Homework (fair game for quiz).

§6.7: Exercises 1-12, §6.8: Exercises 1-6, §6.4: Exercises 1-6

Challenge problems (won't be on the quiz)!

1. 🌶️ §6.8, Exercise 10

2. 🌶️ §6.8, Project 1

3. 🌶️🌶️🌶️ A mathematical knot is any embedding of the unit circle $\mathbf{S}^1$ into three-dimensional space $\mathbb{R}^3$. Knots are classified by how many times they cross over themselves. The simplest knot is called the **trivial knot** or **unknot**, which has 0 crossings. The simplest nontrivial knot is called the **trefoil knot**, which has three crossings. In this problem, we compute the length of a particular parametric representation of the trefoil knot.

   (a) Write down a definition for "parametric curve in three-dimensional space." *(Hint: In perfect analogy with the two-dimensional case we learned about in class, it's a function into $\mathbb{R}^3$ with some particular type of object for its domain.)*

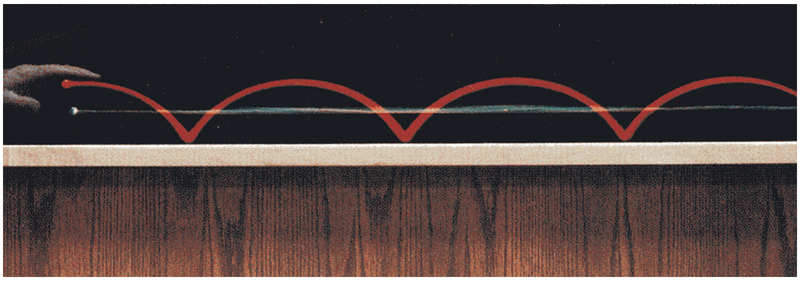
   (b) Given points (x_1, y_1, z_1), (x_2, y_2, z_2) in three-dimensional space, write down a formula that gives the distance between the two points *(Hint: Use your knowledge of the two-dimensional case, and extrapolate)*. Use your formula to find an analogue of §6.8, formula (2) for a curve in three-dimensional space.

   (c) Find a bijective (AKA invertible - you may have to look ahead in chapter 7 to find these definitions) function from $\mathbf{S}^1$ to the half-open interval $[0, 2\pi)$.

   (d) Find the length of the trefoil knot given parametrically by

   $$x(t) = (2 + \cos(3t))\cos(2t),\ y(t) = (2 + \cos(3t))\sin(2t),\ z(t) = \sin(3t)$$

   for $t \in [0, 2\pi)$. *(Highly recommend using CalcPlot3D to graph this: It's very cool looking!)*



The path traced out by a point on the rim of a rolling wheel is a cycloid. For this photograph, taken with camera shutter held open, one light was attached to the rim of the wheel and another light to its center. (*Courtesy of Henry Leap and Jim Lehman*)

EXERCISES 6.7

In Exercises 1–12 first find an equation relating x and y, when possible. Then sketch the curve C whose parametric equations are given, and indicate the direction $P(t)$ moves as t increases.

1. $x = 2 \cos t$ and $y = 2 \sin t$ for $0 \le t \le \pi/2$

2. $x = 3 \sin t$ and $y = 3 \cos t$ for $-\pi/2 \le t \le \pi/2$

3. $x = 2 - \cos t$ and $y = -1 - \sin t$ for $0 \le t \le 2\pi$

4. $x = -1 + \frac{3}{2} \sin t$ and $y = \frac{1}{2} - \frac{3}{2} \cos t$ for $-\pi \le t \le 3\pi$

5. $x = -2 + 3t$ and $y = 2 - 3t$ for all t

6. $x = 5 - t$ and $y = -4$ for $t \ge 0$

7. $x = 3$ and $y = -1 - t$ for $0 \le t \le 1$

8. $x = t$ and $y = \sqrt{1 - t^2}$ for $-1 \le t \le 1$

9. $x = t^3$ and $y = t^2$ for all t

10. $x = e^t$ and $y = e^{-t}$ for all t

11. $x = e^{-t}$ and $y = e^{3t}$ for all t

12. $x = e^t$, $y = \frac{1}{2}(e^t + e^{-t})$ for all t

13. For each pair of curves described parametrically, determine if there is a value of t in $[0, 2\pi]$ for which the corresponding points on the two curves coincide. If so, give the value(s) of t. (*Hint:* Plot the curves sequentially.)

