Alisa Kunimoto || Section B04

MAE3 Clock Report / Spring 2025

Link to Video: https://youtu.be/i_njGC-xke0

Executive Summary

The purpose of this report is to summarize the pendulum analysis conducted for the MAE3 Clock Project, in which CAD, fabrication, and analysis skills were used to predict hardware performance. Additionally, the concept of Design for Manufacturability (DFM) to minimize cost and complexity while maintaining performance and manufacturability was utilized.

Two theoretical methods were used to predict the timing behavior of the pendulum. Those methods consist of the point mass and rigid body analysis. Both methods were utilized to calculate the predicted angular frequency and period, which were then compared to the experimentally measured values. The Point Mass estimation had a 13.54% error in comparison to real values. The rigid body estimation had a 5.62% error from real world calculation.These experimental real-word values were obtained by timing the rotations of the escapement wheel once it was manufactured. The rigid body analysis yielded a lower percent error compared to the point mass analysis, demonstrating its accuracy.

The effective length of the pendulum was calculated using CAD tools, Fusion 360, and validated with physical balancing. Mass and moment of inertia were also calculated using Fusion 360 and compared to physical measurement, with a 10% error in acrylic mass from actual vs. calculated due to manufacturing deviations such as acrylic thickness variation. Additionally, there was a 9.66% error in center of mass with pendulum nuts and bolts estimated.

Overall, the project demonstrates the value of rigid body analysis for precision applications and the importance of validating CAD-based assumptions.

Theoretical Analysis

Overview:

The goal of the theoretical analysis is to predict the natural frequency and period of the pendulum using a point mass and rigid body approximation. Both methods assume small angular oscillations and negligible friction, more details about assumptions can be found below.

The point mass analysis simplifies the pendulum to a single mass concentrated at a specific distance from the pivot. Meanwhile, the rigid body model accounts for more distribution of mass by taking the moment of inertia of the pendulum into account.

Theoretical predictions are important for comparison against the real-world experimental measurements to determine the accuracy of each of the methods to assess how the physical prototypes match the CAD-based calculations.

Point Mass Analysis:

The point mass analysis treats the pendulum as a single mass located at the distance equal to the effective length. This method assumes negligible friction, a small angle of oscillation, mass of the pendulum concentrated at a point, the rope does not stretch, and that the pendulum frequency is not affected by contact with the escapement wheel. Since the pendulum's angular amplitude is small enough, the motion can be approximated as simple harmonic motion.

Calculating Predicted Angular Frequency and Period w/ Point Mass Analysis

Using Fusion 360, the center of mass of the pendulum was determined to be located 6.11 cm from the pivot point as **La**. The values for the distance from the origin of the bolt holes with respect to the y-axis are **L_bolt1** to **L_bolt8**. The effective length of the pendulum **Lcom_meter** in was calculated to be 0.08 meters.

The center of mass was also estimated with the final product, by balancing the pendulum on an equator as intermediate verification. It was conducted by balancing the pendulum on a finger and measuring the distance between the pivot point and center of mass. The estimated effective length **Lcom_est** was found to be 0.09 meters. The percent error, **Lcom_error,** was found by comparing **Lcomm_meter** to **Lcom_est.** This error was found to be 9.66%. Details of these values can be found in the appendix.

With the effective length from the pivot, the angular frequency of the pendulum can be found with the equation. This formula assumes a small angle of rotation:

$$\omega = \sqrt{\frac{g}{L_{com\ meter}}} \tag{1}$$

**Point Mass Angular Frequency Equation from Tutorial 6 pg. 2 in the UCSD MAE3 Canvas Page*

Using the measured effective length of **La** and the equation from (1), the angular frequency, **nat_freq_rad_sec,** was calculated to be 10.80 rad/s. **g** is gravity in meters/sec. This yields a predicted **period** of 0.58 seconds. These results will later be compared to the actual measured timing data and evaluated for accuracy.

