PHYS-E0412 Computational Physics :: Homework 11

Due date 2.4.2019 at 10 am

Can You Turn Back Time?

Time-integration algorithms always consider advancing the solution in time. However, formally it is easy to reverse time by changing the sign of the time step, $\tilde{k} = -k$. While the resulting algorithm is well-defined in some cases the idea is not justified. In this homework we study time-integration methods and their reversibility in time.[1]

a) Consider first the one-dimensional heat equation with Dirichlet boundary conditions:

$$\begin{cases} \frac{\partial}{\partial t} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t), \quad x \in [0,1],\, t > 0 \\ u(0,t) = u(1,t) = 0,\ u(x,0) = u_0(x) = 1.831 \exp\left(-10(x-0.5)^2\right) \end{cases}$$

Choose a suitable discretization for the spatial part (FD or FEM will do fine) and implement implicit Euler and Crank-Nicolson time-integration schemes. Integrate up-to the time $t_f = 0.1$. Then reverse the flow of time and integrate back to $t_0 = 0$. Does either of the methods return to $u_0(x)$?

Entropy of a probability distribution function f can be defined as

$$S(f) = -\int f(x) \ln\left(f(x)\right) dx$$

Using this definition, find out how the entropy of the solutions $u(t,x)$ evolve up to $u_f = 0.1$. (Note that you need to normalize the solution at each time-step.) (3 p.)

b) Consider next the one-dimensional wave equation with periodic boundary conditions

$$\begin{cases} \frac{\partial^2}{\partial t^2} u(x,t) = \frac{\partial^2}{\partial x^2} u(x,t), \quad x \in [0,1],\, t > 0 \\ u(0,t) = u(1,t),\ u(x,0) = u_0(x) = 1.831 \exp\left(-10(x-0.5)^2\right) \end{cases}$$

Using suitable spatial discretization integrate up-to $t_f = 0.2$ using both implicit Euler and Crank-Nicolson methods. Then reverse the flow of time. What do you get at $t_0 = 0$? (2 p.)

c) How many hours did you spend working on this exercise?

[1] For a pop-culture take on the problem, check: `https://www.youtube.com/watch?v=BsKbwR7WXN4`