a. $x = \cos t$ and $y = \sin t$; $x = \sin t$ and $y = \cos t$

b. $x = \cos t$ and $y = \sin t$; $x = -\sin t$ and $y = \cos t$

c. $x = 1 + \cos t$ and $y = \sin t$; $x = \sin t$ and $y = \cos t$

14. Let $a, b > 0$, and consider the ellipse parametrized by

$$x = a \cos t \quad \text{and} \quad y = b \sin t \quad \text{for } 0 \le t \le 2\pi$$

Find a representation of the ellipse in rectangular coordinates.

15. Consider the Folium of Descartes, parametrized by

$$x = \frac{3t}{1 + t^3} \quad \text{and} \quad y = \frac{3t^2}{1 + t^3} \quad \text{for all } t \ne -1$$

a. Find an equation in rectangular coordinates for the folium. (*Hint:* Compute $x^3 + y^3$.)

b. Using a graphics calculator, determine which portion of the folium is traced out when t increases without bound, and when t increases toward -1.

16. Let $b > 0$. Consider the curve $C_{\pi b}$ parametrized on $[0, b]$ by

$$x = \cos t + \cos(\pi t) \quad \text{and} \quad y = \sin t - \sin(\pi t)$$

a. What can you say about the total curve as b increases without bound?

b. Replace π by 2 or 3, or any other positive integer, and see how the graph is altered. How can you account for the change?

17. Using a graphics calculator or computer, plot the graph C_n of

$$x = \sin t \quad \text{and} \quad y = \sin(nt) \quad \text{for } 0 \le t \le 2\pi$$

where $n = 2, 4, 6$, and 8.

a. Determine the behavior of C_n as n increases.

***b.** Why are the outer loops thinner than the inner loops? (Compare dy/dx for $t = 0$ with dy/dx for t near $\pi/2$ or $3\pi/2$.)

18. Let $r > 0$, and consider the curve defined parametrically by

$$x = \cos^r t \quad \text{and} \quad y = \sin^r t \quad \text{for all } 0 \le t \le 2\pi$$

Determine a rational value of r such that the whole graph appearing on the screen

a. lies only in the first quadrant. Explain why this happens.

b. appears to consist of portions of the x and y axes only, even when you zoom in. Explain why this happens.

19. a. Plot the curve parametrized by

$$x = \frac{2t}{1 + t^2} \quad \text{and} \quad y = \frac{1 - t^2}{1 + t^2} \quad \text{for all } t$$

b. By squaring both x and y, find an equation of the graph in rectangular coordinates.

20. a. Plot the curve parametrized by

$$x = \frac{t^2 + 1}{t^2 - 1} \quad \text{and} \quad y = \frac{2t}{t^2 - 1} \quad \text{for all } t$$

b. By squaring both x and y, find an equation of the graph in rectangular coordinates.

21. Write down three sets of parametric equations whose combined graph is the capital letter B.

Project

1. A **Lissajous* figure** is a curve that has a parametrization of the form

$$x = \cos(mt) \quad \text{and} \quad y = \sin(nt) \quad \text{for} \quad 0 \le t \le 2\pi$$

where m and n are positive numbers (normally integers). These figures are named for the French physicist Jules Lissajous (1822–1880). Let L_{mn} denote the Lissajous figure for the given pair m and n.

a. Suppose $m = n$. Prove that L_{mn} is a circle (which is a degenerate Lissajous figure).

b. Suppose m and n are even, and that n divides m. Prove that the origin is not on L_{mn}. (In addition to this case, one can show that any time n is odd, the origin is not on L_{mn}.)

c. Let $m = 4$ and $n = 6$. Prove that the origin is not on L_{mn}.

d. Let $m = 3$ and $n = 6$. Prove that the origin is on L_{mn}.

e. Let m and n be distinct even integers such that n is even and m divides n. By experimenting with your calculator or computer, determine how many loops are in L_{mn}.

f. Consider the figure $L_{3\pi}$, where $m = 3$ and $n = \pi$. (Strictly speaking, $L_{3\pi}$ is not a Lissajous figure. Why?) What happens when you trace out the figure for $0 \le t \le b$ and let b increase without bound? Is a point on the figure ever repeated? Explain why or why not.