CAD of Pendulum

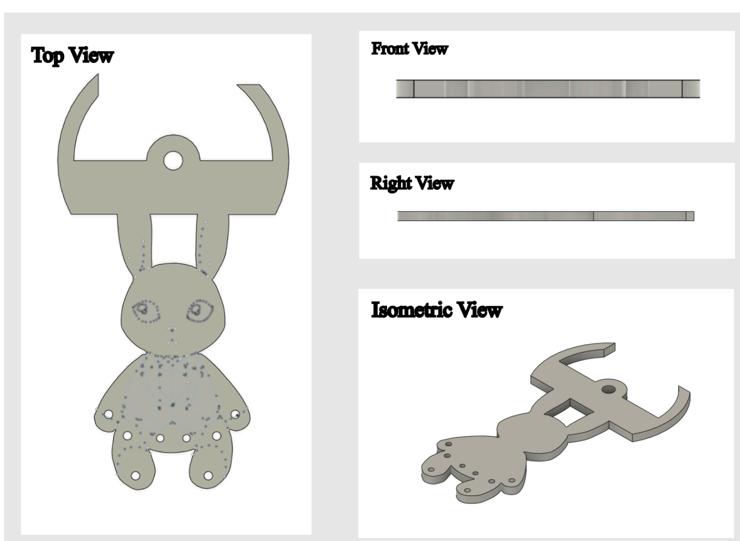

Figure 1: Parts of the Pendulum CAD

Figure 1 shows the CAD model of the pendulum used to determine the center of mass.

The assumptions and simplicity of the point mass model make it useful for initial approximations, but do not account for mass distribution and lead to error when compared to a real world measurement and in the fabrication process.

Rigid Body Analysis:

The rigid body analysis accounts for the distributed mass of the pendulum and requires additional information in the use of the total moment of inertia about the pivot point. The assumptions include negligible friction, small angle of oscillation, the pendulum rope does not stretch, and the frequency is not affected by contact with the escapement wheel. Additionally, the pendulum structure is rigid with the mass uniformly distributed.

Calculating Predicted Angular Frequency and Period w/ Rigid Body Analysis

Fusion 360 was used to obtain relevant parameters such as the surface area, predicted weight, centroid, and moment of inertia values of the pendulum body. The mass of the acrylic was calculated using its thickness and density with its total mass **Mt** of 100.91 grams including the 8 fasteners, consisting of x2 nuts and x1 bolt. The percent error in acrylic mass **M_Error** was found to be 10.03%.

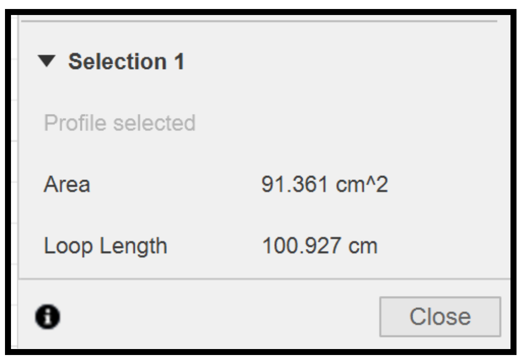

Figure 2: Area of Face from Fusion360

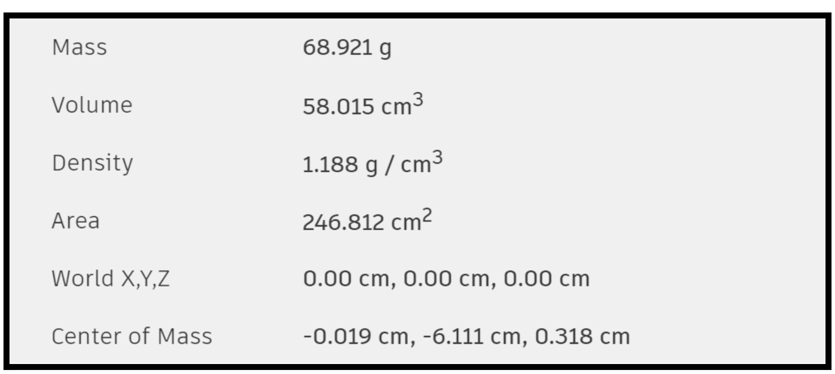

Figure 3: Predicted Weight and Centroid of Pendulum from Fusion360

Moment of Inertia at Origin (g cm^2)	
Ixx	4439.42
Ixy	-15.448
Ixz	0.416
Iyx	-15.448
Iyy	406.958
Iyz	133.721
Izx	0.416
Izy	133.721
Izz	4827.85

