Math 141H - Calculus II Honors **Suggested Problems 3**

Homework (fair game for quiz).

§6.7: Exercises 1-12, §6.8: Exercises 1-6, §6.4: Exercises 1-6

Challenge problems (won't be on the quiz)!

1. 🌶️ §6.8, Exercise 10
2. 🌶️ §6.8, Project 1
3. 🌶️🌶️🌶️ A mathematical knot is any embedding of the unit circle $\mathbf{S}^1$ into three-dimensional space $\mathbb{R}^3$. Knots are classified by how many times they cross over themselves. The simplest knot is called the **trivial knot** or **unknot**, which has 0 crossings. The simplest nontrivial knot is called the **trefoil knot**, which has three crossings. In this problem, we compute the length of a particular parametric representation of the trefoil knot.

   (a) Write down a definition for "parametric curve in three-dimensional space." *(Hint: In perfect analogy with the two-dimensional case we learned about in class, it's a function into $\mathbb{R}^3$ with some particular type of object for its domain.)*

   (b) Given points (x_1, y_1, z_1), (x_2, y_2, z_2) in three-dimensional space, write down a formula that gives the distance between the two points *(Hint: Use your knowledge of the two-dimensional case, and extrapolate)*. Use your formula to find an analogue of §6.8, formula (2) for a curve in three-dimensional space.

   (c) Find a bijective (AKA invertible - you may have to look ahead in chapter 7 to find these definitions) function from $\mathbf{S}^1$ to the half-open interval $[0, 2\pi)$.

   (d) Find the length of the trefoil knot given parametrically by

   $$x(t) = (2 + \cos(3t))\cos(2t),\ y(t) = (2 + \cos(3t))\sin(2t),\ z(t) = \sin(3t)$$

   for $t \in [0, 2\pi)$.