6.8 LENGTH OF A CURVE GIVEN PARAMETRICALLY

In Section 6.2 we derived the integral for the length of the graph of a function on an interval $[a, b]$. Now we will find the lengths of parametrized curves, and in particular we will determine the length of one arch of the cycloid.

Suppose that a curve C is given parametrically by

$$x = f(t) \quad \text{and} \quad y = g(t) \quad \text{for } a \le t \le b$$

where f and g have continuous derivatives on $[a, b]$. Our derivation of the formula for the length L of C will parallel the derivation of the formula given in Definition 6.2. Let $P = \{t_0, t_1, \ldots, t_n\}$ be any partition of $[a, b]$, and for $1 \le k \le n$ let $(x_k, y_k) = (f(t_k), g(t_k))$ be the corresponding point on C (Figure 6.77). Let ΔL_k be the length of the portion of the curve joining (x_{k-1}, y_{k-1}) and (x_k, y_k). If Δx_k is small, ΔL_k is approximately equal to the length of the line segment joining (x_{k-1}, y_{k-1}) and (x_k, y_k). In other words,

$$\begin{aligned}\Delta L_k &\approx \sqrt{(x_k - x_{k-1})^2 + (y_k - y_{k-1})^2}\\ &= \sqrt{[f(t_k) - f(t_{k-1})]^2 + [g(t_k) - g(t_{k-1})]^2}\end{aligned}$$

***Lissajous:** Pronounced "lissajoo."

Next we let

$$u = \cos\frac{t}{2}, \quad \text{so that} \quad du = -\frac{1}{2}\sin\frac{t}{2}\,dt$$

Now if $t = 0$ then $u = 1$, and if $t = 2\pi$ then $u = -1$. Therefore

$$\begin{aligned} S &= 8\pi r^2 \int_0^{2\pi} \overbrace{\left(1 - \cos^2\frac{t}{2}\right)}^{1-u^2} \overbrace{\sin\frac{t}{2}\,dt}^{-2\,du} \\ &= 8\pi r^2 \int_1^{-1} (1 - u^2)(-2)\,du = -16\pi r^2 \left(u - \frac{1}{3}u^3\right)\Big|_1^{-1} \\ &= -16\pi r^2 \left[\left(-1 + \frac{1}{3}\right) - \left(1 - \frac{1}{3}\right)\right] = \frac{64\pi}{3} r^2 \quad \square \end{aligned}$$

EXERCISES 6.8

In Exercises 1–3 find the length L of the curve described parametrically.

1. $x = 1 - t^2$ and $y = 1 + t^3$ for $0 \le t \le 1$
2. $x = e^t \sin t$ and $y = e^t \cos t$ for $0 \le t \le \pi$
3. $x = \sin t - t\cos t$ and $y = t\sin t + \cos t$ for $0 \le t \le \pi/2$

In Exercises 4–6 find the area S of the surface generated by revolving about the x axis the curve with the given parametric representation.

4. $x = \frac{1}{2}t^2$ and $y = t$ for $\sqrt{3} \le t \le 2\sqrt{2}$
5. $x = \frac{2}{3}(1 - t)^{3/2}$ and $y = \frac{2}{3}(1 + t)^{3/2}$ for $-\frac{3}{4} \le t \le 0$
6. $x = \sin^2 t$ and $y = \sin t \cos t$ for $0 \le t \le \pi/2$
7. Let $0 \le \alpha < \beta \le \pi$ and $r > 1$, and consider the circular arc C parametrized by

$$x = r\cos t \quad \text{and} \quad y = r\sin t \quad \text{for } \alpha \le t \le \beta$$

   a. Show that the area S of the surface obtained by revolving C about the x axis does not depend only on the difference $\beta - \alpha$.

   b. Does the result of part (a) contradict the assertion that follows Example 2 of Section 6.3? Explain why or why not.

8. Suppose an object moves in an elliptical orbit parametrized by

$$x = 2\cos t \quad \text{and} \quad y = 3\sin t$$

   where t represents time.

   a. Use (4) and (5) to find a formula for the velocity.

   b. Determine the points on the ellipse at which the object is moving fastest and the points at which it is moving slowest.