Figure 4: Moment of Inertia Values from Fusion 360

The moment of inertia of the acrylic component **I_a** of 4827.85 g/cm^2 was obtained from Fusion 360, and combined with contributions from the bolts in **I_bolt1** to **I_bolt8** to find the total moment of inertia, **I_tot** that came out to 10,697.66 g/m^2. The value for the effective length from the pivot to the center of mass **Lcom_meter** of 0.08 meters was taken from Fusion 360.

$$\omega = \sqrt{\frac{M_t \times g \times L_{com\ meter}}{I_{total}}} \tag{2}$$

**Rigid Body Angular Frequency Equation from Tutorial 7 pg. 2 in the UCSD MAE3 Canvas Page*

Utilizing the measured values and equation (2), the angular frequency, **rb_nat_freq_rad_sec**, was calculated to be 8.81 rad/s and the period, **rb_period,** was 0.71 seconds.

As will be observed shortly, the rigid body analysis showed a more consistent value as experimental results than the point mass model, demonstrating the importance of incorporating more aspects of mass distribution precision timing analysis.

Experimental Results:

To validate the theoretical timing predictions, an experiment was conducted using the fully fabricated clock.

The experiment was performed by winding up the weight behind the pendulum, allowing the escapement wheel on top to rotate. The timer was stopped when a particular tooth had completed one full rotation. To count one full motion, a tooth was marked with a sticky note, and the timer was stopped when the tooth completed one complete rotation. The pendulum was set into motion by raising the attached weight, pushing it slightly to start, and then a stopwatch was used to record the time for one full revolution.

Approximate Timing Results and % Error

This experiment was run 10 times to account for variations in timing, and the mean value was chosen to find the average time it took the pendulum to complete one complete revolution.

Table 1: Experimental Results of Pendulum Period

Trial #	1	2	3	4	5	6	7	8	9	10
Time (s)	9.42	9.34	9.50	9.49	9.38	9.46	9.61	9.33	9.49	9.46

The average timing result was 9.42 seconds per full revolution. Potential sources of error could include human reaction time when starting/stopping the stopwatch, inconsistent starting energy from the initial push, friction in the contact points, and slight misalignments in the escapement mechanism. These discrepancies will be explained more in the discussion. However, experimental values still provide a strong basis for evaluating the accuracy of the calculated predicted analysis values.

Additionally, when Fusion 360 values were compared to actual values, there was a significant percent error difference found. When calculating the error in the acrylic mass, **M_Error**, it was found that there was a 10.03% difference between the Fusion 360 mass and the actually measured value. There was also a difference when finding the center of mass as observed in the **Lcom_error** value of 9.66% when comparing the center of mass from Fusion 360 versus when balancing the pendulum on the finger and measuring the effective length from the pivot point.

Discussion

Table 2: Summary of Hardware Performance Prediction

	Angular Frequency	Period	Percent Error
Point Mass Prediction	10.80 rad/sec	0.58 sec	13.54%
Rigid Body Prediction	8.81rad/sec	0.71sec	5.62%

The results of this analysis highlight the difference in precision between the point mass and rigid body models. The rigid body analysis results in a more accurate estimation of the period when compared to the point mass analysis. The percent error of the estimate from the rigid body analysis was 5.62%, which is lower than the percent error of the point mass analysis, which returned a percent error of 13.54%.

The point mass approximation simplifies the pendulum to a single concentrated mass, which fails to consider the actual shape and mass distribution of the pendulum. While it is useful for rough estimations, the point mass method yielded a higher percent error compared to the rigid body analysis and a higher deviation from the actual performance.

The rigid body analysis produced predictions closer to the observed timing. This method accounted for the pendulum's moment of inertia and distributed mass, providing a more realistic representation of the pendulum and its shape. The lower error percentage demonstrates the superiority of the rigid body analysis for modeling systems where distribution of mass and rotational effects are non-negligible, such as in the case of the pendulum.

These discrepancies make sense because rigid body analysis more accurately depicts the real world. While each object has a center of mass, the entire mass of that object does not concentrate at one point. Rigid body analysis takes this into account and represents the mass distributions along the entire body.

Point mass can still be used in many situations because it still provides a decent approximation of the real world values. For small-scale projects like this pendulum, it's reasonable to apply this method, even if it's less accurate. However, a rigid body analysis will be more accurate than the point mass analysis.

