Section 6.1: Volume

1. A region R is bounded by $y = e^x$ and $y = 0$ on the interval $[-1, 1]$. Set up the integral to find the volume V of the solid formed by rotating R around the x-axis and then find the volume.

2. Let R be the region between the graphs of $f(x) = x + 1$ and $g(x) = x - 1$ on the interval $[1, 4]$. Find the volume V of the solid obtained by revolving R about the x axis.

3. A region R is bounded by $y = \sqrt{x}$ and $y = x^4$. Set up the integral to find the volume V of the solid formed by rotating R around the x-axis and then find the volume.

4. Let R be the region between the graph of $f(x) = \sqrt{x+2}$ and $g(x) = \sqrt{x-2}$ on the interval $[2, 6]$. Set up an appropriate integral to find the volume V of the solid obtained by revolving R about the x axis and find the volume (exact value).

5. Let $f(x) = x^2$ and $g(x) = 3x$, and let R be the bounded region in the xy plane between the graphs of f and g. Set up the integral for the volume V of the solid generated by revolving R around the line $y = -1$. You do **not** have the evaluate the integral.

6. Let R be the region between the graph of $y^2 = x$ and $x = 2y$. Find the volume V of the solid obtained by revolving R about the y axis.

7. The region bounded by the curves $y = x^2 - 2x$ and $y = 3x$ is revolved about the line $y = -1$. Find the volume of the resulting solid.

8. The region bounded by the curves $y = x^3, y = 1$, and $x = 2$ is revolved about the line $y = -3$. Find the volume of the resulting solid.

9. Find the volume V of the solid that has a circular base with radius 1, and the cross sections perpendicular to a fixed diameter of the base are squares.

10. As viewed from above, a swimming pool has the shape of the ellipse

$$\frac{x^2}{400} + \frac{y^2}{100} = 1$$

The cross sections of the pool perpendicular to the ground and parallel to the y axis are squares. If the units are feet, determine the volume V of the pool.

11. The base of a solid is the semicircle $y = \sqrt{9 - x^2}$ centered at the origin with a radius of 3 so that the diameter of the semicircle runs along the x-axis. The cross sections are isosceles right triangles with one leg perpendicular to the diameter and other leg extending vertically upward from the semicircle. Find the volume V of the solid.

12. Find the volume of the described solid S where the base of S is the triangular region with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$ and the cross-sections perpendicular to the x-axis are squares.

Section 6.4: Work

1. A 10-pound bag of groceries is to be carried up a flight of stairs 8 feet tall. Find the work W done on the bag.

2. When a certain spring is expanded 10 centimeters from its natural position and held fixed, the force necessary to hold it is 4×10^6 dynes. Find the work W required to stretch the spring an additional 10 centimeters.

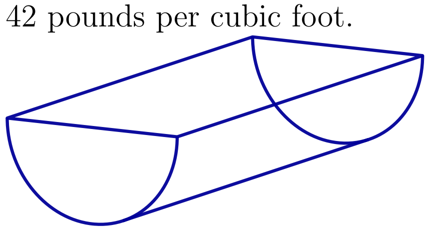
3. If 6×10^7 ergs of work are required to compress a spring from its natural length of 10 centimeters to a length of 5 centimeters, find the work W necessary to stretch the spring from its natural length to a length of 12 centimeters.

4. A bottle of wine has a cork 5 centimeters long. Uncorking the bottle exerts a force to overcome the friction force between the cork and the bottle. The applied force in dynes is given by $F(x) = 2 \times 10^6(5 - x)$ for $0 \leq x \leq 5$ where x represents the length in centimeters of the cork extending from the bottle. Determine the work W done in removing the cork.

5. A swimming pool has the shape of a right circular cylinder with radius 28 feet and height 10 feet. Suppose that the pool is full of water weighing 62.5 pounds per cubic foot. Find the work W required to pump all the water to a platform 2 feet above the top of the pool.

6. A tank in the shape of an inverted cone 12 feet tall and 3 feet in radius is full of water. Calculate the work W required to pump all the water over the edge of the tank.

7. Suppose a large gasoline tank has the shape of a half cylinder 8 feet in diameter and 10 feet long. If the tank is full, set-up the integral to find the work W necessary to pump all the gasoline to the top of the tank. Do NOT evaluate the integral. Assume the gasoline weighs 42 pounds per cubic foot.



8. A tank has the shape of the surface generated by revolving the parabolic segment $y = \frac{1}{2}x^2$ for $0 \leq x \leq 4$ about the y axis. If the tank is full of a fluid weighing 80 pounds per cubic foot, set-up the integral to find the work W required to pump the contents of the tank to a level 4 feet above the top of the tank. Do NOT evaluate the integral.