9. Let $a, b > 0$, and consider the ellipse parametrized by

$$x = a\cos t \quad \text{and} \quad y = b\sin t \quad \text{for } 0 \le t \le 2\pi$$

   a. Find an integral that represents the circumference C_{ab} of the ellipse. (The integral is called an **elliptic integral.**)

   b. Use the formula obtained in (a) to find C_{ab} when $a = b > 0$.

   c. Use the result of (a) to determine $\lim_{b\to 0^+} C_{ab}$.

10. Let $r > 0$. The equations

$$x = r\cos^3 t \quad \text{and} \quad y = r\sin^3 t \quad \text{for } 0 \le t \le 2\pi$$

   parametrize an astroid (see Figure 6.84).

   a. Find the length L of the astroid. (*Hint:* Remember that square roots are nonnegative.)

   b. Show that the astroid is alternatively the graph of

$$x^{2/3} + y^{2/3} = r^{2/3}$$

   c. Find the area S of the surface generated by revolving C about the x axis. (*Hint:* S is equal to twice the surface area of the part for which $0 \le t \le \pi/2$.)

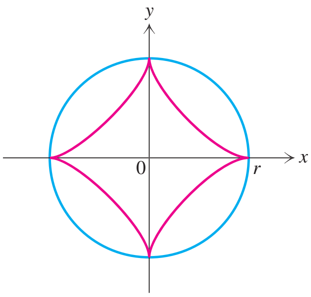


FIGURE 6.84 An astroid.

11. Let C be the curve parametrized by

$$x = \cos^n t \quad \text{and} \quad y = \sin^n t \quad \text{for } 0 \le t \le \pi/2$$

where n is an integer with $n \ge 2$. Let S be the surface area of the surface generated by revolving C about the x axis. Determine the positive integers n for which we can readily carry out the integration given in (12) to obtain the surface area S.

***12.** Using (7) and (10), find the time T_{st} it takes for an ice cube to slide on the straight line from the origin to the bottom point $P = (\pi r, 2r)$ of the cycloid, and show that $T_{st} > T^*$.

Project

1. a. Cornu's spiral (Figure 6.85), which arises in the study of diffraction, is the curve parametrized by

$$x = x(t) = \int_0^t \cos \frac{\pi s^2}{2}\, ds$$

and

$$y = y(t) = \int_0^t \sin \frac{\pi s^2}{2}\, ds \quad \text{for all } t$$

Show that $x(-t) = -x(t)$ and $y(-t) = -y(t)$. How is this fact reflected in Figure 6.85?

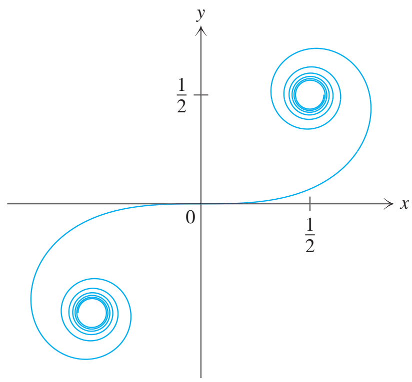


FIGURE 6.85 Cornu's spiral for the project.

b. Let a and b be any real numbers with $a < b$. Find the length L of the part of Cornu's spiral that is traced out for $a \le t \le b$.

REVIEW

Key Terms

Cross-sectional method
Length of a curve
Area of a surface
Work
Moment
Center of gravity
Center of mass
Centroid
Hydrostatic force
Parametric equations
Parameter

Key Theorem

Theorem of Pappus and Guldin

The formula in (8) implies that the work is equal to the change in the kinetic energy of the craft:

$$W(b) = \int_R^b F(r)\,dr = \frac{1}{2}m[v(b)]^2 - \frac{1}{2}m[v(R)]^2 \tag{10}$$

Together (9) and (10) imply that

$$GMm\left(\frac{1}{b} - \frac{1}{R}\right) = \frac{1}{2}m[v(b)]^2 - \frac{1}{2}m[v(R)]^2 \tag{11}$$

When the spacecraft is a distance b from the center of the earth, the **potential energy** of the craft is defined to be $-GMm/b$. Thus we can rewrite (11) in the following form:

$$\overbrace{\frac{1}{2}m\,[v(b)]^2}^{\text{kinetic energy}} + \overbrace{\left(-GMm\,\frac{1}{b}\right)}^{\text{potential energy}} = \frac{1}{2}m\,[v(R)]^2 - GMm\,\frac{1}{R} \tag{12}$$

Notice that the right-hand side of (12) is constant as the spacecraft recedes from earth. Thus we have proved the famous **Law of Conservation of Energy:**

The sum of the kinetic energy and potential energy is constant (for all values of b).