Quantitative analysis of how point mass and rigid body estimations differ and how that impacts accuracy of calculations

1. **Single Point Mass**

For a point mass *m* at a distance *r* from the axis of the rotation, the moment of inertia can be found using the formula below:

$$I = mr^2 \tag{3}$$

**Moment of inertia for a point mass from "L3 - Clock Analysis" slides pg 20 in the UCSD MAE3 Canvas Page*

The point mass model simplifies the pendulum to a single radius from the pivot. Equation (3) is the fundamental idea behind the point mass calculation conducted for the pendulum where the mass is fixed at a set radius *r* from the pivot. However, this formula does not account for the distribution of mass, which means there will be a systematic underestimation of the moment of inertia and an overestimation in angular frequency. This model results in deviations from actual measured values.

2. Body Composed of *N* Point Masses

For a object of *N* number of point masses m_i at a distance of r_i from the axis of rotation, the moment of inertia can be found using the formula below:

$$I = \sum_{i=1}^{N} m_i r_i^2 \qquad (4)$$

**Moment of inertia for a body of N point masses from "L3 - Clock Analysis" pg 20 in the UCSD MAE3 Canvas*

By summing the point masses, equation (4) adds additional details to the single point mass from equation (3). The summation takes different components and incorporates the difference in masses and distances into the estimation. It's useful for calculating non-uniformly distributed masses or discrete segments for individual components by utilizing varying distances to the axis of rotation and incorporating the unequal mass contributions. Regardless of its higher accuracy, it still fails to capture the entirety of the pendulum as it continues to treat it like a group of isolated masses, and not as an entire object.

3. Arbitrary Continuous Rigid Body

For a rigid body with continuous mass distribution, where *r* is the distance from the rotation axis to the element, the moment of inertia can be found using the formula below:

$$I = \int_m r^2 dm \qquad (5)$$

**Moment of inertia of an arbitrary shape from "L3 - Clock Analysis" pg 20 in the UCSD MAE3 Canvas*

This integral sums the contributions of the entire body's mass in equation (5) and is more fit to model complex shapes. (5) takes into account every infinitesimal element, more accurately simulating the continuous geometry of the pendulum. By capturing the entire distribution of mass of the pendulum, it is possible to apply this formula to irregular or non-uniform shapes, providing a more accurate estimate than the point mass methods in (3) and (4).

By taking into account the shape of the pendulum, the estimations will better reflect real world values. The primary reason for this is because real objects have mass spread out over different distances from the pivot, and depending on the distance, there are varying levels of rotational resistance. This is demonstrated in (5) with the r^2 term, reflecting on how points further from the pivot create more resistance. Meanwhile, the point mass fails to take this into account and does not factor in this resistance, leading to an underestimate inertia value.

Additionally, these moments of inertia calculations affect downstream values such as for period and angular frequency, so an inaccurate moment of inertia will result in compounded errors in the final prediction. Therefore, modeling the pendulum with a distributed mass as a rigid body will provide a realistic representation and improve the precision of the estimations.

The discrepancy between predicted and measured results from Fusion 360 to actual fabrication can be attributed to the following factors:

1. Material Variability
    1.1. The thickness of the acrylic used could be slightly different from that predicted in the CAD design.
    1.2. The mass over the area of the pendulum is assumed to be uniform, but manufacturing inconsistencies of the acrylic could mean that the mass is not uniformly distributed, and the moment of inertia would be different.
    1.3. The density provided by Fusion 360 might be slightly different depending on the type of acrylic used.
2. Friction
    2.1. Both estimation models assume negligible friction, but the real world has friction that introduces some damping force. This can impact oscillation timing.
    2.2. If the escapement wheel is placed too tightly against the bearing, it would cause extra fiction.
3. Tolerance
    3.1. Differences in bolt placement and assembly may lead to some shifting of the center of mass.
    3.2. Laser cuts can be inaccurate causing some of the dimensions of the acrylic pendulum component to not match the CAD design.
4. Fastener Mass Estimates
    4.1. Bolt and nut masses were treated as 4g each, but this is not completely accurate to real-world measurements.
    4.2. Each nut and bolt may have more or less mass and their distribution due to manufacturing errors, which will affect the moment of inertia.
5. Assembly
    5.1. Personal manufacturing and assembly of the pendulum could cause errors
        5.1.1. Incorrect usage of tools.
        5.1.2. Damaged parts, possibly from dropping and denting parts.
        5.1.3. Incorrect usage of dimensions due to misinterpretation of instructions.

