

**Homework (fair game for quiz).**

§6.7: Exercises 1-12, §6.8: Exercises 1-6, §6.4: Exercises 1-6

**Challenge problems (won't be on the quiz)!**

1. 🌶️ §6.8, Exercise 10
2. 🌶️ §6.8, Project 1
3. 🌶️ 🌶️ 🌶️ A mathematical [knot](#) is any embedding of the unit circle  $S^1$  into three-dimensional space  $\mathbb{R}^3$ . Knots are classified by how many times they cross over themselves. The simplest knot is called the **trivial knot** or **unknot**, which has 0 crossings. The simplest nontrivial knot is called the **trefoil knot**, which has three crossings. In this problem, we compute the length of a particular parametric representation of the trefoil knot.
  - (a) Write down a definition for "parametric curve in three-dimensional space." (*Hint: In perfect analogy with the two-dimensional case we learned about in class, it's a function into  $\mathbb{R}^3$  with some particular type of object for its domain.*)
  - (b) Given points  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  in three-dimensional space, write down a formula that gives the distance between the two points (*Hint: Use your knowledge of the two-dimensional case, and extrapolate*). Use your formula to find an analogue of §6.8, formula (2) for a curve in three-dimensional space.
  - (c) Find a bijective (AKA invertible - you may have to look ahead in chapter 7 to find these definitions) function from  $S^1$  to the half-open interval  $[0, 2\pi)$ .
  - (d) Find the length of the trefoil knot given parametrically by

$$x(t) = (2 + \cos(3t)) \cos(2t), \quad y(t) = (2 + \cos(3t)) \sin(2t), \quad z(t) = \sin(3t)$$

for  $t \in [0, 2\pi)$ . (*Highly recommend using [CalcPlot3D](#) to graph this: It's very cool looking!*)