The formula in (11) also yields a formula for the escape velocity (which we discussed initially in Section 4.8). If a spacecraft has precisely the escape velocity, then $\lim_{b\to\infty} v(b) = 0$, so that taking the limits of the expression in (11) as b approaches ∞ yields

$$\frac{GMm}{R} = \frac{1}{2}m\,[v(R)]^2, \quad \text{or equivalently,} \quad v(R) = \sqrt{2GM/R}$$

Consequently the initial velocity shortly after liftoff must be $\sqrt{2GM/R}$. This is the same as the formula for the escape velocity derived in Section 4.8.

EXERCISES 6.4

1. Determine the work W done on the wheelbarrow of Example 1 if it is pushed only 60 meters.
2. An elevator in the Empire State Building weighs 1600 pounds. Find the work W required to raise the elevator from ground level to the 102nd story, some 1200 feet above ground level.
3. A 10-pound bag of groceries is to be carried up a flight of stairs 8 feet tall. Find the work W done on the bag.
4. Suppose a 120-pound person carries the bag in Exercise 3 up the stairs. Find the total work W done by the person in walking up the stairs with the bag.
5. A sailboat is stationary in the middle of a lake until a strong gust of wind blows it along a straight line. Suppose the force in newtons exerted on the sails by the wind when the boat is x kilometers from its starting point is

$$F(x) = 10^4 \sin x \quad \text{for } 0 \le x \le \pi$$

Find the work W done on the sails by the gust of wind.

6. Suppose a person pushes a 1/2-centimeter-long thumbtack into a bulletin board and the force (in dynes) exerted when the depth of the thumbtack in the bulletin board is x centimeters is given by

$$F(x) = 10{,}000(1 + 2x)^2 \quad \text{for } 0 \le x \le \frac{1}{2}$$

Find the work W done in pushing the thumbtack all the way into the board.

7. A bottle of wine has a cork 5 centimeters long. A person uncorking the bottle exerts a force to overcome the force of friction between the cork and the bottle. Suppose the applied force in dynes is given by

$$F(x) = 2 \times 10^6 (5 - x) \quad \text{for } 0 \le x \le 5$$

where x represents the length in centimeters of the cork extending from the bottle. Determine the work W done in removing the cork.

8. When a certain spring is expanded 10 centimeters from its natural position and held fixed, the force necessary to hold it is 4×10^6 dynes. Find the work W required to stretch the spring an additional 10 centimeters.

9. Find the work W required to stretch the spring described in Exercise 8 from 20 to 30 centimeters beyond its natural length.

10. If 6×10^7 ergs of work are required to compress a spring 4 centimeters from its natural length, find the work W necessary to compress the spring an additional 4 centimeters. (*Hint:* Hooke's Law is also valid for compressing springs.)

11. If 6×10^7 ergs of work are required to compress a spring from its natural length of 10 centimeters to a length of 5 centimeters, find the work W necessary to stretch the spring from its natural length to a length of 12 centimeters.

12. Find the work W necessary to pump all the water out of the swimming pool in Example 3.

13. Find the work W required to pump the water out of the swimming pool in Figure 6.44. Assume that the pool is initially full, and that all the water is pumped to a level 1 foot above the top of the pool.

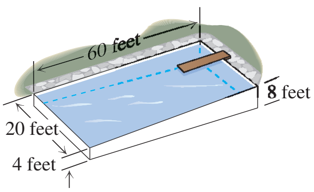


FIGURE 6.44 The swimming pool for Exercise 13.

14. A tank has the shape of the surface generated by revolving the parabolic segment $y = \frac{1}{2}x^2$ for $0 \le x \le 4$ about the y axis. If the tank is full of a fluid weighing 80 pounds per cubic foot, find the work W required to pump the contents of the tank to a level 4 feet above the top of the tank. (*Hint:* Integrate along the y axis.)