Despite these discrepancies, the important outcome is that rigid modeling is more accurate for systems involving complex geometries and distributed mass. While point mass approximation may be sufficient for conceptual modeling, rigid mass approximations are needed for more accurate predictions.

Discrepancies in measuring true value of the pendulum and what could be implemented to improve accuracy of real-world measurements:

1. Human error in timing with stopwatch
    1.1. More accurate stopwatch recording such as a digital sensor to eliminate delay in reaction time
    1.2. High speed video analysis to reduce human error in timing
2. Inaccurate bolt/nut masses
    2.1. Measurement using laboratory scales to find more significant digits of accuracy for weight
    2.2. Manually record each of the multiple mass measurements to identify variance between each of the bolts/nuts
3. Different environmental conditions
    3.1. Conduct experiments in an environment isolated from vibration such as on a solid surface without impacts to the table
    3.2. Constant temperature / airflow
    3.3. Make sure surface flat, and is not slanted or tilted

Experiments to further verify results and try different conditions to see impact on analysis and angular frequency / period:

1. Comparing different pendulum shapes to evaluate applicability of rigid body models
    1.1. Testing designs with differing distributions, lengths, and center of masses
    1.2. Test symmetrical vs. asymmetrical pendulum bodies
    1.3. Utilize different materials such as metal or wood
2. Using a hanging nut with less or more mass
3. Modify fastener (bolt/nut) configuration to vary mass distribution
4. Modification of escapement gear mechanism to a model other than the Graham/Deadbeat Escapement

In conclusion, this reinforced how analysis and estimations must consider real-world factors such as distributed mass and manufacturing variation. Incorporating these factors through estimations such as the rigid body analysis gets results that more closely match fabricated systems, making it the superior method for pendulum timing analysis in regards to precision.

Appendix

Pendulum Timing Analysis

Name: Alisa Kunimoto
Section: Friday @ 2 PM Lab

Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	91.361	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	58.015	cm^3	
Density	p	1.188	grams/cm^3	
Calculated Mass of Acrylic	M_Calc	68.912	grams	
Actual Mass of Acrylic	M_Act	62	grams	measured with physical pendulum
Length to Center of Mass of Acrylic	La	6.111	cm	
Percent Error in Acrylic Mass Calculation	M_Error	10.03018342	%	
Mass of One Bolt with Two Nuts	Mb	4	grams	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	100.912	grams	calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	14.986	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	12.065	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	13.093	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	13.201	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 5	L_bolt5	13.201	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 6	L_bolt6	13.081	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 7	L_bolt7	12.065	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	14.986	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	0.08401708736	meters	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	0.093	meters	measured with physical pendulum balanced on a finger
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	9.659045846	%	
Calculate Natural Frequency and Timing using Point Mass Assumption				
Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	10.80013607	radians/sec	
Natural Frequency in Hz	nat_freq_hz	1.718945737	Hz	
Period of Oscillation	period	0.5817519299	sec	
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	8.144527019	sec	
Measured Time of One Revolution of Escapement Wheel	time_meas	9.42	sec	measured with physical pendulum
Percent Error in Clock Timing	time_error	13.54005288	%	
Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	4827.85	grams cm^2	
Moment of Inertia of Bolt 1	I_bolt1	921.95	grams cm^2	r =15.091
Moment of Inertia of Bolt 2	I_bolt2	698.5449	grams cm^2	13.215
Moment of Inertia of Bolt 3	I_bolt3	703.0983	grams cm^2	13.258
Moment of Inertia of Bolt 4	I_bolt4	619.4125	grams cm^2	12.444
Moment of Inertia of Bolt 5	I_bolt5	698.6506	grams cm^2	13.216
Moment of Inertia of Bolt 6	I_bolt6	909.2637	grams cm^2	15.077
Moment of Inertia of Bolt 7	I_bolt7	702.462	grams cm^2	13.252
Moment of Inertia of Bolt 8	I_bolt8	616.4296	grams cm^2	12.414
Total Moment of Inertia	I_total	10,697.66	grams m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	8.813001168	radians/sec	
Natural Frequency in Hz	rb_nat_freq_hz	1.402674068	Hz	
Period of Oscillation	rb_period	0.712923995	sec	
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	9.980935929	sec	
Percent Error in Clock Timing	rb_time_error	5.620073441	%	