9. The ends of a "parabolic" water tank are the shape of the region inside the graph of $y = x^2$ for $0 \leq y \leq 4$; the cross sections parallel to the top of the tank (and the ground) are rectangles. The tank is 6 feet long. Rain has filled the tank and water is removed by pumping it up to a spout that is 4 feet above the top of the tank. Set up a definite integral to find the work W that is done to lower the water to a depth of 3 feet.

Section 6.5: Moments

1. Calculate the center of gravity of the region R between the graphs of $f(x) = 2x - 1$ and $g(x) = x - 2$ on the interval $[2, 5]$.

2. Find the center of gravity of the region bounded by the graphs of f and g, where $f(x) = x$ and $g(x) = x^2$.

3. Let $f(x) = 23 - x^2$ and $g(x) = x^2 + 5$. Find the center of gravity, $(\bar{x}, \bar{y})$, of the bounded region enclosed by the graphs of f and g.

4. Determine the center of gravity of R where R is bounded by the hexagon with vertices at $(0, 0), (0, 6), (1, 1), (1, 5), (-1, 1)$, and $(-1, 5)$.

5. Find the center of gravity of the region bounded by $y = 2 - x^2$ and $y = x$.

6. Let R be the region between the graphs of $f(x) = (x-1)^2$ and $g(x) = (x+1)^2$ on $[0, 3]$. Find the moments, M_x and M_y and the area A of the region R. Then find the center of gravity, $(\bar{x}, \bar{y})$ of R.

Section 6.7: Parametric Equations

1. First find an equation relating x and y, when possible. Then sketch the curve C whose parametric equations are given, and indicate the direction $P(t)$ moves as t increases.

   (a) $x = 2\cos t$ and $y = 2\sin t$ for $0 \leq t \leq \pi/2$

   (b) $x = 3\sin t$ and $y = 3\cos t$ for $-\pi/2 \leq t \leq \pi/2$

   (c) $x = -2 + 3t$ and $y = 2 - 3t$ for all t

   (d) $x = e^{-t}$ and $y = e^{3t}$ for all t

2. **Fill in the blank:** Consider the curve C parametrized by $x = -1 + 9\sin t$ and $y = \frac{1}{2} - 9\cos t$ for $-\pi \leq t \leq 3\pi$. Then C is a circle of radius BLANK with center $(x, y) =$ (BLANK , BLANK), and is traversed BLANK times in the BLANK direction.

Sections 6.2 and 6.8: Arclength

1. Find the length L of the graph of $f(x) = 2x + 3$ for $1 \leq x \leq 5$.

2. Find the length L of the graph of $f(x) = 2/3x^{3/2}$ for $1 < x < 4$.

3. Find the length L of the graph of $y = \dfrac{(x^2+2)^{3/2}}{3}$ from $x = 0$ to $x = 3$.

4. Find the length L of the graph of $f(x) = \dfrac{1}{3}\sqrt{x}(x - 3)$ for $0 \leq x \leq 3$.

5. Find the length L of the graph of $f(x) = \dfrac{1}{8}x^2 - \ln x$ for $1 \leq x \leq 3$.

6. **Spring 2016 Final Exam** Let $g(x) = x^3 + \dfrac{1}{12x}$, for $1 \leq x \leq 3$. Find the length L of the graph of g.

7. **Spring 2008 Final Exam** Find the length L of the curve described parametrically by $x(t) = 1 - t^2$ and $y(t) = 1 + t^3$ for $0 \leq t \leq 1$.

8. Find the length L of the curve described parametrically by $x(t) = \dfrac{3t^2}{2} + 4$, and $y(t) = 7 + t^3$ for $0 \leqslant t \leqslant \sqrt{3}$.

9. Find the length L of the curve described parametrically by $x(t) = \dfrac{1}{2}t^2$ and $y(t) = \dfrac{1}{9}(6t + 9)^{3/2}$ from $t = 0$ to $t = 4$.

10. Find the length L of the curve described parametrically by $x(t) = \sin t - t\cos t$ and $y(t) = t\sin t + \cos t$ for $0 \leq t \leq \pi/2$.