15. A tank in the shape of an inverted cone 12 feet tall and 3 feet in radius is full of water. Calculate the work W required to pump all the water over the edge of the tank.

16. Suppose that in Exercise 15 just half the water is pumped out the top edge of the tank and the remaining water is pumped to a level 3 feet above the top of the tank. Compute the total work W done.

17. Suppose a large gasoline tank has the shape of a half-cylinder 8 feet in diameter and 10 feet long (Figure 6.45). If the tank is full, find the work W necessary to pump all the gasoline to the top of the tank. Assume the gasoline weighs 42 pounds per cubic foot.

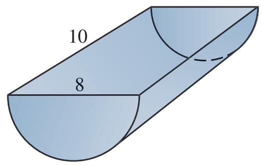
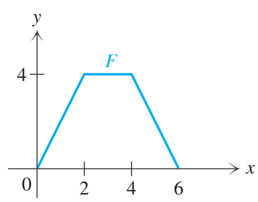


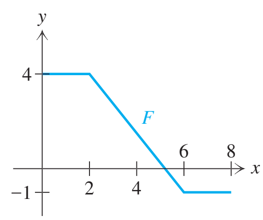
FIGURE 6.45 Figure for Exercise 17.

18. In each of the following, find the work W done by the force F graphed in the figure.

a.



b.



19. Suppose a spring is extended 6 centimeters from its natural length and then compressed to its natural length. How much work is done?

20. A car has run out of gas, and is being pushed on a level road toward a gas station. The force used as the car proceeds toward the station 200 meters away is described in Figure 6.46. Approximate the work W done in moving the car to the station by estimating the area under the graph of F in the figure

a. by counting appropriate rectangles.

b. by calculating the left sum with $n = 10$.

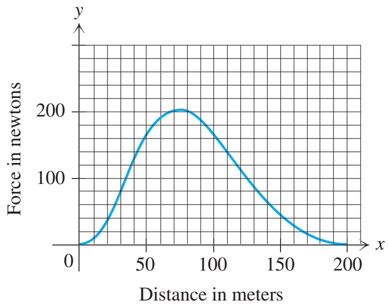


FIGURE 6.46 Graph for Exercise 20.

21. Suppose an object moves a distance of 20 centimeters along the x axis, and that the force (in dynes) acting on the object is measured at intervals of 2 centimeters, with the following results (in order):

1.21, 2.90, 3.01, 3.52, 3.41, 3.19, 2.78, 2.76, 2.83, 2.90, 2.84

By calculating the right sum, approximate the amount of work W done by the force.

22. A stationary proton located at the origin of the x axis exerts an attractive force on an electron located at a point x on the negative x axis. The force is given by

$$F(x) = \frac{a}{x^2}$$

where $a = 2.3 \times 10^{-28}$, x is measured in meters, and $F(x)$ is measured in newtons. Determine the work W done on the electron by this force when the electron moves from $x = -10^{-9}$ to $x = -10^{-10}$.

23. A ball weighing 0.2 pound is thrown vertically upward. Its height after t seconds is given by

$$h(t) = 6 + 8t - 16t^2$$

until the ball strikes the ground again.

a. Find the work W done on the ball by gravity while the ball descends from its maximum height to the ground.

b. Find the work W done on the ball on its descent if it is caught 6 feet above the ground.

24. A rocket lifts off the pad at Cape Canaveral. According to Newton's Law of Gravitation, the force of gravity on the rocket is given by

$$F(x) = \frac{-GMm}{x^2}$$

where M is the mass of the earth, m is the mass of the rocket, G is a universal constant, and x is the distance (in miles) between the rocket and the center of the earth. Take the radius of the earth to be 4000 miles, so that $x \geq 4000$ miles.

a. Find the work W done against gravity when the rocket rises 1000 miles. Express your answer in terms of G, M, and m.

b. Find the work W done against gravity when the rocket rises b miles.

c. Find the limit of the work found in (b) as b approaches ∞, and determine whether it is possible, with a finite amount of work, to send the rocket arbitrarily far away.