Pendulum Timing Analysis

Name: Alisa Kunimoto
Section: Friday @ 2 PM Lab

Variable Description	Variable Name	Values/Equations	Units	Comments
Acrylic Pendulum Specifications				
Area	A	91.361	cm^2	
Thickness	t	0.635	cm	
Volume	Vol	58.015	cm^3	
Density	p	1.188	grams/cm^3	
Calculated Mass of Acrylic	M_Calc	68.912	grams	
Actual Mass of Acrylic	M_Act	62	grams	measured with physical pendulum
Length to Center of Mass of Acrylic	La	6.111	cm	
Percent Error in Acrylic Mass Calculation	M_Error	= (1 - (M_Act/M_Calc)) * 100	%	
Mass of One Bolt with Two Nuts	Mb	4	grams	
Number of Bolts with Two Nuts	Nb	8		
Total Mass of Pendulum with Nuts and Bolts	Mt	=M_Calc + (Nb * Mb)	grams	calculated
Calculate Center of Mass of Pendulum with Bolts				
Length to Center of Mass of Bolt 1	L_bolt1	14.986	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 2	L_bolt2	12.065	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 3	L_bolt3	13.093	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 4	L_bolt4	13.201	cm	distance from pivot point to bolt
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Length to Center of Mass of Bolt 7	L_bolt7	12.065	cm	distance from pivot point to bolt
Length to Center of Mass of Bolt 8	L_bolt8	14.986	cm	distance from pivot point to bolt
Length to Center of Mass **in Meters**	Lcom_meter	= (((M_Calc) * (La/100)) + ((Mb) *((L_bolt1 + L_bolt2 + L_bolt3 +L_bolt4+L_bolt5+L_bolt6+L_bolt7+L_bolt8) / 100))) / Mt	meters	make sure you convert meters
Estimated Center of Mass of Pendulum with Nuts and Bolts	Lcom_est	0.093	meters	measured with physical pendulum balanced on a finger
Percent Error in Pendulum Nuts and Bolts Lcom Estimate	Lcom_error	= (1 -(Lcom_meter/Lcom_est)) * 100	%	
Calculate Natural Frequency and Timing using Point Mass Assumption				
Gravitational Constant	g	9.8	m/s^2	
Natural Frequency in radians/sec	nat_freq_rad_sec	= SQRT(g/Lcom_meter)	radians/sec	
Natural Frequency in Hz	nat_freq_hz	= nat_freq_rad_sec / (2*3.1415)	Hz	
Period of Oscillation	period	=1/nat_freq_hz	sec	
Number of Teeth on Escapement Wheel	nteeth	14		
Calculated Time of One Revolution of Escapement Wheel	time_calc	= period * nteeth	sec	
Measured Time of One Revolution of Escapement Wheel	time_meas	9.42	sec	measured with physical pendulum
Percent Error in Clock Timing	time_error	= (1 - (time_calc/time_meas)) * 100	%	
Calculate Natural Frequency and Timing using Rigid Body Assumption				
Moment of Inertia of Pendulum	I_a	4827.85	grams cm^2	
Moment of Inertia of Bolt 1	I_bolt1	921.95	grams cm^2	r =15.091
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Moment of Inertia of Bolt 7	I_bolt7	702.462	grams cm^2	13.252
Moment of Inertia of Bolt 8	I_bolt8	616.4296	grams cm^2	12.414
Total Moment of Inertia	I_total	= sum(C45:C53)	grams m^2	
Natural Frequency in radians/sec	rb_nat_freq_rad_sec	= SQRT(((Mt * 100) * g * (Lcom_meter* 100)) / I_total)	radians/sec	
Natural Frequency in Hz	rb_nat_freq_hz	= C55 / (2*3.1415)	Hz	
Period of Oscillation	rb_period	= 1 / rb_nat_freq_hz	sec	
Calculated Time of One Revolution of Escapement Wheel	rb_time_calc	= rb_period * 14	sec	
Percent Error in Clock Timing	rb_time_error	= (1 -(time_meas/ rb_time_calc))*100	%	