Answers and Hints

Section 6.1: Volume

1. $\frac{\pi}{2}\left(e^2 - e^{-2}\right)$

2. 30π

3. $\frac{7\pi}{18}$

4. 16π

5. $\frac{19\pi}{4}$

6. $\frac{64}{15}\pi$

7. *Hint: Curves intersect at $x = 0$ and $x = 5$.*
   250π

8. $\pi\left[\left(\frac{128}{7} + 24 - 14\right) - \left(\frac{1}{7} + \frac{3}{2} - 7\right)\right] = \frac{471\pi}{14}$

9. $\frac{16}{3}$

10. $\frac{32,000}{3}$ (cubic feet)

11. 18

12. *Hint: Draw the region and give the region some structure (a formula)*
    $\frac{1}{3}$

Section 6.4: Work

1. 80 (foot-pounds)

2. 6×10^7 (ergs)

3. $\frac{24}{25} \times 10^7 = 9.6 \times 10^6$ (ergs)

4. 2.5×10^7(ergs)

5. $3,430,000\pi$ (ft-pounds)

6. $\frac{62.5\pi}{16}\left(4x^3 - \frac{1}{4}x^4\right)\Big|_0^{12} = 6750\pi$ (ft-pounds)

7. $W = \int_{-4}^{0} 42(0 - x)20\sqrt{16 - x^2}\, dx$

8. *Hint: Integrate along the y axis.* $W = \int_0^8 80(12-y)2\pi y\,dy$

9. *Hint: Integrate along the y axis.* $W = \int_3^4 62.5(8-y)12\sqrt{y}dy$

Section 6.5: Moments

1. $A = \frac{27}{2}, \quad M_x = 54, \quad M_y = \frac{99}{2}, \quad (\bar{x}, \bar{y}) = \left(\frac{11}{3}, 4\right)$

2. $A = \frac{1}{6}, \quad M_x = \frac{1}{15}, \quad M_y = \frac{1}{12}, \quad (\bar{x}, \bar{y}) = \left(\frac{1}{2}, \frac{2}{5}\right)$

3. *Hint: draw it out and use symmetry.* $(\bar{x}, \bar{y}) = (0, 14)$

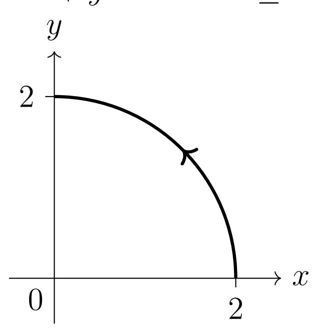
4. *Hint: draw it out and use symmetry.* $(\bar{x}, \bar{y}) = (0, 3)$

5. $(\bar{x}, \bar{y}) = \left(-\frac{1}{2}, \frac{2}{5}\right)$

6. $A = 18, \quad M_x = 99, \quad M_y = 36, \quad (\bar{x}, \bar{y}) = \left(2, \frac{11}{2}\right)$

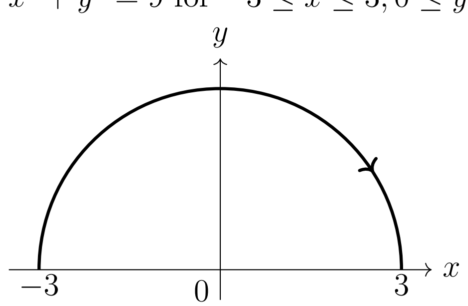
Section 6.7: Parametric Equations

1. (a) $x^2 + y^2 = 4$ for $0 \le x \le 2, 0 \le y \le 2$



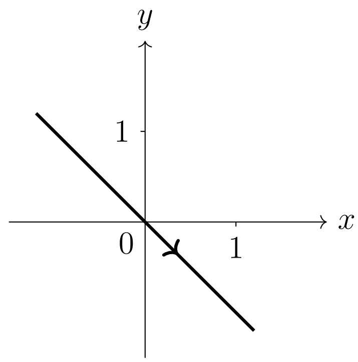
Counter clockwise

(b) $x^2 + y^2 = 9$ for $-3 \le x \le 3, 0 \le y \le 3$

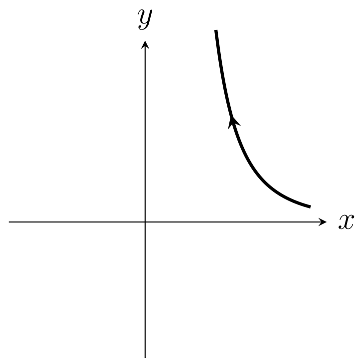


Clockwise

(c) $x = -y$



(d) $y = 1/x^3$ for $x > 0$



2. $9, -1, \frac{1}{2}, 2$, counterclockwise

Sections 6.2 and 6.8: Arclength

1. $4\sqrt{5}$
2. $\frac{2}{3}(5\sqrt{5} - 2\sqrt{2})$
3. 12
4. *Hint: Get a perfect square under the square root*
   $2\sqrt{3}$
5. *Hint: Get a perfect square under the square root*
   $1 + \ln 3$
6. *Hint: Get a perfect square under the square root*
   $26 + \frac{1}{18} = \frac{469}{18}$
7. $\frac{1}{27}(13\sqrt{13} - 8)$ or $\frac{1}{27}(13^{3/2} - 8)$
8. 7
9. 20
10. $\frac{\pi^2}{8}$