25. A bucket of cement weighing 200 pounds is hoisted by means of a windlass from the ground to the tenth story of an office building, 80 feet above the ground.

a. If the weight of the rope used is negligible, find the work W required to make the lift.

b. Assume that a chain weighing 1 pound per foot is used in (a), instead of the lightweight rope. Find the work W required to make the lift. (*Hint:* As the bucket is raised, the length of chain that must be lifted decreases.)

26. A container is lifted vertically at the rate of 2 feet per second. Water is leaking out of the container at the rate of $\frac{1}{2}$ pound per second. If the initial weight of water and container is 20 pounds, find the work W done in raising the container 10 feet. (*Hint:* For $0 \leq x \leq 10$, determine the total weight of water and container when the container has been raised x feet.)

27. A bucket containing water is raised vertically at the rate of 2 feet per second. Water is leaking out of the container at the rate of $\frac{1}{2}$ pound per second. If the bucket weighs 1 pound and initially contains 20 pounds of water, determine the amount of work W required to raise the bucket until it is empty.

***28.** A building demolisher consists of a 2000-pound ball attached to a crane by a 100-foot chain weighing 3 pounds per foot. At night the chain is wound up and the ball is secured to a point 100 feet high. Find the work W done by gravity on the ball and the chain when the ball is lowered from its nighttime position to its daytime position at ground level.

29. The Great Pyramid of Cheops had square cross sections and was (approximately) 482 feet high and 754 feet square at the base. If the rock used to build the pyramid weighed 150 pounds per cubic foot, find the work W required to lift the rock into place as the pyramid was built.

***30.** A cylindrical tank has a radius of 3 feet and height of 10 feet. Water is to be pumped from a lake into the base of the tank, which is 20 feet above the lake. Assuming that the lake is so large that its water level does not change during the pumping, write an integral for the work W required to fill the tank.

***31.** A thin steel plate in the shape of an isosceles triangle with base 6 feet and height 4 feet is lying flat on the ground (Figure 6.47). Suppose the weight (in pounds) of any part of the plate equals its area (in square feet). How much work is required in order to raise the plate to a vertical position, assuming the base remains in contact with the ground? (*Hint:* Let the x axis be perpendicular to the base of the triangle, with the origin on the base. Consider any partition P of $[0, 4]$, and approximate the work by a Riemann sum.)

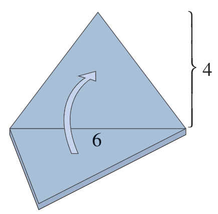


FIGURE 6.47 Figure for Exercise 31.

Project

1. A 5-pound bucket initially containing 20 pounds of water is hoisted by chain from the ground to the top of a bungee jumping tower, 120 feet above ground. Suppose that the basket is leaking water at a steady rate of 0.2 pounds per second, that the bucket is raised at the rate of 3 feet per second, and that the chain weighs 4 ounces per foot of length.

a. Find the number of seconds it takes for the bottom of the bucket to rise x feet from the ground (with $x \leq 120$).

b. How much water has leaked from the bucket by the time that the bucket has risen x feet?

c. Find the work W required to raise the bucket to the top of the tower.

d. Suppose that instead of leaking 0.2 pounds per second the bucket leaks at a steady rate of 0.6 pounds per second until there is no more water in the bucket. Determine the work W_0 required to raise the bucket from the ground to the top of the tower. Is your answer different from your answer to part (c)? Explain why, or why not.

e. In addition to the hypotheses (including 0.6 pound per second leak), suppose that the chain is tapered in a linear fashion, so it weighs 4 ounces per foot at the bottom and 2 ounces per foot at the top. Find the work W_1 required to raise the bucket from the ground to the top of the tower.

6.5 MOMENTS AND CENTER OF GRAVITY

Children playing on a seesaw quickly learn that a heavier child has more effect on the rotation of the seesaw than does a lighter child and that a lighter child can balance a heavier one by moving farther away from the axis of rotation (Figure 6.48). In this section we define a quantity called "moment," which measures the tendency of a mass to produce rotation. We will use moments to define a point called the "center of gravity" of a set of points in the plane.

Moments of Point Masses

Let us first consider the idealized situation in which an object of positive mass m is concentrated at a point (x, y) in the plane. Such an object is called a **point mass.** The **moment** of the point mass **about the *y* axis** is defined to be mx; we may think of mx as a measure of the tendency of the point mass to rotate about the y axis