and s functions F_i with the system

$$\begin{cases} F_1(x_1, \dots, x_k, y_1, \dots, y_s) = 0 \\ \vdots \\ F_s(x_1, \dots, x_k, y_1, \dots, y_s) = 0 \end{cases}$$

We can assume $y_i = \varphi_i(x_1, \dots, x_k)$ where $\varphi_i \in C_1$ by the Implicit Function Theorem (we assume $J_{F_i}(x, y) \neq 0$). We can then differentiate these equations relative to x_j to get

$$\begin{cases} \frac{\partial F_1}{\partial x_j} + \sum_{i=1}^{s} \frac{\partial F_1}{\partial y_i} \cdot \frac{\partial y_i}{\partial x_j} = 0 \\ \vdots \\ \frac{\partial F_s}{\partial x_j} + \sum_{i=1}^{s} \frac{\partial F_s}{\partial y_i} \cdot \frac{\partial y_i}{\partial x_j} = 0 \end{cases}$$

And therefore we get that

$$\sum_{i=1}^{s} \frac{\partial F_t}{\partial y_i} \frac{\partial y_i}{\partial x_j} = -\frac{\partial F_i}{\partial x_j}$$

This is equivalent to saying that

$$J_F \begin{pmatrix} \frac{\partial y_1}{\partial x_j} \\ \vdots \\ \frac{\partial y_s}{\partial x_j} \end{pmatrix} = - \begin{pmatrix} \frac{\partial F_1}{\partial x_j} \\ \vdots \\ \frac{\partial F_s}{\partial x_j} \end{pmatrix}$$

Where

$$J_F = \frac{\partial(F_1, \dots, F_s)}{\partial(y_1, \dots, y_s)}$$